Introduction

"*The fundamental law of investing is the uncertainty of the future*[1]."

Since the advent of modern markets, investors have sought determine the true value of assets. The benefits of determining an asset's "true price" are three-fold; firstly, an investor understands that all assets have a risk-reward tradeoff. This principle implies that an investor can determine the risk level associated with an asset solely from its return. By understanding an asset's "true price" an investor can eliminate any extraneous information that would result in the mispricing of the asset. Once this occurs, the investor is free to select assets based on the risk level that they represent, selecting a portfolio of assets that are more representative of their desired risk levels than if the "true price" was unknown. If assets were ranked according to their risk, investors can more easily select assets that have high, moderate and low levels of risk associated with them. Secondly, investors can limit the exposure of unwanted risk associated with unexpected changes in asset prices. If, for example, an investor could determine that an asset's price becomes more volatile in a volatile market, they could adjust their behavior to protect themselves from unexpected changes by selecting assets that are less volatile in this kind of market environment. Thirdly, investors can also benefit from understanding the true value of assets by taking advantage of asset mispricing. If an asset is undervalued for example, investors could profit by purchasing the asset and holding it until the market adjusts the price to its equilibrium level.

The first to introduce a tool that allowed investors to approximate an asset's value was Sharpe (1964), Lintner (1965) and Black (1972) when they published the Capital Asset Pricing Model (CAPM). It was designed to approximate the *expected* return of an asset given its volatility as it related to the market (as measured by the beta coefficient), the return of the market premium and the return of a risk-free asset. Roll (1977) was the first and more recently Fama & French (1992, 1993 & 1996) to criticize the CAPM and have shown that the model is not as accurate as the original authors believed. They found that the relation between average stock returns and beta is weak[2]. A possible explanation for the inaccuracy of the CAPM is that the data that was used to generate and test the model in the first place was limited. More recent tests have used more recent data that was unavailable at the time of the CAPM was initial tested. In any case, the CAPM has generated substantial debate since its introduction as to the best means to recalibrate or refine the model to best explain asset returns. The debate over alternative models and extensions to the basic CAPM model can be broken into two schools: the multi-variable CAPM proponents and the non-stationary beta proponents. Both methods have provided models that have been successful for asset valuation, however these models have had limited success when testing them on other market sectors or evaluating the same problem with new data. The underlying question that needs to be addressed when evaluating the CAPM is: Can investors more reasonably measure expected returns of assets, thereby reducing their risk associated with unexpected changes in asset prices?

To answer this question, the relationship between expected returns and risk must be defined. In turn, the methods that are currently employed to take into account impacts of the market environment through the inclusion of various macro-economic variables and their impact on asset returns will be discussed. In addition, methods used to account for asset-specific characteristics will be evaluated. Finally, the risk coefficient's role as defined by beta and relevance to the CAPM will be evaluated.

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[1] Peter L. Bernstein, Expectations Investing: Reading Stock Prices for Better Returns, p.xiii.
[2] Leusner, Akhavein and Swamy, "Solving an Empirical Puzzle in the Capital Asset Pricing Model," Research in Finance, p.72.

Financial Theory

The traditional risk-reward tradeoff (also known as the risk-return tradeoff) is one of the most basic financial principles that investors must account for when determining their best allocation of assets. According to this principle, investors are willing to take risks only if there is the reward of higher returns. This relationship can be expressed in functional form as the following:

(1) E(R) = Risk Free Rate of Return (r_f) + Risk Premium

The variables used in this equation are: E(R), the expected return of the asset; the observed risk-free rate of return (r_f, usually the 3-month Treasury bill is used as a proxy for analysis); and the risk premium, the additional return over the risk-free rate that is associated with the risk-return tradeoff principle. The downside of this relationship is that the risk premium as defined by the equation is subjective and therefore unique to the individual performing the analysis. In other words, how much the reward should be in relation to the risk undertaken is not concretely defined, it will be different for each individual.

The Capital Asset Pricing Model (CAPM) is a further advancement in understanding the risk-reward tradeoff relationship. It redefines the traditional representation of the risk-reward tradeoff to include a scalar coefficient that adjusts for the variability of an asset's price. In effect, the risk premium is adjusted to reflect the overall riskiness of the asset as it relates to the market. This scalar coefficient as defined by the CAPM is the beta coefficient. The CAPM also attempts to better quantify the risk-reward tradeoff by associating observed variables when defining the risk premium and the beta coefficient. In doing so the model more effectively defines the risk of an asset and eliminates some of the subjectivity that was necessary in previous models. This model for the first time allowed non-experts who were not intimate with the details of an asset to get a close estimate of the potential asset return. Refining the risk-return relationship to include the extra coefficients, the CAPM is written:

(2) $E(r_{it}) = r_f + \beta_t E(r_{Mt})$

The CAPM estimates the expected return of an asset $E(r_{it})$ given the observed risk-free rate of return, r_f, and the expected risk premium, $E(r_{Mt})$, scaled by the beta coefficient, β_t. The risk premium is defined as the difference between the expected market return and the risk-free rate of return. The beta coefficient is a representation of the relationship between the asset and the market. Given that the CAPM has been criticized for its accuracy of representing asset returns, it would be worthwhile to further investigate this variable since it has the greatest impact on the accuracy of the model.

Beta is also known as systematic, portfolio or market risk. It is the amount of risk that cannot be eliminated with diversification. This means it is any risk that affects a large number of assets, each to a greater or lesser degree. A change in the unemployment rate is an example of systematic risk in that all assets have the potential of somehow being affected by this external change.

The beta coefficient measures the asset's responsiveness to broad swings in the market.[3] If an asset's price tends to move up or down twice as much as the market (whose beta is equal to one), the asset's beta is equal to two. If an asset's price moves one half as much as the market, it has a beta of 0.5. The beta coefficient has been the variable most associated with the risk-reward concept of investing – the higher the beta, the greater the risk of the asset, the higher the return the asset must provide to reward investors for the greater risk.

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[3] Investments, p. 930.

The beta coefficient is the one variable that is computed based on historical information, the rest of the variables are based on current observations or expected returns. It is also the one variable that changes little over time and therefore is sometimes assumed to be a fixed variable. Beta is computed by using the equation:

$$(2) \qquad \beta_i = \frac{\operatorname{cov}(r_i, r_M)}{\sigma^2(r_M)}$$

$$\operatorname{cov}(r_i, r_M) = \frac{(R_i - \overline{R}_i)(R_M - \overline{R}_M)}{N}$$

$$\sigma^2(r_M) = \frac{\sum (r_M - \bar{r}_M)^2}{N}$$

where $\operatorname{cov}(r_i, r_M)$ is the covariance between the asset *i* and the return on the market portfolio and N is the number of observations. The variable $\sigma^2(r_M)$ is the variance of the market with N observations.

The CAPM relies very heavily on the beta coefficient. It uses the 60 most recent monthly observations[4] for its derivation and therefore is static in nature in that any large variations will have very little influence on the data set as a whole. It implicitly assumes that the relationship between the asset and the market will remain relatively stable or constant over time. In a market environment that is continuously changing, this stable relationship can be construed as a very big weakness of the CAPM.

[4] Investments, p.293.

Review of Literature

Initial tests of the CAPM that were performed by Black, Jensen and Scholes (1972) found that the data was consistent with the predictions of the CAPM. They used the returns for all the stocks on the NYSE during 1931-1965 to form 10 portfolios with different historical beta estimates. This data showed that the average return on a portfolio of stocks was positively related to the beta of the portfolio.[5]

Pettengill, Sundarum and Mathur (1995) argue that the evidence that is critical of the CAPM is based on observed rather than expected returns. They argue that it is necessary to adjust the model to use realized returns instead of the expected returns that the CAPM was intended to estimate. To test the relationship between portfolio betas and returns the model is modified to account for the conditional relationship between beat and realized returns. They make this adjustment by estimating two betas: the beta during periods of positive market premiums (when the market return is above the risk-free rate of return) and betas during periods of negative market premiums (when the market return is below the risk-free rate of return). They found that the beta estimates that are generated through this technique are highly significant at explaining the relationship between beta and returns in each sub-period they examined.

Over the following years a vast amount of research was conducted to refine the model. This included work by Chen, Roll and Ross in 1986 when they developed a multi-factor model that included macro-variables. This model allowed for the unsystematic risk that remained when assets were diversified. A beta coefficient was calculated for each of the macro-variables to determine the level of risk associated with each variable. If it was determined through the estimation of the model that a variable had a high beta coefficient, it was said to add to the riskiness of the asset. This technique allowed investors to more effectively ascertain the different risks associated with each asset as well as to associate returns relative to the asset's business cycle. The danger of this

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[5] Corporate Finance, p. 269.

technique was that it might identify external elements that could have a negative of positive impact on an asset's return. In practice, however the causal relationship that this model attempted to identify was less clear and therefore the asset's return was still not as predictable as one would have hoped.

As with the CAPM, the multi-variable model that Chen, Roll and Ross developed was placed under scrutiny as other experts in the field tested their model. Hamao (1988) applied the model to the Japanese market and found strong pricing evidence as suggested by the multi-variable model, with the exception that industrial production did not perform as strongly as expected. Poon and Taylor (1991) applied the model to UK market data and found no clear evidence that stock returns were explained by macro-variables. Martinez and Rubio (1989) also found that there was no significant pricing relationship between returns and the macro-variables when applied to the Spanish market.

In an attempt to identify the factors driving stock returns Chan, Karceski and Lakonishok (1998) have taken the Chen, Roll and Ross model a step further by including eight macro-variables. Their goal was to provide some guidance to other researchers and practitioners who want to use factor models by developing a parsimonious set of observable variables that do a good job in capturing the systematic risk. They found that the macroeconomic variables that included in their model did a very poor of explaining portfolio returns.

Kavussanos and Marcoulis (2000) adapted the Chen, Roll and Ross model by adding oil prices and consumption. They applied their new model to the transportation sector including air, rail, trucks and water transportation. In order to provide context for their findings, they also examined the electric, gas petroleum refining and real estate sectors. They found that the other macro-economic variables that were included in the model did have an impact on each of the sectors that they examined to one degree or another. Perhaps the most important finding was that the extent that each of the macro-variables varied for each of the sectors. The market, as defined by the beta coefficient, was not the only variable to influence asset returns.

Another attempt to build more information into the CAPM was proposed by Fama and French in 1993. They first looked at the beta coefficient to determine the effectiveness of the beta coefficient to characterize the relationship between the asset and the market and its impact on the asset's return. They found that during 1941 to 1990 the relationship between beta and average returns was weak. Furthermore, they found that during 1963 to 1990 the relationship was virtually nonexistent. They hypothesized that adding more information about the asset itself was necessary and proposed using characteristics such as price-to-earnings ratios and market-to-book ratios to further identify attributes that were unique to individual assets. In fact, their tests seemed to support their claims, however further analysis of their model could not find such strong evidence.

Howsten and Preston (1998) used the same procedures as Fama and French, including a sorting technique in constructing portfolios. The major difference in their approach was that instead of using the constant-risk model they instead use a dual-beta model that changes the beta of the security depending on the direction of the market. They start by estimating the security's beta when it is in a bull, bear and constant market and apply this beta during the corresponding time period. In this sense, they are using a technique that changes the beta with market conditions. Their research has found that the coefficients when using a bull-market beta are significant and those using a bear-market beta are significant with the exception of January when size was significant. They also found that the book-to-market equity variable was significant in a bear-market and size for the whole year with the exception of January.

Until recently the improvements to the CAPM that were proposed were to explain the relationship that assets have to market characteristics through the inclusion of additional variables. It was not until recently that the relationship of the asset to the market, as defined by the beta coefficient, was called into question. Since the CAPM relies so heavily on a single variable to represent an asset's relation to the market and implies that this relationship must be linear in form, it would be prudent to investigate another functional form given that the market is a continuously changing environment in which to operate. In fact, Swamy in 1995 continued with the findings that Granger (1993) published that suggested that a time-varying parameter model might provide an adequate approximation to non-linear relationships. Swamy has found that the relation between the observable returns on stock and market portfolios is non-linear. It is this change to the functional form of the CAPM that I investigate in this paper.

Chen & Keown (1981) point out that since prior studies have concluded that beta is nonstationary and that estimating the CAPM with an OLS estimation procedure could lead to misleading results. The OLS procedure assumes that the functional form of the CAPM is linear and would be appropriate if the beta coefficient were stationary. If beta is nonstationary the estimate of systematic risk is inaccurate and in turn causes the mismeasurement to be reflected in the estimate of unsystematic risk. They argue that allowing for the nonstationarity of beta results in better estimate of the corresponding risk elements. They also found beta nonstationarity accounts for thirty-three percent of a security's variability while pure residual risk accounts for only nineteen percent. They go on to propose that the impacts of diversification can be seen by these risk values since the measure of beta nonstationarity falls to eight percent and the measure of pure residual risk falls to four percent.

Episcopos (1996) applies a time-varying beta model to the Canadian market and found evidence that the spread between betas of safe and risky sub-index portfolios may increase during periods of increased aggregate volatility. The difference between time varying and constant betas can be significant, however they could find no clear relation between this discrepancy and portfolio type.

Fletcher (1997) examined the conditional relationship between beta and returns in the UK market. He found no significant relationship until a sorting technique was used to estimate beta during periods of positive and negative market returns and during periods when the risk premium is either positive or negative. In addition, he also finds that portfolio size is insignificant until accounting for nonlinearity.

Lin and Lin (2000) have used an extension of the CAPM called the ICAPM that has been adapted to evaluate individual countries within the global market. The ICAPM uses a beta that describes the relation that each country's stock market return have relative to the international market return. They formally define the beta coefficient for a country's capital market as the ratio of the covariance between the expected excess return on the country's market (i.e. the expected return on the country's stock market in excess of the universal risk-free rate) and the expected excess return on the world market portfolio (i.e. the expected return on the world market portfolio in excess of the universal risk-free rate) to the variance on the expected excess return on the world market portfolio. In other words, the beta coefficient is an index of the systematic risk for a country's stock risk with respect to the world market. They agree with previous research that concludes if the beta coefficient is inherently misspecified by remaining static, there is a real possibility that serious pricing errors can occur if the dynamics of the beta coefficient is not accounted for. They found that the beta coefficient for each of the eleven countries they examine was both stochastic and dynamic. They conclude that allowing for movements in the beta increases the accuracy of stock return forecasting in the global capital markets.

Koutmos, Lee and Theodossiou (1994) examine time-varying behavior and volatility persistence of stock market returns for ten industrialized countries including Australia, Belgium, Canada, France, Germany, Italy, Japan, Switzerland, the UK and the US. They chose to incorporate a time-varying beta coefficient into their model. They cite previous study findings that indicate that the performance of international capital pricing models improves substantially when the conditional variance of stock returns is not constrained to remain constant over time. In using this time-varying beta estimation technique they find that markets with higher volatility persistence tend to have non-diversifiable risk during periods of high world market volatility. In other words, countries with lengthy past volatility have a larger probability of lasting future volatility.

Materials and Methods

The returns that were used included nine stocks that were classified by Charles Schwab & Company as oil-related and collected from Yahoo! Finance. These stocks included: Alberta Energy Company (AOG), Amerada Hess Corporation (AHC), British Petroleum Company, PLC (BP), Chevron Texaco Corporation (CVX), Conoco Inc. (XOM), Imperial Oil Limited (IMO), Petro-Canada (PCZ), Royal Dutch Petroleum Company (RD), and Shell Transport & Trading (SC). These stocks were equally represented in a single portfolio (r_i) from September 1995 to October 2001. The portfolio returns together with the market returns are shown in Chart 1.

The first tests of the model were performed with monthly data from five macro-economic variables including: the Producer Price Index (PPI), the Consumer Price Index (CPI), the Unemployment Rate, Industrial Production and the Consumer Confidence Index (CCI). This data is shown in Charts 2 and 3 together with the mean and median, located in Table 3. As seen in Table 3, CPI and Industrial production both have the highest averages, but it is the Market Premium with the highest median value.

A second estimation was performed with the addition of a Gross Domestic Product (GDP) variable. The data was also changed to a quarterly-basis to reflect the availability of GDP data. In an attempt to reduce the number of variables in the model, the Industrial Production and Consumer Confidence Index variables were eliminated from the estimations. This data is shown in Charts 4 and 5 together with the mean and median, located in Table 7. As seen in Table 7, all of the variables have a more significant mean and median value, with the Market Premium remaining the highest.

Chart 6 includes all variables on the same chart, with the exception of the CCI and Industrial Production variables. These variables were excluded as part of the model refinement process. The chart shows that the variables have similar movements for some periods while significantly different movements for other periods.

The Empirical Model: New Multivariable Model with Non-Stationary Beta

In order to incorporate the ideas on the impacts of multivariable coefficient model that Chen (1996) proposed and the changes in the functional form of the model Swamy (1995) presented, we first need to express the model in equation form. Substituting Yt for (r_{it} - r_{ft}), Xt for (r_{Mt} - r_{ft}), and adding an intercept term (β_o), the CAPM equation can be re-written:

(3) $$Y_t = \beta_o + \beta_1 X_t + e_t$$

Arguably the greatest weakness of the traditional CAPM model is that the beta coefficient (β_1) that represents the variability of an asset's return over time is not able to change very much to reflect current market conditions. In order to allow the beta to change over time, or become time varying or non-static in nature, a new time subscript (*t*) has been added. This is the most significant change to the CAPM model, even more important than the addition of macro-variables or the choice of variables to use in the model. The equation that follows builds in the time-varying nature of the betas as well as the incorporation of the macro-variables.

(4) $$Y_t = \beta_{ot} + \beta_{1t} X_t + e_t$$

where $$\beta_{ot} = a_{00} + a_{01}Z_{1t} + a_{02}Z_{2t} + a_{03}Z_{3t} + a_{04}Z_{4t} + a_{05}Z_{5t}$$

and $$\beta_{1t} = a_{10} + a_{11}Z_{1t} + a_{12}Z_{2t} + a_{13}Z_{3t} + a_{14}Z_{4t} + a_{15}Z_{5t}$$

All of the macro-variables have been added into the model as shown by the Zs both as extra variables similar to a multi-variable CAPM model through the β_{ot} equation and as the influencers to the traditional CAPM beta coefficient as represented by the Zs in the β_{1t} equation. It is worth noting at this point that the betas have become nonstationary as reflected by the addition of the time subscript (*t*) on the beta coefficients.

Rewriting (4)

$$(5) \qquad Y_t = [a_{00} + a_{01}Z_{1t} + a_{02}Z_{2t} + a_{03}Z_{3t} + a_{04}Z_{4t} + a_{05}Z_{5t}] + [a_{10} + a_{11}Z_{1t} + a_{12}Z_{2t} + a_{13}Z_{3t} + a_{14}Z_{4t} + a_{15}Z_{5t}]\, X_t + e_t$$

Simplifying (5)

$$(6) \qquad Y_t = [a_{00} + a_{01}Z_{1t} + a_{02}Z_{2t} + a_{03}Z_{3t} + a_{04}Z_{4t} + a_{05}Z_{5t}] + a_{10}X_t + a_{11}Z_{1t}X_t + a_{12}Z_{2t}X_t + a_{13}Z_{3t}X_t + a_{14}Z_{4t}X_t + a_{15}Z_{5t}X_t + e_t$$

Separating the coefficient terms

$$(7) \qquad Y_t = a_{00} + a_{01}Z_{1t} + a_{02}Z_{2t} + a_{03}Z_{3t} + a_{04}Z_{4t} + a_{05}Z_{5t} + a_{10}X_t + a_{11}(Z_{1t}X_t) + a_{12}(Z_{2t}X_t) + a_{13}(Z_{3t}X_t) + a_{14}(Z_{4t}X_t) + a_{15}(Z_{5t}X_t) + e_t$$

Replacing Y_t and X_t with $(r_{it} - r_{ft})$ and $(r_{Mt} - r_{ft})$ yields:

$$(8) \qquad (r_{it} - r_{ft}) = a_{00} + a_{01}Z_{1t} + a_{02}Z_{2t} + a_{03}Z_{3t} + a_{04}Z_{4t} + a_{05}Z_{5t} + a_{10}(r_{Mt} - r_{ft}) + a_{11}(Z_{1t}(r_{Mt} - r_{ft})) + a_{12}(Z_{2t}(r_{Mt} - r_{ft})) + a_{13}(Z_{3t}(r_{Mt} - r_{ft})) + a_{14}(Z_{4t}(r_{Mt} - r_{ft})) + a_{15}(Z_{5t}(r_{Mt} - r_{ft})) + e_t$$

Replacing the Zs with their corresponding variable name:

$$(9) \qquad (r_{it} - r_{ft}) = a_{00} + a_{01}(\%\Delta PPI) + a_{02}(\%\Delta CPI) + a_{03}(\%\Delta UNEMP) + a_{04}(\%\Delta INDPROD) + a_{05}(\%\Delta CCI) + a_{10}(r_{Mt} - r_{ft}) + a_{11}((\%\Delta PPI)(r_{Mt} - r_{ft})) + a_{12}((\%\Delta CPI)\,(r_{Mt} - r_{ft})) + a_{13}((\%\Delta UNEMP)(r_{Mt} - r_{ft})) + a_{14}((\%\Delta INDPROD)(r_{Mt} - r_{ft})) + a_{15}((\%\Delta CCI)(r_{Mt} - r_{ft})) + e_t$$

Where

$Z_{1t} = \%\Delta PPI$

Z_{2t} = %ΔCPI

Z_{3t} = %ΔUNEMP

Z_{4t} = %ΔINDPROD

Z_{5t} = %ΔCCI

The objective of the new macro-variable model as presented above is to determine if the macro-variables have any impact on asset returns, now included with the β_{ot} variable, or the relationship between asset returns and the market as defined by the traditional CAPM beta coefficient, now defined with the β_{1t}. This model asserts that if the relationship between the market and asset returns is as defined by the traditional CAPM beta, allowing it to be non-stationary, then the model will be more accurate than other models that do not allow for beta nonstationarity.

The effectiveness of the new macro-variable will be measured not only with the strength of the variables included, but the interaction of the two beta coefficients. If the two are not equal to zero or each other (as shown in the equations below), then the macro-variables that have been included in the model do have impacts on both asset returns and the relationship between asset returns and the market. In other words, the model has benefited by including the macro-variables and allowing for nonstationarity of beta.

(10) $\beta_{ot} = a_{00} + a_{01}Z_{1t} + a_{02}Z_{2t} + a_{03}Z_{3t} + a_{04}Z_{4t} + a_{05}Z_{5t} + a_{06}Z_{6t} \neq 0,$

$\beta_{1t} = a_{10} + a_{11}Z_{1t} + a_{12}Z_{2t} + a_{13}Z_{3t} + a_{14}Z_{4t} + a_{15}Z_{5t} + a_{16}Z_{6t} \neq 0$

and $\beta_{ot} \neq \beta_{1t}$.

Empirical Results

The first round of regressions for the multi-variable model was estimated using samples that included the both the entire sample of monthly observations and a rolling two-year monthly sample of observations that created a total of six data sets for the analysis. The reason for a rolling 2-year sample was to determine if any of the variables had a changing roll in the explanation of the portfolio returns. In other words, the objective of the analysis was to determine if any of the macro-variables had a greater influence on returns for one sub-period and a weaker influence in another sub-period. The estimation technique that was employed for the analysis of the multi-variable model was an OLS estimation technique.

The initially tests of the model found, as reported in Table 5, indicated that most of the variables were insignificant at the 5% level as measured by the t-statistics (a t-statistic greater than 2 indicates that the variable is statistically significant) and if they were significant for one period, they were not significant for the other periods. The t-statistics are shown in parenthesis directly below each coefficient's estimate. In the first estimation, for example, all of the variables are insignificant at the 5% level while in the second estimation the Industrial Production variable is significant. In the third estimation the market premium variable (r_{Mt}-r_{ft}) is the only variable that is significant. The fourth and fifth estimations show the same as the first estimation with the entire data set – none of the variables are significant. This changes with the sixth estimation as the market premium times the Consumer Price Index and the market premium times the unemployment variable are both significant at the 5% level.

In addition to the varying strengths of each variable, the most interesting finding of these estimations was the changes in the coefficient signs through the different samples. CCI for example was the only variable to retain a positive and consistent sign throughout each sample. The Industrial Production variable kept its negative sign until the last two estimations.

In order to test if the weakness of the model was due to the exclusion of variables, a new variable was introduced to include Gross Domestic Product (GDP). Given that GDP is released on a quarterly basis, all the variables were adjusted to reflect quarterly returns. This new version of the macro-variable CAPM was tested with a second-round estimation. The results of this new estimation can be found in Table 9. As with the sixth estimation in the first-round of estimations above, the market premium times the unemployment variable had the most significance as measured by the t-statistics at the 5% level.

A third-round estimation was performed with the exclusion of the Industrial Production and Consumer Confidence variables. This reduced the number of variables to more manageable levels and set the stage for autocorrelation testing. The results of this estimation are located in Table 13. As can be seen from the table, the results appear to be strong enough to warrant further investigation. Five of the nine variables included in this estimation are significant according to their t-statistic at the 5% level. Given the varying strengths of the variables in each round of estimations above and the strength of this last estimation, a test was conducted to determine if any autocorrelation within the errors existed. If any systematic bias or non-spherical disturbances in the estimation was detected, a Hildreth-Lu search technique could be used to adjust for autocorrelation in the error terms. Such techniques are often used to adjust for autocorrelation when using time-series data[6].

The first step in testing for autocorrelation was to graph each of the error terms against the independent variable. As shown in the graph in Exhibit 14, there appears to be a pattern in the residuals. Given that this is the case, further review of the data was necessary. This is achieved with a Durbin-Watson test for autocorrelation. As shown in Table 15, the Durbin-Watson test resulted in a 1.993 test statistic. The upper and lower bounds for a 1% and 5% level of significance are also listed in this table. The Durbin-

Watson Test statistic results in a "no conclusion" at a 5% level of significance and a "do not reject" at a 1% level of significance. Given the strength of this measure, no additional testing or adjustment for autocorrelation is necessary.

The objective of the new multi-variable CAPM model with beta nonstationarity was to determine if a more effective model could be developed if the characteristics of the beta coefficient were changed, as previously defined by the CAPM, from a static variable to one that was allowed "float" to reflect market conditions. To determine if this holds true for this new model, a final check was preformed with the estimation results to determine if the non-stationary beta coefficient actually adds to the model. To review Equation 10,

if $\beta_{ot} = a_{00} + a_{01}Z_{1t} + a_{02}Z_{2t} + a_{03}Z_{3t} + a_{04}Z_{4t} + a_{05}Z_{5t} + a_{06}Z_{6t} \neq 0,$

$\beta_{1t} = a_{10} + a_{11}Z_{1t} + a_{12}Z_{2t} + a_{13}Z_{3t} + a_{14}Z_{4t} + a_{15}Z_{5t} + a_{16}Z_{6t} \neq 0$

and $\beta_{ot} \neq \beta_{1t}$,

then the independent variable is better explained with the incorporation of the non-stationary beta. In other words, if the two betas are both not zero and are not equal then the inclusion of the non-stationary beta adds to the strength of the new model. The results of the estimation that includes the autocorrelation adjustment indicates that $\beta_{ot} = -176.45$ and $\beta_{1t} = -880.23$. Since both beta coefficients are not equal to zero and are not equal to one another the independent variable has been explained more effectively with the inclusion of the non-stationary beta coefficient.

The signs for each of the estimated coefficients for each of the included variables yield some surprising results. The Producer Price Index variable appears to be in line with what was expected with a positive sign. The coefficients on the Consumer Price Index, Unemployment and Gross Domestic Product variables however show that these variables move in opposite directions to the oil industry. It was expected that these coefficients

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[6] Kennedy, p.122.

would have positive signs. Prior to the estimations, I would have thought that each of these variables would have had a positive sign, especially for the GDP variable. I expected that as the economy, as measured by GDP, experienced positive return the same would have occurred for the oil industry. A possible explanation could be that as GDP declines, more energy is expended to try to improve or change the direction of the economy.

Conclusions

The work that Fama and French have done on this topic has been similar to what has been presented here. The exception, however is that this new model included macro-variables to mimic the market environment in which stocks operate and Fama and French have included variables that are stock specific. For example, they have included variables like size (stock price times the number of shares), leverage, earnings/price (E/P) and book-to-market equity (the ratio of the book value of a firm's common stock to its market value). The common characteristic between both models is that the beta coefficient is allowed to change over the duration of the sample. As seen in equation 4, this is reflected in the time subscript on the beta coefficient.

Despite the differences in the Fama & French and this new model, the point of both is that more information can be included into the models to more accurately represent the returns of stocks and portfolios. I have gone a step further, illustrating that a nonstationary multi-variable CAPM model is more beneficial during times of economic change. Since the beta coefficient is allowed to change over time, the model is able to represent the returns more effectively. This said, these types of nonstationary multi-variable CAPM models are better than the initial CAPM developed by Lintner and Sharpe in the 1960s.

The results for this new multi-variable CAPM with the non-stationery beta coefficient are consistent with the findings of prior studies. A potential avenue for future research would be to apply this model with other available data or apply it to other market sectors.

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Table 1: Data Sources

Abbreviation	Source
PPI	U.S. Department of Labor, Bureau of Labor Statistics
CPI	U.S. Department of Labor, Bureau of Labor Statistics
UNEMPLOY	U.S. Department of Labor, Bureau of Labor Statistics
IndProd	Federal Reserve Board, Washington, D.C.
CCI	The Conference Board, Inc.
GDP	U.S. Department of Commerce, Bureau of Economic Analysis
r_{Mt}	Charles Schwab & Co.
r_{ft}	Federal Reserve Board of Governors: Three-Month Treasury Bill Rate, Secondary Market
r_{it}	Yahoo.com

Table 2: Definition of Variables

Abbreviation	Variable Name	Variable Definition
PPI	Z_{1t}	Percent change in the monthly Producer Price Index
CPI	Z_{2t}	Percent change in the monthly Consumer Price Index
UNEMPLOY	Z_{3t}	Percent change in the monthly Unemployment Rate
IndProd	Z_{4t}	Percent change in the monthly Industrial Production
CCI	Z_{5t}	Percent change in the monthly Consumer Confidence Index
$(r_{Mt}-r_{ft})$	X_t	Percent change in the monthly Market Premium
$(r_{Mt}-r_{ft})_t$*PPI	X_t*Z_{1t}	Percent change in the monthly Market Premium* Percent change in the monthly Producer Price Index
$(r_{Mt}-r_{ft})$*CPI	X_t*Z_{2t}	Percent change in the monthly Market Premium* Percent change in the monthly Consumer Price Index
$(r_{Mt}-r_{ft})$* UNEMPLOY	X_t*Z_{3t}	Percent change in the monthly Market Premium* Percent change in the monthly Unemployment Rate
$(r_{Mt}-r_{ft})$*IndProd	X_t*Z_{4t}	Percent change in the monthly Market Premium* Percent change in the monthly Industrial Production
$(r_{Mt}-r_{ft})$*CCI	X_t*Z_{5t}	Percent change in the monthly Market Premium* Percent change in the monthly Consumer Confidence Index

Table 3: Data Sample Statistics: First Round (Percent Change)

Variable	*Mean*	*Median*
Z_{1t}	0.06%	0.08%
Z_{2t}	0.20%	0.19%
Z_{3t}	0.12%	-0.01%
Z_{4t}	0.24%	0.33%
Z_{5t}	-0.05%	0.20%
X_t	0.55%	1.04%
X_t*Z_{1t}	0.00%	0.00%
X_t*Z_{2t}	0.00%	0.00%
X_t*Z_{3t}	0.00%	0.00%
X_t*Z_{4t}	0.00%	0.00%
X_t*Z_{5t}	0.02%	0.01%

Chart 1: Observed Market and Portfolio Returns (monthly)

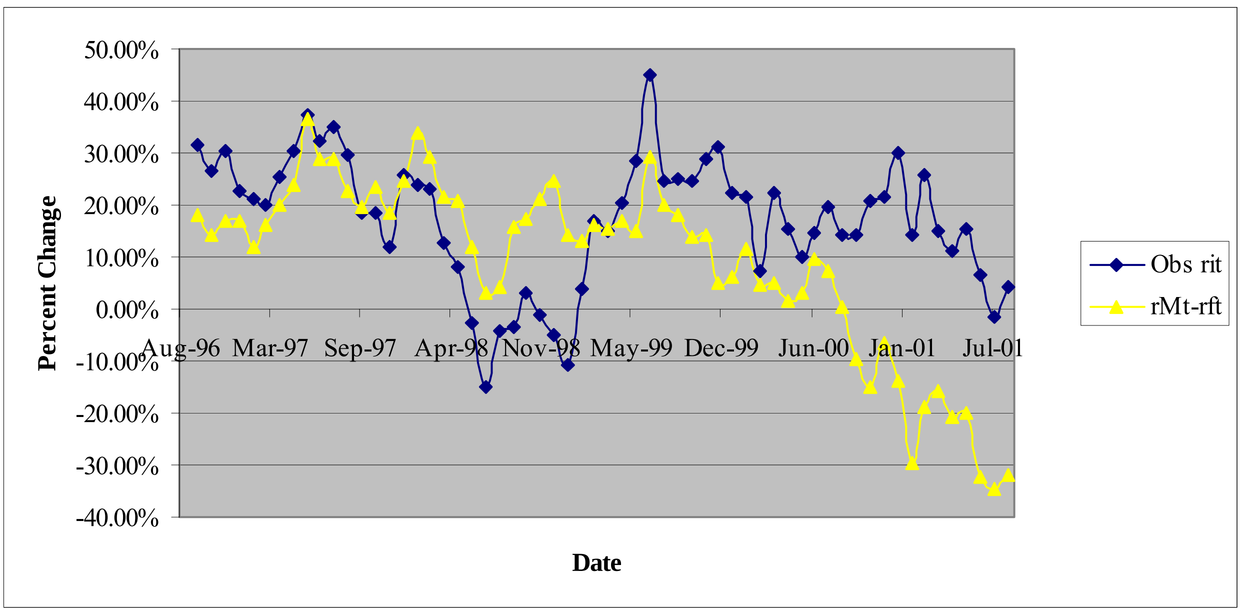

Chart 2: CPI and Industrial Production (monthly changes)

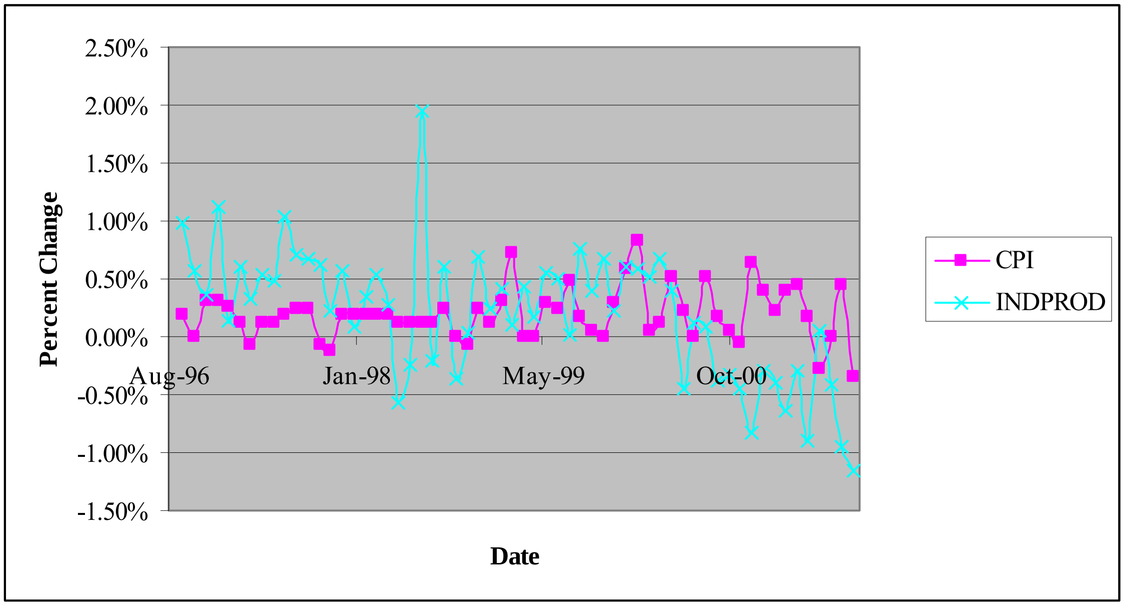

Chart 3: PPI, Unemployment and CCI (monthly changes)

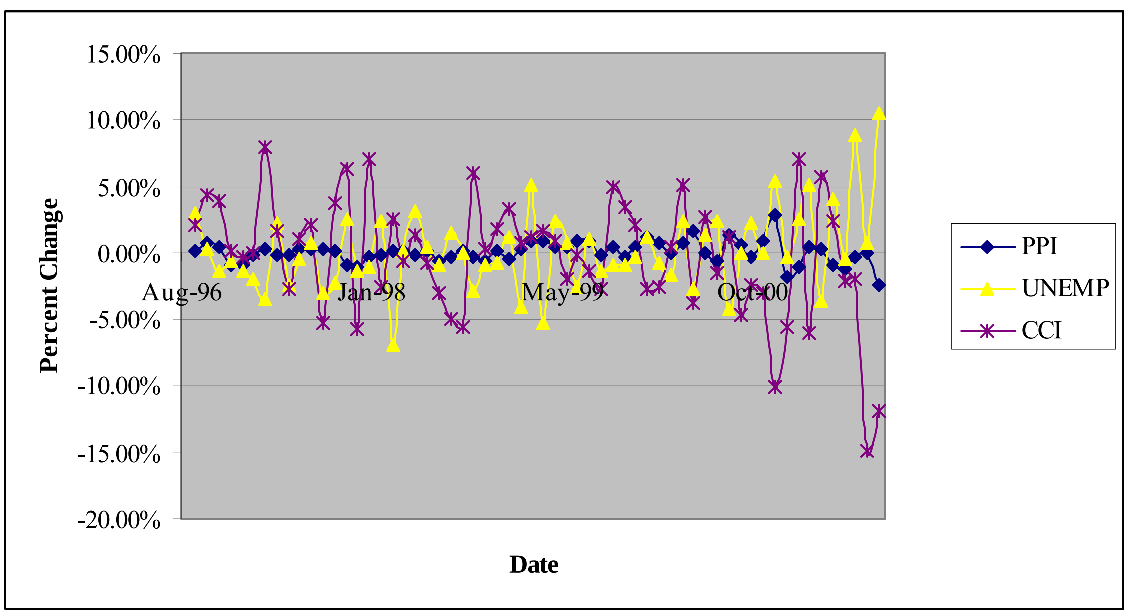

Table 4: Sample Time Period for each Estimation

Regression Number	***Sample Time Period***	***Number of Observations***
Regression 1	October 1995-September 2001	72 observations
Regression 2	October 1995-September 1997	24 observations
Regression 3	October 1996-September 1998	24 observations
Regression 4	October 1997-September 1999	24 observations
Regression 5	October 1998-September 2000	24 observations
Regression 6	October 1999-September 2001	24 observations

Table 5: OLS Regression Results: First Round

Independent Variable Name	Regression 1	Regression 2	Regression 3	Regression 4	Regression 5	Regression 6
Intercept	0.01	0.03	-0.02	-0.03	-0.02	0.01
	(0.88)	*(2.29)*	*(-0.63)*	*(-0.74)*	*(-0.74)*	*(0.62)*
PPI	-0.40	2.10	1.45	0.02	-0.78	0.14
	(-0.42)	*(1.47)*	*(0.53)*	*(0.01)*	*(-0.20)*	*(0.08)*
CPI	0.68	-0.75	13.97	20.20	9.59	-5.72
	(0.19)	*(-0.18)*	*(0.82)*	*(1.13)*	*(0.88)*	*(-0.87)*
Unemp	0.02	-0.28	-0.52	-0.16	0.15	0.68
	(0.08)	*(-0.85)*	*(-0.62)*	*(-0.17)*	*(0.21)*	*(1.06)*
IndProd	-1.21	-4.59	-2.99	-3.89	1.66	3.87
	(-0.90)	*(2.51)*	*(-1.05)*	*(-0.78)*	*(0.38)*	*(1.28)*
CCI	0.19	0.23	0.51	0.73	0.73	0.33
	(1.37)	*(1.26)*	*(0.95)*	*(0.86)*	*(1.30)*	*(0.81)*
(rMt-rft)	0.23	0.71	1.70	0.88	0.83	-0.56
	(0.97)	*(1.28)*	*(1.99)*	*(0.97)*	*(1.05)*	*(-1.08)*
(rMt-rft)*PPI	4.99	13.16	-11.07	-10.21	-5.31	8.65
	(0.24)	*(0.27)*	*(-0.17)*	*(-0.09)*	*(-0.05)*	*(0.32)*
(rMt-rft)*CPI	127.50	-181.80	-393.70	66.29	107.10	249.40
	(1.68)	*(-1.01)*	*(-0.98)*	*(0.15)*	*(0.48)*	*(2.06)*
(rMt-rft)* Unemp	8.23	15.50	23.83	-4.45	7.97	29.77
	(1.47)	*(1.53)*	*(1.18)*	*(-0.15)*	*(0.48)*	*(2.02)*
(rMt-rft)* IndProd	22.78	27.69	-43.59	-72.60	-125.70	81.95
	(1.22)	*(0.52)*	*(-1.04)*	*(-1.08)*	*(-1.03)*	*(0.95)*
(rMt-rft)* CCI	0.36	-1.30	-7.97	-18.59	-15.09	2.77
	(0.13)	*(-0.20)*	*(-0.85)*	*(-1.14)*	*(-0.90)*	*(0.36)*

Table 6: Definition of Variables (Inclusion of New Variable: GDP)

Abbreviation	Variable Name	Variable Definition
PPI	Z_{1t}	Percent change in the quarterly Producer Price Index
CPI	Z_{2t}	Percent change in the quarterly Consumer Price Index
UNEMPLOY	Z_{3t}	Percent change in the quarterly Unemployment Rate
IndProd	Z_{4t}	Percent change in the quarterly Industrial Production
CCI	Z_{5t}	Percent change in the quarterly Consumer Confidence Index
GDP	Z_{6t}	Percent change in the quarterly Gross Domestic Product
$(r_{Mt}-r_{ft})$	X_t	Percent change in the quarterly Market Premium
$(r_{Mt}-r_{ft})$*PPI	X_t*Z_{1t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Producer Price Index
$(r_{Mt}-r_{ft})$*CPI	X_t*Z_{2t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Consumer Price Index
$(r_{Mt}-r_{ft})$* UNEMPLOY	X_t*Z_{3t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Unemployment Rate
$(r_{Mt}-r_{ft})_t$*IndProd	X_t*Z_{4t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Industrial Production
$(r_{Mt}-r_{ft})$*CCI	X_t*Z_{5t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Consumer Confidence Index
$(r_{Mt}-r_{ft})$*GDP	X_t*Z_{6t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Gross Domestic Product

Table 7: Data Sample Statistics: Second Round

Variable	*Mean*	*Median*
Z_{1t}	1.11%	1.03%
Z_{2t}	0.10%	0.18%
Z_{3t}	0.53%	0.50%
Z_{4t}	1.35%	-0.83%
Z_{5t}	0.23%	0.58%
Z_{6t}	-1.74%	-0.34%
X_t	-4.29%	-24.22%
X_t*Z_{1t}	0.03%	-0.25%
X_t*Z_{2t}	-3.50%	-0.52%
X_t*Z_{3t}	-0.50%	-0.08%
X_t*Z_{4t}	-0.01%	0.42%
Z_{5t}	-1.34%	-0.04%
Z_{6t}	7.35%	0.76%

Chart 4: PPI, CPI and Industrial Production (quarterly changes)

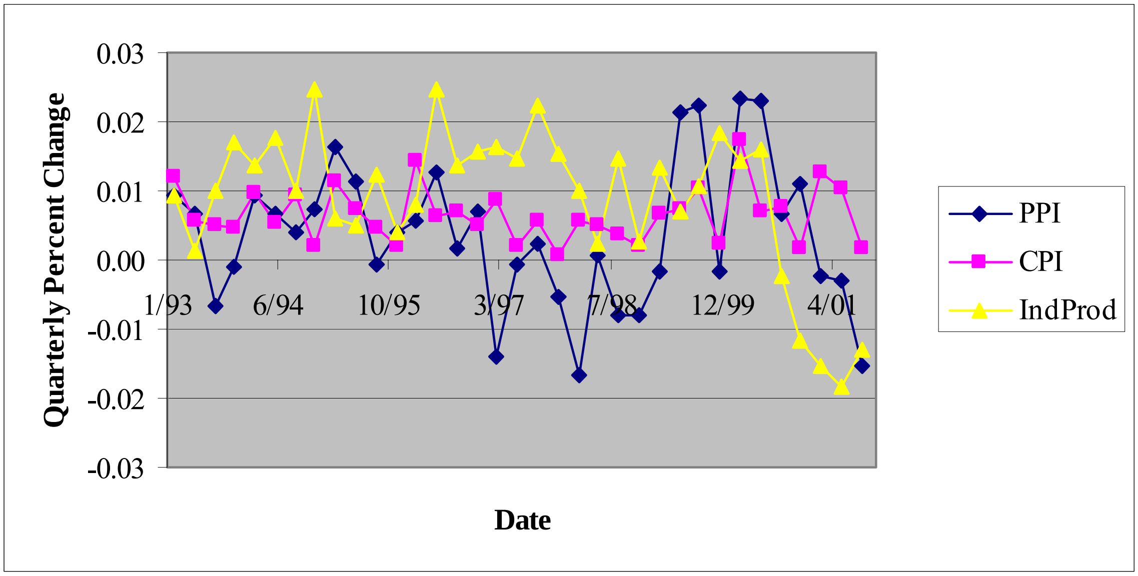

Chart 5: Unemployment, GDP and CCI (quarterly changes)

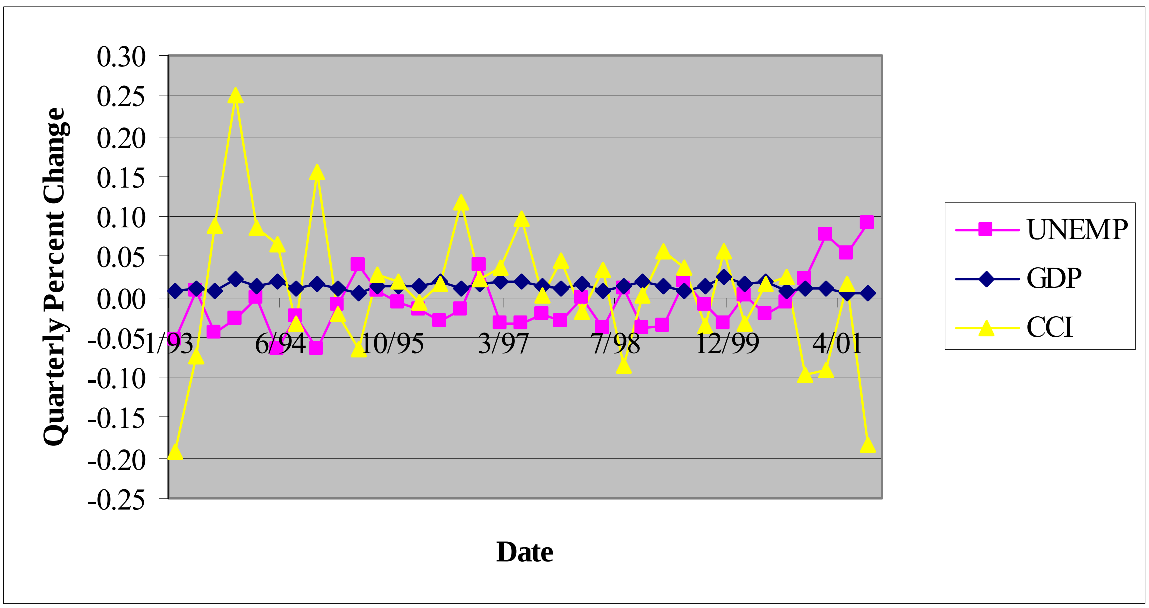

Table 8: Sample Time Period for each Estimation

Regression Number	***Sample Time Period***	***Number of Observations***
Regression 1	December 1992-September 2001	36 observations
Regression 2 (Hildreth-Lu autocorrelation search technique)	March 1993-September 2001	35 observations

Table 9: OLS Regression Results: Second Round

Independent Variable Name	*Coefficient (Intercept forced to Zero)*	*Coefficient*
Intercept		-93.24
		(-0.33)
PPI	0.09	0.19
	(0.08)	*(0.17)*
CPI	1.61	2.33
	(1.11)	*(0.88)*
UNEMPLOY	-0.01	-0.01
	(-1.78)	*(-1.24)*
IndProd	-2.18	-1.89
	(-1.70)	*(-1.20)*
CCI	-0.26	-0.26
	(-0.93)	*(-0.92)*
GDP	0.01	0.00
	(0.81)	*(0.07)*
Xt=rMt-rft	-16.94	-12.06
	(-1.21)	*(-0.58)*
Xt*PPI	0.09	0.08
	(1.86)	*(1.52)*
Xt*CPI	-0.17	-0.21
	(-1.05)	*(-1.04)*
Xt*UNEMPLOY	0.00	0.00
	*(3.35)**	*(3.10)**
Xt*IndProd	0.05	0.03
	(0.51)	*(0.25)*
Xt*CCI	0.03	0.03
	(1.41)	*(1.42)*
Xt*GDP	0.00	0.00
	(0.79)	*(0.80)*

Table 10: Definition of Variables (Exclusion of IndProd & CCI)

Abbreviation	Variable Name	Variable Definition
PPI	Z_{1t}	Percent change in the quarterly Producer Price Index
CPI	Z_{2t}	Percent change in the quarterly Consumer Price Index
UNEMPLOY	Z_{3t}	Percent change in the quarterly Unemployment Rate
GDP	Z_{4t}	Percent change in the quarterly Gross Domestic Product
$(r_{Mt}-r_{ft})$	X_t	Percent change in the quarterly Market Premium
$(r_{Mt}-r_{ft})$*PPI	X_t*Z_{1t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Producer Price Index
$(r_{Mt}-r_{ft})$*CPI	X_t*Z_{2t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Consumer Price Index
$(r_{Mt}-r_{ft})$* UNEMPLOY	X_t*Z_{3t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Unemployment Rate
$(r_{Mt}-r_{ft})$*GDP	X_t*Z_{4t}	Percent change in the quarterly Market Premium* Percent change in the quarterly Gross Domestic Product

Table 11: Data Sample Statistics – Exclusion of IndProd & CCI

Variable	*Mean*	*Median*
$\mathbf{Z_{1t}}$	0.37%	0.40%
$\mathbf{Z_{2t}}$	0.66%	0.56%
$\mathbf{Z_{3t}}$	-0.82%	-1.41%
$\mathbf{Z_{4t}}$	1.32%	1.30%
$\mathbf{X_t}$	79.68%	-17.31%
$\mathbf{X_t*Z_{1t}}$	80.18%	-16.85%
$\mathbf{X_t*Z_{2t}}$	80.89%	-16.12%
$\mathbf{X_t*Z_{3t}}$	77.06%	-17.53%
$\mathbf{X_t*Z_{4t}}$	81.79%	-16.2%

Table 12: Sample Time-Period for each Estimation

Exclusion of IndProd & CCI

Regression Number	*Sample Time Period*	*Number of Observations*
Regression 1	March 1993 -September 2001	35 observations
Regression 2 (Hildreth-Lu autocorrelation search technique)	June 1993-September 2001	34 observations

Table 13: OLS Regression Results: Reduced Variables

Exclusion of IndProd & CCI

Independent Variable Name	*Coefficient*
Intercept	-2.47
	*(-2.17)**
PPI	34.40
	(0.78)
CPI	-170.54
	(-1.65)
UNEMPLOY	-57.37
	*(-3.15)**
GDP	-108.53
	(-1.14)
Xt=rMt-rft	-454.33
	*(-3.37)**
Xt*PPI	-13.82
	(-0.38)
Xt*CPI	237.52
	*(2.99)**
Xt*UNEMPLOY	32.73
	*(3.46)**
Xt*GDP	194.83
	*(3.15)**

Chart 6: Unemployment, GDP, PPI and CPI
(exclusion of CCI and Industrial Production)

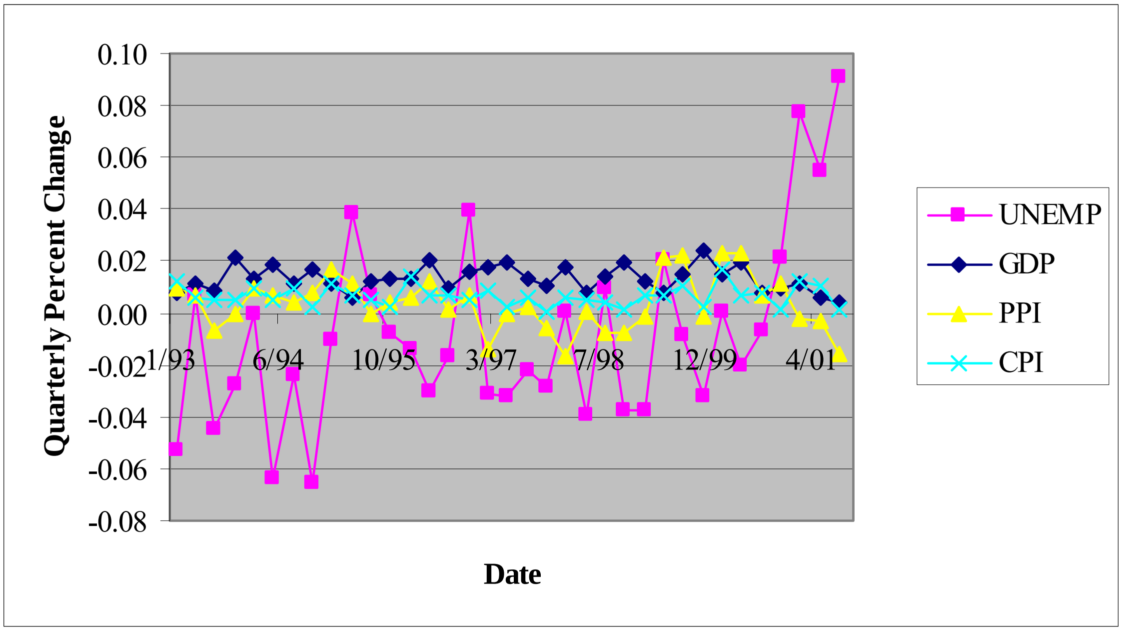

Exhibit 14: Visual Inspection of the Residuals

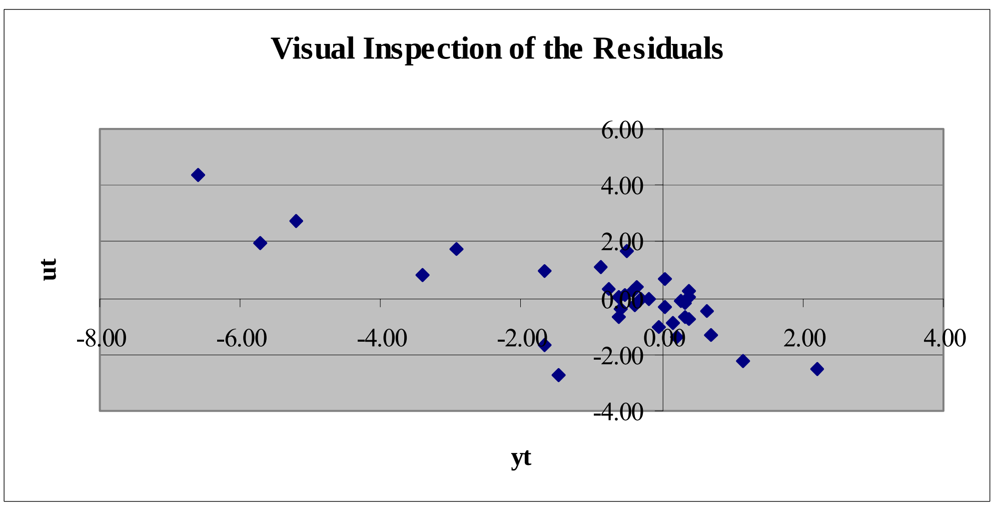

Table 15: Durbin-Watson Test for the Presence of Autocorrelation

$d=\frac{\sum_{i=2}^{n}(r_i - r_{i-1})^2}{\sum_{i=1}^{n} r_i^2}$	do not reject H_0 if $d > d_{*u}$, reject H_0 if $d < d_{*l}$ No conclusion is drawn if $d_{*l} \le d \le d_{*u}$	
Calculated Durbin-Watson Statistic d = 1.993	n = 35	k = 9
Percent of Significance Points	$\boldsymbol{d_{*l}}$	$\boldsymbol{d_{*u}}$
5%	0.908	2.144
1%	0.744	1.94

Testing for the presence of autocorrelation with the Durbin-Watson Test statistic results in a “no conclusion” at a 5% level of significance and a “do not reject” at a 1% level of significance.