

SOME DYNAMIC MECHANICAL PROPERTIES OF APPLE FRUITS
AND THEIR USE IN THE SOLUTION OF AN IMPACTING CONTACT
PROBLEM OF SPHERICAL FRUIT

by

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DEFINITION OF SYMBOLS

t, τ	time
ω	angular frequency
$J(t)$	creep compliance
$\epsilon(t)$	time dependent strain
σ_0	step function stress
$Y(t)$	creep modulus
$Y(i\omega), Y^*$	complex modulus in axial compression
Y_1	real part of the complex modulus $Y(i\omega)$
Y_2	imaginary part of the complex modulus $Y(i\omega)$
$G(t)$	relaxation modulus
$\sigma(t)$	time dependent stress
ϵ_0	step function strain
$e_{ij}(t)$	deviatoric strain tensor
$S_{ij}(t)$	deviatoric stress tensor
$\epsilon_{ii}(t)$	strain invariants
$\sigma_{ii}(t)$	stress invariants
$I(t)$	dilatational creep function
$K(t)$	dilatational relaxation function
$G(i\omega), G^*$	complex modulus in shear
G_1	real part of the complex modulus $G(i\omega)$
G_2	imaginary part of the complex modulus $G(i\omega)$
$i = (-1)^{\frac{1}{2}}$	imaginary coordinate in the complex plane
$J(i\omega), J^*$	complex compliance in axial compression
J_1	real part of the complex compliance $J(i\omega)$

J_2	imaginary part of the complex compliance $J(i\omega)$
θ	phase angle by which strain lags the stress, or a cylindrical coordinate
Y_{1a}	apparent real part of the complex modulus
h	length of the cylindrical specimen
ρ	density
L	distance between two points, special variable
ν, ν^*	Poisson's ratio
D^*	reciprocal of the complex shear modulus
B^*	reciprocal of the complex bulk modulus
M', N'	special dimensions of an ellipse
$\sigma_1, \sigma_2, \sigma_3, \sigma_4$	levels of stress
$E_1, E_3, \eta_2, \eta_3, \eta, E$	mechanical model element constants
M, N	equation constants, or general functions
G_e	equilibrium compression modulus
a, b	equation constants, or semiaxes of an ellipsoid
τ_1	retardation time
G_o	elastic modulus
z_1, z_2, x, y	rectangular coordinates
A, B	constants
R_1, R_1', R_2, R_2'	principal radii of curvature
ϕ	the angle between the normal sections, or a cylindrical coordinate
w_1, w_2	surface deflections
$\alpha, \alpha(t)$	approach of the centers of the two bodies
$p, p(x, y, t), p(r, t)$	contact surface pressure

$k, k(t), k_1, k_2$	functions characterizing material properties
λ, μ, ψ	coefficients
$g(x, y, t), g(r, t)$	a function for the contact pressure solution
$R_p(t)$	kernel for pressure
$R_{w1}(t), R_{w2}(t)$	kernels for displacements
$A(t), A \tau$	areas
$f(x, y, t)$	a function for the displacement solution
r, s, c	radial coordinates
R	radius of a spherical body
P	point load
$x_1, x_2, x_3, x'_1, x'_2, x'_3$	cylindrical coordinates
σ_{ij}	Cartesian stress tensor
a_{ij}	direction cosines
γ	angle relating primed and unprimed coordinates
$a(t)$	radius of circular contact surface
$\ddot{\alpha}, \dot{\alpha}$	approach acceleration and velocity
$\dot{V}_1, \dot{V}_2$	contact velocities
M_1, M_2	masses
P'	force between apples
W	weight
C_1, C_2, C_3	constants
Y, D	special variables
σ_{ss}, σ_{cc}	radial stresses
$\sigma_{\phi\phi}, \sigma_{\theta\theta}$	stresses in angular coordinate directions
σ_{zz}	axial stress

σ_{sz}, σ_{cz}	shear stresses
T	impact duration
e_{zz}	strain in the z direction
w	displacement

Certain other symbols and meanings are used briefly as defined in the text and appendices.

1. INTRODUCTION

1.1 General

Labor shortages and increased labor costs in recent years have focused attention on the mechanization of fruit and vegetable harvest. Removal of tree fruits has been a particularly difficult problem and only recently has significant progress been made. One of the most promising methods for harvesting tree fruits consists of shaking the tree with a mechanical shaker and catching the fruit which falls from the tree [1].* As the fruit falls, it is likely to strike limbs or other fruit and must impact the catching device provided. If the fruit is processed in bulk and dumped and handled mechanically, other impacts will occur. Any of the fruit collisions will cause bruising if the failure level of the fruit is reached.

Bruising in apples is characterized by a discoloration of the edible portion of the apple under the skin usually to a depth of about 30 per cent of the diameter of the bruise [2]. Pears, which appear to have flesh quite similar to apples, exhibit an internal bruise called brown spot which is not visible from the surface and extends to greater depths. Mattus, et al., [3] state that, in their testing, compression forces up to 100 pounds placed on individual pears for short periods did not produce brown spot while free fall of six inches onto a hard surface did. Thus, it is suggested that impact stresses in fruits may reach very high levels even for short drop heights, and since bruises show up differently in apples and pears, different failure criteria may be involved. The

*Numbers in brackets refer to appended references.

failure criteria for bruising will not be considered further in this thesis, but it is evident that considerable effort could be exerted on the subject and this work is a first step in that direction.

The present work consisted of determining two independent dynamic mechanical properties for the flesh of five groups of apples and solution of the apple impact problem for stresses at the surface and in the interior of the apple body.

Much use was made of the digital computer in this research with part of this effort being applied to the statistical analysis of experimental data. The major part of the computer use, however, was the solution of a nonlinear differential indentation equation by the Runge-Kutta-Gill method and the evaluation of definite integrals by Simpson's rule and a rather unique special program for integrating over the contact surface at arbitrary times during indentation to compute interior stresses. Combining several of the developed programs it was possible to compute surface indentations, surface pressures, and internal stresses at arbitrary times from initial contact to maximum indentation with the only input being the apple radius at the contact surface, the weight, the Maxwell stress-strain relation for both impacting bodies, Poisson's ratio, and the drop height.

1.2 Material Response

The mechanism of response of structural materials may be studied on any of three basic levels: the molecular, the structural, and the phenomenological. Fruit flesh is made up of cells which are composed of molecules so all three levels seem to hold for apple flesh as well. The

structural level, which would be based upon the cellular structure of the fruit, seems particularly enticing since the fruit cells are biological structures which possibly would be amenable to an engineering structural analysis. In order to prevent becoming involved in detail early in the study, however, it was decided to use the phenomenological level where the material is assumed to be microscopically homogeneous and isotropic such that the behavior of any part of it, in any direction, is the same as that of the whole. The validity of these assumptions is discussed in Section 1.4.

The classical theory of elasticity deals with mechanical properties of perfectly elastic solids which obey Hooke's law which states that stress is directly proportional to strain and independent of other phenomena. When this relation holds the dynamic and static stress-strain ratios are identical, if it is assumed that isothermal and isentropic moduli are the same. On the other hand, in the theory of hydrodynamics which deals with perfectly viscous liquids stress is proportional to the rate of strain in accordance with Newton's law. Both types of behavior are idealizations and in actual materials both time and stress anomalies may exist. In fact, time anomalies could be expected in apple flesh which contains a large percentage of water in and around the cellular structures. For this reason, in this study the apple flesh was treated as a linear viscoelastic material possessing both fluid and elastic properties. In such materials the physical properties are independent of the level of stress but time anomalies are evident so that stress depends on strain and the higher time derivatives of strain.

The experimental problem in any study of viscoelastic behavior is to determine the relations among stress, strain, and time for a particular type of deformation and a particular stress history. Ideally adequate information could be obtained from a creep test where the compression, tension, or shear stress is applied instantly and held constant while strain is measured as a function of time. The creep compliance can be written as

$$J(t) = \frac{\epsilon(t)}{\sigma_0} \quad (1.1)$$

where $\epsilon(t)$ is the time dependent strain and σ_0 the constant stress. The reciprocal of the creep compliance is the creep modulus written as

$$Y(t) = \frac{\sigma_0}{\epsilon(t)} \quad (1.2)$$

Experimentally, however, it is impossible to apply an instantaneous stress; so if the above relations are obtained by the direct method outlined, they will probably not represent a material's response to rapidly applied stress such as would be encountered during impact. The same experimental impossibility exists for stress relaxation where the sample is brought instantly to a given displacement and the stress required to maintain the constant displacement measured as a function of time.

Since the present work was concerned primarily with the impact problem, both creep and relaxation methods were rejected experimentally except for the determination of Poisson's ratio for which a dynamic method could not be developed.

To provide stress-strain data corresponding to very short loading times, the stress is varied periodically. For linear viscoelastic

behavior, strain will also vary periodically but will be out of phase with the stress. The ratio of stress to strain is then normally written as a complex number called the complex modulus with the form

$$Y(i\omega) = Y_1 + i Y_2 \quad (1.3)$$

where Y_1 is the real part of the complex modulus and Y_2 the imaginary part. This is the form used in this study. It was convenient, however, in the impact problem to transform dynamic results into the form of a relaxation modulus given as

$$G(t) = \frac{\sigma(t)}{\epsilon_0} . \quad (1.4)$$

This will be discussed in Section 4.2.

1.3 The Contact Problem for Impact

The stress analysis problem in continuum theory consists of determining the stress distribution throughout a body when surface tractions, body forces, and prescribed surface motions are given. When the surface displacement is prescribed over a region S_1 of the surface and tractions over the remaining region S_2 , the problem is called a mixed boundary value problem and more involved methods of solution are generally required than when either displacements or tractions are prescribed over the entire surface. For the contact problem a contact region S_1 must satisfy displacement conditions that the two surfaces are in contact and the remainder of the surface S_2 is free of surface traction.

In the classical solution of the elastic contact problem given by Hertz [4], wave propagation (dynamic effects) are neglected. This has

been the general practice for spheres impacting since it has been shown that the time during which the spheres remain in contact is very long in comparison with the lowest mode of vibration of the spheres so that vibrations can be neglected [5]. This should also be true for viscoelastic materials that are nearly elastic. Thus the impact problem reduces to a quasistatic viscoelastic problem with the contact force given by Newton's second law.

The usual approach to solving viscoelastic stress analysis problems is to take the Laplace transform of the field equations and boundary conditions with respect to time thus reducing the problem to a corresponding elastic one [6]. This is not possible for the contact problem, however, since the regions S_1 and S_2 over which the boundary conditions are specified are changing in size so that the integral of the boundary conditions cannot be carried out. To overcome this difficulty, Lee and Radok [7] assume the applicability of a relation developed for the elastic problem and arrive at a solution for restricted viscoelastic contact problems. The validity of this solution was then verified. Yang [8] has generalized this work somewhat by utilizing integral operator relations to represent the stress-strain relation rather than the differential operator relations associated with spring-dashpot models. It is the approach of Yang that the writer takes in developing the contact problem portion of this thesis. Yang's work is extended to include the impact of spherical bodies and the determination of interior stresses for arbitrary loads.

1.4 Specific Objectives and Basic Assumptions

Specific objectives of this research were:

- (1) To experimentally determine dynamic compression modulus values and shear modulus or Poisson's ratio values for selected varieties of apples at harvest and at a selected time during storage.
- (2) To develop relations for and compute surface indentations, surface pressures, and internal stresses for impact of two apples and for impact of an apple with a rigid surface.

Assumptions necessary to simplify the experimental testing and the solution of the contact problem were as follows:

- (1) The body was assumed homogeneous. It is well known that this is not true throughout the entire cross section of fruits such as apples and pears since the core and seed area are obviously anisotropic. However, when one considers that common bruises and extreme stresses do not extend to the core area, the assumption was reasonable. This assumption also neglected the effect of the skin on impact deformations and stresses. The skin is very thin (approximately 0.02 inch) and by itself has very little resistance to bending or compression. Comparison of tensile modulus values for apple skin subjected to slow rates of loading reported by Clevenger and Hamann [9] with compression modulus values for apple flesh tested in creep reported by Chappell [10] revealed that the two moduli are of the same order of magnitude. Thus very little error should be introduced by assuming the fruit homogeneous to the outer surface.

- (2) The body was assumed isotropic. The validity of this assumption was subject to question since fruit flesh is composed of living cells which seem to be roughly ellipsoidal in shape with their long axis parallel to the outer surface of the fruit. It has been estimated that 25 per cent of the volume between the cells is composed of intercellular air [11].

A very limited number of preliminary tests similar to those in the primary study were conducted on cylindrical specimens taken from large apples with the axis of the cylinder parallel to the skin. Results were compared with those obtained from samples taken in such a manner that their axis was perpendicular to the skin. Although an insufficient number of specimens of first type were tested for a statistical analysis, it was shown that if the material was anisotropic the differences in material properties when loaded in the two directions were not large. The assumption of isotropy was therefore retained.

The assumption of isotropy was desirable for both the experimental and theoretical part of the work. Experimentally, it was quite difficult to obtain a sufficiently large diameter cylindrical specimen with its axis parallel to the skin and theoretically the stress problem was simplified by the assumption.

- (3) It was assumed that the stress-strain relations are linearly viscoelastic. This included purely elastic action as a special

case and allowed for time dependency in the general relation to be determined experimentally. Thus experimentally it was only necessary to keep compressive stress levels low with respect to failure and measure dynamic stress and strain changes, the exact stress or strain not being known.

- (4) Poisson's ratio was considered constant for the impact problem and specific values were obtained from the work of Chappell [10] which was done under the direction of the writer.
- (5) The shape of the apple was assumed to be spherical near the region of contact resulting in a circular contact area. Apples are not true spheres especially near the core ends, but it is likely that most contact areas would be approximately circular in shape.
- (6) The indentation and the area of contact were considered to be monotonically increasing with respect to time. With this limitation solutions are valid through maximum indentation but fail to represent rebound.
- (7) Small deformation theory was assumed to be valid in the solution of the contact problem.
- (8) Friction between the impacting bodies was neglected.

Several other assumptions were made during the investigation and will be stated where pertinent.

2. REVIEW OF LITERATURE

2.1 Viscoelastic Representation

A number of references have already been cited in introducing the problem. There are, however, a number of others that were of value in the course of the work. These first established a firm base of knowledge and then helped in solving the many major and minor difficulties encountered.

Viscoelasticity comprises an irreversible energy transformation where the relation between stress and strain is governed by time effects. Modern materials that are viscoelastic in nature include asphalt, plastics, fiber, and solid rocket propellants. Williams [12] suggests that conceptually the easiest way to incorporate rate effects into the mathematical model representation of a stress-strain relation is to provide dashpot elements in addition to the elastic spring elements. This theory is well known and given in detail in a number of sources [13, 14, 15]. In general, the stress-strain relation for real materials can only be represented by models with a large number of elements and the resulting high order linear differential equation, when introduced into a stress analysis problem, causes the problem to become untractable. Therefore, the use of mechanical model representation in the solution of practical problems is usually limited to simple models even though actual materials apart from elastic solids and viscous liquids rarely have a stress-strain law which in differential form contains only a few terms. Simple models, however, are often adequate to represent material behavior in a relatively narrow time or frequency range. Bland [13],

pp. 113-124, gives equations and a procedure for using dynamic data in the form of complex moduli to determine the best simple model representations. His procedure, though sound theoretically, requires well grouped data and when used for the present work on apples was successful for only the Maxwell model representation where a single spring and single dashpot are placed in series.

A time dependent stress-strain formulation that is often more convenient mathematically than the differential form is the hereditary integral representation which Ferry [14], pp. 15-16, shows is an integral statement of the principle of superposition. Numerous references reviewed [8, 13, 14, 16, 17, 18, 19] utilize this formulation which can be written as

$$e_{ij}(t) = \int_0^t J(t-\tau) \frac{\partial}{\partial \tau} s_{ij}(\tau) d\tau$$

$$\epsilon_{ii}(t) = \int_0^t I(t-\tau) \frac{\partial}{\partial \tau} \sigma_{ii}(\tau) d\tau$$

or

(2.1)

$$s_{ij}(t) = \int_0^t G(t-\tau) \frac{\partial}{\partial \tau} e_{ij}(\tau) d\tau$$

$$\sigma_{ii}(t) = \int_0^t K(t-\tau) \frac{\partial}{\partial \tau} \epsilon_{ii}(\tau) d\tau.$$

$J(t)$ and $G(t)$ are respectively the shear creep and relaxation functions relating the deviator stress tensor, s_{ij} , and the deviatoric strain tensor, e_{ij} . $I(t)$ and $K(t)$ are respectively the dilatational creep and relaxation functions relating the stress and strain invariants, σ_{ii} and

ϵ_{ii} . Although the integral relations are written above for shear and dilation they are equally valid for representing the tensile or compressive stress-strain relationship.

An advantage of the hereditary integral form of operator is that viscoelastic theory can be divorced from spring-dashpot models and creep or relaxation functions developed from experimental data utilized directly. A technique for treating this type of measured function has been described by Lee and Rogers [17]. Since equations 2.1 are convolution type integrals, they lend themselves to the Laplace transform method of manipulation.

2.2 Experimental Determination of Viscoelastic Moduli

A very comprehensive review of experimental methods for determining viscoelastic moduli is given by Ferry [13], pp. 82-150. He classifies methods according to the type of material tested and gives methods for viscoelastic liquids, soft viscoelastic solids, and hard viscoelastic solids. For each of these categories, various transient and dynamic methods are briefly explained. The transient tests described are basically creep and relaxation tests for various shaped specimens mounted in a variety of ways and loaded in shear or tension. Dynamic tests described include applying a sinusoidal force to obtain the complex modulus and wave propagation methods. Sinusoidal forcing was the method used in this study.

Apple flesh fits best in the category of soft solids. Estimates of apple impact durations can be made based on estimated or known modulus

values using the elastic solution given by Timoshenko and Goodier [5], pp. 383-384. Impact durations of a few thousandths of a second were thus estimated. If impact and rebound are considered to be one half of a sine wave cycle, dynamic testing at frequencies up to a few hundred cycles per second are required to adequately represent apple behavior during such impacts. Sinusoidal dynamic testing of rubbers at such frequencies is reported as early as 1941 by Gehman [20] and since has been used by a number of investigators [15, 21, 22]. Sinusoidal methods have been used to determine complex bulk modulus values as well as complex moduli in tension, compression, and shear [23, 24]. The bulk methods did not seem usable for apple flesh, however, since they require immersion of a small specimen in a small quantity of liquid. Apple flesh, being permeable to some fluids, is likely to allow the compressing fluid to occupy some of its intercellular air space.

The moduli determined from sinusoidal testing are in complex form as given in equation 1.3. In order to introduce the moduli into the stress-strain relations given in equations 2.1 they must be transformed into the form of a relaxation function or a creep function. Ferry [14], pp. 41-81, gives a survey of exact and approximate interrelations between the various moduli. In the research to be described by the writer it was necessary to obtain the relaxation function from the complex moduli. An approximate method given in detail by Ninomiya and Ferry [25] was used for this purpose and will be described in Section 4.2.

2.3 Experimental Determination of the Mechanical Properties of Agricultural Fruits and Vegetables

The statement that soon fruits and vegetables which cannot be harvested and handled mechanically will not be grown typifies current thought among growers and processors of these products. This thinking has caused some researchers to consider the physical properties of fruits and vegetables and attempt to use these in developing equipment for the fruit and vegetable industry.

Apple firmness tests are rather standard in the evaluation of apples intended for fresh or processing use. Mattus.[26] describes these tests and gives some results of testing. Such tests help indicate the need for picking, storage life potential, need for removal from storage, and influence of cultural or other factors on apple flesh firmness. The common tester consists of a spring loaded cylindrical plunger 7/16 inch in diameter which is pressed against skinned sections of fruit. Spring displacement is scaled in pounds and a force reading is made when the plunger enters the fruit to a marked depth. Results are compared and the harder apples generally have a longer storage life. A modification of the standard tester which indents only a short distance into an unskinned apple is used for nondestructive testing. This tester is called a mechanical thumb tester and was used in this research as a method of selecting apples of approximately equal hardness. It, of course, gives no indication of time effects and the relation between the modulus determined with the plunger and the Young's modulus is needed before results could be compared with results from other load application methods.

Mohsenin and his associates [27, 28, 29, 30, 31, 32] have considered the time dependent deformation of apples and some other biological materials. Much of this research utilized cylindrical plungers acting on intact fruit. Plunger displacement rates varied from relaxation and creep to about 200 inches per second. Force-displacement curves drawn from results of these tests were time dependent and could be approximated by spring-dashpot models. Using the whole apple and applying stress by means of a plunger requires that the solution of the stress problem be known if Young's modulus is to be determined. This solution is known for an elastic material and given in most elasticity textbooks [5]. Mohsenin and Morrow [32] used the elastic solution of this stress problem and the time dependent results of plunger tests to determine a time dependent Young's modulus for selected apples. Poisson's ratio needed for this computation was assumed constant and found by solving the stress problem simultaneously with results of hydrostatic compression tests. Using plunger loadings, Timbers, et al., [33] assumed elastic response in the biological material and determined some numerical modulus values for the potato. Finney [34] using plungers, plates, and a liquid under pressure has also determined certain time dependent properties of the potato including the bulk modulus. He found the potato to be nearly incompressible which is in contrast with what one would expect for an apple which is approximately 25 per cent air space [11].

Other investigators [35, 36, 37] have similarly found the stress-strain relation for fruits and vegetables to be time dependent. Thus far, however, no one has used sinusoidal forcing on accurately formed specimens of fruit or vegetable flesh.

It is interesting to compare Poisson's ratio for apple skin from Clevenger's work [9] with Poisson's ratio for apple flesh from Chappell's work [10]. The ratio proved to be a constant for apple skin under slow rates of loading while the ratio decreased with time for apple flesh under constant stress. Initial values for flesh at harvest were from about 0.2 to slightly over 0.3 depending on the variety while values for the skin were somewhat higher. The values for apple flesh are compatible with the fact that intercellular air is present in apple flesh so it should be somewhat compressible. Chappell's results were used in accomplishing the second objective of this research and will be discussed in more detail in Section 3.8.

2.4 The Contact Problem for Viscoelastic Bodies

The problem of determining the stresses within the area of contact when one solid elastic body is in contact with a second solid elastic body has long been considered to have great importance. This is primarily because of application in calculating the stresses encountered in ball and roller bearings, bridge rollers, railroad wheels and tracks, and similar applications.

A shortcoming of the classical Hertz solution is that it gives only the surface stresses. Explicit solutions for the internal stresses and strains, based on the Hertz solution, have been developed by Huber [38] for the case where the contact area is circular and by Thomas and Hoersch [39] for the more general case where the area of contact is elliptical. Davies [40] used the Huber solution to determine dynamic

yield stresses for steel plate from data obtained by impacting the plate with a steel ball. Stresses shown in graphical form by Davies and tabular form by Thomas and Hoersch were compared with the viscoelastic solutions obtained in this study as an approximate check for correctness and to determine the effect of the viscous response of the material on internal stresses.

The contact problem for viscoelastic materials has been considered by Lee and Radok [7], Hunter [41], Graham [42], and Yang [8]. Partial solutions are obtained by all of these investigators with the recent work of Yang, which was cited in the introduction, being the most general. A common shortcoming of all of these solutions is that they are for surface displacements and stresses only and not interior stresses. In general, solutions fail for times after maximum indentation. However, Hunter was able to formulate rebound for the case where a monotonically increasing contact radius possesses a single maximum and thereby estimates duration of contact and the coefficient of restitution for the impact of a rigid spherical indenter on a viscoelastic solid whose stress-strain relation can be represented by a Maxwell model. Yang's work, which was an extension of that of Lee and Radok, proved to be of the most value in this investigation and will be cited frequently.

In the course of the investigation books on numerical analysis and mathematical methods for digital computers were consulted and will be cited in appropriate places.

3. DETERMINATION OF SOME DYNAMIC MODULI OF APPLE FRUITS

3.1 Theoretical Basis and Experimental Design

To accomplish the objective of determining the dynamic moduli of the fruit, a method somewhat similar to those cited [20, 21, 22] but with considerably simpler instrumentation was used. Basically the method consisted of applying a sinusoidally varying force to a specimen of fruit flesh and sensing the magnitude and phase angle of the deformation with a suitable transducer. The stress could then be decomposed vectorially into two components one of which was in phase with the strain and the other 90 degrees out of phase. These components could then be divided by the magnitude of the strain vector with the result that the modulus had been separated into an in-phase (real) component and an out-of-phase (imaginary) component. Thus, the modulus could be written in the form previously given in equation 1.3 as

$$Y(i\omega) = Y_1 + i Y_2. \quad (1.3)$$

The above notation denotes the compression modulus and the notation

$$G(i\omega) = G_1 + i G_2 \quad (3.1)$$

denotes the shear modulus. For ease in evaluating the real and imaginary complements from test data, the complex compression modulus was written in polar form as

$$Y(i\omega) = |Y(i\omega)| (\cos\theta + i \sin\theta) \quad (3.2)$$

where θ is the phase angle and $| \cdot |$ indicates the absolute value, then

$$Y_1 = | Y(i\omega) | \cos\theta \quad (3.3)$$

and

$$Y_2 = | Y(i\omega) | \sin\theta. \quad (3.4)$$

Equations 3.3 and 3.4 were suitable forms for computation of the real and imaginary parts of the modulus. The complex compliance written as

$$J(i\omega) = \frac{1}{Y(i\omega)} = J_1 - i J_2 \quad (3.5)$$

was also convenient for determining mechanical model representations. Equations similar to 3.2 through 3.5 can of course be written for the shear modulus as well.

The complex modulus in axial compression was chosen as one of the two independent moduli to be determined experimentally for two primary reasons. First, apples are normally bruised by compressive forces so it seemed desirable to know more about this type of loading than any other. Second, developing a suitable specimen shape and suitable grips would be complicating factors in a tension test while two flat end surfaces were all that were required for compression testing. However, the effect of end restraint on diametral strain during compression testing was an important consideration and was unknown for apple flesh. Therefore, several preliminary tests were conducted using different length/diameter ratios the results of which are discussed in Section 3.4.

Analyses of direct measurements of force and deformation from a sinusoidal test are usually made assuming that the specimens are thin

enough so that wave motion may be neglected; that is, that samples (in shear or compression) are deformed as a linear function of the distance from the fixed plates. An estimate of the accuracy of this assumption was determined for one-half inch thick specimens of apple flesh and is given in Appendix A. This was accomplished by developing an expression for the complex modulus taking into account the wave motion. It was assumed that this motion was that of a plane compression wave and by using the first two terms of the series solution the form of the complex modulus obtained is

$$Y(i\omega) = Y_{1a} + \frac{h^2 \omega^2 \rho}{3} + i Y_2$$

where Y_{1a} is the apparent real part of the complex modulus calculated without considering the plane wave, h is the length of the cylindrical specimen, ω is the angular frequency, and ρ is the density of the material. For one-half inch thick specimens of apple flesh the $h^2 \omega^2 \rho / 3$ correction for the real part only amounts to approximately one per cent for the highest frequency used in the testing and therefore was not used.

As a second independent modulus it was decided to attempt to determine the shear modulus by using a sinusoidal shearing force on a disk of apple flesh. The general concept of this test is shown in Figure 3.1. Once the shear modulus and compression modulus were known it would be possible to compute the bulk modulus and formulate the stress-strain relations as given in equations 2.1. From the theory of continuous media for a homogeneous isotropic material the bulk modulus (K^*), the compression modulus (Y^*), the shear modulus (G^*), and Poisson's ratio (ν^*) are

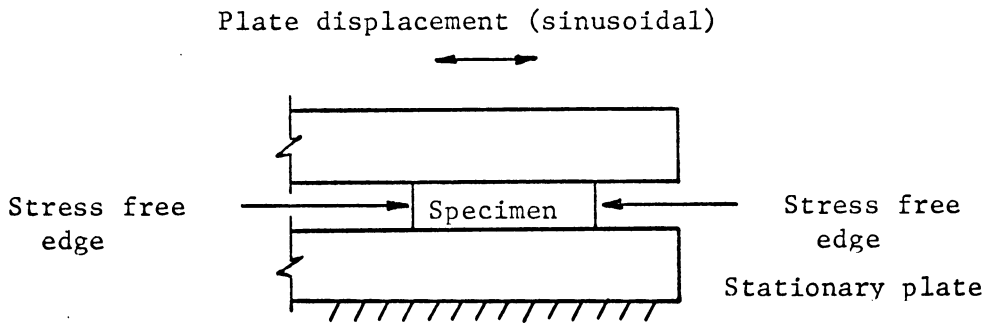


Figure 3.1 Concept of the Shear Test.

related by the equations

$$K^* = \frac{Y^*}{3[1-2\nu^*]} \quad (3.6)$$

and

$$G^* = \frac{Y}{2[1+\nu^*]} \quad (3.7)$$

where the asterisk indicates that the quantity is in general complex.

Substituting the relation for ν^* from equation 3.6 into equation 3.7 the result can be written as

$$\frac{1}{K^*} = \frac{9}{Y^*} - \frac{3}{G^*} \quad (3.8)$$

Then if $B^* = \frac{1}{K^*}$, $J^* = \frac{1}{Y^*}$, and $D^* = \frac{1}{G^*}$ equation 3.8 can be written as

$$B^* = 9J^* - 3D^* \quad (3.9)$$

Equations 3.8 or 3.9 can be used to determine the bulk modulus when the shear and compression moduli or compliances are known. Ferry [14], p. 137, states that the subtractions are unfavorable for achieving a precise result. This is certainly true for materials that are nearly incompressible so that $\nu \approx 0.5$ and $1/K$ are very small. In the case of apple flesh, however, the material is compressible and equations 3.8 and 3.9 appeared to be appropriate if both the compression and shear moduli could be accurately determined.

Although shear test results were very convenient theoretically, experimentally there were several difficulties. First, a pure shear state could only be approximated because of the stress free edges indicated in Figure 3.1. Read [43] considers this type of stress and shows that for an incompressible specimen in the form of a rectangular parallelepiped with a length to thickness ratio of three the actual shear modulus is about seven per cent larger than that computed neglecting the edge effect. Use of a very thin specimen would provide a close approximation of pure shear but could not be accomplished for apple flesh because of the method of holding the specimen. The specimen could not be bonded because of the presence of moisture and rapid material deterioration with time. Therefore, a knurled surface was used to provide a sufficiently high coefficient of friction between the specimen and the surface. The shearing strain near the knurled surface would be affected by the local conformation of the surface; therefore, the specimen thickness should be great enough that the penetrating effect of the knurled surface would be minor.

Second, it was not possible to determine if there actually was zero slip between the specimen and the contact surface. The instrumentation used did reveal when considerable slip occurred but the per cent slip had to be quite large before it became noticeable.

Because of the experimental limitations it was decided to use relatively thick specimens to minimize slip effects and to use shear results only as order of magnitude evaluation.

Since dynamic shear modulus results were not considered sufficiently precise, it was decided to use a creep test with very rapid force application to approximate the step change. The force was applied axially to a cylindrical specimen with continuous recording of force, axial deformation, and lateral deformation. Thus, a time dependent Poisson's ratio was determined in addition to a creep modulus. The experimental creep testing and analysis were performed by Mr. Thomas Chappell under the direction of the writer as his thesis for the Master of Science degree [10]. His equipment and procedure are described briefly in Section 3.2. Certain Poisson's ratio values resulting from his work are given in Section 3.8 and used in the analysis of the impact problem.

A prime consideration in all the experimental work was that biological materials such as apples are quite variable in their hardness, color, and strength since these characteristics are affected by growth and ripening conditions. Such characteristics even vary from one side of an apple to the other. For this reason, and since each specimen could be tested only once, it was necessary to use a large number of tests and analyze the results statistically.

Sinusoidal compression and shear tests were conducted on two varieties, Winesap and Golden Delicious, during February and March of 1966. These apples, furnished by the V.P.I. Horticulture Department, were taken from storage and held at room temperature for 5 to 10 hours before testing. In order to assure that the apples within each variety were typical and consistent in hardness, apples were selected on the basis of the mechanical thumb test described in Section 2.3. Seven frequencies, 50, 80, 110, 140, 170, 200, and 230 cycles per second were used except where minor changes in the test frequencies were made when resonance became a problem. Each test was replicated 10 times so that 140 apples were tested in shear and 140 apples in compression.

Three varieties, Winesap, Golden Delicious, and Red Delicious were tested in sinusoidal compression and creep during October and November 1966 when the apples were at harvest condition. Again 10 replications were made for each set of test conditions but frequencies for the sinusoidal compression tests were spaced 50 cycles per second apart to use frequencies from 50 to 350 cycles per second.

In summary, for the test groupings, a total of five groups of apples were tested in sinusoidal compression. Two of these groups were also tested in sinusoidal shear and three groups in creep so that each group of apples had two independent tests performed on it.

3.2 Apparatus and Instrumentation

Design of the test apparatus was based on the criteria that it was necessary to obtain reasonable accuracy with enough operational simplicity

so that very little time would elapse between the time the specimen was placed in the fixture until the test was complete. This was required so the apple flesh would still be in its fresh state rather than partially dehydrated or oxidized before the test was complete.

An MB Electronics Model SD electrodynamic shaker was used as the sinusoidal force generator. This shaker had the capability of exerting a peak force of 10 lbs which was more than adequate. Figure 3.2 shows the test end of the shaker, less test fixture, properly oriented with respect to a special mount which was designed to support the stationary test fixture and transducer. The special mount consisted of a one inch thick steel base in which a three inch wide by 1/8 inch deep channel was milled to serve as a guide for maintaining alignment as the specimen was placed in position and removed. The vertical fixture support shown was attached to a rectangular base which was guided by the channel as the support was moved toward or away from the shaker by rotation of the threaded shaft. Thus, when appropriate, matched specimen holding fixtures were mounted on the shaker shaft and the vertical support, specimens could be inserted and necessary clamping or positioning accomplished by rotating the threaded shaft after which the movable support was fastened rigidly in place by tightening the two 3/8 inch bolts shown in front of the letters A in Figure 3.2. These bolts were located in slots so that approximately one inch of support movement was possible when the bolts were simply loosened. The entire apparatus was securely fastened to a heavy cast iron lathe bed which served as a rigid foundation and prevented foreign vibrations from being picked up by the instrumentation transducers.

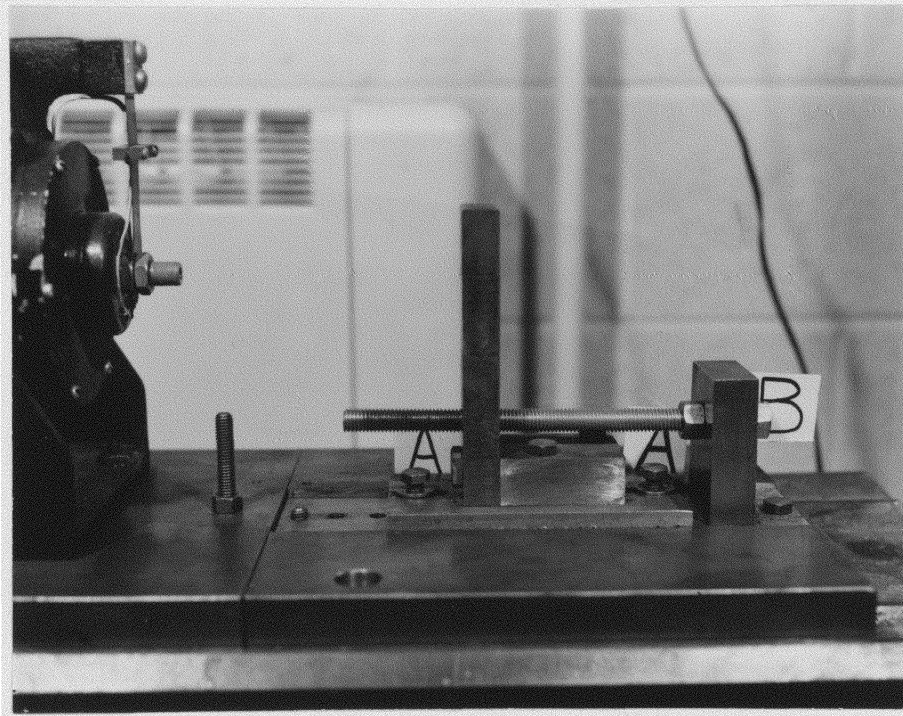


Figure 3.2 Rigid Base and Movable Fixture Support.

The specimens to be tested were short cylinders which for compression testing required two flat compression surfaces. These surfaces were provided by two disks, one mounted on the end of the shaker shaft and the other mounted on the vertical support. Figure 3.3 shows these disks in position holding a specimen of apple flesh. A force transducer was placed between the right hand disk and the vertical support. This disk was mounted to the transducer by a small diameter threaded shaft which was flexible enough to provide a small amount of stress equalization when an imperfectly shaped specimen was placed in position and clamped between the two disks by static force.

Preliminary static compression tests were made to determine the ultimate strength of apple flesh. Depending on the variety, the ultimate strength varied from about 30 psi to over 60 psi. A static plus dynamic level of about eight psi was then selected as that to be used.

Figure 3.4 shows the overall view of the test apparatus with the specimen in position for the sinusoidal compression testing. It was necessary to restrain the inward movement of the shaker shaft with a restraint approximately equal in stiffness to that of the apple flesh specimen if true sinusoidal motion was to be obtained. To accomplish this, a second specimen was cut from each test apple and placed in position similar to the test specimen at the opposite end of the shaker. This restraint specimen is shown at the left end of Figure 3.4 and a top view shows this in detail in Figure 3.5. It was not necessary to have a force transducer at the restraint end since this specimen was not being tested. The fixture mounted on the shaker at this end was

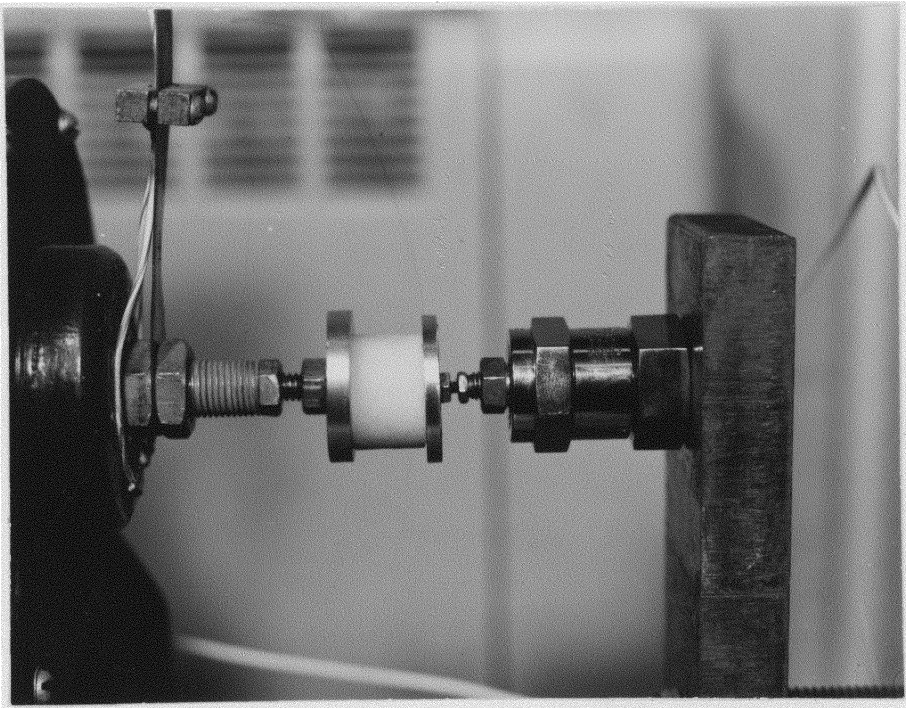


Figure 3.3 Specimen in Position for a Sinusoidal Compression Test.

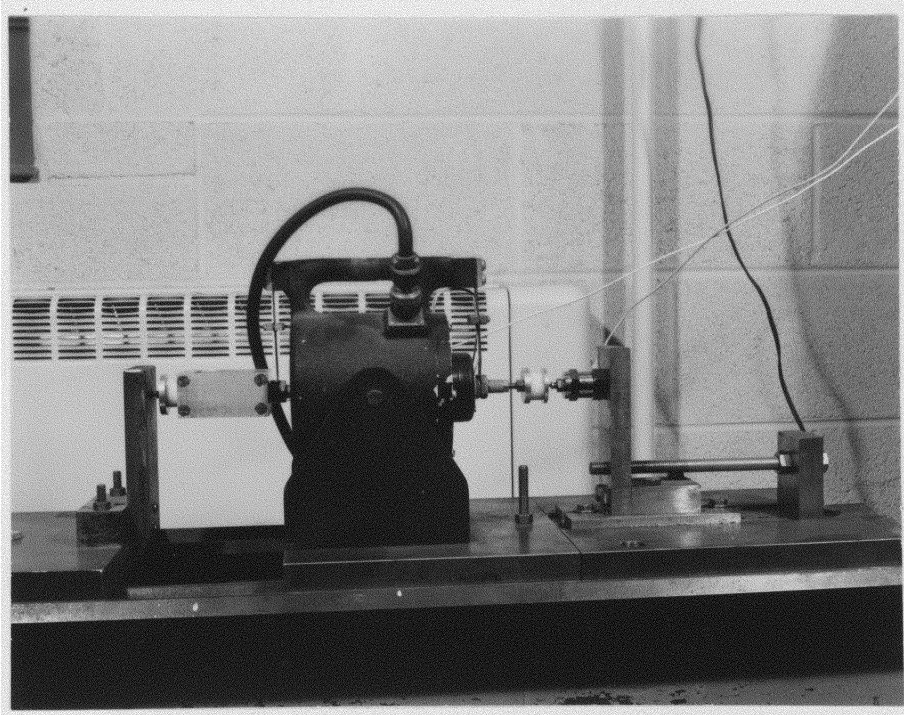


Figure 3.4 Sinusoidal Compression Test Apparatus.

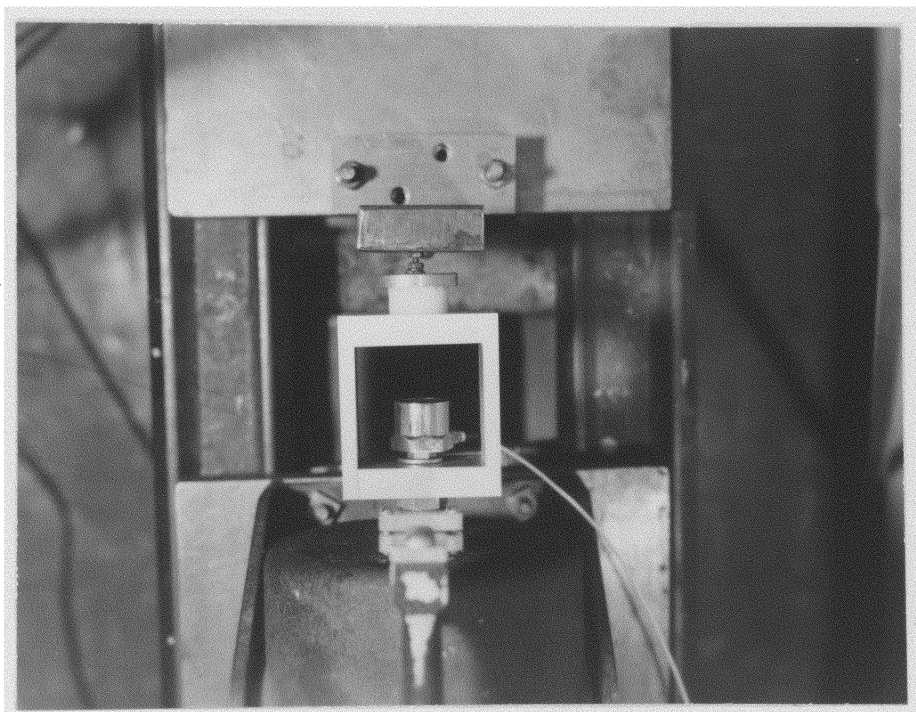


Figure 3.5 Top View of Restraint Specimen in Position.

constructed as shown in Figure 3.5 rather than as a disk to allow for the mounting of an accelerometer directly to the end of the shaker shaft.

This accelerometer was used to obtain data on the shaker shaft displacement and will be discussed further later in this section.

The sinusoidal shear tests required a test fixture quite different from that just described. This is shown with specimens in place in Figure 3.6. The member attached to the shaker was roughly a U in shape and will be referred to as the U-shaped member. Seven-eighths of an inch from the end of each arm of this member a one inch diameter bore $\frac{3}{4}$ of an inch deep was precisely machined. Into each of these bores was fitted a cylinder with clearance between the bore and cylinder being a tight press fit. To control the position of the cylinder in the bore an adjustment screw was provided which pushed against the inner end of the cylinder. Thus, when the adjustment screws on opposite sides of the U were moved inward, the specimens were compressed between the cylinders thus holding them in the test position. Diametral motion of the cylinders in the bores during testing was prevented by set screws at the end of each arm which pressed against the sides of the cylinders. These are the set screws with wooden knobs shown in Figure 3.6. The specimen end of each cylinder was knurled to prevent the specimens from slipping, and the plate located between the specimens was grooved for the same purpose. This plate was fastened to the force transducer by a small diameter threaded shaft and remained stationary during testing. Shear stress was applied to the two specimens simultaneously by sinusoidal motion of the U-shaped member attached to the shaker, and since the shear specimens

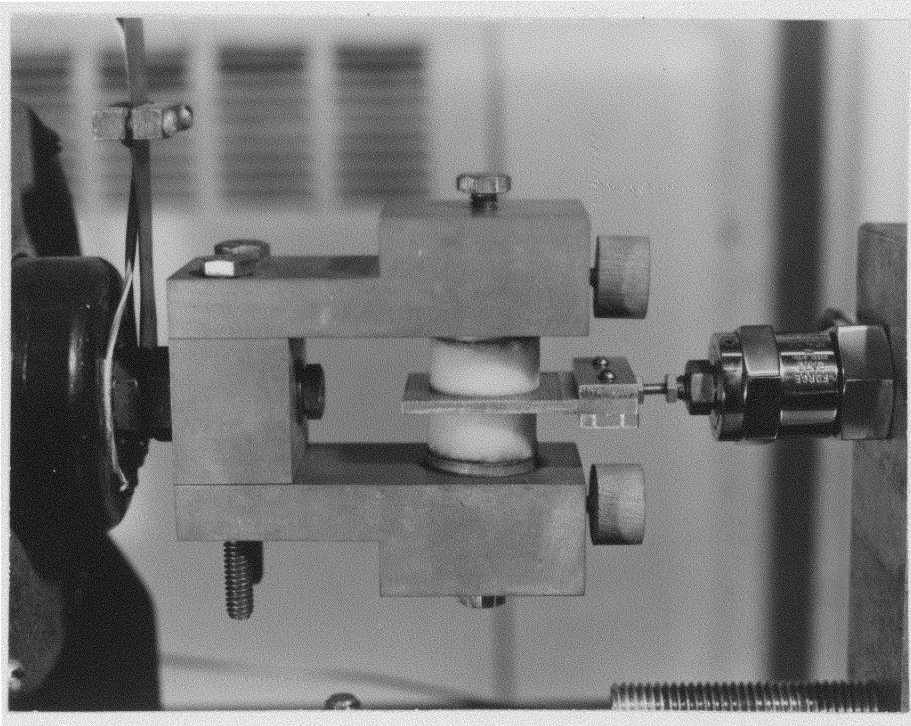


Figure 3.6 Specimens in Position for a Sinusoidal Shear Test.

restrained shaker movement in both directions, it was not necessary to use a restraining sample at the opposite end of the shaker as was done in the compression testing. Figure 3.7 shows an overall view of the shear test apparatus.

The axial stress used to hold the shear specimens in position should not have affected results if the assumptions made were correct and stress levels were low. It was not possible to control this axial stress precisely since adjustment screw displacements were used in holding the specimens in position rather than individually measured forces. The adjustment screws were, however, calibrated so that approximately 3 to 5 psi axial stress was applied during shear testing.

The force applied to the specimen or specimens was sensed by an Endevco model 2103-100 piezoelectric force transducer with a sensitivity of 118 picocoulombs per lb mounted as previously described. This transducer was very stiff so that transducer displacement could be neglected and had a very high natural frequency so that transducer resonance was not a problem. The transducer signal was amplified by an MB Electronics model N504 amplifier and transmitted to the vertical channel of a Hewlett Packard model 120B oscilloscope.

Displacement was obtained indirectly by sensing the acceleration of the shaker shaft and using the relation

$$\text{displacement} = - \frac{1}{\omega^2} [\text{acceleration}] \quad (3.10)$$

for sinusoidal motion. Thus the amplitude of displacement could be calculated by multiplying the amplitude of acceleration by the constant

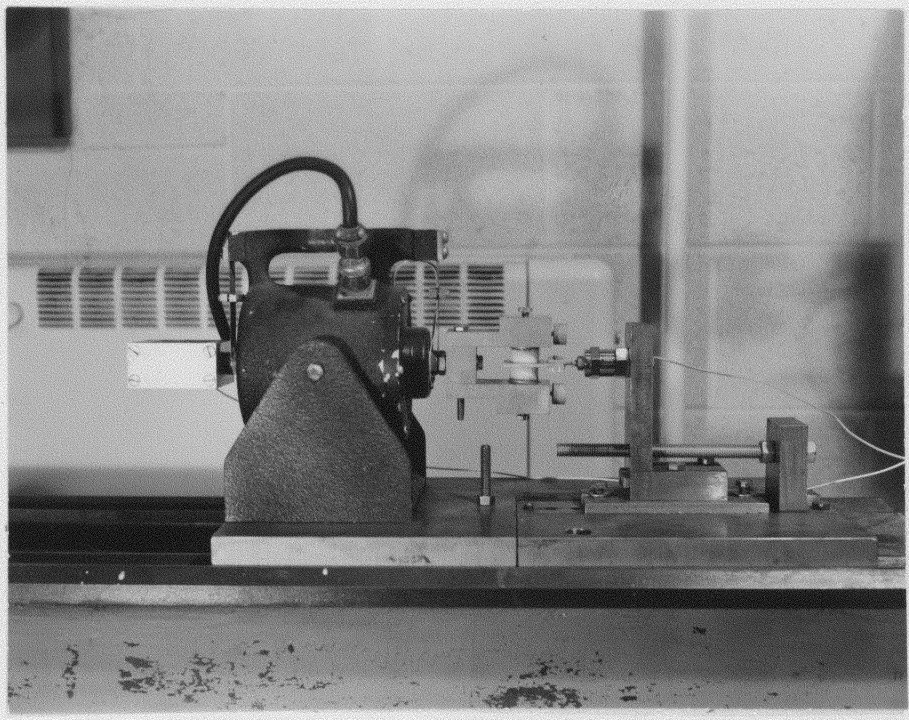


Figure 3.7 Sinusoidal Shear Test Apparatus.

$1/\omega^2$ for a given frequency. An Endevco model 2219E accelerometer with a sensitivity of 93.0 picocoulombs per g of acceleration was used mounted as shown in Figure 3.5. The accelerometer signal was amplified by a Kistler model 504 charge amplifier and transmitted to the horizontal channel of the oscilloscope. Examination of acceleration on the oscilloscope verified that it was in general of good sinusoidal wave form when specimens were being tested thus validating the method used.

The result of showing force as a vertical displacement and acceleration as a horizontal displacement on the oscilloscope screen was a Lissajous figure in the form of an ellipse. The phase angle between force and acceleration was then calculated from the ellipse dimensions using the relation

$$\sin \theta = \frac{M'}{N'} \quad (3.11)$$

where θ is the phase angle, M' is the vertical distance across the ellipse at the center and N' is the vertical dimension of the ellipse from lowest to highest point. Calculation of the phase angle without corrections required that lag times from sensing the force or acceleration until the displacements were seen on the oscilloscope screen were not significant. This was verified by inserting a stiff steel specimen into the test fixture so all displacement would be that caused by strain in the metal parts. Using maximum instrumentation sensitivity, a Lissajous figure was formed which was very nearly a straight line. It was thus concluded that the elliptical shape of the force-displacement curve with the sample in position was caused by the material. It was also concluded from the

stiff specimen testing that displacements caused by strains in the testing apparatus would not be significant when compared to specimen displacements so no displacement corrections were necessary.

It is very unlikely that apple fruits would be subjected to many cycles of impact and quite likely that the material properties would change after a large number of cycles. Therefore, it was desirable to record the first few cycles of stress. This was accomplished by taking pictures of the stress-strain ellipse with an oscilloscope camera immediately after cycling began. This procedure also made it possible to test a large number of specimens in a relatively short time.

Test frequencies were accurately set and monitored by a frequency counter, the input of which was connected to the power amplifier driving the shaker.

Figure 3.8 is a complete view of the apparatus and instrumentation for conducting the sinusoidal tests.

The equipment used by Mr. Thomas Chappell in conducting the creep tests is described in his thesis [10] and will only be briefly described here. Figure 3.9 shows two cylindrical specimens, one on top of the other, in position for testing. These specimens were compressed between the two disks shown. The lower disk was mounted on a force transducer and remained stationary while the upper disk was mounted on the piston rod of a small pneumatic cylinder. When an electrically controlled air valve was actuated, the piston applied force to the specimens. Force was then sensed by the force transducer, axial displacement of the piston rod by the beam transducer shown at the upper left in Figure 3.9, and

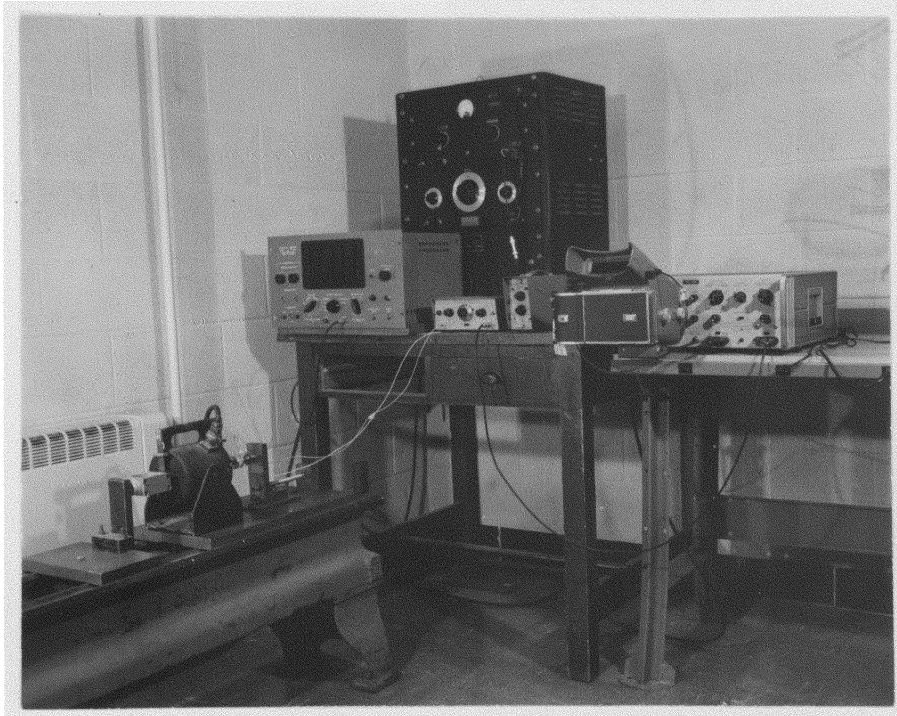


Figure 3.8 Equipment and Instrumentation for Conducting the Sinusoidal Tests.

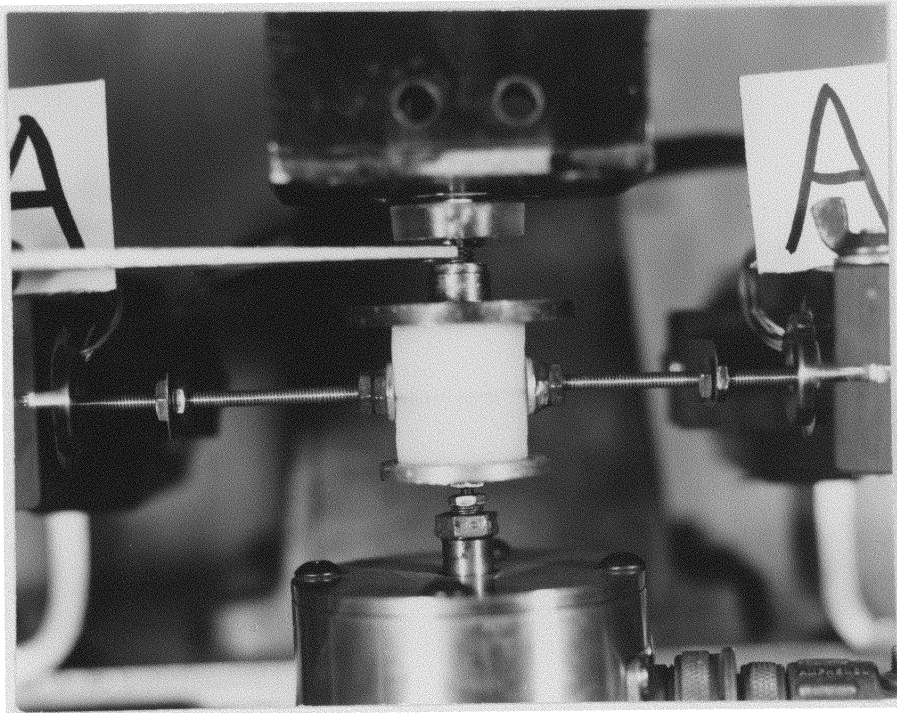


Figure 3.9 Specimens in Position for Creep Testing.

diametral specimen displacement by the two linear variable differential transformer type displacement transducers located on opposite sides of the specimens. All quantities sensed were recorded on appropriate instrumentation.

3.3 Sampling Technique

The specimen shape and size decided upon after considering the geometry and flesh of the apple and dynamic wave propagation was a cylinder approximately one inch in diameter and one-half inch in length. For many apples it was impossible to use a longer cylinder length without taking flesh from the core area. Special equipment and a specific procedure were used to insure that specimens would be as nearly alike as possible and that a small amount of time would be required to obtain a specimen from an apple.

Equipment used in the first step consisted of a saw with fine in-line teeth and a right angle cutting box. The apple was positioned in the box and a thin portion of apple cheek sawn off as shown in Figure 3.10. The apple was next rotated 180 degrees, the cut cheek positioned against the movable guide, and a thin portion of the opposite cheek removed. Next the apple was pushed under the saw by the guide until restrained by an adjustable stop. In this position a one-half inch slice was removed as shown in Figure 3.11. The apple was then again rotated 180 degrees and a slice obtained from the opposite cheek.

The final step in specimen preparation consisted of cutting the cylinders from the two slices obtained in the previous step. This was

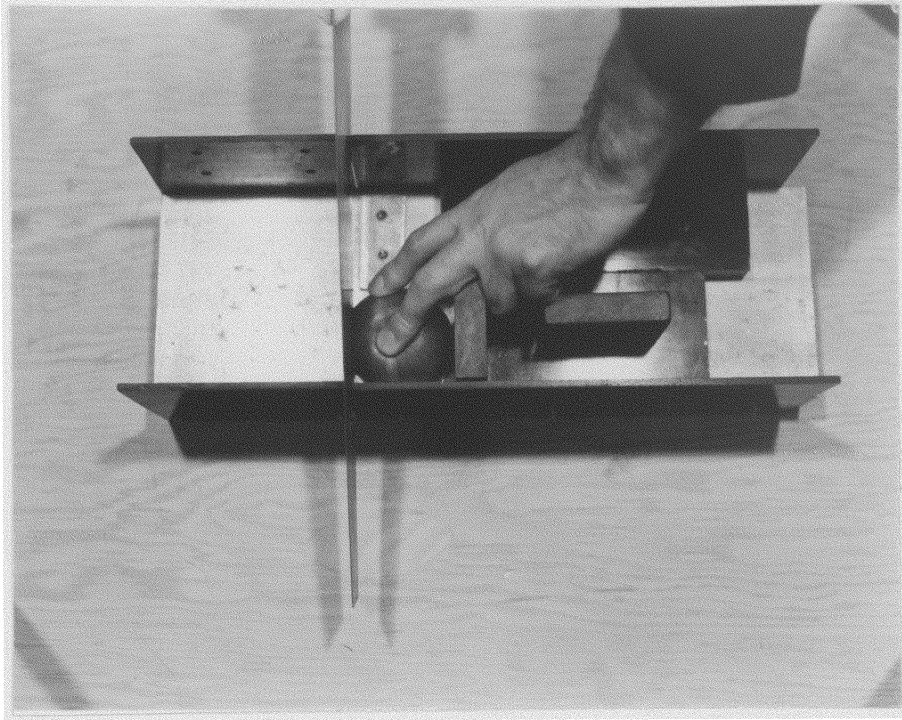


Figure 3.10 First Cut in Specimen Removal.

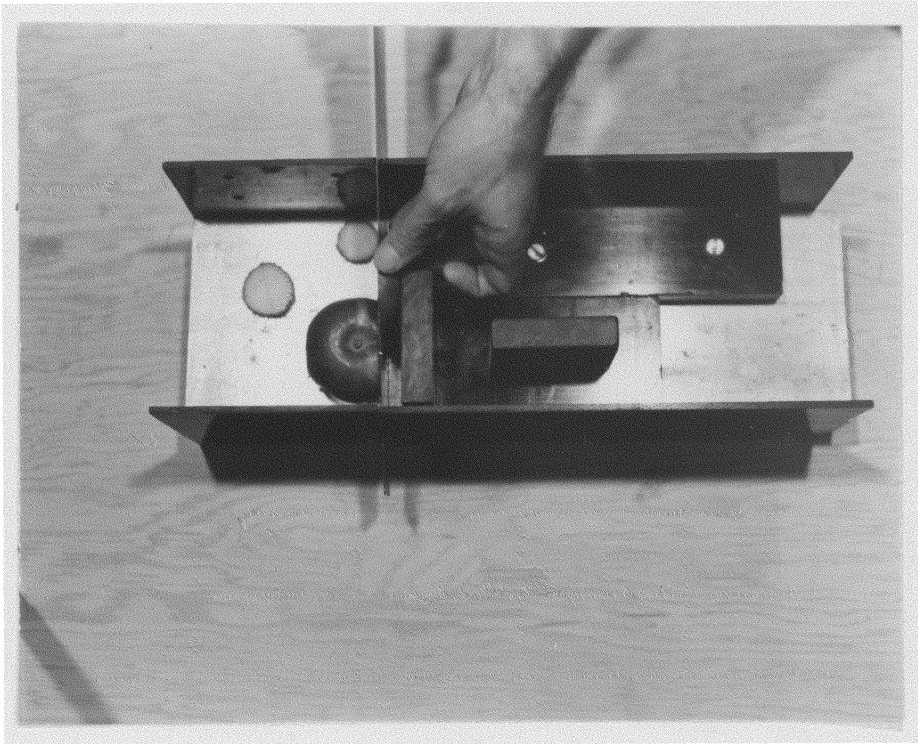


Figure 3.11 Third Cut in Specimen Removal.

accomplished by using a cork borer mounted in a slow rpm drill press. The cork borer used was specially adapted by machining the inside so that it had a true circular cross-section and a thin wall, the final inside diameter being 15/16 of an inch. Figure 3.12 shows the borer being used to cut a specimen. Since the specimen would often stick inside the borer, a 3/4-inch metal disk was placed inside the borer prior to cutting the specimen. Then with the borer removed from the drill press, the specimen could then be pressed from the borer by means of a small shaft which fit through a hole in the borer shank and against the disk.

By the method described, two specimens were obtained from each apple. Both were required for each test in sinusoidal compression, sinusoidal shear, or creep.

3.4 Specimen End Effects

Restraint of the Poisson expansion near the ends of the specimen had to be considered for compression testing of the short cylindrical specimens used. To evaluate this effect, a series of preliminary tests were conducted using single thickness and double thickness specimens to provide two different length to diameter ratios. The double thickness specimens were obtained by using two single thickness specimens from the same apple in series. The results of these tests are shown in Table 3.1 and indicate that there was no appreciable difference between the two L/D ratios.



Figure 3.12 Cutting a Cylindrical Specimen from an Apple Slice.

Table 3.1 Sinusoidal Compression Test Results for Winesap Apples Using Two Different L/D Ratios.

Apple number	Frequency (cps)	L/D Ratio	Absolute value of the complex modulus (psi)	Phase angle (degrees)
1	170	1.13	1285	6.2
1	170	.57	1228	7.0
2	170	1.13	1380	8.4
2	170	.586	1348	7.2
3	170	1.17	1084	5.9
3	170	.586	1080	6.7
4	170	1.13	932	7.2
4	170	.586	923	7.4
5	170	1.13	1070	5.9
5	170	.58	1085	8.2
6	170	1.13	1060	7.2
6	170	.58	1050	8.2
Average for large L/D ratio		1.14	1130	6.80
Average for small L/D ratio		.58	1119	7.45

To confirm the test procedure and equipment Hysol 4485 flexible photoelastic casting resin was tested using four length to diameter ratios. The results of these tests are shown in Table 3.2. It is evident that the length/diameter ratio for this material was significant in affecting modulus values and that the equipment was capable of showing the effect. The absolute modulus values at the 1.05 L/D ratio is about 10 per cent higher than the approximate modulus given by the manufacturer for a strain rate of 63.6 in/in/sec.

It was concluded from the tests discussed that the equipment and test procedure for sinusoidal compression testing were producing results that were at least approximately correct and that the end effects when testing apple flesh could be neglected. This last conclusion was not considered unreasonable since stresses were very low by engineering standards and the juice from the specimen would provide some inherent lubrication.

Table 3.2 Sinusoidal Compression Test Results for Hysol Resin Using Four Different L/D Ratios.

Test number	Frequency (cps)	L/D Ratio	Absolute value of the complex modulus (psi)	Phase angle
1	140	0.266	1462	12.7
2	140	0.528	1058	10.7
3	140	0.789	938	10.4
4	140	1.05	880	11.4

3.5 Testing Procedures

Prior to each sinusoidal test, the variety, mechanical thumb pressure, and approximate diameter were recorded for the apple. Instrumentation settings were recorded and the shaker frequency adjusted. Specimens were then obtained by the procedure outlined in the previous section and placed in the test positions.

For the compression tests, one sample was spaced between the two disks at the right of the shaker and the other in position at the left end of the shaker as was shown in Figure 3.4. The only purpose of the

specimen on the left was to restrain the shaker shaft as it applied force to the left thus maintaining some stress on the test specimen. The apple flesh restraint specimen allowed approximately the same displacement to the left as the test specimen did to the right thus providing a well shaped ellipse on the oscilloscope. With the specimens in position, the vertical support, which held the force transducer, was moved toward the shaker with the threaded shaft until a static force of approximately five pounds was applied to the specimens. This static force was only approximate because the compression was applied in terms of displacement on the threaded shaft and five pounds was an average value determined by preliminary testing. The amplitude adjustment on the shaker controls was then moved quickly from zero to the predetermined test position and the picture of the oscilloscope screen taken immediately. This completed the actual testing and at this time the specimen length was measured using a caliper. Specimen diameters were spot checked but did not vary by a measurable amount.

The procedure for conducting shear tests was identical to that for compression testing except for location of the specimens. In this case, both specimens were being sheared and the resistance to shaker displacement is the sum of that caused by the individual specimens.

The procedure for creep testing consisted of inserting double thickness samples between the disks as was shown in Figure 3.9, adjusting the instrumentation, and actuating the solenoid valve which allowed air to pass to the pneumatic cylinder. Meaningful values of force, axial displacement, and diametral displacement response could be read from the

recorder charts starting approximately 0.02 second after initial needle deflections.

3.6 Complex Modulus Results

The complex modulus for each sinusoidal test was computed from the specimen dimensions and force amplitude, displacement amplitude, and phase angle data taken from the force-displacement ellipse. A typical ellipse is shown in Figure 3.13. Force and acceleration amplitudes were computed directly from ellipse height and width respectively and instrumentation sensitivities. The displacement amplitude was then calculated using equation 3.10. Stress and strain amplitudes were readily computed from force and displacement respectively and the specimen dimensions. The phase angle was computed using equation 3.11. The stress amplitude divided by the strain amplitude then gave the absolute value of the complex modulus so that the real and imaginary parts could be computed using equations 3.3 and 3.4 respectively.

The above procedure was programed for the IBM 7040 digital computer and all computations either computed initially or verified by the computer.

The results of the approximately 500 tests conducted are tabulated in Appendix B. These results are subdivided into seven groupings depending on the variety of apple, whether the tests were shear or compression, and whether the apples were in harvest condition or taken from storage. Mechanical thumb force, frequency, absolute value of the modulus, phase angle, and complex modulus are given for each test.

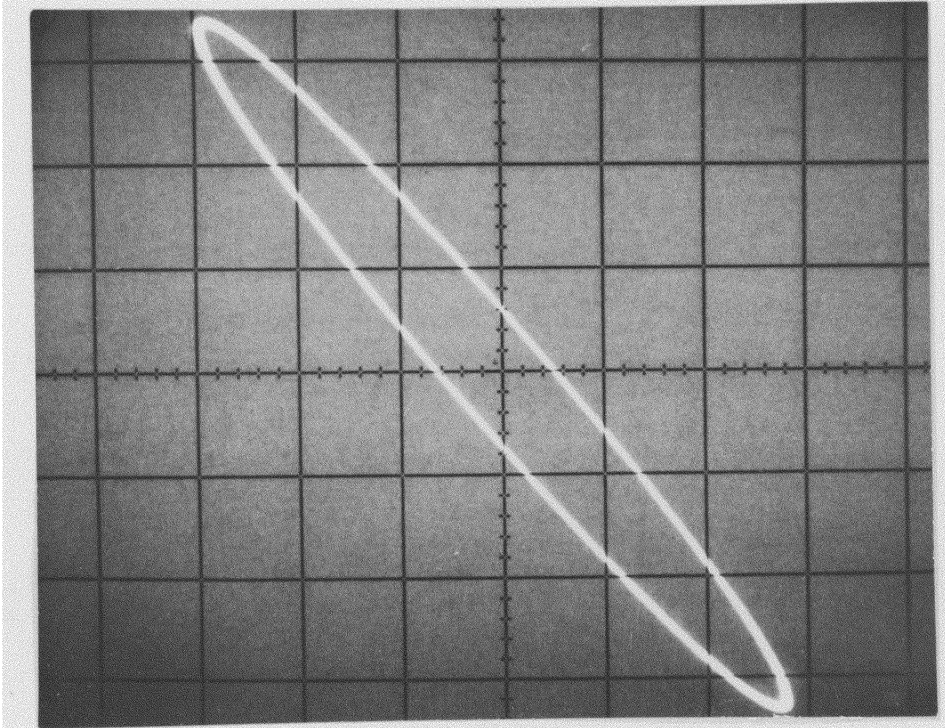


Figure 3.13 Typical Ellipse Obtained from Sinusoidal Testing.

3.7 Statistical Analysis of Results

The results compiled in Appendix B are quite lengthy and the dependency of the complex modulus on frequency is not evident because variation between apples often masks the variation between test results caused by changes in frequency. To summarize the results and develop them into a useful form, two variable statistical correlation analyses were performed on each of the seven groupings of data. Both linear and second degree regression equations were developed to show the dependence of Y_1 and Y_2 (or G_1 and G_2) on the angular frequency. The second degree equations fit the data better in more than half of the cases and were used except where there was nearly an equally good fit by the linear relation. Actual computation involved in the correlations were performed on the IBM 7040 digital computer using a standard program of the V.P.I. Statistics Department.

Figures 3.14a and 3.14b show the data points and second degree regression equations for Y_1 versus ω and Y_2 versus ω from compression testing of Red Delicious apples at harvest. These results are typical of those found for the various groupings.

Table 3.3 gives the results of the statistical correlation for all seven groupings of data. Equations relating Y_1 and Y_2 (or G_1 and G_2) to ω are given in column six. It is interesting to note that the real part of the complex modulus could be related to ω by a first degree equation for both shear and compression for the apples taken from storage. Column seven gives the per cent variation in the data explained by regression on ω . In only three of the 14 correlations is more than

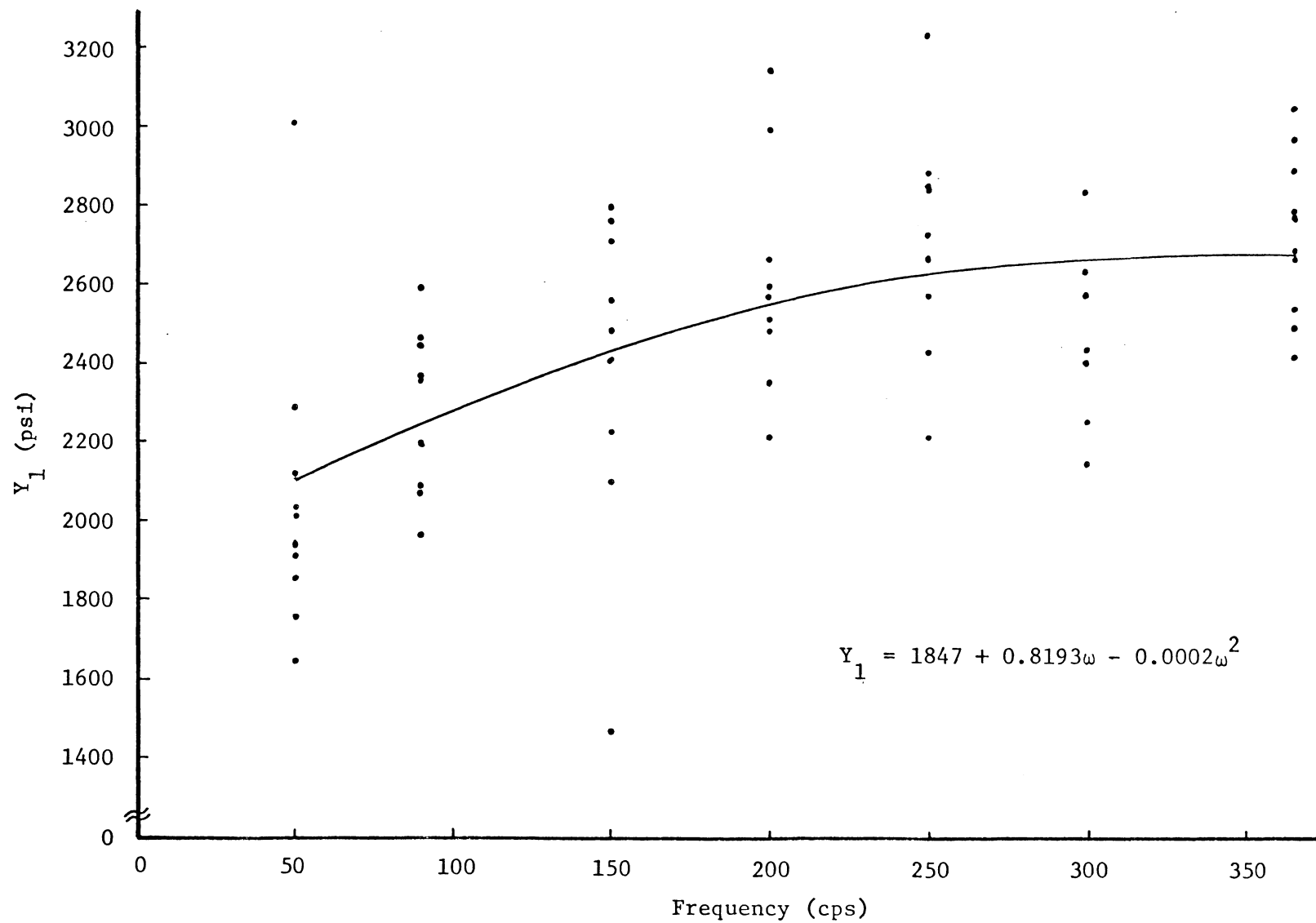


Figure 3.14a Results of Sinusoidal Compression Testing on Red Delicious Apples at Harvest.

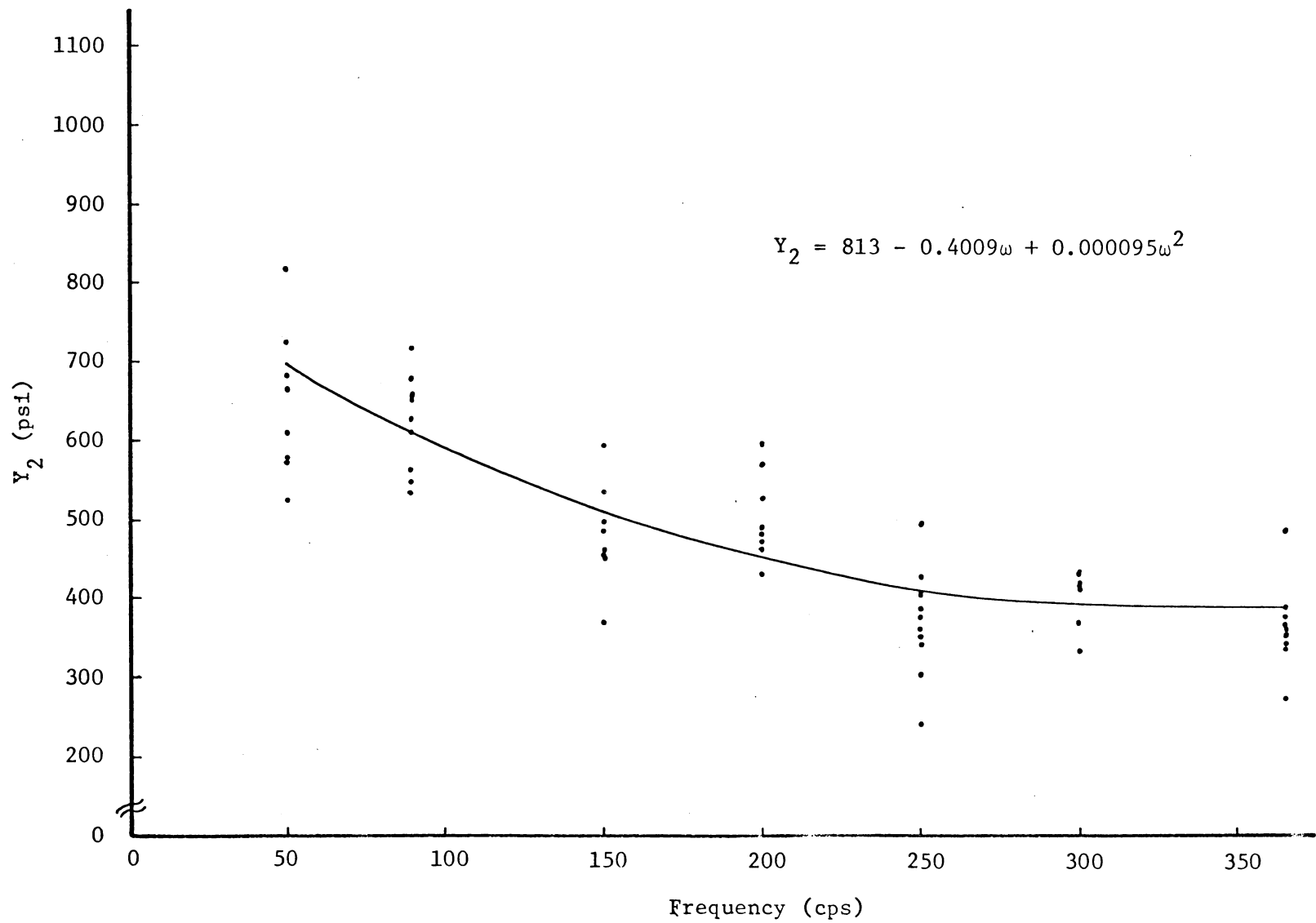


Figure 3.14b Results of Sinusoidal Compression Testing on Red Delicious Apples at Harvest.

Table 3.3 Dependence of the Real and Imaginary Parts of the Complex Moduli on ω

Data group	Type of test	Variety	Approximate time in storage (months)	Average mechanical thumb force(lbs)	Equations relating the real and imaginary parts of the complex moduli to ω	Variation explained by regression (%)	Average deviation(%)
1C	Compression	Red Delicious	0	19	$Y_1 = 1847 + 0.8193\omega - 0.00020\omega^2$	33.5*	9.17
					$Y_2 = 813.4 - 0.4009\omega + 0.000095\omega^2$	63.6*	11.18
2C	Compression	Winesap	0	23	$Y_1 = 1464 + 1.594\omega - 0.00044\omega^2$	62.1*	7.73
					$Y_2 = 447.2 + 0.1083\omega - 0.000065\omega^2$	14.1**	15.31
3C	Compression	Golden Delicious	0	16	$Y_1 = 2072 + 0.2855\omega$	22.1 *	12.06
					$Y_2 = 734.3 - 0.3593\omega + 0.000095\omega^2$	30.8*	20.65
4C	Compression	Winesap	4	17	$Y_1 = 1341 + 0.1899\omega$	14.9*	8.72
					$Y_2 = 143.4 - 0.3796\omega - 0.00019\omega^2$	14.8**	16.08
5C	Compression	Golden Delicious	4	11	$Y_1 = 629.2 + 0.1291\omega$	8.7**	12.73
					$Y_2 = 175.6 - 0.0413\omega$	14.2*	19.64

*There was a significant dependence on ω at the 1.0% level.

**There was a significant dependence on ω at the 2.5% level.

Table 3.3 (Continued)

Data group	Type of test	Variety	Approximate time in storage (months)	Average mechanical thumb force(lbs)	Equations relating the real and imaginary parts of the complex moduli to ω	Variation explained by regression (%)	Average deviation(%)
4S	Shear	Winesap	4	17	$G_1=311.3+0.1080\omega$	39.3	9.03
					$G_2=52.54+0.02892\omega-0.000025\omega^2$	41.4*	9.02
5S	Shear	Golden Delicious	4	11	$G_1=105.5+0.1091\omega$	58.1*	13.54
					$G_2=30.08+0.02124\omega-0.000015\omega^2$	7.0***	16.25

***There was a significant dependence on ω at the 10.0% level.

half of the variation explained by ω . The cause of the large unexplained variation is not known but evidently is common in biological materials as noted by other investigators [9, 10, 32]. The explained variation was in general higher for the compression tests on harvest fruit than for the stored fruit. In all but three of the 14 regressions there was statistical significance at the one per cent level. Two of those not dependent at the one per cent level were significantly dependent at the 2.5 per cent level and the other was significant at the 10 per cent level.

Column eight gives the average per cent deviation of the individual data points from the regression line. This is probably the best indication of how well the equation fits the data since if the plot of Y_1 versus ω has a very small slope, the explained variation may be low even when the equation fits the data quite well. It can be seen that the Golden Delicious data groups had the highest average deviations for both Y_1 and Y_2 thus indicating that there was more scatter in these data.

3.8 Poisson's Ratio for Apple Fruits

The companion work of Chappell [10] had as its primary objective the determination of an accurate mathematical representation of Poisson's ratio for apple fruit. Tests were conducted on three varieties in harvest condition. These varieties were identical to groups 1C through 3C in Table 3.3.

Creep tests were conducted at four levels of stress which were 9.81 psi, 14.78 psi, 19.83 psi, and 23.96 psi. The analysis of results indicated that there was some dependence of Poisson's ratio on stress

level. This was most evident between the lowest stress levels with little dependence on stress evident between the two highest stress levels for Winesap and Red Delicious apples.

Results also indicated that Poisson's ratio was time dependent. This was observed in plots of Poisson's ratio versus time such as that shown in Figure 3.15. The time dependency was a result of the diametral strain becoming constant during the test and axial strain continuing to increase with time.

Impact problems, such as the one considered in this work, require Poisson's ratio for very short loading times, and equations based on comparatively long-time creep curves such as Figure 3.15 are not sufficiently accurate at the short-time end. Therefore, the short-time ends of the graphs were expanded. Figure 3.16 shows this expanded form for the Red Delicious apple flesh. It can be seen that Poisson's ratio at times under 0.05 second departs from the general curve. This phenomenon was observed for the Winesap apples as well. It was also noted that the slope of the curves is not great. Thus, since impact durations of concern are considerably less than 0.05 second it was decided that the most reasonable way to introduce values of Poisson's ratio into the impact problem would be to use constants rather than time dependent relations and that they should be equal to the average of the initial Poisson's ratios for the two highest stress levels for each variety. The initial Poisson's ratio is taken arbitrarily as that at 0.04 second or at 0.05 second if an 0.04 second data point was not recorded. Average initial values for all four stress levels are given in Table 3.4 along with the average for the two highest stress levels.

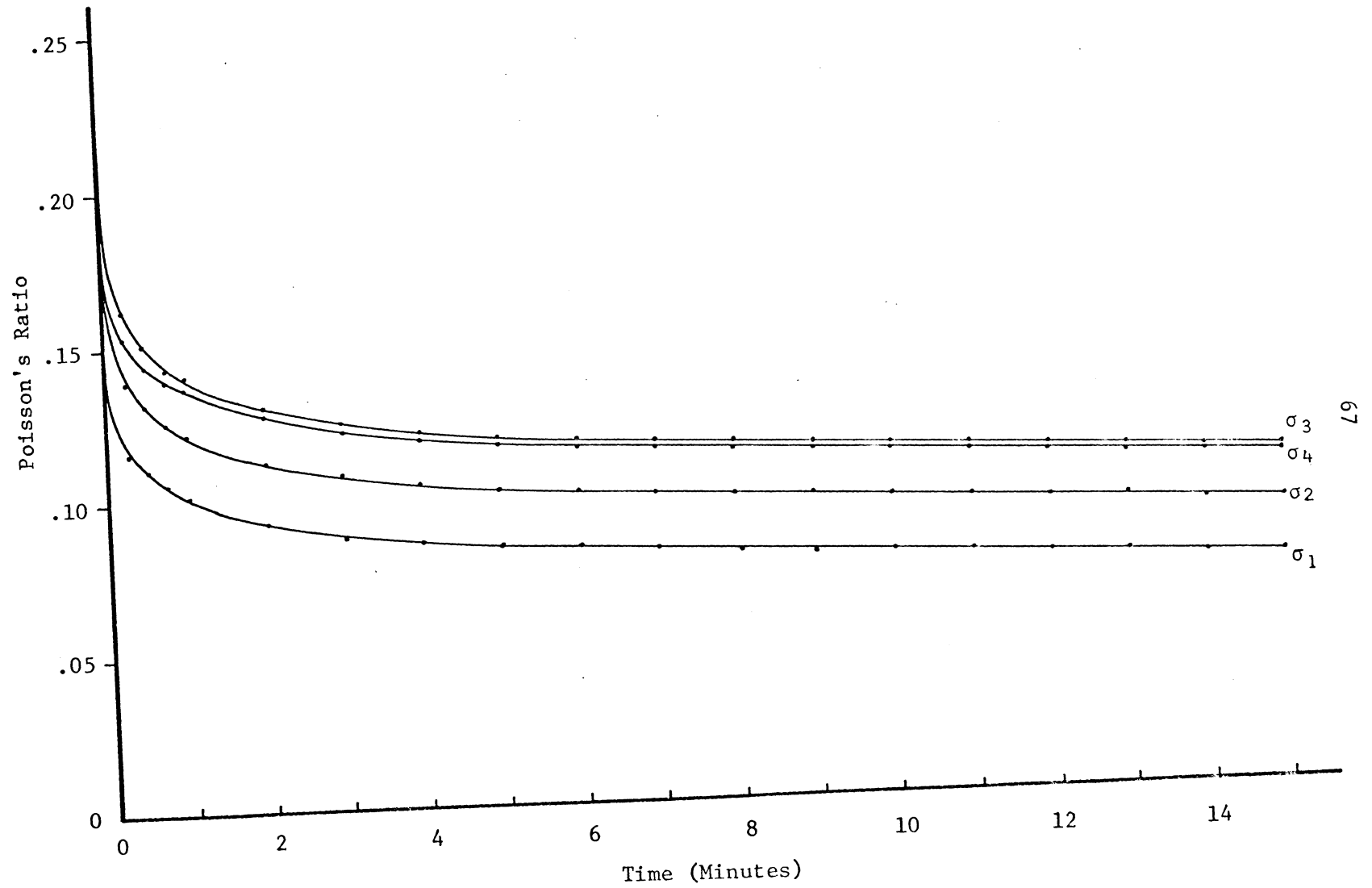


Figure 3.15 Poisson's Ratio Versus Time in Minutes for the Four Stress Levels on Red Delicious Apple Flesh.

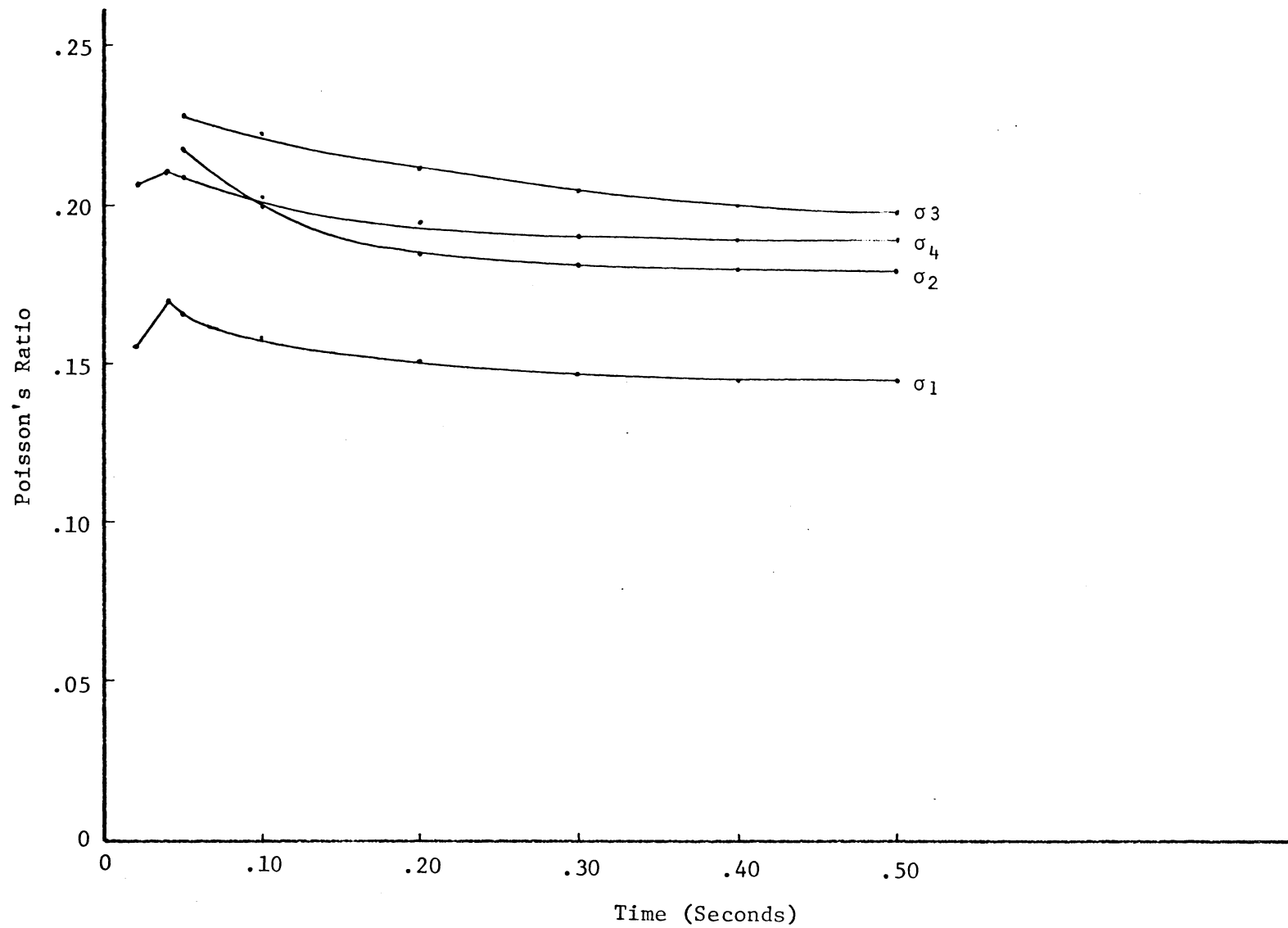


Figure 3.16 Poisson's Ratio Versus Time in Seconds for the Four Stress Levels on Red Delicious Apple Flesh.

Table 3.4 Average Initial Values of Poisson's Ratio

Stress (psi)	Red Delicious	Winesap	Golden Delicious
9.81	0.169	0.193	0.169
14.78	0.216	0.219	0.223
19.83	0.228	0.312	0.192
23.96	0.210	0.288	0.244
Average for the two highest stress levels	0.219	0.300	0.218

Etter [44] conducted a small number of tests similar to those of Chappell except that a weight was brought into contact with the specimen and released electromagnetically. He used apples that had been stored about five months and found an average initial Poisson's ratio for Winesap apples to be 0.410. This differs considerably from Chappell's value of 0.300 for Winesap apples. It could also be noticed in the apples taken from storage that they were more resilient and more easily deformed than harvest apples. Such differences are possibly caused by dehydration of the apple. The condition of Etter's apples would correspond to groups 4C and 4S in Table 3.3.

4. VISCOELASTIC REPRESENTATION OF THE RESPONSE OF APPLE FLESH TO DYNAMIC FORCES

4.1 Model Representation

It is quite often possible to represent the stress-strain relationship for a material over a narrow band of time or frequency by a relatively simple spring-dashpot model even though the model is unsuitable outside the band of time or frequency of interest. An attempt was made to represent the groups of data obtained by simple models using the method of Bland [13], pp. 113-124.

Bland names the model shown in Figure 4.1 a four-element model of the second type and for the complex modulus gives

$$Y_1 = E_1 + E_3 - \frac{E_3^2}{E_3^2 + \omega^2 \eta_3^2} \quad (4.1)$$

$$Y_1 = \eta_2 + \frac{\eta_3 E_3^2}{E_3^2 + \omega^2 \eta_3^2} \quad (4.2)$$

where E_1 , E_3 , η_2 , and η_3 are constants associated with the elements of Figure 4.1. Equations 4.1 and 4.2 can be derived by determining the complex moduli for a spring and dashpot in series (Maxwell model) and a spring and dashpot in parallel (Kelvin model) and adding the two since Figure 4.1 is essentially the Maxwell and Kelvin models in parallel.

Equations 4.1 and 4.2 are related by

$$Y_1 + \frac{E_3}{\eta_3} \frac{Y_2}{\omega} = E_1 + E_3 + \frac{E_3 \eta_2}{\eta_3} \quad (4.3)$$

In slope-intercept form if

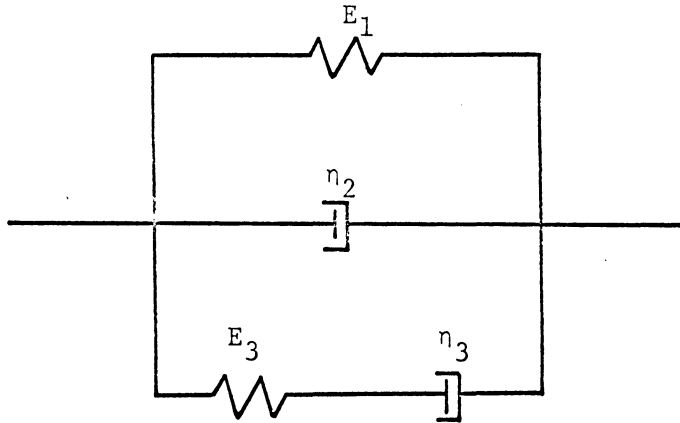


Figure 4.1 Four Element Model of the Second Type.

$$M = \frac{E_3}{\eta_3} \quad (4.4)$$

and

$$N = E_1 + E_3 + \frac{E_3 \eta_2}{\eta_3}, \quad (4.5)$$

equation 4.3 can be written as

$$Y_1 + M \frac{Y_2}{\omega} = N. \quad (4.6)$$

From equation 4.6 it can be seen that if a plot of Y_1 versus Y_2/ω is a straight line, it should be possible to represent the material response by this type of four element model. A graph of these two quantities for any group of complex moduli will be designated as plot number one.

Graphs of Y_1 versus Y_2/ω (plot number one) were made for the complex modulus groups obtained from tests on stored apples. These groups will

be named 4C, 5C, 4S, and 5S to correspond with Table 3.3. Points were generally grouped along straight lines and it seemed likely that a suitable four-element model could be developed. Figure 4.2 shows plot one for data group 4C as an example. Linear regression equations were computed by the method of least squares and from the slopes and Y_1 intercepts M and N values determined.

Equation 4.1 can be rewritten as

$$Y_1 = E_1 + E_3 - E_3 \left[1 + \left(\frac{\eta_3}{E_3} \omega \right)^2 \right]^{-1} \quad (4.7)$$

so a graph of Y_1 versus $\left[1 + \left(\frac{\eta_3}{E_3} \omega \right)^2 \right]^{-1}$

will have $E_1 + E_3$ as the Y_1 intercept and $-E_3$ as the slope. Figure 4.3 shows this graph which will be referred to as plot two for the complex moduli of data group 4C. From both plots involving Y_1

$$E_3 = - \text{slope of plot 2,}$$

$$E_1 = [Y_1 \text{ intercept of plot 2}] - E_3, \quad (4.8)$$

$$\eta_3 = -E_3 / \text{slope of plot 1, and}$$

$$\eta_2 = [\eta_3 / E_3] [Y_1 \text{ intercept of plot 1} - Y_1 \text{ intercept of plot 2}].$$

The last of equations 4.8 reveals the weakness of the procedure. The value of η_2 is dependent on the difference between Y_1 intercepts of plot one and plot two which, for materials that exhibit a large degree of elasticity, will be nearly the same magnitude. Regression lines of plots

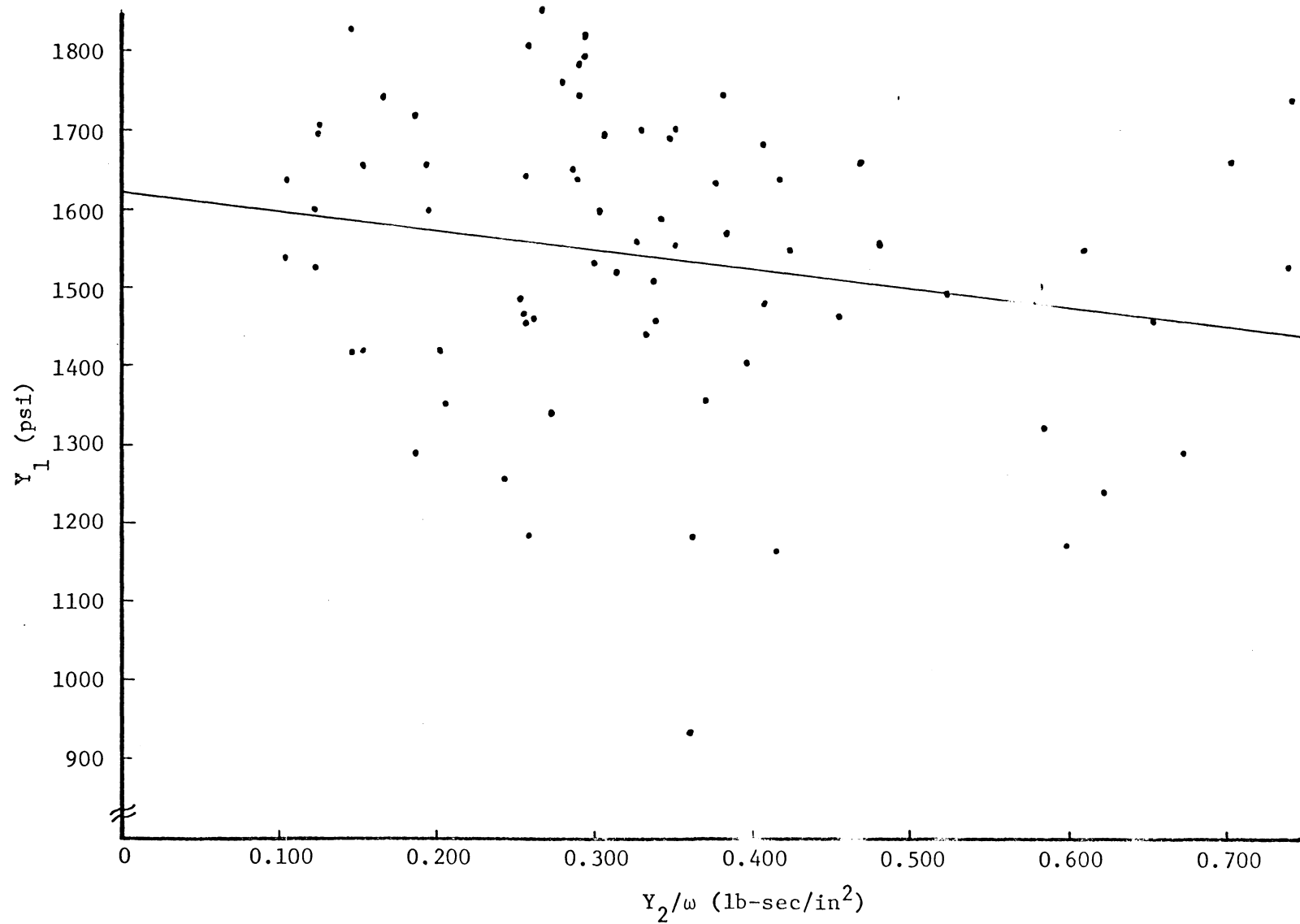


Figure 4.2 Y_1 Versus Y_2/ω for Data Group 4C.

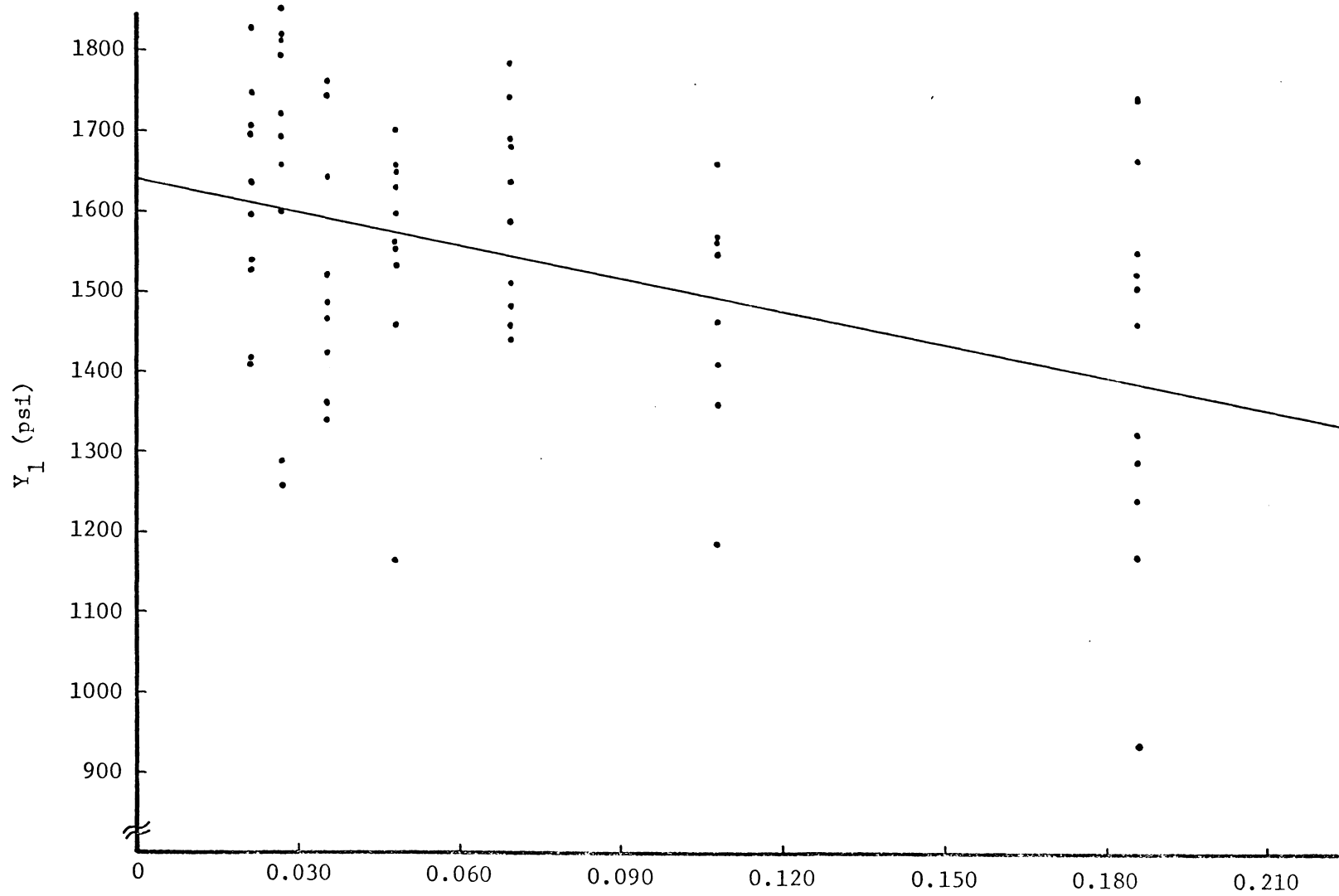


Figure 4.3 Y_1 Versus $\left[1 + \left(\frac{\omega \eta_3}{E_3}\right)^2\right]^{-1}$ for Data Group 4C.

one and two were not sufficiently compatible to provide accurate values of η_2 . In fact, some of the η_2 values calculated were negative thus illustrating the need for very consistent data in plots one and two.

Since data for plots one and two were not sufficiently well grouped for a four-element representation, a three-element representation was attempted. The quantity η_2 was dropped from Figure 4.1 giving the relations

$$E_3 = -\text{slope of plot 2},$$

$$E_1 = [Y_1 \text{ intercept of either plot 1 or 2}] - E_3, \text{ and} \quad (4.9)$$

$$\eta_3 = -E_3 / \text{slope of plot 1}.$$

Element constants computed from equations 4.9 were more reasonable than those computed from equations 4.8 but E_1 would vary considerably depending on whether the Y_1 intercept of plot one or plot two was used. Also, the average deviation of Y_1 and Y_2 computed from the three element model compared to the average data Y_1 and Y_2 for test frequencies was greater than the average deviation for Y_1 and Y_2 computed on the basis of the simpler Maxwell model. It was therefore concluded that the data were not sufficiently consistent to develop a spring-dashpot model more complex than a two-element model.

The real part of the complex compliance (J_1) for a Maxwell model is given by $1/E$ and the imaginary part (J_2) by $1/[\omega\eta]$ while for a Kelvin model $Y_1 = E$ and $Y_2 = \omega\eta$. As already discussed, plots of Y_1 versus Y_2/ω for apple flesh were in general linear suggesting that the Kelvin model would not give a good representation. Graphs of J_1 versus ωJ_2 were not

linear but roughly grouped in a circular pattern suggesting the Maxwell model to be appropriate. Model element constants were computed from

$$E = \frac{1}{J_1} \quad (4.10)$$

and

$$\eta = \frac{1}{J_2} \quad (4.11)$$

for all seven groups of complex moduli and are given in Table 4.1.

Computations consisted of first using the relations

$$J_1 = Y_1 / [Y_1^2 + Y_2^2] \quad (4.12)$$

and

$$J_2 = Y_2 / [Y_1^2 + Y_2^2] \quad (4.13)$$

Table 4.1 Maxwell Model Element Constants

Complex Modulus group	E (psi)	η (psi/sec)
1C	2530	11.79
2C	2615	12.18
3C	2462	12.00
4C	1498	7.56
5C	760.0	4.49
4S	426.0	3.05
5S	199.5	1.38

to compute J_1 and J_2 from Y_1 and Y_2 for each test. The J_1 and J_2 were averaged for the group and E and η computed from equations 4.10 and 4.11. These computations were done on the IBM 7040 digital computer.

The complex moduli for the models were computed for each data group at each test frequency and compared with the average experimental Y_1 and Y_2 to evaluate material representation by the Maxwell model. Model values of Y_1 and Y_2 were computed by

$$Y_1 = \frac{\omega^2 \eta^2 E^2}{E^2 + \omega^2 \eta^2}$$

and

(4.14)

$$Y_2 = \frac{\omega \eta E^2}{E^2 + \omega^2 \eta^2}$$

taken from Bland [13], p. 114. Table 4.2 indicates how well the models fit the data for the seven groups by giving the per cent deviation of the model values from the data values for each test frequency. Generally, the model Y_1 values are slightly lower than the data values and the model Y_2 values are considerably higher than the data values at low frequencies and considerably lower at high frequencies. This makes evident a major difficulty in Maxwell model representation of apple flesh. Data Y_2 values are much less variable with frequency than can be represented by the model if the model is to vary Y_1 sufficiently. Nevertheless, the overall representation is probably a sufficiently accurate approximation considering the scatter in the original data.

Table 4.2 Comparison of Model and Data Moduli

Data group	Frequency (cps)	Y_1 from model (psi)	Y_1 from experimental average (psi)	Deviation (%)	Y_2 from model (psi)	Y_2 from experimental average (psi)	Deviation (%)
1C	50	1725.1	2058.4	-16.19	1178.5	703.7	67.47
1C	90	2211.6	2271.8	- 2.65	839.4	613.3	36.87
1C	150	2405.4	2393.1	0.514	547.8	486.1	12.69
1C	200	2458.4	2654.0	- 7.37	419.9	499.6	-15.95
1C	250	2483.8	2686.7	- 7.55	339.4	374.7	- 9.42
1C	300	2497.8	2475.8	0.89	284.4	402.2	-29.29
1C	365	2508.2	2730.6	- 8.14	234.7	367.6	-36.15
1C	Average*			6.19			29.69
2C	50	1782.6	1863.5	- 4.34	1217.9	459.8	164.88
2C	95	2315.3	2283.0	1.41	832.6	585.8	42.13
2C	150	2485.8	2733.7	- 9.07	566.1	539.8	4.87
2C	200	2540.6	2673.0	- 4.95	434.0	530.8	-18.24
2C	250	2566.8	2791.9	- 8.06	350.8	410.5	-14.54
2C	300	2581.2	2932.1	-11.97	293.9	534.2	-44.98
2C	365	2592.0	2801.8	- 7.49	242.6	413.2	-41.29
2C	Average			6.76			47.28
3C	50	1725.8	1986.6	-13.13	1127.2	618.5	82.23
3C	90	2175.5	2316.0	- 6.07	789.4	581.5	35.66
3C	150	2350.6	2508.1	- 6.28	511.7	469.6	8.97
3C	200	2398.0	2412.3	- 0.60	391.6	451.0	13.17

*The average per cent deviations were computed using absolute values of the per cent deviations for each frequency.

Table 4.2 (Continued)

Data group	Frequency (cps)	Y ₁ from model (psi)	Y ₁ from experimental average (psi)	Deviation (%)	Y ₂ from model (psi)	Y ₂ from experimental average (psi)	Deviation (%)
3C	250	2420.7	2497.2	- 3.06	316.2	331.2	4.53
3C	300	2433.1	2648.6	- 8.14	264.9	443.0	-40.20
3C	365	2442.4	2657.0	- 8.08	218.5	383.1	32.53
3C	Average			6.48			32.53
4C	80	1296.6	1398.7	- 7.30	511.2	314.6	62.49
4C	110	1384.4	1443.7	- 4.11	396.9	284.9	39.31
4C	140	1425.8	1217.4	17.12	321.2	223.6	43.65
4C	170	1448.3	1565.4	- 7.48	268.7	337.2	-20.31
4C	200	1461.8	1539.9	- 5.07	230.5	329.7	-30.09
4C	230	1470.5	1649.1	-10.83	201.7	351.2	-42.57
4C	260	1476.4	1612.3	- 8.43	179.1	214.9	-16.66
4C	Average			7.55			36.44
5C	80	682.5	685.5	- 0.44	229.9	163.7	40.44
5C	110	716.9	717.8	- 0.12	175.6	140.3	25.16
5C	140	732.8	748.9	- 2.15	141.0	122.6	15.01
5C	170	741.3	799.1	- 7.23	117.5	160.2	-26.65
5C	200	746.4	757.6	-11.48	100.6	125.4	-19.78
5C	230	749.7	820.8	- 8.66	87.8	108.7	-19.23
5C	Average			3.35			24.38

Table 4.2 (Continued)

Data group	Frequency (cps)	Y ₁ from model (psi)	Y ₁ from experimental average (psi)	Deviation (%)	Y ₂ from model (psi)	Y ₂ from experimental average (psi)	Deviation (%)
4S	80	395.4	351.3	12.55	109.9	61.5	78.70
4S	110	409.2	381.3	7.32	82.7	65.2	26.84
4S	140	415.5	424.0	- 2.00	66.0	63.3	4.27
4S	170	418.88	449.6	- 6.85	54.8	56.4	- 2.84
4S	200	420.8	449.8	- 6.45	46.8	62.3	-24.87
4S	230	422.0	433.9	- 2.74	40.8	53.4	-23.60
4S	260	422.9	496.9	-14.89	36.2	47.5	-23.79
4S	Average			7.54			26.42
5S	50	164.6	146.4	12.43	75.9	38.3	98.17
5S	80	184.2	145.0	27.03	53.1	32.8	61.89
5S	110	191.1	229.3	-16.66	40.1	40.0	0.25
5S	140	194.3	201.0	- 3.33	32.0	39.5	-18.99
5S	170	195.9	216.3	- 9.43	26.6	33.7	-21.07
5S	200	196.9	263.4	-25.25	22.7	39.3	-42.24
5S	230	197.6	249.4	-20.77	19.8	28.4	-30.28
5S	Average			16.41			38.98

4.2 Hereditary Integral Representation

It was stated in the introduction that it was not necessary to use spring-dashpot representation of material response if recourse was made to hereditary integrals of the convolution type. The hereditary integral representations for uniaxial compression or tension are

$$\epsilon(t) = \int_0^t J(t - \tau) \frac{\partial}{\partial \tau} \sigma(\tau) d\tau \quad (4.15)$$

and

$$\sigma(t) = \int_0^t G(t - \tau) \frac{\partial}{\partial \tau} \epsilon(\tau) d\tau \quad (4.16)$$

where $\sigma(t)$, $\epsilon(t)$, $G(t)$, and $J(t)$ are respectively the time dependent stress, strain, relaxation modulus, and creep compliance. Any mathematical form of relaxation modulus or creep compliance that is amenable to treatment by Laplace transform techniques or other suitable methods is appropriate for insertion into equations 4.15 and 4.16. Such a suitable form was sought for the experimental results of this work.

The complex moduli for the seven test groups were determined as functions of ω and have been given in Table 3.3. It was necessary to transform from these forms to relaxation moduli in order to use equation 4.16. Theoretically the relaxation modulus can be determined from the real or imaginary parts of the complex modulus by inverse Fourier transforms which are given by Ferry [14], p. 55. Using the notation of this thesis

$$G(t) = G_e + \frac{2}{\pi} \int_0^\infty \{[Y_1 - G_e] / \omega\} \sin \omega t \, d\omega \quad (4.17)$$

or

$$G(t) = G_e + \frac{2}{\pi} \int_0^{\infty} [Y_2/\omega] \cos \omega t \, d\omega, \quad (4.18)$$

where G_e is the equilibrium compression modulus. In principle, these integrals can be performed numerically or graphically if the starting function is known over a sufficiently wide range of frequency. In practice, however, the function $Y_1(\omega)$ or $Y_2(\omega)$ is often not available over a wide enough range of frequencies to give convergence of the integral. This was likely true for the cases being considered since $Y(i\omega)$ was known only for a small range of frequencies.

An approximate equation relating $G(t)$ and the complex moduli is given by Ninomiya and Ferry [25] as

$$G(t) = Y_1(\omega) - 0.40 Y_2(0.40\omega) + 0.014 Y_2(10\omega) \quad \left| \omega = \frac{1}{t} \right. \quad (4.19)$$

where $Y_2(0.40\omega)$ means Y_2 calculated at 0.40ω when $\omega = \frac{1}{t}$, $Y_2(10\omega)$ means Y_2 calculated at 10ω when $\omega = \frac{1}{t}$ and similarly for $Y_1(\omega)$. *at 10ω when $\omega = \frac{1}{t}$* Ninomiya and Ferry developed this approximation in detail and it will be used here without additional justification for its use. Equation 4.19 was used to compute $G(t)$ at $t = \frac{1}{\omega}$ for the seven test frequencies used within each group of data. The regression equations of Table 3.3 were first used to compute $Y_1(\omega)$, $Y_2(0.40\omega)$, and $Y_2(10\omega)$ after which these values were substituted directly into equation 4.19. The term $Y_2(10\omega)$ proved to be less than 0.4 per cent of $G(t)$ and was questionable since 10ω was usually considerably outside the test range. Therefore, it was dropped from the computations.

Table 4.3 gives $G(t)$ at times equal to the reciprocals of test angular frequencies for all seven groups of moduli, and graphs of $\log G(t)$ versus t for the moduli of the Red Delicious apples (group 1C) and Wine-sap apples (group 2C) are shown in Figure 4.4 and 4.5 respectively. It was necessary to relate $G(t)$ and t by a mathematical relation suitable for substitution into equation 4.16. A straight line approximation such as that shown in Figure 4.4 has the form

$$\log G(t) = a + bt$$

where a is the $\log G(t)$ intercept and b the slope. This equation can be written as

$$G(t) = 10^{a + bt}$$

or

$$G(t) = G_0 e^{-\frac{t}{\tau_1}} \quad (4.20)$$

where $G_0 = 10^a$ and $-\frac{1}{\tau_1} = 2.3026b$. Equation 4.20 is actually the form for the relaxation modulus of a Maxwell model where $G_0 = E$, the spring constant, and $\tau_1 = \frac{\eta}{E}$ which is known as the retardation time. Thus, although model theory was not used to develop $G(t)$, the form is identical to that for the Maxwell model. The element constants differ from those computed by the method of Section 4.1, however, because of the difference in method.

Equations of the form given in equation 4.20 were developed for all seven groups of relaxation moduli using the method of averages [45].

Table 4.3 G(t) for Times Taken From Test Frequencies for the Seven Groups

ω (rad/sec)	$t = \frac{1}{\omega}$ (sec x 10^4)	G(t) (psi)						
		Group 1C	Group 2C	Group 3C	Group 4C	Group 5C	Group 4S	Group 5S
314.16	31.83	1779	1725	1886			323	127
502.66	19.89				1351	627	343	147
628.32	15.91	1996	2092	1992				
691.15	14.46				1379	653	362	167
879.65	11.36				1406	678	382	187
942.48	10.61	2172	2372	2097				
1068.14	9.362				1435	704	403	207
1256.64	7.957	2307	2566	2200	1465	730	423	228
1445.14	6.919				1495	755	443	248
1570.80	6.366	2401	2675	2302				
1633.63	1.121				1527	781		
1884.96	5.305	2455	2697	2403				
2293.37	4.360	2464	2597	2533				

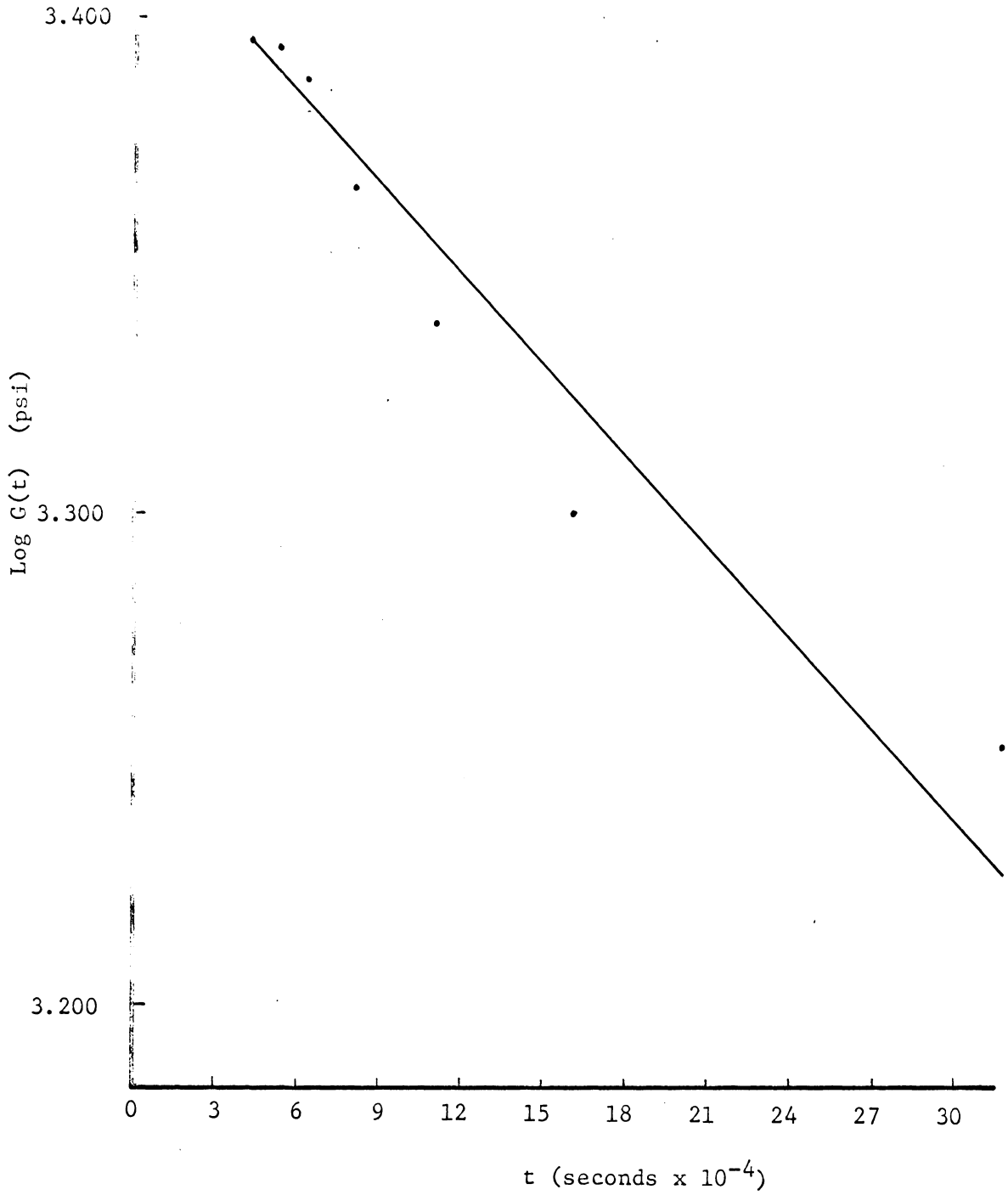


Figure 4.4 $\text{Log } G(t)$ Versus t for Data Group 1C.

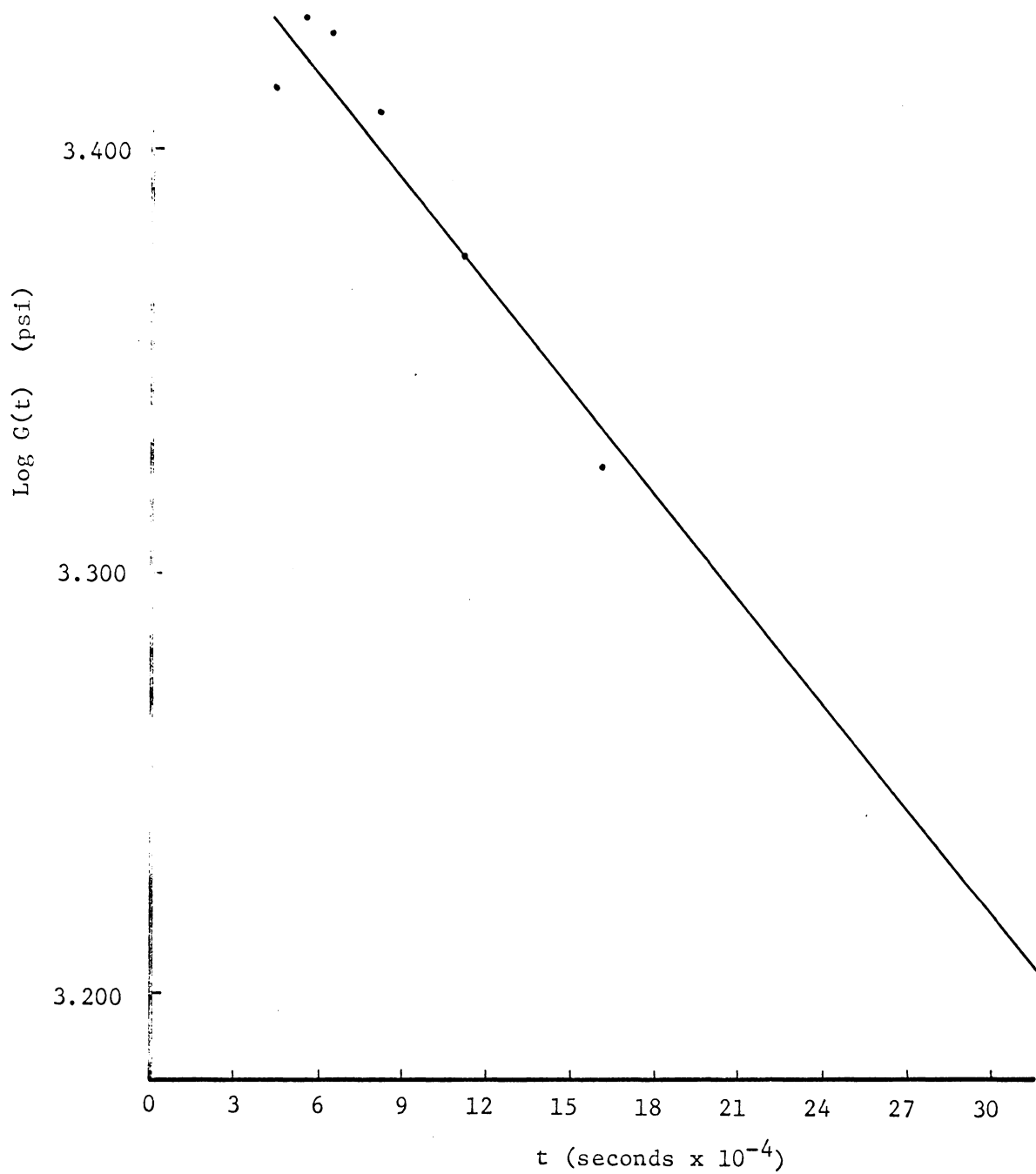


Figure 4.5 $\text{Log } G(t)$ Versus t for Data Group 2C.

Values of G_0 , $\frac{1}{\tau_1}$, and n for these equations are listed in Table 4.4 along with the average per cent deviation of the computed $G(t)$ values from the values of Table 4.3. Comparison of Table 4.4 with Table 4.1 reveals that the equivalent Maxwell models of this section are stiffer than those of Section 4.1. To determine how well the equations of this section represented the original data, Y_1 and Y_2 were computed for these models and are tabulated in Table 4.5. The per cent deviation of these model values from the average experimental values are again given so that Table 4.5 can be compared with Table 4.2. In general, Y_1 values computed from the model of this section are higher than the experimental Y_1 values rather than lower as in the case of the model of Section 4.1. The average deviations for some groups were smaller for the model of Section 4.1 and for some groups the average deviations were smaller for the model of this section.

Equations other than equation 4.20 gave a better representation of the moduli of certain groups in Table 4.3. For example, a graph of $\log G(t)$ versus $\log t$ was more linear for group 1C. This leads to an equation of the form

$$G(t) = at^b \quad (4.21)$$

where a is the antilog of the $\log G(t)$ intercept and b the slope.

Figures 4.6 and 4.7 show graphs of $\log G(t)$ versus $\log t$ for groups 1C and 2C.

The improvement of equation 4.21 over equation 4.20 was not studied further in this work primarily because of the convenience of using equation 4.20 in the impact problem.

Table 4.4 Equation Constants and Deviations for the Seven Groups of Moduli

Moduli group	G_o (psi)	$\frac{1}{\tau_1}$ (sec)	η (psi/sec)	Average per cent deviation of model $G(t)$ values from $G(t)$ values of Table 4.3
1C	2627	146.5	17.93	2.56
2C	3013	199.1	15.13	2.77
3C	2541	125.9	20.19	4.68
4C	1589	93.1	17.06	1.28
5C	842.6	168.0	5.02	2.04
4S	467.1	140.9	3.31	3.87
5S	282.5	264.9	1.07	7.13

Table 4.5 Comparison of Model and Data Moduli

Data group	Frequency (cps)	Y_1 from model (psi)	Y_1 from experimental average (psi)	Deviation (%)	Y_2 from model (psi)	Y_2 from experimental average (psi)	Deviation (%)
1C	50	2157.70	2058.40	4.82	1006.29	703.70	43.00
1C	90	2461.74	2271.80	8.36	637.82	613.30	4.00
1C	150	2565.01	2393.10	7.18	398.75	486.10	-17.97
1C	200	2591.77	2654.00	- 2.34	302.18	499.60	-39.52
1C	250	2604.34	2686.70	- 3.07	242.92	374.70	-35.17
1C	300	2611.22	2475.80	5.17	202.97	402.20	-49.54
1C	365	2616.32	2730.60	- 4.19	167.15	367.60	-54.53
1C	Average*			5.02			34.82
2C	50	2149.42	1863.50	15.34	1362.58	459.80	196.34
2C	95	2711.37	2283.00	18.76	904.64	585.80	54.43
2C	150	2884.41	2733.70	5.51	609.50	539.80	12.91
2C	200	2939.37	2673.00	9.97	465.84	530.80	-12.24
2C	250	2965.53	2791.90	6.22	375.99	410.50	- 8.41
2C	300	2979.94	2932.10	1.63	314.84	534.20	-41.06
2C	365	2990.65	2801.80	6.74	259.71	413.20	-37.15
2C	Average			9.17			51.79
3C	50	2189.72	1986.60	10.22	877.29	618.50	41.84
3C	90	2421.25	2316.00	4.54	538.91	581.90	- 7.39
3C	150	2596.67	2508.10	- 0.46	333.42	469.60	-29.00
3C	200	2515.96	2412.30	4.30	252.00	451.00	-44.12

*The average per cent deviations were computed using absolute values of the per cent deviations for each frequency.

Table 4.5 (Continued)

Data group	Frequency (cps)	Y ₁ from model (psi)	Y ₁ from experimental average (psi)	Deviation (%)	Y ₂ from model (psi)	Y ₂ from experimental average (psi)	Deviation
3C	250	2524.99	2497.20	1.11	202.32	331.20	-38.91
3C	300	2529.92	2648.60	- 4.48	168.93	443.00	-61.87
3C	365	2533.57	2657.00	- 4.65	139.05	383.10	-63.70
3C	Average			4.25			40.98
4C	80	1535.89	1398.70	9.81	284.53	314.60	- 9.56
4C	110	1560.28	1443.70	8.08	210.22	284.90	-26.21
4C	140	1571.00	1217.40	29.05	166.30	223.60	-25.63
4C	170	1576.62	1565.50	0.72	137.45	337.20	-59.28
4C	200	1579.92	1539.90	2.60	117.07	329.70	-64.49
4C	230	1582.03	1649.10	- 4.07	101.94	351.20	-70.97
4C	260	1583.46	1612.30	- 1.79	90.26	214.90	-58.00
4C	Average			8.02			44.88
5C	80	758.07	685.50	10.59	253.14	163.70	54.64
5C	110	795.67	717.80	10.85	193.23	140.30	37.73
5C	140	813.00	748.90	8.56	155.13	122.60	26.53
5C	170	822.29	799.10	2.90	129.22	160.20	-19.34
5C	200	827.83	757.60	9.27	110.57	125.40	-11.83
5C	230	831.38	820.80	1.29	96.56	108.70	-11.17
5C				7.24			26.87

Table 4.5 (Continued)

Data group	Frequency (cps)	Y ₁ from model (psi)	Y ₁ from experimental average (psi)	Deviation (%)	Y ₂ from model (psi)	Y ₂ from experimental average (psi)	Deviation (%)
4S	80	432.97	351.30	23.25	121.56	61.50	97.66
4S	110	448.41	381.30	17.60	91.55	65.20	40.41
4S	140	455.38	424.00	7.40	73.05	63.30	15.40
4S	170	459.09	449.60	2.11	60.65	56.40	7.54
4S	200	461.28	449.80	2.55	51.80	62.30	-16.85
4S	230	462.69	433.90	6.64	45.18	53.40	-15.39
4S	260	463.64	496.90	- 6.69	40.05	47.50	-15.68
4S	Average			9.46			29.84
5S	50	165.57	146.40	13.09	139.14	38.30	263.29
5S	80	221.41	145.00	52.70	116.30	32.80	254.57
5S	110	246.53	229.30	7.51	94.17	40.00	135.43
5S	140	259.15	201.00	28.93	77.78	39.50	96.91
5S	170	266.23	216.30	23.08	65.81	33.70	95.28
5S	200	270.56	263.40	2.72	56.84	39.30	44.63
5S	230	273.38	249.40	9.62	49.94	28.40	75.84
5S	Average			19.66			137.99

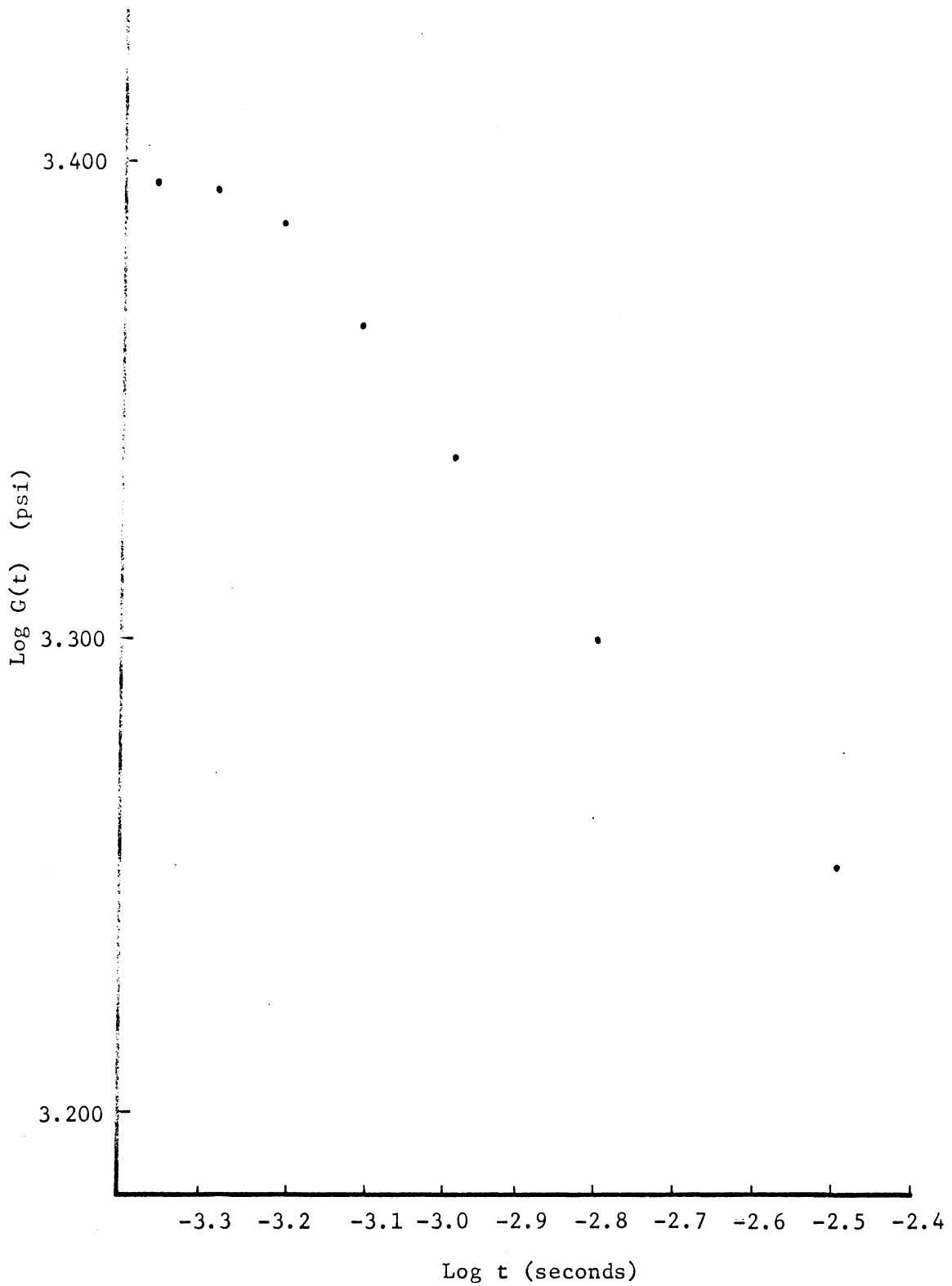


Figure 4.6 $\text{Log } G(t)$ Versus $\text{Log } t$ for Data Group 1C.

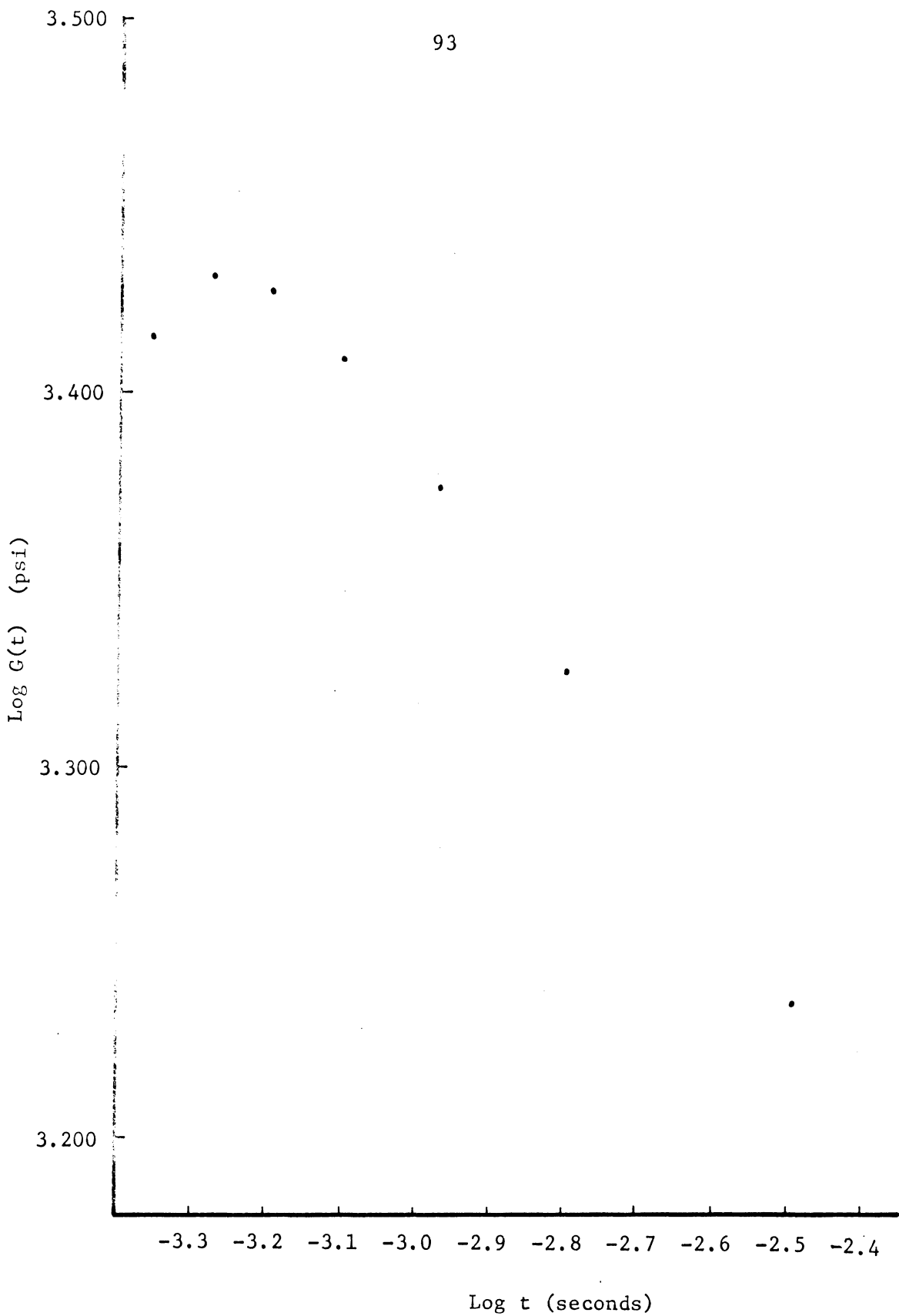


Figure 4.7 $\text{Log } G(t)$ Versus $\text{Log } t$ for Data Group 2C

5. IMPACT OF SPHERICAL FRUIT

5.1 Governing Equations

This solution of the contact problem is based on the work of Yang [8] which will be briefly outlined before extending it. Yang did not consider internal stresses or impact so the extension was made in these areas. In addition to formulating internal stresses, a complete digital computer program was written which reduced the computation of internal stresses caused by impact to reading in physical characteristics of the apple and drop height and using several hours of IBM 7040 computer time.

The main assumptions were discussed in Section 1.4 and only the most pertinent ones will be restated here. It was assumed that the impacting bodies are isotropic and homogeneous. They were assumed linearly viscoelastic in the sense of a uniaxial compression or tension modulus. Poisson's ratio was assumed constant. Thus, the material properties are characterized by a creep relaxation function and a constant in the form

$$\sigma(t) = \int_0^t G(t-\tau) \frac{\partial}{\partial \tau} \epsilon(\tau) d\tau \quad (5.1)$$

$$\left\{ \begin{array}{l} \nu = \text{a constant} \end{array} \right.$$

where ν is Poisson's ratio. Strains and stresses are expressed in Cartesian coordinates at a point. The functions $G(t)$ can be deduced from spring-dashpot model representations or obtained by analysis of direct measurements as done in Section 4.2. Initially the bodies were considered to be arbitrary in shape with smooth surfaces so that friction could

be neglected. The area of contact was considered small in comparison to the dimensions of the bodies and was considered a simply connected region. Indentation of the contact surfaces was assumed monotonically increasing with time.

The Riemann-Stieltjes convolution between two functions $M(t)$ and $N(t)$ is defined as [46]

$$M * N = \int_0^t M(t - \tau) \frac{\partial N}{\partial \tau} d\tau. \quad (5.2)$$

The relaxation functions considered here are considered to be zero for t less than zero. This satisfies situations where the material is undisturbed prior to $t = 0$. The division sign (/) will serve to indicate the inverse operation of $*$, thus,

$$M/N = C$$

signifies

$$M = N * C.$$

Following Timoshenko and Goodier [5], pp. 377-378, surfaces of the undeformed bodies near the center of contact can be described to second-order approximation by

$$\begin{aligned} z_1 &= a_{11} x^2 + a_{12} xy + a_{22} y^2 \\ z_2 &= b_{11} x^2 + b_{12} xy + y^2 \end{aligned} \quad (5.3)$$

where x and y are coordinates lying in the common tangent plane of the

center of contact, and a_{ij} and b_{ij} are constants depending on the curvatures of the bodies. The quantities z_1 and z_2 are coordinates perpendicular to the tangent plane with the positive direction pointing into each respective body. Then by the proper choice of coordinates and adding z_1 and z_2 in equations 5.3, it can be written that

$$z_1 + z_2 = Ax^2 + By^2 \quad (5.4)$$

where A and B are constants depending on the principal radii of curvature (R_1, R'_1 and R_2, R'_2) of the two bodies at the center of contact in such a manner that

$$2 [A + B] = \frac{1}{R_1} + \frac{1}{R'_1} + \frac{1}{R_2} + \frac{1}{R'_2}$$

and

(5.5)

$$4 [A - B]^2 = \left[\frac{1}{R_1} - \frac{1}{R'_1} \right]^2 + \left[\frac{1}{R_2} - \frac{1}{R'_2} \right]^2 + \left[\frac{1}{R_1} - \frac{1}{R'_1} \right] \left[\frac{1}{R_2} - \frac{1}{R'_2} \right] 2 \cos 2\phi.$$

The angle ϕ is between the normal planes containing the curvatures $1/R_1$ and $1/R_2$.

Using the designations that w_1 and w_2 are the deformations of the two bodies due to contact pressure and α is the total distance of approach of the centers of the two bodies after initial contact, it can be written from Figure 5.1 that

$$w_1 + w_2 = \alpha - [z_1 + z_2]. \quad (5.6)$$

Substituting $z_1 + z_2$ from equation 5.4 into equation 5.6, the result is

$$w_1 + w_2 = \alpha - Ax^2 - by^2. \quad (5.7)$$

The assumption of a small contact area then makes it possible to solve the problem approximately by obtaining the solution for surface deflection of a half space with pressure distributed on a portion of the

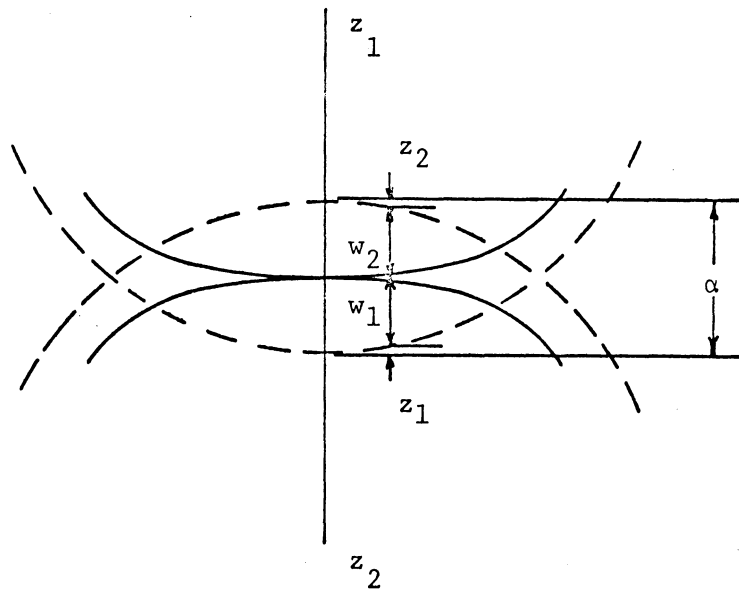


Figure 5.1 Deformed Configurations

surface. For an elemental force, pdA , applied at a point $P(x,y)$ on the surface of an elastic body, the deflection of a point $P(x,y)$ on the surface is given by Yang as

$$dw(x,y) = \frac{1}{\pi} \frac{k \, pdA}{L} \quad (5.8)$$

where k is an elastic constant and L is the distance between P and P' . The boundary of the area dA is fixed for all times within the restriction of infinitesimal deformation theory, therefore, the viscoelastic solution can be obtained by the Laplace transform as

$$dw = \frac{1}{\pi} k * \frac{pdA}{L} \quad (5.9)$$

where k characterizes the material properties of the viscoelastic body.

In terms of the relaxation modulus in axial compression and Poisson's ratio, k is written as

$$k(t) = \frac{1 - \nu^2}{G(t)}. \quad (5.10)$$

For two bodies of different materials, the surface displacements for monotonically increasing indentation are respectively

$$w_1(x, y, t) = \frac{1}{\pi} \iint_{A(t)} k_1 * p \frac{dA'}{L} \quad (5.11)$$

and

$$w_2(x, y, t) = \frac{1}{\pi} \iint_{A(t)} k_2 * p \frac{dA'}{L}$$

where the primed dA indicates an element in a primed coordinate system.

Substituting equations 5.11 into 5.7 yields

$$\frac{1}{\pi} \iint_{A(t)} [k_1 + k_2] * \frac{pdA'}{L} = \alpha(t) - Ax^2 - By^2. \quad (5.12)$$

Yang proceeded to use the Hertz method of solving equation 5.12 by analogy to the potential of a uniformly charged ellipsoid whose principal semi-axes are a , b , and c with the third semiaxis c approaching zero. He thus reduced the problem to two simultaneous integral equations which he solved by direct quadratures using a digital computer. He left his solution in the form

$$a^2(t) = \frac{1}{\lambda \psi} \alpha(t), \quad (5.13)$$

$$b^2(t) = \frac{\lambda}{\psi} \alpha(t), \quad (5.14)$$

and

$$\lambda = \frac{b(t)}{a(t)} \quad (5.15)$$

where ψ is a coefficient depending on A and B and a constant multiplier μ . Yang includes a nomograph giving values of ψ/B , μ/B , and λ for B/A values from 0.0 to 1.0. Thus, A and B can be determined from equations 5.5 after which ψ , μ , and λ can be determined from Yang's nomograph, and $a(t)$ and $b(t)$ can be written in terms of $\alpha(t)$ from equations 5.13 and 5.14. This defines the elliptic contact region in terms of $\alpha(t)$.

In making a term by term equivalence between equation 5.12 and the equation for an uniformly charged ellipsoid, there was obtained the equation for surface pressure in the form

$$p(x,y,t) = \mu \int_0^t R_p(t-\tau) \frac{\partial}{\partial \tau} g(x,y,\tau) d\tau \quad (5.16)$$

where

$$R_p(t) = \frac{1}{k_1 + k_2}, \quad (5.17)$$

$$g(x,y,t) = [a^2(t) - x^2 - \frac{1}{\lambda^2} y^2]^{\frac{1}{2}}, \quad (5.18)$$

and it was assumed that discontinuities at time = 0.0 do not exist. This form is suitable for solving for the pressure distribution on the surface if $a(t)$ is known or if $\alpha(t)$ is known since $a(t)$ can be found from $\alpha(t)$ by equation 5.13.

Displacements on the contact surface were found by substituting equation 5.16 into equations 5.11 after introducing

$$R_{w_1} = \frac{k_1}{k_1 + k_2}$$

and (5.19)

$$R_{w_2} = \frac{k_2}{k_1 + k_2}.$$

The result for w_1 was

$$w_1(x,y,t) = \frac{\mu}{\pi} \iint_{A(t)} \int_0^t R_{w_1}(t-\tau) \frac{\partial}{\partial \tau} g(x',y',\tau) d\tau \frac{dA'}{L}. \quad (5.20)$$

The expression for w_2 is identical except R_{w_2} replaces R_{w_1} . Integration over $A(t)$ can be replaced by integration over $A(\tau)$ since $g(x,y,\tau)$ is equal to zero outside the contact area $A(\tau)$ at time τ . Using equations developed from his previous analogy Yang was able to write

$$\iint_{A(\tau)} g(x', y', \tau) \frac{dA'}{L} = \alpha(\tau) - Ax'^2 - By'^2 = f(x, y, \tau), \quad (5.21)$$

so finally

$$w_1(x, y, t) = \int_0^t R_{w_1}(t - \tau) \frac{\partial}{\partial \tau} f(x, y, \tau) d\tau \quad (5.22)$$

and

$$w_2(x, y, t) = \int_0^t R_{w_2}(t - \tau) \frac{\partial}{\partial \tau} f(x, y, \tau) d\tau.$$

Equations 5.22 along with equation 5.21 are suitable for solving for the displacement if $\alpha(t)$ is known.

It remained to determine $\alpha(t)$ in terms of the total compressive force if contact problems were to be solved. Yang was able to write, from the analogy with the charged ellipsoid,

$$\begin{aligned} \iint_{A(t)} [k_1 + k_2] * p(x', y', t) dx' dy' \\ = \mu \iiint_{A(t)} \left[a^2(t) - x'^2 - \frac{1}{\lambda^2} y'^2 \right]^{\frac{1}{2}} dx' dy'. \end{aligned} \quad (5.23)$$

The integral on the right side of equation 5.23 is equal to one half the volume of an ellipsoid with axes $a(t)$, $a(t)$, and $\lambda a(t)$. The left side can be simplified if it is noted that

$$\iint_{A(t)} p(x', y', t) dx' dy' = P(t)$$

where $P(t)$ is the total contact force on the body. Thus, equation 5.23

can be written as

$$[k_1 + k_2] * P(t) = \frac{2}{3} \pi \lambda \mu [a(t)]^3$$

or in integral form

$$[a(t)]^3 = \frac{3}{2\pi\lambda\mu} \int_0^t [k_1(t-\tau) + k_2(t-\tau)] \frac{\partial}{\partial \tau} P(\tau) d\tau. \quad (5.24)$$

Equations 5.24, 5.22, and 5.16 are suitable equations for obtaining the approach of the bodies $\alpha(t)$, the displacements w_1 and w_2 , and the surface pressure $p(t)$. It remains to formulate expressions for the internal stresses.

The formulation of expressions for internal stresses was based on the equations for the internal stresses in a half space when subjected to a point load. Timoshenko and Goodier [5] give these equations

$$\sigma_{zz} = -\frac{3P}{2\pi} z^3 [s^2 + z^2]^{-\frac{5}{2}}, \quad (5.25)$$

$$\sigma_{ss} = \frac{P}{2\pi} \left\{ [1 - 2\nu] \left[\frac{1}{s^2} - \frac{z}{s^2} (s^2 + z^2)^{-\frac{1}{2}} \right] - 3s^2 z (s^2 + z^2)^{-\frac{5}{2}} \right\} \quad (5.26)$$

$$\sigma_{\phi\phi} = \frac{P}{2\pi} [1 - 2\nu] \left\{ -\frac{1}{s^2} + \frac{z}{s^2} [s^2 + z^2]^{-\frac{1}{2}} + z [s^2 + z^2]^{-\frac{3}{2}} \right\} \quad (5.27)$$

and

$$\sigma_{sz} = -\frac{3P}{2\pi} \cdot s z^2 \left[s^2 + z^2 \right]^{-\frac{5}{2}} \quad (5.28)$$

where s is the radial cylindrical coordinate, ϕ is the angular cylindrical coordinate, z is the axial coordinate positive into the half space, and P is a point load.

Equations 5.25 through 5.28 were suitable for computing the stresses since only equations 5.26 and 5.27 depend on the elastic properties of the material and this dependency is on Poisson's ratio which has been considered constant in the present work. To apply these equations, it was necessary to replace the point load, P , by an elemental load and then integrate over the contact surface. This necessitated a coordinate transformation since the origin of the coordinate system for equations 5.25 through 5.28 is the point load and the origin of the desired system is the center of the contact circle. Figure 5.2 shows the geometry of the problem. The point M is where the stresses are to be computed at some depth z , $r dr d\theta$ is the elemental area, s is the distance from the elemental area to point M , and c is the distance from the center of the contact surface to M . The σ_{zz} stress can be formulated by replacing the point load P , in equation 5.25, by the elemental load $P(r,t) r dr d\theta$ where $P(r,t)$ is the solution of equation 5.16, and by writing s in terms of r , c , and θ using the law of cosines so that

$$s = [r^2 + c^2 - 2rc \cos\theta]^{\frac{1}{2}} \quad (5.29)$$

No coordinate transformation is needed in this case since the z direction

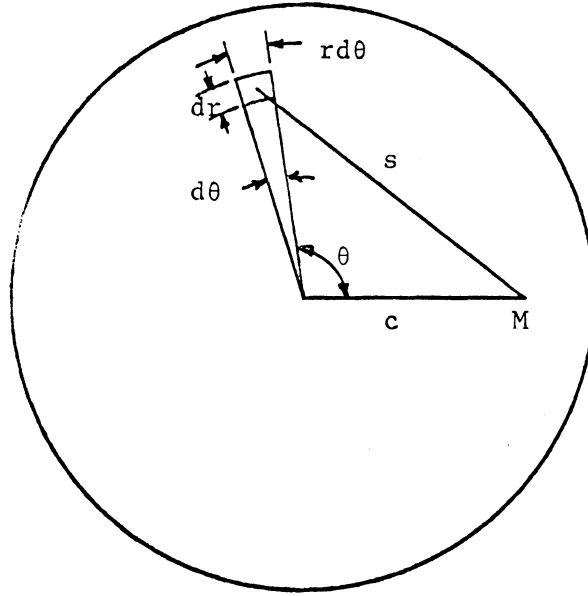


Figure 5.2 Geometry of the Elemental Loading

is the same in both systems. For a given stress location, c and z are constants and the stress equation can be written as

$$\sigma_{zz} = - \frac{3z^3}{2\pi} \int_0^{\text{Bdry}} \int_0^{2\pi} p(r,t) r \left[r^2 + c^2 \cos\theta + z^2 \right]^{-\frac{5}{2}} d\theta dr \quad (5.30)$$

where Bdry is the boundary of the contact surface. For stresses on the z axis, c equals zero and the equation reduces to

$$\sigma_{zz} = - 3z^3 \int_0^{\text{Bdry}} p(r,t) r \left[r^2 + z^2 \right]^{-\frac{5}{2}} dr. \quad (5.31)$$

Suitable formulations of the other stresses can be obtained by considering the coordinate system with its origin at the elemental load as an unprimed coordinate system and the coordinate system with its origin at the center of the contact surface as a primed coordinate system. The designations then are

$$\begin{aligned}x_1 &= s, \\x_2 &= \phi \\x_3 &= z,\end{aligned}\tag{5.32}$$

and

$$\begin{aligned}x'_1 &= c, \\x'_2 &= \theta, \\x'_3 &= z.\end{aligned}\tag{5.33}$$

The coordinate transformation is then given by the transformation equations for Cartesian tensors [47] written as

$$\sigma_{ij}' = a_{ir} a_{js} \sigma_{rs}$$

where a_{ij} are the direction cosines. They can be written from Figure 5.3 as

$$a_{ij} \Rightarrow \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} .\tag{5.35}$$

Inspection of equations 5.25 through 5.28 reveals that

$$\sigma_{12} = \sigma_{21} = \sigma_{23} = \sigma_{32} = 0,$$

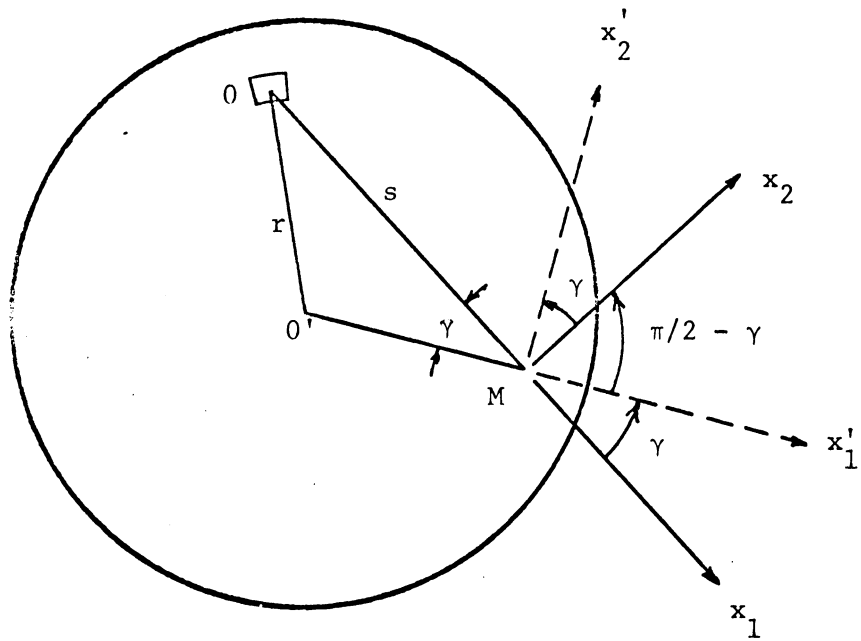


Figure 5.3 The Primed and Unprimed Coordinate Systems.

so expansion of the tranformation equation, equation 5.24, results in

$$\sigma_{11}' = \cos^2 \gamma \sigma_{11} + \sin^2 \gamma \sigma_{22},$$

$$\sigma_{22}' = \sin^2 \gamma \sigma_{11} + \cos^2 \gamma \sigma_{22},$$

$$\sigma_{12}' = \sin \gamma \cos \gamma [\sigma_{22} - \sigma_{11}], \quad (5.36)$$

$$\sigma_{13}' = \cos \gamma \sigma_{13},$$

$$\sigma_{23}' = \sin \gamma \sigma_{13}, \text{ and}$$

$$\sigma_{33}' = \sigma_{33}.$$

The last equation verifies the reasoning used in developing equation 5.30 for σ_{zz} . The expressions for σ_{11} , σ_{22} , σ_{33} , and σ_{13} from equations 5.25 through 5.28 can be substituted into equations 5.36 if the point force P is again replaced by the elemental force. Then, if the c , θ , z notation is again used, the results are

$$\sigma_{cc} = \frac{1}{2\pi} \int_0^{\text{Bdry}} \int_0^{2\pi} rp(r,t) \left\{ \left[1 - 2\nu \right] \left[\frac{1}{s^2} - \frac{z}{s^2(s^2 + z^2)^{\frac{1}{2}}} \right] - \frac{3s^2 z}{[s^2 + z^2]^{\frac{5}{2}}} \right\} \cos^2 \gamma$$

$$\begin{aligned}
 & + (1 - 2\nu) \left\{ -\frac{1}{s^2} + \frac{z}{s^2 [s^2 + z^2]^{\frac{1}{2}}} + \frac{z}{[s^2 + z^2]^{\frac{3}{2}}} \right\} \\
 & \left. \sin^2 \gamma \right\} d\theta dr, \quad (5.37) \\
 \sigma_{\theta\theta} = & \frac{1}{2\pi} \int_0^{\text{Bdry}} \int_0^{2\pi} rp(r,t) \left\{ [1 - 2\nu] \left[\frac{1}{s^2} - \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \frac{z}{s^2 (s^2 + z^2)^{\frac{1}{2}}} \right] - \frac{3s^2 z}{[s^2 + z^2]^{\frac{5}{2}}} \left. \right\} \sin^2 \gamma \\
 & + [1 - 2\nu] \left\{ -\frac{1}{s^2} + \frac{z}{s^2 [s^2 + z^2]^{\frac{1}{2}}} + \frac{z}{[s^2 + z^2]^{\frac{3}{2}}} \right\} \\
 & \left. \cos^2 \gamma \right\} d\theta dr, \quad (5.38) \\
 \sigma_{cz} = & -\frac{3}{2\pi} \int_0^{\text{Bdry}} \int_0^{2\pi} rp(r,t) \frac{sz^2}{[s^2 + z^2]^{\frac{5}{2}}}
 \end{aligned}$$

$$\cos \gamma d\theta dr, \quad (5.39)$$

and similar expressions for $\sigma_{\theta z}$ and $\sigma_{c\theta}$. The angle γ can be written in terms of r , s , and θ using the law of sines. Referring to Figure 5.3, this expression is

$$\gamma = \sin^{-1} \left[\frac{r}{s} \sin \theta \right]. \quad (5.40)$$

The quantity s is given by equation 5.29 in terms of r , c , and θ . Equations 5.29 and 5.40, along with any one of the stress equations is a suitable set of equations for solving for that stress provided that the boundary of the contact surface at time t is known along with the surface pressure $p(r,t)$.

5.2 Impact of Two Identical Apples Considered as Maxwell Spheres

The geometry of two bodies in contact has been shown in Figure 5.1.

For $R_1 = R'_1 = R_2 = R'_2 = R$, equations 5.5 become

$$A + B = 2/R$$

and

$$A - B = 0,$$

therefore,

$$A = B = 1/R. \tag{5.41}$$

From the nomograph of Yang for $B/A = 1$,

$$\lambda = 1,$$

$$\psi/B = 2, \text{ and} \tag{5.42}$$

$$\mu/B = 1.2732.$$

Substituting B from equation 5.51 into equation 5.26, the result is

$$\lambda = 1,$$

$$\psi = 2/R, \text{ and} \tag{5.43}$$

$$\mu = 1.2732/R.$$

From equation 5.13,

$$a^2(t) = \frac{R}{2} \alpha(t). \quad (5.44)$$

The properties of the apple flesh are given in terms of k_1 and k_2 by equation 5.10 as

$$k_1 = k_2 = \frac{1 - v^2}{G(t)}. \quad (5.45)$$

Then, R_{w_1} and R_{w_2} can be determined from equations 5.19 using the Laplace transform. Equations 5.19 can be written as

$$[k_1 + k_2] * R_{w_1} = k_1$$

$$\text{and} \quad (5.46)$$

$$[k_1 + k_2] * R_{w_2} = k_2,$$

but

$$k_1 = k_2 = k,$$

so equations 5.46 become

$$2k * R_{w_1} = k$$

$$\text{and} \quad (5.47)$$

$$2k * R_{w_2} = k.$$

Then, using the Laplace transform, the first of equations 5.47 becomes

$$2\bar{k}(s) * s\bar{R}_{w_1}(s) = k(s) \quad (5.48)$$

where $\bar{k}(s)$ indicates the Laplace transform of k when s is the transform variable and similarly for $\bar{R}_{w_1}(s)$. Rearranging equation 5.48 and taking the inverse transform the result is

$$R_{w_1} = \frac{1}{2},$$

and similarly

$$R_{w_2} = \frac{1}{2}.$$

In preparation for writing the surface pressure distribution equation, $R_p(t)$ can be written from equation 5.17 as

$$R_p(t) = \frac{1}{2} \frac{G(t)}{1 - v^2},$$

and for a Maxwell material, $G(t)$ is given by equation 4.20, so the above equation can be written as

$$R_p(t) = \frac{G_o}{2[1 - v^2]} e^{-\frac{t}{\tau_1}}. \quad (5.49)$$

For a sphere, equation 5.18 reduces to

$$g(r,t) = \left[\frac{R}{2} \alpha(t) - r^2 \right]^{\frac{1}{2}} \quad (5.50)$$

where $r^2 = x^2 + y^2$. Substituting equations 5.43, 5.49, and 5.50 into equation 5.16, the surface pressure distribution is given as

$$p(r,t) = \frac{0.637 G_o}{R [1 - v^2]} \int_0^t e^{-\frac{t - \tau}{\tau_1}} \frac{\partial}{\partial \tau} \left[\frac{R}{2} \alpha(\tau) - r^2 \right]^{\frac{1}{2}} d\tau. \quad (5.51)$$

The surface displacements can be written from equations 5.22 as

$$w_1 = w_2 = \frac{1}{2} \left[\alpha(t) - \frac{r^2}{R} \right]. \quad (5.52)$$

Equation 5.24 relates total force and the contact radius, $a(t)$, and therefore is the place to introduce the impact force and weight. For the present problem, it is assumed that one apple falls onto another, which is stationary, supported by other apples or some other means beneath. The total contact force can be computed following Timoshenko and Goodier [5], p. 383. Designating the apple in motion by the subscript one and the stationary apple by the subscript two, the equations of motion are

$$M_1 \dot{V}_1 = - P',$$

and

$$M_2 \dot{V}_2 = - P'$$

where M indicates mass, $\dot{V}$ is the time derivative of velocity, and P' is the force between the apples due to acceleration. Then,

$$\ddot{\alpha} = \dot{V}_1 + \dot{V}_2 = - P' \left[\frac{1}{M_1} + \frac{1}{M_2} \right],$$

but, in effect, the apple that is stationary has infinite mass so that

$$P' = - M_1 \ddot{\alpha}.$$

The weight, W , of the falling apple will have a small effect on the force between the apples so the total force between apples is given by

$$P = - M_1 \ddot{\alpha} + W. \quad (5.53)$$

Equation 5.24 contains the expression

$$k_1(t - \tau) + k_2(t - \tau)$$

which, in this problem, reduces to $2k$. For a Maxwell material, k can be determined using the Laplace transform on equation 5.10 with $G(t)$ replaced from equation 4.20. Thus,

$$\frac{1}{k} = \frac{G_0}{1 - \nu^2} e^{-\frac{t}{\tau_1}},$$

or

$$\frac{1}{s} = \bar{k}(s) * \frac{G_0}{1 - \nu^2} \frac{s}{s + \frac{1}{\tau_1}},$$

or

$$\bar{k}(s) = \frac{1 - \nu^2}{G_0} \left[\frac{1}{s} + \frac{1}{\tau_1} / s^2 \right],$$

or, finally,

$$k(t) = \frac{1 - \nu^2}{G_0} \left[1 + \frac{t}{\tau_1} \right]. \quad (5.54)$$

Substituting equations 5.53 and 5.54 into equation 5.24 and replacing $[a(t)]^3$ by $Ra(t)/2$ from equation 5.44, the result is

$$-\left[\alpha(t)\right]^{\frac{3}{2}} = C_1' \int_0^t \left[1 - \frac{t - \tau}{\tau_1} \right] \frac{\partial}{\partial \tau} \left[\frac{M_1 \ddot{\alpha}}{2} - W \right] d\tau \quad (5.55)$$

where

$$C_1' = \frac{3[1 - v^2]^{\frac{1}{2}}}{G_0[2R]}. \quad (5.56)$$

Equation 5.55 reduces to

$$-\left[\alpha(t)\right]^{\frac{3}{2}} = C_1' \left\{ M_1 [\dot{\alpha}(t) - \dot{\alpha}(0)] + \int_0^t \frac{t - \tau}{\tau_1} \frac{\partial}{\partial \tau} [M_1 \dot{\alpha}(\tau) - W] d\tau \right\}. \quad (5.57)$$

The terms in a Riemann-Stieltjes convolution integral can be interchanged if, for time equal to zero, the terms are zero. This is true for the problem at hand since

$$\left. \frac{t}{\tau_1} \right|_{t=0} = 0 \text{ and}$$

$$\left. [M_1 \dot{\alpha}(t) - W] \right|_{t=0} = M_1 g - W = 0.$$

The integral term of equation 5.57 can then be written as

$$\int_0^t \frac{M_1 \dot{\alpha}(t - \tau) - W}{\tau_1} d\tau$$

which, upon integrating, becomes

$$\frac{M_1 \dot{\alpha}(t)}{\tau_1} - \frac{M_1 \dot{\alpha}(0)}{\tau_1} - Wt.$$

Initial conditions are $\dot{\alpha}(0) = g$ and $\ddot{\alpha}(0) = V$ where V is the impact

velocity. Insertion of the initial conditons into equation 5.57 and rearranging gives the result

$$\ddot{\alpha}(t) + \frac{\dot{\alpha}(t)}{\tau_1} + C_1 \left[\alpha(t) \right]^{\frac{3}{2}} = \frac{V}{\tau_1} + \frac{g}{\tau_1} [\tau_1 + t] \quad (5.58)$$

where

$$C_1 = \frac{1}{C_1' M_1} = \frac{[2R]^{\frac{1}{2}} G_o}{3M_1 [1 - \nu^2]} \quad (5.59)$$

The spherical impact problem then consists of solving equation 5.58 for the approach of the apple centers $\alpha(t)$ after which equation 5.52 can be used to solve for surface displacements and equation 5.51 used to solve for the surface pressure. Internal stresses can then be obtained from the stress equations 5.30, 5.37, 5.38, and 5.39 since $p(r,t)$ is known and the radius to the circular contact boundary is given by equation 5.44 as

$$a(t) = \left[\frac{R}{2} \alpha(t) \right] \quad (5.60)$$

Equation 5.58 is a nonlinear second order differential equation with the nonlinearity being due to the exponent of $\alpha(t)$. Since it was not possible to obtain a closed form solution of this equation, numerical methods were used. The solution was obtained on the V.P.I. IBM 7040 digital computer using the Runge-Kutta-Gill technique [48] for the approximate solution of an initial value problem involving N first order differential equations. The actual program was taken from the IBM 1620

General Program Library and modified for the present problem. The modified program is given in Appendix C.1.

Equation 5.58 can be written as two simultaneous equations in the form

$$\begin{aligned}\dot{Q}(t) &= -\frac{1}{\tau_1} Q - c_1 L^{\frac{3}{2}} + \frac{V}{\tau_1} + \frac{g}{\tau_1} [\tau_1 + t] \\ \dot{L} &= Q\end{aligned}\tag{5.61}$$

where

$$Q = \dot{\alpha}(t) \text{ and}$$

$$L = \alpha(t).$$

This was the form introduced into the computer program. The output was modified so that $\alpha(t)$, velocity, and acceleration were given for specified times after initial contact. Throughout the discussion of the contact problem, it has been assumed that $\alpha(t)$ is monotonically increasing. Therefore, the solution is valid only to the time of maximum indentation after which $\alpha(t)$ is negative. This change in sign terminates the program. An example of the program output is shown in Appendix D.1. These results are for the Red Delicious apple group, 1C, with an apple of 0.342 pound weight and 1.44 inch radius falling from rest two inches onto another apple. All output was stored in the form of subscripted variables for recall and use in computing surface pressures and stresses.

The second phase in the solution was solving equation 5.51 for the surface pressure distribution over the contact area. Since $\alpha(t)$ was stored and available, it was convenient to integrate the right side of the equation numerically using the general Simpson rule [49]. First,

however, it was necessary to write equation 5.51 in a form more amenable to numerical computation. To do this, the variable Y was defined as

$$Y = \left[\frac{R}{2} \alpha(\tau) - r^2 \right]^{\frac{1}{2}}. \quad (5.62)$$

Equation 5.51 then could be written as

$$p(r,t) = C_2 e^{-\frac{t}{\tau_1}} \int_0^{\left[\frac{R}{2} \alpha(t) - r^2 \right]^{\frac{1}{2}}} e^{\frac{\tau}{\tau_1}} dY \quad (5.63)$$

where

$$C_2 = \frac{0.637 G_0}{R [1 - v^2]}. \quad (5.64)$$

The lower limit of integration is taken as zero as result of the restriction that only real values of $p(r,t)$ be computed.

The program written to integrate equation 5.63 is given in Appendix C.2. The integration was accomplished by first selecting the integrating interval, H , the initial radius value, $r(1)$, and using the lower limit $Y = 0.0$. Then, using the definition of Y (equation 5.62), $\alpha(\tau)$ was computed by

$$\alpha(\tau) = \frac{2[Y^2 + r(i)^2]}{R} \quad (5.65)$$

where i takes on the values 1, 2, 3, etc., and indicates the magnitude of the radius at which the pressure is being computed. For the $\alpha(\tau)$ thus computed, τ was obtained from the results of the Runge-Kutta-Gill computation using DO and IF statements and interpolation. The reasoning process

will be outlined here. First, it was necessary to inspect the first value of $\alpha(t)$ stored. If $\alpha(\tau)$ was greater than $\alpha(t)$, the next stored $\alpha(t)$ value was observed. This process was repeated until $\alpha(\tau)$ was less than the stored $\alpha(t)$ value being considered. In this situation, interpolation between the two $\alpha(t)$ values bracketed began and the value of τ corresponding to $\alpha(\tau)$ found. The value of τ thus found was then substituted into equation 5.63 and the first term of the Simpson rule computed. The value of Y was then increased by the integration interval and the process repeated to obtain the second term of the Simpson rule. Thus, by repeating this process, all the terms of the general Simpson rule were computed and the surface pressure at time, $t(1)$, and radius, $r(1)$, determined. The whole process was repeated for new values of $r(i)$ and time so that the surface pressure was computed for all specified radii within the contact circle at all specified times up to maximum indentation. Pressures for all specified radii and times were again stored as subscripted variables so they would be accessible for use in succeeding operations. Example results of the program output are given in Appendix D.2 for the same apple as described for Appendix D.1.

Internal stresses were computed from equations 5.30, 5.37, 5.38, and 5.39 by writing a special double numerical integration program for addition to the programs already discussed. Values of time, r , and $p(r,t)$ are called for by their subscripted variable name and the radius to the outer boundary is given by equation 5.60 as $[R/2\alpha(t)]^{\frac{1}{2}}$. The flow chart for this program is shown in Figure 5.4. For initial time, z , and c values specified, the program computes the stress due to the

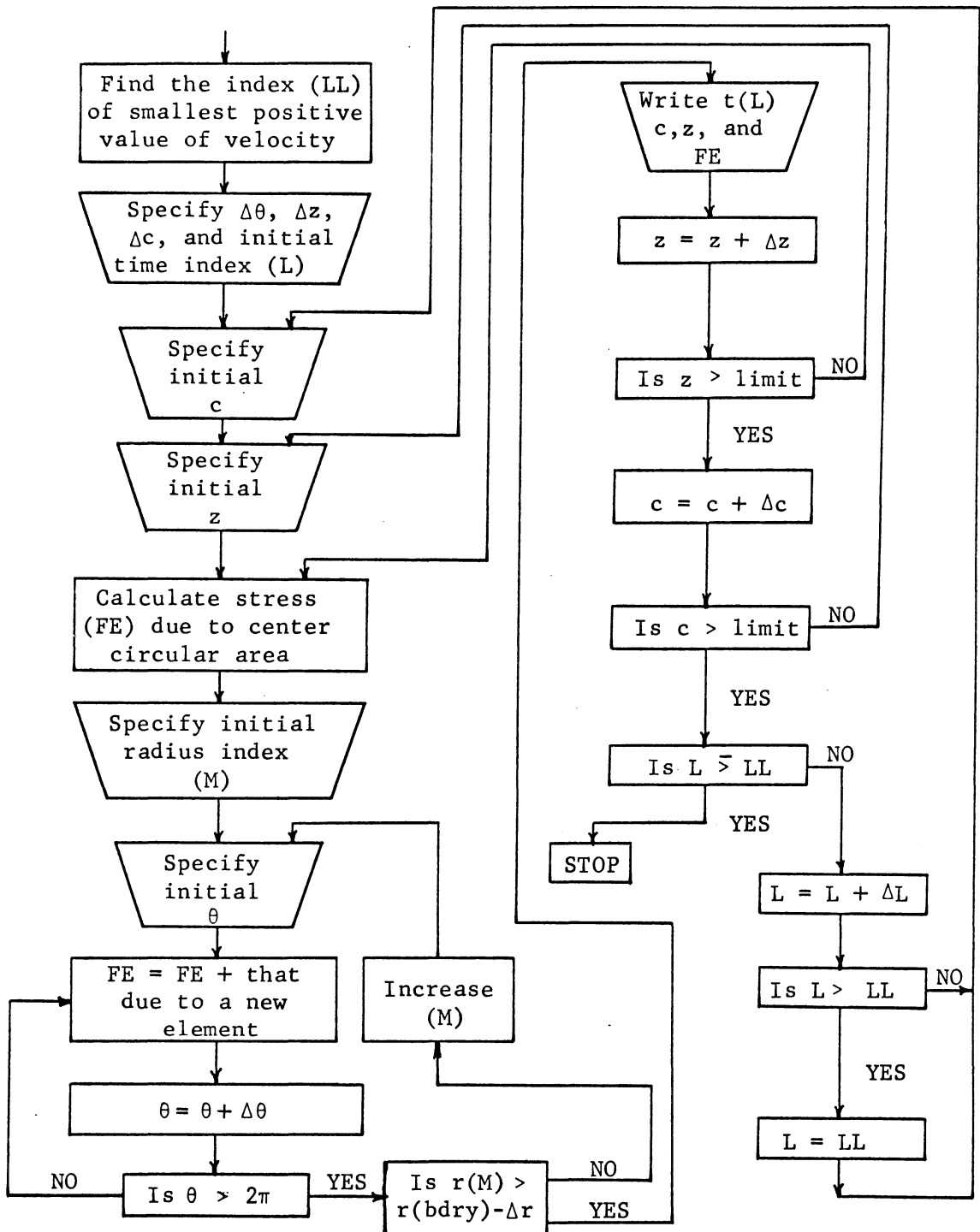


Figure 5.4 Flow Chart for Integration of the Stress Equations.

pressure on the circular area from r equals zero to r equals one half the distance to the first radius value, $r(1)$. Then going to $r(1)$ and $\theta(1)$, the effect of the element $r(1) \Delta r \Delta \theta$ is added in. Proceeding around the annulus by increasing θ , the stress due to each element is computed and added to the total stress. The radius is then increased to $r(2)$ and the process repeated with each stress being added to the previous total. Increasing r and integrating around the annulus continues until r equals, or exceeds, the radius of the contact area at which time the stress is printed. A new depth z is then specified and the whole previous process repeated. New z values are used until the stress is computed to a sufficient depth at which time z returns to its original value and the whole previous process repeated using a larger radius, c . Thus, by continuing in this manner and increasing c and then t , the result is a listing of the stress for each time, depth, and radius specified. Appendix C.3 gives the program for computing σ_{cz} as an example. Results of computing σ_{zz} , σ_{cc} , $\sigma_{\theta\theta}$, and σ_{cz} for the apple and conditions specified for Appendix D.1 are given in Appendix D.3.

The $\sigma_{\theta z}$ and $\sigma_{c\theta}$ stresses were not computed for the impact of two spheres since the pressure on the contact surface is symmetric in the sense that the contact pressure is not dependent on θ . This would lead one to suspect that $\sigma_{\theta z}$ and $\sigma_{c\theta}$ equal zero at all points. Such can be verified by inspecting the angle γ . This reveals that stress expressions multiplied by $\sin\gamma$ or $\sin\gamma \cos\gamma$ will go to zero when summed over the area of integration. From $\gamma = 0.0$ to $\gamma = \pi$, $\sin\gamma$ is positive. For each positive value multiplied by the distance to an elemental area, there

is an equal negative $\sin\gamma$ multiplied by an equal distance to an elemental area that has an identical surface pressure. Therefore, the effects of the two elemental areas cancel each other. The effect of $\sin\gamma \cos\gamma$ is the same except that the signs alternate as γ is increased through the four quadrants. Equations 5.36 show that the expression for $\sigma_{c\theta}$ is multiplied by $\sin\gamma \cos\gamma$ and the expression for $\sigma_{\theta z}$ by $\sin\gamma$ thus indicating that these stresses are equal to zero for a symmetric contact pressure distribution.

Surface displacements on the contact surface are given by equation 5.52 which is readily solved by simple algebra when values of $\alpha(t)$ are introduced from the computer solution.

5.3 Impact of a Maxwell Sphere With a Rigid Half Space

The analysis of the impact of a Maxwell sphere with a rigid half space is nearly identical with that for two spheres so only the differences will be discussed. The sphere will be denoted by the subscript one and the half space by the subscript two. The radius of the sphere will be designated as R .

The geometry of this problem is somewhat different from that of two spheres and equations 5.5 reduce to

$$A + B = \frac{1}{R}$$

and

$$A - B = 0$$

with the result

$$A = B = \frac{1}{2R}. \quad (5.66)$$

this, then, determines values of ψ and μ as

$$\psi = \frac{1}{R} \quad \text{and}$$

$$\mu = \frac{.637}{R},$$

so that equation 5.13 reduces to

$$a^2(t) = R \alpha(t). \quad (5.67)$$

Equations 5.19 give

$$R_{w_1}(t) = \frac{k_1(t)}{k_1(t)} = 1$$

and

$$R_{w_2}(t) = 0.$$

From equation 5.17,

$$R_p(t) = \frac{1}{k_1} = \frac{G_o}{2[1 - \nu^2]} e^{-\frac{t}{\tau_1}}. \quad (5.68)$$

Following the procedure used for two spheres, the equation for surface displacements of the sphere in the contact region is

$$w_1(r, t) = \alpha(t) - \frac{r^2}{2R}, \quad (5.69)$$

and the equation for the surface pressure is

$$p(r,t) = C_2 \int_0^t e^{-\frac{t-\tau}{\tau_1}} \frac{\partial}{\partial \tau} [R\alpha(\tau) - r^2]^{\frac{1}{2}} d\tau. \quad (5.70)$$

Using the change in variable

$$D = \sqrt{R\alpha(\tau) - r^2},$$

equation 5.70 can be written as

$$p(r,t) = C_2 e^{-\frac{t}{\tau_1}} \int_0^{\sqrt{R\alpha(t) - r^2}} e^{\frac{D}{\tau_1}} dD. \quad (5.71)$$

The approach of the center of the sphere toward the rigid half space is equal to the indentation of the center of the contact surface, and can be expressed in terms of a differential equation identical to equation 5.58 except that a different constant C_3 replaces C_1 or

$$\ddot{\alpha}(t) + \frac{\dot{\alpha}(t)}{1} + C_3 \left[\alpha(t) \right]^{\frac{3}{2}} = \frac{V}{\tau_1} + \frac{g}{\tau_1} [\tau_1 + t] \quad (5.72)$$

where

$$C_3 = \frac{4 G_o R^{\frac{1}{2}}}{3M_1 [1 - \nu^2]}.$$

Equations 5.72 and 5.71 can be solved numerically in the same manner as that used for identical spheres and, except for the new boundary for the contact surface (equation 5.67), the equations and method of solving for the interior stresses are the same as for two spheres.

6. RESULTS FOR TWO RED DELICIOUS APPLES IMPACTING DURING HARVEST

The problem solution will be demonstrated by using the results of experimental testing on the Red Delicious apples. The relaxation modulus obtained using the statistical regression equation and the approximate relation of Ninomiya and Ferry for relating $G(t)$ and the complex modulus appeared to represent the material behavior adequately. Maxwell model constants for this representation were $G_0 = 2627$ psi and $\eta = 17.93$ psi/sec. Poisson's ratio from Table 3.4 was 0.219. A typical apple had a representative diameter of 2.88 inches and weighed 0.342 lb, so these values were used. Results were computed for drop heights of two inches and 12 inches. The program for the two-inch drop height has been mentioned previously and is given in Appendices C.1 through C.3 with only the σ_{cz} stress program being given in Appendix C.3 as an example. Partial results of the computation are given in Appendices D.1 through D.3.

The effect of drop height on $\alpha(t)$ is shown graphically in Figure 6.1 where $\alpha(t)$ is plotted against time. It is evident that an increased drop height decreases impact duration and increases $\alpha(t)$ with $\alpha(t)$ for a 12-inch drop height being approximately twice that for a two-inch drop height.

Surface pressure increases as $\alpha(t)$ increases and is plotted versus the radial distance from the center of the contact surface for maximum indentation when the apple is dropped from two inches and 12 inches in Figure 6.2. The correctness of these solutions was verified by numerically integrating the surface pressure over the contact surface and

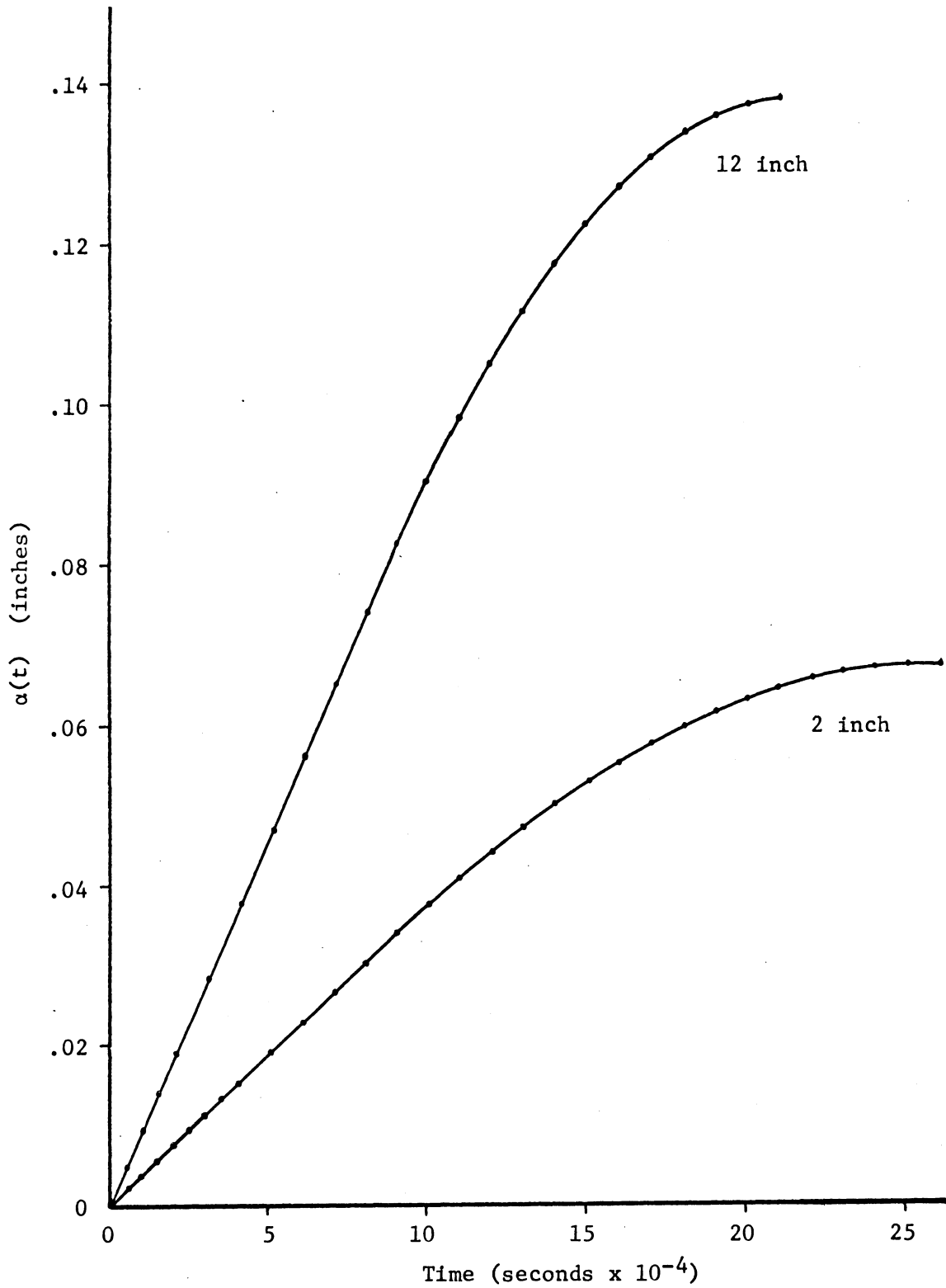


Figure 6.1 $\alpha(t)$ Versus Time for Impact of Two Apples at Two-Inch and Twelve-Inch Drop Heights.

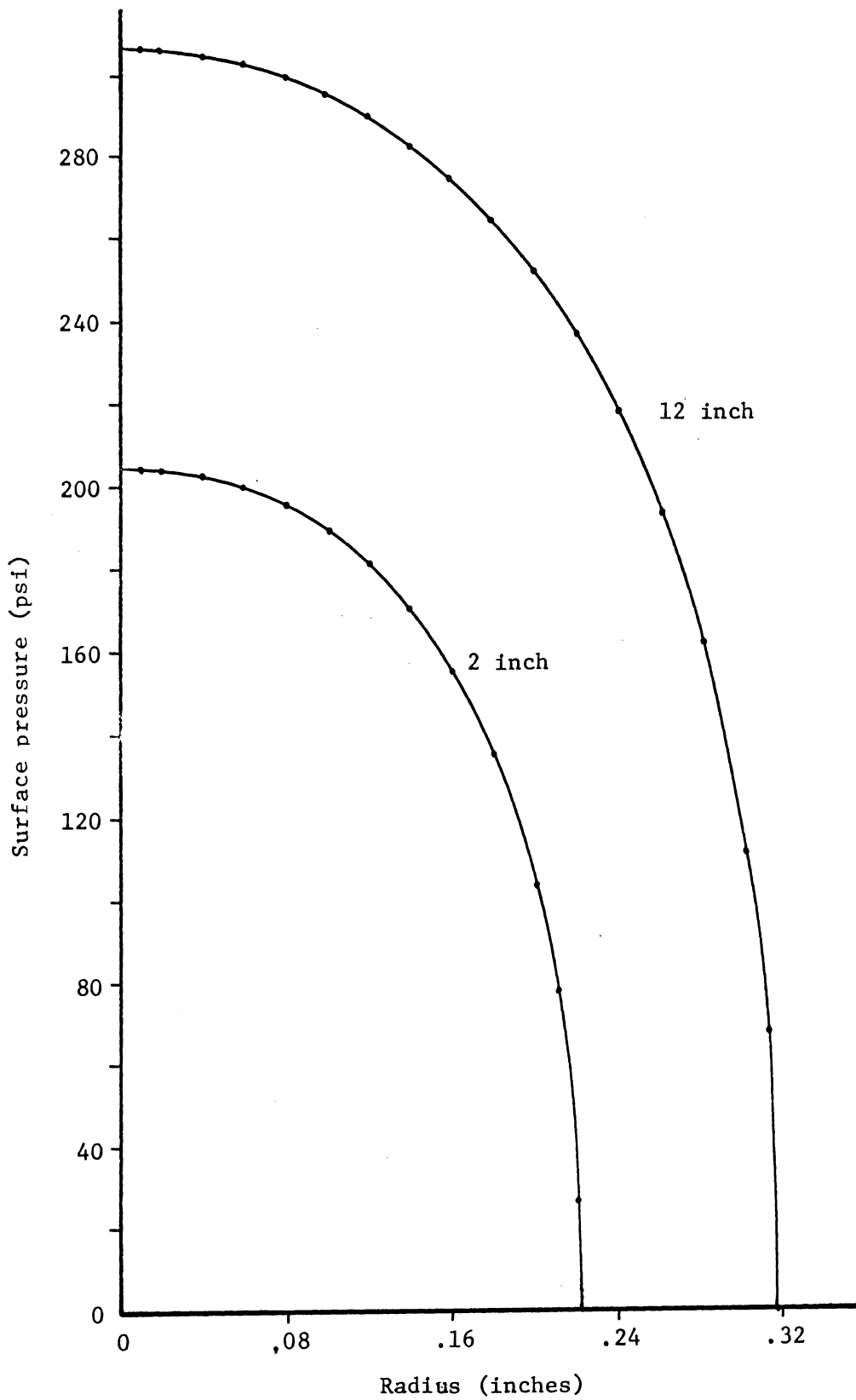


Figure 6.2 Surface Pressure Distribution at Maximum Indentation for Two-Inch and Twelve-Inch Drop Heights.

comparing this force with that obtained using Newton's second law and acceleration at maximum indentation from the Runge-Kutta-Gill solution. The graph shows that the pressure distribution is approximately elliptical. The pressure distribution would not fit an elliptical equation, however, as pressure remained higher as r increased than an elliptical equation would predict. The maximum surface pressure for the two-inch drop height was about 204 psi which was approximately two-thirds of the maximum for the 12-inch drop height. Thus, high surface pressures are created by relatively low drop heights.

Internal stresses in the Red Delicious apples for the condition of maximum indentation are shown graphically in Figures 6.3 through 6.10. Figures 6.3 through 6.6 are for the two-inch drop height and Figures 6.7 through 6.10 for the 12-inch drop height. Comparison of the stress for both drop heights reveals that the stress is large to a considerably greater depth for the 12-inch drop height even though the maximum value is only one-half again as large as that for the two-inch drop height. Thus, if bruising is highly dependent on the σ_{zz} stress, the bruise should be considerably larger for the 12-inch drop height. The σ_{cc} stresses and $\sigma_{\theta\theta}$ stresses are, of course, identical at c equals zero for a specific drop height. They differ, however, as c increases with σ_{cc} being greater except near the surface where $\sigma_{\theta\theta}$ is larger. Relative magnitudes for the two drop heights are similar to those for σ_{zz} . The σ_{cz} shear stress reaches its maximum value near the boundary of the contact surface and at a depth of roughly half the contact radius. This stress is, of course, zero at the center of loading where c equals zero.

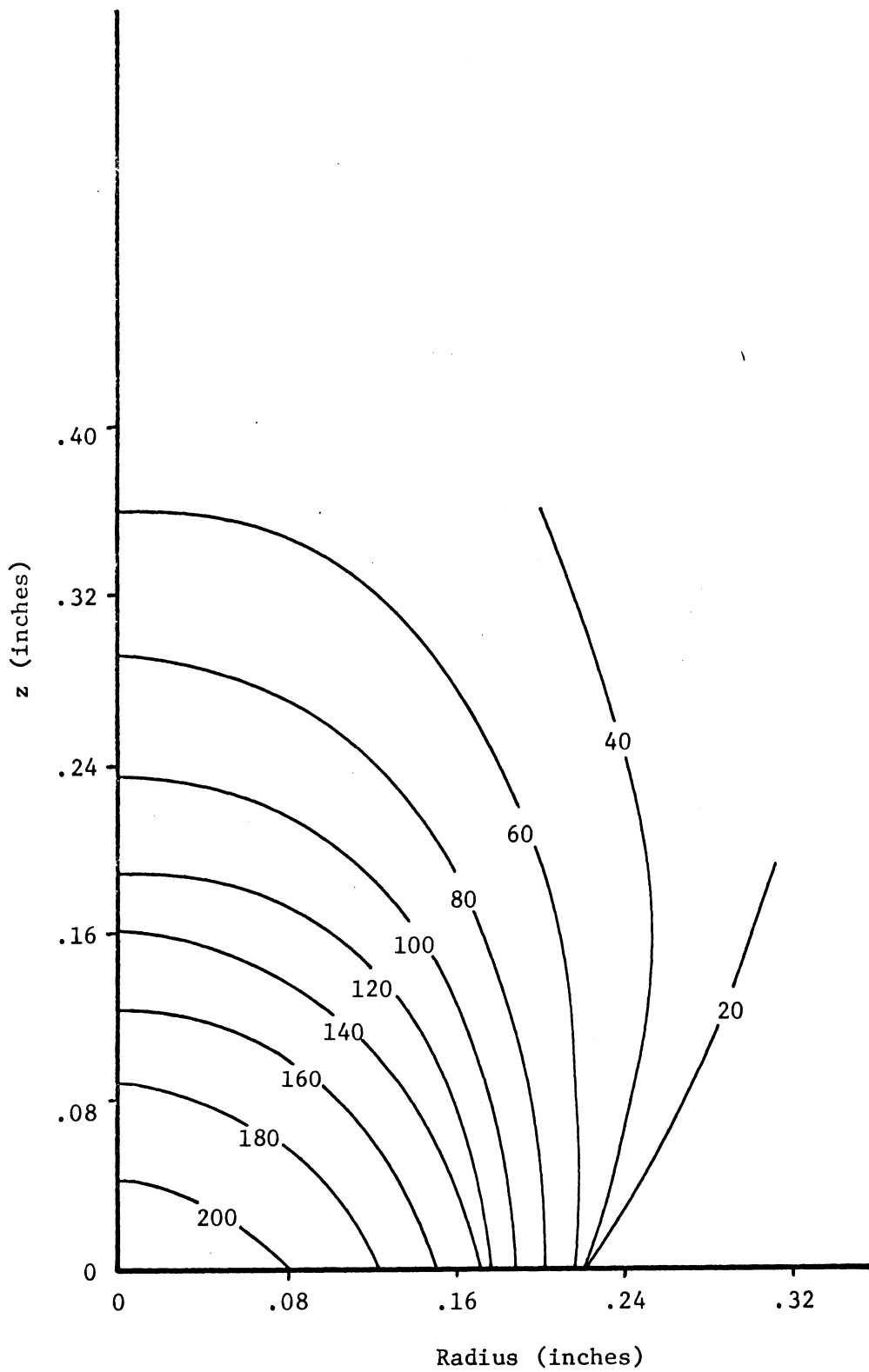


Figure 6.3 σ_{zz} Stress at Maximum Indentation for a Two-Inch Drop Height.

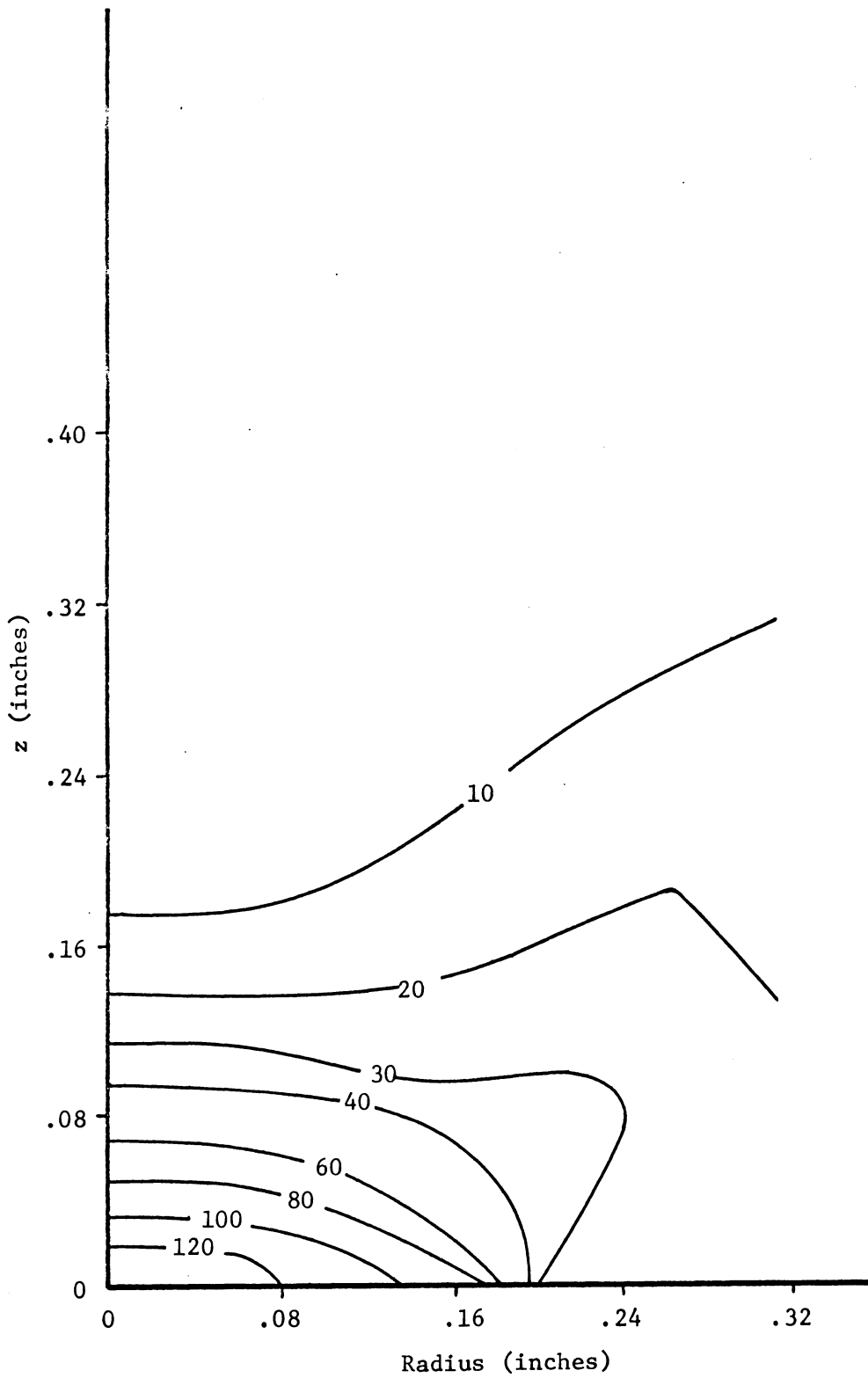


Figure 6.4 σ_{cc} Stress at Maximum Indentation for a Two-Inch Drop Height.

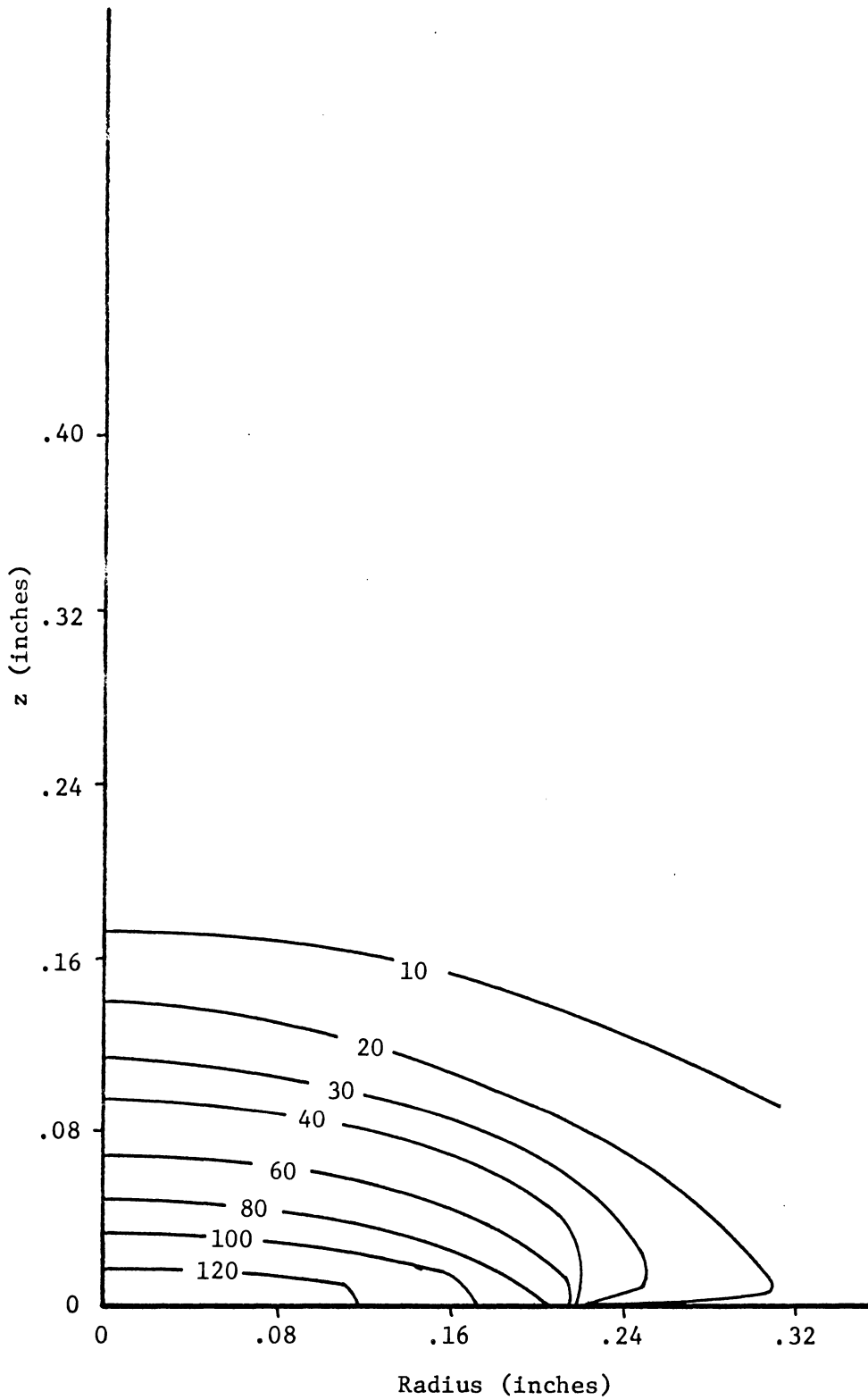


Figure 6.5 $\sigma_{\theta\theta}$ Stress at Maximum Indentation for a Two-Inch Drop Height.

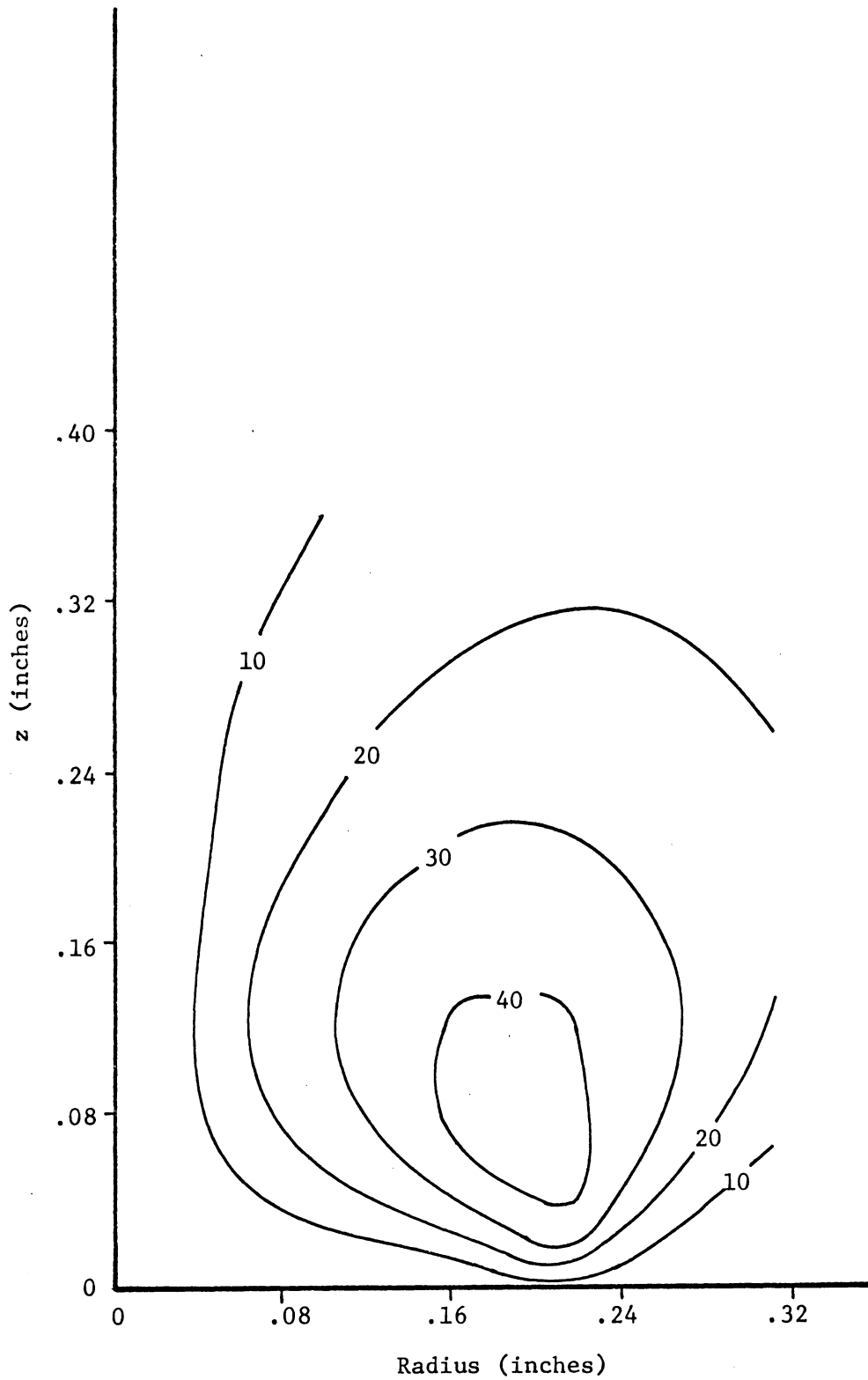


Figure 6.6 σ_{cz} Stress at Maximum Indentation for a Two-Inch Drop Height.

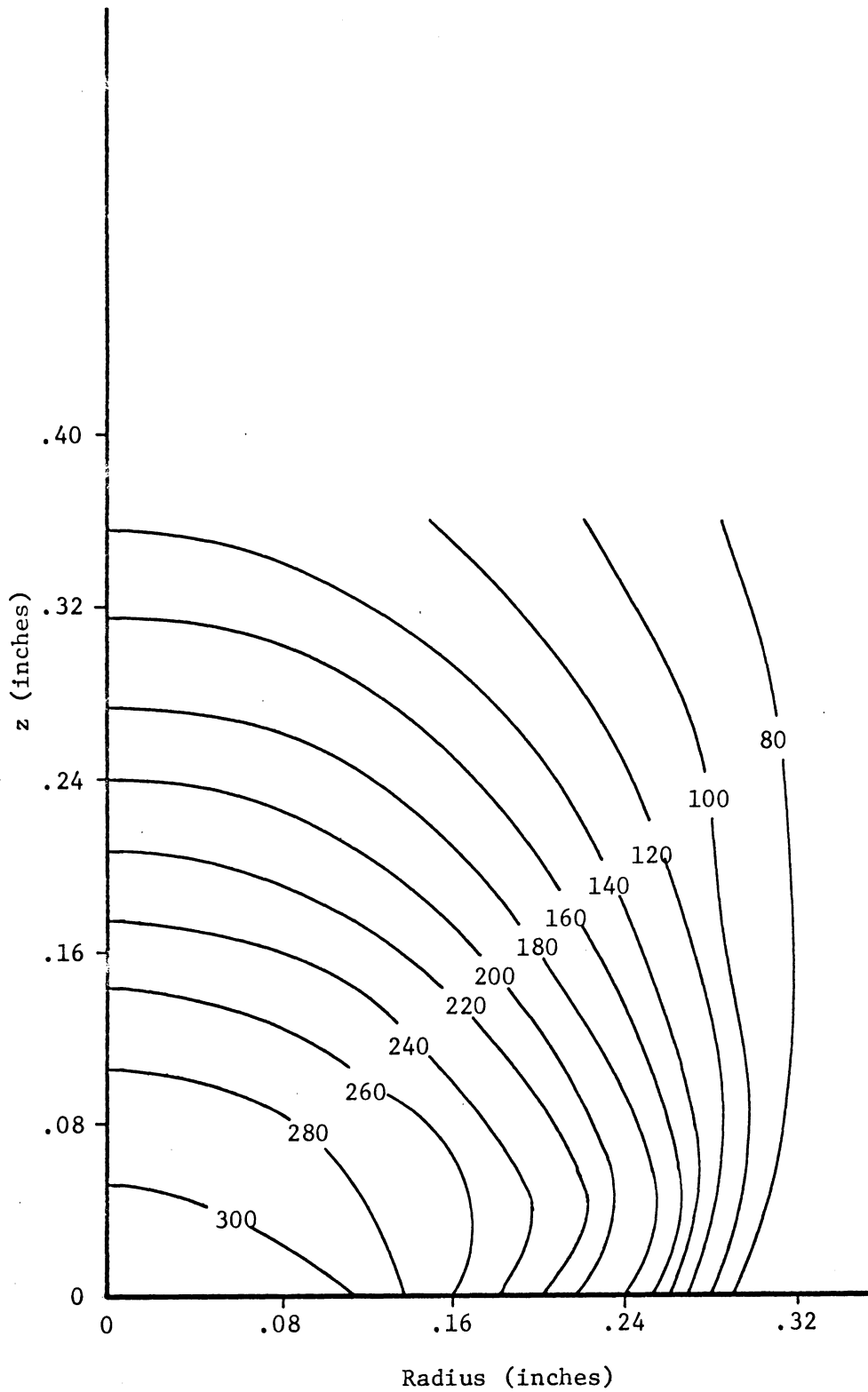


Figure 6.7 σ_{zz} Stress at Maximum Indentation for a Twelve-Inch Drop Height.

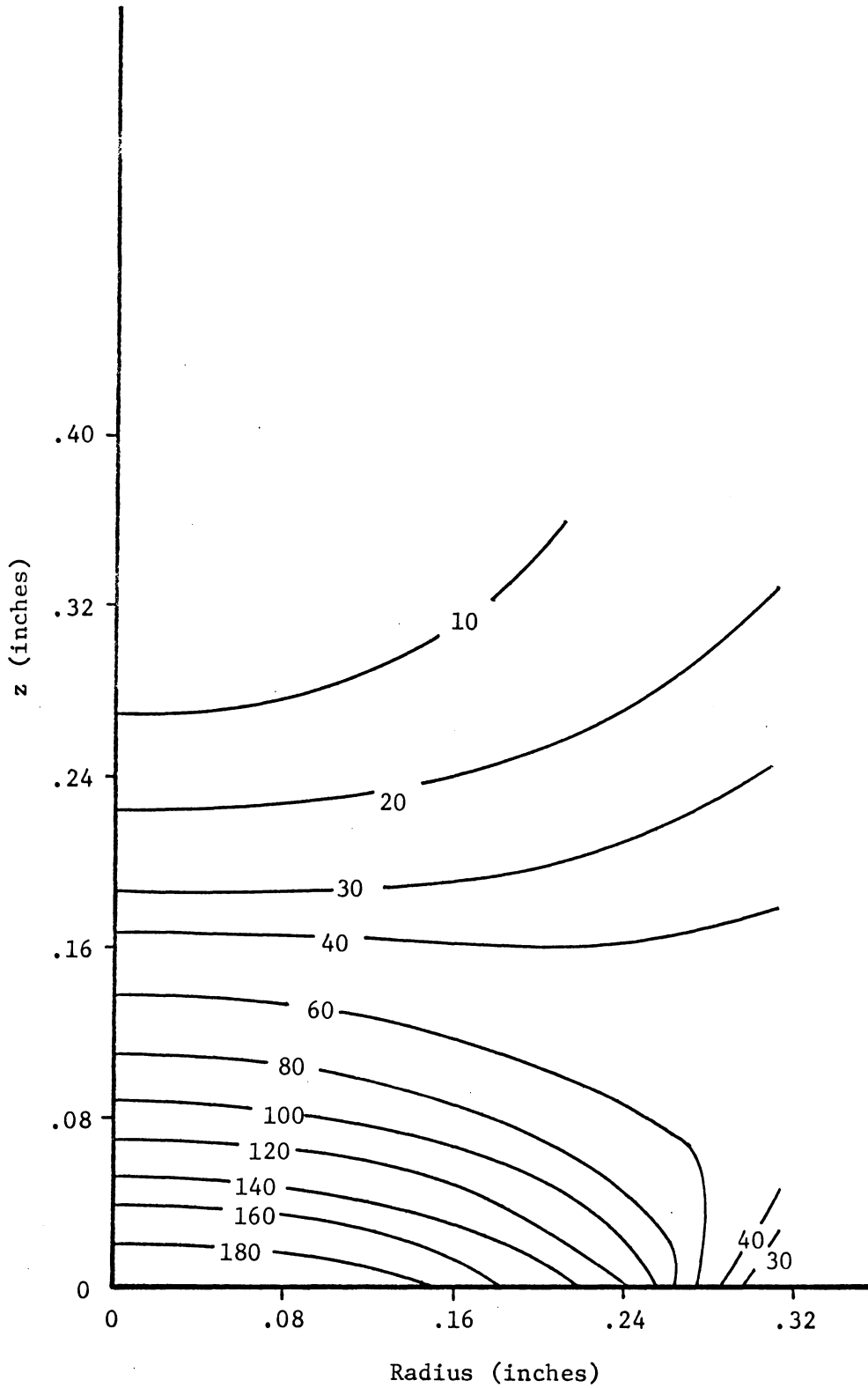


Figure 6.8 σ_{cc} Stress at Maximum Indentation for a Twelve-Inch Drop Height.

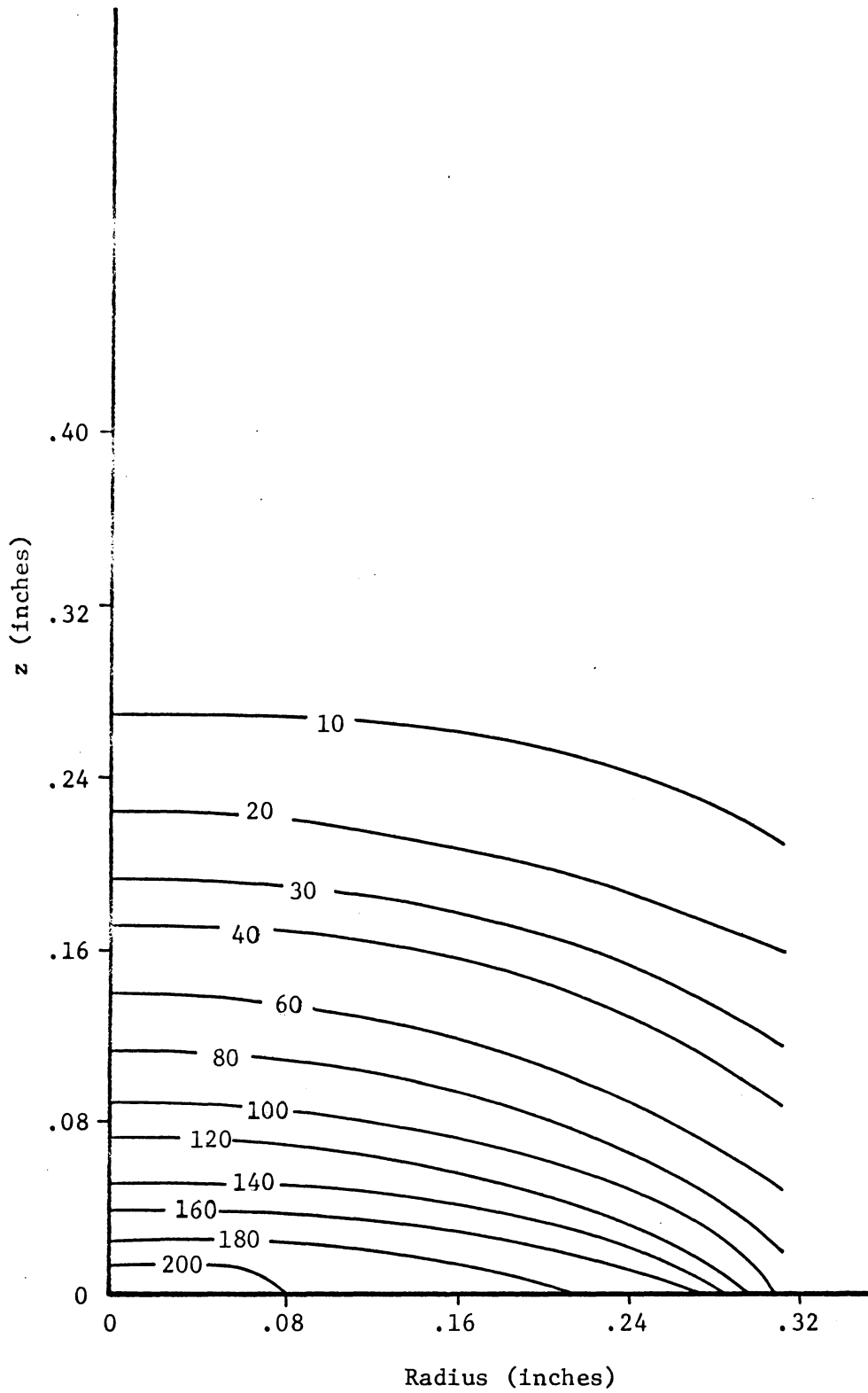


Figure 6.9 $\sigma_{\theta\theta}$ Stress at Maximum Indentation for a Twelve-Inch Drop Height.

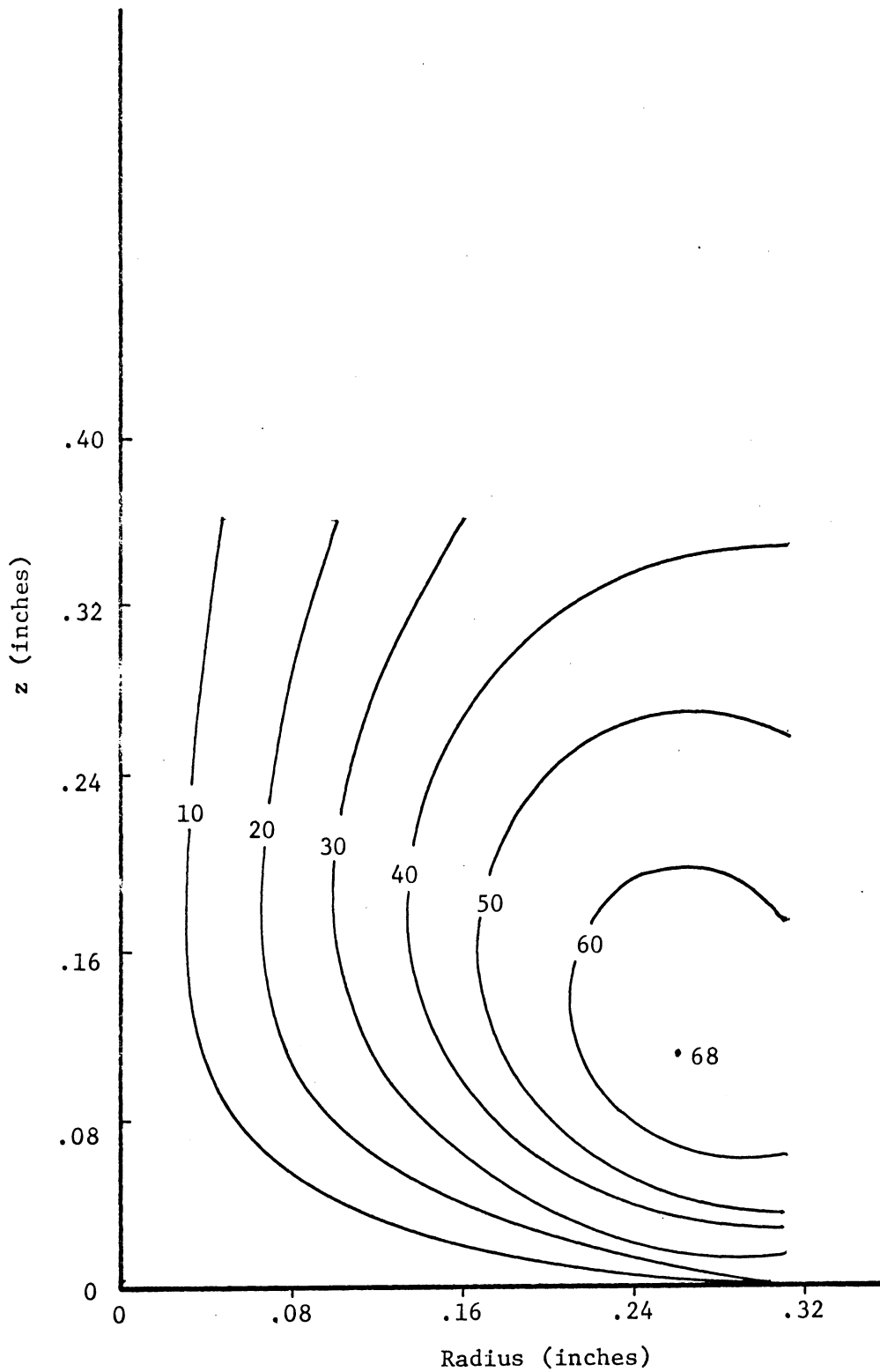


Figure 6.10 σ_{cz} Stress at Maximum Indentation for a Twelve-Inch Drop Height.

All of the above stress distributions are in general agreement with those given by Thomas and Hoersch [39] and Davies [40] for purely elastic problems. Various comparisons were made with the results of these authors and all stress distributions and stress ratios were close to the same with the differences present being those one would expect from the differences in Poisson's ratio and the viscoelastic stress-strain relation. For example, ratios of σ_{cc}/σ_{zz} and $\sigma_{\theta\theta}/\sigma_{zz}$ were lower than those reported by the above authors since this is affected by the Poisson's ratio value used which was lower for the Red Delicious apples.

7. DISCUSSION AND RECOMMENDATIONS

There has been considerable discussion along with presentation of the results of this investigation. Several points, however, should be discussed here and recommendations made where appropriate.

The first point of discussion deals with comparing the example solution to an elastic solution. The approach taken in this work to represent a time-dependent Young's modulus was to assume that the material was linearly viscoelastic. This seemed to add a minimum of complication above that for the elastic solution of pertinent problems, and eliminated the necessity of selecting a constant Young's modulus corresponding to the rate of loading as would be required in quasielastic solutions. An additional complication in the quasielastic solution to the impact problem is that no single constant value of Young's modulus would be applicable during the entire problem. For this reason, the example viscoelastic solution was not compared numerically with an elastic solution. It is likely, however, that some elastic compression modulus could be inserted into the impact problem and numerical values not very different from those for the Maxwell material obtained. A good first try would be to use the modulus at $\omega = \pi/T$ where T is the estimated impact duration. However, since it is even unnecessary to estimate impact durations in the viscoelastic problem, it is easier to use a viscoelastic approach once adequate material representation is known.

A second point of discussion is the accuracy of the sinusoidal testing. Although a large number of sinusoidal compression tests were conducted and the experimental method was successful by every indication,

it would be desirable to have an independent check on dynamic moduli using a different method. This is an area needing further research.

The sinusoidal shear test apparatus operated well, but shear moduli obtained are believed to be low by an unknown percentage due to the fact that a pure shear state could not be very closely approximated and it was not possible to insure that there was no specimen slip in the test fixture. In general, the absolute values of shear complex moduli and computed shear relaxation moduli were slightly greater than one-fourth the magnitude of the compression moduli for the same group of apples. Theoretically, this is impossible for an isotropic and homogeneous material, thus indicating that if initial assumptions were reasonably correct the actual shear moduli should be higher. Further research should be conducted using specimens with more favorable thickness to length or diameter ratios and possibly better methods of securing the specimen in the test position. The sinusoidal shear test is probably the most promising method of obtaining a truly dynamic second independent moduli for materials that are quite compressible and should be worthy of additional investigation. However, it can be stated that the shear moduli obtained were approximately the magnitude expected based on the knowledge of the compression moduli and the imperfect pure shear state, thus tending to validate the compression moduli.

Third, a failure theory for apple flesh was not considered in this thesis but is an avenue for further investigation. A contribution was made to this area, however, since the solution to the impact problem developed in this work will compute stresses at any time prior to maximum indentation and should be useful in studies of apple flesh failure.

An example of research on a failure theory would be to drop a series of apples onto other apples with succeeding apples dropped at increasing heights starting with a very low height. The bruised area of each apple could then be compared in size and shape with the stress distribution computed by the method of this thesis. The actual stress distribution used for the comparison could be the maximum shearing stress or any other stress relation thought to be pertinent. If, for example, the maximum shear stress theory was considered, it would be easy to compute the maximum shearing stress at each point for which σ_{zz} , σ_{cc} , $\sigma_{\theta\theta}$, and σ_{cz} had been computed. The minimum height at which bruising occurred would likely be of major interest since for higher drop heights initial failure probably takes place at some time during indentation after which the damaged material acts to distribute the load over a larger area and possibly dissipate some energy by liquid motion. The shallowness of the bruises noted by Dedolph and Austin [2] would tend to substantiate this. Certainly, at this time, no failure theory for apple flesh has been proved, so any quality contribution in this area would be valuable.

A final point for discussion here is the validity of the small displacement assumption. It was assumed in developing the solution to the impact problem that small displacement theory was applicable. This means that the displacements are so small that the squares and products of first differentials of the displacements with respect to the coordinates can be neglected in comparison with their first powers. The validity of this assumption should be verified.

The maximum stress for a two-inch drop of a typical Red Delicious apple at harvest as solved in this thesis was the σ_{zz} stress at about 200 psi near the surface of contact. Using the small displacement theory, the e_{zz} strain is

$$e_{zz} = \frac{\partial w}{\partial z}$$

where e_{zz} is the strain in the z direction and w is the displacement in the z direction. The e_{zz} strain can also be estimated by using an elastic assumption and applying Hooke's law,

$$e_{zz} = \frac{1}{E} [\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})] ,$$

where the absolute value of the complex modulus at an appropriate frequency is used as E . For the Red Delicious apples near the center of contact, the σ_{zz} , σ_{xx} , and σ_{yy} stresses can be rounded off so that e_{zz} for this specific case is approximately

$$e_{zz} = \frac{1}{2600} [200 - .22 (120 + 120)] \approx 0.06.$$

Thus, the square of $\frac{\partial w}{\partial z}$ would be about six per cent of the first power value. The e_{zz} strain for greater values of z would be slightly higher but would correspondingly only slightly change the ratio of the squared or product strains to first power strains. Even for the twelve-inch drop height, this ratio does not exceed approximately 0.085.

The ratios of the squared strains to the first power strains given in the previous paragraph are quite high if a person is contemplating

precise solutions for well behaved materials. However, for apple flesh, such as considered in the present problem, variability between apples is such that the error introduced by using the small displacement theory is not likely to be important.

8. SUMMARY AND CONCLUSIONS

8.1 Experimental

The purpose of the experimental phase of this investigation was to determine the dynamic compressive stress-strain relation and the dynamic shear moduli or Poisson's ratio values for various groups of apples. This work was at least moderately successful.

The experimental method and equipment used for determination of the complex axial compressive modulus gave reproducible results between frequencies of 50 to 365 cps which, by every indication, seemed to be correct. A large amount of scatter in the data apparently was due to variation between apples. Statistical analyses of the data showed the stress-strain relations for the various groups of apples to be frequency dependent. Due to scatter in the data, Maxwell model approximations were the best mechanical model representations that could be developed directly. These, however, appeared to be sufficiently good approximations for the data. A statistical curve fitting procedure resulted in relaxation moduli expressions that were equivalent to Maxwell model representations with element constants slightly larger than those obtained by the direct method. Elastic element constants varied from approximately 3000 psi for the hardest apples at harvest to approximately 750 psi for the softest apples from storage. Similarly, viscous element constants varied from about 20 psi/sec to about 4.5 psi/sec depending on the variety and storage condition.

The sinusoidal shear test apparatus operated well, but shear moduli obtained were believed to be low by an unknown percentage due to the fact

that a pure shear state could not be very closely approximated and it was not possible to insure that there was no specimen slip in the test fixture. It can be stated, however, that the complex shear moduli obtained were approximately the magnitude expected based on the knowledge of the compression moduli and the imperfect pure shear state thus tending to validate the compression moduli.

Poisson's ratio values for the three groups of apples tested at harvest condition were obtained by Chappell [10] and were of reasonable magnitudes based on the knowledge of air space in the apples. Initial values (time = 0.04 sec) ranged from approximately 0.218 to approximately 0.300. Values decreased with time because diametral deformation ceased after a short period of time while axial deformation continued to increase at least through time equal to 15 minutes. The initial values of Poisson's ratio were used along with relaxation moduli calculated by transformation of complex moduli obtained from the sinusoidal compression tests for the solution of the impact problem.

8.2 The Impacting Contact Problem

Contact problems involving one apple falling onto another supported underneath and an apple falling onto a rigid surface were solved for the approach of the bodies, surface indentations, surface pressures, and internal stresses from time of initial contact to the time of maximum indentation. A computer program was written which computed these with the input being the element constants of the Maxwell material, Poisson's ratio, the radius of the apple at the contact point, the apple weight,

and the drop height. Computer time using an IBM 7040 digital computer was short (about four minutes) except for computation of internal stresses which took several hours, depending on the number of points at which stress was being computed. Comparison of computer results and published elastic solutions validated the method. An example problem worked to illustrate the method revealed that high internal stresses (200 psi) are caused by a two-inch drop of a representative apple. Higher drop heights would likely produce a larger bruise since the high stress region is much larger. The problem solutions and procedure developed should be valuable in future work to develop a bruise criterion for apples and other fruits which should in turn be valuable as a basis for the design of equipment for mechanizing fruit harvest and processing.

In conclusion, the experimental equipment and techniques described and the solution to the impact problem described are not limited to apple flesh or biological materials and possibly will be even more useful for other materials.

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12. APPENDICES

Appendix A. Analysis of Compression Waves in a Viscoelastic Specimen Subjected to Sinusoidal Axial Compression

The following analysis is similar to one given by Marvin [22] for sinusoidal shear stress.

A complex viscosity is defined as the ratio of stress to rate of strain in the form

$$\eta^* = \eta' - i\eta'' = \frac{\sigma^*}{\dot{\epsilon}^*} \quad (\text{A.1})$$

where σ^* is the sinusoidal stress and $\dot{\epsilon}^*$ the strain rate for sinusoidal strain. The relationship between the complex modulus and complex viscosity is then

$$\begin{aligned} Y_1 &= \omega\eta'' \\ Y_2 &= \omega\eta'. \end{aligned} \quad (\text{A.2})$$

The complex modulus can thus be written as

$$Y(i\omega) = Y_1 + i\omega\eta' \quad (\text{A.3})$$

which is the form for a Kelvin model. This shows that writing test results in the form of a complex modulus is equivalent to assuming that for a given frequency the stress-strain relation can be represented by a Kelvin model.

Comparison of equations for electrical circuits with those for spring dashpot mechanical systems shows that the following analogy can

be made:

voltage represents force,

current represents velocity,

electrical resistance represents viscous resistance,

inductance represents mass, and

$1.0/\text{capacitance}$ represents elastic reactance.

It can be shown that the elastic reactance equals $Y_1 A/dx$, and the viscous resistance equals $\eta' A/dx$ where A is the cross section area of the specimen, and dx is an elemental increment of specimen height. To do this, reference is made to Figure A.1. For an elastic bond

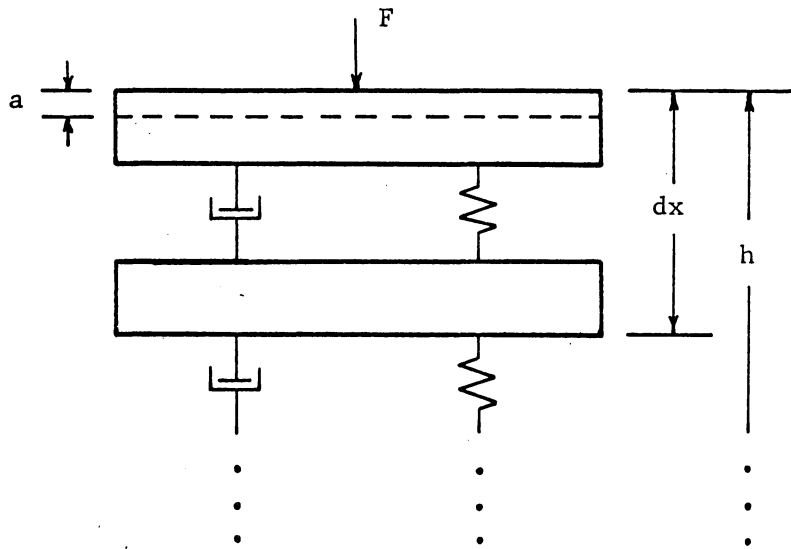


Figure A.1 Specimen Divided into Elemental Thicknesses.

between elements, the elastic constant is given by

$$\frac{F}{a} = Y_1 \frac{1}{dx} A \quad (\text{A.4})$$

where a is the distance dx is shortened by the stress application.

Similarly, for a viscous bond, the viscous constant is

$$\frac{F}{\dot{a}} = \eta' \frac{1}{dx} A . \quad (\text{A.5})$$

The elemental masses of Figure A.1 are in series and the elastic and viscous bonds in parallel between elemental masses. For the electrical analogy, parallel springs and dashpots are drawn as series capacitors and resistances in series with the inductance representing mass. This is shown in Figure A.2 where F_0 is the forcing on the first element.

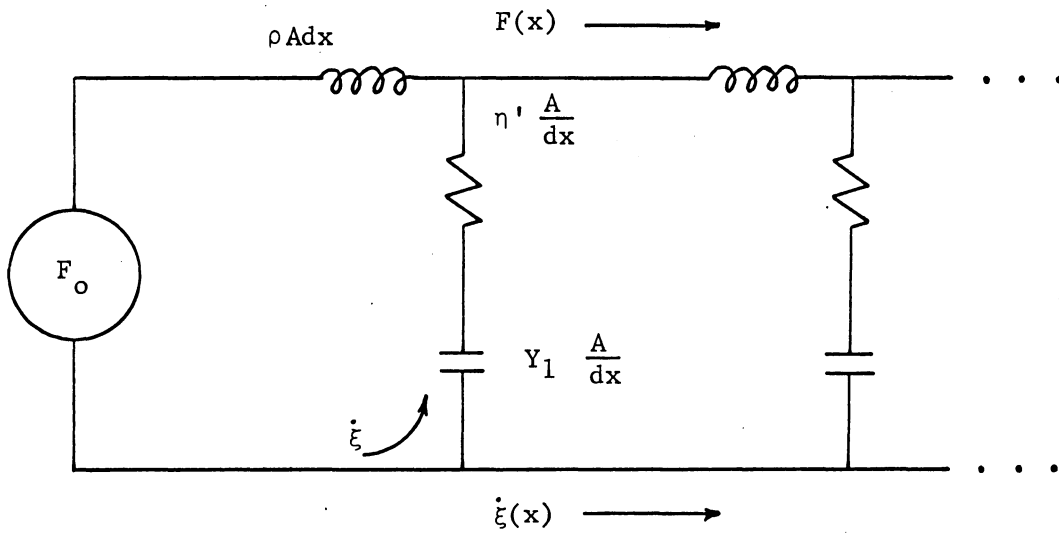


Figure A.2 Electrical Analogy of an Elemental Mass.

Succeeding elemental masses are treated the same way with the forcing (voltage) reduced only by the effect of the elemental masses (inductive

impedance) through which the force has been transmitted. The displacement velocity is represented by $\dot{\xi}(x)$ and the forcing by $F(x)$. The force decrease per element is

$$dF = -i\omega\rho A\dot{\xi}dx = -z\dot{\xi}dx \quad (A.6)$$

where $z = i\omega\rho A$. The inductive reactance leads the resistive reactance by 90 degrees, capacitive reactance lags resistive reactance by 90 degrees, and

$$\text{inductive reactance} = x_L = \omega L \Rightarrow \omega\rho A dx,$$

$$\text{resistance reactance} = x_R \Rightarrow \eta' A/dx, \text{ and}$$

$$\text{capacitive reactance} = x_C = 1.0/(\omega c) \Rightarrow Y_1 A/(\omega dx)$$

where L is the inductance and C is the capacitance. The velocity (current) through the first capacitor and resistance is then

$$d\dot{\xi} = -yFdx \quad (A.7)$$

where $1/y = [\eta' - i Y_1/\omega]A$. Differentiating equation A.7 with respect to x and substituting for dF/dx from equation A.6, the result is

$$\frac{d^2 \dot{\xi}}{dx^2} = yz \dot{\xi}. \quad (A.8)$$

The solution of equation A.8 is

$$\dot{\xi} = M e^{x[yz]^{\frac{1}{2}}} + N e^{-x[yz]^{\frac{1}{2}}} \quad (A.9)$$

where M and N are constants. Equation A.7 can then be written as

$$F = - M[z/y]^{\frac{1}{2}} e^{x(yz)^{\frac{1}{2}}} + N[z/y]^{\frac{1}{2}} e^{-x(yz)^{\frac{1}{2}}} . \quad (A.10)$$

The boundary conditions are $\dot{\xi} = 0$ at $x = h$ and $F = F_0$ at $x = 0$. Using the boundary conditions, equations A.9 and A.10, the velocity $\dot{\xi}$ and force F can be written after some mathematical manipulation as

$$\dot{\xi} = F_0 [y/z_0]^{\frac{1}{2}} \frac{\sinh\{[h - x] [yz]^{\frac{1}{2}}\}}{\cosh\{h [yz]^{\frac{1}{2}}\}} \quad (A.11)$$

and

$$F = F_0 \frac{\cosh\{[h - x] [yz]^{\frac{1}{2}}\}}{\cosh\{h [yz]^{\frac{1}{2}}\}} . \quad (A.12)$$

The apparent stress in the specimen is F_0/A and the apparent strain rate is $\dot{\xi}/h$, thus, the apparent complex viscosity is

$$\eta_a^* = \frac{F_0 h}{A \dot{\xi}} . \quad (A.13)$$

Substituting equation A.11 into A.13, the result is

$$\eta_a^* = \frac{1}{A y} \frac{\{h [yz]^{\frac{1}{2}}\} \cosh\{h [yz]^{\frac{1}{2}}\}}{\sinh\{h [yz]^{\frac{1}{2}}\}} . \quad (A.14)$$

Expanding the hyperbolic sine and cosine terms into series and dividing the numerator by the denominator, retaining the first two terms of the resulting series, the apparent viscosity becomes

$$\eta_a^* = \frac{1}{A} \left[\frac{1}{y} + \frac{h^2 z}{3} \right] . \quad (A.15)$$

Replacing $\frac{1}{y}$ and z with their definitions,

$$\eta_a^* = \eta' - i' \left[\frac{Y_1}{\omega} - \frac{h^2 \omega \rho}{3} \right] . \quad (A.16)$$

From equations A.1 and A.2 neglecting wave theory

$$n_a^* = n' - i \frac{Y_{1a}}{\omega} \quad (A.17)$$

where Y_{1a} is the apparent real part of the complex modulus. Equating the imaginary parts of equations A.16 and A.17, the result is

$$Y_1 = Y_{1a} + \frac{h^2 \omega^2 \rho}{3} . \quad (A.18)$$

The last term of equation A.18 is then the second order correction to be applied to the apparent real part of the complex modulus.

The magnitude of the correction term in equation A.18 was calculated at 350 cycles per second using $h = \frac{1}{2}$ inch and $\rho = 78 \times 10^{-6}$ lb sec²/in.⁴. For the compression testing, this term was less than one per cent of the apparent real part of the modulus and thus was neglected in test computations. Examination of the third and fourth terms of the series from which equation A.15 was obtained revealed that they were much less significant.

Equation A.18 is also valid for shear waves if appropriate notation is used. Marvin [22] derives an equation similar to equation A.18 but in a somewhat different form for the case of shear.

APPENDIX B. Results of Dynamic Testing of Apple Fruits Using Sinusoidal Forcing*

B.1 Compression Testing of Red Delicious Apples at Harvest

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
10/13/66	18-19	50	2055.8	18.86	1945.4 + i664.5
	20-21		1726.0	17.76	1643.7 + i526.6
	18-19		1849.5	18.21	1756.8 + i578.0
	20-21		2001.9	16.60	1918.5 + i572.0
	18-19		2141.8	18.61	2029.8 + i683.5
10/14/66	18-19	50	3234.1	21.28	3013.5 + i1173.8
	18-19		2230.3	17.77	2123.8 + i680.8
	16-17		2104.2	16.83	2014.2 + i609.1
	16-17		2425.9	19.80	2282.5 + i821.7
	20-21		1993.5	21.38	1856.3 + i726.8
10/13/66	18-19	90	2549.0	13.94	2473.9 + i614.2
	20-21		2185.8	17.56	2083.9 + i659.5
	18-19		2269.0	14.31	2198.5 + i560.9
	20-21		2144.9	14.48	2076.7 + i536.2
	18-19		2501.8	12.53	2442.2 + i542.5
10/14/66	18-19	90	2035.8	15.26	1964.1 + i535.7
	18-19		2481.2	16.79	2375.4 + i716.8
	16-17		2269.1	17.37	2165.6 + i677.3
	16-17		2670.4	13.75	2594.0 + i634.6
	20-21		2434.1	15.62	2344.2 + i655.3
10/13/66	18-19	150	2531.3	11.09	2484.0 + i486.8
	20-21		2759.2	9.93	2717.9 + i475.7
	18-19		2821.2	12.10	2758.5 + i591.5
	20-21		2134.2	9.97	2102.0 + i369.4
	18-19		2850.3	10.81	2799.7 + i534.4
10/14/66	18-19	150	2453.1	11.37	2405.0 + i483.5
	18-19		2616.0	11.09	2567.2 + i503.1
	16-17		1530.5	17.16	1462.3 + i451.6
	16-17		2275.2	11.54	2229.2 + i455.0
	20-21		2459.0	11.97	2405.5 + i510.0

*Each line of this appendix gives the results from testing a specific apple. Only one test was possible on most apples.

B.1 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
10/13/66	18-19	200	2527.6	10.81	2482.8 + i473.9
	20-21		3199.0	10.67	3143.7 + i592.4
	18-19		2622.4	10.41	2579.2 + i473.9
	20-21		2559.2	10.67	2514.9 + i473.9
	18-19		3039.0	9.99	2992.9 + i527.2
10/14/66	18-19	200	2404.7	11.79	2354.0 + i491.5
	18-19		3049.3	10.75	2995.8 + i568.5
	16-17		2255.0	10.98	2213.7 + i429.5
	16-17		2640.8	10.48	2596.8 + i480.2
	20-21		2710.0	10.32	2666.2 + i485.4
10/13/66	18-19	250	2445.0	5.60	2433.3 + i238.5
	20-21		2231.9	7.81	2211.2 + i303.1
	18-19		2765.7	7.84	2739.9 + i377.1
	20-21		2699.8	8.75	2668.4 + i410.8
	18-19		3275.8	8.63	3238.8 + i491.4
10/14/66	18-19	250	2874.9	7.75	2848.6 + i387.6
	18-19		2909.5	7.93	2881.7 + i401.3
	16-17		2452.9	8.08	2428.5 + i344.9
	16-17		2601.8	7.96	2576.7 + i360.2
	20-21		2872.8	8.66	2840.1 + i432.5
10/13/66	18-19	300	2477.6	9.93	2440.5 + i427.2
	20-21		2866.9	8.41	2836.0 + i419.5
	18-19		2288.7	10.48	2250.6 + i416.1
	20-21		2599.2	8.21	2572.5 + i371.3
	18-19		2172.2	8.85	2146.3 + i334.2
10/14/66	18-19	300	2325.7	9.21	2259.8 + i372.1
	18-19		2653.3	8.95	2621.0 + i412.7
	16-17		2647.5	9.41	2611.9 + i432.8
	16-17		2610.3	9.23	2576.5 + i418.6
	20-21		2443.1	9.85	2407.1 + i417.9

B.1 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
10/13/66	18-19	365	2435.3	6.38	2420.3 + i270.6
	20-21		2792.7	7.49	2768.8 + i364.3
	18-19		3092.5	8.99	3054.5 + i483.2
	20-21		2816.5	7.58	2791.9 + i371.4
	18-19		2999.1	7.56	2973.0 + i394.6
10/13/66	18-19	365	2718.5	7.41	2695.7 + i350.8
	18-19		2523.5	8.10	2498.3 + i355.4
	16-17		2689.2	8.12	2662.3 + i380.0
	16-17		2568.8	7.70	2545.6 + i344.3
	20-21		2918.0	7.11	2895.6 + i361.0

B.2 Compression Testing of Winesap Apples at Harvest

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
11/3/66	23-25	50	1909.6	12.84	1861.8 + i424.3
	23-25		1962.1	12.23	1917.6 + i415.5
	23-25		2006.2	15.42	1933.9 + i533.6
	20-22		1710.5	14.06	1659.2 + i415.6
	26-28		1807.9	14.02	1754.1 + i437.9
11/4/66	23-25	50	1875.5	13.00	1827.4 + i422.0
	23-25		2142.2	17.07	2047.8 + i628.7
	23-25		1862.0	12.84	1815.5 + i413.8
	20-22		2028.0	13.89	1968.7 + i486.7
	18-19		1896.1	12.79	1849.1 + i419.6
11/3/66	23-25	95	2315.2	13.99	2246.6 + i559.7
	23-25		2057.3	17.33	1963.9 + i612.8
	23-25		2273.2	15.56	2189.9 + i609.9
	20-22		2564.1	14.36	2484.0 + i635.9
	26-28		2021.1	16.45	1938.3 + i572.3
11/4/66	23-25	95	3115.3	7.74	3086.9 + i419.4
	23-25		2342.9	16.60	2245.2 + i669.4
	23-25		2368.5	16.13	2275.3 + i657.9
	20-22		2499.7	15.83	2404.9 + i681.7
	18-19		2042.9	12.42	1995.1 + i439.3
11/3/66	23-25	150	2293.6	11.92	2244.2 + i473.7
	23-25		2786.3	9.70	2746.5 + i469.3
	23-25		2785.7	10.95	2735.0 + i528.9
	20-22		2706.8	11.75	2650.1 + i551.4
	26-28		3014.8	10.84	2961.0 + i566.9
11/4/66	23-25	150	2388.5	11.32	2342.1 + i468.8
	23-25		3285.9	11.33	3221.9 + i645.4
	23-25		2794.6	11.16	2741.8 + i540.9
	20-22		2434.0	10.81	2390.8 + i456.4
	18-19		3376.4	11.90	3303.8 + i696.2

B.2 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
11/3/66	23-25	200	2830.3	12.70	2761.1 + i622.0
	23-25		2457.4	11.41	2408.8 + i486.1
	23-25		2330.1	10.75	2289.3 + i434.4
	20-22		2718.7	10.30	2674.9 + i486.3
	26-28		2845.3	11.99	2783.2 + i590.9
11/4/66	23-25	200	2991.2	11.02	2936.0 + i571.8
	23-25		3125.8	11.54	3062.7 + i625.2
	23-25		2591.4	10.27	2549.9 + i462.0
	20-22		2768.3	11.28	2714.8 + i541.6
	18-19		2569.4	10.93	2522.8 + i487.3
11/3/66	23-25	250	2717.2	8.77	2685.4 + i414.5
	23-25		2855.4	7.77	2829.2 + i385.9
	23-25		3272.2	8.27	3238.1 + i470.8
	20-22		2920.8	7.78	2893.9 + i395.5
	26-28		2635.4	7.00	2615.7 + i321.4
	26-28		2551.4	8.10	2525.9 + i359.3
11/4/66	23-25	250	2737.5	7.52	2713.9 + i358.5
	23-25		3098.7	8.31	3066.2 + i447.6
	23-25		3151.7	9.44	3109.0 + i517.1
	20-22		2738.2	9.59	2699.9 + i456.4
	18-19		2366.1	9.46	2333.9 + i389.0
11/3/66	23-25	300	2745.0	9.72	2705.6 + i463.4
	23-25		3216.4	10.63	3161.2 + i593.3
	23-25		3061.1	10.95	3005.5 + i581.2
	20-22		3205.8	11.14	3145.3 + i619.6
	26-28		3212.3	9.95	3164.1 + i554.9
11/4/66	23-25	300	2859.9	10.37	2813.2 + i514.8
	23-25		2764.6	10.42	2719.0 + i500.0
	23-25		2755.6	9.59	2717.1 + i459.3
	20-22		3173.0	10.84	3116.3 + i596.9
	19-19		2811.3	9.38	2773.3 + i458.4

B.2 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
11/3/66	23-25	365	2940.1	12.20	2873.8 + i621.2
	23-25		2419.1	7.18	2400.1 + i302.4
	23-25		2609.6	8.54	2580.7 + i387.6
	20-22		2782.8	8.80	2750.0 + i425.9
	26-28		2836.8	9.40	2798.7 + i463.2
11/4/66	23-25	365	2689.0	6.96	2669.2 + i325.9
	23-25		2989.8	7.73	2962.6 + i402.0
	23-25		3410.3	7.96	3377.4 + i472.2
	20-22		2864.6	7.62	2839.3 + i380.0
	18-19		2788.8	7.25	2766.5 + i351.7

B.3 Compression Testing of Golden Delicious Apples at Harvest

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus lb/in ²	Phase angle (degrees)	Complex modulus
10/20/66	17-19	50	2547.2	21.02	2277.7 + i913.7
	17-19		2822.3	20.55	2642.7 + i990.8
	15-16		1609.3	18.61	1525.2 + i513.6
	15-16		1864.0	15.64	1794.9 + i502.6
	13-14		2376.9	19.27	2243.8 + i784.4
	17-19	50	1787.3	15.12	1725.4 + i466.2
	17-19		2309.4	19.26	2180.1 + i761.9
	15-16		2217.3	16.40	2127.0 + i626.0
	15-16		1915.1	14.79	1851.7 + i489.0
	13-14		1403.9	5.60	1397.2 + i137.0
	17-19	90	2335.2	13.73	2268.5 + i554.1
	17-19		2606.1	13.95	2529.2 + i628.4
	15-16		2173.6	15.39	2095.7 + i576.7
	15-16		1777.4	12.13	1737.7 + i373.4
	13-14		2888.0	13.89	2803.6 + i693.1
	17-19	90	2096.3	13.95	2034.5 + i505.4
	17-19		2587.6	14.34	2507.0 + i640.7
	15-16		2656.9	15.12	2564.9 + i693.1
	15-16		2778.1	13.53	2701.0 + i649.7
	13-14		1983.1	14.74	1917.9 + i504.5
	17-19	150	2706.0	11.33	2653.3 + i531.5
	17-19		2977.2	9.77	2934.1 + i505.1
	15-16		2544.8	10.62	2501.2 + i468.8
	15-16		3305.1	10.64	3248.3 + i610.2
	13-14		2575.1	10.86	2528.9 + i485.2
	17-19	150	2167.6	10.19	2133.4 + i383.7
	17-19		2485.6	10.93	2440.5 + i471.4
	15-16		2148.3	10.48	2112.5 + i390.6
	15-16		2139.9	11.42	2097.5 + i423.7
	13-14		2468.1	9.93	2431.1 + i425.5

B.3 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
10/20/66	17-19	200	3231.6	10.92	3173.1 + i612.0
	17-19		2936.6	11.45	2878.2 + i582.8
	15-16		2265.9	10.42	2228.6 + i409.8
	15-16		2008.9	9.73	1980.0 + i339.5
	13-14		2068.9	10.74	2032.7 + i385.4
	17-19	200	2403.2	11.09	2358.3 + i462.1
	17-19		2417.5	10.58	2376.4 + i444.0
	15-16		2572.1	10.16	2531.7 + i453.9
	15-16		2382.2	10.29	2353.9 + i425.4
	13-14		2255.4	10.09	2220.5 + i395.3
	17-19	250	2808.4	7.18	2786.4 + i351.1
	17-19		2474.0	6.68	2475.2 + i288.2
	15-16		2457.4	8.21	2432.2 + i351.1
	15-16		2325.7	7.51	2305.8 + i304.1
	13-14		2355.5	7.63	2334.6 + i312.8
	17-19	250	3067.8	8.96	3030.4 + i477.8
	17-19		2439.4	7.83	2419.2 + i313.3
	15-16		2419.9	6.64	2403.7 + i280.0
	15-16		1998.4	8.60	1976.0 + i298.8
	13-14		2845.8	6.76	2826.1 + i334.8
	17-19	300	3017.6	9.59	2975.4 + i502.9
	17-19		3223.6	10.98	3164.6 + i614.0
	15-16		3050.9	10.40	3000.8 + i550.9
	15-16		2916.9	9.08	2880.0 + i460.5
	13-14		2992.0	8.90	2659.6 + i416.3
	17-19	300	2135.0	8.68	2110.6 + i322.3
	17-19		2317.0	8.71	2290.2 + i351.1
	15-16		2625.3	10.12	2584.5 + i461.2
	15-16		2858.0	9.02	2822.6 + i448.2
	14-13		2020.0	8.63	1997.2 + i303.0

B.3 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
10/20/66	17-19	365	3079.1	6.66	3058.3 + i357.0
	17-19		3030.7	9.59	2988.3 + i505.1
	15-16		2754.3	7.95	2727.9 + i380.9
	15-16		2462.1	6.24	2447.6 + i267.6
	14-13		2627.3	7.52	2604.7 + i343.8
	17-19	365	2284.4	4.88	2276.1 + i194.4
	17-19		2758.8	9.37	2722.0 + i449.1
	15-16		2239.0	5.74	2227.7 + i223.9
	15-16		3035.2	13.98	2945.3 + i733.5
	14-13		2599.8	8.31	2572.6 + i375.5

B.4 Compression Testing of Winesap Apples After Approximately Four Months' Storage

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/4/66	17	80	1525.7	10.98	1497.8 + i290.6
	17		1207.8	14.90	1167.2 + i310.6
	18		1779.8	12.10	1739.7 + i373.1
	18		1565.4	13.73	1520.7 + i371.4
	19		1495.7	12.66	1459.4 + i327.8
	20		1333.5	14.75	1289.5 + i339.4
3/8/66	16	80	1278.9	14.12	1240.3 + i311.9
	16		952.6	10.98	935.1 + i181.4
	16		1693.8	11.98	1656.9 + i351.5
	16-		1582.0	11.21	1551.8 + i307.6
	15+		1360.0	12.56	1327.4 + i295.6
3/4/66	17	110	1380.4	10.67	1356.5 + i255.6
	17		1540.8	13.61	1497.5 + i362.5
	18		1587.8	12.08	1552.6 + i332.3
	19		1691.8	10.98	1660.8 + i322.3
	20		1595.1	9.59	1572.8 + i265.9
3/8/66	16	110	1209.3	11.94	1183.1 + i250.2
	16		1501.5	12.15	1467.8 + i316.1
	16		1201.5	8.57	1188.1 + i178.9
	16-		1430.6	10.98	1404.4 + i272.5
	15+		1580.5	10.67	1553.2 + i292.7
3/4/66	17	140	1471.1	11.54	1441.3 + i294.2
	17		1488.5	11.54	1458.4 + i297.7
	18		1722.6	12.02	1684.8 + i358.9
	19		1614.9	10.67	1587.0 + i299.1
	20		1525.2	13.61	1482.4 + i358.9
3/8/66	16	140	1676.9	12.62	1636.4 + i366.4
	16		1538.1	11.16	1509.0 + i297.7
	16		1475.3	8.85	1457.7 + i227.0
	16-		1719.8	10.21	1692.6 + i304.8
	16-		1787.9	7.73	1771.7 + i240.4
	15+		1810.2	8.21	1791.6 + i258.6

B.4 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/4/66	17	170	1485.3	10.89	1458.5 + i280.7
	17		1634.1	11.54	1601.1 + i326.8
	18		1604.3	13.61	1559.3 + i377.5
	19		1576.0	12.60	1538.0 + i343.9
	20		1688.0	10.48	1659.8 + i306.9
3/8/66	16	170	1241.8	20.83	1160.6 + i441.5
	16		1743.4	12.56	1701.7 + i379.0
	16		1668.1	6.04	1658.8 + i175.6
	16		1728.2	11.67	1692.5 + i349.5
	16-		1572.9	11.78	1539.8 + i321.0
	15+		1673.3	13.89	1624.4 + i401.6
3/4/66	17	200	1445.7	10.10	1423.3 + i253.6
	17		1389.1	14.36	1345.7 + i344.4
	18		1683.3	12.49	1643.7 + i364.0
	19		1520.5	12.09	1487.0 + i317.7
	20		1381.3	10.81	1356.8 + i259.0
3/8/66	16	200	1571.6	14.48	1521.7 + i392.9
	16		1805.0	11.29	1770.1 + i353.4
	16		1674.8	11.16	1643.1 + i324.1
	16-		1498.6	12.37	1463.8 + i321.1
	15+		1782.1	11.88	1743.9 + i366.9
3/4/66	17	230	1317.1	11.84	1289.0 + i270.2
	17		1746.5	14.75	1689.0 + i444.6
	18		1688.6	9.59	1664.9 + i281.4
	19		1301.9	15.56	1254.2 + i349.3
	20		1869.7	13.21	1820.2 + i427.4
3/8/66	16	230	1891.0	11.74	1851.5 + i384.6
	16		1847.0	11.74	1808.4 + i375.7
	16		1736.1	8.85	1715.5 + i267.1
	16-		1846.2	13.34	1796.4 + i426.0
	15+		1626.9	10.10	1601.7 + i285.4

B.4 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/4/66	17	260	1437.4	9.59	1417.3 + i239.9
	17		1557.4	6.38	1547.7 + i173.0
	18		1602.7	7.18	1590.1 + i200.3
	19		1843.1	7.49	1827.4 + i240.4
	20		1428.3	9.87	1407.2 + i244.9
3/8/66	16	260	1720.2	6.38	1709.6 + i191.1
	16		1759.9	8.95	1738.5 + i273.8
	16		1720.7	6.99	1707.9 + i209.3
	16-		1547.5	7.54	1534.1 + i202.9
	15+		1652.1	6.04	1643.0 + i173.9

B.5 Compression Testing of Golden Delicious Apples After Approximately Four Months' Storage

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/2/66	11+	80	594.7	14.90	574.7 + i152.9
	11+		691.4	11.78	676.9 + i141.1
	10+		856.9	13.34	833.8 + i197.7
	10+		644.5	14.75	623.3 + i164.1
	10+		678.7	11.74	664.5 + i138.0
3/3/66	12	80	584.0	19.93	549.0 + i199.1
	12		640.9	8.85	633.3 + i 98.6
	12		670.1	18.28	636.3 + i210.2
	11		820.3	10.98	805.3 + i156.2
	11		876.5	11.80	858.0 + i179.3
3/2/66	11+	110	635.4	11.09	623.5 + i122.2
	11+		530.2	14.15	514.1 + i129.6
	10+		739.4	10.02	728.2 + i128.6
	10+		834.3	14.33	808.4 + i206.4
	10+		877.4	10.98	861.3 + i167.1
3/3/66	12	110	625.1	12.84	609.5 + i138.9
	12		693.7	9.97	683.3 + i120.1
	12		756.0	10.48	743.4 + i137.5
	11		734.1	9.87	723.3 + i125.9
	11		862.3	8.07	853.8 + i121.0
3/2/66	10+	140	755.9	9.40	745.7 + i123.4
	10+		760.5	7.93	753.3 + i104.9
	10+		833.0	9.59	821.4 + i138.8
	10+		804.2	9.17	793.9 + i128.2
	10+		536.2	43.22	760.0 + i100.0
3/2/66	10+	140	1088.6	9.02	1075.1 + i170.8
	10+		865.4	6.96	859.0 + i104.9
	10+		545.5	11.09	535.3 + i104.9
	10+		864.5	8.52	854.9 + i128.1
	10+		666.4	7.87	660.1 + i 91.3

B.5 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/3/66	12	140	711.7	7.00	706.4 + i 86.8
	12		675.8	11.09	663.2 + i 130.0
	12		677.5	9.73	667.8 + i 114.5
	11		761.4	10.81	747.9 + i 142.8
	11		614.2	16.01	590.3 + i 169.4
3/2/66	11+	170	680.9	12.02	666.0 + i 141.9
	11+		660.2	8.21	653.9 + i 94.3
	10+		757.1	11.78	741.1 + i 154.5
	10+		981.8	10.37	965.8 + i 176.7
	10+		843.6	11.94	825.4 + i 174.5
3/3/66	12	170	696.8	12.84	679.4 + i 154.9
	12		820.8	12.71	800.7 + i 180.6
	12		665.5	12.21	650.4 + i 140.8
	11		868.5	10.36	854.3 + i 156.1
	11		1177.2	11.16	1154.9 + i 227.8
3/2/66	11+	200	744.7	6.76	739.5 + i 87.6
	11+		801.1	11.33	785.5 + i 157.4
	11+		644.8	10.91	633.2 + i 122.0
	11+		733.2	9.70	722.7 + i 123.5
	11+		780.8	6.96	775.0 + i 94.6
3/3/66	12	200	740.0	8.63	731.6 + i 111.0
	12		807.0	10.72	792.9 + i 150.1
	12		1024.8	8.52	1013.5 + i 151.8
	11		825.1	10.91	810.2 + i 156.1
	11		580.3	9.93	571.6 + i 100.0
3/2/66	11+	230	801.3	5.74	797.3 + i 80.1
	11+		841.6	6.21	836.6 + i 91.0
	11+		1113.2	5.04	1108.8 + i 97.9
	10+		773.0	6.96	767.3 + i 93.7
	10+		1163.6	8.07	1152.1 + i 163.3

B. 5 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/3/66	12	230	658.6	7.18	653.5 + i 82.3
	12		701.1	12.15	685.4 + i147.6
	12		762.2	8.12	754.4 + i108.9
	11		762.1	7.18	756.1 + i 95.3
	11		708.5	10.29	697.1 + i126.5

B.6 Shear Testing of Winesap Apples After Approximately Four Months' Storage

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/5/66	17	80	328.8	12.02	321.6 + i 68.5
	17		447.3	8.21	442.8 + i 63.9
	18		348.9	10.31	343.5 + i 61.4
	19		341.9	11.33	335.2 + i 67.2
	20		431.8	7.78	427.9 + i 58.5
3/7/66	16	80	268.4	11.91	262.6 + i 55.4
	16		327.4	10.43	321.9 + i 59.3
	16		334.0	10.08	328.8 + i 58.4
	16-		375.1	9.21	370.3 + i 60.0
	15+		357.4	9.93	352.1 + i 61.6
3/5/66	17	110	351.2	10.81	345.0 + i 65.9
	17		400.1	10.20	393.8 + i 70.9
	18		377.3	10.20	371.4 + i 66.8
	19		435.3	9.15	429.8 + i 69.3
	20		420.7	7.49	417.1 + i 54.9
3/7/66	16	110	384.8	8.88	308.1 + i 59.4
	16		396.3	8.95	391.5 + i 61.7
	16		370.4	8.52	366.3 + i 54.9
	16-		363.6	11.87	355.8 + i 74.8
	15+		369.5	11.43	362.2 + i 73.2
3/5/66	17	140	427.1	9.28	421.5 + i 68.9
	17		515.5	7.18	511.5 + i 64.4
	18		443.9	6.84	440.7 + i 52.8
	19		475.6	7.45	471.6 + i 61.6
	20		415.8	8.99	410.7 + i 65.0
3/7/66	16	140	416.3	8.08	412.1 + i 58.5
	16		409.5	8.35	405.2 + i 59.4
	16		418.4	10.13	411.9 + i 73.6
	16-		326.7	10.16	321.6 + i 57.7
	15+		438.5	9.33	432.7 + i 71.1

B.6 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/5/66	17	170	445.3	7.09	441.9 + i 55.0
	17		548.3	6.04	545.3 + i 57.7
	18		429.3	6.38	426.7 + i 47.7
	19		541.5	7.00	537.4 + i 66.0
	20		488.5	6.38	485.4 + i 54.3
3/7/66	16	170	380.9	7.84	377.4 + i 51.9
	16		434.9	7.98	430.7 + i 60.4
	16		425.3	7.66	421.5 + i 56.7
	16-		400.2	7.18	397.1 + i 50.0
	15+		437.4	8.46	432.6 + i 64.3
3/5/66	17	200	512.4	7.66	507.9 + i 68.3
	17		497.9	7.90	493.1 + i 68.5
	18		449.8	7.27	446.2 + i 56.9
	19		465.2	7.41	461.3 + i 60.0
	20		495.1	7.77	490.5 + i 66.9
3/7/66	16	200	484.4	8.29	479.3 + i 69.9
	16		406.4	8.41	402.0 + i 59.5
	16		411.6	8.08	407.5 + i 57.9
	16-		493.8	7.37	390.6 + i 50.5
	15+		424.5	8.75	419.5 + i 64.6
3/5/66	17	230	551.4	6.00	548.4 + i 57.6
	17		458.8	5.74	456.5 + i 45.9
	18		432.5	6.96	429.4 + i 52.4
	19		461.1	6.29	458.3 + i 50.5
	20		477.8	7.09	474.1 + i 59.0
3/7/66	16	230	409.3	8.36	405.0 + i 59.5
	16		365.1	7.03	362.3 + i 44.7
	16		425.9	7.41	422.4 + i 55.0
	16-		406.1	6.38	403.6 + i 45.1
	15+		384.7	9.59	379.3 + i 64.1

B.6 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
3/5/66	17	260	625.5	4.65	623.5 + i 50.7
	17		549.5	5.30	547.2 + i 50.7
	18		465.0	4.41	463.7 + i 35.8
	19		501.9	5.64	499.5 + i 49.4
	20		486.9	6.15	484.1 + i 52.2
3/7/66	16	260	523.0	5.84	520.3 + i 53.2
	16		403.7	6.84	400.8 + i 48.1
	16		488.5	5.65	486.2 + i 48.1
	16-		502.7	4.78	500.9 + i 41.9
	15+		445.4	5.74	443.1 + i 44.5

B.7 Shear Testing of Golden Delicious Apples After Approximately Four Months' Storage

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
2/23/66	10-11	50	192.6	10.76	189.2 + i 36.0
	10-11		185.5	16.26	178.1 + i 51.9
	10-11		140.1	17.50	133.6 + i 42.1
	10-11		145.7	17.13	139.2 + i 42.9
	10-11		137.9	16.33	132.3 + i 38.8
2/24/66	10+	50	115.0	14.48	111.3 + i 28.7
	10+		119.0	16.85	113.8 + i 34.7
	10+		178.3	13.57	173.4 + i 41.9
	10+		112.5	13.16	108.5 + i 29.4
	10+		188.6	11.16	185.0 + i 36.5
2/23/66	10-11	80	134.6	13.67	130.8 + i 31.8
	10-11		157.9	12.84	153.9 + i 35.1
	10-11		177.6	13.34	172.9 + i 41.0
	10-11		195.3	12.66	190.6 + i 42.8
	10-11		153.8	11.90	150.5 + i 31.7
2/24/66	10+	80	128.7	12.84	125.5 + i 28.6
	10+		125.7	11.70	123.1 + i 25.5
	10+		153.6	13.25	149.5 + i 35.2
	10+		120.9	12.25	118.1 + i 25.6
	10+		138.7	12.75	135.3 + i 30.6
2/23/66	10-11	110	192.6	10.48	189.4 + i 35.0
	10-11		246.4	10.98	241.9 + i 46.9
	10-11		206.9	10.36	203.5 + i 37.2
	10-11		192.4	11.21	188.8 + i 37.4
	10-11		227.9	10.62	224.0 + i 42.0
2/24/66	10+	110	166.5	12.05	162.8 + i 34.8
	10+		166.5	11.41	163.2 + i 32.9
	10+		188.8	11.75	184.8 + i 38.5
	10+		149.2	12.49	145.7 + i 32.3
	10+		161.2	10.98	158.2 + i 30.7

B.7 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
2/23/66	10-11	140	190.6	10.08	187.7 + i 33.4
	10-11		184.0	12.91	179.3 + i 41.1
	10-11		313.5	10.02	210.3 + i 37.1
	10-11		197.3	11.28	193.5 + i 38.6
	10-11		188.0	11.38	184.3 + i 37.1
2/24/66	10+	140	192.0	22.62	177.3 + i 73.9
	10+		187.3	16.60	174.2 + i 52.1
	10+		216.0	10.84	212.2 + i 40.6
	10+		177.2	16.60	169.8 + i 50.6
	10+		134.7	12.06	131.7 + i 28.1
2/23/66	10-11	170	187.0	7.41	185.5 + i 24.1
	10-11		216.2	6.47	214.9 + i 24.4
	10-11		226.0	7.49	224.1 + i 29.5
	10-11		227.3	8.36	224.9 + i 33.1
	10-11		206.7	7.49	204.9 + i 27.0
2/24/66	10+	170	272.1	9.98	267.9 + i 47.2
	10+		233.3	9.71	230.0 + i 39.4
	10+		216.7	11.69	212.2 + i 43.9
	10+		249.4	9.02	246.3 + i 39.1
	10+		154.8	10.81	152.0 + i 29.0
2/23/66	10-11	200	315.1	9.33	310.9 + i 51.1
	10-11		228.2	9.05	225.4 + i 35.9
	10-11		296.7	8.21	293.7 + i 42.4
	10-11		328.6	8.79	324.7 + i 50.2
	10-11		320.7	9.02	316.8 + i 50.3
2/24/66	10+	200	260.9	6.93	259.0 + i 31.5
	10+		236.2	8.58	233.5 + i 35.2
	10+		226.5	8.85	223.8 + i 34.8
	10+		234.0	7.59	232.0 + i 30.9
	10+		216.6	8.05	214.5 + i 30.3

B.7 (Continued)

Testing date	Mechanical thumb force (lbs)	Frequency (cycles/sec)	Absolute value of modulus (lb/in ²)	Phase angle (degrees)	Complex modulus
2/23/66	10-11	230	266.4	5.61	265.1 + i 26.1
	10-11		240.4	6.24	238.9 + i 26.1
	10-11		290.8	6.04	289.1 + i 30.6
	10-11		240.8	7.56	238.7 + i 31.7
	10-11		232.5	6.87	230.8 + i 27.8
2/24/66	10+	230	266.5	6.76	264.6 + i 31.3
	10+		285.2	6.38	283.4 + i 31.7
	10+		247.1	5.85	245.9 + i 25.2
	10+		212.8	7.33	211.0 + i 27.2
	10+		227.6	6.72	226.0 + i 26.6
2/28/66	11	140	241.4	7.45	239.4 + i 31.3
	11		254.8	10.16	250.8 + i 45.0
	11		259.1	8.01	256.5 + i 36.1
	11		247.5	9.02	244.5 + i 38.8
	11		258.9	8.99	255.7 + i 40.4
	11		199.0	8.41	196.9 + i 29.1
	11		188.0	9.08	185.7 + i 29.7
	11		222.3	11.01	218.2 + i 42.5
	11		204.5	10.36	201.1 + i 36.8
	11		179.0	8.63	177.0 + i 26.8

Appendix C. Digital Computer Program

This program is composed of three sections which, when put together, computes the approach of the two spheres toward each other, contact surface pressures, and interior stresses respectively at specified time increments after initial contact. If only the approach of the spheres and surface pressure are required, the third section can be omitted. Similarly, if only the approach of the spheres is desired, both section two and section three can be omitted.

C.1 Section I ... Solution of an Initial Value Problem Involving Two First Order Differential Equations

This program from the IBM 1620 General Program Library was originally written in Fortran II for N first order differential equations. It was rewritten in Fortran IV and modified by the writer for the two equations solved so that the resulting time, approach displacement, velocity, and acceleration values are stored as subscripted variables accessible to succeeding operations in the overall program. This program, based on Romanelli [48], computes $\alpha(t)$, $\dot{\alpha}(t)$, and $\ddot{\alpha}(t)$ for equations 5.61 at specified times.

```

C  FORTRAN WITH SUBROUTINE TECHNIQUE USED FOR RUNGE KUTTA GILL METHOD OF
C  INITIAL VALUE PROBLEMS INVOLVING 2 FIRST ORDER DIFFERENTIAL EQUATIONS
C  ADAPTED TO FRUIT IMPACT
C  HPR = PRINTING INCREMENT, H = STEPPING INCREMENT,
C  XO = INITIAL TIME AT IMPACT, R = APPLE RADIUS,
C  ETA = VISCOUS ELEMENT CONSTANT, EO = ELASTIC ELEMENT CONSTANT,
C  PRTIO = POISSON'S RATIO, WEIGHT = APPLE WEIGHT,
C  XF (1) = TIME, YF (I) = APPROACH OF THE SPHERES,
C  YO(1) = INITIAL APPROACH DISTANCE, YO(2) = DROP HEIGHT;INITIAL VELOCITY
      DIMENSION XF(60), YF(60), YH(60), D(2), W(2)
      DIMENSION YO(2), YN(2), Q(2), V(2), F(2), C(2)
1  READ (5,900) HPR,H,XO,R,ETA
      WRITE (6,900) HPR,H,XO,R,ETA
      READ (5,902) EO,PRTIO,WEIGHT
      WRITE (6,902) EO,PRTIO,WEIGHT
      T1=ETA/EO
      C1=((2.*R)**.5*EO*12.0*32.1739)/(3.0*WEIGHT*(1.0-PRTIO**2.0))
1005 DO 2 I=1,2
      READ(5,904)YO(I)
      WRITE(6,904)YO(I)
2  CONTINUE
      YO(2)=(2.0*32.17*YO(2)/12.0)**0.5*12.0
      WRITE (6,904)YO(2)
      XPR=HPR
C  END DATA INPUT
C  BEGIN RKG
1111 X=XO
      DO 3 I=1,2
      YN(I)=YO(I)

```

```

3 Q(I)=0.0
  J=2
  XF(1)=0.00
  YF(1)=0.00
  YG(1)=YO(2)
  YH(1)=32.17

```

```
1112 U=X
```

```
C U REPRESENTS TIME
```

```

  DO4 I=1,2
4 V(I)=YN(I)
  I1=1
  GO TO 100
5 DO 6 I=1,2
  C(I)=H*F(I)
  D(I)=.5*(C(I)-2.0*Q(I))
  W(I)=YN(I)+D(I)
  Q(I)=Q(I)+3.0*D(I)-.5*C(I)
6 V(I)=W(I)
  U=X+.5*H
  I1=2
  GO TO 100
8 DO 9 I=1,2
  C(I)=H*F(I)
  D(I)=.29289325*(C(I)-Q(I))
  W(I)=W(I)+D(I)
  Q(I)=Q(I)+3.0*D(I)-.29289325*C(I)
9 V(I)=W(I)
  I1=3
  GO TO 100
11 DO 12 I=1,2
  C(I)=H*F(I)
  D(I)=1.7071067*(C(I)-Q(I))
  W(I)=W(I)+D(I)
  Q(I)=Q(I)+3.0*D(I)-1.7071067*C(I)
12 V(I)=W(I)
  U=X+H
  I1=4
  GO TO 100
14 DO 15 I=1,2
  C(I)=H*F(I)
  D(I)=.16666667*(C(I)-2.0*Q(I))
  YN(I)=W(I)+D(I)
15 Q(I)=Q(I)+3.0*D(I)-.5*C(I)
  X=X+H
  IF (X-XPR) 1112,200,200
200 XF(J)=X
  YF(J)=YN(1)
  YG(J)=YN(2)
  YH(J)=(2)
  J=J+1

```

```

      IF (YN(2)) 202,203,203
203  XPR=XPR+HPR
      GO TO 1112
100  F(1)=V(2)
      F(2)=- (1.0/T1)*V(2)-C1*V(1)**1.5+(1.0/T1)*YD(2)+(32.1739/T1)*U+
1    32.1739
      GO TO(5,8,11,14),I1
202  WRITE (6,901)
      JIP=J-1
      DO 204 K=1,JIP
204  WRITE (6,905)XF(K),YF(K),YG(K),TH(K),K
900  FORMAT(1X,F6.5,F6.5,F917,F6.4,F5.2)
901  FORMAT(1H1,14X,4HTIME,7X,9HALPHA (T),7X,8HVELOCITY,6X,12HACCELERAT
2ION//)
902  FORMAT(1X,F7.2,F5.3,F5.3)
904  FORMAT(1X,F11.6)
905  FORMAT(12X,F9.6,1X,E15.8,1X,E15.8,1X,E15.8,20X,12)
906  FORMAT(12X,F9.6,1X,E15.8,1X,E15.8,1X,E15.8)

```

C.2 Section II ... Integration by the General Simpson Rule With Interpolation

This program was written by the author for the purpose of computing contact surface pressures from the results of Section I. The program based on Simpson's rule as given by Stanton [49] and on ordinary methods of interpolation computes $p(r,t)$ for equation 5.63.

```

C   FINITE DIFFERENCE INTEGRATION BY THE GENERAL SIMPSON RULE
C   RA(M)=RADIUS TO POINT OF STRESS, P(I,J)=SURFACE PRESSURE,
C   FF(MM)=F TERMS OF THE RULE, DT=TIME INTERVAL, DR=RADIUS INTERVAL
C   T1=TAU 1 CONSTANT OF EQUATION
      DIMENSION RA(60), P(60,60),FF(21)
      DT=0.000050
      DR=0.01
      RA(1)=0.01
C   START COMPUTATION ONE INTERVAL FROM RA( )=0.0
      L=2
C   L=TIME INDEX
      WRITE (6,911)
305  T=XF(L)
      M=1
C   M=RADIUS INDEX
310  MM=1
      Y=0.0
      IF (R*YF(L)/2.0-RA(M)**2.0)403,311,311
311  UPLIM=(R/2.0*YF(L)-(RA(M))**2.0)**0.5
C   USING 20 INTERVALS
      H=UPLIM/20.0
400  YE=(Y**2.0+(RA(M))**2.0)/R*2.0
      DO 301 JJ=1,JIP
      IF (YF(JJ).GT.YE) GO TO 302
301  CONTINUE
302  JA=JJ-1
      A=YF(JJ)-YF(JA)
      B=YF(JJ)-YE
      TA=XF(JJ)-(B/A)*DT
      FF(MM)=EXP(TA/T1)
      MM=MM+1
      IF(MM-21)401,401,402
401  Y=Y+H
      GO TO 400
402  P(L,M)=H/3.0*(FF(1)+4.0*FF(2)+2.0*FF(3)+4.0*FF(4)+2.0*FF(5)+4.0*
1FF(6)+2.0*FF(7)+4.0*FF(8)+2.0*FF(9)+4.0*FF(10)+2.0*FF(11)+4.0*FF
1(12)+2.0*FF(13)+4.0*FF(14)+2.0*FF(15)+4.0*FF(16)+2.0*FF(17)+
14.0*FF(18)+2.0*FF(19)+4.0*FF(20)+FF(21))*(1.0/EXP(XF(L)/T1))*C2

```

```
      N=M
      M=M+1
      RA(M)=RA(N)+DR
      GO TO 310
403 DO 404 KK=1,N
      WRITE (6,912) XF(L), RA(KK), P(L,KK), L, KK
404 CONTINUE
      L=L+1
      IF (L-JIP) 305,430, 430
430 CONTINUE
911 FORMAT(/21X,4HTIME,9X,6HRADIUS,8X,16HSURFACE PRESSURE/)
912 FORMAT(18X,F9.6,3X,E15.8,3X,E15.8,20X,12,2X,12)
```

C.3 Section III ... Double Finite Difference Integration of the Stress Equations

This program was written by the author for the specific purpose of integrating the stress equations. The program considers the product of the local surface pressure and the area of the corresponding polar coordinate element as a point load. The effect of each elemental area is added to the previously calculated stress as the element is considered. The program computes σ_{cc} , σ_{zz} , $\sigma_{\theta\theta}$, and σ_{cz} stresses at specified locations in the body at specified times. The program for computing σ_{cz} for equation 5.39 is given here as an example.

```

C    COMPUTER PROGRAM ADDITION FOR CALCULATING STRESS R Z
C    RZ STRESS IS THE CZ STRESS,
C    S=C=RADIUS TO POINT M,DS=RADIUS INCREMENT,
C    RAD=RADIUS TO CONTACT BOUNDARY
    WRITE (6,924)
C    PHI=THETA
    CON=3.0/6.28318
    LL=JIP-1
    ANGDIV=90.0
    DPHI=6.28318/ANGDIV
    PHII=DPHI/2.0
    DZ=0.025
    DS=0.05
C    DR = 0.01 FROM STATEMENT 905+7
    L=21
    RAD=(R/2.0*YF(L))**0.5
1208 S=0.01
    SS=S*S
1201 Z=0.01
    ZZ=Z*Z
    SSZZ=SS+ZZ
1207 M=1
    RAA=RA(M)**2.0
C    INTEGRATE
C    CONSIDER CIRCULAR AREA IN THE CENTER
C    FE=STRESS
    FE=-0.0000375*P(L,M)*S*ZZ/SSZZ**2.5
1203 PHI=PHII
C    AA = A**2
1201 AA=RAA+SS-2.0*RA(M)*S*COS(PHI)

```

```

GAMMA=ARSIN(RA(M)/(AA**0.5)*SIN(PHI))
AAZZ=AA+ZZ
COSGAM=COS(GAMMA)
IF(S.GT.RAD) GO TO 1221
PHI1=ARCOS(S/RAD)
IF(PHI.LT.PHI1) GO TO 1216
PHI2=6.28318-PHI
IF(PHI2.GT.PHI1) GO TO 1221
1216 IF(RA(M).LT.(S/COS(PHI))) GO TO 1221
COSGAM=-COSGAM
1221 FE=FE-CON*RA(M)*P(L,M)*AA**0.5*ZZ/AAZZ**2.5*COSGAM*DPHI*DR
PHI=PHI+DPHI
IF(PHI.GT.6.28318) GO TO 1202
GO TO 1201
1202 IF(RA(M).GT.(RAD-DR)) GO TO 1204
M=M+1
IF (M.GT.%) GO TO 1250
RAA=RA(M)**2.0
GO TO 1203
1204 WRITE (6,906) XF(L),S,Z,FE
WRITE (7,906) XF(L),S,Z,FE
Z=Z+DZ
IF (Z.GT.0.36) GO TO 1206
ZZ=Z*Z
SSZZ=SS+ZZ
GO TO 1207
1206 S=S+DS
CALL CHKPT(28)
IF(S.GT.1.00) GO TO 1250
IF (S.GT.0.31) GO TO 1209
SS=S*S
GO TO 1210
1209 IF(LL-L)1250,1250,1211
1211 L=L+35
IF(L.GT.LL) GO TO 1251
RAD=(R/2.0*YF(L))**0.5
GO TO 1208
1251 L=LL
RAD=(R/2.0*YF(L))**0.5
GO TO 1208
924 FORMAT(1H1,13X,4HTIME,10X,6HRADIUS,12X,1HZ,11X,9HRZ STRESS//)
1250 CONTINUE
STOP
END

```

Appendix D. Computer Results for Impact of Two Red Delicious Apples

D.1 Section I for a Two-Inch Drop Height

Results are given for the computation of the approach, $a(t)$, the approach velocity, $\dot{a}(t)$, and the approach acceleration, $\ddot{a}(t)$, when one apple falls two inches onto an identical apple. Time intervals of 0.00005 seconds are used and the program terminated at maximum indentation.

TIME	ALPHA (T)	VELOCITY	ACCELERATION
0.000000	0.	0.39295802E 02	0.32170000E 02
0.000050	0.19647850E-02	0.39294330E 02	-0.12082289E 03
0.000100	0.39292441E-02	0.39281717E 02	-0.39912625E 03
0.000150	0.58926889E-02	0.39253091E 02	-0.75759880E 03
0.000200	0.78542265E-02	0.39204901E 02	-0.11795788E 04
0.000250	0.98128038E-02	0.39134241E 02	-0.16550718E 04
0.000300	0.11767234E-01	0.39038619E 02	-0.21770034E 04
0.000350	0.13716213E-01	0.38915855E 02	-0.27398692E 04
0.000400	0.15658334E-01	0.38764022E 02	-0.33391079E 04
0.000450	0.17592101E-01	0.38581400E 02	-0.39707645E 04
0.000500	0.19515934E-01	0.38366460E 02	-0.46312917E 04
0.000550	0.21428184E-01	0.38117839E 02	-0.53174229E 04
0.000600	0.23327136E-01	0.37834337E 02	-0.60260925E 04
0.000650	0.25211018E-01	0.37514897E 02	-0.67543760E 04
0.000700	0.27078010E-01	0.37158611E 02	-0.74994543E 04
0.000750	0.28926251E-01	0.36764710E 02	-0.82585844E 04
0.000800	0.30753842E-01	0.36332557E 02	-0.90290809E 04
0.000850	0.32558860E-01	0.35861649E 02	-0.98083015E 04
0.000900	0.34339354E-01	0.35351617E 02	-0.10593639E 05
0.000950	0.36093362E-01	0.34802218E 02	-0.11382515E 05
0.001000	0.37818915E-01	0.34213340E 02	-0.12172379E 05
0.001050	0.39514037E-01	0.33584997E 02	-0.12960705E 05
0.001100	0.41176757E-01	0.32917329E 02	-0.13744992E 05
0.001150	0.42805117E-01	0.32210598E 02	-0.14522772E 05
0.001200	0.44397170E-01	0.31465192E 02	-0.15291600E 05
0.001250	0.45950998E-01	0.30681619E 02	-0.16049076E 05
0.001300	0.47464705E-01	0.29860505E 02	-0.16792833E 05

TIME	ALPHA (T)	VELOCITY	ACCELERATION
0.001350	0.48936433E-01	0.29002595E 02	-0.17520553E 05
0.001400	0.50364362E-01	0.28108747E 02	-0.18229969E 05
0.001450	0.51746722E-01	0.27179933E 02	-0.18918872E 05
0.001500	0.53081789E-01	0.26217230E 02	-0.19585115E 05
0.001550	0.54367898E-01	0.25221826E 02	-0.20226618E 05
0.001600	0.55603446E-01	0.24195007E 02	-0.20841386E 05
0.001650	0.56786896E-01	0.23138158E 02	-0.21427497E 05
0.001700	0.57916784E-01	0.22052759E 02	-0.21983128E 05
0.001750	0.58991720E-01	0.20940377E 02	-0.22506542E 05
0.001800	0.60010397E-01	0.19802663E 02	-0.22996110E 05
0.001850	0.60971591E-01	0.18641350E 02	-0.23450308E 05
0.001900	0.61874166E-01	0.17458241E 02	-0.23867723E 05
0.001950	0.62717081E-01	0.16255208E 02	-0.24247060E 05
0.002000	0.63499385E-01	0.15034185E 02	-0.24587152E 05
0.002050	0.64220229E-01	0.13797161E 02	-0.24886946E 05
0.002100	0.64878864E-01	0.12546175E 02	-0.25145529E 05
0.002150	0.65474644E-01	0.11283306E 02	-0.25362121E 05
0.002200	0.66007027E-01	0.10010672E 02	-0.25536071E 05
0.002250	0.66475580E-01	0.87304179E 01	-0.25666870E 05
0.002300	0.66879974E-01	0.74447106E 01	-0.25754151E 05
0.002350	0.67219993E-01	0.61557327E 01	-0.25797682E 05
0.002400	0.67495526E-01	0.48656747E 01	-0.25797367E 05
0.002450	0.67706574E-01	0.35767279E 01	-0.25753263E 05
0.002500	0.67853250E-01	0.22910779E 01	-0.25665554E 05
0.002550	0.67935769E-01	0.10108971E 01	-0.25534569E 05
0.002600	0.67954463E-01	-0.26166130E 00	-0.25360765E 05

D.2 Section II for a Two-Inch Drop Height

Results given here are for five times selected from the actual program results to show how surface pressure varies with time and with the radius from the center of the contact surface. Actual program results were given for times from 0.00005 second through 0.00255 second in increments of 0.00005 second. It is interesting to note that the pressure at the center of the contact surface reaches a maximum prior to the time of maximum indentation.

<u>Time</u>	<u>Radius</u>	<u>Surface Pressure</u>
0.000050	0.10000000E-01	0.44036930E 02
0.000050	0.20000000E-01	0.38727487E 02
0.000050	0.30000000E-01	0.27629018E 02
0.000100	0.10000000E-01	0.63139454E 02
0.000100	0.20000000E-01	0.59629662E 02
0.000100	0.30000000E-01	0.53230992E 02
0.000100	0.40000000E-01	0.42591845E 02
0.000100	0.50000000E-01	0.22106719E 02
0.001000	0.10000000E-01	0.18227571E 03
0.001000	0.20000000E-01	0.18145093E 03
0.001000	0.30000000E-01	0.18005969E 03
0.001000	0.40000000E-01	0.17807590E 03
0.001000	0.50000000E-01	0.17546144E 03
0.001000	0.59999999E-01	0.17216335E 03
0.001000	0.69999999E-01	0.16811077E 03
0.001000	0.79999999E-01	0.16320883E 03
0.001000	0.89999999E-01	0.15733150E 03
0.001000	0.99999999E-01	0.15030618E 03
0.001000	0.11000000E 00	0.14189125E 03

<u>Time</u>	<u>Radius</u>	<u>Surface Pressure</u>
0.001000	0.12000000E 00	0.13173166E 03
0.001000	0.13000000E 00	0.11926799E 03
0.001000	0.14000000E 00	0.10351705E 03
0.002000	0.10000000E-01	0.21223215E 03
0.002000	0.20000000E-01	0.21175335E 03
0.002000	0.30000000E-01	0.21094821E 03
0.002000	0.40000000E-01	0.20980504E 03
0.002000	0.50000000E-01	0.20830682E 03
0.002000	0.59999999E-01	0.20643152E 03
0.002000	0.69999999E-01	0.20415096E 03
0.002000	0.79999999E-01	0.20142956E 03
0.002000	0.89999999E-01	0.19822192E 03
0.002000	0.99999999E-01	0.19447235E 03
0.002000	0.11000000E 00	0.19011034E 03
0.002000	0.12000000E 00	0.18505023E 03
0.002000	0.13000000E 00	0.17917864E 03
0.002000	0.14000000E 00	0.17234761E 03
0.002000	0.15000000E 00	0.16436129E 03
0.002000	0.16000000E 00	0.15493954E 03
0.002000	0.17000000E 00	0.14367141E 03
0.002000	0.18000000E 00	0.12989503E 03
0.002000	0.18999999E 00	0.11241662E 03
0.002000	0.20000000E 00	0.88576113E 02
0.002000	0.20999999E 00	0.48448003E 02
0.002550	0.10000000E-01	0.20440653E 03
0.002550	0.20000000E-01	0.20399161E 03
0.002550	0.30000000E-01	0.20329359E 03
0.002550	0.40000000E-01	0.20230263E 03
0.002550	0.50000000E-01	0.20100476E 03
0.002550	0.59999999E-01	0.19938205E 03

<u>Time</u>	<u>Radius</u>	<u>Surface Pressure</u>
0.002550	0.69999999E-01	0.19741016E 03
0.002550	0.79999999E-01	0.19506002E 03
0.002550	0.89999998E-01	0.19229371E 03
0.002550	0.99999999E-01	0.18906450E 03
0.002550	0.11000000E 00	0.18531930E 03
0.002550	0.12000000E 00	0.18098487E 03
0.002550	0.13000000E 00	0.17597290E 03
0.002550	0.14000000E 00	0.17017158E 03
0.002550	0.15000000E 00	0.16342958E 03
0.002550	0.16000000E 00	0.15554309E 03
0.002550	0.17000000E 00	0.14622252E 03
0.002550	0.18000000E 00	0.13502830E 03
0.002550	0.18999999E 00	0.12123499E 03
0.002550	0.20000000E 00	0.10346817E 03
0.002550	0.20999999E 00	0.78361562E 02
0.002550	0.21999999E 00	0.27055807E 02

D.3 Section III for Two- and Twelve-Inch Drop Heights

Interior stresses are given here for both drop heights, two and twelve inches, for time equal to 0.001 second and time equal to that for maximum indentation. The CC stress is here labeled the RR stress and the CZ stress is labeled the RZ stress.

TWO INCH DROP HEIGHT

TIME	RADIUS	Z	ZZ STRESS
0.001000	0.10000000E-01	0.10000000E-01	-0.19194269E 03
0.001000	0.10000000E-01	0.34999999E-01	-0.17678044E 03
0.001000	0.10000000E-01	0.59999999E-01	-0.16329687E 03
0.001000	0.10000000E-01	0.84999999E-01	-0.14666961E 03
0.001000	0.10000000E-01	0.11000000E 00	-0.12898741E 03
0.001000	0.10000000E-01	0.13500000E 00	-0.11198600E 03
0.001000	0.10000000E-01	0.16000000E 00	-0.96628483E 02
0.001000	0.10000000E-01	0.18499999E 00	-0.83263994E 02
0.001000	0.10000000E-01	0.20999999E 00	-0.71877242E 02
0.001000	0.10000000E-01	0.23499999E 00	-0.62279952E 02
0.001000	0.10000000E-01	0.25999999E 00	-0.54225087E 02
0.001000	0.10000000E-01	0.28499999E 00	-0.47465652E 02
0.001000	0.10000000E-01	0.30999999E 00	-0.41779626E 02
0.001000	0.10000000E-01	0.33499999E 00	-0.36977630E 02
0.001000	0.10000000E-01	0.35999999E 00	-0.32902727E 02
0.001000	0.59999999E-01	0.10000000E-01	-0.17619108E 03
0.001000	0.59999999E-01	0.34999999E-01	-0.16392570E 03
0.001000	0.59999999E-01	0.59999999E-01	-0.15009499E 03
0.001000	0.59999999E-01	0.84999999E-01	-0.13345984E 03
0.001000	0.59999999E-01	0.11000000E 00	-0.11672285E 03
0.001000	0.59999999E-01	0.13500000E 00	-0.10129737E 03
0.001000	0.59999999E-01	0.16000000E 00	-0.87707850E 02
0.001000	0.59999999E-01	0.18499999E 00	-0.76014229E 02
0.001000	0.59999999E-01	0.20999999E 00	-0.66070922E 02
0.001000	0.59999999E-01	0.23499999E 00	-0.57659754E 02
0.001000	0.59999999E-01	0.25999999E 00	-0.50552997E 02
0.001000	0.59999999E-01	0.28499999E 00	-0.44540486E 02
0.001000	0.59999999E-01	0.30999999E 00	-0.39439110E 02

TIME	RADIUS	Z	ZZ STRESS
0.001000	0.59999999E-01	0.33499999E 00	-0.35094206E 02
0.001000	0.59999999E-01	0.35999999E 00	-0.31377352E 02
0.001000	0.11000000E 00	0.10000000E-01	-0.14356777E 03
0.001000	0.11000000E 00	0.34999999E-01	-0.12903864E 03
0.001000	0.11000000E 00	0.59999999E-01	-0.11365841E 03
0.001000	0.11000000E 00	0.84999999E-01	-0.99567998E 02
0.001000	0.11000000E 00	0.11000000E 00	-0.87476455E 02
0.001000	0.11000000E 00	0.13500000E 00	-0.77106352E 02
0.001000	0.11000000E 00	0.16000000E 00	-0.68150259E 02
0.001000	0.11000000E 00	0.18499999E 00	-0.60381702E 02
0.001000	0.11000000E 00	0.20999999E 00	-0.53631271E 02
0.001000	0.11000000E 00	0.23499999E 00	-0.47762190E 02
0.001000	0.11000000E 00	0.25999999E 00	-0.42657464E 02
0.001000	0.11000000E 00	0.28499999E 00	-0.38214567E 02
0.001000	0.11000000E 00	0.30999999E 00	-0.34343214E 02
0.001000	0.11000000E 00	0.33499999E 00	-0.30964259E 02
0.001000	0.11000000E 00	0.35999999E 00	-0.28008843E 02
0.001000	0.16000000E 00	0.10000000E-01	-0.44861391E 02
0.001000	0.16000000E 00	0.34999999E-01	-0.48157877E 02
0.001000	0.16000000E 00	0.59999999E-01	-0.50387992E 02
0.001000	0.16000000E 00	0.84999999E-01	-0.50476561E 02
0.001000	0.16000000E 00	0.11000000E 00	-0.49139178E 02
0.001000	0.16000000E 00	0.13500000E 00	-0.46934732E 02
0.001000	0.16000000E 00	0.16000000E 00	-0.44246879E 02
0.001000	0.16000000E 00	0.18499999E 00	-0.41336212E 02
0.001000	0.16000000E 00	0.20999999E 00	-0.38376868E 02
0.001000	0.16000000E 00	0.23499999E 00	-0.35481621E 02
0.001000	0.16000000E 00	0.25999999E 00	-0.32719671E 02
0.001000	0.16000000E 00	0.28499999E 00	-0.30129828E 02
0.001000	0.16000000E 00	0.30999999E 00	-0.27730183E 02
0.001000	0.16000000E 00	0.33499999E 00	-0.25525131E 02
0.001000	0.16000000E 00	0.35999999E 00	-0.23510358E 02
0.001000	0.21000000E 00	0.10000000E-01	-0.14111401E 00
0.001000	0.21000000E 00	0.34999999E-01	-0.38833772E 01
0.001000	0.21000000E 00	0.59999999E-01	-0.10581615E 02
0.001000	0.21000000E 00	0.84999999E-01	-0.16472269E 02
0.001000	0.21000000E 00	0.11000000E 00	-0.20583776E 02
0.001000	0.21000000E 00	0.13500000E 00	-0.23094807E 02
0.001000	0.21000000E 00	0.16000000E 00	-0.24383708E 02
0.001000	0.21000000E 00	0.18499999E 00	-0.24793335E 02
0.001000	0.21000000E 00	0.20999999E 00	-0.24592590E 02
0.001000	0.21000000E 00	0.23499999E 00	-0.23982599E 02
0.001000	0.21000000E 00	0.25999999E 00	-0.23110419E 02
0.001000	0.21000000E 00	0.28499999E 00	-0.22081902E 02
0.001000	0.21000000E 00	0.30999999E 00	-0.20972050E 02
0.001000	0.21000000E 00	0.33499999E 00	-0.19833101E 02

TIME	RADIUS	Z	ZZ STRESS
0.001000	0.21000000E 00	0.35999999E 00	-0.16700612E 02
0.001000	0.26000000E 00	0.10000000E-01	-0.16101673E-01
0.001000	0.26000000E 00	0.34999999E-01	-0.60598430E 00
0.001000	0.26000000E 00	0.59999999E-01	-0.23999047E 01
0.001000	0.26000000E 00	0.84999999E-01	-0.50336836E 01
0.001000	0.26000000E 00	0.11000000E 00	-0.78272629E 01
0.001000	0.26000000E 00	0.13500000E 00	-0.10310913E 02
0.001000	0.26000000E 00	0.16000000E 00	-0.12282784E 02
0.001000	0.26000000E 00	0.18499999E 00	-0.13711520E 02
0.001000	0.26000000E 00	0.20999999E 00	-0.14647911E 02
0.001000	0.26000000E 00	0.23499999E 00	-0.15172604E 02
0.001000	0.26000000E 00	0.25999999E 00	-0.15369994E 02
0.001000	0.26000000E 00	0.28499999E 00	-0.15316763E 02
0.001000	0.26000000E 00	0.30999999E 00	-0.15077912E 02
0.001000	0.26000000E 00	0.33499999E 00	-0.14706301E 02
0.001000	0.26000000E 00	0.35999999E 00	-0.14243729E 02
0.001000	0.30999999E 00	0.10000000E-01	-0.42201559E-02
0.001000	0.30999999E 00	0.34999999E-01	-0.16993365E 00
0.001000	0.30999999E 00	0.59999999E-01	-0.75579959E 00
0.001000	0.30999999E 00	0.84999999E-01	-0.18074958E 01
0.001000	0.30999999E 00	0.11000000E 00	-0.31852257E 01
0.001000	0.30999999E 00	0.13500000E 00	-0.46857798E 01
0.001000	0.30999999E 00	0.16000000E 00	-0.61336637E 01
0.001000	0.30999999E 00	0.18499999E 00	-0.74143883E 01
0.001000	0.30999999E 00	0.20999999E 00	-0.84712963E 01
0.001000	0.30999999E 00	0.23499999E 00	-0.92896670E 01
0.001000	0.30999999E 00	0.25999999E 00	-0.98805256E 01
0.001000	0.30999999E 00	0.28499999E 00	-0.10268393E 02
0.001000	0.30999999E 00	0.30999999E 00	-0.10483177E 02
0.001000	0.30999999E 00	0.33499999E 00	-0.10555346E 02
0.001000	0.30999999E 00	0.35999999E 00	-0.10513286E 02
0.002550	0.10000000E-01	0.10000000E-01	-0.21558968E 03
0.002550	0.10000000E-01	0.34999999E-01	-0.20228657E 03
0.002550	0.10000000E-01	0.59999999E-01	-0.19375414E 03
0.002550	0.10000000E-01	0.84999999E-01	-0.18251859E 03
0.002550	0.10000000E-01	0.11000000E 00	-0.16917470E 03
0.002550	0.10000000E-01	0.13500000E 00	-0.15478131E 03
0.002550	0.10000000E-01	0.16000000E 00	-0.14028503E 03
0.002550	0.10000000E-01	0.18499999E 00	-0.12636492E 03
0.002550	0.10000000E-01	0.20999999E 00	-0.11342978E 03
0.002550	0.10000000E-01	0.23499999E 00	-0.10167548E 03
0.002550	0.10000000E-01	0.25999999E 00	-0.91151027E 02
0.002550	0.10000000E-01	0.28499999E 00	-0.81816208E 02
0.002550	0.10000000E-01	0.30999999E 00	-0.73582739E 02
0.002550	0.10000000E-01	0.33499999E 00	-0.66341530E 02
0.002550	0.10000000E-01	0.35999999E 00	-0.59978939E 02

TIME	RADIUS	Z	ZZ STRESS
0.002550	0.59999999E-01	0.10000000E-01	-0.20450364E 03
0.002550	0.59999999E-01	0.34999999E-01	-0.19512147E 03
0.002550	0.59999999E-01	0.59999999E-01	-0.18684426E 03
0.002550	0.59999999E-01	0.84999999E-01	-0.17518332E 03
0.002550	0.59999999E-01	0.11000000E 00	-0.16155910E 03
0.002550	0.59999999E-01	0.13500000E 00	-0.14724747E 03
0.002550	0.59999999E-01	0.16000000E 00	-0.13316814E 03
0.002550	0.59999999E-01	0.18499999E 00	-0.11988588E 03
0.002550	0.59999999E-01	0.20999999E 00	-0.10769117E 03
0.002550	0.59999999E-01	0.23499999E 00	-0.96690068E 02
0.002550	0.59999999E-01	0.25999999E 00	-0.86875973E 02
0.002550	0.59999999E-01	0.28499999E 00	-0.78180259E 02
0.002550	0.59999999E-01	0.30999999E 00	-0.70504649E 02
0.002550	0.59999999E-01	0.33499999E 00	-0.63741025E 02
0.002550	0.59999999E-01	0.35999999E 00	-0.57782210E 02
0.002550	0.11000000E 00	0.10000000E-01	-0.16865243E 03
0.002550	0.11000000E 00	0.34999999E-01	-0.17930120E 03
0.002550	0.11000000E 00	0.59999999E-01	-0.16879936E 03
0.002550	0.11000000E 00	0.84999999E-01	-0.15563844E 03
0.002550	0.11000000E 00	0.11000000E 00	-0.14177155E 03
0.002550	0.11000000E 00	0.13500000E 00	-0.12833600E 03
0.002550	0.11000000E 00	0.16000000E 00	-0.11585402E 03
0.002550	0.11000000E 00	0.18499999E 00	-0.10450680E 03
0.002550	0.11000000E 00	0.20999999E 00	-0.94309356E 02
0.002550	0.11000000E 00	0.23499999E 00	-0.85202780E 02
0.002550	0.11000000E 00	0.25999999E 00	-0.77098234E 02
0.002550	0.11000000E 00	0.28499999E 00	-0.69897832E 02
0.002550	0.11000000E 00	0.30999999E 00	-0.63504550E 02
0.002550	0.11000000E 00	0.33499999E 00	-0.57826743E 02
0.002550	0.11000000E 00	0.35999999E 00	-0.52780494E 02
0.002550	0.16000000E 00	0.10000000E-01	-0.15015838E 03
0.002550	0.16000000E 00	0.34999999E-01	-0.14498381E 03
0.002550	0.16000000E 00	0.59999999E-01	-0.13144959E 03
0.002550	0.16000000E 00	0.84999999E-01	-0.11866995E 03
0.002550	0.16000000E 00	0.11000000E 00	-0.10758508E 03
0.002550	0.16000000E 00	0.13500000E 00	-0.97975145E 02
0.002550	0.16000000E 00	0.16000000E 00	-0.89512327E 02
0.002550	0.16000000E 00	0.18499999E 00	-0.81956556E 02
0.002550	0.16000000E 00	0.20999999E 00	-0.75150687E 02
0.002550	0.16000000E 00	0.23499999E 00	-0.68990570E 02
0.002550	0.16000000E 00	0.25999999E 00	-0.63402432E 02
0.002550	0.16000000E 00	0.28499999E 00	-0.58329092E 02
0.002550	0.16000000E 00	0.30999999E 00	-0.53722823E 02
0.002550	0.16000000E 00	0.33499999E 00	-0.49541352E 02
0.002550	0.16000000E 00	0.35999999E 00	-0.45746084E 02
0.002550	0.21000000E 00	0.10000000E-01	-0.62637758E 02

TIME	RADIUS		Z	ZZ	STRESS
0.002550	0.21000000E 00	00	0.34999999E-01	-0.65843228E	02
0.002550	0.21000000E 00	00	0.59999999E-01	-0.65335192E	02
0.002550	0.21000000E 00	00	0.84999999E-01	-0.64725390E	02
0.002550	0.21000000E 00	00	0.11000000E 00	-0.63443865E	02
0.002550	0.21000000E 00	00	0.13500000E 00	-0.61568899E	02
0.002550	0.21000000E 00	00	0.16000000E 00	-0.59259310E	02
0.002550	0.21000000E 00	00	0.18499999E 00	-0.56662422E	02
0.002550	0.21000000E 00	00	0.20999999E 00	-0.53898963E	02
0.002550	0.21000000E 00	00	0.23499999E 00	-0.51063404E	02
0.002550	0.21000000E 00	00	0.25999999E 00	-0.48227219E	02
0.002550	0.21000000E 00	00	0.28499999E 00	-0.45442893E	02
0.002550	0.21000000E 00	00	0.30999999E 00	-0.42747575E	02
0.002550	0.21000000E 00	00	0.33499999E 00	-0.40166225E	02
0.002550	0.21000000E 00	00	0.35999999E 00	-0.37714315E	02
0.002550	0.26000000E 00	00	0.10000000E-01	-0.24709174E	00
0.002550	0.26000000E 00	00	0.34999999E-01	-0.60316719E	01
0.002550	0.26000000E 00	00	0.59999999E-01	-0.15123030E	02
0.002550	0.26000000E 00	00	0.84999999E-01	-0.22695691E	02
0.002550	0.26000000E 00	00	0.11000000E 00	-0.28043995E	02
0.002550	0.26000000E 00	00	0.13500000E 00	-0.31540800E	02
0.002550	0.26000000E 00	00	0.16000000E 00	-0.33639260E	02
0.002550	0.26000000E 00	00	0.18499999E 00	-0.34706480E	02
0.002550	0.26000000E 00	00	0.20999999E 00	-0.35019983E	02
0.002550	0.26000000E 00	00	0.23499999E 00	-0.34787362E	02
0.002550	0.26000000E 00	00	0.25999999E 00	-0.34164473E	02
0.002550	0.26000000E 00	00	0.28499999E 00	-0.33268792E	02
0.002550	0.26000000E 00	00	0.30999999E 00	-0.32189047E	02
0.002550	0.26000000E 00	00	0.33499999E 00	-0.30992148E	02
0.002550	0.26000000E 00	00	0.35999999E 00	-0.29728363E	02
0.002550	0.30999999E 00	00	0.10000000E-01	-0.23794227E-01	
0.002550	0.30999999E 00	00	0.34999999E-01	-0.88442492E	00
0.002550	0.30999999E 00	00	0.59999999E-01	-0.34437026E	01
0.002550	0.30999999E 00	00	0.84999999E-01	-0.71310565E	01
0.002550	0.30999999E 00	00	0.11000000E 00	-0.11025112E	02
0.002550	0.30999999E 00	00	0.13500000E 00	-0.14535733E	02
0.002550	0.30999999E 00	00	0.16000000E 00	-0.17419460E	02
0.002550	0.30999999E 00	00	0.18499999E 00	-0.19635027E	02
0.002550	0.30999999E 00	00	0.20999999E 00	-0.21234045E	02
0.002550	0.30999999E 00	00	0.23499999E 00	-0.22301953E	02
0.002550	0.30999999E 00	00	0.25999999E 00	-0.22930041E	02
0.002550	0.30999999E 00	00	0.28499999E 00	-0.23203420E	02
0.002550	0.30999999E 00	00	0.30999999E 00	-0.23196656E	02
0.002550	0.30999999E 00	00	0.33499999E 00	-0.22972882E	02
0.002550	0.30999999E 00	00	0.35999999E 00	-0.22584415E	02

TIME	RADIUS	Z	RR STRESS
0.001000	0.10000000E-01	0.10000000E-01	-0.11017966E 03
0.001000	0.10000000E-01	0.34999999E-01	-0.72177161E 02
0.001000	0.10000000E-01	0.59999999E-01	-0.44326710E 02
0.001000	0.10000000E-01	0.84999999E-01	-0.25915706E 02
0.001000	0.10000000E-01	0.11000000E 00	-0.14308655E 02
0.001000	0.10000000E-01	0.13500000E 00	-0.72350191E 01
0.001000	0.10000000E-01	0.16000000E 00	-0.30276917E 01
0.001000	0.10000000E-01	0.18499999E 00	-0.57737025E 00
0.001000	0.10000000E-01	0.20999999E 00	0.81400618E 00
0.001000	0.10000000E-01	0.23499999E 00	0.15727649E 01
0.001000	0.10000000E-01	0.25999999E 00	0.19558122E 01
0.001000	0.10000000E-01	0.28499999E 00	0.21170749E 01
0.001000	0.10000000E-01	0.30999999E 00	0.21489333E 01
0.001000	0.10000000E-01	0.33499999E 00	0.21069523E 01
0.001000	0.10000000E-01	0.35999999E 00	0.20245638E 01
0.001000	0.59999999E-01	0.10000000E-01	-0.99088643E 02
0.001000	0.59999999E-01	0.34999999E-01	-0.64564626E 02
0.001000	0.59999999E-01	0.59999999E-01	-0.39804336E 02
0.001000	0.59999999E-01	0.84999999E-01	-0.24316289E 02
0.001000	0.59999999E-01	0.11000000E 00	-0.14678080E 02
0.001000	0.59999999E-01	0.13500000E 00	-0.86265425E 01
0.001000	0.59999999E-01	0.16000000E 00	-0.47939514E 01
0.001000	0.59999999E-01	0.18499999E 00	-0.23575151E 01
0.001000	0.59999999E-01	0.20999999E 00	-0.81179533E 00
0.001000	0.59999999E-01	0.23499999E 00	0.16036354E 00
0.001000	0.59999999E-01	0.25999999E 00	0.76105255E 00
0.001000	0.59999999E-01	0.28499999E 00	0.11204239E 01
0.001000	0.59999999E-01	0.30999999E 00	0.13229322E 01
0.001000	0.59999999E-01	0.33499999E 00	0.14236988E 01
0.001000	0.59999999E-01	0.35999999E 00	0.14588954E 01
0.001000	0.11000000E 00	0.10000000E-01	-0.72967745E 02
0.001000	0.11000000E 00	0.34999999E-01	-0.45974923E 02
0.001000	0.11000000E 00	0.59999999E-01	-0.31709797E 02
0.001000	0.11000000E 00	0.84999999E-01	-0.23067417E 02
0.001000	0.11000000E 00	0.11000000E 00	-0.16881386E 02
0.001000	0.11000000E 00	0.13500000E 00	-0.12226367E 02
0.001000	0.11000000E 00	0.16000000E 00	-0.87140030E 01
0.001000	0.11000000E 00	0.18499999E 00	-0.60883556E 01
0.001000	0.11000000E 00	0.20999999E 00	-0.41458716E 01
0.001000	0.11000000E 00	0.23499999E 00	-0.27215854E 01
0.001000	0.11000000E 00	0.25999999E 00	-0.16849902E 01
0.001000	0.11000000E 00	0.28499999E 00	-0.93552934E 00
0.001000	0.11000000E 00	0.30999999E 00	-0.39729196E 00
0.001000	0.11000000E 00	0.33499999E 00	-0.13784909E-01
0.001000	0.11000000E 00	0.35999999E 00	0.25665650E 00
0.001000	0.16000000E 00	0.10000000E-01	-0.17757586E 02

TIME	RADIUS		Z	RR STRESS	
0.001000	0.16000000E 00	0.34999999E-01	-0.29837262E 02		
0.001000	0.16000000E 00	0.59999999E-01	-0.28734678E 02		
0.001000	0.16000000E 00	0.84999999E-01	-0.24818974E 02		
0.001000	0.16000000E 00	0.11000000E 00	-0.20435683E 02		
0.001000	0.16000000E 00	0.13500000E 00	-0.16373363E 02		
0.001000	0.16000000E 00	0.16000000E 00	-0.12885548E 02		
0.001000	0.16000000E 00	0.18499999E 00	-0.10009516E 02		
0.001000	0.16000000E 00	0.20999999E 00	-0.76945145E 01		
0.001000	0.16000000E 00	0.23499999E 00	-0.58593889E 01		
0.001000	0.16000000E 00	0.25999999E 00	-0.44190880E 01		
0.001000	0.16000000E 00	0.28499999E 00	-0.32960080E 01		
0.001000	0.16000000E 00	0.30999999E 00	-0.24240139E 01		
0.001000	0.16000000E 00	0.33499999E 00	-0.17488860E 01		
0.001000	0.16000000E 00	0.35999999E 00	-0.12272165E 01		
0.001000	0.21000000E 00	0.10000000E-01	0.99854378E 01		
0.001000	0.21000000E 00	0.34999999E-01	-0.11464642E 02		
0.001000	0.21000000E 00	0.59999999E-01	-0.19552517E 02		
0.001000	0.21000000E 00	0.84999999E-01	-0.20829350E 02		
0.001000	0.21000000E 00	0.11000000E 00	-0.19377098E 02		
0.001000	0.21000000E 00	0.13500000E 00	-0.16978923E 02		
0.001000	0.21000000E 00	0.16000000E 00	-0.14404007E 02		
0.001000	0.21000000E 00	0.18499999E 00	-0.11980891E 02		
0.001000	0.21000000E 00	0.20999999E 00	-0.98362429E 01		
0.001000	0.21000000E 00	0.23499999E 00	-0.80015150E 01		
0.001000	0.21000000E 00	0.25999999E 00	-0.64640498E 01		
0.001000	0.21000000E 00	0.28499999E 00	-0.51926132E 01		
0.001000	0.21000000E 00	0.30999999E 00	-0.41502149E 01		
0.001000	0.21000000E 00	0.33499999E 00	-0.33003965E 01		
0.001000	0.21000000E 00	0.35999999E 00	-0.26100852E 01		
0.001000	0.26000000E 00	0.10000000E-01	0.95583878E 01		
0.001000	0.26000000E 00	0.34999999E-01	-0.12587972E 01		
0.001000	0.26000000E 00	0.59999999E-01	-0.86593680E 01		
0.001000	0.26000000E 00	0.84999999E-01	-0.12566329E 02		
0.001000	0.26000000E 00	0.11000000E 00	-0.13938555E 02		
0.001000	0.26000000E 00	0.13500000E 00	-0.13777211E 02		
0.001000	0.26000000E 00	0.16000000E 00	-0.12803458E 02		
0.001000	0.26000000E 00	0.18499999E 00	-0.11467896E 02		
0.001000	0.26000000E 00	0.20999999E 00	-0.10031611E 02		
0.001000	0.26000000E 00	0.23499999E 00	-0.86364545E 01		
0.001000	0.26000000E 00	0.25999999E 00	-0.73528217E 01		
0.001000	0.26000000E 00	0.28499999E 00	-0.62096345E 01		
0.001000	0.26000000E 00	0.30999999E 00	-0.52125736E 01		
0.001000	0.26000000E 00	0.33499999E 00	-0.43549991E 01		
0.001000	0.26000000E 00	0.35999999E 00	-0.36243639E 01		
0.001000	0.30999999E 00	0.10000000E-01	0.75886159E 01		
0.001000	0.30999999E 00	0.34999999E-01	0.15732016E 01		

TIME	RADIUS	Z	RR STRESS
0.001000	0.30999999E 00	0.59999999E-01	-0.33280345E 01
0.001000	0.30999999E 00	0.84999999E-01	-0.67812181E 01
0.001000	0.30999999E 00	0.11000000E 00	-0.88441267E 01
0.001000	0.30999999E 00	0.13500000E 00	-0.97882041E 01
0.001000	0.30999999E 00	0.16000000E 00	-0.99347846E 01
0.001000	0.30999999E 00	0.18499999E 00	-0.95665703E 01
0.001000	0.30999999E 00	0.20999999E 00	-0.88993851E 01
0.001000	0.30999999E 00	0.23499999E 00	-0.80844456E 01
0.001000	0.30999999E 00	0.25999999E 00	-0.72212119E 01
0.001000	0.30999999E 00	0.28499999E 00	-0.63713495E 01
0.001000	0.30999999E 00	0.30999999E 00	-0.55704139E 01
0.001000	0.30999999E 00	0.33499999E 00	-0.48366650E 01
0.001000	0.30999999E 00	0.35999999E 00	-0.41772991E 01
0.002550	0.10000000E-01	0.10000000E-01	-0.12953020E 03
0.002550	0.10000000E-01	0.34999999E-01	-0.96710541E 02
0.002550	0.10000000E-01	0.59999999E-01	-0.69128524E 02
0.002550	0.10000000E-01	0.84999999E-01	-0.48006507E 02
0.002550	0.10000000E-01	0.11000000E 00	-0.32402591E 02
0.002550	0.10000000E-01	0.13500000E 00	-0.21207916E 02
0.002550	0.10000000E-01	0.16000000E 00	-0.13353751E 02
0.002550	0.10000000E-01	0.18499999E 00	-0.79355798E 01
0.002550	0.10000000E-01	0.20999999E 00	-0.42475007E 01
0.002550	0.10000000E-01	0.23499999E 00	-0.17668794E 01
0.002550	0.10000000E-01	0.25999999E 00	-0.11951042E 00
0.002550	0.10000000E-01	0.28499999E 00	0.95664871E 00
0.002550	0.10000000E-01	0.30999999E 00	0.16429589E 01
0.002550	0.10000000E-01	0.33499999E 00	0.20640771E 01
0.002550	0.10000000E-01	0.35999999E 00	0.23053787E 01
0.002550	0.59999999E-01	0.10000000E-01	-0.12239928E 03
0.002550	0.59999999E-01	0.34999999E-01	-0.91863776E 02
0.002550	0.59999999E-01	0.59999999E-01	-0.65061598E 02
0.002550	0.59999999E-01	0.84999999E-01	-0.45205240E 02
0.002550	0.59999999E-01	0.11000000E 00	-0.30920264E 02
0.002550	0.59999999E-01	0.13500000E 00	-0.20807671E 02
0.002550	0.59999999E-01	0.16000000E 00	-0.13707801E 02
0.002550	0.59999999E-01	0.18499999E 00	-0.87439506E 01
0.002550	0.59999999E-01	0.20999999E 00	-0.52826894E 01
0.002550	0.59999999E-01	0.23499999E 00	-0.28758845E 01
0.002550	0.59999999E-01	0.25999999E 00	-0.12092524E 01
0.002550	0.59999999E-01	0.28499999E 00	-0.62874964E-01
0.002550	0.59999999E-01	0.30999999E 00	0.71722788E 00
0.002550	0.59999999E-01	0.33499999E 00	0.12390969E 01
0.002550	0.59999999E-01	0.35999999E 00	0.15787811E 01
0.002550	0.11000000E 00	0.10000000E-01	-0.10937473E 03
0.002550	0.11000000E 00	0.34999999E-01	-0.78686589E 02
0.002550	0.11000000E 00	0.59999999E-01	-0.55088340E 02

TIME	RADIUS	Z	RR STRESS
0.002550	0.11000000E 00	0.84999999E-01	-0.39216682E 02
0.002550	0.11000000E 00	0.11000000E 00	-0.28348449E 02
0.002550	0.11000000E 00	0.13500000E 00	-0.20661582E 02
0.002550	0.11000000E 00	0.16000000E 00	-0.15065819E 02
0.002550	0.11000000E 00	0.18499999E 00	-0.10912867E 02
0.002550	0.11000000E 00	0.20999999E 00	-0.77977253E 01
0.002550	0.11000000E 00	0.23499999E 00	-0.54500946E 01
0.002550	0.11000000E 00	0.25999999E 00	-0.36791657E 01
0.002550	0.11000000E 00	0.28499999E 00	-0.23450235E 01
0.002550	0.11000000E 00	0.30999999E 00	-0.13428591E 01
0.002550	0.11000000E 00	0.33499999E 00	-0.59332276E 00
0.002550	0.11000000E 00	0.35999999E 00	-0.36062058E-01
0.002550	0.16000000E 00	0.10000000E-01	-0.82279552E 02
0.002550	0.16000000E 00	0.34999999E-01	-0.56169135E 02
0.002550	0.16000000E 00	0.59999999E-01	-0.42032780E 02
0.002550	0.16000000E 00	0.84999999E-01	-0.33613736E 02
0.002550	0.16000000E 00	0.11000000E 00	-0.27382252E 02
0.002550	0.16000000E 00	0.13500000E 00	-0.22284805E 02
0.002550	0.16000000E 00	0.16000000E 00	-0.18006650E 02
0.002550	0.16000000E 00	0.18499999E 00	-0.14421430E 02
0.002550	0.16000000E 00	0.20999999E 00	-0.11443342E 02
0.002550	0.16000000E 00	0.23499999E 00	-0.89933248E 01
0.002550	0.16000000E 00	0.25999999E 00	-0.69948129E 01
0.002550	0.16000000E 00	0.28499999E 00	-0.53759867E 01
0.002550	0.16000000E 00	0.30999999E 00	-0.40721187E 01
0.002550	0.16000000E 00	0.33499999E 00	-0.30267114E 01
0.002550	0.16000000E 00	0.35999999E 00	-0.21917081E 01
0.002550	0.21000000E 00	0.10000000E-01	-0.24467053E 02
0.002550	0.21000000E 00	0.34999999E-01	-0.34400309E 02
0.002550	0.21000000E 00	0.59999999E-01	-0.35123472E 02
0.002550	0.21000000E 00	0.84999999E-01	-0.32621926E 02
0.002550	0.21000000E 00	0.11000000E 00	-0.28995585E 02
0.002550	0.21000000E 00	0.13500000E 00	-0.25123857E 02
0.002550	0.21000000E 00	0.16000000E 00	-0.21405889E 02
0.002550	0.21000000E 00	0.18499999E 00	-0.18019201E 02
0.002550	0.21000000E 00	0.20999999E 00	-0.15029210E 02
0.002550	0.21000000E 00	0.23499999E 00	-0.12442688E 02
0.002550	0.21000000E 00	0.25999999E 00	-0.10236516E 02
0.002550	0.21000000E 00	0.28499999E 00	-0.83736314E 01
0.002550	0.21000000E 00	0.30999999E 00	-0.68120921E 01
0.002550	0.21000000E 00	0.33499999E 00	-0.55101828E 01
0.002550	0.21000000E 00	0.35999999E 00	-0.44290441E 01
0.002550	0.26000000E 00	0.10000000E-01	0.13414703E 02
0.002550	0.26000000E 00	0.34999999E-01	-0.14813977E 02
0.002550	0.26000000E 00	0.59999999E-01	-0.25342795E 02
0.002550	0.26000000E 00	0.84999999E-01	-0.27840844E 02

TIME	RADIUS	Z	RR STRESS
0.002550	0.26000000E 00	0.11000000E 00	-0.27048935E 02
0.002550	0.26000000E 00	0.13500000E 00	-0.24902726E 02
0.002550	0.26000000E 00	0.16000000E 00	-0.22263830E 02
0.002550	0.26000000E 00	0.18499999E 00	-0.19544643E 02
0.002550	0.26000000E 00	0.20999999E 00	-0.16946694E 02
0.002550	0.26000000E 00	0.23499999E 00	-0.14563893E 02
0.002550	0.26000000E 00	0.25999999E 00	-0.12432407E 02
0.002550	0.26000000E 00	0.28499999E 00	-0.10557058E 02
0.002550	0.26000000E 00	0.30999999E 00	-0.89259299E 01
0.002550	0.26000000E 00	0.33499999E 00	-0.75188220E 01
0.002550	0.26000000E 00	0.35999999E 00	-0.63121697E 01
0.002550	0.30999999E 00	0.10000000E-01	0.14334283E 02
0.002550	0.30999999E 00	0.34999999E-01	-0.97683902E 00
0.002550	0.30999999E 00	0.59999999E-01	-0.11548634E 02
0.002550	0.30999999E 00	0.84999999E-01	-0.17419372E 02
0.002550	0.30999999E 00	0.11000000E 00	-0.19921778E 02
0.002550	0.30999999E 00	0.13500000E 00	-0.20342139E 02
0.002550	0.30999999E 00	0.16000000E 00	-0.19579295E 02
0.002550	0.30999999E 00	0.18499999E 00	-0.18198042E 02
0.002550	0.30999999E 00	0.20999999E 00	-0.16539316E 02
0.002550	0.30999999E 00	0.23499999E 00	-0.14804241E 02
0.002550	0.30999999E 00	0.25999999E 00	-0.13108198E 02
0.002550	0.30999999E 00	0.28499999E 00	-0.11514111E 02
0.002550	0.30999999E 00	0.30999999E 00	-0.10052804E 02
0.002550	0.30999999E 00	0.33499999E 00	-0.87356389E 01
0.002550	0.30999999E 00	0.35999999E 00	-0.75624092E 01

TIME	RADIUS	Z	RR STRESS
0.001000	0.10000000E-01	0.10000000E-01	-0.11166181E 03
0.001000	0.10000000E-01	0.34999999E-01	-0.72149942E 02
0.001000	0.10000000E-01	0.59999999E-01	-0.44334925E 02
0.001000	0.10000000E-01	0.84999999E-01	-0.25891270E 02
0.001000	0.10000000E-01	0.11000000E 00	-0.14259177E 02
0.001000	0.10000000E-01	0.13500000E 00	-0.71749143E 01
0.001000	0.10000000E-01	0.16000000E 00	-0.29676611E 01
0.001000	0.10000000E-01	0.18499999E 00	-0.52296882E 00
0.001000	0.10000000E-01	0.20999999E 00	0.86076573E 00
0.001000	0.10000000E-01	0.23499999E 00	0.16118551E 01
0.001000	0.10000000E-01	0.25999999E 00	0.19878961E 01
0.001000	0.10000000E-01	0.28499999E 00	0.21432420E 01
0.001000	0.10000000E-01	0.30999999E 00	0.21702641E 01
0.001000	0.10000000E-01	0.33499999E 00	0.21243495E 01
0.001000	0.10000000E-01	0.35999999E 00	0.20387978E 01

TIME	RADIUS	Z	00 STRESS
0.001000	0.59999999E-01	0.10000000E-01	-0.10603399E 03
0.001000	0.59999999E-01	0.34999999E-01	-0.66737910E 02
0.001000	0.59999999E-01	0.59999999E-01	-0.40216795E 02
0.001000	0.59999999E-01	0.84999999E-01	-0.23238364E 02
0.001000	0.59999999E-01	0.11000000E 00	-0.12762390E 02
0.001000	0.59999999E-01	0.13500000E 00	-0.64382635E 01
0.001000	0.59999999E-01	0.16000000E 00	-0.26763321E 01
0.001000	0.59999999E-01	0.18499999E 00	-0.47040769E 00
0.001000	0.59999999E-01	0.20999999E 00	0.79662508E 00
0.001000	0.59999999E-01	0.23499999E 00	0.14983430E 01
0.001000	0.59999999E-01	0.25999999E 00	0.18601953E 01
0.001000	0.59999999E-01	0.28499999E 00	0.20184813E 01
0.001000	0.59999999E-01	0.30999999E 00	0.20559517E 01
0.001000	0.59999999E-01	0.33499999E 00	0.20230656E 01
0.001000	0.59999999E-01	0.35999999E 00	0.19506700E 01
0.001000	0.11000000E 00	0.10000000E-01	-0.89155364E 02
0.001000	0.11000000E 00	0.34999999E-01	-0.52101158E 02
0.001000	0.11000000E 00	0.59999999E-01	-0.30433996E 02
0.001000	0.11000000E 00	0.84999999E-01	-0.17546317E 02
0.001000	0.11000000E 00	0.11000000E 00	-0.97890439E 01
0.001000	0.11000000E 00	0.13500000E 00	-0.50728513E 01
0.001000	0.11000000E 00	0.16000000E 00	-0.21944162E 01
0.001000	0.11000000E 00	0.18499999E 00	-0.44351309E 00
0.001000	0.11000000E 00	0.20999999E 00	0.60814394E 00
0.001000	0.11000000E 00	0.23499999E 00	0.12230889E 01
0.001000	0.11000000E 00	0.25999999E 00	0.15642937E 01
0.001000	0.11000000E 00	0.28499999E 00	0.17339410E 01
0.001000	0.11000000E 00	0.30999999E 00	0.17966084E 01
0.001000	0.11000000E 00	0.33499999E 00	0.17933658E 01
0.001000	0.11000000E 00	0.35999999E 00	0.17504402E 01
0.001000	0.16000000E 00	0.10000000E-01	-0.47674917E 02
0.001000	0.16000000E 00	0.34999999E-01	-0.28875635E 02
0.001000	0.16000000E 00	0.59999999E-01	-0.18349649E 02
0.001000	0.16000000E 00	0.84999999E-01	-0.11457819E 02
0.001000	0.16000000E 00	0.11000000E 00	-0.69011481E 01
0.001000	0.16000000E 00	0.13500000E 00	-0.38823153E 01
0.001000	0.16000000E 00	0.16000000E 00	-0.18872313E 01
0.001000	0.16000000E 00	0.18499999E 00	-0.57753124E 00
0.001000	0.16000000E 00	0.20999999E 00	0.27192448E 00
0.001000	0.16000000E 00	0.23499999E 00	0.81191251E 00
0.001000	0.16000000E 00	0.25999999E 00	0.11438583E 01
0.001000	0.16000000E 00	0.28499999E 00	0.13362161E 01
0.001000	0.16000000E 00	0.30999999E 00	0.14353129E 01
0.001000	0.16000000E 00	0.33499999E 00	0.14725695E 01
0.001000	0.16000000E 00	0.35999999E 00	0.14693321E 01
0.001000	0.21000000E 00	0.10000000E-01	-0.20201524E 02

TIME	RADIUS	Z	00 STRESS
0.001000	0.21000000E 00	0.34999999E-01	-0.15508356E 02
0.001000	0.21000000E 00	0.59999999E-01	-0.11295651E 02
0.001000	0.21000000E 00	0.84999999E-01	-0.78681959E 01
0.001000	0.21000000E 00	0.11000000E 00	-0.52405696E 01
0.001000	0.21000000E 00	0.13500000E 00	-0.32920681E 01
0.001000	0.21000000E 00	0.16000000E 00	-0.18772242E 01
0.001000	0.21000000E 00	0.18499999E 00	-0.86618107E 00
0.001000	0.21000000E 00	0.20999999E 00	-0.15438421E 00
0.001000	0.21000000E 00	0.23499999E 00	0.33846477E 00
0.001000	0.21000000E 00	0.25999999E 00	0.67250981E 00
0.001000	0.21000000E 00	0.28499999E 00	0.89215104E 00
0.001000	0.21000000E 00	0.30999999E 00	0.10298767E 01
0.001000	0.21000000E 00	0.33499999E 00	0.11093290E 01
0.001000	0.21000000E 00	0.35999999E 00	0.11476449E 01
0.001000	0.26000000E 00	0.10000000E-01	-0.13323735E 02
0.001000	0.26000000E 00	0.34999999E-01	-0.10647029E 02
0.001000	0.26000000E 00	0.59999999E-01	-0.82167202E 01
0.001000	0.26000000E 00	0.84999999E-01	-0.61309959E 01
0.001000	0.26000000E 00	0.11000000E 00	-0.44159906E 01
0.001000	0.26000000E 00	0.13500000E 00	-0.30492719E 01
0.001000	0.26000000E 00	0.16000000E 00	-0.19850477E 01
0.001000	0.26000000E 00	0.18499999E 00	-0.11711855E 01
0.001000	0.26000000E 00	0.20999999E 00	-0.55822172E 00
0.001000	0.26000000E 00	0.23499999E 00	-0.10314810E 00
0.001000	0.26000000E 00	0.25999999E 00	0.22963213E 00
0.001000	0.26000000E 00	0.28499999E 00	0.46870313E 00
0.001000	0.26000000E 00	0.30999999E 00	0.63656808E 00
0.001000	0.26000000E 00	0.33499999E 00	0.75071188E 00
0.001000	0.26000000E 00	0.35999999E 00	0.82459081E 00
0.001000	0.30999999E 00	0.10000000E-01	-0.94897898E 01
0.001000	0.30999999E 00	0.34999999E-01	-0.78760268E 01
0.001000	0.30999999E 00	0.59999999E-01	-0.63660800E 01
0.001000	0.30999999E 00	0.84999999E-01	-0.50113414E 01
0.001000	0.30999999E 00	0.11000000E 00	-0.38379499E 01
0.001000	0.30999999E 00	0.13500000E 00	-0.28502028E 01
0.001000	0.30999999E 00	0.16000000E 00	-0.20374798E 01
0.001000	0.30999999E 00	0.18499999E 00	-0.13809723E 01
0.001000	0.30999999E 00	0.20999999E 00	-0.85874105E 00
0.001000	0.30999999E 00	0.23499999E 00	-0.44888062E 00
0.001000	0.30999999E 00	0.25999999E 00	-0.13125015E 00
0.001000	0.30999999E 00	0.28499999E 00	0.11177132E 00
0.001000	0.30999999E 00	0.30999999E 00	0.29510703E 00
0.001000	0.30999999E 00	0.33499999E 00	0.43111618E 00
0.001000	0.30999999E 00	0.35999999E 00	0.52987265E 00
0.002550	0.10000000E-01	0.10000000E-01	-0.13113069E 03
0.002550	0.10000000E-01	0.34999999E-01	-0.96654338E 02

TIME	RADIUS	Z	00 STRESS
0.002550	0.10000000E-01	0.59999999E-01	-0.69148390E 02
0.002550	0.10000000E-01	0.84999999E-01	-0.48017288E 02
0.002550	0.10000000E-01	0.11000000E 00	-0.32395783E 02
0.002550	0.10000000E-01	0.13500000E 00	-0.21186147E 02
0.002550	0.10000000E-01	0.16000000E 00	-0.13322475E 02
0.002550	0.10000000E-01	0.18499999E 00	-0.78997961E 01
0.002550	0.10000000E-01	0.20999999E 00	-0.42109218E 01
0.002550	0.10000000E-01	0.23499999E 00	-0.17318051E 01
0.002550	0.10000000E-01	0.25999999E 00	-0.87357150E-01
0.002550	0.10000000E-01	0.28499999E 00	0.98538622E 00
0.002550	0.10000000E-01	0.30999999E 00	0.16682438E 01
0.002550	0.10000000E-01	0.33499999E 00	0.20860681E 01
0.002550	0.10000000E-01	0.35999999E 00	0.23243798E 01
0.002550	0.59999999E-01	0.10000000E-01	-0.12846820E 03
0.002550	0.59999999E-01	0.34999999E-01	-0.93485917E 02
0.002550	0.59999999E-01	0.59999999E-01	-0.66168670E 02
0.002550	0.59999999E-01	0.84999999E-01	-0.45577755E 02
0.002550	0.59999999E-01	0.11000000E 00	-0.30584987E 02
0.002550	0.59999999E-01	0.13500000E 00	-0.19944740E 02
0.002550	0.59999999E-01	0.16000000E 00	-0.12531222E 02
0.002550	0.59999999E-01	0.18499999E 00	-0.74342900E 01
0.002550	0.59999999E-01	0.20999999E 00	-0.39664951E 01
0.002550	0.59999999E-01	0.23499999E 00	-0.16298815E 01
0.002550	0.59999999E-01	0.25999999E 00	-0.72840040E-01
0.002550	0.59999999E-01	0.28499999E 00	0.94917464E 00
0.002550	0.59999999E-01	0.30999999E 00	0.16048808E 01
0.002550	0.59999999E-01	0.33499999E 00	0.20102641E 01
0.002550	0.59999999E-01	0.35999999E 00	0.22449809E 01
0.002550	0.11000000E 00	0.10000000E-01	-0.12069012E 03
0.002550	0.11000000E 00	0.34999999E-01	-0.84667387E 02
0.002550	0.11000000E 00	0.59999999E-01	-0.58498257E 02
0.002550	0.11000000E 00	0.84999999E-01	-0.39636931E 02
0.002550	0.11000000E 00	0.11000000E 00	-0.26371161E 02
0.002550	0.11000000E 00	0.13500000E 00	-0.17162189E 02
0.002550	0.11000000E 00	0.16000000E 00	-0.10813212E 02
0.002550	0.11000000E 00	0.18499999E 00	-0.64540842E 01
0.002550	0.11000000E 00	0.20999999E 00	-0.34720444E 01
0.002550	0.11000000E 00	0.23499999E 00	-0.14417654E 01
0.002550	0.11000000E 00	0.25999999E 00	-0.69682890E-01
0.002550	0.11000000E 00	0.28499999E 00	0.84662172E 00
0.002550	0.11000000E 00	0.30999999E 00	0.14469015E 01
0.002550	0.11000000E 00	0.33499999E 00	0.18279128E 01
0.002550	0.11000000E 00	0.35999999E 00	0.20568660E 01
0.002550	0.16000000E 00	0.10000000E-01	-0.10783358E 03
0.002550	0.16000000E 00	0.34999999E-01	-0.68254101E 02
0.002550	0.16000000E 00	0.59999999E-01	-0.45698760E 02

TIME	RADIUS		Z	00 STRESS
0.002550	0.16000000E 00	0.84999999E-01	-0.30672466E 02	
0.002550	0.16000000E 00	0.11000000E 00	-0.20504197E 02	
0.002550	0.16000000E 00	0.13500000E 00	-0.13521605E 02	
0.002550	0.16000000E 00	0.16000000E 00	-0.86779755E 01	
0.002550	0.16000000E 00	0.18499999E 00	-0.52983935E 01	
0.002550	0.16000000E 00	0.20999999E 00	-0.29350002E 01	
0.002550	0.16000000E 00	0.23499999E 00	-0.12840043E 01	
0.002550	0.16000000E 00	0.25999999E 00	-0.13600371E 00	
0.002550	0.16000000E 00	0.28499999E 00	0.65505791E 00	
0.002550	0.16000000E 00	0.30999999E 00	0.11919574E 01	
0.002550	0.16000000E 00	0.33499999E 00	0.15475264E 01	
0.002550	0.16000000E 00	0.35999999E 00	0.17736638E 01	
0.002550	0.21000000E 00	0.10000000E-01	-0.69491568E 02	
0.002550	0.21000000E 00	0.34999999E-01	-0.42240642E 02	
0.002550	0.21000000E 00	0.59999999E-01	-0.29721363E 02	
0.002550	0.21000000E 00	0.84999999E-01	-0.20981185E 02	
0.002550	0.21000000E 00	0.11000000E 00	-0.14697983E 02	
0.002550	0.21000000E 00	0.13500000E 00	-0.10129156E 02	
0.002550	0.21000000E 00	0.16000000E 00	-0.67903540E 01	
0.002550	0.21000000E 00	0.18499999E 00	-0.43465722E 01	
0.002550	0.21000000E 00	0.20999999E 00	-0.25593311E 01	
0.002550	0.21000000E 00	0.23499999E 00	-0.12560805E 01	
0.002550	0.21000000E 00	0.25999999E 00	-0.31073390E 00	
0.002550	0.21000000E 00	0.28499999E 00	0.36943484E 00	
0.002550	0.21000000E 00	0.30999999E 00	0.85292788E 00	
0.002550	0.21000000E 00	0.33499999E 00	0.11905168E 01	
0.002550	0.21000000E 00	0.35999999E 00	0.14199290E 01	
0.002550	0.26000000E 00	0.10000000E-01	-0.28392613E 02	
0.002550	0.26000000E 00	0.34999999E-01	-0.23590327E 02	
0.002550	0.26000000E 00	0.59999999E-01	-0.18694679E 02	
0.002550	0.26000000E 00	0.84999999E-01	-0.14337196E 02	
0.002550	0.26000000E 00	0.11000000E 00	-0.10724750E 02	
0.002550	0.26000000E 00	0.13500000E 00	-0.78303073E 01	
0.002550	0.26000000E 00	0.16000000E 00	-0.55534117E 01	
0.002550	0.26000000E 00	0.18499999E 00	-0.37828921E 01	
0.002550	0.26000000E 00	0.20999999E 00	-0.24178328E 01	
0.002550	0.26000000E 00	0.23499999E 00	-0.13731624E 01	
0.002550	0.26000000E 00	0.25999999E 00	-0.57959823E 00	
0.002550	0.26000000E 00	0.28499999E 00	0.18225019E-01	
0.002550	0.26000000E 00	0.30999999E 00	0.46406889E 00	
0.002550	0.26000000E 00	0.33499999E 00	0.79228079E 00	
0.002550	0.26000000E 00	0.35999999E 00	0.10296970E 01	
0.002550	0.30999999E 00	0.10000000E-01	-0.19905760E 02	
0.002550	0.30999999E 00	0.34999999E-01	-0.16660236E 02	
0.002550	0.30999999E 00	0.59999999E-01	-0.13615869E 02	
0.002550	0.30999999E 00	0.84999999E-01	-0.10881360E 02	

TIME	RADIUS	Z	ΘΘ STRESS
0.002550	0.309999999E 00	0.110000000E 00	-0.85120408E 01
0.002550	0.309999999E 00	0.135000000E 00	-0.65140943E 01
0.002550	0.309999999E 00	0.160000000E 00	-0.48623203E 01
0.002550	0.309999999E 00	0.184999999E 00	-0.35166000E 01
0.002550	0.309999999E 00	0.209999999E 00	-0.24325892E 01
0.002550	0.309999999E 00	0.234999999E 00	-0.15675506E 01
0.002550	0.309999999E 00	0.259999999E 00	-0.88301981E 00
0.002550	0.309999999E 00	0.284999999E 00	-0.34573510E 00
0.002550	0.309999999E 00	0.309999999E 00	0.72361329E-01
0.002550	0.309999999E 00	0.334999999E 00	0.39455147E 00
0.002550	0.309999999E 00	0.359999999E 00	0.63993676E 00

TIME	RADIUS	Z	RZ STRESS
0.001000	0.100000000E-01	0.100000000E-01	-0.65978925E 01
0.001000	0.100000000E-01	0.349999999E-01	-0.23646359E 01
0.001000	0.100000000E-01	0.599999999E-01	-0.30769611E 01
0.001000	0.100000000E-01	0.849999999E-01	-0.35024465E 01
0.001000	0.100000000E-01	0.110000000E 00	-0.35164756E 01
0.001000	0.100000000E-01	0.135000000E 00	-0.32606983E 01
0.001000	0.100000000E-01	0.160000000E 00	-0.28811632E 01
0.001000	0.100000000E-01	0.184999999E 00	-0.24752892E 01
0.001000	0.100000000E-01	0.209999999E 00	-0.20944979E 01
0.001000	0.100000000E-01	0.234999999E 00	-0.17599033E 01
0.001000	0.100000000E-01	0.259999999E 00	-0.14760354E 01
0.001000	0.100000000E-01	0.284999999E 00	-0.12396539E 01
0.001000	0.100000000E-01	0.309999999E 00	-0.10445976E 01
0.001000	0.100000000E-01	0.334999999E 00	-0.88417628E 00
0.001000	0.100000000E-01	0.359999999E 00	-0.75220916E 00
0.001000	0.599999999E-01	0.100000000E-01	-0.40529114E 01
0.001000	0.599999999E-01	0.349999999E-01	-0.12747537E 02
0.001000	0.599999999E-01	0.599999999E-01	-0.18574486E 02
0.001000	0.599999999E-01	0.849999999E-01	-0.20754988E 02
0.001000	0.599999999E-01	0.110000000E 00	-0.20340430E 02
0.001000	0.599999999E-01	0.135000000E 00	-0.18548781E 02
0.001000	0.599999999E-01	0.160000000E 00	-0.16242532E 02
0.001000	0.599999999E-01	0.184999999E 00	-0.13907358E 02
0.001000	0.599999999E-01	0.209999999E 00	-0.11770992E 02
0.001000	0.599999999E-01	0.234999999E 00	-0.99143656E 01
0.001000	0.599999999E-01	0.259999999E 00	-0.83445929E 01
0.001000	0.599999999E-01	0.284999999E 00	-0.70364326E 01
0.001000	0.599999999E-01	0.309999999E 00	-0.59537446E 01
0.001000	0.599999999E-01	0.334999999E 00	-0.50596474E 01
0.001000	0.599999999E-01	0.359999999E 00	-0.43207841E 01

TIME	RADIUS	Z	RZ STRESS
0.001000	0.11000000E 00	0.10000000E-01	-0.93163686E 01
0.001000	0.11000000E 00	0.34999999E-01	-0.27367335E 02
0.001000	0.11000000E 00	0.59999999E-01	-0.34719951E 02
0.001000	0.11000000E 00	0.84999999E-01	-0.35177724E 02
0.001000	0.11000000E 00	0.11000000E 00	-0.32560765E 02
0.001000	0.11000000E 00	0.13500000E 00	-0.28868360E 02
0.001000	0.11000000E 00	0.16000000E 00	-0.25028699E 02
0.001000	0.11000000E 00	0.18499999E 00	-0.21446345E 02
0.001000	0.11000000E 00	0.20999999E 00	-0.18273131E 02
0.001000	0.11000000E 00	0.23499999E 00	-0.15539920E 02
0.001000	0.11000000E 00	0.25999999E 00	-0.13222251E 02
0.001000	0.11000000E 00	0.28499999E 00	-0.11273610E 02
0.001000	0.11000000E 00	0.30999999E 00	-0.96418639E 01
0.001000	0.11000000E 00	0.33499999E 00	-0.82770459E 01
0.001000	0.11000000E 00	0.35999999E 00	-0.71345948E 01
0.001000	0.16000000E 00	0.10000000E-01	-0.23632023E 02
0.001000	0.16000000E 00	0.34999999E-01	-0.36183445E 02
0.001000	0.16000000E 00	0.59999999E-01	-0.37581724E 02
0.001000	0.16000000E 00	0.84999999E-01	-0.35957379E 02
0.001000	0.16000000E 00	0.11000000E 00	-0.33057759E 02
0.001000	0.16000000E 00	0.13500000E 00	-0.29696004E 02
0.001000	0.16000000E 00	0.16000000E 00	-0.26295842E 02
0.001000	0.16000000E 00	0.18499999E 00	-0.23077367E 02
0.001000	0.16000000E 00	0.20999999E 00	-0.20144831E 02
0.001000	0.16000000E 00	0.23499999E 00	-0.17535603E 02
0.001000	0.16000000E 00	0.25999999E 00	-0.15249287E 02
0.001000	0.16000000E 00	0.28499999E 00	-0.13265539E 02
0.001000	0.16000000E 00	0.30999999E 00	-0.11554830E 02
0.001000	0.16000000E 00	0.33499999E 00	-0.10084729E 02
0.001000	0.16000000E 00	0.35999999E 00	-0.88234087E 01
0.001000	0.21000000E 00	0.10000000E-01	-0.11243560E 01
0.001000	0.21000000E 00	0.34999999E-01	-0.96421645E 01
0.001000	0.21000000E 00	0.59999999E-01	-0.17274707E 02
0.001000	0.21000000E 00	0.84999999E-01	-0.21239440E 02
0.001000	0.21000000E 00	0.11000000E 00	-0.22566136E 02
0.001000	0.21000000E 00	0.13500000E 00	-0.22332386E 02
0.001000	0.21000000E 00	0.16000000E 00	-0.21240815E 02
0.001000	0.21000000E 00	0.18499999E 00	-0.19720138E 02
0.001000	0.21000000E 00	0.20999999E 00	-0.18027954E 02
0.001000	0.21000000E 00	0.23499999E 00	-0.16316290E 02
0.001000	0.21000000E 00	0.25999999E 00	-0.14671578E 02
0.001000	0.21000000E 00	0.28499999E 00	-0.13139440E 02
0.001000	0.21000000E 00	0.30999999E 00	-0.11740304E 02
0.001000	0.21000000E 00	0.33499999E 00	-0.10479363E 02
0.001000	0.21000000E 00	0.35999999E 00	-0.93529320E 01
0.001000	0.26000000E 00	0.10000000E-01	-0.24788117E 00

TIME	RADIUS	Z	RZ STRESS
0.001000	0.26000000E 00	0.34999999E-01	-0.27136838E 01
0.001000	0.26000000E 00	0.59999999E-01	-0.64768092E 01
0.001000	0.26000000E 00	0.84999999E-01	-0.99797412E 01
0.001000	0.26000000E 00	0.11000000E 00	-0.12500329E 02
0.001000	0.26000000E 00	0.13500000E 00	-0.13963964E 02
0.001000	0.26000000E 00	0.16000000E 00	-0.14560721E 02
0.001000	0.26000000E 00	0.18499999E 00	-0.14530216E 02
0.001000	0.26000000E 00	0.20999999E 00	-0.14081786E 02
0.001000	0.26000000E 00	0.23499999E 00	-0.13376202E 02
0.001000	0.26000000E 00	0.25999999E 00	-0.12529171E 02
0.001000	0.26000000E 00	0.28499999E 00	-0.11620478E 02
0.001000	0.26000000E 00	0.30999999E 00	-0.10703085E 02
0.001000	0.26000000E 00	0.33499999E 00	-0.98106998E 01
0.001000	0.26000000E 00	0.35999999E 00	-0.89635453E 01
0.001000	0.30999999E 00	0.10000000E-01	-0.93174824E-01
0.001000	0.30999999E 00	0.34999999E-01	-0.10788297E 01
0.001000	0.30999999E 00	0.59999999E-01	-0.28341356E 01
0.001000	0.30999999E 00	0.84999999E-01	-0.48659225E 01
0.001000	0.30999999E 00	0.11000000E 00	-0.67565899E 01
0.001000	0.30999999E 00	0.13500000E 00	-0.82673617E 01
0.001000	0.30999999E 00	0.16000000E 00	-0.93204955E 01
0.001000	0.30999999E 00	0.18499999E 00	-0.99387393E 01
0.001000	0.30999999E 00	0.20999999E 00	-0.10191441E 02
0.001000	0.30999999E 00	0.23499999E 00	-0.10160505E 02
0.001000	0.30999999E 00	0.25999999E 00	-0.99229620E 01
0.001000	0.30999999E 00	0.28499999E 00	-0.95439261E 01
0.001000	0.30999999E 00	0.30999999E 00	-0.90750687E 01
0.001000	0.30999999E 00	0.33499999E 00	-0.85556140E 01
0.001000	0.30999999E 00	0.35999999E 00	-0.80143124E 01
0.002550	0.10000000E-01	0.10000000E-01	-0.70713391E 01
0.002550	0.10000000E-01	0.34999999E-01	-0.15626143E 01
0.002550	0.10000000E-01	0.59999999E-01	-0.19817073E 01
0.002550	0.10000000E-01	0.84999999E-01	-0.24914596E 01
0.002550	0.10000000E-01	0.11000000E 00	-0.28087519E 01
0.002550	0.10000000E-01	0.13500000E 00	-0.29171060E 01
0.002550	0.10000000E-01	0.16000000E 00	-0.28610228E 01
0.002550	0.10000000E-01	0.18499999E 00	-0.26971418E 01
0.002550	0.10000000E-01	0.20999999E 00	-0.24747589E 01
0.002550	0.10000000E-01	0.23499999E 00	-0.22296348E 01
0.002550	0.10000000E-01	0.25999999E 00	-0.19855424E 01
0.002550	0.10000000E-01	0.28499999E 00	-0.17551655E 01
0.002550	0.10000000E-01	0.30999999E 00	-0.15452119E 01
0.002550	0.10000000E-01	0.33499999E 00	-0.13579420E 01
0.002550	0.10000000E-01	0.35999999E 00	-0.11931550E 01
0.002550	0.59999999E-01	0.10000000E-01	-0.22813303E 01
0.002550	0.59999999E-01	0.34999999E-01	-0.71884605E 01

TIME	RADIUS	Z	RZ STRESS
0.002550	0.59999999E-01	0.59999999E-01	-0.11871250E 02
0.002550	0.59999999E-01	0.84999999E-01	-0.15169291E 02
0.002550	0.59999999E-01	0.11000000E 00	-0.16923353E 02
0.002550	0.59999999E-01	0.13500000E 00	-0.17357452E 02
0.002550	0.59999999E-01	0.16000000E 00	-0.16849123E 02
0.002550	0.59999999E-01	0.18499999E 00	-0.15766548E 02
0.002550	0.59999999E-01	0.20999999E 00	-0.14397147E 02
0.002550	0.59999999E-01	0.23499999E 00	-0.12936885E 02
0.002550	0.59999999E-01	0.25999999E 00	-0.11505916E 02
0.002550	0.59999999E-01	0.28499999E 00	-0.10169951E 02
0.002550	0.59999999E-01	0.30999999E 00	-0.89591656E 01
0.002550	0.59999999E-01	0.33499999E 00	-0.78821523E 01
0.002550	0.59999999E-01	0.35999999E 00	-0.69353153E 01
0.002550	0.11000000E 00	0.10000000E-01	-0.44550854E 01
0.002550	0.11000000E 00	0.34999999E-01	-0.15118538E 02
0.002550	0.11000000E 00	0.59999999E-01	-0.23815558E 02
0.002550	0.11000000E 00	0.84999999E-01	-0.28981530E 02
0.002550	0.11000000E 00	0.11000000E 00	-0.30966710E 02
0.002550	0.11000000E 00	0.13500000E 00	-0.30713885E 02
0.002550	0.11000000E 00	0.16000000E 00	-0.29117109E 02
0.002550	0.11000000E 00	0.18499999E 00	-0.26832008E 02
0.002550	0.11000000E 00	0.20999999E 00	-0.24284458E 02
0.002550	0.11000000E 00	0.23499999E 00	-0.21729198E 02
0.002550	0.11000000E 00	0.25999999E 00	-0.19306597E 02
0.002550	0.11000000E 00	0.28499999E 00	-0.17085090E 02
0.002550	0.11000000E 00	0.30999999E 00	-0.15089830E 02
0.002550	0.11000000E 00	0.33499999E 00	-0.13321226E 02
0.002550	0.11000000E 00	0.35999999E 00	-0.11766556E 02
0.002550	0.16000000E 00	0.10000000E-01	-0.90365835E 01
0.002550	0.16000000E 00	0.34999999E-01	-0.28135684E 02
0.002550	0.16000000E 00	0.59999999E-01	-0.38741057E 02
0.002550	0.16000000E 00	0.84999999E-01	-0.42369962E 02
0.002550	0.16000000E 00	0.11000000E 00	-0.42130312E 02
0.002550	0.16000000E 00	0.13500000E 00	-0.39968040E 02
0.002550	0.16000000E 00	0.16000000E 00	-0.36936083E 02
0.002550	0.16000000E 00	0.18499999E 00	-0.33599707E 02
0.002550	0.16000000E 00	0.20999999E 00	-0.30263912E 02
0.002550	0.16000000E 00	0.23499999E 00	-0.27089939E 02
0.002550	0.16000000E 00	0.25999999E 00	-0.24157032E 02
0.002550	0.16000000E 00	0.28499999E 00	-0.21496733E 02
0.002550	0.16000000E 00	0.30999999E 00	-0.19113005E 02
0.002550	0.16000000E 00	0.33499999E 00	-0.16994388E 02
0.002550	0.16000000E 00	0.35999999E 00	-0.15121420E 02
0.002550	0.21000000E 00	0.10000000E-01	-0.24209653E 02
0.002550	0.21000000E 00	0.34999999E-01	-0.41551617E 02
0.002550	0.21000000E 00	0.59999999E-01	-0.44755348E 02

TIME	RADIUS	Z	RZ STRESS
0.002550	0.21000000E 00	0.84999999E-01	-0.44523373E 02
0.002550	0.21000000E 00	0.11000000E 00	-0.42721622E 02
0.002550	0.21000000E 00	0.13500000E 00	-0.40129878E 02
0.002550	0.21000000E 00	0.16000000E 00	-0.37171324E 02
0.002550	0.21000000E 00	0.18499999E 00	-0.34098545E 02
0.002550	0.21000000E 00	0.20999999E 00	-0.31066197E 02
0.002550	0.21000000E 00	0.23499999E 00	-0.28167266E 02
0.002550	0.21000000E 00	0.25999999E 00	-0.25454444E 02
0.002550	0.21000000E 00	0.28499999E 00	-0.22953553E 02
0.002550	0.21000000E 00	0.30999999E 00	-0.20672774E 02
0.002550	0.21000000E 00	0.33499999E 00	-0.18608960E 02
0.002550	0.21000000E 00	0.35999999E 00	-0.16751994E 02
0.002550	0.26000000E 00	0.10000000E-01	-0.17119031E 01
0.002550	0.26000000E 00	0.34999999E-01	-0.13585802E 02
0.002550	0.26000000E 00	0.59999999E-01	-0.23310415E 02
0.002550	0.26000000E 00	0.84999999E-01	-0.28470737E 02
0.002550	0.26000000E 00	0.11000000E 00	-0.30629293E 02
0.002550	0.26000000E 00	0.13500000E 00	-0.30994857E 02
0.002550	0.26000000E 00	0.16000000E 00	-0.30297489E 02
0.002550	0.26000000E 00	0.18499999E 00	-0.28980730E 02
0.002550	0.26000000E 00	0.20999999E 00	-0.27322577E 02
0.002550	0.26000000E 00	0.23499999E 00	-0.25501174E 02
0.002550	0.26000000E 00	0.25999999E 00	-0.23631500E 02
0.002550	0.26000000E 00	0.28499999E 00	-0.21787194E 02
0.002550	0.26000000E 00	0.30999999E 00	-0.20014109E 02
0.002550	0.26000000E 00	0.33499999E 00	-0.18339273E 02
0.002550	0.26000000E 00	0.35999999E 00	-0.16776949E 02
0.002550	0.30999999E 00	0.10000000E-01	-0.35405865E 00
0.002550	0.30999999E 00	0.34999999E-01	-0.38511602E 01
0.002550	0.30999999E 00	0.59999999E-01	-0.91303839E 01
0.002550	0.30999999E 00	0.84999999E-01	-0.14056204E 02
0.002550	0.30999999E 00	0.11000000E 00	-0.17718806E 02
0.002550	0.30999999E 00	0.13500000E 00	-0.20042634E 02
0.002550	0.30999999E 00	0.16000000E 00	-0.21257392E 02
0.002550	0.30999999E 00	0.18499999E 00	-0.21641229E 02
0.002550	0.30999999E 00	0.20999999E 00	-0.21436108E 02
0.002550	0.30999999E 00	0.23499999E 00	-0.20831274E 02
0.002550	0.30999999E 00	0.25999999E 00	-0.19968176E 02
0.002550	0.30999999E 00	0.28499999E 00	-0.18949986E 02
0.002550	0.30999999E 00	0.30999999E 00	-0.17850664E 02
0.002550	0.30999999E 00	0.33499999E 00	-0.16722324E 02
0.002550	0.30999999E 00	0.35999999E 00	-0.15600968E 02

TWELVE INCH DROP HEIGHT

TIME	RADIUS	Z	ZZ STRESS
0.001000	0.10000000E-01	0.10000000E-01	-0.29765078E 03
0.001000	0.10000000E-01	0.34999999E-01	-0.27995513E 03
0.001000	0.10000000E-01	0.59999999E-01	-0.26989659E 03
0.001000	0.10000000E-01	0.84999999E-01	-0.25695860E 03
0.001000	0.10000000E-01	0.11000000E 00	-0.24147240E 03
0.001000	0.10000000E-01	0.13500000E 00	-0.22441226E 03
0.001000	0.10000000E-01	0.16000000E 00	-0.20675137E 03
0.001000	0.10000000E-01	0.18499999E 00	-0.18927634E 03
0.001000	0.10000000E-01	0.20999999E 00	-0.17254229E 03
0.001000	0.10000000E-01	0.23499999E 00	-0.15689165E 03
0.001000	0.10000000E-01	0.25999999E 00	-0.14249853E 03
0.001000	0.10000000E-01	0.28499999E 00	-0.12941706E 03
0.001000	0.10000000E-01	0.30999999E 00	-0.11762225E 03
0.001000	0.10000000E-01	0.33499999E 00	-0.10704277E 03
0.001000	0.10000000E-01	0.35999999E 00	-0.97582687E 02
0.001000	0.59999999E-01	0.10000000E-01	-0.28299665E 03
0.001000	0.59999999E-01	0.34999999E-01	-0.27104849E 03
0.001000	0.59999999E-01	0.59999999E-01	-0.26189348E 03
0.001000	0.59999999E-01	0.84999999E-01	-0.24862596E 03
0.001000	0.59999999E-01	0.11000000E 00	-0.23313363E 03
0.001000	0.59999999E-01	0.13500000E 00	-0.21607894E 03
0.001000	0.59999999E-01	0.16000000E 00	-0.19869183E 03
0.001000	0.59999999E-01	0.18499999E 00	-0.18171467E 03
0.001000	0.59999999E-01	0.20999999E 00	-0.16562362E 03
0.001000	0.59999999E-01	0.23499999E 00	-0.15068384E 03
0.001000	0.59999999E-01	0.25999999E 00	-0.13700927E 03
0.001000	0.59999999E-01	0.28499999E 00	-0.12461358E 03
0.001000	0.59999999E-01	0.30999999E 00	-0.11344878E 03
0.001000	0.59999999E-01	0.33499999E 00	-0.10343312E 03
0.001000	0.59999999E-01	0.35999999E 00	-0.94468631E 02
0.001000	0.11000000E 00	0.10000000E-01	-0.26405887E 03
0.001000	0.11000000E 00	0.34999999E-01	-0.25324474E 03
0.001000	0.11000000E 00	0.59999999E-01	-0.24249610E 03
0.001000	0.11000000E 00	0.84999999E-01	-0.22813148E 03
0.001000	0.11000000E 00	0.11000000E 00	-0.21190800E 03
0.001000	0.11000000E 00	0.13500000E 00	-0.19519395E 03
0.001000	0.11000000E 00	0.16000000E 00	-0.17887503E 03
0.001000	0.11000000E 00	0.18499999E 00	-0.16344922E 03
0.001000	0.11000000E 00	0.20999999E 00	-0.14915487E 03
0.001000	0.11000000E 00	0.23499999E 00	-0.13607298E 03
0.001000	0.11000000E 00	0.25999999E 00	-0.12419440E 03
0.001000	0.11000000E 00	0.28499999E 00	-0.11346139E 03

TIME	RADIUS	Z	ZZ STRESS
0.001000	0.11000000E 00	0.30999999E 00	-0.10379255E 03
0.001000	0.11000000E 00	0.33499999E 00	-0.95096680E 02
0.001000	0.11000000E 00	0.35999999E 00	-0.87281230E 02
0.001000	0.16000000E 00	0.10000000E-01	-0.22103834E 03
0.001000	0.16000000E 00	0.34999999E-01	-0.21975750E 03
0.001000	0.16000000E 00	0.59999999E-01	-0.20627005E 03
0.001000	0.16000000E 00	0.84999999E-01	-0.19066932E 03
0.001000	0.16000000E 00	0.11000000E 00	-0.17517311E 03
0.001000	0.16000000E 00	0.13500000E 00	-0.16068147E 03
0.001000	0.16000000E 00	0.16000000E 00	-0.14742498E 03
0.001000	0.16000000E 00	0.18499999E 00	-0.13538024E 03
0.001000	0.16000000E 00	0.20999999E 00	-0.12444967E 03
0.001000	0.16000000E 00	0.23499999E 00	-0.11452610E 03
0.001000	0.16000000E 00	0.25999999E 00	-0.10551087E 03
0.001000	0.16000000E 00	0.28499999E 00	-0.97316664E 02
0.001000	0.16000000E 00	0.30999999E 00	-0.89866060E 02
0.001000	0.16000000E 00	0.33499999E 00	-0.83089824E 02
0.001000	0.16000000E 00	0.35999999E 00	-0.76925236E 02
0.001000	0.21000000E 00	0.10000000E-01	-0.14735344E 03
0.001000	0.21000000E 00	0.34999999E-01	-0.15653135E 03
0.001000	0.21000000E 00	0.59999999E-01	-0.14230573E 03
0.001000	0.21000000E 00	0.84999999E-01	-0.13108660E 03
0.001000	0.21000000E 00	0.11000000E 00	-0.12206241E 03
0.001000	0.21000000E 00	0.13500000E 00	-0.11431327E 03
0.001000	0.21000000E 00	0.16000000E 00	-0.10732438E 03
0.001000	0.21000000E 00	0.18499999E 00	-0.10084124E 03
0.001000	0.21000000E 00	0.20999999E 00	-0.94744607E 02
0.001000	0.21000000E 00	0.23499999E 00	-0.88981529E 02
0.001000	0.21000000E 00	0.25999999E 00	-0.83530118E 02
0.001000	0.21000000E 00	0.28499999E 00	-0.78381505E 02
0.001000	0.21000000E 00	0.30999999E 00	-0.73531011E 02
0.001000	0.21000000E 00	0.33499999E 00	-0.68974014E 02
0.001000	0.21000000E 00	0.35999999E 00	-0.64704034E 02
0.001000	0.26000000E 00	0.10000000E-01	-0.12611796E 02
0.001000	0.26000000E 00	0.34999999E-01	-0.41680812E 02
0.001000	0.26000000E 00	0.59999999E-01	-0.54132871E 02
0.001000	0.26000000E 00	0.84999999E-01	-0.61053520E 02
0.001000	0.26000000E 00	0.11000000E 00	-0.64865982E 02
0.001000	0.26000000E 00	0.13500000E 00	-0.66673977E 02
0.001000	0.26000000E 00	0.16000000E 00	-0.67106717E 02
0.001000	0.26000000E 00	0.18499999E 00	-0.66569968E 02
0.001000	0.26000000E 00	0.20999999E 00	-0.65345487E 02
0.001000	0.26000000E 00	0.23499999E 00	-0.63637973E 02
0.001000	0.26000000E 00	0.25999999E 00	-0.61600067E 02
0.001000	0.26000000E 00	0.28499999E 00	-0.59347197E 02
0.001000	0.26000000E 00	0.30999999E 00	-0.56967072E 02

TIME	RADIUS	Z	ZZ STRESS
0.001000	0.26000000E 00	0.33499999E 00	-0.54526214E 02
0.001000	0.26000000E 00	0.35999999E 00	-0.52074599E 02
0.001000	0.30999999E 00	0.10000000E-01	-0.11629080E 00
0.001000	0.30999999E 00	0.34999999E-01	-0.36526306E 01
0.001000	0.30999999E 00	0.59999999E-01	-0.11476824E 02
0.001000	0.30999999E 00	0.84999999E-01	-0.19944487E 02
0.001000	0.30999999E 00	0.11000000E 00	-0.27163320E 02
0.001000	0.30999999E 00	0.13500000E 00	-0.32741263E 02
0.001000	0.30999999E 00	0.16000000E 00	-0.36805917E 02
0.001000	0.30999999E 00	0.18499999E 00	-0.39606144E 02
0.001000	0.30999999E 00	0.20999999E 00	-0.41388872E 02
0.001000	0.30999999E 00	0.23499999E 00	-0.42367628E 02
0.001000	0.30999999E 00	0.25999999E 00	-0.42718690E 02
0.001000	0.30999999E 00	0.28499999E 00	-0.42585138E 02
0.001000	0.30999999E 00	0.30999999E 00	-0.42082145E 02
0.001000	0.30999999E 00	0.33499999E 00	-0.41302093E 02
0.001000	0.30999999E 00	0.35999999E 00	-0.40318815E 02
0.002100	0.10000000E-01	0.10000000E-01	-0.32309373E 03
0.002100	0.10000000E-01	0.34999999E-01	-0.30573818E 03
0.002100	0.10000000E-01	0.59999999E-01	-0.29845755E 03
0.002100	0.10000000E-01	0.84999999E-01	-0.28927340E 03
0.002100	0.10000000E-01	0.11000000E 00	-0.27784605E 03
0.002100	0.10000000E-01	0.13500000E 00	-0.26458648E 03
0.002100	0.10000000E-01	0.16000000E 00	-0.25007396E 03
0.002100	0.10000000E-01	0.18499999E 00	-0.23488676E 03
0.002100	0.10000000E-01	0.20999999E 00	-0.21953119E 03
0.002100	0.10000000E-01	0.23499999E 00	-0.20440998E 03
0.002100	0.10000000E-01	0.25999999E 00	-0.18981891E 03
0.002100	0.10000000E-01	0.28499999E 00	-0.17595655E 03
0.002100	0.10000000E-01	0.30999999E 00	-0.16294187E 03
0.002100	0.10000000E-01	0.33499999E 00	-0.15083142E 03
0.002100	0.10000000E-01	0.35999999E 00	-0.13963699E 03
0.002100	0.59999999E-01	0.10000000E-01	-0.31024336E 03
0.002100	0.59999999E-01	0.34999999E-01	-0.29923883E 03
0.002100	0.59999999E-01	0.59999999E-01	-0.29323244E 03
0.002100	0.59999999E-01	0.84999999E-01	-0.28412959E 03
0.002100	0.59999999E-01	0.11000000E 00	-0.27250181E 03
0.002100	0.59999999E-01	0.13500000E 00	-0.25903820E 03
0.002100	0.59999999E-01	0.16000000E 00	-0.24442328E 03
0.002100	0.59999999E-01	0.18499999E 00	-0.22926826E 03
0.002100	0.59999999E-01	0.20999999E 00	-0.21407324E 03
0.002100	0.59999999E-01	0.23499999E 00	-0.19921569E 03
0.002100	0.59999999E-01	0.25999999E 00	-0.18496015E 03
0.002100	0.59999999E-01	0.28499999E 00	-0.17147510E 03
0.002100	0.59999999E-01	0.30999999E 00	-0.15885408E 03
0.002100	0.59999999E-01	0.33499999E 00	-0.14713480E 03

TIME	RADIUS	Z	ZZ STRESS
0.002100	0.59999999E-01	0.35999999E 00	-0.13631577E 03
0.002100	0.11000000E 00	0.10000000E-01	-0.29782661E 03
0.002100	0.11000000E 00	0.34999999E-01	-0.28850527E 03
0.002100	0.11000000E 00	0.59999999E-01	-0.28156830E 03
0.002100	0.11000000E 00	0.84999999E-01	-0.27147863E 03
0.002100	0.11000000E 00	0.11000000E 00	-0.25901390E 03
0.002100	0.11000000E 00	0.13500000E 00	-0.24502048E 03
0.002100	0.11000000E 00	0.16000000E 00	-0.23025956E 03
0.002100	0.11000000E 00	0.18499999E 00	-0.21533545E 03
0.002100	0.11000000E 00	0.20999999E 00	-0.20068483E 03
0.002100	0.11000000E 00	0.23499999E 00	-0.18659991E 03
0.002100	0.11000000E 00	0.25999999E 00	-0.17325878E 03
0.002100	0.11000000E 00	0.28499999E 00	-0.16075661E 03
0.002100	0.11000000E 00	0.30999999E 00	-0.14913024E 03
0.002100	0.11000000E 00	0.33499999E 00	-0.13837762E 03
0.002100	0.11000000E 00	0.35999999E 00	-0.12847151E 03
0.002100	0.16000000E 00	0.10000000E-01	-0.26588568E 03
0.002100	0.16000000E 00	0.34999999E-01	-0.26939744E 03
0.002100	0.16000000E 00	0.59999999E-01	-0.26081048E 03
0.002100	0.16000000E 00	0.84999999E-01	-0.24903952E 03
0.002100	0.16000000E 00	0.11000000E 00	-0.23539692E 03
0.002100	0.16000000E 00	0.13500000E 00	-0.22097764E 03
0.002100	0.16000000E 00	0.16000000E 00	-0.20653992E 03
0.002100	0.16000000E 00	0.18499999E 00	-0.19254536E 03
0.002100	0.16000000E 00	0.20999999E 00	-0.17924541E 03
0.002100	0.16000000E 00	0.23499999E 00	-0.16675823E 03
0.002100	0.16000000E 00	0.25999999E 00	-0.15512296E 03
0.002100	0.16000000E 00	0.28499999E 00	-0.14433399E 03
0.002100	0.16000000E 00	0.30999999E 00	-0.13436105E 03
0.002100	0.16000000E 00	0.33499999E 00	-0.12516126E 03
0.002100	0.16000000E 00	0.35999999E 00	-0.11668553E 03
0.002100	0.21000000E 00	0.10000000E-01	-0.21176543E 03
0.002100	0.21000000E 00	0.34999999E-01	-0.23743137E 03
0.002100	0.21000000E 00	0.59999999E-01	-0.22602372E 03
0.002100	0.21000000E 00	0.84999999E-01	-0.21228610E 03
0.002100	0.21000000E 00	0.11000000E 00	-0.19822340E 03
0.002100	0.21000000E 00	0.13500000E 00	-0.18479902E 03
0.002100	0.21000000E 00	0.16000000E 00	-0.17233606E 03
0.002100	0.21000000E 00	0.18499999E 00	-0.16086792E 03
0.002100	0.21000000E 00	0.20999999E 00	-0.15032548E 03
0.002100	0.21000000E 00	0.23499999E 00	-0.14061614E 03
0.002100	0.21000000E 00	0.25999999E 00	-0.13165260E 03
0.002100	0.21000000E 00	0.28499999E 00	-0.12335993E 03
0.002100	0.21000000E 00	0.30999999E 00	-0.11567550E 03
0.002100	0.21000000E 00	0.33499999E 00	-0.10854684E 03
0.002100	0.21000000E 00	0.35999999E 00	-0.10192904E 03

TIME	RADIUS	Z	ZZ STRESS
0.002100	0.26000000E 00	0.10000000E-01	-0.14205242E 03
0.002100	0.26000000E 00	0.34999999E-01	-0.18088991E 03
0.002100	0.26000000E 00	0.59999999E-01	-0.16669954E 03
0.002100	0.26000000E 00	0.84999999E-01	-0.15453452E 03
0.002100	0.26000000E 00	0.11000000E 00	-0.14465005E 03
0.002100	0.26000000E 00	0.13500000E 00	-0.13634906E 03
0.002100	0.26000000E 00	0.16000000E 00	-0.12906826E 03
0.002100	0.26000000E 00	0.18499999E 00	-0.12245872E 03
0.002100	0.26000000E 00	0.20999999E 00	-0.11631883E 03
0.002100	0.26000000E 00	0.23499999E 00	-0.11053509E 03
0.002100	0.26000000E 00	0.25999999E 00	-0.10504477E 03
0.002100	0.26000000E 00	0.28499999E 00	-0.99813687E 02
0.002100	0.26000000E 00	0.30999999E 00	-0.94823547E 02
0.002100	0.26000000E 00	0.33499999E 00	-0.90064328E 02
0.002100	0.26000000E 00	0.35999999E 00	-0.85529958E 02
0.002100	0.30999999E 00	0.10000000E-01	-0.39052525E 02
0.002100	0.30999999E 00	0.34999999E-01	-0.68783048E 02
0.002100	0.30999999E 00	0.59999999E-01	-0.75551999E 02
0.002100	0.30999999E 00	0.84999999E-01	-0.79938164E 02
0.002100	0.30999999E 00	0.11000000E 00	-0.82489341E 02
0.002100	0.30999999E 00	0.13500000E 00	-0.83681223E 02
0.002100	0.30999999E 00	0.16000000E 00	-0.83855464E 02
0.002100	0.30999999E 00	0.18499999E 00	-0.83260124E 02
0.002100	0.30999999E 00	0.20999999E 00	-0.82081053E 02
0.002100	0.30999999E 00	0.23499999E 00	-0.80461495E 02
0.002100	0.30999999E 00	0.25999999E 00	-0.78514072E 02
0.002100	0.30999999E 00	0.28499999E 00	-0.76328527E 02
0.002100	0.30999999E 00	0.30999999E 00	-0.73976946E 02
0.002100	0.30999999E 00	0.33499999E 00	-0.71517362E 02
0.002100	0.30999999E 00	0.35999999E 00	-0.68996426E 02

TIME	RADIUS	Z	RR STRESS
0.001000	0.10000000E-01	0.10000000E-01	-0.18079668E 03
0.001000	0.10000000E-01	0.34999999E-01	-0.13963496E 03
0.001000	0.10000000E-01	0.59999999E-01	-0.10416546E 03
0.001000	0.10000000E-01	0.84999999E-01	-0.76078431E 02
0.001000	0.10000000E-01	0.11000000E 00	-0.54417779E 02
0.001000	0.10000000E-01	0.13500000E 00	-0.38094983E 02
0.001000	0.10000000E-01	0.16000000E 00	-0.26021672E 02
0.001000	0.10000000E-01	0.18499999E 00	-0.17221937E 02
0.001000	0.10000000E-01	0.20999999E 00	-0.10882603E 02
0.001000	0.10000000E-01	0.23499999E 00	-0.63598169E 01
0.001000	0.10000000E-01	0.25999999E 00	-0.31613993E 01

TIME	RADIUS	Z	RR STRESS
0.001000	0.10000000E-01	0.28499999E 00	-0.92058382E 00
0.001000	0.10000000E-01	0.30999999E 00	0.63172320E 00
0.001000	0.10000000E-01	0.33499999E 00	0.16908444E 01
0.001000	0.10000000E-01	0.35999999E 00	0.23975684E 01
0.001000	0.59999999E-01	0.10000000E-01	-0.17189151E 03
0.001000	0.59999999E-01	0.34999999E-01	-0.13429951E 03
0.001000	0.59999999E-01	0.59999999E-01	-0.99593327E 02
0.001000	0.59999999E-01	0.84999999E-01	-0.72645266E 02
0.001000	0.59999999E-01	0.11000000E 00	-0.52245872E 02
0.001000	0.59999999E-01	0.13500000E 00	-0.37060627E 02
0.001000	0.59999999E-01	0.16000000E 00	-0.25880757E 02
0.001000	0.59999999E-01	0.18499999E 00	-0.17708150E 02
0.001000	0.59999999E-01	0.20999999E 00	-0.11762074E 02
0.001000	0.59999999E-01	0.23499999E 00	-0.74513362E 01
0.001000	0.59999999E-01	0.25999999E 00	-0.43367529E 01
0.001000	0.59999999E-01	0.28499999E 00	-0.20955359E 01
0.001000	0.59999999E-01	0.30999999E 00	-0.49175279E 00
0.001000	0.59999999E-01	0.33499999E 00	0.64662999E 00
0.001000	0.59999999E-01	0.35999999E 00	0.14450410E 01
0.001000	0.11000000E 00	0.10000000E-01	-0.15737202E 03
0.001000	0.11000000E 00	0.34999999E-01	-0.11995406E 03
0.001000	0.11000000E 00	0.59999999E-01	-0.88009119E 02
0.001000	0.11000000E 00	0.84999999E-01	-0.64640444E 02
0.001000	0.11000000E 00	0.11000000E 00	-0.47666051E 02
0.001000	0.11000000E 00	0.13500000E 00	-0.35260779E 02
0.001000	0.11000000E 00	0.16000000E 00	-0.26088466E 02
0.001000	0.11000000E 00	0.18499999E 00	-0.19226439E 02
0.001000	0.11000000E 00	0.20999999E 00	-0.14044953E 02
0.001000	0.11000000E 00	0.23499999E 00	-0.10107850E 02
0.001000	0.11000000E 00	0.25999999E 00	-0.71056080E 01
0.001000	0.11000000E 00	0.28499999E 00	-0.48129957E 01
0.001000	0.11000000E 00	0.30999999E 00	-0.30628017E 01
0.001000	0.11000000E 00	0.33499999E 00	-0.17290783E 01
0.001000	0.11000000E 00	0.35999999E 00	-0.71601838E 00
0.001000	0.16000000E 00	0.10000000E-01	-0.13043355E 03
0.001000	0.16000000E 00	0.34999999E-01	-0.95590156E 02
0.001000	0.16000000E 00	0.59999999E-01	-0.70585511E 02
0.001000	0.16000000E 00	0.84999999E-01	-0.54356515E 02
0.001000	0.16000000E 00	0.11000000E 00	-0.43015624E 02
0.001000	0.16000000E 00	0.13500000E 00	-0.34481665E 02
0.001000	0.16000000E 00	0.16000000E 00	-0.27737043E 02
0.001000	0.16000000E 00	0.18499999E 00	-0.22270695E 02
0.001000	0.16000000E 00	0.20999999E 00	-0.17795335E 02
0.001000	0.16000000E 00	0.23499999E 00	-0.14123066E 02
0.001000	0.16000000E 00	0.25999999E 00	-0.11113952E 02
0.001000	0.16000000E 00	0.28499999E 00	-0.86552731E 01

TIME	RADIUS	Z	RR STRESS
0.001000	0.16000000E 00	0.30999999E 00	-0.66528669E 01
0.001000	0.16000000E 00	0.33499999E 00	-0.50273283E 01
0.001000	0.16000000E 00	0.35999999E 00	-0.37117991E 01
0.001000	0.21000000E 00	0.10000000E -01	-0.81345684E 02
0.001000	0.21000000E 00	0.34999999E -01	-0.61527942E 02
0.001000	0.21000000E 00	0.59999999E -01	-0.53003973E 02
0.001000	0.21000000E 00	0.84999999E -01	-0.47155875E 02
0.001000	0.21000000E 00	0.11000000E 00	-0.41599933E 02
0.001000	0.21000000E 00	0.13500000E 00	-0.36195470E 02
0.001000	0.21000000E 00	0.16000000E 00	-0.31104100E 02
0.001000	0.21000000E 00	0.18499999E 00	-0.26454139E 02
0.001000	0.21000000E 00	0.20999999E 00	-0.22307102E 02
0.001000	0.21000000E 00	0.23499999E 00	-0.18673227E 02
0.001000	0.21000000E 00	0.25999999E 00	-0.15530643E 02
0.001000	0.21000000E 00	0.28499999E 00	-0.12839721E 02
0.001000	0.21000000E 00	0.30999999E 00	-0.10552948E 02
0.001000	0.21000000E 00	0.33499999E 00	-0.86208831E 01
0.001000	0.21000000E 00	0.35999999E 00	-0.69958066E 01
0.001000	0.26000000E 00	0.10000000E -01	-0.68996334E 01
0.001000	0.26000000E 00	0.34999999E -01	-0.37828936E 02
0.001000	0.26000000E 00	0.59999999E -01	-0.44362702E 02
0.001000	0.26000000E 00	0.84999999E -01	-0.44301712E 02
0.001000	0.26000000E 00	0.11000000E 00	-0.41599146E 02
0.001000	0.26000000E 00	0.13500000E 00	-0.37817682E 02
0.001000	0.26000000E 00	0.16000000E 00	-0.33699989E 02
0.001000	0.26000000E 00	0.18499999E 00	-0.29625173E 02
0.001000	0.26000000E 00	0.20999999E 00	-0.25787130E 02
0.001000	0.26000000E 00	0.23499999E 00	-0.22278095E 02
0.001000	0.26000000E 00	0.25999999E 00	-0.19132182E 02
0.001000	0.26000000E 00	0.28499999E 00	-0.16350219E 02
0.001000	0.26000000E 00	0.30999999E 00	-0.13914414E 02
0.001000	0.26000000E 00	0.33499999E 00	-0.11797393E 02
0.001000	0.26000000E 00	0.35999999E 00	-0.99676441E 01
0.001000	0.30999999E 00	0.10000000E -01	0.21808315E 02
0.001000	0.30999999E 00	0.34999999E -01	-0.94869100E 01
0.001000	0.30999999E 00	0.59999999E -01	-0.26213102E 02
0.001000	0.30999999E 00	0.84999999E -01	-0.33177941E 02
0.001000	0.30999999E 00	0.11000000E 00	-0.34990130E 02
0.001000	0.30999999E 00	0.13500000E 00	-0.34185398E 02
0.001000	0.30999999E 00	0.16000000E 00	-0.32083067E 02
0.001000	0.30999999E 00	0.18499999E 00	-0.29390418E 02
0.001000	0.30999999E 00	0.20999999E 00	-0.26498626E 02
0.001000	0.30999999E 00	0.23499999E 00	-0.23626378E 02
0.001000	0.30999999E 00	0.25999999E 00	-0.20893739E 02
0.001000	0.30999999E 00	0.28499999E 00	-0.18362446E 02
0.001000	0.30999999E 00	0.30999999E 00	-0.16059196E 02

TIME	RADIUS	Z	RR STRESS
0.001000	0.30999999E 00	0.33499999E 00	-0.13989591E 02
0.001000	0.30999999E 00	0.35999999E 00	-0.12146854E 02
0.002100	0.10000000E-01	0.10000000E-01	-0.20058792E 03
0.002100	0.10000000E-01	0.34999999E-01	-0.16449838E 03
0.002100	0.10000000E-01	0.59999999E-01	-0.13125925E 03
0.002100	0.10000000E-01	0.84999999E-01	-0.10321609E 03
0.002100	0.10000000E-01	0.11000000E 00	-0.80014914E 02
0.002100	0.10000000E-01	0.13500000E 00	-0.61171381E 02
0.002100	0.10000000E-01	0.16000000E 00	-0.46106777E 02
0.002100	0.10000000E-01	0.18499999E 00	-0.34221619E 02
0.002100	0.10000000E-01	0.20999999E 00	-0.24946547E 02
0.002100	0.10000000E-01	0.23499999E 00	-0.17773024E 02
0.002100	0.10000000E-01	0.25999999E 00	-0.12266006E 02
0.002100	0.10000000E-01	0.28499999E 00	-0.80655507E 01
0.002100	0.10000000E-01	0.30999999E 00	-0.48805660E 01
0.002100	0.10000000E-01	0.33499999E 00	-0.24798116E 01
0.002100	0.10000000E-01	0.35999999E 00	-0.68200377E 00
0.002100	0.59999999E-01	0.10000000E-01	-0.19342856E 03
0.002100	0.59999999E-01	0.34999999E-01	-0.16096013E 03
0.002100	0.59999999E-01	0.59999999E-01	-0.12785292E 03
0.002100	0.59999999E-01	0.84999999E-01	-0.10021359E 03
0.002100	0.59999999E-01	0.11000000E 00	-0.77633808E 02
0.002100	0.59999999E-01	0.13500000E 00	-0.59485950E 02
0.002100	0.59999999E-01	0.16000000E 00	-0.45086180E 02
0.002100	0.59999999E-01	0.18499999E 00	-0.33773783E 02
0.002100	0.59999999E-01	0.20999999E 00	-0.24954046E 02
0.002100	0.59999999E-01	0.23499999E 00	-0.18117444E 02
0.002100	0.59999999E-01	0.25999999E 00	-0.12842007E 02
0.002100	0.59999999E-01	0.28499999E 00	-0.87865324E 01
0.002100	0.59999999E-01	0.30999999E 00	-0.56796520E 01
0.002100	0.59999999E-01	0.33499999E 00	-0.33080065E 01
0.002100	0.59999999E-01	0.35999999E 00	-0.15050722E 01
0.002100	0.11000000E 00	0.10000000E-01	-0.18439995E 03
0.002100	0.11000000E 00	0.34999999E-01	-0.15114846E 03
0.002100	0.11000000E 00	0.59999999E-01	-0.11883875E 03
0.002100	0.11000000E 00	0.84999999E-01	-0.92725870E 02
0.002100	0.11000000E 00	0.11000000E 00	-0.71991099E 02
0.002100	0.11000000E 00	0.13500000E 00	-0.55698869E 02
0.002100	0.11000000E 00	0.16000000E 00	-0.42958700E 02
0.002100	0.11000000E 00	0.18499999E 00	-0.33006328E 02
0.002100	0.11000000E 00	0.20999999E 00	-0.25223379E 02
0.002100	0.11000000E 00	0.23499999E 00	-0.19125434E 02
0.002100	0.11000000E 00	0.25999999E 00	-0.14338417E 02
0.002100	0.11000000E 00	0.28499999E 00	-0.10574686E 02
0.002100	0.11000000E 00	0.30999999E 00	-0.76127515E 01
0.002100	0.11000000E 00	0.33499999E 00	-0.52812927E 01

TIME	RADIUS	Z	RR STRESS
0.002100	0.11000000E 00	0.35999999E 00	-0.34471539E 01
0.002100	0.16000000E 00	0.10000000E-01	-0.16828852E 03
0.002100	0.16000000E 00	0.34999999E-01	-0.13447685E 03
0.002100	0.16000000E 00	0.59999999E-01	-0.10414556E 03
0.002100	0.16000000E 00	0.84999999E-01	-0.81221346E 02
0.002100	0.16000000E 00	0.11000000E 00	-0.63966082E 02
0.002100	0.16000000E 00	0.13500000E 00	-0.50844437E 02
0.002100	0.16000000E 00	0.16000000E 00	-0.40693150E 02
0.002100	0.16000000E 00	0.18499999E 00	-0.32696703E 02
0.002100	0.16000000E 00	0.20999999E 00	-0.26299991E 02
0.002100	0.16000000E 00	0.23499999E 00	-0.21123808E 02
0.002100	0.16000000E 00	0.25999999E 00	-0.16902478E 02
0.002100	0.16000000E 00	0.28499999E 00	-0.13443201E 02
0.002100	0.16000000E 00	0.30999999E 00	-0.10600915E 02
0.002100	0.16000000E 00	0.33499999E 00	-0.82629590E 01
0.002100	0.16000000E 00	0.35999999E 00	-0.63396748E 01
0.002100	0.21000000E 00	0.10000000E-01	-0.14087567E 03
0.002100	0.21000000E 00	0.34999999E-01	-0.10888263E 03
0.002100	0.21000000E 00	0.59999999E-01	-0.83921486E 02
0.002100	0.21000000E 00	0.84999999E-01	-0.67357745E 02
0.002100	0.21000000E 00	0.11000000E 00	-0.55705597E 02
0.002100	0.21000000E 00	0.13500000E 00	-0.46869691E 02
0.002100	0.21000000E 00	0.16000000E 00	-0.39746899E 02
0.002100	0.21000000E 00	0.18499999E 00	-0.33779661E 02
0.002100	0.21000000E 00	0.20999999E 00	-0.28678508E 02
0.002100	0.21000000E 00	0.23499999E 00	-0.24279906E 02
0.002100	0.21000000E 00	0.25999999E 00	-0.20478408E 02
0.002100	0.21000000E 00	0.28499999E 00	-0.17195912E 02
0.002100	0.21000000E 00	0.30999999E 00	-0.14368116E 02
0.002100	0.21000000E 00	0.33499999E 00	-0.11938954E 02
0.002100	0.21000000E 00	0.35999999E 00	-0.98582985E 01
0.002100	0.26000000E 00	0.10000000E-01	-0.94246599E 02
0.002100	0.26000000E 00	0.34999999E-01	-0.72951777E 02
0.002100	0.26000000E 00	0.59999999E-01	-0.62072196E 02
0.002100	0.26000000E 00	0.84999999E-01	-0.55936620E 02
0.002100	0.26000000E 00	0.11000000E 00	-0.50796152E 02
0.002100	0.26000000E 00	0.13500000E 00	-0.45849247E 02
0.002100	0.26000000E 00	0.16000000E 00	-0.41031910E 02
0.002100	0.26000000E 00	0.18499999E 00	-0.36423175E 02
0.002100	0.26000000E 00	0.20999999E 00	-0.32102109E 02
0.002100	0.26000000E 00	0.23499999E 00	-0.28119588E 02
0.002100	0.26000000E 00	0.25999999E 00	-0.24499317E 02
0.002100	0.26000000E 00	0.28499999E 00	-0.21244068E 02
0.002100	0.26000000E 00	0.30999999E 00	-0.18342315E 02
0.002100	0.26000000E 00	0.33499999E 00	-0.15773577E 02
0.002100	0.26000000E 00	0.35999999E 00	-0.13512255E 02

TIME	RADIUS	Z	RR STRESS
0.002100	0.30999999E 00	0.10000000E-01	-0.14841512E 02
0.002100	0.30999999E 00	0.34999999E-01	-0.42908872E 02
0.002100	0.30999999E 00	0.59999999E-01	-0.50124388E 02
0.002100	0.30999999E 00	0.84999999E-01	-0.51166955E 02
0.002100	0.30999999E 00	0.11000000E 00	-0.49446414E 02
0.002100	0.30999999E 00	0.13500000E 00	-0.46392375E 02
0.002100	0.30999999E 00	0.16000000E 00	-0.42727964E 02
0.002100	0.30999999E 00	0.18499999E 00	-0.38854055E 02
0.002100	0.30999999E 00	0.20999999E 00	-0.35002030E 02
0.002100	0.30999999E 00	0.23499999E 00	-0.31305844E 02
0.002100	0.30999999E 00	0.25999999E 00	-0.27840014E 02
0.002100	0.30999999E 00	0.28499999E 00	-0.24642058E 02
0.002100	0.30999999E 00	0.30999999E 00	-0.21725699E 02
0.002100	0.30999999E 00	0.33499999E 00	-0.19089624E 02
0.002100	0.30999999E 00	0.35999999E 00	-0.16723195E 02

TIME	RADIUS	Z	RR STRESS
0.001000	0.10000000E-01	0.10000000E-01	-0.18299560E 03
0.001000	0.10000000E-01	0.34999999E-01	-0.13954272E 03
0.001000	0.10000000E-01	0.59999999E-01	-0.10418336E 03
0.001000	0.10000000E-01	0.84999999E-01	-0.76093268E 02
0.001000	0.10000000E-01	0.11000000E 00	-0.54416902E 02
0.001000	0.10000000E-01	0.13500000E 00	-0.38078605E 02
0.001000	0.10000000E-01	0.16000000E 00	-0.25993714E 02
0.001000	0.10000000E-01	0.18499999E 00	-0.17186753E 02
0.001000	0.10000000E-01	0.20999999E 00	-0.10843992E 02
0.001000	0.10000000E-01	0.23499999E 00	-0.63205591E 01
0.001000	0.10000000E-01	0.25999999E 00	-0.31236022E 01
0.001000	0.10000000E-01	0.28499999E 00	-0.88533565E 00
0.001000	0.10000000E-01	0.30999999E 00	0.66390920E 00
0.001000	0.10000000E-01	0.33499999E 00	0.17197551E 01
0.001000	0.10000000E-01	0.35999999E 00	0.24232629E 01
0.001000	0.59999999E-01	0.10000000E-01	-0.17988512E 03
0.001000	0.59999999E-01	0.34999999E-01	-0.13549607E 03
0.001000	0.59999999E-01	0.59999999E-01	-0.10082232E 03
0.001000	0.59999999E-01	0.84999999E-01	-0.73233779E 02
0.001000	0.59999999E-01	0.11000000E 00	-0.52161204E 02
0.001000	0.59999999E-01	0.13500000E 00	-0.36405952E 02
0.001000	0.59999999E-01	0.16000000E 00	-0.24819918E 02
0.001000	0.59999999E-01	0.18499999E 00	-0.16406710E 02
0.001000	0.59999999E-01	0.20999999E 00	-0.10356895E 02
0.001000	0.59999999E-01	0.23499999E 00	-0.60416192E 01
0.001000	0.59999999E-01	0.25999999E 00	-0.29867926E 01

TIME	RADIUS	Z	00 STRESS
0.001000	0.59999999E-01	0.28499999E 00	-0.84213392E 00
0.001000	0.59999999E-01	0.30999999E 00	0.64786472E 00
0.001000	0.59999999E-01	0.33499999E 00	0.16681293E 01
0.001000	0.59999999E-01	0.35999999E 00	0.23518937E 01
0.001000	0.11000000E 00	0.10000000E-01	-0.17123905E 03
0.001000	0.11000000E 00	0.34999999E-01	-0.12633634E 03
0.001000	0.11000000E 00	0.59999999E-01	-0.92238723E 02
0.001000	0.11000000E 00	0.84999999E-01	-0.66202231E 02
0.001000	0.11000000E 00	0.11000000E 00	-0.46779595E 02
0.001000	0.11000000E 00	0.13500000E 00	-0.32517445E 02
0.001000	0.11000000E 00	0.16000000E 00	-0.22151808E 02
0.001000	0.11000000E 00	0.18499999E 00	-0.14669021E 02
0.001000	0.11000000E 00	0.20999999E 00	-0.92934182E 01
0.001000	0.11000000E 00	0.23499999E 00	-0.54476990E 01
0.001000	0.11000000E 00	0.25999999E 00	-0.27089309E 01
0.001000	0.11000000E 00	0.28499999E 00	-0.76998661E 00
0.001000	0.11000000E 00	0.30999999E 00	0.59118430E 00
0.001000	0.11000000E 00	0.33499999E 00	0.15348893E 01
0.001000	0.11000000E 00	0.35999999E 00	0.21768821E 01
0.001000	0.16000000E 00	0.10000000E-01	-0.16021935E 03
0.001000	0.16000000E 00	0.34999999E-01	-0.10933287E 03
0.001000	0.16000000E 00	0.59999999E-01	-0.77897484E 02
0.001000	0.16000000E 00	0.84999999E-01	-0.55117830E 02
0.001000	0.16000000E 00	0.11000000E 00	-0.38741686E 02
0.001000	0.16000000E 00	0.13500000E 00	-0.26971246E 02
0.001000	0.16000000E 00	0.16000000E 00	-0.18490748E 02
0.001000	0.16000000E 00	0.18499999E 00	-0.12364926E 02
0.001000	0.16000000E 00	0.20999999E 00	-0.79324920E 01
0.001000	0.16000000E 00	0.23499999E 00	-0.47242568E 01
0.001000	0.16000000E 00	0.25999999E 00	-0.24053219E 01
0.001000	0.16000000E 00	0.28499999E 00	-0.73507238E 00
0.001000	0.16000000E 00	0.30999999E 00	0.46038450E 00
0.001000	0.16000000E 00	0.33499999E 00	0.13073978E 01
0.001000	0.16000000E 00	0.35999999E 00	0.18982195E 01
0.001000	0.21000000E 00	0.10000000E-01	-0.13592875E 03
0.001000	0.21000000E 00	0.34999999E-01	-0.82309093E 02
0.001000	0.21000000E 00	0.59999999E-01	-0.57799842E 02
0.001000	0.21000000E 00	0.84999999E-01	-0.41157337E 02
0.001000	0.21000000E 00	0.11000000E 00	-0.29401295E 02
0.001000	0.21000000E 00	0.13500000E 00	-0.20897698E 02
0.001000	0.21000000E 00	0.16000000E 00	-0.14661678E 02
0.001000	0.21000000E 00	0.18499999E 00	-0.10052471E 02
0.001000	0.21000000E 00	0.20999999E 00	-0.66315515E 01
0.001000	0.21000000E 00	0.23499999E 00	-0.40887875E 01
0.001000	0.21000000E 00	0.25999999E 00	-0.22001234E 01
0.001000	0.21000000E 00	0.28499999E 00	-0.80132856E 00

TIME	RADIUS	Z	66 STRESS
0.001000	0.21000000E 00	0.30999999E 00	0.22918295E 00
0.001000	0.21000000E 00	0.33499999E 00	0.96209170E 00
0.001000	0.21000000E 00	0.35999999E 00	0.15253918E 01
0.001000	0.26000000E 00	0.10000000E-01	-0.58600176E 02
0.001000	0.26000000E 00	0.34999999E-01	-0.47667585E 02
0.001000	0.26000000E 00	0.59999999E-01	-0.36995339E 02
0.001000	0.26000000E 00	0.84999999E-01	-0.28208866E 02
0.001000	0.26000000E 00	0.11000000E 00	-0.21246111E 02
0.001000	0.26000000E 00	0.13500000E 00	-0.15791041E 02
0.001000	0.26000000E 00	0.16000000E 00	-0.11538174E 02
0.001000	0.26000000E 00	0.18499999E 00	-0.82326629E 01
0.001000	0.26000000E 00	0.20999999E 00	-0.56702082E 01
0.001000	0.26000000E 00	0.23499999E 00	-0.36893916E 01
0.001000	0.26000000E 00	0.25999999E 00	-0.21633974E 01
0.001000	0.26000000E 00	0.28499999E 00	-0.99285742E 00
0.001000	0.26000000E 00	0.30999999E 00	-0.99989295E-01
0.001000	0.26000000E 00	0.33499999E 00	0.57606705E 00
0.001000	0.26000000E 00	0.35999999E 00	0.10829272E 01
0.001000	0.30999999E 00	0.10000000E-01	-0.35438276E 02
0.001000	0.30999999E 00	0.34999999E-01	-0.30212538E 02
0.001000	0.30999999E 00	0.59999999E-01	-0.25018667E 02
0.001000	0.30999999E 00	0.84999999E-01	-0.20208296E 02
0.001000	0.30999999E 00	0.11000000E 00	-0.15996302E 02
0.001000	0.30999999E 00	0.13500000E 00	-0.12432515E 02
0.001000	0.30999999E 00	0.16000000E 00	-0.94791488E 01
0.001000	0.30999999E 00	0.18499999E 00	-0.70641937E 01
0.001000	0.30999999E 00	0.20999999E 00	-0.51079867E 01
0.001000	0.30999999E 00	0.23499999E 00	-0.35349042E 01
0.001000	0.30999999E 00	0.25999999E 00	-0.22777982E 01
0.001000	0.30999999E 00	0.28499999E 00	-0.12791401E 01
0.001000	0.30999999E 00	0.30999999E 00	-0.49064777E 00
0.001000	0.30999999E 00	0.33499999E 00	0.12767126E 00
0.001000	0.30999999E 00	0.35999999E 00	0.60865957E 00
0.002100	0.10000000E-01	0.10000000E-01	-0.20294333E 03
0.002100	0.10000000E-01	0.34999999E-01	-0.16437922E 03
0.002100	0.10000000E-01	0.59999999E-01	-0.13126982E 03
0.002100	0.10000000E-01	0.84999999E-01	-0.10323523E 03
0.002100	0.10000000E-01	0.11000000E 00	-0.80027409E 02
0.002100	0.10000000E-01	0.13500000E 00	-0.61174051E 02
0.002100	0.10000000E-01	0.16000000E 00	-0.46100048E 02
0.002100	0.10000000E-01	0.18499999E 00	-0.34207012E 02
0.002100	0.10000000E-01	0.20999999E 00	-0.24926057E 02
0.002100	0.10000000E-01	0.23499999E 00	-0.17748550E 02
0.002100	0.10000000E-01	0.25999999E 00	-0.12239497E 02
0.002100	0.10000000E-01	0.28499999E 00	-0.80383410E 01
0.002100	0.10000000E-01	0.30999999E 00	-0.48536344E 01

TIME	RADIUS	Z	00 STRESS
0.002100	0.10000000E-01	0.33499999E 00	-0.24539050E 01
0.002100	0.10000000E-01	0.35999999E 00	-0.65757208E 00
0.002100	0.59999999E-01	0.10000000E-01	-0.20128808E 03
0.002100	0.59999999E-01	0.34999999E-01	-0.16211588E 03
0.002100	0.59999999E-01	0.59999999E-01	-0.12889633E 03
0.002100	0.59999999E-01	0.84999999E-01	-0.10101280E 03
0.002100	0.59999999E-01	0.11000000E 00	-0.78079731E 02
0.002100	0.59999999E-01	0.13500000E 00	-0.59550945E 02
0.002100	0.59999999E-01	0.16000000E 00	-0.44803512E 02
0.002100	0.59999999E-01	0.18499999E 00	-0.33209787E 02
0.002100	0.59999999E-01	0.20999999E 00	-0.24185990E 02
0.002100	0.59999999E-01	0.23499999E 00	-0.17219070E 02
0.002100	0.59999999E-01	0.25999999E 00	-0.11875915E 02
0.002100	0.59999999E-01	0.28499999E 00	-0.78015550E 01
0.002100	0.59999999E-01	0.30999999E 00	-0.47112282E 01
0.002100	0.59999999E-01	0.33499999E 00	-0.23800141E 01
0.002100	0.59999999E-01	0.35999999E 00	-0.63223801E 00
0.002100	0.11000000E 00	0.10000000E-01	-0.19652450E 03
0.002100	0.11000000E 00	0.34999999E-01	-0.15575125E 03
0.002100	0.11000000E 00	0.59999999E-01	-0.12275937E 03
0.002100	0.11000000E 00	0.84999999E-01	-0.95449568E 02
0.002100	0.11000000E 00	0.11000000E 00	-0.73302003E 02
0.002100	0.11000000E 00	0.13500000E 00	-0.55634160E 02
0.002100	0.11000000E 00	0.16000000E 00	-0.41719673E 02
0.002100	0.11000000E 00	0.18499999E 00	-0.30868258E 02
0.002100	0.11000000E 00	0.20999999E 00	-0.22468321E 02
0.002100	0.11000000E 00	0.23499999E 00	-0.16003208E 02
0.002100	0.11000000E 00	0.25999999E 00	-0.11050041E 02
0.002100	0.11000000E 00	0.28499999E 00	-0.72703812E 01
0.002100	0.11000000E 00	0.30999999E 00	-0.43973942E 01
0.002100	0.11000000E 00	0.33499999E 00	-0.22228676E 01
0.002100	0.11000000E 00	0.35999999E 00	-0.58544521E 00
0.002100	0.16000000E 00	0.10000000E-01	-0.19419825E 03
0.002100	0.16000000E 00	0.34999999E-01	-0.14473742E 03
0.002100	0.16000000E 00	0.59999999E-01	-0.11236784E 03
0.002100	0.16000000E 00	0.84999999E-01	-0.86297379E 02
0.002100	0.16000000E 00	0.11000000E 00	-0.65675566E 02
0.002100	0.16000000E 00	0.13500000E 00	-0.49559071E 02
0.002100	0.16000000E 00	0.16000000E 00	-0.37059477E 02
0.002100	0.16000000E 00	0.18499999E 00	-0.27410402E 02
0.002100	0.16000000E 00	0.20999999E 00	-0.19983183E 02
0.002100	0.16000000E 00	0.23499999E 00	-0.14277147E 02
0.002100	0.16000000E 00	0.25999999E 00	-0.99000935E 01
0.002100	0.16000000E 00	0.28499999E 00	-0.65477961E 01
0.002100	0.16000000E 00	0.30999999E 00	-0.39854652E 01
0.002100	0.16000000E 00	0.33499999E 00	-0.20323335E 01

TIME	RADIUS	Z	RR STRESS
0.002100	0.16000000E 00	0.35999999E 00	-0.54934116E 00
0.002100	0.21000000E 00	0.10000000E-01	-0.18924951E 03
0.002100	0.21000000E 00	0.34999999E-01	-0.12754425E 03
0.002100	0.21000000E 00	0.59999999E-01	-0.96883154E 02
0.002100	0.21000000E 00	0.84999999E-01	-0.73353031E 02
0.002100	0.21000000E 00	0.11000000E 00	-0.55417729E 02
0.002100	0.21000000E 00	0.13500000E 00	-0.41743315E 02
0.002100	0.21000000E 00	0.16000000E 00	-0.31285876E 02
0.002100	0.21000000E 00	0.18499999E 00	-0.23259871E 02
0.002100	0.21000000E 00	0.20999999E 00	-0.17080529E 02
0.002100	0.21000000E 00	0.23499999E 00	-0.12311567E 02
0.002100	0.21000000E 00	0.25999999E 00	-0.86255691E 01
0.002100	0.21000000E 00	0.28499999E 00	-0.57750164E 01
0.002100	0.21000000E 00	0.30999999E 00	-0.35715465E 01
0.002100	0.21000000E 00	0.33499999E 00	-0.18709359E 01
0.002100	0.21000000E 00	0.35999999E 00	-0.56218401E 00
0.002100	0.26000000E 00	0.10000000E-01	-0.16657005E 03
0.002100	0.26000000E 00	0.34999999E-01	-0.10130588E 03
0.002100	0.26000000E 00	0.59999999E-01	-0.75555547E 02
0.002100	0.26000000E 00	0.84999999E-01	-0.57132345E 02
0.002100	0.26000000E 00	0.11000000E 00	-0.43497331E 02
0.002100	0.26000000E 00	0.13500000E 00	-0.33166237E 02
0.002100	0.26000000E 00	0.16000000E 00	-0.25217997E 02
0.002100	0.26000000E 00	0.18499999E 00	-0.19042620E 02
0.002100	0.26000000E 00	0.20999999E 00	-0.14214308E 02
0.002100	0.26000000E 00	0.23499999E 00	-0.10424281E 02
0.002100	0.26000000E 00	0.25999999E 00	-0.74425964E 01
0.002100	0.26000000E 00	0.28499999E 00	-0.50947166E 01
0.002100	0.26000000E 00	0.30999999E 00	-0.32464005E 01
0.002100	0.26000000E 00	0.33499999E 00	-0.17933474E 01
0.002100	0.26000000E 00	0.35999999E 00	-0.65394872E 00
0.002100	0.30999999E 00	0.10000000E-01	-0.92581221E 02
0.002100	0.30999999E 00	0.34999999E-01	-0.64390650E 02
0.002100	0.30999999E 00	0.59999999E-01	-0.51287325E 02
0.002100	0.30999999E 00	0.84999999E-01	-0.40761954E 02
0.002100	0.30999999E 00	0.11000000E 00	-0.32278242E 02
0.002100	0.30999999E 00	0.13500000E 00	-0.25431463E 02
0.002100	0.30999999E 00	0.16000000E 00	-0.19899180E 02
0.002100	0.30999999E 00	0.18499999E 00	-0.15425508E 02

TIME	RADIUS	Z	RZ STRESS
0.001000	0.10000000E-01	0.10000000E-01	-0.97203480E 01
0.001000	0.10000000E-01	0.34999999E-01	-0.19215026E 01

TIME	RADIUS	Z	RZ STRESS
0.001000	0.10000000E-01	0.59999999E-01	-0.22943904E 01
0.001000	0.10000000E-01	0.84999999E-01	-0.28720592E 01
0.001000	0.10000000E-01	0.11000000E 00	-0.32890200E 01
0.001000	0.10000000E-01	0.13500000E 00	-0.35025444E 01
0.001000	0.10000000E-01	0.16000000E 00	-0.35371274E 01
0.001000	0.10000000E-01	0.18499999E 00	-0.34372963E 01
0.001000	0.10000000E-01	0.20999999E 00	-0.32487529E 01
0.001000	0.10000000E-01	0.23499999E 00	-0.30099921E 01
0.001000	0.10000000E-01	0.25999999E 00	-0.27498003E 01
0.001000	0.10000000E-01	0.28499999E 00	-0.24878135E 01
0.001000	0.10000000E-01	0.30999999E 00	-0.22362897E 01
0.001000	0.10000000E-01	0.33499999E 00	-0.20020930E 01
0.001000	0.10000000E-01	0.35999999E 00	-0.17884171E 01
0.001000	0.59999999E-01	0.10000000E-01	-0.28035568E 01
0.001000	0.59999999E-01	0.34999999E-01	-0.82148734E 01
0.001000	0.59999999E-01	0.59999999E-01	-0.13411053E 02
0.001000	0.59999999E-01	0.84999999E-01	-0.17326565E 02
0.001000	0.59999999E-01	0.11000000E 00	-0.19801495E 02
0.001000	0.59999999E-01	0.13500000E 00	-0.20936268E 02
0.001000	0.59999999E-01	0.16000000E 00	-0.20992332E 02
0.001000	0.59999999E-01	0.18499999E 00	-0.20280175E 02
0.001000	0.59999999E-01	0.20999999E 00	-0.19083864E 02
0.001000	0.59999999E-01	0.23499999E 00	-0.17628379E 02
0.001000	0.59999999E-01	0.25999999E 00	-0.16074977E 02
0.001000	0.59999999E-01	0.28499999E 00	-0.14529995E 02
0.001000	0.59999999E-01	0.30999999E 00	-0.13057844E 02
0.001000	0.59999999E-01	0.33499999E 00	-0.11693364E 02
0.001000	0.59999999E-01	0.35999999E 00	-0.10451683E 02
0.001000	0.11000000E 00	0.10000000E-01	-0.49866031E 01
0.001000	0.11000000E 00	0.34999999E-01	-0.16217523E 02
0.001000	0.11000000E 00	0.59999999E-01	-0.25984433E 02
0.001000	0.11000000E 00	0.84999999E-01	-0.32792353E 02
0.001000	0.11000000E 00	0.11000000E 00	-0.36547451E 02
0.001000	0.11000000E 00	0.13500000E 00	-0.37775571E 02
0.001000	0.11000000E 00	0.16000000E 00	-0.37187913E 02
0.001000	0.11000000E 00	0.18499999E 00	-0.35437447E 02
0.001000	0.11000000E 00	0.20999999E 00	-0.33032004E 02
0.001000	0.11000000E 00	0.23499999E 00	-0.30329885E 02
0.001000	0.11000000E 00	0.25999999E 00	-0.27566103E 02
0.001000	0.11000000E 00	0.28499999E 00	-0.24865124E 02
0.001000	0.11000000E 00	0.30999999E 00	-0.22368366E 02
0.001000	0.11000000E 00	0.33499999E 00	-0.20055637E 02
0.001000	0.11000000E 00	0.35999999E 00	-0.17960436E 02
0.001000	0.16000000E 00	0.10000000E-01	-0.87000062E 01
0.001000	0.16000000E 00	0.34999999E-01	-0.27572186E 02
0.001000	0.16000000E 00	0.59999999E-01	-0.41550938E 02

TIME	RADIUS	Z	RZ STRESS
0.001000	0.16000000E 00	0.84999999E-01	-0.49332035E 02
0.001000	0.16000000E 00	0.11000000E 00	-0.52259751E 02
0.001000	0.16000000E 00	0.13500000E 00	-0.52000581E 02
0.001000	0.16000000E 00	0.16000000E 00	-0.49854358E 02
0.001000	0.16000000E 00	0.18499999E 00	-0.46696588E 02
0.001000	0.16000000E 00	0.20999999E 00	-0.43082572E 02
0.001000	0.16000000E 00	0.23499999E 00	-0.39352665E 02
0.001000	0.16000000E 00	0.25999999E 00	-0.35708305E 02
0.001000	0.16000000E 00	0.28499999E 00	-0.32262424E 02
0.001000	0.16000000E 00	0.30999999E 00	-0.29072089E 02
0.001000	0.16000000E 00	0.33499999E 00	-0.26159605E 02
0.001000	0.16000000E 00	0.35999999E 00	-0.23526153E 02
0.001000	0.21000000E 00	0.10000000E-01	-0.16047847E 02
0.001000	0.21000000E 00	0.34999999E-01	-0.44988392E 02
0.001000	0.21000000E 00	0.84999999E-01	-0.61586583E 02
0.001000	0.21000000E 00	0.11000000E 00	-0.61132631E 02
0.001000	0.21000000E 00	0.13500000E 00	-0.58632436E 02
0.001000	0.21000000E 00	0.16000000E 00	-0.55136163E 02
0.001000	0.21000000E 00	0.18499999E 00	-0.51201849E 02
0.001000	0.21000000E 00	0.20999999E 00	-0.47147726E 02
0.001000	0.21000000E 00	0.23499999E 00	-0.43161016E 02
0.001000	0.21000000E 00	0.25999999E 00	-0.39351025E 02
0.001000	0.21000000E 00	0.28499999E 00	-0.35778384E 02
0.001000	0.21000000E 00	0.30999999E 00	-0.32472338E 02
0.001000	0.21000000E 00	0.33499999E 00	-0.29441850E 02
0.001000	0.21000000E 00	0.35999999E 00	-0.26683033E 02
0.001000	0.26000000E 00	0.10000000E-01	-0.20361574E 02
0.001000	0.26000000E 00	0.34999999E-01	-0.43292842E 02
0.001000	0.26000000E 00	0.59999999E-01	-0.50972569E 02
0.001000	0.26000000E 00	0.84999999E-01	-0.53718881E 02
0.001000	0.26000000E 00	0.11000000E 00	-0.53869852E 02
0.001000	0.26000000E 00	0.13500000E 00	-0.52513164E 02
0.001000	0.26000000E 00	0.16000000E 00	-0.50265505E 02
0.001000	0.26000000E 00	0.18499999E 00	-0.47511683E 02
0.001000	0.26000000E 00	0.20999999E 00	-0.44503457E 02
0.001000	0.26000000E 00	0.23499999E 00	-0.41408673E 02
0.001000	0.26000000E 00	0.25999999E 00	-0.38339059E 02
0.001000	0.26000000E 00	0.28499999E 00	-0.35367562E 02
0.001000	0.26000000E 00	0.30999999E 00	-0.32539859E 02
0.001000	0.26000000E 00	0.33499999E 00	-0.29882349E 02
0.001000	0.26000000E 00	0.35999999E 00	-0.27407914E 02
0.001000	0.30999999E 00	0.10000000E-01	-0.11496372E 01
0.001000	0.30999999E 00	0.34999999E-01	-0.11018056E 02
0.001000	0.30999999E 00	0.59999999E-01	-0.22338287E 02
0.001000	0.30999999E 00	0.84999999E-01	-0.30416332E 02
0.001000	0.30999999E 00	0.11000000E 00	-0.35225055E 02

TIME	RADIUS	Z	RZ STRESS
0.001000	0.309999999E 00	0.135000000E 00	-0.37638677E 02
0.001000	0.210000000E 00	0.599999999E-01	-0.57809676E 02
0.001000	0.309999999E 00	0.160000000E 00	-0.38416693E 02
0.001000	0.309999999E 00	0.184999999E 00	-0.38102524E 02
0.001000	0.309999999E 00	0.209999999E 00	-0.37072591E 02
0.001000	0.309999999E 00	0.234999999E 00	-0.35587976E 02
0.001000	0.309999999E 00	0.259999999E 00	-0.33830926E 02
0.001000	0.309999999E 00	0.284999999E 00	-0.31929037E 02
0.001000	0.309999999E 00	0.309999999E 00	-0.29971351E 02
0.001000	0.309999999E 00	0.334999999E 00	-0.28019444E 02
0.001000	0.309999999E 00	0.359999999E 00	-0.26115004E 02
0.002100	0.100000000E-01	0.100000000E-01	-0.10387084E 02
0.002100	0.100000000E-01	0.349999999E-01	-0.15235791E 01
0.002100	0.100000000E-01	0.599999999E-01	-0.16169210E 01
0.002100	0.100000000E-01	0.849999999E-01	-0.20702524E 01
0.002100	0.100000000E-01	0.110000000E 00	-0.24867485E 01
0.002100	0.100000000E-01	0.135000000E 00	-0.27970815E 01
0.002100	0.100000000E-01	0.160000000E 00	-0.29884725E 01
0.002100	0.100000000E-01	0.184999999E 00	-0.30699049E 01
0.002100	0.100000000E-01	0.209999999E 00	-0.30599873E 01
0.002100	0.100000000E-01	0.234999999E 00	-0.29803240E 01
0.002100	0.100000000E-01	0.259999999E 00	-0.28516258E 01
0.002100	0.100000000E-01	0.284999999E 00	-0.26917028E 01
0.002100	0.100000000E-01	0.309999999E 00	-0.25147590E 01
0.002100	0.100000000E-01	0.334999999E 00	-0.23314480E 01
0.002100	0.100000000E-01	0.359999999E 00	-0.21493245E 01
0.002100	0.599999999E-01	0.100000000E-01	-0.20195376E 01
0.002100	0.599999999E-01	0.349999999E-01	-0.53838426E 01
0.002100	0.599999999E-01	0.599999999E-01	-0.91572998E 01
0.002100	0.599999999E-01	0.849999999E-01	-0.12437764E 02
0.002100	0.599999999E-01	0.110000000E 00	-0.15042809E 02
0.002100	0.599999999E-01	0.135000000E 00	-0.16877589E 02
0.002100	0.599999999E-01	0.160000000E 00	-0.17954071E 02
0.002100	0.599999999E-01	0.184999999E 00	-0.18362533E 02
0.002100	0.599999999E-01	0.209999999E 00	-0.18232506E 02
0.002100	0.599999999E-01	0.234999999E 00	-0.17701317E 02
0.002100	0.599999999E-01	0.259999999E 00	-0.16894707E 02
0.002100	0.599999999E-01	0.284999999E 00	-0.15917636E 02
0.002100	0.599999999E-01	0.309999999E 00	-0.14851893E 02
0.002100	0.599999999E-01	0.334999999E 00	-0.13757619E 02
0.002100	0.599999999E-01	0.359999999E 00	-0.12676912E 02
0.002100	0.110000000E 00	0.100000000E-01	-0.32574673E 01
0.002100	0.110000000E 00	0.349999999E-01	-0.10316117E 02
0.002100	0.110000000E 00	0.599999999E-01	-0.17538130E 02
0.002100	0.110000000E 00	0.849999999E-01	-0.23684707E 02
0.002100	0.110000000E 00	0.110000000E 00	-0.28322364E 02

TIME	RADIUS	Z	RZ STRESS
0.002100	0.11000000E 00	0.13500000E 00	-0.31358884E 02
0.002100	0.11000000E 00	0.16000000E 00	-0.32933238E 02
0.002100	0.11000000E 00	0.18499999E 00	-0.33302460E 02
0.002100	0.11000000E 00	0.20999999E 00	-0.32756171E 02
0.002100	0.11000000E 00	0.23499999E 00	-0.31565115E 02
0.002100	0.11000000E 00	0.25999999E 00	-0.29956995E 02
0.002100	0.11000000E 00	0.28499999E 00	-0.28109780E 02
0.002100	0.11000000E 00	0.30999999E 00	-0.26155711E 02
0.002100	0.11000000E 00	0.33499999E 00	-0.24188112E 02
0.002100	0.11000000E 00	0.35999999E 00	-0.22269773E 02
0.002100	0.16000000E 00	0.10000000E-01	-0.53924072E 01
0.002100	0.16000000E 00	0.34999999E-01	-0.16945233E 02
0.002100	0.16000000E 00	0.59999999E-01	-0.28084195E 02
0.002100	0.16000000E 00	0.84999999E-01	-0.36843296E 02
0.002100	0.16000000E 00	0.11000000E 00	-0.42800829E 02
0.002100	0.16000000E 00	0.13500000E 00	-0.46155046E 02
0.002100	0.16000000E 00	0.16000000E 00	-0.47392783E 02
0.002100	0.16000000E 00	0.18499999E 00	-0.47057990E 02
0.002100	0.16000000E 00	0.20999999E 00	-0.45636507E 02
0.002100	0.16000000E 00	0.23499999E 00	-0.43517540E 02
0.002100	0.16000000E 00	0.25999999E 00	-0.40993498E 02
0.002100	0.16000000E 00	0.28499999E 00	-0.38274594E 02
0.002100	0.16000000E 00	0.30999999E 00	-0.35506444E 02
0.002100	0.16000000E 00	0.33499999E 00	-0.32786181E 02
0.002100	0.16000000E 00	0.35999999E 00	-0.30175585E 02
0.002100	0.21000000E 00	0.10000000E-01	-0.87975814E 01
0.002100	0.21000000E 00	0.34999999E-01	-0.26749453E 02
0.002100	0.21000000E 00	0.59999999E-01	-0.42039749E 02
0.002100	0.21000000E 00	0.84999999E-01	-0.52145414E 02
0.002100	0.21000000E 00	0.11000000E 00	-0.57634100E 02
0.002100	0.21000000E 00	0.13500000E 00	-0.59705348E 02
0.002100	0.21000000E 00	0.16000000E 00	-0.59467666E 02
0.002100	0.21000000E 00	0.18499999E 00	-0.57755651E 02
0.002100	0.21000000E 00	0.20999999E 00	-0.55152147E 02
0.002100	0.21000000E 00	0.23499999E 00	-0.52052287E 02
0.002100	0.21000000E 00	0.25999999E 00	-0.48720125E 02
0.002100	0.21000000E 00	0.28499999E 00	-0.45329959E 02
0.002100	0.21000000E 00	0.30999999E 00	-0.41994949E 02
0.002100	0.21000000E 00	0.33499999E 00	-0.38786263E 02
0.002100	0.21000000E 00	0.35999999E 00	-0.35746376E 02
0.002100	0.26000000E 00	0.10000000E-01	-0.14403071E 02
0.002100	0.26000000E 00	0.34999999E-01	-0.42854400E 02
0.002100	0.26000000E 00	0.59999999E-01	-0.58971716E 02
0.002100	0.26000000E 00	0.84999999E-01	-0.65979548E 02
0.002100	0.26000000E 00	0.11000000E 00	-0.68035960E 02
0.002100	0.26000000E 00	0.13500000E 00	-0.67430294E 02

TIME	RADIUS	Z	RZ STRESS
0.002100	0.26000000E 00	0.16000000E 00	-0.65348377E 02
0.002100	0.26000000E 00	0.18499999E 00	-0.62441318E 02
0.002100	0.26000000E 00	0.20999999E 00	-0.59092376E 02
0.002100	0.26000000E 00	0.23499999E 00	-0.55540389E 02
0.002100	0.26000000E 00	0.25999999E 00	-0.51939078E 02
0.002100	0.26000000E 00	0.28499999E 00	-0.48388798E 02
0.002100	0.26000000E 00	0.30999999E 00	-0.44954613E 02
0.002100	0.26000000E 00	0.33499999E 00	-0.41677457E 02
0.002100	0.26000000E 00	0.35999999E 00	-0.38581350E 02
0.002100	0.30999999E 00	0.10000000E-01	-0.28358190E 02
0.002100	0.30999999E 00	0.34999999E-01	-0.53282531E 02
0.002100	0.30999999E 00	0.59999999E-01	-0.60269485E 02
0.002100	0.30999999E 00	0.84999999E-01	-0.63143522E 02
0.002100	0.30999999E 00	0.11000000E 00	-0.63733625E 02
0.002100	0.30999999E 00	0.13500000E 00	-0.62904542E 02
0.002100	0.30999999E 00	0.16000000E 00	-0.61161387E 02
0.002100	0.30999999E 00	0.18499999E 00	-0.58830960E 02
0.002100	0.30999999E 00	0.20999999E 00	-0.56137088E 02
0.002100	0.30999999E 00	0.23499999E 00	-0.53237884E 02
0.002100	0.30999999E 00	0.25999999E 00	-0.50246756E 02
0.002100	0.30999999E 00	0.28499999E 00	-0.47245247E 02
0.002100	0.30999999E 00	0.30999999E 00	-0.44291474E 02
0.002100	0.30999999E 00	0.33499999E 00	-0.41426115E 02
0.002100	0.30999999E 00	0.35999999E 00	-0.38676668E 02

**SOME DYNAMIC MECHANICAL PROPERTIES OF APPLE FRUITS
AND THEIR USE IN THE SOLUTION OF AN IMPACTING CONTACT
PROBLEM OF SPHERICAL FRUIT**

by

Donald Dale Hamann

ABSTRACT

Complex compression moduli were determined for five groups of apples by subjecting cylindrical specimens to sinusoidal forcing at cyclic rates of 50 to 365 cycles per second. The five groups of apples consisted of Red Delicious, Winesap, and Golden Delicious at harvest and Winesap and Golden Delicious after about four months' storage. Ten tests at each of seven frequencies were conducted for each group of apples.

Statistical analyses of the complex compression moduli showed the stress-strain relations for the various groups of apples to be frequency dependent. Due to scatter, Maxwell model approximations were the best mechanical model representations that could be developed directly. These, however, appeared to be sufficiently good approximations for the data. A statistical curve fitting procedure resulted in expressions for relaxation moduli which were equivalent to Maxwell model representations with element constants slightly larger than those obtained by the direct method. Several tables and curves are given to summarize and clarify these experimental results.

The two groups of apples taken from storage were also tested in sinusoidal shear but shear moduli obtained were believed to be low by an

unknown percentage due to the fact that a pure shear state could not be very closely approximated and it was not possible to insure that there was no specimen slip in the test fixture. Shear moduli were, however, roughly the magnitude expected based on the knowledge of the compression moduli and the imperfect shear state.

Poisson's ratio values for the three groups of apples tested at harvest condition were determined by Chappell* and used in the contact problem portion of this thesis.

Contact problems involving one apple falling onto another supported underneath and an apple falling onto a rigid surface were solved for the approach of the bodies, surface indentations, surface pressures, and internal stresses from time of initial contact to the time of maximum indentation. A theoretical analysis was made and a computer program written which computed these quantities with the input being the element constants of the Maxwell material, Poisson's ratio, the radius of the apple at the contact point, the apple weight, and the drop height. Computer time using an IBM 7040 digital computer was short (about four minutes) except for computation of internal stresses which took several hours depending on the number of points at which stress was being computed. Comparison of computer results and published elastic solutions validated the method. Example problems worked to illustrate the method revealed that high internal stresses (about 200 psi) are caused by a two-inch

*Chappell, T. W. Poisson's ratio for apple flesh under compressive loading, Unpublished M. S. Thesis, Virginia Polytechnic Institute, 1967.

drop of a representative apple. Higher drop heights produce a somewhat higher maximum stress and a much larger high stress region. A number of curves are presented to illustrate the approach of the impacting bodies, contact stress, and internal stresses for the example problems.

The experimental equipment and techniques described and the solution to the impact problem described are not limited to apple flesh or biological materials and possibly will be even more useful for other materials.