PROJECT SELECTION AND TIMING INTEGRATING PROJECT DEFERRAL, CASH FLOW TRANSFERALS, FINANCING AND INVESTMENT FUNCTIONS

by

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(ABSTRACT)

The success of a firm depends on the overall performance of all projects undertaken, whether they are production, sales, maintenance, or other types of projects. Correctly choosing promising projects becomes one of the most significant prerequisites for the success of the firm. The "correct" decision, in this research, is identified as the selection of the right projects at the right time.

While there are various restrictions that limit the selection decision, this study focuses on situations in which the funds available to the candidate projects are limited. Unlike typical project selection problems, which assume that the disposable capital all comes from the specified budget, this research considers four additional methods to increase the funds available for the projects. They are project deferral, cash flow transferals, financing, and investment functions. The purpose is to increase the firm's wealth by selecting worthy projects which would otherwise be rejected.

To make the right selection under this new situation, a modified version of Weingartner's 0-1 integer programming model is developed. By introducing artificial variables, the modified model can be maintained in a linear programming (LP) form (mixed integer). The advantage of the formulation is that, the relatively convenient LP software package can be used to manipulate the tedious calculations and then derive the optimal answer. No new solution techniques are necessary.

The revised model is developed under an imperfect capital market condition in which an n-step function is used to describe the dependency between the cost of capital and capital supplied. The workings of the revised model are illustrated by numerical examples. Future research topics, including the uncertainty consideration, are recommended at the end of this thesis.

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Chapter I

INTRODUCTION

One of the fundamental and essential decisions a firm has to make is the allotment of its scarce resources to the candidate projects, so that overall benefit to the firm is maximized. The limited resources could be manpower, capital funds, materials, land, machines, etc. This study concentrates on the allocation problem in which the competition is for scarce capital. The nature of this decision is usually twofold; that is, which project will be taken, and how much money will be appropriated for it ? This research focuses on the first problem and assumes the capital expenditures associated with each project are deterministic, once it is implemented, as is often the case in reality. Therefore this capital allocation problem, more precisely, should be called the deterministic project selection problem and will be referred to in brief as the "project selection problem" hereafter in this study.

Having recognized the scope of the research in this aspect, four methods are proposed to increase the availability of the funds, and hopefully, the wealth of the firm. Those propositions together stimulate the theme of this chapter, which identifies the implications and difficulties inherent in the problem and the accompanying objective of research.

1.1 Statement of Problem

In the traditional project selection process, in which the selected project starts from the initial period, one decides the best combination of candidate projects based on the total funds given. In other words, the decision maker wants to use as much capital as available to support as many worthy projects as possible. This implies that the use of money is free. Regardless the correctness of this implication,[1] it also indicates that the decision maker plays a passive role in his contribution to the firm. The fact that the funds are external and assigned to the decision maker, and that he is thus unable to influence them, makes the nature of his decision "not-to-make-a-mistake," rather than to increase the firm's wealth. There are of course various methods to compensate for this "passiveness" deficiency, for example, improving the project's profitability, productivity, cutting down its capital expenditures, and others. The strategies proposed by this research go back to the cause of the compensation: the fixed nature of funds. Four instruments are suggested to augment the amount of funds available for the projects, either explicitly or implicitly: (1) delaying the implementation of one or more of the candidate projects, (2) allowing cash flow transferals among projects, (3) employing financing, and (4) investment opportunities.

The project deferral consideration automatically relaxes the initial-year-starting restriction on the accepted project. However, it also introduces a new dimension of decision, the timing of the implementation. A project could be shifted

[1] If the decision maker, for example, a middle level manager, selects projects simply to maximize his value [38], this implication is correct.

to later years because of the following reasons: it is short of funds this year, it requires a big portion of funds this year (thus other small projects are prohibited), it has a negative wealth to the firm (which could be reduced by the delay),[2] and future economic or technology factors might be in favor of later implementation. However, this strategy is not without cost. The trade-off comes from the fact that the value of a project, originally positive, will possibly be decreased when the project is delayed. If this deduction exceeds the gain in the firm's wealth, there is no advantage in deferring the project.

While the deferral strategy does not explicitly increase the funds available to the projects, the cash flow transferal consideration does.[3] The extra funds are cash flows withdrawn from the projects undertaken. Under this policy several previously rejected projects are likely to be accepted and hence increase the firm's benefit. Moreover the project with a negative benefit, which is usually precluded from the candidate projects, may still be desirable as illustrated in Table 1.

Assume a firm engages in only two projects A and B, as illustrated in Table 1. It can be calculated that project A has a net present value (NPV) of $1000, and B has an NPV of $-50. If project B is excluded from consideration, the net present worth to the firm is zero. However, if the cash flow in year one is shifted from B to A, the net present value becomes $950. Therefore both projects A and B should be selected in this case.

[2] A project with negative benefit is not necessarily excluded from the candidate set of projects; for example, must-fund projects or cash-flow-transferring projects will be discussed soon.

[3] The cash flows are transferred only in the same year. This will be further discussed in Chapter II.

Table 1. Example of Cash Flow Transferals from Negative Worth Project

net cash flows (MARR = 20%)			
year	project A	project B	capital available
0	-800	-200	1000
1	-650	180	600
2	3372	0	0
unit: $			

Two other methods, financing and investment functions, are also commonly used to raise the funds to support more projects. The financing function, which borrows money from outside sources, is usually accompanied by some kind of cost charged by the lender, primarily in the form of borrowing rate. Therefore, a higher borrowing rate is likely to make the adding of other projects less profitable.

The investment function is conducted for two purposes: increasing the funds to be used for the projects and/or improving the firm's wealth, both by receiving interest gained from the investment. The funds used in this function can be unused budget surplus, cash flows after transferring, and/or even borrowed capital.[4] Because of the introduction of investment opportunity, the more the capital is tied to the projects, through the manipulation of project deferral, cash flows transferals, and borrowing, the less return will be gained from investment, and vice versa. These interdependencies will be discussed in more detail later.

The difficulty of this augmented problem is first observed by the size of the project selection combinations (portfolios). If each of the n projects is deferrable to k years, the optimal solution will be brought about from $(k + 2)^n$ portfolios of projects. The number of the alternatives then grows exponentially. This situation is made more difficult if the cash flow transferals and the financing/investment functions are considered. Further calculations are required since financing costs and investment returns have to be evaluated separately.

The project deferral, cash flow transferals, financing, and investment decisions are certainly not made in isolation. This simultaneous decision process, no

[4] For example, international investment.

doubt, increases the complexity of the problem and can be described as shown in Figure 1. As stated previously, the project selection decision, which purposes to increase the firm's wealth, is influenced by the given capital funds and the four blocks in Figure 1, i.e., investment, project deferral, cash flow transferals, and financing. The difficulty of the decision process arises from the competition among the blocks within the second and the third level respectively.

In Figure 1, in the second level, if more projects are intended (pros to the firm) fewer funds will be used for investment (cons); if fewer funds are invested in this period, less interest will be generated to support these projects; if the funds are not sufficient, after other approaches have been attempted, some of the projects must be discarded (cons) and the funds released can be used for investment (pros). What is critical then is the finding of the balance point that maximizes these trade-offs.

In Figure 1, in the third level, none of the blocks can be decided without considering other blocks. The decision process relevant to them can be described as follows. At the beginning, the trade-offs of these strategies are compared with one another for the whole time span. Once a policy is determined, the rest compete again with each other by supplementing the first decision. For instance, if cash flow transferal of project "j" is preferred most in the first year, then the project cannot be delayed and excludes the deferral strategy. Moreover the capital probably will not be enough if this project is not delayed; therefore some funds must be borrowed to support this project. Various kinds of scenarios can be created in similar fashion.

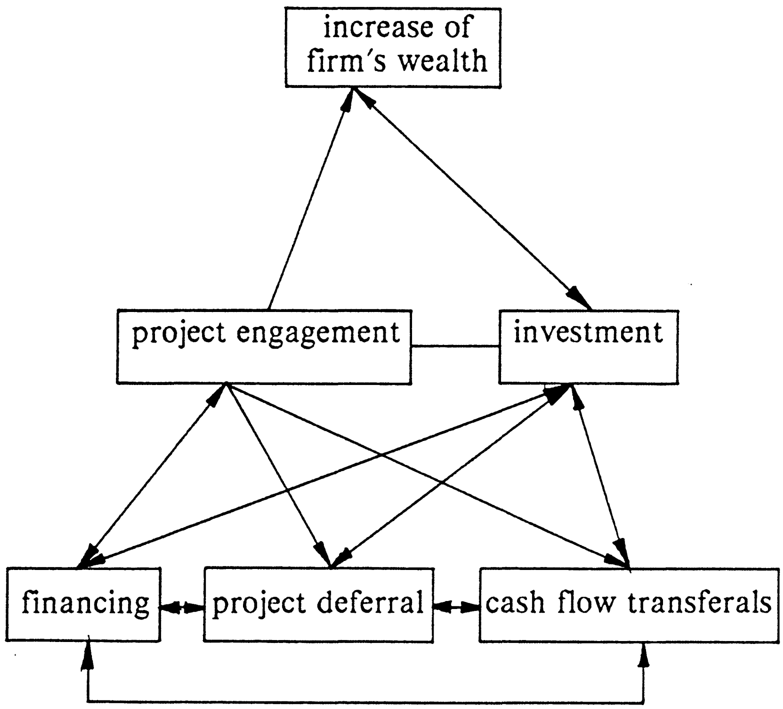

Figure 1. The Relationships among the Firm's Wealth, Project Engagement and the Proposed Strategies

The financing opportunity also creates another type of problem. It is already known that the attractiveness of a borrowing action is dependent upon its associated costs. The dilemma arises when this cost also depends on the capital borrowed. Therefore one cannot decide in advance the optimal amount of borrowing without knowing the cost of capital; in turn, the cost of capital is not known unless the borrowing amount is specified. The existence of this recursive relationship greatly enhances the complexity of the problem.

The entire project selection process is further complicated when the system information is uncertain. The system information includes the amount of funds, the number of candidate projects, the interest factors, the cash inflows and outflows of a project, the project life, etc. Under the conditions of uncertainty, two questions are prompted. They are:

1. How can the best estimation of the above information be obtained ?
2. What adjustments will be needed when the actual results deviate greatly from those expected ?

As an example of the latter case, there are occasions when abandoning a relatively poor project (divestment) is more beneficial to the firm than continuing it. Much work thus has to be done to answer these two questions.

1.2 Objective of Research

The goal of this research is to broaden an industrial engineer's (project manager) role in a firm in the project selection aspect. He is expected to actively interact with the accounting or the finance department by discussing the strategies for investment, cash flow transferals, and financing associated with the projects undertaken. This participation, plus his background in the operational side of the projects, should make his contribution to the firm significant.

To reach this goal, this study aims to formulate a model that optimizes the project selection problem mentioned earlier. That is, the model should be able to describe the recursive relationships and find out the trade-off points identified in the previous section, which emerges from the interactions of project selection, cash flow transferals, financing, and investment actions. In other words, this model should provide the answers to the questions of choice of projects, starting time of projects, the annual amount to be borrowed or invested, and the size of the cash flow transferred from each project. The uncertain information case, however, will not be explored for the time being; it will be left for future research.

Since financing and investment opportunities are offered, the model should allow the existence of different borrowing rates and lending rates (investment returns) in each year. To increase the generality, these differences are not required to be held constant over the time span. The model should also permit flexibility for varying net cash flows and capital expenditures associated with each project when it is started in different years. This kind of design not only approximates reality but also provides the basis for the consideration of uncertain information, which varies yearly.

At the solution technique level, the model should be presented in an understandable and executable manner for the user. The possibility of using available software packages is preferred. As was seen before, the task of enumerating the portfolios is large and tedious; therefore the model to be developed should also handle the computation efficiency problem.

As a summary of previous discussions, the firm's wealth cannot be maximized if the project mangers or finance managers make their decisions separately. What can be maximized is simply the wealth of the firm, conditioned on either project selections or finance decisions. Therefore interactive and simultaneous decision process is required, thus this research entitled "Project Selection and Timing Integrating Project Deferral, Cash Flow Transferal, Financing, and Investment Functions" is presented.

1.3 Overview of Research

While previous sections identified the research problem and discussed the difficulties and objectives associated with it, the overall scope of this problem is yet to be well defined. Chapter II describes the basic concepts, assumptions, and definitions on which the deterministic project selection problem is developed. Along with these, the relevant literature review is presented. This knowledge aids in the identification of the specific environment to which the research problem is oriented. This arrangement attempts to provide for a comprehensive preliminary study as a basis to facilitate the modeling process developed in Chapter III. The modeling process begins with a review of the specific problem environment, which can be regarded as the summary of Chapter II. Efforts are spent on modifying a basic mathematical programming model, so that it will maintain the LP form. The revised model should integrate the project timing and finance decisions into consideration. An imperfect capital market is also approximated by an n-step function to substantialize this model. The workings of the revised model are then presented by numerical examples in Chapter IV. The overall conclusions and suggestions for future research are presented in Chapter V. This model is verified as shown in Appendix A. In determining the mathematical expression of operational dependency relationships, a rule of thumb is given in Appendix B.

Chapter II

MODEL DEVELOPMENT PRELIMINARIES AND LITERATURE REVIEW

As was pointed out in the first chapter, the project selection requires all projects to compete for capital funds not only against each other, but, more importantly, against the overall benefits that will accrue to the firm. Concerns that must be addressed include the following: the scope of the firm, the projects, and the capital markets; the benefits and how they are created and measured; the integrated project selection process and how the decisions are made. This chapter presents an extensive examination and literature review of these concerns to gain further insight into the project selection problem to provide a preliminary basis for the development of the model proposed in Chapter I. Discussions are divided into two parts, basic concepts and specifications of the model environment and those of the selection process.

2.1 Basic Concepts and Specifications of the Model Environment

The model environment is concerned with three basic entities: the firm, the projects, and the capital markets. These elements form the environment under which the project selection is processed.

2.1.1 Considerations of the Firm

A firm is "an entity, be it sole proprietorship; partnership; corporation; joint venture; or any other social organizations, that engages in some activities for its considerations." [5].[5] A firm may be in the private or public sector, with the basic difference that "private activities are evaluated in terms of profit whereas public activities are evaluated in terms of the general welfare of the society" [44]. The general welfare could include factors, for example, education, national security, and charities, that are intangible per se and not easily quantified. Once the public welfare can be expressed in monetary terms, these projects can be evaluated by benefit-cost analysis and selected by traditional approaches [38, 44].

This research focuses on the private sector because of the significant difference in the nature of the projects between the public and the private sector. In most cases, the public sector first decides the projects to be taken according to the needs of the society; if the funds are insufficient, they can be raised by taxing,

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[5] By this definition, a firm can be a department in a real organization. Throughout this paper, if the terms "firm" and "department" appear together, a hierarchical relationship is implied. Thus there should not be any conflicts in using this definition.

borrowing and/or services charging [44]. Thus the problem is more of finding the most efficient combinations of these methods, and the project selection problem is trivial. On the other hand, the private sectors, in addition to the the fund-raising consideration, still have to face the project selection problem, the topic of this study.

2.1.2 Considerations of the Projects and the Capital Markets

It was previously mentioned that the firm's wealth is increased by executing projects and employing investment opportunities. It now becomes important to distinguish between the terms "project" and "investment". In this research, a project is defined as an opportunity to generate a return on a contemplated capital disbursement, with the characteristic that the opportunity is executable only in its entirety. An investment , on the other hand, is the opportunity in which fractional execution is permissible. It should be noted that a project is also an investment, but not vice versa. Further examination of the nature of a project are discussed below, along with the necessary capital-fund implications.

The assumptions that underlie most deterministic project analysis are the essential assumptions of certainty [5, 18, 26]. In those terms projects are viewed as indivisible, as defined above, and independent; the capital market is regarded as perfect. The financial implication of the indivisibility assumption is that firms must commit funds by deterministic amounts. If a large project cannot be executed as an entity, the project should be broken down into smaller discrete and separate portions to meet the requirement, if possible.

The next consideration of the project, continuity, is related to the divisibility assumption. In investment, for example the purchasing of Chrysler stock, the investor invests as much as he can and he wants. He can stop and resume the activity at any time for any reason without losing his accessibility to the investment. In contrast, with a project, capital funds are restricted. Table 2 illustrates this nature of projects and its related implications. If abandoning a project in progress is allowed, Row (2) in Table 2, the project's life becomes a variable and the decision maker must decide when the project should be terminated. This is more like a replacement timing problem [3], in which one can decide, once and for all, the end point of each project; or he can review the performance and make the decision period by period. The latter case arises from the realization of uncertain environment, which might indicate that the to-be-abandoned project actually was underestimated. Moreover, if a project is abandonable during the time span, it implies that this project is an existing activity. Therefore the selection scheme should take existing projects into consideration. There is however no need to explicitly specify a project's life since it is a variable itself.

The other possibility in the "abandonable" case is the decision of whether or not to replace a project, rather than when to replace. This is the traditional replacement analysis problem where the project life should be specified, since it is sensitive to the decision [44].

Table 2. The Continuity Consideration of a Project

	implications status	decision pattern	inclusion of existing projects	specification of project life
(1)	continuous	single period	no	yes
(2)	abandonable	single/ multiple	yes	yes/no
(3)	interruptible	multiple	yes	yes

Similar to the above situation is the interruptible case, Row(3) in Table 2, where a project can be halted temporarily and restored later at an appropriate time. To select projects, Skipper [38] suggested a multiple period decision process in which the projects can be funded or unfunded in any period during the time span. Compared to the abandonable case, an existing project should still be included in the selection scheme; but there is the need to specify a project's life, since it is resumable even though it is stopped. In summary, both abandonable and interruptible considerations are required because of the assumption that, for a project in the optimal set, later alterations of that project are possible, even if the optimal project is selected.

The continuity situation, on the other hand, selects projects only once since each one of them is required to complete its life. The inclusion of an existing project depends on its competition for the designated capital funds. If it does not compete, there is no need to take the existing project into consideration. The project life, however, has to be specified in this case. Because this research simply aims to present an approach to select projects, one-shot decisions for future projects would be sufficient to satisfy this objective. Therefore the projects under analysis are new opportunities and should not be terminated midway. "No alteration of a project in the optimal set" is also assumed to further support the continuity assumption.

Even though each project's life is specified, any two of them can be equal or unequal-lived. The importance of this consideration lies in comparative evaluation of projects. In measuring the worth of unequal lived projects, different criteria are usually inherent in different assumptions. For example, the annual equivalent value method (AEV), when used to compare the worth of any two projects, implicitly assumes a least common multiple life span for each project [12, 48]. Therefore, if two projects have different lives, say two and three years respectively, using this method will require the former project to repeat itself three times and the latter twice, as is usually seen in the replacement analysis problem. Since replacement decisions are not the focus and a two-year project repeating twice can be regarded as a nonrecurring project with four years' life, this study assumes that the project is nonrecurring. This assumption is regarded by the author to be sufficient for the purposes of analysis.

The independency assumption requires that the implementation of one project does not in any way affect the implementation and profitability of other projects. There are two types of dependencies, the operational dependency and the statistical dependency. Operational dependencies such as mutually exclusive, complementary, contingent, priorities, etc. are not assumed in the model development process; however, they will be illustrated by a numerical example to show the richness of the structure of the model. The statistical dependency cases are usually treated as a stochastic process [17], in which the correlation between any two projects is studied. Since this research deals with the certainty condition, statistical independency can be assumed without degrading the value of the research work. This assumption implies that the sum of returns or costs from any two projects is equal to that of their corresponding portfolio; that is, linearity holds.

One of the essential assumptions of certainty is the perfect capital market. In such a market all firms are able to borrow and lend on the same terms, so that funds can be borrowed as needed at the known interest rate. Therefore any size project can be undertaken provided the indivisibility requirement is not violated.

However in reality where the imperfect capital market exists, the borrowing transactions is not so ideal. In Figure 2, an increasing cost of capital curve indicates that the firm cannot raise an unlimited amount of capital at a constant cost [5]; the interest rate depends on the amount of supplied (borrowed) capital when it is beyond B_{crit} . The implication of this situation is that the cost of capital is an ex post product of the optimal solution, whose financing method cannot be decided in advance without knowing the cost, as concluded by Hirshleifer [18].

Based on realistic considerations, an n-step function is used to approximate the curve beyond B_{crit} , as indicated by the broken lines in Figure 2. In that market a higher borrowing rate will be imposed, cumulatively, on each borrowing which exceeds the jump points J_i's. The jump points and extra borrowing rates, however, are predetermined by the financing sources and open to the firm or the public. Since multiple period capital consumptions are possible, curves similar to Figure 2 should be established for each period considered. These curves are not required to be identical, however.

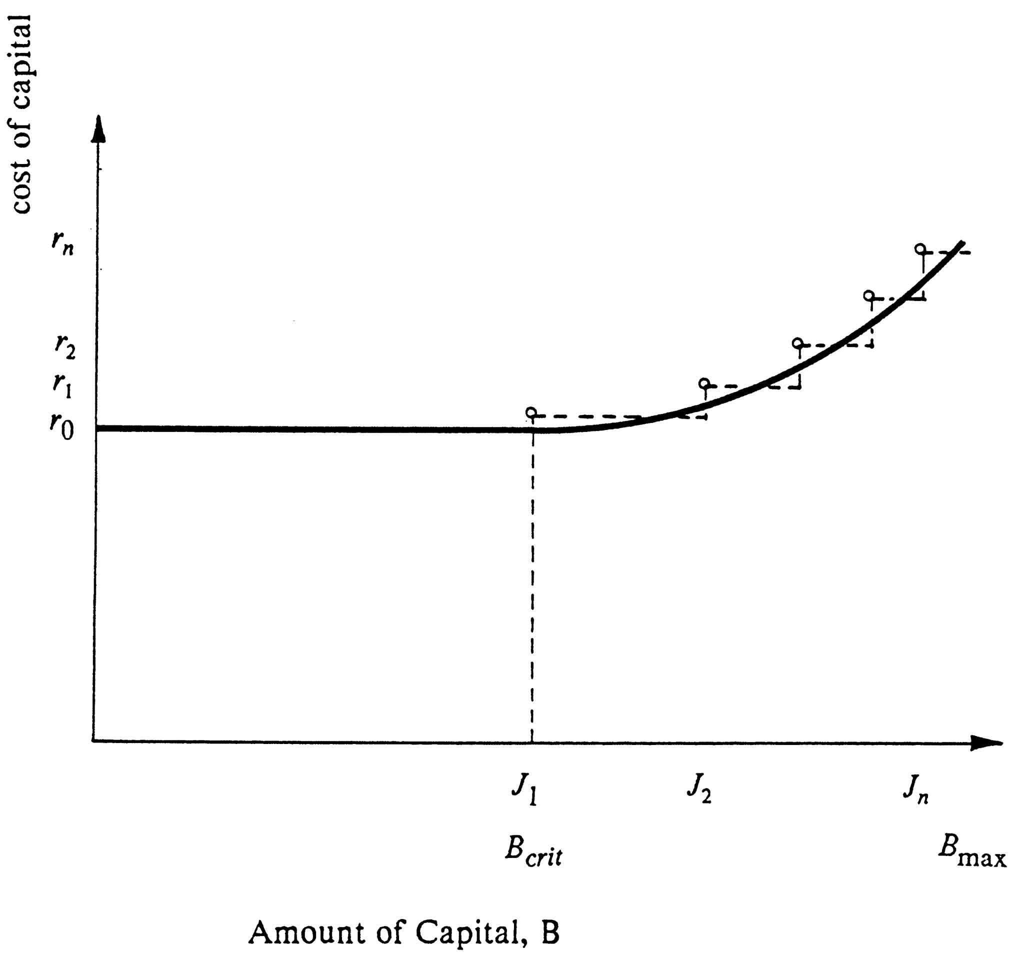

Figure 2. Capital Supply Curve

2.2 Basic Concepts and Specifications of the Project Selection Process

The above section has provided a fundamental framework in which the project selection proceeds. The four stages of the selection approach can be identified as project generation and data collection, project evaluation, budgeting strategies identification, and portfolio selection.

2.2.1 Project Generation and Data Collection

The very beginning of a project comes from an idea or a goal that is good for a firm. This idea or goal then activates some fundamental planning tasks, including R&D, design, test, market analysis, and economic analysis, etc. A project is proposed when it is proved technically feasible; however, its final acceptance has to rely on the associated economic analysis and the subsequent project selection scheme. To conduct the economic analysis, three categories of basic data are collected for each project. They are revenues, costs, and project life.

While it is agreed that net cash flows (after-tax) are better indicators of the revenues and costs of a project than accounting numbers [9, 16, 23, 41], the measurement of their magnitudes and timing are still an issue under debate. The argument is whether they should be estimated as discrete point estimates at the end or beginning of a period, or combinations of continuous functions over time. The continuous function could be any of the following: uniform, trigonometric, exponential, stepwise, ramp, or other types. Advocaters of this approach argue that the daily operation nature of a project makes it more realistic. As an example, Buck and Hill [4] develop a "simplified model" and an "extensive model," where uniform, ramp, decay, and growth functions are used to estimate cash flows of revenues and costs. One result they have claimed is that the deterioration of a project can be closely approximated by ramp or gradient functions. More detailed information about this measuring approach is available in [4, 43, 50].

Since no strong evidence so far has shown that the continuous method is better than the discrete method, this research will assume that the estimated cash flows comprise receipts and disbursements that are dispersed in time, as most studies have done. Also, no attention will be paid to the question of "how to estimate." All cash inflow and outflow estimates (and other data) in this paper will be treated as the best discrete point estimates available. Also all relevant cash flows attributable to a capital project have been included. Transactions are assumed to be occurring at the end of each period, which implies that the payments can be delayed till the end of the period. This assumption facilitates the discussions of this research since all calculations are manipulable at the same time point in each period. Research regarding daily or continuous timing is scattered in open literature such as [42]. It will not be investigated here, however.

In economic analysis there are usually four types of project life, namely depreciation, physical, service, and economic life [41]. The depreciation life is used for tax purposes and subject to tax regulations; therefore it will not be discussed here. The physical life is straightforward and often needs to be estimated. However, it is not frequently used in the evaluation process. On the other hand, the service life is usually adopted in cost comparison, and the economic life is used in replacement analysis or when the service period is infinite [41, 44]. Because this study assumed the project to be nonrecurring, and because, in reality, the assets associated with a project need not be deteriorated or obsolete to be disposable, the service life point of view is adopted in this study.

In addition to preparing this data, the operational and statistical dependency relationships have to be studied. As other tasks initiated at this stage, specific knowledge about technologies and statistics are usually needed. In section 2.1.2 this dependency issue has been specified and will not be repeated here.

2.2.2 Project Evaluation (Investment Worth Measurement)

After the above data have been estimated, the desirability of each project can be measured by a certain criterion. Two more items of information must be provided at this stage, either by estimation or specification. They are cost of capital of the firm, and planning horizon.[6] The planning horizon usually is not equal to the project life unless there is only one project under analysis.

There are two major viewpoints in evaluating the worth of a project: the traditional methods and the finance-oriented methods. The traditional methods of project evaluation can be divided into two categories: those ignoring the time value of money and those recognizing it. Examples of the first category include

[6] These data of course can be collected at the first stage. They are, however, positioned at the second stage because of their significance to the choice of the measurement criterion.

payback period, naive rate of return, and average return on average income (ARAI). Although these methods are popular in practice [21], they are not theoretically sound. The other category principally involves measurements such as net present value (NPV), annual equivalent value (AEV), future value (FV), profitability index (PI), and internal rate of return (IRR). The first three methods are called the equivalence-based measure [38] and require the inputs of cash flow, cost of capital, and project life data. The PI measure is the ratio of the present value of the after-tax cash inflows to the outflows. The IRR measure is defined as the interest rate that causes the net present value of a project to be zero. While these methods are equally applicable in measuring the attractiveness of a single project, care must be taken when there are two or more projects in comparison. Which one of them should be chosen is not a trivial problem as will be discussed below.

The first clue in making the decision is the multiple interest rates phenomenon of IRR. In some types of cash flow streams, in which the cash flow signs change more than once, there is no guarantee that a unique internal rate of return exists; the IRR can be multiple or even does not exist. Second, even if a unique IRR exists, there is still a ranking inconsistency problem. In the following example projects, each has net cash flows of (-1, 3, 5) and (-1000, 3000, 5000) respectively. Using the IRR criterion, they are equally valuable whereas the second one in fact is more profitable. Therefore, the IRR method only considers the rate of return on projects but not the scale of profits. Researchers do recommend an "incremental analysis" approach to compensate for this deficiency; however, it is time consuming if there are more than two projects to be considered [49]. Based on the above discussion, the IRR method will be ruled out from further consideration.

The profitability index (PI) method ranks projects of varying costs and expected lives in the order of their profitability. This method is able to avoid the problem of multiple or non-existent rate(s) of return; however it still suffers the ranking conflicts, as can be seen in [9]. Also, in a case similar to the above numerical example, the PI method might conclude that investment in a calculator appears better than investment in the whole production line of personal computer. Therefore, the PI measurement will not be adopted either.

The selection of a proper criterion from the rest of the measurements relates to the concept of planning horizon and cost of capital. In comparing two projects, one needs to specify a planning horizon on which the projects are compared at the same time point. There is no ambiguity when the projects are equal-lived; however, if they are not, care must be taken in specifying a planning horizon. Usually two assumptions are made in dealing with the unequal life situation; one presumes that, after the end of a project, it will be replaced by the same project; the other assumption asserts that the funds from the short-lived project will be reinvested by the firm in a relevant rate of return. The correct assumption is the one that is believed to be the most accurate representative of future actions. As was pointed out by Solomon, White, et al. [39, 48] the annual worth implicitly assumes that "each project will be replaced at the end of its respective useful life with another asset having the same profitability." Thus the

AEV method matches the first assumption and the planning horizon will be a least common multiple life for the set of all feasible, mutually exclusive projects. Since the projects in this study have been specified to be nonrecurring, the AEV method is not an appropriate one for use and consequently, the planning horizon should not be the common multiple life.

Under the reinvestment situation, if the perfect capital markets and perfect certainty assumptions hold, the NPV and FV methods will give the same evaluation for a project. However, with an imperfect capital market, as Hirshleifer indicates [18], the NPV criterion is not valid. To correct this deficiency, earlier researchers such as Porterfield [32] and Weingartner [47] have suggested the future value approach as an alternative. This point of view has been accepted in the capital budgeting area by investigators such as Swalm and Lopez-Leautaud [43], Bernhard [2], Bierman [3], and Quirin [34]. Recalling the capital market assumed in Section 2.1.2, where borrowing rates and lending rates differ in each period, this author recognizes that employing the future interest rates is a proper approach and will make the evaluation easier.

The interest rate is usually interpreted as the cost of capital of the firm. The cost of capital is an opportunity cost concept. It is the rate of return on the best alternative investment opportunity. Therefore if funds are released from a project, they can be invested at the return rate of that alternative. Although this concept is clear, complications still arise in arriving at an exact value of the cost of capital, even if risk is not considered. This is difficult since the calculation is related to the overall financial structure of the firm. However, Baumol [1] points out a rule that becomes useful in finding the cost of capital. His rule says, if borrowing rates and lending rates are different, and if the firm can lend or borrow as much as it wants at these rates, then

1. "if money to be used on a contemplated project would otherwise be lent, the lending rate determines the cost of capital;
2. if the money would otherwise be used to pay off a debt, the borrowing rate applies."

However, if disposal options for funds are limited, the cost of capital becomes the most profitable marginal yield of money available [1]. In conclusion, the determination of cost of capital depends on the future reinvestment opportunities available to the firm; thus, this is basically an uncertainty problem [40] . Since the uncertainty problem is not the concern here, this study assumes two types of scenarios for investment: (1) the best short-term (one year) lending, and (2) other promising investments, either short-term or long-term. In the first case, the lending rate is always smaller than the borrowing rate; but in the second case, this is not necessarily true. The cost of capital of the firm then is the lending or investment rate, and/or the borrowing rate depending on the following rule:

1. If net cash flow < 0, $i_{xt} = i_{bt}$.
2. Otherwise, $i_{xt} = i_{lt}$.

where i_{xt} is the cost of capital for each year t

i_{lt} is the lending or investment rate

i_{bt} is the borrowing rate.

There is also no restriction on the amount for lending or investment.

The last element to be specified is the planning horizon. White, et al. [48] have identified some commonly used methods for determining the horizon:

1. least common multiple of lives
2. shortest life among alternatives (T_S)
3. longest life among alternatives (T_L)
4. some standard planning horizon T.

where $T < T_s$ or $T > T_L$, or $T < T_s < T_L$.

The first method has been rejected previously and the fourth method is problem dependent; therefore the decision is between 2 and 3. The shortest life method, however, does not consider the reinvestment possibility; therefore the longest life method will be adopted. Furthermore, due to the project deferral consideration, the planning horizon, H, is extended to

$$H = T_L + \mathrm{k}$$

where k is the maximum delay permissible.

All calculations of cash flows should be done up to the planning horizon. In case there is the need to borrow in the last period, it seems to be realistic to carry forward the debt from time H to H+1, at an interest factor $(1 + i_{bH})$. Other post-horizon cash flows are assumed to be nonexistent.

To calculate the future value of a project, two approaches are available: the transformation technique and the discounted cash flow (DCF) technique. The first approach utilizes zeta-transform and difference equation methods to model cash flows and is argued as more robust and convenient for application than the DCF approach [22]. The DCF technique uses interest factors to discount cash flows to present time or to carry them to future time, and so on. Because surveys have shown that the use of DCF technique is dominant in academia and is increasing in practice [36, 45], the DCF method is adopted. Therefore given the best estimates of net cash flows, a_{tj}, project life n_j for each project j, and the cost of capital i_{xt}, the desirability of a project can be calculated over the planning horizon, H, by the following formula:

$$FV_j = \sum_{t=0}^{n_j} [a_{tj} \prod_{s=t}^{n_j - 1} (1 + i_{xs})] \prod_{s_1 = n_j}^{H-1} (1 + i_{xs_1})$$

$$= \mathrm{FV}(a_{tj}) \qquad \mathrm{j} = 1 \ldots \mathrm{m}$$

where i_{xs} is either borrowing or lending rate depending on the sign of a_{tj}

i_{xs_1} depends on the sign of $\sum_{t=0}^{n_j} [a_{tj} \prod_{s=t}^{n_j - 1} (1 + i_{xs})]$

$\prod_{n_j}^{n_j - 1} (1 + i_{xt}) = 1.$

The FV criterion measures the increase of the firm's terminal wealth due to the undertaking of projects. This assertion however is criticized by researchers who take a finance-oriented stance [2, 30, 39]. One of the earliest arguments comes from Bernhard [2]. He alleges that the firm, in planning its productive financing, and investment policy, has as its objective "the maximization of some function of all anticipated dividend payments to the firm's present share holders." Alternatively, the value of a project is measured by a function f, which reflects the project's contribution to the firm by the dividends paid each year. The objective function set in Bernhard's model thus becomes

Maximize

$$\mathrm{f}(w_1, w_2, \ldots, w_T, G)$$

where w_t is cash dividend to be paid to owners at time t

G is the firm's time T terminal wealth, after payment of W_T

and f is a nondecreasing function.

In this approach, G is considered to include the future value of all dividends, that is, G represents the terminal shareholder's equity. The difficulties inherent in this function are:

1. f may not be linear and is difficult to determine, since it is the utility function specified by shareholders.
2. G is difficult to measure.

Bernhard's formulation certainly is more complicated and closer to the behavior of many big firms. In most cases, however, the traditional approach is sufficient for project selection purposes. Specifically, the traditional approach considers f as a dollar function, rather than a utility function. Since borrowing and lending can be regarded as "divisible projects" that generate future cash flows.[7] This study uses the FV criterion as the measurement of project desirability and follows the traditional approach in defining the firm's wealth.

2.2.3 Identification of Budgeting Strategies

The economic analysis so far is not sufficient to serve the project selection purposes of this study. Specifically, due to budget constraints, positive FV alone is not enough reason to accept a project and a project with negative FV will not

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[7] A borrowing can be regarded as a "negative divisible project."

always be rejected. It is thus imperative at this stage to identify the budgeting system that supplies the funds.

In the traditional project selection context, the budgeting system is merely a series of numbers designated from some upper level in the organization. There is no need to further identify the compositions of these figures since, to the department, no matter whether the funds are from retained earnings, issued bonds, bank loaning, or other sources, their use is completely free. However, it was suggested in the first chapter that the selections be conducted on a firm-wide basis; it is thus necessary to investigate the budgeting system of the firm. In other words, the financial consideration is now integrated into the project selection problem.

Figure 3 is the cash flow input/output analysis diagram that illustrates the cash flow I/O relationship of a firm. The output aspect consists of functions f_2, lending or investment opportunities, and f_3, cash disbursements. The investment opportunity case has been specified when the cost of capital issue was discussed and will not be repeated here. The cash disbursement case primarily includes capital outlays, debts, dividends, and liquidity requirements. Capital outlays of a project have been assumed to be deterministic. The debt in each period is dependent on its corresponding financing decision. The dividend policy and liquidity specification, however, are usually decided based on other considerations irrelevant to this study; therefore no further discussion is intended.

To the firm there is seldom a free source of capital. In general, capital funds are either externally financed or internally generated. Functions f_1 and f_2 in Fig-

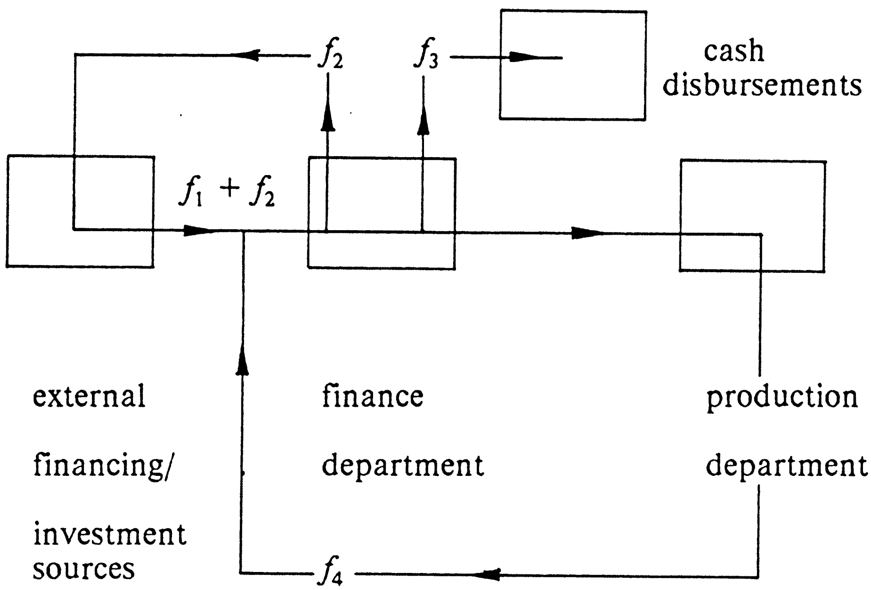

Figure 3. Cash Flow Input/Output Analysis Diagram of a Firm

ure 3 shows the external aspect of gathering money. To acquire money through f_1 function, the firm can sell bonds, stocks, or other types of securities or borrow from a bank directly. These financing tools are usually divided into long-term or short-term borrowing. Often, the former costs more than the latter because the lender has to undergo a longer risk. Furthermore, due to risk, credit, and other economic and noneconomic considerations, the firm is usually limited in its ability to obtain additional funds. For these reasons, the financing opportunity which a firm is willing to take is assumed to be one year long and constrained by an upper bound for borrowing, w_t, in each year t. The borrowing opportunity will be extended by including a long-term financing example later in Chapter IV. Since the f_1 financing approach incurs heavy costs, a firm should first consider the other alternative to alleviate the temporary shortage of capital funds.

The other alternative refers to the internally generated funds. They are earnings from projects undertaken. The uses of these earnings are various; the firm can simply invest them in some activities, or retain the earnings in the production department[8] for projects that are short of capital, etc., to name only a few. In this study, internally generated earnings are assumed to be retained and processed as follows:

> A cash flow transferal function appropriates retained earnings to the projects requesting funds for optimal benefit to the firm and returns the surplus funds to the firm immediately after appropriation.

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[8] The difference between the finance and production department in this study is that the latter does not deal with capital funds.

This type of funds is preferred by the firm if otherwise it has to borrow to support projects. However, if there are other promising investment opportunities available to the firm, cash flow transferal policy might not be so attractive. In conclusion, the firm can adjust the size of the f_4 function (returned earnings) to optimize the wealth of the firm, as is implied in Figure 3.

One important effect of introducing the cash flow transferal strategy lies in the measurement of the firm's wealth. Previously, each individual project's worth was measured by FV(a_{tj}), for j=1...m. The firm's wealth, through undertaking project, investment, and borrowing is therefore the sum of each action's worth:

$$\sum_{j=1}^{m} FV(a_{tj})$$

The cash flow transferal consideration has now made it possible to rewrite this expression as:

$$\mathrm{FV}\left(\sum_{j=1}^{m} a_{tj}\right)$$

This is actually the future value of a portfolio, which comprises the actions undertaken. The measurement of a portfolio's wealth is preferred, as will be seen in Chapters III and IV.

Another strategy that will possibly influence the budgeting system is project deferral, which is imbedded in the production department block in Figure 3. This strategy is a tool that allows the firm to utilize its capital funds more efficiently. Other factors that necessitate timing consideration include business variations, capital market conditions and technology limitations. This research considers all these conditions, either explicitly or implicitly, except technology limitations, which is a totally different topic. Also, each project is assumed to be deferrable to a maximum of k years.[9] This value is, as with other data in this study, an estimated value.

The investment timing issue has received attention as early as the 1960's. One of the earliest systematic treatments of this problem is given by Marglin [28]. He uses a dynamic programming approach to optimize the following function:

$$Y(t) = \int_t^{\infty} P(u)e^{-ru}du - \sum_{i=0}^{\infty} Ce^{-r}(t + n)$$

where Y(t) is the net present value at t

P(u) is the price at time u

C is the capital cost

r is the rate of interest

n is project life

t is the timing variable to be decided.

As a result, Marglin shows that projects with the highest positive NPV need not necessarily be dominant. The investment timing problem is further explored by Manne [27], and Dore [11]. In their models, external factors such as market demand, plant size, etc. can be taken into consideration. These are important research topics, though, under the scope of this study, all information regarding this research field has been assumed to be given; therefore no further discussion is intended.

If enough information is given, the timing decision under constraints is actually not a complicated problem. Byrne, et al. [6] have originated a "Group

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[9] The maximal delay, k, can be different for each project. To simplify the calculation, it is assumed to be same for all projects.

Payback Restriction" approach to allow for the starting of projects at times other than zero,

$$- \sum_{t=1}^{t'} Y_{tj} X_j \leq 0 \qquad j = 1 \dots m$$

where Y_{tj} = net cash flow obtained from a unit of project j at time t

X_j = number of units of project j to be undertaken

t′ = a specified integer, such that $1 \leq t' \leq T$

T = planning horizon.

Therefore if a project requires huge capital disbursements, the project may be delayed at maximum t′ years depending on when this condition is satisfied. The other approach, which is usually given in text books as an illustration example [9, 31, 34], simply assumes mutual exclusiveness between a project and its corresponding delayed possibilities. This concept can even be extended to define multiple dimension decision variables. For example, Martin [29] in his "capacity expansion investments" research defines z_{ilt} as "process option l installation status at location i at the beginning of year t." The mutual exclusiveness treatment seems to be more logical and easier to use; therefore it will be adopted.

The cash flow transferal policy has been an important research topic in the finance area where the concerns are the determination of timing, and the amounts of funds to be transferred between a company's depository bank and concentration bank [42]. Since timing of transfer has been assumed earlier to occur at a single point in each period, only the amount aspect will be studied. Also, no cash flow transfer cost has been assumed.

Research combining financial policies with project selections can be traced back to Weingartner's basic horizon models [47]. His first model, the basic horizon model, uses a mathematical programming format to maximize the firm's value as of the future terminal horizon. In that model, he allows the intertemporal transfer of funds at an assumed cost and finds in the optimal solution some projects with negative NPV. This model is later extended by relaxing the assumptions of a perfect capital market so that short-term borrowing is considered.

Although Weingartner has great contributions to the extensions of the theory of optimal resource allocation, his basic horizon models are actually special cases of an outstanding model developed by Bernhard later [2]. In his model, the objective function is expressed by cash dividends and the firm's post-horizon terminal wealth. Due to the financing and investment opportunities, he also establishes the cash balance restriction constraint which includes liquidity requirements. The terminal wealth restriction constraint, which defines the expression of terminal wealth, and the terminal wealth horizon posture restriction constraint, which defines the range of terminal wealth, together determine the firm's terminal wealth. The specialty of Bernhard's model is his building of an objective function and terminal wealth constraints. Although he perceives the firm's wealth from the point of view of shareholders, his model is not necessarily a linear model and hence will be quite difficult to solve because it is nonlinear.

Because of this practical dilemma, it is felt a simpler model should be developed to serve the needs of this research. This assertion can be supported by recent developments in the financial planning area. Myers and Pogue [30] formulate a model that can be solved by the mixed integer programming approach. Their model evaluates the firm's wealth as the value of the shareholder's equity. Consequently, Myers and Pogue formulate their objective function as a linear combination of the firm's projects, investments, tax savings, and other costs due to market imperfections. In contrast to Bernhard's model, they do not consider dividend policy and post-horizon value of the firm. These two factors are important to the structure of the finance-integrated models. The linearity of an objective function, however, heavily depends on the investor's perception about the utility of shareholder's equity. Other models surveyed [19, 24, 37], which successfully formulate linear programming structures in this field, have constructions similar to that of Myers and Pogue.

Another difficulty in formulating a linear structured model lies in the imperfect capital market behavior. Howe and Patterson [19] study the flotation costs of a long term-financing tool, issuing equity. They assume that flotation costs vary as a percentage of the amount of capital raised, following a decreasing step function. A direct formulation will result in a quadratic programming scheme. By substituting variables, Howe and Patterson are able to approximate the original formulation by a mixed integer programming model. Sharp and Garza [37], on the other hand, let the borrowing rate vary from period to period as a function of the debt-equity ratio. In his study, a linear programming structure can be derived only when the equity of the firm remains constant over the horizon. This implies a loss of generality. It should be noted that this study does not attempt to build an algorithm to transform nonlinear models in this field into linear models; instead it tries to develop a linear model under the predetermined environment.

Several major differences can be observed between the models mentioned above and the model in this research.

1. While most of the models use NPV to measure a firm's wealth, this research suggests FV, which will be shown to be more appropriate.
2. Compared to Bernhard's Model, this research sets a different definition of the firm's wealth and aims to formulate a linear structured model.
3. Compared to Myers and Pogues' model, this research considers a varying cost of capital case.
4. Compared to the Howe and Patterson, and the Sharp and Garza models, the short-term financing opportunity in this research follows an increasing step function.

2.2.4 Selection of Portfolio

This section is concerned with the solution techniques that solve the project selection problem. Basically, three approaches are available: the heuristic, the explicit enumeration (zero-one matrix), and the implicit enumeration (mathematical programming) approaches.

The heuristic approach was popularized by Dean [10] in the 1950s. His model selects projects based on a ranking of candidate projects using their respective indices of net present value relative to their capital requirement. As was pointed out before, the ranking inconsistency phenomenon occurs when the NPV and IRR criteria are used to rank the same set of projects. Recent research has suggested the "incremental analysis approach" to resolve this shortcoming of ranking [48]; however, this approach is time consuming [49] and only applicable to conventional cash flows [3, 14, 33]. The ranking approach is also inadequate when there are multiple projects under budget constraints. This problem arises from the indivisibility assumption of a project. Specifically, a large project under a capital budget constraint may exclude the acceptance of several smaller projects that would have more aggregate value to the firm. Lorie and Savage [25] first pointed out the above flaws and attempted to solve them by using "Generalized Lagrange Multipliers" technique. Their method, however, was unsuccessful in dealing with multiple period capital budgeting problems [8, 46]. Because the ranking approach generally requires three assumptions : (1) parity of size (2) few limiting constraints, and (3) small number of projects, it will not be adopted in this study.

The enumeration method lists combinations of all projects under analysis. After eliminating the infeasible solutions from the exhaustive list, the optimal solution can be obtained by a sorting scheme. This method is rather straightforward; however, it is inefficient since $(k + 2)^n$ portfolios have to be considered. As a result, computer work is required to handle the enormous calculations. Even using computers, as many researchers have indicated, this method should be used only when less than four or five projects are available [7, 35]. Flexibility is another problem associated with this method. In the event that the decision domain is augmented, for example, to the integration of financial policies, economic dependencies, and others, the user has to modify the source program according to his needs. However, it would be unwise to reject this approach before considering its advantages. As pointed out by Skipper [38], this methodology allows the decision makers to see the intermediate results and thus interact correspondingly. Based on the objective of this research, however, this method is not suggested.

The project selection process becomes more complex as the number of project increases, as the decision periods multiply, and as more constraints are considered. In these situations, the use of mathematical programming models seems more natural and attractive. Mathematical programming techniques employed in the deterministic project selection problem include linear programming, integer programming, goal programming, nonlinear programming, and dynamic programming [9]. The application of these operations research techniques depends on the assumptions made about the environment of the system being analyzed. For example, if multiple objectives are incorporated into the model, the goal programming approach should be used [20]. Because this research seeks an easily understandable model for practical use and, because there is only one objective function, the linear programming and integer programming approaches will be applicable.

The linear programming model for allocating resources among competing projects was introduced by Weingartner [46] in the 1960s. Strictly speaking, his model is simply an application in the more general linear programming frame;

however, he incorporates the DCF concept into the objective function and has solved the second Lorie and Savage problem (capital rationing) [25]. Weingartner's model is so theoretically sound that it remains unchallenged, and is the most often referred one in this field. Later developments such as integer programming and mixed integer programming facilitate the solutions to Weingartner's model. Even if the financing, and investment decisions are taken into consideration, the general structure of his model still proves to be valid, as can be seen in the previous section. Although surveys have pointed out that the mathematical programming techniques are not commonly used in practice [15, 36, 45] and other approaches have been proposed [13, 24, 38], this author believes the mathematical programming approaches are superior in that they are : (1) relatively efficient optimization techniques, and (2) capable in describing multiple period constraints and other real world problem settings.

Chapter III

PROJECT SELECTION MODEL DEVELOPMENT

This chapter presents the project selection modeling process. The environment on which the model is developed has been thoroughly discussed and will be summarized as an initiative to facilitate the discussion. Based on the summary, a model that simultaneously copes with the project timing, cash flow transferals, financing, and investment issues will be built. This model is verified by a numerical example as in Appendix A.

3.1 Assumptions Revisited

A department, which belongs to a firm in the private sector, is considering the undertaking of m projects. The firm asks this department to make decisions from the firm's point of view. The decisions are one-shot; that is, they are made at the beginning of the planning horizon only. The department suggests the following strategies to maximize the portfolio's future value derivable from implementing projects and disposing funds:

1. All projects are permitted to be delayed to a maximum of k years.
2. The department is allowed to retain earnings at the current period and transfer them to support projects that lack funds.

The firm lends surplus funds and borrows from external sources when it is out of money.

The financing and investment environment offered to the department have the following characteristics:

1. All borrowing and lending are one-year transactions.
2. The borrowing opportunity is the one that, if the department borrows bears the cheapest cost, i_{bt}.
3. The lending opportunity is the one that, if the department lends, bears the highest return, i_{lt}.
4. The borrowing rate and lending rate are known.
5. The borrowing rate is always higher than the lending rate.
6. The department is limited in its borrowing ability by an upper bound w_t.
7. The department is able to lend all funds available.
8. The capital market is an increasing n-step function.

The projects under consideration have the following characteristics:

1. These are future investment opportunities.
2. The projects are indivisible.
3. All projects, once implemented, cannot be abandoned midway.
4. All projects, once implemented, cannot be altered midway.

5. The capital requirement for each project is deterministic and multi-period.
6. Neither operational nor statistical dependency exists between any pair of projects.
7. The projects are nonrecurring.
8. All projects have equal risk.

Two sets of financial information, yearly disposable fund (I_t) and cost of capital (i_{xt}), are given by the firm. The yearly disposable fund is regarded as the initial financial condition of the firm in each year. The cost of capital is equal to i_{bt}, if the project's net cash flow is negative; otherwise, it equals i_{lt}.

Best estimates regarding the project's yearly after tax cash flow and project lives are also given. These data have the form of point estimates and are provided for each project, as it is started in different years. The capital disbursements and cash revenues, along with other transactions such as loan, interest repayment or receiving, and cash flow transferal are all assumed to occur at the end of each period. The life of a project is measured by its service life, and the planning horizon is thus defined as the longest project life plus the maximum project postponement permissible. In other words, cash flows are assumed to be reinvested at the firm's cost of capital for short-lived projects.

3.2 Model Development

In this section, Weingartner's 0-1 integer programming model is first given as a starting point, and is then modified to meet the model specifications and research objectives mentioned previously.

Maximize

$$\text{NPV} = \sum_{j=1}^{m} \sum_{t=0}^{H} y_{tj}(1 + i)^{-t} x_j$$

Subject to

$$\sum_{j=1}^{m} c_{tj} x_j \leq b_t \qquad t=0...H$$

$$x_j = 0,1 \qquad j=1...m$$

where y_{tj} = net cash flow in period t with project j

i = given marginal investment rate of the firm

c_{tj} = capital required in period t for project j

b_t = available budget in period t

x_j = decision variable for project j (1 = accept, 0 = reject)

m = the number of projects under consideration

H = the planning horizon.

Modifications to Weingartner Model are presented in the following sections.

3.2.1. Use of FV instead of NPV

The objective function (Section 3.2) is first revised, since NPV is not an adequate measurement criterion when borrowing and lending rates differ. Using the

FV measurement, as was discussed in Section 2.2.2, the objective function is re-written as follows:

$$FV = \sum_{j=1}^{m} \sum_{t=0}^{n_j - 1} [y_{tj} \prod_{s=t}^{n_j - 1} (1 + i_{xs})] \prod_{s_1 = n_j}^{H-1} (1 + i_{xs_1}) x_j$$

Since y_{tj}, and i_{xt} are given, the above function is calculable.

3.2.2 Project Deferral

Next, to reflect the second decision dimension, timing, the decision variable x_j is redefined as

$$x_{ij} = \begin{cases} 1, & \text{if project j is started from year i} \\ 0, & \text{otherwise} \qquad i=0...k \quad j=1...m \end{cases}$$

where k is maximum delay permissible.

Since each project can be executed at most once during the planning horizon, this definition automatically induces a mutually exclusive constraint

$$\sum_{i=0}^{k} x_{ij} \leq 1 \qquad j=1...m$$

Since one of the purposes of this research is to clarify the budget decision process, the budget amount in each year t, b_t , is no longer a constant. It is re-placed here by a new term I_t, representing the disposable funds of the firm in each year t, that is,

$$\sum_{j=1}^{m} \sum_{i=0}^{k} c_{tij} x_{ij} = b_t \leq I_t \qquad t=0...H$$

Although I_t is a series of *a priori* numbers, it is still unknown what portion of the fund should be used to support projects. From the department's point of view, b_t will be the best if it equals I_t. However, from the firm's point of view, substituting I_t into the model is likely to result in a distorted solution. Therefore more revisions are expected; however, this problem still maintains an integer programming structure.

3.2.3 Cash Flow Transferals

The purpose of transferring cash flows is to increase the available funds for projects. The basic model developed here assumes that funds are retained in the production department only for the current period. Therefore the cash amount of the firm at the end of each period will be

$$y_{tp} = \sum_{j=1}^{m} \sum_{i=0}^{k} a_{tij} x_{ij} - \sum_{j=1}^{m} \sum_{i=0}^{k} c_{tij} x_{ij} + I_t \qquad t=0...H$$

where y_{tp} = net cash flow of the firm in period t

a_{tij} = positive net cash flow of project j, i year(s) delayed, in year t

c_{tij} = negative net cash flow of project j, i year(s) delayed, in year t.

Several characteristics thus are noted in this expression as follows:

1. If $a_{tij}x_{ij} < c_{tij}x_{ij}$, it means that the amount y_{tp} has been taken from I_t.
2. On the other hand, if $a_{tij}x_{ij} \geq c_{tij}x_{ij}$, then the positive cash flows generated from some projects can support all other projects requesting funds.
3. y_{tp}, the firm's cash position at the end of each period, can be interpreted as the net cash flow of a portfolio, which comprises j projects and a "sinking fund project" that has cash flows ranging from 0 to I_t.
4. All y_{tp} are greater than zero. Therefore the objective function can be

rewritten as

Maximize

$$FV = \sum_{t=0}^{H} [y_{tp} \prod_{s=t}^{H-1} (1 + i_{ts})]$$

By introducing the cash flow transferal strategy, capital funds available for the department in each year t can be increased by a maximum amount of $a_{tij}x_{ij}$. Therefore the resource constraint can be expanded as the following :

$$\sum_{j=1}^{m} \sum_{i=0}^{k} c_{tij}x_{ij} \leq I_t + \sum_{j=1}^{m} \sum_{i=0}^{k} a_{tij}x_{ij} \qquad t=0...H$$

During the whole formulation process up to this stage, no other variables have been introduced or redefined. There is no need to investigate the detailed sources and destinations of the transferred cash flows, since only aggregate values are important. For example, it makes no difference whether project 1 receives funds totally from project 2, or one third of them from project 2 and the rest from project 3.

3.2.4 Financing and Investment Opportunities

Unless the firm is not involved in financial activities, the optimal solution obtained so far does not guarantee that the firm's wealth is also maximized. Material presented below integrates the financial considerations into the project selection process.

The formulation in this stage starts with the "independent cost of capital" assumption (the curve up to B_{crit} in Figure 2). This assumption will then be re-

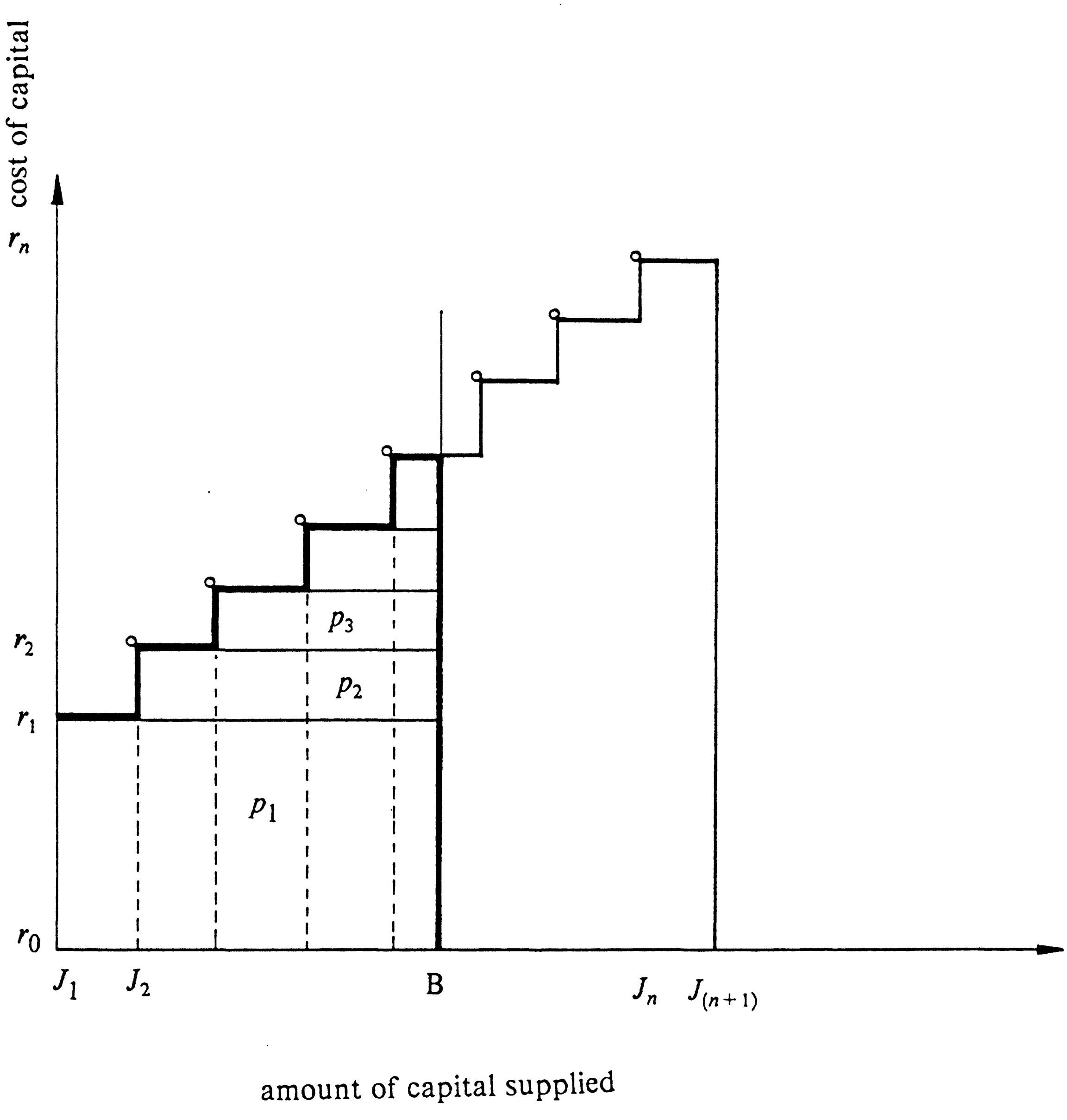

Figure 4. N-Step Function Capital Supply Curve

laxed by extending the curve to $B_{\max}$. An n-step function is used to approximate the segmental curve B_{crit} to $B_{\max}$ as shown in Figure 4. Define

B_t = the amount of funds borrowed at year t

L_t = the amount of funds lent at year t.

In the first period, the firm is offered a borrowing or lending opportunity. By borrowing, the firm's funds are increased by the amount B_0 ; lending, it needs to withdraw L_0 from its cash bank. In the subsequent period t, the firm receives a payment of $i_{l(t-1)}L_{t-1}$, if it engaged in lending in the previous period. Or it has to repay a debt of $i_{b(t-1)}B_{t-1}$, when the borrowing function was utilized in the last period. The funds available for the projects in each period t now become:

$$\sum_{j=1}^{m}\sum_{i=0}^{k} c_{tij}x_{ij} \le I_t + \sum_{j=1}^{m}\sum_{i=0}^{k} a_{tij}x_{ij} + B_t + i_{l(t-1)}L_{t-1} - i_{b(t-1)}B_{t-1} - L_t \qquad t = 0 \ldots H$$

Rearranging the terms the expression becomes:

$$-\sum_{j=1}^{m}\sum_{i=0}^{k} c_{tij}x_{ij} + \sum_{j=1}^{m}\sum_{i=0}^{k} a_{tij}x_{ij} + B_t + i_{l(t-1)}L_{t-1} - i_{b(t-1)}B_{t-1} - L_t + I_t \ge 0 \qquad (1)$$

It can be interpreted as the firm's "cash balance constraint" in which the cash status of a firm is prohibited from being negative. Therefore the capital budget constraint of a department and the cash balance constraint of a firm are actually two sides of the same coin. The above formulation, however, seems to conflict with the previous specification of the borrowing and lending opportunities (borrowing rate > lending rate) in which the mutual exclusiveness (in the same year) of borrowing and lending actions is implied. But there is actually no inconsistency as can be observed in the following objective function:

Maximize

$$\sum_{j=1}^{m}\sum_{i=0}^{k} a_{Hij}x_{ij} - \sum_{j=1}^{m}\sum_{i=0}^{k} c_{Hij}x_{ij} + i_{l(H-1)}L_{H-1} - i_{b(H-1)}B_{H-1} - B_H(1 + i_H) + I_H \qquad (2)$$

Since B_{H-1} and B_H are associated with negative terms, the firm should always avoid borrowing, if possible. If borrowing is inevitable, and an intention is also made to lend in H-1, the firm must borrow an extra amount, δ_{H-1}, and thus ends up with a net benefit associated with previous borrowing and lending as:

$$\delta_{H-1}(i_{l(H-1)} - i_{b(H-1)})$$

In the situation where the borrowing rate is always greater than the lending rate, concurrent borrowing and lending are not advisable. However, if $i_{b(H-1)}$ is smaller than $i_{l(H-1)}$, where $i_{l(H-1)}$ indicates a promising investment opportunity opened to the firm, using borrowed funds to invest is preferred. The same reasoning can be extended to each period t. Therefore the model developed so far automatically decides the use of concurrent or non-concurrent borrowing and lending depending on the interest rates given.

The above objective function is derived from Equation (1). In discussing the cash flow transferal case, it has been pointed out that the net cash flow of the portfolio of projects undertaken in each year, y_{tp}, is always greater than zero. If the amount is negative, the corresponding portfolio cannot be a feasible solution. This fact holds when borrowing and lending opportunities are included, as reflected by Equation (1). Moreover, if there are surplus funds at the end of each period, the lending function is always conducted; therefore the real cash the firm possesses at the end of each period t, $(t < H)$ is actually zero. The same reasoning holds if the firm carries debts, which will be paid back at the end of each period. The firm's wealth thus is accumulated by the lending or borrowing function up to the planning horizon. The future value criterion, expressed by Equation (2), thus is attractive in that one need only look at the cash status of a firm at the planning horizon to judge its wealth, regardless of its previous cash flow values.

An upper bound constraint can be added into the model to reflect the limited financing capability of the firm. An inequality equation suits this constraint:

$$B_t \leq w_t$$

The model formulated so far has switched the concerns of the project undertaking from a departmental level to a firm level. The structure of the model is slightly changed from integer programming to mixed integer programming. The revision following is the allowance of varying cost of capital with reference to capital borrowed.

3.2.5 Increasing Cost of Capital

The imperfect capital market in this paper is described by an n-step function as shown in Figure 4. The extra cost incurred, due to borrowing exceeding B_{crit}, follows these conditions:

$$p_s = \begin{cases} 0, & \text{if } B \leq J_s \\ (r_s - r_{(s-1)})(B - J_s) & \text{if } J_s < B \end{cases} \qquad s = 1 \ldots n \qquad (3)$$

where $s =$ index indicating the starting of the sth step interval

p_s = the incremental borrowing cost in each step interval s

$B =$ the borrowing amount, $0 \leq B \leq w_t$

$J_s =$ the jumping point at which an extra borrowing rate is about to occur

r_s = the increased borrowing rates associated with each step interval s

Also

$J_1 = B_{crit}$

$J_{(n+1)} = w_t$

r_0 = original borrowing rate.

Referring to Figure 4, the total extra borrowing cost is actually the area under the bold lines and can be expressed as:

$$\sum_{i=1}^{s-1}(J_{(i+1)} - J_i)r_i + (B - J_s)r_s$$

This expression is conceptually correct; however, it is not convenient for the use of the formulation purposes. Using this equation, three conditions have to be judged when B_x and J_s are compared:

$$B \leq J_s$$

$$J_s < B \leq J_{(s+1)}$$

$$B > J_{(s+1)}$$

A substitute can be found by partitioning Figure 4 from a horizontal direction. It is thus found that the total extra borrowing cost is equivalent to the sum of areas p_1, p_2, and so on. The value of p_i is actually defined by Equation (3); therefore the total extra borrowing cost can be rewritten as:

$$\sum_{i=1}^{s} p_i = \sum_{i=1}^{s}(r_i - r_{i-1})(B - J_i)$$

This equation, however, still cannot be directly substituted into the model since B is an unknown variable and s is an ex post product of B. The conditional situation inherent in Equation (3) can be eliminated by the following constraints:

$$(r_s - r_{s-1})(J_s - B) + p_s \geq 0$$

$$p_s \geq 0$$

The proof follows:

1. if $J_s \geq B$ then p_s ranges from $(0,\infty)$;
2. since p_s represents the penalty, it should be minimized; therefore $p_s = 0$;
3. if $J_s < B_x$ then p_s ranges from $(B - J_s,\infty)$;
4. for the same minimization reason, p_s is taken as $B - J_s$;
5. therefore these constraints form conform with Equation (3) defined earlier and thus solve the conditional problem . #

The determination of the *s*th interval can be overcome by extending the above constraints into the whole n step intervals. Examining the above proof item one, one can find that if the borrowing amount is smaller than J_s, all p_i's $(i > s)$ will be set to zero, thus the penalty will be accumulated to the *s*th step interval automatically.

The above discussion has dropped the time script, t, in each variable simply for the purposes for making the notations briefer. There is, however, no restriction on extending the above result into multi-period and no need to hold the borrowing conditions identical in each period.

3.2.6 Summary

In light of the above discussion, the model developed can be summarized as the following:

Maximize

$$\sum_{j=1}^{m}\sum_{i=0}^{k} a_{Hij}x_{ij} - \sum_{j=1}^{m}\sum_{i=0}^{k} c_{Hij}x_{Hj} + (1+ i_{l(H-1)})L_{(H-1)} - (1+i_{b(H-1)})B_{H-1} - \sum_{s=1}^{n} p_{(H-1)s} - (1+i_{bH})B_H + I_H$$

Subject to

cash balance/resource constraint

$$\sum_{j=1}^{m}\sum_{i=0}^{k} c_{tij}x_{ij} - \sum_{j=1}^{m}\sum_{i=0}^{k} a_{tij}x_{ij} + L_t - B_t - (1+i_{l(t-1)})L_{(t-1)} + (1+i_{b(t-1)})B_{t-1} + \sum_{s=1}^{n} p_{(t-1)s} - I_t \le 0 \quad t=0...H$$

mutually exclusive constraint

$$\sum_{i=0}^{k} x_{ij} \le 1 \qquad j=1...m$$

borrowing upper bound constraint

$$B_t \le w_t \qquad t=0...H$$

conditional penalty constraint

$$(J_{st} - B_t) + p_{st} \ge 0$$

$$p_{st} \ge 0 \qquad s=1...n, \quad t=0...H$$

nonnegativity constraint

$$x_{ij} = 0 \text{ or } 1$$

$$L_t, B_t \ge 0.$$

Where $(1 + i_{(-1)})L_{-1}$, $(1 + i_{b(-1)})B_{-1}$, p_{-1} are zero. Other notations are defined as in the previous sections.

Chapter IV

MODEL DEMONSTRATION AND APPLICATION

The model developed in Chapter III enables the consideration of more realistic and complex problem settings than those handled by the traditional project selection context. The manipulations of this model are now illustrated by numerical examples. Also, several scenarios are represented by these examples to show the practicability of the model. A special operational dependency scenario is first created to show the richness of the mathematical formulation. To allow for adjustments in the f_1, and f_2 functions (Figure 3), multiple financing, and investment opportunities, both short-term and long-term, are studied. All the calculations are done by running the LINDO-MIP/370 package.

4.1 Manipulation of the Model

The relevant information, such as the project parameters and the financial status, is gathered and summarized in Table 3 and 4. A preliminary study shows that, without considering project deferral and other strategies, a project manager would like to have projects A, C, and D accepted, since they have positive future values \$100.62k, \$108.93k, and \$12.27k respectively. It is yet unknown how many budgets he will be assigned. Any early specifications of these figures will result in erroneous project selection. For example, if the project manager treats disposable funds I_t as the budgets, he will end up with a "do nothing" decision. Having considered the project postponement and cash flow transferal possibilities, the project manager should work with the finance people and together formulate their project selection/capital budgeting model, as shown in Table 5. The computer results given in Table 6 indicate the final acceptance should be projects A and C (both with one year delay). Other valuable information such as financing and lending amount can also be read from the printout. Thus by manipulating this model, both departments submit the following report:

> The production department implements project A and C in the next year. The finance department borrows \$380k, \$473k, and \$250.06k in years 1, 2 and 3, and lends \$200k, \$92.43k, and \$904.44k in years 0, 4 and 5 respectively. The capital budget should be assigned to the production department as needed, that is, \$600k, and \$20k in years 0 and 1 respectively. Cash generated from the projects undertaken can be retained and transferred in year 2 in the amount of \$80k. By these actions, the firm will have \$994.88k at the end of the sixth year.

This model is attractive in that one can set up the formulation directly, without any need of preliminary calculations. While the number of constraints may be huge, the objective function is only relevant to the data at the end of the planning horizon. In fact, it is the cash balance expression in the last period, year 6. The problem of large number of constraints can be solved by interfacing the software package with a computer program. This program should generate coefficients of the variables in the objective function and constraints, and automatically enter these inputs into the model. However, such programs are commercially available; therefore no attempt is made to write this kind of program. In summary, this model handles the functions of project selection, budget determination, funds acquisition, investment specification, and cash flows transmission.

4.2 Inclusion of Operational Dependency

The specific operational dependency to be studied is "priority". Priority is different from contingency. The former is basically a management decision; while the latter arises from physical characteristics of the projects. Following the above example, the priority specified here is "projects C and D have highest priority, and between them project C has higher priority than D." This relationship is described by the following equations:

$$x_{31} - x_{i1} \geq 0 \qquad i = 1, 2, 4$$

$$x_{41} - x_{j1} \geq 0 \qquad j = 1, 2$$

These inequalities thus are added into the previous formulation (Table 5) to obtain another set of solutions. Since the purpose of this discussion is to demonstrate the power of the mathematical programming approach, the output will not be provided here. The operational dependencies are numerous, and there is no

Table 3. Project Net Cash Flows for the Demonstration Example

net cash flows												
	(no delay)				(1 year delay)				(2 year delay)			
time t	A	B	C	D	A	B	C	D	A	B	C	D
0	-400	-300	-250	-500								
1	50	100	-100	50	-350	-350	-250	-450				
2	150	100	150	-50	80	100	-100	80	-380	-330	-270	-400
3	150	100	200	100	170	80	150	-50	60	95	-110	50
4	450	-50	350	900	150	120	250	120	140	100	120	-60
5					450	-50	350	900	140	110	210	80
6									460	-60	340	860

unit: \$1000

Table 4. Financial Data for the Demonstration Example

time t	disposable funds (I_t)	borrowing upper bound (w_t)
0	200	650
1	0	500
2	0	550
3	0	450
4	0	300
5	0	350
6	0	200

time	0	1	2	3	4	5	6
borrowing rate(%)	15	19	20	23	18	17	14
lending rate	10	15	15	18	13	10	

capital market	J_1	J_2	J_3	J_4	J_5
	300	400	500	600	700
extra borrowing rate: 1%					
unit: $1000					

Table 5. Project Selection/Financial Planning Model Formulation

Maximize
0.001 X01 + 0.001 X02 + 0.001 X03 + 0.001 X04 + 0.001 X11 + 0.001 X12 + 0.001 X13 + 0.001 X14 + 460 X21 - 60 X22 + 340 X23 + 860 X24 + 1.1 L5 - 1.17 B5 - P51 - P52 - P53 - P54 - 1.14 B6

Subject to

2) 400 X01 + 300 X02 + 250 X03 + 500 X04 + L0 - B0 <= 200

3) - 50 X01 - 100 X02 + 100 X03 - 50 X04 + 350 X11 + 350 X12 + 250 X13 + 450 X14 - 1.1 L0 + 1.15 B0 + L1 - B1 + P01 + P02 + P03 + P04 <= 0

4) - 150 X01 - 100 X02 - 150 X03 + 50 X04 - 80 X11 - 100 X12 + 100 X13 - 80 X14 + 380 X21 + 330 X22 + 270 X23 + 400 X24 - 1.15 L1 + 1.19 B1 + L2 - B2 + P11 + P12 + P13 + P14 <= 0

5) - 150 X01 - 100 X02 - 200 X03 - 100 X04 - 170 X11 - 80 X12 - 150 X13 + 50 X14 - 60 X21 - 95 X22 + 110 X23 - 50 X24 - 1.15 L2 + 1.2 B2 + L3 - B3 + P21 + P22 + P23 + P24 <= 0

6) - 450 X01 + 50 X02 - 350 X03 - 900 X04 - 150 X11 - 120 X12 - 250 X13 - 120 X14 - 140 X21 - 100 X22 - 120 X23 + 60 X24 - 1.18 L3 + 1.23 B3 + L4 - B4 + P31 + P32 + P33 + P34 <= 0

7) - 450 X11 + 50 X12 - 350 X13 - 900 X14 - 140 X21 - 110 X22 - 210 X23 - 80 X24 + L5 - B5 - 1.13 L4 + 1.18 B4 + P41 + P42 + P43 + P44 <= 0

8) - 460 X21 + 60 X22 - 340 X23 - 860 X24 - 1.1 L5 + 1.17 B5 + P51 + P52 + P53 + P54 - B6 + L6 <= 0

9) X01 + X11 + X21 <= 1
10) X02 + X12 + X22 <= 1
11) X03 + X13 + X23 <= 1
12) X04 + X14 + X24 <= 1
13) B0 <= 650
14) B1 <= 500
15) B2 <= 550
16) B3 <= 450
17) B4 <= 300
18) B5 <= 350
19) B6 <= 200
20) - 0.01 B0 + P01 >= - 3
21) - 0.01 B0 + P02 >= - 4
22) - 0.01 B0 + P03 >= - 5
23) - 0.01 B0 + P04 >= - 6
24) - 0.01 B1 + P11 >= - 3
25) - 0.01 B1 + P12 >= - 4

(continued)

26) - 0.01 B1 + P13 > = - 5
27) - 0.01 B1 + P14 > = - 6
28) - 0.01 B2 + P21 > = - 3
29) - 0.01 B2 + P22 > = - 4
30) - 0.01 B2 + P23 > = - 5
31) - 0.01 B2 + P24 > = - 6
32) - 0.01 B3 + P31 > = - 3
33) - 0.01 B3 + P32 > = - 4
34) - 0.01 B3 + P33 > = - 5
35) - 0.01 B3 + P34 > = - 6
36) - 0.01 B4 + P41 > = - 3
37) - 0.01 B4 + P42 > = - 4
38) - 0.01 B4 + P43 > = - 5
39) - 0.01 B4 + P44 > = - 6
40) - 0.01 B5 + P51 > = - 3
41) - 0.01 B5 + P52 > = - 4
42) - 0.01 B5 + P53 > = - 5
43) - 0.01 B5 + P54 > = - 6

Table 6. Model Output of the Demonstration Example

OBJECTIVE FUNCTION VALUE

1) 994.887939

VARIABLE	VALUE	REDUCED COST
X01	0.000000	0.000000
X02	0.000000	237.541077
X03	0.000000	-2.269234
X04	0.000000	-59.184006
X11	1.000000	-111.641663
X12	0.000000	380.404053
X13	1.000000	0.000000
X14	0.000000	-204.706467
X21	0.000000	24.577240
X22	0.000000	284.985352
X23	0.000000	130.620758
X24	0.000000	-203.768219
L5	904.442383	0.000000
B5	0.000000	0.070000
P51	0.000000	1.000000
P52	0.000000	1.000000
P53	0.000000	1.000000
P54	0.000000	1.000000
B6	0.000000	1.139999
L0	200.000000	0.000000
B0	0.000000	0.111913
L1	0.000000	0.093262
B1	380.000000	0.000000
P01	0.000000	2.238286
P02	0.000000	2.238286
P03	0.000000	2.238286
P04	0.000000	2.238286
L2	0.000000	0.107024
B2	472.999512	0.000000
P11	0.800000	0.000000
P12	0.000000	1.865241
P13	0.000000	1.865241
P14	0.000000	1.865241
L3	0.000000	0.062150
B3	250.059433	0.000000

(continued)

P21	1.729995	0.000000
P22	0.729995	0.000000
P23	0.000000	1.528887
P24	0.000000	1.528887
L4	92.427002	0.000000
B4	0.000000	0.055000
P31	0.000000	1.242998
P32	0.000000	1.242998
P33	0.000000	1.242998
P34	0.000000	1.242998
P41	0.000000	1.099999
P42	0.000000	1.099999
P43	0.000000	1.099999
P44	0.000000	1.099999
L6	994.885986	0.000000

NO. ITERATIONS = 486
BRANCHES = 19 DETERM. = 1.000E 0
BOUND ON OPTIMUM: 996.3003
ENUMERATION COMPLETE. BRANCHES = 19 PIVOTS = 512

LAST INTEGER SOLUTION IS THE BEST FOUND

guarantee that all of them can be expressed by a simple linear equation or equations. Nonetheless, a rule of thumb has been found in locating the mathematical expression and is summarized in Appendix B at the end of this paper.

4.3 Other Realistic Considerations

The model developed and illustrated so far is still in its fundamental stage. It simply states the possibility of integrating the project selection (with delay permissible) with a financial decision process, in which the external financial environment is represented by a series of one-year borrowing and lending opportunities. It is not uncommon, however, for a company to be offered other financing, and investment opportunities, either short-term or long-term. In adapting to this more realistic situation, the above lending opportunity is extended by a long-term investment (bond purchasing), and a one year mixed-investment; similar extension is made to borrowing where the long-term case is for bond-issuing. The company is required to purchase/issue bonds in year 0, if it wants to, and keep the purchased bond until its given maturity. There is no limitation on the amount of issuing bonds. The short-term mixed-investment or financing is a series of yearly return rates which are the best choices in the corresponding year as in Table 7. This mixing is possible since the deterministic assumption is made. Extra terms IL_t and IB_t are introduced into the fundamental model such that

investment amount in year t $= IL_t + L_t$

financing amount in year t $= IB_t + B_t$

where IL is the amount of investment in bond

IB is the amount of borrowing by issuing bond

L, B are interpreted as the mixed investment or financing respectively;

$IL_t, IB_t = 0$, if $t > 0$.

The objective function and the first eight constraints in Table 5 are rewritten as:

Maximize

460 X21 - 60 X22 + 340 X23 + 860 X24 + 1.1 L5 - 1.17 B5 - P51 - P52 - P53 - P54 - 1.14 B6

Subject to

2) 400 X01 + 300 X02 + 250 X03 + 500 X04 + L0 - B0 + IL0 - IB0 < = 200

3) - 50 X01 - 100 X02 + 100 X03 - 50 X04 + 350 X11 + 350 X12
+ 250 X13 + 450 X14 - 1.1 L0 + 1.15 B0 + L1 - B1 + P01
+ P02 + P03 + P04 - 0.20 IL0 + 0.20 IB0 < = 0

4) - 150 X01 - 100 X02 - 150 X03 + 50 X04 - 80 X11 - 100 X12
+ 100 X13 - 80 X14 + 380 X21 + 330 X22 + 270 X23 + 400 X24
- 1.25 L1 + 1.19 B1 + L2 - B2 + P11 + P12 + P13 + P14
-0.20 IL0 + 0.20 IB0 < = 0

5) - 150 X01 - 100 X02 - 200 X03 - 100 X04 - 170 X11 - 80 X12
- 150 X13 + 50 X14 - 60 X21 - 95 X22 + 110 X23 - 50 X24
- 1.25 L2 + 1.2 B2 + L3 - B3 + P21 + P22 + P23 + P24
-1.20 IL0 + 1.20 IB0 < = 0

6) - 450 X01 + 50 X02 - 350 X03 - 900 X04 - 150 X11 - 120 X12

$- 250\ X13 - 120\ X14 - 140\ X21 - 100\ X22 - 120\ X23 + 60\ X24$

$- 1.25\ L3 + 1.23\ B3 + L4 - B4 + P31 + P32 + P33 + P34$

$<= 0$

7) $- 450\ X11 + 50\ X12 - 350\ X13 - 900\ X14 - 140\ X21 - 110\ X22$

$- 210\ X23 - 80\ X24 + L5 - B5 - 1.13\ L4 + 1.18\ B4 + P41$

$+ P42 + P43 + P44 <= 0$

8) $- 460\ X21 + 60\ X22 - 340\ X23 - 860\ X24 - 1.1\ L5 + 1.17\ B5$

$+ P51 + P52 + P53 + P54 - B6 + L6 <= 0$

Running this model, one can obtain the results as in Table 8. The results show that projects 1 and 3 are still started from year one. No extra projects are selected, although funds can be accrued from more financing tools. In year zero, $200k is invested, however, $89k of that is diversified into the purchase of a four year maturity bond. In year one, because the investment return is higher than the borrowing rate, $40.22k is borrowed for investment. Consequently, it has to borrow to its full extent, $550k, in year two to pay back its debt. Year three is again a booming year, so that the company wants to borrow to invest. From year four on, the investment opportunities dim so that the company simply carries the surplus funds in the mixed investment rate and ends up with a terminal value of $1015.92k. This increase of $21k (compared to the original case, Table 5) is the result of a long-term investment and high intermediate short-term return rates. The results also imply that other project possibilities are not promising enough to justify the losses due to long-term financing and opportunity cost of short-term investment.

Including these new conditions is not difficult; the essence is that this model can really be extended to more realistic situations, for instance, the above example eliminates the possibilities of issuing or purchasing bonds at times other than the initial year. This restriction can be relaxed following the logic of project deferral formulation mentioned before. While the above realistic considerations focus on the f_1 and f_2 functions (Figure 3), the fundamental model can also be extended to considerations when the f_3 function is varied. For instance, if liquidity requirements are required to be at least $5k each year, they can be added to the left hand side of the cash balance constraint, and thus obtain another set of solutions. Since the liquidity requirements and dividend policies are usually dependent on other decision systems in the company, no examples will be given to show this determination process. However, conceptually, it can be arrived at by substituting different sets of dividend and liquidity numbers into the fundamental model and combining its objective value with that of other decision systems. Therefore the fundamental model can be a decision support system to other functions of the firm.

Table 7. Short-term and Long-term Financing and Investment Data

year	0	1	2	3	4	5
mixed borrowing rate(%)	15	19	20	23	18	17
mixed investment rate	10	25	25	25	13	10
bond investment yield (%)	20	20	20			
bond issuing yield	20	20	20			

Table 8. Model Output of the Multiple Financing and Investment Example

OBJECTIVE FUNCTION VALUE

1) 1015.92212

VARIABLE	VALUE	REDUCED COST
X01	0.000000	0.000000
X02	0.000000	284.711914
X03	0.000000	-98.288223
X04	0.000000	86.503250
X11	1.000000	-112.861282
X12	0.000000	461.828125
X13	1.000000	-92.331665
X14	0.000000	-88.911774
X21	0.000000	-3.165894
X22	0.000000	332.265869
X23	0.000000	0.000000
X24	0.000000	-144.838821
L5	923.564209	0.000000
B5	0.000000	0.070000
P51	0.000000	1.000000
P52	0.000000	1.000000
P53	0.000000	1.000000
P54	0.000000	1.000000
B6	0.000000	1.139999
L0	111.350739	0.000000
B0	0.000000	0.125977
L1	40.215591	0.000000
B1	500.000000	0.000000
P01	0.000000	2.519588
P02	0.000000	2.519588
P03	0.000000	2.519588
P04	0.000000	2.519588
L2	0.000000	0.073491
B2	550.000000	0.000000
P11	1.999998	0.000000
P12	0.999999	0.000000
P13	0.000000	2.015672
P14	0.000000	2.015672
L3	211.879196	0.000000
B3	450.000000	0.000000
P21	2.499998	0.000000
P22	1.499998	0.000000

(continued)

P23	0.499999	0.000000
P24	0.000000	1.553746
L4	109.349014	0.000000
B4	0.000000	0.055000
P31	1.499999	0.000000
P32	0.499999	0.000000
P33	0.000000	1.242998
P34	0.000000	1.242998
P41	0.000000	1.099999
P42	0.000000	1.099999
P43	0.000000	1.099999
P44	0.000000	1.099999
L6	1015.920170	0.000000
IL0	88.649261	0.000000
IB0	0.000000	0.000000

NO. ITERATIONS = 217
BRANCHES = 13 DETERM. = 0.925E 0
BOUND ON OPTIMUM: 1059.012
ENUMERATION COMPLETE. BRANCHES = 13 PIVOTS = 242

LAST INTEGER SOLUTION IS THE BEST FOUND

Chapter V

CONCLUSIONS AND RECOMMENDATIONS

The conclusions of the study are presented in this chapter. An overview of the research is first presented to facilitate the discussion of subsequent conclusions. To give an overall perspective of the project analysis problem, several areas are suggested for future research.

5.1 Summary

To increase the firm's wealth, the traditional project selection problem has been concentrated on a firm level by integrating financial planning decisions. At the departmental level, the project manger is also allowed to shift the starting time of a project to use capital funds efficiently. The capital market the firm is facing is imperfect, in that the cost of capital is an n-step function of the capital supplied. The research objective is to find an easily understandable and implementable model that determines the optimal project selection and financial policy, simultaneously and efficiently.

The project deferral consideration is solved by (1) augmenting the implementation variable, x_j to the second dimension, x_{ij}, and (2) including a mutually exclusive relationship. The cash flow transferal consideration synchronizes the capital resource and cash balance constraint, and makes it more convenient the measurement of the firm's wealth by "portfolio", using future value criterion. The financing, and investment considerations are directly solvable by introducing two sets of variables B_t, and L_t for each year t. The adjustment of extra borrowing cost, due to the "step-behaved capital market", is accomplished by (1) adding new variables p_{st} for each step s and time t, and (2) establishing two constraints: "penalty condition" and nonnegativity for p_{st}. The model is maintained in a linear structure and can be solved by the mixed integer programming technique.

Extensions of this model fall in these categories: (1) inclusion of operational dependencies, (2) inclusion of other borrowing and investment opportunities, both long-term and short-term. All the above applications are demonstrated by numerical examples.

5.2 Conclusions

The project selection problem is constrained by two factors: (1) the borrowing upper bound, w_t, of the firm and (2) the budget assignment to the production department, b_t. This research argues that b_t should not be predetermined at any rate, as indicated in Chapter IV. Therefore, it is suggested that (1) the project selection process be integrated with the financial decision process, and consequently (2) budget determinations make no differences between project selections. These conclude the nature of the project selection problem.

A mixed integer programming model is formulated to reach the research objective reviewed above. Although the formulation is confined to the model assumptions, the settings specified in this model are considered to be realistic. The model is easily used because one can simply read relevant data to formulate the model. Part of this ease of use should be attributed to the future value portfolio measurement criterion, which enables the objective function to be formulated on the basis of the horizon period. Using two separate variables B_t and L_t to denote borrowing and lending amount in each year t also simplifies the modeling work. Had one variable been used, that is, S_t such that

$$\text{debt or return in } t+1 = s_t(1 + i_{xt})$$

where $i_{xt} = i_{bt}$, if $s_t > 0$

$i_{xt} = i_{lt}$, if $s_t \geq 0$

$S_t =$ borrowing or lending depending on its sign.

the model would become complicated, since extra constraints and recursive calculations are required to justify the above conditions. The simultaneousness and efficiency considerations are achievable due to the mathematical programming structure. Therefore, ease of use, reality, and efficiency are regarded as the features and contributions of this model.

Numerical examples presented in Chapter IV illustrate several extensions in applying this model. Although not comprehensive, those examples do indicate possibilities and directions in expanding the model. This capability again is attributed to the mathematical programming structure; therefore, although this method is not yet popular in industry [15, 45], this author believes the mathematical programming approach will be recognized as time goes on.

Finally, it should be noted that a model like this is hardly for direct use in the complex real world. As was suggested before, a computer code should be written to handle the large volume of data processing that is usually confronted in practice. Other possible extensions are suggested for future research as follows.

5.3 Recommendations

The project analysis problem, starting from the project formation to the project postaudit, still leaves much room for research. Figure 5 gives an overall picture of the project analysis problem. Many production type projects in the first block are technology-oriented. The technical considerations of a specific project are dependent on the researcher's background; therefore no future research will be recommended in relation to the current study. However, in many cases, this stage is recognized by most as critical in that it substantializes a "real" project.

In the second block of Figure 5, important research topics related to the current study are uncertainty considerations and risk analysis. The explicit considerations of uncertainty lead to the estimation of the relevant information that is originally assumed to be given in this study. Consequently, the independency assumption is prone to be relaxed and thus makes the maintenance of a linear structure more difficult. The estimated parameters could be micro-economic variables, for example, power supply life length and its maintenance cost, or macro-

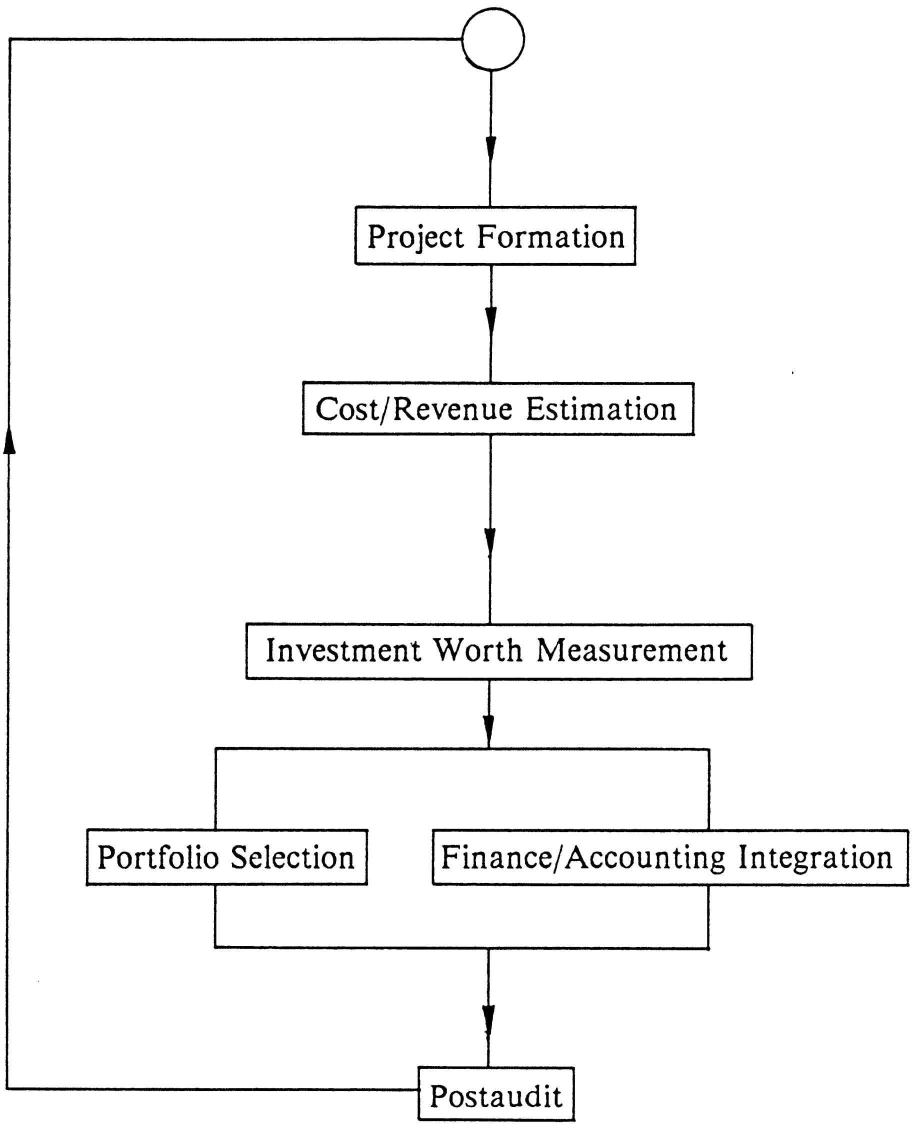

Figure 5. Overall Picture of the General Project Analysis Process

economic variables, for example, taxes and inflation rates. The estimated function could be continuous or discrete. This study also assumes equal risk, which, in an uncertain environment is not a valid assumption; therefore the study on the risk issue is necessary.

The measurement criteria of investment worth also have effects on the objective function and constraints as were discussed earlier. Although this research adopts the future value of cash flows of actions undertaken as the value of the project and the firm, controversies between the modern financial theory should be studied. Multiple-objective function is also an interesting subject that incurs attention in many applications; thus it is suggested for future research.

The fourth block, integration of portfolio evaluation and financial planning, is the focus of this study. Although an exhaustive and comprehensive effort has been spent on this area, there are many more factors to be considered, for example, the flotation costs consideration [19], equity-debt ratio consideration [37], etc. Including major financial decisions expands the original project selection model to an overall financial planning model; it is thus an attractive field to be explored.

Another assumption in this study is the "no-alteration" assumption. Postaudit work, however, is usually required under the conditions of uncertainty. At this stage, a project has to be reviewed and updated constantly. Since this task is significant in application, postaudit is also suggested for future research.

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Appendix A

Model Verification

The model developed in this chapter is verified by enumerating a two-projects-one-year-delay example. Financial information and project parameters are provided in Table 9. The complete model formulation is listed below in its primal form. Table 10 shows the result of enumeration. Table 11 presents the computer output obtained from running the LINDO package. As one can observe from Table 10 and 11, both approaches arrive at the same project selection; that is, project one is delayed for one year and project two starts from the current year. This is the optimal combination with which the firm has a maximum future value of $323.22 million at the end of its planning horizon (the fourth year). With this comparison, and the fact that no logical error is found in the model development process, this author tends to conclude that the model developed is a valid one, at least under its own environment.

In the objective function below, variables x_{01},x_{02}, and x_{12} have the same small coefficients 0.001. They are dummy coefficients because the LINDO package requires that all integer variables appear first in the model.

Maximize

0.001 X01 + 0.001 X02 + 150 X11 + 0.001 X12 + 1.1 L3 - 1.5 B3 - P31 - P32

Subject to

2) 100 X01 + 300 X02 + L0 - B0 <= 100

3) 100 X01 - 200 X02 + 50 X11 + 300 X12 - 1.1 L0 + 1.15 B0 + L1 - B1 + P01 + P02 <= 0

4) - 150 X01 - 250 X02 + 100 X11 - 50 X12 - 1.1 L1 + 1.15 B1 + L2 - B2 + P11 + P12 <= 0

5) - 200 X01 - 100 X11 - 300 X12 + L3 - B3 - 1.1 L2 + 1.15 B2 + P21 + P22 <= 0

6) - 150 X11 - 1.1 L3 + 1.15 B3 + P31 + P32 + L4 - B4 <= 0

7) B0 <= 250

8) B1 <= 200

9) B2 <= 300

10) B3 <= 200

11) B4 <= 100

12) X01 + X11 <= 1

13) X02 + X12 <= 1

14) - 0.05 B0 + P01 >= - 5

15) - 0.05 B0 + P02 >= - 10

16) - 0.05 B1 + P11 >= - 5

17) - 0.05 B1 + P12 >= - 10

18) - 0.05 B2 + P21 >= - 5

19) - 0.05 B2 + P22 >= - 10

20) - 0.05 B3 + P31 >= - 5

21) - 0.05 B3 + P32 >= - 10

Table 9. Project Parameters and Financial Information for the Verification Example

	(no delay)		(1 year delay)		funds available	borrowing upper bound
year	1	2	1	2		
0	-100	-300			100	250
1	-100	200	- 50	-300	0	200
2	150	250	-100	- 50	0	300
3	200		100	300	0	200
4			150		0	100

lending rate = 10 %

borrowing rate	borrowing amount, B_x
15 %	$B_x \leq 100$
20 %	$100 < B_x \leq 200$
25 %	$200 < B_x \leq 200$

unit: $ million

Table 10. Enumeration Result for Verification Example.

combinations	results
(-,-)	146.41
(-,0)	253.80
(0,-)	262.35
(0,0)	infeasible
(0,1)	infeasible
(1,-)	216.99
(1,0)*	323.22
(1,1)	infeasible
unit: $ million	

Table 11. Model Output of Verification Example

OBJECTIVE FUNCTION VALUE

1) 323.223877

VARIABLE	VALUE	REDUCED COST
X01	0.000000	-18.989014
X02	1.000000	-58.989761
X11	1.000000	0.000000
X12	0.000000	26.947830
L3	157.475433	0.000000
B3	0.000000	0.400001
P31	0.000000	1.000000
P32	0.000000	1.000000
L0	0.000000	0.208722
B0	200.000000	0.000000
L1	0.000000	0.060499
B1	84.999725	0.000000
P01	4.999999	0.000000
P02	-0.000001	0.000000
L2	52.250412	0.000000
B2	0.000000	0.054999
P11	0.000000	1.209998
P12	0.000000	1.209998
P21	0.000000	1.099999
P22	0.000000	1.099999
L4	0.000000	0.000000
B4	0.000000	0.000000

NO. ITERATIONS = 9
BRANCHES = 1 DETERM. = 1.000E 0
BOUND ON OPTIMUM: 324.1311
FLIP X01 TO 1 WITH BOUND 324.1311
DELETE X01 AT LEVEL 1
ENUMERATION COMPLETE. BRANCHES = 1 PIVOTS = 17

LAST INTEGER SOLUTION IS THE BEST FOUND

Appendix B
Operational Dependency Mathematical Specification -- Rule of Thumb

For any two projects (decision variables x, y) with some kind of operational dependency, the following procedures can be followed to locate a proper mathematical description, if it is not straightforward:

1. Construct a binary table (4×2).
2. Mark the infeasible combination(s).
3. Perform addition and subtraction in each row of the table.
4. Find out the minimum and maximum from the feasible results in 3.
5. Test expression by "$x+y$" $\geq$ ($>$) min.
6. If infeasible row(s) satisfy this expression then test "$x+y$" $\leq$ ($<$)max.
7. Stop until infeasible row(s) do not satisfy, correct expression found.

Example (y is contingent on x):

binary table

x	y	(x+y)	(x-y)	
1	0	1	1(max.)	
1	1	2(max.)	0	
0	0	0(min.)	0(min.)	
0	1	1	-1	* infeasible

test 1. $x + y \geq 0$, row 4 satisfied;

2. $x + y \leq 2$, row 4 satisfied;

3. $x - y \geq 0$, only row 4 not satisfied, stop.

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