The Effects of Social Support from Parent, Teacher, and Peers on High School Students’ Math Achievement: The Mediational Role of Motivational Beliefs

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ABSTRACT

The present study explored the direct influences of contextual social support, including parental involvement, perceived teacher support, and peer influence, on 11th grade students’ math achievement. The study also examined the indirect influences of these contextual social support factors on students’ achievement through their math motivation in math courses. The first follow-up year data of High School Longitudinal Study of 2012 (HSLS: 09) was used for this study. Structural equation modeling (SEM) served as the main statistical technique to examine the relationship among variables. The results of this study showed three sets of important findings. The first set showed that students’ perception of teacher support and peer influence were significantly and directly related to students’ math achievement, with the relationship between peer influence and math achievement being positive and the relationship between perceived teacher support and math achievement being negative. Controlling for other variables in the model, parental involvement was not significantly related to student math achievement. The second set of findings demonstrated that math motivation indeed plays a significant role in mediating the relationships of social support (from teachers and peers, but not from parental involvement) and student math achievement in high school. The third set of findings indicated that both family SES and prior math achievement influenced student social support and math achievement. Furthermore, two main deviations were found between White/Asian and African-American/Hispanic student models. Perceived teacher support negatively and significantly influenced White/Asian students’ math achievement, but it had no significant influence on African-American/Hispanic students. In addition, math motivation had a stronger influence on the math achievement for White/Asian students than African-American/Hispanic students. The present study makes significant theoretical and practical contributions to the body of knowledge on the role of parental involvement, perceived teacher support, and peer influence on math achievement at the high school level using nationally representative data.
The current study investigated how parental involvement, student perception of teacher support, and peer influence relate to high school students’ math achievement. In addition, this study examined whether or not math motivation acts as a mediator in the relationships between social contextual factors and math achievement. Last, this study examined whether all constructs and relationships used in the study vary across White/Asian students (higher-achieving group) and African-American/Hispanic students (lower achieving group).

The current study found that peer influence had a positive influence on students’ math achievement directly and indirectly through motivation. This relationship suggests that students who have peers with higher grades and higher educational aspirations are more likely to be high-achieving themselves. In addition, this study found that students’ perception of teacher support was significantly related to students’ math achievement, but surprisingly, this effect was negative and small, after controlling for other variables in the model. After a closer examination, this study found that for those students who had a lower motivation level in learning math, perceived teacher support was positively related to math achievement. By contrast, for those students at the medium and higher motivation levels, the relationship between perceived teacher support and math achievement was negative. Although perceived teacher support did not influence student math achievement directly, it supported the mediation hypothesis that motivation plays an essential role in the relationship between perceived teacher support and students’ math achievement. This pattern suggests that receiving high quality support from teachers does not automatically translate into better math achievement for high school students, but perceived teacher support encourages them to demonstrate higher levels of math motivation, which in turn contributes to their improved math achievement. Overall, this study highlighted the important effects of teachers and peers during adolescence. Therefore, it contributes important knowledge for educators and policy makers to develop early interventions, create strategies and policies, and allocate resources to improve the success in math and narrow the achievement gap.
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## Table of Contents

ABSTRACT .............................................................................................................................. iv

ACKNOWLEDGEMENTS .......................................................................................................... iv

CHAPTER ONE ......................................................................................................................... 1

INTRODUCTION ....................................................................................................................... 1
  Background .......................................................................................................................... 1
  Rationale ............................................................................................................................. 5

Purpose Statement and Research Questions ........................................................................... 11

Conceptual Framework .......................................................................................................... 12

Overview of the Methodology ............................................................................................... 14

Significance of the Study ........................................................................................................ 15

Organization of the Study ........................................................................................................ 17

CHAPTER 2 ............................................................................................................................... 18

LITERATURE REVIEW ........................................................................................................... 18
  Mathematics Achievement ................................................................................................. 18
  Motivation and Academic Achievement ............................................................................. 21
    Definition of Motivation .................................................................................................... 21
    The Expectancy-Value Theory ......................................................................................... 23
    Measurement of Motivation Beliefs .................................................................................. 27
    Effects of Motivation on Student Learning and Math Achievement ................................ 28
  Social Context and Academic Achievement ...................................................................... 30
    Vygotsky’s Sociocultural Theory .................................................................................... 31
  Parental involvement .......................................................................................................... 33
    Definition of Parental Involvement ................................................................................ 33
    Measurement of Parental Involvement .......................................................................... 34
    Effects of Parental Involvement on Academic Learning and Math Achievement .......... 36
    Effects of Parental Involvement on Student Motivation .................................................. 39
  Perceived teacher support .................................................................................................. 40
    Definition of Teacher Support ........................................................................................ 40
    Measurement of Teacher Support ................................................................................... 41
    Effects of Teacher Support on Student Learning and Math Achievement ...................... 41
    Effects of Teacher Support on Academic Motivation ...................................................... 43
  Peer influence ...................................................................................................................... 45
    Definition of Peer Influence ............................................................................................ 45
    Measurement of Peer Influence ....................................................................................... 46
    Effects of Peer Influence on Student Learning and Math Achievement .......................... 46
    Effects of Peer influence on Motivation .......................................................................... 50

Effects of Three Social Support Sources on Student Learning and Motivation ................. 52
The relationship of SES to Academic Success .......................................................... 56
Ethnic Disparity in Academic Achievement and Math Achievement .......................... 58
Summary/Literature Gap ......................................................................................... 60

CHAPTER 3 ............................................................................................................ 65

METHODS ............................................................................................................ 65
  Data Sources ...................................................................................................... 65
  Sample ............................................................................................................... 67
  Research Questions ........................................................................................... 69
  Measures ........................................................................................................... 70
    Family SES .................................................................................................... 70
    Prior Achievement ......................................................................................... 70
    Parental involvement ..................................................................................... 71
    Perception of Teacher Support ..................................................................... 72
    Peer Influence ............................................................................................... 72
    Mathematics Identity .................................................................................... 73
    Mathematics Self-Efficacy .......................................................................... 74
    Interest in Mathematics Courses ............................................................... 74
    Mathematics Utility ....................................................................................... 75
    Dependent Variable: 11th Grade Mathematics Achievement ....................... 75

Method: Structural Equation Modeling ................................................................ 76
  Data Analysis Steps ............................................................................................ 76
    Descriptive and Correlation Analyses ............................................................ 77
    Exploratory Factor Analyses ......................................................................... 77
    Confirmatory Factor Analysis ...................................................................... 78
    Structural Model Analysis ........................................................................... 80
    Multi-Group SEM Analyses ........................................................................... 83

Summary ............................................................................................................... 84

Chapter 4 .............................................................................................................. 85

Results ................................................................................................................... 85
  Descriptive Statistics ....................................................................................... 85
    Mathematics Achievement ........................................................................... 87
    Parental involvement ..................................................................................... 89
    Perceived Teacher Support ......................................................................... 90
    Peer influence ............................................................................................... 91
    Mathematics Identity .................................................................................... 92
    Mathematics Self-Efficacy .......................................................................... 93
    Interest in Mathematics ............................................................................. 94
    Mathematics Utility ...................................................................................... 95

Exploratory Factor Analyses ............................................................................. 96
Structural Equation Modeling ................................................................. 98
  Measurement Model ............................................................................ 100
  Structural Equation Models ................................................................. 110
  Suppression Effect .............................................................................. 116
Multi-Group SEM Analyses ................................................................. 120
  Measurement Invariance Testing ............................................................ 122
  Structural model invariance ................................................................. 125

Chapter 5 ............................................................................................... 129

Discussion and Conclusion .................................................................. 129
Summary of Findings ............................................................................ 129
Discussion of Findings ........................................................................ 131
  Parental Involvement ......................................................................... 132
  Perceived Teacher Support ................................................................. 135
  Peer Influence ..................................................................................... 139
  Effects of Motivation on Math Achievement ....................................... 141
  Effects of Family Socio-Economic Status (SES) and Prior Achievement ................................................... 143
  Differences between White/Asian and African-American/Hispanic students ............................................... 146

Implications .......................................................................................... 150

Contribution of the Study .................................................................... 154
Limitations of the Study ....................................................................... 155
Directions for Future Research ............................................................. 156
Conclusions .......................................................................................... 158

Reference .............................................................................................. 160
LIST OF TABLES

Table 3.1. Distributions of the Sample of the HSLS: 09 First Follow-up Year Data and the Sample of the Present Study .................................................................68
Table 4.1 Unweighted and weighted data of 11th Grade Students’ demographic items ..........87
Table 4.2 Descriptive Analysis of 11th and 9th Grade Students’ Mathematics Achievement in the Full Sample and by Gender, Ethnicity, and SES groups .............................................................88
Table 4.3 Descriptive Statistics of Parental Involvement Items (weighted) ......................89
Table 4.4 Descriptive Statistics of Perceived Teacher Support Items ..........................91
Table 4.5 Descriptive Statistics of peer influence Items ..............................................92
Table 4.6 Descriptive Statistics of Mathematics Identity Items ..................................93
Table 4.7 Descriptive Statistics of Mathematics Self-Efficacy Items ............................94
Table 4.8 Descriptive Statistics of Mathematics Interest Items .................................95
Table 4.9 Descriptive Statistics of Mathematics Utility Items ....................................95
Table 4.10 Factor Loadings for Exploratory Factor Analysis with Promax Rotation of the expectancy-value motivation theory .........................................................97
Table 4.11 Factor Loadings for Exploratory Factor Analysis with Promax Rotation of the social support factors and motivation factors ...........................................98
Table 4.12 Correlations of Latent Factors ..................................................................100
Table 4.13 Goodness-of-Fit Summary Table for Measurement Models of parental involvement items ..........................................................................................................101
Table 4.14 Goodness-of-Fit Summary Table for Measurement Models of Perceived Teacher Support ............................................................................................102
Table 4.15 Properties of the Measurement Model of Math Motivation .......................105
Table 4.16 Goodness-of-Fit Summary Table for the Full Measurement Models ............107
Table 4.17 Properties of the Final Measurement Model ............................................109
Table 4.18 Overall Item Correlation Matrix ............................................................111
Table 4.19 Standardized Direct, Indirect, Total Effects of Social Support Factors on Student Math Achievement and motivation ...............................................113
Table 4.20 Test of Measurement Invariance of the Effect of Social Support on Math Achievement across White/Asian and African-American/Hispanic groups ..................122
Table 4.21 CFA loading for White/Asian and African-American/Hispanic for the Configural Baseline Model ..............................................................................124
Table 4.22 Fit Indices for Structural Models ................................................................126
Table 4.23 Standardized and Unstandardized Direct Effects (Final Partially-Constrained Model) .................................................................................................127
LIST OF FIGURES

Figure 1.1 Conceptual Model .........................................................................................14
Figure 2.1 Eccles et al. Expectancy-Value Model of Achievement Motivation ...........25
Figure 3.1 Hypothesized Structural Model ......................................................................81
Figure 4.1 Measurement Model for Motivation Construct with standardized loading coefficients .................................................................................................................105
Figure 4.2 Full Measurement Model with standardized loadings ..................................108
Figure 4.4 Scatter diagram relating 11th graders’ math achievement after controlling math learning motivation ..................................................................................................................119
Figure 4.5 Scatter diagram relating 11th graders’ math achievement after controlling math learning motivation ..................................................................................................................119
Figure 4.6 Scatter diagram relating 11th graders’ math achievement after controlling math learning motivation ..................................................................................................................120
CHAPTER ONE
INTRODUCTION

This research study explores how social support impacts students’ mathematics achievement at the high school level in the United States. More specifically, it examines the effect of significant people in the life of adolescents on their math learning. These are parents, teachers, and peers of the students. The study further examines how the effect of parents, teachers, and peers operates. In other words, the study explores the mechanism of how parents, teachers, and peers affect learning; it hypothesized that supportive parents, effective teachers, and similar peers influence math learning by increasing student motivation. The first chapter reports the background of the study, specifies the research rationale of the study, explains its research purposes and questions, demonstrates the conceptual framework, describes the significance of the study and presents an overview of the methodology used.

Background

The topic of how to promote mathematics learning has concerned educators and policy makers for decades. The low math achievement of U.S. students is a matter of grave concern to educators, parents, and policy makers. A strong mathematical background is important for admission to many college majors, most professional occupations, and many computer-based technical occupations as well. More importantly, mathematics as the most basic pillar of STEM fields (science, technology, engineering, and math) is associated with the global competitiveness and economic leadership of a country. However, the U.S. education system is not producing the level of math proficiency skills required for global technology-based economic leadership and national security. The National Center for Education Statistics (NCES) reported that in 2015, the average math scores of fourth- and eighth-graders were lower than the average scores in the
previous assessment years and that twelfth-grade average math scores have not significantly increased since 2005. Moreover, the latest findings from the National Assessment of Educational Progress (NAEP, 2015) rated 40% of fourth-graders, 33% of eighth-graders and just 25% of 12th-graders as “proficient” or “advanced” in math. Although 82% of high school seniors graduated on time in 2015, just 37% of them were academically well prepared for college math courses, which is 2% lower than the year of 2013 (NAEP, 2015). These facts reveal that American students’ math learning outcomes have declined in recent years and that high school students’ performance is particularly low and thus, worrisome.

Looking at international comparisons, a large-scale study of the Trends in International Mathematics and Science Study (TIMSS) has revealed that American school students are falling behind East Asian students in math proficiency. The Program of International Student Assessment (PISA), which measures math literacy, reported in 2015 that students in the U.S. did not measure up to the worldwide average in mathematics literacy and problem-solving skills. TIMSS study also showed that U.S. students ranked 38th in math and 24th in science out of 71 countries and that the average math scores have been on the decline since 2009.

Considering the importance of STEM education and the fact that American students are not mathematically proficient compared to their international counterparts, the federal government has authorized policies, such as No Child Left Behind (NCLB), to promote math learning. Under NCLB, the National Science Foundation and the U.S. Department of Education invested $1 billion over five years “for results-oriented partnerships between local districts and universities to bring urgency, tested methods, and high-level expertise to rebuilding math excellence” (Cited from Gabbard & Ross, 2004). This program provided funds to states so that
they can increase student participation in advanced math and science courses and ensure that they pass the advanced placement tests. NCLB also requires knowledgeable and experienced math teachers in the classroom and support teachers with high salaries. Furthermore, NCLB made provisions and provided detailed guidance in order to effectively involve parents in their children’s education because parental involvement plays a critical role in assisting children to achieve high academic standards. This aligns with the words of former secretary Paige (2002), “schools can’t improve without the help of parents.” In sum, math proficiency is a priority both at the national and state level. States and school systems have engaged in various ways to improve instruction and math learning outcomes. Overall, there is a consensus that student math learning can be improved by increasing teacher quality and parental involvement.

Other persistent challenges that U.S. education has been facing are racial and socioeconomic disparities in educational opportunities and outcomes. Researchers have argued that the equality of educational opportunity in mathematics would consequently influence the quality and quantity of human resources in the United States (Heckman, 2006; Ladson-Billings, 2006; Gamoran & Long, 2007). However, the racial achievement gap remains substantially large. In math, only 7% of African-American students and 12% of Hispanic students scored proficient or above compared to 32% of White and 47% of Asian students. White 17-year-old students have generally scored about one standard deviation above African-American students (NAEP, 2015). Autor and Handel (2013) reported that African-American students are less likely than their white peers to report that they need to use high school level math in their job on a weekly basis. According to NAEP, the Asian student subgroup was the only one that reported a significant increase in math proficiency. This strikingly high achievement gap negatively
influences the economic success and social status in the adult life of African-American and Hispanic students in the United States (Fryer & Levitt 2006).

Broken down by socio-economic status (SES), students’ math performance also leaves much to be desired. In the U.S., around 45% of SES disadvantaged students achieved low mathematics performance in 2012, compared to 9% of SES advantaged students (PISA, 2012). PISA also reported that students’ socio-economic status explained 15% of the variation in student performance in the United States. Reardon (2013) found that the SES achievement gap grew 40% from 1970 to 2000, and now it is larger than the African-American/White achievement gap. The author also pointed out that the family income achievement gap influences children’s cognitive skills when they enter kindergarten and the gap changes little during the K-12 years. The National Assessment of Educational Progress (2015) revealed that children who received free or reduced lunch showed lower proficiency levels in fourth and eighth grade math and reading compared to their higher-income peers.

Moreover, the achievement gap between the higher- and lower- academic performing students has been widening. The 2015 assessment findings from NAEP demonstrated that low academic achievers displayed a decline in math, but top students did not show any improvement. According to Peggy Carr (cited from Brown, 2016), the acting commissioner of the National Center for Education Statistics, “The students at the lower end are getting worse …that’s something we need to think about.”

In sum, mathematics education is the cornerstone of national economic progress and is specifically critical for the knowledge economy. Math achievement is highly linked to interest and preparation for STEM majors and is a strong determinant of entry into STEM majors in
college. However, an immediate concern exists because the U.S. students have not performed as well as expected in mathematics. Additionally, disparities in educational opportunities and outcomes due to poverty and racial and ethnic backgrounds persist. Therefore, there is a strong need to explore factors which can influence math performance. This would be beneficial for educators and policy makers to develop early interventions, create strategies and policies, and allocate resources to improve the math performance of students and narrow the achievement gap. This study examines the effect of social factors on math achievement and how the social influences of parents, teachers, and peers operate to promote math learning.

**Rationale**

Learning is a dynamic process for obtaining knowledge or skills through study, experience, or instruction. How to promote students’ learning and achievement has been a longstanding concern for educators, policy-makers, and researchers. Research has determined that students’ supportive social relatedness is a strong force in shaping students’ academic achievement and motivation (e.g., Ahmed, et al. 2010; Furrer & Skinner, 2003). Children learn from the environment and community around them, especially from family, teachers, and peer groups who they interact and communicate with daily (Singh, Granville, & Dika, 2002; Callanan & Braswell, 2006; Rogoff, 2003). That means quality support from important social agents in children’s social context strongly impact their development and educational outcomes (Bronfenbrenner, 1979; Wentzel, 1998). Vygotsky’s sociocultural human learning theory (1978) explains that social interaction plays a large role in children’s cognitive development. This integrated approach posits that interactions with more experienced, skilled, and knowledgeable social helpers, especially parents, teachers, and peers, help children to understand concepts faster and improve their problem-solving skills.
Guided by theoretical frameworks and previous empirical findings, education researchers have conducted extensive studies to examine factors that may explain students’ educational outcomes such as math achievement. For the last several decades, many studies have focused on school environment since these factors can be quickly manipulated by administrators and teachers. These studies documented that school environment (such as resources, teacher qualifications, activities, culture, location, staffing, etc.) can positively affect students’ learning by meeting their needs, providing safe environment, increasing their sense of belonging, and then enhancing children’s engagement in school activities (e.g., Wang & Holcombe, 2010; Kweon, et al, 2017; Stewart, 2007; Dupper & Meyer-Adams, 2002).

However, a growing body of research has argued that teachers contribute a substantial portion to the quality of school environment, which means teacher support has a more powerful influence on students’ development compared to the structural characteristics of schools. Coleman et al. (1966) stated that "schools bring little influence to bear upon a child's achievement that is independent of his background and general social context.” Stewart’s (2007) hierarchical linear modeling study revealed that schools’ structural characteristics (including school size, location, cohesion, etc.) have a smaller influence on 10th-graders’ academic achievement in comparison with individual-level characteristics such as positive peers, parent-children discussion, SES, etc.

A long history of research has suggested that the support from teachers can make a difference in students’ learning (e.g., Roorda et al., 2011; Hughes & Kwok, 2007; Desimone & Long, 2010). Classrooms are fundamental social contexts for formal learning, and teachers provide an important source of support for students' academic endeavors through showing care
and interest for students, providing clear instruction and positive feedback, as well as encouraging autonomy (e.g., Roorda, 2011; Wentzel, 2002). Previous research has found that teacher-personal factors, such as “empathy,” “care,” and “warmth” are strongly related to students’ engagement, behaviors, feeling of school belongingness, and academic achievement (Cornelius-White, 2007; Tucker et al., 2002).

Another significant factor in students’ academic success is the parents’ involvement in their children’s schooling. Although children spend a substantial amount of time in school, it is no surprise that they spend more time in their home, neighborhood, and their communities. According to Schools and Staffing Survey (SASS), American students spend less than 15% of their time in school. Since children spend much more time with their families, their parents have a more powerful influence on children’s overall development, such as personality, emotional development, and behavioral habits (Dubow, Boxer & Huesmann, 2009; Morris et al., 2007). There is substantial evidence suggesting that parents’ involvement is consistently associated with higher levels of achievement (e.g., Davis-Kean, 2005; Linver, Brooks-Gunn, & Kohen, 2002; Conger et al., 2002; Hughes, J., & Kwok, O.-m., 2007). Professors of Economics, Houtenville and Conway (2008), have found that schools need to increase individual-student spending by more than $1,000 to reach the same outcomes as students who benefit from parental involvement. Although many types of parental involvement are related to students’ achievement, some studies have showed that parental involvement in education-related discussion with their children have a greater influence on children’s educational success, and this influence is even stronger than parents’ involvement in schools, such as attending meetings and volunteering at students’ schools (Hill & Taylor, 2004; Stewart, 2007).
Even though the majority of educational literature has suggested that parents and teachers strongly influence students’ outcomes, peer support is critical too during adolescence (e.g., Hanushek et al., 2003; Ryan, 2002). Peer influences peak during adolescence in a child’s life since adolescents spend more than half of their time interacting with their peers, place significance on peer approval and suggestions, and consider peers as an important source of identity (Updegraff et al., 2002; Bukowski, Sippola, & Newcomb, 2000). Peers are increasingly becoming important social attachments that offer a sense of belonging and acceptance, as well as assisting adolescents to develop a strong sense of self-concept and self-identity (Rubin, Bukowski, & Parker, 2006). Peer influence can have both positive and negative effects on an adolescent’s development depending on what kind of peer groups adolescents join. Academically engaged students have an influence on their peers and actually help boost their peers’ academic performance (Fuligni et al., 2001; Nichols & White, 2001). In contrast, students in negative peer groups would disparage academic achievement, decrease the learning interest, and enhance the risk for misconduct (French & Conrad, 2001; Langer-Osuna, 2016; Horvat & Lewis, 2003).

Although it is clear from the pervious studies that student achievement is influenced by social supportive systems, several issues are still unresolved. First, most studies have examined the influence of student academic outcomes by focusing on an individual social support factor. However, only a few researchers (Chen, 2005, 2008; Wang & Eccles, 2012; Wentzel, 1998; Estell, & Perdue, 2013; Wentzel, Russell & Baker, 2016; Wang & Neihart, 2015; King & Ganotice Jr, 2014; Vedder-Weiss, & Fortus, 2013; Ma, 2001; Boucey, 2004; King, 2016; Willson & Hughes, 2006; Kindermann & Vollet, 2014; Lam et al, 2012; Domagala-Zysk, 2006) have incorporated all three social supportive systems simultaneously in a single study to investigate students’ learning performance. Further, with the exception of a few studies (Chen,
the influence of parents, teachers, and peers have not been investigated in a single model to examine which social factor or combinations of factors have the most influence on students’ academic achievement. Moreover, among these studies that involved three support systems in a single model, just three of them (Wang & Eccles, 2012; Wentzel, 1998; Ma, 2001) used the U.S. students as sample participants, while the rest of the studies focused on students in Asian and European countries.

Second, the processes that explain links between social support and academic achievement have not been well examined by previous studies. The current study explores a mediating role of motivation in explaining the relationship between social support factors and student academic achievement. Motivation has been recognized as a powerful predictor of students’ academic achievement, for current and future success (e.g., Singh et al., 2002; Turner et al., 2009; Griffin & Neal, 2000). However, few studies (Wentzel, 1998; King & Ganotice Jr, 2014) have investigated the role of motivation in explaining links between social support and academic outcomes. The expectancy-value theory of achievement motivation was extended by Eccles and her colleagues (1983) to understand adolescents' performance, persistence, and choices in the mathematics performance domain. In this theory based model, students’ academic achievement, choice of achievement tasks, and persistence are predicted by their two motivational beliefs: expectancies of success and subjective task values. Specifically, students’ math achievement is determined by how much they value and are interested in mathematics, as well as by the degree to which they expect to succeed in mathematics learning. Eccles and her colleagues emphasize that learners’ social and cultural environment, demographic characteristics, and prior experiences affect achievement indirectly through their expectancies.
and values. For instance, students would be more likely to have an interest in mathematics if the teachers explain how math relates to real life situations. Also, teachers’ encouragement and care can increase students’ confidence in learning math (Zins, et al., 2004). Parents influence children’s motivation through conversations which in turn can transfer parents’ values and beliefs about education, as well as provide knowledge and experiences (Finn-Stevenson, 2014). Peers enhance students’ interest in school by providing friendship, feelings of belonging, and emotional support (Hamm & Zhang, 2010; Nichols & White, 2001).

Third, most of the literature that engaged these three social agents’ support and their effect on math achievement have looked at students in elementary schools (Wentzel, 1998; Wentzel, Estell, & Perdue, 2013; Russell & Baker, 2016; Willson & Hughes, 2006; Kindermann & Vollet, 2014) and middle schools (Wang, & Neihart, 2015; Vedder-Weiss, & Fortus, 2013; Bouchey, 2004; King, 2016; Lam et al., 2012); only a few of them (Wang & Eccles, 2012; King & Ganotive Jr, 2014; Ma, 2001; Chen, 2005, 2008) have focused on high schools. Of these studies, only two studies (Wang & Eccles, 2012; Ma, 2001) examined the U.S. high school students’ math learning in relation to influence from parents, teachers, and peers. Adolescence has been reported as a particularly transformative and challenging development period in all areas of an individual’s life. Adolescents’ future aspirations form and develop in this period. They need more help and guidance from adults; on the other hand, they seek independence to develop their sense of responsibility and identity. Adolescents’ social environment is created by important social interactions and influences from parents, teachers, and peers. In their different ways, parents, teachers, and peers affect adolescents’ social and academic growth and decisions.
Lastly, in general, traditional attempts to explain the achievement gap in academic achievement have either failed to consider the direct and indirect influence of important social agents’ support or simply focused on the results of the racial group differences. There is limited knowledge on how these three sources of social support influence students’ motivation and achievement and how these factors vary across White/Asian and African-American/Hispanic groups (Wang & Eccles, 2012; Cheng & Starks, 2002).

In sum, although a few studies have examined these three social sources of support simultaneously, even fewer studies have looked at how the social support from parents, teachers, and peers influence students’ math achievement directly and indirectly through motivation beliefs among older adolescents. Thus, this study attempts to fill this research gap by including three sources of social support simultaneously in a single model to examine the relative and additive influence of social support on high school students’ math achievement directly and as mediated by their math motivation. The study further compares whether math achievement differences exist in the inter-relationships of those constructs across White/Asian and African-American/Hispanic groups.

Purpose Statement and Research Questions

The purpose of this study is threefold. First, it explores how support from parents, teachers, and peers impact adolescents’ math achievement. More specifically, the study aims to investigate how parental involvement, student perception of teacher support, and peer influence relate to high school students’ math achievement. Second, this research examines whether or not academic motivation acts as a mediator in the relationships between social contextual factors and math achievement. Last, the study examines whether all constructs and relationships used in the
study vary across two groups (White/Asian and African-American/Hispanic). The specific research questions are presented below:

1. What is the effect of each social support factor (parental involvement, student perceptions of teacher support, and peer influence) on high school students’ math achievement after controlling for SES and prior math achievement, and which support factor has a stronger effect?

2. What is the effect of each social support factor (parental involvement, student perceptions of teacher support, and peer influence) on high school students’ motivation in mathematics after controlling for SES and prior math achievement, and which support factor has a stronger effect?

3. Do students’ motivation beliefs (math self-efficacy, math interest, math utility value, and math identity) mediate the relationships between social support factors (parental involvement, student perceptions of teacher support, and peer group) and students’ mathematics achievement?

4. Does the strength of the associations among social support systems, motivational beliefs, and math performance differ by ethnicity?

**Conceptual Framework**

To answer the research questions, a conceptual model was hypothesized, which is rooted in Vygotsky’s sociocultural theory and expectancy-value theory wherein important social agents have the potential to influence students’ math achievement. The expectancy-value theory also suggests that students’ motivation beliefs directly influence their learning outcomes. In the model in Figure 1, which is based on these two theories, family social-economic background and
previous math achievement serve as exogenous variables which influence students’ academic motivation and math learning (See Figure 1 for details).

This conceptual model examines two possible influences on students’ math learning. First, this study tests and compares the extent to which each social support system, including parental involvement, perceived teacher support, and peer influence, contributes to the variation in adolescents’ math achievement, after controlling for SES background (the first research question). A second possibility is that socially supportive relationships directly promote motivation (the second research question) and subsequently impact math outcomes, which addresses the third research question. According to the expectancy-value theory, academic motivation was measured by math self-efficacy, math interest, math utility value, and math identity. Furthermore, this study examines whether all constructs and relationships vary across white/Asian and African-American/Hispanic ethnicity groups which addresses the last research question. Besides, the conceptual model hypothesized that SES background is related to students’ social factors and math achievement. It also hypothesized that prior math achievement is associated with students’ academic motivation. This conceptual model was applied to the HSLS: 09 data set and it guided the analyses undertaken to confirm if the data fit the model.
Overview of the Methodology

To achieve the research purpose and address the research questions, this research used the High School Longitudinal Study of 2009 (HSLS: 09) and applied a quantitative approach that incorporates structural equation modeling (SEM) and related statistical analysis. The HSLS: 09 seeks to gain insight into students’ educational trajectories from the beginning of high school into the postsecondary options, their career choices, and early adult life. Particularly, the HSLS: 09 investigates the paths into and out of STEM fields, especially mathematics, as well as the educational and social experiences that affect these changes (Ingels et al., 2011). It is a nationally representative study, which collected data across all 50 states and the District of Columbia, increasing the validity of findings based on these data.

In the current research, the primary statistical method, SEM, is a confirmatory approach to data analysis of a structural theory based on the hypothesized relationships of factors of
interest (Byrne, 1998). In general, the data analysis in the current study is a five-stage process. First, descriptive statistics and correlation matrices for all variables of interest were summarized to give an overview of the data. Second, exploratory factor analysis (EFA) was conducted to identify the underlying factor structure on a small (about 5%) subset of the data. Third, confirmatory factor analysis (CFA) was conducted to determine whether the observed items measure the corresponding latent variables. Fourth, the structural model was analyzed to empirically test and estimate the proposed hypothesized relationships between latent variables. Lastly, multi-group SEM analyses were performed in this study to examine whether relationships among latent variables differed across two groups (White/Asian and African-American/Hispanic).

**Significance of the Study**

This research makes significant theoretical and practical contributions to the body of knowledge on the role of parental involvement, perceived teacher support, and peer influences on the math achievement at the high school level using national representative data. First, few previous studies have examined the effect of support from parents, teachers, and peers on math achievement in a single study. However, this research incorporates all these three social factors as well as each factor and explores the differences in their impact on adolescents’ math learning motivation and achievement outcomes. It produces a deeper understanding of factors that impact math learning in high school years.

Second, the processes that explain links between social support and math achievement was not thoroughly examined in previous research. (Rosenfeld, Richman, & Bowen, 2000). This study provides an in-depth understanding of how social support agents influence high school
students’ math performance through promoting students’ academic motivation. It supports that the role of motivation in explaining links between social support and learning outcomes, which can benefit policy makers and teachers to make more effective strategies and programs to prompt students’ academic motivation.

Third, substantial research on social factors influence have been conducted in elementary and middle schools, but much less is known about high school classrooms. this research fills the gap in how various social support resources impact older adolescents’ math achievement.

Lastly, this study presents a more complex view of interpreting the different effect of three important social support sources on math achievement and motivation between White/Asian and African-American/Hispanic groups. The findings provide deeper insight into achievement gaps, which can help to narrow the achievement gap of African American and Hispanic students. Further, this study contributes to the literature by identifying the mediating role of math motivation in the relationship between the support of significant people for learning math and mathematics achievement. It also adds to the body of literature on social support factors, as well as strengthens the Vygotsky’s sociocultural and expectancy-value theories by providing empirical evidence in their support.

Overall, this research serves an exploratory role to inform the field of math learning, as well as presents new and complex insights into the relationship of social support and academic motivation. The results of this study may further inform the reform efforts towards increasing student success in high school math courses.
**Organization of the Study**

Chapter 2 provides a comprehensive review of the literature and theories supporting the relationships among social contextual factors, academic motivation, and math achievement. This chapter concludes with a description of the limitations of the reviewed literature. Chapter 3 presents a detailed explanation of research methods to answer the research questions. It describes the research questions, the data source, sample, the measures of the variables, and data analysis strategies in detail. Chapter 4 presents a detailed account of the preliminary data analysis results. It includes descriptive results, correlation matrixes, the CFA measurement testing results, the structural model results, and multi-group SES analysis results. Chapter 5 provides a summary of the findings and then presents the discussion and implications of the results. This chapter also discusses the limitations for practice and provides recommendations for future research in this area of study.
CHAPTER 2
LITERATURE REVIEW

This chapter surveys the theoretical literature and empirical research on parental involvement, perceived teacher support, peer influence, academic motivation, math achievement, as well as the relationships and influences among these factors. Literature presented in this chapter is in the following sections: 1) mathematics achievement; 2) academic motivation and Expectancy-Value Framework on students’ learning and math achievement; (3) the definitions and measures of student motivation, including interest in math, math identity, math self-efficacy, math utility; (4) Vygotsky Sociocultural Theory; (5) the definitions and measures of social support systems including parental involvement, student perception of teacher support and peer influence, as well as the relationships of social support systems to students’ learning and math achievement; (6) socioeconomic and ethnic disparities in social support and learning outcomes; and (7) summary of literature review and gaps.

Mathematics Achievement

Mathematics achievement is a longstanding topic of interest for educators, parents, policy makers, and researchers since the strong mathematical background is highly associated with admission to many college majors, most professional occupations, and many occupations in computer and technology fields as well. Beyond the fact that high school math achievement is directly related to attending college, high school math attainment is the most accurate predictor of whether a student will finish a four-year college degree (James, 2013; Zelkowski, 2011). Students with more advanced math courses in high school are three times more likely to earn a Bachelor’s degree (James, 2013). Additionally, it is surprising to know that high school math
course taking predicts individuals’ future career outcomes: more high-level math courses taking, the better labor market outcomes. Rose and Betts’s (2004) study showed that high school students’ math achievement is a strong indicator of students’ earnings around 10 years later after controlling for demographic, family, school characteristics, highest educational degree acquired, college major, and career. These researchers also pointed out that more advanced math courses (e.g., algebra/geometry) have a stronger influence on students’ earnings than those less advanced ones. Similar results in James (2013) showed that among individuals who have the same level of education degree, those who take more math courses, especially advanced ones in high school, are more likely to obtain a job, and have higher salaries on average. These findings provide insights for policy makers to narrow the earnings gap among racial and socioeconomic groups by enriching the math curriculum and encouraging low-income parents and students to invest in more math courses in high school. Further, mathematics, as the most basic pillar of STEM fields (science, technology, engineering, and math), is associated with the global competitiveness of the U.S.A., since the U.S. economic leadership has become increasingly reliant on math-, science-, and technology-related fields.

However, an immediate concern exists because U.S. students have not performed as well as expected in mathematics. U.S. High school students’ performance is particularly low and thus, worrisome. The National Center for Education Statistics (NCES) reported that in 2015, the average math scores of fourth- and eighth-graders were lower than the average scores in the previous assessment years and that twelfth-grade average math scores have not increased since 2005. Moreover, the latest findings from the National Assessment of Educational Progress (NAEP, 2015) rated 40% of fourth-graders, 33% of eighth-graders and just 25% of 12th-graders as “proficient” or “advanced” in math. Although 82% of high school seniors graduated on time
in 2015, just 37% of them were academically well prepared for college math courses, which is 2% lower than the year of 2013 (NAEP, 2015). Looking at international comparisons, a large-scale study of the Trends in International Mathematics and Science Study (TIMSS) has revealed that American school students are falling behind East Asian students in math proficiency. The Program of International Student Assessment (PISA), which measures math literacy, reported in 2015 that students in the U.S. did not measure up to the worldwide average in mathematics literacy and problem-solving skills. TIMSS study also showed that U.S. students ranked 38th in math and 24th in science out of 71 countries and the average math scores have been on the decline since 2009. Therefore, there is a strong need to explore and identify factors which can influence students’ math performance.

The majority of the literature in this area suggested that secondary students’ math learning can be influenced by a range of factors, including 1) demographic factors (family structure, gender, socio-economic status), 2) social contextual factors (parental involvement, teacher instructional strategies and techniques, peer influence, curriculum, school resources and facilities), 3) individual factors (attitude towards mathematics, mathematics self-efficacy and interest, previous achievement). Many previous studies have focused on school environment since these factors can be manipulated by administrators. However, considerable evidence supported that teachers contribute a substantially to the quality of the school environment, which means teacher support has a more powerful influence on students’ development compared to the structural characteristics of schools (such as school size, location, cohesion, etc.) (Hattie, 2009; Stronge, 2013). Not just teacher support, parental and peer support has also been shown to have powerful influence on students’ academic success at schools. Besides social support factors, motivational constructs such as self-efficacy, interest, and value beliefs are also key factors that
affect students’ academic achievement. In order to achieve a better understanding of students’
math learning and performance, important social agents’ support and academic motivation
should be taken into consideration in a single model to figure out the relationships on students’
success in math learning. These factors and relationships are elaborated in detail in the following
paragraphs.

Motivation and Academic Achievement

Definition of Motivation

The term motivation is derived from the Latin verb *movere*, which means to move.
Motivation is a concept that is used to explain people’s thoughts and behaviors. The nature of
motivation is about needs, desires, or wants, which drive people to start, keep working on, and
complete tasks. The definition of motivation by Pintrich and Schunk (2002) is a “process
whereby goal-directed activity is instigated and sustained” (p. 5). Kleinginna and Kleinginna
(1981) defined the term as an “internal state or condition that serves to activate or energize
behavior and give it direction.” Maehr and Zusho (2009) claimed that “motivation is viewed as a
personality trait exhibited to varying degrees by individuals......It is typically also assumed that it
is a relatively stable trait: a pattern of feeling, personal orientation, and behaviors” (p. 82).
Academic motivation comes from a student’s desire or need (as reflected in approach,
persistence, and level of interest) to succeed in academic subjects based on his/her comparison to
certain standards (Wigfield & Eccles, 2002; DiPerna & Elliott, 1999; McClelland, 1961;
Gresham, 1988).

These motivation definitions show some disagreement due to their broad and less precise
nature. Thus, theorists have developed various theories to explore motivation. As an educational
psychological state construct, motivation and its influences on learning performance, persistence, and choice can be explained in five major theories: the expectancy-value theory (Atkinson, 1957; Eccles et al., 1983; Wigfield, 1994; Wigfield & Eccles, 1992), the achievement goal theory (Dweck & Leggett, 1988; Nicholls, 1984), the interest theory (Hidi, 1990), the self-determination theory (Deci & Ryan, 1985; Deci, Vallerand, Pelletier, & Ryan, 1991), and the self-efficacy theory (Bandura, 1997).

Compared with other popular motivation theories, the expectancy–value theory provides a more comprehensive perspective through some overlap with other theories. For example, in this theory, expectancy beliefs are emphasized as the students’ perception of their abilities and competence on certain tasks (Wigfield & Eccles, 2000). These beliefs are tied to the self-efficacy theory, which has been defined as an individual's belief in his or her ability to achieve intended results (Bandura, 1997). The two subcategories of subjective task values in the expectancy–value theory, intrinsic values (enjoyment or interest) and utility value (usefulness), are constructs similar to the constructs of the interest theory and the achievement goal theory in an academic context. The self-determination theory distinguishes motivation into intrinsic (inherent drive) and extrinsic (external sources) motivation; the expectancy–value theory highlights the influence of the social contextual environment (external) on individuals’ inherent tendency (internal) to development. In addition, the construct of utility value relates to the usefulness of the task for short- or long-term goals; thus this construct can be tied to the construct of extrinsic motivation.

In the current study, the modern model of the expectancy-value theory is used to investigate high school students’ math achievement for five specific reasons: 1) this theory emphasizes a variety of socialization influences. It highlights how and why the important
socializers’ beliefs and behaviors impact students’ allocation of effort, activity, and career choices. 2) Cultural milieu (e.g., economic social background) and academic experiences (e.g., students’ prior achievement) are also involved in the model, which can enhance the rigor of the study. 3) The theory was originally developed as a way to understand adolescents’ motivation for learning performance and choice in the specific domain of mathematics. 4) The survey concept map of nationally representative data HSLS:09, which is used in the current study, is closely linked to the expectancy-value model. 5) The expectancy–value framework as a prominent and long-standing motivation theory is used extensively in educational research to articulate students’ math achievement (Wigfield, 1994; Simpkins, Davis-Kean, & Eccles, 2006; Wigfield & Eccles, 2000; Steinmayr & Spinath, 2009). Collectively, the expectancy-value model represents a valuable framework to study social support context, psychological variables, and their relationship for adolescents’ performance in math-related motivation and performance, which is also the main purpose of this study. In the following paragraphs, this theory is articulated in detail.

The Expectancy–Value Theory

The expectancy-value theory was developed initially by Atkinson (1957) to understand achievement motivation. Green (2002) asserted this theory could be explained as "the amount of effort that people are willing to expend on a task is the product of the degree to which they expect to succeed at the task, and the degree to which they value the task and value success on the task" (p. 990).

Eccles and her colleagues (1983) extended this theory to the field of education and developed a cognitive framework to understand adolescents' performance, persistence, and
choices in the mathematics performance domain. In this theoretical model, students’ academic outcomes and achievement related choices are predicted by their two motivational beliefs: expectancies for success and subjective task values. These two motivational components can be thought of as somewhat domain specific, which refers to the idea that people have varied value beliefs for various tasks (Wigfield & Eccles, 2000). Sociocultural contexts (e.g., SES) and socialization agents (e.g., parents, teachers, and peers) shape motivation beliefs by directly and interactively affecting expectancies and task value beliefs.

Eccles and her colleagues (1983, 1984, 1985) elaborated expectancy-value theory in a dynamic process. The full scope of their recent theory model is depicted in Figure 2.1. The framework shows that achievement-related choices and performances are directly impacted by expectancies for success and subjective task values, which are traced by a set of psychological factors including interest/enjoyment value, attainment value, utility value, and relative cost (Eccles, et al., 1983). The two motivation constructs of expectancies and task values are influenced by a set of social cultural factors, such as socializers’ behaviors and beliefs, cultural norms, SES, previous achievement related experiences, etc (Eccles, et al., 1983).
Expectancy for success

Expectancies for success were defined as individuals’ beliefs of the performance of certain tasks (Eccles et al., 1983). It is a specific belief that refers to an individual’s competence beliefs and self-efficacy expectations regarding their abilities to succeed in a task in the short-term or long-term future. For instance, if students are confident they can achieve a high score in math, they are more likely to enroll in more math courses and participate in more advanced cognitive strategies in math, which together allow an increase in math achievement (Wigfield & Eccles, 1992, 2002). Whereas, when students think that they are really struggling on a math test,
they will feel they are performing poorly on the test, which leads to a decrease in their actual math performance. Studies have shown that when individuals feel competent and confident in the math domain, they will be more motivated to decrease anxiety, start tasks, continue the tasks, and put more work into tasks (e.g., Fast, et al., 2010; Galla & Wood, 2012).

**Subjective task values**

The definition of values is broad as well as task-specific. Task values were assumed to take into account the tasks’ qualities, and these qualities increase or decrease the probability of an individual’s desire to do the task (Eccles et al., 1983; Wigfield & Eccles, 1992). In addition, these values are subjective because different students allocate different values to the same tasks. For instance, some students appreciate math learning, but other children value English more. The subjective task values were defined by four components: attainment value, intrinsic or interest value, utility value, and cost (Eccles et al., 1983; Eccles & Wigfield, 1995).

Based on Battle’s (1965, 1966) work, Eccles and her colleagues defined attainment value as social or personal identity, which means that the value of participating in or performing well in an activity is consistent with an individual’s self-image, or allows an individual to express important aspects of self. Research has found that children who identified more with an academic domain tend to demonstrate increased confidence, motivation, effort, participation, and perseverance (McGee & Martin, 2011).

Intrinsic or interest value is represented as the perceived enjoyment that an individual’s gains from participating in an activity or doing a task. Eccles and her colleagues suggested that interest value beliefs affect students’ academic or career choices by offering positive meaning to these behaviors. Utility value, in terms of whether a task is useful, fits into one’s anticipated
plans and goals. For example, when a student enrolls in an advanced algebra course to fulfill a requirement for college admission even though the student is not interested in algebra knowledge. Cost, the last component of subjective task values, is what the individuals are willing to give up in order to participate in a task. Cost is effected by many factors, such as loss of time, fear of failure, loss of valued alternatives, rejection by peers, or negative psychological feelings and experiences like stress (Covington, 1992; Eccles & Wigfield, 2002).

Measurement of Motivation Beliefs

As discussed above, the central idea of the expectancy-value theory model is that students’ expectancies for success in tasks and the subjective values they attach to those tasks impact students’ learning outcomes (Eccles et al., 1983; Wigfield & Eccles, 1992). Theorists explain this academic motivation theory using a multiplicative formula: Expectancy x Value = Motivation (e.g., Shah & Higgins, 1997; Bong, 2001; Trautwein et al., 2012; Fishbein & Ajzen, 1975; Trautwein et al., 2012). The equation describes that achievement motivation is determined by how highly a goal is valued and the degree to which a person expects to succeed. Motivation is abstract since it is an internal process that people are not able to observe directly; hence, researchers measured motivation beliefs through assessing a full or partial set of psychological factors, including achievement goals, interest, attainment value, utility, and cost value (e.g., Conley, 2012; Chouinard & Roy, 2008; Trautwein et al., 2012; Cole, Bergin, & Whittaker, 2008; Liem, Lau & Nie, 2008).

Although some researchers explored the influence of expectancy and task values separately on educational outcomes and choices (e.g., Wang & Degol, 2013; Thomas & Strunk, 2017; Fan, 2011; Simpkins, Fredricks & Eccles, 2012; Chouinard, Karsenti, & Roy, 2007; Guo
et al., 2015). In the current study expectancy and task values are combined together as a composite measure of motivation. Four reasons explain the use of a composite motivation measure: 1) Based on the expectancy-value theory, achievement motivation in nature is a combination of expectancy and task value believes. 2) Previous researchers demonstrated that these two motivation constructs of expectancy and task values have a very similar influence on students’ math achievement (e.g., Nagy et al., 2006; Nagengast, 2011). 3) A composite measurement would enhance the generalizability of motivation research. 4) This composite measure of motivation was used in past research endeavors (e.g., Leaper, Farkas, & Brown, 2012; Simpkins, Davis-Kean, & Eccles, 2006).

**Effects of Motivation on Student Learning and Math Achievement**

It is well-established that motivation plays a major role in students’ academic performance, adjustment, attitudes, and well-being. Motivation beliefs are reflected in students’ choices of learning and achievement tasks; in their interest on the tasks; in their persistence and confidence on the tasks; in the time and effort they allocate to these tasks; in the goals they pursue; and in their facing of learning difficulties, pressure, and anxiety (Wigfield & Eccles, 2002; Meece, Anderman, & Anderman, 2006; Kusurkar et al., 2013; Allen, & Robbins, 2010; Turner, Chandler, & Heffer, 2009; Pekrun, Elliot, & Maier, 2009). “The absence of academic motivation can lead to feelings of frustration and discontentment and can encumber productivity and well-being” (Legault, Green-Demers, & Pelletier, 2006, p.567).

Studying motivation, particularly in the mathematics domain, is essential. Traditionally, academic achievement, especially, mathematics achievement is often thought to be associated with intelligence (Kuncel, Hezlett, & Ones, 2004; Busato, Prins, Elshout, & Hamaker, 2000).
However, Kuncel and her colleagues (2004) found that students’ intelligence ability just explained 25% of the variance in their academic performance and that general intelligence has a stronger correlation relationship with students’ verbal ability than math ability. Busato et al.’s (2000) correlational analyses revealed that intellectual ability and achievement motivation has a positive relationship positive with academic outcomes, and motivation has a stronger association with academic achievement than intellectual ability.

Additionally, a study involving 342 11th and 12th grade students conducted by Steinmayr and Spinath (2009) revealed that most of the motivational constructs, including self-concepts about ability, self-perceptions, and task values contributed to the prediction of students’ math achievement over and above intelligence. The authors explained these results are because mathematics is highly cognitive in nature, and interest is a critical predictor for high school advanced math courses performance.

The math self-concept variable was used to evaluate students' expectancy of success (e.g., Guo et al., 2015; Simpkins et al., 2006; Marsh & Yeung, 1998). A longitudinal study by Simpkins et al.’s (2006) showed that 6th grade children’s self-concept significantly predicted their 10th math grades, which were related to the number of math courses they would take in future grades. Similar findings by Guo et al. (2015), which involved 5,179 Hong Kong Grade 8 students in three waves, demonstrated that self-concept positively predicted children’s math performance at different levels of utility value, and self-concept was a stronger predictor of educational aspirations and outcomes for students who have lower utility values. In addition, analysis results suggested that higher self-concept, higher educational utility value, and their interaction, all significantly lead to students’ higher math performance and educational
aspiration. Marsh and Yeung (1998) found that the influence of expectancies varied in different subject domains. They argued that self-concepts positively associate with math learning outcomes, including students’ school grades, coursework selection, and standardized test scores, but not in English learning outcomes.

Considering academic motivation for academic achievement is of great importance; educational researchers explored the effective factors that impact motivation in the past decades. They found that social support as well as personality are strongly related to academic motivation (e.g., Wentzel, 1998; Ma, 2001; Legault, Green-Demers, & Pelletier, 2006; Major, Turner, & Fletcher, 2006; Naquin, & Holton, 2002;). Since personality is not easily manipulated or changed, this research focuses on the influence of social support recourses, especially support from significant social agents: parents, teachers, and peers.

**Social Context and Academic Achievement**

The natural process of human development and learning is interactive and dynamic and embedded within a complex social environment (Bronfenbrenner, 1979; Vygotsky, 1978; Anderman & Kaplan, 2008). By this logic, children are influenced by the people with whom they associate and socialize daily. This influence is considerable, beginning at the very start of formal and informal education, and covering a broad range of issues in the students’ development, including educational achievement, behaviors, cognition, values, and relationships (Ryan, 2000; Fan et al., 2009; Wang & Eccles, 2012; King & Ganotice Jr, 2014; Furrer & Skinner, 2003; Patrick, Ryan, & Kaplan, 2007). Adolescence has been identified as a particularly transformative and challenging development period in many areas of an individual’s life. An adolescent’s future aspirations form and develop in this period (Patrick, Ryan, & Kaplan, 2007; Brown, 2004,
Updegraff et al., 2001; Steinberg, & Monahan, 2007). Intentionally and unintentionally, adolescents seek assistance to develop their self-beliefs, goals, and confidence to face difficulty and failures when they are growing (Ryan, 2000). Therefore, quality social support and relationships are necessary and beneficial for adolescents’ learning outcomes.

Social support refers to the perception and actuality that an individual receives care, comfort, and assistance from other people and that an individual belongs to a supportive social network (Newman, 2000). Chen (2005) classified academic support resources into “interpersonal (quality relationships), cognitive (interpretations of expectations), emotional (care and encouragement), behavioral (social control and monitoring), and instrumental (e.g., direct assistance with schoolwork) resources” (p.79). Adolescents’ learning processes and outcomes are increased by obtaining more support from important social agents in their social contextual environment, particularly parents, teachers, and peers, with whom they interact daily in their networks (e.g., Wentzel, 1997, 1998; Wang, 2005, 2008; Ryan, 2001; Rice et al., 2013).

**Vygotsky’s Sociocultural Theory**

For the last several decades, researchers have conceptualized a number of social support theories, but Vygotsky's sociocultural theory is the model that is most relevant to this research. The main argument of Vygotsky's theoretical framework is that social interaction is the basis for children’s cognition development, in particular interactions with more experienced, skilled, and knowledgeable social helpers, such as parents, teachers, and peers who help children to build an understanding of concepts and solve problems (Bransford, Brown, & Cocking, 2000). For Vygotsky, the social contextual environment in which students grow up will impact their thoughts and beliefs. In the beginning, parents are the main players and helpers in the social
context surrounding children; teachers and peers become more important when children enter school.

Vygotsky (1978) proposed that the potential for cognitive improvement is limited to a "zone of proximal development" (ZPD), which is defined as “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (p. 86). American psychologist Bruner (1982) describes this zone as “the child’s ability to recognize the value of hinges and props even before he is conscious of their full significance” (p. 852). The notion of scaffolding refers to assistance or tasks which adults or more knowledgeable peers provide to develop learners’ zones of proximal development. Scaffolding generally includes various forms of coaching, questioning, collaborating, and guided participation that promotes the learners’ understanding of knowledge domains, development of advanced skills and facilitation of intentional learning. Scaffolds are moved out when students have an ability to master knowledge and finish tasks independently.

Berger (2006) suggested that teachers, parents, peers, and mentors should adapt to learners and identify where they are within the zone of proximal development by questioning to understand the learners’ individual learning styles. Then adults and knowledgeable peers are able to identify the learner’s immediate needs and remedy academic misunderstandings, which allows for learners to shift developmental status and master difficult tasks in the future (Vygotsky, 1978). In addition, through these learning experiences, the learners gradually increase cognition skills and come to develop their own way of learning (Rogoff, 1990; Wertsch, 1985; Vygotsky, 1978).
Based on Vygotsky Sociocultural theory, the three important social support systems: parental involvement, teacher support, and peer influence, are elaborated in the following sections.

**Parental involvement**

In the last two decades, a voluminous body of research demonstrated that parental involvement is a critical factor to children’s academic achievement. Policy makers, school principals, teachers, parents, and students themselves all generally agree that parental involvement is beneficial for children’s success, both socially and academically. Parents are thought to be children’s first teachers, monitors, motivators, and resource sponsors, and thus parental involvement is an important indicator of students’ school competence, attitude, engagement, learning performance, aspirations, and behaviors. (Young, Austin, and Growe, 2013; Domina, 2005; Griffin & Galassi, 2010, McNeal, 1999; Epstein, 2005; Epstein & Sheldon 2002). Children are more motivated to work hard to succeed in academic context when their parents are meaningfully engaged in their education (Singh et al., 1995; Chang et al., 2009; Gonzalez-DeHass, 2005; Fan, 2001; Sheldon & Epstein, 2005; Sirvani, 2007).

**Definition of Parental Involvement**

The definitions of parental involvement are numerous and broad. Larocque, Kleiman, and Darling (2011) stated that parent involvement “can be generally defined as the parents’ investment in the education of their children (p.116).” It is also defined as “the participation of parents in regular, two-way, and meaningful communication, involving student learning and other school activities” (the United Code of Law (USCS 7801 32). The National Parent Teacher Association (PTA) defines parent involvement as “the participation of parents in every facet of
children’s education and development from birth to adulthood, recognizing that parents are the primary influence on children’s lives” (National PTA, 2000, p.8). Nokali, Bachman, and Votruba-Drzal (2010) defined it as “parents’ behavior in home and school setting meant to support their children’s educational progress” (p 989). Brewster & Bowen (2004) claimed that it is “the degree to which parents are involved in and promote their children’s education”. Christenson et al., (1992) and Walberg (1986) define parental involvement as parents' communication with children about school matters and evens as well as future plans, which is the most relevant definition for this study (also cited by Fan & Chen, 2001).

Measurement of Parental Involvement

The framework of parental involvement is multidimensional and multifaceted in nature due to varied parents’ behavioral patterns and practices (Fan & Chen, 2001; Li & Hung, 2012). Scholars have usually discussed parental involvement in two parts: school-based involvement and home-based involvement strategies (e.g., Hayes, 2011; Epstein, 1995, 2005; Strickland, 2015). School-based involvement strategies include conferences with teachers, volunteering at school, attending school functions, engaging in school governance, and so on (Hill & Tyson, 2009). Home-based involvement occurs when parents communicate with their children about education, school assignments and events, reinforce students to learn in school, visit museums and libraries, and increase motivation (Strickland, 2015; Hill & Tyson, 2009; Li & Hung, 2012).

Epstein (1992) defined six school-related strategies for parental involvement: “1) assisting parents in child-rearing skills; 2) school-parent communication; 3) involving parents in school volunteer opportunities; 4) involving parents in home-based learning; 5) involving parents in school decision-making; and 6) involving parents in school-community collaborations” (p.3).
Epstein’s theory emphasizes on collaborations between schools and parents; her research mostly focuses on how to stimulate active parental involvement (Fan & Chen, 2001). Fan (2001) identified seven types of parental involvement activities: communication, interaction with the school, parent–teacher interaction, supervision, educational expectations, and house rules.

Grodnick and Slowiaczek (1994) articulated a hypothesized three-dimensional framework of parental involvement consisting of three components: behavioral, intellectual/cognitive, and personal. Parental behavior included actions such as communicating between home and school as well as assisting with homework. The intellectual/cognitive component refers to a parental role in creating experiences for children to participate in educationally stimulating activities (e.g., trips to the museum). The final component, personal, is related to students’ attitudes and expectations about education as a result of parent influence. The researchers further argued that personal aspects referred to parental value and utility about education and emphasized the communication between parents and students regarding school.

Researchers have suggested that studies should measure different aspects of parental engagement separate instead of adding all dimensions into a general composite, since the influences of parental involvement on academic achievement are varied depending on which dimension of parental involvement were examined and what kind of domain achievement were explored (Chen & Fan, 2001; Fan & Williams, 2010; Jeynes, 2005, 2007). The evidence demonstrated that certain aspects of parental involvement have more impact than others on students' math success (Jeynes, 2005, 2007). In the current study, home-based involvement, especially parent-child communication, is measured and explored.
Effects of Parental Involvement on Academic Learning and Math Achievement

An abundance of studies has attested to the positive effects of school-based parental involvement to student academic success (e.g., Domina, 2005; Fan, 2001; Gonzalez, Holbein, & Quilter, 2002; Henderson & Mapp, 2002), but the effects of home-based involvement, especially for parent-child communication, have been conducted very less frequently. Students spend more time at home than at school, which offers parents endless opportunities to influence the everyday lives of children. Parents can improve students’ learning at home by creating a conducive learning environment, providing learning resources, serving as supervisors of family routines and schedules, and displaying an interest in education (Bakker et al., 2007; Jeynes, 2005; Hill & Tyson, 2009; Baker, 2013). Fantuzzo et al.’s (2004) study found that home-based parental involvement is a stronger predictor than some aspects of school-based involvement; it is significantly associated with students’ learning motivation, classroom attention, task persistence, receptive vocabulary skills, and fewer classroom behavior problems.

Two Meta-Analysis studies by Jeynes (2005, 2007) examined the effect of whole and specific components of parental involvement on urban elementary and secondary school students’ academic outcomes. The results showed that there is a strong positive relationship between parental involvement and academic outcomes among urban students. Looking at secondary schools, parents’ expectation is the strongest predictor, and parents-children communication has a positive effect, but other measures, such as parental school attendance and house-hold rules do not statistically influence students’ academic achievement. The author also pointed out that parental involvement has a stronger influence on students’ academic success at the elementary school level than the secondary school level, since parents are more involved when students in their childhood, and children are generally more likely to be effected compared
to adolescents (have their own convince about themselves) (Dubas & Gerris, 2002; Hagger, Biddle, & Wang, 2005). Fan’s (2001) meta-analysis study also confirmed that the effects of parental involvement varied by students’ ages, and the influence of parental involvement seems stronger for younger students.

Parental involvement is likely to influence student academic outcomes directly and indirectly through parental expectations, values, and modeling. According to Vygotsky, a learner’s ongoing process is interpersonal and fluid through communicative exchanges between active participants. Hill and Craft (2003) found that parents who value and appreciate a quality education often have children who value academics as well. This lasting impact is probably due to the direct transmission of values between parents and children through frequent conversations. These researchers argued that daily conversations between parents and children about the value of school and education further improve the efforts of parents to be positive role models for their children. Similar findings demonstrated by Marchant et al. (2001) showed that children are more likely to perceive higher academic competence and put their academic ability, effort, and grades at a high priority when their parents appreciated the importance of hard work and academic achievement. Inkelas and McCarron (2006) analyzed a sample of 1,879 students from the National Educational Longitudinal Study (NELS:88/2000) and found that parent-children discussions about school events were clearly the best predictor of postsecondary educational aspirations for non first-generation students. These research implied that frequent and regular communication between parents and their children contributed to academic growth in their students, because involved parents take more time and effort to transmit the value of education, guidance, and expectations through regular conversations.
The effects of parental involvement also apply to students’ mathematics skills. Yan and Lin (2005) analyzed a national sample of 19,368 students using NELS:88 data to examine three aspects of parental involvement of family obligation, family norms, and parent information networks with 12th-grade students' math achievement across four ethnic groups. The results suggested that the dimension of parent’s obligations, which includes the composition of home discussion, positively and significantly predicts 12th-grade mathematics achievement for Caucasian students. Based on the results, the researchers suggested that parents, teachers, and community leaders should increase parent-child interactions and communications to motivate students as well as raise students’ self-esteem and expectations. Another study by Valadez (2002) found that parent-child discussions about school also have a positive relationship with enrollment in Algebra and other advanced math classes for Latino students. Using the base-year data of NELS:88, Sui-Chu and Willams (1996) and Pong (1997) found that parent-child discussions of school matters had the strongest positive relationship with 8th graders' mathematics and reading achievement.

A longitudinal study by Ma (1999) demonstrated that home discussion between parents and children had no effect on students’ choice of enrollment in advanced mathematics in Grade 8, but had strong effects on their decisions to take advanced mathematics courses in Grades 10 and 11. Hill et al. (2004) also found that parent academic involvement predicts students’ math and language achievement across middle and high school years. Pomerantz, Moorman, and Litwack (2007) pointed out that parents may teach their children useful strategies for learning mathematics through conversations, which can increase students’ mathematical skills and enhance their beliefs in their competence in mathematics. However, Fan and Chen’s (2001) meta-analysis research involving hundreds of studies across a ten-year span indicated that
Effects of Parental Involvement on Student Motivation

Researchers have reported the associations of parental involvement with student achievement motivation. (Wentzel, 1998; Marchant, Paulson, & Rothlisberg, 2001; Gonzalez, 2002). Fan & Williams’s (2010) study explored and compared the influence of eight aspects of parental involvement on 10th graders’ academic self-efficacy, intrinsic motivation and engagement in math and English. These researchers found that differential dimensions of parental involvement can predict positive or negative student outcomes. Parental advising and conversation were positively linked to high school adolescents’ academic self-efficacy and intrinsic motivation in English, but not to math. The authors explained the reason for these findings could be that the items measuring parental advice are too general instead of focusing on a specific subject domain. The variable of parental aspiration for students’ postsecondary education has been demonstrated as the only variable positively associated with all outcome variables.

Gonzalez-DeHass, Willems, & Holbein, (2005) summarized the reviewed literature and concluded parental involvement from elementary school to high school years have a positive relationship with the following motivational constructs: “school engagement, intrinsic and extrinsic motivation, perceived competence, perceived control, self-regulation, mastery goal orientation, and motivation to read (p.100).” Trusty & Lampe (1997) extended the relationship among school performance, motivation, and parental involvement by taking into account the
literature on a locus of control, which refers to “how students attribute the causes of events” (p.111). Internal locus of control identified internal factors, such as ability, effort, and confidence. External locus of control identified external factors, such as “luck or teacher favoritism” (p.112). They examined more than ten thousand high-school seniors’ responses about how often parents discussed school, career plans, current events, and troubling things with adolescents. The findings demonstrated that parent-child communication positively predicted high school adolescents’ internal locus of control, which allowed students to have more positive self-worth and be more willing to communicate with peers and adults about personal and interpersonal problems.

**Perceived teacher support**

**Definition of Teacher Support**

The existing body of educational research discussed the beneficial influence of warm, supportive, high-quality teacher support on students’ outcomes, such as academic, social, behavioral, and emotional adjustment (e.g., Fredricks, Blumenfeld, & Paris, 2004), but just a few of them have provided a clear definition for teacher support. Brewster and Bowen (2004) defined teacher support “as the degree to which teachers listen to, encourage, and respect students.” Wang (2009, p. 242) stated that teacher support is “the extent to which [students feel their] teachers are supportive, responsive, and committed to [their] well-being”. Danielsen et. al (2010) defined it as “teacher provision of friendliness and fairness.” Werner (1990) pointed out that teachers are responsible for “provid[ing] emotional support, reward[ing] competence, and promot[ing] self-esteem” (p.110). Ryan and Patrick (2001) claimed that teacher support “refers to the extent to which students believe teachers value and establish personal relationships with
them” (p. 440). They also summarized that all teacher support definitions share some similarities, such as “caring, friendliness, understanding, dedication, and dependability” (p. 440).

Measurement of Teacher Support

A majority of studies rely on student perceptions to measure teacher support, which represents their perceptions of the extent to which their teachers are available to provide social support, such as care and encouragement. In the current study, the researcher also uses student perceptions of support to measure teacher support. Perceptions of teacher support have been measured as one-dimensional or multidimensional in different studies. For example, researchers measured experiences of teacher support through the extent of care, fair treatment, expectations, interest, respect, concern or encouragement (Klem & Connell, 2004). Multidimensional measurement studies revealed various dimensions of teacher support. These dimensions are slightly different and overlap. For instance, House (1981) proposed that social support, especially in the context of teacher support, includes emotional, instrumental, informational, and appraisal support. He explained that emotional support refers to concern, trust, care, and empathy provided by teachers; instrumental support relates to teachers offers time, assistance, and materials to their students; information support includes teacher’s advice and suggestions; and appraisal support consists of teacher’s evaluative feedback and affirmation for their students. Pitzer and Skinner (2017, p.184) measured teacher support on three scales: “warmth versus rejection, structure versus chaos, and autonomy support versus coercion”.

Effects of Teacher Support on Student Learning and Math Achievement

A majority of the literature has focused on investigating the relationship between teacher support and students’ school-related performance. However, more studies are needed to examine
the complex mechanisms of how teacher support affects child outcomes. Within the school setting, teacher support has been documented as an integral and critical context for students’ academic, behavioral, and emotional (function) development, since students spend a large amount of time in the classroom and receive instruction facilitated by their teachers (Bronfenbrenner & Morris, 2006; Eccles & Roeser, 2010). Researchers also found that teacher support significantly influences student outcomes from the very early stages of schooling and persists to students’ adolescence period (Davis, 2003; Roeser, Midgley, & Urdan, 1996; Turner & Meyer, 2004). Wang (2009) cited that students who are in a psychological supportive and caring classroom environment are more willing to openly express themselves, more likely to meet teachers’ expectations, and tend to decrease misconduct and problem behaviors (Patrick, Ryan, & Kaplan, 2007).

A study employing longitudinal designs with a sample of 2,585 students found that adolescents who perceive an increase of teacher support, which includes caring, interest in, respect for, and concern for students, presented a corresponding growth in self-esteem and a decrease in depressive symptoms (Reddy, Rhodes, & Mulhall, 2003). Metheny, McWhirter, and O’Neil (2008) conducted two studies using a sample of senior students in high school and suggested that perceived teacher support (such as interest and care in students’ future, expectations, and listening) has a significant and positive relationship with career decision-making, self-efficacy, and vocational outcome expectations.

A longitudinal evaluation in a sample of 1,036 students in 7th through 12th graders showed that students who reported high autonomy support and clear expectations from their teachers will have the most positive pattern of outcomes, whereas students who received fewer
autonomy support and vague expectations displayed the most negative pattern of learning outcomes (Vansteenkiste et al., 2012). The authors explained that supportive teachers provide intrinsic reasons to promote students’ learning, believe students are able to master the knowledge, and then set up mutually agreed on expectations for the students. Students who received such affective support are more likely to seek assistance, and less likely to self-handicap and display disruptive behaviors compared to students who came from non-supportive groups.

There is some research specifically exploring the effect of teacher support on student’s math achievement. Newman and Schwager (1993) found that students improve their learning and are more likely to ask questions and seek assistance when they look to their teachers as helpers instead of classmates. A study of more than twenty thousand 10th to 12th grade high school students by Muller (2001) revealed that students’ perceptions of teachers’ care and commitment are associated with higher mathematics proficiency. The findings suggested that 12th graders, who are at risk of dropping out from school, may increase their mathematics proficiency and test scores if they feel teachers care about them when compared to other students who experience lower levels of teacher care. The reason behind this is that caring teachers motivate at-risk children in a different way, and are more responsive to students’ effort.

Effects of Teacher Support on Academic Motivation

Strong links between motivation-related beliefs and academic achievement have been explored by extensive studies, but less is known regarding how perceived teacher support interacts with these beliefs to influence learning outcomes. Teachers can intrinsically motivate students to learn by helping students meet their needs, offering meaningful and challenging work, allowing students to take ownership over their work, providing a safe, caring and
supportive classroom environment, and supporting mastery and understanding of content in their classrooms (Urdan & Schoenfelder, 2006; Ames, 1992; Meece et al., 2006).

A short-time longitudinal study with 1,020 children by Pitzer and Skinner (2017) examined whether teacher support could rebuild students’ established motivational patterns. They found that teacher support plays a crucial role in shaping the “motivational dynamics of the classroom by interrupting existing detrimental feedback loops and reestablishing positive motivational pathways” to promote student’s development of self-appraisals (i.e., relatedness, competence, and autonomy) (p.18). The findings revealed that students who reported being motivationally at-risk at the beginning of the year, but received high teacher support, were able to increase their motivational resilience by the end of the year; whereas low-risk students who experienced low levels of teacher support lost their motivational advantages at the end of the year.

A study by Sakiz, Pape, and Hoy (2012) examined the importance of perceived teacher support in middle school mathematics classrooms. They demonstrated that strong links exist between students’ perceived teacher affective support and “sense of belonging, higher academic enjoyment, lower academic hopeless, and greater academic self-efficacy” (p.247). These outcomes were all expected to lead to greater achievement gains. Yıldırım (2012) also found that perceived teacher support is positively associated with math self-efficacy, intrinsic value, instrumental value and negatively associated with math anxiety. In contrast, Domagala-Zysk (2006) found that students, who reported their teachers as being unhelpful in motivating and offering assistance are more likely to lose confidence and interest in learning as well as experience school failures.
Peer influence

Definition of Peer Influence

In the context of social support, traditionally, peers were established as a crucial influence on the development of an adolescent. Some scholars suggested that a positive peer relationship helps in shaping a supportive and safe classroom learning environment by delivering more help, instruction, and advice (Wentzel, Battle, Russell, & Looney, 2010; Nichols & White, 2001; Burke & Sass, 2013). However, researchers have pointed out that the concept of peer influence does not have conceptual clarity, partly because there are various definitions for it (Ryan, 2001). Hence, just a few studies provide a clear definition for peer influence, explaining that they are people of similar age whom children meet and contact often (e.g., schoolmates, classmates), and these peers have an intangible and strong power to modify students’ way of thinking, value, and behavior (Gardner, & Steinberg, 2005; Prinstein & Dodge, 2008; Wentzel & Caldwell, 1997). Another researcher states that “peer influence is when a child chooses to do something [the] child wouldn’t otherwise do, because [the] child wants to feel accepted and valued by her/his friends. It isn’t just or always about doing something against a child’s will” (Raising Children Network, 2015).

There are some constructs that are very similar or have a certain degree of overlap with peer influence. For example, peer pressure is defined as “pressure to think or behave along certain peer-prescribed guidelines – is regarded as a prominent attribute of adolescence (p. 452)” (Clasen, & Brown, 1985). The authors also stated that “peer pressure is a primary mechanism of transmitting group norms and maintaining loyalties among group members (p.452).” Peer influence and peer pressure can be both positive or negative, but peer pressure is widely used as a major contributor to unhealthy and unsafe behaviors, such as the initiation of drug use,
smoking, and drinking, particularly in adolescence (Bahr et al., 2005). Peer influence is used often as a better way to depict how children’s performance is developed by the feeling that they are owned by a group of peers (Raising Children Network, 2015).

Peer group is another similar construct to peer influence. A peer group refers to people’s interactions as well as ties with their best friends and cohort who have the same age, background, or social status (Brown, 1990). Peer groups assist adolescents in seeking independence from their supervising adults. In a peer group, students learn how to shape relationships and share thoughts, ideas, and interests that parents and teachers may not discuss with them.

Measurement of Peer Influence

Different studies have employed different measurements of peer influence. For instance, in a one-dimensional study, North and Ryan (2017) evaluated peer influence resulting from peer academic reputation by measuring friends’ grades on math/science class. Many studies have used multidimensional measures to study peer influence. For example, Wentzel and Caldwell (1997) measured peer effect from three perspectives: reciprocated friendships, peer acceptance, and group members. Lubbers et al. (2006) focused on peer acceptance and a number of friends in their research. Manski (1993, p.60) lists three types of peer influence: “endogenous effects, exogenous effects, and correlated effects”. Molloy, Gest, and Rulison (2011, p.15) explored three distinct types of peer relationships: “reciprocated friendships, frequent interactions, and shared group membership”.

Effects of Peer Influence on Student Learning and Math Achievement

Strong research evidence suggests that peers have considerable influence over problem/risk behaviors and substance use such as students’ antisocial behavior, drug use,
smoking, and delinquency (e.g., Prinstein, Boergers, & Spirito, 2001; Nash, McQueen, & Bray, 2005; Kerr, Van Zalk, & Stattin, 2012). However, there is a lack of systematic examinations on the relationships between peer influence and adolescents’ academic achievement (Ryan, 2001). In addition, most studies have emphasized the role of teachers and parents, as opposed to peers, as socializing agents of motivation and academic success. However, when children are in school, classrooms and schools become the main places where children socialize. Furthermore, students’ behaviors, values, and performance are influenced by peers through peer groups, providing emotional support, friendship, sense of identity, and entertainment (Bukowski, Sippola, & Newcomb, 2000; Dennis, Phinney, & Chuateco, 2005; Bierman, 2004; Dishion, & Tipsord, 2011). The literature notes that peer influence impacts social, emotional, behavioral, and academic development. These peer influences take place at the very start of formal education, become more important as time passes, and peak during adolescence. Peer influence is more important then than at any other time in an individual’s life because adolescents spend more than half of their time interacting with their peers and less time with their parents and family (Brown, 2004, Updegraff et al., 2001). Wentzel and Caldwell (1997) cited that in the adolescence period, friends seem to impact general levels of involvement in school as well as decisions about how much time should be spent on learning instead of social activities (Berndt & Keefe, 1995; Berndt, Laychak, & Park, 1990).

Empirical research links the peer group effect to academic performance. A study by Zimmerman (2003) examined the relationship between student roommates’ SAT scores and their academic achievement. The findings revealed that students with average SAT scores may decrease their grades if they have roommates who scored in the bottom 15% of verbal SAT distribution. Stewart (2008) analyzed a sample of 11,999 U.S. 10th-grade students using the
National Center for Education Statistics (NCES) dataset and found that positive peer relationships significantly predicted adolescents’ academic outcomes. The researcher pointed out that students were more likely to feel more engaged with their schools when their friends valued school and education. Nichols and White’s (2001) evidence suggested that students tend to achieve higher levels of academic performance when they are affiliated with clique groups compared to an unaffiliated students’ performance; clique group affiliation has potential to impact students’ academic outcomes.

Consistent with the notion that peer group context influences academic achievement, many academics have demonstrated that intelligent and hard-working students, who have high educational aspirations and standards, influence their peers to improve their grades, continue to strive to get good grades, and maintain disciplinary standards in the classroom (Altermatt & Pomerantz, 2005; Ammermueller & Pischke, 2009; Carrell, Fullerton, & West, 2009; Sacerdote, 2001). Vardardottir’s (2013) study explored the effects of high-achieving peers on students’ academic achievement using data for five cohorts of 16-year-old high school students. Evidence from his research showed that students who were assigned in classes with peers of higher academic ability increased their own academic performance. Similarly, another 3-year longitudinal study agreed with these findings by showing that students who have high-achieving friends increase their own grades compared to other similar students who did not have higher achieving friends (Mounts & Steinberg, 1995). Other researchers have confirmed previous research findings that students who have effective cooperation, interaction, and engagement with high-achieving friends significantly increase their learning performance (Rivers et al., 2013; Durlak et al., 2011). These findings are consistent with Vygotsky’s socio-cognitive theory that suggests students ask for assistance from a more knowledgeable peer for answering difficult
questions. This allows learners to promote their cognition skills through positive cooperation and supportive peer interactions.

Empirical research also links peer group effect to math learning and outcomes. Carrell, Fullerton, and West (2009) demonstrated that peer influence in math and science courses are most significant, while there is no statistically significant influence in physical education and foreign language courses. The authors’ reasoned that this was due to fewer chances for such interactions in physical education and foreign language courses. In other recent research on students’ mathematics learning, Martin et al. (2015) pointed out that students’ mathematics self-concepts are building on social comparisons with other classmates’ math achievement. In the context of math domain, students may have a relatively clear understanding about other students’ performance and ability which make comparison easier than in other subjects. In contrast, language arts or social studies courses often focus on writing and evaluating information, which is not easy to make a comparison due to varied assessment rules. Additionally, research by North and Ryan (2017) demonstrated that students’ peer academic reputation in math and science improve their academic beliefs and behaviors across the school year. The authors reasoned that this could be explained by more interaction and specific feedback provided by high-achieving classmates. Sullivan et al. (2006) also found that peer influence impacts students’ response and attitude toward school mathematics and engagement to a large extent. The researchers also evaluated that the effect of classroom culture on students’ school outcomes and found that peer influence plays a critical role in students’ math achievements and engagement compared to the curriculum and other related factors.
A longitudinal study by Burke and Sass (2008) examined students in grades 3–10 over a 6-year period controlling for the fixed inputs of students, teachers, and schools to measure and compare peer influence on students’ mathematics and reading achievement across an elementary school, middle school, and high school. These researchers found that a positive and highly significant peer influence exists within all three types of schools for students’ math achievement. The study revealed that students’ academic achievement in middle and high school depends on the average achievement level of their peer groups. For example, the improvement level of students belonging to high-achieving groups was two to three times larger than those from middle-achieving groups. In addition, the results showed that the lowest scoring students perform worse when collaborating with top-level students than when working with middle-level students, suggesting that teachers should assign lowest-achieving students with peers of average ability for classroom work. In addition, this analysis demonstrated that peer influence is significant at the classroom level, but is rarely significant at the general grade level. Moreover, it also found that estimated peer effects are generally weaker when controlled for teacher influence in the model than when they are excluded.

Effects of Peer influence on Motivation

The effects of peer influence are usually nonlinear and indirect; it influences academic outcomes by affecting students’ motivation, aspiration, attitude, and engagement. A study by Nelson and DeBacker (2008) found that the achievement-related values of a student’s best friends and classroom climate impact students’ self-efficacy beliefs, social goals, and achievement goals. The results revealed that class belongingness and the positive perceptions about their best friends’ achievement-related values motivate adolescents to learn. Other researchers found that adolescents who have friends with stronger aspirations promote their own
motivational orientations, reflected in their learning goals and values, which lead to successful academic achievement (Coleman et al., 1966, Wentzel, 1998). Zimmerman (2003) stated that peer groups impact how much a student enjoys learning by exploring ideas, sharing insights and inspirations, investigating teachers’ lectures, as well as campus and world events.

You’s (2011) longitudinal study involving 13,825 high school students using NELS: 88 showed that peer academic values and aspirations exerted significant influence on adolescents’ locus of control and educational expectation. According to these findings, the researcher implied that peer support motivated students to increase their academic competency and autonomy, which in turn influence students’ future behaviors for success. In addition, the author pointed out that adolescents modify their own thoughts and behaviors based on their peers’ values and expectations to acquire their peer group acceptance. Thus, peer influence may considerably impact students’ learning and outcomes.

Ryan’s (2001) study explored how peer groups impact adolescents’ motivation and academic achievement using Expectancy x Value framework and employing multilevel analysis techniques. This researcher found that peer group plays a crucial role in students’ scores and motivation change processes. From the sixth to seventh grade, students within higher-achieving groups showed less of a decline in scores, while students who hung out with lower-achieving groups showed a greater decline. In terms of motivation changes, students who spend more time with groups of children who like school are more likely to enjoy school; and students who are in groups that don’t like school decrease their enjoyment of school over an academic year. However, peer groups did not significantly impact students’ utility value for school and peer group influence on students’ expectancies for success was small. The author reasoned this is
because parents and teachers have relatively more influence on students’ value beliefs of
education and their future plans. Moreover, the author pointed out that this smaller influence of
peer groups may become greater in the future, since this study only investigated changes over a
period of 8 months.

The literature indicates that students in the negative peer groups are less likely to have
positive values, attitudes, beliefs, and behaviors towards school and learning achievement
(Goldsmith, 2004). For example, adolescents are more likely to drop out from school if they have
negative peer influence or pressure and peer disapproval (Nichols & White, 2001). Some peers
apply negative peer pressure to high achievers. Researchers argued that students in the negative
peer groups would disparage academic achievement and label high academic performance
students as *nerds*. In many schools, nerd groups were classified as one of the lower status
crowds, and status among students is a power relationship (Brown, Mory, & Kinney, 1994;
Langer-Osuna, 2016). High achievers are likely to feel pressure and reduce the academic
performance to cater to social goals rather than academic goals (Rentzsch, Schutz, & Schroder-
Abe, 2013).

**Effects of Three Social Support Sources on Student Learning and Motivation**

Ryan (2000) stated that teachers, parents, and peers offer adolescents expectations,
suggestions, and ways to think about and engage in school. The author also pointed out that the
reactions and evaluations of teachers, parents, and peers “serve to affirm, sustain, or change
adolescents’ motivation and engagement” (p. 101). Although it is clear from the literature that
student achievement is influenced by social support from parents, teachers, and peers, most
studies have examined student academic outcomes and motivation only in relation to individual
social support factors, such as parental involvement. Just a few studies have involved all three 
support systems simultaneously in a single model to examine which support factor, or 
combination of factors, has the most influence on students’ academic achievement, especially for 
high school students. Moreover, the role of motivation in explaining links between social support 
and academic achievement has not been well examined in previous research. A few studies 
presented below are good examples of engaging three social support sources in a single study. 
Although some of them explored the mediation role of motivation, none of them examined the 
relationships among social support, academic motivation, and math achievement on high school 
students.

A longitudinal study by Wentzel (1998) employing 167 sixth-graders examined whether 
students' perceived support from parents, teachers, and peers associated with their multiple 
motivation beliefs, including interest in school and class, social goal pursuit, and goal 
orientations. The regression analysis indicated that peer support was related to students’ 
prosocial goal pursuit, teacher support was associated with class and school interest as well as 
social responsibility goal pursuit, and parent support significantly predicted school-related 
interest and goal orientations. These results are consistent with previous research that teachers 
have the most proximal influence in classrooms, students’ future orientations are driven by their 
parents, and peer groups impact students’ social adjustment to school. Ahmed et al.’s (2010) 
study involving 238 seventh-grade students examined whether the effect of perceived social 
support from parents, teachers, and peers on achievement is transferred through students’ 
motivation beliefs. The SEM results revealed that the three individual perceived social supports 
facilitated students’ motivational and emotional beliefs, which in turn increased students’ 
academic outcomes. These two studies explored how the motivation beliefs mediate the effect of
three social support constructs on students’ academic achievement, but they did not put these three constructs in a single model to explore and compare how the strength of the relationships varied by each support systems on high school math achievement.

Another study by Fan et al., (2009) using Educational Longitudinal Study of 2002 (ELS:2002), which involved 15,362 students in 10th grade, investigated whether students’ interpersonal support relations impact student academic self-efficacy and whether student academic self-efficacy is associated with student math and English achievement. Through SEM approach, the findings showed that all three support systems are positively linked with the sense of academic self-efficacy in both math and English and that students' academic self-efficacy predicted students' academic outcomes. The researcher further found that the perception of teacher support had the greatest influence on students’ math self-efficacy compared to close friends' academic values and parent-student communication. However, the researchers did not examine whether students' supportive relationships affect student academic achievement directly instead of through motivation.

Chen’s (2005) study, engaging 270 Hong Kong students from grades 9 to 11, explored whether adolescents’ academic outcomes would benefit from parental, teacher, and peer support using structural equation modeling. Findings showed that perceived teacher support to academic achievement was the strongest, and while perceived parental and teacher support both significantly predicted academic achievement, perceived peer support did not. However, the relationship is negative between parental support and students’ performance. One of the reasons the author provided for this is that perceived higher levels of parental support may lead to restriction and social control, which often occurs in Chinese culture. The author explained that
this control and restriction is an obstacle for children to seek independence from their family. Thus, more control from parents leads to negative results on their academic outcomes.

A follow-up study by Chen (2008) explored grade level differences in the influence of adolescents’ supportive relationships with teachers, peers, and parents on academic achievement directly and indirectly through students’ engagement. The results revealed that students relied less on their parent support for academic achievement, perceived teacher support only positively predicted 9th graders’ academic achievement, and perceived teacher support does not have any significant relationship on 11th grade math achievement. However, the results also showed that there is no significant direct or indirect relationship between student achievement and perceived peer support at any grade level. The author explained that younger adolescents seek more support from adults, but older adolescents with higher levels of cognitive abilities ask for more independence and autonomy, especially for U.S. students. The author also mentioned that older adolescents may like to seek more approval and advice from peers rather than adults.

Involving 1,479 students (Grades 7 through 11) from a longitudinal study, Wang and Eccles’s (2012) study investigated the relationships of students’ perceived social support from parents, teachers, and peers to academic achievement directly and indirectly through different dimensions of school engagement. HLM analysis results showed that all three support variables significantly predict students’ sense of school identification; teachers and parents are a critical support source in the reduction of declines in school compliance; and peer influence is positively or negatively related to school compliance, depending on types of peer groups. Furthermore, these authors concluded that peer social support has a relatively stronger influence on adolescents’ behavioral outcomes than both teacher and parental social support. The author
pointed out that parents still play an important role in the educational endeavors of youth, since parental social support positively predicted all indicators of school engagement.

Ma’s (2011) study analyzed six waves of data (Grades 7 through 12) from the Longitudinal Study of American Youth (LSAY) to explore how expectations and influence of peers, teachers, and parents are linked to participation in advanced mathematics. According to survival analysis results, peer influence and teacher expectation did not significantly predict student participation. However, the effects of parental expectation and parental college planning on children were relatively strong. Thus, the researcher suggested placing more emphasis on parental involvement. Additionally, the author suggested that students’ future expectations and attitudes toward mathematics played important roles in explaining students’ participation in more advanced mathematics.

Overall, the literature discussed above demonstrated that social support has a differential effect on various outcomes and participants. There has not been another study that has examined how social support (from parents, teachers, and peers) influences high school students’ academic achievement both directly and indirectly through motivation beliefs all within a single model. Thus, this study will address this research gap by involving three social support systems simultaneously within a single model. It will examine the relative and additive influence of social support on high school students’ math achievement directly and indirectly by mediating their academic motivation beliefs.

**The relationship of SES to Academic Success**

The SES variable has long been studied as an important role in students’ development. Duncan, Featherman, and Duncan (1972) determined that SES is a combination of parental
income, parental education, and parental occupation. Numerous studies have indeed shown that one’s educational success depends strongly on their high SES, where the parents are advantaged socially, educationally and economically; in contrast, students living in poverty are more susceptible to poor academic achievement than students not from lower SES family (e.g., Considine & Zappala, 2002a, 2002b; Hecht & Greenfield, 2001; Okpala et al., 2000). Families from high-SES communities are more likely to have the financial resources or time to support their children’s education, whereas low-SES families are unable to provide students with various academic resources; parents may not have the financial resources available to afford academic materials, such as books, computers, or tutors (Orr, 2003). More importantly, the high-SES families stimulate active parental involvement, and active parents increase positive attitudes of children regarding school, classes, and academic achievement (Stevenson & Baker, 1987). In addition, the school systems in low-SES communities suffer from a lack of resources and the loss of well-qualified teachers, which negatively impacts students’ academic performance, increases dropout rates, and perpetuates the low-SES status of the community (Aikens & Barbarin, 2008). Moreover, students from low-SES neighborhoods are more likely to face negative peer pressure to drop out of school, engage in sex or drugs, and conform to a group or gang.

Lower SES background places children at risk of failure in school, but social support from parents, teachers, and peers can reduce this risk. In other words, social support plays a protective-stabilizing role in the relationship between SES background and learning outcomes in those disadvantaged students. Malecki and Demaray (2006) found that students’ GPAs remained unchanged regardless of their SES background when students received higher levels of social support. Additionally, these authors pointed out that when students receive lower support from
their parents and peers, the high SES students still had a higher GPA, but low SES students performed at a lower GPA.

From the explanation provided above, the generally low levels of academic outcomes on the part of lower SES students could be explained by the lack of social support, which is necessary to reduce students’ risk of failure in education. Thus, identifying how and which students’ social support systems are significantly affected by students’ SES background is beneficial for policymakers and educators. This can help to design and implement early intervention programs for preventing negative educational outcomes for students from lower SES status.

**Ethnic Disparity in Academic Achievement and Math Achievement**

Previous research has demonstrated that students’ academic achievement and levels of social support varied among racial and ethnic groups (Yan & Lin, 2005; Jeynes, 2003; Suizzo, & Stapleton, 2007). The racial achievement gap remains substantially large. In math, only 7% of African-American students and 12% of Hispanic students scored proficient or above compared to 47% of White and 32% of Asian students. White 17-year-old students have generally scored about one standard deviation above African-American students (NAEP, 2015). Autor and Handel (2013) reported that African-American students are less likely than their white schoolmates to report that they need to use high school level math in their job on a weekly basis. According to NAEP, the Asian student subgroup was the only one that reported a significant increase in math proficiency.

Thus, social support analyses should consider racial and ethnic variations to understand the impact of social support on student achievement. Inequality in family background existed
between the White/Asian and African-American/Hispanic groups. A longitudinal study from NELS: 88 involving 19,386 high school students by Yan and Lin (2005) reported that Caucasian parents are more involved in their students’ schooling (such as attending school programs about teenager's future planning) compared to Asian, African, and Hispanic American parents. They also reported that parental involvement is the most effective way for Caucasian students rather than other minority groups to improve learning performance. African American parents had contact with school and teachers about their children’s performance more often, but the influence of parental involvement for this group is not as strong as other groups. Asian American parents had the highest educational expectations for their students' academic success; the reason may be that they value education as the best resource to achieve social mobility. Hispanic parents seemingly had the lowest level of involvement possibly because they were unable to effectively engage and interact with their children’s school and teachers. Along with these different levels of parental involvement and assistance, the analysis showed that from 8th to 12th grade, Asian students reported the highest mathematics achievement, Caucasian students followed, and Hispanic and African American students had the lowest math scores. Suizzo and Stapleton (2007) showed that Asian American parents have significantly higher expectations than all three other groups, and Latinos had higher expectations than both African Americans and European Americans. In this research, European Americans reported the lowest expectations for their children’s educational attainment and were the least involved in parent-child discussions, but European Americans reported the highest SES background and the safest communities.

Besides parental involvement, teachers play another vital role in shaping students’ school outcomes. Researchers stated that teachers’ support and expectations may be systematically biased, which will counteract or reinforce traditionally disadvantaged students’ school outcomes.
Using a nationally representative survey of U.S. 10th graders, Gershenson and his colleagues (2016) found that non-African American teachers have lower expectations on the educational outcomes of African American students than African American teachers do. Pigott and Cowen (2000) documented that African American children were judged by their teachers to have “more serious school adjustment problems, fewer competencies, more negatively stereotypic personality qualities, and poorer educational prognoses” than their white peers (p.190). The authors continued to mention that the stereotypic views and negative judgments of African American students may reduce these students’ level of accomplishment, learning effort, and motivation, and increase their self-doubt as well. A study by Brewster and Bowen (2004) showed that teacher support is more efficient than parental support for Latino middle and high school students to increase school engagement and decrease problem behaviors.

As social support is beneficial in shaping student outcomes and reducing systematic biases for disadvantaged minority students, it is important to explore how students from different ethnic groups perceive social support as well as relationships between these social supports and educational outcomes. Such information and knowledge contribute useful practical cues about how to enhance educational success for disadvantaged minority students.

Summary/Literature Gap

As reviewed above, Vygotsky's sociocultural theory and previous empirical research demonstrated that social support from parents, teachers, and peers positively influence students’ school outcomes. In addition, Expectancy-Value theory provides a comprehensive framework to understand adolescents’ learning performance, predicated by achievement motivation beliefs that are determined by students’ social context. In the current study, the following relationships are
explored: 1) how social support systems including parents, teachers, and peers affect 11th grader’s math achievement; 2) how motivation beliefs including interest, utility, value, and self-efficacy predict students’ math outcomes; 3) how family SES impacts students’ social support; 4) how the relationships among social support systems, motivational beliefs, and math performance differ by ethnicity.

Although previous studies demonstrated the importance of social support and motivation on students’ math achievement, there are still limitations in existing literature. First, studies are limited in that few researchers have included all of the three important social support systems simultaneously as academic support sources, in order to investigate the extent of their effect on math achievement and academic motivation. It is important to evaluate all three social support systems simultaneously because adolescents’ cognitive development differs in terms of the nature and quality of their relationships and communication with significant social agents.

Secondly, while many researchers and theorists acknowledge the importance of social helpers, the mechanisms by which and how different social support factors arouse students’ motivation and enhance learning outcomes are not well understood. King and Ganotice (2014) explained that the reason for this may be because many researchers have identified academic motivation as an intrinsic characteristic of the students. In the current study, it explores how parents, teachers, and peers, as social agents, help adolescents increase their learning interest and value beliefs toward education. Furthermore, the current study is likely to highlight how social support factors differ in how they affect children’s achievement motivation. Knowing the role of motivation in explaining links between social support and math learning outcomes can help
guardians and educators to develop more effective strategies for assisting students to improve their math learning outcomes.

Thirdly, high school period for most adolescents is a period of transition. Adolescence has been defined as a particularly transformative and challenging development period in all areas of an individual’s life. Adolescents’ future aspirations form and develop in this period. In this period, they need more informational and emotional support from significant adults to establish a positive self; on the other hand, they seek psychological independence to develop their sense of responsibility and identity. Adolescents think about questions such as “what do I want in the future? Should I go to college or find a job?” “How do I manage my time?” and “What can I do for my goals and plans?” Adolescents’ important social players: parents, teachers, and peers, all affect answers to these questions and children’s choices. There is limited research on how social agents influence adolescents’ development during their challenging high school period. Therefore, it is crucial to conduct research that examines simultaneously the complex and varied factors that affect academic outcomes. Such research helps to understand which support system is more salient, which support system is becoming weaker, and combined effects of the three social support resources on adolescents’ motivation and math learning during their high school years. Furthermore, such research is valuable to educators and policy-makers in making academic policies and producing better support systems.

Fourthly, many studies that have investigated the effects of the three social support resources on adolescents’ academic achievement either used county administrated data (Wang & Eccles, 2012; Wentzel, 1998; Bouchey, 2004; Kindermann & Vollet, 2014; Domagala-Zysk, 2006) or focused on student samples in Asian and European countries (Chen, 2005, 2008; King
& Ganotive Jr, 2014; Vedder-Weiss, & Fortus, 2013; King, 2016; Lam et al, 2012); very few studies have used the recent U.S. nationwide data sets. Therefore, the HSLS:09 national representative dataset used in the current study can help to increase the likelihood of obtaining accurate and representative results.

Lastly, most existing studies examining the influence of each of the three social supports have used a composite measurement instead of focusing on a specific dimension. The current study contributes more precise knowledge by examining specific aspects of each support system rather than adding all dimensions into a general composite. For example, for parental involvement, most previous studies did not distinguish between home-based involvement and school-based involvement. Furthermore, those studies that distinguished home-based involvement from school-based involvement have highlighted the importance of school-based parental involvement. However, the effects of home-based involvement, especially for parent-child communication, have been much less frequently examined. In addition, strong links between teacher support and academic achievement have been explored by extensive studies, but little is known about how teacher support interacts with motivational beliefs to influence students’ math achievement. Lastly, despite a large number of prior studies examining peer influence, most research has focused on examining how peer pressure is related to substance use and risk behavior, rather than on identifying peer influence in explaining adolescents’ academic achievement and motivation.

In sum, Chapter two presents an extensive review of the literature on the social and motivational factors and their inter-relationships in determining the math achievement. It also points out certain areas where more research would lead to deeper understanding of math
learning. Based on the above review of the literature, this study provides an analysis of social support influences, students’ motivation to learn math and achieve math success using a hypothesized structural model, which is explicated in Chapter three.
CHAPTER 3

METHODS

The purpose of this study is to examine the influence of social context support factors (including parental involvement, perceived teacher support, and peer influence) on math outcomes among 11th graders, as well as to explore whether academic motivation acts as a mediator in the relationships between social contextual support factors and math achievement. Structural equation modeling (SEM) was the primary statistical analysis technique used in this study. This chapter includes a detailed description of the data, study participants, variables, research methods and design, and the hypothesized theoretical model of the study.

Data Sources

For the current research, data were drawn from the National Center of Education Statistics (NCES) High School Longitudinal Study of 2009 (HSLS: 09), which was the fifth in a series of longitudinal studies. HSLS:09 provides insights into students’ educational trajectories from the beginning of high school to the postsecondary options, to the work choices, and to early adult life. Particularly, it investigates the paths into and out of STEM fields as well as the educational and social experiences that affects these changes (Ingels et al., 2011). HSLS: 09 is a nationally representative study, which collected data across all 50 states and the District of Columbia. The HSLS:09 base year data were collected in the 2009-10 school year, which randomly sampled more than 23,000 ninth graders in 944 public and private schools (Ingels et al., 2011). The first follow-up wave took place in the Spring of 2012 when most sampled students were in their junior year (11th grade). This 2012 data were used in the current study because the 11th grade is a very important high school year for college admission, as well as for
exploring the dynamics of educational and career decision-making. A 2013 postsecondary update provides the cohort’s college major choices and plans.

A two-stage sampling process was used for data collection in the HSLS:09 study. In the first stage, 1,889 sampled schools were recruited from 50 states and the District of Columbia using stratified sampling, which finally resulted in a total of 944 schools. In the second stage, students were randomly sampled with more than twenty thousand participants. On average, about 28 ninth-grade students were selected from each participating schools. During the student selection stage, students were recruited by the student’s race/ethnicity (Hispanic, Asian, Black, and Other) specified by the school using stratum-specific sampling. Asian students were a little oversampled from each participating school (Ingels et al., 2011). During the year of 2012, student data were collected in 904 of the 939 high schools and included responses from a 35-minute questionnaire and a 40-minute online mathematics assessment. Parents, principals, mathematics and science teachers, and schools’ lead counselors also took surveys via web or phone.

The HSLS dataset is very well suited for this research because: 1) it provides rich information and clear measurements on the variables of interest, such as students’ motivation beliefs, which contain specific measured scales of students’ math self-efficacy beliefs, intrinsic or interest values, identities, and utility values. Furthermore, social context is described in detail, such as parent-child involvement, parent-school involvement practices, perceived teacher support, teaching practices, extracurricular activities, peer influence, student academic outcomes in mathematics and science, and so on. These data are valuable and effective for addressing the current research questions. 2) HSLS is the most recent national representative data focusing on
high school students’ mathematics achievement. It was chosen to help increase the current study’s external validity, extend its generalizability, and reflect new changes. 3) Many researchers have explored 9th graders math achievement and the mechanism of its influence, but only a few studies have investigated 11th grade students’ math learning. Therefore, it is necessary to conduct an upper adolescents’ math study that can be used for cross-cohort comparisons to capture the changes in mathematical proficiency and its important correlation.

Sample

HSLS:09 is a longitudinal study and follows its cohort over time, beginning with 9th graders. In the first follow-up year study, participants were comprised of 23,503 eleventh-graders randomly selected from 944 public and private high schools. In the current study, after dropping out the missing values on the variables of interest, a final nationally representative analytical sample of 4,418 student participants was used. The sample distribution is summarized in Table 1. A random split-sample EFA was conducted with five percent (n = 238) of the cases, randomly selected from the samples of 4,418 students, and the remaining ninety-five percent of cases (n=4,118) were used for the confirmatory factor analyses. A full sample data has been used for the structural model analyses (n=4,418). The HSLS:09 data is a national survey, which is designed to provide estimates for the target population. Therefore, the relative sample weight method was used to reflect the unbiased estimates of the corresponding population values.

A large number of nonresponse (or missing data) is often encountered in large-scale surveys (Madden et al., 2017). For the HSLS:09 data, although the data collectors made great efforts to collect responses, some survey items remain unanswered, especially for parent level reports, which is severely under-reported (Ingels et al., 2011). In the base-year survey, there are
25,206 students who are eligible and 16,995 parents participated the parent questionnaire. For the first follow-up year of 2012, all study-eligible students sampled in the base year were all included for the parent questionnaire regardless of their base-year response status, but only 8,651 parents participated the parent survey (Ingels et al., 2013). The reasons for the larger missing values include: data collectors were not able to locate parents, some participated schools closed, and 9th grade participants were no longer enrolled in 11th grade. HSLS:09 authors imputed missing values for some variables, such as SES, ethnicity, gender, and mathematical ability, but most variables did not receive this treatment (Ingels et al., 2011). Since the missing values are unrelated to the variables of interest, complete case deletion (listwise deletion) was used in the current study to remove all data for a case that has one or more missing values.

Table 3.1. Distributions of the Sample of the HSL: 09 First Follow-up Year Data and the Sample of the Present Study

<table>
<thead>
<tr>
<th></th>
<th>Sample of the HSLS: 09 First Follow-up Year Data</th>
<th>Sample of the Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>23,503 (100%)</td>
<td>4,418 (100%)</td>
</tr>
<tr>
<td>School type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public</td>
<td>18,261 (77.7%)</td>
<td>3,423 (77.5%)</td>
</tr>
<tr>
<td>Private</td>
<td>3,625 (15.4%)</td>
<td>995 (22.5%)</td>
</tr>
<tr>
<td>Region</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northeast</td>
<td>3,460 (14.7%)</td>
<td>720 (16.3%)</td>
</tr>
<tr>
<td>Midwest</td>
<td>5,807 (24.7%)</td>
<td>1,230 (27.8%)</td>
</tr>
<tr>
<td>South</td>
<td>8,908 (37.6%)</td>
<td>1,728 (39.1%)</td>
</tr>
<tr>
<td>West</td>
<td>3,704 (15.8%)</td>
<td>740 (16.7%)</td>
</tr>
<tr>
<td>Locale</td>
<td></td>
<td></td>
</tr>
<tr>
<td>City</td>
<td>6,179 (26.3%)</td>
<td>1,343 (30.4%)</td>
</tr>
<tr>
<td>Suburban</td>
<td>6,709 (28.5%)</td>
<td>1,686 (38.2%)</td>
</tr>
<tr>
<td>Town</td>
<td>2,816 (12%)</td>
<td>443 (10.0%)</td>
</tr>
<tr>
<td>Rural</td>
<td>6,178 (26.3%)</td>
<td>946 (21.4%)</td>
</tr>
<tr>
<td>`Gender</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>11,975 (51%)</td>
<td>2,140 (48.4%)</td>
</tr>
<tr>
<td>Female</td>
<td>11,528 (49%)</td>
<td>2,278 (51.6%)</td>
</tr>
<tr>
<td>Race/Heritage</td>
<td>No. (%)</td>
<td>No. (%)</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td>Amer. Indian/Alaska Native, non-Hispanic</td>
<td>181 (.8%)</td>
<td>23 (.5%)</td>
</tr>
<tr>
<td>Asian, non-Hispanic</td>
<td>1,922 (8.2%)</td>
<td>391 (8.9%)</td>
</tr>
<tr>
<td>Black/African-American, non-Hispanic</td>
<td>2,448 (10.4%)</td>
<td>453 (10.3%)</td>
</tr>
<tr>
<td>Hispanic, no race specified</td>
<td>250 (1.1%)</td>
<td>20 (.5%)</td>
</tr>
<tr>
<td>Hispanic, race specified</td>
<td>3,612 (15.4%)</td>
<td>619 (14.0%)</td>
</tr>
<tr>
<td>More than one race, non-Hispanic</td>
<td>2,021 (8.6%)</td>
<td>536 (9.2%)</td>
</tr>
<tr>
<td>Native Hawaiian/Pacific Islander, non-Hispanic</td>
<td>118 (.5%)</td>
<td>17 (.4%)</td>
</tr>
<tr>
<td>White, non-Hispanic</td>
<td>12,951 (55.1%)</td>
<td>2,490 (56.4%)</td>
</tr>
</tbody>
</table>

**Research Questions**

The study proposes four research questions based on the conceptual model as presented below:

1. **What is the effect of each social support factor (parental involvement, student perceptions of teacher support, and peer influence) on high school students’ math achievement after controlling for SES and prior math achievement, and which support factor has a stronger effect?**

2. **What is the effect of each social support factor (parental involvement, student perceptions of teacher support, and peer influence) on high school students’ motivation in mathematics after controlling for SES and prior math achievement, and which support factor has a stronger effect?**

3. **Do students’ motivation beliefs (math self-efficacy, math interest, math utility value, and math identity) mediate the relationships between social support factors (parental involvement, student perceptions of teacher support, and peer group) and students’ mathematics achievement?**
4. Does the strength of the associations among social support systems, motivational beliefs, and math performance differ by ethnicity?

Measures

All variables, scales, and measures employed in this study were acquired from student and parent self-report survey data from the first follow-up year of 2012. The 11th graders’ standardized math test scores, their family SES, and previous math achievement were administered by the HSLS authors. These variables were used directly in the current study. Eight latent variables were created in the current study by using exploratory and confirmation factor analyses: parental involvement, student’s perception of teacher support, peer influence, motivation beliefs, mathematics self-efficacy, mathematics identity, mathematics utility value, and mathematics interest.

Family SES

The index of X2SES is a scale of socio-economic status composite, which is acquired from parent self-reported data and calculated using guardians’ education, occupation, and family income. This composite variable is standardized and a higher value indicates a higher socioeconomic status.

Prior Achievement

The base year mathematics standardized theta scores are used to indicate students’ previous math achievement. The unique 73 items were selected by the HSLS:09 authors from the field-test pool of 264 items. These math items present a balance throughout the six math content domains, the four algebraic processes, and different difficulty levels (Ingels et al., 2013). The
math assessment scores are used to represent students’ prior math ability based on IRT (Item Response Theory) scaled scores.

Parental involvement

There are seven sections of parent survey in the HSLS:09 data set, such as parents’ race/ethnicity and immigration status, parents’ involvement in students’ education and plans and preparations for their postsecondary education. In the current study, parent home-based involvement was focused on exploring its influence on students’ math achievement. As discussed in Chapter 2, parental involvement is a multidimensional construct due to the fact that there are various types of parental involvement practices and patterns. The parent questionnaire in HSLS:09 detailed several parental involvement strategies, such as parental expectations, parent-child activities and events, parent-child communication, and so on. In the current study, the parent-child communication was measured and explored. In the survey, parents answered the question: “Since the start of the 2011-2012 school year, about how often have you discussed the following with your teenager?” Parents answered on a 4-point Likert scale with 1= Never, 2= Once or twice, 3= Three or four times, and 4= More than four times.

(1) How often discussed selecting courses or programs at school.
(2) How often discussed preparing for college entrance exams.
(3) How often discussed applying to college/other schools after high school.
(4) How often discussed careers he/she might be interested in.
(5) How often discussed job that he/she might want to take after high school.
(6) How often discussed community/national/world events.
(7) How often discussed things that were troubling him/her.
Perception of Teacher Support

In the present study, perceived teacher support for learning math is defined as the students’ perception of their mathematics teachers’ support. Students answered the question: “How much do you agree or disagree with the following statements about your teacher?” The level of agreement on the following 5 items was measured using a 4-point Likert scale where 1=strongly agree, 2=agree, 3=disagree, and 4=strongly disagree. For item 1, a lower score means that students received more support from their math teachers. For the rest of items, a higher score indicates lower teacher support. In order to align with other social support scales and keep items semantically in the positive direction, items 2, 3, 4, and 5 have been reversely coded where 1=strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. After recoding, a higher score means students received more teacher support whereas a lower score means students received less teacher support.

(1) Your teacher treats some kids better than other kids.

(2) Your teacher makes math interesting.

(3) Your teacher makes math easy to understand.

(4) Your teacher wants students to think, not just memorize things.

(5) Your teacher doesn't let people give up when the work gets hard.

Peer Influence

Peer influence was established as a crucial factor on the development of adolescent. It is defined as “pressure to think or behave along certain peer-prescribed guidelines – is regarded as a prominent attribute of adolescence (p. 452)” (Clasen, & Brown, 1985). Under the HSLS:09
data set, students reported their peers’ academic achievement, educational pursuits, and career choices by answering the following questions. All questions are rated on a 5-point Likert scale where 0=None of them; 1=Less than half; 2=About half; 3=More than half; 4=All of them. In order to align with other scales and keep items semantically in the positive direction, item 4 has been reverse coded where 1= All of them; 2= More than half; 3=About half; 4= Less than half; 5= None of them. After recoding, a higher score means students who had no friends never dropped out of high school.

(1) How many of your close friends get good grades?

(2) How many of your close friends have taken the PSAT, SAT, PLAN or ACT?

(3) How many friends plan to attend 4-year college?

(4) How many of your close friends have ever dropped out of high school?

Mathematics Identity

There are two items employed to measure 11th grade students’ beliefs about how they perceive themselves and how they think others perceive them in regard to mathematics. Students were asked to indicate their level of agreement on a 4-point Likert scale where 1=strongly agree, 2=agree, 3=disagree, and 4=strongly disagree. For keeping items semantically in the positive direction, these two items have been reverse coded where 1= strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. After recoding, a higher score represents a stronger mathematics identity.

(1) You see yourself as a math person.

(2) Others see you as a math person.
Mathematics Self-Efficacy

Four items are used to measure a student’s belief in his or her ability to succeed in mathematics learning in the 11th grade. The scales used to measure these four items are: 1=strongly agree, 2=agree, 3=disagree, and 4=strongly disagree. To align them with other scales and keep items semantically in the positive direction, these items have been reverse coded where 1= strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. After recoding, a higher score means students have a more confident perception of their ability to do well in mathematics.

(1) You are confident that you can do an excellent job on math tests.

(2) You are certain that you can understand the most difficult material presented in math textbooks.

(3) You are certain that you can master math skills.

(4) You are confident that you can do an excellent job on math assignments.

Interest in Mathematics Courses

There are three items employed to measure 11th grade students’ interest or enjoyment in mathematics. Students were asked to indicate their level of agreement on a 4-point Likert scale where 1=strongly agree, 2=agree, 3=disagree, and 4=strongly disagree. For keeping items semantically in the positive direction, the first item has been reverse coded where 1= strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. After recoding, a higher score represents a higher level of interest in mathematics classes.

(1) You enjoy mathematics classes very much.
(2) You think mathematics classes are a waste of your time.

(3) You think mathematics classes are boring.

**Mathematics Utility**

There are three items employed to measure 11th grade students’ utility values about whether mathematics is useful for their future goals and plans. Students were asked to indicate their level of agreement on a 4-point Likert scale where 1=strongly agree, 2=agree, 3=disagree, and 4=strongly disagree. To align them with other scales and keep items semantically in the positive direction, all three items have been reverse coded where 1= strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. After recoding, a higher score represents a higher level of mathematics utility values.

(1) Math is useful for everyday life.

(2) Math is useful for college.

(3) Math is useful for a future career.

**Dependent Variable: 11th Grade Mathematics Achievement**

The first follow-up mathematics composite standardized theta scores on an Algebra test is the dependent variable used to measure students’ algebraic skills, understanding, and problem solving (Ingels et al., 2011). The first follow-up math assessment extended the base year assessment from 73 items to 118 items. These assessment scores are used to represent 11th graders’ math ability based on IRT scaled scores. The IRT three-parameter logistic (3PL) model examines the likelihood of a student correctly answering an item by estimating the item’s difficulty, discriminating ability, and guessing factor (Hambleton & Swaminathan, 1991).
Method: Structural Equation Modeling

The reason for choosing structural equation modeling (SEM) as the main statistical technique in the present study is due to its three unique advantages: 1) SEM is a theory-based technique that focuses on the confirmatory instead of exploratory analyses. SEM allows a researcher to determine the causal relationships using existing research as well as to decide whether a certain model is valid using parameter estimates and fit indices. 2) SEM is widely used in the social sciences because it emphasizes latent constructs. It defines latent variables (unobserved) using observed variables. Furthermore, it explores the causal relationships between dependent and independent latent variables, accounting for measurement errors. 3) SEM is a combination of multiple regression analysis, factor analysis, and path analysis. Therefore, it allows a researcher to graphically present a series of complex relationships among multiple and interrelated latent variables simultaneously. The path analysis allows for direct, indirect, and total causal effects estimation in the hypothesis model.

Data Analysis Steps

The data analysis in the current study is a five-stage process. First, descriptive statistics and correlation matrices for all variables of interest were summarized to give an overview of the data. Second, exploratory factor analysis (EFA) was conducted to identify the underlying factor structure. Third, confirmatory factor analysis (CFA) was conducted to determine whether the observed items measure the corresponding latent variables. Fourth, the structural model was analyzed to empirically test and estimate the proposed hypothesized relationships between latent variables. Lastly, multi-group SEM analyses were conducted in this study to explore whether relationships among latent variables differed across groups (White/Asian and African-American/Hispanic). Mplus 8 was used to conduct the structural equation modeling (Muthen &
SPSS 21 was applied to conduct the descriptive and EFA analyses (IBM Corp, 2012).

**Descriptive and Correlation Analyses**

First, a series of descriptive statistics such as the mean, standard deviation, skewness and kurtosis, the frequency for all social contextual support factors, motivation beliefs, and mathematics theta scores were generated. The differences in student math achievement across student demographic status were also presented in Chapter 4. These descriptive statistics provide simple summaries about the data and the measures. Second, correlation matrices were used to determine whether the items are related to each other and also to indicate the strength of these relationships.

**Exploratory Factor Analyses**

Factor analysis was used to better identify the items that load on a single factor, to identify latent variables, and reduce the number of indicators in a scale (Thompson, 2004). There are two main types of factor analysis: EFA and CFA. According to Kim and Mueller (1978), EFA is used when the researcher makes no a priori assumptions about how many underlying dimensions there are for the given data. EFA should be used before CFA since EFA in order to provide important information for the researcher to make decisions about how to conduct the analysis.

In the current study, a random split-samples EFA was conducted with five percent (n = 238) of the cases, randomly selected from the sample of 4,418, to identify and reduce the number of items to form the initial latent constructs. Principal axis factoring (PAF) and maximum likelihood (ML) are two extraction methods that are usually used, since both of them can
produce best results (Fabrigar et al., 1999). In the current study, maximum likelihood method was used because “it allows for the computation of a wide range of indexes of the goodness of fit of the model [and] permits statistical significance testing of factor loadings and correlations among factors and the computation of confidence intervals” (Cudeck & O'Dell, 1994, cited by Fabrigar et al., 1999, p 277). Factor rotation is carried out to interpret factor matrixes. In order to achieve a solution with an equivalently well-fitting structure, factors should be rotated in multidimensional space. In this study, the oblique rotation method as used since the factor correlation matrixes for values of interest are over ±0.32 (Tabachnick & Fiddell, 2007). Furthermore, for oblique rotations, “the promax rotation has the advantage of being fast and conceptually simple” (Abdi, 2003, p 6). Overall, the Maximum Likelihood method with a promax rotation was used to identify the latent constructs that best characterized a set of variables. Furthermore, Kaiser's (1960) eigenvalue-greater-than-one rule and Cattell's (1966) scree plot rule were used to build the optimal number of underlying factors to retain in EFA. More specifically, how many factors are remained in EFA that are decided based on the number of eigenvalues that are larger than 1. An observed item is kept when its factor loading is larger than or equal to 0.3. In addition, the researcher also needs to consider the theoretical and empirical background and the reasonableness of the factor structure (Meyers et al., 2006). Lastly, reliability analysis was used to estimate the internal consistency of a whole construct as well as each dimension within a construct, using Cronbach’s alpha.

**Confirmatory Factor Analysis**

Structural equation modeling (SEM) analysis is carried out in two steps (Schumacker & Lomax, 2010). Confirmatory factor analysis (CFA) as a prior step was used to determine whether the observed items measure the corresponding latent variables. In other words, the goal of CFA
is to examine whether the data fit a hypothesized measurement model. CFA is commonly used in the social science research. It requires that the researcher understands the relevant theory and previous literature first in order to postulate the relationship pattern. In the current study, the initial hypothesized measurement model consisted of four latent constructs and their respective observed variables: parental involvement with seven indicators; teacher support with five indicators, peer influence with six indicators, and motivation beliefs with four indicators.

Although there are multiple choices for evaluating model fit, Kline (2010) strongly recommended reporting the Chi-squared test, the Root mean square error of approximation (RMSEA), the comparative fit index (CFI), and the standardized root mean square residual (SRMR).

It is necessary to examine the validity and reliability of latent constructs. Hair et al. (2006) define construct validity as “the extent to which a set of measured items actually reflect the theoretical latent construct those items are designed to measure” (p. 776). The construct validity can be evaluated using standardized factor loadings, which evaluate the variance that is accounted for by the latent factors. “The construct validity intended to see the extent to which indicator converge or shares in a single construct” (Ye, 2016, p. 64). A high value of factor loading represents high convergent validity. Items with a low factor loading (< 0.40) should be removed from the model due to the fact that they do not fit the measurement model. Reliability is the extent of consistency between individual measures for the intended latent constructs, which can be examined by Cronbach’s Alpha. Cronbach’s alpha values that are above .7 can be considered to achieve reliability for a construct (Churchill, 1979).
Structural Model Analysis

Structural Equation Modeling consisted of five steps: 1) model specification, 2) model identification, 3) model estimation, 4) model testing, and 5) model modification (Schumacker & Lomax, 2010). SEM model specification requires researchers to decide which variables to involve, the relationship of these variables, and parameters in the hypothesis model based on the available relevant theories, research, and information a priori to any data collection or analysis (Schumacker & Lomax, 2010). The hypotheses were stated at this step. Based on the previous literature and relevant theories review, the hypothesized relationships among variables were represented diagrammatically (Figure 3.2). There are 16 hypothesized paths that were estimated simultaneously in the hypothesized model: 1) Family SES is related to parental involvement. 2) Family SES is related to students’ perceptions of teacher support. 3) Family SES is related to peer influence. 4) Family SES, prior math achievement, parental involvement, students’ perceptions of teacher support, and peer influence are related to student mathematics motivation beliefs. 5) Prior math achievement is related to parental involvement. 6) Prior math achievement is related to students’ perceptions of teacher support. 7) Prior math achievement is related to peer influence. 8) Family SES, prior math achievement, parental involvement, students’ perceptions of teacher support, peer influence, and mathematics motivation beliefs are related to student mathematics achievement. As shown in the figure, the large circles represent the latent constructs and the rectangles are the observed or manifest variables.
Figure 3.1 Hypothesized Structural Model

Model identification was used to determine whether or not there is a unique set of parameters to be estimated in the hypothesized model (Schumacker & Lomax, 2010). The model might be underidentified, identified, or overidentified depending on the number of degrees of freedom in the model. According to Brown (2006), underidentification occurs when the number of observation is less than the model parameters in the covariance matrix. The model is considered as identified when the number of observation equals the model parameters, then “a unique set of parameter estimates exactly reproduce the observed covariance matrix” (Carry, 1999, p.199). Overidentified model is preferred, which indicates the number of observation is more than the model parameters in the covariance matrix, in this case the model is sufficient and can be evaluated for fit.

Parameter estimation was obtained by comparing the actual variance-covariance matrices and the estimated variance-covariance matrices of the best fitting model. The focus of the
estimation is to minimize the model residuals. Maximum Likelihood (ML) estimation was used in the current study to test the fit of the hypothesized model, because it is consistent and asymptotically efficient in large samples.

Once the parameter estimates are done, tests of the model fit would be reviewed, which investigate the extent of the theoretical model supported by the obtained sample data. The following goodness-of-fit indices are standard and were employed to assess the model fit of the data: the chi-square value ($\chi^2$), Comparative Fit Index (CFI), the root mean squared error of approximation (RMSEA), and the Standardized Root Mean Square Residual (SRMR), and Tucker-Lewis Index (TLI). Researchers provide cutoff criteria for interpreting that these goodness-of-fit indices are indicative of good fit: (1) CFI and TLI values are is to .90 or greater (Hu and Bentler, 1999); (2) RMSEA and SRMR values are close to .05 or below (MacCallum et al, 1996); and (3) an insignificant level of $\chi^2$ value at a .05 threshold. However, the Chi-Square statistic was not used to judge model fit in this study because of its sensitivity to the current study’s large sample size. When a large sample is employed in the study, the significance test of the Chi-Square always closely rejects the model (Jöreskog and Sörbom, 1993; Bentler and Bonnet, 1980). Schreiber, Nora, Stage, Barlow, & King, (2006) suggested that there is a good fit when the vast majority of indices meet the predetermined criteria.

Model modification was the last step of considering changes when the specified model is a poor fit. Two approaches help for better model modification: releasing constraints and imposing constraints (Chou & Bentler, 2002). More specifically, in order to achieve a better model to data fit, the researcher may choose to add free parameters based on the modification indices, or delete free parameters when the parameters are not statistically significant from 0.
Multi-Group SEM Analyses

In order to answer research question 4 (i.e., whether the relationships between the latent factors are consistent between White/Asian and African-American/Hispanic students), the invariance of the structural parameters between White/Asian and African-American/Hispanic students, multi-group SEM analyses were performed in this study. The rationale for the ethnic grouping in the current study is based on the previous research (e.g., Singh et al., 1995; Yan & Lin, 2005; Autor and Handel 2013; NAEP). These researchers found that the racial achievement gap is substantially large in mathematics. White/Asian students have generally higher academic performance than African-American/Hispanic students. The current study found that White/Asian students had a better performance than that of African-American/Hispanic students in math learning (\( \bar{x} \) for White/Asian students = 55.03, \( \bar{x} \) for African-American/Hispanic students = 49.59; \( p < .01 \)). Therefore, although these groups are culturally and in family context different, they were grouped together for being in higher and lower achievement group. The full sample (\( N=3,910 \)) was used for the multi-group analysis. There were 2,881 White/Asian participants and 1,029 African-American/Hispanic participants. The equivalence of the measurement model is the prerequisite to make statistical comparisons across these two groups. Measurement equivalence is also called measurement invariance (MI), which assumes that the observed items and theoretical constructs are being measured equally across all subgroups. If MI is violated, the comparisons of variables used in the analysis would not be acceptable, and meaningful interpretations would not be yielded. There are some different types of measurement characteristics of indicators that can be used to test for the invariance, such as factor loadings, intercepts, and residual variances (Baumgartner & Steenkamp, 1998; Lubke & Muthén, 2004). If the \( \chi^2 \) difference between the constrained and unconstrained models is statistically significant,
the less restrictive model is approved to have a better fit to the data (Dansie, 2011).

After measurement invariance was satisfied, the invariance of the structural parameters between White/Asian and African-American/Hispanic students was tested to examine whether relationships among latent variables differed across groups (White/Asian and African-American/Hispanic). The loadings and intercepts invariance of all the indicators between these two groups remained under the structural models. Following the Anderson and Gerbing (1988) two-step approach for assessing the structural model, two nested structural models were estimated first. The first model with no equality constraints for the structural paths between these two groups works as the baseline model. The second model is the fully-constrained structural model, which constrained all structural paths equally between these two groups. A χ2 difference test was used to assess whether the invariance of the structural parameters is met or not. If the difference in χ2 values between the baseline model and full-constrained structural model is statistically significant (p < .05), it indicates that there some or all structural paths are not equivalent between these two groups. Therefore, sequential chi-square difference tests were conducted by freeing equality constraints on the structural paths one by one, in order to figure out which structural path constraints should be freely estimated between the two models.

**Summary**

Overall, this chapter outlines a description of the data source, participants, variables, measures, and analysis procedures of the study. A detailed framework of structural equation modeling for examining the underlying relationships is presented in Chapter Two. The findings are presented in Chapter Four, followed by the discussion and conclusion in Chapter Five.
Chapter 4
Results

This chapter presents the results of the data analysis, which is divided into three sections. The first section outlines the descriptive statistics for all the variables of this study and correlations among them. The second section presents the results of the exploratory factor analysis. The third section presents the structural equation modeling analyses, including the measurement models, structural models, and the multi-group analysis related to the effect of three important social support sources on students’ math achievement and motivation between White/Asian and African-American/Hispanic groups.

Descriptive Statistics

HSLS:09 is one of the largest and most recent national longitudinal datasets. It provides rich information and clear measurement items on the variables of interest, such as students’ math self-efficacy beliefs, intrinsic or interest values, identities, and utility values. Furthermore, social context is described in detail in HSLS:09, such as parent-child involvement, parent-school involvement practices, perceived teacher support, teaching practices, peer influence, student academic outcomes in mathematics and science, and so on. These data are valuable and effective for addressing this study’s research questions. In order to understand these data in detail, descriptive statistics, including mean, standard deviation, and correlation matrices, for each variable in the hypothesized model, are shown in Table 4.1 to 4.9. The descriptive statistics of all the latent variables, including students’ prior math achievement, parental involvement, perceived teacher support, peer influence, and students’ mathematics self-efficacy, identity, interest, and utility in mathematics courses are displayed.
All results presented in this chapter are based on relative sample weights so that they can reflect the unbiased estimates of the target population values. Sampling weight is the probability for the individuals in a sample to be selected (Lee & Forthofer, 2006; 2005). In large-scale surveys, complex sampling is frequently used. HSLS:09 data is a national survey, which is designed to provide estimates for the corresponding population. It has five types of weights: student weights, school weights, science and mathematics course enrollee contextual weights, and student home-life contextual weights. Since social support was a focus of this study, the student home-life contextual weights, which also weighted parent/guardian responses, were chosen to create relative sample weights. The relative sample weight is the scaled version of expansion weight and used to reflect the sample size correctly. It was created by dividing the raw weights (student home-life contextual weights) by the mean of the raw weights. In HSLS:09, “the base-year target population was fall-term 9th-graders in all regular public and private schools with 9th and 11th grades in the 50 states and the District of Columbia” (Ingels, & Dalton, 2013, p c21).

Table 4.1 presents descriptive statistics on gender, ethnicity, and SES composition before and after the relative sample weight. As shown in this Table, males and females are not much different before and after the weights are applied, which means they have equal representation in the survey as well as in the target population. Asian students are much over-sampled, and Hispanic students are under-represented in the survey. After being weighted, the target population consists for 4.1% of Asian students and 18.2% of Hispanic students. As for the SES background, the 5th quintile (highest) are over-represented and 1st quintile (lowest) are under-represented in the response. After being weighted, all five SES quintile groups are equal to the percentage of SES levels in the population.
Table 4.1 *Unweighted and weighted data of 11th Grade Students’ demographic items*

<table>
<thead>
<tr>
<th>Total</th>
<th>Percent</th>
<th>Unweighted</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gender</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>48.4</td>
<td>48.5</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>51.6</td>
<td>51.5</td>
<td></td>
</tr>
<tr>
<td><strong>Ethnicity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amer. Indian/Alaska Native, non-Hispanic</td>
<td>0.5</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>Asian, non-Hispanic</td>
<td>8.9</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>Black/African-American, non-Hispanic</td>
<td>10.3</td>
<td>12.2</td>
<td></td>
</tr>
<tr>
<td>Hispanic, no race specified</td>
<td>0.5</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Hispanic, race specified</td>
<td>14</td>
<td>18.2</td>
<td></td>
</tr>
<tr>
<td>More than one race, non-Hispanic</td>
<td>9.2</td>
<td>7.7</td>
<td></td>
</tr>
<tr>
<td>Native Hawaiian/Pacific Islander, non-Hispanic</td>
<td>0.4</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>White, non-Hispanic</td>
<td>56.4</td>
<td>55.8</td>
<td></td>
</tr>
<tr>
<td><strong>SES</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st quintile (Lowest)</td>
<td>13.9</td>
<td>18.6</td>
<td></td>
</tr>
<tr>
<td>2nd quintile</td>
<td>14.1</td>
<td>14.6</td>
<td></td>
</tr>
<tr>
<td>3rd quintile</td>
<td>15.7</td>
<td>16.8</td>
<td></td>
</tr>
<tr>
<td>4th quintile</td>
<td>20.8</td>
<td>20.3</td>
<td></td>
</tr>
<tr>
<td>5th quintile (Highest)</td>
<td>35.5</td>
<td>29.8</td>
<td></td>
</tr>
</tbody>
</table>

**Mathematics Achievement**

The eleventh grade mathematics composite standardized theta scores on an Algebra test were used as the dependent variable in this study. It was a test to measure students’ algebraic skills, understanding, and math problem solving (Ingels et al., 2011). The theta scores represent normal distribution and could be utilized to measure students’ achievement growth over time when longitudinal data are available (Ingels et al., 2011). Table 4.2 presents the mean and standard deviation of 11th graders’ math score by different gender, ethnicity, and SES groups. The 11th grade students have an average score of 50 with a standard deviation of 10. Male and female students had very close mathematics scores (Mean for males =53.13, Mean for females
In terms of ethnicity, Asian American students had the highest mean mathematics score (Mean for Asian students = 60.16); White students were the second (Mean for White students = 54.65); Native Hawaiian/Pacific Islander and African American students had the lowest scores (Mean for Native Hawaiian/Pacific Islander students = 45.49; Mean for African-American students = 47.90). With regard to SES, students in the higher quintile SES groups attained higher mathematics scores than students from lower SES quintile groups. The same trend was shown in Table 4.2 for students’ previous math performance in the 9th grade. In short, there is no difference between male and female students; Asian students achieved highest math scores while African-American students’ math scores were the lowest; and better the SES background, higher the math scores.

Table 4.2 Descriptive Analysis of 11th and 9th Grade Students’ Mathematics Achievement in the Full Sample and by Gender, Ethnicity, and SES groups

<table>
<thead>
<tr>
<th></th>
<th>11th Mathematics Achievement</th>
<th>9th Mathematics Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M (SD)</td>
<td>M (SD)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>53.13 (10.48)</td>
<td>52.99 (9.79)</td>
</tr>
<tr>
<td>Female</td>
<td>52.88 (9.27)</td>
<td>52.59 (9.06)</td>
</tr>
<tr>
<td>Ethnicity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amer. Indian/Alaska Native, non-Hispanic</td>
<td>48.8 (10.79)</td>
<td>49.0 (9.26)</td>
</tr>
<tr>
<td>Asian, non-Hispanic</td>
<td>60.16 (10.27)</td>
<td>59.62 (9.39)</td>
</tr>
<tr>
<td>Black/African-American, non-Hispanic</td>
<td>47.9 (8.1)</td>
<td>47.81 (8.51)</td>
</tr>
<tr>
<td>Hispanic, no race specified</td>
<td>48.91 (5.94)</td>
<td>50.63 (6.29)</td>
</tr>
<tr>
<td>Hispanic, race specified</td>
<td>50.75 (9.36)</td>
<td>52.47 (9.07)</td>
</tr>
<tr>
<td>More than one race, non-Hispanic</td>
<td>51.92 (10.34)</td>
<td>52.06 (9.70)</td>
</tr>
<tr>
<td>Native Hawaiian/Pacific Islander, non-Hispanic</td>
<td>45.49 (8.73)</td>
<td>49.40 (6.43)</td>
</tr>
<tr>
<td>White, non-Hispanic</td>
<td>54.65 (9.6)</td>
<td>54.34 (9.04)</td>
</tr>
<tr>
<td>SES</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st quintile (lowest)</td>
<td>47.26 (8.73)</td>
<td>48.08 (8.01)</td>
</tr>
<tr>
<td>2nd quintile</td>
<td>50.23 (9.25)</td>
<td>50.11 (9.70)</td>
</tr>
<tr>
<td>3rd quintile</td>
<td>51.39 (8.71)</td>
<td>51.23 (8.53)</td>
</tr>
</tbody>
</table>
Parental involvement

As discussed in Chapter 2, Christenson et al. (1992) and Walberg (1986) defines parental involvement as parents' communication with children about school matters and events as well as future plans. There are 7 items from the parents’ survey that pertain to the effect of parental involvement on students’ math achievement in the HSLS:09 study. The selected items focused on the parent-child interactions about school, such as college application, future careers, and school events. Parents were asked to indicate their level of communication with their children on a 4-point Likert scale: 1= Never, 2= Once or twice, 3= Three or four times, and 4= More than four times. The mean scores of all items were higher than 3, indicating that parents believed they were discussing education plans and school events with their children often during the last school year. The correlations between these items were moderate to high ranging from .333 to .619. Skewness ranged between -1.551 and -0.63, and kurtosis ranged between -0.898 and 1.31. Although there is no clear consensus for the values of skewness and kurtosis to assume normal distribution, George and Mallery (2010) suggested that the values for skewness and kurtosis between -2 and +2 are considered acceptable for normality. The detailed descriptive statistics and a correlation matrix are shown in Table 4.3.

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>.508**</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>.438**</td>
<td>.619**</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3 Descriptive Statistics of Parental Involvement Items (weighted)
Perceived Teacher Support

Perceived teacher support is “the extent to which [students feel their] teachers are supportive, responsive, and committed to [their] well-being” (Wang, 2009, p. 242). Table 4.4 shows 5 items from the student survey that pertain to the students’ perception of their math teachers’ support in the HSLS:09 study. Items 1, 2, 3, and 4 have been reverse coded where 1 = strongly disagree, 2 = disagree, 3 = agree, and 4 = strongly agree. After recoding, a higher score means students received more teacher support whereas a lower score means students received less teacher support. Table 4.4 shows that the mean of Item 3 was slightly higher than 3; and the means of Items 1, 2, 4, and 5 were lower than 3. This indicates that students did not feel that they received adequate support from their math teachers. The correlations between all these items were moderate to high ranging from .319 to .713. Skewness ranged between -.805 and -.302, and kurtosis ranged between -.751 and .288. The values for skewness and kurtosis are considered acceptable in assuming normal distribution. The detailed descriptive statistics and a correlation matrix are shown in Table 4.4.
Table 4.4 Descriptive Statistics of Perceived Teacher Support Items

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Teen's spring 2012 math teacher makes math interesting</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Teen's spring 2012 math teacher makes math easy to understand</td>
<td>.713**</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Teen's spring 2012 math teacher wants students to think, not memorize</td>
<td>.535**</td>
<td>.535**</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Teen's spring 2012 math teacher doesn't let students give up</td>
<td>.535**</td>
<td>.527**</td>
<td>.570**</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5. Teen's spring 2012 math teacher treats some kids better than others</td>
<td>.347**</td>
<td>.415**</td>
<td>.319**</td>
<td>.358**</td>
<td>1</td>
</tr>
<tr>
<td>M</td>
<td>2.7</td>
<td>2.77</td>
<td>3.14</td>
<td>2.97</td>
<td>2.75</td>
</tr>
<tr>
<td>SD</td>
<td>0.925</td>
<td>0.925</td>
<td>0.811</td>
<td>0.837</td>
<td>0.949</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.302</td>
<td>-0.403</td>
<td>-0.805</td>
<td>-0.505</td>
<td>-0.376</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.73</td>
<td>-0.647</td>
<td>0.288</td>
<td>-0.311</td>
<td>-0.751</td>
</tr>
</tbody>
</table>

1=strongly disagree, 2=disagree, 3=agree, and 4=strongly agree for item 1, 2, 3, 4
1=strongly agree, 2=agree, 3=disagree, and 4=strongly disagree for item 5
N=4,418

Note: *, p< .05; **, p< .01

Peer influence

Peers have an intangible and strong power to modify students’ ways of thinking, values, and behavior. In the current study, four items shown in Table 4.5 reflect peer influence based on students’ self-reports about their peers’ academic achievement, educational pursuits, and career choices. Students were asked to indicate their peers’ academic performance and future plans on a 5-point Likert scale: 1=None of them; 2=Less than half; 3=About half; 4=More than half; 5=All of them. In order to align with other scales and keep items semantically in the positive direction, item 4 has been reverse coded where 1= All of them; 2= More than half; 3=About half; 4= Less than half; 5= None of them. After recoding, a higher score means students had no friends who dropped out of high school. The means of all four items were higher than 3, indicating that most of the surveyed students’ friends plan to go to a four-year college instead of going to job training or community college after high school graduation. These 4 items had low to moderate
correlation with each other and ranged from .123 to .422. Skewness ranged between -2.718 and -.497, and kurtosis ranged between -.675 and 10.398. However, the skewness of item 4 was over 2, and its kurtosis was over 7, which indicate that normal distribution was slightly violated for item 4. However, researchers have argued that ordinal scale variables rarely follow normal distribution and that this situation often occurs in social science studies (Kinnear & Taylor, 1991; Stewart et al., 2001; Hancock, 2014). In addition, the Maximum Likelihood (ML) estimation method, which was used in this study, can still produce sound results when the normal distribution is not severely violated (Bollen, 1989).

Table 4.5 Descriptive Statistics of peer influence Items

<table>
<thead>
<tr>
<th>Items</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 How many friends get good grades</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How many friends have taken PSAT, SAT, PLAN or ACT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 How many friends plan to attend 4-year college</td>
<td>.250**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>or ACT</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 How many friends have ever dropped out of high school</td>
<td>.422**</td>
<td>.308**</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>3.9</td>
<td>3.69</td>
<td>3.65</td>
<td>4.73</td>
</tr>
<tr>
<td>SD</td>
<td>.779</td>
<td>1.165</td>
<td>1.04</td>
<td>0.556</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.497</td>
<td>-0.556</td>
<td>-0.535</td>
<td>-2.718</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.172</td>
<td>-0.675</td>
<td>-0.407</td>
<td>10.398</td>
</tr>
</tbody>
</table>

N=4,418

Note: *, p< .05; **, p< .01

Mathematics Identity

Eccles and her colleagues (1983) have defined social or personal identity as the value of participating in or performing well in an activity. It is consistent with an individual’s self-image, or allows an individual to express important aspects of self. There were two items employed to measure 11th grade students’ beliefs about how they perceive themselves and how they think others perceive them in regard to mathematics. For keeping items systematically in the positive
direction, these items have been reverse coded where 1= strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. After recoding, a higher score represents a stronger mathematics identity. The mean scores of the two items were lower than 3, indicating that on average students don’t identify themselves as math persons. The correlations between these two items were very strong (.784), which shows they can measure a single factor. Skewness ranged between -.114 and .034, and kurtosis ranged between -.886 and 1.096. These values are considered acceptable for the assumption of normal distribution.

Table 4.6 Descriptive Statistics of Mathematics Identity Items

<table>
<thead>
<tr>
<th>Items</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Teenager sees himself/herself as a math person</td>
<td>1</td>
<td>.784**</td>
</tr>
<tr>
<td>2 Others see teenager as a math person</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>2.45</td>
<td>2.51</td>
</tr>
<tr>
<td>SD</td>
<td>1.012</td>
<td>0.94</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.034</td>
<td>-0.114</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-1.096</td>
<td>-0.886</td>
</tr>
</tbody>
</table>

1=strongly disagree, 2=disagree, 3=agree, and 4=strongly agree
N=4,418
Note: *, p< .05; **, p< .01

Mathematics Self-Efficacy

Students’ self-efficacy expectations measure his or her ability to achieve success in a task in the short-term or long-term future. Four items were used to measure a student’s math self-efficacy in the 11th grade. All items have been reverse coded where 1= strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. After recoding, a higher score means students are more confident of their math ability and have a positive perception about doing well in mathematics. All means of the 4 items were lower than 3, indicating that on average students are less confident of their ability to do well in mathematics. These four items were highly correlated with each other and ranged from .620 to .756, which shows these items measure a single factor.
Skewness ranged between -.555 and .179, and kurtosis ranged between -.656 and .21. These values are considered acceptable to assume normality.

Table 4.7 Descriptive Statistics of Mathematics Self-Efficacy Items

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Teen certain can understand math textbook</td>
<td>.638**</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Teen certain can master skills taught in math course</td>
<td>.666**</td>
<td>.735**</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3 Teen confident can do an excellent job on math tests</td>
<td>.620**</td>
<td>.700**</td>
<td>.756**</td>
<td>1</td>
</tr>
<tr>
<td>4 Teen confident can do excellent job on math assignments</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>2.6</td>
<td>2.9</td>
<td>2.82</td>
<td>2.986</td>
</tr>
<tr>
<td>SD</td>
<td>0.878</td>
<td>0.768</td>
<td>0.817</td>
<td>0.7605</td>
</tr>
<tr>
<td>Skewness</td>
<td>-.179</td>
<td>-.439</td>
<td>-.379</td>
<td>-.555</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.656</td>
<td>-0.02</td>
<td>-0.292</td>
<td>0.21</td>
</tr>
</tbody>
</table>

1=strongly disagree, 2=disagree, 3=agree, and 4=strongly agree

N=4,418

Note: *, p<.05; **, p<.01

Interest in Mathematics

As shown in Table 4.8, there are three items employed to measure 11th grade students’ interest or enjoyment in math learning. Item 1 has been reverse coded where 1= strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. For item 2 and 3, the original coding was kept where 1= strongly agree, 2=agree, 3=disagree, and 4=strongly disagree. After recoding, a higher score represents a higher level of interest in mathematics classes. The mean score of item 2 was 3.05, indicating that students tended to disagree with the statement that math course is a waste of time. The mean scores of item 1 and 3 were 2.52 and 2.46, indicating that students were not enjoying their 11th math courses very much. These 3 items had moderate to high correlation with each other and ranged from .411 to .604. Skewness ranged between -.926 and -.092, and kurtosis ranged between -.84 and 0.333. These values are considered acceptable to assume normality.
Table 4.8 Descriptive Statistics of Mathematics Interest Items

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Teen is enjoying (spring 2012) math course</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Teen thinks (spring 2012) math course is a waste of time</td>
<td>.411**</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3 Teen thinks (spring 2012) math course is boring</td>
<td>.604**</td>
<td>.525**</td>
<td>1</td>
</tr>
<tr>
<td>M</td>
<td>2.52</td>
<td>3.05</td>
<td>2.46</td>
</tr>
<tr>
<td>SD</td>
<td>0.926</td>
<td>0.802</td>
<td>0.872</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.136</td>
<td>-0.734</td>
<td>-0.092</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.84</td>
<td>0.333</td>
<td>-0.707</td>
</tr>
</tbody>
</table>

N=4,418

Note: *, p< .05; **, p< .01

Mathematics Utility

There were three items used to measure 11th grade students’ utility values about whether mathematics is useful for their future goals and plans. All three items have been reverse coded where 1= strongly disagree, 2=disagree, 3=agree, and 4=strongly agree. After recoding, a higher score represents a higher perception of mathematics utility values. All the means of 3 items were higher than 3, indicating that on average students believe that mathematics is useful for their future goals and career plans. These three items were highly correlated with each other ranging from .553 to .611. Skewness for these items ranged between -.836 and -.771, and kurtosis ranged between .612 and .99. These values are considered acceptable to assume normality.

Table 4.9 Descriptive Statistics of Mathematics Utility Items

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Teenager thinks math is useful for everyday life</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Teenager thinks math will be useful for college</td>
<td>.553**</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3 Teenager thinks math is useful for future career</td>
<td>.611**</td>
<td>.651**</td>
<td>1</td>
</tr>
<tr>
<td>M</td>
<td>3.16</td>
<td>3.48</td>
<td>3.28</td>
</tr>
<tr>
<td>SD</td>
<td>0.736</td>
<td>0.58</td>
<td>0.712</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.771</td>
<td>-0.836</td>
<td>-0.823</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.714</td>
<td>0.99</td>
<td>0.612</td>
</tr>
</tbody>
</table>

1=strongly disagree, 2=disagree, 3=agree, and 4=strongly agree
N=4,418

Note: *, p< .05; **, p< .01
Exploratory Factor Analyses

Exploratory factor analysis (EFA) is a data-driven approach. It was used to uncover the underlying structure of all the indicators used in this study. Screen plots and total explained variance were conducted to identify the number of factors. A random split-samples EFA was conducted with five percent (n = 238) of the all the cases, randomly selected from the sample of 4,418, to identify and reduce the number of items for forming the initial latent constructs. Maximum likelihood method was used to extract factors for each scale in this study. The results of the final exploratory factor analysis of each scale are presented in Table 4.10 and Table 4.11.

First, 12 motivation items were analyzed with promax rotation via SPSS. The oblique rotation method was used since the values in factor correlation matrixes are over ±0.32 (Tabachnick & Fiddell, 2007). Furthermore, for oblique rotations, “the promax rotation has the advantage of being fast and conceptually simple” (Abdi, 2003, p 6). The EFA results demonstrated the existence of four components, which fit the expectancy-value theory. Although there are many theories of motivation, in the current study, motivation was explained by the expectancy-value theory. It is theorized that academic motivation is comprised of self-efficacy, interest, identity, and utility (Eccles et al., 1983). The four components explained 68.4% of the variance. Table 4.10 shows the factor loadings of 12 items of the expectancy-value theory model. Based on expectancy-value theory and EFA results, the following four motivation composite variables were created: mathematics self-efficacy, mathematics utility, mathematics identity, and interest in mathematics. These four motivation composite variables were used as indicators to measure students’ mathematics motivation in the SEM models. All reliability coefficients of motivation factors were acceptable, which were larger than the cut-off value of .7 (Lance, 2006; Henson, 2001).
Table 4.10 Factor Loadings for Exploratory Factor Analysis with Promax Rotation of the expectancy-value motivation theory

<table>
<thead>
<tr>
<th>Items</th>
<th>Factor Loading</th>
<th>Cronbach’s alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics Self-Efficacy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Teen confident can do excellent job on (spring 2012) math assignments</td>
<td>0.948</td>
<td>0.933</td>
</tr>
<tr>
<td>2 Teen confident can do an excellent job on (spring 2012) math tests</td>
<td>0.929</td>
<td></td>
</tr>
<tr>
<td>3 Teen certain can master skills taught in (spring 2012) math course</td>
<td>0.920</td>
<td></td>
</tr>
<tr>
<td>4 Teen certain can understand (spring 2012) math textbook</td>
<td>0.752</td>
<td></td>
</tr>
<tr>
<td><strong>Mathematics Utility</strong></td>
<td></td>
<td>0.793</td>
</tr>
<tr>
<td>1 Teenager thinks math is useful for future career</td>
<td>0.849</td>
<td></td>
</tr>
<tr>
<td>2 Teenager thinks math will be useful for college</td>
<td>0.731</td>
<td></td>
</tr>
<tr>
<td>3 Teenager thinks math is useful for everyday life</td>
<td>0.598</td>
<td></td>
</tr>
<tr>
<td><strong>Mathematics Identity</strong></td>
<td></td>
<td>0.841</td>
</tr>
<tr>
<td>1 Teenager sees himself/herself as a math person</td>
<td>0.942</td>
<td></td>
</tr>
<tr>
<td>2 Others see teenager as a math person</td>
<td>0.641</td>
<td></td>
</tr>
<tr>
<td><strong>Interest in Mathematics</strong></td>
<td></td>
<td>0.794</td>
</tr>
<tr>
<td>1 Teen thinks (spring 2012) math course is boring</td>
<td>0.948</td>
<td></td>
</tr>
<tr>
<td>2 Teen is enjoying (spring 2012) math course</td>
<td>0.720</td>
<td></td>
</tr>
<tr>
<td>3 Teen thinks (spring 2012) math course is a waste of time</td>
<td>0.522</td>
<td></td>
</tr>
</tbody>
</table>

Second, for social support items, each analysis was run separately. The results of the analysis showed that one single factor with 7 items explains parental involvement, one single factor with 5 items explains students’ perception of teacher support, and one single factor with 4 items explains peer influence. For each scale, more than 50% of the total variance was explained by the extracted factors. Factors loadings for the all items under the four-factors were acceptable and meaningful (> .4) as shown in Table 4.11. Generally, a factor loading of .40 or above is acceptable (Floyd & Widaman, 1995). In addition, all reliability coefficients of motivation factors were acceptable, which were larger than the cut-off value of .7 (Lance, 2006).
Table 4.11 Factor Loadings for Exploratory Factor Analysis with Promax Rotation of the social support factors and motivation factors.

<table>
<thead>
<tr>
<th>Items</th>
<th>Factor Loading</th>
<th>Cronbach’s alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parental Involvement</td>
<td></td>
<td>0.889</td>
</tr>
<tr>
<td>1</td>
<td>How often discussed careers he/she might be interested in</td>
<td>0.797</td>
</tr>
<tr>
<td>2</td>
<td>How often discussed community/national/world events</td>
<td>0.786</td>
</tr>
<tr>
<td>3</td>
<td>How often discussed applying to college/other schools after high school</td>
<td>0.738</td>
</tr>
<tr>
<td>4</td>
<td>How often discussed selecting courses or programs at school</td>
<td>0.723</td>
</tr>
<tr>
<td>5</td>
<td>How often discussed preparing for college entrance exams</td>
<td>0.713</td>
</tr>
<tr>
<td>6</td>
<td>How often discussed job that he/she might want to take after high school</td>
<td>0.711</td>
</tr>
<tr>
<td>7</td>
<td>How often discussed things that were troubling him/her</td>
<td>0.677</td>
</tr>
<tr>
<td>Students’ Perception of Teacher</td>
<td></td>
<td>0.835</td>
</tr>
<tr>
<td>1</td>
<td>Teen's spring 2012 math teacher makes math easy to understand</td>
<td>0.885</td>
</tr>
<tr>
<td>2</td>
<td>Teen's spring 2012 math teacher doesn't let students give up</td>
<td>0.651</td>
</tr>
<tr>
<td>3</td>
<td>Teen's spring 2012 math teacher makes math interesting</td>
<td>0.829</td>
</tr>
<tr>
<td>4</td>
<td>Teen's spring 2012 math teacher wants students to think, not memorize</td>
<td>0.698</td>
</tr>
<tr>
<td>5</td>
<td>Teen's spring 2012 math teacher treats some kids better than others</td>
<td>0.497</td>
</tr>
<tr>
<td>Peer Influence</td>
<td></td>
<td>0.704</td>
</tr>
<tr>
<td>1</td>
<td>How many friends plan to attend 4-year college</td>
<td>0.740</td>
</tr>
<tr>
<td>2</td>
<td>How many friends get good grades</td>
<td>0.527</td>
</tr>
<tr>
<td>3</td>
<td>How many friends have taken PSAT, SAT, PLAN or ACT</td>
<td>0.519</td>
</tr>
<tr>
<td>4</td>
<td>How many friends have ever dropped out of high school</td>
<td>0.487</td>
</tr>
<tr>
<td>Motivation Beliefs</td>
<td></td>
<td>0.786</td>
</tr>
<tr>
<td>1</td>
<td>Scale of student's mathematics identity</td>
<td>0.821</td>
</tr>
<tr>
<td>2</td>
<td>Scale of student's mathematics self-efficacy</td>
<td>0.678</td>
</tr>
<tr>
<td>3</td>
<td>Scale of student's interest in math course</td>
<td>0.665</td>
</tr>
<tr>
<td>4</td>
<td>Scale of student's mathematics utility</td>
<td>0.636</td>
</tr>
</tbody>
</table>

Structural Equation Modeling

Structural equation modeling (SEM) analysis is carried out in two steps (Schumacker & Lomax, 2010). Confirmatory factor analysis (CFA) as a prior step was conducted to test the relationships between individual items and hypothesized latent factors. After estimating the adequacy of the measurement models, the structural model analyses in the second step presented
the relationships among latent factors in the hypothesized model (Schumacker & Lomax, 2012). In other words, the latent model has two parts in SEM: the structural model showing hypothesized causal relationships among latent factors, and the measurement model presenting the relations between latent factors and their indicators. To evaluate plausibility of proposed models, fit indices from the three major index classes were selected including absolute fit index, parsimonious fit index, and incremental fit index. In the current study, these three fit indices were represented by standardized root mean square residual (SRMR), root mean square error of approximation (RMSEA), Tucker-Lewis Index (TLI), and comparative fit index (CFI), respectively. RMSEA and CFI were given more importance. RMSEA has been noted as “one of the most informative fit indices” (Diamantopoulos & Siguaw, 2000, p 85) because it is sensitive to the number of estimated parameters that are identified in the model. CFI is another of the most popularly reported fit indices since it is one of the measures that is least affected by sample size (Fan et al., 1999).

Lastly, in order to make comparisons between Asian/White and African-American/Hispanic students, multiple-group SEM models were estimated to assess if there are any differences in the effects of the latent factors on the math achievement in the two groups. Mplus 7 was used in this study to conduct the structural equation modeling analysis (Muthen & Muth, 2017). A sample of 95% data has been used for the measurement model analyses (n=4,118) in the first step of this process, and another 5% data has been used for exploratory factor analyses. A full sample data has been used for the structural model analyses (n=4,418). The maximum likelihood estimation with robust standard errors (MLR) was used for both the measurement and structure models. MLR is suitable for those cases when the multivariate normality assumption is not severely violated (Hair et al., 1998).
Measurement Model

The measurement model analysis is also called confirmatory factor analysis (CFA), which was conducted following the results of the exploratory factor analysis. CFA requires that the researcher understands the relevant theory and previous literature first in order to postulate the relationship pattern among latent variables and the indicators of the latent factors. (Brown, 2006). In the current study, the initial hypothesized measurement model identified four latent constructs with a total of 20 indicators: parental involvement with seven indicators; perceived teacher support with five indicators, peer influence with seven indicators, and motivation beliefs with four indicators. These four latent variables significantly correlate with each other, except the correlation relationship between parental involvement and perceived teacher support (see Table 4.12). There are two steps to test the CFA hypothesized model. First, the measurement model for each individual latent variable was tested separately. Second, the full measurement model was tested for all latent variables simultaneously.

<table>
<thead>
<tr>
<th>Latent factor</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Parental involvement</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Perceived teacher support</td>
<td>.032</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Peer influence</td>
<td>.218**</td>
<td>.114**</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4 Math motivation</td>
<td>.082**</td>
<td>.576**</td>
<td>.244**</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: *, p < .05; **, p < .01

Measurement of Parental Involvement

Parental involvement was measured by seven indicators and all these loadings were significant and strong (> .5) However, the modification indices suggested that if the error terms of the “PDISAPP” and “PDISEXAM” indicators of parental involvement were correlated, the value of chi-square will decrease by 58. Saris et al., (2009) stated that in order to find an
acceptable model to represent the relations that exist among observed and latent variables, some model modifications can be conducted. Further, Brown (2006) suggested that correlated errors can be when the items are “very similarly worded, reverse worded, or differentially prone to social desirability and so forth” (p. 181). Therefore, correlating these two error terms seem reasonable, because “How often discussed preparing for college entrance exams (PDISEXAM)” and “How often discussed applying to college/other schools after high school (PDISAPP)” are highly correlated and their errors are likely to have some common variance in measuring something in common (such as applying to college and taking college entrance exams) other than parental involvement. The final model with one modification provided a better fit. The chi-square went down by 51.912 with a one-degree freedom decrease, and RMSEA decreased by .001 and CFI increased by .03 compared to the initial model. Although modification indices demonstrated that there was some room for improvement in this measurement model, only a few and theoretically sound modifications should be undertaken. Some researchers have noted that when a model that provides a good fit to data is re-specified, there may be a better fit obtained but then there could be the likely increase in “fitting small idiosyncratic characteristics of the sample” (MacCallum, Roznowski, and Necowitz, 1992, p. 501). The factor loadings of this measurement model ranged between .594 and .815 and are presented in Table 4.17. All factor loadings were statistically significant and the standardized errors were low at .019 to .03, which indicate that these items are reasonable measures of the latent factor of parental involvement.

Table 4.13 Goodness-of-Fit Summary Table for Measurement Models of parental involvement items

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2$</th>
<th>df</th>
<th>$\Delta \chi^2$</th>
<th>$\Delta df$</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
<th>SRMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Model</td>
<td>136.185</td>
<td>14</td>
<td></td>
<td></td>
<td>.928</td>
<td>.892</td>
<td>.046</td>
<td>.043</td>
</tr>
<tr>
<td>Final model</td>
<td>84.273</td>
<td>13</td>
<td>51.912</td>
<td>1</td>
<td>.958</td>
<td>.932</td>
<td>.036</td>
<td>.036</td>
</tr>
</tbody>
</table>
Measurement of Perceived Teacher Support

Students’ Perception of Teacher Support was measured by five indicators and all these loadings were significant. However, the modification indices showed that if the error terms of the “MTEASY” and “MTINTRES” indicators of perceived teacher support were correlated, the value of chi-square will decrease by 133.270. Correlating these two error terms is reasonable, because “Teen’s spring 2012 math teacher makes math easy to understand (MTEASY)” and “Teen’s spring 2012 math teacher makes math interesting (MTINTRES)” are highly correlated and their errors had some common variances in measuring something in common other than the perceived teacher support latent variable. The final model with modification was much improved by freeing this pair of error covariance of indicators. The chi-square went down by 126.562 with a one degree freedom decrease, and RMSEA decreased by .049 and CFI was increased by .04 compared to the initial model. The factor loadings of this measurement model ranged between .482 and .859 and are presented in Table 4.17. All factor loadings were statistically significant and the standardized errors were low at .009 to .02, which indicate that these items are reasonable measures of the latent factor of perceived teacher support.

Table 4.14 Goodness-of-Fit Summary Table for Measurement Models of Perceived Teacher Support

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2$</th>
<th>$df$</th>
<th>$\Delta \chi^2$</th>
<th>$\Delta df$</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
<th>SRMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Model</td>
<td>150.117</td>
<td>5</td>
<td>0.954</td>
<td>0.083</td>
<td>0.954</td>
<td>0.907</td>
<td>0.083</td>
<td>0.034</td>
</tr>
<tr>
<td>Final model</td>
<td>23.555</td>
<td>4</td>
<td>126.562</td>
<td>1</td>
<td>0.994</td>
<td>0.984</td>
<td>0.034</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Measurement of Peer Influence

There were four measures of peer influence. These 4 indicators showed a well-fitting model ($\chi^2$=2.884; CFI=.998; TLI=.994; RMSEA=.010; SRMR=.009). Modification indices and
Chi-square difference test indicated that the initial model would not be improved significantly by freeing any pair of covariance of item indicators. The factor loadings of this measurement model ranged between .482 and .859 and are presented in Table 4.17. All factor loadings were statistically significant and the standardized errors were low at .009 to .02, which indicate that these items are reasonable measures of the latent factor of peer influence. Due to the good fit statistics and the fact that all factor loadings were statistically significant, no modifications were made to this measurement model.

Measurement of Math Motivation

Bollen (1989) noted that each latent construct must include at least two indicators in order for a model with two or more latent factors to be identified. Furthermore, for increasing content validity and reliability of each latent factor, all latent factors should include at least two or more indicators (Anderson & Gerbing, 1988). There were four items to measure students’ math self-efficacy. These 4 indicators showed a well-fitting model ($\chi^2=3.005; \text{CFI}=1; \text{TLI}=.999; \text{RMSEA}=.011; \text{SRMR}=.004$). All four loadings were strong and significant and ranged from .765 to .885, which indicate that these items are good measures of students’ math self-efficacy.

Bollen (1989) further noted that in order for the model to be identified, the knowns must exceed the unknowns. More specifically, the distinct elements of the moment matrix being analyzed must exceed the parameters to be estimated. In addition, Davis (1993) and Reilly (1995) stated that "three measure rule" and "two measure rule" for measurement models. The “three measure rule” suggests that a congeneric measurement model can be identified when every latent factor is measured by at least 3 indicators. The "two measure rule" suggests that a
congeneric measurement model can be identified if every latent factor is measured by at least 2 indicators and every latent factor is correlated with at least one other latent factor. In the current study, since there were only three items to measure students’ math interest and only three items to measure math utility, these two measurement models are just identified whose degree of freedom=0, and model fit is perfect ($\chi^2=0; \text{CFI}=1; \text{TLI}=1; \text{RMSEA}=0; \text{SRMR}=0$). Thus, their fit can not be evaluated, such models being just identified. There were only two items to measure mathematics identity; therefore, the model could not be identified. In this case, as suggested by the "two measure rule", these four motivation factors were measured simultaneously so that they can be identified by correlating to each other. The four-factor motivation measurement model shows a well-fitting model ($\chi^2=413.779; \text{CFI}=.961; \text{TLI}=.946; \text{RMSEA}=0.043; \text{SRMR}=0.037$), and all four loadings were strong (> .6) and significant. The modification indices and Chi-square difference test indicated that the initial model would not be improved significantly by freeing any pair of error covariance of item indicators. Therefore, due to the good fit statistics and the fact that all factor loadings were statistically significant, no modifications were made to this measurement model. The standardized item loadings, item reliability, and the $R^2$ estimate of each item in the motivation latent construct are presented in Table 4.15. Overall, the standardized loadings ranged from .579 to .931 and error variances ranged from .008 to .024. $R^2$ ranged from .335 to .866, which indicates that 33.5% to 86.6% of these items’ variance can be explained by this model.
Figure 4.1 Measurement Model for Motivation Construct with standardized loading coefficients

Table 4.15 Properties of the Measurement Model of Math Motivation

<table>
<thead>
<tr>
<th>Observed items</th>
<th>Standardized loading</th>
<th>Error</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics Self-Efficacy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teen confident can do excellent job on math assignments</td>
<td>0.885</td>
<td>0.011</td>
<td>0.705</td>
</tr>
<tr>
<td>Teen confident can do an excellent job on math tests</td>
<td>0.840</td>
<td>0.008</td>
<td>0.783</td>
</tr>
<tr>
<td>Teen certain can master skills taught in math course</td>
<td>0.841</td>
<td>0.011</td>
<td>0.706</td>
</tr>
<tr>
<td>Teen certain can understand math textbook</td>
<td>0.765</td>
<td>0.013</td>
<td>0.586</td>
</tr>
<tr>
<td><strong>Mathematics Utility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teenager thinks math is useful for future career</td>
<td>0.848</td>
<td>0.014</td>
<td>0.719</td>
</tr>
<tr>
<td>Teenager thinks math will be useful for college</td>
<td>0.765</td>
<td>0.016</td>
<td>0.585</td>
</tr>
<tr>
<td>Teenager thinks math is useful for everyday life</td>
<td>0.732</td>
<td>0.015</td>
<td>0.536</td>
</tr>
<tr>
<td><strong>Mathematics Identity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teenager sees himself/herself as a math person</td>
<td>0.931</td>
<td>0.010</td>
<td>0.866</td>
</tr>
<tr>
<td>Others see teenager as a math person</td>
<td>0.837</td>
<td>0.016</td>
<td>0.700</td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Interest in Mathematics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teen thinks math course is boring</td>
<td>0.689</td>
<td>0.019</td>
<td>0.474</td>
</tr>
<tr>
<td>Teen is enjoying math course</td>
<td>0.859</td>
<td>0.015</td>
<td>0.738</td>
</tr>
<tr>
<td>Teen thinks math course is a waste of time</td>
<td>0.579</td>
<td>0.024</td>
<td>0.335</td>
</tr>
</tbody>
</table>

**Full Measurement Model**

After testing each latent variable separately, the full hypothesized measurement model was tested with 20 observed items, which were loaded under 4 latent variables (See Figure 4.2). The full measurement model was established including all variables that were retained after single-factor analyses. The chi-square of the full measurement model was large with a value of 878.553, which is significant at the .05 level. The significant and large chi-square demonstrated that the input covariance matrix was not equal to the reproduced covariance matrix. However, chi-square is sensitive to the sample size (Bentler and Bonnet, 1980; Jöreskog and Sörbom, 1993). Due to the large sample size of the current study, it is better to rely more on other fit indices, such as CFI, RMSEA to assess plausibility of the proposed measurement model.

The fit indices for the full initial measurement model were acceptable, indicating an acceptable-fitting model in which data fit well to the hypothesized measurement model. However, there is a notable modification index of the initial model that suggested it was possible to achieve the highest decrease in chi-square value by cross-loading “interest in mathematics” on “perceived teacher support” factor. An item under the perceived teacher support, “Teen's spring 2012 math teacher makes math interesting,” conceptually overlaps with students’ math interest construct. In addition, factor loadings revealed that “interest in mathematics” loaded significantly on both factors (loading on motivation factor .486 and on the perceived teacher support factor .389). Therefore, it is theoretically and statistically reasonable to add a path from
perceived teacher support to students’ interest in math. This may have some theoretical implications which are addressed in the discussion. This modification was made in the next round of analysis (final model).

Results of the initial model and final model were presented in Table 4.16, specifically their fit indices and chi-square difference. In the final model, for the loss of one degree of freedom, the chi-square went down by 140.60, RMSEA was less by .004, and CFI was greater by .013 compared to the baseline model. Therefore, the final measurement model explained the data better than the baseline model.

Table 4.16 Goodness-of-Fit Summary Table for the Full Measurement Models

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2$</th>
<th>df</th>
<th>$\Delta \chi^2$</th>
<th>$\Delta df$</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
<th>SRMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Model</td>
<td>878.553</td>
<td>162</td>
<td></td>
<td></td>
<td>0.927</td>
<td>0.910</td>
<td>0.033</td>
<td>0.045</td>
</tr>
<tr>
<td>Final model</td>
<td>724.543</td>
<td>161</td>
<td>140.60</td>
<td>1</td>
<td>0.940</td>
<td>0.929</td>
<td>0.029</td>
<td>0.041</td>
</tr>
</tbody>
</table>
Figure 4.2 Full Measurement Model with standardized loadings

The standardized item loadings, item reliability, and $R^2$ estimate of each item are presented in Table 4.17. The standardized loadings ranged from .379 to .859 and error variances ranged from .009 to .035. $R^2$ ranged from .144 to .737, which indicates that 14% to 73.7% of the these items’ variance can be explained by this model. Given the significant factor loadings, the
The model was retained as the final measurement model.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Observed variables</th>
<th>Standardized loading</th>
<th>Error</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parental Involvement</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PA1</td>
<td>How often discussed careers he/she might be interested in</td>
<td>0.815</td>
<td>0.018</td>
<td>0.664</td>
</tr>
<tr>
<td>PA2</td>
<td>How often discussed community/national/world events</td>
<td>0.61</td>
<td>0.021</td>
<td>0.372</td>
</tr>
<tr>
<td>PA3</td>
<td>How often discussed applying to college/other schools after high school</td>
<td>0.653</td>
<td>0.021</td>
<td>0.427</td>
</tr>
<tr>
<td>PA4</td>
<td>How often discussed selecting courses or programs at school</td>
<td>0.621</td>
<td>0.021</td>
<td>0.386</td>
</tr>
<tr>
<td>PA5</td>
<td>How often discussed preparing for college entrance exams</td>
<td>0.649</td>
<td>0.019</td>
<td>0.421</td>
</tr>
<tr>
<td>PA6</td>
<td>How often discussed job that he/she might want to take after high school</td>
<td>0.654</td>
<td>0.03</td>
<td>0.428</td>
</tr>
<tr>
<td>PA7</td>
<td>How often discussed things that were troubling him/her</td>
<td>0.594</td>
<td>0.022</td>
<td>0.353</td>
</tr>
<tr>
<td><strong>Students’ Perception of Teacher</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TS1</td>
<td>Teen's spring 2012 math teacher makes math interesting</td>
<td>0.851</td>
<td>0.01</td>
<td>0.724</td>
</tr>
<tr>
<td>TS2</td>
<td>Teen's spring 2012 math teacher makes math easy to understand</td>
<td>0.859</td>
<td>0.009</td>
<td>0.737</td>
</tr>
<tr>
<td>TS3</td>
<td>Teen's spring 2012 math teacher wants students to think, not memorize</td>
<td>0.624</td>
<td>0.017</td>
<td>0.389</td>
</tr>
<tr>
<td>TS4</td>
<td>Teen's spring 2012 math teacher doesn't let students give up</td>
<td>0.626</td>
<td>0.017</td>
<td>0.391</td>
</tr>
<tr>
<td>TS5</td>
<td>Teen's spring 2012 math teacher treats some kids better than others</td>
<td>0.482</td>
<td>0.02</td>
<td>0.233</td>
</tr>
<tr>
<td>MO3</td>
<td>Scale of student's interest in math course</td>
<td>0.389</td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td><strong>Peer Influence</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PE1</td>
<td>How many friends get good grades</td>
<td>0.574</td>
<td>0.027</td>
<td>0.329</td>
</tr>
<tr>
<td>PE2</td>
<td>How many friends have taken PSAT, SAT, PLAN or ACT</td>
<td>0.445</td>
<td>0.035</td>
<td>0.198</td>
</tr>
<tr>
<td>PE3</td>
<td>How many friends plan to attend 4-year college</td>
<td>0.695</td>
<td>0.029</td>
<td>0.483</td>
</tr>
<tr>
<td>PE4</td>
<td>How many friends have ever dropped out of high school</td>
<td>0.379</td>
<td>0.034</td>
<td>0.144</td>
</tr>
<tr>
<td><strong>Motivation Beliefs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MO1</td>
<td>Scale of student's mathematics self-efficacy</td>
<td>0.775</td>
<td>0.018</td>
<td>0.601</td>
</tr>
<tr>
<td>MO2</td>
<td>Scale of student's interest in math course</td>
<td>0.486</td>
<td>0.024</td>
<td>0.604</td>
</tr>
<tr>
<td>MO3</td>
<td>Scale of student's mathematics identity</td>
<td>0.713</td>
<td>0.017</td>
<td>0.508</td>
</tr>
<tr>
<td>MO4</td>
<td>Scale of student's mathematics utility</td>
<td>0.546</td>
<td>0.021</td>
<td>0.298</td>
</tr>
</tbody>
</table>
Structural Equation Models

Structural models were used to test the hypothesis that the relationships of social support from parents, teachers, and peers to math achievement were mediated by students’ mathematics motivation. With these structural models, the research questions 1-3 were addressed (i.e. How does support from parents, teachers, and peers impact adolescents’ math achievement and motivation? Do students’ motivation beliefs mediate the relationships between social support factors and students’ mathematics achievement?). A full sample size (n = 4,418) was used for the structural model analysis. The measurement part of the model was established based on the final measurement model, as presented earlier. The goodness of fit indices showed a good fit of the hypothesized model ($\chi^2=1044.338$, df=213, CFI=.926, TLI=.912, RMSEA=.03, SRMR=.05).

The hypothesized structural model with standardized path coefficients are presented in Figure 4.3. The results showed that not all the hypothesized relationships among the latent variables were significant. Structural equation coefficients and z-test statistics indicated that three hypothesized paths were not supported, after controlling for other variables in the model: SES to perceived teacher support, parental involvement to motivation, and parental involvement to math achievement. These results indicate that perceived teacher support is not related to social-economic background of the students. The lack of effect of parental involvement on motivation and math achievement was surprising, although previous research showed that the effect of parental involvement decreases for high school students’ academic motivation and achievement. Although modification indices demonstrated that there was some room for improvement in this structural model, researchers suggest that it is not necessary to make changes to the initial model unless conducted in accordance with substantive theoretical
justification (Schumacker and Lomax, 2010). Considering that, no modification was made to the structural model, Table 4.18 presents the overall correlation matrix of all the observed variables in the model.

The direct, indirect, and total effects of the model are shown in Table 4.19. Previous math achievement had the strongest positive direct effect on students’ math achievement ($\beta = .576$). Among the three social support factors, parental involvement had no significant effect on students’ math achievement and motivation ($p > .05$), after controlling other variables in the model. Peer influence had the strongest direct influence on students’ math achievement ($\beta = .12$). In addition, the direct effect of peer influence is around eight times larger than its indirect effect. The total effect of peer influence is the strongest ($\beta = .134$), which is almost three times larger than that of perceived teacher support ($\beta = .046$).

Although perceived teacher support had a very small, but significant negative influence on math achievement, its indirect effect was the strongest on students’ math achievement ($\beta = .102$), and is about seven times larger than the indirect effect of peer influence ($\beta = .014$). Among all variables, perceived teacher support had a very strong positive influence on students’ math motivation ($\beta = .528$), and is about seven times larger than the direct effect of peer influence and 1.5 times larger than the direct effect of previous math achievement. It supported the mediation hypothesis that math motivation plays a very important mediating role in the relationship between teacher support and math achievement.

<table>
<thead>
<tr>
<th>Table 4.18 Overall Item Correlation Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
Table 4.18 Overall Item Correlation Matrix (continued)

<table>
<thead>
<tr>
<th></th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
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<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>13</td>
<td>0.619</td>
<td></td>
<td>1</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>14</td>
<td>0.514</td>
<td>0.55</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.376</td>
<td>0.356</td>
<td>0.589</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>16</td>
<td>0.436</td>
<td>0.444</td>
<td>0.465</td>
<td>0.404</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.333</td>
<td>0.349</td>
<td>0.503</td>
<td>0.416</td>
<td>0.456</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.110</td>
<td>0.08</td>
<td>0.068</td>
<td>0.035</td>
<td>0.055</td>
<td>0.001</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>0.071</td>
<td>0.055</td>
<td>0.033</td>
<td>0.008</td>
<td>-0.019</td>
<td>0.034</td>
<td>0.545</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.122</td>
<td>0.093</td>
<td>0.066</td>
<td>0.013</td>
<td>0.067</td>
<td>0.005</td>
<td>0.557</td>
<td>0.50</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0.064</td>
<td>0.049</td>
<td>0.074</td>
<td>0.018</td>
<td>0.019</td>
<td>0.020</td>
<td>0.385</td>
<td>0.41</td>
<td>0.44</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>0.231</td>
<td>0.205</td>
<td>0.102</td>
<td>0.035</td>
<td>0.197</td>
<td>0.072</td>
<td>0.215</td>
<td>0.18</td>
<td>0.393</td>
<td>0.098</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>0.354</td>
<td>0.288</td>
<td>0.216</td>
<td>0.171</td>
<td>0.371</td>
<td>0.175</td>
<td>0.071</td>
<td>0.026</td>
<td>0.106</td>
<td>-0.034</td>
<td>0.375</td>
<td></td>
</tr>
</tbody>
</table>

Note: SES=family SES, MAS=mathematics achievement. For descriptions of other items, see table 4.17.
Table 4.19 *Standardized Direct, Indirect, Total Effects of Social Support Factors on Student Math Achievement and motivation*

<table>
<thead>
<tr>
<th></th>
<th>Math Achievement</th>
<th>Motivation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total effect</td>
<td>0.167**</td>
<td>0.16</td>
</tr>
<tr>
<td>Total indirect</td>
<td>0.044**</td>
<td>0.16</td>
</tr>
<tr>
<td>Direct</td>
<td>0.123**</td>
<td></td>
</tr>
<tr>
<td><strong>Pervious math achievement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total effect</td>
<td>0.677**</td>
<td>0.365**</td>
</tr>
<tr>
<td>Total indirect</td>
<td>0.101**</td>
<td>0.070**</td>
</tr>
<tr>
<td>Direct</td>
<td>0.576**</td>
<td>0.295**</td>
</tr>
<tr>
<td><strong>Peer influence</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total effect</td>
<td>0.134**</td>
<td>0.074*</td>
</tr>
<tr>
<td>Total indirect</td>
<td>0.014*</td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td>0.120**</td>
<td>0.074*</td>
</tr>
<tr>
<td><strong>Teacher support</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total effect</td>
<td>0.046**</td>
<td>0.528**</td>
</tr>
<tr>
<td>Total indirect</td>
<td>0.102**</td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td>-0.055**</td>
<td>0.528**</td>
</tr>
<tr>
<td><strong>Parental involvement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total effect</td>
<td>0.001</td>
<td>0.012</td>
</tr>
<tr>
<td>Total indirect</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td>-0.001</td>
<td>0.012</td>
</tr>
</tbody>
</table>

*Note: *, p < .05, ** p < .01*
The results of the relationships between social support factors, student motivation, and student mathematics achievement are summarized in three parts: the effects of student background (including SES and prior achievement), the effects of social support factors (including parental involvement, perceived teacher support, and peer influence), and the effects of student math learning motivation.

**Effects of SES and prior achievement.** As shown in Figure 4.3, SES had significant positive influences on parental involvement ($\beta = .338$) and peer influence ($\beta = .330$). The results indicated that students with higher levels of SES were more likely to have parents who communicate with them more about school and future education plans. Students with higher levels of SES tend to have more friends who have higher test scores and plan to go to four-year colleges instead of community colleges or job training schools. In addition, students with higher levels of family SES were also more likely to attain higher mathematics achievement in their
11th grade. These relationships and effects are well known in the literature (e.g., Considene & Zappala, 2002a, 2002b; Hecht & Greenfield, 2001; Okpala et al., 2000).

Prior achievement was positively related to all variables in the model, such as parental involvement ($\beta = .16$), perceived teacher support ($\beta = .091$), peer influence ($\beta = .296$), motivation ($\beta = .295$), and mathematics achievement ($\beta = .576$). These results indicate that students who obtained higher mathematics scores in 9th grade were more likely to perceive higher levels of teacher support. Their parents tend to show more school related involvement, such as, discussing school events and future educational plans with them, and they tend to have more friends who have higher academic achievement in school tests. In addition, students who achieved better scores in mathematics in 9th grade tended to keep good academic records in their 11th grade and displayed higher motivation in mathematics learning.

*Effects of parental involvement, students’ perception of teacher support, and peer influence.* Peer influence had direct significant influence on student mathematic achievement ($\beta = .12$) and indirect influence through motivation ($\beta = .014$), which support the hypothesis that peers play an important role in adolescents’ math motivation and achievement. Students’ perception of teacher support was significantly related to students’ math achievement, but surprisingly, this effect was negative and small ($\beta = -.05$), after controlling for other variables in the model. Although some researchers think that effects of .05 or lower are not substantively significant (Keith, 2014), for sake of clarity and better understanding, this statistically significant direct negative effect of teacher support is analyzed and discussed further under the suppression effect section. The indirect effect of the perceived teacher support on math achievement through motivation was positive and significant ($\beta = .102$), which confirms that the primary meditation
hypothesis of this study.

Results showed that parental involvement had no significant influence on students’ math achievement directly and indirectly through math motivation, after controlling for other variables in the model. These results confirm the decreasing effect of parental involvement during high school years which is also found in other studies (e.g., Chen, 2007; Fan, 2001; Jeynes, 2005). In regard to the effect of social support on math motivation, after controlling for family SES, prior achievement, and other variables in the model, perceived teacher support had the strongest influence in explaining students’ math motivation ($\beta = .528$), followed by peer influence which had a positive significant relationship with math motivation ($\beta = .07$). These results will be further elaborated and discussed under the discussion section.

**Effects of math motivation.** After controlling for SES, prior math achievement and social support factors, math motivation had significant positive influences on student math achievement ($\beta = .193$), which support the hypothesized relationship of motivation and academic achievement in the study. The results indicated that students with higher levels of math motivation, who are interested in math, have a positive view of doing well in math, and believe that math is useful for their future goals and plans, were more likely to attain higher mathematics achievement in math learning.

**Suppression Effect**

As mentioned above, an unexpected result was that student perception of teacher support had a negative but significantly direct influence on students’ math achievement after the mediating effect of the variable through motivation. However, when the motivation variable was removed from the model, perceived teacher support had a small, but positive and significant
influence on math achievement. After including motivation into the model as the mediator, the validity of the model increased and the indirect effect was two times larger than the total effect. These results indicated that motivation may be acting as the suppressor variable. Although there are several suppression definitions, in this study, the following definition from MacKinnon, Krull, & Lockwood (2000) was used: “within a mediation model, a suppression effect would be present when the direct and mediated effects of an independent variable on a dependent variable have opposite signs” (p.174).

In order to better understand the reason why perceived teacher support is negatively related to students’ math achievement after controlling the motivation variable, the students’ motivation scores were divided into three groups based on students’ motivation levels: the lower (n=1,177), medium (n=2,186), and higher (n=1,055) level to test the partial effect of motivation on the relationship between perceived teacher support and math achievement. Three scatter plots displayed the relationships between perceived teacher support and math achievement after controlling for the three motivation levels. Based on the regression equations in Figures 4.4, 4.5, and 4.6, perceived teacher support positively was related to math achievement for the students who had a lower motivation level in learning math. However, for those students with medium and higher motivation levels, the relationship between perceived teacher support and math achievement was negative, rather than positive. The results showed that when a student’s level of motivation was high, the perceived teacher support was negative. Students with a medium or higher levels of motivation seemed to be more independent of their teachers’ support, and were probably more likely to be internally motivated. This is discussed further in Chapter 5.
The graph shows the relationship between teacher support and X2 Mathematics standardized theta score. The equation $y = 48.66 + 0.1x$ is overlaid on the scatter plot.
Figure 4.4 Scatter diagram relating 11th graders' math achievement after controlling math learning motivation.

Figure 4.5 Scatter diagram relating 11th graders' math achievement after controlling math learning motivation.
Multi-Group SEM Analyses

To address the research question 4 about ethnicity-based difference in math achievement and motivation, multi-group SEM analyses were performed in this study to examine whether relationships among latent variables differed across groups (White/Asian and African-American/Hispanic). The rationale for the ethnic grouping in the current study is based on the previous research (e.g., Singh et al., 1995; Yan & Lin, 2005; Autor and Handel 2013; NAEP). These researchers found that the racial achievement gap remains substantially large in mathematics. White/Asian students have generally higher academic performance than African-
American/Hispanic students. Previous studies also demonstrated that White/Asian students seem to receive more social support compared with that of African-American/Hispanic students. For example, Caucasian and Asian parents were reported as having more parental involvement and higher expectations of students’ academic success compared to that of other minority groups (Yan & Lin, 2005; Suizzo & Stapleton, 2007). In addition, another reason for the ethnic grouping is based on the mean of math achievement levels in the groups. The current study found that White/Asian students had a better performance than that of African-American/Hispanic students in math learning ($\bar{x}$ for White/Asian students = 55.03, $\bar{x}$ for African-American/Hispanic students = 49.59; $p < .01$). Therefore, although these groups are culturally and in family context, different, they were grouped together in higher and lower achievement group to explore how the relationships among social support factors, motivational beliefs, and math performance differ by higher-achieving and lower-achieving ethnicity groups.

The full sample ($N=3,910$) was used for the multi-group analysis. There were 2,881 White/Asian participants and 1,029 African-American/Hispanic participants. The equivalence of the measurement model is the prerequisite to make statistical comparisons across these two groups. Measurement equivalence also is called measurement invariance (MI), which assumes that the observed items and theoretical constructs are being measured equally across all subgroups. If MI is violated, the comparisons of variables used in the analysis would not be acceptable, and meaningful interpretations would not be yielded. There are some different types of measurement characteristics of indicators that can be used to test for the invariance, such as factor loadings, intercepts, and residual variances (Baumgartner & Steenkamp, 1998; Lubke & Muthén, 2004).
Measurement Invariance Testing

In this section, an iterative process of comparing the Measurement Invariance (MI) across groups in the measurement model is presented first. Then the similarities and differences of individual paths across two groups are presented. More specifically, comparisons with how White/Asian and African-American/Hispanic students are influenced by different theorized CFA models are tested in the order of least restrictive invariance models to more restrictive invariance models. First, the separate models were estimated for each group to identify whether the model fits the data across both groups. Second, all parameters were estimated freely between these two groups (configural invariance). Third, the indicators’ individual factor loadings were estimated equal for both groups (metric invariance). Fourth, all indicators’ intercepts were constrained to equality for both groups. The last step was the invariant residual variances, which estimate the invariance of measurement errors. Mean differences of latent factors also were tested between these two groups.

### Table 4.20 Test of Measurement Invariance of the Effect of Social Support on Math Achievement across White/Asian and African-American/Hispanic groups

<table>
<thead>
<tr>
<th>Measurement Invariance</th>
<th>(\chi^2)</th>
<th>df</th>
<th>(\Delta\chi^2)</th>
<th>(\Delta\ df)</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
<th>SRMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 0_Asian/White (n=2881)</td>
<td>805.707</td>
<td>161</td>
<td>0.938</td>
<td>0.927</td>
<td>0.037</td>
<td>0.044</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 0_AfricanAmer/His (n=1092)</td>
<td>328.084</td>
<td>161</td>
<td>0.92</td>
<td>0.906</td>
<td>0.031</td>
<td>0.052</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1 (baseline-freely estimated)</td>
<td>1021.021</td>
<td>323</td>
<td>0.933</td>
<td>0.921</td>
<td>0.033</td>
<td>0.046</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 2 (Equal Factor Loadings)</td>
<td>1029.205</td>
<td>339</td>
<td>8.18</td>
<td>16</td>
<td>0.933</td>
<td>0.925</td>
<td>0.032</td>
<td>0.05</td>
</tr>
<tr>
<td>Model 3 (Equal Factor Loadings &amp; Intercepts)</td>
<td>1119.59</td>
<td>355</td>
<td>98.569*</td>
<td>32</td>
<td>0.926</td>
<td>0.921</td>
<td>0.033</td>
<td>0.053</td>
</tr>
<tr>
<td>Model 4 (Equal Factor loading and partial intercept invariance)</td>
<td>1064.969</td>
<td>352</td>
<td>43.948*</td>
<td>29</td>
<td>0.931</td>
<td>0.926</td>
<td>0.032</td>
<td>0.052</td>
</tr>
<tr>
<td>Model 5 (Invariant Residual Variances)</td>
<td>1193.468</td>
<td>372</td>
<td>172.447*</td>
<td>49</td>
<td>0.921</td>
<td>0.919</td>
<td>0.033</td>
<td>0.093</td>
</tr>
</tbody>
</table>

*Note: *, p < .05

As presented in Table 4.20, the goodness of fit indices for both groups showed good fit of
the hypothesized models. In regard to configural invariance, the Chi-square difference was not statistically different between the baseline model 1 and model 2, which indicates that the configural model had an adequate fit. Next, model 3 with both factor loadings and item intercepts were constrained to be equal. The results showed that intercept invariance was not supported, since the Chi-square difference test was statistically significant between the baseline model and model 3. Then, the intercepts of three items (MO3, PE2, PA2) were estimated freely one by one based on modification indices to evaluate partial equality. The results of model 4 showed that the partially constrained model and the baseline unconstrained model still differ significantly. However, although the Chi-square difference statistic did not support the invariant residual variances, $\Delta$CFI = .002 and $\Delta$RMSEA = .001 were small between model 4 and the baseline unconstrained model. Cheung & Rensvold (2002) argued if $\Delta$CFI under .01 between nested models, then the measurement invariance will not be violated. If $\Delta$CFI is larger than .01, then there will be an impactful change for estimating invariance. Therefore, the results from the model indicate that the intercept invariance was accepted (Model 3).

Then, model 5 (invariant residual variances) was conducted, and the Chi-square difference between model 5 and the baseline unconstrained model was significant. Furthermore, $\Delta$CFI is .012, which is larger than .01. These results indicate that the assumption of the invariance of measurement errors was not satisfied. However, researchers have stated that the last step is very restrictive and rarely attainable (Dimitrov, 2006). In order to estimate the significant differences in the latent means between White/Asian and African-American/Hispanic groups, the latent mean scores of the White/Asian group were fixed to zero and the latent mean scores of the African-American/Hispanic group were allowed to freely estimated. Results showed that the African-American/Hispanic group reported lower mean scores in peer influence ($M= - .168$, $p$
< .05) and parental involvement (M=.206, p<.05). No significant mean difference was found for perceived teacher support factor and motivation factor between these two groups.

Overall, the measurement invariance of the hypothesized measurement model between White/Asian and African-American/Hispanic groups was met. Table 4.21 presents the factor loadings of the indicators on the latent constructs for both White/Asian and African-American/Hispanic models. In sum, there are no major differences between the factor loadings of White/Asian and African-American/Hispanic groups when they were freely estimated using the nested models.

<table>
<thead>
<tr>
<th>Labels</th>
<th>Observed variables</th>
<th>White/Asian</th>
<th>African-American/Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parental Involvement</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PA1</td>
<td>How often discussed careers he/she might be interested in</td>
<td>0.631</td>
<td>0.635</td>
</tr>
<tr>
<td>PA2</td>
<td>How often discussed community/national/world events</td>
<td>0.619</td>
<td>0.684</td>
</tr>
<tr>
<td>PA3</td>
<td>How often discussed applying to college/other schools after high school</td>
<td>0.628</td>
<td>0.692</td>
</tr>
<tr>
<td>PA4</td>
<td>How often discussed selecting courses or programs at school</td>
<td>0.828</td>
<td>0.816</td>
</tr>
<tr>
<td>PA5</td>
<td>How often discussed preparing for college entrance exams</td>
<td>0.685</td>
<td>0.608</td>
</tr>
<tr>
<td>PA6</td>
<td>How often discussed job that he/she might want to take after high school</td>
<td>0.590</td>
<td>0.639</td>
</tr>
<tr>
<td>PA7</td>
<td>How often discussed things that were troubling him/her</td>
<td>0.545</td>
<td>0.658</td>
</tr>
<tr>
<td><strong>Students’ Perception of Teacher</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TS1</td>
<td>Teen's spring 2012 math teacher makes math interesting</td>
<td>0.849</td>
<td>0.831</td>
</tr>
<tr>
<td>TS2</td>
<td>Teen's spring 2012 math teacher makes math easy to understand</td>
<td>0.860</td>
<td>0.808</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>TS3</td>
<td>Teen's spring 2012 math teacher wants students to think, not memorize</td>
<td>0.644</td>
<td>0.615</td>
</tr>
<tr>
<td>TS4</td>
<td>Teen's spring 2012 math teacher doesn't let students give up</td>
<td>0.614</td>
<td>0.653</td>
</tr>
<tr>
<td>TS5</td>
<td>Teen's spring 2012 math teacher treats some kids better than others</td>
<td>0.494</td>
<td>0.389</td>
</tr>
<tr>
<td>MO3</td>
<td>Scale of student's interest in math course</td>
<td>0.392</td>
<td>0.428</td>
</tr>
</tbody>
</table>

### Peer Influence

| PE1 | How many friends get good grades | 0.595 | 0.621 |
| PE2 | How many friends have taken PSAT, SAT, PLAN or ACT | 0.486 | 0.283 |
| PE3 | How many friends plan to attend 4-year college | 0.706 | 0.692 |
| PE4 | How many friends have ever dropped out of high school | 0.399 | 0.457 |

### Motivation Beliefs

| MO1 | Scale of student's mathematics self-efficacy | 0.519 | 0.391 |
| MO2 | Scale of student's interest in math course | 0.777 | 0.737 |
| MO3 | Scale of student's mathematics identity | 0.753 | 0.667 |
| MO4 | Scale of student's mathematics utility | 0.571 | 0.532 |

**Structural model invariance**

In order to answer research question 4 (i.e. whether the relationships between the latent factors are consistent between White/Asian and African-American/Hispanic students), the invariance of the structural parameters between White/Asian and African-American/Hispanic students was tested after measurement invariance was satisfied. The loadings and intercepts invariance of all the indicators between these two groups remained under the structural models (Model 3). Following the Anderson and Gerbing (1988) two-step approach for assessing the structural model, two nested structural models were estimated first. The first model with no equality constraints for the structural paths between these two groups works as the baseline model. The second model is the fully-constrained structural model, which constrained all structural paths equally between these two groups. A $\chi^2$ difference test was used to assess
whether the invariance of the structural parameters is met or not. Table 4.22 presents fit indices for the baseline, full-constrained, and partially constrained structural models.

### Table 4.22 Fit Indices for Structural Models

<table>
<thead>
<tr>
<th>Measurement Invariance</th>
<th>$\chi^2$</th>
<th>df</th>
<th>$\Delta\chi^2$</th>
<th>$\Delta df$</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
<th>SRMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Model</td>
<td>1628.666</td>
<td>459</td>
<td></td>
<td></td>
<td>0.910</td>
<td>0.901</td>
<td>0.036</td>
<td>0.061</td>
</tr>
<tr>
<td>Fully-Constrained model</td>
<td>1656.916</td>
<td>475</td>
<td>28.25*</td>
<td>16</td>
<td>0.909</td>
<td>0.904</td>
<td>0.035</td>
<td>0.064</td>
</tr>
<tr>
<td>Free path from motivation to math achievement</td>
<td>1653.974</td>
<td>474</td>
<td>25.308*</td>
<td>15</td>
<td>0.909</td>
<td>0.904</td>
<td>0.035</td>
<td>0.064</td>
</tr>
<tr>
<td>Free path from perceived teacher support to math achievement (Final)</td>
<td>1652.176</td>
<td>473</td>
<td>23.51</td>
<td>14</td>
<td>0.909</td>
<td>0.904</td>
<td>0.035</td>
<td>0.064</td>
</tr>
</tbody>
</table>

*Note:*, $p < .05$

As shown in Table 4.22, the difference in $\chi^2$ values between the baseline model and full-constrained structural model was statistically significant ($p < .05$), which indicates that the some or all structural paths are not equivalent between these two groups. Therefore, sequential chi-square difference tests were conducted by freeing equality constraints on the structural paths one by one, in order to figure out which structural path should be kept and which paths should be freely estimated between two the models. Two direct paths between the following latent factors were freely estimated in the partially constrained structural model: (a) motivation to math achievement, (b) perceived teacher support to math achievement. The non-significant Chi-square difference ($\Delta\chi^2 = 23.51$, $df = 14$, $p > .05$) between the baseline model and partially constrained model indicates that the partially-constrained structural model was satisfactory. Table 4.23 presents the partially-constrained structural models’ standardized direct effects for the White/Asian and African-American/Hispanic students, respectively. As shown in Table 4.23, two main deviations were found between White/Asian and African-American/Hispanic student models. Perceived teacher support negatively and significantly influences White/Asian students’
math achievement, but it had no significant influence on African-American/Hispanic students. In addition, math motivation had a stronger influence on the math achievement for White/Asian students than African-American/Hispanic students. These results have theoretical and substantive underpinnings and are discussed further in Chapter 5.

Table 4.23 Standardized and Unstandardized Direct Effects (Final Partially-Constrained Model)

<table>
<thead>
<tr>
<th>Path</th>
<th>White/Asian</th>
<th></th>
<th>African-American/Hispanic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unstandardized</td>
<td>Standardized</td>
<td>Unstandardized</td>
<td>Standardized</td>
</tr>
<tr>
<td>Teacher support to math achievement</td>
<td>-0.897**</td>
<td>-0.073**</td>
<td>-0.897</td>
<td>-0.006</td>
</tr>
<tr>
<td>Motivation on math achievement</td>
<td>2.171**</td>
<td>0.235**</td>
<td>1.023*</td>
<td>0.105*</td>
</tr>
<tr>
<td>Peer influence to math achievement</td>
<td>2.915**</td>
<td>0.124**</td>
<td>2.915**</td>
<td>0.139**</td>
</tr>
<tr>
<td>Parental involvement to math achievement</td>
<td>0.021</td>
<td>0.001</td>
<td>0.021</td>
<td>0.001</td>
</tr>
<tr>
<td>SES to math achievement</td>
<td>1.220**</td>
<td>0.094**</td>
<td>1.220**</td>
<td>0.102**</td>
</tr>
<tr>
<td>SES to peer influence</td>
<td>.179**</td>
<td>0.325**</td>
<td>.179**</td>
<td>0.312**</td>
</tr>
<tr>
<td>SES to teacher support</td>
<td>-0.008</td>
<td>-0.008</td>
<td>-0.008</td>
<td>-0.008</td>
</tr>
<tr>
<td>SES to parental involvement</td>
<td>.216**</td>
<td>0.297**</td>
<td>.216**</td>
<td>0.278**</td>
</tr>
<tr>
<td>Prior math to math achievement</td>
<td>0.594**</td>
<td>0.565**</td>
<td>0.594**</td>
<td>0.576**</td>
</tr>
<tr>
<td>Prior math to motivation</td>
<td>.033**</td>
<td>0.294**</td>
<td>.033**</td>
<td>0.317**</td>
</tr>
<tr>
<td>Prior math to peer influence</td>
<td>.014**</td>
<td>0.303**</td>
<td>.014**</td>
<td>0.276**</td>
</tr>
<tr>
<td>Prior math to teacher support</td>
<td>.008**</td>
<td>0.097**</td>
<td>.008**</td>
<td>0.100**</td>
</tr>
<tr>
<td>Prior math to parental involvement</td>
<td>.004*</td>
<td>0.060*</td>
<td>.004*</td>
<td>0.053*</td>
</tr>
<tr>
<td>Peer influence to motivation</td>
<td>.247**</td>
<td>0.097**</td>
<td>.247**</td>
<td>0.116**</td>
</tr>
<tr>
<td>Teacher support to motivation</td>
<td>.667**</td>
<td>0.502**</td>
<td>.667**</td>
<td>0.527**</td>
</tr>
<tr>
<td>Parental involvement to motivation</td>
<td>0.031</td>
<td>0.016</td>
<td>0.031</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Note: *, p < .05, ** p < .01

In summary, these results show that perceived teacher support has substantial influence on student math leaning motivation, and motivation has a positive and significant impact on
students’ math achievement, regardless of the overall model or each group. In addition, family SES and student prior achievement were significantly related to math achievement and motivation. The overall summary of findings and implications are discussed in Chapter 5.
Chapter 5
Discussion and Conclusion

The purpose of this study was to explore how contextual social support from parents, teachers, and peers impact adolescents’ math achievement. More specifically, the study aimed to investigate how parental involvement, student perception of teacher support, and peer influence relate to high school students’ math achievement. The current research also examined whether or not math motivation acts as a mediator in the relationships between social support factors and math achievement. Finally, the study examined whether all constructs and relationships used in the study vary across two groups (White/Asian and African-American/Hispanic). Structural equation modeling (SEM) served as the main statistical technique to examine the relationship among variables. This chapter presents the summary of findings, discussion, implications, significance and limitations of the study, directions for future research, and conclusions.

Summary of Findings

Based on the expectancy value theory and Vygotsky’s sociocultural theory, the current research displayed a complex model of mathematics achievement, which presents the relationships among social support, math motivation, and math achievement. EFA and CFA were used to create four reliable and valid latent variables: parental involvement, student perception of teacher support, peer influence, and math motivation. Structural models estimated the hypothesized relationships among latent variables and tested how the model fit with the data.

Results from Chapter 4 supported 13 of the 16 hypothesized relationships of this study. Among the 13, the most notable finding is that the mediation hypothesis was supported: social support (from teachers and peers) influences math achievement by affecting students’ math
motivation. In other words, motivation acts as a mediator variable in the relationship of social support factors and math achievement. Significant indirect effects from perceived teacher support to math achievement via motivation confirms that when students perceive that teachers are supportive, they are more likely to be more motivated to learn math, which, in turn, affects their math achievement positively. In addition, the effect of peers also operated by influencing the motivation and math achievement, which further confirms the mediational role of motivation.

In terms of direct effects, students’ perception of teacher support and peer influence were significantly and directly related to students’ math achievement, with the relationship between peer influence and math achievement being positive ($\beta = .120, p < .05$) and the relationship between perceived teacher support and math achievement being negative ($\beta = -.055, p < .05$). In addition, among social support factors, peer influence had the strongest direct effect ($\beta = .120, p < .05$) and the strongest total effect ($\beta = .134, p < .05$) on students’ math achievement, perceived teacher support had the strongest indirect effect ($\beta = .102, p < .05$) on math achievement through math motivation. Moreover, the results showed that parental involvement was not significantly related to students’ math achievement and motivation ($p > .05$).

The negative relationships of teacher support on math achievement among more motivated students is unexpected. After a close examination, partial effect analysis indicated that perceived teacher support had a positive and significant influence on students with lower levels of math motivation. For those students with higher and medium motivation levels, perceived teacher support was negatively related to their math achievement. Furthermore, as expected, math motivation was also significantly and directly related to 11th graders’ math achievement. Additionally, students’ family SES background was significantly and positively related to their
math achievement, parental involvement, and peer influence. Students’ previous math achievement was positively related to all variables in the model, such as parental involvement, students’ perception of teacher support, peer influence, motivation, and math achievement.

Multi-group SEM techniques were used to address research question 4 (i.e., whether the relationships among the latent factors are consistent between White/Asian and African-American/Hispanic students). Results of the sequential measurement invariance tests supported that the items used to assess latent constructs for the two groups functioned similarly between groups. Testing the structural paths among latent constructs indicated that there are indeed differences in the relationships among social support, motivation, and math achievement between white/Asian and African-American/Hispanic students. The current study found that math motivation significantly influenced math achievement for both groups, but it had a larger effect for White/Asian students.

**Discussion of Findings**

The results of this study reveal three sets of important findings. The first shows the effects of parental involvement, perceived teacher support, and peer influence on students’ math achievement and motivation. The second set of findings indicates that both family SES and prior achievement influence students’ social support factors and math achievement. Lastly, the different relationships among the latent factors between White/Asian and African-American/Hispanic students are discussed.

Effects of parental involvement, perceived teacher support, and peer influence.

The first set of findings demonstrated that perceived teacher support and peer influence, but not parental involvement, were directly related to 11th graders’ math achievement and
motivation, suggesting that both peer influence and perceived teacher support play essential roles in students’ math learning and performance, after controlling for other variables in the model. These results also confirmed that math motivation plays a significant role in mediating the relationships of social support (from teachers and peers) and student math achievement in high school. Furthermore, the current study explored the reasons why parental involvement was not significantly related to students’ math motivation and achievement, after controlling for other variables in the model. In addition, the direct negative relationship between perceived teacher support and math achievement, although very small, is discussed. The three relationships are discussed below.

**Parental Involvement**

The framework of parental involvement is multidimensional and multifaceted in nature due to varied parental behavioral patterns and parenting practices (Singh, et al., 1995; Fan & Chen, 2001; Li & Hung, 2012). Researchers have usually discussed parental involvement in two parts: school-based involvement and home-based involvement strategies. In the current study, home-based involvement, especially parent-child communication, was used to explore the effect on the students’ math achievement. However, it is interesting to note that parental involvement had no significant influence on students’ math achievement and motivation. Although these results were not completely surprising, these need to be examined and explained further.

The role of parents in students’ academic achievement has been well documented, but research findings on this topic are not uniform. This is so because the researchers have focused on different sample groups, such as participants’ age, race, and cultural background; and they have focused on different types of parental involvement, which lead to different results. For
example, researchers found that parental involvement, especially home-based involvement, has a relatively strong influence on elementary students’ academic achievement, but its influence becomes weaker for middle school students, and it is sometimes non-significant or negatively associated for high school students. For example, Fan (2001) and Jeynes’s (2005, 2007) meta-analysis studies confirmed that the influence of parental involvement on students’ academic outcomes varied by students’ ages. These authors found that parental involvement has a stronger influence on students’ achievement at the elementary school level than the secondary school level. They reasoned that parents are more engaged when students are in their childhood, and as a result, young children are generally more likely to be affected. Similarly, Chen’s (2007) study also found that grade-level differences occur in the influence of students’ academic support (from parents, teachers and peers) on students’ academic achievement. Her findings showed that parental support was negatively related to 11th graders’ academic achievement, but it was positively linked to academic achievement through engagement for 9th-grade students. These findings aligned with Vygotsky’s sociocultural theory (1978) which suggested that parents are the main players and helpers in the social context surrounding children when their children are young, but teachers and peers become much more important when children are in higher grades.

One interpretation for diminishing effect for high school students could be that adolescence has been determined as a particularly transformative and challenging development period for developing children’s physical, cognitive, and emotional characteristics. During adolescence, students seek independence, autonomy, and detachment from their parents to develop their strong sense of responsibility and identity, and at the same time, adolescents pursue more guidance and support from other people outside of home (Murray, 2009). Particularly in Western cultures, it is essential for adolescents to build independence on their own efforts.
(Twenge, 2006). In this stage, more frequent discussions with children to check their career plans, school events, and college applications may conflict with adolescents’ independence pursuits, because adolescents may treat this kind of frequent discussion as monitoring and controlling behaviors about their school life. Pomerantz et al., (2007) substantiated this aspect since they mentioned that “when parents’ involvement is controlling, children do not have the experience of solving problems challenges on their own,” and, “when parents are controlling, they may deprive children of feeling that they are autonomous, effective agents” (p. 383). Therefore, constant or very frequent parental communication may conflict with the adolescents’ own views about school learning. As a result, higher levels of parental involvement may not be an effective vehicle for improving high school students’ math achievement. Another interpretation for diminishing effect would be that it is challenging for parents to assist high school students’ math education. When students enter high grade levels, the math schoolwork, curricula, and content become more difficult and complex. Some parents may lack confidence, knowledge, and competence to guide and help their children in learning math.

Besides the diminishing effect of parental involvement on students’ motivation and math achievement due to age, there could be other reasons that could explain this non-significant relationship. One is based on the McNeal’s (1999) reactive hypothesis, which is usually used to explain the inconsistent relationships between parental involvement and children’s academic performance. The reactive hypothesis claims that “any negative correlation or relationship between parent involvement and academic achievement stems from a reactive parent involvement strategy whereby when a student is having academic or behavioral difficulties at school”, a parent is more likely to show involvement and communicate with the child more often about school which would lead to a negative or neutral association between parental involvement
and math achievement (McNeal, 1999, p.79). Often parents get involved in higher grades only when a child is having some academic difficulty, resulting in a negative relationship. This pattern of significant negative results may indicate that low-achieving students attract more involvement from parents than do more high-achieving students.

Another reason for this non-significant relationship could be measurement related. There were only limited items of parental involvement that were used in the current study. This study focused on parent-children communication rather than other home-based parental involvement and school-based parental involvement (e.g., attending parent–teacher conferences and school activities). Parent-children communication is just one type of home-based involvements, which focuses on the discussion about school events, college applications and exams, future career plans, and world events. Jeynes’s (2007, p.101) meta-analysis study found that the effect sizes for “parent-child family communication about school were smaller than for either parental style or expectations”. There are several other home-based involvement types that should be examined for future research. In addition, parental involvement survey was completed by parents and motivation beliefs were reported by students. The associations between factors that were reported by different participant groups often are not as strong as the associations between the measures from the same participant group.

Perceived Teacher Support

In the current study, perceived teacher support was conceptualized as the emotional content of the relationships and interactions between teachers and students. Perceived teacher support was hypothesized to impact student math motivation and achievement. Perceived teacher support was measured by the following items: treat all students equally, encourage students to
think rather than memorize, do not let students give up, make mathematics interesting, and make math easy to understand. In contrast to expected results, a negative but very small significant relationship between student perception of teacher support and student achievement was found. Some previous research has found somewhat similar results for a link between teacher support and students’ academic achievement. For example, Yu (2015) found that teacher support had substantial influence on high school students’ math motivation, but no significant direct effect on these students’ math achievement. Similarly, King’s (2016) model showed that teacher support had no significant influence on students’ academic achievement. It is possible that controlling for other variables, the direct effect of teacher support is only small.

This surprising result seems counterintuitive. However, after a deeper exploration and further analysis, some explanations were discovered, which leads to some meaningful interpretations. To take a closer look, the relationship was also analyzed by the level of motivation to examine if the relationship varied for different levels of motivation. The partial effect findings produced a somewhat mixed influence, with changes in students’ math achievement depending on the different motivation levels students have. The relationship between perceived teacher support and math achievement differed when the data was divided into three groups based on their motivation levels: lower, medium, and higher. For those students who had a lower motivation level in learning math, perceived teacher support was positively related to math achievement. By contrast, for those students at the medium and higher motivation levels, the relationship between perceived teacher support and math achievement was negative. Thus, the unexpected finding of a direct negative relationship between perceived teacher support and math achievement reflects a strong influence of different levels of students’ math motivation on the relationship between perceived teacher support and math achievement. This means that
for students at the lower motivation levels teacher support was more important, whereas students at the medium or higher levels of math motivation will be less concerned or will be less affected by teacher support.

After reviewing the items of perceived teacher support in this study, one interpretation would be that students who demonstrated higher motivation have higher aspirations to achieve math proficiency and less likely to need psychological support from teachers. They may have more interest in learning math, as well as a higher utility value for learning math, such as believing math outcomes will benefit their college application and future career. Therefore, it is possible that highly motivated students are less affected by teacher support. On the contrary, students at lower motivation levels are more sensitive to teacher support. That is, unlike their higher motivated counterparts, students at low motivation levels would benefit from teacher support if their teachers put more time to increase students’ math interest, encourage students to think more, and decrease the difficulties of math school work. A study by Koutsoulis (2003) examined that students in different achievement groups have different expectations for their teachers. Koutsoulis found that students who are in the low achieving group appreciated their teachers’ human characteristics (such as understanding and friendly behaviors), but for those students who are in high-achieving groups preferred teachers “to be knowledgeable, clever, and not to spend time on comments for students' behavior” (p. 10). This suggests that students at different levels of motivation or academic achievement are differentially affected by psychological support from their math teachers.

Although perceived teacher support did not influence student math achievement directly, it supported the mediation hypothesis that motivation plays an essential role in the relationship
between perceived teacher support and students’ math achievement. This relationship support that receiving high quality support from teachers could not automatically translate into higher math achievement for high school students, but perceived teacher support encourages them to demonstrate higher levels of math motivation, which in turn contributes to their improved math achievement.

This finding is consistent with previous research results. For example, Chirkov and Ryan (2001) found that perceiving greater autonomy-support from teachers is related to greater well-being and more autonomous motivation in the United States and Russian samples. These authors concluded that teachers have a greater effect on high school students’ experiences of interest and challenge in the academic domain compared with parent support. Filak and Sheldon (2008) found that teacher support predicted students’ self-determined motivation and psychological need satisfaction, both of which increased students’ course grades. Some scholars have reasoned why teacher support is vital and unique to students’ academic performance. They pointed out that teacher support intrinsically motivates students to learn by helping students meet their needs, offering meaningful and challenging work, allowing students to take ownership over their work, providing a safe, caring, and supportive classroom environment, and supporting mastery and understanding of content in their classrooms (Urdan & Schoenfelder, 2006; Ames, 1992; Meece et al., 2006).

More specifically, based on the current study, these supportive teachers encourage student learning by increasing students’ motivation. They treat students equally and utilize different ways of teaching to make the subject more interesting and easier to understand. Additionally, they provide students opportunities to think critically while learning math to help
students’ understanding. These activities help to build a positive relationship between teachers and students as well as establish a supportive classroom environment. When students feel close to their teachers in a positive learning environment, increase their sense of belonging in the classroom, tend to have more interest in school, and have more confidence to face challenges, which all contribute to enhancing students’ math outcomes. In addition, these findings also aligned with the expectancy-value theory (Eccles, 1983), which assumes that the relationship between supportive teachers and student academic achievement is mediated by students’ motivational beliefs, such as efficacy expectations and task values, which in turn improves students’ academic performance. In other words, students would not achieve success in math learning without math motivation, which can be improved by positive teacher support.

Moreover, this finding is consistent with the previous studies on teacher support that were examined in elementary and middle schools in the U.S. Previous research demonstrated the important role teachers play in students’ math learning. (Patrick et al., 2007; Sakiz et al., 2012). In sum, the direct effect of teacher support seemed small and varied by the motivation levels of students, but the indirect effect, confirming the mediational hypothesis, was stronger and positive.

Peer Influence

With regards to the relationship among peer influence, students’ math achievement, and motivation, the current study found that peer influence had a positive influence on students’ math achievement directly and indirectly through motivation. When students have peers who have higher aspirations and have good grades, they are likely to have higher math scores. These results indicate that peer influence plays a vital role in explaining high school students’ academic performance and motivation. This relationship suggests that students who have peers with higher
grades and higher educational aspirations are more likely to be high-achieving themselves. These findings also are consistent with the results of previous research studies. For example, Hanushek et al. (2003) found that peer groups’ average test scores were strongly associated with students’ test score distribution. They found that a 0.1 standard deviation growth in average test scores among peers leads to a roughly 0.02 growth in students’ academic achievement. Their findings indicate that students benefit from their higher achieving peers. Similarly, Burke and Sass (2008) revealed that students’ academic achievement in middle and high school depends on the average achievement level of their peer groups. In addition, Ryan (2001) argued that peer groups impact adolescents’ motivation and engagement in a sharing context. Adolescents share their experiences and exchange information “with regard to the norms, values, and standards that concern academic motivation and achievement” through daily interaction (p. 1136). These findings confirmed that adolescents’ peer groups have a positive effect on the students’ academic outcomes and motivation.

The reasons for why peer influence is becoming more important, even stronger than parents and teachers during high school years is that peer influence peaks more strongly during adolescence than at any other time in a child’s life. Adolescents spend more than half of their time interacting with their peers, place significance on peer approval and suggestions, and consider peers as an important source of identity, and treat peers as a source of social attachment that provides a sense of belonging and acceptance (Updegraff et al., 2002; Bukowski, Sippola, & Newcomb, 2000; Rubin, Bukowski, & Parker, 2006). These findings align with Vygotsky’s Social Development Theory. Vygotsky (1978) found that social interaction is the basis for children’s cognition development, in particular interactions with more experienced, skilled, and knowledgeable social helpers (i.e. peers) who help children to build an understanding of
concepts and solve problems (Bransford, Brown, & Cocking, 2000). For Vygotsky, the social contextual environment in which students grow up will impact their thoughts and beliefs. Teachers and peers become much more important when children enter school and move to higher grades. Piaget (1952) also confirmed that peer interaction is a vital factor to promote students’ cognitive development. He emphasized that the more positive the students’ peer interactions are, the greater they will progress toward mastery of cognitive skills.

In sum, a positive peer relationship is beneficial because it can shape a supportive and safe classroom learning environment by delivering more help, instruction, and advice (Nichols & White, 2001; Burke & Sass, 2013). These studies show that the school value, college pursuits, and career plans of students’ friends are related to high school students’ own academic beliefs, studying patterns, and college aspirations, which in turn improve students’ math achievement. Therefore, given the importance of peer influence during adolescence, the finding that peer influence plays a critical role in explaining achievement outcomes is reasonable and meaningful. Not only did the peer influence have a direct positive effect on math achievement, it also had an indirect positive effect, operating through motivation. Thus, it had the strongest total effect. More academically oriented the peers, the higher the motivation of the students. Higher performing peers enhanced the motivation of the students. This further illustrates the mediational path for peer influence through motivation.

Effects of Motivation on Math Achievement

Motivation is a concept that is used to explain people’s thoughts and behaviors. It was defined as a “process whereby goal-directed activity is instigated and sustained” (Pintrich & Schunk, 2002, p. 5). Although various theories were developed to explore motivation, the
expectancy–value theory was used in the current study. This motivation theory was explained as "the amount of effort that people are willing to expend on a task is the product of the degree to which they expect to succeed at the task, and the degree to which they value the task and value success on the task" (Green 2002, p. 990). In this theoretical model, students’ math outcomes are predicted by their two motivational beliefs: expectancies for success and subjective task values. Sociocultural contexts (e.g., SES) and socialization agents (e.g., parents, teachers, and peers) shape motivation beliefs by directly and interactively affecting expectancies and task value beliefs.

The results of the current study supported the expectancy value theory that academic motivation is positively associated with students’ math achievement. Furthermore, students’ academic motivation was driven by the teacher and peer support. This result is consistent with previous research findings that motivation plays a major role in students’ academic performance, adjustment, attitudes, and well-being, since motivation beliefs are reflected in students’ choice of learning and achievement tasks, in their interest in the tasks, in their persistence and confidence on the tasks, in the time and effort they allocate to these tasks, in the goals they pursue, and in their facing of learning difficulties, pressure, and anxiety (Wigfield & Eccles; 2002; Meece, Anderman, & Anderman, 2006; Kusurkar, et al., 2013; Allen, & Robbins, 2010; Turner, Chandler, & Heffer, 2009; Pekrun, Elliot, & Maier, 2009). The current study revealed that when students have higher motivation, they will earn higher grades in their math learning. This improvement may reflect that students tend to enroll in more math courses and participate in more advanced cognitive strategies in math when they feel confident in achieving a good score in a math test (Wigfield & Eccles, 1992, 2002). Students with more math interest would demonstrate more enjoyment and persistence in learning math, and students who value this
subject may invest long hours and more effort in math tests for realizing their college aspirations and future career choices. Moreover, motivation makes students persist in the face of hardship and difficulties in learning math to perform well in school. Overall, this study confirms that math motivation (including self-efficacy, interest, utility, identity) has a strong association with students’ math achievement and supports the expectancy-value theory and other motivation theories.

Effects of Family Socio-Economic Status (SES) and Prior Achievement

*Family SES.* The socio-economic status (SES) variable has long been studied and widely used as having an important role in student development, especially in the education field. Numerous studies have shown that one’s educational success depends strongly on the socio-economic background, meaning the parents are advantaged socially, educationally and economically (e.g., Considine & Zappala, 2002a, 2002b; Hecht & Greenfield, 2001; Okpala et al., 2000). In this study, the hypothesis that students with higher levels of family SES are more likely to attain good scores in math courses was supported.

Moreover, students with higher levels of family SES are more likely to obtain their parents’ involvement and tend to have more friends who plan to attend colleges instead of community college or job training schools. The current study showed that family SES had the largest effect on parental involvement, followed by peer influence and math achievement. These results confirm earlier findings that family SES is positively associated with the level of parental involvement at home. The reason would be that parents with a higher education background and income demonstrated greater involvement strategies than parents from a lower family SES background would because they have adequate knowledge to assist their children’s schoolwork...
as well as have more skills and experiences to involve in children’s education (Vellymalay, 2012). On the contrary, parents with lower SES are constrained by their life circumstances to pay attention to their children’s human needs and have limited time and resources to attend to their education needs for academic success (Rockwell, 2011). In addition, family SES background impacts students sorting and grouping, which results in students with a lower family SES losing positive influences from attending school with higher family SES peers while higher SES students continue benefiting from “getting a ‘better’ peer group” (van Ewijk, & Sleegers, 2010, p. 135). Overall, unequal social support based on their family SES would lead to unequal educational resources and opportunities, which may continue to widen the achievement gap between both groups of students.

However, after controlling for other variables, no significant relationship was found between family SES and student perception of teacher support. It shows that teachers did not have any explicit SES bias. In other words, the findings from the current study indicated that students perceived that their teachers were helpful and did not differentiate students by their family SES background. This is a good sign for narrowing the achievement gap between high SES and low SES students because teacher support can strongly affect children’s academic outcomes. Within the school setting, teacher support has an integral and critical context for students’ academic, behavioral, and emotional development, since students spend a significant amount of time in the classroom and receive instruction facilitated by their teachers (Bronfenbrenner & Morris, 2006; Eccles & Roeser, 2010). Independent of SES, the more effective support from teachers, the more opportunities there are to succeed in schools.
**Prior math achievement.** Prior academic achievement is usually the strongest predictor of future academic achievement. In the current study, prior math achievement was measured by the 9\(^{th}\) grade students’ math achievement. The present study hypothesized that prior achievement was related to the social support (from parents, teachers, and peers), academic motivation, and 11\(^{th}\) grade math achievement. The findings support all these hypothesized paths. Specifically, the results show that the higher the students’ prior achievement in their 9\(^{th}\) grade, the more they will have parental involvement, perceived teacher support received, positive peer influence, higher math motivation levels, and higher math achievement in their 11\(^{th}\) grade. This study found that prior math achievement was the strongest influence on students’ 11\(^{th}\) math achievement, after controlling for other variables. This result is consistent with previous research. For example, Yu (2015) demonstrated that students who had higher academic performance in 8th grade generally were more likely to achieve math success in 9\(^{th}\) grade. She explained that previous achievement compresses all the factors (e.g., family SES background, social continual support, personal characteristics) that may have led to previous success and will continue to impact students’ later academic success.

Additionally, the results showed that prior achievement had significant and positive influence on students’ math motivation, which indicated that students who had a higher math achievement in 9th grade generally were more likely to report higher levels of math motivation. For example, Pajares, Britner, and Valiante, (2000) showed that students at lower prior achievement levels in writing were more likely to hold a performance-avoidance goal orientation. The findings of this study indicate that students would increase their confidence, interest, and self-identity in learning math when they achieve success in their previous math courses.
With regard to social support from parents, teachers, and peers, this study reports that the higher the prior math achievement, the higher the support students would receive from their parents, teachers, and peers. It is possible that students who had higher previous math achievement were more likely to have more conversations with their parents about future career and educational goals. Those parents are more likely to discuss college applications and future career plans more often with their children. These parents also communicate more with children about school events and encourage their children to achieve. As for higher perceived teacher support, teachers may hold positive views for students who have good records in previous school years. These positive views may impact teachers’ attitudes, which may lead to more support to students who had higher prior math achievement by teachers. Another interpretation would be that students who succeed in prior math courses may have higher evaluation for their teachers or positive relationship with their teachers. Therefore, they reported that they received a higher level of teacher support, since it is a perception survey. This study found that students who obtained higher mathematics scores in 9th grade tend to have more friends who have higher academic achievement in school tests and higher educational aspirations. This finding is consistent with Ryan’s (2001) contention of peer selection, which is that students selected peer groups who had academic performance and behaviors similar to their own.

Differences between White/Asian and African-American/Hispanic students

It has been well documented that students’ math achievement and levels of social support vary among racial and ethnic groups (e.g., Yan & Lin, 2005; Jeynes, 2003; Suizzo, & Stapleton, 2007). Despite many reform efforts, the racial achievement gap remains substantially large. In math, White/Asian students have generally better performance than African-American/Hispanic students (NAEP, 2015). Multi-group SEM techniques were used to examine whether
relationships among latent variables differed across groups (White/Asian and African-American/Hispanic). Although these groups are culturally and in family context different, they were grouped together for being in higher and lower achievement group. Tests of measurement invariance supported the equivalence of the measurement model in two groups, indicating that the items used to assess the hypothesized CFA models functioned similarly between these two groups. Then, structural paths between these two groups were tested. Allowing two structural paths to be freely estimated for both groups resulted in a non-significant Δχ² between the unconstrained structural model and the partially-constrained structural model. In addition, latent mean differences were tested to determine if White/Asian students or African-American/Hispanic students reported significantly different mean values on the four latent constructs.

In terms of latent mean differences between White/Asian and African-American/Hispanic students, this study found that the African-American/Hispanic group reported lower mean scores in peer influence (M=-.168, p < .05) and parental involvement (M=-.206, p <.05), but no significant mean difference was found for perceived teacher support factor and motivation factor between these two groups. These findings are consistent with previous research. For example, a longitudinal study from NELS: 88 involving 19,386 high school students by Yan and Lin (2005) reported that Caucasian parents are more involved in their students’ schooling (such as attending school programs about teenagers' future planning) compared to Asian, African, and Hispanic American parents. Asian American parents had the highest educational expectations for their children's academic success; the reason for this may be that they value education as the best and the only resource to achieve social mobility. Hispanic parents seemingly had the lowest level of
involvement in school possibly because they were unable to effectively engage and interact with their children’s school and teachers.

As for peer influence, the current study found that in general peer influence had a positive influence on students’ math motivation and achievement. However, African-American/Hispanic students reported that their friends had lower grades compared to their white/Asian peers. Moreover, African-American/Hispanic students had fewer friends who planned to pursue a college degree rather than community colleges or job training schools. As discussed earlier, peer influence becomes more powerful and influential during adolescence. Previous research has shown that when African-American/Hispanic adolescents’ have more peers who show anti-achievement behaviors and values, they may be more likely to report lower achievement motivational beliefs and academic grade performance (Butler-Barnes et al., 2015). These findings of the current study suggest that positive achievement values of peers serve as helpful resources for adolescents, and negative achievement values of peers serve as a risk. The present study found that there was a significant mean difference in the two latent variables of parent involvement and peer influence in the two groups; White/Asian group reported higher parent involvement and higher number of academically oriented peers. On the other hand, no mean difference was found in the level of teacher support and motivation. The two groups reported similar levels of teacher support and motivation.

Following the analysis of the mean differences, the differences in the structural paths were analyzed. Structural model invariance analysis showed that there was a difference in the effects of teacher support on math achievement between these two groups. Perceived teacher support was negatively associated with White/Asian students’ math achievement, but it had no
significant influence on African-American/Hispanic students. It showed that students of different racial groups are differentially affected by psychological support from their math teachers. White/Asian students, as the high-achieving group, may be less affected by their teachers’ providing psychological support. However, African-American/Hispanic students, as the lower achieving group, also did not benefit significantly from their teacher support. The reason could be lack of a direct effect of teacher support on math achievement for the African-American/Hispanic students. It could also be due to the limited measures of teacher support.

With regard to the relationship between motivation and math achievement, the structural model invariance analysis showed that math motivation had a positive influence on students’ math achievement for both groups, but it had a stronger influence on the math achievement for White/Asian students than African-American/Hispanic students. This means that both groups of students are more likely to achieve success in math learning when they have a strong sense of their mathematics abilities, show more interest in learning the course content, and have a high math self-identity and math utility value. The difference is that White/Asians can better translate motivation into their academic performance compared to African-American/Hispanic students. Similar differential effects of motivation on learning outcomes have been found in other studies. For example, Singh et al. (2010) found that self-concept had a significant positive influence on academic effort for Caucasian groups while it was not significantly related to African-American students, although they reported higher self-concept scores.

The current study found that motivation plays an important role in explaining students’ math performance, but differential effects of motivation on math achievement exist across these two groups. This finding could mean that White/Asian students might have had other
opportunities and experiences, likely not available to African-American/Hispanic students, to help them to translate their academic motivation to math achievement. Stevens et al. (2004) pointed out that “Hispanic students may not have the same opportunities enjoyed by Caucasian students to receive verbal persuasion and to view models that encourage participation in mathematics related activities” (p. 219). Cokley (2000) found that African Americans do not lack academic motivation and value academic achievement similar to their white peers. The lower academic performance of African Americans may be due to the fact that generally the educational environment is less favorable for African American students compared to their white peers. Additionally, the reason that White/Asian students are able to translate their motivation into math achievement could be due to other resources in the family and social environment. For example, they probably have more opportunities to learn, attend relatively higher resource schools, and develop better study habits.

This finding about the higher effect of motivation on math achievement for White/Asian students could also be related to response bias. Motivation survey was reported by students and was not reported by teachers: thus, the student motivation reports may contain desirability bias. Students might have over-reported their motivation levels. In sum, for both groups, the higher the motivation, the higher the math achievement. Future studies should explore more reasons for such differential effect of motivation on math achievement.

Implications

In the face of declining math achievement during high school, the findings of the present study have both theoretical and practical implications. The findings support that math learning is a complex process for high school students, which is impacted by social support (from teachers...
and peers) and mediated by students’ motivational beliefs. The current study confirms most of
the hypothesized relationships of this study. It supports the mediation hypothesis that social
support (from teachers and peers) influences math achievement by affecting students’ math
motivation. In other words, motivation acts as a mediator variable in the relationship of social
support factors and math achievement. In addition, this study identified the important role of peer
influence in explaining students’ math motivation and achievement. Besides, the current study
confirms previous findings, such as family SES and prior achievement being strongly related to
student academic achievement, and motivation beliefs being positively associated with students’
math outcomes. Given that the racial achievement gap remains substantially large, the current
study also provides comparisons between high-achieving White/Asian students and low-
achieving African/American students. These comparisons will help contribute insights into
theories regarding the role of social support in adolescents’ math achievement across racial
groups. Besides theoretical implications, several practical implications are presented below.

First, results about the important mediation role of motivation in explaining the
relationships between social support (from teachers and peers) and math achievement lead to
implications for educational practice. It suggests that students’ math motivation can be increased
by quality teacher support and positive peer influence. The current study found that teacher
support had substantial influence on student academic motivation in learning math. Therefore,
policy makers and educators need to continue providing more resources for professional
development of teachers whereby teachers can be provided training and continued pedagogical
support on how to encourage and support different groups of students. Increased professional
opportunities to learn how to be supportive in the classroom will lead to higher motivation to
learn and excel in math. To increase students’ math motivation, teachers could pay more
attention to students’ psychological needs and communication skills that promote motivation beliefs in students. Thus, professional development for teachers should include pedagogy of caring and nurturing students. For instance, by encouraging open communication and approving critical thinking, teachers can help students build confidence. Being enthusiastic in teaching, teachers can increase students’ recognition of the value of math in future educational pursuits, and inspire students to be engaged in learning math. Based on the current study, when teachers treat students equally, make math content easier and interesting, encourage students to think rather than memorize content, and don't let students give up, students will increase math interest and feel confident about their ability in math courses, which in turn will improve students’ math achievement (e.g., Sakiz, Pape, & Hoy, 2012). This finding has practical implications for professional development and for in service education of math teachers.

Secondly, the results of the study show that teacher support seems to be more valuable for low performing and low motivation students, and its effect tended to be more beneficial for minority and low SES students. To support teachers in their efforts, this finding is theoretically important in developing and testing more complex theories and models, but also leads itself to practical implications. More the teachers are supportive, and encouraging the greater the probability of better math learning outcomes. Understanding that students at different levels of academic motivation or achievement are differentially affected by teacher support would be beneficial to students’ math learning outcomes. Based on the findings that perceived teacher support was positively related to low-motivation students, teachers should understand the different developmental needs of their adolescents and develop special tutoring programs to meet different educational needs of adolescents. In addition, policy makers and administrators need to understand that teachers’ support is an effective vehicle for less motivated students for
improving their math achievement. Moreover, this study found that no relationship between family SES and teacher support, which indicates that teacher support can play a protective-stabilizing role in the relationship between SES and academic achievement in students from lower SES. This is another good reason for policy makers to invest in teachers’ professional development for narrowing the achievement gap between high SES and low SES students.

Thirdly, among the three social support resources, peer influence had the largest total effect on students’ math achievement. Considering this finding, policy makers and researchers need to understand the important role of peers and invest in the exploration of peer influence by creating mixed ability groups in math courses. Teachers should encourage students to work together and to form multi-ability groups. Educators and parents should provide assistance in forming peer groups and supporting group interactions as well as develop grouping strategies, such as teaching students social-emotional skills, observing and assisting in solving conflicts, and stepping in and guiding the interactions for students to build positive relationships and work successfully with peers.

In addition, this study shows that parental involvement and peer influence were strongly affected by family SES background, which indicates that policymakers and educators need to design and implement early intervention programs to encourage and promote better math outcomes in lower grades. In sum, this study provides valuable theoretical insights into the relationships among social support factors, math motivation, and math achievement as well as provides some practical implications to educators and policy makers.
Contribution of the Study

The present study makes significant theoretical and practical contributions to the body of knowledge on the role of parental involvement, perceived teacher support, and peer influence on math achievement at the high school level using national representative data. First, in the past, few previous studies have examined the effect of support from parents, teachers, and peers on math achievement in a single study. This research incorporates all these three social factors simultaneously to explore the differences in their influence on adolescents’ math learning motivation and achievement outcomes. Controlling for other relevant factors, such as family SES background and previous math achievement, this study presents a more complex and comprehensive explanation of adolescents’ math achievement and the role of contextual factors on math learning. More specifically, it produces a deeper understanding of how perceived teacher support and peer influence were associated with adolescents’ math achievement in high school years.

Secondly, building on previous research, this study extends the understanding about the processes that explain links between social support and math achievement, which was not thoroughly examined in previous research, supporting a mediation hypothesis. This study presents a dynamic picture of the process of how motivation can mediate social support factors and high school students’ math achievement. It provides a deeper understanding of how social support agents influence high school students’ math outcomes through promoting students’ academic motivation. It confirms that motivation plays an important role in explaining links between social support (from teachers and peers) and math learning outcomes, which can benefit policy makers and teachers to make more effective strategies and programs to nurture and promote students’ academic motivation.
Thirdly, this study presents a more complex view of interpreting the different effects of three important social support sources on math achievement and motivation between White/Asian and African-American/Hispanic groups. The findings provide deeper insight into achievement gaps by highlighting the different relationships among these constructs in the two groups, which can help in developing and implementing strategies to reduce the achievement gap of African American and Hispanic students. Finally, this research attempts to fill the gap in the current literature on how various social support resources impact older adolescents’ math achievement, since most of the previous research on social factors has been conducted in elementary and middle schools, but much less is known about high school classrooms.

Further, this study contributes to the literature by identifying the mediating role of math motivation in the relationship between social support (from teachers and peers) and mathematics achievement. It also adds to the body of literature on social support factors, as well as supports Vygotsky’s sociocultural and expectancy-value theories by providing empirical evidence in their support. Moreover, by using the HSLS: 09 data, which is one of the most recent nationally representative data set, the findings of the study have significant implications for practice as mentioned above and provide valid insights in student achievement patterns.

**Limitations of the Study**

Although this study makes significant theoretical and practical contributions to existing knowledge in the role of teachers and peers in adolescents’ math achievement, there are some limitations in the study. First, the current research has a cross-sectional design, which presents a snapshot in time. A strong analysis of causal assumptions can’t be made by cross-sectional data. Cross-sectional data provides correlational relationships. Therefore, one depends on previous
theories for hypothesizing causal relationships and it is hard to determine the direction of influence using cross-sectional data.

Secondly, most of the variables in the current study were measured using self-report data (except for math academic achievement). Self-report measures may contain social desirability bias. Participants tend to answer questions with intention of doing so in a socially acceptable manner, which affects the validity of the measures. For example, motivation was reported by students rather than asking teachers to report on students’ motivation. This report of perceived motivation may not be very accurate, since participants use their own estimation and values, which can lead to underestimated or overestimated factors. Similarly, peer performance was also reported by participants, which led to many missing values since participants could not remember or did not know their friends’ grades or future plans. Furthermore, parental involvement was reported by parents, while motivation beliefs, teachers and peer items were reported by students. The associations between factors that are reported by different participant groups often are not as strong as the associations between the measures from the same participant group.

Thirdly, due to the use of extant dataset, the measures are chosen from the available items, they may not reflect a comprehensive conceptualization of a latent factor. For example, the math motivation items capture certain aspects of motivation and do not reflect behavioral aspects of motivation, such as effort and engagement in math learning.

Directions for Future Research

This study provides ideas for future research. First, in later waves of HSLS:09 data, longitudinal studies could be used to clarify the causal pathways and long-term influence of
latent constructs. In the current study, all variables are measured at the same time period, so the direction of influence was determined by previous theory only. Moreover, students’ feelings and their contextual factors may change with time. Moreover, the effect of parental involvement, perceived teacher support, and peer influence may take time to become influential on students’ math motivation and achievement. Therefore, a longitudinal design for future studies will be appropriate for examining the developmental effect of social support on student academic motivation and achievement. Such design will strengthen the validity of the hypothesized relations and claims of causal relationships.

Secondly, in the current study, it was found that perceived teacher support had a direct negative relationship, although very small, on students at middle and higher motivation levels, while it was positively related to students with low motivation levels. Although the current study has discussed and explored some putative reasons, qualitative (e.g., interview, observations) research and more research should be conducted to gain deeper insights into such relationships. Qualitative research could further help to explain the non-significant relationships between parental involvement and math achievement during adolescence. Results from qualitative research studies can provide more specific strategies to meet students’ needs and concerns, which would assist policy makers, parents, and educators.

Thirdly, the relationships among different aspects of social support (from parents, teachers, and students) and adolescents’ math achievement should be explored further. For instance, the influence of home-based parental involvement is non-significant in this study. Future researchers could examine the effect of school-based parental involvement on math achievement and motivation in order to identify which type of parental involvement is more
effective for high school students’ academic achievement and motivation. In addition, different samples and different groups based on SES, language, and ethnicity can be used to deliver more useful information for policy makers.

Fourthly, different analysis methods could be employed to gain more comprehensive understanding of the postulated relationships. For example, hierarchical linear modeling method could be used to explore the hierarchical structures, which can be helpful for drawing out the causal inferences. The HSLS:09 dataset has a nested structure in which student math outcome was measured at the individual level as well as school level (i.e. school type, math activities). Therefore, a hierarchical linear modeling study could be useful to explore how school effects contribute towards explaining math achievement differences.

Lastly, different components of motivation beliefs could be explored in future research. Based on expectancy-value theory (Eccles, 1983), motivation beliefs include five components: expectancies for success, attainment value, intrinsic or interest value, utility value, and cost. These motivation components are all related to math achievement, but, at same time, they may have their own specific influence which can be modeled and tested in further studies. Therefore, future research could explore the relationship between different components of motivation and math outcomes to understand the relationship between motivation and academic outcomes from multidimensional perspective.

Conclusions

Math education is the cornerstone of nationally economic progress and is specifically critical for knowledge economy. Given the fact that a large number of U.S. high school students are not mathematically proficient (NCES, 2013), there is a strong need to explore how to
improve high school students’ math achievement. The present study examines how support from parents, teachers, and peers impact adolescents’ math achievement directly and indirectly through math motivation. The results reveal that teacher support played an important role in influencing students in math motivation, which in turn improved students’ math achievement. Peer influence also affected directly and indirectly students’ math achievement. Overall, this study explored the relationships among these constructs, highlighted the important effects of teachers and peers during adolescence, and examined important relationships that vary across White/Asian and African-American/Hispanic student groups. Therefore, this study contributes important knowledge for educators and policy makers to develop early interventions, create strategies and policies, and allocate resources to improve the success in math and narrow the achievement gap.


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