

Reconfigurable Resonant Cubic HF Phased Array for In-Space Assembly Operation

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Abstract

Conventional two-dimensional phased arrays face two major shortcomings: the presence of ambiguities in direction of arrival measurements and beam broadening endfire effects. The literature provides methods for addressing and minimizing these problems on conventional planar phased array structures, but there has been no investigation into solving these issues with three-dimensional geometries. In this thesis, the design and performance of a cubic phased array that can eliminate endfire effects and dramatically improve direction of arrival ambiguity resolution is investigated. Both beamforming and direction of arrival simulations are performed in MATLAB and 4nec2 simulation environments for cubic phased arrays of various sizes and at different frequencies and demonstrate that the endfire effects are eliminated and direction of arrival ambiguity resolution is dramatically improved. These findings are expected to lead to new designs of high fidelity three-dimensional phased arrays.

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General Audience Abstract

Conventional two-dimensional, flat, plane antenna arrays have revolutionized how sensing and detection systems perform. These systems, however, face two major shortcomings due to their “flat” geometry. The computation that determines the direction from which an object is approaching or a signal has been transmitted will have two solutions that are opposite each other in the same way that the polynomial expression $x^2 = +2$ or -2 has two solutions that are opposite each other. This is known as the ambiguity problem and presents major uncertainty in direction finding or direction of arrival measurements. The second major shortcoming has to do with transmitting a signal at different directions. The antenna elements in the array are stationary, but the beams that each element transmits can be aimed in specific directions by controlling the phase of the voltage sources for each respective antenna. This is why it is called a phased array. When every element is transmitting directly forward, it is known as broadside. As the voltage sources for the elements are shifted, or steered, away from this direction, it is known as beam steering. When the beam is steered 90 degrees from the broadside direction, the beams of one column of elements are actually transmitting into the next column of elements, effectively transmitting out of a one-dimensional line array. This is known as endfire and has significant negative effects that are often desired to be avoided.

Current scientific literature provides methods for addressing and minimizing these problems on conventional two-dimensional planar phased array structures, but there has been no

investigation into solving these issues with three-dimensional geometries. In this thesis, the design and performance of a cubic phased array is presented. The cubic phased array eliminates endfire effects entirely because each face of the cube is identical; when transmitting at 90 degrees off broadside, the transmit area of the cube is identical to that of the broadside direction. The cubic geometry also dramatically improves the direction-finding process. By introducing a third dimension, the mathematics can more precisely determine the direction from which the object or the signal is coming, thus dramatically decreasing the ambiguity simply as a function of geometry.

Both beam steering and direction of arrival simulations are performed in MATLAB and 4nec2 simulation environments for cubic phased arrays of various sizes and at different frequencies. This demonstrates that the endfire effects are eliminated and direction of arrival performance is dramatically improved. These findings are expected to lead to new designs of high fidelity three-dimensional phased arrays for a multitude of applications, especially for space applications where the three-dimensional geometry has the added benefit of resolving the requirements for compensation for the tumbling motion of objects in orbit.

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Table 4.13 Simulation H-Plane Pattern Parameters for 4nec2 Plots in Figure 4.8 at $(\phi, \theta) = (45,0)$

Table 4.14 Simulation Pattern Parameters for MATLAB Plots in Figure 4.10 $(\phi, \theta) = (0,0)$

Table 4.15 Simulation Pattern Parameters for MATLAB Plots in Figure 4.10 $(\phi, \theta) = (90,0)$

Table 4.16 Simulation Pattern Parameters for MATLAB Plots in Figure 4.10 (ϕ, θ) = (45,0)

Table 4.17 Simulation Pattern Parameters for 4nec2 Plots in Figure 4.11 (ϕ, θ) = (0,0)

Table 4.18 Simulation Pattern Parameters for 4nec2 Plots in Figure 4.11 (ϕ, θ) = (90,0)

Table 4.19 Simulation Pattern Parameters for 4nec2 Plots in Figure 4.11 (ϕ, θ) = (45,0)

Table 5.1 Simulation Pattern Parameters for Ideal Plots in Figure 5.1

Table 5.2 Simulation Pattern Parameters for Ideal Plots in Figure 5.2

Table 5.3 Simulation Pattern Parameters for Ideal Plots in Figure 5.3

Table 5.4 Simulation Pattern Parameters for Ideal Plots in Figure 5.4

Table A.1 Simulation Pattern Parameters for Ideal Plots in Figure A.1

Table A.2 Simulation Pattern Parameters for 4nec2 Plots in Figure A.2

Table A.3 Simulation Pattern Parameters for the Ideal MATLAB Plots in Figure A.3

Table A.4 Simulation Pattern Parameters for the Mutual Coupling 4nec2 Plots in Figure A.4

Table A.5 Simulation Pattern Parameters for Ideal MATLAB Plots in Figure A.5

Table A.6 Simulation Pattern Parameters for Mutual Coupling 4nec2 Plots in Figure A.6

Table A.7 Simulation Pattern Parameters for the Ideal MATLAB Plots in Figure A.7

Table A.8 Simulation Pattern Parameters for the Mutual Coupling 4nec2 Plots in Figure A.8

Table A.9 Simulation Pattern Parameters for plots in Figure A.9

Table A.10 Simulation Pattern Parameters for plots in Figure A.10

Table A.11 Simulation Pattern Parameters for plots in Figure A.11

Table A.12 Simulation Pattern Parameters plots in Figure A.12

Table A.13 Simulation Pattern Parameters for 4nec2 Plots in Figure A.13

Table A.14 Simulation Pattern Parameters for 4nec2 Plots in Figure A.14

Table A.15 Simulation Pattern Parameters for 4nec2 Plots in Figure A.15

Table A.16 Simulation Pattern Parameters for 4nec2 Plots in Figure A.16

Table A.17 Simulation Pattern Parameters for MATLAB Plots in Figure A.17

Table A.18 Simulation Pattern Parameters for 4nec2 Plots in Figure A.18

Table A.19 Simulation Pattern Parameters for MATLAB Plots in Figure A.19

Table A.20 Simulation Pattern Parameters for 4nec2 Plots in Figure A.20

Table A.21 Simulation Pattern Parameters for MATLAB Plots in Figure A.21

Table A.22 Simulation Pattern Parameters for 4nec2 Plots in Figure A.22

Table A.23 Simulation Pattern Parameters for MATLAB Plots in Figure A.23

Table A.24 Simulation Pattern Parameters for 4nec2 Plots in Figure A.24

Chapter 1 – Motivation and Introduction

This chapter presents the motivation for the research presented in thesis. Some background information providing the context of the research is provided in the first section. The second section identifies the problem statement for the research. In the third section, the overall approach for how the research will be accomplished is presented. The final section of the chapter provides the outline of the thesis.

1.1 Motivation of the Research

Space observatories provide the scientific community with a plethora of information about astronomical bodies and atmospheric effects. One such observatory of national recognition is the Arecibo Observatory. The Arecibo Observatory was built in 1963 by Cornell University and the National Science Foundation and was the largest single aperture radio telescope in the world until 2016 [1]-[2]. Scientists utilized the Arecibo Observatory to perform radio astronomy, detect near-earth objects, study pulsars, monitor space weather events, and perform atmospheric studies [3]-[4]. The Arecibo Observatory featured a 305 m diameter spherical reflector dish. A collection of sensors could be moved around the stationary dish, fixing the focal point of the radio telescope to specific regions of the sky, allowing the Arecibo telescope to perform scans and tracking activities of astronomical bodies of interest [1], [5]. The frequency range of the Arecibo telescope was between 50 MHz to 10 GHz [6]-[7].

Unfortunately, the Arecibo facility was damaged in 2017 during Hurricane Maria. Over time, the damage became worse until the instrument collapsed in December of 2020. The loss of the Arecibo Observatory has had an impact on the scientific community, and demand for a replacement has been high [8].

While an incredibly useful radio telescope, Arecibo was more akin to a traditional antenna. Another nationally recognized observatory that utilizes the advantages of electronically steerable phased arrays is the Long Wavelength Array (LWA). LWA, located in New Mexico, utilizes a phased array approach to perform radio astronomy and other measurements. LWA consists of two separate array stations, with more planned for construction, that operate independently as radio telescopes, but can also be operated together as a very large collection area for lower frequency operations. The current operational frequency range of LWA is 10 MHz to 88 MHz [9]-[10].



Figure 1.1 The (a) Arecibo Observatory radio telescope and (b) Long Wavelength Array. Figures taken from [9] and [11].

A phased array implementation of Arecibo will still face a major limitation in performance. In fact, every ground-based radio telescope faces the same challenge; electromagnetic propagation through the ionosphere becomes significantly more challenging at frequencies approaching and below HF band [12]. Rather than attempting to model trans ionospheric propagation, the concept in this thesis is to place the array in orbit. In this way, the ionosphere no longer acts as an obstacle for radio telescope operations. This is similar to the idea behind the Hubble telescope and the obstacles to optical imagery presented by atmospheric effects [13]. The motivation for this thesis is utilize a similar approach to that used in LWA to create an in-orbit electrically steerable phased array that can act as a replacement for the Arecibo Observatory.

There is an additional benefit to putting the array into orbit. The ionosphere no longer impacts the operation of the array as a radio telescope for radio astronomy. However, the array could also be directed to transmit into the ionosphere from above rather than below like ground-based radio telescopes. This operation paves the way for further advances in atmospheric studies and for calibration purposes for existing ground-based radio telescopes.

1.2 Problem Statement for the Thesis

An electrically steerable phased array with the same aperture size of the Arecibo radio telescope would be excessively large. Launching such an array into orbit would be unfeasible. However, advancements in in-space robotic assembly are leading to realizable techniques for launching smaller components into orbit and integrating them to form larger overall structures [14]-[15].

Objects in orbit also experience a tumbling motion as they move around the earth [18]. A conventional, planar array undergoing this kind of motion requires attitude control to keep the array directed where desired [17]. Attitude control, however, may be challenging to implement on large structures. Therefore, a three-dimensional array design is proposed as an alternative to account for the tumbling motion of the in-orbit environment. As the array tumbles, electrical steering can be applied to the array elements to maintain operation in the desired angular direction because one face of the array will always be facing in the desired direction.

1.3 Approach

The ratio of the highest to lowest operational frequencies of the Arecibo radio telescope is 200:1. Rather than designing an array that can replicate this full bandwidth, this thesis focuses on the design and simulation performance of a resonant structure array at the HF band with the ability to tune to different frequencies within the HF band. HF is selected because the ionosphere

layer of the atmosphere is usually studied using signals of HF frequencies, [16] but the concept can be extended to any frequency band.

1.4 Thesis Outline

The outline of this thesis is as follows. Chapter 2 explores the theoretical foundation of antenna theory, phased arrays, and the HF frequency band. Chapter 3 then describes the geometry considerations, the design of the three-dimensional in-space assembled phased array. In Chapter 4, simulation tests are performed to characterize the array design performance. Chapter 5 discusses the results from the simulations in Chapter 4 and extends the principles to a full-scale model of the proposed array. The thesis then concludes in Chapter 6 with a summary and a discussion of future work that could be done on this project. An appendix is added at the end of the thesis which provides further simulation results for a more comprehensive view of the array performance, and practical design considerations for a prototype of the array design for laboratory measurements.

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Chapter 2 – Background Information and Literature Review

This chapter covers the background information necessary for the principles implemented in this thesis. The literature review begins with basic concepts of electromagnetic wave propagation. The key principles of antenna theory relevant to this thesis are then introduced. A discussion of phased array theory and how it differs from general antenna theory then follows. Different challenges encountered in array theory as well as the theory behind applications such as beam steering and direction of arrival are presented. The literature review then shifts to the challenges faced by HF frequency transmission and propagation.

2.1 Basic Principles and Terminology

The high-level concepts of electromagnetic propagation are introduced in this section. Understanding the terminology and concepts in this section paves the way for an understanding of antenna theory and the analysis presented in the following sections.

2.1.1 Coordinate Systems

A number of different coordinate systems that can be used in antenna and phased array systems. In most coordinate systems for antenna applications, spherical coordinates are used with the geometric center of the antenna at the origin [1]-[4]. The coordinate system used for this thesis is shown in Figure 2.1 [5]. The x-axis is defined as the direction normal to the antenna centered at the origin. Starting at the x-axis and rotating counterclockwise to the positive y-axis is known as ϕ or the azimuth angle. The upward angle from the xy plane is known as θ or elevation. This is not the standard system used in engineering and physics, but it is consistent with the simulation software used for this thesis. θ and ϕ are used interchangeably with elevation and azimuth respectively throughout this thesis.

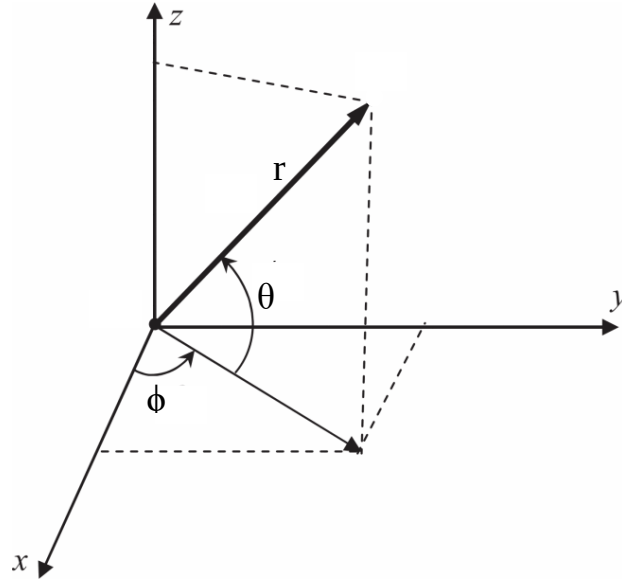


Figure 2.1 Coordinate System for azimuth (ϕ) and elevation (θ) used for this thesis. Image was adapted from [5] with (ϕ) and (θ) swapped.

2.1.2 Electromagnetic Wave Basics

Electromagnetic waves consist of an electric field and a magnetic field that propagate together in space and time [6]. The electric and magnetic fields are orthogonal to each other and propagate in the same direction, known as the propagation direction [7]. An example of an electromagnetic propagating wave is shown in Figure 2. This wave can be viewed either as propagating over space at a given time or as propagating over time at a given point in space.

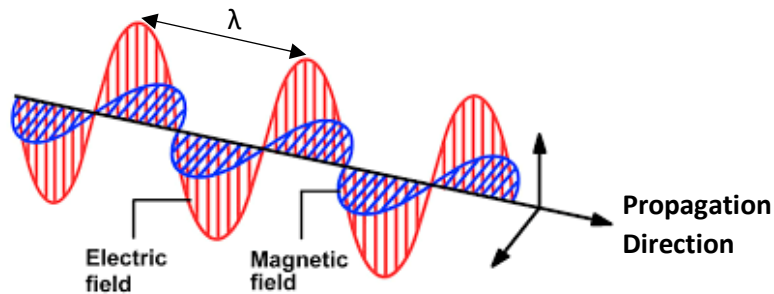


Figure 2.2 Electromagnetic wave propagation. Image taken from [8].

Electromagnetic waves can be characterized their properties: wavelength, frequency, phase, and polarization. The wavelength λ is defined by the distance between two consecutive maxima peaks. Frequency is inversely proportional to wavelength and is defined as a v/λ where v is the propagation speed of the electromagnetic wave [9]. Typically, v is taken to be the speed of light, c [7]. Phase can be thought of as the angular offset of the sinusoidal wave [10]. Phase is directly related to applying a time delay in the propagation of the electromagnetic wave as seen in equation (100) or equation (104).

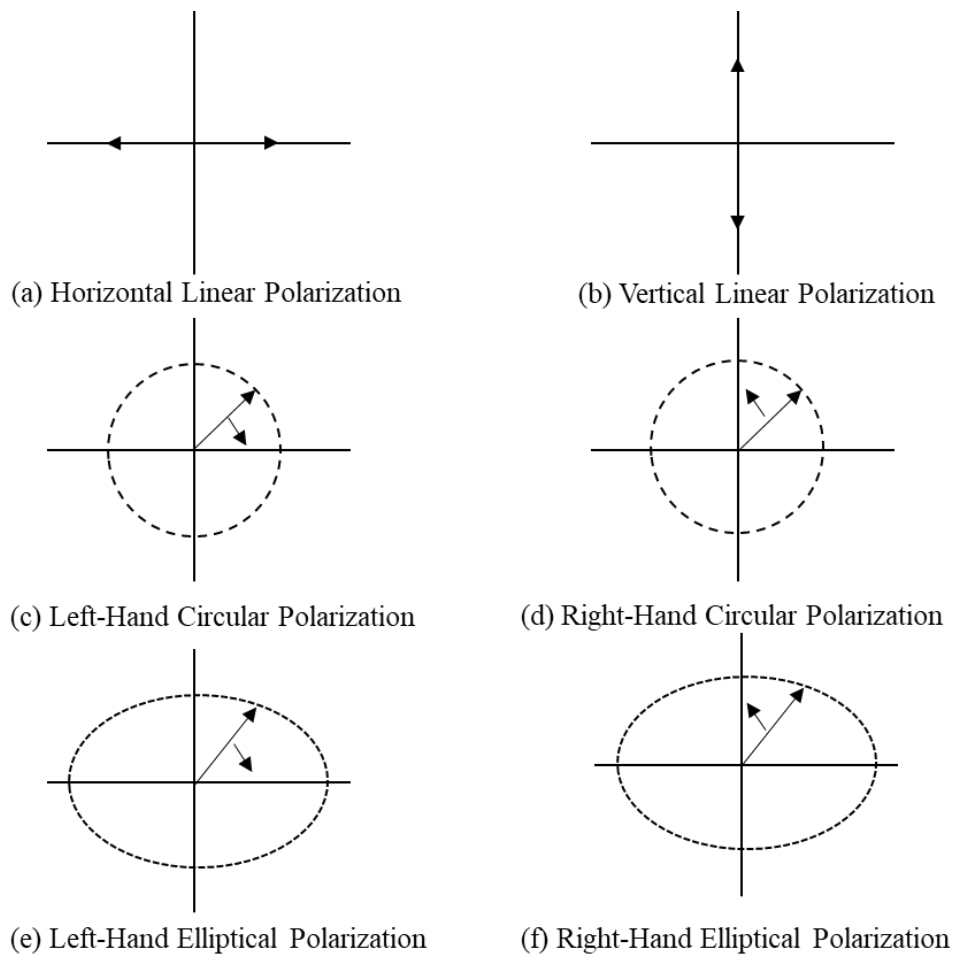


Figure 2.3 Polarizations of electromagnetic waves with propagation direction out of the page. Image reproduced from [11].

The polarizations of the electric and magnetic fields are orthogonal to each other and determined by the oscillation of the fields at a fixed point in the observation area over time.

These polarizations can be linear, circular, or elliptical [7], [11]-[12].

The different polarizations are shown in Figure 2.3. Note that there are left- and right-hand polarizations for both circular and elliptical polarizations. This is determined using the left-hand rule. If the electromagnetic wave is traveling towards the observer and the field vector rotates clockwise, it is left-hand polarized; if the field vector rotates counterclockwise, it is right-hand polarized [11].

While each of the three polarizations is considered separately, it is important to note that all electromagnetic polarizations are elliptical. The circular polarization is the special case where the semi-major and semi-minor axes of the ellipse are equal. Linear polarization occurs when the ellipse collapses into a line [11]-[12]. It is also worth mentioning that there are electromagnetic waves that have no polarization. Because antennas cannot generate these electromagnetic waves, they are disregarded [11].

2.1.3 Plane Wave and Phase Front

When an electromagnetic wave is transmitted from a source, it can be thought of as a spherical wave propagating radially outward [9], [11]. As the electromagnetic wave propagates over large distances radius r gets larger. If a detector is placed far enough away from the source, the curvature of the electromagnetic wave in the localized spatial region of the detector will be small such that the wave can be treated as a plane [9], [11]. This is known as a plane wave. The electromagnetic analysis for antenna theory predominately assumes plane wave behavior [7], [11]. These planar electromagnetic regions near the detector are also considered in approximately constant phase and are also known as a phase fronts [11]-[12].

2.1.4 Near Field vs Far Field

In antenna theory there are two basic regions of operation for electromagnetic waves. These are known as the near field region and the far field region. The far field region is of most interest and is generally used for analysis. In the far field region, the antenna radiation pattern discussed in Section 2.2.3 becomes independent of distance from the source [11]. The far field distance r_{ff} for an element of length L is [4], [9], [11], [13]-[15]

$$r_{ff} = \frac{2L^2}{\lambda} \quad (1)$$

The far field region then exists for $r \geq r_{ff}$. Three conditions must also be met for distance r to be considered the far field region. These three conditions are shown mathematically as [11]

$$r > \frac{2L^2}{\lambda} \quad (2)$$

$$r \gg L \quad (3)$$

$$r \gg \lambda \quad (4)$$

Generally, equation

(1) is sufficient enough for determining the far field region for antennas operating in the UHF region, defined in Section 2.4, or above. For lower frequencies, however, the far field distance may be greater than that defined in equation

(1) to ensure that all three conditions are met [11].

The near field is the space surrounding the antenna before reaching the far field distance. The near field is broken into two regions. The reactive near field region is the region closest to the antenna. The reactive field caused by the reactive component of antenna element impedance dominates the radiated fields of the antenna. This region exists to distance $r \leq 0.62\sqrt{L^3/\lambda}$ [1], [9], [11], [15]. Between the far field and the near field reactive region is the near field radiating

region. Within the near field radiating region, the radiated fields dominate the reactive fields, but vary with respect to distance. This is sometimes referred to as the Fresnel region [9], [11], [13]-[16]. A summary of the field regions is provided in Table 2.1 [12].

Table 2.1 Near Field and Far Field Regions [1], [11], [13]-[15]

Region	Distance r from antenna
Reactive near field region	$0 \leq r \leq 0.62\sqrt{L^3/\lambda}$
Radiating near field region	$0.62\sqrt{L^3/\lambda} \leq r \leq \frac{2L^2}{\lambda}$
Far field region	$\frac{2L^2}{\lambda} \leq r \leq \infty$

2.2 Antenna Theory

This section introduces the basics of antenna theory from Maxwell's equations to radiation patterns. Much of the material in this section was primarily taken from Stutzman [11] and the sequence of topics largely follows the same sequence as [11].

2.2.1 Electromagnetic Radiation

Antenna theory and performance metrics are based upon the radiation of electromagnetic fields from current distributions on conductive elements. This section provides the understanding of electromagnetic radiation for antenna theory based on Maxwell's equations.

2.2.1.1 Maxwell's Equations

Maxwell's equations are a set of well-known equations in engineering and physics that describe the propagation and interaction of electromagnetic fields. The electromagnetic fields are assumed to be time-harmonic fields, meaning that they vary sinusoidally over time at angular frequency ω [6], [11]-[12]. This assumption is reasonable because in most cases of interest, the bandwidth of the signal is small enough that the spread of frequencies around the carrier formed

by modulation techniques can be ignored [11]. Phasor fields can be substituted into the time domain versions of Maxwell's equations to derive the following phasor form [11]-[12]

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{M}' \quad (5)$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \mathbf{J} \quad (6)$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon} \quad (7)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (8)$$

where $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{J}$ represent the electric field, magnetic field, current density complex phasor vectors respectively and ρ is charge density complex phasor. $\mathbf{M}'$ represents a fictitious magnetic current density that can be useful as a substitute for complicated electric fields in calculations. ϵ and μ denote the permittivity and permeability of the propagation medium. It should be noted that the time derivatives have been replaced with the $j\omega$ factor for the phasor form of the equations.

A fifth equation that is often taken into consideration with Maxwell's equations is the continuity equation given by [11]

$$\nabla \cdot \mathbf{J} = -j\omega\rho \quad (9)$$

The boundary conditions for the time-harmonic forms of Maxwell's equations between two homogeneous media m_1 and m_2 are given by [11]-[12]

$$\hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s \quad (10)$$

$$(\mathbf{E}_2 - \mathbf{E}_1) \times \hat{\mathbf{n}} = \mathbf{M}_s \quad (11)$$

where $\mathbf{J}_s$ and $\mathbf{M}_s$ represent the surface electric and magnetic currents on the boundary between m_1 and m_2 . Unless an equivalent magnetic sheet is used in this condition, $\mathbf{M}_s = 0$. $\hat{\mathbf{n}}$ represents the unit vector normal to the boundary surface that is directed from m_1 into m_2 . The cross product with $\hat{\mathbf{n}}$ forms components that are tangential to the boundary surface [1], [11]-[12]

$$H_{\tan 2} = H_{\tan 1} + J_s \quad (12)$$

$$E_{\tan 2} = E_{\tan 1} + M_s \quad (13)$$

If conductor 1 is a perfect conductor, the boundary conditions simplify to [11]-[12]

$$H_{\tan} = J_s \quad (14)$$

$$E_{\tan} = 0 \quad (15)$$

These equations apply for the fields at every point along the boundary.

Another important concept that can be derived from Maxwell's equations and the continuity equation is the conservation of power equation, also known as Poynting's theorem [6], [11]-[12]. This theorem states that the complex power P_s delivered by a volume v enclosed by a surface s equals the sum of the power P_{out} flowing out of s , the time-averaged power P_{avg} dissipated in v , and the time-average stored power in v . Stated mathematically, the conservation of power is [11]-[12]

$$P_s = P_{\text{out}} + P_{\text{avg}} + j2\omega(W_{\text{avg}_M} - W_{\text{avg}_E}) \quad (16)$$

The complex power that flows out through closed surface s is given by [1], [11]-[12]

$$P_{\text{out}} = \frac{1}{2} \oint\oint_s \mathbf{E} \times \mathbf{H}^* \cdot d\mathbf{s} \hat{\mathbf{n}} \quad (17)$$

where $\mathbf{H}^*$ represents the complex conjugate of the magnetic field vector.

The inside of the integrand of the above equation is defined as the Poynting Vector [6], [11]-[12]. The Poynting Vector represents the complex power density at a given point. Therefore, the total complex power flowing out through any surface s can be determined by taking the integral of the Poynting Vector over the surface. Of particular interest is the real power. The real power flowing out of any surface s is given by [11]-[12]

$$P_{\text{out}} = \text{Re} \left(\iint_s \mathbf{S} \cdot d\mathbf{s} \right) = \frac{1}{2} \text{Re} \left(\iint_s \mathbf{E} \times \mathbf{H}^* \cdot d\mathbf{s} \hat{\mathbf{n}} \right) \quad (18)$$

2.2.1.2 Maxwell's Equations and Radiation Fields

For antenna theory, the electromagnetic fields $\mathbf{E}$ and $\mathbf{H}$ need to be determined. In antennas, these fields are caused by an excitation current along the conductive element. Therefore, a method is needed for determining $\mathbf{E}$ and $\mathbf{H}$ from a given $\mathbf{J}$. This is accomplished using scalar potentials and vector potentials in Maxwell's equations.

From equation

(8) the divergence of $\mathbf{H} = 0$ indicating that $\mathbf{H}$ only has curl. For this reason, $\mathbf{H}$ is known as a solenoidal field and can be represented by [1], [11]-[12]

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \quad (19)$$

where $\mathbf{A}$ represents the magnetic vector potential. By substituting equation (19) into equation (5) the electric scalar potential can be found as [11]-[12]

$$\mathbf{E} + j\omega\mathbf{A} = -\nabla\Phi \quad (20)$$

where Φ is the electric scalar potential.

$\mathbf{E}$ and $\mathbf{H}$ can be expressed in terms of the scalar and vector potentials by substituting equations (19) and (20) into equation (6). Simplifying using a Lorentz condition results in the following equation known as the vector wave equation [11]-[12].

$$\nabla^2 \mathbf{A} + \omega^2 \mu \epsilon \mathbf{A} = -\mu \mathbf{J} \quad (21)$$

It should be noted that the scalar and vector potentials have been decoupled and the vector wave equation is only in terms of the magnetic vector potential $\mathbf{A}$. Assuming that $\mathbf{J}$ is known, the

vector wave equation can be solved for $\mathbf{A}$ by decomposition into the rectangular coordinates and solving for a point source. This solution takes the form of equation (22) [7], [11].

$$\nabla^2 \psi + \beta^2 \psi = \delta(x)\delta(y)\delta(z) \quad (22)$$

where ψ represents the response to a point source located at the origin and δ is Dirac delta function. β represents the phase constant of the plane wave and is defined as [6]-[7], [11]-[12]

$$\beta = \omega \sqrt{\mu \epsilon} = \frac{2\pi}{\lambda} \quad (23)$$

If the point source is assumed to be z-directed [11]

$$\psi = A_z \quad (24)$$

This implies that the point source is 0 everywhere except at the origin. Applying this simplification to equation (22) yields [11]

$$\nabla^2 + \beta^2 \psi = 0 \quad (25)$$

everywhere except for the origin.

This equation is known as the complex scalar wave equation, or the Helmholtz equation [7], [11]-[12]. The Laplacian is applied in spherical coordinates to maintain consistency with the coordinate frame from Section 2.1.1 This means that ψ varies only with radius r and equation (25) has two solutions: $e^{-j\beta r}/r$ and $e^{+j\beta r}/r$ which represent waves propagating radially outward and inward respectively. The only feasible solution is for waves propagating outwardly from the point source. Therefore, the solution for the Helmholtz equation is [11]

$$\psi = \frac{e^{-j\beta R}}{4\pi R} \quad (26)$$

where R represents the distance between the arbitrary source location and the observation point.

If the point source is z-directed, then current density, and therefore the vector potential, will also be z-directed. Furthermore, if the current density is treated as an accumulation of point

sources, then the vector potential can be thought of as the sum of the individual responses for each point source in a given source volume v' [11]-[12]. This is expressed mathematically as [11]-[12]

$$\mathbf{A} = \iiint_{v'} \mu \mathbf{J} \frac{e^{-j\beta R}}{4\pi R} dv' \quad (27)$$

where $\mathbf{R}$ is the vector $\mathbf{r}_p - \mathbf{r}'$, $\mathbf{r}_p$ is the vector from the origin to a field point P, and $\mathbf{r}'$ is the vector from the origin to the source point. A pictorial representation of the vector representations is shown in Figure 2.4.

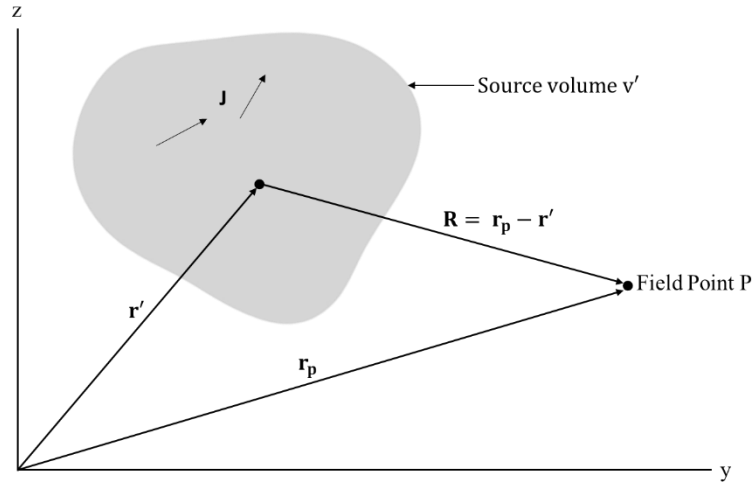


Figure 2.4 Vectors for radiation problems with Maxwell's Equations. Figure recreated from [11]-[12].

The solution for $\mathbf{H}$ can be solved by substituting equation (27) above into equation (19). The solution for $\mathbf{E}$, however, can be found by substituting equation (27) into the vector wave equation (21), which gives [11]-[12]

$$\mathbf{E} = -j\omega \mathbf{A} - j \frac{\nabla(\nabla \cdot \mathbf{A})}{\omega \mu \epsilon} \quad (28)$$

More conveniently, equation (21) can be substituted into equation (19), which gives [11]-[12]

$$\mathbf{E} = \frac{1}{j\omega\epsilon}(\nabla \times \mathbf{H} - \mathbf{J}) \quad (29)$$

Another approach for solving the vector and scalar potentials is to solve the following scalar wave equation [11]-[12]

$$\nabla^2 \Phi + \omega^2 \mu \epsilon \Phi = -\frac{\rho}{\epsilon} \quad (30)$$

In this approach, both the scalar and vector wave equations need to be solved. Note that $\mathbf{J}$ is related to ρ by the continuity equation (5). In this approach $\mathbf{E}$ is determined by equation (20).

2.2.2 Element Types

While there are many types of antenna elements that can be used [9], [11]-[12], [17]-[19] this thesis focuses on line sources, specifically dipoles. There are three dipole models that are used in practice: the short dipole, the half-wave dipole, and the long dipole [11], [17]-[19]. It should be noted that far field approximations are used for all calculations in this section. All dipole elements are considered centered at the origin with the power supply connected to the geometric center. This is known as a center fed element [11].

2.2.2.1 Line Sources

The easiest way to model an antenna element is as a z-directed line source. A line source is a finite thin wire antenna and can be thought of as a line current. The vector potential from equation (27) becomes one-dimensional in the z-axis and takes the form [1], [11]

$$\mathbf{A} = \mu \int I(z') \frac{e^{-j\beta R}}{4\pi R} dz' \quad (31)$$

Due to the symmetry of the source, an observer can move around the source with respect to ϕ , assuming r and z are kept constant. From the observer's perspective, the appearance of the source does not change. Therefore, the radiation fields do not vary with ϕ , and ϕ can be considered a fixed value. ϕ is chosen as 0° for simplicity, fixing the observation to the yz plane.

The scenario is visualized in Figure 2.5. From Figure 2.5 [11]

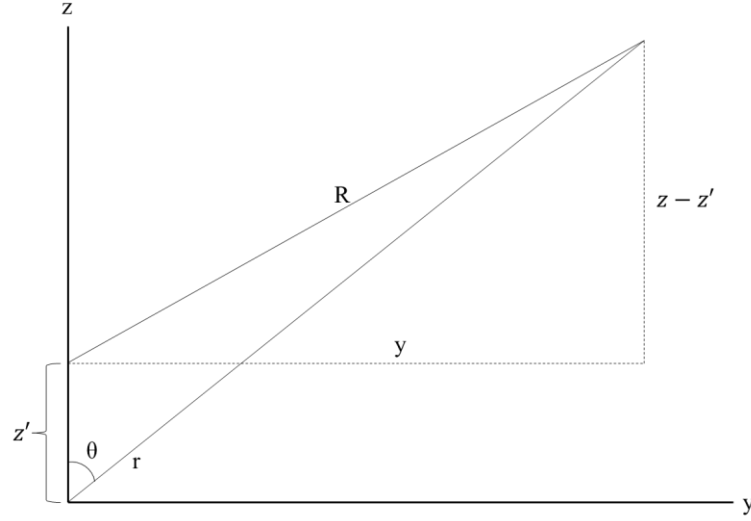


Figure 2.5 Geometry for far field calculations of a z-directed line source. Figure adapted from [11].

$$r^2 = y^2 + z^2 \quad (32)$$

$$z = r \cos(\theta) \quad (33)$$

$$y = r \sin(\theta) \quad (34)$$

Applying the $\mathbf{R}$ definitions from equation (27) and Figure 2.4, $\mathbf{r} = y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ and $\mathbf{r}' = z'\hat{\mathbf{z}}$.

Therefore, $\mathbf{R} = y\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}$. The magnitude of $\mathbf{R}$ is then [11]

$$R = \sqrt{r^2 + (-2r \cos(\theta)z' + z'^2)} \quad (35)$$

Applying a binomial expansion for R gives [11]

$$R = r - z' \cos(\theta) + \frac{z'^2 \sin^2(\theta)}{2r} + \frac{z'^3 \sin^3(\theta)}{2r^2} \dots \quad (36)$$

Since $z' \ll r$, the second and following terms of R rapidly decrease to 0. Therefore [1], [11]

$$R \approx r - z' \cos(\theta) \quad (37)$$

The denominator of equation (31) only impacts the amplitude of the vector potential, so R can be further simplified as $R \approx r$ because r is very large compared to the length of the line source. However, in the phase term, path length differences can be comparable sizes to the

wavelength and a more precise approximation is necessary [1], [11]. The substitution into equation (31) for the vector potential gives [1], [11]

$$\mathbf{A}_z = \mu \frac{e^{-j\beta r}}{4\pi r} \int I(z') e^{-j\beta z' \cos(\theta)} dz' \quad (38)$$

Substituting $\mathbf{A}_z$ into equations (19) and (29) gives $\mathbf{H}$ and $\mathbf{E}$ [1], [11]

$$\mathbf{H} = \hat{\Phi} \frac{j\beta}{\mu} \sin(\theta) \mu \frac{e^{-j\beta r}}{4\pi r} \int I(z') e^{-j\beta z' \cos(\theta)} dz' = \hat{\Phi} \frac{j\beta}{\mu} \sin(\theta) A_z \quad (39)$$

$$\mathbf{E} = -j\omega A_\theta \hat{\theta} = -j\omega \sin(\theta) A_z \hat{\theta} \quad (40)$$

Equations (39) and (40) define the magnetic and electric fields in the far field region of the line source antenna. They are known as the radiation fields of the line source and are denoted as E_θ and H_ϕ [11]-[12], [20].

2.2.2.2 Ideal Dipole Elements

The ideal dipole is functionally an infinitesimally small line source with a uniform current distribution. This is represented as a dipole with length $\Delta z \ll \lambda$ and a current that is constant in both magnitude and phase along the entire length Δz [11]. This is shown pictorially in Figure 2.7.

A z-directed element of length Δz centered at the origin excited with current of constant amplitude I is used for the derivation in this section. The vector potential for this element defined in equation (27) is simplified to a one-dimensional integral of the form [11], [21]

$$\mathbf{A} = \hat{z} \mu I \int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} \frac{e^{-j\beta R}}{4\pi R} dz \quad (41)$$

Because Δz is very small, R and r are approximately equal, as seen in Figure 2.6. This allows the simplification of equation (41) to [11], [21]

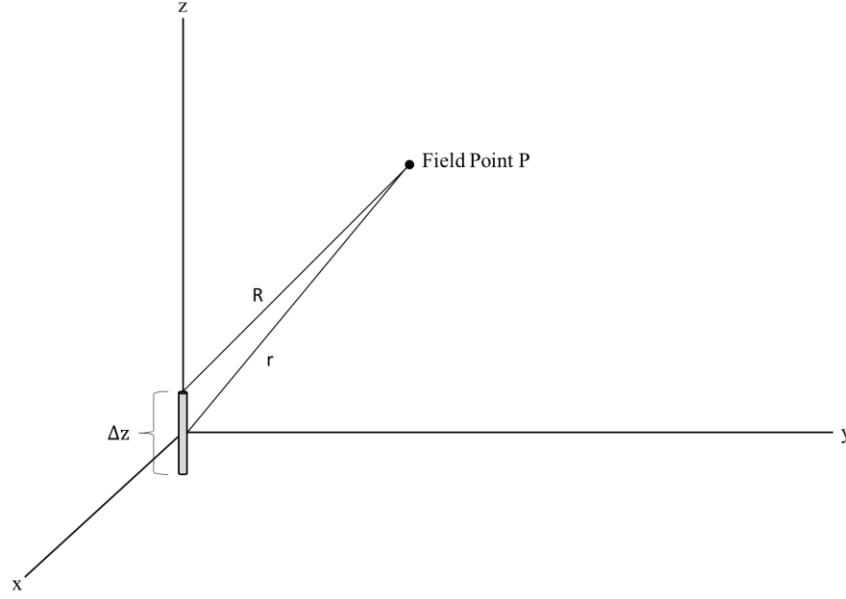


Figure 2.6 Geometry used for field calculations of the ideal dipole. Figure adapted from [11].

$$\mathbf{A} = \hat{\mathbf{z}} \frac{\mu I e^{-j\beta r}}{4\pi r} \Delta z \quad (42)$$

Substituting $\mathbf{A}$ from equation (42) into equations (19) and (29) and simplifying in spherical coordinates gives the radiation fields $\mathbf{H}_\phi$ and $\mathbf{E}_\theta$ as [11], [20]-[21]

$$\mathbf{H}_\phi = \frac{I\Delta z}{4\pi} \left[\frac{j\beta}{r} + \frac{1}{r^2} \right] e^{-j\beta r} \sin(\theta) \hat{\boldsymbol{\Phi}} \quad (43)$$

$$\begin{aligned} \mathbf{E}_\theta = & \frac{I\Delta z}{4\pi} \left[\frac{j\omega\mu}{r} + \sqrt{\frac{\mu}{\epsilon}} \frac{1}{r^2} + \frac{1}{j\omega\epsilon r^3} \right] e^{-j\beta r} \sin(\theta) \hat{\boldsymbol{\theta}} \\ & + \frac{I\Delta z}{2\pi} \left[\sqrt{\frac{\mu}{\epsilon}} \frac{1}{r^2} + \frac{1}{j\omega\epsilon r^3} \right] e^{-j\beta r} \cos(\theta) \hat{\mathbf{r}} \end{aligned} \quad (44)$$

If the medium surrounding the dipole is free space, then the above equations simplify to [11], [20]-[21]

$$\mathbf{H}_\phi = \frac{I\Delta z}{4\pi} j\beta \left[1 + \frac{1}{j\beta r} \right] \frac{e^{-j\beta r}}{r} \sin(\theta) \hat{\boldsymbol{\Phi}} \quad (45)$$

$$\mathbf{E}_\theta = \frac{I\Delta z}{4\pi} j\omega\mu \left[1 + \frac{1}{j\beta r} - \frac{1}{(\beta r)^2} \right] \frac{e^{-j\beta r}}{r} \sin(\theta) \hat{\theta} + \frac{I\Delta z}{2\pi} \eta \left[\frac{1}{r} - j\frac{1}{\beta r^2} \right] \frac{e^{-j\beta r}}{r} \cos(\theta) \hat{r} \quad (46)$$

In the far field region, $r \gg \lambda$, or $\beta \gg 1$, and every term with a β in the denominator is much less than 1. In this case, the equations (45) and (46) reduce to [11]-[12], [21]

$$\mathbf{H}_\phi = \frac{I\Delta z}{4\pi} j\beta \frac{e^{-j\beta r}}{r} \sin(\theta) \hat{\phi} \quad (47)$$

$$\mathbf{E}_\theta = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-j\beta r}}{r} \sin(\theta) \hat{\theta} \quad (48)$$

$$\frac{E_\theta}{H_\phi} = \sqrt{\frac{\mu}{\epsilon}} = \eta \quad (49)$$

It should be noted that the radiation fields vary inversely with $1/r$. The ratio of these fields is defined by the variable η seen in equations (46) and (49) which is equal to the intrinsic impedance of the medium, a property of plane waves [11]-[12], [21].

Substituting $\mathbf{E}_\theta$ and $\mathbf{H}_\phi$ into the Poynting Vector integrating as defined in equation (18) gives the real power flowing out of a sphere with radius r surrounding the ideal dipole [11], [21]

$$\mathbf{P}_{\text{out}} = \iint \mathbf{S} \cdot d\mathbf{s} = \frac{1}{2} \left(\frac{I\Delta z}{4\pi} \right)^2 \omega\mu\beta \int_0^{2\pi} d\phi \int_0^\pi \sin^3(\theta) d\theta = \frac{\omega\mu\beta}{12\pi} (I\Delta z)^2 \quad (50)$$

In contrast to the radiation fields, the output power does not vary with radius r of the sphere. This indicates that the power is the same regardless of the size of the sphere surrounding the dipole. This is known as the far field radiated power. As the distance is taken closer to the dipole the simplifications from equations (43) and (44) do not apply. Another set of equations known as the near field equations must be used [11], [20].

The input impedance for an ideal dipole is given by equation (51) [11], [22]-[23].

$$\mathbf{Z}_A = \mathbf{R}_A + j\mathbf{X}_A \quad (51)$$

R_A represents the radiated power losses caused by radiative resistance R_r and the heating of the antenna caused by ohmic losses R_{ohmic} . The input reactance X_A represents the stored power in the near field region. The input resistance can be solved with Ohm's Law as [11], [22]-[23]

$$R_A = \frac{2P_{in}}{|I_A|^2} \quad (52)$$

where $|I_A|$ represents the magnitude of the time current at the input of the antenna element and P_{in} represents the input power of the dipole which is defined as the sum of the radiated power and the ohmic power losses [11], [23]

$$P = P_{in} + P_{ohmic} \quad (53)$$

The radiated resistance is found by substituting the radiated power P_{out} from equation (18) into equation (52). The radiated resistance for the ideal dipole is [11]

$$R_r = 80 \pi^2 \left(\frac{\Delta z}{\lambda} \right)^2 \Omega \quad (54)$$

Because $\Delta z \ll \lambda$, $R_r \ll 1$. The ohmic losses for an ideal dipole of length Δz and radius a can be approximated as [11], [23]

$$R_{ohmic} \approx \frac{\Delta z}{2\pi a} \sqrt{\frac{\omega\mu}{2\sigma}} \Omega \quad (55)$$

where σ is the conductivity of the dipole element. Because Δz is infinitesimally small, the input reactance X_A for an ideal dipole is infinite [24]. This is shown in equation (59) in the next section.

2.2.2.3 Short Dipole Elements

Ideal dipoles are non-realizable elements. However, a very small line source could be fabricated that is much shorter than a wavelength. In this way, the same $\Delta z \ll \lambda$ condition from the ideal dipole can be met. This is known as a short dipole and can also be described using Figure 2.6. Figure 2.6 shows the electrical length of the short dipole as half of its physical length.

Unlike the ideal dipole, the current distribution for the short dipole is not uniform but is instead triangular and is given by [11], [25]

$$\mathbf{I}(\mathbf{z}) = \mathbf{I}_A \left(1 - \frac{2|\mathbf{z}|}{\Delta z} \right), \quad |\mathbf{z}| \leq \frac{\Delta z}{2} \quad (56)$$

This is shown in Figure 2.7. However, if proper terminal loading is applied, the charge, and therefore the current, reduces to 0 at the ends of the dipole. This restricts the radiation fields to the peak of the current distribution which is approximately uniform. Because the short dipole meets these conditions, the ideal dipole is approximated. Therefore, the radiation fields of the short dipole are those of the ideal dipole [11].

The radiative and ohmic resistances and input reactance for a short dipole of length Δz and radius a are given by [11], [23]

$$\mathbf{R}_r = 20 \pi^2 \left(\frac{\Delta z}{\lambda} \right)^2 \Omega \quad (57)$$

$$\mathbf{R}_{\text{ohmic}} = \frac{2P_{\text{ohmic}}}{|\mathbf{I}_A|^2} = \frac{\Delta z}{2\pi a} \sqrt{\frac{\omega\mu}{2\sigma}} \int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} |\mathbf{I}(\mathbf{z})|^2 d\mathbf{z} = \frac{\Delta z}{6\pi a} \sqrt{\frac{\omega\mu}{2\sigma}} \Omega \quad (58)$$

$$\mathbf{X}_A = \frac{-120}{\pi \frac{\Delta z}{\lambda}} \left(\ln \left(\frac{\Delta z}{2a} \right) - 1 \right) \Omega \quad (59)$$

Equation (59) shows that electrically short dipoles have very high input reactance. It should be noted that the radiative resistance of the short dipole is less than that of the ideal dipole. This is because radiation resistance is proportional to the radiated power, discussed in Section 2.2.3, which is proportional to the square of the radiated electric field. Because the short dipole has an electrical length that is half its physical length, the radiation resistance is $\frac{1}{4}$ that of the ideal dipole [11]. It should be noted that the $\lambda/4$ curve in Figure 2.9 looks very similar to that of the short dipole in Figure 2.7.

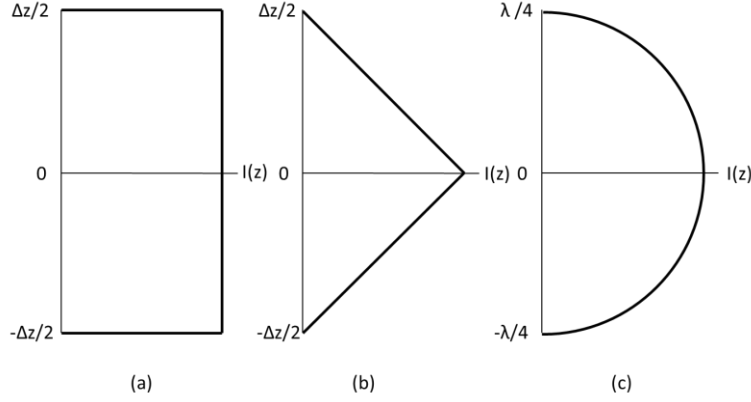


Figure 2.7 Current distributions for the ideal dipole (a), the short dipole (b), and the half-wave dipole (c). Figure adapted from [11].

2.2.2.4 Half-Wave Dipole Elements

A half-wave dipole is a line source with a current amplitude that varies as one-half of a sine wave with a maximum exactly in the center [11], [26]. This is shown in Figure 2.7. It should be noted from Figure 2.7 that the current distributions for the ideal, short, and half-wave dipoles are all in phase. The radiation fields from these dipoles will be strongest in the direction normal to the dipole, where the peak of the current distribution is, and weakest along the axis of the dipole [11]. The length of a half-wave dipole is $\frac{1}{2}$ the wavelength of the operational frequency. This means that half-wave dipoles can only be operated at one frequency or at very narrow bandwidths as discussed at the end of this section [22], [27].

The half sine wave current distribution for the z-directed half-wave dipole from Figure 2.7 is mathematically defined as [11]-[12]

$$I(z) = I_m \sin \left[\beta \left(\frac{\lambda}{4} - |z| \right) \right], \quad |z| \leq \frac{\lambda}{4} \quad (60)$$

As stated, this current goes to 0 at the ends with its maximum in the center. Taking the half-wave dipole as a z-directed line source and substituting equation (60) into equation (38) gives the electric radiation field as [11]-[12], [23]

$$E_{\theta} = j\omega\mu \sin(\theta) \frac{e^{-j\beta r}}{4\pi} \int I(z') e^{-j\beta z' - \cos(\theta)} dz' \quad (61)$$

which simplifies to

$$E_{\theta} = j\omega\mu \frac{2I_m}{\beta} \frac{e^{-j\beta r}}{4\pi} \sin(\theta) \frac{\cos[(\pi/2) \cos(\theta)]}{\sin^2(\theta)} \quad (62)$$

Half-wave dipoles have an input impedance is found using the induced IMF method [11], [23], [28]. The simplified form of the input resistance and reactance of a dipole of length L can be approximated as [23]

$$R_A = \frac{R_r}{\sin^2\left(\beta \frac{L}{2}\right)} \quad (63)$$

$$X_A = \frac{X_{A \text{ Short}}}{\sin^2\left(\beta \frac{L}{2}\right)} \quad (64)$$

where $X_{A \text{ Short}}$ is the reactance of the short dipole found in equation (59). The ohmic resistance for the half-wave dipole is given by [11]

$$\frac{\lambda}{8\pi a} \sqrt{\frac{\omega\mu}{2\sigma}} \Omega \quad (65)$$

The input impedance for the half-wave dipole can be approximated as $73 + j42.5 \Omega$ with an inductive reactance [11]. This impedance for varies slightly with respect length and thickness as illustrated in Table 2.2 [11], [22], [27].

One major advantage of half-wave dipoles is that they are resonant structures. The reactance of the half-wave dipole varies with frequency as is seen in equation (64). This is shown visually in Figure 2.8. As the frequency is increased, the reactance of the half-wave dipole transitions from capacitive to inductive. The peak of the curve indicates where the capacitive reactance and the inductive reactance cancel, giving a total reactance of 0. This is known as resonance, and it significantly simplifies impedance matching at the input of the half-wave dipole antenna [11],

[22]-[23]. Under resonant conditions, the input impedance of the half-wave dipole becomes 70Ω [11], [22]-[23], [26].

Table 2.2 Impact of Dipole Thickness on Resonance Length. Table adapted from [11].

Length: Diameter Ratio ($L/2a$)	Shortening Required	Resonant Length (L)	Dipole Thickness
5000	2%	0.49λ	Very thin
50	5%	0.475λ	Thin
10	9%	0.455λ	Thick

Half-wave dipoles also have a bandwidth [11], [22]. The bandwidth of an antenna element is defined as the range of frequencies in which antenna performance meets a desired threshold [11], [22]. Resonance and bandwidth are usually measured over the S_{11} parameter. The bandwidth for half-wave dipoles is usually narrow [11].

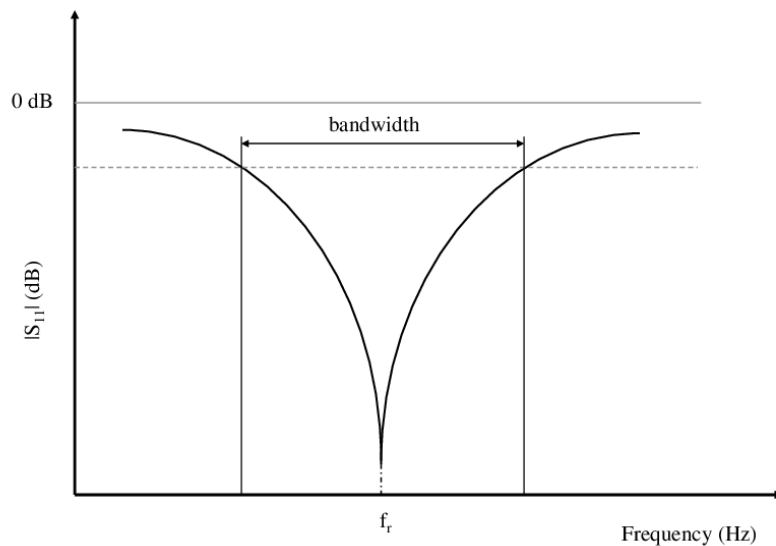


Figure 2.8 Example resonance curve for half-wave dipole antenna. Figure adapted from [29].

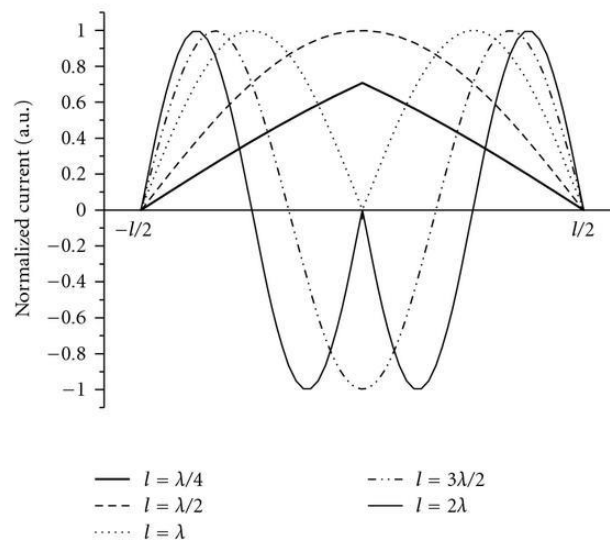
2.2.2.5 Long Dipole Elements

Dipoles that are longer than a half-wavelength can also be used in practice. The current distribution for these dipoles is also a sine wave with a value of 0 at the ends of the dipole [11], [23], [30]. Figure 2.9 shows the current distributions for dipoles of various lengths. As the length

Table 2.3 Radiation Resistance of Dipole vs Length L [11]

Length L	Input Radiation Resistance [Ω]
$0 < L < \frac{\lambda}{4}$	$20\pi^2\left(\frac{L}{\lambda}\right)^2$
$\frac{\lambda}{4} < L < \frac{\lambda}{2}$	$24.7\left(\pi\frac{L}{\lambda}\right)^{2.4}$
$\frac{\lambda}{2} < L < 0.637\lambda$	$11.14\left(\pi\frac{L}{\lambda}\right)^{4.7}$

is increased, the current distribution in some sections of the dipole drifts out of phase with the current in other sections of the dipole [11]. This causes cancellations in the radiation fields in the direction normal to the dipole and reinforcement of the radiation fields along the dipole axis. This is generally considered a negative effect in antenna theory, and dipole lengths are typically limited to one wavelength [11]. The impedance for long dipoles can be approximated using equations (63) and (64) [23]. A summary table of the approximate input radiation resistance for a dipole of length L is given in Table 2.3.

**Figure 2.9** Current distributions on dipole antenna of various lengths. Figure taken from [30].

2.2.3 Antenna Patterns

One of the most important characteristics of any antenna is the radiation pattern. The radiation pattern in simple terms is the angular variation of the far field radiation intensity around the antenna at a given spatial coordinate [11], [23].

In the case of the z-directed ideal dipole, the fields E_θ and H_ϕ radiate from the dipole over a sphere of radius r in the far field region. From equations (47) and (48), E_θ and H_ϕ vary angularly over the sphere as $\sin(\theta)$. Any plane through the sphere containing the z-axis will be parallel to, and contain, E_θ , the electric field vector in the direction of maximum radiation. These are known as E-planes. The radiation patterns in all of these E-planes are identical because there is no ϕ dependence. These are known as E-plane patterns. The plane perpendicular to the E-plane that cuts through the dipole is known as the H-plane and contains H_ϕ , the magnetic field in the direction of maximum radiation. In the z-directed dipole case, the H-plane is the xy plane. Patterns taken in the H-plane are known as H-plane patterns. The E- and H-planes are known as principal patterns or principal sweeps [11], [23].

It is important to note that the full radiation pattern is a three-dimensional pattern covering the entire sphere surrounding the antenna, while the E- and H-planes patterns are two-dimensional cross sections of that pattern [11], [23]. However, much of the information necessary for analysis can be obtained from the two-dimensional principal patterns [23]. In this thesis, the dominant radiation pattern metric will be on the two-dimensional principal patterns. Figure 2.10 shows the magnitude of the E- and H- principal patterns and the three-dimensional radiation pattern for a half-wave dipole. It should be noted that the E- and H-plane patterns are taken for the spatial range for which the radiation pattern is optimal. However, in the three-dimensional pattern, as the elevation is increased (or decreased) from the optimal range, the

radiation pattern begins to decrease in strength. When the elevation reaches $\pm 90^\circ$ along the z-axis, the radiation pattern is taken across the axis of the dipole and the magnitude dramatically decreases. This was predicted from the current distribution of the half-wave dipole in Section 2.2.2.4. This rapid decrease in the radiation power pattern is known as a null [27].

In practice, the radiation fields are normalized to a maximum value of unity. The phase is also set to zero at a chosen pattern maximum, because only relative phase information is needed [11]. The normalized field pattern for a z-directed line source element is [11]

$$F(\phi, \theta) = \frac{E_\theta}{E_\theta(\max)} \quad (66)$$

E_θ can be complex valued because it has a magnitude and a phase term. Therefore, $F(\phi, \theta)$ can also be complex valued. Note that there is no ϕ dependence for the z-directed element [11].

The normalized pattern for the uniform line source from equations (38) and (40) in Section 2.2.2.1 is given by [11]

$$F(\theta) = \sin(\theta) \frac{\sin\left(\frac{\beta L}{2} \cos(\theta)\right)}{\frac{\beta L}{2} \cos(\theta)} \quad (67)$$

where $\sin(\theta)$ is the maximum of E_θ , β is the phase constant, and L is the length of the line source in terms of λ . The appearance of the sinc function $\sin(x)/x$ in equation (67) should be noted. The sinc function occurs very frequently in antenna theory [11].

In general, the normalized field pattern can be written as the following product [11]

$$F(\phi, \theta) = g(\phi, \theta)f(\phi, \theta) \quad (68)$$

where $g(\phi, \theta)$ represents the element factor and $f(\phi, \theta)$ is the pattern factor. The element factor can be thought of as the effect on the radiation pattern from individual points along the current distribution of the dipole. The pattern factor, however, is the normalized sum of all of the

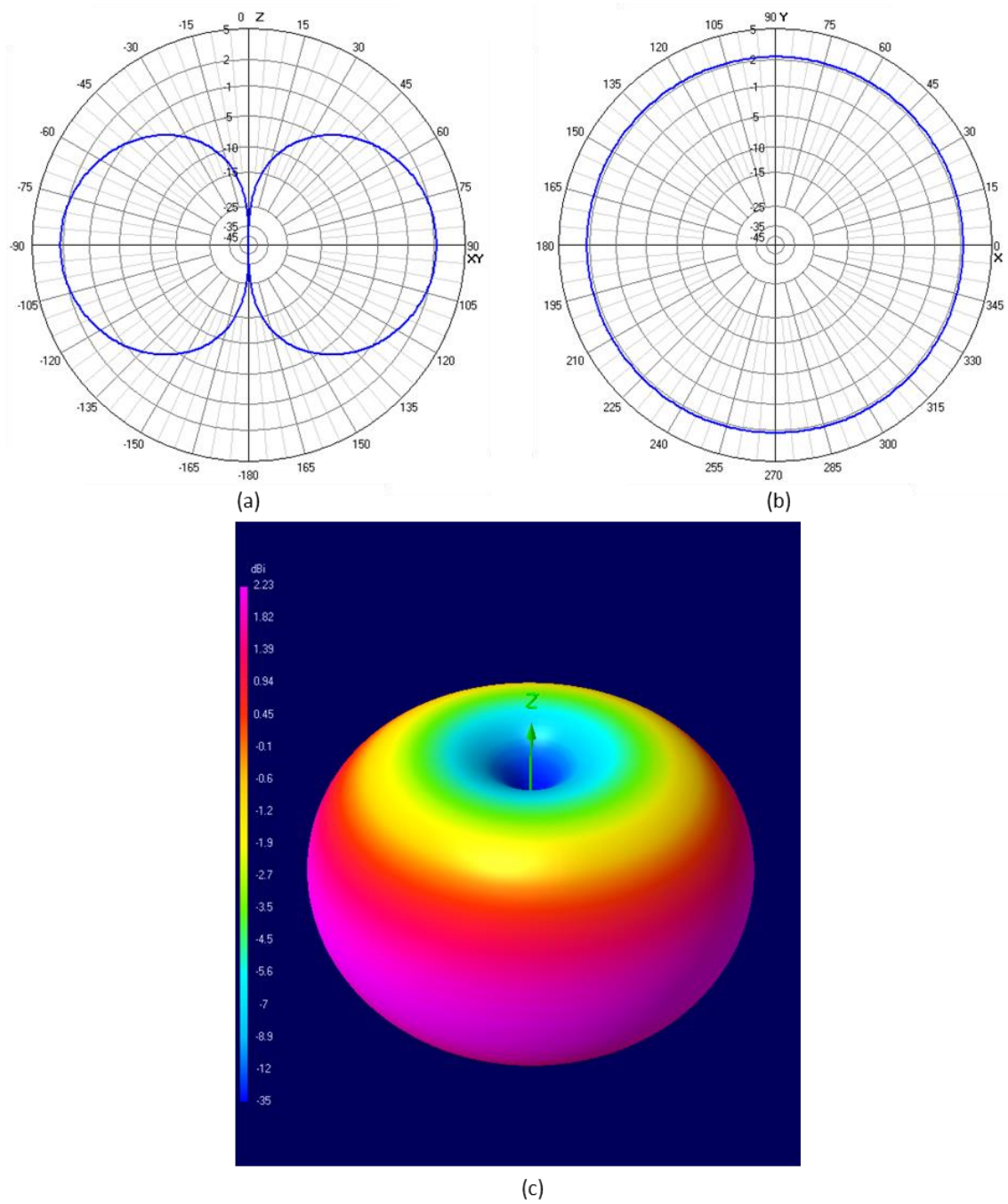


Figure 2.10 The (a) E-plane radiation pattern, (b) H-plane radiation pattern, and (c) three-dimensional radiation pattern of a single half-wave dipole in 4nec2 simulation software.

radiation contributions from the infinitesimally small current distributions. This is represented by the integration given in equation (38) [11].

E_θ for a half-wave dipole from equation (62) can be normalized as shown in equation (66) to give [11]

$$E_\theta = \sin(\theta) \frac{\cos[(\pi/2) \cos(\theta)]}{\sin^2(\theta)} \quad (69)$$

The element factor $g(\phi, \theta)$ from equation (61) is $\sin(\theta)$ and the pattern factor $f(\phi, \theta)$ is [11]

$$f(\phi, \theta) = \frac{\cos[(\pi/2) \cos(\theta)]}{\sin^2(\theta)} \quad (70)$$

The complete normalized far field pattern for a half-wave dipole is then [11]

$$F(\phi, \theta) = \frac{\cos[(\pi/2) \cos(\theta)]}{\sin(\theta)} \quad (71)$$

Typically, in antenna theory, the power of the radiation pattern is of most interest. The power pattern, as it is called, is [11]

$$P(\phi, \theta) = |F(\phi, \theta)|^2 \quad (72)$$

The power pattern is generally calculated in log scale as decibels (dB). It should be noted that the radiation magnitude pattern and the power pattern are the same in decibels [11], [23].

$$20\log |F(\phi, \theta)| = 10\log |P(\phi, \theta)| = 10\log |F(\phi, \theta)|^2 \quad (73)$$

Power patterns can be taken in either polar or in rectangular coordinates. An example power pattern for both coordinate systems is shown in Figure 2.11. The power pattern has a several different parameters. The most notable and important are the lobes of the pattern. The lobe with the greatest power is known as the main lobe, major lobe or main beam. This lobe points in the direction of maximum radiation for the antenna. The radiation in this direction arrives in the far field more nearly in phase than in other directions. It is possible for power patterns to have more than one main lobe. There are also special cases known as back lobes that direct energy in the opposite direction as the main lobes. Sometimes back lobes have the same magnitude as the main lobe. The other lobes are known as minor lobes or side lobes [11], [22]-[23], [27].

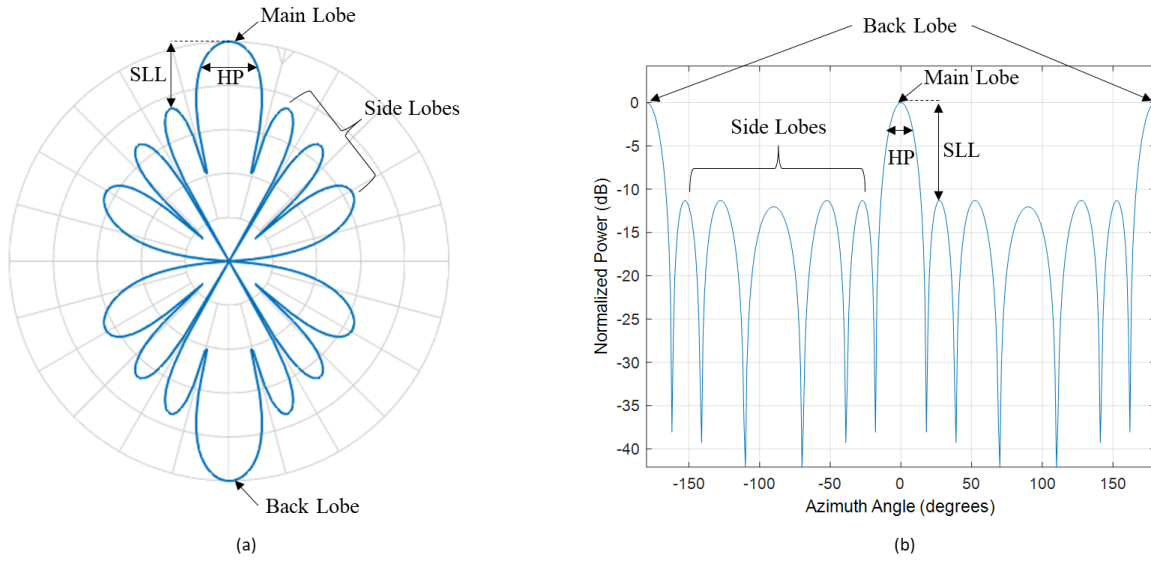


Figure 2.11 Radiation power pattern of a 4x4 rectangular array in (a) polar coordinates and (b) cartesian coordinates. Figure generated in MATLAB.

A metric of how well the power is concentrated in the main lobe can be obtained by taking the ratio of the peak of the maximum side lobe to the main lobe. This is known as the relative side lobe level (SLL) ratio and is defined as [11], [23], [27]

$$\text{SLL}_{\text{dB}} = 20 \log \left| \frac{F(\text{SLL})}{F(\text{max})} \right| \quad (74)$$

For a normalized power pattern, $F(\text{max}) = 1$.

Another important metric is the width of the main beam. This is quantified as the half-power beamwidth HP [11], [23], [27]

$$\text{HP} = |\theta_{\text{HP left}} - \theta_{\text{HP right}}| \quad (75)$$

The values of $\theta_{\text{HP left}}$ and $\theta_{\text{HP right}}$ are the points that have a value of $1/\sqrt{2}$ or 3dB down on the main lobe of the power pattern. In three-dimensional pattern plots the HP contour becomes a continuous curve rather than a single value. If the HP curve forms an ellipse, pattern cuts that contain the major and minor axes of the ellipse are referred to by IEEE as principal half-power beamwidths [11].

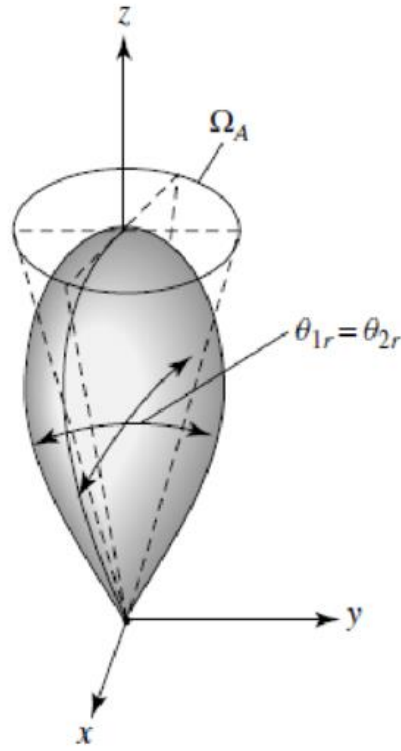


Figure 2.12 Visual representation of beam solid angle Ω_A with respect to antenna pattern directivity. Figure taken from [31].

2.2.3.1 Directivity and Gain

A very important concept for antenna theory is directivity. Directivity is the measure of how much energy the antenna can concentrate in a given direction and is equal to the antenna gain if 100% efficiency is achieved. Directivity is determined in terms of the radiation intensity in a given direction and the beam solid angle Ω_A [11], [23]. The beam solid angle is the solid angle through which all power would be radiated if the radiation intensity equaled the maximum value over the entire beam area. Mathematically this is defined as [11]

$$\Omega_A = \frac{P}{U_m} \quad (76)$$

where U_m is the maximum radiation intensity. A visualization for beam solid angle is provided in Figure 2.12. Expressed in more relevant terms, beam scan angle can be defined by [11]

$$\Omega_A = \iint |F(\phi, \theta)|^2 d\Omega \quad (77)$$

Directivity D can be expressed as [11], [23]

$$D(\theta, \phi) = \frac{4\pi}{\Omega_A} |F(\phi, \theta)|^2 \quad (78)$$

It should be noted that both beam scan angle and directivity are completely dependent on the pattern shape. It should also be noted that when the pattern is normalized, the maximum directivity is $D = 4\pi/\Omega_A$ [11].

Another important concept to consider in antenna analysis is the gain. Gain is directly related to directivity. Whereas directivity considers the maximum possible gain in a specified direction without taking into consideration parameters that can affect the efficiency of the antenna, gain takes into account the efficiency of the antenna element. Thus, while gain and directivity are referring to the same concept, directivity assumes perfect performance while gain gives a realizable value. Gain is mathematically defined as [11], [23]

$$G = e_r D \quad (79)$$

where D is the directivity and e_r is the radiation efficiency defined as P/P_{in} . P is radiated power and P_{in} is the input power to the antenna. Radiation efficiency can also be defined in terms of the input impedance of the antenna [11], [23]

$$e_r = \frac{R_r}{R_r + R_{ohmic}} \quad (80)$$

Units for both directivity D and gain G are generally in dB.

$$D_{dB} = 10\log(D) \quad (81)$$

$$G_{dB} = 10\log(G) \quad (82)$$

The values for all of these parameters are tabulated for the ideal dipole, short dipole and half-wave dipole in Table 2.4.

Table 2.4 Parameters for Different Dipole Elements [11]

Dipole Type	Length L	Current Distribution	Pattern F(θ, φ)	HP (degrees)	D (dB)	R _r (Ω)	R _{ohmic} (Ω)
Ideal	L << λ	Uniform	sin(θ)	90	1.76	$80 \pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$	$\frac{\Delta z}{2\pi a} \sqrt{\frac{\omega\mu}{2\sigma}}$
Short	L << λ	Triangular	sin(θ)	90	1.76	$20 \pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$	$\frac{\Delta z}{6\pi a} \sqrt{\frac{\omega\mu}{2\sigma}}$
Half-Wave	L = λ/2	Sinusoidal	$\frac{\cos[(\pi/2) \cos(\theta)]}{\sin(\theta)}$	78	2.15	~ 70	$\frac{\lambda}{8\pi a} \sqrt{\frac{\omega\mu}{2\sigma}}$

2.2.3.2 Polarization

The polarization of the antenna is the same as the polarization of the electromagnetic wave radiated by the antenna. The polarization of the radiation from the antenna can vary with direction but is constant over the main lobe of the antenna's radiation pattern. Therefore, the polarization of the main lobe is considered the polarization of the antenna. By convention, the polarization is taken to be the polarization of the radiated electric field [11], [23].

At a given instant, the electric field $\mathbf{E}$ propagating along the z-axis is a vector in the xy plane with both x- and y-components E_x and E_y . If E_y is leading E_x by phase δ , then the vector representation of $\mathbf{E}$ is given by [11]

$$\mathbf{E} = |E_x|\hat{\mathbf{x}} + |E_y|e^{j\delta}\hat{\mathbf{y}} \quad (83)$$

or [11]

$$\mathbf{E} = \sqrt{|E_x|^2 + |E_y|^2} (\cos(\gamma)\hat{\mathbf{x}} + \sin(\gamma)e^{j\delta}\hat{\mathbf{y}}) \quad (84)$$

where γ is defined as [11]

$$\gamma = \tan^{-1} \left(\frac{|E_x|}{|E_y|} \right), \quad 0^\circ \leq \gamma \leq 90^\circ \quad (85)$$

In this thesis, only the vertical linear polarizations are utilized. However, if the antenna is not perfectly oriented in the desired vertical z-axis direction, then the electric field will not be perfectly vertically polarized with respect to the z-axis. Therefore, only a component of the electric field will interact with the antenna as desired. Equations (84) and (85) can be used to compute that vector component for more accurate results.

2.3.3.3 Reciprocity

One final, but very powerful concept in antenna theory is known as reciprocity. The reciprocity concept simply states that the characterizations of the antenna are the same when the antenna is transmitting and when it is receiving. Therefore, antennas are generally characterized in one mode and assumed to operate identically in the other [11], [23].

2.3 Electrically Steered Arrays

The focus of the literature review thus far has been on single array element analysis. This thesis focuses on phased arrays. An array antenna consists of several antennas of the same orientation placed in a certain configuration to form a larger antenna. A phased array or an electronically steered array (ESA) is an array antenna that has digital control on each element that allows phase changing of the currents that excite each element in the array. In this way, the array can digitally scan through space while the physical array is mechanically stationary [11], [23]. This section discusses the nuanced changes of antenna theory for an array of elements and some applications for these arrays.

2.3.1 Array Radiation and Pattern

The theory behind a phased array is mostly identical to standard antenna theory with a few changes. Because arrays are formed by bringing multiple antenna elements together, the array properties are determined by the cumulative effects of each individual element [11], [23].

Fortunately, Maxwell's equations are linear, so the radiation analysis can be performed for each element and summed together to obtain the overall behavior for the array.

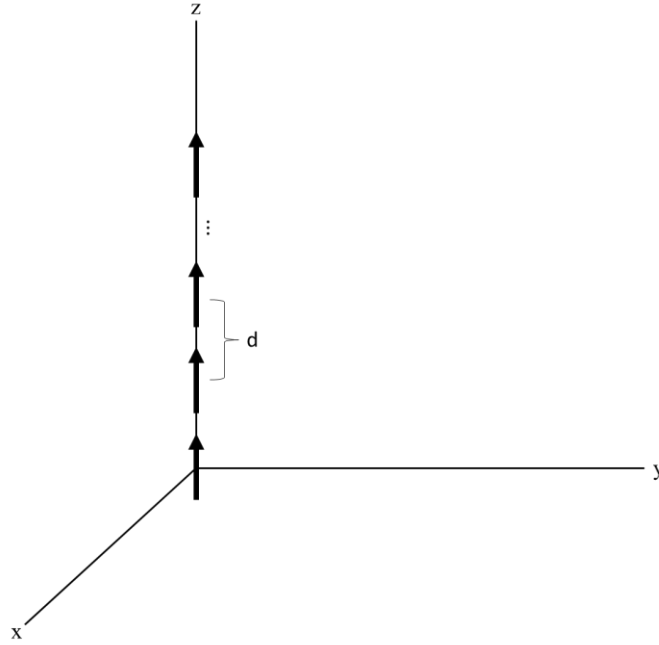


Figure 2.13 A line array of n z -directed line sources. The first element is centered at the origin. The distance between the centers of adjacent line sources is d . Figure adapted from [11].

This process is performed by summing the vector potentials and radiation fields for each element. This shown below for n z -directed line sources placed in a line as shown in Figure 2.13

Each element has current distribution [11]

$$I(z') = \sum_{n=0}^{N-1} i_n(z') \quad (86)$$

The vector potential and far field electric field are then determined as [11]

$$A_z = \mu \frac{e^{-j\beta r}}{4\pi r} \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} i_n(z') e^{j\beta z' \cos(\theta)} dz' \quad (87)$$

$$E_{\theta} = j\omega\mu \frac{e^{-j\beta r}}{4\pi r} \sum_{n=0}^{N-1} \sin(\theta) \int_{-\infty}^{\infty} i_n(z') e^{j\beta z' \cos(\theta)} dz' \quad (88)$$

The above analysis assumes that there is no symmetry or equal spacing d between elements. If there is equal spacing d between elements, symmetry simplifies the analysis. The vector potential and the electric field for an equally spaced linear array of z -directed line sources are [11]

$$A_z = \mu \frac{e^{j\beta r}}{4\pi r} \Delta z (I_0 + I_1 e^{j\beta d \cos(\theta)} + \dots + I_{N-1} e^{j\beta(N-1)d \cos(\theta)}) \quad (89)$$

$$= \mu \frac{e^{j\beta r}}{4\pi r} \Delta z \sum_{n=0}^{N-1} I_n e^{j\beta n d \cos(\theta)} \quad (90)$$

$$E_\theta = j\omega\mu \frac{e^{-j\beta r}}{4\pi r} \Delta z \sin(\theta) \sum_{n=0}^{N-1} I_n e^{j\beta n d \cos(\theta)} \quad (91)$$

Normalizing E_θ gives

$$E_\theta(\text{normalized}) = \sin(\theta) \sum_{n=0}^{N-1} I_n e^{j\beta n d \cos(\theta)} \quad (92)$$

This is a much simpler form. Array elements can be spaced equally to take advantage of this symmetry and computational simplicity. The arrays in this thesis will be regarded as equally spaced. From equation (40) it can be shown that the element pattern for a line source is $\sin(\theta)$ [11]. The remaining term is what is known as the array factor (AF) [3], [11]-[12], [23]

$$AF = \sum_{n=0}^{N-1} I_n e^{j\beta n d \cos(\theta)} \quad (93)$$

The array factor represents the sum of the radiation fields if each element in the array were replaced by an ideal point source that radiates equally in all directions [11], [23]. These theoretical ideal point sources are called isotropic elements and are located at the center of each element [11], [27]. In this way, the isotropic elements maintain the positions and current magnitudes and phases of the array elements. The element pattern $g_e(\phi, \theta)$ contains the radiation

pattern of the individual element taking into account current distribution and spatial orientation [3], [11]-[12], [23]. The array factor can be extended to arrays of multiple dimensions as [11], [23]

$$AF(\phi, \theta) = \sum_{n=0}^N I_n e^{j\beta \bar{P}_n \cdot r(\phi, \theta)} \quad (94)$$

where $\bar{P}_n$ is the position vector of each element in the cartesian coordinates and $r(\phi, \theta)$ is the far field direction of interest in spherical coordinates. The dot product represents the projection of the element coordinates into spherical coordinates relative to far field direction of interest. This is shown explicitly in equations (101) to (103) and Figure 2.17.

For a single antenna, the pattern was determined as the product of a pattern factor and an element factor. For an array, the radiation pattern is the product of the array factor with the element factor. This is known as the principle of pattern multiplication and is shown below [3], [11], [23], [32]

$$F(\phi, \theta) = g_e(\phi, \theta) AF(\phi, \theta) \quad (95)$$

where g_e is the element pattern. Note the similarity between equation (95) and equation (68). Pattern multiplication is a very good approximation, given the elements in the array have identical current distributions and are oriented in the same direction [11].

Similarly, the normalized power pattern is given by [11]

$$P(\phi, \theta) = |F(\phi, \theta)|^2 \quad (96)$$

and is measured in dB. The principles discussed in Section 2.2 for the single antenna are also applicable to phased arrays. In this thesis, SLL for an array pattern is taken at the first sidelobe.

2.3.2 Array Conventions and Properties

This section discusses common terminologies and properties that apply to phased arrays that do not necessarily apply to single element antennas. These topics include standard array geometries, important spatial directions for phased arrays, and element spacing considerations for phased arrays. Understanding these concepts is crucial for the discussion for the rest of the thesis.

2.3.2.1 Conventional Array Geometries

Phased arrays exist in various forms. Several examples of standard configurations are shown in Figure 2.14. The simplest array is known as a line array. In a line array, individual antenna elements are spaced in a one-dimensional line. These arrays can scan in one dimension, either azimuth or elevation, depending on whether the elements are placed in the azimuth or the elevation direction [11], [33]. If the array elements are equally spaced, the structure is known as a uniform line array or ULA [34]-[35].

Another popular array design is the extension of the line array into two-dimensions to form a rectangular grid. This is known as a rectangular array. If the spacing between elements is the same the rectangular array is known as a uniform rectangular array or URA. URAs do not have to have equal spacing in both dimensions. Consider a URA with a normal to the positive x-axis. The array elements lie in the yz plane. As long as the y-spacing between elements is consistent and z-spacing between elements is consistent, the array is considered a URA. If there is equal spacing in both dimensions, the URA takes the form of a square [36]-[37].

There are also two-dimensional arrays that take different geometries. Hexagonal arrays and circular arrays are not uncommon. If uniform spacing is implemented on these structures, they are known as uniform hexagonal arrays (UHAs) and uniform circular arrays (UCAs) respectively

[38]-[40]. Finally, there are what are known as conformal arrays that have an arbitrary geometry. These are generally used when the array needs to fit a contour of an arbitrary surface [41]. Any of these two-dimensional arrays is capable of scanning in both azimuth ϕ and elevation θ [33].

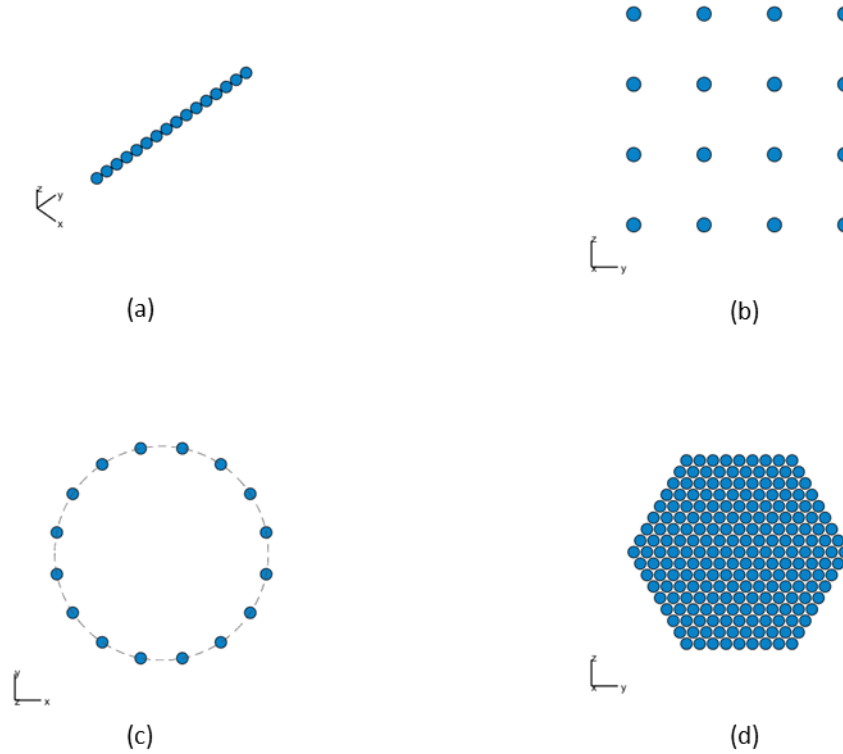


Figure 2.14 Examples of array configurations: (a) line arrays, (b) rectangular arrays, (c) circular arrays, and (d) hexagonal arrays. Figure generated in MATLAB.

2.3.2.2 Broadside and Endfire

There are two special orientations and directions in phased array theory. Broadside is the term used to refer to the direction that is perpendicular to the plane of the array elements [23]. Using z -directed dipoles in the coordinate system from Section 2.1.1, the positive x -axis is broadside. Broadside can also be considered the array normal. At broadside, the antenna or array will have optimum performance with respect to beamwidth, directivity, and gain. As soon as the transmitted or received signal is shifted away from the broadside direction, array performance

will continue to degrade until the worst performance is reached [3], [11], [23]. This occurs at endfire.

Endfire is the direction that is orthogonal to the broadside direction and the dipole axis [11]. For an array of z-directed dipoles in the coordinate system used from Figure 2.1 the endfire direction is the positive y-axis. At endfire, the beamwidth is at its maximum and directivity is at its minimum [11]. While there are arrays specifically designed to operate at endfire, [3], [11], [23] this thesis focuses on broadside antennas. Therefore, endfire effects are considered something to be avoided. Figure 2.15 illustrates the difference in performance of an 8x8 URA with half-wavelength spacing at broadside and endfire. The very broad beamwidth at endfire should be noted. This is the undesired effect that is sought to be avoided in this thesis.

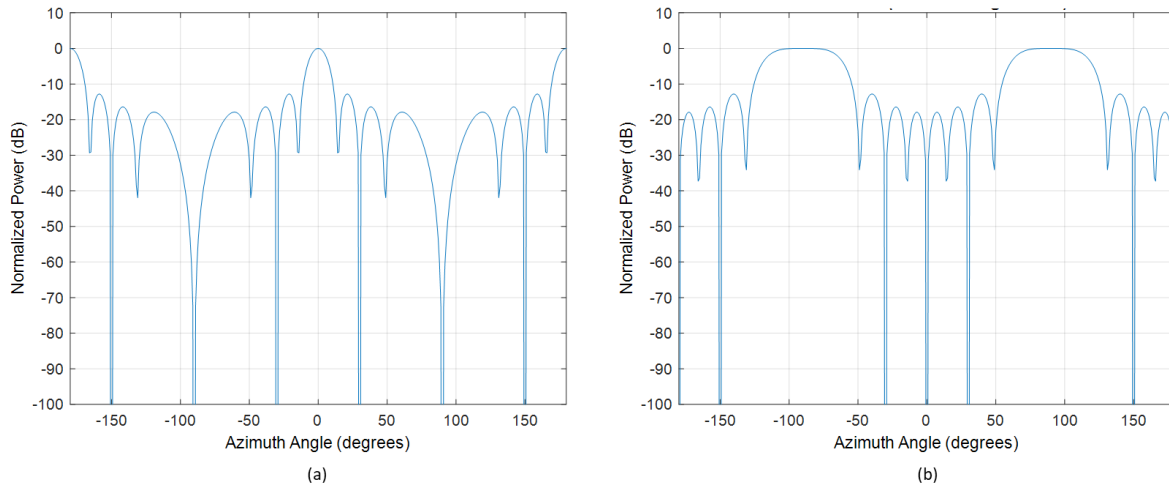


Figure 2.15 8x8 URA normalized radiation power pattern at (a) broadside and (b) endfire in rectangular coordinates. Figure generated in MATLAB.

2.3.2.3 Element Spacing, Grating Lobes and Aperture Size

One of the most important concepts to consider when designing an array is the spacing between the individual elements. Jonson [42], Van Trees [43] and Balanis [23] apply the theory proposed by Woodward [44] to phased array spacing. According to this theory, element spacing in an array is analogous to sampling theory in the spatial domain. As such, sampling theory

concepts like the Nyquist limit apply. Just like too few samples in frequency can result in a frequency alias, taking too few samples by spacing the elements too far apart can lead to a spatial alias [26]. From the geometry in Figure 2.17 the time delay between two adjacent elements is given as $\Delta t = \beta d \sin(\theta')/c$ in equation (100). More about the time delay and its derivation are explored in the next section. The phase difference between adjacent elements must be kept below π to avoid incorrect time delay calculations [46]. This imposes the restriction $(2\pi)(\Delta t)(f) \leq \pi$. Therefore, the distance between elements is given by [3]

$$d \leq \frac{c}{2f \sin(\theta')} = \frac{\lambda}{2 \sin(\theta')} \quad (97)$$

This means that the maximum possible spacing to avoid aliasing in the spatial domain is [3], [11], [23], [46]

$$d \leq \frac{\lambda}{2 \sin(90)} = \frac{\lambda}{2} \quad (98)$$

If spacing larger than or $\lambda/2$ is implemented, a spatial alias will appear in the antenna pattern as a second major lobe that has equal gain as the main lobe. This is what is known as a grating lobe. To avoid grating lobes, spacing must be kept at or below $\lambda/2$ [1], [11], [23].

The aperture of the array is the effective area over which the electromagnetic wave can be collected [23]. Figure 2.16 illustrates the relationship between element spacing, aperture size, HP, and grating lobes. Aperture size can be increased by adding more elements with constant spacing. Figure 2.16a and c show that as the aperture size is increased HP decreases, and array performance is improved [11]. This increase in aperture, however, comes at the SWAP-C cost of adding extra elements to the array. Another way to increase aperture size is by increasing spacing as in Figure 2.16b and d. As element spacing is increased, the HP of the array decreases but with

increasing presence of grating lobes. If the element spacing is decreased, the grating lobes will disappear, but the HP will increase. This is a significant tradeoff in array design [45].

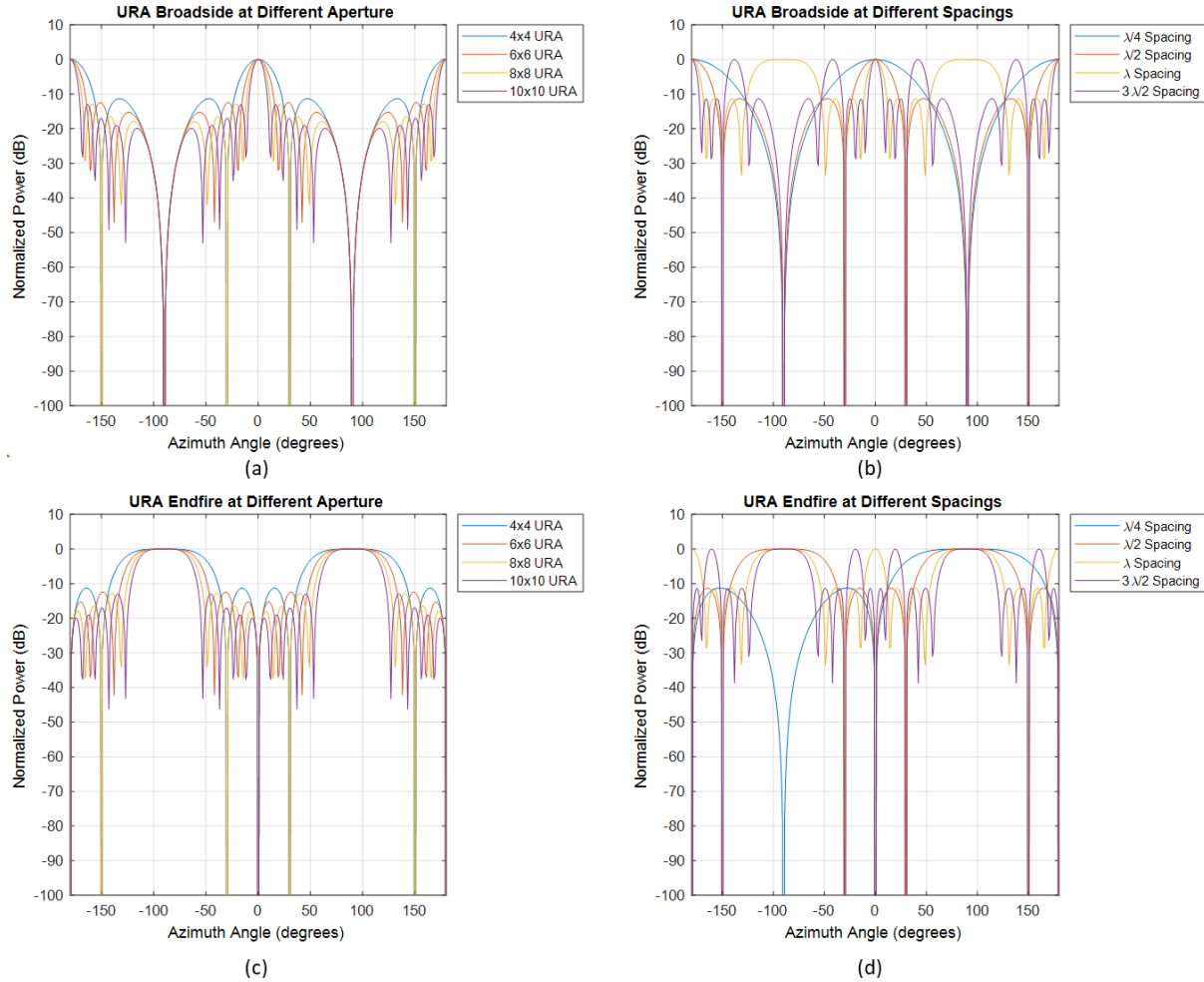


Figure 2.16 Radiation power patterns at (a) broadside with fixed $\lambda/2$ spacing and differing number of elements, (b) broadside with fixed 4x4 array size and differing element spacing, (c) endfire with fixed $\lambda/2$ spacing and differing number of elements, (d) endfire with fixed 4x4 array size and differing element spacing. Figure generated in MATLAB.

2.3.3 Practical Phased Array Operation

This section introduces applications of phased arrays and introduces some details about their operation. For the scope of this thesis, the phased array is used for direction of arrival, beam steering and beamforming. These are discussed in detail in this section. In order to understand

how the array can be used for these applications, an understanding of how an electromagnetic wave interacts with the array elements is first necessary.

2.3.3.1 Wave Propagation through Array Elements

An example of an electromagnetic wave propagating through an array is shown in Figure 2.17. As the wave front propagates through the array, it will encounter element 1 before encountering element 2 Δt later in time, element 3 Δt_2 later in time etc.

Simple trigonometry similar to that performed in Section 2.2.1 and Figure 2.5 can be used to compute a phase difference between when two elements interact with a present wave front. Considering only element 1 and element 2, a right triangle can be formed consisting of the

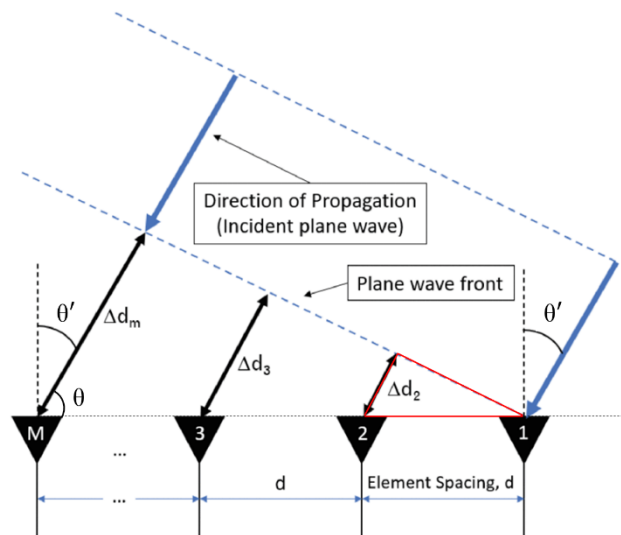


Figure 2.17 Incident plane wave propagating through an m-element uniform linear array arranged in the θ direction. Figure adapted from [47]

distance d between the elements, the distance Δd_2 from element 2 to the wave front and the length of the wave front between element 1 and element 2. The wave front forms an angle θ' with respect to the array. It should be noted that the angle θ' is equal to $90 - \theta$, where θ is the elevation variable in the coordinate system given in Figure 2.1. Because the array is assumed to be in the far field region with respect to the source, every point on the wave front over the area of

the array is assumed to be at the same phase [11]. Therefore, the angle formed between the wave front and each element can be considered to be approximately equal to θ [11]. The length Δd_2 is then $d\cos(\theta')$ which represents the path length difference between the two elements [48].

The phase difference between adjacent elements can be found by multiplying this path length difference by the phase constant β . However, there may be an intentional phase shift δ applied between elements. Therefore, the total phase shift ψ between elements is given by [48]

$$\psi = \beta d\cos(\theta') + \delta = \beta d\sin(\theta) + \delta \quad (99)$$

The time delay Δt between when the wave front interacting with element 1 and element 2 is defined as [49]

$$\Delta t = \frac{\text{Path Length Difference}}{\text{Propagation Velocity}} = \frac{\beta \Delta d_2}{c} = \frac{\beta d\cos(\theta')}{c} = \frac{\beta d\sin(\theta)}{c} \quad (100)$$

The above procedure was performed on a one-dimensional ULA, but the concept can be extended to higher dimensional arrays by including spacing and distances between elements in other dimensions.

The cartesian coordinates of the element positions must be projected onto spherical coordinates in terms of azimuth ϕ and elevation θ using the following equations [50]

$$x = \sin(\theta) \cos(\phi) \quad (101)$$

$$y = \sin(\theta) \sin(\phi) \quad (102)$$

$$z = \cos(\theta) \quad (103)$$

The time shift Δt for this case is represented by [50]

$$\Delta t = \frac{\beta(x + y + z)}{c} \quad (104)$$

The procedure as presented here is performed on receive and can be utilized in direction of arrival (DOA) calculations. However, this same process can be applied to the array on transmit

for beam steering and beamforming. Beamforming, beam steering, and DOA are discussed in the following sections.

2.3.3.2 Beamforming vs Beam Steering

The pattern discussed in Section 2.3.1 is the overall radiation pattern when every element in the array is transmitting simultaneously when no steering is applied. As defined, a phased array allows for the phase shifting of the excitation currents on each element to steer the main lobe or beam in a desired direction [11]. This is known as beam steering [51]. An important feature in beam steering is that only the phase of each element is affected; the magnitudes of the currents remain unchanged [23], [50], [52].

Beamforming is a more advanced version of beam steering where both the magnitudes and the phases of the excitation current for each element are manipulated. In this way multiple beams can be formed in multiple directions simultaneously [52]. Beamforming can be utilized in conjunction with null forming, forming nulls in specified directions, to filter out interference signals and improve the signal-to-interference-plus-noise ratio (SINR) [33], [53]-[54].

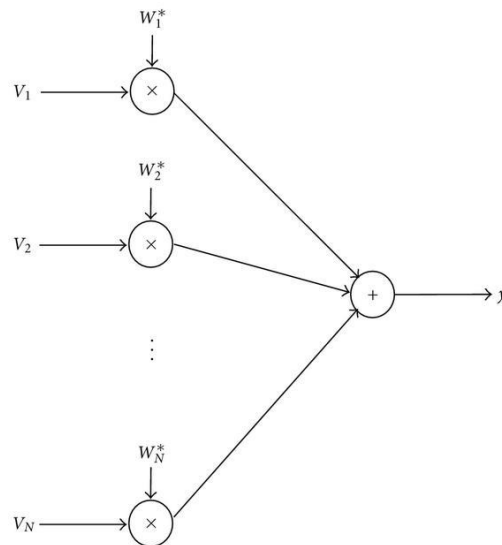


Figure 2.18 Illustration of beamforming. Figure taken from [55].

Figure 2.18 shows a visual representation of beamforming with a ULA of n elements. Every element has a weight w_n applied which will manipulate the phase and amplitude of the current such that the radiative response x_n of the elements combine into an output signal in the desired direction (ϕ, θ) . The radiative response signals of each element x_n are assumed to be identical except for a phase shift. This phase shift is computed from the previous section and converted into a time delay Δt_n as in equation (100) or equation (104). The effective response of each element x_n' is [56]

$$x_n'^{(t)} = x_n(t)e^{-j\omega\Delta t_n} \quad (105)$$

The total output signal y is [50], [56]-[57]

$$y(t) = \sum_{n=0}^{N-1} w_n^* x_n(t)e^{-j\omega\Delta t_n} = \mathbf{w}^H \mathbf{X} + \text{noise} \quad (106)$$

where $\mathbf{X}$ is the element response vector, $\mathbf{w}^H$ is the Hermitian conjugate transpose of the weighting vector, and the noise is assumed as white Gaussian noise. It should be noted in equation (106) and Figure 2.18 that the conjugate of the weight of each element w_n is used for this calculation.

The variance of the output signal is determined by the value of the power of the signal. This is expressed mathematically as [50]

$$P(y) = \sigma^2 = E\{|y(t)|^2\} \quad (107)$$

$$= E\{(\mathbf{w}^H \mathbf{X}) (\mathbf{w}^H \mathbf{X})^H\} \quad (108)$$

$$= E\{(\mathbf{w}^H \mathbf{X} \mathbf{X}^H \mathbf{w})\} \quad (109)$$

$$= \mathbf{w}^H E\{\mathbf{X} \mathbf{X}^H\} \mathbf{w} \quad (110)$$

$$= \mathbf{w}^H \mathbf{R} \mathbf{w} \quad (111)$$

where $\mathbf{R}$ is the covariance or correlation matrix of the element responses and is defined as [43], [50]

$$\mathbf{R} = E\{\mathbf{X}\mathbf{X}^H\} = \frac{\mathbf{X}\mathbf{X}^H}{n} \quad (112)$$

Equation (111) represents the output signal $y(t)$ in the specific steered direction. The challenge is to determine the exact values for the weight vector $\mathbf{w}$. This is accomplished by solving for the variance of the signal as a function of steering direction [43], [50], [57].

The phase shift information is stored in a time shift vector as $e^{-j\omega\Delta t_n}$ for each term and is known as the steering vector $\mathbf{sv}$ [50]. The steering vector represents the phase information for each element at each steering direction. The covariance matrix $\mathbf{R}$ at an arbitrary direction will have phase shifts associated with the steering direction. This indicates that $\mathbf{R}$ is a function of steering direction $\mathbf{R}(\mathbf{sv}) = \mathbf{sv}^H \mathbf{R} \mathbf{sv} + \text{noise}$ [50].

The power can be calculated with respect to steering vector $\mathbf{sv}$ as [50]

$$P(\mathbf{sv}) = \mathbf{w}^H \mathbf{R}(\mathbf{sv}) \mathbf{w} = \mathbf{w}^H (\mathbf{sv}^H \mathbf{R} \mathbf{sv}) \mathbf{w} \quad (113)$$

This is known as the steered response of the array at a given steering direction and gives the power spectrum of the array in a given direction in the frequency domain [50]. When uniform spacing is implemented in the array and all elements have been given equal weight, the normalized steered response reduces to [43], [50], [58]

$$P_{\text{Bartlett}}(\mathbf{sv}) = \mathbf{sv}^H \mathbf{R} \mathbf{sv} \quad (114)$$

The similarity between equation (114) and equation (111) should be noted. This implies that the normalized weight vector $\mathbf{w}$ for an equally weighted uniform array is simply the steering vector $\mathbf{sv}$ [50]. Because all the sensors have equal weight $|\mathbf{w}| = 1$, [50], [59] the only factor that is changing is the phase. Therefore, this solution can be thought of as a phase-only beamformer or a beam steerer. It is also known as the delay and sum beamformer, conventional beamformer and Bartlett beamformer [43], [50]. In this thesis, it will be referred to as a Bartlett beamformer.

More advanced beamforming methods are also be used in practice. One such beamformer is the Minimum Variance Distortion-less Response (MVDR). The MVDR beamformer changes the weights in magnitude and phase for each element based on the element response in a given direction. For this reason, MVDR is known as an adaptive beamformer [50], [60]. The goal of MVDR beamforming is to minimize the power in all directions except for the direction of interest. This is achieved by applying the following constraint [50], [57]

$$\begin{aligned} \min P(y) &= \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{subject to } &\mathbf{w}^H \mathbf{s}_v = 1 \end{aligned} \quad (115)$$

The constraint given in equation (115) is a Lagrange optimization problem and can be solved using Lagrange multipliers [60]. The solution for $\mathbf{w}$ can be found as [43], [57], [60]

$$\mathbf{w} = \frac{\mathbf{R}^{-1} \mathbf{s}_v}{\mathbf{s}_v^H \mathbf{R}^{-1} \mathbf{s}_v} \quad (116)$$

If the spacing of the array is such that grating lobes will be present, they will manifest in the beamforming pattern as well. Back beams may also appear in the opposite direction as the main beam, such as in Figure 2.11. Back beams may also form in the opposite direction any grating lobes that are present in the pattern as in Figure 2.16

2.3.3.3 Direction of Arrival

The same principles that were implemented for beamforming can be applied to determine the direction from which a given signal was transmitted. This direction-finding process is known as a direction of arrival or DOA measurement. It should be noted that for a DOA measurement the power spectrum in the frequency domain is utilized rather than a beam pattern [50]. Therefore, maxima are referred to as peaks rather than lobes or beams. All noise in these measurements is assumed to be white Gaussian noise [57].

The simplest form of DOA measurement is to perform a beam scan of space. This is achieved using the Bartlett beamformer to form a beam for each direction in θ and ϕ for a given scanning area and compute the power in each direction using equation (114). The direction for which maximum power is found is the direction from which the signal was transmitted. Bartlett is a special case where the DOA power spectrum is identical to the beam pattern [50], [59].

MVDR can also be used for DOA measurements. MVDR sweeps through the directional area of interest one direction at a time, minimizing the power in all directions except for the desired direction. It is important to note that for a DOA measurement, MVDR does not form a beam; MVDR only computes a power for a given direction while minimizing power elsewhere. This allows for much narrower peaks in the power spectrum than the Bartlett DOA calculation. The power spectrum for the MVDR DOA calculation is found by solving equation (111) using the weights w from equation (115). The normalized MVDR steering response is [43], [50], [57]

$$P_{\text{MVDR}}(sv) = \frac{1}{sv^H \mathbf{R}^{-1} sv} \quad (117)$$

It should be noted that the covariance matrix $\mathbf{R}$ is required to compute the power spectrum for MVDR. Therefore, MVDR is more computationally expensive than the Bartlett DOA approach.

The final DOA approach considered in this thesis is the multiple signal classification (MUSIC) technique. In the music technique, the covariance matrix undergoes eigenvalue decomposition for both signal and noise subspaces. This is expressed mathematically as [57]

$$\mathbf{R}(sv) = sv^H \mathbf{R} sv + \text{noise} = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \mathbf{U}_n \Lambda_n \mathbf{U}_n^H \quad (118)$$

where $\mathbf{U}_s$ and $\mathbf{U}_n$ are the matrices of eigenvectors that span the signal subspace and the noise subspace respectively and Λ_s and Λ_n are the diagonal vectors of eigenvalues of the signal and noise subspaces respectively. Because the noise is assumed to be white Gaussian noise, the signal and noise subspaces, and therefore Λ_s and Λ_n , are assumed to be orthogonal to each other.

The eigenvalues of Λ_s and Λ_n are positive, real values arranged from greatest to least. If there are n signals and s sources present, then the largest n eigenvalues of Λ_s and Λ_n correspond to the signal while the smaller $n-s$ eigenvalues correspond to the noise [57].

The objective of MUSIC is very similar to that of MVDR. MUSIC sweeps through the directional area of interest one direction at a time and computes a power for each steering direction. Instead of finding where the maximum power is, however, MUSIC looks where the noise is minimal. This is achieved by projecting onto the noise subspace $\mathbf{U}_n \mathbf{U}_n^H$. Substituting $\mathbf{U}_n \mathbf{U}_n^H$ for $\mathbf{R}^{-1}$ in equation (117) gives the MUSIC power spectrum [43], [57]

$$P_{\text{MUSIC}}(\mathbf{sv}) = \frac{1}{\mathbf{sv}^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{sv}} \quad (119)$$

The desired the peak power from equation (119) occurs when the noise is at a minimum. Like MVDR, the MUSIC approach requires the covariance matrix $\mathbf{R}$, but must also perform eigenvalue decomposition. MUSIC is therefore the most computationally expensive approach out of the three.

All three DOA algorithms are presented for a 4x4 URA with $\lambda/2$ and $3\lambda/4$ spacing in Figure 2.19. Bartlett has a very wide main peaks with side peaks throughout the spectrum. These are the two major disadvantages to Bartlett as a DOA approach. If a second signal is present but at a lower SNR the power level in the spectrum will be lower. If the power level of the weaker signal is lower than the side peak level of the Bartlett power spectrum, then the signal will be masked and unresolvable. Similarly, because the width of the main peak, Bartlett is said to have low resolution [43], [59]. Resolution refers to how spatially distant two signals can be and still be resolvable [43], [57].

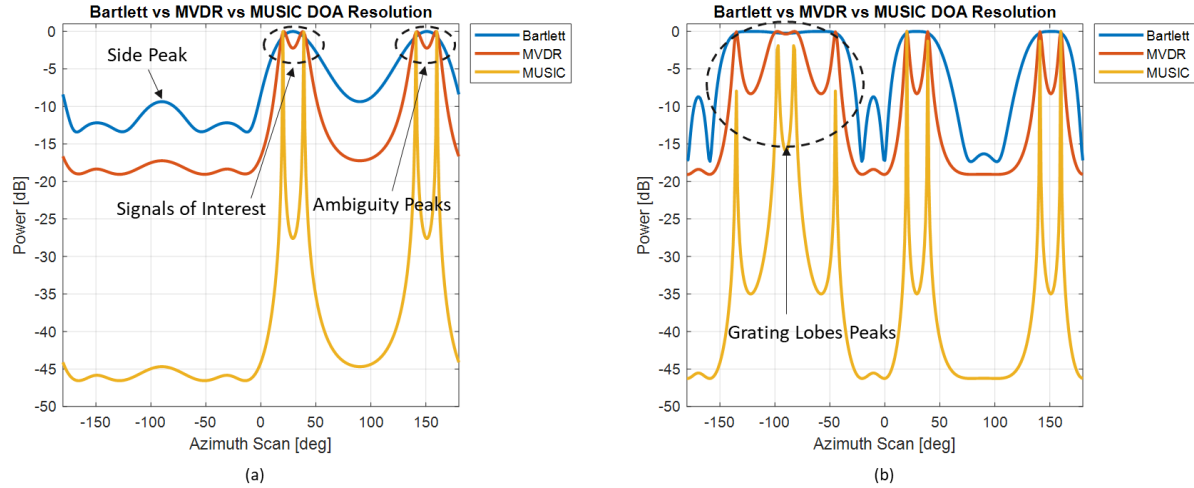


Figure 2.19 DOA spectra comparisons for a 4x4 URA with (a) $\lambda/2$ spacing and (b) $3\lambda/4$ spacing. Figure generated in MATLAB.

MVDR and MUSIC do not have either of these issues. From Figure 2.19 MVDR and MUSIC have negligibly high side peaks. Therefore, secondary signals of lower power can easily be resolved as peaks of lower power level in the spectrum and will not be masked. Furthermore, Signals can be spatially close together for both MVDR and MUSIC and still be resolved. MUSIC exhibits the best resolution of the three [43], [57] and will be used exclusively for the simulations in Chapter 4.

As aperture size is increased, the performance of all three DOA algorithms improves [57], [61]. This is illustrated in Figure 2.19b. When element spacing increases beyond $\lambda/2$, however, the effects of grating lobes are introduced into the spectrum as extra spectrum peaks. Grating lobes form a kind of ambiguity because without external information identifying which peak in the spectrum is the real signal of interest cannot be determined. There are, however, other ambiguities that can occur. These ambiguities are discussed in the next section. All of these ambiguities will manifest as peaks in the DOA power spectrum.

2.3.3.4 Ambiguities in Direction of Arrival

If there exists a set of distinct DOA calculations for which the steering vector is rank deficient, there will be ambiguities in the spectrum [62]. In other words, if there is more than one steering vector that result in the same DOA calculation, an ambiguity is present [63]. Consider a URA in place of the ULA in Figure 2.17 performing a DOA measurement for a signal incident at broadside $(\phi, \theta) = (0, 0)$ for the coordination system in Figure 2.1. The plane wave will excite every element in the array at the same time. Therefore, the time delay Δt from equation (100) or equation (104) will be 0. However, a plane wave incident on the array at $(\phi, \theta) = (180, 0)$ will also excite every element in the array at time t with no time delay. Therefore, a DOA measurement taken with the steering vector for either direction will result in detection of two signals: one signal from each direction. This means an ambiguity will be present at 180 when the plane wave is at broadside and vice versa. Every incident direction for two-dimensional URAs arrays will inherently result in an ambiguity [64].

These ambiguities converge with the incident plane wave signal at endfire [52]. This convergence of the two power spectrum peaks or beam patterns causes the increase in width for both the power spectrum and the beam pattern in Figure 2.15 [52]. This convergence is shown for the DOA spectrum of a 4x4 URA with half-wavelength spacing in Figure 2.20. It should be noted that a grating lobe also forms at endfire for this array, and the DOA spectrum detects a second peak.

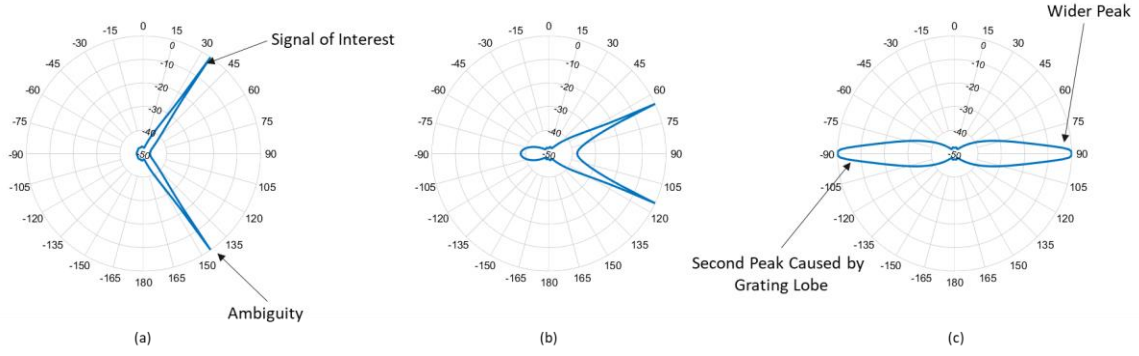


Figure 2.20 DOA Power spectrum of a 4x4 URA with $\lambda/2$ spacing performing a MUSIC DOA measurement. The incident signal is located at an azimuth of (a) 35 (b) 65 and (c) at 90 (endfire). The ambiguity and the incident signal converge at endfire.

These ambiguities exist for all linear and two-dimensional planar arrays as result of their planar geometry [64]. Effort has been made to utilize uniform circular arrays and arrays with no phase centers to eliminate and resolve ambiguities [65]-[66]. Rectangular planar arrays, however, are the focus of this thesis and the ambiguities for these geometries cause significant operational concerns.

2.3.4 Mutual Coupling Effects

Antenna operation is defined by the current distribution, and the magnitude and phase of the currents on the radiating elements. When elements are spaced in close proximity to one another as in an array, the radiated fields from one element will interact with those of the other elements. This interaction is known as mutual coupling and leads to deviations from ideal behavior. As the current distribution on each element changes, the impedance on each respective element will also change. Therefore, mutual coupling effects can be computed by looking at the impedances of the array elements. This is conveniently done by modeling each element as having its own generator with generator impedance. An array of n elements can be modeled as n -port network using circuit analysis as [11], [67]-[68]

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \cdots Z_{1n}I_n \quad (120)$$

$$V_2 = Z_{12}I_1 + Z_{22}I_2 + \cdots Z_{2n}I_n \quad (121)$$

$$\vdots$$

$$V_n = Z_{1n}I_1 + Z_{2n}I_2 + \cdots Z_{nn}I_n \quad (122)$$

where V_n and I_n are the applied voltages and currents to each element and Z_{nn} is the self-impedance of each element when the other elements are open-circuits. Z_{mn} represents the mutual impedance between elements in the array [11].

Dividing equation (122) by I_n gives the input impedance of any element in the array with all mutual coupling effects included. This is known as active impedance and is given by [11]

$$Z_n = Z_{n1} \frac{I_1}{I_n} + Z_{n2} \frac{I_2}{I_n} + \cdots Z_{nm} \frac{I_m}{I_n} \quad (123)$$

Mutual coupling effects impact more than just the input impedance of the array elements. As discussed, the radiating behavior of one element impacts the radiating behavior of another element in the array. This has significant impacts on the radiation pattern and beam steering behavior of the array as a whole [11], [67], [69]. It should be noted that the active impedance depends on the current in each element which contains phase information. Therefore, at different steering directions, different mutual coupling effects occur [11]. Mutual coupling also impacts DOA measurements [68], [70].

To compensate for mutual coupling, the mutual impedance terms from equations (122) and (123) must be determined. Once these mutual impedances are found, the mutual coupling matrix for the array can be generated. Applying the inverse of this mutual coupling matrix to the array excitation sources will offset the mutual coupling effects for the specified direction and approximate the ideal performance. There are several ways that the mutual coupling matrix can be calculated [55], [71]-[73].

2.4 HF Fundamentals

This section of the literature review transitions into HF propagation. The basic principles and operation of electromagnetic waves is maintained at HF as well as at other frequencies.

However, HF signals interact with the ionosphere in ways that higher frequency signals do not. These interactions present an additional challenge for ground-based radio telescopes. An orbiting array, such as the one proposed in this thesis, would avoid many of these challenges.

2.4.1 Frequency Spectrum

To understand the fundamentals of HF, the basics of the electromagnetic spectrum must first be discussed. The electromagnetic spectrum is the range of frequencies and wavelengths of all electromagnetic radiation propagating in space [74]. A visual representation of the electromagnetic spectrum is shown in terms of wavelength below in Figure 2.21.

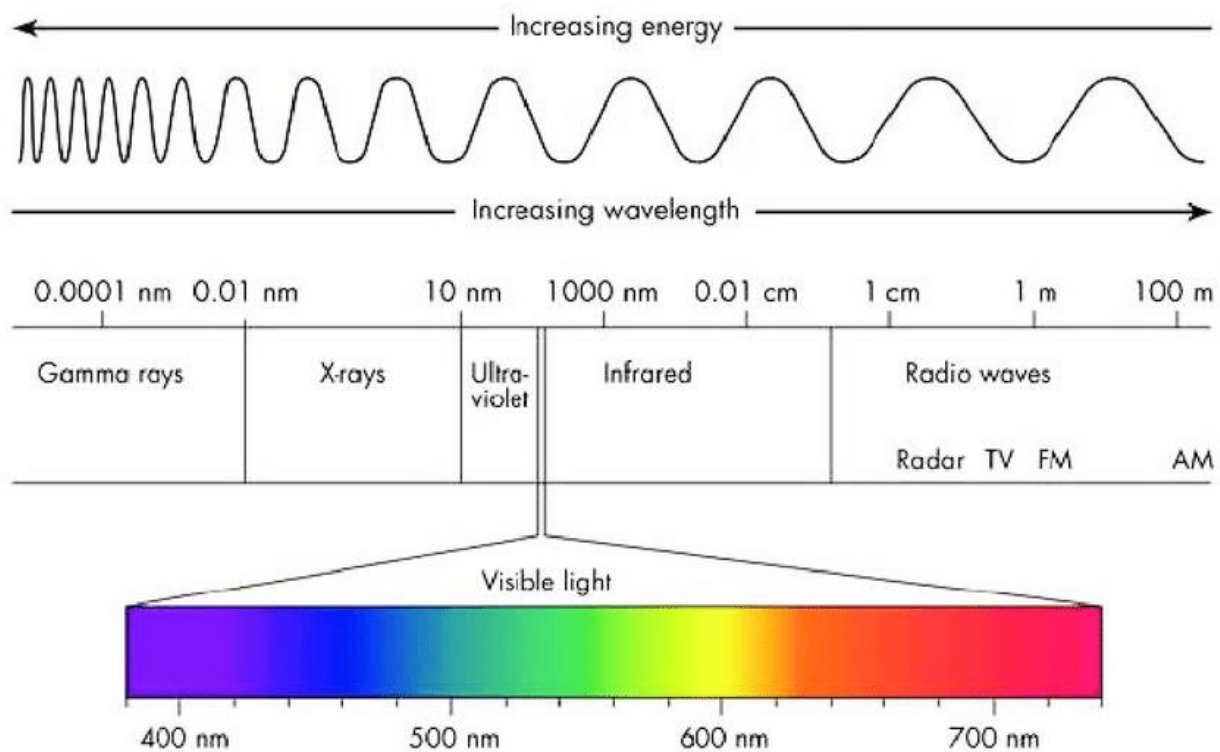


Figure 2.21 The electromagnetic spectrum. Figure taken from [75].

As can be seen in Figure 2.21, the electromagnetic spectrum covers a wide range of wavelengths and frequencies for an equally wide variety of purposes. The part of the electromagnetic spectrum that is of interest to this thesis is the radio wave spectrum, also called the radio frequency (RF) spectrum. The RF spectrum is defined as the subset of the electromagnetic spectrum with frequencies between 30 Hz and 300 GHz [76]. This is the section of the electromagnetic spectrum over which wireless communication and broadcasting takes place. The RF spectrum is broken down into even further subsets listed in Table 2.5.

From Table 2.5, HF is found to have frequencies ranging from 3 – 30 MHz, which corresponds to wavelengths of 10 – 100 m. Arrays built to operate at HF with half-wavelength spacing have element spacings on the order of 5 – 50 m for optimal performance. This is one major constraint imposed by HF operation. The other major constraints of the HF operating frequency band are given in terms of the ionospheric environment discussed in the next section.

Table 2.5 RF Spectrum Breakdown. Table taken from [76]

Frequency	λ	Band	Description
30 – 300 Hz	$10^4 - 10^3$ km	ELF	Extremely Low Frequency
300 – 3000 Hz	$10^3 - 10^2$ km	VF	Voice Frequency
3 – 30 kHz	100 – 10 km	VLF	Very Low Frequency
30 – 300 kHz	10 – 1 km	LF	Low Frequency
0.3 – 3 MHz	1 – 0.1 km	MF	Medium Frequency
3 – 30 MHz	100 – 10 m	HF	High Frequency
30 – 300 MHz	10 – 1 m	VHF	Very High Frequency
300 – 3000 MHz	100 – 10 cm	UHF	Ultra-High Frequency
3 – 30 GHz	10 – 1 cm	SHF	Super-High Frequency
30 – 300 GHz	10 – 1 mm	EHF	Extremely High Frequency

2.4.2 Ionospheric Effects on HF Signals and Electromagnetic Propagation

While many signals will reflect off the ionosphere and bounce back to earth, signals near the HF band can penetrate into the ionosphere and propagate through it. However, the ionosphere is not a static medium for radio signal propagation. Rather, it is split into layers based on the distribution of the density of charged particles. Furthermore, the density in each of these layers is time varying and is highly dependent on the geomagnetic field of the earth and can be affected by solar activity from the sun. This constant shifting of ionization causes electromagnetic waves to experience dispersive effects as they propagate through the ionosphere [7], [76]-[77].

Another effect experienced by electromagnetic waves in the ionosphere is known as Faraday rotation. Faraday rotation is the gradual rotation of the polarization of an electromagnetic wave as it propagates through the plasma medium of the ionosphere. This effect is usually experienced by linearly polarized signals [7]. A change in the polarization of a signal has a significant impact on the receiving array, especially if the array is designed to operate only at a specific polarization.

Similarly, the charged particles in the ionosphere interfere with the electromagnetic wave causing the propagating velocity to decrease. This ionosphere induced time delay depends on the frequency of the electromagnetic wave and the density of the charged particles in the ionosphere [7], [77]. This can have significant impacts on DOA measurements, as the time delay used for those calculations assumes a propagation speed equal to the speed of light. If a high enough density of charged particles is present for a transmitted signal at a given frequency in the ionosphere, then the propagation velocity may be delayed enough to cause errors in the DOA calculations.

Finally, a third major impact caused by the ionosphere is known as scintillation. Because different distributions of the ionosphere have different ionization densities, the propagation speed, the amplitude, and the phase can be varied by different amounts across the spatial distribution of the propagating wave [7]. When this wave encounters the receiving phased array, the electromagnetic wave that illuminates the array is no longer in phase and is therefore not a phase front. Thus, the DOA calculations from Section 2.3.3.3 no longer identify the exact position of the source of the electromagnetic wave, and additional, more complex analysis must be performed.

This thesis does not provide solutions for these issues and instead focuses exclusively on the design and operation of the three-dimensional array. In orbit, the proposed array will only be faced with these challenges when it is implemented for ionospheric and atmospheric studies and calibration for ground-based systems. For studies beyond the atmosphere, the ionosphere no longer acts as an obstacle and the array will avoid the challenges presented in this section.

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Chapter 3 – Three-Dimensional Array Design and Geometry

This chapter presents the design considerations for the space-assembled three-dimensional phased array. Geometry considerations for the design are presented with corresponding solutions. The components for the array design are then discussed. The chapter transitions to a discussion of geometric terms for the array that will be used in the rest of the thesis. Finally, a theoretical analysis is presented with the expected results due to the cubic geometry of the array.

3.1 Geometry Selection

The first decision regarding the three-dimensional array design was which geometry to implement. There were two major factors to be considered in selecting a geometry. The first factor to be considered was the simplicity of the design for simulation models and prototyping fabrication. Simplicity in the geometry can not only simplify the design but can also lead to simpler analysis. The second factor to be considered was practical application of the geometry to the end use-case of a space-assembled HF radio telescope. Ideally, a simple geometry with a regular, modular structure was desired for this use case. A cubic structure met both requirements and was readily selected for the end design and the prototypes developed for this thesis.

There are additional benefits to using a cubic structure for the array geometry. A cubic structure is inherently symmetric on all sides. This symmetry will make analysis far simpler than arbitrary three-dimensional geometries. Secondly, a cubic array is the natural extension of a two-dimensional URA. Thus, the performance of a cubic array can be easily compared to that of a conventional URA of the same number of elements.

3.2 Cubic Three-Dimensional Geometry Design

The end goal is for the array design to be easy to simulate and simple to fabricate, especially when considering the in-space assembled HF radio telescope end use-case. For this reason, the design presented in this thesis consists of small pieces that are integrated together in a regular structure.

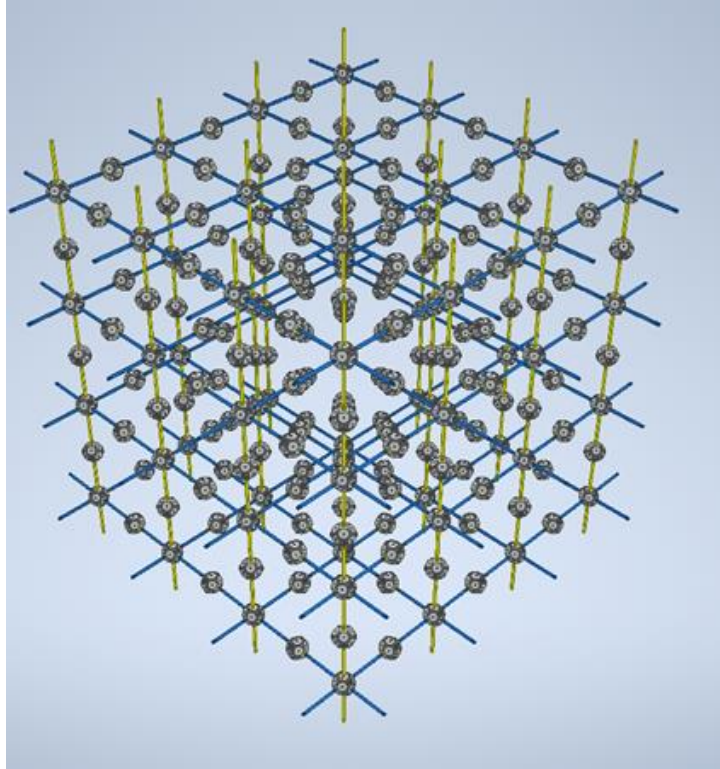


Figure 3.1 Proposed array design for in-space assembled HF radio telescope

The entire array structure consists of four major components: conductive rods for array dipole elements, nonconductive rods for spacing and support, spacers to connect the rods together, and feed-points (or excitation sources) for the array elements. The integration of these components to form the cubic design is shown in Figure 3.1. The vertical yellow rods represent the conductive rods for the array half-wave dipole elements, the horizontal blue rods represent the nonconductive rods that provide the spacing between the elements and structural support, and

the spacers are represented by the hexagonal-sphere shapes. It should be noted that the spacers are modeled with a size of 10 cm for easy implementation with 1U CubeSats.

This design implements the desired regular, modular structure with the added capability of tunability. Radio frequency integrated circuit (RFIC) switches within each spacer would allow for opening or shorting the spacer itself to connect or disconnect conductive elements. Through this process, dipole element sizes and spacings can be reconfigured for operation at different frequencies within the HF band. RFIC switches are not implemented in this thesis, however, the design presented herein intentionally includes the RFIC tunability as a feature that can be explored as future work on the project. A more detailed discussion is presented in Chapter 6.

One shortcoming of the design in Figure 3.1 is the limited polarization capability. The z-directed nature of the dipoles in the design limits the performance of the array to linear polarizations. As seen in Chapter 2, the polarization of an electromagnetic wave can be captured by two orthogonal vector components. To operate at all polarization states for antenna and array operation, two orthogonal sets of conductive rods are required. However, as the array tumbles, one of those sets of rods may align with the propagation direction of the electromagnetic wave. Therefore, a third set of orthogonal rods is needed. With three sets of orthogonal rods at each feed-point, no matter what the orientation, both vector components of the electromagnetic wave for any polarization will be measured. This is known as a hexapole and would be implemented by replacing the blue, nonconductive rods with conductive rods. More information on the hexapole is provided in Chapter 6.

The dipole element and the resulting polarization limitation was chosen intentionally for simplicity in simulation and analysis. The simulation environments assume co-polarized conditions; therefore, orthogonal sets of rods result in unnecessary computational complexity.

Additional rods also present scaling limitations for the array in the 4nec2 simulation discussed in Chapter 4. Finally, the dipole simplification also makes a prototype fabrication and laboratory measurement much more practical.

3.2.1 Detailed Analysis of Cubic Array Structure Design

The array elements for the cubic geometry design are z-directed half-wave dipoles. As mentioned in Chapter 2, these are dipoles that have a length that is approximately half of the wavelength, $\lambda/2$, of the dipole's resonant frequency. For the design in Figure 3.1, the smallest half-wave dipole element consists of a single feed-point in the center of two conductive rods. The regular structure of conductive rod, spacer, conductive rod, spacer, etc. is utilized for the purpose of tunability. If the spacers are shorted, rather than serving as feed-points for smaller dipole elements, they can contribute the length of the dipole element and connect conductive rods to form larger dipole elements. Regardless of operational frequency, dipole elements cannot physically touch or overlap in space. Therefore, a hollow spacer is required between dipole elements in the z-direction.

Another representation of the array design in a 4x4x4 configuration is presented in Figure 3.2. This is the same representation from Figure 3.1 with some helpful visualization. There are four different colored hexagonal-like shapes and two different colored rods. The dark colored vertical rods represent the z-directed half-wave dipole conductive rods. The orange hexagonal solids at the intersections of vertical dark rods and the horizontal blue rods represent spacers or CubeSats that act as feed-points for the half-wave dipoles. The horizontal blue rods represent nonconductive structural components which are used to provide the desired spacing in the x- and y-directions between array elements. The white hexagonal shapes between the blue rods represent hollow spacers or CubeSats which are used to connect the spacing/structural rods to

maintain the x- and y-spacing between array elements. Finally, the maroon-colored hexagonal shapes placed between the vertical dark rods represent hollow, empty spacers that maintain the z-spacing between array feed-points. It should be noted that the hollow and empty spacers along the z-axis would still contain RFIC switches in a full implementation of the design.

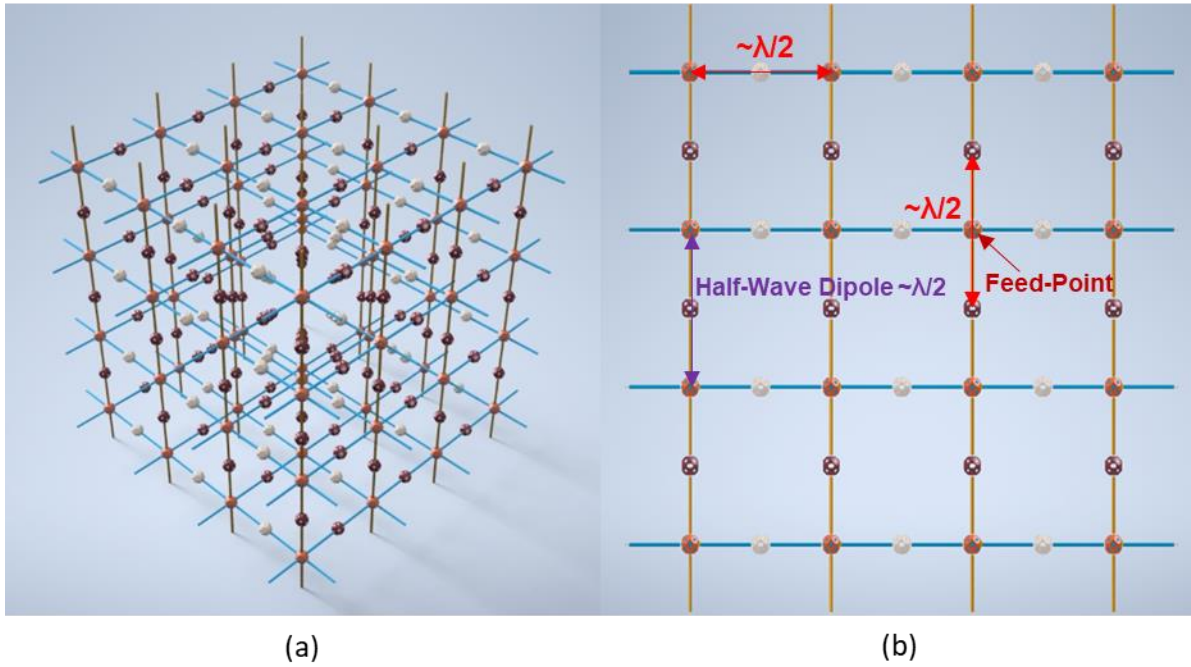


Figure 3.2 4x4x4 64-element array design on (a) all x-, y-, and z-axes and (b) on the YZ plane

Figure 3.2b shows the YZ two-dimensional plane of this three-dimensional array design. The half-wave dipoles are directed vertically along the z-axis. Between two vertically adjacent feed-points, there are two conductive rods and one hollow spacer. However, the actual feed-point connection is the exact center of the spacer with respect to the z-axis. Therefore, there is an additional spacer length between two feed-points. The vertical feed-point to feed-point distance is then: length of dipole + length of spacer. The horizontal distance is computed the same way. The spacing in the x-direction is identical to the spacing in the y-direction due to the symmetrical geometry. In this configuration, the spacing in all three directions is identical and the cubic geometry is maintained.

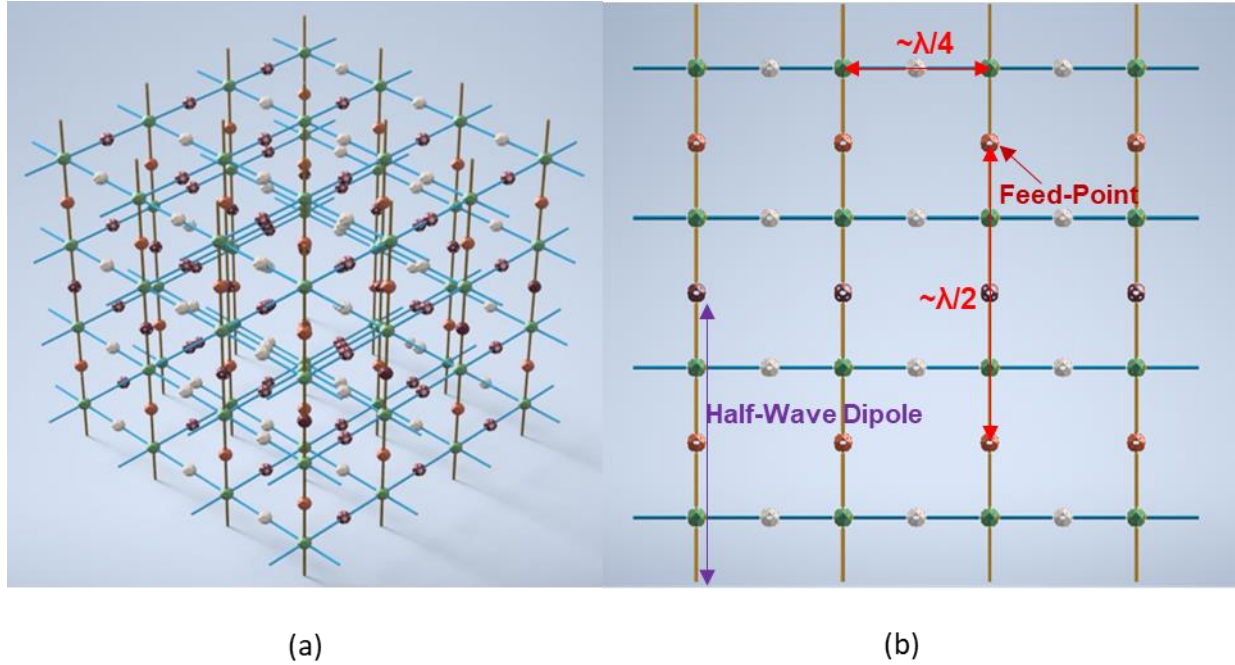


Figure 3.3 4x2x4 32-element array reconfiguration of the 4x4x4 64-element array from Figure 3.2 on (a) all x-, y-, and z-axes and (b) on the YZ plane

The RFIC switches inside the spacers between conductive elements could be opened or closed to reconfigure the array into other forms such as the one shown in Figure 3.3. In this configuration, the array operates at approximately half the frequency as that in Figure 3.2. The green hexagonal shapes for this configuration represent spacers or CubeSats that have been shorted to connect conductive elements together to form larger dipoles. In this configuration, the z-distance from feed-point to feed-point is maintained around approximately one half-wavelength. Likewise, the dipole length itself is kept at approximately one half-wavelength. However, the x- and y-spacings have not changed and are approximately one quarter-wavelength spacing for this configuration. Because the spacing is not symmetric in all three dimensions, the cubic nature of the geometry is lost. This is an important consequence that will be discussed in the next section.

It should be noted that the physical dimensions of the array in Figure 3.2 and Figure 3.3 have

not changed. Only the size of the dipole elements and the spacing between the dipole elements is changed. It is also worth re-emphasizing that RFIC switches are not implemented in this thesis, however, the reconfiguration of the array can be implemented in simulation by restructuring a model as was done in Figure 3.3.

3.2.2 HF Cubic Geometry Considerations

As mentioned in Chapter 2, an HF array operates between 3 and 30 MHz. This frequency range corresponds to wavelengths between 10 and 100 m. For half-wave dipoles, the element size and the spacing between elements is limited to between 5 and 50 m. The design for the array approximates that of a cubic geometry. The goal is to maintain that approximate cubic geometry at different frequencies within the HF band. As seen in the previous section and in Figure 3.3, certain reconfigurations can cause the cubic geometry to be lost. For the purposes of this thesis, such reconfigurations are not useful. To maintain cubic geometry for the HF frequency band, a cube must be designed at the lowest frequency of 3 MHz. The goal is to implement elements that are sized to operate at the highest frequency of 30 MHz such that a cube that operates 30 MHz can be reconfigured to an approximate cube that operates at 3 MHz.

The smallest cube possible is an 8-element 2x2x2 configuration. As mentioned, at 3 MHz, the element size and spacing required is 50 m. The element size for 30 MHz is approximately 5 m. Therefore, ten 30 MHz-elements can be combined to form a single 3 MHz-element, as shown in Figure 3.4. To form two vertical 3 MHz-elements would then require approximately twenty 30 MHz-elements. Furthermore, these elements cannot physically overlap in space, therefore additional empty spacers or CubeSats between each of the 30 MHz elements as well as one to separate the 3 MHz elements are required. A total of 19 spacers per 3 MHz element and one spacer between 3 MHz elements results in 39 spacers for 2 z-directed 3 MHz half-wave dipoles

in a single column. To maintain the cubic geometry, a 39x39x39 spacer structure is required.

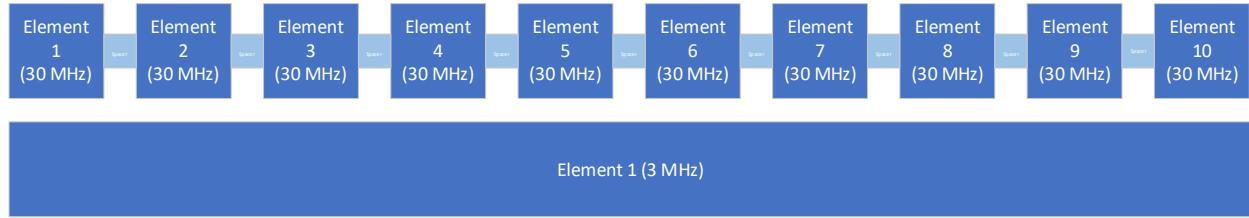


Figure 3.4 Ten 30 MHz elements combine with nine spacers to form a single element at 3 MHz

In Figure 3.5, all the dots represent the spacers or CubeSats. Figure 3.5a shows 19 spacers being combined to form individual 3 MHz elements. It should be noted in Figure 3.5a where the elements are formed; the yellow lines represent where the elements are formed in the 3 MHz configuration whereas the red boxes provide useful illustration for the reader. There are enough spacers to form additional 3 MHz elements. However, by forming these elements, the cubic geometry of the array is lost and a rectangular solid is formed as was shown in Figure 3.3. Because this thesis focuses specifically on cubic geometry, these extra elements are non-desirable. Therefore, the additional 3 MHz elements are excluded. For symmetry purposes, the central elements in Figure 3.5a are selected to form the 3 MHz cube.

At 30 MHz, there are a lot more elements that can be implemented. The 2x2x2 3 MHz cube becomes a 20x20x20 30 MHz cube with each element consisting of a single spacer acting as a feed-point and 2 conductive rods, as in Figure 3.2. Every other spacer acts as an open circuit. This structure is shown in Figure 3.5b. Therefore, the design shown in Figure 3.1 can be implemented to form a regular structure that can be in-space assembled that could be tunable to operate at 3 MHz and 30 MHz in the HF band with while retaining cubic array geometry with various aperture sizes.

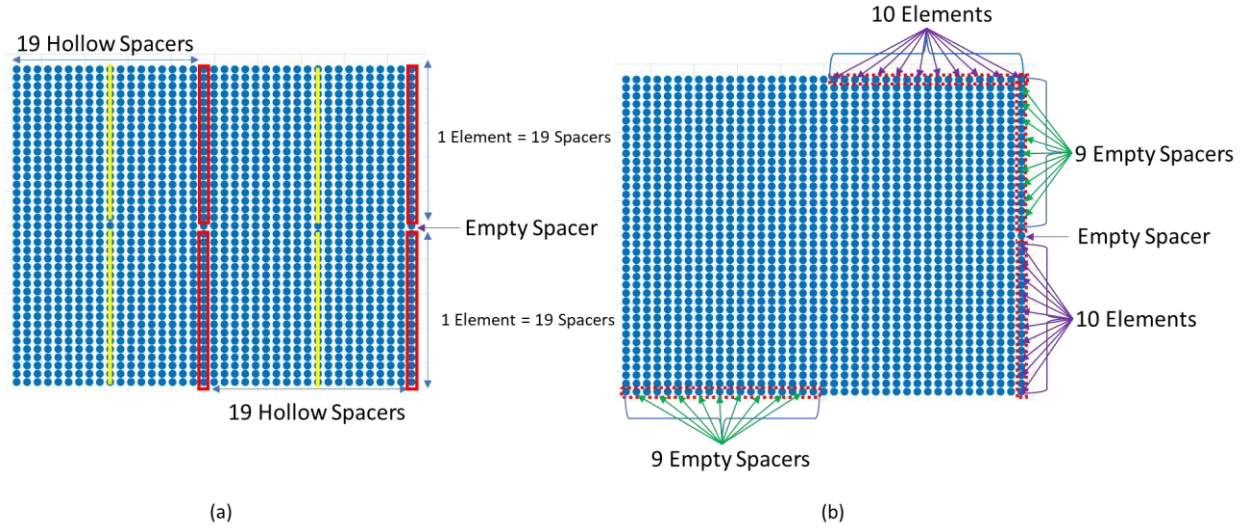


Figure 3.5 A single face of a 39x39x39 spacer cube that can be configured into (a) a 2x2x2 8-element 3 MHz cube or (b) a 20x20x20 8000-element 30 MHz cube. The yellow lines in (a) represent the actual location of the array elements.

One final note regarding the example in Figure 3.5 is that of the mutual coupling effects. The same amount of metal is present in the array regardless of the frequency of operation. At 30 MHz, every conductive rod in the array is being used by an array element. At 3 MHz, however, there are spacers with conductive elements that are not in use by the array that are placed between and around the larger elements that are being used in the array. This excess metal from the elements that are not in use could have substantial effects on the mutual coupling performance at different frequencies.

3.2.3 Array Element Specifications

As mentioned, the ideal component to implement as a spacer would be a 1U CubeSat that is a 10 cm cube. The overall size of a single dimension of the cubic array structure from Figure 3.1 for N 30 MHz elements is

$$\text{Size} = 2N * (\text{Length of Conductive Rod}) + (N + (N - 1)) * 0.1 \text{ m} \quad (124)$$

At 3 MHz, the size of the half-wave dipole needs to be slightly shorter than 50 m. Using Figure

3.4 and substituting 50 m for Size in (124) results in

$$50 \text{ m} = 2 * 10 * (\text{Length of Conductive Rod}) + (10 + (10 - 1)) * 0.1 \text{ m} \quad (125)$$

$$(\text{Length of Conductive Rod}) = 2.405 \text{ m} \quad (127)$$

Additionally, in order to avoid grating lobes, the spacing between elements in all three dimensions is kept under 0.5λ . Therefore, the length of the conductive rod components needs to be reduced slightly to ensure that grating lobes are avoided. For this reason, the length of the conductive and nonconductive rod lengths are identically selected as 2.3 m. The results for the 3 MHz and 30 MHz elements are shown in Table 3.1

Table 3.1 Array Element Specifications

Frequency	Wavelength	Element Length	Element Spacing	% Shorter than $\lambda/2$
3 MHz	100 m	47.9 m	48 m	4.2
30 MHz	10 m	4.7 m	4.8 m	6

3.2.4 Design Pros and Cons

The major benefit to the design in Figure 3.1 is the regular, modular structure for an in-space assembly process. The solid cube geometry provides symmetry for the analysis and fabrication process as well as allowing for tunability to different frequencies.

However, in order to achieve the desired cubic geometry that is desired, a substantial number of elements is required. The high element count increases the cost of the system as well as introducing complexity in the assembly process. Furthermore, high number of elements leads to increase power consumption, increased data processing requirements, and an increase in mutual coupling effects.

3.3 Geometric Terms

Throughout this thesis there are references to different groups of elements within the array structure. Some definitions for these terms and visual representations for each are provided:

- *Panel* is used to refer to a singular face of 16 elements of the cubic array. The panels are listed in terms of letters A, B, C, and D with A being the outermost and broadside face. Using the coordinate system in Figure 2.1, Panel A is the furthest along the +x-axis. The panels are represented in Figure 3.6a.
- *Stack* is a vertical grouping of 4 elements in the same panel. There are 4 stacks in each panel. The first stack of each panel A1, B1, C1, and D1 is represented in Figure 3.6b.
- *Plane* refers to a grouping of stacks from different panels. Stacks A1, B1, C1, and D1 from Figure 3.6b form the plane shown Figure 3.6c.
- *Cut* refers to a cross section that is not defined by a plane, panel, or stack. There are four cuts in the array structure. Figure 3.6d represents cut 1. The other cuts proceed vertically downward.

Each element in the array also has a corresponding index number. Panel A consists of elements 1-16, Panel B consists of elements 17-32, etc. Therefore, Stack A1 consists of elements 1-4, Stack A2 consists of elements 5-8, etc. From these notations any grouping of any size of elements within the array can be properly identified.

3.4 Predictions of Cubic Three-Dimensional Geometry Performance

This section briefly presents the hypothesis and expectations of the proposed array performance based on its three-dimensional geometry.

As seen in Chapter 2, two-dimensional arrays face two major shortcomings: the ambiguity seen in DOA measurements and the broadening of the beam and DOA spectrum peak when

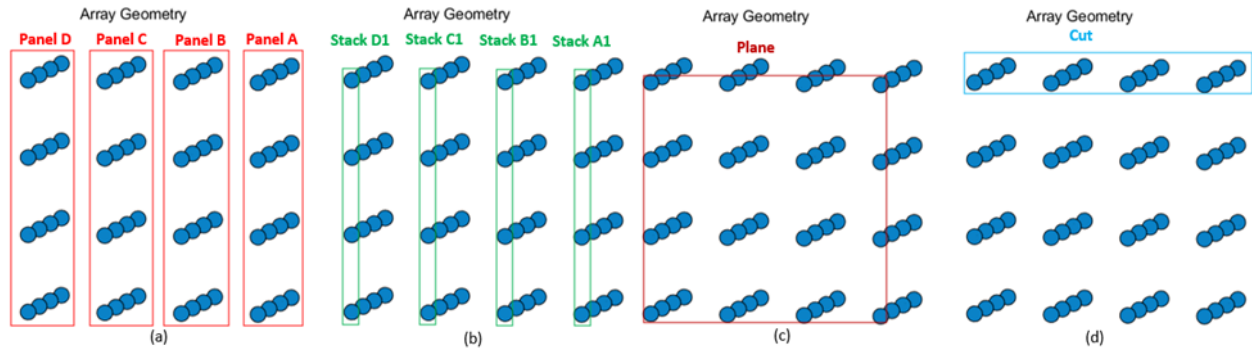


Figure 3.6 Geometric terms for cubic geometry: (a) panels, (b) stacks, (c) planes, and (d) cuts operating near endfire. Consider again the URA in Section 2.3 with the plane wave incident at broadside. The reason the ambiguity is observed at 180 is because the time delay between when the plane wave interacts with all the elements in the array is identical from both directions. For a cube, however, this is not the case. If a plane wave is incident at broadside for the cube in Figure 3.7, the elements in Panel A will be excited before the elements in Panel B, which will be excited before the elements in Panel C, etc. However, if the plane wave is incident from 180, then the elements in Panel D will be excited before the elements in Panel C, etc. Therefore, by adding the third dimension to the array, an incident signal from any direction in space should have a unique DOA solution. Therefore, it is hypothesized that these ambiguities are not seen by the cubic structure DOA measurements.

It should be noted that there could still be ambiguities caused by grating lobes. These ambiguities are brought about by the spatial aliasing caused by an insufficient spacing of elements. A third geometric dimension should not remove these effects. Therefore, the element spacing effects for two-dimensional arrays are expected to be maintained at three-dimensions. However, the spacing in this design is intentionally decreased in order to remove grating lobes from the analysis altogether.

Furthermore, because the proposed design is a cube, each face of the array is identical. At 90 degrees off broadside (endfire), the signal will be incident on an identical surface as that of a signal incident at broadside. Therefore, the cubic geometry should completely remove the beam broadening endfire effects as there is no endfire for the array. Rather, the array performance at “endfire” should be identical to that at broadside shifted by 90. It is hypothesized, therefore, that the cubic geometry should solve both major shortcomings of two-dimensional URAs.

It was shown in Chapter 2 that performance of an array is best at broadside and gradually decreases until endfire. Because endfire for the cubic array is another broadside face, it is anticipated that the array performance will degrade until about 45° off broadside, and then increase until “endfire” where another broadside face is reached. However, as Figure 3.7 shows, the aperture area of the cubic array at 45° off broadside is larger than the aperture at broadside. Additionally, the spacing between elements as viewed at azimuth = 45, is smaller than when viewed at broadside. Therefore, it is possible that azimuth = 45 could have the best performance of any orientation of the cube.

The cubic design from Figure 3.1 is a filled cube. It is not anticipated that the additional elements in the interior of the cube will increase performance. As seen in Chapter 2, the aperture of the array is determined by the area of the array, not its volume. Therefore, the aperture area of the 4x4x4 configuration is the same as that for a 4x4 URA on a single face at broadside. Therefore, a two-dimensional URA with the same number of elements should out-perform the cubic structure at broadside in terms of HP and SLL. However, a two-dimensional URA will exhibit ambiguities and endfire effects, whereas the cubic geometry will not.

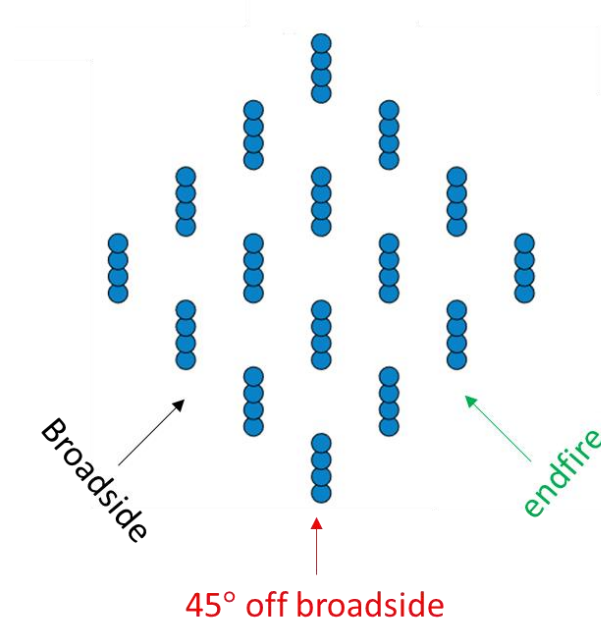


Figure 3.7 Near-top view of cubic geometry with wave fronts incident at different angles

As mentioned, the interior elements are present in the array to allow a tunable structure that can be reconfigured to different dipole sizes and spacings at different frequencies. It is highly anticipated that these elements will have significant impact on mutual coupling effects.

Chapter 4 – Array Design Simulations

The focus of this chapter is the simulation behavior of the three-dimensional cubic array from Chapter 3. All simulations are run assuming the design given in Figure 3.1. Each array configuration is simulated to determine the performance of beamforming and the performance of (DOA) measurements. Simulations are performed using MATLAB 2022a and 4nec2.

Of specific interest in this chapter is the performance of the cubic phased array structure at 30 MHz at broadside, endfire, and the azimuth = 45 out of the corner edge of the cube as the array increases in size. The focus then shifts to tunability and reconfiguration, performing the same simulations at 10.34 MHz. The mutual coupling effects at both frequencies are also investigated. Additional figures for each section are provided in Appendix A1.

4.1 Modeling Software Selection and Assumptions

4.1.1 MATLAB

MATLAB conveniently includes a phased array toolbox. This toolbox allows rapid modeling and characterization capabilities of arbitrary array geometries in terms of pattern generation, beamforming, DOA measurements and much more. However, these models ignore mutual coupling effects and only present ideal behavior. One useful feature of these toolboxes is user control over element size, type, and material. It is important to note that MATLAB will model the dipoles as open-ended hollow cylinders and assumes a co-polarized environment.

4.1.2 4nec2

The 4nec2 software includes the mutual coupling in array pattern generation through the method of moments numerical approach. The method of moments is a numerical technique for approximating electromagnetic integrals in the frequency domain with a system of linear

simultaneous equations [1]-[2]. The analysis in Chapter 2 was performed assuming specific current distributions. With the method of moments, however, any arbitrary current distribution can be used because electromagnetic radiation can always be stated in the form of the following integral [1]

$$-\int I(z')K(z,z')dz' = E^i(z) \quad (127)$$

where $I(z')$ is the unknown current distribution, $E^i(z)$ represents the inhomogeneous source term, and $K(z,z')$ is a kernel that depends on the integration that is performed. A popular form used for this integral kernel is the Pocklington form [1]. The integral in equation (127) can be solved as a set of linear simultaneous equations for $I(z')$. Once $I(z')$ is found, analysis from Chapter 2 can be used to find the impedance terms. For more information on the method of moments and how it can be used to solve radiation problems, see [1] and [2].

In contrast to MATLAB, 4nec2 models the array element exclusively as a solid cylinder. 4nec2 also includes ability to perform beamforming operations by implementing phase and magnitude shifts on the excitation sources if the required beamforming weights are computed from an external source. Therefore, 4nec2 can be used to investigate mutual coupling effects on patterns and beamforming applications. However, 4nec2 cannot be used to perform direct DOA calculations. Similarly to MATLAB, 4nec2 also assumes a co-polarized environment.

Because both software packages fall short in one way or another, both will be used in conjunction to perform the simulations in this thesis. Because certain assumptions such as the hollow vs filled cylinder are inconsistent between the software packages, results may vary slightly. Finally, the precision for the MATLAB simulations is far greater than the precision permitted in the 4nec2 simulations. As a result, there is a margin of error in the 4nec2 simulation results.

4.1.3 4nec2 Limitations

To perform the integration in equation (127), the individual dipole antenna elements must be broken into segments, and the analysis is performed per segment per element. The more segments per element, the more accurate the simulations. However, there are limitations to the size of the segments that be used in 4nec2 as well as a maximum number of segments that be implemented in a system, approximately 11,000. Furthermore, 4nec2 restricts the number of excitation sources, and therefore elements in a system, to 256.

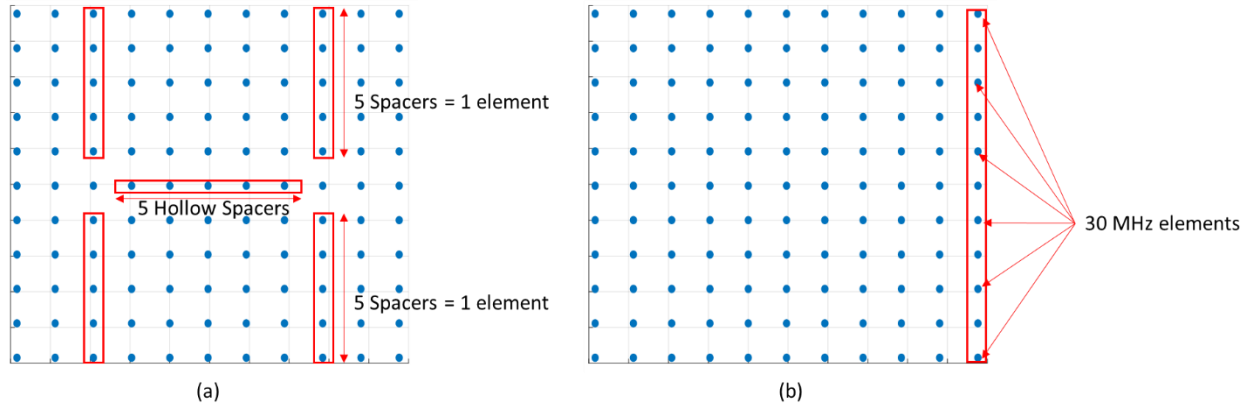


Figure 4.1 Single face of an 11x11x11 spacer cube that corresponds to (a) a 2x2x2 8-element 10.34 MHz cube that can be reconfigured to a (b) 6x6x6 216-element 30 MHz cube. The blue dots represent every spacer in the array configuration. In (b) every second dot represents a 30 MHz array element.

4.2 Three-Dimensional Array Configuration Software Models

Due to the constraints imposed by 4nec2, the largest array that can be modeled is a 6x6x6 216-element cube. This cube consists of 11x11x11 spacer components and is not nearly large enough to be reconfigured to a cube that operates at 3 MHz. The simulations in this chapter will focus on the frequency range that is achievable through the simulation tools. The simulation models presented in this chapter are a 4x4x4, a 5x5x5 and a 6x6x6 cube at 30 MHz. Each of these models is designed to operate at 30 MHz and be reconfigured to a smaller cube that operates at a lower frequency in the HF band. The process from Chapter 3 is used to determine

the minimum operational frequency for each configuration. The 6x6x6 216-element 30 MHz cube can be reconfigured to a 2x2x2 10.34 MHz cube as illustrated below in Figure 4.1. The results for all the models are presented in Table 4.1.

Table 4.1 Operational Parameters for Simulation Models

Spacers	Elements (30 MHz)	Maximum Length	Lowest Frequency	Number of Elements at Lowest Frequency
7x7x7	4x4x4	9.5 m	15.8 MHz	2x2x2
9x9x9	5x5x5	11.9 m	12.5 MHz	2x2x2
11x11x11	6x6x6	14.3 m	10.34 MHz	2x2x2
39x39x39	20x20x20	47.9 m	3 MHz	2x2x2

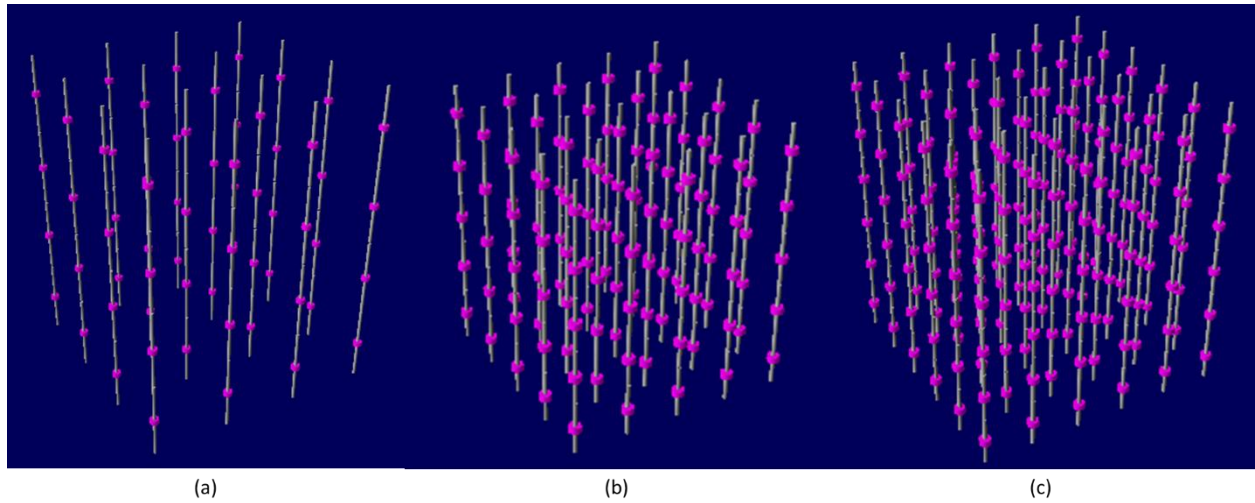


Figure 4.2 4nec2 software models of the (a) 4x4x4 64-element configuration, (b) 5x5x5 125-element configuration, and (c) 6x6x6 216-element configuration. All models consist of identically sized components that are identically spaced and operated at 30 MHz. The pink solids represent the excitation sources and indicate the number of elements in each configuration.

The half-wave dipole conductive rods for each simulation model are identical and are equal to the 2.3 m as determined in Chapter 3. Furthermore, the conductive rods are chosen as 0.25-inch-thick aluminum rods. The nonconductive rods are modeled as free space, as they have no direct impact on the electrical performance of the array. Likewise, the spacers that are hollow are represented as free space, whereas the spacers that act as feed-points are represented by the

voltage excitation sources. Spacers that are shorted together to form longer dipole elements are represented in the longer dipole element length. In all cases, the spacer is modeled as a 10 cm increase in length, or gap of free space. Each of the models is presented in Figure 4.2.

4.3 30 MHz Simulations

Each of the array configurations shown in Figure 4.2 is simulated at 30 MHz to determine the beamforming performance and the performance of DOA measurements. These simulations are presented in their own respective sections below. General array patterns and more comprehensive plots are presented in Appendix A1.1.

4.3.1 Beamforming Simulations

Each of the array configurations undergoes beamforming simulations. The MATLAB Phased Array Toolbox permits the user to calculate the beamformer weights for the elements in an arbitrary array to form a beam in a specified direction with respect to azimuth ϕ and elevation θ in the coordinate system from Figure 2.1. These weights are determined per element using a conventional beamformer in MATLAB. To accurately reflect the beamformer weights from the MATLAB model in the 4nec2 software model, the indices of each element were designed with a 1 to 1 mapping between simulation tools as defined in Section 3.3. Therefore, the weights computed in MATLAB could be directly applied as real and imaginary components to the excitation sources in the 4nec2 software.

The beam directions chosen for this thesis are the broadside (azimuth, elevation) = (0,0) direction, the endfire (azimuth, elevation) = (90,0) direction, and out of the corner edge (azimuth, elevation) = (45,0) direction. The plots for the ideal MATLAB model and the mutual coupling model are provided for each direction. Because the elevation is constant, the main observation between broadside and endfire is observed in the H-plane pattern. Therefore, only the H-plane

patterns are presented in this chapter. The E-plane patterns are presented in Appendix A1. All of the plots in are normalized, but the pattern parameters of interest are tabulated for each angular direction following each figure. Both rectangular and polar plots are provided for each simulation.

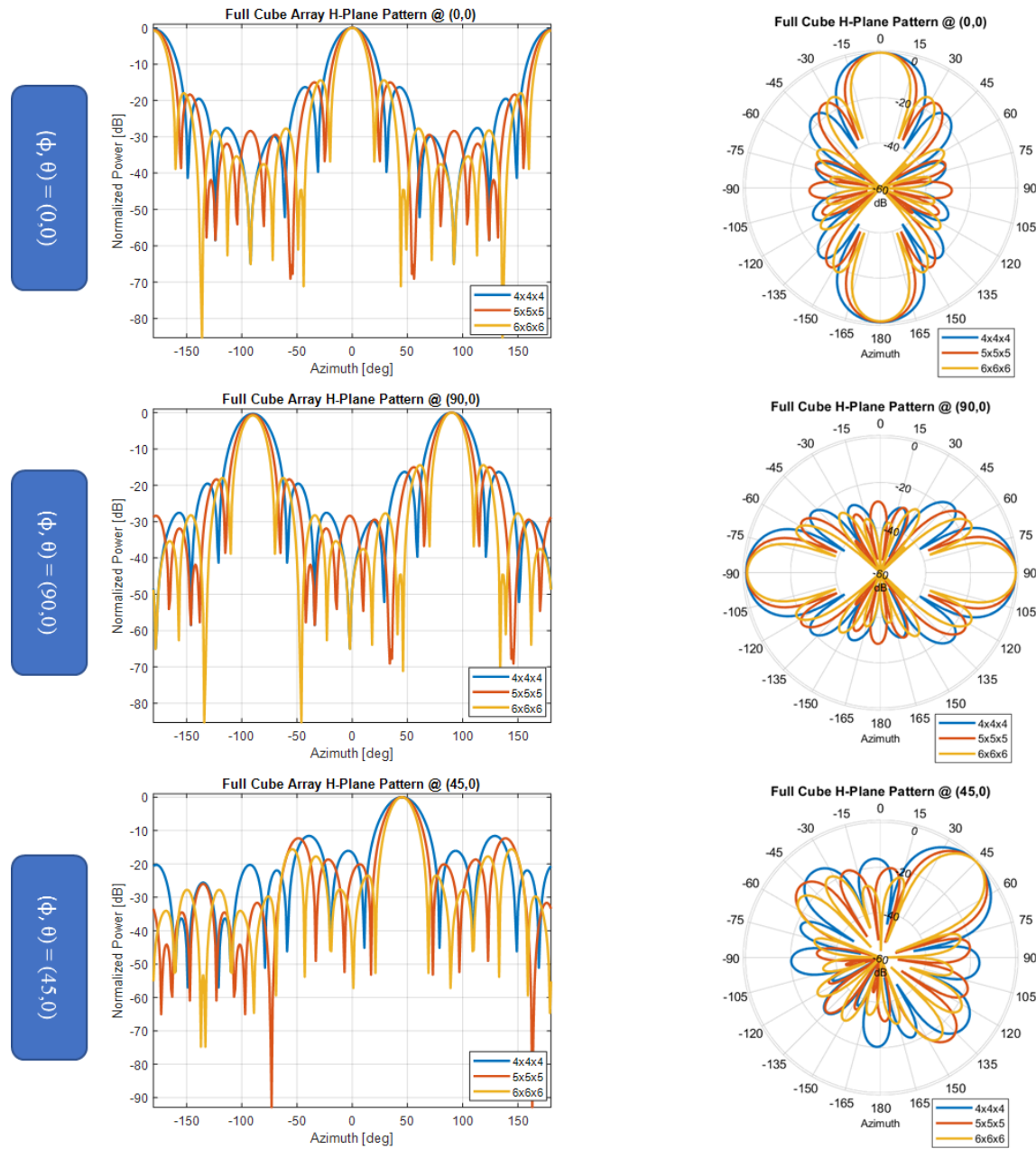


Figure 4.3 H-Plane pattern beamforming behavior for ideal cubic model in rectangular and polar coordinates at 30 MHz

Table 4.2 Simulation H-Plane Pattern Parameters for the Ideal MATLAB Plots in Figure 4.3 at $(\phi, \theta) = (0,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	27.22	-16.3127	14.6668
5x5x5	21.54	-15.0124	16.6552
6x6x6	17.84	-14.4211	18.3098

Table 4.3 Simulation H-Plane Pattern Parameters for Ideal MATLAB Plots in Figure 4.3 at $(\phi, \theta) = (90,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	27.22	-16.3127	14.6668
5x5x5	21.54	-15.0124	16.6552
6x6x6	17.84	-14.421	18.3098

Table 4.4 Simulation H-Plane Pattern Parameters for the Ideal MATLAB Plots in Figure 4.3 at $(\phi, \theta) = (45,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	27.72	-16.0769	16.7763
5x5x5	21.92	-20.0887	18.772
6x6x6	18.16	-23.4624	20.5108

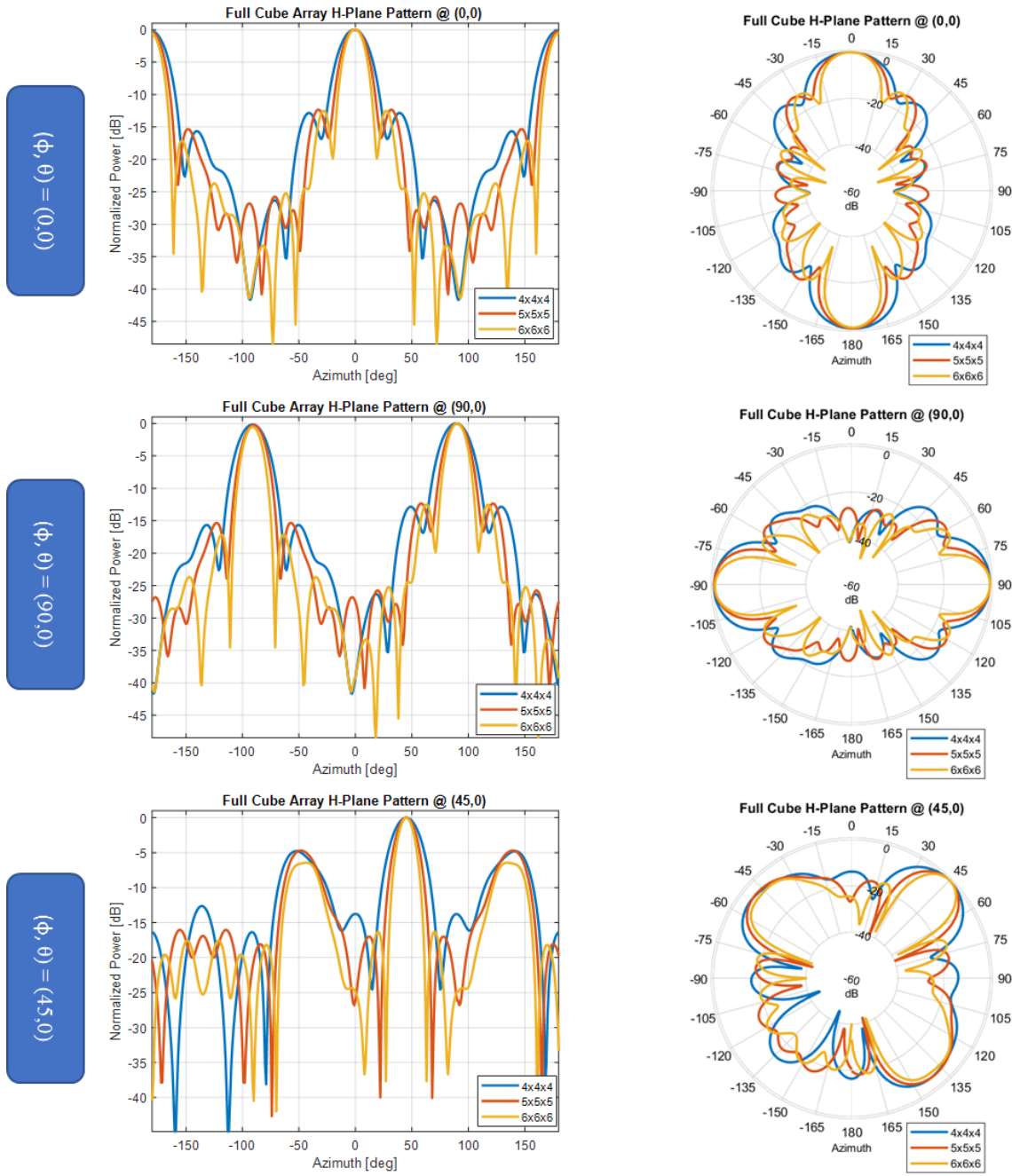


Figure 4.4 H-Plane pattern beamforming behavior for cubic model with mutual coupling effects in rectangular and polar coordinates at 30 MHz

Table 4.5 Simulation H-Plane Pattern Parameters for the Mutual Coupling 4nec2 Plots in Figure 4.4 at $(\phi, \theta) = (0,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	26	-13.77	14.86
5x5x5	20	-16.98	16.46
6x6x6	16	-16.29	18.45

Table 4.6 Simulation H-Plane Pattern Parameters for the Mutual Coupling 4nec2 Plots in Figure 4.4 at $(\phi, \theta) = (90,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	28	-12.84	114.86
5x5x5	21	-12.34	16.47
6x6x6	18	-12.51	18.45

Table 4.7 Simulation H-Plane Pattern Parameters for Mutual Coupling 4nec2 Plots in Figure 4.4 at $(\phi, \theta) = (45,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	26	-12.84	14.86
5x5x5	20	-12.34	16.47
6x6x6	18	-12.51	18.45

4.3.2 Direction of Arrival Simulations

Additionally, each configuration undergoes DOA simulations with simulated sources located at the same angles as the beamforming directions in the previous section. All DOA simulations are performed using MATLAB and represent ideal performance. When the Bartlett DOA

algorithm is used, identical behavior to that of the beamformer results from Section 4.3.1 is found. Therefore, reciprocity is maintained for the three-dimensional cubic geometry. However, as mentioned in Chapter 2, the Bartlett DOA spectrum is not high resolution. Therefore, the MUSIC algorithm is used. The MUSIC DOA simulation results are shown in Figure 4.5. The plots have been normalized for this simulation as well.

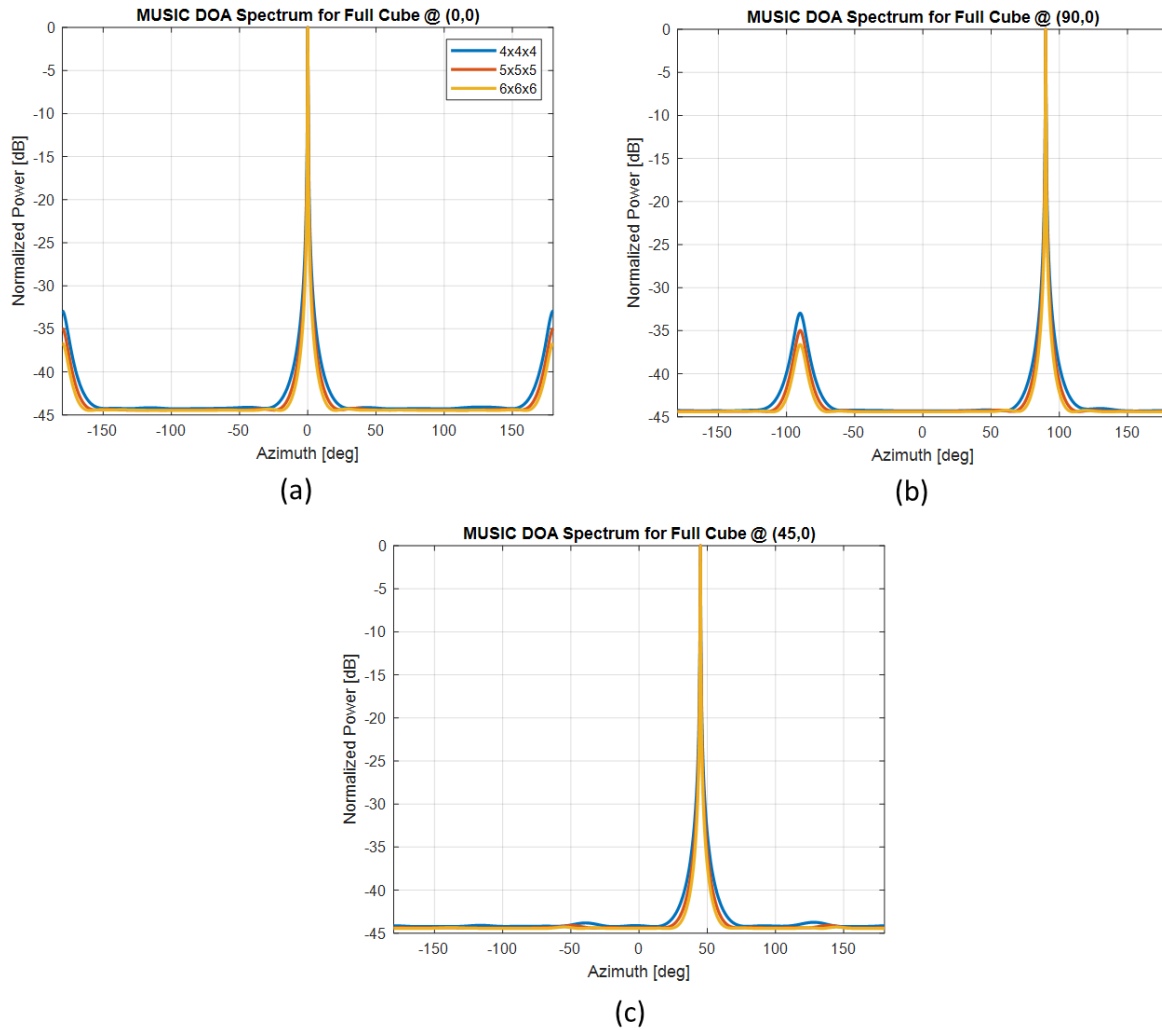


Figure 4.5 The MATLAB ideal DOA power spectrum plots for the full cube simulation models at (a) (azimuth, elevation) = (0,0) (broadside), (b) (azimuth, elevation) = (90,0) (endfire), and (c) (azimuth, elevation) = (45,0) at 30 MHz

4.3.3 Mutual Coupling Effects

Up until this point, each array model has been compared within the respective simulation environments. In this section, the ideal and non-ideal models are compared. Because the 6x6x6 full cube is the largest array and most closely approximates the in-space assembly array, it will

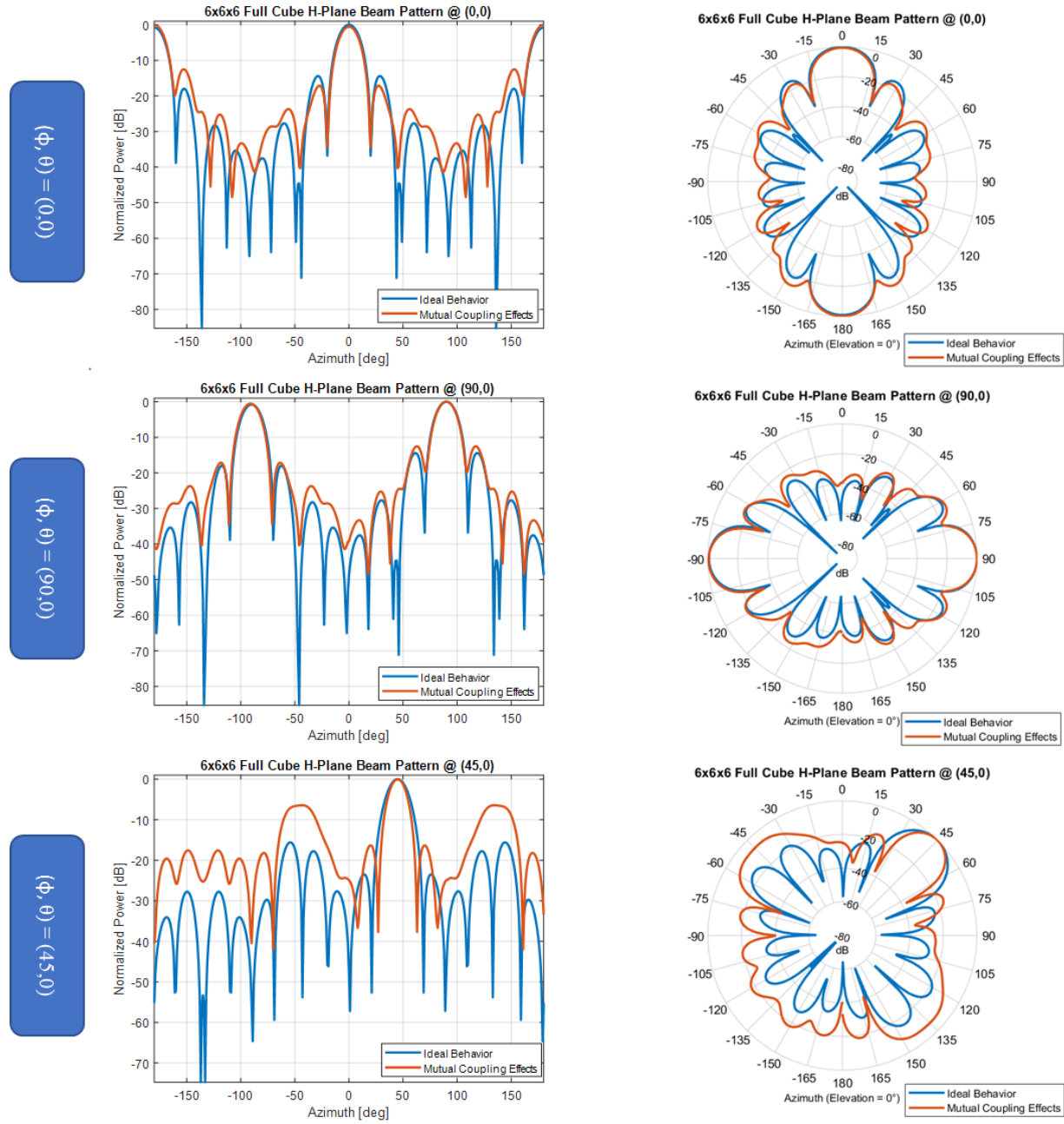


Figure 4.6 Ideal behavior vs mutual coupling effects of the 6x6x6 full cube model at 30 MHz

be used for the mutual coupling comparisons. The plots for the 6x6x6 array from the previous sections are repeated comparing the ideal MATLAB model to the effects of mutual coupling in 4nec2. Once again, these plots are normalized and the parameters for each angular direction are provided in separate tables. The E-plane patterns are provided in Appendix A1.

Table 4.8 Simulation Pattern Parameters for plots in Figure 4.6 at $(\phi, \theta) = (0,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
Ideal	17.84	-14.4211	18.3098
Mutual Coupling	16	-12.51	18.45

Table 4.9 Simulation Pattern Parameters for plots in Figure 4.6 at $(\phi, \theta) = (90,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
Ideal	17.84	-14.4211	18.3098
Mutual Coupling	18	-12.51	18.45

Table 4.10 Simulation Pattern Parameters plots in Figure 4.6 at $(\phi, \theta) = (45,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
Ideal	18.16	-23.4624	20.5108
Mutual Coupling	15	-16.29	18.45

4.4 10.34 MHz Simulations

Because 4nec2 limits the number of elements to 256, the largest cubic array that can be simulated is a 6x6x6 216-element array at 30 MHz. As discussed in Section 4.2, the lowest frequency cubic structure that can be reconfigured from the 6x6x6 30 MHz cube is a 2x2x2 cube that operates at 10.34 MHz. This new configuration is shown below in Figure 4.7. In this configuration, the 10 cm spacers for the 10.34 MHz elements would be shorted together to form the 14.3 m long elements while the spacers for the surrounding 30 MHz elements remain open. Therefore, surrounding each 10.34 MHz element are stacks of four 30 MHz elements that have their feed-point excitation sources turned off. While 10.34 MHz is not the lower bound of the HF band of interest, the concept of a reconfigurable resonant cubic array that operates in the HF band is still demonstrated. The same simulations from Section 4.3 are performed for this new array configuration at 10.34 MHz.

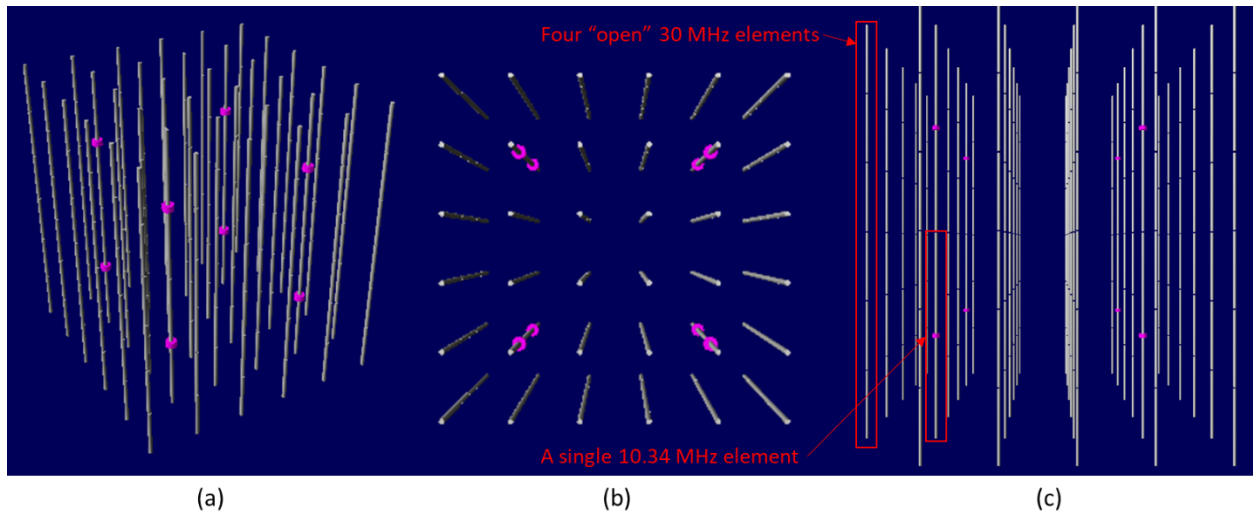


Figure 4.7 The 6x6x6 30 MHz full cube model from Figure 4.7 reconfigured into a 2x2x2 10.34 MHz cube surrounded by 30 MHz elements that have been turned off in (a) isometric view, (b) top view, and (c) side view.

4.4.1 Beamforming Simulations

The beam patterns for the 10.34 MHz configuration of the array at broadside, endfire and 45 degrees off broadside are presented below. The E-plane patterns are presented in Appendix A1.

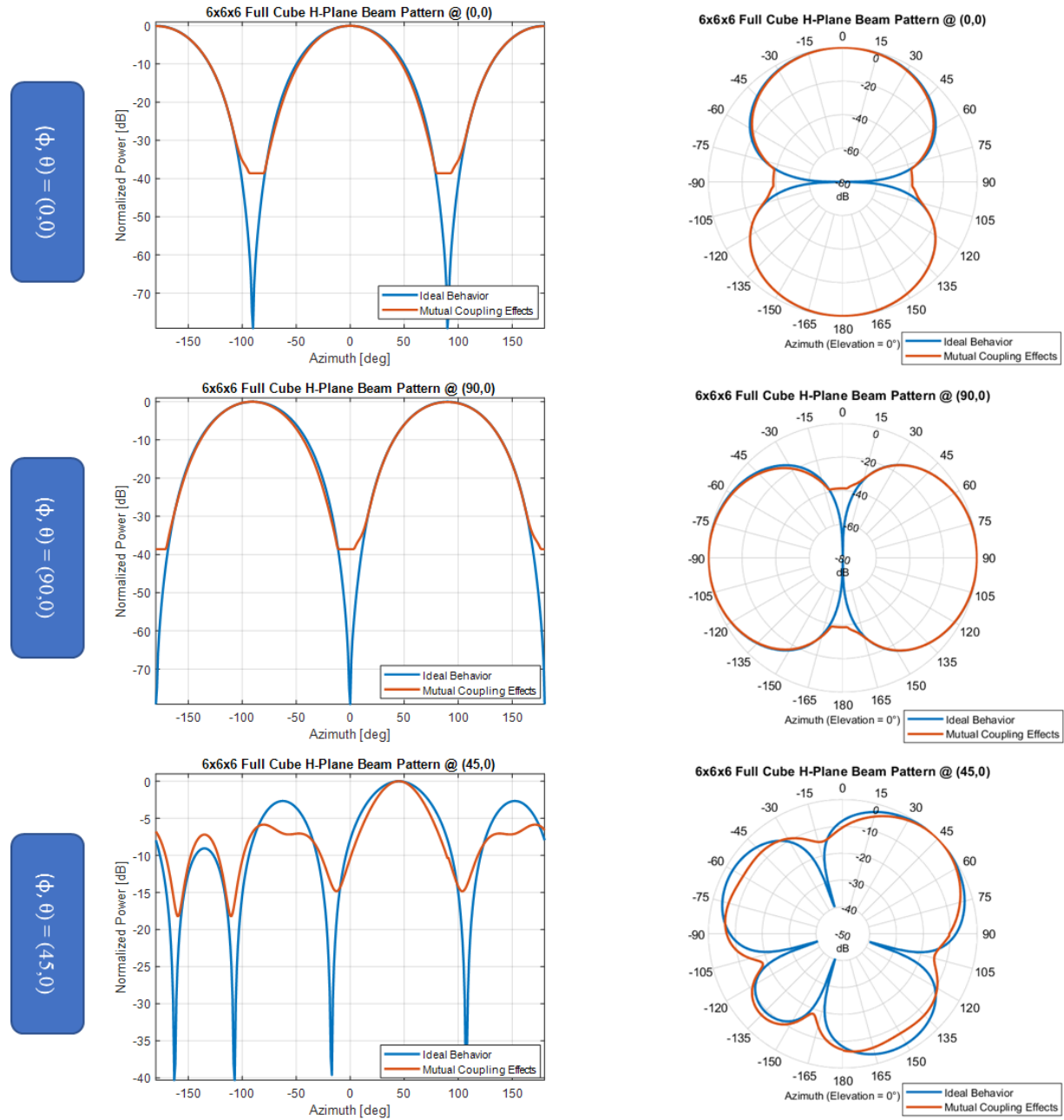


Figure 4.8 H-Plane pattern beamforming behavior for ideal and mutual coupling models in rectangular and polar coordinates at 10.34 MHz

Table 4.11 Simulation H-Plane Pattern Parameters for 4nec2 Plots in Figure 4.8 at $(\phi, \theta) = (0,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
MATLAB	58	--	10.6875
4nec2	56	--	8.61

Table 4.12 Simulation H-Plane Pattern Parameters for 4nec2 Plots in Figure 4.8 at $(\phi, \theta) = (90,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
MATLAB	58	--	10.6875
4nec2	58	--	8.61

Table 4.13 Simulation H-Plane Pattern Parameters for 4nec2 Plots in Figure 4.8 at $(\phi, \theta) = (45,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
MATLAB	60	-2.6629	11.0369
4nec2	56	-5.85	8.61

4.4.2 Direction of Arrival Simulations

The same MUSIC DOA analysis from Section 4.3.2 is performed on the 10.34 MHz configuration. The sources are located at broadside, endfire, and 45 degrees off broadside respectively. The results of these simulations are presented in Figure 4.12.

4.4.3 Mutual Coupling Effects

The mutual coupling effects and comparison to the ideal performance for the 10.34 MHz configuration are captured in the results from the simulations in Section 4.4.1.

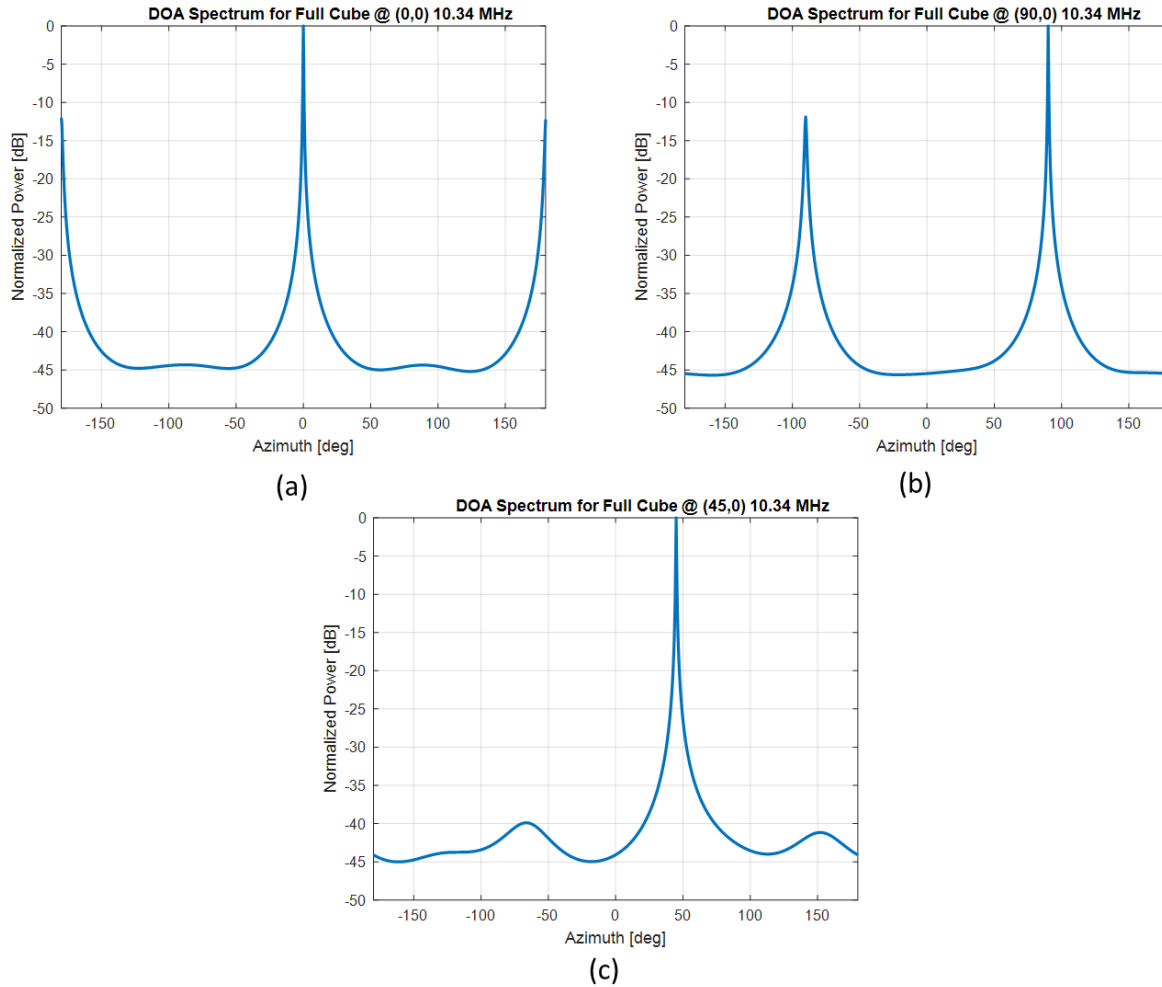


Figure 4.9 The MATLAB ideal DOA power spectrum plots for the 6x6x6 30 MHz cube reconfigured to the 2x2x2 cube (a) (azimuth, elevation) = (0,0) (broadside), (b) (azimuth, elevation) = (90,0) (endfire), and (c) (azimuth, elevation) = (45,0) at 10.34 MHz

4.5 Comparison of the 30 MHz and the 10.34 MHz Configurations

Section 4.3 and Section 4.4 investigate the performance of the array at 30 MHz and 10.34 MHz individually. The goal of this chapter is to present the performance of the array structure that can be reconfigured to operate at both 30 MHz and 10.34 MHz. It is useful, therefore, to compare the performance of the same array at these different frequencies. The comparison of the array performance at 30 MHz and 10.34 MHz is presented in this section.

4.5.1 Beamforming Simulations

This section overlays both the ideal and the mutual coupling results from Section 4.3.1 and Section 4.4.1 to compare the beamforming performance at the specified angular directions. Once again, the plots are normalized, and the parameters are tabulated per direction. The E-plane

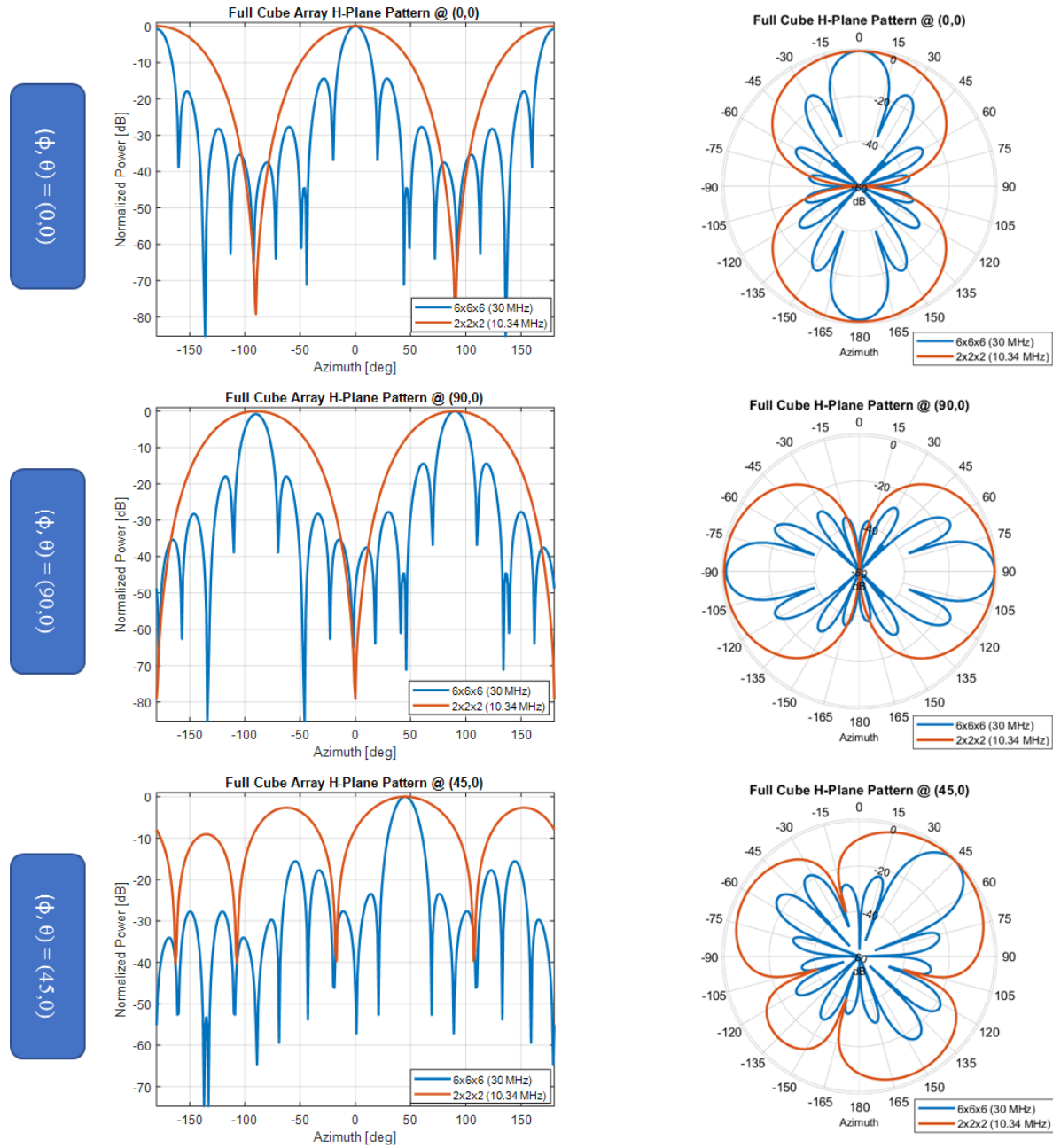


Figure 4.10 H-Plane pattern beamforming behavior for ideal models at 30 MHz and 10.34 MHz in rectangular and polar coordinates

results are available in Appendix A1.

Table 4.14 Simulation Pattern Parameters for MATLAB Plots in Figure 4.10 (ϕ, θ) = (0,0)

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	17.84	-14.4211	18.3098
10.34 MHz	58	--	10.6875

Table 4.15 Simulation Pattern Parameters for MATLAB Plots in Figure 4.10 (ϕ, θ) = (90,0)

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	17.84	-14.421	14.6668
10.34 MHz	58	--	10.6875

Table 4.16 Simulation Pattern Parameters for MATLAB Plots in Figure 4.10 (ϕ, θ) = (45,0)

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	18.16	-23.4624	20.5108
10.34 MHz	60	-2.6629	11.0369

Table 4.17 Simulation Pattern Parameters for 4nec2 Plots in Figure 4.11 (ϕ, θ) = (0,0)

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	22	-12.51	18.45
10.34 MHz	54	--	8.61

Table 4.18 Simulation Pattern Parameters for 4nec2 Plots in Figure 4.11 (ϕ, θ) = (90,0)

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	18	-12.51	18.45
10.34 MHz	58	--	8.61

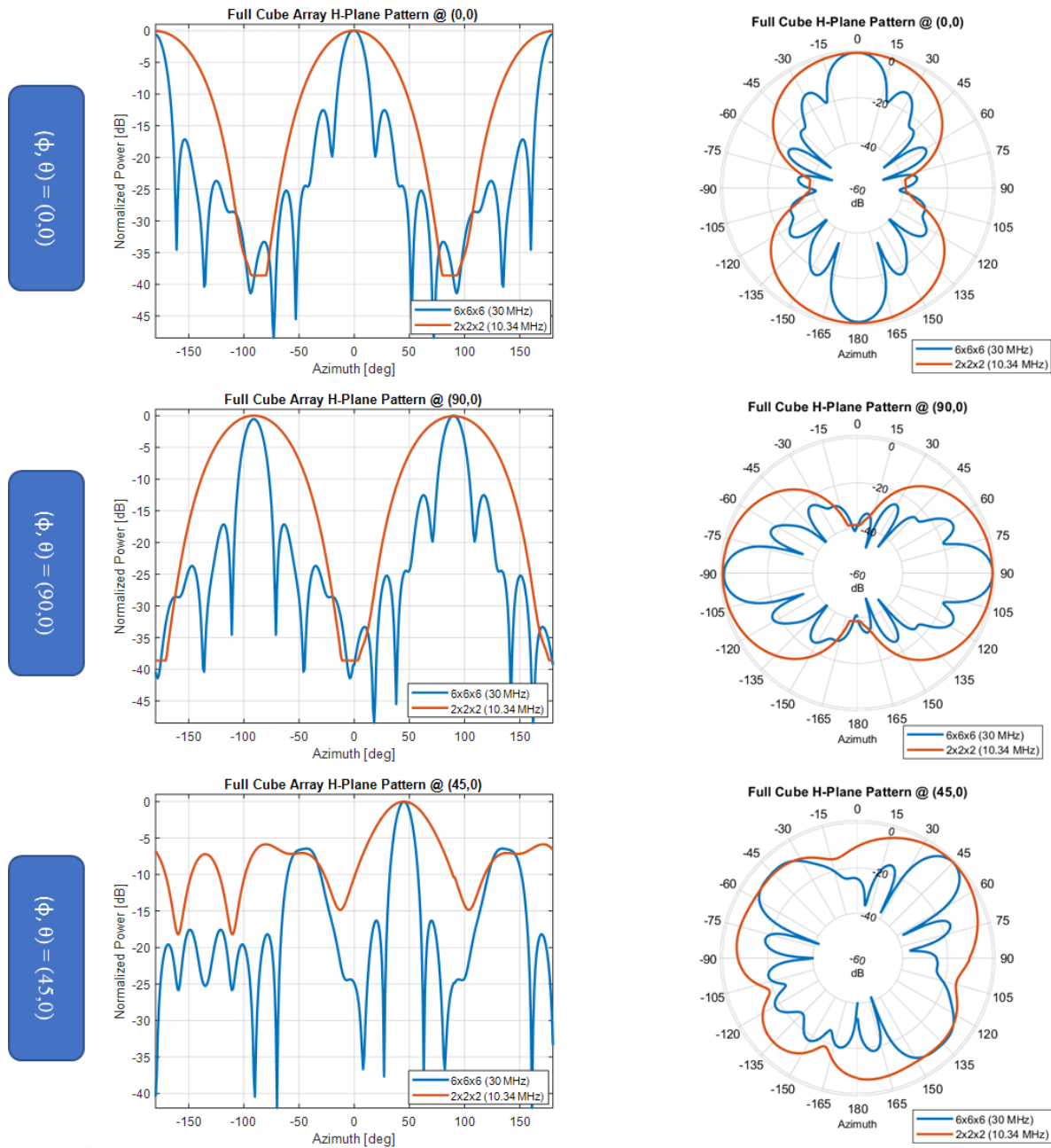


Figure 4.11 H-Plane pattern beamforming behavior for mutual coupling models at 30 MHz and 10.34 MHz in rectangular and polar coordinates

Table 4.19 Simulation Pattern Parameters for 4nec2 Plots in Figure 4.11 $(\phi, \theta) = (45,0)$

Configuration	HP [deg]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	15	-16.29	18.45
10.34 MHz	54	-5.85	8.61

4.5.2 Direction of Arrival Simulations

The direction of arrival simulations from Section 4.3.2 and Section 4.4.2 are also overlaid and compared in this section.

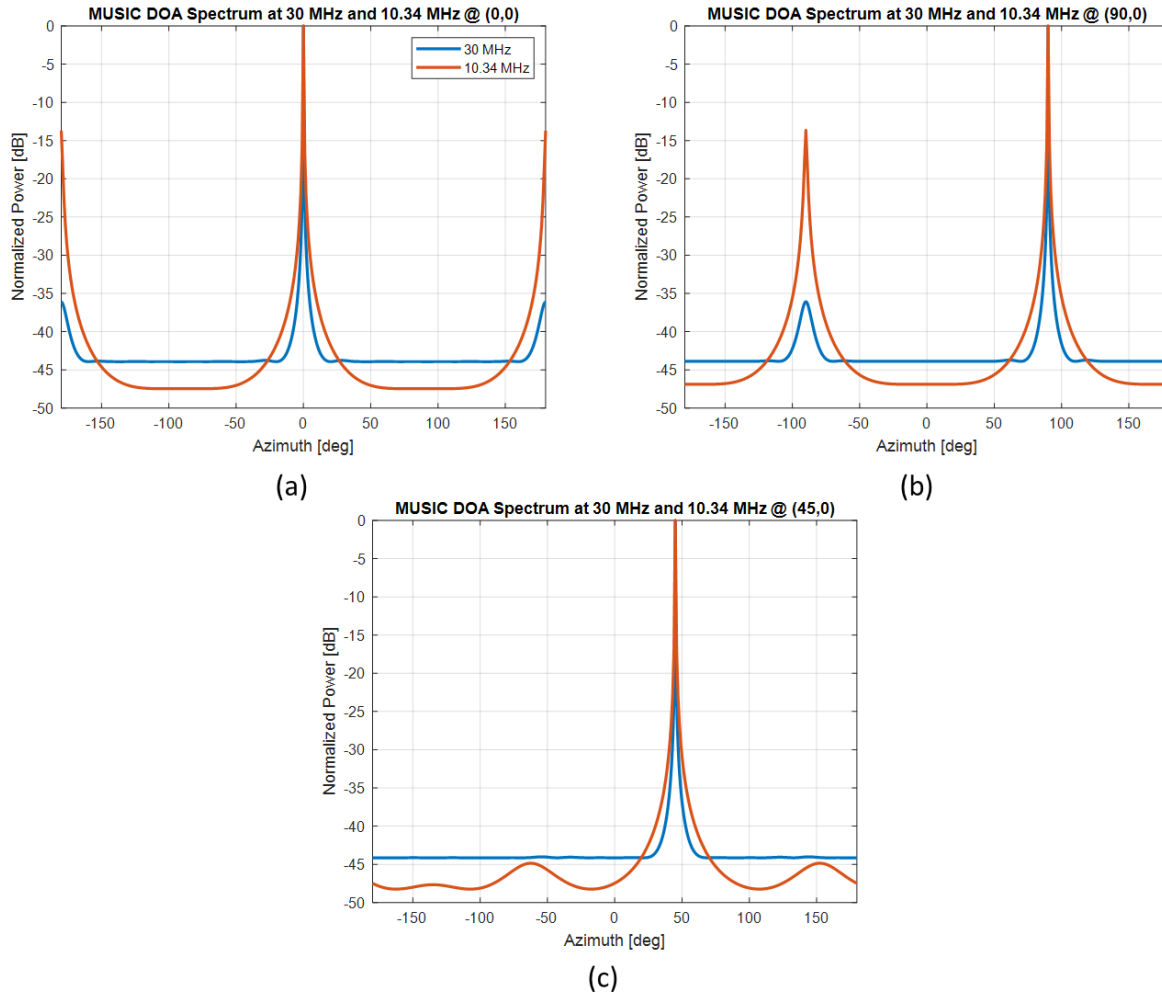


Figure 4.12 The MATLAB ideal DOA power spectrum plots for the 6x6x6 cube at 30 MHz and the reconfigured 10.34 MHz cube at (a) (azimuth, elevation) = (0,0) (broadside), (b) (azimuth, elevation) = (90,0) (endfire), and (c) (azimuth, elevation) = (45,0)

4.5.3 Mutual Coupling Effects

The mutual coupling effects for the 30 MHz and the 10.34 MHz configurations have been established in Section 4.3.5 and Section 4.4.5 respectively.

References

- [1] W. L. Stutzman and G. A. Thiele, “Antenna Theory and Design,” 2nd Edition, John Wiley and Sons, Inc., Hoboken, 1998.
- [2] W. C. Gibson, *The method of moments in electromagnetics*. Philadelphia, PA: Chapman & Hall/CRC, 2007.

Chapter 5 – Results and Discussion

This chapter presents a discussion of the key findings from the simulations in Chapter 4. After a discussion of the important results from Chapter 4, the full-scale proposed structure is simulated in MATLAB at 30 MHz, 15.46 MHz, and 3 MHz.

5.1 30 MHz Simulations

The results from Section 4.3 verify several key concepts. One obvious result is that as the 30 MHz cube increases in size from a 4x4x4 to a 6x6x6, the HP and gain performance of the array improves. In all cases in both the MATLAB and the 4nec models, increasing the number of elements decreases the 3-dB half-power beamwidth HP and increases the gain. This is not a surprising result as increasing the size of the cube increases the aperture area of the array, which in turn decreases HP. At the same time, increasing the number of elements in the aperture increases the gain.

5.1.1 Beam Pattern Results

5.1.1.1 Broadside vs Endfire

The results from Section 4.3.1 illustrate that the beam patterns at broadside and endfire are identical. Not only is the shape of the beam pattern the same, the SLL for the H-plane patterns are identical as are the gains of the beams and the HP. These results are maintained in the E-plane results in Appendix A1 and confirm the hypothesis presented in Chapter 3 regarding the broadside and endfire behavior of the cubic geometry. Therefore, the cubic geometry successfully eliminates endfire effects.

The 4nec2 mutual coupling results agree with these results as well with regard to E- and H-plane pattern SLL and gain. The HP at broadside and endfire, however, do not perfectly match.

This could be caused by two factors. One factor is the mutual coupling effects introduced by the 4nec2 method of moments calculation. However, the coupling environment at broadside is the same as the coupling environment at endfire. Therefore, mutual coupling is not likely the primary source of this discrepancy. The more likely cause of this discrepancy is the difference in precision between the two simulation environments. The 4nec2 simulations are run with 1-degree increments, whereas the MATLAB simulation could get within 2 decimal places of a degree of precision. As such, the HP calculations in 4nec2 are estimated and inherently have a margin of error. A combination of both effects explains the minor discrepancy in the data between broadside and endfire effects in the 4nec2 data.

5.1.1.2 Corner Edge: (Azimuth, Elevation) = (45, 0)

The results from Section 4.3.1 at the angular direction of 45 degrees off broadside illustrate the results for the case where the beam is formed out of the corner edge of the cube. In the ideal model, both the HP and the gain increase. The increase in gain is not surprising as at azimuth = 45, there are more elements in the aperture than at broadside or endfire, as was illustrated in Figure 3.7, and with an increase of elements comes an increase in gain. However, the increase in HP indicates that the aperture is smaller than at broadside. This is contrary to the result predicted in Chapter 3. At broadside and endfire, all of the elements in the aperture are in the same plane of space. At azimuth = 45, however, the elements in the aperture are no longer in the same plane of space. This is the expected cause of the result. Further investigation into three-dimensional aperture where elements are distributed in different planes of space is necessary to fully explain this result.

The 4nec2 data is mostly consistent with these results as well. The gain in the 4nec2 data increases, consistent with the ideal model behavior. However, the HP in the 4nec2 models is

either identical, or even slightly improved over that of the broadside and endfire cases. Once again, this is likely explained by a combination of mutual coupling effects and the difference in precision between the two simulation environments. Furthermore, as illustrated in Table 4.2 and Table 4.4, the difference in HP for the ideal models is between 0.32 and 0.5 degrees. This is well within the margin of error caused by the imprecision of the 4nec2 simulation.

One final observation regarding the corner at 45 degrees off broadside and the HP is the effect of adding more elements. As the cube is increased from a 4x4x4 to a 6x6x6 the decrease in performance at the corner edge steadily declines. Therefore, with a large enough cube structure, the corner edge performance may approximate equivalent performance than at the broadside face. Further investigation into this effect is necessary in order to make that determination.

5.1.2 Direction of Arrival Results

The DOA results from Section 4.3.2 further confirm the identical behavior at broadside and at endfire. However, at broadside and at endfire, there is still the formation of an ambiguity. The ambiguity is reduced by 33 or more dB, and therefore resolvable. However, an ambiguity is still present in the spectrum. This, too, is contrary to the hypothesis from Chapter 3. By adding more elements, the ambiguity is further reduced. Therefore, a large enough cube may also remove the ambiguity from the spectrum entirely.

The DOA spectrum for source located out of the corner edge at azimuth = 45 degrees illustrates another interesting result. At azimuth = 45, the ambiguity practically disappears from the spectrum. This is likely caused by the fact that viewed from that direction, the spacing between elements is reduced. This can also be seen from Figure 3.7.

5.2 10.34 MHz Simulations

The results at 10.34 MHz mirror those from the 30 MHz simulations. The broadside and endfire behavior are identical. Furthermore, the azimuth = 45 corner edge case demonstrates greater HP and greater gain. Similarly, the DOA behavior follows the same trend as that of the 30 MHz results.

The 10.34 MHz configuration has only 8 elements compared to the 216 elements in the 30 MHz configuration. Therefore, it is not surprising that the performance at 10.34 MHz is substantially worse than at 30 MHz. The ambiguity in the DOA spectrum at broadside and endfire, for example, is only reduced by approximately 11 dB as opposed to 33-35 dB in the 30 MHz configurations. This lends credibility to the idea that increasing the array size at the same spacing will resolve the ambiguity entirely.

5.3 Mutual Coupling Effects and Tunability/Reconfiguration

The mutual coupling effects for each configuration can be seen in the respective simulations from Chapter 4. Of note, however, is the change in the mutual coupling effects at 10.34 MHz versus the effects of mutual coupling at 30 MHz. At 30 MHz, there is noticeable distortion in the beamforming patterns and shifts in the SLL and gain. However, at 10.34 MHz, while there is reduction in the SLL and the gain, there is almost no distortion apart from the nulls for the broadside and endfire patterns. Therefore, the extra metal from the 30 MHz elements that are not in use, does not substantially affect the behavior of the array at 10.34 MHz. This is not surprising as the 30 MHz elements are unlikely to resonate at 10.34 MHz. This result shows a major advantage of narrowband resonant dipole elements for the array implementation.

5.4 Full System Simulation

The verification of the results at 30 MHz and at 10.34 MHz with the predictions of the array behavior from Section 3.4 indicate that a reconfigurable resonant cubic array could be implemented that operates at frequencies across the HF band. A full model of the array is simulated in MATLAB that consists of a 20x20x20 8000-element array that operates at 30 MHz that can be reconfigured to a 10x10x10 1000-element array that operates at 15.46 MHz and a 2x2x2 8-element array that operates at 3 MHz. In this way, simulations can be run to predict performance at both ends and near the middle of the HF band.

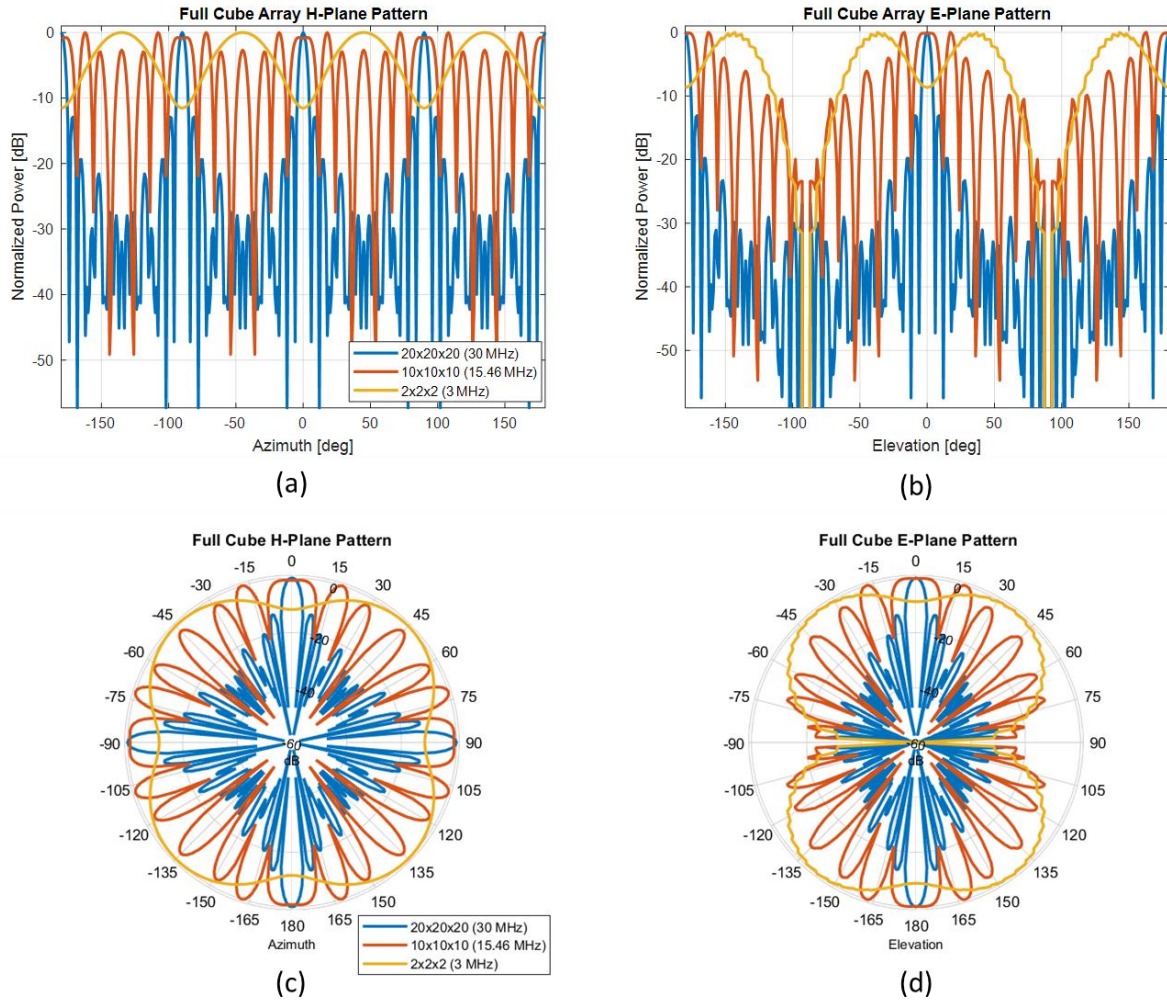


Figure 5.1 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the proposed structure at 30 MHz, 15.46 MHz, and 3 MHz

Table 5.1 Simulation Pattern Parameters for Ideal Plots in Figure 5.1

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
20x20x20	7	-13.0879	-13.6009	51.5609
10x10x10	5.5	-4.0361	-0.7729	24.689
2x2x2	38	--	--	5.4099

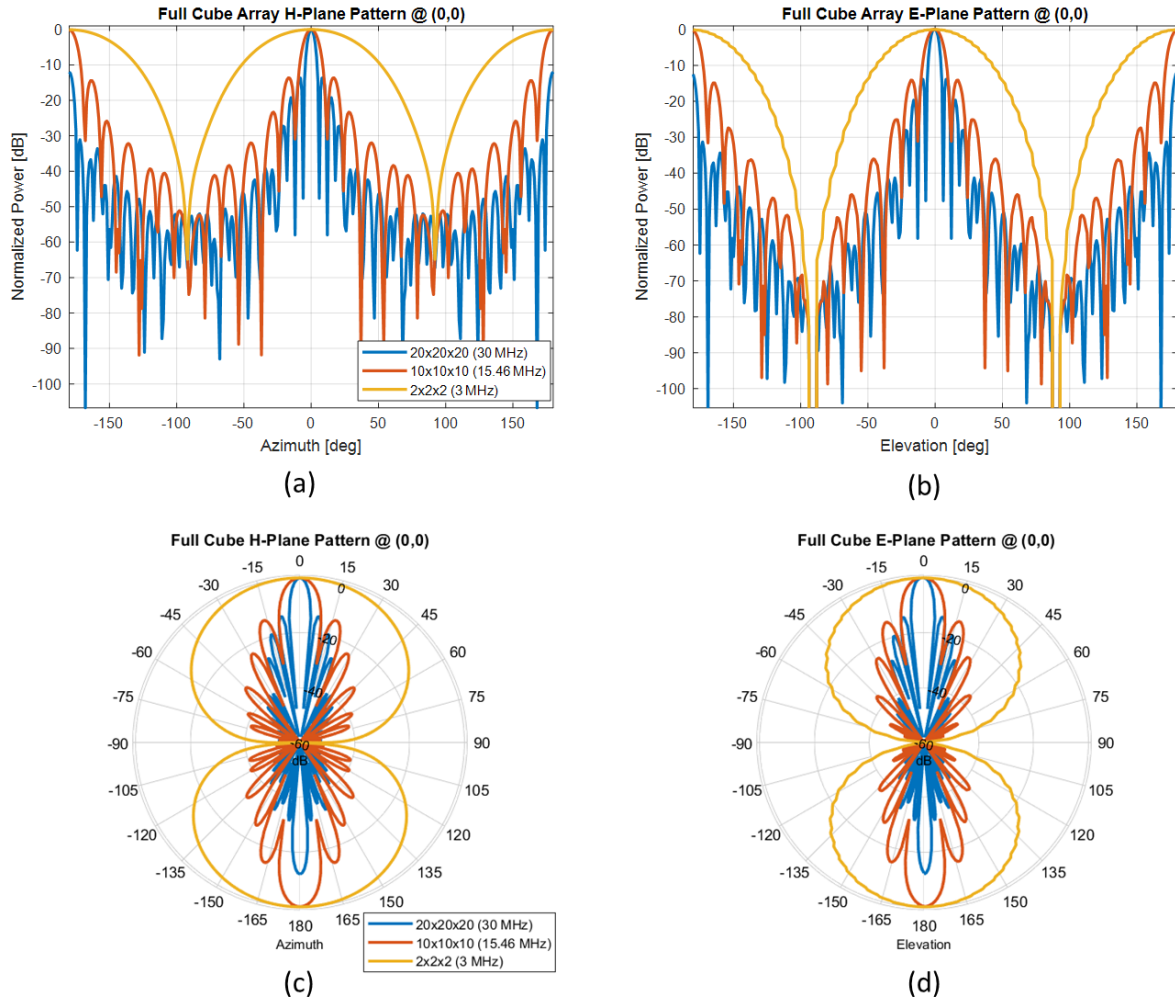


Figure 5.2 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the proposed structure at 30 MHz, 15.46 MHz, and 3 MHz with beamforming coefficients at (azimuth, elevation) = (0,0)

Table 5.2 Simulation Pattern Parameters for Ideal Plots in Figure 5.2

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
20x20x20	5	-13.7908	-13.6009	30.7641
10x10x10	10	-14.0838	-13.645	22.4687
2x2x2	60	--	--	8.3537

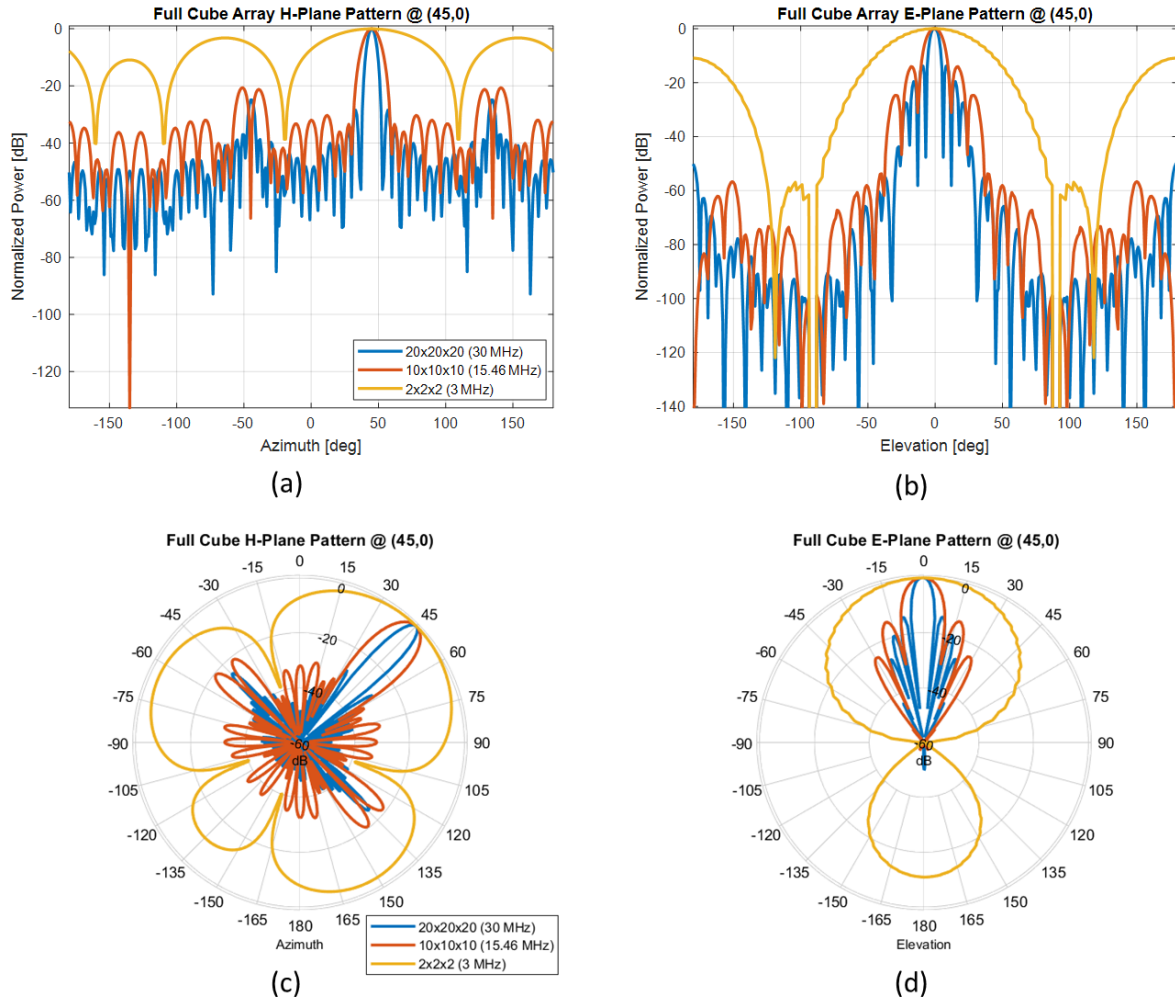


Figure 5.3 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the proposed structure at 30 MHz, 15.46 MHz, and 3 MHz with beamforming coefficients at (azimuth, elevation) = (45,0)

Table 5.3 Simulation Pattern Parameters for Ideal Plots in Figure 5.3

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
20x20x20	5	-13.7903	-28.4493	31.0628
10x10x10	10	-14.0784	-33.4141	25.3036
2x2x2	62	-61.5922	-3.1994	9.2898

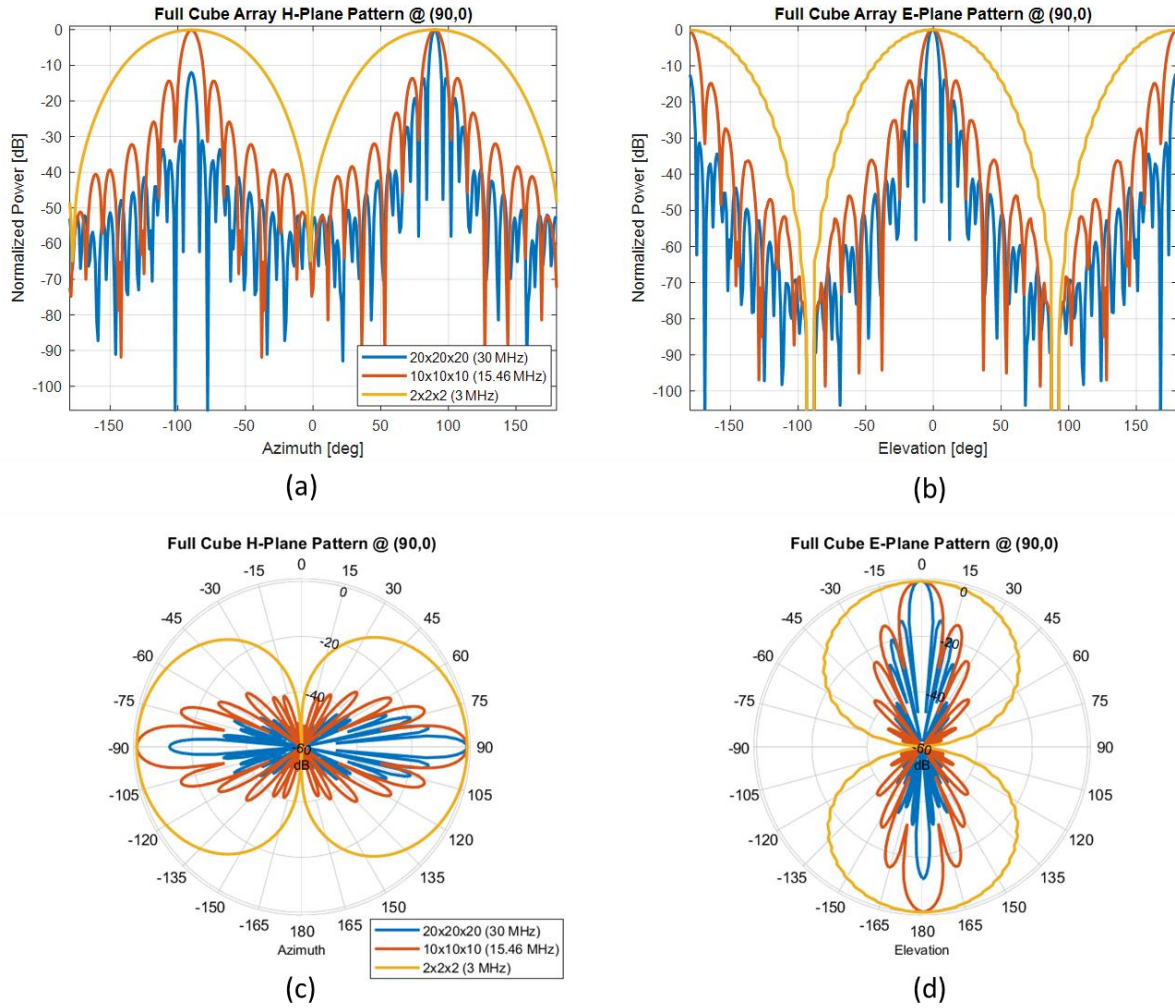


Figure 5.4 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the proposed structure at 30 MHz, 15.46 MHz, and 3 MHz with beamforming coefficients at (azimuth, elevation) = (90,0)

Table 5.4 Simulation Pattern Parameters for Ideal Plots in Figure 5.4

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
20x20x20	5	-13.7908	-13.6009	30.7641
10x10x10	10	-14.0838	-13.645	22.4867
2x2x2	60	--	--	8.3537

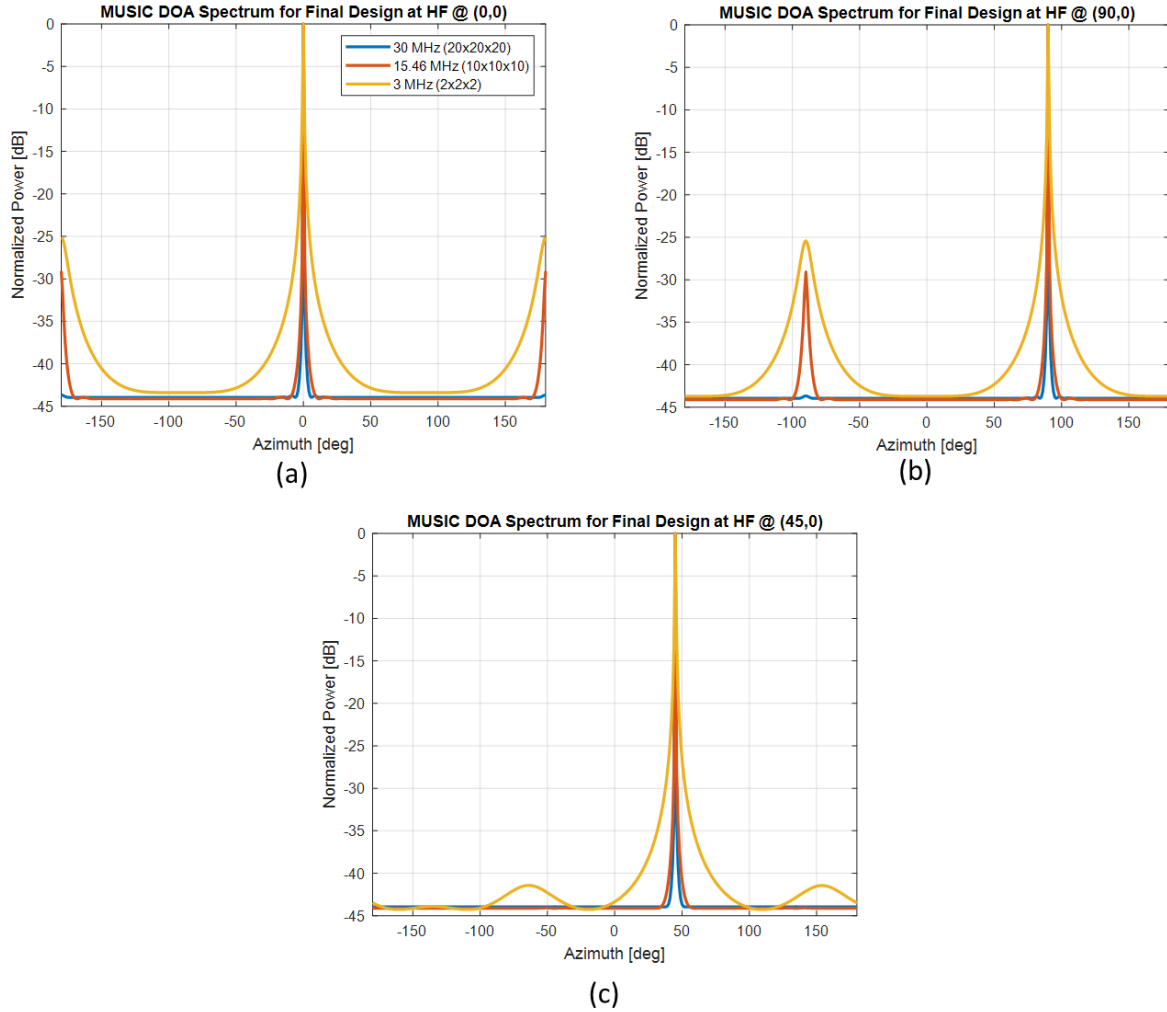


Figure 5.5 The MATLAB ideal DOA power spectrum plots for proposed structure at 30 MHz, 15.46 MHz, and 3 MHz at (a) (azimuth, elevation) = (0,0) (broadside), (b) (azimuth, elevation) = (90,0) (endfire), and (c) (azimuth, elevation) = (45,0)

5.5 Full System Results and Discussion

To save on computational cost, the precision for the full system simulation was reduced to approximately 1-degree steps. This introduces some margin of error into SLL and HP calculations. Nonetheless, the desired behavior is maintained for each configuration of the array. Broadside and endfire behavior are identical at each respective frequency.

An interesting result occurs for the 20x20x20 and the 10x10x10 configuration beamformer at 45 degrees off broadside. In this case, the HP is approximately the same. This could be a result

of the reduction in precision for the model. However, from the 30 MHz simulations in Section 5.1.1, as the array is increased, the increase in HP at the corner edge is reduced. It is possible that by forming a $10 \times 10 \times 10$ or a $20 \times 20 \times 20$, the cube is large enough that the HP performance approximates the performance at broadside and endfire. Similarly, the broadside and endfire DOA spectra for the $20 \times 20 \times 20$ configuration show that the ambiguity is reduced to just above the noise floor.

Chapter 6 – Summary and Future Work

This chapter concludes the thesis with a summary and a discussion of opportunities for future work on the project.

6.1 Summary

This thesis investigated the performance of a three-dimensional cubic resonant phased array to demonstrate the concept of an in-space assembled HF radio telescope that can be reconfigured to operate at different frequencies. A modular structure for the design of the array that permits tunability and is conducive to robotic in-space assembly was designed. The cubic array was simulated to perform beamforming operations in both ideal and mutual coupling software environments as well as ideal direction of arrival measurements at various frequencies within the HF band. Of particular interest was the performance at broadside, endfire, and the azimuth = 45 corner edge. Also of interest was the mutual coupling effects that elements that were not in use for configurations at lower frequencies had on the array performance. After a smaller scale model was thoroughly investigated, a full-scale cubic array capable of operating at 3 MHz, 15.46 MHz, and 30 MHz was simulated.

6.2 Results Summary

The key results from this thesis are as follows. The cubic geometry provides identical performance with regards to beamforming and DOA at broadside and at endfire. This effectively removes endfire effects from the array performance. The DOA spectra for both broadside and

endfire show the presence of a suppressed ambiguity. The larger the cube, the greater the suppression in this ambiguity.

At the corner edge of the cube, the gain of the beam increases due to the increased number of elements in the aperture. However, the HP increases as well. This indicates that the effective aperture when the beam is directed out of the corner edge is smaller than at broadside or at endfire. As the size of the cube increases, the change in HP decreases. The DOA spectra for the corner edge of the cube have significantly more suppression in the ambiguity than at broadside or endfire. This is likely a result of the smaller element spacing that is seen from the viewpoint of the corner.

The mutual coupling effects at 10.34 MHz were significantly reduced compared to those seen at 30 MHz. This indicates that the metal from the inactive 30 MHz elements that surround the larger elements at lower frequencies do not significantly impact the pattern and beamforming behavior of the lower frequency configurations of the array.

6.3 Future Work

The work presented in this thesis provides ample opportunity for continued effort. The following topics are just a few of the many ways in which the research could be continued.

6.3.1 Utilize Other Geometries

The geometry chosen for this thesis was a cube due to the ease of design and fabrication as well as symmetry that a cube provides. As mentioned, the inner array elements of the cube do not impact the effective aperture size of the array at broadside or at endfire and do not provide much benefit at 45 degrees off broadside. Therefore, the internal elements only provide the benefit of tunability at cost of significant increase in power consumption, mutual coupling effects, and design complexity.

A modification for the structural design would be to remove the interior elements and create a hollow cube as shown in Figure 6.1. A hollow cube array structure maintains the aperture size and the three-dimensional cubic symmetry while reducing the element count, array weight, power consumption, and mutual coupling. However, the x-spacing in the hollow cube design is not equal to the y- and z-spacing between elements. As the hollow cube gets larger, this x-spacing very quickly exceeds 1 wavelength, and grating lobes will form in the patterns. A thorough investigation to the hollow cube's performance compared to the full cube is highly recommended.

The physical structure of the cube design also required a lot of external support for rigidity. Other geometries might provide similar RF behavior while having a more inherently rigid structure. An octahedral or icosahedral geometry, for example, would be much more structurally stable than the cube designs.

Other more arbitrary geometries might be tested as well. One such example is shown in Figure 6.1b. This "Space-Jack" design significantly reduces the number of elements in the array with a much simpler structure. Studying array geometries such as this can result in an optimum three-dimensional design that maximizes aperture size and performance while minimizing the number of elements.

6.3.2 Implement More Advanced DOA Algorithms

This thesis focused exclusively on three very commonly used DOA algorithms: Bartlett, Capon (MVDR) and MUSIC. There are, however, many others that could be utilized to compare three-dimensional geometric arrays to more classical two-dimensional arrays. ESPRIT, Root-MUSIC, [1], [2] the Linear Prediction Method, [3] and the Expectation and Maximization

Method [4] are just a few of the algorithms that could be adapted and applied to a three-dimensional array for future study.

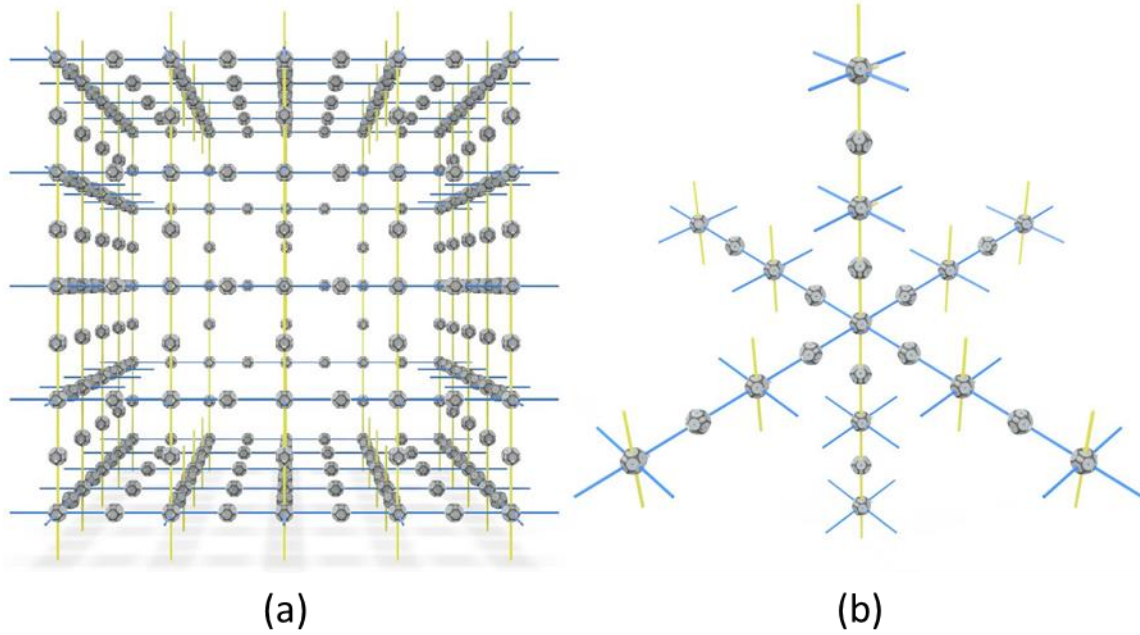


Figure 6.1 (a) Hollow cube design and (b) “Space-Jack” geometry design. The yellow rods represent the conductive aluminum standoffs while the blue rods represent the nonconductive nylon standoffs for spacing and support.

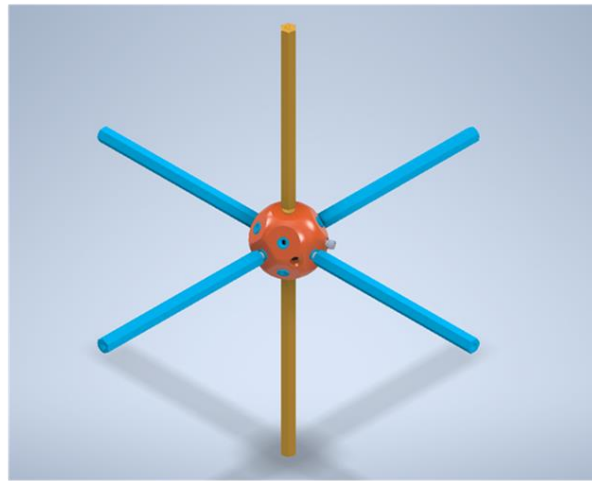


Figure 6.2 Single Hexapole Element

6.3.3 Hexapole Element and Polarization

The arrays in this thesis focused exclusively on z-directed linearly polarized dipoles. Therefore, if the array is upright, only dominantly vertically polarized signals will be detected,

whereas if the array is on its side, only dominantly horizontally polarized signals will be detected. When the array is tilted, only a vector component is interacting with the array element, and the analysis from Chapter 2 must be performed. By replacing all of the nonconductive rods in the cubic design, or the “Half-Jack” design in Figure 6.1, with conductive rods, each array element acts as a hexapole rather than a dipole. This was discussed in Chapter 3. A single hexapole is identified in Figure 6.2 where all the rods, blue and yellow, represent conductive elements. There are two conductive rods in each of the, x-, y- and z-directions. Therefore, there are six rods per element, hence the name hexapole. A hexapole element structure allows for detection of both vector components of any polarization from any direction in a tumbling environment. A study implementing the hexapole design in a three-dimensional geometry is highly recommended and would be of great value.

6.3.4 RFIC Switch Implementation for Tunability

As mentioned, the initial design in Chapter 3 includes interior elements for the purpose of tunability. Utilization of RFIC switches in the array design would permit reconfiguration of array elements to different sized dipoles for operation at a broader bandwidth of frequencies. While this thesis investigated multiple fixed configurations of a single design to model the tunability of the array, the actual RFIC switches were not implemented. One major reason for choosing not to implement these switches has to do with the challenges of impedance matching caused by RFIC switches. Open switches in the spacers act as capacitors which influence the impedance of the individual elements, which has impacts on the radiation behavior of the array. Therefore, to fully implement the RFIC switch system, impedance matching networks also must be designed and implemented which significantly increases the complexity of the array. A full implementation of the design with the RFIC switches built in would be a great next step for the research.

6.3.5 Implementing Different Antenna Elements

The work in this thesis was limited to half-wave dipoles. One major shortcoming to half-wave dipoles is their narrow-band behavior. Even with the tunability and implementation of the RFIC switches, the array is not operable over the entire HF band. Rather, it is operable at many discrete frequencies within the HF band. Therefore, the tunability of the RFIC switches gives the array behavior that is more akin to multiband than broadband. Replacing the passive elements and RFIC switches with active elements, however, converts the array to a broadband array. Active elements, however, come with their own set of challenges [5].

Furthermore, the array elements could be implemented as short dipoles instead of half-wave dipoles. As seen in Chapter 2, short dipoles approximate ideal dipoles. Therefore, substituting short dipoles into the array design can result in more ideal antenna radiation behavior.

6.3.6 Ionospheric Modeling and Effects

Chapter 2 presented a few examples of the challenges faced by HF propagation in the ionosphere. This thesis did not address or model these ionospheric effects on the array design in any way as it was out of scope of the current research goal. However, if a replacement for the Arecibo radio telescope is to be placed into orbit and used for ionospheric and atmospheric studies, then the study of these effects on the array design and performance is necessary.

6.3.7 Chamber Measurement and Characterization

A prototype design is implemented in Appendix A2. This prototype was taken to a chamber and characterized. However, the post-processing and calibration of the data from the chamber is a very time-consuming task. Time constraints on the project prevented the data analysis from being completed and verified. As a next step in the research, this data should be thoroughly analyzed and used to verify the simulations and discussion from Chapter 4 and Chapter 5.

Additionally, mutual coupling compensation can be performed on experimental measurements to characterize the real-world compensated performance as compared to the simulated ideal performance.

6.3.8 Three-Dimensional Effective Aperture Study

It is suspected that the strange HP behavior at the corner edge of the cube 45 degrees off broadside is a result of the effective aperture of the array. Because not all of the elements in the array exist in the same plane of space, the effective aperture of the three-dimensional geometry may change as the signal propagates through the array. A study of this effect and quantifying the relationship between this effect and the size of the cube would be a valuable addition to the research.

References

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- [3] E. Grosicki, K. Abed-Meraim and Yingbo Hua, "A weighted linear prediction method for near-field source localization," in *IEEE Transactions on Signal Processing*, vol. 53, no. 10, pp. 3651-3660, Oct. 2005, doi: 10.1109/TSP.2005.855100.
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- [5] Y. Taachouche, M. Abdallah, F. Colombel, G. Le Ray, and M. Himdi, "Active Compact Antenna for Broadband Applications," *Progress in Compact Antennas. InTech*, Sep. 10, 2014. doi: 10.5772/58839.

Appendix A – Simulation Results

This appendix provides supplementary information to the simulation results in Chapter 4. Also included are the general patterns which refers to the array when every element's excitation source is equal to $1 + j0$. These patterns can provide additional characterization information.

A.1 30 MHz Simulations

A.1.1 Array Pattern Simulations

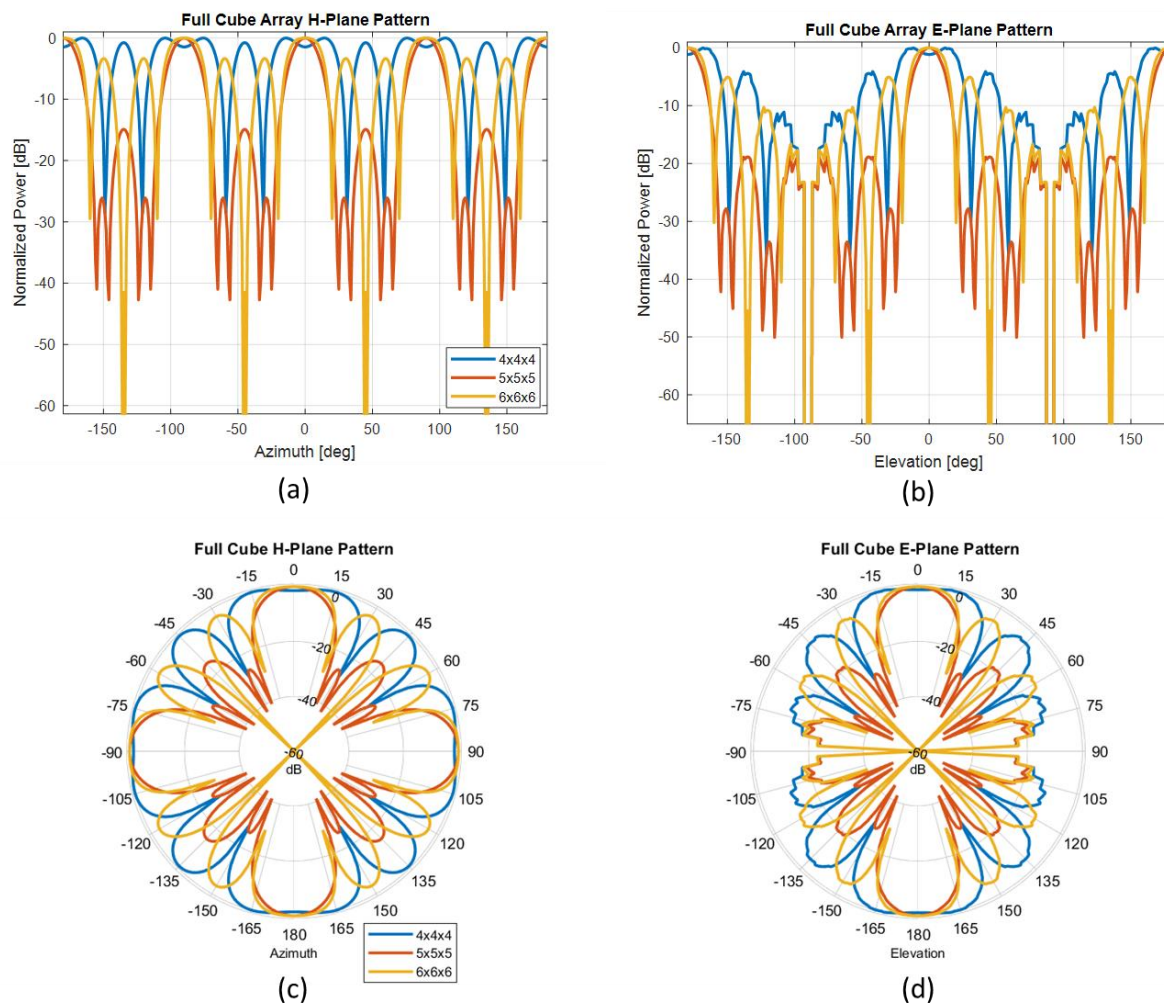


Figure A.1 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the full cube configurations models at 30 MHz

Table A.1 Simulation Pattern Parameters for Ideal Plots in Figure A.1

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	47.00	-4.1005	-0.7447	13.3164
5x5x5	20.28	-27.8052	-26.0805	27.5541
6x6x6	25.84	-5.043	-3.3184	17.8777

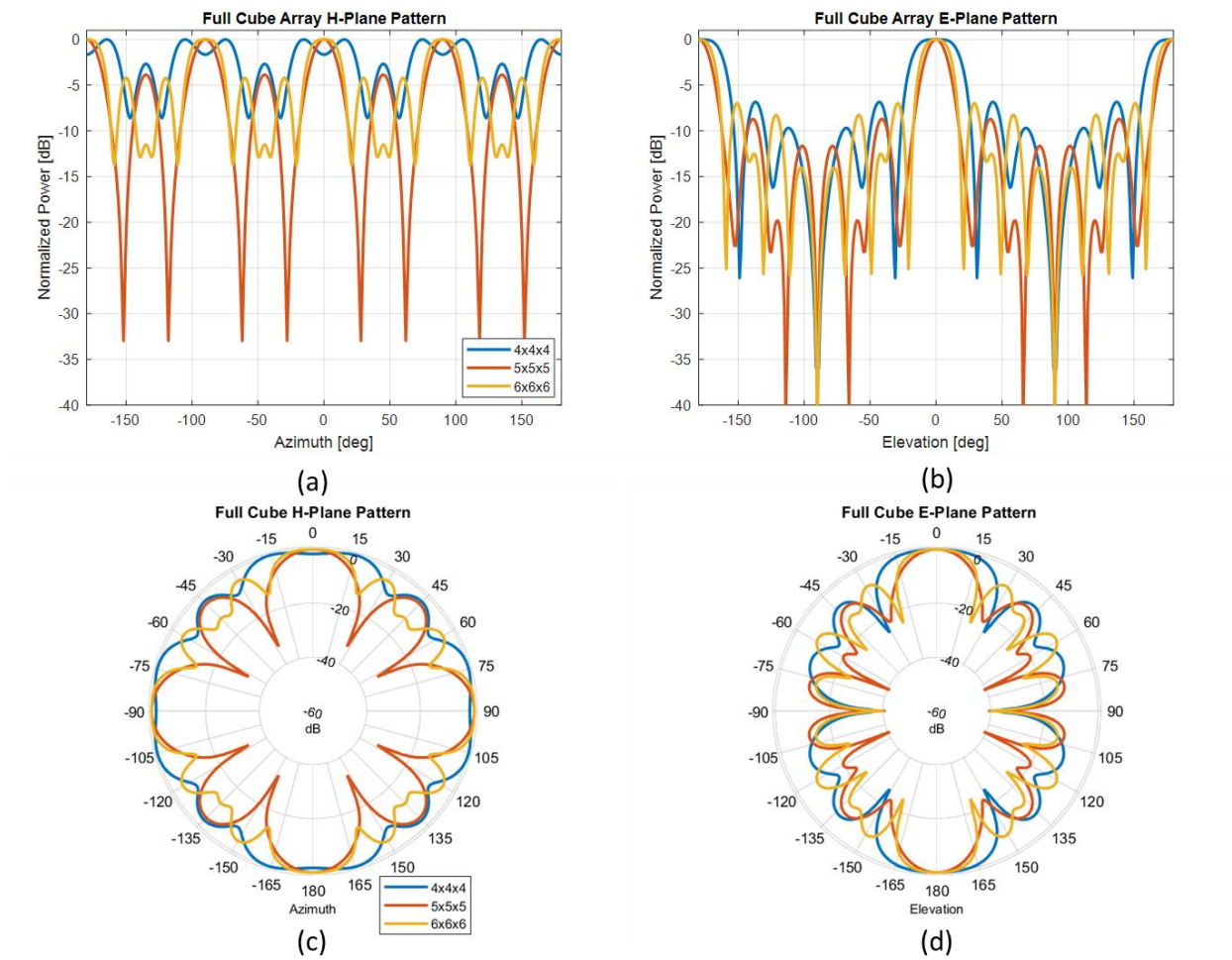


Figure A.2 The 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the full cube configurations models at 30 MHz

Table A.2 Simulation Pattern Parameters for 4nec2 Plots in Figure A.2

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	55	-6.83	-2.68	6.01
5x5x5	22	-8.7	-3.86	10.91
6x6x6	27	-6.97	-4.19	10.37

A.1.2 Beamforming Simulations

A.1.2.1 Beamforming Simulations at (Azimuth, Elevation) = (0,0)

Table A.3 Simulation Pattern Parameters for the Ideal MATLAB Plots in Figure A.3

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	27.22	-19.4317	-16.3127	14.6668
5x5x5	21.54	-17.1669	-15.0124	16.6552
6x6x6	17.84	-15.879	-14.4211	18.3098

Table A.4 Simulation Pattern Parameters for the Mutual Coupling 4nec2 Plots in Figure A.4

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	28	-10.79	-12.84	114.86
5x5x5	21	-11.86	-12.34	16.47
6x6x6	18	-12.75	-12.51	18.45

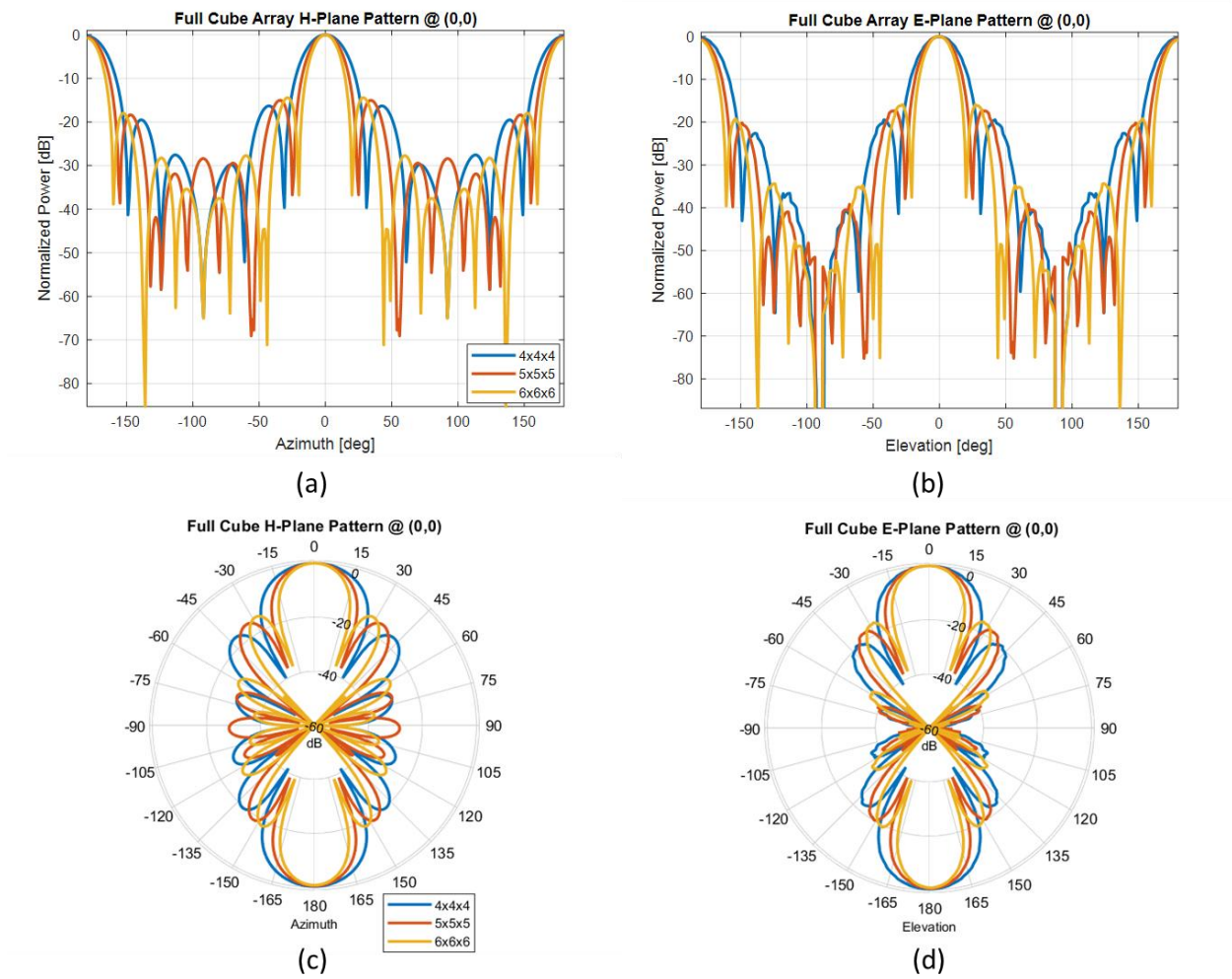
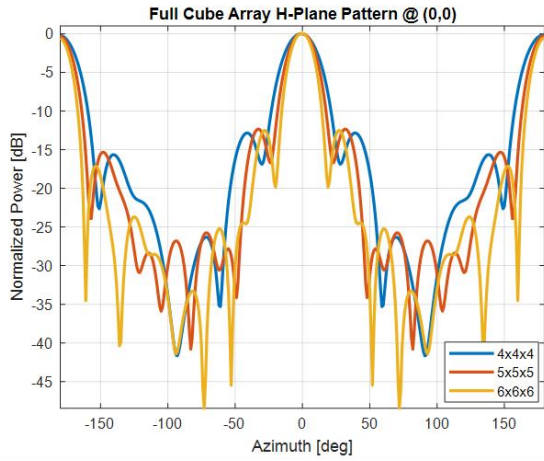
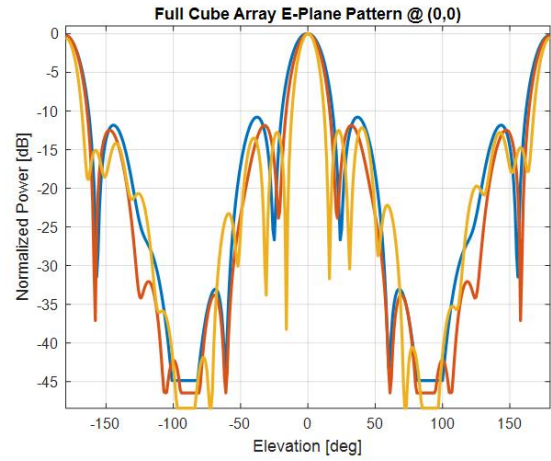


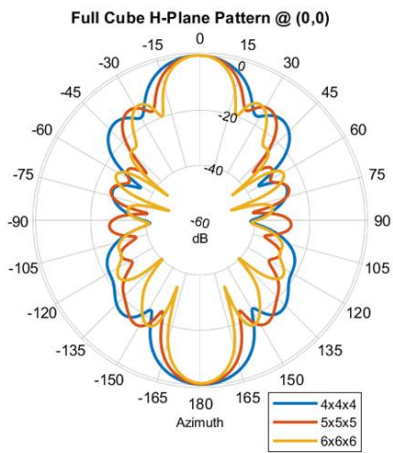
Figure A.3 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the full cube configurations models with beamforming coefficients at (azimuth, elevation) = (0,0) at 30 MHz



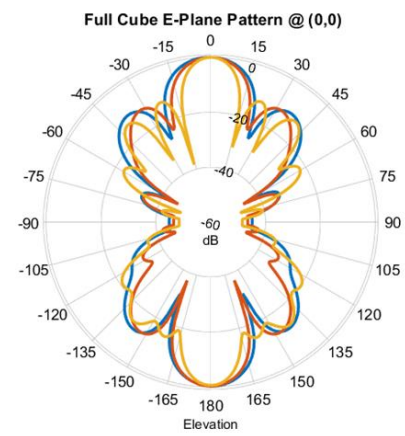
(a)



(b)



(c)



(d)

Figure A.4 The 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the full cube configurations models from with beamforming coefficients at (azimuth, elevation) = (0,0) at 30 MHz

A.1.2.2 Beamforming Simulations at (Azimuth, Elevation) = (90,0)

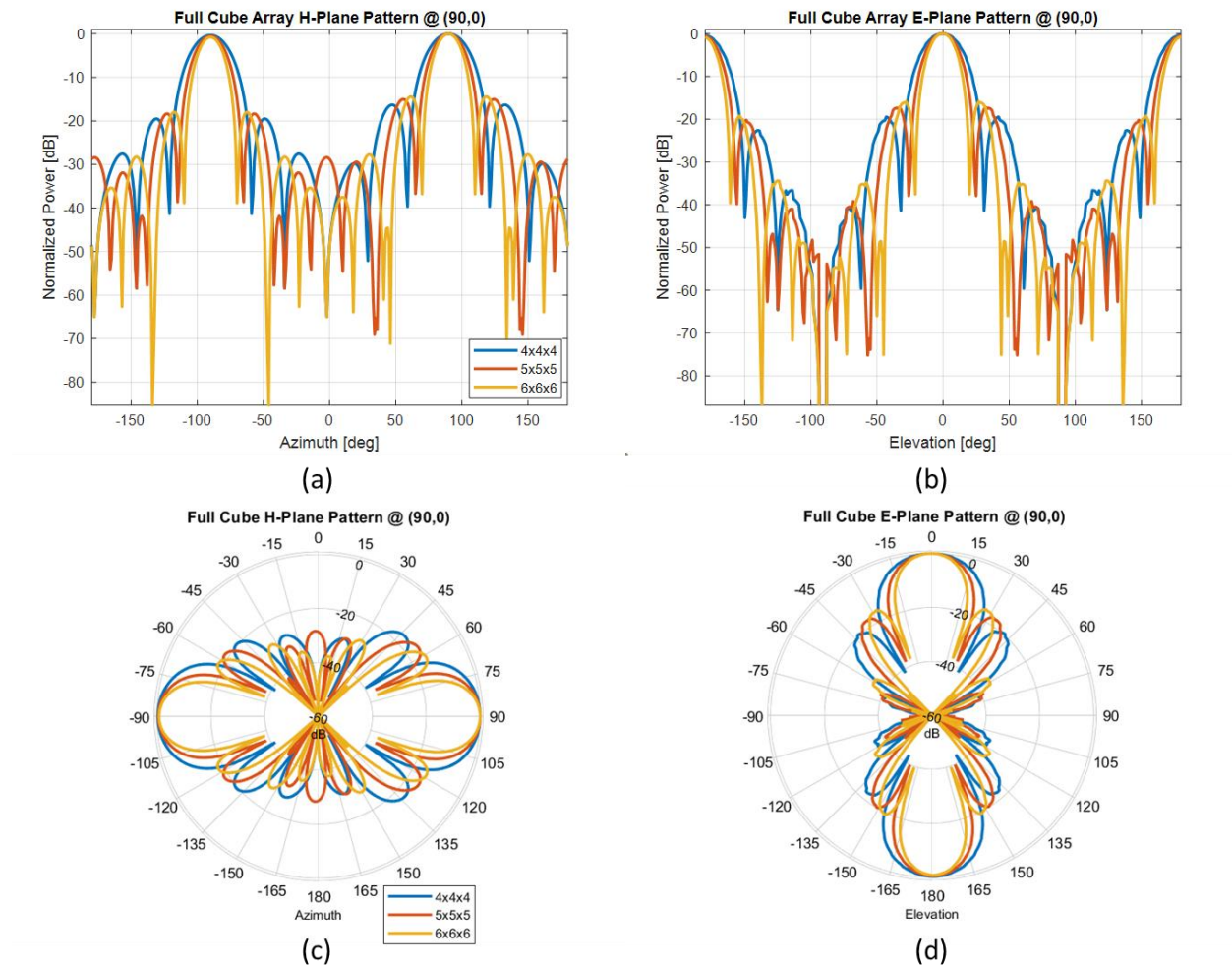


Figure A.5 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the full cube configurations models with beamforming coefficients at (azimuth, elevation) = (90,0) at 30 MHz

Table A.5 Simulation Pattern Parameters for Ideal MATLAB Plots in Figure A.5

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	27.22	-19.4317	-16.3127	14.6668
5x5x5	21.54	-17.1669	-15.0124	16.6552
6x6x6	17.84	-15.879	-14.421	18.3098

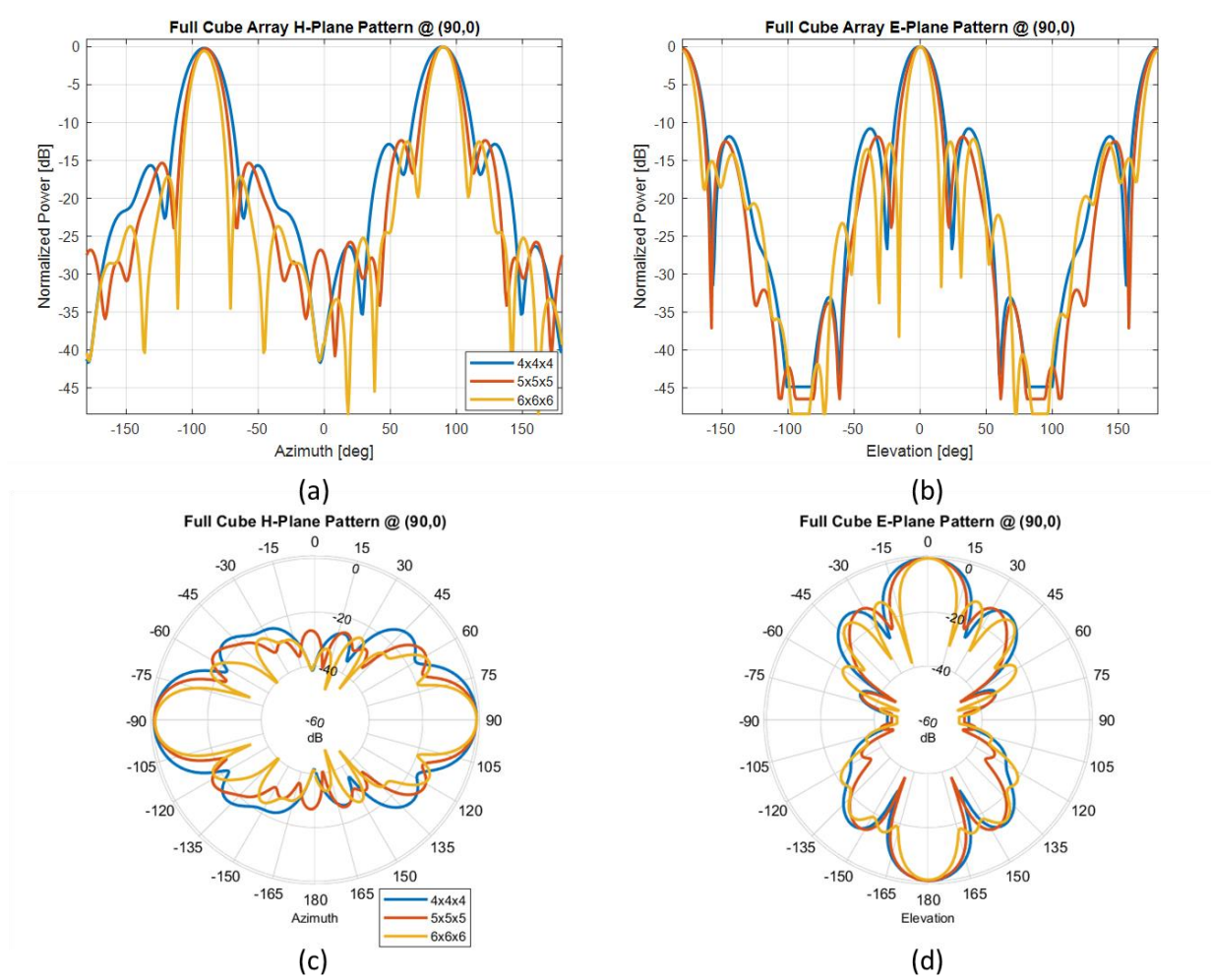


Figure A.6 The 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the full cube configurations models with beamforming coefficients at (azimuth, elevation) = (90,0) at 30 MHz

Table A.6 Simulation Pattern Parameters for Mutual Coupling 4nec2 Plots in Figure A.6

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	26	-10.79	-12.84	14.86
5x5x5	20	-11.86	-12.34	16.47
6x6x6	18	-12.75	-12.51	18.45

A.1.2.3 Beamforming Simulations at (Azimuth, Elevation) = (45,0)

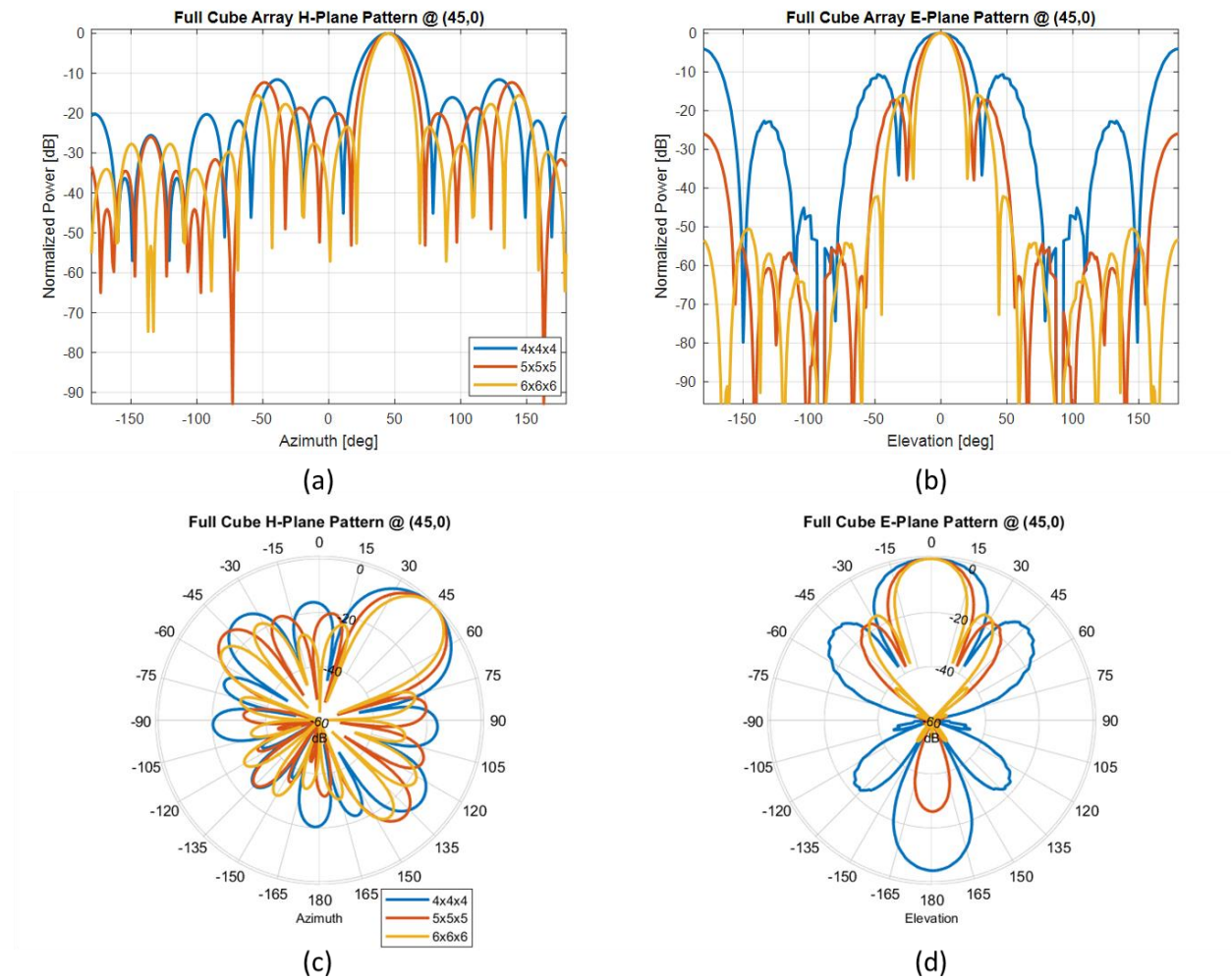


Figure A.7 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the full cube configurations models with beamforming coefficients at (azimuth, elevation) = (45,0) at 30 MHz

Table A.7 Simulation Pattern Parameters for the Ideal MATLAB Plots in Figure A.7

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	27.72	-11.0696	-16.0769	16.7763
5x5x5	21.92	-17.1193	-20.0887	18.772
6x6x6	18.16	-15.8538	-23.4624	20.5108

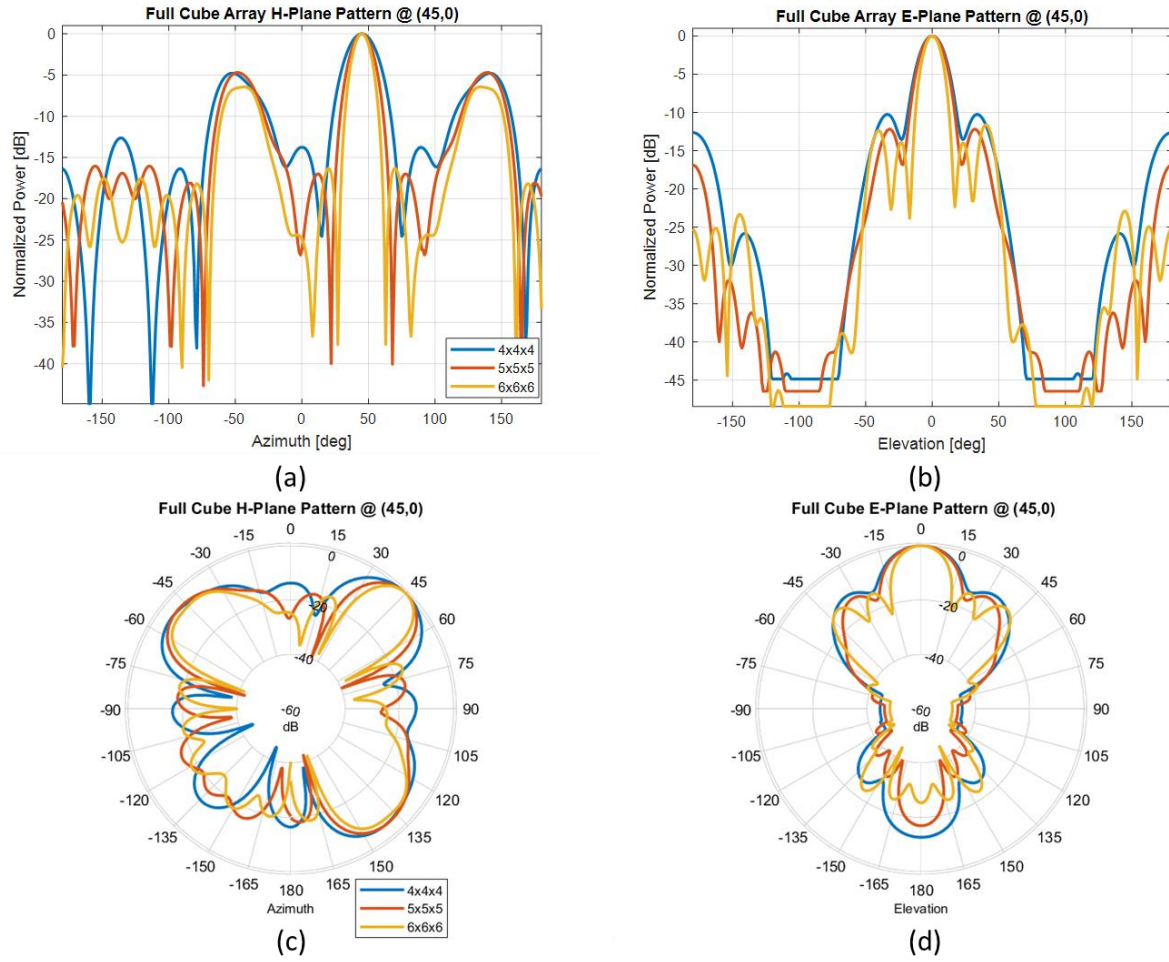


Figure A.8 The 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of the full cube configurations models with beamforming coefficients at (azimuth, elevation) = (45,0) at 30 MHz

Table A.8 Simulation Pattern Parameters for the Mutual Coupling 4nec2 Plots in Figure A.8

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
4x4x4	26	-10.26.	-13.77	14.86
5x5x5	20	-12.19	-16.98	16.46
6x6x6	16	-13.94	-16.29	18.45

A.1.3 Mutual Coupling Effects

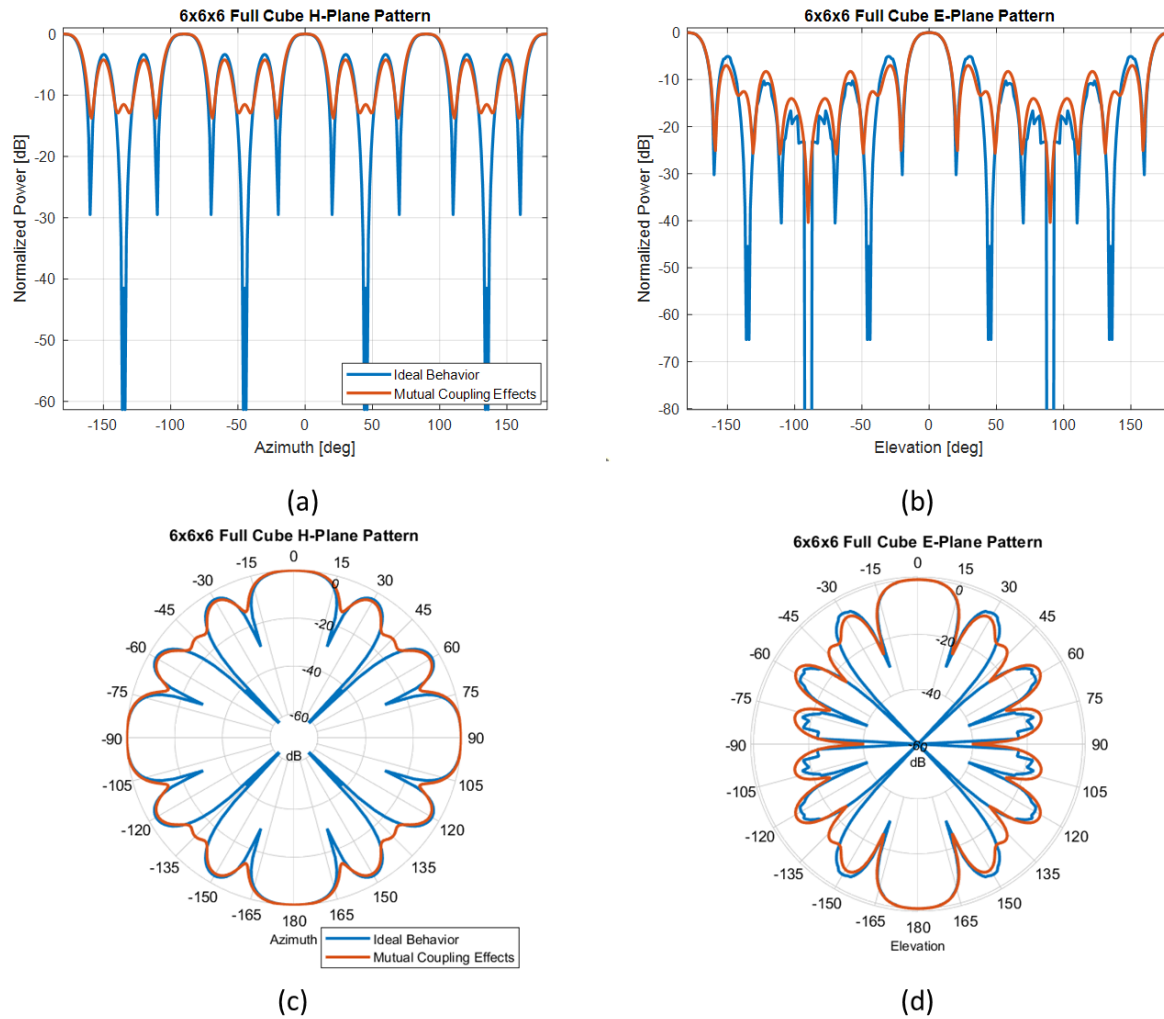
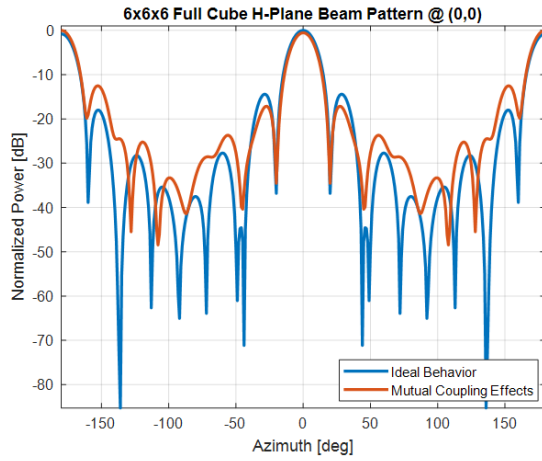


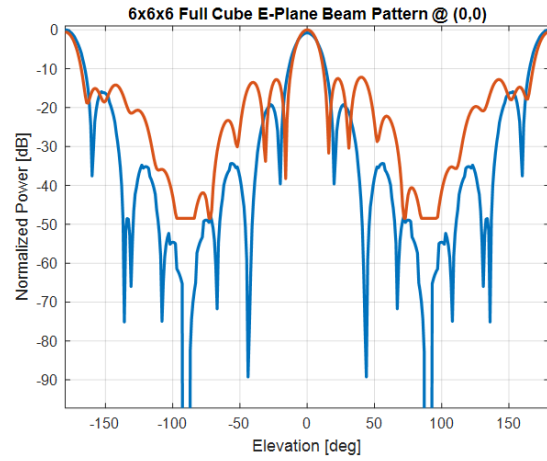
Figure A.9 The MATLAB ideal plot vs the 4nec2 mutual coupling plot for the general array pattern for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of 6x6x6 model at 30 MHz

Table A.9 Simulation Pattern Parameters for plots in Figure A.9

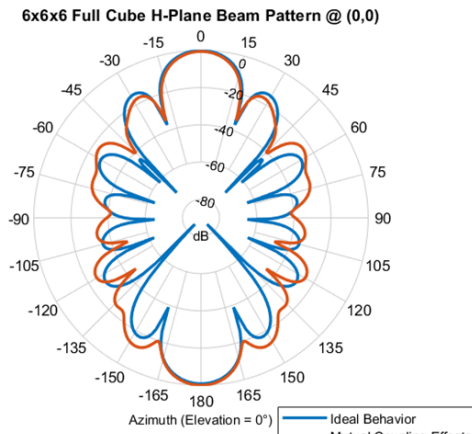
Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
Ideal	25.84	-5.043	-3.3184	22.3239
Mutual Coupling	27	-8.7	-3.86	10.37



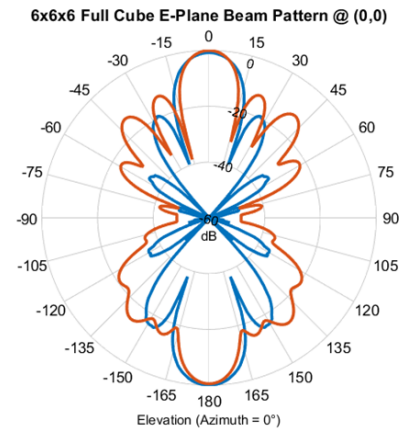
(a)



(b)



(c)



(d)

Figure A.10 The MATLAB ideal plot vs the 4nec2 mutual coupling plot for the beamformer at (0,0) for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of 6x6x6 model at 30 MHz

Table A.10 Simulation Pattern Parameters for plots in Figure A.10

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
Ideal	17.84	-15.879	-14.4211	18.3098
Mutual Coupling	16	-12.75	-12.51	18.45

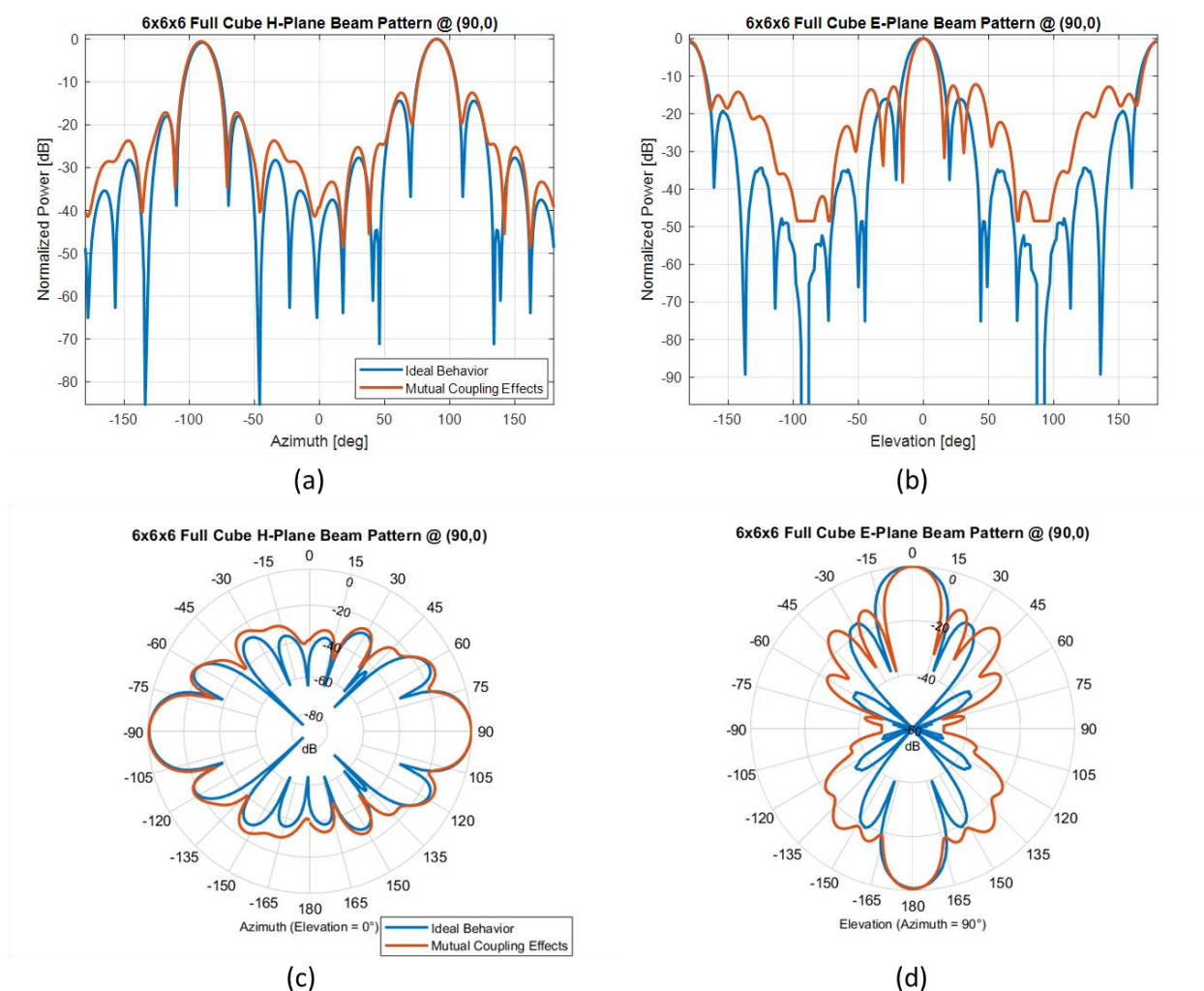
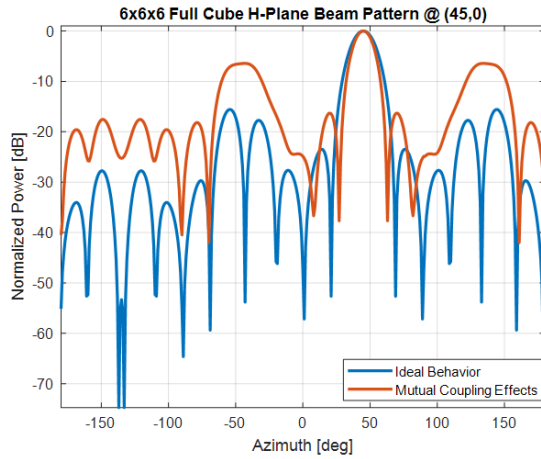


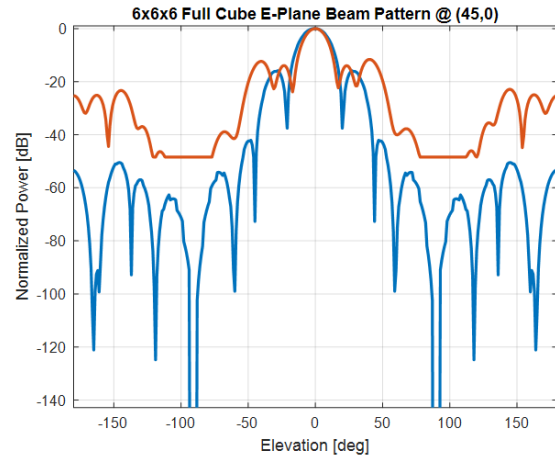
Figure A.11 The MATLAB ideal plot vs the 4nec2 mutual coupling plot for the beamformer at (90,0) for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of 6x6x6 model at 30 MHz

Table A.11 Simulation Pattern Parameters for plots in Figure A.11

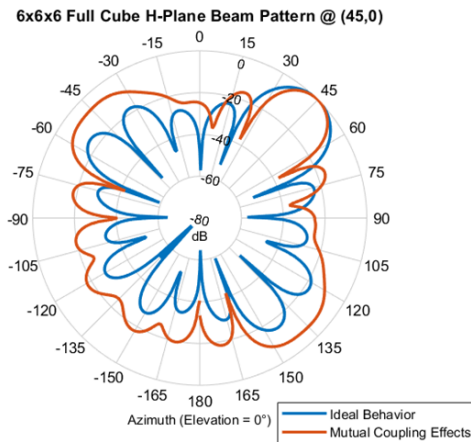
Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
Ideal	17.84	-15.879	-14.4211	18.3098
Mutual Coupling	18	-12.75	-12.51	18.45



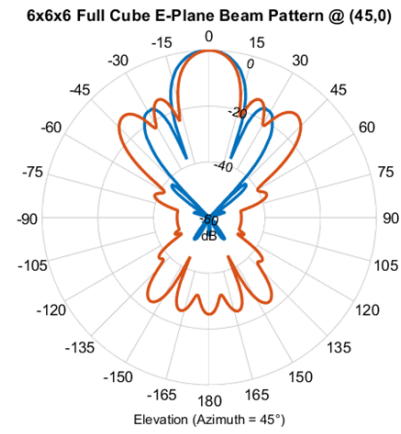
(a)



(b)



(c)



(d)

Figure A.12 The MATLAB ideal plot vs the 4nec2 mutual coupling plot for the beamformer at (45,0) for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern of 6x6x6 model at 30 MHz

Table A.12 Simulation Pattern Parameters plots in Figure A.12

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
Ideal	18.16	-15.8538	-23.4624	20.5108
Mutual Coupling	15	-13.94	-16.29	18.45

A.2 10.34 MHz Simulations

A.2.1 Array Pattern Simulations

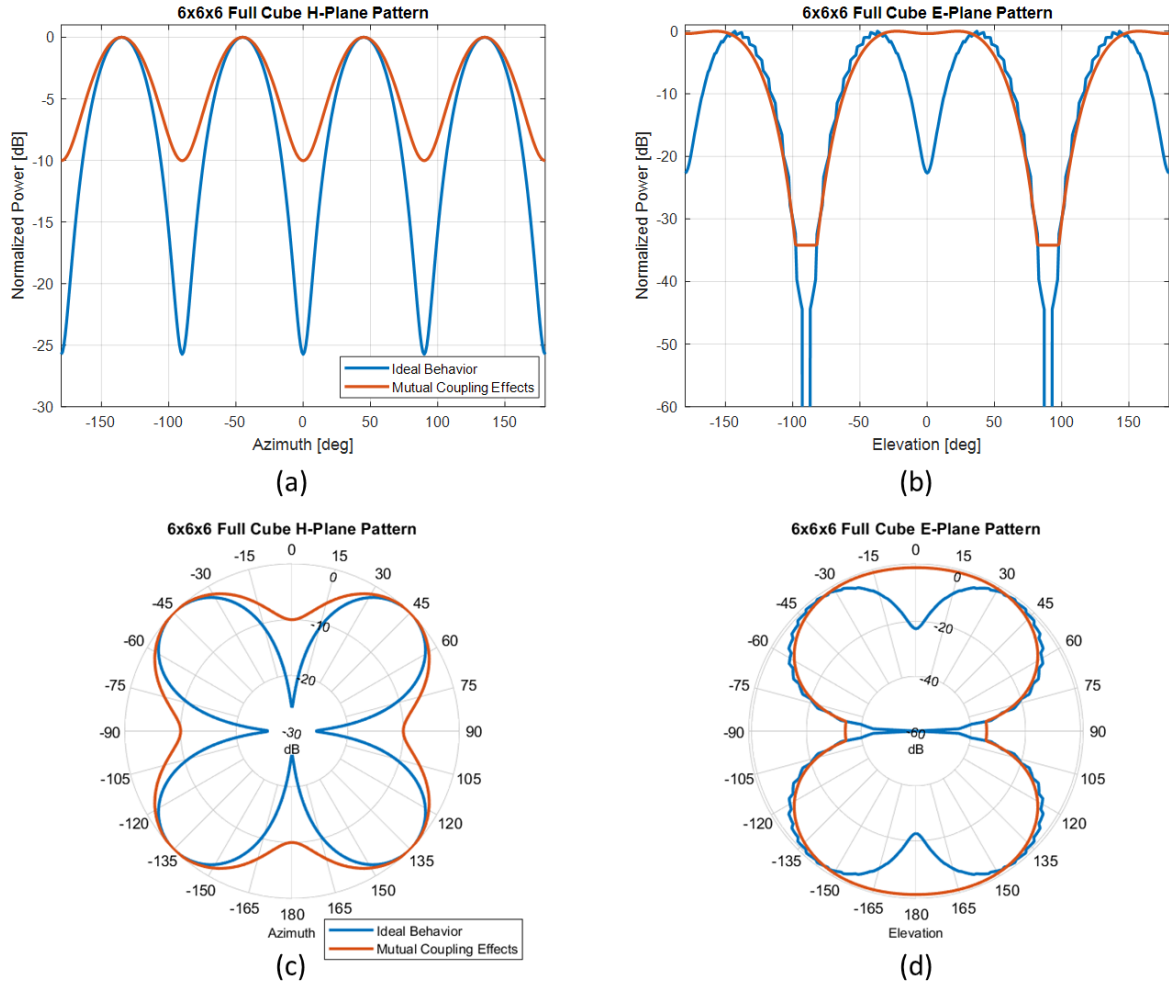


Figure A.13 Ideal MATLAB and 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the general pattern of the 6x6x6 Full Cube reconfigured to 2x2x2 Full Cube at 10.34 MHz

Table A.13 Simulation Pattern Parameters for 4nec2 Plots in Figure A.13

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
MATLAB	34	--	--	15.2716
4nec2	40	--	--	4.21

A.2.2 Beamforming Simulations

A.2.2.1 Beamforming Simulations at (Azimuth, Elevation) = (0,0)

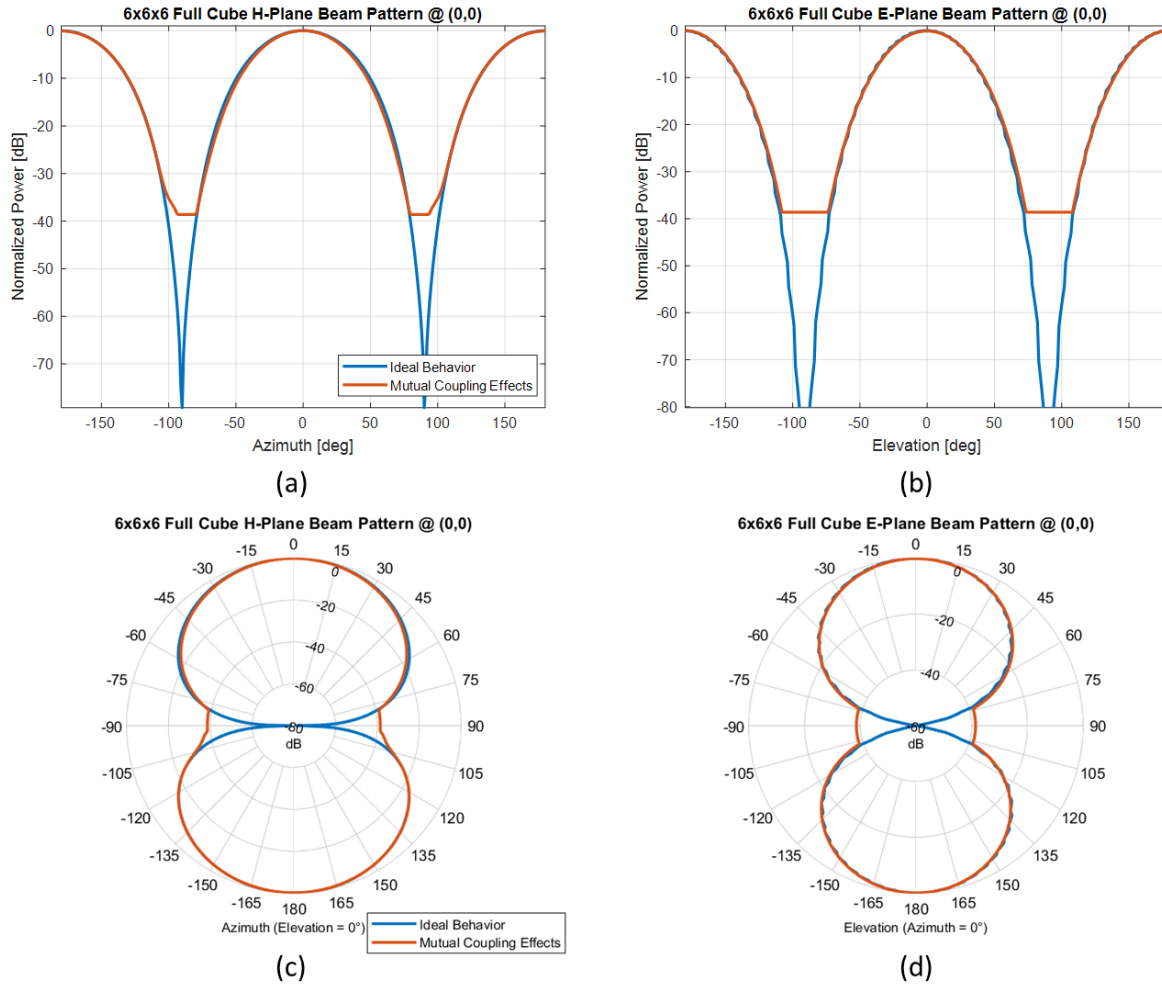


Figure A.14 Ideal MATLAB and 4nec2 mutual coupling (0,0) beamforming plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 Full Cube reconfigured to 2x2x2 Full Cube at 10.34 MHz

Table A.14 Simulation Pattern Parameters for 4nec2 Plots in Figure A.14

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
MATLAB	58	--	--	10.6875
4nec2	56	--	--	8.61

A.2.2.2 Beamforming Simulations at (Azimuth, Elevation) = (90,0)

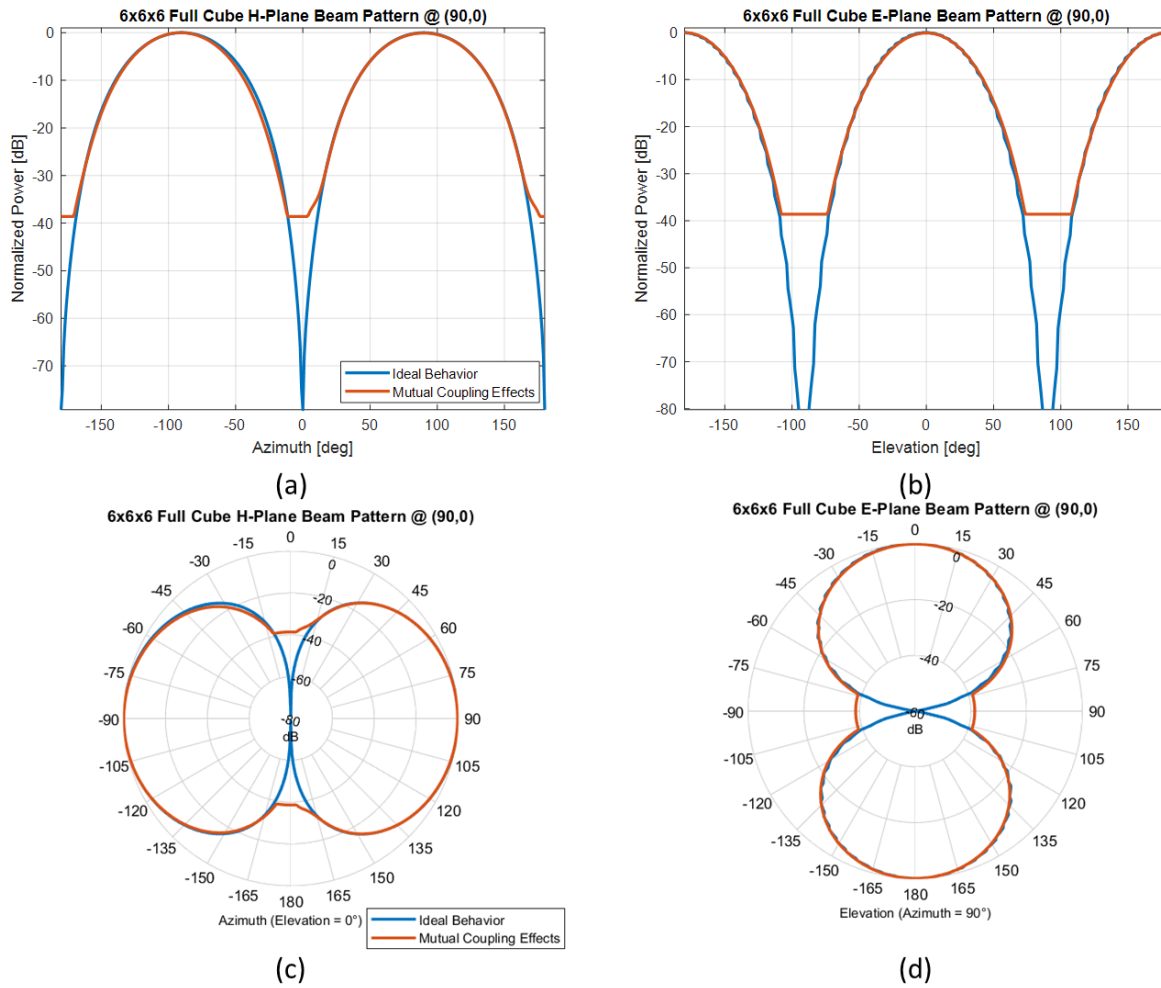


Figure A.15 Ideal MATLAB and 4nec2 mutual coupling (90,0) beamforming plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 Full Cube reconfigured to 2x2x2 Full Cube at 10.34 MHz

Table A.15 Simulation Pattern Parameters for 4nec2 Plots in Figure A.15

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
MATLAB	58	--	--	10.6875
4nec2	58	--	--	8.61

A.2.2.3 Beamforming Simulations at (Azimuth, Elevation) = (45,0)

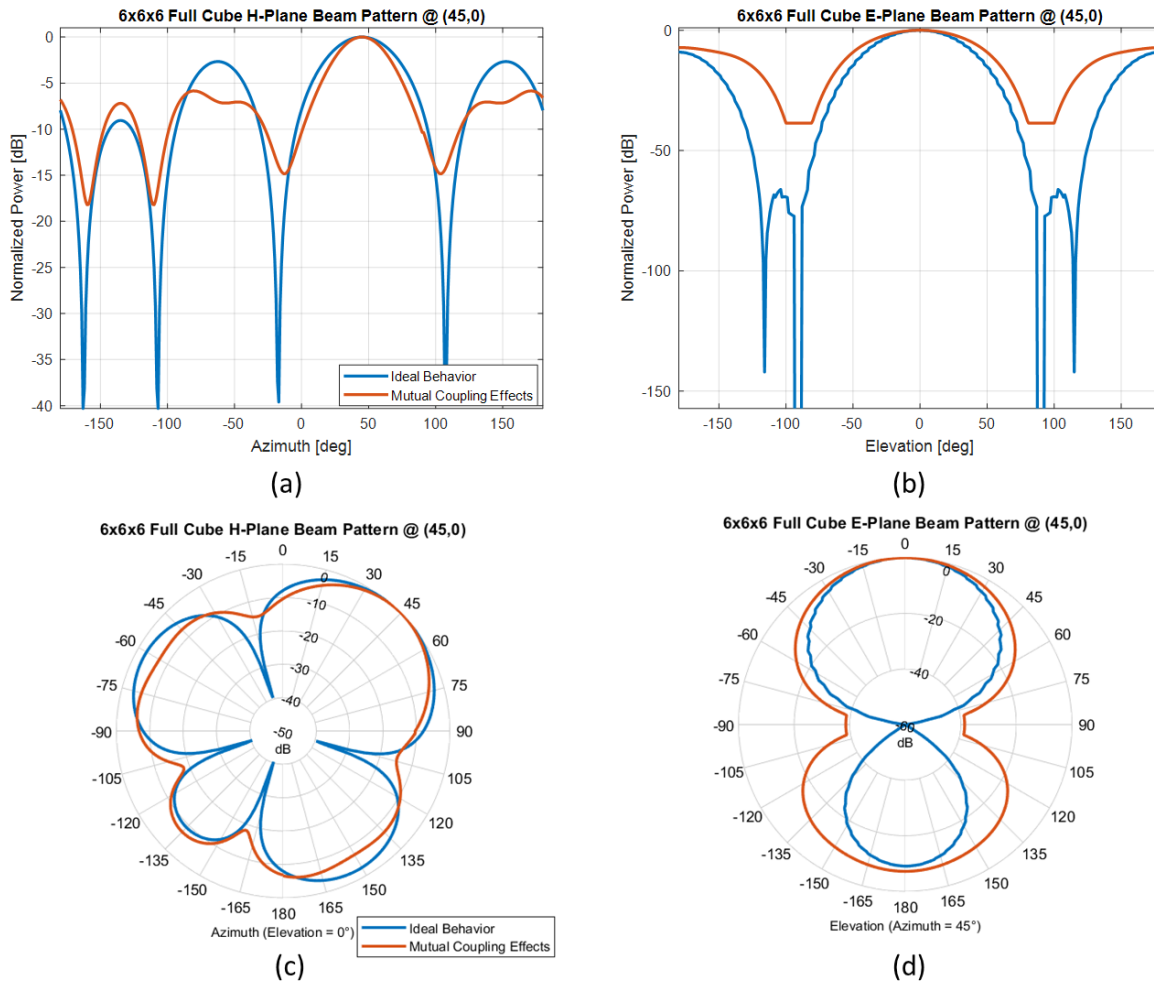


Figure A.16 Ideal MATLAB and 4nec2 mutual coupling (45,0) beamforming plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 Full Cube reconfigured to 2x2x2 Full Cube at 10.34 MHz

Table A.16 Simulation Pattern Parameters for 4nec2 Plots in Figure A.16

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
MATLAB	60	-9.08197	-2.6629	11.0369
4nec2	46	-7.2	-5.85	8.61

A.2.3 Mutual Coupling Effects

The mutual coupling effects and comparison to the ideal performance for the 10.34 MHz configuration are captured in the results from the simulations in Appendix A1.2.2.1 – A1.2.2.3

A.3 Comparison of the 30 MHz and the 10.34 MHz Configurations

A.3.1 Array Pattern Simulations

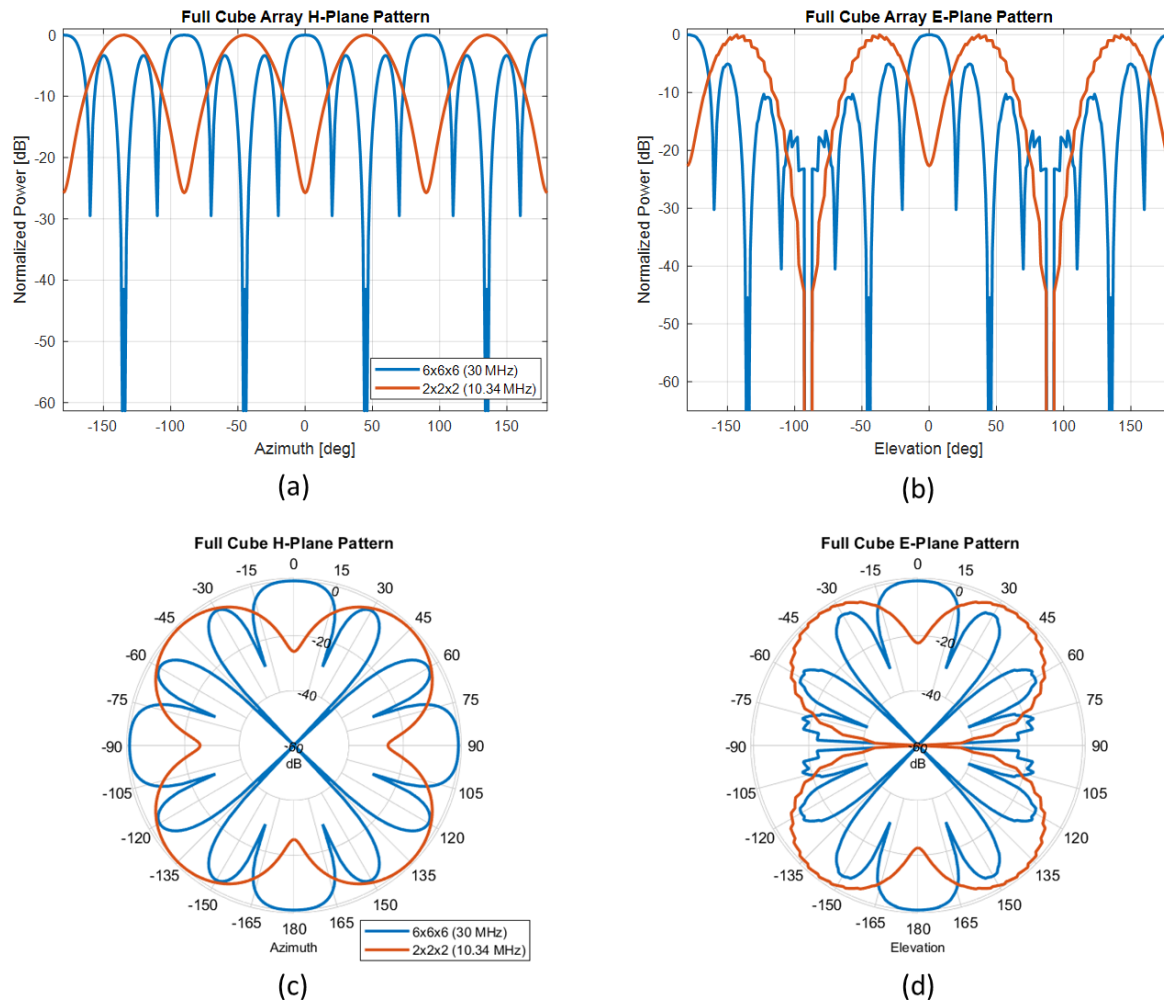


Figure A.17 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 cube at 30 MHz and reconfigured to the 2x2x2 cube at 10.34 MHz

Table A.17 Simulation Pattern Parameters for MATLAB Plots in Figure A.17

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	25.84	-5.043	-3.3184	22.3239
10.34 MHz	34	--	--	15.2716

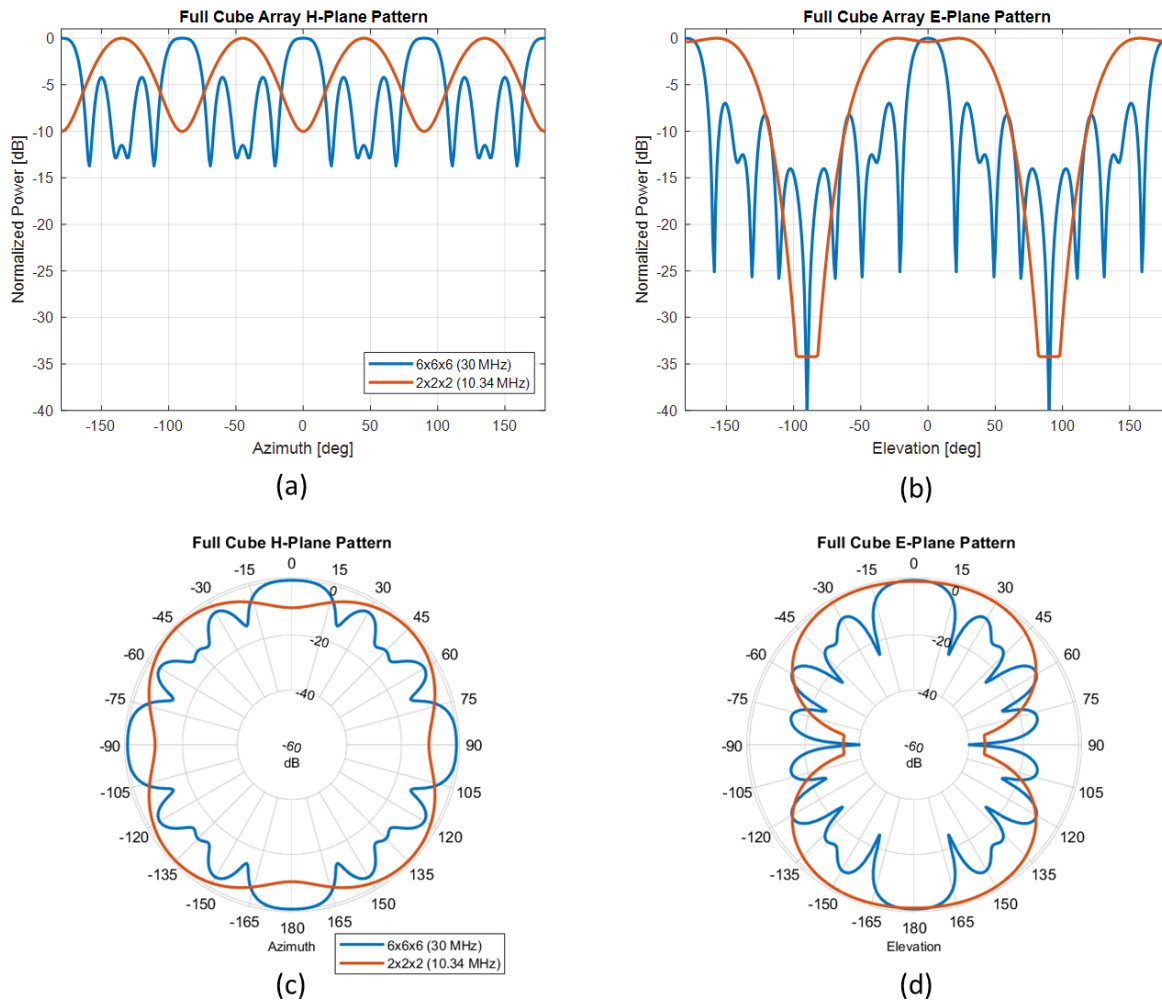


Figure A.18 The 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 cube at 30 MHz and reconfigured to the 2x2x2 cube at 10.34 MHz

Table A.18 Simulation Pattern Parameters for 4nec2 Plots in Figure A.18

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	27	-12.75	-12.51	10.37
10.34 MHz	40	--	--	4.21

A.3.2 Beamforming Simulations

A.3.2.1 Beamforming Simulations at (Azimuth, Elevation) = (0,0)

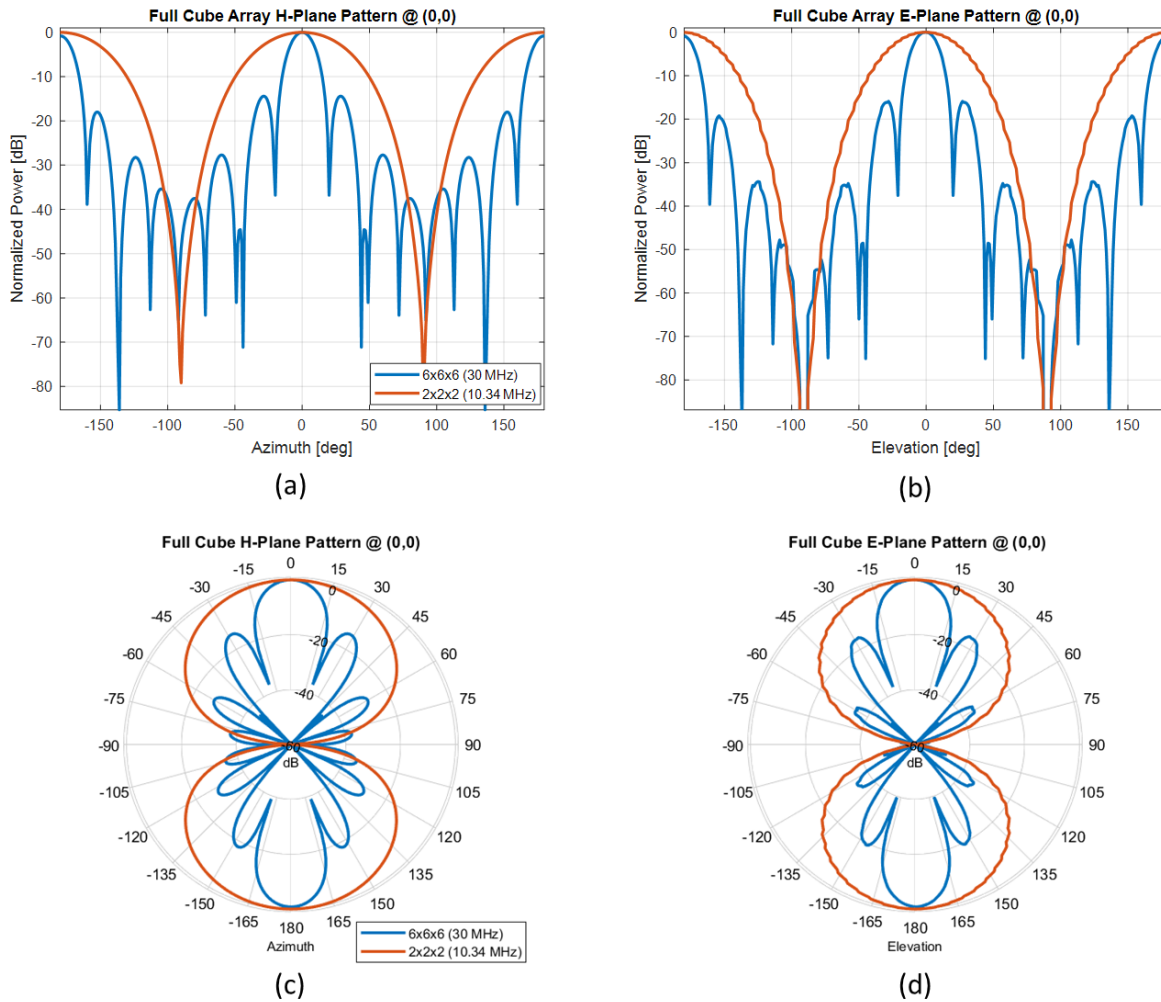
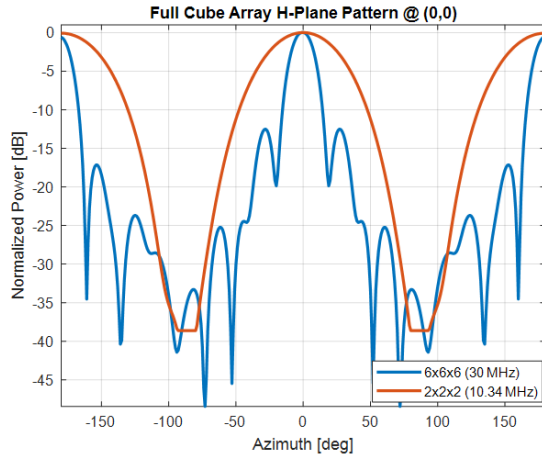


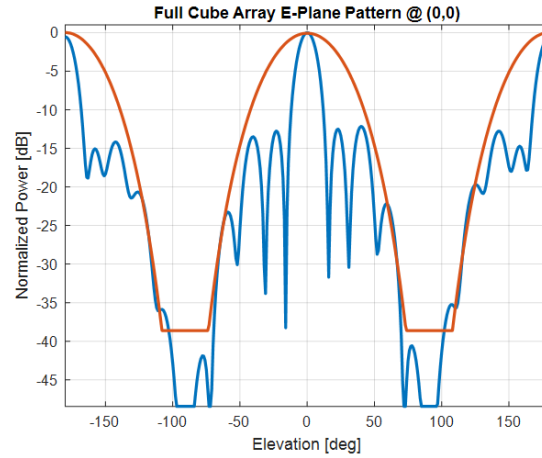
Figure A.19 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 cube at 30 MHz and reconfigured to the 2x2x2 cube at 10.34 MHz with beamforming coefficients at (azimuth, elevation) = (0,0)

Table A.19 Simulation Pattern Parameters for MATLAB Plots in Figure A.19

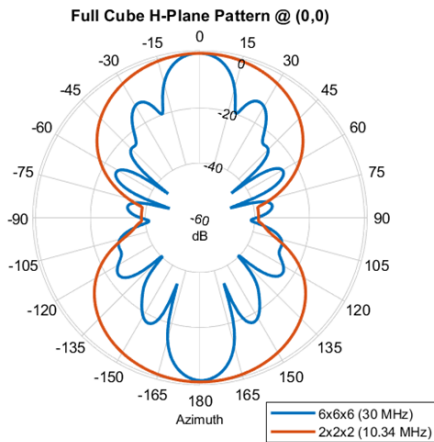
Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	17.84	-15.879	-14.4211	18.3098
10.34 MHz	58	--	--	10.6875



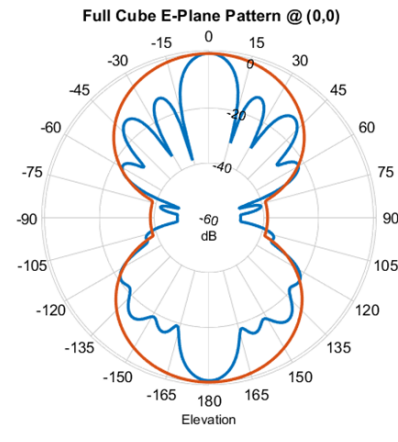
(a)



(b)



(c)



(d)

Figure A.20 The 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 cube at 30 MHz and reconfigured to the 2x2x2 cube at 10.34 MHz with beamforming coefficients at (azimuth, elevation) = (0,0)

Table A.20 Simulation Pattern Parameters for 4nec2 Plots in Figure A.20

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	22	-12.75	-12.51	18.45
10.34 MHz	54	--	--	8.61

A.3.2.2 Beamforming Simulations at (Azimuth, Elevation) = (90,0)

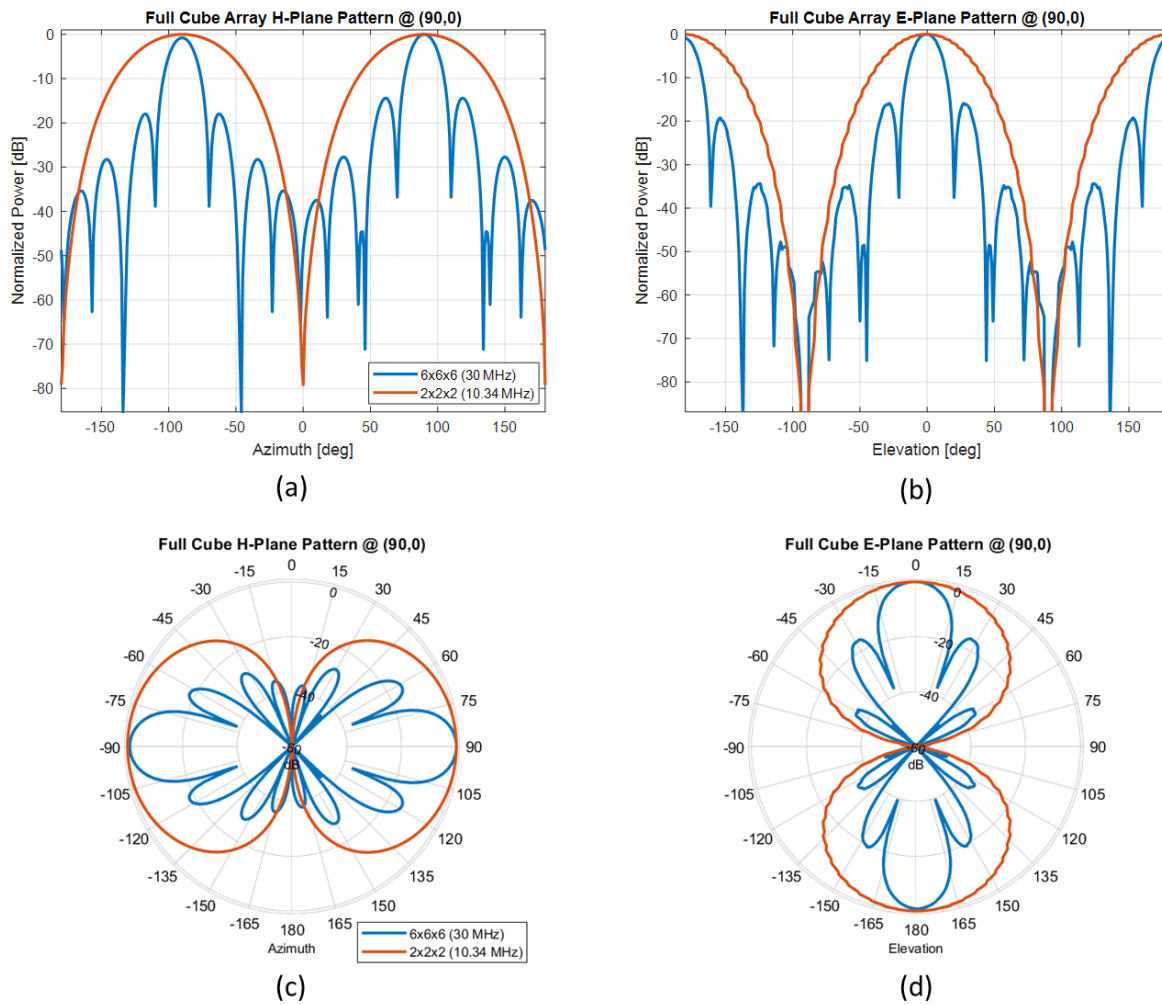
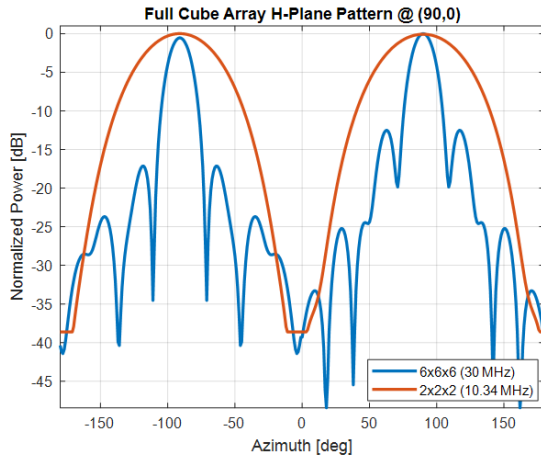


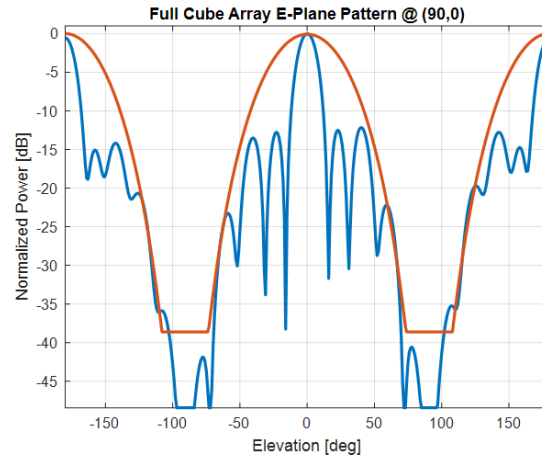
Figure A.21 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 cube at 30 MHz and reconfigured to the 2x2x2 cube at 10.34 MHz with beamforming coefficients at (azimuth, elevation) = (90,0)

Table A.21 Simulation Pattern Parameters for MATLAB Plots in Figure A.21

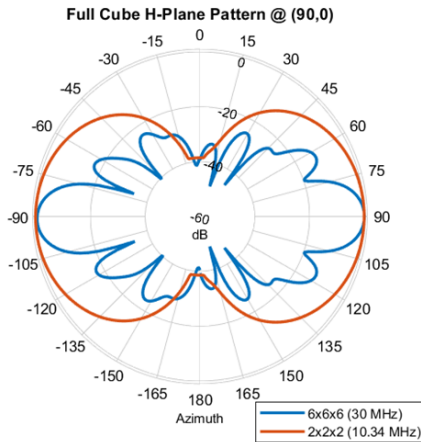
Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	17.84	-15.879	-14.421	14.6668
10.34 MHz	58	--	--	10.6875



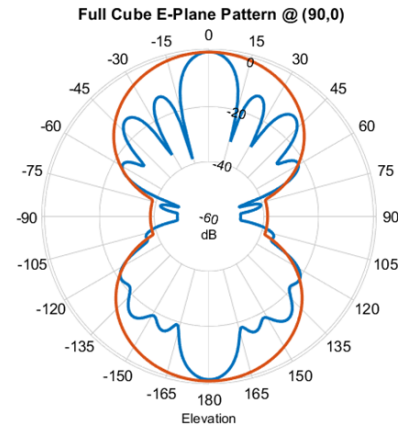
(a)



(b)



(c)



(d)

Figure A.22 The 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 cube at 30 MHz and reconfigured to the 2x2x2 cube at 10.34 MHz with beamforming coefficients at (azimuth, elevation) = (90,0)

Table A.22 Simulation Pattern Parameters for 4nec2 Plots in Figure A.22

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	18	-12.75	-12.51	18.45
10.34 MHz	58	--	--	8.61

A.3.2.3 Beamforming Simulations at (Azimuth, Elevation) = (45,0)

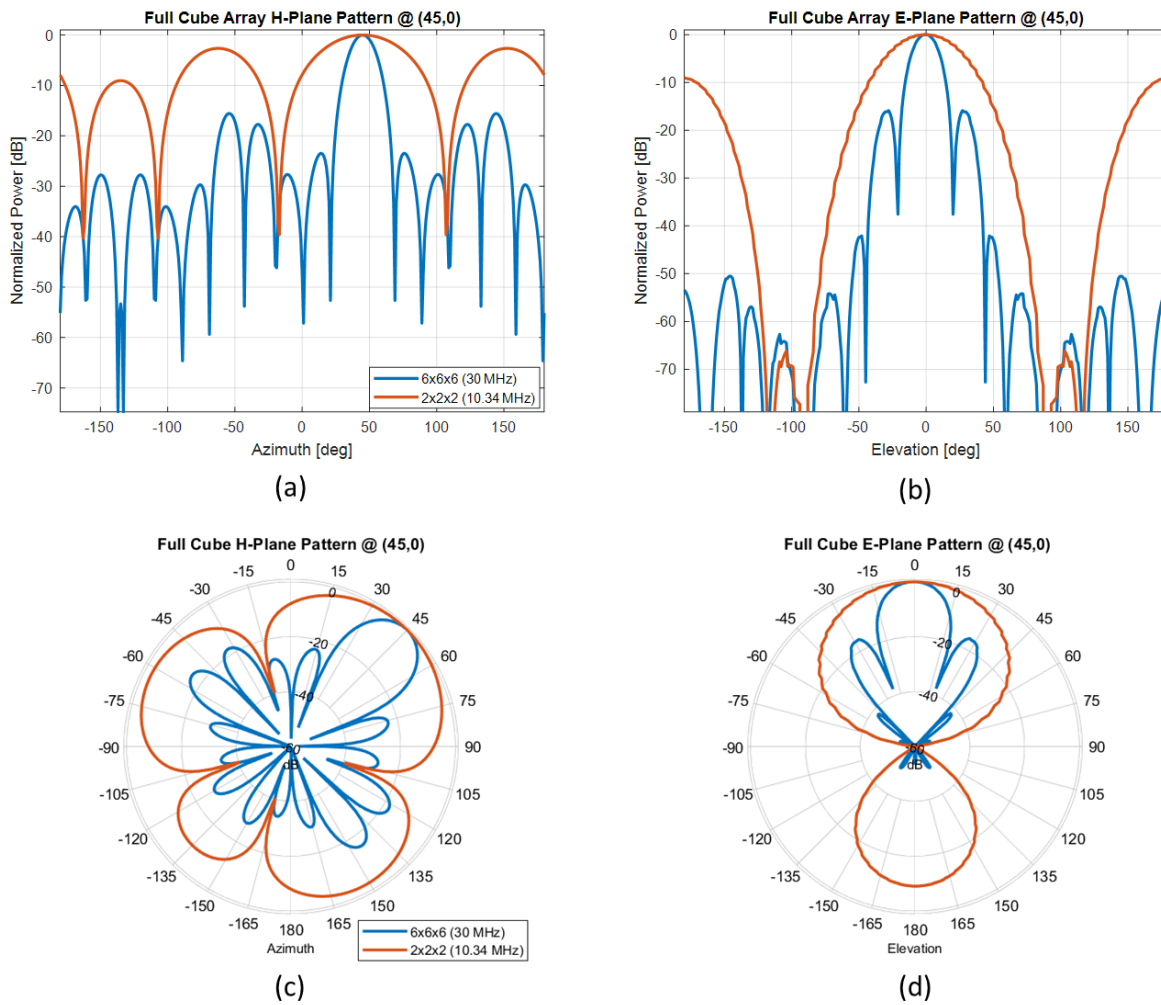
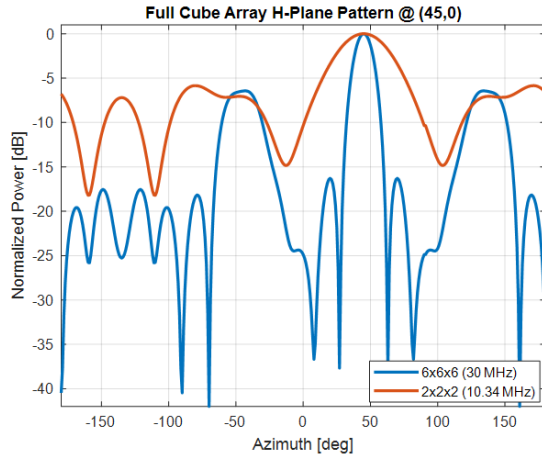


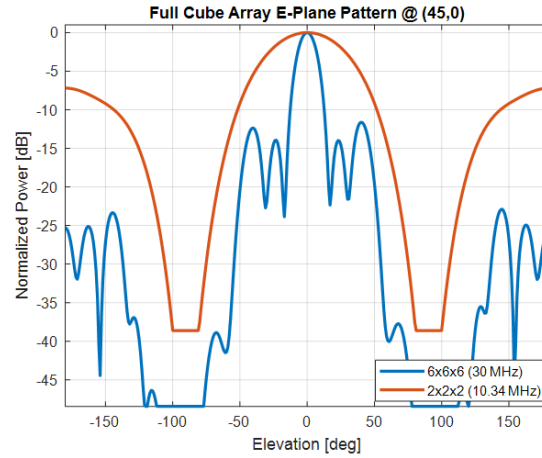
Figure A.23 The MATLAB ideal plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 cube at 30 MHz and reconfigured to the 2x2x2 cube at 10.34 MHz with beamforming coefficients at (azimuth, elevation) = (45,0)

Table A.23 Simulation Pattern Parameters for MATLAB Plots in Figure A.23

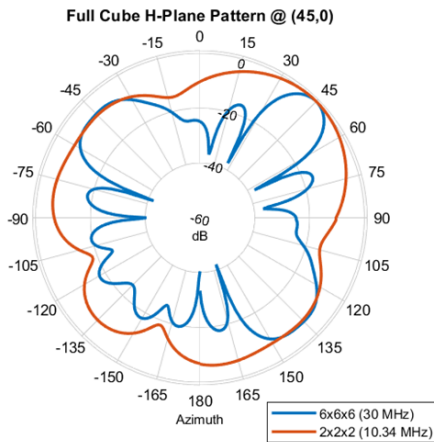
Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	18.16	-15.8538	-23.4624	20.5108
10.34 MHz	60	-9.08197	-2.6629	11.0369



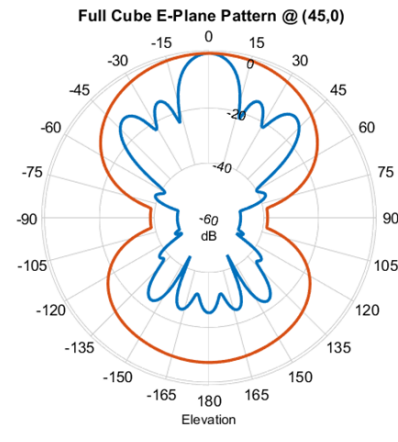
(a)



(b)



(c)



(d)

Figure A.24 The 4nec2 mutual coupling plots for the (a) rectangular coordinate H-plane pattern, (b) rectangular coordinate E-plane plot, (c) polar coordinate H-plane pattern and (d) polar coordinate E-plane pattern for the 6x6x6 cube at 30 MHz and reconfigured to the 2x2x2 cube at 10.34 MHz with beamforming coefficients at (azimuth, elevation) = (45,0)

Table A.24 Simulation Pattern Parameters for 4nec2 Plots in Figure A.24

Configuration	HP [deg]	SLL (E-Plane) [dB]	SLL (H-Plane) [dB]	Gain [dB]
30 MHz	15	-13.94	-16.29	18.45
10.34 MHz	54	-7.2	-5.85	8.61

Appendix B – Prototype of Cubic Array

This chapter discusses the design and fabrication of the prototype version of the array design. A brief discussion into the frequency scaling for experimental environment is provided. The discussion then shifts to the individual components for the prototype design and their respective sizes and scales. The data processing architecture of the prototype is then discussed. Modifications to the simulations due to environmental effects are provided in the next section. The chapter concludes with a discussion of how the reconfiguration can be implemented on the prototype design.

B.1 Prototype Constraints and Approach

A full-scale array prototype that operates at HF would be approximately $882,000 \text{ m}^3$ in size. This is far beyond the practical scale for a feasible laboratory environment. In order to verify the array design in a laboratory setting, the array is scaled down in size by scaling the operational frequency up to an arbitrary value of 500 MHz for convenient experimental operation and measurement. Due to timing and material constraints, the largest prototype that was fabricated was a 4x4x4 64-element array.

B.2 Prototype Design and Fabrication

The prototype for the array utilizes the same design concept as presented in Figure 3.1. The sizes of the individual components for the design, however, are scaled down for the new laboratory environment operational frequency. At 500 MHz, the wavelength should be approximately 0.6 m. Therefore, the length of the individual elements for the prototype should be slightly less than approximately 0.3 m. Furthermore, for simplicity, the spacing between elements in all three dimensions for the prototype is $\lambda/2$. This spacing is slightly larger from the

design in Chapter 3 that was simulated in Chapter 4; however, the performance of the prototype should not change much other than the occurrence of a grating lobe in analysis.

Like the design in Figure 3.1, each array element consists of two conductive rods and a spacer and is separated from other elements by an empty spacer. The spacers are custom 3D printed hexagonal-sphere shapes similar to those in Figure 3.1. Each spacer is composed of 2 identical halves that fit together such that a SMA feed-point can be placed inside of it in either x, y, or z. As such, the spacers need to be big enough to contain SMA feed-point connectors which are approximately 0.04572 m in length. The individual conductive rods were selected as 4-inch aluminum standoffs. The overall length of a single element then is 0.24892 m, which meets the desired requirement of slightly shorter than $\lambda/2$. Finally, the spacing between elements in all three dimensions is 0.29464 m, approximately $\lambda/2$ as desired. The aluminum standoffs are connected to the feed-point using brass screws. A step-by-step illustration of how the feed-points are assembled is provided below in Figure B.1.

An S11 measurement is performed on a single element. The results of that measurement are shown in Figure B.2. The resonant frequency of the array element is found to be 505 MHz, very near the desired 500 MHz.

Ideally, the nonconductive rods from Figure 3.1 would be implemented as 4-inch nylon standoffs that are identical to the aluminum standoffs. However, due to a supply chain issue, the nylon standoffs were not able to be acquired. The nonconductive rods in the design serve as structural support and as a way to maintain spacing between elements. For the prototype, an alternative means was implemented to provide both spacing and structural support.

The support was implemented using a multi-beam fiberglass L-bracket cage design with tension cables to prevent sag. The element spacing was achieved in several ways. The z-spacing

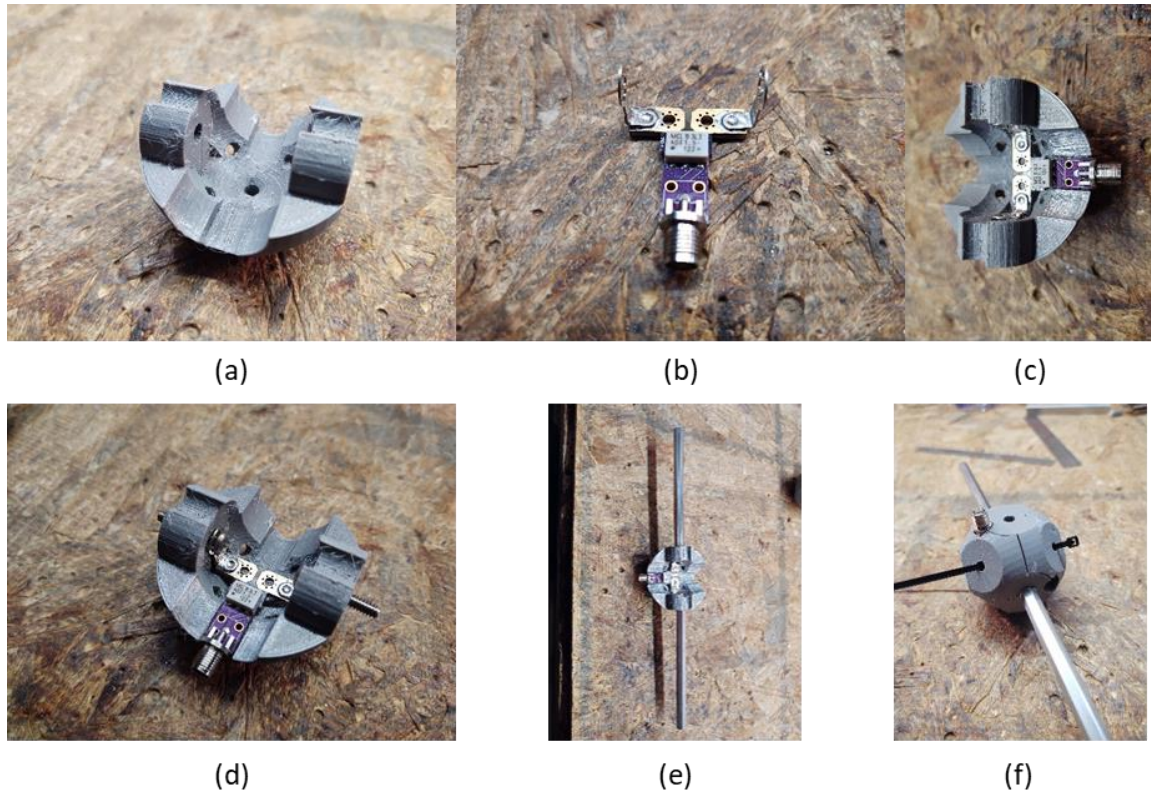


Figure B.1 The array components and process for constructing a half-wave dipole at 500 MHz: (a) half of a spacer component, (b) SMA feed-point for a single array element, (c) feed-point placed inside the spacer, (d) brass screws to connect the feed-point to the aluminum standoffs, (e) complete half-wave dipole of length is 0.24892 m, (f) half-wave dipole with feed-point completely encased by the spacer

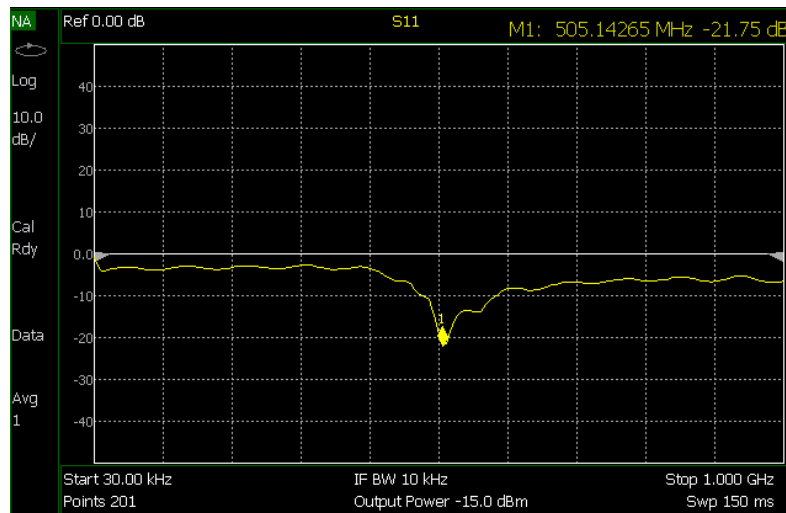


Figure B.2 S11 measurement on a single prototype half-wave dipole element consisting of two 4-inch aluminum standoffs and one 1.8-inch spacer showing that resonance occurs at 505 MHz

was maintained as in the original design with dipole sizing and spacers. The y-spacing was maintained by securing PVC pipes to the spacers in each plane and securing them at the desired spacing apart. The PVC also served the secondary purpose of providing additional structural support. The x-spacing was maintained by the spacing between the fiberglass beams in the cage design. All connections were done using nylon nuts and bolts to prevent electromagnetic interference.

The prototype is shown side-by-side with the design from Chapter 3 in Figure B.3. While the design and the physical prototype appear to be very different, it is important to mention that the electrical performances of the two are identical. As the conductive array elements are the same, the only factor that must be maintained is the element spacing. How that spacing is achieved should not perturb performance results.

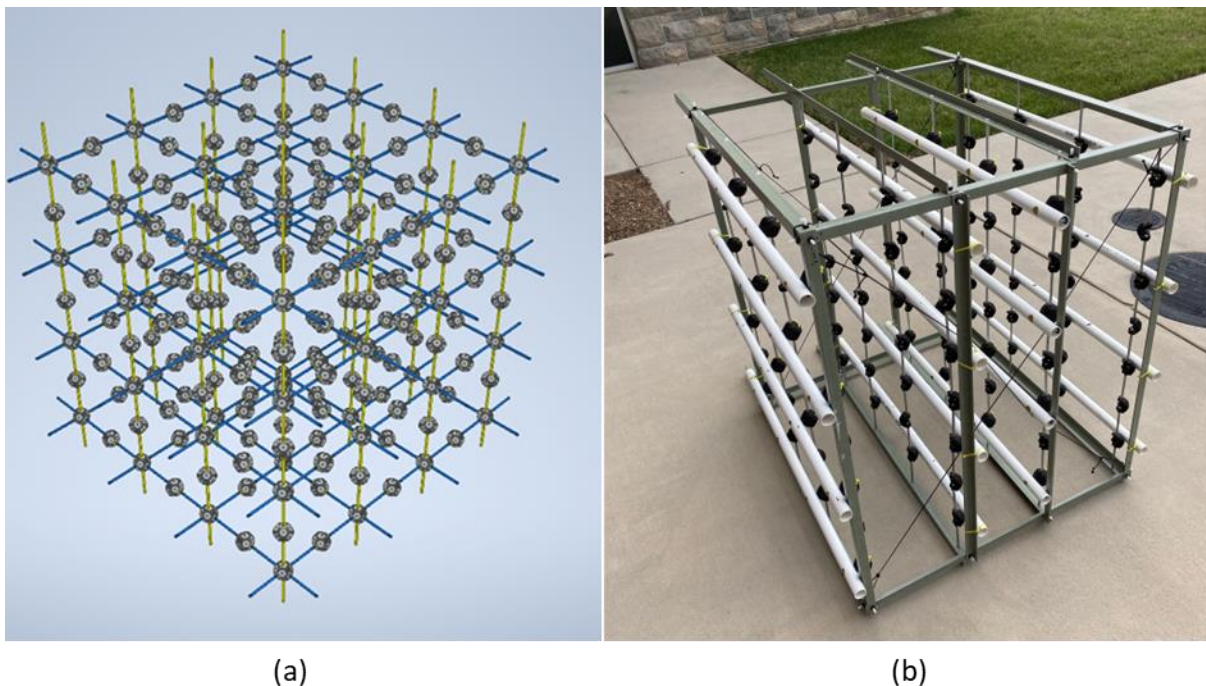


Figure B.3 (a) The design of the array for the in-space assembly of a three-dimensional cubic HF radio telescope phased array vs (b) the electrically equivalent scaled down prototype three-dimensional cubic phased array for testing and characterization

B.2.1 Prototype Resonance Performance

Once the 4x4x4 64-element array prototype was built, a 65th half-wave dipole was placed in the center of the prototype and an S11 measurement was performed. This measurement revealed that the resonance of the entire prototype structure was 505.6 MHz, practically identical to the 505 MHz from the initial test Figure B.2. The results of both of this S11 test is shown in Figure B.4.

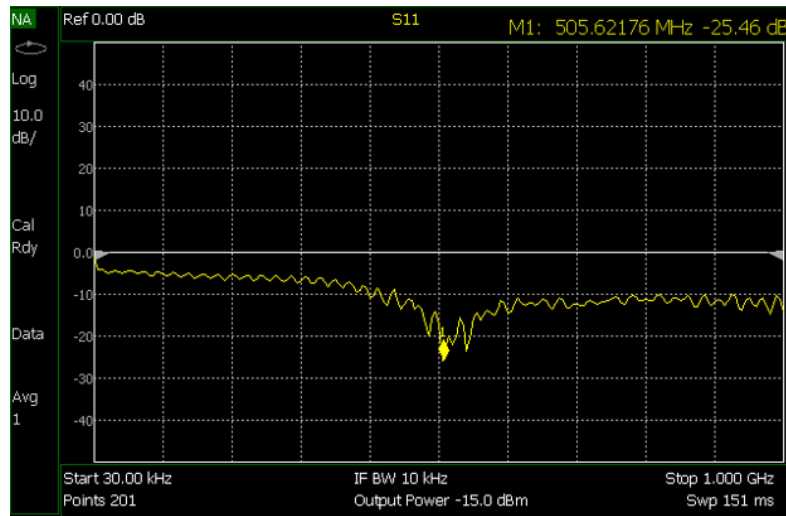


Figure B.4 S11 measurements for a single half-wave dipole in the center of the three-dimensional cubic phased array at the (a) 4x4x4 64-element configuration and (b) 4x2x4 32-element configuration

B.2.2 Digital Processing Architecture

The digital architecture for data acquisition, storage, and processing for the array prototype is provided in Figure B.5. Each feed-point is implemented as an RTL SDR. Every row of each panel of the array is processed by a single raspberry pi4; the first four elements of panel A (A1-A4) interface with the first raspberry pi, elements A2-A8 interface with the second raspberry pi, etc. In total there are 16 raspberry pi4 components and 64 RTL SDR components. Each of the raspberry pi4 components is connected to the central digitizer unit for data acquisition and storage for post processing. The RTL SDR components are not phase coherent and will

experience a phase drift throughout the chamber measurement. This can be addressed by utilizing a calibration signal to cohere the phases of the RTL SDRs with respect to a known reference point.

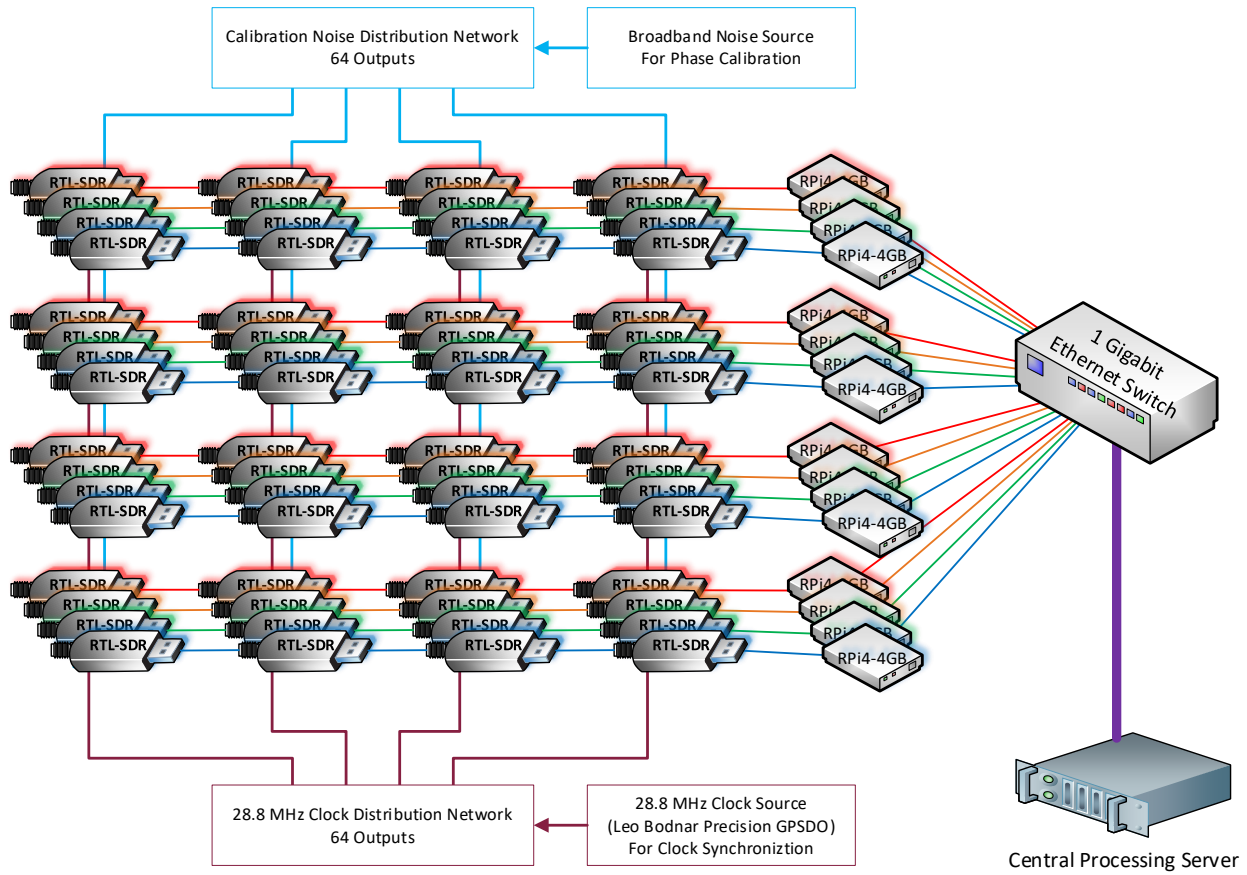


Figure B.5 Digital architecture for the array prototype

B.3 Laboratory Modifications

In order to take measurements in a chamber, the array must be placed on a tower. Therefore, a mechanism must be implemented to affix the array to the tower as well as connecting the digitizer, which contains the RTL SDRs and the raspberry pi4s, to the opposite side of the tower as the array prototype. This is achieved through the use of 80/20 aluminum frames. By implementing a metal frame in close proximity to the array elements, the mutual coupling effects will be influenced, and a modification to the simulation model is required to more accurately

compare simulation results to chamber measurement results. The aluminum frame is added to the 4nec2 model of the 4x4x4 Full Cube scaled to 505 MHz. This modified 4nec2 model is shown in Figure B.6.

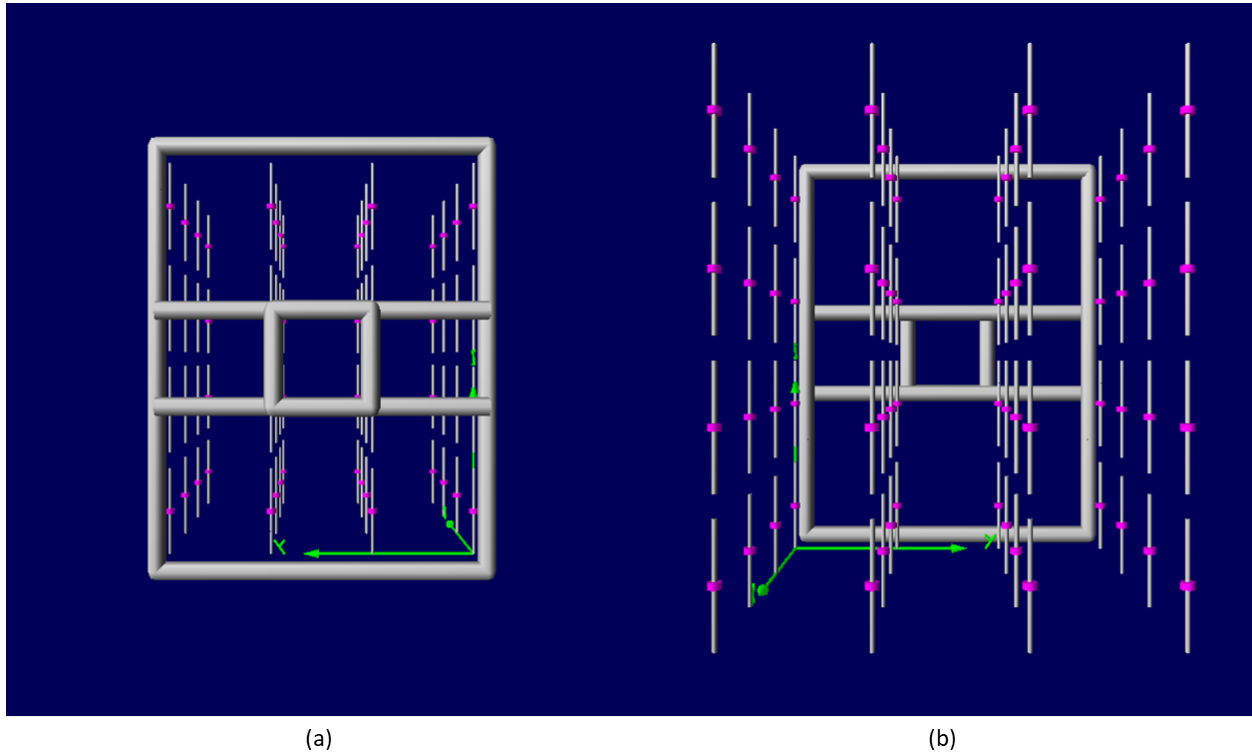


Figure B.6 (a) Back view and (b) front view of the array showing the aluminum mounting frame on the 4x4x4 64-element configuration in the 4nec2 simulation software

It should be noted that the aluminum mounting frame in 4nec2 is not an exact recreation of the actual mounting plate. However, the model implemented in Figure B.6 is the limit of the 4nec2 simulation's capability and should adequately represent the mutual coupling effects of the mounting plate. There are other sources of mutual coupling in a chamber that are not accounted for in this updated model. The digitizer also has an 80/20 aluminum frame and is essentially an aluminum box itself. In addition, the tower in the chamber acts as an extra source of mutual coupling effects. Finally, the SMA cables that connect the feed-points in the prototype to the digitizer will also have an impact on the mutual coupling performance. None of these effects are accounted for in the updated 4nec2 model. However, the dominant source of extra mutual

coupling should be the prototype's 80/20 frame, which is approximated in the updated simulation model.

B.4 Reconfiguration of Prototype Design

The array prototype as discussed thus far is designed to operate at 505 MHz. Reconfiguration of this prototype can be implemented by moving the SMA feed-points into different spacers and using brass threading to connect aluminum standoffs through empty spacers. The brass threading mimics the closed RFIC switch. The new configuration of the prototype forms a 2x2x2 8-element cubic array that operates at 252.5 MHz. In this fashion, a prototype has been built that can be used to experimentally measure the RF performance of the cubic three-dimensional geometry array and reconfiguration performance.

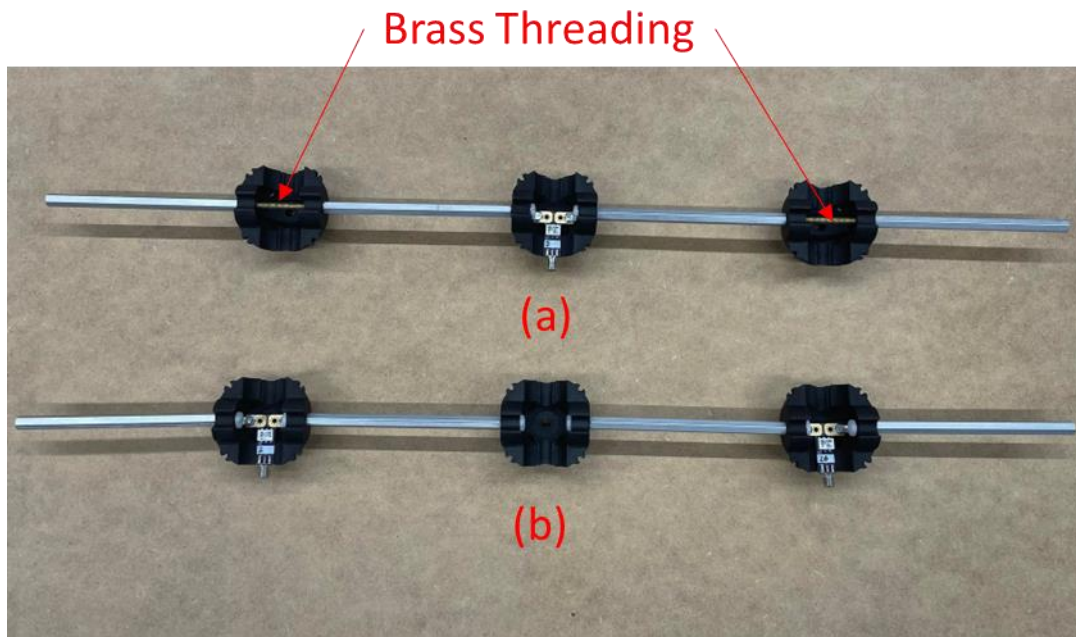


Figure B.7 (a) A single 252.5 MHz element by using brass threading to short together 505 MHz elements and (b) Two 505 MHz elements separated by an empty spacer