AN ENGINEERING EVALUATION OF THE CONTINUITY METHOD

FOR PREDICTING THE FORM AND LOCATION OF DETACHED

SHOCK WAVES FOR TWO-DIMENSIONAL FLOW

by

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II. SYMBOLS

A = area

a = speed of sound

B = $\sigma\left(\frac{P_o}{P_s}\right)_c$

C = $\beta\left(\beta \tan \phi_S - \sqrt{\beta^2 \tan^2 \phi_S - 1}\right)$

L = distance between vertex of detached shock wave and x_{SB}

M = Mach Number = $\frac{\text{velocity of flow}}{\text{speed of sound}}$

P = stagnation pressure

V = local velocity

x = coordinate in stream direction

x_o = distance from foremost point of detached shock to intercept of its asymptote on x-axis

y = coordinate perpendicular to stream direction

β = $\sqrt{M_o^2 - 1}$

γ = ratio of specific heats (1.4 for air)

η = angle between sonic line and normal to free-stream direction

θ = half-angle of wedge

λ = angle of streamline relative to x-axis

δ = distance from leading edge of wedge to foremost point of shock

λ_d = wedge half-angle for which shock becomes detached

ρ = density

σ = isentropic contraction ratio from free stream to sonic velocity

ϕ = local inclination of detached shock relative to x-axis

Subscripts:

o = free-stream conditions

c = centroid of stream tube passing sonic line

S = sonic point of detached shock

SB = sonic point of body

s = conditions along sonic line

III. INTRODUCTION

One of the most difficult and least understood problems in aerodynamics occurs when a region of subsonic flow exists in a supersonic flow field. One of the most important, and perhaps one of the simplest conditions where a mixed flow field exists, is the case of the detached shock wave for simple two-dimensional bodies. Although this condition can occur for any Mach Number greater than unity, depending upon the shape of the body, it is referred to as the transonic region. Many attempts (1, 2, 3, 4, 5) have been made to find a theoretical method that predicts the location of detached shock waves ahead of two-dimensional bodies in this transonic region. It is well known that bodies with detached shock waves are, in effect, immersed in an imbedded non-uniform subsonic flow. The difficulty is in determining the actual velocity distribution, entropy, pressure, and density changes that occur immediately downstream from the detached shock wave.

It is important to the practicing aeronautical engineer that a method of analysis for detached shock waves be available and it is even more important to know the accuracy of the method, and the limiting range in which the method is applicable. The Continuity Method, a theoretical approximate method for predicting the form and location of detached shock waves (6), appears to be most promising. The Continuity Method is based on the assumptions that: 1) the form of the shock wave between its foremost part and its sonic point is adequately represented by an hyperbola asymptotic to the free stream Mach lines; and 2) the sonic line between the shock and the body is straight and inclined at an angle that depends only on the free stream Mach Number.

Unfortunately, at the time of the publication of the Continuity Method, very few reliable experimental results for two-dimensional bodies were available for comparison with the theory. Further, the experimental data available was, of necessity, for relatively bluff-type bodies at Mach Numbers considerably greater than unity. With the advent of new interferometric techniques and shock tubes, a great deal of experimental data for supersonic airfoil shapes in the Mach Number range slightly greater than unity is now available. With the existence of this extensive and more reliable detached shock data, it is now possible to investigate more thoroughly the reliability of the Continuity Method.

The purpose of this thesis is: 1) to present Moeckel's formulation of the Continuity Method (6) for two-dimensional, sharp-nosed wedge-type airfoils; 2) to calculate the form and location of the detached shock waves for the specific ranges of Mach Number experimentally investigated recently in references 7 and 8; and 3) to evaluate the Continuity Method for engineering usage.

IV. REVIEW OF LITERATURE

Moeckel, W. E., "Approximate Method for Predicting Form and Location of Detached Shock Waves Ahead of Plane or Axially Symmetric Bodies". NACA TN 1921, July 1949.

An approximate method, called the "Continuity Method", is developed for predicting the form and location of detached shock waves ahead of plane or axially symmetric bodies; and for estimating the drag of the portion of the body upstream of the theoretical sonic point by investigating the change of momentum of the air passing the sonic line.

Comparison of the theory is made with existing experimental data, namely for blunt, axially symmetric bodies, with the exception of a cone. The comparison of the form of the detached wave is limited to the axially symmetric case at a single Mach Number, and the comparison of the detachment distance is for axially symmetric blunt bodies and one two-dimensional wedge at a single Mach Number.

The comparison of the theory for only one wedge at one Mach Number seems to be inadequate for a proper evaluation from an engineering standpoint. An investigation should be made for a series of Mach numbers in the detachment range.

Griffith, Wayland, "Shock-Tube Studies of Transonic Flow Over Wedge Profiles", Journal of the Aeronautical Sciences, Vol. 19, No. 4, April, 1952, pp. 249-257.

The flow around wedges in the transonic range is studied experimentally using a shock-tube and Mach-Zehnder interferometer.

The results of these studies, made to determine the location and shape of detached bow waves on wedges of 20°, 30°, 40°, 60°, and 180° total angle, are presented in the form of drawings showing the detached shock waves, sonic lines, and lines of constant density.

The experimental procedure and results are discussed along with implications from various detached shock theories. In addition, the pressure distributions on the front surface of the wedges are computed and presented.

Bryson, A. E., Jr., "An Experimental Investigation of Transonic Flow Past Two-Dimensional Wedge and Circular-Arc Sections Using a Mach-Zehnder Interferometer", NACA TN 2560, November 1951.

Interferometer measurements for the flow fields near two-dimensional wedge and circular-arc sections at zero angle of attach at high subsonic and low supersonic velocities are discussed. The experimental procedures are discussed at length and theoretical studies of transonic flow past wedge profiles are reviewed.

Drawings accurately scaled from interferograms show the shocks, sonic lines, and constant density lines for a 10° wedge at various Mach Numbers from 1.207 to 1.465. Also included are similar drawings for a biconvex airfoil.

The pressure variation with Mach Number over these wedges is also discussed.

V. THEORY OF THE CONTINUITY METHOD

It is considered necessary first to present briefly Moeckel's formulation of the Continuity Method for two-dimensional, wedge-type airfoils.

A. BASIC ASSUMPTIONS

The Continuity Method is an approximate one, aimed at predicting expediently the approximate form and location of detached shock waves. Many simplifying assumptions are required to make theoretical analysis feasible for a region of subsonic flow occurring in a supersonic flow field, as is the case with detached shock waves. Without such assumptions, the flow can be constructed only by means of lengthy numerical methods based on the differential equations of fluid mechanics. With such methods, general trends and important parameters are not easily discernible.

The assumptions of the Continuity Method concern the form of the boundaries of the subsonic region rather than the nature of the flow variables. Basically, the form of the detached shock wave is assumed to be only secondarily influenced by the form of the body ahead of the sonic point, and the sonic line between the shock and the body is assumed to be straight and inclined at an angle that depends only on the free-stream Mach number. The continuity relation is then applied to this simplified picture to obtain the location of the shock wave relative to the body sonic point.

B. METHOD OF ANALYSIS

The representation of flow with detached shock waves to be used in Moeckel's analysis is shown in Figure 1. The detached shock wave is located upstream a distance L from the body sonic point, SB. Somewhere along the shock wave, there is also a sonic point, S. The locus of all points between S and SB where the velocity is sonic is called the sonic curve, and for this analysis is assumed to be straight and inclined at an angle dependent only on the free-stream Mach Number, M_o. The boundaries of the subsonic flow field then are: 1) the shock wave to the sonic point, S; 2) the sonic line from S to SB; and 3) the portion of the body ahead of the body sonic point, SB. If the form of the detached shock wave is known, the location of S on the wave can be determined immediately, because for a given free-stream Mach Number, M_o, the local shock angle, ϕ_S, for which sonic velocity exists behind the shock is known from the so-called exact shock theory (11). The flow direction, λ_S, at this point is then also known.

Concerning the assumption that the sonic curve is a straight line, more exact computations, as well as experimental results, indicate that the form may depend considerably on the shape of the nose or leading edge (9, 10); but Moeckel states that to the degree of approximation of the Continuity Method, such variations appear to be unimportant. With this simplified picture, approximate expressions are derived for the form and location of the shock relative to the body sonic point.

C. LOCATION OF THE BODY SONIC POINT

It is immediately evident that the location of the body sonic point, SB, must be known. Evidence is available to show that for bodies with sharp or well defined shoulders, the sonic point is located at the shoulder (10). For more gradually curved bodies, the location of the shoulder can be estimated as being at the point where the contour of the body is inclined at the wedge angle corresponding to shock wave detachment. For purposes of analysis, the body sonic point and the shoulder are assumed to coincide, the differences being insignificant, according to Moeckel.

D. DERIVATION

The derivation of the expression for the location of the detached wave will now be briefly presented.

First, consider the most typical characteristics of detached waves, namely: 1) they are normal to the free-stream at their foremost point, and 2) they are asymptotic to the free-stream Mach lines at a large distance from their foremost point. A simple curve that has these characteristics is an hyperbola represented by

$$\beta y = \sqrt{x^2 - x_o^2} \tag{1}$$

where

$$\beta = \text{cot. Mach angle} = \sqrt{M_o^2 - 1}$$

and x_o is the distance from the vertex of the wave to the intersection of its asymptotes (Figure 1). For the purpose of the analysis, the hypothesis is made that to the degree of approximation required, all detached shock waves in the region between the axis and the sonic point, S, may be represented by Equation (1).

With this form of detached wave, the angle between the stream direction and the tangent to the shock at any point is obtained from Equation (1) by taking the derivative with respect to x. Thus,

$$\frac{dy}{dx} = \tan \phi = \frac{x}{\beta\sqrt{x^2 - x_o^2}} = \frac{\sqrt{x_o^2 + \beta^2 y^2}}{\beta^2 y} \tag{2}$$

The location of S is then

$$y_S = \frac{x_o \cot \phi_S}{\beta \sqrt{\beta^2 - \cot^2 \phi_S}} \tag{3}$$

and

$$x_S = \frac{\beta\, x_o}{\sqrt{\beta^2 - \cot^2 \phi_S}} \tag{3a}$$

If the y-coordinate of the body sonic point is used as a reference dimension, then the form of the shock wave is given by

$$\frac{y}{y_{SB}} = \frac{1}{\beta} \sqrt{\left(\frac{x}{y_{SB}}\right)^2 \quad \left(\frac{x_o}{y_{SB}}\right)^2} \tag{4}$$

from Equation (1) where, from equation (3)

$$\frac{x_o}{y_{SB}} = \beta \frac{y_S}{y_{SB}} \sqrt{\beta^2 \tan^2 \phi_S - 1} \tag{5}$$

From Equation (3a), the dimensionless location of the shock wave sonic point is

$$\frac{x_S}{y_{SB}} = \frac{\beta \dfrac{x_o}{y_{SB}}}{\sqrt{\beta^2 - \cot^2 \phi_S}} \tag{6}$$

The distance from the foremost point of the wave to the x-coordinate of the body-sonic point is

$$\frac{L}{y_{SB}} = \frac{x_{SB}}{y_{SB}} - \frac{x_o}{y_{SB}} \tag{7}$$

where, from Figure 1,

$$\frac{x_{SB}}{y_{SB}} = \frac{x_S}{y_{SB}} + \left(\frac{y_S}{y_{SB}} - 1\right) \tan \eta \tag{8}$$

If equations (5), (6), and (8) are combined to eliminate all unknown coordinates except y_S and L, equation (7) becomes

$$\frac{L}{y_{SB}} = \frac{y_S}{y_{SB}} (C + \tan \eta\,) - \tan \eta \tag{9}$$

where

$$C = \beta\ (\beta \tan \phi_S - \sqrt{\beta^2 \tan^2 \phi_S - 1}\ ) \tag{10}$$

Since β and ϕ_S are known for any given M_o, only the quantities $\frac{y_S}{y_{SB}}$ and η remain to be determined to predict the relation between the sonic point on a body and the location of its detached wave.

In order to determine the quantity $\frac{y_S}{y_{SB}}$, the continuity relation is applied to the fluid that passes the sonic line. An appropriate average value of the stagnation pressure distribution immediately behind the shock is that existing along the streamline which represents the mass centroid of the fluid passing the sonic line. This centroid streamline enters the shock wave at $y_c = \frac{y_S}{2}$ for plane flow. Equations (2) and (3) give the shock angle, ϕ_c, corresponding to this value of y. Since the stagnation pressure remains constant along each streamline behind the shock, the value of $P_{s,c}$ will remain unchanged between the shock and the sonic line. Because the total temperature is constant, the simplified continuity equation may be written as

$$\frac{A_o}{A_s} = \frac{(\rho_s V_s)_c}{\rho_o V_o} = \frac{(\rho_s \frac{V_s}{a_s} a_s)_c}{\rho_o \frac{V_o}{a_o} a_o} = \frac{(\rho_s M_s \sqrt{T_s})_c}{\rho_o M_o \sqrt{T_o}} \tag{11}$$

Applying the thermodynamic relationships between T, M, ρ, and P, equation (11) becomes

$$\frac{A_O}{A_s} = \frac{P_s}{P_O})_c \frac{1}{\sigma} \tag{12}$$

where

$$\sigma = \frac{M_o}{1}\left[\frac{1 + \frac{\gamma - 1}{2}}{1 + \frac{\gamma-1}{2} M_o^2}\right]^{-\frac{(\gamma+1)}{2(\gamma-1)}} \tag{13}$$

σ = the contraction ratio required to decelerate the free-stream to sonic velocity isentropically.

In terms of the coordinates of the sonic points, equation (12), becomes

$$\frac{y_S - y_{SB}}{\cos \eta} = \left(\frac{P_o}{P_s}\right)_c \sigma\, y_S = B\, y_S \tag{14}$$

The appropriate value of η to be used remains to be established. In order to do this, the sonic line is assumed to be normal to the average flow direction in its vicinity. At S, the inclination of the flow is known to be λ_S; whereas at SB the inclination is assumed to be λ_d (the wedge half-angle for which a shock becomes attached at a given M_o). Using the arithmetic mean of the inclination at the two extremities, the expression for η becomes

$$\eta = \frac{1}{2}(\lambda_d + \lambda_S) \tag{15}$$

Because λ_S differs only slightly from λ_d, the inclination of the sonic line for plane flow is assumed to be simply $\eta = \lambda_S$.

To use the Continuity Method for a given M_o, it is necessary, therefore, to know ϕ_S, $\eta = \lambda_S$, and $\left(\frac{P_S}{P_o}\right)_c$. These values are found from exact shock theory either from curves (11) or by calculations. With these quantities, the values of the various parameters involved in the method can be calculated.

The equations actually used in applying the Continuity Method are listed as follows:

$$\frac{1}{\sigma} = \frac{1}{M_o}\left[\frac{1 + \frac{\gamma-1}{2} M_o^2}{1 + \frac{\gamma-1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{13}$$

$$\beta = \sqrt{M_o^2 - 1}$$

$$B = \sigma\left[\frac{1}{\frac{P_s}{P_o})_c}\right]$$

$$C = \beta^2 \tan \phi_S - \beta \sqrt{\beta^2 \tan^2 \phi_S - 1} \tag{10}$$

$$\frac{y_S}{y_{SB}} = \left[1 - B \cos \eta\right]^{-1} \qquad \text{from (12) and (14)}$$

$$\frac{x_o}{y_{SB}} = \beta \frac{y_S}{y_{SB}} \sqrt{\beta^2 \tan^2 \phi_S - 1} \tag{5}$$

$$\frac{x_S}{y_{SB}} = \frac{\beta \frac{x_o}{y_{SB}}}{\sqrt{\beta^2 - \cot^2 \phi_S}} \tag{6}$$

$$\frac{L}{y_{SB}} = \frac{y_S}{y_{SB}} \left[C + \tan \eta\right] - \tan \eta \tag{9}$$

Equation of shock: $\beta\, y = \sqrt{x^2 - x_o^2}$ (1)

VI. EXPERIMENTAL DATA USED FOR EVALUATING THE CONTINUITY METHOD

The experimental data used in this evaluation was taken from some recent works by A. E. Bryson, Jr., (7) and Wayland Griffith (8). As part of his report, Bryson presents several drawings showing the form and location of shock waves at various free stream Mach numbers for a 10° wedge at zero degrees angle of attack, the data being obtained from a transonic wind tunnel and a Mach-Zehnder interferometer. Several of these were selected for use in the evaluation.

In order to make the evaluation as general as possible, use was made of Griffith's article which contained drawings of the form and location of shocks for various wedge angles at various Mach numbers. This data was obtained using a Mach-Zehnder interferometer in conjunction with a shock tube.

The geometry of the wedges selected for the evaluation, and their corresponding Mach numbers and origin are given in Table I.

VII. METHOD OF EVALUATION

It was ascertained from the author of the report that the sketches of the shock waves in reference 7 were accurately scaled drawings. The shock waves selected for use in the evaluation were then redrawn to twice their original size, all measurements being to the nearest hundredth of an inch. The shock waves selected from reference 8 were traced directly from the original drawings. Great care was exercised in the reconstruction of the shocks, and it is felt that the shocks as presented in references 7 and 8 are truly represented in this evaluation. Thus, the basis for the evaluation was formed.

It will be recalled that according to the assumptions used in the Continuity Method, the shock wave location and form is completely determined by the free stream Mach Number - the wedge angle exerting no influence. Accordingly, Moeckel has calculated the various parameters used in applying the Continuity Method, and presents them in his report in the form of curves of the various parameters and quantities involved versus the free stream Mach Number. These include σ, $\left.\frac{P_s}{P_o}\right)_c$, ϕ_S, λ_S, ϕ_c, C, $\frac{x_o}{y_{SB}}$, and finally $\frac{L}{y_{SB}}$. Thus, all that is necessary to determine the location of the shock wave is to pick off the value of $\frac{L}{y_{SB}}$ for the corresponding free stream Mach Number (knowing, of course, the wedge semi-thickness, y_{SB}). The only additional parameter necessary for one to be able to plot the shock wave is $\frac{x_o}{y_{SB}}$ which also can be read directly from Moeckel's curves. However, it was observed that values could not be read from the curves with any consistency for several reasons: 1) several of the curves merged at very acute angles, thus making accurate reading in that region very difficult; 2) the scale was small due to the wide range of Mach numbers covered (effectively, Mach numbers 1.15 to 3.6); 3) for the range of Mach numbers under investigation, the slopes of the curves were for the most part very steep, or very shallow; and 4) the grid employed was very inconvenient, i.e., the grid lines were 0.37 inches apart.

Therefore, in order to assure that exact theoretical values of the parameters were used in the evaluation, it was decided to calculate accurately all the parameters for the Mach numbers under investigation. This was done, the calculations being carried to the fourth decimal place. It was further decided that since most practical engineering problems concerning detached shock waves would fall in the range of Mach numbers from 1.1 to 1.7, it would be appropriate to include in the evaluation a set of curves of the various parameters for that Mach number range, drawn to a larger and more convenient scale. Accordingly, a table of some of the calculated variables and a table of the important parameters (Tables 2 and 3, respectively) is presented along with curves of ϕ_S, C, σ, λ_S, ϕ_c, $\frac{P_s}{P_o})_c$, $\frac{x_o}{y_{SB}}$, $\frac{y_S}{y_{SB}}$, and $\frac{L}{y_{SB}}$ (Figures 2 through 8).

Using the calculated values of $\frac{L}{y_{SB}}$ and $\frac{x_o}{y_{SB}}$, the theoretical shock waves were plotted for each Mach number with the corresponding experimental shock. These are presented in Figures 9 through 17.

A comparison of the theoretical and experimental values of $\frac{L}{y_{SB}}$ and δ along with the corresponding percentage error is presented in Table 4.

VIII. DISCUSSION

The accuracy of the Continuity Method is clearly indicated in Figures 9 through 17. The theoretical shock waves are seen to approximate closely the forms of the corresponding experimental waves except at Mach numbers close to the attachment Mach number, and at large distances from the wedge axis of symmetry.

A large discrepancy appears in the location of the sonic line. The theoretical sonic line is seen to be inclined, in almost every case, upstream from the experimental sonic line by 9 to 14 degrees. Also, except for the 30 degree wedge at $M_o = 1.36$, the experimental sonic line touches the body forward of the shoulder, the distance appearing to be independent of wedge angle. Although not apparent from the figures included here (because the Mach numbers are not close enough to the attachment Mach numbers), the base of the sonic line moves forward as the shock approaches attachment (7). This is due to the boundary layer changing the "effective shape" of the wedge. That is, as the shock wave gets close to attachment, the velocities in the subsonic region behind it are getting very close to sonic velocity and hence the flow in this region is very sensitive to any slight curvature of the "revised shape" of the wedge.

As a projected consideration, it was decided to determine the theoretical $\frac{L}{y_{SB}}$ by considering L as being the distance along the x-axis from the apex of the theoretical shock wave to the <u>actual</u> body sonic point instead of to the shoulder; and the y_{SB} as being the corresponding distance from the wedge axis of symmetry along the y-axis to the <u>actual</u>

body sonic point. The purpose was to ascertain if this would improve the agreement between the theory and experiment. Agreement was improved in almost every case. That is, considering the $\frac{L}{y_{SB}}$ as obtained from the Continuity Method to follow the above definitions of L and y_{SB}, the theoretical shock wave is placed further from the body thus making it, in most cases, more nearly approach the experimental shock wave. Of course, in a practical problem the actual location of the body sonic point is not known so this consideration is of little aid in the application of the Continuity Method. However, in most cases the "rounding off" of the shoulder (due to viscosity) would allow the investigator to know which way his prediction is off.

The agreement between the theoretical and experimental location of the shock sonic points is seen to be very poor, particularly when the wave is far from attachment. Consequently, the theoretical subsonic region is generally much smaller than indicated by experiment, and Moeckel's method for determining the drag by considering the change of momentum of the air that passes the sonic line would, therefore, be somewhat underestimated.

The theoretical shock detachment distance, as given by the Continuity Method, is generally less than the experimental values, the difference being less as the shock approaches attachment, which would be expected. Accordingly, the Continuity Method tends to indicate shock attachment prematurely. As seen from Table 4, the Continuity Method gives an $\frac{L}{y_{SB}} = 5.425$ for a free stream Mach number of 1.356, whereas a value of 5.68 would correspond to an attached shock, i.e., the nose of the wedge has penetrated the theoretical shock location. Actually, experiment indicates the shock for this Mach number is still detached.

The distance from the detached shock to the shoulder is seen to vary little with wedge angle but, of course, the actual detachment distance from the leading edge to the shock does vary with wedge angle. From Table 4, it can be seen that the percent error based on $\frac{L}{y_{SB}}$ is much less than that based on the actual detachment distance, δ. The parameter $\frac{L}{y_{SB}}$ is somewhat misleading. It should be noted that $\frac{L}{y_{SB}}$ does not approach zero as the shock wave approaches attachment, but the parameter $\frac{\delta}{L}$ does (where L is the distance from the leading edge to the shoulder along the axis of symmetry). The parameter $\frac{\delta}{L}$ is now used almost exclusively, and from a practical point of view is a much better parameter.

IX. CONCLUSIONS

The various parametric curves in reference 6 were found to be very difficult to read accurately. Therefore, a set of more convenient curves of the various quantities and parameters for the lower range of Mach numbers has been plotted and is included for the reader's ready reference.

The Continuity Method has been evaluated using available experimental data, and the results of the evaluation presented.

The theoretical and experimental locations of the sonic line are seen to be in poor agreement, since the experimental sonic line is inclined downstream from the theoretical sonic line. As a result, the theoretical subsonic region is much smaller in extent than indicated by experiment.

The theoretical shock detachment distance agrees very closely with experiment in some cases, but is at variance in others. In general the Continuity Method appears to predict shock wave detachment distances most accurately within a limited range. That is, the inaccuracy appears to increase as the free stream Mach number approaches unity, and as the free stream Mach number approaches the attachment Mach number. However, the experimental data available for this evaluation is insufficient to permit this range of accuracy of the shock wave detachment distance to be determined.

Although the shock detachment distance is important, a much <u>more</u> important aspect of the Continuity Method is the prediction of the form of the detached shock wave. When the form of a shock wave is known, the flow variables across the wave at any point can be determined. That is, the local pressure, density, and temperature behind the wave, and the flow deflection and change of entropy across the wave are determined by the free stream Mach number and the inclination of the shock wave. It is seen from Figures 9 through 17 that the form of the detached shock wave is very closely approximated in almost every case, particularly for the portion close to the body. However, the accuracy is better for some cases than others. Too few cases have been considered, in this evaluation, to permit establishing limitations on the range of accuracy. Further investigation would be necessary as more experimental data becomes available. As mentioned before, however, for the cases here considered, the predicted shock wave forms are very good approximations. This would indicate that, within limitations, the flow variables obtained as a consequence of the Continuity Method would very closely approximate those in the actual flow.

In conclusion, it may be stated that, within limitations, the Continuity Method represents a satisfactory engineering approach to the problem of determination of the form and location of detached shock waves.

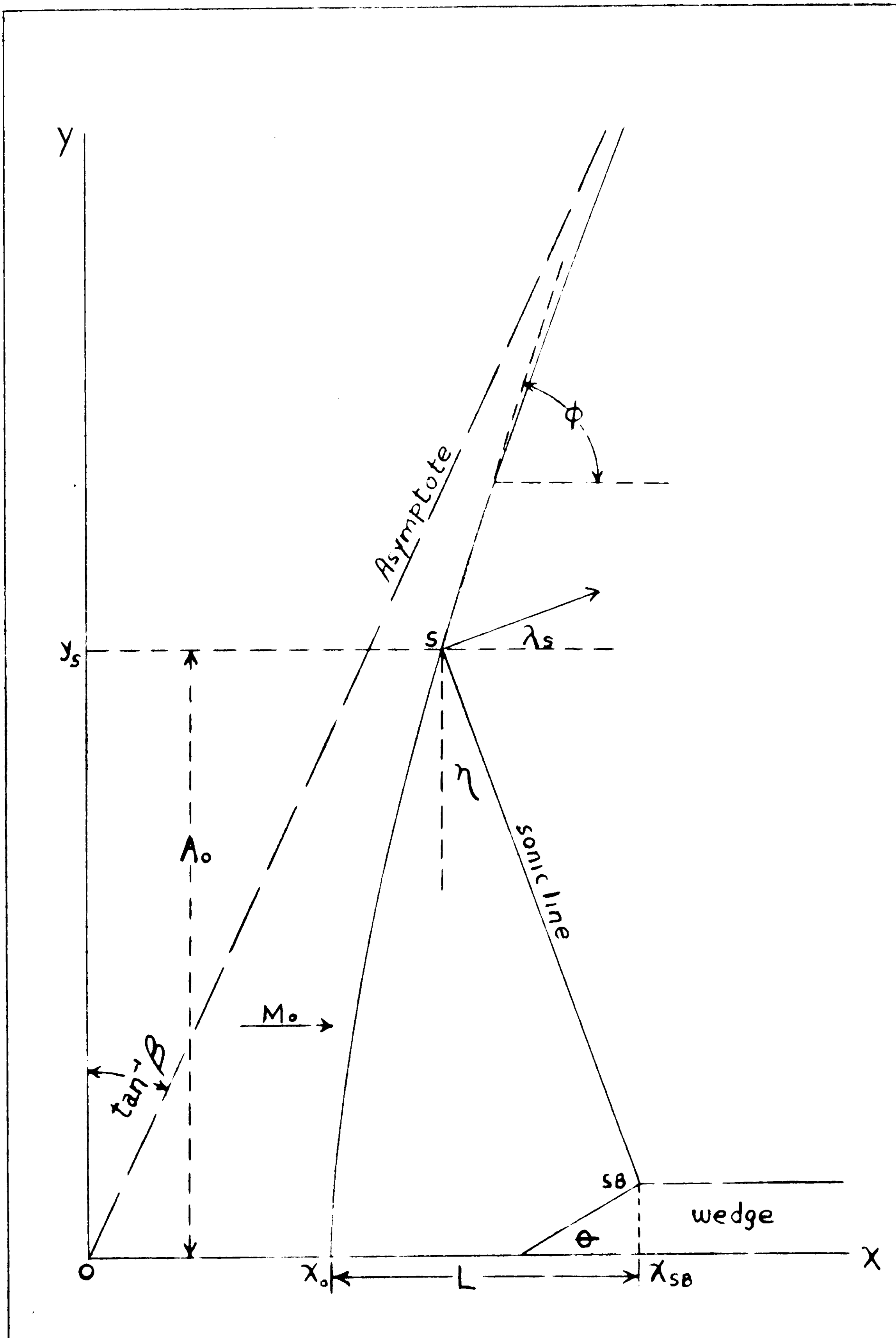


Fig. 1 Representation of Flow with Detached Shock Wave Notation Used in Analysis.

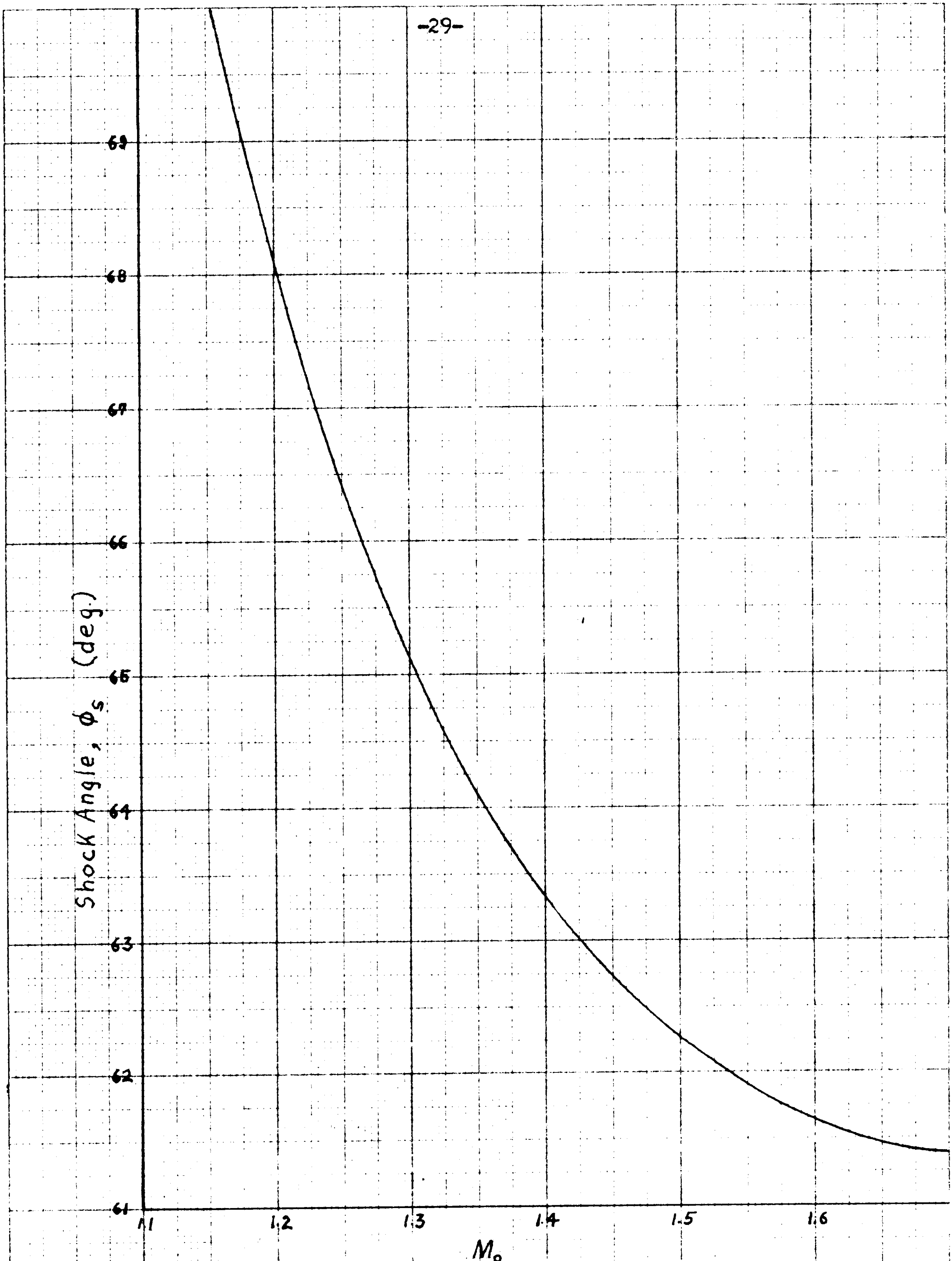


Fig. 2 - Variation of Shock Angle, ϕ_S, With Mach Number

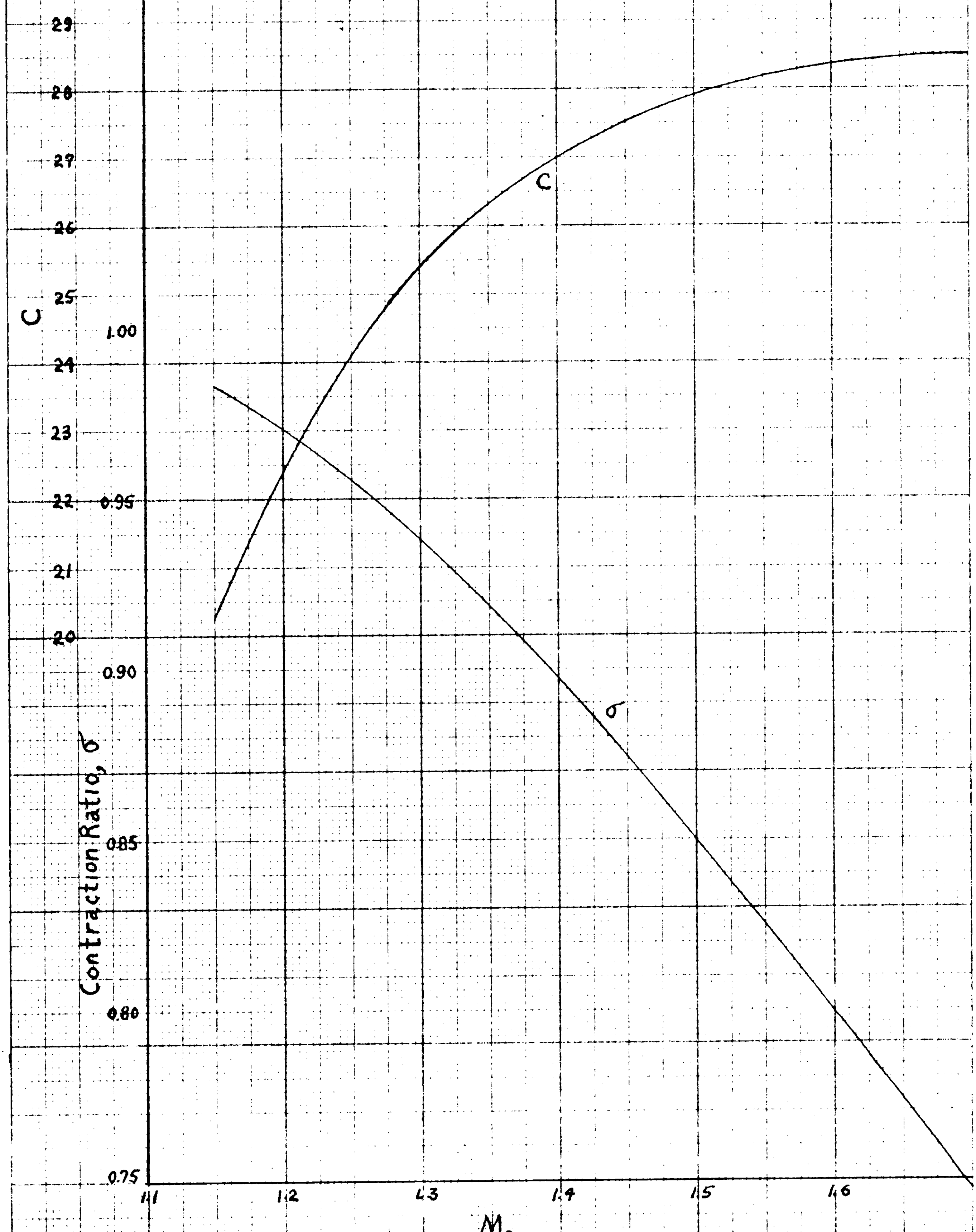


Fig. 3 - Variation of σ and C with Mach Number

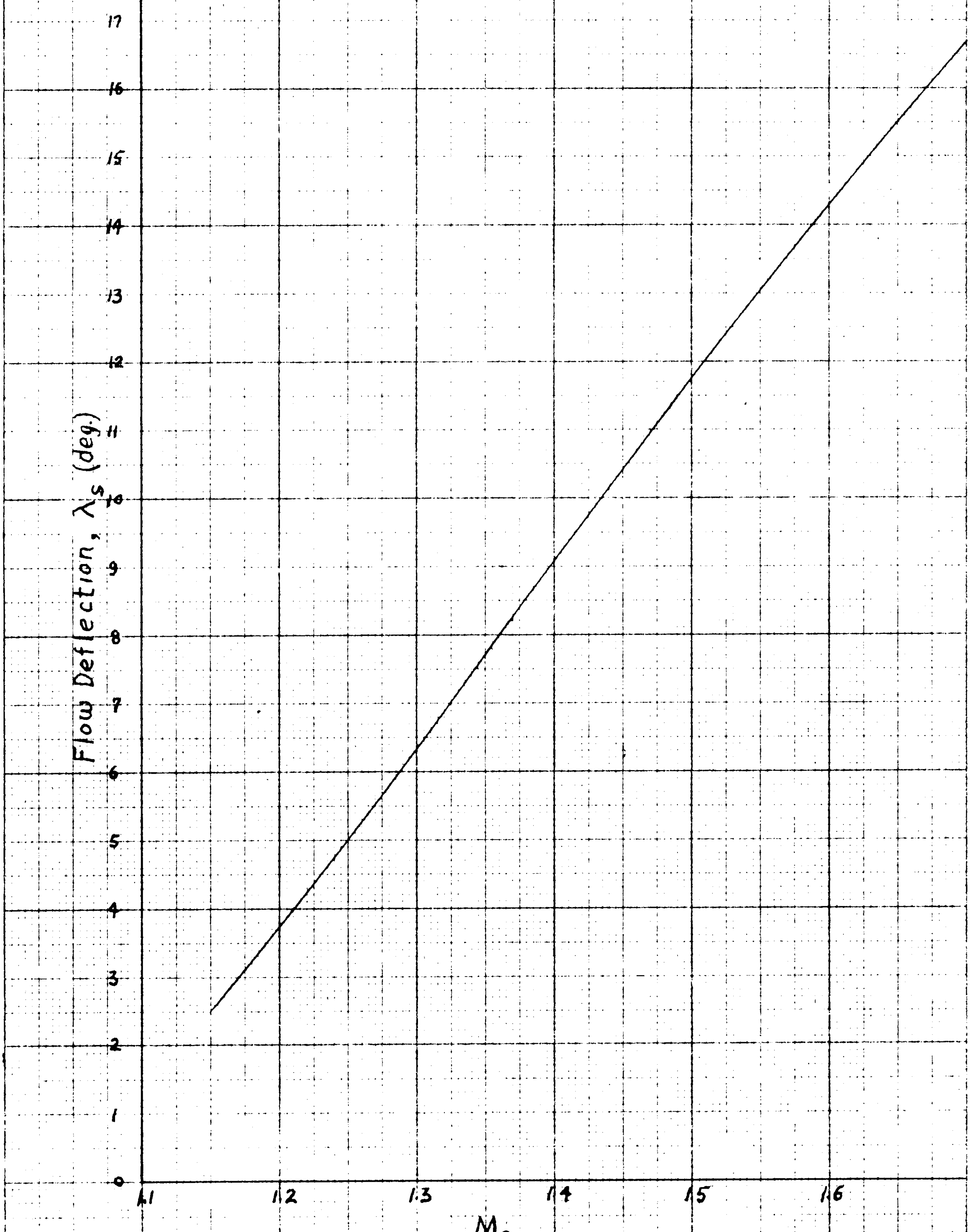


Fig. 4 - Variation of Flow Deflection, λ_S, With Mach Number

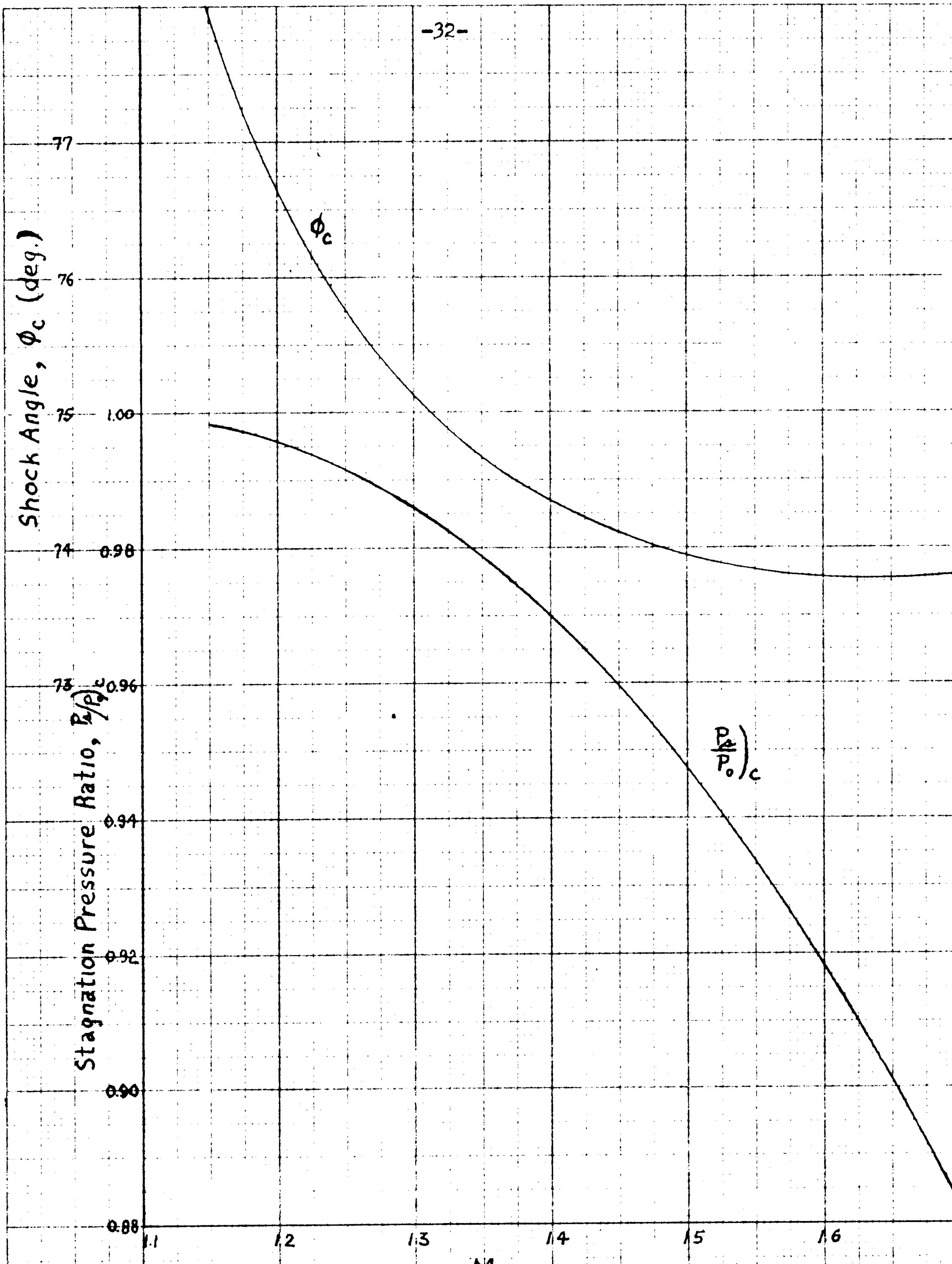


Fig. 5 - Variation of $\frac{P_s}{P_o})_c$ and ϕ_c With Mach Number

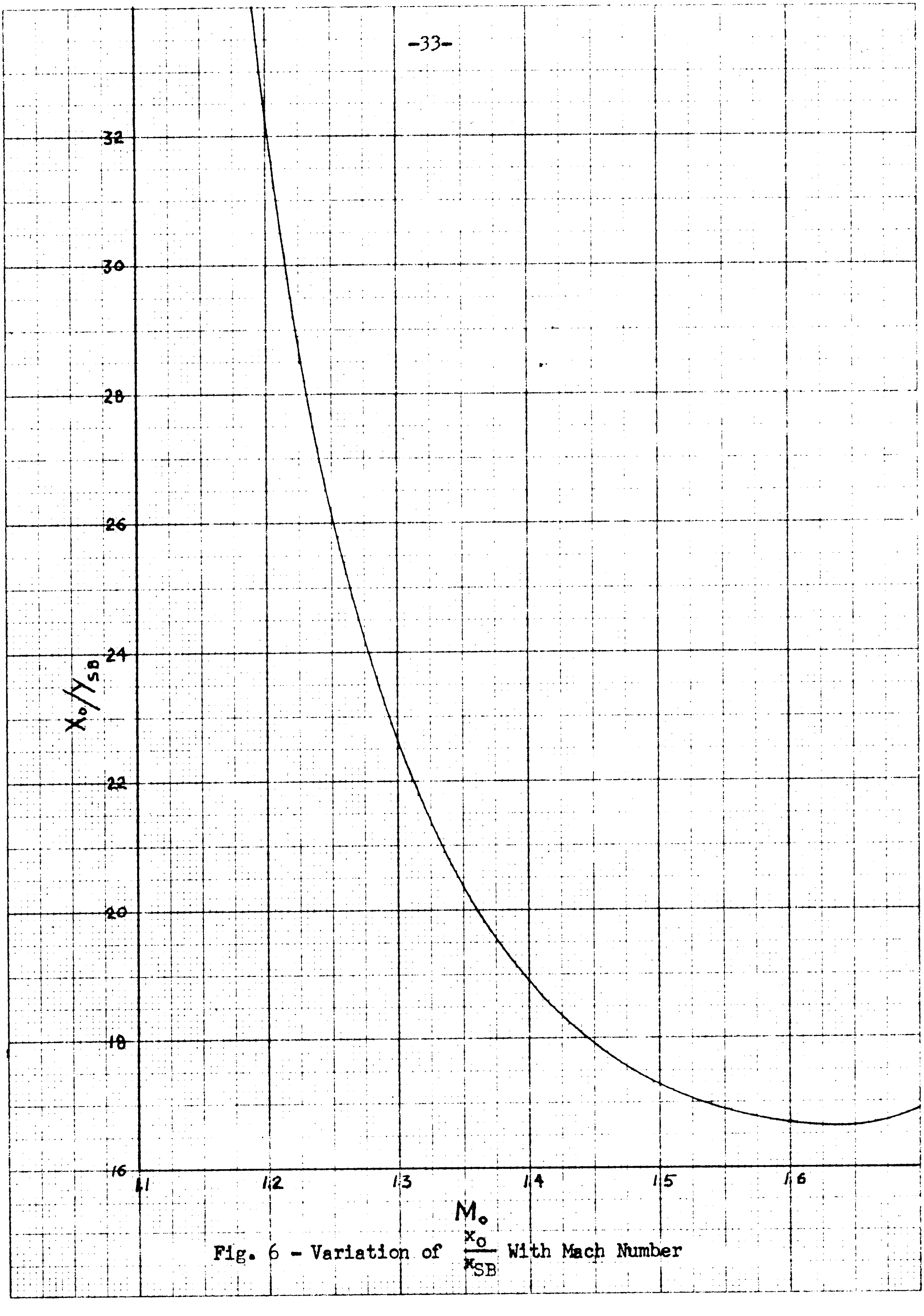


Fig. 6 - Variation of $\frac{x_o}{x_{SB}}$ With Mach Number

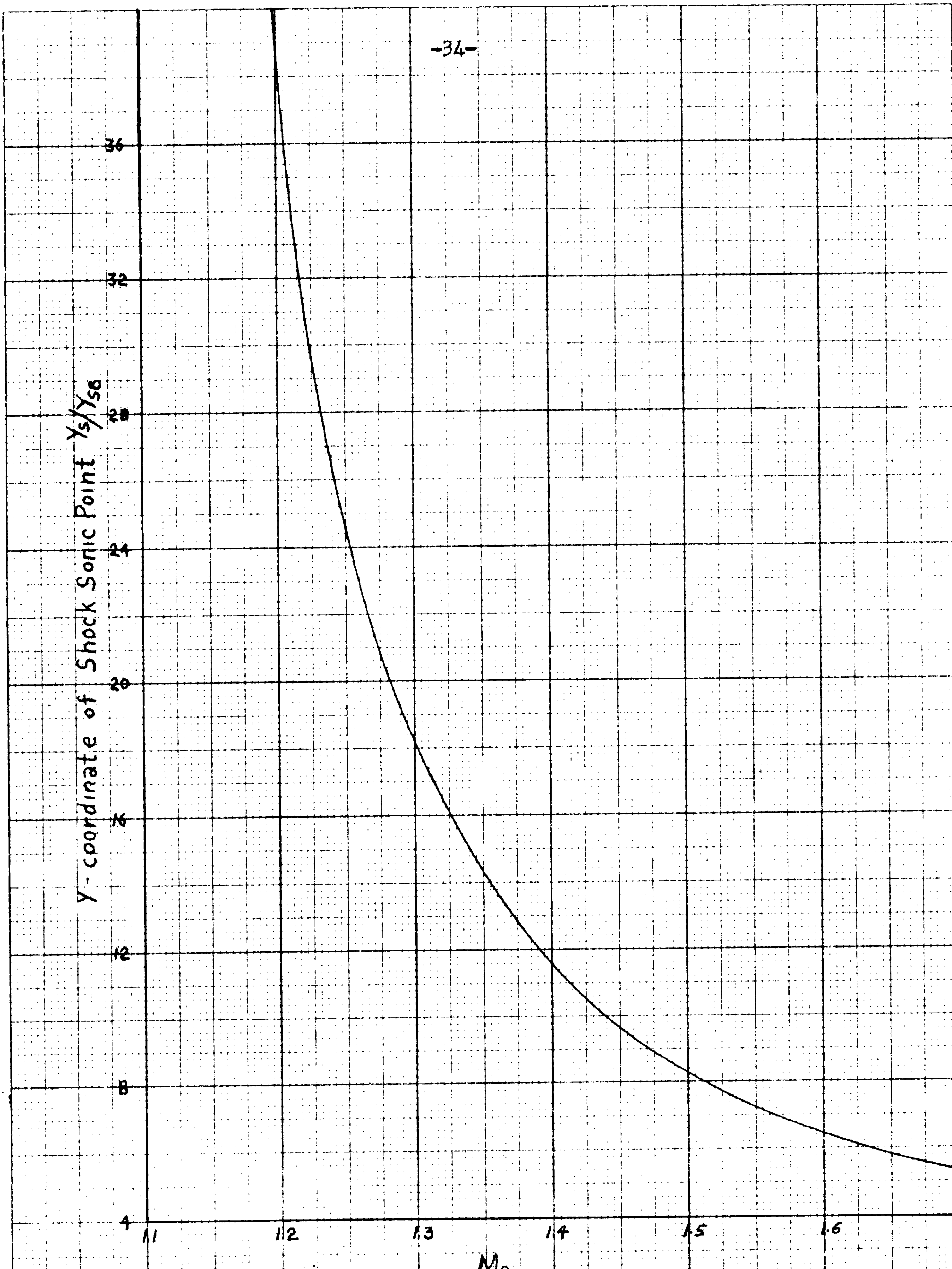


Fig. 7 - Variation of y-coordinate of Shock Wave Sonic Point With Mach Number

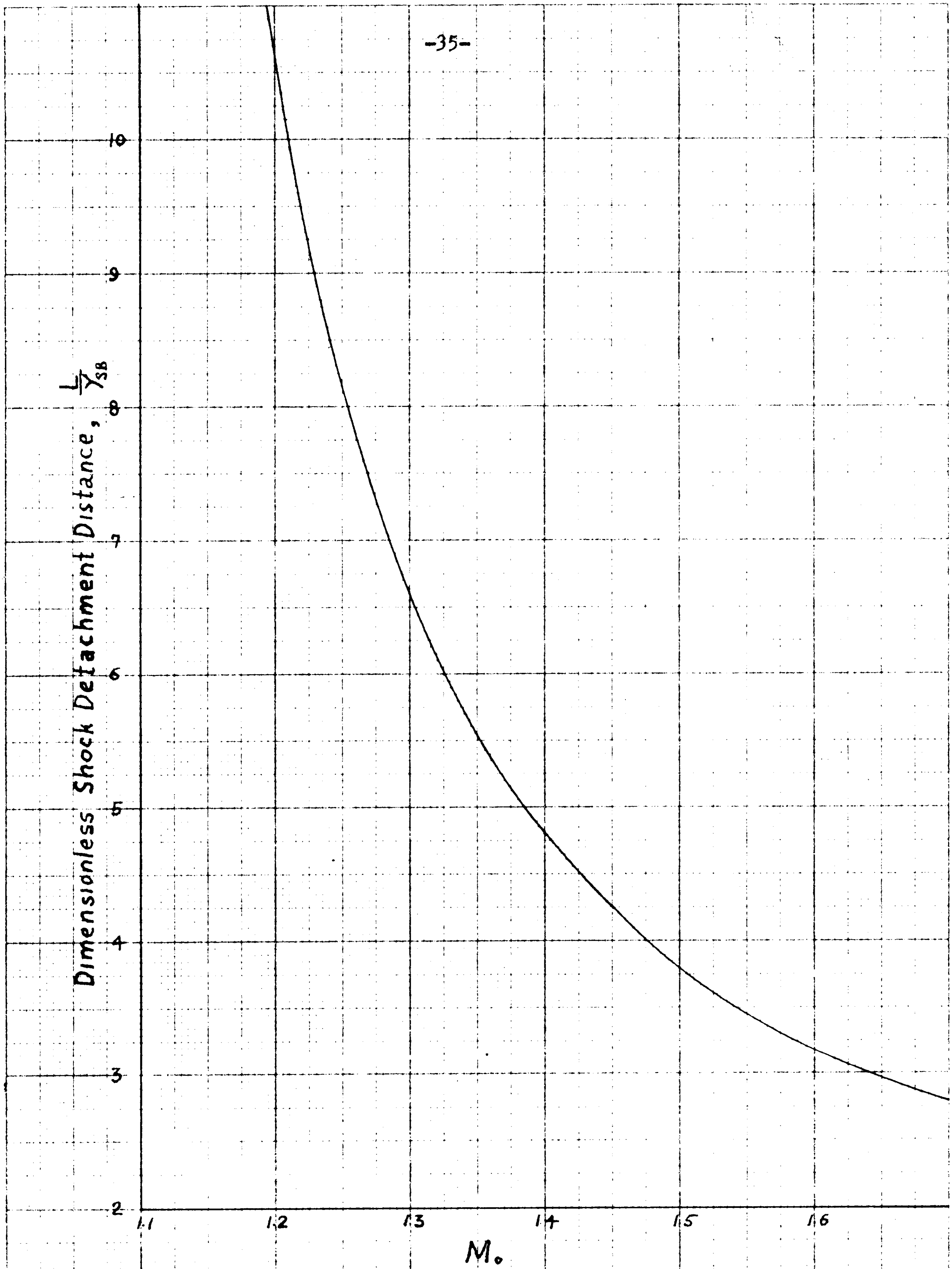
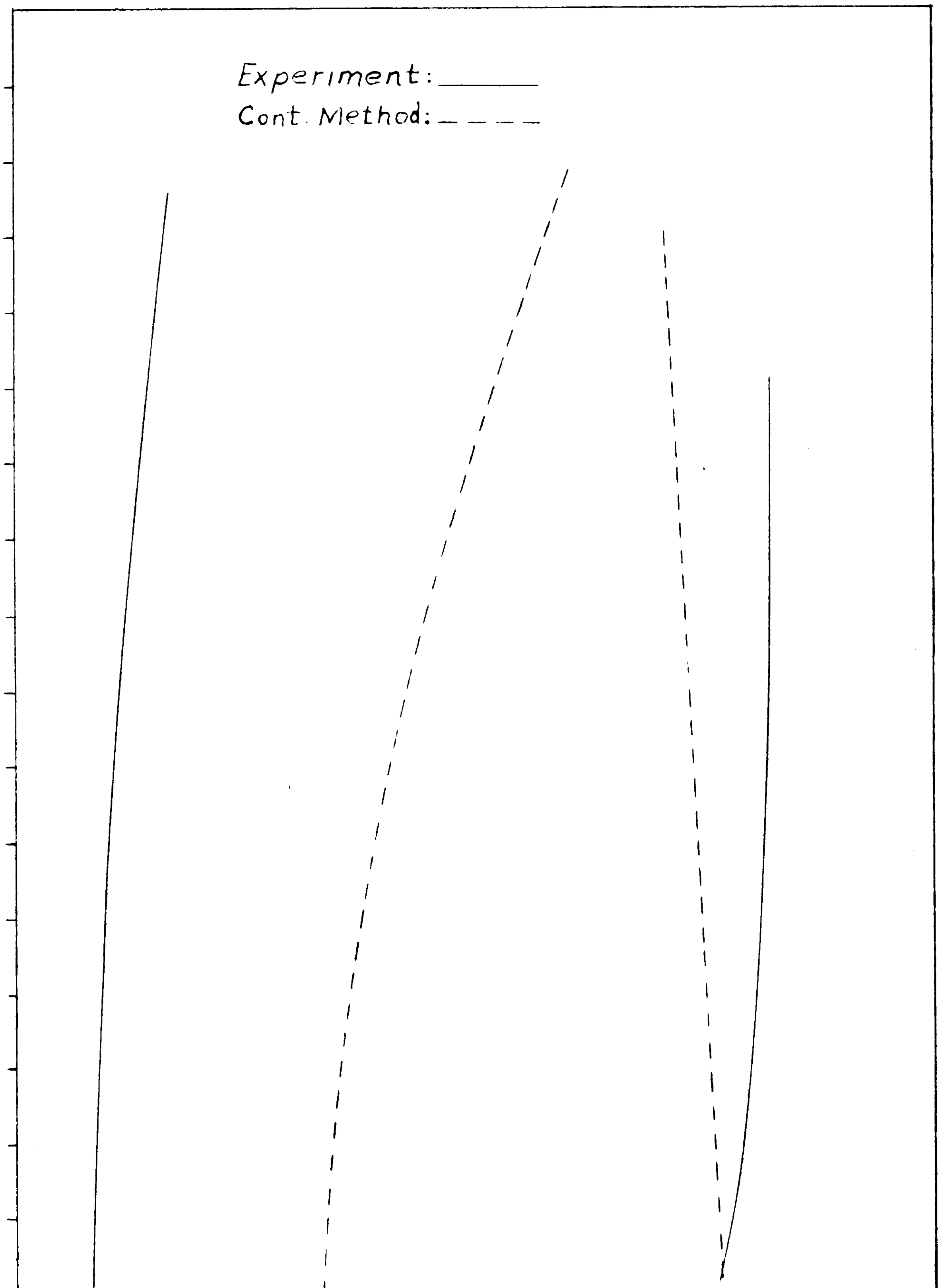
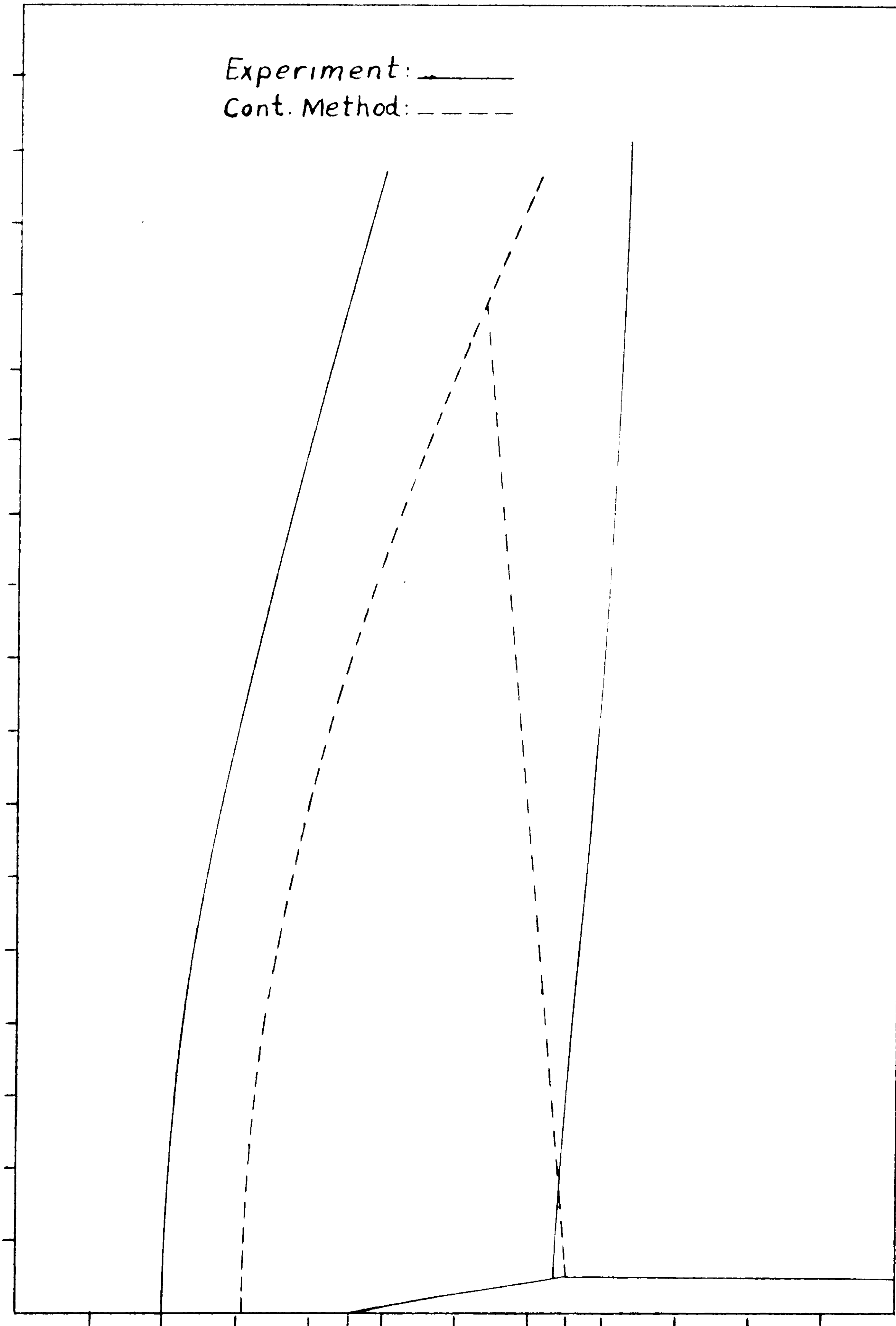


Fig. 8 - Variation of Shock Wave Detachment Distance, $\frac{L}{y_{SB}}$, with Mach Number



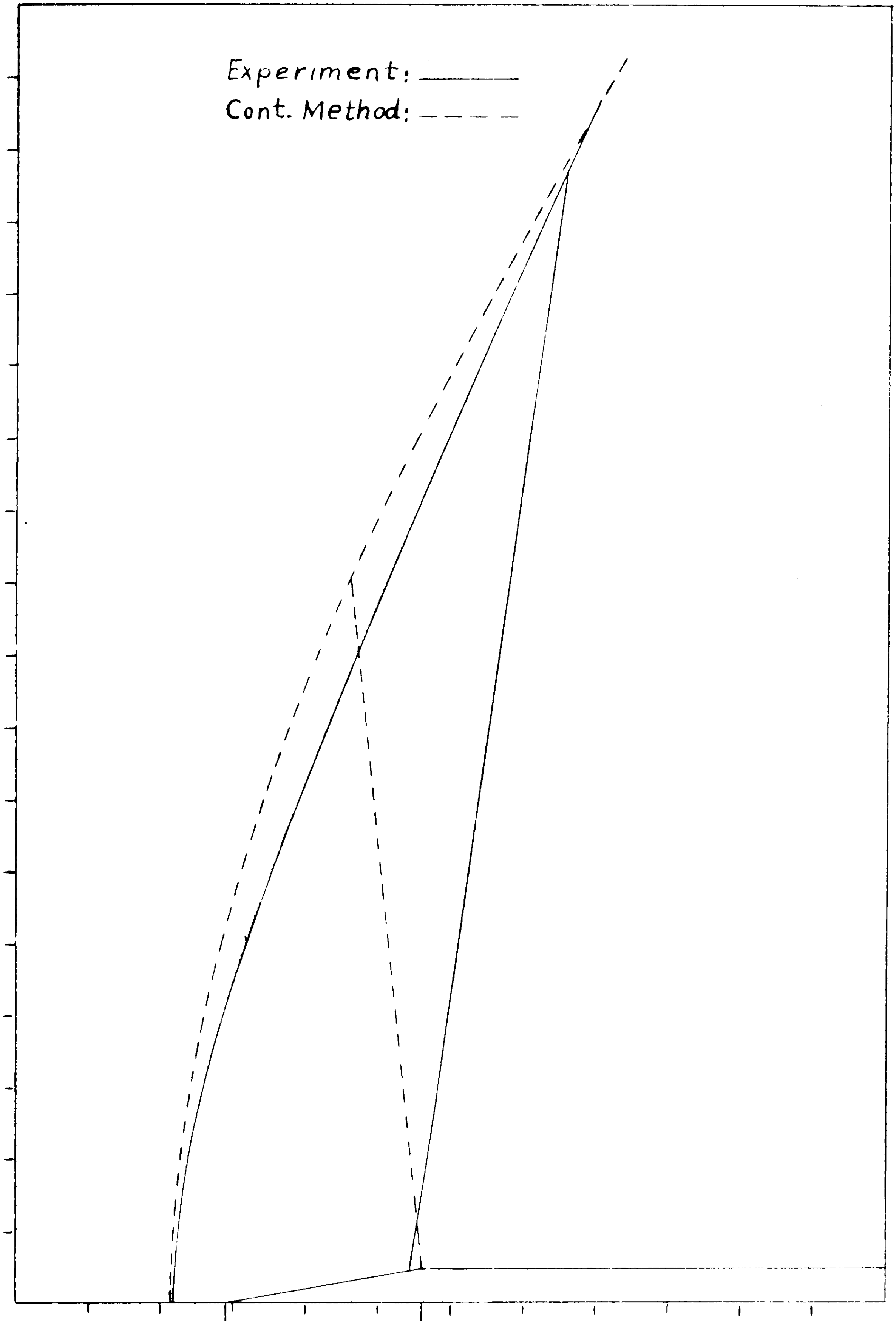
5.59 x actual size

Fig. 9 Experimental and Theoretical Shock Waves for a 10° Wedge at $M_0 = 1.207$.



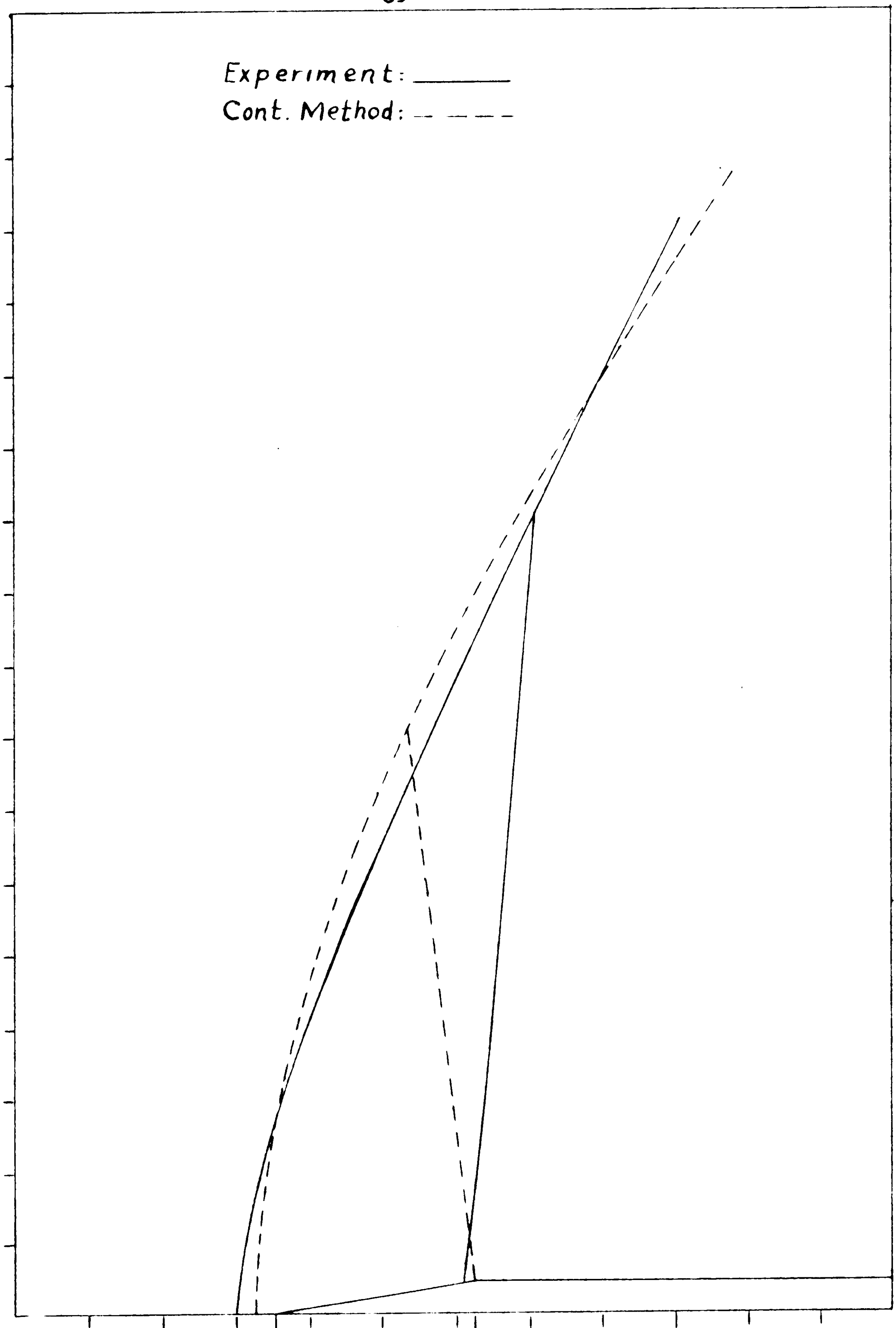
5.59 x actual size

Fig. 10 Experimental and Theoretical Shock Waves for a 10° Wedge at $M_0 = 1.240$.



5.16 x actual size

Fig 11 Experimental and Theoretical Shock Waves for a 10° Wedge at $M_0 = 1.278$.



5.16 x actual size

Fig. 12 Experimental and Theoretical Shock Waves for a 10° Wedge at $M_o = 1.315$

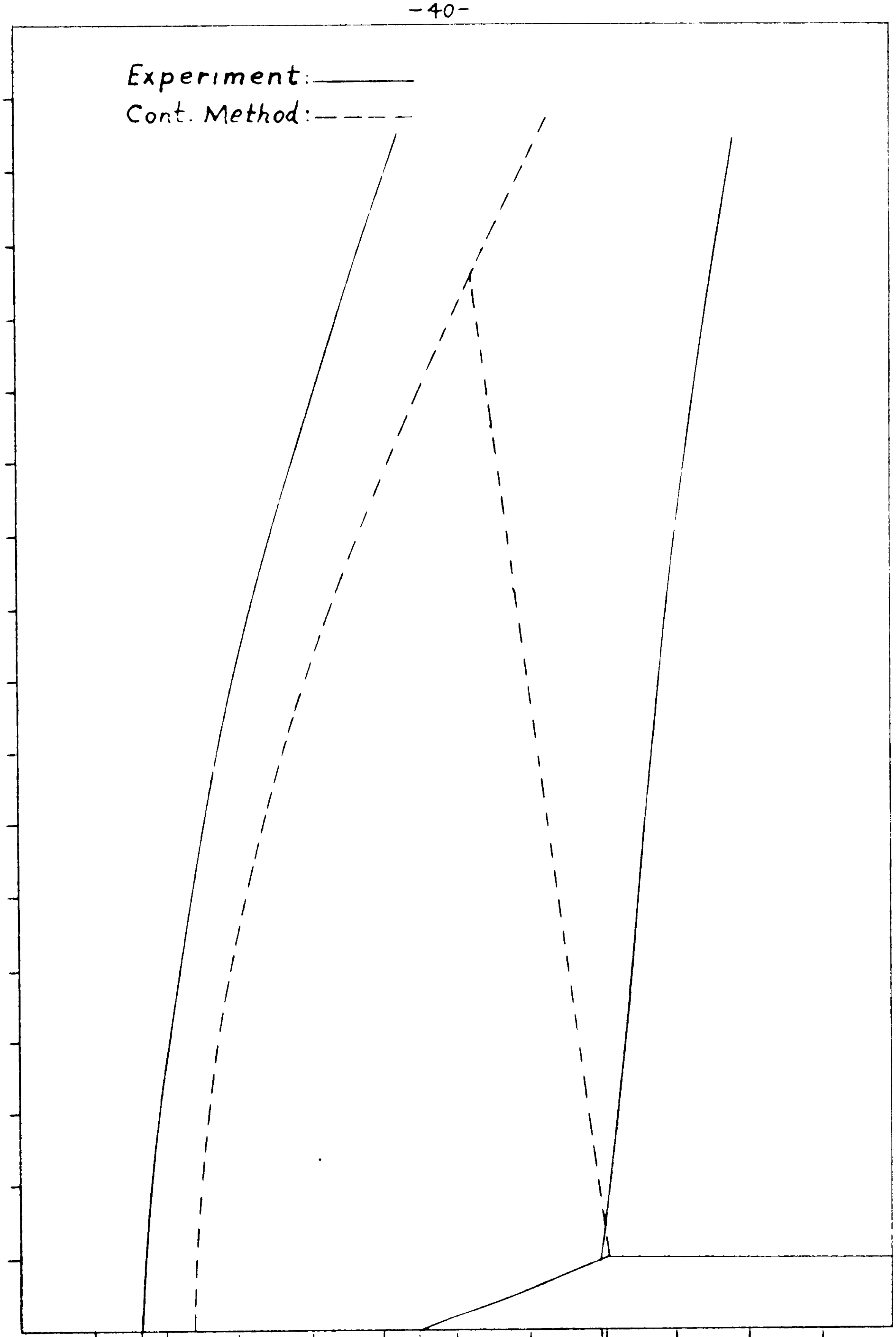


Fig. 13 Experimental and Theoretical Shock Waves for a 20° Wedge at $M_0 = 1.350$.

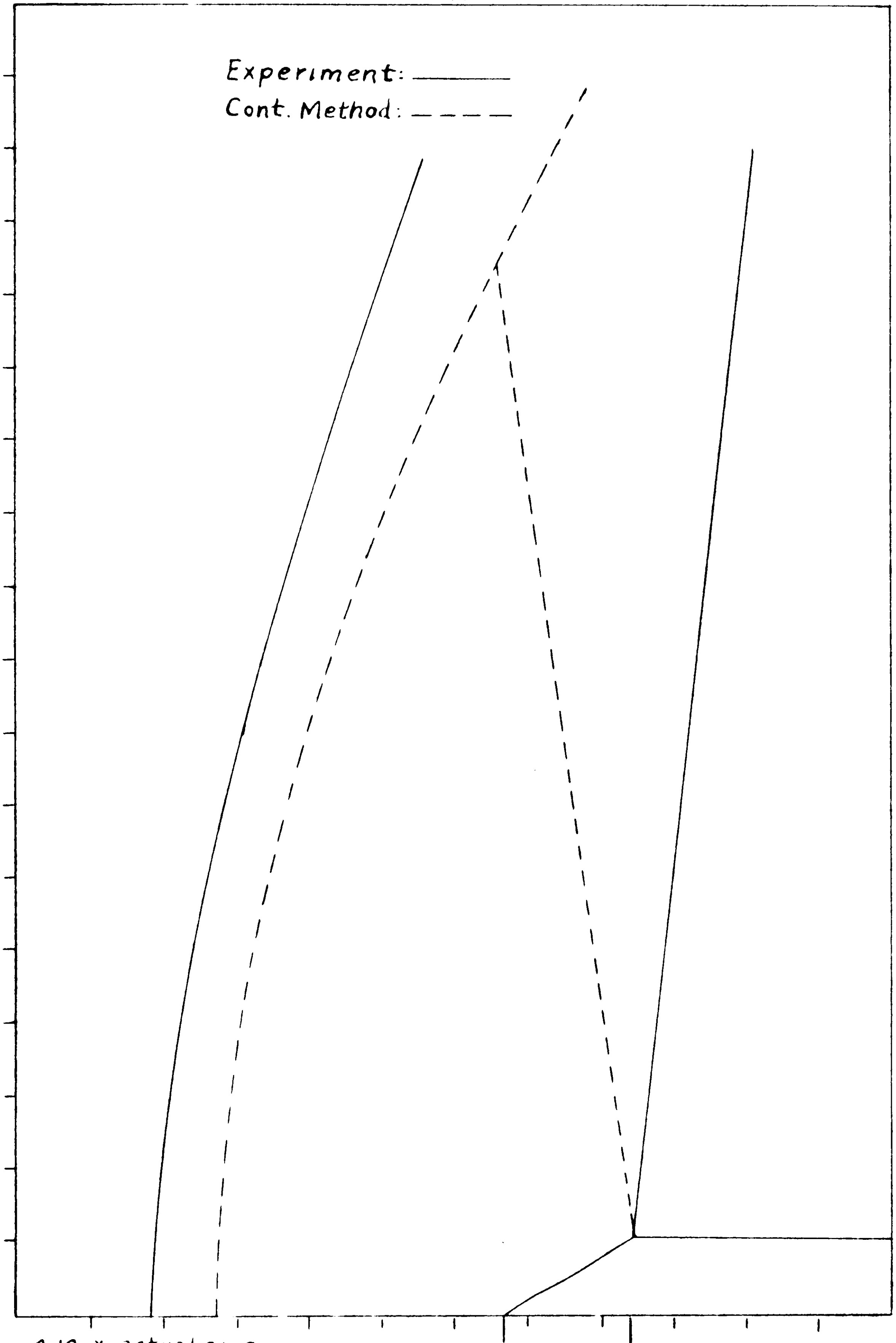


Fig. 14 Experimental and Theoretical Shock Waves for a 30° Wedge at $M_0 = 1.360$.

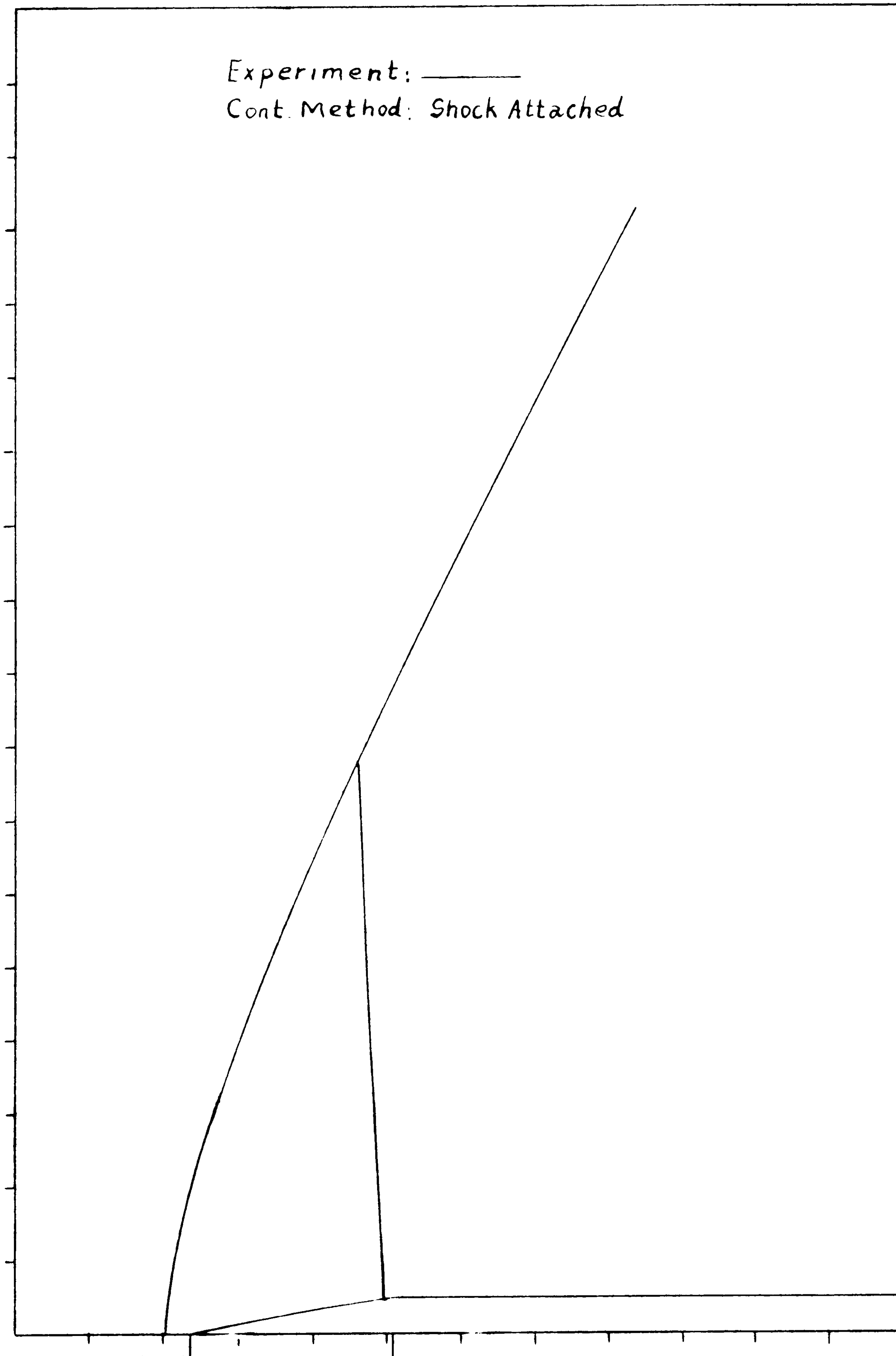
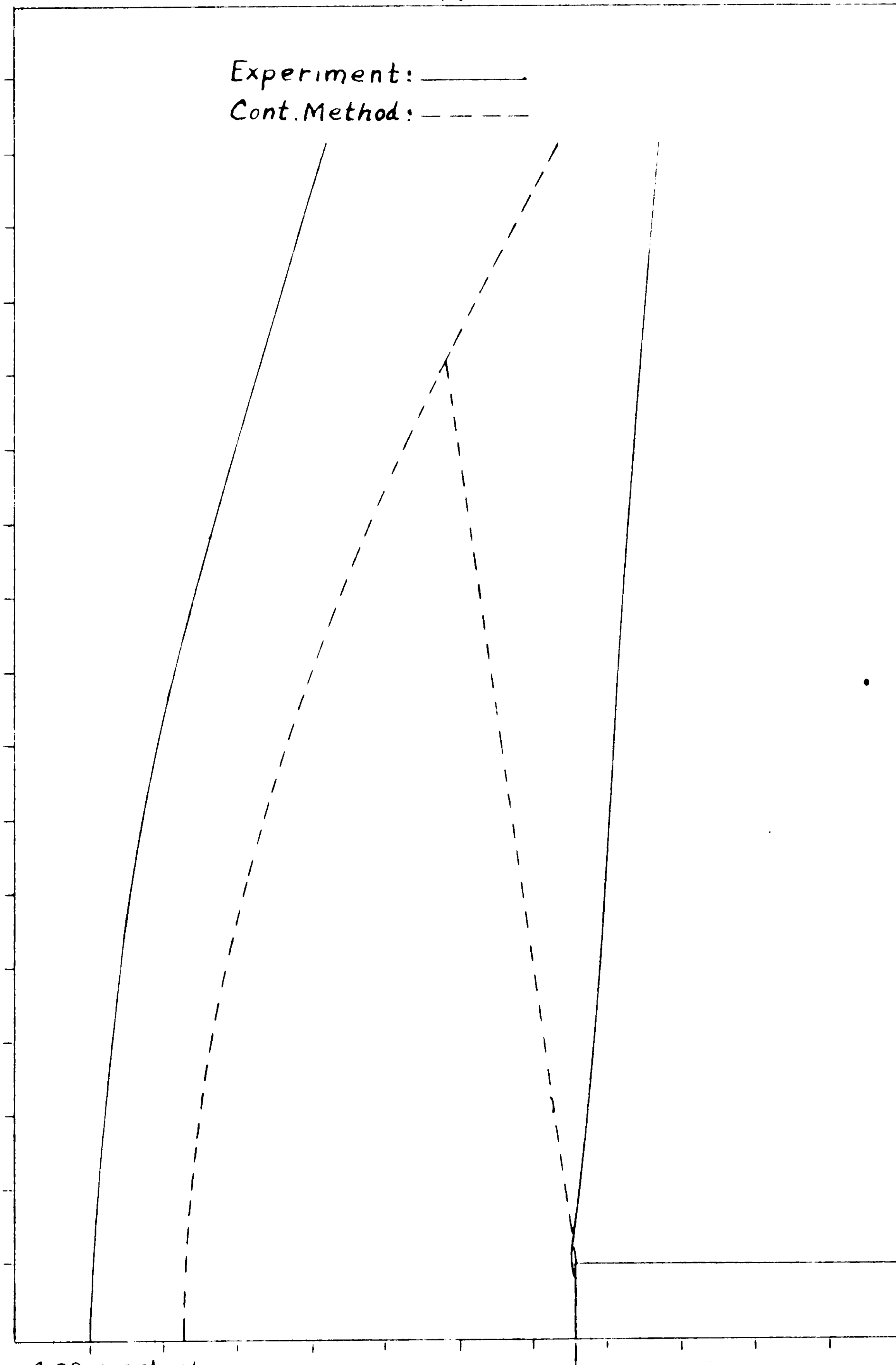
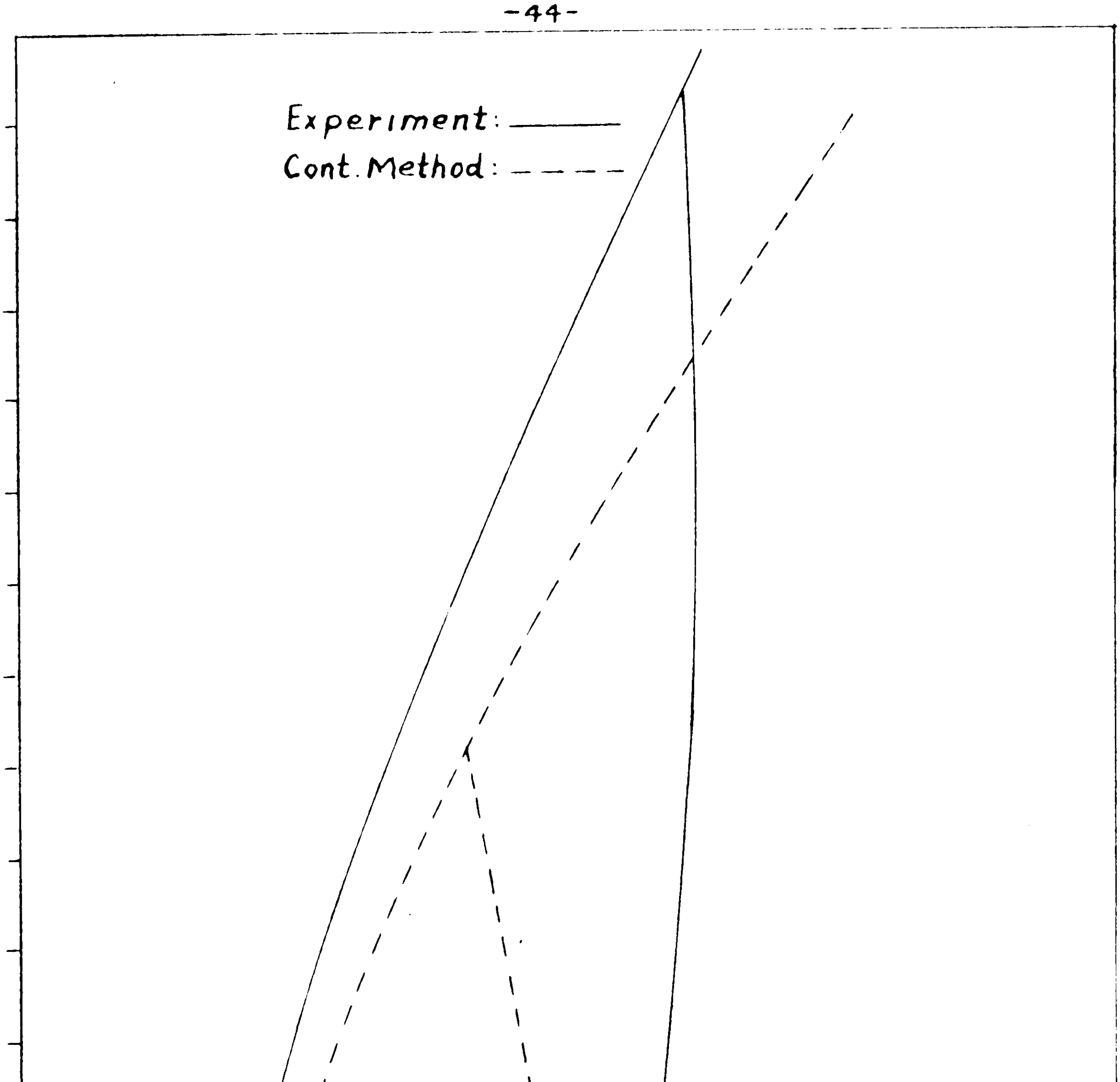


Fig. 15 Experimental and Theoretical Shock Waves for a 10° Wedge at $M_0 = 1.356$.



4.08 x actual size

Fig. 16 Experimental and Theoretical Shock Waves for a 90° Wedge at $M_0 = 1.370$.



4.08 x actual size

Fig. 17 Experimental and Theoretical Shock Waves for a 26.6° Wedge at $M_0 = 1.440$.

Table 1

GEOMETRY OF WEDGES USED IN THE EVALUATION

Mach Number	Wedge Semi-Angle	Wedge Semi-Thickness	Report From Which Taken
1.207	10°	0.0465"	Ref. 7
1.240	10°	0.0465"	Ref. 7
1.278	10°	0.0465"	Ref. 7
1.315	10°	0.0465"	Ref. 7
1.350	20°	0.125"	Ref. 8
1.356	10°	0.0465"	Ref. 7
1.360	30°	0.125"	Ref. 8
1.370	90°	0.125"	Ref. 8
1.440	26.6°	0.125"	Ref. 8

Table 2

CALCULATED QUANTITIES USED IN ANALYSIS FOR VARIOUS MACH NUMBERS

M_o	θ_ω	σ	ϕ_S	λ_S	ϕ_c	C	$P_s/P_o)_c$
1.150	-	0.9829	70.25	2.48	77.88	0.2023	0.9983
1.207	10	0.9683	67.80	3.86	76.53	0.2263	0.9947
1.240	10	0.9586	66.70	4.73	75.96	0.2380	0.9926
1.278	10	0.9456	65.65	5.73	75.30	0.2486	0.9879
1.315	10	0.9322	64.78	6.72	75.00	0.2571	0.9842
1.350	20	0.9182	64.10	7.67	74.66	0.2633	0.9787
1.356	10	0.9158	64.00	7.80	74.63	0.2641	0.9779
1.360	30	0.9141	63.93	7.94	74.60	0.2649	0.9771
1.370	90	0.9097	63.77	8.22	74.53	0.2659	0.9757
1.440	26.6	0.8789	62.83	10.12	74.20	0.2744	0.9614
1.500	-	0.8502	62.30	11.75	73.96	0.2790	0.9478
1.600	-	0.7999	61.65	14.25	73.78	0.2837	0.9188
1.700	-	0.7476	61.38	16.63	73.80	0.2844	0.8826

Table 3

CALCULATED PARAMETERS FOR VARIOUS MACH NUMBERS

M_o	θ_ω	y_S/y_{SB}	x_o/y_{SB}	L/y_{SB}
1.150	-	61.3497	42.6935	15.0302
1.207	10	34.7222	31.0107	10.1439
1.240	10	26.6667	26.9414	8.4693
1.278	10	20.9644	24.1700	7.2223
1.315	10	16.8634	21.7825	6.2059
1.350	20	14.2450	20.3732	5.5348
1.356	10	13.8504	20.1551	5.4248
1.360	30	13.6054	20.0230	5.3625
1.370	90	12.9702	19.6242	5.1773
1.440	26.6	10.0000	18.1800	4.3487
1.500	-	8.2169	17.2716	3.7936
1.600	-	6.4020	16.6926	3.1878
1.700	-	5.3135	16.8957	2.7996

Table 4

COMPARISON OF THEORETICAL AND EXPERIMENTAL SHOCK DETACHMENT DISTANCES

		L/y_{SB}			δ''		
M_o	θ_ω	Calc.	Exp.	% Error	Calc.	Exp.	% Error
1.207	10°	10.144	15.95	36.40	1.16	2.66	56.4
1.240	10°	8.469	10.59	20.00	0.72	1.27	43.3
1.278	10°	7.222	7.21	0.17	0.39	0.37	5.4
1.315	10°	6.206	6.80	8.74	0.13	0.27	51.8
1.350	20°	5.535	6.25	11.43	1.43	1.78	19.6
1.356	10°	5.425	6.42	15.50	0	0.38	100
1.360	30°	5.363	6.18	13.20	1.95	2.39	18.4
1.370	90°	5.177	6.40	19.11	2.64	3.26	19.0
1.440	26.6°	4.349	4.57	4.83	1.19	1.29	7.8

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