Chapter 5

Collaborative Decision-Making and Equity Considerations

5.1. Cost Model

5.1.1. Fuel Cost

Fuel consumption is an objective metric typically used to assess the cost of executing a flight plan. The APCDM utilizes the Base of Aircraft Data (BADA) Operations Performance Model, developed by the Eurocontrol Experimental Center [17], to calculate flight fuel costs for any flight plan that is proposed for execution. The BADA database specifies a set of aircraft performance factors, airline procedure parameters, and performance statistics for 151 distinct aircraft types. The model includes ground movement (e.g. taxiing and parking) costs as well as airborne costs. The fuel cost of executing a particular flight plan p plan of flight f, is hence given by a function $\mathfrak{F}_{fuel\text{-}cost}(\bullet)$ as defined by this database:

$$F_{fp} = \mathfrak{F}_{fuel-\cos t}\begin{pmatrix}\text{aircraft type, starting mass, operating speed, aerodynamics,}\\ \text{engine thrust, fuel consumption, ground movement}\end{pmatrix}. \qquad (5.1)$$

5.1.2. Flight Arrival Delay Costs

Let τ_f^* be the earliest arrival time of the best flight plan, and τ_{fp} be that for flight plan p, of a given flight f. Then the arrival delay time can be expressed as

$$t_{fp}^{d} = \max\left\{0, \tau_{fp} - \tau_{f}^{*}\right\}. \tag{5.2}$$

Let us consider a flight whose arrival at the destination airport is delayed by some $t_{fp}^{d} \neq 0$. If the flight has only passengers who are traveling to their terminal destination, then the delay cost can be expressed simply as some function of t_{fp}^{d}. However, if there are passengers who must connect to other flights, then the delay cost must include a consideration of any impact to downstream flights. For example, if a passenger misses a connection due to a late arrival, then the planned outbound flight departs with an empty seat, and the airline must reschedule the passenger to depart on a subsequent flight (which perhaps has already been oversold). This phenomenon can cascade through the system, where, a single delayed flight can impact a number of other flights as the affected passengers continue towards their respective terminal destinations.

Let us differentiate the costs associated with delays on the basis of the flight's passenger profile (i.e. the proportion of passengers arriving at their terminal destinations). For example, a flight inbound to a regional airport would incur a lower per-passenger delay cost than a flight inbound to a major hub airport.

One possible approach might be to examine each flight and determine the actual proportion of passengers who are enroute to their terminal destination. This method, however, would require a detailed data specification for a large number of flights that might not be known, or suitable, for this strategic planning stage. Alternatively, we may classify flights according to the respective destination airport's most common (or average) inbound passenger profile. While this approach only approximates flight profiles, it has the advantage that once each airport has been classified, any combination of flights or surrogates may be considered in the model without the need for meticulous data collection. Observe that such an approach includes an inherent estimation of downstream cost impacts so that each flight plan may be considered independently.

Accordingly, we assign to each airport a *connection delay cost factor*, based on the most common passenger profile for inbound flights. For simplicity, we group airports into one of three categories:

- Low Connection Rate. Most or all passengers are assumed to be arriving at their terminal destination in this case. This would apply to small regional airports, where nearly all inbound flights are commuter aircraft (e.g., the Santa Maria Regional Airport, CA). Unit delay costs are lowest with respect to these destinations.
- Medium Connection Rate. A mix of passengers are assumed to be arriving at their terminal destination, which would be typical of small to medium hub airports. The per unit delay costs are median-valued in this case.
- High Connection Rate. A high percentage of passengers are assumed to be making connections in this case, which is typical of major hub and international airports. This category includes the four high-density airports (John F. Kennedy International, LaGuardia, O'Hare International, and Ronald Reagan Washington National). Delays at these destinations have the greatest impact on the overall NAS, and are therefore assigned the highest per-passenger delay costs.

These connection delay cost factors can be used to inflate the related consequences of delays. For example, we can take the low, medium, and high connection rate factors to be 1.0, 1.5, and 2.0, respectively. Let d_f^c denote this value corresponding to the terminal airport for flight f. Table 5-1 displays the lists of high and medium connection rate airports [2]. All other airports are assumed to have low connection rates.

HIGH CONNECTION RATE AIRPORTS		MEDIUM CONNECTION RATE AIRPORTS	
ATL	THE WILLIAM B HARTSFIELD AT	ABQ	ALBUQUERQUE INTL SUNPORT
BOS	GENERAL EDWARD LAWRENCE LOG	ANC	TED STEVENS ANCHORAGE INTL
BWI	BALTIMORE-WASHINGTON INTL	AUS	AUSTIN-BERGSTROM INTL
CLT	CHARLOTTE/DOUGLAS INTL	BDL	BRADLEY INTL
CVG	CINCINNATI/NORTHERN KENTUCK	BNA	NASHVILLE INTL
DCA	REAGAN WASHINGTON NATIONAL	BUF	BUFFALO NIAGARA INTL
DEN	DENVER INTL	BUR	BURBANK-GLENDALE-PASADENA
DFW	DALLAS/FORT WORTH INTERNATI	CLE	CLEVELAND-HOPKINS INTL
DTW	DETROIT METROPOLITAN WAYNE	CMH	PORT COLUMBUS INTL
EWR	NEWARK INTL	DAL	DALLAS LOVE FIELD
FLL	FORT LAUDERDALE/HOLLYWOOD I	HOU	WILLIAM P HOBBY
HNL	HONOLULU INTL	IND	INDIANAPOLIS INTL
IAD	WASHINGTON DULLES INTERNATI	JAX	JACKSONVILLE INTL
IAH	GEORGE BUSH INTERCONTINENTA	MCI	KANSAS CITY INTL
JFK	JOHN F KENNEDY INTL	MDW	CHICAGO MIDWAY
LAS	MC CARRAN INTL	MEM	MEMPHIS INTL
LAX	LOS ANGELES INTL	MKE	GENERAL MITCHELL INTERNATIO
LGA	LA GUARDIA	MSY	NEW ORLEANS INTL/MOISANT FL
MCO	ORLANDO INTL	OAK	METROPOLITAN OAKLAND INTL
MIA	MIAMI INTL	OGG	KAHULUI
MSP	MINNEAPOLIS-ST PAUL INTL/WO	OMA	EPPLEY AIRFIELD
ORD	CHICAGO O'HARE INTL	ONT	ONTARIO INTL
PHL	PHILADELPHIA INTL	PBI	PALM BEACH INTL
PHX	PHOENIX SKY HARBOR INTL	PDX	PORTLAND INTL
PIT	PITTSBURGH INTERNATIONAL	PVD	THEODORE FRANCIS GREEN STAT
SAN	SAN DIEGO INTL-LINDBERGH FL	RDU	RALEIGH-DURHAM INTL
SEA	SEATTLE-TACOMA INTL	RNO	RENO/TAHOE INTERNATIONAL
SFO	SAN FRANCISCO INTERNATIONAL	RSW	SOUTHWEST FLORIDA INTL
SLC	SALT LAKE CITY INTL	SAT	SAN ANTONIO INTL
STL	LAMBERT-ST LOUIS INTL	SDF	LOUISVILLE INTL-STANDIFORD
TPA	TAMPA INTL	SJC	SAN JOSE INTERNATIONAL
		SJU	LUIS MUNOZ MARIN INTL
		SMF	SACRAMENTO INTERNATIONAL
		SNA	JOHN WAYNE AIRPORT-ORANGE C
		TUS	TUCSON INTL

Table 5-1: High and Medium Connection Rate Airports

Naturally, the per-minute time delay cost is a function of the number of passengers affected. Rather than count individual passengers on each flight (such information may not be available *a priori* at this strategic planning stage), we assign a *passenger load estimate*, l_f, for each flight f, that depends on the type of aircraft used and the estimated load factor.

The delay cost for a surrogate p for flight f can now be written as

$$D_{fp} = \left(t_{fp}^{d} d_{f}^{c}\right)\left(l_{f}\right)\left(\gamma\right), \tag{5.3}$$

where γ is the average delay cost per passenger-minute across all airlines and their respective flights.

5.1.3. Total Flight Plan Cost

Finally, the total cost of executing a flight plan is given by

$$c_{fp} = F_{fp} + D_{fp}. \tag{5.4}$$

Observe that the flight plan having the lowest fuel cost may not be the same flight plan that has the lowest delay cost. Moreover, there is a potential cost tradeoff between the fuel efficiency of the flight trajectory and the timeliness of the flight's arrival at the destination airport.

The foregoing analysis pertains to the actively considered potential flight plans. We will also need to prescribe a cost factor for the zeroth (p=0) plan of each flight f, which corresponds to the flight being cancelled. (For each flight f, we have a constraint $\sum_{p \in P_f} x_{fp} \leq 1$, where the slack variable, x_{f0}, takes on a value of 1 if the flight f is cancelled, and is 0 otherwise.) Define P_{f0} to be the set of surrogates that includes this cancellation surrogate, that is, $P_{f0} = P_f \cup \{0\}$. Let us designate the cost corresponding to canceling flight f to be c_{f0}. Since a cancelled flight is the least preferred surrogate, we should expect that

$$c_{f0} > \max_{p \in P_f} \left\{c_{fp}\right\}. \tag{5.5}$$

Let $t_{f0}^{d} > \max_{p \in P_f} \left\{t_{fp}^{d}\right\}$ be an estimated penalty ascribed to canceling flight f (which might depend on average turnaround times for reestablishing connections, or some upper bound on this in the absence of reliable data). Such a penalty estimate might be based on the destination airport of flight f. Using historical airport data, we might take this penalty to be the average passenger waiting time required to obtain an alternative connecting flight. The cancellation penalty should additionally include a consideration of the number of passengers affected, as well as the connecting flights that may be impacted as discussed previously. Accordingly, consistent with (5.3) and (5.4), we define the cost of a cancelled flight to be

$$c_{f0} = \max_{p \in P_f} \left\{ F_{fp} \right\} + \left(t_{f0}^{d} d_{f}^{c} \right) \left(l_f \right) \left(\gamma \right), \tag{5.6}$$

where the first term in (5.6) reflects a conservative estimate of the net additional fuel cost attributed to the rescheduled passengers. (Note that (5.5) holds true.)

Hanson et al. [24] concluded that schedule disruptions (e.g. flight cancellations) is a better cost metric as opposed to traditional modeling that uses average flight delay times. Such schedule disruption costs can also be readily included in this approach, provided that the relevant data for estimating these costs exists.

Remark: Observe that we can use the foregoing cost estimation approach to scale the flight costs onto [0,1]. Using this approach, airlines might rank order the set of surrogates for each of their respective flights, where for the most preferred flight plan, c_f* = 0, and for a flight that is cancelled, c_{fp} = 1. The remaining surrogates would then be assigned a cost, $c_{fp} \in (0,1)$. This allows the airlines to rank order surrogates according to their own prioritization scheme, rather than use an imposed cost metric that may or may not be suitable. Such a ranking method is consistent with MAUT concepts and allows airlines to express preference intensities with respect to alternatives.

Such a formulation must include a mechanism to account for gaming strategies that might be employed by competing airlines. For example, an airline may choose to distribute the costs assigned to various surrogates in such a way so as to increase the likelihood that preferred sets of flight plans are selected by the optimization model. One approach is to ensure that the distributions of cost factors that are assigned to surrogates are similar for each of the participating airlines. (We could perform some statistical analysis, and test the null hypothesis that the distributions are the same with, say, a 0.9 level of certainty.) Alternatively, we might constrain the assignment of c_{fp} for each of the various surrogates. For example, if each airline is to choose five surrogates for each flight (including one corresponding to flight cancellation), we might assign the airline's rank ordered set (from most preferred to least preferred) the costs 0, 0.4, 0.7, 0.9, and 1, respectively. In this manner, selecting a flight's third-most preferred option, for example, will yield the same cost in the objective, without regard to any airline's particular pricing scheme. However, this would be oblivious to the relative sizes of the aircraft and the actual cost impact. Hence, we prefer using the approach proposed above, with the proviso that the airlines are required to submit surrogate plans that are uniform (across flights) with respect to their extents of delays (e.g., a surrogate for each 15-30 minutes of delay up to some limit). This way, airlines can optimize the individual flight plan trajectories, given the enforced departure delay and the existing environmental (wind, weather, SUAs, etc.) situation.

5.2. Collaboration Efficiencies

For each of the F flights, in the absence of constraints, airlines would select the least-cost flight plan, c_f^*, from among the available surrogates, where,

$$c_f^* = \min\{c_{fp} : p \in P_{f0}\}, \ \forall f = 1,...,F . \tag{5.7}$$

However, such a selection may not be feasible when subject to collaborative decision considerations. We define the *airline collaboration cost* function as

$$d_\alpha(x) = \frac{\sum_{f \in A_\alpha} \sum_{p \in P_{f0}} c_{fp} x_{fp}}{\sum_{f \in A_\alpha} c_f^*}, \tag{5.8}$$

where A_α is the set of flights f, with respective surrogates $p \in P_{f0}$, that belong to airline α, and where the cost term c_{fp} has been motivated and developed in Section 5.1.

We now define the *airline collaboration efficiency*, $E_\alpha(x)$, as follows. Let us designate $D_{\max}>1$ as the maximum allowable ratio for any airline of its cost pertaining to the set of surrogates selected through the CDM process to its individually optimized set of surrogates. Hence, we impose

$$d_\alpha(x) \le D_{\max}, \quad \forall \alpha = 1,\ldots,\bar{\alpha}. \tag{5.9}$$

Accordingly, we then compose $E_\alpha(x)$ as the linear function that achieves

$$E_\alpha(x) = \begin{cases} 1 & \text{if} \quad d_\alpha(x) = 1 \\ 0 & \text{if} \quad d_\alpha(x) = D_{\max} \end{cases}, \quad \forall \alpha = 1,\ldots,\bar{\alpha}. \tag{5.10}$$

This yields

$$E_\alpha(x) = \frac{D_{\max} - d_\alpha(x)}{D_{\max} - 1} = \frac{D_{\max} \sum_{f \in A_\alpha} c_f^* - \sum_{f \in A_\alpha} \sum_{p \in P_{f0}} c_{fp} x_{fp}}{\left(D_{\max} - 1\right) \sum_{f \in A_\alpha} c_f^*}, \quad \forall \alpha = 1,\ldots,\bar{\alpha}, \tag{5.11}$$

where note from (5.7) and (5.9) that $E_\alpha(x) \in [0,1]\ \forall \alpha$. Observe that in general, the ratio $D_{\max}$ used is the same for all airlines. However, we might choose to use a value $E_{\max}^\alpha$ specific to each airline α in (5.11). For example, we might wish to give some airline α_1 preferential treatment by letting $D_{\max}^{\alpha_1} < D_{\max}^\alpha \quad \forall \alpha \ne \alpha_1$. This might be desirable if airline α_1 is a small or new entrant airline that requires lower operating costs to initially enter a market or to establish a new service that is beneficial to the overall airline industry. Notice that the ratio $D_{\max}$ provides a minimal degree of cost efficiency for all the airlines, i.e.,

$$\frac{1}{d_\alpha(x)} \ge \frac{1}{D_{\max}}, \quad \forall \alpha. \tag{5.12}$$

When used in concert with a policy that would require the airlines to offer surrogates for each flight that conform with some uniform distribution of flight plan costs, this would obviate gaming strategies on the part of airlines that might tend to bias the collaborative decision towards individually optimized surrogates or to prevent cancellations of particular flights, at the expense of other airlines. Furthermore, since the efficiency measure (5.11) is relative to the individual airline's optimized operating costs, we alleviate the concern that a more efficient airline might subsidize a less efficient airline in the model.

Accordingly, we define the *ω-mean collaboration efficiency* as the weighted sum of the individual airline collaboration efficiencies:

$$\sum_{\alpha} \omega_{\alpha} E_{\alpha}(x) = \sum_{\alpha} \omega_{\alpha} \left[\frac{D_{\max} \sum_{f \in A_{\alpha}} c_f^* - \sum_{f \in A_{\alpha}} \sum_{p \in P_{f0}} c_{fp} x_{fp}}{\left(D_{\max} - 1\right) \sum_{f \in A_{\alpha}} c_f^*} \right], \tag{5.13}$$

where

$$\omega_{\alpha} = \frac{|A_{\alpha}|}{F} \ \forall \alpha, \text{ so that } \sum_{\alpha=1}^{\bar{\alpha}} \omega_{\alpha} = 1 \,. \tag{5.14}$$

5.3. Collaboration Equities: A Preview

Let us begin by recalling the equity-related objective terms and constraints as put forth in the APM model [47]. This formulation minimizes the maximal airline collaboration disutility as well as the maximum differential between these disutility measures, as composed via the following terms in the model formulation, where $U_{\alpha}(x)$ is defined as before in (2.11).

$$\text{Minimize} \quad ... + \mu_e(x_u^e - x_l^e) + \mu_u^e x_u^e \tag{5.15}$$

$$\text{subject to:} \quad x_l^e \le U_\alpha(x) \le x_u^e \quad \forall \alpha = 1,...,\overline{\alpha}$$

$$x_l^e \ge 0,\ x_u^e \le v_e.$$

A problem with (5.15) is that it does not sufficiently distinguish between solutions in terms of the level of equity achieved. To illustrate this issue, consider an example with $\overline{\alpha} = 6$, and suppose that the model permits the following two feasible solutions, S1 and S2, having respective disutility values as stated below.

S1: $U_1(x) = 0.5,\ U_2(x) = 0.5,\ U_3(x) = 0.8,\ U_4(x) = 0.8,\ U_5(x) = 1.0,\ U_6(x) = 1.0$;

S2: $U_1(x) = 0.5,\ U_2(x) = 0.6,\ U_3(x) = 0.7,\ U_4(x) = 0.8\ ,U_5(x) = 0.9,\ U_6(x) = 1.0$.

For both the above solutions, we see that $x_l^e = 0.5,\ x_u^e = 1.0$, and $(x_u^e - x_l^e) = 0.5$. Hence, the two solutions are indistinguishable in the model APM in that they both yield an objective function component in (5.15) of $0.5\mu_e + 1.0\mu_u^e$. However, observe that S1 is inferior in that airlines α=1 and 2 appear to receive preferential treatment with respect to the other airlines, while S2 exhibits a relatively more uniform distribution of cost measures.

Mulvey, Vanderbei, and Zenios [33] make a strong case against modeling equity exclusively in terms of mean-value or worst-case (i.e. maximum spread) measures. They propose constructs that additionally include a consideration of cost distribution measures such as standard deviation.

Gopalan, Kolluri, Batta, and Karwan [22] offer an alternative integer programming formulation to address equity. Suppose that we modify (5.15) as follows to consider cost distribution, rather than the cost range, using appropriately defined parameters μ_d^e, μ_u^e, and v_u^e.

$$\text{Minimize} \quad ... + \mu_d^e x_d^e + \mu_u^e x_u^e \tag{5.16}$$

$$\text{subject to:} \quad \sum_{\substack{i=1 \\ i \neq \alpha}}^{\bar{\alpha}} \left| U_\alpha(x) - U_i(x) \right| \le x_d^e \quad \forall \alpha = 1,...,\overline{\alpha}$$

$$U_\alpha(x) \le x_u^e \quad \forall \alpha = 1,...,\overline{\alpha}$$

$$x_u^e \le \nu_u^e .$$

Here, the variable x_d^e represents the largest value, over the various airlines, of the sum of absolute differences in disutility of each individual airline from that of the other airlines. Using this formulation, we obtain for S1 the solution $x_d^e = 1.6$, while for S2, we get $x_d^e = 1.5$. Hence, (5.16) distinguishes between the two alternative solutions with S2 being the preferred solution, i.e., the respective cost components for S1 and S2 in (5.16) are given by $1.6\mu_d^e + \mu_u^e$ and $1.5\mu_d^e + \mu_u^e$.

5.4. Equity Model Formulation

With the above motivation, let us define a *collaboration equity* function for each airline as the following relative deviation of the corresponding airline's collaboration efficiency from the ω-mean collaboration efficiency:

$$E_\alpha^e(x) = E_\alpha(x) - \left(\sum_{\alpha=1}^{\bar{\alpha}} \omega_\alpha E_\alpha(x) \right) \quad \forall \alpha . \tag{5.17}$$

Observe that when $E_\alpha^e(x) > 0$, we have the case where airline α has achieved an individual collaborative efficiency that is better than the ω-mean collaboration efficiency defined in (5.13). We then accommodate this linear equity representation into our model as follows, where μ^e, μ^D, ν^e, $D_{\max}$, and $E_{\max}^e$ are suitable parameters:

Minimize $$\ldots + \mu^e x^e + \mu^D \left[1 - \sum_{\alpha=1}^{\bar{\alpha}} \omega_\alpha E_\alpha(x) \right] \tag{5.18a}$$

subject to: $$\sum_{\alpha=1}^{\bar{\alpha}} \omega_\alpha \left| E_\alpha^e(x) \right| = x^e \tag{5.18b}$$

$$E_\alpha^e(x) = E_\alpha(x) - \left(\sum_{\alpha=1}^{\bar{\alpha}} \omega_\alpha E_\alpha(x) \right) \quad \forall \alpha \tag{5.18c}$$

$$E_\alpha(x) \; = \; \frac{D_{\max} \sum_{f \in A_\alpha} c_f^* - \sum_{f \in A_\alpha} \sum_{p \in P_{f0}} c_{fp} x_{fp}}{\left(D_{\max} - 1 \right) \sum_{f \in A_\alpha} c_f^*} \quad \forall \alpha \tag{5.18d}$$

$$x^e \le \nu^e, \quad \left| E_\alpha^e(x) \right| \le D_{\max}^e \;\forall \alpha, \quad E_\alpha(x) \ge 0 \;\; \forall \alpha \,. \tag{5.18e}$$

Observe that by using the collaboration equity function developed in (5.17), as in (5.18b), we have eliminated the $\frac{\bar{\alpha}(\bar{\alpha}-1)}{2}$ pairwise absolute terms required by Gopalan, et al.'s formulation in (5.16). Furthermore, we require each airline's absolute collaboration equity to be less than a maximal limit $E_{\max}^e$. Note that the upper bound ν^e is implied if we choose a value greater (or equal) to $E_{\max}^e$. However, if we choose $\nu^e < E_{\max}^e$, we impose the additional restriction that at least one airline α must achieve a collaboration equity that is better than the "worst" allowable.

To implement the constraint (5.18b) in our model, we must first linearize the absolute value term. This can be done in a straightforward manner as follows. In (5.18b) and (5.18e) we dispense with the absolute term by making the substitution

$$\nu_\alpha \;=\; \left| E_\alpha^e(x) \right| \quad \forall \alpha \,, \tag{5.19a}$$

where we additionally impose

$$\nu_\alpha \;\ge\; E_\alpha^e(x) \;\; \text{and} \;\; \nu_\alpha \;\ge\; -E_\alpha^e(x), \quad \forall \alpha \,. \tag{5.19b}$$

Let us consider an example to illustrate the foregoing formulation. Suppose that the model permits the following two feasible solutions, S1 and S2, having respective efficiency values as stated below.

S1: $E_1(x)=1.0,\ E_2(x)=1.0,\ E_3(x)=0.8,\ E_4(x)=0.7,\ E_5(x)=0.5,\ E_6(x)=0.5$;

S2: $E_1(x)=1.0,\ E_2(x)=0.9,\ E_3(x)=0.8,\ E_4(x)=0.7\ ,E_5(x)=0.6,\ E_6(x)=0.5$.

If we assume equal weights for each airline (i.e., $\omega_\alpha = 1/6\ \forall\alpha$), the corresponding objective function terms in (5.18) for S1 are $0.18\mu^e + 0.25\mu^D$, while for S2 they are $0.15\mu^e + 0.25\mu^D$. Observe that both solutions have the same total (weighted) collaboration efficiency. However, the model formulation (5.18) identifies S2 to be a more equitable solution because it has a more uniform distribution of efficiencies as compared with that for S1.

5.5. Alternative Equity Model Formulation

In the foregoing formulation (5.18a), observe that (assuming a constant penalty factor μ^D) an ω-mean collaboration efficiency of, say, 0.5, yields an objective function penalty that is twice as large as when the ω-mean collaboration efficiency is 0.75. This linear relationship might not adequately reflect airline decision maker attitudes (i.e. risk tolerance) with respect to costs incurred via the collaborative process.

Suppose instead that we assign a utility function value to each level of collaboration efficiency achieved by a given airline, based on the respective airline decision maker's revealed preference structure. As discussed in Chapter 2, Kirkwood's [27] exponential utility function formulation provides an excellent representation of such preference structures, given a constant risk tolerance. Accordingly, we shall use this methodology to develop an alternative utility-based equity formulation.

A generalized exponential collaboration efficiency utility function is given in Figure 5-1. A relatively greater value of efficiency is considered an "economic good," hence the utility curve is convex, reflecting a typical risk-prone attitude with respect to increasing wealth.

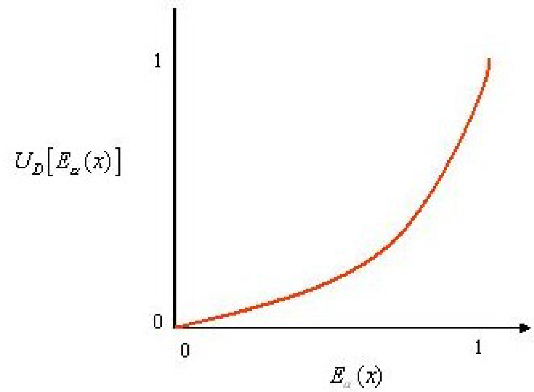

Figure 5-1: Utility Function for Collaboration Efficiency

In a likewise fashion, Figure 5-2(a) shows the expected generalized utility function for ω-mean absolute collaboration equity. Since we regard increasing dispersion of collaboration equities to be an "economic bad," the utility curve is concave, reflecting the typical risk-aversion. The APCDM objective function increasingly penalizes ω-mean absolute collaboration equity as it increases from zero (where collaboration efficiencies for all airlines are equal) to its upper bound v^e. We therefore transform the function as given by Figure 5-2(b), which yields a convex structure.

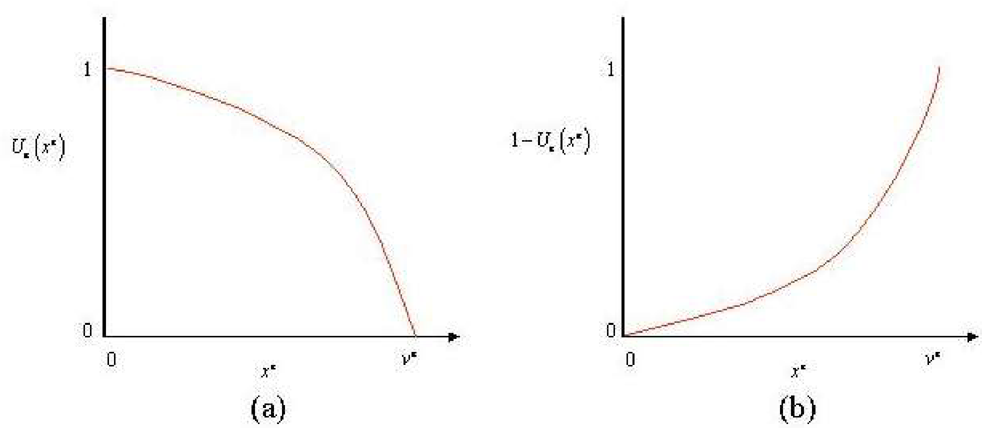

Figure 5-2: Utility Functions for Collaboration Equity

With the foregoing utility function constructs, we modify (5.18a) to be given as

$$\text{Minimize} \qquad \ldots + \mu^e \left[1 - U_e\left(x^e\right)\right] + \mu^D \left[1 - \sum_{\alpha=1}^{\bar{\alpha}} \omega_\alpha U_D\left(E_\alpha(x)\right)\right]. \tag{5.20}$$

Observe that the first term of (5.20) is an exponential group utility function, where the individual component airline utility functions have not been specified. Alternatively, we could explore these individual utility functions, and generate the appropriate modeling construct by modifying (5.18b) to be given as

$$x^e = \sum_{\alpha=1}^{\bar{\alpha}} \omega_\alpha U_\alpha^e \left[E_\alpha^e(x)\right], \tag{5.21}$$

and then penalizing the value of x^e in the objective as in (5.18a). Since collaboration equities may take both negative (increasing "cost") and positive (increasing "wealth") values, the corresponding utility curve would have the form shown in Figure 5-3, where the function is concave for negative collaboration equities and convex for positive collaboration equities. Such a formulation is significantly more complex than the one offered in (5.20), and we shall leave this alternative for later study.

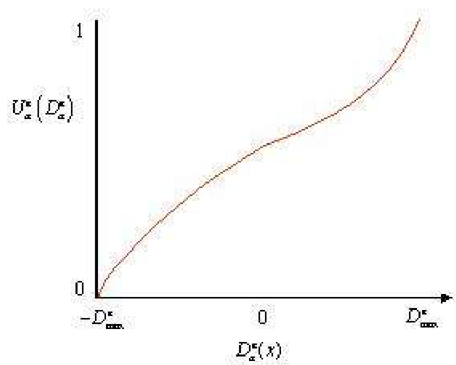

Figure 5-3: Alternative Collaborative Equity Utility Function

The second term of (5.20) also reflects a group utility function. In this case, the individual airline utility functions are identically specified, and are aggregated into an additive utility function using weights that correspond to the proportion of total flights to be scheduled by each respective airline. Notice that we could identify a unique preference structure for each airline decision maker (thus deriving unique exponential utility functions), however, this would require an extensive decision-maker interview process that is beyond the scope of our present study. Furthermore, airline decision makers could identify extremely risk-averse preference structures in an effort to bias the collaborative decision towards one that reflects their own individually optimized selection of flight plans. Hence, we prefer a single preference structure (and the resulting utility function) that is agreed upon by all airline participants.

A weakness in these utility function formulations is with respect to their inherent non-linearity, which dramatically increases the overall model's computational complexity. To accommodate these utility functions without significantly compromising the model's solvability, we shall create piecewise linear approximations for each of them.

Referring to Figure 5-2(b), let us describe $x^e \in \left[0, v^e\right]$ as the convex combination of some $\bar{g}$ points, i.e.,

$$x^e = \sum_{g=1}^{\bar{g}} a_g \lambda_g, \text{ where } \lambda_g \geq 0 \ \forall g \text{ and } \sum_{g=1}^{\bar{g}} \lambda_g = 1, \tag{5.22}$$

with the corresponding utility function found in the first term of (5.20) being approximated by the piecewise linear function

$$1 - U_e\left(x^e\right) \approx \sum_{g=1}^{\bar{g}} \lambda_g \left[1 - U_e\left(a_g\right)\right]. \tag{5.23}$$

As noted in Section 4.2, the convex structure of this utility function ensures us that, at optimality, at most two λ_g-variables will be non-zero, and any two such non-zero variables shall be adjacent.

However, since we are seeking to maximize the ω-mean collaboration efficiency (or utility thereof), the second term of (5.20) takes on a concave structure. Here, we use Sherali's [44] locally ideal (partial convex hull) methodology to formulate a suitable piecewise linear approximation as follows. Referring to Figure 5-1, we make the substitution

$$E_\alpha(x) = \sum_{d=1}^{\bar{d}} \left[a_{d-1}\lambda_d^L + a_d \lambda_d^R \right], \tag{5.24}$$

with the corresponding utility function found in the second term of (5.20) being approximated by the piecewise linear function

$$U_D\left[E_\alpha(x)\right] \approx \sum_{d=1}^{\bar{d}} \left[U_D\left(a_{d-1}\right)\lambda_d^L + U_D\left(a_d\right)\lambda_d^R \right], \tag{5.25a}$$

where

$$\lambda_d^L + \lambda_d^R = v_d \quad \forall d = 1,...,\bar{d} \tag{5.25b}$$

$$\sum_{d=1}^{\bar{d}} v_d = 1 \tag{5.25c}$$

$$\left(\lambda^L, \lambda^R, v\right) \geq 0, \quad v \text{ binary}, \tag{5.25d}$$

and where λ^L, λ^R, and v are vectors of the corresponding subscripted variables indexed by $d = 1,...,\bar{d}$. Note that this representation adds the complication of involving additional binary variables into the model formulation. We shall leave this piecewise linear equity representation for future investigation.