Diameter Distributions and Yields Of Thinned Loblolly Pine Plantations

Publication No. FWS-1-82
School of Forestry and Wildlife Resources
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061
1982
DIAMETER DISTRIBUTIONS AND YIELDS
OF THINNED LOBLOLLY PINE PLANTATIONS

by

Quang V. Cao
Harold E. Burkhart
Ronald C. Lemin, Jr.

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ABSTRACT

A growth and yield model for thinned loblolly pine plantations was developed using data from 128 0.2-acre permanent plots in the Virginia Piedmont and Coastal Plain. The Weibull function, used to characterize stand diameter distributions, was searched to insure that the resulting total basal area and average dbh estimates were identical to those predicted from stand variables using regression equations. Program WTHIN was written in standard FORTRAN to provide stand and stock tables for thinned old-field loblolly pine plantations.

Trials with different thinning intensities indicated reasonable trends, as compared with published studies.
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INTRODUCTION

Growth and yield predictions are essential to forest management planning. Reliable growth and yield models assist managers in analyzing alternative management strategies. For loblolly pine (Pinus taeda L.), a myriad of yield information for unmanaged stands has accumulated over the years. On the other hand, yield models for thinned loblolly pine plantations still seem inadequate, and flexible models that supply information about diameter distributions are needed.

Different probability density functions (pdf's) have been used to characterize diameter distributions; most recently the beta, Weibull, and Johnson's S_2 distributions have been employed to develop yield estimates. The so-called probability density function approach to yield modeling involves predicting the pdf parameters from stand variables (age, site, and density) using regression techniques, and then calculating the number of trees and yield per acre in each dbh class. The drawback of this approach is that the regression models for predicting the pdf parameters usually account for only a small percentage of the variation (i.e. low R^2 values). Recently, research has been conducted to develop methods for approximating the parameters in a theoretical diameter distribution (e.g. the beta or Weibull) from overall stand values such as total basal area and mean diameter (Hyink 1980, Frazier 1981, Matney and Sullivan 1982).

The objectives of this study were: (1) to develop a whole stand model for thinned loblolly pine plantations using regression techniques, and (2) to derive diameter distributions from the predicted stand attributes by assuming that the underlying dbh distribution is Weibull distributed.

PREVIOUS WORK

Whole Stand and Diameter Distribution Models

MacKinney and Chaiken (1939) used multiple linear regression techniques to predict the logarithm of yield as a function of stand variables (age, site, density, and composition). This approach, with
certain modifications, has been employed in more recent models for loblolly pine (such as Schumacher and Coile 1960, Coile and Schumacher 1964, Goebel and Warner 1969, Burkhart et al. 1972a, 1972b).

Growth and yield are not two separate attributes but are closely related to one another. Buckman (1962) developed a yield model for red pine where yield is obtained by mathematically integrating the growth equation over time. Clutter (1963) discussed this concept in detail and introduced a compatible growth and yield model which was later refined by Sullivan and Clutter (1972). A similar approach has been used by several other researchers including Brender and Clutter (1970), Bennett (1970), Beck and Della-Bianca (1972), Sullivan and Williston (1977), Murphy and Sternitzke (1979), and Murphy and Beltz (1981).

Diameter distributions in even-aged stands have been modeled with various probability density functions, among them the Gram-Charlier series (Meyer 1928, 1930; Schumacher 1928, 1930; Schnur 1934), the modified Pearl-Reed growth curve (Osborne and Schumacher 1935, Nelson 1964), Pearsonnian curves (Schnur 1934), and the log-normal distribution (Bliss and Reinker 1964).

Bennett and Clutter (1968) developed a yield model to predict multiple-product yields for slash pine plantations by using the stand table generated from a beta pdf via the Clutter and Bennett (1965) diameter distribution model. In this yield model, the parameters of the beta function that approximated the diameter distribution were predicted from stand variables (age, site, and density). The number of trees and volume per acre in each diameter class were calculated and per acre yield estimates were obtained by summing over diameter classes of interest. A similar approach was applied to loblolly pine plantations by Lenhart and Clutter (1971), Lenhart (1972), and Burkhart and Strub (1974).

The main drawback of using the beta distribution is that its cumulative distribution function (cdf) does not exist in closed form. As a result, the proportion of trees in each diameter class has to be solved by numerical integration techniques. Bailey and Dell (1973) pointed out that the Weibull distribution fits diameter data well and its cdf exists in closed form. The Weibull function was applied in plantation yield models for loblolly pine (Smalley and Bailey 1974a, Feduccia et al. 1979), slash pine (Clutter and Belcher 1978, Dell et al. 1979), and shortleaf pine (Smalley and Bailey 1974b).

Strub and Burkhart (1975) presented a class-interval-free method for predicting whole stand yield per unit area from diameter distribution models:
\[
TV = N \int_{L}^{U} g(D) f(D) \, dD
\]

where

- \( TV \) = expected stand volume per unit area,
- \( N \) = number of trees per unit area,
- \( D \) = diameter at breast height,
- \( g(D) \) = individual tree volume equation,
- \( f(D) \) = pdf for \( D \), and
- \((L, U)\) = merchantability limits for the product described by \( g(D) \).

Using this relationship, Hyink (1980) introduced a method of solving for the parameters of the pdf approximating the diameter distribution, using attributes predicted from a whole stand model. The same concept was employed by Matney and Sullivan (1982) in their model for loblolly pine plantations. In the first phase of Matney and Sullivan's study, stand volume and basal area were predicted using compatible growth and yield equations. The second phase involved solving for two parameters of the Weibull pdf which characterized the diameter distribution such that the resulting stand volume and basal area per acre would be identical to those predicted in the first phase. Frazier (1981) investigated alternative formulations for estimating parameter values in the beta and Weibull distributions from stand attributes.

**Modeling Thinned Loblolly Pine Stands**

Coile and Schumacher (1964) included amount of thinning as input in their model. Different types of thinning (thinning by rows, from below, or by a combination of both) can be specified in Daniels and Burkhart's (1975) and Daniels et al.'s (1979) individual tree models. Other models based on data from thinned loblolly pine stands include Clutter (1963), Brender and Clutter (1970), Sullivan and Clutter (1972), and Sullivan and Williston (1977).

The Weibull function was used by Bailey et al. (1981) to describe diameter distribution of slash pine plantations before and after thinning. Matney and Sullivan (1982) also used the Weibull distribution to produce compatible stand and stock tables for thinned loblolly pine plantations. In addition to the models mentioned above, growth and yield of thinned loblolly pine stands have been reported by many researchers (such as Bassett 1966, Bruner and Goebel 1968, Andrulot et al. 1972, Shepard 1974, Goebel et al. 1974, Feduccia and Mann 1976, Burton 1980).
DEVELOPING THE THINNED-STAND MODEL

Data

The growth and yield model for thinned loblolly pine plantations developed in this study was based on data from the Virginia Division of Forestry (VDF). This data set consists of 128 0.2-acre permanent plots from old-field plantations in the Virginia Piedmont and Coastal Plain. Number of remeasurements varied from plot to plot, ranging from 1 to 7. There were a total of 490 plot measurements.

Diameter at breast height (dbh) was recorded to the nearest inch and total height was measured to the nearest foot. Trees in the 1- and 2-inch classes were not tallied separately but combined to form one class whose midpoint was arbitrarily set at 1.5 inches. In each plot, measurements of dbh of all trees were taken but only some tree heights were measured. Height corresponding to each dbh class was predicted for each plot measurement using a regression equation of the form

\[ \log_e(H) = b_0 + \frac{b_1}{D}, \]

where \( H \) = total tree height in feet,
\( D \) = diameter at breast height in inches,
\( b_0, b_1 \) = regression coefficients.

Site index was determined from the average height of the dominants and codominants in each plot, using a site index equation developed by Devan (1979). Total cubic-foot volume outside bark per acre was computed using Burkhart et al.'s (1972b) individual tree volume equation.

The stands were thinned up to 3 times and, for the most part, thinnings were from below. However, some codominants and dominants were removed to improve the quality of the leave stand. The thinnings carried out were done during routine, operational thinnings of the plantations in which the plots were located. Table 1 presents a description of plots in this data set immediately before and after thinning. The distribution of all observations by site index, age, basal area, and number of trees per acre is presented in Table 2.

Model for
Thinned Loblolly Pine Plantations

The model for thinned loblolly pine plantations developed in this study consisted of two stages. In the first stage, stand-level
Table 1. Description of plots immediately before and after thinning and amount of thinning. a/

<table>
<thead>
<tr>
<th>Variable</th>
<th>First thinning</th>
<th>Subsequent thinnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before</td>
<td>Amount</td>
</tr>
<tr>
<td>Number of trees/acre</td>
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</tr>
<tr>
<td>Minimum</td>
<td>355</td>
<td>165</td>
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<tr>
<td>Mean</td>
<td>774</td>
<td>459</td>
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<tr>
<td>Maximum</td>
<td>1305</td>
<td>770</td>
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<tr>
<td>Basal area (sq.ft./acre)</td>
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<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>107</td>
<td>29</td>
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<tr>
<td>Mean</td>
<td>174</td>
<td>87</td>
</tr>
<tr>
<td>Maximum</td>
<td>227</td>
<td>148</td>
</tr>
<tr>
<td>Total outside-bark volume (cu.ft./acre)</td>
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</tr>
<tr>
<td>Minimum</td>
<td>1700</td>
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<tr>
<td>Maximum</td>
<td>6235</td>
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<tr>
<td>Average DBH (inches)</td>
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<td>10.1</td>
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<tr>
<td>Maximum</td>
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</table>

a/ Discrepancies in the plot description (e.g., the means of a stand attribute after thinning and amount of thinning do not sum to the mean of that attribute before thinning as expected) are due to missing observations either before or after thinning.
Table 2. Distribution of all observations by site index (base age 25 years), age, basal area, and number of trees per acre.

<table>
<thead>
<tr>
<th>Site Index (feet)</th>
<th>Basal Area (sq.ft./acre)</th>
<th>Age (years)</th>
<th>Number of trees per acre ≤ 300</th>
<th>301- 500</th>
<th>501- 700</th>
<th>701- 900</th>
<th>901- 1100</th>
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Table 2. Distribution of all observations by site index (base age 25 years), age, basal area, and number of trees per acre (continued).

<table>
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<tr>
<th>Site Index</th>
<th>Age (feet)</th>
<th>Basal Area (sq.ft. /acre)</th>
<th>Number of trees per acre</th>
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</tr>
<tr>
<td>TOTAL</td>
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<td>276</td>
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</table>
attributes were predicted using regression techniques. The second stage involved determining the Weibull parameters so that the resulting diameter distribution would produce stand basal area and average dbh estimates identical to those predicted from regression equations in the first stage. By linking these two stages, the size-class distribution information produced is conditioned to provide aggregate values that are consistent with the predicted overall stand attributes.

**Stand-Level Model**

The stand-level model consisted of regression equations that predict (1) stand attributes (such as number of trees, basal area, minimum, and average diameters), and (2) density of a stand in the future (age $A_2$) based on stand information at present (age $A_1$). Also needed was a mean height equation that predicts total height corresponding to a given dbh. Table 3 shows the equations that form a whole stand model for thinned loblolly pine plantations.

Individual tree volume equations developed by Burkhart et al. (1972b) and Burkhart's (1977) volume ratio model were employed for estimating merchantable volumes. The site index equation developed by Devan (1979) was used to predict the average height of the dominants and codominants (HD) from site index and stand age, or to estimate site index from HD and stand age.

**Deriving Diameter Distribution from Stand Attributes**

The three-parameter Weibull pdf employed here to approximate diameter distribution is:

$$f(x) = \frac{c}{b} \left(\frac{x-a}{b}\right)^{c-1} \exp \left(-\left(\frac{x-a}{b}\right)^{c}\right), \quad x \geq a,$$

where $b$, $c$ = positive scale and shape parameters, respectively,

$a$ = nonnegative location parameter,

$x$ = diameter random variable.

The location parameter was predicted from a regression equation. The scale and shape parameters were searched for such that the resulting Weibull distribution would produce stand basal area and arithmetic mean dbh estimates identical to those predicted from regression equations. In other words, $b$ and $c$ were solutions of the following system of two equations:
Table 3. Regression equations that form a whole stand model for thinned loblolly pine plantations.

<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Equation a/</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \ln(B_2) = 5.40816 + 0.0032121 S - \left( \frac{A_1}{A_2} \right) \left[ 5.40816 + 0.0032121 S - \ln(B_1) \right] )</td>
</tr>
<tr>
<td></td>
<td>( n = 207; \quad \overline{\ln(B_2)} = 4.7230; \quad s_{y,x} = 0.0860 )</td>
</tr>
<tr>
<td></td>
<td>( R^2 = 99.34%; \quad R^2(B_2) = 80.47% )</td>
</tr>
<tr>
<td>2</td>
<td>( N_2 = \left[ N_1 ^{-0.65808 + 0.0000075795 \left( A_2^{1.78019} - A_1^{1.78019} \right)} \right] ^{-1/0.65808} )</td>
</tr>
<tr>
<td></td>
<td>( n = 207; \quad \overline{N_2} = 253.02; \quad s_{y,x} = 18.64 )</td>
</tr>
<tr>
<td></td>
<td>( R^2 = 97.07%; \quad R^2(N_2) = 97.07% )</td>
</tr>
<tr>
<td>3</td>
<td>( \ln(B) = -4.39181 + 0.19054 /A + 1.34753 \ln(HD) + 0.63902 \ln(N) )</td>
</tr>
<tr>
<td></td>
<td>( n = 490; \quad \overline{\ln(B)} = 4.7149; \quad s_{y,x} = 0.1407 )</td>
</tr>
<tr>
<td></td>
<td>( R^2 = 75.48%; \quad R^2(B) = 77.01% )</td>
</tr>
<tr>
<td>4</td>
<td>( \ln(N) = 7.79805 + 2.10495 /A - 1.89908 \ln(HD) + 1.16744 \ln(B) )</td>
</tr>
<tr>
<td></td>
<td>( n = 490; \quad \overline{\ln(N)} = 5.6732; \quad s_{y,x} = 0.1902 )</td>
</tr>
<tr>
<td></td>
<td>( R^2 = 87.19%; \quad R^2(N) = 85.78% )</td>
</tr>
<tr>
<td>5</td>
<td>( \ln(H) = 0.46152 + 0.43275 /A + 0.93333 \ln(HD) - 0.08583 \ln(B) )</td>
</tr>
<tr>
<td></td>
<td>( + 0.07596 \ln(N) - 2.15312 /D )</td>
</tr>
<tr>
<td></td>
<td>( n = 3559; \quad \overline{\ln(H)} = 4.0404; \quad s_{y,x} = 0.0422 )</td>
</tr>
<tr>
<td></td>
<td>( R^2 = 96.76%; \quad R^2(H) = 97.62% )</td>
</tr>
</tbody>
</table>
Table 3. Regression equations that form a whole stand model for thinned loblolly pine plantations (continued).

<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$\ln(D_{\text{min}}) = 1.10835 + 5.10755/A + 0.50531 \ln(HD)$</td>
</tr>
<tr>
<td></td>
<td>$+ 0.28544 \ln(B) - 0.57131 \ln(N)$</td>
</tr>
<tr>
<td></td>
<td>$n = 427; \overline{\ln(D_{\text{min}})} = 1.5253; s_{y.x} = 0.2972$</td>
</tr>
<tr>
<td></td>
<td>$R^2 = 46.84%; R^2(D_{\text{min}}) = 51.02%$</td>
</tr>
<tr>
<td>7</td>
<td>$\ln(D_{q-D}) = -9.05733 + 0.89274 \ln(HD) + 0.58151 \ln(N)$</td>
</tr>
<tr>
<td></td>
<td>$n = 489; \overline{\ln(D_{q-D})} = -2.1316; s_{y.x} = 0.6206$</td>
</tr>
<tr>
<td></td>
<td>$R^2 = 11.50%; R^2(D_{q-D}) = 99.80%$</td>
</tr>
</tbody>
</table>

\(^a/\) Notation:

- $\ln(x)$ = Natural logarithm of $x$,
- $R^2(x)$ = Percent variation of $x$ explained by the model,
- $A$ = Stand age in years,
- $B$ = Basal area in square feet per acre,
- $D$ = Tree diameter at breast height (dbh) in inches,
- $\overline{D}$ = Arithmetic mean dbh in inches,
- $D_{\text{min}}$ = Minimum dbh in inches,
- $D_{q}$ = Quadratic mean dbh in inches,
- $H$ = Total height in feet of a tree having dbh $D$,
- $H_D$ = Average height in feet of the dominants and codominants,
- $N$ = Number of surviving trees per acre,
- $S$ = Site index in feet (base age 25 years).

Subscript $i$ denotes that the measurement is taken at time $i$. 
where

\[ \hat{D} = \int_{a}^{\infty} x f(x) \, dx \]  \hspace{1cm} (8) \\

\[ \hat{B} = 0.005454 \, N \int_{a}^{\infty} x^2 f(x) \, dx \]  \hspace{1cm} (9)

where \( \hat{D} \) = predicted arithmetic mean dbh in inches, \\
\( \hat{B} \) = predicted basal area in square feet per acre, \\
\( N \) = number of surviving trees per acre, \\
\( f(x) \) = Weibull pdf with parameters a, b, and c.

Equation (8) can be rewritten as

\[ \hat{D} = a + b \, \Gamma(1 + 1/c) \]  \hspace{1cm} (10)

or

\[ b = (\hat{D} - a) / \Gamma(1 + 1/c) \]  \hspace{1cm} (11)

where \( \Gamma(x) \) = gamma function evaluated at \( x \).

In most diameter distribution models, stand volume and basal area are often obtained by first computing these attributes for each dbh class and then summing over diameter classes of interest. Equation (9) can be approximated in a similar manner by replacing the integral sign with a summation sign:

\[ B = 0.005454 \, N \sum_{x_i=1}^{\infty} x_i^2 \, f_i \]  \hspace{1cm} (12)

where \( x_i \) = midpoint of the \( i \)th dbh class, \\
\( f_i = F(x_i+0.5) - F(x_i-0.5) \) = proportion of trees in the \( i \)th dbh class, \\
\( F(x) = 1 - \exp \left\{ -[(x-a)/b]^c \right\} \) = Weibull cumulative distribution function with parameters a, b, and c.

Starting with a guess for c, parameter b can be computed from (11) given a and c. All three parameters (a, b, and c) then specify a Weibull distribution. If equation (12) is not satisfied, a refined estimate for c will be computed and the procedures are repeated until both sides of equation (12) are almost equal. This method reduces the problem to that of solving one nonlinear equation (equation 12) whose unknown is the shape parameter c of the Weibull pdf.
RESULTS AND DISCUSSION

Program WTHIN

All of the techniques described earlier were incorporated into program WTHIN, which was written in standard FORTRAN. This program can generate stand and stock tables for different combinations of site, stand age, and density. It is also able to simulate a loblolly pine stand for a specified period during which thinning options are available at any point in time.

Prediction of the Present Stand

The inputs needed are:

1. age of the present stand,
2. site index (or average height of the current dominants and codominants),
3. two measures of density (total basal area and number of trees per acre).

If only one measure of density is available, the other can be estimated by employing the appropriate equation (3 or 4) of Table 3. Equations (6, 7) of Table 3 predict the minimum and arithmetic mean dbh of the stand. The Weibull location parameter \( a \) is computed from \( D_{\text{min}} \) as follows:

\[
a = \text{FLOOR} \left( D_{\text{min}} - 0.5 \right) - 0.49,
\]

where \( \text{FLOOR} (x) \) = integer portion of \( x \).
This adjustment simply sets \( D_{\text{min}} \) at the lower end of its 1-inch dbh class and then decreases it by 1 inch.

The Weibull parameters \( b \) and \( c \) are obtained by solving equation (12). As a result, number of trees and basal area per acre for each dbh class can be computed. The mean height equation (equation 5 of Table 3) predicts total height corresponding to the midpoint of each dbh class. Total volumes outside and inside bark can be obtained from the individual tree volume equations published by Burkhart et al. (1972b). Merchantable volumes can also be calculated using the volume ratio methods developed by Burkhart (1977) and Cao and Burkhart (1980).
Thinning

Inputs for the thinning option include age of the stand when thinning occurs and type of thinning. Thinning can be carried out by rows, from below, or a combination of both.

It is assumed that the diameter distribution does not change due to row thinning. Thus the number of trees, basal area, and volume per acre in each dbh class are reduced by the proportion of trees removed in thinning.

Thinning from below is defined here as removing all trees with dbh values less than a specified diameter. Input for this type of thinning can be either this diameter limit or a residual basal area. A combination of row and low thinning involves first a row thinning followed by a thinning from below.

Alternative thinning algorithms can be easily substituted for those included in this model if one has information on removal patterns for the operations of interest.

Projection

Basal area and number of trees per acre at some age in the future can be projected using equations (1) and (2) of Table 3 for thinned stands, or the following equations from Coile and Schumacher (1964) for unthinned loblolly pine plantations:

\[
\log_{10}(N) = \log_{10}(N_0) + [2.1346 - 1.1103 \log_{10}(N_0) + 0.1384 (OF)] A/100
\]

\[
\log_{10}(B) = 1.4366 \log_{10}(S) - 0.7084 (10/A) + 0.4888 \log_{10}(N) + 0.0585 (OF) - 1.4436
\]

where

- A = age in years,
- B = stand basal area in square feet per acre at age A,
- N = number of surviving trees per acre at age A,
- \( N_0 \) = number of trees planted per acre,
- OF = +1 if old-field origin, and -1 otherwise,
- S = site index in feet (base age 25 years).

Procedures similar to those for predicting the present stand are then employed to produce stand and stock tables for the future stand.
Diameter Distribution of a Previously Low-Thinned Stand

Suppose that in a previous thinning from below, all trees having dbh below Dthin were cut. If the predicted Weibull location parameter (a) for the present stand is greater than or equal to Dthin, then the complete Weibull function is used to characterize the current diameter distribution. On the other hand, when a is less than Dthin, a left-truncated Weibull pdf is more appropriate where Dthin is the truncation point.

When the truncated Weibull is employed, equation (10) is replaced with:

\[
\hat{a} = a + \int_{(D\text{thin}-a)/b}^{\infty} \frac{x(c/b)(x/b)^{c-1} \exp[-(x/b)^c]}{1 - F(D\text{thin})} \, dx
\]

or

\[
\hat{b} = a + \frac{b}{1 - F(D\text{thin})} \int_{(D\text{thin}-a)/b}^{\infty} y^{1/c} \exp(-y) \, dy
\]

or

\[
\hat{b} = a + \frac{b}{1 - F(D\text{thin})} \left[ (1 + 1/c) - \int_{0}^{(D\text{thin}-a)/b} y^{1/c} \exp(-y) \, dy \right]
\]

(13)

where \( F(x) = 1 - \exp\left\{-[(x-a)/b]^c\right\} \).

The procedures for deriving the parameters of the truncated Weibull pdf are similar to those of the complete Weibull described earlier. The shape parameter c is solved from equation (12); for each estimated value of c, the scale parameter b is obtained from equation (13) (instead of from equation (11) as in the case of the complete Weibull pdf). The proportion of trees in the ith dbh class of the truncated distribution is given by:

\[
f_i = \frac{F(i+0.5) - F(i-0.5)}{1 - F(D\text{thin})}
\]
Effect of Thinning Regimes on Yield

In order to demonstrate the effect of thinning type and intensities on yield, different thinning options were applied to loblolly pine plantations on site index 60 soil. These hypothetical stands had 800 trees and 130 sq.ft. per acre of basal area at age 15, and would be harvested at age 30. Option D was the control where no thinning was applied. In the rest of the thinning options, the stands were thinned repeatedly at ages 15, 20, and 25 to a specified residual basal area. Residual basal areas were arbitrarily set at 80, 95, and 110 sq.ft. per acre for options A, B and C, respectively. Three types of thinning were considered for each residual density: (1) row thinning, (2) low thinning, and (3) a combination of row and low thinnings, where 25% of the basal area removed was first cut in a row thinning and then the remainder from a thinning from below. Option B1, for example, means row thinning to 95 sq.ft./acre of residual basal area.

Yields of these stands under different regimes are presented in Table 4. Total cubic-foot volume production (amount removed in thinnings plus final harvest volume) did not differ much from row to low thinning for a given thinning level. Note that thinning level is to a specified residual basal area and that number of trees remaining therefore varies by thinning type. Stand average diameter, however, was lowest in row thinning, highest in low thinning, and somewhere between these two extremes in the combination of row and low thinnings, as expected. As found by other researchers (such as Feduccia and Mann 1976, Sullivan and Williston 1977), cubic-foot volume production increased with higher residual basal area. On the other hand, average dbh increased as the thinnings were more severe, which implies an increase in board-foot volume production.

Although only total cubic-foot volume is presented in Table 4, users can readily develop yield tables in other units (cords, board feet, pounds, etc.) and for any specified portion of the stand by substituting appropriate volume or weight equations and specifying desired threshold diameters in the model.

Comparison with Published Information on Thinning

Coile and Schumacher's (1964) Model

Program WTHIN was compared with the model for thinned loblolly pine plantations developed by Coile and Schumacher (1964); results
Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trees (sq.ft.)</td>
<td>Basal Area (inches)</td>
<td>Average DBH</td>
<td>Total Volume (cu.ft.)</td>
<td>Number of trees (sq.ft.)</td>
</tr>
<tr>
<td><strong>OPTION A1:</strong> Row thinning -- Residual basal area = 80 sq.ft./acre</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>20</td>
<td>466</td>
<td>108</td>
<td>6.4</td>
<td>2375</td>
</tr>
<tr>
<td>25</td>
<td>326</td>
<td>102</td>
<td>7.4</td>
<td>2643</td>
</tr>
<tr>
<td>30</td>
<td>242</td>
<td>98</td>
<td>8.5</td>
<td>2860</td>
</tr>
<tr>
<td><strong>OPTION A2:</strong> Low thinning -- Residual basal area = 80 sq.ft./acre</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>20</td>
<td>335</td>
<td>108</td>
<td>7.6</td>
<td>2376</td>
</tr>
<tr>
<td>25</td>
<td>202</td>
<td>102</td>
<td>9.5</td>
<td>2652</td>
</tr>
<tr>
<td>30</td>
<td>134</td>
<td>98</td>
<td>11.5</td>
<td>2868</td>
</tr>
<tr>
<td><strong>OPTION A3:</strong> 25% row thinning and 75% low thinning -- Residual basal area = 80 sq.ft./acre</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>20</td>
<td>351</td>
<td>108</td>
<td>7.4</td>
<td>2376</td>
</tr>
<tr>
<td>25</td>
<td>212</td>
<td>102</td>
<td>9.3</td>
<td>2652</td>
</tr>
<tr>
<td>30</td>
<td>143</td>
<td>98</td>
<td>11.1</td>
<td>2868</td>
</tr>
</tbody>
</table>
Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed</th>
<th>Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trees</td>
<td>Basal Area (sq.ft.)</td>
<td>Average DBH (inches)</td>
<td>Total Volume (cu.ft.)</td>
<td>Number of trees</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>20</td>
<td>550</td>
<td>123</td>
<td>6.3</td>
<td>2700</td>
</tr>
<tr>
<td>25</td>
<td>398</td>
<td>117</td>
<td>7.2</td>
<td>3028</td>
</tr>
<tr>
<td>30</td>
<td>304</td>
<td>113</td>
<td>8.1</td>
<td>3294</td>
</tr>
</tbody>
</table>

**OPTION B1:** Row thinning -- Residual basal area = 95 sq.ft./acre

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed</th>
<th>Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trees</td>
<td>Basal Area (sq.ft.)</td>
<td>Average DBH (inches)</td>
<td>Total Volume (cu.ft.)</td>
<td>Number of trees</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>20</td>
<td>430</td>
<td>123</td>
<td>7.1</td>
<td>2700</td>
</tr>
<tr>
<td>25</td>
<td>261</td>
<td>117</td>
<td>9.0</td>
<td>3038</td>
</tr>
<tr>
<td>30</td>
<td>180</td>
<td>113</td>
<td>10.6</td>
<td>3305</td>
</tr>
</tbody>
</table>

**OPTION B2:** Low thinning -- Residual basal area = 95 sq.ft./acre

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed</th>
<th>Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trees</td>
<td>Basal Area (sq.ft.)</td>
<td>Average DBH (inches)</td>
<td>Total Volume (cu.ft.)</td>
<td>Number of trees</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>20</td>
<td>446</td>
<td>123</td>
<td>7.0</td>
<td>2699</td>
</tr>
<tr>
<td>25</td>
<td>279</td>
<td>117</td>
<td>8.6</td>
<td>3037</td>
</tr>
<tr>
<td>30</td>
<td>192</td>
<td>113</td>
<td>10.3</td>
<td>3305</td>
</tr>
</tbody>
</table>

**OPTION B3:** 25% row thinning and 75% low thinning -- Residual basal area = 95 sq.ft./acre

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed</th>
<th>Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trees</td>
<td>Basal Area (sq.ft.)</td>
<td>Average DBH (inches)</td>
<td>Total Volume (cu.ft.)</td>
<td>Number of trees</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>20</td>
<td>446</td>
<td>123</td>
<td>7.0</td>
<td>2699</td>
</tr>
<tr>
<td>25</td>
<td>279</td>
<td>117</td>
<td>8.6</td>
<td>3037</td>
</tr>
<tr>
<td>30</td>
<td>192</td>
<td>113</td>
<td>10.3</td>
<td>3305</td>
</tr>
</tbody>
</table>
Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed</th>
<th>Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>Basal Area</td>
<td>Average</td>
<td>Total Volume ob</td>
<td>Volume</td>
</tr>
<tr>
<td>of trees</td>
<td>(sq.ft.)</td>
<td>DBH (inches)</td>
<td>(cu.ft.)</td>
<td>(cu.ft.)</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>20</td>
<td>632</td>
<td>138</td>
<td>6.2</td>
<td>3013</td>
</tr>
<tr>
<td>25</td>
<td>472</td>
<td>132</td>
<td>7.0</td>
<td>3401</td>
</tr>
<tr>
<td>30</td>
<td>368</td>
<td>128</td>
<td>7.8</td>
<td>3717</td>
</tr>
<tr>
<td>OPTION Cl:</td>
<td>Row thinning --</td>
<td>677</td>
<td>110</td>
<td>5.3</td>
</tr>
<tr>
<td></td>
<td>Residual basal area = 110 sq.ft./acre</td>
<td>504</td>
<td>110</td>
<td>6.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>394</td>
<td>110</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>368</td>
<td>128</td>
<td>7.8</td>
</tr>
<tr>
<td>OPTION C2:</td>
<td>Low thinning --</td>
<td>564</td>
<td>110</td>
<td>5.9</td>
</tr>
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<td></td>
<td>Residual basal area = 110 sq.ft./acre</td>
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<td>110</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>246</td>
<td>110</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>234</td>
<td>128</td>
<td>9.9</td>
</tr>
<tr>
<td>OPTION C3:</td>
<td>25% row thinning and 75% low thinning -- Residual basal area = 110 sq.ft./acre</td>
<td>573</td>
<td>110</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>372</td>
<td>110</td>
<td>7.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>264</td>
<td>110</td>
<td>8.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>250</td>
<td>128</td>
<td>9.6</td>
</tr>
</tbody>
</table>
Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed</th>
<th>Volume Production</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of trees</td>
<td>Basal area (sq.ft.)</td>
<td>Average DBH (inches)</td>
<td>Total Volume ob (cu.ft.)</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>30</td>
<td>540</td>
<td>186</td>
<td>7.8</td>
<td>5387</td>
</tr>
</tbody>
</table>

OPTION D: No thinning
are presented in Table 5. Both row and low thinning options were tried, for the thinning in practice would likely be somewhere between these two cases. Care was taken such that cord volume removed in each thinning was identical to that specified by Coile and Schumacher. Examination of the residual stands at age 30 revealed that the number of surviving trees from Coile and Schumacher's model was between the predicted values from the two types of thinning of program WTHIN. Residual basal area, quadratic mean dbh, and volume from Coile and Schumacher's predictions were consistently higher than those from WTHIN.

Coile and Schumacher's predicted total volume production of thinned stands far exceeded that of unthinned counterparts. On the other hand, total volume predictions (i.e., volume removed in thinnings plus residual volume) of thinned stands at age 30 from program WTHIN were close to volumes of unthinned stands at age 30 from Coile and Schumacher's model. This agrees well with what other investigators have found, namely, that total cubic-foot volume production is generally little affected by thinning (Smith 1962, Andrulot et al. 1972, Goebel et al. 1974).

Yields Reported by Goebel et al. (1974)

Goebel et al. (1974) reported yields of 9 old-field loblolly pine stands; each had been thinned 4 to 5 times to a specified residual basal area per acre. Site indices were determined from curves developed by Goebel and Shipman (1964). Goebel and Warner (1969) recognized a significant site-age bias in these site index curves and revised their yield model using Clutter and Lenhart's (1968) polymorphic site index curves. Devan's (1979) site index equation was replaced with that of Clutter and Lenhart (1968) in program WTHIN when simulating the stands based on the guidelines set forth by Goebel et al. (1974). Data for total cubic-foot volumes reported by Goebel et al. (1974) were based on volume tables prepared by MacKinney and Chaiken (1939). Thus MacKinney and Chaiken's (1939) individual tree volume equation was used in this simulation.

The observed number of trees per acre and average dbh in each plot fell between values predicted from WTHIN using the row and low thinning options (Table 6). Comparison of total volume production in these 9 stands shows that the mean relative difference between observed and predicted yields (averages of yields from the row and low thinning options) is -2.52%.
Table 5. Comparison of predicted yields of Coile and Schumacher (1964) and those from program WTHIN on a per acre basis for thinned loblolly pine plantations.

<table>
<thead>
<tr>
<th>Source</th>
<th>Site Index</th>
<th>Number of trees</th>
<th>Basal Area (sq.ft.) at age 5</th>
<th>Age when thinned (years)</th>
<th>Basal Area (sq.ft.)</th>
<th>Volume (cords)</th>
<th>Residual stand at age 30</th>
<th>Total Volume Production (cords)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C&amp;S</td>
<td>50</td>
<td>600</td>
<td>9.9</td>
<td>20</td>
<td>68</td>
<td>10</td>
<td>13.2</td>
<td>140 (26.7) 135 (26.7) 28.7 38.7</td>
</tr>
<tr>
<td>Row</td>
<td>50</td>
<td>600</td>
<td>11.4</td>
<td>20</td>
<td>82</td>
<td>12</td>
<td>13.4</td>
<td>146 (26.7) 142 (26.7) 30.3 42.3</td>
</tr>
<tr>
<td>Low</td>
<td>50</td>
<td>600</td>
<td>12.9</td>
<td>17.22</td>
<td>45.36</td>
<td>7.7</td>
<td>12.6</td>
<td>168 (29.8) 170 (29.8) 43.7 47.7</td>
</tr>
<tr>
<td>C&amp;S</td>
<td>60</td>
<td>600</td>
<td>14.8</td>
<td>17.22</td>
<td>58.47</td>
<td>9.9</td>
<td>14.6</td>
<td>159 (42.9) 185 (42.9) 47.1 65.1</td>
</tr>
<tr>
<td>Row</td>
<td>60</td>
<td>600</td>
<td>16.1</td>
<td>15.20.25</td>
<td>37.37</td>
<td>6.8,10</td>
<td>15.1</td>
<td>158 (47.2) 196 (47.2) 60.6 84.6</td>
</tr>
<tr>
<td>Low</td>
<td>60</td>
<td>600</td>
<td>18.5</td>
<td>15.20.25</td>
<td>43.47</td>
<td>7.8,13</td>
<td>14.7</td>
<td>189 (70.0) 222 (70.0) 68.2 98.2</td>
</tr>
</tbody>
</table>

* Site index at base age 25 years.
* Cord volume to a 4-inch top, converted from cubic-foot volume outside bark to a 4-inch top, using ratios from Burkhart et al. (1972b).
* Coile and Schumacher (1964).
* Row thinning, program WTHIN.
* Low thinning, program WTHIN.
* Residual stand at age 30. Numbers in parentheses are for unthinned stands.
Table 6. Comparison of observed yields of Goebel et al. (1974) and predicted yields from program WTHIN on a per acre basis for thinned loblolly pine plantations.

<table>
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<tr>
<th>Source</th>
<th>Site Index (feet)</th>
<th>Age (years)</th>
<th>Number of trees (sq.ft.)</th>
<th>Average DBH (inches)</th>
<th>Total Volume (cu.ft.)</th>
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<td></td>
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<tr>
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<td>13</td>
<td>790</td>
<td>121</td>
<td>5.3</td>
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<tr>
<td>Row</td>
<td>60</td>
<td>5.2</td>
<td>1491</td>
<td></td>
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<tr>
<td>Low</td>
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<tr>
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<td>51</td>
<td>13</td>
<td>800</td>
<td>116</td>
<td>5.0</td>
</tr>
<tr>
<td>Row</td>
<td>60</td>
<td>5.0</td>
<td>14/22</td>
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<td></td>
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<tr>
<td>Low</td>
<td>60</td>
<td>5.0</td>
<td>14/22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>51</td>
<td>13</td>
<td>800</td>
<td>116</td>
<td>5.4</td>
</tr>
<tr>
<td>Row</td>
<td>60</td>
<td>5.4</td>
<td>16/00</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>Observed</td>
<td>51</td>
<td>13</td>
<td>1016</td>
<td>124</td>
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<tr>
<td>Row</td>
<td>60</td>
<td>4.6</td>
<td>14/94</td>
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<td>4.6</td>
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<td>1004</td>
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<td>28/80</td>
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<td>Low</td>
<td>61</td>
<td>5.0</td>
<td>28/80</td>
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<table>
<thead>
<tr>
<th>After periodic thinnings</th>
<th>After periodic thinnings</th>
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<tr>
<td>Row</td>
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<td>Row</td>
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<td>Observed</td>
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<td>Row</td>
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<td>Low</td>
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</tr>
<tr>
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</tr>
<tr>
<td>Row</td>
<td>30</td>
</tr>
<tr>
<td>Low</td>
<td>30</td>
</tr>
</tbody>
</table>

---

Note:
- a/ Site index (base age 25 years) from Goebel and Shipman (1964).
- b/ Row thinning, program WTHIN.
- c/ Low thinning, program WTHIN.
- d/ Site index (base age 25 years) from Clutter and Lenhart (1968).
Possible Modifications and Refinements

In this study, a growth and yield model for thinned loblolly pine plantations was developed in which the parameters of the Weibull function that characterized the diameter distribution were searched for to insure that the resulting stand basal area and average dbh estimates were identical to those predicted from stand variables using regression techniques. Although the model gave logical results that agreed well with past work on thinning, there is still room for improvement.

Two specific areas for further investigation are:

1. Various methods for deriving a dbh distribution from stand attributes for thinned stands need to be more fully evaluated.

2. More realistic removal patterns for thinning from below should be developed. One possibility is to establish stochastic models in which trees in each dbh class are assigned probabilities of being removed, and are cut or left in each thinning operation depending on values of the random numbers generated.
LITERATURE CITED


<table>
<thead>
<tr>
<th>Appendix</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>A numerical example.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>36</td>
</tr>
<tr>
<td>Input variable formats and description for program WTHIN.</td>
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</tr>
<tr>
<td>2a. Subprogram identification card (first card).</td>
<td></td>
</tr>
<tr>
<td>2b. Subprogram INPUT1.</td>
<td>37</td>
</tr>
<tr>
<td>2c. Subprogram INPUT2.</td>
<td>40</td>
</tr>
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<td>3</td>
<td>41</td>
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<tr>
<td>Input example for program WTHIN.</td>
<td></td>
</tr>
<tr>
<td>3a. Simulate a stand through time.</td>
<td></td>
</tr>
<tr>
<td>3b. Stand and stock tables for specified combinations of site index, age, and density.</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>43</td>
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<tr>
<td>Generalized flowchart of program WTHIN.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>45</td>
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<tr>
<td>Source listing for program WTHIN.</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 1. A numerical example.

The following example is chosen to illustrate the techniques employed in program WTHIN. Consider a loblolly pine plantation on soil of site index 60 feet (base age 25 years), with 600 trees and 150 sq.ft. of basal area per acre at age 20. The stand is thinned to 100 sq.ft. per acre at age 20; the thinning method is a combination of 25% row thinning and 75% low thinning (i.e. a row thinning removes 25% of the total basal area scheduled to be thinned, and then a thinning from below removes the remaining 75%). No minimum diameter for removal in the low thinning is specified in this example. The stand is then left to grow until it is harvested at age 40. The card input needed by program WTHIN to simulate this particular stand is presented in Appendix 3a. Figures A1 to A4 show the outputs of this simulation from program WTHIN. The computational steps (on a per acre basis) are outlined as follows.

**Step 1: Yield prediction of the stand before thinning.**

Stand variables: Site index = 60 feet, A = 20 years, N = 600 trees, B = 150 sq.ft. (variable names are defined in Table 3).

From Devan's (1979) site index equation, average height of the dominants and codominants at age 20 is 49.55 feet. Substituting the values into the appropriate stand variables in equations (6, 7) of Table 3 gives: $D_{min} = 3.04$ inches and $\bar{D} = 6.61$ inches.

The Weibull location parameter is adjusted from $D_{min}$ as follows:

$$a = \text{FLOOR}(D_{min}-0.5) - 0.49 = 1.51,$$

where $\text{FLOOR}(x) = \text{integer portion of } x$.

The remaining parameters defining a Weibull distribution which produces a total basal area of 150 sq.ft./acre and an average dbh of 6.61 inches are found to be

$$b = 5.6274 \quad \text{and} \quad c = 4.0385.$$

Per acre number of trees, basal area, and volume for each dbh class can be computed. For example, number of trees in the 6-inch class is $600 \times F(6.5) - F(5.5) = 143.3$ trees, where $F(x)$ is the Weibull cdf evaluated at $x$. Basal area in the 6-inch class:

$$143.3 \times 0.005454 \times 6^2 = 28.1 \text{ sq.ft.}$$

Average height of a tree with a 6-inch dbh in this plantation is calculated from equation (5) of Table 3 to be 45.7 feet. Burkhart et al.'s (1972b) tree volume equation is applied on 143.3 trees of dbh 6 inches and total height 45.7 feet, resulting in a volume of 597.4 cu.ft. outside bark in the 6-inch dbh class. Summing volume
25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/acre. HARVEST AGE = 40.

**INPUTS**

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<th>SITE</th>
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<tr>
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<tr>
<td>BASAL AREA</td>
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**PREDICTED**

<table>
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<th>NUMBER OF TREES</th>
<th>AVERAGE HEIGHT</th>
<th>BASAL AREA</th>
<th>TOTAL VOLUME O.B.</th>
<th>TOTAL VOLUME I.B.</th>
<th>VOLUME TO 4.1 IN O.B.</th>
<th>VOLUME TO 4.1 IN I.B.</th>
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AVERAGE DBH = 6.61

CORD VOLUME TO 4.1 IN = 31.43

WEIBULL PARAMETERS

A = 1.5100
B = 5.6274
C = 4.0385

CONVERGENCE ATTAINED

---

Figure A1. Example output from program WTHIN --
Step 1: Yield prediction of the stand before thinning.
25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

ROW THINNING AT AGE 20.
8.33% OF TREES IN ALL DIAMETER CLASSES ARE CUT

BEFORE ROW THINNING

<table>
<thead>
<tr>
<th>SITE</th>
<th>60.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGE</td>
<td>20.00</td>
</tr>
<tr>
<td>NUMBER OF TREES</td>
<td>500.00</td>
</tr>
<tr>
<td>BASAL AREA</td>
<td>150.00</td>
</tr>
<tr>
<td>AVERAGE DBH</td>
<td>6.61</td>
</tr>
</tbody>
</table>

AFTER ROW THINNING

<table>
<thead>
<tr>
<th>DBH CLASS</th>
<th>NUMBER OF TREES</th>
<th>AVERAGE HEIGHT</th>
<th>BASAL AREA</th>
<th>TOTAL VOLUME O.B.</th>
<th>TOTAL VOLUME I.B.</th>
<th>VOLUME TO 4. IN</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.5</td>
<td>22.3</td>
<td>0.0</td>
<td>0.3</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>7.7</td>
<td>31.9</td>
<td>0.4</td>
<td>7.8</td>
<td>5.0</td>
<td>108.9</td>
</tr>
<tr>
<td>4</td>
<td>33.0</td>
<td>28.2</td>
<td>2.9</td>
<td>58.3</td>
<td>41.1</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>80.2</td>
<td>42.6</td>
<td>10.9</td>
<td>226.1</td>
<td>167.4</td>
<td>151.9</td>
</tr>
<tr>
<td>6</td>
<td>131.4</td>
<td>45.7</td>
<td>25.8</td>
<td>547.6</td>
<td>415.5</td>
<td>436.1</td>
</tr>
<tr>
<td>7</td>
<td>145.4</td>
<td>48.1</td>
<td>38.8</td>
<td>846.3</td>
<td>651.4</td>
<td>734.8</td>
</tr>
<tr>
<td>8</td>
<td>102.0</td>
<td>50.0</td>
<td>35.6</td>
<td>793.4</td>
<td>616.3</td>
<td>720.7</td>
</tr>
<tr>
<td>9</td>
<td>40.9</td>
<td>51.6</td>
<td>18.1</td>
<td>410.8</td>
<td>321.0</td>
<td>383.4</td>
</tr>
<tr>
<td>10</td>
<td>8.2</td>
<td>52.8</td>
<td>4.5</td>
<td>103.5</td>
<td>81.2</td>
<td>98.3</td>
</tr>
<tr>
<td>11</td>
<td>0.7</td>
<td>53.8</td>
<td>0.5</td>
<td>10.9</td>
<td>8.6</td>
<td>10.5</td>
</tr>
<tr>
<td>12</td>
<td>0.0</td>
<td>54.7</td>
<td>0.0</td>
<td>0.4</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

550.0
137.5
3005.4
2308.0
2538.1
1930.0

AMOUNT REMOVED IN ROW THINNING

| NUMBER OF TREES | 50.00 |
| BASE AREA       | 12.50 |
| TOTAL CU FT VOLUME O.B. | 273.22 |
| CU FT VOLUME O.B. TO 4. IN | 230.73 |
| CORD VOLUME TO 4. IN | 2.62 |

Figure A2. Example output from program WTHIN --
Step 2: Row thinning at age 20.
25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

LOW THINNING AT AGE 20.
THIN TO 100.00 SQ.FT. RESIDUAL BASAL AREA

BEFORE LOW THINNING

---

<table>
<thead>
<tr>
<th>SITE</th>
<th>60.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGE</td>
<td>20.00</td>
</tr>
<tr>
<td>NUMBER OF TREES</td>
<td>550.00</td>
</tr>
<tr>
<td>BASAL AREA</td>
<td>137.50</td>
</tr>
<tr>
<td>AVERAGE DBH</td>
<td>6.61</td>
</tr>
</tbody>
</table>

AFTER LOW THINNING

---

<table>
<thead>
<tr>
<th>DBH CLASS</th>
<th>NUMBER OF TREES</th>
<th>AVERAGE HEIGHT</th>
<th>AVERAGE BASE AREA</th>
<th>TOTAL VOLUME O.B.</th>
<th>TOTAL VOLUME I.B.</th>
<th>VOLUME O.B. TO 4.1N</th>
<th>VOLUME I.B. TO 4.1N</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12.7</td>
<td>45.7</td>
<td>2.5</td>
<td>53.1</td>
<td>40.3</td>
<td>42.4</td>
<td>31.6</td>
</tr>
<tr>
<td>7</td>
<td>145.4</td>
<td>48.1</td>
<td>38.0</td>
<td>846.3</td>
<td>651.4</td>
<td>734.8</td>
<td>558.1</td>
</tr>
<tr>
<td>8</td>
<td>102.0</td>
<td>50.0</td>
<td>35.6</td>
<td>793.4</td>
<td>615.3</td>
<td>720.7</td>
<td>554.4</td>
</tr>
<tr>
<td>9</td>
<td>40.9</td>
<td>51.6</td>
<td>18.1</td>
<td>410.8</td>
<td>321.0</td>
<td>383.4</td>
<td>297.4</td>
</tr>
<tr>
<td>10</td>
<td>0.7</td>
<td>52.8</td>
<td>4.5</td>
<td>103.5</td>
<td>81.2</td>
<td>98.3</td>
<td>76.7</td>
</tr>
<tr>
<td>11</td>
<td>4.5</td>
<td>63.8</td>
<td>0.5</td>
<td>10.9</td>
<td>8.6</td>
<td>10.5</td>
<td>8.2</td>
</tr>
<tr>
<td>12</td>
<td>0.0</td>
<td>54.7</td>
<td>0.0</td>
<td>0.4</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

310.0 100.0 2218.4 1719.0 1990.5 1526.7

SITE  = 60.00
AGE  = 20.00
NUMBER OF TREES = 309.97
BASAL AREA = 100.00
AVERAGE DBH = 7.64 BASED ON 1-INCH DBH CLASSES

AMOUNT REMOVED IN LOW THINNING

---

| NUMBER OF TREES = 240.03 |
| BASAL AREA = 37.50 |
| TOTAL CU.FT. VOLUME O.B. = 787.06 |
| CU.FT. VOLUME O.B. TO 4.1N = 547.57 |
| CORD VOLUME TO 4.1N = 6.46 |

Figure A3. Example output from program WITHIN

---

Step 3: Low thinning at age 20.
25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

**INPUTS**

- **SITE** = 60.00
- **AGE** = 40.00
- **NUMBER OF TREES** = 245.26
- **BASAL AREA** = 164.52

**PREDICTED**

- **HD** = 81.14
- **AVERAGE DBH** = 10.95
- **MINIMUM DBH** = 5.87

**THIS STAND WAS PREVIOUSLY THINNED FROM BELOW**
**ALL TREES UNDER 5.5 INCHES IN DBH WERE CUT**

```
<table>
<thead>
<tr>
<th>DBH CLASS</th>
<th>NUMBER OF TREES</th>
<th>AVERAGE HEIGHT</th>
<th>BASEAL AREA</th>
<th>TOTAL VOLUME O.B.</th>
<th>TOTAL VOLUME I.B.</th>
<th>VOLUME TO 4. IN O.B.</th>
<th>VOLUME TO 4. IN I.B.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.3</td>
<td>66.5</td>
<td>0.2</td>
<td>7.4</td>
<td>5.7</td>
<td>5.9</td>
<td>4.5</td>
</tr>
<tr>
<td>7</td>
<td>5.7</td>
<td>69.9</td>
<td>1.5</td>
<td>47.0</td>
<td>36.5</td>
<td>40.8</td>
<td>31.3</td>
</tr>
<tr>
<td>8</td>
<td>15.1</td>
<td>72.7</td>
<td>5.3</td>
<td>168.6</td>
<td>132.0</td>
<td>153.1</td>
<td>118.7</td>
</tr>
<tr>
<td>9</td>
<td>29.6</td>
<td>74.9</td>
<td>13.1</td>
<td>426.9</td>
<td>335.6</td>
<td>398.4</td>
<td>310.9</td>
</tr>
<tr>
<td>10</td>
<td>45.0</td>
<td>76.7</td>
<td>24.5</td>
<td>816.5</td>
<td>643.8</td>
<td>775.6</td>
<td>608.1</td>
</tr>
<tr>
<td>11</td>
<td>53.3</td>
<td>78.2</td>
<td>35.1</td>
<td>1188.0</td>
<td>938.8</td>
<td>1142.1</td>
<td>898.3</td>
</tr>
<tr>
<td>12</td>
<td>47.6</td>
<td>79.5</td>
<td>37.4</td>
<td>1281.1</td>
<td>1013.9</td>
<td>1241.9</td>
<td>979.1</td>
</tr>
<tr>
<td>13</td>
<td>30.5</td>
<td>80.6</td>
<td>28.1</td>
<td>975.8</td>
<td>773.2</td>
<td>951.8</td>
<td>751.8</td>
</tr>
<tr>
<td>14</td>
<td>13.2</td>
<td>81.6</td>
<td>14.1</td>
<td>492.4</td>
<td>390.6</td>
<td>482.5</td>
<td>381.7</td>
</tr>
<tr>
<td>15</td>
<td>3.5</td>
<td>82.4</td>
<td>4.3</td>
<td>152.3</td>
<td>120.9</td>
<td>149.8</td>
<td>118.6</td>
</tr>
<tr>
<td>16</td>
<td>0.5</td>
<td>83.2</td>
<td>0.7</td>
<td>26.4</td>
<td>21.0</td>
<td>26.0</td>
<td>20.6</td>
</tr>
<tr>
<td>17</td>
<td>0.0</td>
<td>83.8</td>
<td>0.1</td>
<td>2.3</td>
<td>1.8</td>
<td>2.3</td>
<td>1.8</td>
</tr>
<tr>
<td><strong>-----</strong></td>
<td><strong>-----</strong></td>
<td><strong>-----</strong></td>
<td><strong>-----</strong></td>
<td><strong>-----</strong></td>
<td><strong>-----</strong></td>
<td><strong>-----</strong></td>
<td><strong>-----</strong></td>
</tr>
<tr>
<td><strong>245.3</strong></td>
<td><strong>164.5</strong></td>
<td><strong>5584.7</strong></td>
<td><strong>4413.8</strong></td>
<td><strong>5370.3</strong></td>
<td><strong>4225.5</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

**AVERAGE DBH** = 10.95

**Cord volume to 4. in** = 57.51

**WEIBULL PARAMETERS**

- **A** = 4.5100
- **B** = 7.0872
- **C** = 4.1068

**CONVERGENCE ATTAINED**

Figure A4. Example output from program WTHIN --
Step 4: Project to age 40.
estimates over dbh classes gives a stand volume value of 3279 cu.ft. per acre.

**Step 2: Row thinning at age 20.**

In this example, 25% of the basal area removed is due to row thinning. Total basal area removed in two thinnings: 150 - 100 = 50 sq.ft. Residual basal area after row thinning:

\[
150 - 0.25 \times 50 = 137.5 \text{ sq.ft.}
\]

Let \( Q \) be the ratio of basal area after row thinning and basal area before thinning, \( Q = 137.5 / 150 = 0.9167 \). The stand and stock table after row thinning is constructed by multiplying the residual ratio \( Q \) by the entries in the stand and stock table before row thinning.

Number of trees in the 6-inch class: \( 0.9167 \times 143.3 = 131.4 \) trees. Basal area in the 6-inch class: \( 0.9167 \times 28.1 = 25.76 \) sq.ft. Volume in the 6-inch class: \( 0.9167 \times 597.4 = 547.6 \) cu.ft.

**Step 3: Low thinning at age 20.**

Basal area removed in low thinning: \( 0.75 \times 50 = 37.5 \) sq.ft. The diameter limit (\( D_{\text{thin}} \)) is searched for by summing basal area in each dbh class, starting from the lowest class, until the total is closest to but not greater than 37.5 sq.ft. Basal area of cut trees having dbh's of 5.5 inches and below:

\[
0.4 + 2.9 + 10.9 = 14.2 \text{ sq.ft.}
\]

Basal area of trees in the 6-inch class that are removed in low thinning: \( 37.5 - 14.2 = 23.3 \) sq.ft., which corresponds to:

\[
131.4 \times (23.3) / 25.76 = 118.7 \text{ trees.}
\]

Residual number of trees in the 6-inch class: \( 131.4 - 118.7 = 12.7 \) trees/acre. Trees in the 7-inch class and above are left in this low thinning.

**Step 4: Project to age 40.**

Stand attributes at age 40 are predicted from those at age 20 after thinning. The procedures for constructing the stand and stock table are similar to those described earlier in Step 1, except that a Weibull distribution left-truncated at a diameter of 5.5 inches is used in this case.
Appendix 2a. Input variable formats and description for program

WITHIN -- Subprogram identification card (first card).

<table>
<thead>
<tr>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I1</td>
<td>IPROG</td>
<td>= 1 = Call INPUT1: project a stand through time.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 2 = Call INPUT2: stand and stock tables for specified combinations of age, site, and density.</td>
</tr>
</tbody>
</table>
Appendix 2b. Input variable formats and descriptions for program WTHIN -- Subprogram INPUT1.

<table>
<thead>
<tr>
<th>Card Type</th>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>STAND DESCRIPTION CARD</td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>F3.0</td>
<td>SI1</td>
<td>Site index in feet (base age 25 years).</td>
<td></td>
</tr>
<tr>
<td>4-6</td>
<td>F3.0</td>
<td>AGE1</td>
<td>Age in years of the present stand.</td>
<td></td>
</tr>
<tr>
<td>7-10</td>
<td>F4.0</td>
<td>XN1</td>
<td>Number of trees per acre at AGE1.</td>
<td></td>
</tr>
<tr>
<td>11-16</td>
<td>F6.2</td>
<td>BA1</td>
<td>Basal area in square feet per acre at AGE1. (Either XN1 or BA1 has to be specified).</td>
<td></td>
</tr>
<tr>
<td>17-18</td>
<td>I2</td>
<td>INDEX</td>
<td>= 1 = XN1 and BA1 are both inputs.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 2 = Only XN1 is input for density.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 3 = Only BA1 is input for density.</td>
<td></td>
</tr>
<tr>
<td>19-23</td>
<td>F5.2</td>
<td>DTHIN1</td>
<td>= 0 = This stand has never been thinned from below.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 0 = All trees having dbh below DTHIN1 were cut in a previous low thinning.</td>
<td></td>
</tr>
<tr>
<td>24-26</td>
<td>F3.0</td>
<td>AGE2</td>
<td>Age at the next input or decision period.</td>
<td></td>
</tr>
<tr>
<td>27-28</td>
<td>I2</td>
<td>NDEC</td>
<td>Number of decision cards, each card describes management routine (thinning or not) at a specified age.</td>
<td></td>
</tr>
<tr>
<td>29-30</td>
<td>I2</td>
<td>IOPT</td>
<td>= 0 = No title card for this stand.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 = Title card immediately follows this card.</td>
<td></td>
</tr>
<tr>
<td>31-32</td>
<td>I2</td>
<td>MORE</td>
<td>= 0 = No other stand. Stop when this stand is finished.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 = Another stand follows.</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 2b. Input variable formats and descriptions for program
WTHIN -- Subprogram INPUT1 (continued).

<table>
<thead>
<tr>
<th>Card Type</th>
<th>Column Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>F3.0</td>
<td>AGE1</td>
<td>Current stand age, equal to AGE2 specified in the previous card.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-6</td>
<td>F3.0</td>
<td>AGE2</td>
<td>Age at the next input or decision period (harvest age if this is the last decision card of this stand).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7-8</td>
<td>I2</td>
<td>ITHIN</td>
<td>= 1 = No thinning at AGE1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 2 = Row thinning at AGE1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 3 = Low thinning at AGE1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 4 = Row thinning followed by low thinning at AGE1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9-10</td>
<td>I2</td>
<td>JOPT</td>
<td>(Needed only when IROW=2 or ILOW=2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 = BTHIN is specified.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 2 = BRESR or BRES is specified.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11-12</td>
<td>I2</td>
<td>IROW</td>
<td>(Needed only when ITHIN=2 or 4).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 = Specify residual ratio (Q).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 2 = Residual ratio not specified.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13-17</td>
<td>F5.2</td>
<td>Q</td>
<td>= Residual ratio (after / before thinning), when ITHIN=2 and IROW=1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= Ratio of basal area removed in row thinning and total basal area removed, when ITHIN=4 and IROW=2.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18-23</td>
<td>F6.2</td>
<td>BRESR</td>
<td>(Needed only when JOPT=2 and IROW=2) Residual basal area per acre after row thinning.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24-29</td>
<td>F6.2</td>
<td>BTHINR</td>
<td>(Needed only when JOPT=1 and IROW=2) Basal area per acre removed in row thinning.</td>
</tr>
</tbody>
</table>
Appendix 2b. Input variable formats and descriptions for program WTHIN -- Subprogram INPUT1 (continued).

<table>
<thead>
<tr>
<th>Card Type</th>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
</table>
| 2         | 30-31  | I2     | ILOW     | (Needed only when ITHIN=3 or 4)  
            |        |        |          | = 1 = All trees below a specified diameter limit (DTHIN) are cut.  
            |        |        |          | = 2 = Thin to a specified residual basal area (BRES). |
|           | 32-36  | F5.2   | DTHIN    | (Needed only when ILOW=1)  
            |        |        |          | All trees having dbh below DTHIN are cut. |
|           | 37-42  | F6.2   | BRES     | (Needed only when JOPT=2 and ILOW=2)  
            |        |        |          | Residual basal area per acre after low thinning. |
|           | 43-48  | F6.2   | BTHIN    | (Needed only when JOPT=1 and ILOW=2)  
            |        |        |          | Basal area per acre removed in low thinning. |
Appendix 2c. Input variable formats and description for program
WTHIN -- Subprogram INPUT2.

<table>
<thead>
<tr>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>I4</td>
<td>ISB</td>
<td>Site index: Begin</td>
</tr>
<tr>
<td>5-8</td>
<td>I4</td>
<td>ISE</td>
<td>End</td>
</tr>
<tr>
<td>9-12</td>
<td>I4</td>
<td>ISI</td>
<td>Increment</td>
</tr>
<tr>
<td>13-16</td>
<td>I4</td>
<td>IAB</td>
<td>Stand age: Begin</td>
</tr>
<tr>
<td>17-20</td>
<td>I4</td>
<td>IAE</td>
<td>End</td>
</tr>
<tr>
<td>21-24</td>
<td>I4</td>
<td>IAI</td>
<td>Increment</td>
</tr>
<tr>
<td>25-28</td>
<td>I4</td>
<td>INB</td>
<td>Trees/acre: Begin</td>
</tr>
<tr>
<td>29-32</td>
<td>I4</td>
<td>INE</td>
<td>End</td>
</tr>
<tr>
<td>33-36</td>
<td>I4</td>
<td>INI</td>
<td>Increment</td>
</tr>
<tr>
<td>37-40</td>
<td>I4</td>
<td>IBB</td>
<td>Basal area: Begin</td>
</tr>
<tr>
<td>41-44</td>
<td>I4</td>
<td>IBE</td>
<td>End</td>
</tr>
<tr>
<td>45-48</td>
<td>I4</td>
<td>IBI</td>
<td>Increment</td>
</tr>
<tr>
<td>49-52</td>
<td>I4</td>
<td>INDEX</td>
<td>= 1 = Number of trees (IN) and basal area (IB) per acre are both inputs.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 2 = Only IN is input for density.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 3 = Only IB is input for density.</td>
</tr>
<tr>
<td>53-56</td>
<td>I4</td>
<td>IOPT</td>
<td>= 0 = No title card.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 = Title card immediately follows this card.</td>
</tr>
</tbody>
</table>
Appendix 3a. Input example for program WTHIN -- simulate a stand through time.

Stand 1:
Site index = 60 feet (base age 25 years).
Density at age 5 = 600 trees/acre.
Thinning: Age = 17. Amount = 38 sq.ft./acre. Type = ROW.
Age = 22. Amount = 29 sq.ft./acre. Type = ROW.
Harvest age = 30 years.
Title: COILE AND SCHUMACHER (1964)

Stand 2:
Site index = 60 feet (base age 25 years).
Density at age 20 = 600 trees and 150 sq.ft./acre.
Thinning: Age = 20. Thin to 100 sq.ft./acre. Type = 25% ROW, 75% LOW.
Harvest age = 40 years.
Title: 25% ROW, 75% LOW THINNING

Card Input:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Column: 1234567890...5...0...5...0...5...0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>5</td>
<td>600</td>
<td>2</td>
</tr>
<tr>
<td>COILE AND SCHUMACHER (1964)</td>
<td>17</td>
<td>22</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>22</td>
<td>30</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>60</td>
<td>20</td>
<td>600150.00</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>25% ROW, 75% LOW THINNING</td>
<td>20</td>
<td>40</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
Appendix 3b. Input example for program WTHIN -- stand and stock tables for specified combinations of site index, age, and density.

**Combinations:**

Site index = 50 feet (base age 25 years).

Stand age = 10, 15, 20, 25, 30 years.

Number of trees = 200, 400, 600, 800 trees/acre.

Basal area = 50, 100, 150, 200 sq.ft./acre.

No title wanted.

**Card input:**

<table>
<thead>
<tr>
<th>Column</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site index</td>
<td>50</td>
<td>50</td>
<td>10</td>
<td>10</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>Stand age</td>
<td>200</td>
<td>800</td>
<td>200</td>
<td>50</td>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>Number of trees</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 4. Generalized flowchart of program WTHIN.
Appendix 4. Generalized flowchart of program WTHIN (continued).

Appendix 4. Generalized flowchart of program WTHIN (continued).
Appendix 5. Source listing of program WTHIN.

CALL ERRSET(208,256,-1,1)
CALL ERRSET(207,256,-1,1)
CALL ERRSET(209,256,-1,1)
CALL ERRSET(262,256,-1,1)
CALL ERRSET(263,256,-1,1)
READ(5,500) IPROG
500 FORMAT(11)
IF(IPROG.EQ.1) CALL INPUT1
IF(IPROG.EQ.2) CALL INPUT2
RETURN
END

SUBROUTINE INPUT1

READ STAND DESCRIPTION CARD.
READ(5,500,END=999) SI,AGE1,XN1,BA1,INDEX,ITHIN1,AGE2,INDEX,MORE,
	: DATA (1BLANK/)

C----- READ STAND DESCRIPTION CARD.
C
1 READ(5,500,END=999) SI,AGE1,XN1,BA1,INDEX,ITHIN1,AGE2,NDEC,IOPT
: MORE
500 FORMAT(2F3.0,F4.0,F6.2,I12,F5.2,F3.0,3I2)
ITHIN=1
JJJ=0
Appendix 5. Source listing of program WTHIN (continued).

C----- READ TITLE CARD IF ANY.
C
DO 2 II=1,20
  2 ITITLE(II)=IBLANK
  IF(IOPT.EQ.1) READ(5,501) (ITITLE(II),II=1,20)
501 FORMAT(20A4)
CALL GROW
C----- READ DECISION CARDS.
C
IF(MORE.EQ.1.AND.NDEC.EQ.0) GO TO 1
IF(MORE.NE.1.AND.NDEC.EQ.0) RETURN
DO 3 II=1,NDEC
  READ(5,502) AGE2,ITHIN,JOPT,IROW,Q,BRESR,BTHINR,ILOW,DTHIN,BRES
  BTHIN
  IF(ITHIN.NE.1) JJJ=1
  AGE1=AGE
  XN1=XN
  B1=BA
  IF(JOPT.EQ.1.AND.IROW.EQ.2) BRESR=BA1-BTHINR
  IF(JOPT.EQ.1.AND.ILOW.EQ.2) BRES=BA1-BTHIN
  IF(JOPT.EQ.1.AND.ILOW.EQ.2.AND.ITHIN.EQ.4) BRES=BRESR-BTHIN
  INDEX=1
  CALL GROW
  IF(MORE.EQ.1) GO TO 1
999 RETURN
END
SUBROUTINE INPUT2

**************************************************************************************
* SUBROUTINE INPUT2 READS THE NECESSARY INPUTS FOR SUBROUTINE YIELD. *
**************************************************************************************
IMPLICIT REAL*8 (A-H,O-Z)
COMMON /ONE/ Si,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ Sl1,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN,
  INDEX,ITHIN,ILOW,IROW
COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41,
  CVOB41,IOPT,JJJ
COMMON /FOUR/ A,B,1BMIN,C,CONST,CINV,GAMMA
DATA IBLANK/'/
1 READ(5,500,END=999) ISB,ISE,ISI,1AB,IAE,I1A,INB,INE,INI,IBB,
  ,1BE,IBI,INDEX,IOPT
500 FORMAT(14I4)
DO 2 II=1,20
  2 ITITLE(II)=IBLANK
  IF(IOPT.EQ.1) READ(5,501) (ITITLE(II),II=1,20)
501 FORMAT(20A4)
Appendix 5. Source listing of program WITHIN (continued).

C  ---- DO LOOPS. CHECK INDEX FOR INPUTS FOR STAND DENSITY.
C
DO 40 IS=ISB,ISE,ISI
SI=DFLOAT(IS)
DO 30 IA=IAB,IAE,IAI
AGE=DFLOAT(IA)
AINV=1.00/AGE
CALL HEIGHT
AHD1=AINV/HD
GO TO (13,11,12), INDEX
11 IBB=100
IBE=IBB
IBI=181
GO TO 13
12 INB=100
INE=INB
INI=50
GO TO 13
13 DO 20 IN=INB,INE,INI
GO TO (21,22,23), INDEX
21 XN=DFLOAT(IN)
XNLOG=DLOG(XN)
GO TO 23
22 XN=DFLOAT(IN)
XNLOG=DLOG(XN)
BLOG=-4.39180687DO*AINV
+ 0.19054366DO*HDLOG
+ 0.63902092DO*XNLOG
BA=DEXP(BLOG)
23 DO 10 IB=IBB,IBE,IBI
GO TO (31,33,32), INDEX
31 BA=DFLOAT(IB)
BLOG=DLOG(BA)
GO TO 33
32 BA=DFLOAT(IB)
BLOG=DLOG(BA)
XNLOG=7.79805237DO*AINV
+ 2.10495039DO*HDLOG
XNLOG=7.79805237DO*AINV
+ 2.10495039DO*HDLOG
+ 1.67143646DO*BLOG
BA=DEXP(XNLOG)
CONTINUE

C  ---- SOLVE FOR DIAMETER CDF.
C
CALL YIELD
CONTINUE
CONTINUE
CONTINUE
CONTINUE
GO TO 1
999 RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

SUBROUTINE GROW

*******************************************************************************
*
* SUBROUTINE GROW PRODUCES A STAND AND STOCK TABLE AT AGE1. THE STAND IS THEN SUBJECT TO *
* THINNING (OR NO THINNING), AND THEN PROJECTED TO AGE2. *
*
*******************************************************************************

IMPLICIT REAL*8 (A-H,O-Z)
COMMON /ONE/ S1,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ S11,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
COMMON /THREE/ INDEX,ITHIN,ILOW,IROW
COMMON /FOUR/ ITITLE(20),AINV,XNLOG,BLOG,HOLOG,TVOB1,TVOB41

DATA 81/0.02273D0/, 82/-0.01110300/ WTH01740
IF(AGE1.EQ.AGE.AND.XN1.EQ.XN.AND.BA1.EQ.BA) GO TO WTH01780
IDTHIN=DTHIN1+0.500 WTH01790
DTHIN1=DFLOAT( IOTHIN)-0.5D0 WTH01800
S1=S11 WTH01810
AGE=AGE1 WTH01820
AINV=1,DO/AGE WTH01830
CALL HEI GHT WTH01840
GO TO (1,2,3), INDEX WTH01850
WTH01860
INDEX= 1 = BOTH XN1 AND BA1 ARE INPUTS FOR STAND DENSITY.
1 XNLOG=DLOG(XN1)
BLOG=DLOG(BA1)
GO TO 4 WTH01930
WTH01940
INDEX= 2 = ONLY XN1 IS INPUT FOR STAND DENSITY.
2 XNLOG=DLOG(XN1)
IF(JJJ.EQ.0) BLOG=DLOG(10.DO)*[(1.4366DO*DLOG10(S1)-7.0840DO*AINV] WTH01970
+ 0.488800*DLOG10(XN1)-1.3851DO)
IF(JJJ.EQ.1) BLOG=-4.39180687D0 + 0.19054366DO*AINV WTH01980
+ 1.34753473DO*DLOG10(XN1)-1.3851D0)/(-0.488800)
XN1=DEXP(XNLOG)
GO TO 4 WTH02030
WTH02040
INDEX= 3 = ONLY BA1 IS INPUT FOR STAND DENSITY.
3 BLOG=DLOG(BA1)
IF(JJJ.EQ.0) XNLOG=DLOG(10.DO)*[(1.4366DO*DLOG10(S1)-7.0840DO*AINV WTH02070
-0.488800)/(1-1.3851DO)
IF(JJJ.EQ.1) XNLOG=7.79052370D0 + 2.10495039DO*AINV WTH02080
+ 1.89908311DO*1.16743646D0*LOG(XNLOG)
GO TO 4 WTH02100
WTH02100
Appendix 5. Source listing of program WTHIN (continued).

C----- SOLVE FOR DIAMETER CDF.
C
4 BA=BA1
XN=XN1
CALL YIELD
C----- THINNING AT AGE1.
C
5 CALL THIN
IF(AGE.EQ.AGE2) RETURN
C----- PROJECT TO AGE2.
C
AGE=AGE2
AINV=1.0D/AGE2
CALL HEIGHT
C1=5.40815546D0 + 0.321208D-2*SI
XNLOG=DLOG10(XN1)
XNLOG=(XNLOG - B1*AGE1)/(1.DO + B2*AGE1)
IF(JJJ.EQ.0) XNLOG=DLOG(10.DO)*(XNLOG + AGE*
$ (B1 + B2*XNLOG))
IF(JJJ.EQ.1) XNLOG=-DLOG(DEXP(-0.658083DO*XNLOG)+0.75795D-5
$ *(AGE2**1.78018705D0-AGE1**1.78018705D0))/0.658083D0
XN=DEXP(XNLOG)
IF(JJJ.EQ.0) BLOG=DLOG(10.DO)*(1.4366DO*DLOG10(SI)-7.084DO
$ +0.4888DO*DLOG10(XN) -1.385100)
IF(JJJ.EQ.1) BLOG=C1 + (BLOG-C1)*AGE1/AGE2
BA=DEXP(BLOG)
C----- SOLVE FOR DIAMETER CDF.
C
CALL YIELD
RETURN
END
SUBROUTINE YIELD

*******************************************************************************
*
* SUBROUTINE YIELD PRODUCES A STAND AND STOCK TABLE FOR A SPECIFIED
* COMBINATION OF AGE, SITE, AND DENSITY.
*
*******************************************************************************

CALL MODEL
CALL DIST
CALL OUTPUT(1)
RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

```
SUBROUTINE HEIGHT

******************************************************************************
*                             * 
* SUBROUTINE HEIGHT COMPUTES HEIGHT OF THE                              *
* DOMINANTS AND CODEDOMINANTS OF A STAND, GIVEN                            *
* SITE INDEX AND AGE.                                                    *
* FROM J I M D E V A N ' S THESIS (1979).                                *
******************************************************************************

IMPLICIT REAL*8 (A-H, O-Z)
COMMON /ONE/ SI, AGE, XN, BA, HD, DMIN, DMED, DMAX, DBAR, IMAX, IMIN
COMMON /THREE/ ITITLE(20), AINV, XNLOG, BLOG, HDLOG, TVOB1, TVOB41
 DATA X0/0.04D0/, XL/0.20D0/, A/8.96178D0/, 
     B1/-5.27794D0/, B2/19.90047D0/, B3/-58.76122D0/
 X=AINV
 Z=DEXP(A*(X-X0))
 XOZ=XO*Z
 YO=DLOG(SI)
 HDLOG=YO*Z + B1*(Z-1.DO) + B2*(XOZ-X) + B3*(XOZ*XO-X*X)
 HD=DEXP(HDLOG)
 RETURN
END

SUBROUTINE MODEL

******************************************************************************
*                             * 
* SUBROUTINE MODEL PREDICTS FROM THE STAND                                *
* CHARACTERISTICS MINIMUM AND AVERAGE DIAMETERS.                         *
******************************************************************************

IMPLICIT REAL*8 (A-H, O-Y)
COMMON /ONE/ SI, AGE, XN, BA, HD, DMIN, DMED, DMAX, DBAR, IMAX, IMIN
COMMON /THREE/ ITITLE(20), AINV, XNLOG, BLOG, HDLOG, TVOB1, TVOB41
 DQ=(BA/(0.545415D-2*XN))**0.5D0
 DMIN=DEXP(1.10834919D0 + 5.10754613D0*AINV + 0.505350582*HDLOG 
     + 0.28543547D0*BLOG - 0.57131133D0*XNLOG)
 DBAR=DQ - DEXP(-9.05733080D0 + 0.89273788D0*HDLOG 
     + 0.56851114D0*XNLOG)
 RETURN
END
```
Appendix 5. Source listing of program WTHIN (continued).

```plaintext
SUBROUTINE DIST

***************************************************

SUBROUTINE DIST SOLVES FOR WEIBULL PARAMETERS * FOR DBH, GIVEN BA, N, MINIMUM AND AVERAGE DBH.*

***************************************************

IMPLICIT REAL*8 (A-H,0-Y)

COMMON /ONE/ Sl,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN

COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOBl,TVOB41,CVOB41, IOPT,JJJ

COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA

EXTERNAL FCN

DATA TOL/0.005/

c----- INITIALIZE VARIABLES.

CONST=-DLOG(0.5DO/XN)

l=DMIN-0.5DO

A=l-0.4900

IF(A.LT.O.DO) A=0.00

W1=-0.BDO

IMIN=0.5DO+A

IF( IMIN.LE.O) IMIN=l

SOLVE EQUATION: FCN(C) = 0, USING THE SECANT METHOD.

CALL SECAN1(FCN,TOL,W1,ITER,IER)

C=10. DO*( 1. DO+DERF(W1))

RETURN

END

SUBROUTINE SECAN1(F,ERROR,SOL, ITER, IER)

***************************************************

SECANT METHOD

FIND A ROOT OF A NONLINEAR EQUATION F(X) = 0.

INPUTS : F = FUNCTION.

ERROR = PROCEDURE IS STOPPED WHEN IF(X) < ERROR.

SOL = A GUESS OF THE SOLUTION TO F(X) = 0.

OUTPUTS : SOL = SOLUTION TO F(X) = 0.

ITER = NUMBER OF ITERATIONS.

IER = 0 = A ROOT IS FOUND.

1 = NO ROOT IS FOUND AFTER 50 ITERATIONS.

***************************************************

IMPLICIT REAL*8 (A-H,0-Z)

COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
```

WTH03120
WTH03130
WTH03140
WTH03150
WTH03160
WTH03170
WTH03180
WTH03190
WTH03200
WTH03210
WTH03220
WTH03230
WTH03240
WTH03250
WTH03260
WTH03270
WTH03280
WTH03290
WTH03300
WTH03310
WTH03320
WTH03330
WTH03340
WTH03350
WTH03360
WTH03370
WTH03380
WTH03390
WTH03400
WTH03410
WTH03420
WTH03430
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WTH03570
WTH03580
WTH03590
WTH03600
WTH03610
WTH03620
WTH03630
WTH03640
WTH03650
WTH03660
WTH03670
WTH03680
WTH03690
WTH03700
WTH03710
WTH03720
Appendix 5. Source listing of program WTHIN (continued).

C----- INITIALIZATION.
C
IER=0
ITER=0
XO=SOL
F0=F(X0)
B0=B
X1=X0+.5D0
F1=F(X1)
AFMIN=ABS(F1)
XMIN=X1
BMIN=B
IF(AFMIN.LT.DABS(F0)) GO TO 1
C1=X0
C2=F0
X0=X1
F0=F1
X1=C1
F1=C2
AFMIN=ABS(F1)
XMIN=X1
BMIN=B
IF(AFMIN.LT.DABS(F0)) GO TO 1
ITER=ITER+1
SOL=(X0*F1-X1*F0)/(F1-FO)
IF(DABS(SOL).GT.5.D0) GO TO 3
F2=F(SOL)
AF2=ABS(F2)
IF(AF2.GE.AFMIN) GO TO 2
AFMIN=AF2
XMIN=SOL
BMIN=B
C----- START THE ITERATIVE PROCEDURE.
C
1 ITER=ITER+1
SOL=(XO*F1-X1*F0)/(F1-FO)
IF(DABS(SOL).GT.5.5D0) GO TO 3
F2=F(SOL)
AF2=ABS(F2)
IF(AF2.GE.AFMIN) GO TO 2
AFMIN=AF2
XMIN=SOL
BMIN=B
C----- CHECK CONVERGENCE.
C
2 IF(AF2.LE.ERROR) RETURN
IF(ITER.LE.50) GO TO 3
C----- REINITIALIZE VARIABLES.
C
X0=X1
F0=F1
X1=SOL
F1=F2
GO TO 1
C----- NO SOLUTION AFTER 50 ITERATIONS.
C
3 IER=1
SOL=XMIN
B=BMIN
RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

DOUBLE PRECISION FUNCTION FCN(W1)

* * * * * *
FUNCTION FCN IS CALLED BY SUBROUTINE SECAN1 TO EVALUATE THE LEFT-HAND SIDE OF EQUATION:
* FCN(C) = 0.
* *
* ***************************************************

IMPLICIT REAL*8 (A-H,O-Y)
COMMON /ONE/ Sl,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ Sl1,AGE1,XN1,BA1,DTHIN1,AGE2,A,DTHIN,BRES,BRESR,INDEX,THIN,LOW,ROW
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA

C----- INITIALIZATION.
C
C=10.D0*(1.DO+DERF(W1))
CINV=1.DO/C
GAMMA=DGAMMA(1.DO+CINV)
IMAX=1.500+A+B*CONST**(CINV)
FCN=0.DO
IF(A.LT.DTHIN1) GO TO 2
F1=0.DO

DO 1 l=IMIN, IMAX
Xl=DFLOAT(l)
F2=CDF(Xl+0.5DO)
F=F2-F1
IF(F.LT.0.DO) F=0.DO
IF(1.EQ.IMAX) F=1.DO-F1
F1=F2
FCN=FCN+F*Xl*Xl
FCN=FCN*0.545415D-2*XN-BA
RETURN
1 FCN=FCN+Xl*Xl*F
FCN=FCN*0.54545D-2*XN-BA
RETURN

C----- WHEN THE LOCATION PARAMETER (A) IS LOWER THAN DTHIN1.
C
CALL FINDB
F1=CDF(DTHIN1)
FRES=1.DO-F1
IMIN1=DTHIN1+0.51DO
DO 3 l=IMIN1, IMAX
Xl=DFLOAT(l)
F2=CDF(Xl+0.5DO)
F=(F2-F1)/FRES
IF(F.LT.0.DO) F=0.DO
IF(1.EQ.IMAX) F=(1.DO-F1)/FRES
F1=F2
FCN=FCN+F*Xl*Xl
FCN=FCN*0.54545D-2*XN-BA
RETURN
3 END
Appendix 5. Source listing of program WTHIN (continued).

SUBROUTINE FINDS

******************************************************************
* * SUBROUTINE FINDS SEARCHES FOR THE WEIBULL * *
* * PARAMETER B, GIVEN A AND C, IN CASE OF LEFT- * *
* * TRUNCATION DUE TO LOW THINNING. * *
* * ******************************************************************

IMPLICIT REAL*8 (A-H,0-Y)
COMMON /ONE/ S1,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ S11,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
EXTERNAL FF
DATA TOL/0.50-2/,W2, •O. 600 W

CALL SECAN2(FF,TOL,W2,ITER,IER)
B=10.DO*(1.DO+DERF(W2))
RETURN
END

SUBROUTINE SECAN2(F,ERROR,SOL,ITER,IER)

******************************************************************
* * SECANT METHOD * *
* * FIND A ROOT OF A NONLINEAR EQUATION F(X) = 0. * *
* * INPUTS : F = FUNCTION. * *
* * ERROR = PROCEDURE IS STOPPED WHEN * *
* * IF(X) < ERROR. * *
* * SOL = A GUESS OF THE SOLUTION TO * *
* * F(X) = 0. * *
* * OUTPUTS : SOL = SOLUTION TO F(X) = 0. * *
* * ITER = NUMBER OF ITERATIONS. * *
* * IER = 0 = A ROOT IS FOUND. * *
* * = 1 = NO ROOT IS FOUND AFTER * *
* * 50 ITERATIONS. * *
* * ******************************************************************

RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

```
IMPLICIT REAL*8 (A-H,O-Z)

C----- INITIALIZATION.
C
IER=0
ITER=0
XO=SOL
F0=F(X0)
X1=XO+0.5D0
F1=F(X1)
AFMIN=DABS(F1)
XMIN=X1
IF(AFMIN.LT.DABS(F0)) GO TO 1
C1=X0
C2=F0
X0=X1
F0=F1
X1=C1
F1=C2
AFMIN=DABS(F1)
XMIN=X1
C----- START THE ITERATIVE PROCEDURE.
C
1  ITER=ITER+1
   SOL=(X0*F1-X1*F0)/(F1-F0)
   IF(DABS(SOL).GT.5.DO) GO TO 3
   F2=F(SOL)
   AF2=DABS(F2)
   IF(AF2.GE.AFMIN) GO TO 2
   AFMIN=AF2
   XMIN=SOL
C----- CHECK CONVERGENCE.
C
2  IF(AF2.LE.ERROR) RETURN
   IF(ITER.GE.50) GO TO 3
C----- REINITIALIZE VARIABLES.
C
X0=X1
F0=F1
X1=SOL
F1=F2
GO TO 1
C----- NO SOLUTION AFTER 50 ITERATIONS.
C
3  IER=1
   SOL=XMIN
   RETURN
END
```
Appendix 5. Source listing of program WTHIN (continued).

DOUBLE PRECISION FUNCTION FF(W2)

**************************************************************************
* * *
FUNCTION FF IS CALLED BY SUBROUTINE SECAN2 TO * * *
* EVALUATE THE LEFT-HAND SIDE OF THE EQUATION: * *
* FF(B) = 0. * *
* *
**************************************************************************

IMPLICIT REAL*8 (A-H,0-Y)
COMMON /ONE/ Sl,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ Sl1,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
, INDEX, ITHIN, ILOW, IROW
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
EXTERNAL Y
B=(0.00*(1.DO+DERF(W2))
FRES=1.DO-CDF(DTHIN1)

CALL GAUSS(Y,ZA,ZB,S)

EVALUATE FF( B).

FF=A+B*(GAMMA-S)/FRES-DBAR
RETURN
END

SUBROUTINE GAUSS(F,XA,XB,S)

**************************************************************************
* * *
GAUSS QUADRATURE METHOD
* * *
INPUTS: F = FUNCTION TO BE INTEGRATED. *
* XA AND XB = LOWER AND UPPER LIMITS OF *
* INTEGRATION.
* *
OUTPUT: S = VALUE OF THE INTEGRAL.
* *
**************************************************************************

ZA=0.DO
ZB=((DTHIN1-A)/B)**C
CALL GAUSS(Y,ZA,ZB,S)

C----- EVALUATE THE INCOMPLETE GAMMA INTEGRAL.
C
C----- EVALUATE FF(B).
C
FF=A+B*(GAMMA-S)/FRES-DBAR
RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

```fortran
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION Y(5), W(5)
DATA Y/1468743390D0,
: .3339539410D0,
: .6794095683D0,
: .850633667D0,
: .9739065285D0,
: 2952422470D0,
: .2692667193D0,
: .2190636250D0,
: .189451392D0,
: .0666713443D0/, M/5/.
C1=0.5DO*(XB+XA)
C2=0.5DO*(XB-XA)
S=0.0D0
DO 2 1=1,M
C3=C2*Y(1)
2 S=S+W(1)*(F(C1+C3)+F(C1-C3))
S=S*C2
RETURN
END
DOUBLE PRECISION FUNCTION Y(X)
IMPLICIT REAL*8 (A-H, O-Z)
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
Y=X**CINV*DEXP(-X)
RETURN
END
DOUBLE PRECISION FUNCTION CDF(XX)
***************************************************
* * *
FUNCTION CDF EVALUATES THE WEIBULL CDF.
* *
***************************************************
IMPLICIT REAL*8 (A-H, O-Y)
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
CDF=0.0D0
IF(XX.LE.A) RETURN
C1=CDLOG((XX-A)/B)
C2=0.0D0
IF(C1.GT.-50.0D0.AND.C1.LT.50.0D0) C2=-DEXP(C1)
IF(C1.GT.50.0D0) C2=-1.0D8
C3=0.0D0
IF(C2.GT.-50.0D0) C3=DEXP(C2)
CDF=1.0D0-C3
RETURN
END
```
Appendix 5. Source listing of program WTHIN (continued).

```
SUBROUTINE OUTPUT(111)

*** SUBROUTINE OUTPUT PRINTS THE STAND AND STOCK ***
*** TABLE. ***
***

C IMPLICIT REAL*8 (A-H,0-Z) C
DIMENSION CF(20),ROB(3),RIB(3),BOB(2),BIB(2),BH(2) C
COMMON /ONE/ Sl,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN C
COMMON /TWO/ Sl1,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN C
COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HLOG,TV081,TV0841 C
COMMON /FOUR/ A,8,BMIN,C,CONST,CINV,GAMMA C
DATA ROB/-0.32354D0, 3.1579D0, -2.711500/ C
DATA RIB/-0.35206DO, 3.076300, -2.6540DO/ C
DATA BOB/ 0.34864D0, 0.0023200/ C
DATA BIB/ 0.11691D0, 0.00185D0/ C
DATA TOP/4.DO/,KROW/'ROW '/,KLOW/'LOW '/,KTYPE/' '/ C
DATA CF/0.,0.,0.,0.,84.,85.,87.,90.,91.,92.,93.,94.,95.,95.,95.,95.,95.,95./ C
BH(1)=0.46151540D0 + 0.43274521DO*AINV + 0.93333081DO*HDLOG C
- 0.08583288DO*BLOG + 0.07596439*XNLOG C
TOPOB=TOP**ROB(2) C
TOPIB=TOP**RIB(2) C
IF (111 .EQ. 2) GO TO 11 C
WRITE(6 666) ( !TITLE(11), I1=1,20) C
WRITE(6,599) Sl,HD,AGE,DBAR,XN,DMIN,BA C
GO TO 12 C
KTYPE=KLOW C
IF(ITHIN.NE.3) KTYPE=KROW C
WRITE(6,600) KTYPE C
GO TO 12 C
IF(A.LT.DTHIN1) WRITE(6,602) DTHIN1 C
IF(111.EQ.2) WRITE(6 603) KTYPE C
WRITE(6,604) TOP, TOP C
```

Appendix 5. Source listing of program WTHIN (continued).

C----- INITIALIZATION.
C
C      F1=0.DO
BB=0.DO
XNRES=0.DO
DAVG=0.DO
TVOB=0.DO
TVIB=0.DO
TVOB4=0.DO
TVIB4=0.DO
CVOB4=0.DO

XNT=XN
IMIN=MIN
IF(I,I.EQ.2) GO TO 13
IF(A.GE.DTHIN1) GO TO 3
F1=CDF(DTHIN1)
IMIN1=DTHIN1+0.51DO
XNT=XN/(1.DO-F1)
GO TO 3
13 IF(I,I.EQ.1) GO TO 3
IF(I,I.EQ.3) GO TO 2
IF(A.LT.DTHIN1) GO TO 1
C----- ROW THINNING. NO PREVIOUS LOW THINNING.
C
C      XNT=XN*Q
GO TO 3
C----- ROW THINNING. PREVIOUS LOW THINNING.
C
1 F1=CDF(DTHIN1)
XNT=XN*Q/(1.DO-F1)
IMIN1=DTHIN1+0.51DO
GO TO 3
C----- LOW THINNING.
C
2 F1=CDF(DTHIN1)
IF(A.LT.DTHIN1) XNT=XN/(1.DO-CDF(DTHIN1))
IMIN1=DTHIN1+0.51DO
C
C----- LOOP OVER DBH CLASSES.
C
3 CONTINUE
DO 5 I=IMIN1,IMAX
X1=DFLOAT(I)
F2=CDF(X1+0.5DO)
F=XNT*(F2-F1)
IF(I,EQ.IMIN1.AND.,III,EQ.2) F=F*QTHIN
IF(F,LT.0.DO) F=0.DO
F1=F2
X12=X1*X1
BASAL=0.545415D-2*X12*F
H=DEXP(BH(1)+BH(2)/X1)
D2H=X12*H
VOB=F*(BOB(1)+BOB(2)*D2H)
VIB=F*(BIB(1)+BIB(2)*D2H)
VOB4=0.DO
VIB4=0.DO
IF(I,L.T.5) GO TO 4
VOB4=VOB*(1.DO+ROB(1)*TOPOB*X1**ROB(3))
VIB4=VIB*(1.DO+RIB(1)*TOPIB*X1**RIB(3))
GO TO 5
4
Appendix 5. Source listing of program WTHIN (continued).

4 IF(1.LE.20) CV084=CV0B4+V084/CF(1)
   DAVG=DAVG+F*X1
   BB=BB+BASAL
   XNRES=XNRES+F
   TVOB=TVOB+V0B
   TVIB=TVIB+VIB
   TVOB4=TVOB4+V0B4
   TVIV4=TVIV4+V1B4
   WRITE(6,605) I,F,H,BASAL,V0B,V1B,V0B4,V1B4
   DAVG=DAVG/XNRES
   FORMAT(111,7F11.1)
   END LOOP.
   IF(I.I.EQ.2) GO TO 7
   WRITE(6,608) XNRES,BB,TVOB,TVIB,TVOB4,TVIV4,DAVG,TO,P,CV0B4,A,B,C
   FORMAT(16X,6('-'),11X,5F11.1)
   'AVERAGE DBH = ,F7.2,CORD VOLUME TO',F3.0,'IN =',F7.2,'WEIBULL PARAMETERS'
   'A=',F7.4,'B=',F7.4,'C=',F7.4)
   C1=DABS(BA-BB)
   IF(C1.LT.0.05) IER=0
   IF(IER.EQ.0) WRITE(6,609)
   FORMAT(/35X, 'CONVERGENCE ATTAINED')
   IF(IER.NE.0) WRITE(6,610)
   FORMAT(23X, 'DIFFERENCE IN BASAL AREA >0.05 SQ.FT./ACRE')
   GO TO 8
   WRITE(6,606) XNRES,BB,TVOB,TVIB,TVOB4,TVIV4
   FORMAT(16X,6('-'),11X,5F11.1)
   WRITE(6,611) SI,AGE1,XNRES,BB,DAVG
   FORMAT(/42X, 'SITE=' ,F7.2/43X, 'AGE=' ,F7.2/31X, 'NUMBER OF ' ,F7.2/36X, 'BASAL AREA=' ,F7.2)
   XNTHIN=XN-XNRES
   BATHIN=BA-BB
   TVTHIN=TVOB1-TVOB
   TV4=TVOB41-TVOB4
   CV4=CV0B41-CV0B4
   WRITE(6,607) KTYPE,XNTHIN,BATHIN,TVTHIN,TO,P,TV4,TOP,TV4,TOP,CV4T
   FORMAT(/15X, 'AMOUNT REMOVED IN ',F7.2,'THINNING'
   : /15X,6('-'),1X,7('-'),1X,3('-'),1X,3('-'),1X,3('-'),1X,3('-')
   : /31X, 'NUMBER OF TREES =',F7.2
   : /36X, 'BASAL AREA =',F7.2
   : /22X, 'TOTAL CU.FT. VOLUME 0.8. =',F7.2
   : /20X, 'CU.FT. VOLUME 0. B. TO',F3.0,'IN =',F7.2
   : /27X, 'CORD VOLUME TO',F3.0,'IN =',F7.2)
   XN=XNRES
   BA=BB
   TVOB1=TVOB
   TVOB41=TVOB4
   CV0B41=CV0B4
   RETURN
Appendix 5. Source listing of program WTHIN (continued).

SUBROUTINE THIN

*******************************************************************
* * * SUBROUTINE THIN TAKES CARE OF THE THINNING * * *
* * OPTIONS AT AGE1. * * *
*******************************************************************
IMPLICIT REAL*8 (A-H, O-Z)
COMMON /ONE/ S1, AGE, XN, BA, HD, DMIN, DMED, DMAX, DBAR, IMAK, IMIN
COMMON /TWO/ S1, AGE1, XN1, BA1, DTHIN1, AGE2, Q, DTHIN, BRES, BRESR, QTHIN
COMMON /THREE/ INDEX, ITHIN, ILW, IROW
COMMON /FOUR/ A, B, BMIN, C, CONST, CINV, GAMMA

C THIN = 1 = NO THINNING AT AGE1.
1 RETURN

C THIN = 2 = ROW THINNING AT AGE1. EVERYTHING IS REDUCED
C BY A FACTOR Q.
2 IF(IROW.EQ.2.AND.ITHIN.EQ.2) Q=BRESR/BA
   IF(IROW.EQ.2.AND.ITHIN.EQ.4) Q=1.0-Q*(1.0-BRESR/BA)
   Q1=100.0-Q*100.0
   WRITE(6,666) (ITITLE(11), I=1,20)
   WRITE(6,600) AGE1, Q1
   FORMAT(//32X, 'ROW THINNING AT AGE', F4.0//23X, '% OF TREES IN ALL DIAMETER CLASSES ARE CUT')
   IF(Q1.GE.100.00) RETURN
   CALL OUTPUT(2)
   IF(THIN.EQ.2) GO TO 10
   ITHIN=3

C THIN = 3 = LOW THINNING AT AGE1.
3 GO TO (4, 5), ILW

C ILW = 1 = ALL TREES HAVING DBH LESS THAN DTHIN ARE CUT.
4 (DTHIN=DTHIN+0.500
   DTHIN=DFLOAT(DTHIN)-0.500
   IF(DTHIN.LT.A.OR.DTHIN.LT.DTHIN1) RETURN
   WRITE(6,666) (ITITLE(11), I=1,20)
   WRITE(6,601) AGE1, DTHIN
   FORMAT(//32X, 'LOW THINNING AT AGE', F4.0//23X, 'ALL TREES UNDER', F5.1, ' INCHES DBH ARE CUT')
   CALL OUTPUT(2)
   DTHIN=DTHIN
   GO TO 10

WTH08690
WTH08700
WTH08710
WTH08720
WTH08730
WTH08740
WTH08750
WTH08760
WTH08770
WTH08780
WTH08790
WTH08800
WTH08810
WTH08820
WTH08830
WTH08840
WTH08850
WTH08860
WTH08870
WTH08880
WTH08890
WTH08900
WTH08910
WTH08920
WTH08930
WTH08940
WTH08950
WTH08960
WTH08970
WTH08980
WTH08990
WTH09000
WTH09010
WTH09020
WTH09030
WTH09040
WTH09050
WTH09060
WTH09070
WTH09080
WTH09090
WTH09100
WTH09110
WTH09120
WTH09130
WTH09140
WTH09150
WTH09160
WTH09170
WTH09180
WTH09190
WTH09200
WTH09210
WTH09220
WTH09230
WTH09240
WTH09250
Appendix 5. Source listing of program WTHIN (continued).

```
C----- ILOW = 2 = THIN TO A SPECIFIED RESIDUAL BASAL AREA (BRES).
C
5 BTHIN=BA-BRES
   BB=0.DO
   IF(A.LT.DTHIN1) GO TO 6
   F1=0.DO
   XNT=XN
   IMIN1=IMIN
   GO TO 7
   6 F1=CDF(DTHIN1)
      XNT=XN/(1.DO-F1)
      IMIN1=DTHIN1+0.51DO
      GO TO 7
C----- FIND DTHIN CORRESPONDING TO BRES.
C
7 DO 8 I=IMIN1,IMAX
   XI=DFLOAT(I)
   F2=CDF(XI+0.5DO)
   F=XNT*(F2-F1)
   IF(F,LT.0.00) F=O.DO
   F1=F2
   BASAL=0.545415D-2*F*X1*XI
   BB=BB+BASAL
   IF(BB,GT.BTHIN) GO TO 9
   CONTINUE
8 CONTINUE
C----- QTHIN IS THE RESIDUAL PROPORTION (AFTER / BEFORE THINNING)
C
9 QTHIN=(BB-BTHIN)/BASAL
   DTHIN=XI-0.5DO
   WRITE(6,666) (I1=1,20)
   WRITE(6,602) AGE1,BRES
602 FORMAT(//32X,'LOW THINNING AT AGE' ,F4.0
   //23X,'THIN TO',F7,2,' SQ.FT. RESIDUAL BASAL AREA')
   IF( BRES. LE. 0. DO) RETURN
   CALL OUTPUT(2)
   DTHIN1=DTHIN
   XNLOG=DLOG(XN)
   BLOG=DLOG(BA)
   RETURN
END
```

```
C----- BLOCK DATA
IMPLICIT REAL*8 (A-H,O-Z)
COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DMED,DMAX, DBAR,IMAX,IMIN
COMMON /TWO/ SI1,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN,
   INDEX,ITHIN,ILOW,IRON
COMMON /THREE/ ITITLE,AINV,XNLOG,BLOG,HDLOG,TV081,TVOB41,
   CV0B41,1OPT,JJJ
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
DATA AGE/O.DO/,XN/0.DO/,BA/0.DO/,DTHIN1/0.DO/,ITHIN/1/ END
```

```
WTH09260
WTH09270
WTH09280
WTH09290
WTH09300
WTH09310
WTH09320
WTH09330
WTH09340
WTH09350
WTH09360
WTH09370
WTH09380
WTH09390
WTH09400
WTH09410
WTH09420
WTH09430
WTH09440
WTH09450
WTH09460
WTH09470
WTH09480
WTH09490
WTH09500
WTH09510
WTH09520
WTH09530
WTH09540
WTH09550
WTH09560
WTH09570
WTH09580
WTH09590
WTH09600
WTH09610
WTH09620
WTH09630
WTH09640
WTH09650
WTH09660
WTH09670
WTH09680
WTH09690
WTH09700
WTH09710
WTH09720
WTH09730
WTH09740
WTH09750
WTH09760
WTH09770
WTH09780