RELATIONSHIPS BETWEEN TREE CROWN, STEM AND STAND CHARACTERISTICS IN LOBLOLLY PINE PLANTATIONS

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(ABSTRACT)

Empirical and theoretical relationships between tree crown, stem and stand characteristics for unthinned and thinned stands of planted loblolly pine were investigated. The individual tree crown measurements of crown diameter (CD) and crown projection area (CPA), and stand level measurement of the sum of crown projection areas (SCPA) were particularly important in contributing to high levels of model fit and prediction abilities of common stem and stand characteristics. As these crown measures developed over time so did corresponding stem and stand attributes. The results were similar for trees and stands located in unthinned or thinned situations; however, a limited range of data may have accounted for these similarities.

The stem attributes modeled included basal area and diameter and associated growth, while the stand attributes modeled were basal area and mean dbh and associated growth. Models were also developed for the individual tree crown characteristics of CD and growth, CPA, and height to crown diameter, and for the stand level crown attributes of SCPA and growth, mean crown projection area and mean height to crown diameter.

Several common competition indices were adapted to include crown information and various structural changes. The most effective competition measures in helping to predict basal area growth were point in time crown measures of SCPA and CD for trees located in unthinned stands and SCPA and CPA for trees located in thinned stands.

Lastly, the effects of planting rectangularity on stem basal area growth were investigated. Two measures of rectangularity were calculated: one dealing with the ratio of distances between adjacent competing trees and another involving the ratio of the major and minor axis distances of a subject tree's crown diameter. Depending on the age of the stand, these measures of rectangularity were found to be significant in negatively affecting stem basal area growth of trees located in unthinned stands.

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1.0 INTRODUCTION

There are four basic types of prediction models found in forestry today: whole stand, diameter distribution, individual tree distance dependent and individual tree distance independent. The basis for these models are quantified relationships of certain stand and stem characteristics at a point in time and over time, and as affected by silvicultural practices. In terms of modeling conifer growth and yield, stand and stem attributes that have not been thoroughly investigated have been tree crown dimensions and development.

In the past, tree crowns have not been measured due to difficulties in defining and measuring their changing dimensions and shapes (Spurr 1952, Honer 1972, Honer and Collins 1974). However, recent applications of low-level aerial photographs for measuring crown dimensions (Mitchell 1975a, Mitchell 1980, Burton and Shoulders 1982) suggest cheaper and more accurate methods are available. Measurements from aerial photographs of tree and stand level crown dimensions can provide accurate and precise estimates of individual tree attributes (e.g., dbh, dbh growth, or volume) and stand level attributes (e.g., basal area, basal area growth, surviving trees or volume per acre). This latter application has particular use in inventory through the reduction of ground cruising.

An obvious advantage of using crown characteristics of conifers as a basic modeling unit is the cause and effect relationship between crown and stem in terms of growth. It was recognized in the mid 1900's that certain tree crown dimensions were strongly related to various stem parameters and that definite growth patterns over time due to differing stocking levels existed. Physiologists have qualified the internal mechanisms, and the cause and effect relationships of why and how crown development influences stem development. Some of these relationships, such as the correlation between diameter at breast height (dbh) and crown diameter, have been equated and used directly or indirectly for modeling purposes. The relationships existing between crown and stem dimensions and development should also be similar on a stand level, although the former have not been well quantified.

The implications from integrating crown information into modeling stem and stand characteristics are numerous. For example, level of stocking, degree of rectangularity (or configuration) of spacing, pruning, and timing, intensity and method of thinning directly affect crown dimensions and development, and thus indirectly affect stem and stand characteristics. If the effects of various cultural treatments on crown development are quantified, then better estimates of treatment effects on stem and stand characteristics can be made. The characteristics of interest can be in terms of the quantity of wood produced, such as total and merchantable volume for stems and stands, and quality of wood, such as early-wood and late-wood, and size-number-distribution of knots, since each are directly associated with crown development.

The point of this work was to quantify crown, stem and stand relationships for thinned and unthinned loblolly pine (Pinus taeda L.) plantations.

1.1 OBJECTIVES

The specific objectives were to:

1.) Quantify individual tree crown and stem relationships over time for trees located in unthinned and thinned stands of planted loblolly pine.

2.) Quantify stand level crown and stand relationships over time for unthinned and thinned stands of planted loblolly pine.

3.) Quantify and test adaptations of individual tree and stand level measures of competition for use in modeling individual tree growth for trees located in unthinned and thinned stands of planted loblolly pine.

4.) Quantify the effects that spacing rectangularity has on crown and stem relationships for trees located in unthinned stands of planted loblolly pine.

2.0 LITERATURE REVIEW

2.1 QUALIFYING CROWN AND STEM RELATIONSHIPS

For shade intolerant species and when light is the limiting factor for tree growth, competition for light results in different patterns of crown formation. Open-grown trees free from intraspecific competition develop full, deep crowns generally to the ground. In contrast, the crowns of forest-grown trees recede in vertical length and decrease in horizontal development as competition for light increases.

Stem growth consists of cylindrical sheaths of wood being superimposed on top of each other in an annual sequence. In 1864, Pressler determined that the crown affected stem taper by observing ring growth within the crown increased downward. Hartig (1870, 1871, 1891) agreed and also stated three growth laws for the branch-free bole:

- Open-grown trees have downward increasing ring basal area and width growth.
- Dominant and codominant trees in a closed stand have equal ring basal area growth but downward decreasing ring width growth.
- Intermediate and suppressed trees have downward decreasing ring basal area and width growth.

In general, the longitudinal pattern of stem growth of a tree under medium competition consists of a sheath widening from the top of the tree to the widest portion of the crown: thereafter, sheath width is fairly constant until near the base of the tree where it widens. General agreement is in evidence that the widest part of the crown is the most productive part of the crown. However, ring width, not ring basal area, is maximized at that point. These trends were noticed for red pine[1] (Duff and Nolan 1953, Duff and Nolan 1957, Farrar 1961), loblolly pine (Labyak and Schumacher 1954, Reukema 1961), Norway spruce (Ilvessalo 1950), jack pine (Shea and Armson 1972), Pinus thunbergii (Yoshida and Kanamitsu 1979), larix (Larson 1965) and others (Wilcox 1962, Larson 1962a, Larson 1962b). As age progresses, the point of maximum radial increase will shift upwards (Duff and Nolan 1953).

Point in time measurements of crown or stem dimensions do not reflect the significance of what stem growth trends will occur (Reukema 1957, Reukema 1961, Hall 1965). The complete history of how crowns were formed determines how diameter increment was formed and how future increment, especially in the upper part of the stem, will be formed.

Crown and stem growth relationships are influenced directly by factors

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[1] See Table A1 in Appendix A for a list of corresponding scientific names.

affecting crown development as in the case of intraspecific competition. Ring width and stem basal area in open-grown trees will not only increase downward within the crown, but due to the proximity of the crown to the base of the tree, they will continue increasing to ground level. In contrast, a suppressed tree will follow a similar pattern within the crown area, but the sheath widths will decrease downward below the crown.

As crown and stem growth are cause and effect related, cultural treatments, such as thinning, may directly affect crown development and thus indirectly affect stem development. Thinning a closed stand can release trees from intraspecific competition enabling selected crop trees to grow as open-grown trees until crown closure occurs. The resultant crop trees grow as dominants with associated increases in crown size (length and width) and vigor with similar growth patterns as discussed for open-grown trees (Farrar 1961, Larson 1962a, Larson 1963, Cown 1974). As crown closure begins a few years after thinning, crown dimensions are once again restricted and stem growth takes on the patterns as discussed with closed-grown trees.

The effects of thinning on crown development have been questioned. It is generally accepted that thinning causes increases in stem radial growth basipetally within a tree; however, Reukema (1964) found thinning reduced crown development of red pine during the first few years after being released. He concluded crown expansion was not the major cause for increased growth of thinned trees, and cited improved aerial environment, reduced root competition, and increased crown surface area as the primary factors.

2.2 QUANTIFYING CROWN AND STEM RELATIONSHIPS

A wide spectrum of equations and models have been developed to quantify the basic processes and relationships described in the previous section. The main interest here is to review the quantified crown and stem relationships and how they have been employed in describing individual tree relationships and developing models.

An alphabetical list and graphical representation of general abbreviations and terminology used in this paper are presented in Appendix B.

2.2.1 TREE RELATIONSHIPS

Most reported findings are based on unreplicated studies. Researchers describing the relationships between crown dimensions and stem growth have emphasized horizontal and vertical crown dimensions.

2.2.1.1 Horizontal Crown Relationships

The theoretical relationship between crown diameter (CD) and dbh is non-linear (Spurr 1952, Dawkins 1963); most of the data available, because of associated range limitations, suggest a linear trend (e.g., Honer 1972). Generally, the CD-dbh relationship for open-grown trees or trees found in repeatedly thinned stands was of the following linear trend:

$$CD = \alpha_0 + \alpha_1 dbh \ .$$

Tropical species from a variety of management regimes (Dawkins 1963), thinned stands of loblolly pine (Minor 1951), open-grown loblolly pine (Strub *et al*. 1975), red pine from a wide range of stocking levels (Stiell 1966), lodgepole pine and Douglas-fir (Smith and Bailey 1964), and *Eucalyptus obliqua* (Curtin 1964) have followed this basic trend. In unthinned stands, however, height differentiation due to competition and effects from constant stocking levels altered this CD-dbh relationship to necessitate the inclusion of some measure of stocking. Reports have shown that CD-dbh relationships for forest-grown trees differed significantly in slope from that in open-grown stands. Total height proved to be a significant variable with a negative contribution, when modeling forest-grown stands (Minor 1960, Bonner 1964, Curtin 1964, Smith and Bailey 1964).

Two hypotheses were confirmed by these reports: 1) crown compaction (i.e., for a given diameter, tall trees have narrower crowns than short trees) existed for some species and thus crown diameter ratio (CDR) was independent of stocking levels; and, 2) increasing height caused a reduction in the relative growing space regardless of stocking level. Height thus acted as a link between the two-dimensional stocking level indices, and age or site quality characteristics. Briegleb (1952) analyzing Douglas-fir data and Ilvessalo (1950) working with Norway spruce, also confirmed the above trend of stocking effects on the CD-dbh relationship. In addition, early findings (Munger 1946, Eversole 1955) from a Douglas-fir spacing study indicated as initial spacings increased (i.e., 4x4 to 12x12 feet), CD significantly increased (a similar trend was noticed for red pine by Stiell and Berry 1977). Quantification of the exact trends did not occur until these stands were 42-years-old when it was reported that CD, as a linear function of dbh, was significantly different between spacings (Curtis and Reukema 1970).

2.2.1.2 Vertical Crown Relationships

A variety of models describing the relationships between CL or height to the live crown (HLC) and other stem variables have been reported. In general, as stocking levels increased, CL decreased or HLC increased (Ilvessalo 1950, Briegleb 1952, Smith et al. 1961, van Laar 1963, Beekhuis 1965, Kramer 1966, Stiell 1966, Curtin 1970, Siemon et al. 1976, Harms and Langdon 1976, and Stiell and Berry 1977). In addition, Cole and Jensen (1982) presented a non-linear model describing the HLC of dominant trees as a function of the average height of the dominant trees and either stand basal area or CCF.

2.2.1.3 Other Relationships

Curtis and Reukema (1970) mentioned for a given age, dominant trees of a particular dbh class in a highly stocked stand were identical in dimensions as dominated trees of the same dbh class in a lower stocked stand. This trend suggested that the concept of crown class can vary with stocking level. Others (e.g., Lane-Poole 1936, Minor 1951, Beekhuis 1965, Stiell 1966) found that with constant stocking, crown size became constant, suggesting crown dimensions may not be well correlated with stem characteristics.

The majority of the above discussion centered around open or unthinned stands; however, thinned stands generally follow the same relationships as unthinned stands. For example, high correlations between CLR and CD to basal area growth of thinned stands of loblolly pine were found by MacKinney (1933); while, Hamilton (1968) found crown projection area and crown surface area were highly correlated with individual volume increment of thinned stands of Sitka spruce.

Crown and stem relationships have also been applied in establishing intensity of thinning regimes. Beekhuis (1965) and Kramer (1966) questioned the wisdom of heavy thinning aimed at the retention of CL extremes under the premise that maximum CL's would produce maximum volumes (e.g., as suggested by Guttenberg 1953). CLR was also not recommended as an indicator for thinning by Smith and Dubow (1960) who analyzed loblolly pine data. They found crown length ratio was not related to vigor in terms of diameter growth or need for thinning. If CLR was indeed related to these variables, then diameter growth should be negatively correlated to stem length and positively correlated, in similar magnitude, to CLR. As this was not the case they suggested using direct measures of crown dimensions for cultural decisions. In contrast, Reukema (1964) found stem radial growth was not directly related to an immediate crown expansion following thinning. He also did not find significant relationships between stem radial growth and CD, CLR, crown surface area increment and crown diameter ratio for thinned stands of Douglas-fir.

2.2.2 MODEL APPLICATION OF CROWN AND STEM RELATIONSHIPS

While relationships between crown dimensions and stem characteristics have been published as discussed, various authors have incorporated these findings in models by: use of direct relationships, as in the estimation of dbh or total height through functions of crown variables; or, use of competition indices based on relative or absolute measures of crown and/or stem development.

In modeling even-aged stands of white spruce, Mitchell (1969) used branch development as the basic modeling unit. He equated crown diameter to an adjusted branch length while crown length was allometrically related to crown diameter. The final stem-variable models were total height as a function of height of the dominants and crown diameter; dbh as related to CD and total height; and, stem volume as a function of CPA and total height.

Using a similar approach, Mitchell (1975b) also modeled Douglas-fir yields. Two modifications from the white spruce model were a cumulated branch growth function and quantity of foliage of a particular shell (FVi) function. He estimated total photosynthesis production (FV) by accumulating the total number of FVi. Bole increment was estimated as an anabolic function of FV, while crown length was estimated using CPA and a crown curvature value.

Loblolly pine plantation yields were modeled by Daniels and Burkhart (1975) using a modified competition index based on Hegyi's (1974) work and estimates of clear bole length (CBL). CBL was estimated using a linear function of stand characteristics. The estimate of CBL was used to form crown length ratios which were used with the competition index to adjust potential height and diameter growth, and to calculate a probability of survival in a mortality function. Similar relationships were also used to develop a growth and yield model for seeded stands of loblolly pine (Daniels _et_ _al_. 1979).

Unthinned loblolly pine yields were estimated by Feduccia _et_ _al_. (1979)

using crown length ratio. CLR was inversely related to age, and for a given age, inversely related to site index. The ratio was calculated using dbh of the subject tree, mean dbh of the stand, mean CLR of the stand and surviving trees. Volumes were found to vary by CLR classes, thus refined taper equations were used to predict inside and outside volumes per tree.

Lastly, a generalized model developed for hardwoods and pine used CLR as a variable (Hahn and Leary 1979). Using equation forms containing anabolic and catabolic terms developed by Richards (1959), changes in diameter over time were estimated using dbh, site index, and CLR variables. In this case mean CLR was related to age, arithmetic mean dbh and basal area.

2.2.3 TREE AND STAND DENSITY INDICES

Crown and stem relationships have been used directly and indirectly in a wide variety of competition indices. Munro (1974) reviewed different types of models and measures of competition and separated them into three model types based on common usage: 1) tree distance dependent, 2) tree distance independent, and 3) stand distance independent models.

2.2.3.1 Tree Distance Dependent

A review of distance dependent indices according to Alemdag's (1978) classification follows.

Crown or Adjusted Crown Overlap. These indices are based on the idea that a tree has a circular growing space (influence circle) proportional to some stem characteristic, such as dbh. Also, the amount of competitive stress affecting tree growth is determined by the amount of overlap of a subject tree's (i.e., the tree whose competitive stress is being evaluated) influence circle with adjacent or competing tree influence circles. The differences between overlap indices are found in how the influence circle is defined and overlap measured.

The measure of competition for this type of index has generally been based on the amount of theoretical crown overlap that exists when estimated open-grown crown areas of competing and subject trees are substituted for the actual crown areas (Newnham 1964, 1966). Competitive stress has also been measured by summing the overlap areas and weighting them by either a subject tree's influence zone (Gerrard 1969, Opie 1969), or by ratios of subject to competing tree's dbh (Bella 1971), crown radius and total height (Ek and Monserud 1974), or crown volume (Matney 1976).

This overlap measure of competitive stress increases as the size of competing trees increase, and as the amount crown overlap between competing and subject trees increases. The rationale in regards to this index measuring competitive stress is not as direct as other indices since theoretical overlap is the measure of competition. Like most indices, this index form suggests that competition is additive in that interaction between competing trees does not exist; however, it does allow for consideration of crown and height competition and in 3 dimensions. As individual tree distances and additional calculations beyond simple ratios are required, this index is more expensive to obtain and calculate than the distance independent or size and/or distance of neighboring tree indices.

Available Growing Space. Brown (1965) determined the amount of competition a tree was subjected to was inversely proportional to the area potentially available (APA) for that tree to use for growth. APA was the area of a closed irregular polygon, the sides of which were defined as lines perpendicular to a point bisecting lines connecting the subject tree with its competitors (i.e., a subject tree had half the distance between its neighbor for crown and root expansion). The major problem with this index was large trees with small, close competitors would be assigned a low APA (APA is given in area per tree or is the reciprocal of trees per unit area). Adaptations to this procedure redefined dividing points on the lines connecting the subject and competing trees: Moore *et al*. (1973) indicated polygons should be drawn at points proportional to the basal area of the trees and also suggested using a variable to reflect the change in competitive status over time. Alemdag (1978) proposed a ratio of dbh's of the subject and competing trees; while, Pelz (1978) found height, dbh and height, or dbh^2 and height should be used to define the dividing points and thus account for relative competition of trees in 3-dimensions. In addition to defining the size of the polygon, Mead (1966) used eccircularity (measure of how elliptical the area compares to a circle) and abcentricity (measure of how central the tree is in the polygon) in determining competition between trees.

Generally, the growing space potentially available decreases (i.e., competitive stress increases) as competing trees increase in diameter and decrease in distance from a subject tree. However, the index is not sensitive to the number of competitors and unit area is divided so that every square foot of space is accounted for in some area potentially available. On a practical basis tree crowns can only develop in a balanced manner and can only utilize open areas readily available. In many respects this seems like an effective index in that a potential growing space is defined, and it is based on the size and distances of competing trees. The obvious drawback is the high cost involved in obtaining and calculating the index.

Diameter and/or Distances of Neighboring Trees. An example of an index of interest here is one by Hegyi (1974), who used a single ratio of dbh's of the subject and competing trees, weighted by the distance between them. One shortcoming with this index was only trees within 10 feet of the subject tree were used; Daniels (1976) modified Hegyi's index by using a ratio of dbh or dbh^2 of the subject and competing trees plus a basal area factor to select competing trees.

With this type of index form, the measure of competitive stress increases as the number of competing trees increases, diameter of competing trees increases, and distance between competing and subject trees decreases. This index form also suggests that competition is additive and that competition, as it affects individual tree basal area growth at breast height, is measurable by ratios of diameters and distances. Since this type of index requires individual tree distances, there is a cost involved in knowing distances and calculating the index.

Crown Surface Area and Height. An index in this category is one by Hatch *et al*. (1975) which includes a measure of the spatial pattern of competing trees plus vertical development of the subject tree relative to other competing trees. This index is based on the ratio of directly exposed crown surface area to the height of the live crown of the subject tree, modified by the ratio of basal areas of the subject and competing trees.

2.2.3.2 Tree Competition Index Comparisons

Competition indices have generally been compared on the basis of variation explained in diameter or basal area increment. Most authors found marginal results in regard to one index explaining a significant increase in diameter or basal area growth compared to another. For example, Alemdag (1978), and Noone and Bell (1980) each compared eight indices and found none were clearly superior in explaining additional amounts of variation of diameter growth. Common occurrences, however, were as follows: simple diameter ratio indices were least costly to generate; and, the larger the influence circle or the larger the number of trees counted as competitors, the higher the R^2. This last relationship is possibly due to inclusion of trees not directly competing but indirectly competing through root grafts.

2.2.3.3 Tree Distance Independent

There are various distance independent indices that fall between tree and stand level measures. Alder (1979) formulated a stand measure using current basal area of the stand in comparison to the maximum potential basal area. In addition, he used an individual tree index based on the ratio of the subject tree's dbh to the mean dbh of the stand. A similar ratio, but using a comparison of the subject tree's dbh to the average stand diameter, was employed by Cole and Stage (1972). Glover and Hool (1979) developed a basal area index for determining loblolly pine plantation mortality based on the ratio of a subject tree's basal area in comparison to the basal area of the tree with mean dbh. In a similar fashions, Daniels (1981) used a ratio of a subject tree's basal area to the tree of average basal area, while Harrison (1984) weighted Daniels' index by stand basal area.

With these types of indices, the measure of competitive stress increases as the diameter of a subject tree decreases and as the stand basal area increases (i.e., average stand dbh increases). Since tree distances are not required, these indices need less information and computer time than the distance dependent indices; however, they may not be as reflective of the immediate levels of competition surrounding a subject tree. If there exists uneven levels of stocking or competition, stand averages may not be helpful in determining the placement of a subject tree in relation to the stand. As with the size and/or distance type of index, diameters of competing trees are assumed to indicate competitive stress and help predict future stem development of a subject tree.

2.2.3.4 Stand Density Measures

Stand density measures are used to indicate degree of crowding by relating the number of trees per unit area to various stem characteristics. The actual use of these indices is for prediction of growth and yield (Beck 1974) and prescription of silvicultural activities. Stand density indices can be grouped into two general categories: absolute and relative measures (see Spurr 1952 and Bickford et al. 1957 for general discussions on the older indices). Only the absolute measures are of interest and they include basal area, merchantable volume and trees per unit area. In conjunction with other stem characteristics, especially age, they offer the simplest, easiest and most widely used measures in terms of silvicultural prescriptions and as growth and yield variables.

2.3 DISCUSSION

Certain crown and stem variables and relationships described in the literature warrant further discussion. Depending on the type of radial development (i.e., diameter and basal area for a tree or stand level, at one point in time or over time) the important variables in predicting stem and stand development have been site index, age, and initial stand density and/or stem size. Certain crown dimensions appear logical for improving the prediction of radial development since crown characteristics indirectly measure photosynthetic area and distance that photosynthate must travel to other parts of the tree. The following individual tree and stand level crown variables may have a significant impact in predicting radial development:

2.3.1 INDIVIDUAL TREE CROWN VARIABLES

Crown Diameter (CD) and Crown Projection Area (CPA). Horizontal crown development can be measured by the diameter (CD) or area (CPA) of the widest portion of the effective crown. These are indirect measures of photosynthetic area, in that large values of these variables indicate large crown volumes and large areas of photosynthate production potential. Age and immediate stocking levels surrounding a tree affect the size and growth of CD and CPA; however, within a density and age class, CD or CPA should be highly related to diameter or basal area, respectively. Also, as CD or CPA develop, there should be an associated development with tree diameter or basal area growth.

Effective Live Crown (ELC). This height measurement from the top of the tree to the widest part of the crown provides a vertical measurement of crown development. Given other factors being constant, such as stocking level and age, as ELC increases, photosynthetic area increases. Within stocking levels, ELC is generally constant, while with increasing stocking levels, ELC decreases.

Height to Crown Diameter (HCD). As stocking levels do not generally affect total height, HCD is the antithesis of ELC. As such, as stocking levels increase, HCD increases.

Crown Volume (CV). In a sense, this represents the total cubic feet of crown available for photosynthate production. Thus, as CV increases, photosynthate production should increase.

Crown Surface Area (CSA). The entire crown volume does not have the potential for photosynthate production. As an attempt to reflect that condition, CSA is a measure of the surface area of CV, excluding the base. Crown Volume Weighted (CVW) and Crown Surface Area Weighted (CSW). In order to weight an indirect measure of photosynthetic area with the distance it must travel for lower stem growth, the variables CV and CSA were weighted by HCD. Thus, given two equal CV (or CSA) measures, the CV (or CSA) closer to dbh would have a larger weighted value and positive impact on stem development than the CV (or CSA) further away.

2.3.2 STAND LEVEL CROWN VARIABLES

Sum of Crown Projection Area (SCPA). This is analogous to basal area per acre where basal area is the horizontal development of crowns rather than stems. While having the same properties and trends as basal area, SCPA also indicates the rate of crown expansion on a stand level. An additional property of SCPA lies in its asymptotic characteristic found with intolerant species such as loblolly pine - as SCPA approaches unit area (i.e., 43,560 sq ft/ac), horizontal crown development decreases and vertical crown development increases. Depending on the stand structure and age, stem diameter growth will continue to increase depending on the rate of mortality. This is a similar relationship as expressed in Reineke's (1933) stand density index and the reason why CD as a function of dbh should be curvilinear.

For very intolerant species, SCPA will approach an asymptote at unit area and as the tolerance and efficiency of the species increases so will this level. Exact levels of asymptotes are not known; however, for a species like loblolly, the asymptote should not be much greater than unit area. For convenience of discussion, 43,560 sq ft/ac will be referred to as loblolly's natural asymptote with the realization that it is actually somewhat higher.

Sum of Crown Volume (SCV) and Sum of Crown Surface Area (SCSA). These are similar measures as above in that totals of crown volume and surface area are being calculated on an acre basis. The same implications of the sum of these variables can not be as readily applied as SCPA, however. These variables represent estimates of total photosynthetic area and should be highly related to stem development.

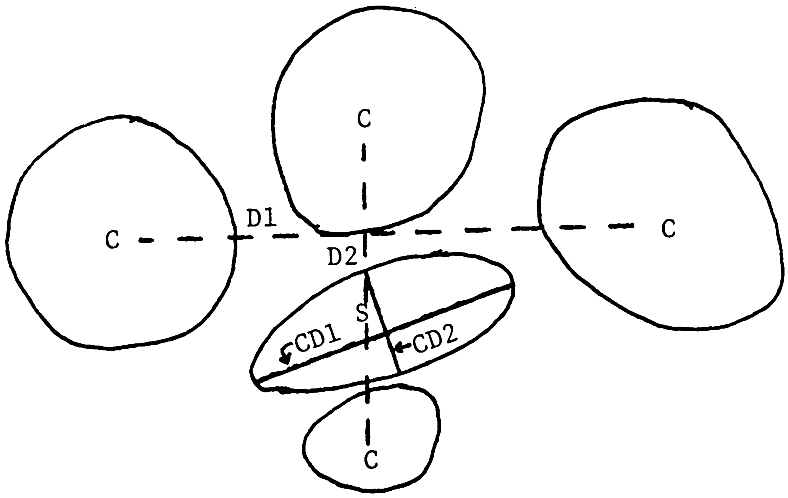
Other important measures of crown dimensions are mean crown projection area (MCPA) and mean height to crown diameter (MHCD). The MCPA and MHCD are, respectively, arithmetic means using the sum of crown projection area (SCPA) and heights to crown diameter (HCD) each divided by trees surviving.

2.3.3 SPACING RECTANGULARITY

Rectangular spacing of trees is common in operational planting. In terms of efficient growing space arrangements, square spacing (i.e., 1:1

degree of rectangularity) is more desirable than rectangular spacing for individual tree growth when trees are aligned within and across rows. A 1:1 degree of rectangularity promotes symmetric crown development which is the most efficient system for production and translocation of photosynthate. Branch development in the major axis of an elliptical crown requires more photosynthate for branch growth and support than minor axis branch development. In contrast, the branches in the minor axis direction of an elliptical crown are generally not efficient branches. That is, since competition from surrounding trees have limited the development of minor axis branches, they may represent net photosynthate users. While there is a theoretical justification for square spacing, determination of practical effects of degrees of rectangularity on stem growth needs to be made.

Two approaches of determining degrees of rectangularity seem available: they represented cause and effect situations. Rectangularity, representing cause, is the area available from surrounding trees expressed in terms of distances between competing trees (across and within rows) (Figure 1). The four nearest competing trees, one in each quadrant surrounding the subject tree, may represent the trees defining the size and shape (degree of rectangularity) of area potentially available for crown and stem growth of a subject tree. In this approach, the available degree of rectangularity is the ratio of the distances between across and within row competing trees in relation to the subject



where,

S = subject tree
C = competing tree
D_i = horizontal distances between competing trees
CD_i = major and minor axis distances of crown diameter of a subject tree.

The ratios are defined as:

Ratio of tree distances

$$RTD = d1 / d2$$

Ratio of crown diameters

$$RCD = CD1 / CD2$$

Figure 1: Graphical representation of tree distance and crown diameter ratios describing spacing rectangularity.

tree. For simplicity, the degree of rectangularity calculated in this manner will be referred to as the ratio of tree distances.

In the other approach, the degree of rectangularity is the measure of the effect on tree crown shape caused from competing tree placement. In order to measure this relationship, major and minor axis distances of individual crown diameters can be a used for this type of ratio (Figure 1). The size and growth of a subject tree should be influenced by the size and location of competing trees, when light is the limiting factor. The immediate influence of competing tree placement should be indicated by the size and shape of the crown of the subject tree. The degree of rectangularity calculated in this manner will be referred to as the ratio of crown diameters.

3.0 METHODS AND PROCEDURES

The general objectives of this study were to investigate relationships between crown, stem and stand characteristics, and to consider integration of crown information into various stem and stand models. After information on the data base is given, methods and procedures will be given for each of the following areas of interest:

Modeling crown and stem dimensions of trees.

Modeling crown and stand dimensions of stands.

Modeling stem development through crown-based competition indices.

Assessing the effects of rectangularity on stem growth.

3.1 CROWN, STEM AND STAND DATA BASE

This study was based on tree and plot data from unthinned and thinned loblolly pine plantations measured at two time periods. In the summer of 1978, crown, stem and stand information from unthinned loblolly pine plantations was collected. Similar information was collected from thinned loblolly pine plantations during the fall and winter of 1978 and 1979. The measurements included the following: dbh, total height, X and Y coordinates of individual tree location, crown class, crown diameter across and within rows, height to live crown and height to crown diameter. In the late summer of 1982, a portion of the original plots was remeasured with similar information collected.

3.1.1 DATA COLLECTED

3.1.1.1 Measurement Details

The details and instructions established for both sets of measurements were as follows:

dbh	measured to the nearest 0.1 in. with a d-tape at 4.5 ft above the ground.
total height	measured to the nearest 0.1 ft with a height pole in stands age 10 and younger. In older stands, a Spiegel Relaskop was used with estimates recorded to the nearest 0.1 ft.
X,Y coordinates	measured to the nearest 1.0 ft.
crown class	recorded according to SAF definitions (Society of American Foresters 1971).
crown diameter	across and within row measurements of crown diameter were measured to the nearest 0.1 ft with a tape. Ocular estimates of the live crown edge were made; branches protruding beyond the main crown were measured, depending on their size and position in the crown.

height to live crown	same procedures and instruments as with total height. Persistent branches beyond the main crown were measured as described under crown diameter.
height to crown diameter	similar details as described under height to the live crown.

3.1.1.2 Plot and Tree Details

The plot locations included a mixture of old-field and cutover sites located in North Carolina, Arkansas, Oklahoma and Louisiana (Figure 2). Plots in the first three states were located in research studies on Weyerhaeuser land; the Louisiana plots came from a spacing-thinning study managed by the Southern Forest Experiment Station (Feduccia and Mann 1976). In general, the data represented research plots located in stands that had received site preparation, hardwood control, and no burning.

The thinned stands were thinned from below with consideration given to spatial arrangement. The 1978 and 1982 measurements taken in the thinned stands did not always coincide with a thinning operation. Thus, growth estimates were not always directly related to a level or time of thinning as much as being related to thinned stands.

With the completion of the second measurement, 89 plots with 3421 indi-

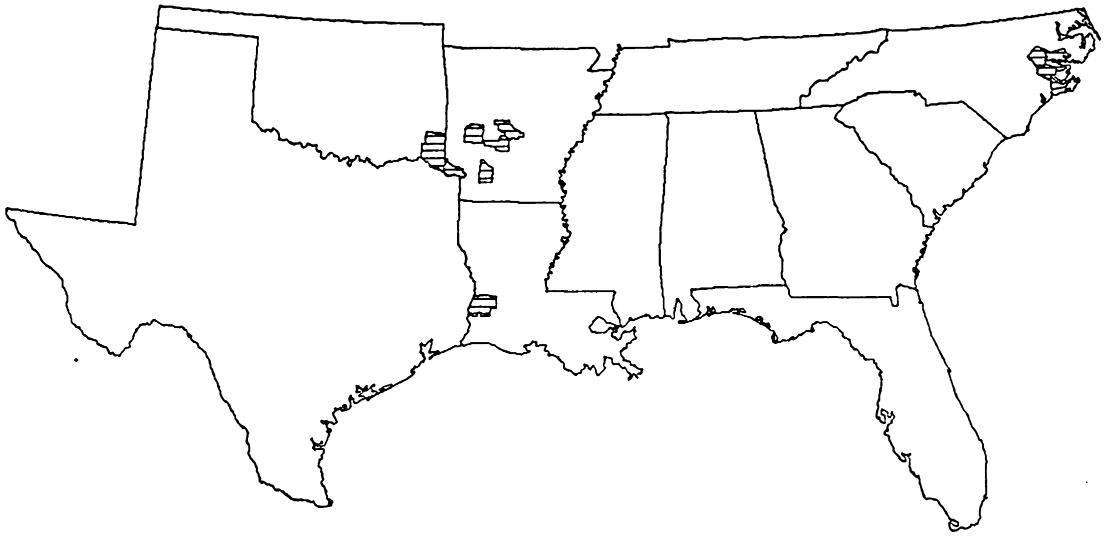


FIGURE 2: GENERAL LOCATION OF THINNED AND UNTHINNED LOBLOLLY PINE SAMPLE PLOTS USED IN THIS STUDY.

vidual tree observations were available for analysis. However, due to a variety of reasons, such as subsampling and incorrect measurements during the first measurement period, only a portion of these plots were available for certain analyses. Only 35 thinned plots and 27 unthinned plots were suitable for a main data base (Tables A2-A4). Analyses requiring a complete set of stem and crown measurements for both periods of time restricted the thinned plots to 647 tree observations and the unthinned plots to 1442 tree observations (Tables A5, A6). In cases where distances between a subject tree and competing trees were required, the original measurement plots were subdivided into a buffer and interior measurement plot. Use of the interior plots resulted in 202 tree observations on 30 thinned plots and 471 tree observations on 26 unthinned plots (Tables A7-A10).

3.1.2 PREDICTED DATA

Site index, missing total heights and certain crown variables were predicted. Site index values were calculated with Devan and Burkhart's (1982) model using a base age of 25 years, and old-field combined piedmont and coastal plain coefficients.

In the first measurement period, total heights were at times subsampled on the Louisiana plots. Two methods to estimate total height for the subsampled plots were investigated:

$$H_i = \alpha_0 + \alpha_1 H_{(i+4)}$$

$$\ln(H_i) = \alpha_0 + \alpha_1/dbh_i$$

where,

H_i = total height of a tree at age i

$H_{(i+4)}$ = total height of a tree 4 years after age i

dbh_i = diameter at breast height of a tree at age i

ln = natural logarithm

α_i = regression coefficients

The above two models were fitted to each plot to avoid complications from differences in site index and management practices. The first method was dropped from consideration when negative slope coefficients were calculated for some of the plots. Using the second method, plots were grouped into four site class categories as some thinned plots did not have enough observations for regression analysis. The regressions by site class were tested for similarities using full and reduced models (Tables A11, A12). While there was no significant differences in the intercept and slope coefficients between site classes (0.05 probability level), separate regressions were still used.

The predicted crown variables and formulas used in their calculation are given below:

CROWN DIAMETER (ft)

$$CD = \frac{\text{CD minor axis} + \text{CD major axis}}{2}$$

CROWN PROJECTION AREA (sq ft)

$$CPA = \frac{\pi \times CD^2}{4}$$

CROWN VOLUME (cu ft) as defined by a cone

$$CV = \frac{CPA \times ELC}{3}$$

CROWN SURFACE AREA (sq ft) as defined by a cone

$$CSA = \frac{\pi \times CD \times \sqrt{ELC^2 + (CD/2)^2}}{2}$$

Tree and stand averages of crown variables were calculated as above; however, stand totals involving crown projection area were adjusted. Crown projection area on an acre basis (SCPA) was initially calculated by summing the tree crown projection areas and dividing by the plot acreage. Trees with crown areas extending over plot boundaries had to be adjusted so that the total crown projection area would not be over estimated. Individual crown areas were adjusted for calculating SCPA by the formulas given in Table A13.

3.2 MODELING CROWN AND STEM DIMENSIONS OF TREES

The purpose of this section is to outline the methods for achieving the following objectives: how stocking levels, in relation to crown characteristics, affect tree stem and crown development; and, how crown information can be integrated into stem and crown models.

The basic stem dimensions modeled were individual tree basal area, diameter, basal area growth, diameter growth, crown diameter, crown diameter growth, crown projection area and height to crown diameter. Initially, linear and intrinsically linear models were emphasized with transformations of both the dependent and independent variables. Two basic model types were fitted to the data in an attempt to determine variable relationships.

$$Y_i = \alpha_0 + \alpha_1 SI + \alpha_2 A + \Sigma(\alpha_i X_i)$$

$$Y_i = \exp(\alpha_0 + \alpha_1 SI + \alpha_2 A + \Sigma(\alpha_i X_i))$$

where,

Y_i = dependent variable (e.g., diameter of tree i at age A)

SI = site index base age 25

A = age

X_i = other independent stand and/or crown variables (e.g., crown diameter of tree i at age A)

α_i = regression coefficients

With the range of age and site index values associated with the trees on unthinned and thinned plots, all models had age and site index initially included as part of the linear function. Other basic independent stem and stand variables, and crown variables, such as stem basal area or diameter, total height, stand basal area and crown variables previously mentioned, were included in the variable screening process where appropriate. Retention of independent variables was determined on the basis of the contribution of the variable to the fit and prediction of the dependent variable. The fit criteria included the standard error of the estimate ($S_{y.x}$) and coefficient of determination; while, the prediction criteria included PRESS, VIF, and variable sign and significance (Montgomery and Peck 1982). Only variables significant at the 0.05 probability level were included unless multicollinearity was apparent.

3.3 MODELING CROWN AND STAND DIMENSIONS OF STANDS

The methods and procedures given in this section are to achieve the following objectives: investigate how stocking levels, in relation to crown characteristics, affect stand development; and, determine what and how crown information can be integrated into the modeling of stand level attributes.

Stand level attributes modeled were stand basal area, basal area growth, crown projection area, mean crown projection area, mean height to crown diameter, mean dbh and mean dbh growth. Similar model types, variables, and selection procedures and criteria used to model individual tree attributes were used to model stand level attributes.

Other model forms were also tested, using similar criteria, after several well-defined non-linear relationships were noticed with the above dependent variables and SCPA. It was mentioned before that SCPA had similar attributes as stand basal area, plus an important asymptotic property. Using this relationship, several non-linear model types, such as power, hyperbola and exponential functions, were also tested.

3.4 MODELING STEM DEVELOPMENT THROUGH CROWN-BASED COMPETITION INDICES

The objective of this section is to outline the investigation into determining what and how crown information can be effectively integrated into an individual tree or stand level competition index.

While many competition indices have been published, five different groups of indices were selected for analysis. An interactive FORTRAN IV program, called COMP FORTRAN, was written to compute the various indices and adaptations, and to time the calculations. The groups were size and/or distance of neighboring trees, distance independent, available growing space, overlap and miscellaneous. One or two indices from each group were analyzed in their original form and then adapted to reflect the physiological and ecological relationships existing within and between trees.

1.) Size and/or Distance of Neighboring Trees. Two competition indices selected from this group were from Daniels (1976). One index involves the sum of the ratios of competing tree diameters to a subject tree's diameter, weighted by the distances between competing and subject trees.

$$C1 = \Sigma^{n} [Dc / (Ds \times DSsc)]$$

where,

C1 = competition index

n = number of competing trees

Dc = dbh of competing tree

Ds = dbh of subject tree

DSsc = distance between competing and subject trees.

The other index is similar to the above, but involves a ratio of squared diameters. Both indices selected competing trees using a BAF 10.

2.) Distance Independent. Glover and Hool (1979) and Daniels (1981) presented similar distance independent indices and were selected for analysis using crown information. Glover and Hool's index compares the squared diameter of a subject tree in relation to the average diameter squared of the stand,

$$C2 = Ds^2 / \bar{D}^2$$

where,

Ds^2 = dbh squared of subject tree

$\bar{D}^2$ = average dbh squared.

A variation presented by Daniels (1981) compares the squared diameter of a subject tree in relation to the average squared diameter of the stand (i.e., tree of average basal area).

$$C3 = Ds^2 / \overline{D^2}$$

where,

$\overline{D^2}$ = average squared dbh.

3.) Available Growing Space. An area potentially available index selected for analysis was one by Moore et al. (1973), who calculated the line division by the following:

$$C4 = f(\, DSsc \times Ds^2 / (Ds^2 + Dc^2))$$

where,

DSsc = distance between subject and competing trees

Ds = dbh of a subject tree

Dc = dbh of a competing tree.

4.) Overlap. Ek and Monserud's (1974) index was selected for this analysis. Their index is as below with the size characteristics of the trees based on crown radius and total height.

$$C5 = \Sigma^{n} [(Osc / As) \times (Sc / Ss)]$$

where,

n = number of competing trees or overlap occurrences

As = theoretical CPA of an open-grown tree with Ds

Ds = actual dbh of a subject tree

Osc = area of overlap between competing and subject tree's CPA assuming open-grown conditions

Sc = size of competing trees

Ss = size of subject tree.

Open-grown CPA was estimated using Strub et al. (1975) equation.

$$CD = 3.5607 + 1.6073 \text{ dbh}$$

where,

CD = crown diameter (ft) of an open-grown tree with dbh (in.).

5.) Miscellaneous. A last group of indices available for analysis includes some variations of the previous mentioned measures of competition. One general index type analyzed was suggested by Harrison (1984), which consists of a ratio of a subject tree's dbh squared compared to the mean squared dbh of the stand, but weighted by stand basal area (i.e., tree of average basal area squared).

$$C6 = Ds^2 / (\overline{D^2} \times B)$$

where,

Ds^2 = dbh squared of a subject tree

$\overline{D^2}$ = average squared dbh of the stand

B = stand basal area.

Stand level characteristics form another index type that was analyzed. Competitive stress surrounding a subject tree as indicated by a stand measure, such as stand basal area, may lack the sensitiveness necessary to indicate the growth potential of a subject tree as the competitive stresses surrounding a subject tree may or may not be the same as measured on a stand level. Another index approach is to subsample immediate levels of stand competition particular to a subject tree. Thus for this index type, partial measures of stand basal area were computed by selecting competing trees through a BAF value and the basal areas of the selected trees summed.

$$C7 = \Sigma^n b$$

where,

n = number of competing trees selected using a BAF

b = stem basal area.

If all trees in a plot were selected as being competitors, normal stand measures of competition would be calculated; however, if different BAF values were used to select competitors, various levels of stand measures of competition would be calculated. The BAF values that were tested were 3, 5, 10, 20, and 30.

Several other adaptations in terms of variables and structure were made to the five groups of indices. In addition to calculating the indices based on their published or suggested forms, which usually used tree dbh and/or stand basal area, indices using tree and stand level crown information were also tested.

The individual crown dimensions available for modeling stand level attributes were crown diameter (CD), crown projection area (CPA), crown volume (CV), crown surface area (CSA), height to crown diameter (HCD) and effective length of crown (ELC). In addition, two ratios were also investigated: ratio of tree basal area to crown projection area (CBR) and ratio of length of live crown to total height (CLR). Stand level measures involving crown information that were also included in the modeling process were the sum of crown projection area (SCPA), sum of crown volume (SCV) and sum of crown surface area (SCSA).

Several structural changes were made in some of the index groups. Theory suggests that competition is not additive and that competition between competing trees should be considered. Thus, one adaptation to reflect the competition between competing trees was to weight the calculated index value by the number of competing trees.

$C8 = Ci / n$

where,

Ci = competition indices such as C1 (size and/or distance type), C4 (APA) or C5 (overlap)

n = number of competing trees.

In the indices using the distances between competing and subject trees, the distances were a horizontal measure from stem to stem. A subject tree's crown expansion is based on the open distance between competing and subject tree crown edges, and its ability to grow into that space. It seems that a better measure of competition is the distance available between two competing crowns rather than the distance between stems. Thus, another adaptation to the size and/or distance or APA type indices was to use the distance between crown edges instead of stems as a measure of competition. For the size and/or distance type of index,

$C9 = \Sigma^n [Dc^2 / (Ds^2 \times DCsc)]$

where,

n = number of competing trees

Dc = dbh of competing tree

Ds = dbh of subject tree

DCsc = distance between crown edges of competing and subject trees.

For the APA type of index,

$$C10 = f(CDs/2 + (DCsc \times Ds^2 / (Ds^2 + Dc^2))$$

where,

CDs = crown diameter of a subject tree

DCsc = distance between crowns of subject and competing trees

Ds = dbh of a subject tree

Dc = dbh of a competing tree.

There was one other adaptation to the APA index which involved adjusting the polygon area to reflect what was readily available to a tree. The adaptation made was to move distant points to some reasonable distance beyond the subject tree's crown edge, but on the same line joining the tree and the original point. Different maximum distances, such as 3, 5, 10 and 20 feet, beyond the crown edge were tested to determine if an appropriate distance was suggested.

$$C11 = f(CDs/2 + DAsc)$$

where,

CDs = crown diameter of a subject tree

DAsc = maximum distance beyond the subject tree's crown edge, when DCsc was greater than DAsc, otherwise, DCsc was used.

Some authors found that the method of determining what trees competed with a subject tree was important in the subsequent indices relationship with a subject tree's growth. Six BAF values, 3, 5, 10, 20, 30 and 40 were used to select competing trees for the calculation of the size and/or distance of neighboring tree indices.

$$C12 = \Sigma^n [\, Dc / (Ds \times DXsc) \,]$$

where,

n = number of competing trees selected by a different BAF value

DXsc = DSsc or DCsc

and others as previously defined.

The effectiveness of each competition index was measured by analyzing the contribution the index made in explaining and predicting basal area growth at breast height over a 4-year period of individual trees. Various basal area growth models were constructed using site index, age, tree basal area, and stand basal area.

Using the previously mentioned fit and prediction criteria, a basic form of the basal area growth model was selected. For trees in unthinned stands, stem basal area growth was a function of site index, age and natural logarithms of initial stem basal area and stand basal area; while, for trees located in thinned stands, stem basal area growth was a function of site index over age, and natural logarithms of initial stem basal area and stand basal area. Stand basal area measures average competition, while a competition index measures competition at a point. The latter should more accurately and precisely reflect competitive stresses for an individual tree. Thus in analyzing the effectiveness of an index in predicting basal area growth, the stand basal area term was eliminated from the above models, and various competition indices and transformations at a point in time and change over time were substituted. Using the above fit and prediction criteria, indices were ranked on their contribution in increasing model fit and prediction of stem basal area growth.

It seemed logical that stand basal area and an effective index would be highly correlated to each other and given that a competition index was in the growth model, the addition of stand basal area would be non-significant. In terms of fit and prediction, one standard to judge the growth model containing a competition index against is the growth model containing stand basal area. An effective index must do as well as stand basal area in predicting stem basal area growth; and given the information that an index may have, it should theoretically do better and it must do better to justify the cost.

3.5 ASSESSING THE EFFECTS OF RECTANGULARITY ON STEM GROWTH

In this last section, methods and procedures are given to determine the effects that planted spacing or available space configuration has on the lower stem development of trees in unthinned stands.

The effects of rectangularity were measured by analyzing the contribution the ratios of tree distances and crown diameters made in explaining and predicting stem development of an individual tree. Initially, correlation coefficients and graphical trends between the two ratios and stem development (basal area and diameter at each measurement period, and the respective growth between periods) were made to determine if a simple relationship existed. Since the relationship was found to be more complicated, the importance of the degree of rectangularity was tested by its contribution in a multiple regression model in predicting basal area growth. Various basal area growth models were constructed using site index, age, tree basal area, and stand basal area. A model was selected according to previously mentioned fit and prediction criteria. Basal area growth was best modeled as a function of site index, age and logarithms of initial tree basal area and stand basal area. Contributions of the ratio terms in the above growth model toward explaining and predicting basal area growth were also judged by the same criteria.

Each method of calculating rectangularity addressed the relationship between rectangularity and stem development (i.e., basal area or basal area growth). However, given a degree of rectangularity, associated stem development varied not only with age and site index, but also with available growing space. Stocking level effects on individual tree growth were eliminated by grouping individual trees by or including a variable of available growing space. While there were many ways of defining available area, it was calculated using the area potentially available (APA) index with the division of the lines that define the polygon calculated by the following:

$$DSsc \times CPAs^2 / (CPAs^2 + CPAc^2)$$

where,

DSsc = horizontal distance between subject and competing trees (ft)
CPAs = crown projection area of the subject tree (sq ft)
CPAc = crown projection area of a competing tree (sq ft).

As available growing space was important in analyzing the effects of rectangularity on basal area growth, then additional analyses were made with the consideration of growing space. Graphs of the ratios and basal area growth by available growing space were made. Also, a term was added to the basic basal area growth model in place of stand basal area to compensate for a tree's available growing space.

Stand basal area and area potentially available measure competition on an overall stand level basis and individual tree basis, respectively. Stand basal area was then dropped from the model as this measures competition on a stand basis, while available growing space measures immediate competition surrounding a particular tree. A basal area growth model with site index, age and stem basal area should require only the point density measure as this measures the immediate competition surrounding a particular tree in question. An overall stand competition measure of stocking should only be important in predicting growth, for example, of a subject tree when it represents a similar condition of competition as the intensive measure.

The most suitable plots for both measures of rectangularity were the North Carolina unthinned plots. The constant age (6 years), exact spacing, and low mortality in these plots offered the most precise way of looking at the effects of rectangularity with the data available. However, with these plots, extremes in degrees of rectangularity did not exist nor had the trees grown for a long time under the influence of the rectangularity. The Louisiana unthinned plots also offered a constant age (28 years), exact spacing (square by initial planting distances) and mortality consistent with its age. Again, extremes in rectangularity were not available, but the trees had grown under the rectangularity influence for a considerable time. A problem with trees located in old stands with the measurements available was that the time of competing tree mortality was unknown. Thus, it was not known how long a particular tree had grown under the influence of a calculated ratio of tree distances.

4.0 RESULTS AND DISCUSSION

4.1 MODELING CROWN AND STEM DIMENSIONS OF TREES

4.1.1 UNTHINNED STANDS

For the following stem and crown models, 1442 individual tree observations from unthinned plots were available for analysis (see Table A6 for individual tree characteristics and Table A4 for plot characteristics).

Basal Area. Initial screening of variables indicated that the following crown and plot characteristics were important in terms of fit and prediction in a tree basal area linear model containing age and site index (except the * model which used 1/A and site index):

VARIABLE	$S_{y.x}$	R^2	VIF	PRESS
ln(B)	0.0947	0.70	3.4	13.03
SCPA	0.0917	0.72	2.8	12.22
* e(ln(CPA))	0.0490			

The horizontal crown variables, CPA and SCPA, improved the fit and prediction abilities of the basal area model compared to the model containing stand basal area. Additional variables were tested in combination with site index, age and CPA.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
CPA B	0.0551	0.90	6.0	4.44

The inclusion of stand basal area marginally improved model fit and prediction of tree basal area compared to a linear model containing CPA, but not compared to the non-linear model containing ln(CPA).

Diameter. Modeling diameters as a linear function of site index, age and various characteristics indicated the following:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
ln(B)	1.2112	0.81	3.5	2128.96
CD	0.7257	0.93	3.6	763.92
CD B	0.6380	0.95	5.6	591.68

As noticed with tree basal area, the single most important characteristic in describing diameter was also a crown dimension - in this case, CD. Given a site index and age, CD by itself across wide ranges of stocking levels improved model fit and prediction compared to the best non-crown variable, stand basal area. In this case, however, stocking levels were additionally important with CD in predicting tree diameters.

Basal Area Growth. Growth models can take on elaborate forms; however, using a linear model with site index and age seemed sufficient for variable screening. The initial variable selection indicated the following:

VARIABLE	$S_{y.x}$	R^2	VIF	PRESS
CPAg	0.0360	0.35	2.7	1.87
ln(b)	0.0343	0.41	3.7	1.70
ln(CPA)	0.0332	0.45	3.2	1.59
e(ln(CPA))	0.0298			

Interestingly, the best variable in describing basal area growth was not the initial basal area of the tree but its crown projection area. Stem growth is dependent on initial stem size; however, stem growth is also dependent, perhaps more so, on what is going to determine the extent of growth - horizontal crown measures, such as CPA. While not a good initial variable in this growth model, crown projection area growth (CPAg) still indicated that a relationship existed between crown growth and stem growth at breast height.

Subsequent selections lead to the following model development:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
ln(CPA) SCPA	0.0291	0.58	3.2	1.23
ln(CPA) CDg	0.0284	0.60	3.3	1.17

ln(b) SCPA CD	0.0273	0.63	6.9	1.08
ln(CPA) SCPA CDg	0.0269	0.64	3.8	1.05
ln(b) SCPA CD CDg	0.0260	0.66	7.3	0.98

All variables were significant at the 0.0001 probability level.

The importance of knowing the following items becomes evident in modeling tree growth: initial size of the tree, level of stocking surrounding the tree, size of the crown and rate of crown development. Each of these is expressed in the last model. The only other variable of importance would be a measure of distance between the crown and diameter at breast height; however, while this may be an important variable on a stand level basis, its inclusion on a tree level basis is questionable. As HCD and ELC are constant within a stocking level they should not significantly help in the model fit and prediction of tree basal area or diameter.

Diameter Growth. Diameter growth followed a similar progression of variable screening as found with basal area growth.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
SCPA	0.5468	0.72	2.8	432.04
CDg	0.5368	0.73	2.6	416.79
SCPA CD	0.4884	0.78	3.6	345.04
ln(b) SCPA	0.4863	0.78	5.1	342.51
ln(b) SCPA CD	0.4667	0.80	6.9	315.67

ln(b) SCPA CDg	0.4637	0.80	5.6	311.83
ln(b) SCPA CD CDg	0.4452	0.81	7.3	287.75

All variables were significant at the 0.0001 probability level.

With site index and age, CDg was the single most important tree or stand variable in terms of model fit and prediction of stem diameter growth. As the crown developed so did the stem. It appeared that in describing stem growth (basal area or diameter), the important variables were initial size of the stem, level of stocking, size of the crown and rate of crown development. It is noteworthy that in both cases of basal area and diameter growth that the stocking level of importance was not basal area but crown projection area per acre (SCPA).

Crown Diameter. The following variables indicated an important contribution to the fit and prediction of crown diameter in a linear model containing age and site index:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
ln(B)	2.2794	0.51	3.5	7529.64
SCPA	2.1075	0.58	2.8	6436.56
dbh	1.2772	0.85	5.7	2363.89
SCPA Ts	1.9235	0.65	3.5	5362.74
dbh B	1.2070	0.86	6.4	2114.90

Of the stand level measures, SCPA contributed the most to model fit and prediction of crown diameter; however, the strong relationship between CD and dbh was evident. Using just stand parameters, SCPA and surviving trees in a linear model proved to predict CD better than others - but then given SCPA and Ts, an average CD could be directly calculated. While crown diameter and dbh were strongly related, site index, age and a measure of stocking level were required for a suitable crown diameter model. The relationship between CD and dbh with total height was not independent of stocking levels as basal area was a significant variable in the model.

Crown Diameter Growth. As CDg was an important variable in the stem growth models, variables important in predicting it were also investigated. The progression of variable screening in a model containing site index and age, lead to the following:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
ln(B)	1.6772	0.23	3.4	4069.34
SCPA	1.6384	0.27	2.8	3884.59
SCPA ln(CD)	1.6026	0.30	3.2	3719.11
SCPA dbh	1.5269	0.36	6.2	3377.81
SCPA ln(CD) Ts	1.5804	0.32	3.6	3619.36
SCPA dbh CPA	1.5059	0.38	12.8	3291.80

The single most important variable in predicting CDg was SCPA rather than tree or stand basal area. With the addition of other variables, dbh with SCPA and CPA provided a better model in predicting CDg than with just SCPA. However, if only stand conditions were known then a model containing SCPA with ln(CD) and Ts was also acceptable.

Crown Projection Area. The following characteristics were important, in addition to site index and age, in predicting crown projection area:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
SCPA	35.6936	0.48	2.8	1,848,902.90
b	22.2209	0.80	5.9	719.532.97
b ln(B)	21.9467	0.80	6.4	702,355.20
b SCPA	21.5889	0.81	5.9	679,735.08

The variables important in predicting CPA were similar to those found for predicting CD. While tree basal area was the most effective variable within a linear model in predicting tree CPA, a stocking level measure, described by SCPA, was the next most important variable rather than stand basal area. Stand level variables, such as B and Ts, were not effective by themselves in model prediction of CPA as much as just SCPA.

Height to Crown Diameter. As some heights to crown diameter were not measured, only 1350 tree observations were available for analysis. The following variables, in addition to site index and age (except in the * models where the models are as written), were important in predicting height to crown diameter.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
base	3.7621	0.96	2.5	19200.09
dbh	3.1784	0.97	6.0	13755.95
H	2.5721	0.98	20.1	9007.68
* A H	2.5715	0.98	17.4	8991.40
* H SCPA	2.8868	0.97	2.0	11293.01
* H SCPA CD	2.5737	0.98	2.7	9001.33

A high level of model fit was obtained using just site index and age; however, suitable prediction of HCD necessitated a linear model containing total height with SCPA and CD. These variables had an intuitive appeal in predicting height to crown diameter. As stand density, measured by crown projection area (SCPA), increased, HCD increased; and, as H increased, there was a proportional change in HCD.

4.1.2 THINNED STANDS

The data available for the following analyses included 647 trees found in thinned stands (see Table A5 for individual tree characteristics and Table A3 for plot characteristics).

Basal Area. The basal area of a tree in a thinned stand was regressed against site index, age and a variety of stand and crown characteristics. The single most important crown and stand variables were as follows:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
SCPA	0.1681	0.67	3.2	18.40
B	0.1634	0.71	6.7	16.28
CD	0.0913	0.90	3.0	5.42

Stand characteristics were less important than either of the crown dimensions, CPA and CD, in terms of model fit and prediction of tree basal area. Additional variables beyond site index, age and CPA or CD did not improve the fit or prediction abilities of the tree basal area model.

Diameter. The outcome of variable selection for diameter prediction, with a linear model containing site index and age, indicated similar results as with basal area.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
SCPA	1.5484	0.81	3.2	1560.56
B	1.4950	0.82	6.7	1454.52
CD	0.8072	0.95	3.0	424.22
CD ln(B)	0.7899	0.95	10.5	406.76

Residual stand basal area or crown projection area (SCPA) were not as important as individual crown characteristics, such as CD, in predicting tree diameter. While individual CD was important in predicting tree diameters in thinned stands, residual basal area with CD improved the prediction ability of the model.

Basal Area Growth. Using similar procedures as before, variable selection indicated the following variables were important, along with site index and age, in predicting tree basal area growth:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
CPAg	0.0375	0.59	2.3	0.92
B	0.0368	0.60	6.7	0.88
ln(b)	0.0331	0.68	9.1	0.71
ln(CPA)	0.0322	0.69	3.4	0.69
ln(CD) SCPA	0.0312	0.71	4.1	0.63
ln(CD) CDg	0.0298	0.74	3.4	0.58
ln(CD) SCPA CDg	0.0295	0.75	4.2	0.57

An individual crown dimension, specifically CPA, was the single most important variable in a model with age and site index describing tree basal area growth. Stem size or stocking level measures were not as important in terms of basal area growth prediction as CPA. Crown growth and stem growth were well correlated and the former proved to be an important variable, in terms of CDg, in the final model selection. Tree basal area growth seemed to be related to a crown measure of stocking, crown dimension and crown dimension growth.

<u>Diameter Growth.</u> Diameter growth in thinned and unthinned stands indicated that stand characteristics and crown growth were as important as crown characteristics. Variable selection, along with site index and age in a linear model, indicated the following:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
dbh	0.3781	0.90	6.3	93.03
ln(CD)	0.3655	0.91	3.4	87.01
CDg	0.3610	0.91	2.3	84.92
ln(B)	0.3437	0.92	7.9	77.01
ln(CD) CDg	0.3324	0.92	3.4	72.11
ln(B) ln(CD)	0.3290	0.92	10.5	70.69
ln(CD) ln(B) CDg	0.3135	0.93	11.6	64.30
ln(CD) ln(B) CDg b	0.3100	0.93	12.6	63.02

The most important variable next to stand basal area in predicting diameter growth was crown diameter growth. Thus, as the crown diameter expanded with age so did the tree diameter. An adequate diameter growth model for trees found in thinned and unthinned stands required the same variables: measure of stocking, initial tree size and crown size, and crown growth.

Crown Diameter. Variable selection using a linear model containing site index and age (except * model which contained only age) to predict CD was investigated. The important variables for estimating crown diameter were as follows:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
B	3.5310	0.60	6.7	8116.01
dbh	2.0113	0.87	6.2	2635.97
SCPA Ts	3.2900	0.65	9.0	7063.12
* B Ts	3.2405	0.66	16.5	6836.01
dbh B	1.8875	0.89	14.1	2327.28

Variable selection indicated that dbh was the best variable in predicting CD and that a measure of stocking, basal area, was also needed. Of the stand characteristics, SCPA along with Ts seemed to provide a more useful model in terms of prediction, given the levels of the variance inflation factors. However, given SCPA and Ts, an average value of CD could be directly calculated.

Crown Diameter Growth. The following variables, in a linear model containing site index and age, were important in predicting crown diameter growth:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
ln(B)	1.9304	0.21	7.9	2424.52
ln(B) CD	1.9147	0.23	10.5	2389.27
ln(B) CD b	1.8184	0.30	12.0	2159.09
ln(B) CD dbh	1.8077	0.31	19.9	2133.75

Trees found in thinned stands required stand basal area, initial crown diameter and stem diameter for an adequate model to predict crown diameter growth. Even with these variables, a low amount of variation was explained as was also found with trees in unthinned stands.

<u>Crown Projection Area.</u> Screening of crown and plot variables indicated that the following were important in terms of fit and prediction in a CPA linear model containing site index and age (except * models which contained only age):

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
b	56.5701	0.82	3.0	2,094,886
* B Ts	88.4070	0.57	16.5	5,088,213
b B	54.4070	0.84	11.8	1,943,290

Stem and stand basal area were significant variables in predicting CPA. Using stand level information, only stand basal area with surviving number of trees were important in a model designed to predict CPA.

Height to Crown Diameter. Only 569 tree observations were available for this analysis as some heights to crown diameter were not measured. HCD was best predicted by a linear model containing site index and age (except * models where the models are as written) and the following variables:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
base	4.6596	0.95	2.7	12407.33
dbh	4.5180	0.96	5.0	11696.31
H	4.1587	0.96	33.5	9916.05
* A H	4.1866	0.96	20.9	10020.47
* H SCPA ln(CD)	4.4947	0.96	3.5	11572.19

The base model of site index and age was acceptable in predicting HCD given the variance inflation factors and PRESS statistics of the other models. It seemed logical that some form of density measure would be important in predicting HCD of trees in unthinned stands as increasing degrees of crown closure would result in increasing HCD. The thinning regimes of the data available indicated that thinning was done repeatedly so that severe crown closure would not seem likely to occur; however, residual stocking levels, in addition to total height and crown diameter, were still important in predicting height to crown diameter of trees in thinned stands.

4.1.3 DISCUSSION

The indirect measures of the amount of photosynthetic area and distance of the translocation process were important factors in describing tree stem and crown development of trees located in unthinned and thinned stands. These two factors were important in predicting point in time stem dimensions of basal area and diameter. The amount of photosynthetic area was indicated by horizontal measures of the crown, such as CPA or CD and SCPA, while the distance of translocation was indicated by age and stand density measures. Prediction of stem dimensions over time required two additional factors. Tree basal area and diameter growth also required indirect measures of the change in available photosynthate through the variable crown diameter growth (more or as important than the above) and initial stem size (except basal area growth of trees in thinned stands).

Prediction of individual tree crown dimensions and development was possible using a variety of crown, stem and stand information. Depending on the type of information available, be it from ground or aerial photograph measurements, models with acceptable levels of fit and prediction abilities were found. Using ground information, such as diameter or basal area at breast height along with stand CPA (for trees located in unthinned stands) and stand basal area (for trees located in thinned stands), provided important variables in predicting CD, CDg and CPA. Models containing just stand level information, as obtainable from aerial photographs, had lower levels of prediction ability than those containing stem and stand information. In these cases, stand CPA was important in predicting CD, CDg, CPA and HCD for trees located in unthinned and thinned stands (except CDg and CPA for trees in thinned stands required stand basal area instead of SCPA).

The sets of untransformed variables that were best in terms of model fit and prediction of various stem and crown dimensions are given below. The first independent variable contributed the most to model prediction, and site index and age are assumed in the model unless otherwise noted. Two sets of variables are listed for some crown dimensions; the first set includes significant stem variables, while the second set includes only stand level variables.

Dimension	Unthinned	Thinned
Basal area	CPA	CD
Diameter	CD B	CD B
Basal area growth	CD CDg b SCPA	CD CDg SCPA
Diameter growth	CDg CD b SCPA	B CDg CD b
Crown diameter	dbh B	dbh B
	SCPA Ts	SCPA Ts
Crown diameter growth	SCPA dbh CPA	B dbh CD
	SCPA CD Ts	B CD

Crown projection area	b SCPA	b B
	SCPA	B TS (no SI)
Height to CD	H SCPA CD	H SCPA CD
	(no SI or A)	(no SI or A)

A list of the respective models with coefficients is given in Table A14.

While the analyses and results were presented separately for trees found in unthinned and thinned stands, the important variables in fitting and predicting a particular stem or crown dimension were generally similar for both stand types. A possible reason for this occurrence is that the majority of the trees from the thinned stands were 28-years-old and were at a stage in development similar to the same aged trees located in unthinned stands.

All tree stem and crown characteristics were better fitted and predicted with some form of crown information included in the model. Horizontal dimensions of the tree crown, such as CPA, CD and CDg, best characterized the amount of photosynthate available as these crown dimensions were key variables in predicting tree basal area, diameter and their growth. Since crown dimensions indirectly indicated the amount of photosynthate that had been produced and the amount that could be produced, as these horizontal measures developed, so did the stem. Horizontal development and vertical dimensions of crowns were influenced by stocking levels, such as SCPA or B, and age or related variables, such as total height. The vertical development of the crown was probably closely related to the proximity that SCPA of a stand was to unit area. As SCPA approached 43,560 sq ft/ac, HCD increased in height (translocation process became longer) and CPA or CD slowed in growth, stem development decreased. Diameter and basal area growth and HCD were related to SCPA rather than B, except for diameter growth of trees in thinned stands.

The failure of certain crown variables, such as crown volume and surface area or the weighted versions, in terms of defining basic relationships of stem characteristics, was not completely understood. While they seemed to represent theoretically important measures, the actual field measurements and simplified computations may have provided too much variability in their values.

4.2 MODELING CROWN AND STAND DIMENSIONS OF STANDS

4.2.1 UNTHINNED STANDS

The data from unthinned stands provided only 25 remeasured plot observations as one plot was suspect (Table A9).

Basal Area. For the range of the data available, basal area increased with age until age 20-21 and then decreased (Figure 3). Prior to crown closure, basal area also increased as SCPA increased (Figure 4); however,

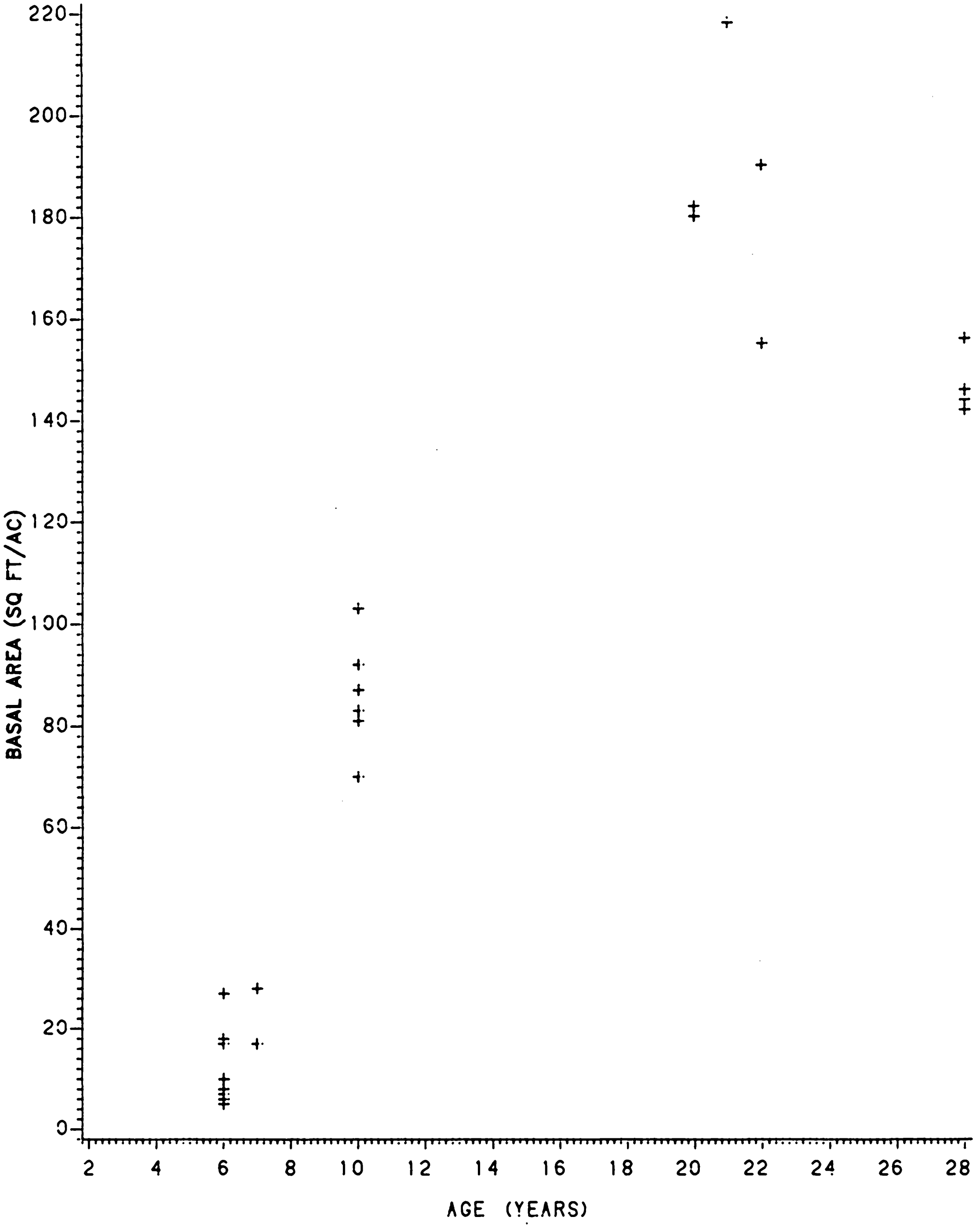


FIGURE 3: COMPARISON OF BASAL AREA AND AGE OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

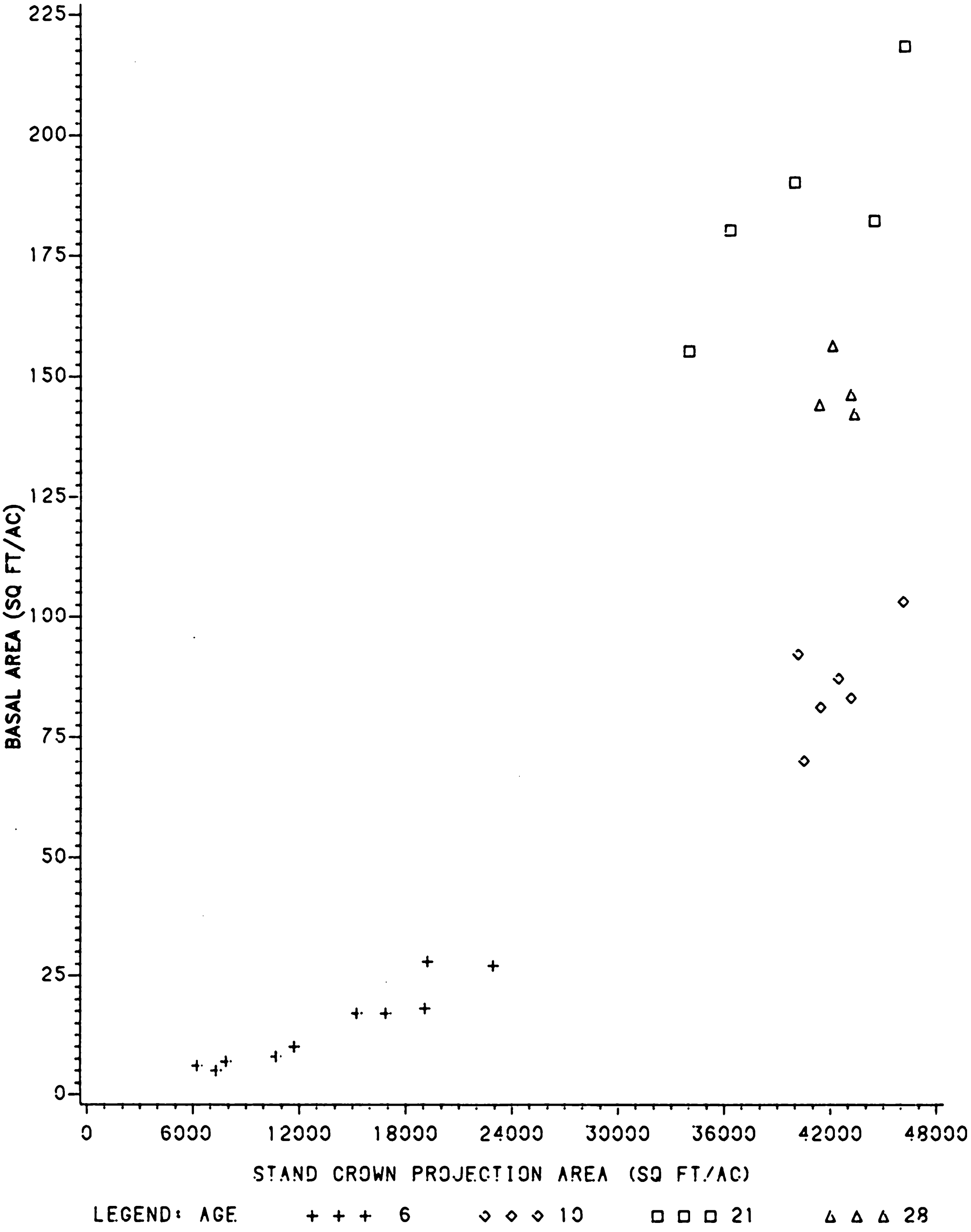


FIGURE 4: COMPARISON OF BASAL AREA AND STAND CROWN PROJECTION AREA OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

after closure the rate of development between basal area and SCPA within an age class seemed higher than before crown closure. This rate of development seemed constant with respect to age, but the level of development appeared related to age. In addition, as the number of trees increased within an age class, basal area increased, and the rate of increase increased with age (Figure 5). Basic linear and intrinsically linear models with the following stand and crown variables indicated suitable fit and/or prediction of stand basal area:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
1/A MCPA	16.6739	0.95	2.4	8590.36
e(1/A SI ln(Ts) ln(SCPA))	9.9455			
e(1/A SI ln(MCPA) ln(SCPA))	7.9024			

The importance of crown information was indicated, in terms of fit and prediction of basal area, by the inclusion of stand CPA and mean CPA in the last model, with MCPA being the single most important variable. It was expected that SCPA and basal area would be related as the crown projection area measured the cross sectional area of the crowns, while basal area measured the cross sectional area of the trees. As SCPA indicated the amount of photosynthate that had been and would be produced by the crown biomass, it therefore was an indication of the level of basal area development.

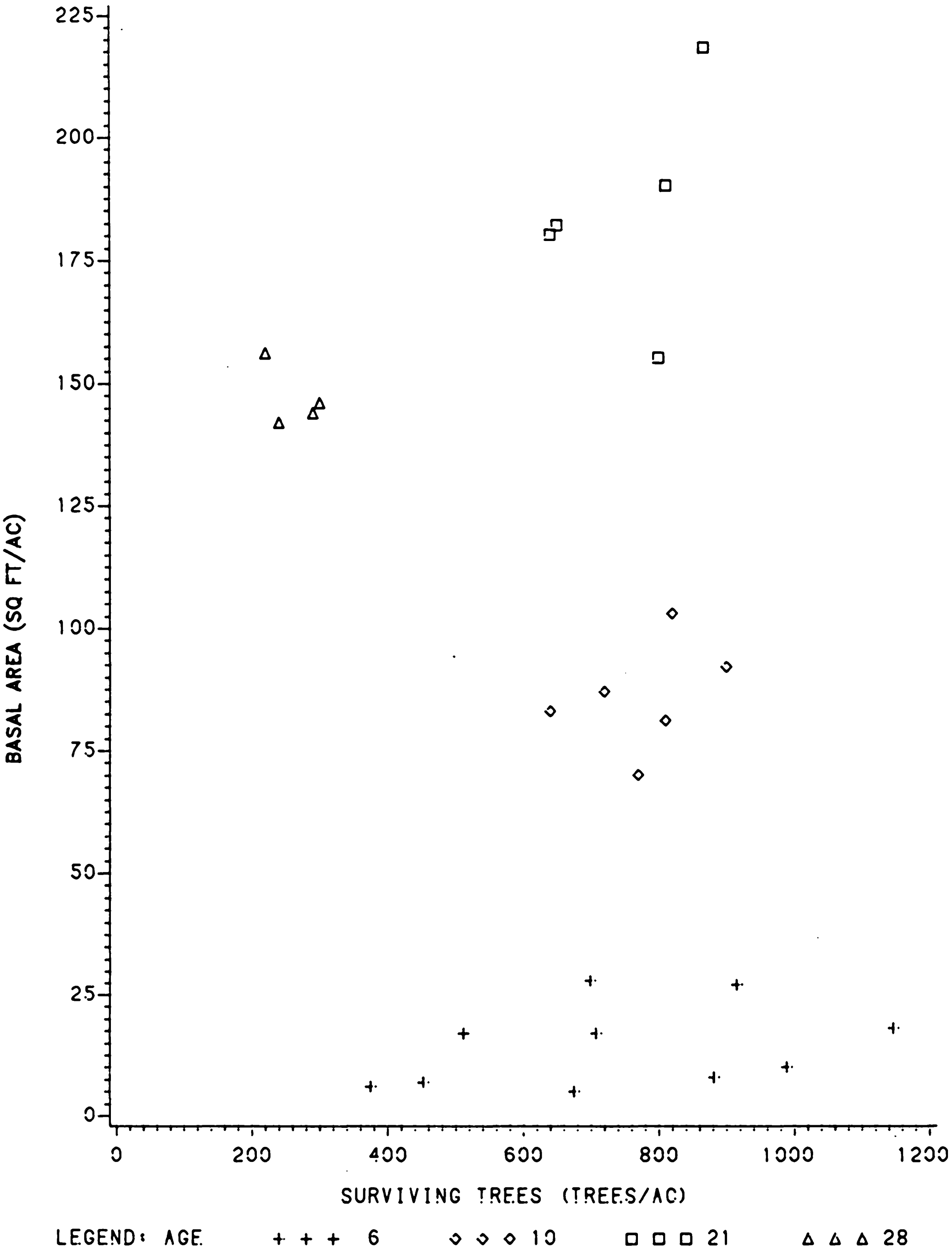


FIGURE 5: COMPARISON OF BASAL AREA AND SURVIVING TREES OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

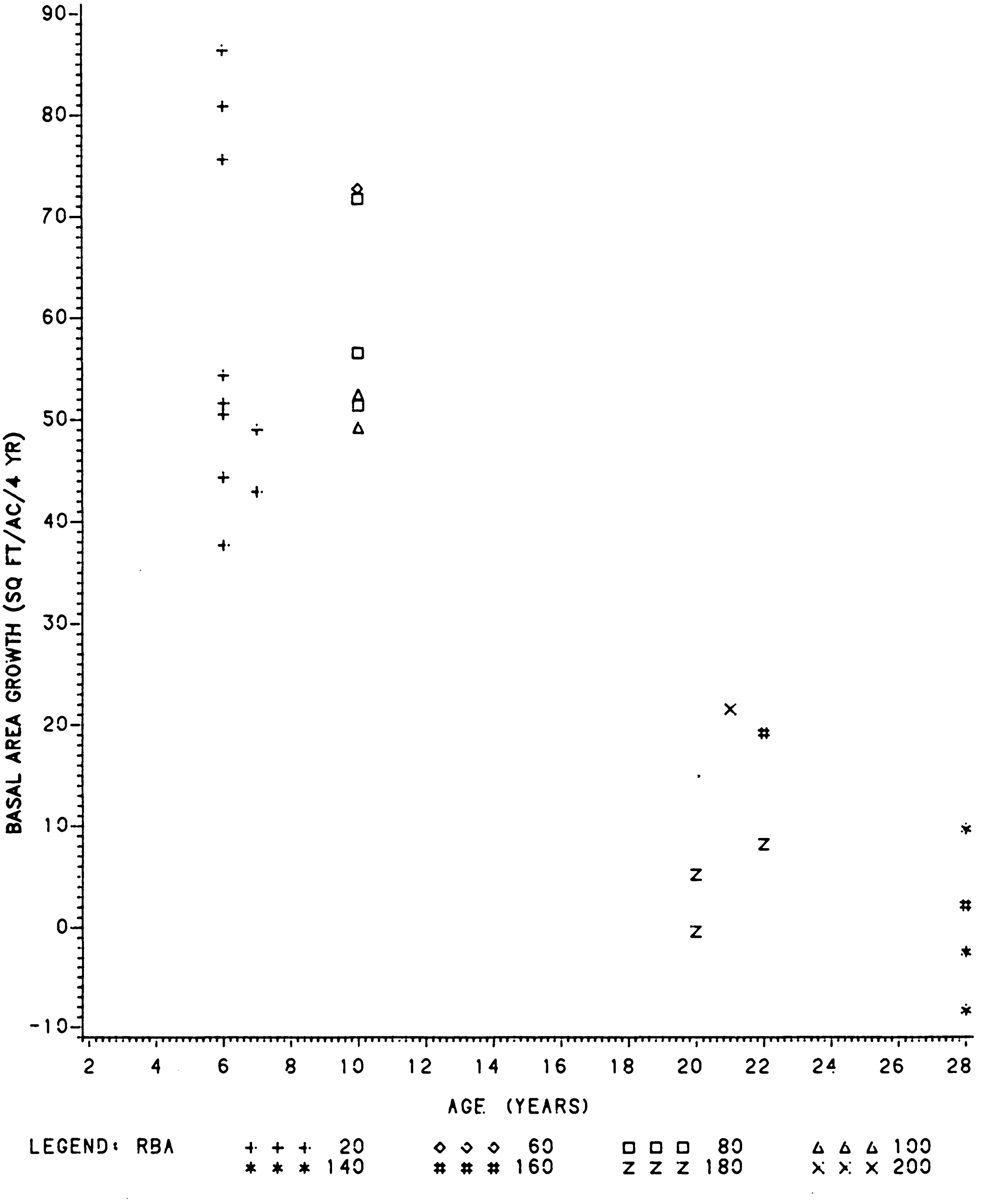
It was noticed that given an age and site index, basal area increased with increasing SCPA; specifically, the level of basal area development seemed to be directly related to age, while the rate of basal area development was similar across ages and dependent on SCPA (Figure 4). In the cases of age classes having enough data, such as 6-7, 10, 20-21-22, and 28 years, a linear relationship between basal area and SCPA within an age class was suggested. Also, the slopes of this relationship between classes appeared constant, except in the youngest age class. Full and reduced models were used to determine if a common slope and different intercept values were indicated between the age classes. The full model represented separate intercepts and slopes for each age class; while, the reduced model represented separate intercept but common slope values (Table A15). Results from the test suggested that the null hypothesis of equal slopes and different intercepts could not be rejected at the 0.05 probability level. While there was no statistical difference between the slope coefficients for the four age classes, there is justification in terms of physiological development to separate the model components describing basal area development prior to stand closure. The results from testing common slope and different intercept values for the 10, 20-21-22 and 28 age classes indicated similar findings as above. Overall, this latter model of describing basal area development prior to stand closure and then having a reduced model for after closure predicted better than the other combinations attempted. Thus, it was not disproved that between age classes in stands having attained a degree of stand closure, basal area and crown projection area development were similar in rate but different in level by age.

Basal Area Growth. As age increased, basal area growth decreased (Figure 6); however, well defined trends with initial basal area, SCPA or SCPAg were not evident. The sets of important variables for fit and prediction of basal area growth for a 4-year period were as follows:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
1/A ln(SCPA)	9.9606	0.89	3.1	2838.51
e(1/A SCPA B)	9.7507			

The important variables in predicting basal area growth were, besides age, initial stand basal area and stand CPA. Stand basal area indicated the initial size of basal area on which growth occurred; while, stand CPA indirectly indicated how much photosynthate had been produced and what would be produced in the future for basal area growth. Stand CPA was the single most important stand variable in predicting basal area growth. However, basal area growth was primarily related to age; that is, as age increased, basal area growth decreased.

In contrast to the reported relationships between tree basal area growth and crown dimensions in the previous section, the change in SCPA between the two measurement periods was not a significant variable in pre-



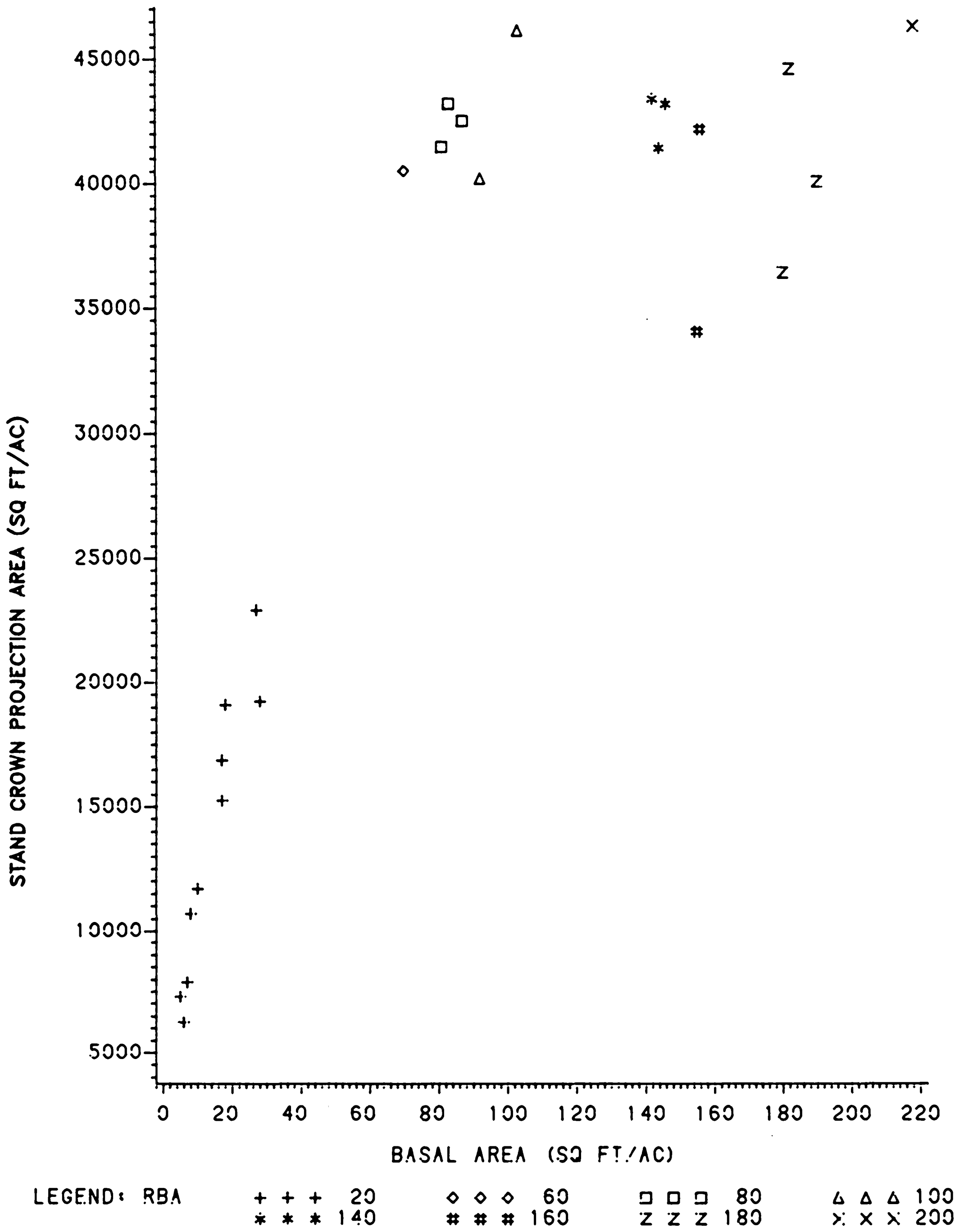
WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 6: COMPARISON OF BASAL AREA GROWTH AND AGE OF UNTHINNED STANDS OF PLANTED LOBOLLY PINE.

dicting stand basal area growth. Changes in crown structure were not as related to changes in stand structure due to differences in development and response. After crown closure and with no mortality, SCPAg is theoretically zero (for intolerant species), while basal area growth may continue depending on the age, mortality and distributions. With mortality, SCPA will drop and then SCPAg will decrease back to zero; however, stand basal area growth may not drop at all since the individual trees and stand depend on the individual and aggregate crown mass, respectively, for growth. Mortality creates an instant loss and eventual gain in SCPA, but stand basal area, especially with suppressed trees, may not be affected.

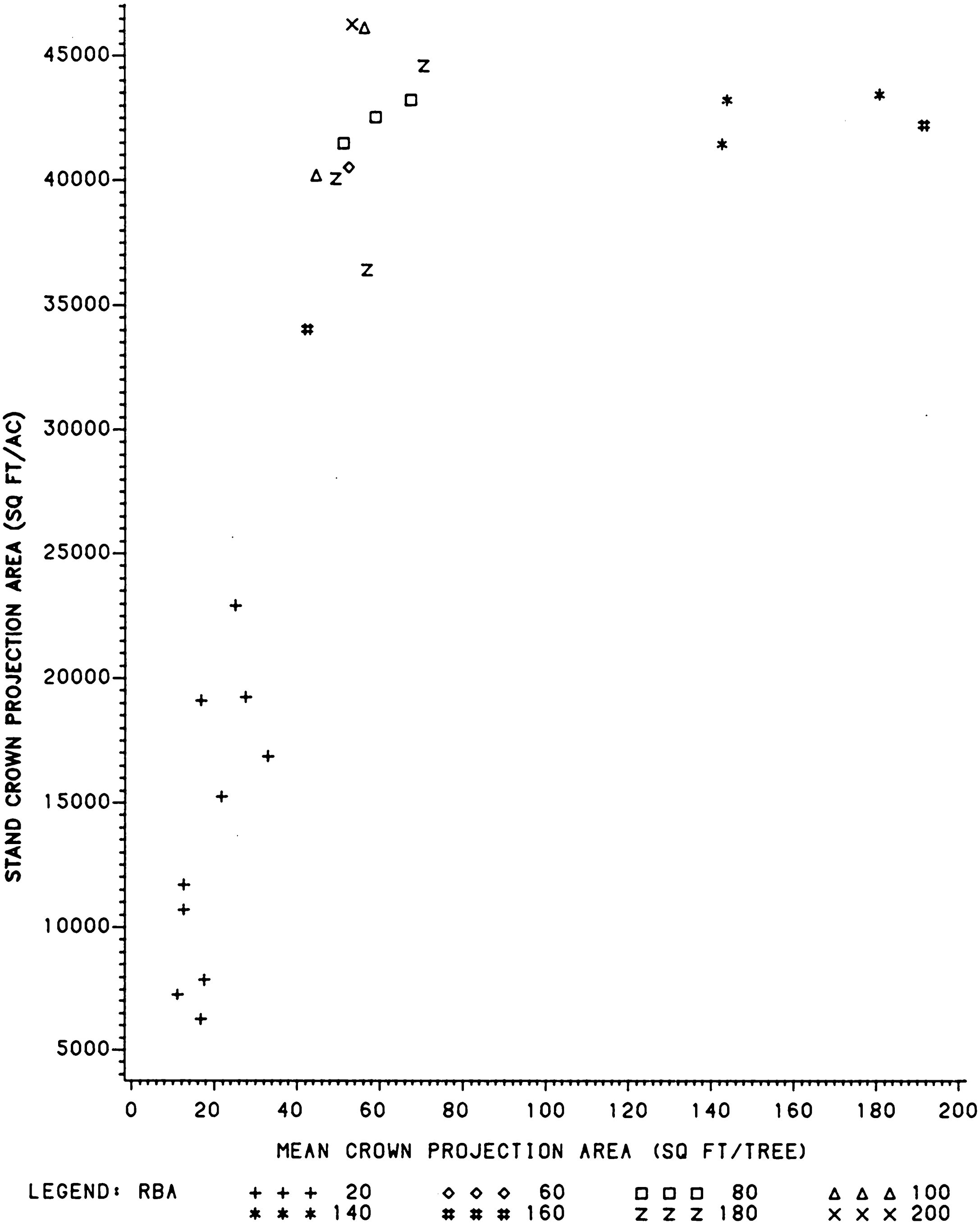
<u>Sum of Crown Projection Area.</u> It was a contention of this work that CPA and SCPA can readily be measured and sampled from low-level aerial photography. If prediction of SCPA is needed, it was indicated that SCPA prior to crown closure was related to stand basal area, age and MCPA; and, after crown closure, SCPA was constant with respect to those variables, excepting for variations due to mortality (Figures 7,8). SCPA can be predicted using the following sets of variables:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
A ln(B)	4137.7309	0.91	2.7	472,524,907
A ln(B) ln(Ts)	3819.6753	0.94	6.4	416,294,755
e(A ln(B) ln(Ts))	3573.9860			



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 7: COMPARISON OF STAND CROWN PROJECTION AREA AND BASAL AREA OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 8: COMPARISON OF STAND AND MEAN CROWN PROJECTION AREA OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

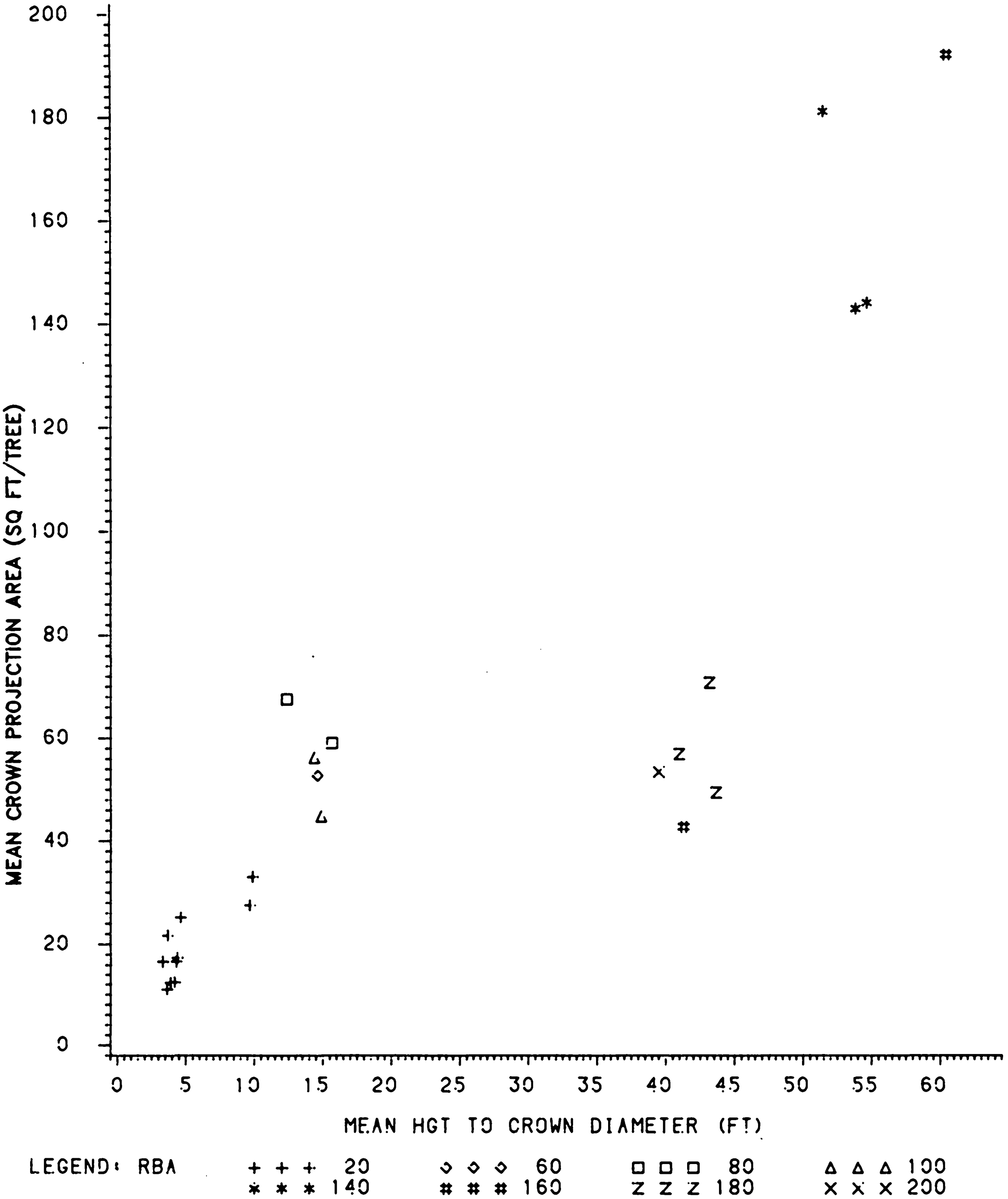
The relationships between SCPA and basal area, and vise versa were similar. Prior to crown closure, SCPA and stand basal area were positively correlated. After a stand reached 50 sq ft/ac of basal area or age 10, stand CPA was at approximately unit area excepting for slight fluctuations in mortality (Figure 8). An appropriate model for prediction of SCPA contained basal area and the number of surviving trees. This model or other non-linear models that asymptote at unit area could adequately predict SCPA prior to and at crown closure. These models would not be very sensitive to SCPA fluctuations due to mortality since basal area can develop at different rates and levels compared to SCPA.

Growth of the sum of crown projection area was not an important variable in regressions for unthinned stand variables. Obvious relationships with SCPA, age, basal area growth and dbh growth were not indicated. The lack of relationships between SCPAg and other logical variables were found in the stands having attained some degree of crown closure. The level and rate of SCPAg in these stands were dependent on the size of new openings resulting from mortality and in particular, the number and response of the trees surrounding that space. Thus, SCPAg may or may not be well related to the level and rate of stand development which is based on the whole stand and all the trees.

Mean Crown Projection Area. Only 24 observations were available for analysis of MCPA due to one plot not having measurements on height to crown diameter. Mean CPA was related to MHCD (likewise age, as will be shown later), and SCPA prior to crown closure and Ts after crown closure (Figures 9-11). While MCPA can be directly calculated from surviving trees after crown closure, it can also be predicted using the following sets of variables:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
SI MHCD	25.6225	0.78	2.0	18268.12
MHCD B	25.2216	0.79	4.8	17551.04
e(SI MHCD ln(SCPA))	19.9545			
1/A SI 1/Ts	14.9567	0.93	4.1	6758.78

Prior to crown closure, mean CPA was independent of the number of surviving trees or other measures of stocking as intraspecific competition had not begun. In this case, mean CPA was a function of site index, age and SCPA. Once crown closure occurred, however, mean CPA was directly related to the number of surviving trees as SCPA was at unit area, excepting for slight fluctuations due to mortality. In stands having obtained a degree of crown closure, and the number of surviving trees is unknown, then stocking and vertical crown measures were needed to predict mean CPA. Stocking level and MHCD affected horizontal crown development in different fashions. As stocking levels increased, horizontal crown development (i.e., MCPA) decreased due to less open area for crown development; and, as MHCD (or age) increased, horizontal development increased as surviving



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 9: COMPARISON OF MEAN CROWN PROJECTION AREA AND HEIGHT TO CROWN DIAMETER OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

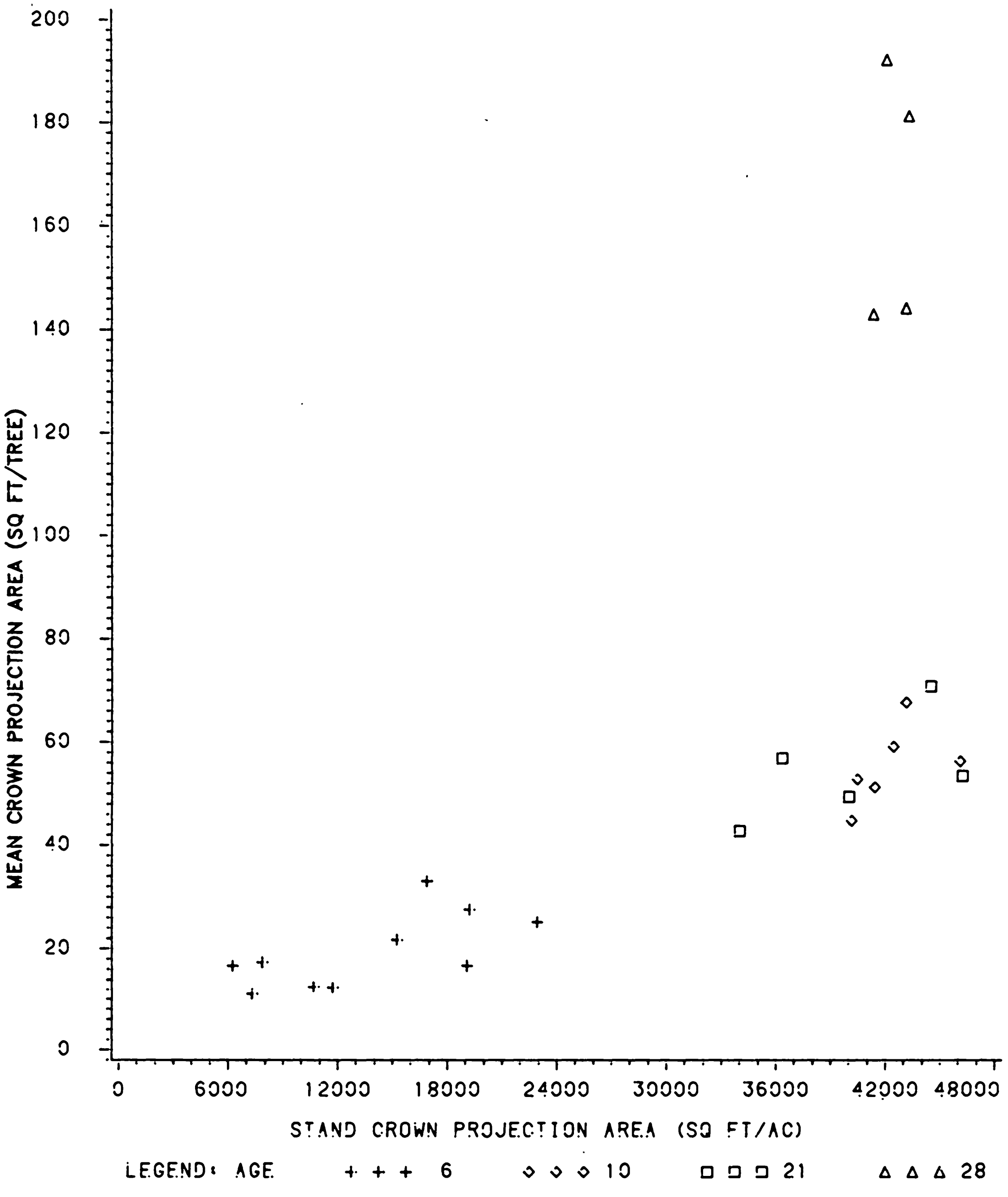
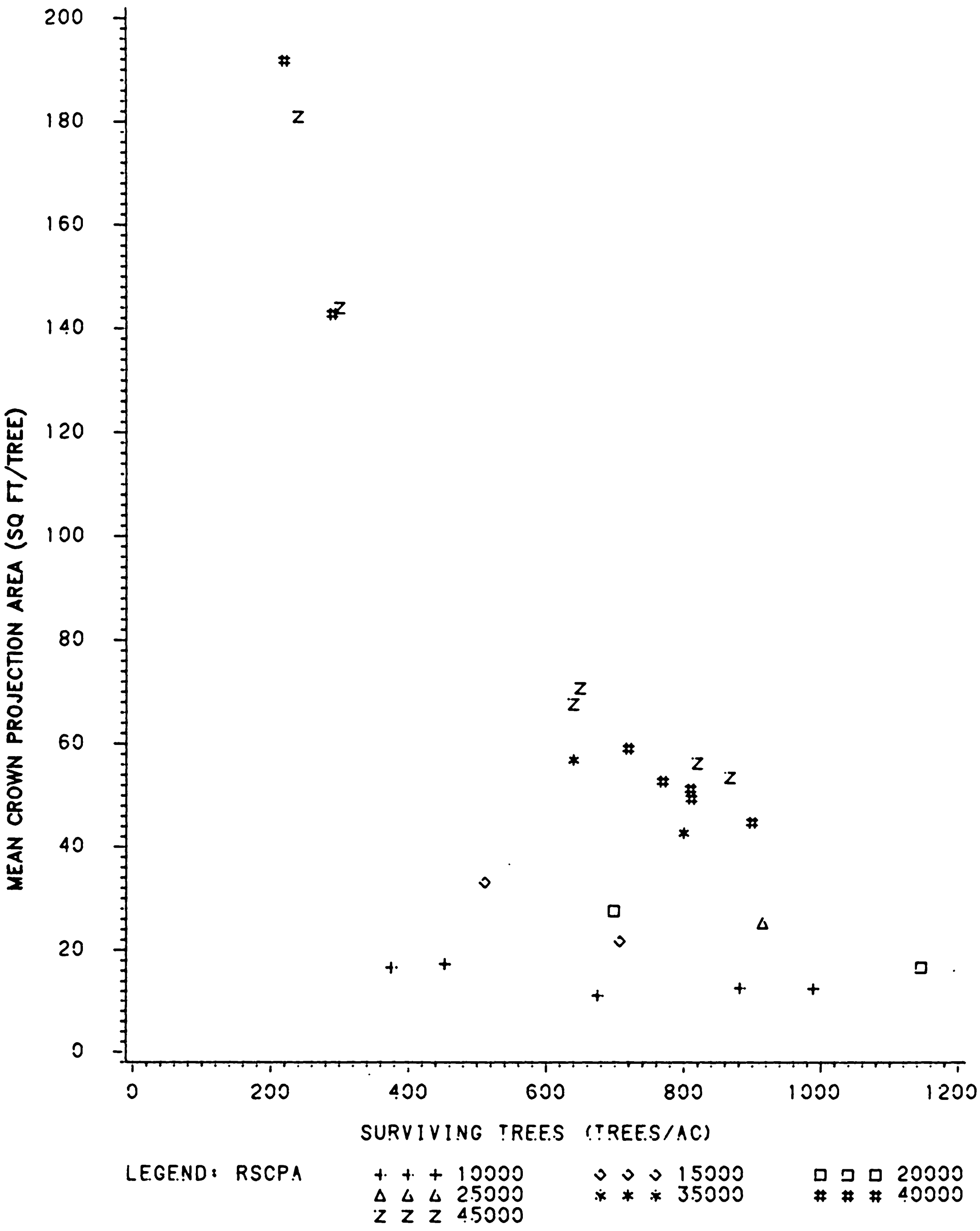


FIGURE 10: COMPARISON OF MEAN AND STAND CROWN PROJECTION AREA OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RSCPA IS A STAND CROWN PROJECTION AREA CLASS (SQ FT/AC).

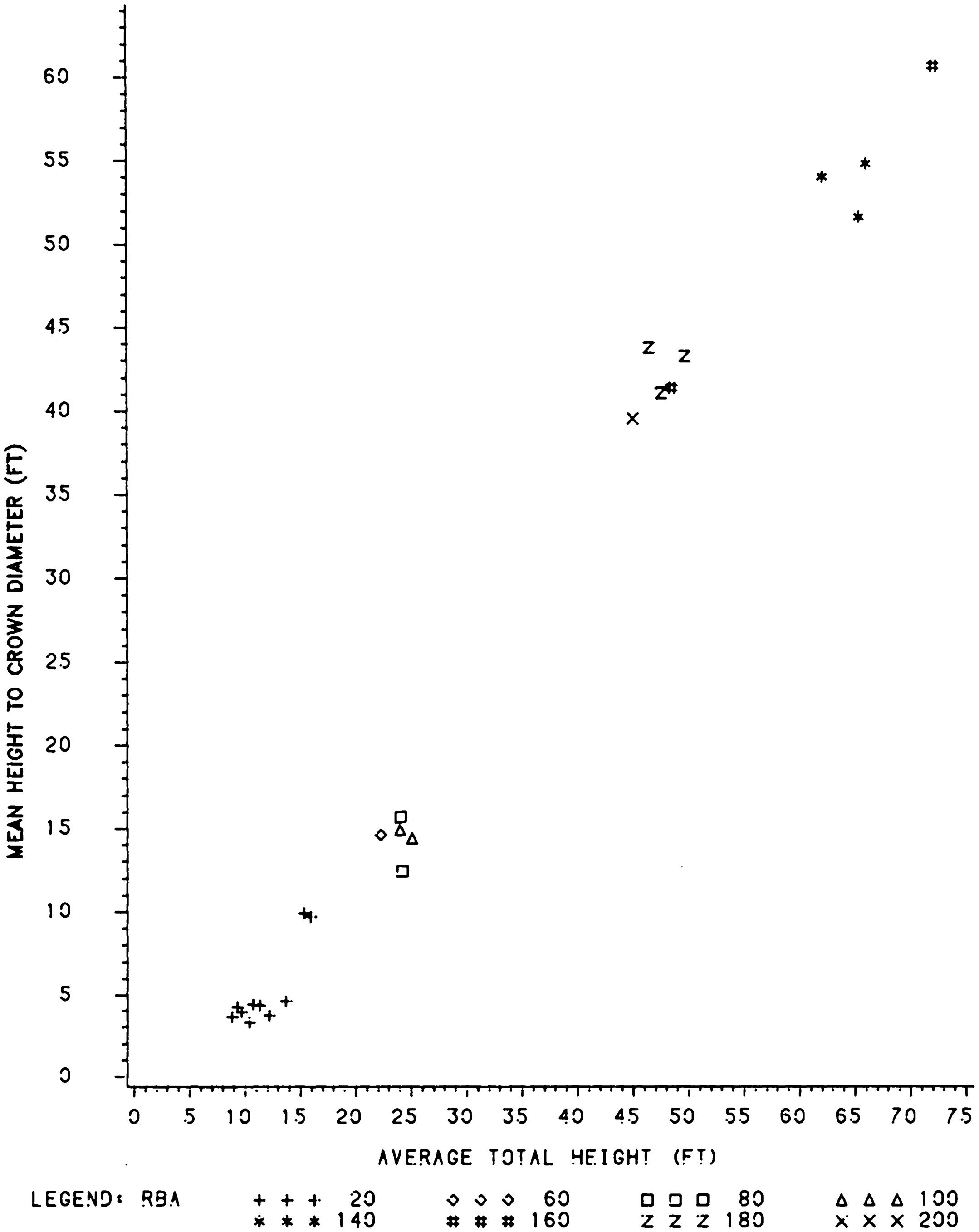
FIGURE 11: COMPARISON OF MEAN CROWN PROJECTION AREA AND SURVIVING TREES OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

tree crowns grew and expanded to open areas. The relationship between mean CPA and surviving trees was evident in the resultant model containing surviving trees; however, the strength of this relationship was a result of crown closure with MCPA then being a function of SCPA and Ts.

Mean Height to Crown Diameter. Height to crown diameter was best expressed as a linear relationship with mean total height of the stand (MHt) (Figure 12); also, given an age and stand basal area or SCPA, MHCD appeared constant (Figure 13). With only 24 observations available, the important variables in predicting MHCD were as follows:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
MHt	2.5719	0.99	1.0	172.07
MHt B	2.3354	0.99	4.2	142.42

Age was not a significant variable in the above models as age and mean total height were related and explained the same area of variation. The relationship between MHCD with MHt and B was understandable. As the stand developed in height (or age), the level of competition increased as crown expansion occurred, resulting in an increase in MHCD; also, as stocking levels increased, crown competition naturally increased resulting in an increase in MHCD. The low tolerance level of loblolly pine was the reason that MHCD increased in the above situations and was found to be constant within a given set of stand conditions.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 12: COMPARISON OF HEIGHT TO CROWN DIAMETER AND TOTAL HEIGHT OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

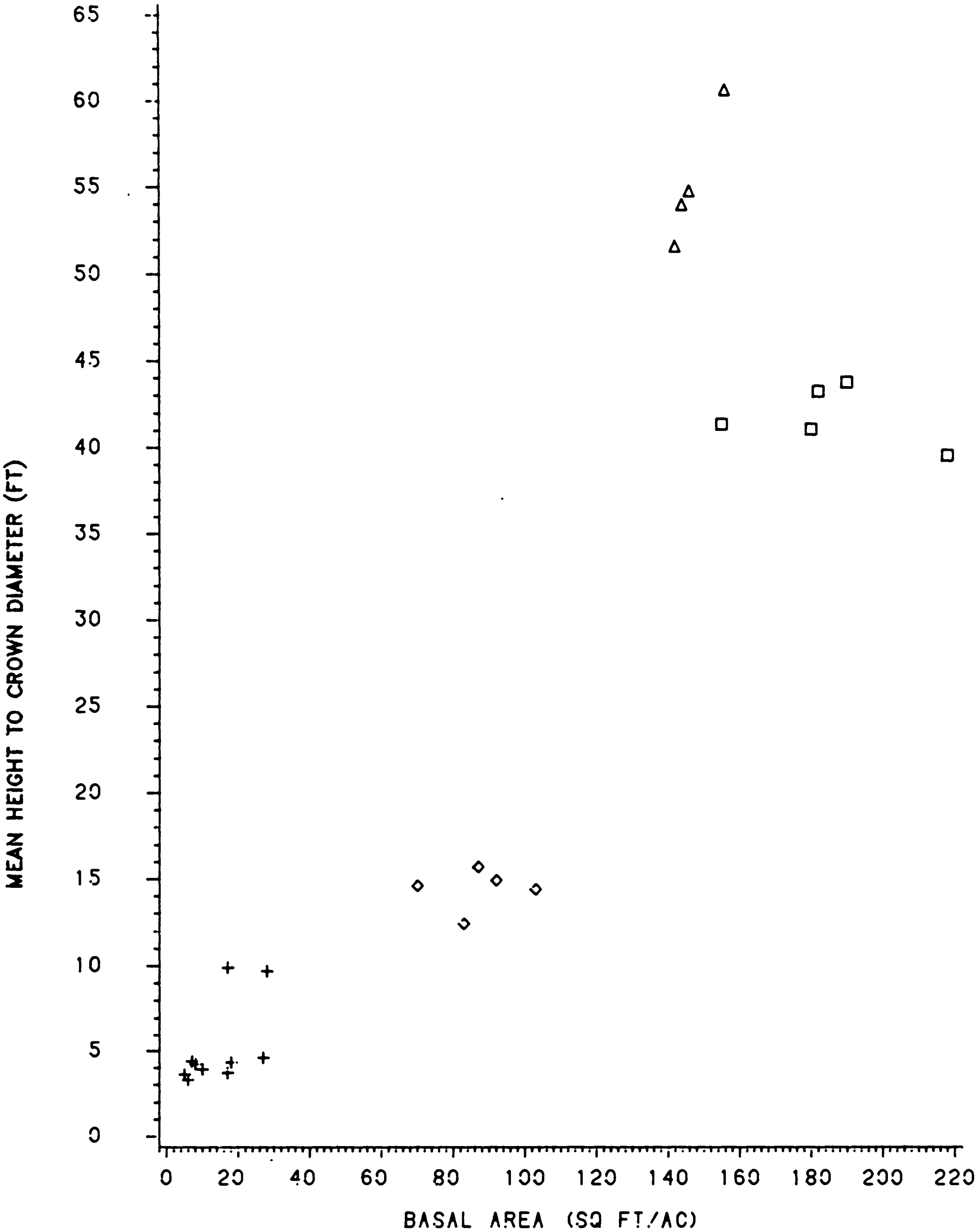


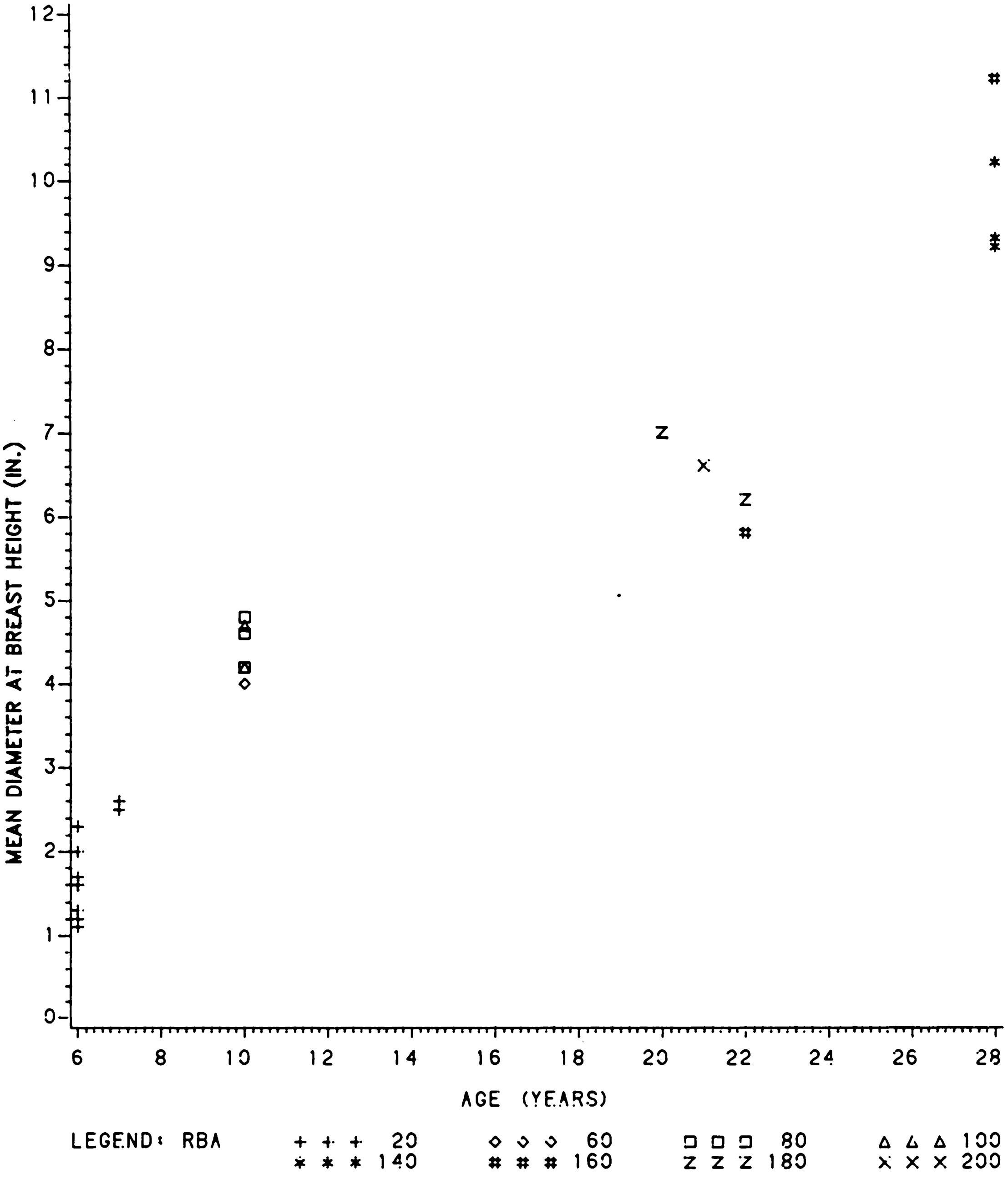
FIGURE 13: COMPARISON OF HEIGHT TO CROWN DIAMETER AND BASAL AREA OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

Mean dbh. Mean dbh increased with age, and for a given age, it decreased with increasing levels of residual stocking (Figure 14). Also, mean dbh increased with mean CPA, but the rate of increase appeared to decrease with age (Figure 15). As one plot did not have MHCD measured, the following sets of variables were important in predicting mean dbh when 24 observations were used:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
A SI SCPA	0.4813	0.98	2.6	7.35
A SI ln(MCPA)	0.3435	0.99	6.0	3.84
MHCD ln(MCPA)	0.3554	0.99	3.8	3.62

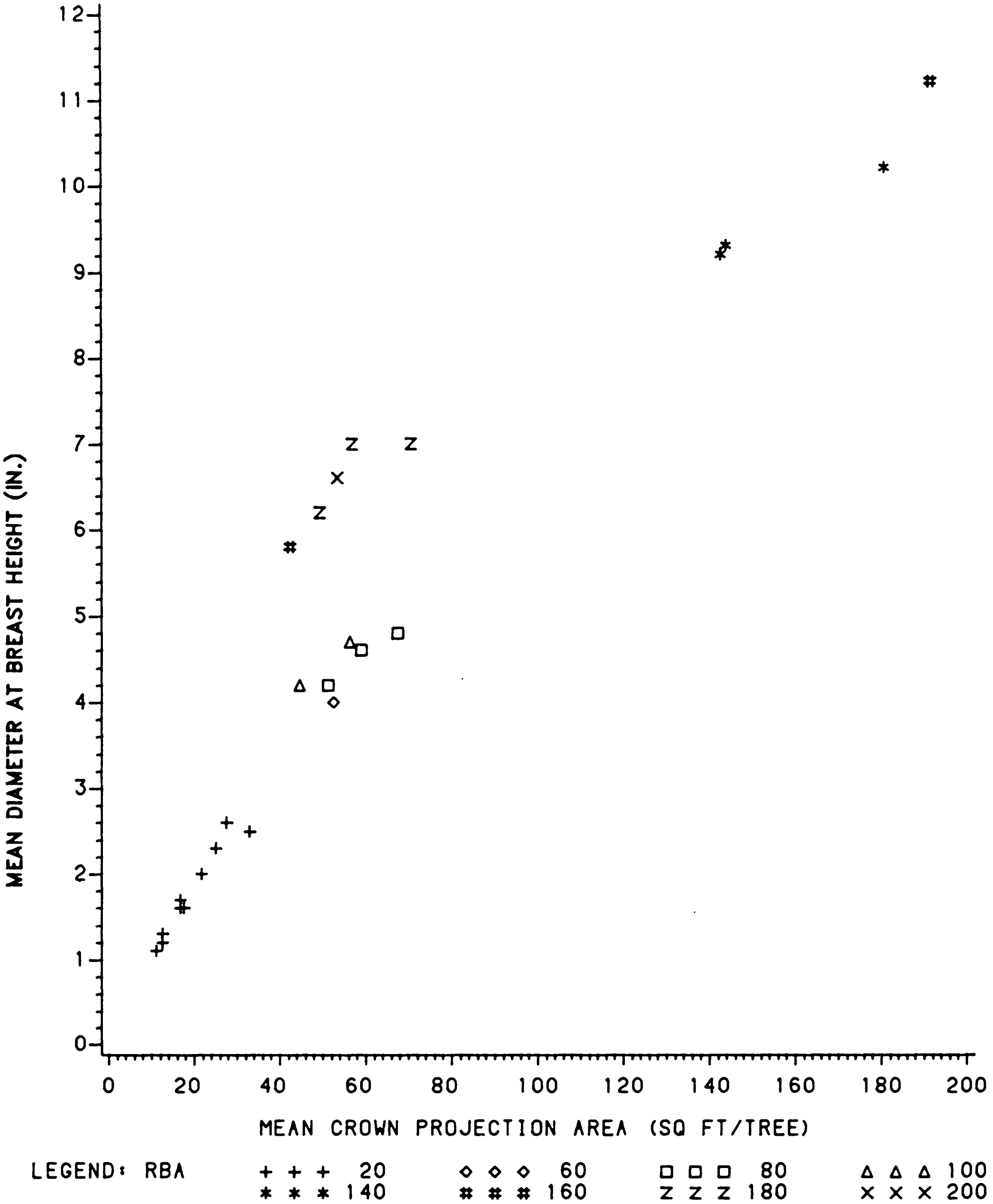
The two main expressions for crown development (horizontal crown development in the form of mean CPA and vertical crown development as expressed by MHCD) were the most important variables in predicting dbh. The MCPA was strongly correlated with mean dbh within an age class: as the mean CPA increased, there was a linear increase in mean dbh within an age class. As the average size of the crown increased, so did the average size of the stem. Thus, the importance and strength of the relationship between an individual tree crown and stem dimension (i.e., CPA and dbh) were also indicated on a stand level basis between the means of those variables.

Mean dbh Growth. As age, initial dbh, and mean CPA increased, dbh growth



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 14: COMPARISON OF DIAMETER AT BREAST HEIGHT AND AGE OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 15: COMPARISON OF DIAMETER AT BREAST HEIGHT AND MEAN CROWN PROJECTION AREA OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

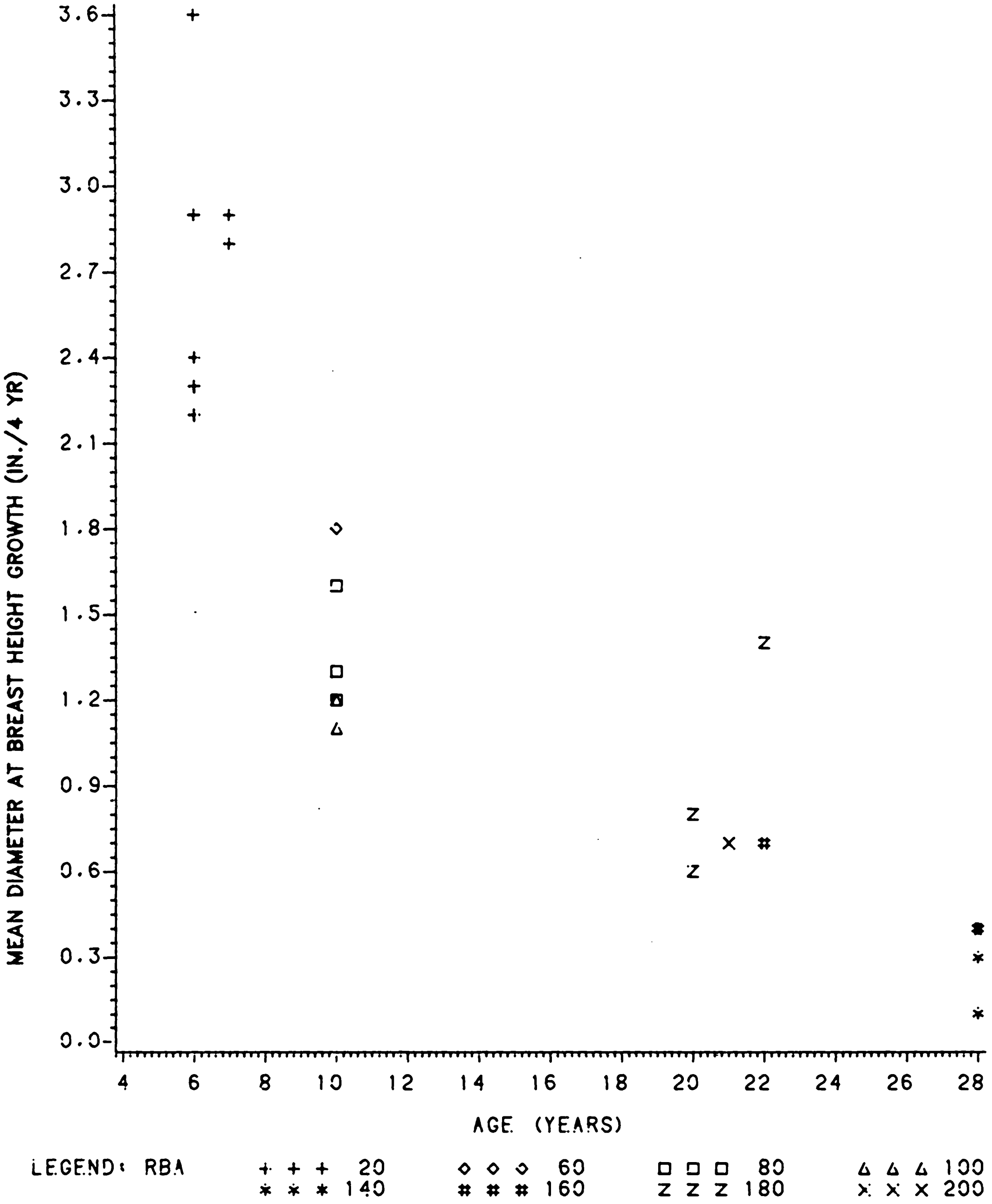
over a 4-year period decreased (Figures 16-18). There was a positive trend between mean dbh growth and SCPAg (Figure 19). The important variables in predicting mean dbh growth were as follows:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
1/A	0.4061	0.84	1.0	4.50
A SCPA	0.3503	0.88	1.9	3.64
A SCPA ln(dbh)	0.3131	0.91	25.2	2.87

A model containing just age fitted and predicted mean dbh growth better than models containing other single variables. The only important stand variable found for the prediction of mean dbh growth was SCPA; other variables, such as basal area or number of surviving trees did not help in prediction. The SCPA had a direct relationship with mean dbh since it was an indirect measure of the amount of photosynthate presently and potentially available for diameter growth.

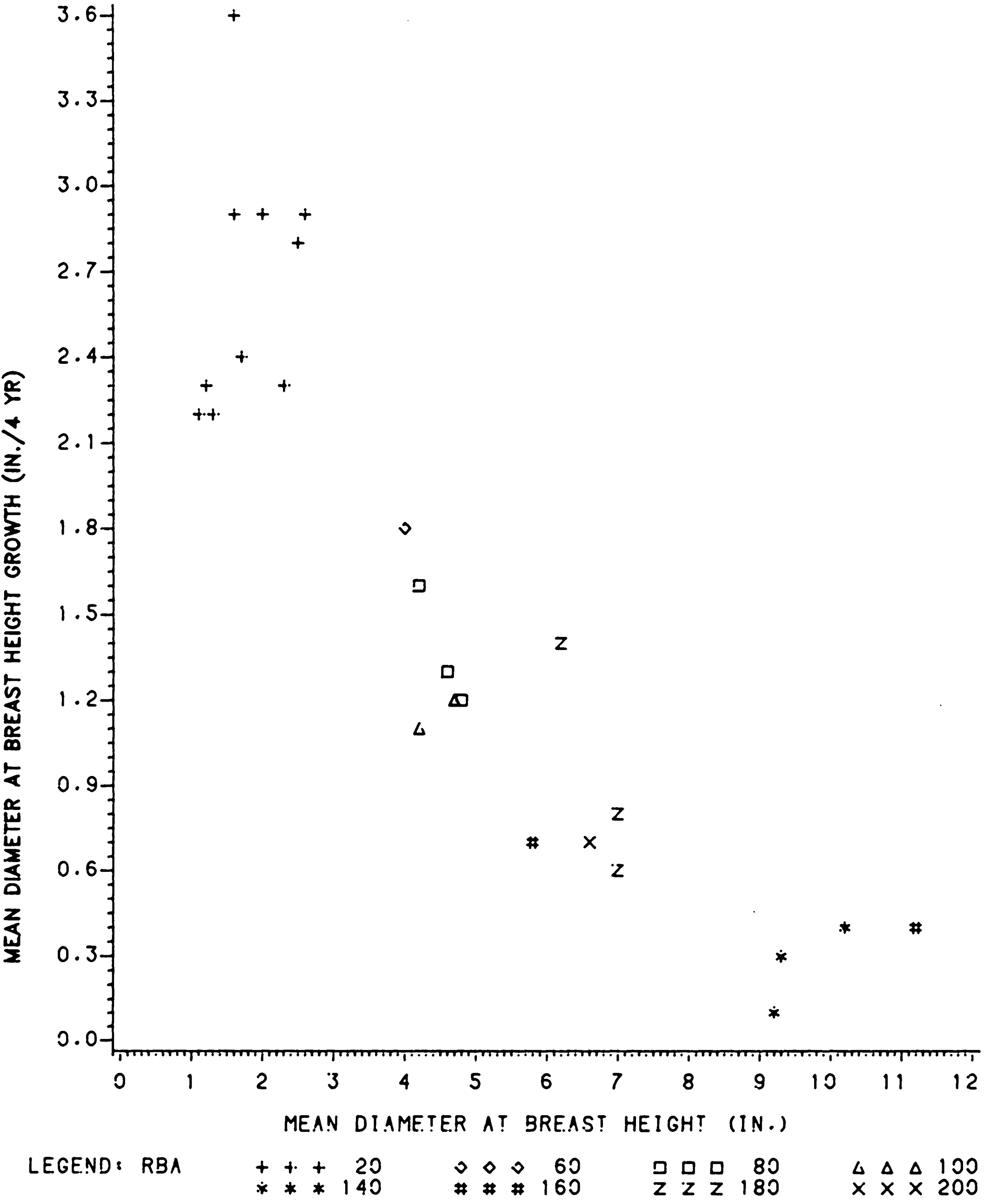
4.2.2 THINNED STANDS

The available data provided 35 remeasured plot observations from thinned stands (Table A3). Of the 35 plots, 29 were 28-years-old, while the remaining were either 7-, 10- or 20-years-old. While a balanced matrix of plots with age, site index, and level and timing of thinning was not available, some trends and relationships were evident.



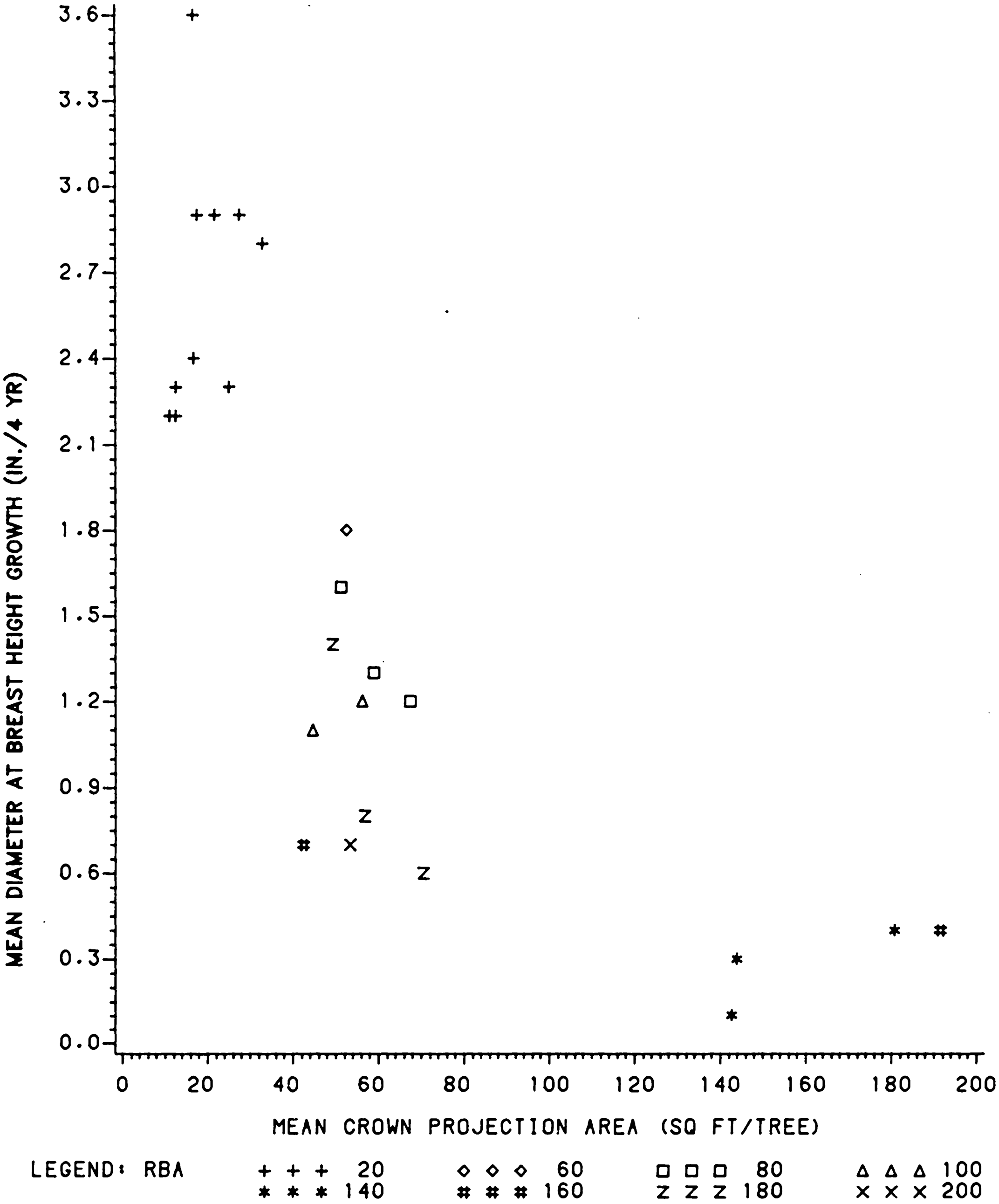
WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 16: COMPARISON OF DIAMETER AT BREAST HEIGHT GROWTH AND AGE OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.



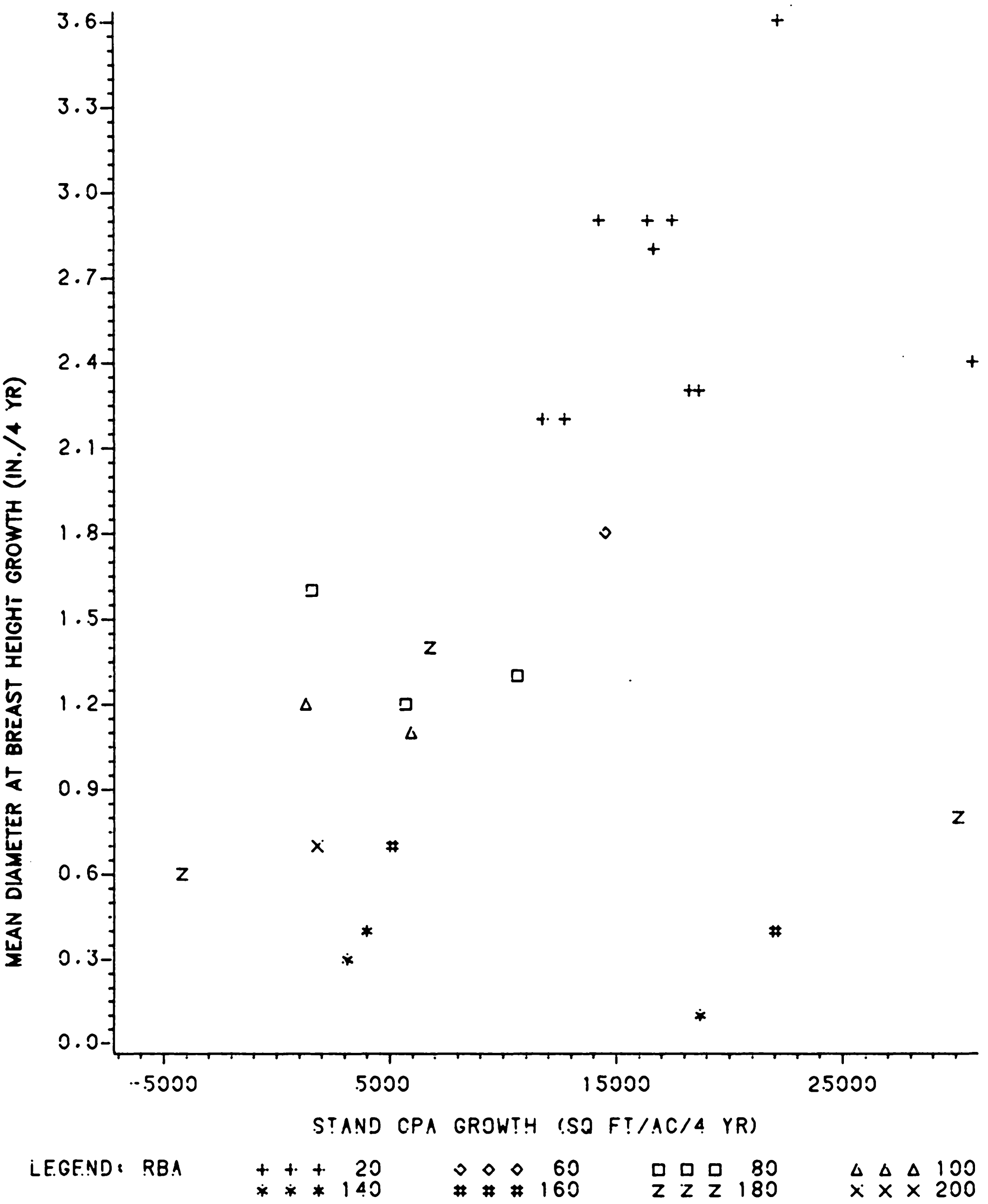
WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 17: COMPARISON OF DIAMETER AT BREAST HEIGHT GROWTH AND INITIAL DIAMETER OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 18: COMPARISON OF DIAMETER AT BREAST HEIGHT GROWTH AND MEAN CROWN PROJECTION AREA OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 19: COMPARISON OF DIAMETER AT BREAST HEIGHT GROWTH AND STAND CROWN PROJECTION AREA GROWTH OF UNTHINNED STANDS OF PLANTED LOBLOLLY PINE.

Basal Area. Simple relationships suggested that basal area and SCPA were correlated, although variability was high (Figure 20). Also, for the older thinned stands, as the number of surviving trees increased, so did basal area (Figure 21). Further analyses indicated the variables important in predicting basal area of unthinned stands were also important in thinned stands.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
1/A SI ln(Ts) SCPA	7.0515	0.94	4.1	1973.55
e(1/A SI ln(Ts) ln(SCPA))	7.0486			
e(1/A SI ln(MCPA) ln(SCPA))	6.1512			

Basal area of a tree or stand, regardless of thinning, basically depended on crown size of the tree or mean crown size of the trees, respectively, and stocking levels, such as crown projection area and surviving trees. Basal area development on a stand basis was related to the aggregate development of the crown canopy as each measured a cross sectional area that was also highly related on a tree basis. However, number of trees or in a similar sense, mean CPA, was also needed to reflect the average number and size of trees making up SCPA. While a SCPA value reflected a possible range of predicted basal area values, surviving number of trees indicated the level of basal area. That is, given a SCPA value, as the number of trees increased, basal area increased in the older stands.

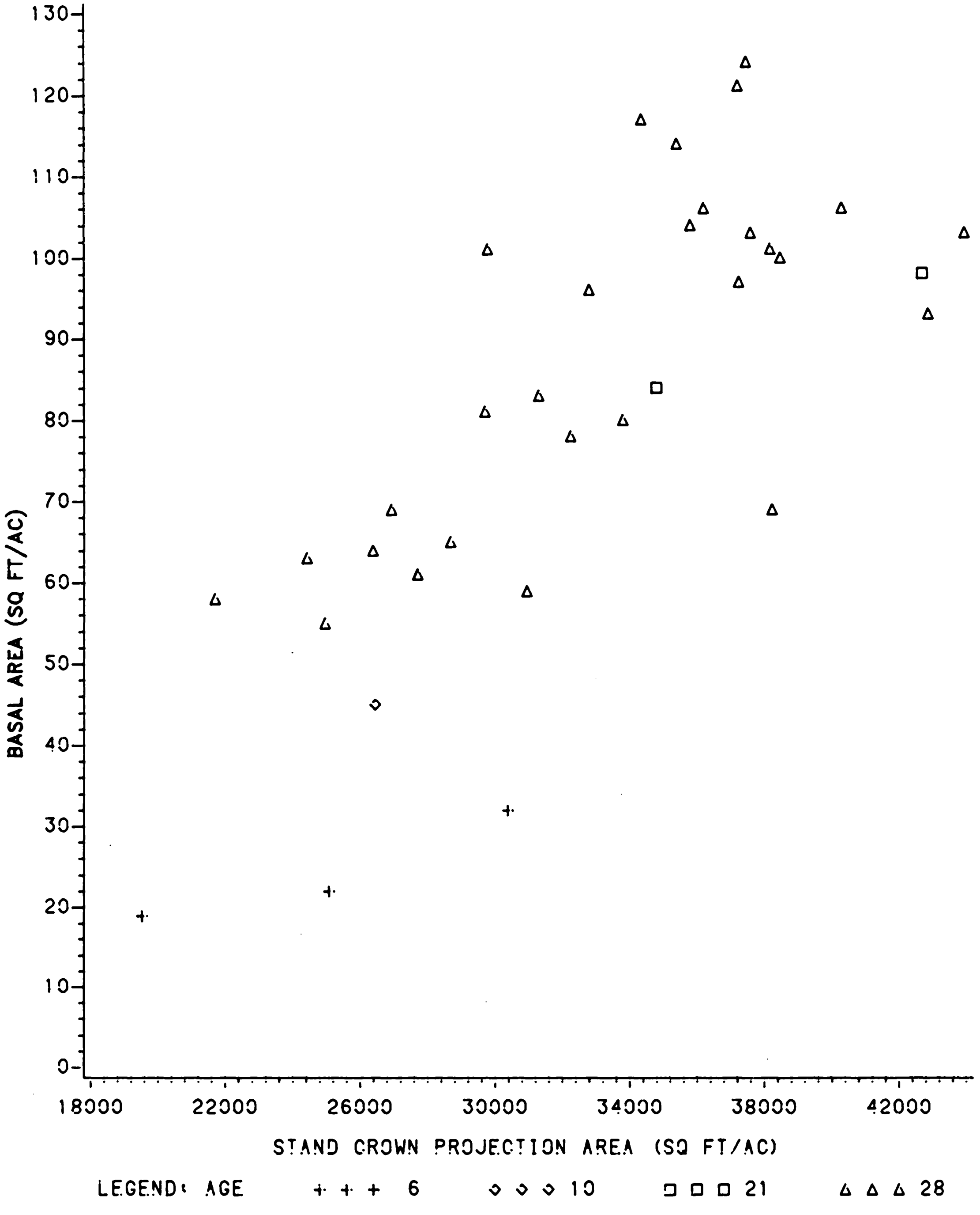
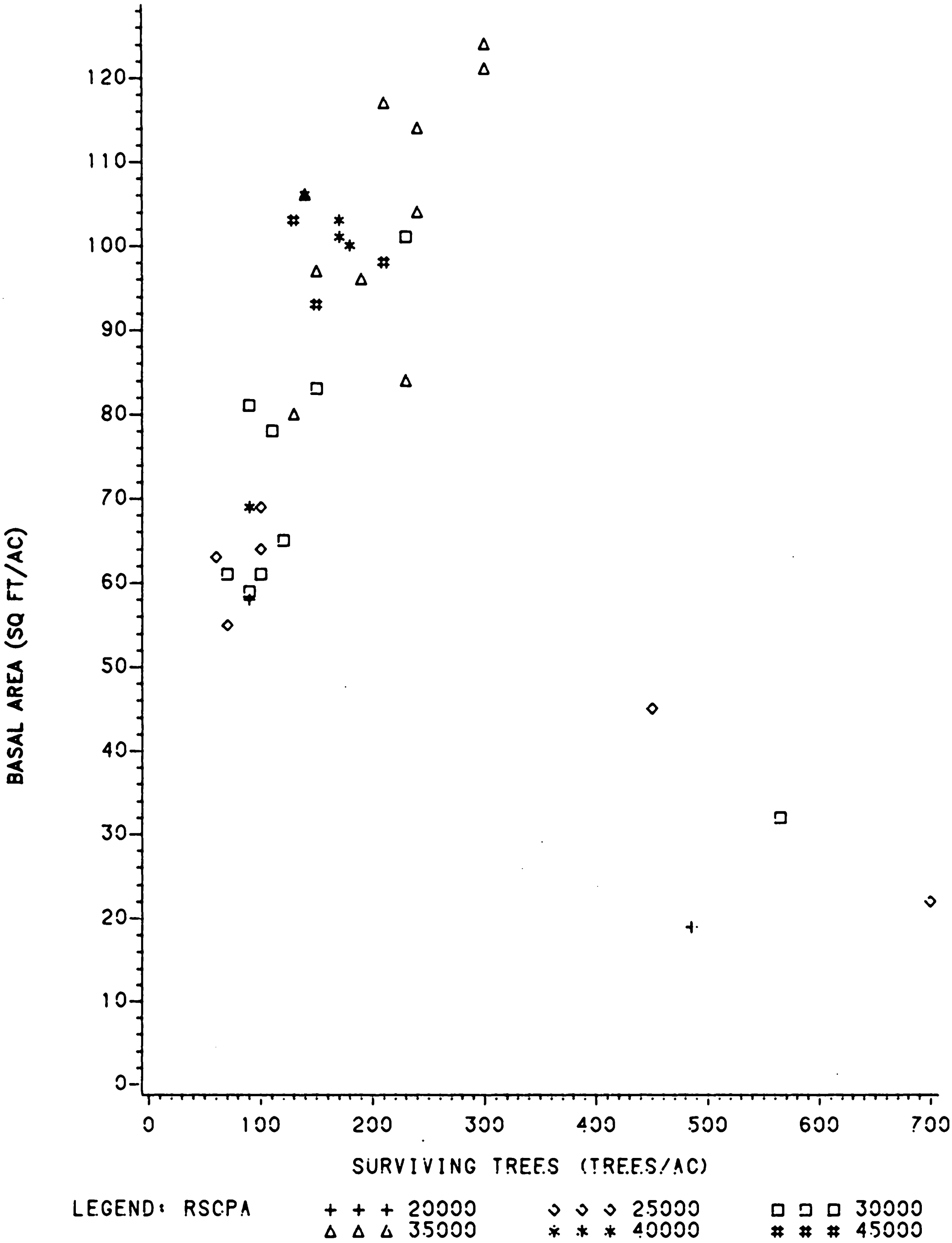


FIGURE 20: COMPARISON OF BASAL AREA AND STAND CROWN PROJECTION AREA OF THINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RSCPA IS A STAND CROWN PROJECTION AREA CLASS (SQ FT/AC).

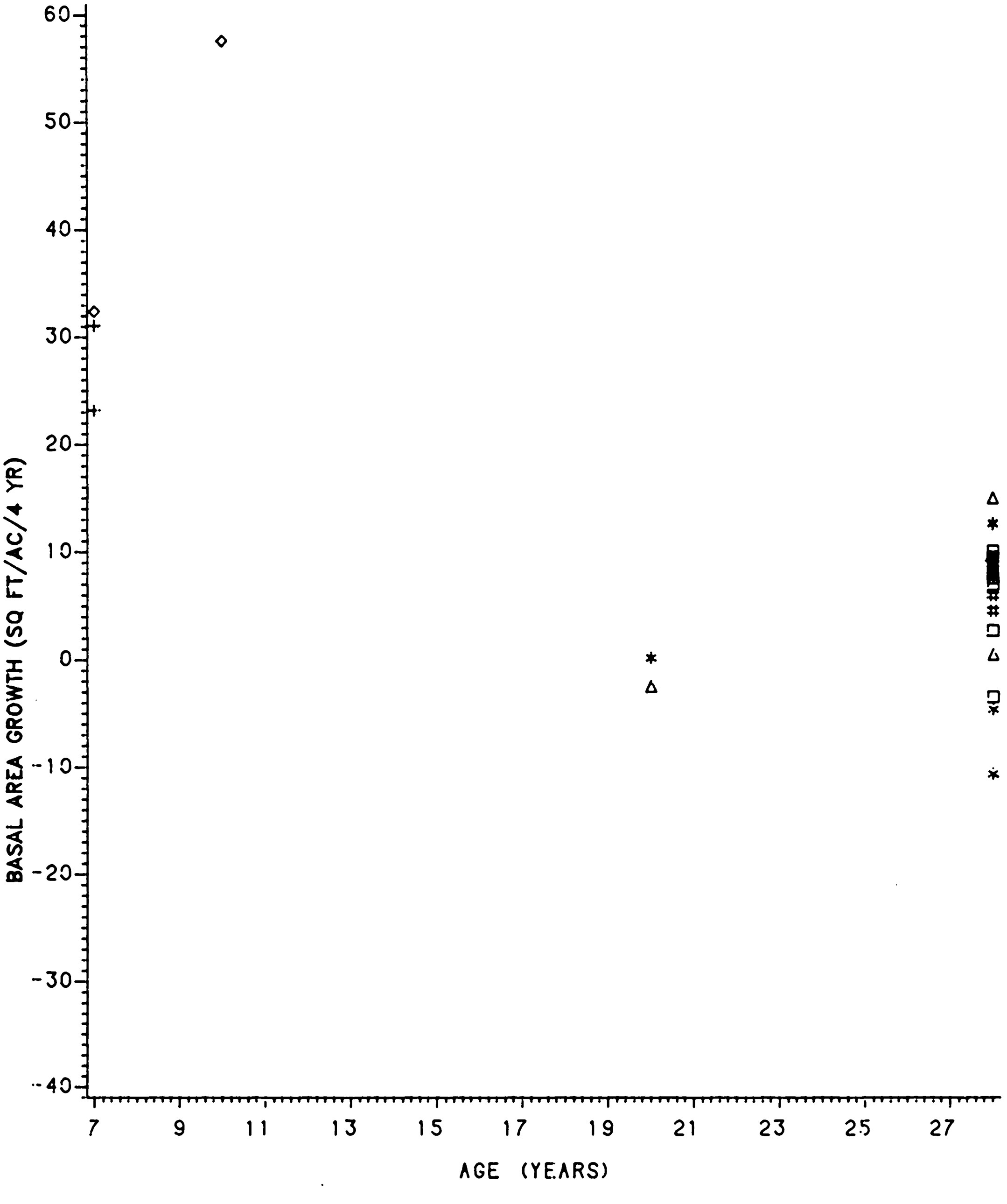
FIGURE 21: COMPARISON OF BASAL AREA AND SURVIVING TREES OF THINNED STANDS OF PLANTED LOBLOLLY PINE.

Basal Area Growth. This variable represented growth over a 4-year period and was inversely related to age (Figure 22), fairly constant regardless of thinning intensity for stands 10-years and older (Figure 23), and was slightly related to SCPAg in the older stands (Figure 24). Basal area growth of unthinned and thinned stands depended on SCPA, basal area and age.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
B SCPAg	9.3366	0.40	1.0	3504.90
1/A ln(B) SCPA	7.8241	0.60	8.1	2534.17
1/A SCPAg	6.6695	0.70	1.0	1908.63
e(1/A SCPA ln(B))	6.5925			

The inclusion of SCPA and basal area in the basal area growth model were understandable. SCPA provided an indirect measure of photosynthate area and indicated the extent of possible growth, while stand basal area provided a measure of initial conditions on which growth could occur. While variability was high, there was an indication that basal area growth remained fairly constant for stands 10-years and older, regardless of the residual basal area or SCPA.

There was a strong relationship between tree basal area growth and crown diameter growth in unthinned and thinned stands. It was of interest that while SCPAg was positively related to stand basal area growth in the



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 22: COMPARISON OF BASAL AREA GROWTH AND AGE OF THINNED STANDS OF PLANTED LOBOLLY PINE.

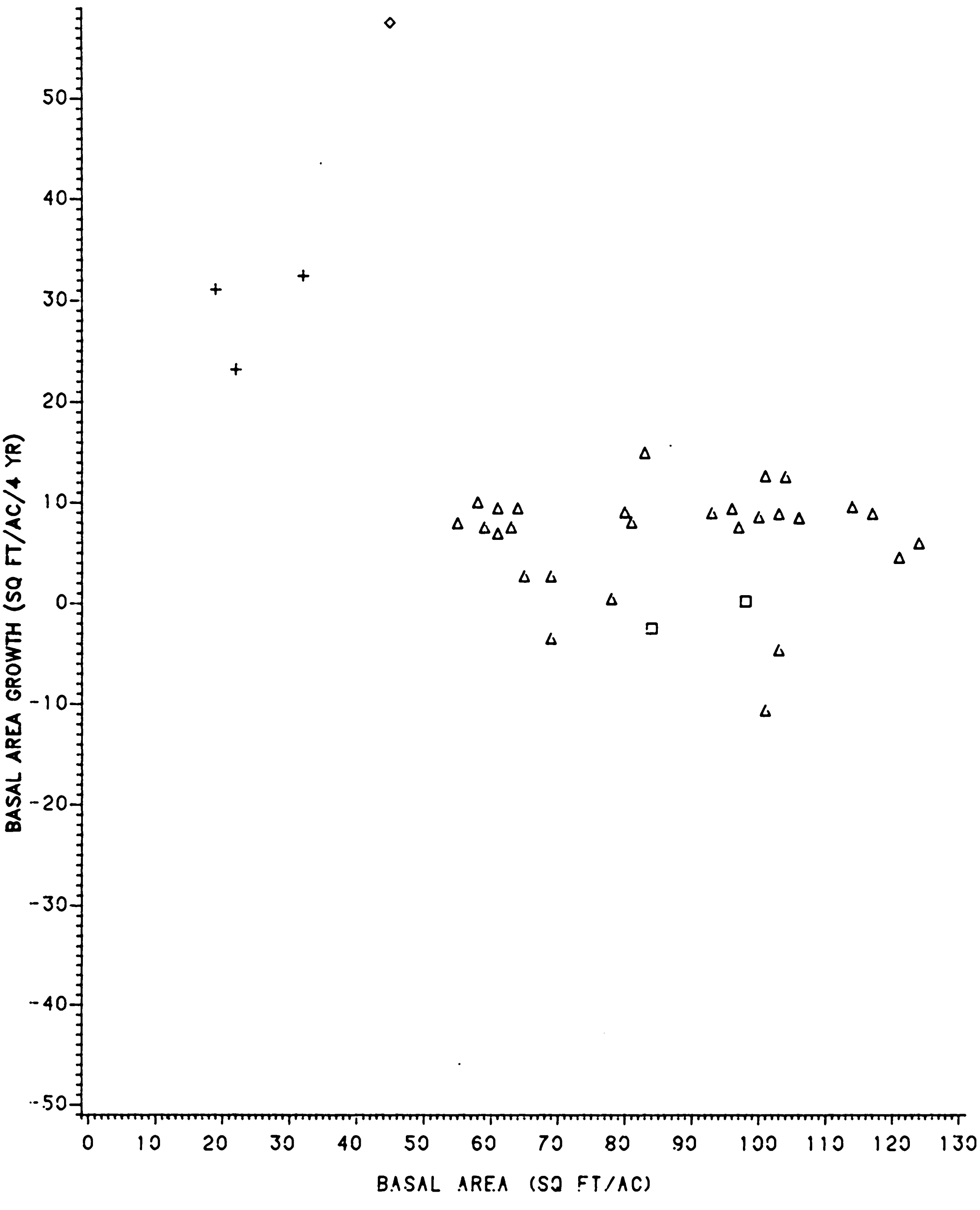
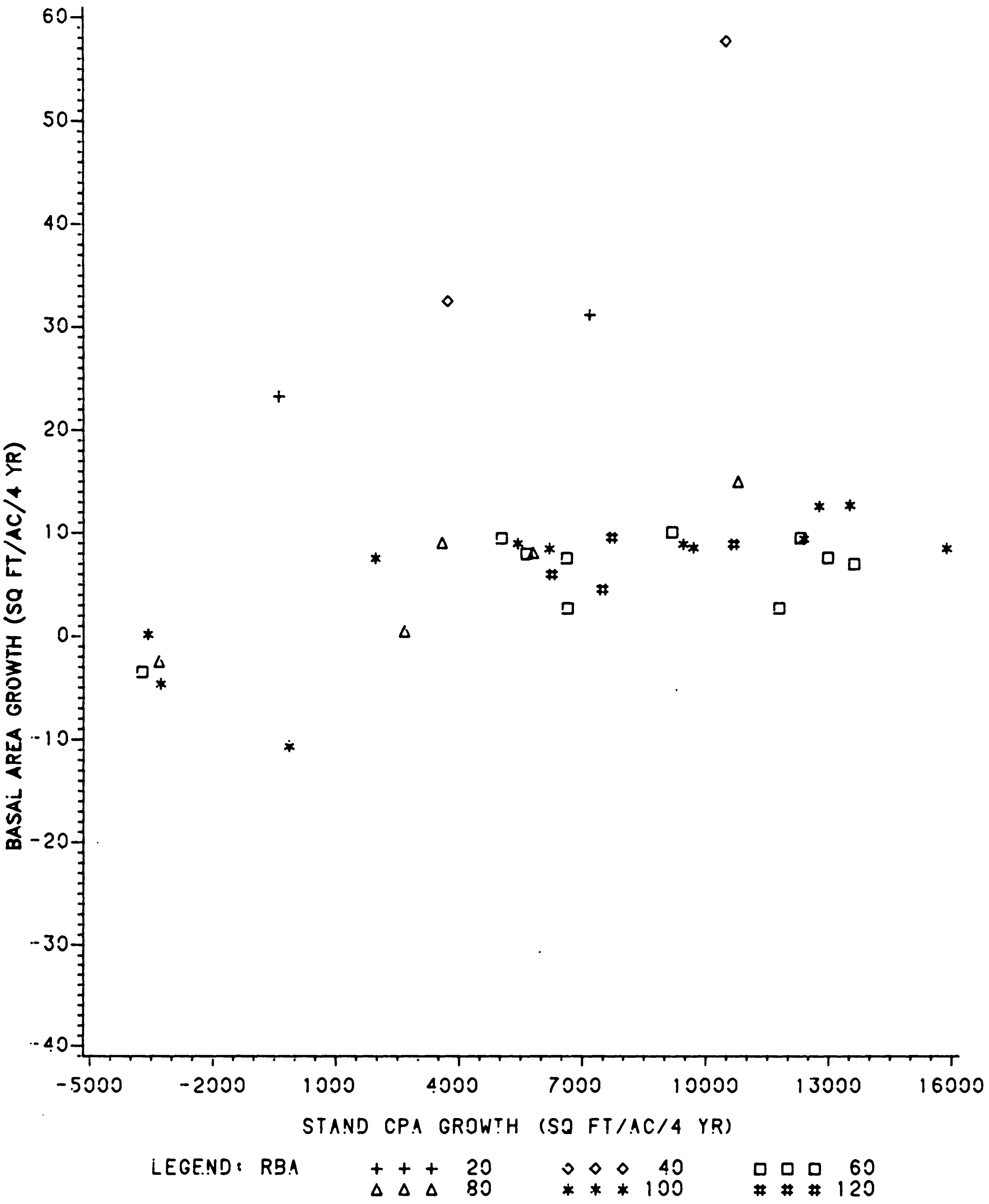


FIGURE 23: COMPARISON OF BASAL AREA GROWTH AND BASAL AREA OF THINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 24: COMPARISON OF BASAL AREA GROWTH AND STAND CROWN PROJECTION AREA GROWTH OF THINNED STANDS OF PLANTED LOBLOLLY PINE.

older thinned stands, it was not as a significant of a variable in terms of model fit or prediction of the other stand variables in the thinned stands as was found on an individual tree basis. In unthinned stands, mortality and degrees of crown closure provided situations where a lack of a relationship between basal area growth and SCPA growth was reasonable. Since these situations did not seem to apply to thinned stands with less than unit area of crown area, it seemed that an increase in mean CPA should be directly related to an increase in basal area as long as crown closure has not occurred.

Sum of Crown Projection Area. The relationship between stand basal area and SCPA, and vise versa were similar. The following sets of variables were important in predicting SCPA:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
e(1/A SI ln(B))	3546.8248			
1/A SI ln(B)	3468.6251	0.71	5.0	477,132,416
1/A SI ln(B) ln(Ts)	3167.5528	0.77	30.0	422,661,269

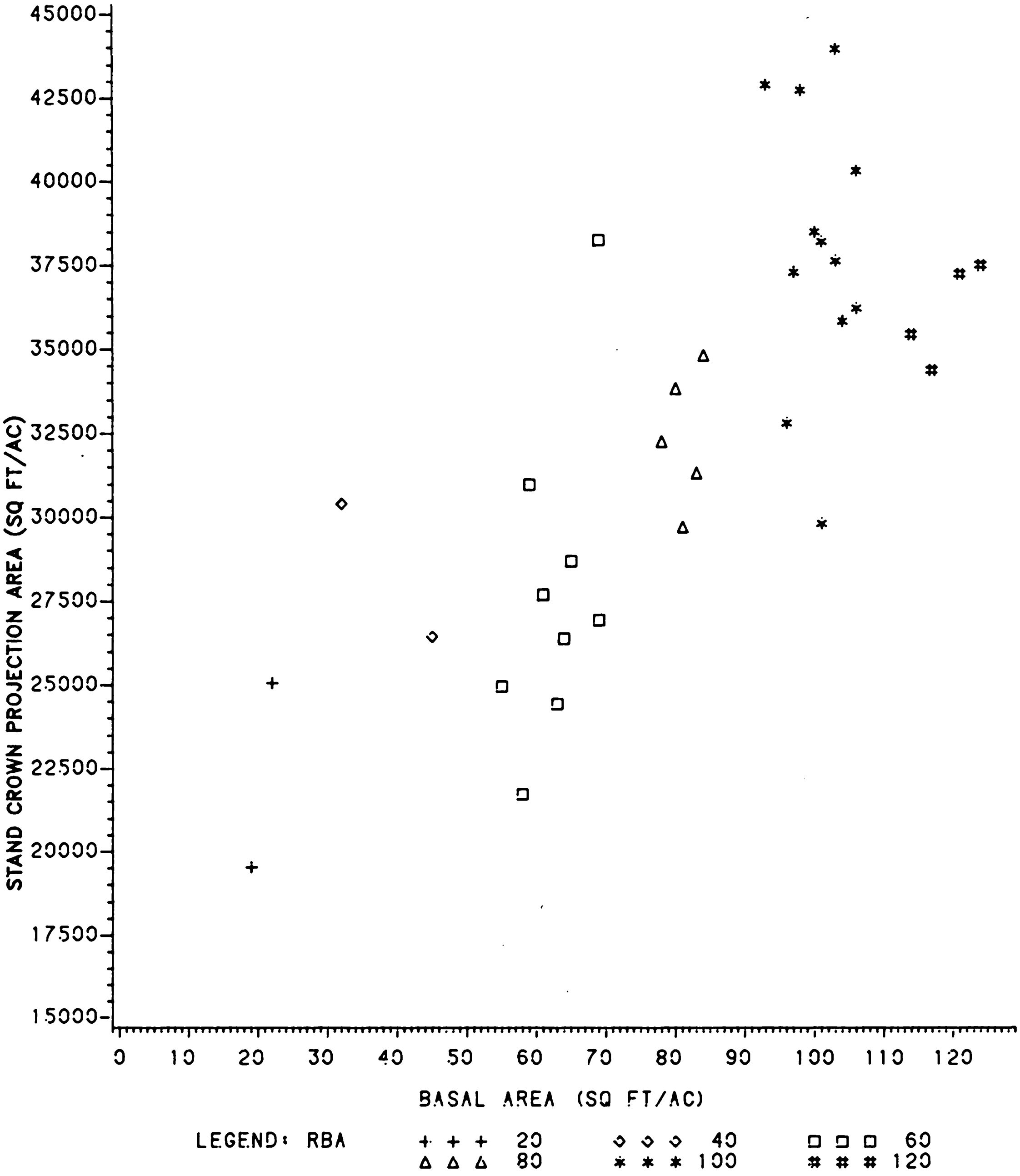
These results were similar to those found in the unthinned case and understandable. The years after thinning represented a similar situation of open crown area as found in the unthinned stands prior to crown closure. Due to age, however, the relationship between SCPA and basal area in thinned stands followed that of the older unthinned stands that had attained crown closure or because of mortality were re-approaching crown unit area. Thus, stand basal area and surviving trees were found to be important variables in predicting SCPA (Figures 25, 26).

SCPAg over a 4-year period was not well related to initial SCPA, stand basal area, surviving trees, basal area growth or dbh growth in thinned stands. With 35 observations, the following sets of variables were found to be significant in the prediction of SCPAg:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
A SI SCPA	4552.4999	0.35	1.8	858,695,920
A SI SCPA ln(MCPA)	4287.9956	0.44	4.0	787,361,551
SCPA B	4306.0111	0.40	2.4	701,656,308

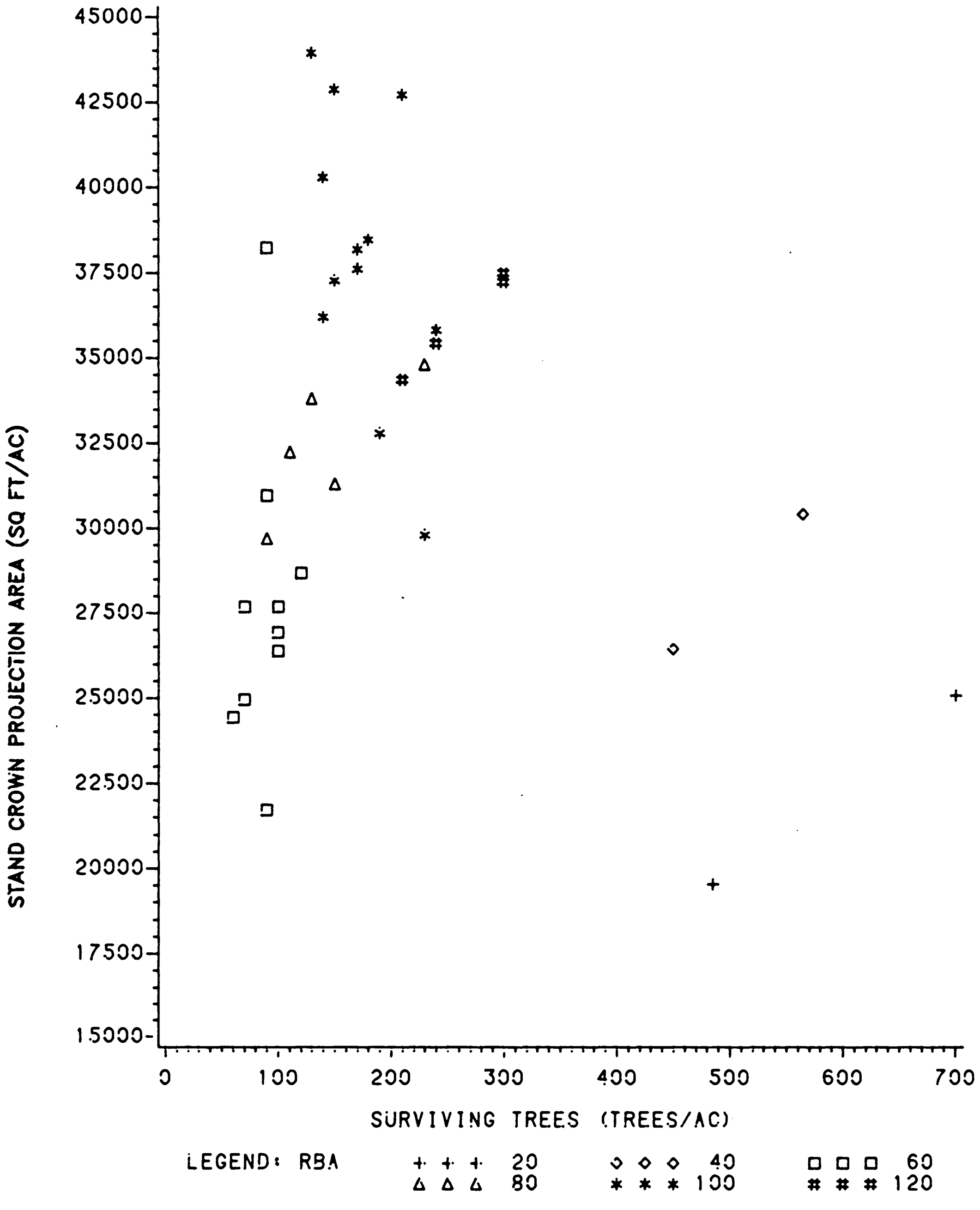
The growth of SCPA was related to its initial base, SCPA, and stand basal area. Linear and non-linear models containing other stand characteristics did not predict SCPAg as well as the above models.

<u>Mean Crown Projection Area.</u> Three thinned plots did not have measurements taken on height to crown diameter, so only 32 plot observations were available for this and the next analysis (this eliminated the 10- and 20-year-old stands). In the 28-year-old stands, as basal area (Figure 27) or surviving trees (Figure 28) increased, MCPA decreased.



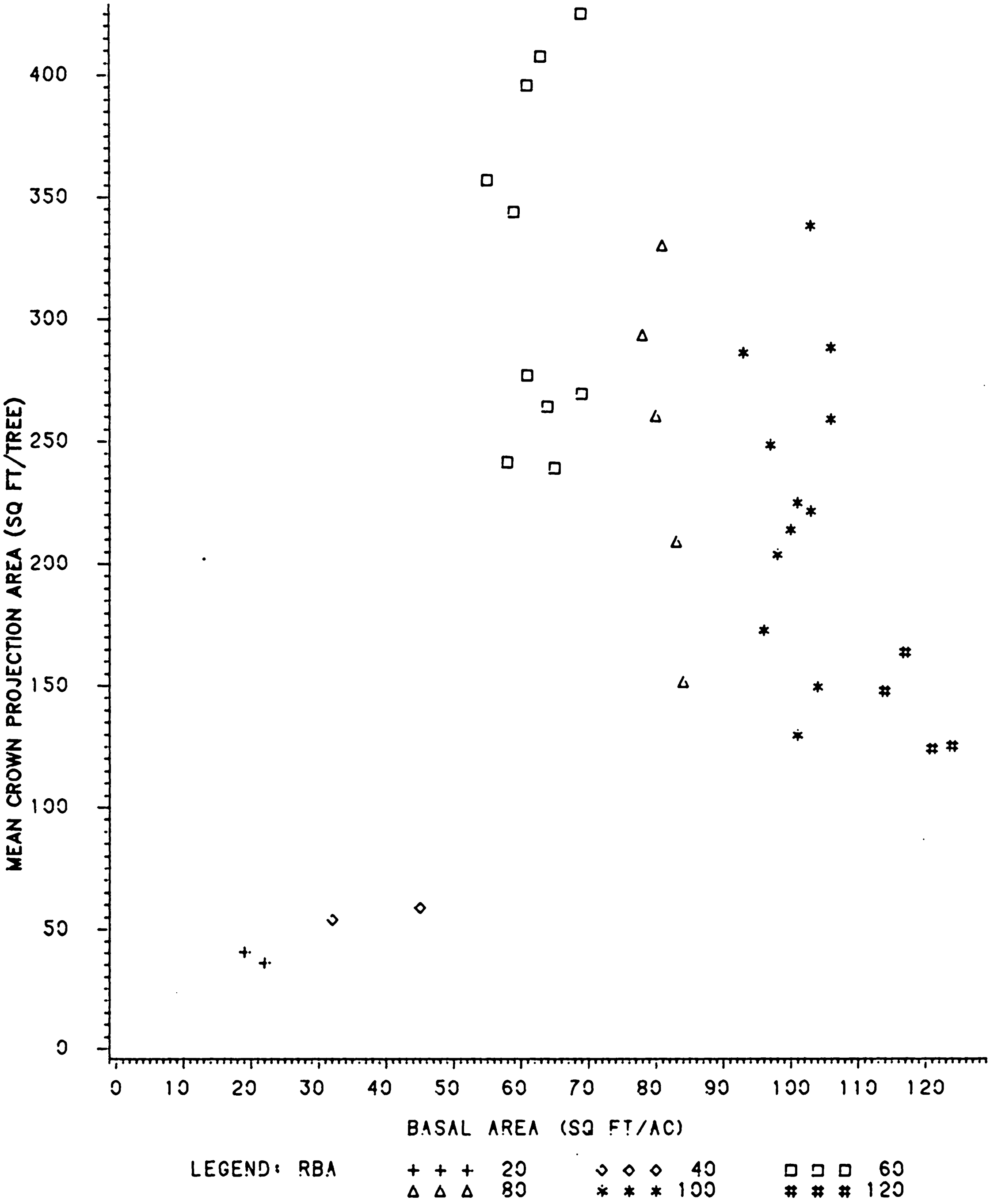
WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 25: COMPARISON OF STAND CROWN PROJECTION AREA AND BASAL AREA OF THINNED STANDS OF PLANTED LOBLOLLY PINE.



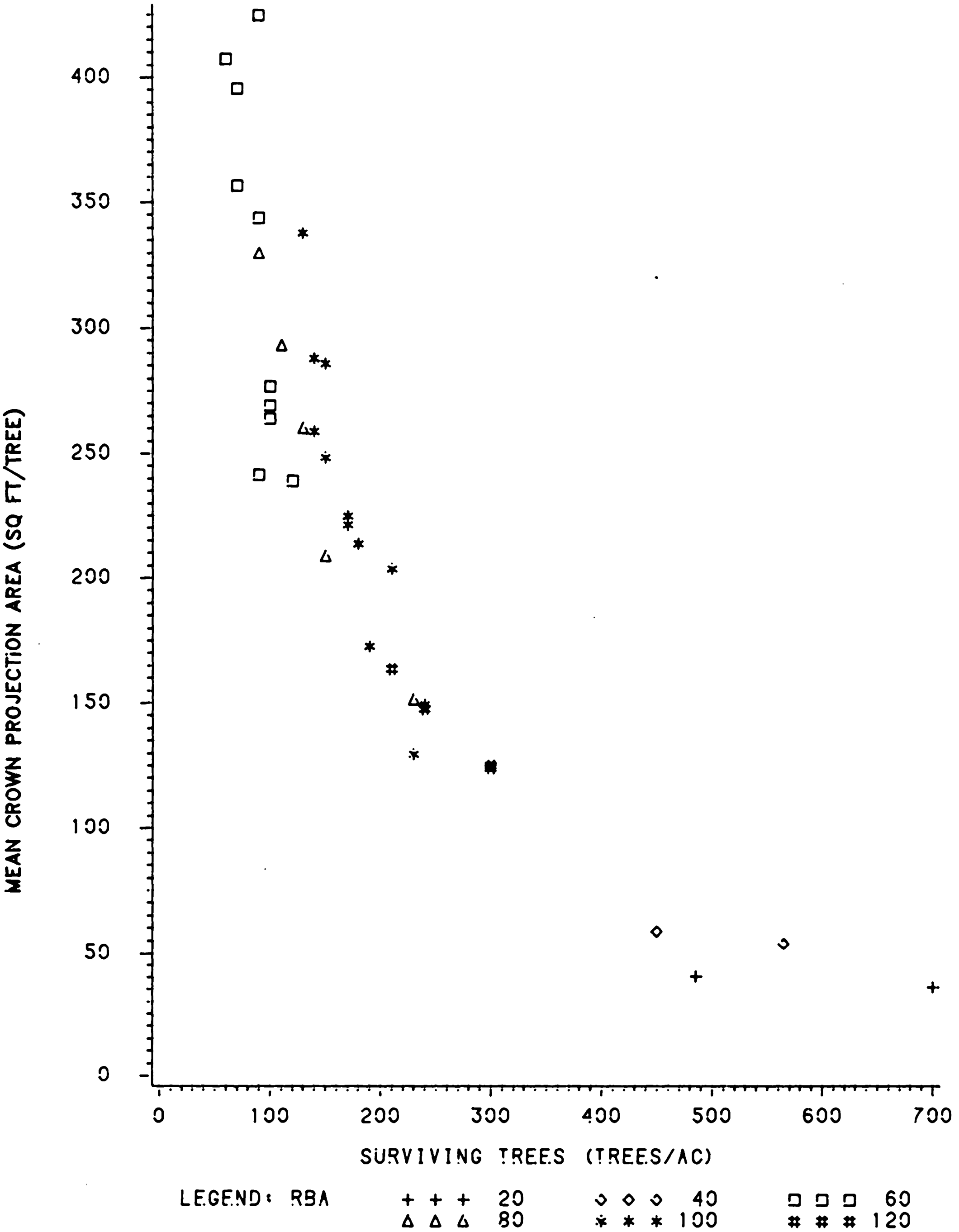
WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 26: COMPARISON OF STAND CROWN PROJECTION AREA AND SURVIVING TREES OF THINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 27: COMPARISON OF MEAN CROWN PROJECTION AREA AND BASAL AREA OF THINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 28: COMPARISON OF MEAN CROWN PROJECTION AREA AND SURVIVING TREES OF THINNED STANDS OF PLANTED LOBLOLLY PINE.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
MHCD	80.2565	0.40	1.0	208087.21
e(SI MHCD)	76.2143			
MHCD B	60.9384	0.67	1.7	125828.61
1/A 1/Ts	38.4453	0.87	1.4	54082.32

Mean CPA in thinned stands can be directly measured from aerial photographs or calculated using the number of surviving trees. The calculation of MCPA using the number of surviving trees was possible in the unthinned stands that had attained a degree of crown closure. This similar situation with the older thinned stands suggested that horizontal crown development in these stands was still related to the number of trees regardless of the residual basal area or that a degree of crown closure had not been attained. If the number of surviving trees is unknown, then horizontal development, in terms of MCPA, was affected by the residual level of basal area and was also related to MHCD - a vertical measure of crown development also influenced by residual level of basal area. If the number of surviving trees is known, then due to the relationship between MCPA and SCPA with Ts, a model containing 1/Ts fitted and predicted MCPA better than the other models.

Mean Height to Crown Diameter. MHCD was linearly related to total height (Figure 29), and for the 28-year-old plots, it appeared constant regardless of basal area (Figure 30), MCPA or SCPA. With only 32 observations

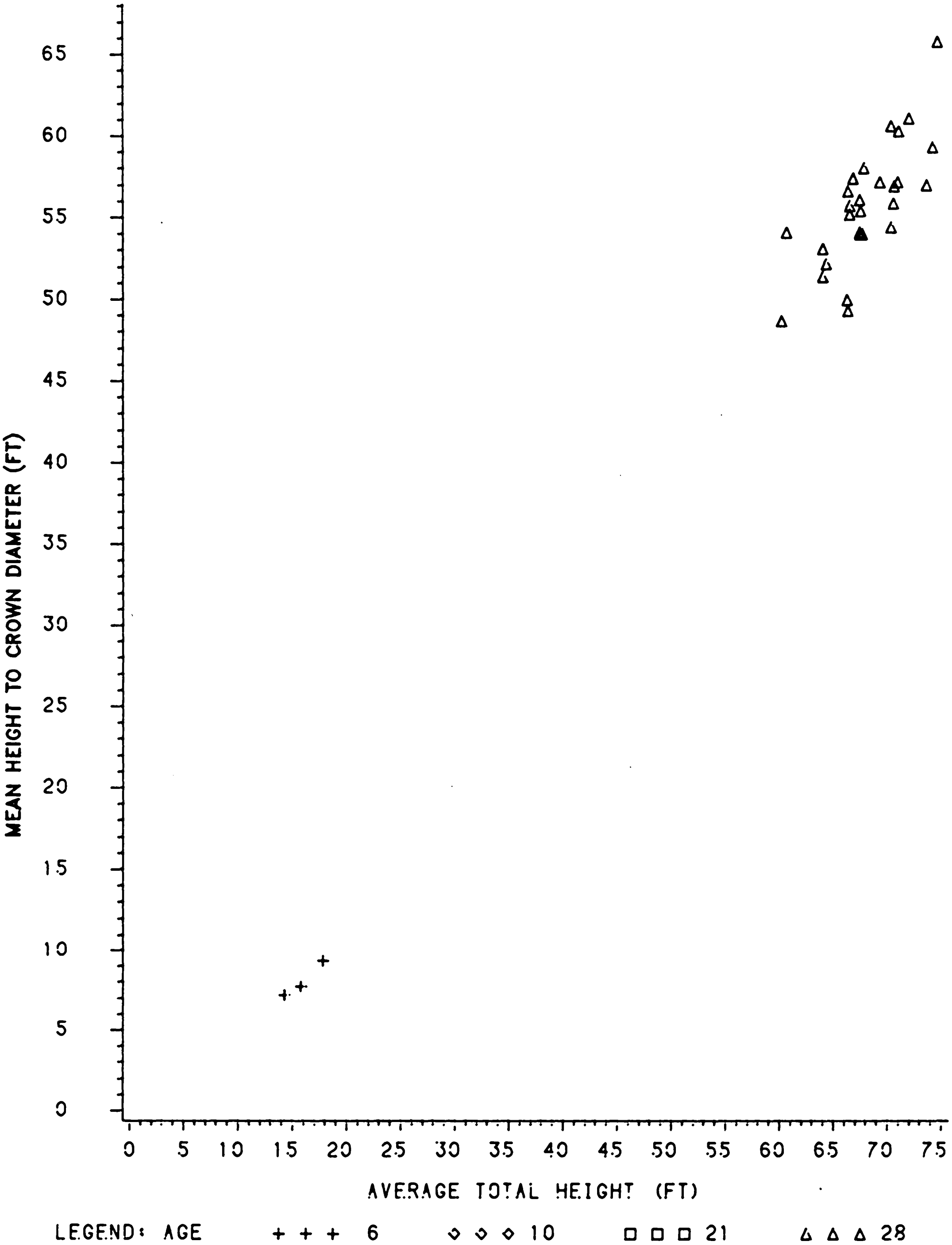


FIGURE 29: COMPARISON OF HEIGHT TO CROWN DIAMETER AND TOTAL HEIGHT OF THINNED STANDS OF PLANTED LOBLOLLY PINE.

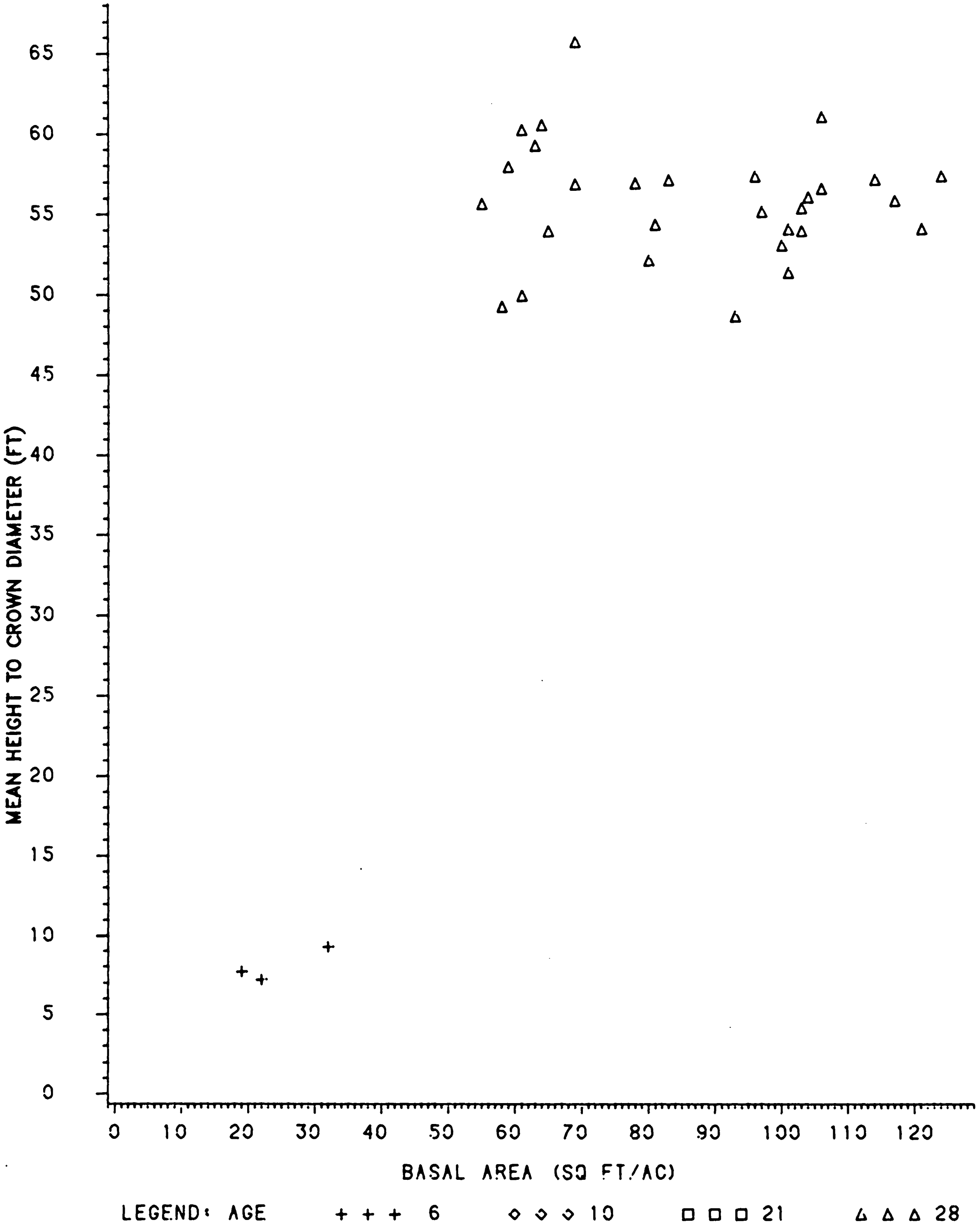


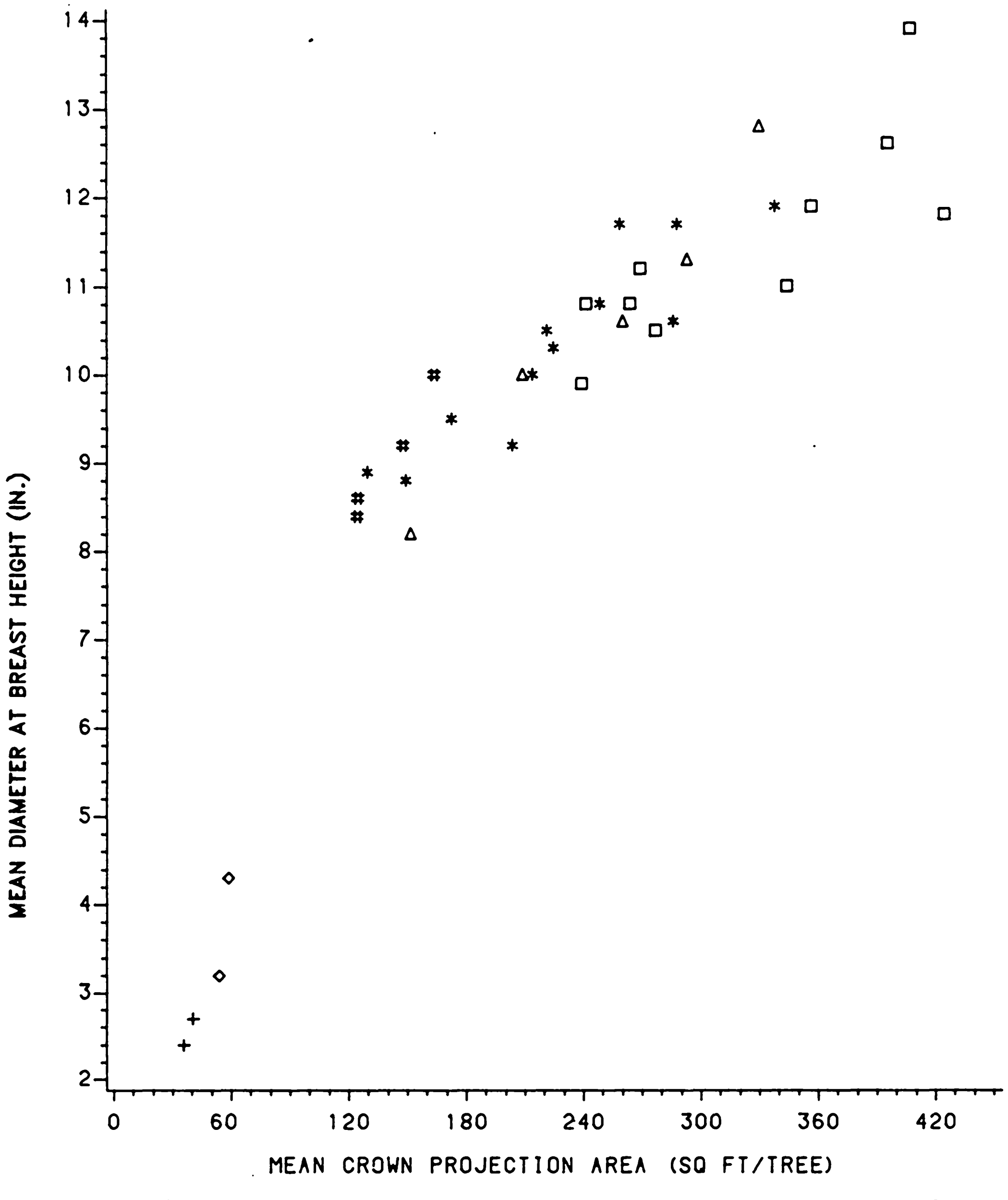
FIGURE 30: COMPARISON OF HEIGHT TO CROWN DIAMETER AND BASAL AREA OF THINNED STANDS OF PLANTED LOBLOLLY PINE.

available, representing 2 age classes, mean total height and basal area were the important variables in fitting and predicting MHCD.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
MHt	2.3943	0.97	1.0	186.31
e(MHt ln(B))	2.2657			

These results were identical with those found with unthinned stands. The recession of the effective crown was dependent on the stands average height, which was directly related to age, and the level of competition between trees, as expressed by basal area. Increases in the independent variables suggested that as the level of intraspecific competition through basal area increased, height to crown diameter would increase. Likewise, as age or total height increased, horizontal crown dimensions would increase thus causing an increase in intraspecific competition resulting in an increase in MHCD. Regardless of the reason for increased levels of intraspecific competition, MHCD for an intolerant species like loblolly will increase as the lowest branches are shaded and become part of the non-effective part of the crown.

Mean dbh. As MCPA increased, mean dbh increased; however, the rate of this development within an older age class varied with the residual basal area (Figure 31). The following set of variables was important, in terms of fit and prediction of mean dbh:



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

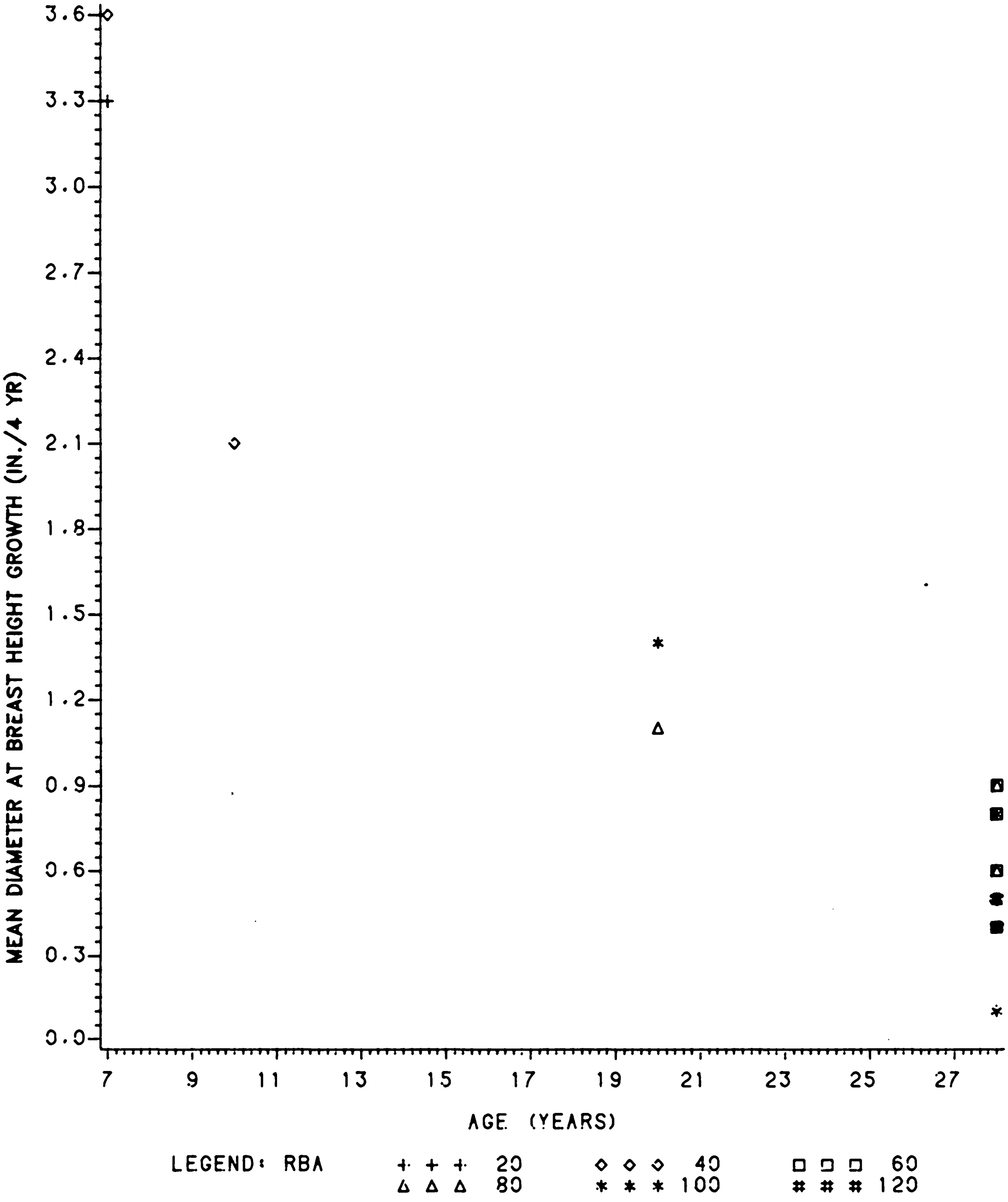
FIGURE 31: COMPARISON OF DIAMETER AT BREAST HEIGHT AND MEAN CROWN PROJECTION AREA OF THINNED STANDS OF PLANTED LOBLOLLY PINE.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
A SI ln(MCPA)	0.4393	0.97	3.8	8.50

Other stand variables, such as SCPA, number of surviving trees or basal area were not significant before or after the inclusion of MCPA. As mean CPA increased or age increased, mean dbh increased. Within the older age class, it was suggested that the rate of development between dbh and MCPA decreased with decreasing levels of residual basal area. This was understandable in that in the higher stocked stands, MCPA was physically restricted in development due to a limited amount of open space, while dbh development was not so restricted. Thus, MCPA developed at a slower rate compared to dbh under high intensities of thinning than under low intensities of thinning.

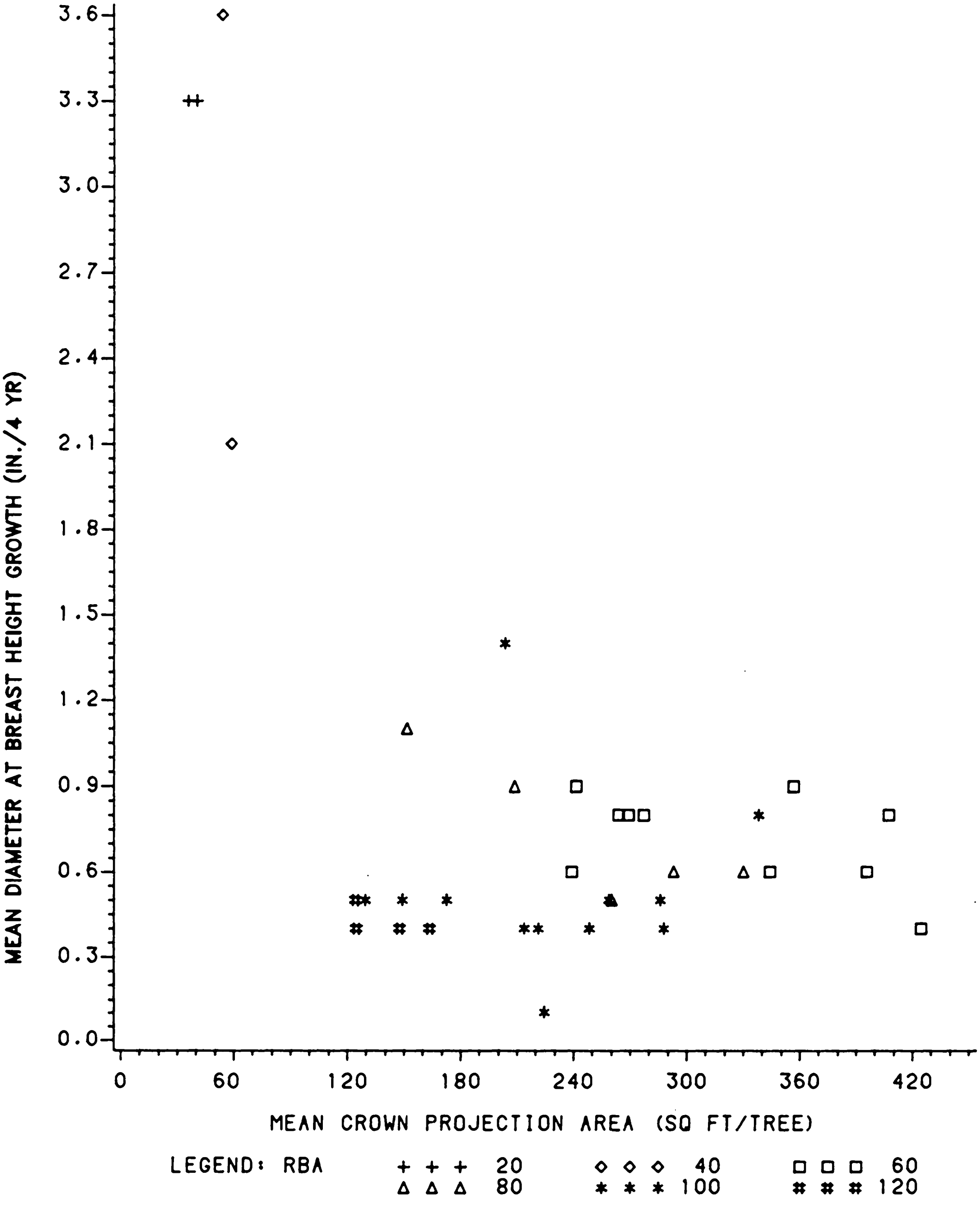
Mean dbh Growth. As age, mean CPA (Figures 32, 33), and stocking level measures increased, mean dbh growth decreased. The following sets of variables were significant in fitting and predicting dbh growth over a 4-year period:

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
1/A	0.2054	0.94	1.0	1.55
1/A SI ln(MCPA)	0.1897	0.96	3.8	1.36
1/A 1/MCPA	0.1861	0.96	9.7	1.24
1/A SI B	0.1710	0.96	3.2	1.18



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 32: COMPARISON OF DIAMETER AT BREAST HEIGHT GROWTH AND AGE OF THINNED STANDS OF PLANTED LOBLOLLY PINE.



WHERE RBA IS A RESIDUAL BASAL AREA CLASS (SQ FT/AC).

FIGURE 33: COMPARISON OF DIAMETER AT BREAST HEIGHT GROWTH AND MEAN CROWN PROJECTION AREA OF THINNED STANDS OF PLANTED LOBLOLLY PINE.

Mean dbh growth was strongly related to just age; however, the inclusion of mean CPA improved the fit and prediction of the growth model. Variables usually associated with dbh growth, such as initial dbh, stand basal area growth or mean CPA, were not significant in terms of model fit or prediction of dbh growth after stand basal area was included. Also, SCPAg was not a significant variable before or after basal area was included in the linear model containing age and site index.

4.2.3 DISCUSSION

Crown information was important in terms of increasing model fit and prediction of certain stand attributes of unthinned and thinned stands. On a stand level, as also on an individual tree level, two physiological factors associated with crown and stand development were evident in the final variable selection for stand models. The factors were photosynthate area, as described by horizontal crown measures of stand or mean CPA, and distance the photosynthate traveled, as expressed by vertical crown measures of age or mean HCD. Stem and stand dimensions were found to be related to crown dimensions, while stem and stand development was found to be related to crown development.

The stand level crown variables that were modeled can be measured from aerial photographs, excepting SCPAg. If estimation of SCPA, SCPAg, MCPA or MHCD is required, adequate models were found using a variety of stand information, such as age, site index, stand basal area, average total height, height to crown diameter and trees surviving.

The following summarizes variable combinations used in a linear model that fitted and predicted selected stand and crown characteristics better than other variable combinations (the following variables are not transformed). In some cases two sets of variables are given as stand and crown attributes can be modeled using different levels of information. One set represents information readily available from aerial photographs, while the second set requires a measurement from the ground, for example, dbh or stand basal area.

ATTRIBUTE	UNTHINNED	THINNED
B	A SI MCPA SCPA	A SI MCPA SCPA
Bg	A SCPA	A SCPAg
	A SCPA B	A SCPA B
SCPA	A B Ts	A SI B Ts
SCPAg		A SI SCPA MCPA
		SCPA B
MCPA	SI MHCD SCPA	SI MHCD
		MHCD B
	A SI Ts	A Ts
MHCD	MHt	MHt
	MHt B	MHt B

dbh	A SI MCPA	A SI MCPA
	MHCD MCPA	
dbhg	A SCPA	A MCPA
	A SCPA dbh	A SI B

The corresponding models with coefficients are given in Table A16.

The models developed for particular unthinned and thinned stand characteristics contained similar average stem and stand parameters (except in some cases where age or site index terms were not significant). One reason for the similarities may be that the majority of the thinned plots were in one age class, 28-years-old, and were apparently developing as the older unthinned stands developed. Combining unthinned and thinned plot data indicated that some stand and crown relationships were similar and could possibly be modeled together. For example, combined unthinned and thinned plot information indicated that the slopes between stand basal area and surviving trees were related to age regardless of thinning activity (Figure 34). Also, the slopes between stand basal area and stand CPA varied by age and not thinning intensity (Figure 35). Lastly, mean HCD was still linearly related to mean total height of the stand (Figure 36), and mean dbh was non-linearly related to mean CPA (Figure 37) regardless of thinning. However, as the range of data was limited for the thinned stands, actual similarities observed may be data specific.

Horizontal crown measures were an indication of present and a base for

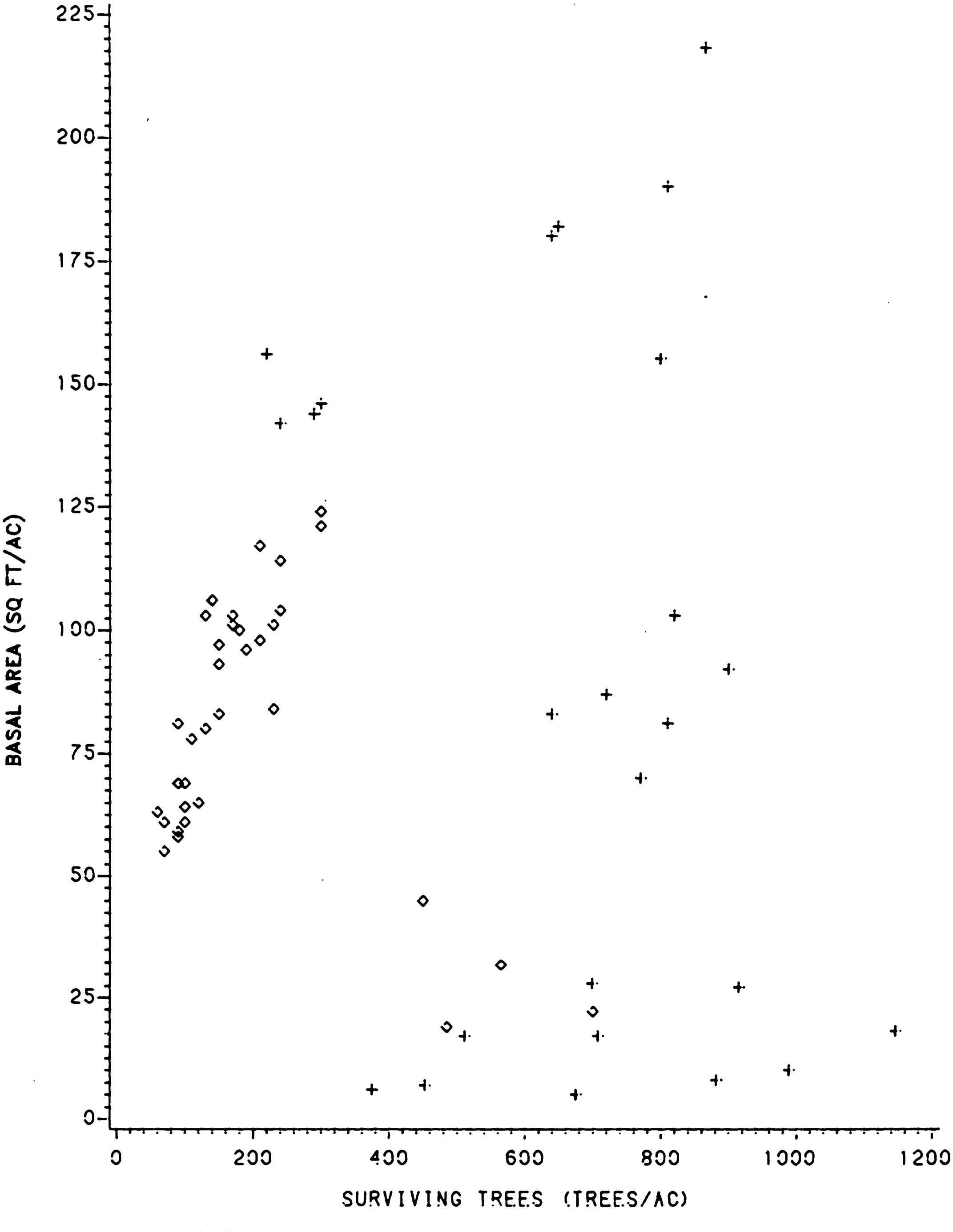


FIGURE 34: COMPARISON OF BASAL AREA AND SURVIVING TREES OF UNTHINNED AND THINNED STANDS OF PLANTED LOBLOLLY PINE.

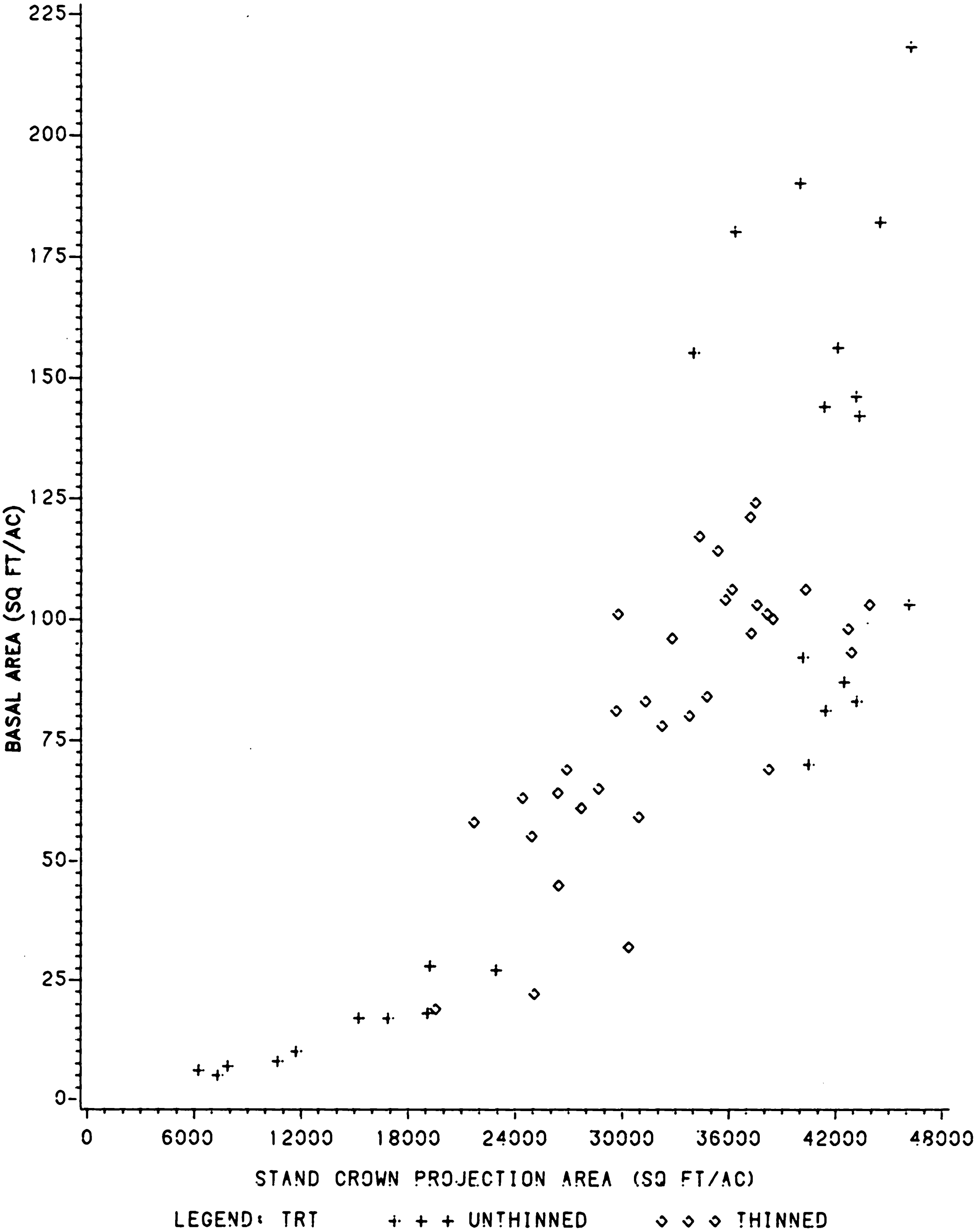


FIGURE 35: COMPARISON OF BASAL AREA AND STAND CROWN PROJECTION AREA OF UNTHINNED AND THINNED STANDS OF PLANTED LOBLOLLY PINE.

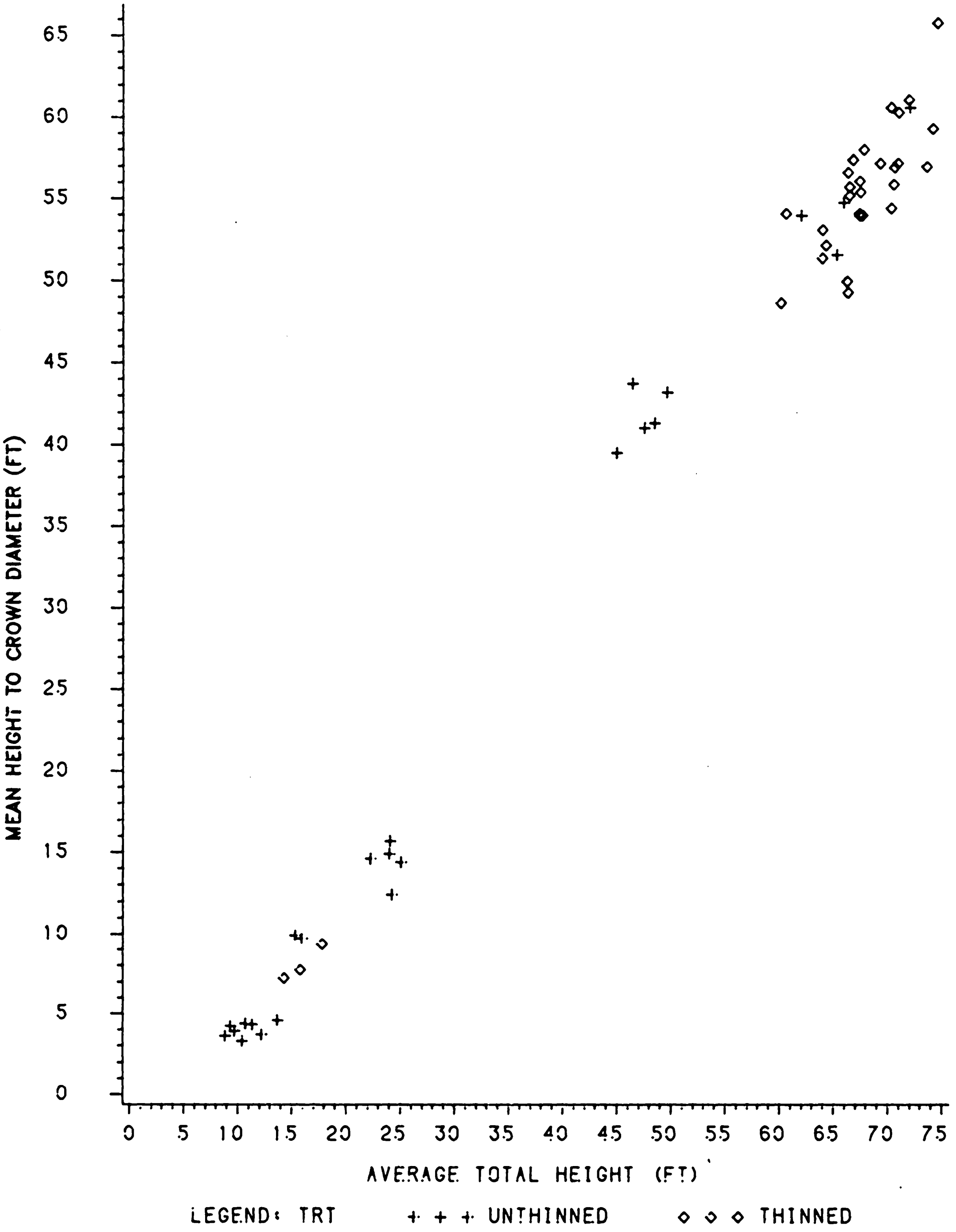


FIGURE 36: COMPARISON OF HEIGHT TO CROWN DIAMETER AND TOTAL HEIGHT OF UNTHINNED AND THINNED STANDS OF PLANTED LOBLOLLY PINE.

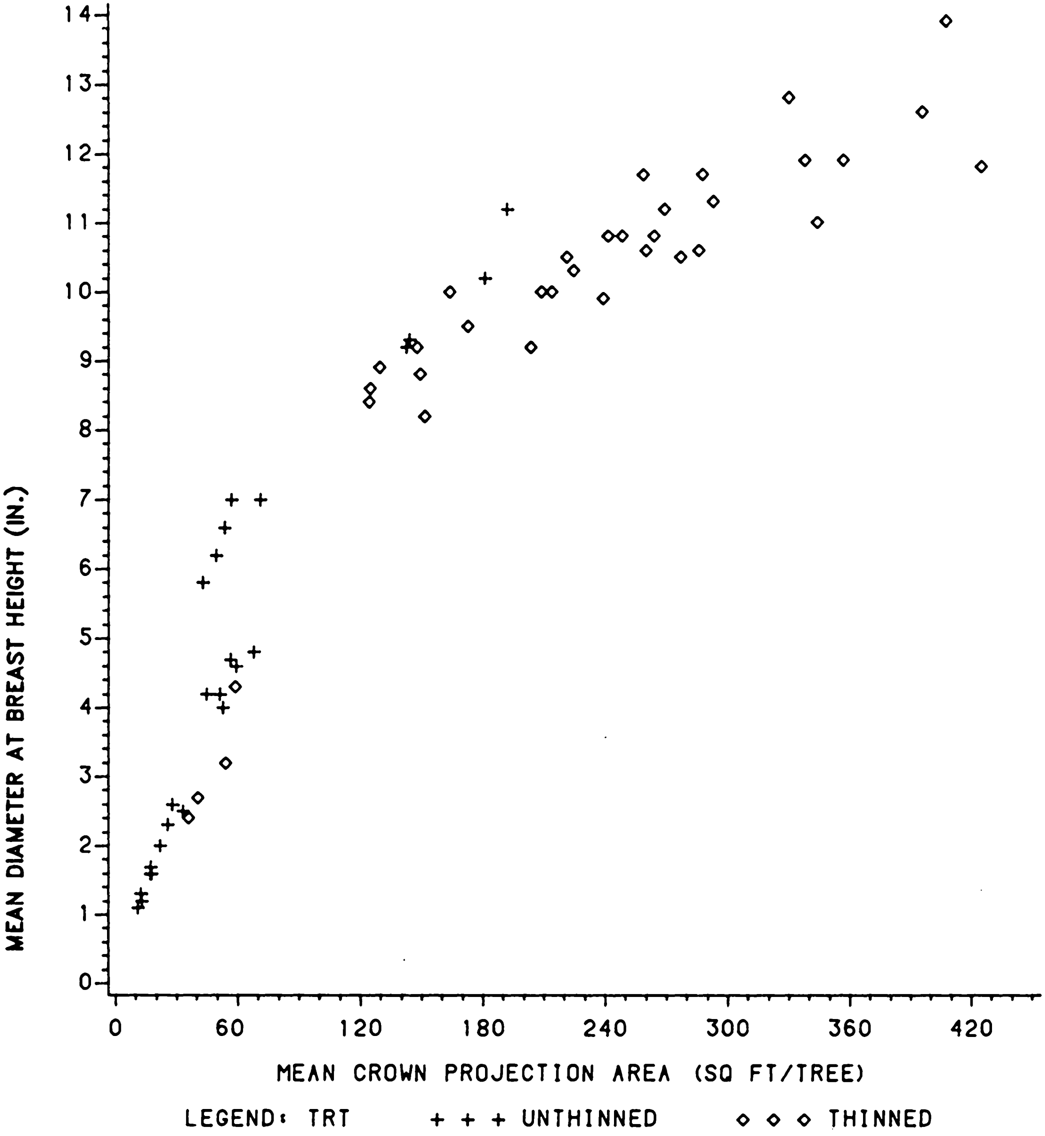


FIGURE 37: COMPARISON OF DIAMETER AT BREAST HEIGHT AND MEAN CROWN PROJECTION AREA OF UNTHINNED AND THINNED STANDS OF PLANTED LOBLOLLY PINE.

future photosynthate production. These dimensions, measured by mean or stand CPA, were important in describing all the stand attributes including stand basal area and growth, and mean dbh and growth. This was true regardless whether the stands were unthinned or thinned. A vertical measure of crown information that was also important in modeling stand characteristics was mean HCD. Mean HCD was linearly related to mean total height (and such, also indirectly related to age) and secondly to stand basal area. Of the stand attributes modeled, age was a significant variable except when modeling mean CPA (with surviving trees unknown) and mean dbh, in which cases mean HCD was significant. These variable relationships were also found on a tree level and thus explains why they probably existed on a stand level.

A strong relationship was noticed between stand basal area growth and stand CPA growth with the thinned stands. While SCPAg was not as strongly related to stand and crown characteristics as some other attributes, it did in this case adequately describe stand basal area development. As the crown basal area developed so did stand basal area.

Crown measures that were not significant in terms of improving model fit or prediction ability were total crown surface area and total crown volume. While these variables would seem to have a theoretical basis for a relationship with certain stand characteristics, their lack of significance was not understood. However, the results reported in this context on a stand level were also found at the individual tree level.

4.3 MODELING STEM DEVELOPMENT THROUGH CROWN-BASED COMPETITION INDICES

4.3.1 UNTHINNED STANDS

There were 471 trees located in unthinned stands available for selection of a stem basal area growth model and analyzing competition indices (see Table A10 for individual tree characteristics and Table A9 for plot characteristics). Various stem basal area growth models were constructed using age, site index, tree basal area and stand basal area, and transformations. A linear model containing age, site index, and natural logarithms of tree and stand basal area was selected, based on the fit and prediction criteria mentioned, for predicting basal area growth for a 4-year period for trees found in unthinned stands.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
A SI ln(b)	0.0346	0.36	3.7	0.5663
A SI ln(b) ln(B)	0.0284	0.57	6.1	0.3871
A SI ln(b) ln(B) ΔB	0.0274	0.60	8.3	0.3602

It was indicated that the addition of stand basal area improved the fit and prediction of the basal area growth model; also, the inclusion of the

4-year change in stand basal area (ΔB) improved to a lesser degree the model fit and prediction of stem basal area growth.

Using the following linear combination as a base:

$$bg = f(A, SI, \ln(b))$$

different competition indices with variable and structural adaptations, and the change of the indices over a 4-year period were then included. The results and discussion from this procedure are presented by index group.

1.) Size and/or Distances of Neighboring Trees. Competition indices in this group that were initially investigated were ratios of competing and subject tree diameters and ratios of competing and subject tree squared diameters (Daniels 1976). Both indices were weighted by the distance between competing and subject trees, and competing trees were selected using a BAF 10. These indices were significant point in time variables and helped in the model prediction of basal area growth, although not as much as stand basal area (Table 1). While the model R^2 increased with the inclusion of these indices and changes over time, the prediction ability of the models was not improved except in the model using the index composed of a ratio of squared diameters.

Table 1: Size and/or distance of neighboring trees indices based on BAF 10: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in unthinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0284	0.57	6.1	0.3871	0.0274	0.60	8.3	0.3602
C1	D,DSsc[a/]	0.0300	0.52	3.8	0.4309	0.0290	0.55	3.8	0.4571
C1	D^2,DSsc[a/]	0.0310	0.49	3.7	0.4625	0.0293	0.54	3.7	0.4168
C1	CPA,DSsc	0.0309	0.49	3.7	0.4625	0.0301	0.52	3.8	0.4585
C1	CD,DSsc	0.0301	0.52	3.8	0.4347	0.0294	0.54	3.9	0.4231
C9	D,DCsc	0.0293	0.54	3.8	0.4117	0.0290	0.55	3.8	0.4016
C9	D^2,DCsc	0.0305	0.51	3.7	0.4484	0.0288	0.56	3.7	0.3998
C9	CPA,DCsc	0.0294	0.54	3.7	0.4174	0.0285	0.57	3.8	0.3919
C9	CD,DCsc	0.0285	0.57	3.8	0.3869	0.0283	0.57	3.9	0.3831
C9	CD-H,DCsc	0.0279	0.59	3.7	0.3722	0.0275	0.60	3.8	0.3620
C8	D,DSsc,N	0.0302	0.51	3.8	0.4366	0.0292	0.55	3.9	0.4429
C8	D,DCsc,N	0.0292	0.54	3.8	0.4078	0.0287	0.56	3.8	0.3946
C8	D-H,DCsc,N	0.0283	0.57	3.7	0.3826	0.0277	0.59	3.8	0.3682

[a/] Daniels' (1976) index.

Improvement in model fit and prediction of stem basal area growth was found in most cases when the 4-year change in a competition index was included (Table 1). For example, with the point in time and change over time indices based on squared diameters included in the growth model, model R^2 increased by 0.05 and PRESS decreased by 0.0457 compared to the model containing just the point in time index. The ability of point in time and change over time index variables in helping model prediction varied with the type of variables and index structure. While model fit improved with the inclusion of the change over time index based on a ratio of diameters, the prediction ability of the model decreased. Therefore, consistent results on the use of an indices change over time in a model designed for prediction were not found.

The substitution of crown information into this index type indicated that ratios of certain crown variables provided the growth model with similar fit and prediction abilities compared to the models containing the diameter-based indices (index type C1 in Table 1). For example, the indices composed of a ratio of diameters and ratio of crown diameters provided the basal area growth model with similar R^2 and PRESS statistics when just point in time values weighted by the distances between stems were used. Also, with the addition of the change over time of an index, the indices based on ratios of squared diameters or ratios of crown diameters provided the growth model with similar R^2 and PRESS values.

The substitution of the open distance between crowns (DCsc) rather than distance between stems (DSsc) improved all the indices' contributions to model fit and prediction of stem basal area growth (index type C9 in Table 1). With this change, the point in time and change over time crown diameter based indices improved model fit and prediction by increasing R^2 by 0.03 and decreasing PRESS by 0.0400 compared to the model with the crown diameter based index using stem distances. When point in time and change over time values of an index containing a ratio of total height with crown diameter and weighted by the distance between crowns were added to the growth model, model fit and prediction abilities were similar to those indicated when stand basal area and its change over time were used.

Weighting by the number of competing trees did not help the prediction ability of the models containing the index composed of a ratio of squared diameters nor indices that used crown variables and combinations (index type C8 in Table 1). While the prediction ability of some models was marginally improved through the inclusion of a weighted index, other indices unweighted still contributed more to the fit and prediction of the basal area growth model than weighted versions.

Selection of competing trees using BAF 3, 5, 10, 20, 30 and 40 indicated that the resultant indices' contributions to model fit and prediction of stem basal area growth varied with the index form (index type C12 in Table 2). For example, when the diameter based indices using the open

Table 2: Size and/or distance of neighboring trees indices based on different BAF values: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in unthinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0284	0.57	6.1	0.3871	0.0274	0.60	8.3	0.3602
C12	D,DCsc,BAF3	0.0289	0.56	3.8	0.3981	0.0286	0.57	3.9	0.3906
C12	D,DCsc,BAF5	0.0291	0.55	3.8	0.4040	0.0288	0.56	3.8	0.3961
C12	D,DCsc,BAF10	0.0293	0.54	3.8	0.4117	0.0290	0.55	3.8	0.4016
C12	D,DCsc,BAF20	0.0286	0.57	3.7	0.3896	0.0280	0.58	3.9	0.3746
C12	D,DCsc,BAF30	0.0291	0.55	3.9	0.4035	0.0272	0.61	4.0	0.3530
C12	D,DCsc,BAF40	0.0302	0.51	4.0	0.4354	0.0275	0.60	4.0	0.3603
C12	D^2,DCsc,BAF3	0.0298	0.53	3.7	0.4265	0.0285	0.57	3.7	0.3907
C12	D^2,DCsc,BAF5	0.0301	0.52	3.7	0.4355	0.0286	0.57	3.7	0.3946
C12	D^2,DCsc,BAF10	0.0305	0.51	3.7	0.4484	0.0288	0.56	3.7	0.3998
C12	D^2,DCsc,BAF20	0.0299	0.52	3.7	0.4340	0.0285	0.57	3.8	0.3949
C12	D^2,DCsc,BAF30	0.0292	0.55	3.8	0.4079	0.0278	0.59	3.9	0.3761
C12	D^2,DCsc,BAF40	0.0305	0.51	3.9	0.4468	0.0283	0.57	4.0	0.3917
C12	CD,DCsc,BAF3	0.0279	0.59	3.9	0.3714	0.0278	0.59	3.9	0.3706
C12	CD,DCsc,BAF5	0.0381	0.58	3.8	0.3771	0.0280	0.58	3.9	0.3761
C12	CD,DCsc,BAF10	0.0285	0.57	3.8	0.3869	0.0283	0.57	3.9	0.3831
C12	CD,DCsc,BAF20	0.0281	0.58	3.8	0.3766	0.0277	0.59	4.0	0.3670
C12	CD,DCsc,BAF30	0.0292	0.55	3.9	0.4066	0.0274	0.60	4.1	0.3582
C12	CD,DCsc,BAF40	0.0305	0.51	4.1	0.4413	0.0278	0.59	4.1	0.3706
C12	CD-H,DCsc,BAF3	0.0271	0.61	3.8	0.3525	0.0269	0.61	3.8	0.3475
C12	CD-H,DCsc,BAF5	0.0274	0.60	3.8	0.3598	0.0272	0.61	3.8	0.3538
C12	CD-H,DCsc,BAF10	0.0279	0.59	3.7	0.3722	0.0275	0.60	3.8	0.3620
C12	CD-H,DCsc,BAF20	0.0278	0.59	3.8	0.3701	0.0272	0.61	3.9	0.3541
C12	CD-H,DCsc,BAF30	0.0289	0.55	3.9	0.3993	0.0269	0.61	4.0	0.3479
C12	CD-H,DCsc,BAF40	0.0303	0.51	4.0	0.4390	0.0276	0.60	4.1	0.3648

distance between crown edges and a BAF 30 (i.e., few trees selected as competitors) were included in the growth model, the resultant models had the highest model R^2 and lowest PRESS compared to other BAF values. This was true for models containing point in time and change over time index values and just point in time index values. There was one exception when the index based on a ratio of diameters and a BAF 20 produced a model with better fit and prediction ability than the model containing the same index based on a BAF 30.

A range of BAF values used with similarly constructed indices based on crown diameter and crown diameter with total height gave slightly different results than above. The growth models containing just point in time measures that had high R^2 and low PRESS statistics were generally those that contained these crown based indices using a BAF 3. By including point in time and change over time index values in the growth model, the best models for prediction were those containing the crown diameter based index using a BAF 3, and the crown diameter with total height based index using either a BAF 3 or 30. This crown diameter and total height based index was the best index, in terms of increasing model fit and prediction abilities, of this group regardless of BAF compared to indices with other variables and structural changes.

2.) Distance Independent. Two indices were used for comparison within this type of index: Glover and Hool's (1979) index, which compared the basal area of a subject tree to the tree of average dbh squared, and Daniels (1981) index, which compared the basal area of a subject tree to the tree of average basal area. While both of these indices helped in the model fit and prediction of stem basal area growth, Daniels' index, when included in the growth model as a point in time and change over time measure increased R^2 by 0.15 and decreased PRESS by 0.1355 compared to the model containing stand basal area and its change over time (Table 3). The best index with crown information incorporated was one containing crown diameter in the form of Daniels' index (index type C3 in Table 3). It was indicated that Daniels' index as a point in time and change over time value helped fit and predict stem basal area growth better than similar indices with crown variables. However, the crown diameter based index as just a point in time measure, when included in the growth model, resulted in a higher model R^2 and lower PRESS than the model containing Daniels's point in time index.

3.) Available Growing Space. When Moore's (1973) index as a point in time and change over time measure of competition was included in the growth model, model prediction of basal area growth was similar to the prediction ability of the model containing stand basal area and its change over time (Table 4).

Using the same structural form as Moore's, the best crown based index for helping model fit and prediction of basal area growth involved crown

Table 3: Distance independent indices: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in unthinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0284	0.57	6.1	0.3871	0.0274	0.60	8.3	0.3602
C2	$Dc^2/\overline{D}^2$ a/	0.0294	0.54	4.0	0.4180	0.0246	0.68	4.0	0.2926
C3	$Dc^2/\overline{D^2}$ b/	0.0285	0.57	4.0	0.3940	0.0215	0.75	4.1	0.2247
C3	$CDc^2/\overline{CD^2}$	0.0283	0.57	3.8	0.3841	0.0265	0.63	3.8	0.3401

a/ Glover and Hool's (1979) index.

b/ Daniels' (1981) index.

Table 4: Available growing space indices: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in unthinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0284	0.57	6.1	0.3871	0.0274	0.60	8.3	0.3602
C4	D^2,DSsc [a]	0.0293	0.54	3.7	0.4107	0.0274	0.60	3.9	0.3676
C4	CPA^2,DSsc	0.0295	0.54	3.7	0.4156	0.0290	0.55	3.7	0.4032
C8	D^2,DSsc,N	0.0295	0.54	3.7	0.4155	0.0271	0.61	3.8	0.3553
C8	CD^2,DSsc,N	0.0291	0.55	3.8	0.4030	0.0281	0.58	3.8	0.3794
C10	D^2,DCsc	0.0299	0.52	3.9	0.4262	0.0287	0.56	3.9	0.3967
C10	CPA^2,DCsc	0.0302	0.51	3.8	0.4343	0.0291	0.55	4.4	0.4055
C10	CV^2,DCsc	0.0298	0.53	3.9	0.4226	0.0288	0.56	4.4	0.3991
C11	D^2,DCsc,10'	0.0297	0.53	3.9	0.4210	0.0287	0.56	4.6	0.3953
C11	CPA^2,DCsc,10'	0.0300	0.52	3.8	0.4303	0.0291	0.55	4.4	0.4047
C11	CV^2,DCsc,10'	0.0296	0.53	4.0	0.4192	0.0289	0.56	4.3	0.4012
C11	CV^2,DCsc,N,10'	0.0296	0.53	4.4	0.4174	0.0287	0.56	4.5	0.3956

[a] Moore *et al.*'s (1973) index.

projection area squared as the means for dividing the lines (index type C4 in Table 4). However, this crown based index was not as good as Moore's index in terms of fitting and predicting stem basal area growth when included in the growth model as point in time and change over time values. The models containing point in time estimates of this index type and Moore's were similar in fit and prediction abilities.

In general, weighting these types of indices by the number of competitors increased the model fit and prediction abilities of stem basal area growth, but not in all cases (index type C8 in Table 4). For example, Moore's index and similar indices containing CPA squared and CD squared were improved when weighted by the number of competing trees. The increases, however, in the level of fit and prediction ability of the models containing these weighted indices were fairly small.

Mixed results were found when using the distance between crowns as the line length to divide for determination of area available (index type C10 in Table 4). For example, Moore's index with this change did not result in increasing model fit and prediction abilities; however, there were other indices based on this distance causing model increases in those abilities. In general, if there were increases in the fit and prediction levels of models containing these indices based on distances between crowns, models using weighted versions of these indices provided only marginal results.

The last structural change was to eliminate acute areas not readily available for crown expansion (index type C11 in Table 4). Besides not removing the acute areas, maximum distances of 5,10, 15 and 20 feet beyond the crown edge of the subject tree were adaptations made on this index type. With the indices investigated there were marginal benefits in model fit and prediction of basal area growth in restricting the distances beyond the crown edge to 10 feet. This distance resulted in a slightly better level of prediction ability for the growth model than the models containing a similar index based on no restriction or other maximums. While there were slightly better model fit and prediction abilities with this change incorporated with indices using dbh squared, CPA squared and CV squared, the increases were marginal. Also, there were no improvements in model fit and prediction abilities when these types of indices were weighted by the number of competing trees.

4.) Overlap. Ek and Monserud's (1974) index was computed with the ratio of tree sizes based on crown radius and total height as originally published. With a point in time measure or point in time and change over time measures added to the basic growth model, model fit and prediction abilities were not as high compared to the model containing just stand basal area (Table 5).

Crown variables did not produce an index that helped the growth model in terms of fit and prediction of stem basal area growth better than using

Table 5: Overlap indices: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in unthinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0284	0.57	6.1	0.3871	0.0274	0.60	8.3	0.3602
C5	CR-H [a]	0.0310	0.49	3.7	0.4583	0.0304	0.51	4.9	0.4432
C5	CD-H	0.0292	0.55	3.7	0.4068	0.0289	0.56	4.6	0.3997
C5	D-H	0.0287	0.56	3.7	0.3927	0.0280	0.58	4.7	0.3758
C5	D^2-H	0.0291	0.55	3.8	0.4076	0.0275	0.60	4.1	0.3705
C8	CD-H,N	0.0288	0.56	3.9	0.3968	0.0285	0.56	4.5	0.3904
C8	D-H,N	0.0284	0.57	3.8	0.3849	0.0277	0.59	4.5	0.3658

[a] Ek and Monserud's (1974) index.

dbh or dbh^2 (index type C5 in Table 5). For a point in time measure of competition, the index using dbh and total height produced a model with a high R^2 and low PRESS. The best combination of point in time and change over time index values was an index using dbh squared and total height. However, neither of the resultant models based on these indices did better in fitting and predicting basal area growth than the model containing stand basal area.

Weighting this index type by the number of trees that overlapped with the subject tree's influence zone produced marginal results (index type C8 in Table 5). While weighted indices using dbh squared or crown variables did not sufficiently improve model prediction ability, the weighted indices using dbh and total height noticeably improved the growth model R^2 and PRESS.

5.) Miscellaneous. Other types of indices that were calculated and used in the stem basal area growth model were adaptations of the previous forms. Ratios of tree to average stand characteristics were calculated using crown, stem and stand information. These weighted versions provided the growth model with reasonable fit and prediction abilities (Table 6). When the model contained this index form using a ratio of subject tree crown diameter squared to the tree of average CPA as point in time and change over time variables, it indicated an increase in R^2 of 0.02 and decrease in PRESS of 0.0138 compared to the model containing just a point

Table 6: Miscellaneous indices - weighted versions of distance independent indices: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in unthinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0284	0.57	6.1	0.3871	0.0274	0.60	8.3	0.3602
C6	$CSA^2, \overline{SCSA^2}$	0.0284	0.57	3.7	0.3849	0.0277	0.59	3.8	0.3685
C6	$D^2, \overline{B^2}$	0.0283	0.57	3.8	0.3841	0.0270	0.61	3.9	0.3530
C6	$CPA, \overline{B^2}$	0.0267	0.62	3.8	0.3409	0.0250	0.67	8.7	0.3017
C6	$D^2, \overline{SCPA}$	0.0307	0.50	5.3	0.4500	0.0241	0.69	7.0	0.2729
C6	$CD^2, \overline{SCPA}$	0.0264	0.63	3.7	0.3337	0.0258	0.65	3.7	0.3199

in time version of the same index. A similarly constructed index with dbh instead of crown diameter resulted in a further model increase in R^2 of 0.04 and decrease in PRESS of 0.0470 when point in time and change over time values were included compared to the above model with point in time and change over time values.

Another type of index investigated was partial stand values of basal area and crown projection area. Using different BAF values for selection of competing trees, this index type represented the sum of their basal area or CPA. For comparison, the resultant model characteristics using the stand values of basal area and crown projection area are given in Table 7. The results indicated that with the logarithm of the sum of dbh, as the BAF values increased (i.e., selecting fewer trees as competitors) the resultant models with point in time or point in time and change over time index values increased in fit and prediction abilities. Within the range of BAF values tested, the model containing stand basal area still had a higher model R^2 and lower PRESS than any of the models with a subsample of tree basal areas.

Similar results were found with models containing subsampled stand CPA information. In this case, the indices based on low BAF values (i.e., more tree selected as competitors) provided the growth model with high fit and prediction abilities; in addition, the model with the whole stand CPA variable had a higher R^2 and lower PRESS than any of the subsampled ver-

Table 7: Miscellaneous indices - partial measures of stand attributes using different BAF values: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in unthinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0284	0.57	6.1	0.3871	0.0274	0.60	8.3	0.3602
BASE	SCPA	0.0284	0.57	5.8	0.3842	not significant			
C7	ΣD^2,BAF3	0.0324	0.44	4.1	0.5024	0.0320	0.45	9.2	0.4909
C7	ΣD^2,BAF5	0.0332	0.41	3.8	0.5229	0.0327	0.43	8.3	0.5083
C7	ΣD^2,BAF10	0.0328	0.42	4.0	0.5106	0.0320	0.46	5.7	0.4877
C7	ΣD^2,BAF20	0.0310	0.49	4.1	0.4574	0.0308	0.50	4.8	0.4538
C7	ΣD^2,BAF30	0.0305	0.50	4.3	0.4438	0.0303	0.51	4.3	0.4403
C7	ΣCPA,BAF3	0.0313	0.48	4.9	0.4669	not significant			
C7	ΣCPA,BAF5	0.0321	0.45	6.5	0.4916	not significant			
C7	ΣCPA,BAF10	0.0329	0.42	7.6	0.5143	not significant			
C7	ΣCPA,BAF20	0.0329	0.42	7.9	0.5148	0.0328	0.43	7.9	0.5142
C7	ΣCPA,BAF30	0.0333	0.41	8.1	0.5269	0.0331	0.42	8.3	0.5214
Base	SCPA,CD	0.0254	0.66	7.6	0.3116	not applicable			
Base	SCPA,CD,CDg	not applicable				0.0247	0.68	11.7	0.2952

sions. It is worthwhile to note that this last model containing just stand CPA as a point in time competition index resulted in a similar R^2 and decrease of 0.0029 in PRESS compared to the model with stand basal area.

While the above indices attempted to measure the competitive stress surrounding a subject tree, the effects of stress can also be measured by analyzing the stem and stand variables associated with a subject tree. In this case, stand variables, such as basal area and SCPA, and stem variables, such as CD and CPA, were included in the basic growth model. With point in time information of stand SCPA and subject tree crown diameter, the resultant growth model indicated a higher R^2 and lower PRESS than the model containing just stand basal area (Table 7). Combining point in time and change over time measures of stand CPA, and subject tree CD and CDg in the growth model, increased model R^2 by 0.02 and decreased PRESS by 0.0164 compared to the model containing SCPA and CD.

4.3.1.1 Discussion

Comparison of all the index groups, and variable and structural changes indicated that the distance independent types, when included as point in time, and point in time and change over time variables in the basic stem basal area growth model, resulted in higher fit and prediction abilities than the models containing any other form of indices. The model with the best fit and prediction abilities was found when just stand CPA and subject tree crown diameter were included in the growth model. For comparison of models containing point in time indices that provided growth models with better fit and prediction abilities than the model containing stand basal area, the following summary for trees located in unthinned stands is given:

Point In Time Measures

INDEX	CHANGES	$S_{y.x}$	R^2	VIF	PRESS
Base	B	0.0284	0.57	6.1	0.3871
Base	SCPA CD	0.0254	0.66	7.6	0.3116
C6	CD^2,$\overline{CPA}$	0.0264	0.63	3.7	0.3337
C6	CPA,$\overline{B}^2$	0.0267	0.62	3.8	0.3409
C12	CD-H,DCsc,BAF3	0.0271	0.61	3.8	0.3525
Base	SCPA	0.0284	0.57	5.8	0.3842

If point in time measurements and change over time estimates are available, it was indicated that Daniels' (1981) index of a ratio of dbh squared to the tree of average basal area provided the growth model with the highest R^2 of 0.75 and lowest PRESS of 0.2247. Predicting this change over time index is some what analogous to predicting stem basal area growth since stem basal area is in the numerator of the index. The best set of competition indices without this problem were stand CPA and subject tree CD and CDg. Other comparable indices were ratios of subject tree CPA to the tree of average basal area squared or ratios of subject tree crown diameter squared to the tree of average CPA. A summary of point in time and change over time competition measures that provided the growth model with better fit and prediction abilities than basal area is given below.

PIT & Change Over Time Measures

INDEX	CHANGES	$S_{y.x}$	R^2	VIF	PRESS
Base	B,ΔB	0.0274	0.60	8.3	0.3602
C3	$D^2/\overline{D^2}$	0.0215	0.75	4.1	0.2247
C6	$D^2,\overline{SCPA}$	0.0241	0.69	7.0	0.2729
C2	$D^2/\bar{D}^2$	0.0246	0.68	4.0	0.2926
Base	SCPA,CD,CDg	0.0247	0.68	11.7	0.2952
C6	$CPA,\overline{B^2}$	0.0250	0.67	8.7	0.3017
C6	$CD^2,\overline{SCPA}$	0.0258	0.65	3.7	0.3199
C2	$CD^2/\overline{CD^2}$	0.0265	0.63	3.8	0.3401
C12	CD,H,DCsc,BAF3	0.0269	0.61	3.8	0.3475
C12	CD,H,DCsc,BAF30	0.0269	0.61	4.0	0.3479

Mixed results in improving the fit and prediction of a stem basal area growth model by the inclusion of various measures of competition were found. Indices containing crown information did not always improve the growth model's fit and prediction abilities; however, of the indices that provided the model with the highest levels of fit and prediction, crown information was generally involved. Crown diameter or crown projection area and stand CPA were the important crown variables in the final analysis.

Depending on the index group, weighting, open distance between crowns, selection of competing trees through different BAF values and other structural changes did not make large differences in an indices' contribution to model prediction of stem basal area growth. In almost all cases, however, the inclusion of an indices change over time measure of competition did improve model fit and prediction of stem basal area growth.

In general the growth models containing a point in time and change over time measure of competition had lower standard error of estimates and PRESS values than models containing just point in time measures. The reason why some change over time indices did better than others is that the estimation of the change of some indices over time is almost analogous to the estimation of the dependent variable. For example, Daniels' (1981) index has basal area of the subject tree in the numerator; by using this measure as a change over time variable, there is an implicit estimate of the dependent variable, stem basal area growth, being made as an independent variable. While there are better fit and prediction statistics associated with models containing these type of point in time and change over time measures of competition, prediction of the change over time measure of competition may be difficult as predicting stem basal area growth.

4.3.2 THINNED STANDS

There were 202 trees available from thinned stands for selection of a stem basal area growth model and analysis of competition indices (see Table A8 for individual tree characteristics and Table A7 for plot characteristics). A stem basal area growth model for a 4-year period was selected using a linear model containing variables and transformations of age, site index, and tree and stand basal area. The criteria for variable and model selection was based on the fit and prediction criteria mentioned. A different basal area growth model was required for trees found in thinned stands than unthinned stands due to the lack of variable significance of stand basal area. The base model selected for determining stem basal area growth of trees located in thinned stands was a linear function of site index over age, and logarithms of tree and stand basal area.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
SI/A ln(b)	0.0293	0.78	9.3	0.1762
SI/A ln(b) ln(B)	0.0286	0.80	14.6	0.1686

In this model form, stand basal area was a significant variable, variance inflation factors were relatively high and the change in stand basal area for a 4-year period was not a significant variable, nor significant in terms of prediction ability of stem basal area growth. A possible reason for the lack of significance was that in the previous section on modeling whole stand attributes, it was indicated stem basal area growth was fairly constant in thinned stands regardless of the residual stand basal area.

1.) Size and/or Distance of Neighboring Trees. The indices containing ratios of diameters or squared diameters (Daniels 1976) were not significant point in time variables, nor did they help in the model prediction of stem basal area growth (Table 8). Only when point in time and change over time indices were included in the growth model were these variables significant and predictions improved. Of the published forms, the index using dbh squared, distance between stems and a BAF 10 for competing tree selection was the best index of this type in aiding model prediction of stem basal area growth compared to indices with other variables and structural changes. This index and its change over time provided as good of a model fit and prediction ability as the model containing just stand basal area.

Using similarly structured indices as above, but with crown information incorporated, it was indicated that point in time index significance and resultant increases in model prediction ability were not always evident (index type C1 in Table 8). Indices with ratios of crown projection area or crown diameter with competing tree selection based on a BAF 10, did provide adequate model fit and prediction ability when point in time

Table 8: Size and/or distance of neighboring trees indices based on BAF 10: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in thinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0286	0.80	14.6	0.1686	not significant			
C1	D,DSsc [a]	not significant				0.0286	0.80	13.1	0.1703
C1	D^2,DSsc [a]	not significant				0.0282	0.80	14.6	0.1659
C1	CPA,DSsc	not significant				0.0287	0.79	13.0	0.1736
C1	CD,DSsc	not significant				0.0288	0.79	12.7	0.1729
C9	D,DCsc	0.0292	0.79	9.8	0.1748	not significant			
C9	D^2,DCsc	0.0291	0.79	10.8	0.1738	0.0287	0.80	11.8	0.1699
C9	D-HCW,DCsc	0.0290	0.79	9.9	0.1726	not significant			
C9	CPA,DCsc	0.0292	0.79	10.9	0.1748	0.0289	0.79	11.7	0.1726
C9	CD,DCsc	0.0292	0.79	10.0	0.1749	0.0291	0.79	12.1	0.1720
C9	CPA-H,DCsc	0.0292	0.79	11.4	0.1755	0.0288	0.79	12.1	0.1720
C9	CD-H,DCsc	0.0291	0.79	10.6	0.1739	0.0290	0.79	11.0	0.1733
C8	D,DSsc,N	not significant				0.0290	0.79	12.1	0.1749
C8	D,DCsc,N	0.0292	0.79	10.7	0.1755	not significant			

[a] Daniels' (1976) index.

and change over time values were used.

Structural changes in the indices did make a difference in variable significance and prediction ability of some growth models. By substituting the open distance between crown edges rather than stems in the indices mentioned, point in time measures of competition were significant variables in the growth model (index type C9 in Table 8). However, in most cases this structural change did not increase an indices' contribution to model fit or prediction when its point in time and change over time value were used.

Weighting by the number of competitors was ineffective as an aid to increase an indices' ability to help the growth model predict stem basal area growth. For the indices already mentioned, weighted versions were not significant variables, nor did they help in model prediction of stem basal area growth (index type C8 in Table 8).

The choice of a BAF value for competing tree selection was not as critical with trees found in thinned stands as those in unthinned stands. Five indices based on dbh, dbh squared, crown diameter and crown diameter with total height weighted by the distance between crown edges, plus dbh squared weighted by the distance between stems were calculated using BAF 3, 5, 10, 20, 30 and 40. In general, the results indicated that regardless of index type and BAF value, model fit and prediction of stem basal area growth remained similar (index type C12 in Table 9). Combinations of variables and BAF values existed, however, such that certain indices were not significant variables in the model.

Some marginal improvements were made in model prediction with certain BAF values. The model containing the point in time and change over time index using dbh squared, distance between stems (DSsc) and BAF 5 had the highest R^2 and lowest PRESS compared to models containing other indices from this group. Even with the ability to change the number of competing trees used in the calculation of these type of indices, the base model with stand basal area fitted and predicted basal area growth almost as well.

2.) Distance Independent. Indices in their original form as suggested by Glover and Hool (1979) and Daniels (1981) helped in model prediction of stem basal area growth (Table 10). For these two types, the inclusion of point in time and change over time measures of competition were not significant variables in the growth model. However, by just adding the change over time value of Daniels' (1981) index (i.e., ratio of subject tree dbh squared compared to the tree of average basal area) to the growth model helped fit and predict stem basal area growth better than the model with just stand basal area. The only similarly structured index with crown information incorporated that was of benefit to the prediction ability of the growth model, but still not as good as Daniels', was one using a

Table 9: Size and/or distance of neighboring trees indices based on different BAF values: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in thinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0286	0.80	14.6	0.1686	not significant			
C12	D,DCsc,BAF3	0.0292	0.79	9.5	0.1754	not significant			
C12	D,DCsc,BAF5	0.0292	0.79	9.5	0.1749	not significant			
C12	D,DCsc,BAF10	0.0292	0.79	9.8	0.1748	not significant			
C12	D,DCsc,BAF20	0.0292	0.79	11.9	0.1756	0.0288	0.79	12.0	0.1712
C12	D,DCsc,BAF30	0.0290	0.79	11.6	0.1734	0.0289	0.79	11.6	0.1723
C12	D,DCsc,BAF40	0.0290	0.79	11.1	0.1742	not significant			
C12	D^2,DCsc,BAF3	0.0290	0.79	10.7	0.1734	0.0286	0.80	11.8	0.1692
C12	D^2,DCsc,BAF5	0.0290	0.79	10.5	0.1731	0.0286	0.80	11.8	0.1695
C12	D^2,DCsc,BAF10	0.0291	0.79	10.8	0.1738	0.0287	0.80	11.8	0.1699
C12	D^2,DCsc,BAF20	0.0292	0.79	12.9	0.1751	0.0283	0.80	13.1	0.1660
C12	D^2,DCsc,BAF30	0.0290	0.79	12.2	0.1737	0.0287	0.80	12.3	0.1701
C12	D^2,DCsc,BAF40	0.0291	0.79	11.7	0.1740	not significant			
C12	D^2,DSsc,BAF3	0.0294	0.78	10.6	0.1734	0.0292	0.79	10.8	0.1692
C12	D^2,DSsc,BAF5	0.0291	0.79	12.8	0.1738	0.0281	0.80	14.4	0.1629
C12	D^2,DSsc,BAF10	0.0292	0.79	12.8	0.1756	0.0282	0.80	14.6	0.1659
C12	D^2,DSsc,BAF20	not significant				0.0282	0.80	14.0	0.1651
C12	D^2,DSsc,BAF30	not significant				not significant			
C12	D^2,DSsc,BAF40	not significant				not significant			
C12	CD,DCsc,BAF3	0.0292	0.79	9.5	0.1751	not significant			
C12	CD,DCsc,BAF5	0.0292	0.79	9.5	0.1747	not significant			
C12	CD,DCsc,BAF10	0.0292	0.79	10.0	0.1749	0.0291	0.79	10.4	0.1751
C12	CD,DCsc,BAF20	0.0292	0.79	12.2	0.1759	0.0288	0.79	12.2	0.1715
C12	CD,DCsc,BAF30	0.0291	0.79	11.8	0.1739	0.0289	0.79	11.8	0.1728
C12	CD,DCsc,BAF40	0.0292	0.79	11.2	0.1749	not significant			
C12	CD-H,DCsc,BAF3	0.0290	0.79	10.2	0.1733	not significant			
C12	CD-H,DCsc,BAF5	0.0290	0.79	10.1	0.1731	0.0289	0.79	10.8	0.1728
C12	CD-H,DCsc,BAF10	0.0291	0.79	10.6	0.1739	0.0290	0.79	11.0	0.1733
C12	CD-H,DCsc,BAF20	0.0292	0.79	12.6	0.1750	0.0286	0.80	12.6	0.1690
C12	CD-H,DCsc,BAF30	0.0290	0.79	11.9	0.1733	0.0288	0.79	12.0	0.1718
C12	CD-H,DCsc,BAF40	0.0291	0.79	11.5	0.1746	not significant			

Table 10: Distance independent indices: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in thinned stands of planted loblolly pine.

INDEX	VARIABLES	Point In Time				PIT & Change Over Time			
		$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0286	0.80	14.6	0.1686	not significant			
		Point In Time				JUST Change Over Time			
C2	$Dc^2/\bar{D}^2$ a/	0.0289	0.79	17.9	0.1712	0.0276	0.81	10.8	0.1551
C3	$Dc^2/\overline{D^2}$ b/	0.0289	0.79	18.5	0.1711	0.0254	0.84	11.4	0.1314
		Point In Time				PIT & Change Over Time			
C3	$CDc^2/\overline{CD^2}$	0.0290	0.79	12.2	0.1727	0.0277	0.81	12.8	0.1590

a/ Glover and Hool's (1979) index.

b/ Daniels' (1981) index.

ratio of the subject tree's crown diameter squared to the tree of average crown diameter squared (index type C3 in Table 10).

3.) Available Growing Space. Moore's et al. (1973) original index and change in index over time helped model prediction of stem basal area growth better than models using similarly constructed indices with dbh or crown variables. If only the above indices at one point in time were used, there was no discernible increase in model prediction ability over using just stand basal area. However, when Moore's index at a point in time and change over time was included in the growth model, the resultant model fitted and predicted basal area growth better than the model containing stand basal area (Table 11). Crown projection area, when incorporated in a similarly structured index as Moore's, resulted in the best crown based index; however, that index and change over time did not improve the model fit and prediction abilities compared to the model containing Moore's indices (index type C4 in Table 11).

Weighting by the number of competitors generally helped in the fit and and prediction of the stem basal area growth model except in the form Moore originally developed. There was marginal benefit in prediction of models containing indices using dbh, CPA and CPA squared: the benefits were such that Moore's index unweighted still provided a higher model R^2 and lower PRESS than the above weighted versions (index type C8 in Table 11).

Table 11: Available growing space indices: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in thinned stands of planted loblolly pine.

INDEX	VARIABLES	Point In Time				PIT & Change Over Time			
		$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0286	0.80	14.6	0.1686	not significant			
C4	D^2,DSsc [a]	0.0291	0.79	14.2	0.1750	0.0267	0.82	14.3	0.1501
C4	D,DSsc	0.0292	0.79	13.1	0.1756	0.0280	0.80	13.2	0.1646
C4	CPA^2,DSsc	0.0291	0.79	13.3	0.1752	0.0280	0.80	13.5	0.1648
C8	D^2,DSsc,N	0.0291	0.79	14.7	0.1743	0.0271	0.82	15.2	0.1551
C8	D,DSsc,N	0.0292	0.79	13.4	0.1756	0.0279	0.81	13.5	0.1629
C8	CPA^2,DSsc,N	not significant				0.0280	0.80	13.9	0.1633
C8	CPA,DSsc,N	0.0291	0.79	13.6	0.1744	0.0279	0.81	14.0	0.1625
C10	D^2,DCsc	not significant				0.0271	0.82	17.8	0.1524
C10	D,DCsc	0.0291	0.79	16.9	0,1746	0.0274	0.81	17.3	0.1549
C10	CPA^2,DCsc	0.0290	0.79	17.3	0.1719	0.0276	0.81	17.6	0.1583
C10	CPA,DCsc	0.0290	0.79	16.9	0.1733	0.0276	0.81	17.2	0.1582
C11	D^2,DCsc,10'	0.0291	0.79	17.2	0.1755	0.0264	0.83	19.9	0.1457
C11	CPA^2,DCsc,20'	0.0288	0.79	17.2	0.1709	0.0265	0.83	17.4	0.1452
C11	D,DCsc,10'	not significant				0.0272	0.82	15.1	0.1537
C11	D,DCsc,15'	not significant				0.0274	0.81	14.9	0.1566
C11	D,DCsc,20'	not significant				0.0257	0.84	16.6	0.1385
		Point In Time				JUST Change Over Time			
C11	D,DCsc,20'	not applicale				0.0261	0.83	10.7	0.1404

[a] Moore et al.'s (1973).

The replacement of distance between stems (DSsc) with between crowns (DCsc), benefited the prediction ability of the models containing the indices that used dbh, CPA and CPA squared, but not dbh^2 (index type C10 in Table 11). The growth model containing the index and its change over time with line division based on dbh and crown distances indicated a model increase in R^2 of 0.01 and decrease in PRESS of 0.0097 compared to the model with indices using dbh and based on stem distances.

Mixed results were also indicated when the distance between crown edges was used and acute points of the polygons were adjusted. There was some benefit in model prediction when the index with dbh or CPA squared and its change over time were used if distant points were brought back closer to the crown edge (index type C11 in Table 11). However, this maximum distance beyond which points were changed and the resultant model fit and prediction abilities varied some what with the variables in the index. For example, with dbh as a variable in this type of index, there was an increase in model fit and prediction when points were dropped beyond 20 feet from the crown edge compared to other distances. With no maximums and maximums at 10, 15 and 20 feet, the respective PRESS statistics of the models containing these point in time and change over time indices were 0.1549, 0.1537, 0.1566, and 0.1385. The model containing an index with squared dbh and change over time indicated a maximum of 10 feet was desirable for increased prediction ability, while with CPA squared, the maximum distance was 20 feet.

4.) Overlap. As a base for comparison, Ek and Monserud's (1974) index, which used crown radius and total height, was calculated and included in the stem basal area growth model. This index as a point in time or point in time and change over time measure of competition was not a significant variable in the model (Table 12).

Other stem and crown variables and weighting by the number of competing trees produced some indices that helped the prediction of the growth model. The best combination of adjustments resulted in an unweighted version of Ek and Monserud's index with the ratio of tree sizes based on dbh squared and total height (index type C5 in Table 12). However, the base model with stand basal area fitted and predicted basal area growth better than the models containing point in time or point in time and change over time indices as described above.

There was a slight improvement in model PRESS by including this type of index when it was weighted by the number of competing trees (index type C8 in Table 12). For example, a weighted index containing dbh and total height produced a model with lower VIF but similar R^2 and PRESS as compared to the model containing an unweighted index containing dbh squared and total height. Comparing these results with those found with the model containing just the point in time measure of stand basal area, indicated that stand basal area produced a model with higher R^2 and lower PRESS than any of these type of indices.

Table 12: Overlap indices: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in thinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0286	0.80	14.6	0.1686	not significant			
C5	CR-H [a]	not significant				not significant			
C5	D^2-H	0.0292	0.79	12.0	0.1750	0.0289	0.79	12.0	0.1730
C8	D-H,N	not significant				0.0289	0.79	10.1	0.1728
C8	CPA-H,N	0.0294	0.78	10.6	0.1781	0.0289	0.79	10.7	0.1729

[a] Ek and Monserud's (1974) index.

5.) Miscellaneous. Similar weighted indices as described in the unthinned section were also tested with trees found in thinned stands. With point in time information, the index containing a ratio of a subject tree's crown diameter squared compared to the tree of average crown projection area provided a model with the best fit and prediction abilities in light of VIF (Table 13). The combination of point in time and change over time information included in the growth model suggested that a ratio of subject tree dbh squared with the tree of average squared basal area or subject tree dbh with the tree of average crown projection area provided the model with high levels of fit and prediction. The VIF's were relatively large for the model containing the dbh squared ratio suggesting that the prediction ability of that model was not as high as suggested by the PRESS statistic. An alternative index based on subject tree dbh and the tree of average CPA when used as a point in time and change over time measure of competition resulted in a model with much lower VIF and slightly higher PRESS statistics compared to the model with the above index.

Another index adaptation that was investigated was based on selecting competing trees by different BAF values and summing their basal area or CPA values as a measure of competition. Change in time values of any of these stand or subsampled stand characteristics were not significant variables in the growth model (Table 14). Subsampled information for basal area and CPA point in time indices resulted in model PRESS decreasing and VIF increasing as BAF values decreased (i.e., more trees were selected as

Table 13: Miscellaneous indices - weighted versions of distance independent indices: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in thinned stands of planted loblolly pine.

INDEX	VARIABLES	Point In Time				PIT & Change Over Time			
		$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0286	0.80	14.6	0.1686	not significant			
C6	$CSA,\overline{SCSA^2}$	0.0290	0.79	10.3	0.1727	0.0288	0.79	10.4	0.1710
C6	$D^2,\overline{B^2}$	0.0286	0.80	14.3	0.1679	0.0242	0.85	45.2	0.1204
C6	$CPA,\overline{SCPA^2}$	0.0287	0.79	11.1	0.1691	0.0275	0.81	11.2	0.1562
C6	$CPA,\overline{B^2}$	0.0285	0.80	18.5	0.1677	0.0286	0.80	20.1	0.1691
C6	$D^2,\overline{SCPA^2}$	0.0289	0.79	40.5	0.1724	0.0274	0.81	11.4	0.1553
C6	$D^2,\overline{SCPA}$	0.0294	0.78	19.1	0.1773	0.0261	0.83	20.7	0.1412
C6	$CD^2,\overline{SCPA}$	0.0286	0.80	11.7	0.1680	0.0275	0.81	11.8	0.1558

Table 14: Miscellaneous indices - partial measures of stand attributes using different BAF values: fit and prediction statistics of stem basal area growth model with point in time, and point in time and change over time measures included, for trees in thinned stands of planted loblolly pine.

		Point In Time				PIT & Change Over Time			
INDEX	VARIABLES	$S_{y.x}$	R^2	VIF	PRESS	$S_{y.x}$	R^2	VIF	PRESS
BASE	B	0.0286	0.80	14.6	0.1686	not significant			
BASE	SCPA	0.0285	0.80	9.8	0.1606	not significant			
C7	ΣD^2,BAF3	0.0281	0.80	18.3	0.1629	not significant			
C7	ΣD^2,BAF5	0.0286	0.80	15.6	0.1683	not significant			
C7	ΣD^2,BAF10	0.0284	0.80	14.9	0.1658	not significant			
C7	ΣD^2,BAF20	0.0286	0.80	12.8	0.1680	not significant			
C7	ΣD^2,BAF30	0.0285	0.80	10.4	0.1678	not significant			
C7	ΣCPA,BAF3	0.0279	0.80	14.7	0.1606	not significant			
C7	ΣCPA,BAF5	0.0287	0.79	12.9	0.1692	not significant			
C7	ΣCPA,BAF10	0.0288	0.79	12.4	0.1703	not significant			
C7	ΣCPA,BAF20	0.0290	0.79	11.1	0.1733	not significant			
C7	ΣCPA,BAF30	0.0291	0.79	9.9	0.1694	not significant			
Base	SCPA,CPA	0.0270	0.82	12.8	0.1519	not applicable			
Base	SCPA,CDg	not applicable				0.0274	0.81	10.0	0.1536

competitors). Any of the models containing a subsample of basal area had lower PRESS than the base model with stand basal area. Also, the model containing stand CPA and especially the subsampled form with a BAF 3, indicated a lower PRESS than the base model with stand basal area.

Using just subject tree and stand level information in the basic linear growth model, indicated that stand CPA and subject tree CPA when added as point in time measures provided high fit and prediction abilities to the model. Similar model fit and prediction abilities as above were noticed when point in time and change over time information in the form of SCPA and CDg, or stand basal area and CDg were added to the basal area growth model.

4.3.2.1 Discussion

In comparing the index types and various variable and structural changes, the indices based on crown information generally provided the stem basal area growth model with the highest fit and prediction abilities. In general, however, the results from the point in time measures of competition indicated that regardless of the type of measure, model fit and prediction abilities were fairly similar. The measures that created the best model fit and prediction abilities were stand CPA, or stand CPA with subject tree CPA. A summary of the point in time indices that provided the growth model with higher fit and prediction abilities than the model containing just stand basal area is given below:

Point In Time Measures

INDEX	CHANGES	$S_{y.x}$	R^2	VIF	PRESS
Base	B	0.0286	0.80	14.6	0.1686
Base	SCPA,CPA	0.0270	0.82	12.8	0.1519
Base	SCPA	0.0285	0.80	9.8	0.1606
C7	ΣCPA,BAF3	0.0279	0.80	14.7	0.1606
C6	$CD^2,\overline{SCPA}$	0.0286	0.80	11.7	0.1680

With point in time and change over time measures of competition available, the ratio of subject tree dbh squared to the tree of average basal area squared when included in the growth model provided a model with the highest R^2 and VIF, and lowest PRESS. The models with the highest fit and prediction abilities contained change over time indices with stem basal area in the numerator. Prediction of these type of indices is somewhat similar to the prediction of the dependent variable. The next best set of measures without stem basal area in the index was the available growing space index with stem CPA, distance between crowns and maximum distance of 20' beyond the subject tree's crown edge set for polygon points. Other indices that were also beneficial in the growth model fit and prediction were as follows:

PIT & Change Over Time Measures

INDEX	CHANGES	$S_{y.x}$	R^2	VIF	PRESS
C6	$D^2,\overline{B^2}$	0.0242	0.85	45.2	0.1204
C3	$\Delta\ D^2/\ \overline{D^2}$	0.0254	0.84	11.4	0.1314
C11	D,DSsc,20'	0.0257	0.84	16.6	0.1385
C11	Δ D,DSsc,20'	0.0261	0.83	10.7	0.1404
C6	$D^2,\overline{CPA}$	0.0261	0.83	20.7	0.1412
C11	Δ CPA,DSsc,20'	0.0265	0.83	17.4	0.1452
Base	B,CDg	0.0270	0.82	15.3	0.1513
Base	SCPA,CDg	0.0274	0.81	10.0	0.1536
C3	$CD^2/\overline{CD^2}$	0.0277	0.81	12.8	0.1590

Modeling stem basal area growth of trees found in thinned stands using a competition index did not produce the gains in fit and prediction as found with trees in unthinned stands. In general, of the final indices selected as contributing the most to model fit and prediction, crown information was important, especially with point in time measures, while weighting, open distance between crown edges, competing tree selection with different BAF values and other structural changes gave mixed results. Also, point in time and change over time measures of competition increased model fit and prediction compared to the models containing just point in time information.

In general, the combination of point in time and change over time measures of competitive stress provided lower model standard error of estimate and PRESS values than just point in time measures. The same concerns expressed in the unthinned section with change over time measures of competition are also relevant here. These measures of changes in competition are valid, but in some cases as with Daniels' (1981) index, where basal area of the tree is the numerator, its use as a change over time measure of competition is similar to the dependent variable. Although the index value itself is not the same as the independent variable, concern is stated in the problem of its estimation.

4.3.3 SUMMARY DISCUSSION

Various variable and structural changes based on crown information produced competition indices that when included in a stem basal area growth model increased the model's fit and prediction abilities. Certain trends and characteristics of effective index adaptations were common for models describing growth of trees located in unthinned and thinned stands. For example, the highest levels of model fit and prediction using point in time competition information for trees in both stand types involved knowing tree and stand level horizontal crown measures. The tree level measures involved crown diameter (for trees located in unthinned stands) and crown projection area (thinned stands), while the stand level measure was the sum of crown projection area (SCPA). The competition measures included in the growth model that produced the highest model R^2 and lowest PRESS involved SCPA and CD for trees located in unthinned stands and SCPA and CPA for trees located in thinned stands. Other measures of competition that helped predict basal area growth of trees in unthinned stands involved ratios of crown diameter and tree of mean CPA, or crown projection area and tree of mean basal area squared. In thinned stands, besides the above mentioned indices, measures of just SCPA helped predict growth better than any other index regardless of variable or structural adaptations.

There was a difference between the stand conditions in the effectiveness of an indices' contribution to modeling stem growth. The best set of point in time competition indices for trees in unthinned stands included SCPA and CD: growth model R^2 increased by 0.09 and PRESS decreased by 0.0755 compared to the model containing just stand basal area. While the best set of indices for trees located in thinned stands included SCPA and CPA, the resultant model increased in R^2 by 0.02 and decreased in PRESS by 0.0167 compared to the model containing stand basal area. A possible reason for this occurrence is that competition is not as important in terms of growth for trees located in thinned stands as those in unthinned stands. Stem growth of trees in thinned stands depends less on sizes and distances of competing trees than just the potential of the tree to respond to the open area and available resources. That is, the growth of a tree in a thinned stand depends on its crown dimensions and amount of net photosynthate available for lower stem growth.

In general, it was observed that with the inclusion of an indices' change over time measure of competition that model fit and prediction abilities increased. Disregarding the change over time indices containing basal area of a subject tree, the set of important measures of competition included a linear combination of SCPA, CD and CDg for trees in unthinned stands and an APA index using CPA (similar results were found using linear combinations of B with CDg and SCPA with CDg) for trees in thinned stands. Again, it was indicated that individual tree basal area growth was better predicted using tree and stand level measures of horizontal crown dimensions and development than other distance dependent indices or those distance independent indices with other variable or structural changes.

Comparing the results of models with point in time and change over time measures of competition to models with just point in time measures, indicated that while there were some improvements in the former model's fit and prediction, the gains were not large. For example, using SCPA and CD in the growth model for the unthinned case, resulted in an increase in model $S_{y.x}$ of 0.0007 and PRESS of 0.0164 compared to the model containing SCPA, CD and CDg. There was only one other index of those tested with its change over time measure that did slightly better in increasing model prediction than just the above point in time measure. Similarly, for trees in thinned stands, the point in time measure did as well as point in time and change over time measures in predicting growth. The model containing the point in time values of SCPA and CPA increased model $S_{y.x}$ by 0.0005 and PRESS by 0.0067 compared to the model containing the best set of point in time and change over time measures (disregarding those indices with subject tree basal area) of an APA index with CPA. The former model had a VIF of 12.8, while the latter had one of 17.4. Other point in time and change over time measures (e.g., linear combinations of B and CDg, or SCPA and CDg), when included in the growth model that produced lower VIF than the above, resulted in similar fit and prediction abilities as just the point in time measures of SCPA and CPA.

In addition to the effectiveness of an index in helping to predict growth, costs involved for data collection and index calculation are also factors to be concerned with. Information on individual stem and crown characteristics (e.g., dbh, crown diameter, X-Y tree coordinates, etc.) and stand level attributes have a cost that will vary depending on whether the information is available from aerial photograph or ground measurements, and the accuracy and precision of the measurements. Another cost is the execution time required for the calculation of each index. This cost varied mainly by index group, as the addition or substitution of a variable or structural change added little to the calculation time within a group. Using the two remeasurement periods and trees located in both unthinned and thinned stands, 134,400 calculations were made for each index type, and variable and structural change. Assuming a cost of \$400/hour of CPU time, the costs of calculating selected point in time and change over time indices are given in Table A17.

Depending on the exact information and its form, some of those values will change. It is clear, however, that the distance dependent type indices were the least expensive to calculate. Even less expensive would be the subject tree and stand level measures of CD or CPA and SCPA that proved to be the most effective indices in helping to predict basal area growth of trees in unthinned and thinned stands. The numerous calculations required of APA and overlap indices resulted in the highest execution times and costs. Likewise, as the number of competing trees that were selected for an index increased, so did its cost as seen with the indices using different BAF values.

With the results from above, the most effective indices were linear combinations of SCPA and CD for trees located in unthinned stands and SCPA and CPA for trees located in thinned stands. For trees in unthinned stands, the addition of SCPA and CD improved model R^2 by 0.09 and PRESS by 0.0755 compared to a similar model containing just stand basal area. Since intraspecific competition in repeatedly thinned stands is not as severe as found in unthinned stands, the results from incorporating a competition index in a stem basal area growth model for trees in thinned stands were not as impressive. In this case, the addition of SCPA and CPA resulted in an improvement of model R^2 of 0.02 and PRESS of 0.0167 compared to the model containing just stand basal area.

The horizontal measures of individual tree and stand level crown areas were not only empirically effective indices, but also theoretically effective. For a competition index to be effective it should reflect vital areas of competition, be accurately and precisely measured, reflect changes in area and level of competition, and be cost effective. An effective index should measure the vital areas of competition that will affect the dependent variable of interest - in this case, stem basal area growth. The horizontal crown measure of a tree indicates the amount of photosynthate that may be available and might be available for lower stem growth. Competition from other trees will affect this measure and its development - thus CD or CPA is the resultant effect from causes of competition. The addition of SCPA as a measure of horizontal stand crown area gives an indication of average crown closure or average degree of competitive stress. Once the stand closes or re-opens and closes, average CPA or CD of individual trees will not develop depending on its crown position and subsequent mortality.

Measurement of SCPA and CPA or CD can be taken accurately and precisely from ground or aerial photograph measurements. If estimates of these variables are needed, they were found to be related to a variety of other stem and stand characteristics.

The response of these horizontal crown measures to subsequent changes in competition was evident. As mortality of competing trees, crown expansion of subject and/or competing trees, or ingrowth of competing trees occurred subject tree and stand level horizontal crown area were directly affected. It was also indicated that the degree to which these measures were affected, subsequently affected stem growth.

Lastly, the costs involved in obtaining information on SCPA and CPA or CD and subsequent calculations were minimal. If measurements of these variables are needed, they can be obtained from ground or aerial photograph measurements. Different costs will be associated with the two methods, but indications are that aerial photographs can be effectively used to obtain horizontal measures of crown area on an individual tree and stand level. The execution time required for the inclusion of a stem and stand dimension is of little consequence compared to any of the other types of indices.

4.4 ASSESSING THE EFFECTS OF RECTANGULARITY ON STEM GROWTH

In assessing the effects of rectangularity on stem development, two measures of rectangularity were investigated. Rectangularity expressed as the ratio of distances between competing trees (across and within rows) was termed the ratio of tree distances (RTD), while the ratio of major to minor axis distances of the subject tree's crown diameter was termed the ratio of crown diameters (RCD) (Figure 2).

Of the 471 trees found on unthinned plots, only 459 had complete sets of measurements for this analysis. Trees found on North Carolina and Louisiana plots were analyzed initially since they represented data uncomplicated by differences in site index, age, and management practices. Information from North Carolina indicated degrees of rectangularity of 1.01 to 2.95 by the RTD method and 1.00 to 1.55 by the RCD method (Table A18). The simple correlations of tree basal area (age 6) and basal area growth (ages 6 to 10) with RTD were significant and negative. However, RCD was positively correlated with basal area at age 6, but not significantly correlated with basal area growth. The correlation between the two ratios was also not significant. Graphical trends between the stem variables and ratios did not indicate any well-defined non-linear trends. Similar results were found with basal area at the time of the second measurement, diameter (both measurements) and diameter growth.

The trees located on the Louisiana plots at ages 28 and 32 indicated even less significant correlations and graphical trends than the North Carolina plots. While the inclusion of all trees found on unthinned plots increased the range of ratio values, they did not provide any trends different from above (Table A18).

Graphs by available growing area did not indicate strong trends between the stem variables (basal area and diameter at both measurement periods, and growth between periods) and rectangularity ratios. In the case of the data set with mixed ages, similar graphs by age and growing space did not indicate different trends from above.

Various basal area growth models were tested on the 471 trees found on unthinned plots. Of the candidate models available, the following was chosen for its over-all fit and prediction ability of basal area growth (Table A19):

$$bg = \alpha_0 + \alpha_1 A + \alpha_2 \ln(b) + \alpha_3 \ln(B)$$

where,

bg = basal area growth of an individual tree (sq ft/tree/4 yrs)

A = age

b = basal area of an individual tree (sq ft), at A

B = basal area of the stand (sq ft), at A

$\ln$ = natural logarithm

α_i = regression coefficients

The significance of the ratio terms was then tested using the two basal area growth models, one with basal area and the other with available growing space (C). Regression analyses based on trees located on the North Carolina or Louisiana plots did not include an age variable, as age was constant. With trees located on North Carolina plots, RTD, in conjunction with stand basal area, seemed to benefit fit and prediction ability of tree basal area growth more than RCD (Table A20). The basic model with RTD that was suggested for the trees on the North Carolina plots was as follows:

$$bg = 0.2897 + 0.0337\ln(b) - 0.0173\ln(B) - 0.0102RTD$$

where the variables were defined as before. This indicated that as the degree of rectangularity increased, basal area growth of an individual tree decreased. While the APA index significantly increased fit and prediction of basal area growth compared to stand basal area, neither ratio variable was significant after its inclusion.

The regressions using the trees from plots in Louisiana still indicated a problem of variable significance in light of VIF and over-all ability of the ratio variables to aid in fit and prediction. Additional models were tested with the exclusion of age and stand basal area. The results suggested that while RCD significantly improved the model fit of basal area growth, it did not improve model prediction of basal area growth (Table A20).

Using all the trees found on unthinned plots, it was indicated that RCD was significant and improved model fit and prediction abilities even after the inclusion of the individual stocking level measure (Table A20). For both types of basal area growth models the regressions were as follows:

$$bg = 0.4192 - 0.0049A + 0.0440\ln(b) - 0.0384\ln(B) - 0.0180RCD$$

$$bg = 0.2060 - 0.0066A + 0.0168\ln(b) + 0.0016C - 0.0324RCD$$

The degree of rectangularity, as measured by RCD, indicated a negative contribution toward basal area growth.

4.4.1 DISCUSSION

The degree of rectangularity may have a minor effect on basal area growth of a tree compared to other measurable (e.g., stand basal area) and unmeasurable (e.g., microsite) factors. In this analysis, the degree of rectangularity, as measured within a tree crown or between competing trees that define the growing space of a subject tree, did, in general, have a significant influence in fit and prediction of basal area growth over a range of data. In the case of trees 6-years-old, growing on similar sites in North Carolina, the degree of rectangularity as measured by competing trees had a significant negative effect on basal area growth for a 4-year period. Basal area growth of trees found in unthinned stands covering a range of site index, age and stocking levels was negatively affected by the degree of rectangularity as measured by the ratio of the major and minor axis distances of crown diameters. Lastly, neither ratio was significant in a linear model for basal area growth using the Louisiana data. The trees in this case reflected a common age (28 years), similar site index and consistent levels of mortality with its age.

The differences in results were probably attributable to differences in age and mortality. For example, the RTD method of calculating the degree of rectangularity would be related more to stem development in stands with no mortality, as in the case of the North Carolina data. This ratio is a one point in time measurement of competing tree placement that is suppose to reflect basal area growth for a four year period. If one of the competing trees used in calculating the ratio dies in that period of time, basal area growth of the subject tree would be greater than the ratio would suggest. Crown ratio, on the other hand, reflects a measure of rectangularity over time and independent of competing tree mortality in its calculation. This variable was not significant in the regressions using the North Carolina data probably due to the young age and lack of extremes in rectangularity. Lastly, the trees from the Louisiana plots seemed to be suspect: stand basal area and combinations of individual tree basal area and available growing space were not significant in the basal area growth models. The late age, small amount of growth and small sample probably contributed to the lack of significant relationships with these data.

5.0 SUMMARY AND CONCLUSIONS

The general objective of this work was to investigate the relationships between tree crown, stem and stand dimensions and development for unthinned and thinned stands of planted loblolly pine. Specific objectives were to: model crown and stem dimensions and development of trees; model crown and stand dimensions and development of stands; model stem development through crown-based competition indices; and, assess the effects of rectangularity on stem growth.

Individual tree and stand level information from unthinned and thinned stands of planted loblolly pine were available for analysis. The data posed some restrictions on the type and extent of the subsequent analyses. First, a complete matrix of age, site index and basal areas was not available; for example, of the 35 thinned plots, 29 were in one age class. Secondly, in the case of the thinned plots, the measurements were taken at times not corresponding to thinning operations. Thus, analyses regarding timing and level of thinnings were not possible.

A tree is an integrated system where the component parts of roots, stem and crown balance structurally and physiologically if the tree is to survive and grow. Consequently, there is a relationship between the tree crown as the component that produces photosynthate and lower stem as the component that utilizes the photosynthate for radial expansion. Factors which influence the gross amount of available photosynthate and distance it must travel, have a indirect influence on lower stem growth. These influencing factors can be age or silvicultural operations, such as stocking level or thinning.

The relationships between crown and stem dimensions and development were similar for trees located in unthinned and thinned stands. Point in time dimensions that were modeled included stem basal area, diameter, crown diameter, crown projection area and height to crown diameter. In modeling stem dimensions, crown information increased model fit and prediction abilities compared to models not containing crown information. There were two types of crown information that were particularly important: horizontal and vertical dimensions of the crown. Horizontal measures, such as the sum of the crown projection area (SCPA) and crown diameter (CD) or crown projection area (CPA), represented an estimate of photosynthetic area; while, vertical measures, such as age and height to crown diameter (HCD), indicated the distance of photosynthate translocation. The strongest relationships occurred between the stem and horizontal crown dimensions since each was measuring a cross sectional area that were related in a cause and effect manner.

Crown and stem development, which included stem basal area growth, diameter growth and crown diameter growth (CDg), were modeled. The development of the stem was strongly related not only to the point in time horizontal and vertical measures of the crown just mentioned, but also crown diameter growth. As an individual tree crown grew so did the stem at breast height.

Tree crown dimensions (i.e., CD, CPA) indirectly indicated the amount of photosynthate production that was and is present for lower stem development. It also provided an indication of the amount potentially available for future development. The growth of crowns, particularly CDg, provided a better indication of how the lower stem was going to develop, since this represented an estimated increase in photosynthate available. Additional information on stand density and its effects on the stem and crown relationships was indicated through the use of SCPA rather than stand basal area. In most cases of modeling stem dimensions and development, the inclusion of SCPA in the model resulted in higher levels of fit and prediction compared to the models containing stand basal area.

Crown dimensions and development were found to be related to a variety of stem and stand attributes. If stem information is available, higher levels of model fit and prediction were found compared to the models based on just stand information.

When modeling individual tree and stand level attributes, similar models were found for unthinned and thinned situations. Trees or stands in a thinned situation can be thought of an extension of those in an unthinned situation. The former represents levels of stocking that are ordered and regulated, while the latter are not. In many cases the relationships between variables were similar and probably could have been modeled together; however, since the range of the data was limited, these observed relationships may have been data specific. For example, depending on the analysis and data available, a large portion of the trees and plots in the thinned stands were 28-years-old. Generally, trees and stands at this age will show little difference in response compared to trees and plots of the same age in unthinned stands.

In modeling the relationships between crown and stand attributes, models with similar independent variables were found for unthinned and thinned stands. The importance of crown information was particularly evident in modeling stand attributes for both situations. The physiological relationships between crown and stem development at the tree level were still applicable at the stand level since stand level attributes were just an aggregate of individual tree attributes.

While the basic relationships between individual tree crown and stem dimensions and development were similar on an aggregate basis, there were other factors to be considered in modeling stand level attributes. These factors included number of surviving trees and crown and stem distributions. Also, some of the relationships between stand and crown dimensions changed with the occurrence of crown closure (e.g., stand basal area and SCPA). In modeling stand attributes, age seemed to act as a physiological indicator of the rate of growth, while the distribution of crown sizes indicated the level of development - the change from rate to level depended on the occurrence of crown closure.

The point in time stand measures that were modeled included stand basal area, SCPA, mean CPA, mean HCD and mean dbh. In the cases of stand basal area and mean dbh, the models with the best fit and prediction abilities contained horizontal stand level measures of crown area, such as SCPA and mean CPA, and vertical measures of crown distance, such as age and height to crown diameter. Even on a stand level, indications of photosynthetic area and distance traveled were important in the prediction of stand basal area and dbh.

Change over time attributes that were modeled included basal area growth, SCPAg and mean dbh growth. Crown variables, such as SCPA, SCPAg and mean CPA, were important in contributing to high levels of model prediction of stand basal area and mean dbh growth. SCPAg was not an important variable in predicting basal area growth of unthinned stands due to differences in crown and stem development in response to mortality after crown closure; however, as mortality and crown closure were not factors in thinned stands, SCPAg was important in predicting basal area growth in thinned stands. As the cross sectional crown area in thinned stands developed so did the cross sectional stem area.

The point in time and change over time crown characteristics were modeled using stem and stand attributes. While point in time crown variables, such as SCPA and MCPA, can be easily measured from low-level aerial photographs, it was found that acceptable levels of model fit and prediction of these crown characteristics existed. Higher levels of model fit and prediction abilities were found when stem characteristics were used as independent variables compared to those models using just stand characteristics.

The importance of crown information was also indicated when modeling stem basal area growth as a function of individual tree competition indices for trees located in unthinned and thinned stands. The most effective indices were point in time horizontal crown measures of stand SCPA and CD for trees in unthinned stands and stand SCPA and CPA for trees in thinned stands. Compared to the basal area growth model containing stand basal area, the former set of variables for trees in unthinned stands improved model R^2 by 0.09 and PRESS by 0.0755, while the latter set for trees in thinned stands improved model R^2 by 0.02 and PRESS by 0.0167. These sets of competition indices proved to be theoretically and empirically effective as they reflected vital areas of competition, were capable of being accurately and precisely measured, reflected changes in area and level of competition and were inexpensive to calculate.

Other index types that were investigated included size and/or distance of neighboring trees, distance independent, available growing space, overlap and others. A variety of variable and variable combinations based on crown, stem and stand information were substituted in these index groups, but none did as well as just the horizontal crown measures of the subject tree and stand.

Structural changes to the indices included weighting by the number of competing trees, distances between crowns rather than stems, adjustments of acute areas not readily available for crown expansion and selection of competing trees using different BAF values. In addition, point in time and change over time values of the indices were included in the growth model.

Increases in model prediction depended on the index type, variable and structural changes. Generally, when indices with these structural changes were included in the growth model, no improvement or marginal improvement in model fit or prediction resulted. Consistent improvements in model fit and prediction were noticed with the inclusion of point in time and change over time index values. Indices containing subject tree basal area (e.g., Daniels 1981) provided high levels of model fit and prediction when included as point in time and change over time measures. However, prediction of the change over time values of this type of index was analogous to the problem of predicting stem basal area growth. Comparing the results of models containing indices other than those with subject tree basal area indicated that the models with just horizontal crown dimensions of SCPA and CD or CPA did as well as models containing any of the other point in time and change over time indices.

The last area of investigation involved assessing the effects of rectangularity on stem basal area growth. Two measures of rectangularity were defined: rectangularity of the growing space surrounding a subject tree and rectangularity of the subject tree's crown. The degree of rectangularity of the growing space was a ratio of the distances across and within rows between competing trees. The second measure was a ratio of major and minor axis distances of the subject tree's crown diameter. In a cause and effect situation, the former represents cause, while the latter represents effect.

Sets of trees in unthinned stands representing different age groups were analyzed to determine if stem basal area growth for 4 years was adversely affected by increasing degrees of rectangularity. In the case of trees 6-years-old growing on similar sites, the rectangularity of growing space indicated a deleterious affect on stem basal area growth; while, in a mixture of trees covering a range of ages, mortality and sites, stem basal area growth was negatively affected by the rectangularity of the individual crowns.

The relationships between crown, stem and stand dimensions and development for unthinned and thinned stands of planted loblolly pine were investigated in a number of ways. Crown information, especially horizontal crown measures at an individual tree (CD or CPA) and stand level (SCPA), were particularly important in contributing to high levels of model fit and prediction abilities of common stem and stand attributes. As these crown measures grew so did the corresponding stem and stand measures. In addition, SCPA and CD or CPA were more effective competition indices in helping to predict stem basal area growth than others that were investigated and adapted. The results from these analyses were similar for trees and stands located in unthinned or thinned situations; however, a limited range of data may have accounted for these similarities. Lastly, certain measures of spacing rectangularity were found to be significant in negatively affecting stem basal area growth of trees located in unthinned stands.

The results of the analyses indicated that individual tree and stand level crown dimensions are related to certain individual tree and stand level dimensions. The results also suggested that as these crown characteristics developed over time, so did the tree and stand measures develop. The relationships quantified here provide a better means to model the effects of many silvicultural practices, aid in management and economic decisions and obtain additional information from existing remote sensing material. Regardless of the model resolution, the qualification and quantification of the relationships between crown, stem and stand characteristics are important and useful.

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A.0 TABLES

Table A1: List of common and scientific names.

Douglas-fir	Pseudotsuga menziesii
jack pine	Pinus banksiana
larix	Larix larcina
loblolly pine	Pinus taeda
lodgepole pine	Pinus contorta
northern red oak	Quercus rubra
Norway spruce	Picea abies
radiata pine	Pinus radiata
red pine	Pinus resinosa
Sitka spruce	Picea sitchensis
white spruce	Picea glauca

Table A2: Frequency table by age and state of plots located in unthinned and thinned stands.

	Age Class	Arkansas	Louisiana	Oklahoma	North Carolina
Thinned		6	3		
	10	1			
	21	2			
	28		29		
	total	6	29		
Unthinned		6	2		8
	10	2		4	
	21	6			
	28	1	4		
	total	11	4	4	8

Table A3: Frequency table of 35 plots located in thinned stands.

SI_{25} Class (ft)	Age Class	Basal Area Classes (sq ft/ac)				
		20	60	100	140	Total
50	30		1	1		2
60	10		1			1
	20			1		1
	30		10	12	2	24
	Total		11	13	2	26
70	5	3				3
	20			1		1
	30		1	2		3
	Total	3	1	3		7
TOTAL		3	13	17	2	35

Summary statistics of 35 plots located thinned stands.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age1, 1st measurement	7	25.2	28
Age2, 2nd measurement	11	29.2	32
dbh1[a] (in.)	2.4	9.7	13.9
dbh2[b] (in.)	5.7	10.6	14.7
CD1 (ft)	6.6	16.8	24.2
CD2 (ft)	10.0	19.4	29.7
B1 (sq ft/ac)	19.0	80.9	124.0
B2 (sq ft/ac)	45.2	90.4	129.9
CPA1 (sq ft/ac)	19524	32607	43942
CPA2 (sq ft/ac)	24666	39271	56170
Ts1 (trees/ac)	60	198.6	700
SI_{25} (ft)	54	62.4	74

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[a]/ Initial measurement taken at Age1.

[b]/ Subsequent measurement taken 4 years after Age1.

Table A4: Frequency table of 27 plots located in unthinned stands.

SI_{25} Class (ft)	Age Class	Basal Area Classes (sq ft/ac)						Total
		20	60	100	140	180	220	
60	5	1						1
	10			3				3
	20				1	3	2	6
	30				3	1		4
	Total	1		3	4	4	2	14
70	5	9						9
	10		1	2				3
	30				1			1
	Total	9	1	2	1			13
Total		10	1	5	5	4	2	27

Summary statistics of 27 plots located in unthinned stands.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age1, 1st measurement	6	14.3	28
Age2, 2nd measurement	10	20.3	32
dbh1[a] (in.)	1.1	4.8	11.2
dbh2[b] (in.)	3.3	6.3	11.6
CD1 (ft)	3.7	7.8	15.7
CD2 (ft)	5.5	9.8	20.0
B1 (sq ft/ac)	5.0	95.1	218.0
B2 (sq ft/ac)	42.7	131.7	239.5
CPA1 (sq ft/ac)	6256	31363	55293
CPA2 (sq ft/ac)	20050	43125	66511
Ts1 (trees/ac)	220	685.4	1146
SI_{25} (ft)	56	65.5	72

---

[a] / Initial measurement taken at Age1.

[b] / Subsequent measurement taken 4 years after Age1.

Table A5: Frequency table of 647 trees with complete measurements located in the buffer and measurement plots of thinned stands.

SI_{25} Class (ft)	Age Class	Basal Area Classes (sq ft/ac)				Total
		20	60	100	140	
50	30		8	15		23
60	10		45			45
	20			17		17
	30		95	201	56	352
	Total		140	218	56	414
70	5	150				150
	20			16		16
	30		6	38		44
	Total	150	6	54		210
Total		150	154	287	56	647

Summary statistics of 647 trees with complete measurements located in the buffer and measurement plots of thinned stands.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age1, 1st measurement	7	21.5	28
Age2, 2nd measurement	11	25.5	32
dbh1[a] (in.)	1.8	8.1	15.9
dbh2[b] (in.)	4.3	9.3	16.3
CD1 (ft)	4.4	14.2	34.8
CD2 (ft)	6.1	16.7	40.2
HCD1 (ft)	4.0	43.0	75.5
HCD2 (ft)	7.9	44.6	70.2
H1 (ft)	11.9	52.0	81.5
H2 (ft)	19.8	57.8	86.8

---

[a]/ Initial measurement taken at Age1.

[b]/ Subsequent measurement taken 4 years after Age1.

Table A6: Frequency table of 1442 trees with complete measurements located in the buffer and measurement plots of unthinned stands.

SI_{25} Class (ft)	Age Class	Basal Area Classes (sq ft/ac)						Total
		20	60	100	140	180	220	
60	5	53						53
	10			224				224
	20				18	131	119	268
	30				79	45		124
	Total	53		224	97	176	119	669
70	5	518						518
	10		74	160				234
	30				21			21
	Total	518	74	160	21			773
Total		571	74	384	118	176	119	1442

Summary statistics of 1442 trees with complete measurements located in the buffer and measurement plots of unthinned stands.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age1, 1st measurement	6	12.2	28
Age2, 2nd measurement	10	16.2	32
dbh1[a] (in.)	0.2	4.3	15.7
dbh2[b] (in.)	1.0	5.8	16.4
CD1 (ft)	1.8	7.3	22.4
CD2 (ft)	2.6	8.9	29.2
HCD1 (ft)	1.2	19.9	72.4
HCD2 (ft)	5.6	28.0	78.4
H1 (ft)	4.7	27.6	77.8
H2 (ft)	10.8	36.7	82.6

---

[a]/ Initial measurement taken at Age1.

[b]/ Subsequent measurement taken 4 years after Age1.

Table A7: Frequency table of 30 plots containing 202 trees with complete measurements located in the measurement plots of thinned stands.

SI_{25} Class (ft)	Age Class	Basal Area Classes (sq ft/ac) 20	60	100	140	Total
50	30		1	1		2
60	30		8	12	2	22
70	5	3				3
	30		1	2		3
	Total	3	1	2		6
TOTAL		3	9	14	2	30

Summary statistics of 30 plots containing 202 trees with complete measurements located in the measurement plots of thinned stands.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age1, 1st measurement	7	25.9	28
Age2, 2nd measurement	11	29.9	32
dbh1[a] (in.)	2.4	9.8	13.9
dbh2[b] (in.)	5.7	10.7	14.7
CD1 (ft)	6.6	16.8	24.2
CD2 (ft)	11.1	19.5	29.7
B1 (sq ft/ac)	19.0	83.0	124.0
B2 (sq ft/ac)	45.2	91.8	129.9
CPA1 (sq ft/ac)	19524	32713	43942
CPA2 (sq ft/ac)	24666	39959	56170
Ts1 (trees/ac)	60	196.7	700
SI_{25} (ft)	54	62.2	74

---

[a]/ Initial measurement taken at Age1.

[b]/ Subsequent measurement taken 4 years after Age1.

Table A8: Frequency table of 202 trees with complete measurements located in the measurement plots of thinned stands.

SI_{25}	Age	Basal Area Classes (sq ft/ac)				
Class (ft)	Class	20	60	100	140	Total
50	30		2	8		10
60	30		22	63	18	103
70	5	77				77
	30		1	11		12
	Total	77	1	11		89
TOTAL		77	25	82	18	202

Summary statistics of 30 plots containing 202 trees with complete measurements located in the measurement plots of thinned stands.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age1, 1st measurement	7	20.0	28
Age2, 2nd measurement	11	24.0	32
dbh1[a] (in.)	1.8	7.5	15.3
dbh2[b] (in.)	4.6	9.1	15.9
CD1 (ft)	5.0	13.6	34.8
CD2 (ft)	8.2	16.8	40.2
HCD1 (ft)	5.5	37.6	70.5
HCD2 (ft)	7.9	40.5	70.0
H1 (ft)	11.9	48.7	80.5
H2 (ft)	19.8	55.0	86.1

---

[a]/ Initial measurement taken at Age1.

[b]/ Subsequent measurement taken 4 years after Age1.

Table A9: Frequency table of 26 plots containing 471 trees with complete measurements located in the measurement plots of unthinned stands.

SI_{25} Class (ft)	Age Class	Basal Area Classes (sq ft/ac)						Total
		20	60	100	140	180	220	
60	5	1						1
	10			3				3
	20				1	3	2	6
	30				3	1		4
	Total	1		3	4	4	2	14
70	5	9						9
	10		1	1				2
	30				1			1
	Total	9	1	1	1			12
Total		10	1	4	5	4	2	26

Summary statistics of 26 plots containing 471 trees with complete measurements located in the measurement plots of unthinned stands.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age1, 1st measurement	6	14.5	28
Age2, 2nd measurement	10	18.5	32
dbh1[a] (in.)	1.1	4.8	11.2
dbh2[b] (in.)	3.3	6.4	11.6
CD1 (ft)	3.7	7.8	15.7
CD2 (ft)	5.5	9.8	20.0
B1 (sq ft/ac)	5.0	95.7	218.0
B2 (sq ft/ac)	42.7	130.9	239.5
CPA1 (sq ft/ac)	6256	30974	55293
CPA2 (sq ft/ac)	20050	43130	66511
Ts1 (trees/ac)	220	680.7	1146
SI_{25} (ft)	56	65.5	72

---

[a]/ Initial measurement taken at Age1.

[b]/ Subsequent measurement taken 4 years after Age1.

Table A10: Frequency table of 471 trees with complete measurements located in the measurement plots of unthinned stands.

SI_{25} Class (ft)	Age Class	Basal Area Classes (sq ft/ac)						Total
		20	60	100	140	180	220	
60	5	11						11
	10			84				84
	20				6	20	36	62
	30				30	11		41
	Total	11		84	36	31	36	198
70	5	191						191
	10		37	35				72
	30				10			10
	Total	191	37	35	10			273
Total		202	37	119	46	31	36	471

Summary statistics of 471 trees with complete measurements located in the measurement plots of unthinned stands.

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age1, 1st measurement	6	11.7	28
Age2, 2nd measurement	10	15.7	32
dbh1[a] (in.)	0.2	4.2	13.7
dbh2[b] (in.)	1.7	5.8	14.1
CD1 (ft)	2.2	7.4	22.0
CD2 (ft)	2.9	9.0	29.2
HCD1 (ft)	1.2	18.5	72.4
HCD2 (ft)	9.2	26.7	67.2
H1 (ft)	5.0	26.6	76.6
H2 (ft)	12.8	35.9	81.2

---

[a]/ Initial measurement taken at Age1.

[b]/ Subsequent measurement taken 4 years after Age1.

Table A11: Basic hypotheses and test for analysis of full and reduced model comparisons.

H_o: $\alpha_0^1=\alpha_0^2=. \ . \ .=\alpha_0^n$.

H_1: $\alpha_1^1=\alpha_1^2=. \ . \ .=\alpha_1^n$

where,

Full Model is as follows:

$$y_1 = \alpha_0^1 + \alpha_1^1 x_1$$
$$y_2 = \alpha_0^2 + \alpha_1^2 x_2$$
$$y_3 = \alpha_0^3 + \alpha_1^3 x_3$$
$$\cdot$$
$$\cdot$$
$$y_n = \alpha_0^n + \alpha_1^n x_n$$

Reduced model is as follows:

$$y_i = \alpha_0 + \alpha_1 x_i \qquad i=1,2,\ldots,n$$

The test statistic is a F statistic as follows:

$$F = \frac{(\text{Full SSerror} - \text{Reduced SSerror})/(\text{Full df} - \text{Reduced df})}{\text{Full SSerror}/\text{Full df}}$$

having degrees of freedom (Full df-Reduced df),Full df

where,

$$F_{(\text{Full df-Reduced df,Full df})} \sim F_{\alpha,(\text{Full df-Reduced df,Full df})}$$

under the H_o.

Table A12: Full and reduced model comparison of levels of site index classes on the relationships of total height and dbh.

H_o: slope and intercept coefficients are equal

H_1: at least one slope or intercept coefficient is not equal

where,

Full Model is as follows:

$H_1 = \alpha_0{}^1 + \alpha_1{}^1/dbh_1$ for site class 1
$H_2 = \alpha_0{}^2 + \alpha_1{}^2/dbh_2$ for site class 2
$H_3 = \alpha_0{}^3 + \alpha_1{}^3/dbh_3$ for site class 3
$H_4 = \alpha_0{}^4 + \alpha_1{}^4/dbh_4$ for site class 4

Reduced model is as follows:

$H_i = \alpha_0 + \alpha_1/dbh_i$ i=1,2,3,4

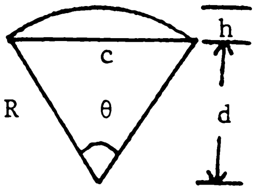
Model	SI Class	df	SSE
Full	1	99	856.676
	2	51	284.037
	3	112	1083.555
	4	22	181.935
	total	284	2406.203
Reduced		290	2481.886

where,

$F_{6,284} = 1.49$ and $F_{\alpha=.05,(6,\infty)}=2.10$

Table A13: Formulas used in adjusting individual tree crown projection areas that extended over plot boundaries.

Segment of a Circle



where,

$$\text{Area} = R^2 \cos^{-1}((R-h)/R) - (R-h)\sqrt{2Rh-h^2}$$

Sector of a Circle

$$\text{Area} = 0.5\, R^2\theta$$

Table A14: Final models for selected stem and crown attributes.

TREES LOCATED IN UNTHINNED STANDS

Basal Area (sq ft)

$$b = e^{[\,-3.5556 + 0.0129SI - 16.3300/A + 0.5475\ln(CPA)\,]}$$

Diameter (in.)

$$dbh = -3.9064 + 0.0375SI + 0.1110A + 0.4607CD + 0.0117B$$

or

$$= 0.2654 - 0.0252SI + 0.1736A + 0.4894CD$$

Basal Area Growth (sq ft/4 years)

$$bg = 0.2977 - 1.1701E\text{-}03SI - 6.5687E\text{-}03A + 5.9313E\text{-}03CD + 5.4218E\text{-}03CDg - 1.7177E\text{-}06SCPA + 0.0252\ln(b)$$

Diameter Growth (in./4 years)

$$dg = 3.3167 + 8.8522E\text{-}03SI - 0.0989A + 0.1962\ln(b) - 4.2039E\text{-}05SCPA + 0.0726CD + 0.0895CDg$$

Crown Diameter (ft)

$$CD = 2.9622 + 1.0151E\text{-}03SI - 0.1238A + 1.6485dbh - 1.5111E\text{-}02B$$

or

$$= -7.1076 + 0.1837SI + 0.1605A + 1.2107E\text{-}04SCPA - 4.6020E\text{-}03Ts$$

Crown Diameter Growth (ft/4 years)

$$CDg = 1.8131 + 0.0287SI + 0.0295A - 8.9902E\text{-}05SCPA - 1.5561E\text{-}03Ts + 0.7976\ln(CD)$$

or

$$= 0.3701 + 0.0480SI - 0.0804A - 1.1929E\text{-}04SCPA + 0.7688dbh - 0.0105CPA$$

Crown Projection Area (sq ft)

$$CPA = -43.2944 + 0.8912SI - 1.9906A + 309.051b + 4.8008E\text{-}04SCPA$$

or

$$= -383.281 + 5.0835SI + 4.8643A + 1.2002E\text{-}03SCPA$$

Height to Crown Diameter (ft)

$$HCD = -3.8995 + 1.0522H - 2.9008E\text{-}05SCPA - 0.6334CD$$

Table A14: continued.

TREES LOCATED IN THINNED STANDS

Basal Area (sq ft)

$b = -0.6324 + 4.3962\text{E-}03SI + 1.1902\text{E-}02A + 3.6410\text{E-}02CD$

Diameter (in.)

$dbh = -5.6336 + 0.0361SI + 0.1611A + 0.3558CD + 0.6904\ln(B)$

Basal Area Growth (sq ft/4 years)

$bg = -0.0730 + 1.2907\text{E-}03SI - 6.4476\text{E-}03A + 9.3028\text{E-}02\ln(CD)$
$\quad - 9.2596\text{E-}07SCPA + 5.2287\text{E-}03CDg$

Diameter Growth (in./4 years)

$dg = 2.2425 + 0.0216SI - 0.0903A - 0.4861\ln(B) + 0.6472\ln(CD)$
$\quad + 0.0587CDg - 0.4529b$

Crown Diameter (ft)

$CD = 8.8842 - 0.0712SI - 0.1393A + 1.9983dbh - 0.0433B$
or
$= 29.3634 - 0.1012SI - 0.1724A + 9.8704\text{E-}05SCPA - 2.7842\text{E-}02Ts$

Crown Diameter Growth (ft/4 years)

$CDg = 6.9019 + 0.1207SI + 0.0878A - 3.7153\ln(B) - 0.3550CD$
$\quad + 0.8039dbh$
or
$= 2.3729 + 0.1497SI + 0.2173A - 3.1602\ln(B) - 6.8953\text{E-}02CD$

Crown Projection Area (sq ft)

$CPA = 236.228 - 2.4800SI - 1.2123A + 487.339b - 0.9899B$
or
$= 333.891 + 5.3822A - 1.6532B - 0.4902Ts$

Height to Crown Diameter (ft)

$HCD = -3.2896 + 0.9461H + 1.5155\text{E-}04SCPA - 3.7485\ln(CD)$

Table A15: Full and reduced model comparison of levels of age classes on the relationships of basal area and sum of the crown projection area of unthinned loblolly pine stands.

Case 1.

H_o: slope coefficients are equal

H_1: at least one slope coefficient is not equal

where,

Full Model is as follows:

$B_1 = \alpha_0{}^1 + \alpha_1{}^1 SCPA_1$ for age class 6
$B_2 = \alpha_0{}^2 + \alpha_1{}^2 SCPA_2$ for age class 10
$B_3 = \alpha_0{}^3 + \alpha_1{}^3 SCPA_3$ for age class 21
$B_4 = \alpha_0{}^4 + \alpha_1{}^4 SCPA_4$ for age class 28 .

Reduced model is as follows:

$B_i = I_1 + I_2 + I_3 + I_4 + \alpha_i SCPA \quad i=1,2,3,4$

where,

I_i are indicator variables for the above specified age classes.

Model	Age Class	SSE	df	R^2	PRESS
Full	6	65.85	8	0.90	102.06
	10	330.04	4	0.46	982.49
	21	683.39	3	0.69	1970.66
	28	106.62	2	0.08	611.65
	total	1185.90	17		3666.86
Reduced		1624.36	20		2867.32

where,

$$F_{3,17} = 2.10 \quad \text{and} \quad F_{\alpha=.05,(3,17)} = 3.20$$

Thus,

$$B = -13.6203 I_6 - 0.1018 I_{10} + 103.0830 I_{21} + 60.4743 I_{28} + 0.0020 SCPA$$

where,

B = basal area (sq ft/ac)
I_i = indicator variables for age classes 6, 10, 21 and 28 years
SCPA = sum of crown projection area (sq ft/ac)

Table A15: continued.

Case 2.

H_o: slope coefficients are equal

H_1: at least one slope coefficient is not equal

where,

Full Model is as follows:

$B_1 = \alpha_0{}^1 + \alpha_1{}^1 SCPA_1$ for age class 6
$B_2 = \alpha_0{}^2 + \alpha_1{}^2 SCPA_2$ for age class 10
$B_3 = \alpha_0{}^3 + \alpha_1{}^3 SCPA_3$ for age class 21
$B_4 = \alpha_0{}^4 + \alpha_1{}^4 SCPA_4$ for age class 28

Reduced model is as follows:

$B_i = I_1 + \alpha_i SCPA$ $i=1$
$B_i = I_2 + I_3 + I_4 + \alpha_i SCPA$ $i=2,3,4$

where,
I_i are indicator variables for the above specified age classes.

Model	Age Class	SSE	df	PRESS
Full	total	1185.90	17	3666.86
Reduced	6	65.85	8	102.06
	10,21,28	1151.37	11	2549.91
	total	1217.23	19	2651.97

where,

$$F_{1,20} = 6.36 \quad \text{and} \quad F_{\alpha=.01,(1,20)} = 8.10$$

Thus,

$B = -4.7390 + 0.0014SCPA$ for age class 6

$B = -61.3355I_{10} + 44.8258I_{21} - 1.0609I_{28} + 0.0035SCPA$

where,

B = basal area (sq ft/ac)
I_i = indicator variables for age classes 10, 21 and 28 years
SCPA = sum of crown projection area (sq ft/ac)

Table A16: Selected models for various stand and crown attributes.

TREES LOCATED IN UNTHINNED STANDS

Basal Area (sq ft/ac)

$$B = e^{[\,-\,6.0118 + 0.0245SI - 18.1737/A - 0.5307\ln(MCPA) + 1.2011\ln(SCPA)\,]}$$

Basal Area Growth (sq ft/4 years/ac)

$$Bg = -436.608 + 815.163/A + 38.5071\ln(SCPA)$$

or

$$= e^{[\,1.2806 + 12.5072/A + 7.0140E\text{-}05SCPA - 1.7044E\text{-}02B\,]}$$

Sum of Crown Projection Area (sq ft/ac)

$$SCPA = e^{[\,10.6989 - 0.0434A + 0.6604\ln(B) - 0.3898\ln(Ts)\,]}$$

Mean Crown Projection Area (sq ft)

$$MCPA = e^{[\,-\,8.9500 + 0.0536SI + 3.0583E\text{-}02MHCD + 0.8287\ln(SCPA)]}$$

or

$$= -\,125.894 + 3.1319SI - 703.137/A + 28053.836/Ts$$

Mean Height to Crown Diameter (ft)

$$MHCD = -6.2419 + 0.9497H$$

or

$$= -6.1846 + 0.8642H + 0.0305B$$

Diameter at Breast Height (in.)

$$dbh = -7.5418 + 0.0438SI + 0.2153A + 1.7174\ln(MCPA)$$

or

$$= -3.8249 + 0.0823MHCD + 1.7781\ln(MCPA)$$

Diameter at Breast Height Growth (in./4 years)

$$dbhg = 3.4718 - 0.0599A - 3.4601E\text{-}05SCPA$$

or

$$= 3.6084 - 0.1139A - 6.8544E\text{-}05SCPA + 1.2408\ln(dbh)$$

Table A16: continued.

TREES LOCATED IN THINNED STANDS

Basal Area (sq ft/ac)

$$B = e^{[\, -3.8937 + 0.0107SI - 15.8240/A - 0.3496\ln(MCPA) + 0.9777\ln(SCPA) \,]}$$

Basal Area Growth (sq ft/4 years/ac)

$$Bg = -11.2693 + 295.740/A + 1.0080\text{E-}03SCPAg$$

or

$$= e^{[\, -15.4801 + 61.6771/A - 2.5817\text{E-}04SCPA + 5.2221\ln(B) \,]}$$

Sum of Crown Projection Area (sq ft/ac)

$$SCPA = -49036.830 - 488.227SI + 411886.0/A + 29835.944\ln(B) - 7113.436\ln(Ts)$$

Sum Crown Projection Area Growth (sq ft/ac/4 years)

$$SCPAg = 1008.767 + 379.570SI + 892.932A - 0.4476SCPA - 4917.168\ln(MCPA)$$

or

$$= 21401.690 - 0.8581SCPA + 163.655B$$

Mean Crown Projection Area (sq ft)

$$MCPA = e^{[\, 6.1942 - 0.0368SI + 2.8551\text{E-}02MHCD \,]}$$

or

$$= 107.498 - 741.697/A + 21651.666/Ts$$

Mean Height to Crown Diameter (ft)

$$MHCD = -6.1494 + 0.9085H$$

or

$$= e^{[\, 1.7743 + 0.0254H + 0.1141\ln(B) \,]}$$

Diameter at Breast Height (in.)

$$dbh = -16.9330 + 0.0799SI + 0.1262A + 3.5027\ln(MCPA)$$

Diameter at Breast Height Growth (in./4 years)

$$dbhg = -2.7276 + 0.0146SI + 29.1132/A + 0.2508\ln(MCPA)$$

or

$$= -0.9268 + 0.0212SI + 20.7478/A - 6.0468\text{E-}03B$$

Table A17: Estimated computing costs of calculating selected point in time and change over time competition indices.

Size and/or Distance of Neighboring Trees.		
Daniels (1976) with	DSsc, BAF 3	$ 7.67
	DSsc, BAF 10	5.89
	DSsc, BAF 30	5.11
	DCsc, BAF 3	9.11
	DCsc, BAF 10	8.00
	DCsc, BAF 30	7.44
Distance Independent		
Glover and Hool (1979)		.08
Daniels (1981)		.05
Available Growing Space		
Moore *et al*. (1973)	DSsc	107.22
	DCsc	109.67
Overlap		
Ek and Monserud (1974)		45.56
Miscellaneous		
D/B^2		.03
Σb	BAF 3	6.78
	BAF 10	5.44
	BAF 30	5.11

Table A18: Summary statistics of trees located in the measurement plots of unthinned stands for analysis of spacing rectangularity.

North Carolina: 170 observations

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age		6	
SI_{25} (ft)	65	69.9	72
b (sq ft/tree)	0.0003	0.0170	0.0668
bg (sq ft/tree/4 yr)	0.0097	0.0786	0.1185
RTD	1.01	1.85	2.95
RCD	1.01	1.13	1.55

CORRELATION COEFFICIENTS

	b	bg	RCD
RTD	-0.1690	-0.2407	0.0038*
RCD	0.1559	0.0101*	

* not significant at α=0.05 level

Louisiana: 40 observations

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age		28	
SI_{25} (ft)	62	63.4	67
b (sq ft/tree)	0.2672	0.6290	1.0237
bg (sq ft/tree/4 yr)	0.0000	0.0402	0.1665
RTD	1.00	1.29	2.06
RCD	1.00	1.14	1.51

CORRELATION COEFFICIENTS

	b	bg	RCD
RTD	0.0310*	-0.0080*	0.0637*
RCD	-0.0415*	0.2133*	

* not significant at α=0.05 level

Table A18: continued.

All Trees: 459 observations

VARIABLE	MINIMUM	MEAN	MAXIMUM
Age	6	11.7	28
SI_{25} (ft)	56	66.6	72
b (sq ft/tree)	0.0003	0.1449	1.0237
bg (sq ft/tree/4 yr)	0.0000	0.0678	0.2216
RTD	1.00	1.74	4.67
RCD	1.00	1.20	2.52

CORRELATION COEFFICIENTS

	b	bg	RCD
RTD	-0.2632	0.0166^*	0.1904^*
RCD	0.0326^*	-0.1487^*	

* not significant at $\alpha=0.05$ level

Table A19: Tree basal area growth models for a 4-year period based on 471 trees located on the interior measurement plots in unthinned stands.

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
A ln(b) ln(B)	0.0284	0.57	5.9	0.3853
1/A SI ln(b) ln(B)	0.0293	0.54	15.4	0.4091
1/A SI ln(b) B	0.0288	0.56	18.9	0.3955

Table A20: Statistics associated with ratios of tree and crown distances in fitting and predicting tree basal area growth for a 4-year period based on trees located in unthinned stands.

North Carolina: 170 observations

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
ln(b) ln(B)	0.0241	0.57	1.4	0.1022
ln(b) ln(B) RTD	0.0238	0.59	1.5	0.0995
ln(b) ln(B) RCD	not significant			
ln(b) C[a/]	0.0237	0.59	1.3	0.0988
ln(b) C RTD	not significant			
ln(b) C RCD	not significant			

Louisiana: 40 observations

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
ln(b)	0.0284	0.29	1.0	0.0329
ln(b) RTD	not significant			
ln(b) RCD	0.0272	0.36	1.0	0.0364

All Trees: 459 observations

VARIABLES	$S_{y.x}$	R^2	VIF	PRESS
A ln(b) ln(B)	0.0285	0.57	5.9	0.3772
A ln(b) ln(B) RTD	not significant			
A ln(b) ln(B) RCD	0.0283	0.58	5.9	0.3737
A ln(b) C	0.0311	0.49	2.8	0.4517
A ln(b) C RTD	not significant			
A ln(b) C RCD	0.0306	0.51	3.1	0.4366

[a/] C is a available growing space variable based on an adaptation of Moore *et al*. (1973).

B.0 ABBREVIATIONS AND TERMINOLOGY

An alphabetical list and graphical representation (Figure A1) of general abbreviations and terminology used in this paper are presented in this section.

B.1.1 CROWN NOTATION

CBL	Clear Bole Length: same as HLC.
CBR	Crown Basal Area Ratio: ratio of crown projection area to stem basal area.
CD	Crown Diameter: diameter of the effective crown at the widest horizontal point.
CDg	Crown Diameter Growth: growth of crown diameter.
CDR	Crown Diameter Ratio: ratio of crown diameter to dbh.
CL	Crown Length: vertical length of the live crown.
CLR	Crown Length Ratio: ratio of crown length to total height.
CPA	Crown Projection Area: circular area of a crown using crown

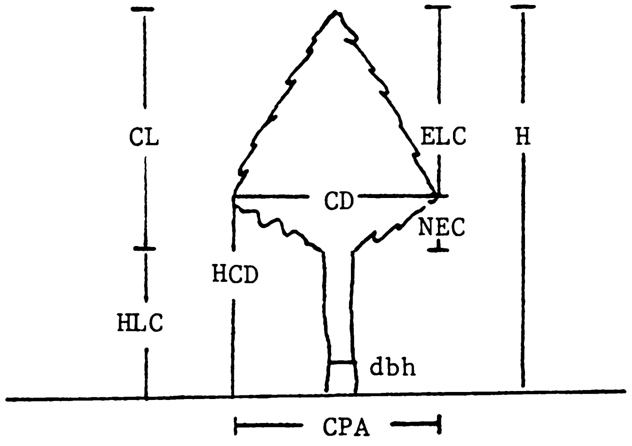
	diameter.
CPAg	Crown Projection Area Growth: growth of crown projection area.
CSA	Crown Surface Area: surface area of the effective crown, not including the base.
CSW	Crown Surface Area Weighted: crown surface area divided by HCD.
CV	Crown Volume: cubic content of the effective crown.
CVW	Crown Volume Weighted: crown volume divided by HCD.
DBC	Diameter at the Base of the Live Crown: diameter of the stem below the live crown.
EC	Effective Crown: portion of the crown above crown diameter, normally associated with branches being net photosynthate-auxin producers.
ELC	Effective Live Crown: distance from the top of the tree to crown diameter.
HCD	Height to Crown Diameter: distance from the ground to crown diameter.
HLC	Height to the Live Crown: height from the ground to the base of the live crown.
LC	Live Crown: live portion of the crown from

	the first major live branch to the top of the tree.
MCPA	Mean Crown Projection Area: sum of crown projection area (SCPA) divided by the number of surviving trees.
MHCD	Mean Height to Crown Diameter: sum of heights to crown diameter (HCD) of all trees on a plot divided by the number of surviving trees.
NEC	Non-effective Crown: portion of the crown below crown diameter, normally associated with branches being net photosynthate-auxin users.
SCPA	Sum of Crown Projection Area: crown projection area (CPA) of a stand expressed on an acre basis.
SCSA	Sum of Crown Surface Area: crown surface area (CSA) of a stand expressed on an acre basis.
SCV	Sum of Crown Volume: crown volume (CV) of a stand expressed on an acre basis.

B.1.2 OTHER NOTATION

A	age
B	basal area of a stand
b	basal area at breast height of an individual tree
BAF	basal area factor
Bg	basal area growth of a stand
bg	basal area growth of an individual tree
Ci	competition index
D	diameter at breast height
dbh	diameter at breast height
Dc	dbh of competing tree
Ds	dbh of subject tree
DCsc	distance between crown edges of competing and subject trees
DSsc	distance between stems of competing and subject trees
H	total height of an individual tree
Hd	average total height of the dominant and codominant trees
ln	natural logarithm
log	logarithm to the base 10
MHt	mean total height of a stand
PIT	point-in-time

PRESS	prediction error sum of squares
RCD	ratio of crown diameters
RTD	ratio of tree distances
SI	site index base age 25
Ts	surviving number of trees
VIF	variance inflation factor



where,

CD = crown diameter
CL = crown length
CPA = crown projection area
dbh = diameter at breast height
ELC = effective live crown
H = total height
HCD = height to crown diameter
HLC = height to live crown
NEC = non-effective crown

Figure A1: Graphical representation of crown and stem terminology.

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