

A STOCHASTIC SIMULATION STUDY
OF THE BIOCHEMICAL OXYGEN DEMAND
AND DISSOLVED OXYGEN IN STREAMS
AND ESTUARIES

by

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I INTRODUCTION

1.1 General Problem

The pollution of the rivers and estuaries of this nation are finally receiving the attention not only of the experts but of the general public as well. Many of our rivers are so grossly polluted that they have become not only unsightly and unproductive but also a serious health hazard.

Stream pollution is considered to be any degradation of the stream waters by run-off from inhabited areas, eroding soils, industrial waste, domestic sewage, and by liquid or other material not naturally present in the stream. A more legalistic definition is any degradation to the water which puts constraints on its future use.

The problem of water pollution lies in the proper utilization of the stream resources. Since these resources are used for many purposes, such as municipal water supply, irrigation, commercial fishing, boating, and swimming, a high degree of water quality must be maintained. Rivers have a capacity for assimilating pollution and thus saving on the cost of waste treatment and are, therefore, additionally used as a means of disposal for sewage and industrial waste. Attempts are being made by the governing bodies to

allocate the rivers' resources among its various uses. In order to do this successfully a means of determining a river's assimilative capacity is necessary.

The infamous properties associated with gross stream pollution are known only too well. Some of the characteristics such as putrid odors and the destruction of major fish and green-plant life are the direct result of exhausting a stream's oxygen reserve. For this reason, the effect of organic pollution upon the stream's oxygen reserve, or dissolved oxygen concentration, must be understood. With the proper understanding, water quality standards can be set which are both rational and practical.

1.2 Stream Characteristics and Parameters

Organic pollutants can be measured in terms of the total oxygen requirements and the rate of oxygen demand of the residual unoxidized organic matter at the point of discharge. The dissolved oxygen (DO) is measured in terms of its concentration. The units of the concentration are parts per million (ppm) or, equivalently, milligrams per liter (mg/l). The oxygen consuming pollutants are measured in terms of biochemical oxygen demand (BOD). The BOD is the amount of oxygen consumed by the bacteria in stabilizing the organic matter under given conditions. BOD can be

determined by extrapolation from a laboratoryly determined BOD₅. The BOD₅ determined in a laboratory test is the amount of oxygen consumed in five days by the aerobic bacteria in stabilizing the organic matter. The temperature, amount of sample and other conditions must be specified in the test. Thus a quantitative measure of the ultimate oxygen demand of organic pollutants provides a measure of the streams liability.

In determining a stream's liability the BOD can be considered as that which is owed and must be paid at a later time. The dissolved oxygen can be thought of as that which is on hand at the present time. Consequently, a high DO is meaningless if the BOD is exceedingly high. It can be seen that both quantities must be considered when evaluating stream liability. Any change in DO or BOD, such as an increase in pollution from an outside source, would be measured in terms of its effect on the BOD and the DO in the main stream.

As pollutants enter a river the bacteria present in the river oxidize the organic pollutants and in so doing utilize the dissolved oxygen. The rate of usage of the dissolved oxygen is directly proportional to the amount of pollutants present and the water temperature. A stream

resists the reduction of dissolved oxygen basically by the processes of reaeration and photosynthesis of the water plants.

The processes of dissolved oxygen replenishment can be overtaxed by pollution, resulting in unsatisfactory river conditions. The saturation level of the dissolved oxygen is only about 9 ppm at 70°F and standard barometric pressure. The most serious conditions arise during the warmer months, since the solubility decreases with a rise in water temperature. The seriousness of this reduction in DO level is exemplified by noting the narrow DO range. To maintain most fish life a minimum DO level of 4ppm must be sustained. Unfortunately the saturation level of the dissolved oxygen is seldom obtained due to the presence of either residual pollution or the small amounts of pollution added by general run-off. Consequently, the DO range to sustain fish life is quite small. The difference between the saturation level and the actual DO level attained in the river is called the oxygen deficit.

The reduction of dissolved oxygen in a stream below 4ppm will result in the death of many fish and, if the DO is essentially depleted, will result in the production of hydrogen sulfide and obnoxious odors. In addition, sludge

banks may form in polluted streams by the settling of suspended solids near the point of pollution. Anaerobic bacteria act on the sludge bank material resulting in the formation of methane, carbon dioxide, hydrogen sulfide, and other noxious gases.

With natural purification forces at work in a stream the effects of any degree of pollution can be eliminated provided the stream is long enough and not recontaminated. Thus time is a very important factor in the analysis of a river's assimilative capacities.

1.3 Stochastic Simulation

Stochastic simulation has only recently found extensive use in fields of research where a variety of problems exist which are beyond the available resources of theoretical mathematics. Stochastic simulation can be quite helpful in the field of statistics. For example, it can be used to obtain properties of estimators whose distributions have been virtually impossible to obtain.[5] With the advancements in computers a good approximation of an estimator's distribution can be found by using the estimator repeatedly and making frequencies of its outcomes. In particular, by observing the estimator's average value for a large sequence of values, its expectation can be determined with reasonable

accuracy.

Hammersly and Handscomb [4] define Stochastic simulation or Monte Carlo methods, as that branch of experimental mathematics which is concerned with experiments on random numbers. The greatest successes of this method have been in areas where the basic mathematical problem itself consists of the investigation of some random process, this is generally considered a probabilistic problem. Monte Carlo methods are also used for deterministic problems. Generally the underlying structure of a deterministic problem also describes some seemingly unrelated random process.

Stochastic simulation answers are, by the nature of the process, uncertain and of course do not prove anything conclusively. But, if Stochastic simulation is used properly its answers can serve a useful purpose in showing something to be reasonably certain.

II PREVIOUS DEVELOPMENTS

2.1 Deterministic Models

The basis for all deterministic models used in analyzing a stream's assimilative capacity has been the classical theory of Streeter and Phelps [8] in 1925. Streeter and Phelps assume that the dissolved oxygen concentration in a stream is governed by two independent reactions, bacterial deoxygenation and reaeration. By definition the BOD is decreased by the first reaction at the same rate the DO is depleted. The DO is increased at a rate proportional to the oxygen deficit. From this they determine the differential equations:

$$\frac{dL}{dt} = -K_1 L \quad (2.1.1)$$

$$\frac{dD}{dt} = K_1 L - K_2 D \quad (2.1.2)$$

The first equation expresses the rate at which the residual BOD (L) is decreased by the bacteria oxidizing the organic pollutant. The second equation states that the rate of increase in the oxygen deficit is proportional to the algebraic sum of the two terms. The first term of equation (2.1.2) represents the deoxygenation reaction and has the proportionality constant K_1 which is a function of the

temperature, nature of the waste and other variables. The second term represents reaeration and the negative sign indicates that the oxygen deficit is decreased by the reaction. The proportionality constant, K_2 , is a function of water temperature, stream velocity and other physical characteristics of the stream.

Under the initial conditions $L = L_0$ and $D = D_0$ at $t = 0$ the differential equations have the following solutions:

$$L = L_0 e^{-K_1 t} \quad (2.1.3)$$

$$D = \frac{K_1 L_0}{K_2 - K_1} (e^{-K_1 t} - e^{-K_2 t}) \quad (2.1.4)$$

L_0 and D_0 are the initial BOD level and oxygen deficit level, respectively. D is the oxygen deficit level at time (t). L denotes the BOD level at time (t). If D_0 , L_0 , and the K 's are known, the oxygen deficit and BOD level may be computed for any time downstream.

In order to apply these equations to a stream

Streeter and Phelps [8] made the following assumptions:

1. The stream flow is steady and uniform
2. The process for the stretch as a whole is a steady state process, the conditions at every cross-section being unchanged with time.
3. The BOD and DO are uniformly distributed over each cross-section, thus permitting the equations to be written in the usual one-dimensional form.

In a 1964 publication W. E. Dobbins modified the Streeter-Phelps equations to account for additional sources of oxygen supply and demand [3]. Dobbins took into account longitudinal dispersion, oxygen removal by benthic demand and plant respiration, oxygen addition by photosynthesis, and the addition of BOD from the benthic layer or from local run-off.

Dobbins, in determining the equations for the BOD and DO profiles along a river's stretch added the following assumptions to those of Streeter and Phelps:

1. The removal of BOD by both the bacterial oxidation and the sedimentation or absorption or both are first order reactions, the rate of removal at any section being proportional to the amount present.
2. The removal of oxygen by the benthic demand and by plant respiration, the addition of oxygen by photosynthesis, and the addition of BOD from the benthic layer or the local runoff are all uniform along the stretch.

Dobbins determined an equation for the biochemical oxygen demand profile and an equation for dissolved-oxygen profile, both taking in the effects of longitudinal dispersion. The longitudinal diffusion coefficient is often uncertain. Dobbins showed by using extreme values for the stream parameters that the ratio of the true concentration to the concentration obtained by neglecting longitudinal

diffusion is .996. Lynch in a later article [6] corrected the figure to .960, a corrected value of 4% probable maximum error but however drawing the same conclusions as Dobbins "that the effect of the longitudinal dispersion is negligible".

$$\frac{dL}{dt} = L_a - (K_1 + K_3)L \quad (2.1.5)$$

The uniform rate of addition of pollution, commonly known as pollution due to land run-off, is denoted by L_a . The rate of decrease of pollution due to bacterial action is represented by K_1L , and that due to sedimentation of solids is represented by K_3L . The K_3L also includes other losses of pollution which do not cause a corresponding loss in DO. For an initial condition of L_0 ppm at time $t = 0$, the differential equation has the solution:

$$L = L_0 e^{-(K_1 + K_3)t} + \frac{L_a}{K_1 + K_3} (1 - e^{-(K_1 + K_3)t}) \quad (2.1.6)$$

The L describes the mean value of pollution for a given time of travel.

Similarly according to Dobbins [3] the rate of change of the dissolved oxygen concentration C , with respect to time of travel is given by the differential equation:

$$\frac{dC}{dt} = K_2(C_s - C) - K_1L - D_B \quad (2.1.7)$$

The C_s expresses the saturation level under given conditions of temperature and barometric pressure, with its concentration measured in ppm. The dissolved oxygen is increased at the rate $K_2(C_s - C)$ due to reaeration of the stream. Its rate is in units per day. The K_1L as discussed before represents the rate of decrease of dissolved oxygen due to bacterial action. The D_B represents the rate of decrease of dissolved oxygen due to benthic demand and also includes the effects of algae.

$$C(t) = C_s(1 - e^{-K_2 t}) - \frac{K_1(L_0 - \frac{L_a}{K_1 + K_3})}{K_2 - (K_1 + K_3)} (e^{-(K_1 + K_3)t} - e^{-K_2 t}) - \left(\frac{D_B}{K_2} + \frac{K_1 L_a}{K_2(K_1 + K_3)} \right) (1 - e^{-K_2 t}) + C_0 e^{-K_2 t} \quad (2.1.8)$$

where $K_2 \neq K_1 + K_3$

The t in these equations is the travel time in days given by $t = x/u$; where x represents the distance along the stretch in miles and u denotes the average stream velocity in miles per day.

2.2 Stochastic Model (Streams)

So far we have considered only the deterministic models which yield the average conditions of a stream. Although Dobbins was able to predict the BOD and DO concen-

trations at a given point along a river quite accurately, he gave no consideration to the inherent variability of these values. Because of this shortcoming in the deterministic models, Thayer and Krutchkoff developed a stochastic model which considered the actual probability distribution of the values as a function of time of travel. The mean BOD and mean DO of the stochastic model are compatible with those of Dobbins' deterministic model.

As mentioned in Dobbins' work, Thayer and Krutchkoff [10] assumed the factors which cause changes in BOD concentration and dissolved oxygen concentration to be as follows:

1. Pollution added at the rate L_a from an outside source.
2. Pollution decreased at the rate K_3L from sedimentation and absorption.
3. Pollution and oxygen decreased at the rate K_1L from bacterial action.
4. Oxygen decreased at the rate D_B from the benthic demand.
5. Oxygen increased at the rate $K_2(C_s - C)$ from the physical process of reaeration.

The model used the usual assumption's of steady and uniform flow, steady state process, and one-dimensional flow,

but additionally considered the BOD and DO discretized. In other words changes in both pollution and dissolved oxygen were assumed to occur in small units of size delta (Δ), a measurable parameter of the individual stream. Thayer and Krutchkoff viewed the problem as a stochastic birth and death process with BOD and DO being increased and decreased by small amounts (Δ) in very short time intervals (h).

In the stochastic model the term state is used to describe a BOD or dissolved oxygen condition. A change of size Δ in the concentration constitutes a change of one state. The pollution state M corresponds to a concentration of L_M and the relationship between the two is

$$\text{State } M = L_M / \Delta.$$

Similarly, the dissolved oxygen state N corresponds to a concentration C_o such that

$$\text{State } N = C_o / \Delta$$

Thayer and Krutchkoff made the assumption that the probability of a change of one state is proportional to h and that a change of more than one state by a factor in time h has probability $O(h)$, which is negligible. The probability of one change in state for each of the five factors assumed for the increasing and decreasing BOD and DO concentrations are as follows:

1. The probability that pollution increases by an amount Δ due to local run-off in time interval h is $\frac{L_1 h}{\Delta}$
2. The probability that pollution decreases by an amount Δ due to all non-oxygen consuming processes in time interval h is $\frac{K_3 L_m h}{\Delta}$
3. The probability that pollution and dissolved oxygen decreases by an amount Δ in time interval h is $\frac{K_1 L_m h}{\Delta}$
4. The probability that dissolved oxygen decreases by an amount Δ due to benthic demand in time interval h is $\frac{D_b h}{\Delta}$
5. The probability that dissolved oxygen increases by an amount Δ due to reaeration in time interval h is $\frac{K_2 (C_s - C) h}{\Delta}$

By utilizing the above probabilities and considering the initial condition of a point source of pollution and dissolved oxygen, Thayer and Krutchkoff determined the marginal distribution, the mean, and the variance, of both BOD and DO for some time of travel.

For our analysis, we will concern ourselves only with

the mean and variance of pollution and dissolved oxygen.

Thayer and Krutchkoff obtained for the mean amount of pollution present,

$$L(t) = L_0 e^{-(K_1+K_3)t} + \frac{L_a}{K_1+K_3} (1 - e^{-(K_1+K_3)t}) \quad (2.2.1)$$

which is equivalent to Dobbins' equation (2.1.6). For the variance of the amount of pollution, Thayer and Krutchkoff obtained

$$\begin{aligned} \text{BOD variance} = \Delta & \left[\frac{L_a}{K_1+K_3} + L_0 e^{-(K_1+K_3)t} \right] \\ & \cdot [1 - e^{-(K_1+K_3)t}] \quad (2.2.2) \end{aligned}$$

The dissolved oxygen was treated in a similar manner to that of the BOD and the mean amount of dissolved oxygen determined to be

$$\begin{aligned} C(t) = C_s - \frac{L_a K_1}{(K_1+K_3)[K_2-(K_1+K_3)]} (1 - e^{-(K_1+K_3)t}) \\ + \frac{L_a K_1}{K_2[K_2-(K_1+K_3)]} (1 - e^{-K_2 t}) - \frac{D_B}{K_2} (1 - e^{-K_2 t}) \\ - (C_s - C_0) e^{-K_2 t} - L_0 \frac{K_1}{K_2 - (K_1 + K_3)} (e^{-(K_1+K_3)t} - e^{-K_2 t}) \quad (2.2.3) \end{aligned}$$

and the variance of the amount of dissolved oxygen present to be

$$\begin{aligned}
\text{DO Variance} = \Delta \{ & (C_s - C_o)(1 - e^{-K_2 t})e^{-K_2 t} + \frac{D_B}{K_2}(1 - e^{-K_2 t}) \\
& + \frac{L_a K_1}{(K_1 + K_3)[K_2 - (K_1 + K_3)]}(1 - e^{-(K_1 + K_3)t}) \\
& - \frac{L_a K_1}{K_2[K_2 - (K_1 + K_3)]}(1 - e^{-K_2 t}) \\
& + L_o \frac{K_1}{K_2 - (K_1 + K_3)}(e^{-(K_1 + K_3)t} - e^{-K_2 t}) \\
& \cdot \left[1 - \frac{K_1}{K_2 - (K_1 + K_3)} \cdot (e^{-(K_1 + K_3)t} - e^{-K_2 t}) \right] \} \quad (2.2.4)
\end{aligned}$$

Since no previous theoretical work had given predicted variances, Thayer and Krutchkoff checked their variances with actual river and laboratory data. It should be noted that the mean BOD and DO are independent of the choice of Δ but that the variance of BOD and DO are directly proportional to Δ and thus calculated with it.

The quantity Δ was estimated from the measurements made of the variance. First a stream survey was conducted, making repeated observations of both BOD and DO at various points along a given stretch. From the survey, the stream parameters were estimated and the BOD and DO sample variance (S^2), were calculated. For each point there are predicted BOD and DO variances which, having estimated the

parameters, are functions of Δ . The estimate for Δ was then determined by:

$$\hat{\Delta} = S^2(t) / \sigma^2(t) / \Delta$$

2.3 Purpose: Stream

This work purports to examine the properties of the average BOD and DO as calculated from Dobbins deterministic equations with the values of K_1 , K_2 and K_3 randomized to simulate their expected variation for a river. The mean of both the BOD and DO are calculated using the means of K_1 , K_2 and K_3 in Dobbins deterministic equations. The standard error, i.e. the standard deviation of the average, is calculated for each time of travel. Using the standard error we will determine if the average BOD and average DO for the randomization procedure are sufficiently close to their means.

This work will also investigate the parameter delta (Δ) as established by the work of Thayer and Krutchkoff. More precisely, we are interested in whether the variation in the values of K_1 , K_2 and K_3 for a river account for the variance of the average BOD and DO or if this variance is in the stochastic process.

We will also investigate the source of variance of the

average BOD and DO by randomizing the initial BOD (L_0), the land run-off (L_a), and the benthic demand (D_B), while holding the K values constant. This will be used to determine if the variance can be accounted for by the variation in the values of L_0 , L_a and D_B or if the variance of the average BOD and DO is in the stochastic process. By the same criterion as that used in the process with K_1 , K_2 , and K_3 randomized, we will determine if the average BOD and DO for the randomization of L_0 , L_a and D_B are sufficiently close to their mean.

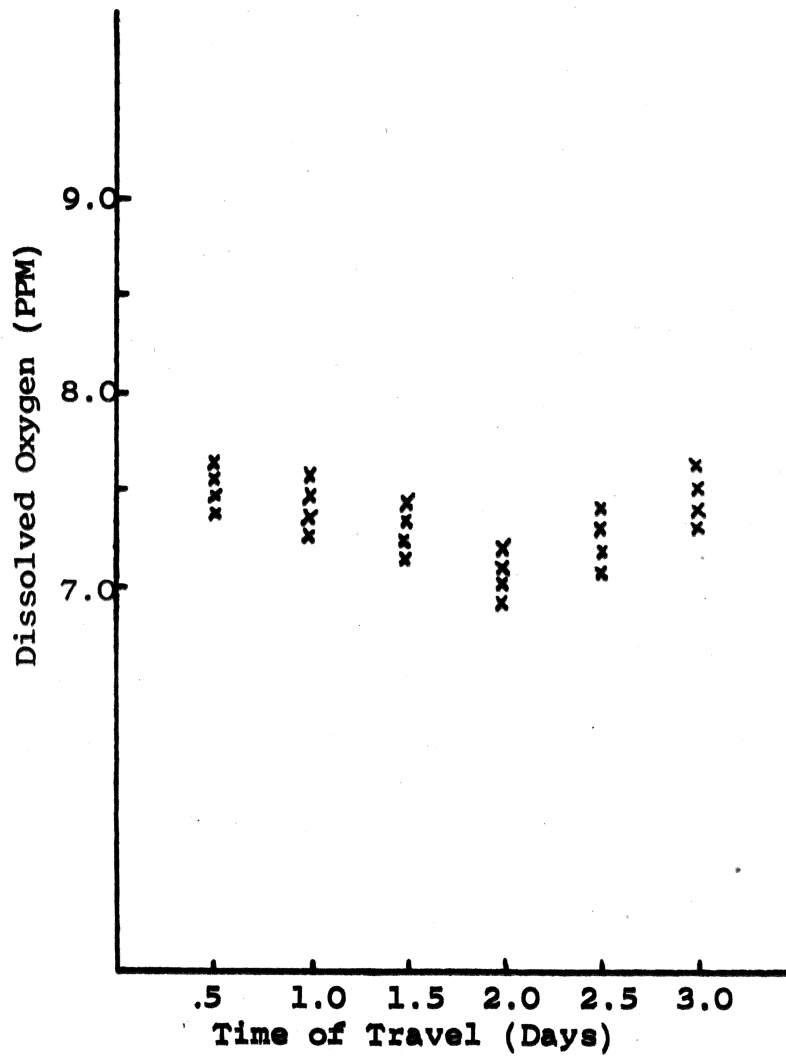
III PROCEDURE AND RESULTS

3.1 Description of the Program

Dobbins' deterministic equations in section (2.1) and those of Thayer and Krutchkoff in section (2.2) were used as the basis for this work. Using a high speed computer, and Stochastic simulation methods, the values of K_1 (coefficient defining the rate of deoxygenation), K_2 (coefficient defining the rate of reaeration), and K_3 (coefficient defining the rate of all non-oxygen consuming processes) are generated as normal pseudo-random variables about their respective means and with suitable variation to represent that found in the K values. Since the ranges of the individual K values are somewhat uncertain, several different variances were tried using a standard deviation of about 10% of the range. Dobbins' equations for BOD and DO were calculated two hundred times for several times of travel and for a given set of river parameters with initial BOD load and initial DO level. The times of travel were chosen in order to capture the critical dissolved oxygen level. The following graph will illustrate this.

The x's on the graph (Figure 1) represent the dissolved oxygen levels calculated by using Dobbins' deterministic DO equations. The x's over the time .5 days illustrate the

Figure 1
Dissolved Oxygen Levels



grouping of the DO values calculated by using the normal pseudo-random K values. The same is true for the other time values. A similar graph can be drawn for the BOD. From what we have so far, we can calculate the average BOD and DO levels, as well as, the variance of the BOD and DO for each time of travel for our given set of river parameters. If we now use the mean values of K_1 , K_2 and K_3 in Dobbins' deterministic equations we can calculate the mean values of the BOD and DO for specified times of travel. Since we have already calculated the variance of the BOD and DO values for the specified times of travel we can calculate the standard error by dividing the variance by the square of the number of times Dobbins' equation is calculated for each specified time of travel and then taking the square root. For our case,

$$\text{squared error} = \sqrt{\text{Var}/(200)^2}$$

The squared error will give us a measurement of how close our average BOD and DO are to their mean values.

Since, in Thayer and Krutchkoff's stochastic model, the variance of BOD and DO are directly proportional to Δ , we calculate $\sigma^2(t)/\Delta$ for both BOD and DO. An estimate for Δ can then be determined by

$$\hat{\Delta} = S^2(t) / \sigma^2(t) / \Delta$$

where $S^2(t)$ in this case is the variance of the DO or BOD calculated by the method just discussed. $\hat{\Delta}$ can be calculated for each specified time of travel and a mean Δ for the BOD and DO determined. The standard deviation for the Δ 's is also calculated to indicate how close the Δ values are about their mean.

The predicted variances of the BOD and DO can be calculated by

$$\sigma^2(t) = \bar{\Delta} \cdot \frac{\sigma^2(t)}{\Delta}$$

for small standard deviation of the Δ 's. The variance of the BOD and DO computed by the use of Stochastic Simulation and Dobbins' deterministic equations can be compared with the predicted variances for specified times of travel and the set of river parameters chosen.

This procedure was done for fifteen stream situations and the whole process repeated for values of K_1 , K_2 and K_3 generated as uniform pseudo-random variables. Fifteen stream situations were used and are given in Table I. For the fifteen stream situations the values of L_a , D_B , C_s and C_o were held constant at .2, .1, 9.0 and 8.6 respectively.

A similar procedure was used to randomize the values of L_o , L_a , and D_B about their respective means while holding

the values of K_1 , K_2 and K_3 constant at .35, .75 and .2 respectively. Both normal and uniform random variables were used in generating the values of L_0 , L_a , and D_B . Again fifteen stream situations were chosen and are given in Table XXXV. For the fifteen situations the values of C_s and C_0 were held at 9.0 and 8.6 respectively.

3.2 Results: Streams

The results of the study are presented in the form of tables and graphs. Tables II, III and IV gives the resulting mean deltas of the BOD and DO with K_1 , K_2 and K_3 randomized. The calculated mean deltas and their respective standard deviations are shown for the fifteen situations. These tables give a comparison of the mean deltas for K_1 , K_2 and K_3 normally and uniformly generated as well as a comparison between situations. A similar presentation of the resulting values are given for the procedure with L_0 , L_a and D_B randomized in Tables XXXVI, XXXVII, and XXXVIII.

In viewing Tables II, III, and IV it should be noted that the mean delta of the BOD and its standard deviation increases from Table II thru IV, that is for increases in the initial BOD. The mean delta of the DO and its standard deviation remain approximately the same size from Tables II thru IV. Consequently, the difference in the

mean delta of the BOD and that of the DO for a particular situation is considerably larger in Table IV than in Table II. The effect of changing the mean values of the K's on the mean delta is illustrated in the first four situations in each of the tables and the difference noted to be small. When the variance of the K values are increased, the mean deltas of both the BOD and DO with their respective standard deviations increase as illustrated in the last situation in each of the tables. All of the noted relationships just discussed remain true for both normally and uniformly generated K values since the mean deltas of both the BOD and the DO with their respective standard deviations are virtually the same in either case.

Figures 2-7 show how the calculated BOD variances fit the predicted BOD variance curves; similarly, figures 8-13 show how the calculated DO variances fit the predicted DO variance curves. The figures for both the BOD and DO show how much better the fit is for the smaller variances. Similarly the fit is better for the smaller initial pollution of six ppm as opposed to the larger initial pollution of sixteen ppm.

Each of the Tables V thru XXXIV presents, for each time of travel, the mean, average and standard error of the

BOD as well as the mean, average and standard error of the DO. These tables are the result of using the procedure of randomizing the K values both normally and uniformly. In all thirty tables the average BOD and average DO values are within three standard errors of their respective means. Tables XXXIX thru LII present the same items for the normal randomization of the values L_0 , L_a and D_B . Similarly, in the resulting fifteen tables the average BOD and DO values are within three standard errors of their respective means.

Tables XXXVI, XXXVII and XXXVIII give the resulting mean deltas of the BOD and DO with L_0 , L_a and D_B randomized. In viewing these tables it should be noted that the mean deltas of both the BOD and DO with their respective standard deviations are virtually the same for L_0 , L_a and D_B whether normally or uniformly randomized. The effect of changing the mean values of the L_a and D_B on the mean delta is illustrated in the first three situations in each of the tables and the difference in the mean delta noted to be small. Likewise, the effect of changing the mean value of L_0 is small which is illustrated by comparing the values of Tables XXXVI, XXXVII and XXXVIII. When the variance of L_0 is increased the mean deltas of the BOD and their standard deviations increase but the mean deltas of the DO and their

standard deviations remain approximately the same size. This effect is noted in the next to last situation of the three tables and recalling that the same effect was shown when the initial BOD increased for K_1 , K_2 and K_3 randomized. The last situation of the three tables illustrates an increase in both the BOD and DO mean deltas and, likewise, for their respective standard deviations when the variances of L_a and D_B are increased.

Figures 14-19 show, for randomized L_o , L_a and D_B , how the calculated BOD variances fit the predicted BOD variance curves; similarly, figures 20-25 show how the calculated DO variances fit the predicted DO variance curves. The figures for the BOD show how much better the fit is for the smaller variance of L_o than for the larger variance while the DO fit seems unaffected. The figures similarly show how much better the fit is for smaller variances of L_a and D_B than for larger variances in the case of both the BOD and DO.

TABLE I

Values Used in Program with
 K_1 , K_2 , and K_3 Randomized

Situation	LO	MK1	MK2	MK3	SD1	SD2	SD3
1A	6	.25	.75	.05	.01	.016	.01
2A	6	.35	.75	.05	.01	.016	.01
3A	6	.35	1.50	.05	.01	.016	.01
4A	6	.35	.75	.20	.01	.016	.01
5A	6	.35	.75	.20	.05	.066	.05
6A	10	.25	.75	.05	.01	.016	.01
7A	10	.35	.75	.05	.01	.016	.01
8A	10	.35	1.50	.05	.01	.016	.01
9A	10	.35	.75	.20	.01	.016	.01
10A	10	.35	.75	.20	.05	.066	.05
11A	16	.25	.75	.05	.01	.016	.01
12A	16	.35	.75	.05	.01	.016	.01
13A	16	.35	1.50	.05	.01	.016	.01
14A	16	.35	.75	.20	.01	.016	.01
15A	16	.35	.75	.20	.05	.066	.05

TABLE II

Program Results for Initial BOD of Six PPM
with K_1 , K_2 and K_3 Randomized

Situation		Normally Distributed		Uniformly Distributed	
		Mean Delta	Standard Deviation	Mean Delta	Standard Deviation
1A	BOD	.0046	.00136	.0044	.00119
	DO	.0031	.00069	.0031	.00059
2A	BOD	.0031	.00073	.0033	.00072
	DO	.0024	.00052	.0024	.00044
3A	BOD	.0031	.00073	.0033	.00072
	DO	.0027	.00037	.0027	.00046
4A	BOD	.0020	.00035	.0021	.00036
	DO	.0024	.00046	.0025	.00051
5A	BOD	.0439	.01799	.0421	.01637
	DO	.0196	.00471	.0195	.00417

TABLE III

Program Results for Initial BOD of Ten PPM
with K_1 , K_2 , and K_3 Randomized

Situation		Normally Distributed		Uniformly Distributed	
		Mean Delta	Standard Deviation	Mean Delta	Standard Deviation
6A	BOD	.0075	.00268	.0072	.00241
	DO	.0032	.00033	.0032	.00024
7A	BOD	.0051	.00150	.0053	.00151
	DO	.0023	.00029	.0023	.00018
8A	BOD	.0051	.00150	.0053	.00151
	DO	.0021	.00027	.0022	.00020
9A	BOD	.0033	.00075	.0034	.00079
	DO	.0023	.00021	.0023	.00025
10A	BOD	.0732	.02697	.0750	.02892
	DO	.0341	.00800	.0313	.00689

TABLE IV

Program Results for Initial BOD of Sixteen PPM
with K_1 , K_2 , and K_3 Randomized

Situation		Normally Distributed		Uniformly Distributed	
		Mean Delta	Standard Deviation	Mean Delta	Standard Deviation
11A	BOD	.0117	.00474	.0114	.00430
	DO	.0040	.00072	.0039	.00059
12A	BOD	.0079	.00271	.0083	.00275
	DO	.0027	.00053	.0028	.00043
13A	BOD	.0079	.00271	.0083	.00275
	DO	.0020	.00013	.0021	.00008
14A	BOD	.0051	.00141	.0053	.00149
	DO	.0027	.00047	.0027	.00030
15A	BOD	.1237	.04901	.1208	.04554
	DO	.0507	.01196	.0511	.01272

TABLE V

Time (days)	Stream Situation 1A			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.6146	5.6137	.0028	8.3151	8.3148	.0031
.50	5.2571	5.2584	.0033	8.1001	8.1001	.0033
.75	4.9254	4.9261	.0041	7.9414	7.9412	.0035
1.00	4.6177	4.6170	.0052	7.8280	7.8266	.0037
1.25	4.3322	4.3295	.0056	7.7509	7.7503	.0040
1.50	4.0673	4.0779	.0062	7.7027	7.7020	.0037
1.75	3.8216	3.8204	.0068	7.6772	7.6769	.0040
2.00	3.5937	3.5997	.0069	7.6695	7.6726	.0039
2.25	3.3822	3.3947	.0074	7.6757	7.6748	.0041
2.50	3.1860	3.1829	.0081	7.6924	7.6933	.0041

TABLE VI

Stream Situation 1A				Uniform Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.6146	5.6156	.0027	8.3151	8.3148	.0030
.50	5.2571	5.2552	.0034	8.1001	8.0998	.0033
.75	4.9254	4.9204	.0041	7.9414	7.9432	.0035
1.00	4.6177	4.6175	.0050	7.8280	7.8280	.0038
1.25	4.3322	4.3329	.0058	7.7509	7.7521	.0039
1.50	4.0673	4.0679	.0062	7.7027	7.7065	.0039
1.75	3.8216	3.8139	.0066	7.6772	7.6754	.0040
2.00	3.5937	3.5791	.0070	7.6695	7.6693	.0039
2.25	3.3822	3.3811	.0069	7.6757	7.6735	.0041
2.50	3.1860	3.1837	.0077	7.6924	7.6935	.0040

TABLE VII

Stream Situation 2A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.4766	5.4754	.0026	8.1858	8.1868	.0031
.50	5.0030	5.0005	.0032	7.8766	7.8743	.0032
.75	4.5745	4.5754	.0040	7.6513	7.6532	.0035
1.00	4.1868	4.1928	.0042	7.4926	7.2957	.0036
1.25	3.8359	3.8387	.0050	7.3864	7.3818	.0039
1.50	3.5185	3.5268	.0057	7.3214	7.3191	.0038
1.75	3.2312	3.2338	.0059	7.2882	7.2863	.0041
2.00	2.9713	2.9699	.0054	7.2795	7.2795	.0038
2.25	2.7361	2.7452	.0059	7.2894	7.2911	.0040
2.50	2.5233	2.5185	.0057	7.3129	7.3181	.0039

TABLE VIII

Time (days)	Stream Situation 2A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.4766	5.4768	.0027	8.1888	8.1891	.0030
.50	5.0030	5.0022	.0033	7.8875	7.8871	.0032
.75	4.5745	4.5792	.0040	7.6736	7.6748	.0034
1.00	4.1868	4.1886	.0048	7.5288	7.5308	.0036
1.25	3.8359	3.8330	.0049	7.4382	7.4351	.0037
1.50	3.5185	3.5191	.0055	7.3896	7.3925	.0039
1.75	3.2312	3.2154	.0055	7.3734	7.3725	.0038
2.00	2.9713	2.9863	.0060	7.3818	7.3843	.0038
2.25	2.7361	2.7333	.0060	7.4084	7.4059	.0039
2.50	2.5233	2.5377	.0060	7.4483	7.4468	.0037

TABLE IX

Stream Situation 3A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.4766	5.4754	.0026	8.2870	8.2855	.0030
.50	5.0030	5.005	.0032	8.1082	8.1086	.0033
.75	4.5745	4.5754	.0040	8.0180	8.0179	.0033
1.00	4.1868	4.1928	.0042	7.9857	7.9845	.0033
1.25	3.8359	3.8387	.0050	7.9904	7.9890	.0033
1.50	3.5185	3.5268	.0057	8.0179	8.0167	.0033
1.75	3.2312	3.2338	.0059	8.0587	8.0578	.0032
2.00	2.9713	2.9699	.0054	8.1067	8.1080	.0032
2.25	2.7361	2.7452	.0059	8.1576	8.1573	.0031
2.50	2.5233	2.5185	.0057	8.2089	8.2102	.0032

TABLE X

Stream Situation 3A				Uniform Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.4766	5.4768	.0027	8.2870	8.2879	.0031
.50	5.0030	5.0022	.0033	8.1082	8.1085	.0031
.75	4.5745	4.5792	.0040	8.0180	9.0184	.0033
1.00	4.1868	4.1886	.0048	7.9857	7.9879	.0033
1.25	3.8359	3.8330	.0049	7.9904	7.9883	.0033
1.50	3.5185	3.5191	.0055	8.0179	8.0181	.0033
1.75	3.2312	3.2154	.0055	8.0587	8.0569	.0033
2.00	2.9713	2.9863	.0060	8.1067	8.1075	.0032
2.25	2.7361	2.7333	.0060	8.1576	8.1570	.0031
2.50	2.5233	2.5377	.0060	8.2089	8.2092	.0033

TABLE XI

Stream Situation 4A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.2759	5.2754	.0027	8.1973	8.1956	.0030
.50	4.6449	4.6458	.0030	7.9174	7.9183	.0032
.75	4.0949	4.0955	.0035	7.7322	7.7292	.0033
1.00	3.6155	3.6178	.0037	7.6196	7.6210	.0034
1.25	3.1978	3.1991	.0041	7.5619	7.5644	.0037
1.50	2.8337	2.8295	.0041	7.5452	7.5484	.0037
1.75	2.5164	2.5160	.0043	7.5583	7.5600	.0035
2.00	2.2398	2.2412	.0042	7.5929	7.5916	.0036
2.25	1.9988	1.9941	.0042	7.6421	7.6430	.0038
2.50	1.7887	1.7915	.0043	7.7008	7.7007	.0035

TABLE XII

Time (days)	Stream Situation 4A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.2759	5.2759	.0025	8.1973	8.1979	.0031
.50	4.6449	4.6464	.0029	7.9174	7.9153	.0032
.75	4.0949	4.0994	.0037	7.7322	7.7312	.0034
1.00	3.6155	3.6136	.0041	7.6196	7.6174	.0035
1.25	3.1978	3.1946	.0042	7.5619	7.5656	.0036
1.50	2.8337	2.8361	.0047	7.5452	7.5446	.0037
1.75	2.5164	2.5210	.0044	7.5583	7.5554	.0036
2.00	2.2398	2.2416	.0041	7.5929	7.5938	.0036
2.25	1.9988	2.0042	.0040	7.6421	7.6432	.0037
2.50	1.7887	1.7932	.0043	7.7008	7.7023	.0036

TABLE XIII

Stream Situation 5A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.2759	5.2759	.0066	8.1973	8.1965	.0052
.50	4.6449	4.6529	.0109	7.9174	7.9185	.0073
.75	4.0949	4.1023	.0148	7.7322	7.7331	.0093
1.00	3.6155	3.6208	.0187	7.6196	7.6160	.0107
1.25	3.1978	3.1975	.0194	7.5619	7.5623	.0117
1.50	2.8337	2.8826	.0107	7.5452	7.5415	.0102
1.75	2.5164	2.5273	.0214	7.5583	7.5588	.0114
2.00	2.2398	2.2740	.0207	7.5929	7.6037	.0111
2.25	1.9988	2.0248	.0212	7.6421	7.6390	.0115
2.50	1.7887	1.8027	.0219	7.7008	7.7037	.0118

TABLE XIV

Time (days)	Stream Situation 5A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.2759	5.2814	.0064	8.1973	8.1965	.0051
.50	4.6449	4.6389	.0112	7.9174	7.9173	.0078
.75	4.0949	4.0785	.0145	7.73 2	7.7429	.0089
1.00	3.6155	3.6224	.0180	7.6196	7.6210	.0106
1.25	3.1978	3.2112	.0205	7.5619	7.5685	.0114
1.50	2.8337	2.8479	.0205	7.5452	7.5627	.0109
1.75	2.5164	2.5058	.0208	7.5583	7.5548	.0117
2.00	2.2398	2.2120	.0208	7.5929	7.5951	.0112
2.25	1.9988	2.0119	.0195	7.6421	7.6358	.0112
2.50	1.7887	1.8027	.0207	7.7008	7.7035	.0113

TABLE XV

Stream Situation 6A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	9.3256	9.3241	.0038	8.0957	8.0953	.0032
.50	8.6999	8.7021	.0051	7.7147	7.7148	.0037
.75	8.1195	8.1207	.0066	7.4331	7.4329	.0043
1.00	7.5810	7.5799	.0083	7.2315	7.2293	.0048
1.25	7.0814	7.0769	.0091	7.0939	7.0929	.0053
1.50	6.6179	6.6353	.0100	7.0072	7.0064	.0047
1.75	6.1878	6.1858	.0110	6.9607	6.9603	.0052
2.00	5.7889	5.7988	.0112	6.9458	6.9510	.0051
2.25	5.4188	5.4230	.0120	6.9553	6.9541	.0055
2.50	5.0754	5.0704	.0131	6.9835	6.9852	.0055

TABLE XVI

Stream Situation 6A				Uniform Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	9.3256	9.3273	.0038	8.0957	8.0953	.0031
.50	8.6999	8.6968	.0051	7.7147	7.7143	.0037
.75	8.1195	8.1112	.0064	7.4331	7.4362	.0042
1.00	7.5810	7.5808	.0081	7.2315	7.2318	.0048
1.25	7.0814	7.0826	.0095	7.0939	7.0959	.0051
1.50	6.6179	6.6188	.0101	7.0072	7.0133	.0050
1.75	6.1878	6.1752	.0108	6.9607	6.9580	.0054
2.00	5.7889	5.7651	.0114	6.9458	6.9456	.0052
2.25	5.4188	5.4171	.0112	6.9553	6.9517	.0053
2.50	5.0754	5.0717	.0125	6.9835	6.9855	.0054

TABLE XVII

Stream Situation 7A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	9.9060	9.0940	.0037	7.8855	7.8848	.0031
.50	8.2779	8.2738	.0049	7.3617	7.3589	.0037
.75	7.5378	7.5393	.0063	6.9895	6.9878	.0040
1.00	6.8680	6.8780	.0069	6.7370	6.7382	.0041
1.25	6.2620	6.2667	.0082	6.5785	6.5768	.0046
1.50	5.7137	5.7275	.0093	6.4930	6.4878	.0045
1.75	5.2176	5.2218	.0095	6.4637	6.4647	.0049
2.00	4.7686	4.7663	.0088	6.4770	6.4796	.0047
2.25	4.3624	4.3772	.0097	6.5221	6.5256	.0046
2.50	3.9949	3.9870	.0092	6.5902	6.6008	.0053

TABLE XVIII

Stream Situation 7A				Uniform Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	9.0960	9.0963	.0038	7.8855	7.8861	.0032
.50	8.2779	8.2767	.0051	7.3617	7.3611	.0036
.75	7.5378	7.5457	.0063	6.9895	6.9916	.0042
1.00	6.8680	6.8711	.0078	6.7370	6.7404	.0045
1.25	6.2620	6.2573	.0079	6.5785	6.5735	.0048
1.50	5.7137	5.7148	.0089	6.4930	6.4975	.0050
1.75	5.2176	5.1916	.0090	6.4637	6.4621	.0048
2.00	4.7683	4.7932	.0097	6.4770	6.4810	.0047
2.25	4.3624	4.3578	.0097	6.5221	6.5181	.0050
2.50	3.9949	4.0183	.0098	6.5902	6.5879	.0047

TABLE XIX

Time (days)	Stream Situation 8A			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	9.0960	9.0940	.0037	8.0102	8.0078	.0032
.50	8.2779	8.2738	.0049	7.6674	7.6683	.0034
.75	7.5378	7.5393	.0063	7.4884	7.4882	.0037
1.00	6.8680	6.8780	.0069	7.4166	7.4146	.0037
1.25	6.2620	6.2667	.0082	7.4136	7.4114	.0035
1.50	5.7137	5.7275	.0093	7.4535	7.4519	.0035
1.75	5.2176	5.2218	.0095	7.5189	7.5176	.0034
2.00	4.7686	4.7663	.0088	7.5982	7.6003	.0036
2.25	4.3624	4.3772	.0097	7.6837	7.6834	.0035
2.50	3.9949	3.9870	.0092	7.7707	7.7727	.0035

TABLE XX

Time (days)	Stream Situation 8A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	9.0960	9.0963	.0038	8.0102	8.0117	.0032
.50	8.2779	8.2767	.0051	7.6674	7.6679	.0034
.75	7.5378	7.5457	.0063	7.4884	7.4891	.0037
1.00	6.8680	6.8711	.0078	7.4166	7.4201	.0038
1.25	6.2620	6.2573	.0079	7.4136	7.4103	.0038
1.50	5.7137	5.7148	.0089	7.4535	7.4540	.0038
1.75	5.2176	5.1916	.0090	7.5189	7.5161	.0037
2.00	4.7683	4.7932	.0097	7.5982	7.5995	.0037
2.25	4.3624	4.3578	.0097	7.6837	7.6828	.0035
2.50	3.9949	4.0183	.0098	7.7707	7.7711	.0035

TABLE XXI

Stream Situation 9A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.7621	8.7612	.0038	7.8998	7.8969	.0030
.50	7.6831	7.6847	.0046	7.4114	7.4129	.0036
.75	6.7428	6.7539	.0055	7.0867	7.0821	.0039
1.00	5.9233	5.9271	.0059	6.8875	6.8899	.0043
1.25	5.2091	5.2113	.0065	6.7833	6.7877	.0047
1.50	4.5866	4.5798	.0066	6.7501	6.7553	.0046
1.75	4.0441	4.0436	.0070	6.7688	6.7716	.0044
2.00	3.5713	3.5736	.0069	6.8247	6.8229	.0045
2.25	3.1592	3.1516	.0068	6.9062	6.9077	.0048
2.50	2.8001	2.8046	.0069	7.0044	7.0043	.0044

TABLE XXII

Stream Situation 9A				Uniform Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.7621	8.7620	.0036	7.8998	7.9009	.0034
.50	7.6831	7.6858	.0045	7.4114	7.4078	.0036
.75	6.7428	6.7504	.0058	7.0867	7.0853	.0040
1.00	5.9233	5.9201	.0066	6.8875	6.8842	.0043
1.25	5.2091	5.2040	.0069	6.7833	6.7894	.0045
1.50	4.5866	4.5906	.0076	6.7501	6.7494	.0046
1.75	4.0441	4.0518	.0071	6.7688	6.7645	.0045
2.00	3.5713	3.5743	.0067	6.8247	6.8263	.0044
2.25	3.1592	3.1681	.0064	6.9062	6.9081	.0047
2.50	2.8001	2.8074	.0070	7.0044	7.0068	.0045

TABLE XXIII

Time (days)	Stream Situation 10A			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.7621	8.7654	.0114	7.8998	7.9021	.0079
.50	7.6831	7.6605	.0197	7.4114	7.4009	.0133
.75	6.7428	6.7642	.0270	7.0867	7.0922	.0163
1.00	5.9233	5.9223	.0283	6.8875	6.8758	.0168
1.25	5.2091	5.2153	.0340	6.7833	6.7998	.0191
1.50	4.5866	4.5723	.0316	6.7501	6.7758	.0170
1.75	4.0441	4.0410	.0331	6.7688	6.7684	.0166
2.00	3.5713	3.5816	.0331	6.8247	6.7949	.0189
2.25	3.1592	3.1569	.0342	6.9062	6.8987	.0191
2.50	2.8001	2.8604	.0344	7.0044	7.0066	.0188

TABLE XXIV

Time (days)	Stream Situation 10A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.7621	8.7851	.0113	7.8998	7.9099	.0081
.50	7.6831	7.7120	.0191	7.4114	7.4321	.0120
.75	6.7428	6.7576	.0269	7.0867	7.0797	.0146
1.00	5.9233	5.9256	.0298	6.8875	6.9101	.0168
1.25	5.2091	5.2311	.0319	6.7833	6.7760	.0179
1.50	4.5866	4.6053	.0338	6.7501	6.7521	.0179
1.75	4.0441	4.0804	.0343	6.7688	6.7560	.0172
2.00	3.5713	3.6015	.0344	6.8247	6.8261	.0185
2.25	3.1592	3.2030	.0325	6.9062	6.9452	.0184
2.50	2.8001	2.8305	.0361	7.0044	7.0120	.0162

TABLE XXV

Time (days)	Stream Situation 11A			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	14.8921	14.8897	.0053	7.7667	7.7659	.0037
.50	13.8642	13.8677	.0075	7.1366	7.1368	.0047
.75	12.9106	12.9126	.0101	6.6707	6.6706	.0059
1.00	12.0259	12.0242	.0129	6.3366	6.3332	.0068
1.25	11.2051	11.1980	.0143	6.1083	6.1068	.0076
1.50	10.4436	10.4714	.0158	5.9639	5.9630	.0067
1.75	9.7372	9.7340	.0173	5.8860	5.8854	.0073
2.00	9.0818	9.0975	.0175	5.8602	5.8685	.0072
2.25	8.4737	8.4805	.0189	5.8747	5.8729	.0079
2.50	7.9096	7.9018	.0206	5.9201	5.9230	.0079

TABLE XXVI

Stream Situation 11A				Uniform Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	14.8921	14.8948	.0053	7.7667	7.7661	.0037
.50	13.8642	13.8593	.0076	7.1366	7.1359	.0049
.75	12.9106	12.8975	.0099	6.6707	6.6756	.0056
1.00	12.0259	12.0256	.0127	6.3366	6.3375	.0067
1.25	11.2051	11.2071	.0149	6.1083	6.1116	.0072
1.50	10.4436	10.4452	.0158	5.9639	5.9735	.0070
1.75	9.7372	9.7172	.0170	5.8860	5.8818	.0077
2.00	9.0818	9.0441	.0179	5.8602	5.8600	.0073
2.25	8.4737	8.4712	.0177	5.8747	5.8690	.0075
2.50	7.9096	7.9038	.0196	5.9201	5.9233	.0075

TABLE XXVII

Stream Situation 12A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard BOD	Mean DO	Average DO	Standard Error
.25	14.5250	14.5220	.0052	7.4307	7.4297	.0036
.50	13.1903	13.1837	.0073	6.5731	6.5685	.0046
.75	11.9827	11.9851	.0098	5.9633	5.9608	.0054
1.00	10.8900	10.9059	.0106	5.5493	5.5514	.0056
1.25	9.9012	9.9086	.0128	5.2889	5.2862	.0066
1.50	9.0066	9.0285	.0146	5.1480	5.1398	.0062
1.75	8.1971	8.2038	.0150	5.0991	5.1009	.0070
2.00	7.4646	7.4609	.0138	5.1198	5.1239	.0065
2.25	6.8018	6.8253	.0152	5.1925	5.1981	.0062
2.50	6.2021	6.1897	.0144	5.3031	5.2196	.0075

TABLE XXVIII

Time (days)	Stream Situation 12A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	14.5250	14.5256	.0052	7.4307	7.4316	.0037
.50	13.1903	13.1883	.0078	6.5731	6.5721	.0046
.75	11.9827	11.9953	.0097	5.9633	5.9667	.0056
1.00	10.8900	10.8950	.0121	5.5493	5.5548	.0062
1.25	9.9012	9.8938	.0123	5.2889	5.2812	.0067
1.50	9.0066	9.0084	.0140	5.1480	5.1550	.0072
1.75	8.1971	8.1559	.0141	5.0991	5.0965	.0067
2.00	7.4646	7.5036	.0153	5.1198	5.1260	.0065
2.25	6.8018	6.7945	.0152	5.1925	5.1863	.0071
2.50	6.2021	6.2392	.0154	5.3031	5.2996	.0064

TABLE XXIX

Stream Situation 13A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	14.5250	14.5220	.0052	7.5948	7.5912	.0034
.50	13.1903	13.1837	.0073	7.0061	7.0077	.0042
.75	11.9827	11.9851	.0098	6.6939	6.6937	.0047
1.00	10.8900	10.9059	.0106	6.5629	6.5598	.0047
1.25	9.9012	9.9086	.0128	6.5485	6.5449	.0044
1.50	9.0066	9.0285	.0146	6.6070	6.6045	.0045
1.75	8.1971	8.2038	.0150	6.7092	6.7072	.0042
2.00	7.4646	7.4609	.0138	6.8354	6.8387	.0045
2.25	6.8018	6.8253	.0152	6.9729	6.9724	.0042
2.50	6.2021	6.2897	.0144	7.1132	7.1165	.0042

TABLE XXX

Time (days)	Stream Situation 13A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	14.5250	14.5256	.0052	7.5948	7.5975	.0035
.50	13.1903	13.1883	.0078	7.0061	7.0072	.0042
.75	11.9827	11.9953	.0097	6.6939	6.6951	.0046
1.00	10.9800	10.8950	.0121	6.5629	6.5684	.0048
1.25	9.9012	9.8938	.0123	6.5485	6.5434	.0048
1.50	9.0066	9.0084	.0140	6.6070	6.6079	.0048
1.75	8.1971	8.1559	.0141	6.7092	6.7049	.0046
2.00	7.4646	7.5036	.0153	6.8354	6.8375	.0045
2.25	6.8018	6.7945	.0152	6.9729	6.9715	.0043
2.50	6.2021	6.2392	.0154	7.1132	7.1140	.0042

TABLE XXXI

Stream Situation 14A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	13.9913	13.9900	.0054	7.4535	7.4489	.0034
.50	12.2406	12.2413	.0070	6.6524	6.6548	.0045
.75	10.7148	10.7166	.0085	6.1185	6.8113	.0052
1.00	9.3850	9.3910	.0093	5.7894	5.7932	.0057
1.25	8.2261	8.2297	.0102	5.6154	5.6227	.0067
1.50	7.2160	7.2052	.0104	5.5575	5.5657	.0065
1.75	6.3357	6.3349	.0110	5.5845	5.5891	.0060
2.00	5.5685	5.5722	.0108	5.6724	5.6698	.0061
2.25	4.8999	4.8879	.0107	5.8024	5.8048	.0067
2.50	4.3171	4.3243	.0109	5.9598	5.9598	.0058

TABLE XXXII

Time (days)	Stream Situation 14A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	13.9913	13.9912	.0051	7.4535	7.4554	.0036
.50	12.2406	12.2440	.0067	6.6524	6.6466	.0046
.75	10.7148	10.7269	.0090	6.1185	6.1163	.0053
1.00	9.3850	9.3799	.0103	5.7894	5.7845	.0057
1.25	8.2261	8.2180	.0107	6.6154	5.6252	.0062
1.50	7.2160	7.2224	.0119	5.5575	5.5565	.0064
1.75	6.3357	6.3479	.0112	5.5845	5.5782	.0063
2.00	5.5685	5.5733	.0105	5.6724	5.6750	.0060
2.25	4.8999	4.9139	.0101	5.8024	5.8055	.0064
2.50	4.3171	4.3287	.0110	5.9598	5.9636	.0060

TABLE XXXIII

Stream Situation 15A				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	13.9913	13.9754	.0183	7.4535	7.4606	.0121
.50	12.2406	12.2923	.0306	6.6524	6.6719	.0179
.75	10.7148	10.6780	.0401	6.1185	6.0851	.0238
1.00	9.3850	9.4775	.0500	5.7894	5.7894	.0263
1.25	8.2261	8.2177	.0522	5.6154	5.5804	.0283
1.50	7.2160	7.2608	.0494	5.5575	5.6464	.0279
1.75	6.3357	6.3687	.0598	5.5845	5.5884	.0279
2.00	5.5685	5.5710	.0563	5.6724	5.6776	.0275
2.25	4.8999	4.9303	.0531	5.8024	5.7466	.0283
2.50	4.3171	4.3599	.0538	5.9598	5.9350	.0274

TABLE XXXIV

Time (days)	Stream Situation 15A			Uniform Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	13.9913	13.9765	.0170	7.4535	7.4338	.0114
.50	12.2406	12.2959	.0305	6.6524	6.6518	.0190
.75	10.7148	10.7298	.0431	6.1185	6.1258	.0242
1.00	9.3850	9.4652	.0461	5.7894	5.8053	.0276
1.25	8.2261	8.3500	.0514	5.6154	5.6333	.0283
1.50	7.2160	7.2332	.0563	5.5575	5.5424	.0265
1.75	6.3357	6.3679	.0563	5.5845	5.5838	.0276
2.00	5.5685	5.6133	.0550	5.6724	5.6795	.0303
2.25	4.8999	4.9536	.0538	5.8024	5.8488	.0262
2.50	4.3171	4.3313	.0495	5.9598	5.9858	.0270

Figure 2
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 4A

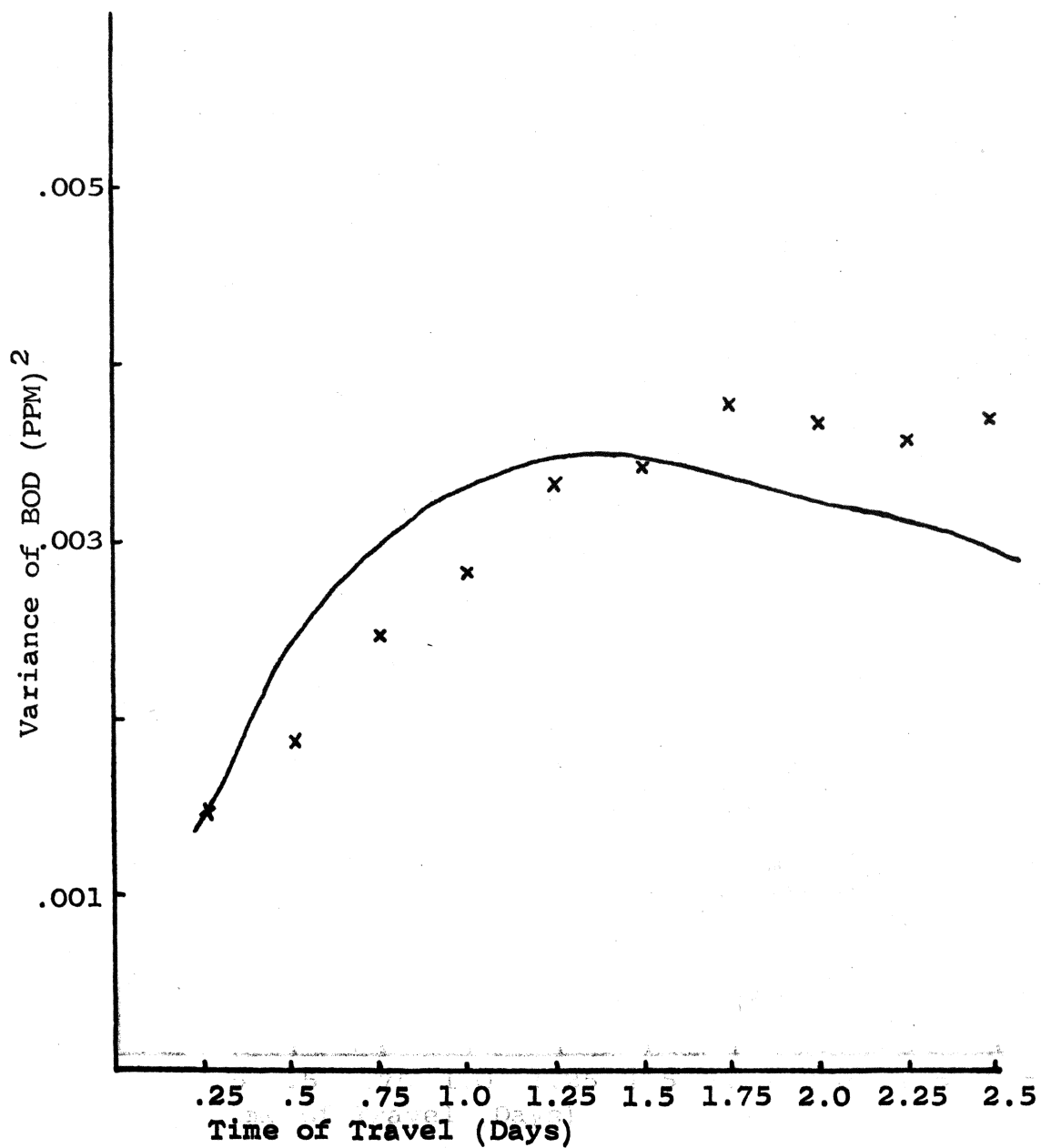


Figure 3
Predicted and Observed Variance
Biochemical Oxygen Demand - Situation 5A

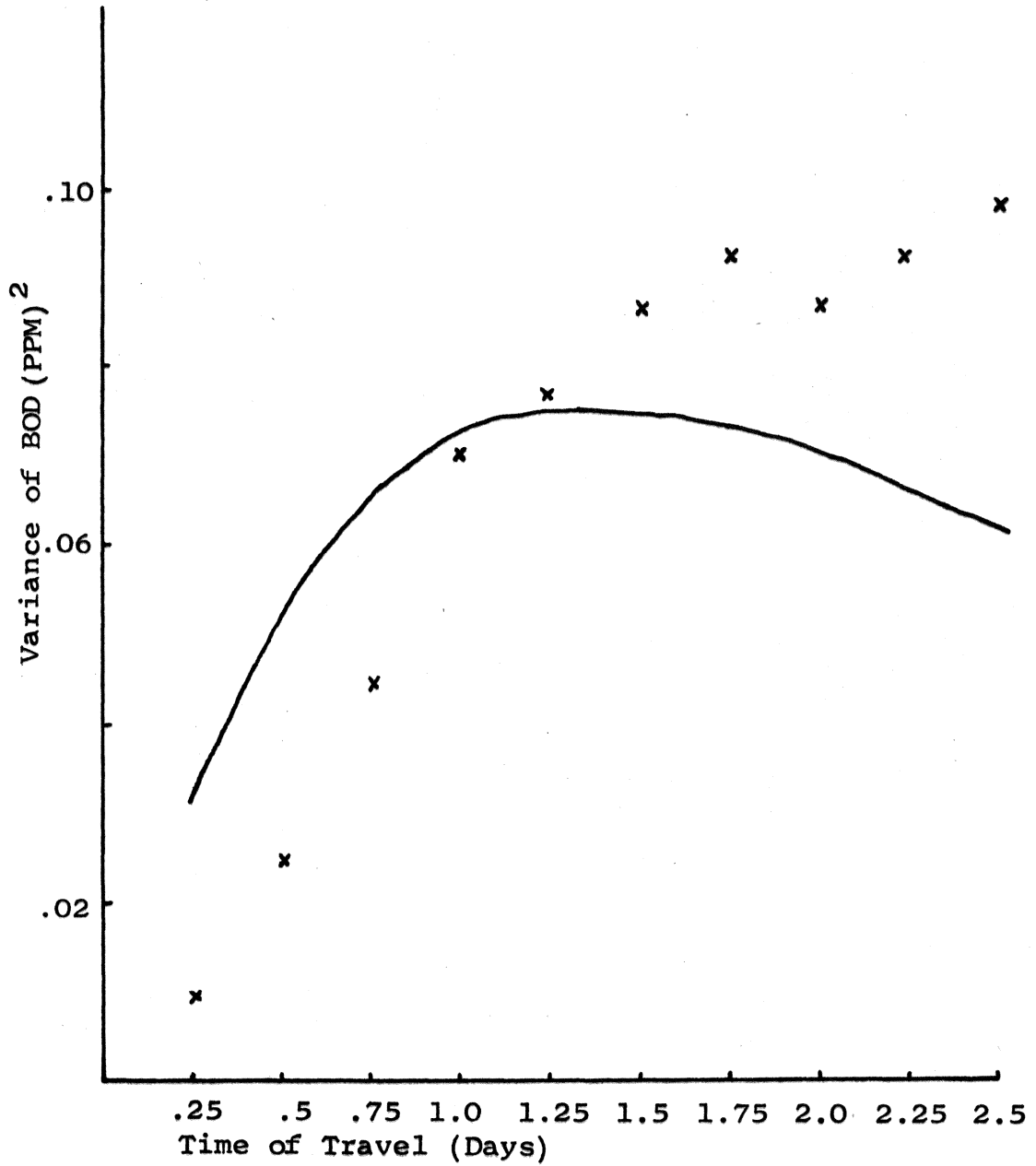


Figure 4
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 9A

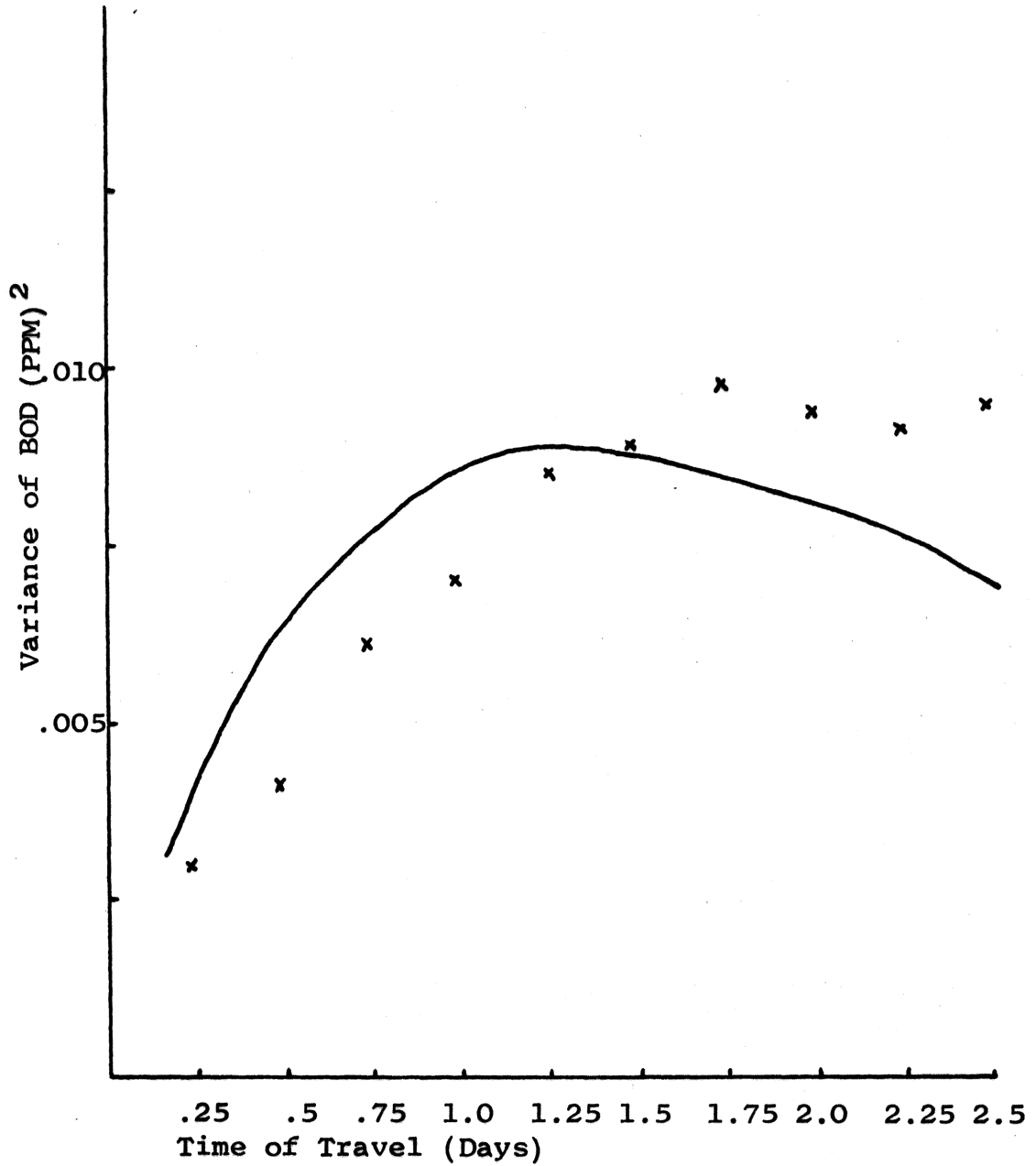


Figure 5
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 10A

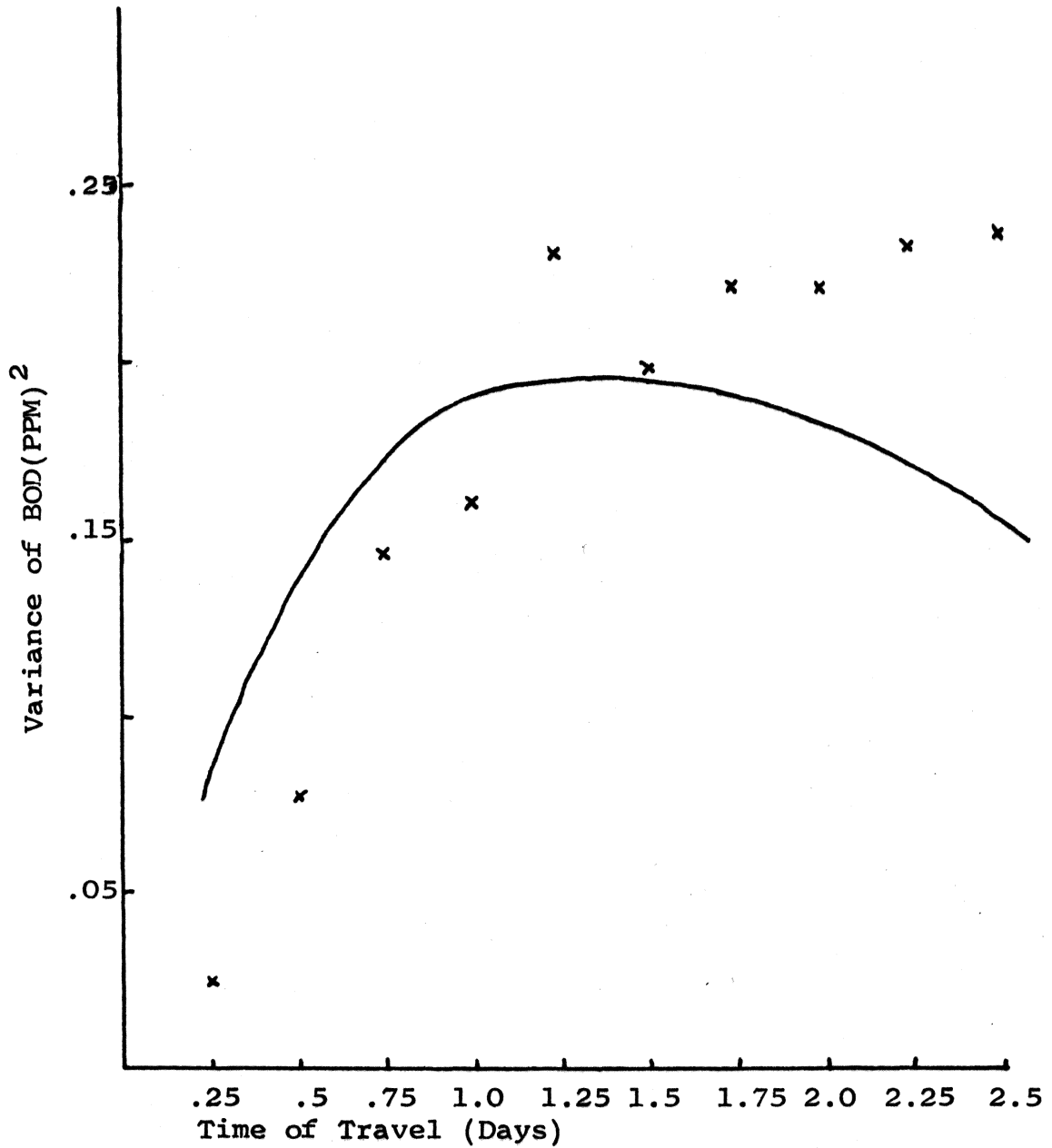


Figure 6
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 14A

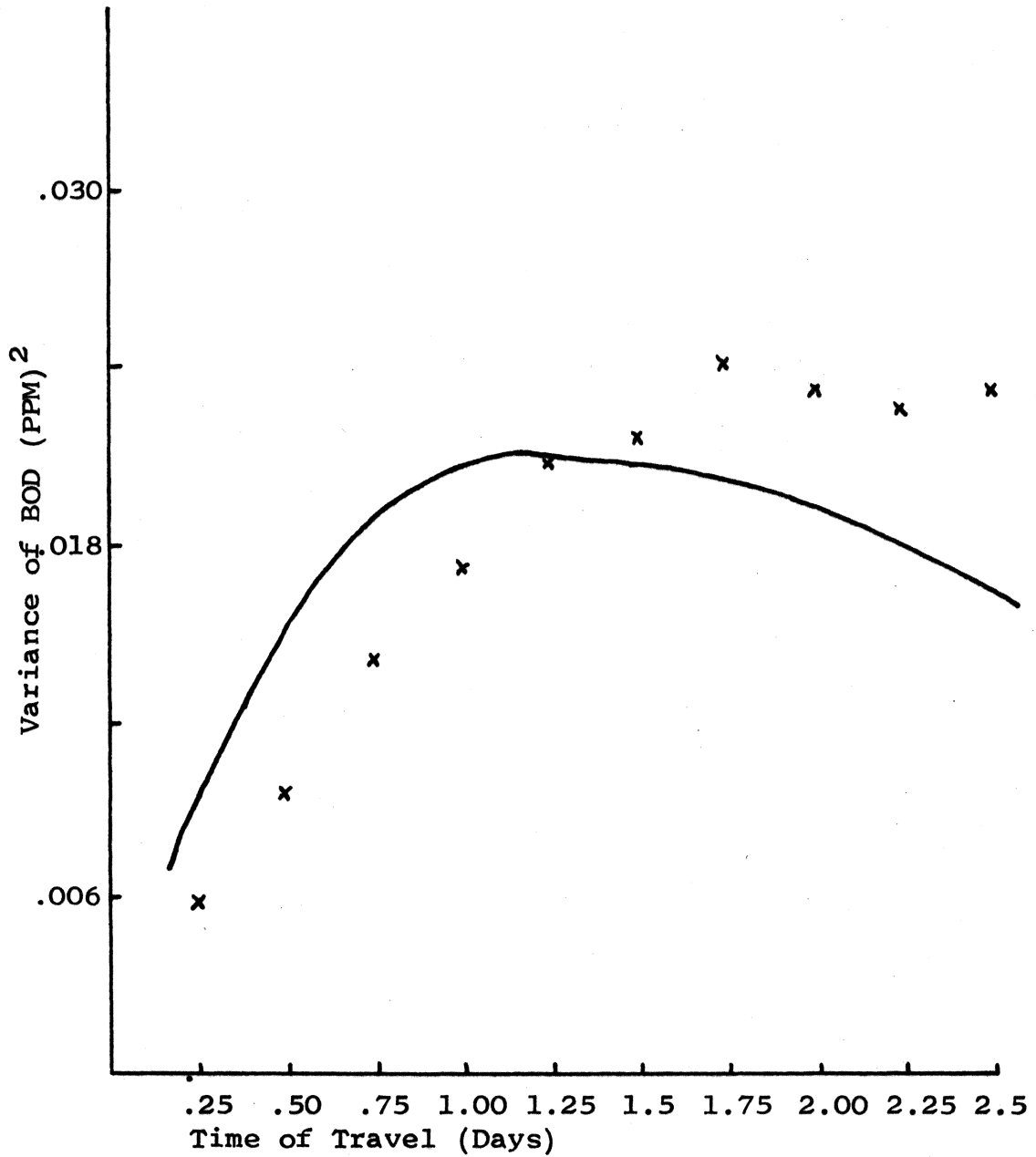


Figure 7
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 15A

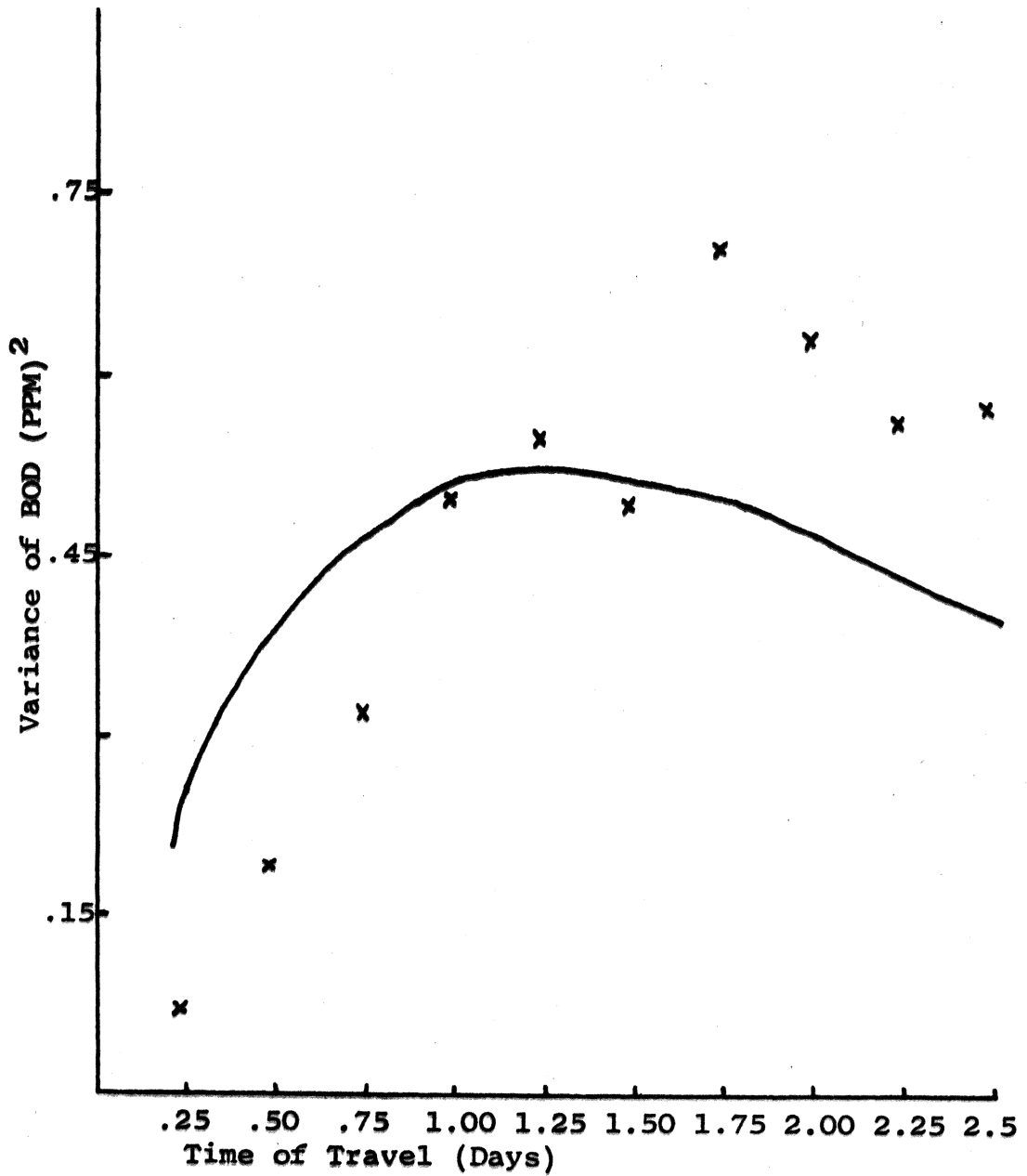


Figure 8
Predicted and Observed Variances
Dissolved Oxygen - Situation 4A

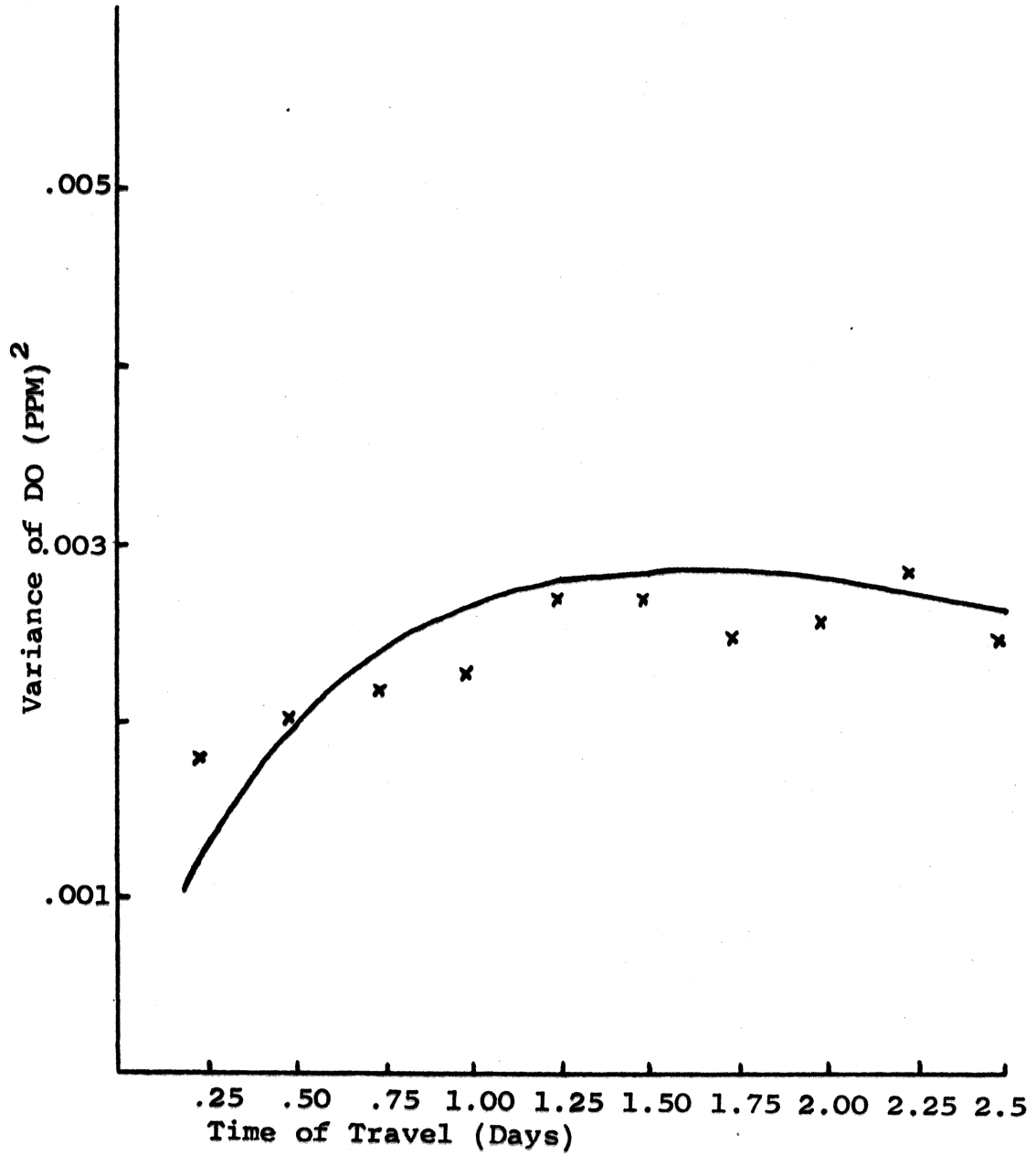


Figure 9
Predicted and Observed Variances
Dissolved Oxygen - Situation 5A

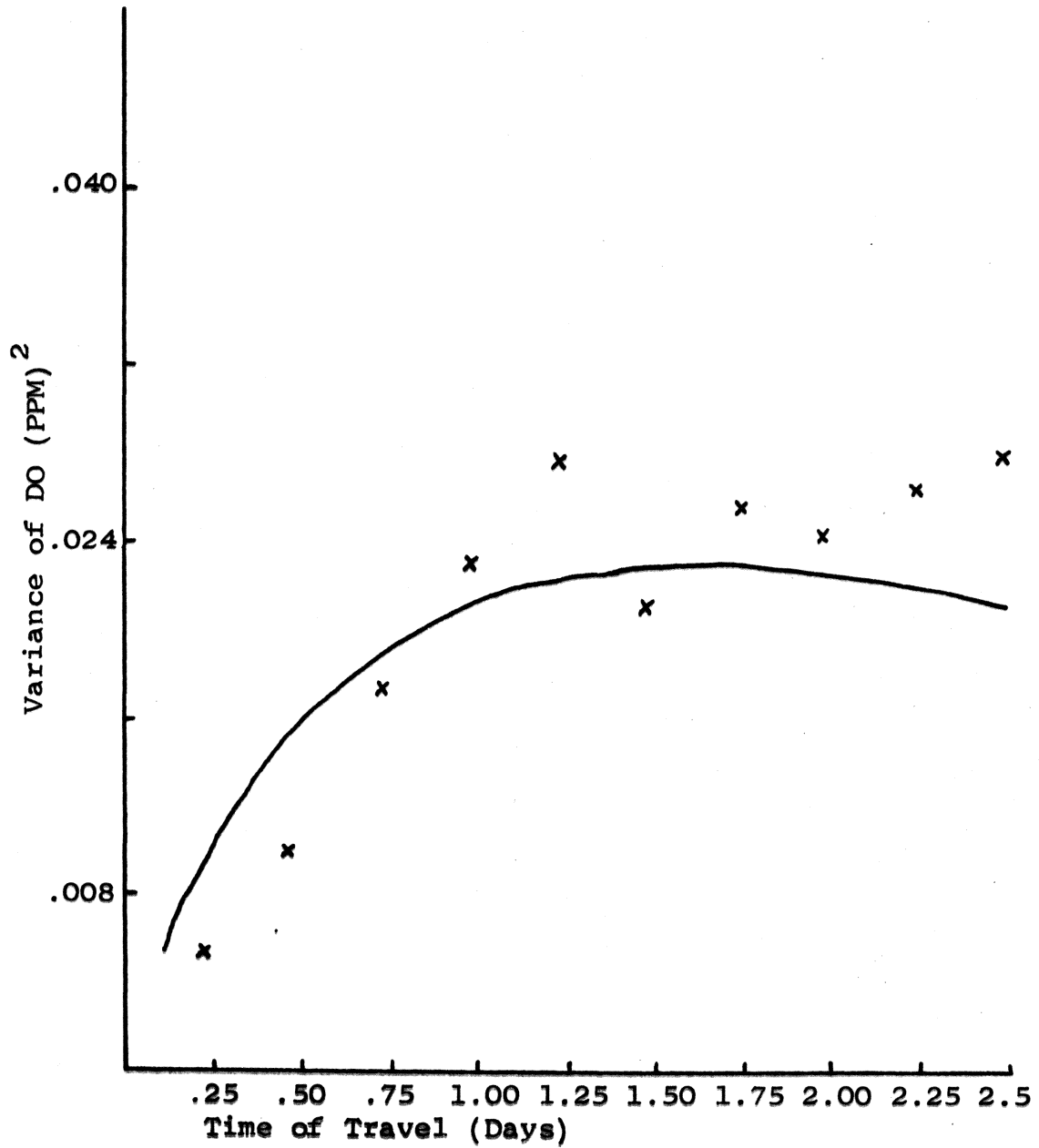


Figure 10
Predicted and Observed Variances
Dissolved Oxygen - Situation 9A

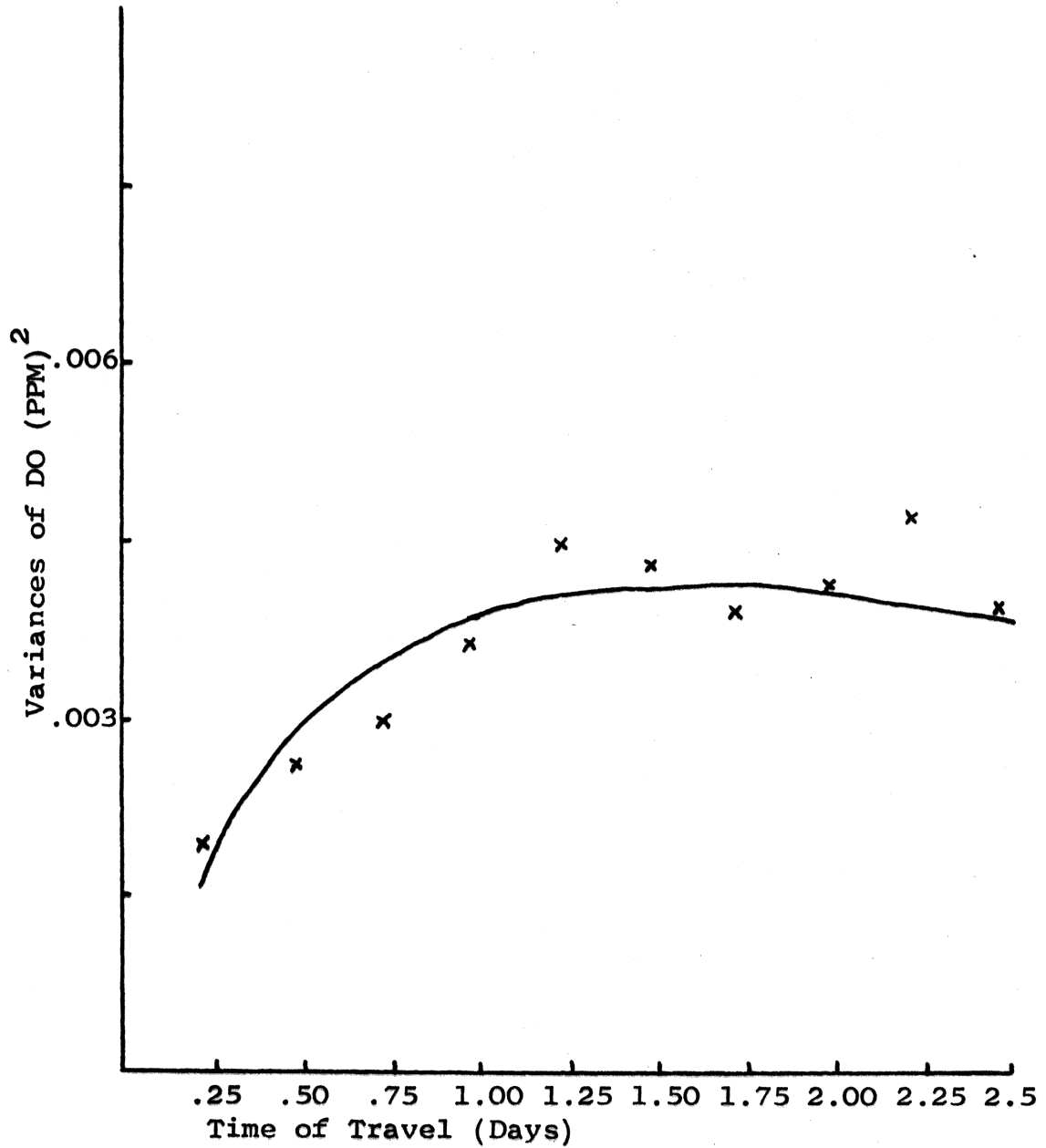


Figure 11
Predicted and Observed Variances
Dissolved Oxygen - Situation 10A

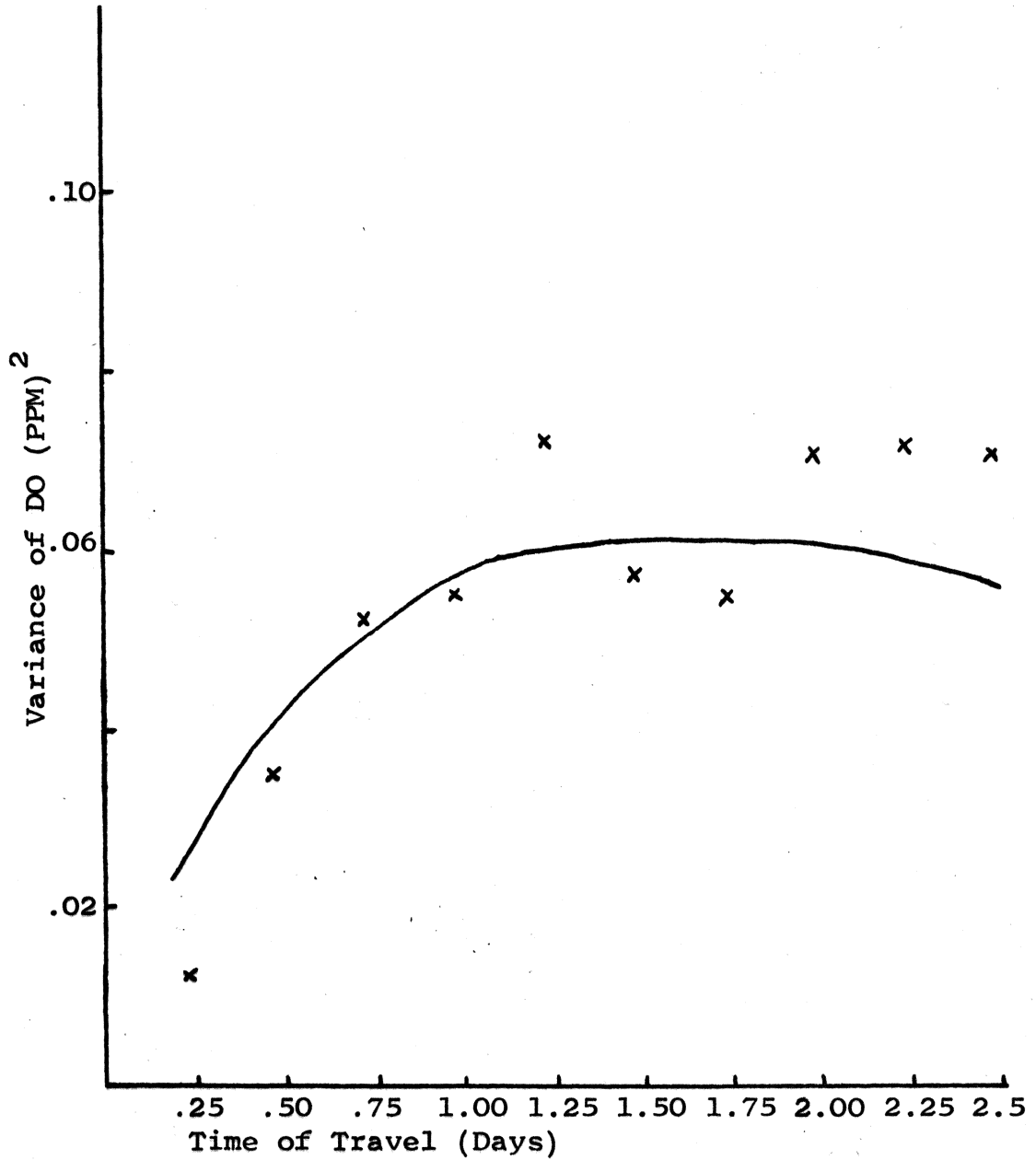


Figure 12
Predicted and Observed Variances
Dissolved Oxygen - Situation 14A

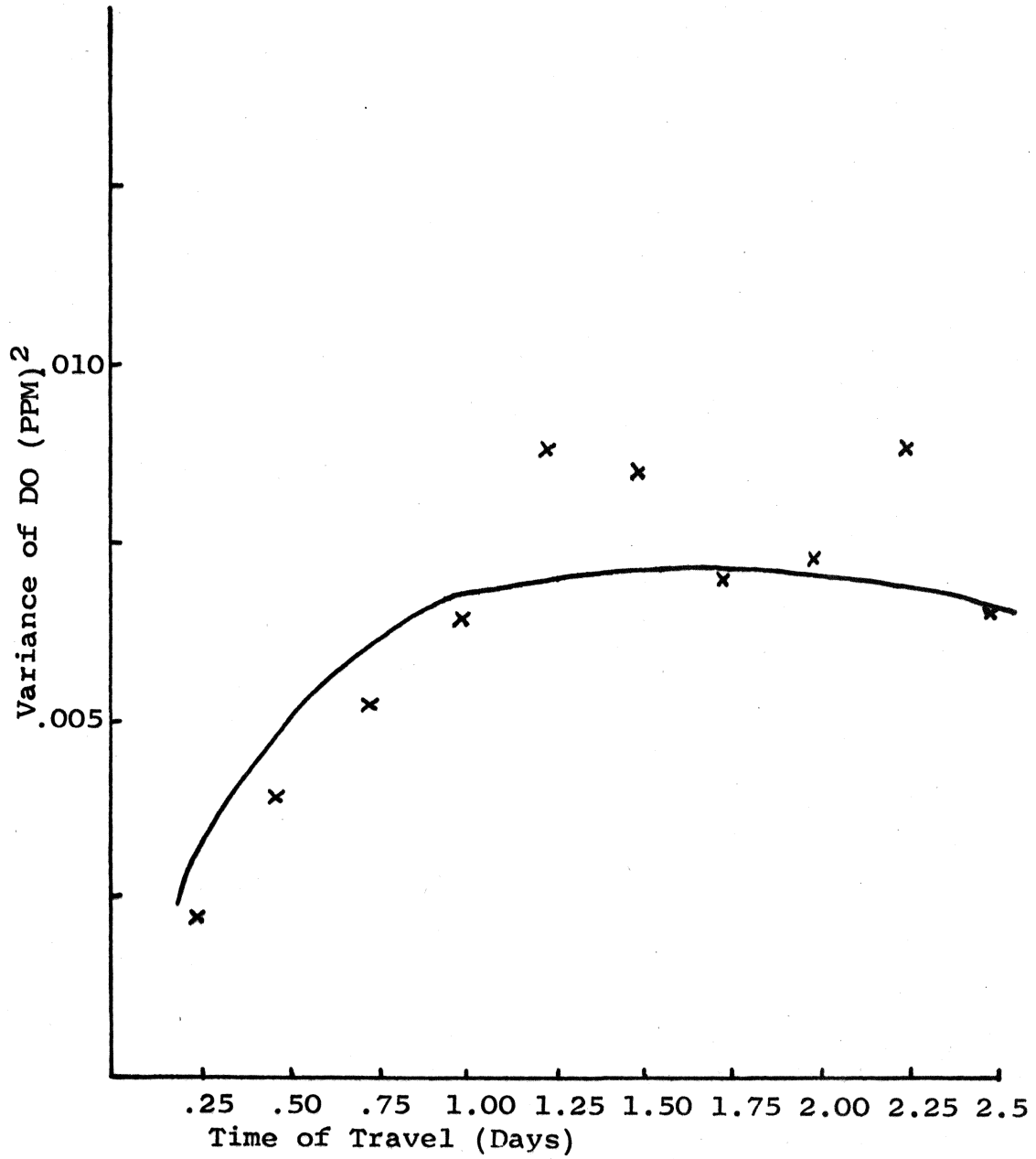


Figure 13
Predicted and Observed Variances
Dissolved Oxygen - Situation 15A

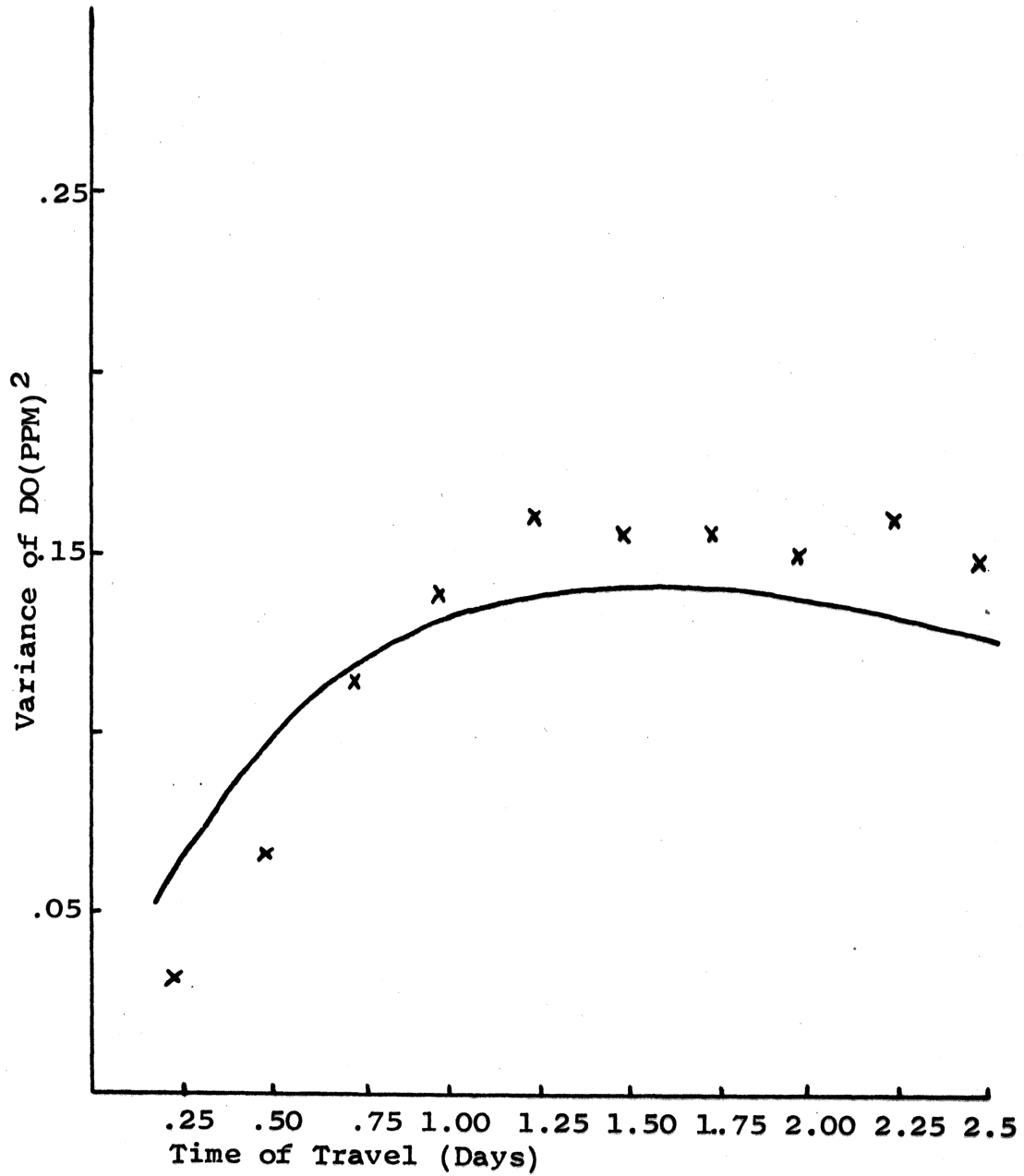


TABLE XXXV

Values Used in Program with
 L_O , L_a , and D_B Randomized

Situation	ML_O	ML_a	MD_B	SD1	SD2	SD3
1B	6	.2	.2	.05	.01	.05
2B	6	.2	.6	.05	.01	.05
3B	6	.6	.6	.05	.01	.05
4B	6	.6	.6	.05	.10	.05
5B	6	.6	.6	.10	.01	.10
6B	10	.2	.2	.05	.01	.05
7B	10	.2	.6	.05	.01	.05
8B	10	.6	.6	.05	.01	.05
9B	10	.6	.6	.05	.10	.05
10B	10	.6	.6	.10	.01	.10
11B	16	.2	.2	.05	.01	.05
12B	16	.2	.6	.05	.01	.05
13B	16	.6	.6	.05	.01	.05
14B	16	.6	.6	.05	.10	.05
15B	16	.6	.6	.10	.01	.10

TABLE XXXVI

Program Results for Mean Initial BOD of Six PPM
with L_0 , L_a , and D_B Randomized

Situation		Normally Distributed		Uniformly Distributed	
		Mean Delta	Standard Deviation	Mean Delta	Standard Deviation
1B	BOD	.0020	.00089	.0019	.00071
	DO	.0031	.00068	.0030	.00057
2B	BOD	.0020	.00089	.0019	.00071
	DO	.0024	.00044	.0024	.00045
3B	BOD	.0016	.00051	.0016	.00048
	DO	.0022	.00039	.0023	.00040
4B	BOD	.0036	.00219	.0038	.00246
	DO	.0025	.00042	.0024	.00037
5B	BOD	.0050	.00268	.0051	.00260
	DO	.0056	.00196	.0056	.00201

TABLE XXXVII

Program Results for Mean Initial BOD of Ten PPM
with L_0 , L_a , and D_B Randomized

Situation		Normally Distributed		Uniformly Distributed	
		Mean Delta	Standard Deviation	Mean Delta	Standard Deviation
6B	BOD	.0015	.00053	.0015	.00044
	DO	.0020	.00048	.0020	.00047
7B	BOD	.0015	.00053	.0015	.00044
	DO	.0016	.00033	.0016	.00034
8B	BOD	.0013	.00038	.0013	.00038
	DO	.0016	.00034	.0015	.00029
9B	BOD	.0027	.00158	.0027	.00152
	DO	.0016	.00032	.0017	.00027
10B	BOD	.0037	.00184	.0039	.00210
	DO	.0040	.00157	.0040	.00150

TABLE XXXVIII

Program Results for Mean Initial BOD of Sixteen PPM
with L_o , L_a , and D_B Randomized

Situation		Normally Distributed		Uniformly Distributed	
		Mean Delta	Standard Deviation	Mean Delta	Standard Deviation
11B	BOD	.0013	.00039	.0012	.00032
	DO	.0012	.00034	.0012	.00033
12B	BOD	.0013	.00039	.0012	.00032
	DO	.0010	.00026	.0010	.00029
13B	BOD	.0011	.00032	.0011	.00029
	DO	.0010	.00024	.0010	.00025
14B	BOD	.0020	.00136	.0021	.00134
	DO	.0011	.00022	.0011	.00024
15B	BOD	.0027	.00123	.0027	.00120
	DO	.0028	.00118	.0028	.00117

TABLE XXXIX

Stream Situation 1B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.2759	5.2756	.0025	8.1745	8.1738	.0029
.50	4.6449	4.6470	.0026	7.8757	7.8752	.0031
.75	4.0949	4.0945	.0031	7.6748	7.6757	.0034
1.00	3.6155	3.6170	.0033	7.5492	7.5521	.0037
1.25	3.1978	3.1961	.0038	7.4808	7.4835	.0040
1.50	2.8337	2.8320	.0036	7.4551	7.4630	.0046
1.75	2.5164	2.5186	.0038	7.4609	7.4603	.0049
2.00	2.2398	2.2439	.0045	7.4893	7.4866	.0046
2.25	1.9988	2.0024	.0050	7.5334	7.5367	.0048
2.50	1.7887	1.7879	.0051	7.5879	7.5813	.0050

TABLE XL

Stream Situation 2B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.2759	5.2756	.0025	8.0833	8.0838	.0030
.50	4.6449	4.6470	.0026	7.7089	7.7101	.0031
.75	4.0949	4.0945	.0031	7.4454	7.4454	.0034
1.00	3.6155	3.6170	.0033	7.2678	7.2707	.0037
1.25	3.1978	3.1961	.0038	7.1563	7.1562	.0040
1.50	2.8337	2.8320	.0036	7.0940	7.1034	.0042
1.75	2.5164	2.5186	.0038	7.0711	7.0725	.0045
2.00	2.2398	2.2439	.0045	7.0750	7.0755	.0046
2.25	1.9988	2.0024	.0050	7.0987	7.0959	.0048
2.50	1.7887	1.7879	.0051	7.1364	7.1303	.0050

TABLE XLI

Time (days)	Stream Situation 3B			Normal Distribution		
	Mean BOD	Average BOD	Standard BOD	Mean DO	Average DO	Standard Error
.25	5.3693	5.3677	.0025	8.0794	8.0779	.0029
.50	4.8197	4.8212	.0026	7.6947	7.5970	.0032
.75	4.3407	4.3396	.0031	7.4167	7.4183	.0034
1.00	3.9232	3.9201	.0031	7.2219	7.2226	.0036
1.25	3.5594	3.5628	.0037	7.0914	7.0892	.0039
1.50	3.2422	3.2474	.0038	7.0103	7.0153	.0043
1.75	2.9659	2.9719	.0044	6.9666	6.9629	.0044
2.00	2.7250	2.7244	.0041	6.9510	6.9582	.0049
2.25	2.5151	2.5108	.0047	6.9559	6.9587	.0047
2.50	2.3321	2.3325	.0050	6.9757	6.9733	.0050

TABLE XLII

Stream Situation 4B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.3693	5.3874	.0062	8.0794	8.0787	.0031
.50	4.8197	4.8273	.0058	7.6947	7.6951	.0035
.75	4.3407	4.3461	.0057	7.4167	7.4155	.0035
1.00	3.9232	3.9230	.0051	7.2219	7.2196	.0040
1.25	3.5594	3.5618	.0053	7.0914	7.0089	.0041
1.50	3.2422	3.2340	.0048	7.0103	7.0176	.0043
1.75	2.9659	2.9652	.0049	6.9666	6.9644	.0048
2.00	2.7250	2.7211	.0048	6.9510	6.9494	.0049
2.25	2.5151	2.5139	.0056	6.9559	6.9559	.0049
2.50	2.3321	2.3340	.0052	6.9757	6.9679	.0053

TABLE XLIII

Stream Situation 5B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	5.3693	5.3668	.0028	8.0794	8.0765	.0033
.50	4.8197	4.8233	.0036	7.6947	7.6995	.0040
.75	4.3407	4.3375	.0050	7.4167	7.4200	.0049
1.00	3.0232	3.9168	.0055	7.2219	7.2235	.0057
1.25	3.5594	3.5665	.0068	7.0914	7.0869	.0063
1.50	3.2422	3.2527	.0070	7.0103	7.0207	.0072
1.75	2.9659	2.9777	.0085	6.9666	6.9593	.0075
2.00	2.7250	2.7235	.0078	6.9510	6.9654	.0086
2.25	2.5151	2.5066	.0091	6.9559	6.9616	.0082
2.50	2.3321	2.3326	.0098	6.9757	6.9709	.0088

TABLE XLIV

Stream Situation 6B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.7621	8.7617	.0032	7.8770	7.0763	.0029
.50	7.6831	7.6852	.0033	7.3697	7.3692	.0031
.75	6.7428	6.7424	.0036	7.0294	7.0303	.0033
1.00	5.9233	5.9248	.0037	6.8172	6.8201	.0036
1.25	5.2091	5.2074	.0040	6.7022	6.7049	.0039
1.50	4.5866	4.5850	.0039	6.6600	6.6680	.0044
1.75	4.0441	4.0463	.0041	6.6714	6.6708	.0048
2.00	3.5713	3.5753	.0047	6.7211	6.7184	.0044
2.25	3.1592	3.1628	.0052	6.7975	6.8108	.0048
2.50	2.8001	2.7992	.0052	6.8915	6.8849	.0050

TABLE XLV

Time (days)	Stream Situation 7B			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.7621	8.7617	.0032	7.7858	7.7863	.0030
.50	7.6831	7.6852	.0033	7.7029	7.2041	.0031
.75	6.7428	6.7424	.0036	6.7999	6.7999	.0034
1.00	5.9233	5.9248	.0037	6.5358	6.5386	.0036
1.25	5.2091	5.2074	.0040	6.3777	6.3776	.0039
1.50	4.5866	4.5850	.0039	6.2998	6.3083	.0042
1.75	4.0441	4.0463	.0041	6.2816	6.2829	.0044
2.00	3.5713	3.5753	.0047	6.3068	6.3073	.0045
2.25	3.1592	3.1628	.0052	6.3629	6.3600	.0047
2.50	2.8001	2.7992	.0052	6.4400	6.4339	.0049

TABLE XLVI

Stream Situation 8B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.8555	8.8539	.0034	7.7819	7.7803	.0031
.50	7.5580	7.8595	.0031	7.1888	7.1911	.0030
.75	6.9887	6.9875	.0035	6.7713	6.7729	.0033
1.00	6.2310	6.2279	.0036	6.4898	6.4906	.0036
1.25	5.5707	5.5741	.0040	6.3128	6.3106	.0038
1.50	4.9952	5.0003	.0040	6.2152	6.2203	.0042
1.75	4.4936	4.4996	.0046	6.1771	6.1734	.0043
2.00	4.0565	4.0558	.0043	6.1828	6.1900	.0047
2.25	3.6755	3.6712	.0049	6.2200	6.2228	.0046
2.50	3.3435	3.3539	.0051	6.2793	6.2769	.0049

TABLE XLVII

Stream Situation 9B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.8555	8.6606	.0066	7.7819	7.7811	.0029
.50	7.8580	7.8583	.0060	7.1888	7.1858	.0032
.75	6.9887	6.9930	.0058	6.7713	6.7729	.0034
1.00	6.2310	6.2358	.0061	6.4898	6.4904	.0038
1.25	5.5707	5.5737	.0055	6.3128	6.3121	.0038
1.50	4.9952	4.9960	.0047	6.2152	6.2108	.0041
1.75	4.4936	4.4857	.0054	6.1771	6.1769	.0047
2.00	4.0565	4.0579	.0054	6.1828	6.1848	.0046
2.25	3.6755	3.6727	.0050	6.2200	6.2140	.0051
2.50	3.3435	3.3428	.0054	6.2793	6.2708	.0049

TABLE XLVIII

Stream Situation 10B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	8.8555	8.8529	.0036	7.7819	7.7790	.0032
.50	7.8580	7.8616	.0041	7.1888	7.1935	.0041
.75	6.9887	6.9855	.0053	6.7713	6.7745	.0048
1.00	6.2310	6.2246	.0057	6.4898	6.4915	.0056
1.25	5.5707	5.5778	.0070	6.3128	6.3083	.0062
1.50	4.9952	5.0057	.0071	6.2152	6.2256	.0072
1.75	4.4936	4.5054	.0086	6.1771	6.1697	.0074
2.00	4.0565	4.0550	.0080	6.1828	6.1972	.0085
2.25	3.6755	3.6670	.0092	6.2200	6.2257	.0082
2.50	3.3435	3.3440	.0098	6.2793	6.2745	.0088

TABLE XLIX

Stream Situation 11B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	13.9913	13.9909	.0042	7.4307	7.4300	.0028
.50	12.2406	12.2427	.0041	6.6107	6.6103	.0027
.75	10.7148	10.7144	.0042	6.0611	6.0620	.0032
1.00	9.3850	9.3865	.0042	5.7190	5.7220	.0034
1.25	8.2261	8.2244	.0045	5.5343	5.5370	.0037
1.50	7.2160	7.2143	.0043	5.4674	5.4754	.0042
1.75	6.3357	6.3379	.0044	5.4871	5.4865	.0046
2.00	5.5685	5.5726	.0049	5.5688	5.5661	.0043
2.25	4.8999	4.9034	.0054	5.6937	5.6969	.0047
2.50	4.3171	4.3162	.0054	5.8469	5.8403	.0048

TABLE L

Time (days)	Stream Situation 12B			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	13.9913	13.9909	.0042	7.3395	7.3400	.0029
.50	12.2406	12.2427	.0041	6.4439	6.4451	.0030
.75	10.7148	10.7144	.0042	5.8317	5.8317	.0031
1.00	9.3850	9.3865	.0042	5.4376	5.4405	.0034
1.25	8.2261	8.2244	.0045	5.2099	5.2097	.0036
1.50	7.2160	7.2143	.0043	5.1072	5.1157	.0040
1.75	6.3357	6.3379	.0044	5.0973	5.0987	.0042
2.00	5.5685	5.5726	.0049	5.1545	5.1550	.0044
2.25	4.8999	4.9034	.0054	5.2590	5.2562	.0046
2.50	4.3171	4.3162	.0054	5.3954	5.3893	.0048

TABLE LI

Stream Situation 13B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	14.0847	14.0831	.0042	7.3356	7.3340	.0029
.50	12.4154	12.4169	.0039	6.4298	6.4321	.0030
.75	10.9606	10.9595	.0042	5.8030	5.8047	.0031
1.00	9.6927	9.6896	.0042	5.3917	5.3924	.0034
1.25	8.5877	8.5911	.0045	5.1449	5.1427	.0036
1.50	7.6246	7.6297	.0044	5.0226	5.0277	.0040
1.75	6.7852	6.7922	.0049	4.9928	4.9891	.0041
2.00	6.0537	6.0530	.0046	5.0305	5.0378	.0046
2.25	5.4162	5.4118	.0050	5.1162	5.1190	.0044
2.50	4.8605	4.8609	.0052	5.2347	5.2323	.0048

TABLE LII

Stream Situation 14B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	14.0847	14.0781	.0074	7.3356	7.3333	.0028
.50	12.4154	12.4121	.0065	6.4298	6.4315	.0030
.75	10.9606	10.9689	.0062	5.8030	5.8058	.0032
1.00	9.6927	9.6923	.0058	5.3917	5.3923	.0036
1.25	8.5877	8.5896	.0057	5.1449	5.1440	.0039
1.50	7.6246	7.6287	.0053	5.0226	5.0260	.0043
1.75	6.7852	6.7934	.0054	4.9928	4.9892	.0043
2.00	6.0537	6.0553	.0049	5.0305	5.0389	.0047
2.25	5.4162	5.4113	.0053	5.1162	5.1189	.0046
2.50	4.8605	4.8638	.0056	5.2347	5.2326	.0048

TABLE LIII

Stream Situation 15B				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean DO	Average DO	Standard Error
.25	14.0847	14.0821	.0044	7.3356	7.3327	.0032
.50	12.4154	12.4190	.0047	6.4298	6.4346	.0038
.75	10.9606	10.9574	.0057	5.8030	5.8063	.0047
1.00	9.6927	9.6863	.0061	5.3917	5.3933	.0055
1.25	8.5877	8.5948	.0072	5.1449	5.1405	.0062
1.50	7.6246	7.6351	.0073	5.0226	5.0330	.0071
1.75	6.7852	6.7970	.0088	4.9928	4.9854	.0073
2.00	6.0537	6.0522	.0081	5.0305	5.0450	.0084
2.25	5.4162	5.4076	.0093	5.1162	5.1219	.0081
2.50	4.8605	4.8610	.0099	5.2347	5.2299	.0087

Figure 14
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 3B

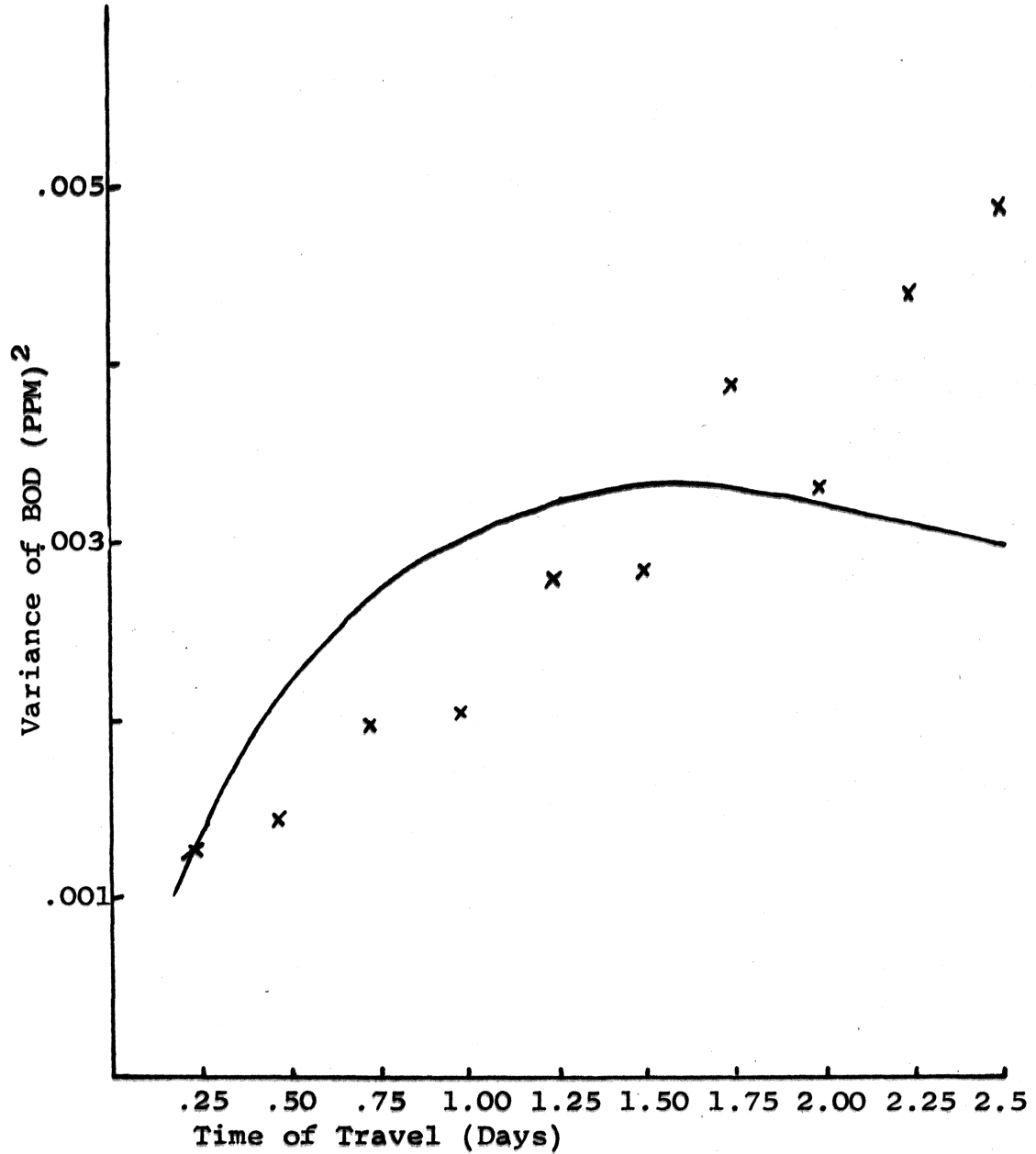


Figure 15
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 4B

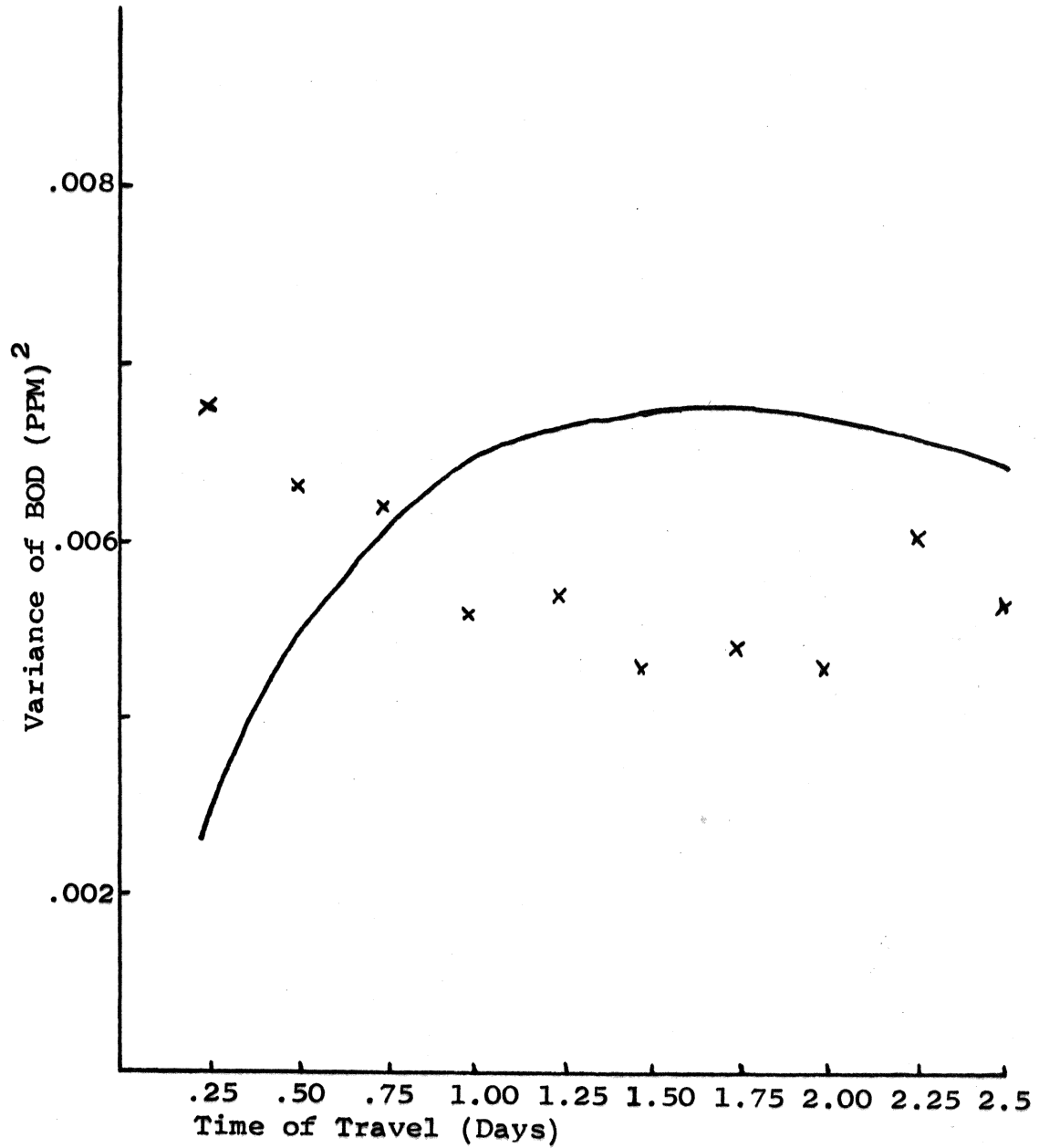


Figure 16
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 5B

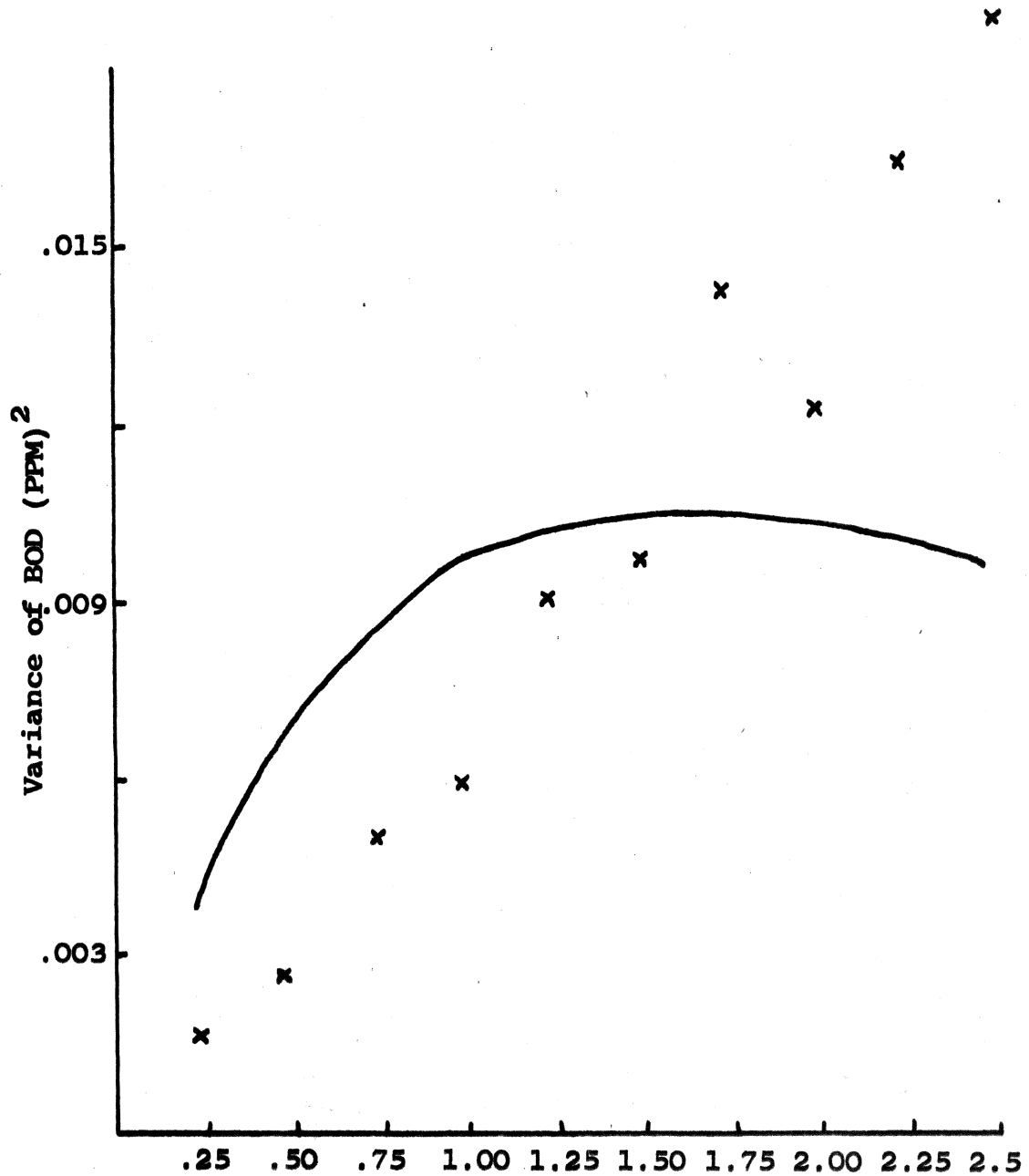


Figure 17
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 13B

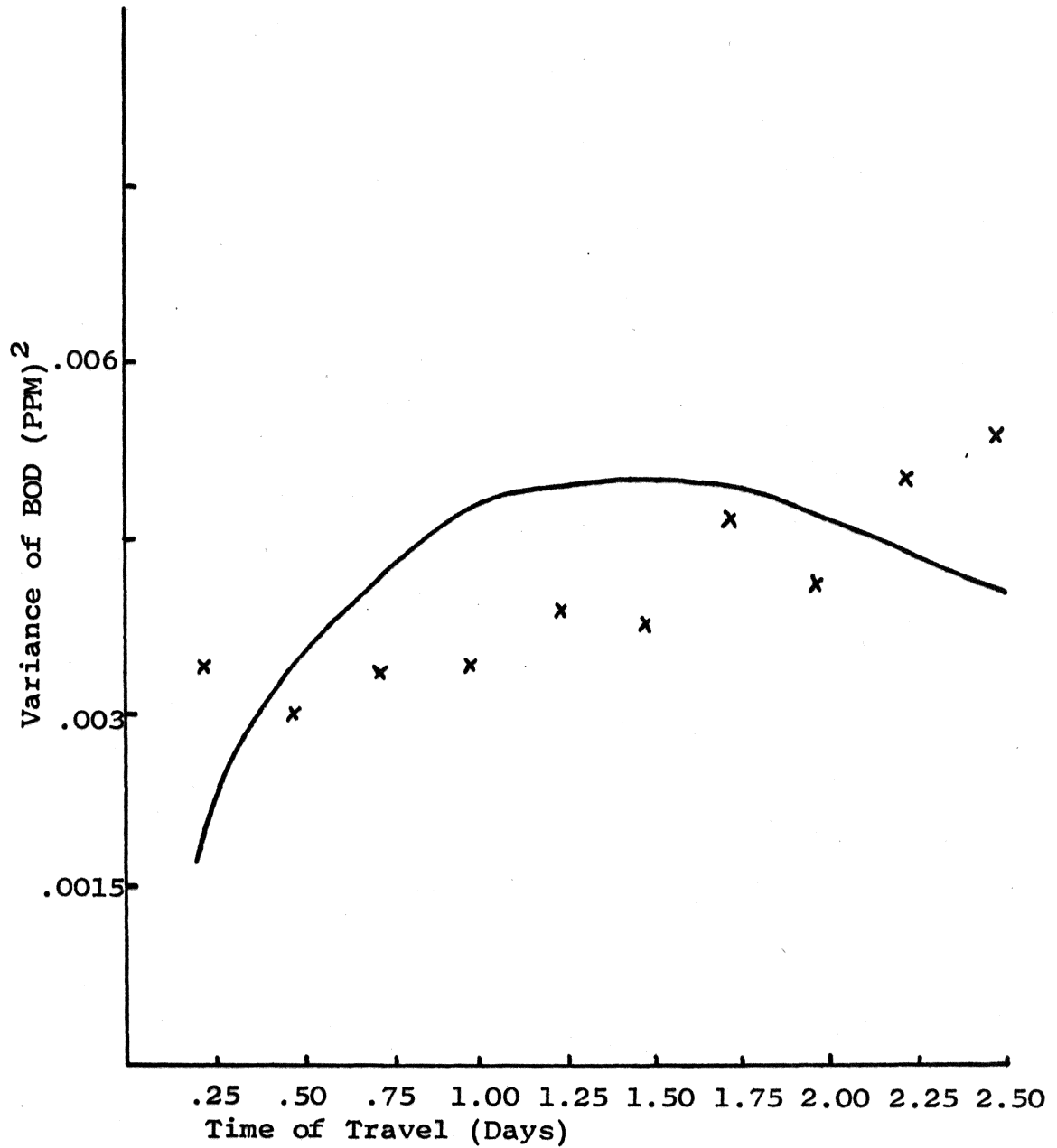


Figure 18
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 14B

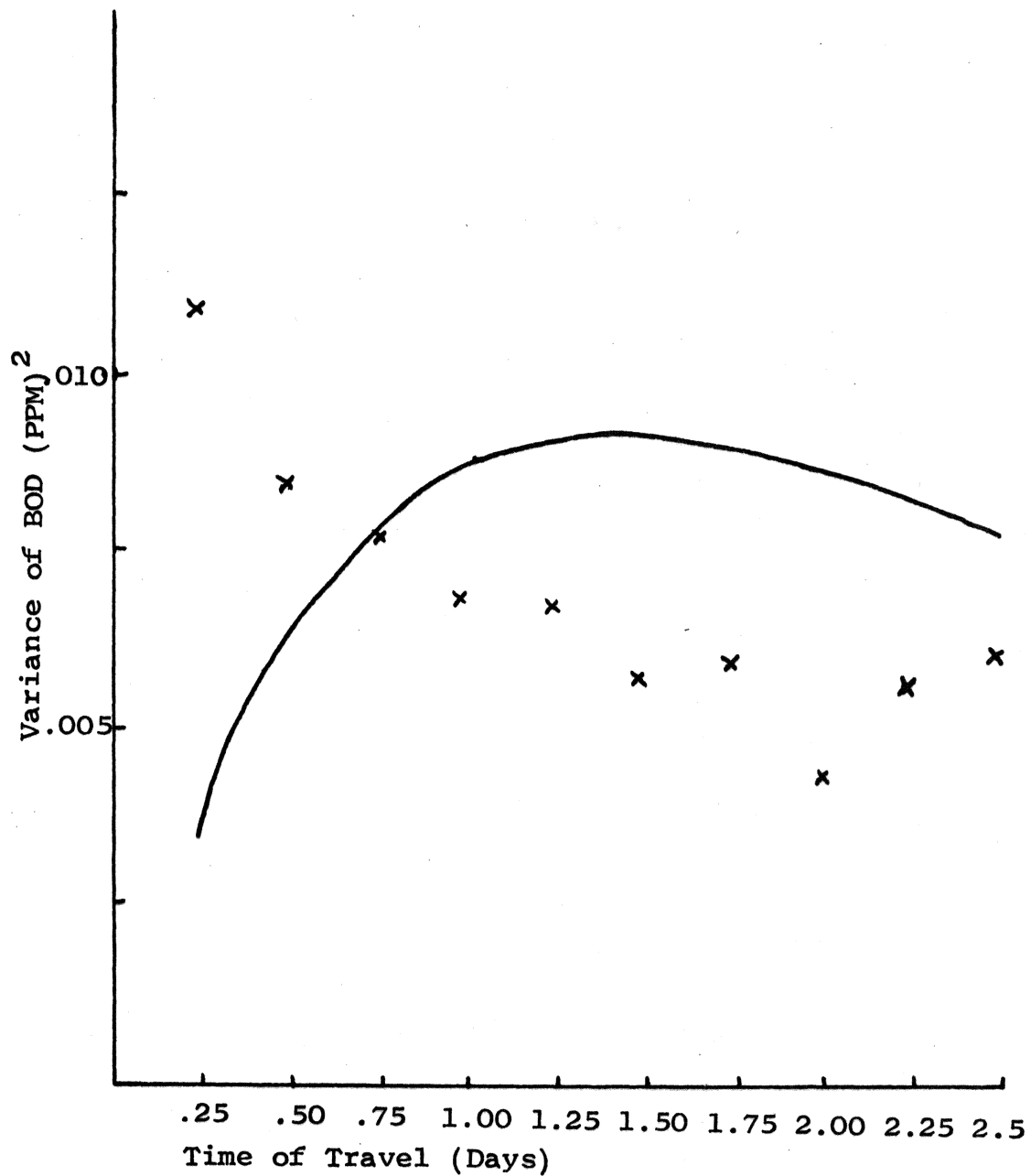


Figure 19
Predicted and Observed Variances
Biochemical Oxygen Demand - Situation 15B

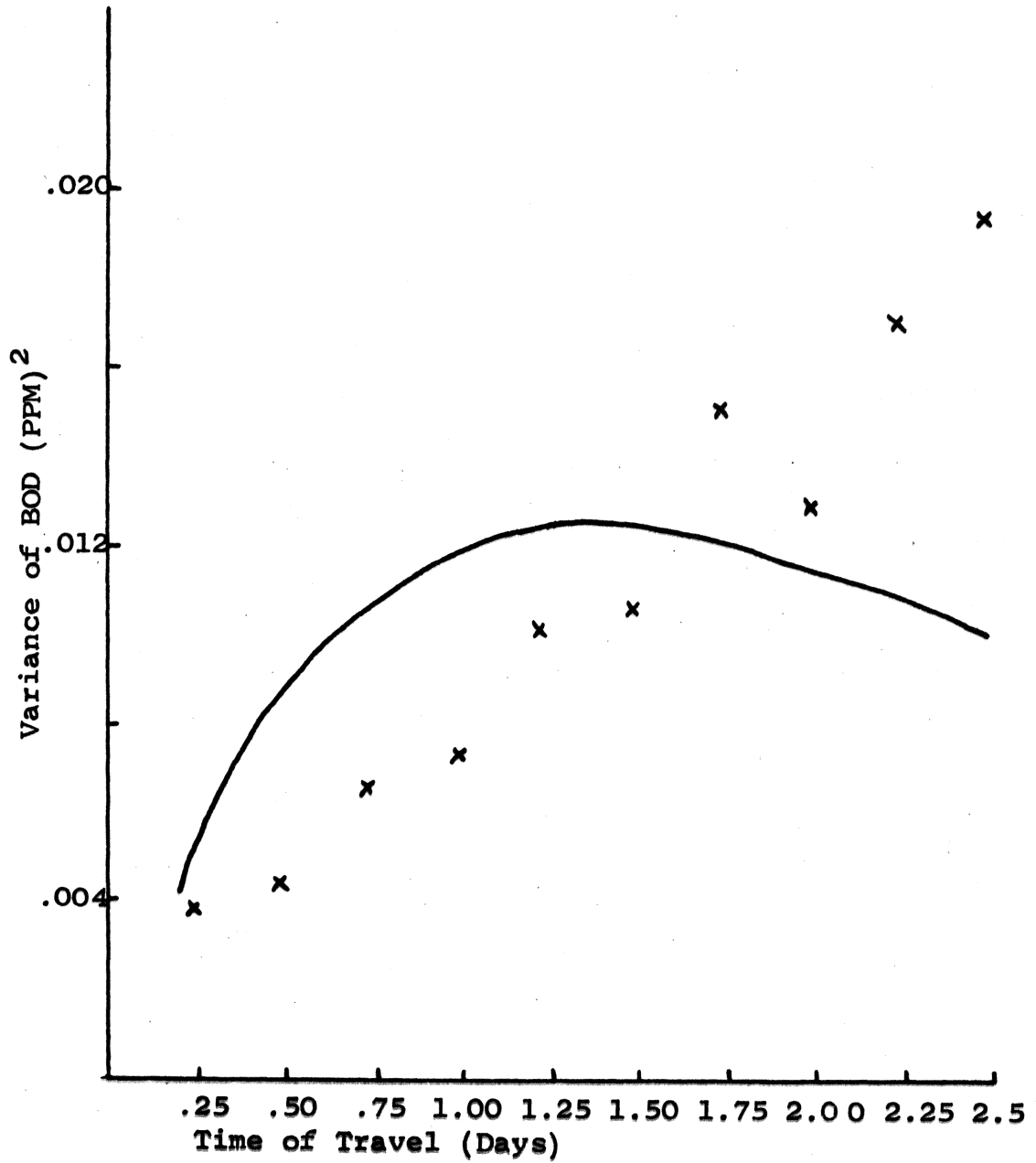


Figure 20
Predicted and Observed Variances
Dissolved Oxygen - Situation 3B

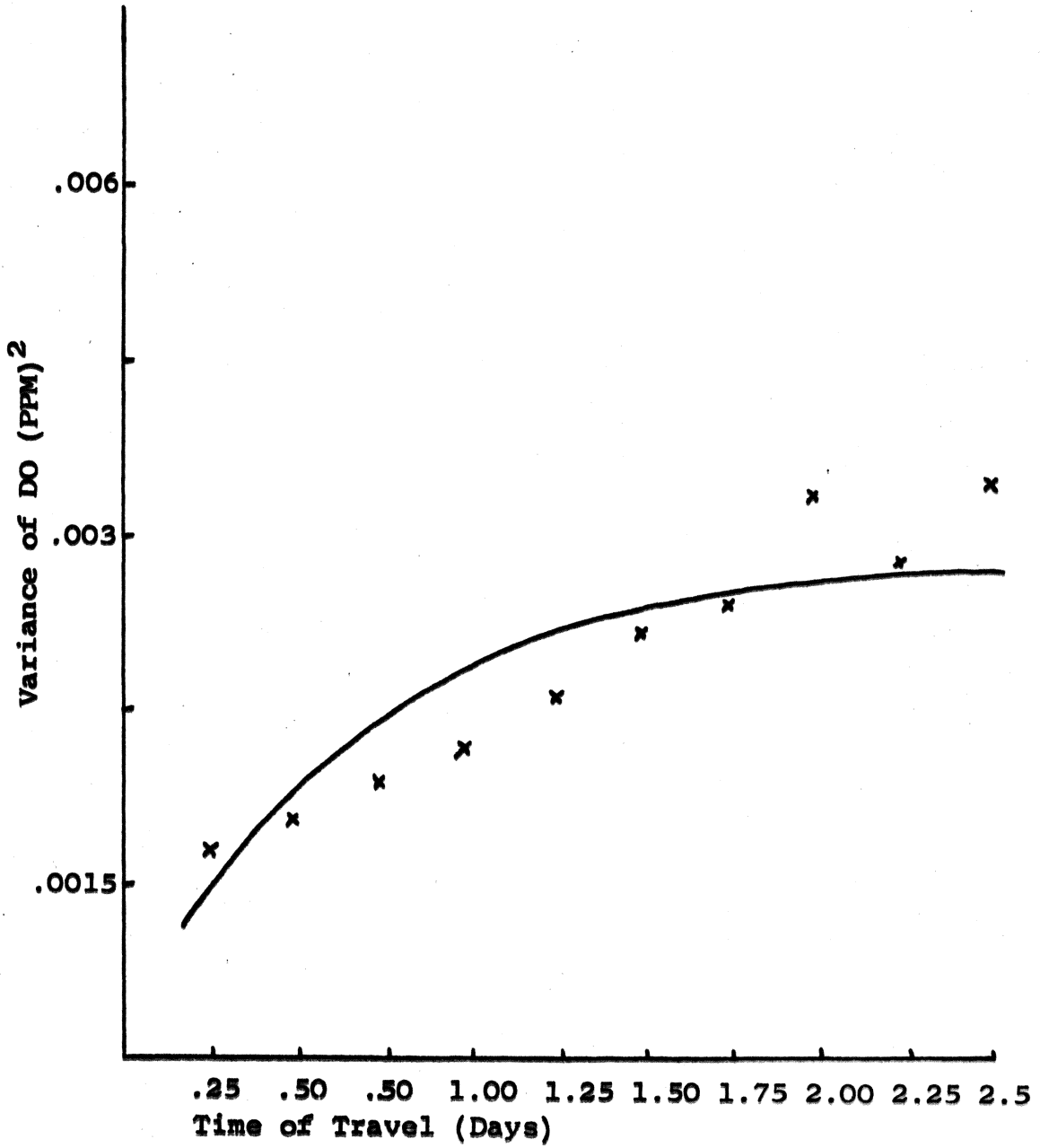


Figure 21
Predicted and Observed Variances
Dissolved Oxygen - Situation 4B

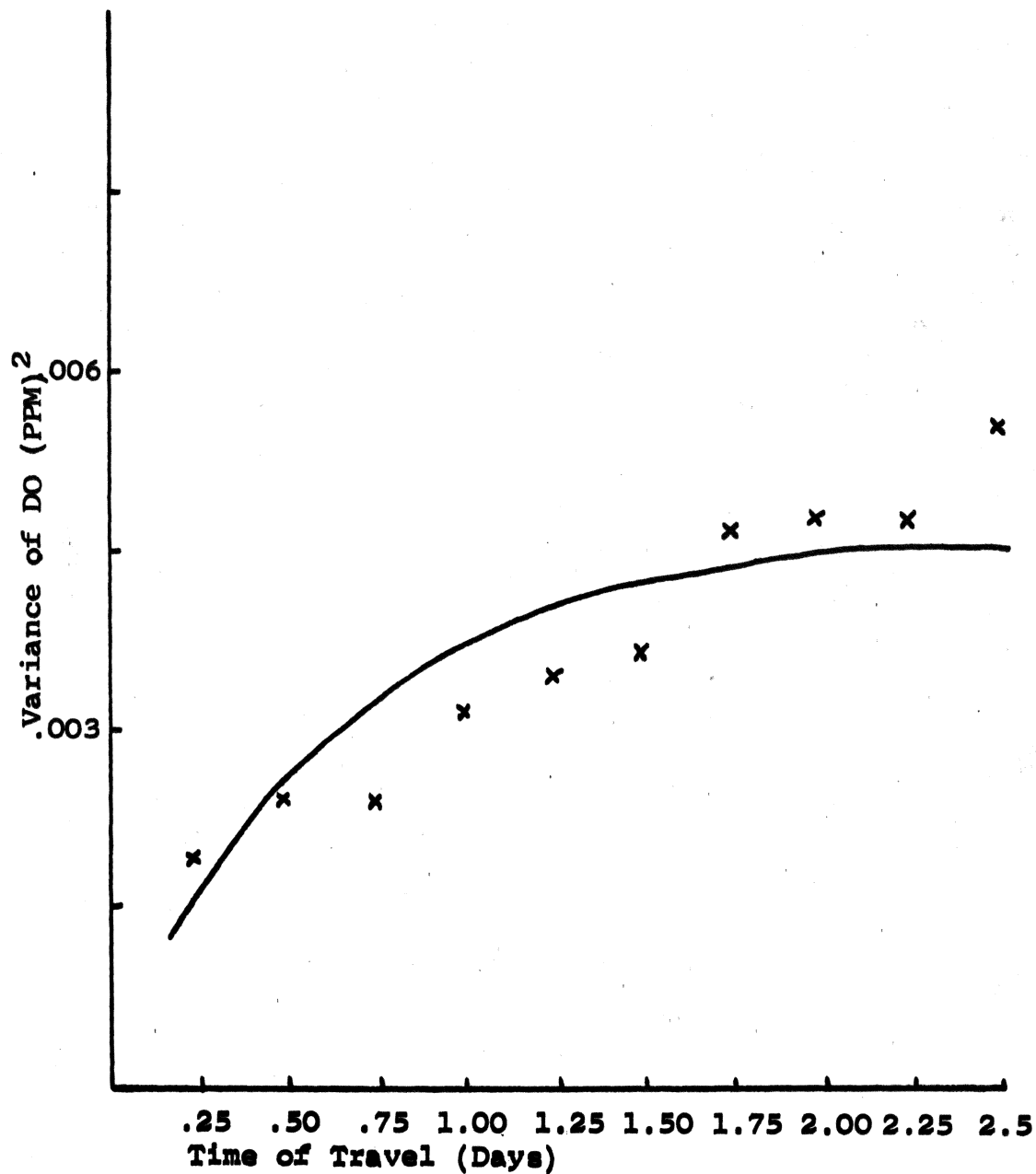


Figure 22
Predicted and Observed Variances
Dissolved Oxygen - Situation 5B

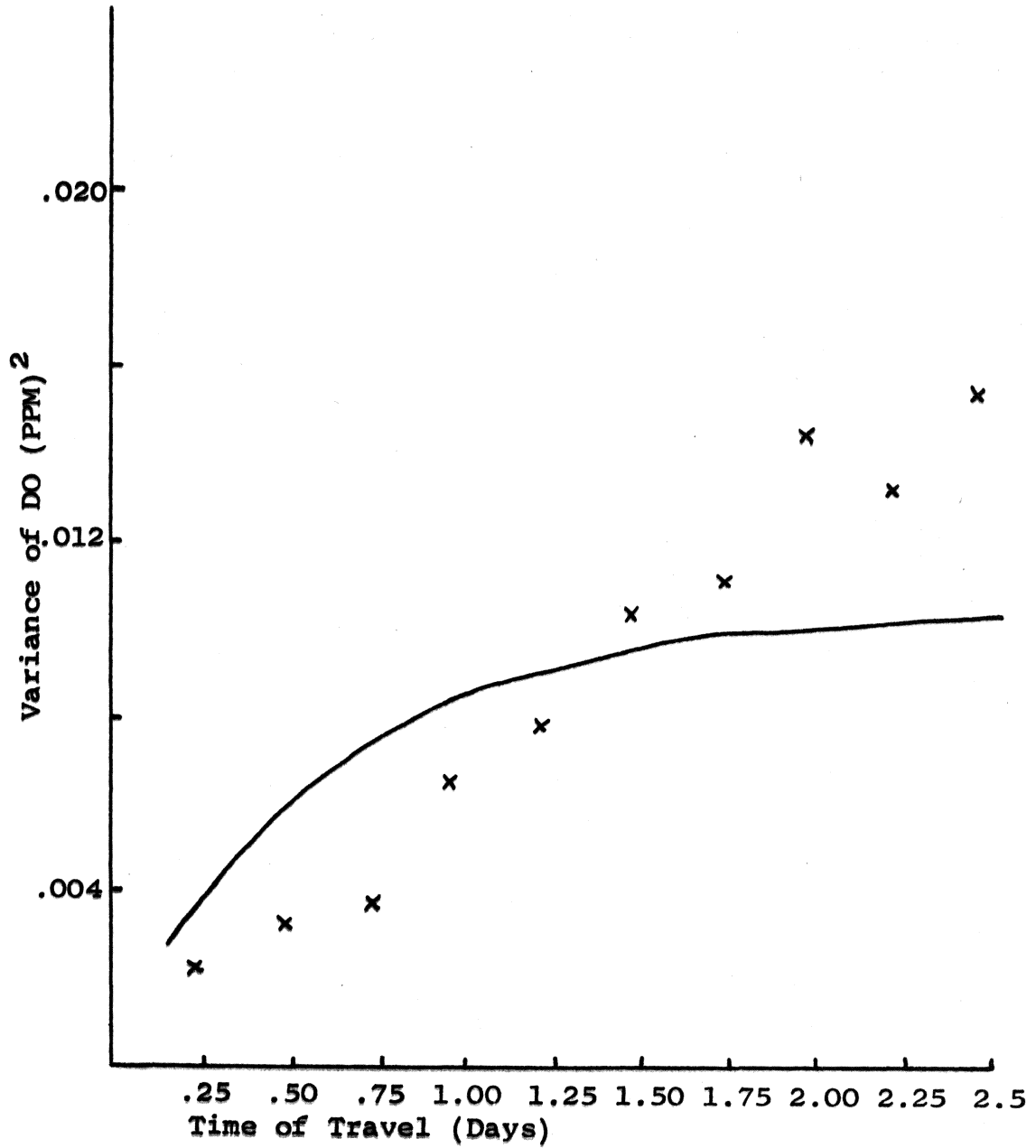


Figure 23
Predicted and Observed Variances
Dissolved Oxygen - Situation 13B

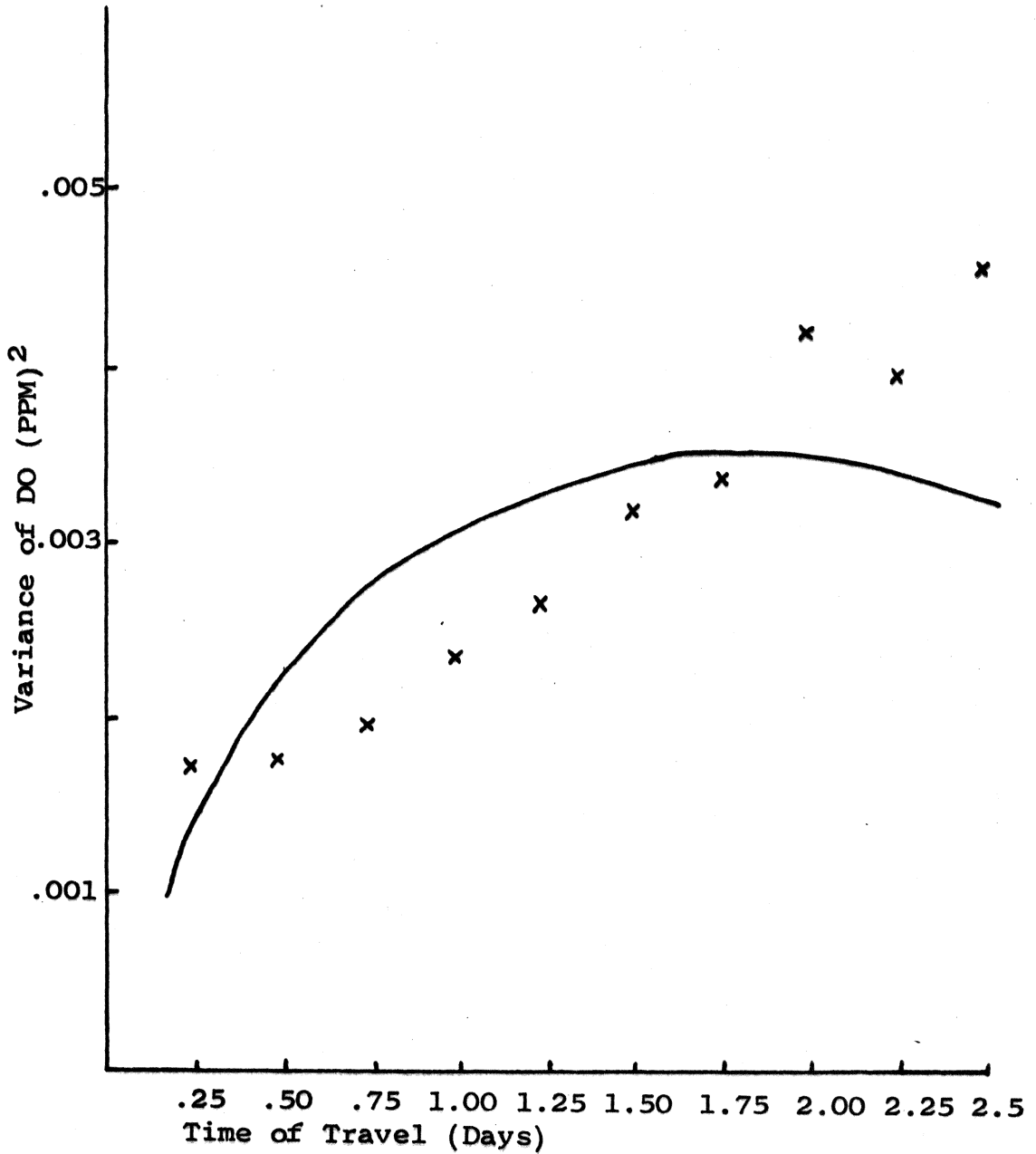


Figure 24
Predicted and Observed Variances
Dissolved Oxygen - Situation 14B

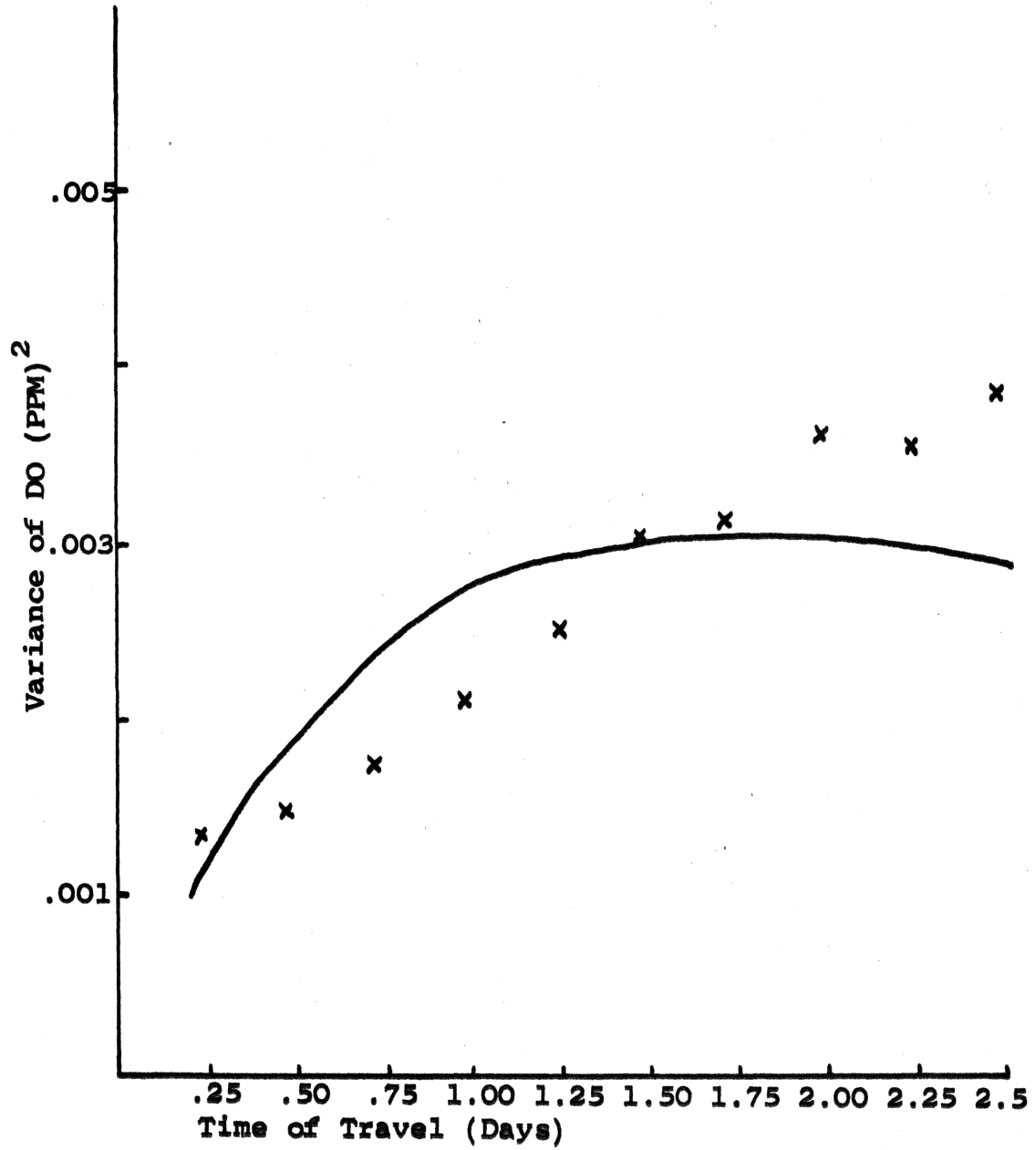
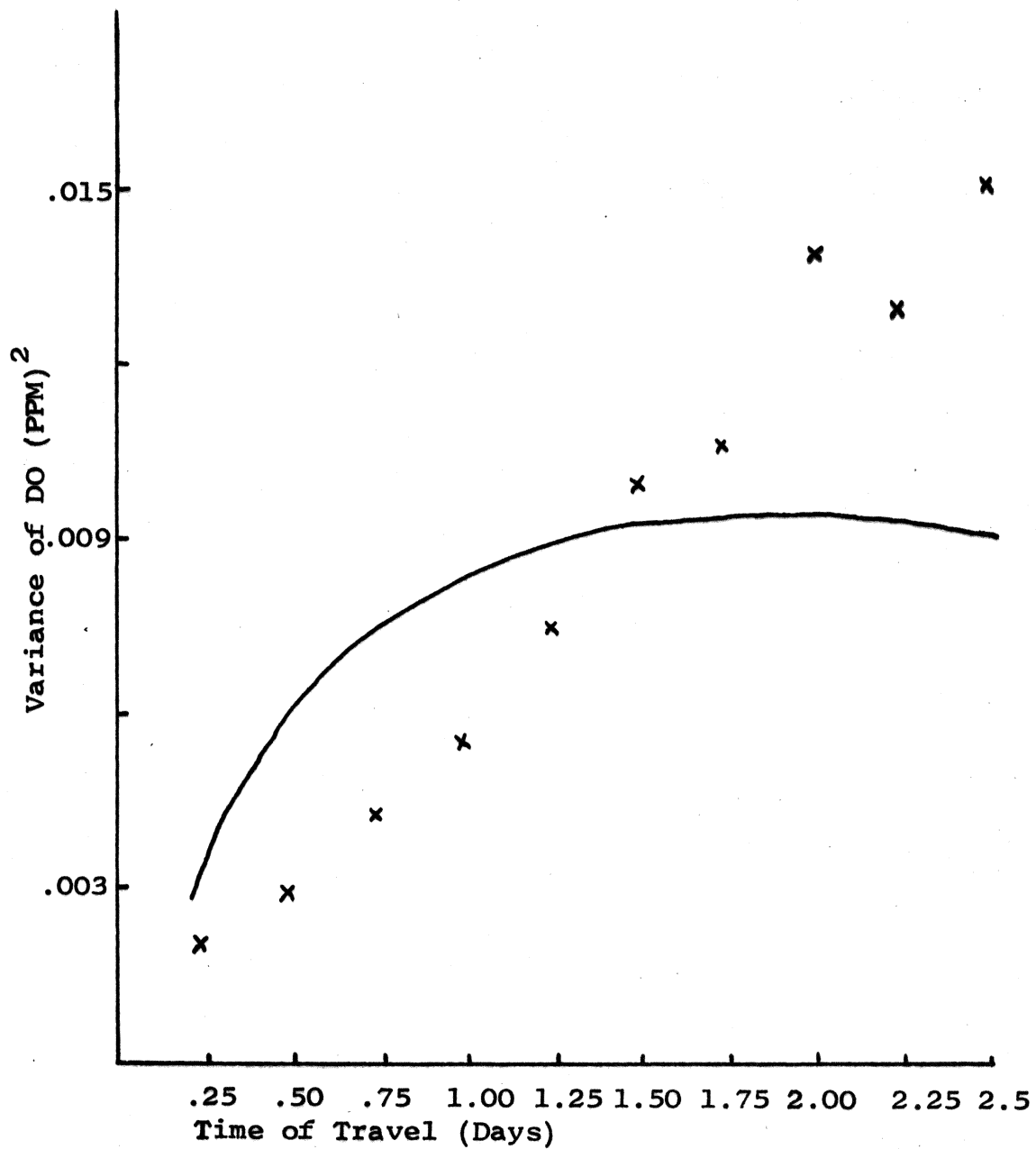


Figure 25
Predicted and Observed Variance
Dissolved Oxygen - Situation 15B



IV CONCLUSION: STREAMS

This study indicates that the average BOD and DO values calculated from uniformly and normally randomizing the K values in Dobbins' deterministic equations lies within three standard errors of their respective means. This can also be surmised for the normal randomization of the parameters L_0 , L_a and D_B in Dobbins' equations.

The deltas calculated in this work are only a fraction of the size of those calculated from stream data. This indicates that only a small part of the variance of the BOD and DO can be attributed to the variation in the non-stochastic parameters K_1 , K_2 and K_3 or the parameters L_0 , L_a and D_B . The majority of the variance of the BOD and DO must then be found in the stochastic process.

The deltas calculated in this study indicate that for small amounts of initial BOD, such as six ppm, there is little difference in the deltas calculated from the BOD or DO variances but for larger amount of initial BOD, such as sixteen ppm, the difference is quite significant.

V ESTUARIES

5.1 General Characteristics

An estuary, which has been considered the lower tidal reaches of a river, is one of our most valuable natural resources. A more encompassing definition is that given by Donald W. Pritchard [10].

"An estuary is a semi-enclosed coastal body of water which has a free connection with the open sea and within which sea water is measurably diluted with fresh water derived from land drainage."

Estuaries play a significant part in the welfare of the human race. They have served man not only in their role of transportation, but additionally they have furnished us with a bountiful food supply, wonderful recreational facilities, and they have also provided a sink for industrial and human waste.

Estuaries, due to their size and turbulence, give the appearance of being indestructable, but in actuality many estuaries have less fresh water flow than streams. It is for this reason that particular attention must be paid to the pollutional load in order that this use of the estuary does not usurp the many other benefits derived from it.

5.2 Stochastic Model: Estuaries

The diffusion model developed by Custer and Krutchkoff [2] considers diffusion as an additional random process and also includes the varying stream velocity inherent in an estuary. This model takes into account all the factors considered by Thayer and Krutchkoff except for:

1. The uniform increase of BOD by land runoff
2. The decrease in DO due to benthic demand.

These are handled separately by Custer and Krutchkoff and their effect added to the results by the principle of superposition.

Custer and Krutchkoff made this basic assumption for the diffusion process: that it is certain that the particle will move in any interval of time. The two components of the forces acting on the particle are the diffusion and non-symmetric drift. Since BOD is a non-conservative pollutant, the possibility of absorption must be considered.

The motion of a BOD unit is considered as a random walk with decay on the real axis. In each time lapse of length δt , the unit moves forward on the positive axis a distance δx with probability $p(t)$ or backwards the distance δx with probability $q(t)$. Since this is the only motion possible only absorption is left to consider. At each time

lapse of length δt , the unit may be absorbed with a constant probability r . With this Custer and Krutchkoff obtained a difference equation which gives the joint probability of the unit being at any location x after any time t , given that the unit was at the origin at t_0 .

The diffusion process of a unit of oxygen deficit (OD) is handled in a similar manner to that of BOD. The initial conditions are slightly modified for the oxygen deficit (OD). A unit of OD is formed simultaneously with the absorption of a unit of BOD as opposed to the uniform rate at which the BOD units originate. The oxygen deficit unit is then absorbed from the system by reaeration in the same manner that BOD is absorbed by the bacteria.

The parameter δt is defined as the length of time required for one unit of size Δ to enter the estuary. In order to obtain the effect of a continuous source, units of size Δ are released at each time interval δt . The units act independently and their behavior can be studied individually by the random walk mechanism. Therefore, the probability of a given concentration at location x after time t is the convolution of the probabilities associated with the individual units.

Custer and Krutchkoff showed the convolution could be

derived by considering the situation as a set of individual Bernoulli trials. Then showing that as δt approaches zero, the BOD and DO distributions approach Poisson distributions, Custer and Krutchkoff determined that the mean (BOD state) approaches

$$\frac{2W}{A\Delta} \int_0^t f(x, t | \tau) d\tau \text{ as } \delta t \rightarrow 0$$

where W is the rate at which pollution enters the estuary and A is the cross-sectional area. $f(x, t | \tau)$ is the joint frequency function for units of BOD being at any location x after any time t , given that the unit was at the origin at t_0 .

Consequently, they showed the distribution of the BOD state approaches a Poisson distribution with parameter

$$m_1 = \frac{2W}{A\Delta} \int_0^t f(x, t | \tau) d\tau$$

Similarly, the distribution of the OD state approaches a Poisson distribution with parameter

$$m_2 = \frac{2W}{A\Delta} \int_0^t g(x, t | \tau) d\tau .$$

5.3 Purpose: Estuaries

This work purports to examine the properties of the average BOD and OD as calculated from Custer and Krutchkoff's stochastic model with the values of K_2 and K_d randomized to simulate their expected variation for an estuary. The mean

of both the BOD and OD are calculated using the mean of K_2 and K_d in Custer and Krutchkoff's stochastic model. We will use the standard error as a measurement for closeness of the averages to their respective means. This examination is aimed at determining if the variation in the K values will adversely affect the mean state poisson distribution.

VI PROCEEDURE AND RESULTS

6.1 Description of Program

Custer and Krutchkoff's stochastic model was used as the basis for this work. Using a high speed computer, and Stochastic simulation methods, the values of K_2 and K_d are generated as normal pseudo-random variables about their means. Custer and Krutchkoff's stochastic model for BOD and DO was calculated one hundred and fifty times for five times of travel.

This proceedure was done for eight estuarial situations given in table LIV. The mean values of K_2 and K_d were varied and their variances increased. The following parameters were held constant:

Rate BOD enters	240,000 lbs/day
Cross-sectional area	100,000 sq. ft.
Delta	.1 mg/l
Fresh water velocity	1.0 mi/day
Tidal velocity amp.	25.0 mi/day
Tidal frequency	12.57 rad./day
Diffusion coefficient	2 sq. mi./day
Distance from source	5 mi.

6.2 Results: Estuaries

The results of this study are presented in the form of tables. Each of the tables LV thru LXII presents, for five times in tidal cycle, the mean, average and standard error of the BOD as well as the mean, average and standard error of the DO.

TABLE LIV

Values used in Estuarial Program with
 K_2 and K_d Randomized

Situation	MK2	MKD	SK1	SD2
1	.3	.50	.02	.02
2	.3	.75	.02	.02
3	.3	1.00	.02	.02
4	.3	1.00	.04	.04
5	.4	.50	.02	.02
6	.4	.75	.02	.02
7	.4	1.00	.02	.02
8	.4	1.00	.04	.04

TABLE LV

Estuarial Situation 1C				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean OD	Average OD	Standard Error
.1	4.3989	4.4129	.0181	2.3717	2.3813	.0073
.2	3.7146	3.7170	.0181	2.2527	2.3021	.0070
.3	2.2728	2.2675	.0140	1.8148	1.8078	.0063
.4	1.9780	1.9984	.0152	1.6887	1.6618	.0060
.5	2.9250	2.9224	.0175	2.0948	2.0677	.0060

TABLE LVI

Estuarial Situation 2C				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean OD	Average OD	Standard Error
.1	4.3989	4.4155	.0207	1.6804	1.6947	.0040
.2	3.7146	3.7811	.0176	1.5683	1.5859	.0034
.3	2.2728	2.2722	.0148	1.1748	1.1708	.0029
.4	1.9780	1.9620	.0166	1.0656	1.0520	.0028
.5	2.9250	2.8861	.0167	1.3985	1.3862	.0032

TABLE LVII

Estuarial Situation 3C				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean OD	Average OD	Standard Error
.1	4.3989	4.4262	.0185	1.2971	1.3034	.0028
.2	3.7146	3.7403	.0200	1.1045	1.2049	.0022
.3	2.2728	2.2911	.0186	.8540	.8512	.0018
.4	1.9780	1.9470	.0151	.7640	.7536	.0016
.5	2.9250	2.0253	.0182	1.0378	1.0309	.0017

TABLE LVIII

Time (days)	Estuarial Situation 4C			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean OD	Average OD	Standard Error
.1	4.3989	4.4375	.0365	1.2971	1.2933	.0058
.2	3.7146	3.7718	.0413	1.1945	1.2045	.0045
.3	2.2728	2.3755	.0406	.8540	.9603	.0036
.4	1.9780	1.9627	.0309	.7640	.7529	.0031
.5	2.9250	2.9549	.0367	1.0378	1.0247	.0035

TABLE LIX

Time (days)	Estuarial Situation 5C			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean OD	Average OD	Standard Error
.1	3.4868	3.4721	.0131	2.6303	2.5490	.0070
.2	2.8543	2.8311	.0136	2.4998	2.4108	.0075
.3	1.5517	1.5180	.0139	1.8941	1.7887	.0064
.4	1.2788	1.2619	.0080	1.6758	1.6145	.0069
.5	2.0877	2.0497	.0105	2.1966	2.1161	.0067

TABLE LX

Time (days)	Estuarial Situation 6C			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean OD	Average OD	Standard Error
.1	3.4868	3.4850	.0132	1.8372	1.8048	.0036
.2	2.8543	2.8086	.0115	1.7035	1.6671	.0034
.3	1.5517	1.5176	.0087	1.1881	1.1476	.0031
.4	1.2788	1.2541	.0087	1.0272	.9979	.0028
.5	2.0877	2.0636	.0127	1.4395	1.4063	.0032

TABLE LXI

Time (days)	Estuarial Situation 7C			Normal Distribution		
	Mean BOD	Average BOD	Standard Error	Mean OD	Average OD	Standard Error
.1	3.4868	3.4542	.0130	1.4096	1.3868	.0025
.2	2.8543	2.8359	.0128	1.2845	1.2619	.0021
.3	1.5517	1.5075	.0098	.8476	.8199	.0016
.4	1.2788	1.2500	.0082	.7223	.7062	.0018
.5	2.0877	2.0593	.0114	1.0561	1.0357	.0020

TABLE LXII

Estuarial Situation 8C				Normal Distribution		
Time (days)	Mean BOD	Average BOD	Standard Error	Mean OD	Average OD	Standard Error
.1	3.4868	3.4570	.0256	1.4086	1.3789	.0051
.2	2.8543	2.8709	.0264	1.2845	1.2645	.0042
.3	1.5517	1.5258	.0208	.8476	.8249	.0033
.4	1.2788	1.2553	.0168	.7223	.7065	.0037
.5	2.0877	2.0736	.0232	1.0561	1.0346	.0040

VII CONCLUSION: ESTUARIES

This study indicates that the average BOD and OD values calculated from normally randomizing K_2 and K_d in Custer and Krutchkoff's stochastic model lies close to their respective means although not always within three standard errors. This indicates that the variation in the K values does not adversely affect mean of the BOD and OD mean state distributions.

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A STOCHASTIC SIMULATION STUDY
OF THE BIOCHEMICAL OXYGEN DEMAND
AND DISSOLVED OXYGEN IN STREAMS
AND ESTUARIES

by

John George Barile, Jr.

ABSTRACT

Using Dobbins' deterministic equations Stochastic simulation was used to determine if, for K_1 , K_2 and K_3 randomized, the average BOD and DO would be close to its mean. The BOD and DO was determined by using the mean K values in Dobbins' equations. Similar calculations were performed where L_0 , and L_a , and D_B were randomized and the average BOD and DO determined.

The same procedure was used with Custer and Krutchkoff's stochastic model randomizing K_2 and K_d .

Tables were calculated for each situation. These indicate that the average BOD and DO values calculated for streams lie within three standard errors of their respective means. The average BOD and DO values of estuaries lie close to their respective means. This indicates that the variation in the K values of the estuarial model does not adversely effect the mean state Poisson distribution of the BOD and DO.

A study was also made to determine if the variation in the previously discussed parameters found in the stream equations account for the variance in the BOD and DO or whether this variance is in the stochastic process.

Tables and graphs were constructed showing the observed and predicted variances of the BOD and DO for each stream situation. These indicate that only a small part of the variance can be attributed to the variation in the non-stochastic parameters. Therefore the majority of the variance must be found in the stochastic process.