



Examining the elements of culturally relevant pedagogy captured and missed in a measure of high-quality mathematics instruction

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Abstract

Mathematics instruction is not race or culture neutral. For students who have been historically marginalized in mathematics classrooms, high-quality mathematics instruction, instruction that helps students build conceptual understanding, on its own might not be enough to disrupt inequities. These students might also need instruction that is culturally relevant, with teachers who demonstrate cultural competence, build critical consciousness, and support student learning. Our goal in this study was to understand which components of culturally relevant pedagogy (CRP) are captured and which are missed in a typical U.S. framework of high-quality mathematics instruction. To find the overlaps and gaps, we analyzed the mathematics lessons of three elementary teachers through both the lens of CRP and the Mathematics-Scan, a mathematics observation tool. We found the strongest overlap between the two frameworks in the patterns of strengths and weaknesses in the teachers' lessons. When the teachers were delivering high-quality instruction, they were also often supporting students' learning or showing cultural competence. When the teachers were delivering lower quality instruction, they were also often missing opportunities to enact CRP. At the same time, key elements of CRP including linguistic support for students, high expectations, critical consciousness, and nuances within cultural competence, were missed by the high-quality instruction framework. High-quality instruction was the foundation for CRP in the teachers' classrooms, but CRP was more than just high-quality instruction. We conclude with recommendations for increasing the alignment between the frameworks and implications for international educators also grappling with equity in their own frameworks of mathematics instruction.

Keywords Culturally relevant pedagogy · Mathematics · Culturally responsive teaching · Standards-based mathematics · High-quality mathematics instruction

1 Introduction

Mathematics can be a tool that students use to make sense of and create change in the world, as well as a gateway to future academic opportunities (Aguirre & Zavala, 2013;

Boaler, 2016; Vithal et al., 2024). Mathematics classrooms should be places where all students have opportunities to excel while drawing upon the rich resources they bring with them, including their funds of cultural and linguistic knowledge (Gutiérrez, 2018). Too often, however, mathematics classrooms are inequitable (Leonard et al., 2010; Wilson et al., 2019).

Mathematics instruction is not race, power, or culture neutral (Leonard et al., 2010; Robertson & Graven, 2020). In far too many countries, including the United States (U.S.), how students are spoken to in their mathematics classrooms, the methods used to teach them, the expectations to which they are held, and even the content they learn depends on students' position in society (Boaler & Staples, 2008; Robertson & Graven, 2020; Vithal et al., 2024; Wilson et al., 2019). In the U.S., many Black and brown students learn mathematics through whole-group lectures and time spent

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working through textbook problems with few opportunities for discussion, concept development, or group work (Boaler & Staples, 2008; Wilson et al., 2019). The inequities in mathematics instruction reflect societal beliefs about who is a “doer” of mathematics, as well as broader disparities in school funding and access to rigorous courses (Martin, 2009; Shah, 2017). While the axes of discrimination vary from nation to nation, from knowledge of the English language in Singapore to level of assimilation in Germany, educational inequities are found in far too many nations (Civitillo et al., 2019; Lim et al., 2019; Vithal et al., 2024).

All students, regardless of race or privilege deserve high-quality mathematics instruction (Leinwand et al., 2014). In high-quality mathematics lessons, teachers create opportunities for students to learn, to understand the *whys* of mathematics, to grapple with concepts, and to build understanding (Leinwand et al., 2014; Praetorius & Charalambous, 2018; Walkowiak et al., 2018). How teachers do this might vary from country to country, with an emphasis on discussion in some locales, on choral responses in others, and silence to promote thinking in others (Leinwand et al., 2014; Xu & Clarke, 2019). While definitions of high-quality mathematics instruction vary globally, most include elements missing in many classrooms serving marginalized learners (Robertson & Graven, 2020; Vithal et al., 2024; Wilson et al., 2019; Xu & Clarke, 2019).

For marginalized learners, however, high-quality mathematics instruction on its own might not be enough to disrupt injustice. A high-quality mathematics classroom is not necessarily an equitable one (Melhuish et al., 2022; Wilson, 2022). After one group of teachers built their expertise through intensive professional development, they became better at delivering high-quality instruction. The benefits to students, however, were uneven. The professional development had omitted coverage of cultural responsiveness and social justice, and that omission appeared to show in students’ unequal growth (Melhuish et al., 2022).

In a study of mathematics classrooms where marginalized students were exceeding expectations, researchers found that the teachers were both delivering high-quality instruction and attending to students’ racial and ethnic identities (Wilson et al., 2019). Given the inequities in society and in mathematics instruction, truly supporting marginalized learners in mathematics might require attending to their identities and histories in addition to providing them with high-quality mathematics instruction (Aguirre & Zavala, 2013; Leonard et al., 2010). Similar realizations in countries like South Africa, Germany, and New Zealand has led to an increased focus on equity-focused pedagogies across the world (Averill et al., 2009; Civitillo et al., 2019; Vithal et al., 2024). One way for teachers to attend to students’ histories and identities is to enact culturally relevant pedagogy

(CRP), a strengths-based approach to culturally responsive instruction (Ladson-Billings, 2021).

In this study, our goal was to understand which aspects of CRP are addressed through a typical framework of high-quality mathematics instruction in the U.S. and which are missed. To do this, we focused on the mathematics lessons of three teachers who had participated in professional development on CRP. We analyzed their lessons through the lenses of CRP and high-quality instruction to find overlap and omissions, with the ultimate goal of finding better ways to support teachers in delivering high-quality, culturally relevant mathematics instruction.

2 Literature review

2.1 Unpacking high-quality mathematics instruction

There is no one international definition of high-quality mathematics instruction (Praetorius & Charalambous, 2018; Xu & Clarke, 2019). In the U.S., the Common Core State Standards and the National Council of Teachers of Mathematics (NCTM) process standards both uplift teaching practices that support conceptual understanding through problem solving, reasoning and proof, communication, connections, and representation (Berry et al., 2017; Leinwand et al., 2014; Walkowiak et al., 2018). To develop a broader, international understanding of high-quality mathematics instruction, researchers have looked across different observational tools used to assess the quality of instruction and to support teachers’ professional development (Praetorius & Charalambous, 2018). Mathematics observation tools vary widely in their scope from broad enough to be used in mathematics or social studies to narrow enough to capture only mathematics pedagogy, possibly omitting even classroom climate (Charalambous & Praetorius, 2018). Each tool also reflects the cultural beliefs and priorities of the region where it was created, with many Western tools emphasizing student-to-student and teacher-to-student discussion and de-emphasizing other modes of active participation and ways to grapple with ideas (Xu & Clarke, 2019). Many of the math-focused tools also emphasize cognitive activation, the extent to which teachers choose challenging tasks, ask students to reason, explain, and generate ideas, and support them in accessing challenging tasks and ideas (Praetorius & Charalambous, 2018).

In this study we define high-quality mathematics instruction as instruction that builds students’ conceptual understanding of mathematics content. Teachers might enact high-quality instruction by assigning ambitious tasks, asking students to reason, explain, or generate ideas, or by

building connections between and across content. Each of these practices is high-quality to the extent it creates opportunities for all students, including those who have historically been marginalized, to build a deep understanding of the content.

2.2 Understanding culturally relevant pedagogy in mathematics

One way that educators in the U.S. and other countries have attempted to make high-quality mathematics instruction equitable is through the integration of CRP (Averill et al., 2009; Civitillo et al., 2019; Ladson-Billings, 2021). CRP is built on a foundation of educator self-awareness and knowledge of societal injustices, awareness and knowledge that can translate into teachers' classroom practices (Ladson-Billings, 1995, 2021; Lim et al., 2019). Knowledge of social injustices can come from books, lived experiences, and various other forms of exposure (Ladson-Billings, 2021; Lim et al., 2019). A number of studies have examined how teachers' sense of awareness or their beliefs about equity translate into their teaching practice. For example, for one teacher in Singapore, it was his experiences as a non-native English speaker that shaped his desire to enact CRP, while for a peer, it was his non-dominant cultural background (Lim et al., 2019). In another case study of secondary teachers, German researchers found that teachers with more equity-focused beliefs were more likely to enact elements of CRP than their peers with "color evasive" ideologies (Civitillo et al., 2019, p. 348).

When teachers enact CRP, they draw on at least some of its three key pillars. The first pillar is student learning. Culturally relevant educators hold students to high standards, build positive relationships, and provide students with rigorous instructional opportunities. In mathematics classrooms, teachers might ingrate mathematics tools into lessons or support student inquiry (Thomas & Berry, 2019). Mathematics teachers might also support student learning by holding students accountable, asking them challenging questions, or requiring that students revise and improve their work (Thomas & Berry, 2019). The second pillar of CRP is cultural competence. Culturally relevant educators use students' culture as a bridge to the curriculum. In mathematics classrooms, teachers might learn about their students' communities or their funds of knowledge and incorporate that information into lessons (Thomas & Berry, 2019). Teachers in New Zealand, for example, incorporated Māori legends into word problems (Averill et al., 2009). The third pillar is critical consciousness, the awareness and challenging of sociopolitical inequities. CRP is a model "that not only addresses student achievement but also helps students to accept and affirm their cultural identity while developing

critical perspectives that challenge inequities that schools (and other institutions) perpetuate" (Ladson-Billings, 2021, p.18). The third pillar is the least commonly found in mathematics classrooms (Thomas & Berry, 2019). Some culturally responsive teachers report feeling afraid of wading into more uncomfortable topics and limited in their enactment of CRP by the pressures of their schools and their assigned curriculum (Averill et al., 2009). Mathematics teachers might treat students as experts and empower them within the classroom, but only rarely connect mathematical concepts to social justice concerns or empower students in the broader world (Thomas & Berry, 2019).

Given the findings of prior researchers and the framework of CRP, in this study we define culturally relevant mathematics instruction as instruction that (1) provides students with high-quality learning opportunities while holding them to high expectations and providing them with support; (2) builds connections between the content and students' lives and identities; and (3) empowers students within the classroom and community through power sharing and the inclusion of community and societal problems and solutions.

2.3 Bridging high-quality mathematics and culturally relevant pedagogy

High-quality mathematics instruction is foundational to CRP in mathematics. The first pillar of CRP is student learning (Ladson-Billings, 1995, 2021). Ladson-Billings began her work by looking at mathematics classrooms where Black students were excelling. Those were classrooms taught by expert teachers, teachers who often taught conceptually, encouraged discussion, brought in multiple representations of content, or made connections across the curriculum (Ladson-Billings, 2021). When Bonner (2014) looked at the culturally relevant practices of upper elementary and middle school mathematics teachers, she found that they often asked students to work in small groups and to discuss concepts. When Wilson (2022) looked at the culturally relevant practices of strong mathematics educators, she found practices such as asking students to explain their thinking, to problem solve, and to justify their reasoning. In these classrooms, high-quality instruction formed the foundation of the teacher's CRP, the backbone that they used to maintain high expectations, to make connections to students' lives, and to build students' power.

While high-quality mathematics education is foundational to implementation of CRP, the quality of mathematics instruction is also strengthened and made more equitable by integrating CRP practices. As CRP is not content specific, mathematics educators have tried to operationalize CRP in mathematics education, including by developing subject-specific CRP tools. One approach is the Culturally

Responsive Mathematics Tool (CRMT; Aguirre & Zavala, 2013). The CRMT was developed to heighten in-service and pre-service teachers' attention to students' funds of knowledge and the teacher's own pedagogical content knowledge. The eight dimensions of the tool include intellectual support, mathematical discourse, and use of funds of knowledge. A second tool that has been developed is the Equity and Access Rubrics for Mathematics Instruction (EAR-MI; Wilson, 2022). The EAR-MI was developed based on observations of conceptually focused classrooms where Black students were excelling. In earlier work, Wilson and colleagues (2019) found seven practices that were up to 2.5 times as common in classrooms where Black students excelled. The seven practices were: explicitly stating expectations; coaching students; supporting connections and engagement between student context and mathematics learning environment; attending to language; attributing responsibility to students in responses to their requests for assistance; positioning students as competent; and supporting a nurturing environment (Wilson, 2022, p.195). Both the EAR-MI and the CRMT begin to bridge the gap between CRP and high-quality mathematics instruction by operationalizing CRP for mathematics.

The relationship between high-quality mathematics instruction and CRP can be synergistic. In Wilson's (2022), Bonner's (2014), and Ladson-Billings's (1995, 2021) work, high-quality instruction was at the heart of CRP and CRP was at the heart of the teachers' high-quality instruction. At the same time, high-quality instruction is not currently defined in the U.S. as culturally relevant. By better understanding what aspects of CRP are captured in or omitted within a typical U.S. framework of high-quality mathematics instruction, we can further bridge the two frameworks and unlock the power of their synergy. This work is inherently culturally specific as definitions of high-quality instruction vary by nation, as do the nuances of inequities in mathematics classrooms. At the same time, given the global prevalence of marginalization, projects like these are essential as educators seek to create just and equitable mathematics classrooms across the world.

3 Research purpose and questions

Students who have historically been marginalized in mathematics classrooms deserve high-quality, culturally relevant mathematics instruction. In this study, we sought to understand the overlap and distinctions between high-quality instruction and CRP in order to better support U.S. teachers in implementing culturally relevant mathematics instruction and to provide an example for global educators for how

these frameworks can be bridged. Specifically, we set out to understand:

What aspects of culturally relevant pedagogy are captured or missed in a typical observation measure of high-quality instruction in the U.S.?

4 Methods

To answer our research question, we drew on classroom observations of three teachers in the Southeast of the U.S., analyzing their lessons through the frameworks of both high-quality instruction and CRP.

4.1 Sample and data

The three teachers in Wayne School District (all names are pseudonyms) were purposively sampled due to their expressed interest in CRP, their participation in a district-created professional development on culturally responsive practices in the 2018 school year (for more on the district professional development see Thomas & Sebastian, 2023), and their willingness to participate in this IRB approved study. The first teacher in the study is Sophia, a pre-kindergarten teacher at River Elementary. Sophia identifies as Black and has been teaching for six years. Sophia is moderately fluent in Spanish. In 2018, Sophia had 18 students in her classroom and one full-time aide. Her classroom primarily served lower-income students and was mixed ethnically and racially, with 11 students identifying as Latino, five as Black, and two as of Middle Eastern descent. The second teacher is Ava, a third-grade teacher at the same school. Ava identifies as Black and has been teaching for five years. She, too, is moderately fluent in Spanish and has been nominated by her district as a distinguished teacher. She had 20 students in her co-taught, multi-age classroom. Almost half of her students received English for Speakers of Other Languages (ESOL) support. Her classroom was also diverse, with nine students identifying as Black, seven as Latino, two as multi-racial, one as Asian, and one as White. The third teacher is Chris, a fourth-grade teacher at a Spanish immersion program at Willow Elementary. Chris identifies as White and Jewish and has been teaching for 11 years. He is bilingual in English and Spanish. His classroom had 20 students, 13 of whom identified as Latino and seven who identified as White. Half of his class received ESOL support, and 13 students came from lower-income homes.

This study draws on observations of these three teachers collected as part of a larger study on the district-created professional development. In the fall of 2019, the first author

spent 15 to 16.5 h in each teacher's classroom, observing approximately one unit of mathematics instruction in each classroom over the course of two consecutive weeks. Each observation followed an observation protocol and, in addition to taking field notes, the first author video recorded each lesson for further analysis.

4.2 Data analysis

We analyzed 31 classroom lessons from three teachers using the frameworks of CRP and of high-quality instruction. The CRP focused analysis began during the observations with reflections and analytic memos (Miles et al., 2014). Then, after data collection was complete, we coded the field notes in Dedoose, a qualitative data analysis program, and used analytic memos to synthesize the codes and identify patterns in the data (Miles et al., 2014). All materials were separately line by line coded using a combination of a priori codes, generated from work on CRP and the research questions, and emergent codes that came from the observations (Miles et al., 2014). Colleagues with expertise in CRP in mathematics reviewed each phase of the data collection process and analysis, including the initial codes. The final codes consisted of care, critical consciousness (with students and parents), cultural competency, teacher awareness and reflection, gaining knowledge of learners, partnerships (with learners and families), power, and support for student learning in mathematics (expectations, explanation, connections, discourse, feedback, tools, demand, applications). Coding was at the stanza level and each stanza could receive multiple codes (Saldaña, 2013).

The high-quality instruction focused analysis utilized the Mathematics-Scan (M-Scan) tool. The M-Scan is a research-validated narrow observational tool, focused on uncovering how teachers enact or fail to enact high-quality mathematics instruction within their lessons (Berry et al., 2017; Litke et al., 2021; Walkowiak et al., 2018). M-Scan was selected for this study due to its focus on elementary education and the overlap between its key domains and the definition of high-quality mathematics instruction used in this study.

M-Scan has four key domains that are subdivided into a total of nine scoring dimensions. The four domains are: task selection and enactment, representations, mathematical discourse, and lesson coherence (Berry et al., 2017). Task selection and enactment includes three dimensions: cognitive demand (the extent to which the teacher focuses on underlying concepts and the promotion of complex thinking in the teaching and tasks), problem solving (the extent to which students are given tasks that encourage multiple solutions and that encourage them to wrestle with concepts), and connections and applications (the extent to which the teacher makes connections between mathematical concepts

and between the concepts and the world, as well as how students are asked to apply those concepts to the real world). The second domain of representations includes two dimensions for mathematical tools (whether the lesson enables the use of tools and whether those tools help the student develop conceptual understanding) and the use of multiple mathematics representations (how teachers create and translate between different representations like pictures and symbols, and how they ask students to do the same). The third domain of mathematical discourse has two dimensions for mathematics discourse community (this includes the teacher's discourse and opportunities for peer-to-peer discussion) and explanation and justification (both whether the teacher asks for them and whether the students provide more than procedural explanations). The fourth and final domain of lesson coherence includes the final two dimensions of lesson structure (the extent to which the lesson is logically organized, coherent, and designed to promote deeper understanding) and mathematical accuracy (both the accuracy with which teachers present content and how they respond to student misconceptions). Each dimension is coded on a scale of 1 to 7 with descriptors of low (1–2), medium (3–5), and high (6–7), and qualitative notes to justify each score. We scored each of the 31 lesson observations using the M-Scan, with eight lessons double scored to ensure reliability.

After the lessons had been analyzed through the lenses of CRP and high-quality instruction, we then shifted our analysis to understanding where the two frameworks overlapped and diverged. We built visual organizers including color-coded flow charts to help make sense of the patterns we were seeing (Miles et al., 2014). We also wrote memos throughout the process and looked for rival explanations (Yin, 2018). To build credibility, we engaged in member checking, maintained a research database, and consulted with expert colleagues (Yin, 2018).

4.3 Researcher as instrument statement/ reflexivity

As qualitative researchers, we cannot be written out of the research process. Who we are shapes what questions we ask and what we see in the data. We share a commitment to equity in education and, as former teachers, a belief in the power of teachers to create new opportunities for their students. Our personal beliefs and identities shaped what we were predisposed to notice during data analysis and the questions we asked. While we took care to stay close to the data, to confer with expert colleagues, and to attend to our own biases, our own "assumptions, interest, and theoretical commitments" (Emerson et al., 1995, p.167) inevitably informed what we found.

5 Results

Across the 31 lessons we observed in the teachers' classrooms, we found patterns in which aspects of CRP were captured by the high-quality mathematics instruction tool and which were missed. To explain the patterns we found, we draw on example lessons from each teacher's classroom followed by brief explanations of each.

5.1 Areas of overlap between CRP and a measure of high-quality instruction

In these three teachers' classrooms, high-quality mathematics instruction formed the foundation of the teachers' culturally relevant instruction. When we saw the teachers delivering high-quality instruction, they were also likely enacting aspects of CRP. When we saw gaps in the teachers' high-quality instruction, we also often saw missed opportunities for CRP.

5.1.1 Instructional examples

Chris. As students sit on the front carpet, Chris explains their lesson on problem solving and the need for norms for problem solving. He and the students identify ways to show someone that you are listening. Next, they discuss baking and what the students like to bake, with terms presented in both English and Spanish. Then, Chris introduces the first task in English, "On Monday, the baker baked 234 cookies. On Tuesday, she baked 158 cookies. How many cookies did the baker bake?" A student reads the task aloud in Spanish. Chris then asks for predictions, telling the students to stand if the number will be more than 234 and to sit if it will be less. Students share their thoughts with partners, after which Chris brings the class back together. He asks a student, "What do you think?" The student responds, "More," and Chris asks, "How do you know?" The student says, "Because you are adding." Chris follows up, "Share why you think adding." The student responds, "*suma*."

Chris then explains the packet of mathematics tasks that the students are receiving. Tasks 2 and 3 extend the practice to more digits and place value knowledge while Task 4 is open-ended, with students creating and solving their own word problems. Each task is on a separate page and Chris talks about how students are expected to use the space on the pages to show their strategies for solving the problems. He also encourages students to use different strategies and representations for each task. Then, Chris gives the students a few minutes to think on their own before they move into small groups. During small groups, Chris interrupts once to share his expectations for their work and to address misconceptions he heard in the small groups. He shares what it

means to be "doers of mathematics," and tells students they should feel challenged by the task. He then encourages them to continue working in small groups.

When they return to the whole group on the carpet, Chris asks students to share the strategies they used and why they chose each. He makes connections between the strategies, bringing out base-10 blocks, showing pictorial representations, and using the "cone strategy" to help learners to visualize the tasks. He then connects those to the traditional algorithm. Students share that they generally only baked a dozen cookies at a time and that the first problem had a larger than average amount of cookies.

Ava. Ava's lesson begins with a discussion of the word "routines." To help students understand the concept, Ava brings up fire drills, a routine that students know well. Ava discusses other routines for group talk and instructs the students to get into groups of four. Ava shares that students will be given two mathematics tasks and that they are expected to talk about how the tasks are the same, different, and related. The two tasks are:

Task 1: Frogs usually have four legs. In a pond, there are eight frogs. How many frog legs are there?

Task 2: Frogs usually have four legs. In a pond, there are 16 frog legs altogether. How many frogs are there?

Students move into groups and begin to talk as Ava circulates. She interrupts one group to say, "If you are not sure what he did, ask why, and if you are not listening to your partners the way that you should've been then you need to."

After small groups, the students transition back to the carpet. Ava asks the students about the experience of talking with their group members; "What are some things that could help us to be successful? Everyone in our room matters. If we don't show someone that we are listening then we are not showing someone that they matter." Then, as a whole group, the students engage in a mathematics discussion about the relationship between the tasks. Later, Ava tells the students that they will be doing two, 20-minute rotations; one with her and one of their choice (either working in their mathematics journals, engaging in a skill practice or working on ST Math). During her rotation, she helps students examine Task 2, spending the first part of their time unpacking the language of the task and posing questions. They then explore students' differing representations of the task and work on similar tasks.

After the rotations, Ava asks students to share how they might solve $10/2 = \underline{\quad}$ and $2 \times \underline{\quad} = 10$. When one student tells her the answer to the second problem is five, Ava asks, "How do you know?" The student says he "skip counted by 2... 2, 4, 6, 8, 10," putting up a finger for each number. Another student adds that, for the first problem, "It's just

going a different direction.” Ava follows up, asking, “How [do] you know?” The student grabs tiles and tells the class, “If I start with two tiles and keep adding two [tiles] until I get to 10, I have 5 groups of 2. If I start with 10 tiles and take 2 tiles away at a time, I have to do that 5 times. So, multiplication is the same as repeated addition and division as repeated subtraction.” A third student speaks up, asking the student, “So, when you are doing that multiplication problem, you are really doing a division problem at the same time?”

Sophia. The students are seated on the carpet facing a SmartBoard. They watch a catchy video that introduces different types of patterns with examples of color patterns (i.e. red, blue, red, blue...), number patterns (i.e. 6,3,3,6,3,3,6,3...), letter patterns (i.e. AABBAABBA...), and shape patterns (i.e. triangle, circle, square, triangle, circle, square...). Following each pattern type, Sophia pauses the video to discuss the examples and to pose questions. As the video comes to an end, Sophia starts to point out patterns on some of the students’ clothing. The learners get excited and start to share examples of patterns they see in the classroom, like the blue and white tiles on the floor. Sophia then breaks the class into two groups, one with her and one with the paraprofessional, Mrs. Traugutt.

Sophia’s group learns that they will be working on “spooky patterns” as it is close to Halloween. The first pattern is on the SmartBoard. A student stands up and Sophia has him put his finger on the pumpkin. She says, “pumpkin, cat, pumpkin, cat...” as he touches each picture. After he finishes, Sophia asks him, “What comes next?” The student immediately responds “Pumpkin!” When Sophia asks how he knows, the student responds, “Because it’s pumpkin, cat, pumpkin, cat, pumpkin” as he tags the pictures again from left to right. He adds that “a pumpkin comes after a cat.”

Following his explanation, Sophia introduces a cut and paste spooky patterns activity sheet. As students work on the activity on the carpet, Sophia reminds them to talk to each other, sings patterns from the video, and encourages them to sing too. The students help each other with the activity, sing, and make connections to patterns around them. One student notices that the act of using his scissors makes a pattern as the scissors open, close, open, close. Sophia moves around the carpet checking in with the learners, switching between English and Spanish. When Sophia hears students saying, “I can’t,” she jumps in and has them say, “I can’t yet, but I will.” After the timer goes off, the students switch groups.

Later, Sophia draws upon some of the learners’ cultural practices by having the students make beaded necklaces with color patterns of at least three colors. When each student in the group has threaded five or more beads, she has them pause and share their patterns. Each learner in turn lays down their necklace and points at the beads as they

share their color pattern. Sophia often jumps in to ask the learners questions like “What color will come next?” and, “How do you know?”

5.1.2 The overlap

A key overlap between CRP and high-quality instruction was in how the teachers’ high and low scores in some domains reflected where they had incorporated, or failed to incorporate, aspects of CRP. One place where this was clear was in lesson coherence. On a scale of one to seven, the teachers scored high in this area with a 5.55 on average on the dimension of structure and a 5.62 on the dimension of accuracy. In the lessons above, the teachers presented lessons that were designed to build students’ deep understanding of the content and effectively addressed students’ needs. The teachers’ high scores on these dimensions indicate that they had strong content knowledge of mathematics and were able to flexibly use their knowledge to support student understanding by correcting and responding to students’ misconceptions. The teachers’ high scores on coherence also captured strengths in their CRP-aligned support for student learning, specifically through the feedback they provided students.

Similarly, the teachers’ strong scores on their use of representations also captured key components of their CRP. The teachers scored a 5.16 average in this dimension, with individual average scores ranging from a 5.09 to a 5.20. In the examples, the teachers leveraged representations from beaded patterns to mathematics tiles to build relevance for students and to create multiple entry points. The teachers’ high scores also captured both their support for student learning through representations and their cultural competence as they drew on students’ funds of knowledge and honored their perspectives. The teachers also scored high in discourse community (average of 5.16, range of 4.90 to 5.50). During Sophia’s discussions on the rug about patterns, Ava’s on frogs, and Chris’s on baking, the students talked about mathematics with the teacher and with each other and were free to express their ideas. The teachers’ scores also captured how they supported student learning and demonstrated cultural competence as they listened to students, learned from them, and honored their ways of knowing.

Lower scores on the measure of high-quality instruction also often indicated less evidence of CRP. Many of the mathematics tasks, like Chris’s first baking task, Ava’s frog tasks, and Sophia’s spooky pattern were less cognitively demanding and required fewer problem solving skills. The tasks were also generic, without clear connections to students’ unique experiences. The teachers, however, then enacted each task in a way that increased their quality and that demonstrated cultural competence. The teachers extended

the tasks, adding more cognitively demanding elements and creating more problem solving opportunities, resulting in their final moderate average scores of 4.40 in cognitive demand and 4.47 in problem solving. The extensions also built the cultural competence of the tasks, as when Chris connected cookies to students' home lives or Sophie connected patterns to the clothing students wore.

5.2 Areas missed in CRP by a measure of high-quality instruction

While high-quality instruction was the foundation for the teachers' culturally relevant instruction, the measure of high-quality instruction also missed key elements of CRP from high expectations to critical consciousness.

5.2.1 Instructional examples

Chris. The lesson begins with Chris talking about musical artists. He asks the students to hypothesize which artist, Taylor Swift, Lil Nas X, or Panic at the Disco, is most popular. Students look up information about the artists. As they analyze the data, students begin to explore how popularity impacts the money the artists earn and, briefly, to how their earnings relate to wages from other jobs and income inequalities. Eventually, Chris moves into the goals for the lesson of ordering and comparing numerical values using greater than, less than, equal to, or not equal to. Chris sits on the carpet with students and asks them, "When might it be important to compare numbers?" The students give examples about money, races, and numbers of pages read in class. One student says, "The weight of something," and Chris asks for an example. The student proceeds to say, "at a grocery store," and Chris asks him to share more. The student goes into an explanation of how you put vegetables on a scale to weigh them, and how the weight determines the cost, like how a larger tomato will cost more than one that weighs less. The students then solve a series of word problems and play a number comparison dice game, as Chris provides bilingual support.

Ava. The day after a test, and following individual meetings with students, Ava places a graph on the carpet. She asks the students, "What do you notice or wonder?" The students have two minutes to think on their own and then find a partner to share what they noticed and wondered. Ava walks around the room listening to students. One student says, "There's 2 and 4 and one is 9, so I wonder how many there are in all?" Another wonders what the color means while a third comments on the graph's scale. Ava brings the class back together and previews upcoming lessons on graphs; "When you analyze things, you notice and wonder. How many of you wondered where I got this from?"

Ava adds a graph title of "Our First Test Results." She then asks the students, "Pretend that you are the teacher. What decision would you make? Would you move on based on the test results?" One student says he would not move on. Ava asks, "Why wouldn't you move on based on this?" The student responds that "most of my students are in the red." Ava agrees and says, "Today, we are going to go back and look." She asks students to look at their tests and determine if the problems they missed were because of carelessness, computation mistakes, or conceptual challenges. She goes over each term with the students, putting up examples of questions that students might have missed due to computational mistakes. She asks students, "What did the person do on number one that shows that it's wrong?" She then puts up examples of conceptual mistakes and mistakes from carelessness and has the students analyze them. Once the students have worked through and discussed the examples, they analyze their own tests as she provides support.

Sophia. The students are all seated on the carpet. Class begins with a read aloud of *The Apple Seed*. As the story comes to an end, Sophia connects the story to their upcoming field trip to the pumpkin patch and apple orchard. She points to the date on the calendar and counts how many days they have left until the trip. Then students transition to the back two half-moon tables, where Mrs. Traugutt has been cutting apples. The students are each given a slice of a green, Granny Smith apple to try. Then, they are each given a slice of a red Macintosh apple followed by a yellow Golden Delicious. Sophia and Mrs. Traugutt pass out paper cut-outs of apples to all of the students and tell the students to color their apple the color of their favorite apple of the three. Sophia pulls out a laminated pictograph where each colored apple is represented on the bottom axis. Then Sophia and Mrs. Traugutt collect the students' colored apples and place them on the graph. The students chorally count the apples on the graph, discuss voting, and analyze their class results to see which was the class's favorite and least favorite apples. As the conversation comes to an end, Sophia checks in with several students in Spanish.

5.2.2 Aspects of CRP missed by the high-quality instruction Tool

High-quality instruction was the foundation of the teachers' culturally relevant instruction, but their CRP also included elements missed by the measure of high-quality assessment. First, the measure missed high expectations, which were central to the teachers' CRP. In 5.1.1, we shared how Chris told students that they should expect to feel challenged by the task. We also shared how Sophia jumped in when she heard students saying "I can't," reminding them to say, "I can't yet, but I will." In 5.2.1 we shared how Ava made

students go back to their old tests to analyze them and learn from the mistakes they made. Each of these showed the teachers holding students to high expectations, but none was captured by the M-Scan. Second, the measure missed linguistic support. In both 5.1.1 and 5.2.1 we shared examples of the teachers switching between Spanish and English. The teachers also encouraged the students to talk to each other in Spanish, honoring students' home languages. The teachers' use of and respect for Spanish was a key way that they demonstrated cultural competence while teaching mathematics, and it was not captured by the M-Scan. Third, the measure missed nuances in cultural competence. Sophia connected apples to the student's field trip. Her lesson was relevant to the students' lives at school. Chris connected his lesson to the musical artists that students listened to at home. Those nuances in relevance were missed in the M-Scan which operationalizes relevance, in the connections dimension, broadly. Similarly, the measure missed instructional practices that honored students' home lives from clapping games to songs in the classroom. While strengths in teachers' CRP aligned instruction were broadly captured by the measure of high-quality instruction, the nuances that were missed were often essential ways the teachers created a welcoming, responsive classroom for students.

Fourth, the M-Scan missed critical consciousness entirely, with no codes to capture it when observed and no codes to note when it was missing. In Ava's classroom, students were consistently presented as leaders and empowered, as seen in 5.1.1 in her reprimands when students were not listening to each other and in 5.2.1 when she asked the students what they would do if they were the teacher. In Sophia's classroom, students were able to vote and analyze their own data, beginning to build power as four-year-olds. These instructional moves are building blocks to critical consciousness and student empowerment that were missed by the M-Scan. So, too, was Chris's discussion of musical artists that began to touch on inequalities in society. That discussion was the closest to full critical consciousness seen in the teachers' instruction and the high-quality instruction tool had no component that captured that important moment, nor any tool that could be used to give the other teachers' feedback on where consciousness was lacking or how it could be brought in.

6 Discussion

In 1995, Ladson-Billings titled her article on CRP, "But that's just good teaching!" In the article, she describes CRP as pedagogical excellence plus a little more; a little bit more that begins to support the excellence of students who have historically been marginalized in mathematics

instruction (Boaler & Staples, 2008; Leonard et al., 2010; Wilson et al., 2019). Across the globe, from New Zealand to South Africa, from Germany to the U.S., there are students who are taught to become doers of mathematics and students who are marginalized in mathematics instruction (Civitillo et al., 2019; Ladson-Billings, 2021; Lim et al., 2019; Robertson & Graven, 2020; Vithal et al., 2024). How and why students are marginalized might vary from country to country (Civitillo et al., 2019; Lim et al., 2019), but what does not change is our need to support the excellence of all students, including those who have been left out in the past. In each of these countries, there are mathematics educators seeking to disrupt the status quo (Civitillo et al., 2019; Ladson-Billings, 2021; Lim et al., 2019; Robertson & Graven, 2020; Vithal et al., 2024). To get there, we need to understand where CRP is just good mathematics teaching and where it is something more, something missed in traditional models of pedagogical excellence. In this study, we set out to understand which aspects of CRP were captured or missed in a typical American observation measure of high-quality instruction. We looked at teachers' actual instruction rather than abstract frameworks because doing so let us see beyond theory and to learn how expert teachers enact both high-quality and culturally relevant mathematics instruction. Our goal in this work is not to create a universal tool for equitable mathematics instruction, but instead to demonstrate one way to begin making a culturally specific measure of high-quality instruction more equitable.

Mathematics has the potential to be a tool students draw on to create the world they want to see (Aguirre & Zavala, 2013; Vithal et al., 2024). High-quality mathematics instruction helps students learn to use the tools of mathematics to become doers of mathematics and instruments of change (Aguirre & Zavala, 2013; Gutiérrez, 2018). What high-quality mathematics looks like in Singapore might be different than in the U.S., with more silence or more discussion, but in both the students will be building conceptual understanding of mathematics and learning to become doers (Leinwand et al., 2014; Praetorius & Charalambous, 2018; Xu & Clarke, 2019). The challenge for mathematics educators and researchers is discovering how to enact that high-quality instruction for all students in a society (Robertson & Graven, 2020; Vithal et al., 2024; Wilson et al., 2019). What we found is that high-quality mathematics instruction was foundational to teachers' enactment of equity-focused CRP in mathematics. This was in alignment with our view that high-quality mathematics instruction is instruction that builds students' conceptual understanding of mathematics content while also creating equitable opportunities for learners.

When teachers were enacting high-quality mathematics instruction, they were also engaging in practices that were

supportive of student learning, a key pillar of CRP (Ladson-Billings, 1995, 2021). They were also often engaging in practices that demonstrated cultural competence, using students' culture as a bridge to the curriculum or drawing on students' funds of knowledge (Ladson-Billings, 1995, 2021). When teachers were engaged in lower-quality instruction, they were also often missing opportunities for culturally relevant instruction. For example, when classroom tasks were less ambitious and rigorous, they were also often less customized for the students. High-quality instruction was at the heart of CRP in these teachers' classrooms, a finding that echoes what Ladson-Billings (1995, 2021) found in her studies of mathematics teachers and what other researchers have since found in other strong, culturally relevant mathematics classrooms (Bonner, 2014; Wilson, 2022).

At the same time, in line with Ladson-Billings's (1995) argument that CRP is something more than just what is traditionally captured in models of "good teaching," we found that CRP was more than just high-quality instruction. The teachers' enactment of CRP also included elements that were missing from our framework for high-quality instruction. Some of the omissions were small. For example, the framework we used for high-quality instruction missed high expectations, a key component of student learning in CRP and missed linguistic supports, a key component of cultural competence (Ladson-Billings, 1995, 2021). The framework also missed some nuances, like the differences between instruction that is relevant to students' lives at school and instruction that is relevant to students' lives outside of school.

Most importantly, the model of high-quality instruction missed critical consciousness. Critical consciousness, the development of "critical perspectives that challenge inequities that schools (and other institutions) perpetuate" (Ladson-Billings, 2021, p.18) is integral to CRP. Critical consciousness is also a key element of mathematics for social justice (Aguirre & Zavala, 2013; Gutiérrez, 2018). Rather than being a lifeless subject, mathematics has the potential to be a tool that students can use to become change makers and to make sense of the world (Aguirre & Zavala, 2013; Vithal et al., 2024). All students need opportunities to grapple with concepts in math, to build conceptual understanding – and to use those foundations to become doers of mathematics who draw on the tools of mathematics to create better futures. Getting there requires critically analyzing how we think about high-quality mathematics instruction and finding new ways to align that culturally-specific framework with frameworks focused on equity.

6.1 Implications

It sounds complex to change entrenched frameworks of high-quality mathematics instruction to be more culturally relevant. However, we found that many of the changes needed with just one, research-validated observation tool for high-quality instruction (Litke et al., 2021; Walkowiak et al., 2018) were small. The first minor change the M-Scan tool would need to be culturally responsive is in discourse communities. Currently, the dimension does not capture home language and linguistic support. Thus, we suggest that the dimension for mathematics discourse community include separate indicators within *teacher role* for how the teacher encourages and/or engages in use of students' home language and/or linguistic supports when applicable as well as an additional indicator under *sense of community through student talk* for learners' comfort with using home language when applicable to student populations. The second minor change is to the connections and applications dimension. Right now, the indicators for *connections* ask whether the content is broadly "relevant to students' lives" and whether "meaningful connections" are made between the world or between school domains. We propose adding to these indicators language that discerns between relevance to students' lived experiences at school and outside of school to increase the depth of the dimension. We also suggest indicators to distinguish between relevance of the mathematics *task selection* versus the *teacher's enactment* in making the mathematics relevant.

The third change we propose is larger. Applications is currently a small indicator within connections and applications of "students are... asked to apply the math they learn to the world around them" (Berry et al., 2017, p.16). Applications are too important in CRP to be part of another indicator and currently the language falls short of critical consciousness. We propose making applications its own dimension separate from connections. The new applications dimension would have the current indicator as well as an indicator on the focus of the task, and the extent to which it is building social justice with distinctions made between the teacher's enactment of the task and students' engagement. The third indicator would be on the development of critical consciousness (continuum of learning something new, developing critical knowledge, using math as a tool, and/or using math as a tool to enact change) with additional attention on whose questions are being addressed (teacher directed versus student directed).

The fourth change we propose is also larger. The domain of discourse is "central to what students learn about mathematics as a domain of human inquiry" (Berry et al., 2017, p.4). The two current dimensions of it are discourse community and explanation and justification. We propose adding a

third dimension of learners' identities. M-Scan is currently missing indicators for how teachers use conversations in their classrooms to build students' identities as "doers" of mathematics, for how teachers empower students, and for how teachers affirm and acknowledge students' identities in the classroom. The indicators would focus on teachers' use of language to position students as empowered mathematicians in their classrooms.

Our suggested changes to the M-Scan would not create a perfect, universal tool. We acknowledge that teachers face competing goals in many lessons that impact both their enactment of high-quality instruction and CRP. Our suggestions, though, show that it is possible to bring high-quality instruction and CRP into closer alignment. M-Scan is just one tool for operationalizing high-quality mathematics instruction and, if we were to truly integrate high-quality instruction and CRP, we would need to make similar changes to each tool and to draw on the work of others who have developed CRP focused observation tools (i.e., Aguirre & Zavala, 2013; Wilson, 2022). While suggested changes to other frameworks fall outside of the scope of this manuscript, we hope that mathematics educators internationally, who are developers and/or experts with other tools, might be compelled to examine the extent to which their preferred measures for capturing high-quality mathematics instruction are also capturing equitable practices like those demonstrated in CRP within their unique cultural context.

6.2 Limitations

Despite our care in building trustworthiness, this study has limitations. First, we did not interview or speak to students or their families and we need more research that brings in their voices. Second, the tool that we used to operationalize high-quality mathematics instruction is culturally specific and we need more work across nations using different measures and comparisons of high-quality mathematics instruction, especially in the global south (Vithal et al., 2024). Additionally, our qualitative observations of CRP reflected our own interpretation of the framework. Third, our study focused on a small and biased sample of teachers invested in CRP and we need more research with broader populations of teachers. We need more research across international contexts using different measures and comparisons of high-quality mathematics instruction so that we can develop stronger, more holistic tools for supporting marginalized learners in mathematics.

7 Conclusion

All students deserve empowering, enabling mathematics instruction that prepares them to create change in the world and to succeed academically. For marginalized students in the U.S. and elsewhere in the world who face inequities in society and, too often, in mathematics classrooms, high-quality mathematics instruction alone might not be enough to disrupt patterns of injustice. Students need mathematics instruction that is high-quality *and* culturally relevant. In this study, we found that a typical U.S. framework of high-quality instruction captured overall strengths and missed opportunities in teachers' culturally relevant mathematics instruction. The framework, however, missed key components of high expectations, linguistic supports, and critical consciousness and overlooked nuances that could alter students' mathematics experiences. To create high-quality mathematics instruction for all students, we need to begin weaving those elements into our frameworks and expanding what it means for mathematics instruction to be high-quality.

Declarations

Conflict of interest There are no conflicts of interest.

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