Chapter 7

Conclusion

Lean-premixed combustion has the advantage of low emissions for modern gas turbines, but it is susceptible to thermoacoustic instabilities, which can result in large amplitude pressure oscillation in the combustion chamber. The thermoacoustic limit cycle is generated by the unsteady heat release dynamics coupled to the combustor acoustics.

In this research, we focused on reduced-order modeling of the dynamics of a laminar premixed flame. From first principles of combustion dynamics, a physically-based, reduced-order, nonlinear model was developed based on the proper orthogonal decomposition technique and generalized Galerkin method. Experimentally, the describing function for the flame was measured and used to identify an empirical nonlinear flame model. Furthermore, a linear acoustic model was developed and identified for the Rijke tube experiment. Closed-loop thermoacoustic modeling using the first principles flame model coupled to the linear acoustics successfully reproduces the linear instability and predicts the thermoacoustic limit cycle amplitudes. With the measured experimental flame data and the modeled linear acoustics, the describing function technique was applied for limit cycle analysis. The thermoacoustic limit cycle amplitude is predicted accurately for a low equivalence ratio case, and the closed-loop model also predicts the performance for a phase shift controller.

7.1 Summary of Modeling Work

Combustion instability is an active research field with a long history. Recently, linear stability analysis has been applied to predict thermoacoustic instabilities. However, the limit cycle amplitudes are not predicted due to the lack of a simple but effective nonlinear model of flame dynamics.

To begin understanding the requirements of a nonlinear flame model, the simplified chemical kinetics for a laminar premixed flame were examined. Starting from the first-principles of conservation equations, the flame dynamics can be modeled by a set of partial differential equations. The shooting method was able to obtain the steady-state temperature and species mass fraction profiles. The finite difference method was then applied for time-dependent simulations.

Using the time-domain simulation data as snapshots, the proper orthogonal decomposition technique was used to extract the dominant basis functions, and the Galerkin procedure was applied to convert the partial differential equations (PDE) into a set ordinary differential equations (ODE). We discussed the importance of the input signal used to generate the simulation data. Using a pseudo random binary sequence (PRBS) as input, a low order weakly nonlinear model was identified to describe the flame system for small perturbations. Further order reduction using balancing created a 2nd order nonlinear model. The model accurately described the linear part of the system. However, the weakly nonlinear model was not able to account for the output response if the input signal was large, because neither the snapshot data nor the basis functions contained information for large input system response. Due to the complexity of the flame dynamics, a 10th order nonlinear model was developed to capture the describing functions for inputs with various amplitudes within the frequency range between 160-200 Hz, which was the frequency range related to the actual combustor instability frequencies. The accuracy of the model in conjunction with the model order selection criteria was discussed. The challenges to reduced-order modeling were presented based on our experience with this first-principles flame modeling.

For the purpose of identifying a nonlinear flame model from experiments, we proposed a nonlinear system identification technique in the frequency domain based on harmonic balancing. A nonparametric nonlinear system identification procedure was presented with an example of identifying a 4th order nonlinear flame model from experiments. To resolve the stability issues of high-order nonlinear systems, a subsystem technique, which breaks the system into two 2nd order blocks, was developed to fit the data. For certain nonlinear systems we showed that the input-output differential equation form might not be able to describe the system. A nonlinear state-space form might be necessary for these systems. Consequently, a simple iterative algorithm was developed to identify state-space nonlinear systems and an example of identifying a Wiener-Hammerstein system is given. The objective of this exploration of nonlinear system identification techniques and algorithms was to apply them to create an empirical nonlinear flame model from experiments.

The describing function technique is very useful for nonlinear system limit cycle analysis. To capture the describing function of a flame model, a flat flame burner was used to measure OH* fluctuations, which were assumed proportional to the heat release rate changes, due to sinusoidal velocity perturbations at various amplitudes within the frequency range of 160-200Hz. An empirical nonlinear model with cubic nonlinearities was identified to fit the measured describing functions. Numerical simulation was performed to validate the identified model and reproduce the measured describing functions.

A Rijke tube combustor in our laboratory was used for experiments and thermoacoustic limit cycle measurements. The data of pressure oscillation frequency and amplitude for each limit cycle condition were used for our closed-loop modeling comparison. A theoretical linear acoustic model was developed based on wave equations and boundary conditions, followed by identification of the damping ratio from experiments using an inverse frequency response technique.

One of the most important goals of this research was to build a closed-loop model that predicts the thermoacoustic instability in both amplitude and frequency. With the first-principles flame model that we developed coupled by the linear acoustic model, the linear stability analysis of the closed-loop system clearly showed the existence of instability from root locus analysis. Using the nonlinear flame model, it predicted the limit cycle frequencies as well as amplitudes. The prediction matched well with the actual experimental data.

The describing function data that we measured from experiments was also used together with the linear acoustic model to investigate the limit cycle based on the describing function technique. A proportionality constant was determined that related the dynamic heat release rate *q'* to the changes of *OH** response. From the linear data, the stability analysis graphically showed the existence of instability for different flow conditions. The nonlinear analysis was able to predict the limit cycle amplitude. Finally, for a controlled system, the limit cycle changes, while changing the control gain of a phase-shift controller, was studied using the describing function model.

This work also pointed out problems with the linear experimental data for high equivalence ratios ($\phi>0.55$) and flow rate (Q>160cc/s), in that linear instability is not correctly predicted in this regime. There are numerous potential causes of this inconsistency, chief being the relationship between OH* and heat release rate *q'*, that are under investigation at this time.

7.2 Contributions

We began our investigation of thermoacoustic instabilities by assuming we could create simple 1st order or 2nd order nonlinear models for laminar premixed flames. The only dynamic flame model in the literature was a 1st order global linear flame model based on flame area theory with many assumptions and simplifications. Unfortunately, our investigation showed that a global nonlinear model for the laminar premixed flame was not so straightforward. The modal function extraction from the first-principles model requires more than 2 modes to ensure linear stability, while the experimental frequency response measurements suggested at least a 4th order system. The flame dynamics also showed great complexity in the process of nonlinear identification. The difficulties of reduced-order modeling for complex nonlinear systems and the challenges of prediction with experimental data drove this research work. This dissertation contributes in the following aspects:

1. It developed a flame model from first principles. To the author's knowledge, there is no such nonlinear model existing in the field. This model introduces the POD technique to the flame modeling area and obtains reliable and productive results.

2. Experimentally, the describing function for a laminar premixed flame was measured for the first time. Due to the existence of limit cycle oscillations, it is practically impossible to directly measure the flame dynamics in the combustor itself. This work utilized a specially designed (by V. Khanna) flat flame burner to avoid any acoustic resonance. It measures the describing function from velocity perturbations to OH* fluctuations as heat release dynamics.

3. It proposed a nonparametric technique to identify the nonlinear flame model from harmonic data. This work uses the frequency domain techniques for the flame dynamics experiments because of the noisy combustion environment. The fundamental frequency is considered here to identify an empirical nonlinear model, and the technique is expandable to account for higher harmonics and gain more nonlinearity information.

4. High order nonlinear models have a large set of possible nonlinear terms and are often susceptible to instabilities. It requires a large amount of data to identify the system correctly. A subsystem technique is proposed here to break the higher order nonlinear model into subsystems. It speeds up the identification process and enhances the stability of the identified model.

5. It investigated the problems of obtaining a global nonlinear flame model. Both the first-principles analysis and experimental identification demonstrated the difficulties of obtaining a global nonlinear flame model. A describing function technique was introduced to resolve the issue from a practical viewpoint. It captures the important dynamics around the instability frequencies while discarding low frequency nonlinear dynamics to maintain the model simplicity.

6. This dissertation successfully implemented nonlinear closed-loop analysis for thermoacoustic systems. This goes beyond the linear stability analysis in the literature, and is able to predict the nonlinear limit cycle frequencies and amplitudes. By including a linear acoustic model, the first-principles nonlinear

flame model gives a good prediction in comparison with the Rijke tube experiments. The describing function technique was applied for the first time to experimental flame modeling, and is able to predict both linear instability and nonlinear limit cycle characteristics for low equivalence ratio cases.

7.3 Future Work

The research efforts described in this dissertation were aimed at the development of a reduced-order nonlinear flame model to predict thermoacoustic limit cycles. During the process of this research work, a number of issues have arisen where a better measurement of flame dynamics and combustor hot acoustics would be helpful to obtain a more accurate model for the thermoacoustic system.

It is important to understand the relationship between OH* and heat release rate. Research shows that OH* is a good indicator of heat release rate for steady state. Since our nonlinear model is trying to describe the dynamic response from velocity perturbations to heat release rate fluctuations, work needs to be done to investigate and validate the proportionality relationship between OH* and heat release rate fluctuation. In the application of describing function analysis to the flame data, the proportionality constant k was chosen such that the limit cycle frequency matched up with experiments. If the proportionality constant can be obtained accurately from experiments and computations, it will help to predict the limit cycle frequencies accurately. This will also validate the modeling work from first principles by the experimental flame measurements.

As we pointed out earlier, based on the current flame describing functions and acoustic model, there remains a problem in predicting instability for high equivalence ratio ($\phi>0.55$) and flow rate ($Q>160$cc/s). There are two things that could lead to the problem. The flame describing function measurement using OH* may not be accurate when the heat release rate to the burner is large. Another possible factor is the applicability of the experimentally measured flame data from the flat flame burner to the actual Rijke tube that exhibits the thermoacoustic limit cycles. Our preliminary investigation shows the linear gain and phase effects with different chimneys. While it is difficult to measure the flame describing function directly from the Rijke tube due to the presence of the limit cycle, further research is recommended to look at the similarities and differences of heat release dynamics between the flat flame burner and the Rijke tube. Understanding the effects of physical parameters with regard to the applicability of experimental flame data between two different rigs would be helpful in extending our reduced-order modeling results.

In addition, the acoustic model used in the closed-loop modeling work is based on wave equations and boundary conditions. The outlet radiation impedance, temperature gradients, and the flame holder effects were not considered in detail. Further work on the acoustics would be beneficial to understanding the combustor acoustic model and provide a more accurate closed-loop model for predicting thermoacoustic limit cycles. To extend this modeling research from laminar premixed flames to turbulent flames and liquid fuel combustion will provide interesting and attractive research topics to be investigated. Eventually this work could provide a modular model for the gas turbine combustor designer and for control applications as well.