THE SYNTHESIS OF FOUR-BAR LINKAGE COUPLER CURVES USING DERIVATIVES OF THE RADIUS OF CURVATURE

by

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Table of Contents

List of Figures

List of Tables

Nomenclature

S	Slider position measured from origin of x-y coordinate system in the x-direction
ϕ	Crank angle of input or driving crank
K	Constant
ϕ_s, S_s	Starting or initial values of crank angle and slider position
ϕ_f, S_f	Final values of crank angle and slider position
ϕ_1, ϕ_2, ϕ_3	Precision positions of crank
S_1, S_2, S_3	Precision positions of slider
ρ	Magnitude of the radius of curvature of coupler curve
r	Radius of driving crank
a_3	Slider path offset from the x-axis
l	Length of connecting rod
z	Displacement of coupler point along the coupler curve
M,N	Constants in the cubic of stationary curvature
s,t	Coordinate system at the link pole aligned with the centrode tangent
ψ	Angle in the s-t coordinate system measured from the centrode tangent
Point T	Point on the centrode tangent
r_ψ	Radius vector from the pole of the s-t coordinate system to points on the cubic of stationary curvature
b_s	Intercept of the s-axis by the connecting rod
m	Slope of the connecting rod in the s-t coordinate system
x,y	Coordinate system with origin at the pole of the driving crank. The x-axis is parallel to the slider path.

x', x'', etc.	Derivatives of the x coordinate position with respect to the input crank angle ϕ
y', y'', etc.	Derivatives of the y coordinate position with respect to the input crank angle ϕ
xx,yy	Coordinate system fixed to connecting rod with the origin at the input crank end of the rod and the xx axis aligned with the rod
λ	xx/l dimensionless coordinate in the xx direction
l_x	Horizontal or x-component of the connecting rod length
θ	Included angle of connecting rod at the driving crank end
γ	Angle between the connecting rod and the x-axis
x_c, y_c	Coordinates, in the x-y system, of the center of curvature or link pole
RR	Length of follower crank
ϕ_D	Design crank angle
a	Length of driving crank
u,v	Coordinate system fixed to the coupler with the origin at the input crank end of the coupler and the u axis aligned with the coupler
β	Angle between the coupler and the x-axis
$x_{co}, x'_{co}, x'''_{co}$, etc.	Constant coefficients in x, x', x'', etc.
x_u, x'_u, x''_u, etc.	Coefficients of u in x, x', x'', etc.
x_v, x'_v, x''_v, etc.	Coefficients of v in x, x', x'', etc.
$y_{co}, y'_{co}, y''_{co}$, etc.	Constant coefficients in y, y', y'', etc.
y_u, y'_u, y''_u, etc.	Coefficients of u in y, y', y'', etc.
y_v, y'_v, y''_v, etc.	Coefficients of v in y, y', y'', etc.
Z,W,U,T,S,R, Z',U',T'	Dummy variables used for simplification

a_{jk}	Constant
f_1	Numerator of $d\rho/d\phi$
f_2	Numerator of $d^2\rho/d\phi^2$
F,G,H,J	Dummy variables used for simplification
b	Length of coupler or connecting rod
μ	yy/l, dimensionless coordinate in the yy direction
Point A	Connection point between the driving crank and the coupler
Point B	Solution coupler point that is constrained
Point I	Pole of the coupler
Point C	Revolute-coupler attachment in original linkage configuration
c	Length of follower crank in original four-bar configuration
d	Length of fixed link in original four-bar configuration
V,D,E,P,Q,L,HH, JJ,UU,AA,BB	Dummy variables used for simplification
KK,MM,NN	Dummy variables used for simplification
XB,YB	Center of curvature of coupler point path in x-y coordinate system
XF,YF	Location of coupler point in the original configuration in the x-y coordinate system
IA,IB	Length of line segments from I to A and B
ψ_A, r_A	Coordinates of Point A in polar coordinate system at the coupler pole
ψ_C, r_C	Coordinates of Point C in polar coordinate system at the coupler pole

Introduction

If a link as shown in Fig. 1 is in plane motion with two points on the link constrained to particular paths, the nature of the paths of all other points on the link is known. In particular, the functional behavior of the radius of curvature of the path of any point on the link and the derivatives of the radius of curvature with respect to some displacement parameter may be ascertained. Given this determination of the radius of curvature of the path of the points, it is possible to approximate the motion of the link by approximating the behavior of the derivatives of the radius of curvature of the paths of points on the link.

In pin-connected planar linkages, there are two broad classifications for the links. The first classification includes those links which are constrained to rotate about a fixed point. These links are called cranks or revolutes. It should be noted that, though the center of rotation may be at some infinite distance from the link, the link remains a revolute and it may be referred to as a slider because the link, in such a case, is in translation. The second classification, called couplers or connecting rods, includes those links which are used to connect a pair of revolutes or a revolute and another coupler. Points on the coupler, or coupler points, may be varied paths or curves depending upon the location of the point on the coupler and upon the constraints imposed upon the coupler by its revolutes. The concern of this discussion is the location of the coupler point in the plane of the coupler with freedom being allowed in one or two coordinate directions, assuming that revolute constraints have been defined.

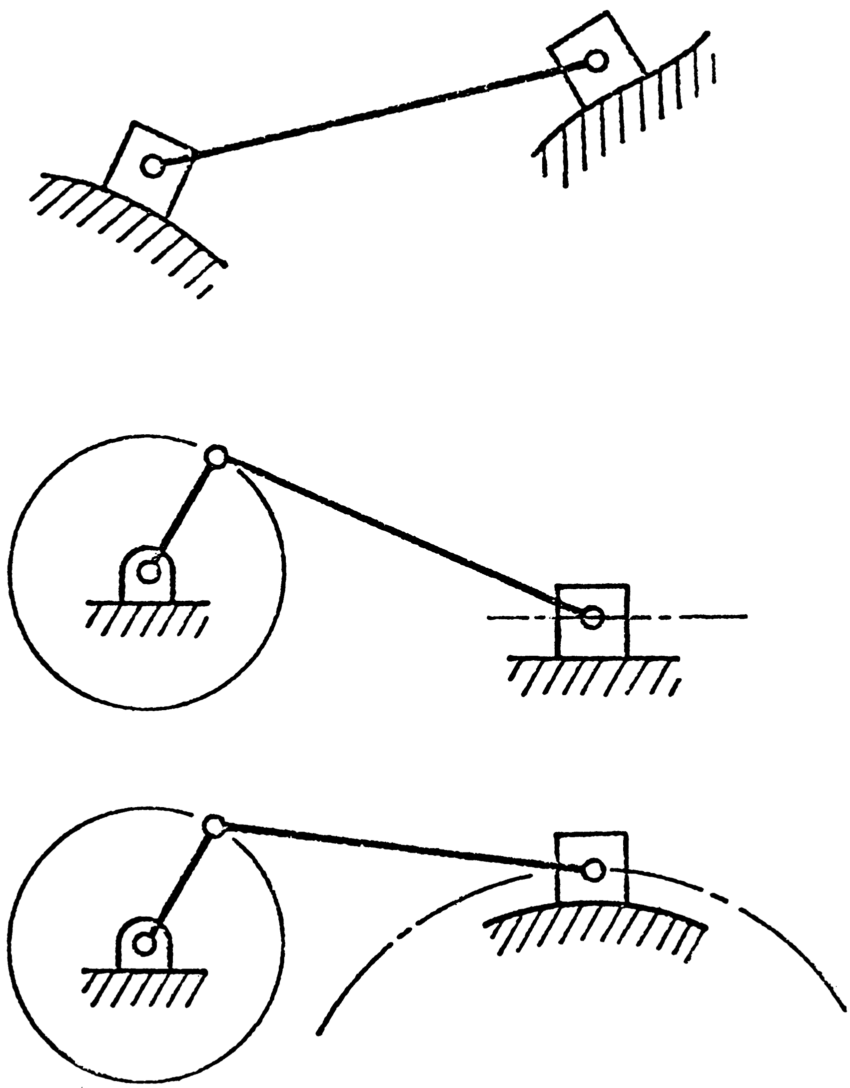

Figure 1. A Constrained Link with Variations of the Four-Bar Linkage

The discussion focuses upon the coupler or connecting rod of the four-bar linkage in two common configurations as shown in Fig. 1. One end of the coupler is constrained to move in a circular path, and the other end of the coupler is constrained to move in either a straight path or a circular path. The straight path case is investigated using the first derivative of the radius of curvature of the coupler point path with respect to the coupler point displacement. In this straight path case, the coupler point location has one degree of freedom. Additionally the straight path case and the circular path case, each with two degrees of freedom in the coupler point location, are studied using the behavior of both the first and second derivatives of the radius of curvature of the coupler point path with respect to the input crank angle.

The four-bar linkage with either a straight-line or circular coupler curve finds wide usage in the design of mechanisms for a variety of applications. Materials handling, graphic arts, agricultural, earth moving, and business machines are but a few of the broad classes of machinery employing four-bar linkages with specific coupler curves.

The synthesis of linkages with distinctive coupler curves has long been a topic of kinematic interest. Recent and older investigations have developed a number of methods for the synthesis of particular four-bar linkage coupler curves. A search of the literature reveals that these methods are predominately graphical or analytical techniques which focus upon planar characteristics other than the radius of curvature.

A literature search disclosed no synthesis procedure based upon derivatives of the radius of curvature other than those governed by the behavior of the radius of curvature at some extreme values. It should be pointed out that until recent times, the analytical tools such as the computer and relevant software necessary for a comprehensive attack of the problem did not exist.

The discussion that follows focuses upon the development of the equations essential to this synthesis procedure in the order of increasing complexity of the original mechanism, of the equations, and of the solution methods. Attention is first given to the single degree of freedom case, then to the two degrees of freedom cases. The single degree of freedom case is developed only with respect to the straight-line path. For the two degrees of freedom situation the general equations and their solutions are developed and particularized with respect to straight-line, circular, and arbitrary paths.

Literature Review

The problem of synthesizing a multi-linked mechanism with either a straight-line, circular, or generalized coupler curve has been treated extensively in the literature. This discussion will focus upon the recent works for each of the coupler curve types mentioned above.

The generalized coupler curve may be approximated by using a least squares synthesis technique as proposed by Levitskii [1] and Sarkisyan [2] with extensions by Bagci [3] and Southerland [4]. Essentially, this technique requires the description of points in the fixed plane through which the sixth-order coupler curve of the four-bar linkage should pass. Equations, with the size and position of the mechanism links as variables, are determined such that the sum of the squares of the deviations of the fixed-plane points from the coupler curve is minimized. A set of linear simultaneous equations in the linkage parameters results and this set of equations may be solved to yield the best least squares approximation. This procedure is not a method that guarantees displacement precision at any point and errors in velocities and accelerations are inevitable. However, the procedure can accommodate up to nine degrees of freedom in the specification of the four-bar linkage.

The problem of synthesizing a four-bar linkage with a straight-line coupler curve has been treated most recently for the case of adjustable linkages. Tao and Amos [5] and Tesar and Watts [6] developed procedures, both graphical and analytical, using the Ball point for the linkage synthesis. The Ball point is that point of intersection of the locus of points with an infinite radius of curvature and of the locus of points whose radius of curvature is momentarily invariant. The emphasis in these two investigations was upon the adjustable nature of the resulting linkage. In either case though the procedures presented do result in no displacement and velocity errors, they do not permit the designer to specify the straight path initially. Krishnamoorthy and Tao [7], Beaudrot [8], and McGovern and Sandor [9] have contributed to the literature in the synthesis of adjustable mechanisms for straight-line paths. The work of McGovern and Sandor is notable in that a complex number technique was employed.

Hoekzema and others [10] presented a method by which an adjustable four-bar linkage with a variable radius of curvature of the coupler curve may be synthesized. This method is based upon a graphical procedure whereby the instant center of the coupler is made to coincide with a non-adjustable fixed pivot. Krishnamoorthy and Tao [11] have extended the work of Hoekzema but their procedures are graphical in nature and concerned primarily with the adjustable behavior of the linkage.

The method of synthesis of straight and circular coupler curves, as presented in this dissertation, allows the linkage designer to specify the coupler point path and the driving crank and coupler. This flexibility permits other synthesis procedures to be applied beforehand in order to assure some approximate functional relationship between the driving or input crank position and the coupler point position. Through the use of the equations presented herein one is able to locate points of constraint on the coupler such that the approximate coupler curve is realized and such that, at the design position, errors in position, velocity, and acceleration are avoided.

Chapter 1

Straight Line Path - First Derivative

Attention is given to the synthesis of a four-bar linkage with a straight line coupler point path utilizing the first derivative of the radius of curvature with respect to a displacement of the coupler point. A procedure is presented by which a four-bar linkage may be synthesized such that a coupler point of the four-bar linkage will retain its functional relationship with the crank and will also have approximately straight line motion. Thus, having selected a slider-crank mechanism for a particular application, one may use this procedure to determine a four-bar linkage that is suitable for the same application.

A slider-crank mechanism may be designed to approximately satisfy virtually any functional relationship between the crank position and the slider position. As an example, a slider-crank mechanism with the slider displacement proportional to the crank rotation will be synthesized using the method of Freudenstein [12]. The slider displacement S can be expressed as a function of the crank rotation ϕ as follows:

$$\Delta S = K\ (\Delta\phi)$$

where K is a constant of proportionality.

Assuming

ϕ_s = 30 degrees	S_s = 10.00 cm
ϕ_f = 90 degrees	S_f = 6.00 cm
$\Delta\phi$ = 60 degrees	ΔS = 4.00 cm

Using Chebyshev spacing of the crank positions for three accuracy points as shown by Hartenberg and Denavit [13]

ϕ_1 = 34.02 degrees

ϕ_2 = 60.00 degrees

ϕ_3 = 85.98 degrees

Therefore,

S_1 = 9.7321 cm

S_2 = 8.0000 cm

S_3 = 6.2679 cm

As a result,

r = 3.1762 cm	Crank Radius
l = 8.2918 cm	Connecting Rod Length
a_3 =-2.5068 cm	Slider Path Offset

This slider-crank mechanism which is shown in Fig. 2 satisfies approximately the required functional relationship between the crank and slider positions and provides exactly a straight line motion of the slider, Point C. Attention will now be given to the synthesis of the straight line motion using a four-bar linkage, with particular emphasis on the retention of the approximate functional relationship.

Given a generalized slider-crank mechanism as shown in Fig. 3, points exist on the connecting rod where, the change in the radius of curvature of the coupler point path, dρ, for each infinitesimal displacement, dz, of such points is zero or

$$\frac{d\rho}{dz} = 0 \qquad (1)$$

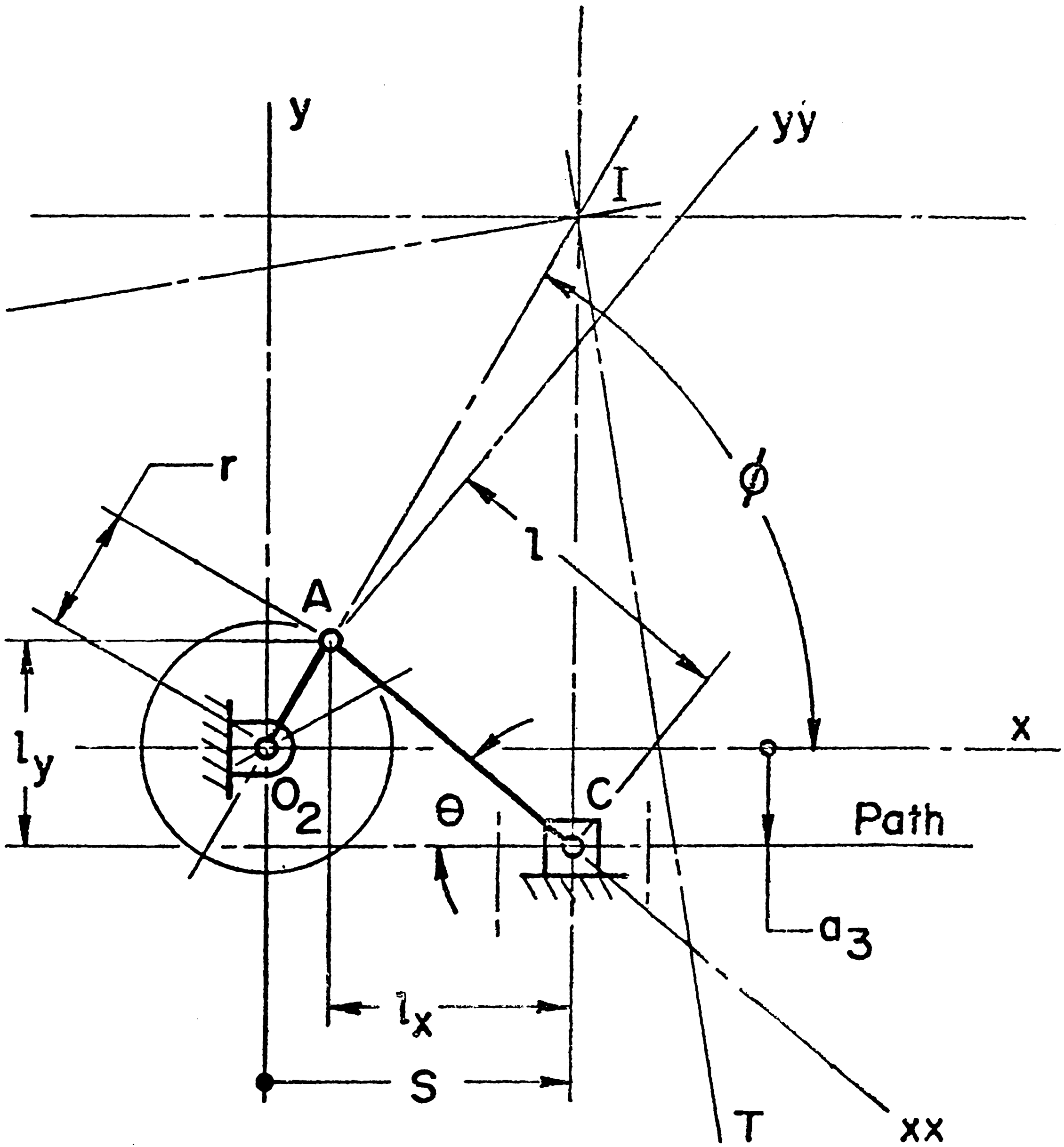

Figure 2. The Slider-Crank Mechanism

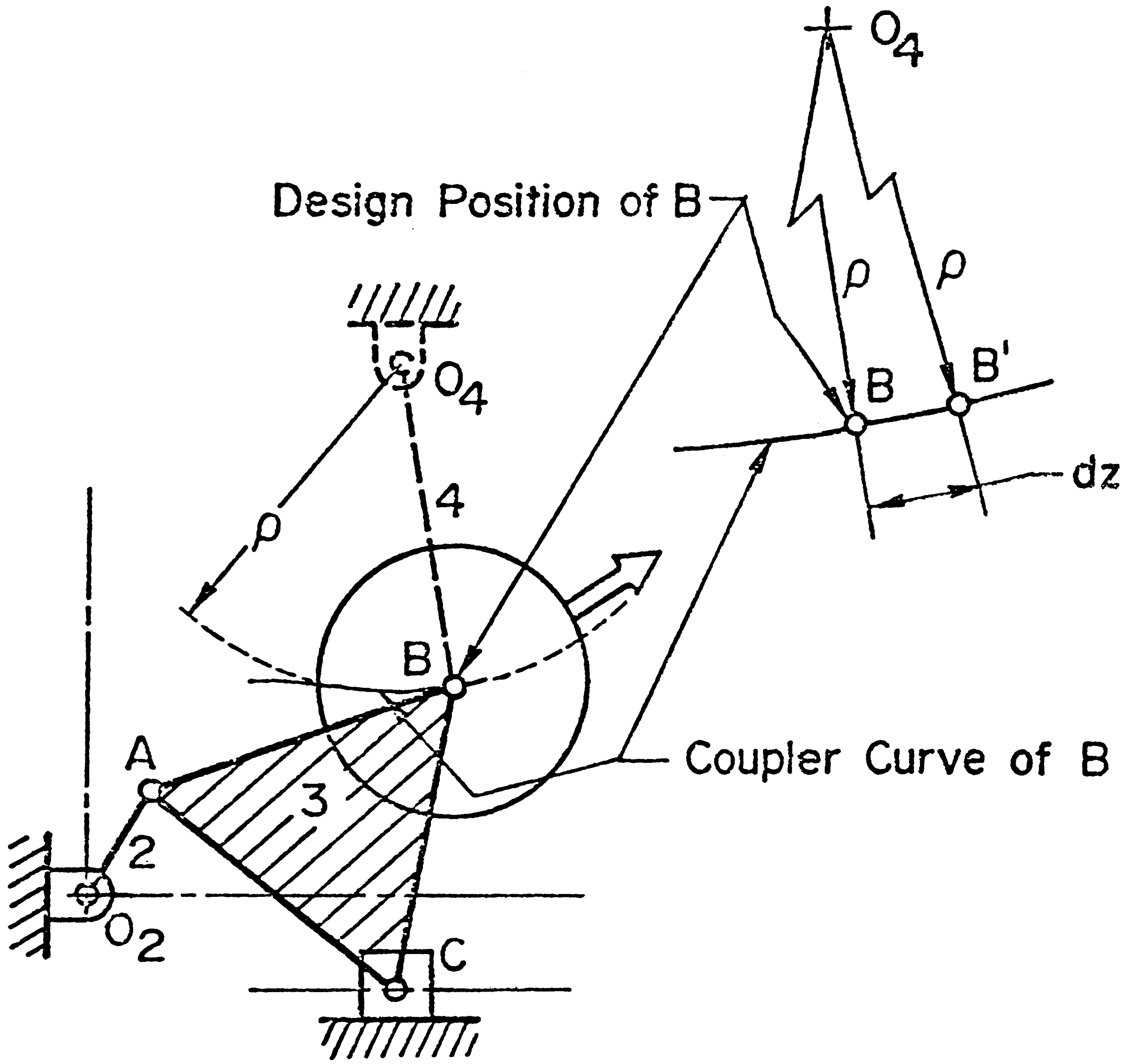

Figure 3. A Slider-Crank Mechanism with a Change in Coupler Constraints

The object of this synthesis procedure is the location of a point on the coupler whose path is approximately circular. Such a point is Point B in the slider-crank mechanism shown in Fig. 3. If the crank in Fig. 3 is permitted a small rotation such that Point B is displaced an infinitesimal distance, dz, along its coupler curve, Eq. 1 will be satisfied if the radius of curvature is invariant. Thus Point B may be constrained with a revolute since the revolute will enforce the condition that $d\rho/dz = 0$. Figure 3 shows a revolute connecting B with the center of curvature of the coupler curve of Point B. Because the motion of the coupler at the design position has not been substantially altered, the revolute at C may be removed and the coupler curve of Point C should approximate the coupler curve of Point C in the original slider crank configuration.

At this stage of the development of the method, no conditions are attached to higher derivatives of the radius of curvature of the coupler curve as only one degree of freedom shall be permitted. Thus, at these slider-crank coupler points, the radius of curvature of the coupler curve is invariant for small displacements of the slider on each side of the position for which the radius is to be specified.

The locus of points satisfying Eq. 1 is given by

$$(s^2 + t^2)\ (Mt + Ns) - st = 0 \qquad (2)$$

where M and N are constants to be defined below.

This third degree equation, called the "cubic of stationary curvature", is derived by Hartenberg and Denavit [13]. Equation 2 will not necessarily describe points whose coupler paths are circular arcs because of the lack of conditions on the higher derivatives of $d\rho/dz$. The cubic of stationary curvature for the slider-crank mechanism of the preceding example is plotted in Fig. 4. To simplify the solution, the s-t coordinate system is defined and displaced as shown. Because crank Point A and slider Point C are points where $d\rho/dz$ is zero, the cubic must pass through these points.

Equation 2 may be written alternatively in polar form as

$$\frac{M}{\sin \psi} + \frac{N}{\cos \psi} = \frac{1}{r_\psi} \qquad (3)$$

The pole of the coordinate system corresponds to the pole[1] of the link and the angle ψ is measured from the common centrode[2] tangent at the pole to the radius vector described by r_ψ, where r_ψ is a radius vector from the pole to a point on the cubic. Fig. 5 shows the moving and fixed centrodes of the connecting rod, and the common tangent.

The direction of the common centrode tangent may be established by locating the inflection circle[3] by the Euler-Savary construction as shown by Hartenberg and Denavit [13]. The centrode tangent is located by rotating the line through the pole, Point I, and the inflection

---

[1] Pole - the instantaneous center of velocities of points on the link with respect to a fixed reference plane.

[2] Centrode - the path of the instant center of a link as it moves relative to another link. The fixed centrode is the path of the pole of a link as that link moves relative to the frame, or ground. The moving centrode is the path of the pole of the frame as the frame moves relative to the link for which the centrode is developed.

[3] Inflection Circle - the locus of points on a link for which the radius of curvature of the path of such points is infinite.

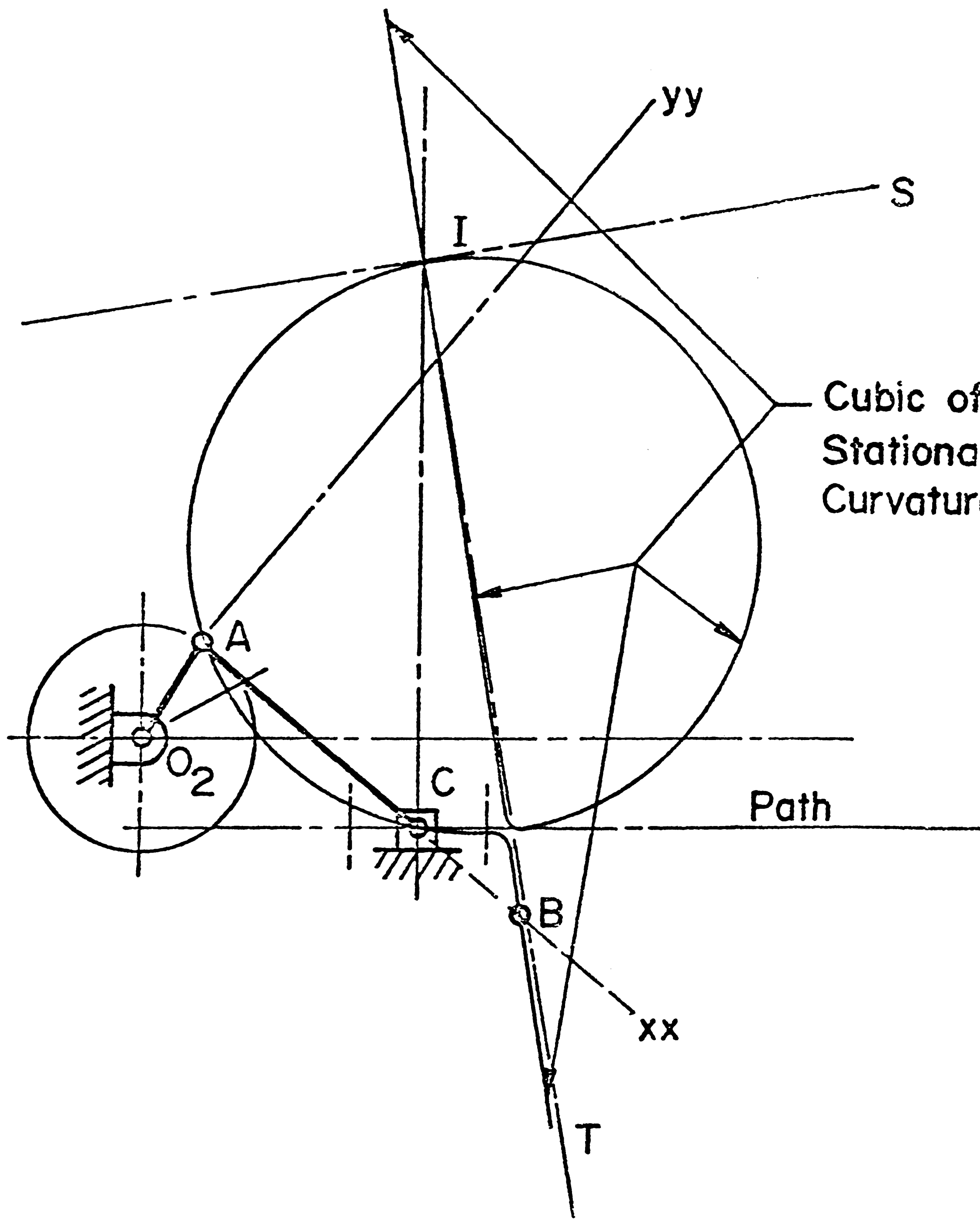

Figure 4. The Slider-Crank Mechanism with the Cubic-of-Stationary-Curvature

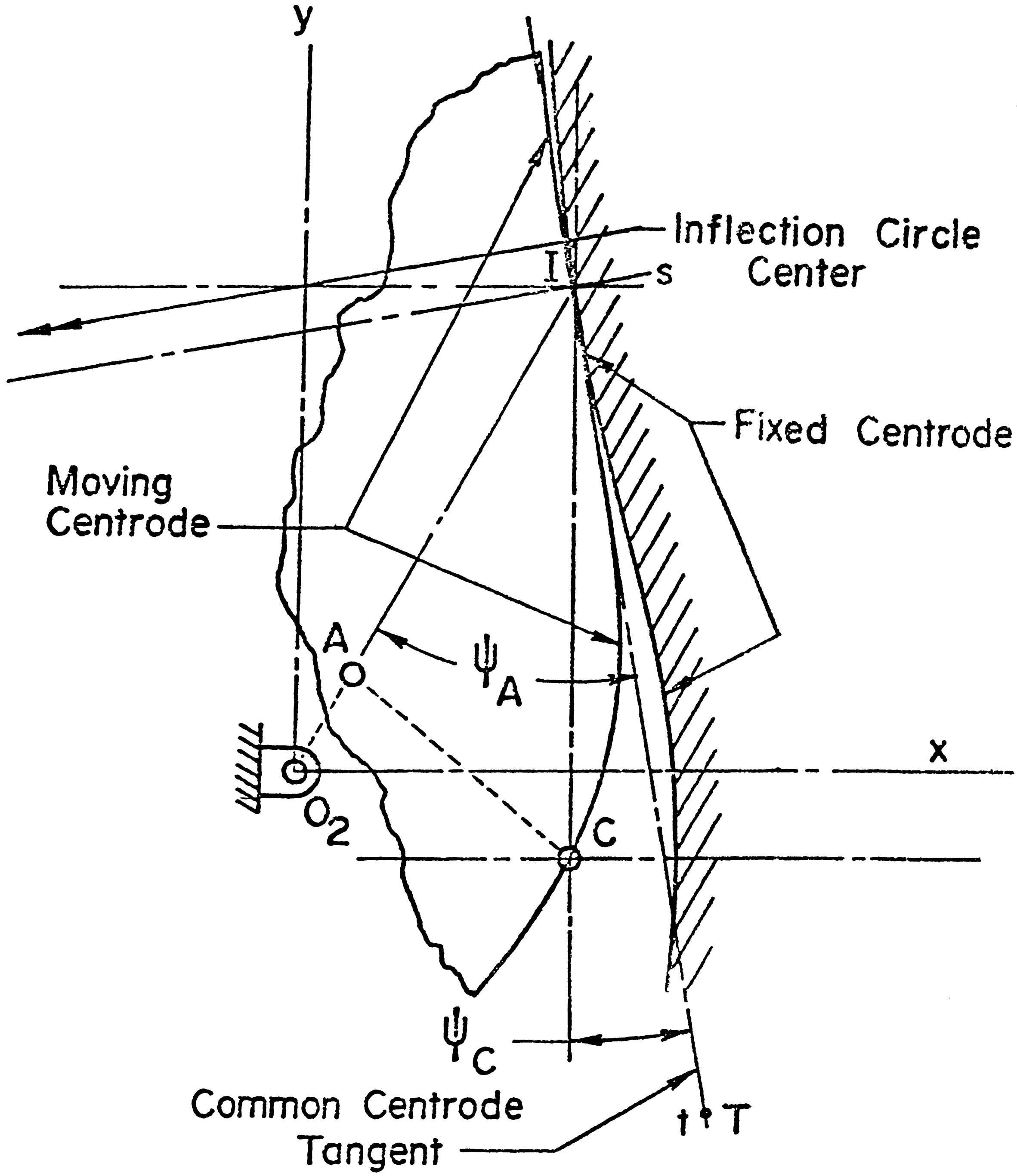

Figure 5. The Connecting Rod Centrodes of the Slider-Crank Mechanism

circle center, 90 degrees in a counter-clockwise direction. The centrode tangent is line IT in Fig. 4.

Using a coordinate system with the origin at the poles and aligned with the centrode tangent, it is possible to solve for the constants, M and N, in Eqs. 2 and 3. It is known that Points A and C in Fig. 4 lie on the plot of the cubic of stationary curvature. Thus

$$\begin{cases} M \csc \psi_A + N \sec \psi_A = 1/IA \\ M \csc \psi_C + N \sec \psi_C = 1/IC \end{cases} \tag{4}$$

Solving this system of equations will characterize M and N for the link in question.

Denoting s and t as distances in the coordinate directions in the s-t coordinate system shown in Fig. 6,

$$r_\psi^2 = s^2 + t^2 \tag{5}$$

$$t = r_\psi \cos \psi \tag{6}$$

$$s = r_\psi \sin \psi \tag{7}$$

Let Point B be chosen as a coupler point such that Point A, B, and C lie on a straight line as shown in Fig. 4. Denoting the slope of the coupler AC as m and the s-axis intercept of AC as b_s (refer to Fig. 6), the equation of the defining line of the coupler may be given by

$$s = mt + b_s \tag{8}$$

Now using Eq. 3,

$$\frac{M}{s} + \frac{N}{t} = \frac{1}{s^2 + t^2}$$

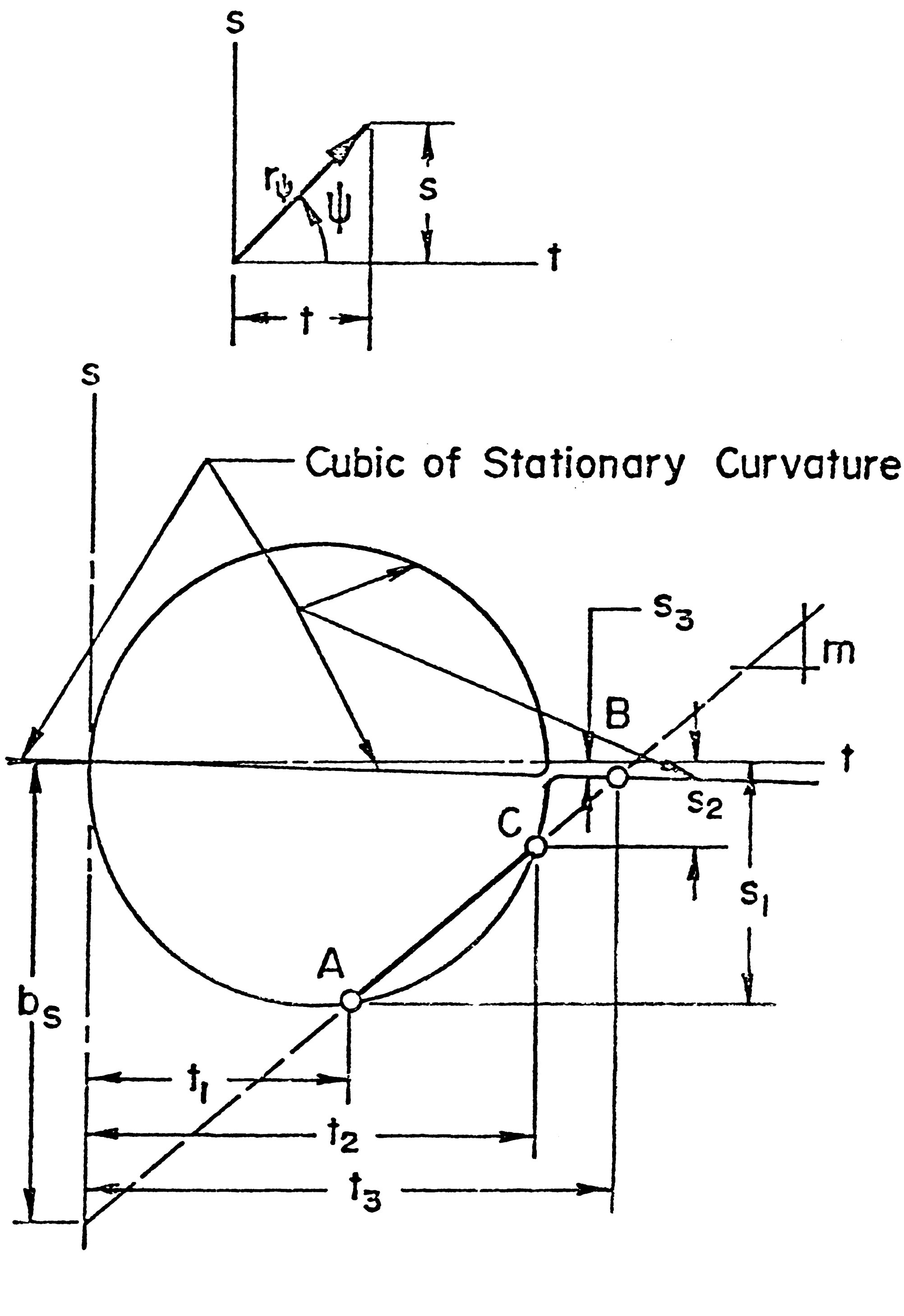

Figure 6. The Cubic of Stationary Curvature and the Connecting Rod in the S-T Plane

$$(Mt + Ns)(t^2 + s^2) = st$$

$$Mt^3 + Ns^3 + Mts^2 + Nt^2s - st = 0 \qquad (9)$$

Combining Eq. 8 and Eq. 9 to eliminate s,

$$(M + Nm^3 + Mm^2 + Nm)\ t^3 + (3Nm^2b_s + 2Mmb_s + Nb_s - m)\ t^2$$
$$+\ 3(Nmb_s 2 + Mb_s^{\,2} + Mb_s^{\,2} - b_s)\ t + Nb_s^{\,3} = 0 \qquad (10)$$

Or combining Eq. 8 and Eq. 9 to eliminate t,

$$\left(\frac{M}{m^3} + N + \frac{M}{m} + \frac{N}{m^2}\right) s^3 + \left(-3\,\frac{b_sM}{m} - \frac{b_sM}{m} - 2\,\frac{b_sN}{m} - \frac{1}{m}\right) s^2$$
$$+ \left(3\,\frac{b_s^{\,2}M}{m^3} + \frac{b_s^{\,2}N}{m^2}\right) s - \left(\frac{b_s^{\,3}M}{m}\right) = 0 \qquad (11)$$

Since M, N, m, and b_s are all defined, the coefficients in Eqs. 10 and 11 are defined. The roots of either or both of these equations may be determined. The roots correspond to the points of intersection of the cubic and the line defined by the coupler. See Fig. 6. Of these roots, one will correspond to Point A, one to Point C, and the remaining root to the required coupler point, Point B.

It is now necessary to determine the radius of curvature and the center of curvature of the coupler curve for the design position of the slider-crank mechanism. Crossley [14] shows that the parametric equations for the coupler point position may be expressed as

$$x = r\cos\phi + \lambda l_x$$
$$y = r\sin\phi - \lambda l_y \qquad (12)$$

In these equations $l_x = l \cos\theta$, $l_y = r \sin\theta - a_3$ as shown in Fig. 2 and $\lambda = xx/l$ where xx is measured in the xx–yy coordinate system and is the distance from Point A to the coupler point. From the derivatives with respect to the crank angle, ϕ, of Eqs. 12 it is possible to evaluate the expression for the radius of curvature using

$$\rho = \frac{(x'^2 + x'^2)^{3/2}}{(x' \, y'' - y' \, x'')} \tag{13}$$

Substituting the derivatives of Eqs. 12 into Eq. 13 yields Eq. 14 (p. 21) which defines the radius of curvature of the coupler point path in terms of the slider crank linkage dimensions and the parameter λ.

The location of the center of curvature in the fixed plane, coordinates (x_c, y_c) is given by

$$x_c = x - y' \, \frac{(x'^2 + y'^2)}{x'y'' - y'x''} \tag{15}$$

$$y_c = y + x' \, \frac{(x'^2 + y'^2)}{x'y'' - y'x''} \tag{16}$$

Substituting the derivatives of Eqs. 12 into Eqs. 15 and 16 yields Eqs. 17 and 18. Equations 17 and 18 (pp. 22 and 23) locate the center of curvature of the coupler point path in the fixed plane. Note that x and y in Eqs. 15 and 16 are the coordinate positions of the coupler point, Point B, in the fixed plane. Figure 7 shows the center of curvature, radius of curvature, and the coupler curve which is generated by Point B.

VARIABLES USED IN EQUATIONS 14, 17, AND 18

X	=	X COORDINATE OF SLIDER-CRANK COUPLER POINT
Y	=	Y COORDINATE OF SLIDER-CRANK COUPLER POINT
PHI	=	CRANK ANGLE
LAMBDA	=	XX/L WHERE XX IS MEASURED IN THE COORDINATE SYSTEM ON THE CONNECTING ROD
L	=	CONNECTING ROD LENGTH
A3	=	SLIDER PATH OFFSET
R	=	CRANK RADIUS
XC,YC	=	COORDINATES OF THE CENTER OF ROTATION OF THE FOLLOWER LINK
RR	=	RADIUS OF FOLLOWER LINK; RADIUS OF CURVATURE OF THE SLIDER CRANK COUPLER CURVE

```
             2                                 2   2
RR = ( COS    ( PHI ) (  - LAMBDA + 1. )       R    + (  - COS ( PHI )
---------------------------------------------------------------------------
                                                 2
(  - A3 + SIN ( PHI ) R ) R LAMBDA /  ( L      - (  - A3 + SIN (
---------------------------------------------------------------------------
         2    1/2                            2    3/2
PHI ) R )      )       - SIN ( PHI ) R )       )       /  ( SIN ( PHI ) (
---------------------------------------------------------------------------
                                                                  2
 - COS ( PHI ) (  - A3 + SIN ( PHI ) R ) R LAMBDA /  ( L      - (
---------------------------------------------------------------------------
                                 2    1/2
 - A3 + SIN ( PHI ) R )      )       - SIN ( PHI ) R ) ( LAMBDA - 1.
---------------------------------------------------------------------------
                                                            2         2
 ) R - COS ( PHI ) ( (  (  - SIN ( PHI ) A3 R + SIN  ( PHI ) R
---------------------------------------------------------------------------
       2          2          2                                 2    1/2
 - COS  ( PHI ) R    ) / ( L    - (  - A3 + SIN ( PHI ) R )      )
---------------------------------------------------------------------------
       2                                        2   2         2
 - COS  ( PHI ) (  - A3 + SIN ( PHI ) R )       R    /  ( L    - (  -
---------------------------------------------------------------------------
                          2    3/2
A3 + SIN ( PHI ) R )       )       ) LAMBDA - COS ( PHI ) R ) (  -
---------------------------------------------------------------------------
LAMBDA + 1. ) R )
-----------------
```

EQUATION 14

```
                      2                            2   2
YC = Y + ( COS   ( PHI ) (   - LAMBDA + 1. )     R     /   (   - COS (
--------------------------------------------------------------------------
                                                       2
PHI ) (   - A3 + SIN ( PHI ) R ) R LAMBDA /   ( L     - (   - A3 +
--------------------------------------------------------------------------
                      2   1/2                         2
SIN ( PHI ) R )     )       - SIN ( PHI ) R )     + 1. ) (   - COS (
--------------------------------------------------------------------------
                                                       2
PHI ) (   - A3 + SIN ( PHI ) R ) R LAMBDA /   ( L     - (   - A3 +
--------------------------------------------------------------------------
                      2   1/2                          3
SIN ( PHI ) R )     )       - SIN ( PHI ) R )     /   ( SIN ( PHI ) (
--------------------------------------------------------------------------
                                                               2
 - COS ( PHI ) (   - A3 + SIN ( PHI ) R ) R LAMBDA /   ( L     - (
--------------------------------------------------------------------------
                            2   1/2
 - A3 + SIN ( PHI ) R )     )       - SIN ( PHI ) R ) ( LAMBDA - 1.
--------------------------------------------------------------------------
                                                       2                 2
 ) R - COS ( PHI ) ( ( (   - SIN ( PHI ) A3 R + SIN   ( PHI ) R
--------------------------------------------------------------------------
       2         2          2                               2   1/2
 - COS   ( PHI ) R    ) / ( L     - (   - A3 + SIN ( PHI ) R )     )
--------------------------------------------------------------------------
       2                                   2   2         2
 - COS   ( PHI ) (   - A3 + SIN ( PHI ) R )    R    / ( L     - (   -
--------------------------------------------------------------------------
                        2   3/2
A3 + SIN ( PHI ) R )    )       ) LAMBDA - COS ( PHI ) R ) (   -
--------------------------------------------------------------------------
LAMBDA + 1. ) R )
------------------
```

EQUATION 18

```
                                            2                                 2   2
XC = X - COS ( PHI ) ( COS  ( PHI ) ( - LAMBDA + 1. )    R   /
------------------------------------------------------------------
                                                                  2
( - COS ( PHI ) ( - A3 + SIN ( PHI ) R ) R LAMBDA / ( L   -
------------------------------------------------------------------
                                  2   1/2                    2
( - A3 + SIN ( PHI ) R )    )     - SIN ( PHI ) R )     + 1. ) (
------------------------------------------------------------------
                                                              2
 - COS ( PHI ) ( - A3 + SIN ( PHI ) R ) R LAMBDA / ( L    - (
------------------------------------------------------------------
                                2   1/2                     2
 - A3 + SIN ( PHI ) R )    )     - SIN ( PHI ) R )     ( - LAMBDA
------------------------------------------------------------------
 + 1. ) R / ( SIN ( PHI ) ( - COS ( PHI ) ( - A3 + SIN ( PHI
------------------------------------------------------------------
                          2                                 2   1/2
 ) R ) R LAMBDA / ( L    - ( - A3 + SIN ( PHI ) R )    )      -
------------------------------------------------------------------
SIN ( PHI ) R ) ( LAMBDA - 1. ) R - COS ( PHI ) ( ( ( - SIN (
------------------------------------------------------------------
                   2              2          2            2              2
PHI ) A3 R + SIN    ( PHI ) R     - COS    ( PHI ) R     ) / ( L    - (
------------------------------------------------------------------
                               2   1/2        2
 - A3 + SIN ( PHI ) R )    )      - COS    ( PHI ) ( - A3 + SIN (
------------------------------------------------------------------
             2    2           2                                 2   3/2
PHI ) R )     R     / ( L    - ( - A3 + SIN ( PHI ) R )    )       )
------------------------------------------------------------------
LAMBDA - COS ( PHI ) R ) ( - LAMBDA + 1. ) R )
----------------------------------------------
```

EQUATION 17

By the use of Eqs. 14, 17, and 18, the synthesis of the four-bar linkage equivalent of the slider-crank mechanism has been accomplished. The length of the follower link is made equal to the radius of curvature of the slider-crank coupler curve of Fig. 7 and rotates about the center of curvature. It is attached to the connecting rod at Point B and the slider is eliminated. Figure 8 shows the resulting four-bar linkage where the path of Point C is an approximate straight line along the original path of the slider. Figures 9 and 10 show enlarged plots of the four-bar coupler curve for Point C with Fig. 9 showing the complete coupler curve and Fig. 10 an enlargement of the straight-line portion. Figure 11 shows a comparison between the actual values and the theoretical values of the displacement S as a function of the crank angle ϕ for the four-bar linkage. The amount of error in the y-direction of the path of Point C for the four-bar linkage can be determined from Fig. 10.

The results of this procedure indicate that a four-bar linkage can be determined such that the cubic of stationary curvature of the four-bar coupler is identical to that of the slider-crank coupler. The difference in each case is the location of the links which provide for constant curvature. The accuracy of the coupler point path resulting from the conversion of the slider-crank into a four-bar linkage is illustrated by the coupler curve charts in Figs. 12 to 15. In particular, the accuracy of the path was investigated with respect to the crank design angle, the ratio of the connecting rod length to

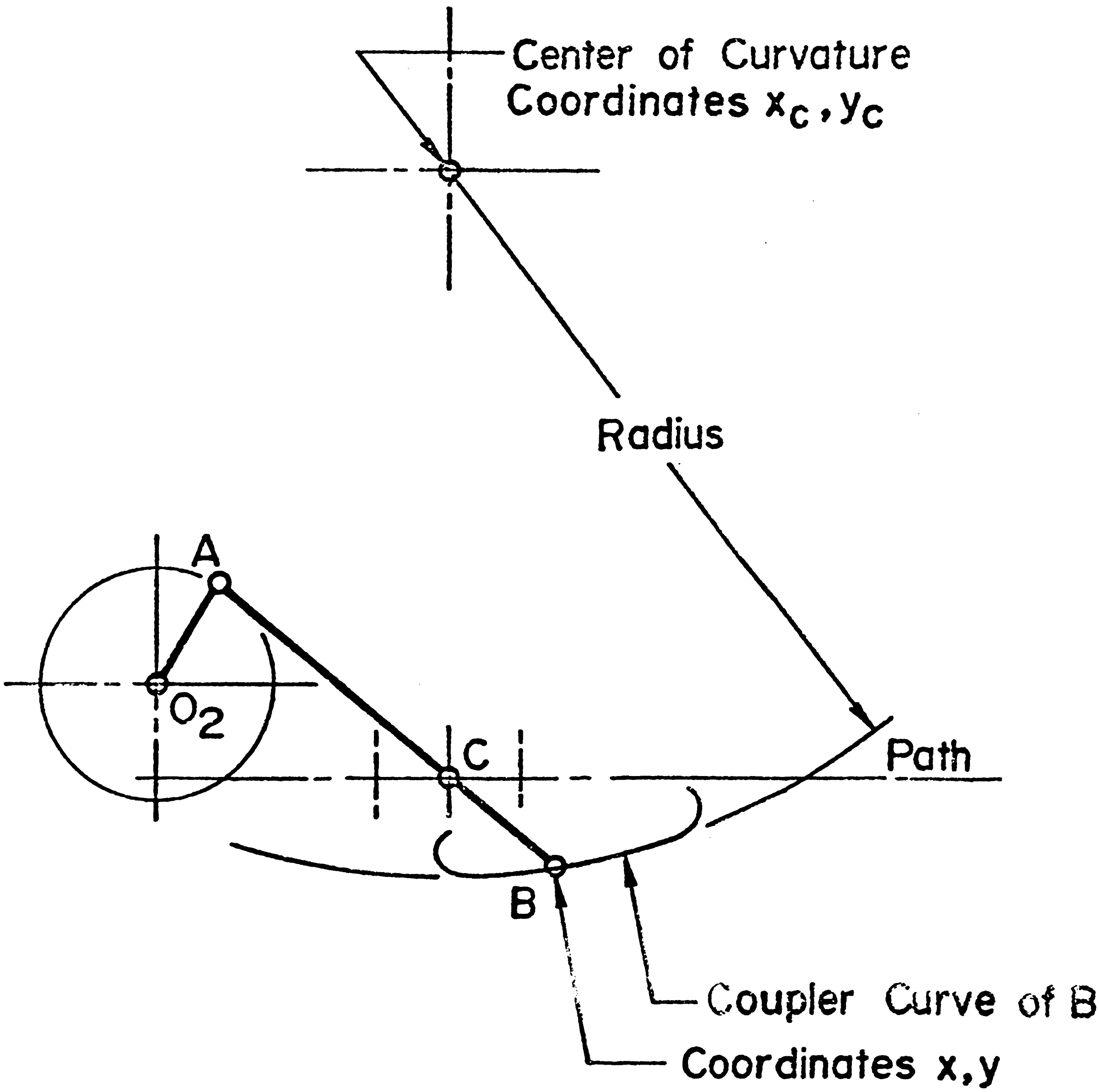

Figure 7. The Slider-Crank Couple Curve

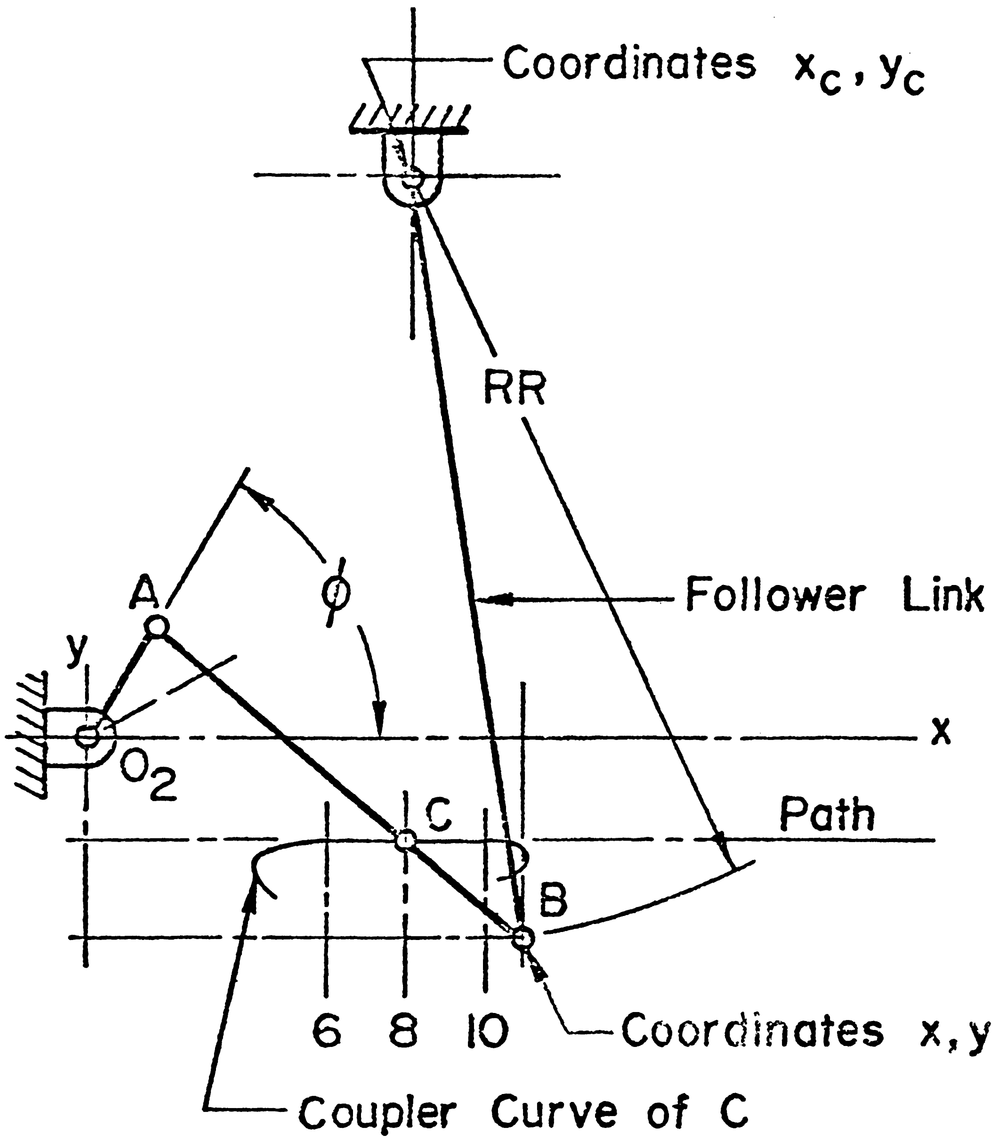

Figure 8. The Four-Bar Linkage with the Coupler Curve

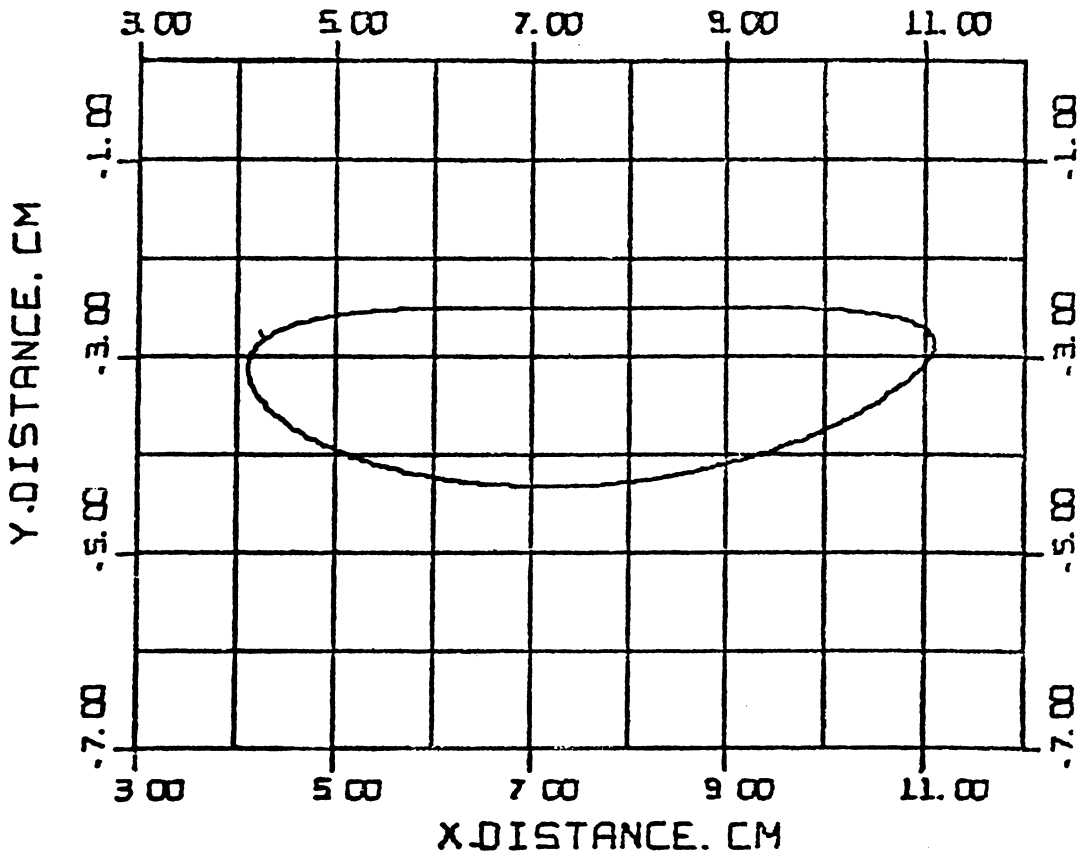

Figure 9. The Four-Bar Linkage Coupler Curve of Point C

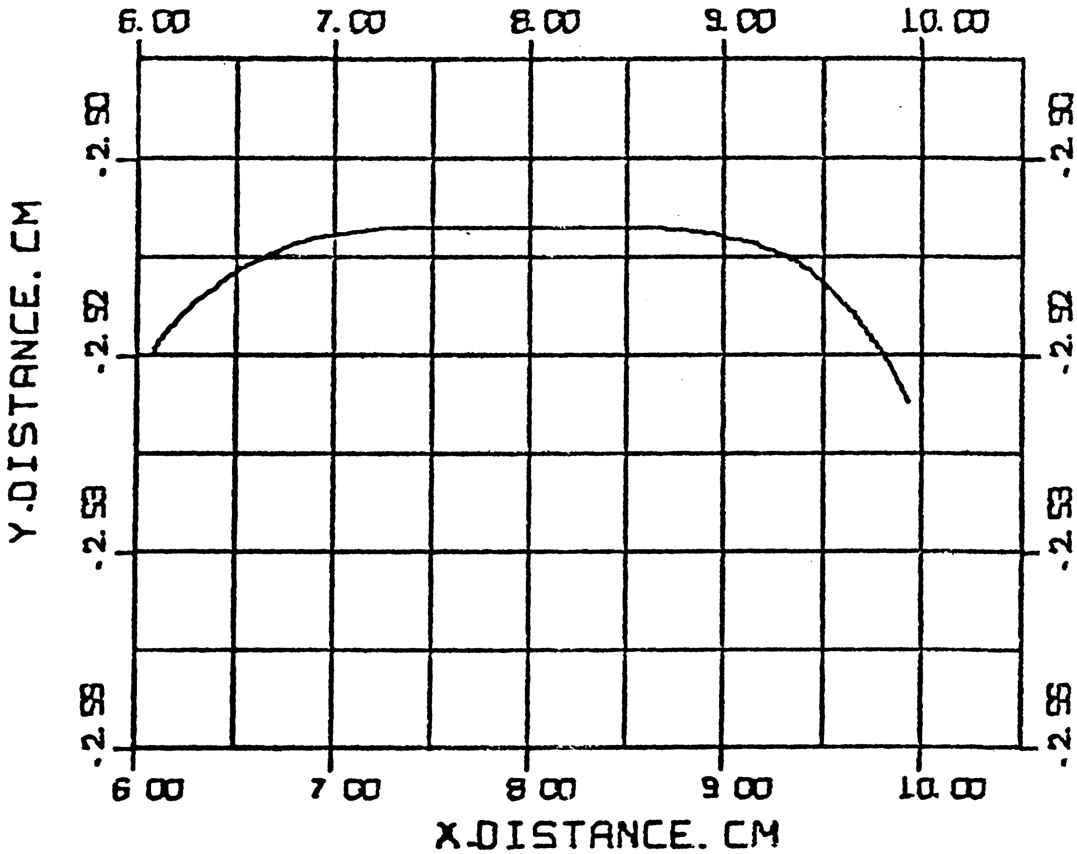

Figure 10. The "Straight" Portion of the Four-Bar Linkage Coupler Curve of Point C

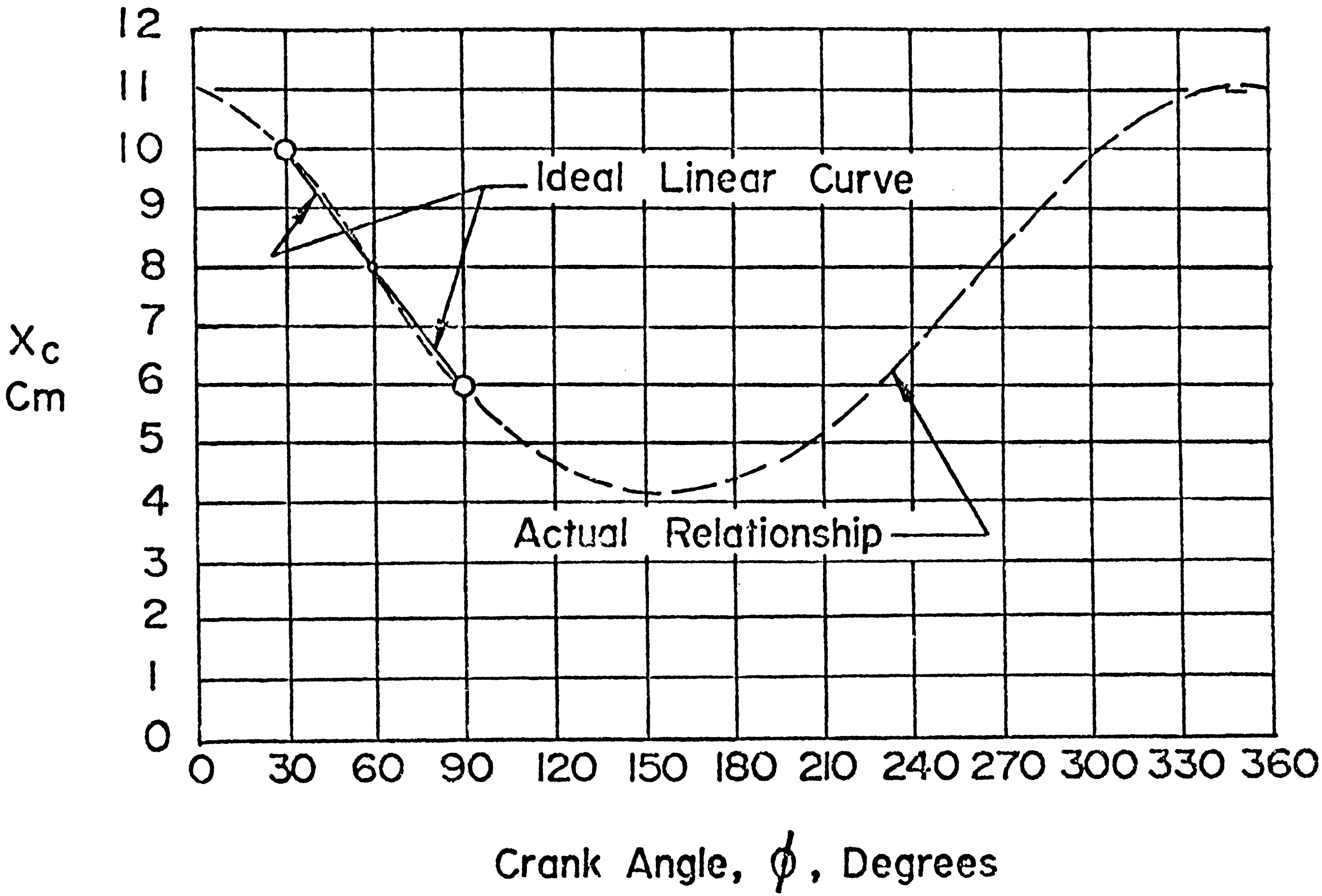

Figure 11. Comparison of Actual with Theoretical Values of Horizontal Displacement Relative to Crank Angle for the Four-Bar Linkage

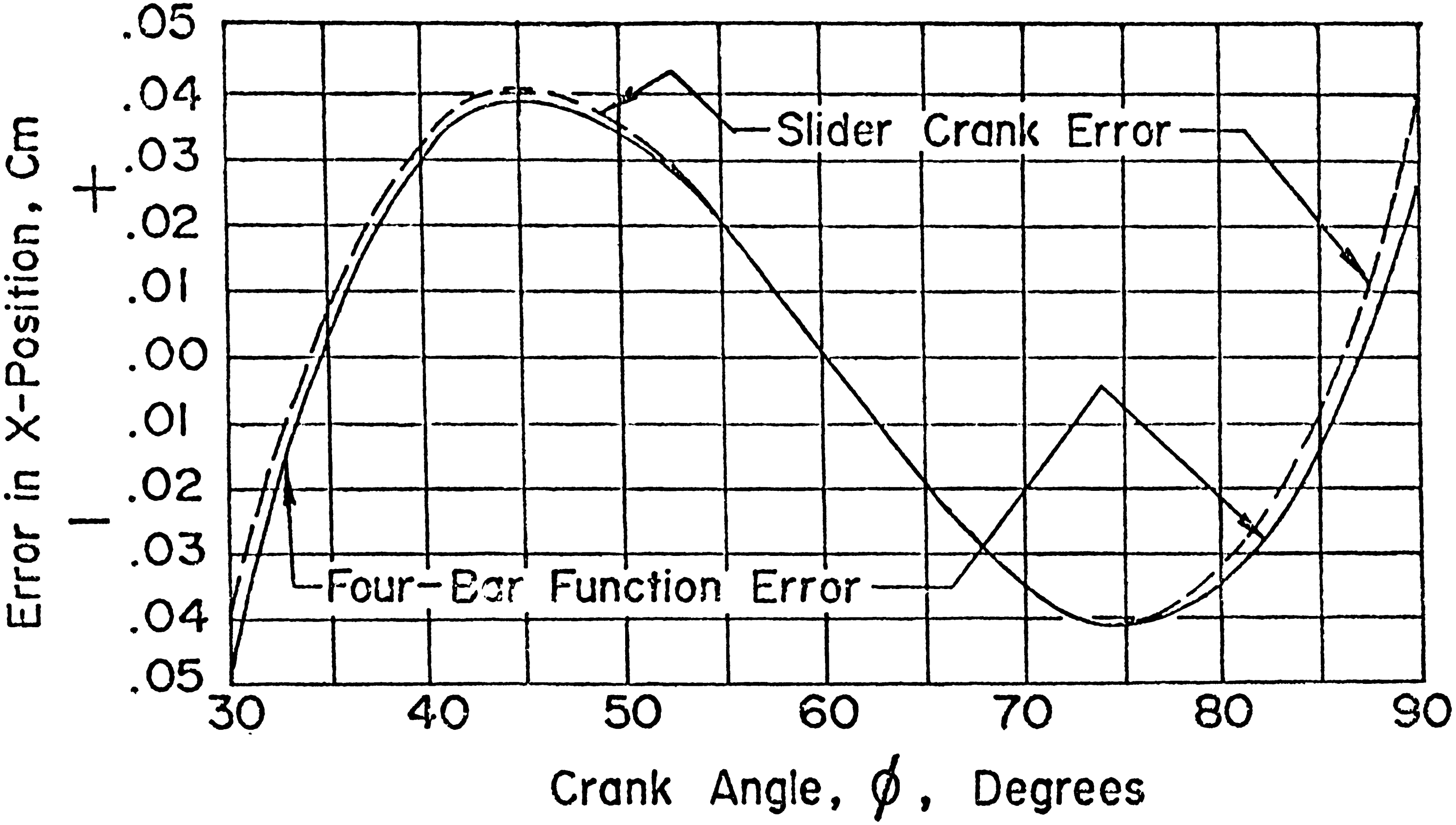

Figure 12. Comparison of Structural Errors for Slider-Crank Mechanism and Four-Bar Linkage

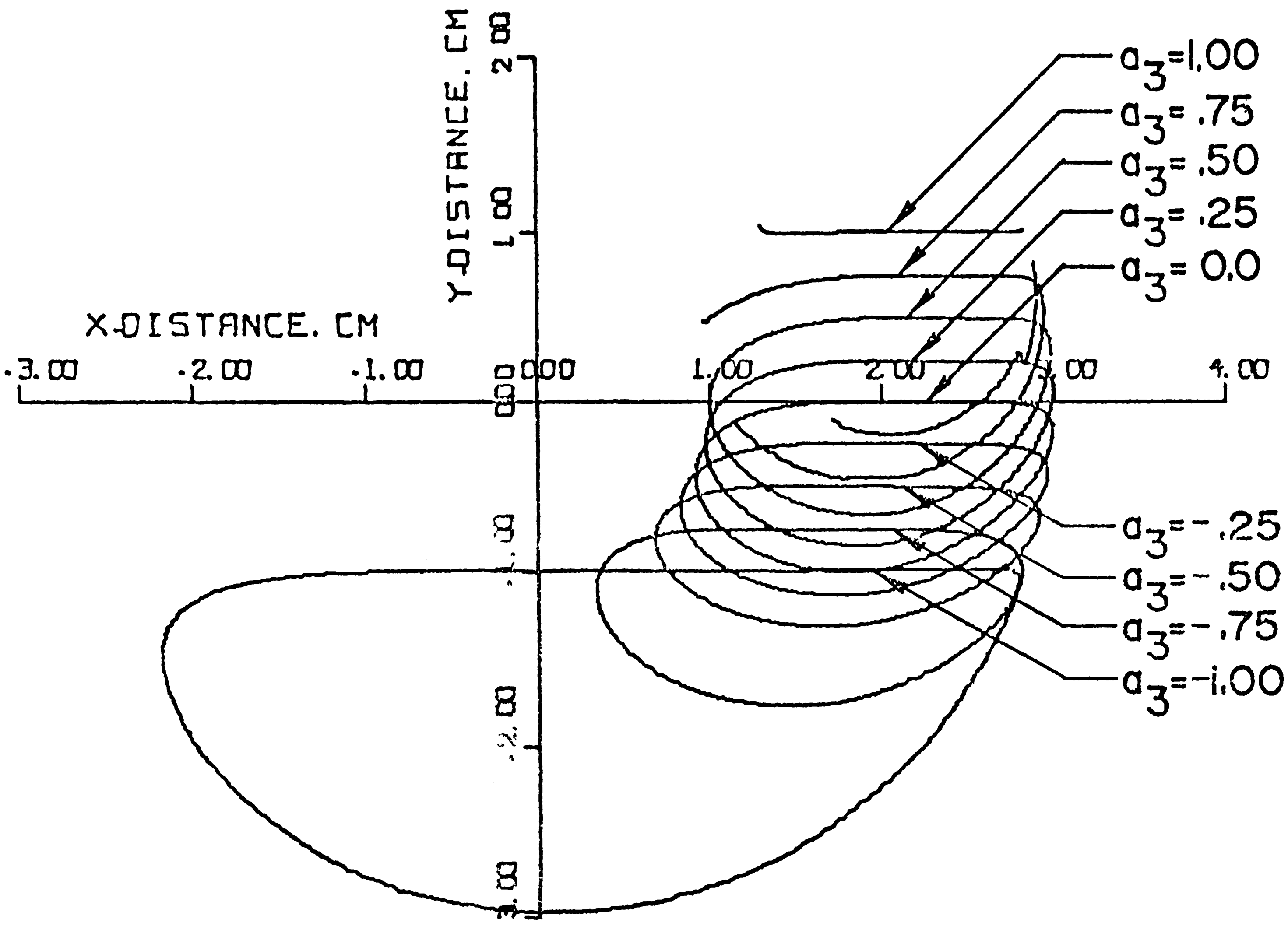

Figure 13. Four-Bar Coupler Curves from Slider-Cranks with Varying Path Offsets. Slider Crank r = 1.00 cm, l = 2.00 cm, $\phi = 60^0$

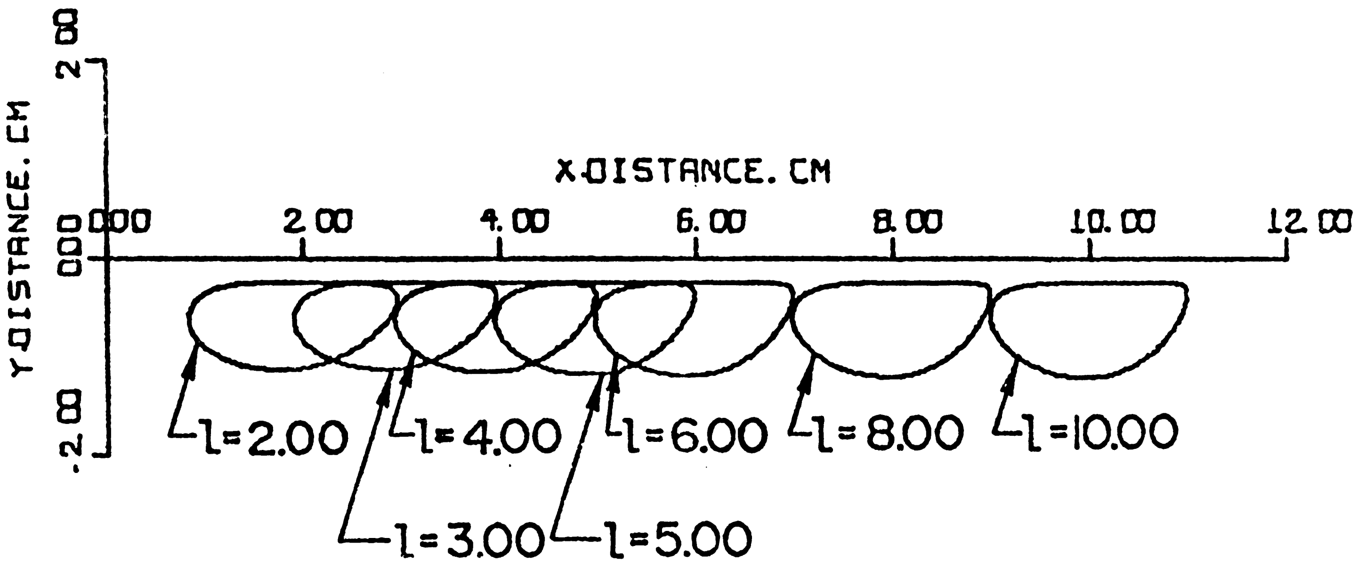

Figure 14. Four-Bar Coupler Curves from Slider-Cranks with Varying Connecting Rod Lengths. Slider Crank r = 1.00 cm, $a_3 = -.25$ cm, $\phi = 60^0$

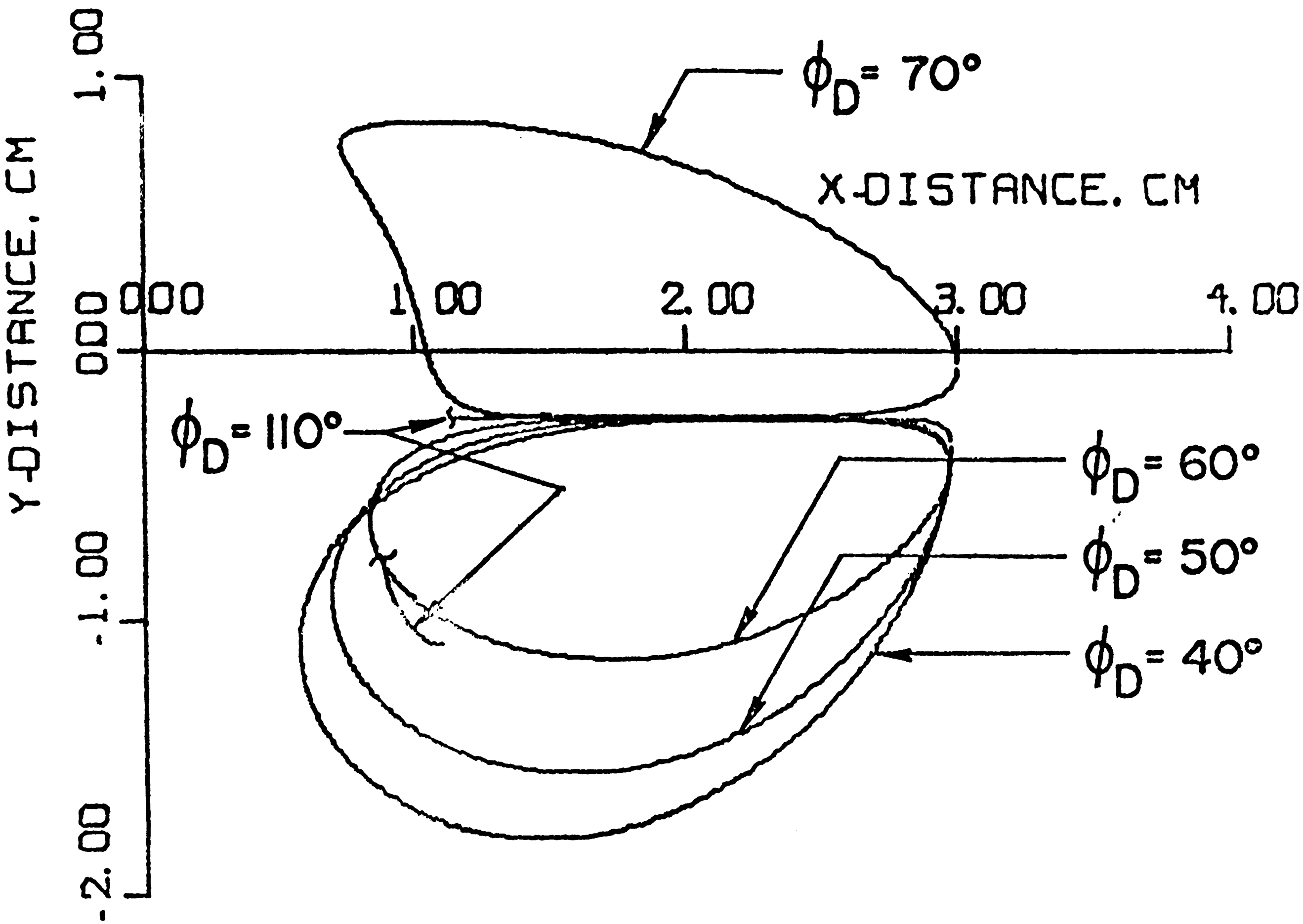

Figure 15. Four-Bar Coupler Curves from Slider-Cranks with Varying Design Crank Angles. Slider Crank r = 1.00 cm, l = 2.00 cm, a_3 = -.25 cm

crank length, and the ratio of the crank length to the offset distance. These charts exhibit relative insensitivity to changes in the crank angle and the ratios listed above. However, the cases of negative slider path offset appear to give better straight line performances.

This discussion, procedure, and example have been concerned with satisfying Eq. 1. For the four-bar linkage of Fig. 8, with the follower link RR, the same conditions may be satisfied by

$$\frac{d}{d\psi} (RR) = 0 \tag{19}$$

The satisfaction of this condition is necessary for an acceptable mechanism, but in many situations it will not be sufficient. The following discussion will focus upon the establishment of these sufficient conditions, namely

$$\frac{d^2}{d\phi^2} (RR) = 0 \quad (20$$

and

$$\frac{d^3}{d\phi^3} (RR) \rightarrow 0 \tag{21}$$

Chapter 2

General Equations - Three Derivatives

The general equations and their solution for three derivatives of the radius of curvature are presented such that the nature of the coupler constraints remains undefined at this point. The position of Point P on the dyad[4], shown in Fig. 16, in the x-y coordinate system may be shown to be

$$x = a \cos \phi + u \cos \beta - v \sin \beta \tag{22}$$

$$y = a \sin \phi + u \sin \beta - v \cos \beta \tag{23}$$

The magnitude and sign of the angle β will be dependent upon the size and position of the dyad and the constraints placed upon the position of Point B.

Equations 22 and 23 are of the form

$$x = x_{co} + x_u u + x_v v \tag{24}$$

$$y = y_{co} + y_u u + y_v v \tag{25}$$

where

$x_{co} = a \cos \phi$

$y_{co} = a \sin \phi$

$x_u = \cos \beta = x_u (a,\phi,b,\ldots)$

$x_v = - \sin \beta = x_v (a,\phi,b,\ldots)$

$y_u = \sin \beta = y_u (a,\phi,b,\ldots)$

$y_v = \cos \beta = y_v (a,\phi,b,\ldots)$

From Eqs. 24 and 25, derivatives of x and y with respect to the

---

[4] Dyad - An open chain of two links.

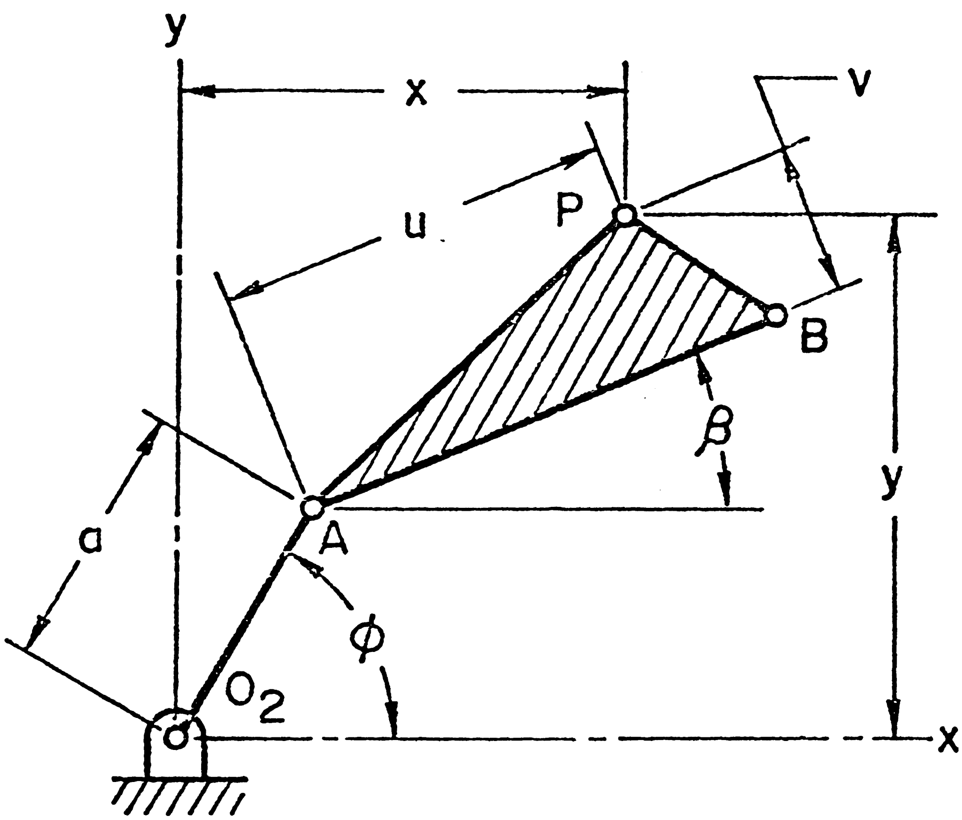

Figure 16. A Generalized Dyad

crank angle ϕ, may be formed

$$\frac{dx}{d\phi} = x' = x'_{co} + x'_u u + x'_v v \tag{26a}$$

$$\frac{dy}{d\phi} = y' = y'_{co} + y'_u u + y'_v v \tag{27a}$$

$$x'' = x''_{co} + x''_u u + x''_v v \tag{26b}$$

$$x''' = x'''_{co} + x'''_u u + x'''_v v \tag{26c}$$

$$x^{iv} = x^{iv}_{co} + x^{iv}_u u + x^{iv}_v v \tag{26d}$$

$$x^{v} = x^{v}_{co} + x^{v}_u u + x^{v}_v v \tag{26e}$$

$$y'' = y''_{co} + y''_u u + y''_v v \tag{27b}$$

$$y''' = y'''_{co} + y'''_u u + y'''_v v \tag{27c}$$

$$y^{iv} = y^{iv}_{co} + y^{iv}_u u + y^{iv}_v v \tag{27d}$$

$$y^{v} = y^{v}_{co} + y^{v}_u u + y^{v}_v v \tag{27e}$$

The radius of curvature of the path of Point P is given by

$$\rho = \frac{(x'^2 + y'^2)^{3/2}}{(x'y'' - y'x'')} = (x'^2 + y'^2)^{3/2}(x'y'' - y'x'')^{-1}$$

Evaluating the derivatives of the radius of curvature of Point P with respect to the parameter ϕ

$$\frac{d\rho}{d\phi} = \frac{3(x'^2+y'^2)(x'x''+y'y'')(x'y''-y'x'') - (x'^2+y'^2)(x'y'''-y'x''')}{\sqrt{x'^2+y'^2}\,(x'y'' - y'x'')^2} \tag{28}$$

$$\frac{d^2\rho}{d\phi^2} = \frac{1}{\sqrt{x'^2 + y'^2}\,(x'y'' - y'x'')^3}\Big[3(x'x''+y'y'')^2\,(x'y'' - y'x'')^2$$

$$-9(x'x'' + y'y'')(x'y''' - y'x''')\,(x'y'' - y'x'')(x'^2 + y'^2)$$

$$+2(x'y''' - y'x''')^2\,(x'^2 + y'^2) + 3(x''^2 + y'' \;\; + x'x''' + y'y''')$$

$$(x'y'' - y'x'')^2\,(x'^2 + y'^2) - (x''y''' + x'y^{iv} - y''x''' - y'x^{iv})$$

$$(x'y'' - y'x'')\,(x'^2 + y'^2)^2\Big] \qquad (29)$$

$$\frac{d^3\rho}{d\phi^3} = \frac{1}{2Z^{3/2}W^6}\Big[2ZW^3\;\; 6RW^2T + 6R^2WS - 6TSWZ$$

$$-6RUWZ - 6RS^2Z - 6RSWZ' + 4SUZ^2$$

$$+4S^2ZZ' + 3T'W^2Z + 6TWSZ + 3TW^2Z'$$

$$-U'WZ^2 - USZ^2 - 2UWZZ' - (Z'W^3 + 6ZW^2S)$$

$$\left(3R^2W^2 - 6RSWZ + 2S^2Z^2 + 3TW^2Z - UWZ^2\right)\Big] \qquad (30)$$

where

$$Z = x'^2 + y'^2$$

$$W = x'y'' - x''y'$$

$$U = x''y''' + x'y^{iv} - y''x''' - y'x^{iv}$$

$$T = x''^2 + y''^2 + x'x''' + y'y'''$$

$$S = x'y''' - y'x'''$$

$$R = x'x'' - y'y''$$

$$Z' = 2x'y'\;(y'x'' + x'y'')$$

$$U' = 2x''y^{iv} + x'y^{v} - 2y''x^{iv} - y'x^{v}$$

$$T' = 3x''x''' + 3y''y''' + x'x^{iv} + y'y^{iv}$$

Consider now the case of the first derivative. If Eq. 28 is set equal to zero, or the numerator of Eq. 28 is set equal to zero, it may be shown that the resulting expression will be of the form

$$\sum_{j=0}^{6} \sum_{k=0}^{6} a_{jk} u^j v^k = 0 \tag{31}$$

The locus of the real roots of this equation describes points for which the radius of curvature is invariant with respect to infinitesimal changes in the parameter ϕ. It should be noted that a description of the linkage through the definition of both the link lengths and crank angle will define each of the a_{jk}'s of Eq. 31. Solution of the equation will also yield a locus of roots in the imaginary plane. Having described a real linkage, the imaginary roots are of no interest.

The denominator of Eq. 28 may be set to zero and may be expanded to result in an expression of the form

$$\sum_{j=0}^{4} \sum_{k=0}^{4} a_{jk} u^j v^k = 0 \tag{32}$$

Coincident roots of Eqs. 31 and 32 locate points for which the first derivative of the radius of curvature is undefined. Generally, it may be shown that such coincident roots are not in the vicinity of the coupler, with the lone exception of the ever-present coincident root pair at the pole of the link. As such the definition of the first derivative of the radius of curvature presents no real problem except at the pole. The nature of this difficulty will be discussed later.

The numerator of the equation for the second derivative of the radius of curvature, Eq. 29, may be set equal to zero and expanded to yield an equation of the form

$$\sum_{j=0}^{8} \sum_{k=0}^{8} a_{jk} u^j v^k = 0 \qquad (33)$$

The locus of the real roots of Eq. 33 describes points for which the second derivative of the radius of curvature is invariant with respect to infinitesimal changes in the parameter ϕ. Again, for a defined linkage, each a_{jk} is defined. The denominator of Eq. 27 may be set to zero and expanded, and will yield the same results as in the case of the first derivative denominator. That result is an indeterminate definition of the derivative at the pole.

The premise of this synthesis procedure is that the link may undergo a change in constraints and retain approximately the same characteristics of the derivatives. Practically, the circular constraint of a path is the easiest of all constraints to realize and control. In the case of a point constrained to move in a circular path, the radius of curvature of the path of such a point is constant and all derivatives of the radius of curvature with respect to a displacement parameter are zero.

The location of Point P in Fig. 16 is defined with respect to two parameters, u and v. As there are two degrees of freedom in this case, solutions satisfying two equations (Eqs. 31 and 32) may be found. The simultaneous solution of the equations defines points for which the first and second derivatives of the radius of curvature (in the fixed plane) with respect to the driving angle are equal to zero.

It should be noted that the result of the entire procedure will be to constrain a point, at which the first and second derivatives of ρ are zero, such that ALL derivatives are zero. Because the higher derivatives are not zero in the original linkage configuration, the procedure represents an approximation of the third and higher derivatives of the radius of curvature. With only two degrees of freedom being involved, only the first two derivatives of the radius of curvature may be specified exactly. However, the solution of Eqs. 31 and 33 should yield multiple combinations of u and v, each pair of which has a different value of the third derivative of the radius of curvature. Selection from these multiple solutions on the basis of the minimization of the absolute value of the third derivative should yield the most desirable approximation.

Equations corresponding to Eqs. 26 through 33 with the exception of Eq. 32 are presented in Appendix A. Super- and subscripts are not used. X1 corresponds to x', Y4 corresponds to y^{iv}, X3U corresponds to x_u''', etc. Also, rather than being equated to zero, the appended equations are equated to a dummy variable. Additionally, all equations are unexpanded. The reason for this is that Eq. 31, for example, when expanded, will consist of thousands of terms and require tens of pages for its listing.

The Solution Technique

The loci of the zeroes of the first and second derivatives of the radius of curvature of coupler point curves are defined by

$$\sum_{j=0}^{6} \sum_{k=0}^{6} a_{jk} u^j v^k = 0 \qquad (31) \text{ Repeated}$$

$$\sum_{j=0}^{8} \sum_{k=0}^{8} a_{jk} u^j v^k = 0 \qquad (32) \text{ Repeated}$$

The above pair of non-linear equations may be solved simultaneously using the Newton-Raphson technique as shown by Carnahan [15].

Assume

$$f_1 = \sum_{j=0}^{6} \sum_{k=0}^{6} a_{jk} u^j v^k = 0$$

$$f_2 = \sum_{j=0}^{8} \sum_{k=0}^{8} a_{jk} u^j v^k = 0$$

Expanding these relationships in a Taylor Series and truncating second order and higher order terms

$$\frac{\partial f_1}{\partial u} \Delta u + \frac{\partial f_1}{\partial v} \Delta V = -f_1$$

$$\frac{\partial f_2}{\partial u} \Delta u + \frac{\partial f_2}{\partial v} \Delta V = -f_2$$

The determinant of the coefficient matrix (the Jacobian) becomes

$$\begin{vmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} \end{vmatrix} = \frac{\partial f_1}{\partial u}\frac{\partial f_2}{\partial v} - \frac{\partial f_1}{\partial v}\frac{\partial f_2}{\partial u} = D$$

Then

$$\Delta u = \frac{f_2 \frac{\partial f_1}{\partial v} - f_1 \frac{\partial f_2}{\partial v}}{D} \quad (34)$$

$$\Delta v = \frac{f_1 \frac{\partial f_2}{\partial u} - f_2 \frac{\partial f_1}{\partial u}}{D} \quad (35)$$

Thus, after choosing satisfactory initial values of u and v, Eqs. 34 and 35 may be solved repeatedly, updating u and v, until Δu and Δv vanish.

An essential difficulty in formulating a Newton-Raphson Solution is the determination of the expressions for the derivatives.

Let

$$F = -x''_v y' + x'_v y'' + y''_v x' - y'_v x''$$

$$G = 2(y'_v y' + x'_v x')$$

$$H = 2(y'_u y' + x'_u x')$$

$$J = -x''_u y' + x'_u y'' + y''_u x' - y'_u x''$$

$$R = x'x'' + y'y''$$

$$S = x'y''' - y'x'''$$

$$T = x''^2 + y''^2 + x'x''' + y'y'''$$

$$U = x''y''' + x'y^{iv} - y''x''' - y'x^{iv}$$

$$W = y''x' - y'x''$$

$$Z = x'^2 + y'^2$$

Then

$$f_1 = \text{numerator of}\left(\frac{d\rho}{d\phi}\right) = 3ZRW - SZ^2 \tag{36}$$

$$f_2 = \text{numerator of}\left(\frac{d^2\rho}{d\phi^2}\right) = 3R^2W^2 - 6RSWZ + 2S^2Z^2 + 3TW^2Z - UWZ^2 \tag{37}$$

Further

$$\frac{\partial f_1}{\partial v} = 3(y_v'y'' + y_v''y' + x''x_v' + x'x_v'')\ WZ - 2GSZ + 3FRZ + 3GRW - Z^2\ (x'y_v''' - y_v'x''' - x_v'''y' + x_v'y''') \tag{38}$$

$$\frac{\partial f_1}{\partial u} = 3WZ(x''x_u' + x'x_u'' + y'y_u'' + y'y'') - 2HSZ + 3JRZ + 3HRW - (y'''x_u' - y'x_u''' + x'y_u''' - x'''y_u')Z^2 \tag{39}$$

$$\begin{aligned}\frac{\partial f_2}{\partial u} = {} & -2WUZH + 4S^2ZH - TRSWH + 3TW^2H + 6TWZJ - 6RSZJ \\ & - UZ^2J - 6(x''x_u' + x'x_u'' + y'y_u'' + y_u'y'')\ SWZ - 6 \\ & (y'''x_u' - y'x_u''' + x'y_u''' - x'''y_u')\ RWZ \\ & + 3(x'x_u''' + x'''x_u' + 2x''x_u'' + 2y''y_u'' + y'y_u''' + y'''y_u')ZW^2 \\ & - (y^{iv}x_u' - y''x_u''' + y'''x_u'' - x'''y_u'' + x'y_u^{iv} - y'x_u^{iv} - y_u'x^{iv} \\ & + x''y_u''')\ WZ + 4(\ y'''x_u' - y'x_u''' + x'y_u''' - x'''y_u')SZ^2 + 6 \\ & (x''x_u' + x'x_u'' + y'y_u'' + y_u'y'')\ RW^2\end{aligned} \tag{40}$$

$$\frac{\partial f_2}{\partial v} = -2WUGZ + 4S^2GZ-6(y_v'y'' + y_v''y' + x''x_v' + x'x_v'')SWZ$$
$$+6TFWZ-6(x'y_v''' - y_v'x''' - x_v'''y' + x_v'y''')\ RWZ-6RFSZ$$
$$+3(2y_v''y'' + y'y_v''' + x_v'x''' + y'''y_v' + x'x_v''' + 2x''x_v'')ZW^2$$
$$-6RSWG + 3TW^2G + 6R^2FW-FUZ^2 -(x'y_v^{iv} - x_v'''y'' - y_v'x^{iv}$$
$$-y'x_v^{iv} + x_v'y^{iv} + x''y_v''' - y_v''x''' + x_v''y''')\ WZ^2 + 4(x'y_v'''$$
$$-y_v'x''' - x_v'''y' + x_v'y''')SZ^2 + 6(y_v'y'' + y_v''y' + x''x_v'$$
$$+ x'x_v'')RW^2 \qquad (41)$$

It should be noted that solution difficulties are encountered at points where the Jacobian vanishes. For the case of Eqs. 31 and 32, such a point is the pole of the link. At the pole the first and second derivatives of the radius of curvature are zero and as such, the coordinates of the pole cause the numerators of Eqs. 28 and 29 to be zero. But at the pole, the Jacobian vanishes and ideally Δu and Δv are not defined at this point. The practical considerations of finite arithmetic, however, does yield a solution at the pole, though convergence may be difficult to achieve.

Further, generally at one of the link ends, the derivatives of f_2 with respect to u and v are double-valued. This contributes to convergence difficulties at the link end. However, solutions defining a link end are of no interest, thus the lack of convergence is inconsequential.

Success in obtaining solutions using the Newton-Raphson method depends primarily on the suitability of the initial parameters chosen. In this problem, it is particularly important that all solutions in the vicinity of the linkage be revealed. Thus, it is important that the initial values of u and v be chosen such that the convergence to the proper solutions is assured. These initial values may be chosen such that they satisfy the cubic of stationary curvature.

The cubic of stationary curvature defines the locus of points such that

$$\frac{d\rho}{dz} = 0 \qquad \text{(1) Repeated}$$

Equation 28 defines the locus of points in the real plane such that

$$\frac{d\rho}{d\phi} = 0$$

Asuming

$$\frac{d\rho}{d\phi} \; \frac{d\phi}{dz} = 0$$

the two functions should, and in fact do, describe the same set of points. Thus, using the cubic of stationary curvature, a set of points or initial values may be determined to satisfy one of the two equations of concern. Provided that these points are adequately spaced along the locus of points satisfying the first derivatives equation, the crossing of the first and second derivative equations should be adequately bracketed such that convergence to every crossing in the vicinity of the linkage is assured. Solutions at infinity are of no interest.

The Computer Program

The functions describing the zeroes of the derivatives and their solution are generated numerically using standard numerical techniques. An ANSI FORTRAN program for each of the two cases, straight path and circular path, has been developed. The listing of each program is listed in Appendix B.

Essentially the two programs are the same; however, the derivatives of the coordinate positions in the fixed plane are formed differently. The curved path program uses dimensioned coordinate positions, u and v, in the coupler coordinate system while the straight path program uses non-dimensional coordinates, μ and λ. Both programs provide for the point-by-point determination of the original and synthesized coupler curves and for the generation of a printer plot of these curves. For a CALCOMP plot of the coupler curves, it is suggested that the user may direct the point-by-point coordinates to an auxiliary file that may be read by a simple plotting program for the plot generation.

The programs consist of a number of special purpose subroutines, each of which performs a well defined function in the synthesis. These routines are orchestrated by a small main program that provides for the calling sequence and conditioning of the input and output arguments of the subroutines. All routines used, including commonly available scientific subroutines and functions, are shown in the appended listings.

The order of calling of the major subroutines and their principal functions are as follows:

SUBROUTINE TRIAL - This subroutine performs an Euler-Savary analysis to fix the inflection circle and the common centrode tangent of the link. The constants in the cubic of stationary curvature are determined. The asymptotic direction for the cubic is fixed. Fifty trial solutions are generated. Forty of these solutions are evenly distributed angularly around the cubic. Ten solutions are distributed along the asymptote.

SUBROUTINE NEWRAP - For the original linkage the coefficients of the expressions for the derivatives of the coordinate positions are generated. Using each of the trial solutions as a starting point, Newton-Raphson iterations are performed until the changes in the variables are less than some epsilon or until lack of convergence is apparent. If either or both variables proceed to infinity, lack of convergence is indicated. If the number of iterations exceeds 100, the values of the next 10 iterations are averaged and lack of convergence is assumed.

SUBROUTINE SOL - This routine inspects the results of the Newton-Raphson iterations. Solutions are deleted if

1. Lack of convergence is indicated;
2. Solution indicates a link end; or

3. Solution duplicates an existing solution within 0.5%.

SUBROUTINE RANK - This subroutine computes, for all confirmed solutions, the absolute value of $d^3\rho/d\phi^3$. The solutions are then ranked in increasing value of the absolute value of $d^3\rho/d\phi^3$.

SUBROUTINE STRLIN - For each solution, this routine locates the coupler point in the fixed plane and determines the radius of curvature and center of curvature of the coupler curve for this point in the design position.

SUBROUTINE ANALZE - For integral degree positions of the driving crank, the coordinates of the coupler point in both the original linkage and the synthesized linkage are computed and printed along with the differences in the x and y directions. SUBROUTINE DRAW may be called to provide a printer plot of the coupler curves.

AUXILIARY ROUTINES

SUBROUTINE SIMQ - IBM Scientific Subroutine which solves a set of simultaneous linear equations.

SUBROUTINE CIRCLE - This routine locates the center of a circle and determines the circle radius given the coordinates of three points of the circle.

Computer Program Specifications

Table 1 shows the size and time characteristics of both the straight path and curved path programs. These programs have been executed on an IBM System 370 and a Control Data Corporation 3300 System. Specifications for both systems are given.

Table 1

Computer Program Specifications

	Program			
	Straight Path		Curved Path	
Computer	IBM 370/158	CDC 3300	IBM 370/158	CDC 3300
Compiler	FORTG	MSOS	FORTG	MSOS
Core Req'd*	40.1	50	62.3	65
Compile Time Secs.	20	62	33	104
Execute Time† Secs.	14	159	26	192

* Core requirements are given in kilobytes for IBM and quarter-pages for CDC.

† Includes central processor and channel time for examples presented herein. Includes printer plot of coupler curves.

Discussion of the Accuracy of Computer Programs

The equations developed in previous and subsequent portions are exact and represent no approximations or compromises. However the use of these equations in a computer program that is executed with finite mathematics represents a compromise in the exactness of solutions.

The original versions of both programs were written such that the Newton-Raphson solution and the location of the coupler point and its center and radius of curvature were performed in double precision. The appended listings show single precision programs. With an epsilon of 10^{-7} for the Newton-Raphson solution, and double-precision calculations with 16 and 24 significant digits, and single precision calculations with 8 and 12 significant digits, it may be shown that no material differences will result in the solutions as a function of precision.

If the solutions obtained by inputting a desired linkage configuration are themselves input to the program, it may be shown that the original linkage configuration is given as the best solution. With a solution epsilon of 10^{-7}, the solutions obtained are recursive to within 6 to 7 significant digits.

The existing programs, as shown in the Appendix B, employ single precision arithmetic only. The recursion checks indicate that within the normal range of link length ratios the accuracy of the solutions is far greater than normal requirements. Link length ratios greater than

200 will cause problems because of the large number of exponentiations required by the solution technique.

Ideally, the use of a very large radius path (>10^{50}cm) in the circular path procedure should duplicate the straight path procedure. However, because of the aforementioned limits on link length ratios, the results are meaningless. With large length ratios within the 200 limit, the two procedures do approach each other.

The data processing software will affect the behavior of the routines with respect to underflows and overflows. The standard fix-ups taken in the cases of underflows and overflows will preserve the accuracy of the procedures. It is nonetheless bothersome to have the occurrences of over- and underflows printed, thus, the detection of these errors should be masked-off to avoid notification and to avoid termination of execution if applicable.

Chapter 3

Straight Line Path - Three Derivatives

Having established the general equations for the synthesis procedure with three derivatives, the particularized equations for the straight line path case are developed. A slider-crank mechanism is shown in Fig. 17 in which the doordinates of Point C and the fixed coordinate system are given by

$$x = r\cos\phi + b\cos(\Theta - \gamma)$$
$$y = r\sin\phi + b\sin(\Theta - \gamma) \tag{42}$$

or

$$x = r\cos\phi + xx\cos\gamma + yy\sin\gamma$$
$$y = r\sin\phi + yx\cos\gamma - xx\sin\gamma \tag{43}$$

But

$$\cos\gamma = \frac{l_x}{l} \qquad \sin\gamma = \frac{l_y}{l}$$

where $l_y = r\sin\phi - a_3$

Let $\lambda = xx/l \qquad \mu = yy/l$

Then

$$x = r\cos\phi + \lambda l_x + \mu l_y$$
$$y = r\sin\phi + \mu l_x - \lambda l_y \tag{44}$$

Or, expressing x and y in terms of r, l, a_3, and ϕ

$$x = r\cos\phi + \lambda l\cos\left[\sin^{-1}\left(\frac{r\sin\phi - a_3}{l}\right)\right] + \mu(r\sin\phi - a_3) \tag{45}$$

$$y = r\sin\phi - \lambda(r\sin\phi - a_3) + \mu l\cos\left[\sin^{-1}\left(\frac{r\sin\phi - a_3}{l}\right)\right] \tag{46}$$

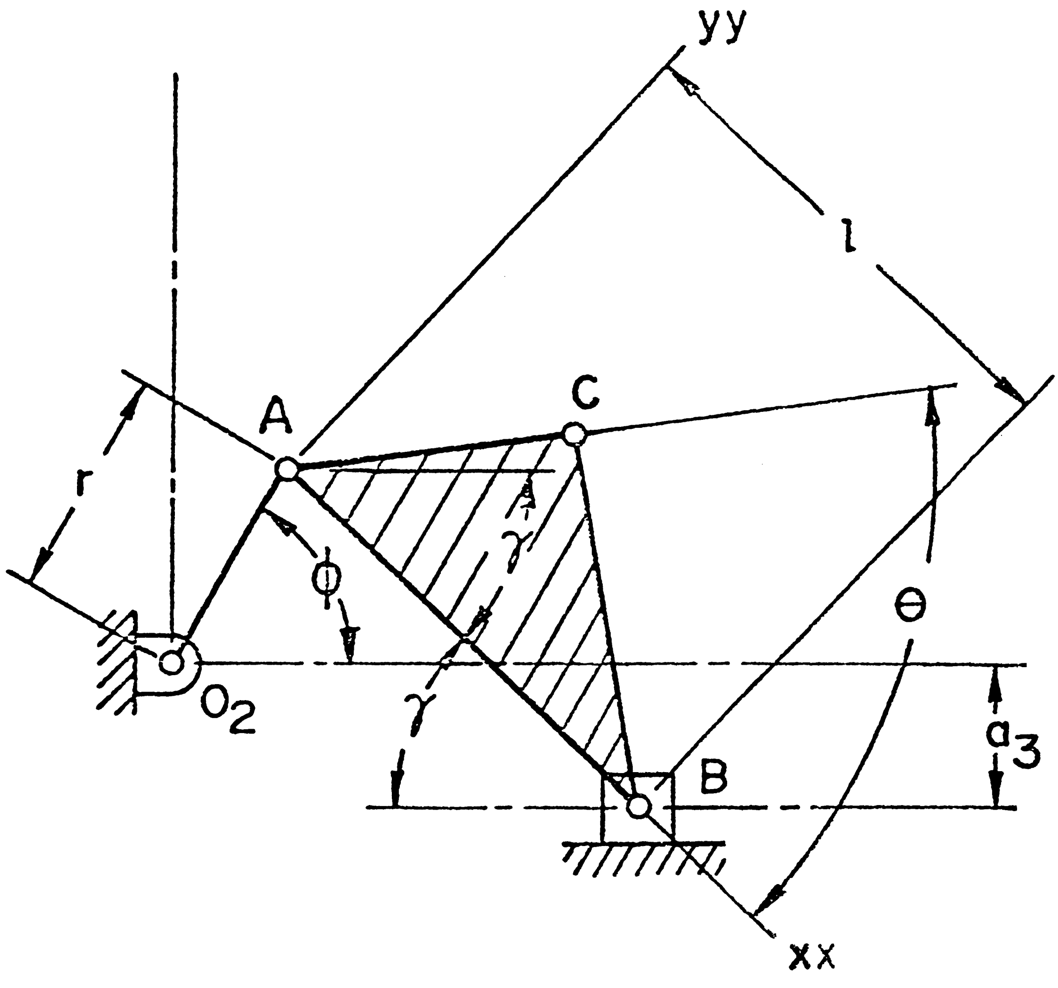

Figure 17. A Slider-Crank Mechanism

The derivatives of Eqs. 45 and 46 with respect to the displacement parameter ϕ may now be taken. These derivatives were taken on the computer using a formula manipulator, FORMAC. The resulting expressions are shown in the Appendix A as a part of the Straight Path Equations. The expressions are shown as X1L, Y2M, X4L, etc. as described previously. X1L indicates the lambda coefficient in $dx/d\phi$; Y2C indicates the constant coefficient of $d^2y/d\phi^2$.

It should be noted that the coefficients that make up these derivatives are functions of r, l, a_3 and ϕ only. If the original slider crank has been described in terms of these parameters, the nature of the derivatives throughout the moving plane can be determined, and these derivatives will vary linearly with respect to the coordinate positions λ and μ.

Straight-Line Path Example

An illustration of a straight-line path synthesis example follows. Assume that the slider crank mechanism used in the previous examples were to be the object of this two degrees of freedom synthesis procedure utilizing the first, second, and third derivatives of the radius of curvature of a coupler point path.

The slider crank linkage is described by

Crank radius = 3.176162 cm

Connecting rod length = 8.291779 cm

Slider path offset = 2.50682 cm

Crank angle = 60.000^0 = 1.047198 radians

Table 2 shows those characteristics of the slider-crank mechanism used for the generation of trial solutions.

At this point, ψ in Eq. 3 may be incremented, solving for r, and in turn for s and t in Eqs. 6 and 7. Thus the forty trial solutions on the cubic of stationary curvature are generated. The remaining ten trial solutions are distributed along the asymptote. Figure 18 shows a plot of the trial solutions in the $t - s$, $\mu - \lambda$, and $x - y$ coordinate systems.

Next, each of the coefficients of the derivatives of x and y are evaluated. Then using each of the trial solutions and Eqs. 34 through 41, Newton-Raphson iterations are continuously made until the procedure converges to a solution or lack of convergence is indicated. Figure 19 shows the original trial solutions and the solutions to which each trial converges by the Newton-Raphson method.

Table 2

Characteristics of the Original Linkage
Used in the Straight Path Example

Point A is at	(1.58808, 2.75068) cm
Point C is at	(8.00000, -2.506821) cm
Connecting rod pole is at	(8.00000, 13.85642) cm
IA	12.8239 cm
IB	16.3632 cm
Points on the inflection circle	(-24.3002, -42.0893) cm (8.00000, -2.506821) cm (8.00000, 13.85642) cm
Inflection circle centered at	(-42.4296, 5.6748) cm
Inflection circle radius	51.0889 cm
Angle between the common centrode and the horizontal	-1.40995 cm
ψ_A	5.59875 radians
r_A	12.8239 cm
ψ_C	-1.60837 radians
r_C	16.3632 cm
M	5311.99
N	16.54536
Angle between centrode tangent and asymptote	-0.0003115 radians

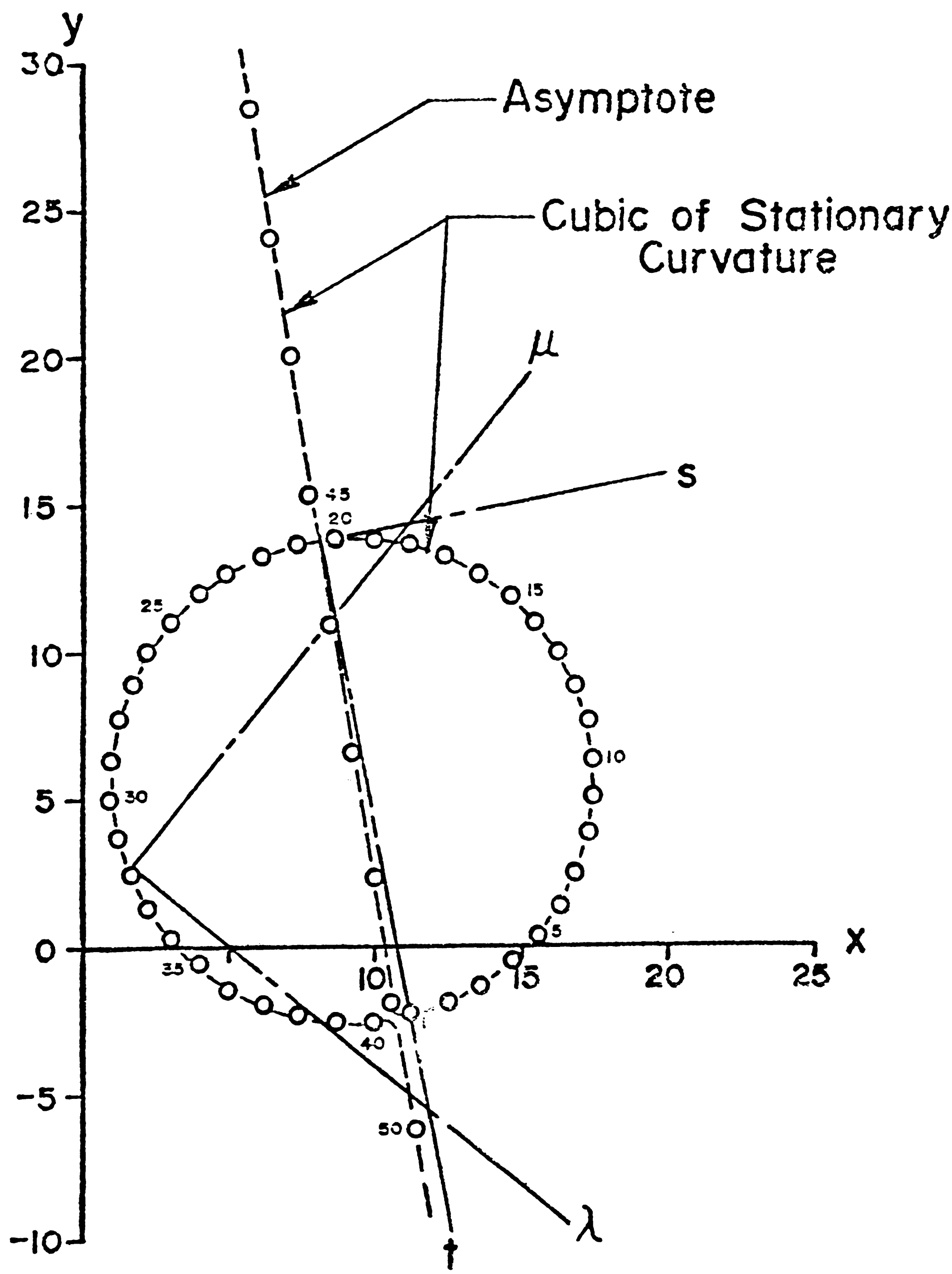

Figure 18. Trial Solutions in the Fixed Plane

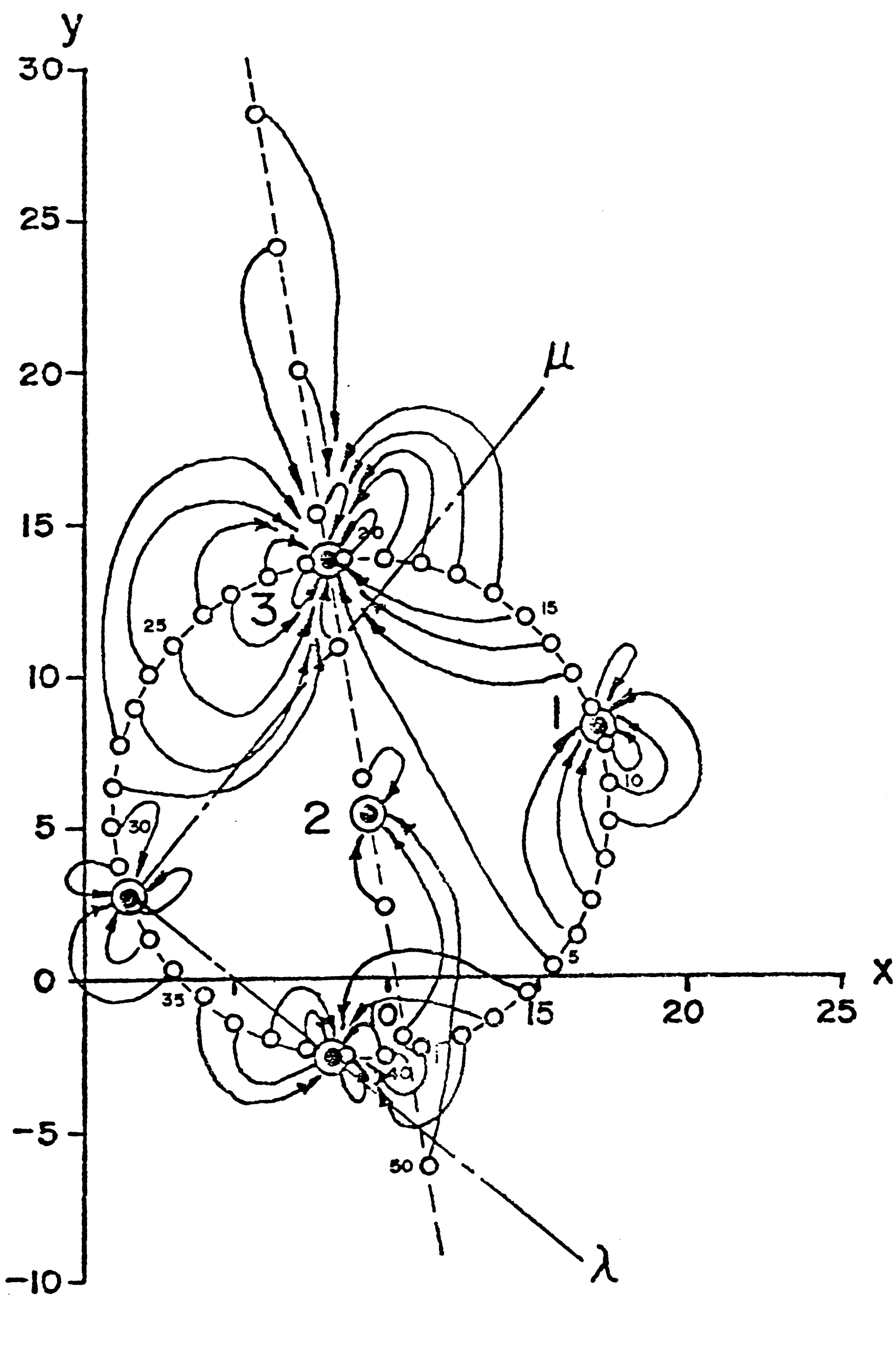

Figure 19. Migration of the Trial Solutions in a Newton-Raphson Procedure

Figure 20 shows the loci of zeroes of the first and second derivatives of the radius of curvature of coupler points in the fixed plane. These curves were obtained by a procedure independent of the solution technique described herein. Note that the coincident zeroes in Fig. 20 correspond to the indicated solutions in Fig. 19.

The coupler points whose first and second derivatives of the radius of curvature of the coupler curve are equal to zero are shown in Table 3.

Figure 21 shows the linkage in the slider-crank configuration with coupler curves plotted in part for each of the solutions. Note that Solution 3 indicates the pole of the coupler and as such is of no interest.

Solution 1

$\mu = 1.69360$

$\lambda = 1.05318$

Coupler point position in the fixed plane = (17.2450, 8.07279) cm

Radius of curvature of coupler path = 1.3550 cm

Center of curvature = (16.0959, 8.79170) cm

The four-bar linkage indicated by this solution is shown in Fig. 22.

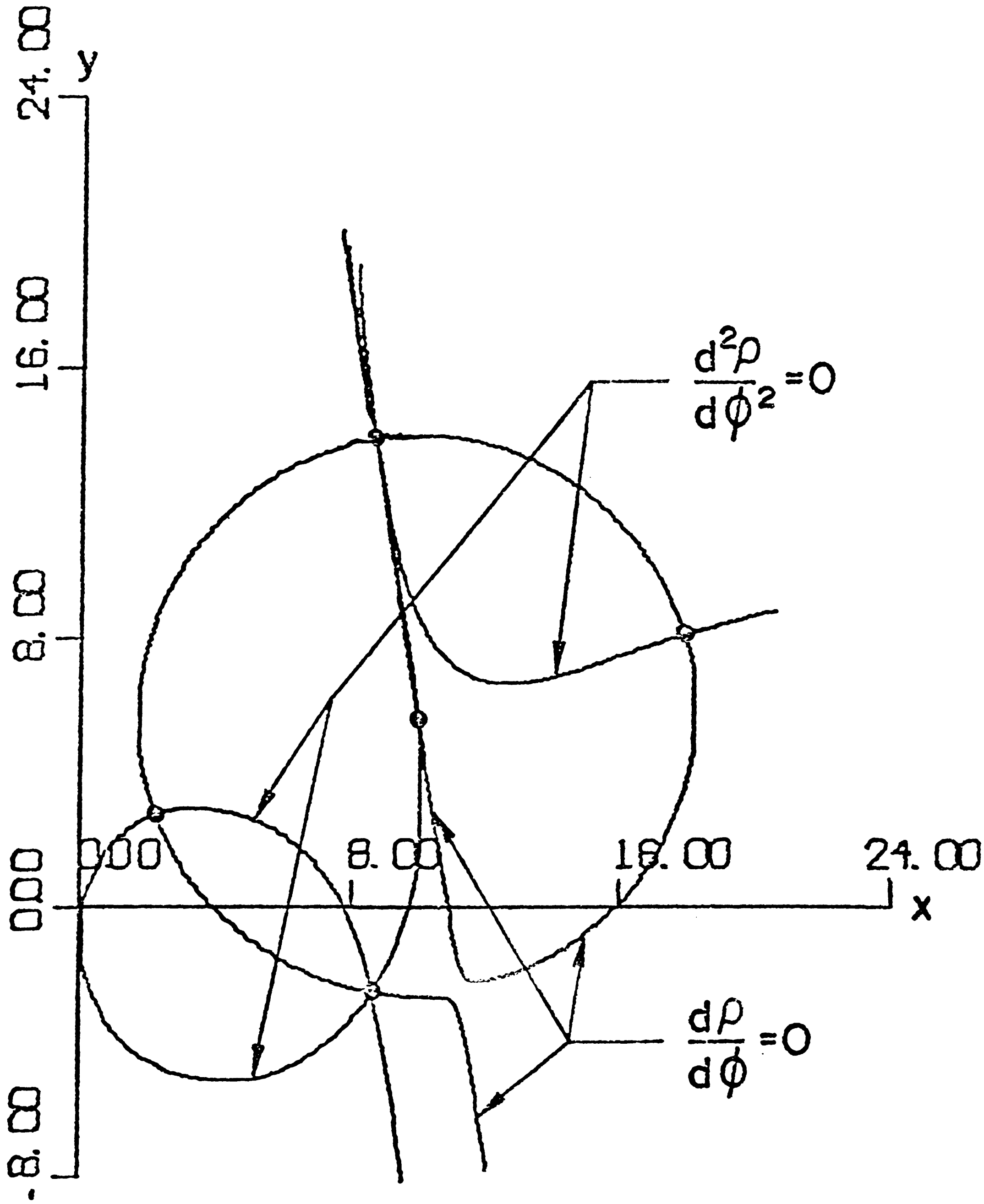

Figure 20. The Loci of the Zeroes of the First and Second Derivatives of the Radius of Curvature for the Slider-Crank Mechanism

Table 3

Solutions for the Straight Path Example

Solution Number	Coupler Point B Coordinates		$\left\lvert \frac{d^3\rho}{d\phi^3} \right\rvert$
	λ	μ	
1	1.05318	1.69360	42.425
2	.52381	.1842330	521.03
3	-.25127	1.52603	2.5865×10^{13}

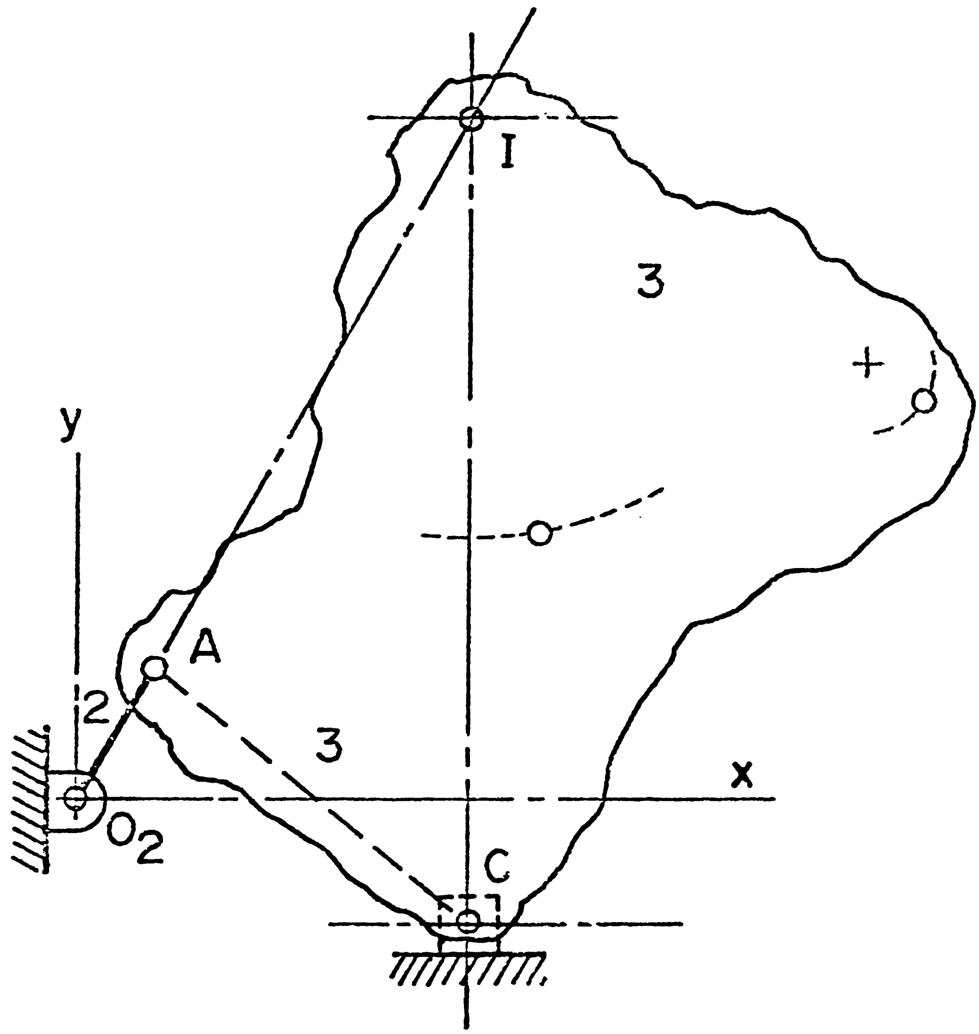

Figure 21. The Behavior of Solution Points in the Coupler Plane of the Slider-Crank Mechanism

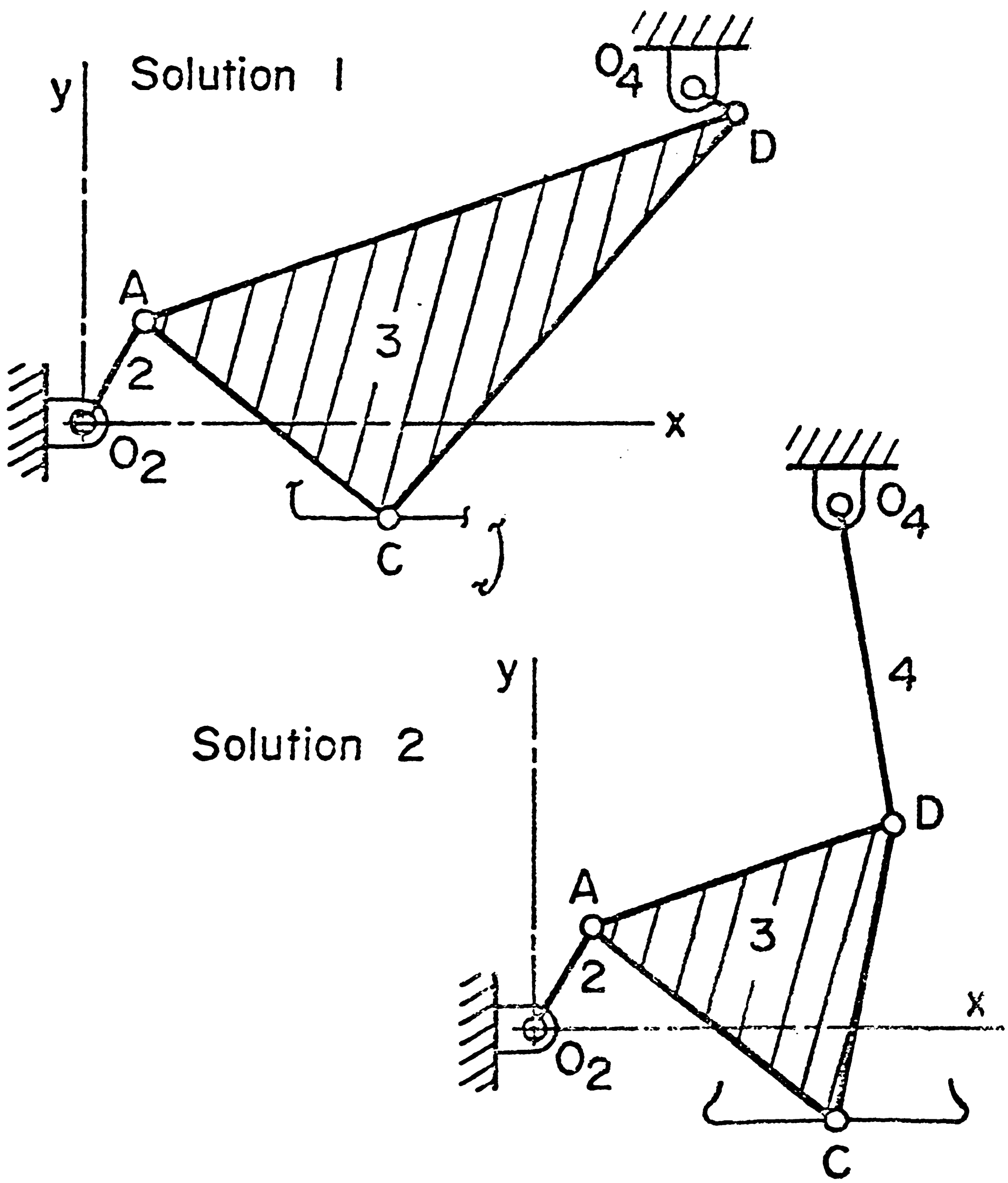

Figure 22. The Configurations of the Four-Bar Solutions

Solution 2

$\mu = 0.842330$

$\lambda = 0.52381$

Coupler point position in the fixed plane = (9.3752, 5.39767) cm

Radius of curvature of coupler path = 8.5358 cm

Center of curvature = (8.0055, 13.8228) cm

The four-bar linkage indicated by this solution is also shown in Fig. 22.

Figure 23 shows a comparison of the coupler curves of Point C in the four-bar configurations of the two solutions and the original slider path and stroke.

Assuming ω_2 = 100 radians/sec clockwise and $\alpha_2 = 0$, a velocity analysis of the original slider-crank mechanism and of each of the solutions shown in Fig. 22, results in the angular velocity of the coupler, ω_3, being 24.7676 radians/sec in each case. An acceleration analysis of each of the same linkages with the same assumptions, determines the angular acceleration of the coupler, α_3, to be 3786.9 radians/sec/sec in each case. Since no first or second order approximations have been made, it is to be expected that the first and second order displacement functions for the coupler of each solution would be exact.

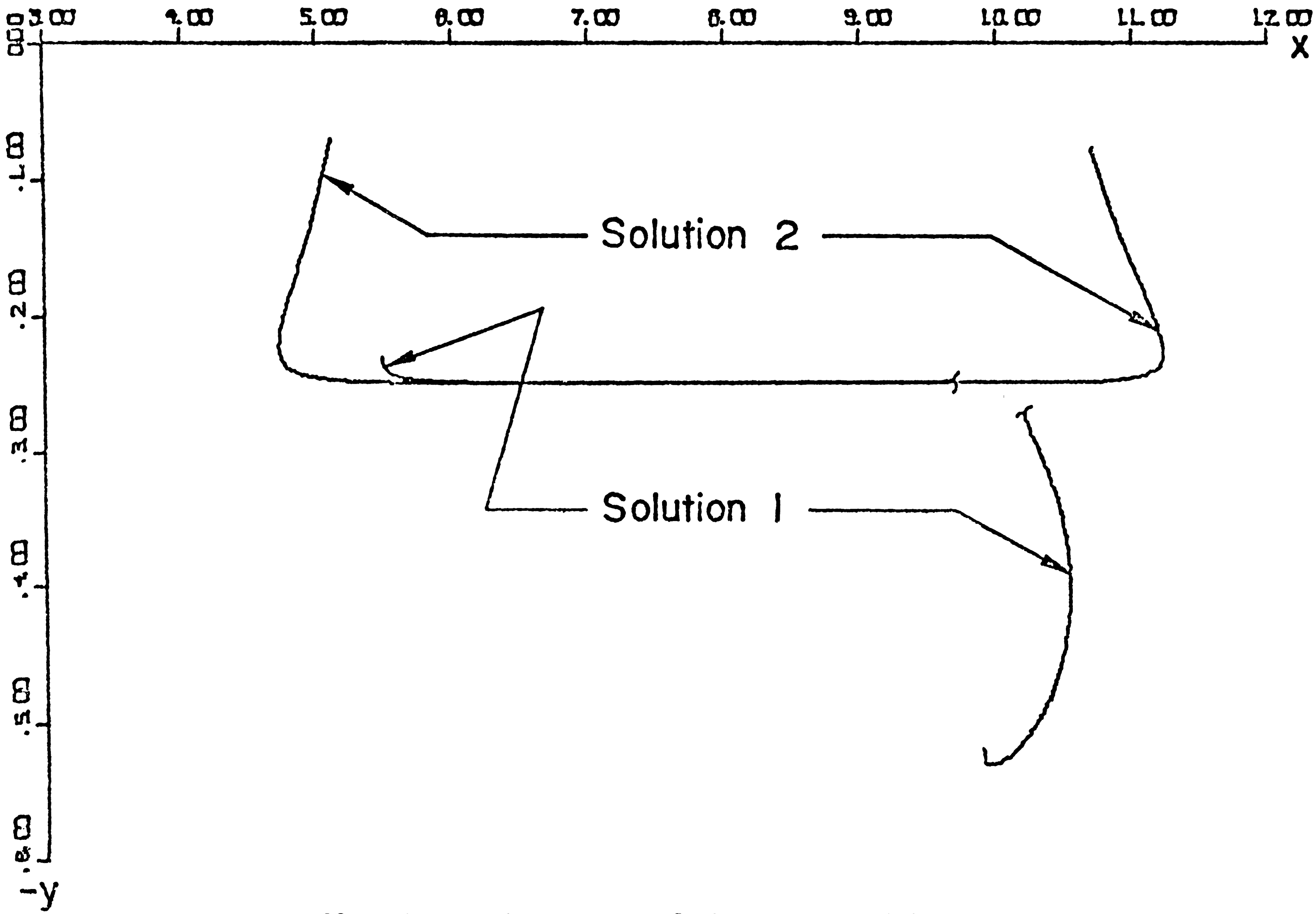

Figure 23. The Coupler Curves of the Four-Bar Solutions

Chapter 4

Circular Path - Three Derivatives

Given a four-bar linkage, as shown in Fig. 24, the vector loop equation may be written

$$\bar{a} + \bar{b} + \bar{c} = \bar{d} \qquad (47)$$

From Fig. 25, Kersten [16] shows that

$$c^2 = \left[(d - a\cos\phi) - b\cos\beta\right]^2 + \left[a\sin\phi + b\sin\beta\right]^2$$

$$c^2 = (d - a\cos\phi)^2 - 2b(d - a\cos\phi)\cos\beta + b^2\cos^2\beta + (a\sin\phi)^2 + 2ab\sin\beta\sin\phi + b^2\sin\beta$$

Now using

$$P = d - a\cos\phi$$

$$Q = a\sin\phi$$

$$c^2 = P^2 - 2bP\cos\beta + Q^2 + 2bQ\sin\beta + b^2$$

or

$$Q\sin\beta = \frac{c^2 - b^2 - P^2 - Q^2 + 2bP\cos\beta}{2b}$$

$$= P\cos\beta - \frac{P^2 + Q^2 + b^2 - c}{2b}$$

Using

$$V = \frac{P^2 + Q^2 + b^2 - c^2}{2b}$$

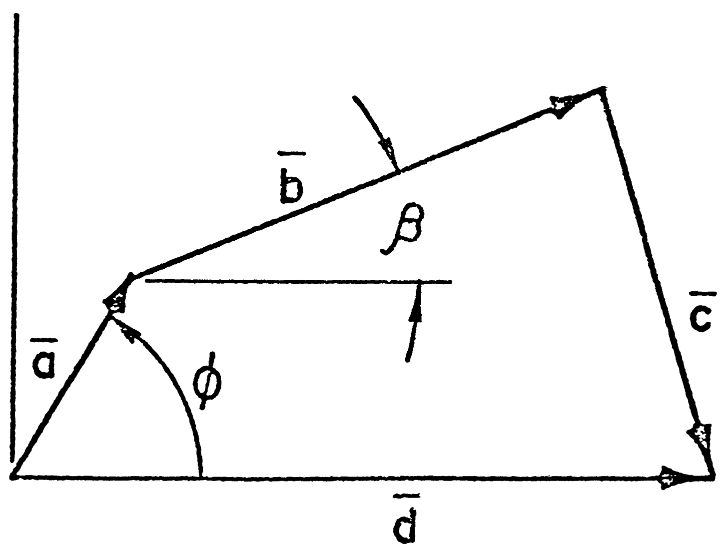

Figure 24. The Vector Loop for a Four-Bar Linkage

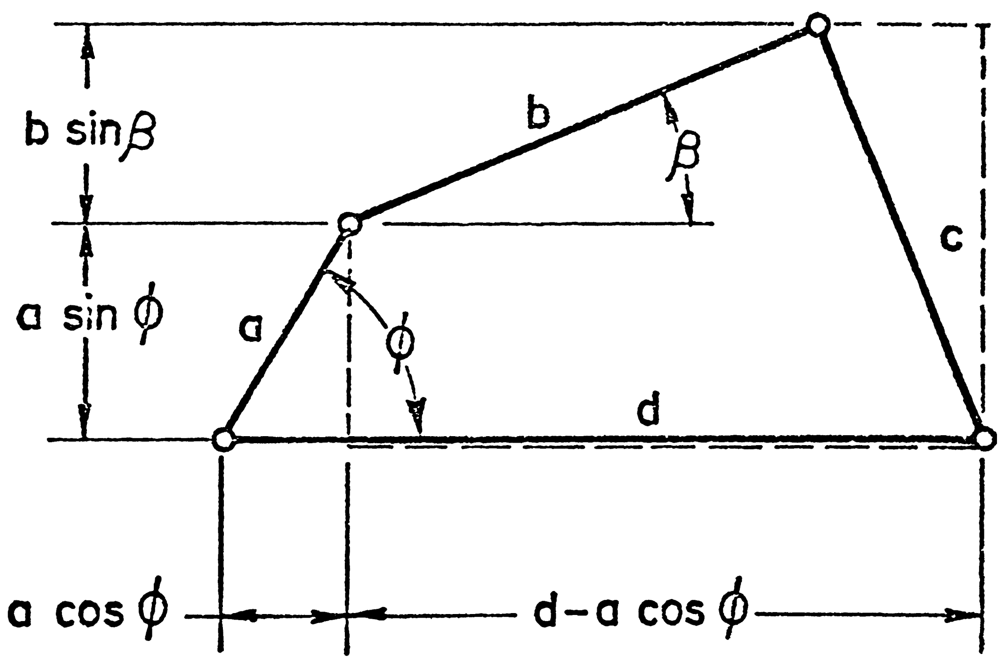

Figure 25. The Geometry of the Four-Bar Vector Loop

then

$$Q \sin \beta = P \cos \beta - V$$

$$P \cos \beta = V + Q \sin \beta$$

$$P^2 \cos^2 \beta = V^2 + 2VQ \sin \beta + Q^2 \sin^2 \beta$$

$$P^2 - P^2 \sin \beta = V^2 + 2VQ \sin \beta + Q^2 \sin \beta$$

$$\sin^2 \beta + 2 \frac{VQ}{P^2 + Q^2} \sin \beta + \frac{V^2 - P^2}{P^2 + Q^2} = 0$$

Using

$$D = \frac{VQ}{P^2 + Q^2} \quad \text{and}$$

$$E = \frac{V^2 - P^2}{P^2 + Q^2}$$

Note: $(P^2 + Q^2) = 0$ only when $P^2 = 0$ and $Q^2 = 0$, for which $\phi = 0$, $a = d$, and $b = c$

Then

$$\sin^2 \beta + 2D (\sin \beta) + E = 0$$

Therefore

$$\sin \beta = -D \pm \sqrt{D^2 - E} \qquad (48)$$

Also

$$\cos \beta = \frac{V + Q \sin \beta}{P} \qquad (49)$$

The double solution is a result of the dual geometric inversions possible for a four-bar linkage described in terms of link length and one crank angle. Therefore, in dealing with a definite linkage, it is necessary to particularize the sign of the radical. Figure 26 shows the geometric inversions for a four-bar linkage.

Then

$$\beta = \sin^{-1}\left[- D \pm \sqrt{D^2 - E} \right]$$

Having defined the angle β for a four-bar linkage as shown in Fig. 27, the parametric equations indicating the position of a coupler point in the fixed plane are

$$\begin{aligned} x &= a \cos \phi + u \cos \beta - v \sin \beta \\ y &= a \sin \phi + u \sin \beta + v \cos \beta \end{aligned} \qquad (50)$$

Then

$$\begin{aligned} x' &= \frac{dx}{d\phi} = -a \sin \phi - u \sin \beta \frac{d\beta}{d\phi} - v \cos \beta \frac{d\beta}{d\phi} \\ y' &= \frac{dy}{d\phi} = a \cos \phi + u \cos \beta \frac{d\beta}{d\phi} - v \sin \beta \frac{d\beta}{d\phi} \end{aligned} \qquad (51)$$

where

$$\frac{d\beta}{d\phi} = \frac{d}{d\phi}\left[\sin^{-1} \left(- D \pm \sqrt{D^2 - E} \right) \right] \qquad (52)$$

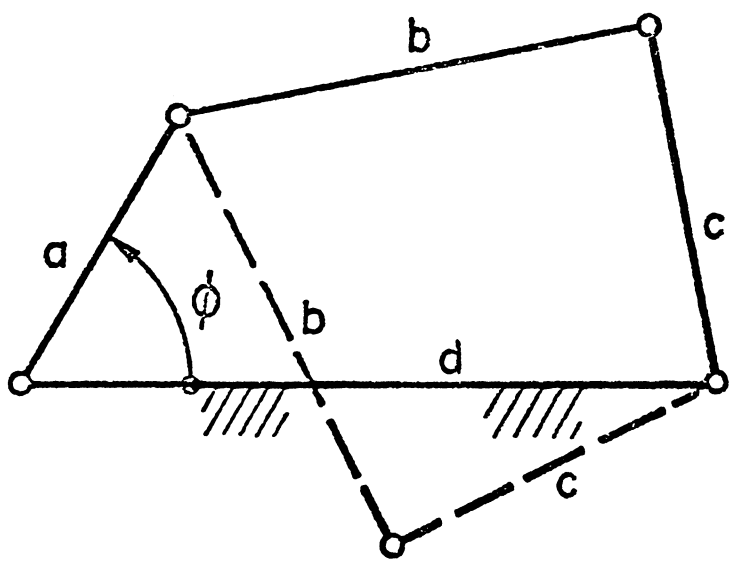

Figure 26. Geometric Inversions of a Four-Bar Linkage

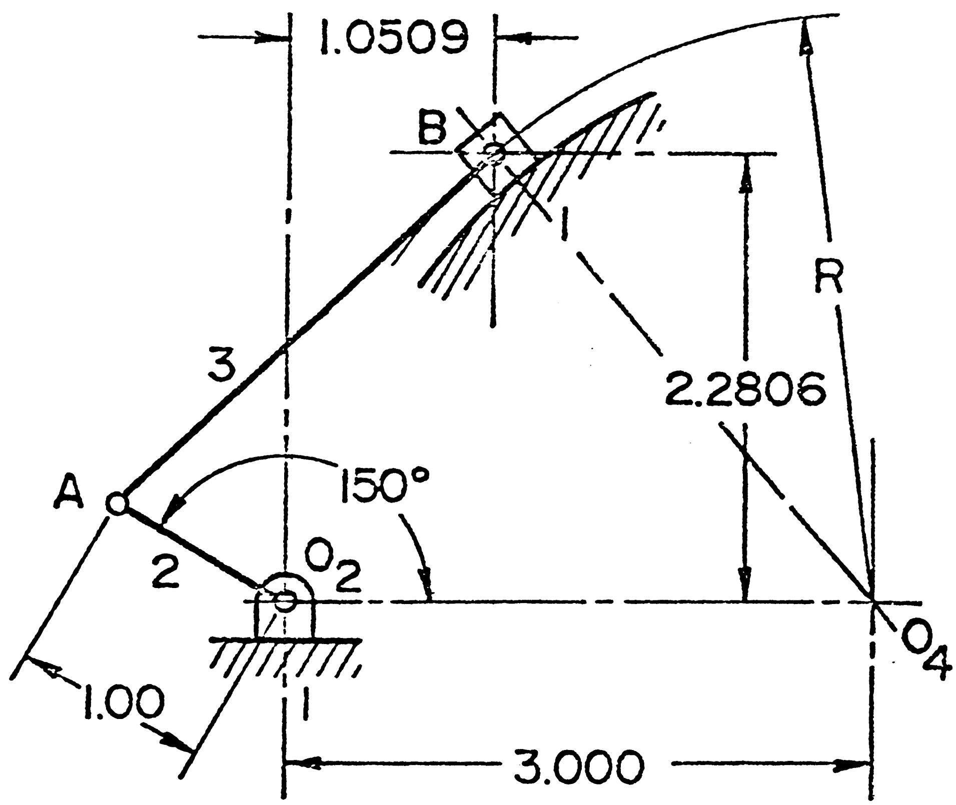

Figure 27. The Four-Bar Linkage

Then

$$\frac{d\beta}{d\phi} = \frac{d}{d\phi}\left\{ \sin^{-1}\left[\frac{1}{P^2 + Q^2} \left(-VQ \pm P\sqrt{(P^2 + Q^2) - V^2}\right)\right]\right\}$$

$$= \frac{d}{d\phi}\left\{ \sin^{-1}\left[\frac{1}{(a^2 + d^2 - 2\,ad\cos\phi)} \left\{\left(\left(\frac{a^2 d}{2b} \sin 2\phi - \left[\frac{a^2 + ad^2 + ab^2 - ac^2}{2b}\right]\sin\phi\right) \pm (d - a\cos\phi)\left[a^2 + d^2 - 2ad\cos\phi\right) - \frac{a^4 + 2a^2d^2 + 2a^2b^2 - 2a^2c^2 - 2c^2b^2 + c^4 + d^4}{4b^2} - \frac{2b^2d^2 - 2c^2d^2 + b^4}{4b^2} \quad \frac{a^2d^2}{b^2} - \frac{a^3d + ad^3 - ab^2d - ac^2d}{b^2}\right]^{\frac{1}{2}} \cos\phi\right\}\right]\right\}$$

Let

$KK = a^2 + d^2$

$L = 2ad$

$HH = (a^3 + ad^2 + ab^2 - ac^2)\,/2b$

$$JJ = \frac{a^2 d}{2b}$$

$MM = (a^4 + 2a^2d^2 + 2a^2b^2 - 2a^2c^2 - 2c^2b^2 + c^4 + d^4 + 2b^2d^2 - 2c^2d^2 + b^4)/(4b^2)$

$$NN = \frac{a^2 d^2}{b^2}$$

$$UU \quad \frac{a^3 d + ad^3 - ab^2 d - ac^2 d}{b^2}$$

Then

$$\frac{d\beta}{d\phi} = \frac{d}{d\phi}\left\{\sin^{-1}\left[\frac{1}{KK-L\cos\phi}\left(-\left[HH\sin\phi - JJ\sin 2\phi\right] \pm \left[d-a\cos\phi\right]\left[(KK-L\cos\phi)-(MM+NN\cos^2\phi-UU\cos\phi)\right]^{\frac{1}{2}}\right)\right]\right\}$$

Let

$$AA = KK - MM$$

$$BB = UU - L$$

$$\frac{d\beta}{d\phi} = \frac{d}{d\phi}\left\{\sin^{-1}\left[\frac{JJ\sin 2 \;-HH\sin\phi}{KK - L\cos\phi} \pm \frac{\left[(d-a\cos\phi)^2\,(AA+BB\cos\phi - NN\cos^2\phi)\right]^{\frac{1}{2}}}{KK - L\cos\phi}\right]\right\} \tag{53}$$

Now $d\beta/d\phi$ (Eq. 53) may be substituted into Eqs. 51 in order to define x' and y' for the circular path case. The problem of the plus or minus sign of the radical in Eq. 53 may be solved by introducing a variable of unit magnitude. The sign of this variable may be determined such that the size of angle β as given by Eq. 48 corresponds to that size of β on the original four-bar linkage.

The expressions for x' and y' may be successively differentiated with respect to the crank angle ϕ such that Eqs. 26 and 27 are defined for the circular path case. Now the general equations of Chap. 3 may be used to search for coupler points such that the first and second derivatives of the radius of curvature with respect to the crank angle are zero.

Circular Path Example

An example of the two degrees of freedom as it applies to the synthesis of a circular path is presented. Assume that the synthesis of an approximately circular path corresponding to that of Point C in Fig. 27 is desired. Using the two degrees of freedom synthesis procedure, points on the coupler (Link 3 in Fig. 27) shall be determind such that the first and second derivatives of the radius of curvature with respect to the crank angle of the path of such points are zero.

The characteristics of the original linkage of Fig. 27 used for the generation of trial solutions are shown in Table 4.

With M and N having been defined, the cubic of stationary curvature may be used to generate the set of 50 trial solutions. These trial solutions are plotted in Fig. 28. At this point the Newton-Raphson procedure may be applied to each of the trial solutions. Figure 29 shows the behavior of each of the trial solutions with respect to convergence to a solution or a point in the moving plane such that $d\rho/d\phi=0$ and $d^2\rho/d\phi^2=0$.

The unique solutions result from culling the Newton-Raphson results are shown in Table 5.

Solution 1

u = 6.205584 cm

v = .468489 cm

Coupler point position in the fixed plane=(3.36187, 5.055578)cm

radius of curvature of coupler path = 2.39977 cm

Center of curvature = (2.66866, 7.36404) cm

Table 4

Characteristics of the Original Linkage
Used in the Circular Path Example

Crank radius	1.0000 cm
Crank angle	2.617994 cm
Center of curvature of coupler point path	(3.0000, 0.0000) cm
Coupler point at	(1.050888, 2.280562) cm
Connecting rod pole at	(5.922297, -3.419239) cm
IA	7.838479 cm
IB	7.497890 cm
Points on the inflection circle	(52.34410, -30.22087) cm (13.22598, -11.96493) cm (5.922297, -3.419239) cm
Inflection circle center at	(58.58922,34.19929) cm
Inflection circle radius	64.72217 cm
Angle between the centrode tangent and the horizontal	2.19104 radians
ψ_A	0.426957 radians
r_A	7.838479 cm
ψ_C	.0869542 radians
r_C	7.49789 cm
M	553.8348
N	8.916274
Angle between centrode tangent and asymptote	-0.016097 radians

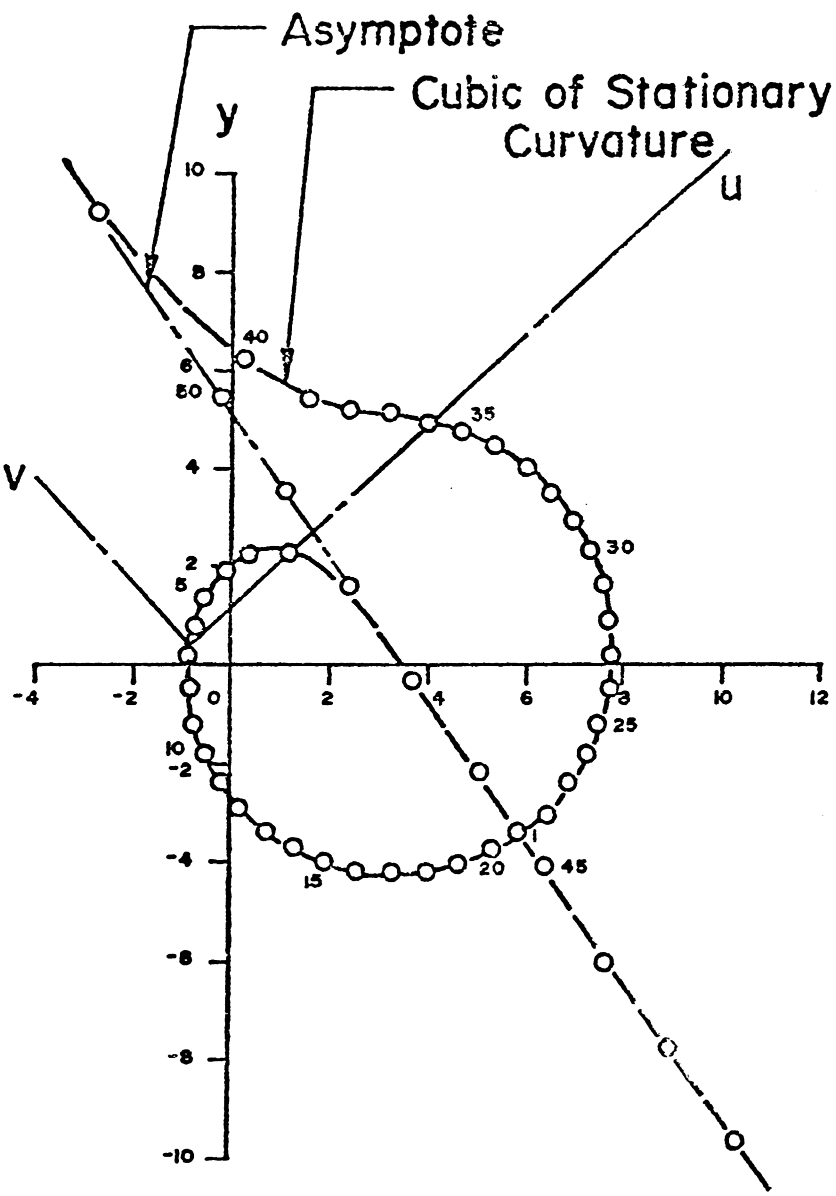

Figure 28. Trial Solutions in the Fixed Plane

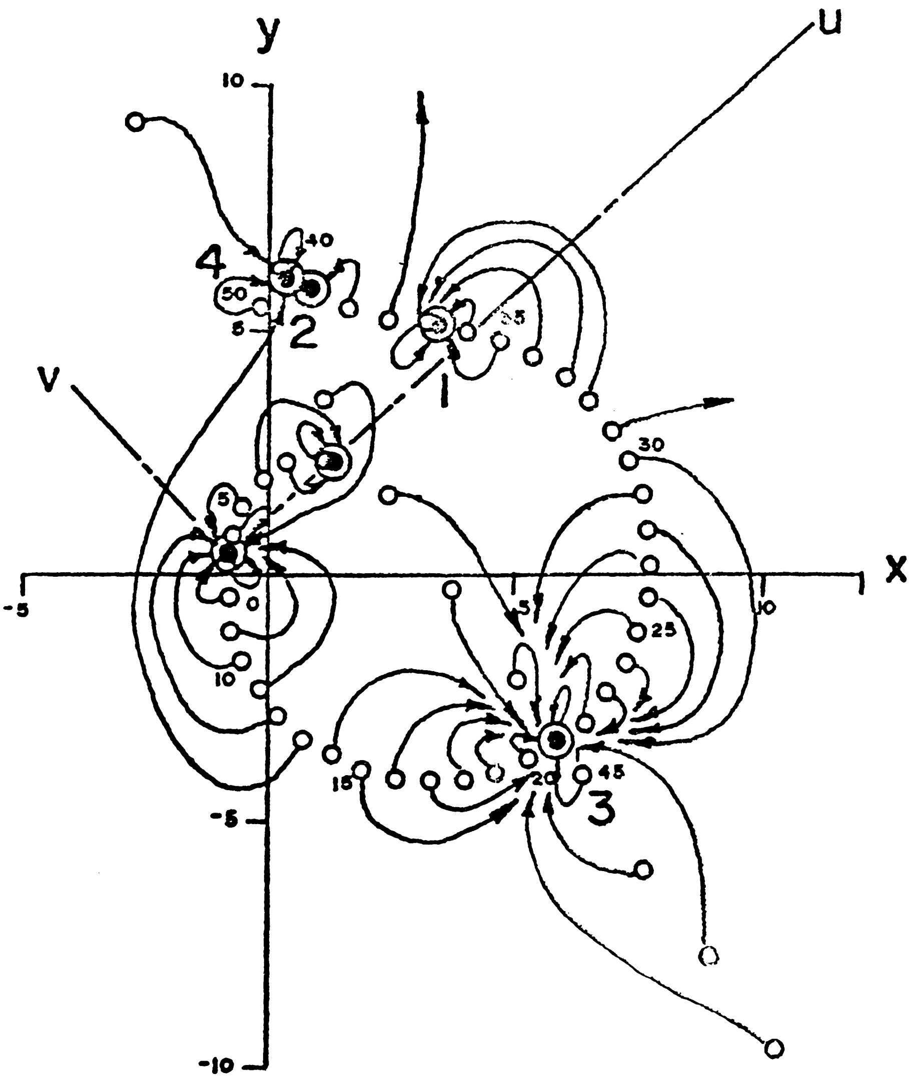

Figure 29. Migration of the Trial Solutions in a Newton-Raphson Procedure

Table 5

Solutions for the Circular Path Example

Solution Number	Coupler Point B Coordinates		$\left\vert \frac{d^3\rho}{d\phi^3} \right\vert$
	u, cm	v, cm	
1	6.205584	.468489	32.2341
2	4.705078	2.771973	2095.03
3	2.306390	-7.491483	7.7639×10^{12}
4	4.624201	3.221359	2.4706×10^{22}

The four-bar linkage indicated by this solution is shown in Fig. 30

Solution 2

$u = 4.705078$ cm

$v = 2.771973$ cm

Coupler point position in the fixed plane = (.694796, 5.73311) cm

Radius of curvature of coupler path = 43.5759 cm

Center of curvature = (-20.9174, 43.5719) cm

The four-bar linkage indicated by this solution is shown in Fig. 31

The third solution is at the coupler pole, and as such is of no interest.

Solution 4

$u = 4.624201$ cm

$v = 3.221359$ cm

Coupler point position in the fixed plane = (.3297006, 6.00732) cm

Radius of curvature of coupler path = 1.8789×10^{11} cm

Center of curvature = $(9.5868 \times 10^{10}, - 1.6159 \times 10^{11})$ cm

The four-bar linkage indicated by this solution is shown in Fig. 32. In the discussion that follows, it is assumed that the radius of the coupler path in this case is infinite.

Figure 33 shows the plot of the coupler curve of Point B in the configuration shown in Fig. 30. Figure 34 shows the plot of the

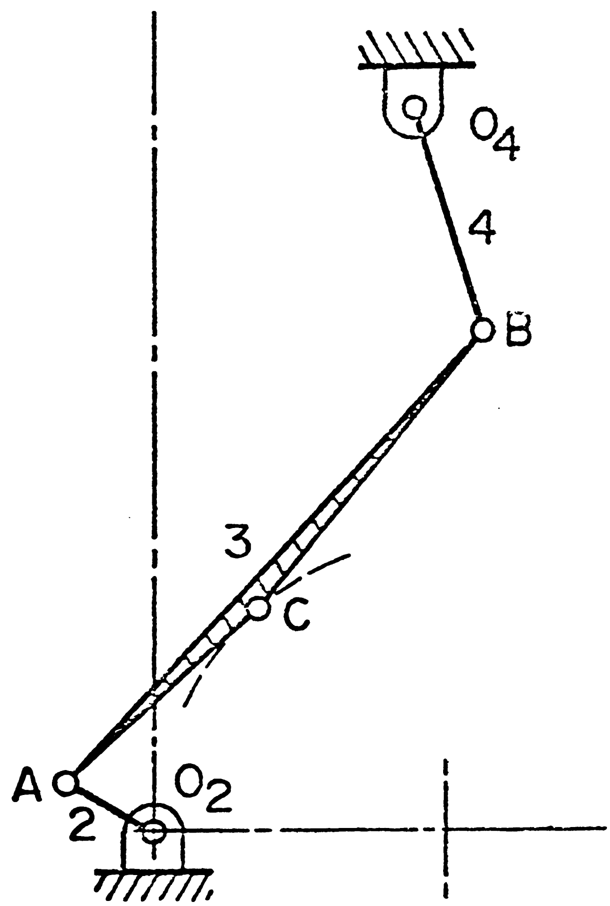

Figure 30. Solution 1, a Four-Bar Linkage with a Circular Coupler Curve

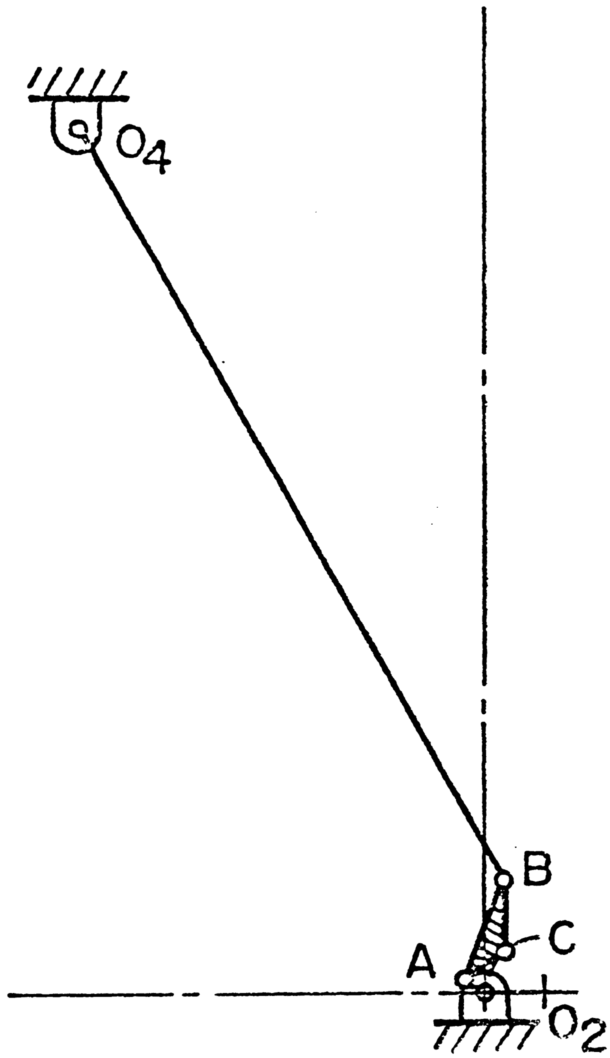

Figure 31. Solution 2, a Four-Bar Linkage with a Circular Coupler Curve

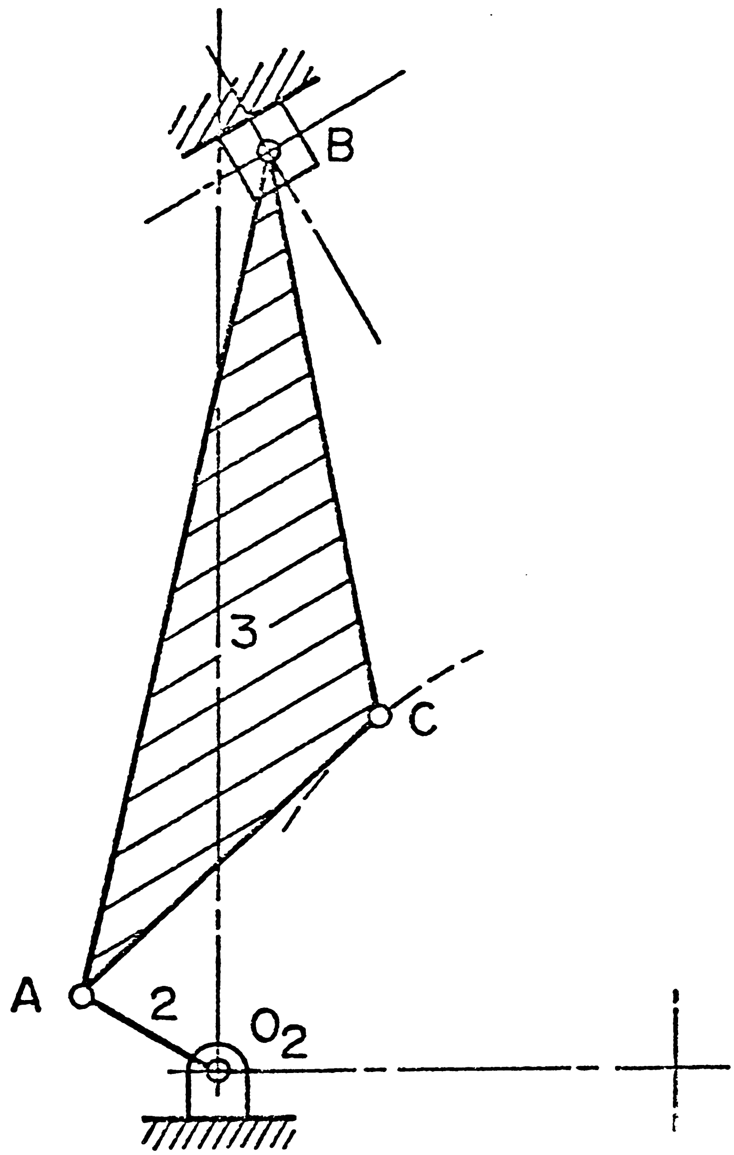

Figure 32. Solution 4, a Four-Bar Linkage with a Circular Coupler Curve

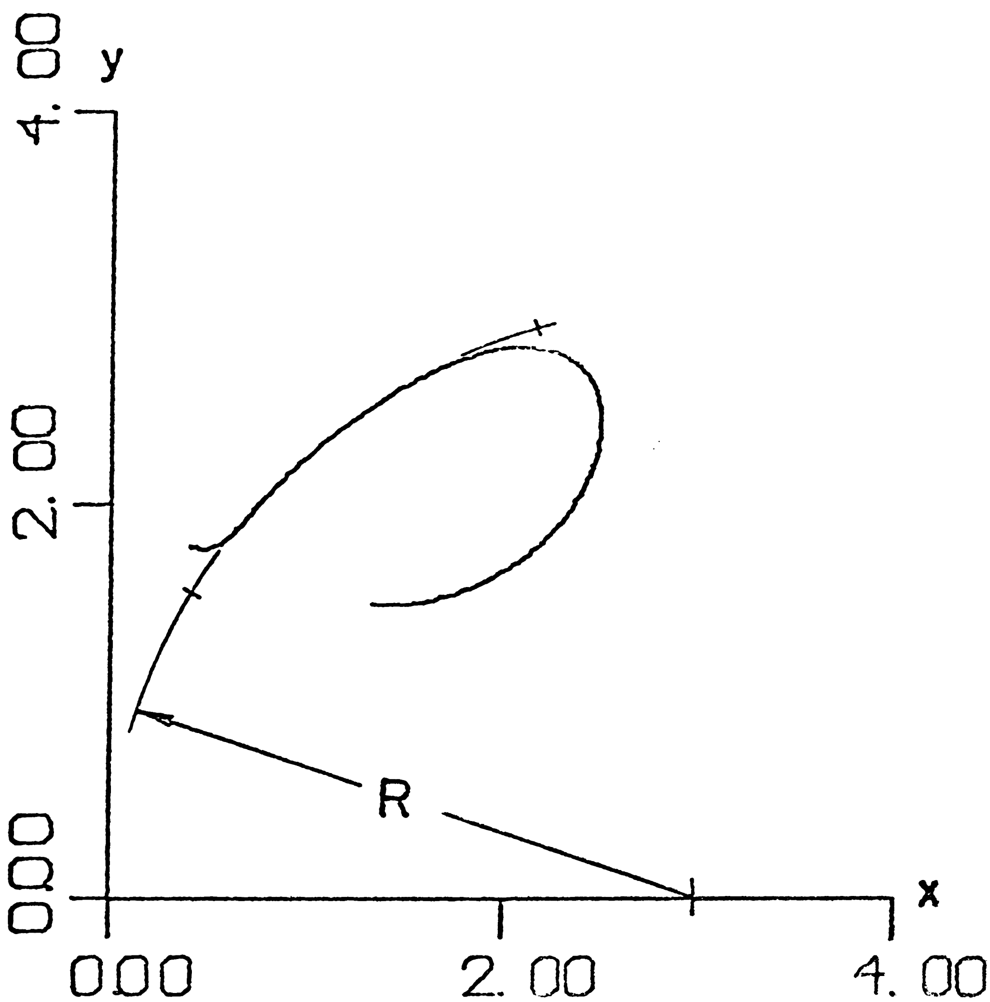

Figure 33. The Coupler Curve of Solution 1

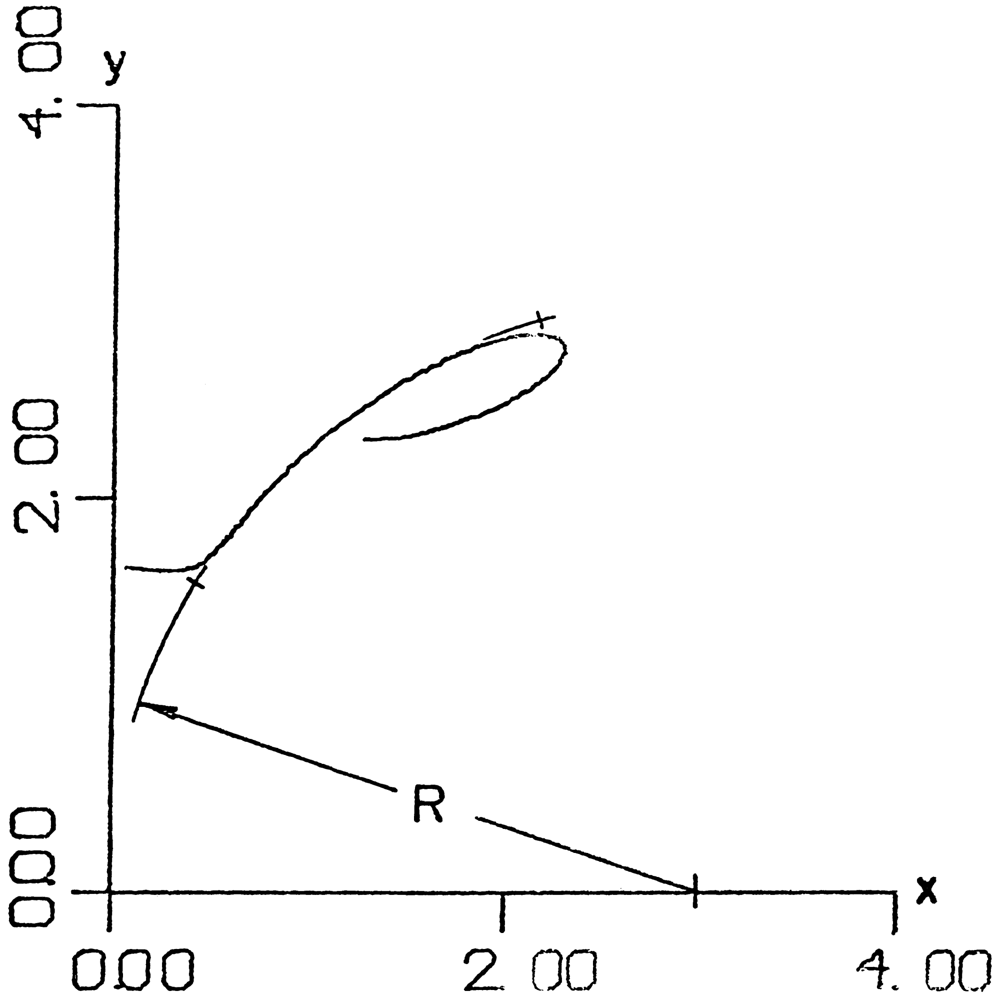

Figure 34. The Coupler Curve of Solution 2

coupler curve of Point C in the configuration shown in Fig. 31. In both Figs. 33 and 34, the path of Point C in the original mechanism, Fig. 27, is shown as an arc of radius R. Because the ratio of link lengths is large, a plot of the coupler curve for Solution 4 is omitted.

The loci of the zeroes of the first and second derivatives of the radius of curvature in the moving plane are shown in Fig. 35 for the original mechanism of Fig. 27. The transformation of this mapping into the fixed plane is shown in Fig. 36.

Figures 37 and 38 show the loci of the zeroes of the first and second derivatives for Solution 1 (Fig. 30) in the moving and fixed planes respectively. Figures 39 and 40 show the loci of the zeroes of the first and second derivatives for Solution 2 (Fig. 31) in the moving and fixed planes respectively.

A comparison of Figs. 36, 38, and 40 reveals that in the fixed plane the original linkage and its synthesized solutions have identical loci of the zeroes of the first and second derivatives of the radius of curvature. Correspondence is noted not only at the coincident zeroes, but throughout the fixed plane. In the case of the first derivative, constraints on the constants in the cubic of stationary curvature indicate that coincident cubics will result. Because the expanded form of the second derivative equation transformed to the fixed plane is unobtainable, no definitive explanation of the second derivative zeroes can be given.

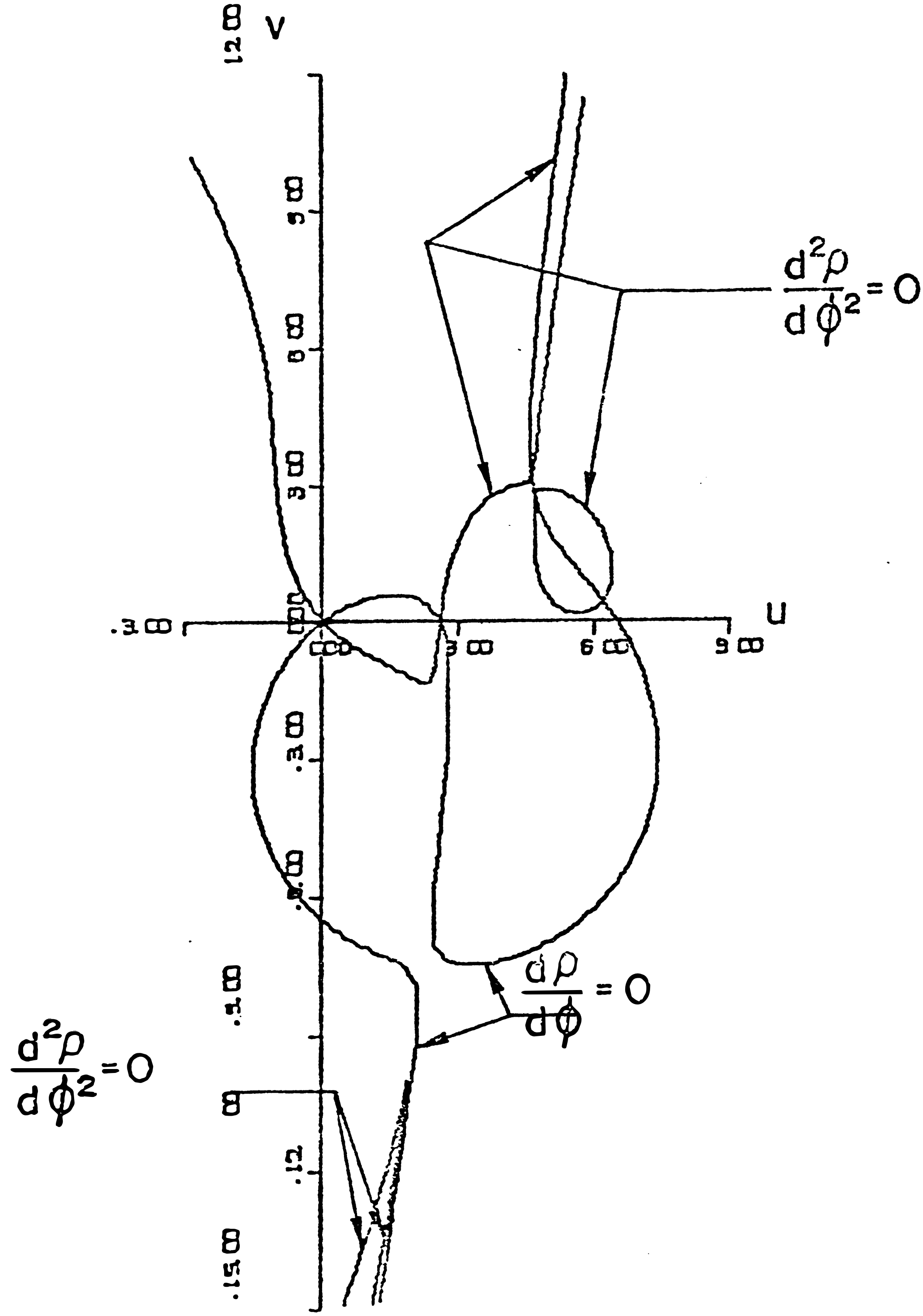

Figure 35. The Loci of the Zeroes of the First and Second Derivatives of the Radius of Curvature in the Coupler of the Original Four-Bar Linkage in the Moving Plane

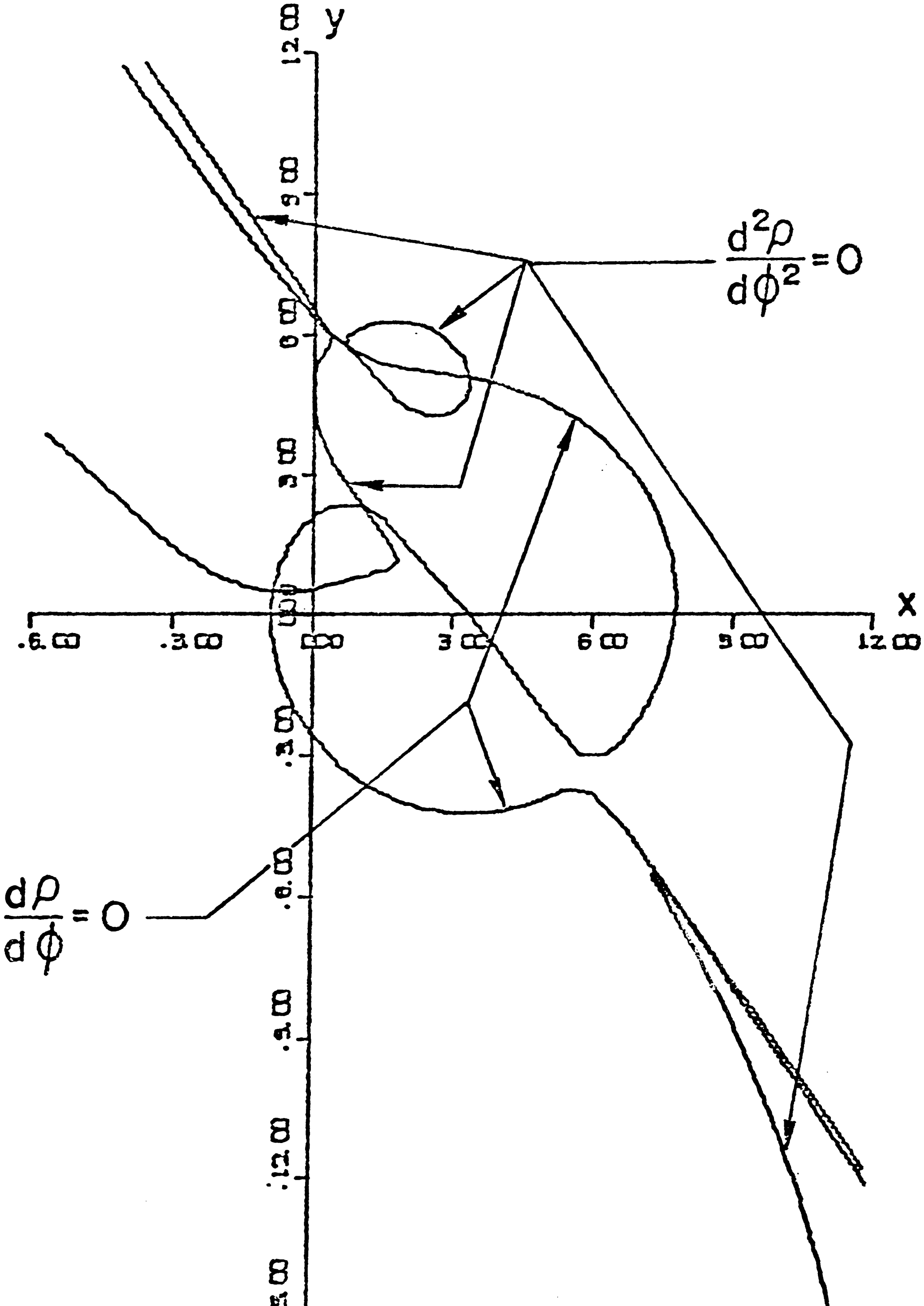

Figure 36. The Loci of the Zeroes of the First and Second Derivatives of the Radius of Curvature in the Coupler of the Original Four-Bar Linkage in the Fixed Plane

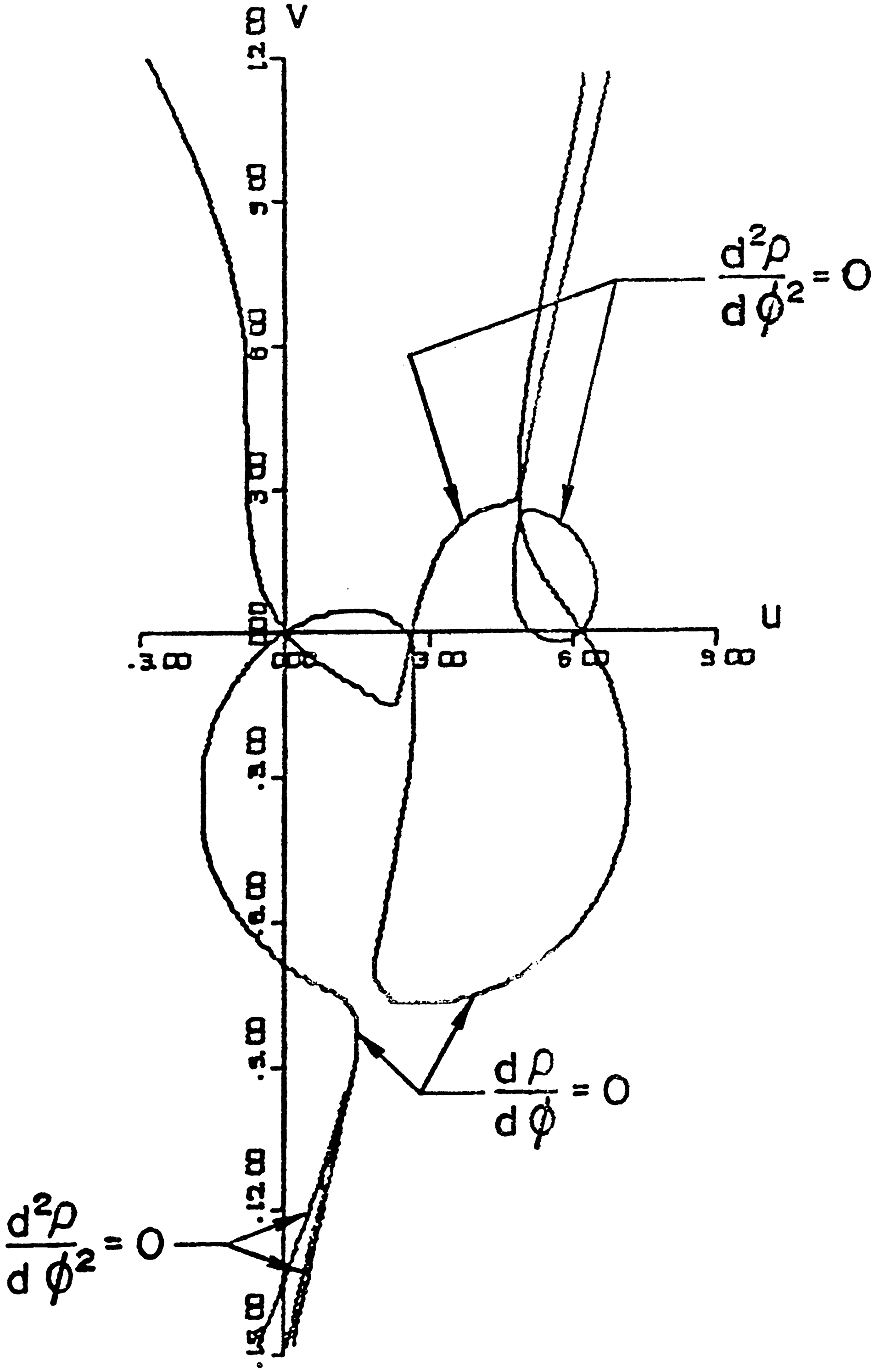

Figure 37. The Loci of the Zeroes of the First and Second Derivatives of the Radius of Curvature in the Coupler of Solution 1 in the Moving Plane

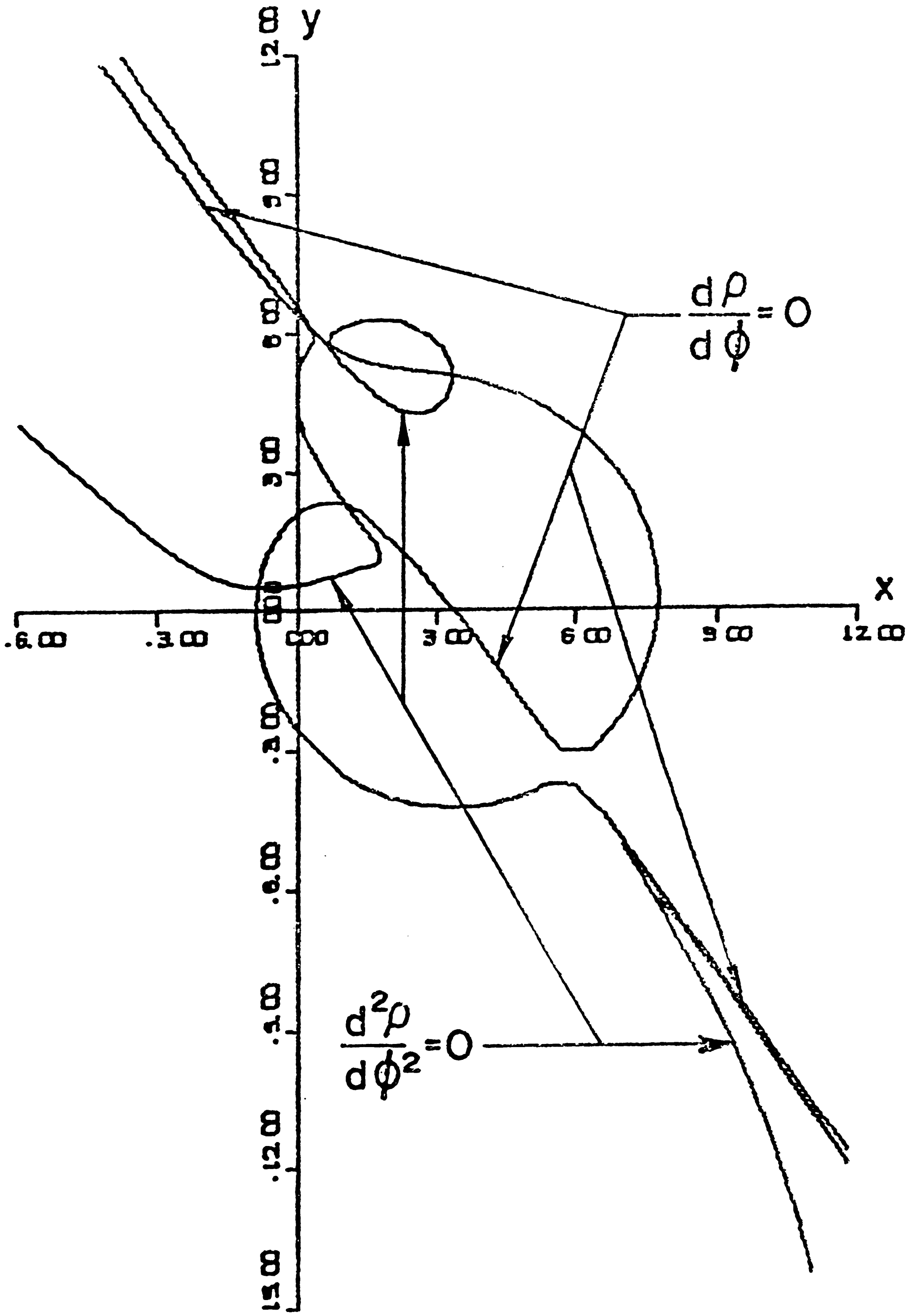

Figure 38. The Loci of the Zeroes of the First and Second Derivatives of the Radius of Curvature in the Coupler of Solution 1 in the fixed Plane

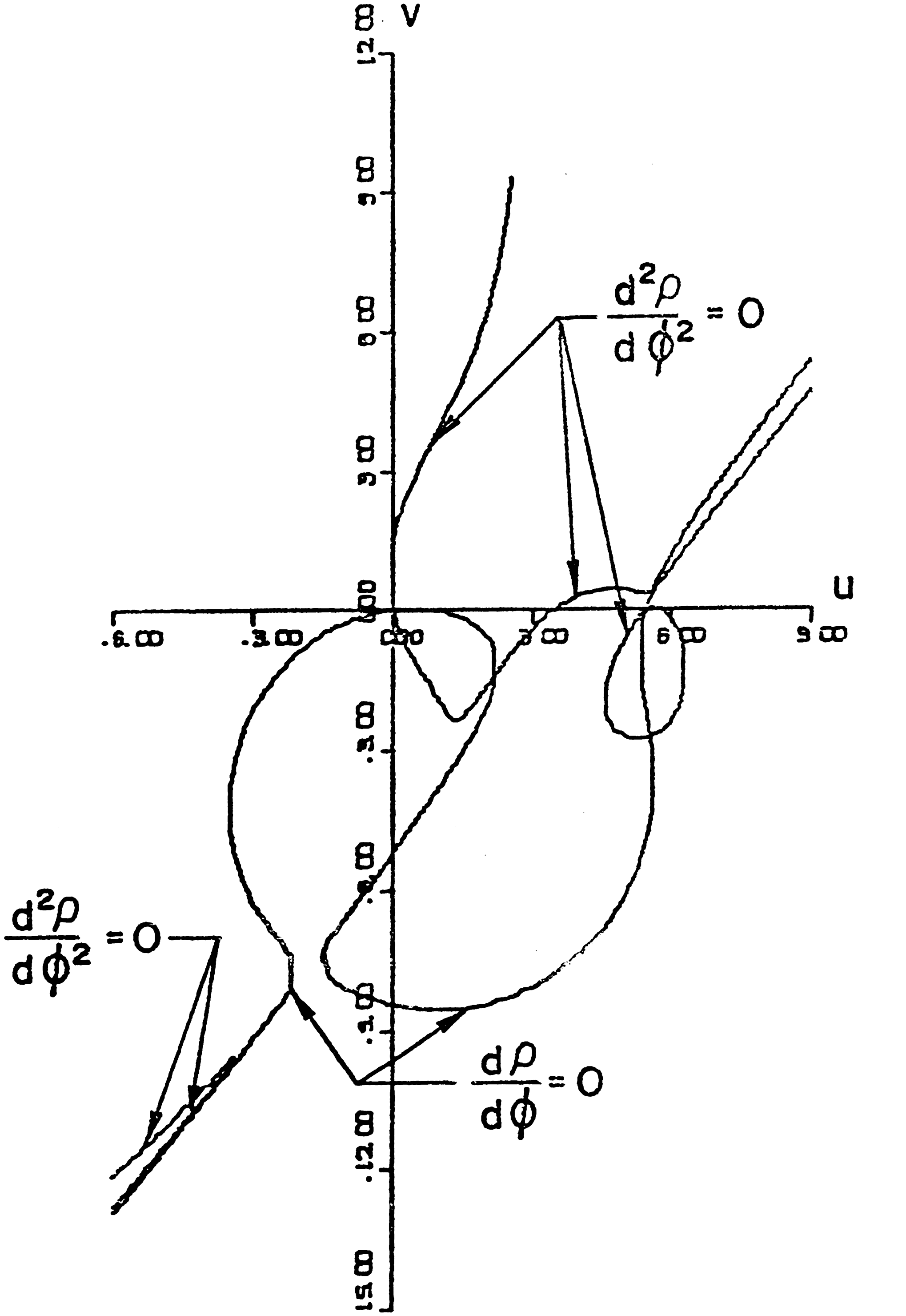

Figure 39. The Loci of the Zeroes of the First and Second Derivatives of the Radius of Curvature in the Coupler of Solution 2 in the Moving Plane

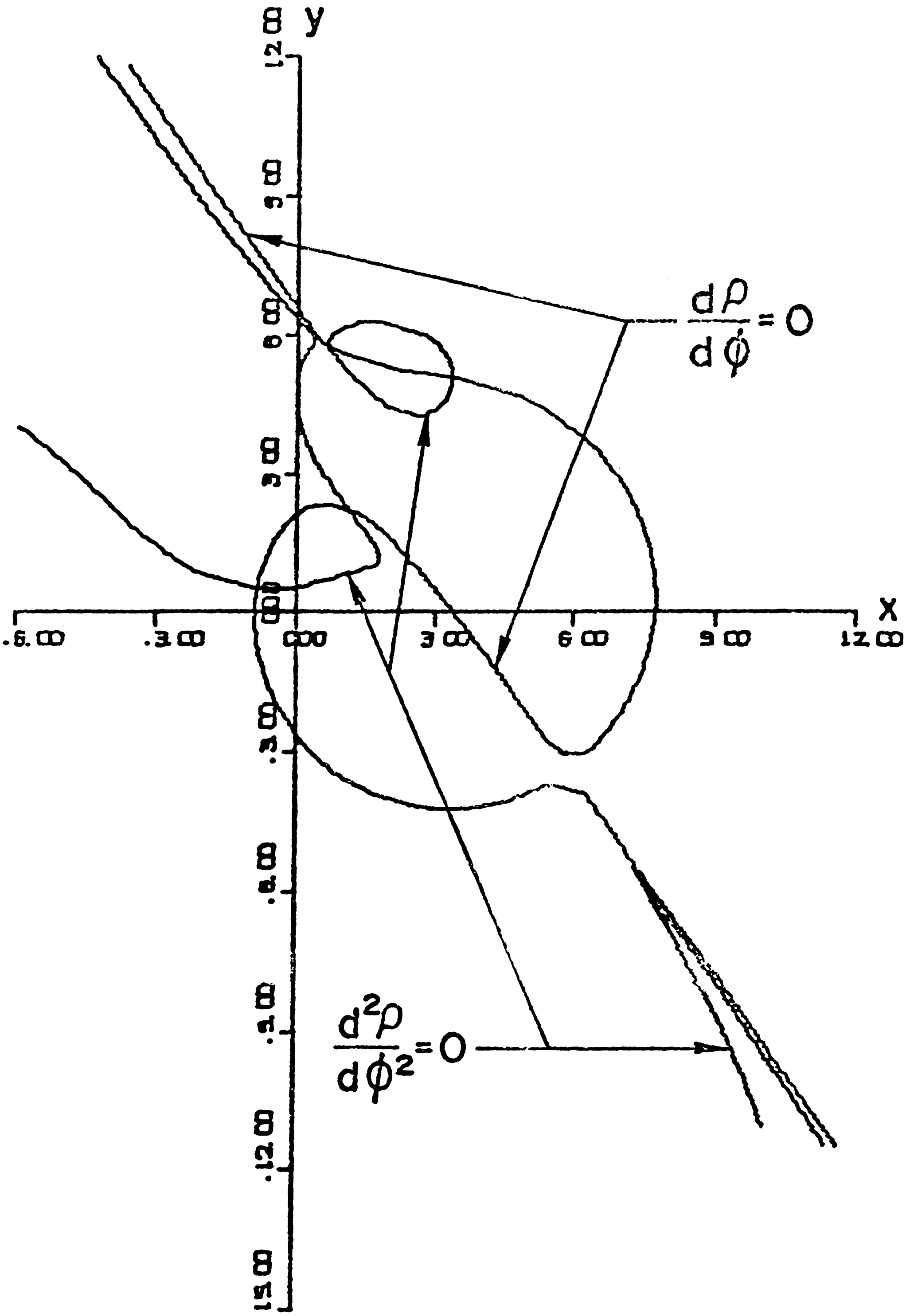

Figure 40. The Loci of the Zeroes of the First and Second Derivatives of the Radius of Curvature in the Coupler of Solution 2 in the Fixed Plane

It is apparent, however, that having defined the constraints on a link in a manner similar to those constraints of Fig. 27, a family of constraints has been determined. Figure 41 shows all of these constraints for the coupler of this example.

These constraints of Fig. 41 when taken two at a time will necessarily impose all of the others. It is implied that all of the constraints apply only to first and second order displacement functions. However, because of the requirement that the functional relationship between driving crank angle and coupler point position be maintained, the constraint at Point A must always be observed.

Velocity and acceleration analyses of the original mechanism and of each of the solutions will confirm that the angular velocities and accelerations of the coupler of each mechanism will agree with those velocities and accelerations of every other case because of the second-order exactness in displacement functions.

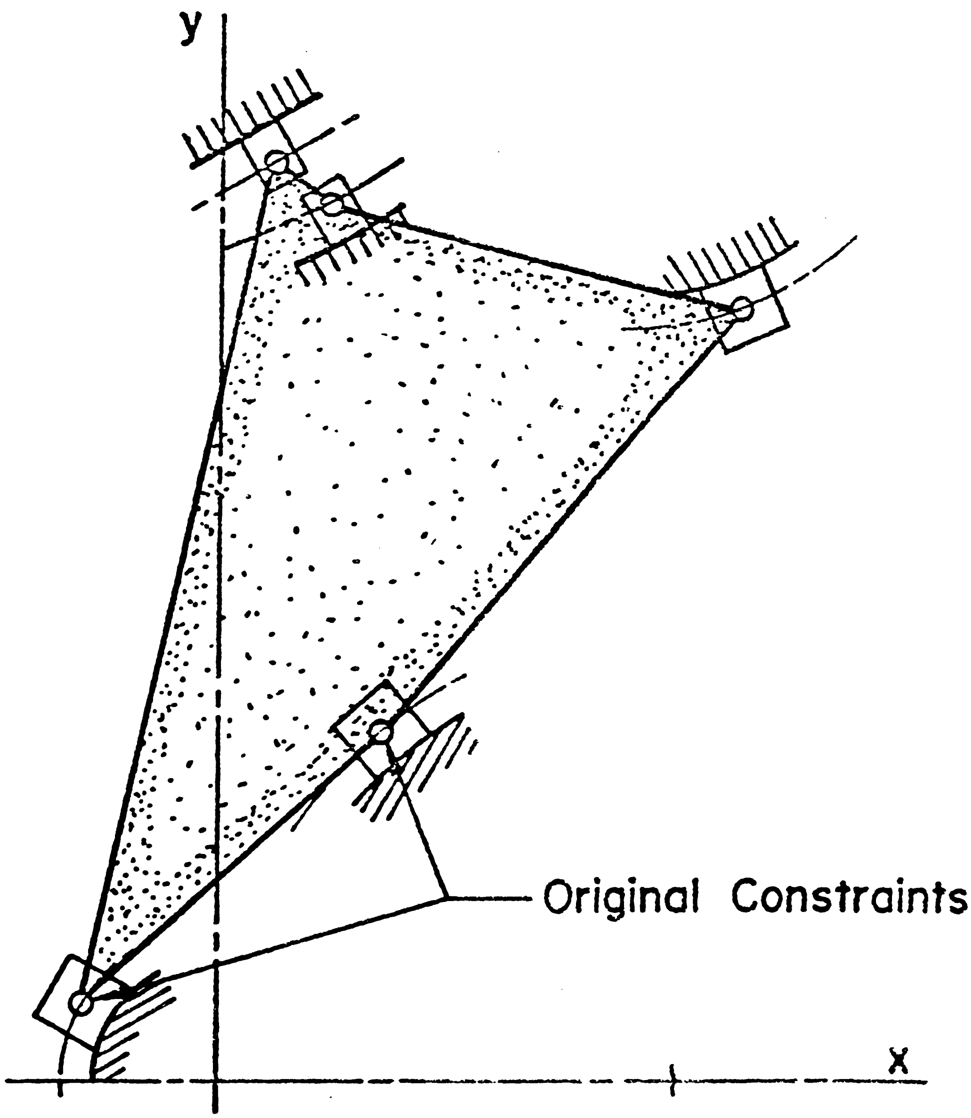

Figure 41. The Constrained Coupler of the Four-Bar Linkage

Chapter 5

The Arbitrary Path

Coupler curves approximating paths that are arbitrary in the sense that these paths are neither straight nor circular are often desired. If one end of a link is constrained to a circular path and the other end is constrained to a path that is defined in the fixed plane by some relationship, $y = f(x)$, it is possible to synthesize a four-bar linkage such that the coupler point motion will approximate $y = f(x)$. The first five derivatives of the function must exist in the range of the variables for which the synthesis is desired.

Figure 42 shows a linkage in which the coupler is subject to the constraints above. Applications of loop equations yield

$$y = a \sin \phi + b \sin \beta \tag{54}$$

$$x = a \cos \phi + b \cos \beta \tag{55}$$

But

$$y = f(x) \tag{56}$$

Then

$$a \sin \phi + b \sin \beta = f \quad a \cos \phi + b \cos \beta \tag{57}$$

Provided that the functional relationship between x and y is defined, Eq. 57 may be expanded and simplified to yield an equation in $\sin \beta$. It should be noted that if the functional relationship is in the form of a polynomial, the order of the polynomial must be four or less as Eq. 57 must be evaluated for an explicit function for $\sin \beta$. Any other functional form that will not yield an explicit solution for $\sin \beta$ cannot be synthesized.

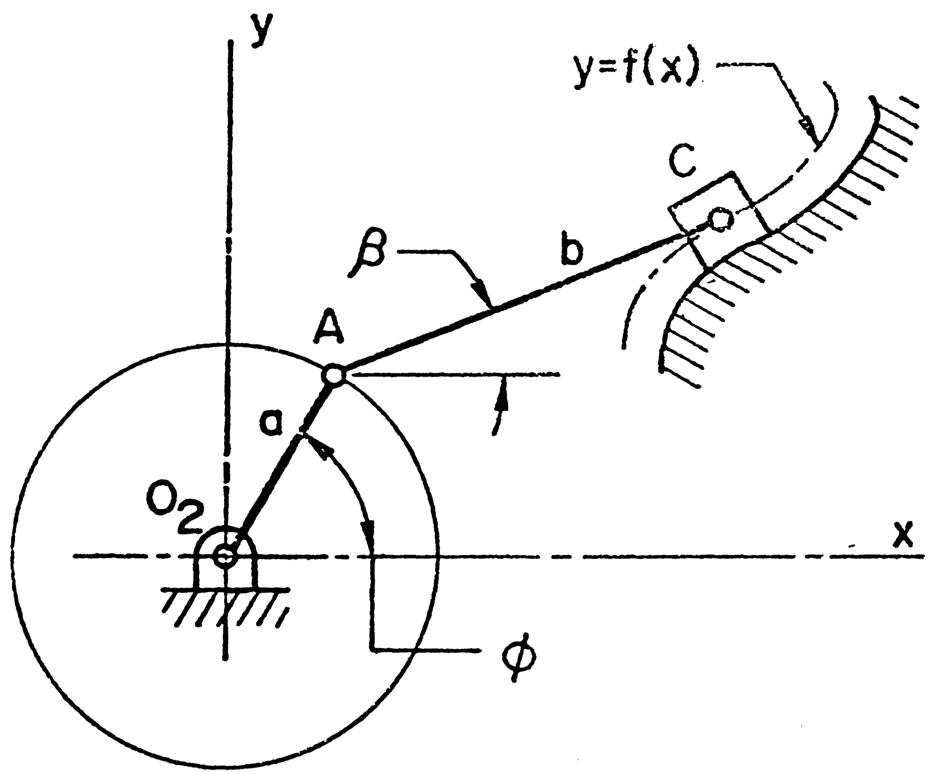

Figure 42. A Linkage with an Arbitrarily Constrained Coupler

$$\sin \beta = g(\phi,a,b,.....) \tag{58}$$

Having determined a representation for sin β as shown in Eq. 58, β may be evaluated as

$$\beta = \sin^{-1} g(\phi,a,b,.....) \tag{59}$$

Equation 59 may be differentiated and substituted along with Eq. 58 into Eq. 51 for the definition of x' and y'. Then x' and y' may each be successively differentiated to form the terms of Eqs. 36 through 41. The general solution technique may then be followed to make the first and second derivatives of the radius of curvature of the coupler point path zero.

Implicit to the previously outlined procedure of using the cubic of stationary curvature for the generation of trial solutions is the requirement that two points in the moving plane with constant curvature couple path must be known. In the case of the arbitrary path, only one such point can be identified. Because of this, relatively precise trial solutions cannot be determined. Trial solutions can only be a collection of uniformly distributed points in the vicinity of the linkage.

The evaluation of dβ/dφ, x', y', and successive derivatives may be, and perhaps must be, evaluated using a formula translator such as FORMAC. The feasibility and possibility of accomplishing this task is dependent upon the complexity of the functional relationship as shown in Eq. 56. For the case of the circular coupler point path, this task required hours of computer time and up to 1.2 million bytes of core storage for the algebraic manipulations.

Conclusion

It has been demonstrated that, given a description of the coupler constraints, it is possible to approximate the behavior of the derivatives of the radius of curvature of points on the coupler. This approximation may be in the form of exactness in the first derivative, approximation or exactness of the second derivative, and approximations of third and higher derivatives. The mechanism of the approximations involve duplicating the loci of the zeroes of the derivative(s) of the original mechanism coupler in the coupler plane of the solution mechanism. Implicit in the synthesis procedures outlined herein is the requirement that the coupler in the original linkage configuration must not be in translation only. The pole of the link must exist at a finite location. If the link translates only, the only coupler points suitable for attachment of a link are at some infinity. As finite mathematics is employed in the calculations, failure is assured.

The cubic of stationary curvature defines points on a coupler such that the derivative of the radius of curvature of the path of such points with respect to a displacement along the coupler curve is zero. If the coupler is in a dwell position, the cubic does exist, but it may define points whose radius of curvature is double-valued. The coupler curves of such points will contain cusps or crunodes.

In the two degree of freedom synthesis procedure the derivatives of the radius of curvature of the path of coupler points were formed with respect to the angle of the driving crank. If the coupler is in a dwell position the derivatives of the radius of curvature through-out the coupler plane are zero. As such, the procedure is not useful for the dwell position of the coupler.

This discussion is limited to the development of one and two degrees of freedom synthesis procedures. Those degrees of freedom involve the number of coordinate dimensions that may be specified in the coupler point location. It has been demonstrated that the procedure is sound and it would appear that higher orders of exactness with more degrees of freedom would be attractive.

The approximation of the third derivative of the radius of curvature required the determination of the expression for the third derivative and the specification of the expressions for the fifth derivatives of the coordinate positions in the fixed plane. Thus, it appears that a three degrees of freedom synthesis procedure is attainable provided that the derivatives essential to the Newton-Raphson method may be formed. The third degree of freedom may be the link length of the coupler or the driving crank angle in the original configuration. It must be recognized that the cubic of stationary curvature cannot be used for the generation of trial solutions as a complete specification of the coupler is required for the definition of the cubic's constants.

While the procedure may appear to be extendable to fourth and higher orders of exactness, the untractability of the successively higher derivatives will undoubtedly prove to be more than a typical modern computer configuration may handle. The sizes of the equations handled so far are staggering and these expressions were manipulated with considerable difficulty. There may exist, however, other mathematical techniques as well as other formula manipulation techniques or manipulators which will make equation size much less of a limitation of the scope of the synthesis procedure.

To this point, emphasis has been placed upon the task of locating coupler points whose lower derivatives of the radius of curvature coupler curve are zero. While this is necessary for the approximation of some coupler path, it is not the only procedure available. For instance, in the coupler plane those points possessive of an infinite radius of curvature and a zero first derivative will have an approximately straight path. Thus, this coupler point may be located through the use of a similar procedure utilizing equations that have been developed. The radius of curvature, the first, or the second derivative of the radius of curvature may each or in pairs be given nonzero values.

It is believed that the procedures outlined herein are useful and that these procedures provide for the advancement of planar kinematic synthesis. However, the equations required by these procedures may prove to be of greater usefulness as other synthesis procedures may employ these relationships.

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APPENDIX A

Straight Path Equations

and

Circular Path Equations

Some of the names of intermediate variables in this Appendix do not correspond to the names of the same variables in the main body of the text. In all cases, all variables are completely defined. Initial and final variables utilize the same names as are found in the main body of the text or these variables are named using the naming schemes described in the text.

STRAIGHT PATH EQUATIONS

$$SP = SIN(PHI)$$

$$CP = COS(PHI)$$

$$Z = (2\ R\ A3\ SP - R^{2}\ SP^{2} - A3^{2} + L^{2})^{1/2}$$

$$X1C = -R\ SP$$

$$X1L = (-R^{2}\ SP\ CP + R\ A3\ CP)\ /\ Z$$

$$X1M = R\ CP$$

$$X2C = -X1M$$

$$X2L = (2\ R^{3}\ A3\ SP\ CP^{2} - R^{4}\ SP^{2}\ CP^{2} - R^{2}\ A3^{2}\ CP^{2} + (-R\ A3\ SP - R^{2}\ CP^{2} + R^{2}\ SP^{2})\ Z^{2})\ /\ Z^{3}$$

$$X2M = X1C$$

$$X3C = -X1C$$

$$X3L = (3\ R^{2}\ Z^{2}\ A3^{2}\ SP\ CP + 4\ R^{2}\ Z^{4}\ SP\ CP - R\ Z^{4}\ A3\ CP - 6\ R^{3}\ Z^{2}\ A3\ SP^{2}\ CP + 3\ R^{4}\ Z^{2}\ SP^{3}\ CP - 9\ R^{4}\ A3^{2}\ SP\ CP^{3} - 3\ R^{4}\ Z^{2}\ SP\ CP^{3} + 3\ R^{3}\ Z^{2}\ A3\ CP^{3} + 9\ R^{5}\ A3\ SP^{2}\ CP^{3} - 3\ R^{6}\ SP^{3}\ CP^{3} + 3\ R^{3}\ A3^{3}\ CP^{3})\ /\ Z^{5}$$

STRAIGHT PATH EQUATIONS

X3M = X2C

X4C = X1M

$$X4L = (R Z^{6} A3 \, SP - 26 R^{3} Z^{4} A3 \, SP \, CP^{2} - 18 R^{3} Z^{2} A3^{3} SP \, CP^{2} + 54 R^{4} Z^{2} A3^{2} SP^{2} CP^{2} + 22 R^{4} Z^{4} SP^{2} CP^{2} - 54 R^{5} Z^{2} A3 \, SP^{3} CP^{2} + 18 R^{6} Z^{2} SP^{4} CP^{2} + 4 R^{2} Z^{4} A3^{2} CP^{2} + 4 R^{2} Z^{6} CP^{2} + 36 R^{5} Z^{2} A3 \, SP \, CP^{4} + 60 R^{5} A3^{3} SP \, CP^{4} - 90 R^{6} A3^{2} SP^{2} CP^{4} - 18 R^{6} Z^{2} SP^{2} CP^{4} + 60 R^{7} A3 \, SP^{3} CP^{4} - 15 R^{8} SP^{4} CP^{4} - 18 R^{4} Z^{2} A3^{2} CP^{4} - 15 R^{4} A3^{4} CP^{4} - 3 R^{4} Z^{4} CP^{4} - 3 R^{2} Z^{4} A3^{2} SP^{2} - 4 R^{2} Z^{6} SP^{2} + 6 R^{3} Z^{4} A3 \, SP^{3} - 3 R^{4} Z^{4} SP^{4}) / Z^{7}$$

X4M = X3C

X5C = X1C

STRAIGHT PATH EQUATIONS

$$X5L = (1 - 15\ A3^{2}\ R^{2}\ Z^{6}\ SP\ CP - 16\ R^{2}\ Z^{8}\ SP\ CP + 45\ A3^{3}\ R^{3}$$

$$Z^{4}\ SP^{2}\ CP + 75\ A3\ R^{3}\ Z^{6}\ SP^{2}\ CP - 135\ A3^{2}\ R^{4}\ Z^{4}\ SP^{3}\ CP$$

$$- 60\ R^{4}\ Z^{6}\ SP^{3}\ CP + 135\ A3\ R^{5}\ Z^{4}\ SP^{4}\ CP - 45\ R^{6}\ Z^{4}\ SP^{5}$$

$$CP + A3\ R\ Z^{8}\ CP + 150\ A3^{4}\ R^{4}\ Z^{2}\ SP\ CP^{3} + 270\ A3^{2}\ R^{4}\ Z^{4}$$

$$SP\ CP^{3} + 60\ R^{4}\ Z^{6}\ SP\ CP^{3} - 600\ A3^{3}\ R^{5}\ Z^{2}\ SP^{2}\ CP^{3} - 450$$

$$A3\ R^{5}\ Z^{4}\ SP^{2}\ CP^{3} + 900\ A3^{2}\ R^{6}\ Z^{2}\ SP^{3}\ CP^{3} + 210\ R^{6}\ Z^{4}$$

$$SP^{3}\ CP^{3} - 600\ A3\ R^{7}\ Z^{2}\ SP^{4}\ CP^{3} + 150\ R^{8}\ Z^{2}\ SP^{5}\ CP^{3} -$$

$$30\ A3^{3}\ R^{3}\ Z^{4}\ CP^{3} - 30\ A3\ R^{3}\ Z^{6}\ CP^{3} - 450\ A3^{2}\ R^{6}\ Z^{2}\ SP$$

$$CP^{5} - 45\ R^{6}\ Z^{4}\ SP\ CP^{5} - 525\ A3^{4}\ R^{6}\ SP\ CP^{5} + 450\ A3\ R^{7}\ Z^{2}$$

$$SP^{2}\ CP^{5} + 1050\ A3^{3}\ R^{7}\ SP^{2}\ CP^{5} - 150\ R^{8}\ Z^{2}\ SP^{3}\ CP^{5} -$$

$$1050\ A3^{2}\ R^{8}\ SP^{3}\ CP^{5} + 525\ A3\ R^{9}\ SP^{4}\ CP^{5} - 105\ R^{10}\ SP^{5}$$

$$CP^{5} + 150\ A3^{3}\ R^{5}\ Z^{2}\ CP^{5} + 45\ A3\ R^{5}\ Z^{4}\ CP^{5} + 105\ A3^{5}\ R^{5}$$

$$CP^{5}\ ) / Z^{9}$$

$$X5M = X1M$$

STRAIGHT PATH EQUATIONS

$$Y1C = X1M$$

$$Y1L = X2C$$

$$Y1M = (- R^{2} SP\ CP + R\ A3\ CP) / Z$$

$$Y2C = X1C$$

$$Y2L = X3C$$

$$Y2M = (- R\ Z^{2} A3\ SP + 2 R^{3} A3\ SP\ CP^{2} - R^{4} SP^{2} CP^{2} - R^{2}$$
$$A3^{2} CP^{2} - R^{2} Z^{2} CP^{2} + R^{2} Z^{2} SP^{2}) / Z^{3}$$

$$Y3C = X2C$$

$$Y3L = X1M$$

$$Y3M = (3 R^{2} Z^{2} A3^{2} SP\ CP + 4 R^{2} Z^{4} SP\ CP - R\ Z^{4} A3\ CP - 6$$
$$R^{3} Z^{2} A3\ SP^{2} CP + 3 R^{4} Z^{2} SP^{3} CP - 9 R^{4} A3^{2} SP\ CP^{3} - 3$$
$$R^{4} Z^{2} SP\ CP^{3} + 3 R^{3} Z^{2} A3\ CP^{3} + 9 R^{5} A3\ SP^{2} CP^{3} - 3 R^{6}$$
$$SP^{3} CP^{3} + 3 R^{3} A3^{3} CP^{3}) / Z^{5}$$

$$Y4C = X3C$$

$$Y4L = X1C$$

STRAIGHT PATH EQUATIONS

$$Y4M = (R Z^{6} A3\ SP - 26 R^{3} Z^{4} A3\ SP\ CP^{2} - 18 R^{3} Z^{2} A3^{3} SP\ CP^{2} + 54 R^{4} Z^{2} A3^{2} SP^{2} CP^{2} + 22 R^{4} Z^{4} SP^{2} CP^{2} - 54 R^{5} Z^{2} A3\ SP^{3} CP^{2} + 18 R^{6} Z^{2} SP^{4} CP^{2} + 4 R^{2} Z^{4} A3^{2} CP^{2} + 4 R^{2} Z^{6} CP^{2} + 36 R^{5} Z^{2} A3\ SP\ CP^{4} + 60 R^{5} A3^{3} SP\ CP^{4} - 90 R^{6} A3^{2} SP^{2} CP^{4} - 18 R^{6} Z^{2} SP^{2} CP^{4} + 60 R^{7} A3\ SP^{3} CP^{4} - 15 R^{8} SP^{4} CP^{4} - 18 R^{4} Z^{2} A3^{2} CP^{4} - 15 R^{4} A3^{4} CP^{4} - 3 R^{4} Z^{4} CP^{4} - 3 R^{2} Z^{4} A3^{2} SP^{2} - 4 R^{2} Z^{6} SP^{2} + 6 R^{3} Z^{4} A3\ SP^{3} - 3 R^{4} Z^{4} SP^{4}) / Z^{7}$$

$$Y5C = Y1C$$

$$Y5L = Y1L$$

$$Y5M = X5L$$

$$X1 = X1C + X1L\ LAMBDA + X1M\ MU$$

$$X2 = X2C + X2L\ LAMBDA + X2M\ MU$$

$$X3 = X3C + X3L\ LAMBDA + X3M\ MU$$

$$X4 = X4C + X4L\ LAMBDA + X4M\ MU$$

$$X5 = X5C + X5L\ LAMBDA + X5M\ MU$$

$$Y1 = Y1C + Y1L\ LAMBDA + Y1M\ MU$$

$$Y2 = Y2C + Y2L\ LAMBDA + Y2M\ MU$$

$$Y3 = Y3C + Y3L\ LAMBDA + Y3M\ MU$$

STRAIGHT PATH EQUATIONS

$$Y4 = Y4C + Y4L\ LAMBDA + Y4M\ MU$$

$$Y5 = Y5C + Y5L\ LAMBDA + Y5M\ MU$$

$$F = X1M\ Y2 - X2\ Y1M - X2M\ Y1 + X1\ Y2M$$

$$G = 2\ (\ Y1\ Y1M + X1\ X1M\ )$$

$$H = 2\ (\ X1\ X1L + Y1\ Y1L\ )$$

$$J = Y2\ X1L - Y1\ X2L + X1\ Y2L - X2\ Y1L$$

$$R = Y1\ Y2 + X1\ X2$$

$$S = -\ Y1\ X3 + X1\ Y3$$

$$T = X1\ X3 + Y3\ Y1 + Y2^{2} + X2^{2}$$

$$U = -\ X3\ Y2 - Y1\ X4 + X1\ Y4 + X2\ Y3$$

$$W = X1\ Y2 - X2\ Y1$$

$$ZZ = Y1^{2} + X1^{2}$$

$$D1 = 3\ R\ W\ ZZ - S\ ZZ^{2}$$

$$D1L = 3\ (\ X2\ X1L + X1\ X2L + Y1\ Y2L + Y1L\ Y2\ )\ W\ ZZ - 2\ (\ 2\ X1\ X1L + 2\ Y1\ Y1L\ )\ S\ ZZ + 3\ (\ Y2\ X1L - Y1\ X2L + X1\ Y2L - X2\ Y1L\ )$$
$$R\ ZZ + 3\ (\ 2\ X1\ X1L + 2\ Y1\ Y1L\ )\ R\ W - (\ -\ Y1\ X3L + Y3\ X1L + X1\ Y3L - X3\ Y1L\ )\ ZZ^{2}$$

$$D1M = 3\ (\ Y1M\ Y2 + Y2M\ Y1 + X2\ X1M + X1\ X2M\ )\ W\ ZZ - 2\ (\ 2\ Y1\ Y1M + 2\ X1\ X1M\ )\ S\ ZZ + 3\ (\ X1M\ Y2 - X2\ Y1M - X2M\ Y1 + X1\ Y2M\ )$$
$$R\ ZZ + 3\ (\ 2\ Y1\ Y1M + 2\ X1\ X1M\ )\ R\ W - (\ X1\ Y3M - Y1M\ X3 - X3M\ Y1 + X1M\ Y3\ )\ ZZ^{2}$$

STRAIGHT PATH EQUATIONS

$$D2 = -6 R S W ZZ + 3 T W^{2} ZZ - W U ZZ^{2} + 2 S^{2} ZZ^{2} + 3 R^{2}$$

$$W^{2}$$

$$D2L = -2 W U ZZ H + 4 S^{2} ZZ H - 6 R S W H + 3 T W^{2} H + 6 T W$$

$$ZZ J - 6 R S ZZ J + 6 R^{2} W J - U ZZ^{2} J - 6 (X2 X1L + X1 X2L$$

$$+ Y1 Y2L + Y1L Y2) S W ZZ - 6 (- Y1 X3L + Y3 X1L + X1 Y3L -$$

$$X3 Y1L) R W ZZ + 3 (X1 X3L + X3 X1L + 2 X2 X2L + 2 Y2 Y2L + Y1$$

$$Y3L + Y3 Y1L) W^{2} ZZ - (- Y2 X3L + Y4 X1L + Y3 X2L - X3 Y2L$$

$$+ X1 Y4L - Y1 X4L - Y1L X4 + X2 Y3L) W ZZ^{2} + 4 (- Y1 X3L +$$

$$Y3 X1L + X1 Y3L - X3 Y1L) S ZZ^{2} + 6 (X2 X1L + X1 X2L + Y1 Y2L$$

$$+ Y1L Y2) R W^{2}$$

$$D2M = -2 W U G ZZ + 4 S^{2} G ZZ - 6 (Y1M Y2 + Y2M Y1 + X2 X1M$$

$$+ X1 X2M) S W ZZ + 6 T F W ZZ - 6 (X1 Y3M - Y1M X3 - X3M Y1$$

$$+ X1M Y3) R W ZZ - 6 R F S ZZ + 3 (2 Y2M Y2 + Y1 Y3M + X1M X3$$

$$+ Y3 Y1M + X1 X3M + 2 X2 X2M) W^{2} ZZ - 6 R S W G + 3 T W^{2} G$$

$$+ 6 R^{2} F W - F U ZZ^{2} - (- X3M Y2 + X1 Y4M - Y1M X4 - Y1 X4M$$

$$+ X1M Y4 + X2 Y3M - Y2M X3 + X2M Y3) W ZZ^{2} + 4 (X1 Y3M - Y1M$$

$$X3 - X3M Y1 + X1M Y3) S ZZ^{2} + 6 (Y1M Y2 + Y2M Y1 + X2 X1M +$$

STRAIGHT PATH EQUATIONS

```
                   2
X1 X2M ) R W
-------------
TP = 2 ( X3 X2 + Y3 Y2 ) + X3 X2 + Y4 Y3 + X4 X1 + Y1 Y4
------------------------------------------------------------
ZP = 2 ( Y1 X2 + X1 Y2 ) Y1 X1
------------------------------------
UP = X1 Y5 - Y1 X5 + 2 Y4 X2 - 2 X4 Y2
---------------------------------------------

                                                  2
D3 = 1/2 ( 2 (  - 6 U R W ZZ - 2 ZP U W ZZ - 6 S   R ZZ + 3 TP W
---------------------------------------------------------------------
2              2                           2            2
   ZZ + 4 ZP S    ZZ - 6 ZP S R W + 6 S R   W - UP W ZZ   + 3 U S
---------------------------------------------------------------------
   2            2           2    3            2          3
 ZZ   + 6 T R W   + 3 ZP T W   ) W   ZZ - ( 6 S W   ZZ + ZP W
---------------------------------------------------------------------
                           2             2     2    2          2
 ) (  - 6 S R W ZZ + 3 T W   ZZ - U W ZZ   + 2 S   ZZ   + 3 R
---------------------------------------------------------------------
 2            6    3/2
W   ) ) / (  W   ZZ      )
---------------------------
```

CIRCULAR PATH EQUATIONS

$$L = 2\,A\,D$$

$$K = D^{2} + A^{2}$$

$$H = 1/2\,(A\,D^{2} + A\,B^{2} - A\,C^{2} + A^{3})\,/\,B$$

$$J = 1/2\,A^{2}\,D\,/\,B$$

$$AA = 1/2\,C^{2}\,D^{2}\,/\,B^{2} - 1/2\,A^{2}\,D^{2}\,/\,B^{2} - 1/4\,D^{4}\,/\,B^{2} + 1/2\,A^{2}\,C^{2}\,/\,B^{2} - 1/4\,C^{4}\,/\,B^{2} - 1/4\,A^{4}\,/\,B^{2} + 1/2\,D^{2} - 1/4\,B^{2} + 1/2\,C^{2} + 1/2\,A^{2}$$

$$BB = -A\,D - A\,C^{2}\,D\,/\,B^{2} + A^{3}\,D\,/\,B^{2} + A\,D^{3}\,/\,B^{2}$$

$$N = A^{2}\,D^{2}\,/\,B^{2}$$

$$CP = COS\,(PHI)$$

$$SP = SIN\,(PHI)$$

$$S2P = SIN\,(2\,PHI)$$

$$C2P = COS\,(2\,PHI)$$

$$R2 = D - CP\,A$$

$$R5 = (AA + CP\,BB - CP^{2}\,N)^{1/2}$$

$$R7 = K - CP\,L$$

CIRCULAR PATH EQUATIONS

$$R3 = R5\ S\ R2 - SP\ H + S2P\ J$$

$$R4 = - SP\ BB + S2P\ N$$

$$R6 = 1/2\ R4\ S\ R2\ /\ R5 - CP\ H + 2\ J\ C2P + SP\ R5\ S\ A$$

$$R8 = \left(- R3^{2}\ /\ R7^{2} + 1 \right)^{1/2}$$

$$R9 = - CP\ BB + 2\ CP^{2}\ N - 2\ SP^{2}\ N$$

$$R10 = - SP\ L\ R3\ /\ R7^{2} + R6\ /\ R7$$

$$R11 = SP\ BB - 4\ S2P\ N$$

$$R12 = 2\ R6\ R3\ /\ R7^{2} - 2\ SP\ L\ R3^{2}\ /\ R7^{3}$$

$$R14 = - 1/4\ R4^{2}\ S\ R2\ /\ R5^{3} + 1/2\ R9\ S\ R2\ /\ R5 + SP\ H + CP\ R5\ S\ A + SP\ R4\ S\ A\ /\ R5 - 4\ S2P\ J$$

$$R15 = 2\ SP^{2}\ L^{2}\ R3\ /\ R7^{3} - CP\ L\ R3\ /\ R7^{2} - 2\ SP\ R6\ L\ /\ R7^{2} + R14\ /\ R7$$

CIRCULAR PATH EQUATIONS

$$R16 = 2\ R3\ R14 / R7^{2} - 8\ SP\ L\ R6\ R3 / R7^{3} - 2\ CP\ L\ R3^{2} / R7^{3} + 6\ SP^{2}\ L^{2}\ R3^{2} / R7^{4} + 2\ R6^{2} / R7^{2}$$

$$R17 = 1/2\ S\ R2\ R11 / R5 + 3/8\ R4^{3}\ S\ R2 / R5^{.5} - 3/4\ R9\ R4\ S\ R2 / R5^{3} + CP\ H - 8\ J\ C2P - SP\ R5\ S\ A - 3/4\ SP\ R4^{2}\ S\ A / R5^{3} + 3/2\ R4\ CP\ S\ A / R5 + 3/2\ SP\ R9\ S\ A / R5$$

$$R18 = -6\ SP^{3}\ L^{3}\ R3 / R7^{4} + 3\ S2P\ L^{2}\ R3 / R7^{3} + SP\ L\ R3 / R7^{2} + 6\ SP^{2}\ R6\ L^{2} / R7^{3} - 3\ CP\ R6\ L / R7^{2} - 3\ SP\ R14\ L / R7^{2} + R17 / R7$$

$$R19 = -1/2\ R10\ R12\ R3 / (R8^{3}\ R7) + SP\ R10\ L\ R3 / (R8\ R7^{2}) - R15\ R3 / (R8\ R7) - R10\ R6 / (R8\ R7)$$

$$R20 = SP\ BB - 8\ SP\ CP\ N$$

$$R23 = CP\ BB - 8\ CP^{2}\ N + 8\ SP^{2}\ N$$

CIRCULAR PATH EQUATIONS

$$\mathrm{R21 = 1/2\ S\ R2\ R20\ /\ R5 + 3/8\ R4^{3}\ S\ R2\ /\ R5^{5} - 3/4\ R9\ R4\ S}$$

$$\mathrm{R2\ /\ R5^{3} + CP\ H - 8\ J\ C2P - SP\ R5\ S\ A - 3/4\ SP\ R4^{2}\ S\ A\ /\ R5^{3}}$$

$$\mathrm{+ 3/2\ R4\ CP\ S\ A\ /\ R5 + 3/2\ SP\ R9\ S\ A\ /\ R5}$$

$$\mathrm{R22 = - R4\ S\ R2\ R20\ /\ R5^{3} + 2\ SP\ S\ A\ R20\ /\ R5 + 1/2\ S\ R23\ R2}$$

$$\mathrm{/\ R5 - 15/16\ R4^{4}\ S\ R2\ /\ R5^{7} + 9/4\ R9\ R4^{2}\ S\ R2\ /\ R5^{5} - 3/}$$

$$\mathrm{4\ R9^{2}\ S\ R2\ /\ R5^{3} - SP\ H - CP\ R5\ S\ A + 3/2\ SP\ R4^{3}\ S\ A\ /\ R5^{5}}$$

$$\mathrm{- 3/2\ R4^{2}\ CP\ S\ A\ /\ R5^{3} - 3\ SP\ R9\ R4\ S\ A\ /\ R5^{3} + 3\ R9\ CP\ S}$$

$$\mathrm{A\ /\ R5 - 2\ SP\ R4\ S\ A\ /\ R5 + 16\ S2P\ J}$$

CIRCULAR PATH EQUATIONS

$$\mathrm{R24} = -12\,\mathrm{R3}\,\mathrm{R6}\,\mathrm{CP}\,\mathrm{L} / \mathrm{R7}^3 - 12\,\mathrm{R6}^2\,\mathrm{SP}\,\mathrm{L} / \mathrm{R7}^3 - 12\,\mathrm{R3}\,\mathrm{R14}\,\mathrm{SP}\,\mathrm{L} / \mathrm{R7}^3 + 2\,\mathrm{R3}^2\,\mathrm{SP}\,\mathrm{L} / \mathrm{R7}^3 + 6\,\mathrm{R14}\,\mathrm{P6} / \mathrm{R7}^2 + 2\,\mathrm{R3}\,\mathrm{R21} / \mathrm{R7}^2 + 18\,\mathrm{R3}^2\,\mathrm{SP}\,\mathrm{CP}\,\mathrm{L}^2 / \mathrm{R7}^4 + 36\,\mathrm{R3}\,\mathrm{R6}\,\mathrm{SP}^2\,\mathrm{L}^2 / \mathrm{R7}^4 - 24\,\mathrm{R3}^2\,\mathrm{SP}^3\,\mathrm{L}^3 / \mathrm{R7}^5$$

$$\mathrm{R25} = -3\,\mathrm{R6}\,\mathrm{CP}\,\mathrm{L} / \mathrm{R7}^2 - 3\,\mathrm{R14}\,\mathrm{SP}\,\mathrm{L} / \mathrm{R7}^2 + \mathrm{R3}\,\mathrm{SP}\,\mathrm{L} / \mathrm{R7}^2 + \mathrm{R21} / \mathrm{R7} + 6\,\mathrm{R3}\,\mathrm{SP}\,\mathrm{CP}\,\mathrm{L}^2 / \mathrm{R7}^3 + 6\,\mathrm{R6}\,\mathrm{SP}^2\,\mathrm{L}^2 / \mathrm{R7}^3 - 6\,\mathrm{R3}\,\mathrm{SP}^3\,\mathrm{L}^3 / \mathrm{R7}^4$$

$$\mathrm{R26} = -6\,\mathrm{R14}\,\mathrm{CP}\,\mathrm{L} / \mathrm{R7}^2 + \mathrm{R3}\,\mathrm{CP}\,\mathrm{L} / \mathrm{R7}^2 + 4\,\mathrm{P6}\,\mathrm{SP}\,\mathrm{L} / \mathrm{R7}^2 - 4\,\mathrm{R21}\,\mathrm{SP}\,\mathrm{L} / \mathrm{R7}^2 + \mathrm{R22} / \mathrm{R7} + 24\,\mathrm{R6}\,\mathrm{SP}\,\mathrm{CP}\,\mathrm{L}^2 / \mathrm{R7}^3 + 6\,\mathrm{R3}\,\mathrm{CP}^2\,\mathrm{L}^2 / \mathrm{R7}^3 + 12\,\mathrm{R14}\,\mathrm{SP}^2\,\mathrm{L}^2 / \mathrm{R7}^3 - 8\,\mathrm{R3}\,\mathrm{SP}^2\,\mathrm{L}^2 / \mathrm{R7}^3 - 36\,\mathrm{R3}\,\mathrm{SP}^2\,\mathrm{CP}\,\mathrm{L}^3 / \mathrm{R7}^4 - 24\,\mathrm{R6}\,\mathrm{SP}^3\,\mathrm{L}^3 / \mathrm{R7}^4 + 24\,\mathrm{R3}\,\mathrm{SP}^4\,\mathrm{L}^4 / \mathrm{R7}^5$$

$$\mathrm{R27} = 32\,\mathrm{CP}\,\mathrm{SP}\,\mathrm{N} - \mathrm{SP}\,88$$

CIRCULAR PATH EQUATIONS

$R28 = - 48\ R6\ R14\ SP\ L\ /\ R7^{3} - 16\ R3\ R21\ SP\ L\ /\ R7^{3} + 16\ R6$

$R3\ SP\ L\ /\ R7^{3} - 24\ R3\ CP\ R14\ L\ /\ R7^{3} + 2\ R3^{2}\ CP\ L\ /\ R7^{3}$

$- 24\ R6^{2}\ CP\ L\ /\ R7^{3} + 144\ R6\ R3\ CP\ SP\ L^{2}\ /\ R7^{4} + 72\ R3$

$R14\ SP^{2}\ L^{2}\ /\ R7^{4} - 24\ R3^{2}\ SP^{2}\ L^{2}\ /\ R7^{4} + 72\ R6^{2}\ SP^{2}\ L$

$^{2}\ /\ R7^{4} + 18\ R3^{2}\ CP^{2}\ L^{2}\ /\ R7^{4} - 144\ R3^{2}\ CP\ SP^{2}\ L^{3}\ /$

$R7^{5} - 192\ R6\ R3\ SP^{3}\ L^{3}\ /\ R7^{5} + 120\ R3^{2}\ SP^{4}\ L^{4}\ /\ R7^{6} +$

$6\ R14^{2}\ /\ R7^{2} + 8\ R6\ R21\ /\ R7^{2} + 2\ R3\ R22\ /\ R7^{2}$

$R29 = - 5\ SP\ R4\ S\ A\ R20\ /\ R5^{3} + 5\ CP\ S\ A\ R20\ /\ R5 + 15/4\ R2$

$R4^{2}\ S\ R20\ /\ R5^{5} - 5/2\ R9\ R2\ S\ R20\ /\ R5^{3} + 5/2\ SP\ S\ A\ R23\ /$

$R5 - 5/4\ R2\ R4\ S\ R23\ /\ R5^{3} + 1/2\ R2\ S\ R27\ /\ R5 + SP\ R5\ S\ A$

$- 75/16\ SP\ R4^{4}\ S\ A\ /\ R5^{7} + 45/4\ SP\ R9\ R4^{2}\ S\ A\ /\ R5^{5} + 15/$

$4\ CP\ R4^{3}\ S\ A\ /\ R5^{5} - 15/2\ CP\ R9\ R4\ S\ A\ /\ R5^{3} + 5/2\ SP\ R4^{2}$

$S\ A\ /\ R5^{3} - 15/4\ SP\ R9^{2}\ S\ A\ /\ R5^{3} - 5/2\ CP\ R4\ S\ A\ /\ R5 - 5$

$SP\ R9\ S\ A\ /\ R5 + 105/32\ R2\ R4^{5}\ S\ /\ R5^{9} - 75/8\ R9\ R2\ R4^{3}\ S$

$/\ R5^{7} + 45/8\ R9^{2}\ R2\ R4\ S\ /\ R5^{5} - CP\ H + 32\ C2P\ J$

CIRCULAR PATH EQUATIONS

$$
\begin{aligned}
R30 = {} & R29 / R7 + 10\ R14\ SP\ L / R7^{2} - 5\ R22\ SP\ L / R7^{2} - R3 \\
& SP\ L / R7^{2} - 10\ R21\ CP\ L / R7^{2} + 5\ R6\ CP\ L / R7^{2} + 60\ CP \\
& R14\ SP\ L^{2} / R7^{3} - 30\ R3\ CP\ SP\ L^{2} / R7^{3} + 20\ R21\ SP^{2}\ L^{2} \\
& / R7^{3} - 40\ R6\ SP^{2}\ L^{2} / R7^{3} + 30\ R6\ CP^{2}\ L^{2} / R7^{3} - 90 \\
& R3\ CP^{2}\ SP\ L^{3} / R7^{4} - 180\ R6\ CP\ SP^{2}\ L^{3} / R7^{4} - 60\ R14\ SP^{3} \\
& L^{3} / R7^{4} + 60\ R3\ SP^{3}\ L^{3} / R7^{4} + 240\ R3\ CP\ SP^{3}\ L^{4} / R7^{5} \\
& + 120\ R6\ SP^{4}\ L^{4} / R7^{5} - 120\ R3\ SP^{5}\ L^{5} / R7^{6}
\end{aligned}
$$

$Y1C = CP\ A$

$Y1U = R10$

$Y1V = -\ R10\ R3 / (R7\ R8)$

$Y2C = -\ SP\ A$

$Y2U = R15$

$Y2V = R19$

$Y3C = -\ Y1C$

$Y3U = R18$

CIRCULAR PATH EQUATIONS

$$Y3V = - R10\ R14 / (R7\ R8) - 3/4\ R10\ R12^{2}\ R3 / (R7\ R8^{5})$$

$$- R15\ R12\ R3 / (R7\ R8^{3}) + SP\ L\ R10\ R12\ R3 / (R7^{2}\ R8^{3})$$

$$- 1/2\ R10\ R16\ R3 / (R7\ R8^{3}) - R18\ R3 / (R7\ R8) + 2\ SP\ L$$

$$R15\ R3 / (R7^{2}\ R8) + CP\ L\ R10\ R3 / (R7^{2}\ R8) - 2\ SP^{2}\ L^{2}$$

$$R10\ R3 / (R7^{3}\ R8) - R10\ R12\ R6 / (R7\ R8^{3}) - 2\ R15\ R6 /$$

$$(R7\ R8) + 2\ SP\ L\ R10\ R6 / (R7^{2}\ R8)$$

$$Y4C = - Y2C$$

$$Y4U = (- 6\ CP\ R7^{3}\ L\ R14 + 12\ R7^{2}\ SP^{2}\ L^{2}\ R14 + CP\ R7^{3}\ L\ R3$$

$$- 8\ R7^{2}\ SP^{2}\ L^{2}\ R3 + 6\ CP^{2}\ R7^{2}\ L^{2}\ R3 - 36\ CP\ R7\ SP^{2}\ L^{3}$$

$$R3 + 24\ SP^{4}\ L^{4}\ R3 + 4\ R7^{3}\ SP\ L\ R6 + 24\ CP\ R7^{2}\ SP\ L^{2}\ R6 - 24$$

$$R7\ SP^{3}\ L^{3}\ R6 - 4\ R21\ R7^{3}\ SP\ L + R22\ R7^{4}) / R7^{5}$$

CIRCULAR PATH EQUATIONS

$$
\begin{aligned}
&Y4V = 3/2\ R3\ R10\ SP\ L\ R16 / (\ R8^{3}\ R7^{2}\ ) - 3/2\ R10\ R6\ R16 / (\\
&R8^{3}\ R7\ ) - 3/2\ R3\ R15\ R16 / (\ R8^{3}\ R7\ ) - 9/4\ R3\ R12\ R10\ R16 \\
&/ (\ R8^{5}\ R7\ ) + 3\ R10\ R6\ CP\ L / (\ R8\ R7^{2}\ ) + 3\ R3\ R15\ CP\ L \\
&/ (\ R8\ R7^{2}\ ) + 3/2\ R3\ R12\ R10\ CP\ L / (\ R8^{3}\ R7^{2}\ ) + 6\ R15 \\
&R6\ SP\ L / (\ R8\ R7^{2}\ ) + 3\ R12\ R10\ R6\ SP\ L / (\ R8^{3}\ R7^{2}\ ) + 3 \\
&R3\ R25\ SP\ L / (\ R8\ R7^{2}\ ) + 3\ R3\ R12\ R15\ SP\ L / (\ R8^{3}\ R7^{2}\ ) \\
&+ 9/4\ R3\ R12^{2}\ R10\ SP\ L / (\ R8^{5}\ R7^{2}\ ) + 3\ R14\ R10\ SP\ L / (\\
&R8\ R7^{2}\ ) - R3\ R10\ SP\ L / (\ R8\ R7^{2}\ ) - 3\ R25\ R6 / (\ R8\ R7\ ) \\
&- 3\ R12\ R15\ R6 / (\ R8^{3}\ R7\ ) - 9/4\ R12^{2}\ R10\ R6 / (\ R8^{5}\ R7\ ) \\
&- 1/2\ R3\ R10\ R24 / (\ R8^{3}\ R7\ ) - 3/2\ R3\ R12\ R25 / (\ R8^{3}\ R7\ ) \\
&- R3\ R26 / (\ R8\ R7\ ) - 9/4\ R3\ R12^{2}\ R15 / (\ R8^{5}\ R7\ ) - 3\ R14 \\
&R15 / (\ R8\ R7\ ) - R21\ R10 / (\ R8\ R7\ ) - 15/16\ R3\ R12^{3}\ R10 / \\
&(\ R8^{7}\ R7\ ) - 3/2\ R12\ R14\ R10 / (\ R8^{3}\ R7\ ) - 6\ R3\ R10\ SP\ CP\ L^{2} \\
&/ (\ R8\ R7^{3}\ ) - 6\ R10\ R6\ SP^{2}\ L^{2} / (\ R8\ R7^{3}\ ) - 6\ R3\ R15 \\
&SP^{2}\ L^{2} / (\ R8\ R7^{3}\ ) - 3\ R3\ R12\ R10\ SP^{2}\ L^{2} / (\ R8^{3}\ R7^{3}\ ) \\
&+ 6\ R3\ R10\ SP^{3}\ L^{3} / (\ R8\ R7^{4}\ )
\end{aligned}
$$

CIRCULAR PATH EQUATIONS

Y5C = Y1C

$$Y5V = (- 9\ R7^{4}\ R12\ R8^{4}\ R3\ R16\ R15 + 6\ L\ SP\ R7^{3}\ R8^{6}\ R3\ R16$$

$$R15 - 6\ R6\ R7^{4}\ R8^{6}\ R16\ R15 + 6\ L\ R7^{3}\ R12\ R8^{6}\ R3\ CP\ R15 - 24$$

$$L^{2}\ SP\ R7^{2}\ R8^{8}\ R3\ CP\ R15 + 12\ R6\ L\ R7^{3}\ R8^{8}\ CP\ R15 - 4\ R7^{4}$$

$$R8^{8}\ R21\ R15 - 2\ R7^{4}\ R8^{6}\ R3\ R24\ R15 - 15/2\ R7^{4}\ R12^{3}\ R8^{2}\ R3$$

$$R15 + 9\ L\ SP\ R7^{3}\ R12^{2}\ R8^{4}\ R3\ R15 - 12\ L^{2}\ SP^{2}\ R7^{2}\ R12\ R8^{6}$$

$$R3\ R15 + 24\ L^{3}\ SP^{3}\ R7\ R8^{8}\ R3\ R15 - 4\ L\ SP\ R7^{3}\ R8^{8}\ R3\ R15 -$$

$$9\ R6\ R7^{4}\ R12^{2}\ R8^{4}\ R15 + 12\ R6\ L\ SP\ R7^{3}\ R12\ R8^{6}\ R15 - 6\ R14$$

$$R7^{4}\ R12\ R8^{6}\ R15 - 24\ R6\ L^{2}\ SP^{2}\ R7^{2}\ R8^{8}\ R15 + 12\ R14\ L\ SP$$

$$R7^{3}\ R8^{8}\ R15 + 3\ L\ R7^{3}\ R8^{6}\ R3\ CP\ R10\ R16 - 45/4\ R7^{4}\ R12^{2}\ R8^{2}$$

$$R3\ R10\ R16 + 9\ L\ SP\ R7^{3}\ R12\ R8^{4}\ R3\ R10\ R16 - 6\ L^{2}\ SP^{2}\ R7^{2}$$

$$R8^{6}\ R3\ R10\ R16 - 9\ R6\ R7^{4}\ R12\ R8^{4}\ R10\ R16 + 6\ R6\ L\ SP\ R7^{3}$$

$$R8^{6}\ R10\ R16 - 3\ R14\ R7^{4}\ R8^{6}\ R10\ R16 - 3\ R7^{4}\ R8^{6}\ R3\ R25\ R16$$

$$+ 6\ R14\ L\ R7^{3}\ R8^{8}\ CP\ R10 - 2\ R7^{4}\ R12\ R8^{6}\ R21\ R10 + 4\ L\ SP$$

$$R7^{3}\ R8^{8}\ R21\ R10 - 3\ R7^{4}\ R12\ R8^{4}\ R3\ R24\ R10 + 2\ L\ SP\ R7^{3}\ R8^{6}$$

$$R3\ R24\ R10 - 2\ R6\ R7^{4}\ R8^{6}\ R24\ R10 - R7^{4}\ R8^{8}\ R22\ R10 + 15/2$$

CIRCULAR PATH EQUATIONS

```
       3     3     2                      2     2     2       2     4
L SP R7   R12   R8    R3 R10 - 9 L   SP    R7    R12    R8    R3 R10
----------------------------------------------------------------------
     6     2     4                          4       4
 + R7   R12   R8    R3 R10 - 105/16 R7    R12    R3 R10 - 9/2 R14
----------------------------------------------------------------------
  4     2     4                       3         6                   2
R7   R12   R8    R10 + 6 R14 L SP R7    R12 R8    R10 - 12 R14 L
----------------------------------------------------------------------
  2     2     8                 3     8                     4      2
SP   R7    R8    R10 + 6 L R7    R8    R3 CP R25 - 9/2 R7   R12    R3
----------------------------------------------------------------------
4                      3         6                 2     2     2    8
   R3 R25 + 6 L SP R7    R12 R8    R3 R25 - 12 L   SP    R7    R8
----------------------------------------------------------------------
                    4         6                     3     8
R3 R25 - 6 R6 R7    R12 R8    R25 + 12 R6 L SP R7    R8    R25 - 6
----------------------------------------------------------------------
      4     8             4         6                      3     8
R14 R7   R8    R25 - 2 R7    R12 R8    R3 R26 + 4 L SP R7    R8    R3
----------------------------------------------------------------------
                  4     8                5     9
 R26 - 4 R6 R7    R8    R26 ) / ( R7    R8    )
------------------------------------------------------
           5                 4                  4     3
Y5U = ( R7    R29 - 10 L R7    R21 CP + 240 L   SP    R7 R3 CP - 30
----------------------------------------------------------------------
 2     3                      3     2     2                 2       3
L   SP R7    R3 CP - 180 R6 L   SP    R7    CP + 60 R14 L   SP R7
----------------------------------------------------------------------
                  4          2     2     3                   4
 CP + 5 R6 L R7    CP + 20 L   SP    R7    R21 - 5 L SP R7    R22 +
----------------------------------------------------------------------
     3     3     2              4             5     5                4
60 L   SP    R7    R3 - L SP R7    R3 - 120 L   SP    R3 + 120 R6 L
----------------------------------------------------------------------
  4          3        2       2             2     3     2
 SP    R7 - 90 L   SP R7   R3 CP    + 30 R6 L   R7    CP    - 60 R14
----------------------------------------------------------------------
  3     3     2            2     2     3                   4          6
 L   SP    R7    - 40 R6 L   SP    R7    + 10 R14 L SP R7    ) / R7
----------------------------------------------------------------------
X5U = Y5V
---------
X5V = -Y5U
----------
```

CIRCULAR PATH EQUATIONS

X1C = Y2C

X1U = Y1V

X1V = - R10

X2C = Y3C

X2U = R19

X2V = - R15

X3C = - Y2C

X3U = Y3V

X3V = - R18

X4C = Y1C

X4U = Y4V

X4V = - Y4U

X5C = X1C

X1 = X1C + X1U U + X1V V

X2 = X2C + X2U U + X2V V

X3 = X3C + X3U U + X3V V

X4 = X4C + X4U U + X4V V

X5 = X5C + X5U U + X5V V

Y1 = Y1C + Y1U U + Y1V V

Y2 = Y2C + Y2U U + Y2V V

Y3 = Y3C + Y3U U + Y3V V

Y4 = Y4C + Y4U U + Y4V V

Y5 = Y5C + Y5U U + Y5V V

CIRCULAR PATH EQUATIONS

$$F = X1V\ Y2 - X2\ Y1V - X2V\ Y1 + X1\ Y2V$$

$$G = 2\ (\ Y1\ Y1V + X1\ X1V\ )$$

$$H = 2\ (\ X1\ X1U + Y1\ Y1U\ )$$

$$J = Y2\ X1U - Y1\ X2U + X1\ Y2U - X2\ Y1U$$

$$R = Y1\ Y2 + X1\ X2$$

$$S = -\ Y1\ X3 + X1\ Y3$$

$$T = X1\ X3 + Y3\ Y1 + Y2^{2} + X2^{2}$$

$$UU = -\ X3\ Y2 - Y1\ X4 + X1\ Y4 + X2\ Y3$$

$$W = X1\ Y2 - X2\ Y1$$

$$ZZ = Y1^{2} + X1^{2}$$

$$TP = 2\ (\ X3\ X2 + Y3\ Y2^{2}\ ) + X3\ X2 + Y4\ Y3 + X4\ X1 + Y1\ Y4$$

$$ZP = 2\ (\ Y1\ X2 + X1\ Y2\ )\ Y1\ X1$$

$$UP = X1\ Y5 - Y1\ X5 + 2\ Y4\ X2 - 2\ X4\ Y2$$

$$D1 = 3\ R\ W\ ZZ - S\ ZZ$$

$$D1U = 3\ (\ X2\ X1U + X1\ X2U + Y1\ Y2U + Y1U\ Y2\ )\ W\ ZZ - 2\ (\ 2\ X1\ X1U + 2\ Y1\ Y1U\ )\ S\ ZZ + 3\ (\ Y2\ X1U - Y1\ X2U + X1\ Y2U - X2\ Y1U\ )\ R\ ZZ + 3\ (\ 2\ X1\ X1U + 2\ Y1\ Y1U\ )\ R\ W - (\ -\ Y1\ X3U + Y3\ X1U + X1\ Y3U - X3\ Y1U\ )\ ZZ^{2}$$

CIRCULAR PATH EQUATIONS

$$D1V = 3\,(Y1V\;Y2 + Y2V\;Y1 + X2\;X1V + X1\;X2V)\,W\;ZZ - 2\,(2\;Y1\;Y1V + 2\;X1\;X1V)\,S\;ZZ + 3\,(X1V\;Y2 - X2\;Y1V - X2V\;Y1 + X1\;Y2V)\,R\;ZZ + 3\,(2\;Y1\;Y1V + 2\;X1\;X1V)\,R\;W - (X1\;Y3V - Y1V\;X3 - X3V\;Y1 + X1V\;Y3)\,ZZ^2$$

$$D2 = -6\;R\;S\;W\;ZZ + 3\;T\;W^2\;ZZ - W\;UU\;ZZ^2 + 2\;S^2\;ZZ^2 + 3\;R^2\;W^2$$

$$D2U = -2\;W\;UU\;ZZ\;H + 4\;S^2\;ZZ\;H - 6\;R\;S\;W\;H + 3\;T\;W^2\;H + 6\;T\;W\;ZZ\;J - 6\;R\;S\;ZZ\;J + 6\;P^2\;W\;J - UU\;ZZ^2\;J - 6\,(X2\;X1U + X1\;X2U + Y1\;Y2U + Y1U\;Y2)\,S\;W\;ZZ - 6\,(-Y1\;X3U + Y3\;X1U + X1\;Y3U - X3\;Y1U)\,R\;W\;ZZ + 3\,(X1\;X3U + X3\;X1U + 2\;X2\;X2U + 2\;Y2\;Y2U + Y1\;Y3U + Y3\;Y1U)\,W^2\;ZZ - (-Y2\;X3U + Y4\;X1U + Y3\;X2U - X3\;Y2U + X1\;Y4U - Y1\;X4U - Y1U\;X4 + X2\;Y3U)\,W\;ZZ^2 + 4\,(-Y1\;X3U + Y3\;X1U + X1\;Y3U - X3\;Y1U)\,S\;ZZ^2 + 6\,(X2\;X1U + X1\;X2U + Y1\;Y2U + Y1U\;Y2)\,F\;W^2$$

CIRCULAR PATH EQUATIONS

$$
\begin{aligned}
D2V = {} & -2\,W\,UU\,G\,ZZ + 4\,S^{2}\,G\,ZZ - 6\,(Y1V\,Y2 + Y2V\,Y1 + X2\,X1V \\
& + X1\,X2V)\,S\,W\,ZZ + 6\,T\,F\,W\,ZZ - 6\,(X1\,Y3V - Y1V\,X3 - X3V\,Y1 \\
& + X1V\,Y3)\,R\,W\,ZZ - 6\,R\,F\,S\,ZZ + 3\,(2\,Y2V\,Y2 + Y1\,Y3V + X1V\,X3 \\
& + Y3\,Y1V + X1\,X3V + 2\,X2\,X2V)\,W^{2}\,ZZ - 6\,R\,S\,W\,G + 3\,T\,W^{2}\,G \\
& + 6\,R^{2}\,F\,W - F\,UU\,ZZ^{2} - (-X3V\,Y2 + X1\,Y4V - Y1V\,X4 - Y1 \\
& X4V + X1V\,Y4 + X2\,Y3V - Y2V\,X3 + X2V\,Y3)\,W\,ZZ^{2} + 4\,(X1\,Y3V - \\
& Y1V\,X3 - X3V\,Y1 + X1V\,Y3)\,S\,ZZ^{2} + 6\,(Y1V\,Y2 + Y2V\,Y1 + X2\,X1V \\
& + X1\,X2V)\,R\,W^{2}
\end{aligned}
$$

$$
\begin{aligned}
D3 = {} & 1/2\,(2\,(-6\,R\,W\,UU\,ZZ - 2\,ZP\,W\,UU\,ZZ - 6\,S^{2}\,R\,ZZ + 3\,TP \\
& W^{2}\,ZZ + 4\,ZP\,S^{2}\,ZZ - 6\,ZP\,S\,R\,W + 6\,S\,R^{2}\,W + 3\,S\,UU\,ZZ^{2} - \\
& UP\,W\,ZZ^{2} + 6\,T\,R\,W^{2} + 3\,ZP\,T\,W^{2})\,W^{3}\,ZZ - (6\,S\,W^{2}\,ZZ + ZP \\
& W^{3})\,(-6\,S\,R\,W\,ZZ + 3\,T\,W^{2}\,ZZ - W\,UU\,ZZ^{2} + 2\,S^{2}\,ZZ^{2} + 3 \\
& R^{2}\,W^{2}))\,/\,(W^{6}\,ZZ^{3/2})
\end{aligned}
$$

APPENDIX B

Listing of Straight Path Program

and

Listing of Circular Path Program

```
C     PROGRAM STRGHT
C
C
C     PROGRAM STRGHT
C
C     THE MAIN PROGRAM PROVIDES FOR THE INPUT OF THE DESCRIPTION OF
C     THE ORIGINAL FOUR-BAR CONFIGURATION.  THEN, THROUGH SUBROUTINE
C     CALLS, TRIAL SOLUTIONS ARE GENERATED, UNIQUE SOLUTIONS DETER-
C     MINED, AND THE SOLUTIONS ARE FURTHER PROCESSED.
C
C        INPUT VARIABLES
C         A1     = DRIVING CRANK RADIUS
C         PHI1   = DRIVING CRANK ANGLE
C         A2     = CONNECTING ROD LENGTH
C         OFST   = SLIDER PATH OFFSET
C         EPS4   = CONVERGENCE CRITEREA
C         START  = INTIAL ANGLE FOR DRIVING CRANK FOR COUPLER CURVE
C                  PLOT
C         ENDD   = FINAL ANGLE FOR DRIVING CRANK FOR COUPLER CURVE PLOT
C         IPRINT = PRINT LEVEL
C
C        INPUT VARIABLES MUST BE IN THE ORDER ABOVE AND IN THE FORMAT
C
C        (7F10.0,I1)
C
C        THE FINAL DATA CARD SHOULD BE BLANK TO TERMINATE EXECUTION
C
C
      DIMENSION LX(50),MX(50),ITER(50),ICONV(50),DERIV(10,3),D3(50)
      REAL LX,MX
   40 WRITE(6,20)
   20 FORMAT(1H1)
```

```
      READ(5,30) A1,PHI1,A2,OFST,EPS4,START,ENDD,IPRINT
   30 FORMAT(7F10.0,I1)
      IF(A1.EQ.0.0) GO TO 50
      IF(ENDD.EQ.0.0) ENDD=359.
      IF(EPS4.EQ.0.0) EPS4=1.E-7
      CALL TRIAL(A1,A2,OFST,PHI1,LX,MX,IPRINT,ICHK)
      IF(ICHK.EQ.1) GO TO 40
      CALL NEWRAP(MX,LX,ICONV,EPS4,ITER,PHI1,A1,A2,OFST,DERIV,IPRINT)
      CALL SOL(LX,MX,NNN,IPRINT,ICONV)
      CALL RANK(NNN,LX,MX,DERIV,D3,IPRINT)
      DO 10 I=1,NNN
      X1=DERIV(1,1)+DERIV(1,2)*LX(I)+DERIV(1,3)*MX(I)
      X2=DERIV(2,1)+DERIV(2,2)*LX(I)+DERIV(2,3)*MX(I)
      Y1=DERIV(5,1)+DERIV(5,2)*LX(I)+DERIV(5,3)*MX(I)
      Y2=DERIV(6,1)+DERIV(6,2)*LX(I)+DERIV(6,3)*MX(I)
      CALL STRLIN(MX(I),LX(I),A1,A2,OFST,PHI1,X1,X2,Y1,Y2,RHO,XC,YC,X,Y)
      IF(RHO.LT.1.E-1) GO TO 10
      IPLOTR=0
      IF(IPRINT.GE.1) IPLOTR=1
      CALL ANALZE(PHI1,START,ENDD,A1,A2,OFST,X,Y,XC,YC,RHO,IPLOTR,0,MX(I
     1),LX(I))
   10 CONTINUE
      GO TO 40
   50 WRITE(6,20)
      STOP
      END
```

```
      SUBROUTINE RANK(NNN,LX,MX,DERIV,D3,IPRINT)
C
C
C     SUBROUTINE RANK
C
C     GIVEN THE UNIQUE NEWTON-RHAPSON SOLUTIONS IN THE UPPER NNN
C     SPACES OF THE ARRAYS LX AND MX, THIS ROUTINE WILL COMPUTE,
C     FOR EACH SOLUTION, THE ABSOLUTE VALUE OF THE THIRD DERIVATIVE
C     OF THE RADIUS OF CURVATURE OF THE COUPLER POINT PATH WITH
C     RESPECT TO THE CRANK ANGLE.  THE VALUES OF THE DERIVATIVE ARE
C     STORED IN ARRAY D3.  THEN, THE SOLUTIONS ARE REARRANGED IN
C     ORDER OF INCREASING VALUE OF D3 IN THE UPPER NNN SPACES OF
C     LX AND MX.
C
C        INPUT ARGUMENTS
C         LX,MX = UNIQUE SOLUTIONS
C         NNN   = NUMBER OF SOLUTIONS
C         DERIV = ARRAY OF COEFFICIENTS OF DERIVATIVES OF X AND Y
C         IPRINT = IF NOT EQUAL ZERO, INPUT AND OUTPUT ARRAYS PRINTED
C
C        OUTPUT ARGUMENTS
C         LX,MX = REARRANGED UNIQUE SOLUTIONS
C         D3    = ARRAY OF ABSOLUTE VALUES OF THE THIRD DERIVATIVE
C                  OF THE RADIUS OF CURVATURE OF THE COUPLER CURVE
C
      DIMENSION LX(50),MX(50),D3(50),DERIV(10,3)
      REAL LX,MX,J
      IF(IPRINT.NE.0) WRITE(6,50) (LX(K),MX(K),K=1,NNN)
   50 FORMAT(/////,10X,'SUBROUTINE RANK*****',//,10X,'INPUT SOLUTIONS',/
     1/,20X,'LAMBDA',15X,'MU',//,50(10X,2E20.7,/),///)
      DO 10 I=1,50
   10 D3(I)=0.
```

```
      DO 20 I=1,NNN
      X1=DERIV(1,1)+DERIV(1,2)*LX(I)+DERIV(1,3)*MX(I)
      X2=DERIV(2,1)+DERIV(2,2)*LX(I)+DERIV(2,3)*MX(I)
      X3=DERIV(3,1)+DERIV(3,2)*LX(I)+DERIV(3,3)*MX(I)
      X4=DERIV(4,1)+DERIV(4,2)*LX(I)+DERIV(4,3)*MX(I)
      X5=DERIV(9,1)+DERIV(9,2)*LX(I)+DERIV(9,3)*MX(I)
      Y1=DERIV(5,1)+DERIV(5,2)*LX(I)+DERIV(5,3)*MX(I)
      Y2=DERIV(6,1)+DERIV(6,2)*LX(I)+DERIV(6,3)*MX(I)
      Y3=DERIV(7,1)+DERIV(7,2)*LX(I)+DERIV(7,3)*MX(I)
      Y4=DERIV(8,1)+DERIV(8,2)*LX(I)+DERIV(8,3)*MX(I)
      Y5=DERIV(10,1)+DERIV(10,2)*LX(I)+DERIV(10,3)*MX(I)
      F=-DERIV(2,3)*Y1+DERIV(1,3)*Y2+DERIV(6,3)*X1-DERIV(5,3)*X2
      G=2.*(DERIV(5,3)*Y1+DERIV(1,3)*X1)
      H=2.*(DERIV(5,2)*Y1+DERIV(1,2)*X1)
      J=-DERIV(2,2)*Y1+DERIV(1,2)*Y2+DERIV(6,2)*X1-DERIV(5,2)*X2
      R=X1*X2+Y1*Y2
      S=X1*Y3-Y1*X3
      T=X2**2+Y2**2+X1*X3+Y1*Y3
      U=X2*Y3+X1*Y4-Y2*X3-Y1*X4
      W=Y2*X1-Y1*X2
      Z=X1**2+Y1**2
      IF(IPRINT.GE.5) WRITE(6,70) Z,I
   70 FORMAT(10X,'Z = ',E20.7,10X,'I = ',I5,/)
      TP=2.*(X2*X3+Y2*Y3)+X2*X3+X1*X4+Y4*Y3+Y1*Y4
      ZP=2.*X1*Y1*(Y1*X2+X1*Y2)
      UP=2.*X2*Y4+X1*Y5-2.*Y2*X4-Y1*X5
      D3(I)=ABS((2.*Z*W**3*(6.*R*W**2*T+6.*R**2*W*S-6.*T*S*W*Z-6.*R*U*W*
     1Z-6.*R*S**2*Z-6.*R*S*W*ZP+4.*S*U*Z**2+4.*S**2*Z*ZP+3.*TP*W**2*Z
     2+6.*T*W*S*Z+3.*T*W**2*ZP-UP*W*Z**2-U*S*Z**2-2.*U*W*Z*ZP)-(ZP*
     3W**3+6.*Z*W**2*S)*(3.*R**2*W**2-6.*R*S*W*Z+2.*S**2*Z**2+3.*T*
     4W**2*Z-U*W*Z**2))/(2.*Z**1.5*W**6))
   20 CONTINUE
```

```
      DO 30 I=1,NNN
      DO 30 K=I,NNN
      IF(D3(K).LT.D3(I)) GO TO 40
      GO TO 30
   40 TEMP=D3(I)
      D3(I)=D3(K)
      D3(K)=TEMP
      TEMP=LX(I)
      LX(I)=LX(K)
      LX(K)=TEMP
      TEMP=MX(I)
      MX(I)=MX(K)
      MX(K)=TEMP
   30 CONTINUE
      IF(IPRINT.NE.0) WRITE(6,60) (LX(K),MX(K),D3(K),K=1,NNN)
   60 FORMAT(///,10X,'OUTPUT SOLUTIONS',//,20X,'LAMBDA',15X,'MU',17X,'D3
     1',//,50(10X,3E20.7,/),///)
      RETURN
      END
```

```
      SUBROUTINE CIRCLE(XH,XK,R,X1,Y1,X2,Y2,X3,Y3,ICIRCL)
C
C
C     SUBROUTINE CIRCLE
C
C
C        SUBROUTINE CIRCLE DETERMINES THE CENTER AND RADIUS OF THE
C     CIRCLE PASSING THROUGH THREE DESCRIBED POINTS
C
C
C        ARGUMENTS
C
C
C     SUPPLIED BY THE CALLING ROUTINE
C
C     X1,Y1,X2,Y2,X3,Y3 = X,Y COORDINATES OF THREE POINTS THROUGH WHICH
C                         THE CIRCLE MUST PASS
C
C
C     SUPPLIED BY CIRCLE
C
C     R = RADIUS OF CIRCLE
C     XH,XK = X,Y COORDINATES OF THE CENTER OF THE CIRCLE
C
C
C
      DIMENSION A(9),B(3)
      ICIRCL=0
      EPS=.001
      XK1=ABS(X1-X2)
      XK2=ABS(X2-X3)
      XK3=ABS(X1-X3)
```

```
      YK1=ABS(Y1-Y2)
      YK2=ABS(Y2-Y3)
      YK3=ABS(Y1-Y3)
      IF(XK1.LT.EPS.AND.YK1.LT.EPS) GO TO 10
      IF(XK2.LT.EPS.AND.YK2.LT.EPS) GO TO 10
      IF(XK3.LT.EPS.AND.YK3.LT.EPS) GO TO 10
      A(1)=-2.*X1
      A(2)=-2.*X2
      A(3)=-2.*X3
      A(4)=-2.*Y1
      A(5)=-2.*Y2
      A(6)=-2.*Y3
      A(7)=1.
      A(8)=1.
      A(9)=1.
      B(1)=-(X1**2+Y1**2)
      B(2)=-(X2**2+Y2**2)
      B(3)=-(X3**2+Y3**2)
      K=3
      L=9
      M=0
      CALL SIMQ(A,B,K,M)
      IF(M.EQ.1) GO TO 10
      XH=B(1)
      XK=B(2)
      R=SQRT(XH**2+XK**2-B(3))
      RETURN
   10 ICIRCL=1
      WRITE(6,20)  X1,Y1,X2,Y2,X3,Y3
   20 FORMAT(/,47X,'UNABLE TO RESOLVE INFLECTION CIRCLE.',/,51X,
     1'CHECK FOR COINCIDENT POINTS.',//,40X,'P1 = (',2E20.6,' )',//,
     240X,'P2 = (',2E20.6,' )',//40X,'P3 = (',2E20.6,' )')
```

```
      RETURN
      END
```

```
      SUBROUTINE TRIAL(A1,A2,A3,PHI,X,Y,IPRINT,ICHK)
C
C     SLIDER CRANK VERSION
C
C
C     SUBROUTINE TRIAL, GIVEN THE ARGUMENTS BELOW, WILL GENERATE 50
C     TRIAL SOLUTIONS FOR A NEWTON-RHAPSON ANALYSIS.  FORTY OF THESE
C     POINTS ARE EVENLY DISTRIBUTED, ANGULARLY, AROUND THE CUBIIC-OF-
C     STATIONARY CURVATURE, TEN ARE DISTRIBUTED ALONG THE CUBIC'S
C     ASYMPTOTE.  AN EULER-SAVARY ANALYSIS IS PERFORMED TO LOCATE
C     THE INFLECTION CIRCLE AND, IN TURN TO LOCATE THE COMMON
C     CENTRODE TANGENT, OR THE INSTANT CENTER VELOCITY DIRECTION.
C     THEN, USING A COORDINATE SYSTEM ALIGNED WITH THE TANGENT, WITH
C     THE ORIGIN AT THE INSTANT CENTER, M AND N ARE DETERMINED FOR
C     THE CUBIC.  USING POLAR NOTATION, R AND PSI ARE DETERMINED,
C     YIELDING X AND Y IN THE ORIGINAL COORDINATE SYSTEM.  FINALLY
C     THE X'S AND Y'S ARE TRANSFORMED INTO MU AND LAMBDA.
C
C           INPUT ARGUMENTS
C            A1   = CRANK RADIUS
C            A2   = CONNECTING ROD LENGTH
C            A3   = SLIDER PATH OFFSET
C            PHI  = CRANK ANGLE, RADIANS
C            IPRINT = IF NOT EQUAL ZERO, INTERNAL VARIABLES PRINTED
C
C           OUTPUT
C
C            X,Y = ARRAYS OF TRIAL SOLUTIONS, DIMENSIONED 50
C            ICHK = ALARM IF NOT EQUAL ZERO, INFLECTION CIRCLE NOT FIXED
C
      DIMENSION A(4),B(2),X(50),Y(50)
      REAL IX,IY,IA,IB,JAA,JAAP,JAX,JAY
```

```
      PI=3.141593
      HALFPI=1.570796
      IF(IPRINT.NE.0) WRITE(6,50)
   50 FORMAT(////,10X,'SUBROUTINE TRIAL*****',//,10X,'POINT O = CRANK CE
     1NTER',/,10X,'POINT A = CRANK END',/,10X,'POINT B = SLIDER',/,10X,
     2'POINT I = INSTANT CENTER',/,10X,'POINT T = POINT ON TANGENT',/,
     310X,'THETA = ANGLE X-AXIS AND I-T',/,10X,'ALPHA = ANGLE I-T AND I-
     4A',/,10X,'BETA = ANGLE I-T AND I-B',/,10X,'PSI ASM = ANGLE I-T AND
     5 ASYMPTOTE'/,10X,'J - POINTS ON INFLECTION CIRCLE',//)
      DO 5 I=1,50
      X(I)=0.
    5 Y(I)=0.
      AX=A1*COS(PHI)
      AY=A1*SIN(PHI)
      BX=AX+A2*COS(ARSIN((A3-AY)/A2))
      BY=A3
      IY=BX*TAN(PHI)
      IX=BX
      IB=ABS(IY-BY)
      IA=SQRT((IY-AY)**2+(IX-AX)**2)
      IF(IPRINT.NE.0) WRITE(6,60) A1,A2,A3,PHI,AX,AY,BX,BY,IX,IY,IA,IB
   60 FORMAT(//,10X,'A1 = ',E20.8,/,10X,'A2 = ',E20.8,/,10X,'A3 = ',E20.
     18,/,10X,'PHI = ',E20.8,/,10X,'A AT ',2E20.8,/,10X,'B AT ',2E20.8,
     2/,10X,'I AT ',2E20.8,/,10X,'I-A = ',E20.8,/,10X,'I-B = ',E20.8,//)
C
C     EULER-SAVARY ANALYSIS
C
      JAA=IA**2/A1
      JAAP=A1-JAA
      JAX=JAAP*COS(PHI)
      JAY=JAAP*SIN(PHI)
      CALL CIRCLE(XH,XK,R,IX,IY,JAX,JAY,BX,BY,ICHK)
```

```
      IF(ICHK.EQ.1) GO TO 40
      IF(IPRINT.NE.0) WRITE(6,70) JAX,JAY,BX,BY,ICHK,XH,XK,R
   70 FORMAT(///,10X,'POINTS ON THE INFLECTION CIRCLE',/,10X,2E20.8,/,
     110X,2E20.8,///,10X,' ICHK = ',I10,/,10X,'INFLECTION CIRCLE AT ',2E2
     20.8,/,10X,'INFLECTION CIRCLE RADIUS = ',E20.8,//)
C
C     TRANSFORM TO RECTANGULAR COORDINATES AT INSTANT CENTER
C
      CXX=XH-IX
      CYY=XK-IY
      TXX=-CYY
      TYY=CXX
      AXX=AX-IX
      AYY=AY-IY
      BXX=BX-IX
      BYY=BY-IY
      SIGN=1.
      IF(BYY.LT.0.) SIGN=-1.
      THETA=ATAN2(TYY,TXX)
      ALPHA=ATAN2(AYY,AXX)-THETA
      BETA=SIGN*HALFPI-THETA
C
C     SOLVE FOR CONSTANTS IN CUBIC, SEE HARTENBERG & DENAVIT,P.209
C
      A(1)=1./SIN(ALPHA)
      A(2)=1./SIN(BETA)
      A(3)=1./COS(ALPHA)
      A(4)=1./COS(BETA)
      B(1)=1./SQRT(AXX**2+AYY**2)
      B(2)=1./SQRT(BXX**2+BYY**2)
      CALL SIMQ(A,B,2,KS)
      XM=1./B(1)
```

```
      XN=1./B(2)
C
C     DETERMINE TRIAL SOLUTIONS
C
      DO 10 I=1,40
      PSI=(I-1)*.07653982+.0392699
      R=(XN*XM*SIN(2.*PSI))/(2.*(XN*COS(PSI)+XM*SIN(PSI)))
      X(I)=R*COS(PSI)*COS(THETA)-R*SIN(PSI)*SIN(THETA)+IX
   10 Y(I)=R*COS(PSI)*SIN(THETA)+R*SIN(PSI)*COS(THETA)+IY
      PSIASM=ATAN(-XN/XM)
      XAVG=(IA+IB)/2.
      IF(IPRINT.NE.0) WRITE(6,80) THETA,ALPHA,BETA,XM,XN,PSIASM,XAVG
   80 FORMAT(///,10X,'THETA = ',E20.8,/,10X,'ALPHA = ',E20.8,/,10X,
     1'BETA = ',E20.8,/,10X,'M = ',E20.8,/,10X,'N = ',E20.8,/,10X,
     2'PSI ASM = ',E20.8,/,10X,'XAVG = ',E20.8,//)
      X(41)=2.*XAVG*COS(PSIASM)*COS(THETA)-2.*SIN(PSIASM)*XAVG*SIN(THETA
     1)+IX
      Y(41)=2.*XAVG*COS(PSIASM)*SIN(THETA)+2.*XAVG*SIN(PSIASM)*COS(THETA
     1)+IY
      DO 20 I=42,50
      XX=(I-42)*XAVG/3.333-XAVG
      X(I)=XX*COS(PSIASM)*COS(THETA)-XX*SIN(PSIASM)*SIN(THETA)+IX
   20 Y(I)=XX*COS(PSIASM)*SIN(THETA)+XX*SIN(PSIASM)*COS(THETA)+IY
      IF(IPRINT.NE.0) WRITE(6,90) (X(I),Y(I),I=1,50)
   90 FORMAT(///,10X,'ORIGINAL TRIAL SOLUTIONS',//,25X,'X',19X,'Y',///,5
     10(10X,2E20.8,/),///)
C
C     TRANSFORM INTO MU AND LAMBDA
C
      ETA=ARSIN((AY-A3)/A2)
      DO 30 I=1,50
      XT=X(I)-AX
```

```
      YT=Y(I)-AY
      X(I)=(XT*COS(ETA)-YT*SIN(ETA))/A2
   30 Y(I)=(XT*SIN(ETA)+YT*COS(ETA))/A2
      IF(IPRINT.NE.0) WRITE(6,100) (X(I),Y(I),I=1,50)
  100 FORMAT(///,10X,'TRANSFORMED TRIAL SOLUTIONS',//,20X,'LAMBDA',15X,'
     1MU',//,50(10X,2E20.8,/),///)
   40 RETURN
      END
```

```
      SUBROUTINE SOL(X,Y,NN,IPRINT,ICONV)
C
C
C     SUBROUTINE SOL
C
C     GIVEN ALL NEWTON-RHAPSON SOLUTIONS IN ARRAYS X AND Y (OF DIMENSION
C     50 AND DETERMINED WITHIN EPS), SOL WILL :
C
C           1) EXCLUDE ALL SOLUTIONS THAT DUPLICATE EACH OTHER WITHIN
C              0.5 PER CENT
C
C           2) EXCLUDE SOLUTIONS FOR WHICH LACK OF CONVERGENCE IS
C              INDICATED
C
C           3) EXCLUDE SOLUTIONS AT EITHER END OF THE COUPLER
C
C           4) ARRANGE UNIQUE SOLUTIONS WITHIN THE UPPER NN SPACES OF
C              X AND Y, ALL OTHER SPACES SET TO ZERO
C
C
C        INPUT ARGUMENTS
C         X,Y = ARRAYS OF NEWTON-RHAPSON SOLUTIONS
C         N   = NUMBER OF SOLUTIONS (50)
C         IPRINT = IF NOT EQUAL TO ZERO, INPUT AND OUTPUT ARRAYS
C                 PRINTED
C         ICONV = ARRAY INDICATING CONVERGENCE,I.E. ICONV=0
C
C        OUTPUT ARGUMENTS
C         NN= NUMBER OF UNIQUE SOLUTIONS
C         X,Y = ARRAYS OF NN UNIQUE SOLUTIONS
C
      DIMENSION X(50),Y(50),ICONV(50)
```

```
      NN=1
      EPSN=.005
      IF(IPRINT.NE.0) WRITE(6,70)   EPS5,(X(I),Y(I),I=1,50)
   70 FORMAT(////,10X,'SUBROUTINE SOL*****',//,10X,'SOL EPSILON = ',E20.
     18,//10X,'ORIGINAL SOLUTIONS',//,25X,'X',19X,'Y',//,100(10X,2E20.8,
     2/))
      DO 10 I=1,50
      IF(ICONV(I).EQ.0) GO TO 15
      X(I)=0.
      Y(I)=0.
   15 CONTINUE
      IF(X(I).LT.1.E-4.AND.Y(I).LT.1.E-4) GO TO 16
      IF(X(I).LT.1.01.AND.X(I).GT..99.AND.ABS(Y(I)).LT.1.E-8) GO TO 16
      GO TO 17
   16 X(I)=0.
      Y(I)=0.
   17 CONTINUE
      IF(X(I).EQ.0..AND.Y(I).EQ.0.) GO TO 10
      DO 20 J=1,50
      IF(I.EQ.J) GO TO 20
      EPSX=ABS((X(I)-X(J))/((ABS(X(I))+ABS(X(J)))/2.))
      EPSY=ABS((Y(I)-Y(J))/((ABS(Y(I))+ABS(Y(J)))/2.))
      IF(EPSX.LE.EPSN.AND.EPSY.LE.EPSN) GO TO 30
      GO TO 20
   30 X(J)=0.
      Y(J)=0.
   20 CONTINUE
   10 CONTINUE
      IF(IPRINT.NE.0) WRITE(6,80) (X(I),Y(I),I=1,50)
   80 FORMAT(////,10X,'UNIQUE SOLUTIONS',///,25X,'X',19X,'Y',///,100
     1(10X,2E20.8,/))
      DO 40 I=1,50
```

```
      IF(X(I).NE.0..AND.Y(I).NE.0.) GO TO 40
      J=I+1
      IF(J.EQ.51) GO TO 40
      DO 50 K=J,50
      IF(X(K).NE.0..AND.Y(K).NE.0.) GO TO 60
      GO TO 50
   60 X(I)=X(K)
      Y(I)=Y(K)
      X(K)=0.
      Y(K)=0.
      NN=I
      GO TO 40
   50 CONTINUE
   40 CONTINUE
      IF(IPRINT.NE.0) WRITE(6,90) NN,(X(I),Y(I),I=1,NN)
   90 FORMAT(////,10X,'RETURNED SOLUTIONS = ',I10 ,///,15X,'X',19X,'Y',/
     1/,100(10X,2E20.8,/))
      RETURN
      END
```

```
      SUBROUTINE NEWRAP(MX,LX,ICONV,EPS,ITER,PHI,A1,A2,OFST,DERIV,
     1IPRINT)
C
C
C     SUBROUTINE NEWRAP
C
C     SUBROUTINE NEWRAP, GIVEN A DESCRIPTION OF THE COUPLER CON-
C     STRAINTS, WILL FORM THE RELATIONSHIPS NECESSARY TO LOCATE
C     SIMULTANEOUS ZEROES OF THE FIRST AND SECOND DERIVATIVES OF THE
C     RADIUS OF CURVATURE OF A COUPLER POINT PATH.  THEN, GIVEN TRIAL
C     SOLUTIONS, THE ROUTINE WILL EXECUTE A NEWTON-RHAPSON ITERATION
C     PROCEDURE UNTIL THE SOLUTION CONVERGES TO WITHIN SOME EPSILON.
C     CONVERGENCE IS PRESUMED TO HAVE FAILED IF
C
C        1) EITHER MU OR LAMBDA EXCEED 200 (ICONV=2) OR
C        2) THE NUMBER OF ITERATIONS EQUAL OR EXCEED 110. (ICONV=1)
C
C     IN THE CASE OF (2) ABOVE, THE FINAL 10 VALUES OF U AND V ARE
C     AVERAGED AND REPORTED AS MU AND LAMBDA.
C
C        INPUT ARGUMENTS
C         MX,LX = ARRAYS OF TRIAL SOLUTIONS. DIMENSIONED 50
C         EPS = RELATIVE CONVERGENCE CRITEREA
C         PHI = DRIVING CRANK ANGLE
C         A1  = DRIVING CRANK RADIUS
C         A2  = CONNECTING ROD LENGTH
C         OFST = SLIDER PATH OFFSET
C         IPRINT = PRINT COMMAND
C                = 0, NO PRINTED OUTPUT
C                > 5, RESULTS OF EACH ITERATION STEP PRINTED PLUS
C                > 3, RESULTS OF ITERATIONS ON EACH TRIAL SOLUTION
C                     PRINTED PLUS
```

```
C                 NE 0, INPUT TRIAL SOLUTIONS AND OUTPUT SOLUTIONS
C                       PRINTED
C          OUTPUT ARGUMENTS
C           UX,VX = ARRAYS OF ITERATED SOLUTIONS
C           ICONV = ARRAY INDICATING CONVERGENCE OR LACK OF IT
C                 = 0, CONVERGENCE
C                 = 1, ITERATIONS EXCEEDED 110
C                 = 2, SOLUTIONS EXCEEDED 200 IN VALUE
C            ITER = ARRAY INDICATING NUMBER OF ITERATIONS FOR EACH
C                   SOLUTION
C           DERIV = ARRAY CONTAINING COEFFICIENTS OF DERIVATIVES OF X
C                   AND Y WITH RESPECT TO PHI, THE DRIVING CRANK ANGLE
C
C
C
      REAL MX,LX,L,MU,MP,LP,J,LAMBDA
      DIMENSION LX(50),MX(50),ITER(50),ICONV(50),DERIV(10,3)
      A3=OFST
      R=A1
      L=A2
      CP=COS(PHI)
      SP=SIN(PHI)
      Z=SQRT(2.00*SP*A3*R-SP**2*R**2-A3**2+L**2)
      X1C=-R*SP
      X1L=(R*A3*CP-CP*SP*R**2)/Z
      X1M=R*CP
      X2C=-X1M
      X2L=(Z**2*(-R*A3*SP+R**2*SP**2-R**2*CP**2)-R**2*A3**2*CP**2+2.0
     10*CP**2*SP*A3*R**3-CP**2*SP**2*R**4)/Z**3
      X2M=X1C
      X3C=-X1C
      X3L=(3.00*CP*SP*A3**2*R**2*Z**2-6.00*CP*SP**2*A3*R**3*Z**2+3.00*CP
```

```
1**3*A3*R**3*Z**2+3.00*CP*SP**3*R**4*Z**2-3.00*CP**3*SP*R**4*Z**2   NWR  650
2-CP*A3*R*Z**4+4.00*CP*SP*R**2*Z**4+3.00*CP**3*A3**3*R**3-9.00*     NWR  660
3CP**3*SP*A3**2*R**4+9.00*CP**3*SP**2*A3*R**5-3.00*CP**3*SP**3*R**6NWR  670
4)/Z**5                                                             NWR  680
 X3M=X2C                                                            NWR  690
 X4C=X1M                                                            NWR  700
 X4L=(-18.00*CP**2*SP*A3**3*R**3*Z**2+54.00*CP**2*SP**2*A3**2*R**4 NWR  710
1*Z**2-18.00*CP**4*A3**2*R**4*Z**2-54.00*CP**2*SP**3*A3*R**5*Z**2  NWR  720
2+36.00*CP**4*SP*A3*R**5*Z**2-18.00*CP**4*SP**2*R**6*Z**2+18.00*   NWR  730
3CP**2*SP**4*R**6*Z**2-3.00*SP**2*A3**2*R**2*Z**4+4.00*CP**2*A3**2 NWR  740
4*R**2*Z**4)/Z**7                                                   NWR  750
 X4L=X4L+(6.*SP**3*A3*R**3*Z**4-26.*CP**2*SP*A3*R**3*Z**4+          NWR  760
122.00*CP**2*SP**2*R**4*Z**4-3.00*SP**4*R**4*Z**4-3.00*CP**4*R**4  NWR  770
2*Z**4+SP*A3*R*Z**6-4.00*SP**2*R**2*Z**6+4.00*CP**2*R**2*Z**6-     NWR  780
315.00*CP**4*A3**4*R**4+60.00*CP**4*SP*A3**3*R**5-90.00*CP**4*SP   NWR  790
4**2*A3**2*R**6+60.00*CP**4*SP**3*A3*R**7-15.00*CP**4*SP**4*R**8)/ NWR  800
5Z**7                                                               NWR  810
 X4M=X3C                                                            NWR  820
 X5C=X1C                                                            NWR  830
 X5M=X1M                                                            NWR  840
 X5L =(-CP*SP*Z**6*R**2*A3**2*15.-CP*SP*Z**8*R**2*16.+CP*SP**2*Z**4NWR  850
1*R**3*A3**3*45.+CP*SP**2*Z**6*R**3*A3*75.-CP*SP**3*Z**4*R**4*A3**2NWR  860
2*135.-CP*SP**3*Z**6*R**4*60.+CP*SP**4*Z**4*R**5*A3*135.-CP*SP**5*ZNWR  870
3**4*R**6*45.+CP*Z**8*R*A3 +CP**3*SP*Z**2*R**4*A3**4*150.+CP**3*SP*NWR  880
4Z**4*R**4*A3**2*270.)/Z**9                                         NWR  890
 X5L=X5L+(CP**3*SP*Z**6*R**4*60.-CP**3*SP**2*Z**2*R**5*A3**3*600.-CNWR  900
1P**3*SP**2*Z**4*R**5*A3*450.+CP**3*SP**3*Z**2*R**6*A3**2*900.+CP**NWR  910
23*SP**3*Z**4*R**6*210.-CP**3*SP**4*Z**2*R**7*A3*600.+CP**3*SP**5*ZNWR  920
3**2*R**8*150.-CP**3*Z**4*R**3*A3**3*30.-CP**3*Z**6*R**3*A3*30.-CP*NWR  930
4*5*SP*Z**2*R**6*A3**2*450.-CP**5*SP*Z**4*R**6*45.-CP**5*SP*R**6*A3NWR  940
6**4*525.+CP**5*SP**2*Z**2*R**7*A3*450.)/Z**9                       NWR  950
 X5L=X5L+(CP**5*SP**2*R**7*A3**3*1050.-CP**5*SP**3*Z**2*R**8*150.   NWR  960
```

```
     1-CP**5*SP**3*R**8*A3**2*1050.+CP**5*SP**4*R**9*A3*525.-CP**5*SP**
     25*R**10*105.+CP**5*Z**2*R**5*A3**3*150.+CP**5*Z**4*R**5*A3*45.
     3+CP**5*R**5*A3**5*105.)/Z**9
      Y5C=X1M
      Y5L=X5M
      Y5M=X5L
      Y1C=X1M
      Y1L=X2C
      Y1M=X1L
      Y2C=X1C
      Y2L=X3C
      Y2M=X2L
      Y3C=X2C
      Y3L=X1M
      Y3M=X3L
      Y4C=X3C
      Y4L=X1C
      Y4M=X4L
      DO 10 I=1,50
   10 ICONV(I)=0
      DO 30 I=1,50
      SUMM=0.
      SUML=0.
      MU=MX(I)
      LAMBDA=LX(I)
      ZMU=MU
      ZLM=LAMBDA
      IF(IPRINT.GE.5) WRITE(6,120) MU,LAMBDA
  120 FORMAT(////,10X,'SUBROUTINE NEWRAP',//,10X,'TRIAL MU = ',  E20.8,'
     1 , TRIAL LAMBDA = ',  E20.8,///,10X,'ITERATION', 8X,'MU',16X,'LAMB
     2DA',14X,'EPS MU',12X,'EPS LAMBDA',//)
      N=0
```

```
   40 CONTINUE
      LP=LAMBDA
      MP=MU
      X1=X1C+X1L*LAMBDA+X1M*MU
      X2=X2C+X2L*LAMBDA+X2M*MU
      X3=X3C+X3L*LAMBDA+X3M*MU
      X4=X4C+X4L*LAMBDA+X4M*MU
      Y1=Y1C+Y1L*LAMBDA+Y1M*MU
      Y2=Y2C+Y2L*LAMBDA+Y2M*MU
      Y3=Y3C+Y3L*LAMBDA+Y3M*MU
      Y4=Y4C+Y4L*LAMBDA+Y4M*MU
      F=-X2M*Y1+X1M*Y2+Y2M*X1-Y1M*X2
      G=2.00*(Y1M*Y1+X1M*X1)
      H=2.00*(Y1L*Y1+X1L*X1)
      J=-X2L*Y1+X1L*Y2+Y2L*X1-Y1L*X2
      R=X1*X2+Y1*Y2
      S=X1*Y3-Y1*X3
      T=X2**2+Y2**2+X1*X3+Y1*Y3
      U=X2*Y3+X1*Y4-Y2*X3-Y1*X4
      W=Y2*X1-Y1*X2
      ZZ=X1**2+Y1**2
      D1=3.00*ZZ*R*W-ZZ**2*S
      D1L =ZZ*R*(X1*Y2L-X2*Y1L-Y1*X2L+Y2*X1L)*3.00+ZZ*W*(X1*X2L+X2*X1L+Y
     11*Y2L+Y2*Y1L)*3.00-ZZ*S*(X1*X1L*2.00+Y1*Y1L*2.00)*2.00+R*W*(X1*X1L
     2*2.00+Y1*Y1L*2.00)*3.00-ZZ**2*(X1*Y3L-X3*Y1L-Y1*X3L+Y3*X1L)
      D1M =ZZ*R*(X1*Y2M-X2*Y1M-Y1*X2M+Y2*X1M)*3.00+ZZ*W*(X1*X2M+X2*X1M+Y
     11*Y2M+Y2*Y1M)*3.00-ZZ*S*(X1*X1M*2.00+Y1*Y1M*2.00)*2.00+R*W*(X1*X1M
     2*2.00+Y1*Y1M*2.00)*3.00-ZZ**2*(X1*Y3M-X3*Y1M-Y1*X3M+Y3*X1M)
      D2=3.00*R**2*W**2-6.00*R*S*W*ZZ+2.00*S**2*ZZ**2+3.00*T*W**2*ZZ
     1-U*W*ZZ**2
      D2L =-ZZ*H*U*W*2.00+ZZ*H*S**2*4.00-ZZ*J*R*S*6.00+ZZ*J*W*T*6.00-ZZ*
     1R*W*(X1*Y3L-X3*Y1L-Y1*X3L+Y3*X1L)*6.00-ZZ*W*S*(X1*X2L+X2*X1L+Y1*Y2
```

```
     2L+Y2*Y1L)*6.00+ZZ*W**2*(X1*X3L+X2*X2L*2.00+X3*X1L+Y1*Y3L+Y2*Y2L*2.
     300+Y3*Y1L)*3.00-H*R*W*S*6.00+H*W**2*T*3.00+J*R**2*W*6.00+R*W**2*(X
     41*X2L+X2*X1L+Y1*Y2L+Y2*Y1L)*6.00-ZZ**2*J*U-ZZ**2*W*(X1*Y4L+X2*Y3L-
     5X3*Y2L-X4*Y1L-Y1*X4L-Y2*X3L+Y3*X2L+Y4*X1L)+ZZ**2*S*(X1*Y3L-X3*Y1L-
     6Y1*X3L+Y3*X1L)*4.
      D2M =-ZZ*G*U*W*2.00+ZZ*G*S**2*4.00-ZZ*F*R*S*6.00+ZZ*F*W*T*6.00-ZZ*
     1R*W*(X1*Y3M-X3*Y1M-Y1*X3M+Y3*X1M)*6.00-ZZ*W*S*(X1*X2M+X2*X1M+Y1*Y2
     2M+Y2*Y1M)*6.00+ZZ*W**2*(X1*X3M+X2*X2M*2.00+X3*X1M+Y1*Y3M+Y2*Y2M*2.
     300+Y3*Y1M)*3.00-G*R*W*S*6.00+G*W**2*T*3.00+F*R**2*W*6.00+R*W**2*(X
     41*X2M+X2*X1M+Y1*Y2M+Y2*Y1M)*6.00-ZZ**2*F*U-ZZ**2*W*(X1*Y4M+X2*Y3M-
     5X3*Y2M-X4*Y1M-Y1*X4M-Y2*X3M+Y3*X2M+Y4*X1M)+ZZ**2*S*(X1*Y3M-X3*Y1M-
     6Y1*X3M+Y3*X1M)*4.
      N=N+1
      DJAC=D1M*D2L-D2M*D1L
      DTEMP1=D2*D1L-D1*D2L
      DTEMP2=D1*D2M-D2*D1M
      DELM=DTEMP1/DJAC
      DELL=DTEMP2/DJAC
      MU=MU+DELM
      LAMBDA=LAMBDA+DELL
      EPSM=(MU-MP)/MU
      EPSL=(LAMBDA-LP)/LAMBDA
      IF(ABS(EPSM).LE.EPS.AND.ABS(EPSL).LE.EPS) GO TO 70
      IF(ABS(MU).GE.200..AND.ABS(LAMBDA).GE.200.) GO TO 75
      IF(IPRINT.GE.5) WRITE(6,110) N,MU,LAMBDA,EPSM,EPSL
  110 FORMAT(10X,I5,4(  E20.8))
      IF(N.GE.101) GO TO 20
      GO TO 40
   75 MX(I)=MU
      LX(I)=LAMBDA
      ITER(I)=N
      ICONV(I)=2
```

```
      GO TO 30
   20 SUMM=SUMM+MU
      SUML=SUML+LAMBDA
      IF(N.NE.110) GO TO 40
      LAMBDA=SUML/10.
      MU=SUMM/10.
      ICONV(I)=1
   70 CONTINUE
      MX(I)=MU
      LX(I)=LAMBDA
      ITER(I)=N
      IF(IPRINT.GE.3) WRITE(6,130) ZMU,ZLM,N,MU,LAMBDA,EPSM,EPSL
  130 FORMAT(////,10X,'SUBROUTINE NEWRAP',//,10X,'TRIAL MU = ',  E20.8,'
     1 , TRIAL LAMBDA = ',  E20.8,///,10X,'ITERATION', 8X,'MU',16X,'LAMB
     2DA',14X,'EPS MU',12X,'EPS LAMBDA',/,10X,I5,4(  E20.8),///)
   30 CONTINUE
      IF(IPRINT.NE.0) WRITE(6,100) (MX(I),LX(I),ITER(I),ICONV(I),I=1,50)
  100 FORMAT(////,10X,'SUBROUTINE NEWRAP',///,17X,'MU SOLUTION',7X,'LAMB
     1DA SOLUTION',3X,'ITERATIONS',1X,'CONVERGENCE',//,100(10X,2E20.8,2I
     210,/))
      DERIV(1,1)=X1C
      DERIV(1,2)=X1L
      DERIV(1,3)=X1M
      DERIV(2,1)=X2C
      DERIV(2,2)=X2L
      DERIV(2,3)=X2M
      DERIV(3,1)=X3C
      DERIV(3,2)=X3L
      DERIV(3,3)=X3M
      DERIV(4,1)=X4C
      DERIV(4,2)=X4L
      DERIV(4,3)=X4M
```

```
      DERIV(5,1)=Y1C
      DERIV(5,2)=Y1L
      DERIV(5,3)=Y1M
      DERIV(6,1)=Y2C
      DERIV(6,2)=Y2L
      DERIV(6,3)=Y2M
      DERIV(7,1)=Y3C
      DERIV(7,2)=Y3L
      DERIV(7,3)=Y3M
      DERIV(8,1)=Y4C
      DERIV(8,2)=Y4L
      DERIV(8,3)=Y4M
      DERIV(9,1)=X5C
      DERIV(9,2)=X5L
      DERIV(9,3)=X5M
      DERIV(10,1)=Y5C
      DERIV(10,2)=Y5L
      DERIV(10,3)=Y5M
      IF(IPRINT.NE.0) WRITE(6,50)((DERIV(K,I),I=1,3),K=1,10)
   50 FORMAT(////,27X,'CONST',14X,'LAMBDA',17X,'MU',//,10X,'X1',5X,3E20.
     18,/,10X,'X2',5X,3E20.8,/,10X,'X3',5X,3E20.8,/,10X,'X4',5X,3E20.
     28,/,10X,'Y1',5X,3E20.8,/,10X,'Y2',5X,3E20.8,/,10X,'Y3',5X,3E20.
     38,/,10X,'Y4',5X,3E20.8,/,10X,'X5',5X,3E20.8,/,10X,'Y5',5X,3E20.8,/
     4/////)
      RETURN
      END
```

```
      SUBROUTINE SIMQ(A,B,N,KS)
C
C        ..................................................................
C
C        SUBROUTINE SIMQ
C
C        PURPOSE
C           OBTAIN SOLUTION OF A SET OF SIMULTANEOUS LINEAR EQUATIONS,
C           AX=B
C
C        USAGE
C           CALL SIMQ(A,B,N,KS)
C
C        DESCRIPTION OF PARAMETERS
C           A - MATRIX OF COEFFICIENTS STORED COLUMNWISE.  THESE ARE
C               DESTROYED IN THE COMPUTATION.  THE SIZE OF MATRIX A IS
C               N BY N.
C           B - VECTOR OF ORIGINAL CONSTANTS (LENGTH N). THESE ARE
C               REPLACED BY FINAL SOLUTION VALUES, VECTOR X.
C           N - NUMBER OF EQUATIONS AND VARIABLES. N MUST BE .GT. ONE.
C           KS - OUTPUT DIGIT
C                0 FOR A NORMAL SOLUTION
C                1 FOR A SINGULAR SET OF EQUATIONS
C
C        REMARKS
C           MATRIX A MUST BE GENERAL.
C           IF MATRIX IS SINGULAR , SOLUTION VALUES ARE MEANINGLESS.
C           AN ALTERNATIVE SOLUTION MAY BE OBTAINED BY USING MATRIX
C           INVERSION (MINV) AND MATRIX PRODUCT (GMPRD).
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
```

```
C
C        METHOD
C           METHOD OF SOLUTION IS BY ELIMINATION USING LARGEST PIVOTAL
C           DIVISOR. EACH STAGE OF ELIMINATION CONSISTS OF INTERCHANGING
C           ROWS WHEN NECESSARY TO AVOID DIVISION BY ZERO OR SMALL
C           ELEMENTS.
C           THE FORWARD SOLUTION TO OBTAIN VARIABLE N IS DONE IN
C           N STAGES. THE BACK SOLUTION FOR THE OTHER VARIABLES IS
C           CALCULATED BY SUCCESSIVE SUBSTITUTIONS. FINAL SOLUTION
C           VALUES ARE DEVELOPED IN VECTOR B, WITH VARIABLE 1 IN B(1),
C           VARIABLE 2 IN B(2),........., VARIABLE N IN B(N).
C           IF NO PIVOT CAN BE FOUND EXCEEDING A TOLERANCE OF 0.0,
C           THE MATRIX IS CONSIDERED SINGULAR AND KS IS SET TO 1. THIS
C           TOLERANCE CAN BE MODIFIED BY REPLACING THE FIRST STATEMENT.
C
C     ..................................................................
C
      DIMENSION A(1),B(1)
C
C        FORWARD SOLUTION
C
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0
      IT=JJ-J
      DO 30 I=J,N
C
C        SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN
```

```
C
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
C
C        TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)
C
      IF(ABS(BIGA)-TOL) 35,35,40
   35 KS=1
      RETURN
C
C        INTERCHANGE ROWS IF NECESSARY
C
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
C
C        DIVIDE EQUATION BY LEADING COEFFICIENT
C
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
C
C        ELIMINATE NEXT VARIABLE
```

```
C
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
C
C        BACK SOLUTION
C
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

```
      SUBROUTINE STRLIN(MU,LAMBDA,A1,A2,A3,PHI,X1,X2,Y1,Y2,R,XC,YC,X,Y)
C
C
C     SUBROUTINE STRLIN
C
C     SUBROUTINE STRLIN, GIVEN THE ARGUMENTS BELOW, WILL LOCATE THE
C     CENTER OF CURVATURE OF THE COUPLER POINT CURVE AND WILL DETER-
C     MINE THE RADIUS OF CURVATURE OF THE CURVE
C
C        INPUT ARGUMENTS
C         MU,LAMBDA = COORDINTES OF THE COUPLER POINT IN THE MOVING
C                     PLANE
C         A1   = DRIVING CRANK RADIUS
C         PHI  = DRIVING CRANK ANGLE
C         OFST = SLIDER PATH OFFSET
C         A2   = CONNECTING ROD LENGTH
C         X1,X2 = FIRST AND SECOND DERIVATIVES OF X WITH RESPECT TO PHI
C         Y1,Y2 = FIRST AND SECOND DERIVATIVES OF Y WITH RESPECT TO PHI
C
C        OUTPUT ARGUMENTS
C         R = RADIUS OF CURVATURE OF THE COUPLER CURVE
C         XC,YC = CENTER OF CURVATURE OF THE COUPLER CURVE
C         X,Y = COORDINATES OF THE COUPLER POINT IN THE FIXED PLANE
C
      REAL MU,LAMBDA,LX,LY
      PSI=ARSIN((A3-A1*SIN(PHI))/A2)
      LX=A2*COS(PSI)
      LY=A1*SIN(PHI)-A3
      X=A1*COS(PHI)+LAMBDA*LX+MU*LY
      Y=A1*SIN(PHI)+MU*LX-LAMBDA*LY
      R=ABS((X1**2+Y1**2)**1.5/(X1*Y2-Y1*X2))
      YP=Y1/X1
```

```
      YPP=(X1*Y2-Y1*X2)/X1**3
      XC=X-(YP+YP**3)/YPP
      YC=Y+(1.+YP**2)/YPP
      WRITE(6,10) MU,LAMBDA,A1,A2,A3,PHI,X,Y,R,XC,YC,X1,X2,Y1,Y2,YP,YPP
   10 FORMAT(/////,10X,'SUBROUTINE STRLIN******',//,10X,'MU = ',E15.8,/,
     110X,'LAMBDA = ',E15.8,/,10X,'R = ',E15.8,/,10X,'L = ',E15.8,/,10X,
     2'OFST = ',E15.8,/,10X,'PHI = ',E15.8,/,10X,'COUPLER POINT AT ',2E1
     35.8,/,10X,'RADIUS OF CURVATURE = ',E15.8,/,10X,'CENTER OF CURVATUR
     4E AT ',2E15.8,///,10X,'X1 = ',E15.8,/,10X,'X2 = ',E15.8,/,10X,'Y1
     5= ',E15.8,/,10X,'Y2 = ',E15.8,/,10X,'YP = ',E15.8,/,10X,'YPP = ',
     6E15.8,////////)
      RETURN
      END
```

```
      SUBROUTINE ANALZE(PHI1,START,ENDD,A1,A2,A3,X,Y,XC,YC,RC,IPLOTR,ICH
     1K,MU,LMBDA)
C
C
C
C     SUBROUTINE ANALZE
C
C
C
C        SUBROUTINE ANALZE PROVIDES AN ANALYSIS, FOR INTEGRAL DEGREES,
C     OVER A SPECIFIED RANGE FOR THE FOUR BAR - COUPLER CONFIGURATION.
C
C
C
C        ARGUMENTS
C
C     PHI1 = DESIGN ANGLE OF CRANK, DEGREES
C     START = INITIAL VALUE OF CRANK ANGLE
C     ENDD = FINAL VALUE OF CRANK ANGLE
C     A1,A2,A3 = LENGTHS- CRANK, CONN ROD, OFFSET
C     X,Y, = COORDS OF COUPLER POINT IN S-C CONFIGURATION
C     XC,YC = COORDS OF CENTER OF FOLLOWER CRANK ROTATION
C     RC = RADIUS OF FOLLOWER CRANK
C     IPLOTR    = 1,    PRINTER PLOT OF COUPLER CURVES PROVIDED
C               = 2,    NO PRINTER PLOT OF COUPLER CURVES
C     ICHK      = 0,  ANALYSIS OF COUPLER CURVES PROVIDED
C               = 1,  NO ANALYSIS OF COUPLER CURVES
C     MU,LMBDA = DMNLESS COORDS OF COUPLER POINT IN 4-BAR CONFIG
C
C     NO OUTPUT ARGUMENTS PROVIDED
C
C
```

```
      DIMENSION XZ(360),YZ(360),IFAULT(360),AA(720)
      REAL MU,LMBDA
      PI=3.141593
      HALFPI=1.570796
      IF(ICHK.NE.0) CALL DRAW(XZ,YZ,N,A1,A2,A3,1,IFAULT,PHI1,START,ENDX,
     1AA,IVPI)
      IF(ICHK.NE.0) RETURN
      ENDX=ENDD*PI/180.
      PHI=PHI1
      IF(PHI.LT.0.) PHI=PHI+2.*PI
      WRITE(6,20)
   20 FORMAT(1H1,////,15X,'THETA',9X,'X4',13X,'X3',13X,'Y4',13X,'Y3',
     113X,'DX',13X,'DY',//)
      ALPHA=ATAN2(MU,LMBDA)
      A=SQRT(XC**2+YC**2)
      CORR=ATAN2(YC,XC)
      B=A1
      C=SQRT((A1* SIN(PHI)-Y)**2+(A1*COS(PHI)-X)**2)
      D=ABS(RC)
C
C     DETERMINE THE VALUE OF THE VARIABLE, SIGN, +1 OR -1
C
      ETA=ATAN2((Y-A1*SIN(PHI)),(X-A1*COS(PHI)))-CORR
      ETA=ARSIN(SIN(ETA))
      P=A-B*COS(PHI-CORR)
      Q=B*SIN(PHI-CORR)
      R=(P**2+Q**2+C**2-D**2)/(2.*C)
      S=R*Q/(P**2+Q**2)
      T=(R**2-P**2)/(P**2+Q**2)
      EPSP=ABS(ETA-ARSIN(-S+SQRT(ABS(S**2-T))))
      EPSM=ABS(ETA-ARSIN(-S-SQRT(ABS(S**2-T))))
      SIGN=-1.
```

```
      IF(EPSP.LE.EPSM) SIGN=1.
C
C     INCREMENT LINKAGE THROUGH RANGE
C
C     IFAULT = 1  FOR IMPOSSIBLE LINKAGE POSITIONS
C
      DELTA=PI/180.
      THETA=START*PI/180.-DELTA
      J=0
      NX=50
      DO 120 I=1,360
      IFAULT(I)=0
      XZ(I)=0
  120 YZ(I)=0
      DO 10 I=1,360
  100 THETA=THETA+DELTA
      IF(THETA.GT.ENDX) GO TO 110
      ZLNGTH=ABS(C+D)
      YLNGTH=ABS(C-D)
      XLNGTH=SQRT((A1*SIN(THETA)-YC)**2+(A1*COS(THETA)-XC)**2)
      IF(XLNGTH.GT.ZLNGTH.OR.XLNGTH.LT.YLNGTH) GO TO 140
      GO TO 130
  140 IFAULT(I)=1
      GO TO 100
  130 N=I-1
      J=J+1
      EPS=THETA-CORR
      P=A-B*COS(EPS)
      Q=B*SIN(EPS)
      R=(P**2+Q**2+C**2-D**2)/(2.*C)
      P2Q2=P**2+Q**2
      S=R*Q/P2Q2
```

```
      T=(R**2-P**2)/P2Q2
      BETA=ARSIN(-S+SIGN*SQRT(ABS(S**2-T)))-ALPHA
      XK=B*COS(EPS)+A2*COS(BETA)
      YK=B*SIN(EPS)+A2*SIN(BETA)
      XZ(I)=XK*COS(CORR)-YK*SIN(CORR)
      YZ(I)=XK*SIN(CORR)+YK*COS(CORR)
      Y3=A3
      THETAX=THETA*180./PI
      X3=B*COS(THETA)+A2*COS(ARSIN((B*SIN(THETA)-A3 )/A2))
      DX=XZ(I)-X3
      DY=YZ(I)-Y3
      IF(J.EQ.NX) GO TO 30
      GO TO 40
   30 J=0
      NX=50
      WRITE(6,70)
   70 FORMAT(1H1,////,15X,'THETA',9X,'X4',13X,'X3',13X,'Y4',13X,'Y3',
     113X,'DX',13X,'DY',//)
   40 WRITE(6,50) THETAX,XZ(I),X3,YZ(I),Y3,DX,DY
   50 FORMAT(15X,F6.2,2X,6(F10.5,5X))
   10 CONTINUE
  110 IVPI=2*N
      IF(IPLOTR.EQ.1) CALLDRAW(XZ,YZ,N,A1,A2,A3,0,IFAULT,PHI1,START,ENDX
     1,AA,IVPI)
      RETURN
      END
```

```
      SUBROUTINE DRAW(X,Y,N,A1,A2,A3,ICHK,IFAULT,PHI,START,ENDX,A,IVPI) DRW
C                                                                       DRW
C                                                                       DRW
C     SUBROUTINE DRAW                                                   DRW
C                                                                       DRW
C     SUBROUTINE DRAW WILL PROVIDE A PRINTER PLOT OF ALL COUPLER        DRW
C     CURVES DESCRIBED IN TERMS OF COORDINATES OF POINTS ALONG THE      DRW
C     CURVE.                                                            DRW
C                                                                       DRW
C                                                                       DRW
C     X,Y      = ARRAYS OF DATA TO BE PLOTTED                           DRW
C     A        = WORKING VECTOR OF SIZE IVPI (N*2)                      DRW
C     N        = NUMBER OF ELEMENTS IN X (OR Y)                         DRW
C     IVPI     = LENGTH OF VECTOR A,N*2.                                DRW
C     IFAULT   = ARRAY INDICATING DISCONTINUITY IN COUPLER CURVE        DRW
C     ICHK =1,  PAGE EJECT ONLY                                         DRW
C          =0,  COUPLER CURVE PLOT                                      DRW
C                                                                       DRW
C                                                                       DRW
C     ALL OTHER ARGUMENTS ARE VARIABLES PROVIDED FOR REFERENCE PRINTING DRW
C     ONLY.                                                             DRW
C                                                                       DRW
C                                                                       DRW
C     A1,A2,A3 = CRANK,RADIUS,CONNECTING ROD LENGTH,OFFSET              DRW
C     START,ENDX = STARTING AND ENDING ANGLES OF CRANK ROTATION         DRW
C                                                                       DRW
C                                                                       DRW
      DIMENSION CUT(101),YPR(11),A(720),X(360),Y(360),IFAULT(360)       DRW
      INTEGER CUT,BLANK,DOT,STROKE,USC,STAR                             DRW
C     DATA (BLANK=1H ),(DOT=1H.),(STROKE=1HI),(USC=1H-),(STAR=1H*)      CDC
      DATA BLANK/1H /,DOT/1H./,STROKE/1H|/,USC/1H-/,STAR/1H*/           IBM
      IF(ICHK.EQ.1) WRITE(6,95)                                         DRW
```

```
   95 FORMAT(1H1,///)
      IF(ICHK.EQ.1) RETURN
      WRITE(6,1) A1,A2,A3,PHI,START,ENDX
    1 FORMAT(1H1,//,50X,'COUPLER CURVE ILLUSTRATING THE',/,32X,'PERFORMA
     1NCE OF A FOUR-BAR LINKAGE SYNTHESIZED FROM A SLIDER CRANK',//,50X,
     2'CRANK RADIUS = ',F15.8,/,45X,'CONNECTING ROD LENGTH = ',F15.8,/,
     347X,'SLIDER PATH OFFSET = ',F15.8,/,47X,'DESIGN CRANK ANGLE = ',
     4F15.8,/,35X,'CRANK ROTATED FROM ',F15.8,' TO ',F15.8,' RADIANS',//
     5/)
      DO 100 J=1,N
      A(J)=X(J)
      JTEMP=J+N
  100 A(JTEMP)=Y(J)
      DO 14 I=1,N
      DO15J=1,N
      IF(A(J).LT.A(I))GOTO15
      DO17K=1,2
      KK=K-1
      ITP=I+KK*N
      JTP=J+KK*N
      F=A(ITP)
      A(ITP)=A(JTP)
      A(JTP)=F
17    CONTINUE
   15 CONTINUE
14    CONTINUE
      NLL=61
      XSCAL=(A(N)-A(1))/60.
      M1=N+1
      YMIN=A(M1)
      YMAX=YMIN
      M2=2*N
```

```
      DO40J=M1,M2
      IF(A(J)-YMIN)28,26,26
26    IF(A(J)-YMAX)40,40,30
28    YMIN=A(J)
      GOTO40
30    YMAX=A(J)
   40 CONTINUE
      DELY =ABS(YMAX-YMIN)
      YSCAL=DELY/100.
      YPR(1)=YMIN
       DO 90 KN=1,10
   90 YPR(KN+1)=YPR(1)+KN*YSCAL*10.
      WRITE(6,8) (YPR(IP),IP=1,11)
    8 FORMAT(1H ,9X,11F10.4,/)
      XB=A(1)
      L=1
      LX=1
      MY=1
      I=1
45    F=I-1
      XPR=XB+F*XSCAL
      XPRHI=XPR+XSCAL/2.
      XPRLO=XPR-XSCAL/2.
      IF(A(L).LT.XPRLO.OR.A(L).GT.XPRHI) GO TO 70
50    DO55IX=1,101
55    OUT(IX)=BLANK
      IF(LX.NE.1)GOTO300
      DO301IX=1,101
301   OUT(IX)=USC
300   CONTINUE
      DO56IX=1,101,10
56    OUT(IX)=STROKE
```

```
  220 IS=L+N
      JP=(A(IS)-YMIN)/YSCAL+1.5
      OUT(JP)=STAR
      IF(A(L+1).GE.XPRLO.AND.A(L+1).LE.XPRHI) GO TO 221
      GO TO 200
  221 L=L+1
      GO TO 220
  200 CONTINUE
      WRITE(6,2)XPR,(OUT(IZ),IZ=1,101)
2     FORMAT(1H ,F11.4,5X,101A1)
      LX=LX+1
      IF(LX.EQ.7)LX=1
      L=L+1
      GOTO80
   70 CONTINUE
      DO 71 IX=1,101
   71 OUT(IX)=BLANK
      DO 72 IX=1,101,10
   72 OUT(IX)=STROKE
      IF(LX.NE.1) GO TO 74
      DO 76 IX=1,101
   76 OUT(IX)=USC
      DO 77 IX=1,101,10
   77 OUT(IX)=STROKE
   74 WRITE(6,73) OUT
   73 FORMAT(17X,101A1)
      LX=LX+1
      IF(LX.EQ.7) LX=1
   80 I=I+1
      IF(I-NLL)45,84,86
84    XPR=A(N)
      GOTO50
```

```
86    WRITE(6,6)                                                        DRW 1290
6     FORMAT(1H1)                                                       DRW 1300
      RETURN                                                            DRW 1310
      END                                                               DRW 1320
```

```
      FUNCTION ATAN2(Y,X)
      C=0.0
      IF(X.LT.0.) C=3.141593
      ATAN2=ATAN(Y/X)+C
      RETURN
      END
C
C
C
      FUNCTION TAN(X)
      TAN=SIN(X)/COS(X)
      RETURN
      END
C
C
C
      FUNCTION ARSIN(X)
      ARSIN=ATAN(X/SQRT(1.-X**2))
      RETURN
      END
```

```
C     PROGRAM CIPCLR
C
C
C     PROGRAM CIRCLR
C
C     THE MAIN PROGRAM PROVIDES FOR THE INPUT OF THE DESCRIPTION OF
C     THE ORIGINAL FOUR-BAR CONFIGURATION.  THEN, THROUGH SUBROUTINE
C     CALLS, TRIAL SOLUTIONS ARE GENERATED, UNIQUE SOLUTIONS DETER-
C     MINED, AND THE SOLUTIONS ARE FURTHER PROCESSED.
C
C        INPUT VARIABLES
C         A1     = DRIVING CRANK RADIUS
C         PHI1   = DRIVING CRANK ANGLE
C         XF,YF  = COORDINATES OF THE ROD END OF THE FOLLOWER CRANK
C         XB,YB  = COORDINATES OF THE FOLLOWER CRANK CENTER
C         EPS4   = CONVERGENCE CRITEREA
C         IPRINT = PRINT LEVEL
C         START  = INITIAL ANGLE FOR DRIVING CRANK FOR COUPLER CURVE
C                  PLOT
C         ENDD   = FINAL ANGLE FOR DRIVING CRANK FOR COUPLER CURVE PLOT
C
C        INPUT VARIABLES MUST BE IN THE ORDER ABOVE AND IN THE FORMAT
C
C        (6F10.0,F5.0,I1,2F7.0)
C
C        THE FINAL DATA CARD SHOULD BE BLANK TO TERMINATE EXECUTION
C
C
      DIMENSION UX(50),VX(50),ICONV(50),ITER(50),DERIV(10,3),D3(50)
   25 CONTINUE
      WRITE(6,20)
      READ(5,5) A1,PHI1,XF,YF,XB,YB,EPS4,IPRINT,START,ENDD
```

```
    5 FORMAT(6F10.0,F5.0,I1,2F7.0)
      IF(A1.EQ.0.) GO TO 15
      IF(ENDD.EQ.0.) ENDD=359.
      IF(EPS4.EQ.0.) EPS4=1.E-7
      CALL TRIAL(A1,PHI1,XB,YB,XF,YF,UX,VX,IPRINT,ICHK)
      IF(ICHK.EQ.1) GO TO 25
      CALL NEWRAP(UX,VX,ICONV,EPS4,ITER,PHI1,A1,XF,YF,XB,YB,DERIV,
     1BLEN,IPRINT)
      CALL SOL(UX,VX,NN,IPRINT,BLEN,ICONV)
      CALL RANK(NN,UX,VX,DERIV,D3,IPRINT)
      DO 10 J=1,NN
      X1=DERIV(1,1)+DERIV(1,2)*UX(J)+DERIV(1,3)*VX(J)
      X2=DERIV(2,1)+DERIV(2,2)*UX(J)+DERIV(2,3)*VX(J)
      Y1=DERIV(5,1)+DERIV(5,2)*UX(J)+DERIV(5,3)*VX(J)
      Y2=DERIV(6,1)+DERIV(6,2)*UX(J)+DERIV(6,3)*VX(J)
      CALL STRLIN(UX(J),VX(J),A1,PHI1,XF,YF,XB,YB,RHO,XC,YC,X,Y,X1,X2,Y1
     1,Y2)
      IF(RHO.LT.1.E-2) GO TO 10
      IPLOTR=0
      IF(IPRINT.GE.1) IPLOTR=1
      CALL ANALZE(PHI1,START,ENDD,A1,XF,YF,XB,YB,X,Y,XC,YC,RHO,IPLOTR,
     1ICHK,U,V)
   10 CONTINUE
      GO TO 25
   15 WRITE(6,20)
   20 FORMAT(1H1)
      STOP
      END
```

```
      SUBROUTINE RANK(NNN,UX,VX,DERIV,D3,IPRINT)
C
C
C     SUBROUTINE RANK
C
C     GIVEN THE UNIQUE NEWTON-RHAPSON SOLUTIONS IN THE UPPER NNN
C     SPACES OF THE ARRAYS UX AND VX, THIS ROUTINE WILL COMPUTE,
C     FOR EACH SOLUTION, THE ABSOLUTE VALUE OF THE THIRD DERIVATIVE
C     OF THE RADIUS OF CURVATURE OF THE COUPLER POINT PATH WITH
C     RESPECT TO THE CRANK ANGLE.  THE VALUES OF THE DERIVATIVE ARE
C     STORED IN ARRAY D3.  THEN, THE SOLUTIONS ARE REARRANGED IN
C     ORDER OF INCREASING VALUE OF D3 IN THE UPPER NNN SPACES OF
C     UX AND VX.
C
C        INPUT ARGUMENTS
C         UX,VX = UNIQUE SOLUTIONS
C         NNN   = NUMBER OF SOLUTIONS
C         DERIV = ARRAY OF COEFFICIENTS OF DERIVATIVES OF X AND Y
C         IPRINT = IF NOT EQUAL ZERO, INPUT AND OUTPUT ARRAYS PRINTED
C
C        OUTPUT ARGUMENTS
C         UX,VX = REARRANGED UNIQUE SOLUTIONS
C         D3    = ARRAY OF ABSOLUTE VALUES OF THE THIRD DERIVATIVE
C                    OF THE RADIUS OF CURVATURE OF THE COUPLER CURVE
C
      DIMENSION UX(50),VX(50),D3(50),DERIV(10,3)
      REAL J
      IF(IPRINT.NE.0) WRITE(6,50) (UX(K),VX(K),K=1,NNN)
   50 FORMAT(/////,10X,'SUBROUTINE RANK*****',//,10X,'INPUT SOLUTIONS',/
     1/,22X,'U',18X,'V',//,50(10X,2E20.7,/),///)
      DO 10 I=1,50
   10 D3(I)=0.
```

```
      DO 20 I=1,NNN
      X1=DERIV(1,1)+DERIV(1,2)*UX(I)+DERIV(1,3)*VX(I)
      X2=DERIV(2,1)+DERIV(2,2)*UX(I)+DERIV(2,3)*VX(I)
      X3=DERIV(3,1)+DERIV(3,2)*UX(I)+DERIV(3,3)*VX(I)
      X4=DERIV(4,1)+DERIV(4,2)*UX(I)+DERIV(4,3)*VX(I)
      X5=DERIV(9,1)+DERIV(9,2)*UX(I)+DERIV(9,3)*VX(I)
      Y1=DERIV(5,1)+DERIV(5,2)*UX(I)+DERIV(5,3)*VX(I)
      Y2=DERIV(6,1)+DERIV(6,2)*UX(I)+DERIV(6,3)*VX(I)
      Y3=DERIV(7,1)+DERIV(7,2)*UX(I)+DERIV(7,3)*VX(I)
      Y4=DERIV(8,1)+DERIV(8,2)*UX(I)+DERIV(8,3)*VX(I)
      Y5=DERIV(10,1)+DERIV(10,2)*UX(I)+DERIV(10,3)*VX(I)
      F=-DERIV(2,3)*Y1+DERIV(1,3)*Y2+DERIV(6,3)*X1-DERIV(5,3)*X2
      G=2.*(DERIV(5,3)*Y1+DERIV(1,3)*X1)
      H=2.*(DERIV(5,2)*Y1+DERIV(1,2)*X1)
      J=-DERIV(2,2)*Y1+DERIV(1,2)*Y2+DERIV(6,2)*X1-DERIV(5,2)*X2
      R=X1*X2+Y1*Y2
      S=X1*Y3-Y1*X3
      T=X2**2+Y2**2+X1*X3+Y1*Y3
      U=X2*Y3+X1*Y4-Y2*X3-Y1*X4
      W=Y2*X1-Y1*X2
      Z=X1**2+Y1**2
      IF(IPRINT.GE.5) WRITE(6,70) Z,I
   70 FORMAT(10X,'Z = ',E20.7,10X,'I = ',I5,/)
      TP=2.*(X2*X3+Y2*Y3)+X2*X3+X1*X4+Y4*Y3+Y1*Y4
      ZP=2.*X1*Y1*(Y1*X2+X1*Y2)
      UP=2.*X2*Y4+X1*Y5-2.*Y2*X4-Y1*X5
      D3(I)=ABS((2.*Z*W**3*(6.*R*W**2*T+6.*R**2*W*S-6.*T*S*W*Z-6.*R*U*W*
     1Z-6.*R*S**2*Z-6.*R*S*W*ZP+4.*S*U*Z**2+4.*S**2*Z*ZP+3.*TP*W**2*Z
     2+6.*T*W*S*Z+3.*T*W**2*ZP-UP*W*Z**2-U*S*Z**2-2.*U*W*Z*ZP)-(ZP*
     3W**3+6.*Z*W**2*S)*(3.*R**2*W**2-6.*R*S*W*Z+2.*S**2*Z**2+3.*T*
     4W**2*Z-U*W*Z**2))/(2.*Z**1.5*W**6))
   20 CONTINUE
```

```
      DO 30 I=1,NNN
      DO 30 K=I,NNN
      IF(D3(K).LT.D3(I)) GO TO 40
      GO TO 30
   40 TEMP=D3(I)
      D3(I)=D3(K)
      D3(K)=TEMP
      TEMP=UX(I)
      UX(I)=UX(K)
      UX(K)=TEMP
      TEMP=VX(I)
      VX(I)=VX(K)
      VX(K)=TEMP
   30 CONTINUE
      IF(IPRINT.NE.0) WRITE(6,60) (UX(K),VX(K),D3(K),K=1,NNN)
   60 FORMAT(///,10X,'OUTPUT SOLUTIONS',//,22X,'U',18X,'V',18X,'D3',//,
     150(10X,3E20.7,/),///)
      RETURN
      END
```

```
      SUBROUTINE STRLIN(U,V,A,PHI,XF,YF,XB,YB,R,XC,YC,X,Y,X1,X2,Y1,Y2)
C
C     SUBROUTINE STRLIN
C
C     SUBROUTINE STRLIN, GIVEN THE ARGUMENTS BELOW, WILL LOCATE THE
C     CENTER OF CURVATURE OF THE COUPLER POINT CURVE AND WILL DETER-
C     MINE THE RADIUS OF CURVATURE OF THE CURVE
C
C        INPUT ARGUMENTS
C         U,V = COORDINATES OF THE COUPLER POINT IN THE MOVING PLANE
C         A   = DRIVING CRANK RADIUS
C         PHI = DRIVING CRANK ANGLE
C         XF,YF = COORDINATES OF THE ROD END OF THE FOLLOWER CRANK
C         XB,YB = COORDINATES OF THE FOLLOWER CRANK CENTER
C         X1,X2 = FIRST AND SECOND DERIVATIVES OF X WITH RESPECT TO PHI
C         Y1,Y2 = FIRST AND SECOND DERIVATIVES OF Y WITH RESPECT TO PHI
C
C        OUTPUT ARGUMENTS
C         R = RADIUS OF CURVATURE OF THE COUPLER CURVE
C         XC,YC = CENTER OF CURVATURE OF THE COUPLER CURVE
C         X,Y = COORDINATES OF THE COUPLER POINT IN THE FIXED PLANE
C
      ETA= ATAN2((YF-A* SIN(PHI)),(XF-A* COS(PHI)))
      Y=A* SIN(PHI)+U* SIN(ETA)+V* COS(ETA)
      X=A* COS(PHI)+U* COS(ETA)-V* SIN(ETA)
      TEMP=X1*Y2-Y1*X2
      R=ABS((X1**2+Y1**2)**1.5/TEMP)
      YP=Y1/X1
      YPP=TEMP/X1**3
      ALPHA=ATAN2(YB,XB)
      YC=Y+(-(YP+YP**3)* SIN(ALPHA)+(1.E0+YP**2)* COS(ALPHA))/YPP
      XC=X+(-(YP+YP**3)* COS(ALPHA)-(1.E0+YP**2)* SIN(ALPHA))/YPP
```

```
      WRITE(6,10) U,V,A,PHI,XB,YB,XF,YF,ETA,X,Y,R,XC,YC,X1,X2,Y1,Y2,YP,
     1YPP
   10 FORMAT(/////,10X,'SUBROUTINE STRLIN*****',//,10X,'U = ',E15.8,/,
     110X,'V = ',E15.8,/,10X,'CRANK RADIUS = ',E15.8,/,10X,'CRANK ANGLE
     2= ',E15.8,/,10X,'FOLLOWER CRANK CENTER AT ',2E15.8,/,10X,'FOLLOWER
     3 CRANK END AT ',2E15.8,/,10X,'COUPLER ANGLE = ',E15.8,/,10X,'COUPL
     4ER POINT AT ',2E15.8,/,10X,'RADIUS OF CURVATURE = ',E15.8,/,10X,'C
     5ENTER OF CURVATURE AT ',2E15.8,/,10X,'X1 = ',E15.8,/,10X,'X2 = ',E
     615.8,/,10X,'Y1 = ',E15.8,/,10X,'Y2 = ',E15.8,/,10X,'YP = ',E15.8,
     7/,10X,'YPP = ',E15.8,//////)
      RETURN
      END
```

```
      SUBROUTINE SOL(X,Y,NN,IPRINT,BLEN,ICONV)
C
C     SUBROUTINE SOL
C
C
C     GIVEN ALL NEWTON-RHAPSON SOLUTIONS IN ARRAYS X AND Y (OF DIMENSION
C     50 AND DETERMINED WITHIN EPS), SOL WILL :
C
C           1) EXCLUDE ALL SOLUTIONS THAT DUPLICATE EACH OTHER WITHIN
C              0.5 PER CENT
C
C           2) EXCLUDE SOLUTIONS FOR WHICH LACK OF CONVERGENCE IS
C              INDICATED
C
C           3) EXCLUDE SOLUTIONS AT EITHER END OF THE COUPLER
C
C           4) ARRANGE UNIQUE SOLUTIONS WITHIN THE UPPER NN SPACES OF
C              X AND Y
C
C        INPUT ARGUMENTS
C         X,Y = ARRAYS OF NEWTON-RHAPSON SOLUTIONS
C         BLEN = LENGTH OF COUPLER
C         IPRINT = IF NOT EQUAL TO ZERO, INPUT AND OUTPUT ARRAYS
C                  PRINTED
C         ICONV = ARRAY INDICATING CONVERGENCE,I.E. ICONV=0
C
C        OUTPUT ARGUMENTS
C         NN= NUMBER OF UNIQUE SOLUTIONS
C         X,Y = ARRAYS OF NN UNIQUE SOLUTIONS
C
C
      DIMENSION X(50),Y(50),ICONV(50)
```

```
      NN=1
      BMX=1.01*BLEN
      BMN=.99*BLEN
      EPSN=.005E0
      IF(IPRINT.NE.0) WRITE(6,70)   EPSN,(X(I),Y(I),I=1,50)
   70 FORMAT(////,10X,'SUBROUTINE SOL*****',//,10X,'SOL EPS = ',E20.6,//
     1,10X,'ORIGINAL SOLUTIONS',//,25X,'X',19X,'Y',//,100(10X,2E20.8,/))
C
      DO 10 I=1,50
      IF(ICONV(I).EQ.0) GO TO 15
      X(I)=0.
      Y(I)=0.
   15 CONTINUE
      IF(X(I).LT.1.E-4.AND.Y(I).LT.1.E-4) GO TO 16
      IF(X(I).LT.BMX.AND.X(I).GT.BMN.AND. ABS(Y(I)).LT.1.E-5) GO TO 16
      GO TO 17
   16 X(I)=0.
      Y(I)=0.
   17 CONTINUE
      IF(X(I).EQ.0..AND.Y(I).EQ.0.) GO TO 10
      DO 20 J=1,50
      IF(I.EQ.J) GO TO 20
      EPSX= ABS((X(I)-X(J))/(( ABS(X(I))+ ABS(X(J)))/2.))
      EPSY= ABS((Y(I)-Y(J))/(( ABS(Y(I))+ ABS(Y(J)))/2.))
      IF(EPSX.LE.EPSN.AND.EPSY.LE.EPSN) GO TO 30
      GO TO 20
   30 X(J)=0.
      Y(J)=0.
   20 CONTINUE
   10 CONTINUE
      IF(IPRINT.NE.0) WRITE(6,80) (X(I),Y(I),I=1,50)
   80 FORMAT(////,10X,'UNIQUE SOLUTIONS',///,25X,'X',19X,'Y',///,100
```

```
  1(10X,2E20.8,/))
   DO 40 I=1,50
   IF(X(I).NE.0..AND.Y(I).NE.0.) GO TO 40
   J=I+1
   IF(J.EQ.51) GO TO 40
   DO 50 K=J,50
   IF(X(K).NE.0..AND.Y(K).NE.0.) GO TO 60
   GO TO 50
60 X(I)=X(K)
   Y(I)=Y(K)
   X(K)=0.
   Y(K)=0.
   NN=I
   GO TO 40
50 CONTINUE
40 CONTINUE
   IF(IPRINT.NE.0) WRITE(6,90) NN,(X(I),Y(I),I=1,NN)
90 FORMAT(////,10X,'RETURNED SOLUTIONS = ',I10 ,///,15X,'X',19X,'Y',/
  1/,100(10X,2E20.8,/))
   RETURN
   END
```

```
      SUBROUTINE TRIAL(A1,PHI,XB,YB,XF,YF,U,V,IPRINT,ICHK)
C
C     FOUR-BAR VERSION
C
C
C     SUBROUTINE TRIAL, GIVEN THE ARGUMENTS BELOW, WILL GENERATE 50
C     TRIAL SOLUTIONS FOR A NEWTON-RHAPSON ANALYSIS.  FORTY OF THESE
C     POINTS ARE EVENLY DISTRIBUTED, ANGULARLY, AROUND THE CUBIIC-OF-
C     STATIONARY CURVATURE, TEN ARE DISTRIBUTED ALONG THE CUBIC'S
C     ASYMPTOTE.  AN EULER-SAVARY ANALYSIS IS PERFORMED TO LOCATE
C     THE INFLECTION CIRCLE AND, IN TURN TO LOCATE THE COMMON
C     CENTRODE TANGENT, OR THE INSTANT CENTER VELOCITY DIRECTION.
C     THEN, USING A COORDINATE SYSTEM ALIGNED WITH THE TANGENT, WITH
C     THE ORIGIN AT THE INSTANT CENTER, M AND N ARE DETERMINED FOR
C     THE CUBIC.  USING POLAR NOTATION, R AND PSI ARE DETERMINED,
C     YIELDING X AND Y IN THE ORIGINAL COORDINATE SYSTEM.  FINALLY
C     THE X'S AND Y'S ARE TRANSFORMED INTO U AND V.
C
C            INPUT ARGUMENTS
C             A1  = CRANK RADIUS
C             PHI = CRANK ANGLE, RADIANS
C             XB,YB = CENTER OF FOLLOWER CRANK
C             XF,YF = ROD END OF FOLLOWER CRANK
C             IPRINT = IF NOT EQUAL ZERO, INTERNAL VARIABLES PRINTED
C
C            OUTPUT
C
C             U,V = ARRAYS OF TRIAL SOLUTIONS, DIMENSIONED 50
C             ICHK = ALARM IF NOT EQUAL ZERO, INFLECTION CIRCLE NOT FIXED
C
      DIMENSION A(4),B(2),U(50),V(50)
      REAL IX,IY,IA,IB,JAA,JAAP,JAX,JAY,JBB,JBBP,JBX,JBY
```

```
      PI=3.141593                                                       TRL
      HALFPI=1.570796                                                   TRL
      IF(IPRINT.NE.0) WRITE(6,50)                                       TRL
   50 FORMAT(////,10X,'SUBROUTINE TRIAL*****',//,10X,'POINT O = DRIVING TRL
     1CRANK CENTER',/,10X,'POINT A = DRIVING CRANK END',/,10X,'POINT B =TRL
     2 FOLLOWER CRANK END',/,10X,'POINT I = INSTANT CENTER',/,10X,'POINTTRL
     3 T = POINT ON TANGENT',/,10X,'THETA = ANGLE X-AXIS AND I-T',/,10X,TRL
     4'ALPHA = ANGLE I-T AND I-A',/,10X,'BETA = ANGLE I-T AND I-B',/,10XTRL
     5,'PSI ASM = ANGLE I-T AND ASYMPOOTE',/,10X,'J - POINTS ON THE INFLTRL
     6ECTION CIRCLE',/,10X,'POINT OB = FOLLOWER CRANK CENTER',///)      TRL
      DO 5 I=1,50                                                       TRL
      U(I)=0.                                                           TRL
    5 V(I)=0.                                                           TRL
      AX=A1*COS(PHI)                                                    TRL
      AY=A1*SIN(PHI)                                                    TRL
      IY=(YB*XF-XB*YF)/(XF-XB-(YF-YB)/TAN(PHI))                         TRL
      IX=IY/TAN(PHI)                                                    TRL
      IB=SQRT((IY-YF)**2+(IX-XF)**2)                                    TRL
      IA=SQRT((IY-AY)**2+(IX-AX)**2)                                    TRL
      IF(IPRINT.NE.0) WRITE(6,60) A1,PHI,XB,YB,XF,YF,IX,IY,IA,IB        TRL
   60 FORMAT(//,10X,'A1 = ',E20.8,/,10X,'PHI = ',E20.8,/,10X,'XB,YB = ',TRL
     22E20.8,/,10X,'XF,YF = ',2E20.8,/,10X,'I AT ',2E20.8,/,10X,'I-A = 'TRL
     3,E20.8,/,10X,'I-B = ',E20.8,///)                                  TRL
C                                                                       TRL
C     EULER-SAVARY ANALYSIS                                             TRL
C                                                                       TRL
      JAA=IA**2/A1                                                      TRL
      JAAP=A1-JAA                                                       TRL
      JAX=JAAP*COS(PHI)                                                 TRL
      JAY=JAAP*SIN(PHI)                                                 TRL
      B1=SQRT((XB-XF)**2+(YB-YF)**2)                                    TRL
      JBB=IB**2/B1                                                      TRL
```

```
      JBBP=81-JBB
      SLP4=ATAN2((YF-YB),(XF-XB))
      JBX=JBBP*COS(SLP4)+XB
      JBY=JBBP*SIN(SLP4)+YB
      CALL CIRCLE(XH,XK,R,IX,IY,JAX,JAY,JBX,JBY,ICHK)
      IF(ICHK.EQ.1) GO TO 40
      IF(IPRINT.NE.0) WRITE(6,70) JAX,JAY,JBX,JBY,ICHK,XH,XK,R
   70 FORMAT(///,10X,'POINTS ON THE INFLECTION CIRCLE',/,10X,2E20.8,/,
     110X,2E20.8,///,10X,'ICHK = ',I10,/,10X,'INFLECTION CIRCLE AT ',2E2
     20.8,/,10X,'INFLECTION CIRCLE RADIUS = ',E20.8,//)
C
C     TRANSFORM TO RECTANGULAR COORDINATES AT INSTANT CENTER
C
      CXX=XH-IX
      CYY=XK-IY
      TXX=-CYY
      TYY=CXX
      AXX=AX-IX
      AYY=AY-IY
      BXX=XF-IX
      BYY=YF-IY
      THETA=ATAN2(TYY,TXX)
      ALPHA=ATAN2(AYY,AXX)-THETA
      BETA=ATAN2(BYY,BXX)-THETA
C
C     SOLVE FOR CONSTANTS IN CUBIC, SEE HARTENBERG & DENAVIT,P.209
C
      A(1)=1./SIN(ALPHA)
      A(2)=1./SIN(BETA)
      A(3)=1./COS(ALPHA)
      A(4)=1./COS(BETA)
      B(1)=1./SQRT(AXX**2+AYY**2)
```

```
      B(2)=1./SQRT(BXX**2+BYY**2)
      CALL SIMQ(A,B,2,KS)
      XM=1./B(1)
      XN=1./B(2)
C
C     DETERMINE TRIAL SOLUTIONS
C
      DO 10 I=1,40
      PSI=(I-1)*.07853982
      R=(XN*XM*SIN(2.*PSI))/(2.*(XN*COS(PSI)+XM*SIN(PSI)))
      U(I)=R*COS(PSI)*COS(THETA)-R*SIN(PSI)*SIN(THETA)+IX
   10 V(I)=R*COS(PSI)*SIN(THETA)+R*SIN(PSI)*COS(THETA)+IY
      PSIASM=ATAN(-XN/XM)
      XAVG=(IA+IB)/2.
      IF(IPRINT.NE.0) WRITE(6,80) THETA,ALPHA,BETA,XM,XN,PSIASM,XAVG
   80 FORMAT(///,10X,'THETA = ',E20.8,/,10X,'ALPHA = ',E20.8,/,10X,
     1'BETA = ',E20.8,/,10X,'M = ',E20.8,/,10X,'N = ',E20.8,/,10X,
     2'PSI ASM = ',E20.8,/,10X,'XAVG = ',E20.8,//)
      U(41)=2.*XAVG*COS(PSIASM)*COS(THETA)-2.*SIN(PSIASM)*XAVG*SIN(THETA
     1)+IX
      V(41)=2.*XAVG*COS(PSIASM)*SIN(THETA)+2.*XAVG*SIN(PSIASM)*COS(THETA
     1)+IY
      DO 20 I=42,50
      XX=(I-42)*XAVG/3.333-XAVG
      U(I)=XX*COS(PSIASM)*COS(THETA)-XX*SIN(PSIASM)*SIN(THETA)+IX
   20 V(I)=XX*COS(PSIASM)*SIN(THETA)+XX*SIN(PSIASM)*COS(THETA)+IY
      IF(IPRINT.NE.0) WRITE(6,90) (U(I),V(I),I=1,50)
   90 FORMAT(///,10X,'ORIGINAL TRIAL SOLUTIONS',//,25X,'X',19X,'Y',///,5
     10(10X,2F20.8,/),///)
C
C     TRANSFORM INTO U AND V
C
```

```
      ETA=-ATAN2((YF-AY),(XF-AX))                                       TRL 1290
      DO 30 I=1,50                                                      TRL 1300
      XT=U(I)-AX                                                        TRL 1310
      YT=V(I)-AY                                                        TRL 1320
      U(I)= XT*COS(ETA)-YT*SIN(ETA)                                     TRL 1330
   30 V(I)= XT*SIN(ETA)+YT*COS(ETA)                                     TRL 1340
      IF(IPRINT.NE.0) WRITE(6,100) (U(I),V(I),I=1,50)                   TRL 1350
  100 FORMAT(///,10X,'TRANSFORMED TRIAL SOLUTIONS',//,25X,'U',19X,'V',  TRL 1360
     2//,50(10X,2F20.8,/),///)                                          TRL 1370
   40 RETURN                                                            TRL 1380
      END                                                               TRL 1390
```

```
      SUBROUTINE CIRCLE(XH,XK,R,X1,Y1,X2,Y2,X3,Y3,ICIRCL)
C
C
C     SUBROUTINE CIRCLE
C
C
C
C        SUBROUTINE CIRCLE DETERMINES THE CENTER AND RADIUS OF THE
C     CIRCLE PASSING THROUGH THREE DESCRIBED POINTS
C
C
C        ARGUMENTS
C
C
C     SUPPLIED BY THE CALLING ROUTINE
C
C     X1,Y1,X2,Y2,X3,Y3 = X,Y COORDINATES OF THREE POINTS THROUGH WHICH
C                         THE CIRCLE MUST PASS
C
C     SUPPLIED BY CIRCLE
C
C     R = RADIUS OF CIRCLE
C     XH,XK = X,Y COORDINATES OF THE CENTER OF THE CIRCLE
C
C
C
      DIMENSION A(9),B(3)
      ICIRCL=0
      EPS=.0001
      XZ1=ABS(X1-X2)
      XZ2=ABS(X2-X3)
      XZ3=ABS(X3-X1)
```

```
      YZ1=ABS(Y1-Y2)
      YZ2=ABS(Y2-Y3)
      YZ3=ABS(Y3-Y1)
      IF(XZ1.LT.EPS.AND.YZ1.LT.EPS) GO TO 10
      IF(XZ2.LT.EPS.AND.YZ2.LT.EPS) GO TO 10
      IF(XZ3.LT.EPS.AND.YZ3.LT.EPS) GO TO 10
      A(1)=-2.*X1
      A(2)=-2.*X2
      A(3)=-2.*X3
      A(4)=-2.*Y1
      A(5)=-2.*Y2
      A(6)=-2.*Y3
      A(7)=1.
      A(8)=1.
      A(9)=1.
      B(1)=-(X1**2+Y1**2)
      B(2)=-(X2**2+Y2**2)
      B(3)=-(X3**2+Y3**2)
      K=3
      L=9
      M=0
      CALL SIMQ(A,B,K,M)
      IF(M.EQ.1) GO TO 10
      XH=B(1)
      XK=B(2)
      R=SQRT(XH**2+XK**2-B(3))
      RETURN
   10 ICIRCL=1
      WRITE(6,20)  X1,Y1,X2,Y2,X3,Y3
   20 FORMAT(/,47X,'UNABLE TO RESOLVE INFLECTION CIRCLE.',/,51X,
     1'CHECK FOR COINCIDENT POINTS.',//,40X,'P1 = (',2E20.6,' )',//,
     240X,'P2 = (',2E20.6,' )',//40X,'P3 = (',2E20.6,' )')
```

```
      RETURN                                                            CIR  650
      END                                                               CIR  660
```

```
      SUBROUTINE NEWRAP(UX,VX,ICONV,EPS,ITER,PHI,A,XF,YF,XB,YB,DERIV,
     1BLEN,IPRINT)
C
C
C     SUBROUTINE NEWRAP
C
C     SUBROUTINE NEWRAP, GIVEN A DESCRIPTION OF THE COUPLER CON-
C     STRAINTS, WILL FORM THE RELATIONSHIPS NECESSARY TO LOCATE
C     SIMULTANEOUS ZEROES OF THE FIRST AND SECOND DERIVATIVES OF THE
C     RADIUS OF CURVATURE OF A COUPLER POINT PATH.  THEN, GIVEN TRIAL
C     SOLUTIONS, THE ROUTINE WILL EXECUTE A NEWTON-RHAPSON ITERATION
C     PROCEDURE UNTIL THE SOLUTION CONVERGES TO WITHIN SOME EPSILON.
C     CONVERGENCE IS PRESUMED TO HAVE FAILED IF
C
C        1) EITHER U OR V EXCEED 200  (ICONV=2)  OR
C        2) THE NUMBER OF ITERATIONS EQUAL OR EXCEED 110. (ICONV=1)
C
C     IN THE CASE OF (2) ABOVE, THE FINAL 10 VALUES OF U AND V ARE
C     AVERAGED AND REPORTED AS U AND V.
C
C        INPUT ARGUMENTS
C         UX,UX = ARRAYS OF TRIAL SOLUTIONS. DIMENSIONED 50
C         EPS = RELATIVE CONVERGENCE CRITEREA
C         PHI = DRIVING CRANK ANGLE
C         A   = DRIVING CRANK RADIUS
C         XF,YF = COORDINATES OF THE ROD END OF THE FOLLOWER CRANK
C         XB,YB = COORDINATES OF THE FOLLOWER CRANK CENTER
C         IPRINT = PRINT COMMAND
C               = 0, NO PRINTED OUTPUT
C               > 5, RESULTS OF EACH ITERATION STEP PRINTED PLUS
C               > 3, RESULTS OF ITERATIONS ON EACH TRIAL SOLUTION
C                    PRINTED PLUS
```

```fortran
C               NE O, INPUT TRIAL SOLUTIONS AND OUTPUT SOLUTIONS
C                     PRINTED
C        OUTPUT ARGUMENTS
C         UX,VX = ARRAYS OF ITERATED SOLUTIONS
C         ICONV = ARRAY INDICATING CONVERGENCE OR LACK OF IT
C               = 0, CONVERGENCE
C               = 1, ITERATIONS EXCEEDED 110
C               = 2, SOLUTIONS EXCEEDED 200 IN VALUE
C         ITER = ARRAY INDICATING NUMBER OF ITERATIONS FOR EACH
C                 SOLUTION
C         DERIV = ARRAY CONTAINING COEFFICIENTS OF DERIVATIVES OF X
C                 AND Y WITH RESPECT TO PHI, THE DRIVING CRANK ANGLE
C         BLEN  = LENGTH OF COUPLER
C
      REAL L,K,J,N
      DIMENSION ICONV(50),ITER(50),UX(50),VX(50),DERIV(10,3)
      B=SQRT((YF-A*SIN(PHI))**2+(XF-A*COS(PHI))**2)
      C=SQRT((YF-YB)**2+(XF-XB)**2)
      D=SQRT(XB**2+YB**2)
      BLEN=B
      CORR=ATAN2(YB,XB)
      ETA=ATAN2((YF-A*SIN(PHI)),(XF-A*COS(PHI)))-CORR
      ETA=ARSIN(SIN(ETA))
      P=D-A*COS(PHI-CORR)
      Q=A*SIN(PHI-CORR)
      R=(P**2+Q**2+B**2-C**2)/(2.*B)
      SS=R*Q/(P**2+Q**2)
      T=(R**2-P**2)/(P**2+Q**2)
      EPSM=ABS(ETA-ARSIN(-SS-SQRT(ABS(SS**2-T))))
      EPSP=ABS(ETA-ARSIN(-SS+SQRT(ABS(SS**2-T))))
      S=-1.E0
      IF(EPSP.LE.EPSM) S=1.E0
```

```
      L=2.E0*A*D
      K=A**2+D**2
      H=(A*B**2-C**2*A+A*D**2+A**3)/(2.E0*B)
      J=A**2*D/(2.E0*B)
      AA=A**2+D**2-.5E0*A**2*D**2/B**2+.5E0*C**2*D**2/B**2-.25E0*D**4/
     1B**2+.5E0*C**2*A**2/B**2-.25E0*A**4/B**2-.25E0*C**4/B**2-.5E0*
     2D**2-.5E0*A**2-.25E0*B**2+.5E0*C**2
      BB=-1.E0*A*D-C**2*A*D/B**2+A**3*D/B**2+A*D**3/B**2
      N=A**2*D**2/B**2
      CP=COS(PHI-CORR)
      SP=SIN(PHI-CORR)
      C2P=COS(2.*(PHI-CORR))
      S2P=SIN(2.*(PHI-CORR))
      R2=D-A*CP
      TEMP=AA+BB*CP-N*CP**2
      R5= SQRT(AA+BB*CP-N*CP**2)
      R7=K-L*CP
      R3=J*S2P+R2*R5*S-H*SP
      R4=N*S2P-BB*SP
      R6=-H*CP+2.E0*J*C2P+.5E0*R4*R2*S/R5+A*SP*R5*S
      R8= SQRT(1.E0-R3**2/R7**2)
      R9=-BB*CP+2.E0*CP**2*N-2.E0*N*SP**2
      R10=-L*SP*R3/R7**2+R6/R7
      R11=-4.E0*N*S2P+BB*SP
      R12=2.E0*R6*R3/R7**2-2.E0*L*SP*R3**2/R7**3
      R14=-4.E0*J*S2P+A*SP*S*R4/R5+A*CP*S*R5+H*SP-.25E0*R2*S*R4**2/R5**3
     1+.5E0*R9*R2*S/R5
      R15=R14/R7+2.E0*L**2*SP**2*R3/R7**3-L*CP*R3/R7**2-2.E0*L*SP*R6/R7
     1**2
      R16=2.E0*R3*R14/R7**2-8.E0*L*SP*R6*R3/R7**3+6.E0*L**2*SP**2*R3**2/
     1R7**4-2.E0*L*CP*R3**2/R7**3+2.E0*R6**2/R7**2
      R17=-8.E0*J*C2P-.75E0*R2*S*R9*R4/R5**3+1.5E0*A*CP*S*R4/R5-A*SP*S*
```

```
     1R5+H*CP-.75E0*A*SP*S*R4**2/R5**3+.375E0*R2*S*R4**3/R5**5+1.5E0*A* NWR
     2SP*S*R9/R5+.5E0*R2*S*R11/R5                                       NWR
      R18=-3.E0*L*SP*R14/R7**2+R17/R7+3.E0*L**2*R3*S2P/R7**3-6.E0*L**3* NWR
     1SP**3*R3/R7**4+L*SP*R3/R7**2+6.E0*L**2*SP**2*R6/R7**3-3.E0*L*CP*  NWR
     2R6/R7**2                                                          NWR
      R19=-.5E0*R10*R3*R12/(R7*R8**3)-R3*R15/(R7*R8)+L*SP*R10*R3/(R7**2 NWR
     1*R8)-R10*R6/(R7*R8)                                               NWR
      R20=-8.E0*N*SP*CP+BB*SP                                           NWR
      R23=BB*CP-8.E0*N*CP**2+8.E0*N*SP**2                               NWR
      R21=1.5E0*A*S*R4*CP/R5+H*CP-8.E0*J*C2P-.75E0*R2*S*R9*R4/R5**3-A*  NWR
     1SP*S*R5-.75E0*A*SP*S*R4**2/R5**3+.375E0*R2*S*R4**3/R5**5+1.5E0*A* NWR
     2SP*S*R9/R5+.5E0*R20*R2*S/R5                                       NWR
      R22=-A*S*R5*CP-1.5E0*A*S*R4**2*CP/R5**3+3.E0*A*S*R9*CP/R5+16.E0*J NWR
     1*S2P-3.E0*A*SP*S*R9*R4/R5**3-R20*R2*S*R4/R5**3-2.E0*A*SP*S*R4/R5  NWR
     2-H*SP+2.25E0*R2*S*R9*R4**2/R5**5+1.5E0*A*SP*S*R4**3/R5**5-.9375E  NWR
     30*R2*S*R4**4/R5**7-.75E0*R2*S*R9**2/R5**3+.5E0*R2*S*R23/R5+2.E0   NWR
     4*R20*A*SP*S/R5                                                    NWR
      R24=2.E0*R3*R21/R7**2-12.E0*L*SP*R3*R14/R7**3+6.E0*R6*R14/R7**2-  NWR
     112.E0*L*R6*R3*CP/R7**3+18.E0*L**2*SP*R3**2*CP/R7**4+36.E0*L**2*   NWR
     2SP**2*R6*R3/R7**4-24.E0*L**3*SP**3*R3**2/R7**5+2.E0*L*SP*R3**2/   NWR
     3R7**3-12.E0*L*SP*R6**2/R7**3                                      NWR
      R25=R21/R7-3.E0*L*SP*R14/R7**2+6.E0*L**2*SP*R3*CP/R7**3-3.E0*L*   NWR
     1R6*CP/R7**2-6.E0*L**3*SP**3*R3/R7**4+L*SP*R3/R7**2+6.E0*L**2*SP   NWR
     2**2*R6/R7**3                                                      NWR
      R26=-4.E0*L*SP*R21/R7**2+R22/R7-6.E0*L*CP*R14/R7**2+12.E0*L**2*   NWR
     1SP**2*R14/R7**3-36.E0*L**3*SP**2*R3*CP/R7**4+L*R3*CP/R7**2+24.E0  NWR
     2*L**2*SP*R6*CP/R7**3+24.E0*L**4*SP**4*R3/R7**5-8.E0*L**2*SP**2*   NWR
     3R3/R7**3-24.E0*L**3*SP**3*R6/R7**4+4.E0*L*SP*R6/R7**2+6.E0*L**2   NWR
     4*R3*CP**2/R7**3                                                   NWR
      R27 =N*SP*CP*32.-BB*SP                                            NWR
      R28 =(-L*SP*R14*R7**3*R6*48.-L*SP*R7**3*R21*R3*16.+L*SP*R7**3*R3*RNWR
     16*16.-L*R14*R7**3*CP*R3*24.+L*R7**3*CP*R3**2*2.-L*R7**3*CP*R6**2*2NWR
```

```
24.+L**2*SP*R7**2*CP*R3*R6*144.+L**2*SP**2*R14*R7**2*R3*72.-L**2*SP
3**2*R7**2*R3**2*24.+L**2*SP**2*R7**2*R6**2*72.+L**2*R7**2*CP**2*R3
4**2*18.-L**3*SP**2*R7*CP*R3**2*144.-L**3*SP**3*R7*R3*R6*192.+L**4*
5SP**4*R3**2*120.+R14**2*R7**4*6.+R7**4*R21*R6*8.+R7**4*R22*R3*2.)/
6R7**6
 R29 =(-R20*A*S*R5**6*R4*SP*5.+R20*A*S*R5**8*CP*5.+R20*S*R5**4*R4**
12*R2*3.75 -R20*S*R5**6*R2*R9*2.5 +R23*A*S*R5**8*SP*2.5 -R23*S*R5**
26*R4*R2*1.25 +R27*S*R5**8*R2*.50 -A*S*R5**2*R4**4*SP*4.6875 +A*S*R
35**4*R4**2*R9*SP*11.25 +A*S*R5**4*R4**3*CP*3.75 )/R5**9
 R29 =R29 +(-A*S*R5**6*R4*R9*CP*7.5 +A*S*R5**6*R4**2*SP*2.5 -A*S*R5
1**6*R9**2*SP*3.75 -A*S*R5**8*R4*CP*2.5 -A*S*R5**8*R9*SP*5.+A*S*R5*
2*10*SP-S*R5**2*R4**3*R2*R9*9.375 +S*R5**4*R4*R2*R9**2*5.625 +S*R4*
3*5*R2*3.28125 -R5**9*H*CP+R5**9*J*C2P*32.)/R5**9
 R30 =(R29*R7**5+L*SP*R14*R7**4*10.-L*SP*R7**4*R22*5.-L*SP*R7**4*R3
1 -L*R7**4*CP*R21*10.+L*R7**4*CP*R6*5.+L**2*SP*R14*R7**3*CP*60.-L**
22*SP*R7**3*CP*R3*30.+L**2*SP**2*R7**3*R21*20.-L**2*SP**2*R7**3*R6*
340.+L**2*R7**3*CP**2*R6*30.-L**3*SP*R7**2*CP**2*R3*90.-L**3*SP**2*
4R7**2*CP*R6*180.-L**3*SP**3*R14*R7**2*60.+L**3*SP**3*R7**2*R3*60.+
5L**4*SP**3*R7*CP*R3*240.+L**4*SP**4*R7*R6*120.-L**5*SP**5*R3*120.)
6/R7**6
 Y1C=A*CP
 Y1U=R10
 Y1V=-R10*R3/(R7*R8)
 Y2C=-A*SP
 Y2U=R15
 Y2V=R19
 Y3C=-Y1C
 Y3U=R18
 Y3V=-R3*R18/(R7*R8)-R3*R15*R12/(R7*R8**3)+L*SP*R10*R3*R12/(R7**2*
1R8**3)-R10*R6*R12/(R7*R8**3)-R10*R14/(R7*R8)+2.E0*L*SP*R3*R15/(R7
2**2*R8)-2.E0*R6*R15/(R7*R8)-.5E0*R10*R3*R16/(R7*R8**3)-2.E0*L**2
3*SP**2*R10*R3/(R7**3*R8)+L*CP*R10*R3/(R7**2*R8)+2.E0*L*SP*R10*R6
```

```
4/(R7**2*R8)-.75E0*R10*R3*R12**2/(R7*R8**5)                          NWR 1610
 Y4C=-Y2C                                                            NWR 1620
 Y4U=(-4.E0*L*SP*R7**3*R21+R7**4*R22-6.E0*L*R7**3*CP*R14+12.E0*L**  NWR 1630
12*SP**2*R7**2*R14-36.E0*L**3*SP**2*R7*R3*CP+L*R7**3*R3*CP+24.E0*   NWR 1640
2L**2*SP*R7**2*R6*CP-8.E0*L**2*SP**2*R7**2*R3+24.E0*L**4*SP**4*R3   NWR 1650
3-24.E0*L**3*SP**3*R7*R6+4.E0*L*SP*R7**3*R6+6.E0*L**2*R7**2*R3*CP   NWR 1660
4**2)/R7**5                                                          NWR 1670
 Y4V=-R10*R21/(R7*R8)-1.5E0*R14*R12*R10/(R7*R8**3)-2.25E0*R3*R16*   NWR 1680
1R12*R10/(R7*R8**5)+1.5E0*L*R3*CP*R12*R10/(R7**2*R8**3)-3.E0*L**2*  NWR 1690
2SP**2*R3*R12*R10/(R7*R8)**3+3.E0*L*SP*R6*R12*R10/(R7**2*R8**3)+    NWR 1700
33.E0*L*SP*R14*R10/(R7**2*R8)+1.5E0*L*SP*R3*R16*R10/(R7**2*R8**3)   NWR 1710
4-1.5E0*R6*R16*R10/(R7*R8**3)-6.E0*L**2*SP*R3*CP*R10/(R7**3*R8)     NWR 1720
 Y4V=Y4V+3.*L*R6*CP*R10/(R7**2*R8)-.5*R24*R3*R10/(R7*R8**3)+6.*L**3NWR 1730
1*SP**3*R3*R10/(R7**4*R8)-L*SP*R3*R10/(R7**2*R8)-6.E0*L**2*SP**2*   NWR 1740
2R6*R10/(R7**3*R8)+2.25E0*L*SP*R3*R12**2*R10/(R7**2*R8**5)-2.25E0   NWR 1750
3*R6*R12**2*R10/(R7*R8**5)-1.875E0*R3*R12**3*R10/(R7*R8**7)+3.E0*   NWR 1760
4L*SP*R3*R15*R12/(R7**2*R8**3)-3.E0*R6*R15*R12/(R7*R8**3)           NWR 1770
 Y4V=Y4V-1.5E0*R25*R3*R12/(R7*R8**3)-3.E0*R15*R14/(R7*R8)-1.5E0*    NWR 1780
1R3*R16*R15/(R7*R8**3)+3.E0*L*R3*CP*R15/(R7**2*R8)-6.E0*L**2*SP**   NWR 1790
22*R3*R15/(R7**3*R8)+6.E0*L*SP*R6*R15/(R7**2*R8)+3.E0*R25*L*SP*R3   NWR 1800
3/(R7**2*R8)-R26*R3/(R7*R8)-3.E0*R25*R6/(R7*R8)-2.25E0*R3*R15*R12   NWR 1810
4**2/(R7*R8**5)                                                      NWR 1820
 Y5C=Y1C                                                             NWR 1830
 Y5U =(R29*R7**5-CP*R21*R7**4*L*10.+CP*R3*R7*SP**3*L**4*240.-CP*R3*NWR 1840
1R7**3*SP*L**2*30.-CP*R7**2*SP**2*L**3*R6*180.+CP*R7**3*SP*L**2*R14NWR 1850
2*60.+CP*R7**4*L*R6*5.+R21*R7**3*SP**2*L**2*20.-R22*R7**4*SP*L*5.+RNWR 1860
33*R7**2*SP**3*L**3*60.-R3*R7**4*SP*L-R3*SP**5*L**5*120.+R7*SP**4*LNWR 1870
4**4*R6*120.-CP**2*R3*R7**2*SP*L**3*90.+CP**2*R7**3*L**2*R6*30.-R7*NWR 1880
5*2*SP**3*L**3*R14*60.-R7**3*SP**2*L**2*R6*40.+R7**4*SP*L*R14*10.)/NWR 1890
6R7**6                                                               NWR 1900
 Y5V =(-R15*R16*R3*R8**4*R12*R7**4*9.+R15*R16*R3*R8**6*R7**3*SP*L*6NWR 1910
1.-R15*R16*R8**6*R7**4*R6*6.+R15*CP*R3*R8**6*R12*R7**3*L*6.-R15*CP*NWR 1920
```

```
2R3*R8**8*R7**2*SP*L**2*24.+R15*CP*R8**8*R7**3*L*R6*12.-R15*R21*R8*
3*8*R7**4*4.-R15*R24*R3*R8**6*R7**4*2.-R15*R3*R8**2*R12**3*R7**4*7.
45 +R15*R3*R8**4*R12**2*R7**3*SP*L*9.-R15*R3*R8**6*R12*R7**2*SP**2*
5L**2*12.+R15*R3*R8**8*R7*SP**3*L**3*24.-R15*R3*R8**8*R7**3*SP*L*4.
6-R15*R8**4*R12**2*R7**4*R6*9.+R15*R8**6*R12*R7**3*SP*L*R6*12.)/(R8
7**9*R7**5)
 Y5V =Y5V+(-R15*R8**6*R12*R7**4*R14*6.-R15*R8**8*R7**2*SP**2*L**2*R
16*24.+R15*R8**8*R7**3*SP*L*R14*12.+R16*R10*CP*R3*R8**6*R7**3*L*3.-
2R16*R10*R3*R8**2*R12**2*R7**4*11.25+R16*R10*R3*R8**4*R12*R7**3*SP*
3L*9.-R16*R10*R3*R8**6*R7**2*SP**2*L**2*6.-R16*R10*R8**4*R12*R7**4*
4R6*9.+R16*R10*R8**6*R7**3*SP*L*R6*6.-R16*R10*R8**6*R7**4*R14*3.-R1
56*R25*R3*R8**6*R7**4*3.+R10*CP*R8**8*R7**3*L*P14*6.)/(R8**9*R7**5)
 Y5V =Y5V+(-R10*R21*R8**6*R12*R7**4*2.+R10*R21*R8**8*R7**3*SP*L*4.-
1R10*R24*R3*R8**4*R12*R7**4*3.+R10*R24*R3*R8 **6*R7**3*SP*L*2.-R10*
2R24*R8**6*R7**4*R6*2.-R10*R22*R8**8*R7**4+R10*R3*R8**2*R12**3*R7**
33*SP*L*7.5 -R10*R3*R8**4*R12**2*R7**2*SP**2*L**2*9.+R10*R3*R8**4*R
412**2*R7**6-R10*R3*R12**4*R7**4*6.5625 -R10*R8**4*R12**2*R7**4*R14
5*4.5 +R10*R8**6*R12*R7**3*SP*L*R14*6.)/(R8**9*R7**5)
 Y5V =Y5V+(-R10*R8**8*R7**2*SP**2*L**2*R14*12.+R25*CP*R3*R8**8*R7**
13*L*6.-R25*R3*R8**4*R12**2*R7**4*4.5 +R25*R3*R8**6*R12*R7**3*SP*L*
26.-R25*R3*R8**8*R7**2*SP**2*L**2*12.-R25*R8**6*R12*R7**4*R6*6.+R25
3*R8**8*R7**3*SP*L*R6*12.-R25*R8**8*R7**4*R14*6.-R26*R3*R8**6*R12*R
47**4*2.+R26*R3*R8**8*R7**3*SP*L*4.-R26*R8**8*R7**4*R6*4.)/(R8**9*R
57**5)
 X1C=Y2C
 X1U=Y1V
 X1V=-R10
 X2C=Y3C
 X2U=R19
 X2V=-R15
 X3C=-Y2C
 X3U=Y3V
```

```
      X3V=-R18
      X4C=Y1C
      X4U=Y4V
      X4V=-Y4U
      X5C=X1C
      X5U=Y5V
      X5V=-Y5U
      DO 10 I=1,50
   10 ICONV(I)=0
      DO 30 I=1,50
      SUMU=0.
      SUMV=0.
      U=UX(I)
      V=VX(I)
      UZ=U
      VZ=V
      IF(IPRINT.GE.5) WRITE(6,120) U,V
  120 FORMAT(///,10X,'SUBROUTINE NEWRAP******',//,10X,'TRIALU = ',  E20.
     18,' , TRIAL V = ',  E20.8,////,10X,'ITERATION',8X,'U',19X,'V',17X,
     2'EPS U',14X,'EPSV',//)
      NC=0
   40 CONTINUE
      NC=NC+1
      UP=U
      VP=V
      X1=X1C+X1U*U+X1V*V
      X2=X2C+X2U*U+X2V*V
      X3=X3C+X3U*U+X3V*V
      X4=X4C+X4U*U+X4V*V
      Y1=Y1C+Y1U*U+Y1V*V
      Y2=Y2C+Y2U*U+Y2V*V
      Y3=Y3C+Y3U*U+Y3V*V
```

```
      Y4=Y4C+Y4U*U+Y4V*V
      F=-X2V*Y1+X1V*Y2+Y2V*X1-Y1V*X2
      G=2.E0*(Y1V*Y1+X1V*X1)
      H=2.E0*(Y1U*Y1+X1U*X1)
      J=-X2U*Y1+X1U*Y2+Y2U*X1-Y1U*X2
      R=X1*X2+Y1*Y2
      S=X1*Y3-Y1*X3
      T=X2**2+Y2**2+X1*X3+Y1*Y3
      UU=X2*Y3+X1*Y4-Y2*X3-Y1*X4
      W=Y2*X1-Y1*X2
      ZZ=X1**2+Y1**2
      E1=3.E0*ZZ*R*W-ZZ**2*S
      E1U =ZZ*R*(X1*Y2U-X2*Y1U-Y1*X2U+Y2*X1U)*3.E0+ZZ*W*(X1*X2U+X2*X1U+Y
     11*Y2U+Y2*Y1U)*3.E0-ZZ*S*(X1*X1U*2.E0+Y1*Y1U*2.E0)*2.E0+R*W*(X1*X1U
     2*2.E0+Y1*Y1U*2.E0)*3.E0-ZZ**2*(X1*Y3U-X3*Y1U-Y1*X3U+Y3*X1U)
      E1V =ZZ*R*(X1*Y2V-X2*Y1V-Y1*X2V+Y2*X1V)*3.E0+ZZ*W*(X1*X2V+X2*X1V+Y
     11*Y2V+Y2*Y1V)*3.E0-ZZ*S*(X1*X1V*2.E0+Y1*Y1V*2.E0)*2.E0+R*W*(X1*X1V
     2*2.E0+Y1*Y1V*2.E0)*3.E0-ZZ**2*(X1*Y3V-X3*Y1V-Y1*X3V+Y3*X1V)
      E2=3.E0*R**2*W**2-6.E0*R*S*W*ZZ+2.E0*S**2*ZZ**2+3.E0*T*W**2*ZZ
     1-UU*W*ZZ**2
      E2U=-ZZ*H*UU*W*2.E0+ZZ*H*S**2*4.E0-ZZ*J*R*S*6.E0+ZZ*J*W*T*6.E0-ZZ*
     1R*W*(X1*Y3U-X3*Y1U-Y1*X3U+Y3*X1U)*6.E0-ZZ*W*S*(X1*X2U+X2*X1U+Y1*Y2
     2U+Y2*Y1U)*6.E0+ZZ*W**2*(X1*X3U+X2*X2U*2.E0+X3*X1U+Y1*Y3U+Y2*Y2U*2.
     3E0+Y3*Y1U)*3.E0-H*R*W*S*6.E0+H*W**2*T*3.E0+J*R**2*W*6.E0+R*W**2*(X
     41*X2U+X2*X1U+Y1*Y2U+Y2*Y1U)*6.E0-ZZ**2*J*UU-ZZ**2*W*(X1*Y4U+X2*Y3U
     5-X3*Y2U-X4*Y1U-Y1*X4U-Y2*X3U+Y3*X2U+Y4*X1U)+ZZ**2*S*(X1*Y3U-X3*Y1U
     6-Y1*X3U+Y3*X1U)*4.E0
      E2V=-ZZ*G*UU*W*2.E0+ZZ*G*S**2*4.E0-ZZ*F*R*S*6.E0+ZZ*F*W*T*6.E0-ZZ*
     1R*W*(X1*Y3V-X3*Y1V-Y1*X3V+Y3*X1V)*6.E0-ZZ*W*S*(X1*X2V+X2*X1V+Y1*Y2
     2V+Y2*Y1V)*6.E0+ZZ*W**2*(X1*X3V+X2*X2V*2.E0+X3*X1V+Y1*Y3V+Y2*Y2V*2.
     3E0+Y3*Y1V)*3.E0-G*R*W*S*6.E0+G*W**2*T*3.E0+F*R**2*W*6.E0+R*W**2*(X
     41*X2V+X2*X1V+Y1*Y2V+Y2*Y1V)*6.E0-ZZ**2*F*UU-ZZ**2*W*(X1*Y4V+X2*Y3V
```

```
     5-X3*Y2V-X4*Y1V-Y1*X4V-Y2*X3V+Y3*X2V+Y4*X1V)+ZZ**2*S*(X1*Y3V-X3*Y1V
     6-Y1*X3V+Y3*X1V)*4.E0
      DJAC=E1V*E2U-E2V*E1U
      DTEMP1=E2*E1U-E1*E2U
      DTEMP2=E1*E2V-E2*E1V
      DELV=DTEMP1/DJAC
      DELU=DTEMP2/DJAC
      U=U+DELU
      V=V+DELV
      EPSU=(U-UP)/U
      EPSV=(V-VP)/V
      IF( ABS(EPSV).LE.EPS.AND. ABS(EPSU).LE.EPS) GO TO 70
      IF( ABS(U).GE.2.E2.AND. ABS(V).GE.2.E2) GO TO 75
      IF(IPRINT.GE.5) WRITE(6,110)NC,U,V,EPS,EPSV
  110 FORMAT(10X,I5,4(  E20.8))
      IF(NC.GE.101) GO TO 20
      GO TO 40
   75 VX(I)=V
      UX(I)=U
      ITER(I)=NC
      ICONV(I)=2
      GO TO 30
   20 SUMU=SUMU+U
      SUMV=SUMV+V
      IF(NC.NE.110) GO TO 40
      V=SUMV/10.
      U=SUMU/10.
      ICONV(I)=1
   70 CONTINUE
      UX(I)=U
      VX(I)=V
      ITER(I)=NC
```

```
      IF(IPRINT.GE.3) WRITE(6,130) UZ,VZ,NC,U,V,EPSU,EPSV
  130 FORMAT(////,10X,'SUBROUTINE NEWRAP******',//,10X,'TRIAL U = ',E20.
     18,', TRIAL V = ',E20.8,///,10X,'ITERATION',9X,'U',19X,'V',16X,'EPS
     2 U',15X,'EPS V',/,10X,I5,4(  E20.8),///)
   30 CONTINUE
      IF(IPRINT.NE.0) WRITE(6,100)(UX(I),VX(I),ITER(I),ICONV(I),I=1,50)
  100 FORMAT(////,10X,'SUBROUTINE NEWRAP',///,17X,' U SOLUTION',7X,'  V
     1SOLUTION   ',3X,'ITERATIONS',1X,'CONVERGENCE',//,100(10X,2E20.8,2I
     210,/))
      DERIV(1,1)=X1C
      DERIV(1,2)=X1U
      DERIV(1,3)=X1V
      DERIV(2,1)=X2C
      DERIV(2,2)=X2U
      DERIV(2,3)=X2V
      DERIV(3,1)=X3C
      DERIV(3,2)=X3U
      DERIV(3,3)=X3V
      DERIV(4,1)=X4C
      DERIV(4,2)=X4U
      DERIV(4,3)=X4V
      DERIV(5,1)=Y1C
      DERIV(5,2)=Y1U
      DERIV(5,3)=Y1V
      DERIV(6,1)=Y2C
      DERIV(6,2)=Y2U
      DERIV(6,3)=Y2V
      DERIV(7,1)=Y3C
      DERIV(7,2)=Y3U
      DERIV(7,3)=Y3V
      DERIV(8,1)=Y4C
      DERIV(8,2)=Y4U
```

```
      DERIV(8,3)=Y4V
      DERIV(9,1)=X5C
      DERIV(9,2)=X5U
      DERIV(9,3)=X5V
      DERIV(10,1)=Y5C
      DERIV(10,2)=Y5U
      DERIV(10,3)=Y5V
      IF(IPRINT.NE.0) WRITE(6,50)((DERIV(I,JJ),JJ=1,3),I=1,10)
   50 FORMAT(////,27X,'CONST',14X,'  U   ',17X,' V',//,10X,'X1',5X,3E20.
     18,/,10X,'X2',5X,3E20.8,/,10X,'X3',5X,3E20.8,/,10X,'X4',5X,3E20.
     28,/,10X,'Y1',5X,3E20.8,/,10X,'Y2',5X,3E20.8,/,10X,'Y3',5X,3E20.
     38,/,10X,'Y4',5X,3E20.8,/,10X,'X5',5X,3E20.8,/,10X,'Y5',5X,3E20.8,/
     4/////)
      RETURN
      END
```

```
      SUBROUTINE SIMQ(A,B,N,KS)
C
C     ..................................................................
C
C        SUBROUTINE SIMQ
C
C        PURPOSE
C           OBTAIN SOLUTION OF A SET OF SIMULTANEOUS LINEAR EQUATIONS,
C           AX=B
C
C        USAGE
C           CALL SIMQ(A,B,N,KS)
C
C        DESCRIPTION OF PARAMETERS
C           A - MATRIX OF COEFFICIENTS STORED COLUMNWISE.  THESE ARE
C               DESTROYED IN THE COMPUTATION.  THE SIZE OF MATRIX A IS
C               N BY N.
C           B - VECTOR OF ORIGINAL CONSTANTS (LENGTH N). THESE ARE
C               REPLACED BY FINAL SOLUTION VALUES, VECTOR X.
C           N - NUMBER OF EQUATIONS AND VARIABLES. N MUST BE .GT. ONE.
C           KS - OUTPUT DIGIT
C                0 FOR A NORMAL SOLUTION
C                1 FOR A SINGULAR SET OF EQUATIONS
C
C        REMARKS
C           MATRIX A MUST BE GENERAL.
C           IF MATRIX IS SINGULAR , SOLUTION VALUES ARE MEANINGLESS.
C           AN ALTERNATIVE SOLUTION MAY BE OBTAINED BY USING MATRIX
C           INVERSION (MINV) AND MATRIX PRODUCT (GMPRD).
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
```

```
C
C        METHOD
C           METHOD OF SOLUTION IS BY ELIMINATION USING LARGEST PIVOTAL
C           DIVISOR. EACH STAGE OF ELIMINATION CONSISTS OF INTERCHANGING
C           ROWS WHEN NECESSARY TO AVOID DIVISION BY ZERO OR SMALL
C           ELEMENTS.
C           THE FORWARD SOLUTION TO OBTAIN VARIABLE N IS DONE IN
C           N STAGES. THE BACK SOLUTION FOR THE OTHER VARIABLES IS
C           CALCULATED BY SUCCESSIVE SUBSTITUTIONS. FINAL SOLUTION
C           VALUES ARE DEVELOPED IN VECTOR B, WITH VARIABLE 1 IN B(1),
C           VARIABLE 2 IN B(2),........, VARIABLE N IN B(N).
C           IF NO PIVOT CAN BE FOUND EXCEEDING A TOLERANCE OF 0.0,
C           THE MATRIX IS CONSIDERED SINGULAR AND KS IS SET TO 1. THIS
C           TOLERANCE CAN BE MODIFIED BY REPLACING THE FIRST STATEMENT.
C
C     ...................................................................
C
      DIMENSION A(1),B(1)
C
C        FORWARD SOLUTION
C
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0
      IT=JJ-J
      DO 30 I=J,N
C
C        SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN
```

```
C
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
C
C        TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)
C
      IF(ABS(BIGA)-TOL) 35,35,40
   35 KS=1
      RETURN
C
C        INTERCHANGE ROWS IF NECESSARY
C
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
C
C        DIVIDE EQUATION BY LEADING COEFFICIENT
C
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
C
C        ELIMINATE NEXT VARIABLE
```

```
C
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
C
C        BACK SOLUTION
C
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

```
      FUNCTION ATAN2(Y,X)
      C=0.0
      IF(X.LT.0.) C=3.141592654
      ATAN2=ATAN(Y/X)+C
      RETURN
      END
C
C
C
      FUNCTION TAN(X)
      TAN=SIN(X)/COS(X)
      RETURN
      END
C
C
C
      FUNCTION ARSIN(X)
      ARSIN=ATAN(X/SQRT(1.-X**2))
      RETURN
      END
```

```
      SUBROUTINE ANALZE(PHI1,START,ENDD,A1,XF,YF,XB,YB,X,Y,XC,YC,RC,IPLO
     1TR,ICHK,U,V)
C
C
C        SUBROUTINE ANALZE PROVIDES AN ANALYSIS, FOR INTEGRAL DEGREES,
C     OVER A SPECIFIED RANGE FOR THE FOUR BAR - COUPLER CONFIGURATION.
C
C
C    INPUT ARGUMENTS
C
C     PHI1 = DESIGN ANGLE OF CRANK, DEGREES
C     START = INITIAL VALUE OF CRANK ANGLE
C     ENDD = FINAL VALUE OF CRANK ANGLE
C     A1 = DRIVING CRANK RADIUS
C     XF,YF = CONNECTING ROD END OF FOLLOWER CRANK
C     XB,YB = CENTER OF FOLLOWER CRANK
C     X,Y = COORDINATES OF THE COUPLER POINT SOLUTION
C     XC,YC = CENTER OF CURVATURE FOR SOLUTION
C     RC = RADIUS OF CURVATURE OF THE SOLUTION
C     U,V = COORDINATES, IN THE MOVING PLANE, OF THE SOLUTION
C
C
C     NO OUTPUT ARGUMENTS PROVIDED
C
C
      DIMENSION AA(1450)
      DIMENSION XZ(360),YZ(360),IFAULT(360),JFAULT(360),X3(360),Y3(360)
      PI=3.141593
      HALFPI=1.570796
      IF(ICHK.NE.0) CALL DRAW(XZ,YZ,N1,N2,A1,PHI,XF,YF,XB,YB,XC,YC,RC,
     1ICHK,       START,ENDX,AA,IVPI,X,Y,X3,Y3)
      IF(ICHK.NE.0) RETURN
```

```
      ENDX=ENDD*PI/180.
      PHI=PHI1
      IF(PHI.LT.0) PHI=PHI+2.*PI
      WRITE(6,20)
   20 FORMAT(1H1,////,15X,'THETA',9X,'X4',13X,'X3',13X,'Y4',13X,'Y3',
     113X,'DX',13X,'DY',//)
      ALPHA=ATAN2(V,U)
      A3=SQRT((YF-A1*SIN(PHI))**2+(XF-A1*COS(PHI))**2)
      A=SQRT(XC**2+YC**2)
      CORR=ATAN2(YC,XC)
      B=A1
      C=SQRT((A1* SIN(PHI)-Y)**2+(A1*COS(PHI)-X)**2)
      D=ABS(RC)
C
C     DETERMINE THE VALUE OF THE VARIABLE, SIGN, +1 OR -1
C
      ETA=ATAN2((Y-A1*SIN(PHI)),(X-A1*COS(PHI)))-CORR
      ETA=ARSIN(SIN(ETA))
      P=A-B*COS(PHI-CORR)
      Q=B*SIN(PHI-CORR)
      R=(P**2+Q**2+C**2-D**2)/(2.*C)
      S=R*Q/(P**2+Q**2)
      T=(R**2-P**2)/(P**2+Q**2)
      EPSP=ABS(ETA-ARSIN(-S+SQRT(ABS(S**2-T))))
      EPSM=ABS(ETA-ARSIN(-S-SQRT(ABS(S**2-T))))
      SIGN=-1.
      IF(EPSP.LE.EPSM) SIGN=1.
      COR1=ATAN2(YB,XB)
      B=A1
      A2=SQRT(YB**2+XB**2)
      C1=SQRT((YF-B*SIN(PHI))**2+(XF-B*COS(PHI))**2)
      E1=SQRT((XF-XB)**2+(YF-YB)**2)
```

```
      ETA1=ATAN2((YF-B*SIN(PHI)),(XF-B*COS(PHI)))-COR1
      ETA1=ARSIN(SIN(ETA1))
      P1=A2-B*COS(PHI-COR1)
      Q1=B*SIN(PHI-COR1)
      R1=(P1**2+Q1**2+C1**2-E1**2)/(2.*C1)
      S1=R1*Q1/(P1**2+Q1**2)
      T1=(R1**2-P1**2)/(P1**2+Q1**2)
      EPSP1=ABS(ETA1-ARSIN(-S1+SQRT(ABS(S1**2-T1))))
      EPSM1=ABS(ETA1-ARSIN(-S1-SQRT(ABS(S1**2-T1))))
      SIGN1=-1.
      IF(EPSP1.LE.EPSM1) SIGN1=1.
C
C     INCREMENT LINKAGE THROUGH RANGE
C
C     IFAULT = 1  FOR IMPOSSIBLE LINKAGE POSITICNS
C
      DELTA=PI/180.
      THETA=START*PI/180.-DELTA
      N2=0
      N1=0
      J=0
      NX=51
      DO 120 I=1,360
      IFAULT(I)=0
      JFAULT(I)=0
      X3(I)=0.
      Y3(I)=0.
      XZ(I)=0
  120 YZ(I)=0
  100 THETA=THETA+CELTA
      IP1=0
      IP2=0
```

```
      IF(THETA.GT.ENDX) GO TO 110
      ZLNGTH=ABS(C+D)
      YLNGTH=ABS(C-D)
      XLNGTH=SQRT((A1*SIN(THETA)-YC)**2+(A1*COS(THETA)-XC)**2)
      IF(XLNGTH.GT.ZLNGTH.OR.XLNGTH.LT.YLNGTH) GO TO 140
      GO TO 130
  140 IF(N1.EQ.0) GO TO 150
      IFAULT(N1)=1
      GO TO 150
  130 N1=N1+1
      EPS=THETA-CORR
      P=A-B*COS(EPS)
      Q=B*SIN(EPS)
      R=(P**2+Q**2+C**2-D**2)/(2.*C)
      P2Q2=P**2+Q**2
      S=R*Q/P2Q2
      T=(R**2-P**2)/P2Q2
      BETA=ARSIN(-S+SIGN*SQRT(ABS(S**2-T)))-ALPHA
      XK=B*COS(EPS)+A3*COS(BETA)
      YK=B*SIN(EPS)+A3*SIN(BETA)
      XZ(N1)=XK*COS(CORR)-YK*SIN(CORR)
      YZ(N1)=XK*SIN(CORR)+YK*COS(CORR)
      IP1=1
  150 ZL=ABS(C1+E1)
      YL=ABS(C1-E1)
      XL=SQRT((B*SIN(THETA)-YB)**2+(B*COS(THETA)-XB)**2)
      IF(XL.GT.ZL.OR.XL.LT.YL) GO TO 160
      N2=N2+1
      EPS1=THETA-COR1
      P=A2-B*COS(EPS1)
      Q=B*SIN(EPS1)
      R=(P**2+Q**2+C1**2-E1**2)/(2.*C1)
```

```
      P2Q2=P**2+Q**2
      S=R*Q/P2Q2
      T=(R**2-P**2)/P2Q2
      BETA=ARSIN(-S+SIGN1*SQRT(ABS(S**2-T)))+COR1
      X3(N2)=B*COS(THETA)+C1*COS(BETA)
      Y3(N2)=B*SIN(THETA)+C1*SIN(BETA)
      IP2=1
      GO TO 170
  160 IF(N2.EQ.0) GO TO 165
      JFAULT(N2)=1
  165 IF(IP1.EQ.0.AND.IP2.EQ.0) GO TO 100
  170 CONTINUE
      THETAX=THETA*180./PI
      IF(N1.EQ.0.OR.N2.EQ.0) GO TO 45
      IF(IP1.EQ.0.OR.IP2.EQ.0) GO TO 45
      DX=XZ(N1)-X3(N2)
      DY=YZ(N1)-Y3(N2)
      GO TO 40
   45 DX=0.
      DY=0.
   40 J=J+1
      IF(J.EQ.NX) GO TO 30
      GO TO 41
   30 J=0
      NX=51
      WRITE(6,70)
   70 FORMAT(1H1,////,15X,'THETA',9X,'X4',13X,'X3',13X,'Y4',13X,'Y3',
     113X,'DX',13X,'DY',//)
   41 IF(IP1.EQ.1.AND.IP2.EQ.1) WRITE(6,50)THETAX,XZ(N1),X3(N2),YZ(N1),
     1Y3(N2),DX,DY
   50 FORMAT(15X,F6.2,2X,6(F10.5,5X))
      IF(IP1.EQ.1.AND.IP2.EQ.0) WRITE(6,51) THETAX,XZ(N1),YZ(N1)
```

```
 51 FORMAT(15X,F6.2,2X,F10.5,20X,F10.5)                                 ANL 1610
    IF(IP1.EQ.0.AND.IP2.EQ.1) WRITE(6,52) THETAX,X3(N2),Y3(N2)          ANL 1620
 52 FORMAT(15X,F6.2,17X,F10.5,20X,F10.5)                                ANL 1630
    GO TO 100                                                           ANL 1640
110 IVPI=2*(N1+N2)                                                      ANL 1650
    IF(IPLOTR.EQ.1) CALL DRAW(XZ,YZ,N1,N2,ICHK,AA,IVPI,X,Y,X3,Y3)       ANL 1660
    RETURN                                                              ANL 1670
    END                                                                 ANL 1680
```

```
      SUBROUTINE DRAW(X,Y,N1,N2,ICHK,A,IVPI,XX,YY,X3,Y3)                DRW
C                                                                       DRW
C                                                                       DRW
C     SUBROUTINE DRAW                                                   DRW
C                                                                       DRW
C     SUBROUTINE DRAW WILL PROVIDE A PRINTER PLOT OF ALL COUPLER        DRW
C     CURVES DESCRIBED IN TERMS OF COORDINATES OF POINTS ALONG THE      DRW
C     CURVE.                                                            DRW
C                                                                       DRW
C                                                                       DRW
C     INPUT ARGUMENTS                                                   DRW
C                                                                       DRW
C     X3,Y3    = ARRAYS OF PTS IN ORIG COUPLER CURVE                    DRW
C     X,Y      = ARRAYS OF PTS IN SYNTHESIZED COUPLER CURVE             DRW
C     N1,N2    = NUMBER OF POINTS IN THE FIRST AND SECOND COUPLER       DRW
C                CURVES RESPECTIVELY                                    DRW
C     XX,YY    = COORDINATES OF COUPLER PT SOLUTION IN THE FIXED PLANE  DRW
C     A        = WORKING VECTOR OF SIZE IVPI (N*2)                      DRW
C     IVPI     = LENGTH OF VECTOR A,N*2.                                DRW
C     ICHK =1,  PAGE EJECT ONLY                                         DRW
C          =0,  COUPLER CURVE PLOT                                      DRW
C                                                                       DRW
C                                                                       DRW
C                                                                       DRW
C                                                                       DRW
      DIMENSION OUT(101),A(1080),X(360),Y(360),X3(360),Y3(360),ITYPE(720DRW
     1),YPR(11)                                                         DRW
      INTEGER BLANK,DOT,OUT,STROKE,USC,STAR,OH,EKS                      DRW
C     DATA (BLANK=1H ),(DOT=1H.),(STROKE=1HI),(USC=1HI),(STAR=1H*),(OH=1CDC
C    1HO),(EKS=1HX)                                                     CDC
      DATA BLANK/1H /,DOT/1H./,STROKE/1HI/,USC/1H-/,STAR/1H*/,OH/1HO/,  IBM
     1EKS/1HX/                                                          IBM
```

```
      IF(ICHK.EQ.1) WRITE(6,95)
   95 FORMAT(1H1,///)
      IF(ICHK.EQ.1) RETURN
      WRITE(6,1)
    1 FORMAT(1H1,//,50X,'COUPLER CURVE ILLUSTRATING THE',/,32X,'PERFORMA
     1NCE OF A FOUR-BAR LINKAGE SYNTHESIZED FROM A FOUR-BAR LINKAGE',///
     2//)
      N=N1+N2
      NT=N1+1
      DO 100 J=1,N1
      ITYPE(J)=0
      A(J)=X(J)
      KTP=J+N
  100 A(KTP)=Y(J)
      DO 110 J=NT,N
      ITYPE(J)=1
      KTP=J-N1
      KTX=J+N
      A(J)=X3(KTP)
  110 A(KTX)=Y3(KTP)
      DO 14 I=1,N
      DO 15 J=1,N
      IF(A(J).LT.A(I))GOTO15
      DO17K=1,2
      KK=K-1
      ICD=I+KK*N
      JCD=J+KK*N
      IF(K.EQ.2) GO TO 13
      ITEMP=ITYPE(ICD)
      ITYPE(ICD)=ITYPE(JCD)
      ITYPE(JCD)=ITEMP
   13 CONTINUE
```

```
      F=A(ICD)
      A(ICD)=A(JCD)
      A(JCD)=F
17    CONTINUE
15    CONTINUE
14    CONTINUE
      NLL=61
      XSCAL=(A(N)-A(1))/60.
      M1=N+1
      YMIN=A(M1)
      YMAX=YMIN
      M2=2*N
      DO40J=M1,M2
      IF(A(J)-YMIN)28,26,26
26    IF(A(J)-YMAX)40,40,30
28    YMIN=A(J)
      GOTO40
30    YMAX=A(J)
   40 CONTINUE
      DELY =ABS(YMAX-YMIN)
      YSCAL=DELY/100.
      YPR(1)=YMIN
       DO 90 KN=1,10
   90 YPR(KN+1)=YPR(1)+KN*YSCAL*10.
      WRITE(6,8) (YPR(IP),IP=1,11)
    8 FORMAT(1H ,9X,11F10.4,/)
      XBZ=A(1)
      L=1
      LX=1
      MY=1
      I=1
45    F=I-1
```

```
      XPR=XBZ+F*XSCAL
      XPRHI=XPR+XSCAL/2.
      XPRLO=XPR-XSCAL/2.
      IF(A(L).LT.XPRLO.OR.A(L).GT.XPRHI) GO TO 70
50    DO55IX=1,101
55    OUT(IX)=BLANK
      IF(LX.NE.1)GOTO300
      DO301IX=1,101
301   OUT(IX)=USC
300   CONTINUE
      DO56IX=1,101,10
56    OUT(IX)=STROKE
  220 IS=L+N
      IZ=L
      JP=(A(IS)-YMIN)/YSCAL+1.5
      IF(OUT(JP).EQ.EKS) GO TO 400
      IF(OUT(JP).EQ.USC) GO TO 410
      IF(OUT(JP).EQ.STROKE) GO TO 410
      IF(OUT(JP).EQ.BLANK) GO TO 410
      IF(OUT(JP).EQ.STAR) GO TO 450
      IF(ITYPE(IZ).EQ.1) GO TO 400
      GO TO 420
  450 IF(ITYPE(IZ).EQ.0) GO TO 400
      GO TO 420
  410 IF(ITYPE(IZ).EQ.1) GO TO 430
      GO TO 440
  430 OUT(JP)=OH
      GO TO 400
  440 OUT(JP)=STAR
      GO TO 400
  420 OUT(JP)=EKS
  400 CONTINUE
```

```
      IF(A(L+1).GE.XPRLO.AND.A(L+1).LE.XPRHI) GO TO 221                 DRW 1290
      GO TO 200                                                         DRW 1300
  221 L=L+1                                                             DRW 1310
      GO TO 220                                                         DRW 1320
  200 CONTINUE                                                          DRW 1330
      WRITE(6,2)XPR,(CUT(IZ),IZ=1,101)                                  DRW 1340
2     FORMAT(1H ,F11.4,5X,101A1)                                        DRW 1350
      LX=LX+1                                                           DRW 1360
      IF(LX.EQ.7)LX=1                                                   DRW 1370
      L=L+1                                                             DRW 1380
      GOTO80                                                            DRW 1390
   70 CONTINUE                                                          DRW 1400
      DO 71 IX=1,101                                                    DRW 1410
   71 OUT(IX)=BLANK                                                     DRW 1420
      DO 72 IX=1,101,10                                                 DRW 1430
   72 OUT(IX)=STROKE                                                    DRW 1440
      IF(LX.NE.1) GO TO 74                                              DRW 1450
      DO 76 IX=1,101                                                    DRW 1460
   76 OUT(IX)=USC                                                       DRW 1470
      DO 77 IX=1,101,10                                                 DRW 1480
   77 OUT(IX)=STROKE                                                    DRW 1490
   74 WRITE(6,73) OUT                                                   DRW 1500
   73 FORMAT(17X,101A1)                                                 DRW 1510
      LX=LX+1                                                           DRW 1520
      IF(LX.EQ.7) LX=1                                                  DRW 1530
   80 I=I+1                                                             DRW 1540
      IF(I-NLL)45,84,86                                                 DRW 1550
84    XPR=A(N)                                                          DRW 1560
      GOTO50                                                            DRW 1570
86    WRITE(6,6)                                                        DRW 1580
6     FORMAT(1H1)                                                       DRW 1590
      RETURN                                                            DRW 1600
```

END

APPENDIX C

Sample Output of Straight Path Program

and

Sample Output of Circular Path Program

```
SUBROUTINE TRIAL*****

POINT O = CRANK CENTER
POINT A = CRANK END
POINT B = SLIDER
POINT I = INSTANT CENTER
POINT T = POINT ON TANGENT
THETA = ANGLE X-AXIS AND I-T
ALPHA = ANGLE I-T AND I-A
BETA = ANGLE I-T AND I-B
PSI ASM = ANGLE I-T AND ASYMPTOTE
J - POINTS ON INFLECTION CIRCLE


A1 =        0.31761618E 01
A2 =        0.82917786E 01
A3 =       -0.25068207E 01
PHI =        0.10471973E 01
A AT        0.15880804E 01       0.27506361E 01
B AT        0.80000029E 01      -0.25068207E 01
I AT        0.80000029E 01       0.13856404E 02
I-A =        0.12823838E 02
I-B =        0.16363220E 02


POINTS ON THE INFLECTION CIRCLE
     -0.24300201E 02     -0.42089188E 02
      0.80000029E 01     -0.25068207E 01


ICHK =              0
INFLECTION CIRCLE AT      -0.42429413E 02       0.56747894E 01
INFLECTION CIRCLE RADIUS =        0.51088791E 02


THETA =       -0.14099579E 01
ALPHA =        0.55987473E 01
BETA =       -0.16083717E 00
M =       0.53239875E 05
N =       0.16545410E 02
PSI ASM =       -0.31077070E-03
XAVG =       0.14593529E 02
```

ORIGINAL TRIAL SOLUTIONS

X	Y
0.11260493E 02	-0.22191572E 01
0.12507535E 02	-0.18991699E 01
0.13664946E 02	-0.13231554E 01
0.14713740E 02	-0.56401825E 00
0.15629179E 02	0.35283089E 00
0.16389023E 02	0.14029274E 01
0.16974686E 02	0.25596590E 01
0.17371811E 02	0.37941713E 01
0.17570648E 02	0.50758581E 01
0.17566330E 02	0.63730412E 01
0.17358963E 02	0.76536922E 01
0.16953674E 02	0.88862123E 01
0.16360443E 02	0.10040216E 02
0.15593906E 02	0.11087241E 02
0.14672909E 02	0.12001493E 02
0.13620146E 02	0.12760428E 02
0.12461540E 02	0.13345340E 02
0.11225645E 02	0.13741804E 02
0.99428644E 01	0.13940051E 02
0.86447983E 01	0.13935178E 02
0.73634157E 01	0.13727291E 02
0.61302681E 01	0.13321493E 02
0.49757214E 01	0.12727761E 02
0.39282084E 01	0.11960701E 02
0.30135221E 01	0.11039180E 02
0.22541952E 01	0.99858742E 01
0.16689196E 01	0.88266954E 01
0.12721157E 01	0.75901670E 01
0.10735607E 01	0.63067112E 01
0.10781422E 01	0.50078859E 01
0.12857618E 01	0.37256393E 01
0.16913147E 01	0.24914742E 01
0.22848291E 01	0.13357086E 01
0.30516968E 01	0.28667068E 00
0.39730902E 01	-0.63003635E 00
0.50263729E 01	-0.13921919E 01
0.61857347E 01	-0.19817972E 01
0.74229317E 01	-0.23862257E 01
0.87086020E 01	-0.26023054E 01
0.10020852E 02	-0.26850386E 01
0.12665234E 02	-0.14955394E 02

0.56673880E 01	0.28262299E 02
0.63672428E 01	0.23940094E 02
0.70670967E 01	0.19617889E 02
0.77669506E 01	0.15295697E 02
0.84668045E 01	0.10973505E 02
0.91666584E 01	0.66513042E 01
0.98665142E 01	0.23290882E 01
0.10566368E 02	-0.19931040E 01
0.11266223E 02	-0.63152876E 01

TRANSFORMED TRIAL SOLUTICNS

LAMBDA	MU
0.12820740E 01	0.27615070E 00
0.13739033E 01	0.40135151E 00
0.14377966E 01	0.54357529E 00
0.14775562E 01	0.69457132E 00
0.14928198E 01	0.85007793E 00
0.14833841E 01	0.10061131E 01
0.14495497E 01	0.11587734E 01
0.13921843E 01	0.13042707E 01
0.13127203E 01	0.14390049E 01
0.12131243E 01	0.15596495E 01
0.10958557E 01	0.16632252E 01
0.96381104E 00	0.17471781E 01
0.82024217E 00	0.18094358E 01
0.66869146E 00	0.18484659E 01
0.51288861E 00	0.18633013E 01
0.35667413E 00	0.18535767E 01
0.20389622E 00	0.18195286E 01
0.58320373E-01	0.17619963E 01
-0.76470554E-01	0.16823921E 01
-0.19715488E 00	0.15826778E 01
-0.30075920E 00	0.14653053E 01
-0.38473135E 00	0.13331642E 01
-0.44700211E 00	0.11895075E 01
-0.48603702E 00	0.10378704E 01
-0.50087315E 00	0.88198584E 00
-0.49114317E 00	0.72569072E 00
-0.45708555E 00	0.57283151E 00
-0.39953625E 00	0.42717063E 00
-0.31990999E 00	0.29229319E 00
-0.22016406E 00	0.17151570E 00
-0.10275066E 00	0.67810535E-01
0.29445183E-01	-0.16275175E-01
0.17317516E 00	-0.78676164E-01
0.32491070E 00	-0.11786807E 00
0.48093814E 00	-0.13290250E 00
0.63744730E 00	-0.12343818E 00
0.79065472E 00	-0.89770257E-01
0.93696088E 00	-0.32881018E-01
0.10733843E 01	0.45280267E-01
0.12020912E 01	0.13790989E 00
0.23869944E 01	-0.80420536E 00
-0.15703926E 01	0.26911373E 01

-0.11746149E 01	0.23415689E 01
-0.77883619E 00	0.19919987E 01
-0.38305879E 00	0.16424313E 01
0.12718987E-01	0.12928629E 01
0.40849733E 00	0.94329345E 00
0.80427700E 00	0.59372258E 00
0.12000542E 01	0.24415392E 00
0.15958319E 01	-0.10541391E 00

SUBROUTINE NEWRAP

MU SOLUTION	LAMBDA SOLUTION	ITERATIONS	CONVERGENCE
0.20378655E-43	0.10000000E 01	50	0
0.20054801E-43	0.10000000E 01	51	0
0.20054801E-43	0.10000000E 01	51	0
0.20141399E-43	0.10000000E 01	51	0
0.15260258E 01	-0.25126582E 00	98	0
0.16935987E 01	0.10531836E 01	12	0
0.16935987E 01	0.10531836E 01	7	0
0.16935987E 01	0.10531836E 01	6	0
0.16935978E 01	0.10531836E 01	6	0
0.16935987E 01	0.10531836E 01	5	0
0.16935987E 01	0.10531836E 01	4	0
0.16935987E 01	0.10531836E 01	5	0
0.16935978E 01	0.10531836E 01	8	0
0.15260277E 01	-0.25126588E 00	43	0
0.15260258E 01	-0.25126439E 00	45	0
0.15260258E 01	-0.25126445E 00	45	0
0.15260258E 01	-0.25126439E 00	45	0
0.15260258E 01	-0.25126445E 00	44	0
0.15260258E 01	-0.25126445E 00	43	0
0.15260258E 01	-0.25126439E 00	39	0
0.15260229E 01	-0.25126565E 00	40	0
0.15260229E 01	-0.25126559E 00	44	0
0.15260229E 01	-0.25126582E 00	45	0
0.15260229E 01	-0.25126582E 00	46	0
0.15260229E 01	-0.25126576E 00	47	0
0.15260229E 01	-0.25126570E 00	48	0
0.15260229E 01	-0.25126582E 00	48	0
0.15260229E 01	-0.25126559E 00	51	0
0.45871616E-07	-0.28109682E-07	110	1
0.25972724E-07	-0.18274646E-08	110	1
0.48544720E-07	0.54498770E-08	110	1
0.12546536E-07	0.15292365E-07	11	0
0.50358452E-07	0.49217590E-08	110	1
0.36537333E-07	-0.22934408E-07	110	1
0.20326311E-43	0.10000000E 01	61	0
0.92930960E-44	0.10000000E 01	51	0
0.88050705E-44	0.10000000E 01	51	0
0.87903169E-44	0.10000000E 01	50	0
0.20316347E-43	0.10000000E 01	48	0
0.20051124E-43	0.10000000E 01	50	0
0.89086455E-44	0.10000000E 01	55	0
0.15260258E 01	-0.25126582E 00	64	0
0.15260229E 01	-0.25126451E 00	63	0
0.15260258E 01	-0.25126576E 00	61	0

0.15260258E 01	-0.25126570E 00	55	0
0.15260229E 01	-0.25126439E 00	59	0
0.15260258E 01	-0.25126576E 00	72	0
0.84233135E 00	0.52381194E 00	8	0
0.84233153E 00	0.52381182E 00	11	0
0.84233129E 00	0.52381194E 00	12	0

	CONST	LAMBDA	MU
X1	-0.27506361E 01	-0.13021469E 01	0.15880804E 01
X2	-0.15880804E 01	0.15976114E 01	-0.27506361E 01
X3	0.27506361E 01	0.43192835E 01	-0.15880804E 01
X4	0.15880804E 01	-0.19075298E 01	0.27506361E 01
Y1	0.15880804E 01	-0.15880804E 01	-0.13021469E 01
Y2	-0.27506361E 01	0.27506361E 01	0.15976114E 01
Y3	-0.15880804E 01	0.15880804E 01	0.43192835E 01
Y4	0.27506361E 01	-0.27506361E 01	-0.19075298E 01
X5	-0.27506361E 01	-0.24220062E 02	0.15880804E 01
Y5	0.15880804E 01	0.15880804E 01	-0.24220062E 02

SUBROUTINE SOL*****

SOL EPSILON = 0.49999990E-02

NUMBER OF SOLUTIONS = 50

ORIGINAL SOLUTIONS

X	Y
0.10000000E 01	0.20378655E-43
0.10000000E 01	0.20054801E-43
0.10000000E 01	0.20054801E-43
0.10000000E 01	0.20141399E-43
-0.25126582E 00	0.15260258E 01
0.10531836E 01	0.16935987E 01
0.10531836E 01	0.16935987E 01
0.10531836E 01	0.16935987E 01
0.10531836E 01	0.16935978E 01
0.10531836E 01	0.16935987E 01
0.10531836E 01	0.16935987E 01
0.10531836E 01	0.16935987E 01
0.10531836E 01	0.16935978E 01
-0.25126588E 00	0.15260277E 01
-0.25126439E 00	0.15260258E 01
-0.25126445E 00	0.15260258E 01
-0.25126439E 00	0.15260258E 01
-0.25126445E 00	0.15260258E 01
-0.25126445E 00	0.15260258E 01
-0.25126439E 00	0.15260258E 01
-0.25126565E 00	0.15260229E 01
-0.25126559E 00	0.15260229E 01
-0.25126582E 00	0.15260229E 01
-0.25126582E 00	0.15260229E 01
-0.25126576E 00	0.15260229E 01
-0.25126570E 00	0.15260229E 01
-0.25126582E 00	0.15260229E 01
-0.25126559E 00	0.15260229E 01
-0.28109682E-07	0.45871616E-07
-0.18274646E-08	0.25972724E-07
0.54498770E-08	0.48544720E-07
0.15292365E-07	-0.12546536E-07
0.49217590E-08	0.50358452E-07
-0.22934408E-07	0.36537333E-07
0.10000000E 01	0.20326311E-43
0.10000000E 01	-0.92930960E-44
0.10000000E 01	-0.88050705E-44
0.10000000E 01	-0.87903169E-44

0.10000000E 01	0.20316347E-43
0.10000000F 01	0.20051124E-43
0.10000000E 01	-0.89086455E-44
-0.25126582F 00	0.15260258E 01
-0.25126451E 00	0.15260229E 01
-0.25126576E 00	0.15260258E 01
-0.25126570E 00	0.15260258E 01
-0.25126439E 00	0.15260229E 01
-0.25126576F 00	0.15260258E 01
0.52381194E 00	0.84233135E 00
0.52381182E 00	0.84233153E 00
0.52381194E 00	0.84233129E 00

UNIQUE SOLUTIONS

X	Y
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
-0.25126582E 00	0.15260258E 01
0.10531836E 01	0.16935987E 01
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0

0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0
0.52381194E 00	0.84233135E 00
0.0	0.0
0.0	0.0

RETURNED SOLUTIONS = 3

X	Y
-0.25126582E 00	0.15260258E 01
0.10531836E 01	0.16935987E 01
0.52381194E 00	0.84233135E 00

SUBROUTINE RANK*****

INPUT SOLUTIONS

LAMBDA	MU
-0.2512658E 00	0.1526026E 01
0.1053184E 01	0.1693599E 01
0.5238119E 00	0.8423313E 00

Z = 0.1818989E-11 I = 1

Z = 0.7295014E 01 I = 2

Z = 0.4505151E 01 I = 3

OUTPUT SOLUTIONS

LAMBDA	MU	D3
0.1053184E 01	0.1693599E 01	0.4172296E 02
0.5238119E 00	0.8423313E 00	0.5406426E 03
-0.2512658E 00	0.1526026E 01	0.2424665E 09

```
SUBROUTINE STRLIN******

MU =  0.16935987E 01
LAMBDA =  0.10531836E 01
R =  0.31761618E 01
L =  0.82917786E 01
OFST = -0.25068207E 01
PHI =  0.10471973E 01
COUPLER POINT AT  0.17245026E 02 0.80727930E 01
RADIUS OF CURVATURE =  0.13554964E 01
CENTER OF CURVATURE AT  0.16095871E 02 0.87916937E 01

X1 = -0.14324646E 01
X2 = -0.45639763E 01
Y1 = -0.22897730E 01
Y2 =  0.28520002E 01
YP =  0.15984840E 01
YPP =  0.49452600E 01




SUBROUTINE STRLIN******

MU =  0.84233135E 00
LAMBDA =  0.52381194E 00
R =  0.31761618E 01
L =  0.82917786E 01
OFST = -0.25068207E 01
PHI =  0.10471973E 01
COUPLER POINT AT  0.93752413E 01 0.53976812E 01
RADIUS OF CURVATURE =  0.85357466E 01
CENTER OF CURVATURE AT  0.80054665E 01 0.13822803E 02

X1 = -0.20950260E 01
X2 = -0.30681791E 01
Y1 = -0.34061396E 00
Y2 =  0.35897255E-01
YP =  0.16258216E 00
YPP =  0.12182993E 00
```

```
SUBROUTINE STRLIN******

MU =  0.15260258E 01
LAMBDA = -0.25126582E 00
R =  0.31761618E 01
L =  0.82917786E 01
OFST = -0.25068207E 01
PHI =  0.10471973E 01
COUPLER POINT AT  0.79999981E 01 0.13856414E 02
RADIUS OF CURVATURE =  0.49630829E-12
CENTER OF CURVATURE AT  0.79999981E 01 0.13856414E 02

X1 =  0.95367432E-06
X2 = -0.61870461E 01
Y1 =  0.95367432E-06
Y2 = -0.10037804E 01
YP =  0.10000000E 01
YPP =  0.56990609E 13
```

```
SUBROUTINE TRIAL*****

PCINT O = DRIVING CRANK CENTER
PCINT A = DRIVING CRANK END
PCINT B = FOLLOWER CRANK END
POINT I = INSTANT CENTER
PCINT T = POINT CN TANGENT
THETA = ANGLE X-AXIS AND I-T
ALPHA = ANGLE I-T AND I-A
BETA = ANGLE I-T AND I-B
PSI ASM = ANGLE I-T AND ASYMPCOTE
J - PCINTS ON THE INFLECTION CIRCLE
POINT OB = FOLLOWER CRANK CENTER

A1 =         1.00000000E 00
PHI =         2.61799400E 00
XB,YB =         3.00000000E 00                  0
XF,YF =         1.05088700E 00       2.28056100E 00
I AT         5.92230240E 00      -3.41924192E 00
I-A =         7.83848528E 00
I-B =         7.49789579E 00

PCINTS ON THE INFLECTICN CIRCLE
      5.23441826E 01      -3.02209194E 01
      1.32260087E 01      -1.19649485E 01

ICHK =             0
INFLECTION CIRCLE AT        5.85893616E 01       3.41994286E 01
INFLECTION CIRCLE RADIUS =        6.47223570E 01

THETA =         2.19103755E 00
ALPHA =         4.26956453E-01
BETA =         8.69542147E-02
M =         5.53836367E 02
N =         8.91627864E 00
PSI ASM =        -1.60977303E-02
XAVG =         7.66819054E 00


ORIGINAL TRIAL SOLUTIONS

              X                   Y

          5.92230240         -3.41924192
          1.17531411          2.23054391
           .31556592          2.27880873
         -0.21322792          1.90730246
         -0.57562581          1.38244540
```

-0.80491898	.77832098
-0.91099542	.13230997
-0.89845933	-0.52922608
-0.77151484	-1.18435432
-0.53562818	-1.81355254
-0.19813325	-2.39911755
.23159847	-2.92509332
.74220658	-3.37740000
1.32051957	-3.74403299
1.95181866	-4.01527092
2.62016231	-4.18386004
3.30875394	-4.24515692
4.00033842	-4.19721801
4.67761478	-4.04082984
5.32365287	-3.77947617
5.92230293	-3.41924154
6.45858674	-2.96865212
6.91906058	-2.43845715
7.29214011	-1.84135550
7.56837891	-1.19167317
7.74069305	-0.50499889
7.80452515	.20221464
7.75794272	.91307612
7.60166592	1.61070019
7.33902048	2.27866576
6.97581012	2.90148654
6.52009928	3.46510009
5.98188626	3.95739999
5.37262116	4.36887553
4.70445379	4.69352726
3.98889513	4.93052031
3.23389445	5.08802903
2.43545234	5.19490299
1.54255822	5.35089698
.16069990	6.15558330
-2.78964215	9.20243018
10.27827467	-9.73007797
8.97135230	-7.83663781
7.66442992	-5.94319765
6.35750755	-4.04975749
5.05058517	-2.15631733
3.74366280	-0.26287717
2.43674042	1.63056298
1.12981805	3.52400314
-0.17710433	5.41744330

TRANSFORMED TRIAL SOLUTIONS

U	V
2.30639207	-7.49148898
2.67341157	-0.12132790
2.07633476	.49915234
1.43606024	.58683571
.81333580	.44891750
.23418829	.16233446
-0.28318689	-0.23879565
-0.72422234	-0.73202462
-1.07707156	-1.29842144
-1.33245337	-1.91996230
-1.48369298	-2.57868514
-1.52679757	-3.25652131
-1.46050841	-3.93542315
-1.28630614	-4.59763101
-1.00835891	-5.22600501
-0.63341017	-5.80438116
-0.17060636	-6.31792606
.36873269	-6.75347242
.97139552	-7.09982200
1.62260674	-7.34800522
2.30639271	-7.49148907
3.00597651	-7.52632686
3.70419251	-7.45124503
4.38391060	-7.26766389
5.02845972	-6.97965098
5.62204037	-6.59380680
6.15011627	-6.11908439
6.59977581	-5.56654456
6.96005501	-4.94904908
7.22221490	-4.28089279
7.37996831	-3.57737211
7.42965347	-2.85427876
7.37035673	-2.12728747
7.20399475	-1.41115915
6.93538643	-0.71855828
6.57239752	-0.05793059
6.12641554	.57130315
5.61414428	1.19300243
5.06609901	1.91497225
4.60127660	3.44500252
4.51319355	7.68528972
1.20299132	-15.07987833
1.53404465	-12.80313385
1.86509798	-10.52638937
2.19615131	-8.24964490

2.52720464	-5.97290042
2.85825797	-3.69615594
3.18931129	-1.41941146
3.52036462	.85733302
3.85141795	3.13407750

SUBROUTINE NEWRAP

U SOLUTION	V SOLUTION	ITERATIONS	CONVERGENCE
2.30639207E 00	-7.49148898E 00	1	0
2.61628577E 00	-2.47695079E-10	110	1
2.61628577E 00	-2.30689668E-10	110	1
2.61628577E 00	-2.31455302E-10	110	1
1.57785022E-11	1.25958115E-12	110	1
-3.76150924E-12	2.28000334E-11	10	0
-3.46610073E-11	-1.09794287E-11	18	0
8.02256673E-12	-1.42326048E-12	110	1
7.58116832E-12	-7.45550214E-12	110	1
-3.27660954E-11	1.06335999E-11	7	0
-2.66506005E-11	-1.07053429E-11	10	0
4.62419569E 00	3.22136057E 00	16	0
2.30639146E 00	-7.49148922E 00	55	0
2.30639155E 00	-7.49148909E 00	55	0
2.30639144E 00	-7.49148905E 00	54	0
2.30639151E 00	-7.49148900E 00	54	0
2.30639145E 00	-7.49148898E 00	53	0
2.30639143E 00	-7.49148896E 00	52	0
2.30639149E 00	-7.49148894E 00	51	0
2.30639155E 00	-7.49148893E 00	49	0
2.30639255E 00	-7.49148905E 00	1	0
2.30639259E 00	-7.49148907E 00	49	0
2.30639265E 00	-7.49148909E 00	51	0
2.30639270E 00	-7.49148912E 00	52	0
2.30639268E 00	-7.49148914E 00	53	0
2.30639262E 00	-7.49148914E 00	54	0
2.30639269E 00	-7.49148920E 00	54	0
2.30639275E 00	-7.49148928E 00	54	0
2.30639263E 00	-7.49148931E 00	55	0
2.30639264E 00	-7.49148944E 00	57	0
1.32293972E 03	-3.00108712E 03	2	2
6.20558138E 00	4.68490361E-01	9	0
6.20558138E 00	4.68490361E-01	7	0
6.20558138E 00	4.68490361E-01	7	0
6.20558138E 00	4.68490361E-01	6	0
6.20558138E 00	4.68490361E-01	5	0
6.20558138E 00	4.68490361E-01	4	0
-3.20774007E 02	3.89571145E 02	1	2
4.70507335E 00	2.77197258E 00	5	0
4.62419569E 00	3.22136057E 00	5	0
4.62419569E 00	3.22136057E 00	11	0
2.30639185E 00	-7.49149070E 00	79	0
2.30639229E 00	-7.49148729E 00	76	0
2.30639182E 00	-7.49149093E 00	75	0

2.30639180E 00	-7.49149098E 00	46	0
2.30639247E 00	-7.49148718E 00	45	0
2.30639235E 00	-7.49148681E 00	73	0
2.30639234E 00	-7.49148688E 00	74	0
2.61628577E 00	-2.30304784E-10	110	1
4.62419569E 00	3.22136056E 00	10	0

	CCNST	U	V
X1	-4.99999894E-01	-8.68239531E-02	-9.34727359E-02
X2	8.66025465E-01	-1.78423556E-01	-1.68172181E-01
X3	4.99999894E-01	-5.02832990E-02	8.34465206E-02
X4	-8.66025465E-01	-7.16240506E-02	1.67706877E-01
Y1	-8.66025465E-01	9.34727359E-02	-8.68239531E-02
Y2	-4.99999894E-01	1.68172181E-01	-1.78423556E-01
Y3	8.66025465E-01	-8.34465206E-02	-5.02832990E-02
Y4	4.99999894E-01	-1.67706877E-01	-7.16240506E-02
X5	-4.99999894E-01	1.32506047E 00	2.84027803E-01
Y5	-8.66025465E-01	-2.84027803E-01	1.32506047E 00

SUBROUTINE SOL*****

SOL EPS = 5.00000000E-03

ORIGINAL SOLUTIONS

X	Y
2.30639207E 00	-7.49148898E 00
2.61628577E 00	-2.47695079E-10
2.61628577E 00	-2.30689668E-10
2.61628577E 00	-2.31455302E-10
1.57785022E-11	1.25958115E-12
-3.76150924E-12	2.28000334E-11
-3.46610073E-11	-1.09794287E-11
8.02256673E-12	-1.42326048E-12
7.58116832E-12	-7.45550214E-12
-3.27660954E-11	1.06335999E-11
-2.66506005E-11	-1.07053429E-11
4.62419569E 00	3.22136057E 00
2.30639146E 00	-7.49148922E 00
2.30639155E 00	-7.49148909E 00
2.30639144E 00	-7.49148905E 00
2.30639151E 00	-7.49148900E 00
2.30639145E 00	-7.49148898E 00
2.30639143E 00	-7.49148896E 00
2.30639149E 00	-7.49148894E 00
2.30639155E 00	-7.49148893E 00
2.30639255E 00	-7.49148905E 00
2.30639259E 00	-7.49148907E 00
2.30639265E 00	-7.49148909E 00
2.30639270E 00	-7.49148912E 00
2.30639268E 00	-7.49148914E 00
2.30639262E 00	-7.49148914E 00
2.30639269E 00	-7.49148920E 00
2.30639275E 00	-7.49148928E 00
2.30639263E 00	-7.49148931E 00
2.30639264E 00	-7.49148944E 00
1.32293972E 03	-3.00108712E 03
6.20558138E 00	4.68490361E-01
6.20558138E 00	4.68490361E-01
6.20558138E 00	4.68490361E-01
6.20558138E 00	4.68490361E-01
6.20558138E 00	4.68490361E-01
6.20558138E 00	4.68490361E-01
-3.20774007E 02	3.89571145E 02
4.70507335E 00	2.77197258E 00
4.62419569E 00	3.22136057E 00

4.62419569E 00	3.22136057E 00
2.30639185E 00	-7.49149070E 00
2.30639229E 00	-7.49148729E 00
2.30639182E 00	-7.49149093E 00
2.30639180E 00	-7.49149098E 00
2.30639247E 00	-7.49148718E 00
2.30639235E 00	-7.49148681E 00
2.30639234E 00	-7.49148688E 00
2.61628577E 00	-2.30304784E-10
4.62419569E 00	3.22136056E 00

UNIQUE SOLUTIONS

X	Y
2.30639207E 00	-7.49148898E 00
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
4.62419569E 00	3.22136057E 00
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
6.20558138E 00	4.68490361E-01

0	0
0	0
0	0
0	0
0	0
0	0
4.70507335E 00	2.77197258E 00
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0

RETURNED SOLUTIONS = 4

X	Y
2.30639207E 00	-7.49148898E 00
4.62419569E 00	3.22136057E 00
6.20558138E 00	4.68490361E-01
4.70507335E 00	2.77197258E 00

SUBROUTINE RANK*****

INPUT SOLUTIONS

U	V
2.3063921E 00	-7.4914890E 00
4.6241957E 00	3.2213606E 00
6.2055814E 00	4.6849036E-01
4.7050734E 00	2.7719726E 00

OUTPUT SOLUTIONS

U	V	D3
6.2055814E 00	4.6849036E-01	3.2234013E 01
4.7050734E 00	2.7719726E 00	2.0950131E 03
2.3063921E 00	-7.4914890E 00	1.6150553E 11
4.6241957E 00	3.2213606E 00	3.5556049E 22

```
SUBROUTINE STRLIN*****

U =  6.20558138E 00
V =  4.68490361E-01
CRANK RADIUS =  1.00000000E 00
CRANK ANGLE =  2.61799400E 00
FOLLOWER CRANK CENTER AT  3.00000000E 00            0
FOLLOWER CRANK END AT  1.05088700E 00 2.28056100E 00
COUPLER ANGLE =  7.48537967E-01
COUPLER POINT AT  3.36186917E 00 5.06657744E 00
RADIUS OF CURVATURE =  2.39976831E 00
CENTER OF CURVATURE AT  2.66865412E 00 7.36403931E 00
X1 = -1.08258408E 00
X2 = -3.19983477E-01
Y1 = -3.26648980E-01
Y2 =  4.60016546E-01
YP =  3.01730819E-01
YPP =  4.74889921E-01



SUBROUTINE STRLIN*****

U =  4.70507335E 00
V =  2.77197258E 00
CRANK RADIUS =  1.00000000E 00
CRANK ANGLE =  2.61799400E 00
FOLLOWER CRANK CENTER AT  3.00000000E 00            0
FOLLOWER CRANK END AT  1.05088700E 00 2.28056100E 00
COUPLER ANGLE =  7.48537967E-01
COUPLER POINT AT  6.94793020E-01 5.73310536E 00
RADIUS OF CURVATURE =  4.35759142E 01
CENTER OF CURVATURE AT -2.09173170E 01 4.35716862E 01
X1 = -1.16761682E 00
X2 = -4.39639127E-01
Y1 = -6.66903003E-01
Y2 = -2.03322650E-01
YP =  5.71165977E-01
YPP =  3.50496911E-02
```

```
SUBROUTINE STRLIN*****

U =  2.30639207E 00
V = -7.49148898E 00
CRANK RADIUS =  1.00000000E 00
CRANK ANGLE =  2.61799400E 00
FOLLOWER CRANK CENTER AT  3.00000000E 00                    0
FOLLOWER CRANK END AT  1.05088700E 00 2.28056100E 00
COUPLER ANGLE =  7.48537967E-01
COUPLER POINT AT  5.92230240E 00-3.41924192E 00
RADIUS OF CURVATURE =  3.07865423E-16
CENTER OF CURVATURE AT  5.92230240E 00-3.41924192E 00
X1 =  1.45519152E-11
X2 =  1.71437083E 00
Y1 = -7.27595761E-11
Y2 =  1.22452919E 00
YP = -5.00000000E 00
YPP =  4.62621125E 22
```

```
SUBROUTINE STRLIN*****

U =  4.62419569E 00
V =  3.22136057E 00
CRANK RADIUS =  1.00000000E 00
CRANK ANGLE =  2.61799400E 00
FOLLOWER CRANK CENTER AT  3.00000000E 00                    0
FOLLOWER CRANK END AT  1.05088700E 00 2.28056100E 00
COUPLER ANGLE =  7.48537967E-01
COUPLER POINT AT  3.29695988E-01 6.00732228E 00
RADIUS OF CURVATURE =  1.87892461E 11
CENTER OF CURVATURE AT -9.58684572E 10 1.61590160E 11
X1 = -1.20260023E 00
X2 = -5.00783205E-01
Y1 = -7.13480501E-01
Y2 = -2.97105425E-01
YP =  5.93281528E-01
YPP =  8.36674069E-12
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Page 1 of 2

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Page 2 of 2

THE SYNTHESIS OF FOUR-BAR LINKAGE
COUPLER CURVES USING DERIVATIVES
OF THE RADIUS OF CURVATURE

by

Reginald Glennis Mitchiner

(ABSTRACT)

Procedures for the synthesis of four-bar kinematic linkages with approximate portions of their coupler point curves specified are developed. The necessary equations are derived and computer programs using these equations which have been integrated into a complete synthesis procedure are set forth.

If a body, or a mechanism link, is in plane motion with two points on the body constrained to particular paths, the nature of the paths of all other points on the link is known. The functional behavior of the radius of curvature of the path of any point on the link and the derivatives of the radius of curvature with respect to some displacement parameter may be ascertained. It is then possible to approximate the motion of the link by approximating the behavior of the derivatives of the radius of curvature.

A procedure allowing one degree of freedom in locating the coupler point, such that the zeroes of the first derivative of the radius of curvature are approximated, is presented for the case of an approximately straight coupler point path. Another procedure permitting two degrees of freedom in the coupler point specification is shown for both straight and circular coupler curves. In the case of the two degrees

of freedom procedure, both the first and second derivatives of the radius of curvature are specified with respect to the loci of the zeroes of the derivatives.

For each synthesis procedure, examples are presented. The computer program listings and sample outputs for each example are shown.