Computational Labs in Calculus: Examining the Effects on Conceptual Understanding and Attitude Toward Mathematics

Brielle Tinsley Spencer-Tyree

Dissertation submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
In
Curriculum and Instruction

Bradley D. Bowen, Chair
Aaron C. Clark
Catherine L. Ulrich
Thomas O. Williams

October 16, 2019
Blacksburg, VA

Keywords: STEM Education, Computational Labs, Calculus, Mathematics Education, College and University Calculus, Undergraduate Mathematics, Integrative-STEM Education

Computational Labs in Calculus: Examining the Effects on Conceptual Understanding and Attitude Toward Mathematics

Brielle Tinsley Spencer-Tyree

ABSTRACT

This study examined the effects of computational labs in Business Calculus classes used at a single, private institution on student outcomes of conceptual understanding of calculus and attitudes towards mathematics. The first manuscript addresses the changes in conceptual understanding through multiple-method research design, a quantitative survey given pre and post study and qualitative student comments, found no significant gains in conceptual knowledge as measured by a concept inventory, however, student comments revealed valuable knowledge demonstrated through reflection on and articulation of how specific calculus concepts could be used in real world applications. The second manuscript presents results to the effects on attitudes toward mathematics, studied through multiple-method research design, using a quantitative survey given at two intervals, pre and post, and analysis of student comments, which showed that students that participated in the labs had a smaller decline in attitude, although not statistically significant, than students that did not complete the labs and the labs were most impactful on students that had previously taken calculus; student comments overwhelmingly demonstrate that students felt and appreciated that the labs allowed them to see how calculus could be applied outside the classroom. Overall students felt the labs were beneficial in the development of advantageous habits, taught some a skill they hope to further develop and study, and provided several recommendations for improvement in future implementation. Collectively, this research serves as a foundation for the effectiveness of computational tools employed in general education mathematics courses, which is not currently a widespread practice.

Computational Labs in Calculus: Examining the Effects on Conceptual Understanding and Attitude Toward Mathematics

Brielle Tinsley Spencer-Tyree

GENERAL AUDIENCE ABSTRACT

Students from a variety of majors often leave their introductory calculus courses without seeing the connections and utility it may have to their discipline and may find it uninspiring and boring. To address these issues, there is a need for educators to continue to develop and research potentially positive approaches to impacting students' experience with calculus. This study discusses a method of doing so, by studying students' understanding of and attitude toward calculus in a one-semester Business Calculus course using computational labs to introduce students to calculus concepts often in context of a business scenario. No significant gains in conceptual knowledge were found as measured by a concept inventory; however, student comments revealed valuable knowledge demonstrated through articulation of how specific calculus concepts could be used in real world applications. Students that participated in the labs also had a smaller decline in attitude than students that did not complete the labs. Student comments overwhelmingly demonstrate that students felt and appreciated that the labs allowed them to see how calculus could be applied outside the classroom. The labs were most impactful on students that had previously taken calculus. Overall students felt the labs were beneficial in the development of advantageous habits such as persistence, utilizing resources, and precision, introduced them to coding, a skill they hope to further develop and study, and students provided several recommendations for improvement in future implementation. This research provides a foundation for the effectiveness of computational tools used in general education mathematics courses.

Dedication

I would like to dedicate this to Cody. Your never-ending support got me through this process. Thanks for always going above and beyond to make sure I could get everything I needed to done. Thanks for always keeping the laundry and dishes done and keeping me well stocked in beverages. Also thanks for endlessly reading something in which you had no real interest.

Also I dedicate this to Grandma Marfs, who saw me start this process but unfortunately didn't get to see me through to the end. Wish you could have been here for this!

Acknowledgements

I would like to express my thanks to my advisor, Dr. Bradley Bowen, for always providing support, feedback, guidance, answering my endless emails, scheduling Zoom calls at all hours, and being a calming presence when I had my minor freakouts. Your support and guidance were extremely valuable during this PhD adventure. I would like to thank my committee members, Dr. Aaron Clark, Dr. Catherine Ulrich, and Dr. Thomas Williams. Thank you for always providing feedback, making me clarify my thoughts, asking me questions that pushed me in directions I didn't even realize I needed to go, and helping me through this entire process.

Thank you to the High Point University Department of Mathematical Sciences for your never-ending support during this process. You all inspired me long ago and continue to do so today. Thank you for always inquiring about how I was doing. A special thank you to Dr. Rob Harger and Dr. Jenny Fuselier for being willing to work around my crazy scheduling these last few years to allow me to attend classes, even when I requested those last minute changes. I literally couldn't have done this without you guys accommodating me.

Thank you to Dr. Aaron Titus for introducing me to computational tools in the classroom years ago and impacting my education. Even more importantly thanks for inspiring me to take this on in my classroom and study this innovative teaching method. Thank you also to introducing me to the work of Dr. Jacob Koehler. Thank you Dr. Jacob Koehler for taking your time to share your expertise on Jupyter notebooks, your ideas of content that works well with computational tools, your wealth of knowledge of math history and through lines that connect things I never realized, and meeting this random stranger to share your thoughts.

Thank you to my family for continuously supporting me in so many ways!

Table of Contents

Abstract .... **ii**

General Audience Abstract .... **iii**

Dedication .... **iv**

Acknowledgements .... **v**

List of Figures .... **x**

List of Tables .... **xi**

CHAPTER 1: Introduction **1**

Overview **1**

Definition of Terms **12**

CHAPTER 2: Literature Review **15**

Introduction **15**

Calculus **15**

Pedagogy of Calculus 22

Calculus Related to STEM 34

Attitudes Toward Mathematics **48**

Calculus and Business Calculus Attitudes 49

Attitude and Self-Efficacy 50

Instruments Used in this Study **64**

Calculus Concept Inventory 64

Mathematics Attitudes and Perceptions Survey 74

CHAPTER 3: Manuscript 1 Computational Labs and Conceptual Understanding **85**

Abstract **85**

Introduction **85**

Literature Review **87**

Calculus as a Filter 88

Recommendations for Improving the Courses 91

Research Question **98**

Participants 99

Methodology **101**

Description of Intervention 101

Procedures 105

Results **113**

Equivalence of Groups to Start the Semester 114

Normalized Gains Results 115

Comparing Calculus Concept Inventory Scores 118

Qualitative Results and Discussion from Student Comments 123

Discussion **144**

Improving this Class 150

Limitations and Future Research 152

General Conclusions 156

CHAPTER 4: Manuscript 2 Computational Labs and Attitude Toward Mathematics **158**

Abstract **158**

Introduction **158**

Literature Review **160**

Attitudes Toward Mathematics 161

Attitude and Pedagogy 165

Research Question 176

Participants 176

Methodology 178

Description of Intervention 179

Procedures 182

Results 190

Group Equivalence at the Beginning of the Semester 190

Differences from Pre- to Post-MAPS 191

MAPS Categories Analysis 194

Qualitative Results from Student Comments 200

Discussion 216

Limitations and Future Research 224

Improving this Class 227

General Conclusions 231

CHAPTER 5: Conclusions and Recommendations 234

Conclusions and Implications 234

Recommendations 238

Recommendations for Practice 239

Recommendations for Research 251

References 256

Appendices 292

Appendix A: Course Timelines ... 292

Appendix B: Lab Excerpts ... 293

Appendix C: Online Consent Statement ... 298

Appendix D: Consent Form ... 299

List of Figures

Manustript 1 Figure 1: Model of Career Choice... 58

Manuscript 1 Figure 2: Frequency of Individual Normalized Gains Both Groups. 117

Manuscript 1 Figure 3: Scatterplots of Pre-CCI Percent and Normalized Gain Each Group..... 118

Manuscript 1 Figure 4: Change in Percentage of CCI Scores. 119

List of Tables

Manuscript 1 Table 1: Student Demographic Characteristics 100

Manuscript 1 Table 2: Student Major and Concentrations 100

Manuscript 1 Table 3: Experimental and Control Group CCI Percentage Score Statistics........ 116

Manuscript 1 Table 4: Normalized Gains Computed from Group Averages 117

Manuscript 2 Table 5: Student Demographic Characteristics 177

Manuscript 2 Table 6: Student Major and Concentration... 178

Manuscript 2 Table 7: Control and Experimental Group MAPS Statistics 193

Manuscript 2 Table 8: MAPS Constructs Differences 195

Manuscript 2 Table 9: MAPS Constructs Correlations for the Experiemental Group 196

Manuscript 2 Table 10: MAPS Constructs Correlations for the Control Group 196

CHAPTER 1: Introduction

Overview

Some version of introductory calculus is a requirement in the college careers of numerous students from many different majors, and the experience in a calculus classroom can have a great impact on students' conceptual understanding and attitudes. Too often students leave their introductory calculus course without seeing the connections and utility the techniques may have to their chosen discipline, may find the course to be uninspiring and boring, and may actually develop a negative attitude toward mathematics and their abilities because of such a course. To address the needs of the vast amount of students tasked with taking calculus, their necessity of gaining knowledge of the subject, and the impact the course may have on their attitudes, there is a need for educators to continue to develop, test, and research potentially positive approaches to teaching and helping students experience calculus.

The President's Council of Advisors on Science and Technology (PCAST, 2012) reports that college students complain that their introductory STEM courses are unwelcoming from faculty, provide insufficient math support, and deliver uninspiring environments; there is a need to reform STEM classes, including calculus, at the post-secondary level to address some of these complaints (Frechtling, Merlino, & Stephenson, 2015). There is a "long overdue reconsideration of the appropriate intellectual content of calculus," (Kaput, 1997, p. 731). There have been major changes in who is now expected to complete calculus, including most STEM majors and most business majors, but there have not been grand changes to how or what is taught in a calculus course leading to an increasingly poor fit for today's students (Kaput, 1997). The way calculus is taught only serves about 10% of the population, which includes the "socio-economic and intellectual elite" (Kaput, 1997, p. 731); rethinking the way calculus is taught could open it up to the ignored 90% and allow for deeper understanding for the 10% to whom calculus has traditionally been catered (Kaput, 1997).

Research reveals that beyond finding their introductory calculus classes dull and uninspiring, students around the world also have problems with basic conceptual understanding of calculus (Epstein, 2013). Students are finishing first semester calculus courses with little to no basic conceptual knowledge that university faculty assumed they were all developing (Epstein, 2013). Students often learn to answer questions involving limits and derivatives through routines, algebraic skills, and rules, which does not equate to them understanding the concepts of limits and derivatives (Muhundan, 2005). Many calculus classes become overloaded with rules and algebraic manipulation with little time spent on concepts (Ferrini-Mundy & Graham, 1991). Knowing how to find derivatives and integrals is meaningless unless students can use them and interpret results in real life contexts (Gordon, 2005). Numerous studies have found that in introductory mathematics courses, students are not learning the intended material (Breidenbach et al., 1992; Carlson, 1998; Tallman et al. 2016; Thompson, 1994), students are leaving the courses unprepared for other courses (Carlson, 1998; Selden, Selden, Hauk, & Mason, 2000; Thompson, 1994), and students lose interest in mathematics after completing the course (Bressoud, Mesa, & Rasmussen, 2015; Seymour, 2006), all of which have been seen in both small, localized studies but also in national studies of introductory college mathematics courses (Bressoud, Mesa, & Rasmussen, 2015). It could be hypothesized that these issues with conceptual understanding may come from a variety of sources including the poor attitudes students have towards mathematics, lack of foundational skills before entering the course, pedagogy, and other classroom factors. It appears students are coming away from calculus without appreciating its connections to other disciplines, connections to real applications, and viewing it as yet another mathematics course focused on rote memorization and algebraic manipulation and thus unproductive; this must be addressed and taken up by mathematics educators.

In addition to content knowledge issues, Bressoud & Rasmussen (2015) reveal that post-secondary calculus, as currently taught, "is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics." Instructors of undergraduate calculus are keenly aware that students find calculus to be a challenging course and are often fearful of it (Sonnert, Sadler, Sadler, & Bressoud, 2015). Factors that may be contributing to these student attitudes toward mathematics include the instruction students experience including: pedagogical decisions and use of technology, environmental factors, and personal factors including previous interaction with and attitudes toward mathematics and the choice of college major (Sonnert, Sadler, Sadler, & Bressoud, 2015).

With calculus lowering confidence and enjoyment of mathematics, it is often a filter that forces students out of future careers including STEM careers and business (PCAST, 2012; Mosina, 2014). The negative attitudes that are developed or perpetuated in many calculus courses may be doing more than simply filtering students out of career fields but may also be affecting performance in the classroom since that and learning outcomes can be affected by students' attitudes toward math (Boaler, 2016). Negative attitudes toward mathematics may be severely hindering students' successes, so developing a more positive attitude toward mathematics is vital for potentially increasing success. Andersson, Valero, and Meaney (2015) acknowledged that negative attitudes are influenced by the students' beliefs and experiences and an interaction of classroom and pedagogical experiences, which implies that educators and how they run their classrooms can have a major impact on students' attitudes toward mathematics. Students' mindset toward math can be changed and affected by the context of the teaching (Andersson, Valero, & Meaney, 2015); since attitudes can be changed, there is a need to create an environment that fosters more positive attitudes toward mathematics to nurture mathematical skills, which could include more relevant contexts, more modern topics, more modern approaches, and more technology since research supports that teaching mathematics within relevant contexts and with real world connections can impact student attitude positively (Cornell, 1999; Andersson, Valero, & Meaney, 2015).

These issues of minimal conceptual knowledge, leaving the course with marginal understanding of how calculus can be applied to other disciplines, recognition by students that calculus courses are often dull, and the detrimental effects the courses seem to be having on students' attitudes, it is imperative to consider changes to calculus courses that could more positively impact students' attitudes and understanding. Calculus courses have remained relatively stable in their content and pedagogy because change has been inhibited by a desire to make changes while stakeholders in education continue to rely on traditions and are not adapting to fit the changing world outside of school (Kaput, 1997). Recommendations for changes to post-secondary STEM classes include moving away from lecture courses that focus on facts and that "instruction needs to be more engaging, deliberately structured to involve a range of cognitive processes, and oriented toward deeper understanding," (Frechtling, Merlino, & Stephenson, 2015, p. 29). To truly reflect some of these recommendations, changes must be made to curriculum and pedagogies (Frechtling, Merlino, & Stephenson, 2015).

Calculus reforms of the late 1980's and 1990's emphasize similar recommendations calling for technology to be used to help students move away from as much algebraic manipulation, reduce the amount of tedious calculations, use visualization, and to explore mathematical situations and call attention to calculus needing to include more modern mathematical ideas, more realistic applications from various disciplines, and more conceptual understanding (Muhundan, 2005). There have been repeated calls to reform mathematics education to improve student understanding and performance including making use of calculators and computers (National Council of Teachers of Mathematics [NCTM], 1974, 1980, 1989, 2000). The change requires mathematics educators to be open to the use of technology, organization of topics, and what counts as mathematical thinking (Kaput, 1997).

Technology in the mathematics classroom has the potential to improve students' learning and students' attitudes and motivation levels toward mathematics (Muhundan, 2005). "Mathematics programs must take full advantage of the power of calculators and computers at all levels" (NCTM, 1980, p. 8). "Computer technology is changing the ways we use mathematics; consequently, the content of mathematics programs and methods by which mathematics is taught are changing" (NCTM, 1989, p. 2); "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student's learning" (NCTM, 2000, p. 24). Technology has the potential to move mathematics instruction away from manipulative skills and toward "developing concepts, relationships, structures, and problem-solving" (Corbitt, 1985, p. 244). To affect students' understanding and attitudes, the inclusion of technology must be considered and as advances in technology have been made this consideration must go beyond the use of a graphing calculator, which is the dominant form of technology in calculus classes (Selinski & Milbourne, 2015). When considering technology, it is important to consider technology that could be beneficial outside of the classroom as well. "With mathematics seeing an increasing focus on computation, mathematics education should not be far behind in its pursuit to understand the teaching and learning of computing within mathematics" (Lockwood, DeJarnette, & Thomas, 2019, p. 1). "We are living in an ever-evolving computerized age, and we are seeing trends in mathematical research and mathematics education research that reflect our society" (Lockwood, DeJarnette, & Thomas, 2019, p. 1). With the importance of such computational tools growing within mathematics, there has also been a rise in research on these tools in mathematics education (e.g., Cline et al., 2019; Jones & Hopkins, 2019; Kilty & McAllister, 2019) and must continue to be studied. diSessa (2018), Lockwood, DeJarnette, and Thomas (2019), and others encourage mathematics educators to research how their ideas of how computers and computational tools can be used in mathematics learning and assert that there is still much to be studied and developed in effective learning and learning of computing within mathematics.

The inclusion of technology and computing into the calculus classroom, however, requires more than simply adding it to how students can solve problems. To best serve students, including technology in the mathematics classroom calls for revamping curriculum and modifying instruction (Dossey, Mullis, Lindquist, & Chambers, 1988). In 1987 a national colloquium was held to discuss improving calculus and a report was produced titled *Calculus for a New Century: A Pump Not a Filter*, which described support for reforming calculus and to modernize it; "We need to teach calculus in a way that facilitates complex and sophisticated numerical computation in an age of computers. Somehow or other we have to make calculus exciting to students," (The Mathematical Association of America, 1988, p. 9). More than thirty years after this recommendation, this is still important to consider.

The 1970s and 1980s saw rise to publications on a finite approach to calculus incorporating computational tools (Gordon, 1979; Ralston, 1984; Hoffman, 1989), and during that time it was identified that computers were only going to grow in importance and use in daily life for all and in all facets of life, including mathematics (Gordon, 1979). The computing power at the time was a limiting factor in the implementation in the classroom. In the early 1990s, in response to the mathematics education community's call for improving calculus, there was a flurry of research on innovative ideas of teaching calculus, many of which incorporated computers, mathematical programming languages, and other computational tools in calculus courses (Tucker, 1990). There is promise in this prior research on how interventions such as these could positively impact students' understanding of and attitude toward calculus. Research on similar technologies has continued to the present, evolving as technology progresses, reporting similar results (e.g., Cline et al., 2019; Cetin & Dubinsky, 2017; Fenton & Dubinsky, 1996; Heid, 1988; Jones & Hopkins, 2019; Kilty & McAllister, 2019).

Motivated in part by previous research and the researcher's own classroom experiences, one potential intervention that could have an impact on student achievement and attitude toward mathematics, specifically calculus, is to teach calculus, at least for some students, using computational labs to introduce students to calculus topics, which could take full advantage of today's computational tools and update how students learn calculus. The computational labs suggested in this project include topics that connect calculus to other disciplines, often include real data sets, frequently introduce students to topics through finite and discrete elements, and introduce students to new problem-solving techniques with technology new to them. Analyzing real world problems using computational tools is more reflective of how math and other professionals, including STEM and non-STEM professionals, use mathematical knowledge outside of the classroom. Computing can bring mathematics education more in line with professional practices (Weintrop et al., 2015). There is a level of authenticity and real world use, both mathematically and for computational tools, when they are applied to mathematics and science activities (Weintrop et al., 2015). Many who use mathematics in their careers or in their workplace are not doing it with paper and pencil, especially not for complex scenarios that arise from real world situations, but changes have not been reflected in mathematics curricula and pedagogy, while changes have arisen in other disciplines including physics, chemistry, and economics (The Mathematical Association of America, 1988).

Demands on today's college-educated professionals require them to be "creative, confident, competent problem-solvers, and clear, critical thinkers", which can be developed in part in their exposure to undergraduate mathematics that includes modeling, inquiry, and using technological tools to solve problems from all disciplines (Arney, 2009, pp. 94-95). When looking at student attitudes of mathematics, one major complaint is the lack of relevance and connection to the outside world. "Students see mathematical tools for the life sciences and social sciences as useful, interesting and beautiful when they learn to use them in realistic applications and when computers do the calculations" (Hoffman, 1989). The use of modern technology can be extremely beneficial for students to learn calculus with true business applications, especially for students with weak math skills (Liang & Pan, 2009) and negative attitudes. The use of computational labs could provide the relevance and real world applications students are craving.

Research reveals that there is tension between how calculus can be taught at the post-secondary level to deal with the diverse needs of students from different majors that are all required to take calculus (Rasmussen, Marrongelle, & Borba, 2014) and what those students need to be exposed to and learn in their undergraduate calculus course. Students from economics or business do not necessarily need the same calculus as mathematics or engineering majors. The President's Council of Advisors on Science and Technology (2012) details that in response to the negative impact these introductory mathematics courses can have on students, client disciplines are no longer satisfied with how students are emerging from calculus courses and recommends the faculty from the client disciplines develop curricula for and teach the mathematics courses. Mathematics educators must take note of this and make appropriate changes. Perhaps a more modern approach that reflects technology used in the workforce, including coding and applications to real life scenarios, would allow students to see math come alive, improve their attitudes toward it, and impact their conceptual knowledge of calculus.

The current study is an attempt to empirically determine the extent to which using computational labs with business applications to introduce students to calculus topics in an undergraduate Business Calculus class affects students' conceptual knowledge gains in calculus and their attitudes toward mathematics. Business Calculus is an ideal course for implementation of computational labs to introduce students to calculus topics because it is often extremely daunting to undergraduate business students (Depaolo & Mclaren, 2006). "Business students, although able, are often math phobic. Courses should strive to lessen math phobia, enable students to be more comfortable with mathematics, and help students appreciate the relevance of mathematics," (Lamoureux, Beach, & Hallet, 2000, p. 19). Many universities report Business Calculus as a course with high D, F, and withdraw rates, describe it as a course where a high level of resistance to mathematics is present, and note that it is often comprised of students that are unprepared for and unexcited to take the required course (Depaolo & Mclaren, 2006; Liang & Pan, 2009). Hoffman (1989) points out that calculus has been inappropriate for social and life sciences since teachers use concocted real life examples, but students still perceive much of what they have been learning in these calculus classes as something they would never use outside of the classroom. Challenges to what is taught and how it is taught also come from students asking when they will ever use any of what they are learning in their future careers and ask why they cannot work the problems on the computer like they do in their business classes and as reflected by professionals. Calculus can be reconceived with the computer, which will allow for development of courses that can allow for more applications outside of the concocted examples (Hoffman, 1989). Motivation, participation, and interest are all shown to improve in business calculus classes when computers are allowed as an aid and more emphasis is placed on understanding and application (Judson, 1990). The age-old student question of "why do I have to learn this stuff?" is gone because students are allowed to see the calculus in action with real applications and are less mired by the stress of by-hand skills and techniques (Judson, 1990).

While there is promise in previous research and recently there has been an increase in research on computational tools in mathematics education, there is need for further study of such tools in various introductory settings to examine how these can affect students' experiences in understanding and attitude of calculus. More empirical research must be done if mathematics educators are to be convinced to use computational tools in calculus to modernize how students learn it, which may potentially contribute to gains in understanding and improve students' attitude toward mathematics.

Purpose of the Study

This study investigates students' conceptual understanding of calculus and attitudes toward mathematics in a one-semester Business Calculus course where students are introduced to calculus topics using computational labs. Investigating student gains in conceptual understanding from the beginning of the semester to the end of the semester provides empirical evidence about the effectiveness of computational lab activities on conceptual knowledge. Students' conceptual understanding of calculus is measured through the administration of the

Calculus Concept Inventory, an assessment tool that focuses only on conceptual topics covered in first semester calculus, as a pre-test and post-test, and through analysis of student comments on labs. Studying students' attitudes of mathematics determines if using computational labs in Business Calculus has an impact on students' attitude toward mathematics. Students' mathematical attitudes are considered through the Mathematics Attitudes and Perceptions Survey and student comments. Findings from this study have implications for mathematics educators who are looking to find ways to make mathematics, specifically calculus, more modern, relevant, applicable, understandable, and enjoyable for the masses that are now required to take it.

Definition of Terms

The following definitions are given to familiarize the reader with key terminology used in this research.

Business Calculus. At the university where this study took place Business Calculus is a 1000-level mathematics course that is a requirement for all business majors and can fulfill the general education mathematics requirement for some other majors as well. The course is called Calculus for Business and Social sciences. The description of this course is: "An introduction to the concepts of differentiation and integration with emphasis on their applications to solving problems that arise in business, economics, and social sciences." Students that take this class often fulfill their mathematics requirement with this course and beyond this they may also be required to take a statistics class for business majors.

Computational. The use of the word computational or computing follows Lockwood et al. (2019) definition of computing within mathematics, which is "the practice of using tools to perform mathematical calculations or to develop or implement algorithms in order to accomplish a mathematical goal" and "from a calculation perspective, computing involves using a tool to complete numerical or symbolic calculations." Here the tool used is a computer on which Jupyter notebooks running Python were employed. The 2005 Joint Task Force for Computing Curricula report defines computing stating, "In a general way, we can define computing to mean any goal-oriented activity requiring, benefiting from, or creating computers" (p. 9).

Computational labs. Computational labs in this project were teacher-generated Jupyter notebooks, running Python, in which students were presented with notes, formulas, some teacher generated functions and pre-written code, and visuals. Students were asked to follow along through the lab in class as it was projected onto the big screen. Students had to add lines of code as the teacher did to get the results they wanted. The teacher and students worked through problems together starting with a mathematics problem and having to type out code to work toward a solution. Students also had anywhere from 6-9, often multi-part, problems at the end of each lab that they had around one and a half weeks to complete. Completing each problem often involved students executing multiple steps and multiple lines of code. Some problems involved importing data sets from internet sources such as the CDC or financial data from Yahoo, and in other problems students were given smaller data sets and asked to input them into a format the computer could understand to then work with. Students employed commands from symbolic and numerical libraries as well as the data science library, pandas. The majority of the problems students were asked to complete at the end of labs were presented in context of a business or economic problem.

Conceptual calculus knowledge. Conceptual knowledge is defined as “implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain” (Rittle-Johnson et al., 2001, p. 346) and “thus, there is general consensus that conceptual knowledge should be defined as knowledge of concepts”, (Rittle-Johnson & Schneider, 2015). Using these components of the definition of conceptual knowledge and considering the concepts that are almost universally covered in introductory calculus classes, which are limits and continuity, derivatives, and integration (Burn & Mesa, 2015), conceptual calculus knowledge would include the underlying concepts of these, not including the computing of these by hand (i.e. computing derivatives or integrals using the rules). Epstein, the developer of the CCI, uses the term conceptual calculus knowledge in describing what the CCI assesses, which he states to have domains of functions, derivatives, and limits/ratios/the continuum.

Visualization. Discussion of visualization emerges from student comments on how they thought the labs helped them learn. The way the term visualization is used in this study is in accordance with Zimmermann and Cunningham (1991), Hershkowitz et al. (1989), Arcavi's (2003) definition of visualization as follows: "visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings" (p. 217). Rosken and Rolka (2006) state:

> This definition emphasizes that, in mathematics learning, visualization can be a powerful tool to explore mathematical problems and to give meaning to mathematical concepts and the relationship between them. Visualization allows for reducing complexity when dealing with a multitude of information. (p. 458)

CHAPTER 2: Literature Review

Chapter 2 provides a review of literature in the areas of rationale for studying college calculus, calculus and I-STEM education, business calculus, computing technology in mathematics education, and attitude. The literature review also provides an overview of the instruments used in this study.

Introduction

Students across most STEM majors and business majors are required to finish calculus, however, the report of the President's Council of Advisors on Science and Technology (PCAST) reported that college students "describe the teaching methods and atmosphere in introductory STEM classes as ineffective and uninspiring" (PCAST, 2012, p. 5), which may mean that what is taught and how it is taught may be a poor fit for the vast amount of students tasked with completing calculus. The implications of these findings are a need for better teaching of introductory STEM classes such as calculus. The changes that could have an impact require mathematics educators to be more open to the use of technology, organization of topics, what counts as mathematical thinking (Kaput, 1997), and to continue to pursue and research ways of improving the courses.

Calculus

Introductory calculus has long been a required course for most STEM majors and business majors as well. Mathematics educators have continuously been developing and studying ways to make this course more effective for their students. While there have been changes, overall the content and pedagogy have remained relatively stagnant and may be contributing factors for students lamenting that introductory STEM courses, such as calculus, are "over-stuffed" with material that is taught too quickly, typically with lecturing, lacking "application, illustration, or discussion of conceptual material" (Seymour, 2006, p. 4), and seemingly disconnected from their interests or intended discipline.

Introductory calculus classes are affecting students' attitude and understanding of calculus. Calculus is still a filter causing many students to change majors or career paths. In any given calculus class around 25% of students achieve a D, F, or withdraw from the course (Bressoud, Carlson, Pearson, & Rasmussen, 2012). A national study of calculus found that of those students that do complete the course only about two-thirds self-report being able to correctly compute derivative and integrals, only 40% feel confident in their ability to use ideas of calculus, and less than 30% felt that the course increased their interest in mathematics (Bressoud, 2015). This same study found that introductory calculus is exceptionally effective in lowering students' confidence in mathematics, enjoyment of mathematics, and desire to take any more mathematics classes (Bressoud, 2015). Such findings indicate an assiduous need to continue to study ways in which calculus may be taught to impact student learning and attitude and research suggests that finding more innovative and engaging teaching methods could improve such student outcomes. This study contributes to this literature by examining the effects on students' learning and attitude by one particular innovative strategy in teaching calculus.

Calculus Content Covered

The content of a traditional introductory college calculus course covers basic differential calculus and some integral calculus (Burn & Mesa, 2015). A national study found that, in general, instructors felt they had enough time to help students understand difficult concepts and did not feel rushed to get through all of the required material (Burn & Mesa, 2015), however, students do not seem to feel the same way, often reporting feeling rushed, the course is packed with too much material, and perceptions of not having enough opportunities to learn the more difficult ideas of calculus in the course (Hagman, Johnson, & Fosdick, 2017). The content covered must be considered:

> The content of Calculus I has remained relatively stable over the decades regardless of calls for a 'lean and lively calculus' (Douglas, 1986), the calculus reform movement of the 1990s (Hughes-Hallett, n.d.), or the general trend toward reducing content in mathematics courses (Hillel, 2001). To be sure, course content in Calculus I has changed over time. Delta-epsilon proofs are no longer considered a standard part of the Calculus I curriculum. The 'rule of four' calling for a mix of graphical, numerical, symbolic, and verbal approaches has had a lasting impact on current textbooks. The availability of technology has enabled increased emphasis on visualizing graphs of functions and their derivatives and other graphical connections. (Burn & Mesa, 2015, p. 46)

It is not to say that calculus has not changed at all, because indeed it has, but one must continue to consider how it is impacting students and what content can benefit the particular population.

A traditional calculus course is typically targeted toward supporting students of majors such as mathematics, chemistry, physics, or engineering, however, students of many other disciplines may be tasked with taking an introductory calculus course and thus the traditional content may not be wholly appropriate. There has been some fracturing of calculus into courses targeting specific disciplines, however, tailoring specific calculus courses for the various types of needs of different disciplines may not be possible at all institutions because of staffing and enrollment numbers. When courses are fractured into different courses based on the discipline they are serving some content is changed based on what that discipline needs, and one major change is the applications presented, such as the course in this project, but the content largely remains the same.

The content of first semester calculus courses nationally has four major content areas, which cover basic differential and some integral calculus (Burn & Mesa, 2015). These content areas include: limits and continuity, derivatives, integrals, and sequences and series (Burn & Mesa, 2015). This has remained relatively stable over the last several decades (Burn & Mesa, 2015). Typically Calculus I includes the "rule of four" – graphical, numerical, symbolic, and verbal- and can be seen in current textbooks (Burn & Mesa, 2015, p. 46). The traditional Business Calculus courses in this study aligns with the aforementioned and cover limits, continuity, the derivative, differentiation rules, applications of the derivative, exponential and logarithmic functions, the definite and indefinite integral, and basic methods of integration much like Narasimhan (1993) reports calculus courses for students of non-science client disciplines.

Limits and Continuity. Concepts of the limit are seen as important in first semester calculus; yet limits are often a stumbling block for students in first semester calculus (Parameswaran, 2007). Students are often "expected to develop an intuitive understanding of the limiting process, calculate limits using algebra, estimate limits from graphs or tables, and find limits at infinity" (Burn & Mesa, 2015, p. 46).

> The concept of limit is the cornerstone of several related concepts such as continuity, differentiability, integration, convergence of sequences and series, etc. The precise, formal definition of the concept of limit is so complex and counterintuitive that it fails to bring out readily the simple and intuitively obvious ideas, which led to it in the first place. The definition of a limit involving universal and existential quantifiers is designed to solve mathematical difficulties and not psychological ones. (Williams, 2001, p. 342)

Limits are used in the limit definition of the derivative and moving from Riemann sum to the integral. Continuity is also often covered and involves an intuitive understanding of it and being able to determine it in terms of limits using the definition (Burn & Mesa, 2015). All of these are covered in the traditional sections of the Business Calculus course in this study. One main change in content in the experimental course in this study is less of a focus on limits, especially computing them algebraically. Students will not be expected to do so, but will need an intuitive understanding of limits to extend from finite beginnings to the study of continuous.

The formal definition is not taught in either variant of the course. The alternate course places less of a focus on limits than the traditional course; the finding of limits algebraically is not taught. Finding limits graphically and an intuitive understanding of limits are taught. Continuity is not formally taught, as in using the definition of continuity; continuity was not determined to be critical content by calculus experts (Burn & Mesa, 2015), and thus is an area that is not stressed. Continuity also has a lesser place in this section because many of the problems will not start as continuous, but rather be situated in finite, discrete data points. For example in the income in equality lab example, the data is five data points, to which later students fit a curve.

Derivatives. Calculus courses typically cover four areas related to derivatives: the concept of derivatives, computation of the derivative, graphical connections and visualization of the derivative, and applications of the derivative (Burn & Mesa, 2015). Teaching the derivative often begins with the concept as instantaneous rate of change, the limit of the difference quotient, and then progresses into derivatives of basic functions and derivative rules for computing the derivative (Burn & Mesa, 2015). The traditional Business Calculus course covers these areas. Derivative rules include the power, sum, product, quotient, and chain rules. Derivatives of exponentials and logarithmic functions are covered, but trigonometric functions are not taught, which is standard for Business Calculus at the university where the study took place and similar to Narasiamhan's (1993) discussion of a calculus course for non-science client disciplines. Implicit differentiation and related rates are not covered in this course, although typically covered in a standard Calculus I course. Derivatives and their graphical connections are studied including approximating rates of change from graphs and tables and used in the analysis of f, f', and f''. Applications of derivatives are covered including interpreting rates of change and optimization. Applications of the derivative in the Business Calculus course typically revolve around marginal analysis and optimization problems. In the alternate version of the course, as in the traditional class, students are shown how to compute derivatives and learn the rules for finding derivatives because at the time of implementation these are required outcomes of the course. Overall less time is spent on computing these to allow time for labs and more time for applications of derivatives. The concept of derivative is developed first through finite differences. Applications of differentiation are covered. Similar to the traditional course these include optimization and marginal analysis.

Integrals. Burn and Mesa (2015) found that content involving integrals includes the indefinite integral as an antiderivative, the definite integral as a limit of Riemann sums, interpretation and applications of the definite integral, and techniques of integration with basic functions and substitution. These are covered in the traditional course. The traditional Business Calculus course in this study tends to include applications of integration related to business such as consumer and producer surplus. In the alternate version of the course some time is spent on techniques for integration, as this is still part of the required course content, but overall less time is spent on this than in the traditional course. Time is also spent on definite integrals and their applications.

Sequences and Series. While sequences and series were reported to be one of the major content areas of calculus courses in Burn and Mesa (2015), these are not typically covered in the Business Calculus course at the university where the study took place and are not taught to the traditional group in the proposed study. While sequences and series are given no attention in the traditional course, they are briefly touched on in the course with the labs. Sequences are used in a lab to make connections to finite points and functions, connecting arithmetic sequences to linear functions and geometric sequences to exponential functions.

Types of Questions. In addition to the content covered, the national study of college calculus surveyed instructors about the types of questions faculty choose to include on assignments or exams of which 50% reported being comprised of skills and methods for computation, 20% on graphical interpretations of central ideas, 20% on solving standard word problems, 20% on solving complex or unfamiliar word problems, and 10% reported proofs or justification (Burn & Mesa, 2015, p. 48). The highest percentage of problems were centered on skills and methods for carrying out computations, which aligned with the fact that two-thirds of surveyed faculty believed that understanding concepts in calculus comes after achieving procedural fluency (Burn & Mesa, 2015).

This, however, could be called into question as we are living in a world where students have seemingly instant access to technology that can quickly perform some of these computations for them. The time has arrived to question what content needs to be taught and how it can be taught in ways that may not compete with the computer but rather compliment it at least for certain populations tasked with taking the course where understanding of how the calculus concepts can be used in their selected field may be more beneficial. The labs in this project remove some of the focus off of by-hand computations, allowing the computer to perform these, and instead focus on using the concepts and uses of them. Burn and Mesa (2015) recommend, "faculty modify assignments to more frequently include questions that require students to apply knowledge, make interpretations, or provide explanations related to the calculus I concepts" (p. 53). These labs have elements that do these things; one such example being giving students data and simply asking them to create a model and help a company determine how to maximize their profits.

Pedagogy of Calculus

The relatively stable content in calculus tends to be taught using lecture, as lecture is still the dominant style of teaching for Calculus I (Larsen, Glover, & Melhuish, 2015). Despite lecture being shown to be less-effective pedagogy, it is still college faculty's primary pedagogical strategy (Seymour, 2006). Research suggests that during lecture, students are only passively engaged and very few are actively engaged in constructing mathematics during it (Lew et al., 2016). Recently, concern has risen as to undergraduate STEM course quality and instructor engagement because many are taught with traditional lecture driven methods (Frechtling, Merlino, & Stephenson, 2015). There is voluminous research on other teaching practices in undergraduate calculus such as flipped classroom (e.g., Zack et al., 2015), homogenous grouping (e.g., Carnell et al., 2018), modeling approaches (e.g., Kilty & McAllister, 2019), and many more ambitious teaching methods, but overall there have not been wide spread changes in pedagogy.

The traditional sections of Business Calculus in this study will be taught using predominantly lecture. Outside of lecture, students in this course will also have the opportunity to work during most class meetings with classmates on problem sets and during lecture or group work times students will be allowed and encouraged to ask questions. The class also often allows for the popular exchange of IRE/F (initiation, response, evaluation/feedback) (Cazden, 1986; Hicks, 1995-1996). All of which are described as typical student teacher interactions in the national study of college calculus and overall the course follows the five practices related to good teaching of introductory college calculus, which include creating a positive atmosphere where students are encouraged to ask questions, maintaining positive attitudes toward students' mistakes, setting high, clear, and attainable expectations, having availability to answer student questions, and ensuring reasonable pacing of lectures and time for group work (Mesa, Burn, & White, 2015).

Computational Labs. The classes that receive the intervention in this project include lecture but also include computational lab activities. There were six labs during the fifteen-week semester. Examples of lab activities in calculus courses of Kowalczyk and Hausknecht (1994) and Basson, Krantz, and Thorton (2006) have five to six labs in the fifteen-week semester. The labs were done using Jupyter notebooks. The lab activities were used to introduce topics. This structure is done because the labs introduce students to calculus topics often using discrete data points, elements such as finite differences, and business applications and then extends them to the more traditional continuous approach to calculus covered more in the lecture portion of the course.

Labs in a calculus course are not new and are a suggestion of the Mathematical Association of America's Committee on the Undergraduate program in Mathematics' subcommittee Curriculum Renewal Across the First Two Years (MAA CRAFTY) project's recommendations for business to support student learning (Lamoureux, Beach, & Hallet, 2000). A review of literature revealed that labs in calculus courses take on a variety of forms and cover a variety of content (Leinbach, 1991). Some labs are supplemental and optional for extra help, some include a separate lab section, some courses are completely laboratory, the content of the labs varies, and the technology varies (e.g., Leinhach, 1991). A calculus lab can be used as a "learning device to see how calculus applies to other courses and disciplines" and "helps students relate the rather abstract ideas of mathematics to non-mathematical ideas they have encountered in other courses" (Basson, Krantz, & Thorton, 2006, p. 346). An example of a successful implementation of a calculus lab entailed enhancing the existing calculus course and having the main goal of the lab not be to teach additional material but to teach students to make connections (Basson, Krantz, & Thorton, 2006). Successful calculus labs also seem to use real data (e.g., Basson, Krantz, & Thorton, 2006; Kowalczyk & Hausknecht, 1994), which this project also aims to do.

Technology. A pedagogical consideration that all teachers must make is the use of technology. This is a distinguishing factor between the traditional and alternative Business Calculus courses that was studied. Selinski and Milbourne (2015) assert that technology in the calculus classroom has increased as availability has increased and tends to include graphing calculators, computer algebra systems (abbreviated: CAS; i.e., Mathematica, Maple, and MATLAB), clickers, online homework systems, and computer simulations. The graphing calculator, however, is the dominant form of technology used in the undergraduate calculus classroom (Selinski & Milbourne, 2015). The graphing calculator was allowed in the traditional Business Calculus classes in this study. The graphing calculator was not used to intentionally teach calculus but was used by students for arithmetic computations and graphing. The national study of calculus makes note that some students reported their instructors using computer algebra systems (CAS – such as MAPLE, Mathematic, or MATLAB) to demonstrate mathematical ideas and some students were expected to use them as well (Sonnert & Sadler, 2015), but the graphing calculator was still the dominant form of technology students were expected to use. Studies on technology in the calculus classroom show mixed results (Selinski & Milbourne, 2015). It is important to consider how different technologies could impact students' learning and attitude in a calculus course.

The Jupyter notebooks running Python are the technology employed in this study. Koehler and Kim (2018) support the use of Jupyter notebooks and Python in the mathematics classroom and provide information as to how it can be used. Koehler and Kim (2018) assert that Jupyter notebooks and Python are easy to use even for teachers with little to no background in coding and computing and students can easily discover how technology can help them solve mathematics problems and communicate solutions to challenging and real problems. Positive results were seen in a course designed for biology and chemistry majors using Jupyter notebooks because they can have many different types of cells, easy to use, and are easily shared (Smith, 2016). Python is a popular, commonly used programming language for scientific computing and data science that has a large focus on ease of use and readability (Meurer et al., 2016). SymPy, an open source computer algebra system written in Python, can also provide the basic operations of calculus (Meurer et al., 2016). Technology is playing a growing role in the financial and business industries, and Python, with many open source financial libraries, is growing in importance (Hilpisch, 2016), so exposing business majors to this computing language could teach them a valuable skill they may be expected to use in industry.

Computational Tools. The national study of calculus (Bressoud, Mesa, & Rasmussen, 2015) does not mention the use of coding and computing in a programming language as will be used in this study, however, the use of software in mathematics has grown of late and it is known that these tools are useful for professional mathematicians, so students should be exposed to these tools. Often software can make mathematical computation and inquiry quicker and more accessible to those not advanced in their mathematical careers (Quinlan, 2016). A recent study aimed to look at mathematicians' attitudes toward software to teach and learn mathematics and sought to find any software that mathematicians recommended (Quinlan, 2016). There is a commonly held stereotype that all mathematicians insist on doing calculations with pencil and paper although that may not be the case and these misconceptions may even have implications for teacher education. Many university mathematics professors, surveyed in several recent studies, reported that technology was significantly important in mathematics and mathematics teaching (Lockwood et al., 2019; Quinlan, 2016). University professors recommended software including MATLAB, Maple, and Python as well as others (Quinlan, 2016). Overall there was support for technology in post-secondary mathematics courses, responses varied for technology inclusion in K-12, and recommended tools included Python, Maple, and MATLAB (Quinlan, 2016). There seems to be an overall lack of research in mathematics education journals connecting computational tools to practice, although there is recent growth in this area. There is some research on mathematical programming languages such as Maple, Mathematica, and MATLAB but not as much on other tools such as Python that could more easily be used in other disciplines outside of mathematics. With an influx of computational tools used in mathematics, it is an area that has seen a rise in research in the mathematics, and specifically calculus, classroom and where this project is focused. "With mathematics seeing an increasing focus on computation, mathematics education should not be far behind in its pursuit to understand the teaching and learning of computing within mathematics" (Lockwood, DeJarnette, & Thomas, 2019, p. 1). diSessa (2018) encourages mathematics educators to empirically and theoretically flesh out their ideas of how computers and computational tools can be used in mathematics learning and calls for a clearer vision of how it will progress into the future. This project is one ideation of how computational tools could be used in a specific calculus classroom.

Research on the use of Computational Tools. The use of computational tools in mathematics education is not a new concept. Discussion of computational tools in the classroom must include discussion of Seymour Papert, who was part of the creation of the computer programming language LOGO, which let students use mathematics to construct on the computer. Papert is behind constructionism and asserts that better learning comes from giving the learner better opportunities to construct (Harel & Papert, 1991). In constructionism, learning happens when students are constructing a meaningful product, which could be a computer program or writing working code. The theory supports that when students construct things concretely they are also constructing knowledge in their minds, and the new knowledge allows for them to construct more complexities outside the classroom, which continues to perpetuate the cycle of knowledge construction (Harel & Papert, 1991). By using primitive rules that connect to what they currently know, learners can construct, or program, artifacts through which they can reorganize, reconstruct, and build on previous knowledge to produce new ideas (Wilkerson-Jerde, Wagh, & Wilensky, 2015). In *Mindstorms,* it is put forth:

> the child programs the computer. And in teaching the computer how to think, children embark on an exploration about how they themselves think. The experience can be heady: Thinking about thinking turns the child into an epistemologist, an experience not even shared by most adults. (Papert, 1980, p. 19)

Papert uses the word computation to emphasize a thinking process not necessarily a sequence of linear steps (Jahnke, 1983). Papert also supports that what was taught in mathematics was dependent on the technology available at the time (Jahnke, 1983).

Other investigations of computational tools in mathematics, and specifically calculus, classrooms have revealed mixed results both in understanding and attitude. Computational tools could allow for mathematics to focus on the concepts rather than a large emphasis on the skills (Heid, 1988). Heid, Blume, Hollebrands, and Piez, (2002) find many benefits to incorporating technology into the mathematics classroom including in conceptual understanding. Analyzing previous research, Heid, Blume, Hollebrands, and Piez (2002) find that students perform just as well on test items that require computation and procedural skills as students who did not use computer algebra systems, CAS, in their class. Studies have shown that students using CAS have overall conceptual understanding at or above a level of those not using CAS and students using CAS better understood concepts (Heid, Blume, Hollebrands,& Piez, 2002). "CAS use allows more time for developing conceptual understanding and for enabling students to understand real world quantitative situations" (Heid, Blume, Hollebrands,& Piez, 2002, p. 588).

There is promise in research of the early 1990s that communicating with the computer could potentially enhance the student experience in a calculus course. A project was undertaken at Dartmouth in calculus courses where students explored calculus topics through programming in BASIC (Crowell & Prosser, 1991), which did not find overall improvement in understanding as measured by final exam scores, found ease of implementation, found mixed results on students attitudes toward the computer enhancing calculus, and still asked the questions of what is the computer's place in calculus with CAS systems or programming and how would the traditional curriculum and pedagogy be revised to best incorporate the power computers could provide (Crowell & Prosser, 1991) both of which are still questions of today. Fenton and Dubinsky (1996) developed ISETL language to help students more effectively learn mathematics beginning with the argument that "communicating with a computer requires a level of precision that will help illuminate important mathematical ideas for students" (Lockwood, DeJarnette, & Thomas, 2019, p. 17). In studying using Excel and solving algebra problems, a student responded that they improved from pre-test to post-test "because you have to think before you type it into the computer anyway . . . so it's just like thinking with your brain" (Sutherland, 1994, p. 183).

After calls for turning calculus into a pump rather than the filter in the late 1980s, an MAA report on *Priming the Calculus Pump: Innovations and Resources* discussed numerous projects on improving calculus at a variety of universities, many of which included the use of some type of technology and computing (Tucker, 1990). These projects ranged from using BASIC, ISETL, Mathematica, Maple, and other mathematical programming languages and programming languages and in general found that these projects did no harm and did see a variety of benefits (Tucker, 1990). While overall none of them found grand shifts in student learning or attitude because of such technology, many of them found overall positive results based on students' comments and did not see negative effects on performance and many of these projects had not been long implemented at the time of reporting. These projects were implemented in a time when not every student had his or her own computer, let alone a laptop, to bring to class with them daily. In the early 1990s, it was recognized that the computing power of the future could empower students to do mathematics, however, it must be shown to them, which is demonstrated by:

> In the future, students are likely to have their own portable computer, which will be powerful enough to support a range of programming environments. The majority of students will not spontaneously use their computers for mathematical experimentation

unless this is supported by the culture of the school mathematics classroom. (Sutherland, 1994, p. 186)

These projects were the start but did not spark wide spread adoption of the use of computers and computational tools in calculus education, as currently the graphing calculator is still the leading form of technology used in introductory calculus.

The questions of using coding in mathematics, and overall STEM, education must continue to be studied considering the computing power is now more accessible and powerful than ever. There seems to be another wave in researching how these tools can be used in mathematics classrooms, perhaps because the technology is now more accessible to all students. In teaching an upper level mathematics course for mathematics majors, Lovric (2018) used programming to investigate math problems and encourages more research in integrating programming and mathematics. Berkeley Science Books claims to be “calculus without tears” and claims that computational calculus is easy compared to traditional analytic methods and does so through examples of physics and can be used for a wide variety of physical systems (Flannery, 2013). An example in pre-calculus was found that is said to use “introductory programming (in Python) as a vehicle for strengthening student intuition and confidence in pre-calculus concepts via hands-on simulation of physical phenomena, and thereby stimulate interest in more advanced study within these technical areas” (Freudenthal, et al., 2009, p. T4J-1). Koeher (2018a) has anecdotally reported positive results in the classroom getting students to investigate calculus concepts in Jupyter notebook. Cetin and Dubinsky (2017) found that students learned concepts such as functions more effectively using ISETL; by writing and running their code students had to think about what the computer is doing with the code. They explain that students had to define the function correctly in the program and then students really begin to reflect when having to enter "f(2)", see the result, and have to think about how the computer got the result (Cetin & Dubinsky, 2017, p. 74). Benakli, Kostadinov, Satyanarayana, and Singh (2017) also report similar results when using hands on computer programming in *R* to solve problems of calculus, probability, statistics, and data analysis. Using computational tools, like R, improves conceptual understanding of many difficult concepts from complex and abstract problems and improves problem-solving skills (Benakli, Kostadinov, Satyanarayana, & Singh, 2017). Cline et al. (2019) discuss integrating programming across an undergraduate mathematics curriculum that allows students to "actively engage in problem solving and collaboration on large-scale problems beyond those that are possible with traditional by-hand techniques" with benefits appearing especially for upper level students and upon entering the workforce. Jones and Hopkins (2019) present results of a sophomore-level course that introduces students to mathematics and some programming, which include resistance, feeling over burdened trying to learn new mathematics and a new way of doing so, and frustration with syntax errors. They also discuss the need for in class time to work on the projects and a buddy system for debugging and the importance of having "interesting and demanding problems". Kostadinov, Thiel, and Singh (2019) report positive student comments from learning to use RStudio with R and Python, which note that students felt it helped with visualization, students want to understand it better, and students support beginning to learn to solve problems this way as early as possible. They also note that students, even those with some previous programming experience, are fearful of programming, and they face common challenges such as syntax and not loading required packages (Kostadinov, Thiel, & Singh, 2019).

Many mathematics majors get exposed to some form of technology such as Maple, MATLAB, and LaTex, but it is important to also consider what the impacts of similar technologies could be in a general education course, a course not designed as an upper level course, or one primarily composed of non-mathematics majors. Using RStudio in a mathematics modeling and applied calculus course designed primarily for non-mathematics majors, Kilty and McAllister (2019) found improvement in "students' self-perceived understanding of mathematical models and calculus" and gains in "students' self-perceived abilities to recognize when the tools of mathematics can be used to describe a situation" (p. 16). They also found that students appreciated the study of mathematics in ways relevant to their discipline and felt more comfortable using mathematics and mathematical software for real–life settings (p. 17). Kilty and McAllister (2019) also note that students appreciate being able to study calculus and mathematics topics in ways that connect to other courses in their major and that instructors from those non-mathematics majors report students being more capable of using mathematics concepts in their courses.

It is important to draw upon the findings of this previous research to further develop how computational tools can be effectively used in mathematics education. It is also important to consider how these can be used in general education classes not just classes for STEM majors or upper level mathematics classes.

In addition to research from mathematics education, one can also look to physics education for some inspiration. Chabay and Sherwood's (2008) and Titus' (2018) work in Physics also serves as inspiration for this project with use of computational tools and computational modeling. Computation is a central tool for theory and experiment in physics (Chabay & Sherwood, 2008) and the same could become the case for mathematics. Introductory undergraduate physics classes have traditionally introduced students to both theory and experiment and some are now evolving to include computation as well (Chabay & Sherwood, 2008). The inclusion of computation brings the course up to date and makes it more in touch with real world practice and students' lives (Chabay & Sherwood, 2008), as could similarly be said about traditional introductory undergraduate mathematics classes. Just as conceptual understanding and skills of problem solving are goals for introductory mathematics courses, this is true of introductory physics courses, and the inclusion of computational activities can aid student learning of both of these (Chabay & Sherwood, 2008). Also similar to introductory mathematics courses that are limited in their real world applications by students' mathematical abilities, physics courses have the same problem; traditional courses were shaped by the limited technology and mathematical tools of the time (Chabay & Sherwood, 2008). Some of the benefits of using computational tools in an introductory physics course are using multiple representations, combining knowledge to correctly write the concepts into the programs, exploration changing parameters, modeling and visualization of complex scenarios can be developed by producing simulations for idealized situations and moving to more complicated situation, and using a language like Python to write programs from scratch allows for no black boxes so students must describe the situations correctly mathematically and apply physics principles correctly (Chabay & Sherwood, 2008). Titus (2018) does similar work getting students to use Jupyter notebooks and requiring introductory calculus-based physics classes to write programs using GlowScript. In physics many believe that programming, even at an introductory level, is an important component of general education for students today (Chabay & Sherwood, 2008). "Writing a program to solve a problem is a useful skill… programming offers practice in algorithmic thinking, which is a powerful intellectual tool" (Chabay & Sherwood, 2008, p. 308). Chabay and Sherwood acknowledge that the inclusion of a computational component to introductory courses requires a rethinking of the curriculum, but support that that would provide the opportunity to truly evaluate the goals of the course, not assume the traditional methods and content are ideal, and to reshape courses since there are no longer such limitations in technology and mathematical tools as when traditional courses were conceived (2008). The same can be said if computation is to be included in introductory mathematics course, but it can also not be assumed that traditional content and practices in calculus courses are ideal now with data from student performance and attitudes supporting that the traditional practices are not. My own experiences in physics classes taught with computational tools, throughout an entire physics major, inspired me to implement such tools in my classroom in a mathematics course.

Calculus Related to STEM

Much of the attention given to calculus and STEM education relates to calculus being seen as a challenging course that can force students out of STEM majors as well as out of the STEM career pipeline. STEM fields typically require their majors to complete at least one semester of calculus. Students on the "STEM path should be prepared to apply their understanding of various calculus concepts to their STEM fields. Therefore, current calculus students should have the ability to connect and apply their understanding of rate of change to another STEM domain" (Zeeuw, Craig, & You, 2013). Calculus, for many STEM and some non-STEM majors, is the course that prevents them from proceeding toward their intended career and can often be seen as a filter for weeding students out of that field. "Calculus is a critical filter in this pipeline, blocking access to professional careers for the vast majority of those who enroll" (The Mathematical Association of America, 1988, p. xi). In 1987 a national colloquium was held to discuss calculus as a pump not a filter; the attention paid to this was to prepare the next generation of scientists, engineers, and an acknowledgement to sustain American businesses, academia, and industry (The Mathematical Association of America, 1988). This does not seem to have changed much as Bressoud et al. (2013) emphasized the importance of

Calculus I for retention in STEM, finding that each fall semester around 300,000 college and university students, many in their first post-secondary semester, take Calculus I; Calculus I for many of these students remains a filter, which can dissuade even some of the strongest students from pursing careers in science or engineering. The President's Council of Advisors on Science and Technology (PCAST, 2012) found that students often leave STEM degrees because of courses, such as calculus, being taught in an unwelcoming and uninspiring manner with students referring to them as "frequently uninspiring, relying on memorization and rote learning while avoiding richer mathematical ideas" (PCAST, 2012, p. 28). Undergraduate STEM retention rates have been correlated with success in first calculus courses, which is seen as a gateway course for all STEM majors and minors (Mosina, 2014).

While typically thought of as the filter for STEM degree attainment, calculus can often serve as a screening course for business and social science students as well (Brito & Goldberg, 1988). Even in 1987 in discussions of calculus for business and social sciences, leaders in the field called upon mathematics departments to collaborate with other disciplines including interdisciplinary seminars, co-teaching of courses, and revision of content for mutual interest (Egerer & Cannon, 1988). It seems these were calls for interdisciplinary work similar to integrative-STEM education.

Calculus and STEM Integration. In searching for how calculus has been used for integrative-STEM education, it can be seen that this is an area that needs to be developed. Calculus is a branch of mathematics that has applications across numerous disciplines, however, many students leave the course seeing it as algebraic manipulation, miss the beauty of its connections to other disciplines, and only about forty percent report a level of confidence in being able to use the ideas of calculus (Bressoud, 2015). Calculus was developed to solve real, practical problems. Calculus, including many formal definitions and procedures, developed out of investigations of practical problems, which were interdisciplinary in nature such as astronomy, probability theory, fluid dynamics, and electromagnetism (Knill, n.d.). Calculus has applications across numerous disciplines from most sciences such as physics, chemistry, and biology, to statistics, engineering, and economics, so it should be used as an integrator. Research suggests that when students engage in math problems that are situated in real world contexts, it helps reveal students' conceptual understanding and difficulties with certain math topics (Mkhatshwa, 2017); this is much like the intentions of Integrative-STEM (abbreviated: I-STEM) education.

Gravemeijer et al. (2017) assert that mathematics is underemphasized in the current push for STEM integration, and the National Council of Teachers of Mathematics emphasizes that mathematics cannot be trivialized in integrative-STEM education (Larson, 2017). These may be contributors as to why there is limited research on calculus being used as an integrator for STEM.

There is literature on needing to get STEM majors successfully through calculus (e.g., Carver et al., 2017; Norton et al., 2018), but there is limited information as to how calculus can be used for integration. Examples of calculus and integrative-STEM include using e-pathways to improve student retention in calculus for STEM majors (Mosina, 2014), use of interactive tools such as graphical user interface to get STEM majors to see fundamental concepts in calculus and provide them with visual and intuitive understanding of typically abstract mathematics concepts (Goncalves, Hobbi, & Golnabi, 2016), incorporating problem based learning into mathematics courses for engineering students (Frank & Roeckerath, 2016), use of calculus to model global climate change as part of a multidisciplinary program to show that some questions have to cross disciplines to get answers (Hamilton et al., 2010), use of projects that allow students to incorporate calculus with their particular interests and industry (Fox, et al., 2017), "interdisciplinary lively application projects in calculus courses" that allow for students to experience "the real-world applications of mathematics in science and engineering" (Farrior, et al., 2007, p. 50), and an interdisciplinary collaboration between AP Calculus students and engineering students to develop CAD and produce cross sectional areas (Berkeihiser & Ray, 2013).

The use of calculus as a way to integrate STEM disciplines seems somewhat limited in published research. Some of this may be a result of some students not taking calculus until the tertiary level where collaboration between the STEM disciplines and other disciplines outside of STEM may be limited and other students taking calculus as a high school course where interdisciplinary work may also not be as prominent. With research revealing that calculus is a course that is a gatekeeper and often seen as a challenging course (Bressoud, Mesa, & Rasmussen, 2015), it seems that it is a place where integration of other disciplines to show its connections and relevance would be wholly appropriate and valuable. If a first college calculus course is taught well, it "could be an opportunity to have them leave not hating math, but actually bring them in" and "for those who continue in their chosen non-STEM field, whether business or social work having more people who are STEM- and calculus-literate would be great" (Ellis as cited in Courage, 2016).

There is also limited research on how Business Calculus, specifically, could be an integrator for STEM. The National Council of Teachers of Mathematics highlights that there are mathematical models involving finance that do not necessarily connect the science, engineering, or technology (Larson, 2017) lending to the opportunity for integration of other disciplines as well. There are numerous applications of calculus in the field of business beyond other STEM disciplines, and students of business need to be shown such examples. Several examples of connections between business and STEM include: Brinkmann et al. (2016) suggest that STEM students are increasingly choosing business related classes to fill elective spots in their undergraduate degree to complement their STEM skills, K-12 schools creating programs to facilitate business and technical exploratory programs that have emphasis on business and STEM that foster applications of business, science, and math concepts in technological and business systems (Chase, 2010), and connecting business and STEM education in undergraduate research (Bouldin et al., 2015). "Business is interdisciplinary. The practices, policies, and norms that govern business are grounded in social science, and the goods and services that businesses produce are themselves the fruits of science, engineering, arts, and humanities" (Bouldin et al., 2015, p. 17).

While it is acknowledged that the country needs more STEM professionals (PCAST, 2012), it is also documented that there is an increasing demand for business professionals capable of integrating science and technology into their business operation and management (Ledley, 2012; Ledley & Holt, 2014; Ledley & Oches, 2013; McCann, 2006). Focusing on science, Bouldin et al. (2015) detail that many business majors take an introductory science course that typically fulfills their general education requirement and is the last formal science course they take, which is usually introductory in nature and often has little context for applying the scientific knowledge outside of the science discipline. Because of such, Bouldin et al. (2015) propose interdisciplinary courses and experiences that allow students to see the connections between the disciplines. The same may be, at least in part, said of Business Calculus. Business students required to take calculus often miss seeing the connection between calculus and the rest of their courses leaving many of them "unmotivated and even resentful" (Narasimhan, 1993, p.

254), which this project hopes to remedy by providing more interdisciplinary connections utilizing modern technology.

Connecting this Project to Integrative-STEM. As addressed above both calculus and business have connections across many disciplines. Research, however, is limited on using calculus, specifically Business Calculus, to integrate. This project would provide some research in this area. Using computational labs would allow for the use of technology, specifically coding and real world problems as a method of problem solving and a way to introduce topics in a calculus classroom. The project also aligns with the National Council of Teachers of Mathematics recommendations of keeping a solid commitment to teaching, and not trivializing, mathematics when integrating (Larson, 2017).

Authentic Contexts. Integrative-STEM education calls for learning in authentic contexts. Introducing students to calculus topics situated within context outside of the classroom may allow them to better use the knowledge outside of the specific situation. Being able to use mathematics knowledge, or recontextualize it, is a skill that needs to be developed for all future workers in a variety of industries (FitzSimons & Boistrup, 2017). This is similar to problems or design challenges in Integrative-STEM education that foster learning because students are not only taught the abstract knowledge, but they are also provided with a context in which they can use it, which connects knowing, doing, participation, and authenticity (Sidawi, 2007). Research shows that students are more motivated to learn when they are in integrative classes because the content becomes more relevant and connections can be seen between the STEM subjects and real-life situations (Satchwell & Leopp, 2002). By seeing how calculus concepts can be related to scenarios outside of the specific discipline of mathematics and the classroom setting, perhaps students will be able to transfer and connect calculus concepts or at least have some appreciation of the connections it does have. For example, in a study looking into transfer of knowledge about slope, rate of change, and regression, a student was more able to reason and connect ideas of slope and regression when the problem was framed related to a situation of people and movie ticket prices rather than a problem situated only in classroom context (Nagle, Moore-Russo, & Casey, 2017). Design challenges in I-STEM education are often situated in an authentic context from outside of the classroom (e.g., Wells, 2017).

Authentic Ways of Doing. The labs in the current project are not design challenges, such as in I-STEM education, but they will be situated in context outside of the mathematics classroom and will allow students to do so using tools often employed outside of the classroom. Analyzing real world problems using computational tools is also reflective of how math and other professionals, including STEM and non-STEM professionals, use mathematical knowledge outside of the classroom. Computing can bring mathematics education more in line with professional practices; by 2020 one of every two STEM fields will require some level of computing (Weintrop et al., 2015). There is a level of authenticity and real world use, both mathematically and for computational tools, when they are applied to mathematics and science activities (Weintrop et al., 2015). Including computational tools in mathematics education can also potentially reach a wider audience than typically reached with traditional methods (Weintrop et al., 2015). Some mathematicians argue that computational mathematics is approximation and not pure, but outside of academia many engineering and science problems are solved through approximate solutions arrived at computationally and then implemented in the "real world" (Gordon, 1979, p. 25). Many economic marginal analysis problems and growth and decay problems are artificial when done continuously, but they are much more real in content and practice when done with computational tools (Gordon, 1979). Many who use mathematics in their careers or in their workplace are not doing it with paper and pencil, especially not for their complex scenarios that arise from their real world situation, but changes have not been reflected in mathematics curricula and pedagogy, while changes have arisen in other disciplines including physics, chemistry, and economics (The Mathematical Association of America, 1988). Zevenbergen (2004) asserts that studies of mathematics in the real world and mathematics in school have shown educators' abilities, or lack thereof, to identify real world mathematics and introduce students to it in the classroom and that more of a link between school mathematics and real world mathematics can provide students with more meaningful activities, which this project intends to do. The labs in this project aim to make meaningful connections between calculus and business and given them authentic ways of performing such applications.

Exposure to undergraduate mathematics that includes modeling, inquiry, and using technological tools to solve problems from all disciplines can help students develop into "creative, confident, competent problem-solvers, and clear, critical thinkers" (Arney, 2009). In 1987 the Mathematical Association of America held a national colloquium to discuss improving calculus and a report was produced titled *Calculus for a New Century: A Pump Not a Filter*; in this report there is support for reforming calculus and to modernize it. "We need to teach calculus in a way that facilitates complex and sophisticated numerical computation in an age of computers. Somehow or other we have to make calculus exciting to students" (The Mathematical Association of America, 1988, p. 9). In this report there is also discussion that calculus classes tend to be extremely oriented toward skills and techniques, but that they should be more concept and application driven (The Mathematical Association of America, 1988). There is still great importance placed on wanting students to walk away from first semester calculus with increased conceptual understanding, but what is emphasized and taught in the classroom is driven by students spending large amounts of time using sample problems to compute derivatives and integrals (Heid, 1988). Computational tools could allow for mathematics to focus on the concepts rather than such emphasis on the skills (Heid, 1988). "In the real world we use computers for calculating, almost universally. In education we use people for calculating almost universally" (Wolfram, 2014, p. 1); this changes what mathematics might be of importance to be taught. "In school the professor formulates the [mathematical] problem and you solve it – you hope. In industry, you formulate the [mathematical] problem and the software solves it – you hope" (Keeler & Grandine, 2013, p. 41). Gravemeijer et al. (2017) assert that the previous quote reflects "that we have to shift away from teaching competencies that compete with what computers can do and start focusing on competencies that complement computer capabilities" (p. S107).

The National Council of Teachers of Mathematics state the view, "An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (NCTM, 2014) and advises that students learn about math from problems that arise from outside of mathematics (NCTM, 2000). This also aligns with the National Council of Teachers of Mathematics standards (2010), which call for the use for technology in the classroom, making connections to other mathematical topics and other disciplines, and for representing mathematical ideas in a variety of ways (National Council of Teachers of Mathematics, 2010). Opening the mathematics classroom to different types of thinkers through computational tools and the mathematics that is actually taught could have an impact on students' attitudes and conceptual understanding. Mkhatshwa (2017), in studying Business Calculus students' reasoning about optimization, discusses that students need to be given opportunity to reason about these topics in multiple representations such as algebraically, textually, graphically, and numerically in contexts from outside of the classroom, which could also be applied to other topics in Business Calculus. Since calculus is seen as "efficient at lowering students' confidence, enjoyment of mathematics, and desire to continue in a field that require further mathematics" (Bressoud & Rasmussen, 2015, p. 144), perhaps opening space for the inclusion of modern technology, enabling different ways of knowing and doing could change that for some students required to take it.

Using Technology to Intentionally Teach. The National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* states the view,

> Technology is not a panacea. As with any teaching tool, it can be used well or poorly. Teachers should use technology to enhance their students' learning opportunities or creating mathematical tasks that take advantage of what technology can do efficiently and well. (2000, p. 25)

The technology used in this project is selected to intentionally support learning and aligns with the definition of Integrative-STEM education. Integrative-STEM Education is defined as,

> the application of technological/engineering-design-based approaches to *intentionally* teach content and practices of science and mathematics education concurrently with content and practices of technology/engineering education. Integrative-STEM Education is equally applicable at the natural intersections of learning within the continuum of content areas, educational environments, and academic levels. (Wells & Ernst, 2012)

The last sentence is important as the project goes outside of the traditional integrative-STEM education areas by also having the connection to business. This definition is also connected in that the project uses technology to intentionally teach mathematics concepts. This project is doing more than placing a computer in students' hands to do some computation for them. It is using the computer as a tool to intentionally teach calculus content. Dubinsky and Yiparaki (1996) describe that

> Computer experiences can be an effective way of not only helping students to construct reasonable schemas, but also to get them to reconstruct erroneous or incomplete conceptions. The basic principle is that anytime you construct something on a computer then, whether you are aware of it or not, you construct something in your head. By studying the connections the connections carefully, we have found it possible to induce a considerable amount of learning. (p. 10)

In studying students using ISETL, Dubinsky and Yiparaki (1996) describe that when students are writing code in ISETL they think they are studying the syntax of it but in addition important mathematical ideas are learned because of how they have to type things in and the similarities to mathematical notation. Cetin and Dubinsky (2017) found that students learned concepts such as functions more effectively using ISETL by writing and running their code, students had to think about what the computer is doing with the code. They explain that students had to define the function correctly in the program and then students begin to reflect when having to enter "f(2)", observed the result, and had to think about how the computer got the result (Cetin & Dubinsky, 2017, p. 74). In this project, for example, when a student must type "diffs" to compute finite differences or "sum" for summation in the introduction to integrals they may be thinking about what is going on. Another example is when students are using Riemann Sums to compute area under the curve they must figure out how to type in length times width correctly, which causes them to have to determine how to define the width of the rectangle and correctly define the changing height of the rectangle based on the function they defined and then further developed as students change the number of rectangles to better their approximations. Some of the problems also involve real data, to which the students must determine how to mathematically model this data to the use calculus concepts and answer questions; this is done in part to teach students where those "given functions" come from and show them techniques to handle some data.

The computer is used as more than an auxiliary tool and is used to teach calculus concepts. It will allow for some numerical, iterative approaches rather than predominantly analytic ones, and describe situations and make predictions by considering small steps, which can be done easily on a computer. Felix Klein argued for teaching calculus in such a way that begins with an intuitive and practical approach on which later more abstract concepts can be built (Klein, trans. 2007). Klein discusses that in thinking of teaching calculus, one must consider how infinitesimal calculus was developed as such it was built through considering discrete and finite pieces or in approximating the area of a circle by considering inscribed, circumscribed polygons increasing the number of sides, which is really integration and looking at a curve as a collection of finite points finding secant lines, which is the process of derivation, all done before extending the limit to these processes. In the teaching of calculus, Klein asserts:

> We desire that the concepts which are expressed by the symbols $y=f(x)$, dy/dx, $\int y\,dx$ be made familiar to pupils, under these designations; not, indeed, as a new abstract discipline, but as an organic part of the total instruction; and that one advance slowly, beginning with the simplest examples. (Klein, trans. 2007, p. 223)

Klein also emphasizes introducing these concepts through concrete examples. Klein's thoughts explain why in the classroom students should be able to consider calculus from a finite perspective first before moving to more abstract concepts that are new to them. Klein's text was originally produced in 1932, so it was not explicitly calling for implementation on a computer, but it can be conceived in such a way now. This is what these labs intend to do; the computer serves as the tool to help teach these concepts.

The 1970s and 1980s saw rise to publications on a finite or discrete approach to calculus incorporating computational tools. In the 1970s and 1980s there was a push to move away from the continuous approach and cover the discrete or finite approach to calculus, especially for non-STEM majors (Gordon, 1979; Hoffman, 1989; Ralston, 1984). The driving force behind this thought was to use “finite differences and sums as motivation for infinitesimal calculus and as an appropriate setting for solving real problems by discrete approximations” (Gordon, 1979, p. 24). Findings from Gordon’s (1979) study suggest that students gain more complete understanding of the concepts more quickly than when solely introduced to the continuous approach (Gordon, 1979). Gordon also supports that when students have gained solid understanding of concepts from the finite approach it can be more easily expanded to the continuous approach with the introduction to the limit. Students with weak mathematical foundations can benefit from this approach, which can reach a more diverse group of learners (Gordon, 1979). In studying the implementation of the discrete approach with computational tools Gordon presented the benefits, which include students gaining stronger understanding of and ability to use basic concepts and methods of calculus, students gaining appreciation of the relationship between math and the computer, “the approach provides an ideal context in which to develop several simple, yet useful, numerical algorithms for approximating functions and for actually finding where all those ‘given’ functions come from” (1979, p. 23), and it demonstrates a natural context to demonstrate the use of discrete and continuous mathematical models and how they are developed (Gordon, 1979). Since the 1980s students complained that colleges and universities along with other educational systems have been slow to respond to the need for students to learn and use discrete and finite mathematics (Turner, 1983); it seems as though it still may be said that many have not responded. Ralston (1984) argued that with changing economies moving away from production to service, a need for solving problems using discrete techniques that not only math and computer science students will need, but also students of social and management sciences, physical scientists, and engineers will need to translate the problems in to discrete terms to be solved on computers. It is also thought that computational tools would allow for more active involvement of students in models and simulations thus enhancing their appreciation for mathematics and their capacity to do independent work (Hoffman, 1989). The use of computational tools and a study of some elements of finite calculus will rid students of the need to do some of the work through symbolic manipulation with paper and pencil, thus allowing for the introduction of more advanced topics and scenarios than many students could do by hand (Hoffman, 1989). Many times the classical techniques of calculus require special cases and a lot of time to master for the novice, so teachers must reduce problems to a very simplified version (Hoffman, 1989). Hoffman presents some findings from his combination of calculus and finite mathematics course, which reveal that students finish the course with a broad appreciation of how mathematics can be used in their discipline, gain confidence in their ability to use and understand tools used to work problems, and the ability to use computers for modeling and making estimates (Hoffman, 1989). Echoing a similar sentiment, in 1984 the National Research Council put out a report that summarized how numerical computation and applied mathematics had been a vital part of the United States' national defense and industry and laid claims that the problems of engineering and science were too complex for exact methods, so they needed to be modeled by mathematics and computation and recommended education that aligned with this

(*Computational modeling and mathematics applied to the physical science*, 1984). This calls for an increase in education that reflects the use of computational tools and mathematical models to handle the messy, real-world mathematics that are still not widely reflected in classrooms today but are part of this project. Kaput (1994), too, states that drawing upon Leibniz has implications for curriculum design in beginning with finite differences even in a student's study before a calculus course and discusses using the computer and graphical representation (MathCars) to get students to think about change in position based on change in time in finite steps and then decreasing the step intervals to smaller and smaller units thus approximating the derivative of the position function.

This project connects Business Calculus to I-STEM education by integrating multiple disciplines and doing so in a way with intentional teaching through technology. There is also a lack of research connecting Business Calculus to I-STEM education, but business is interdisciplinary in nature, and this projects aims to fill some of the void. With much of the attention and research relating calculus to STEM education involving calculus being a challenging gatekeeper course that students lament is often taught in an uninspiring, unwelcoming way (PCAST, 2012), it is important to consider how Business Calculus can be an integrative course and potentially increase conceptual knowledge and attitudes toward mathematics.

Attitudes Toward Mathematics

Students' mathematical attitude and beliefs, including self-efficacy, confidence, and self-concept, strongly relate to achievement in mathematics classes (Pajares & Miller, 1995; Carlson, 1999, Schommer-Aikins, Duell, & Hutter, 2005) and have important effects on persisting in problem-solving. Introductory college calculus is a place where negative attitudes toward the course and subject are often present and represents a course that is highly adept at negatively impacting students' attitudes. Literature has demonstrated that students' attitudes are affected by classroom structure and teaching, so it is imperative to continue to develop, study, and document ways in which students can potentially develop more positive attitudes.

Calculus and Business Calculus Attitudes

Undergraduate calculus causes a sharp decrease in students' enjoyment of mathematics and confidence in mathematics ability (Bressoud & Rasmussen, 2015), which raises concern since attitudes related to math have been linked to achievement. Bressoud & Rasmussen (2015) reveal that post-secondary calculus, as currently taught, "is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics." House (1995) demonstrated the positive relationship between attitude and achievement in college calculus courses. Pyzdrowski et al. (2013) found a strong positive correlation between attitude and performance in entry-level college calculus; their findings indicated that attitudes affecting course performance were not based simply on previous mathematics experiences and preparation but also psychological factors such as confidence.

Business Calculus is a course that has been shown to have students that are not excited and often unprepared to take the required course (Liang & Pan, 2009). Business Calculus is a course with a high level of resistance to and negative attitudes toward mathematics is present (Depaolo & Mclaren, 2006; Liang & Pan, 2009). Depaolo and Mclaren (2006) found that attitude towards math was significant in predicting exam scores in Business Calculus courses. Findings also support that attitude had a larger effect on calculus performance than it did on statistics performance for business students (Depaolo & Mclaren, 2006).

Important in Depaolo and Mclaren's (2006) findings is that attitude appeared to have a stronger affect on performance for students that had not taken calculus before than ones that had;

this is important since many students in Business Calculus are being exposed to the material for the first time. "How to convince business students in the classroom that mathematics will later be able to save them time, hence money, remains an open problem" (Nievergelt, 1996, p. 146). It is important to consider how attitude can affect performance, how attitude can be improved, and how attitude affects one's overall willingness to use mathematics, which may translate into their future careers as well.

Attitude and Self-Efficacy

The Mathematical Association of America called for the undergraduate introductory mathematics courses to be "effective in positively affecting student attitudes about mathematics" (Saxe & Braddy, 2015, p. 66), emphasizing the importance of early college mathematics courses positively impacting students' attitudes toward mathematics, which is also a goal of this project. Aiken, (1970) in discussion of attitudes toward mathematics, defines attitude as "a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept, or another person" (p. 551) and Neale (1969) defined attitude toward mathematics as a measure of "liking or disliking mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless" (p. 623).

Links to Achievement. The relationship between attitude toward mathematics and achievement in mathematics is usually positive and practically significant, not always statistically significant, at the elementary and secondary school levels (Aiken, 1976). Negative attitudes toward math have been linked to drop out rates from math courses (Ma & Willms, 1999) and to poor engagement leading to failure (Mayes, Chase, & Walker, 2008). In mathematics, positive attitudes have been associated with higher scores on standardized tests and higher classroom achievement (Aiken, 1976; Aiken & Dreger, 1961; Stankov & Lee, 2014;

Zimmerman, Bandura, & Martinez-Pons, 1992). University students with negative attitudes toward math tend to get lower scores on final exams (Nunez-Pena et al., 2013). A student's belief about if he or she is a "math person" and his or her attitude toward mathematics has an effect on a student's learning outcomes (Boaler, 2016). Students' negative attitudes toward mathematics may be hindering their successes, so developing a more positive attitude toward mathematics is vital for chances of increasing success. In contrast, Ma and Kishor (1997) find that the relationship of attitudes toward math and achievement has been found to be significant and positive but not strong.

The Mathematics Attitudes and Perceptions Survey (abbreviated: MAPS) (Code et al., 2016) was used in this study, and this instrument has seven factors of expert-like behavior in and views of mathematics (Code et al., 2016). The seven categories of expert-like behavior include: confidence in and attitudes towards mathematics, persistence in problem solving, belief about whether mathematical ability is static or developed, motivation and interest in studying mathematics, views of the applicability of mathematics to everyday life, learning mathematics for understanding, and the nature of answers to mathematical problems (Code, et al., 2016, p. 920). While MAPS does not have a specific category for self-efficacy, several of its categories are elements of self-efficacy such as confidence, which the creators define as "a person's perceived ability to successfully engage in mathematical tasks" (Code et al., 2016, p. 920). This is noted because it can be seen that mathematical self-efficacy could be linked to attitude toward mathematics because students lack confidence in their math abilities because of previous experiences, poor grades, a general lack of interest in math, and the inability to relate math to usefulness in everyday life (Peters, 2013) and research has revealed that self-efficacy has a strong connection to mathematics performance. Research on self-efficacy and mathematics performance has been going on since the 1980s with studies on self-efficacy's influence on performance (Williams & Williams, 2010, p. 456). Findings since the 1980s seem to align well with the proposition that self-efficacy in mathematics is positively related to academic performance in mathematics (Ayotola & Adedeji, 2009; Grigg, Perera, McIlveen, & Svetleff, 2018; Hacket, 1985; Cooper & Robinson, 1991; Kaya & Bozdag, 2016; Pajares & Miller, 1994; Pajares & Graham 1999; Pietsch, Walker, & Chapman, 2003; Schober, Schutte, Koller, McElvany, & Gebauer, 2018; Stevens, Olivares, Jr., & Hamman, 2006). The relationship appears at many different grade levels from elementary school through college (Kaya & Bozdag, 2016). Pajares and Miller (1994) report a strong connection of math self-efficacy to mathematics performance for college students. Grigg, Perera, McIlveen, and Svetleff (2018) demonstrated that self-efficacy in mathematics positively predicted mathematics achievement for grade received in the class and standardized test score. Besides final grade in the course as a measure of achievement, Collins (1982) demonstrated that students with high self-efficacy in mathematics are more accurate in the math computations and are more persistent on difficult mathematics items than students with low self-efficacy. Self-efficacy for college students in mathematics had a stronger relationship to mathematics performance and mathematics problem-solving than other variables such as math self-concept, high school level mathematics courses, and math anxiety (Pajares and Miller, 1994). All of these demonstrate that there is an identifiable relationship between self-efficacy and math achievement.

Attitude and Classroom Experiences. Literature has revealed factors that often predict student attitudes toward mathematics include: instruction students experience including pedagogical decisions, use of technology, environmental factors, and personal factors including previous interaction and attitudes toward mathematics and the choice of college major (Sonnert, Sadler, Sadler, & Bressoud, 2015). Students often enter school with a positive mindset toward mathematics but lose this early on and enter into a cycle of lack of success in mathematics (Westenskow, Moyer-Packenham, & Child, 2017). Andersson, Valero, and Meaney (2015) identify that statements such as "I hate math" or "I am bad at math" do not come from only the student but come from an interaction of classroom and pedagogical experiences, implying that understanding of the experiences that shape students' attitudes can help to prevent students from hating math. The math-hating attitudes come attached with very strong emotions reflected by words such as sickening, frustrating, and wanting to cry (Larkin & Jorgensen, 2016), and it seems that these attitudes begin to appear at a very young age (Cornell, 1999). Other factors that appear to influence students' attitudes and success are peers' attitudes (Kotok, 2017), poor content knowledge foundations, negative mindset, lack of number sense (Westenskow, Moyer-Packenham, & Child 2017), finding the subject boring with too much memorization, and their teachers (Cornell, 1999).

Students' mathematical mindsets can be changed and affected by the context of the teaching (Andersson, Valero, & Meaney, 2015); research suggests that the attitude can be changed if math is taught in more relevant contexts and real world connections (Andersson, Valero, & Meaney, 2015; Cornell, 1999). Since mindsets can be changed, there is a need to create an environment that fosters more positive attitudes toward mathematics (Andersson, Valero, & Meaney, 2015) to nurture mathematical skills, which could include more relevant contexts, more modern topics, more modern approaches, and more technology and computing such as in this project. "Students see mathematical tools for the life sciences and social sciences as useful, interesting, and beautiful when they learn to use them in realistic applications and when computers do the calculations" (Hoffman, 1989, p. 65). Not only do students find mathematics more useful when taught in such a way but achievement is also linked to these realistic applications; more specifically research suggests, "mathematical capabilities of children are greatly influenced by whether they are in a real world or a classroom context" (Couch & Haines, 2004, p. 199). Classroom activities that represent math as static, unchallenging, and boring leave students unable to see the usefulness of the mathematics (Wilkins & Ma, 2003). Students that get continuously exposed to mathematics as rote memorization and unchallenging develop a negative attitude toward mathematics and its applicability outside of the classroom (Greenwood, 1984).

Attitude and Computational Tools. In addition to attitude of mathematics being supported by being able to see connections outside of the classroom, the use of modern technologies could also potentially improve students' attitudes. In research from the 1990s on different uses of technology such as BASIC, Mathematica, Maple, and others, revealed mixed results on the impact of student attitude as well. Baumgartner and Shemanske (1990) report that students who had taken calculus before did not like the computing because they were expecting an easy course since they had already been exposed to the material, students new to calculus overall liked the new approach, some students thought computing was a great addition to the class while others did not see the value in adding it to a mathematics class, but found that even students that complained about the computing component did not think that it hindered their learning and others thought it helped. In studying computer labs in a calculus course, Höft and James (1990) found that students in the lab sections reported being more interested in the material, more responsive, asked more questions, and in general had very positive attitudes toward how the computer helped them learn calculus. Schwingendorf and Dubinsky (1990) also report positive impacts on students' attitudes toward calculus when using ISETL and Maple in a calculus course with a lab. The three previously mentioned studies all indicate some positive impacts on students' attitude toward calculus when using some computer program, but all three also make note of some negative feedback from students particularly noting student complaints of an increased workload or having to learn a language in which to communicate with the computer in addition to calculus concepts. Technology, although not specifically the use of computing technology, was not significant in affecting students' attitudes in calculus either positively or negatively in the national study of calculus (Sonner & Sadler, 2015).

Recent research reveals mixed results as well. In a mathematical programming course, Jones and Hopkins (2019) found that students do tend to give up easily because of syntax errors and the fact that it is brand new for most students, but those are also mixed with students commenting that the course proved challenging but extremely rewarding. Kilty and McAllister (2019) found positive impacts on students' attitudes when using RStudio in a Mathematical Modeling and Applied Calculus course with increases in students' self-reported confidence in using tools of calculus for analyzing real world data, using mathematical software to help analyze real world data, using mathematical software in building mathematical models for a real world situation, feeling comfortable in a non-mathematics course answering mathematical questions, and recognizing when a situation could be described with mathematics without being explicitly told; they synthesize that these results show that their approach increases students' self-reported confidence in mathematics and comfort in using software and mathematics in studying real world problems. Kilty and McAllister (2019) also note that students appreciate being able to study calculus and mathematics topics in ways that connect to other courses in their major.

With some promise in previous research, it is important to consider interventions such as this one that may impact students' attitude toward mathematics. Given the role that attitude toward mathematics has on success in mathematics courses and how students' attitudes and perceptions about mathematics play an important role in one's willingness to use this knowledge outside of the classroom, it is important to study interventions, such as this project, so that students may develop more positive attitudes toward mathematics and become more willing to engage in mathematical tasks, perhaps even in their future careers.

Attitude to Career. Anecdotally, when discussing with my students why they chose to major in business or how they think math relates to their future business career, many of my students usually let me know their feelings about math, typically more negative than positive, tell me about their previous perceived failures in math, and then connect those to why they chose business as their major thinking that it may not be as math-intensive as a STEM major. Many of these students are in their first few semesters of college. This anecdote aligns with existing literature on how students decide on college majors and future careers. Research supports that students often have their minds made up about going into STEM careers or not upon exiting high school (Maltese & Tai, 2011). Intent to pursue a STEM degree and career is often affected by interest in math and science, completion of math and science in high school, social background, parental education, self-efficacy in mathematics, and post-secondary support such as financial aid, taking remedial courses, and other time demands such as working (Wang, 2013). "Intent to pursue a STEM major is significantly and positively influenced by 12th grade math self-efficacy" (Wang, 2013, p. 1101), which would imply that those with lower mathematics self-efficacy are less likely to pursue a STEM major or career and may choose other majors and careers such as business or something in the humanities. Wang's (2013) findings indicate that high school science and math preparation are critical in cultivating student interest in STEM careers and majors, and motivation, including mathematics attitude and self-efficacy beliefs, intent to pursue a STEM career, and intent to pursue a graduate degree are all positively linked to starting a STEM degree. Attitude and achievement, especially in mathematics, can be seen as influencers on a students' desire to pursue STEM or not.

Career choice is a process involving many influencers, which are on-going both in and out of school. Attitudes and achievement in the discipline are not the only influencers of students' career choice. Students' educational experiences, interests, and attitudes, not just their success in STEM subjects in K-12 schooling, are predictors in students choosing a STEM major and career (Maltese & Tai, 2011). Dick and Rallis (1991) provide a model of career choice where students make career choices based on beliefs of themselves, their abilities, and different values of careers, which can all be influenced by internal factors such as intellectual interest, interpretation of past experiences, perception of expectations and attitudes of others, and external factors such as length and cost of training or schooling for a career, expected salary, and cultural stereotypes of careers. Dick and Rallis (1991) provide their adaptation of a model of student career choice seen below in Figure 1 (p. 283). As the arrows in Figure 1 display, socializers play a major two-way part in influencing what experiences students have and how they interpret those experiences (Dick & Rallis, 1991). Academically strong students may choose careers different from their academic strengths because of socializers and cultural norms that may affect students' self-concept and valuation of careers (Dick & Rallis, 1991). Maltese and Tai (2011) found that providing students with opportunities to see how math and science are vital and woven into their every day lives is extremely important to their perception of these subjects and thus their decision to pursue them.

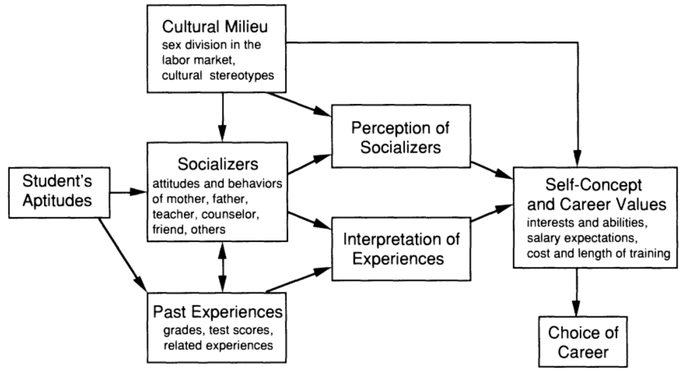


Figure 1: Model of career choice. From Dick and Rallis, 1991, Factors and Influences on High School Student Career Choice. *Journal of Research in Mathematics Education, 22(*4), 281-292.

Mathematics and Career Choice. Attitude toward, self-efficacy in, and past experiences with mathematics all factor in to choice of college major and future career. Attitude and self-efficacy appear to be very strongly related to career choice. Perceived mathematical self-efficacy contributes more significantly to educational and career choices making use of quantitative skills than does the amount of mathematical preparation in high school, level of math ability and past achievement, and anxiety over math activities (Betz & Hackett, 1983; Hackett, 1985; Hackett & Betz 1989). In a study of undergraduates, mathematics self-efficacy was found to be an important factor in career choice (Lent, Lopez, & Bieschke, 1991). "Past success experiences in a particular performance domain may promote self-efficacy; viewing oneself as efficacious likely enhances interest in that domain; and such interest then motivates further exposure to, and choice of, correspondent educational and vocational activities" (Lent, Lopez, & Bieschke, 1991, p. 429). Attitudes toward mathematics are significantly and positively correlated with community college students' perceptions of careers that were mathematically intensive, and students that expressed interest or reported entertaining the idea of a mathematically intensive career had higher average attitude towards math than those that had never thought about pursuing a math intensive career (Dogbey, 2010). It is important to note that educational and social factors influence these, highlighting the importance of positive educational environments for mathematics, such as shown in the Dick and Rallis model (1991).

Calculus for a New Century: A Pump not a Filter (The Mathematical Association of America, 1988) called for calculus to not act as a filter, filtering students out of particular majors and potentially careers but rather as a pump to increase the number entering fields and careers requiring mathematics knowledge. Despite more than 30 years since that report and recommendation, calculus is often still serving as that filter (Bressoud, Mesa, Rasmussen, 2015). Calculus, a requirement for almost all STEM majors and often business majors (Dahl, 2014), can act as a gatekeeper and as an "insurmountable obstacle or discourages from the pursuits of fields that build upon the insights of mathematics" (Bressoud, Mesa, & Rasmussen, 2015, p. v). This is cause for both pause and concern.

Why Business Calculus. Business Calculus is one place that aligns for the placement of computational labs as pointed out by Hoffman (1989) explaining that calculus, as traditionally approached, has been inappropriate for social and life scientists. Teachers use concocted real life examples, but students still perceive much of what they have been learning in these calculus classes as something they would never use outside of the classroom (Hoffman, 1989). Students question what is taught and how it is taught asking when they will ever use any of what they are learning in their future careers and why they cannot work the problems on the computer like they do in their business classes and what is reflected by professionals. Calculus can be reconceived with the computer, which will allow for development of courses that can allow for more applications outside of the concocted examples (Hoffman, 1989). The use of modern technology can be extremely beneficial for students to learn calculus with true business applications, especially for students with weak math skills (Liang & Pan, 2009) and negative attitudes.

Motivation, participation, and interest are all shown to improve in Business Calculus classes when computers are allowed as an aid and more emphasis is placed on understanding and application (Judson, 1990). The age-old student question of "why do I have to learn this stuff?" is gone because students are allowed to see the calculus in action with real applications and are less mired by the stress of algebraic manipulation, skills, and techniques (Judson, 1990, p. 154).

Given that mathematics preparation, past achievement in mathematics, self-efficacy toward mathematics, and attitude toward mathematics have been found to be factors in career choice and that math sometimes serves as a "critical filter" (Bleyer, Pedersen, & Elmore, 1981, p. 46) in the career choice process and that Business Calculus is a course where students tend to be unprepared mathematically and show a dislike toward mathematics (Liang & Pan, 2009), it could be inferred that some of these students' are choosing to major in business and potentially choosing it as a career because of their previous experiences with mathematics. Students' mathematics test scores influence choice of major; students with higher mathematics test scores were more likely to choose a technical major rather than health, business, public service, or liberal arts (Simpson, 2001). The more mathematics preparation a student has from high school is indicative of the student choosing a more technical major than a non-technical one (Simpson, 2001), which would include business (Pritchard, Potter, & Saccucci, 2004). In analyzing basic skills tests for algebra and students majoring or concentrating in different fields of business, Pritchard, Potter, and Saccucci (2004) found that students with higher computational and algebraic skills chose to major in more quantitatively focused business concentrations such as accounting or finance while students with lower scores tended to select a concentration or major in less mathematically focused ones such as management or marketing. Some business students chose less quantitatively focused concentrations because they perceive them to have less demanding quantitative requirements (Pritchard, Potter, & Saccucci, 2004) implying that mathematics does play a role in the selection of major and potentially future career. A quick Google search also reveals forums and question threads where people ask if business majors have to take calculus and why business majors have to take calculus, which indicates mathematics is at least weighing on some students' minds when thinking about majoring in business. Undergraduate calculus classes teach students important skills and concepts in mathematics and also have a large impact on students' attitudes, which can affect their career aspirations and choice of taking future mathematics classes (Sonnert, Sadler, Sadler, & Bressoud, 2015).

Employers and future trends indicate that there is a great need for mathematically proficient individuals. There is a need for mathematically literate individuals for the twenty-first century business world (Yıldırım & Sidekli, 2018), and mathematics knowledge is becoming imperative for many career opportunities (Bureau of Labor Statistics, 2016). A 2013 poll of 200 employers revealed the second most important skill they look for in potential employees is the ability to make decisions and solve problems, and the ability to analyze quantitative data and use computer software programs are also in the top ten (Adams, 2015). The National Network of Business and Industry Association (2014) lists using mathematics to solve problems as one of its necessary employability skills. Employment of mathematics occupations has a projected growth of twenty-eight percent from 2016 to 2022, and growth in the areas of business and government needing mathematicians or people comfortable using math is expected to grow as business and data analytics continue to grow (Bureau of Labor Statistics, 2016). Sales and marketing, research and development, supply chain management, and workplace management are all areas in which analytics, including big data analytics, are growing (Columbus, 2018); the need to have a better disposition toward mathematics or willingness to use it in daily and workplace life will likely increase as analytics becomes an even larger role in the business world and leaders must interact with the outputs to make decisions.

A student's "mathematical disposition is related to his or her beliefs about and attitude toward mathematics may be as important as content knowledge for making informed decisions in terms of willingness to use this knowledge in everyday life" (Wilkins & Ma, 2003, p. 52) which may very well include his or her willingness to use it in the workplace. Because attitude and achievement are linked to career choice and willingness to use math in the workplace, this project aims to improve both, which in turn may increase one's willingness to use mathematics in the workplace, potentially increase confidence to work with mathematics outside of the class room, and potentially choose more mathematically involved careers.

"Business students, although able, are often math phobic. Courses should strive to lessen math phobia, enable students to be more comfortable with mathematics, and help students appreciate the relevance of mathematics" (Lamoureux, Beach, & Hallet, 2000, p. 19). Some of the intentions of this project are to do just that. If the project succeeds in doing so, perhaps it will answer Nievergelt's (1996) question, "how to impart just enough mathematics to business majors, so that they may understand the potential power and limitations of mathematics, decide when to hire mathematicians, and consult with them profitably" (p. 146), but also inspire students to be more appreciative of and willing to work with mathematics in their careers.

Mathematics, being able to interact with technology, and appreciating some of the interdisciplinary nature of problems will be valuable skills for future careers. "Across a wide range of industries and occupations, people are required to use, develop, and communicate mathematical ideas and techniques in a diversity of ways with others who have differing expertise, experience, and interests including in mathematics itself' (FitzSimons & Boistrup, 2017, p. 330). In observations of the workplace, students and teachers find it hard to distinguish mathematical activities that are occurring other than number and measurement because they are looking for the formal mathematics they experienced in the classroom, which is often not how math presents itself in the workplace (FitzSimons & Boistrup, 2017). Aspects of theoretical math such as geometry and algebra can be seen in the workplace in spreadsheets, machining, and quality control systems, yet hidden, but still very much functioning in the work place, is formal mathematics that are the foundation for management and production technologies and enable predictive modeling and analysis of business analytics and financial math (FitzSimons & Boistrup, 2017). Students that participate in this project may emerge from the class with a more expert-like disposition toward mathematics that will allow them to see mathematics as a valuable tool in the work place. "Professional and skilled workers are able to integrate relevant disciplinary domain knowledge, mathematical and vocational, as well as knowledge of professional or vocational contexts developed through formal and informal learning, including social and cultural knowledges" (FitzSimons & Boistrup, 2017, p. 331). Gravemeijer et al. (2017) take the position that part of mathematics education is preparing students to apply math to "all sorts of work- and everyday- life situations" (p. S108). Perhaps students that are participants in this project will immerge with a more expert-like disposition towards mathematics and thus be more willing to use it in everyday life.

With the boom of big data and the growing use of data analytics, there will be an increasing need for professionals to interpret and appreciate the outputs. Hilgers et al. (2015) discuss the need for creating courses with content within computing and business preparing students for big data work with companies. While the labs in this project certainly will not prepare students for careers in big data or analytics, being exposed to tackling concrete mathematics problems using the computer and coding could influence attitudes of some students creating an interest in or appreciation of careers such as ones in big data or business analytics. Students by no means will be proficient but they will have experienced working with some very introductory coding in Python. “Employees’ understanding of both technology and business processes are becoming valuable commodities” (McBane, 2003, p. 2). Many of the world’s leading financial institutions are requiring Python, and other computing languages, courses for their employees since the way they do business is evolving (Rayome, 2019). This is happening across other sectors as well, so as a business major having some knowledge of computational tools may become a marketable, if not even required, skill for the future.

Students may walk away from the course with the appreciation and willingness to problem solve using computational tools and to mathematize real problems they encounter outside of the classroom. This project could perhaps inspire some business majors to add other mathematics, data analytics, business analytics, or computer science courses to their undergraduate coursework, such as Brinkmann et al. (2016) say that some STEM students are doing with business courses currently, which may in turn affect their career aspirations or future careers.

Instruments Used in this Study

The Calculus Concept Inventory and the Mathematics Attitudes and Perceptions Survey were the instruments selected for this study. Development, use, psychometric properties, and limitations of each instrument are discussed below.

Calculus Concept Inventory

The Calculus Concept Inventory was the first instrument developed to define the basic understanding from a first semester differential calculus course (Epstein, 2013). The CCI was developed to measure conceptual understanding of differential calculus and designed to measure foundational knowledge of that subject (Epstein, 2007). Conceptual knowledge is defined as "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson et al., 2001, p. 346).

Students are finishing first semester calculus courses with little to no basic conceptual knowledge that university faculty assumed they were all developing (Epstein, 2013, p. 1019). Evidence such as this demanded the need for a validated tool to determine conceptual content gains to gather hard, scientific evidence that students can gain basic concepts of first semester calculus that can be used to determine the impact of different teaching methodologies on students' conceptual understanding (Epstein, 2013).

To measure conceptual knowledge, concept inventories have emerged in many disciplines and are usually multiple-choice tests given as a pre-test and post-test involving little or no computation. The Force Concept Inventory (abbreviated: FCI) sparked a large amount of research in physics education and began to cause reform (Epstein, 2013). The FCI started the movement of studying and analyzing students' conceptual understanding of basic principles in a variety of STEM disciplines through concept inventories (Gleason et al., 2015a). Research in undergraduate physics has shown that an increase in basic conceptual knowledge has been strongly dependent on the teaching methodology employed by the teacher, and classes that were taught in a nontraditional way with more interactive engagement by students had greater growth in conceptual knowledge from the beginning to the end of the semester (Epstein, 2013). Subsequent concept inventories have followed in many STEM disciplines such as in statistics (Allen, 2006), pre-calculus including the Calculus Concept Readiness Instrument and the Pre-

Calculus Concept Assessment (Carlson, Madison, & West, 2010; Carlson, Oehrtman, & Engelke, 2010), the Biological Concepts Instrument (Klymkowsky, Underwood, & Garvin-Doxas, 2010), the Chemical Concepts Inventory for chemistry (Mulford & Robinson, 2002), astronomy such as the Star Properties Concept Inventory (Bailey et al., 2011), and many others. The FCI has taken physics education and research ahead of many others disciplines in using concept inventories to analyze effectiveness of teaching practices and has inspired change in teaching methods (Epstein, 2013; Gleason et al., 2015a). The Calculus Concept Inventory was an attempt to do the same to calculus education.

Much like Hake (1998), Epstein (2013) began using the Calculus Concept Inventory to compare traditional teaching methods with interactive engagement methods. Using Hake's (1998) definition of interactive engagement:

> Interactive engagement methods are those designed at least in part to promote conceptual understanding through interactive engagement of students in heads-on (always) and hands-on (usually) activities which yield immediate feedback through discussion with peers and/or instructors (p. 65),

large studies were done in calculus courses at the University of Michigan in 2008 where many sections were taught using interactive engagement, with the highest normalized gains occurring in the classes that were reported to have the most interactive engagement (University of Michigan Department of Mathematics, 2009). Thomas (2013) studied conceptual knowledge gains in interactively engaged classrooms measuring gains using the CCI. Studies using the CCI that involve studying IE classrooms typically contrast them to traditional classrooms with passive student learning involving lectures (Bagley, 2014; Epstein, 2013; Thomas, 2013).

Many studies in mathematics using the Calculus Concept Inventory use it to study the effectiveness of a specific type of interactive engagement, a flipped classroom, which is similar to how the instruments of other STEM disciplines are using their concept inventories (Freeman et al., 2014). Anderson and Brennan (2015), Macejewski (2016), Schroeder, McGiveny-Burelle, and Fue (2015), and Ziegelmeier and Topaz (2015) all use the CCI in studies of flipped calculus classrooms. These studies all had populations of undergraduate students in calculus classrooms; some were classrooms with a variety of STEM and non-STEM majors (e.g., Schroeder, McGiveny-Burelle, & Fue, 2015; Anderson & Brennan, 2015), and Macejewski (2016) studied calculus exclusively for life sciences majors.

The Calculus Concept Inventory has also been used to study differences in different types of Calculus I classrooms such as in Bagley's (2014) study of traditional lecture, lecture with discussion, lecture with discussion and technology, and an inverted classroom. Thomas (2013) used the CCI to investigate student gains in interactively engaged calculus classrooms and what influence individual variables, such as gender and previous math exposure, and instructor level variables had on CCI gain scores. The CCI was also used in gathering nationwide data on the impact of ambitious teaching, such as active learning and student centered teaching, and found that students that had graduate student instructors that had training in ambitious teaching had students that scored just as well on the CCI as students of faculty members teaching the course (Larsen, Glover, & Melhuish, 2015). In this, the proponents of the ambitious teaching program used CCI scores to defend their use of non-traditional teaching practices to the administration pointing to positive results of students conceptual understanding as demonstrated by their CCI scores (Larsen, Glover, Melhuish, 2015).

Besides uses for studying teaching using interactive engagement in calculus classrooms, the CCI has been used to study differences in students' performance across different nations.

Chai, Friedler, Wolff, Li, and Rhea (2015) used the CCI to compare student gains between Chinese and American students, studying both gains and normalized gains on the overall test and in subsets of the test. Certain questions on the CCI have also been used in isolation as a component of studying students' understanding of the derivative (Park, 2013).

When used as a pre-post test the CCI is typically analyzed with normalized gains and often overall gains as well (e.g., Chai et al., 2014; Maciejewski, 2016). Besides using the CCI for measuring gains over a period of time, some studies use the CCI as a baseline measure of calculus knowledge (Schroeder, McGiveny-Burelle, & Xue, 2015; Ziegelmeier & Topaz, 2015).

The "Calculus Concept Inventory is a test of conceptual understanding (and only that—there is essentially no computation) of the most basic principles of differential Calculus" (Epstein, 2013, p. 1018). The CCI is a twenty-two question multiple-choice test that is administered as a pre- and post-test with gains analyzed. The CCI has been used to determine conceptual knowledge gains in calculus and used to study differences in calculus gains comparing different interventions or student populations.

The test has undergone development and validation and was funded by the National Science Foundation (Epstein, 2013). "It was developed by a panel of respected calculus educators and a consultant, nationally known for development and validation of standardized tests" (Epstein, 2013, p. 1019). Funding for the CCI development began in 2004 and began pilot testing on its first version in 2005 (Epstein, 2013) administering it to about 1100 students between twelve American universities and one university in Finland (Epstein, 2007). After the pilot testing, the test was revised to bring the questions to a more basic level (Epstein, 2013, p. 1021). The CCI was administered in the fall of 2008 to all of the University of Michigan's Calculus I sections, totaling 1,342 students (Epstein, 2013). The test was administered in a proctored setting with the pre-test taking place the first week of class and the post-test in the last week of class (Epstein, 2013). Cognitive laboratories, which are structured interviews where individual students express their thoughts verbally as they work through a problem and questions are asked to determine the student's mental process, were performed on the CCI in fall 2006 and brought the test to the twenty-two questions that are on it now (Epstein, 2013). Epstein (2013) presents that the "discrimination numbers were all acceptable" (p. 1023). The final test with twenty-two items was said to have two dimensions that "correlated well internally but not as well with each other" with the two dimensions being functions and derivatives, and a smaller dimension on limits, ratios, and the continuum (Epstein, 2007, p. 168). The measure of internal reliability, Cronbach's alpha, was 0.7, which was considered "modestly respectable" but falls short of the 0.8 which professional test developers wish to see (Epstein, 2013, p. 1023). Epstein (2013) reported that there was more data analysis possible with the data they already had and were continuing to receive and expected to report more on validation in the future, however, more reporting on validation from the CCI developers has yet to come. Other studies of its psychometric properties confirm the reported 0.7 Cronbach's alpha, but highlight other issues with the instrument (Bagley et al., 2017; Gleason et al., 2015a; Gleason et al., 2015b).

For internal reliability, Cronbach's alpha of 0.7 as reported by Epstein (2013) was confirmed (Bagley et al., 2017; Gleason et al., 2015a). Bryman and Cramer (1990) suggest that alpha of 0.8 is acceptable. Cohen, Manion, and Morrison's (2009) guidelines for the alpha coefficient rank 0.7 in the reliable category, although just above marginally reliable. This 0.7 level of internal consistency is on the lower end for an acceptable instrument that is created to measure mean differences between groups with twenty-five to fifty members (Gleason et al., 2015b).

Bagley and colleagues' (2017) factor analysis showed issues with internal structure validity. Epstein (2013) claims that the CCI measures calculus in three factors (functions, derivatives, and limits/ratios/the continuum), but other studies reveal that the CCI has at most two components and potentially just one (Bagley et al., 2017; Gleason et al., 2015b). Bagley et al. (2017) revealed "item responses are so closely correlated that the total CCI score is explained by one factor, which appears to be an overall knowledge of calculus content." Analysis by Gleason et al. (2015b) shows that a unidimensional model is assumed to fit the data well. This reveals that the CCI cannot be used to gain information about the conceptual understanding of various parts of calculus (Bagley et al., 2017). Gleason et al. (2015b) highlight that Epstein (2013) and colleagues reporting on the CCI having three dimensions but then using total percent correct to compute normalized gains is contradictory. The use of normalized gains implies measuring a single element, which Bagley et al. (2017) and Gleason et al. (2015a) analyze the CCI to have, which is overall calculus knowledge, over the twenty-two items.

While reporting on validity and reliability has not come from the developers, critiques of the instrument have emerged from others. Bagley et al. (2017) present that the CCI is, " a good starting point toward measuring calculus understanding" but some recent uses of it in studies have highlighted some issues of the instrument, but also acknowledge that both formally and informally the CCI is growing in use. One issue of the CCI is that it contains mathematical notation and terminology, such as derivative and *f'(x)*, which would not have been seen by students before an introductory calculus course (Bagley, et al., 2017) and presents issues with content validity raising concerns with whether it is really measuring what it is supposed to be measuring (Gleason, et al., 2015b). Of the twenty-two questions, nine of them have language that is not standard before a calculus course (Gleason et al., 2015b). This presents issues in several ways. One issue is that the CCI is supposed to assess understanding of calculus concepts not explicit vocabulary (Bagley et al., 2017). Another issue is that this notation and terminology may confuse students that have not seen it before and cause answering by random chance, which would bring the validity of pre-test scores into question (Bagley et al., 2017). If normalized gains are used to analyze the pre-post test change, students that have taken calculus before may score higher on the pre-test in spite of any conceptual knowledge and then have lower normalized gain scores (Gleason et al. 2015b) and users may underestimate conceptual understanding at the beginning of the course simply because students that have not taken calculus before are unfamiliar with terminology but may understand the underlying ideas of calculus (Bagley et al., 2017). The specific language that is questionable on the pre-test is common language in a typical calculus course and thus may measure conceptual understanding at the end of a one semester calculus course, but the specific language on the pre-tests brings in the question of normalized gains for evaluation (Gleason et al., 2015b). One proposed change to analysis is to use Item Response Theory (Gleason et al., 2015b; Thomas & Lozano, 2013). Thomas and Lozano (2013) found in comparing normalized gains and item response theory gains the two measures were strongly correlated. Normalized gain scores are related to pre-test score and favor the group with the higher pretest scores (Wallace & Bailey, 2010). The group with the higher pre-test score will have a higher normalized gain score. Normalized gain scores are also test-specific, so normalized gains, especially cut off values from Hake (1998) of high, medium, and low gain scores may not be completely transferable to the CCI (Thomas & Lozano, 2013). Thomas and Lozano (2013) assert that there is no objective way to choose which model to use before the test and each method has its own advantages, which are that normalized gains are much easier to compute and interpret but item response theory results in measure that are test and population independent. The debate over how to analyze student gains, normalized gains or item response theory, is a limitation of the CCI; many published studies found using the CCI used normalized gains (e.g., Anderson & Brennan, 2015; Bagley, 2014; Maciejewski, 2016), and Thomas (2013) developed IRT models for the CCI and was the only study found to detail item response theory of the CCI. Despite being relatively widely used in formal and informal settings (Bagley et al., 2017), the Calculus Concept Inventory has its shortcomings. The 0.7 Cronbach's alpha level of internal consistency is on the lower end for an acceptable instrument that is created to measure mean differences between groups with twenty-five to fifty members (Gleason et al., 2015b).

Pros and Cons of the CCI as Related to this Study. The Calculus Concept Inventory has been selected for use in the current study because it is the only validated instrument currently available for measuring calculus concepts. Despite some of its limitations and Gleason and colleagues' (2015a) calls for a better instrument, the CCI is still used in formal and informal settings (e.g., Anderson & Brennan, 2015; Maciejewski, 2016; Schroeder McGivney-Burelle, & Xue, 2015; Ziegelmier & Topaz, 2015; Peters et al., 2019) and another instrument has not been developed. The CCI was developed and tested on undergraduate students in a first semester calculus courses, which is the same population on which this study will focus. The Calculus Concept Inventory has been found to have only one factor, which can be explained as "overall knowledge of calculus content" (Bagley et al., 2017). While this could be seen as a drawback, the proposed study does not plan to study the effects of the treatment on specific sub-categories of calculus knowledge but rather on conceptual calculus knowledge as a whole, so the one factor of the CCI fits.

The limitations of the Calculus Concept Inventory, as presented previously, are the drawbacks for selecting this instrument for the current study. One concerning element is that the CCI seems more capable of detecting large differences in sample means or when the sample size is large (Gleason et al., 2015b). The sample size for this study will be four sections of undergraduate Business Calculus with each section having twenty-five to thirty students. Gleason et al. (2015b) state that the CCI is able to differentiate between sample means if the sample size reaches about one hundred students each. The total sample is likely to be over or near one hundred students total based on pre-post data, but each group will not have one hundred, so the inventory may not differentiate between the sample means because of small sample size.

Another draw back to using the CCI is that while developed to measure conceptual knowledge, which is defined as, “implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain” (Rittle-Johnson et al. 2001, p. 346), it does have some specific vocabulary of calculus. While this is not thought to present a problem at the end of the course, many students in the population of interest have not previously been exposed to calculus and may not know the terminology, which may underestimate pre-test knowledge of some students and skew the gain results (Bagley et al., 2017). Because of this, in data analysis it is then necessary to compare the results when having previously taken calculus or not is controlled for. Another limitation of the CCI is the debate over how to analyze the results as previously discussed.

The Calculus Concept Inventory has limitations, as discussed above, however, it is the only validated measure of conceptual understanding in calculus and provides a more standard measure on which to compare students rather than final grades or end of course exam scores.

The Calculus Concept Inventory is also an appropriate measurement tool for this project since it is typically used to study different teaching methods. While this study will not use it specifically for interactive engagement following Hake's (1998) definition, which has been studied often through flipped classrooms, the study does have elements of interactive engagement because the CCI will be used to compare the use of computational labs motivated by finite calculus to a traditional lecture course. The treatment is not necessarily the same interactive engagement in previous studies, but it is interactive engagement since "students cannot be passive observers in programming activities; rather they have to be actively engaged in reframing a given math problem, in writing and testing the computer code, and in making necessary adjustments and modifications" (Lovric, 2018, p. 2); it is a hands-on, minds-on process.

Mathematics Attitudes and Perceptions Survey

The tool selected to measure students' attitudes toward mathematics is the Mathematics Attitudes and Perceptions Survey (abbreviated: MAPS). Bressoud & Rasmussen (2015) reveal that post-secondary calculus, as currently taught, "is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics" (p. 144), which is much the opposite of the following goal of what an undergraduate mathematics course should do. "One goal of undergraduate education in mathematics is to help students develop a productive disposition toward mathematics…A way of conceiving of this is as helping mathematical novices transition to more expert-like perceptions of mathematics" (Code et al., 2016, p. 917). The Mathematics Attitudes and Perceptions survey has seven factors of expert-like behavior which were determined to be confidence in and attitudes towards mathematics, persistence in problem solving, belief about whether mathematical ability is static or developed, motivation and interest in studying mathematics, views of the applicability of mathematics to everyday life, learning mathematics for understanding, and the nature of answers to mathematical problems. MAPS is a thirty-two question survey with one filter question on a five point Likert scale. Development of MAPS was done iteratively with interviews of faculty and students, multiple rounds of responses from different populations of students, and factor analysis and model confirmation (Code & Maciejewski, 2017). The Mathematics Attitudes and Perceptions survey is a relatively new instrument published in 2016. Development began in 2010 and the instrument underwent multiple pilot runs and revisions to develop the final version, which had factor analysis and model confirmation to establish the categories that appear on the final version and efforts to establish reliability and concurrent validity were performed (Code et al., 2016). Factor analysis helped to arrive at seven categories with which the creators then attached names to the factors by matching themes with existing constructs in literature (Code et al., 2016). Using the full set of student data, N=3411, Cronbach's alpha value was found to be 0.87 for the whole instrument, without the filter statement (Code et al., 2016). This value indicated good reliability in using guidelines of Cohen, Manion, and Morrison (2009), which would indicate that it has an alpha level that indicates that the instrument is highly reliable.

The creators attempted to establish concurrent validity through patterns in course levels, patterns in correlations with course grades, and comparing findings to results from the Colorado Learning Attitudes about Science Surveys (Code et al., 2016). Concurrent validity, a type of criterion-related validity, provides a measure as to how well the new instrument compares to a previously well-established instrument or instrument measuring the same construct and does so by having the data gathered from the one instrument correlated highly to data from another instrument (Cohen, Manion, & Morrison, 2009). To also establish this validity, in comparing student groups and MAPS expertise score, it was found that students in Calculus I with no previous experience had the lowest mean expertise scores where an introduction to proofs class had the highest mean expertise score (Code et al., 2016); this was believed to be because the students in the introduction to proofs group had taken the most mathematics specific courses, thus developing their expert disposition more, and the students in the Calculus I courses tended to not be mathematics majors (Code et al., 2016). This aligned with longitudinal CLASS-Phys data, which found that students that complete a physics degree tend to have the expert-like orientations toward physics early on in their collegiate physics education (Perkins & Granty, 2010). In correlating MAPS scores with course grades, it was found that overall expertise index is correlated with course grade for the groups observed (Code et al., 2016). All correlations were found to be significant at the $p < 0.01$ level, except mindset, which was the lowest correlation for all groups, and confidence, real world, and sense making categories were not significant at this level for the introduction for proofs students with the creators noting that that was the smallest in sample size (Code et al., 2016). Findings also indicated that confidence was the most highly correlated with course grade in comparing the categories (Code et al., 2016), which aligns with research that confidence and self-efficacy are predictors of course grade (Code et al., 2016). The creators also compared trends over an academic year for students in first year mathematics courses on MAPS expertise score. The results align with results from CLASS-survey instruments where overall students move away from expert orientations in the first year course (Code et al., 2016). However, in comparing students in courses with more active teaching approaches, such as a flipped classroom, the findings indicate that these students decline less than students in a traditional first year calculus class, which aligns with results from studies in physics with CLASS-type instruments (Code et al., 2016). At this time, there are no further psychometric studies available on the Mathematics Attitude and Perceptions Survey.

The Mathematics Attitudes and Perceptions Survey is a new instrument and has limited published results. In the uses of the instrument and reports on new data from the creators, it has yielded similar findings to the Mathematical Association of American's National Study of calculus findings (Sonnert & Sadler, 2015) and results similar to the CLASS surveys with which the creators of this instrument drew inspiration including overall decrease in attitudes over a semester of first year calculus and men reporting higher attitudes in many categories on MAPS than women (Code & Maciejewski, 2017). Validation was done on a large set of student data involving 3,411 students (Code et al., 2016), however, there are no further studies on psychometric properties of this instrument, but the one published study that uses it (Maciejewski, 2016) does find similar results to what the developers found in change in MAPS scores from pre-test to post-test. Because of the limited usage, the MAPS also does not have enough data for comparison across demographic variables or comparison based on teaching method (Code & Maciejewski, 2017).

The Mathematics Attitudes and Perceptions Survey was adapted from the expert/novice instruments for undergraduate STEM education from the group of Colorado Learning Attitudes about Science Surveys (Code et al., 2016). These attitude surveys were originally developed for physics with CLASS-Phys (Adam et al., 2006) and have been extended to other science domains such as biology (Semsar et al., 2011), chemistry (Barbera et al., 2008), earth science (Jolley et al., 2012), and computer science (Dorn & Tew, 2015). Each of these, as is MAPS, is a multidimensional survey with subscales related to expert-like thinking in the specific discipline (Code et al., 2016). Attitude surveys that measure expert-novice perspectives must be domain-specific to reflect expert-like thinking of the respective discipline (Code et al., 2016). Adams et al. (2006) reveal that students cannot decide on an answer if the questions are too general and gives an example when students are asked about science they report having different feelings based on the type of science. Because of the need for more specificity in the questions relating to the domain of interest, MAPS was developed for undergraduate mathematics specifically.

MAPS shares similar statements to and aspects of development of these expert/novice surveys (Code & Maciejewski, 2017). The CLASS statements were written to be as "clear and concise as possible and suitable for use in a wide variety of physics courses," with students responding on a five-point Likert scale (Adams et al., 2006, p. 010101-1). CLASS "was designed to address a wider variety of issues that educators consider important aspects of learning physics" and "the wording of each statement was carefully constructed and tested to be clear and concise and subject to only a single interpretation by both a broad population of students and a range of experts," which "make the survey suitable for use in many different courses covering a range of levels, and also allows most of the statements to be readily adapted for use in other sciences" (Adams et al., 2006, p. 010101-2) on which MAPS then built on for mathematics. Wording for CLASS statements was created by listening to and writing down statements that students said in interviews to word the statements in ways students will easily understand and represent their ideas about wording (Adams et al., 2006). Comparing the statements on the CLASS-Phys (Adams et al., 2006) with statements on MAPS (Code et al., 2016), it can be seen that the wording is similar for many statements but the discipline is changed. CLASS instruments were developed so that statements could be used for students that had never taken physics (Adams et al., 2006), which is also an important consideration for MAPS in the proposed study. CLASS instruments were also designed to be administered quickly, requiring ten minutes or less for thoughtful responses, and ease of administration and scoring (Adams et al., 2006). MAPS follows this same format.

MAPS is also multi-dimensional and can be analyzed as a complete expert index and can also be analyzed by category. MAPS has seven categories, which include growth mindset, real world, confidence, interest, persistence, sense making, and answers (Code et al., 2016). Other mathematics attitudes surveys are unidimensional, such as the Conceptions of Mathematics Survey, which expresses students' views of mathematics as fragmented or cohesive (Crawford et al., 1994); the Fennema-Sherman Mathematics Attitude Scales are multi-dimensional but has over one hundred questions and has dimensions, such as mathematics as a male domain, that may no longer be as applicable based on the way the questions are asked (Forgasz, Leder, & Gardner, 1999). The multi-dimensionality is of interest in the proposed study.

The different categories on the Mathematics Attitudes and Perceptions Survey are defined below.

- Confidence: This category is described as confidence in mathematics. "Confidence in mathematics is a person's perceived ability to successfully engage in mathematical tasks. Confidence is known to affect a student's willingness to engage with a task, the effort they expend in working the task, and the degree to which they persist when encountering setbacks." (Code et al., 2016, p. 920). A statement representative of this category is given by, "No matter how much I prepare, I am still not confident taking math tests" (Code et al., 2016, p. 920).
- Persistence: Persistence is referencing persistence in problem solving. The developers of MAPS describe it as, "How students approach solving a non-routine mathematical problem (i.e., one where they can 'get stuck') is just as important as their ability to solve that problem. It is now well established that experts and novices differ in how they solve problems. Experts have a wealth of knowledge –

in terms of knowledge of facts and definitions, but also of problem types and solution strategies – and this aids in their problem solving. Experts also attend to different features of problems than novices," and "experts grouped the problems according to their deep structure, that is, according to the underlying principles needed to solve them. Novices tended to group the problems according to their surface structure, concerning superficial features of the problem setup. Moreover, experts engage metacognitive skills while solving problems, monitoring their own progress, looking for relevant choices among their broad set of known solution strategies, and are willing to abandon strategies when they are judged to be no longer applicable… We may thus consider perseverance in problem solving as relatively distinct from issues of anxiety or laziness and more in terms of the ability to select appropriately from a sufficiently large set of strategies and to continue selecting and attempting strategies based on one's progress" (Code et al., 2016, p. 921). The statement that represents the persistence category is, "If I get stuck on a math problem, there is no chance that I will figure it out on my own" (Code et al., 2016, p. 921).

- Growth mindset: The representative statement for growth mindset is "'Math ability is something about a person that cannot be changed very much" (Code et al., 2016, p. 921). Code et al. (2016) state, "This category rates students' belief about whether mathematical ability is innate or can be developed" (p. 921).
- Interest: The interest category is described as interest in mathematics with the representative statement given as "I only learn math when it is required" and the developers describe the scale as quantifying students' interest in engaging with

mathematics (Code et al., 2016).

- Real world: The real world category rates students' beliefs in the relationships between mathematics and the real world. The statement given as the representative statement is, "Reasoning skills used to understand mathematics can be helpful to me in my everyday life" (Code et al., 2016, p. 922).
- Sense making: The sense making category is "intended to quantify students' perspectives on the nature of their personal mathematical knowledge. Students tend to structure and apply their mathematical knowledge in two broad ways: as certain tools to solve learned problem types or as a coherent body of knowledge that can be interpreted and applied equally to known and novel problems" (Code et al., 2016, p. 922). Experts have the belief that mathematics is a coherent body of knowledge. The statement that represents this category is, "In math, it is important for me to make sense out of formulas and procedures before I use them" (Code et al., 2016, p. 923).
- Answers: The developers of the MAPS describe this category as the nature of answers with the representative question being, "I expect the answer to math problems to always be numbers" (Code et al., 2016, p. 923). "This category characterizes students' views on the nature of solutions to mathematics problems. Students may view answers in mathematics as being either right or wrong and the solutions supporting these answers as having a certain degree of rigidity" (Code et al., 2016, p 923).

Because of its recent development, the Mathematics Attitudes and Perceptions Survey has not been widely used in published research yet. Maciejewski (2016) uses this instrument, along with the Calculus Concept Inventory, to study flipping a calculus classroom. His findings indicate, similar to CLASS data, there is a decline in MAPS expert scores over the semester, but that students in the flipped section had a smaller decrease in MAPS score than students in a traditional course. Also while there were no statistically significant differences in pre-test MAPS scores, the flipped section had statistically significant higher post-test MAPS scores than the traditional group (Maciejewski, 2016).

Pros and Cons of the MAPS as Related to this Study. The Mathematical Attitudes and Perceptions Survey was selected for this study because of its focus on undergraduate mathematics students. The developers of MAPS state, "one goal of an undergraduate education in mathematics is to help students develop a productive disposition towards mathematics" (Code et al., 2016, p. 917), which this project aims to do. MAPS was developed for the study of and through administration to undergraduate students, which are from whom this study will get its sample, whereas several other instruments were developed on high school students or younger.

MAPS was also selected for this study in part because of its motivation from instruments in other STEM disciplines. MAPS was selected over several other mathematics attitude inventories because of its relative short length compared to other instruments for measuring attitude toward mathematics; longer surveys can cause survey fatigue. The creators of MAPS "intentionally kept the survey brief enough to be used as a pre- and post-test instrument in authentic course settings" (Code et al., 2016, p. 932). MAPS was selected over the Fennema-Sherman Scales because they are 108 items long and take around forty-five minutes to complete making survey fatigue a serious consideration as participants become bored or uninterested and no longer give effort on answering the questions; the Fennema-Sherman Scales are also more than thirty-years old and some of the constructs they measure such as mother and father education scale, math as a male dominant field, and teacher scale (Fennema & Sherman, 1976) are not of interest for this study. MAPS was also chosen over the Attitudes Toward Mathematics Inventory, which is forty questions (Tapia, 1996).

MAPS was selected because it is multidimensional and was developed with mathematicians' input throughout the process (Code et al., 2016). The multidimensionality is fitting for the current study. MAPS, as with other inventories from mathematics and other STEM disciplines, shows that students tend to move away from expert-like thinking throughout the semester (Code et al., 2016). The results from MAPS can be analyzed as a whole but can also be broken down into seven specific categories to see if there is change in sub-categories individually. While there may be no positive effect on the expert-like disposition as a whole, based on previous findings in mathematics (Code et al., 2016) there may be effects on sub-categories, which include growth mindset, real world, confidence, interest, persistence, sense making, and answers. Of particular interest is the real world category. Previous research (Gordon, 1979; Hoffman, 1989; Ralston, 1984) has revealed that using a finite approach to calculus can make mathematics more real and relevant for students as it can be more concrete and can include less concocted examples; in addition, students tend to support the use of technology in the mathematics classroom (Heid et al., 2002; Zevenbergen, 2004) and that technology can make mathematics also more real and relevant, so being introduced to some calculus topics using the labs may enhance student beliefs that mathematics is more connected to the real world outside of the classroom than those that do not use the technology as much. The developers of MAPS encourage others to use it "in any undergraduate mathematics education setting where student beliefs and perceptions are suspected to play a role" (Code et al., 2016, p. 933), which Business Calculus fits with well since students in these classes are thought to have relatively negative attitudes toward mathematics (Depaolo & Mclaren, 2006; Liang & Pan, 2009).

As previously discussed, a limitation of MAPS is that it is a relatively new instrument without much published usage for comparison. Maciejewski (2016) uses both MAPS and the CCI in a study of flipping undergraduate calculus classrooms. Maciejewski's (2016) findings do align with the findings of the validation of the instrument; his findings indicate that overall students move away from expert-like dispositions and did so on all subscales but none were statistically significant comparing the two groups (p. 195). The findings indicate that while there were no statistically significant differences between the two groups in the beginning on MAPS expertise score, there were significant differences on post-test MAPS expertise score at the p=0.01 level with the flipped section having higher overall scores. Maciejewski (2016) does not present data or findings on other demographic variables, which may be considered in this study, and MAPS does not have data on other demographic variables.

CHAPTER 3: Manuscript 1
Computational Labs and Conceptual Understanding

Abstract

This study investigates the impact of computational labs on students' conceptual understanding of calculus in a one-semester Business Calculus course. Investigating students' gains in conceptual understanding from the beginning to the end of the semester provides empirical evidence about and student experiences of the use of computational labs with business applications. Students' conceptual understanding of calculus was measured through the administration of the Calculus Concept Inventory as a pre-test and post-test. Overall gains were not significant and reasons for this are explored. Student comments were also analyzed for students' perceptions of computational labs and calculus; these comments do indicate several important impacts and suggest a number of changes that may improve the impacts.

Introduction

Over 300,000 students enroll in college or university calculus every semester (Bressoud, 2015). There have been major changes in who is now expected to complete calculus, including most STEM majors and most business majors, but what is taught in a calculus course and how it is taught have been relatively stagnant, leading to an increasingly poor fit for today's students (Kaput, 1997). Calculus has long served as a stumbling block and a "critical filter" that is "blocking access to professional careers for the vast majority of those who enroll" (The Mathematical Association of America, 1988, p. xi) and appears to remain as a filter today, even for some strong students (Bressoud et al., 2013) in STEM majors and business majors. The last thirty years have involved mathematics educators discussing pedagogy and curricula for calculus to improve the courses. Many of these changes desire calculus as a pump not a filter for numerous disciplines and helping students get through it with meaningful understanding (Axtell, Doree, & Dray, 2016).

Not only is calculus serving as a weeding-out course for many disciplines, but students that complete calculus are finishing first semester calculus courses with little to no basic conceptual knowledge that university faculty assumed they were all developing (Epstein, 2013). Students very often struggle with calculus and it is thought to be a difficult subject because "it is a coherent theory that builds on all of high school mathematics and then builds on itself. That is one must thoroughly understand what has come before in order to go on" (Douglas, 1985).

To address the issues of lack of conceptual knowledge and the issues of students leaving their intended majors and potential careers because of calculus, recommendations have been made to better serve the vast, diverse population of students that are required to take it. The Mathematical Association of America has two reports on partner disciplines both suggesting that partner disciplines desire calculus to "increase emphasis on conceptual understanding, problem-solving skills, mathematical modeling, and communication of ideas; provide a better balance of perspectives (such as exact and approximate); and make appropriate use of technology" (Axtell, Doree, & Dray, 2016, pp. 33-34). These recommendations also include more modeling, examples from a variety of contexts, and applied projects (Axtell, Doree, & Dray, 2016).

One way these recommendations could be implemented would be through computational tools. "With mathematics seeing an increasing focus on computation, mathematics education should not be far behind in its pursuit to understand the teaching and learning of computing within mathematics" (Lockwood, DeJarnette, & Thomas, 2019, p. 1). This project aims to fulfill some of these recommendations and studies one example of teaching and learning in calculus, by using computational labs, in Jupyter notebooks, in a one-semester Business Calculus course.

Literature Review

Students around the world have problems with basic conceptual understanding of calculus often finishing the course with little to no basic conceptual knowledge that university faculty assumed they were developing (Epstein, 2013). The recent national study of calculus found that in any introductory calculus class about one-fourth of students attain a D, F, or withdraw from the course; of the students that do complete the course about two-thirds self-report being able to correctly compute derivatives and integrals and 40% feel confident in their ability to use ideas of calculus (Bressoud, 2015). Research repeatedly suggests that in introductory mathematics courses, students are not learning the intended material (Breidenbach et al., 1992; Carlson, 1998; Tallman et al. 2016; Thompson, 1994), students are leaving the courses unprepared for other courses (Carlson, 1998; Selden, Selden, Hauk, & Mason, 2000; Thompson, 1994), and students lose interest in mathematics after completing the course (Bressoud, Mesa, & Rasmussen, 2015; Seymour, 2006), all of which have been seen in both small, localized studies but also in national studies of introductory college mathematics courses (Bressoud, Mesa, & Rasmussen, 2015).

To address the needs of the vast amount of students tasked with taking calculus and their need to gain conceptual knowledge of the subject, there is a "long overdue reconsideration of the appropriate intellectual content of calculus," (Kaput, 1997, p. 731). Kaput argues that what is taught and how it is taught in a calculus course have largely remained unchanged because they have "served traditional purposes and populations extremely well" (Kaput, 1997, p. 731). However, there have been major changes in who is now expected to complete calculus, including most STEM majors and most business majors, but there have not been striking changes to how or what is taught in a calculus course leaving it misaligned with the needs of today's students (Kaput, 1997). Client disciplines are also not pleased with students completing calculus courses that have been extremely stagnant, and suggestions have been made to have experts from these client disciplines teach the calculus courses rather than mathematics educators (PCAST, 2012). Rethinking the way calculus is taught and what content is necessary today could make it more accessible to more students, but that that will require mathematics educators to be more open to the use of technology, organization of topics, and what counts as mathematical thinking (Kaput, 1997).

Calculus as a Filter

For many years, calculus has served as a challenging course that is a "critical filter" that is "blocking access to professional careers for the vast majority of those who enroll" (The Mathematical Association of America, 1988, p. xi) and appears to remain as a filter today (Bressoud et al., 2013). The President's Council of Advisors on Science and Technology (PCAST, 2012) found that students often leave STEM degrees because of courses, such as calculus, being taught in an unwelcoming and uninspiring manner with students referring to them as "frequently uninspiring, relying on memorization and rote learning while avoiding richer mathematical ideas" (PCAST, 2012, p. 28) and struggle to get through them. Some students simply cannot successfully get through the barrier of calculus.

Though often thought of as the filter for STEM degrees, calculus can also serve as the screening course for business and social science students (Brito & Goldberg, 1988) thus ways to improve these courses must be explored. While it is acknowledged that the country needs more STEM professionals (PCAST, 2012), it is also documented that there is an increasing demand for business professionals capable of integrating science, technology, and mathematics into their business operations and management (Ledley, 2012; Ledley & Holt, 2014; Ledley & Oches, 2013; McCann, 2006). Leaders in mathematics education have long called upon mathematics departments to collaborate with other disciplines, such as business, for the revision of course content for mutual interest (Egerer & Cannon, 1988). It seems these were calls for interdisciplinary work since "business is inherently interdisciplinary…the practices, policies, and norms that govern business are grounded in social science, and the goods and services that businesses produce are themselves the fruits of science, engineering, arts, and humanities" (Bouldin et al., 2015, p. 17).

> The business leaders of tomorrow, and therefore the business students of today, need to understand the conceptual basis of algebra, calculus, and statistics. They must be able to interpret and use the results of calculations… For business executives to be successful, they need proficiency in the technology that produces the data they need, understanding of the algebra, calculus, and statistics underlying these data, and knowledge of how sensitive the results are to changes in the input data. (Lamoureux, Beach, & Hallet, 2000, p. 20)

Bouldin et al. (2015) detail that many business majors take an introductory science course that usually satisfies a general education requirement and is the last formal science course they take, but this course is commonly purely introductory, often has little context for applying the scientific knowledge outside of the science discipline, and may leave them feeling that the course was pointless and unrelated to anything they will be doing in the future. To combat this, Bouldin et al. (2015) propose interdisciplinary courses and experiences that allow students to see the connections between the disciplines. The same may be, at least in part, said of Business Calculus. Many of these students finish the course feeling that it was simply a mathematics course they had to complete to fulfill a requirement and nothing more, but a more interdisciplinary course with authentic contexts and realistic ways of problem solving could remedy this situation.

The Mathematical Association of America's Curriculum Renewal Across the First Two Years (abbreviated: CRAFTY) subcommittees report The Curriculum Foundations Project: Voices of the Partner Disciplines emphasizes that there is a need for "a shift from business mathematics viewed as a method of weeding out students to business mathematics with the purpose of adding value" (Lamoureux, Beach, & Hallet, 2000, p. 22). It recommends doing so in part by stating

> Calculus in the business mathematics curriculum should emphasize the basic concepts and how they apply to business problems, with more attention to numerical methods and less to techniques of symbolic differentiation and integration. The Business Calculus curriculum should include an introduction to rates of change, and the dynamic nature of real world systems, constrained optimization, and interpretations of area under a graph. (Lamoureux, Beach, & Hallet, 2000, p. 20)

Business students required to take Business Calculus often miss seeing the connection between calculus and the rest of their courses, which causes them to be uninterested in the material, "unmotivated and even resentful" (Narasimhan, 1993, p. 254), which may be hindering the effort they put in and their performance. "Students learn more when they are intensely involved in their education and have opportunities to think about and apply what they are learning in different settings" (Kuh, 2003), which in a business mathematics course needs to be in applications to business. Students cannot be filtered out of their desired field of study because they are unable to get through a challenging course that is often presented in a way that leaves them missing the connections and value for their intended careers. Business mathematics courses should help students appreciate the relevance of mathematics (Lamoureux, Beach, & Hallet, 2000, p. 19), which this project hopes to aid by introducing students to calculus topics with more interdisciplinary connections utilizing modern technology to also help them gain greater understanding.

Recommendations for Improving the Courses

Many recommendations as to how to do improve a Business Calculus course include using technology to show students tools they will use in the work place, to enhance the efficiency of the learning process, and to enrich and maintain student interest (Lamoureux, Beach, & Hallet, 2000). One recommendation calls on instructors to use such technology in the classroom and allow for hands-on experiences for students through introductory data analysis, creating models, and applications such as optimization and other simulations (Lamoureux, Beach, & Hallet, 2000), which the project aims to do through the use of Jupyter notebook labs with business applications to introduce calculus topics.

Labs. The MAA CRAFTY project recommendations for business mathematics also include the use of labs to support student learning (Lamoureux, Beach, & Hallet, 2000). A review of literature revealed that labs in calculus courses take on a variety of forms, use various technologies, and cover a variety of content (Leinbach, 1991). A calculus lab can be used as a "learning device to see how calculus applies to other courses and disciplines" and "helps students relate the rather abstract ideas of mathematics to non-mathematical ideas they have encountered in other courses" (Basson, Krantz, & Thorton, 2006, p. 346). Successful calculus labs are used for enhancing the existing calculus course and having the main goal of the lab not be to teach additional material but to teach students to make connections (Basson, Krantz, & Thorton, 2006), which is a goal of this project. Successful calculus labs also seem to use real data (e.g., Basson, Krantz, & Thorton, 2006; Kowalczyk & Hausknecht, 1994), which this project does as well.

Labs are a suggestion for engaging students in the mathematics they will be learning:

> Another potentially useful method for drawing students into the lecture is to start the lecture with a real-world (or realistic) business problem. If students are convinced that the problem is worthy of their attention, and that they do not know how to solve it, they are much more likely to pay attention and to retain what they learn. It is important to get the buy-in at the beginning. (Lamoureux, Beach, & Hallet, 2000, p. 21)

The labs in this project are used to introduce students to calculus topics and are situated in the context of a business or financial problem. The choice to begin with the labs rather than use them as extension labs aligns with the aforementioned recommendations and is done to introduce students to calculus concepts so that concepts are introduced through simple, concrete examples. "It is essential here to make it clear to the pupil that he is dealing, not with something mystical, but with the simple things that anyone can understand" (Klein, trans. 2007, p. 223). Felix Klein argued for introducing calculus in such a way that,

> We desire that the concepts which are expressed by the symbols $y=f(x)$, dy/dx, $\int y\, dx$ be made familiar to pupils, under these designations; not, indeed, as a new abstract discipline, but as an organic part of the total instruction; and that one advance slowly, beginning with the simplest examples. (Klein, trans. 2007, p. 223)

Research from 1970s and 1980s emphasized them same and suggests doing so on the computer, especially for non-STEM majors (Gordon, 1979; Hoffman, 1989; Ralston, 1984). These studies suggest using "finite differences and sums as motivation for infinitesimal calculus and as an appropriate setting for solving real problems by discrete approximations" (Gordon, 1979, p. 24). Findings from Gordon's (1979) study suggest that students gain more complete understanding of the concepts faster than when solely introduced to the continuous approach and can be easily expanded to the continuous approach with the introduction to the limit. Students with weak mathematical foundations can benefit from this approach (Gordon, 1979). Gordon claims that students gain appreciation of the relationship between math and the computer, which "provides an ideal context in which to develop several simple, yet useful, numerical algorithms for approximating functions and for actually finding where all those 'given' functions come from" (Gordon, 1979, p. 23).

The use of computational tools through these labs will rid students of the need to do some of the work through symbolic manipulation with paper and pencil, thus allowing for the introduction of more advanced topics and scenarios than many students could do by hand (Hoffman, 1989). Teachers often reduce problems to a very simplified version so that they are at a level students can complete with the tools at there disposal, but using computational tools could allow for more advanced examples (Hoffman, 1989). This is also following recommendations of Kaput (1994) for calculus in which "the power of new dynamic interactive technologies should be exploited in ways that reach beyond facilitating the use of traditional symbol systems (algebraic, numeric, and graphical), and especially, in ways that allow controllable linkages between measurable events that are experienced as real by students and more formal mathematical representations of those events" (p. 42). To which Dubinsky (n.d.) expands on and gives an example of that "calculus may be needed to analyze DNA or understand behaviors of the market. In the latter case, we might wish to reverse the traditional use of calculus in which the discrete is an approximation of the continuous and study fluctuation of prices in which the continuous is a model for the discrete" (pp. 6-7). This may help students find the mathematics more relevant, but also may increase achievement as research suggests that learning in a real world context greatly influences mathematical capabilities (Couch & Haines, 2004).

The labs in this project were done using Jupyter notebooks, which allows for coding in Python. Juypter notebook is thought to be an easy to use program even for teachers with little to no background in coding and computing, and students can easily discover how technology can help them solve mathematics problems and communicate solutions to challenging and real problems (Koehler and Kim, 2018). "The Jupyter Notebook is an open-source web application that allows you to create and share documents that contain live code, equations, visualizations and narrative text. Uses include: data cleaning and transformation, numerical simulation, statistical modeling, data visualization, machine learning, and much more" (Project Jupyter, n.d.). Jupyter notebooks support multiple programming languages, can be easily shared, can produce interactive output with images, videos, LaTex, and more, and can support big data tools (Project Jupyter, n.d.). The notebooks used in this project were running Python, which was chosen since Python is a popular, commonly used programming language that has a large focus on ease of use and readability (Meurer et al., 2016). Technology is playing a growing role in the financial and business industries, and many companies, such as big banks, are requiring Python courses for analysts and other employees (Hilpisch, 2016; Rayome, 2019), so exposing business majors to this computing language could introduce them to a valuable skill they may be expected to use in industry.

Computational Tools. The national study of calculus (Bressoud, Mesa, & Rasmussen, 2015) does not highlight the use of computing in a programming language as used in this study, however, the use of software in mathematics has grown recently and it is known that these tools are useful for professional mathematicians, so students should be exposed to these tools. Often software can make mathematical computation and inquiry quicker and more accessible to those not advanced in their mathematical careers (Quinlan, 2016). Many university mathematics professors reported that technology was significantly important in mathematics and mathematics teaching and recommended software including MATLAB, Maple, and Python as well as others (Quinlan, 2016). Lockwood, DeJarnette, and Thomas (2019) found from interviewing mathematicians:

> Computing is an activity that can be performed with a variety of tools, including paper and pencil. At the same time, given that the advances in technology can increase the efficiency, accuracy, and utility of computational work, it is reasonable to expect that computing is particularly salient practice in this era, at least inasmuch as mathematicians may recognize it as an integral part of the work of doing mathematics. (p. 4)

With the importance of such tools growing within mathematics, there has also been a rise in research on computational tools in mathematics education (e.g., Cline et al., 2019; Jones & Hopkins, 2019; Kilty & McAllister, 2019). Lockwood, DeJarnette, and Thomas (2019) call on practice and research to reflect practices of society since we live in a progressingly computerized time. diSessa (2018), Lockwood, DeJarnette, and Thomas (2019), and others encourage mathematics educators to research how their ideas of how computers and computational tools can be used in mathematics learning and that there is still much to be unearthed in effective learning and learning of computing within mathematics.

"In the real world we use computers for calculating, almost universally. In education we use people for calculating almost universally" (Wolfram, 2014, p. 1); this changes what mathematics might be of importance to be taught. Because of such, Gravemeijer et al. (2017) assert "that we have to shift away from teaching competencies that compete with what computers can do and start focusing on competencies that complement computer capabilities" (p. S107). Students tend to align with the research that the inclusion of technology in the classroom can be beneficial for focusing on problem solving and more real situations and being reflective of the work place, but this is not always reflected in the mathematics classroom (Zevenbergen, 2004). "In school the professor formulates the [mathematical] problem and you solve it – you hope. In industry, you formulate the [mathematical] problem and the software solves it – you hope" (Keeler & Grandine, 2013, p. 41). It is often difficult for students to engage in mathematical modeling, or translating a real world situation into a mathematical representation, because they have learned math decontextualized and have a hard time switching between real world and mathematics because of the lack of practice they get with this in school (Couch & Haines, 2004). These, however, are important skills because, "when mathematics is applied in the modern world for a practical purpose, we almost always require a computer to deal with a realistic level of complexity or to manage the data involved" (Cline et al., 2019).

Impacting Learning. Some previous research indicates positive results on impacting understanding using computational tools in the mathematics classroom. Heid, Blume, Hollebrands, and Piez, (2002) report many benefits to incorporating technology into the mathematics classroom including on conceptual understanding and time for more focus on concepts instead of such large emphasis on skills (Heid, 1988). Heid, Blume, Hollebrands, and Piez (2002) also report students perform just as well on test items that require computation and procedural skills as students who did not use computer algebra systems, CAS, in their class; studies have shown that students using CAS have overall conceptual understanding at or above a level of those not using CAS and students using CAS better understood concepts (Heid, Blume, Hollebrands,& Piez, 2002) because it allows more time for developing conceptual understanding and for enabling students to understand real world quantitative situations" (Heid, Blume, Hollebrands,& Piez, 2002, p. 588).

Research from the 1990s in using computers and a variety of programming languages in calculus courses did not find major improvement in student learning because of such technology, but many of them found overall positive results based on students' comments and did not see negative effects on performance (e.g., Schwingendorf & Dubinsky, 1990; Höft & James, 1990). Fenton and Dubinsky (1996) developed ISETL language to help students more effectively learn mathematics beginning with the argument that "communicating with a computer requires a level of precision that will help illuminate important mathematical ides for students" (Lockwood, DeJarnette, & Thomas, 2019, p. 17). In a calculus courses where students explored calculus topics through programming in BASIC, Crowell and Prosser (1991) did not find overall improvement in understanding as measured by final exam scores, but did find ease of implementation, mixed results on students attitudes toward the computer enhancing calculus, and still asked the questions of what is the computer's place in calculus with programming and how would the traditional curriculum and pedagogy be revised to best incorporate the power computers could provide (Crowell & Prosser, 1991), both of which are still questions of today.

Cetin and Dubinsky (2017) found that students learned concepts such as functions more effectively using ISETL by writing and running their code students had to think about what the computer is doing with the code, which can help students internalize concepts. They explain that students had to define the function correctly in the program and then students really begin to reflect when having to enter certain code, see the result, and have to think about how the computer got the result (Cetin & Dubinsky, 2017). Benakli, Kostadinov, Satyanarayana, and Singh (2017) also report similar results when using hands on computer programming in *R* to solve problems of calculus, probability, statistics, and data analysis. Using computational tools can improve conceptual understanding of many difficult concepts from complex and abstract problems and improve problem-solving skills (Benakli, Kostadinov, Satyanarayana, & Singh, 2017). Rich, Bly, and Leatham (2014) in studying the impacts that learning computer programming can have on the way students approach mathematics claim that learning to code "provided participants with context, application, structure, and motivation for mathematics" that was long lasting. Further developing such tools that utilize the computing power that is now at most students' fingertips could provide valuable opportunities for students to learn calculus concepts in a modern way.

Research Question

This study was designed to determine the impact, if any, of the use of computational labs on students' conceptual understanding of calculus in a one-semester undergraduate Business Calculus course. The overarching research question used to guide this study is:

> To what extent will students demonstrate gains in conceptual calculus knowledge when introduced to calculus concepts through computational labs in a Business Calculus course?

This project studies the implementation of computational labs and their impact on conceptual understanding through a quasi-experimental, multi-method design. The Calculus Concept Inventory was used to collect data on students' conceptual calculus knowledge, and the data was statistically examined to determine the impact of the labs on this variable. Students' comments from the end of each lab were collected and analyzed as well. The main purpose of this research is to determine the effectiveness of computational labs as a method to affect students' conceptual knowledge of calculus and further inform teaching practices.

Participants

Participants in this study were undergraduate students at a medium-sized private university, with undergraduate enrollment of around 4,500 students, enrolled during the spring semester of 2019. At this university the liberal arts general education curriculum currently includes a mathematics requirement of which approximately 39% of students meet by taking Calculus I or Business Calculus during their freshman year. Students in both groups ranged from freshmen to seniors at the university, but the majority of students enrolled were freshmen.

There were a total of four sections of Business Calculus participating in this study all taught by the same instructor. Thirty students were enrolled in each section to begin the semester with numbers dropping by a few students in each class as the semester progressed. The semester began with 120 students enrolled across the four sections, with 60 eligible students in each of the experimental and control groups. Of the 120 students enrolled at the beginning of the semester, 113 elected to participate in the first day administration of the Calculus Concept Inventory. By the end of the semester, enrollment across the four sections was down to 106 students, of which 94 participated in the end of semester Calculus Concept Inventory. Despite 94 responses on the post-test, only 79 responses gave consent and were successfully matched to their corresponding pre-test code. Demographic information about the 79 participants for which matched data was acquired and analyzed is presented in the following table.

Table 1

Student Demographic Characteristics

Demographic Characteristic	Number of Respondents		
	Control (n=39)	Experimental (n=40)	TOTAL (n=79)
Gender			
Male	25	21	46
Female	14	19	33
Major			
Business	35	36	71
Undecided	3	4	7
Other	1	0	1
Previous Calculus			
Yes	17	12	29
No	22	28	50

In addition to demographic information, students were asked to provide their intended major and the concentration of their major if applicable. The table below details the different majors that were present among the participants, which included Business and the different concentrations within, non-business, and undecided.

Table 2

Student Major and Concentration

College Major and Concentration	Number of Respondents		
	Control (n=39)	Experimental (n=40)	Total (n=79)
Business	35	36	71
Business Admin.	13	12	26
Marketing	9	6	15
Accounting	2	5	7
Entrepreneurship	5	4	9
Finance	2	4	6
International Bus.	3	4	7
Sales	1	1	2
Non-Business Major	1	0	1
Undecided	3	4	7

Methodology

This project studies the implementation of computational labs and the effects on conceptual understanding through a quasi-experimental, multi-method design including an experimental group and a control group. Gains in students' conceptual understanding of calculus were quantitatively measured using the Calculus Concept Inventory. In addition to the quantitative data, student comments were also gathered and analyzed to provide additional insight into what students learned from the computational labs and how they perceived the labs as impacting their learning. This study used a multi-method Quan + qual design (Morse, 2003). In this design "the description is primarily from the quantitative data with qualitative description enhancing particular aspects of the study" (Morse, 2003, p. 204).

Description of Intervention

Students in the experimental group completed six computational labs throughout the semester. The labs were completed using Jupyter notebook with Python. The lab activities were used to introduce topics. In this study, students worked through the beginnings of the labs with the instructor as a class followed by a problem set that involved some of the techniques covered in the beginning to tackle the problems on their own. Students were typically given one and a half weeks to complete the problem set on the lab. During that time, the topics from the lab were expanded on in the lecture portion of the course. Students were also given time each class meeting to work on the lab problems while the instructor circulated.

The first lab assignment was done on the second-class meeting. The first lab allowed students to download and install the appropriate software and set up a folder where they would store their work. The purpose of the first lab was to introduce students to Jupyter notebooks, demonstrate some of the different types of cells they would need throughout the semester (code or markdown), discover different ways to type text in markdown, insert images, use code cells to do simple mathematics operations, create lists and arrays, use simple loops, append elements, and define and plot functions. Students worked through the lab in class on their personal computers as the instructor projected the notebook on the board and worked through it with them. The end of the notebook had a set of four problems that students were to complete on their own, which asked students to implement what they had learned working through the beginning part and referring back to the beginning part to write the appropriate code to complete the tasks. The tasks included navigating to the appropriate type of cell to answer questions about themselves or insert a picture; performing simple mathematics operations; producing specified arrays; printing numbers from the NumPy library; and translating sentences into mathematical expressions, defining these expressions as functions, and plotting these functions on a specified domain and in a specific format. Students were also asked to write a paragraph at the end of the lab reflecting on what they learned, which occurred on all subsequent labs as well. This notebook had many of the basic commands students would need to progress into future labs and students often referred back to this first assignment.

The second lab was focused on functions with its goals including a review of linear, quadratic, and exponential functions, relating functions to sequences, and using functions to model situations.

The third lab was titled Introduction to the Derivative. In this lab students were tasked with computing limits, slopes and finite differences, working with the definition of the derivative, interpreting the derivative as slope of tangent line, interpreting the derivative as a function, and using Python to symbolically compute derivatives and use these examples to conjecture rules for computing derivatives.

The fourth lab was centered on applications of the derivative. In this lab students explored applications of derivatives to shapes of curves, finding relative and absolute extrema, and optimization. Students used pre-written code to graph the function and first and second derivatives to make conjectures about the connections between them. Students then learned how to find critical points, describe intervals of increase/decrease and concavity, and determine where there were maxima or minima. Students used this knowledge to analyze sales of smokeless tobacco products, complete examples of the law of diminishing returns, and optimization problems relating to maximizing revenue or profit.

Lab five focused on the study of integration. In this lab students had to conjecture how they would determine area under the curve and were then shown how to approximate the area under the curve using rectangles. They then improved their approximation by increasing the number of rectangles, which lead to the introduction of the definite integral. Students used numerical integration to apply this concept to applications including net change and average value.

The concentration of the sixth and final lab was area between curves. Students created graphs of several curves and made determinations about how they would then find the area between the curves. This was extended to examples of income inequality, consumer and producer surplus, and future value of income streams.

Employing the computational labs creates both content and pedagogical differences between the experimental and control groups, which are discussed below.

Overall, since the intervention was a series of labs, the content between the two groups largely overlapped throughout the semester. Some content was changed to allow time for the labs. To maintain more consistency between the courses, the order in which topics were covered mostly remained the same between the two courses. The time spent on each topic was around the same number of class meetings, but the experimental method spent some class meetings on the labs of that topic where the control course had lecture on that topic. The course timelines are shown in Appendix A.

In addition to slight content differences between the courses, there are also pedagogical differences. The control sections of Business Calculus in this study was taught using predominantly lecture, which is still the dominant style of teaching for Calculus I nationwide (Larsen, Glover, & Melhuish, 2015). Outside of lecture, students in the control course had the opportunity to briefly work during most class meetings with classmates on problem sets in addition to receiving lecture. Students in the control course also completed four projects, which included solving a calculus problem related to business and writing a letter about their findings. These projects were teacher-generated; project one covered analysis of cost, revenue, and profit functions, project two was a marginal analysis of cost, revenue, and profit functions, project three was optimization of how to construct a pipeline, and project four was a research paper on how mathematics and calculus could potentially be used in their intended future careers.

Another pedagogical consideration was the use of technology. This was a distinguishing factor between the control and experimental Business Calculus courses that were studied. The control course included the use of a graphing calculator, and the experimental course included the use of Jupyter notebooks. The graphing calculator was allowed in the control Business Calculus classes in this study. The graphing calculator was not used to intentionally teach calculus but was used by students for arithmetic computations and graphing. This differed from the pedagogical decision to use technology, specifically Jupyter notebooks, to intentionally teach calculus topics, not simply support students in computation.

Procedures

Approval from the university Institutional Review Board was obtained before the semester began. The previously described intervention took place over a fifteen-week semester in Business Calculus courses all taught by the same instructor. All four sections of the course were ninety-minute classes that met twice each week. Students self-selected the sections with no prior knowledge as to how the courses were going to be taught. All of the courses had the same instructor, so students were not selecting different sections based on listed instructor, and all of the courses were listed as the same course number with the identical course description. The only variation in the courses as seen by the students when they were registering for classes were the different times the courses were offered. None of the courses were at extremes of the day with none occurring very early or late in the day. To randomly select which courses were to receive the intervention and which were not to account for being at different times of day, the courses were assigned numbers one through four and then selected with a random number generator. Two sections of the course were taught with traditional, currently used practices in Business Calculus, which includes students learning continuous calculus with some business applications using lecture. Two sections of the course were taught using the intervention, which was the use of computational labs that were used to introduce Business Calculus students to calculus concepts.

Students in both groups took the pre-test Calculus Concept Inventory (CCI) on the first day of class. The pre-tests were completed via online test in Qualtrics, a secure survey collection software system, and the post-tests were done in the same format. The pre-test CCI had a consent statement that students signed (see Appendix C online consent statement). In addition to the pre-CCI, on the first day of class survey students also provided a yes/no response to if they had taken a calculus course before, and provided their gender, major and major concentration, and in which section of the course they were enrolled.

After the pre-tests were completed, the semester progressed with the control group receiving lecture and the experimental group receiving labs to introduce calculus topics that were then extended in lecture with tests and labs following the provided timelines. For the most part, the two groups covered material in the same order and around the same time in the semester. The two groups also took tests at the same time, which were identical tests, and took the same final exam. The two groups also had similar grading scales with the only difference being the control group had projects where the experimental groups had labs, but both carried the same weight in the gradebook.

As students progressed through their six lab assignments, they were asked on each one to reflect on what they learned in the lab. Each lab concluded with the following statement, “Think about what you learned and write about it! Write a short paragraph about what you learned in this notebook. This needs to be a thoughtful, reflective paragraph. There should be reflection on the mathematics content you learned. You may want to review the goals of this notebook (listed at the top).” This statement was generated by the researcher and was intended to get students to reflect on the mathematics covered in the lab and inspire them to pull together what they had learned in the assignment. Students used this solicitation to provide what they learned mathematically but also took the opportunity to reflect on the technology, how they felt about doing these labs, and how the labs impacted their learning. Every student that signed a consent form had his or her comments saved verbatim at the end of the semester. These student comments from the end of the labs were analyzed after the semester ended. The lab comments were each studied individually for specific content knowledge on the lab. The comments from all six labs were also analyzed as a whole to determine the themes occurring throughout the comments over the entire semester. The student comments were analyzed using initial coding with in vivo codes (Saldaña, 2016), a second cycle of coding was done with pattern coding (Saldaña, 2016), and then themes emerged from these categories.

On the final-exam day, students in both groups took the post-CCI. The post-CCI was given on final exam day when all students must be present. This was done to attempt to avoid issues of student apathy and missing data as was seen in Anderson and Brennan (2015). On the final-exam day students were given a written consent form to sign as well (see Appendix D for consent form). A helper administrator gathered this consent rather than the instructor-researcher to mitigate any pressure the students might have felt if the instructor collected consent and to maintain more anonymity. Final grades were also collected to compare the classes and compare grades with the CCI scores.

For the pre- and post-CCI students chose a code, following Self-Generated Identification Code procedures (Yurek et al., 2008), to identify themselves. The code was consistent for each student, which allowed for information from the different sources to be linked when the semester was complete. On the final exam day, students wrote their name and code on a notecard when written consent was explained and gathered so that final grade could be matched to their other data sources. The helper administrator collected the codes and matched the data.

Instrument. The Calculus Concept Inventory was used to collect data on conceptual calculus knowledge administered as a pre- and post-test with scores and change in score analyzed. The Calculus Concept Inventory was the first instrument developed to define the basic understanding from a first semester differential calculus course (Epstein, 2013). This instrument has been used to gather evidence to determine the impact of different teaching methodologies on students' conceptual calculus understanding (Epstein, 2013).

Calculus Concept Inventory has often been used to study the effectiveness of a specific type of interactive engagement, a flipped classroom; Anderson and Brennan (2015), Macejewski (2016), Schroeder, McGiveny-Burelle, and Fue (2015), and Ziegelmeier and Topaz (2015) all use the CCI in studies of flipped calculus classrooms. These studies all had populations of undergraduate students in calculus classrooms. The Calculus Concept Inventory has also been used to study differences in different types of Calculus I classrooms such as in Bagley's (2014) study of traditional lecture, lecture with discussion, lecture with discussion and technology, and an inverted classroom. Thomas (2013) used the CCI to investigate student gains in interactively engaged calculus classrooms and what influence individual variables, such as gender and previous math exposure, and instructor level variables had on CCI gain scores. The CCI was also used in gathering nationwide data on the impact of ambitious teaching, such as active learning and student centered teaching, and found that students that had graduate student instructors that had training in ambitious teaching had students that scored just as well on the CCI as students of faculty members teaching the course (Larsen, Glover, & Melhuish, 2015). The CCI has been used to study team-based learning in large calculus classes (Peters et al., 2019). Besides uses for studying teaching using interactive engagement and other interventions in calculus classrooms, the CCI has been used to study differences in students' performance across different nations; Chai, Friedler, Wolff, Li, and Rhea (2015) used the CCI to compare student gains between Chinese and American students, studying both gains and normalized gains on the overall test and in subsets of the test. Certain questions on the CCI have also been used in isolation as a component of studying students' understanding of the derivative (Park, 2013).

The Calculus Concept Inventory has twenty-two items and is said to have two dimensions that "correlated well internally but not as well with each other" with the two dimensions being functions and derivatives, and a smaller dimension on limits, ratios, and the continuum (Epstein, 2007, p. 168). The measure of internal reliability, Cronbach's alpha, was 0.7, which was considered "modestly respectable" but falls short of the 0.8 which professional test developers wish to see (Epstein, 2013) and is confirmed but brought into question by other researchers (Bagley et al., 2017; Gleason et al., 2015a). Bryman and Cramer (1990) suggest that alpha of 0.8 is acceptable, so this instrument does fall short. Cohen, Manion, and Morrison's (2009) guidelines for the alpha coefficient rank 0.7 in the reliable category, although just above marginally reliable. This 0.7 level of internal consistency is on the lower end for an acceptable instrument that is created to measure mean differences between groups with twenty-five to fifty members (Gleason et al., 2015b).

The Calculus Concept Inventory has limitations, however, it is the only validated measure of conceptual understanding in calculus. Limitations of the CCI include the Cronbach's alpha of 0.7 (Gleason et al., 2015b), how to analyze the results (Gleason et al., 2015b; Thomas & Lozano, 2013, and how many dimensions the test has (Bagley et al., 2017; Gleason et al., 2015b). Another major limitation of the CCI is that it contains mathematical notation and terminology, such as derivative and *f'(x)*, which would not have been seen by students before an introductory calculus course (Bagley, et al., 2017) and presents issues with content validity raising concerns with whether it is really measuring what it is supposed to be measuring (Gleason, et al., 2015b). This presents issues in several ways. One issue is that the CCI is supposed to assess understanding of calculus concepts not explicit vocabulary (Bagley et al., 2017). Another issue is that this notation and terminology may confuse students that have not seen it before and cause answering by random chance, which would bring the validity of pre-test scores into question (Bagley et al., 2017). The specific language that is questionable on the pre-test is common language in a typical calculus course and thus may measure conceptual understanding at the end of a one semester calculus, but the specific language on the pre-tests brings in the question of normalized gains for evaluation (Gleason et al., 2015b). Despite some of its limitations and Gleason and colleagues' (2015a) calls for a better instrument, the CCI is still used in formal and informal settings (e.g., Anderson & Brennan, 2015; Maciejewski, 2016; Schroeder McGivney-Burelle, & Xue, 2015; Ziegelmier & Topaz, 2015; Peters et al., 2019) and another instrument has not been developed.

The Calculus Concept Inventory was an appropriate measurement tool for this project since it is typically used to study different teaching methods. While this study did not use it specifically for interactive engagement, following Hake's (1998) definition and motivation of Epstein's development (2007 & 2013) which has been studied often through flipped classrooms, the study did have elements of interactive engagement because the CCI was used to compare the use of computational labs to a traditional lecture course. The intervention was not necessarily interactive engagement as in previous studies, but it was interactive engagement since "students cannot be passive observers in programming activities; rather they have to be actively engaged in reframing a given math problem, in writing and testing the computer code, and in making necessary adjustments and modifications" (Lovric, 2018, p. 2); it was a hands-on minds-on process. The CCI was developed and tested on undergraduate students in first semester calculus courses, which is the same population on which this study focused. The Calculus Concept Inventory has been found to have only one factor, which can be explained as "overall knowledge of Calculus content" (Bagley et al., 2017). While this could be seen as a drawback, this study did not look at effects of the intervention on specific sub-categories of calculus knowledge but rather on conceptual calculus knowledge as a whole, so the one factor of the CCI fit and most of the course focuses on differential calculus.

When used as a pre-post test the CCI is typically analyzed with normalized gains (e.g., Chai et al., 2014; Maciejewski, 2016) and statistical analyses of scores are recommended as well. Besides using the CCI for measuring gains over a period of time, some studies use the CCI as a baseline measure of calculus knowledge (Schroeder, McGiveny-Burelle, & Xue, 2015; Ziegelmeier & Topaz, 2015), which was done in this study.

Additional Information Gathered. In addition to the Calculus Concept Inventory scores, students' gender, college major, and previous calculus exposure were collected, which are displayed in Tables 1 and 2.

Previous Calculus Exposure. An important piece of descriptive data was whether students have taken a calculus course before or not. With the CCI having terminology and notation only used in calculus, this is important. Other studies using the CCI have collected this data as well (e.g., Anderson, & Brennan, 2015; Schroeder, McGivney-Burelle, & Xue, 2015). Information on prior calculus exposure, as previously discussed, was used to compare the two groups at the beginning of the semester to determine if there were significant differences between the groups in the count of students that have previously taken calculus. This data was also used when analyzing the results of the intervention to control for prior calculus exposure (e.g., Anderson & Brennan, 2015).

While the Calculus Concept Inventory baseline score, from the student's first attempt at the test before any calculus instruction had been delivered, factored into analysis of the dependent variable of change in conceptual knowledge of calculus, it also served as a measure to determine if the groups were similar before taking the calculus course. This was used to compare the groups before the intervention in addition to whether they had previous exposure to calculus as even though students may have had previous exposure to calculus their conceptual knowledge may be varied (as in Schroeder, McGivney-Burelle, & Xue, 2015). This measurement was also used in analysis of comparing scores using it as a covariate.

Gender. Literature on STEM students highlights that there may be differences based on gender and success in the discipline. Ellis et al. (2016) find that women are 1.5 times more likely to drop out of STEM majors after taking calculus than men are, and these findings remain true after controlling for preparedness academically, career intentions, and instructional methods (Ellis et al., 2016). With this dramatic number of women changing majors after calculus, it was important to also look at the intervention's impact on gender. Because gender has been seen as having an effect on achievement, it was collected and analyzed as well to see if there are gender differences. There are also studies on the Force Concept Inventory (FCI) detailing differences in gender (Traxler et al., 2018); while this is not the instrument used in the proposed study, the CCI, which was used, was modeled from the FCI, so gender was an important consideration.

Major. The intervention took place in a Business Calculus course, which is required for all business majors at the university where the study took place, however the class sometimes has students from other majors since it can fulfill a general education requirement for some majors and some students place in that class based on SAT or ACT score. Because of this, students' college major and major concentration were collected. This provided information on how the intervention worked based on a student's major and allowed for the data to be analyzed based on what a student's intended business concentration was.

Anecdotally, when discussing with my students why they chose to major in business or how they think math relates to their future business career, many of my students usually let me know their feelings about math, typically more negative than positive, tell me about their previous perceived failures in math, and then connect those to why they chose business as their major thinking that it may not be as math-intensive as a STEM major; these students are not incapable, many simply have negative feelings towards mathematics. This anecdote aligns with existing literature on how students decide on college majors and future careers. Research supports that students often have their minds made up about going into STEM careers or not upon exiting high school (Maltese & Tai, 2011). Pritchard, Potter, and Saccucci (2004), in studying business students and their basic algebraic skills, found that students with higher computational and algebraic skills chose to major in more quantitatively focused business concentrations such as accounting or finance while students with lower scores tended to select a concentration or major in less mathematically focused ones such as management or marketing, and some business students choose less quantitatively focused concentrations because they perceive them to have less demanding quantitative requirements (Pritchard, Potter, & Saccucci, 2004). It was of interest to see if students from more quantitatively focused concentrations or majors differ from their less quantitatively focused counterparts in achievement.

Results

To address the research question as to what extent students demonstrate gains in conceptual calculus knowledge when introduced to calculus concepts through computational labs in a Business Calculus course, demographic information and Calculus Concept Inventory scores were collected and input into SPSS to compile and analyze the data. Normalized gains, independent samples *t*-tests, chi-square test, ANCOVA, and correlation were used to analyze the results. Student comments from the end of the labs were analyzed as well looking for reflection on understanding, students' evaluation of and recommendations for the labs, and common themes that resulted from the students' experiences with computational labs in a Business Calculus course.

Equivalence of Groups to Start the Semester

On the pre-CCI on the first day of the semester, students answered whether or not they had taken calculus before. Responses to this question for the two groups were analyzed using a chi-square test for independence. This was used to compare the two groups at the beginning of the semester to determine if there were significant differences between the groups in count of students that have previously taken calculus (as in Schroeder, McGivney-Burelle, & Xue, 2015). A chi-square was chosen for this analysis because chi-squared tests can be used to determine if there are significant differences between frequencies in the two groups. Chi-square tests can be used to determine statistical independence in the frequency distribution of a variable is the same for all levels of some other variable (Chi-square independence test – What and why?, n.d.), with calculus or not frequency being compared between the experimental and the control group. This test is also appropriate because the observations were independent and all expected counts were greater than five. The chi-square test of independence was calculated comparing the frequency of students that had previously taken calculus in the experimental and control groups and revealed no statistically significant differences between the groups, $\chi^2(1, N=79)=1.570$, $p=0.219$. There was also no statistically significant difference between the groups on gender, $\chi^2(1, N=79)=1.093$, $p=0.296$, or college major, $\chi^2(1, N=79)=1.144$, $p=0.564$, as well.

Pre-CCI scores were also analyzed to determine if there was a difference between the two groups; this was done to compare the groups before the intervention in addition to whether they had previous exposure to calculus as even though students may have had previous exposure to calculus their conceptual knowledge may be varied (i.e.; Schroeder, McGivney-Burelle, & Xue, 2015). An independent samples *t*-test was run for this comparison. The pre-CCI percent scores for the control group (M=29.02, SD=9.48) and the experimental group (M=28.29, SD=10.67) were not significantly different, $t(77)= -0.319$, p=0.75. Based on the responses to whether students had taken calculus before, the comparison of pre-CCI scores, and comparison of frequencies of gender and college major students in the two groups were not significantly different on their previous knowledge of calculus.

Normalized Gains Results

"Normalized gains are almost always used in concept inventory studies measuring gains" (Thomas & Lozano, 2013). Normalized gains, <*g*>, as defined by Hake (1998), are measured by

$$< g >= \frac{\mu_f - \mu_o}{100 - \mu_o}$$

where μ_f is the mean percentage score of the class at the end of the observation period and μ_o is the mean percentage score of the class at the beginning of the period. Normalized gains can be calculated using class averages of pre-test and post-test scores to look at the effect on an entire class or they can be calculated using individual gain scores with pre-test and post-test data for each student (Thomas & Lozano, 2013). Hake (1998) used class averages to compute gain scores. Bao (2006) discussed the differences in class verses individual gain scores; the two ways yielding very close results (Bao, 2006). Using individual normalized gain scores allows for analysis of class level variables and individual student variables as well (Thomas & Lozano, 2013). Individual normalized gains were collected to allow for individual level variables such as previous calculus exposure, gender, and/or major to be analyzed as well.

An example of interpreting gain score is, "if a student correctly answered 50% of the questions on the pre-test and 75% on the post-test, the normalized gain would be <g>=0.5, meaning that student correctly answered half of the 50% of the material they did not know at the beginning of the class" (Thomas & Lozano, 2013). Normalized gains are measured in many published uses of the CCI (e.g., Anderson & Brennan, 2015; Bagley, 2014). Hake (1998) defines levels of gain with greater than 0.7 being high, between 0.7 and 0.3 as medium, and under 0.3 as low gain, which he argued that the normalized gain allows for "consistent analysis over diverse student populations with widely varying initial knowledge states" (p. 66). However, it is unclear that these cut-off gain levels transfer to a different concept inventory (Thomas & Lozano, 2013). Using normalized gains is also problematic when post-test score is not higher than pre-test score and the value of *g* may not have a sensible interpretation (Miller et al., 2010). Normalized gains have been widely used on concept inventories and thus they cannot be completely ignored, but they do not align with practices of the broader fields of social sciences (Nissen et al., 2018). Normalized gains also favor those that have higher pretest scores, so Nissen et al. (2018) recommend that researchers use statistical analyses as well.

The two groups had similar results on both the pre- and post-CCI as shown in Table 3.

Table 3

Experimental and Control Group CCI Percentage Score Statistics

Statistic	Group			
	Control		Experimental	
	Pre-CCI	Post-CCI	Pre-CCI	Post-CCI
n	39	39	40	40
Mean	29.021	28.904	28.29	27.50
Median	27.273	22.727	29.55	27.27
Standard Deviation	9.478	11.399	10.67	10.08
Minimum	13.64	13.64	9.09	13.64
Maximum	59.09	68.18	54.55	50

Both group averages and individual normalized gains were computed. Normalized gains computed from pre- and post-CCI group mean percentage are shown in Table 4 and show that there was no gain in either group.

Table 4

Normalized Gains Computed from Group Averages on Pre- and Post-CCI

CCI Percentage	Group	
	Control	Experimental
Pre-CCI Group Mean Percentage	29.02	28.29
Post-CCI Group Mean Percentage	28.90	27.50
Normalized Gain Computed from Group Mean Percentage	-0.0016	-0.011

Normalized gains computed for each student and then averaged for the group produce similar results with the control group average normalized gain of -0.0068 and the experimental group average normalized gain as -0.0205. Neither group had any improvement in gain score, which will be discussed in a later section.

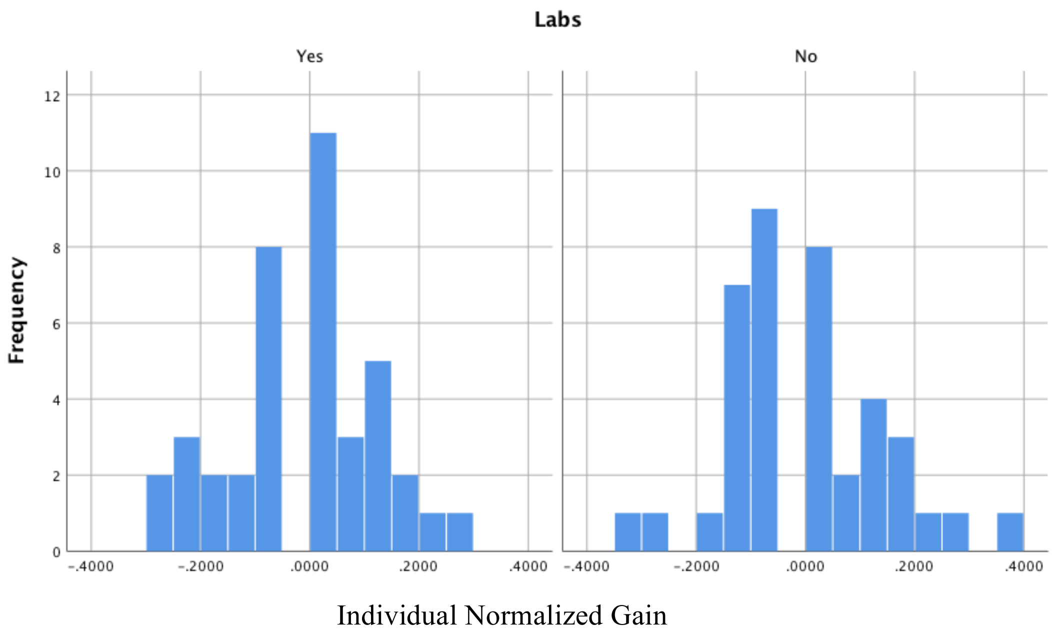


Figure 2. Frequency of individual normalized gains for experimental and control groups.

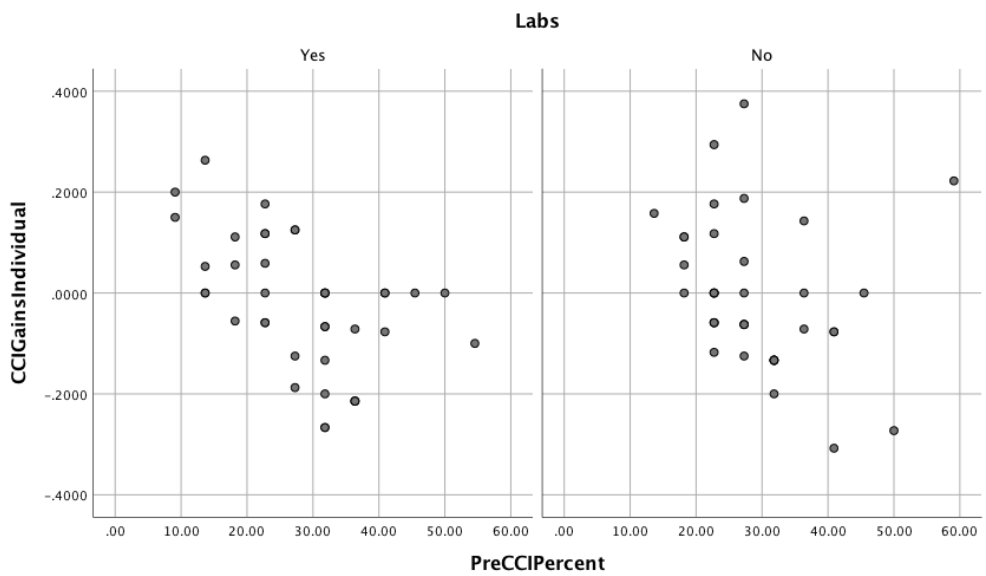


Figure 3. Scatterplots of pre-CCI percent and normalized gain for each group.

Comparing Calculus Concept Inventory Scores

Since normalized gains can be affected by pre-test scores, different methods of computing gain score exist, and normalized gains do not align with the practices of the broader fields of social sciences, statistical analyses were done as well.

Differences from Pre to Post-CCI. To statistically assess the effect on calculus conceptual understanding, an independent samples *t*-test was run to determine the statistical significance of the change in pre-test to post-test scores. The difference between post-test and pre-test was computed and the mean difference between the experimental and control groups was tested. The samples met the assumptions for independent samples *t*-tests as follows.

An independent samples *t*-test was selected for analysis in part because the samples were independent. No person was in both groups and students do not have the option to attend a different section of the course, so the values from one population are not related or linked to values from the other population as needed for an independent samples *t*-test (Bowen, 2016). If the size of each sample is greater than or equal to thirty, the *t*-test for independent groups can be used without much error even if there are moderate violations in the normality or equal variance assumptions (Pagano, 2004, p. 339). The sample size for each group was over thirty, so the normality assumption should be met. Homogeneity of variances was met as assessed by Levene's *F* Test for Equality of Variance, $p>.05$.

With the independent samples *t*-test run, there was no significance difference in Calculus Concept Inventory pre-post percentage difference for the group without labs (M= -0.117, SD=9.92) and the group with labs (M=-0.796, SD=9.59), $t(77)$= -0.309, p=0.758.

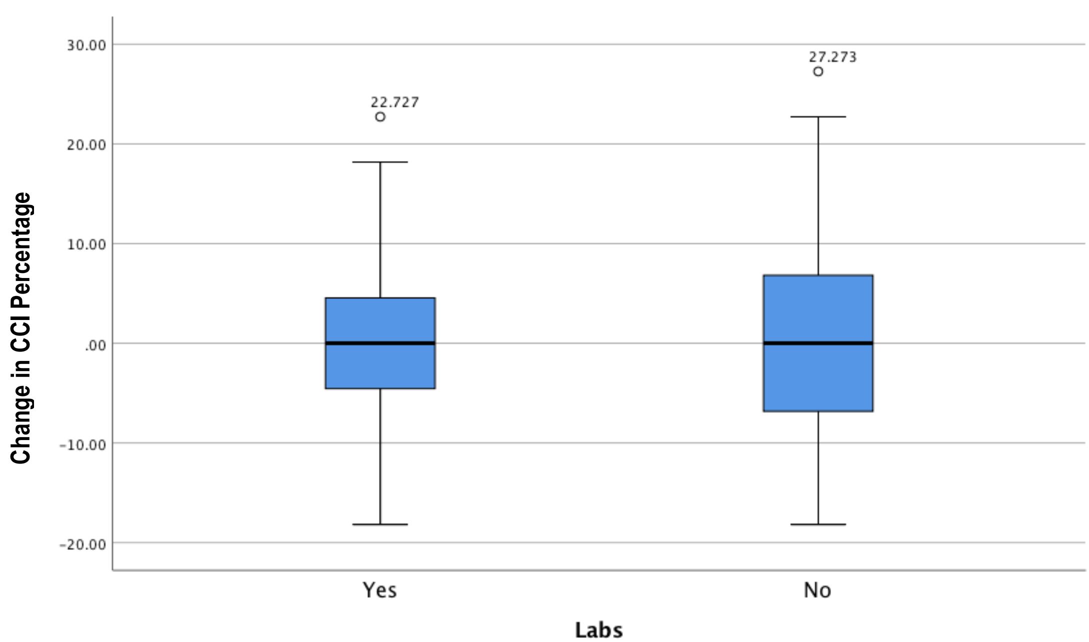


Figure 4. Change in percentage of CCI scores.

Results Considering Post-CCI Only. Since the two groups were roughly equivalent to start the semester in previously acquired calculus knowledge, examined earlier by the yes/no response to prior calculus and the comparison of baseline CCI scores, the differences between the groups on only the post-CCI were examined as well. This was done using an independent samples *t*-test since the samples were independent, the sample size was large enough that a *t*-test can be used even with minor violations to the normality assumption, and homogeneity of variances was met as assessed by Levene's *F* Test for Equality of Variance, $p>.05$. Examining post-CCI scores, also revealed no significant differences between the group with labs (M=27.50, SD=10.08) and the one without (M=28.90, SD=11.40), $t(77)$= -0.580, p=0.563.

Controlling for Prior Calculus Knowledge. For the CCI data, additional analysis was performed because of the need to control for having previously taken calculus or not. This was done on the CCI because of the issues with specific calculus terminology on the inventory. Analysis of covariance (ANCOVA) was done when analyzing the data to control for the covariate of previous calculus exposure as measured by baseline pre-test CCI scores; pre-test scores are often used as a covariate in pre-test post-test experimental design (Huang, n.d.; ANCOVA, n.d.). Covariates should be measured on an interval or ratio scale (Huang, n.d.; ANCOVA, n.d.). The ANCOVA is a type of statistical control, which is using a statistical technique to isolate or subtract the variance in the dependent variable that are attributable to the variables that are not the subject of study (Vogt, 1999). An ANCOVA must meet the following assumptions; this study's data is addressed for these assumptions below:

1. The dependent variable is continuous – The post-CCI score was a continuous variable
2. The independent variable is categorical – The assigned group variable was categorical with two groups
3. The samples are independent – no students are in both groups and students self-selected into the course with no prior knowledge of how the course would be taught
4. The dependent variable is approximately normally distributed – While the Shaprio-Wilk's test was significant, so the post-CCI scores did deviate from a normal distribution,

Norman (2010) states, "both theory and data converge on the conclusion that parametric methods examining differences between means, for sample sizes greater than five, do not require the assumption of normality, and will yield nearly correct answers even for manifestly non-normal and asymmetric distributions like exponentials." The sample sizes are greater than five.

5. Homogeneity of variances – There was homogeneity of variances $F(1, 78)=0.149$, $p=0.701$

6. For each independent variable the relationship between the dependent variable and the covariate is linear, $r(77)=0.565$, the correlation was significant at the 0.01 level

7. Homogeneity of regression slopes was met, $F(1, 78)=0.426$, $p=0.516$

8. The independent variable and the covariate are independent of each other, $F(1, 77)=0.102$, $p=0.75$

A one-way ANCOVA was conducted to determine the statistically significant difference between the group that did not have labs and the group that did have labs on their post-CCI scores controlling for the pre-CCI score. There was not a significant effect of labs on post-CCI score when controlling for pre-CCI score, $F(1, 78)=0.232$, $p=0.632$. There was also not a significant effect of labs on the post-CCI score when controlling for previously taken calculus by the response to whether students had taken calculus or not, $F(1, 78)=0.112$, $p=0.739$. In the experimental group those that had previously had calculus had higher pre and post-CCI scores, only separated by about five percentage points on both, higher average normalized gains, and growth from pre to post-CCI than those without, but none of these were statistically or practically significant. For the control group the results on all of these measures were very similar, also none of which were statistically or practically significant.

Controlling for Other Variables. Other variables such as gender and college major were of interest. These other variables were analyzed using ANCOVA methods as well, which "although covariates are typically measured on a continuous scale, they can also be categorical" (One-way ANCOVA in SPSS Statistics, n.d.), which they would be for gender, major, and concentration.

When controlling for gender, the computational labs had no statistically significant effect on students CCI percent difference, $F(1,76)=0.063$, $p=0.802$. Controlling for the effects of college major the intervention also had no statistically significant results on students' CCI difference, $F(1,76)=0.226$, $p=0.636$. Nor were there statistically significant results when controlling for major concentration, $F(1,76)=0.188$, $p=0.667$.

Final Grades. Comparing the changes in conceptual knowledge as measured by the Calculus Concept Inventory for the groups with and without computational labs, revealed little difference between the groups. Overall the two groups also had very similar final grades. The control group ($M=87.641$, $SD=11.946$) was slightly, but not statistically significantly ($t(77)=-1.155$, $p=0.252$), higher than the experimental group ($M=84.727$, $SD=10.458$). During the semester, however, it was noted by the instructor that within each of the groups, each comprised of two sections of the course, there was clearly a higher and lower performing section, simply by chance since the courses that received the intervention were randomly chosen. In each, the higher performing section had overall higher class averages, had higher office hour attendance, and were overall more engaged in class daily. Comparing the mean final grades of the lower and higher performing classes within each group, they were not statistically significantly different, but they were practically different both of which were separated by more than five percentage points. There were also no statistically significant differences on CCI scores of these groups.

Final grade percentage was positively correlated with the post-CCI score for both groups and slightly stronger for the experimental group ($r(38)=0.430$, $p=0.006$) than the control group ($r(37)=0.399$, $p=0.022$), as in other studies (Bagley, 2014; Epstein, 2007). Pre-CCI and post-CCI score were also significantly correlated with each other ($r(77)=0.565$, $p<0.01$), similar to (Bagley, 2014).

Qualitative Results and Discussion from Student Comments

In an attempt to understand what students' learned and how their experiences with the computational labs helped them learn the calculus content and gain understanding, students' comments were analyzed. Students' comments were first examined for reflection on calculus content, which revealed that overall students did comment on specific elements of calculus that were supposed to be learned in each lab and glaring errors or erroneous statements were minimal.

Students also used the paragraph at the end of each lab to remark on how they felt the lab did or did not help them learn and even how the labs could be improved to better help them understand the calculus. All of the consenting students' comments were inspected through several analytic passes through the data. The comments were coded using initial coding with in vivo coding using participants own language as codes (Saldaña, 2016). A second cycle of coding was done using pattern coding to group the segments of data from the first cycle into a smaller number of categories (Saldaña, 2016). From the categories, two themes emerged that expressed elements of students' experiences with the computational labs. Additionally, students' comments were analyzed using evaluative coding (Saldaña, 2016) to assign judgments about the merit, worth, or significance of programs (Rallis & Rossman, 2003), which can be used to make changes for future implementation.

Student Comments on Content. During the semester as the labs were graded, student comments were read for reflection on content and they received credit for simply writing the paragraph and did not lose points for what was said. Reading through all of the comments, there were minimal glaring errors in students' descriptions of what they learned. Some students provided more detailed accounts of exactly what they learned, whereas others simply stated minimal descriptions of what the lab covered.

While overall students may not have demonstrated measurable gains in conceptual knowledge, as measured by the CCI, students were able to explicitly articulate how calculus concepts could be used outside of the classroom. This in itself highlights a level of understanding. Throughout the student comments there were very specific explanations of how they were able to use a calculus concept and apply it to a real world situation. A comment that exemplifies this is:

> I was able to take a company's profit data, fit a curve to it, find the derivative, and use the derivative and critical points to determine when the profits were increasing, decreasing, and where it was at a maximum. From that I was able to think about under their current business model how many units they should aim to sell to maximize profits.

Similarly another student remarked:

> I liked that we learned about derivatives early on with the real data. Like the smokeless tobacco sales problem where I had to fit a curve to the year and sales, then find the derivative of that curve, and then tell you about the rate of change of sales in certain years and when that was the greatest; that really helped remind me throughout the chapter about the derivative being a rate of change. I just thought back to this example, when the derivative was positive the sales were increasing. It also made me think about like what

else might be going on in those years that would have made sales higher.

These last two student quotes are important because in both the student comments on the calculus content but does so in a way where he or she is also reflecting on what might be going on in the business that could be affecting the sales or profits. In each of these, the student is taking his/her understanding of calculus and connecting it to his/her knowledge about business and acknowledging that perhaps the business model could be changed or the outside factors could be considered, which could affect the mathematics of the situation as well. These students are realizing that their mathematics understanding and their knowledge of business can indeed intersect and affect one another.

Another student commented on how he or she believed that doing some lab problems about the derivative helped him/her learn the material by stating:

> During this notebook, I learned how to apply derivatives to math problems without the question flat out saying that you need to find the derivative. I learned how to work with derivatives when dealing with functions and slope and rate of change. Learning how to apply the equations to real life problems is something that I will be able to use in the future. Importing a table of values and also using that to find the derivative is useful in the sense of being able to apply this to the real world.

Regarding integration, a student stated:

> The goals of this notebook were summation, integration, and applications of integration. I was able to develop more knowledge about these three topics throughout the lab and the various applications. More so, I became familiar with the idea of integration and letting the computer numerically integrate and discover the area under the curve. From that I was able to integrate a population growth function to find the net change in population

> over some years, how to integrate the average workers production rate over different blocks of time and think about what might be happening when the units they made started to drop, and use the average value of different quantities like advertising spending.

Comments such as these and similar ones demonstrate that students did learn calculus concepts and were able to then state how they used these concepts in applications. The Mathematical Association of America calls for calculus in the business math curriculum to highlight basic calculus concepts and how they apply to business problems (Lamoureux, Beach, & Hallet, 2000), which students specifically articulated in their comments and shows valuable understanding they gained.

By the end of the semester, a few students began to use Jupyter notebooks to complete assignments outside of lab questions. One student in particular defaulted to Jupyter notebook over paper and pencil. In observing her work through some problems, I noted she had become comfortable using the technology and the techniques she had learned. In watching her work through an optimization problem, I asked her to talk about what she was doing. She said:

> I just think it is easier on here and to me it reinforces the steps. See like here I defined the function, and then I know I need to take the derivative, so I type the command for that. Defined the derivative as a function so I can use it later. Then I need to find the critical points, well how do you do that you see where it doesn't exist or equals zero so I then used the solve command. Now I can easily plug points in around those to the derivative using the function I defined and make determinations about max or min. I think typing in these commands helps me remember what I am doing.

While this student was an exception rather than the rule and only a few other students got to the point of using Jupyter notebooks outside of labs over the semester, like the quote above others also reiterated that using the commands reminded them of the steps of the calculus problem. A student explained how he or she felt typing the commands in helped understanding of integration by stating:

> We found the area under the curve by writing a function that found the sum of the area of rectangles between the curve and the x-axis. We had to define the width of the rectangles as the width of the interval divided by the number of rectangles and we had to define the height of the rectangle to be the value of the function at those different widths. Then we defined the area as the width times the height that we just defined. Having to define the height this way reminded me that that the height of the rectangle actually came from evaluating the function there. Also if I increase the rectangles to a really big number I can find the area under the curve there, so the definite integral.

The previous two quotes are reflective of additional quotes of this nature and demonstrate that for some students the new technology of the labs aided their understanding by forcing them to think through what they were typing into the computer helped them to reflect on the mathematics. This aligns with previous research that students have to think through the mathematics as they are communicating with the computer (e.g., Benakli et al., 2017; Cetin & Dubinsky, 2017).

This, however, was not true of all students and some comments suggest that for some students the technology may have interfered with the learning of the calculus content. A student remarked:

> I feel like I understand things when it is being taught or explained but once I have to execute something dealing with math, I just lose everything I was taught. In class I felt like I understood about derivatives but then I still don't understand how to use these labs.

> I just can't do them on my own and put together the new math and how to type it in the computer.

Another student said a very similar sentiment:

> It is really like trying to learn a whole new language with how intricate and different it is for someone who has never had experience with it before this class. With already being confused with the "normal" assignments out of the labs, it makes all the derivative rules that much harder. To be honest I really have no idea if what I am plugging in is the right thing or not, i.e. struggling to finish the question in full. I don't think it is a good idea to mix coding with math when kids already struggle enough with the math.

Statements such as these illustrate that while for many students, the labs provided them with a new and effective way to learn the calculus content, for others the work of the labs actually added an extra burden, which they felt may have hindered them from learning the calculus concepts as well.

Aside from specifically stating that the labs seemed to interfere with their learning of calculus, plenty of students made comments with words such as frustrating, difficult, challenging, hard, intimidating, and stressful, which were focused on the technology they were being asked to use. Statements that reflect these are:

- "One thing that I need to work on is to not get frustrated at a single problem and then give up and move on, but to stick with it and try and figure out why it isn't working."
- "At first it was very challenging and I wasn't sure I would be able to finish it. After toying around with it and spending a lot of time on it, most of it was okay. I think in the future I can better my work on the lab by starting right away so I know what I am doing wrong and figuring out why for future labs"

- "Even though I was extremely frustrated when I was beginning, I kept trying to get through it."
- "I found this lab very tedious and requiring extreme focus. I learned to become more patient and that I need to slow down and make sure everything is in the right place…. It is very frustrating and complicated. It takes being very diligent and making sure you make no mistakes or the whole cell will be wrong and error will occur."
- "Even if you get one little punctuation error, then everything can be wrong. Some of these problems were pretty hard and I had to refocus my attention and look at the problems in great detail."
- "Not only that we had to use our notes from last lab to get the correct answer. So it's not just do it and forget it. No, it's do the lab and remember what you did to complete other labs. Using information from the past to complete this lab."
- "I really struggled with some parts but I began to understand better when I went back and kept rereading the instructions and examples."

Many of the more negative statements occurred at the beginning of the semester and the comments became more positive as the semester progressed and students became more comfortable using the technology and doing mathematics problems this way, which shows that in the beginning of the semester mixing the two proved to be challenging and may have impacted understanding. Statements later in the semester for many students were ones such as, "Although this lab was difficult, I think I'm getting better at understanding how to use coding to do math."

Other students, however, remained frustrated and extremely challenged all semester. These students made statements such as:

- "After doing several of these labs I figured out that I do not like this and am not good at it at all."
- "I feel like I know the basic math, but I can't get it with the code and running thing, but I'm gonna need a lot more help to understand this."
- "This lab was hard and required a lot more work and thought putting together different math concepts in a way the computer could understand to get the answers needed."
- "I learned that even if I ask for help I am not very good at this type of thing."

Statements, such as the ones above, show that some students were challenged by the technology and mathematics and were burdened by such all semester. These students may not have shown gains in understanding because they were hampered by trying to understand the technology and did not feel that they could devote enough time to both understanding the technology and the calculus concepts.

How Students Viewed Labs as Beneficial in Impacting Understanding. Aside from reflection on content, several themes emerged as to how students thought the labs helped them learn. There were two main themes, which were real world connections and visualization. Both of these themes help to demonstrate ways in which the labs were beneficial for many students gaining conceptual understanding of calculus.

Theme 1: Real World Connections. The theme that was the most prominent throughout student comments was real world connections. This can be seen in part in how they reflected on what they learned as presented above. The words such as real world, real world applications, outside of the classroom, real life stuff, and business problems occurred throughout the labs. The theme of real world connections was a two-pronged theme. One part is that students reflected on being able to connect the mathematical concepts to real world scenarios or applications. The other prong is that some students felt that how they were problem solving was more realistic, authentic, or how they might approach problems when they are in the working world. Out of the comments analyzed over 50% mentioned something about "real" and many of the others, while not using the word real, stated how they could use the calculus concepts in some other context and talked about a scenario such as supply and demand or cost and revenue for example.

Real Applications. Mentions of real applications, real life scenarios, or explicitly stating these "real" concepts ran throughout students' reflections. These comments were found throughout labs one through six. A student remarked:

> Through this notebook, I have been able to actually see how mathematical models and exponential equations are actually applicable to real world situations since usually random scenario word problems in a textbook don't do a good job of illustrating these things for me.

In response to students being asked to import stock trends from a company of their choice and simply use their knowledge of slope to roughly determine the support and resistance lines and determine if they thought they would buy the stock or not, a student commented, "Seeing how stocks can be analyzed with the help of this program makes the math feel like it is not pointless and I might actually use this math." Another student stated,

> As much as we go over the material in class and homework, I like having a 'real life' example and to see 'real' graphs from the imported data in these labs... I feel like having the opportunity to see the importance of derivatives and how they actually work (in the real world) is something that has definitely improved my understanding and appreciation for the concept.

Many students also commented both in class and in their lab reflections that they liked that some of what we were doing connected directly to content they were learning in some of their business classes. A student remarked, “The questions in this lab have also helped me connect the information I have learned in another class to the math we’re studying, making everything as a whole feel useful since I can form a relationship between the two in my mind.” This student and others felt and expressed that being able to make connections between the material of this class and their business classes helped them form important connections and aided their understanding.

The most salient quote demonstrating this point was:

> Once again, I found this lab to be extremely helpful in ‘driving home’ some of the key concepts that we are working on in the classroom. Specifically, I learned how to graph and predict stocks using elements of coding (which I never thought I would learn in business calc). This could definitely help me as I continue to 'dip my foot' into the stock market and day trading. To be honest, question 6 helped me learn that sometimes a model can ‘over predict’ and give you extremely incorrect information. This lab clearly was designed to challenge students into looking at concepts from class from a different, real-life perspective. I really like doing these labs and trying to figure out how functions, graphs, exponents and other ‘class material’ will be used after college in the ‘business world.’ I look forward to some of the future labs. The truth of the matter is: I am not really fond of math, but these labs help me want to learn more. I have always been a believer that a lot of these concepts will never be used after graduation. I am starting to change my mind!

Making real world connections can be valuable for students and may have aided their understanding as real world connections have been shown to promote understanding. Boaler (1998) indicates that getting students to experience meaningful real world connections makes them better equipped to use their mathematics knowledge and adapt to new situations. The likelihood of being able to transfer mathematics skills is increased if students learn mathematics in real life contexts and "the connections between mathematics in school and real life are made explicit" (Boaler, 1993, p. 12). When students are provided with a context in which they can use their abstract knowledge, it can connect the knowing, doing, participation, and authenticity (Sidawi, 2007). Overall students that participated in these labs made statements that reflect that they felt that the labs and the real world connections and applications did in fact help them learn the material and gain conceptual understanding.

An Authentic Way of Doing. Some students also believed that the labs gave them a more authentic or "real" way of doing mathematics. On the first lab a student recognized that potentially learning more about using a computer could be a valuable skill and connect to his future stating, "This coding thing is going to be hard, but I am excited to learn a new skill. Especially one that might be valuable since everything seems to be done on the computer now. This might actually be helpful later on." In the second lab of the semester a student stated, "I learned there is more to math than just learning how to do problems on paper."

Another student, finding value in this different way of solving problems stated the belief, "If I continue to practice this, I may actually be able to use it in a job in the business world one day." As the semester progressed this same student was observed using Jupyter notebooks almost exclusively when tackling homework problems not from the labs. When she was asked why, she explained that she thought they made the work quicker, easier, and more like she would do if she were solving a problem outside of her math class.

For some students their comments reflect that the labs created a seemingly more authentic way of doing mathematics and afforded them different ways to problem solve. Leaders in mathematics education call for technology in business mathematics to allow students to use realistic data, use technology as an analytic tool, and encourage alternative approaches to solve problems in part because "technology has revolutionized the way in which business is practiced" which has changed what students of today must be able to do and understand (Lamoureux, Beach, & Hallet, 2000). This is especially important since the Mathematical Association of America has recommended:

> The business leaders of tomorrow, and therefore the business students of today, need to understand the conceptual basis of algebra, calculus and statistics…For business executives to be successful, they need proficiency in the technology that produces the data they need, understanding of the algebra, calculus and statistics underlying these data, and knowledge of how sensitive the results are to changes in the input data. (Lamoureux, Beach, & Hallet, 2000, p. 20)

Students' comments demonstrate that, at least some, students did walk away connecting the technology and mathematics as ways to solve problems that they might outside of the classroom.

Other students, however, did not find value in this and may have thought that it was a meaningless addition to the course. A few students stated that they could not understand why coding had anything to do with business and questioned why a business major needed to be exposed to this. These students were very closed to learning how to work with the labs, and for these students the computational labs certainly did not improve their understanding of calculus concepts.

Theme 2: Visualization. A common theme throughout student comments was that doing the labs helped them visualize what was going on. Students used words such as visualize, see, showed, picture, image, and graphs to express a manner in which they thought the labs added to their understanding. The theme of visualization occurred across many of the labs and associated the idea of visualization helping them make connections between real data points and functions, rates of change and differences, applications of the derivative, and integration.

In labs one and two the theme of visualization occurred most when students talked about connecting real data points to models of that data. Several students commented on how fitting a function to these data points and graphing such allowed them to "see" where the "given functions come from". A student remarked:

> I guess I never really thought about where the functions my teacher gave me came from. But doing this I can literally see that I was able to fit a model that goes through or near a lot of the data points, so it did actually come from somewhere.

Along these same lines another student commented on being able to graph the data points and the model on the same axes also made him think about how that model might not predict the specified quantity in the future well because the model looked different from the data at the end. A different student stated, "I also enjoy how I can literally SEE what is happening in equations, functions, and graphs and what inputs create specific outputs when I change them."

Regarding derivatives students also found that visualizing them helped their understanding. Several students relayed that using the prewritten command for graphing the function, first derivative, and second derivative all on the same axes at the same time and the prewritten code for plotting the tangent line aided seeing and connecting the concepts without getting bogged down to start with on computing the derivative and tangent line and figuring out how to plot them. A student stated, "Being able to easily see the derivatives helped me with the increasing, decreasing, max/min, and concavity parts. It was very good for the visualization of that because I was struggling to get those with the lecture in class."

Students also stated similar feelings on visualizing with integration. Students expressed value in being able to quickly see the rectangles under the curve and being able to expediently increase the number of rectangle and observe how that changed the approximation.

Visualization in learning mathematics is valuable for understanding, and students in this project echoed this sentiment. Visualization is at the root of many powerful mathematical ideas such as the development of the ideas of functions, limits, continuity, the fundamental theorem of calculus, among others (Tall, 1991; Perkins, 2012). Visualization can be powerful:

> Before there can be proof, there must be an idea of what theorems are worth proving, or what theorems might be true. This exploratory stage of mathematical thinking benefits from building up an overall picture of relationships and such a picture can benefit from visualization. It is not accident that when we think we understand something we say 'oh, I see!' (Tall, 1991, p. 2)

All of the labs had multiple graphs, plots, or other images, which the students remarked helped them explore and understand different topics, which were made quicker through the technology. Tall (1991) determines that visualizing and using technology to do so can aid students in numerous topics in calculus such as limits, tangent lines, differentiation, and integration, which student comments seem to echo. Tall (1992) suggests research in and use of computer graphics, computer programming, and symbolic manipulators, which can be used to “build up intuitions for later formalizations” (p. 9) and thus the need for the three representations (graphic, numeric, symbolic). Tall (2009) suggests that students, especially those needing extra remediation, may benefit from visual and graphical representations to help make sense of something, but they still may have difficulty linking the visual with the symbolic, which is important since many of the students enrolled in this classes had not taken calculus before and some had not even taken pre-calculus. Galindo’s research (1995), in the midst of the calculus reform in the US in the 1990’s, suggests that technology, including software with multiple representations, can be used to promote conceptual understanding for both college students that tend to be visualizers and those that are more non-visual. Bishop (1989) states, “There is evidence that there is value in emphasizing visual representation in all aspects of the mathematics classroom” (p. 14), and Dreyfus (1994) agrees that “visualization is generally considered helpful in supporting intuition and concept formation in mathematics learning” (p. 33). Kostadinov, Thiel, and Singh (2019) also report that tools similar to those used in this project can be employed as visualization tools. Students’ comments align with research that visualization in mathematics education can help students make valuable connections and aid in learning concepts. Students that participated in this project articulated that they felt that the visuals they were provided, the visuals they were able to create themselves, being able to produce visuals of data sets, and being able to manipulate their created graphical representations helped their understanding of calculus concepts.

Analysis of Student Comments Through an Evaluative Lens. The reflection on content and the themes discussed above represent important impacts these labs had on students’ understanding of calculus concepts and habits they developed. In addition to the themes previously presented, student comments highlighted several ways they evaluated the labs and recommendations for the future to better impact students. Comments of this nature were analyzed through evaluation coding, which assigns judgments about the merit, worth, or significance of programs (Rallis & Rossman, 2003). According to Rallis and Rossman (2003) evaluation data can describe, compare and predict. Describing "focuses on the patterned observations or participant responses of attributes and details that assess quality. Comparison explores how the program measures up to a standard or ideal. Prediction provides recommendation for change, if needed, and how those changes might be implemented" (Saldaña, 2016, p. 141). Comments of this nature provide valuable insight into how the students believed that the labs contributed to them gaining understanding and how the course could be improved to further increase understanding for more students.

Starting with Labs. One defining element of the computational labs in this project was to begin instruction of calculus topics within the lab and then extend the topic in lecture. This was done for several reasons. One reason for this was so that students may not view the labs as an extension activity or something else to do, but rather students might view them as a tool that was helping them learn. This also followed the recommendation of the MAA CRAFTY projects (Lamoureux, Beach, & Hallet, 2000). Another important reason was to introduce students to the calculus concepts beginning with finite elements and discrete data points. This was done in the introduction to derivatives, applications of derivatives, summation and integration, and area between curves labs. The goal of this was to introduce students to these concepts in a concrete way and then extend to more abstraction in lecture. Many students reflected on the placement of the lab at the beginning of new content and voiced their support for the placement on helping them learn. Student comments relating to going through the lab first were present in reflections on labs two through six. These statements on the placement of the labs demonstrate that many students believed that beginning with the labs actually helped them understand the material better when it was extended upon in lecture.

On lab three one student remarked, “I like that we started this lab before learning the material because it made it easier to understand the content when we went to take notes in class. I liked that I got a little experience with these topics, then took ‘paper’ notes on them, and then was able to go back and practice with the questions at the end of the lab.”

On lab 5 another student echoed a similar sentiment stating, “Overall, this lab was helpful because it introduced the ideas of integration before I learned it on paper, which helped me to understand some of the significance of it to the real world.” Lab 5 drew out similar statements from others such as, “I really liked this lab. It helped me learn a lot about chapter 6 (integration) before we went through it in lecture.”

Students also remarked on, in their words, “differences and rectangles” and how starting with these concepts aided their understanding. While yes, either of these can be and often are done with paper and pencil, students recognized that doing these on the computer made these calculations and visualizations quicker and easier, and much simpler to then extend to “a really large number of them to help see what happens going to infinity” or “make the differences really really small”. To start, students were introduced to finite differences. Students first had to plot a given sequence of a few numbers, then find the differences between the terms, and plot the sequence and differences on graphs side by side; students began by looking at one plot that looked linear, then one that looked quadratic, followed by an exponential. During the lesson going through this lab, students quickly remarked that the differences from the linear one were all the same. When we moved to the quadratic we first only plotted a few differences and students did not immediately see a pattern, but when we added many more points, looking at smaller and smaller intervals, they realized that these differences looked linear. In some of the problems students were asked to do on their own they had to investigate other functions and start recognizing patterns. A comment that was similarly echoed throughout student comments on this lab can be summarized by this student comment:

> I was able to spot similarities throughout the problems and was able to recognize the connections between different functions and their differences. Looking at the points and the little differences beside each other helped. I was especially able to recognize the connections to slope.

Students said analogous statements about starting integration with adding up deposits into an account followed by computing area using rectangles and increasing the number of rectangles. A student stated, “We used summation and area with rectangle boxes to find area under the curve, which we then used to determine net change and average value in business problems. With these rectangles we were able to start with 10, get an okay approximation, and then continue to increase the number of rectangles to get a more exact approximation.” One student reflected, “I really liked having the computer do all of those rectangles for me. I wouldn’t do 100 rectangles by hand. I have done these type of Riemann Sum problems by hand, but never had to do it via computer before.” Another student that had taken calculus before remarked:

> In this notebook, I learned an easier way of doing Riemann Sum than by hand. I always enjoyed doing them by hand previously but it didn’t dawn on me until now that to really get accurate with it, you have to draw an insane amount of rectangles, and doing it by computer was the easiest way.

Beginning instruction with the labs, extending the concepts introduced in the labs during lecture, and then allowing the students time to pull this knowledge together to complete the problems at the end of the lab let them see concrete examples of the topics and then continue to practice and apply their acquired knowledge. Student comments revealed support for this and expressed ways in which they believe they helped them learn the calculus concepts.

A Different Way of Problem Solving. Some students come into these Business Calculus classes having previously taken calculus. It is always a goal to challenge these students or to get them to walk away from the class learning something different than they had seen in their high school calculus courses formerly. For this instructor, while the business applications are usually new for students, previous discussions with students have revealed that they had seen most of the content before. The labs alleviated this issue to an extent because most students had not had practice using such tools and the very few that had had not seen them in use within a math class. These students were also better able to step up to the challenge of learning to use the labs and the content because they had previously learned the material and were not as challenged by learning both. Looking at comments from only students that had taken calculus before revealed that they learned a new way of problem solving and were indeed challenged in the course.

These students made mention of the new experiences of seeing the calculus concepts in concert with business applications, but they also remarked on the labs helping clear up confusion they still had and how they were able to solve problems differently. A student that had taken calculus before remarked, “I have learned all of this calculus stuff before but not with these applications and certainly not doing it on the computer. I honestly didn’t even realize you could.” A quote demonstrative of this point was:

> Overall, I feel like the mathematical content from this lab was information I have seen in my previous calc course, but the way the questions in the lab presented the same

> information has helped me to see what the point of learning that mathematical content was. In high school if we did word problems to reinforce the topics we were learning the situations seemed too made up for me to find them relevant. But some of these they just seem more real.

A similar sentiment was reiterated by a student stating:

> I have also enjoyed using more real world data to find the derivatives. When I took calculus before we didn't get to use as much realistic data as we are using in these labs. I thought that the np.polyfit to fit a curve to the data has been very cool. It is nice to know that the computer can do a lot of time consuming or seemingly impossible calculations that we might face elsewhere.

Another student that had taken calculus before commented that doing these labs gave her a different way to problem solve. This student stated that in doing her homework problems she no longer pulled out paper and pencil; instead she opened a Jupyter notebook and worked through the steps. She remarked that to her it actually seemed easier and quicker to do it that way and that was a "totally different way of doing things".

For some students this "totally different way of doing things" seemed to help them to understand some of the content and how it was related to calculations that had previously seen. The labs also showed how the content connected to outside of the classroom, which helped to give the problems more meaning and context and could then help them transfer the knowledge to other settings. This was certainly not all students and seemed most impactful for students that had previously taken calculus because it provided them with a new way of doing mathematics they had already learned, reinforced content they had seen before, provided them with opportunities to explore mathematical situations, and they were most able to focus on the coding and not as burdened by the new mathematics.

Student Challenges with Calculus and Coding. When starting this project, it was known that there would be challenges and a steep learning curve to get students to use the computer in a way many had probably never done before. A concern was getting students to learn calculus concepts while balancing that with learning new skills of coding. This proved true throughout the semester and was reflected in student comments. Students being intimidated by and resistant to the coding element along with thinking an approach similar to this expects too much out of them has been reiterated in other research on computing in mathematics classes (Jones & Hopkins, 2019; Tonkes et al., 2005; Lockwood, DeJarnette, & Thomas, 2019).

A student comment that reflects how some students felt is well expressed by this student's comment on lab 3:

> In this lab, we began to use what we learned in the past two labs and apply it into this lab. It was really tricky having to go back and forth to find exactly what to type in due to how one mistake can mess up a whole strain of coding. It was also tough trying to figure out which exact lines of code were needed because of the lack of labeling on the problems, which was intentional. It is really like trying to learn a whole new language with how intricate and different it is for someone who has never had experience with it before this class. With already being confused with the "normal" assignments out of the labs, it makes all of the derivative rules that much harder. To be honest I really have no idea if what I am plugging in is the right thing or not, i.e. struggling to finish the question in full. I don't think it is a good idea to mix coding with this math when kids already struggle enough with the math. The amount of time that is necessary to be put into the lab really

> equates for more credit hours. This isn't a coding class and it is just making kids struggle that much more. Yes I do admit that I can see the helpfulness of learning this program, I just think it should be offered in a different class. I struggled with understanding what the questions were asking for but after figuring out the necessary formulas and codes for derivatives and functions my codes stopped running to where there would just be a blue * so I did not know if I was doing the problems right.

Other students certainly felt this way too but did not express their thoughts quite so explicitly in their comments. Words such as frustrating, difficult, stressful, intimidating, hard, and time consuming were especially present throughout labs 1-3 as students were getting used to working in Jupyter notebook. These comments decreased greatly in labs 4-6 and were in some instances replaced with sentiments such as "this is still hard but I'm starting to get the hang of it a little" and "I'm beginning to be a little more confident in using this program".

It is important to note that students explicitly stated that learning calculus concepts in conjunction with learning to use Jupyter notebooks proved to be extremely challenging. These comments reflect that undoubtedly many students were challenged by this and could have minimized the impacts on conceptual understanding for some. It is imperative to listen to students' evaluations of this and develop ways in which the balance can be better struck, so that more students can see the technology as useful for developing understanding of the calculus concepts.

Discussion

The data from the Calculus Concept Inventory reveals minimal gains and no significant difference between the groups in conceptual understanding. Several factors and limitations of this study may have impacted the lack of gains and will be discussed in later sections. The qualitative data from student comments, however, does demonstrate that the computational labs did have important impacts on many students' learning of calculus. The qualitative data also reveals the challenges that some students faced that may have also adversely impacted their understanding. The mixed students' comments help to explain why there were no overall gains for students in the group with computational labs. The qualitative data helps to demonstrate that for some students the labs were thought to increase understanding while for others the labs interfered with that. Some students articulated that combining the labs with content was challenging but not impossible. An example of this was,

> This lab was by far the most challenging simply due to the fact that to do it correctly, you have to both recall the previous information that was used in previous labs, as well as understanding the use and right timing from the new math topics at hand. Which is safe to say is one of the more challenging ones. Much like the other labs, while I feel like they are really challenging, I believe that they are very useful in learning this stuff and we can actually see what the real world use of the topics are and in a way that I might use in the future.

Statements such as this one combined with both very positive and very negative comments demonstrate reasons as to why there was overall minimal impact on conceptual calculus understanding as measured by the Calculus Concept Inventory. The mixed results also align with previous research as Crowe and Zand (2000) noted that computer programming helps students explore mathematics often in a constructivist manner but they also note that it can be an extreme challenge for many students and adds a burden to rigorous mathematics courses that can have negative consequences.

The content of the computational labs allowed students to see calculus concepts used in what they deemed “a real world setting” or application. Students were able to articulate how different calculus concepts could be used in settings outside of the classroom, which demonstrated understanding. Students explicitly commented on the real world relevance they could see. Being able to make real world connections can be valuable for students and can aid understanding of the mathematics concepts. Getting students to experience meaningful real world connections makes them better equipped to use their mathematics knowledge and adapt to new situations (Boaler, 1998).

Despite the fact that students were able to articulate and discuss calculus concepts in connection with other situations and that students stated that they thought the real world connections helped them better understand, this knowledge did not come out as assessed with the Calculus Concept Inventory. This may have occurred in part because none of the questions on the Calculus Concept Inventory related to real world situations of business, which is the context in which students experienced the calculus concepts in this class. The qualitative data reveals that students were able to discuss the calculus concepts in context of business situations such as profits, sales, and supply and demand and make valuable connects in regards to these situations, but when asked about calculus concepts on the quantitative assessment the questions did not relate to the field of business. Perhaps this disconnect between how students had seen calculus concepts used in what they deemed as “real world scenarios” and the scenarios in which they were asked on the Calculus Concept Inventory contributed to the lack of gains as quantitatively measured.

Some of the ways in which students were able to discuss calculus concepts in connection to real world scenarios demonstrates that they did gain understanding of calculus concepts and how they can be applied. Some of the valuable connections between the calculus concepts and applications could have been done without the use of the computational labs, however, some applications such as importing live stock data and analyzing rates of change and support and resistance lines were facilitated through the use of the labs. Other applications, such as the analysis of a larger data set of Center of Disease Control data, were also made easier by the use of the labs. In many ways the labs allowed for the use of more real data and "real world problems" beyond the use of a typical word problem. This study of the use of computational labs in a Business Calculus course provides a foundation on which they can be expanded to include even more real and relevant content in connection to calculus concepts.

Students' comments revealed that learning calculus concepts at the same time as learning to do such on the computer proved to be a challenging task. This is not an unexpected result. Other studies integrating elements of coding into mathematics class have found resistance by students stemming from several factors such as the disconnect between computers working numerically while being shown analytic techniques in lecture, missing the connections between the lab exercises and course concepts, difficulty with errors and debugging, and the combination of learning mathematics and coding being perceived as too much work by students (Tonkes et al., 2005; Jones & Hopkins, 2019). Students having to learn new mathematics concepts and learning to communicate with a computer in ways they never have appeared to be a challenge for some students in these courses. Students also complained that they felt like it was time-consuming and a lot of work; some stating that it was too much work and "more work than the other class got". The students did not seem to realize that the workload of other assignments was reduced to create roughly equivalent demands of their time as compared to the other class. This too was not unexpected, as research from the early 1990's on different computational tools in tertiary calculus classrooms often cited complaints from students of workloads being increased (Tucker, 1990).

The qualitative data certainly highlights that balancing learning new technology with new mathematics concepts was a major challenge for numerous students. This challenge combined with that students felt that the course was too much work may have been such hurdles that these students' conceptual understanding may have been hindered by the labs. These students may have been so burdened by trying to learn the technology and the time commitment that required that they might not have been able to devote appropriate time to learning the calculus concepts. Listening to what students said throughout the semester and adjusting as necessary did lead to changes throughout the semester such as allowing more time for work on the labs in class. Student comments on labs and discussions with students will lead to changes in future iterations of this project.

This is also echoed in that the students that discussed how the labs helped them learn were those that had previously taken calculus. These students were not stressed with learning the new calculus concepts with new technology. A reflective statement of this was, "When you incorporate the math knowledge you already have, it makes the lab a lot easier to complete." These students were not as challenged with combining learning new mathematics material and learning a new technology, so they may have learned more of the calculus. These students benefited the most and may have had gains in understanding where as those without previous calculus exposure may have not made gains and exhibited losses, which influenced the overall lack of gains in the calculus concept inventory especially since 70% of these students in the course had not previously taken calculus.

The computational labs used in this project also showed students new ways of solving problems, which was reflected throughout students' comments. For both students who had seen calculus before and those who had not, the labs added a challenging new layer to their business calculus course and an element that may translate to their future careers. Students were able to see other methods of solving mathematics problems aside from paper and pencil and algebraic manipulation. While students were not proficient in coding or programming when they left this class, which was not an intended outcome of the course, they were at least exposed to some very introductory skills and were able to discover some of the power and utility these tools could have. A few students expressed interest in wanting to pursue learning more about programming, acknowledged that they thought it could be a useful skill to develop, and one student mentioned after doing these labs she was thinking about perhaps picking up a data analytics minor. Knowing how to code is becoming an increasingly important and marketable skill for those in business, evidenced by examples such as Citigroup announcing all new analysts would get training in Python, Goldman Sachs traders are expected to know how to code, and Columbia's Business School adding elective courses that teach programming languages (Kurczy, 2018). Some students are finding that having exposure to and strong knowledge of such programming languages are resume builders and "give them a leg up" (Kurczy, 2018). There are now numerous masters degree programs Business Analytics, such as one at the University of Virginia, which include learning about programming languages. For many of the students participating in this project this was their first time using the computer in this manner, which may spark their interests in learning more about this and could end up being valuable.

Improving this Class

A limitation of this study was this was the first iteration of using these labs in this class. While I had used Jupyter notebooks myself in my own work and had been a student where using computational tools had been a large part of my classes and learning, as an instructor I had not yet used them in my classroom. Because of such, the semester proved to be a learning process for all of us involved. Through my experience implementing the labs, the experiences and conversations with students throughout the semester, and in reviewing student comments, there are numerous changes that should be made to the class going forward and informing future research.

One change that should be made in the future is the implementation of the computational tools as labs. The computational tools were employed as labs in this study in part to keep the control and experimental classes more similar and comparable. Before the semester began, in discussing the implementation with a university Physics educator who uses Jupyter notebooks in his classes, he warned that by doing them as labs rather than daily use and fully intertwined in the class, students may see them as something extra, a completely separate entity, and not part of their accessible problem solving methods. He raised these points because he said he had seen them when doing so in physics classes (Titus, 2018). Having more incorporation of Jupyter notebooks throughout his courses rather than an additional piece to the class, he had seen students' views on them evolve and students had grown to use Jupyter notebooks as a way to solve physics problems rather than always pulling out paper, pencil, and a calculator to tackle problems. The issue of students seeing these labs as more assignments to complete and a separate entity was echoed in student comments and was observed in how many students approached problems. One issue was that to keep the two groups similar, students were not allowed to use Jupyter notebooks on their tests, but they could use them any other time. Another issue was while much of the content was introduced using the computational labs and the notes that followed built on and expanded the content covered in the labs, students were shown how to do concepts with paper and pencil as well because this was still an expected outcome of the course. This seemed to cause a bit of a disconnect. With knowledge that they were not going to be used on the test and being shown multiple ways to solve the problems, students tended to gravitate towards traditional methods of paper and pencil. There were exceptions to this, however. By the end of the semester, several students began using Jupyter notebooks to complete their homework assignments. As discussed and demonstrated in the section on students' comments on content, some students found these notebooks to help them more quickly do their homework, reinforce concepts because of the commands they had to employ, and feel more authentic in how they would do math problems outside of the classroom, it can be seen that some students did progress to considering Jupyter notebooks as another way to solve calculus problems. These students were in the minority, however, and most students still resorted to traditional techniques. Because the labs were not used everyday and some students viewed them as separate from the class, this may have minimized the overall impact. Students also commented that they desired more time in class to work on Jupyter notebooks. In future research the computational tool, Jupyter notebook, must be more intertwined throughout the entire class, so that students increase their willingness to use them to solve problems and thus the impact may grow. As the use of these tools grow and evolve in my classroom I will increase more independent practice in the class and getting students to code more of their own functions, which could potentially increase the impact.

Another change that took place throughout the semester and should evolve in the future iterations of the project is allowing more time for students to work in pairs. There was some time for this, however, students remarked that they would have like even more time to work in class with other students. On lab three one student stated, “Working with others really helped me as well, especially given the class time to work on this.” On the same lab another student said, “Being able to work on this in class was a big help to me because I have found that these labs are very frustrating, but it really helped to go through it with someone else. They could help me figure out if I was missing something with the syntax and try to fix it. I also found it helpful to talk to someone else about the math stuff too.” Allowing this time could help students work in pairs to help each other, especially to find syntax errors. Other examples of using computational tools in a mathematics classroom make similar recommendations (e.g., Jones & Hopkins, 2019). Students’ comments reflect that they believe that working with others may help them understand the mathematics and work through the challenges with the computational labs.

Limitations and Future Research

Student comments revealed that the computational labs used in this study did have an impact on what and how they learned, but no impact immerged in the quantitative data. There are several notable limitations in this study.

Some noteworthy limitations to this study include the instrument used and the timing of administration of the post-test. Some issues with the Calculus Concept Inventory results likely happened because of random guessing on the pre-test and apathy on the post-test. As seen in the description of the participants, most students in this study had not taken calculus, at any level before; in the traditionally taught sections of the course 56.4% of students had not taken calculus before and in the sections participating in the computational labs 70.7% of students had not taken calculus before. With most students unfamiliar with the notation and terminology of calculus on the pre-test CCI, random guessing undoubtedly occurred and likely affected, perhaps inflating, pre-test CCI scores. Miller et al. (2010) in studying gains raise the question that do losses come from actual conceptual losses or rather "correct guesses on the pre-test that, by chance, became incorrect on the post-test?" The random guessing on the pre-test is of great importance since in both groups most students had not taken calculus before and the test contains numerous questions with calculus terminology and notation. It is unlikely that the losses came from actual conceptual losses since so few had seen calculus concepts before, and the three tests students took throughout the semester that were not multiple choice did not have large failure rates. Other studies have found difficulty detecting differences between classes when using the CCI and call the instrument's validity into question (e.g., Bagley, 2014).

Another serious limitation was that the post-test CCI was given on final exam day, which did ensure all students were present, however, also presented issues. Students had ample time during the final exam period to complete all that was required of them and there was a block of thirty minutes during the final exam period where they were to work on the CCI if agreeing to participate. Since the CCI did not count towards their grade in anyway many students did not appear to give much effort in completing it. Numerous students appeared to rush through it, closing their computers after less than ten minutes. Understandably students were more concerned about getting to their actual final exam that would affect their final grade. In future studies, the post-test should not occur on the same day as the final exam and other methods of making students take the post-test more seriously must be considered. The apathy from students on the CCI certainly affected the results and was likely part of the reason for the negative gains. This study made the assumption that students would honestly and intentionally answer the questions on the CCI, a violation of this assumption could have impacted the results. Violations to this assumption would call into question the use of normalized gains. Hake's (1998) *g* has the implicit assumption that gains will be positive (Miller et al., 2010). Miller et al. (2010) assert that when losses are normalized with respect to possible gain, the normalized gain does not have a "sensible interpretation". Going forward in future research, normalized gains will not be used, instead Dewello's *G* and *L* will be computed, which according to Miller et al (2010) normalizes gains with respect to potential gains and losses with respect to potential losses rather than *g* normalizing all to gains only.

Some students, including very strong students, remarked how hard they thought the CCI was. Multiple students commented on how they thought the wording of the questions confused them. Even students that had taken calculus before and knew the calculus specific notation and terminology on the pre-test struggled. Wording of many of these questions was different than problems seen by students throughout the semester, so they may have had issues translating their knowledge of calculus concepts and the way they had been asked them during the semester to the types of questions on the CCI. Also students grew accustomed to seeing calculus concepts asked in context of a business problem, which do not appear on the CCI and could have added to the lack of growth seen. Another issue of using the Calculus Concept Inventory was that it did not assess concepts of integration, which were part of the class. While this was known before implementation, with a majority, over two-thirds, of the semester focused on concepts of functions, limits, and differentiation (the topics on the CCI) and only the last bit of the semester centered on integration, it seemed that the CCI would be appropriate. Two of the computational labs did focus on integration, so an instrument that had concepts on integration would be more suitable.

In future research, a different instrument should be used to quantitatively gauge student gains in conceptual knowledge. While it still remains that no new similar instrument has been developed, in future studies a department generated assessment could be used. At the university where this study took place, common finals are not given but a short department-developed assessment is given to randomly selected classes each semester. This assessment was not used in this study, however, this intervention and study did spark conversation about how to modify that assessment to perhaps make it more focused on concepts and applications rather than algebraic manipulations and by-hand computation of derivatives and integrals, which may allow for changes in this assessment that would make it more suitable for use in future research. As changes to this assessment are made, this would also allow for future research to compare this method with students from other instructors. This study further echoed calls of Bagley (2014) and Gleason et al. (2015a) for a better instrument for use in research to assess conceptual calculus knowledge.

Another limitation of this study is the way in which the qualitative data was gathered. The qualitative data in this study was very unstructured. The students reflected on how they felt about the labs without solicitation specifically and thus there may be more to be gleaned from what students thought if they were specifically asked about their thoughts. In the comments from the end of the labs students appeared to be rather open and honest perhaps because they did not feel any pressures since they knew they would not be penalized on their grade for what they said and would receive credit for completing the paragraph. There, however, could still be some bias in their responses as they may have responded more positively since they knew their instructor would be looking at the paragraph. There were certainly negative comments throughout the labs and some students were very open about how they did not think the labs helped, but other students may have kept some of these thoughts to themselves as well. It is recommended that in future research interviews could also be conducted to help to better explain how the computational labs affected students. Interviews may also give students an opportunity to express their thoughts without the solicitation being any part of an assignment in the class. This would also help to differentiate whether students felt that it was the selected technology that was used that helped them learn, if it was the content of the labs that helped, or if it was some combination of both. It was not possible to fully make this determination from the student comments as they were gathered in this study.

General Conclusions

Increasing students' conceptual understanding of calculus is certainly a goal of undergraduate calculus classes and was an objective of using computational labs to introduce students to calculus concepts in this project. Within this study there was no statistically significant difference in calculus conceptual understanding when students participate in these computational labs, however, there was evidence from students' comments that the labs did help many students learn calculus, most importantly making connections of calculus concepts to their chosen field of interest business. These labs helped a majority of students see the applicability of calculus to the "real world" and left them not asking the question of when are we ever going to use this outside of this classroom; there were, however, also negative comments of the labs. Thus I hypothesize that this is an area for future work further developing courses that employ such tools. This project sparked real conversations about ways to engage students in learning calculus, about what we really want them to walk away with having taken the course, and what role technology can play in this venture at the university where this took place.

Many students' comments revealed that the technology did help them learn concepts in the course, helped them make connections with business applications, and discover a different way of approaching problems and new problem solving tools. Others however did express their distaste of the labs, the challenges they felt the technology added, and some even felt that the labs made the class more challenging and did not learn much calculus from them. Because of the mixed feelings and experiences, the use of computational tools in this course should continue to undergo revisions to enhance students' experiences and learning of calculus concepts.

Based on the results of this study and experiences in these classrooms, I will continue to use computational tools in this class and several other instructors at the university where this study took place have expressed interest in further developing this class and others using such tools. How computational tools can be employed in undergraduate general education mathematics classes to help students better understand mathematical concepts is an area that needs further study.

CHAPTER 4: Manuscript 2
Computational Labs and Attitude Toward Mathematics

Abstract

This study investigates the change in students' attitude toward mathematics during a one-semester Business Calculus course taught using computational labs with business applications. Investigating students' attitudes and perceptions of mathematics provides empirical evidence about the intervention of computational labs in Business Calculus having an impact on students' attitude toward mathematics. Students' mathematical attitudes are considered through the Mathematics Attitudes and Perceptions Survey. Overall significant gains in attitude were not found, however, important gains were present in certain populations and reasons for this are discussed. Student comments were also analyzed for effects the labs had on attitudes and perceptions, which revealed several notable impacts and help to explain the quantitative results. Findings from this study may have implications for mathematics educators who are looking to find ways to make mathematics, specifically calculus, more relevant, applicable, understandable, and enjoyable for the masses that are now required to take it.

Introduction

"One goal of an undergraduate education in mathematics is to help students develop a productive disposition towards mathematics" (Code et al., 2016, p. 917). In addition to imparting knowledge of important mathematics skills and concepts on students, undergraduate calculus classes also have an impact on students' attitudes, which can affect their willingness to use mathematics, career aspirations, and choice of taking future mathematics classes (Sonnert, Sadler, Sadler, & Bressoud, 2015). Students often find undergraduate calculus to be challenging and are fearful of it, a fact of which instructors are keenly aware (Bressoud & Rasmussen, 2015). Bressoud & Rasmussen (2015) present that as post-secondary calculus is currently taught it "is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics." This applies to STEM students, but also extends to business students as well, most of whom are required to take calculus. This also has implications in that calculus has long served as a stumbling block and a "critical filter" that is "blocking access to professional careers for the vast majority of those who enroll" (The Mathematical Association of America, 1988, p. xi) and appears to remain as a filter today, even for some strong students (Bressoud et al., 2013). Calculus can be a filter for students because instead of helping students develop a willingness to use mathematics and a productive disposition toward it, it may be lowering their confidence in their mathematical abilities, their desire to take more mathematics courses, and their ability to see its connections beyond the classroom walls and other disciplines.

Considering how courses, including Business Calculus, could impact attitude toward mathematics is important because how students feel about mathematics may have an impact on their performance in the course as well. Attitude toward mathematics and mathematics achievement are often thought to be positively associated and has been demonstrated to be a positive relationship in college calculus courses (House, 1995). Research suggests that attitude can be positively changed if math is taught in more relevant contexts and real world connections (Andersson, Valero, & Meaney, 2015; Cornell, 1999), which is part of this project. The Mathematical Association of America called for introductory undergraduate mathematics courses to be "effective in positively affecting student attitudes about mathematics" (Saxe & Braddy, 2015, p. 66), emphasizing the importance of early college mathematics courses positively impacting students' attitudes toward mathematics, which is a goal of this project. This project uses computational labs, using Jupyter notebooks, making use of connections to business applications, in a one-semester Business Calculus course to introduce students to calculus topics.

Literature Review

"Business students, although able, are often math phobic. Courses should strive to lessen math phobia, enable students to be more comfortable with mathematics, and help students appreciate the relevance of mathematics" (Lamoureux, Beach, & Hallet, 2000, p. 19). Some of the intentions of this project are to do just that. If the project succeeds in doing so, perhaps it will answer Nievergelt's (1996) question, "how to impart just enough mathematics to business majors, so that they may understand the potential power and limitations of mathematics, decide when to hire mathematicians, and consult with them profitably" (p. 146), but also inspire students to be more appreciative of mathematics outside of the classroom and be more willing to use mathematics by impacting their attitudes positively.

Business Calculus is often daunting to undergraduate business students (Depaolo & Mclaren, 2006). Business Calculus tends to have high D, F, and/or withdraw rates even though it is a 1000 level class and a requirement for graduation for all business majors at the small, private, liberal arts university where this study took place. This is the case at other universities as well, many of which report that Business Calculus is a course where students tend to have high levels of resistance to mathematics (Depaolo & Mclaren, 2006; Liang & Pan, 2009). In this course, it is often seen that students are unprepared and not excited to take the required course (Liang & Pan, 2009), and unfortunately business students required to take Business Calculus often miss seeing the connection between calculus and the rest of their courses leaving many of them "unmotivated and even resentful" (Narasimhan, 1993, p. 254). Early college mathematics courses, such as Business Calculus, should help students overcome some of these negative feelings and perceptions about mathematics and positively affect students' attitudes about mathematics, which how to effectively do so is still very much an open question.

Attitudes Toward Mathematics

Aiken (1970), in discussion of attitudes toward mathematics, defines attitude as "a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept, or another person" (p. 551), and Neale (1969) defined attitude toward mathematics as a measure of "liking or disliking mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless" (p. 623). These are the definitions used to guide this study of attitude toward mathematics.

It is important to consider attitudes toward mathematics outside of following recommendations to positively affect them but also because attitudes have been linked to achievement. The relationship between attitude toward mathematics and achievement in mathematics is usually positive and practically significant, not always statistically significant, at the elementary and secondary school levels (Aiken, 1976). Ma and Kishor (1997) find that the relationship of attitudes toward math and achievement has been found to be significant and positive but not strong. Research has suggested that mathematics attitude is a critical construct in learning mathematics (Singh, Granville, & Dika, 2002). In mathematics, positive attitudes have been associated with higher scores on standardized tests and higher classroom achievement (Aiken, 1976; Aiken & Dreger, 1961; Stankov & Lee, 2014; Zimmerman et al., 1992). Negative attitudes toward math have been linked to drop out rates from math courses (Ma & Willms, 1999) and to poor engagement leading to failure (Mayes, Chase, & Walker, 2008, pp. 28-29). University students with negative attitudes toward math tend to get low scores on final exams

(Nunez-Pena et al., 2013). This research implies that a student's attitude toward math and belief in oneself as a "math person" has an effect on a student's learning outcomes. Students' negative attitudes toward mathematics may be hindering their successes, so developing a more positive attitude toward mathematics is vital for chances of increasing success. The degree to which students enjoy math, find and place value on mathematics, and believe it is valuable for success in school and the future affects students' motivation to learn (Ismail, 2009; Middleton & Spanias, 1999). Belief in one's abilities to do math could be linked to attitude toward mathematics because students lack confidence in their math abilities because of previous experiences, poor grades, a general lack of interest in math, and the inability to relate math to usefulness in everyday life (Peters, 2013). Self-efficacy for college students in mathematics had a stronger relationship to mathematics performance and mathematics problem-solving than other variables such as math self-concept, high school level mathematics courses, and math anxiety (Pajares and Miller, 1994). Since students around the world have problems with basic conceptual understanding of calculus (Epstein, 2013), and since attitude has been linked to academic achievement, it is important to consider how a course is impacting students' attitudes.

Calculus and Business Calculus Attitudes. Undergraduate calculus causes a sharp decrease in students' enjoyment of mathematics and confidence in mathematics ability (Bressoud & Rasmussen, 2015), which raises concern since attitudes related to math have been linked to achievement and since we live in a world that requires many professionals to interact with mathematics at some level. The extreme efficiency post-secondary calculus, as currently taught, has in lowering student confidence, enjoyment of math, and likelihood to continue to other math class is much the opposite of the following goal of what an undergraduate mathematics course should do (Bressoud & Rasmussen, 2015). The relationship between calculus attitude and achievement has been seen at the college level. House (1995) demonstrated the positive relationship between attitude and achievement in college calculus courses. Pyzdrowski et al. (2013) found a strong positive correlation between attitude and performance in entry-level college calculus; their findings indicated that attitudes affecting course performance were not based simply on previous mathematics experiences and preparation but also link to psychological factors such as confidence.

Business Calculus is a course that often has students that are not excited to take this required course (Liang & Pan, 2009); this is not to say that the students are incapable, however, some have, over their years of schooling, developed unpleasant feelings towards math. Likely because of perceived negative prior experiences, Business Calculus is a course where students walk in the door with a high level of resistance to and negative attitude toward mathematics (Depaolo & Mclaren, 2006; Liang & Pan, 2009). In studying these students, Depaolo and Mclaren (2006) found that attitude towards math in Business Calculus was significant in predicting exam scores. Findings also support that attitude had a larger effect on calculus performance than it did on statistics performance for business students (Depaolo & Mclaren, 2006). An important point to highlight in Depaolo and Mclaren's (2006) findings is that attitude appeared to have a stronger affect on performance for students that had not taken calculus before than ones that had; this is important since many students in Business Calculus are being exposed to the material for the first time.

Anecdotally, when discussing with my students why they chose to major in business or how they think math relates to their future business career, many of my students usually let me know their feelings about math. They are usually more negative than positive, they tend to identify and hold onto their previous perceived failures in math, and they connect those to why they chose business as their major thinking that it may not be as math-intensive as a STEM major. This anecdote aligns with existing literature on how students decide on college majors and future careers. Research supports that students often have their minds made up about going into STEM careers or not upon exiting high school (Maltese & Tai, 2011). Pritchard, Potter, and Saccucci (2004), in studying business students and their basic algebraic skills, found that students with higher computational and algebraic skills chose to major in more quantitatively focused business concentrations such as accounting or finance, while students with lower scores tended to select a concentration or major in less mathematically focused ones such as management or marketing, and some business students choose less quantitatively focused concentrations because they perceive them to have less demanding quantitative requirements (Pritchard, Potter, & Saccucci, 2004).

Students enter the class with their minds often already made up, so in trying to positively influence a student's attitude it is important to consider from where those attitudes derived. Many students have mathematical biographies littered with perceived negative experiences, which cause students to dislike math and blame their negative experiences on mathematics itself or believe that they are lacking some innate ability to do math. Van Damme et al. (2004) showed that differences in mathematics attitude could be explained related to the way students experience the teaching of mathematics. These attitudes were likely influenced by a combination of student level factors and classroom experiences. The math-hating attitudes also come attached with very strong emotions reflected by words such as sickening, frustrating, and wanting to cry (Larkin & Jorgensen, 2016), appear at a very young age (Cornell, 1999), and may become very much engrained.

Attitude and Pedagogy

Attitudes toward mathematics and calculus are linked to the instruction students experience including pedagogical decisions, use of technology, environmental factors, and personal factors including previous interaction and attitudes toward mathematics and the choice of college major (Sonnert, Sadler, Sadler, & Bressoud, 2015). Other factors that appear to influence students' attitudes and success are peers' attitudes (Kotok, 2017), poor content knowledge foundations, negative mindset, lack of number sense (Westenskow, Moyer-Packenham, & Child 2017), finding the subject boring with too much memorization, and their teachers (Cornell, 1999). Andersson, Valero, and Meaney (2015) identify that statements such as "I hate math" or "I am bad at math" do not come from only the students' internal feelings but come from an interaction of classroom and pedagogical experiences, implying that understanding of the experiences that shape students' attitudes can help to prevent students from hating math.

Research suggests that attitude can be changed if math is taught in more relevant contexts and real world connections (Andersson, Valero, & Meaney, 2015; Cornell, 1999). Challenging, innovative, meaningful, and stimulating classroom activities influence students' satisfaction and growth, which helps a positive attitude toward learning (e.g., Strauss & Volkwein, 2002). There is a need to create an environment that fosters more positive attitudes toward mathematics to nurture mathematical skills, which could include more relevant contexts, more modern topics, more modern approaches, and more technology and computing such as in this project. "Students see mathematical tools for the life sciences and social sciences as useful, interesting, and beautiful when they learn to use them in realistic applications and when computers do the calculations" (Hoffman, 1989). Not only do students find mathematics more useful when taught in such a way but achievement is also linked to these realistic applications; more specifically research suggests, “mathematical capabilities of children are greatly influenced by whether they are in a real world or a classroom context” (Couch & Haines, 2004, p. 199). Classroom activities that represent math as static, unchallenging, and boring leave students unable to see the usefulness of the mathematics (Wilkins & Ma, 2003). Students that get continuously exposed to mathematics as rote memorization and find it unchallenging develop a negative attitude toward mathematics and its applicability outside of the classroom (Greenwood, 1984).

Considering this with a focus on Business Calculus, The Mathematical Association of America *Calculus for a New Century: A Pump Not a Filter* report supports reforming calculus and to modernize it; “We need to teach calculus in a way that facilitates complex and sophisticated numerical computation in an age of computers. Somehow or other we have to make calculus exciting to students” (The Mathematical Association of America, 1988, p. 9). The Mathematical Association of America’s report The Curriculum Foundations Project: Voices of the Partner Disciplines emphasizes a “potentially useful method for drawing students into the lecture is to start the lecture with a real-world (or realistic) business problem. If students are convinced that the problem is worthy of their attention, and that they do not know how to solve it, they are much more likely to pay attention and to retain what they learn. It is important to get the buy-in at the beginning” (Lamoureux, Beach, & Hallet, 2000, p. 21). Gordon asserts that students gain appreciation of the relationship between math and the computer, which “provides an ideal context in which to develop several simple, yet useful, numerical algorithms for approximating functions and for actually finding where all those ‘given’ functions come from” (Gordon, 1979, p. 23). Many times the classical techniques of calculus require special cases and a lot of time to master for the novice with teachers then reducing problems to a very simplified version (Hoffman, 1989), but the labs in this project allow for students to see more realistic, advanced business topics. The labs in this project are used to introduce students to calculus topics and are often situated in context of a business or financial problem. This may help students find the mathematics more relevant and in turn positively affect their attitudes.

One recommendation that the class in this project aims to fulfill is:

> Calculus in the business mathematics curriculum should emphasize the basic concepts and how they apply to business problems, with more attention to numerical methods and less to techniques of symbolic differentiation and integration. The Business Calculus curriculum should include an introduction to rates of change, and the dynamic nature of real world systems, constrained optimization, and interpretations of area under a graph. (Lamoureux, Beach, & Hallet, 2000, p. 20)

Recommendations as to how to do so include using technology to show students tools they will use in the work place, to enhance the efficiency of the learning process, and to enrich and maintain student interest (Lamoureux, Beach, & Hallet, 2000). Suggestions, such as these, call on instructors to use technology in the classroom and allow for hands-on experiences for students potentially through labs to support student learning (Lamoureux, Beach, & Hallet, 2000).

Attitude and Technology. The use of modern technology can be extremely beneficial for students to learn calculus with true business applications, especially for students with weak math skills (Liang & Pan, 2009) and negative attitudes. Motivation, participation, and interest are all shown to improve in Business Calculus classes when computers are allowed as an aid and more emphasis is placed on understanding and application; by doing so the age-old student question of “why do I have to learn this stuff?” is gone because students are allowed to see the calculus in action with real applications and are less mired by the stress of by-hand skills and techniques (Judson, 1990). In a calculus course with Mathematica, Brown, Porta, and Uhl (1990) report that students were excited and ready to experiment and discover using a technology that was new to them. Using *R*, which has some similarities to using Python, students can write programs to model, simulate, analyze data, and more is shown to have produced strong enthusiasm for many students (Benakli, Kostadinov, Satyanarayana, & Singh, 2017, p. 422). These along with other results are promising. Since "how to convince business students in the classroom that mathematics will later be able to save them time, hence money, remains an open problem" (Nievergelt, 1996, p. 146), it is important to consider interventions such as this project that may impact that.

The national study of calculus (Bressoud, Mesa, & Rasmussen, 2015) does not mention the use of coding and computing in a programming language as used in this study, however, the use of software in mathematics has grown of late and it is known that these tools are useful for professional mathematicians, so students should be exposed to these tools. Often software can make mathematical computation and inquiry quicker and more accessible to those not advanced in their mathematical careers (Quinlan, 2016). Lockwood, DeJarnette, and Thomas (2019) found from interviewing mathematicians that:

> Computing is an activity that can be performed with a variety of tools, including paper and pencil. At the same time, given that the advances in technology can increase the efficiency, accuracy, and utility of computational work, it is reasonable to expect that computing is particularly salient practice in this era, at least inasmuch as mathematicians may recognize it as an integral part of the work of doing mathematics. (p. 4)

With the importance of such tools growing within mathematics, there has also been a rise in research on computational tools in mathematics education (e.g., Cline et al., 2019; Jones &

Hopkins, 2019; Kilty & McAllister, 2019). diSessa (2018), Lockwood, DeJarnette, and Thomas (2019), and others encourage mathematics educators to research how their ideas of how computers and computational tools can be used in mathematics learning and that there is still much to be discovered in effective learning and learning of computing within mathematics.

Computational Tools. "The emergence of accessible computational technology enables a wholly different mathematical experience for university students" (Koehler, 2017). Some previous research indicates positive results of using computational tools in the mathematics classroom. Computational tools could allow for mathematics to focus on the concepts rather than a large emphasis on the skills (Heid, 1988) and students in such classes benefit, including in conceptual understanding from incorporating technology (Heid, Blume, Hollebrands,& Piez, 2002). Fenton and Dubinsky (1996) developed ISETL language to help students more effectively learn mathematics beginning with the argument that "communicating with a computer requires a level of precision that will help illuminate important mathematical ides for students" (Lockwood, DeJarnette, & Thomas, 2019). Schwingendorf and Dubinsky (1990) also reported that students felt that the computer labs helped them explore the mathematics in a new way, which helped understanding and was something of which many were appreciative. Cetin and Dubinsky (2017) found that by using ISETL for writing and running code, students learned concepts such as functions more effectively and had to think about what the computer is doing with the code. Using computational tools, like R, improves conceptual understanding of many difficult concepts from complex and abstract problems and improve problem-solving skills (Benakli, Kostadinov, Satyanarayana, & Singh, 2017). Because computational tools can aid students in effectively interacting with and learning mathematical concepts, the tools may be effectual in positively influencing attitudes as students gain confidence in successfully understanding and completing challenging mathematical tasks. When students are successful, they are more motivated and willing to engage in mathematical tasks, which could positively affect how they feel about mathematics;

> Research indicates that success in mathematics is a powerful influence on the motivation to achieve. Students perceive success as reinforcing and they will engage in mathematics if they expect to be successful. In addition, students will not only engage more, they will also tend to enjoy tasks for which they have a moderately high probability of success more than tasks for which the probability of success is near chance. (Middleton and Spanias, 1991, p. 68)

The use of Jupyter notebooks running Python is a pedagogical decision to intentionally teach calculus topics, not simply support students in computation. Schwingendorf and Dubinsky (1990) found through evaluation of student comments that using a mathematical programming language, ISETL, helped them realize that mathematics was more than using a formula or pattern but instead involved a lot of thinking, and these students could articulate that calculus was a method and thought process involved in solving problems. Cetin and Dubinsky (2017) discuss that writing and running code and thinking about what the computer is doing with that code can help students to internalize concepts, specifically understanding of functions. A project at Dartmouth in calculus courses where students explored calculus topics through programming in BASIC (Crowell & Prosser, 1991) found ease of implementation, mixed results on students attitudes toward the computer enhancing calculus, and still asked the questions of what is the computer's place in calculus with CAS systems or programming and how would the traditional curriculum and pedagogy be revised to best incorporate the power computers could provide (Crowell & Prosser, 1991, pp. 151-155) both of which are still questions of today. Rich, Bly, and Leatham (2014), in studying the impacts that learning computer programming can have on the way students approach mathematics, claim that learning to code “provided participants with context, application, structure, and motivation for mathematics” that was long lasting. Kilty and McAllister (2019) found that students in their calculus modeling course using RStudio reported increases in self-confidence in mathematics, comfort in using mathematics and software to study real-life situations, and are appreciative of being able to study calculus in a manner relevant to their other courses.

Technology and Labs in Calculus Courses. A review of literature revealed that labs in calculus courses take on many forms, cover a variety of content, and use a variety of technology (Leinbach, 1991). A calculus lab can be used as a “learning device to see how calculus applies to other courses and disciplines” and “helps students relate the rather abstract ideas of mathematics to non-mathematical ideas they have encountered in other courses” (Basson, Krantz, & Thorton, 2006, p. 346). Numerous examples of labs in a calculus course with some programming or mathematical programming language emerged in the late 1980s and early 1990s, which reported overall positive results (Tucker, 1990). Höft and James (1990) found that students in a calculus course with computer labs were more interested in calculus material, more responsive, and more engaged with questions than students in a traditionally taught course. An example of a successful implementation of a calculus lab entailed enhancing the existing calculus course and having the main goal of the lab as teaching students to make connections not teaching additional material but (Basson, Krantz, & Thorton, 2006). Successful calculus labs also often use real data (e.g., Basson, Krantz, & Thorton, 2006; Kowalczyk & Hausknecht, 1994), which this project will do.

The labs in this project will be done using Jupyter notebooks, which allows for coding in Python and is thought to be an easy to use program (Koehler and Kim, 2018). Koehler and Kim (2018) assert that Jupyter notebooks and Python are easy to use even for teachers with little to no background in coding and computing, and with them students can easily discover how technology can help them solve mathematics problems and communicate solutions to challenging and real problems. Jupyter notebooks are open-source and can include creation and sharing of documents that can contain live code, visualizations, equations, and text (Project Jupyter, n.d.). Jupyter notebooks support multiple programming languages, can be easily shared, can produce interactive output with images, videos, LaTex, and more, and can support big data tools (Project Jupyter, n.d.). The labs in this project ran Jupyter notebook with Python. Python is a popular, commonly used programming language for scientific computing and data science that has a large focus on ease of use and readability (Meurer et al., 2016). Technology is playing a growing role in the financial and business industries, and Python, with many open source financial libraries, is growing in importance (Hilpisch, 2016), and there is an increase in business analytics masters degrees with some coding component, so exposing business majors to a programming language could introduce them to a valuable skill they may be expected to use in industry.

There is promise in the research reported about the use of different computing tools, computer assignments, and computer labs to positively affect attitudes toward mathematics, especially in the research from the late 1980s and early 1990s, but this is an area that must continue to be studied as the computing capabilities are more powerful, accessible, and prevalent than ever before.

Use of Technology for Preparing for the Future. "In the real world we use computers for calculating, almost universally. In education we use people for calculating almost universally" (Wolfram, 2014, p. 1); this changes what mathematics might be of importance to be taught. Gravemeijer et al. (2017) assert that the previous quote reflects "that we have to shift away from teaching competencies that compete with what computers can do and start focusing on competencies that complement computer capabilities" (p. S107). Students tend to align with the research that the inclusion of technology in the classroom can be beneficial for focusing on problem solving and more real situations and being reflective of the work place, but this is not always reflected in the mathematics classroom (Zevenbergen, 2004). Students have difficulty engaging in mathematical modeling, or translating a real world situation into a mathematical representation, because they have learned math decontextualized and have a hard time switching between real world and mathematics because of the lack of practice they get with this in school (Couch & Haines, 2004). These however are important skills because, "when mathematics is applied in the modern world for a practical purpose, we almost always require a computer to deal with a realistic level of complexity or to manage the data involved." (Cline et al., 2019).

Today's college-educated professionals need to be "creative, confident, competent problem-solvers, and clear, critical thinkers," which can be developed in part in their exposure to undergraduate mathematics that includes modeling, inquiry, and using technological tools to solve problems from all disciplines (Arney, 2009, pp. 94-95). Mathematics, being able to interact with technology, and appreciating some of the interdisciplinary nature of problems will be valuable skills for future careers. "Across a wide range of industries and occupations, people are required to use, develop, and communicate mathematical ideas and techniques in a diversity of ways with others who have differing expertise, experience, and interests including in mathematics itself" (FitzSimons & Boistrup, 2017, p. 330). Employers and future trends indicate that there is a great need for mathematically proficient individuals. There is a need for mathematically literate individuals for the twenty-first century business world (Yıldırım & Sidekli, 2018), and mathematics knowledge is becoming imperative for many career opportunities (Bureau of Labor Statistics, 2016). A 2013 poll of 200 employers revealed the second most important skill they look for in potential employees is the ability to make decisions and solve problems, and the ability to analyze quantitative data and use computer software programs are also in the top ten (Adams, 2015). The National Network of Business and Industry Association (2014) lists using mathematics to solve problems as one of its necessary employability skills. Employment of mathematics occupations has a projected growth of twenty-eight percent from 2016 to 2022, and growth in the areas of business and government needing mathematicians or people comfortable using math is expected to grow as business and data analytics continue to grow (Bureau of Labor Statistics, 2016). Sales and marketing, research and development, supply chain management, and workplace management are all areas in which analytics are growing (Columbus, 2018). This project aims to study changes in students' attitudes and perceptions about mathematics because these play an important role in one's willingness to use this knowledge outside of the classroom; "A person's mathematical disposition related to her or his beliefs about and attitude toward mathematics may be as important as content knowledge for making informed decisions in terms of willingness to use this knowledge in everyday life" (Wilkins & Ma, 2003). Perhaps students that are participants in this project will immerge with more expert-like dispositions towards mathematics and thus be more willing to use it in everyday life, which may very well include willingness to use it in the workplace.

Given that mathematics preparation, past achievement in mathematics, and attitude toward mathematics have been found to be factors in career choice and that math sometimes serves as a "critical filter" (Bleyer, Pedersen, & Elmore, 1981, p. 46) in the career choice process and that Business Calculus is a course where students tend to be unprepared mathematically and show a dislike toward mathematics (Liang & Pan, 2009), it could be inferred that some of these students' are choosing to major in business and potentially choosing it as a career because of their previous experiences with mathematics. Students' mathematics test scores influence choice of major; students with higher mathematics test scores were more likely to choose a technical major rather than health, business, public service, or liberal arts (Simpson, 2001). The more mathematics preparation a student has from high school is indicative of the student choosing a more technical major than a non-technical one (Simpson, 2001), which would include business (Pritchard, Potter, & Saccucci, 2004). Some business students chose less quantitatively focused concentrations because they perceive them to have less demanding quantitative requirements (Pritchard, Potter, & Saccucci, 2004) implying that mathematics does play a role in the selection of major and potentially future career. A quick Google search also reveals forums and question threads where people ask if and why business majors have to take calculus and why business majors have to take calculus, which indicates mathematics is at least weighing on some students' minds when thinking about majoring in business.

Undergraduate calculus classes teach students important skills and concepts in mathematics and also have a large impact on students' attitudes, which can affect their career aspirations and choice of taking future mathematics classes (Sonnert, Sadler, Sadler, & Bressoud, 2015), and is thought to be the case in the classes of this study. If a first college calculus course, such as Business Calculus, is taught well, it "could be an opportunity to have them leave not hating math, but actually bring them in" and "for those who continue in their chosen non-STEM field, whether business or social work, having more people who are STEM- and calculus-literate would be great" (Ellis as cited in Courage, 2016). Acknowledging that attitude toward mathematics may play an important role in success in mathematics courses, it is important to consider interventions, such as this project, that aim to make more connections to applications outside of the classroom, so that students' attitudes may improve.

Research Question

This study was designed to determine the impact, if any, of computational labs on students' attitudes towards mathematics in a one-semester undergraduate Business Calculus course. The research question guiding this investigation is as follows:

> To what extent will attitudes toward mathematics change as a result of being introduced to calculus concepts through computational labs in a Business Calculus course?

This project studied the implementation of computational labs and their impact on students' attitudes through a quasi-experimental, multi-method design. The Mathematics Attitudes and Perceptions Survey was employed in this study to gather data on students' attitudes toward mathematics, and the data was statistically examined to measure the effect of the labs on students' attitudes toward mathematics. Students' comments from the end of each lab were collected and analyzed as well. The main purpose of this research is to determine the effectiveness of computational labs as a method to positively affect students' attitude toward mathematics and further inform teaching practices.

Participants

Participants in this study were undergraduate students at a medium-sized private university, with undergraduate enrollment around 4,500 students, enrolled during the spring semester of 2019. At this university the liberal arts general education curriculum currently includes a mathematics requirement of which approximately 39% of students meet by taking Calculus I or Business Calculus during their freshman year. The majority of students in both groups were freshmen, but there were students from freshmen to seniors.

Four sections of Business Calculus all taught by the same instructor participated in this study. Thirty students were enrolled in each section to begin the semester with numbers dropping by a few students in each class as the semester progressed. The semester began with 120 students enrolled across the four sections, with 60 eligible students in each of the experimental and control groups. Of the 120 students enrolled at the beginning of the semester, 113 elected to participate in the first day administration of the Mathematics Attitudes and Perceptions Survey. By the end of the semester, enrollment across the four sections was down to 106 students, of which 94 participated in the end of semester MAPS. Despite 94 responses on the post-test, only 79 responses gave consent and were successfully matched to their corresponding identifying pre-test code. Demographic information about the 79 participants for which matched data was acquired and analyzed is presented in the following table.

Table 5

Student Demographic Characteristics

Demographic Characteristic	Number of Respondents		
	Control (n=39)	Experimental (n=40)	TOTAL (n=79)
Gender			
Male	25	21	46
Female	14	19	33
Major			
Business	35	36	71
Undecided	3	4	7
Other	1	0	1
Previous Calculus			
Yes	17	12	29
No	22	28	50

In addition to demographic information, students were asked to provide their intended major and the concentration of their major if applicable. The table below details the different majors that were present among the participants, which included Business and the different concentrations within, non-business, and undecided.

Table 6

Student Major and Concentration

College Major and Concentration	Number of Respondents		
	Control (n=39)	Experimental (n=40)	Total (n=79)
Business	35	36	71
Business Admin.	13	12	26
Marketing	9	6	15
Accounting	2	5	7
Entrepreneurship	5	4	9
Finance	2	4	6
International Bus.	3	4	7
Sales	1	1	2
Non-Business Major	1	0	1
Undecided	3	4	7

Methodology

This project studies the implementation of computational labs and the effects on attitude toward mathematics through a quasi-experimental, multi-method design including an experimental group and a control group. Students' attitudes were quantitatively measured using the Mathematics Attitude and Perceptions Survey. In addition to the quantitative data, student comments were also gathered and analyzed to provide additional insight into students' attitudes and how they perceived the labs as impacting their attitude. Student comments were also analyzed with evaluative coding to gather students' assessment of the course to help guide improvement to better impact attitude.

This study used a multi-method Quan + qual design (Morse, 2003). In this design "the description is primarily from the quantitative data with qualitative description enhancing particular aspects of the study" (Morse, 2003, p. 204).

Description of Intervention

Students in the experimental group completed six computational labs throughout the semester. The labs were completed using Jupyter notebook with Python. The lab activities were used to introduce topics. This structure was used because the labs introduce students to calculus topics using business applications often starting with discrete data and ideas of finite calculus and then extend them to the more traditional continuous approach to calculus, which was covered more in the lecture portion of the course. The labs were distributed electronically as Juypter notebook files or through Google Colab links. In this study, students worked through the beginnings of the labs with the teacher as a class followed by a problem set that involved some of the techniques covered in the beginning to tackle the problems on their own. Students were typically given one and a half weeks to complete the problem set on the lab. During that time, the topics from the lab were expanded on in the lecture portion of the course. Students were also given time each class meeting to work on the lab problems while the instructor circulated.

The first lab assignment was done on the second-class meeting. The first lab allowed students to download and install the appropriate software and set up a folder where they would store their work. The purpose of the first lab was to introduce students to Jupyter notebooks, demonstrate some of the different types of cells they would need throughout the semester (code or markdown), discover different ways to type text in markdown, insert images, use code cells to do simple mathematics operations, create lists and arrays, use simple loops, append elements, and define and plot functions. Students worked through the lab in class on their personal computers as the instructor projected the notebook on the board and worked through it with them. The end of the notebook had a set of four problems that students were to complete on their own, which asked students to implement what they had learned working through the beginning part and referring back to the beginning part to write the appropriate code to complete the tasks. The tasks included navigating to the appropriate type of cell to answer questions about themselves or insert a picture, performing simple mathematics operations, producing specified arrays, printing numbers from the NumPy library, and translating sentences into mathematical expressions, defining these expressions as functions, and plotting these functions on a specified domain and in a specific format. Students were also asked to write a paragraph at the end of the lab reflecting on what they learned, which occurred on all subsequent labs as well. This notebook had many of the basic commands students would need to progress into future labs and students often referred back to this first assignment.

The second lab was focused on functions with its goals including review of linear, quadratic, and exponential functions, relate functions to sequences, and use exponential functions to model situations.

The third lab was titled “Introduction to the Derivative”. In this lab students were tasked with computing limits, slopes and finite differences, understanding the definition of the derivative, interpreting the derivative as slope of tangent line, interpret the derivative as a function, and using Python to symbolically compute derivatives and using these examples to conjecture rules for computing derivatives.

The fourth lab was centered on applications of the derivative. In this lab, students explored applications of derivatives to shapes of curves, finding relative and absolute extrema, and optimization. Students used pre-written code to graph the function and first and second derivatives to make conjectures about the connections between them. Students then learned how to find critical points, describe intervals of increase/decrease and concavity, and determine where there were maxima or minima. Students used this knowledge to analyze sales of smokeless tobacco products, complete examples of the law of diminishing returns, and optimization problems relating to maximizing revenue or profit.

Lab five focused on the study of integration. In this lab students had to conjecture how they would determine area under the curve and were then shown how to approximate the area under the curve using rectangles. They then improved their approximation by increasing the number of rectangles, which lead to the introduction of the definite integral. Students used numerical integration to apply this concept to applications including net change and average value.

The concentration of the sixth and final lab was area between curves. Students created graphs of several curves and made determinations about how they would then find the area between the curves. This was extended to examples of income inequality, consumer and producer surplus, and future value of income streams.

Employing the computational labs creates both content and pedagogical differences between the experimental and control groups, which are discussed below.

Overall, since the intervention was a series of labs, the content between the two groups largely overlapped throughout the semester. Some content was changed to allow time for the labs. To maintain more consistency between the courses, the order in which topics are covered mostly remained the same for the two courses. The time spent on each topic was around the same number of class meetings, but the experimental sections spent some class meetings on the labs of that topic where the control sections had lecture on that topic. The course timelines are shown in Appendix A and excerpts from several labs are in Appendix B.

In addition to content differences between the courses, there were also pedagogical differences. The control sections of Business Calculus in this study were taught using predominantly lecture, which still the dominant style of teaching for Calculus I nationwide (Larsen, Glover, & Melhuish, 2015). Outside of lecture, students in the control course had the opportunity to briefly work with classmates during most class meetings on problem sets in addition to receiving lecture. Students in the control course also completed four projects, which included solving a calculus problem related to business and writing a letter about their findings.

Use of technology was also a pedagogical difference between the groups. This was a distinguishing factor between the control and experimental Business Calculus courses. The control course included the use of a graphing calculator, and the experimental course included the use of Python to facilitate business application. The graphing calculator was allowed in the control Business Calculus classes in this study but was not used to intentionally teach calculus but was used by students for arithmetic computations and graphing. This differed from the pedagogical decision to use technology, specifically Jupyter notebooks with Python, to intentionally teach calculus topics, not simply support students in computation.

Procedures

Approval from the university Institutional Review Board was obtained before the semester began. The described intervention occurred over a traditional fifteen-week semester in four Business Calculus courses all thought by the same instructor. Each of the four sections of the course was a ninety-minute class meeting twice a week. Students self-selected the sections with no prior knowledge as to how the courses were going to be taught. All of the courses had the same instructor, so students were not selecting different sections based on the listed instructor, and all of the courses were listed as the same course number with the identical course description. The only variation in the courses as seen by the students when they were registering for classes was the different times the courses were offered. None of the courses were at extremes of the day with none occurring very early or late in the day. To randomly select which courses were to receive the intervention and which were not to account for being at different times of day, the courses were assigned numbers one through four and then selected with a random number generator. Two sections of the course were taught with traditional, currently used practices in Business Calculus, which includes students learning continuous calculus with some business applications using lecture. Two sections of the course were taught using the intervention, which was the use of computational labs that were used to introduce Business Calculus students to calculus concepts.

Students in both groups took the pre-test Mathematics Attitudes and Perceptions Survey on the first day of the class. The pre-tests were completed via computer, as were the post-tests. These tests were done through Qualtrics, a secure survey software system. The pre-test had a statement of consent, which the students electronically elected to sign (see Appendix C for the electronic MAPS consent statement). In addition to pre-MAPS, students provided a yes/no response to if they had taken a calculus course before, selected their gender, wrote in their major and major concentration, and designated which section of the course in which they were enrolled.

After the MAPS pre-tests were completed, the semester progressed with the control group receiving lecture and the experimental group receiving labs to introduce calculus topics that were then extended in lecture. For the most part, the two groups covered material in the same order and around the same time in the semester. The two groups also took tests at the same time, were given identical tests throughout the semester, and took the same final exam. The two groups also had similar grading scales with the only difference being the control group had projects where the experimental groups had labs, but both carried the same weight in the gradebook.

Through the six lab assignments, students were asked on each one to reflect on what they learned in the lab. Each lab concluded with the following statement, "Think about what you learned and write about it! Write a short paragraph about what you learned in this notebook. This needs to be a thoughtful, reflective paragraph. There should be reflection on the mathematics content you learned. You may want to review the goals of this notebook (listed at the top)." This was a teacher-generated question that was originally intended solely for data on conceptual understanding. Students used this solicitation to provide what they learned mathematically but also took the opportunity to reflect on the technology and how they felt about doing these labs. Every student that signed a consent form had his or her comments saved verbatim at the end of the semester. These student comments from the end of the labs were analyzed after the semester ended. The student comments were analyzed using initial coding with in vivo codes (Saldaña, 2016), a second cycle of coding was done with pattern coding (Saldaña, 2016), and then themes emerged from these categories. Separately, student comments were analyzed through evaluative coding to make determinations about how students evaluated the labs and what recommendations they would make for future iterations.

On the final-exam day, students in both groups took the post-MAPS test. The post-MAPS was given on final-exam day when all students must be present. This was done to avoid issues of missing data. Final grades were recorded at the end of the semester for comparison. Students were given a written consent form on the final-exam day (see Appendix D). A helper administrator gathered this consent rather than the instructor-researcher to maintain more anonymity and to mitigate any pressure the students might have felt if the instructor collected consent.

For the pre- and post-MAPS students chose a code, following Self-Generated Identification Code procedures (Yurek et al., 2008), to identify themselves. The code was consistent for each student, which allowed for information from the different sources to be linked when the semester was complete. On the final exam day, students wrote their name and code on a notecard when written consent was explained and gathered so that all data sources could be matched. The helper administrator collected the codes and matched the data.

Instrument. The tool used to measure students' attitudes toward mathematics in this study was the Mathematics Attitudes and Perceptions Survey. The MAPS is a thirty-two question survey with one filter question on a five-point Likert scale. The MAPS was adapted from the expert/novice instruments for undergraduate STEM education from the group of Colorado Learning Attitudes about Science Surveys (Code et al., 2016). Factor analysis helped to arrive at seven categories with which the creators then attached names to the factors by matching themes with existing constructs in literature (Code et al., 2016), which include growth mindset, real world, confidence, interest, persistence, sense making, and answers (Code et al., 2016). The MAPS has Cronbach's alpha value of 0.87 for the whole instrument, without the filter statement (Code et al., 2016). This value indicated good reliability using guidelines of Cohen, Manion, and Morrison (2009), which would indicate that it has an alpha level that indicates that the instrument is highly reliable. The creators of MAPS attempted to establish concurrent validity through patterns in course levels, patterns in correlations with course grades, and comparing findings to results from the Colorado Learning Attitudes about Science Surveys (abbreviated: CLASS) (Code et al., 2016). Concurrent validity, a type of criterion-related validity, provides a measure as to how well the new instrument compares to a previously well-established instrument or instrument measuring the same construct and does so by having the data gathered from the one instrument correlated highly to data from another instrument (Cohen, Manion, & Morrison, 2009). At this time, there are no further psychometric studies available on the MAPS.

The MAPS shares similar statements to and aspects of development of these expert/novice surveys (Code & Maciejewski, 2017). The CLASS instruments' statements were written to be as "clear and concise as possible and suitable for use in a wide variety of physics courses," with students responding on a five-point Likert scale (Adams et al., 2006, p. 010101-1). CLASS "was designed to address a wider variety of issues that educators consider important aspects of learning physics" and "the wording of each statement was carefully constructed and tested to be clear and concise and subject to only a single interpretation by both a broad population of students and a range of experts," which "make the survey suitable for use in many different courses covering a range of levels, and also allows most of the statements to be readily adapted for use in other sciences" (Adams et al., 2006, p. 010101-2) on which MAPS then built for mathematics. Wording for CLASS statements was created by listening to and writing down statements that students said in interviews to word the statements in ways students will easily understand and represent their ideas about wording (Adams et al., 2006). Comparing the statements on the CLASS-Phys (Adams et al., 2006) with statements on MAPS (Code et al., 2016), it can be seen that the wording is similar for many statements but the discipline is changed.

The Mathematics Attitudes and Perceptions Survey is a fairly new instrument and has not been as widely used as other mathematics attitudes surveys. The MAPS, unlike some other mathematics attitude surveys, was developed specifically for undergraduate mathematics students and developed with input from mathematicians about what constitutes expert-like thinking (Code et al., 2016). The MAPS was selected for this study in part because of its focus on undergraduate students. The MAPS is shorter than other mathematics attitudes instruments and typically takes less than ten minutes to complete (Code et al., 2016), thus reducing issues of test fatigue that could be present on longer surveys. The authors proposed that MAPS can "usefully be employed in any undergraduate mathematics education setting where student beliefs and perceptions are suspected to play a role" (Code et al., 2016, p. 933), which is indicative of why it was selected for use in this study since students in Business Calculus are thought to have relatively negative attitudes toward mathematics (Depaolo & Mclaren, 2006; Liang & Pan, 2009). The authors of this instrument kept it intentionally short and intend for it to be used as a pre-post instrument (Code et al., 2016).

MAPS is also multi-dimensional and can be analyzed as a complete expert index and can also be analyzed by category. The multi-dimensionality was of interest in this study. The results from MAPS can be analyzed as a whole but can also be broken down into seven specific categories to see if there is change in sub-categories individually. While there may be no positive effect on the expert-like disposition as a whole, based on previous findings in mathematics (Code et al., 2016) there may be effects on sub-categories. Of particular interest was the real world category. Previous research (Gordon, 1979; Hoffman, 1989; Ralston, 1984) has revealed that using a finite approach to calculus with computers can make mathematics more real and relevant for students as it can be more concrete and can include less concocted examples; in addition, students tend to support the use of technology in the mathematics classroom (Heid et al., 2002; Zevenbergen, 2004), and that technology can make mathematics also more real and relevant, so being introduced to some calculus topics using the labs may enhance student beliefs that mathematics is more connected to outside of the classroom than those that do not participate in the labs.

Because of its recent development, the Mathematics Attitudes and Perceptions Survey has not been widely used in published research yet, which is a limitation of this instrument. Maciejewski (2016) uses this instrument, along with the Calculus Concept Inventory, to study flipping a calculus classroom, and Code et al. (2016) use it across numerous undergraduate mathematics classes during the development.

Additional Information Gathered. In addition to the Mathematics Attitudes and Perceptions Survey, students' gender, college major, and previous calculus exposure were collected.

Previous Calculus Exposure. An important piece of descriptive data was whether students have taken a calculus course before or not. Students that are further along in their mathematics careers tend to have better attitudes toward math (Code et al., 2016), and projects on computational tools from the 1990s show mixed results for those with previous calculus exposure (Tucker, 1990). Maciejewski (2016) using MAPS found that higher ability groups tended to have more positive expertise scores. Information on prior calculus exposure, as previously discussed, was used to compare the two groups at the beginning of the semester to determine if there were significant differences between the groups in the count of students that have previously taken calculus.

Gender. Literature on STEM students highlights that there may be differences based on gender and attitude. Ellis et al. (2016) find that women are 1.5 times more likely to drop out of

STEM majors after taking calculus than men are, and these findings remain true after controlling for preparedness academically, career intentions, and instructional methods (Ellis et al., 2016). With this dramatic number of women changing majors after calculus, it was important to also look at the intervention's impact on gender. In a large study of college calculus students, Sonnert and Sadler (2015) found that overall males scored significantly higher than females in attitude toward mathematics composite score. Code and Maciejewski (2017) reference that Maciejewski found that male students overall reported higher attitudes in most categories on the MAPS including confidence in their mathematical abilities. Because gender has been related to attitude, it was collected and analyzed as well to determine if there were gender differences.

Major. The intervention took place in a Business Calculus course, which is required for all business majors at the university where the study took place, however the class sometimes has students from other majors since it can fulfill a general education requirement for some majors and some students place in that class based on SAT or ACT score. Because of this, students' college major and major concentration were collected. Sonnert and Sadler (2015) also collected information on students' intended career path, which was done in this study as intended major, as they acknowledged that students' career path may be related toward their attitude toward mathematics. This provided information on how the intervention worked based on a student's major and also allowed for the data to be analyzed based on what a student's intended business concentration was. Pritchard, Potter, and Saccucci (2004), in studying business students and their basic algebraic skills, found that students with higher computational and algebraic skills chose to major in more quantitatively focused business concentrations such as accounting or finance while students with lower scores tended to select a concentration or major in less mathematically focused ones such as management or marketing, and some business students choose less quantitatively focused concentrations because they perceive them to have less demanding quantitative requirements (Pritchard, Potter, & Saccucci, 2004). There may be differences in students' attitudes from more quantitatively focused concentrations than less quantitatively focused counterparts. It was of interest to see if students from more quantitatively focused concentrations or majors differ from their less quantitatively focused counterparts in achievement.

Results

To address the research question as to what are the effects of computational labs on students' attitudes toward mathematics, MAPS scores and demographic information were collected and input into SPSS to compile and analyze the data. Independent samples *t*-tests, paired samples *t*-tests, chi-square test, ANCOVA, and correlation were used to analyze the results. Students' comments from the end of the labs were analyzed as well looking for reflection on how the labs affected students' attitudes, students' evaluation of and recommendations for the labs, and common themes that resulted from the students' experiences with computational labs in the Business Calculus course.

Group Equivalence at the Beginning of the Semester

On the pre-Mathematics Attitudes and Perceptions Survey on the first day of the semester, students answered whether or not they had taken calculus before. Responses to this question were analyzed using a chi-square test. This was used to compare the two groups at the beginning of the semester to determine if there were significant differences between the groups in count of students that have previously taken calculus (as in Schroeder, McGivney-Burelle, & Xue, 2015). A chi-squared test was chosen for this analysis because chi-squared tests can be used to determine if there are significant differences between frequencies in the two groups.

Chi-square tests can be used to determine statistical independence in the frequency distribution of a variable is the same for all levels of some other variable (Chi-square independence test – What and why?, n.d.), with calculus or no calculus frequency being compared between the experimental and the control group. This test was also appropriate because the observations were independent and all expected frequencies met the requirements (Chi-square independence test- What and why?, n.d.). The chi-square test of independence was calculated comparing the frequency of students that had previously taken calculus in the experimental and control groups and revealed no statistically significant differences between the groups, $\chi^2(1, N=79)=1.570$, $p=0.219$. There was also no statistically significant difference between the groups on gender, $\chi^2(1, N=79)=1.093$, $p=0.296$, or college major, $\chi^2(1, N=79)=1.144$, $p=0.564$, as well. Using an independent samples *t*-test to compare the two groups on the pre-MAPS assessment at the start of the semester, it was determined there was also no statistically significant difference between the control group ($M=42.216$, $SD=19.20$) and the experimental group ($M=41.518$, $SD=20.035$), $t(77)=-0.159$ $p=0.874$. Based on responses to the pre-MAPS and across other variables, students in the two groups were not significantly different at the beginning of the semester.

Differences from Pre- to Post-MAPS

For the Mathematics Attitudes and Perceptions Survey, respondents are scored based on if they align with the expert response. For each response that a student answers in the direction of the expert he or she receives positive one point and for each response in the opposite direction of the expert he or she receives zero points. This scoring creates an overall expertise index that is the average score for all questions. The subscale or categories are scored the same way with the average calculated from the total number of questions in that category. The expertise scores are reported here as percentages.

To determine differences in Mathematics Attitudes and Perceptions Survey expertise scores between the two groups, a two-samples *t*-test was performed for mean difference between pre-test and post-test expertise scores. A variable was computed for the change in attitude, as measured by the MAPS, from pre- to post-assessment by subtracting the pre-MAPS percentage from the post-MAPS percentage. An independent samples *t*-test was selected for analysis in part because the samples are independent. No person was in both groups and students do not have the option to attend a different section of the course, so the values from one population were not related or linked to values from the other population as needed for an independent samples *t*-test (Bowen, 2016). If the size of each sample is greater than or equal to thirty, the *t*-test for independent groups can be used without much error even if there are moderate violations in the normality or equal variance assumptions (Pagano, 2004). The sample size for each group was over thirty, so the normality assumption should be met. To test homogeneity of variances a Levene's *F* Test for Equality of Variance was run, $p>0.05$. The null hypothesis was no change in expertise score and was tested against a two-sided alternative. As Henrich et al. (2016) detail using a two-tailed test is appropriate because this approach allows for detection of significance of improvement while also allowing for the possibility of detecting negative effect if one did occur. Since previous MAPS data and CLASS data has shown a decrease in expertise score over a semester of instruction, a test that allowed for detecting negative effects was necessary.

With the independent samples *t*-test run, there was no significant difference in Mathematics Attitudes and Perceptions Survey pre-post test difference for the group without labs (M= -3.205, SD=18.662) and the group with labs (M= -2.054, SD=20.522), $t(77)=0.261$, $p=0.795$. There was minimal change between pre-MAPS and post-MAPS for both of the groups, both of which, however, had very large standard deviations. Both differences were somewhat negative with the control group of slightly higher magnitude, however, this was not unexpected as previous research indicates that there is often a decrease in attitude when students are taught using an innovative method they have not experienced before (Sonnert et al., 2014) and even over the semester in general (Sonert & Sadler, 2015). Overall, the experimental group had a smaller decline in attitude over the semester.

Table 7

Control and Experimental Group MAPS Statistics

Statistic	Group			
	Control		Experimental	
	Pre-MAPS	Post-MAPS	Pre-MAPS	Post-MAPS
n	39	39	40	40
Mean Percentage	42.216	39.011	41.518	39.464
Median	42.857	32.123	41.071	39.286
Standard Deviation	19.20	42.615	20.035	21.926
Minimum	0	0	3.75	0
Maximum	82.14	89.29	78.57	92.86

Maciejewski (2016) compares MAPS scores between the experimental and control groups by comparing pre-test scores, finding no significant difference between the groups, and then comparing post-test scores of the groups using a two-sample *t*-test finding that the experimental group had higher expertise scores that were statistically significant. This was done in this study as well. With the groups being roughly equivalent at the beginning of the semester as reported in the previous section, post-MAPS scores were compared as well using an independent samples *t*-test. There was no statistically significant difference between the two groups, $t(77)=0.086$, $p=0.932$. Neither group had significant change in attitude over the semester.

Using a paired samples *t*-test for only the experimental group also revealed no significant differences between the pre- to post-MAPS, $t(39)=0.633$, $p=0.531$; the same was determined of the control group, $t(38)=1.073$, $p=0.290$. For the experimental group MAPS percent difference was significantly correlated to post-MAPS percentage $r=0.566$ and for the control group $r=0.650$, so for both groups in general as the change from pre- to post-MAPS got larger the post-MAPS expertise score also went up.

MAPS Categories Analysis

The individual categories on MAPS scores were compared as well. The two groups were not significantly different at the beginning of the semester in any categories of the MAPS assessed through independent samples *t*-tests. Comparing percent difference for the two groups from beginning to end of the semester was performed through independent samples *t*-tests on all categories, shown in Table 8.

Table 8

MAPS Construct Differences

MAPS Construct	Labs	N	Percent Difference	Standard Deviation	*df*	*t*	*p*
Growth Mindset	Yes	40	0.625	34.197	77	0.614	0.541
	No	39	-4.487	39.683			
Real World	Yes	40	-4.375	36.641	77	-0.868	0.388
	No	39	2.564	34.314			
Confidence	Yes	40	7.500	40.112	77	0.891	0.376
	No	39	0	34.412			
Interest	Yes	40	0	31.123	77	0.239	0.812
	No	39	-1.709	32.398			
Persistence	Yes	40	-3.750	29.171	77	-1.088	0.280
	No	39	3.205	27.613			
Sense Making	Yes	40	-7.50	25.894	77	0.769	0.444
	No	39	-12.308	29.599			
Answers	Yes	40	-2.08	30.473	77	0.837	0.405
	No	39	-7.692	29.081			

Individual categories were also analyzed comparing differences between pre- and post-MAPS scores using paired-sample *t*-tests, considering change in individual categories for the students in the each group. This is similar to analyzing changes in pre- and post-test attitude categories as in Ng et al. (2005, p. 66). For the control group, the only statistically significant pair was the sense making category, $t(38)=2.597$, $p=0.013$. For the experimental group none of the categories were statistically significant using a paired samples *t*-test at the 0.05 level. At the 0.1 level, sense making was the only significant difference, $t(39)=1.832$, $p=0.075$. When comparing only the post-MAPS scores using a two-samples *t*-test, none of the categories were significantly different.

Pearson correlation coefficients were also computed between the seven subscales and the overall expertise score since the categories may be related to each other and similarly to attitude subscales in Ng et al. (2005, p. 67). Pearson's correlation assesses linear relationships between two continuous values, which these MAPS construct scores would be. The correlations between categories and post-MAPS expertise score for the experimental group are shown in Table 9. Very similar correlations were found in the control group, which are shown in Table 10.

Table 9

MAPS Constructs Correlations for the Experimental Group

Construct	Growth Mindset	Real World	Confidence	Interest	Persistence	Sense Making	Answers	Post-MAPS Expertise
Growth Mindset	--	0.354*	0.506**	0.441**	0.438**	0.565**	0.231	0.655**
Real World	--	--	0.383**	0.473**	0.580*	0.360*	0.236	0.687**
Confidence	--	--	--	0.581**	0.551**	0.451	0.264	0.769**
Interest	--	--	--	--	0.362*	0.507**	0.337*	0.729*
Persistence	--	--	--		--	0.344*	0.251	0.724**
Sense Making						--	0.414**	0.744*
Answers							--	0.582**

**Correlation is significant at the 0.01 level (2-tailed)

*Correlation is significant at the 0.05 level (2-tailed)

Table 10

MAPS Constructs Correlations for the Control Group

Construct	Growth Mindset	Real World	Confidence	Interest	Persistence	Sense Making	Answers	Post-MAPS Expertise
Growth Mindset	--	0.419**	0.635**	0.353**	0.537**	0.396**	0.481	0.709**
Real World	--	--	0.407**	0.491**	0.292	0.586**	0.104	0.640**
Confidence	--	--	--	0.618**	0.796**	0.504**	0.306	0.803**
Interest	--	--	--	--	0.522* *	0.601**	0.204	0.735**
Persistence	--	--	--		--	0.548*	0.478**	0.806**
Sense Making						--	0.396*	0.823**
Answers							--	0.595**

**Correlation is significant at the 0.01 level (2-tailed)

*Correlation is significant at the 0.05 level (2-tailed)

As displayed in Table 9, all correlations were positive and numerous were significant at the 0.05 level and some at the 0.01 level. This implies that in general as students increased in one category they also went up in the others. For both groups, all categories were statistically significantly correlated to the post-MAPS expertise score all at the 0.01 level. For the experimental group the strongest correlations were between confidence and interest (r=0.581), persistence and real world (r=0.580), and confidence and persistence (r=0.551). For the control group the strongest correlations were between confidence and persistence (r=0.796) and interest and confidence (r=0.618).

Controlling for Previous Calculus Exposure. A piece of descriptive data that was collected was whether students have taken a calculus course before or not. Information on prior calculus exposure was used to compare the two groups at the beginning of the semester and found no significant differences. Previous MAPS data has revealed that students that have taken more advanced math classes tend to have more positive attitudes toward mathematics (Code et al., 2016). As seen in the description of the participants, most students in this study had not taken calculus, at any level before; in the control sections of the course 56.4% of students had not taken calculus before and in the experimental sections 70.7% of students had not taken calculus before.

When controlling for previous calculus exposure using an ANCOVA with previous calculus exposure as the covariate, the difference in post-MAPS expertise score between the two groups was not significant, $F(1,76)=0.205$, $p=0.652$. In both groups there were no significant differences between those with previous calculus exposure and those without on overall pre-MAPS percentage or in any of the categories.

In analyzing only the experimental group for the effects of previous calculus exposure, it can be seen that the MAPS expertise score at the end of the semester was statistically significantly higher for students that had taken calculus before (*N*=12, *M*=57.123, *SD*=23.739) than those that had not taken calculus (*N*=28, *M*=31.888, *SD*=16.321), as assessed by an independent samples *t*-test, *t*(38)=3.899, *p*= 0.00, *d*=1.239. This was not seen in the control group as students that had taken calculus before did have higher post-MAPS scores (*N*=15, *M*=44.524, *SD*=31.073) than those that had not taken calculus before (*N*=24, *M*=35.566, *SD*=19.912) but was not statistically significant, *t*(37)=1.10, *p*=0.278. In the experimental group those with calculus (*M*=8.036, *SD*=13.287) had statistically significant more growth from pre to post test than the group without calculus (*M*=-6.378, *SD*=21.721), *t*(38)=2.125, *p*=0.040, *d*=0.801. This was not seen for the control group, where those with calculus (*M*=0.00, *SD*=18.211) and those without (*M*=-5.208, *SD*=19.043) were not statistically significantly different, *t*(37)=0.845, *p*=0.404.

For the experimental group, those with calculus had more improvement from pre- to post-MAPS in every category with growth mindset (previous calculus: *M*=29.167 *SD*=39.648, no calculus: *M*= -1.786, *SD*=37.223, *t*(38)=2.364, *p*=0.023, *d*=0.805) and sense making (previous calculus: *M*=5.00 *SD*=22.764, no calculus: *M*=-12.857, *SD*=25.655, *t*(38)=2.082, *p*=0.044, *d*=0.736) being statistically different. Whereas for the control group, the real world category was the only statistically significant difference between those previously taken calculus (*M*=16.667 *SD*=33.630) and those that had not (*M*=-6.25 *SD*=32.345), *t*(37)=2.012, *p*=0.041, *d*=0.695.

Additional Variables Analysis. There were not statistically significant differences between the two groups on post-MAPS percentage on neither gender nor college major. Using an ANCOVA, when controlling for gender the results were not statistically different between the two groups, $F(1,76)=0.007$, $p=0.933$. Similar results occur when considering difference from pre- to post-MAPS score. At the beginning of the semester there were no significant differences in the pre-MAPS score of females compared to males in either group. Considering only the experimental group, significant differences in post-MAPS scores or in difference from pre- to post-MAPS based on gender were not found using an independent samples *t*-test, $t(38)=-1.035$, $p=0.307$ and $t(38)=-1.101$, $p=0.278$ respectively, however, females did have a higher mean percentages post-MAPS score but with more variability ($N=21$, $M=43.233$, $SD=27.611$) than males ($N=19$, $M=36.054$, $SD=15.021$). Females ($M=44.388$, $SD=23.006$) also had higher mean percentage post-MAPS expertise score than males ($M=36.00$, $SD=25.711$) in the control group as well. There were not statistically significant differences in percent changes from beginning of the semester to end of the semester when controlling for gender for the experimental group or control group.

While there were not statistically significant differences in overall post-MAPS expertise score between the genders in the experimental group, analysis on the numerous categories of the MAPS was also performed. None of the differences from pre- to post-assessment were significantly different for males and females. Comparing only post-scores from the experimental group, of note were the results that females had higher mean post-MAPS percentage in all categories. None of these were significant at the 0.05 level and only interest was significant at the 0.1 level, (Females: $N=19$, $M=36.842$, $SD=42.882$; Males: $N=21$, $M=15.873$, $SD=24.987$) $t(38)=-1.912$, $p=0.063$, $d=0.598$. Also the growth in the interest category was statistically significant, (Females: $N=21$, $M=10.526$, $SD=31.530$; Males: $N=21$, $M= -9.524$, $SD=28.172$), $t(38)=-2.124$, $p=0.040$, $d=0.671$. For males and females in the control group, no categories revealed difference in post-MAPS score or growth.

Controlling for college major also revealed that there were not statistically significant differences between the two groups, tested using an ANCOVA, $F(1,76)=0.023$, $p=0.880$). There were no significant differences between the group with labs and the one without, when controlling for concentration as well also using an ANCOVA, $F(1,76)= 0.004$, $p=0.950$.

Final Grades. The two groups under consideration had very similar final grades. The control group had slightly higher final averages ($M=87.64$, $SD=11.945$) than the experimental group ($M=84.726$, $SD=10.458$). Previous research using the MAPS found that overall expertise index is correlated with course grade (Code et al., 2016, p. 930), which was found in this study. Pearson's correlation evaluates the linear trend between continuous variables, which course grades as percentages and MAPS score are. For the experimental group the pre-MAPS expertise score was correlated with final grade ($r=0.575$) and for the control group, however it was not ($r=0.044$). For the experimental group, the post-MAPS score and final course grade were correlated with $r=0.634$ and for the control group post-MAPS score and final course grade were also statistically significantly correlated, $r=0.440$, however not as strongly; both were shown to be significant at the 0.01 level, which is similar to results in other uses of the MAPS.

Qualitative Results from Student Comments

To understand the effects of the computational labs on students' attitudes and what students' experiences were with labs, students' comments were analyzed. Students were asked to reflect on the mathematics content of the labs, however, in addition to that they used this solicitation to comment on how they felt about the lab, how the labs did or did not help them learn, and even how the labs could be made better. All of the consenting students' comments were inspected through several analytic passes through the data. The comments were coded using initial coding with in vivo coding using participants' own language as codes (Saldaña, 2016). A second cycle of coding was done using pattern coding to group the segments of data from the first cycle into a smaller number of categories (Saldaña, 2016). From these categories, several themes emerged that expressed elements of students' attitudes that were affected by the computational labs.

Student comments that were of the evaluative nature were analyzed through evaluation coding, which assigns judgments about the merit, worth, or significance of programs (Rallis & Rossman, 2003) and can be used for the further development of the computational labs. These comments are discussed separately. Comments of the evaluative nature provide valuable insight into how students believed that the labs benefited or negatively impacted their attitudes and ways in which they believe the labs could be improved for more impact.

Theme 1: Real World Connections. Throughout student comments were reflections on and mentions of real world connections, which was the most prominent theme. Words such as real world, real world applications, outside of the classroom, real life stuff, and business problems occurred throughout the labs. The theme of real world connections was echoed in two ways. One way was that students reflected on being able to connect the mathematical concepts to real world scenarios or applications. The other way was that some students felt that the way they were problem solving was more realistic, authentic, or even how they may approach problems when they are in the working world. Over 50% of the comments analyzed mentioned something about "real" and many of the others, while not using the word real, stated how they could use the calculus concept in some other context and talked about scenarios such as cost and revenue, supply and demand, or net change.

Real Applications. Mention of real applications, real life scenarios, or explicitly stating these "real" concepts ran throughout many students' reflections. These comments were found throughout labs one through six and were typically stated with affirmative wording. An example of this was, "Overall, I really enjoyed this lab because it tied in several aspects of the real world and emphasized the significance of math within the field of business." Another example was, "This lab was very useful because it showed how a real life business world would use math in their everyday life, and it is very interesting for me because I am a business major." In response to lab three, a student commented:

> The truth of the matter is: I am not really fond of math, but these labs help me want to learn more. I have always been a believer that a lot of these concepts will never be used after graduation. I am starting to change my mind!

Another student remarked:

> Through this notebook, I have been able to actually see how mathematical models and exponential equations are actually applicable to real world situations since usually random scenario word problems in a textbook don't do a good job of illustrating these things for me.

In response to students being asked to import stock trends from a company of their choice and simply use their knowledge of slope to roughly determine the support and resistance lines and determine if they thought they would buy the stock or not, a student commented, "Seeing how stocks can be analyzed with the help of this program makes the math feel like it is not pointless and I might actually use this math." Many students also commented both in class and in their lab reflections that they liked that some of what we were doing connected directly to content they were learning in some of their business classes. One student stated:

> Once again, I found this lab to be extremely helpful in 'driving home' some of the key concepts that we are working on in the classroom. Specifically, I learned how to graph

> and predict stocks using elements of coding (which I never thought I would learn in business calc)…This lab clearly was designed to challenge students into looking at concepts from class from a different, real-life perspective. I really like doing these labs and trying to figure out how functions, graphs, exponents and other 'class material' will be used after college in the 'business world.'

The quotes from students' comments demonstrate that they were clearly able to see how the mathematics they were learning in class was meaningful and actually connected to the "real world". Their comments convey an attitude toward math that shows that they can clearly see some of the value and power of mathematics beyond what they learn in the classroom. Comments on "real world" were almost exclusively expressed in a positive way and exhibit a positive impact on attitude toward mathematics that the labs had. Students, using more positive language, made statements such as "I liked" or "I changed my mind about". There were no student comments that explicitly stated that they could not see the relevance of the content, but not all comments remarked on the perceived realness so some students may not have felt that the labs were beneficial in showing them the relevance. Since some students did not comment on the real applications and appreciating them, some may not have viewed the labs as relevant to business applications as others did.

Being able to articulate that they felt it was real shows a positive benefit on understanding that mathematics has connections to other domains and positive effects on attitude. Seeing connections may "improve their motivation to study mathematics and have longer-term effects on their academic achievement in mathematics" (Code et. al, 2016, p. 923). Research has suggested that authentic situations, which students find relevant, are likely to positively change students' attitudes toward math (Simonson & Maushak, 2001). Tobias and

Weissbrod (1980) find that focusing on concepts and creating meaning and relevance improves students' motivation to learn mathematics. Students were afforded many opportunities to see mathematics in relevant context of business. Students explicitly articulated that the opportunity to interact with the labs helped them see real connections and how some of what they learned could be used outside the classroom. This shows that for many students there was a positive impact on how they viewed mathematics as real.

An Authentic Way of Doing. Some students also felt that the labs gave them a more authentic or "real" way of doing mathematics. A student commented on the first lab that potentially learning more about using a computer could be a valuable skill and connect to his future stating:

> This coding thing is going to be hard, but I am excited to learn a new skill. Especially one that might be valuable since everything seems to be done on the computer now. This might actually be helpful later on.

In the second lab of the semester a student stated, "I learned there is more to math than just learning how to do problems on paper."

Another student, found benefit in this different way of solving problems and learning a skill that may be useful in her future career, stated the belief, "If I continue to practice this, I may actually be able to use it in a job in the business world one day." As the semester progressed this same student was observed using Jupyter notebooks almost exclusively when tackling homework problems not from the labs. When she was asked why, she explained that she thought they made the work quicker, easier, and more like she would do if she were solving a problem outside of her math class, which demonstrated that she felt that using these tools to perform mathematics computations and problem solve was more realistic to how she would approach problems beyond the classroom walls.

There were a few comments that expressed that students did not appreciate this new way of solving mathematics problems and could not understand why this was a requirement of the course. An example of this is, “This isn't a coding class and it is just making kids struggle that much more. Yes I do admit that I can see the helpfulness of learning this program, I just think it should be offered in a different class.” Another instance when this sentiment occurred was a student stating, “I’m not majoring in anything that requires coding, so I cannot understand why these labs were necessary in this class.” Comments, such as these, reflect that for some students the new, authentic way of problem solving did not benefit their attitude and perhaps could have negatively impacted their attitude of how they felt mathematics was real or not.

For some students their comments reflect that the labs created a seemingly more authentic way of doing mathematics and afforded them different ways to problem solve. Leaders in mathematics education call for technology in business mathematics to allow students to use realistic data, use technology as an analytic tool, and encourage alternative approaches to solve problems in part because “technology has revolutionized the way in which business is practiced” which has changed what students of today must be able to do and understand (Lamoureux, Beach, & Hallet, 2000); these labs seem to have done that for many based on students’ comments. Students’ comments demonstrate that at least some did leave the class connecting the technology and mathematics as ways to solve problems that they might encounter outside of the classroom. Fostering this “real” way of problem solving may influence students to view mathematics as more relevant in contexts applicable to their future careers and the world external to the classroom.

Theme 2: Persistence. Examples of persistence occurred throughout student comments. Persistence is defined as "firm or obstinate continuance in a course of action in spite of difficulty or opposition" (Lexico, n.d.). In analyzing students' reflections it can be seen that completing the labs using technology completely new to most of them in conjunction with learning concepts of calculus, demanded a great deal from these students but in facing these challenges they continued to work through the obstacles and challenges. Their comments reflect that many of them acknowledged that the labs were indeed a challenge but with perseverance and work they were able to get through the task. Numerous student comments demonstrate this such as:

- "I worked on this lab for over 12 hours. I doubt you meant for it to take me that long, but it did. I just kept working to get it. One of my estimates was still a little off. I am slowly getting the hang of this. Knowing I have five more of these labs I think I can do this."
- "Even though I was extremely frustrated when I was beginning, I kept trying to get through it."
- "However, I also learned how to overcome my frustration when dealing with something that is new to me. My favorite part of this has been going out of my comfort zone and trying something new, despite the fact that it was very challenging."
- "One thing that I need to work on is to not get frustrated at a single problem and then give up and move on, but to stick with it and try and figure out why it isn't working."
- "At first it was very challenging and I wasn't sure I would be able to finish it. After toying around with it and spending a lot of time on it, most of it was okay. I think in

the future I can better my work on the lab by starting right away so I know what I am doing wrong and figuring out why for future labs"

- "These are hard but I might be getting better at them. Knowing I have four more of these, I think I can do them."
- "Throughout working through this notebook, I struggled in some areas but I learned to overcome my hardships with patience and diligence."
- "It also taught me patience and persistence."
- "This was very hard but also intriguing and interesting. I need to be patient and review my work. But I look forward to improving my skills."
- "This lab was difficult and I tried my best but I feel like I learned a lot out of it."

Many of the students that remarked on being frustrated but demonstrating a willingness to try made statements that got more positive as the semester went on. Running throughout the comments and reviewing comments from lab one through lab six, the comments begin by mentions of students feeling intimidated and this being a stressful process, however, as they progressed through the semester the comments became more positive as they used the program more and students seemed more confident in their abilities. In comments on labs that occurred later in the semester references to frustration were replaced with sentiments of "I'm getting better at these" and "feeling more comfortable doing math this way." Students' comments reflect feeling more confident in their abilities with the labs as they increased their practice. Another student demonstrated her growth in persistence when faced with an error message. This student was working through a problem in my office and when she ran her cell an error message popped up. I had skimmed her code quickly and missed the syntax error, so when the error message appeared I reacted with an "uh-oh, let's see what's wrong." The student chuckled and said:

> Oh these things don't bother me any more. In the beginning they made me so mad and like panic as to what might be wrong, but now it's like they just give me an opportunity to fix what I did wrong and they don't scare me anymore. I can learn from my error and fix it.

Not all comments on persistence were positive however; some students were resistant and put off entirely. Many of the students that echoed these sentiments remained very immobile in their willingness to try throughout the semester. A student remarked on lab one, "After doing this I realize I do not like this and I am not good at this either." This same student later said, "I learned that even if I ask for help I am not very good at this type of thing" and in the next to last lab stated, "What I have learned is that I am still terrible at remembering how to do math." This student's comments reflect both a fixed mindset in learning how to use Jupyter notebooks but also in mathematics ability. A different student commented, "I definitely learned I cannot do this, and this coding thing is going to be the most difficult part of the course."

Some students that expressed comments of negative feelings toward the technology did not present it in a way that demonstrates persistence such as:

- "These labs are extremely stressful."
- "Math is not my strong suit and to be honest I don't feel great."
- "Doing this was an extreme challenge, it was not easy to follow step by step what you had to put and it was hard to make sure everything was exactly right."
- "I learned how bad my computer skills are and that coding is like trying to read another language."
- "I really don't know what to do but I'm trying it mainly through trial and error."

- "Other than that I really have no idea what I am doing on here so I really need to get help on the next one so I don't fail the class."

Student comments, such as these, show that for some students the labs and calculus were extreme challenges and did not help them learn to push through challenges but instead reinforced their perceptions of being "bad" at something or encouraged them to easily give up.

Students that were more open to the idea of trying something new and persisting through the challenges they faced kept this throughout the semester and their comments got more positive as the semester progressed. Students, however, that were immediately opposed to the challenge and made statements aligned with a fixed mindset remained negative and stagnant in their feelings throughout the semester.

Persistence and pushing through challenges is a valuable trait and something we wish for all students to have. Perseverance is a very beneficial characteristic for success in mathematics (Schwartz, 2005). Persistence is also a desirable workplace trait and is listed as one of eight secrets of success (St. John, 2005). Many students reflected that this experience demanded persistence of them and hopefully by developing this they will carry this on through out their future. Others, however, remained opposed to persisting through challenges, and the labs did not positively affect this.

Analysis through Evaluative Coding. The themes discussed above represent important impacts these labs had on students' attitudes. In addition to the themes previously presented, student comments highlighted several other key takeaways that may shed light on how these labs affected their attitudes and recommendations students had that could potentially benefit students' attitudes in the future. Comments of this nature were analyzed thorough evaluation coding, which assigns judgments about the merit, worth, or significance of programs (Rallis & Rossman, 2003). According to Rallis and Rossman (2003) evaluation data can describe, compare, and predict. Describing "focuses on the patterned observations or participant responses of attributes and details that assess quality. Comparison explores how the program measures up to a standard or ideal. Prediction provides recommendation for change, if needed, and how those changes might be implemented" (Saldaña, 2016, p. 141). The perceptions revealed through the evaluative coding provide valuable insight into developing new assignments and structuring the course in the future.

Some general comments that students made about the course overall reflect that there were truly mixed feelings and mixed evaluations of how the labs affected students. These comments ranged from loving the labs, the labs sparking greater interest, enjoying learning about coding to do mathematics, and liking the variety of ways to learn and practice, to the negative end of the spectrum with comments of the labs being insurmountable obstacles, the labs making the class too challenging and time consuming, the labs causing much frustration, and the labs being overwhelming and unfair. Besides these mixed general remarks on labs through analysis of students' evaluative comments, several prominent themes emerged as to how the labs and course structure could be improved to more positively influence students attitudes.

Feeling Like More Work. Students articulated in their comments that they felt like the labs added more work and added extra time demands; this also came out in discussion with them. Students felt as if the labs added "too much work" and thought that more was being expected of them than was of students in the control sections of the course. Students seemed to fail to realize that because they had to complete the computational labs they had less traditional homework; they also seemed to think that so much more was required of them than the other sections even though the other sections had more homework and projects. Despite the fact that this was specifically articulated to them, it still came out in their comments.

Most of the thoughts of the course requiring too much work related to having to work through the challenges and a steep learning curve to get students to use the computer in a way many had never done before. A concern going into this study was getting students to learn calculus concepts while balancing that with learning new skills of coding. This proved true throughout the semester and was reflected in student comments. Students being intimidated by and resistant to the coding element along with thinking an approach similar to this expects too much out of them has been reiterated in other research on computing in mathematics classes (Johns & Hopkins, 2019; Tonkes et al., 2005; Lockwood, DeJarnette, & Thomas, 2019).

A student comment that reflects how some students felt is well expressed by this student's comment on lab 3,

> It is really like trying to learn a whole new language with how intricate and different it is for someone who has never had experience with it before this class. To be honest I really have no idea if what I am plugging in is the right thing or not, i.e. struggling to finish the question in full. I don't think it is a good idea to mix coding with this math when kids already struggle enough with the math. The amount of time that is necessary to be put into the lab really equates for more credit hours. This isn't a coding class and it is just making kids struggle that much more. Yes I do admit that I can see the helpfulness of learning this program, I just think it should be offered in a different class. I struggled with understanding what the questions were asking for but after figuring out the necessary formulas and codes for derivatives and functions my codes stopped running to where there would just be a blue * so I did not know if I was doing the problems right.

Other students certainly felt this way too but did not express their thoughts quite so explicitly in their comments. Words such as difficult, time-consuming, and a lot of work were especially present throughout labs 1-3 as students were getting used to working in Jupyter notebook. These comments decreased greatly in labs 4-6 and were in some instances replaced with sentiments such as “this is still hard but I’m starting to get the hang of it a little” and “I’m beginning to be a little more confident in using this program”.

This is not an unexpected result and the feeling of increased work and time demands may have had negative effects on students’ attitudes. Numerous innovations that involved using some type of computing in a calculus classroom in *Priming the Calculus Pump: Innovations and Resources* cite that students often felt like courses that incorporated these tools were more work and time-demanding even though they were not (Tucker, 1990), which may have an influence on students’ attitudes. In a project at Dartmouth that required computing in calculus classes found that students viewed the computer problems as “add-on” and additional work, despite the fact that they were made more relevant to the course material (Baumgartner & Shemanske, 1990). Baumgartner and Shemanske (1990), Schwingendorf and Dubinsky (1990), and Brown, Porta, and Uhl (1990) report that students thought that the computing component added more work. Kilty and McAllister (2019) note that in their study using RStudio students struggled to get comfortable to the technology, but they note that these struggles are an important part of students’ growth and development. Sonnert and Sadler (2015) also found that college calculus often has a negative effect on students’ attitudes because of the increased rigor and demands compared to pre-college coursework.

The computational labs used in this project were challenging, were an increase in rigor for most of them, and did demand a lot from students, however, the course was modified and other assignments were adjusted to make time requirements roughly equivalent to those in the control sections of the course. Students failed to see this though and felt more was required of them, which likely resulted from them having to learn something entirely new to most of them. This perceived extra burden on time very likely influenced a negative attitude toward the course, which then swayed the lack of growth in attitudes as measured by the Mathematics Attitudes and Perceptions Survey. The labs were undoubtedly challenging for many students because they required learning something completely new to most of them and did require a large time commitment to combine these skills and the calculus knowledge. Student comments of more work are certainly an important factor for demonstrating why some students had negative attitudes and the lack of overall growth in attitude at the end of the semester.

Facilitating Group Work. The need for more work in groups was revealed through analysis of student comments and could be a change that could improve attitudes in the future. Students had challenges combining mathematics knowledge with how to then implement that on the computer. Comments demonstrated that students thought that this would have been made easier by more time for in class work with their classmates. There was some time for this, however, students remarked that they would have liked even more time to work in class with other students. On lab three one student stated, “Working with others really helped me as well, especially given the class time to work on this.” On lab four a student remarked:

> Having time in class to work with other students really helped me because I was able to talk with them about how to translate the problem into symbols and then talk about how it connected to other topics from class. Like talking to someone about the rate of change and connecting that to the derivative helped me and then we talked about how we could put it into the Jupyter notebook. I think that made things easier.

Another challenge faced by students that they evaluated would have been remedied in part by group work was debugging their code and having another set of eyes look through their code to find syntax errors. Many students commented on liking having time in class for "another set of eyes to look through it and even just find typos or missing symbols". On lab three another student said, "Being able to work on this in class was a big help to me because I have found that these labs are very frustrating, but it really helped to go through it with someone else. They could help me figure out if I was missing something with the syntax and try to fix it. I also found it helpful to talk to someone else about the math stuff too." A student stated, "Working with one of my buddies helped because my code wasn't running but his was so we were able to figure out what was wrong in mine. That helped me because previously I had gotten stuck and given up." Students specifically said that group work helped them feel better about the material and the labs. Students' comments certainly reflect that they believed that working with other students benefited them and made them feel better about completing the labs. Other examples of using computational tools in a mathematics classroom make similar recommendations (e.g., Jones & Hopkins, 2019). Based on students' reflections, providing them with more time in class and more group work could help to improve their attitudes.

Students' recommendation of more group work could potentially benefit student attitudes. If students have opportunities to see other students struggling with the labs, they may feel less defeated. All students struggled with learning how to communicate with the computer in a new way, from very strong students to others that did not give much effort to learn this. Having more time for group work in class, could help all students see that this was not something intended to be easy and that everyone makes syntax errors, everyone has to utilize notes and old labs to figure out the problems, and that this is new for almost everyone. Students could see they were not the only ones struggling, which could help improve attitudes. Also having more in-class time for work on the labs could provide students with opportunities to get help from other students and from the instructor, which could improve attitudes because some students were burdened by being unable to find a tutor that could help with this aspect of the course. All of the comments relating to wanting more group work are stated in such a way that the inference can be made that working with others could benefit and facilitate growth in attitude by reducing the frustration of syntax errors, discussion of challenging mathematics concepts, and seeing that others too are having difficulties.

The Use of Data. Students evaluated the use of real data to be a beneficial aspect of the computational labs. Students liked that they saw the data from which the function was written. Students' comments on the use of data seemed to be positive and mention of it mostly went along with liking how they could see the connections to the real world. A student remarked:

> I guess I never really thought about where the functions my teacher gave me came from. But doing this I can literally see that I was able to fit a model that goes through or near a lot of the data points, so it did actually come from somewhere.

Along these same lines another student commented on being able to graph the data points and the model on the same axes also made him think about how that model might not predict the specified quantity in the future well because the model looked different from the data at the end. Another student commented:

> To be honest, question 6 helped me learn that sometimes a model can 'over predict' and give you extremely incorrect information. This lab clearly was designed to challenge students into looking at concepts from class from a different, real-life perspective. I really like doing these labs and trying to figure out how functions, graphs, exponents and other

'class material' will be used after college in the 'business world.'

Overall there were no specific negative comments about the use of data; the negative comments were on the use of coding to handle the data. The numerous positive comments on the use of data, however, went along with the parts of the labs that were most aided by the technology, such as the stock data and large CDC data file. The many positive comments on the use of data and appreciating seeing where functions came from help to emphasize that students viewed this as a positive element of the lab and should be further incorporated to positively impact students' attitudes.

Discussion

The data from the Mathematics Attitude and Perceptions Survey revealed slight decreases in attitude and no significant difference between the groups on overall attitude, as measured as an expertise score. When further analyzing the data, several differences can be seen on individual categories of the survey and across the variables of previous calculus exposure and gender. Overall much of the data aligns with previous uses of the Mathematics Attitudes and Perceptions Survey and other studies of attitude. Several factors and limitations of this study may have impacted the lack of gains and will be discussed in later sections. The qualitative data from student comments does demonstrate that the computational labs did have important impacts on students' attitude toward mathematics for many students while for other students highlight some negative impacts. The mixed comments from students on attitudes in the qualitative data help to explain why overall there were minimal changes in attitudes as shown in the quantitative measure.

The overall lack of improvement in attitude toward mathematics after only a one-semester course was not a surprising result. Previous research suggests that it takes more than a semester to positively affect students' attitudes (Carlson, Buskirk, & Halloun, 1998). Townsend and Wilton (2003) assert that students' beliefs about mathematics are formed through their history of experiences and are built over time, which implies that changing attitudes and beliefs is not something that can be done quickly. The slight shift away from expert-like attitudes and perceptions aligns with previous results from MAPS and related CLASS surveys (Adams et al., 2006; Barbera et al., 2008; Gray et al., 2008; Semsar et al., 2011; Jolley et al., 2012; Code et al., 2016). The change in attitude in this study corroborated results as in Maciejewski (2016) where even though both groups showed a slight decline in expertise score, the experimental group had a smaller decline in this.

The results of this study also align with outcomes from other studies. Sonnert and Sadler (2015) have found that technology was shown to have a non-significant impact on attitude, although they did not include the study of the type of technology used in this project. Final grade was also strongly correlated to end of semester expertise score, which has been seen in other studies (Carlson, Buskirk, & Halloun, 1998), implying that course achievement is certainly linked to expert-like mathematics beliefs. Sonnert and Sadler (2015) found that students in classes with ambitious and innovative teaching tend to have slightly negative shifts in attitude and that ambitious teaching has a more positive influence on students' attitudes for those that had more positive attitudes initially than students starting with more negative attitudes (Sonnert & Sadler, 2015), both of which occurred in this study as well. Sonnert and Sadler (2015) have also found that initial attitude was a powerful predictor in end of semester attitude, which was found in this study for both groups with pre- and post-attitude being significantly correlated (control: r=0.630; experimental: r=0.521).

The qualitative comments also align with data from previous research on computational tools in a calculus classroom (e.g., Baumgartner & Shemanske, 1990) where mixed reviews are very much expected because some students find benefit to learning a new way of problem solving and learning math content in conjunction with new technology while others are opposed to the expectation of learning computing in a mathematics class and find this to be a large burden in addition to learning the calculus content. There were many positive comments but also numerous negative comments, so this demonstrates that some students greatly benefited while others were negatively impacted. This qualitative data helps to explain why there was minimal change in attitudes as measured by the Mathematics Attitudes and Perceptions Survey.

The quantitative data does not reveal overall growth in the real world and persistence categories, however, much of the qualitative data does reveal positive impact the labs had on both of these. Student comments overwhelmingly support that the labs helped students see the connections calculus had to business applications and other real world settings, but the data from the MAPS revealed the opposite with the group with labs having a decline in this category. Elements of persistence ran throughout the comments, but this too did not emerge as a difference between the groups in the MAPS results. Despite the many positive comments on persistence and elements of having a growth mindset, it appears that many of these were in connection to the elements of coding, however, that did not translate over to having a growth mindset or persistence in problem solving in mathematics. In analysis of students' comments, it can be seen that most of the negative comments were related to the coding aspect of the course but were not related to specifically disliking mathematics. The negative comments made mention of being frustrated by the technology not by the mathematics content. Frustration with and challenges caused by the technology may have negatively influenced students' attitudes, which came out in the lack of improvement in attitude as measured by the Mathematics Attitudes and Perceptions Survey. Students may not have been able to separate their feelings toward the technology and their attitude toward mathematics and demonstrated these on the way they responded on the MAPS including in these categories. It should also be noted that attitudes toward mathematics have also previously been positively linked to attitudes toward computers; students with the highest level of mathematics attitude also had the highest scores on a multidimensional computer attitude scale (Jennings & Onwuegbuzie, 2001). Understanding the impact on attitude about mathematics is difficult also because some students make minimal effort to learn the programming language which makes the assignments very challenging and was something seen by Baumgartner and Shemanske (1990) as well. The mixed reviews and the discussion of negative impacts mainly regarding the technology in the qualitative data help illuminate why there was overall minimal change in the quantitative data in overall attitude towards mathematics and these categories.

In this study, students that had previously taken calculus before benefited the most from the computational labs. Students with previous calculus exposure had higher expertise scores in both the control and experimental groups, which aligns with previous uses of the Mathematics Attitude and Perceptions Survey as students further along in their mathematics careers tend to have higher expertise scores (Code et al., 2016: Macejewski, 2016). Students in the experimental group with previous calculus exposure had statistically significantly higher expertise scores at the end of the semester than those without previous calculus; this result was not present in the control group however. For students in the experimental group, those that had taken calculus before had much higher scores in all categories with large differences in the categories of growth mindset, interest, and sense making, with the overall highest difference in the confidence category. The separation of post-MAPS expertise score in the individual categories was much smaller between those that had and had not taken calculus before in the control group. In the experimental group, the change in attitude over the semester was greater for those who had taken calculus with categories of growth mindset and sense making both statistically significant. Their overall change in attitude over the semester was also higher than those without calculus. The comments by students that had previously taken calculus were also very positive and discussed how they were appreciative of the different way of problem solving, the use of real data, and the opportunities to see calculus used in context of the field they have chosen to study, business. These were positive impacts of the computational labs on attitude.

Some students come into these Business Calculus classes having previously taken calculus. Many instructors make it a goal to challenge these students or to get them to walk away from the class learning something different than they had seen in their high school calculus courses formerly. For me, while the business applications are usually new for students, previous discussions with students have revealed that they had seen most of the content before and thus some students leave the class somewhat bored or unchallenged, which may have previously negatively impacted their attitudes. The labs alleviated this issue to an extent because most students had not had practice using such tools and the very few that had had not seen them in use within a math class. Looking at comments from only students that had taken calculus before revealed that they learned a new way of problem solving and were challenged in the course. These students made mention of the new experiences of seeing the calculus concepts in concert with business applications, but they also remarked on the labs helping clear up confusion they still had and how they were able to solve problems differently. A quote demonstrative of this point was:

> Overall, I feel like the mathematical content from this lab was information I have seen in my previous calc course, but the way the questions in the lab presented the same information has helped me to see what the point of learning that mathematical content was. In high school if we did word problems to reinforce the topics we were learning the situations seemed too made up for me to find them relevant. But some of these they just seem more real.

A similar sentiment was reiterated by a student stating:

> I have also enjoyed using more real world data to find the derivatives. When I took calculus before we didn't get to use as much realistic data as we are using in these labs. I thought that the np.polyfit to fit a curve to the data has been very cool. It is nice to know that the computer can do a lot of time consuming or seemingly impossible calculations that we might face elsewhere.

Another student that had taken calculus before remarked, "I have learned all of this calculus stuff before but not with these applications and certainly not doing it on the computer. I honestly didn't even realize you could." Another student that had taken calculus before commented that doing these labs gave her a different way to problem solve. This student stated that in doing her homework problems she no longer pulled out paper and pencil; instead she opened a Jupyter notebook and worked through the steps. She remarked that to her it actually seemed easier and quicker to do it that way and that was a "totally different way of doing things".

Challenging these students with a different way of working through mathematics problems and using problems that allowed them to practice their calculus knowledge in context of business, which they have interest in and are choosing as their major, positively impacted their attitude about mathematics. These students seemed to benefit from experiencing mathematics in a new way. Their attitudes may have been more positively affected because they were not as challenged to learn the calculus content because it was not entirely new to them, which allowed them devote more time to understanding how to interact with the technology. This result is different than previous results where students that had previously taken calculus were actually dissuaded by the technology because they were hoping for an easy class with material they had already seen, thought that they would not be challenged because calculus was simply derivatives and antiderivatives and did not want to be convinced otherwise, or thought the computing component should not be in a math class (Baumgartner & Shemanske, 1990). Some of these students have continued to reach out to me after the semester about having enjoyed the course, the material, and the overall challenge; several of whom have now expressed an interest in learning more about data analytics and potentially picking up a minor in that, which can certainly be beneficial for them going forward.

For these students that had previously taken calculus the quantitative and qualitative data align demonstrating that for this group there was indeed benefit and was reflected in both sources of data, but for those students without previous calculus quantitatively there was no overall benefit on attitudes and their attitudes may have been negatively impacted. These results from students that had previously taken calculus help to explain the quantitative results as well. Upon inspection of the quantitative data that revealed no overall impact for the group with labs as a whole but that students that had taken calculus before had substantial growth in attitude, it was conjectured that this may have occurred because these students were less burdened by the content since they had seen it before and could devote more time to the understanding the technology and having the technology clear up confusion they previously had. The qualitative data helps to substantiate this conjecture as these students' comments were much more positive and explicitly stated that the labs helped resolve confusion they had from that last time they learned calculus, showed them how they could use the calculus, and demonstrated a new way of problem solving, all of which likely contributed to their growth in attitude over the semester. Those students that had not taken calculus before appear to have been encumbered by mixing learning calculus with learning how to do so on the computer thus likely contributing to their decrease in attitude over the semester. There were certainly positive comments from some students that had not taken calculus before, but they were not predominantly positive as the ones from those that had seen the calculus content before were, which indicates that for some there were negative impacts on attitude. Overall, however, there was only a very small decline in attitude for the experimental group as a whole, but over 70% of these students had not previously taken calculus, so it is not likely that all of these students' attitudes were negatively impacted by the labs and that the labs were only beneficial for students had previously taken calculus.

Revealed in the data is also that female students' attitudes appeared to profit more from the computational labs than males'. While there were not significant changes in overall expertise score, females in the experimental group did have higher post-MAPS expertise scores, higher changes from pre- to post expertise score, and more growth in almost all categories of the MAPS. Females had statistically significantly more growth in the interest category. These results need to be further investigated to determine the benefit computational tools may have on females' study of calculus. The positive results on females' attitude toward math that emerged in this study are important because in the national study of calculus Sonnert and Sadler (2105) found that males had significantly higher mathematics attitudes than their female counterparts after taking calculus and Ellis et al. (2016) found that females are 1.5 times more likely to leave

STEM degrees as a result of their calculus course, so it is important to consider interventions that may positively affect the female experience in a calculus class.

Limitations and Future Research

Many student comments revealed that the computational labs used in this study did have effects on their attitudes and perceptions about mathematics, but minimal impact emerged in the quantitative data from the Mathematics Attitudes and Perceptions Survey. There are several notable limitations in this study.

A noteworthy limitation to this study includes the timing of administration of the post-test. Some issues with the Mathematics Attitudes and Perceptions Survey results likely happened because of apathy on the post-test. The post-test MAPS was given on final exam day, which did ensure all students were present, however, also presented issues. Students were given sufficient time during the final exam block to complete all requirements and time was set-aside during the exam period specifically for the survey. Since the MAPS had no influence on their grade in anyway, many students did not appear to give much effort in completing it. Some other studies have suggested giving a very small grade incentive to complete it, which is recommended for future studies. Numerous students appeared to rush through it, closing their computers after only a few minutes; it can reasonably be inferred that students were more concerned about getting to their actual final exam that would affect their final grade. In future studies options should be explored to ensure students take the post-MAPS more seriously, perhaps not giving it the same day as the final exam. The apathy from students on the MAPS likely affected the results and may have influenced the small changes over the semester. This study made the assumption that students would honestly and intentionally answer the questions on the MAPS, a violation of this assumption could have impacted the results.

Another limitation that was exposed especially through the qualitative data was the technology, which was brand new for many students, caused the frustration and challenges. Many of the more negative comments were about being frustrated by the technology and not by the mathematics. When answering the questions on the MAPS, students may not have been able to separate their feelings and attitudes, sometimes negative, of the technology versus how they actually felt about the mathematics. The qualitative and quantitative data are especially conflicting in the "real world" category. Overwhelmingly, student comments revealed that they felt the mathematics they were learning was real and they could see how it could be used outside of the classroom, with over 50% of the comments analyzed mentioning something about "real" and many of the others stated how they could use the calculus concept in some other context and talked about scenarios such as supply and demand or cost and revenue, although there were some comments on not seeing why learning this way was important, which could have negatively impacted some. The voluminous positive comments on seeing the real world connections do not align with the data from the MAPS category of real world, where the group with labs actually saw a decline in this category over the semester and scored lower than the group without labs. The groups that did not have labs did have four projects throughout the semester, as described in the methodology section. These projects all had connections to a business application and required students to write up their findings in a report or letter to their "boss". The students in the control group were also afforded the opportunity to use what they were learning in context, so this certainly may have impacted their positive views of mathematics being applicable in the real world. However, it is important to note that none of these projects in the control group began with data and most included students being given the functions that represented the scenario or were more closely connected to a problem that the students had done before, but the writing component may have influenced the feeling of realness as well. This is certainly an area that requires further investigation.

Because of these results and the need to tease out the differences the technology specifically had on attitude, going forward in future research it is imperative to have more structured qualitative data collection. The qualitative data gathered in this project was very unstructured and was originally intended to provide insight into what students were learning. Students, however, used the opening to discuss how they were feeling and their thoughts about the labs. The solicitation for comments on content at the end of the lab may have induced some bias as students may not have felt completely comfortable making negative comments when they knew their instructor would be reading them during the semester. However, the fact that students knew they would not be penalized for what they wrote and would be given credit for writing the paragraph, appears to have let them give honest responses as evidenced by the numerous negative comments that were present in the data. Having more structured collection of qualitative data going forward would allow for more information to be gleaned and understand how the interaction of the technology used and the content on the labs helped to influence attitudes toward mathematics. Future research designs should include interviews as a method of data collection. Conducting interviews with students would allow for more targeted questions about how the labs contributed to students' attitudes, what content of the labs was most beneficial, how the labs could be improved to increase the benefit, how the course could be best structured to positively impact attitude, and more. Conducting interviews may allow the researcher to discern how overall attitudes toward math were influenced by the new technology and students' struggles with it as well.

Interviews would also provide valuable insight into students' evaluative thoughts on the computational labs and could help guide future development of the course, which could lead to making the intervention more beneficial in positively shaping students' attitudes. In future research students from the control group could be interviewed as well to help triangulate results. In this study, this group was not used to gather qualitative data, which was a limiting factor of this study. Going forward, however, this group should be interviewed to help understand what helped shape their attitudes. Information from these students along with data from the students in the experimental group could shed light on what was most beneficial and how elements of both classes could be combined to improve the course overall.

Also to increase understanding as to the impact of the lab and substantiate the impressions I got as the instructor and from the student comments, a questionnaire, such as ones used by Höft and James (1990), is recommended to get students to respond to statements such as "using a computer in the classroom contributed to my understanding of the course material", "the computer assignments could be completed in a reasonable amount of time", and "using a computer in the laboratory enhanced my interest in the course material" (p. 147).

Improving this Class

A limitation of this study was this was the first iteration of using these labs in this class. I had used Jupyter notebooks myself in my own work and had been a student where using computational tools had been a large part of my classes and learning, but as an instructor I had not yet used them in my classroom. Because of the first iteration this was a learning experience for all of us involved. Through my process implementing the labs, the experiences and conversations with students throughout the semester, and in reviewing student comments, there are numerous changes that could be made to the class going forward and informing future research.

One change that could be made in the future is the implementation of the computational tools more intertwined in the entire class not as labs. The computational tools were employed as labs in this study in part to keep the control and experimental classes more similar and comparable and to fulfill the way the course content had to be structured at the time. Before the semester began, in discussing the implementation with a university physics educator who uses Jupyter notebooks in his classes, he cautioned that by doing them as labs rather than fully intertwined in the class for daily use, students may see them as a completely separate entity, an extra burden on their time, and not part of their accessible problem solving methods. He made such comments because he said he had seen these issues when doing so in physics classes. By having more incorporation of Jupyter notebooks, or other similar tools, throughout his courses rather than an additional piece to the class, he had seen students' views on them develop and students had grown to use Jupyter notebooks as a way to solve physics problems rather than always pulling out paper, pencil, and a calculator to tackle problems. The issue of students seeing these labs as more assignments to complete and a separate entity was echoed in student comments and was observed in how many students approached problems. The students also made it abundantly clear in their comments that they thought there was an increased time demand because of the labs. Another issue was that to keep the two groups similar, students were not allowed to use Jupyter notebooks on their tests, but they could use them any other time. An additional issue was while much of the content was introduced using the computational labs and the notes that followed built on and expanded the content covered in the labs, students were shown how to do concepts with paper and pencil as well because this was still an expected outcome of the course. This seemed to cause a bit of a disconnect. With knowledge that they were not going to be used on the test and being shown multiple ways to solve the problems, students tended to gravitate towards traditional methods of paper and pencil. There were exceptions to this, however. By the end of the semester, several students began using Jupyter notebooks to complete their homework assignments. Some students found these notebooks to help them more quickly do their homework, reinforce concepts because of the commands they had to employ, and a feel of more authenticity in how they would do math problems outside of the classroom. It can be seen that some students did progress to considering Jupyter notebooks as another way to solve calculus problems; these students had positive attitudes toward them and many were students that had previously taken calculus before, which was reflected in their overall attitudes. These students were in the minority, however, and most students still resorted to traditional techniques. Because the labs were not used everyday and some students viewed them as separate from the class, this may have minimized the overall impact. Students also commented that they desired more time in class to work on Jupyter notebooks. In future research the computational tool, Jupyter notebook, should be more intertwined throughout the entire class, so that students increase their willingness to use them to solve problems and thus the impact on attitude may grow. As the use of these tools grow and evolve in my classroom, I will increase more independent practice in the class and getting students to code more of their own functions, which could potentially increase the effects including using students' evaluative comments a guide to help to increase the impact.

Another change that could possibly take place during another iteration of the project is allowing more time for students to work in pairs, which arose from the evaluative analysis of student comments and in discussion with students throughout the semester. There was some time for this, students remarked that they wanted even more time to work in class with other students. Allowing this time could help students pair up to discuss how to translate mathematics problems to something they can approach on the computer and to find syntax errors. Other examples of using computational tools in a mathematics classroom make similar recommendations (e.g., Jones & Hopkins, 2019). Allowing more time for this may increase students' comfort with the technology, give them more confidence as they see others struggling and working through errors, and overall ease some of their frustration, which could positively impact their attitudes.

Following student recommendations that arose through analysis of student comments with evaluative coding, in future iterations of this project even more data should be incorporated. Students' comments revealed their support for the inclusion of data, "seeing where the function came from", and the data adding an element of realness. Going forward, the use of more data to motivate problems must be incorporated. Additionally, the use of larger data sets would be beneficial. Some of the comments revealed that where the labs had the most impact revolved around using stock data imported directly from Yahoo Finance and the large data set from the CDC on smokeless tobacco products. Both of these stood out to students more than the problems with smaller data sets, which could cause one to infer that students could see the benefits of the computational tools in handling a larger data set than they had probably ever tackled using their graphing calculator. By dealing with more and larger data sets, students would likely be able to see the power of the tools and could see that the data sets were not concocted just for the example, which may more positively impact attitudes especially in the real world category. The relevance of the content and connection to the real world would be undeniable.

Even since implementation of computational labs, the Business Calculus course discussed in this project has undergone revisions. Several major changes have occurred which open up more opportunities for the use of computational tools. One major change is the course now begins with an introduction to linear algebra topics such as systems of equations, matrices, and applications of these. Tools, such as those in this project, could open up this study to handling more complicated problems of these applications. An example of this is in input-output models; many of the examples used by the textbook in use involve examining only a few sectors of the economy at a time, but the United States economy has over 500 sectors and even a simplified model still has close to 100 sectors, so using computational tools may allow for handling of problems incorporating more sectors, which, similar to the larger data sets, may help students appreciate the computing power and increase the sense of applicability to the real world. Another change in the course is a smaller focus on computing derivatives and integrals through rules and by-hand manipulation. This was already done in this project; however, it further sparked conversation of how much of that needed to be in this particular course and it was decided that that focus could be lessened. With a decreased focus on computing these by-hand, it was decided that there should be more concentration of applications and interpreting answers. Decisions such as these were made in collaboration with the School of Business, which is a necessity. All of these changes further open up occasions for computational tools and afford more time in the course that they could be used daily, as discussed previously, rather than as isolated labs, which student comments imply could help increase the impact. The results of this study of computational labs in Business Calculus provide a strong foundation on which further revisions of the intervention could be made and studied.

General Conclusions

Increasing students' attitude toward mathematics, or at least not destroying it, is undoubtedly a goal of undergraduate calculus classes and was an objective of using computational labs to introduce students to calculus concepts in this project. Within this study there are no statistically significant differences in attitude towards mathematics when students participate in computational labs, however, there was evidence from students' comments that the labs did help many students make connections of calculus concepts to their chosen field of interest business, see how mathematics can be used in the real world, and develop their willingness to push through challenges. These labs aided many students in seeing the applicability of calculus to the "real world" and left them not asking the question of when are we ever going to use this outside of this classroom. The labs used in this project also demonstrated a different way of problem solving than many students had previously been exposed and positively impacted attitudes of students that had previously taken calculus. Because of the aforementioned results, this is an area for future work further developing courses that can employ such tools.

One area that helped students perceive the mathematics in this course as more "real" or related to their field was the use of data. This cannot be ignored. Students remarked on appreciating seeing where the "given" function came from and being able to fit curves to the data to model the data and make inferences and predictions. In this project Jupyter notebooks with Python were used as the technology to handle the data, however, more traditionally employed technology, such as a graphing calculator, could also be used to handle some data sets and curve fitting. Whatever method of technology is chosen, it is recommended that students have the opportunity to see some problems that arise from data. This project revealed that it appears important and valuable for students to see where the given functions come from and the use of data, modeling that data, and then interpreting answers may allow students to make more sense of what they are learning. Not only is this valuable for students to acknowledge, but it aligns with that fact that business leaders must have an understanding of algebra, calculus, and statistics and be proficient with technology to do so (Lamoureux, Beach, Hallet, 2001).

Many students' comments revealed that the technology did help them make connections with business applications especially with real data and discover a different way of approaching problems and new problem solving tools. Others, however, did express their aversion to the labs, the challenges they felt the technology added, and some even felt that the labs made the class more challenging and thus likely negatively influenced their attitudes. Because of the mixed feelings and experiences, the use of computational tools in this course must continue to undergo revisions to enhance students' experiences using my experiences and student comments as foundations for the changes.

Based on the results of this study and experiences in these classrooms, I will continue to use and develop computational labs in this class and several other instructors at the university where this study took place have expressed interest in further developing this class and others using such tools. The mixed results of this study and the increase in research on computational tools in educational settings, including in mathematics classes, demonstrate this is an area that still needs to be studied and further researched to continue to evaluate how computational tools can be effectively used to increase the student experience in undergraduate mathematics classes such as business calculus.

CHAPTER 5: Conclusions and Recommendations

This chapter discusses the conclusions and recommendations resulting from the data analysis as presented in the previous two chapters and discussed in regard to future practice. Recommendations for applications of the findings to both research and practice follow.

Conclusions and Implications

In describing introductory college calculus, Bressoud (2017) states, “Today we teach a greater numbers of students, who are less prepared, using fewer resources, and with increased expectations for student success” creating a perfect storm and “this is why we can’t keep doing things the way we have always done them”. This perfect storm has lead to college calculus negatively affecting students by decreasing their desire to continue their study of mathematics, lessening their overall enjoyment of mathematics, and lowering their confidence in their mathematical abilities (Bressoud & Rasmussen, 2015).

Students complain that their introductory STEM courses, including calculus, are uninspiring as they deliver material in a traditional manner of lecturing despite that being shown to be ineffective pedagogy (PCAST, 2012; Frechtling, Merlino, & Stephenson, 2015). “It would be difficult to design an educational model that is more at odds with the findings of current research about human cognition than the one being used today at most colleges and universities” because most college faculty teach the way they were taught themselves (Halpern & Hakel, 2003, p. 38). There is a growing body of research that students are switching out of STEM majors and other majors, such as business, where first year mathematics courses, such as calculus, are serving as a critical filter, but not simply because of academic preparedness as one might think (Berrett, 2011; Seymour & Hewitt, 1997) rather because of poor experiences in introductory-level courses (PCAST, 2012; Seymour & Hewitt, 1997). Research repeatedly suggests that in introductory mathematics courses, students are not learning the intended material (Breidenbach et al., 1992; Carlson, 1998; Tallman et al. 2016; Thompson, 1994), students are leaving the courses unprepared for other courses (Carlson, 1998; Selden, Selden, Hauk, & Mason, 2000; Thompson, 1994), and students lose interest in mathematics after completing the course (Bressoud, Mesa, & Rasmussen, 2015; Seymour, 2006), all of which have been seen in both small, localized studies but also in national studies of introductory college mathematics courses (Bressoud, Mesa, & Rasmussen, 2015). In addition to these detriments, other disciplines "are no longer content with seeing their students taking calculus courses that haven't changed over 30, 40, or 50 years" (Bressoud, 2017) and are even considering teaching the required calculus courses themselves rather than mathematicians.

Because of such issues, improving student success in first-year introductory mathematics courses is a growing concern. Bressoud and Rasmussen (2015) and Rasmussen et al. (2019) make numerous recommendations for making more successful introductory mathematics programs by studying currently successful programs, suggesting active learning strategies and challenging, engaging courses that focus more on concepts rather than skills. Mesa, Burn, and White (2015) along with Larsen, Glover, and Melhuish (2015) report that lecturing, in some form, is still the dominant style of teaching in Calculus I; they also report that good teaching in calculus involves a positive atmosphere with ample interactions between students and the instructor, high levels of student engagement, high standards and expectations, and can include ambitious teaching incorporating active learning such as small group collaboration and solving non-routine and application problems. Technology can help calculus courses be successful, but not simply by using technology but rather because of trying innovative teaching with the selected technology and being intentional about how that technology can support learning (Selinski &

Milbourne, 2015). Innovations building upon elements of good teaching of calculus must be developed and researched to improve students' experiences and conceptual understanding in introductory calculus classes.

> If mathematicians really are tired of teaching a calculus of algebraic manipulation and 'inert material,' if mathematicians really are tired of over-stuffed textbooks, if mathematicians really are tired of high failure rates and low retention of what is taught, then it is time to try something new. We owe it to our students. We owe it to ourselves. We owe it to mathematics. And it can be done… There are no obstacles other than our own indifference and the constant lack of fair reward for good undergraduate education. (Tucker, 1990).

We must consider changes to calculus courses that can modernize it and better serve the abundant amount of students from numerous STEM and other client disciplines. Interventions that make use of interdisciplinary content and practices are imperative. At a time when college's value is being questioned it is crucial to create courses that can positively affect conceptual understanding and attitude, has demonstrable value such as making explicit connections to other disciplines, and teaches students to use current technology that could prove to be marketable, valuable skills. This project is one such example. These two manuscripts support the efforts to make college calculus more efficacious at positively impacting attitudes and teaching content with applications from client disciplines with new technology implemented through computational labs.

The first manuscript, *Computational Labs and Conceptual Understanding,* presents a study that examined the impacts of computational labs in a Business Calculus course on conceptual calculus knowledge on college students as compared to college students in traditional sections of the course. Results did not show any significant increase in conceptual understanding as measured by the Calculus Concept Inventory. The two groups were roughly equivalent in conceptual calculus knowledge over the semester using the CCI as a measure. Through analysis of student comments from the students participating in the labs, it can be seen that through the labs many students did learn to make valuable connections between the calculus concepts and applications in business and some students appreciated the new technology they could employ to problem-solve. Students found the mixture of learning a technology new to them and calculus concepts simultaneously to be challenging, however, research indicates that students actually want their introductory STEM courses to provide a challenge, and having challenging courses that had "high expectations for students including engaging, conceptually oriented content beyond an emphasis on procedures and skills related to calculus" (Rasmussen et al., 2019, p. 100) is one of the seven characteristics of a successful calculus program. A majority of students that completed the six computational labs over the semester left the course being able to specifically articulate ways in which calculus concepts can be applied to business situations and can be derived from real data, which is valuable. These students also finished the course learning a bit about a technology that may be beneficial in their future.

The second manuscript, *Computational Labs and Attitude Toward Mathematics,* examines the effects of computational labs on students' attitude towards mathematics of college students in Business Calculus. There were no identifiable differences on attitude improvement between the experimental and control groups as measured as a pre/post test using the Mathematics Attitudes and Perceptions Survey. The attitude trend was similar to literature where new innovative teaching strategies can cause a slight dip in attitude in a semester, although a smaller drop for students completing labs than students in the group without labs, and also aligns with the literature that attitudes are relatively stagnant over one semester and may take longer to change. There were differences when individual categories of the instrument were analyzed, with the students participating in labs having more growth in certain categories including growth mindset and confidence. Students that had previously taken calculus and participated in the labs were the most positively affected by the labs in terms of attitude with overall greater growth during the semester, much higher expertise scores, and very high expertise scores in certain categories. Many students' comments also revealed that the labs helped them realize and acknowledge the usefulness of mathematics beyond the walls of the classroom and the development of persistence characteristics in successfully completing the labs.

Together, these studies suggest that using computational labs positively impacts and has considerable benefits on attitude toward and conceptual understanding of calculus for some students. While results of changes in attitude and conceptual calculus knowledge were not statistically significant, students' comments reveal numerous positive effects the computational labs had on both for many, but certainly not all, students. This research provides a foundation for the effectiveness of computational labs used in a Business Calculus course, which is not currently a common practice and needs to be studied further as mathematics educators seek to modernize calculus courses, make the content more relevant, and reach the vast amounts of students tasked with completing the course.

Recommendations

The study on the use of computational labs in Business Calculus effects on students' attitudes toward mathematics and conceptual calculus knowledge led to recommendations both for practice and for future research. Recommendations for practice include what changes can be made to courses to broaden the use of the computational tools, developing the use of computational tools to handle other topics from business and increase the use of data sets, asking questions of what is the appropriate content for different populations of students to best serve the client disciplines, and how to improve the classroom experience for students tasked with the challenge of learning new technologies and new calculus concepts simultaneously. Recommendations for research include the use of different instruments, longitudinal data collection, gathering of additional data points, gathering of more student comments, growth mindset research, and further breakdown of how students believed the computational labs benefited them. Other recommendations for research include further researching how these tools can be used in other general education mathematics classes.

Recommendations for Practice

This dissertation connects to the on-going body of research of innovative teaching methods to impact the student experience in introductory tertiary calculus classes. It builds on the knowledge that more active learning strategies have benefits for students in such classes by suggesting using computational labs in Business Calculus. This research connects to the growing body of research of computational tools in undergraduate mathematics classes and specifically studies a course not intended for mathematics majors. There are several recommendations for improving upon this use of computational labs for implementation and further development.

The computational labs used in this study were most beneficial to students that had taken calculus before. These students articulated their appreciation of learning a different way of problem solving, seeing calculus concepts they had seen before being used in relation to and in applications of an area they truly had interest, being able to see that functions can be motivated by real data rather than simply being given functions, and being challenged rather than simply relearning what they had already seen before. These students were also most able to specifically state how calculus concepts can be applied to real world scenarios. They also had the most growth in attitude, had attitudes more closely aligned with experts than any other group in this study, and were most willing to step up to the challenge of interacting with new technology. These are all important results because Bressoud (2017) reports that of the roughly 800,000 students that take calculus in high school over 650,000 retake calculus or a lower math, and out of those over 150,00 students take business calculus each semester as their first university mathematics course. This is a valuable result as in practice it is sometimes hard to challenge, motivate, and excite students that have already seen this material. In future practice the labs and the structure of the course need to be retooled, which is discussed below, to better reach those who have not yet been exposed to calculus.

As mentioned in Chapters Three and Four, several changes are recommended for refining the class structure of the course employing computational tools. One significant change would be to allow more time in class for pair and group work. The pair and group work would allow for additional time for help with detecting syntax errors, working together on successfully writing the correct code, the opportunity to discuss the techniques of how one went about doing a problem, and discussion of the relevant calculus concepts used to work through a problem. More time for work on the labs in the class could be an active learning strategy, which is one of the seven characteristics of a successful calculus program (Bressoud & Rasmussen, 2015). Allowing for this time would also create more in-class opportunities for students to get help from the instructor, especially overcoming technology issues they were facing. One major necessary shift would be to allow students to use these tools on tests. Some students felt that the labs were additional work and not fully integrated into the class, part of which came from not allowing them to be used on tests. In the future the entire class could be revamped to use Jupyter notebooks not only as lab assignments but fully integrated into the class daily. In this full integration, students should be allowed to use Jupyter notebooks on assessments, which may help students to not view them as extra work and may increase the overall impact of them.

When using computational tools, instructors must prepare themselves for dealing with technology issues and must be ready for the immense time involved in the development of the assignments, the out of class time required to meet the many demands and questions from students, and the time entailing technological troubleshooting. This is similar to recommendations made from other studies employing similar tools (e.g. Baumgartner & Shemanske, 1990; Höft & James, 1990; Kilty & McAllister, 2019). Students often sited issues with technology as the frustrating element of the class, so it is imperative to create time and opportunities for students to get help. Time must be set aside for technological troubleshooting. This goes beyond helping students use correct syntax, but it also involves issues such as getting the required software, troubleshooting when something is not working, locating files students have misplaced, and navigating to and moving files to appropriate folders. Instructors must also be willing to devote time in-class specifically for teaching about the technology and demonstrating how to successfully write and debug lines of code. It is valuable for students to see an instructor write code live, make typos and syntax errors, and make errors and get error messages; this can help students realize that making such errors is inevitable and one must be willing to work through the errors. Allowing more time in class would give students opportunities to ask questions, which is especially important since getting tutoring from other sources is challenging since so few students have experience with this. This could help with syntax debugging and organizing thoughts of how to write lines of code to get through the problem. Another change that could be implemented in future use of these tools is to provide students with a webpage that is like a quick reference toolkit. This page could have quick glances back at techniques previously used, references for commonly used syntax, and some explanations of the syntax.

It should also be noted that while students seem to think of themselves as technologically advanced, I discovered that in many regards this was not be the case. Issues were seen with students' lack of file organization, challenges downloading and installing the required software, issues figuring out how to name files, and problems uploading and submitting the correct type of file. These are valuable and important things to learn. Some students seemed to get defeated by this, gave up easily, were not willing to troubleshoot, and made comments such as "mine just isn't working". Too many students seemed unable to do little more than Internet browsing and word processing, when a computer can be used for so much more. Many students expressed sentiments that reflected that the way they were being asked to interact with the computer was completely new to them demonstrated in statements such as:

- "Prior to this, I had no idea of coding or how to do any of it. I did not know that it could relate to math either."
- "I learned in this notebook that there is more to math than just learning how to do problems on paper. I mean I knew about the calculator but not this stuff."
- "I thought this activity was very interesting because anything other than shopping online, checking my email, typing a paper, and submitting homework was way past my knowledge and I really wasn't interested. I didn't really realize how much more it could be used for. Even in just a few labs I really have a better appreciation and I really appreciate people who code for a living."

- “I had heard people talk about Terminal and stuff but I didn’t know what that was until now.”
- “At first I couldn’t get my notebook to import the picture. I later realized I didn’t have the picture in the right folder and I had named it something else. I need to keep my folders more organized.”

Instructors must be aware of this and be willing to intentionally teach students about the technology.

In addition to time demands dealing with technological issues, instructors also need to be prepared for time requirements in developing the notebooks and educating oneself of the other disciplines to authentically make use of the tools and the applications. This is a similar note to those in Kostandinov, Thiel, and Singh (2019). Grading the Jupyter notebook lab assignments also requires large amounts of time. Instructors should be prepared for this and should also look for available resources such as the grading that can be done through nbgrader. This grading resource was not used in this iteration of the project but should be considered in future studies.

Another technological change that should be made going forward is the distribution of the notebooks. The notebooks in this project were shared as Jupyter notebook files that students had to download, move to the appropriate folder, and then run locally on their machines. This required students download several programs including the Anaconda suite. Students had trouble with this and then had trouble remembering how to open the notebooks, despite the instructor demonstrating opening notebooks during the in-class lab time. Some students also ran into issues where paths were not properly defined and thus could not successfully run certain code. A recommendation for future iterations of the project is to distribute the notebook files through JupyterHub or the Google Colaboratory. “JupyterHub allows users to interact with a computing environment through a webpage. As most devices have access to a web browser, JupyterHub makes it is easy to provide and standardize the computing environment of a group of people (e.g., for a class of students or an analytics team)" (JupyterHub, n.d.). Google Colaboratory (Google Colab) is a free Jupyter notebook environment that requires no setup and runs entirely in the cloud. "With Colaboratory you can write and execute code, save and share your analyses, and access powerful computing resources, all for free from your browser" (Google Colab, n.d.). This would eliminate issues of downloading the software and running it on their personal machines. Many of the libraries we used are pre-installed and ready to be imported. Using Google Colab allows users to run Jupyter notebooks without having to download, install, or run anything except for an Internet browser. Some students had issues running Jupyter notebooks locally, so we started using Google Colab for them and many commented that it did make things easier and they liked it better. Making this change could help to alleviate some of the technological issues.

Future use of this project would be to go beyond labs and instead re-vamp the course completely using computing as a problem solving tool constantly not simply as introductory labs. This would also require changing more of the content in the course and could allow for even more business applications to be brought in. In research from the 1990s this was recognized as well in that using computing power in the classroom can change how material is taught but also what is taught (Tucker, 1990). Such discussions, of truly evaluating the content needed in this course, are under way at the university where the study took place. Changes have been made to the content in this course even since this study. Much of the study of limits was removed, similar to Kilty and McAllister (2019) who suggest the same for a calculus course focused on modeling that used RStudio. Instead of algebraic manipulation, the study of limits was presented as an intuitive focus on limits at the heart of derivatives and integrals without as much algebraic detail, such as in discussing derivatives understanding the value approached when average rates of change are computed on smaller and smaller intervals and more and more rectangles for Riemann sum. We have also now decreased the focus on computing the derivative through the use of rules, although this is still part of the course. We have also added a linear algebra unit at the request of the School of Business. The expansion of the course would involve more mathematical modeling as well, which is often done in the workplace. Creating and interpreting models in the workplace involves turning a real situation into a mathematized problem to answer a practical question (Gravemeijer et al., 2017), which students enrolled in this course may be more adept to do. Expanding the project and using larger data sets as motivation for some Calculus concepts could expand on this even more such as using customer churn data (Koehler, 2018b). Redesign could also include a blend of data science topics as well as including probability distributions and combinatorics which can relate to integration, optimization, least squares method, and bias variance trade-off (Koehler, 2018b).

Another area that could be developed is to create a statistics course that students could take after the course that was presented in this study that also uses computational tools. Taking two courses where such work is required would improve students' proficiency with these tools, impact their desire to continue to develop these skills, and perhaps impact their attitudes since it is likely that attitude towards mathematics takes longer than one semester to change. Work from courses such as these may also allow students to create portfolios that they could take to future employers (Koehler, 2018a). This implication is similar to one outcome of a study where students learned to program in R, in which many students viewed learning to program in R to solve problems as a benefit and a resume-builder for when they look for internships and jobs

(Benakli, Kostadinov, Satyanarayana, & Singh, 2017). After taking two such courses students would likely be more skilled at using these tools and appreciative of the work that can be done with them, which may help them as the enter the job market or graduate programs as there are numerous graduate programs in business analytics, among others, that have a coding element as part of them, however this needs to be explicitly demonstrated for students.

In the evaluations of the course some students remarked about how they did not understand why a business major should be expected to learn to code or they were not sure how they could ever use it in the future; some students remarked that they did not think that coding had anything to do with business. This is obviously not true, however, students may not have any way of previously knowing how computational tools are used in the business world. Students need to be explicitly shown how tools like these are currently in use in business, and they need to truly see how powerful these tools can be, especially by seeing how they could potentially handle very large data sets. Knowing how to code in business could be a marketable skill, and this iteration of the project did show some students that this is something they may want to pursue; however, to reach more students the benefits of knowing how to interact with such tools must be clearly conveyed. A recommendation would be to bring in an outside expert that can relay to students how such tools are present in the current business world and how these tools are growing. Students may not know it, but many major banks, such as Goldman Sachs and JPMorgan Chase, are teaching their investment bankers to code since technology like artificial intelligence and online lending and trading platforms are reinventing the industry; these banks are requiring mandatory Python trainings for traders and analysts (Rayome, 2019). These business professionals are being asked to learn concepts of coding, data science, machine learning, and cloud computing (Rayome, 2019). Other companies outside of the banking industry are doing so as well. In modernizing a business calculus course to include such topics, we are giving students an introduction to a very valuable skillset and one they may be called upon to use later and expand, but the value must be explicitly conveyed to them.

Another recommendation would be to bring in a representative from a masters degree program in business or business analytics (e.g., The University of Virginia's Master of Science in Business Analytics) that includes learning programming languages. For example, Business Analytics masters degrees currently exist and include learning different programming languages. Looking through these programs online, many of them list course descriptions that include learning Python and other languages. It appears that having had exposure to these types of computational tools, at this point, is not a prerequisite of some of these programs but having even seen some in use and had some practice with such would have given students some beneficial exposure before entering. As previously mentioned, the students that participated in computational labs are not proficient coders, but they may have acquired some skills that others lack simply because they have never seen these or used the computer in such a way. Students may not use Python in their future careers and the language of choice may be up to the employer and the intended use. While another language may be used, having had some exposure to Python may help them get started in learning another language.

Another benefit that needs to be overtly conveyed to students is the growing trend of cross-silo leadership and cross-functional teams.

> The value of horizontal teamwork is widely recognized. Employees who can reach outside their silos to find colleagues with complementary expertise learn more, sell more, and gain skills faster. Harvard's Heidi Gardner has found that firms with more cross-boundary collaboration achieve greater customer loyalty and higher margins. As

> innovation hinges more and more on interdisciplinary cooperation, digitalization transforms business at a breakneck pace, and globalization increasingly requires people to work across national borders, the demand for executives who can lead projects at interfaces keeps rising. (Casciaro, Edmonson, & Jang, 2019).

With this collaboration, while a person might not be required to write much code, it may prove beneficial that they have some level of understanding of such so that when they are working in collaborative teams they can be a benefit for understanding some of the technical resources and language of the other members of the team. The head of JPMorgan Chase Asset Management told the Financial Times:

> Coding is not just for tech people, it is for anyone who wants to run a competitive company in the 21st century. These are skillsets of the future… By better understanding coding, our business teams can speak the same language as our technology teams, which ultimately drives better tolls and solutions for our clients. (Mary Callahan Erdes as quoted in Rayome, 2019).

A recommendation that was reinforced by this project is the need for collaboration between departments. Bressoud (2017) states that the other disciplines often lament that students cannot use the calculus they have been taught back in their home discipline. This, along with student complaints that mathematics is dull and unimaginative, lead to the President's Council of Advisors on Science and Technology (2012) recommending tailoring calculus courses to specific disciplines, suggesting teaching the courses and developing curriculum with discipline experts themselves rather than mathematicians, which is concerning to many mathematics educators. Mathematics educators must consider changes to calculus courses that will better serve the voluminous amounts of students for a variety of disciplines. Perhaps more fragmentation of calculus courses could allow the course to be tailored to specific examples from their discipline, such as in this project. This could also help to both ask and answer the question more deeply as to what students really need to know after leaving their calculus course since if the courses were more segmented by discipline they could be better designed to meet the mathematical needs of these disciplines. The computational labs and structure used in this course would not meet the needs of all students required to take calculus, but as seen in this project for a population of business students they did have important effects on students' attitudes, abilities to make connections to their disciplines, and introduced students to a new way of problem solving. Using this method, especially in fragmented courses, could affect more than math ability but also peak interest and improve attitude since it would be in direct connection to their content areas. This project could be expanded outside of business calculus and used in other versions of a calculus course designed for specific disciplines, especially a calculus course for biology and chemistry or one for engineering. Because of the need to create calculus courses that can truly impact students from disciplines other than mathematics, it is imperative to collaborate with these partner disciplines. Collaboration with the client discipline will help mathematics educators become more knowledgeable of authentic applications and authentic practices in that discipline. These partner disciplines can shed light on the content, applications, and technology that they desire their students know upon existing an introductory calculus course. In creating an introductory calculus course including modeling and using technology Kilty and McAllister (2019) report positive comments from the client discipline that their course served. More collaboration between disciplines is recommended for any calculus courses, but specifically more collaboration is recommended for future iterations of the use of computational labs. Every one involved, students, mathematics educators, and educators from other disciplines, will benefit from collaboration and integration.

The model in this project provides a starting point on which to build how to best use computational tools in a general education mathematics classroom. In future practice, it will be imperative to collaborate with other departments to determine how the course and use of the tools can be further developed to best cover realistic content in connection to calculus, and other subjects, and use authentic ways of problem-solving. Further development of these can create authentic opportunities for knowing and doing.

Some student comments made reference to their thoughts that technology helped them learn. Many students did not expand on what they meant by this and thus it still needs to be studied as to how they felt it helped them learn. Mathematics educators are constantly asking about the appropriate use of technology, dating from before the Math Wars and still to this day. "We are living in an ever-evolving computerized age, and we are seeing trends in mathematical research and mathematics education research that reflect our society" (Lockwood, DeJarnette, & Thomas, 2019, p. 1). This study provides an example of the use of a technology not typically employed in an introductory college mathematics course, especially a course not designed for mathematics majors. While findings from this study and others using similar tools to undergraduate mathematics classrooms reveal mixed results, these undoubtedly need to continue to be developed and researched.

The following suggestions for instructors and teacher educators are also recommended:

- Professional development for educators in how to use these technologies in the classroom
- Develop and disseminate how educators conceive computational tools being used in the classroom

- Education for future STEM educators must include computation so that they can continue to develop how such tools can be used to intentionally teach at other levels of education as well

Recommendations for Research

This research on computational labs used in Business Calculus classes revealed mixed results. The analysis of students' comments showed beneficial aspects of the computational labs on many students' attitudes and on conceptual understanding. The qualitative data also helps to demonstrate that there was a mix of both positive and negative comments, which shows that some students greatly benefited while others struggled, and highlight why there was minimal change scores on both of the instruments used to gather quantitative data. The mixed results that the computational labs had on both attitude and conceptual knowledge of calculus must be researched further.

Some of the limitations of the instruments used in this study were discussed in both Chapters Three and Four. For future research it is recommended that other instruments should be used. The many limitations of the Calculus Concept Inventory have been previously discussed. In future research designs, perhaps a locally created end of semester assessment could be used. This assessment could be made in such a way that the issues of calculus terminology and notation would not be a problem as they were on the CCI. The wording of questions on the CCI were noted as confusing, using a different instrument could alleviate some of this and the questions could be written in such a way that they were more similar to the style of questions students have seen before. If such an instrument were a multiple-choice assessment, it would also not be analyzed using normalized gains. The losses seen in the current study were likely not because of actual decreases in knowledge, but rather because of correct guesses on the pre-test that were then incorrect guesses on the post-test. Normalized gains do not account for losses such as these and make the assumption that there will be improvement from pre-test to post-test. This study assumed that students would honestly and intentionally answer the questions on the Calculus Concept Inventory, however, since no credit in the class was given for completing it there was certainly apathy; a violation of this assumption could have impacted the results. Violations to this assumption would call into question the use of normalized gains. Hake's (1998) *g* has the implicit assumption that gains will be positive (Miller et al., 2010). Miller et al. (2010) assert that when losses are normalized with respect to possible gain, the normalized gain does not have a "sensible interpretation". Going forward in future research, normalized gains could not be used, instead Dewello's *G* and *L* could be computed, which according to Miller et al. (2010) normalizes gains with respect to potential gains and losses with respect to potential losses rather than *g* normalizing all to gains only. Another recommendation would be to use an instrument that is not multiple-choice but is open-ended and can be interpreted as level of understanding rather than simply right or wrong.

A recommendation for data collection on students' attitude would be a more calculus specific instrument or to allow students to self-report how they feel about their abilities such as in Kilty and McAllister (2019). Another recommendation is to consider the XPIPSM, a newly developed instrument, to see what the students thought of the course and their experience (Apkarian, et al., 2019). Another consideration would be to use a survey rating of the impact of the technology such as in Höft and James (1990) that asks questions such as "using a computer in the classroom contributed to my understanding of the course material" and "using a computer in the classroom enhanced my interest in the course material" (p. 147).

It is recommended that in replication of this project, students' comments from the control group would be gathered and analyzed as well. In future research to help triangulate the results, it is recommended that interviews with students be conducted as well. The students reflected on how they felt about the labs without solicitation specifically and thus there may be more to be gleaned from what students thought if they were specifically asked about their thoughts. This would also help to differentiate what students felt were the most beneficial aspects of the labs and further how the labs impact both understanding and attitude. The comments from the end of the labs should still be gathered, as students appeared to be rather open and honest in them perhaps because they did not feel any pressures of an interview, but interviews could help to explain how the computational labs affected students. Students may put up barriers during interviews and not give honest answers, especially if they were being interviewed by the instructor-researcher after the semester or there may be issues getting students to participate after the semester has been completed.

Some previous research has shown that one semester is not typically long enough to see a change in students' attitude and that attitude takes a long time to be changed. Because of such, it would be of interest to gather longitudinal data on students that have had Business Calculus taught with the computational labs. Many of these students go on to take a business statistics course after completing business calculus. If a second course could be developed, such as the course discussed in the recommendations for practices, students' attitudes toward mathematics could be tracked over multiple semesters to see if more exposure to such work could have more of an impact on their attitudes than simply over one semester. This longitudinal data could reveal that using these tools over multiple semesters with mathematics may have an impact on attitude.

The students' comments revealed several traits and habits that they developed by taking the Business Calculus course with computational labs. One of the overwhelming themes was persistence and fighting through challenges. While the measure of growth mindset was not found to be significant as measured by the Mathematics Attitude and Perceptions Survey for the entire group with labs, yet was for students that had previously taken calculus, students' comments often mentioned pushing through errors, being extremely frustrated, and struggles but continuously working through these problems and persisting even when faced with these challenges. Future research on the use of computational labs could be studied as a growth mindset intervention and to see how these can be used to specifically target growth mindset development.

This study took place in a general education mathematics class and demonstrated several important impacts as previously discussed. Much of the research pertaining to computational tools in undergraduate mathematics classes emphasize how these tools can be used in classes for mathematics majors or how it can be incorporated throughout the curriculum for an undergraduate mathematics program (e.g. Cline et al., 2019; Jones & Hopkins, 2019, Kostadinov, Thiel, & Singh, 2019), but there is a lack of research in classes not intended for mathematics majors. The current study provides one such example and Kilty and McAllister (2019) provide another of a general education mathematics course with computational tools designed for non-mathematics majors that reveal positive impacts. Kilty and McAllister (2019) found that their course helped lower-performing students by removing some of the algebraic manipulation and met students where they are. Because of positive findings from that study and this study, it is important to continue to develop and research courses intended for general education mathematics classes that can integrate computational tools.

Based on the results of this study, the growth of the use of computational tools in practice, and the increase in research in computational tools in STEM education, the calls of diSessa (2018) and Lockwood et al. (2019) for mathematics educators to further investigate and empirically research how computational tools can be used in mathematics classrooms are echoed.

The following suggestions for research are also recommended:

- The collection of more data points on understanding of content to compare the groups, such as grades from each test and the final exam
- Larger sample sizes including comparison from sections taught by different instructors
- Study of computational tools across introductory general education classes of other STEM disciplines and use in other mathematics classes

References

Adams, S. (2015). The 10 skills employers most want in 20-something employees. Retrieved from https://www.forbes.com/sites/susanadams/2013/10/11/the-10-skills-employers-most-want-in-20-something-employees/#6017a9766330

Adams, W. K., Perkins, K. K. Podolefsky, N. S., Dubson, M., Finkelstein, N. D., & Wieman, C. E. (2006). New instrument for measuring student beliefs about physics and learning physics: the Colorado learning attitudes about science survey. *Physical Review Physics Education Research*, *2,* 010101.

Aiken, L. R. (1970). Attitudes toward mathematics. *Review of Educational Research, 40,* 551-596.

Aiken, L. R. (1976). Update on attitudes and other affective variables in learning mathematics. *Review of Educational Research, 46,* 293-311.

Aiken, L. R., Jr., & Dreger, R. M. (1961). The effect of attitudes on performance in mathematics. *Journal of Educational Psychology, 52*(1), 19-24. http://dx.doi.org/10.1037/h0041309

Allen, K. (2006). The Statistics Concept Inventory: Development and analysis of a cognitive assessment instrument in statistics. (Unpublished doctoral dissertation). University of Oklahoma.

ANCOVA. (n.d). Retrieved from http://oak.ucc.nau.edu/rh232/courses/eps625/handouts/ancova/understanding%20ancova.pdf.

Anderson, L. & Brennan, J. P. (2015). An experiment in “flipped” teaching in freshman Calculus. *PRIMUS, 25*(9-10), 861-875.

Andersson, A., Valero, P., & Meaney, T. (2015). "I am [not always] a maths hater": Shifting students' identity narratives in context. *Educational Studies in Mathematics, 90*(2), 143-161.

Apkarian, N., Smith, W. M., Vroom, K., Voigt, M., Gehrtz, J., PtC Project Team, & SEMINAL Project Team. (2019). X-PIPS-M Survey Suite. https://www.maa.org/sites/default/files/XPIPSM%20Summary%20Document.pdf.

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, 52, 215-241.

Arney, C. (2009). Discrete math first! *The UMAP Journal, 30*(2), 93-97.

Axtell, M., Doree, S., & Dray, T. (2016). New directions for the Calculus track. *MAA Focus,* 33-34 & 37.

Ayotola, A. & Adedeji, T. (2009. The relationship between mathematics self-efficacy and achievement in mathematics. *Procedia Social and Behavior Sciences, 1,* 953-957.

Bagley, S. (2014). *Improving student success in Calculus: A comparison of four college Calculus classes.* (Doctoral dissertation, UC San Diego). Retrieved from ProQuest Dissertations and Theses. (Accession Order No. 3633127). http://sdsu-dspace.calstate.edu/bitstream/handle/10211.3/123839/Bagley_sdsu_0220D_10443.pdf

Bagley, S., Gleason, J., Rice, L., Thomas, M., & White, D. (2017). Does the Calculus Concept Inventory really measure conceptual understanding of Calculus? Retrieved from https://blogs.ams.org/matheducation/2016/07/25/does-the-Calculus-concept-inventory-really-measure-conceptual-understanding-of-Calculus/

Bailey, J., Johnson, B., Prather, E., & Slater, T. (2011). Development and validation of the star properties concept inventory. 2257-2286. https://doi.org/10.1080/09500693.2011.589869

Bao, L. (2006). Theoretical comparisons of average normalized gain calculations. American Journal of Physics, 74(10), 917. doi:10.1119/1.2213632

Barbera J, Adams WK, Wieman CE, Perkins KK. (2008). Modifying and validating the Colorado Learning Attitudes about Science Survey for use in chemistry. *Journal for Chemistry Education*, *85*, 1435–1439.

Basson, A., Krantz, S. G., & Thornton, B. (2006). A new kind of instructional mathematics laboratory. *PRIMUS, 16*(4), 332-348.

Baumgartner, J. & Shemanske, T. (1990). Teaching calculus with True BASIC. In T. W. Tucker (Ed.) *Priming the Calculus Pump: Innovations and Resources* (pp. 33-50). Washington, DC: Mathematical Association of America.

Benakli, N., Kostadinov, B., Satyanarayana, A., & Singh, S. (2017). Introducing computational thinking through hands-on projects using R with applications to Calculus, probability and data analysis. *International Journal of Mathematical Education in Science and Technology, 48*(3), 393-427. doi:10.1080/0020739x.2016.1254296

Berkeihiser, M. & Ray, D. (2013). Bringing STEM to life. *Technology and Engineering Teacher, 72*(5), 21-24.

Berrett, D. (2011, November 6). What spurs students to stay in college and learn? Good teaching practices and diversity. *The Chronicle of Higher Education.* Retrieved from https://www.chronicle.com/article/What-Spurs-Students-to-Stay-in/129670

Betz, N. E., & Hackett, G. (1983). The relationship of mathematics self-efficacy expectations to the selection of science-based college majors. *Journal of Vocational Behavior, 23*(3), 329-345.

Bishop, A. (1989). Review of research in visualization in mathematics education. *Focus on Learning Problems in Mathematics, 11*(1), 7-16.

Bleyer, D. Pedersen, K., & Elmore, P. (1981). Mathematics: A critical filter for career choice. *Journal of Career Development, 8*(1), 46-56.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education*, 29(1), 41-62.

Boaler, J. (1993). The Role of Contexts in the Mathematics Classroom: Do they make Mathematics More "Real"?, *For the Learning of Mathematics*, *13*(2), 12- 17.

Boaler, J. (2016). *Mathematical Mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching.* San Francisco, CA: Jossey-Bass, a Wiley Brand.

Bouldin, R., Hall, G., Oches, E., Szymanski, D., & Ledley, F. (2015). Connecting business and STEM education through undergraduate research. *Council on Undergraduate Research Quarterly,35*(4), 17-23.

Bowen, C. (2016). *Straightforward statistics*. Los Angeles: SAGE.

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics*, *23*(3), 247–285. doi:10.1007/ BF02309532

Bressoud, D. (2015). The Calculus Student. In D. Bressoud, C. Rasmussen, and V. Mesa (Eds.), *Insights and Recommendations from the MAA National Study of College Calculus* (pp. 1-16). Washington, D.C.: MAA Press.

Bressoud, D. (2017). Teaching calculus now: Current trends and best practices. https://www.youtube.com/watch?v=I97aXeG6C2s

Bressoud, D. & Rasmussen, C. (2015). Seven characteristics of successful Calculus programs. *Notices of American Mathematical Society, 62*(2), 144-146.

Bressoud, D., Carlson, M. P., Mesa, V., & Rasmussen, C. (2013). The Calculus student: insights from the Mathematical Association of America national study. *International Journal of Mathematical Education in Science and Technology, 44*(5), 685–698.

Bressoud, D., Mesa, V., & Rasmussen, C. (2015). *Insights and recommendations from the MAA national study of college Calculus*. Washington, DC: MAA Press.

Bressoud, D., Carlson, M., Pearson, J. M., & Rasmussen, C. (2012). Preliminary results of the study on characteristics of successful programs in college calculus. 12th International Congress on Mathematics Education. Seoul, Korea, July 8-15, 2012. Retrieved from http://www.maa.org/cspcc/TSG13_Bressoud.pdf

Brinkmann, D., Harvey, J. E., Zou, C., Crosthwait, R., Elrod, C., & Flachsbart, B. (2016). Putting a business mindset into STEM education and career planning. *Issues in Information Systems,17*(2), 242-249.

Brito, D., L. & Goldberg, D., Y. (1988). Calculus for business students, In *Calculus for a New Century: A Pump Not a Filter* (pp. 68-70). Washington: Mathematical Association of America.

Brown, D., Porta, H., A., & Uhl, J. J. (1990). Calculus and mathematica. In T. W. Tucker (Ed.) *Priming the Calculus Pump: Innovations and Resources* (pp. 104-120). Washington, DC: Mathematical Association of America.

Bryman, A. and Cramer, D. (1990) *Quantitative data analysis for social scientists*. London: Routledge.

Bureau of Labor Statistics. (2016). *Occupational outlook handbook.* Retrieved from https://www.bls.gov/ooh/

Burn, H. E. & Mesa, V. (2015) The Calculus I curriculum. In D. Bressoud, C. Rasmussen, and V. Mesa (Eds.), *Insights and Recommendations from the MAA National Study of College Calculus* (pp. 45-58). Washington, D.C.: MAA Press.

Carlson, M. P. (1998). A cross-sectional investigation of the development of the function concept. In *CBMS Issues in Mathematics Education: Vol. 7. Research in Collegiate Mathematics Education. III* (pp. 114–162). doi:10.1090/cbmath/007/04

Carlson, M. P, Buskirk, T. and Halloun, I. (1999,). Assessing college students' views about mathematics *with the* Views About Mathematics Survey*, Educational Studies in Mathematics*

Carlson, M. P. (1999). The mathematical behavior of six successful mathematics graduate students: Influences leading to mathematical success. *Educational Studies in Mathematics*, 40, 237-258.

Carlson, M., Madison, B., & West, R. (2015). A study of students' readiness to learn Calculus. *International Journal of Research in Undergraduate Mathematics Education, 1*(2), 209-233.

Carlson, M., Oehrtman, M., & Engelke, N. (2010). The PreCalculus Concept Assessment: A tool for assessing students' reasoning abilities and understandings. Cognition and Instruction, 28(2), 113–145. doi:10.1080/07370001003676587

Carnell, L., Graham-Squire, A., O'Hara, K. & Piechnick, L. (2018). Environmental impacts: How comparative prior knowledge affects students' Calculus experience. *PRIMUS.* doi:10.1080/10511970.2018.1472682

Carver, S. D., Van Sickle, J., Holcomb, J. P., Jackson, D. K., Resnick, A. H., Duffy, S. F., & Quinn, C. M. (2017). Operation STEM: Increasing success and improving retention among mathematically unprepared students in STEM. *Journal of STEM Education: Innovations and Research, 18*(3), 30-39.

Casciaro, T., Edmondson, A. C., & Jang, S. (2019). What Cross-silo leadership looks like. Retrieved from https://hbr.org/2019/05/cross-silo-leadership

Cazden, C. (1986). Classroom discourse. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 432–463). New York, NY: Macmillan.

Cetin, I. & Dubinsky, E. (2017). Reflective abstraction in computational thinking. *The Journal of Mathematical Behavior, 47,* 70-80.

Chabay, R., & Sherwood, B. (2008). Computational physics in the introductory Calculus-based course. *American Journal of Physics,76*(4), 307-313. doi:10.1119/1.2835054

Chai, J., Friedler, L., Wolff, E., Li, J., & Rhea, K. (2015). A cross-national study of Calculus. *International Journal of Mathematical Education in Science and Technology,46*(4), 481-494. doi:10.1080/0020739X.2014.990531

Chase, D. (2010). STEM and career exploratory classes. *Techniques,*34-37. Retrieved from http://www.acteonline.org

Chi-square independence test – What and why? (n.d.). Retrieved from https://www.spss-tutorials.com/chi-square-independence-test/#what-is-it

Cline, K., Fasteen, J., Francis, A. E., Sullivan, E., & Wendt, T. J. (2019). Integrating programming across undergraduate mathematics curriculum. *PRIMUS,* https://doi.org/10.1080/10511970.2019.1616637

Code, W. & Maciejewski, W. (2017, February). *The Mathematics Attitudes and Perceptions Survey: New data and alignment with other recent findings.* Poster abstract presented at the Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education Conference 2017, San Diego, CA.

Code, W., Merchant, S., Maciejewski, W., Thomas, M., & Lo, J. (2016). The Mathematics Attitudes and Perceptions Survey: An instrument to assess expert-like views and dispositions among undergraduate mathematics students. *International Journal of Mathematical Education in Science and Technology,47*(6), 917-937. doi:10.1080/0020739x.2015.1133854

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Lawrence Earlbaum Associates.

Cohen, L., Manion, L., & Morrison, K. (2009). *Research methods in education* (6th ed.). London: Routledge.

Collins, J., L. (1982). Self-efficacy and ability in achievement behavior. Paper presented at the meeting of the American Educational Research Association, New York.

Columbus, L. (2018, May 23). 10 charts that will change your perspective of big data's growth. Retrieved from https://www.forbes.com/sites/louiscolumbus/2018/05/23/10-charts-that-will-change-your-perspective-of-big-datas-growth/#193ef2752926

Computational modeling and mathematics applied to the physical science (pp. 1-105, Publication). (1984). Washington, DC: National Academy Press. (ERIC Document Reproduction Service No. ED247083)

Cooper, S. & Robinson, D. (1991). Thee relationship of mathematics self-efficacy beliefs to mathematics anxiety and performance. *Measurement & Evaluation in Counseling & Development, 24,* 4-11.

Corbitt, M. K., ed. 1985. The impact of computing technology on school mathematics: Report of an NCTM conference. *Mathematics Teacher, 78,* 243-249.

Cornell, C. (1999). I hate math!: I couldn't learn it, and I can't teach it!. *Childhood Education*, *75*(4), 225-230. doi: 10.1080/00094056.1999.10522022

Couch, R., & Haines, C. (2004). Mathematical modeling: transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology, 35*(2), 197-206.

Courage, K. (2016). Calculus I factors women out of STEM degrees, researchers find. Retrieved from https://source.colostate.edu/Calculus-i-factors-women-out-of-stem-degrees-researchers-find/

Crawford, K., Gordon, S., Nicholas, J., & Prossert, M. (1994). Conceptions of mathematics and how it is learned: The perspectives of students entering university. *Learning & Instruction*, *4*(4), 331-345.

Crowe, D. & Zand, H. (2000). Computers and undergraduate mathematics 2: on the desktop. *Computers & Education, 35*(2), 317-344.

Crowell, R. H. & Prosser, R. T. (1991). Computers with Calculus at Dartmouth. *PRIMUS, 1*(2), 149-158.

Dahl, W. (2014). What kind of math courses are required for a business degree? Retrieved from https://www.greatbusinessschools.org/what-kind-of-math-courses-are-required-for-a-business-degree/

Depaolo, C. & Mclaren, G. H. (2006). The relationship between attitudes and performance in business calculus. *INFORMS Transactions on Education 6*(2), 8-22. http://dx.doi.org/10.1287/ited.6.2.8

Dick, T. P. & Rallis, S. F. (1991). Factors and influences on high school students' career choices. *Journal for Research in Mathematics Education, 22*(4), 281-292.

diSessa, A. (2018). Computational literacy and "The Big Picture" concerning computers in mathematics education. *Mathematical Thinking and Learning*, *20*(1), 3-31. https://doi-org.ezproxy.lib.vt.edu/10.1080/10986065.2018.1403544

Dogbey, G. Y. (2010). *Attitudes of community college developmental students toward mathematics and their perception of mathematically intensive careers* (Doctoral Dissertation). Retrieved from https://etd.ohiolink.edu/!etd.send_file?accession=ohiou1273165763&disposition=inline

Dorn, B. & Tew, A. E. (2015). Empirical validation and application of the Computing Attitudes Survey. *Computer Science Education*, *25*(1), 1-36.

Dossey, J., Mullis, I. V. S., Lindquist, M. M., & Chambers, D. L. (1988). *The mathematics report card: Are we measuring up?* Princeton, NJ: Educational Testing Service.

Douglas, R. G. (1985). The importance of calculus in core mathematics. In *Toward a lean and lively calculus,* R. G. Douglas (Ed.), Washington, DC: Mathematical Association of America.

Dreyfus, T. (1994). Imagery and reasoning in mathematics and mathematics education. In D. F. Robitaille, D. H. Wheeler, & C. Kieran (Eds.) *Selected Lectures from the 7th International Congress on Mathematical Education.* (pp. 107-122). Quebec, Canada: Les Presses de l'Université Laval.

Dubinsky, E. (n.d.). Reaction to James Kaput's paper 'Democratizing access to calculus: New route to old roots'. *Informally published manuscript*, Purdue University, Purdue University, Lafayette, IN.

Dubinsky, E. & Yiparaki, O. (1996). Predicate calculus and the mathematical thinking of students. *International Symposium on Teaching Logic and Reasoning in an Illogical World.* Center of Discrete Mathematics and Theoretical Computer Science, Rutgers University.

Egerer, G. & Cannon, R., J. (1988). Calculus for business and social science students, Second discussion, In *Calculus for a new century: A pump not a filter* (pp. 70-71). Washington: Mathematical Association of America.

Ellis, J., Fosdick, B. K., & Rasmussen, C. (2016). Women 1.5 times more likely to leave STEM pipeline after Calculus compared to men: Lack of mathematical confidence a potential culprit. *PlosONE, 11*(7), 1-14. doi:10.1371/journal.pone.0157447

Epstein, J. (2007). Development and validation of the Calculus Concept Inventory. Proceedings of the ninth international conference on mathematics education in a global community (pp. 165–170).

Epstein, J. (2013). The Calculus Concept Inventory–Measurement of the effect of teaching methodology in Mathematics. *Notices of the American Mathematical Society,60*(08), 1018. doi:10.1090/noti1033

Farrior, D., Hamill, W., Keiser, L., Kessler, M., LoPresti, P., McCoy, J., . . . Tapp, B. (2007). Interdisciplinary lively application projects in Calculus courses. *Journal of STEM Education,8*(3 & 4), 50-61.

Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman mathematics attitudes scales: Instruments designed to measure attitudes toward the learning of mathematics by males and females. *Journal for Research in Mathematics Education,* 7(5), 324-326.

Fenton, W. E. & Dubinsky, E. (1996). *Introduction to Discrete Mathematics with ISETL.* Springer-Verlag: New York, NY.

Ferrini-Mundy, J. & Graham, K. G. (1991). An overview of the calculus curriculum reform effort: Issues for learning, teaching, and curriculum development. *The American Mathematical Monthly, 98*(7), 627-635.

FitzSimons, G. E., & Boistrup, L. B. (2017). In the workplace mathematics does not announce itself: Towards overcoming the hiatus between mathematics education and work. *Educational Studies in Mathematics,95*(3), 329-349. doi:10.1007/s10649-017-9752-9

Flannery, W. (2013). *Mathematical modeling and computational Calculus* (Vol. 1). St. Petersburg, FL: Berkeley Science Books.

Forgasz, H. J., Leder, G. C., & Gardner, P. L. (1999). The Fennema-Sherman mathematics as a male domain scale reexamined. *Journal For Research In Mathematics Education*, *30*(3), 342-48.

Fox, G., Campbell, S., Grinshpan, A., Xu, X., Holcomb, J., Beneteau, C., & Ramachandran, K. (2017). Implementing projects in Calculus on a large scale at the University of South Florida. *Journal of STEM Education,18*(3), 30-38.

Frank, M., & Roeckerrath, C. (2016). Augmenting mathematics courses by problem-based learning. *International Journal of Engineering Pedagogy, 6*(1), 50-55. doi:10.3991/ijep.v6i1.5368

Frechtling, J., Merlino, F., & Stephenson, K. (2015). The call to transform postsecondary STEM education practices and institutional policies. *American Journal of Educational Studies, 7*(1), 27-42.

Freeman, S., Eddy, S., McDonnough, M., Smith, M., Okoroafor, N., Jort, H., & Wenderoth, M. (2014). Active learning increases student performance in science, engineering, and mathematics. *PNAS,* 111, 8410-8415. https://doi.org/10.1073/pnas.1319030111

Freudenthal, E., Roy, M., Ogrey, A., Siegel, A., & Gates, A. (2009). A computational introduction to programming, mathematical modeling, and elementary mechanics. In *39th ASEE/IEEE Frontiers in Education Conference*(pp. T4J-1-T4J-6). San Antonio, TX: IEEE.

Galindo, E. (1995). Visualization and students' performance in technology-based calculus. Paper presented at the *17th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Columbus, OH, October 21-24, 1995.

Gleason, J., Thomas, M., Bagley, S., Rice, L., White, D., and Clements, N. (2015a). Analyzing the Calculus Concept Inventory: Content validity, internal structure validity, and reliability analysis, *Proceedings of the 37th International Conference of the North American Chapter of the Psychology of Mathematics Education,* 1291-1297.

Gleason, J., White, D., Thomas, M., Bagley, S., and Rice, L. (2015b) The Calculus Concept Inventory: A psychometric analysis and framework for a new instrument, *Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education,* 135-149.

Goncalves, B., Hobbi, A., & Golnabi, A. H. (2016). A hands-on application-based tool for STEM students to understand differentiation. *2016 IEEE Integrated STEM Education Conference (ISEC),*231-234. doi:10.1109/isecon.2016.7457539

Google Colaboratory. (n.d.). Retrieved September 30, 2019, from https://colab.research.google.com/notebooks/welcome.ipynb.

Gordon, S. (1979). A discrete approach to the Calculus. *International Journal of Mathematics Education in Science and Technology,10*(1), 21-31.

Gordon, S. P. (2005). Preparing Students for Calculus in the Twenty First Century. MAA Notes: A Fresh

Gravemeijer K. & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39*(1/3), 111-129.

Gravemeijer, K., Stephan, M., Julie, C., L in, F., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education,15*(S1), 105-123. doi:10.1007/s10763-017-9814-6

Gray, K. E., Adams, W. K., Wieman, C. E., & Perkins, K. K. (2008) Students know what physicists believe, but they don't agree: a study using the class survey. *Physical Review Physics Education Research, 4*, 020106.

Greenwood, J. (1984). My anxieties about math anxiety. *Mathematics Teacher, 77,* 662-663.

Grigg, S., Perera, H. N., McIlveen, P., & Svetleff, Z. (2018). Relations among math self-effficacy, interest, intentions, and achievement: A social cognitive perspective. *Contemporary Educational Psychology*. https://doi.org/10.1016/j.cedpsych.2018.01.007

Hackett, G. (1985). The role of mathematics self-efficacy in the choice of math-related majors of college men and women: A path analysis. *Journal of Counseling Psychology, 32,* 45-56.

Hackett, G. & Betz, N. (1989). An exploration of the mathematics self-efficacy/mathematics performance correspondence. *Journal of Research in Mathematics Education, 20,* 261-273.

Hagman, J. E., Johnson, E., & Fosdick, B. (2017). Factors contributing to students and instructors experiencing a lack of time in college calculus. *International Journal of STEM Education, 4*(12).

Hake, R. R. (1998). Interactive-engagement versus traditional methods: A six-thousand student survey of mechanics test data for introductory physics courses. *American Journal of Physics*, *66*(1), 64–74. doi:10.1119/1.18809

Halpern, D. F. & Hakel, M. D. (2003) Applying the science of learning to the university and beyond. *Change*, July/August.

Hamilton, J., Rogers, M., Pfaff, T., & Erkan, A. (2010). Multidisciplinary collaborations in the traditional classroom: Wrestling with global climate change to improve science education. *Transformations: The Journal of Inclusive Scholarship and Pedagogy,21*(1), 89-98.

Harel. I. E & Papert, S. (1991). *Constructionism.* Westport, CT: Ablex.

Hastings, N. B. Ed. (2006). *A Fresh Start for Collegiate Mathematics: Rethinking the courses below calculus.* MAA Notes #69, Mathematical Association of America.

Heid M. K. (1988). Resequencing skills and concepts in applied Calculus using the computer as a tool. *Journal for Research in Mathematics Education, 19*(1), 3-25.

Heid, M. K., Blume, G. W., Hollebrands, K., & Piez, C. (2002). Computer algebra systems in mathematics instruction: Implications from research. *The Mathematics Teacher, 95*(8), 586-591.

Henrich, A., Sloughter, J. M., Anderson, J., & Bahuaud. (2016). Addressing negative math attitudes with service learning. *PRIMUS, 26*(8), 788-802. doi:10.1080/101511970.2016.1174967

Hershkowitz, R., Ben-Chaim, D., Hoyles, C., Lappan, G., Mitchelmore, M., & Vinner, S. (1989). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and Cognition*, (pp. 70-95). Cambridge: University Press.

Hicks, D. (1995-1996). Discourse, learning, and teaching. *Review of Research in Education, 21,*49-95.

Hilgers, M. G., Stanely, S. M., Elrod, C. C., & Flachsbart, B. B. (2015). Big data and business analytics in a blended computing-business department. *Issues in Information Systems, 16*(1).

Hillel, J. (2001). Trends in curriculum: A working group report. In D. Holton (ed.), *The Teaching and Learning of Mathematics at University Level,* (pp. 59–69). Dordrecht: Kluwer.

Hilpisch, Y. (2016). Computational finance- Why Python is taking over. *WilmottMagazine, 82*(3), 43-44.

Hoffman, K. R. (1989). Calculus and more: Computers, finite mathematics, and an innovative service course. *New Directions for Teaching and Learning,1989*(38), 65-72. doi:10.1002/tl.37219893810

Höft, M. & James, D. (1990). Computer labs in calculus. In T. W. Tucker (Ed.) *Priming the Calculus Pump: Innovations and Resources* (pp. 142-158). Washington, DC: Mathematical Association of America.

House, J. D. (1995) The predictive relationship between academic self-concept, achievement expectancies, and grade performance in college Calculus. *The Journal of Social Psychology, 135*(1), 111-112. doi:10.1080/00224545.1995.9711411 http://dx.doi.org/10.1006/jvbe.1994.1027

Huang, W. (n.d.). ANCOVA (Analysis of Covariance). Retrieved from https://www.lehigh.edu/~wh02/ancova.html

Hughes-Hallett, H. D. (n.d.) *What have we learned from calculus reform? The road to conceptual understanding.* Retrieved from math.arizona.edu/~dhh/NOVA/calculus-conceptual-understanding.pdf

Ismail, N. A. (2009) Understanding the gap in mathematics achievement of Malaysian students. *The Journal of Educational Research, 102,* 389-394.

Jahnke, H. (1983). Technology and education: The example of the computer. *Educational Studies in Mathematics, 14*(1), 87-100.

Jennings, S. E. & Onwuegbuzie, A. J. (2001). Computer attitudes as a function of age, gender, math attitude, and developmental status. *Journal of Educational Computing Research, 25*(4), 367-384.

Jolley, A., Lane, E., Kennedy, B., & Frappe´-Se´ne´clauze, T-P. (2012). SPESS: A new instrument for measuring students’ perceptions about earth and ocean science. *Journal for Geoscience Education, 60*, 83-91.

Jones, L. B. & Hopkins, B. J. (2019). Teaching a course in mathematical programming. *PRIMUS,* https://doi.org/10.1080/10511970.2019.1619207

Judson, P. (1990). Elementary business calculus with computer algebra. *Journal of Mathematical Behavior, 9*(2), 153-157.

JupyterHub. (n.d.). Retrieved September 30, 2019, from https://jupyterhub.readthedocs.io/en/stable/.

Kaput, J., J. (1994). The representational roles of technology in connecting mathematics with authentic experience. *Didactics of Mathematics as a Scientific Discipline*. Kluwer Academic Publishers, Dordrecht, 379-297.

Kaput, J., J. (1997). Rethinking Calculus: Learning and thinking. *The American Mathematical Monthly,104*(8), 731-737. doi:10.2307/2975238

Kaya, D. & Bozdag, H., C. (2016). Resources of mathematics self-efficacy and perception of science self-efficacy as predictors of academic achievement. *European Journal of Contemporary Education, 18*(4), 438-451.

Keeler, S. P., & Grandine, T. A. (2013). Getting math off the ground: applied mathematics at Boeing. *New ICMI Study Series Educational Interfaces between Mathematics and Industry,*27-42. doi:10.1007/978-3-319-02270-3_3

Kilty, J. M. & McAllister, A. M. (2019). A modeling-based approach to calculus. *PRIMUS,* https://doi.org/10.1080/10511970.2019.1588813

Klein, F., Hedrick, E. R., & Noble, C. A. (2007). *Elementary mathematics from an advanced standpoint*. New York: Cosimo Classics. (Original work published in 1924).

Klymkowsky, M.W., Underwood, S.M., & Garvin-Doxas, R.K. (2010). Biological Concepts Instrument (BCI): A diagnostic tool for revealing student thinking. *arXiv:1012.4501v1*.

Knill, O. (n.d.). On the Harvard Consortium Calculus. Retrieved from http://www.math.harvard.edu/~knill/pedagogy/harvardCalculus/

Koehler, J. F. (2017). Jupyter Notebooks and Undergraduate Mathematics Curricula - JupyterCon in New York 2017. Retrieved from https://conferences.oreilly.com/jupyter/jup-ny-2017/public/schedule/detail/62340.

Koehler, J. F. (2018a, March). Calculus with Python [E-mail to the author].

Koehler, J. F. (2018b, May/June). Calculus with Python [E-mail to the author].

Koehler, J. F., & Kim, S. (2018). Interactive classrooms with Jupyter and Python. *The Mathematics Teacher,111*(4), 304. doi:10.5951/mathteacher.111.4.030

Kostadinov, B., Thiel, J., & Singh, S. (2019). Creating dynamic documents with R and Python as computational and visual tools for teaching differential equations. *PRIMUS, 29*(6), 584-605.

Kotok, S. (2017). Unfulfilled potential: High-achieving minority students and the high school achievement gap in math. *High School Journal, 100*(3), 183-202.

Kowalczyk, R. E. & Hausknecht, A. O. (1994). Our experiences with using visualization tools in teaching Calculus. Presented at the Annual Conference of the American Mathematical Association of Two-Year Colleges (Tulsa, OK, November 3-6, 1994).

Kurczy, S. (2018, September 12). MBAs Who Code. Retrieved from https://www8.gsb.columbia.edu/articles/ideas-work/mbas-who-code

Kuh, G. D. (2003). Built to engage: Liberal arts colleges and effective educational practice. *Invited paper for ACLS Conference on Liberal Arts Colleges in American Higher Education.* Williamstown, Massachusetts, November 15, 2003.

Lamoureux, C., Beach, L., & Hallet, D. (2000). Business and Management CRAFTY Curriculum Foundations Project. In The Mathematical Association of America, *The curriculum foundations project: Voices of the partner disciplines* (19-25).

Larkin, K. & Jorgensen, R. (2016). I hate maths: Why do we need to do maths? Using iPad video diaries to investigate attitudes and emotions towards mathematics in year 3 and year 6 students. *International Journal of Science and Math Education 14,* 925-944. doi:10.1007/s10763-015-9621-x

Larsen, S., Glover, E., & Melhuish, K. (2015). Beyond good teaching: The benefits of ambitious teaching In D. Bressoud, C. Rasmussen, and V. Mesa (Eds.), *Insights and Recommendations from the MAA National Study of College Calculus* (pp. 93-106). Washington, D.C.: MAA Press.

Larson, M. (2017). Math education is STEM education! Retrieved from https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Matt-Larson/Math-Education-Is-STEM-Education!/

Ledley, F. D. (2012). Bridging the boundary between science and business. *International Journal of Science in Society, 3*(3), 171-194.

Ledley, F. D., & Holt, S. S. (2014). Learning objectives and content of science curricula for undergraduate management education. *Journal of Management Education, 38*(1), 86-113.

Ledley, F. D., & Oches, E. (2013). Business education in an age of science and technology. In M. Hardy, M. Deniel, and L. Everett, *Shaping the Future of Business Education: Relevance, Rigor, and Life Preparation,* 162-174. Basingstoke, UK: Palgrave Macmillian.

Leinbach, L. C. (1991). *The laboratory approach to teaching Calculus*. Washington, D.C.: Mathematical Association of America.

Lent, R. W., Brown, S. D., & Hackett, G. (1994). Toward a unifying social cognitive theory of career and academic interest, choice, and performance. *Journal of Vocational Behavior, 45*(1), 79-122.

Lew, K., Fukawa-Connelly, T. P., Mejia-Ramos, P., & Weber, K. (2016). Lectures in advanced mathematics: Why students might not understand the mathematics the professor is trying to convey. *Journal for Research in Mathematics Education, 47*(2), 162-198.

Lexico. (n.d.). Persistence: Definition of Persistence. Retrieved from https://www.lexico.com/en/definition/persistence

Liang, J. & Pan, W. S. Y. (2009). A MATLAB-aided method for teaching Calculus-based business mathematics. *American Journal of Business Education, 2*(9), 15-40.

Lockwood, E., DeJarnette, A. F., & Thomas, M. (2019). Computing as a mathematical disciplinary practice. *Journal of Mathematical Behavior, 54,* doi.org/10.1016/j.jmathb.2019.01.004

Lovric, M. (2018). Programming and mathematics in an upper-level university problem-solving course. *PRIMUS.* doi:10.1080/10511970.2017.1403524

Ma, X. & Willms, J. D. (1999). Dropping out of advanced mathematics: How much do students and schools contribute to the problem? *Educational Evaluation and Policy Analysis, 21*(4), 365-383.

Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A Meta-Analysis. *Journal for Research in Mathematics Education,28*(1), 26-47. doi:10.2307/749662

Maciejewski, W. (2016). Flipping the Calculus classroom: an evaluative study. *Teaching Mathematics and Its Applications, 35*, 187-201.

Maltese, A. V., & Tai, R. H. (2011). Pipeline persistence: Examining the association of educational experiences with earned degrees in STEM among U.S. students. *Science Education*, 95(5), 877. Retrieved from http://libproxy.highpoint.edu/login?url=https://search.proquest.com/docview/889823634?accountid=11411

Mayes, R., Chase, P.N., & Walker, V.L. (2008). Supplemental practice and diagnostic assessment in an applied college algebra course. *Journal of College Reading and Learning, 38*(2), 7-30.

McBane, D. (2003). Getting a horse to drink: Teaching technology to marketing students. *Marketing Education Review, 13*(2), 1-6.

McCann, J. (2006). The Next Economy. *BizEd, 5*(3), 40-44.

Mesa, V., Burn, H., & White, N. (2015). Chapter 7: Good teaching of calculus. In D. Bressoud, C. Rasmussen, and V. Mesa (Eds.) *Insights and Recommendations from the MAA National Study of Calculus.* (pp. 83-91). Washington, DC: MAA Press.

Meurer, A., Smith, C., Paprocki, M., Certik, O., Kirpichev, S., Rocklin, M., et al. (2016). SymPy: symbolic computing in Python. *PeerJ Computer Science.* doi:10.7717/peerj-cos.103

Middleton, J. and Spanias, P. 1991. Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education*, *30*(1): 65–88.

Miller, K., Lasry, N. Reshef, O., Dowd, J., Araujo, I., & Mazur, E. (2010). Losing it: The influence of losses on individual normalized gains. *Conference Paper.* DOI: 10.1063/1.3515208

Mkhatshwa, T. (2017). Undergraduate students' reasoning about marginal change in a profit maximization context: The case of Carlos and Mark. In *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 761-764). Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Morse, J. (2003). Principles of mixed methods and multimethod research design. In *Handbook of Mixed Methods in Social and Behavioral Research* (pp. 189-208). Thousand Oaks, CA: SAGE.

Mosina, N. (2014). Using wiki-based discussion forums in Calculus: E-pathway toward improving students retention and learning in STEM gateway courses (Minority serving two-year college settings). *2014 IEEE Integrated STEM Education Conference*. doi:10.1109/isecon.2014.6891038

Muhundan, A. (2005). *Effects of using graphing calculators with a numerical approach on students' learning of limits and derivatives in an applied calculus course at a community college* (Doctoral dissertation). Retrieved from American Studies Commons, http://scholarcommons.usf.edu/etd/780

Mulford, D. & Robinson, W. (2002) An inventory for alternate conceptions among first-semester general chemistry students. *Journal of Chemistry Education, 79*(6). doi:10.1021/ed079p739

Nagle, C., Moore-Russo, D., & Casey, S. (2017). Slope and line of best fit: A transfer of knowledge case study. *School Science & Mathematics, 111* (1-2), 13-26.

Narasimhan,C. C. (1993). Calculus reform for the non-science client disciplines. *PRIMUS, 3*(3), 254-262.

National Council of Teachers of Mathematics. (1974). NCTM board approves policy statement on the use of minicalculators in the mathematics classroom. *NCTM Newsletter 11*(3).

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics in the 1980s.* Reston, VA.

National Council of Teachers of mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA.

National Council of Teachers of Mathematics. (2000). *Principles of standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2010). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2014). Principles to actions: ensuring mathematical success for all. Reston, VA: NCTM.

National Network of Business and Industry Associations. (2014). Common employability skills. Retrieved from https://www.businessroundtable.org/sites/default/files/Common Employability_asingle_fm.pdf

Neale, D. C. (1969). The role of attitudes in learning mathematics. *The Arithmetic Teacher, 16*(8), 631-640.

Ng, W. L., Choo, K. T., Soon, L. H., Yi-Huak, K., & Sun, Y. Y. (2005). Effects of using a computer algebra system (CAS) on junior college students' attitudes towards CAS and achievement in mathematics. *International Journal for Technology in Mathematics Education, 12*(2), 59-72.

Nievergelt, Y. (1996). Is optimal pricing a myth from Business Calculus? Is Business Calculus an oxymoron? *The American Mathematical Monthly,103*(2), 143-148. doi:10.1080/00029890.1996.12004713

Nissen, J., Talbot, R., Thompson, A., & Ven Dusen, B. (2018). Comparison of normalized gain and Cohen's d for analyzing gains on concept inventories. *Physical Review Physics Education Research ,114,* 1-27.

Norman, G. (2010). Likert scales, levels of measurement and the "laws" of statistics. *Advances in Health Sciences Education, 15*(5), 625–632.

Norton, P., Bridges, W., & High, K. (2018). Impact of course policy changes on Calculus I DFW Rates. *Journal Of STEM Education: Innovations And Research*, *19*(1), 41-47.

Núñez-Peña, M., Suárez-Pellicioni, M., & Bono, R. (2013). Effects of math anxiety on student success in higher education. *International Journal of Educational Research,58*, 36-43. doi:10.1016/j.ijer.2012.12.004

One-way ANCOVA in SPSS Statistics. (n.d.). Retrieved from https://statistics.laerd.com/spss-tutorials/ancova-using-spss-statistics.php

Pagano, R. R. (2004). *Understanding Statistics in the Behavioral Sciences* (7th ed.). Belmont, CA: Thomson/Wadsworth.

Pajares, F. & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary Educational Psychology, 24*(2), 124-139.

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology,86*(2), 193-203. doi:10.1037//0022-0663.86.2.193

Papert, S. (1980). *Mindstorms : Children, computers, and powerful ideas*. New York: Basic Books.

Parameswaran, R. (2007). On understanding the notion of limits and infinitesimal quantities. *International Journal of Science and Mathematics Education, 5*(2), 193-216.

Park, J. (2013). Is the derivative a function? If so, how do students talk about it? *International Journal of Mathematical Education in Science and Technology, 44*(5), 624-640. https://doi.org.libproxy.highpoint.edu/10.1080/0020739X.2013.795248

Perkins, D. N. (2012). *Calculus and its origins*. Washington, D.C.: Mathematical Association of America.

Peters, M. (2013). Examining the relationships among classroom climate, self-efficacy, and achievement in undergraduate mathematics: A multi-level analysis. *International Journal of Science and Mathematics Education, 11*(2), 459-480. doi:10.1007/s10763-012-9347-y

Peters, T., Johnston, E., Bolles, H., Ogilvie, C., Knaub, A., & Holme, T. (2019). Benefits to students of team-based learning in large enrollment calculus. *PRIMUS,* DOI: 10.1080/10511970.2018.1542417

Pietsch, J., Walker, R., & Chapman, E. (2003). The relationship among self-concept, self-efficacy, and performance in mathematics during secondary school. *Journal of Educational Psychology, 95,* 589-603.

President's Council of Advisors on Science and Technology (2012). *Engage to excel: Producing one million additional college graduates with degrees in science, technology,* *engineering, and mathematics.* Washington, D.C.: Office of Science and Technology Policy.

Pritchard, R., Potter, G., & Saccucci, M. (2004). The selection of a business major: elements influencing student choice and implications for outcomes assessment. *Journal of Education for Business,* 152-156.

Project Jupyter. (n.d.). Retrieved from http://jupyter.org/

Pyzdrowski, L., Sun, Y., Curtis, R., Miller, D., Winn, G., & Hensel, R. (2013). Readiness and attitudes as indicators for success in college Calculus. *International Journal of Science and Mathematics Education, 11,* 529-554.

Quinlan, J. (2016). Mathematicians' perspectives on the utility of software. *International Journal of Technology in Mathematics Education*, *23*(3), 103-109. http://dx.doi.org.ezproxy.lib.vt.edu/10.1564/tme_v23.3.02.

Rallis, S. F. & Rossman, G. B. (2003). Mixed methods in evaluation contexts: A pragmatic framework. In A. Tashakkori & C. Teddlie (Eds.), *Handbook of mixed methods in social and behavioral research* (pp. 491-512). Thousand Oaks, CA: Sage.

Ralston, A. (1984). Will discrete mathematics surpass Calculus in importance? *The College Mathematics Journal, 15*(5), 371-373.

Rasmussen, C., Marrongelle, K., Borba, M. (2014). Research on calculus: What we know and where do we need to go? *ZDM: The International Journal on Mathematics Education, 46*(4), 507-515.

Rasmussen, C., Apkarian, N., Hagman, J. E., Johnson, E., Larsen, S., & Bressoud, D. (2019). Characteristics of precalculus through calculus 2 programs: Insights from a national census survey. *Journal for Research in Mathematics Education, 50*(1), 98-112.

Rayome, A. D. N. (2019, January 27). Why big banks are requiring workers to learn coding. Retrieved from https://www.techrepublic.com/article/why-big-banks-are-requiring-workers-to-learn-coding/

Rich, P. J., Bly, N., & Leatham, K. R. (2014). Beyond cognitive increase: Investigating the influence of computer programming on perception and application of mathematical skills. *Journal of Computers in Mathematics and Science Teaching, 33*(1), 103-128.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. Journal of Educational Psychology, 93(2), 346–362.

Rosken, B. & Rolka, K. (2006). A picture is worth a 1000 words- The role of visualization in mathematics learning. *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, PME30*, 4, 457-464

Saldaña, J. (2016). *The coding manual for qualitative researchers*(3rd ed.). Los Angeles, CA: SAGE.

Satchwell, R. E. & Loepp F. (2002). Designing and implementing integrated mathematics, science, and technology curriculum for the middle school. *Journal of Industrial Teacher Education, 39*(3), 41-66.

Saxe, M. & Braddy, L. (2015). *A Common Vision for Undergraduate Mathematical Sciences Programs in 2015.* Washington, Dg: Mathematical Association of America.

Schober, C., Schutte, K., Koller, O., McElvany, N., Gebauer, M. (2018). Reciprocal effects between self-efficacy and achievement in mathematics and reading. *Learning and Individual Differences, 63*, 1-11. https://doi.org/10.1016/j.lindif.2018.01.008

Schroeder, L., McGiveny-Burelle, J., & Xue, F. (2015). To flip or not to flip? An exploratory study comparing student performance in Calculus I. *PRIMUS, 25(*9-10), 876-885. doi:10.1080/10511970.2015.1050617

Schommer-Aikins, M., Duell, O. K., & Hutter, R. (2005). Epistemological beliefs, mathematical problem-solving, and academic performance of middle school students. *The Elementary School Journal, 105*, 289-304.

Schwartz, A. E. (2006). Learning math takes attitude, perseverance, and courage. *The Education Digest, 71*(7), 50-54.

Schwingendorf, K. E. & Dubinsky, E. (1990). Calculus, concepts, and computers: Innovations in learning. In T. W. Tucker (Ed.) *Priming the Calculus Pump: Innovations and Resources* (pp. 175-198). Washington, DC: Mathematical Association of America.

Selden, A., Selden, J., Hauk, S. & Mason, A. (2000). Why can't calculus students access their knowledge to solve non-routine problems? In *CBMS Issues in Mathematics Education: Vol 8. Research in collegiate mathematics education. IV* (pp. 128–153). Providence, RI & Washington, DC: American Mathematical Society & Mathematical Association of America.

Selinski, N. E. & Milbourne, H. (2015). The Institutional Context. In D. Bressoud, C. Rasmussen, and V. Mesa (Eds.), *Insights and Recommendations from the MAA National Study of College Calculus* (pp. 31-44). Washington, D.C.: MAA Press.

Semsar, K., Knight, J. K., Birol, G., & Smith, M. K. (2011). The Colorado learning attitudes about science survey (CLASS) for use in Biology. *CBE Life Sciences Education*, *10*(3), 268–278. http://doi.org/10.1187/cbe.10-10-0133

Seymour, E. (2006). Testimony offered to the Research Subcommittee of the Committee on Science of the U.S. House of Representatives hearing on Undergraduate 231 Science, Mathematics, and Engineering Education: What's Working? Available from http://commdocs.house.gov/committees/science/hsy26481.000/hsy26481_0f.htm

Seymour, E., & Hewitt, N. M. (1997). *Talking about leaving: Why undergraduates leave the sciences.* Boulder, CO: Westview Press.

Sidawi, M. (2007). Teaching Science Through Designing Technology. *International Journal of Technology Design Education*. doi:10.1007/s10798-007-9045-1

Simonson, M., & Maushak, N. (2001). Instructional technology and attitude change. In D. H. Jonassen (Ed.), *Handbook of research for educational communications and technology.* New York: Simon & Schuster Macmillan.

Simpson, J. C. (2001). Segregated by subject: Racial differences in the factors influencing academic major between European Americans, Asian American, and African, Hispanic, and Native Americans. *The Journal of High Education, 72*(1)*,* 63-100.

Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *The Journal of Educational Research, 95*, 323-333.

Smith, A. A. (2016). Teaching computer science to biologists and chemists, using Jupyter notebooks: tutorial presentation*. Journal of Computing Sciences in Colleges, 32* (1), 126—128.

Soft Skills: Definitions and Examples. (n.d.). Retrieved from https://www.indeed.com/career-advice/resumes-cover-letters/soft-skills

Sonnert, G. & Sadler, P. (2015). Chapter 2: The impact of instructor and institutional factors on students' attitudes. In D. Bressoud, C. Rasmussen, and V. Mesa (Eds.), *Insights and recommendations from the MAA national study of college Calculus,* (pp. 17-30). Washington, DC: MAA Press.

Sonnert, G., Sadler, P. M., Sadler, S. M., & Bressoud, D. M. (2015). The impact of instructor pedagogy on college Calculus students' attitude toward mathematics. *International Journal of Mathematical Education in Science and Technology, 46*(3), 370-387. doi:10.1080/0020739x.2014.979898

St. John, R. (2005). Retrieved July 06, 2019, from https://www.ted.com/talks/richard_st_john_s_8_secrets_of_success?utm_campaign=&utm_medium=&utm_source=&utm_content=talk&utm_term=business#t-139930

Stankov, L., & Lee, J. (2014). Quest for the best non-cognitive predictor of academic achievement. Educational Psychology, 34, 1–8. doi:10.1080/01443410.2013.858908

Strauss, L. C. & Volkwein,F.J. (2002). Comparing student performance and growth in 2- and 4-year institutions. *Research in Higher Education, 43*(2), 133-161.

Stevens, T., Olivares, A., Jr., & Hamman, D. (2006). The role of cognition, motivation, and emotion in explaining the mathematics achievement gap between Hispanic and White students. *Hispanic Journal of Behavioral Sciences, 28,* 161-186.

Sutherland, R. (1994). The role of programming In *Didactics of Mathematics as a Scientific Discipline, 177-187* (Biehler, R., Scholz, R. W., Winklemann, S. b. (Eds.) Dordrecht: Kluwer Academic Publishers. Printed in the Netherlands.

Tall, D. (1991). Intuition and rigor: The role of visualization in the calculus. In *Visualization in teaching and learning mathematics* (pp. 105-119). Washington, DC: Mathematical Association of America.

Tall, D. O. (1992). Visualizing differentials in two and three dimensions. *Teaching Mathematics and its Applications, 11*(1), 1-7.

Tall, D. O. (2009). Dynamic mathematics and the blending of knowledge structures in the calculus. *ZDM – The International Journal on Mathematics Education*, *41*(4) 481–492.

Tallman, M. A., Carlson, M. P., Bressoud, D. M., & Pearson, M. (2016). A characterization of Calculus I final exams in U.S. colleges and universities. *International Journal of Research in Undergraduate Mathematics Education*, *2*(1), 105–133. doi:10.1007/s40753-015-0023-9

Tapia, M. (1996). The Attitudes Toward Mathematics Instrument. Paper presented at the annual meeting of the Mid-south Educational Research Association, Tuscaloosa, AL (ERIC Reproduction Service No. ED 404165).

The Joint Task Force for Computing Curricula. (2005). *Computing Curricula: The overview report covering undergraduate degree programs in computer engineering, computer science, information systems, information technology, and software engineering, September, 30, 2005.*

The Mathematical Association of America. (1988). *Calculus for a new century: A pump, not a filter, a national Colloquium, October 28-29, 1987*. Washington, D.C.: Mathematical Association of America.

Thomas, M. (2013). *Analyzing Conceptual Gains in Introductory Calculus with Interactively-Engaged Teaching Styles* (Doctoral Dissertation, Arizona State University). Retrieved from https://eric.ed.gov/?id=ED559494

Thomas, M., & Lozano, G. (2013). Analyzing Calculus Concept Inventory gains in introductory Calculus. Proceedings of the Sixteenth Annual Conference on Research in Undergraduate Mathematics Education. Denver, CO.

Thompson, P. W. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. *Educational Studies in Mathematics*, *26*(2–3), 229–274. doi:10.1007/BF01273664

Titus, A. (2018, November). Jupyter notebooks. [E-mail to the author].

Tobias, S., & Weissbrod, C. (1980). Anxiety and mathematics: An update. *Harvard Educational Review, 50,* 63-70.

Tonkes, E. J., Loch, B. I., & Stace, A. W. (2005). An innovative learning model for computation in first year mathematics. *International Journal of Mathematical Education in Science and Technology, 36*(7), 751-759.

Townsend. M., & Wilton, K. (2003). Evaluating change in attitude towards mathematics using 'then and now' procedure in a cooperative learning programme. *British Journal of Educational Psychology, 73,* 473-487.

Traxler, A., Henderson, R., Stewart, J., Stewart, G., Papak, A., & Lindell, R. (2018). Gender fairness within the Force Concept Inventory. *Physical Review Physics Education Research*, *14*(1), 010103-1-010103-17.

Tucker, T. W. (1990). *Priming the calculus pump: Innovations and resources*. Washington: Math. Assoc. of America.

Turner, J. (1983). 'Discrete' math challenges Calculus as computers alter students' needs. *Chronicle of Higher Education, 26*(17), 1&6.

University of Michigan Department of Mathematics. (2009). Calculus Concept Inventory. *Continuum,*6-7. Retrieved from https://www.math.upenn.edu/~pemantle/active-papers/UM-Calculus.pdf.

Van Damme, J., Opdenakker, M., & Van de Broeck, A. (2004). Do classes and schools have an effect on attitudes? In C. Papanastasious (Ed.), *TIMSS (Proceedings of the IEA International Research Conference (IRC-2004))* (pp 9-15). Lefkosia: Cyprus University Press.

Vandecandelaere, M., Speybroek, S., Vanlaar, G., De Fraine, B., & Van Damme, J. (2012). Learning environment and students' mathematics attitude. *Studies in Educational Evaluation, 38,* 107-120.

Vogt, W. P. (1999). *Dictionary of Statistics and Methodology: A Nontechnical Guide for the Social Sciences* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Wallace, C. S. & Bailey, J. M. (2010). Do concept inventories actually measure anything? *Astronomy Education Review, 9*(1), 010116. http://dx.doi.org/10.3847/AER2010024

Wang, X. (2013). Why students choose STEM majors: motivation, high school learning, and postsecondary context of support. *American Educational Research Journal, 50*(5), 1081-1121.

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2015). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and Technology,25*(1), 127-147. doi:10.1007/s10956-015-9581-5

Wells, J. & Ernst, J. (2012). *Integrative STEM education.* Blacksburg, VA: Virginia Tech: Invent the Future, School of Education. Retrieved from www.soe.vt.edu/istemed/

Wells, J. G. (2017). *Design based biotechnology literacy teaching guide*. Blacksburg, VA: Biosens, Inc.

Westenskow, A., Moyer-Packenham, P. S., & Child, B. (2017). An iceberg model for improving mathematical understanding and mindset or disposition: An individualized summer intervention program. *Journal of Education, 197*(1), 1-9.

Wilkerson-Jerde, M., Wagh, A., & Wilensky, U. (2015). Balancing curricular and pedagogical needs in computational construction kits: Lessons from the DeltaTick Project. *Science Education*, *99*(3), 465-499. doi:10.1002/sce.21157

Wilkins, J. L. M. & Ma, X. (2003) Modeling change in student attitude toward and beliefs about mathematics, *The Journal of Educational Research*, *97*(1), 52-63. doi:10.1080/00220670309596628

Williams, S. (2001). Predications of the limit concept: An application of repertory grids. *Journal for Research in Mathematics Education, 32*(4), 341-367.

Williams, T., & Williams, K. (2010). Self-Efficacy and performance in mathematics: Reciprocal determinism in 33 nations. *Journal Of Educational Psychology*, *102*(2), 453-466.

Wolfram, C. (2014). The UK needs a revolution in the way maths is taught. Here's why… The Guardian, Sunday 23 February 2014: downloaded from https://www.theguardian.com/education/2014/feb/23/maths-teaching-revolution-needed-conrad-wolfram

Yıldırım , B. & Sidekli, S. (2018). STEM applications in mathematics education: The effect of STEM on different dependent variables. *Journal of Baltic Science Education*, *17*(2), 200-214.

Yurek, L. A., Vasey, J., &Sullivan Havens, D. (2008). The use of self-generated identification codes in longitudinal research. *Evaluation Review, 32*(5), 435-452.

Zack, L., Fuselier, J., Graham-Squire, A., Lamb, R., & O'Hara, K. (2015). Flipping freshman mathematics. *PRIMUS, 25*(9-10), 803-813.

Zeeuw, A. D., Craig, T., & You, H. S. (2013). Assessing conceptual understanding in mathematics. *2013 IEEE Frontiers in Education Conference (FIE)*. doi:10.1109/fie.2013.6685135

Zevenbergen, R. (2004) Technologizing Numeracy: Intergenerational differences in working mathematically in new times. *Educational Studies in Mathematics, 56*(1), 91-117.

Ziegelmeier, L., & Topaz, C. (2015). Flipped Calculus: a study of student performance and perceptions. *PRIMUS, 25*(9-10), 847-860. Doi:10.1080/10511970.2015.1031305

Zimmerman, B. J., Bandura, A., & Martinez-Pons, M. (1992). Self-motivation for academic attainment: The role of self-efficacy beliefs and personal goal setting. *American Educational Research Journal, 29*(3), 663-676.

Zimmermann, W., & Cunningham, S. (1991): Editor's introduction: What is mathematical visualization. In W. Zimmermann & S. Cunningham (Eds.). *Visualization in Teaching and Learning Mathematics*, (pp. 1-8). Mathematical Association of America, Washington, DC.

Appendices

Appendix A
Course Timelines

Week	Control Course Timeline	Experimental Course Timeline
Week 1	1: Introduction, Pre-CCI and Pre-MAPS 2: Review of pre-Calculus topics	1: Introduction; Pre-CCI and Pre-MAPS 2: Lab 1: Introduction to Jupyter notebooks
Week 2	1: Cartesian coordinates and linear models 2: Functions and graphing	1: Pre-calculus review; Lab 2: Sequences and functions 2: Review of functions; Algebra of functions
Week 3	1: Algebra of functions and mathematical models 2: Exponential and Logarithmic functions	1: Cartesian coordinates and graphing functions 2: Exponential and Logarithmic functions
Week 4	1: Limits: graphically and algebraically 2: One sided limits and continuity; review	1: Limits graphically/ intuitively 2: One sided limits and continuity; review
Week 5	1: Test 1 2: Intro to the derivative; Limit definition of derivative	1: Test 1 2: Lab 3: Introduction to derivative
Week 6	1: Derivative rules: power, product 2: Quotient rule and chain rule; Higher order derivatives	1: The derivative; Derivative rules: power and product 2: Derivative rules quotient and chain; Higher order derivatives
Week 7	1: Derivatives of exponentials and logarithms 2: Practice computing derivative	1: Derivatives of exponentials and logarithms 2: Practice of computing derivatives
Week 8	1: Applications of the first and second derivative 2: Optimization 1	1: Lab 4: Applications of Derivatives 2: Applications of first and second derivative
Week 9	1: Optimization 2 2: Marginal Analysis	1: Optimization 1 2: Optimization 2
Week 10	1: Finish up derivatives/applications; review 2: Review	1: Marginal Analysis 2: Review
Week 11	1: Test 2 2: Antiderivatives and indefinite integration	1: Test 2 2: Lab 5: Integration
Week 12	1: Integration by substitution 2: Area under the curve and the definite integral	1: Antiderivatives; indefinite integrals 2: Integration by substitution
Week 13	1: Fundamental Theorem of Calculus 2: Area Between curves	1: Area under the curve and the definite integral 2: Lab 6: Area between curves
Week 14	1: Consumer and producer surplus 2: Consumer and producer surplus continued; review	1: Area between curves; Consumer and producer surplus 2: Wrap up integration; review
Week 15	1: Test 3 2: Final Exam Review	1: Test 3 2: Review for final exam
Exam Week	Final Exam; Post-CCI; Post-MAPS	Final Exam; Post-CCI; Post-MAPS

Appendix B
Excerpts from Labs

Excerpts from Functions Lab

Stock Example:

Stocks

Let's follow the link to watch an interesting video. https://www.youtube.com/watch?v=GSsefODDN_s

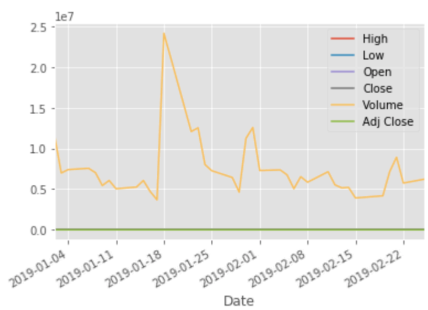
Let's use what we did in lab 1 about importing stock data to try to do an example that involves using lines to predict whether we should buy or not.

In [22]:
```
#libraries, such as pandas, that we need to import to work with the stock data
import datetime as dt
from matplotlib import style
import pandas as pd
import pandas_datareader.data as web
%matplotlib inline
import matplotlib.pyplot as plt


```

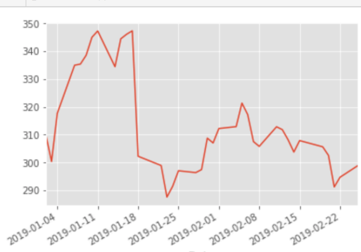
In [23]:
```
style.use('ggplot') #just a style of plot there are others

start = dt.datetime(2019, 1, 1)  #tells where to start looking at the data, year, month, day
end = dt.datetime.now() #stops looking now
df = web.DataReader('TSLA', 'yahoo', start, end)

print(df.head())

```

```
                  High         Low        Open       Close    Volume  \
Date
2019-01-02  315.130005  298.799988  306.100006  310.119995  11658600
2019-01-03  309.399994  297.380005  307.000000  300.359985   6965200
2019-01-04  318.000000  302.730011  306.000000  317.690002   7394100
2019-01-07  336.739990  317.750000  321.720001  334.959991   7551200
2019-01-08  344.010010  327.019989  341.959991  335.350006   7008500

             Adj Close
Date
2019-01-02  310.119995
2019-01-03  300.359985
2019-01-04  317.690002
2019-01-07  334.959991
2019-01-08  335.350006
```

In [24]:
```
df.plot()
plt.show()
```



In [25]:
```
df['Adj Close'].plot()
plt.show()
```



Now let's think back to the video we watched about slope using that to inform buying decisions. For now we are going to do a simplified version of this. Later, perhaps when we know more about the derivative, we will tackle it a little more sophisticatedly.

Let's find the slope of the line between the highs and the slope of the line between the lows.

```
In [26]: df[['Adj Close']]
```

Out[26]:

Date	Adj Close
2019-01-02	310.119995
2019-01-03	300.359985
2019-01-04	317.690002
2019-01-07	334.959991
2019-01-08	335.350006
2019-01-09	338.529999
2019-01-10	344.970001
2019-01-11	347.260010
2019-01-14	334.399994
2019-01-15	344.429993
2019-01-16	346.049988
2019-01-17	347.309998
2019-01-18	302.260010
2019-01-22	298.920013
2019-01-23	287.589996
2019-01-24	291.510010
2019-01-25	297.040009

```
In [27]: (300.359985-287.589996)/(3-23)
Out[27]: -0.6384994500000005

In [28]: (347.260010-347.309998)/(11-17)
Out[28]: 0.008331333333330804
```

Supply and Demand Example:

Example: Restaurants need chicken and Farms need to sell chicken

The weekly demand for chicken at several local farm to table restaurants is given in the following table:

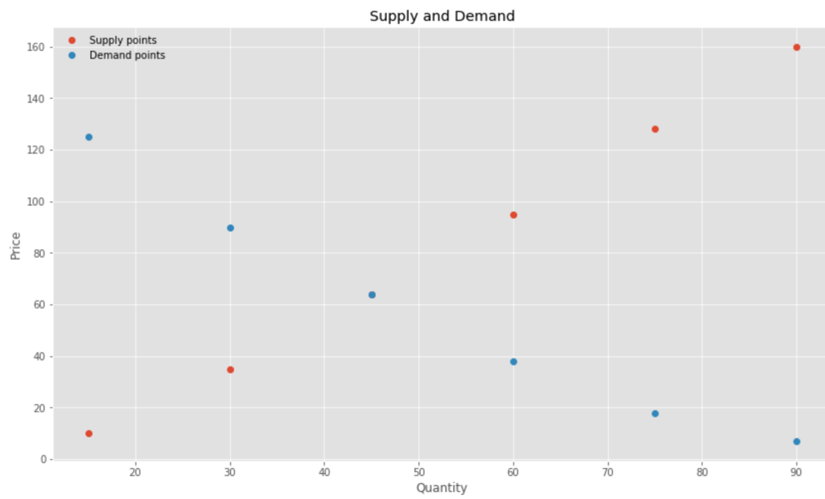
Price per kilogram	Farm Eats	Table Farm	Local Eatz	Yumz	The Chicken	Total Market Demand
15	30	40	10	15	30	125
---------------	----	----	----	----	----	---
30	20	29	8	10	25	90
---------------	----	----	----	----	----	---
45	10	20	7	7	20	64
---------------	----	----	----	----	----	---
60	5	10	5	3	15	38
---------------	----	----	----	----	----	---
75	0	8	1	1	8	18
---------------	----	----	----	----	----	---
90	0	5	0	0	2	7

The weekly supply for two local chicken farms is given in the table below:

Price per kilogram	Cornish Cross	Jersey Giants	Total Market Supply
15	0	10	10
---------------	----	---	----
30	15	20	35
---------------	----	---	----
45	30	34	64
---------------	----	---	----
60	45	50	95
---------------	----	---	----
75	58	70	128
---------------	----	---	----
90	70	90	160

In [43]:
```python
fig = plt.figure(figsize= (14, 8)) #create figure and reset q to be numbers
q = np.linspace(0, 40, 1000)

ax = fig.add_subplot(111)

plt.plot(x, s, 'o', label = "Supply points") #plot supply, demand
plt.plot(x, d, 'o', label = "Demand points")



plt.title("Supply and Demand")#add titles and legend
plt.legend(frameon = False)
plt.xlabel("Quantity")
plt.ylabel("Price")
```

Out[43]: Text(0,0.5,'Price')



This next cell will get the coefficients of the demand curve and the next uses them to write in the form $ax^2 + bx + c$

In [44]:
```python
a, b, c=np.polyfit(x,d,2)
print(a, b, c)
```

```
0.0114285714286 -2.78476190476 164.2
```

In [45]:
```python
#using the coefficients found above defining the demand curve as a function
def P(x):
    return (0.0114285714286*(x**2)-2.78476190476*x+164.2)
```

This following cell is the coefficents for the supply curve and then writes it in the form of a quadratic function

In [46]:
```python
e, f, g=np.polyfit(x,s,2)
print(e ,f, g)
```

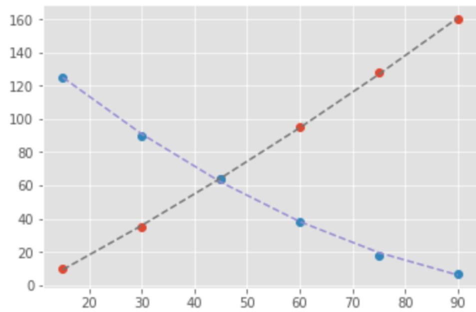
```
0.00404761904762 1.59404761905 -15.5
```

In [47]:
```python
def Z(x):
    return (0.00404761904762*(x**2)+1.59404761905*x+-15.5)
```

In [48]:
```python

x=np.arange(15,105,15) #had to redefine x so that it could be used in the functions for supply and demand.  Here i
print(x)
plt.plot(x, s, 'o', label = "Supply points") #plot supply, demand as the data points
plt.plot(x, d, 'o', label = "Demand points")
plt.plot(x, P(x), '--', label="Demand Curve") #plot the fitted supply and demand curves
plt.plot(x, Z(x), '--', label="Supply Curve")
```

```
[15 30 45 60 75 90]
```

Out[48]: [<matplotlib.lines.Line2D at 0x18163cb898>]



In [49]:

```
x = sy.Symbol('x')
eq = sy.Eq(P(x), Z(x))
sy.solve(eq)
```

Out[49]: [44.3547149654631, 548.903349548511]

Note: Supply and demand functions aren't always quadratic. We we see some other examples in our notes.

Excerpts from Derivative Lab

Example of CDC Data File:

The data in the CSV file on blackboard, has Years, Sales in Pounds, Sales in Dollars, and Advertising and Promotional Expenditures for smokeless tobacco products from 1985-2015.

In [79]:

```
import csv
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from scipy.stats import expon
import datetime as dt
from matplotlib import style
import pandas as pd
import pandas_datareader.data as web
%matplotlib inline
import matplotlib.pyplot as plt
```

In [80]:

```
df=pd.read_csv("smokelesstobacco.csv")
print(df)
```

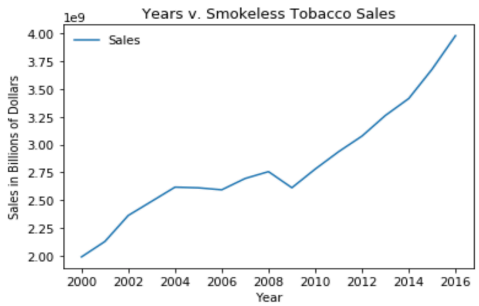
```
    Year       Pounds       Sales  Advertising (dollars)
0   2000  111,741,335  1988875535              224582757
1   2001  112,193,550  2127520387              236676917
2   2002  112,148,366  2362166931              234645000
3   2003  112,924,505  2489501857              242514000
4   2004  116,768,672  2617388686              231084000
5   2005  116,197,005  2611292547              250792000
6   2006  115,818,739  2593436592              354123000
7   2007  118,234,763  2695462138              411239000
8   2008  119,915,125  2757087244              547873000
9   2009  117,693,273  2611908686              493071000
10  2010  120,522,070  2780437133              444494000
11  2011  122,735,863  2936852337              451985000
12  2012  125,496,642  3077403248              435927000
13  2013  128,043,919  3263105347              503161000
14  2014  127,810,528  3415702358              600786000
15  2015  129,363,158  3680195446              684938000
16  2016  131,433,651  3981902822              759348000
```

In [81]:

```
x=df.Year
y=df.Sales
plt.plot(x,y)
plt.title("Years v. Smokeless Tobacco Sales")#add titles and lege
plt.legend(frameon = False)
plt.xlabel("Year")
plt.ylabel("Sales in Billions of Dollars")
```

Out[81]:

```
Text(0,0.5,'Sales in Billions of Dollars')
```



From the graph where does the rate of change seem to be the greatest?

Use the np.polyfit and different degree polynomials to try to fit a curve that models this data. Then find its derivative and use that to estimate the rate of change in sales in the year 2015.

Appendix C
Online Consent Statement

Students coming into Business Calculus at High Point University have a diverse set of previous calculus experiences, varying skill levels, and leave the course with varying degrees of understanding and attitude. In the interest of helping to make this course more effective, the math department is studying how different interventions and teaching methods affect student attitude and calculus understanding. In particular, we are interested in getting information about how using labs to introduce students to calculus topics affects their feelings toward mathematics and their performance in calculus. We will be doing the study regardless, but in order to share our findings with others, we need to have consent from students to use their data.

There are no repercussions (either negative or positive) for either giving or not giving consent. We will not look at any of the data until after final grades have been turned in, and the data is encoded so that the investigators will *not* be able to link individual students to what they put in the surveys.

Appendix D
Consent Form

You have been asked to participate in a research study. Please read this form carefully and ask any questions you have before agreeing to take part in the study.

What the study is about: *The purpose of this study is to investigate the effects computational labs in a one semester Business Calculus course on students' attitudes toward mathematics and on students' calculus content knowledge.*
What you will be asked to do: *You will be asked to participate in administration of pre-test and post-test Calculus Concept Inventory and Mathematics Attitudes and Perceptions Survey along with participating in the activities that are part of your business calculus course. You will also be asked to provide your final grade in the course and demographic information such as gender, previously taken calculus, and college major.*
What good will come from the study: *This study may provide evidence to support a new pedagogical tool to employ in Business Calculus that could make the material and techniques more relevant to outside of the classroom, perhaps improving attitudes about the course and potentially increasing understanding by making connections to business applications and doing so using computing. Students in this study may also learn a new skill and way to problem solve by being introduced to simple programming and data analysis.*

Important Things to Know about Being Part of the Study

1. **You don't have to do this.** Participation is completely voluntary and you can withdraw at any time without penalty, even after you start.

2. **Pay**. There is none for doing this. You are doing it for free.

3. **Risks to you.** I believe that there are *no* risks to you. If you are hurt, you may seek treatment through Student Health Services in Wilson Hall (for full-time day students). Otherwise, you or your health care insurer will have to pay. If you have any questions about what your insurer will pay for, you should contact them.

4. **Your responses will be kept confidential.** Your name will not be stored with your responses and only those involved in the research project will have access to the responses of individuals.

5. **If you have questions about the study.** Contact Brielle Spencer-Tyree, (336)841-9589, email: btyree@highpoint.edu

6. **If you have questions regarding your rights as a subject in this study.** You may contact Dr. Kimberly Wear, IRB Chair, (336) 841-9246, kwear@highpoint.edu.

Statement of Consent: I have read the above information, and have received answers to any questions I asked. I agree to participate in this research study and am at least 18 years of age.

Signature: ____ Date: ___________________
Printed Name: ______________________________________

Person Obtaining Consent: I have explained to the above named individual the nature and purpose, the potential benefits and possible risks associated with participation in this research. I have answered any questions that have been raised and I will provide the participant with a copy of this consent form.

Signature: ____ Date: ___________________
Printed Name: ______________________________________
This consent form will be kept by the researcher for at least three years beyond the end of the study and was approved by the IRB on (date).