TIP LEAKAGE FLOW, HEAT TRANSFER AND BLADE LIFING IN A JET ENGINE TURBINE

by

Udey Chaudhry

Thesis submitted to the Faculty of Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Master of Science in

Mechanical Engineering

Approved:

J. Moore, Chairman

H. L. Moses A. Myklebust

November 1987
Blacksburg, Virginia
TIP LEAKAGE FLOW, HEAT TRANSFER AND BLADE LIFING IN A JET ENGINE TURBINE

by

Udey Chaudhry

J. Moore, Chairman
Mechanical Engineering

ABSTRACT

An existing Navier-Stokes code (MEFP) was used to calculate developing flow and heat transfer in turbine tip gaps. Successful calculations of the heat transfer to a model turbine blade tip were obtained with a Prandtl mixing length turbulence model. The calculations revealed details of the flow development including recirculation and reattachment on the blade tip surface. The calculated heat transfer distributions were in good agreement with experimental data. A combined solution of the energy equation in the tip gap flow and in the rotor blade tip gave tip temperature distributions.

An independent computational study, using the same numerics as MEFP but a separate new computer program, was also performed to investigate the numerical accuracy of heat transfer calculations for fully developed flow.

A literature survey of gas turbine blade materials and factors influencing turbine tip blade life was performed. Approximate temperature ranges for the significant blade life reduction mechanisms, hot corrosion, oxidation, and melting were determined. In the present calculations for typical jet engine conditions, a maximum tip temperature of 1488 K was predicted which would lead to high oxidation rates for present day turbine blade alloys.
ACKNOWLEDGMENTS

I would like to extend my sincere appreciation to Dr. John Moore and Mrs. Joan Moore for their guidance, insight, and constant support for this work. Once again, I thank Dr. Moore for the encouragement and the genuine concern he showed during his supervision of this thesis.

I wish to thank the members of my advisory committee, Dr. H. L. Moses and Dr. Arvid Myklebust, for all the advice and support.

The support of this project, from Rolls Royce plc, under a cooperative agreement with Virginia Tech, is gratefully acknowledged.

I would also like to thank my friends and fellow students - Kumud Ajmani, H. S. Oberoi, Sanjay Chawla, G. K. Maliwae, Chandra and Arun Veeraraghavan for their help and suggestions.

Finally, I would like to thank my parents for their support and encouragement at all stages.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SECTION I - Introduction</td>
<td>1</td>
</tr>
<tr>
<td>SECTION II - Literature Review</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Turbine Tip Heat Transfer</td>
<td>3</td>
</tr>
<tr>
<td>SECTION III - Moore Elliptic Flow Program (MEFP)</td>
<td>11</td>
</tr>
<tr>
<td>3.1 Turbulence Model</td>
<td>12</td>
</tr>
<tr>
<td>3.2 Effective Viscosity</td>
<td>12</td>
</tr>
<tr>
<td>SECTION IV - Discretization for 2-D Fully Developed Flow Calculations</td>
<td>14</td>
</tr>
<tr>
<td>4.1 Control Volumes</td>
<td>14</td>
</tr>
<tr>
<td>4.2 Continuity and Momentum Equations</td>
<td>16</td>
</tr>
<tr>
<td>4.3 Predicted Wall Shear Stress $\tau_w$</td>
<td>21</td>
</tr>
<tr>
<td>4.4 Momentum Equations for Near-Wall Grid Point Using Altered Control Volume</td>
<td>23</td>
</tr>
<tr>
<td>4.5 Fully Developed Laminar Flow</td>
<td>23</td>
</tr>
<tr>
<td>4.5.1 Grid Spacing</td>
<td>23</td>
</tr>
<tr>
<td>4.5.2 Results and Discussion</td>
<td>26</td>
</tr>
<tr>
<td>4.6 Fully Developed Turbulent Flow</td>
<td>29</td>
</tr>
<tr>
<td>4.6.1 Setting Values</td>
<td>29</td>
</tr>
<tr>
<td>4.6.2 Grid Spacing</td>
<td>31</td>
</tr>
<tr>
<td>4.6.3 Main Program</td>
<td>32</td>
</tr>
<tr>
<td>4.6.4 Normalization</td>
<td>33</td>
</tr>
<tr>
<td>4.6.5 Results and Conclusions</td>
<td>33</td>
</tr>
<tr>
<td>4.7 Energy Equation</td>
<td>36</td>
</tr>
<tr>
<td>4.7.1 Nusselt Number</td>
<td>41</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (continued)

4.8 Laminar Heat Transfer ........................................... 42
   4.8.1 Discretized Laminar Heat Transfer
         Equation Results ...................................... 42
4.9 Turbulent Heat Transfer ........................................ 43

SECTION V - Calculations for Developing Flow and Heat Transfer
in Tip Gaps .......................................................... 48

5.1 Gridding of Test Section ....................................... 48
5.2 Flow Calculations .............................................. 51
   5.2.1 Flow Results ........................................... 54
5.3 Heat Transfer Equations ....................................... 61
5.4 Calculated Temperatures ...................................... 63
5.5 Heat Transfer Results ........................................ 63
5.6 Discussion of Results ......................................... 69

SECTION VI - Gas Turbine Blade Materials ....................... 74

6.1 Conventionally Cost Alloys ................................... 74
6.2 Directional Materials ......................................... 77
6.3 Ceramics ......................................................... 78
6.4 Influence of Alloying Materials .............................. 78
   6.4.1 Influence of Chromium Additions ........................ 78
   6.4.2 Influence of Aluminum Additions ........................ 80
   6.4.3 Influence of Tantalum, Molybdenum or Tungsten
         Additions ................................................. 80
   6.4.4 Influence of Rare Earth Additions ...................... 80

SECTION VII - Blade Tip Life Considerations ..................... 81

7.1 High Temperature Oxidation ................................... 81
   7.1.1 Effect of Oxidation on Material ........................ 81
   7.1.2 Effect of Applied Stress ................................ 82
   7.1.3 Effect of High Velocity Environments .................. 84
TABLE OF CONTENTS (continued)

7.2 Hot Corrosion .................................................. 84
   7.2.1 Chronology of Attack .................................. 86
   7.2.2 Effect of Temperature on Hot Corrosion .............. 86

7.3 Erosion .......................................................... 88
   7.3.1 Effect of Particle Velocity ............................ 90
   7.3.2 Effect of Particle Impingement Angle
       (Direction or Angle of Attack) ....................... 90
   7.3.3 Effect of Sample Temperature ......................... 90
   7.3.4 Effect of Sample Material ............................ 93

SECTION VIII - Predictions of Turbine Tip Temperatures ....... 94

SECTION IX - Conclusions ......................................... 97
   9.1 Computational Study of Finite-Difference
       Gridding ....................................................... 97
   9.2 Ability to Calculate Tip Leakage Flow and
       Heat Transfer .............................................. 98
       9.2.1 Flow Results ......................................... 98
       9.2.2 Heat Transfer Results ............................... 98
   9.3 Rotor Blade Tip Life Considerations .................... 99

REFERENCES .......................................................... 101

APPENDIX I - Typical Turbine Inlet and Blade Metal
   Temperatures for Jet Engines ............................... 103

APPENDIX II - Near-Wall Velocity Overshoot with Regular
   Control Volume in an Accelerating Flow .................. 107

APPENDIX III - TDMA Subroutine ................................. 112

APPENDIX IV - Estimates of Fully Developed Heat Transfer for
   Metzger and Bunker Case ..................................... 114

APPENDIX V - Thermal Conductivity and Melting Temperatures
   of Ni-Cr Alloys ................................................. 117
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1</td>
<td>Metzger and Bunker test section with 70° chamfer at the inlet</td>
<td>4</td>
</tr>
<tr>
<td>Fig. 2</td>
<td>Correlation of local heat transfer to a model turbine blade tip near the entrance of a circular tube. Ratio of local Nusselt number to that with fully developed flow [Moore and Tilton]</td>
<td>5</td>
</tr>
<tr>
<td>Fig. 3</td>
<td>Pipe test section with right angled entrance [Boelter]</td>
<td>7</td>
</tr>
<tr>
<td>Fig. 4</td>
<td>Zone of a cooled turbine blade tip, prone to heat damage [Brindon]</td>
<td>8</td>
</tr>
<tr>
<td>Fig. 5</td>
<td>Different grid spacing, with two choices of control volumes</td>
<td>15</td>
</tr>
<tr>
<td>Fig. 6</td>
<td>A 2-D incompressible fully developed flow example</td>
<td>17</td>
</tr>
<tr>
<td>Fig. 7</td>
<td>The momentum control volume for the $i^{th}$ grid point extending from $I - \frac{1}{2}$ to $I + \frac{1}{2}$</td>
<td>19</td>
</tr>
<tr>
<td>Fig. 8</td>
<td>Linear shear stress distribution for fully developed flow</td>
<td>22</td>
</tr>
<tr>
<td>Fig. 9</td>
<td>Regular and altered control volumes for the grid point near the top wall</td>
<td>24</td>
</tr>
<tr>
<td>Fig. 10</td>
<td>Calculated to theoretical wall shear stress ratio vs. total number of grid points for fully developed laminar flow</td>
<td>28</td>
</tr>
<tr>
<td>Fig. 11</td>
<td>Calculated velocity profiles for fully developed laminar flow u vs. y</td>
<td>30</td>
</tr>
<tr>
<td>Fig. 12</td>
<td>Calculated to ideal wall shear stress ratio vs. total number of grid points with regular control volume for fully developed turbulent flow</td>
<td>34</td>
</tr>
<tr>
<td>Fig. 13</td>
<td>Calculated to ideal wall shear stress ratio vs. total number of grid points with altered control volume for fully developed turbulent flow</td>
<td>35</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS (continued)

Fig. 14  Calculated velocity profile for fully developed turbulent flow with regular control volume for non-uniformly spaced grids ........................................ 37
Fig. 15  Calculated velocity profile for fully developed turbulent flow with altered control volumes for non-uniformly spaced grids ........................................ 38
Fig. 16  Heat transfer ratio and shear ratio vs. grid points. Laminar flow, regular and altered control volumes ........... 44
Fig. 17  Heat transfer ratio as a function of total number of grid points in a fully developed turbulent flow ........... 45
Fig. 18  Variations of shear stress and heat flux for fully developed laminar flow ........................................ 47
Fig. 19  A typical turbine rotor blade test section, with 90° corner on the pressure side .................. 49
Fig. 20  The dimensions in terms of tip gap heights of the chosen 70° chamfer test section .................. 50
Fig. 21  The grid point locations for the chosen grid in the tip gap ........................................ 52
Fig. 22  The calculation grid and the blow up of the tip gap entrance corner ........................................ 53
Fig. 23  Velocity vectors for the 70° chamfer test section, from turbulent flow calculations .................. 55
Fig. 24  Velocity vectors for the 90° corner test section, from turbulent flow calculations .................. 56
Fig. 25  Coefficient of the calculated static pressure in the 70° chamfer test section .................. 58
Fig. 26  Coefficient of the calculated total pressure in the 70° corner test section .................. 59
Fig. 27  The velocity vectors and the calculated static pressures near the tip gap corner .................. 60
Fig. 28  Temperature contours from the solution of the energy equation for flow field and blade, contour interval $\Delta T = 5$ ........................ 64
LIST OF ILLUSTRATIONS (continued)

Fig. 29 Normalized Nusselt number on the blade tip as a function of distance, tip gap heights, for the cases 1 and 2 .......................... 66

Fig. 30 Normalized Nusselt number as a function of distance, tip gap heights, for case 1 with both residual and temperature difference method .......................... 67

Fig. 31 Normalized skin friction coefficient comparison with normalized Nusselt number for the case 2 .................. 68

Fig. 32 Normalized Nusselt number for the cases 1 and 2, on the blade tip and the pressure surface with residual method .......................... 70

Fig. 33 Comparison of normalized Nusselt number for the 70° and 90° corners with temperature difference method .......................... 71

Fig. 34 Development of turbine entry and material temperatures [Hennecke] .......................... 75

Fig. 35 Development of turbine blade materials [Wilson] .......................... 76

Fig. 36 Influence of chromium content on the oxidation resistance of nickel [13] .......................... 79

Fig. 37 Critical applied stress vs. temperature [14]. The critical applied stress required to produce accelerated oxidation as a function of temperature. .......................... 83

Fig. 38 Weight change produced due to oxidation in high-velocity environment [14]. Values in parentheses represent the metal loss (mil/side) after 1000 hr exposure .......................... 85

Fig. 39 Plot showing the progression from 'simple oxidation' to 'hot corrosion' for a high chromium Ni-base alloy [14] .......................... 87

Fig. 40 Effect of increased salt concentration in burner-rig tests on the threshold and terminal temperatures of hot corrosion [14] .......................... 89

Fig. 41 Erosion rate vs. sample temperature for Ti 6-4, particle velocity of 500 ft/sec and impingement angles of 25°, 45° and 90° [16] .......................... 91

Fig. 42 Erosion rate vs. sample temperature for INCO 718, for a particle velocity of 500 ft/sec and impingement angles of 25°, 45° and 90° .......................... 92
LIST OF ILLUSTRATIONS (continued)

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>Calculated blade tip temperature as a function of $T_{inlet} - T_{blade tip}$ base for the 70° and 90° corners</td>
<td>95</td>
</tr>
<tr>
<td>44</td>
<td>Blade tip corner temperatures achieved for the 70° and 90° test sections with typical jet engine turbine temperatures</td>
<td>96</td>
</tr>
<tr>
<td>45</td>
<td>Approximate temperature ranges for significant blade life reduction mechanisms</td>
<td>100</td>
</tr>
<tr>
<td>46</td>
<td>Typical turbine inlet and blade metal temperatures for jet engine turbine [Gell and Thomas]</td>
<td>105</td>
</tr>
<tr>
<td>47</td>
<td>A reaction turbine with a rotor velocity triangle</td>
<td>106</td>
</tr>
<tr>
<td>48</td>
<td>Near-wall velocity overshoot with regular control volume in an accelerating flow</td>
<td>108</td>
</tr>
<tr>
<td>49</td>
<td>Discretized altered and regular near-wall control volumes</td>
<td>111</td>
</tr>
<tr>
<td>50</td>
<td>Liquidus and solidus diagrams for Ni-Cr-Fe ternary alloys [20]; temperature in °C</td>
<td>118</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>--------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1.</td>
<td>Flow and Heat Transfer Calculations</td>
<td>72</td>
</tr>
</tbody>
</table>
NOMENCLATURE

A,B,C,D coefficients of velocities in the momentum equation

$c_p$ specific heat capacity

$C_{p_s}$ static pressure loss coefficient

$C_{p_t}$ total pressure loss coefficient

D hydraulic diameter of the channel

EM effective viscosity

$h$ heat transfer coefficient

$H$ tip gap height

$\bar{H}$ rothalpy

$H_R$ heat transfer ratio

$I_{\text{max}}$ total number of points in the grid

$k$ thermal conductivity

$L$ Prandtl mixing length

$m$ mass flow rate

$Nu_\infty$ Nusselt number (fully developed value)

$p$ static pressure

$Pr_l$ laminar Prandtl number

$Pr_t$ turbulent Prandtl number

$q$ heat transfer rate

$Re$ Reynolds number, $= \rho \bar{U} H/\mu$

$S_R$ shear ratio

$T$ static temperature

$T_i$ inlet temperature

$T_{\text{max}}$ peak temperature

$T_w$ wall temperature
NOMENCLATURE (continued)

\( u \)  relative velocity
\( \bar{u} \)  mean velocity
\( U_{\text{max}} \)  maximum velocity
\( W \)  pressure gradient
\( x \)  cartesian coordinate along the blade tip, Fig. 1
\( y \)  cartesian coordinate across the tip gap, Fig. 1
\( \gamma \)  ratio of specific heat capacities
\( \theta \)  dimensionless temperature
\( \mu \)  effective viscosity
\( \mu_l \)  laminar viscosity
\( \mu_t \)  turbulent viscosity
\( \rho \)  density
\( \tau \)  shear stress
\( \Omega \)  rotation rate
SECTION I

Introduction

The quest for lower fuel consumption, higher performance and greater efficiency, has led to jet engines with higher turbine inlet temperatures and cycle pressure ratios. Hence, turbine blade materials with significant improvements in strength and temperature capabilities are required. There has been substantial improvement in the thermal fatigue and creep properties of the materials over the last four decades, however, the same cannot be said of the temperature capabilities. It is therefore important to be able to predict turbine blade temperatures especially in such high temperature regions as unshrouded blade tips, which are becoming increasingly prone to "burnout".

The pressure gradient across an unshrouded turbine rotor blade induces tip leakage through the clearance between the rotor and the casing. This leakage flow exhibits flow separation and reattachment, turbulence and mixing. It also enhances heat transfer between the hot gas and the blade tip surface, which further leads to high turbine blade tip temperatures. These high temperatures may result in failure of turbine blades due to thermal fatigue, oxidation, and corrosion. Furthermore, tip deterioration can reduce blade life and increase tip leakage which in turn can lower the efficiency of the turbine.

Various experimental investigations of essentially incompressible flow and a study of heat transfer to a simulated turbine rotor blade tip have been performed. However, it appears that no attempts have been made to calculate heat transfer to turbine blade tips.
The present thesis is an endeavour to understand through literature survey and calculations, blade tip heat transfer, and blade tip life. A computational study is undertaken to calculate 2-D tip leakage flow and combined heat transfer in the flow and in the blade tip. These calculations of heat transfer from the hot gas (flow) to and within the cooled turbine blade provide an estimate of the maximum blade metal temperature, which can be used to improve the thermal design of unshrouded turbine blade tips for enhanced blade life.

It is hoped that the present work is a contribution towards improved understanding of characteristics of flow and resulting heat transfer at turbine rotor blade tip surfaces.
SECTION II

LITERATURE REVIEW

2.1 Turbine Tip Heat Transfer

The convective heat transfer associated with tip leakage flow has a detrimental effect on the turbine rotor blade life. Together with the suction and pressure side surface area, the blade tip surface area constitutes additional thermal loading on the blade's internal cooling flow. Due to high heat transfer, the blade tip has been an area susceptible to structural damage. Structural damage (loss of material) leads to widening of the tip gap. Heat transfer is further enhanced due to increased fluid flow through the tip gap.

Metzger and Bunker (1985) [1] evaluated heat transfer to a simulated turbine rotor blade tip. The geometry of the test section (without cavity) for these measurements had a 70° chamfer on the pressure side/blade tip corner and three different tip clearances, H, were tested, 0.254, 0.508, and 0.762 cm. The length of the test section was kept 6.98 cm as shown in Fig. 1. A thin layer of coating material was sprayed uniformly over the tip surface and the time required for melting at a given location was determined visually by noting the change from opaque white to transparent upon melting. This melting time was utilized to evaluate the heat transfer rate to the tip surface. The Nusselt number is normalized with fully developed values and plotted as a function of distance along the tip gap, x/H, in Fig. 2. For a tip clearance of 0.254 cm, the Reynolds number of the flow was $1.0 \times 10^4$ and for 0.508 cm and 0.762 cm it was $1.5 \times 10^4$. The results (dotted lines
Fig. 1 Metzger and Bunker test section with 70° chamfer at the inlet
Fig. 2 Correlation of local heat transfer to a model turbine blade tip with heat transfer near the entrance of a circular tube. Ratio of local Nusselt number to that with fully developed flow [Moore and Tilton]. —— Metzger and Bunker — model turbine; Boeltzer et al. — circular tube with right-angle-edge-entrance. H — tip gap height; D — tube diameter.
and symbols in Fig. 2) show that with increasing Reynolds number and tip clearance the heat transfer rate increases.

Boelter (1947) [2] measured heat transfer in a pipe test section with a right-angled edge entrance. Figure 3 shows Boelter's interpretation of the flow with separation and then reattachment at the leading edge. The tests were carried out with Reynolds numbers ranging from $1.3 \times 10^4$ to $2.7 \times 10^4$. He observed that the maximum heat transfer occurred quite close to the leading edge.

Moore and Tilton (1987) [3] compared the results of Metzger and Bunker and Boelter, as shown in Fig. 2. The dashed lines correspond to the results as obtained by Metzger and Bunker and the solid lines correspond to Boelter's data. The data agrees reasonably well for $\frac{X}{H} > 2$ as the Nusselt number ratio approaches unity. It is observed from Boelter's data that there is enhanced heat transfer ($\frac{X}{H} < 2$) by a factor of 2.0-2.5 for Reynolds numbers over $2.0 \times 10^4$. Below this value, the heat transfer rate reduces at $\frac{X}{H} < 2$ which is supported by the data of Metzger and Bunker. Although, it is seen, at $1.5 \times 10^4$ the Nusselt number ratio is a function of tip clearance. Metzger and Bunker measured a maximum Nusselt number ratio of 1.6 for a tip clearance of 0.508 cm at approximately two tip gap heights downstream of the leading edge.

The high convective heat transfer between the hot gas flow and blade tip surface can lead to burning of the tip regions. Near the blade trailing edge, secondary flows [4] in the blade passage tend to bring the hottest portions of the flow up to the blade tip entrance. Figure 4 shows the zone of a cooled turbine blade tip which is prone to
Fig. 3 Pipe test section with right angled entrance [Boelter]
Fig. 4 Zone of a cooled turbine blade tip, prone to heat damage [Bindon]
heat damage. Bindon (1986) [5] reported that the burnout of turbine blades occurred because of the breakdown of heat transfer rate predictions at the tip. Exposure to such high temperatures may cause the blade to fail structurally, thus it is essential to provide coatings so as to protect the base alloy.

However, such protective coatings may be removed by a tip rub or through low cycle fatigue on the unstrained blade tip. Once the coating is removed, metal is exposed to high temperature gases. The blade life can then be determined by various factors such as the thermal fatigue, oxidation and a susceptibility to corrosion and erosion [6].

Since the leakage of hot free stream fluid through the tip gap leads to higher heat transfer, resulting in structural failure of the blade at the tip corner, it is necessary to study and control the turbine tip metal temperatures. However, there is an immediate need for improved and accurate heat transfer rate predictions.

In the present thesis, the plan of investigation and presentation of results is as follows:

(i) Description of the Moore Elliptic Flow Program (MEFP).

(ii) Investigation of the gridding necessary to accurately calculate heat transfer in fully developed flow.

(iii) Computational study for the Metzger and Bunker model turbine tip geometry [1]. Calculation of flow development and heat transfer using MEFP.

In addition, there are two literature survey sections dealing with;

(iv) Gas turbine blade materials. Development of typical turbine blade materials from the 1950's onwards. Improvements in
their creep, thermal fatigue, oxidation and corrosion properties.

(v) Factors affecting turbine blade life:
   a) High temperature oxidation,
   b) Hot corrosion,
   c) Erosion.

(vi) Predictions of turbine tip metal temperatures in a jet engine using the results of the present calculations. Discussion of the practical implications for turbine rotor blade life.
SECTION III

Moore Elliptic Flow Program (MEFP)

MEFP solves the equations for 2-D or 3-D steady compressible flow in rotating or stationary coordinates, including regions of separation and recirculation. The equations used in MEFP [7] are:

Mass

\[ \nabla \cdot \rho \mathbf{u} = 0 \]  \hspace{1cm} (3.1)

Momentum

\[ \rho \mathbf{u} \cdot \nabla \mathbf{u} - (\nabla \cdot \mu) \mathbf{u} = \nabla \cdot \mu \nabla \mathbf{u}^T - \nabla \mathbf{p} - 2 \rho \Omega \times \mathbf{u} - \rho \Omega \times (\Omega \times \mathbf{r}) \quad \text{convection} \hspace{1cm} \text{viscous} \hspace{1cm} \text{pressure} \hspace{1cm} \text{coriolis} \hspace{1cm} \text{centrifugal} \]  \hspace{1cm} (3.2)

Equation of state

\[ p = \rho RT \]  \hspace{1cm} (3.3)

Rothalphy (energy)

\[ \rho \mathbf{u} \cdot \nabla \mathbf{H} - (\nabla \cdot \mu \nabla) \mathbf{H} = 0 \quad \text{convection} \hspace{1cm} \text{viscous} \]  \hspace{1cm} (3.4)

\[ \mathbf{H} = c_p T + \frac{1}{2} (\mathbf{u} \cdot \mathbf{u}) - \frac{1}{2} (\Omega \times \mathbf{r} \cdot \Omega \times \mathbf{r}) \]  \hspace{1cm} (3.5)

The symbols used are

- \( \mathbf{u} \) \hspace{1cm} \text{relative velocity}
- \( \rho \) \hspace{1cm} \text{density}
- \( p \) \hspace{1cm} \text{static pressure}
\[ \Omega \quad \text{rotation rate} \]
\[ \mu \quad \text{viscosity} \]
\[ T \quad \text{static temperature} \]
and \[ \tilde{H} \quad \text{enthalpy} \]

3.1 Turbulence Model

A Prandtl mixing length viscosity model is used in the MEFP, and the turbulent viscosity, \( \mu_t \), is given by

\[
\mu_t = \rho L^2 \frac{\partial u}{\partial y},
\]

where \( L \) is the mixing length. Within the boundary layer, \( L \) is the smaller of 0.08 times the width of shear or boundary layer and 0.41 times the distance to the nearest wall. A van Driest correction is applied in the 0.41 'y' region and the corrected mixing length is given as

\[
L = 0.41 'y' \left[ 1 - \exp (-'y' \sqrt{\rho \tau/26 \mu_x}) \right].
\]

3.2 Effective Viscosity

Away from the wall, the effective viscosity is taken as the simple sum of the laminar and turbulent viscosities. But for the near wall point, a logarithmic mean of the laminar viscosity and the simple sum is used; this makes the resulting calculated skin friction less sensitive to the location of the near wall point. Thus
\[ \mu_{\text{eff}} = \mu_L + \mu_t, \quad \text{away from the wall,} \quad (3.8) \]

\[ \mu_{\text{eff}} = [\mu_L (\mu_L + \mu_t)]^{1/2}, \quad \text{near wall point.} \quad (3.9) \]

A subset of these equations, for 2-D incompressible flow in stationary coordinates, is used for the present calculations, these calculations are described in more detail in Section V.

The calculation procedure uses the pressure correction method as described in AGARD [7].
SECTION IV

Discretization for 2-D Fully Developed Flow Calculations

Numerical accuracy or grid refinement tests were carried out to find an acceptable grid with a moderate number of grid points and high accuracy in a fully developed flow. In particular, the tests show the influence of a) number of points across the grid, b) spacing of these points, and c) choice of regular or altered near-wall control volume on the accuracy of the calculated wall shear stresses and heat transfer rates for two flows,

(i) 2-D fully developed laminar flow,
and (ii) 2-D fully developed turbulent flow.

4.1 Control Volumes

There are two choices of control volume near the wall in MEFP. They are:

a) Regular Control Volume

For the regular control volume, Fig. 5, the wall control volume lies between the wall and half way to the near wall grid point.

b) Altered Control Volume

For the altered control volume, Fig. 5, the wall control volume lies between the wall and the near wall grid point. This type of control volume is useful when the flow is rapidly developing, for example, as it enters the tip gap. With the regular control volume, in an accelerating flow the velocity can overshoot at the near-wall grid point since linear velocity profiles are used between the grid points (see Appendix II). With the altered control volume, the volume between
Fig. 5 Different grid spacing, with two choices of control volumes
the wall and the near-wall point is not used in the momentum balance determining the velocity at the near-wall point.

Two types of grid spacings are investigated in this section, they are:

a) Uniform Spacing

In this type of spacing the grid points are spaced evenly in the grid across the flow.

b) Non-Uniform Spacing

In this type of grid, as the wall is approached, the distance of the grid points from the wall is reduced by a fixed factor, e.g., factors from 2 to 7 are tested in this section. This sort of spacing allows a finer grid in the boundary layer and a coarser one in the free stream region.

4.2 Continuity and Momentum Equations

The continuity and momentum equations are solved for 2-D incompressible fully developed flow, as shown in Fig. 6.

1. The continuity equation is given by:

\[
\frac{H}{2} \int_{-H/2}^{H/2} \rho u \, dy = \dot{m} = \text{mass flow rate/height} \quad (4.1)
\]

2. For fully developed flow \( v = 0 \) and \( \frac{\partial u}{\partial x} = 0 \), therefore, the momentum equation is:

\[
- \frac{\partial}{\partial y} \mu \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} = \text{constant} \quad (4.2)
\]
Fig. 6 A 2-D incompressible fully developed flow example
where,

\[ H = \text{height of channel} \]
\[ \mu = \text{effective viscosity} \]

The continuity equation is integrated in finite-difference form over the channel height as follows:

\[
\frac{H}{2} \int_{-H/2}^{H/2} \rho u \, dy = \dot{m} = \rho \sum_{I=1}^{I_{\text{max}}-1} \left[ \frac{U(I) + U(I + 1)}{2} \right] [y(I + 1) - y(I)]
\]

(4.3)

where \( \left[ \frac{U(I) + U(I + 1)}{2} \right] \) is the average velocity in the interval \( y(I + 1) - y(I) \).

The momentum control volume for the \( I^{th} \) grid point is shown in Fig. 7; in general, it extends from \( I - \frac{1}{2} \), midway between grid points \( I - 1 \) and \( I \), to \( I + \frac{1}{2} \), midway between grid points \( I \) and \( I + 1 \). Discretizing the momentum equation for this regular control volume yields

\[
\left( \mu \frac{\partial u}{\partial y} \right)_I + \frac{1}{2} - \left( \mu \frac{\partial u}{\partial y} \right)_I - \frac{1}{2} = \text{constant}
\]

(4.4)

with

\[
y(I + \frac{1}{2}) = \frac{y(I) + y(I + 1)}{2}
\]

(4.5)

and

\[
y(I - \frac{1}{2}) = \frac{y(I) + y(I - 1)}{2}
\]

(4.6)
Fig. 7 The momentum control volume for the $i^{th}$ grid point extending from $I - \frac{1}{2}$ to $I + \frac{1}{2}$
further expansion gives

\[ \frac{2}{y(I + 1) - y(I - 1)} \left( \mu(I + \frac{1}{2}) \left( \frac{U(I + 1) - U(I)}{y(I + 1) - y(I)} \right) \right) \]

\[ - \mu(I - \frac{1}{2}) \left( \frac{U(I) - U(I - 1)}{y(I) - y(I - 1)} \right) = W = \frac{dP}{dx} \]  

(4.7)

Now defining A(I), B(I), and C(I) as the coefficients of U(I + 1), U(I), and U(I - 1), respectively, and defining EM(I) ≡ \( \mu(I + \frac{1}{2}) \)

\[ A(I) = 2 \frac{EM(I)}{((y(I + 1) - y(I - 1)) (y(I + 1) - y(I)))} \]

\[ B(I) = -2 \frac{EM(I)/(y(I + 1) - y(I)) + EM(I - 1)/(y(I) - y(I - 1))}{(y(I + 1) - y(I - 1))}/(y(I + 1) - y(I - 1)) \]

\[ C(I) = 2 \frac{EM(I - 1)}{((y(I + 1) - y(I - 1)) (y(I) - y(I - 1)))} \]

\[ D(I) = W \]

(4.8)

The momentum equation can then be written as:

\[ A(I) \ast U(I + 1) + B(I) \ast U(I) + C(I) \ast U(I - 1) = D(I) \]  

(4.9)

The velocities at the walls, I = 1 and I = I_{\text{max}}, are zero, by the no slip boundary condition. Equation (4.9), therefore represents I_{\text{max}} - 2 equations for the unknown velocities at I = 2 to I = I_{\text{max}} - 1.
Solution Procedure

The solution procedure for Eqs. (4.3) and (4.9) is

a) guess \( W = \frac{\partial p}{\partial x} \)

b) solve Eq. (4.9) using the Tridiagonal Matrix Algorithm, see Appendix V.

c) evaluate the mass flow rate, \( \dot{m} \), using Eq. (4.3)

d) reset \( W_{\text{new}} = \frac{m_{\text{specified}}}{m_{\text{calculated}}} \) \( W_{\text{old}} \rightarrow \)

e) repeat until the calculated mass flow rate is within tolerance limits of the specified mass flow rate. In the case of fully developed laminar flow, there is no need to iterate, instead the calculated velocities on the first iteration are corrected by the ratio \( \frac{m_{\text{specified}}}{m_{\text{calculated}}} \).

4.3 Predicted Wall Shear Stress, \( \tau_w \)

The shear stress distribution for fully developed flow is linear, as shown in Fig. 8. The wall shear stress \( \tau_w \) and the longitudinal pressure gradient \( W = \frac{\partial p}{\partial x} \) are related by

\[
W = \frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y} = -\frac{2 \tau_w}{H}
\] (4.11)

Thus,

\[
\tau_w = -\frac{W_{\text{calc}} \cdot H}{2}
\] (4.12)

Shear ratio, \( S_R \), is thus defined as
Fig. 8 Linear shear stress distribution for fully developed flow
\[ S_R = \frac{\tau_w \text{ calculated}}{\tau_w \text{ theoretical}}, \quad (4.13) \]
\[ \frac{W \text{ calculated}}{W \text{ theoretical}} \]

4.4 Momentum Equation for Near-Wall Grid Point Using Altered Control Volume

Regular and altered control volumes for the grid point near the top wall are shown in Fig. 9. As discussed earlier, the altered control volume includes none of the volume between the grid point and the wall, instead the "I + \frac{1}{2}" surface is located at the near wall point. In expanding Eq. (4.4), Eq. (4.5) is replaced by

\[ y(I + \frac{1}{2}) = y(I) \quad (4.14) \]

Consistent with the discretization in MEFP, no other change is made.

4.5 Fully Developed Laminar Flow

4.5.1 Grid Spacing

As shown in Fig. 5, two sets of grid spacing were tried, they were:

- a) Uniform Spacing - (i) Regular Control Volumes
  (ii) Altered Near-Wall Control Volume
- b) Non-Uniform Spacing - Each successive grid point a factor of two closer to the wall.

These two sets with different numbers of grid points, ranging from 3 to 19, successive odd integer values, were used.

The basic procedure followed was:
Fig. 9 Regular and altered control volumes for the grid point near the top wall
a) Setting up initial conditions,

(i) density, \( \rho = 1.0 \) (incompressible flow)

(ii) Laminar viscosity = \( 0.00002 \) kg/ms

(iii) \( \frac{dP}{dx} = -320 \) N/m\(^3\) (pressure gradient)

b) Choosing the grid spacing, i.e., uniform or non-uniform spacing.

c) Setting up tridiagonal equations, Eq. (4.9), to solve for the velocities \( U(I) \) with the help of TDMA subroutine (see Appendix III).

The mass flow rate was evaluated with the help of the continuity equation, and the theoretical mass flow rate was calculated as follows:

Tip gap height \( (H) = 0.001 \) m

\[
\text{Reynolds number } \frac{\rho U_{\text{max}} H}{\mu} = 100 \quad \Rightarrow \quad U_{\text{max}} = 2
\]

Parabolic velocity profile \( U = ay^2 + U_{\text{max}} \)

\[
(\text{At } y = H/2, \ U = 0) \quad \Rightarrow \quad a = -U_{\text{max}} \left( \frac{4}{H^2} \right) = -8 \times 10^6
\]

\[
m = \int_{-H/2}^{H/2} \rho U \ dy = \int_{-H/2}^{H/2} (ay^2 + U_{\text{max}}) \ dy
\]

\[
= \left[ ay^3 + U_{\text{max}} y \right]_{-H/2}^{H/2}
\]

\[
= ay^3 + U_{\text{max}} y \bigg|_{-H/2}^{H/2} = \frac{a H^3}{12} + U_{\text{max}} H
\]
\[ = \frac{-8 \times 10^6}{12} \times (0.001)^3 + 2 \times 0.001 \]

\[ = 1.333 \times 10^{-3} \text{ kg/s per meter} \]

The calculated velocities are corrected to this mass flow rate.

Hence

\[ U_{corrected} = U_{calculated} \times \frac{m_{theoretical}}{m_{calculated}} \quad (4.15) \]

The calculated wall shear stress corresponding to the correct mass flow rate scales like the velocity, thus

\[ W_{calculated} = W_{theoretical} \times \frac{m_{theoretical}}{m_{calculated}} \quad (4.16) \]

The calculated wall shear stress approaches the theoretical value \( (W_{theoretical} = -320 \text{ N/m}^3) \) as the number of grid points is increased. The difference between the shear ratio \( \left( \frac{W_{cal}}{W_{th}} \right) \) and unity is known as the discretization error.

4.5.2 Results and Discussion

The discretization error, as explained above, as a function of the number of grid points was evaluated and plotted, as shown in Fig. 10.
Results for uniformly spaced grid with regular control volumes

<table>
<thead>
<tr>
<th>Grid Spacing ($\delta y/H$)</th>
<th>Grid Points</th>
<th>Shear Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>5</td>
<td>1.0666</td>
</tr>
<tr>
<td>0.1667</td>
<td>7</td>
<td>1.0286</td>
</tr>
<tr>
<td>0.125</td>
<td>9</td>
<td>1.0159</td>
</tr>
<tr>
<td>0.1</td>
<td>11</td>
<td>1.01</td>
</tr>
<tr>
<td>0.0833</td>
<td>13</td>
<td>1.0069</td>
</tr>
<tr>
<td>0.07143</td>
<td>15</td>
<td>1.005</td>
</tr>
<tr>
<td>0.0625</td>
<td>17</td>
<td>1.0039</td>
</tr>
<tr>
<td>0.0555</td>
<td>19</td>
<td>1.0031</td>
</tr>
</tbody>
</table>

Uniform spacing with altered near-wall control volume

<table>
<thead>
<tr>
<th>Grid Spacing ($\delta y/H$)</th>
<th>Grid Points</th>
<th>Shear Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>5</td>
<td>1.5238</td>
</tr>
<tr>
<td>0.1667</td>
<td>7</td>
<td>1.1999</td>
</tr>
<tr>
<td>0.125</td>
<td>9</td>
<td>1.108</td>
</tr>
<tr>
<td>0.1</td>
<td>11</td>
<td>1.0684</td>
</tr>
<tr>
<td>0.0833</td>
<td>13</td>
<td>1.0473</td>
</tr>
<tr>
<td>0.07143</td>
<td>15</td>
<td>1.0346</td>
</tr>
<tr>
<td>0.0625</td>
<td>17</td>
<td>1.0265</td>
</tr>
<tr>
<td>0.0555</td>
<td>19</td>
<td>1.021</td>
</tr>
</tbody>
</table>

Non-uniformly spaced (factor of 2 spacing) with altered near-wall control volume

<table>
<thead>
<tr>
<th>Near-Wall Grid Spacing ($\delta y/H$)</th>
<th>Grid Points</th>
<th>Shear Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>5</td>
<td>1.5238</td>
</tr>
<tr>
<td>0.125</td>
<td>7</td>
<td>1.1377</td>
</tr>
<tr>
<td>0.0625</td>
<td>9</td>
<td>1.0617</td>
</tr>
<tr>
<td>0.0313</td>
<td>11</td>
<td>1.0433</td>
</tr>
<tr>
<td>0.01565</td>
<td>13</td>
<td>1.0386</td>
</tr>
<tr>
<td>0.007825</td>
<td>15</td>
<td>1.0374</td>
</tr>
<tr>
<td>0.0039</td>
<td>17</td>
<td>1.0371</td>
</tr>
<tr>
<td>0.002</td>
<td>19</td>
<td>1.037</td>
</tr>
</tbody>
</table>

These results have been plotted in Fig. 10. Obviously regular control volumes give the most accurate wall shear stress and should be used whenever possible in laminar flow. However, other considerations may require the use of altered near-wall control volumes (e.g., in strongly accelerating flow, see Appendix II). From Fig. 10, the number
Fig. 10 Calculated to theoretical wall shear stress ratio vs. total number of grid points
of grid points required to achieve a given accuracy can be found. For example, with altered control volumes, to get the calculated wall shear stress within 10 percent of the theoretical value, more than 9 points must be used with uniform grid spacing and more than 7 points are required with factor of 2 spacing. Note also that as the number of grid points is increased with a factor of 2 spacing, the wall shear stress does not approach the theoretical value, but remains nearly 4 percent above it.

Calculated velocity profiles for fully developed laminar flow are plotted in Fig. 11. The cases shown are for both uniformly and non-uniformly spaced grids with regular near-wall control volumes, the uniformly spaced grids have 5, 9, and 20 grid points and the non-uniformly spaced grid has 9 grid points. All velocities have been corrected to the same mass flow rate which can be verified by the fact that the areas under the curves are the same for all the four cases. It is seen that the 20 point grid approaches the ideal parabolic velocity profile while the 5 point grid is farthest from it and has the largest discretization errors.

4.6 Fully Developed Turbulent Flow

Numerical accuracy tests were made with a specified turbulence model, the Prandtl mixing length model, described in Section III.

4.6.1 Setting Values

(i) Mass flow rate (\dot{m}) = 0.315 \text{ kg/s per meter}
(ii) Density (\rho) = 1.017 \text{ kg/m}^3
Fig. 11 Calculated velocity profiles for fully developed laminar flow $u$ vs. $y$
(iii) Viscosity ($\mu_2$) = 0.000021 kg/ms
(iv) Tip gap height (h) = 0.2 x 0.0254 m
   = 0.00508 m
(v) Mean velocity $\bar{U}$ = 60.97 m/s
(vi) Maximum velocity $\sim \frac{60.97}{0.8} = 76.21$ m/s
(initial estimate)
(vii) Reynolds number ($\frac{\rho \bar{U} H}{\mu}$) = 15,000

4.6.2 Grid Spacing

The purpose of this test was to find a sufficiently accurate grid with a reasonable number of grid points across the tip gap. Different numbers of grid points with different spacings between them were used, they were:

Grid points: 7, 9, 11, 13, 15, 17, 19, 21, 23, 25
Grid spacing: factors of 2, 3, 4, 5, 6, 7 (Non-uniform spacing)
and uniformly spaced 501 points.

The maximum velocity, on the centerline, was estimated to be 1.25 $\bar{U}$. The velocities at other grid points were estimated using

$$U(I - 1) = 0.8 \ U(I) \text{ for } 2 < I < \frac{I_{\text{max}}}{2} \quad (4.17)$$

and

$$U(I + 1) = 0.8 \ U(I) \text{ for } \frac{I_{\text{max}} + 2}{2} < I < \frac{I_{\text{max}} - 1}{2} \quad (4.18)$$

The velocity profile is symmetrical about the centerline which is appropriate for a fully developed flow.

A program was written which inputs the estimated values of the
velocities at the respective grid locations. It also inputs the grid for a given number of grid points with the specified grid spacing. Other input values include the estimated pressure gradient \( W \), the mass flow rate and a tolerance for the calculated mass flow rate, 1%.

### 4.6.3 Main Program

Using the Prandtl mixing length turbulence model, Eqs. (3.6) and (3.7), and applying the near wall modification, Eq. (3.9), the effective viscosity between each pair of grid points is calculated. The "boundary layer" thickness is \( H/2 \) for this fully developed flow. Then the momentum equations, Eq. (4.9), are solved with the variable 'D' set equal to the estimated pressure gradient, \( W \).

Using TDMA (Tridiagonal Matrix Subroutine, see Appendix V), velocities at each location are determined.

Calculated velocities are averaged with the estimated values, the viscosities are recalculated, and this procedure is repeated ten times to obtain consistent distributions of velocity and viscosity. Having evaluated the velocities, the mass flow rate is calculated by integrating the product of density, velocity, and area over the grid, Eq. (4.1). With every iteration, the wall shear stress is updated, i.e.,

\[
W_{\text{new}} = W_{\text{old}} \frac{m}{m_{\text{calculated}}}
\]  

(4.19)

The entire procedure is repeated until the calculated mass flow rate lies within the tolerance limits of the specified mass flow rate.

To determine the limiting shear stress as the number of grid points
is increased for each grid-spacing factor, the shear stresses from successive calculations are compared. If these two values lie within 0.1% of each other, it is taken to mean that shear stress has reached a limiting value.

The velocity distribution for a uniformly spaced grid with 501 grid points was also calculated. The value of the wall shear stress from this calculation was used as a reference "theoretical" value for these fully developed turbulent flow calculations.

4.6.4 Normalization

The computed values of wall shear stress for a particular type of spacing were normalized with the reference value. This gives the variation of shear ratio with the number of grid points for a given non-uniformly spaced grid.

4.6.5 Results and Conclusions

The shear ratio as a function of the total number of grid points for various non-uniformly spaced grids, with regular and altered near-wall control volumes, is plotted in Figs. 12 and 13. The shear ratio is an indication of the accuracy of the grid. It can be seen that the factor of two grid spacing gives the best accuracy when the number of grid points is over ten. For a grid with less than eight points, it is more desirable to use a grid with a spacing factor of four or five.

Figure 12 shows the results of the calculations using regular near-wall control volumes while Fig. 13 shows the results of the altered near-wall control volumes. The results are almost identical in terms of
Fig. 12 Calculated to ideal wall shear stress ratio vs. total number of grid points with regular control volume
Fig. 13 Calculated to ideal wall shear stress ratio vs. total number of grid points with altered control volume
shear ratio. Thus, unlike the laminar flow calculations, the turbulent flow calculations were relatively insensitive to the choice of near-wall control volume.

Figures 14 and 15 show the velocity profiles for different non-uniformly spaced grids. The profiles were all calculated with the same mass flow rate, which can be verified by the fact that the area under the curves are the same. The velocity profile for uniformly spaced 501 points is also plotted. This curve represents the theoretical velocity profile for the mixing length turbulence model used here. Both figures, i.e., for regular and altered control volumes, have similar velocity profiles as expected. It can be observed that factor of two grid spacing gives closest agreement with the 501 point profile.

4.7 Energy Equation

The energy equation, Eq. (3.4), has been written in terms of the rothalpy \( \tilde{H} \). For stationary coordinates,

\[
\tilde{H} = c_p T + \frac{1}{2} \mathbf{u} \cdot \mathbf{u}
\]

Further, for heat transfer in low Mach number flows

\[
\frac{1}{2} u \cdot u < c_p T
\]

so that

\[
\tilde{H} \approx c_p T.
\]

Modifying Eq. (3.4) to include Prandtl number effects then gives

\[
\rho \mathbf{u} \cdot \nabla T - \nabla \cdot \left( \frac{\mathbf{u}}{\Pr_{eff}} \right) \nabla T = 0
\] (4.20)

Therefore for 2-D fully developed flow with a developing temperature
Fig. 14 Calculated velocity profile for fully developed turbulent flow with regular control volume for non-uniformly spaced grids
Fig. 15 Calculated velocity profile for fully developed turbulent flow with altered control volumes for non-uniformly spaced grids
profile and neglecting heat conduction in the x direction, we have,

\[ \rho u \frac{\partial T}{\partial x} - \frac{\partial}{\partial y} \left( \frac{\mu}{Pr_{eff}} \right) \frac{\partial T}{\partial y} = 0 \]  

(4.21)

The wall temperature, \( T_w \), and the peak temperature, \( T_{max} \), are functions of x only.

Defining,

\[ \theta = \frac{T - T_w}{T_{max} - T_w} \]  

(4.22)

\( \theta \) is then zero at the wall and has a maximum value of one at the centerline. For fully developed flow, \( \theta \) is a function of y only. Substituting \( \theta \) for \( T \) in the left hand side of Eq. (4.21), and expanding, we get,

\[ \rho u \frac{\partial}{\partial x} \left( \frac{T - T_w}{T_{max} - T_w} \right) - \frac{\partial}{\partial y} \left( \frac{\mu}{Pr_{eff}} \right) \frac{\partial}{\partial y} \left( \frac{T - T_w}{T_{max} - T_w} \right) \]

\[ = \frac{\rho u}{(T_{max} - T_w)} \frac{\partial T}{\partial x} - \frac{\rho u}{(T_{max} - T_w)} \frac{\partial T_w}{\partial x} - \frac{\rho u (T - T_w)}{(T_{max} - T_w)^2} \frac{\partial}{\partial x} (T_{max} - T_w) \]

\[ - \frac{1}{(T_{max} - T_w)} \frac{\partial}{\partial y} \left( \frac{\mu}{Pr_{eff}} \right) \frac{\partial T}{\partial y} \]  

(4.23)

on rearranging,

\[ \rho u \frac{\partial \theta}{\partial x} - \frac{\partial}{\partial y} \left( \frac{\mu}{Pr_{eff}} \right) \frac{\partial \theta}{\partial y} = \frac{1}{(T_{max} - T_w)} \left( \rho u \frac{\partial T}{\partial x} \frac{\partial}{\partial y} \left( \frac{\mu}{Pr_{eff}} \right) \frac{\partial T}{\partial y} \right) \]

\[ - \frac{\rho u}{(T_{max} - T_w)} \left( \frac{\partial T_w}{\partial x} + \theta \frac{\partial}{\partial x} (T_{max} - T_w) \right) \]  

(4.24)
Kays and Crawford [8] analyse the energy equation for fully developed flow with two wall boundary conditions:

a) Constant wall temperature \( T_w = \text{constant} \);

b) Constant heat flux, so that \( (T_{\text{max}} - T_w) \) is a constant.

For the present fully developed flow calculations, constant heat flux is used. Therefore, Eq. (4.24) becomes

\[
- \frac{1}{\rho u} \frac{\partial}{\partial y} \left( \frac{\mu}{Pr_{\text{eff}}} \right) \frac{\partial \theta}{\partial y} = - \frac{1}{(T_{\text{max}} - T_w)} \frac{dT_w}{dx} \tag{4.25}
\]

The velocity, \( u \), is a function of \( y \) only, hence the left hand side of Eq. (4.25) is a function of \( y \) only and the right hand side is a function of \( x \) only. Therefore, they are equal to a constant, \( c \) say.

The equation,

\[
- \frac{\partial}{\partial y} \left( \frac{\mu}{Pr_{\text{eff}}} \right) \frac{\partial \theta}{\partial y} = - c \rho u \tag{4.26}
\]

was solved for laminar and turbulent flow with heat transfer. In discretized form, the energy equation can then be written as

\[
A(I) \ast \theta(I + 1) + B(I) \ast \theta(I) + C(I) \ast \theta(I - 1) = D(I) \tag{4.27}
\]

similar to the discretized momentum equation, Eq. (4.9). For the evaluation of \( A, B, \) and \( C \), Eq. (4.8) is used with,
\[ EM(I) = \frac{\mu_1}{Pr_1} \text{ for laminar flow,} \]

\[ EM(I) = \frac{\mu_1}{Pr_1} + \frac{\mu_t}{Pr_t} \text{ for turbulent flow, and} \]

\[ EM(I) = \left( \frac{\mu_1}{Pr_1} \left( \frac{\mu_1}{Pr_1} + \frac{\mu_t}{Pr_t} \right) \right)^{1/2} \text{ at } I = 1 \text{ and } I = I_{\text{max}} - 1 \]  

\[ \text{in turbulent flow} \]  

The right hand side, \( D(I) = c \rho U(I) \) where the constant \( c \) is determined by \( \theta_{\text{max}} = 1 \).

4.7.1 Nusselt Number

The Nusselt number for fully developed flow is defined by

\[ Nu_\infty = \frac{h D}{k} \]  

(4.29)

From a heat balance,

\[ q = h(T_{\text{max}} - T_w) = -k \frac{\partial T}{\partial y}_w \]  

(4.30)

or

\[ h = \frac{-k}{(T_{\text{max}} - T_w)} \frac{\partial T}{\partial y}_w \]  

(4.31)

\[ \therefore \]

\[ Nu_\infty = \left( \frac{-k}{(T_{\text{max}} - T_w)} \frac{\partial T}{\partial y}_w \right) \cdot \frac{2 H}{k} \]

\[ = -\frac{\partial \theta}{\partial y}_w \cdot 2 H \]  

(4.32)

where,
h – heat transfer coefficient
k – thermal conductivity of the fluid
D – hydraulic diameter of the channel = 2 H

4.8 Laminar Heat Transfer

For fully developed laminar flow with constant viscosity and Prandtl number, there is a closed form solution for Eq. (4.26).

With \( U = U_{\text{max}} \left( 1 - \left( \frac{2y}{H} \right)^2 \right) \), the solution is

\[
\theta = \frac{16}{5} \left( \frac{y}{H} \right)^4 - \frac{24}{5} \left( \frac{y}{H} \right)^2 + 1
\] (4.33)

therefore,

\[
\frac{\partial \theta}{\partial y} = \frac{-16}{5 \ H}
\] (4.34)

The Nusselt number (theoretical) is evaluated from Eq. (4.32),

\[
\text{Nu}_{\text{\infty (theoretical)}} = -\left( \frac{-16}{5 \ H} \right) \times 2 \times H = 6.4
\] (4.35)

4.8.1 Discretized Laminar Heat Transfer Equation Results

For laminar flow, Eq. (4.27) is solved, and the discretization errors are shown in Fig. 16 in terms of the heat transfer ratio,

\[
H_R = \frac{\text{Nu}_{\text{\infty calculated}}}{\text{Nu}_{\text{\infty theoretical}}}
\] (4.36)

\( H_R \) is shown as a function of the number of grid points across the
channel for uniformly spaced grid points.

For comparison, the corresponding shear ratios from Fig. 10 are also included. As for the shear ratio, the regular control volume gives more accurate heat transfer results, Fig. 16, than the altered near-wall control volume. While the discretized equations overestimate the wall shear, \( S_R > 1 \), they underestimate the wall heat transfer \( H_R < 1 \).

For both the control volumes, the maximum discretization error is observed for the smallest number of grid points; the error reduces as the total number of grid points is increased. From Fig. 16, to obtain the wall heat transfer to within 5%, for the altered near-wall control volume, 13 or more grid points must be used while only 7 or more are required for the regular control volume.

4.9 **Turbulent Heat Transfer**

The discretized equation for turbulent heat transfer was solved using Eq. (4.27) and Eq. (4.28) with a laminar Prandtl number of 0.7 and a turbulent Prandtl number of 1.0.

\[
\text{Nu}_\infty \text{ calculated was evaluated as}
\]

\[
\text{Nu}_\infty = \frac{EM(I)(\theta(2) - \theta(1)) \cdot 2H}{\mu_\lambda/Pr_\lambda(y(2) - y(1))}
\]

The results for turbulent heat transfer ratio with grid spacing factors ranging from 2 to 7 are shown in Fig. 17. Overall, the heat transfer ratio is less sensitive to gridding as compared to wall shear stress ratio, Fig. 12. For all the grids studied, the heat transfer was obtained within 25% of the ideal value. The ideal heat transfer rate
Fig. 16 Heat transfer ratio and shear ratio vs. grid points. Laminar flow, regular and altered control volumes.
Fig. 17 Heat transfer ratio as a function of total number of grid points in a fully developed turbulent flow
was taken as the value calculated for 501 uniformly spaced points.

The wall heat transfer is probably less sensitive to the grid than the wall shear stress because in the near wall region for fully developed flow the heat flux is more uniform than the shear stress. Figure 18 shows this for laminar flow.

A practical consequence of this is that a grid which is adequate to resolve the velocity distribution and the wall shear stress should also be adequate to resolve the temperature distribution and the wall heat transfer.
Fig. 18 Variations of shear stress and heat flux for fully developed laminar flow
SECTION V

Calculations for Developing Flow and Heat Transfer in Tip Gaps

Calculations for heat transfer in developing flow were performed on two sections of different geometry. The first test section is identical to the one used by Metzger and Bunker (see Fig. 1). It has a 70 degree chamfer on the pressure side. The geometry of the second test section had a 90° corner along the pressure side, as shown in Fig. 19.

The dimensions for the tip geometries were:

- Tip gap height \( (H) = 0.508 \text{ cm} \)
- Tip gap length \( = 6.98 \text{ cm} \)
- Blade tip height \( = 5.08 \text{ cm} \)

These dimensions are shown in terms of tip gap heights for the 70 degree chamfer geometry in Fig. 20.

Calculations were performed in two parts:

a) Flow calculations

and b) Heat transfer calculations.

5.1 Gridding of Test Section

The grid covers the blade tip, the tip gap, and the inlet on the pressure side of the blade. After studying the numerical accuracy results, a twenty seven point grid with twenty one grid points uniformly spaced across the tip gap and three additional points, each placed a factor of two closer to the wall was chosen. The points were located at \( y/H = -1, -0.99375, -0.9875, -0.975, -0.95, -0.9, \) by 0.05's to -0.05, -0.025, -0.0125, -0.00625, 0 as shown in Fig. 21. The accuracy of this grid was tested on fully developed turbulent flow. The wall shear stress and the
Fig. 19 A typical turbine rotor blade test section, with 90° corner on the pressure side
Fig. 20  The dimensions in terms of tip gap heights of the chosen 70° chamfer test section
wall heat transfer for this grid had an error of 4.4% and 3.7%, respectively, relative to the 501 point grid result. For fully developed flow with this grid, the near wall point was in the laminar sublayer, in particular at

\[ y^+ = \frac{y}{\nu} \frac{\sqrt{\rho \tau}}{\mu_k} = 5 \]  

(5.1)

Inspection of the computer output for the developing flow calculations showed similar results with \( \mu_t < \mu_k \) for all the near wall viscosities.

Similar grid spacing was used near the pressure side of the blade. The grid and a blow up of it near the entrance to the tip gap is shown in Fig. 22. The total numbers of grid points in the x and y directions were 41 and 48, respectively, for the 70° chamfer test section. For the 90° corner, a 41 by 43 grid was used as points were not required to grid the chamfer corner on the pressure surface.

5.2 Flow calculations

MEFP (see Section III) was used to perform flow calculations for 2-D incompressible fully turbulent flow. In stationary coordinates, Eqs. (3.1) and (3.2) reduce to,

\[ \nabla \cdot \rho \mathbf{u} = 0 \]  

(5.2)

\[ \rho \mathbf{u} \cdot \nabla \mathbf{u} - \nabla \cdot \mu_{\text{eff}} \nabla \mathbf{u} = -\nabla p + \nabla \cdot \mu_{\text{eff}} \overline{\nabla \mathbf{u}^T} \]  

(5.3)

MEFP uses an iterative procedure in which the flow is made to converge
Fig. 21 The grid point locations for the chosen grid in the tip gap
Fig. 22 The calculation grid and an enlargement of the tip gap entrance corner region
to a solution which satisfies the above equations.

Flow conditions were chosen to correspond to the Metzger and Bunker test case with 0.508 cm tip gap height:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number</td>
<td>15000</td>
</tr>
<tr>
<td>Density ($\rho$)</td>
<td>1.017 kg/m$^3$</td>
</tr>
<tr>
<td>Mean velocity in tip gap ($\bar{U}$)</td>
<td>61 m/s</td>
</tr>
<tr>
<td>Laminar viscosity ($\mu_1$)</td>
<td>0.000021 kg/ms</td>
</tr>
<tr>
<td>Mass flow rate ($\dot{m}$)</td>
<td>0.3150 kg/s per meter</td>
</tr>
</tbody>
</table>

Inlet and boundary conditions chosen were as follows:

At inlet, $x = -12 H$, $u = 5.5$ m/s, $v = 0$ m/s

In and on the surface of the blade, the velocity was zero. The velocity was also set to zero on the endwall. The grid line upstream of the blade, at $y/H = 10$, was treated as a symmetry plane.

5.2.1 Flow Results

Velocity vector plots for both test sections (70° chamfer and 90° corner), seen in Figs. 23 and 24, clearly show flow separation at the tip gap entrance. For the 70° chamfer test section, a separation bubble with back flow (recirculation), as shown in Fig. 23, extends up to 2.2 H (tip gaps) from the tip corner. This bubble has a height of 0.27 H, measured from the tip surface. Similarly for the 90° corner test section, Fig. 24, the separation bubble has a length of 2.9 H and a height of 0.41 H.

Static pressure and total pressure loss coefficients are defined as
Fig. 23 Velocity vectors for the 70° chamfer test section, from turbulent flow calculations
Fig. 24 Velocity vectors for the 90° corner test section, from turbulent flow calculations
\[ C_p \equiv 1 - \frac{p_o - p}{\frac{1}{2} \rho U^2} \quad (5.4) \]

and

\[ C_p \equiv \frac{p_o - p_t}{\frac{1}{2} \rho U^2} \quad (5.5) \]

where
- \( p \) = static pressure
- \( p_o \) = upstream total pressure
- \( p_t \) = local total pressure.

Plots of the calculated pressure coefficients for the 70° chamfer test section are shown in Figs. 25 and 26. In Fig. 25, the static pressure is seen to fall as the flow accelerates into the tip gap. Two pressure minima are observed, one with \( C_{p_s} = -1.5 \) is near the center of the recirculating flow, and the other is located at the corner, \( C_{p_s} = -2.0 \). As the flow reattaches, the static pressure becomes uniform across the tip gap height.

The velocity vectors and static pressures near the corner are shown in Fig. 27; the minimum static pressure at the corner is seen to be associated with a second small recirculation zone. This is consistent with the results of Bindon [9], who measured locally low static pressures on the pressure surface corner of his turbine blade tip.

The contours of total pressure loss in Fig. 26 show the large recirculation bubble and the boundary layer development after reattachment on the blade tip, and the relatively smaller boundary layer growth on the endwall.
Fig. 25  Coefficient of the calculated static pressure in the 70° chamfer test section
where

\[ C_{pt} = \frac{p_0 - p_t}{\frac{1}{2} \rho U^2} \]

\[ p_0 = \text{upstream total pressure} \]
\[ p_t = \text{local total pressure}. \]

Fig. 26 Coefficient of the calculated total pressure in the 70° corner test section
Fig. 27 The velocity vectors and the calculated static pressures near the tip gap corner
5.3 Heat Transfer Equations

Using MEFP the energy equation was solved for combined heat transfer in the flow and in the blade tip. Writing the energy equation for the flow, Eq. (4.20), in terms of conductivity, \( k \),

\[
\frac{c_p \rho u \cdot \nabla T}{\nabla \cdot k_{tot} \nabla T = 0}
\] (5.6)

where

\[
k_{tot} = \frac{c_p \left( \frac{\mu}{Pr} \right)}{\text{eff}} = \frac{c_p \left( \frac{\mu_l}{Pr_l} + \frac{\mu_t}{Pr_t} \right)}{\text{eff}}
\] (5.7)

The laminar Prandtl number was taken as 0.7 and the turbulent Prandtl number as 1.0. The equation for heat conduction in the blade is;

\[
\nabla \cdot k_{bl} \nabla T = 0
\] (5.8)

For the calculations, the thermal conductivity in the blade, \( k_{bl} \), was taken as 300 times the laminar thermal conductivity of the gas (fluid); this agrees well with typical blade metal properties (see Appendix V).

Since the velocity is zero in the blade, Eqs. (5.6) and (5.8) may be combined to give

\[
\frac{c_p \rho u \cdot \nabla T}{\nabla \cdot k_{eff} \nabla T = 0}
\]

where \( k_{eff} = k_{tot} \) in flow and \( k_{bl} \) in the blade. This has the same form as Eq. (3.4), the rothalpy equation in MEFP, so that with \( \mu \) evaluated as \( k_{eff}/c_{pgas} \) MEFP may be used to solve Eq. (5.9).

Equation (5.9) was solved with the following boundary conditions:
1. Inlet gas temperature - 74°C
   
   Melting temperature of blade tip surface coating - 23°C

The inlet temperature for the calculation was given as the temperature difference $T_i = 74\degree - 23\degree = 51\degree$.

2. Suction side surface - adiabatic surface

3. Pressure side and blade tip surface - conducting surface

4. Blade metal temperatures,
   a) $T_{\text{blade}} = 0$
   or
   b) $T_{\text{base}} = 0$ (Fig. 19)

5. Endwall, $T = 0$

Three cases with different boundary conditions were considered, they were,

<table>
<thead>
<tr>
<th>Cases</th>
<th>Geometry</th>
<th>Temperature Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70° chamfer</td>
<td>$T_{\text{blade}} = 0$</td>
</tr>
<tr>
<td>2</td>
<td>70° chamfer</td>
<td>$T_{\text{base}} = 0$</td>
</tr>
<tr>
<td>3</td>
<td>90° corner</td>
<td>$T_{\text{base}} = 0$</td>
</tr>
</tbody>
</table>

All the calculations were performed with inlet fluid temperature as 51°C. Two cases used the 70° chamfer test section. For the first one, the blade was kept at 0°C, i.e., the energy equation was not solved in the blade. For the second one, the temperature at the blade tip base was fixed ($T_{\text{base}} = 0$) and the energy equation was solved throughout the flow and the blade tip; this calculation was performed to test the sensitivity of Nusselt number to the boundary conditions.
5.4 Calculated Temperatures

For Case 2, with boundary condition $T_{\text{base}} = 0$, temperature contours are shown in Fig. 28. Sixty percent of the temperature difference takes place in the blade tip. This is due to the high thermal conductivity of the blade metal; the rest, 40 percent, takes place between the freestream flow and the blade tip surface. The peak temperature in the freestream remained above 50°C at the exit.

5.5 Heat Transfer Results

From the temperature distribution, the Nusselt number for heat transfer to the blade tip is evaluated by two different methods.

a) Residual method

MEFP is used to obtain the heat flux to the wall directly as the residual of the discretized form of the energy equation. MEFP gives $\dot{q}/c_p$. The local Nusselt number is given by

$$\text{Nu} = \frac{\dot{q} \left(2 \text{ H} \right)}{c_p \left(T_i - T_w\right) \left(\frac{\mu_l}{\text{Pr}_l}\right)}$$

(5.10)

The Nusselt number is evaluated at the grid points.

b) Temperature difference method

In this method, $\dot{q}$ is evaluated directly from the temperature difference between the near-wall and the wall grid points.

$$\frac{\dot{q}}{c_p} = \left(\frac{\mu_l}{\text{Pr}_l} + \frac{\mu_t}{\text{Pr}_t}\right) \left(T_w - T_{w-1}\right) \frac{1}{\left(y_w - y_{w-1}\right)}$$

(5.11)

where $w$ is the wall grid point and $w - 1$ is the near wall grid point.
Fig. 28 Temperature contours from the solution of the energy equation for flow field and blade contour interval $\Delta T = 5'$
For this method, \( \text{Nu} \) was evaluated between the grid points in the \( x \) direction.

Metzger and Bunker normalized the local Nusselt numbers with the fully developed flow value. Here the local Nusselt numbers are normalized with the fully developed flow value calculated for the current tip gap grid using the program written for the grid tests, Section IV. This value, 60.54, is in the middle of the range of values obtained from the literature (see Appendix IV). Calculated Nusselt numbers normalized by the fully developed value for the blade tip, for Cases 1 and 2, are shown in Fig. 29. These results were obtained with the residual method and are shown together with Metzger and Bunker's data. The results agree well with the Metzger and Bunker data and are also quite insensitive to the blade tip boundary condition.

The Nusselt numbers evaluated by both methods, residual and temperature difference, Fig. 30, are similar. High heat transfer was calculated underneath the strong recirculation bubbles. Between the bubbles the calculated heat transfer fell below the fully developed flow value. At reattachment of the large recirculation bubble, the ratio \( \text{Nu}/\text{Nu}_\infty \) is calculated to be 1.55. After reattachment, the heat transfer decays, but remains above 1.1 at the exit, \( x/H = 13.6 \).

As shown in Fig. 31, the skin friction coefficient on the blade tip normalized by the fully developed flow value is negative in the separation zones and is near zero at reattachment. Further on, it increases to only 70 percent of the fully developed flow value by the exit. The velocity and temperature profiles tend to approach those for fully developed flow, but the wall slopes remain 30% below and 10% above
Fig. 29 Normalized Nusselt number on the blade tip as a function of distance, tip gap heights, for the cases 1 and 2
Fig. 30 Normalized Nusselt number as a function of distance, tip gap heights, for case 1 with both residual and temperature difference method.
Fig. 31 Normalized skin friction coefficient comparison with normalized Nusselt number for the case 2
fully developed values, respectively, at the exit.

Figure 32 shows the normalized Nusselt number, for the 70° chamfer test section, at the pressure surface as well as the blade tip surface. On the pressure surface, it is observed that the heat transfer rises rapidly to its maximum value at the tip gap corner. A slight increase in heat transfer is observed in the region around the chamfer corner on the pressure surface as sudden turning of the fluid causes a low pressure zone, which possibly enhances the heat transfer rate in that region.

Furthermore, normalized Nusselt numbers for both the test sections are compared in Fig. 33. The temperature difference evaluation method was used. The 90° corner profile is not as smooth as the 70° one. This may be attributed to the fact that the separation bubble for the 90° corner is longer and thicker than the 70° chamfer one, which leads to higher peak heat transfer and perhaps a less smooth approach towards the fully developed value.

5.6 Discussion of Results

For both the tests sections, the peak blade tip heat transfer rate has been tabulated in Table 1. For the 70° chamfer test section, Cases 1 and 2, maximum values of 1.7 and 1.68 were calculated, compared with 1.8 for the 90° corner. These peak heat transfer rates occurred upstream of reattachment at $x/h = 1.4$. These peak values and their locations are in good agreement with Metzger and Bunker's data with $\frac{Nu}{Nu_\infty}$, maximum, of 1.6 at $x/H = 1.4$.

All these results are also in reasonable agreement with Vogel and
Fig. 32 Normalized Nusselt number for the cases 1 and 2, on the blade tip and the pressure surface with residual method.
Fig. 33 Comparison of normalized Nusselt number for the 70° and 90° corners with temperature difference method
## CONCLUSION TABLE : (No. 1)

Flow and Heat Transfer Calculations:

<table>
<thead>
<tr>
<th>Angle</th>
<th>Length (Tip Gaps)</th>
<th>Temp. Boundary Cond.</th>
<th>Height Length (Tip Gaps)</th>
<th>Value At (Tip gaps)</th>
<th>CORNER TEMP (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70°</td>
<td>13.6</td>
<td>$T_{\text{blade}} = 0$</td>
<td>0.266 2.21</td>
<td>1.7 1.4</td>
<td>0</td>
</tr>
<tr>
<td>70°</td>
<td>13.6</td>
<td>$T_{\text{base}} = 0$</td>
<td>0.266 2.21</td>
<td>1.677 1.4</td>
<td>0.59</td>
</tr>
<tr>
<td>90°</td>
<td>13.6</td>
<td>$T_{\text{base}} = 0$</td>
<td>0.414 2.86</td>
<td>1.8 1.4</td>
<td>0.64</td>
</tr>
</tbody>
</table>
Eaton's experimental data for a backward facing step [11]. They observed that reattachment causes a local augmentation of the heat transfer coefficient by a factor of about two, and the maximum heat transfer occurs slightly upstream of reattachment.
The quest for lower fuel consumption, higher performance and greater efficiency, has led to engines with higher turbine inlet temperatures and cycle pressure ratios. Hence, turbine blade materials with significant improvements in strength and temperature capabilities are required. There has been substantial improvement in the thermal fatigue and creep properties of the materials over the last four decades [12], however, the same cannot be said of their temperature capabilities.

Figure 34 shows the development of the turbine entry temperature of aero-engines since 1950 and the corresponding increase in the maximum allowable blade material temperature. It can be seen that while the turbine inlet temperature has risen from 1100 K to around 1700 K, the allowable metal operating (bulk mean) temperature has only risen from 1000 K to about 1200 K.

In the last forty years, improvements have been made in the turbine blade materials. The development of materials, Fig. 35, has been from conventional cast alloys to directionally solidified eutectics (DS) and is finally moving towards ceramics [6].

6.1 Conventionally Cast Alloys

Astroloy (Ni - 15 Co - 15 Cr - 5.2 (Mo + W)), and Inconel 718 (Ni - 18 Fe - 19 Cr - 3.1 (Mo + W)), are typical of the conventional alloys which were used in the 1950's. These alloys were uneconomical to produce and had a lower creep rupture and yield strength at higher
Fig. 34 Development of turbine entry and material temperatures [Hennecke]
Fig. 35 Development of turbine blade materials [Wilson]
operating temperatures. Thus, they have been replaced by oxide dispersion strengthened nickel-chromium alloys and directionally solified eutectics.

6.2 Directional Materials

Directional solidification (DS) has been applied recently for the manufacture of conventional superalloy turbine blades. In DS, grains are nucleated at the base of a blade and allowed to grow only longitudinally or along the principal stress axis, thus eliminating grains transverse to the principal axis, which reduces the thermal fatigue of the material. There are two major eutectic systems, both having a ductile matrix reinforced with a brittle fiber or lamellar phase.

(a) $\gamma/\gamma'-\delta$ alloy (Ni - 20 Cb - 6 Cr - 2.5 Al) in which gamma-gamma prime forms the ductile matrix and delta ($\text{Ni}_3 \text{Cb}$) forms the lamellar phase.

(b) $\gamma/\gamma' - \alpha$ eutectic alloy, in which the molybdenum fibers (the $\alpha$ phase) form the lamellar phase in a $\gamma/\gamma'$ matrix.

These alloys provide a higher stress rupture life and greater transverse ductility and tensile shear strength. Nowadays, oxide-dispersion strengthened superalloys have become the leading contenders for turbine blades. These can be formed by combining dispersion strengthening with the gamma prime ($\gamma'$) strengthening mechanism of conventional superalloys [12]. Tungsten fiber reinforced superalloys are also being evaluated as materials for future turbine blades.
6.3 Ceramics

Ceramics represent the ultimate in service temperature capability. Their low cost and low density are the major attractions for aircraft turbines. Two of the most promising ceramics are Silicon Nitride ($\text{Si}_3\text{N}_4$) and Silicon Carbide (SiC). They both exhibit far greater stress rupture strength in the high temperature zone. Despite considerable improvements, the future potential of ceramics rests on the ability to tackle their brittle nature. Ductile ceramics are not feasible, although some success has been achieved by providing cushioning interfaces between ceramic blades and turbine disks.

6.4 Influence of Alloying Materials

The most widely used materials in the making of turbine blades have been nickel and cobalt superalloys. The gamma prime ($\gamma'$) strengthened nickel base alloys are much stronger than the solid solution and carbide strengthened cobalt base materials [13]. The constitutional elements of cobalt and nickel superalloys have a tremendous influence on the hot corrosion, oxidation, and erosion resistance of the material.

6.4.1 Influence of Chromium Additions

Chromium content in the basic alloy influences the oxidation resistance of the alloy. It has been observed, Fig. 36, that below about 10% chromium, the oxidation rate of the basic alloy increases. However, it decreases as chromium percentage starts increasing. This may be due to the formation of $\text{Cr}_2\text{O}_3$ or $\text{NiCr}_2\text{O}_4$ spinel oxide on the surface of the basic alloy. The chromium oxide ($\text{Cr}_2\text{O}_3$) forms a
Fig. 36 Influence of chromium content on the oxidation resistance of nickel [13]
continuous and compact layer. The thickness and compactness of this layer determines the oxidation rate of the basic alloy. The rate of formation of \( \text{Cr}_2\text{O}_3 \) is dependent on the nature of the base alloy. Interdiffusion of the different elements is higher in iron base alloys. Thus \( \text{Cr}_2\text{O}_3 \) forms more easily on iron base alloys. The hot corrosion resistance also increases with the increase in chromium content.

6.4.2 Influence of Aluminum Additions

Addition of aluminum increases the oxidation resistance of the base alloy, by forming a stable layer of \( \text{Al}_2\text{O}_3 \) on the surface.

6.4.3 Influence of Tantalum, Molybdenum or Tungsten Additions

Addition of tantalum to Co-Cr alloys increases their oxidation resistance, however, addition of molybdenum and tungsten has a deleterious effect on their oxidation rate.

6.4.4 Influence of Rare Earth Additions

The influence of rare earth and reactive elements such as Zr, Si, Mn, Y and Th improves the oxidation resistance of Ni or Co base alloys. The rare earths Ce, La and Gd reduce the hot corrosion resistance of the base alloys.
SECTION VII
Blade-Tip Life Considerations

Some of the factors which contribute to the life of turbine blade tips are:

(i) High temperature oxidation,
(ii) Hot corrosion,
(iii) Erosion.

Each one of these is discussed in detail in this section.

7.1 High Temperature Oxidation

To enhance the blade life, it is important for turbine materials (superalloys) to have resistance to oxidizing environments at high temperature. Oxidation involves a reaction of the blade material and oxygen in the gas. Oxidation is sensitive to the gas temperature, composition velocity, flow pattern, and thermal fluctuations of the environment.

7.1.1 Effect of Oxidation on Material

Oxidation reduces the load-bearing cross-sectional area and thus introduces stress concentration. This can occur via:

a) Surface Scaling: As the metal gets converted into oxide, it decreases the cross-sectional area and thus the load carrying capacity.

b) Internal Oxidation: It further reduces the load carrying ability and imposes stress concentrations which reduces fatigue resistance.
c) Oxide Spalling: Heterogenous scale growth resulting in oxide mismatch and thus increasing oxidation rate during thermal cycling.

d) Oxide Vaporization: A layer of protective oxide such as Chromium Oxide (Cr$_2$O$_3$) is vaporized resulting in loss of material and chromium from the alloy, which further enhances the oxidation rate [14].

Due to surface and internal oxidation, the thermal fatigue resistance and the mechanical strength reduces, resulting in the failure of the superalloy. This thermal fatigue resistance can be improved by applying a layer of coating over the surface, since it minimizes internal oxidation.

The effect of oxidation under creep-rupture conditions is not very well understood. The superalloy may fail due to scaling or intergranular penetration. On the other hand, under static load, the dislocation density and distribution in an alloy dictates the primary creep rate. A dispersion strengthening effect may be produced by the interference of internal oxides with grain sliding. The rupture life of Ni-base alloys is increased by the above mechanism.

The effect of oxide spalling and vaporization are deleterious at high temperatures although their mechanism are not well understood.

7.1.2 Effect of Applied Stress

Applied stress does not affect oxidation until a critical stress level is reached, above which the rate of oxidation is rapid and increases appreciably, as shown in Fig. 37. The critical applied stress is defined as the stress required to produce 1% extension in 100 hr.
Fig. 37 Critical applied stress vs. temperature [14]. The critical applied stress required to produce accelerated oxidation as a function of temperature.
Since the metal is being continually deformed by the applied stress, it inhibits the formation of protective and adherent oxide scales. This leads to continuous exposure of fresh metal to oxidation, resulting in a faster oxidation rate. The oxide layer can crack, separate, or even exfoliate if excessive stress is applied at the oxide-metal interface [14].

7.1.3 Effect of High Velocity Environments

Industrial gas turbines and jet engines are exposed to high velocity atmospheres with transonic Mach numbers, containing $O_2$, $N_2$, $CO_2$, $H_2O$, and $SO_2$. Considerable weight loss and internal oxide penetration can occur when metal is exposed to such environments, which may be due to oxidation-vaporization of chromium containing oxides. Thus, high velocity degradation plays a key role in limiting turbine rotor blade life. Figure 38 shows the weight loss produced in typical nickel based superalloys at 1367 K [14]; the values in parentheses represent the metal loss (mil/side) after 1000 hr exposure. A life limit suggested by Swansson et al. [15] is 0.010 inch (10 mil) loss of material; for the materials in Fig. 38 at 1367 K, the blade life would be in excess of 1000 hr. Reference [14] did not give sufficient additional information to predict the reductions in blade life due to further temperature increases or due to thermal cycling.

7.2 Hot Corrosion

Corrosion involves a chemical reaction resulting in damage to the blade. This chemical reaction is sometimes called sulfidation in which
Fig. 38 Weight change produced due to oxidation in high-velocity environment [14]. Values in parentheses represent the metal loss (mil/side) after 1000 hr exposure.
sulfur reacts with sodium to form sodium sulphate. Corrosion rates depends on blade material composition, blade coating, salt content of the air, sulfur content of the fuel, and temperature.

7.2.1 Chronology of Attack

A protective oxide layer of $\text{Cr}_2\text{O}_3$ (chromium oxide) is formed initially. Later a film of sodium sulphate salt is formed over the scale. Now the salt reacts with the protective oxide to form double oxides with sodium. This leads to breakdown of the protective oxide layer. Sulphur released through this reaction combines with chromium to form $\text{Cr}_x\text{S}$ thus depleting the alloy of chromium. This permits a faster rate of oxidation as insufficient chromium is available to maintain the $\text{Cr}_2\text{O}_3$ scale. Oxygen diffusion into the alloy depleted zone causes internal precipitation of $\text{Cr}_2\text{O}_3$ by displacing sulfur from $\text{Cr}_x\text{S}$, and the sulfur diffuses further into the alloy.

The liquid sodium sulfate penetrates the oxide scale and comes in direct contact with the underlying metal. Now, the protective layer cannot be maintained at the surface since it is dissolved by the salt ions and also due to depletion of chromium in the alloy. Thus, scale with a porous mixture of oxides permeated with salt is formed. These stages of corrosion for a high chromium, nickel base superalloy are shown in Fig. 39.

7.2.2 Effect of Temperature on Hot Corrosion

At low temperatures, the salt is solid and oxide scale is compact hence the rate of attack is low. Catastrophic attack occurs at the
Fig. 39 Plot showing the progression from 'simple oxidation' to 'hot corrosion' for a high chromium Ni-base alloy [14]
temperature at which the salt starts to melt. This is the 'threshold temperature' and generally attack increases further and is maximum at around 1253°K (980°C, 1800°F) for most alloys. At higher temperatures, the salt deposition decreases, hence the environment becomes less sulfidizing and more oxidizing. This temperature of transition from hot corrosion to high temperature oxidation is called the 'terminal temperature'.

Figure 40 shows the threshold and terminal temperatures for two salt concentrations in a burner-rig test [14]. Sulfidation is seen to be significant between about 1100 and 1300 K (1550 and 1900 F). Possible solutions for hot corrosion are: development of surface coatings which are resistant to sulfidation, removal of corroding impurities in the gas stream, developing alloys with superior hot corrosion resistance, and providing suitable design modifications to present day gas turbines to avoid the critical temperature range.

7.3 Erosion

Erosion takes place in a gas turbine due to erosive action of high velocity particles. Erosion is one of the major problems when a gas turbine is extensively used in dusty terrains. This reduces engine performance and increases it maintenance cost. Impact of particles can cause severe erosion leading to increased surface roughness and structural failure of blades.

Erosion rate is a function of particle and blade material properties, i.e.,

a) Velocity of impinging particles,
Fig. 40 Effect of increased salt concentration in burner-rig tests on the threshold and terminal temperatures of hot corrosion [14]
b) Direction or angle of attack of particles,
c) Number of striking particles,
d) Geometry and material of the blade row,
e) Material temperature.

The influence of the above factors on erosion are described in detail below.

7.3.1 Effect of particle velocity

From experimentation, the erosion rate has been shown to be proportional to the particle velocity taken to a constant power [16].

This erosion rate versus particle velocity is plotted and the constant power 'n' is the slope of the curve. The values for n depend on the angle of attack and the sample temperature.

7.3.2 Effect of Particle Impingement Angle (Direction or Angle of Attack)

The erosion rate depends on the impingement angle of the particles and increases from zero degrees, is maximum at about a 30° angle, and then decreases to a minimum of a 90° (perpendicular) impingement angle. The maximum rate occurs at different impingement angles for different materials.

7.3.3 Effect of Sample Temperature

As shown in Figs. 41 and 42, erosion rate is a function of sample temperature. As the temperature increases from room temperature to 700-750 K the erosion rate increases slowly, however, once the temperature rises above 700-750 K there is a rapid increase in erosion rate. The
Fig. 41 Erosion rate vs. sample temperature for Ti 6-4, particle velocity of 500 ft/sec and impingement angles of 25°, 45° and 90° [16]
Fig. 42 Erosion rate vs. sample temperature for INCO 718, for a particle velocity of 500 ft/sec and impingement angles of 25°, 45° and 90° [16]
sample materials used for these experiments [16] were Ti 6-4 and Inco 718. Thus at elevated temperatures, erosion may lead to structural failure of the blades. Erosion rate data does not appear to be available at the blade temperature levels associated with enhanced oxidation or corrosion, 1300-1500 K and 1100-1300 K, respectively.

7.3.4 Effect of Sample Material

Erosion rate is a function of sample materials. It is observed that for different materials, as the material temperature increases, the erosion rate increases rapidly but the rate of increase is different for each material.
SECTION VIII

Predictions of Turbine Tip Temperatures

Calculations were performed to evaluate the blade tip surface temperatures for both the 70° chamfer and the 90° corner test sections considered in Section V. A plot of the tip surface temperatures is shown in Fig. 43. The plot indicates relatively uniform tip surface temperatures with blade tip corner temperatures of 59% and 64% of the difference between freestream fluid and blade tip base temperatures for the 70° and 90° corners, respectively.

Turbine rotor blade metal temperatures at the tip on the pressure side can be estimated from these results. Suppose the temperature at the base of the blade tip is 1200 K and the relative total temperature of the gas entering the tip gap is 1650 K (see Appendix I). The corner temperatures will then be 1466 K and 1488 K for the 70° and 90° corner test sections as shown in Fig. 44, respectively. These temperatures are approaching the melting point of modern blade metals (see Appendix V). This may lead to blade tip corner burnout on the pressure surface, as reported by Bindon [5].

These high operating temperatures together with adverse environmental conditions may cause thermal fatigue, high oxidation rates and failure due to corrosion and erosion. Thus enhanced heat transfer to unshrouded turbine blade tips can be a major factor in determining blade life in high temperature jet engines.
Fig. 43 Calculated blade tip temperature as a fraction of \( T_{\text{inlet}} - T_{\text{blade tip base}} \) for the 70° and 90° corners.
Fig. 44 Blade tip corner temperatures achieved for the 70° and 90° test sections with typical jet engine turbine temperatures
SECTION IX

Conclusions

A Navier-Stokes solver, the Moore Elliptic Flow Program, was used to calculate developing flow and heat transfer in turbine tip gaps. Successful predictions of heat transfer to a model turbine blade tip were obtained with a Prandtl mixing length turbulence model. A combined solution of the energy equation in the tip gap flow and in the rotor blade tip gave tip temperature distributions.

9.1 Computational Study of Finite-Difference Gridding

An independent computational study, using the same numerics as MEFP but a separate new computer program, was performed to investigate the numerical accuracy of heat transfer calculations for fully developed flow. Two items were considered: (1) the choice of near-wall control volume, and (2) the number and spacing of grid points.

In terms of the tip gap calculations using MEFP, the significant conclusions from the fully developed flow study were:

a) Grids which are adequate to resolve the velocity distribution and the wall shear stress are also adequate to resolve the temperature distribution and the wall heat transfer.

b) A 27 point grid across the tip gap gave a Nusselt number of 60.5 which lies in the middle of the range of values 60.5 ± 8.3, obtained from the literature.
9.2 Ability to Calculate Tip Leakage Flow and Heat Transfer

Two test sections were considered, a 70° chamfered corner corresponding to the Metzger and Bunker geometry and a 90° corner typical of a turbine rotor blade.

9.2.1 Flow Results

a) Flow separates at the tip gap entrance.

b) For the 70° and the 90° corner, the length of separation bubble are 2.2 H and 2.9 H, respectively, and the height of the bubble are 0.27 H and 0.41 H, respectively.

c) Two pressure minima are observed. For the 70° corner, one with \( C_{p_s} = -1.5 \) is near the center of the recirculating flow, and the other is located at the corner, \( C_{p_s} = -2.0 \).

d) A second, very small recirculation zone is observed near the corner.

9.2.2 Heat Transfer Results

The energy equation was solved to obtain the heat transfer to the blade tip with,

(i) Temperature in the blade fixed, \( T_{\text{blade}} = 0 \).

(ii) Heat conduction in the blade and the blade tip base temperature set to zero, \( T_{\text{base}} = 0 \).

a) The calculated Nusselt numbers were not sensitive to the choice of the boundary conditions.

b) The Nusselt numbers evaluated with the temperature difference method gave smoother results than those evaluated using the residual method.
c) Enhanced heat transfer with $\frac{Nu}{Nu_\infty}$ from 1.7 to 1.8 was calculated underneath the strong recirculation bubble, compared with 1.6, measured by Metzger and Bunker.

d) The calculated heat transfer rates agree well with the Metzger and Bunker data at the location of the peak enhancement ($\frac{x}{H} = 1.4$) and downstream.

e) Neither the velocity nor the temperature profiles were fully developed at the exit ($\frac{x}{H} = 13.6$). At this location, the calculation gave $\frac{Nu}{Nu_\infty} = 1.1$ and $\frac{C_f}{C_{f\infty}} = 0.7$ on the blade tip.

9.3 Rotor Blade Tip Life Considerations

Turbine blades currently in use are typically made of directionally solidified eutectics, for example, $\gamma/\gamma' - \delta$ alloy (Ni - 20 Cb - 6 Cr - 2.5 Al). From the literature review, approximate temperature ranges for significant blade life reduction mechanisms are (Fig. 45):

- Hot corrosion - 1100 - 1300 K
- Oxidation - above 1300 K
- Melting - above 1600 K

Turbine rotor blade temperatures at the tip on the pressure side were estimated from the present calculations. The temperature estimated for the 90° corner test section under typical engine conditions was 1488 K. This temperature is approaching the melting point of modern blade metals and lies in the range where high oxidation occurs. This may lead to blade tip corner burnout on the pressure surface as reported in the literature.
Fig. 45 Approximate temperature ranges for significant blade life reduction mechanisms
REFERENCES


Typical turbine inlet and blade metal temperatures for jet engine turbines are given in Fig. 46.

Consider a reaction turbine with a rotor velocity triangle as shown in Fig. 47. If the turbine inlet total temperature, $T_{t1}$, is 1850 K and $a_1 = 70^\circ$ and $\beta_1 = 45^\circ$

$$\frac{V_1}{\sin 135^\circ} = \frac{W_1}{\sin 20^\circ}$$

$$W_1 = V_1 \frac{\sin 20^\circ}{\sin 135^\circ} = 0.484 V_1$$

Say $M_1 = \frac{V_1}{\sqrt{\gamma R T_1}} = 1.0$ then $\frac{T_1}{T_{t1}} = 0.857$

where $T_1 = \text{static temperature}$

$$T_1 = 1850 \times 0.857 = 1585.5 \text{ K}$$

For combustion gases,

$\gamma = 1.333$

$R \sim 287 \text{ J/kgK}$

$$V_1 = \sqrt{1.333 \times 287 \times 1586} = 778.8 \text{ m/s}$$

$$\therefore \quad W_1 = 0.484 \times 809.9 = 377 \text{ m/s}$$
Fig. 46 Typical turbine inlet and blade metal temperatures for jet engine turbine [Gell and Thomas]
Fig. 47 A reaction turbine with a rotor velocity triangle
we know,

\[ T_{tr_1} = T_1 + \frac{W_1^2}{2 \rho} \]

\( \text{(relative total temperature)} \)

\[ . \quad . \quad T_{tr_1} = 1585.5 + \frac{(377)^2}{2 \times 4 \times 287} = 1647 \text{ K} \]

The relative total temperature, \( T_{tr_1} \), is the adiabatic stagnation temperature seen by the turbine rotor blade.
APPENDIX II

Near-Wall Velocity Overshoot with Regular Control Volume in an Accelerating Flow.

The overshoot is demonstrated for the geometry shown in Fig. 48.

Given: Upstream  Downstream

\[ \rho = 1 \quad \rho = 1 \]
\[ U(1) = 0 \quad U(1) = 0 \]
\[ U(2) = 1 \quad U(2) = ? \]
\[ U(3) = 1 \quad U(3) = 2 \]
\[ P_t(2) = P_t(3) = 0 \quad P_t(3) = 0 \]

Objective: Calculate \( U(2) \) downstream using discretized form of the inviscid momentum equation,

\[ \rho \frac{\partial u}{\partial x} = -\frac{\partial P}{\partial x} \]  \hspace{1cm} (1)

The upstream and downstream static pressures are calculated from the total pressure at \( J = 3 \),

\[ P + \frac{1}{2} \rho U^2 = P_t \]  \hspace{1cm} (2)

Equation (1) is then discretized using linear profile between the grid points for the regular and altered near-wall control volume as shown in Fig. 49.

Equation (1) can then be integrated,
Fig. 48 Near-wall velocity overshoot with regular control volume in an accelerating flow
\[ \int (\rho \ u \ \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x}) \ dv \] (3)

In discretized form,

\[ \rho \ \frac{(U_c + U_a) (U_a - U_c)}{2} \ \delta_x \ \frac{\delta y}{2} \ \text{(top)} \]

\[ + \ \rho \ \frac{(U_b + U_d) (U_b - U_d)}{2} \ \delta_x \ \frac{\delta y}{2} \ \text{(bottom)} \]

\[ = -\frac{\partial p}{\partial x} \ \delta_x \ \delta y \] (4)

where

\[ U_a = \frac{3}{4} \ U(2), \]

\[ U_b = \frac{3}{4} \ U(2) + \frac{1}{4} \ U(3), \]

\[ U_c = \frac{3}{4} \ U(1) \ \text{upstream} \]

and \[ U_d = (\frac{3}{4} \ U(2) + \frac{1}{4} \ U(3)) \ \text{upstream} \]

Substituting these values of \( U_a, U_b, U_c \) and \( U_d \) in Eq. (4), we get,

\[ \frac{(\frac{3}{4} + \frac{3}{4} \ U(2)) (\frac{3}{4} \ U(2) - \frac{3}{4})}{2} \ + \ \frac{(\frac{3}{4} \ U(2) + .5 + 1) (\frac{3}{4} \ U(2) + .5 - 1)}{2} \]

\[ = -(-2 - (-.5)) = 1.5 \] (5)

On solving,

\[ U(2) = 2.24 \]
Calculations for Altered Near-Wall Control Volume

For the altered near-wall control volume, Fig. 49, the discretized form of Eq. (3) can be written as,

\[
\rho \left( \frac{U_b + U_d}{2} \right) \left( \frac{U_b - U_d}{\delta_x} \right) \frac{\delta x \delta y}{2} = -\frac{\delta P}{\delta x} \frac{\delta x \delta y}{2}
\]  

(6)

\[
\left( \frac{3}{4} U(2) + .5 + 1 \right) \left( \frac{3}{4} U(2) + .5 - 1 \right) = \frac{1.5}{2}
\]  

(7)

on solving,

\[ U(2) = 2.0 \]

From the calculation, in an accelerating flow with viscous effects being insignificant, a velocity overshoot is observed with the regular near-wall control volume. This 12\% overshoot gives too high a total pressure at the near wall point. The altered near-wall control volume gives a correct total pressure at the near-wall point.
Fig. 49 Discretized altered and regular near-wall control volumes
APPENDIX III

TDMA Subroutine

Equations (4.9) and (4.27) were solved using the tri-diagonal matrix algorithm (TDMA). The method of solution is described here.

Equations to be solved:

\[ A(J) \times U(J + 1) + B(J) \times U(J) + C(J) \times U(J - 1) = D(J) \]

for \( J = 2 \) to \( J_{\text{max}} - 1 \), with

\[ U(1) = 0 \text{ and } U(J_{\text{max}}) = 0 \]

The equation can be written in matrix form as;

\[
\begin{pmatrix}
B(1) & A(1) & 0 & 0 & \cdots & 0 \\
C(2) & B(2) & A(2) & 0 & \cdots & 0 \\
0 & C(3) & B(3) & A(3) & \cdots & 0 \\
\end{pmatrix}
\begin{pmatrix}
U(1) \\
U(2) \\
U(3) \\
\end{pmatrix}
= 
\begin{pmatrix}
D(1) \\
D(2) \\
D(3) \\
\end{pmatrix}
\]

The first equation, for \( J = 1 \), may be written as,

\[ B(J) \times U(J) + A(J + 1) \times U(J + 1) = D(J) \] (1)

It can also be written as

\[ U(J) = D^R(J) + A^R(J) \times U(J + 1) \] (2)

where

\[ D^R(J) = D(J)/B(J) \] (3)
and

$$A^n(J) = -A(J)/B(J)$$ (4)

The $J + 1$th equation is

$$C(J + 1) U(J) + B(J + 1) U(J + 1) + A(J + 1) U(J + 2) = D(J + 1)$$ (5)

Substituting the equation for $U(J)$, Eq. (2), into Eq. (5) and rearranging Eq. (5) into the form

$$U(J + 1) = D^n(J + 1) + A^n(J + 1) U(J + 2)$$ (6)

gives,

$$D^n(J + 1) = \frac{D(J + 1) - C(J + 1) D^n(J)}{C(J + 1) A^n(J) + B(J + 1)}$$ (7)

$$A^n(J + 1) = \frac{A(J + 1)}{C(J + 1) A(J) + B(J + 1)}$$ (8)

Thus $D^n(J)$ and $A^n(J)$ can be evaluated from Eqs. (7) and (8), from $J = 1, 2, \ldots, J_{\text{max}}$. By backsolving, $U(J)$ is evaluated from Eq. (6), from $J = J_{\text{max}}, \ldots, 2, 1$. 
APPENDIX IV

Estimates of Fully Developed Heat Transfer for Metzger and Bunker Case

Metzger and Bunker normalized the Nusselt number by the fully developed flow value. An estimate of fully developed flow heat transfer in circular channels is given by McAdams formula [18].

\[
\frac{hD}{k_b} = 0.023 \left( \frac{G}{b} \right)^{0.8} \left( \frac{\mu}{\mu_b} \right)^{0.4} \quad (1)
\]

where,

G - mass flow per unit area

b - bulk mean value

and the rest of the symbols have their usual meaning. Equation (5.3) can be written as

\[
Nu_\infty = 0.023 \left( Re_b \right)^{0.8} \left( Pr \right)^{0.4} \quad (2)
\]

It is valid for \( 0.5 < Pr < 120 \) and \( 2300 < Re < 10^7 \). Formulas by Colburn, and Sieder and Tate [18], for the same flow range, may be written in the similar form as

\[
Nu_\infty = 0.023 \ Re^{0.8} \ Pr^{0.33} \quad (3)
\]

(Colburn)

and

\[
Nu_\infty = 0.027 \ Re^{0.8} \ Pr^{0.33} \quad (4)
\]

(Sieder and Tate)
For the Metzger and Bunker test case,

\[ \text{Re}_D = 30,000 \]
\[ \text{Pr} = 0.7 \]

These equations (2-4) give \( \text{Nu}_\infty = 76, 78, \) and \( 91.5 \), respectively.

It is suggested that for turbulent flow in non-circular channels—rectangular, triangular, etc., a good approximation is achieved by multiplying the above equations by a factor of 0.75 and replacing \( D \) by hydraulic diameter [18].

Thus from the equations, the fully developed Nusselt numbers for Metzger and Bunker channel flow are 57, 58.5, and 68.6, respectively.

Another estimate for fully developed turbulent flow in circular tubes for \( 0.5 < \text{Pr} < 1.0 \) is given by Kays and Crawford [8] as

\[ \text{Nu} = 0.022 \text{Re}^{0.8} \text{Pr}^{0.5} \text{ for } \text{Re} < 10^5 \] (5)

Multiplying this by 0.75 gives a Nusselt number of 52.2 for the current test case.

The fully developed value of Nusselt number can be found from the fully developed flow calculation, Section IV, as follows for the 27 point grid.

\[ \text{Nu}_\infty = \frac{2 H}{T_{\text{inlet}}} \left( \frac{T_{\text{wall}} - T_{\text{nearwall}}}{y_{\text{wall}} - y_{\text{nearwall}}} \right) \]

\[ H = 0.00508 \text{ m} \]
\[ y_{\text{w}} - y_{\text{nw}} = 0.317 \times 10^{-4} \text{ m} \]

\[ (T_w - T_{\text{nw}})/T_i = 0.1892 \]
Thus from our calculations for the Metzger and Bunker case, the fully developed value is 60.54, for the 27 point grid; the 501 point grid gave $\text{Nu}_\infty = 58.4$.

The fully developed flow calculation for turbulent flow between parallel plates, with the Prandtl mixing length turbulence model and using the 27 point grid gave a Nusselt number in the middle of the range of the values from Eqs. (2) to (5), 60.5 ± 8.3. Therefore, the calculated Nusselt numbers for the developing flow will be normalized by the calculated fully developed value for the 27 point grid, 60.54.
APPENDIX V

Thermal Conductivity and Melting Temperature of Ni-Cr Alloys

Reference [10] gives thermal conductivities of Ni-Cr alloys, for example, Rene 41, and of air. These compared in the following table.

| Temperature (K) | Rene 41 | 1212 | 24.1 |
| | | 1392 | 26.7 |
| | | 1617 | 30.0 |
| Air | 1300 | 0.0797 |
| | 1500 | 0.0870 |
| on extrapolating - | 1700 | 0.0943 |

For typical turbine rotor blade metal temperatures of 1200-1300 K, and typical gas temperatures of 1500-1700 K (see Appendix 1),

\[
\frac{k_{metal}}{k_{gas}} \sim 250-300.
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>Ref. No.</th>
<th>Melting Point Temperatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr</td>
<td>10</td>
<td>2118</td>
</tr>
<tr>
<td>Ni + Cr + ΣX, Alloys</td>
<td>10</td>
<td>1726</td>
</tr>
<tr>
<td>(Ni + Cr + Fe, Ternary alloy)</td>
<td>20</td>
<td>1668-1700</td>
</tr>
<tr>
<td>(minimum) solidus temperature</td>
<td></td>
<td>1573</td>
</tr>
<tr>
<td>(minimum) liquidus temperature</td>
<td></td>
<td>1623</td>
</tr>
</tbody>
</table>

Figure 50 shows the liquidus and solidus diagrams for Ni + Cr + Fe, ternary alloys[20].
Fig. 50 Liquidus and solidus diagrams for Ni-Cr-Fe ternary alloys [20]; temperatures in °C
The vita has been removed from the scanned document