Chapter 6

Mathematical Modeling of the Test Data

6.1 Scope and Purpose of the Model

The purpose of the mathematical model is to characterize the behavior of a rope during a snap load so that Snapping Cable Energy Dissipators can be accurately simulated in the finite element analysis of a structural system which includes these devices. The model is based upon the force, velocity, and displacement values that were recorded during the Taut Phase of every dynamic test that was run on a particular rope. All of the dynamic tests that were conducted as part of this research were modeled, as well as the dynamic tests from the Follow-Up tests. By conducting a regression analysis of the data, several coefficients will be obtained that will enable a force equation to be established that approximates the dynamic response of these ropes during the Taut Phase.

6.2 Development of the Model

The format of the model was modified several times during its development based on the results of the previous model. The software that was used to perform the regression analysis on the test data is SigmaPlot. This program is capable of performing both linear and non-linear multiple regression analysis on a given data set, and produces the desired coefficients, a measure of the accuracy of the model equation, and a three-dimensional plot of the input data series and the resulting mesh that is produced by the modeling of the data. The results from this analysis were then input in Microsoft Excel to evaluate and compare the results. The progression of the model from its original conception to its final form is detailed as follows.

6.2.1 Initial Model (Model 1)

The first attempt at modeling the test data was done by trying to fit the data with a linear regression equation using a least-squares-fit procedure. The force, velocity, and displacement data from the Taut Phase of each test was taken from Excel and input into SigmaPlot to conduct the investigation. Each series contained between 80 and 150 data points for each quantity, which gives a pulse duration between 0.04 and 0.075 seconds. Figure 6.2.1.1 shows an example of these data sets for one dynamic test.

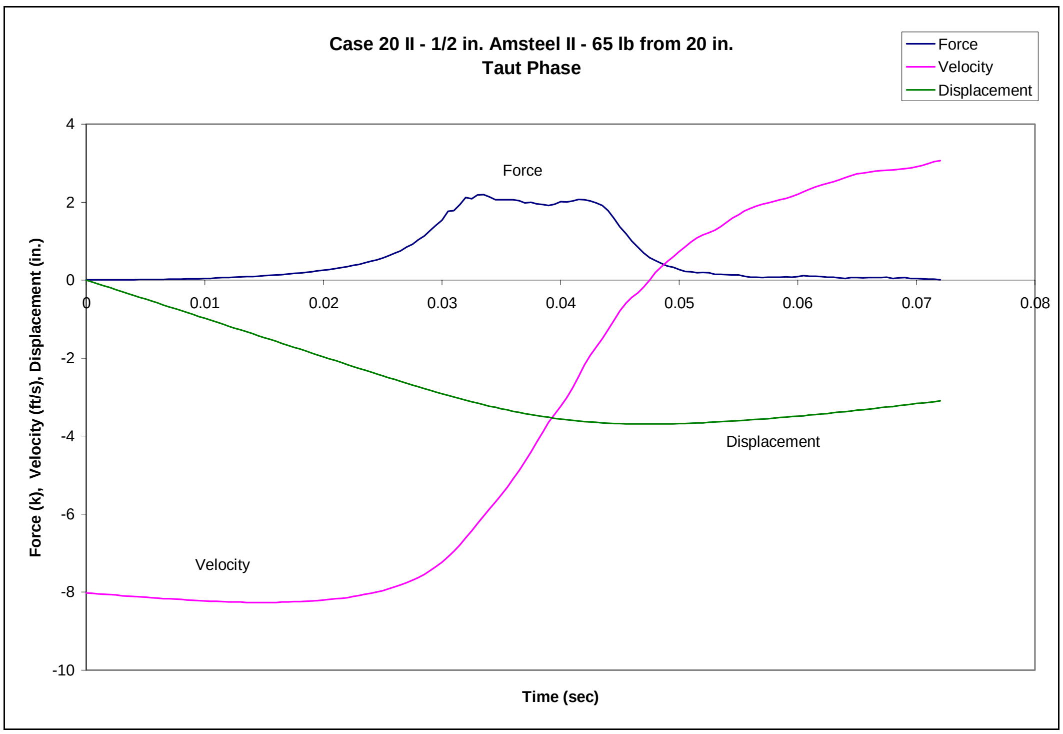

Figure 6.2.1.1: Force, Velocity, and Displacement Values for a Taut Phase

The equation that was used in this first attempt to model the test data is Equation (6-1).

$$\min_{(k,c)} \left\{ \frac{1}{n} \sum_{i=1}^{n} \left[F_i - \left(ky_i + cv_i \right) \right]^2 \right\} \qquad (6\text{-}1)$$

In this equation, the force at a given increment (F_i) is related to the displacement of the plate (y_i) and the velocity of the plate (v_i) at that same increment, but is also dependent on two unknown values. These are the stiffness coefficient (k) and the damping coefficient (c). These values are not unique at a given increment, but constant for the entire data series. The simplified form of Equation (6-1) that was utilized in SigmaPlot is Equation (6-2).

$$F_i = \left(ky_i \right) + \left(cv_i \right) \qquad (6\text{-}2)$$

The goal of this model was to determine the two coefficients that could best characterize the input data for a given test. These values were then to be compared to those of the other tests to determine if similarities existed and to determine what set of coefficients best characterized the data for a sequence of dynamic tests and for an entire rope. However, only one data series was modeled using this equation and the results of this trial were not good. The equation was applied to the data series under the constraint that the stiffness and damping coefficients must be positive. Also, the displacement was taken as positive for the entire data series. The velocity was taken as positive when the plate was moving downward, and was then taken as negative when the plate began to rebound from the snap load. The resulting coefficients were $k = 0.5293$ and $c = 3.5 \times 10^{-10}$. While it was expected that the rope would not provide much damping, the calculated value for c is very small. In addition, the three-dimensional plot of the data produced a nonlinear curve which only crosses the linear fit mesh at three locations. This indicates that the model values do not correspond well at all to the input values. This can be seen in Figure 6.2.1.2.

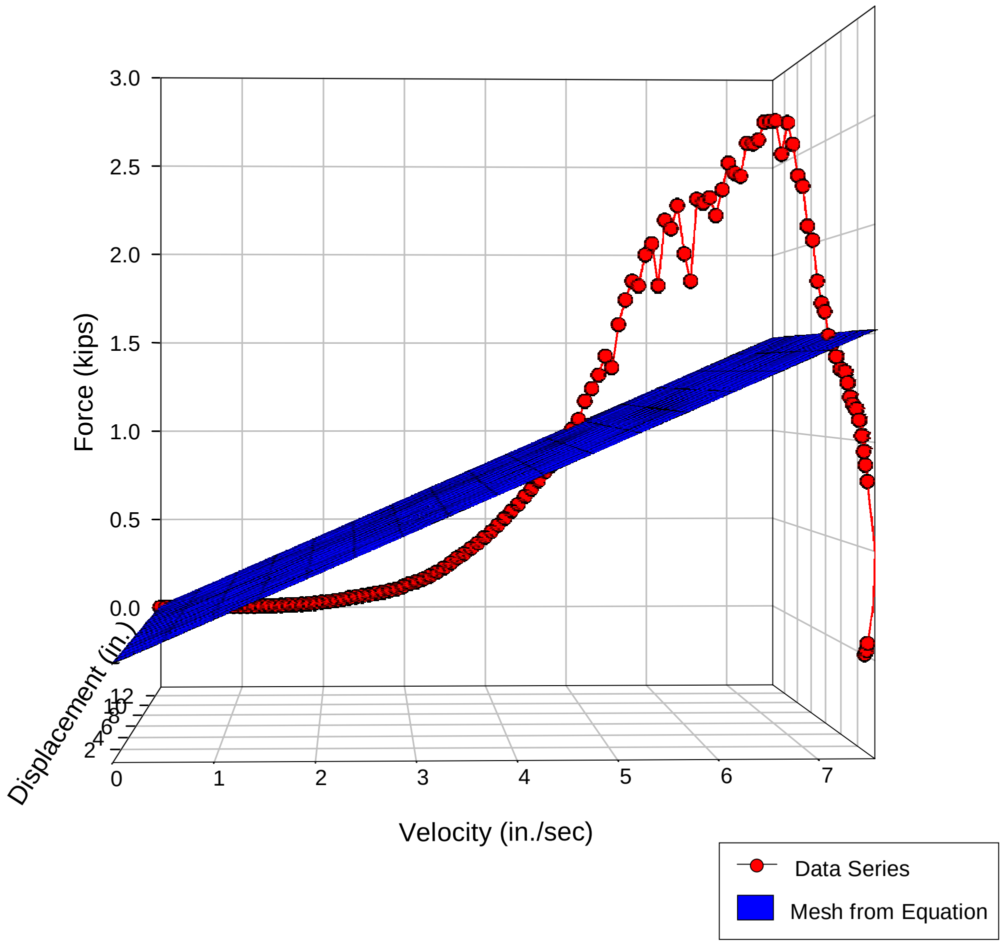

Figure 6.2.1.2: Linear Fit of the Test Data for Model 1

The mesh that is produced by SigmaPlot is a surface that represents every possible value for the force, velocity, and displacement that could be obtained by the model using the inputted range of values. The mesh is a planar surface for Model 1 since it is based on a linear equation. The meshes will be curves in the revised models since those equations will match up better with the test data.

6.2.2 First Model Revision (Model 2)

As a result of the poor correlation between the input data and the results of the Model 1 regression equation, the format of the mathematical model was changed so that it would be based on a nonlinear equation which included squared and cubed terms. For this equation, the stiffness and damping coefficients were replaced by a series of coefficients that have no assigned designation. These coefficients are simply meant to weight the velocity and displacement terms so that the resultant regression values better match the data series. These coefficients are symbolized by the letter a, with a subscript that represents their location within the equation. The equation that was utilized for Model 2 is Equation (6-3).

$$F_i = a_1 y_i + a_2 v_i + a_3 y_i^2 + a_4 y_i v_i + a_5 v_i^2 + a_6 y_i^3 + a_7 y_i^2 v_i + a_8 y_i v_i^2 + a_9 v_i^3 \tag{6-3}$$

The signs of velocity and displacement values were defined the same way for this model as they were before, and the only constraints that were applied were that the values of a_1 and a_2 must be positive. This equation was applied to the same data series as the first model, but this time it produced more favorable results. Three-dimensional plots of the data series and the resultant mesh can be found in Figure 6.2.2.1 and Figure 6.2.2.2.

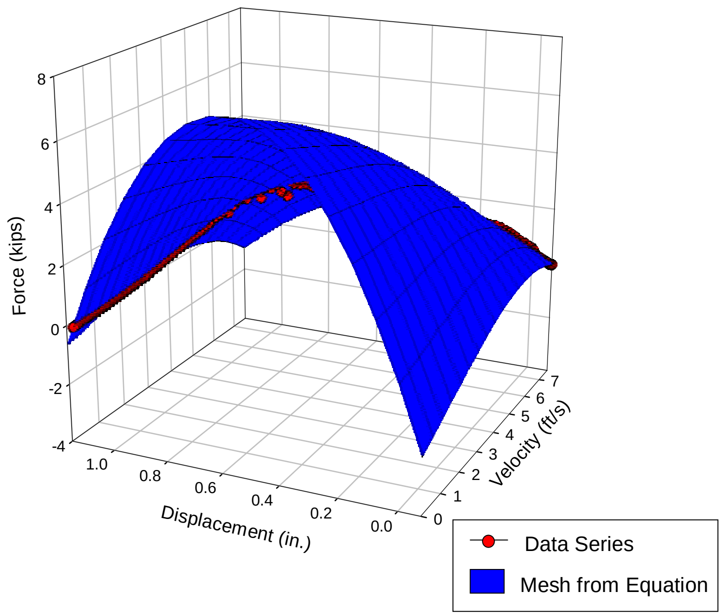

Figure 6.2.2.1: Non-linear Fit of the Test Data for Model 2 (View 1)

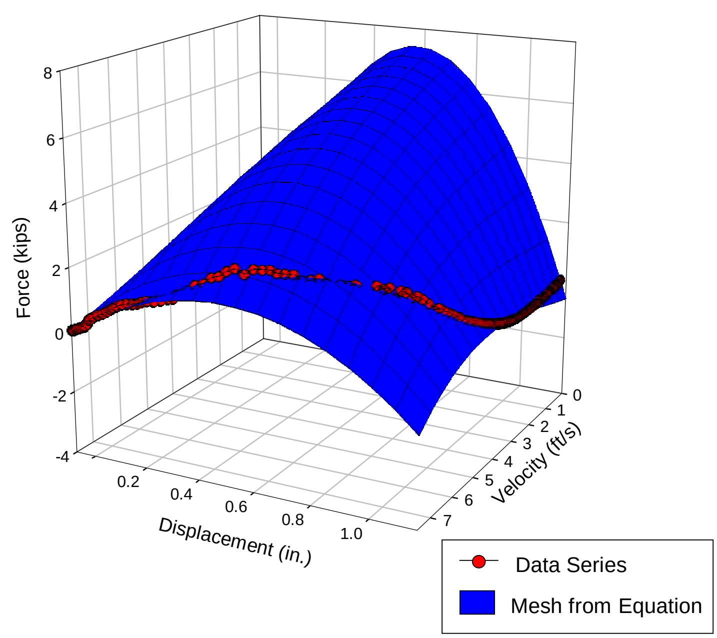

Figure 6.2.2.2: Non-linear Fit of the Test Data for Model 2 (View 2)

As can be seen, the mesh very closely matched the inputted data series. Therefore the model is reasonably accurate. However, this model was not used because of a software conflict that is explained in the next section.

6.2.3 Second Model Revision (Model 3)

The software that was chosen to conduct the finite element analysis of buildings that contain SCEDs for the subsequent research is ABAQUS. This program is a powerful tool, but it cannot handle equations that contain coupled terms such as the $a_4 y_i v_i$ term from Equation (6-3). Therefore, the coupled terms were eliminated and the resulting equation is the basis for Model 3. This is Equation (6.4).

$$F_i = a_1 y_i + a_2 v_i + a_3 y_i^2 + a_4 v_i^2 + a_5 y_i^3 + a_6 v_i^3 \qquad (6.4)$$

This equation produced slightly less accurate results than Model 2, but the difference was very small. The measure of how closely the model matched the data series was determined by the r^2 value. The r^2 value is a descriptive statistic that quantifies the relationship between two groups of values, in this case the inputted data series and the model results. The r^2 value can range between zero and one, and the larger the number is, the more accurate the match. The resulting r^2 values for the individual tests using Model 2 were between 0.985 and 0.996, and the values for Model 3 were between 0.981 and 0.988. This shows that both these models are very accurate and that discarding the coupled terms does not significantly affect the model.

Initially, the individual tests were modeled separately. This was done to ensure that the model equations were capable of accurately characterizing a single data series. Since the equations produced results that had 98-99% correlation with the recorded values, it was concluded that the equations modeled the data well. However, the goal of the model was to develop an equation that could characterize a whole sequence of drop tests or all of the drop tests for an individual rope type. Therefore, the data from all of the taut phases for each sequence of 20 drop tests and the data from every drop test conducted on an individual rope type were also modeled. The coefficients and r^2 values obtained from these regression analyses can be found in Table 6.2.3.1.

Coefficients from the Follow-Up Tests							
Ropes	a_1	a_2	a_3	a_4	a_5	a_6	r^2
Am Blue - Precycled	0	5.223	0.149	-8.651	-0.016	3.031	0.696
Am Blue - New	0	5.639	0.106	-10.300	-0.010	4.513	0.503
Am II - Precycled	0	4.837	0.238	-7.455	-0.033	2.263	0.739
Am II - New	0.334	5.533	0.083	-10.266	-0.014	4.401	0.662

Coefficients from Amsteel Blue Ropes							
Ropes	a_1	a_2	a_3	a_4	a_5	a_6	r^2
Am Blue - Precycled	0	1.548	0.089	-1.323	-0.009	-0.009	0.472
Am Blue - New	0	1.723	0.071	-1.453	-0.006	0.002	0.443

Sequences	a_1	a_2	a_3	a_4	a_5	a_6	r^2
A - 56" - Precycled	0.128	6.177	0.055	-1.926	-0.008	-2.402	0.853
B - 44" - Precycled	0	1.869	0.144	0.409	-0.019	-2.296	0.726
D - 20" - Precycled	0.070	3.456	0.086	0.987	-0.017	-7.223	0.905
E - 8" - Precycled	0	3.374	0.163	-0.603	-0.039	-10.123	0.909
AA - 56" - New	0.450	4.390	-0.059	2.570	0.001	-4.955	0.904
BB - 44" - New	0.096	4.865	0.053	0.321	-0.008	-4.958	0.905
DD - 20" - New	0.747	2.801	-0.208	-0.089	0.014	-6.433	0.809
EE - 8" - New	0	2.248	0.103	-3.556	-0.021	-1.124	0.743

Coefficients from Amsteel II Ropes							
Ropes	a_1	a_2	a_3	a_4	a_5	a_6	r^2
Am II - Precycled	0	4.189	0.162	-7.449	-0.018	3.259	0.603
Am II - New	0	4.290	0.155	-7.753	-0.017	3.444	0.614

Sequences	a_1	a_2	a_3	a_4	a_5	a_6	r^2
F - 56" - Precycled	0.478	6.062	-0.048	1.617	-0.001	-5.869	0.916
G - 44" - Precycled	0	6.642	0.247	-3.769	-0.040	-2.512	0.896
H - 32" - Precycled	0	5.001	0.183	1.911	-0.030	-9.126	0.918
I - 20" - Precycled	0	6.063	0.274	-6.111	-0.059	-4.572	0.912
J - 8" - Precycled	0	4.390	0.258	-4.581	-0.068	-18.051	0.898
FF - 56" - New	0.159	5.457	0.063	3.970	-0.010	-7.321	0.916
GG - 44" - New	0	5.625	0.175	1.116	-0.026	-6.672	0.930
HH - 32" - New	0	5.017	0.093	3.745	-0.011	-10.910	0.928
II - 20" - New	0.101	4.724	0.163	1.617	-0.039	-12.884	0.907
JJ - 8" - New	0	4.804	0.363	-7.484	-0.110	-13.991	0.914

Table 6.2.3.1: Results from Model 3 for the Follow-Up Tests and New Tests

The coefficients that are multipliers of the displacement (a_1, a_3, and a_5) are much smaller than the multipliers of the velocity (a_2, a_4, and a_6). This may be because the coefficients act to equalize the displacement and velocity values so that the best force value is obtained. Since the displacement values are larger than the velocity values, the opposite may be true for their multipliers. While there is variance in the value of a certain coefficient between the different sequences and ropes, the values are generally close to each other and follow the same trend.

The coefficients that were obtained from the regression analyses were then put back into Excel and multiplied by the appropriate data value at each time step. This produced a theoretical force value at each time step, and the series of theoretical forces was plotted against the recorded forces to determine how the curves compared to each other. For clarification purposes, the theoretical curves were named for the type of data on which the regression analyses were conducted. A model equation that utilizes coefficients that were obtained from modeling one drop test is called a *Test Equation*. By that same logic, model equations that are based on coefficients acquired from modeling a sequence of drop tests and all of the drop tests for a particular rope type are called *Sequence Equations* and *Rope Equations*, respectively. Figure 6.2.3.1 is a plot of the recorded force and the three theoretical forces versus the time, Figure 6.2.3.2 is a plot of the forces versus the displacement, and Figure 6.2.3.3 is a plot of the forces versus the velocity.

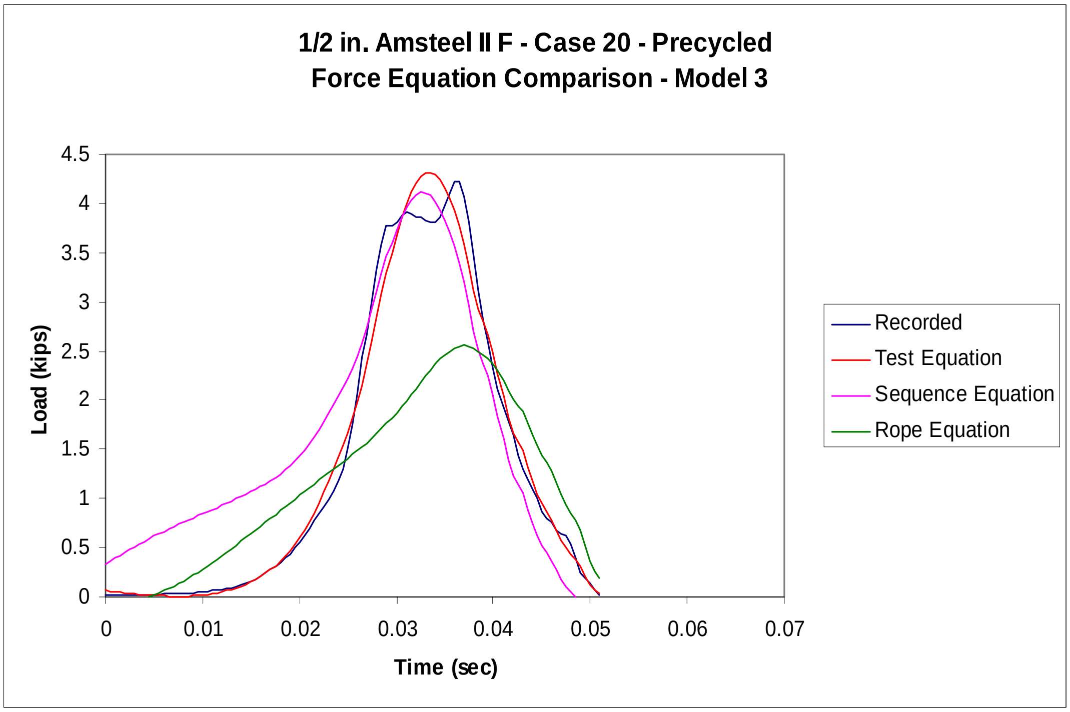

Figure 6.2.3.1: Recorded Force and Theoretical Forces vs. Time – Model 3

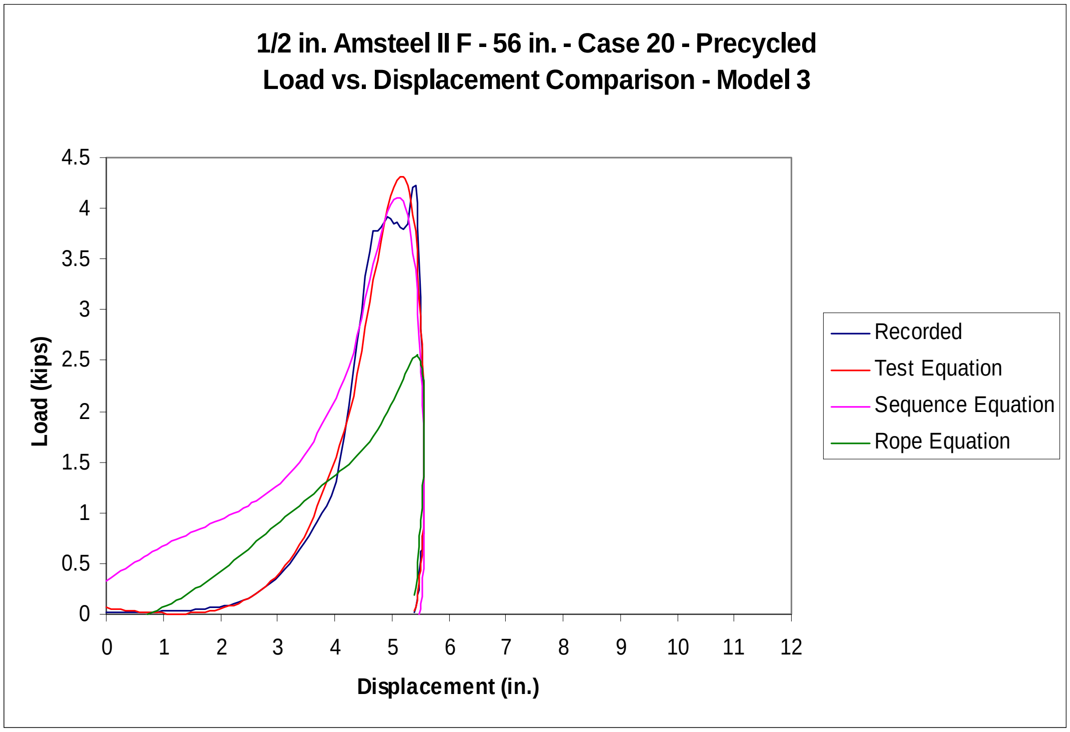

Figure 6.2.3.2: Recorded Force and Theoretical Forces vs. Displacement – Model 3

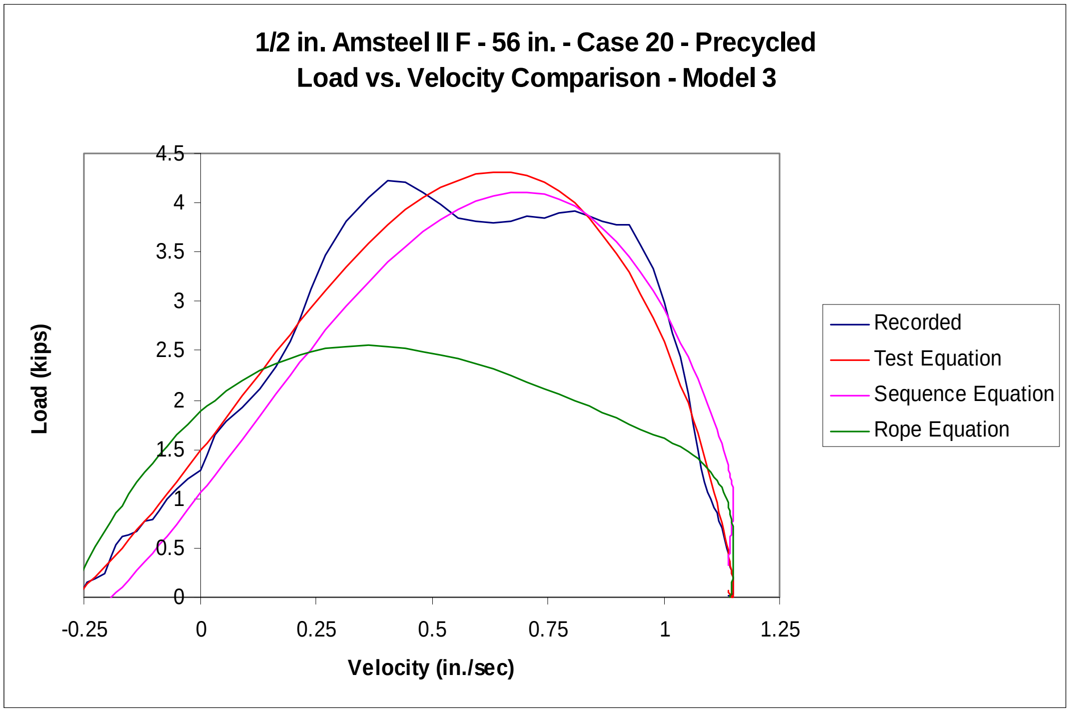

Figure 6.2.3.3: Recorded Force and Theoretical Forces vs. Velocity – Model 3

As can be seen, the Test Equation characterizes the recorded force very well, but is only applicable to one particular drop test. The Sequence Equation also produces a good match and it can be applied to all of the drop tests for a particular height and rope type. The r^2 values for the Amsteel Blue ropes ranged from 0.72 to 0.91, while the r^2 values for the Amsteel II ropes were from 0.89 to 0.93. However, the Rope Equation, which is applicable to all of the drop tests for a given rope type, greatly underestimates the recorded forces. The r^2 values for the Amsteel Blue ropes ranged from 0.44 to 0.47, while the r^2 values for the Amsteel II ropes were around 0.60. The coefficients and r^2 values from the Follow-Up Tests were much lower than those from the New Tests. This is probably due to the varying drop heights throughout the sequences and the large variations in the resulting forces.

6.2.4: Third Model Revision (Model 4)

After an extensive investigation of the dynamic test data using Model 3, it was decided to revise the model one more time to improve the correlation of the Sequence Equation and Rope Equation. The logic behind this revision is associated with the velocity data. When the rope is becoming taut, the drop plate is still falling and has an initial velocity that is greater than zero. At the beginning of the Taut Phase the force and displacement values are essentially zero, but because the velocity values are positive and are being multiplied by non-zero coefficients, there is a small error introduced into the theoretical force curve. To correct this problem, a new coefficient was added to the equation which will act as a constant value that will not be multiplied by the displacement or velocity. This coefficient is called a_0 and is found in Equation (6-5).

$$F_i = a_0 + a_1 y_i + a_2 v_i + a_3 y_i^2 + a_4 v_i^2 + a_5 y_i^3 + a_6 v_i^3 \qquad \text{(6-5)}$$

The data for each drop sequence and rope type was reanalyzed using the revised equation, this time with no sign constraints applied to the coefficients. The resulting coefficients and r^2 values can be found in Table 6.2.4.1.

Coefficients from the Follow-Up Tests							
Ropes	a_0	a_1	a_2	a_3	a_4	a_5	a_6	r^2
Am Blue - Precycled	-0.554	0.245	5.449	0.101	-7.767	-0.013	2.193	0.845
Am Blue - New	-0.889	0.418	6.361	0.012	-10.046	-0.003	3.968	0.737
Am II - Precycled	-0.806	0.380	5.409	0.167	-6.576	-0.028	1.188	0.890
Am II - New	-0.841	1.165	5.796	-0.180	-9.640	0.010	3.750	0.837

Coefficients from Amsteel Blue Ropes								
Ropes	a_0	a_1	a_2	a_3	a_4	a_5	a_6	r^2
Am Blue - Precycled	-0.997	0.021	2.070	0.151	0.221	-0.015	-1.293	0.597
Am Blue - New	-1.104	0.350	2.411	0.031	-0.608	-0.005	-0.892	0.550

Sequences	a_0	a_1	a_2	a_3	a_4	a_5	a_6	r^2
A - 56" - Precycled	0.251	0.143	6.065	0.043	-1.650	-0.007	-2.648	0.924
B - 44" - Precycled	-0.389	-0.151	1.815	0.217	1.484	-0.025	-2.939	0.736
D - 20" - Precycled	-0.142	0.056	3.420	0.106	1.026	-0.019	-6.959	0.906
E - 8" - Precycled	-0.178	-0.174	3.255	0.283	-0.151	-0.054	-9.323	0.912
AA - 56" - New	0.330	0.394	4.250	-0.058	2.895	0.001	-5.221	0.905
BB - 44" - New	-0.083	0.095	4.872	0.057	0.279	-0.008	-4.867	0.905
DD - 20" - New	-0.132	0.804	2.760	-0.217	0.190	0.014	-6.543	0.810
EE - 8" - New	-0.502	0.020	1.916	0.172	0.579	-0.032	-5.132	0.790

Coefficients from Amsteel II Ropes								
Ropes	a_0	a_1	a_2	a_3	a_4	a_5	a_6	r^2
Am II - Precycled	-1.142	-0.008	5.361	0.281	-6.783	-0.033	2.245	0.864
Am II - New	-0.860	-0.388	5.100	0.448	-6.670	-0.054	2.271	0.862

Sequences	a_0	a_1	a_2	a_3	a_4	a_5	a_6	r^2
F - 56" - Precycled	0.511	0.394	6.052	-0.056	1.836	0.001	-6.237	0.959
G - 44" - Precycled	0.166	-0.979	6.666	0.644	-4.430	-0.081	-1.623	0.953
H - 32" - Precycled	0.482	-0.364	5.144	0.333	0.775	-0.050	-8.380	0.961
I - 20" - Precycled	0.106	-0.468	6.080	0.563	-6.929	-0.106	-3.199	0.957
J - 8" - Precycled	-0.202	-0.107	4.213	0.336	-2.264	-0.071	-19.296	0.952
FF - 56" - New	0.143	0.110	5.473	0.073	3.930	-0.011	-7.348	0.957
GG - 44" - New	0.704	-0.338	5.847	0.280	0.199	-0.038	-6.358	0.968
HH - 32" - New	-0.364	-0.064	4.909	0.109	4.186	-0.008	-10.707	0.964
II - 20" - New	0.128	0.118	4.790	0.136	1.437	-0.035	-13.100	0.953
JJ - 8" - New	0.052	-0.264	4.829	0.577	-8.811	-0.157	-10.379	0.957

Table 6.2.4.1: Results from Model 4 for the Follow-Up Tests and New Tests

The theoretical forces were then recalculated for the Sequence and Test Equations and compared to the recorded force values. Figure 6.2.4.1 is a plot of the recorded force and the theoretical forces versus the time, Figure 6.2.4.2 is a plot of the forces versus the displacement, and Figure 6.2.4.3 is a plot of the forces versus the velocity.

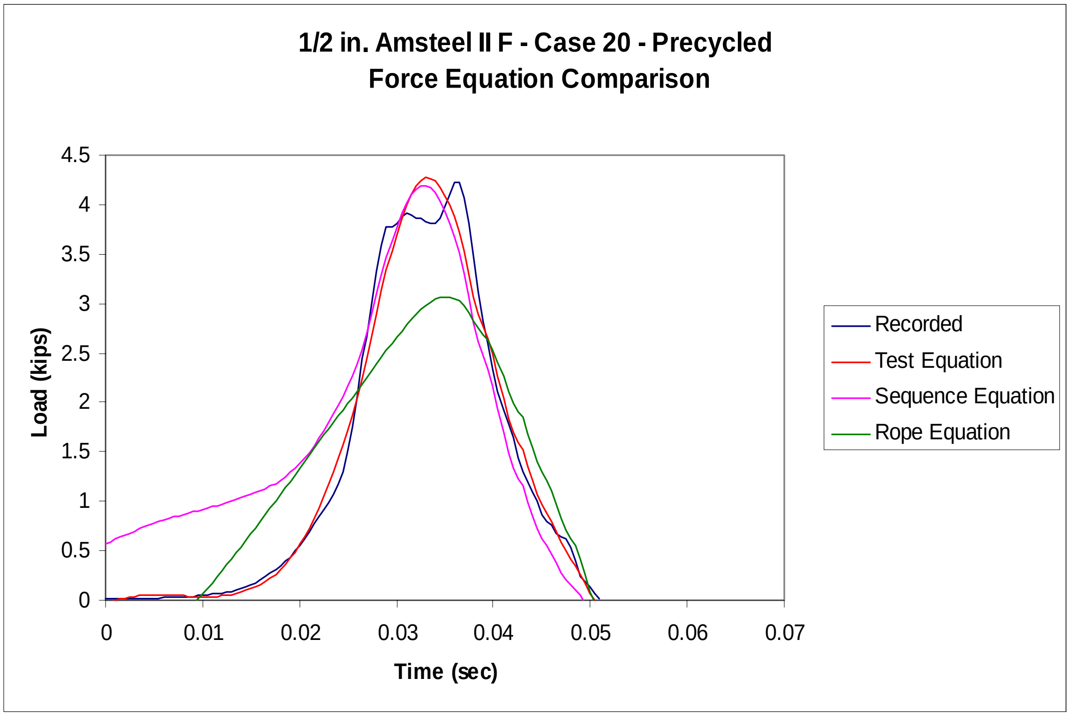

Figure 6.2.4.1: Recorded Force and Theoretical Forces vs. Time – Model 4

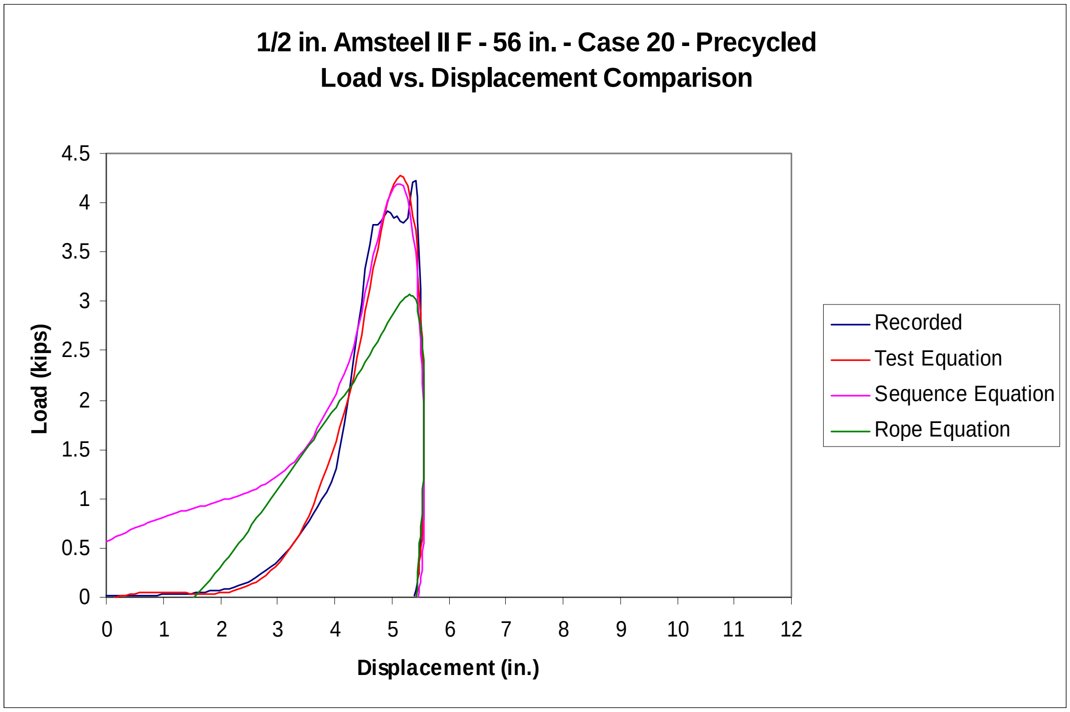

Figure 6.2.4.2: Recorded Force and Theoretical Forces vs. Displacement – Model 4

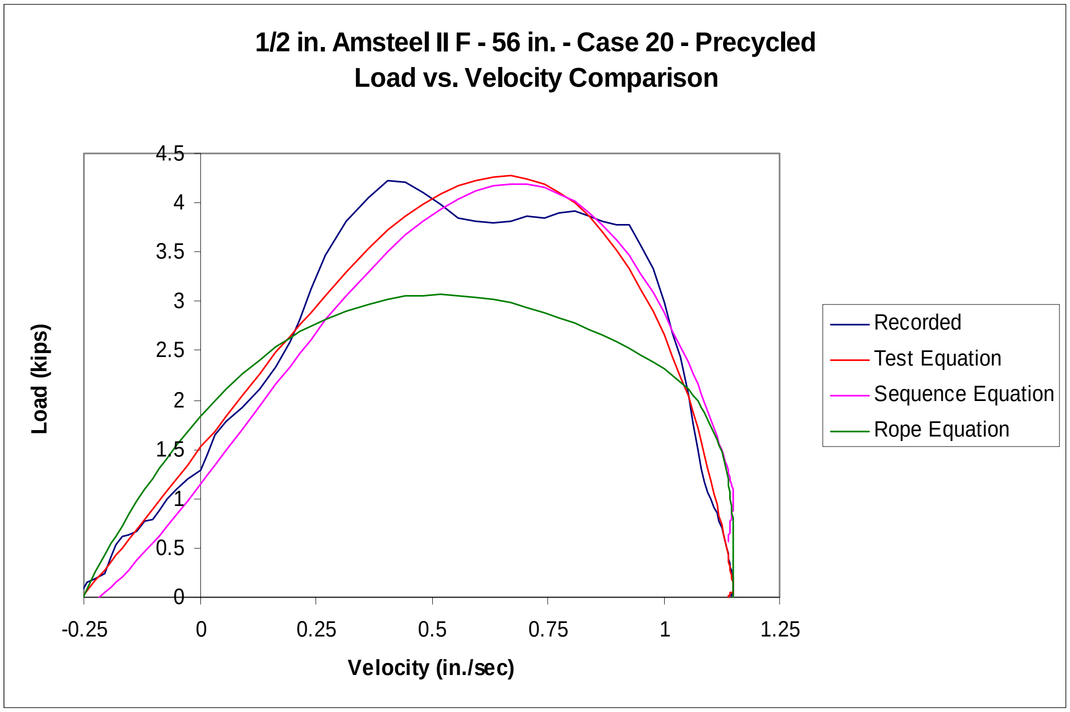

Figure 6.2.4.3: Recorded Force and Theoretical Forces vs. Velocity – Model 4

As can be seen, Model 4 produces more accurate results for both the Sequence Equation and the Rope Equation. The r^2 values for the Amsteel Blue ropes are from 0.73 to 0.93 for the Sequence Equation and 0.55 to 0.59 for the Rope Equation. This translates to a 2% increase in accuracy for the Sequence Equation and a 28% increase for the Rope Equation. The r^2 values for the Amsteel II ropes are from 0.95 to 0.97 for the Sequence Equation and around 0.86 for the Rope Equation. This translates to a 4% increase in accuracy for the Sequence Equation and a 43% increase for the Rope Equation. The accuracy for the Follow-Up Tests also increased, but the r^2 values are still much lower than those for the New Tests. Plots like those found in Figures 6.2.4.2 and 6.2.4.3 were made for every fifth drop test for every sequence in the Follow-Up and the New Tests and are found in Appendix C.

6.3 Model Results

The coefficients that were obtained from the Model 4 regression analyses produce very accurate theoretical force curves that correlate well with the forces that were recorded during the Taut Phase. The curves that were produced by the Test Equation are very accurate, but are only applicable to individual tests. The curves that were obtained from the Sequence Equation are also accurate and are applicable to all of the drop tests for a particular sequence. However, even with the large increase in accuracy, the curves produced by the Rope Equation still underestimate the force values and thus are not of much use in this application. Therefore, the coefficients that were obtained from the regression analysis of the sequence data are the ideal choice for use in this model. These coefficients are only applicable to a particular rope type and a certain drop height, but when a SCED is used in a structural frame, the amount the rope moves when the building shakes will remain relatively constant.

In general, the Amsteel II ropes have higher r^2 values than the Amsteel Blue ropes. This probably occurs because the behavior of the Amsteel II ropes remains nearly constant after a few drop tests whereas the Amsteel Blue ropes continue to stiffen throughout a sequence. However, there is not much difference between r^2 values of the Precycled and New ropes, and the drop height does not seem to distinctly affect the values either. However, there is a lot of variability in the values of the coefficients between the individual sequences. The values do not seem to follow a pattern and it is unclear why this occurs.

Considering the results of the static and dynamic analyses and the mathematical modeling, the equation that should be used to characterize a SCED in the finite element analysis portion of the subsequent research should be Equation (6-5). For the Precycled Amsteel II rope J, the coefficients are given in Equation (6-6).

$$F_i = -0.202 + 4.213 v_i + 0.336 y_i^2 - 2.264 v_i^2 - 0.071 y_i^3 - 19.296 v_i^3 \qquad (6\text{-}6)$$

The Amsteel II ropes showed the most desirable results throughout this research, and since the amount of dynamic elongation must be minimized, the precycling process is necessary. Also, since the amount of slack the SCED will have when it is placed in a building will be small, the coefficients from the lowest drop height should be used.

When a final decision is made on what size, length, and type of rope should be used as a SCED and the details of their configuration within a structural frame have been worked out, it may be necessary to conduct a sequence of drop tests using a large amount of weight and a low drop height to obtain a final force equation, which should be in the form of Equation (6-5).