AGENCY THEORY: A MODEL OF INVESTOR EQUILIBRIUM AND
A TEST OF AN AGENCY COST RATIONALE FOR
CONVERTIBLE BOND FINANCING

by

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To the memory of Greta

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INTRODUCTION

> The known is finite, the unknown infinite; intellectually we stand on an islet in the midst of an inimitable ocean of inexplicability. Our business in every generation is to reclaim a little more land.
>
> T. H. Huxley, 1887

The economic problems associated with the relationships between principals and their agents are far-ranging. This study focuses on a rather narrow aspect of the set of problems arising from "agency" relationships; the conflict that may develop between classes of security holders who act in their respective self-interests. More specifically, the study addresses that arena of conflict between stockholders and bondholders that is characterized by conflicting expected utility-of-wealth maximizing behavior.

The term "agency" applied in this context represents something of a misnomer in that there is virtually never a legal agent - principal relationship between stockholders and bondholders of typical corporations. However, the use of the phrase "agency problems" is ubiquitous in the scholarly literature dealing with stockholder -

bondholder conflicts and will be adopted for use throughout this study.

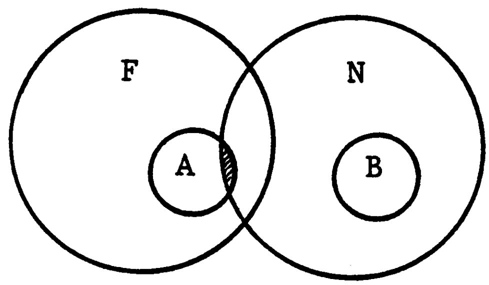
Stockholders of business firms organized as corporations are generally not personally liable for their firms' debts. As such, a firm's management acting exclusively in its stockholders' best interests may direct the resources of the firm in such a way as to modify promised payoffs to bondholders to the advantage of the stockholders. Since the stockholders are not obliged to serve as personal guarantors of the promised payoffs, the potential for conflict between the two classes of security holders is clear.

Does an agency problem really exist between stockholders and bondholders of modern U.S. corporations? Is the problem economically significant?

Given a particular regime of assumptions regarding capital market operations and efficiency, it can be shown that firms' managers will act in the unanimously agreed upon best interests of all security holders, hence no agency problem exists. But, as will be detailed in the following chapters, the observed state of capital markets today strongly suggests that unanimity of economic interest is rare. Indeed, both parties of the conflict are seen to undertake protective measures of substantial sophistication and expense; witness the complexities of the typical bond indenture. The weight of the evidence supports the proposition that agency problems exist in modern capital markets in economically significant proportions, and it follows that any sound economic theory must ultimately recognize this reality.

Definition of the Agency Problem

Before formally defining the agency problem of interest in this study, it is helpful to first define its parameters in light of the entire universe of conflicts arising among classes of security holders. The Venn diagram presented below, helps isolate the agency problem of interest from other types of conflicts.



THE AGENCY PROBLEM IN PERSPECTIVE

The total area of the diagram represents the universe of events that cause transfers of value to take place between equity and debt securities. The set N is that portion of the universe of events that are caused by

Nature; events exogenous to the decision makers of each individual firm. An example might be a change in the riskiness of the firm as measured by the variance of total firm value. If prices of debt and common stock are set according to the Black-Scholes Option Pricing Model, an increase in variance will cause a transfer of value from debt to equity. Such a change in variance, if precipitated by forces exogenous to the firm, would be a member of Set N in the diagram.

Set F encompasses those events that are under the control of individual firm managements. An example is a management-induced change in riskiness of investment projects that increases the probability of default on bonds.[1]

The universe of value transfer events is the union of sets N and F, denoted

$$\underline{N} \cup \underline{F}$$

The area of overlap between N and F is that set of events that are caused or aggravated jointly by Nature and an individual firm. This is denoted

$$\underline{N} \cap \underline{F}$$

[1]For an example of such an event see The Wall Street Journal, July 31, 1981, p. 25. The article is entitled, "Cummins Puts Future On Line for $1 Billion," and reports a major change in production philosophy at Cummins Engine Company.

Note that the two sets thus far have been defined in terms of _value_ transfers between securities, as opposed to _wealth_ transfers between holders of the secuities. The distinction is crucial and an example may help clarify the point. Assume investor i holds 500 dollars in stock (S_j) and 500 dollars in bonds (B_j) issued by the same firm j. If the management of firm j makes a decision in such a way that causes a 100 dollar _value_ transfer from B_j to S_j, investor i then has a portfolio worth the same as before the decision. No _wealth_ loss (or gain) has been experienced by investor i. If, on the other hand, i held only bonds (B_j), a _wealth_ loss of 100 dollars would have been suffered as a result of the 100 dollar _value_ transfer.

Inside Set N is subset B, consisting of those _value_ transfer events caused by Nature that also cause a _wealth_ transfer among security holders, i.e., those who are not somehow protected from wealth erosion. Set A includes wealth transfer events caused in full or in part by firm managements.

Sets A and B, then, represent events in which wealth transfers occur. The theoretical portion of this research will concentrate only on Set A, those wealth transfers caused by conscious management decisions. The exclusion of Set B is somewhat arbitrary, since it can be persuasively argued that Nature can serve as an "agent." Nonetheless, exclusion of Set B effectively narrows the scope of this research to a manageable size and allows the study to focus on those decisions made consciously and with forethought by managers to transfer wealth among individuals.

For the theoretical component of this research, the agency problem is formally defined below:

(1) Stockholder - bondholder agency problem: any adverse wealth effect caused or aggravated by a conscious management decision to expropriate wealth from, or deny the accumulation of wealth by holders of debt securities.

The theoretical focus is on Set A, and this includes the shaded region in the Venn diagram that represents the intersection of A and N, that collection of agency problems caused jointly by managerial decisions and Nature. The empirical portion of this research will not distinguish between wealth transfers caused by management and those caused by Nature, since identification of the source is unnecessary for the empirical analysis.

Throughout this study the assumption is made that the firm's management always acts in the best interests of the stockholders, hence the exclusive focus is the problem defined in definition (1), above.

Purpose of the Research

There are four distinct, yet closely associated purposes of this research effort.

1. Formal Development of an Investor Equilibrium Theoretical Model.

The most recent research in this area featured a single period state preference model which was developed under the assumption that investors who hold equity are distinct and apart from those investors who hold bonds in a particular firm. See Mikkelson (1978). The primary theoretical purpose of this research is the rigorous development of a single period investor equilibrium model of the agency problem as seen from the viewpoint of the individual portfolio investor. This will be derived under the simplifying assumptions of a competitive securities market, costless diversification, and an economic world with no corporate and personal taxes. By investor equilibrium is meant that unique point at which the individual has balanced consumption and savings decisions in such a way as to obtain maximum expected utility of consumption over the period. This analysis will therefore treat the individual investor explicitly.

2. Extension of a Single Period Model to Multi-Period.

Mikkelson's work in 1978 featured a single period state preference model of the consequences of the agency problem on the firm's production decision. In order to examine the problem in a dynamic setting, Mikkelson's model will be extended to multi-period.

3. Definition of the Determinants and Magnitude of the Agency Problem.

The theoretical models developed in items 1 and 2, above, will lead to an enumeration of the possible determinants of the agency problem. Firm characteristics such as probability of default and amount of debt outstanding will be set forth as factors that influence the severity of the agency problem. From these theoretical models some measures of the potential impact of the agency problem on the welfare of the individual investor will be set forth.

4. A Test of Firm Reactions.

An empirical test will be proposed that will examine specific agency cost-reducing behavior of firms. When confronted by a significant agency problem there are a variety of ways in which a firm may act to avoid or reduce the effects on wealth, as will be brought out in detail in the theoretical developments of Chaper Two. The proposed test selects one such method, the issue of convertible debt securities, and examines actual firm data for evidence of the agency cost-reducing behavior.

Justification for the Study

The study of agency theory in general and the study of the narrow aspects of that theory to which this research is confined can be justified on the following grounds.

First, the problems of agency apparently take a large toll in the reduction of economic efficiency in the capital markets, hence the erosion of the efficient allocation of scarce financial resources. As will be pointed out in Chapter Two, mitigation of the stockholder-bondholder conflicts often results in severe constraints on the financial freedom of the firm, particularly in the areas of dividend decisions and investments in capital assets. The recent expansion of academic interest in the problem in general is clearly justified.

Second, since the theoretical developments arising from the agency problem are mostly of recent origin, much work is yet to be done to push the study to a mature state. The first purpose of this dissertation, the development of an investor equilibrium model, is unique in that all previous theoretical agency research has concentrated on problems besetting a pure stockholder or pure bondholder. The portfolio investor has been generally ignored. The effects of agency problems on individual consumption and savings decisions have not been met on a rigorous level. The second and third purposes of the research, the multiperiod analysis and enumeration of agency problem determinants, are also unique and are considered necessary for further research. The final purpose, the empirical test, is not unique, but offers a fresh methodological attempt to test a specific firm reaction to the problem.

The Empirical Research Methodology

Statistical Technique

In theory there are a variety of ways in which both investors and firms may choose to deal with the agency problem. In practice we see much casual evidence as to the popularity of these devices, such as bond convenants which restrict dividend payments and capital investments, as well as conversion features whereby bonds may be converted into shares of common stock.

The empirical arm of this research effort is aimed at identifying statistically the strength of the influence of the agency problem on the firm's decision to issue convertible versus non-convertible bonds. There are several factors which, according to the theoretical developments in Chapter Three, cause or aggravate the agency problem. These include the probability of default, the portion of debt in the capital structure, and other so-called "agency variables." An agency cost rationale is only one reason for convertible financing. There are other reasons popularly cited in the literature, such as the familiar "debt sweetener argument."[1] The task is to econometrically

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[1] See Weston and Brigham (1978), pp. 582-587, for a discussion. Also, most other basic corporate finance textbooks discuss the popular motives for convertible bond financing.

model the firm's decision to issue convertible bonds as a function of these "agency" and "popular" factors.

The econometric model may be expressed compactly as:

$$\tilde{Y} = X\beta + \tilde{\varepsilon},$$

where $\tilde{Y}$ = n x 1 column vector of indicator or decision variables

X = n x K matrix of explanatory variables, including agency and popular variables

β = K x 1 column vector of linear regression coefficients

$\tilde{\varepsilon}$ = n x 1 column vector of linear random error terms

Since the dependent or endogenous variable is dichotomous, this is recognized as a simple linear discriminant function or a linear probability model, and presents some statistical difficulties in estimation. These problems, outlined in Chapter Five, give need to consider estimation of the model in four different ways. These are:

(1) ordinary least squares regression

(2) weighted least squares regression

(3) PROBIT analysis

(4) LOGIT analysis

These methods will be described in detail in Chapter Five where corresponding results of estimation will be set forth and contrasted.

Data Sources

The principal data sources for the empirical test include Moody's Bond Record, Moody's Industrial Manual, and the annual version of Industrial COMPUSTAT tapes.

Limitations of the Study

The analytical and empirical results of this research suffer from drawbacks not atypical of similar efforts in this field. The analytical results are derived under the assumption of competitive capital markets in a taxless economic environment. In reality, of course, some capital markets may deviate from the perfectly competitive model. Also, taxes and transactions costs are pervasive throughout any industrialized economy. These are important limitations to the theoretical development of an agency theory of economic behavior, and should be taken formally into account. However, for the research at hand this is considered a task appropriately left to future research.

Another limitation is that the empirical test, an econometric model of the firm's decision to issue convertible bonds, is not completely general; that is, there are possibly some hypotheses that have not been accounted for that would explain the observed behavior. While the empirical results, though marginally statistically significant, do not offer ironclad substantiation of the agency phenomenon, they are nonetheless evidence that the heretofore popular explanations for convertible bond financing are neither exclusive nor exhaustive.

Organization of the Dissertation

This chapter has set out the agency problem of interest, disclosed the purposes of the research, and cited justification and limitations of the study. Chapter Two includes a survey of relevant theoretical and empirical literature that has been published to date.

The major theoretical developments, including the single period investor equilibrium model and the multiperiod state preference analysis, are in Chapter Three. Chapter Four features a description of the econometric model of convertible bond financing decisions and includes a discussion of the traditional rationale for this form of financing.

In Chapter Five the econometric model results are reported and analyzed. Finally, in Chapter Six, the implications of this study for future research are highlighted.

Chapter Two

SURVEY OF PAST RESEARCH

> William James used to preach the 'will to believe.' For my part, I wish to preach the 'will to doubt...' What is wanted is not the will to believe, but the wish to find out...
>
> Bertrand Russell, 1928

This chapter is organized into two sections, the first dealing with theoretical developments, the second dealing with previous empirical findings. No attempt is made to enumerate all agency-related contributions to the literature; only those contributions that are most recent and immediately relevant to this research are addressed.

Review of Theoretical Contributions

It will probably come as no surprise that the mention of agency problems in the economics literature came as early as 1776, when Adam Smith (1776) described the control problem in "joint stock" companies. Since that time agency problems have been addressed in detail in the economics literature and now represent a major factor in modern law. The theory of agency is, however, a relative newcomer to corporate finance. The recent contribution to the finance literature most often cited is that by Jensen and Meckling (1976), and that is where this literature review shall begin.

Jensen and Meckling (JM) focused on the agency problem arising between stockholders and professional managers, a true agency problem in the sense that in this case there exists a true agent-principal relationship. They sought to integrate elements of agency theory with modern finance and property rights theories in order to prescribe a theory of firm ownership structure. By viewing the firm as a "nexus of contracts"[1] linking a set of human and physical resources, JM demonstrated that the agency problem was really a contracting problem between conflicting parties.

If stockholders could anticipate expropriating behavior on the part of the owner-manager, they would extract a penalty for compensation when pricing the firm's equity. The owner-manager, JM pointed out, might be better off entering into a contract that would guarantee stockholder wealth-maximizing behavior on his part. The contract would be enforced by (1) stockholders monitoring the owner-manager's behavior, and (2) the owner-manager having himself personally bonded. Agency costs were defined by JM as: (1) the loss in selling price for equity

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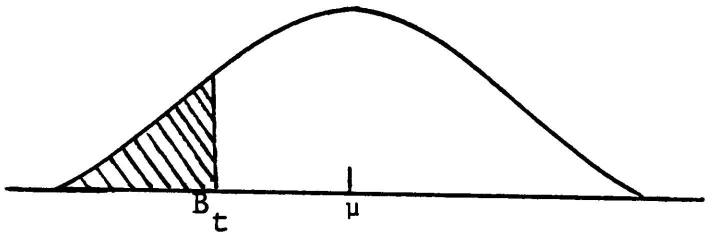
[1]See Jensen and Meckling (1976), p. 311, for a more detailed discussion of the agency theory of the firm.

securities in anticipation of expropriating behavior, (2) monitoring costs, and (3) bonding fees. The relationship between the stockholders and owner-managers presented by JM is directly analogous to that between stockholders and bondholders in a levered corporation, though there is no legal agency relationship in the latter. In the modern corporation, bondholders restrict firm behavior by imposing restrictive covenants in the bond indenture, the primary contract between the parties. Enforcement of the covenants is routinely the task of a trustee, an independent observer charged with serving the bondholders' interests. There is a true agency relationship between the trustee and the bondholders, but that is not of interest in this research. The fee charged by the trustee, borne by the indebted firm, is a monitoring cost. Often the firm's officers must undertake surety bonds as a demonstration of good faith. Expenses associated with these are bonding fees. To make the analogy complete, the bondholders may also elect to penalize the firm <u>ex ante</u> for anticipated agency behavior by paying less for bonds sold by the firm.

JM suggested that the presence of agency costs might justify an optimal capital structure. JM assumed that agency costs arising from the relationships between stockholders and owner-managers were increasing functions of the amount of outside equity financing in the firm. They also assumed that agency costs associated with the stockholder-bondholder conflict were increasing functions of the portion of debt in the capital structure. Given these assumptions, there should be a unique minimum of total agency costs that occurs at an interior debt-equity mix. This was the first demonstration in the finance literature that capital structure and agency costs were related.

Also of interest in this research is another idea put forth by JM dealing with hybrid securities, such as convertible bonds and stock purchase warrants. They suggested that one way of combating the agency problem between stockholders and bondholders was for the firm to issue convertibles. Convertibles would then represent a form of contract that allowed bondholders to exercise contingent claims against equity as a safety valve in the event of expropriation behavior. The use of these securities acts as a force on the firm and on the investor. The firm's management has its incentive to expropriate wealth weakened by the hybrids, and the holders of these securities are less concerned about expropriation due to their claims on equity.

The effect of hybrid securities is made clear by viewing the profits of a firm in the form of a probability distribution, on portions of which the various security holders have claims. In the case of a firm with equity and straight debt maturing at time T, the claims are as depicted in the figure below.



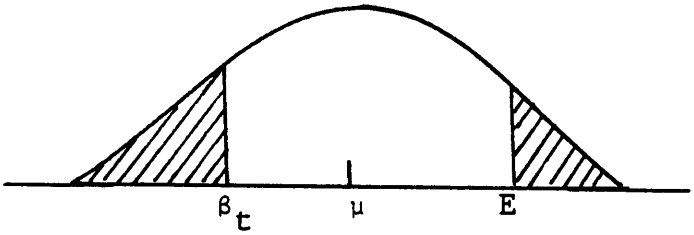
DISTRIBUTION OF CLAIMS IN A FIRM FINANCED WITH STRAIGHT DEBT AND EQUITY

The mean value of firm profits at period T is denoted μ and the face value of debt promised to bondholders at T is denoted B_T. The bondholders have legal claim to the portion of the distribution below B_T, while the stockholders claim all above B_T after paying back the face value of the bonds. Then, if stockholders take certain actions to alter the distribtution of firm value at time T, they can expropriate bond value. This might be accomplished by increasing the dispersion in the distribution.

If bondholders have conversion rights they have claims to both sides of the profit distribution. This is shown in the figure below, where E represents the level of firm value that justifies rational exercise of warrants.

If value were transferred from bonds to stock by a shift in dispersion, the hybrid security holders could exercise their contingent claims against the equity side of the distribution.

Many of the ideas put forth by Jensen and Meckling in their important paper were provocative and have been the seeds of much subsequent analysis.



DISTRIBUTION OF CLAIMS:
EQUITY AND CONVERTIBLE DEBT

Another piece of path-breaking work was published by Galai and Masulis (1976). Their contribution was to unite the option pricing model[1] and the capital asset pricing model.[2] They showed that if common stock were valued in

[1]This is the Black-Scholes Option Pricing Model developed in 1973.

[2]The capital asset pricing model (CAPM) was developed independently by William Sharpe, John Lintner, and Jan Mossin in 1964, 1965, and 1966, respectively.

the market according to the option pricing model, a transfer of value could take place when the firm's variance rate is changed,[1] though overall firm value remained the same. Specifically, if the variance rate were increased, value would be transferred from bonds to stock. If the rate decreased, the transfer would be reversed. Though the option pricing model is restricted to partial equilibrium pricing analysis, Galai and Masulis have illustrated some of the potential determinants of the agency problem in their thorough comparative statics analysis.[2]

The work of Jensen and Meckling has not gone without criticism. Fama (1980) applauded their "striking insight" into the theory of the firm, but asserted that their analysis had not been carried far enough. Fama saw the two main functions of the entrepreneur, management and risk bearing, as separable entities, each with its own market. He argued that if these markets were efficient the entrepreneur would be "disciplined" by the managerial labor market and the capital market, respectively. If the entrepreneur deviated from contract, the managerial labor

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[1]The variance rate is the variance of firm value per unit of time measured on an instantaneous basis.

[2]See cases I through IV, pp. 62-70, Galai and Masulis (1976).

market would adjust in the long run[1] via "ex post settling up," or revisions in the entrepreneur's money wage. It is not clear that the mechanism of "ex post settling up" can be applied to the bond market to resolve the problem between stockholders and bondholders. Whether or not the bond market does in fact fully settle up ex post is, of course, an empirical question. In a multi-period world where trading is allowed, bondholders are not chronologically the same individuals, and this makes identification, hence settling up, difficult. But even if the bond market does settle up, this is not costless to the individual firm. In order to correctly adjust prices paid for bonds, some surveillance is required. Also, an opportunity for a large-scale expropriation could cause an otherwise far-sighted management to forgo long-run considerations, the same as in Fama's case of managerial behavior. This would necessitate bonding costs, at least for very large debt issues.

Fama (1978) had also demonstrated that as long as investors had equal access to the capital markets along with firms, the firm's management decisions could not affect the wealth of investors. This remarkable piece developed the results in the absence of perfect "me-first"

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[1]Fama used an infinite time horizon to fully resolve the problem.

rules. Fama invoked the Coase Theorem (1960) to bring about a unanimous decision in a stable, efficient capital market to "maximize combined stockholder-bondholder wealth." If stockholders and bondholders were different classes of security holders, they would enter into side contracts to reduce expropriation. The Coase Theorem showed that given costless contracting, a farmer and a "cattle raiser" living adjacent to one another would voluntarily enter into agreements to arbitrate the problem of crop damage by straying cows. Unless the contracts can be costlessly let and enforced, however, there is some positive agency cost. The farmers and "cattle raisers" of the Coase example will still enter the contracts as long as the cost of contracting is less than the benefits derived.

Fama's article used the existence of "side-payments" between classes of security holders to resolve the problem. No examples of such payments between stockholders and bondholders appear to be in evidence. This could be because (1) there is no agency problem in the "real world," or (2) there is some other inhibition to side payments. The notion that there are no agency problems is unfounded, but an example of an inhibition to a side payment solution could lie in mistrust among adverse groups of security holders. Fama's argument, therefore, does not convincingly eliminate the agency problem from the real financial markets.

An interesting contribution was made by Myers (1977), wherein he took a fresh approach to the long-standing problem of optimal capital structure. By abandoning the cumbersome and rather dubious assumption of fixed investment policy, he was able to show that the existence of risky debt could induce a sub-optimal investment policy. The difference between the optimal and sub-optimal policies is an agency cost of debt. By viewing the firm as a combination of assets in place and future investment options, he was able to show that the equilibrium value of the firm in perfect capital markets would be affected by the volume of risky debt in the capital structure. Specifically, Myers demonstrated a situation in which the presence of risky debt could induce stockholders to reject investment projects with positive total net present values (NPV). Though it was clearly in both stockholders and bondholders' best interests to maximize total firm value, they generally could not costlessly agree to do so except by shortening the maturity of debt so that it would be retired before investment decisions were made. The remaining ways that bondholders had to insure firm value maximizing behavior on the part of stockholders involved costly contracting, such as restrictive bond convenants on dividends, bonding, and surveillance.

Myers' contribution was extended by Barnea, Haugen, and Senbet (1980) when it was shown that, under similar assumptions, a call provision would achieve the same agency cost reducing results as shortening the maturity.

The most complete single synthesis of modern developments in agency theory to date was also by Barnea, Haugen, and Senbet (1981). These authors classified agency problems in three categories: (1) informational asymmetry whereby managers cannot "reveal the exact nature of the firm to debt and equity financiers (the principals) costlessly," (2) problems associated with the issue of debt financing under limited liability (discussed in detail, below), and (3) excess perquisite consumption by owner-managers.

The category of interest to this research is (2). They divided this further into (a) stockholders' incentives to make wealth-expropriating decisions, (b) incentives to forgo new investments to avoid sharing, and (c) bankruptcy and the costs of settling claims disputes. In a series of arbitrage arguments the authors demonstrated that efficient capital markets would eliminate the agency problems mentioned above.[1] Were capital markets without impediments, the agency problem would cease to be a consideration. The authors describe how various imperfections such as taxes, however, might block the capital market

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[1]See Barnea, Haugen, and Senbet (1981), pp. 11-12, for their arbitrage arguments.

mechanism from solving the problem.[1] Given these impediments they suggested that complex financial contracts might be seen by investors and firms' managers as necessary solutions to the agency problem. This discussion included call provisions on corporate bonds, convertible securities, and "income bonds."[2]

Now that the theoretical developments have been reviewed, the questions that are still unanswered will be summarized. It should be clear that the field is still in adolescence, since the unanswered questions are basic.

First, all of the agency development in the finance literature thus far has assumed a demarcation between stockholders and bondholders. No work has been done on the portfolio investor's problem. Galai and Masulis (1976) mentioned somewhat in passing that the agency problem vanishes when all investors hold the market portfolio. But investors never hold the market portfolio in reality. No diversification strategies short of the market portfolio have been put forth rigorously, or even casually. Also, no examination of the agency problem under investor consumption equilibrium conditions has been attempted.[3]

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[1]See Barnea, Haugen, and Senbet (1981), for examples of blockages.

[2]Barnea, Haugen, and Senbet (1981), pp. 16-18.

[3]Jensen and Meckling (1976), examined the owner-manager's utility problem only.

Second, all the work thus far has been done in a single period environment.[1] New insights may be gained from multi-period analysis. Also, the single period models have called upon full anticipation by bondholders of the possible behavior of stockholders; no informational asymmetry. Is this realistic?

Third, there have been no determinants or measures of the magnitude of the problem set out. What economic variables trigger the agency problem? How large a role does each play? Are all levered corporations having some positive probability of default, even those with identical capital structures and ownership distributions exposed to the same kinds of agency problems?

Finally, no work has been done on optimization in an environment characterized by significant agency problems. What is the optimal portfolio solution given this environment?

This research, of course, does not attempt to resolve all the problems cited above. Instead, a first step is pursued by examining the problem under conditions of an investor equilibrium; that is to be taken up in Chapter Three.

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[1]See Jensen and Meckling (1976) and Mikkelson (1978). Both works dealt with single period horizons.

Review of Empirical Contributions

An in-depth analysis of the provisions of bond indentures was performed by Smith and Warner (1979) to test two competing hypotheses as to how the stockholder-bondholder conflict was resolved. The Irrelevance Hypothesis[1] suggests that stockholder behavior is not altered by the stockholder-bondholder conflict; the choice of financial contracts is irrelevant. The Costly Contracting Hypothses[2] asserts that control of the conflict through contracting can increase firm value. If the latter holds one would expect to see firms undergo costly contractual solutions to the agency problem, such as sinking fund requirements, dividend restrictions, borrowing constraints, investment policy limitations, and so on.

The qualitative work by Smith and Warner lent strong support for the Costly Contracting Hypothesis. One could conclude from these findings that borrowers anticipate agency problems and they often elect to supplement the disciplinary power of the capital markets with separate sets of contracts. This does not necessarily mean that the capital markets are impotent in resolving the problem. It is possible that (1) the markets are better capable of

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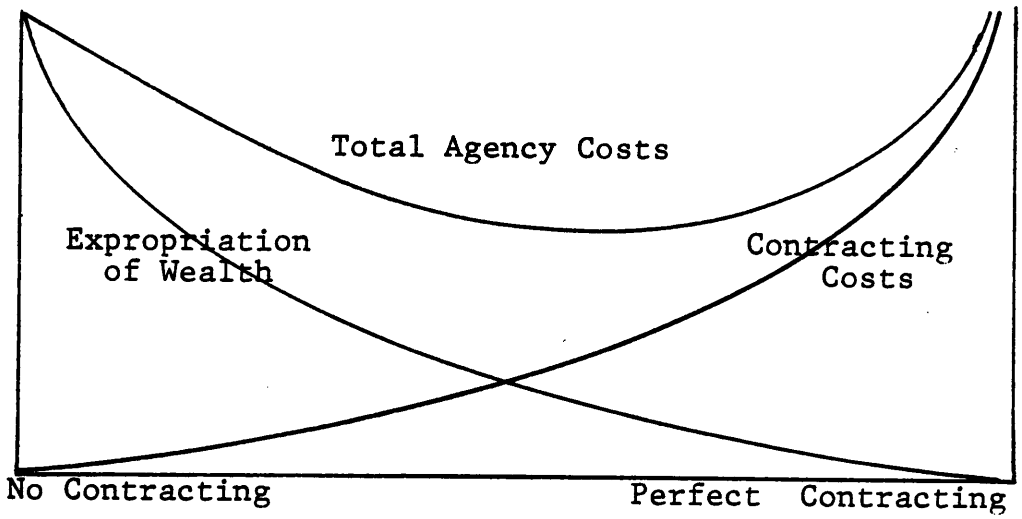
[1]Refer to Smith and Warner (1979), pp. 120-121, for the full definition.

[2]*Ibid.*, pp. 121-122.

resolving the problem now than before, and the contracting mechanism is a lingering device (somehow avoiding social Darwinism), or (2) the mix of contracts and market mechanisms represents a minimum cost solution to the agency problem. The graph presented below shows this. View the contracting decision as a continuous set of possibilities ranging from no contracting at all (allowing the markets to resolve the problem), to perfect "me-first" rules, whereby every contingency is anticipated and neutralized.



MIX OF CONTRACTING AND MARKET DISCIPLINE MEASURES

Contracting costs increase as the parties move to make the contracts "airtight." As they become more encompassing, avenues for wealth expropriation are narrowed until, finally, the contracts perfectly resolve the problem. The shape of the curves is an empirical question, but one might expect the contracting cost curve to increase asymptotically, ruling out perfect "me-first" rules in practice.

Another important contribution was made by Masulis (1980). This work sought to identify the magnitude and direction of value transfers resulting from a reasonably "pure" shift in capital structure--the intrafirm exchange of one security for another. By examining daily returns on common stocks, preferred stocks, and bonds, the author found some evidence of a re-distribution of wealth. The re-distribution effects were generally smaller than the value effects due to changes in leverage tax shields, but they were still significant.[1]

Mikkelson (1978) tested an agency theory of warrant financing.[2] The theoretical model he developed suggested that outstanding warrants exert forces on stockholders to avoid taking actions with adverse wealth effects. If the number of warrants/convertibles is reduced, one would expect to find a positive effect on stock prices. The empirical findings were to the contrary. This and other tests done by Mikkelson are not exhaustive. The results

[1]See Masulis (1980), Sections 5.1.1. for a detailed discussion.

[2]See Mikkelson (1981), for a definition of the methodology.

do not support the theoretical model established in his dissertation,[1] but nor are they convincing evidence that an agency theory of warrant financing is inappropriate.

The empirical work in the area is far from complete. Specifically, more needs to be done in exploring methods for resolving the problem that go beyond bond covenants.

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[1]Mikkelson's dissertation was done in 1978 and the empirical results were updated and reported in 1981. See Mikkelson (1978, 1981).

Chapter Three

THEORETICAL ANALYSIS

As is probably clear from the discussion in Chapter Two, the comprehensive development of agency theory in finance is still in the embryonic stage. The purpose of this chapter is to present two theoretical contributions to the literature.

The first is developed in recognition of the fact that the effects of the agency problem on the most fundamental of economic units, the individual consumer, have not been rigorously set forth. This contribution attempts to model the effects of agency-type wealth expropriations on the consumption and investment decisions of the individual.

The second contribution is the extension of a model of the agency problem developed by Wayne Mikkelson in 1978.[1] His model was derived in a single period state preference world, while the extension takes the analysis into a multi-period setting in which some additional insights are gained.

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[1]See Mikkelson (1978), for the complete work.

An Investor Equilibrium Model in an Agency Setting

This section presents a formally derived mathematical model of the individual's consumption and investment decisions when explicit account is taken of the agency problem. The development proceeds with the fundamental assumption that individual investors faced with competing consumption and investment decisions act in such a way as to maximize utility of consumption of wealth to which they have access during the period of analysis. The utility-of-consumption maximization problem is solved subject to a binding income constraint that forces the individual to optimally divide scarce income between consumption and investment over time.

Assumptions Regarding the Economic World

The economic environment is inhabited by many competitive firms (there are N of them), subscripted "j," and many individuals (n of them), subscripted "i." The lifespan of the world is technically a single period, delimited by two dates, denoted as $t = 0$ and $t = 1$. Each individual enters the economic period of analysis at $t = 0$ with an initial allotment of wealth denoted as y_i for the ith investor. Initial wealth exists in the form of a portfolio of marketable securities, of which there are three types: nondividend paying common stocks, risky pure discount bonds, and securities which are hybrids of the stocks and bonds. No cash is held since liquidation of securities can be accomplished instantaneously and costlessly in the financial markets. The common stocks are included in the model to represent the residual owners' claim to firm output. Bonds are pure discount in that no coupon interest is paid during the period of issue. The existence of bonds provides the potential for expropriation problems. Hybrid securities may be conversion features on the bonds whereby the bonds may be transformed into common stock, or they may be separate instruments that represent firm-issued options on common stock. For consistency throughout this paper, the hybrids will be referred to as "warrants," and the term warrant will be taken to mean any form of hybrid security to include a convertible bond.

Each investor acts in such a way as to maximize expected utility of consumption over the single period, by optimally choosing a balance between immediate consumption and savings for future consumption upon arrival in the period of interest at $t = 0$. The portion of endowed wealth y_i that is not consumed at $t = 0$ is then continued in an investment portfolio. This portfolio of savings is optimally balanced at $t = 0$ in anticipation of expected payoffs on the individual securities at the end of the period at $t = 1$. At this time all portfolio holdings are liquidated and immediately consumed, thus ending the economic world.

Investor i arrives at $t = 0$ with an endowed portfolio of securities in the amounts $\bar{\alpha}_{ij}$, $\bar{\gamma}_{ij}$, and $\bar{\delta}_{ij}$ of stocks (s_j), bonds (b_j), and warrants (w_j), respectively. Since all of the investor's wealth is contained in a portfolio of the three types of securities issued by the N firms, the individual's $t = 0$ initial wealth may be expressed as:

$$Y_i = \sum_{j=1}^{N} \bar{\alpha}_{ij} s_j + \sum_{j=1}^{N} \bar{\gamma}_{ij} b_j + \sum_{j=1}^{N} \bar{\delta}_{ij} w_j \qquad (1)$$

At $t = 0$ each investor consumes some portion of his endowed wealth in the amount C_0^i. What is not consumed at $t = 0$ is, by definition, saved for future consumption by continuing an investment in securities in the revised amounts α_{ij}, γ_{ij}, and δ_{ij} (less the "bars") of stocks, bonds, and warrants, respectively.

The consumption decision at $t = 0$ is thus a reflexive one with respect to the savings or investment decision, since all that is not consumed must be saved. Therefore, individual consumption, individual total demand for securities, and individual income are all bound together in a system such that

$$Y_i = C_0^i + \sum_{j=1}^{N} \alpha_{ij} s_j + \sum_{j=1}^{N} \gamma_{ij} b_j + \sum_{j=1}^{N} \delta_{ij} w_j \qquad (2)$$

for all investors.

The payoffs mentioned above for each security are in effect the market prices of the securities at t = 1, since they are all liquidated and consumed at that time. The payoffs are divided among each jth firm's security holders proportionally to each holder's claim. Payoffs at t = 1, given decisions made by firms at t = 0 are state-contingent. Defining θ as a state parameter, the payoffs are

$S_j(\theta)$ = payoff to firm j stockholders

$B_j(\theta)$ = payoff to firm j bondholders

$W_j(\theta)$ = payoff to firm j warrantholders (or other hybrid security holders)

Since each investor holds the three types of securities in the amounts α_{ij}, γ_{ij}, and δ_{ij}, the total value of the consumable welath (hence the consumption) at t = 1 is given by

$$C_1^i = \sum_{j=1}^{N} \alpha_{ij} S_j(\theta) + \sum_{j=1}^{N} \gamma_{ij} B_j(\theta) + \sum_{j=1}^{N} \delta_{ij} W_j(\theta) \quad (3)$$

Each investor seeks to maximize utility of consumption over the time period subject to the budget constraint. Decisions made by individuals at t = 0 incorporate expected payoffs at t = 1 and known or observed portfolio values at t = 0, hence the total utility function contains a random component for which expectations are formed at t = 0. Investors must then solve an expected utility-of-consumption maximization problem at t = 0, expressed symbolically as

$$\text{MAXIMIZE:} \quad E[U^i(C_0^i, C_1^i] \tag{4}$$

$$\text{Subject To:} \quad Y_i = C_0^i + \sum_{j=1}^{N} \alpha_{ij} s_j + \sum_{j=1}^{N} \gamma_{ij} b_j + \sum_{j=1}^{N} \delta_{ij} w_j \tag{2}$$

Further assume that the total expected utility function is separable into utility of consumption at t = 0 and utility of consumption at t = 1, denoted as V_0^i and V_1^i, respectively. From the perspective at t = 0, utility of consumption at t = 1 is an expectation, therefore total expected utility can be expressed as $E[U^i(C_0^i, C_1^i)] = V_0^i(C_0^i) + EV_1^i(C_1^i)$. Since the function is separable, the utility of consumption at t = 0 does not affect the utility of consumption at t = 1. Each utility component is assumed to be at least twice differentiable with a positive first derivative and a negative second derivative.

The investor's maximization problem is re-written with the utility function expanded and consumption arguments included, below.

$$\text{MAXIMIZE:} \quad V_0^i(C_0^i) + EV_1^i(C_1^i) \tag{5}$$

Subject To: $$Y_i = C_0^i + \sum_{j=1}^{N} \alpha_{ij} s_j + \sum_{j=1}^{N} \gamma_{ij} b_j + \sum_{j=1}^{N} \delta_{ij} w_j \qquad (2)$$

Equating equations (1) and (2) and solving for C_0^i, the following obtains:

$$C_0^i = \sum_{j=1}^{N} (\bar{\alpha}_{ij} - \alpha_{ij}) s_j + \sum_{j=1}^{N} (\bar{\gamma}_{ij} - \gamma_{ij}) bj + \sum_{j=1}^{N} (\bar{\delta}_{ij} - \delta_{ij}) wj \qquad (6)$$

Substituting this expression, as well as that for C_1^i in equation (3) into equation (5), the maximization problem may be written as follows:

MAXIMIZE: $$V_0^i \left[\sum_{j=1}^{N} (\bar{\alpha}_{ij} - \alpha_{ij}) s_j + \sum_{j=1}^{N} (\bar{\gamma}_{ij} - \gamma_{ij}) b_j + \sum_{j=1}^{N} (\bar{\delta}_{ij} - \delta_{ij}) w_j \right] + EV_1^i \left[\sum_{j=1}^{N} \alpha_{ij} S_j(\theta) + \sum_{j=1}^{N} \gamma_{ij} B_j(\theta) + \sum_{j=1}^{N} \delta_{ij} W_j(\theta) \right] \qquad (7)$$

Note that the budget constraint is now embedded in the function to be maximized, thus the investor faces an unconstrained maximization problem.

The Expropriation Problem

At $t = 0$ some unanticipated information arrives in the economic world. The information may pertain to investment and production opportunities available to firms during the period of analysis. Firms may exploit the new information on behalf of their respective shareholders in such a way as to alter the distribution of firm values (payoffs) at $t = 1$, by manipulation at $t = 0$ of some parameter or set of parameters I_j.

The parameter could be the dispersion parameter of the distribution of $t = 1$ security prices. As was mentioned previously, if security prices are set in a Black-Scholes world, a shift in variance could cause a wealth transfer.

The $t = 1$ payoffs on stocks, bonds, and warrants are joint functions of the state of Nature θ and the parameter I_j. For notational convenience, the symbols $S_j(\theta)$, $B_j(\theta)$, and $W_j(\theta)$, with the I_j argument suppressed, will be used to denote payoffs on stocks, bonds, and warrants, respectively.

The changes in I_j are made and announced to all market participants at $t = 0$, hence expectations as to $S_j(\theta)$, $B_j(\theta)$, and $W_j(\theta)$ are revised. These revisions cause a simultaneous change in t = 0 security prices, s_j, b_j, and w_j, for each firm j.

The magnitudes of expected changes in t = 1 payoffs for a given change in the payoff decision variable I_j are:

$$\frac{\partial S_j(\theta)}{\partial I_j} \text{ for stock, firm j} \qquad (8)$$

$$\frac{\partial B_j(\theta)}{\partial I_j} \text{ for bonds, firm j} \qquad (9)$$

$$\frac{\partial W_j(\theta)}{\partial I_j} \text{ for warrants, firm j} \qquad (10)$$

The magnitudes of changes in t = 0 security prices for given changes in I_j are:

$$\frac{\partial s_j}{\partial I_j} \text{ for stock, firm j} \qquad (11)$$

$$\frac{\partial b_j}{\partial I_j} \text{ for bonds, firm j} \qquad (12)$$

$$\frac{\partial w_j}{\partial I_j} \text{ for warrants, firm j} \qquad (13)$$

This representation assumes that the relationship between security prices and the decision variable I_j is a continuous one, at least once differentiable, for each firm j. The decision variable I_j is a parameter (or set of parameters) of a security valuation paradigm and is under the control of the jth firm's management. If the paradigm is such that the signs of equations (8) through (13) are all the same, then there will be unanimous agreement among all security holders that I_j should be changed in response to the new information arriving at t = 0. However, this does not imply that all classes of security holders will agree on the level of I_j. If, on the other hand, manipulation of I_j causes a divergence between equity and bond values, then unanimity among equity holders and bondholders with respect to the direction of change in I_j breaks down, and the conflict is activated.

Expected Utility Maximization

At t = 0 each investor i determines demand for securities by solving the maximization problem in (7). The necessary conditions for an optimum to (7) are given below.

$$\alpha_{ij}: \quad \frac{\partial V_0^i}{\partial C_0^i}(-s_j) + \frac{\partial EV_1^i}{\partial C_1^i}[S_j(\theta)] = 0, \quad j=1,\ldots,N \qquad (14)$$

$$\gamma_{ij}: \quad \frac{\partial V_0^i}{\partial C_0^i}(-b_j) + \frac{\partial EV_1^i}{\partial C_1^i}[B_j(\theta)] = 0, \quad j=1,\ldots,N \quad (15)$$

$$\delta_{ij}: \quad \frac{\partial V_0^i}{\partial C_0^i}(-w_j) + \frac{\partial EV_1^i}{\partial C_1^i}[W_j(\theta)], = 0, \quad j=1,\ldots,N \quad (16)$$

Each set of first order conditions, (14), (15), and (16), includes two terms expressing marginal utility of consumption. The first term in each represents marginal utility of consumption at $t = 0$, while the second term represents marginal utility of consumption at $t = 1$. The optimal portfolio amounts, α_{ij}, γ_{ij}, and δ_{ij}, are those amounts which cause the marginal utility of consumption in both periods to be equal, in effect driving (14), (15), and (16) to zero.

<u>The Agency Problem Affects Utility of Wealth</u>

Were there no agency problem triggered in this model by firms changing I_j at $t = 0$, all investors would act to maximize utility by satisfying equations (14), (15), and (16), for all N firms. The next task is to examine the effects of the agency problem on the individual investor's consumption and investment decisions by formally admitting the agency-type expropriations brought on by changes in I_j.

Before proceeding with the mathematics it may be beneficial to describe the operation verbally. Investor i acts to maximize expected utility at $t = 0$ by satisfying equations (14), (15), and (16). Upon entering the period of economic interest at $t = 0$, investor i holds a portfolio of securities whose prices were derived by all investors based on anticipation of levels of I_j ($j = 1,\ldots,N$) announced by firms prior to $t = 0$. At $t = 0$, each investor i consumes an amount C_0^i that is optimal in terms of expected utility of consumption over the period, based on expectations of payoffs and security prices formed as a result of <u>revisions</u> in I_j announced at $t = 0$ by all firms. The revisions in I_j are motivated by the arrival at $t = 0$ of unanticipated information that describes investment opportunities which may benefit stockholders to the detriment of bondholders.

A change in I_j can alter each investor's decisions regarding optimal consumption and investment from what they would have been in the absence of the new information; these changes come about due to changes in security prices as shown in equations (8) through (13). The changes in I_j thus precipitate a disturbance in the general equilibrium that <u>would</u> have been in existence had the changes not been made. Observe that the equilibrium that would have obtained with no manipulation of I_j is merely an abstraction for the sake of analytical convenience--it never really occurs. Instead, as I_j is shifted, trading occurs and new market clearing prices are set based on changes in I_j announced at t = 0.

Equilibrium Conditions are Formally Disturbed

The effect on the ith individual's expected utility of a change in I_j is found by differentiating the total expected utility function (7) with respect to I_j. This derivative, shown below in (17), reveals the ith investor's preferences regarding changes in I_j. If, for example, an increase in I_j causes a transfer of value from bonds to stock, an investor who holds more of stock of the jth firm than bonds of the same firm will derive positive expected utility from the shift. This disturbance in the ith investor's equilibrium conditions is a measure of the agency cost associated with a change in I_j.

$$\frac{\partial E[U^i]}{\partial I_j} = \frac{\partial V_0^i}{\partial C_0^i}\left[(\bar{\alpha}_{ij} - \alpha_{ij})\frac{\partial s_j}{\partial I_j} - s_j\frac{\partial \alpha_{ij}}{\partial_{Ij}}\right.$$

$$+ (\bar{\gamma}_{ij} - \gamma_{ij})\frac{\partial b_j}{\partial I_j} - b_j\frac{\partial \alpha_{ij}}{\partial I_j} + (\bar{\delta}_{ij} - \delta_{ij})\frac{\partial w_j}{\partial I_j}$$

$$\left. - w_j\frac{\partial \delta_{ij}}{\partial I_j}\right] + \frac{\partial EV_1^i}{\partial C_1^i}\left[\alpha_{ij}\frac{\partial S_j(\theta)}{\partial I_j} + \gamma_{ij}\frac{\partial B_j(\theta)}{\partial I_j}\right.$$

$$+ \delta_{ij}\frac{\partial W_j(\theta)}{\partial I_j} + S_j(\theta)\frac{\partial \alpha_{ij}}{\partial I_j} + B_j(\theta)\frac{\partial \gamma_{ij}}{\partial I_j}$$

$$+ W_j(\theta) \frac{\partial \delta_{ij}}{\partial I_j} \Bigg] , \quad j=1,\ldots,N \tag{17}$$

Equation (17) is re-written below with terms involving portfolio amounts factored out.

$$\frac{\partial E[U^i]}{\partial I_j} = \frac{\partial \alpha_{ij}}{\partial I_j} \left[\frac{\partial V_0^i}{\partial C_0^i} (-s_j) + \frac{\partial EV_1^i}{\partial C_1^i} [S_j(\theta)] \right]$$

$$+ \frac{\partial \gamma_{ij}}{\partial I_j} \left[\frac{\partial V_0^i}{\partial C_0^i} (-b_j) + \frac{\partial EV_1^i}{\partial C_1^i} [B_j(\Theta)] \right]$$

$$+ \frac{\partial \delta_{ij}}{\partial I_j} \left[\frac{\partial V_0^i}{\partial C_0^i} (-w_j) + \frac{\partial EV_1^i}{\partial C_1^i} [W_j(\Theta)] \right]$$

$$+ \alpha_{ij} \left[\frac{\partial V_0^i}{\partial C_0^i} \left(-\frac{\partial s_j}{\partial I_j} \right) + \frac{\partial EV_1^i}{\partial C_1^i} \left(\frac{\partial S_j(\Theta)}{\partial I_j} \right) \right]$$

$$+ \gamma_{ij} \left[\frac{\partial V_0^i}{\partial C_0^i} \left(-\frac{\partial b_j}{\partial I_j} \right) + \frac{\partial EV_1^i}{\partial C_1^i} \left(\frac{\partial B_j(\theta)}{\partial I_j} \right) \right]$$

$$+ \delta_{ij} \left[\frac{\partial V_0^i}{\partial C_0^i} \left(-\frac{\partial w_j}{\partial I_j} \right) + \frac{\partial EV_1^i}{\partial C_1^i} \left(\frac{\partial W_j(\theta)}{\partial I_j} \right) \right]$$

$$+ \bar{\alpha}_{ij} \frac{\partial V_0^i}{\partial C_0^i} \left(\frac{\partial s_j}{\partial I_j} \right) + \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0^i} \left(\frac{\partial b_j}{\partial I_j} \right)$$

$$+ \bar{\delta}_{ij} \frac{\partial V_0^i}{\partial C_0^i} \left(\frac{\partial w_j}{\partial I_j} \right), \quad j=1,\ldots,N \tag{18}$$

Observe that by the first order conditions for a maximum (equations (14), (15), and (16)), the first three terms of equation (18) are zero. There are three separable effects on utility left and these are housed in the remaining terms of equation (18).

First, there is what is termed a "price effect," or the effect on total expected utility precipitated by changes in security prices assessed at t = 0. Intuitively this means that as each firm makes a change in I_j, there is some re-pricing that takes place. Algebriacally, the "price effect" is:

$$\alpha_{ij} \frac{\partial V_0^i}{\partial C_0} \left(-\frac{\partial s_j}{\partial I_j}\right) + \gamma_{ij} \frac{\partial V_0^i}{\partial C_0} \left(-\frac{\partial b_j}{\partial I_j}\right)$$

$$+ \delta_{ij} \frac{\partial V_0^i}{\partial C_0} \left(-\frac{\partial w_j}{\partial I_j}\right), \quad j=1,\ldots,N \qquad (19)$$

Second, there is what is called a "consumption effect," or the effect on expected utility motivated by changes in t = 1 security payoffs for each <u>jth</u> firm. This is expressed algebraically as:

$$\alpha_{ij} \frac{\partial EV_1^i}{\partial C_1} \left(\frac{\partial S_j(\theta)}{\partial I_j}\right) + \gamma_{ij} \frac{\partial EV_1^i}{\partial C_1} \left(\frac{\partial B_j(\theta)}{\partial I_J}\right)$$

$$+ \delta_{ij} \frac{\partial EV_1^i}{\partial C_1} \left(\frac{\partial W_j(\theta)}{\partial I_j}\right), \quad j=1,\ldots,N \qquad (20)$$

Finally, there is what will be termed a "wealth effect," or the change in utility brought on by a change in the value of the original portfolio; that is, the portfolio in effect when investor i entered the period of analysis. This is actually the change in the budget constraint which is now embedded in the optimization equation. Algebraically, the "wealth effect" is:

$$\bar{\alpha}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial s_j}{\partial I_j}\right) + \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial b_j}{\partial I_j}\right)$$

$$+ \bar{\delta}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial w_j}{\partial I_j}\right), \quad j=1,\ldots,N \tag{21}$$

Recognize that equations (19), (20), and (21) together represent the portion of equation (18) remaining after the first order conditions have been met. Thus far, then, it would appear that the magnitude of the effect on expected utility of the ith investor would be equations (19), (20), and (21) summed over all firms. However, this would be an overstatement of the magnitude of the agency problem, since the capital markets should afford some protection to the individual. Indeed, by assuming that firms do not hold monopolistic access to the capital markets, and that security prices are set competitively, it is shown below that the "price effects" and "consumption effects" vanish, leaving only the "wealth" effect."

First, assume that firms cannot create unique securities. That is to say, the financial markets are complete in the sense that changes in payoffs for one firm's securities can be matched by the individual investor by holding a linear combination of claims on payoffs of securities of other firms. This is called the "spanning condition"[1] and is spelled out algebraically below.

$$\frac{\partial S_j(\theta)}{\partial I_j} = \sum_{j=1}^{N} x_{ij}{}^{(s)} S_j(\theta) + \sum_{j=1}^{N} d_{ij}{}^{(s)} B_j(\theta) + \sum_{j=1}^{N} h_{ij}{}^{(s)} W_j(\theta) \qquad (22)$$

$$\frac{\partial B_j(\theta)}{\partial I_j} = \sum_{j=1}^{N} x_{ij}{}^{(b)} S_j(\theta) + \sum_{j=1}^{N} d_{ij}{}^{(b)} B_j(\theta) + \sum_{j=1}^{N} h_{ij}{}^{(b)} W_j(\theta) \qquad (23)$$

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[1] See Grossman and Stiglitz (1978), for a detailed discussion of spanning.

$$\frac{\partial W_j(\theta)}{\partial I_j} = \sum_{j=1}^{N} x_{ij}^{(w)} S_j(\theta) + \sum_{j=1}^{N} d_{ij}^{(w)} B_j(\theta)$$

$$+ \sum_{j=1}^{N} h_{ij}^{(w)} W_j(\theta) \qquad (24)$$

This condition neutralizes the consumption effect on the individual investor, since he is protected against the effects on consumption at t = 1.

Next, assume that prices set at t = 0 efficiently impound the announced changes in I_j. If the pricing mechanism is working properly, changes in payoffs resulting from changes in the decision I_j should be reflected in prices. This is called the "competitivity condition"[1] and is spelled out below.

$$\frac{\partial s_j}{\partial I_j} = \sum_{j=1}^{N} x_{ij}^{(s)} s_j + \sum_{j=1}^{N} d_{ij}^{(s)} b_j$$

$$+ \sum_{j=1}^{N} h_{ij}^{(s)} w_j \qquad (25)$$

[1]See Grossman and Stiglitz (1978), for a detailed discussion.

$$\frac{\partial b_j}{\partial I_j} = \sum_{j=1}^{N} x_{ij}{}^{(b)} s_j + \sum_{j=1}^{N} d_{ij}{}^{(b)} b_j$$

$$+ \sum_{j=1}^{N} h_{ij}{}^{(b)} w_j \tag{26}$$

$$\frac{\partial w_j}{\partial I_j} = \sum_{j=1}^{N} x_{ij}{}^{(w)} s_j + \sum_{j=1}^{N} d_{ij}{}^{(w)} b_j$$

$$+ \sum_{j=1}^{N} h_{ij}{}^{(w)} w_j \tag{27}$$

The competitivity condition neutralizes the price effect in equation (18). By algebraically substituting the spanning and competitivity conditions, equations (22) through (27), into equation (18), the fourth, fifth, and sixth terms cancel, leaving:

$$\frac{\partial E[U^i]}{\partial I_j} = \bar{\alpha}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial s_j}{\partial I_j}\right) + \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial b_j}{\partial I_j}\right)$$

$$+ \bar{\delta}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial w_j}{\partial I_j}\right) \tag{28}$$

In effect, the spanning and competitivity conditions neutralize the price and consumption effects on the individual's expected utility of consumption, leaving only what was earlier termed the wealth effect. The derivation just completed demonstrates formally that at that point where the investor reaches an equilibrium condition with respect to consumption and savings, there can still be an agency effect on utility of consumption, even if the capital markets exhibit the spanning and competitivity conditions.

<u>Proposition 3.1</u>. Under market conditions in which spanning and competitivity conditions hold, in an economic world characterized by a one period planning horizon, those portfolio holdings in existence at the beginning of the period are the only holdings affected by agency-type changes in value.

<u>Proposition 3.2</u>. Under the conditions specified in Proposition 3.1, portfolio holdings undertaken at $t = 0$, after the announcement of payoff changes is made by the firm, are fully insulated from agency-type changes in value.

The sequence of steps leading to the derivation of equation (28) demonstrates the two propositions mathematically. By inspection of equation (28) it is seen that only original portfolio holdings are left. Another interpretation of this development is that the only investors affected by the agency problem are the "old" security holders. The arrival of new information as to the firm's decisions is instantly impounded in prices, so "new" security holders cannot be affected. Also, it is obvious that "new" security holders will be indifferent as to firm decisions to change I_j as long as such changes are allowed only at t = 0, since the market pricing mechanism is such that t = 0 prices fairly reflect expected t = 1 payoffs.

It should be mentioned that the full derivative (17) of expected utility with respect to I_j reduces to the relatively simple expression (28) only to the extent that spanning and competitivity conditions hold. Though beyond the scope of this effort, an analysis of this more complex case, wherein spanning and competitivity fail, would be of interest.

Determinants of the Agency Problem

In the previous section it was shown that portfolios in existence prior to the firms' value transfer decisions were exclusively affected by these decisions. The magnitude and direction of the change in utility for an individual investor depend on the direction of the flow of value from one class of security to another, as well as the amount of each type security held.[1] To this point the effect has been set out in very general terms. At this

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[1]It is possible that the debt and equity securities will show price changes in the same direction, thus reducing the conflict.

time some new assumptions regarding the pricing of securities are brought in so that the specific agency problem of interest in this research can be developed.

The value transfer problem derived in the previous section is triggered by a change in some unspecified decision variable or set of variables that causes an alteration in the distribution of expected t = 1 payoffs. In order to identify the determinants of the agency problem, it will be necessary to specify I_j. This will be done by assuming that firms' t = 1 payoffs on outstanding securities are priced according to a particular valuation function, one that allows divergence of debt and equity values when the parameter I_j is manipulated.

There are probably many types of valuation functions that would serve the purpose at hand. Galai and Masulis[1] formulated the problem in a Black-Scholes Option Pricing world,[2] where a firm's management could cause a shift in the variance of the firm's value,[3] thereby causing a transfer of value from debt to equity or from equity to debt, depending on the nature of the variance shift. The Black-Scholes model, however, depends on some restrictive

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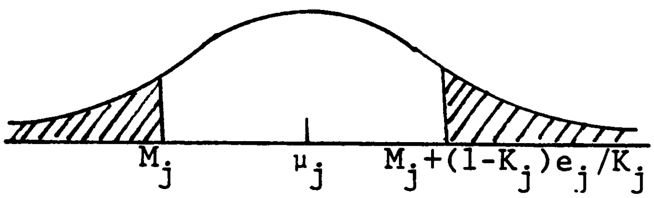
[1]Galai and Masulis (1976), pp. 62-69.

[2]See Black and Scholes (1973).

[3]Instantaneous variance rate is the actual parameter and this is the variance per unit of time for the firm value computed instantaneously.

assumptions as to the stochastic process generating total firm values at $t = 1$,[1] so a more general (and simpler) valuation function is set forth below.

The valuation function will relate random total firm value (X_j) at $t = 1$ to the $t = 0$ value of the control variable I_j. Assume the function is uniquely described by its first two moments. The function is depicted in the figure below.



Firm j is financed with risky bonds, common stock, and stock purchase warrants. The bonds have promised maturity value M_j and the warrants have a total exercise value e_j. The variable X_j is generated by the probability distribution in the figure. Since bondholders have senior claim on income at $t = 1$, they will receive all of X_j if it falls below M_j, and M_j if X_j lies above M_j. The payoff

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[1]See Merton (1973), for a thorough discussion of the necessary assumptions.

to bondholders may then be expressed algebraically as:

$$B_j = \text{MIN}\ (M_j ; X_j) \qquad (29)$$

If X_j is sufficiently high, warrants will have positive value at $t = 1$. The value of X_j will have to be high enough so that the net amount that goes to equity holders, $X_j - M_j$, is sufficient to justify paying the exercise price e_j. Since warrant holders receive the proportion k_j of net equity value at $t = 1$, in exchange for the exercise amount e_j, the warrants will have positive value if the following holds:

$$X_j > M_j + \frac{(1-K_j)e_j}{K_j} \qquad (30)$$

The payoff to warrant holders is expressed as:

$$W_j = \text{MAX}\left[K_j^*(X_j - M_j + e_j^*) - e_j^*;\ 0\right] \qquad (31)$$

where $$K_j^* = \begin{cases} k_j \text{ if exercised} \\ 0 \text{ if not exercised} \end{cases}$$

$$e_j^* = \begin{cases} e_j \text{ if exercised} \\ 0 \text{ if not exercised} \end{cases}$$

The payoff to stockholders may be expressed as:

$$S_j = \text{MAX}\left[(1 - K_j^*)(X_j - M_j + e_j^*);\ 0\right] \qquad (32)$$

For the sake of simplicity assume that only three states of nature are possible at t = 1. The payoff variable X_j will depend jointly on the state of nature and the decisions made by the jth firm at t = 0. The three possible states of nature (N) are:

$$N = \begin{cases} N^*, \text{ if state 1 (no bond default and optimal warrant exercise)} \\ 0, \text{ if state 2 (no default, no warrant exercise)} \\ -N^*, \text{ if state 3 (default, no exercise)} \end{cases}$$

The state of nature will influence the outcome of X_j according to the following:

$$\tilde{X}_j = \mu_j + \tilde{N}\sigma_j \tag{33}$$

where σ_j is a component of the standard deviation of the distribution of X_j and this parameter is under the control of the firm's management. If the probability of state 1 is equal to the probability of state 3, the value of σ_j can be changed without altering the mean of the distribution, since $E(\tilde{N}) = 0$.

State prices for claims on $1 payoffs in each state are denoted as $\emptyset_1$, $\emptyset_2$, and $\emptyset_3$, for states, 1, 2, and 3 respectively.

The jth firm will be able to alter payoffs to security holders by shifting σ_j. In the analytical development of the preceding sections, firms announced changes in a parameter I_j at t = 0. The parameter σ_j will now be substituted for I_j.

Assume that prior to t = 0 the value of σ_j was $\bar{\sigma}_j$ for the j<u>th</u> firm. Each investor's endowed wealth is then a function of $\bar{\sigma}_j$, since this is the parameter used to value securities prior to t = 0. Assume again for simplicity that the exercise values and bond maturity values were set prior to t = 0 such that:

$$\mu_j < M_j + \frac{(1-K_j)e_j}{K_j} < \mu_j + N^*\bar{\sigma} \tag{34}$$

$$\mu_j > M_j > \mu_j - N^*\bar{\sigma} \tag{35}$$

This assumption has the effect of making state 1 a state wherein exercise of warrants is rational and income is sufficient to fully satisfy bondholders' claims, state 2 a state wherein warrant exercise does not occur, and state 3 a default state whrein warrants are not exercised and bondholders are paid less than M_j.

The state payoffs to stockholders, bondholders, and warrant holders, equations (30), (31), and (32), may be expressed in summary form as follows:

$$S_j(\tilde{N}, I_j) = \begin{cases} (1 - K_j)(X_j - M_j + e_j);\ \tilde{N} = N^* \\ X_j - M_j;\ \tilde{N} = 0 \\ 0;\ \tilde{N} = -N^* \end{cases} \tag{36}$$

$$B_j(\tilde{N}, I_j) = \begin{cases} M_j; & \tilde{N} = N^* \\ M_j; & \tilde{N} = 0 \\ X_j; & \tilde{N} = -N^* \end{cases} \quad (37)$$

$$W_j(\tilde{N}, I_j) = \begin{cases} (K_j)(X_j - M_j + e_j) - e_j; & \tilde{N} = N^* \\ 0; & \tilde{N} = 0 \\ 0; & \tilde{N} = N^* \end{cases} \quad (38)$$

Recognizing that prices of stocks, bonds, and warrants (s_j, b_j, and w_j) are discounted payoffs (S_j, B_j, and W_j), and combining equations (29), (31), (32), (36), (37), and (38), the following $t = 0$ price schedule obtains:

$$b_j = (\emptyset_1 + \emptyset_2)M_j + \emptyset_3(\mu_j - N^*\sigma_j) \quad (39)$$

$$s_j = \emptyset_1(1 - K_j)[\mu_j + N^*\sigma_j - M_j + e_j] + \emptyset_2(\mu_j - M_j) \quad (40)$$

$$w_j = \emptyset_1[K_j(\mu_j + N^*\sigma_j - M_j + e_j) - e_j] \quad (41)$$

In this pricing regime it will now be shown that shifting σ_j will cause a transfer of value among securities, while leaving the total value of the firm intact. Assume that $\emptyset_1 = \emptyset_3$.

The effects of changes in σ_j on individual security prices can be seen by differentiating equations (39), (40), and (41) with respect to σ_j.

$$\frac{\partial b_j}{\partial \sigma_j} = \emptyset\ (-\ N^*) < 0 \tag{42}$$

$$\frac{\partial s_j}{\partial \sigma_j} = (1 - K_j)\ \emptyset_1 N^* > 0 \tag{43}$$

$$\frac{\partial w_j}{\partial \sigma_j} = K_j\ \emptyset_1 N^* > 0 \tag{44}$$

Then, if $\emptyset_1 = \emptyset_3$, and $E(N) = 0$, this is shown to be value preserving by recognizing that the value of firm j (V_j) is the sum of its security prices, hence

$$\frac{\partial V_j}{\partial \sigma_j} = \frac{\partial b_j}{\partial \sigma_j} + \frac{\partial s_j}{\partial \sigma_j} + \frac{\partial w_j}{\partial \sigma_j} = 0 \tag{45}$$

The Costs of Agency

The results of the preceding analytical development may now be used to analyze the costs arising from the agency problem. It should be emphasized that the magnitude of agency costs will be described in terms of the loss of utility suffered by a particular investor i, however, this does not imply that investor i bears the costs of agency Rationally, one would expect investor i to act to shift the costs to the firm either *ex ante* by downpricing bonds at t = 0, or *ex post* by requiring that stockholders restore any reductions in expected firm payoffs. Throughout this paper it is assumed that firms act in the best interests of stockholders, so attempted value transfers will generally proceed from debt to equity. Under the assumption that these value transfers leave total firm value unchanged, any negative agency effect in dollars is exactly offset by a positive effect in dollars. However, this does not imply that aggregate agency costs are zero, since costs are defined in terms of individual utility losses and utility functions are generally heterogeneous.

Recall equation (28).

$$\frac{\partial E[U^i]}{\partial I_j} = \bar{\alpha}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial s_j}{\partial I_j}\right) + \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial b_j}{\partial I_j}\right) + \bar{\delta}_{ij} \frac{\partial V_0^i}{\partial C_0}\left(\frac{\partial w_j}{\partial I_j}\right) \tag{28}$$

Now let I_j be replaced by the specific parameter σ_j in the valuation function equation (33). Then, substituting equations (42), (43), and (44) into (28), the following obtains:

$$\frac{\partial E[U^i]}{\partial \sigma_j} = \bar{\alpha}_{ij} \frac{\partial V_0^i}{\partial C_0}\left[(1 - K_j)\ \emptyset_1 N^*\right] + \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0}\left[(\emptyset_3(- N^*)\right] + \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0}\left[(K_j \emptyset_1 N^*)\right] \tag{46}$$

The total effect on the utility of consumption of investor i is equation (46) summed over all firms.

$$\text{Total Effect} = \sum_{j=1}^{n} \bar{\alpha}_{ij} \frac{\partial V_0^i}{\partial C_0} \left[(1 - K_j)\ \emptyset_1 N^*)\right]$$

$$+ \sum_{j=1}^{n} \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0} \left[(\emptyset_3\ (-N^*)\right]$$

$$+ \sum_{j=1}^{n} \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0} \left[(K_j\ \emptyset_1 N^*)\right] \qquad (47)$$

Now that the agency effect on investor i has been described analytically, the next section will feature an analysis of the effects of diversification on the problem.

Portfolio Effects on Agency Costs

As was mentioned in Chapter Two the theoretical research to date has assumed that investors hold only stocks or only bonds in some firm j. Equation (46) permits the analysis to proceed to the portfolio investor. This is done in the context of a single firm j, though the results can easily be extended to a total effect as in equation (47). The analysis will focus on three categories of bondholders of firm j; those who hold bonds only; those who own stock and bonds; and those who hold bonds and warrants. The emphasis is on bondholders because it is they who suffer the utility effects under the assumptions of the analytical development.

(1) Pure Bondholders

The effect of a value transfer brought on by a shift in the variance of firm value on a pure bondholder is seen by considering equation (46) with $\overline{\alpha}_{ij}$ and $\overline{\delta}_{ij}$ constrained to zero.

$$\frac{\partial E[U^i]}{\partial \sigma_j} = \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0} [\emptyset_3 \ (-N^*)] < 0 \qquad (48)$$

In the case of the pure bondholder the expropriating behavior of the firm has the greatest effect, since no equity securities are held to offset the transfer of value.

Proposition 3.3. Given the assumptions of this chapter, the magnitude of agency costs borne by a pure bondholder is a positive function of the amount of bonds held.

Proposition 3.4. Given the assumption of this chapter, the magnitude of agency costs borne by a pure bondholder is a positive function of the increase in dispersion of the valuation function.

These two propositions are obvious from equation (48) and have the following empirical implications.

(a) It is expected that investors who are pure bondholders will hold smaller amounts of bonds of firms exhibiting high risk of expropriation, i.e., cash flow distributions which can be subjected to relatively high variances.

(b) Pure bondholders of firms with high expropriation risk should be observed to demand greater assurances of protection than pure bondholders of other firms.

(2) Bondholders Who Own Stock

The effect of value transfers on investors who own both bonds and stock is seen by constraining $\bar{\delta}_{ij}$ to be zero in equation (46).

$$\frac{\partial E[U^i]}{\partial \sigma_j} = \bar{\alpha}_{ij} \frac{\partial V_0^i}{\partial C_0} (\emptyset_1 N^*) + \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0} (\emptyset_3 (-N^*)$$

$$\gtreqless 0 \qquad (49)$$

Proposition 3.5. Given the assumptions of this chapter, the magnitude of agency costs borne by investor i, who holds bonds and stock of firm j, is determined by the relationship between $\bar{\alpha}_{ij}$ and $\bar{\gamma}_{ij}$.

The proposition is obvious from equation (49).

Proposition 3.6. Given the previous assumptions, a bondholder of firm j who owns γ_{ij} of the bonds, will suffer lower agency costs if, in addition to bonds, he owns some positive amount of stock of firm j.

This proposition must hold since

$$\bar{\alpha}_{ij} \frac{\partial V_0^i}{\partial C_0} (\emptyset_1 N^*) + \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_o} (\emptyset_3 (-N^*) \geq$$

$$\bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0} (\emptyset_3 (-N^*).$$

An empirical implication of these propositions is that bondholders of firms exhibiting severe agency problems may hold offsetting positions in the common stock of those firms.

(3) Holders of Bonds and Warrants

Bondholders may hold either detachable or non-detachable warrants with their bonds.[1]

(a) Detachable Warrants.

This effect may be seen by constraining $\bar{\alpha}_{ij}$ to be zero in equation (46).

$$\frac{\partial E[U^i]}{\partial \sigma_j} = \bar{\gamma}_{ij} \frac{\partial V_0^i}{\partial C_0} (\emptyset_3 (-N^*)) + \bar{\delta}_{ij} (K_j \emptyset_1 N^*) \tag{50}$$

(b) Non-Detachable Warrants.

In this case equation (46) becomes

$$\frac{\partial E[U^i]}{\partial \sigma_j} = \bar{\lambda}_{ij} \frac{\partial V_0^i}{\partial C_0} (\emptyset_1 (K_j - 1) N^*)$$

$$= \bar{\lambda}_{ij} \frac{\partial V_0^i}{\partial C_0} (\emptyset_3 (K_j - 1) N^*) \tag{51}$$

where $\bar{\lambda}_{ij}$ is the amount of warrant - bond combinations held entering $t = 0$. Equations (50) and (51) lead to the following proposition.

---

[1]Detachable warrants may be traded separately from the bonds, while non-detachable warrants must be transferred with their parent bonds.

Proposition 3.7. Given the previous assumptions, the magnitude of the agency problem for holders of hybrid securities is less than or equal to that for pure bondholders.

This is seen by comparing equations (50) and (51) to equation (48).

Some empirical implications of this proposition are:

(1) Firms with severe agency problems will tend to offer convertible debt as opposed to straight debt, given otherwise identical sets of bond convenants.

(2) The issue of convertible bonds should have some disciplinary effect on firms, since the incentive to transfer wealth is clearly eroded when hybrid securities are outstanding.

More About the Dispersion Parameter σ_j

The act of reducing the payoff to bondholders to some value below M_j constitutes default. It is of interest, then, to examine the effect of changes in σ_j on the default characteristics of bonds of firm j, and to examine the effects of changes in parameters other than σ_j on default, since default is the key mechanism by which the agency problem is triggered.

Consider two aspects of default, the severity or magnitude of the reduction in promised payment to bondholders, and the likelihood or probability of the default event occuring. Both aspects are crucial to determining the potency of the agency problem. In the theoretical model developed in this chapter (equation (33)), the parameter σ_j influences the severity of default only. That is to say, the likelihood of state 3, the default state, is exogenous to the individual firm, but the shortfall of X_j, the severity of default, is influenced by the choice of σ_j. This was formalized in Proposition 3.3.

It is important to show that the firm can manipulate the probability of default, as well as the severity of default, and this will be done with the following discussion.

Corollary 3.3. Given the assumptions stated for the preceding propositions, the potential expropriation of value from bonds to equity is a positive function of the probability of default.

To see this, assume that the t = 1 value of firm j, denoted as X_j, is distributed normally with mean μ_j and standard deviation σ_j. Firm j has risky debt maturing at t = 1 with a face value of M_j. This is depicted in Figure 3.2a, below.

The probability of default under the assumption of function $f_j(x)$ is given by (52), below.

$$\int_{-\infty}^{M_j} f_j(x)dx. \qquad (52)$$

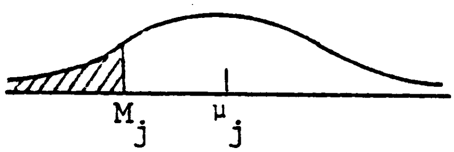


FIGURE 3.1a

NORMAL SITUATION

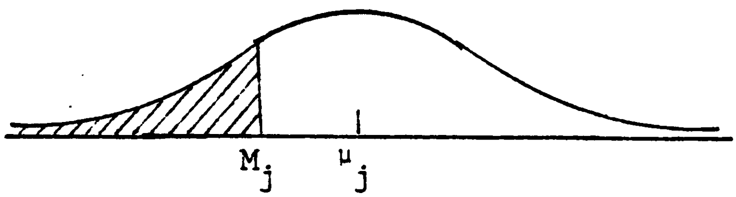


FIGURE 3.1b

INCREASED DISPERSION

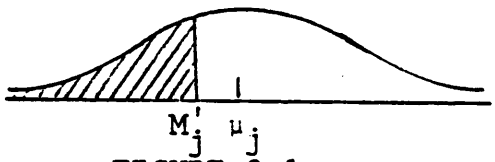


FIGURE 3.1c

INCREASED DEBT

The choice of the normal distribution for this analyisis is not considered to be unduly restrictive. The normal density function provides analytical convenience in that it contains only two moments. Prior to t = 0 the moments, μ_j and σ_j, as well as the parameter M_j are known to the participants in the capital market.

As time t = 0 arrives, the parameter σ_j or M_j may be changed in an effort to transfer wealth from bondholders to stockholders. This presumes that σ_j and M_j are at least partially under the control of the firm's management. It is further assumed that the mean μ_j will not be manipulated. This assumption is made because it is not obvious that such a change in μ_j can simultaneously bring about a wealth transfer and maintain total firm value consistent with the assumption of the preceding theoretical developments.

The firm can increase the probability of default in two distinct ways. First, the firm may choose function $f_k(x)$, with standard deviation $\sigma_k > \sigma_j$ such that

$$\int_{-\infty}^{M_j} f_k(x)d_x > \int_{-\infty}^{M_j} f_j(x)d_x \qquad (53)$$

This is shown in Figure 3.1b.

Second, the firm can increase M_j to M_j', as shown in Figure 3.2c, though continuing the choice of function $f_j(x)$.

$$\int_{-\infty}^{M_j'} f_j(x)\,dx > \int_{-\infty}^{M_j} f_j(x)\,dx \tag{54}$$

Assuming risk averse bondholders, changes in M_j will cause changes in bond prices at t = 0 and t = 1 such that

$$\frac{\partial b_j}{\partial M_j} < 0 \tag{55}$$

and $$\frac{\partial B_j(\Theta, M_j)}{\partial M_j} < 0, \tag{56}$$

where b_j and B_j (Θ, M_j) are prices per pure discount bond at t = 0 and t = 1, respectively. If total firm value is preserved, reductions in b_j and B_j (Θ, M_j) will be offset by changes in equity value, such that

$$\frac{\partial s_j}{\partial M_j} > 0 \tag{57}$$

$$\frac{\partial S_j(\Theta, M_j)}{\partial M_j} > 0 \tag{58}$$

$$\frac{\partial w_j}{\partial M_j} > 0 \tag{59}$$

$$\frac{\partial W_j(\Theta, M_j)}{\partial M_j} > 0 \qquad (60)$$

Without repeating the arguments involving σ_j that led to equation (46) and subsequent equations, it is clear that firms can motivate wealth transfers by manipulation of both the severity and probability of default.

This concludes the analysis in the context of investor equilibrium effects of the agency problem. The findings will be summarized and empirical implications will be highlighted, along with the findings and implications of the multi-period analysis, after the multi-period development is presented in the next section.

Multi-Period Model Development

In this section a multi-period state preference model of the agency problem will be developed. The model is an extension of a previous theoretical contribution and abstracts from asset diversification in assuming that bondholders are necessarily distinct from stockholders [Mikkelson (1978)].

Review of Mikkelson's Single Period Analysis

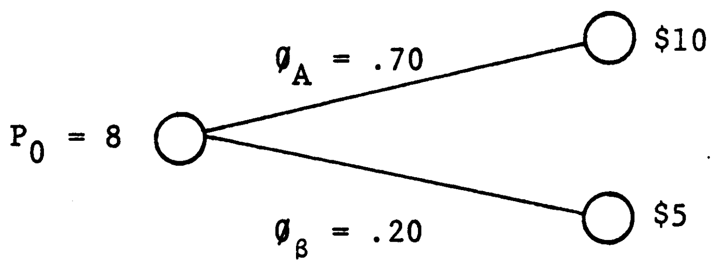
Wayne Mikkelson (1978) developed a two-state, single period model of the agency problem. The key assumptions underlying the model are:

(1) exogenously determined state prices, $\emptyset_A$ and $\emptyset_B$. for states A and B, respectively.

(2) perfect information regarding state payoffs

(3) a capital market that is pricing efficient in that changes in promised state payoffs are translated immediately into revised prices for securities.

The single period is delimited by two time points, t = 0 and t = 1. At t = 0 all investors know the state prices, $\emptyset_A$ and $\emptyset_B$, for promised payoffs at t = 1, and price claims accordingly. An example is presented below in the diagram.



SINGLE PERIOD TWO STATE MODEL

In the example the price of a claim on the state payoffs, denoted as P_o, is formed as follows:

$$P_o = \emptyset_A \text{ (Payoff A)} + \emptyset_B \text{ (Payoff B)}$$

$$= .70\ (10) + .20\ (5) = 8$$

If the state payoffs are altered the prices for claims will change. If the framework adopted by Mikkelson is strictly enforced, the buyer of the claim at t = 0 always anticipates the changes in payoffs and exacts a penalty *ex ante*. In the example above, assume the state B payoff is reduced to zero, while the state A payoff remains the same. Buyers of claims on the security will pay only 7 dollars for the payoffs (.70 (10) + .20 (0) = 7).

Assume the payoffs priced in this regime represent claims arising from a risky bond, hence the owners of the claim are bondholders. If they anticipate the changes in payoffs described above and reduce the t = 0 price of the bonds from $8 to $7, the difference of $1 is the agency cost. Under the assumptions of perfect information and efficient capital markets, the *seller* of the claims (the stockholders) will bear the full costs of agency, thus the original bondholders in the single period context are fully protected by the market pricing mechanism. As will be shown below, this umbrella of protection may no longer be effective when multiple periods are introduced.

Stockholder Reactions

In the example above, it is rational for stockholders to undertake costly bonding and permit costly monitoring of their decisions in order to insure that bondholders receive the originally promised payoffs, as long as the costs of monitoring and bonding are less than the reduction in bond price at t = 0 (\$1).[1] Specifically, stockholders may be personally bonded as guarantors of the state payoffs, or they may demonstrate their intentions by having their decision-making behavior policed by a neutral third party (a trustee). From the bondholders' viewpoint, it should be noted, it does not matter which course the stockholders choose, as long as the economic world is restricted to single period and the other key assumptions remain intact.

Extension of the Model to Two Periods

The theoretical developments that follow will be set forth in a two period world, however the results may be correctly extended to more than two periods.

Assume a two-period world with an initial time point t = 0, an intermediate time point t = 1, and a terminal time point t = 2. There are two possible states of the world at t = 1, and four possible states at t = 2, as shown in the diagram below.

Note that this model features intertemporal dependency between states, that is, state 1 must occur in order for states 11 and 12 to be possible. Market values of some

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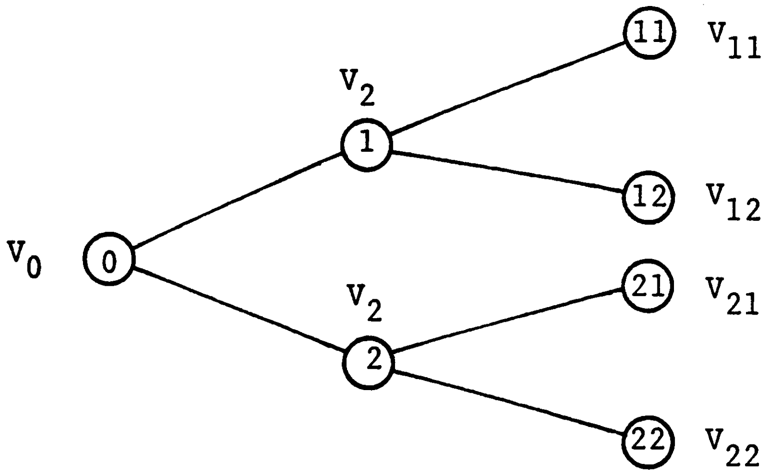
[1]Were the demarcation between bondholders and stockholders relaxed, the incentive of the stockholders to alter state payoffs could be reduced, and the potential harm to bondholders diminished to the extent that investors were diversified between stocks and bonds. That is if $\overline{\alpha}_{ij}$ and $\overline{\gamma}_{ij}$ in equation (49) were equal, the magnitude of ^{ij}(49) would be zero.



TWO PERIOD MODEL

expected payoff at each time period are denoted V_s, where s = 1 for state 1 at t = 1, s = 11 for state 1 at t = 2, and so on.

Assume the payoff scheme above is for a two-period bond that is pure discount in that it does not pay coupon interest. Buyers fully anticipate the payoff scheme and impound this information in the price paid at t = 0 such that

$$V_o = \phi_1\phi_{11}V_{11} + \phi_1\phi_{12}V_{12}$$
$$+ \phi_2\phi_{21}V_{21} + \phi_2\phi_{22}V_{22}, \quad (61)$$

where V_s denotes bond value, given state s, and ϕ_s denotes state price (s = 1,2,...,22). By introducing the intermediate time point t = 1, whereupon a state of nature is revealed, it is shown below that the market pricing mechanism no longer unambiguously serves the interests of a particular class of security holders. In the single period analysis, it is recalled, new security holders were protected and old security holders were affected. In the two-period analysis below, there are two possible patterns of payoffs to bondholders at t = 2, one that was promised at t = 0, and another which may be undertaken by a change in investment decision at t = 1. The originally promised payoffs represent the NON-AGENCY scheme, while the revised payoffs are the AGENCY scenario. There are also two possible reactions of bondholders, in that they may correctly anticipate the AGENCY scenario at t = 0 and price down bonds accordingly (call this ANTICIPATION), or they may be deceived by firms and price bonds at t = 0 based on the originally promised t = 2 payoffs (call this NO ANTICIPATION). There are four possible combinations of these events:

Case	Event
I	ANTICIPATION - AGENCY
II	ANTICIPATION - NON-AGENCY
III	NO ANTICIPATION - AGENCY
IV	NO ANTICIPATION - NON-AGENCY

The analysis will focus on the bondholders who buy at t = 0, since, by assumption, bondholders who buy at t = 1 are aware of firm decisions regarding t = 2 payoffs and price bonds accordingly. It will be shown below, using a numerical example, that Cases I and IV are such that the agency problem vanishes, since actual payoffs are correctly priced in both cases. In Case III the bondholders will suffer agency-type wealth expropriation, and in Case II, the stockholders will be harmed by bondholders who anticipate expropriation, though it never occurs.

Numerical Example

Assume that firm j issues two period risky bonds at t = 0 and that at t = 1 the identity of one of two possible states of nature is revealed. Trading of bonds in an efficient secondary market is permitted.

States are denoted the same as above. Refer to the schedule of equilibrium prices for claims of $1 in various states presented below.

STATE PRICE SCHEME

State	Price
1	.50
2	.40
11	.60
12	.30
21	.20
22	.70

The capital raised from the sale of bonds is used, in conjunction with equity capital, to purchase firm assets that will generate total firm values at t = 2. The t = 2 firm value scheme originally presented to bondholders at t = 0 is as shown in Table 3.1.

This represents the NON-AGENCY payoff scheme. At t = 1 firms may choose to alter the payoff scheme to that indicated in Table 3.2

This pattern reflects greater risk and is deemed the AGENCY scheme. Prices are set at t = 0 based on the scheme that bondholders expect to occur at t = 2, regardless of which scheme actually occurs. If they anticipate the AGENCY scenario, they will price bonds at t = 0 as though the AGENCY scheme will in fact occur at t = 2. If they do not anticipate the agency problem, they will pay t = 0 prices based on payoffs in Table 3.1. The various combinations, Cases I - IV, are presented in Appendices 1 through 4.

In Case I (Appendix 1), bondholders anticipate the expropriation by stockholders and stockholders conform to the expectation by causing the AGENCY scheme to occur. Bondholders therefore buy bonds at t = 0 for $313. If state 1 occurs, the bonds are then worth $450 and bondholders earn a return of 43.77 percent. If state 2 occurs, bonds are worth only $220 and bondholders earn a return of -29.71 percent. Stockholders pay $315 for their claims at t = 0, and if state 1 occurs, they earn a 100 percent return; if state

TABLE 3.1

NON-AGENCY PAYOFF SCHEME

State at t = 2	t = 2 Firm Value	t = 2 Payment to Bondholders	t = 2 Payment to Stockholders
11	1000	500	500
12	1000	500	500
21	600	500	100
22	400	400	0

TABLE 3.2

AGENCY PAYOFF SCHEME

State at t = 2	t = 2 Firm Value	t = 2 Payment to Bondholders	t = 2 Payment to Stockholders
11	1200	500	700
12	1200	500	700
21	400	400	0
22	200	200	0

two they lose 100 percent. The expected single period returns for both bondholders and stockholders are 10 percent.[1]

The same expected return results in Case IV (Appendix 4), wherein no agency problem exists and none is anticipated.

In Case III (Appendix 3), bondholders anticipate the originally promised payoff scheme, but are treated instead to the AGENCY scenario. They pay the full amount for bonds ($377) at t = 0, but at t = 1 they suffer when the AGENCY scenario is revealed. Their state price-weighted return *ex post* is given by:

$$(.50)(.1936) + (.40)(-.4164) = -.0698$$

By successfully deceiving the bondholders, the stockholders need only invest $233 instead of $315.[2] The stockholders' state price-weighted return is:

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[1]This is determined by weighting the state contingent returns by the state prices and summing. For example, bondholders earn 43.77 percent in state 1 and -29.71 percent in state 2. State prices are .50 and .40 for states 1 and 2, respectively.

$$\text{Expected Return} = .50(.4377) + .40(-.2977) = .10$$

[2]The true equilibrium price of equity is $315 and, given many stockholders, this price would ordinarily be obtained in competitive bidding. But, paying more than $233 would signal the AGENCY scenario to the bondholders, therefore the stockholders conspire to hold the price at $233.

TABLE 3.3

CASE PAYOFFS

Payoffs to Bondholders

ANTICIPATION	PRICE PAID AT t = 0	WEIGHTED RETURN NON-AGENCY	WEIGHTED RETURN AGENCY
yes	313	.3045	.1000
no	377	.1000	-.0698

Payoffs to Stockholders

ANTICIPATION	PRICE PAID AT t = 0	WEIGHTED RETURN NON-AGENCY	WEIGHTED RETURN AGENCY
yes	315	-.1603	.1000
no	233	.1000	.4519

.50(1.7038) + .40(-1.00) = .4519

In Case II (Appendix 2), the bondholders price bonds at t = 0 based on the anticipated AGENCY scenario, which does not come to pass. Bondholders pay only $313 (instead of $377) for the originally promised payoff scheme. In order to complete the financing of the firm, the stockholders must pay $315 for equity claims. The weighted reaturns and other relevant information regarding the four cases are summarized in Table 3.3.

The analysis summarized in Table 3.3 leads to the following propositions.

<u>Proposition 3.8</u>. In a two-period world in which investors do not anticiapte agency-type expropriations of wealth in setting initial prices, buyers at t = 0 may be affected. (Case III)

It is important to note that those who buy bonds at t = 0 are "new" bondholders at that time, and if no interim managerial decisions were permitted at t = 1, they would not suffer an erosion of wealth due to the agency problem. This was the result of the single period analysis of the investor's equilibrium problem. The introduction of the interim managerial decision effectively "ages" the bondholders, bringing about the transition from new to old. The two-period model above can easily be transformed into an n-period model, where each period entails a management decision regarding terminal payoffs. Since new bondholders become old as each decision is revealed, then in terms of the final (nth period) payoff on pure discount bonds, the only new bondholders in the period are those who buy at time n - 1. They are the only ones protected by the pricing mechanism. In a continuous time world, wherein states of nature and managerial decisions are revealed continuously, there are no bondholders who are new with respect to the final payoff, because there is always some state or decision that can be revealed at any instant up until t = n.

Proposition 3.9. In a two-period world in which bond investors anticipate agency-type expropriation of wealth in setting initial prices, they may inflict losses upon stockholders by effectively underpricing bonds. This is an agency cost borne directly by stockholders.

Assuming that stockholders are rational, then, upon seeing that bondholders have priced bonds down in anticipation of expropriation, they will oblige the bondholders by then executing the expropriation decisions at t = 1. In effect, Case II will almost certainly never occur, but it is interesting to see that there is a conceivable scenario in which bondholders can reverse the expropriation process.

Summary and Implications of Theoretical Developments

The theoretical results arising from single period investor equilibrium analysis and from multi-period state preference analysis are now summarized. The results are embodied in the following four key points.

(1) The agency problem does not vanish, even in capital markets characterized by conditions of spanning and competitivity. If the capital markets are perfect the magnitude of the derivative of expected utility with respect to I_j, equation (18) is zero, indicating no disturbance in equilibrium. This is true because, in perfect markets, the investor can costlessly contract around the disputes regarding changes in I_j and restore unanimity. If the markets are in disarray, that is if they are pricing inefficient and if firms may have monopolistic access, then equation (18) is non-zero. The key result of the analysis is to locate a degree of efficiency intermediate to the two extremes of perfect and imperfect markets, and to demonstrate that some, not all, of the agency effect on consumption utility is neutralized in the markets. This result is found in equation (28). The single period investor equilibrium analysis also identifies the subset of bondholders who are affected as the "old" bondholders relative to the firm's expropriating decision. The implication is that there exists some middle ground in market

efficiency and completeness in which bond covenants and other forms of costly contracting are theoretically justified.

(2) The costs of agency vanish to the extent that equation (28) approaches zero. However, portfolio adjustments made solely to force equation (28) to zero do not eliminate agency costs. An investor who arranges a portfolio entering t = 0 with the intention of making equation (28) equal to zero will generally fail. The reason is that such a portfolio will generally result in a sub-optimal mix of consumption and savings, causing the "other terms" in equation (18) to be non-zero. The wealth effect may be eliminated, but the "other terms" involving the first order conditions will still be present. This simply represents the substitution of one type of cost for another. The value of hybrid securities is to permit the existence of a greater number of portfolios that are optimal ("other terms" = 0) and, at the same time, offer some reduction in the magnitude of equation (28). This was formalized in Proposition 3.7. Thus the economic rationale for the existence of warrants is not confined to the various popular notions concerning pricing inefficiency in the capial markets.[1]

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[1]Popular reasons for the issuance of warrants and convertible bonds are based on some form of transitory valuation disequilibria, such as a "temporarily depressed" common stock price or a cost of debt that is "too high."

(3) The probability and severity of default are central to the agency problem examined in this research. Thus, if agency problems of this type exist in reality, then security prices must behave as though they were determined by a valuation function having the following properties:

(a) It must have a parameter or a set of parameters that may be manipulated in such a way as to change the probability or severity of default on bonds.

(b) Manipulation of the parameter must cause a divergence between debt and equity values. That is, a function that could be manipulated to increase default probability (and reduce bond values) might also cause a decline in equity value. Ths function would not cause a breakdown in unanimity.

(c) The parameter must be under the control of the firm's management. A parameter such as the rate of interest (risk-free rate in the Black-Scholes model) may cause a wealth transfer, but it is determined exogenously to an individual firm.

The Black-Scholes model meets the three conditions necessary for the agency problem, if the instantaneous variance rate of total firm value can be shifted by the firm's management to transfer wealth from bondholders to stockholders. The simple stochastic function in equation (33) also meets the three conditions, and in this case, there are two parameters, σ_j and M_j, which may be manipulated.

(4) In a multi-period economic world, new bondholders at t = 0 may also suffer expropriation when managerial decisions are permitted at subsequent periods, since the decision event effectively "ages" them. If the arrival of information conducive to expropriation follows a continuous time process, there are no bondholders who can be considered "new" throughout the period. Therefore, all bond investors, not just old investors, will rationally demand protection.

This suggests that the number of decision periods has a positive effect on the agency problem. The longer the time-to-maturity, one would expect, the more possible decision periods there are. Hence, time-to-maturity is, by induction, a factor that influences the severity of the problem.

The preceding summary shows how important underlying assumptions regarding asset pricing and information arrival can be. The problem can be made to vanish, at least for some investors, by assuming a pricing regime that does not conduce to a breakdown in unanimity, or by assuming an information and decision-making structure that is discrete.

This concludes the theoretical development. An empirical test of some of the theoretical implications of this chapter is taken up next.

Chapter Four

AN ECONOMETRIC MODEL OF THE CONVERTIBLE BOND FINANCING DECISION

Bond investors reacting rationally to the agency problem will arrange to shift the costs of agency to firms. Firms, responding rationally, will act to minimize the costs of agency by appealing to one or more of the numerous ways in which bond values can be protected. In the modern U.S. corporation the protective covenants agreed to by bondholders and firms are encoded formally in the original bond indenture.[1] As was brought out in Chapter Three, one of the many vehicles available for agency cost reduction is the issue of hybrid securities of debt and equity. This method is chosen for an empirical test of some of the theoretical developments of Chapter Three for three reasons: (1) an agency cost explanation of the firm's decision to use hybrid securities (convertible bonds) will constitute further empirical evidence of a breakdown of capital structure irrelevance; (2) convertible bond issues are easily observed and data is readily available; and

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[1]See Smith and Warner (1979), for an excellent description of the various covenants found in bond indentures.

(3) the only known previous test of an agency cost motive for hybrid security financing failed to show the hypothesized relationship. Mikkelson (1978) reasoned that if warrants dampened the effects of value transfers from debt to equity, as has been brought out in this research, then a reduction in the amount of warrants outstanding for a given firm should precipitate an increase in stock value. His empirical findings showed a decrease in stock value, contrary to the research hypothesis. Mikkelson made no attempt to control for other factors that may influence the use of warrants, so his results do not represent a strong rejection of the agency cost rationale. In contrast, the research at hand will attempt to control for non-agency causal factors.

Theoretical Model of Agency Costs

In this section the results of Chapter Three are used to parameterize an agency cost function that would face firms given the array of assumptions made regarding asset pricing. Once these parameters are established an econometric model of the firm's agency cost reducing behavior via convertible financing will be derived.

Assumptions

(1) Investors do not hold equally offsetting positions in debt and equity of firms having positive agency costs, hence there exist dichotomous sets of stockholders and bondholders.

(2) The capital markets provide some degree of protection in that spanning and competitivity conditions hold, however contracting is costly.

(3) Disciplinary mechanisms exogenous to the capital markets, such as governmental agencies, fail to fully protect bondholders.[1]

The Model

Recall equation (28) from Chapter Three.

$$\frac{\partial E[U^i]}{\partial I_j} = \bar{\alpha}_{ij}\left(\frac{\partial s_j}{\partial I_j}\right) + \bar{\gamma}_{ij}\left(\frac{\partial b_j}{\partial I_j}\right) + \bar{\delta}_{ij}\left(\frac{\partial w_j}{\partial I_j}\right) \tag{28}$$

It was shown in Chapter Three that the decision variable I_j in (28) could be σ_j in the valuation model in (33), or it could be M_j, the amount of debt outstanding. More specifically the parameter σ_j can be interpreted as the variance of the rate of return on total firm value, since in the single period model (33) total firm value and terminal cash flow are the same. The interpretation of σ_j as the dispersion in total firm value also has the effect of making the analyses compatible with valuation of debt and equity according to the Black - Scholes (1973) Option Pricing Model.

In a multi-period setting firm value in year t and cash flow in year t are not the same. Since σ_j is correctly

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[1]Agencies include the Securities Exchange Commission, Justice Department, Interstate Commerce Commission, and others.

interpreted in terms of firm value and not cash flow, a problem arises in the actual measurement of the parameter. This will be outlined in detail in Chapter Five.

A third factor, arising from the multi-period analysis, is time-to-maturity of debt (T_j). While σ_j and M_j enter equation (28) directly, T_j enters implicitly by increasing the likelihood that firm j will find it profitable to stockholders to change σ_j or M_j. That is, the more intermediate time points included in the multi-period model, the more managerial decisions allowed which could, given appropriate information, result in expropriation.

A fourth agency cost factor is also obvious from equation (28). The degree of diversification of the investor's portfolio with respect to debt and equity of firm j is of much importance since the agency effect diminishes to the extent that equation (28) is zero.[1]

Let C denote the universe of possible contractual methods of achieving reductions in agency costs. The cost of contracting is a function, denoted as Γ, of the contract set C. The cost of contracting is thus $\Gamma(C)$. The agency costs suffered by the firm, namely the erosion of firm value, monitoring costs, and bonding fees, are a function of the four variables σ_j, M_j, T_j, and

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[1]Diversification is not a general solution, but the utility preferences of some investors may lead to greater diversification than others.

diversification, as well as the contracting set chosen to reduce the costs.

$$A_j = f(\sigma_j, M_j, T_j, Div_j, C) \qquad (62)$$

where A_j = dollar amount representing expenses (or loss in value) imposed on firm j as a result of the agency problem

Div_j = degree of diversification of bondholders[1] of firm j

other terms are as defined previously

The firm acts to minimize the total costs associated with the agency problem, equation (62) and $\Gamma(C)$. So the minimization problem is given by:

$$\text{MIN:} \quad f(\sigma_j, M_j, T_j, Div_j, C) + \Gamma(C) \qquad (63)$$

Of interest to this research is a specific subset of C, namely the issuance of convertible securities. This is taken up next.

The Role of Hybrid Securities in Financial Markets

The theoretical notion that hybrid securities can be used to reduce agency costs is firmly established.[2]

One of the difficulties of empirically testing an agency cost rationale for hybrid financing is that, just

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[1]Further specification of the exact nature of this index is unnecessary since this parameter will be deleted from the econometric model.

[2]See, for example, Jensen and Meckling (1976).

as firms may choose among many agency reducing methods, they may issue hybrids for a variety of reasons apart from agency. Indeed, the agency cost rationale is a relative newcomer to the finance literature. The previous arguments that have been proposed, however, seem to stop short of offering any real economic motives; rather they are based on observed conditions which could be symptomatic of a plethora of economic circumstances.

Perhaps the most popular reason traditionally cited is the familiar "debt sweetener" argument,[1] whereby firms issue convertible bonds and warrants with bonds to secure cheaper debt. What is missed in this explanation is whether the firms' debt costs are high because of general market conditions (macroeconomic reason), because of some firm - specific reason such as agency costs (microeconomic), or both. The macroeconomic reason is dependent upon there existing economic conditions in which the cost of debt is relatively high and the cost of equity is simultaneously low. If these conditions exist, then there is some basis for a timing strategy such that firms may reduce the cost of capital by tapping the appropriate market (debt or equity) at the right time. A casual test was done in this research to see if the aggregate level of warrant and convertible financing showed significant responsiveness to the average

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[1]See Weston and Brigham (1977), and other financial management texts.

cost of corporate debt relative to the average return on equity, over time.

The test was aimed at finding some preliminary evidence of the macroeconomic version of the "debt sweetener" argument, and was done by estimating the following model with ordinary least squares.

$$\tilde{y}_t = \beta_0 + \beta_1 x_t + \tilde{\varepsilon}_t \qquad (64)$$

where y_t = portion of corporate debt issued in year t having warrants or conversion features attached

x_t = ratio of average annual corporate bond yield (based on Moody's Corporate Bond Index) to the average rate of return on the equity market portfolio (Dow Jones Industrial average)

$\tilde{\varepsilon}_t$ = random error term

t = 1957 - 1977

The data regarding the portion of debt bearing warrants and conversion features is from the Securities Exchange Commission. The results of the model estimation appear in Table 4.1.

The overall model is statistically significant at the .10 level, and the sign of the estimated slope coefficient is consistent with the hypothesis, and significant at this level (t = 1.7086). While this casual test does not constitute strong support for the debt sweetener argument, it does support the need for including non-agency variables

TABLE 4.1

REGRESSION RESULTS - DEBT SWEETENER TEST

OUTPUT	ANOVA
$\hat{\beta}_0$ = 8.705	Sum of Squares Regression (SSR) = 101.635
$\hat{\beta}_1$ = .9496	Sum of Squares Residual (SSE) = 661.494
	Total Sum of Squares (SST) = 763.129

df_r = degrees of freedom - regression = 1

df_e = degrees of freedom - error = 19

F = 2.919255

in any model of the firm's decision to issue convertibles or warrants.

Another popular rationale is that warrants and convertibles are a method of raising "delayed equity" capital.[1] A firm in need of capital may determine that an issue of stock may be underpriced by the market, so it chooses to issue hybrids in anticipation that they will be exercised when the price of stock is restored. As in a case of the debt sweetener argument, this thesis does not specify whether stock prices in general are "temporarily depressed" (macroeconomic version), the equity of the particular firm is undervalued (microeconomic reason), or both.

One published empirical test failed to support the macroeconomic version of the delayed equity thesis.[2] This test featured a regression of monthly stock market returns against the number of convertible bonds issued during each month from 1965 to 1972. Since it takes several months to clear a major debt issue through the Securities Exchange Commission, and it takes a considerable amount of time to coordinate such a sale with investment bankers, it would seem that monthly data would not provide a fair test. Also, the number of convertible bonds is probably a poor

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[1]See Brigham (1965).

[2]See Alexander, Stover, and Kuhnan (1978), for details of the test.

statistic, since this does not consider the total amount of borrowing in the test period. A similar test was performed in this research using yearly data from 1901 to 1978. The response variable was the ratio of convertible debt to total debt issued each year in order to control for the volume of borrowing. The following model was estimated.

$$\tilde{y}_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \hat{\varepsilon}_t \qquad (65)$$

where y_t = portion of debt issued in year t bearing warrants or conversion features

x_t = return on the Dow Jones Industrial Index, year t

x_{t-1} = market return lagged one year

The results of the regression are contained in Table 4.2.

The results of the regression are significant at the .16 level, but the signs of the slope coefficients are both contrary to the macroeconomic version of the hypothesis.

The debt sweetener and delayed equity theses have emerged mostly from surveys of financial managers, who report that their motives for warrant issue are associated with the delayed equity argument about 70 to 80 percent of the time. They report that the debt sweetener argument is their motive about 25 percent of the time.[1]

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[1]See Brigham (1965); Broman (1963); Hoffmeister (1977).

TABLE 4.2

REGRESSION RESULTS - DELAYED EQUITY THESIS

OUTPUT		ANOVA	
β_0 = 9.1839264	SSR	189.17096	df_r = 2
β_1 = .0518075	SSE	3573.77249	
β_2 = .0517950	SST	3762.94347	df_e = 72
F = 1.90559			

Some of the professional investment literature also suggests that warrants are issued merely because they are popular among investors at the time. Thus, there may be a marketing motivation behaind the choice of warrants and convertible bonds, in addition to the other two arguments, but this argument also falls short in that it does not pentrate the reasons for popularity.

To date, no successful comprehensive test of any of the asserted reasons for issuing warrants and convertibles has appeared in the finance literature, and this includes Mikkelson's (1978) test of an agency cost rationale. Now that the role of hybrid securities in reducing agency costs has been theoretically established, and popular rationale have been examined, a test will be proposed that will include all of the hypotheses concurrently in an econometric model of the firm's decision to issue convertible versus nonconvertible (straight) bonds. This model will include both microeconomic and macroeconomic variables and will be formally set forth in the following section.

The Econometric Model

In this section an econometric model of the convertible bond decision will be developed. The model will include those agency cost parameters from equation (62) that are considered observable, along with variables intended to measure the popular reasons that were discussed above.

Denoting the agency cost parameters of firm j as A_j, and the popular variables as P_j, the firm's decision to issue convertible bonds may be expressed in stochastic form as:

$$\tilde{w}_j = f_1(A_j) + f_2(P_j) + \tilde{\varepsilon}_j \tag{66}$$

where $\tilde{w}_j = \begin{cases} 1 \text{ if convertible} \\ 0 \text{ if straight} \end{cases}$

$\tilde{\varepsilon}_j$ = random error term

The econometric model may be expressed in matrix form as follows:

$$y = x\beta + \varepsilon, \tag{67}$$

where terms are as defined in Chapter One.

Actual measurement of enodogenous and exogenous variables will be treated in Chapter Five.

Chapter Five

MODEL ESTIMATION AND HYPOTHESIS TESTS

This chapter features the results of estimation of the econometric model of the convertible bond financing decision. It will be shown that the exogenous variables as a whole account for a highly significant amount of discrimination between convertible and straight debt financing, while the set of variables designed to measure the agency factors contributes only a marginal amount to explanatory power.

Sample Selection

The model will be estimated using two samples of debt issues: a convertible sample ('CONVERT'), and a nonconvertible sample ('STRAIGHT'). The dependent or endogenous variable will be binary with a value of '1' if the issue is convertible and '0' if straight.

The samples were collected by reviewing issues of Moody's Corporate Bond Survey and are listed in Appendix 5. There are 174 debt issues in the combined 'STRAIGHT' and 'CONVERT' samples. To satisfy data availability constraints only those bonds issued during the period 1962 through 1978 were selected. To control for exogenous factors the issues were restricted to those made by U.S. industrial firms. To make both samples as nearly alike as possible, the list was further reduced to those issues with call provisions, sinking fund requirements, and subordination to existing debt. The intent was to develop two samples that were essentially the same, except for the conversion feature.

Measurement of Variables

The Agency Variables

As was discussed in Chapter Four, there are essentially four distinct factors which trigger or amplify the agency problem: (1) the degree of diversification Div_j of the firm's investors; (2) changes in the maturity value of debt M_j; (3) the variability of the return on total firm value; and (4) the time-to-maturity of the debt issue T_j.

It will be assumed that the degree to which individual investors are diversified between debt and equity of the jth firm is negligible. In terms of direct investment, this assumption has strong support from empirical research.[1] In terms of indirect investment through financial intermediaries it is also apparent that diversifica-

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[1] Blume, Crockett, and Friend (1974) found that investors hold highly undiversified portfolios.

tion is lacking.[1] It is assumed that differences in the levels of investor diversification in the sample firms are insignificant, therefore no variable will be used as a proxy for diversification. This assumption has the effect of forcing the measured effect of diversification into the intercept term. That is, if all of the sample firms experience the same small degree of diversification, this will uniformly bias their financing decision toward convertible bonds, thus increasing the intercept.

The remaining agency factors were measured as described below.

Changes in M_j (Debt-to-Equity Ratio, Variable X_1)

The firm's debt-to-equity ratio is considered to be a valid proxy for M_j, since this normalizes the measure of debt in the capital structure. The behavioral phenomenon of interest is the firm's decision to change M_j and the relevant time at which the decision should be measured is at the issuance of the bonds.

The theoretical motivation for including M_j rests upon the probability of default. If M_j is increased, *ceteris paribus*, the probability of default will increase,

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[1]Casual examination of portfolio holdings of insurance companies, pension funds, and investment firms shows little evidence of diversification between debt and equity of the same firm.

resulting in a wealth transfer. In practice, however, observation of changes in M_j as an agency variable will be obscurred by the fact that existing bondholders will often insist that new debt be issued such that new bondholders' claims are subordinate to old bondholders' claims. Indeed, in the sample of firms in this analysis, all of the bond issues are subordinated. This should have the effect of restricting the probability of default on existing bonds to that which was prevailing prior to the new issue, though the probability of default for the firm in general will increase. The new bondholders, under subordination, must absorb the additional default probability, so there is no erosion of old bondholder wealth. Protection of old bondholders via subordination breaks down, however, to the extent that default by the firm incurs bankruptcy costs or other costs associated with financial distress. If the firm defaults only on new bonds, the old bondholders will suffer to the extent that the resolution of the default erodes total firm value. The significance of bankruptcy costs was estimated to be low by Warner (1977). For the moment it will be assumed that subordination is not a perfect remedy due to bankruptcy costs or other imperfections. Findings on variable X_1 will be interpreted with due caution.

Since the debt-equity ratios for firms vary within industries, the ratio itself would not be comparable from firm to firm, hence the percentage deviation of the debt-equity ratio at the issue date from the previous five-year average is used. The value of debt is the book value of total long-term debt, while the value of equity is the market value of common stock outstanding. For example, Fischbach and Moore, Inc. issued convertible bonds in 1966. The yearly debt-equity ratios for the five-year period preceding the issue date were as indicated below.[1]

DEBT-EQUITY RATIOS FOR
FISCHBACH AND MOORE

Year	Ratio
1962	.0782
1963	.0453
1964	.0957
1965	.0974
1966	.1489

The percentage deviation from the previous average is 59.94, hence the value of X_{j1} is .5994.

Dispersion in the Distribution of Firm Values (Variable X_2)

The theoretical developments of Chapter Three indicated that the firm's ability to manipulate the variance of terminal cash flows was an agency cost factor. In

[1]The source of this data and much of the remaining data is the annual file of Industrial COMPUSTAT Tapes.

Chapter Four, the variance was more specifically defined as the variance of the rate of return on the firm's total value. An empirical measure of such variability is difficult to obtain, since total firm market value, hence the rate of return on that value, is most likely impossible to measure accurately. As a proxy for return on firm value the firm's distribution of cash flows will be used.

An ideal measure of the firm's ability to manipulate the variance of its cash flows would be some index of stationarity of the parameter σ_j over time. Given the number of observations in the available data, however, a stationarity test would be weak. Therefore, as a rough proxy for stationarity, a time series estimate of the standard deviation of cash flows will be used. The relevant cash flow distribution from which to estimate the standard deviation σ_j is that of "unencumbered cash flows" (C_t):[1]

$$C_t = EBIT_t - I_t + Dep_t - \frac{SF}{(1-\tau)} \quad (68)$$

where $EBIT_t$ = earnings before interest and taxes, year t

I_t = interest payments, year t

Dep_t = depreciation charges, year t

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[1]See Martin and Scott (1976), for a complete discussion.

SF_t = sinking fund and other principal obligations, year t

τ = average corporate tax rate for firm j, year t

The value of C_t for each year t is calculated directly from available accounting information included in the annual COMPUSTAT file.

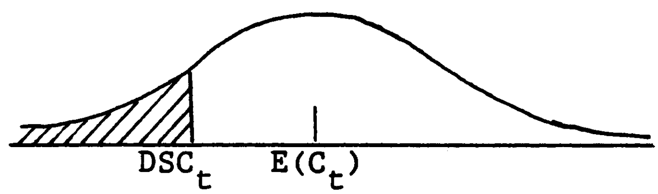
To adjust for differences in scale amoung firms, the estimated value of σ_j is converted to a coefficient of variation by dividing by the sample mean.

For simplicity, the distribution of C_t is assumed to be approximately normal.

Probability of Default (Variable X_3)

Changes in M_j, as well as changes in σ_j will affect the probability and severity of default. Another measure is needed to identify the basic default probability of the firm, and that measure is variable X_3. As Martin and Scott (1976) showed, this may be estimated by first estimating the distribution of "unencumbered cash flows" (equation 68). If the distribution is normal, the probability of default is the portion of the normal density function beneath the minimum required debt service charge (DSC) level.[1] This is shown in the drawing below.

[1]DSC includes interest payments, sinking fund payments, and principal obligations.



DISTRIBUTION OF C_t

The probability of default is the shaded region in the figure. The variable X_3 seeks to measure the probability that the jth firm's average unencumbered cash flow will fall beneath the debt service charge level (DSC). The sample mean and sample standard deviation of the cash flow distribution for each firm were first calculated. If C is normally and independently distributed, as is assumed, then the variance estimate $\hat{\sigma}_j^2$ will have a χ^2 distribution, and the sample mean $\overline{C}$ will have a normal distribution. The value of debt service charges DSC_t is a fixed amount set by the firm's debt contracts, so it is a constant for any period t. Thus, given the distributional and independence assumptions, the probability of default is estimated by evaluating the Student - t density beneath the variable:

$$\frac{\overline{C}_j - \overline{DSC}}{\hat{\sigma}_j / n}$$

where $\overline{DSC}$ = average debt service charge level

n = sample size for estimation of $\overline{C}_j$ and $\hat{\sigma}_j$.

There are three reasons why this variable can be considered an approximation of default probability, at best.

The first is the normality assumption. If C_t comes from a probability distribution with finite mean and variance, then the sampling distribution of $\overline{C}$ (sample mean) will be approximately normal with mean $\overline{C}$ and standard deviation $\hat{\sigma}/ n$, as long as n, the sample size is large.[1] The sample size in most observations in this research is n = 20, so the claim to the Central Limit Theorem is tenuous. Proceeding as though the normality assumption were valid, the Student - t distribution was used for estimating the probability of default. This procedure also requires that the variable $\hat{\sigma}_j$ be distributed independently of $\overline{C}_j$ for all firms j.

The second reason for caution in using X_3 is that the distribution of unencumbered cash flows takes into account flow variables only, ignoring the stock of liquidity that the firm may have, as well as the ability of the firm to borrow or raise equity capital to satisfy debt claims (DSC).

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[1]This is the Lindberg-Levy Central Limit Theorem.

Finally, default on a bond issue may take place at any time during the issue period in which coupon payments or sinking fund payments are due. In order for X_3 to be correctly interpreted as a default probability for time t, the distribution of C_t must be weakly stationary.

Time-to-Maturity (Variable X_4)

The time-to-maturity of the bond issues arose from the multi-period analysis as an agency factor. This particular variable poses a special problem in that it may be endogenous.[1] Bondholders may demand shorter periods of maturity if agency problems are significant, causing X_4 to be dependent upon the indicator variable Y_j, leading to two-way causation. If this is the case, the parameter estimates will be biased and inconsistent.[2] This problem can conceivably be dealt with by using an estimation technique designed to reduce the linear association between X_3 and the random error term.[3] But these methods require that the variable itself (X_3) be explained by a separate model. No such comprehensive model of time-to-maturity is known to

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[1]By endogenous is meant that it not only causes a change in Y, but it itself changes as a result of changes in Y.

[2]See Wonnacott and Wonnacott (1979), pp. 254-255.

[3]Methods such as Two Stage Least Squares and Three Stage Least Squares may be used with the result that the estimates of coefficients will be consistent, though still biased.

exist at this writing, therefore the problem will not be treated.

The Popular Variables

Cost of Debt (Variable X_5)

This variable is intended to measure the strength of the "debt sweetener" argument, mentioned in Chapter Four. The conversion feature is seen as a way to secure cheaper debt, either by a reduction in coupon interest expense, or by a modification of bond covenants. One would expect the firm to consider using convertibles when the cost of debt is relatively high. Moody's Corporate Bond Index is an average yield of selected corporate bonds, so it serves to provide a relative measure of debt costs. Of interest in this research is the relative cost of debt at the time the convertible decision is made. To measure the behavioral phenomenon, it is assumed that firms assess relative debt costs based on immediate past experience. Consequently, X_5 will be measured as the percentage deviation from the previous five-year average cost of debt, as indexed by Moody's Bond Index.

The debt sweetener argument applies to reducing restrictive covenants in the bond indenture, as well as to interest rate reduction, so X_5 may not contain all the relevant information. In anticipation of this, the samples of debt issues were chosen so as to control for the key features of the indentures, in that both 'STRAIGHT' and 'CONVERT' are callable sinking fund debentures.

Cost of Equity (Variable X_6)

According to the "delayed equity" thesis, firms issue convertible bonds when the price of stock is "temporarily depressed." Variable X_6 is intended to gauge the delayed equity motive by measuring the magnitude of a common stock index relative to its previous five-year average. Since all firms in the samples were industrials, the Dow Jones Industrial Average was used.

Market Popularity (Variable X_7)

It is possible that firms issue convertibles because the instruments enjoy good market acceptance at the time. This could be due to some macroeconomic phenomenon such as the personal tax system. In order to test the marketing thesis, the variable X_7 will be the proportion of total corporate debt bearing conversion features issued during the year in which the convertible decision is made.

Research Hypothesis

The major hypothesis to be tested is whether or not the agency variables make a significant contribution to the explanatory power of a model fit with only popular variables. This test will be performed by fitting a model with only the popular variables, then adding the agency variables to assess the contribution to explanatory power.

A set of hypotheses regarding the direction of influence on Y of each of the variables X_1 through X_7 will also be tested. The hypothesized directions of the relationships between the explanatory variables and Y are indicated in Table 5.1.

These hypothesized directions will be tested by examining correlation coefficients between each variable X_1 through X_7 separately with Y.

Estimation Methods

The response variable $\tilde{Y}_j$ may be interpreted as a conditional probability. That is, given the observed values of $X_1,\ldots,X_7$, $\tilde{Y}_j$ is the probability that firm j will choose convertible versus straight bonds. The predicted value of $\tilde{Y}_j$ is, therefore, an estimated conditional probability.

The dichotomous nature of $\tilde{Y}_j$ leads to two nontrivial problems in estimation. First, ordinary least squares regression (OLS) will produce inefficient estimates of the coefficients. This is so because $\tilde{Y}_j$ is a binomially distributed variable, thus the variance of $\tilde{Y}_j$, hence that of the error term ε_j, is a binomial variance and depends upon the particular value of $\tilde{Y}_j$. This will be discussed in detail below. It means that the OLS estimates will be

TABLE 5.1

HYPOTHESIZED RELATIONSHIPS

Variable	Coefficient	Expected Direction of Relationship With Y
Changes in M_j	β_1	> 0
Cash Flow Dispersion	β_2	> 0
Default Probability	β_3	> 0
Time-to-Maturity	β_4	> 0
Cost of Debt	β_5	> 0
Stock Price Index	β_6	< 0
Market Popularity	β_7	> 0

inefficient, though unbiased, due to the absence of homoscedastic error term variance.

The second problem in estimation has to do with the unrestricted nature of the predicted response variable $\hat{Y}_j$. In order for the model to have a logical interpretation, $\hat{Y}_j$ must fall inside the range [0,1], since it is an estimated conditional probability of an event. There are several ways of dealing with this problem, as well as that of heteroscedastic error term variance, and these will be outlined below.

Weighted Least Squares Regression (WLS)

In regression problems involving heteroscedastic error term variance, it is possible to weight the observation on the explanatory variables in such a way as to shrink the estimated standard errors of the coefficient estimates, while retaining the unbiased property. In the case of a binary dependent variable, the weights are the reciprocals of the estimated standard deviation of the error for each observation. That is, for observation j, the variance of $\hat{\varepsilon}_j$ is $Y_j(1 - Y_j)$. The estimated variance is $\hat{Y}_j(1 - \hat{Y}_j)$ and the appropriate weight to use for all the explanatory variables, including the intercept term X_0, is:

$$W_j = \frac{1}{\hat{Y}_j(1-\hat{Y}_j)} \qquad (69)$$

The weights are then multiplied times each observation in the jth observation set. Actual estimation of the $\hat{Y}_j$ values in equation (69) is performed by using a two-stage technique.[1] The estimates of Y_j are first obtained by performing OLS regression on the original data set. This provides unbiased estimates of Y_j to be used in equation (69).

Next the resulting weights, equation (69), are used to weight the explanatory variables in the X matrix, to form the X* matrix. The variable matrix X* is then used in OLS again to form WLS estimates of the regression coefficients. It can be shown that these estimates are "BLUE" and that ordinary statistical hypothesis testing is appropriate.[2]

LOGIT Analysis

There are several ways in which to deal with the problem of $\hat{Y}_j$ falling outside the range [0,1]. An expedient method is to simply truncate the predicted values. More sophisticated alternatives involve transforming the dependent variable in such a way as to force it to lie in the appropriate range.[3]

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[1]See Neter and Wasserman (1974), pp. 326-328.

[2]BLUE estimators are best linear unbiased.

[3]See Pindyck and Rubinfeld (1981), pp. 273-312.

One way to force the predicted value $\hat{Y}_j$ to fall inside the range [0,1] is to use a LOGIT transformation.[1] Recall equation (67).

$$Y = X\beta + \varepsilon \tag{67}$$

Define f as the observed sample frequency of convertible bond issues given particular values of the explanatory variables. Then define $\hat{L} = \ell n(f/1-f)$ as the logit transform of f. Denote the true probabilities of convertible issue given particular values of explanatory variables as P. The logit transform of P is then $L = \ell n(P/1-P)$. The transform of the frequencies $\hat{L} = \ell n(f/1-f)$ is an estimator of the transform of the true probabilities $L = \ell n(P/1-P)$.

The following model is then estimated using a non-linear technique.[2]

$$\ell n \left(\frac{f}{1-f}\right) = X\theta + v \tag{70}$$

where θ = vector of non-linear regression coefficients

v = random error term

---

[1]An alternative to LOGIT and PROBIT, wherein the explanatory variables are assumed to be normally distributed.

[2]The Program used in this work used maximum likelihood estimation.

The expected value of v is asymptotically zero. This is seen by recognizing that

$$v = \ln\left(\frac{f}{1-f}\right) - \ln\left(\frac{P}{1-P}\right)$$

$$= \hat{L} - L \qquad (71)$$

If the observed frequencies f are independent binomial variables,[1] then as the sample size n used to calculate each frequency grows large, v will have a normal distribution with mean zero and variance $1/nf(1-f)$.[2] Since the limiting distribution of v has a mean of zero, there is asymptotic justification for expressing (70) as

$$\frac{f}{1-f} = e^{x\theta} \qquad (72)$$

Therefore,

$$f = \frac{1}{1+e^{-x\theta}} \qquad (73)$$

Equation (73) is then estimated using a nonlinear technique, with the desired result that f will be inside the appropriate range.[3]

---

[1]Actually, f could be drawn from any population having finite moments. See Theil (1971), p. 378.

[2]This follows from Theorem 8.2 in Theil (1971), p. 378.

[3]See Intriligator (1978), pp. 174-175.

In order to justify the LOGIT transformation statistically, the X variables must be stochastic. Indeed, they must be normally distributed.[1]

A second transform of Y is available via the PROBIT model, wherein the transformed value of Y is the cumulative normal density function evaluated at Y from below.

$$P_i = F(Y_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y_i} e^{-y^2/2} dy \qquad (74)$$

where Y_i is standardized to have zero mean and unit variance.

This transform necessarily has the effect of constraining the variable P_i to lie in the range [0,1]. As in the LOGIT model, a nonlinear estimation technique must be used, and the theoretical justification for PROBIT is rather limited.[2]

Estimation Results

Estimation of the econometric model, equation (67), was accomplished using ordinary least squares (LS) regression, weighted least squares (WLS) regression, LOGIT analysis, and PROBIT analysis. The results of model estimation by each technique are summarized in Table 5.2.

[1]See Intriligator (1978), p. 174.

[2]See Pindyck and Rubinfeld (1981), pp. 280-287.

TABLE 5.2

SUMMARY OF ESTIMATION RESULTS

Coefficient	OLS	PROBIT	LOGIT	WLS
β_0	.01022 (.9616)	-1.58020 (.0309)	-2.86280 (.0291)	.01411 (.8280)
β_1	.00499 (.9156)	.05008 (.7718)	.07042 (.8081)	-.02399 (.2932)
β_2	-.00355 (.1988)	-.00993 (.2462)	-.08159 (.2773)	-.00099 (.3296)
β_3	-.11359 (.3524)	-.30332 (.4701)	-.51041 (.4761)	.10111 (.0256)
β_4	.01286 (.1183)	.04141 (.1302)	.07951 (.1068)	.00287 (.3069)
β_5	.63224 (.2719)	3.45200 (.0736)	5.54030 (.0850)	.75395 (.0073)
β_6	1.85457 (.0851)	7.61350 (.0402)	11.45300 (.0726)	1.53136 (.0006)
β_7	2.11974 (.0143)	4.54600 (.1098)	8.06380 (.0958)	-.41332 (.2999)
R^2	.3383	.3405	.3420	.6254

Degrees of Freedom for Partial F-Tests

	OLS	PROBIT	LOGIT	WLS
df(numerator)	1	1	1	1
df(denominator)	166	166	166	163**

*The values in parentheses beneath each estimated coefficient are the significance levels of partial F-tests performed on each coefficient. Each partial F-test provides a significance level for the rejection of the null hypothesis $\beta_i = 0$, given that the remaining variables are in the model. The alternative hypothesis is $\beta_i \neq 0$ $(i=0,\ldots,7)$.

**Three observations were lost since their OLS predicted values were less than zero, hence no appropriate weights could be computed for WLS.

The results of the LOGIT and PROBIT models are quite similar with respect to the portion of total variation accounted for, R^2, and the significance of coefficient estimates. The popular variables, X_5, X_6, and X_7 are statistically significant at levels less than .10, and X_4 is significant at less than .15, in both models. The justification for using LOGIT and PROBIT analysis lies in the fact that $\hat{Y}_j$ can fall out side [0,1]. The LOGIT and PROBIT models do not address the problems of inefficient coefficient estimates. In this particular data set, using WLS and OLS, only 3 of the 174 total predicted observations fell outside [0,1], and the departure was trivial in each case. This weakens the case for PROBIT and LOGIT, and in view of the superior overall fit and the generally lower significance levels, the WLS model is believed to offer the best interpretation of the data.

Analysis

The F-test for overall regression significance for both the OLS and WLS models leads to a rejection of the null hypothesis at better than the .0001 level, therefore it is concluded that some prediction of Y is possible from the variables included in the model. The OLS model accounts for about one-third of the total variation (R^2 = .338259), while the WLS model accounts for about two-thirds (R^2 = .625370). The results of the overall analysis are not surprising. Given the preponderence of survey literature described in Chapter Two regarding motives for issuance of convertible bonds, one would expect to find significant results, particularly with respect to the two leading popular variables, X_5 and X_6. The major question that still remains is whether or not agency costs motivate firms to choose convertible versus non-convertible debt financing.

Of much research interest is whether or not the set of agency variables represents a significant contribution to the explanatory power of the model. The appropriate null hypothesis for this test is:

$$H_0: \ \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$

The appropriate test statistic is:

$$F = \frac{[SSR(full) - SSR(null)]/[df(full) - df(null)]}{MSE(full)}$$

where SSR = sum of squares regression

MSE = mean of square error

df = degrees of freedom; df(full) = 7; df(null) = 3

"Full" denotes the model with all explanatory variables included, and "null" denotes the model with variables X_1 through X_4 deleted.

Substituting the appropriate values,

$$F = \frac{(58.15940827 - 56.7570645)/4}{.21374596} = 1.64$$

The F statistic is distributed with four numerator degrees of freedom and 163 denominator degrees of freedom. The null hypothesis may be rejected at a level of significance of .1666. The null model accounts for a little over 61 percent of the variation in $Y(R^2 = .610291)$, while the full model accounts for about 62.5 percent ($R^2 = .625370$).

The difference in explanatory power as indicated in this test is small. It was anticipated that the agency variables would have made a greater contribution to explanatory power. There are at least three possible reasons for the lackluster results: (1) multicollinearity in the data set; (2) improper selection of proxy variables; and (3) the underlying structure of the convertible bond decision may indicate a non-linear relationship among the variables. Each of these possibilities will be discussed in the following paragraphs.

Multicollinearity

The problem of multicollinearity was examined by determining the correlation matrix for the independent variables. This is presented in Appendix 6. There is clearly some significant interdependence among the variables. In particular, X_5 (average cost of debt) and X_7 (portion of total U.S. debt with conversion features) are highly correlated (r = .91172). Since these are both macroeconomic variables relating to the U.S. bond market over time, the correlation is probably the result of true interdependence between the two. Also significantly correlated are X_4 (time-to-maturity) and X_7 (r = .59713), as well as X_6 (return on the equity marktet) and X_7 (r = .58265). The latter correlation is also likely an indication of a true interdependent relationship, while the former could be unique to this data set. These results indicate that multicollinearity could explain the weak results.

The impact of multicollinearity was further examined by inverting the correlation matrix in Appendix 6. The result is in Appendix 7. The variance of the vector of estimated regression coefficients ($\hat{\underset{\sim}{\beta}}$) is given by:

$$\mathrm{VAR}(\hat{\underset{\sim}{\beta}}) = (x'x)^{-1}\sigma^2$$

Each diagonal element of $(x'x)^{-1}$ is a variance coefficient for a corresponding estimated regression coefficient $\hat{\beta}_i$. If each variable $X_j (j = 1,\ldots,7)$ is standardized by subtracting its sample mean $\overline{X}_j$ and dividing by the estimated standard error $\hat{\sigma}_j$, then the matrix $\underset{\sim}{X}^{*\prime}\underset{\sim}{X}^{*}$, where X^*

is the standardized X matrix, is the correlation matrix of the independent variables. If no multicollinearity exists, the diagonal elements of $(X^{*\prime}X^{*})^{-1}$ will all be 1.0, indicating no inflation in the variance of the estimates $\hat{\underset{\sim}{\beta}}$. To the extent that the diagonal elements exceed 1.0, variance is inflated, and the estimated standard error is inflated by the square root of this amount. The inverse of the correlation matrix in Appendix 7 reveals that there is some significant inflation of variance, particularly among two of the popular variables, X_5 and X_7.

The presence of multicollinearity suggests that (1) the lackluster performance of the agency variables does not necessarily indicate a weakness in an agency cost rationale for convertible bond financing, and (2) signs and magnitudes of individual regression coefficients in the full model do not merit strong interpretation.

To examine the direction of influence that each variable exerts on Y, Pearson correlation coefficients were estimated and the results are presented in Table 5.3.

The variables X_3 through X_7 show significant correlation. Two anomalous results stand out. First, the direction of influence shown for X_6 (return on the equity market) on Y is contrary to the "delayed equity" hypothesis described earlier in Chapter Four. The finding is consistent, however, with the results of the preliminary research done in this effort summarized in Table 4.2, Chapter Four.

TABLE 5.3

ESTIMATION OF CORRELATION COEFFICIENTS

Variable	Estimated Coefficient	Significance Level*
X_1	.01370	.8576
X_2	-.06148	.4203
X_3	-.16075	.0341
X_4	.20711	.0061
X_5	.45588	.0001
X_6	.42855	.0001
X_7	.53735	.0001

*Significance level for rejection of the null hypothesis $\rho_{xy} = 0$. Sample size is n = 174. This is a two-sided test.

The second anomalous finding is the direction of influence on X_3 (probability of default) on Y. The estimated correlation coefficient (r = -.16075) is significant at the .0341 level, but the direction of influence was hypothesized to be positive. The finding may be the result of the weaknesses of the proxy variables which have been discussed earlier in this chpater.

Choice of Proxy Variables

Variables X_1 and X_2 are insignificant and this may be due to weaknesses in proxies. Changes in M_j (variable X_1) should change default probability, thereby resulting in a wealth transfer. As was mentioned earlier, however, subordination of the new issues could preclude the erosion of old bondholder wealth. One, therefore, cannot strictly conclude from Table 5.4 that changes in M_j are unrelated to the agency problem.

The difficulties in measurement of X_2 were discussed in detail earlier in this chapter. Improvement of the choice and measurement of proxy variables could strengthen the results, and this task is left to future, more comprehensive research.

Theoretical Structure

It is also possible that the linear structure examined in this research does not suit the true theoretical structure underlying the convertible financing decision. The variables may influence the decision in non-linear fashion. No attempt was made in the theoretical development to specify a particular causal structure. The linear model was chosen for its simplicity. Structural specification is left as a possible task of future research.

<u>Summary of Estimation Results</u>

(1) The model formally developed in Chapter Four, estimated with variables measured as described in this chapter, accounts for a highly significant amount of the discrimination between convertible and straight bonds.

(2) The cost of debt (variable (X_5)) is highly significant in the model and this supports the survey literature regarding the "debt sweetener" argument.

(3) The set of agency variables contributes only a marginal amount to explanatory power of the model. The time-to-maturity variable (X_4) is positively associated with the decision to issue convertible bonds and this is consistent with the theoretical development. The probability of default (X_3) seems to be negatively associated with the convertible decision when examined independently (see Table 5.4). Either the theoretical development has missed something, or the proxy is bad for variable X_3. The overall results show that there is at least a modicum of evidence supporting an agency cost rationale for convertible financing.

(4) The return on the stock price index (variable X_6) makes a contribution that is in conflict with a priori theoretical judgment. This finding remains unexplained.

(5) The intercept term is not significant in any of the estimated models. As was noted previously, lack of investor diversification should be reflected in the intercept. The result could be due to multicollinearity or it may reflect that the level of investor diversification is heterogeneous among firms.

Chapter Six

SUMMARY AND CONCLUSIONS

Summary of Theoretical and Empirical Findings

The analysis of the agency problem in terms of its effects on the investor's utility of wealth produced some interesting results. As is well known, the firm's decision to manipulate I_j does not enjoy unanimous security holder agreement if the valuation function of which I_j is a parameter allows divergence of debt and equity values. Capital markets in which spanning and competitivity conditions hold are not sufficient to bring about unanimity.

The markets permit a reduction in the magnitude of agency costs, but unless the markets are perfect and complete, there are positive costs as measured by a reduction in utility of wealth for some investors. To the extent that the capital markets may not be spanned, or that they lack competitivity, the agency problem is more severe. Of course, if the markets are complete in that contracting and takeovers are costless, the problem vanishes. The severity of the agency problem may be seen as a step function of capital market efficiency and completeness, given that I_j is a parameter of a valuation function that allows divergence of security values. If capital markets are complete in that contracting around the agency problem is costless, the problem vanishes. That is:

$$\frac{dE[U^i]}{dI_j} = 0$$

If the capital markets are not complete in the strict sense, but exhibit spanning and competitivity conditions, the pricing and consumption effects of equation (18) vanish leaving:

$$\frac{\partial EU^i}{\partial I_j} = \bar{\alpha}_{ij} \frac{\partial VO^i}{\partial CO}\left(\frac{\partial s_j}{\partial I_j}\right) = \bar{\gamma}_{ij} \frac{\partial VO^i}{\partial CO}\left(\frac{\partial b_j}{\partial I_j}\right) + \bar{\delta}_{ij} \frac{\partial VO^i}{\partial CO}\left(\frac{\partial w_j}{\partial I_j}\right) \quad (28)$$

Finally, if the markets are not characterized by competitivity and spanning, at equilibrium, the full differential, equation (18) obtains.

In general, equation (18) is greater in absolute value than equation (28), and both equations are generally nonzero.

Viewed from the perspective of the firm, conditions in which the agency problem of the sort examined in this research may fluorish are:

(1) Existence of competing claims on value.

(2) Positive probability of default on promised payments to one or more classes of security holders.

(3) Limited stockholder liability for promised payments.

(4) Firms act in such a way as to maximize shareholder wealth.

From the perspective of the capital markets, the conditions under which the agency problem will exist are:

(5) Incompleteness in that contracting is not costless.

(6) Securities are valued in such a way that manipulation of some parameter I_j by the firm can cause a divergence in debt and equity values.

The next task in the theoretical development was to describe some valuation function that satisfies condition (6), above. Assuming that firms can control the variance rate of their respective future values, the Black - Scholes model is one such paradigm. Another is the function, equation (33). From this analysis it was shown that changes in M_j or σ_j could trigger the agency problem. Finally, if time periods are interpreted as periods of opportunity in which firms may find it advantageous to alter the parameters, time-to-maturity was also shown to be a factor.

The impact of the effects of the agency problem were analyzed with respect to their effects on the utility of the individual investor, since this is where the costs originate. But, this is not to say that investors bear the costs of agency. The costs are transferred by rational bondholders to stockholders. A way was chosen for empirically testing the agency variables outlined in Chapter Four.

It was shown theoretically that hybrid securities could be used to reduce agency costs, and this led to an econometric model of the firm's decision to issue convertible bonds. The agency variables made a marginal contribution to the explanatory power of the model, providing limited evidence that agency costs influence the firm's convertible bond financing decision, hence the choice of capital structure.

Directions for Future Research

The work that has been completed in this effort suggests some possible extensions.

(1) Extend the investor equilibrium model to multi-period. This could conceivably be accomplished with a dynamic programming approach and might prove useful in that some additional insights could arise.

(2) Incorporate taxes in the analysis. It is quite possible that corporate and personal taxes play a significant role in the agency problem and the method chosen by firms and investors to deal with it. From an empirical viewpoint, taxes might provide additional explanatory power to the econometric model.

(3) Improve the proxy variables in the econometric model. Many of the proxies chosen for this research are *ad hoc*. The measure of default probability, though most significant statistically among the agency variables, lacks strong theoretical support. The measures of the cost of

debt and the price of stock are based on arbitrary assessments as to how managers form opinions. Perhaps a more elaborate model of expectations would improve the overall fit.

(4) Attempt to determine the structural relationship between the convertible decision and the explanatory variables. The linear relationship assumed in this research was chosen for simplicity, in the absence of any competing structure resulting from the theoretical development. Perhaps the best approach in this case would be to estimate the model under a variety of structural assumptions and select the estimated model with the greatest explanatory power.

(5) Examine why X_6 is positively related to the convertible decision. Two possible reasons are: (1) the proxy misses the manner in which managers form their perceptions as to relative equity prices, or (2) perhaps X_6 should represent the relative price of the j*th* firm's stock, as opposed to that of the stock market as a whole.

This effort represents only a small step forward. The suggestions for future research enumerated above represent extensions only of this very narrow examination of the agency problem, and the opportunities for more advanced research in this intriguing area are many.

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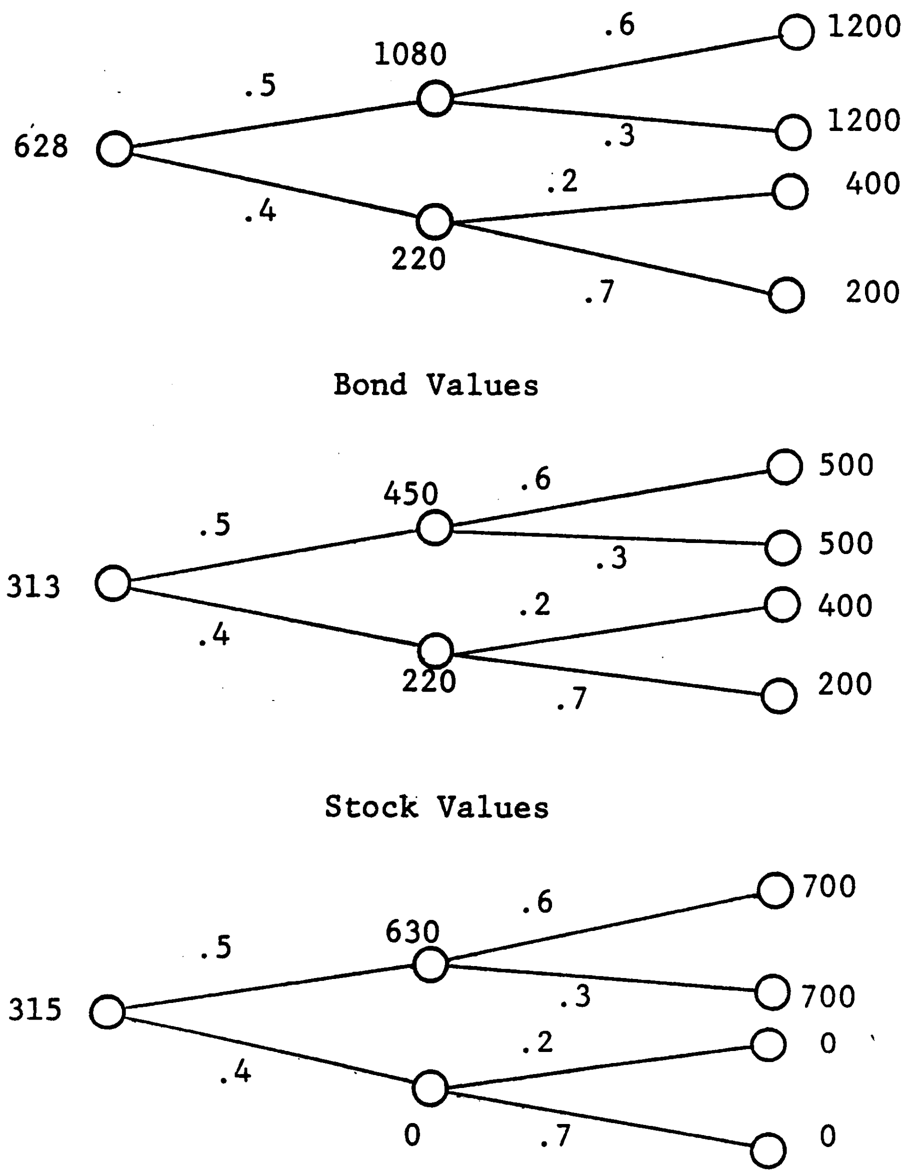
APPENDIX 1

Case I - Stockholders Execute the Expropriation Decision and Bondholders Anticipate the Decision.

Total Firm Values

Bond Values

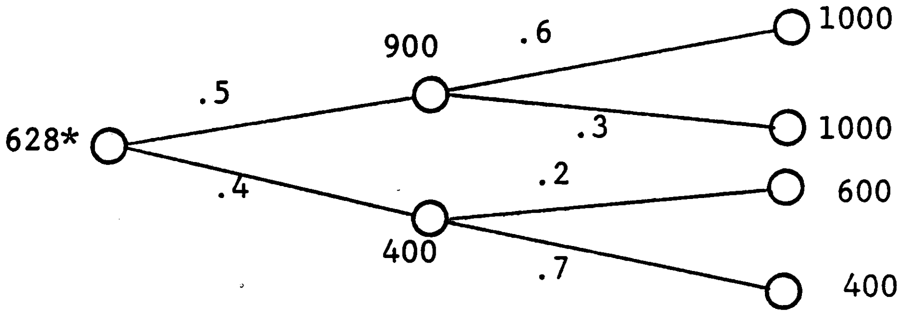
Stock Values



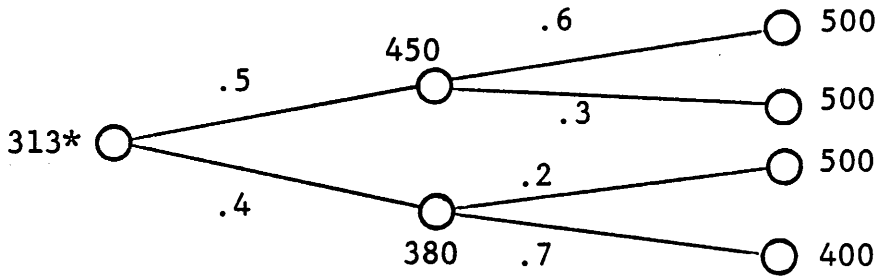
APPENDIX 2

Case II - Stockholders Do Not Execute Expropriation Decision, But Bondholders Price Bonds as Though Expropriation Takes Place.

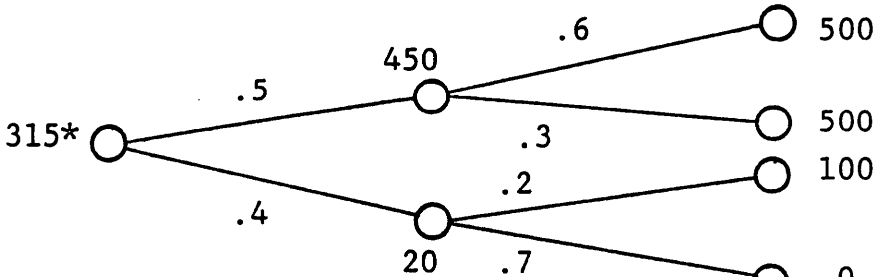
Total Firm Values



Bond Values



Stock Values

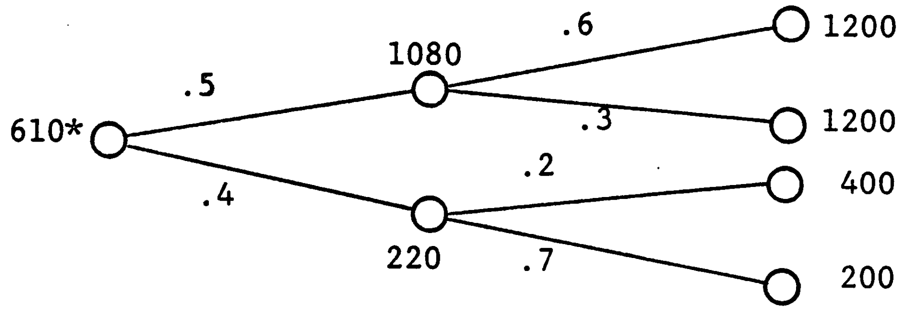


* These prices are set as though payoffs follow the AGENCY Pattern.

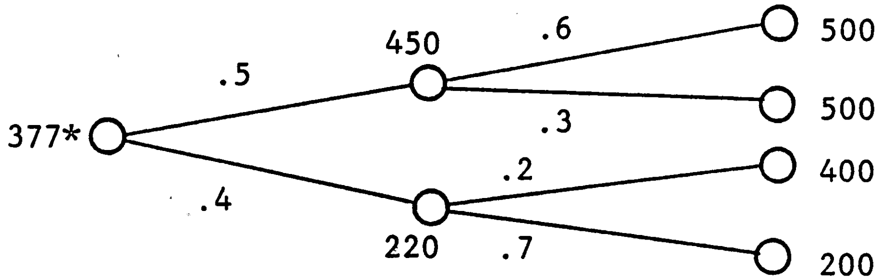
APPENDIX 3

Case III - Stockholders Execute the Expropriation Decision, But Bondholders Expect Original Payoffs.

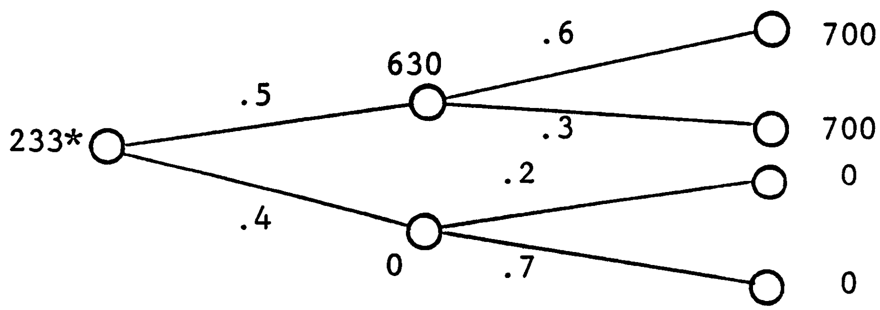
Total Firm Values



Bond Values



Stock Values

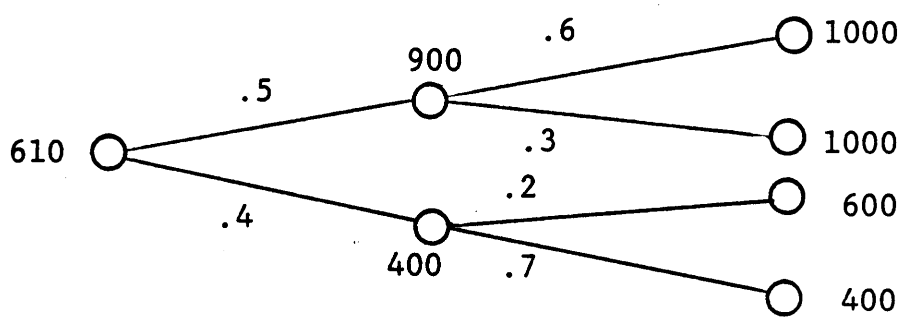


* These prices are set as though the original payoffs are anticipated.

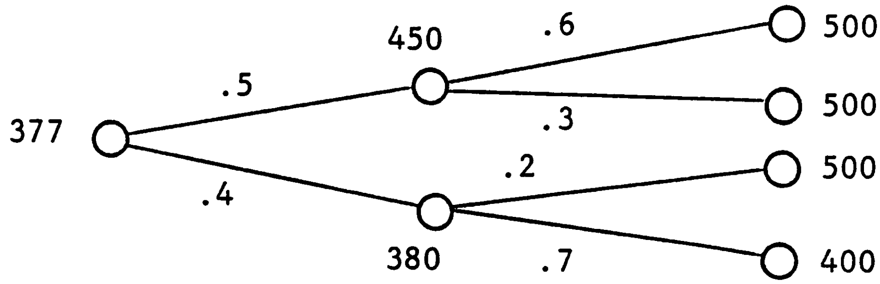
APPENDIX 4

Case IV - Stockholders Do Not Execute Expropriation and Bondholders Anticipate Original Payoffs.

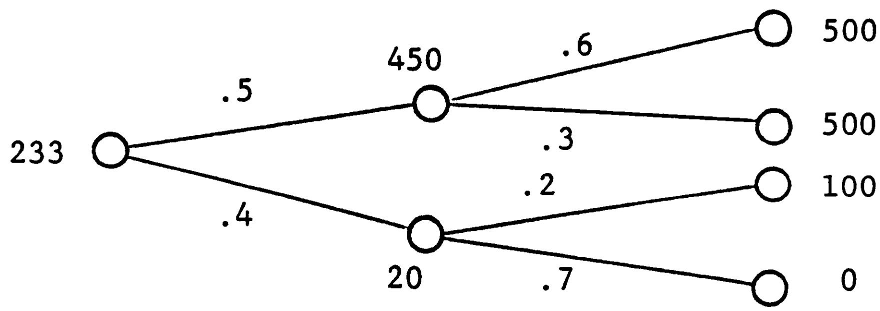
Total Firm Values



Bond Values



Stock Values



APPENDIX 5

DESCRIPTION OF SAMPLES CHOSEN FOR THE ECONOMETRIC MODEL

'STRAIGHT'

No.	Issue	Year	Yr. of Maturity
1	AMAX, Inc.	1970	1986
2	ASARCO, Inc.	1963	1988
3	Gulf Resources & Chemical	1977	1997
4	Buttes Gas & Oil	1977	1997
5	Global Marine	1978	1998
6	Western Company of N. America	1977	1997
7	Western Company of N. America	1970	1998
8	Zapata Corp.	1976	2001
9	Zapata Corp.	1977	1997
10	General Host	1965	1990
11	Iowa Beef Processors	1969	1984
12	Kane-Miller Corp.	1969	1990
13	United Brands	1973	1998
14	Amstar Corp.	1963	1993
15	R. J. Reynolds	1969	1989
16	APL Corp.	1977	1997
17	Technical Tape Inc.	1977	1996
18	Essex Chemical Corp.	1978	1998
19	Amerada Hess	1971	1996
20	Crystal Oil Co.	1978	1990
21	Jim Walter Corp.	1973	1998
22	B. F. Goodrich	1972	1997
23	U.S. Gypsum	1974	2004
24	Athlone Industries	1978	1993
25	Bethlehem Steel	1965	1990
26	U.S. Steel	1966	1996
27	Welded Tube of America	1976	1995
28	Diversified Industries	1971	1991
29	Continental Group	1978	2008
30	Zero Corp.	1974	1989
31	Mite Corp.	1977	1997
32	Allied Products	1968	1984
33	CONDEC Corp.	1977	1997
34	Tyler Corp.	1978	1998
35	Crown Industries	1978	1993

No.	Issues	Year	Yr. of Maturity
36	Esterline Corp.	1974	1995
37	A-T-O Inc.	1968	1988
38	A-T-O Inc.	1978	1998
39	Telex Corp.	1971	1996
40	Fedders Corp.	1974	1994
41	U V Industries	1965	1995
42	Duro-Test	1962	1982
43	Altec Corp.	1968	1988
44	Warner Communications	1969	1974
45	Warner Communications	1976	1996
46	Loral Corp.	1977	1997
47	Oak Industries	1978	1998
48	International Harvester	1963	1988
49	International Harvester	1966	1991
50	Lear Siegler	1974	2004
51	Fairchild Industries	1978	1998
52	Bangor Punta	1967	1992
53	Bangor Punta	1978	1998
54	Bangor Punta	1978	1998
55	United Technologies	1966	1991
56	United Technologies	1967	1992
57	Metromedia, Inc.	1978	1998
58	Arrow Electronics	1978	1998
59	Kay Corp.	1978	1998
60	Culbro Corp.	1975	2005
61	Univar Corp.	1972	1999
62	Ames Department Stores	1975	1995
63	Gamble-Skogmo	1978	1989
64	Rapid-American	1968	1988
65	Rapid-American	1971	1985
66	Rapid-American	1978	1999
67	Rapid-American	1978	2004
68	Circle K Corp.	1966	1981
69	Pneumo Corp.	1978	1998
70	Caesar's World	1974	1990
71	Caesar's World	1977	1997
72	Blessings Corp.	1977	1992
73	Columbia Pictures	1975	1990
74	Twentieth Century-Fox	1978	1998
75	Gulf and Western	1968	1988
76	Gulf and Western	1973	2003
77	LTV	1968	1988
78	Teledyne, Inc.	1969	1999
79	Teledyne, Inc.	1974	2004
80	Whittaker Corp.	1971	1988
81	Whittaker Corp.	1973	1993

'CONVERT'

No.	Issue	Year	Yr. of Maturity
1	Castle and Cook	1969	1994
2	Buttes Gas & Oil	1968	1988
3	Ryan Homes	1971	1977
4	U.S. Home Corp.	1971	1996
5	Dillingham Corp.	1969	1994
6	Dillingham Corp.	1973	1999
7	Elgin National Industries	1968	1988
8	Anthony Industries	1979	1999
9	Fischbach and Moore	1972	1997
10	Greyhound Corp.	1966	1986
11	United Brands	1969	1994
12	Stokely-Van Camp	1962	1982
13	Ralston Purina	1975	2000
14	DPF, Inc.	1967	1987
15	Heublein, Inc.	1972	1997
16	National Distillers and Chemicals	1967	1992
17	Pepsico	1971	1996
18	Wometco Enterprises	1969	1994
19	Burlington Industries	1966	1991
20	Chelsea Industries	1968	1993
21	Chelsea Industries	1974	1999
22	Reeves Brothers, Inc.	1966	1991
23	Riegel Textile Corp.	1968	1993
24	J. P. Stevens	1965	1990
25	Texfi Industries	1971	1996
26	West Point - Pepperell	1975	2000
27	Cluett, Peabody, & Company	1964	1984
28	Farah Manufacturing	1969	1994
29	Georgia-Pacific	1971	1996
30	National Homes Corp.	1971	1996
31	Kirsch Co.	1970	1995
32	G. F. Business Equipment	1968	1988
33	Great Northern Nekoosa	1966	1991
34	St. Regis Paper Company	1972	1997
35	Papercraft Corp.	1969	1994
36	Maryland Cup Corp.	1969	1994
37	Filmways, Inc.	1968	1988
38	Grolier, Inc.	1967	1987
39	McGraw-Hill, In.	1967	1992
40	Celanese Corp.	1965	1990
41	Hercules, Inc.	1974	1999
42	Cooper Laboratories	1970	1991
43	Cooper Laboratories	1972	1992

No.	Issue	Year	Yr. of Maturity
44	Pfizer, Inc.	1972	1997
45	Purex Industries	1969	1994
46	Grow Group, In.	1967	1987
47	Insilco	1968	1993
48	Insilco	1969	1989
49	Insilco	1974	1999
50	SCM Corp.	1968	1988
51	Sherwin-Williams	1970	1995
52	Ashland Oil	1968	1993
53	Belco Petroleum	1968	1988
54	Pennzoil Company	1971	1996
55	Tesoro Petroleum	1969	1989
56	Witco Chemical	1968	1993
57	Jim Walter	1971	1991
58	Amerace Corp.	1967	1992
59	Armstrong Rubber	1967	1987
60	Dayco Corp.	1971	1996
61	Mohawk Rubber	1963	1983
62	Uniroyal	1971	1996
63	Apache Corp.	1970	1990
64	Melville Corp.	1971	1996
65	Suave Shoe, Inc.	1972	1997
66	Owen-Illinois	1967	1992
67	Lone Star Industries	1968	1993
68	Susquehanna Corp.	1968	1988
69	Bliss & Laughlin Industries	1967	1987
70	Crane Company	1968	1993
71	Nortek, Inc.	1969	1989
72	Penn-Dixie, Inc.	1967	1982
73	U.S. Steel	1976	2001
74	ALCOA	1966	1991
75	Reynolds Metal Company	1966	1991
76	Diversified Industries	1967	1987
77	Diversified Industries	1968	1993
78	Phoenix Steel	1967	1987
79	Revere Copper and Brass	1967	1992
80	National Can Corp.	1968	1993
81	CECO Corp.	1968	1988
82	Rusco Industries	1969	1989
83	Rusco Industries	1977	1989
84	Standex International Corp.	1967	1987
85	CONDEC Corp.	1967	1982
86	CONDEC Corp.	1968	1993
87	CONDEC Corp.	1971	1996
88	Deere & Company	1976	2001

No.	Issue	Year	Yr. of Maturity
89	American Hoist & Derrick	1967	1992
90	American Hoist & Derrick	1968	1993
91	Caterpillar Tractor	1975	2000
92	Black & Decker	1967	1992
93	Giddings & Lewis	1967	1987

APPENDIX 6

CORRELATION MATRIX OF EXPLANATORY VARIABLES

	X_1	X_2	X_3	X_4	X_5	X_6	X_7
X_1	1.00000	-.23654	-.31771	-.14498	.13480	.44465	.10457
X_2	-.23654	1.00000	.34311	.23903	-.05926	-.14703	-.02867
X_3	-.31771	.34311	1.00000	.26504	-.13935	-.49926	-.09980
X_4	-.14498	.23903	.26504	1.00000	.50719	-.01876	.59713
X_5	.13480	-.05926	-.13935	.50719	1.00000	.47399	.91172
X_6	.44465	-.14703	-.49926	-.01876	.47399	1.00000	.58265
X_7	.10457	.02867	-.09980	.59713	.91172	.58265	1.00000

APPENDIX 7

INVERSE OF CORRELATION MATRIX OF EXPLANATORY VARIABLES

	X_1	X_2	X_3	X_4	X_5	X_6	X_7
X_1	1.42049	.25253	-.04732	-.22212	-.68030	-.98905	1.18316
X_2	.25352	1.25710	-.40732	-.48461	-.19108	-.45685	.69865
X_3	-.04732	-.40732	1.68293	-.00949	.79603	1.13472	-1.22001
X_4	-.22212	-.48461	-.00949	2.64948	1.07808	1.59131	-3.48378
X_5	-.68030	-.19108	.79603	1.07808	7.22958	2.11393	-8.32168
X_6	-.98905	-.45685	1.13472	1.59131	2.11393	3.82273	-4.90127
X_7	1.18316	.69865	-1.22001	-3.48378	-8.32168	-4.90127	13.29760

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AGENCY THEORY: A MODEL OF INVESTOR EQUILIBRIUM AND A TEST OF AN AGENCY COST RATIONALE FOR CONVERTIBLE BOND FINANCING.

by

William T. Moore

(ABSTRACT)

The conflict that may arise among holders of competing claims on firms' assets is being studied under the heading of "agency theory."

The primary purposes of the research done in this study were to: (1) economically model the individual investor's consumption-investment decision as it is modified by the agency problem, and (2) to econometrically model the firm's decision to issue convertible versus nonconvertible bonds using explanatory variables which measure the extent of the agency problem.

Individual investors are assumed to maximize expected utility of consumption by choosing consumption and investment amounts over a single period. A mathematical model of the investor's consumption-investment decision was derived in an environment characterized by agency problems between stockholders and bondholders. It was demonstrated that if

the capital markets exhibit conditions known as spanning and competitivity, then the only investors affected by the agency problem are those holding the affected securities prior to the act of expropriation. It was also shown that the agency problem does not vanish in general, even if investors attempt to avoid the expropriation by holding balanced portions of all outstanding claims on a firm's assets.

Implications of the theoretical development were then tested by econometrically modelling the firm's choice of convertible versus nonconvertible debt. The explanatory variables included in the model included measures of the more popular reasons for convertible financing, such as the "debt sweetener" hypothesis and the "delayed equity" rationale discussed in most basic finance textbooks. In addition, measures of agency costs were included, since one possible solution to the agency problem is the issuance of convertible bonds. The empirical results showed that the model accounted for a significant portion of the discrimination between convertible and straight debt, and that the variables designed to measure agency costs were marginally significant.