Expected Vibration Performance of Wood Floors As Affected by MSR vs. VSR Lumber E-Distribution

by

Ann C. Wilson

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APPROVED:

Frank E. Woeste, Chairman	J. Dan Dolan
John V. Perumpral	John V. Perumpral

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(ABSTRACT)

A simulation study was done to investigate the effect of the coeffiecient of variation of the modulus of elasticity (Ω_E) on the vibrational performance of joist floor systems. Eight floor cases were studied and two types of lumber were considered: MSR and VSR lumber where Ω_E is 0.11 and 0.25, respectively. The expected floor vibrational performance of MSR versus VSR lumber floors was evaluated by: 1) the probability that the fundamental frequency is less than 10 Hz and 2) the ratio of the first percentile of predicted fundamental frequency of MSR to VSR lumber.

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Introduction

Traditionally, most wood floors were constructed with solid-sawn lumber joists and designed according to the L/360 live-load deflection limit. This deflection criterion was developed to prevent cracks in the ceiling plaster, and probably not to limit vibrations (Percival, 1979). Vibrational performance of floor systems was not a major issue in previous decades because the floor spans were relatively short. However, recent architectural changes have created a demand for longer spans in floor systems. As a result, engineered-wood products such as I-joists and parallel-chord trusses were developed to satisfy this demand for longer spans. These engineered-wood products offer high strength-to-weight and high stiffness-to-weight ratios (Kalkert, 1995). Unfortunately, the switch to longer span floors have decreased the serviceability of floors with regard to annoying vibrations. Most people are sensitive to vibrations in the 8 to 10 Hz range because some human organs have a natural frequency of about 4 to 8 Hz. Therefore, when a floor is vibrating at the same range in frequency, it is perceived as uncomfortable. The unacceptable vibrational performance of some long span wood floors reveal that the L/360 live-load deflection criterion is not alone sufficient in designing a wood floor for 100 percent acceptable vibrational performance.

Currently, U.S. codes do not contain design criterion for controlling floor vibrations. Much research has been conducted on the subject of floor vibrations and several design criteria have been proposed to allow designers to check for acceptable vibrational performance of floors.

However, none are widely accepted due to the complexity of some criteria and the lack of readily available information for the designer (Dolan, 1994). Research conducted by Johnson (1994) at Virginia Tech have resulted in the development of a design criterion that will eliminate most unacceptable and marginally acceptable floor systems (floor systems in or close to the 8 to 10 Hz range) and will aid designers in designing acceptable floors with acceptable vibration performance.

Solid-sawn lumber have different grading systems. Two such grading systems are visually stress rated lumber (VSR) and machine stress rated lumber (MSR). Due to the differences in grading, MSR and VSR lumber have differences in the coefficient of variation of modulus of elasticity (Ω_E). VSR lumber has an Ω_E of 0.25 and MSR lumber has an Ω_E of 0.11 (AF&PA, 1997). This difference in E-variability between MSR and VSR lumber affects the strength and serviceability performance of the structure. Variability of the modulus of elasticity between MSR and VSR lumber is recognized in strength design and addressed in the NDS (buckling capacity of members under compression stress, truss compression chords, and beam stability). Basically, the lower the Ω_E, the higher the capacity of the member with respect to stability issues because the likelihood of a very low E-value is lessened. However, the variability of modulus of elasticity is not addressed in the NDS with regards to serviceability (i.e. deflection and vibrations performance), instead the published average values of modulus of elasticity are used in serviceability calculations.

Objective

The objective of this simulation project was to investigate the effect of the coefficient of variation of the modulus of elasticity (Ω_E) of lumber on the vibration performance of joist floor systems.

Two types of lumber will be considered: machine stress rated lumber (MSR) and visually stress rated lumber (VSR) where the assumed Ω_E's are 0.11 and 0.25, respectively. Expected floor vibrational performance of MSR lumber versus VSR lumber will be evaluated by 1) the probability that the fundamental frequency is less than 10 Hz, and 2) a comparative measure of the ratio of the first percentile of the distribution for fundamental frequency of vibration, *f*, of MSR lumber to the first percentile of the distribution for *f* of VSR lumber:

$$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}} \tag{1}$$

Two floor systems will be considered: 1) joists supported on both ends by concrete blocks (rigid supports) and 2) joists supported by a simple span girder (flexible support) on one end and by concrete blocks on the other end.

This study involved a total of eight cases that are summarized by Table 1. Case 1 involved a floor system of joists supported by rigid supports with the span of the joists designed to the L/360 live-load deflection limit. Case 2 involved a floor system with joists supported by a simple span girder on one end and concrete blocks on the other end. The span of both the girder and joists were designed to the L/360 live-load deflection limit. Case 3 had the same floor system as Case 1, but the joist span was designed to the L/480 live-load deflection limit. The floor system for Case 4 was the same as Case 2, but the joist span was designed to L/480 while the girder span was designed to L/600. Cases 5 through 8 were a repeat of Cases 1 through 4 using an effective joist E based on a load sharing model.

Table 1 **Eight floor systems constructed of either VSR or MSR lumber were studied based on a live-load deflection criteria on rigid versus flexible supports, and no load sharing versus a simple load sharing model.**

Case	Joist Design	Girder Design	Load Sharing Included
1	L/360	NA[1]	No
2	L/360	L/360	No
3	L/480	NA[1]	No
4	L/480	L/600	No
5	L/360	NA[1]	Yes
6	L/360	L/360	Yes
7	L/480	NA[1]	Yes
8	L/480	L/600	Yes

[1]Not Applicable

Distributions of the calculated frequency, *f*, and the modulus of elasticity, E, (with coefficient of variations equal to 0.25 and 0.11) were studied for the above floor system cases. Cases that involved the use of a girder for support were accomplished by determining the distribution of f_{girder} through Monte Carlo simulation using a derived equation for effective modulus of elasticity (E_{eff}). For Cases 1 through 4, Monte Carlo simulation was used to calculate a set of data for effective modulus of elasticity, frequency of the girder, frequency of one joist framing into the girder, and the resulting frequency of the system. The distribution of the system frequency was determined, for both MSR and VSR lumber, using Graphical Distribution Analysis (GDA) (Worley et al., 1990), and the lower distribution tails were studied and compared to one another. For Cases 5 through 8, the procedure was the same, but there were three joists taken into account in a simple load sharing model of the joists. Monte Carlo simulation was used to calculate effective joist E and predicted joist frequency using a model based on the load sharing test results from the Johnson (1994) study.

Literature Review

Research conducted at Virginia Tech resulted in a design criterion that predicts acceptable floor vibrational performance when the calculated frequency of the floor system is 15 Hz or greater. The equation for calculating the fundamental frequency of a joist or a supporting girder is:

$$f = 1.57\sqrt{\frac{386EI}{WL^3}} \qquad (2)$$

where:

f is the fundamental frequency (Hz),

E is the modulus of elasticity (psi),

I is the moment of inertia (in^4),

W is the total *actual* dead weight supported by joist or girder (lb), and

L is the clear span (in.).

If the joist is supported on one end by a girder, then the theoretical frequency of the system can be calculated using the following equation:

$$f_{system} = \sqrt{\frac{f_{joist}^2 * f_{girder}^2}{f_{joist}^2 + f_{girder}^2}} \qquad (3)$$

where f_{joist} is the calculated frequency of the joist, and f_{girder} is the calculated frequency of the girder. According to Equation 3, the system frequency is always less than the frequency of the two parts -- joist plus girder. For example, when

$$f_{\text{joist}} = f_{\text{girder}} = 15 \text{ Hz} \tag{4}$$

the predicted system frequency is 10.6 Hz.

Johnson (1994) developed the 15 Hz design criterion from tests of 15 laboratory-built floors and 73 in-situ floors. The laboratory built floors were 16 ft. x 16 ft. with joist spacing at 24-in. on-center and the ends were rigidly supported (by a heavy steel girder). The sheathing used for the floor system was 23/32 in. thick tongue and groove plywood, nailed and glued to the joists. The 15 laboratory-built floors consisted of:

4 floors with 2x12 No. 2 Southern Pine joists,

1 floor with 2x12 No.1/No.2 Spruce-Pine-Fir joists,

4 floors with 12 inch deep Parallel Chord Trusses,

2 floors with 9-1/2" TJI/25 I-joists,

2 floors with 11-7/8" TJI/25 I-joists, and

2 floors with 11-7/8" TJI/55 I-joists.

All joists spanned 16 feet.

Load sharing tests were performed on the laboratory-built floors to "measure the contribution of each joist in a floor system resisting the applied load" (Johnson, 1994). A 600 lb. load was applied to the centerline of each of the floor systems perpendicular to the joist span and deflection profiles of the floor system were represented by graphs according to joist type. In general, the deflection profiles reveal that the center joist of the floor system supports about 50% of the load while the neighboring joists carry approximately the other 50% (25% each to the two adjacent joists). From this observation, the “apparent stiffness” of the loaded joist in the center of the floor is twice as much as the stiffness predicted by standard deflection formula and the tabulated E for the lumber grade. The center joist of each deflection profile had different deflections (relative to neighboring joists) depending on the joist type. The average deflection of the center joist of both 2x12 No.2 Southern Pine and 2x12 No.1/No.2 Spruce-Pine-Fir solid sawn lumber was 0.15 inches. The average deflection of the center joist of the 12-inch deep parallel chord trusses was 0.16 inches. For I-joists, the deflections for 9-1/2" TJI/25, 11-7/8" TJI/25, and 11-7/8" TJI/55 were 0.19 inches, 0.12 inches, and 0.09 inches, respectively. A conclusion that can be drawn from these deflection profiles is the depth and the type of joist used will affect how much the loaded joist deflects relative to the neighboring joists.

Johnson (1994) tested 73 in-situ floors at various construction sites near Blacksburg, Virginia. The 73 in-situ floors consisted of:

46 floors with 2x10 solid-sawn joists spanning 7 to 23 feet,
4 floors with 2x12 solid-sawn joists spanning 14.5 to 16 feet,
3 floors with 18" deep parallel chord trusses spanning 9 to 25 feet,
13 floors with 20" deep parallel chord trusses spanning 25 to 28 feet,
2 floors with 24" deep parallel chord trusses spanning 30 feet,
2 floors with 11 1/4" deep I-joist floors spanning 20 feet, and
3 floors with 16" deep I-joist spanning 14 to 20 feet.

The joist spacing ranged from 12 to 24 in. on center, with the majority spaced at 16 in. on center. Some of these floors were bare while others were part of nearly completed structures.

The predicted fundamental frequency for the 86 floors was calculated using Equations 2 and 3. Heel drop tests were used to measure the frequency of each floor system, and the vibration performance of each floor system was subjectively rated as *acceptable*, *marginal*, or *unacceptable* (Johnson, 1994). Measured frequencies compared favorably with the predicted frequencies using Equation 2 which validated the equations used for calculating the frequency of the individual joist.

It was determined from the measured data that a frequency of 15 Hz, using Equation 2, provides the distinction between *unacceptable* and *acceptable* floors for all floors evaluated. Johnson (1994) concluded that the frequency of the floor should be calculated based upon only the self weight of the floor since the proposed design criterion was based on tests of bare floors. If the calculated frequency is greater than 15 Hz, then the vibration performance of the floor is expected to be acceptable.

Shue's (1995) study provided further validation of the Johnson study. The difference between the two studies was that Shue tested occupied floors as well as unoccupied floors. The Shue study took into account the effect of partitions or the overall structure itself on the vibration performance of the floor system. Shue tested 106 in-situ wood floors of which 20 were originally in the Johnson study. The result of this study validated the design criterion proposed by the Johnson study for unoccupied floors. However, for occupied floors, Shue concluded that the frequency of the floor system must be greater than 14 Hz in order for it to be considered acceptable.

McGuire (1997) proposed a design rule of thumb that is based on dead load deflection. He used Equation 2 from above and the deflection equation for a simple span joist with uniform dead load in developing this design rule. His proposed vibration design rule causes the fundamental frequency to be greater than 15 Hz when the calculated dead load deflection is less than 0.055 inches. Actual dead load and not the design dead load is to be used in calculating the deflection of the joist. The reader is referred to the article for the mathematics in developing this rule of thumb.

Dolan and Skaggs (1994) stated that one of the main goals in the Johnson study (1994) in developing the criterion was to eliminate the majority of complaints about annoying floor vibrations. The criterion was set at 15 Hz which eliminated the floors judged to be unacceptable and marginally acceptable as well as a few judged to be acceptable. A result of this criteria is the decrease in allowable span for parallel-chord trusses in long-span floors. Dolan and Skaggs stated that the maximum span to depth ratio (l/d) is reduced by 50 percent as the span increases from 10 to 30 feet for parallel-chord trusses spaced at 24 inches on center, with chord modulus of elasticities of 1.6 and 1.9 million psi and sheathed with 23/32" plywood. Dolan and Skaggs also compared the criteria with the L/360 live-load deflection criteria using the parallel-chord truss floor system described above with chord modulus of elasticity of 1.9 million psi. For spans between 22 and 30 feet, the vibration criteria falls between the L/360 and L/480 live-load deflection criteria. The L/360 live-load deflection criteria is sufficient for spans up to 22 feet but after 22 feet the vibration criteria developed by Johnson should be used.

Dolan et al. (1995) studied the effect of imposed load on floor vibrations. Solid-sawn wood joist floors were studied. There were four 16 ft. by 16 ft. laboratory-built floors designed to the L/360 live-load deflection limit and a live load of 40 psf. The joist size was 2x12 Southern Pine solid-sawn joists spaced at 24 inches on center with 23/32 inch tongue and groove plywood nailed and glued to the joists. There were two levels of Ω_E for the joists: 9 percent and 28 percent. Three levels of live load (0 psf, 15 psf, and 40 psf) by means of bags of steel pellets were imposed on the floor system and the dynamic response of the solid-sawn wood joist floors was examined by looking at 3 parameters: natural frequency, damping ratios, and RMS acceleration. The natural frequency of the joist floors decreased as the imposed load increased. No conclusions could be drawn for damping ratio since the results were inconsistent. The frequency weighted RMS acceleration for a joist floor decreased as the imposed load increased. This result was explained by the principle of conservation of momentum – the heavier the floor the more difficult it is to set it into motion compared to a lighter floor. The final conclusion from this study was that Ω_E had little or no effect on the measured vibration responses of the floors tested.

Kalkert et al. (1995) evaluated six design criteria for floor vibrations. Deflection factors (from Span/Deflection Factor) were determined for each criteria. The values for these deflection factors were two to four times larger than the deflection factor of 360 (L/360). The deflection factors ranged from 701 to 1448. The deflection factor of 701 was determined from the Canadian Wood Council (CWC) criterion. The deflection factors calculated excluded the stiffening effects of sheathing. The authors stated that if sheathing were included in the calculations the values for the deflection factors would be lower. Six different floor types (solid-sawn, I-joists, etc.) were then evaluated for acceptable vibrational performance by each criteria. It was found that a floor system would need to be designed to a minimum L/701 live-load deflection limit in order for the predicted vibrational performance of the floor to be acceptable. A consequence of designing to this stricter deflection limit from the traditional L/360 limit is a 20 percent reduction in span. The authors concluded that further research is needed in determining an appropriate deflection limit based on experimental investigations and economic analysis.

Chui (1994) offered suggestions for optimizing the vibrational performance of wood floor systems and retrofitting. A number of variables can affect the vibrational performance of floors. Some of these variables are joist size and spacing, between-joist bridging, composite action between joists and flooring, flooring panel thickness and ceiling boards, and mass distribution. When designing floor systems, Chui suggested selecting a heavier floor. A heavy floor will reduce the amplitude of the vibration. Basically, having a floor vibrate at low amplitudes, high frequency and high damping is not as annoying as a floor vibrating at high amplitudes, low frequency and low damping. Therefore, a heavy floor mass is beneficial because it lowers the amplitude. However, he states that the heavier floor will also reduce the frequency but the reduction is small. An example of a heavier floor design would be to use low-grade joists with large cross-section and narrow joist spacing. Between-joist bridging was also suggested because wood floors are highly orthotropic – the stiffness along the span of the joist is higher than the stiffness across joists. Between-joist bridging increases the capacity of the floor system to share loads across joists. In addition to using between-joist bridging, a continuous bottom strap was suggested to provide continuity on the bottom side of the bridging system. Chui also suggested using rigid glue in addition to nailing to increase the degree of composite action between solid-sawn joists and the flooring. However, Chui added that the improvement in the vibrational performance of floors built with engineered wood products is not significant. Oriented strand board (OSB) was suggested over plywood for floor sheathing due to its higher density which produces a lower vibrational amplitude. Finally, with the exception of very wide floors, Chui suggested an unbalanced, imposed mass distribution on floors. The above recommendations were also offered as suggestions in retrofitting existing floors.

Fridley and Rosowsky (1994) addressed and summarized the service-load behavior of wood floor systems as follows. Load distribution and smoothing of the deflection profile are two actions that take place with two-way action. For load distribution in a floor system, the load carried by each joist is proportional to its stiffness. As a result of load distribution, the deflection of the joists in the floor system is affected. There is a smoothing of the deflection profile which is similar to that of a real floor. Assuming full composite action, a sheathed floor would have less deflection than a bare floor. A composite floor with load sharing has a similar deflection profile to that of a real floor. Gaps in the sheathing can affect two-way and composite action. However, staggering the sheathing lessens this negative affect on two-way action. For composite action under uniform load, they concluded that it is acceptable to not account for gaps and assume full composite action when computing the deflection.

Rosowsky and Fridley (1994) also added that creep in floor systems is significantly less than creep in single members. Fridley and Rosowsky suggested using a system factor for serviceability to address the effects of the behavior of the floor system, versus a single member design approach.

Suddarth et al. (1997) performed a simulation study of the influence of Ω_E on floor performance. The simulation study involved two types of floors, machine stress rated (MSR) and visually stress rated (VSR) joists. There were two sizes for the joists, 2x8 and 2x12. The model used in the simulation included the effect of sheathing and load sharing. The purpose of this study was to compare the deflection performance of MSR floors against the deflection performance of VSR floors using average deflection and soft-spot deflection as measures of performance. Soft-spot deflection was defined as the maximum deflection of three adjacent joists in a floor system. Simulations yielded distributions for average deflection and soft-spot deflection that were reported in terms of L/Δ where L was the span and Δ was the deflection. The 10^{th} percentile of MSR and VSR floors for average deflection and soft-spot deflection were compared to show the difference in expected floor performance between MSR and VSR floors. The results based on average deflection were that a $1.45x10^6$ psi MSR E-grade floor performed equivalently to a $1.6x10^6$ psi VSR E-grade floor. For soft spot deflection, a $1.4x10^6$ psi MSR E-grade floor performed equivalently to a $1.6x10^6$ psi VSR E-grade floor.

Procedure

Variables *E* and *f* in Equation 2 were the main focus in this project. Floor systems studied consisted of eight 2x10 joists spaced at 16 in. on center. For Cases 1, 2, 5, and 6, the span of the joists was 16'-1" and the span of the 3-ply 2x10 girder was 11'-4". For Cases 3, 4, 7, and 8, the span of the joists was 14'-10" and the span of the 3-ply 2x10 girder was 11'-3". The joists and the girder were No. 2 KD19 Southern Pine. An individual joist was studied for Cases 1 through 4 instead of eight joists, and thus load sharing due to floor sheathing was not taken into account. However, in Cases 5 through 8, three joists were studied and thus a measure of load sharing was taken into account as described later.

In order to investigate the effect of Ω_E on the expected vibrational performance of each floor system, the frequency distributions were determined for each floor case.

Case 1

The E was assumed to be lognormally distributed (Suddarth et al., 1975). This assumption was made because it allows us to vary the coefficient of variation easily. It is also a reasonable selection for E because the values for E are positive and the lognormal distribution falls above zero along the x-axis. The parameters for the E-distribution were calculated using the following equations from Ang and Tang (1975):

$$\lambda = \ln \mu - \tfrac{1}{2}\xi^2 = \ln \overline{E} - \tfrac{1}{2}\xi_E^2 \tag{5}$$

$$\xi^2 = \ln\left(1+\Omega^2\right) = \ln\left(1+\Omega_E^2\right) \tag{6}$$

where:

λ is the mean of the logarithms,

ξ^2 is the variance of the logarithms,

μ is the mean,

$\overline{E}$ is the average modulus of elasticity (psi),

ξ_E^2 is the variance of modulus of elasticity,

Ω is the coefficient of variation, and

Ω_E is the modulus of elasticity coefficient of variation.

In order to determine the distribution of the fundamental frequency, *f*, of a single joist, the natural logarithm (ln) is taken on both sides of Equation 2:

$$\ln f = \ln 1.57 + 1/2 \ln 386 + 1/2 \ln I - 1/2 \ln W - 3/2 \ln L + 1/2 \ln E \tag{7}$$

where:

Joist:	I=98.93 in^4	Girder:	I=297 in^4
	W=93.93 lbs.		W=536 lbs.
	L=193 in.		L=136 in.

Equation 7 can be rewritten as:

$$\ln f = k + 1/2 \ln E \tag{8}$$

where:

k is a constant, and

E is the modulus of elasticity (psi) for an individual joist.

Since E is lognormally distributed, then *ln* of E results in E being normally distributed. The ln *f* term would be normally distributed since a constant plus a normally distributed variable is normally distributed. The exponential of both sides of the equation is thus lognormally distributed. The parameters for *f*, using Equation 8 are:

$$\lambda_f = E(\ln f) = k + 1/2\ E(\ln E) \quad (9)$$

$$\lambda_f = k + 1/2\ \lambda_E \quad (10)$$

$$\mathrm{var}(\ln f) = \xi_f^2 = \mathrm{var}\ (k) + (1/2)^2\ \mathrm{var}\ (\ln E) \quad (11)$$

$$\xi_f^2 = 0 + 1/4\ \xi_E^2 = 1/4\ \xi_E^2 \quad (12)$$

$$\xi_f = 1/2\ \xi_E \quad (13)$$

The E and frequency distributions for MSR and VSR lumber were plotted using Equations 5, 6, 10, and 13 to calculate the lognormal density function parameters. The first percentile of the frequency distribution and the probability that the frequency is less than 10 Hz were then calculated from the fitted distributions. Finally, the ratio of the values of the first percentile for each type of lumber were used to compare the vibrational performance to one another using Equation 1.

Case 2

The distributions of the girder modulus of elasticity and the distribution of its frequency needed to be determined first before determining the system frequency. The E-distribution of the girder differs from the E-distribution of the joist since the girder has three members acting together instead of one. If three bending members are connected such that they deflect the same amount, then the effective E (E_{eff}) is:

$$E_{eff} = \frac{E_1 + E_2 + E_3}{3} \tag{14}$$

In this study, it was assumed that the three plys of the girder deflect the same amount due an assumed typical construction nailing between plies.

Using Equations 2 and 14, 5000 values were generated for E_{eff} and girder frequency (f_{girder}) using Monte Carlo simulation. The program, GDA (Worley et al., 1990), was used to determine the best-fit distribution for the data. Using 5000 values of joist E, Equation 2 was used to calculate 5000 values of f_{joist}. Lastly, 5000 values of f_{system} were calculated using Equation 3. GDA was used to determine the best fitting distribution of f_{system} and then the first percentile and the probability of the frequency being less than 10 Hz were calculated for MSR and VSR lumber. The first percentile for both types of lumber were then used to compare the predicted vibrational performance to one another using Equation 1.

Case 3

The procedure for case 3 was the same as the Case 1 procedure. However, the values for the different variables in Equation 7 were different because the joist span was designed to the L/480 live-load deflection limit. The values of the variables are:

Joist:	$I=98.93$ in^4	Girder:	$I=296.79$ in^4
	W=86.63 lbs.		W=502 lbs.
	L=178 in.		L=135 in.

Case 4

The procedure for Case 4 was the same as Case 2, but with differing values for the variables because the joist and girder span were designed for the L/480 and L/600 live-load deflection limits, respectively.

Case 5 to Case 8

Cases 1 through 4 were repeated using an effective joist E, E_j, based and justified by the load sharing results from the Johnson study. The model used was:

$$E_j=0.25E_L + 0.5E_C +0.25E_R \tag{15}$$

where the E's represent three joists in a row in a floor. E_L is the left joist, E_C is the center joist impacted by a footfall, and E_R is the right joist, and the three were considered independent random variables.

However, except for the procedures in obtaining the data for E and f_{joist}, the procedures used for Cases 5 through 8 were the same as Cases 1 through 4. Monte Carlo simulation was used to generate 5000 values for each variable in Equation 15. Then, E_j was used in Equation 2 in place of the variable E to calculate 5000 values of f_{joist}. GDA was used to determine the best-fit distribution for the data for E_j and f_{joist}. For Case 6 and Case 8 the 5000 values for f_{joist} and E_j along with f_{girder} values were then substituted into Equation 3 to produce 5000 values for f_{system}.

Results and Discussion

Case 1

The assumed modulus of elasticity (E) distribution for VSR lumber is shown in Figure 1. The modulus of elasticity was assumed to be lognormally distributed. It ranges from about 700,000 psi to 3,000,000 psi. The modulus of elasticity for MSR lumber was also assumed to be lognormally distributed with a coefficient of variation of 0.11 and it is shown in Figure 2. It ranges from about 1,000,000 psi to 2,250,000 psi. The E-distribution for MSR lumber is dramatically less variable than the E-distribution for VSR lumber by inspecting Figures 1 and 2. This lower variability of the E-distribution in Figure 2 compared to the E-distribution in Figure 1 can be attributed to MSR lumber having a lower coefficient of variation than VSR lumber.

Figure 3 represents the calculated frequency distribution of a joist for VSR lumber and it is lognormally distributed because E was assumed to be lognormal. It ranges from about 10 Hz to 22 Hz. The calculated frequency distribution for MSR lumber is shown in Figure 4. It also is lognormally distributed and is noticeably less variable than f_{joist} depicted in Figure 3.

Based on Equations 10 and 13, the E-distribution affects the predicted fundamental frequency distribution of a joist. For example, the E for MSR lumber is less variable than VSR lumber (0.11 versus 0.25). ξ^2 in Equation 6 was calculated based on Ω_E. For MSR lumber, a lower Ω_E would

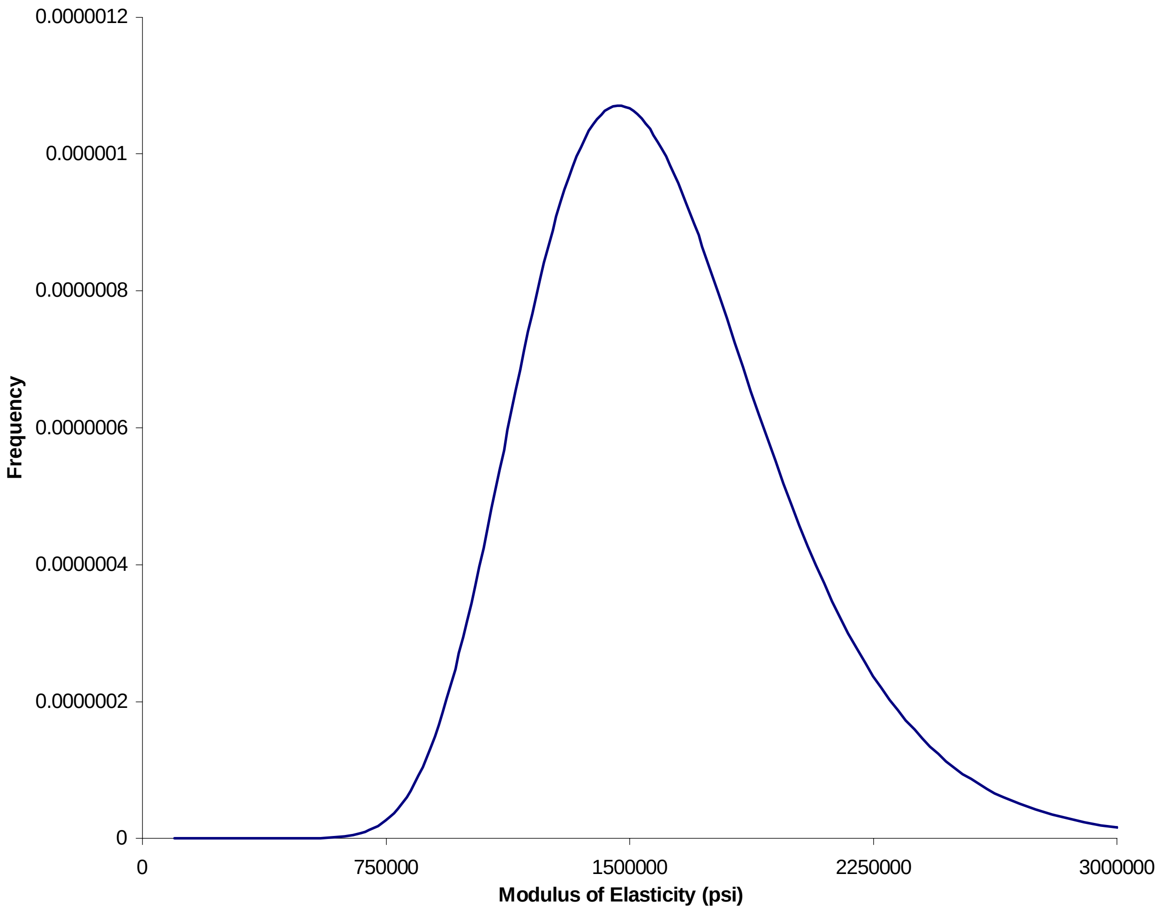

Figure 1 **Two-parameter lognormal distribution was assumed for the modulus of elasticity (E) for VSR lumber (E coefficient of variation = 0.25).**

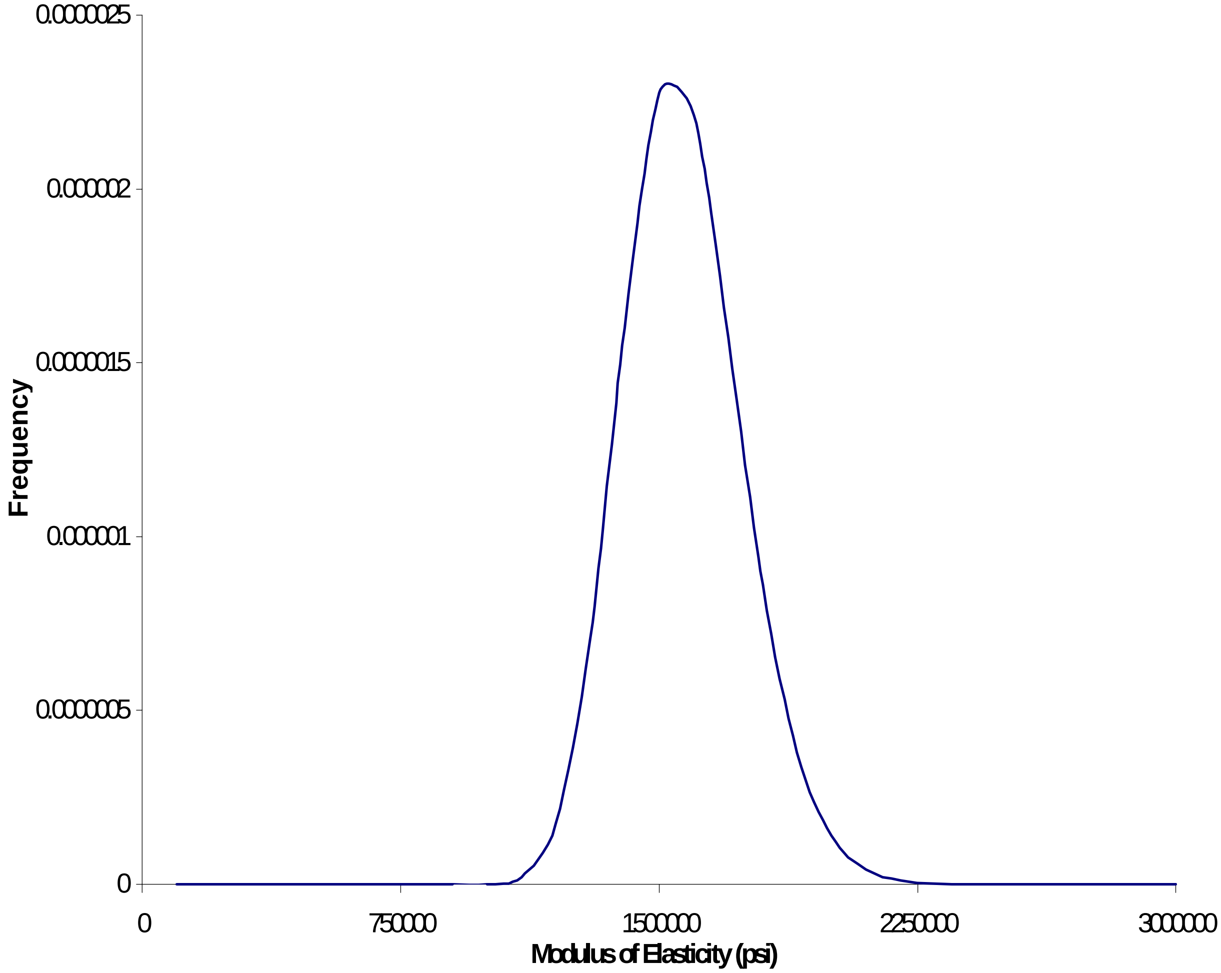

Figure 2 **Two-parameter lognormal distribution was assumed for the modulus of elasticity (E) for MSR lumber (E coefficient of variation = 0.11).**

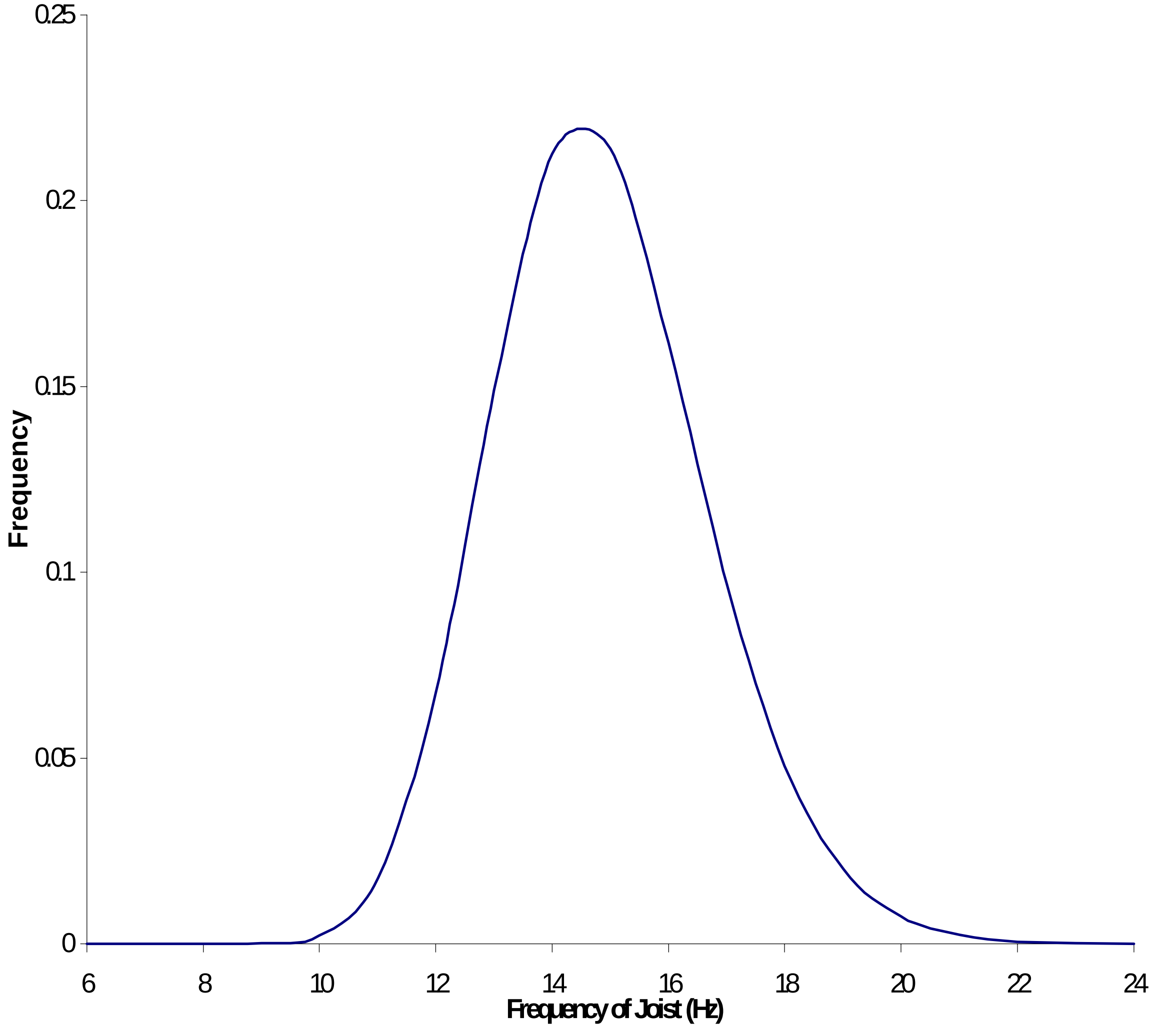

Figure 3 **Two-parameter lognormal distribution and parameters for the frequency of joist vibration with rigid supports were derived for VSR lumber using Equation 2 and Equations 9 through 13.**

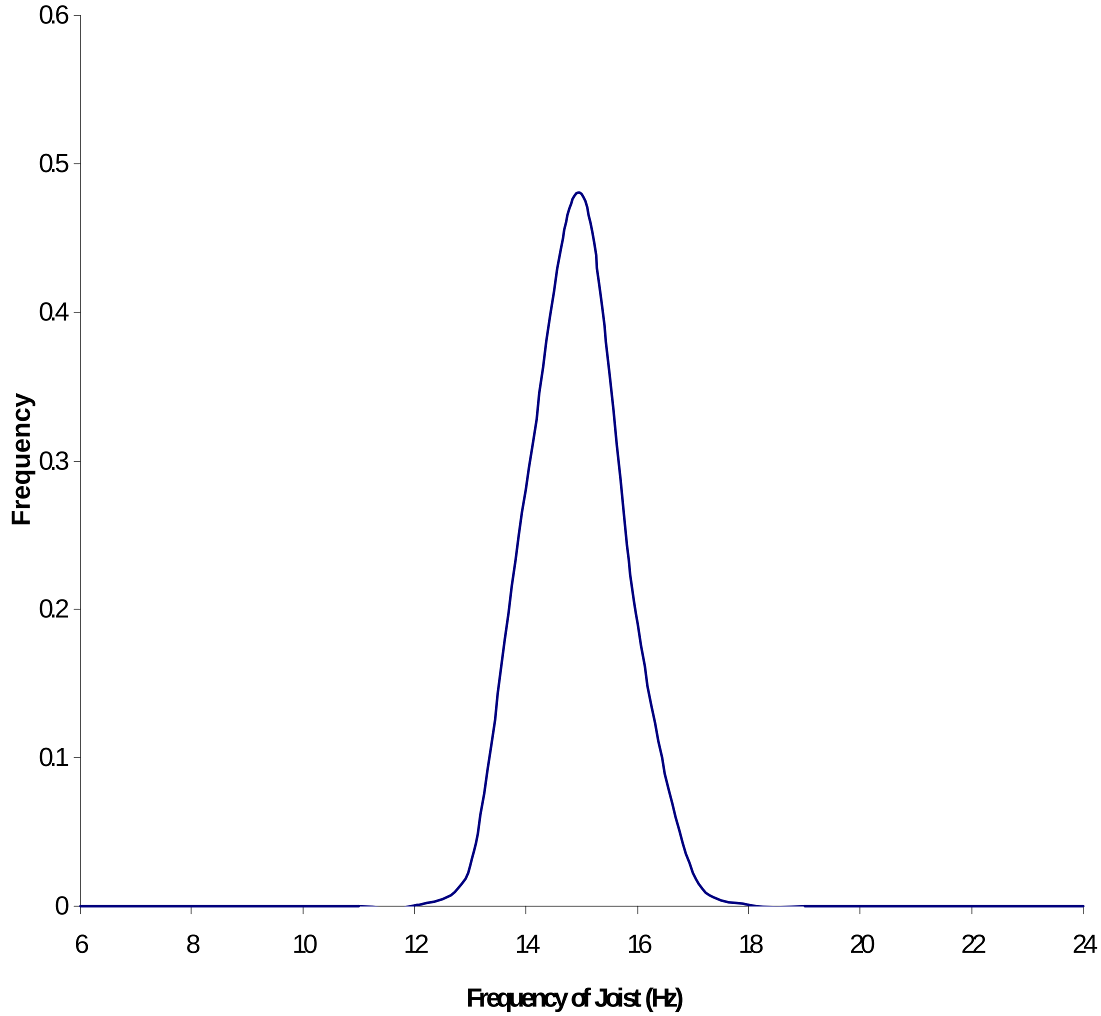

Figure 4 **Two-parameter lognormal distribution and parameters for the frequency of joist vibration for MSR lumber were derived using Equation 2 and Equations 9 through 13.**

cause the standard deviation of the logarithms of E, ξ_E , to be lower. Since ξ_E is the standard deviation for the lognormal density, the variability of the predicted frequency is lower (Figure 4).

The probability that the frequency will be less than 10 Hz in Figure 3 is 0.0008. The first percentile of the distribution is 11.06 Hz. For Figure 4, the probability that the frequency will be less than 10 Hz is approximately zero. The first percentile of the distribution for MSR is 13.09 Hz, which is about 18 percent more than the first percentile for VSR, lumber (Figure 3).

Case 2

In Figure 5 and in Figure 6, the two-parameter lognormal distribution fit the data well based collectively on the visual test, the maximum log-likelihood, and the Chi-Square test. The distribution for the effective girder E, E_{eff}, for MSR lumber (Figure 6) was less variable than the E_{eff} distribution for VSR lumber (Figure 5) and this result was expected. When comparing the E_{eff} distribution for VSR lumber in Figure 5 against the E-distribution in Figure 1, an averaging effect can be observed in Figure 5. This same effect occurs for MSR lumber (Figure 6) when the E_{eff} distribution is compared with Figure 2.

The distribution for E_{eff} has a lower coefficient of variation (Ω) for both MSR and VSR lumber. The standard deviation of E_{eff} can be calculated using Equation 14 and it is given by

$$\frac{\overline{E}\Omega_E}{\sqrt{3}} \tag{16}$$

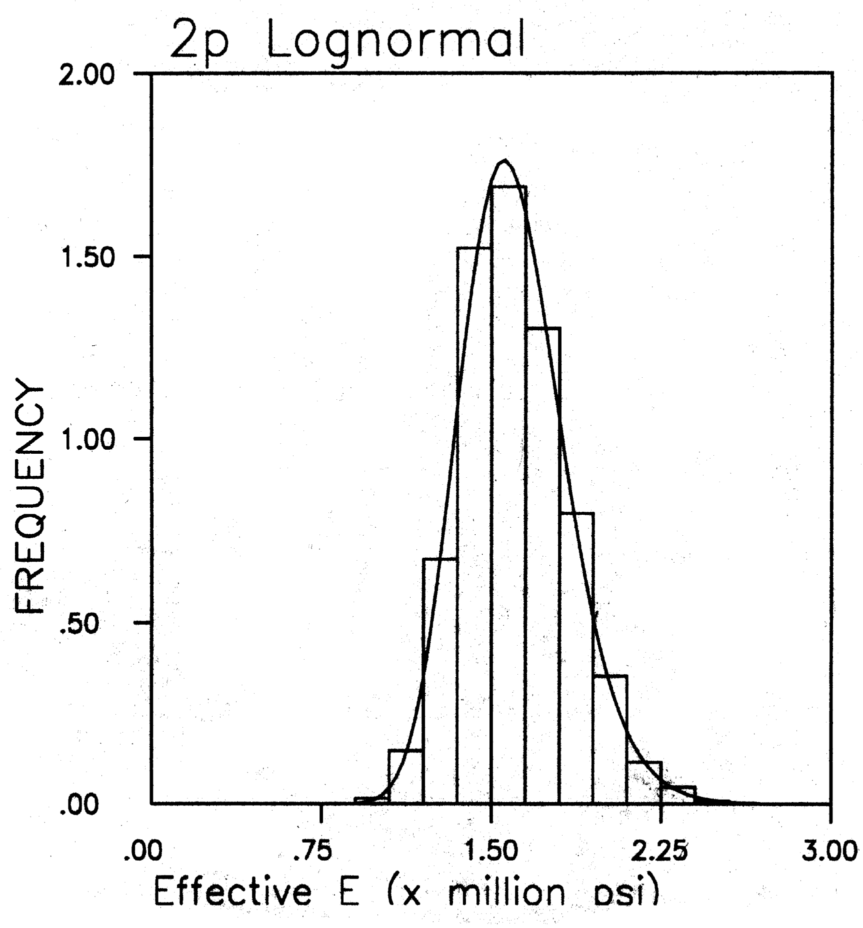

Figure 5 **Five thousand values were generated for the girder effective E (E_{eff}) by Monte Carlo simulation using Equation 14. A two-parameter lognormal distribution was superimposed on the histogram by GDA for the girder E_{eff} for VSR lumber and was determined to be the best-fit distribution for the data.**

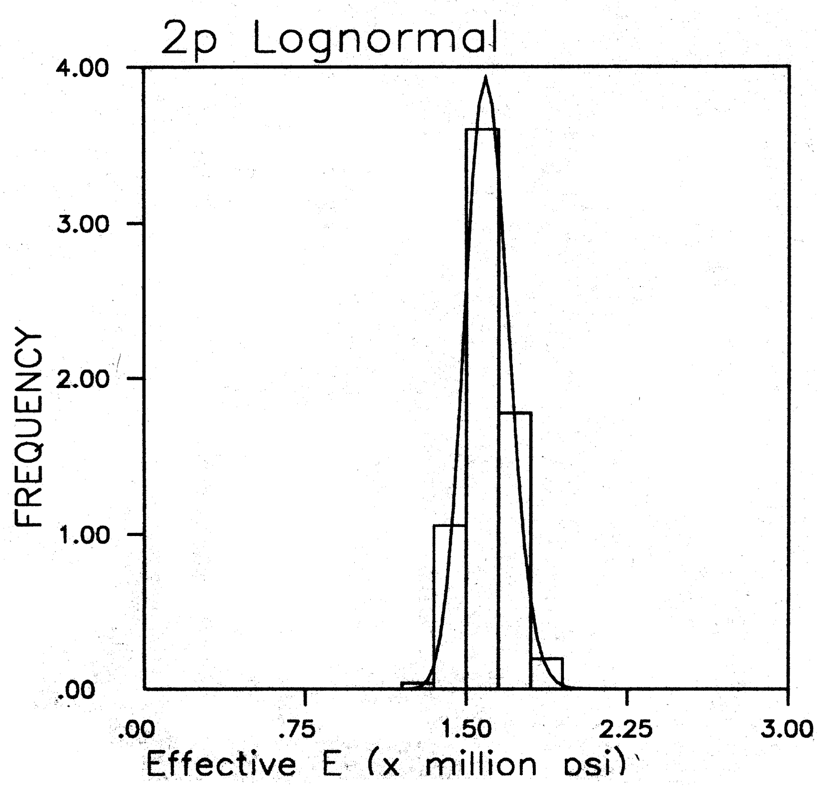

Figure 6 **Five thousand values were generated for the girder effective E (E_{eff}) by Monte Carlo simulation using Equation 14. A two-parameter lognormal distribution was superimposed on the histogram by GDA for the girder E_{eff} for MSR lumber and was determined to be the best-fit distribution for the data.**

The coefficient of variation of E_{eff} is therefore

$$\frac{\left(\frac{\bar{E}\Omega_E}{\sqrt{3}}\right)}{\bar{E}} = \frac{\Omega_E}{\sqrt{3}} \tag{17}$$

The two-parameter lognormal distribution fitted the data well for girder frequency (Figure 7 and Figure 8) based collectively on the visual test, the maximum log-likelihood, the Chi-Squared Test, and the Kolmogorov-Smirnov test. The girder frequency distribution for MSR lumber (Figure 8) was less variable than the distribution for VSR lumber (Figure 7). Upon inspection of Figure 7 and Figure 8, the probability that the frequency of the girder is less than 10 Hz would be zero for each case.

For the frequency of the floor system, the normal distribution fit the data well for VSR lumber (Figure 9) and MSR lumber (Figure 10) based collectively on the visual test, the maximum log-likelihood, the Chi-Squared test, and the Kolmogorov-Smirnov test. The system frequency distribution for MSR lumber (Figure 10) was also less variable than the system frequency distribution for VSR lumber (Figure 9). The probability that the system frequency is less than 10 Hz for VSR lumber (Figure 9) is 0.0537 while the same probability for MSR lumber (Figure 10) is zero. The first percentile of VSR lumber and MSR lumber is 9.34 Hz and 10.60 Hz, respectively. The difference between the first percentile values using Equation 1 is about 13 percent, which is caused by differences in Ω_E.

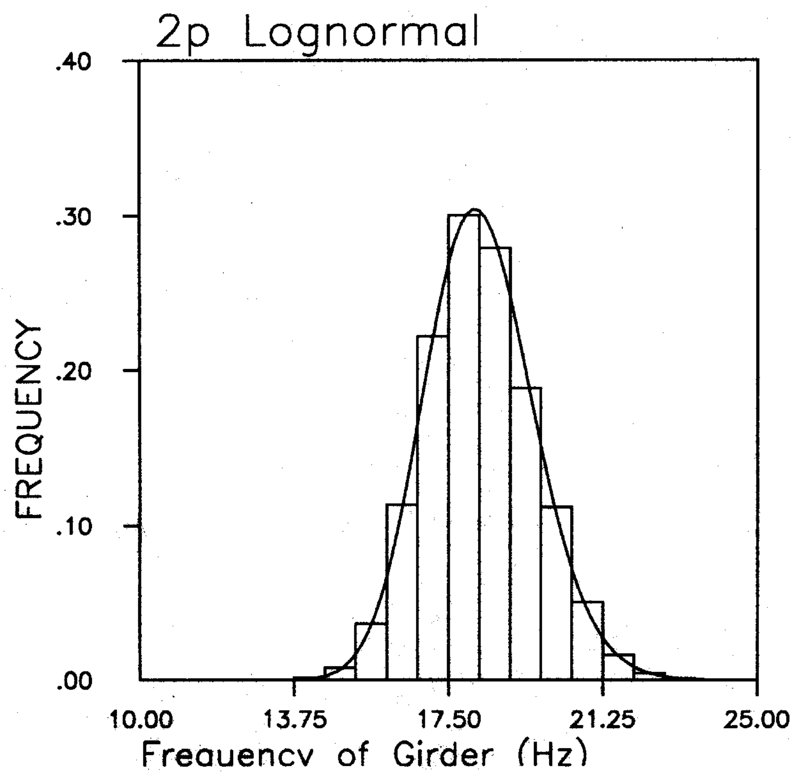

Figure 7 **Five thousand values were generated for the girder frequency (f_{girder}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{girder} for VSR lumber and was determined to be the best-fit distribution for the data.**

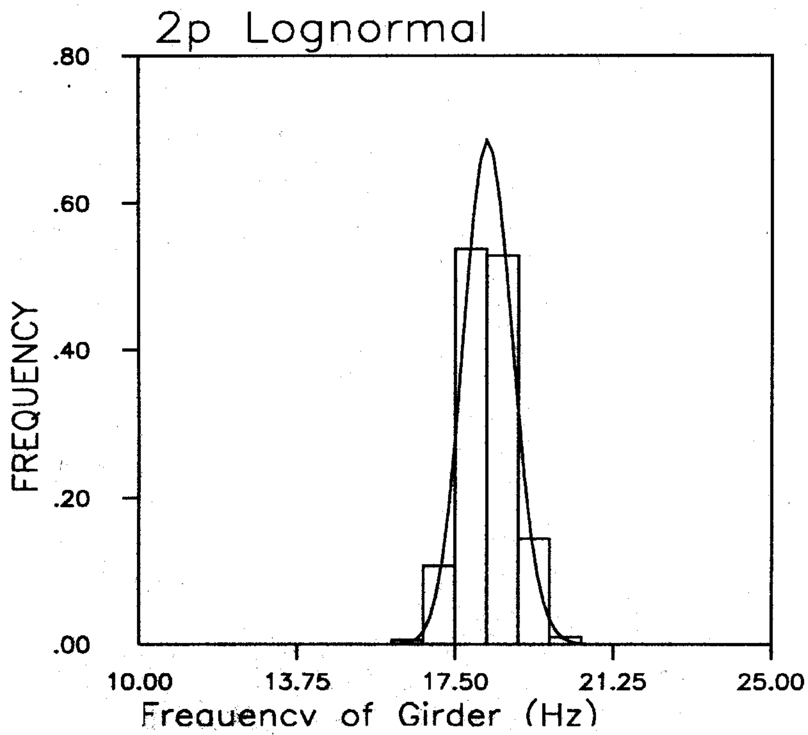

Figure 8 Five thousand values were generated for the girder frequency (f_{girder}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{girder} for MSR lumber and was determined to be the best-fit distribution for the data.

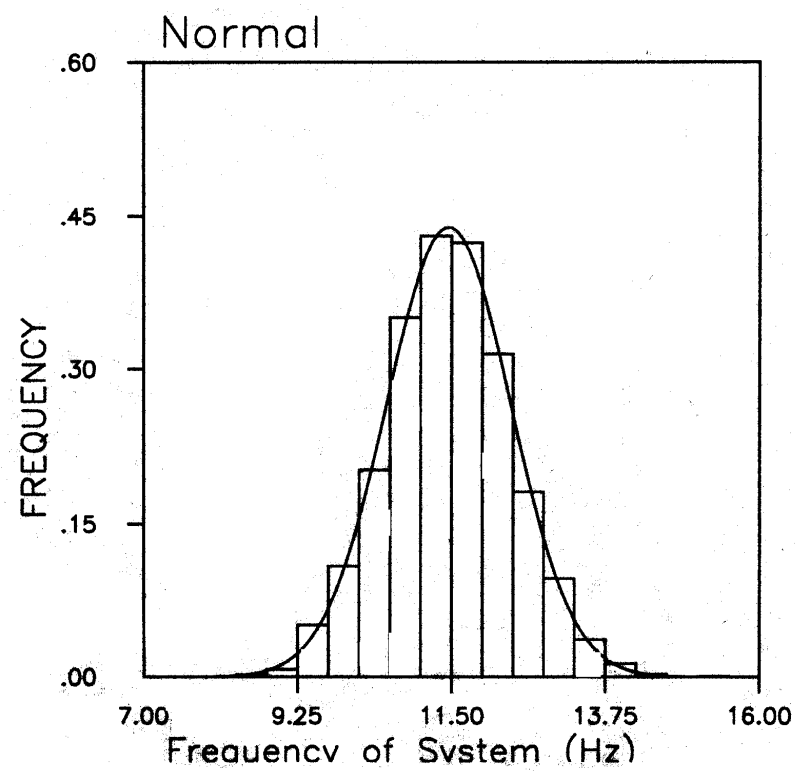

Figure 9 **Five thousand values were generated for the floor system frequency (*f_{system}*) by Monte Carlo simulation using Equation 3. A normal distribution was superimposed on the histogram by GDA for *f_{system}* for VSR lumber and was determined to be the best-fit distribution for the data.**

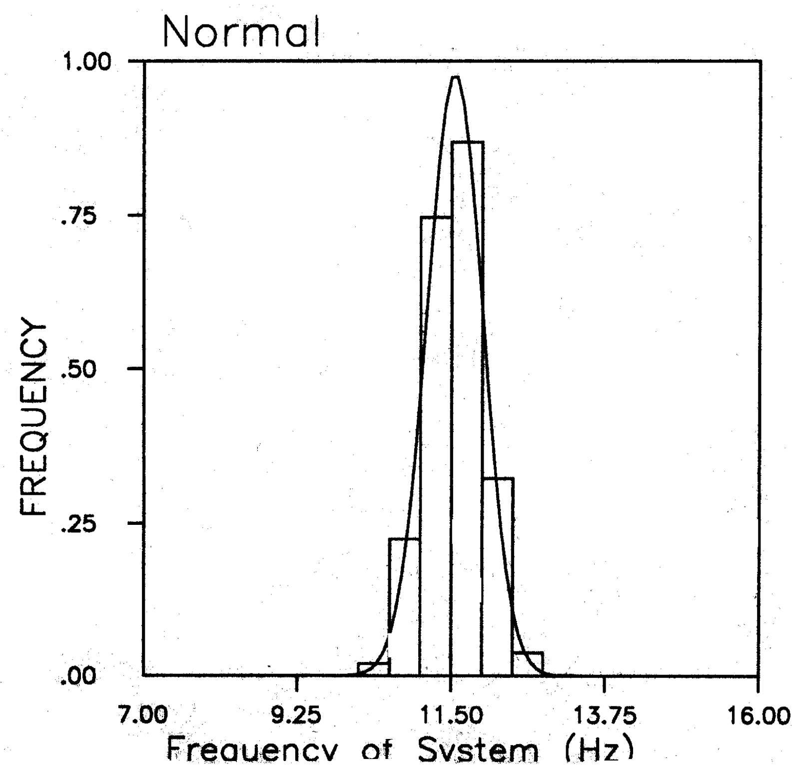

Figure 10 Five thousand values were generated for the floor system frequency (f_{system}) by Monte Carlo simulation using Equation 3. A normal distribution was superimposed on the histogram by GDA for f_{system} for MSR lumber and was determined to be the best-fit distribution for the data.

Case 3

Results for the E-distribution for both MSR and VSR lumber (Figure 11 and Figure 12) are identical to the results given for the E-distribution for Case 1. They are identical to Case 1 because the values used to calculate the parameters for the E-distribution in Equations 5 and 6 did not change across floor cases. These values remained constant throughout the floor cases studied. The only variable that did change was Ω_E.

Results for the frequency distribution differed from the results for the frequency distribution in Case 1 because the joists were designed to a stricter deflection limit (L/480). The frequency distribution for both MSR and VSR lumber shifted to the right which resulted in better expected vibrational performance than in Case 1. The calculated frequency distribution of a joist for VSR lumber is shown in Figure 13. The predicted frequency is lognormally distributed because E was assumed to be lognormal. The calculated frequency distribution for MSR lumber is represented in Figure 14 and it is also lognormally distributed. The frequency distribution in Figure 14 for MSR lumber is also noticeably less variable than the frequency distribution for VSR lumber in Figure 13.

The probability that the frequency is less than 10 Hz in Figure 13 is practically zero. Theoretically, the lognormal density functions starts at zero, thus some very small area exists between 0.0 and 10.0. Hereafter, this case will be referred to as zero area. The first percentile of the distribution is 12.98. For Figure 14, the probability that the frequency is less than 10 Hz is also zero. The first percentile of the frequency distribution for MSR lumber is 15.36 which is 18

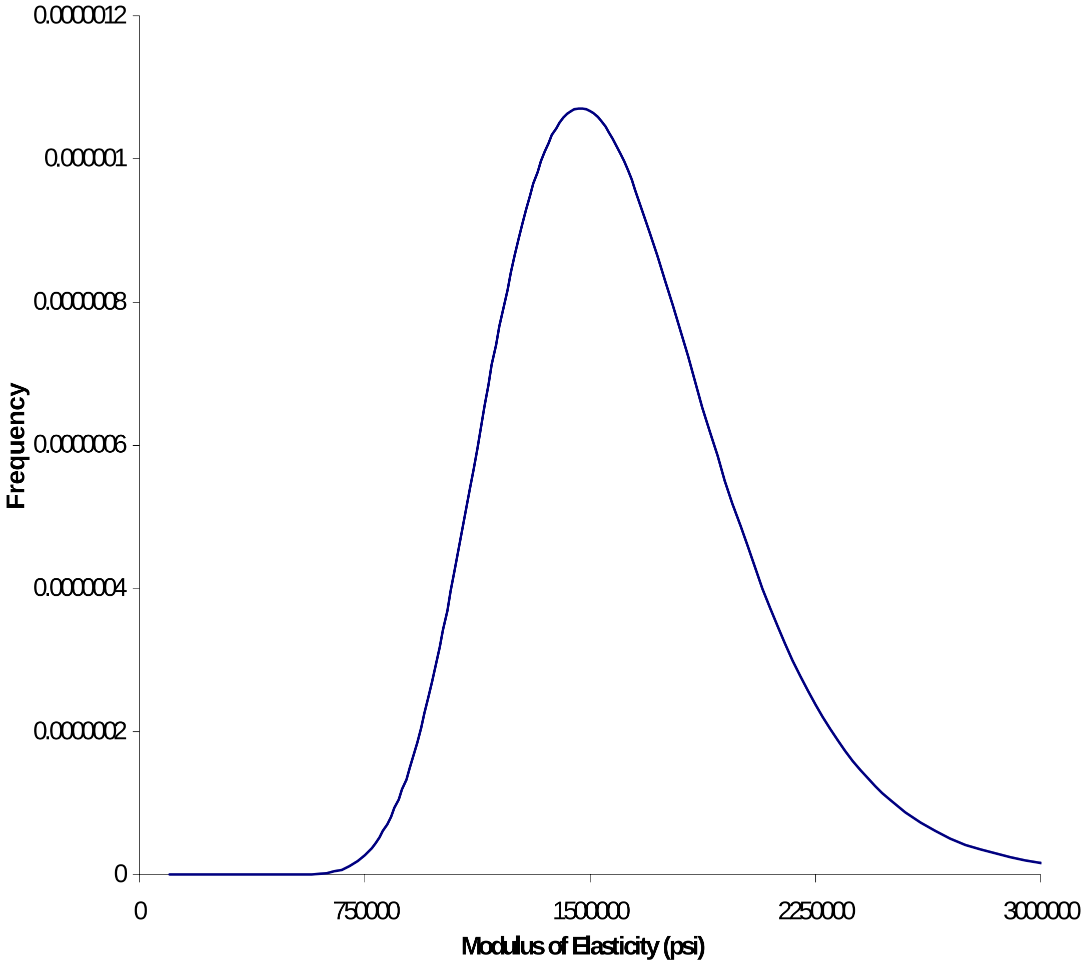

Figure 11 **Two-parameter lognormal distribution was assumed for the modulus of elasticity (E) for VSR lumber (E coefficient of variation=0.25) in Case 3.**

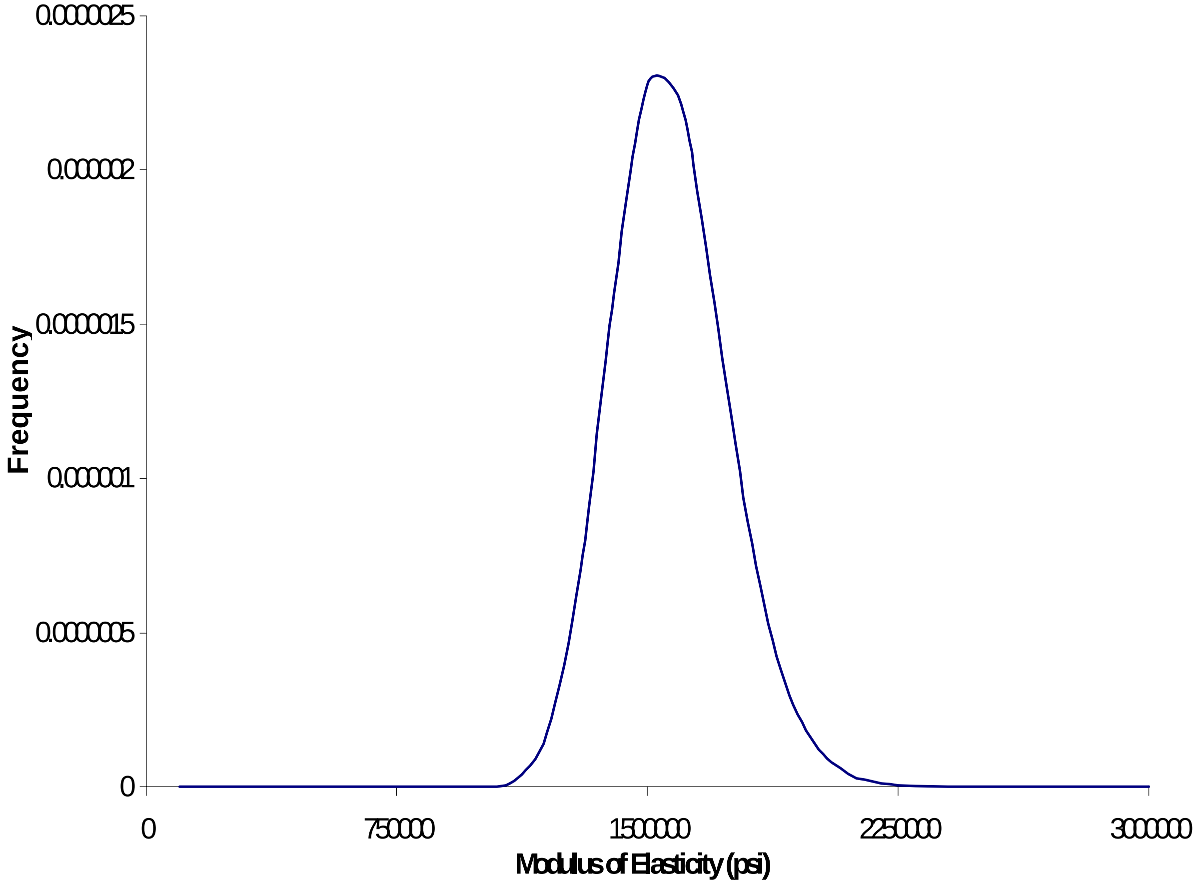

Figure 12 Two-parameter lognormal distribution was assumed for the modulus of elasticity (E) for MSR lumber (E coefficient of variation=0.11) in Case 3.

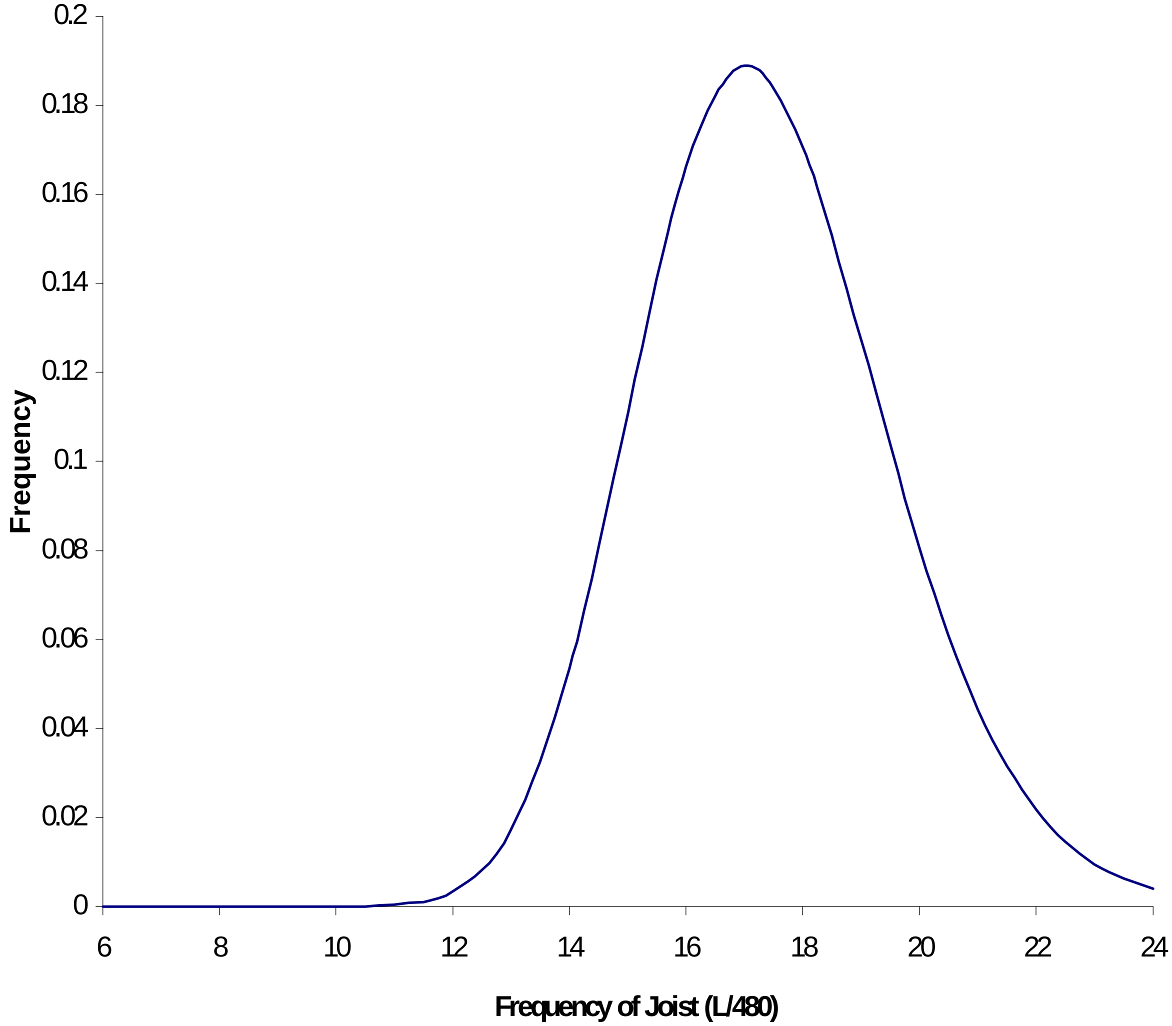

Figure 13 **Two-parameter lognormal distribution and parameters were also derived for VSR lumber with joist deflection limit of L/480 using Equation 2 and Equations 9 through 13.**

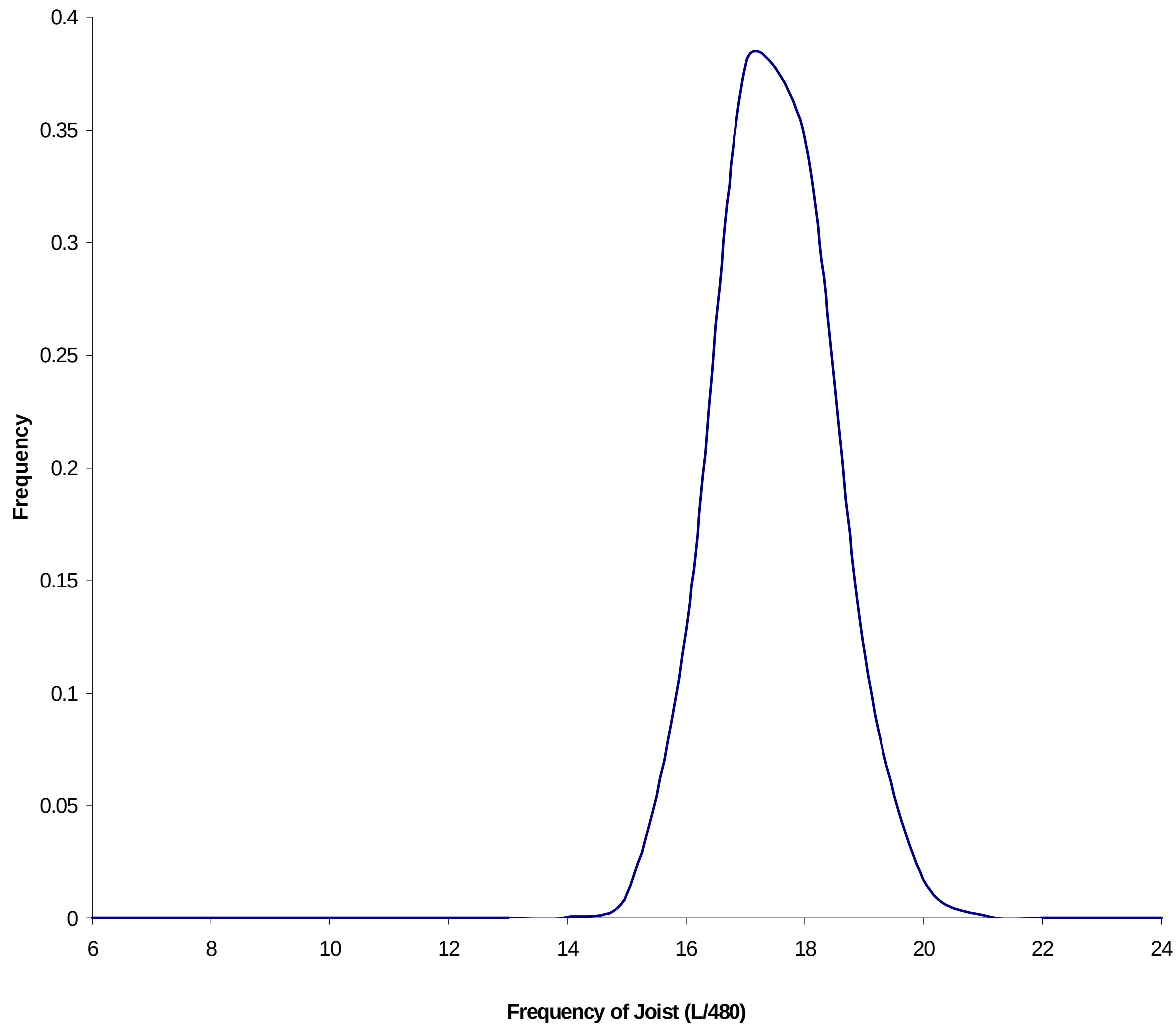

Figure 14 Two-parameter lognormal distribution and parameters for frequency of joist vibration for MSR lumber (with deflection limit of L/480) were also derived using Equation 2 and Equations 9 through 13.

percent more than the first percentile for VSR lumber according to Equation 1. In other words, the expected vibrational performance for MSR lumber in this case is 18 percent better than VSR lumber.

Case 4

The results for effective E, E_{eff}, are similar to the results for E_{eff} in Case 2. The two-parameter lognormal distribution fit the data for both VSR (Figure 15) and MSR (Figure 16) lumber. An averaging effect can also be observed in Figure 15 and Figure 16 when compared to Figure 1 and Figure 2 in Case 1.

The two-parameter lognormal also fit the data well for girder frequency (Figures 17 and 18) based collectively on the visual test, the maximum log-likelihood, the Chi-Squared test, and the Kolmogorov-Smirnov test. The distribution for MSR lumber (Figure 18) is less variable than the distribution for VSR lumber (Figure 17). The probability that the frequency of the girder is less than 10 Hz would be approximately zero for both MSR and VSR lumber.

The normal distribution fit the data for frequency of the floor system (f_{system}) for both VSR and MSR lumber (Figures 19 and 20). This was based collectively on the visual test, maximum log-likelihood, Chi-Squared test, and Kolmogorov-Smirnov test. The distribution for MSR lumber (Figure 20) was also less variable than the distribution for VSR lumber (Figure 19). For MSR lumber, the probability that f_{system} is less than 10 Hz is zero. The probability for VSR lumber is 0.0016. The first percentile of f_{system} for MSR lumber is 11.91 Hz compared to 10.58 Hz for VSR

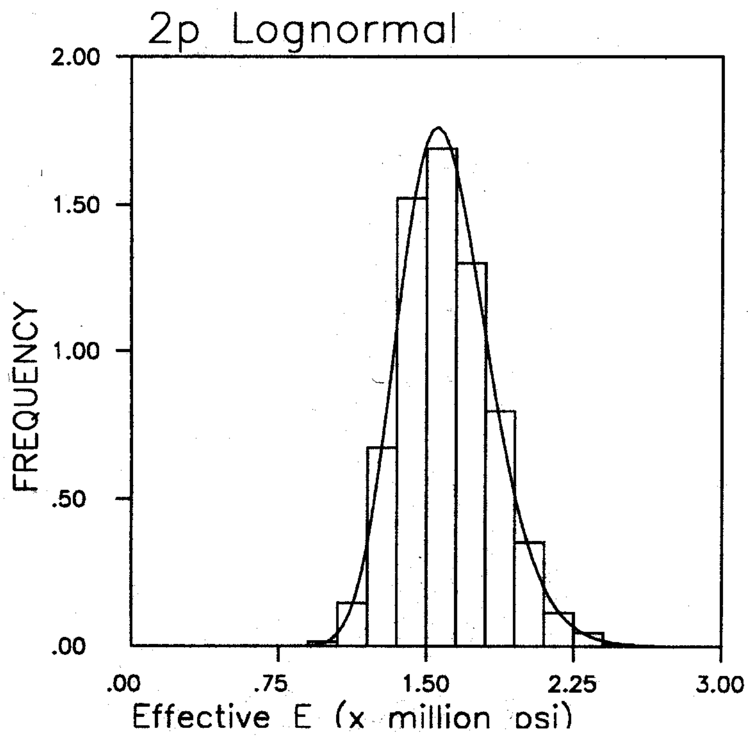

Figure 15 Five thousand values were generated for the girder effective E (E_{eff}) by Monte Carlo simulation using Equation 14. A two-parameter lognormal distribution was superimposed on the histogram by GDA for the girder E_{eff} for VSR lumber and was determined to be the best-fit distribution for the data.

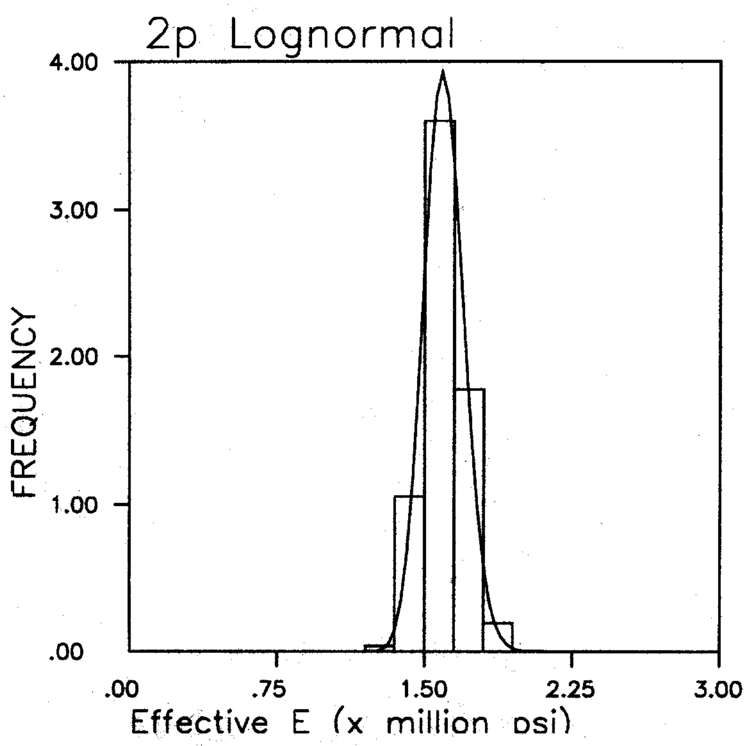

Figure 16 **Five thousand values were generated for the girder effective E (E_{eff}) by Monte Carlo simulation using Equation 14. A two-parameter lognormal distribution was superimposed on the histogram by GDA for the girder E_{eff} for MSR lumber and was determined to be the best-fit distribution for the data.**

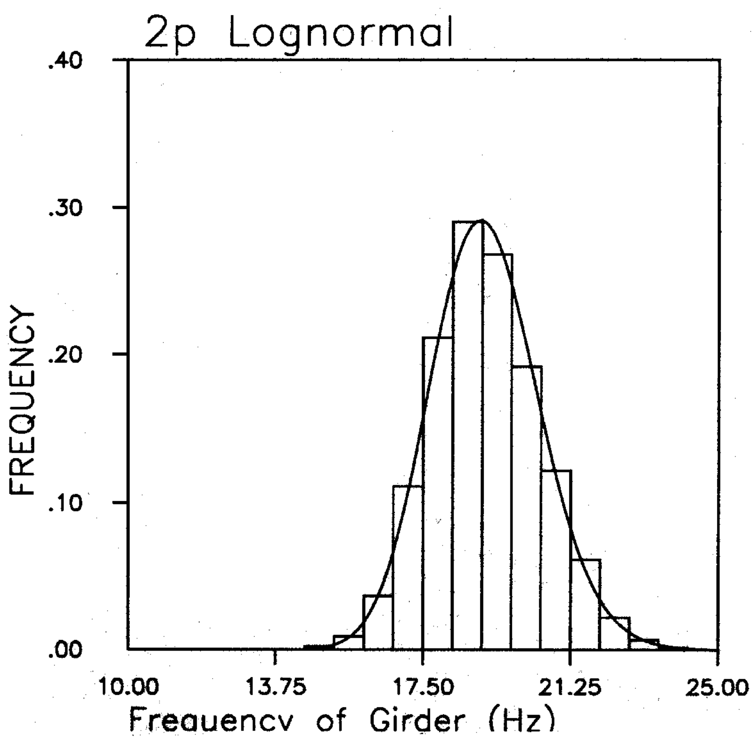

Figure 17 **Five thousand values were generated for the girder frequency (f_{girder}) Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{girder} for VSR lumber and was determined to be the best-fit distribution for the data.**

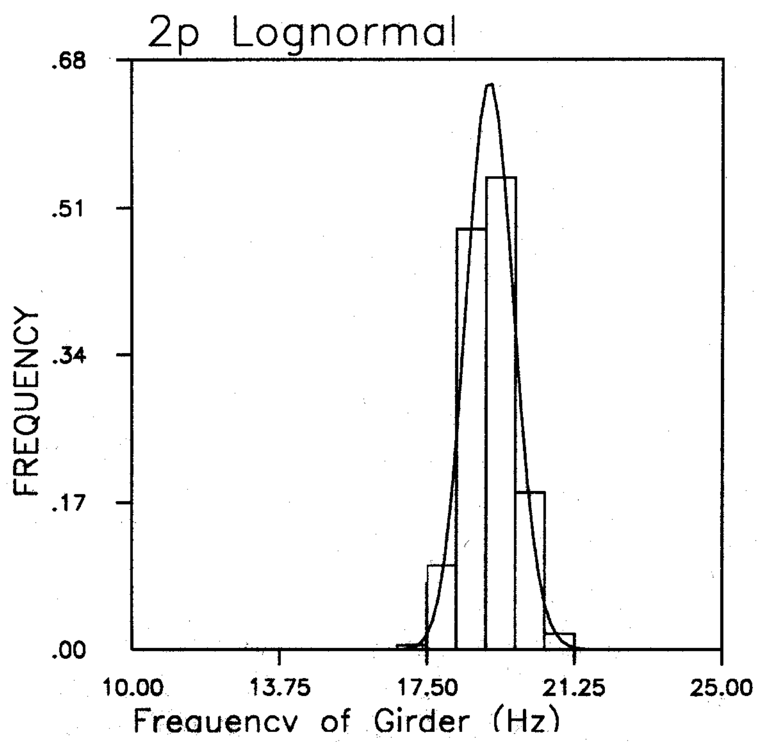

Figure 18 **Five thousand values were generated for the girder frequency (f_{girder}) Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{girder} for MSR lumber and was determined to be the best-fit distribution for the data.**

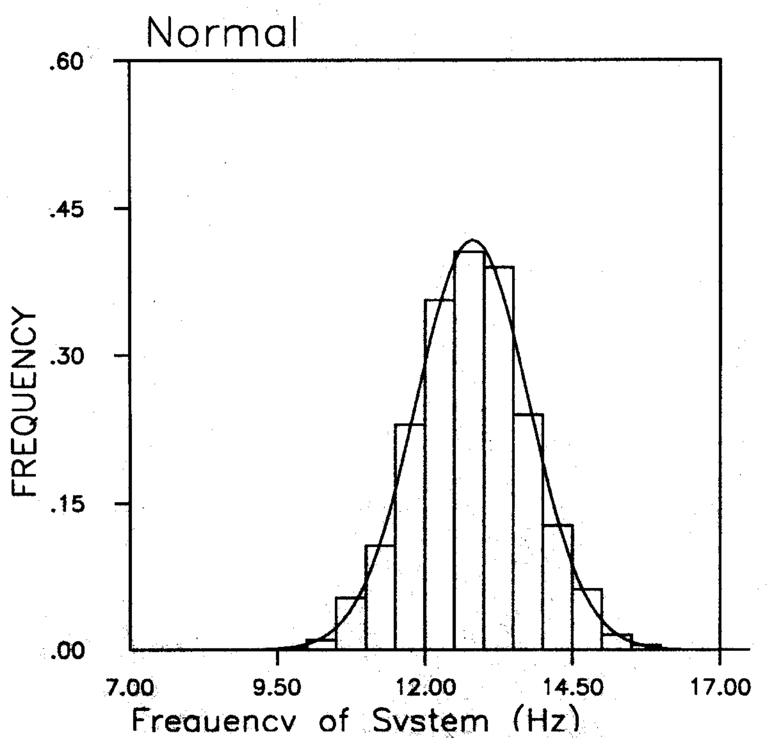

Figure 19 **Five thousand values were generated for the floor system frequency (f_{system}) by Monte Carlo simulation using Equation 3. A normal distribution was superimposed on the histogram by GDA for f_{system} for VSR lumber and was determined to be the best-fit distribution for the data.**

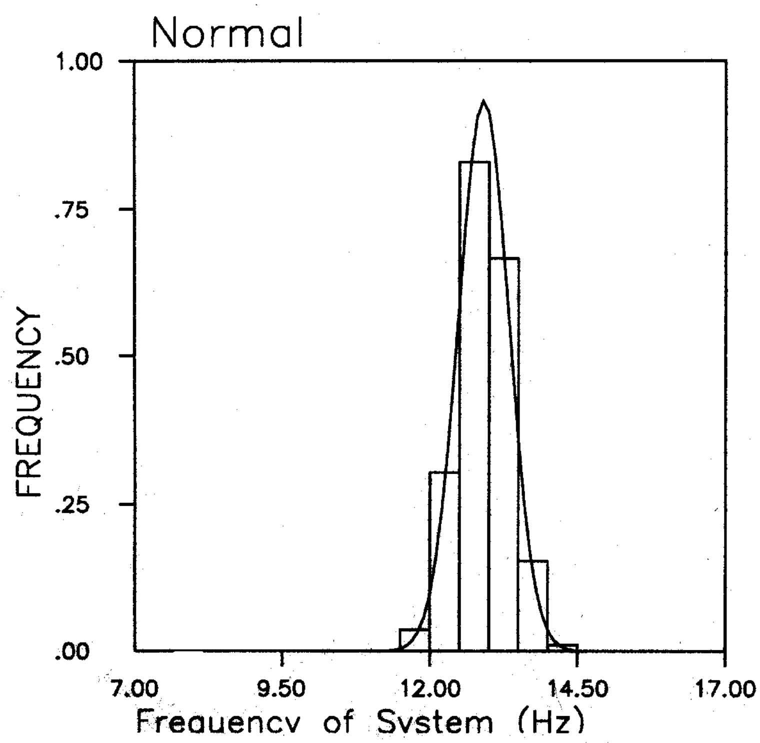

Figure 20 **Five thousand values were generated for the floor system frequency (f_{system}) by Monte Carlo simulation using Equation 3. A normal distribution was superimposed on the histogram by GDA for f_{system} for MSR lumber and was determined to be the best-fit distribution for the data.**

lumber. The ratio of the first percentile of MSR lumber to the first percentile of VSR lumber is 1.13 which indicates that an MSR lumber floor system will have better floor performance than VSR lumber by 13 percent.

A limit on f_{system} for good floor performance has been proposed by Shue (1995) for continuous girders. The 13 percent difference between first percentiles for MSR and VSR lumber in Case 4 may be significant. Based on in-situ tests, he first calculated f_{system} based on the clear span between points of support and the prediction did not correlate well with measured results. He then determined that the span between points of inflection produced results that correlated better with the measured floor vibrational response.

In comparing the girder frequency and system frequency in Case 4 with the corresponding distributions in Case 2, the distributions in Case 4 are slightly shifted more to the right. This shift to the right in Case 4 can be attributed to the joists and girder being designed to a more strict deflection limit (L/480 and L/600, respectively).

Case 5 (Load Sharing)

The two-parameter lognormal distribution fit the data well in Figures 21 and 22 for effective joist E, E_j, based collectively on the visual test, maximum log-likelihood, Chi-Squared test, and Kolmogorov-Smirnov test. Again, MSR lumber effective joist E based on Equation 15 (Figure 22) is less variable than VSR lumber effective joist E (Figure 21).

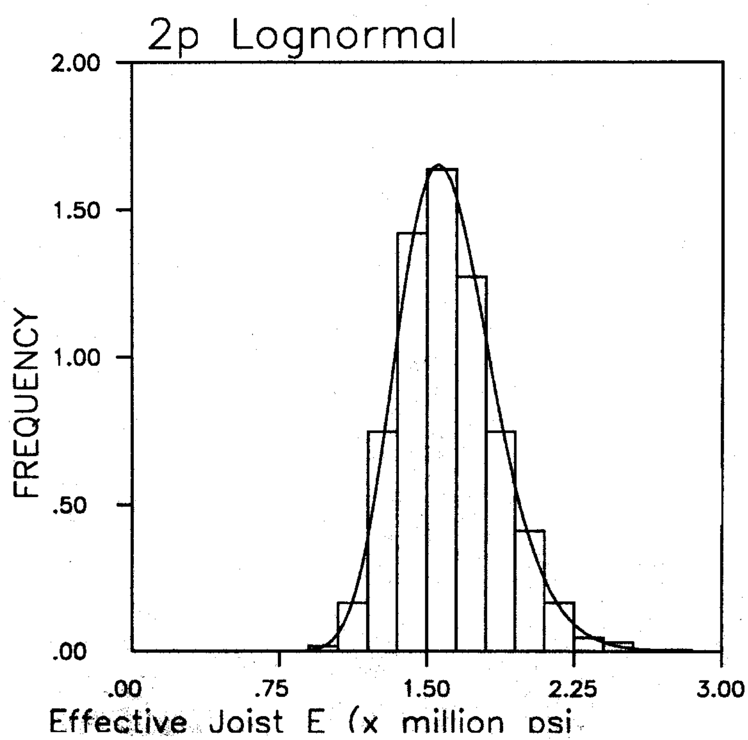

Figure 21 Five thousand values were generated for the effective joist E (E_j) by Monte Carlo simulation using Equation 15. A two-parameter lognormal distribution was superimposed on the histogram by GDA for E_j for VSR lumber and was determined to be the best-fit distribution for the data.

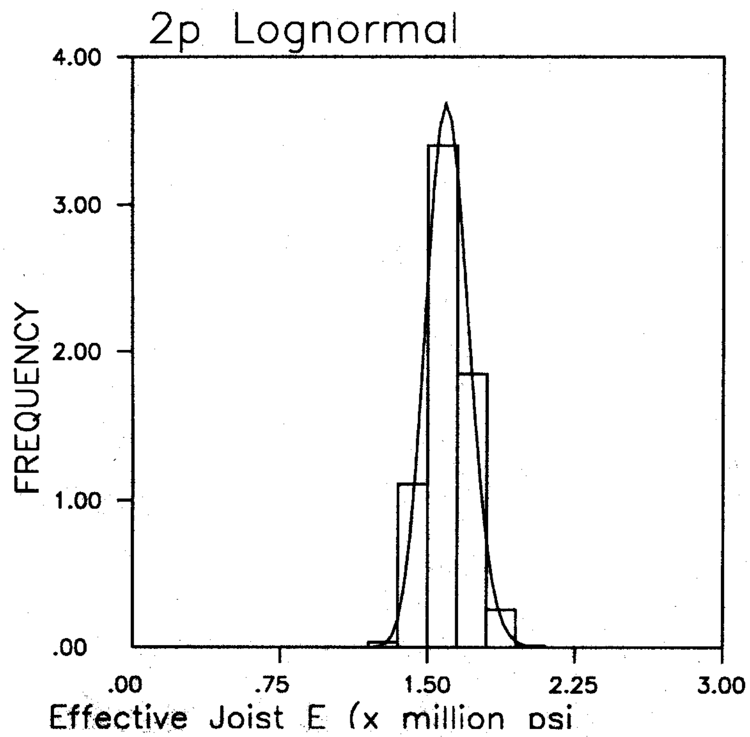

Figure 22 Five thousand values were generated for the effective joist E (E_j) by Monte Carlo simulation using Equation 15. A two-parameter lognormal distribution was superimposed on the histogram by GDA for E_j for MSR lumber and was determined to be the best-fit distribution for the data.

For the predicted joist frequency, f_{joist}, the two-parameter lognormal distribution fit the data as well in Figures 23 and 24 based on the same tests. MSR lumber (Figure 24) was less variable than VSR lumber (Figure 23).

The probability that f_{joist} is below 10 Hz for both VSR and MSR lumber was zero. The first percentile for Figure 24 is 13.79 Hz compared to 12.47 Hz for Figure 23. The ratio of the first percentile of MSR lumber to the first percentile of VSR lumber is 1.11. With load sharing, MSR lumber in case 5 performs 11 percent better than VSR lumber with respect to the first percentile predicted fundamental frequency as the measure of floor performance.

The effective joist E (Figures 21 and 22) for both VSR and MSR lumber in Case 5 is slightly less variable and has a smaller range than the E-distribution in Figure 1 and Figure 2 in Case 1. In Figure 21, the distribution ranges from about $1.0x10^6$ psi to about $2.75x10^6$ psi. The distribution in Figure 1 ranges from about 700,000 psi to about $3.0x10^6$ psi. For MSR lumber in Case 5, the range in Figure 22 is from about $1.2x10^6$ psi to $2.0x10^6$ psi. In Case 1, the distribution in Figure 2 ranges from $1.0x10^6$ psi to $2.25x10^6$ psi. The results in Case 5 for E_j is due to Equation 15 being a weighted average which accounts for the distributions having a smaller range, or lower standard deviation.

The distribution for frequency of joist (Figures 23 and 24) for both VSR and MSR lumber in Case 5 is also slightly less variable and has a smaller range than the distribution for freuqency of joist in Figures 3 and 4 in Case 1. In Figure 23, the distribution ranges from about 12 to 18 Hz compared

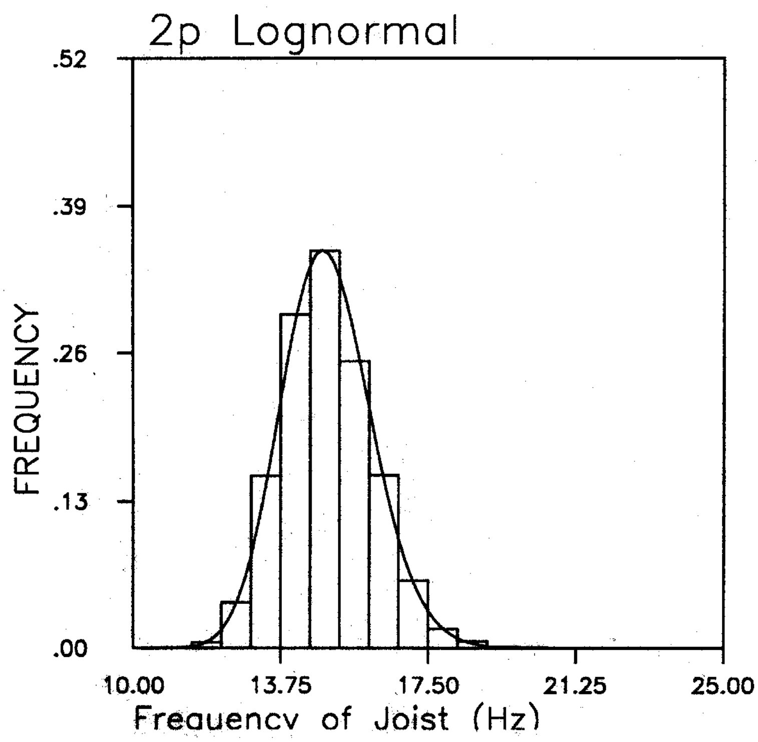

Figure 23 Five thousand values were generated for the joist frequency (f_{joist}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{joist} for VSR lumber and was determined to be the best-fit distribution for the data.

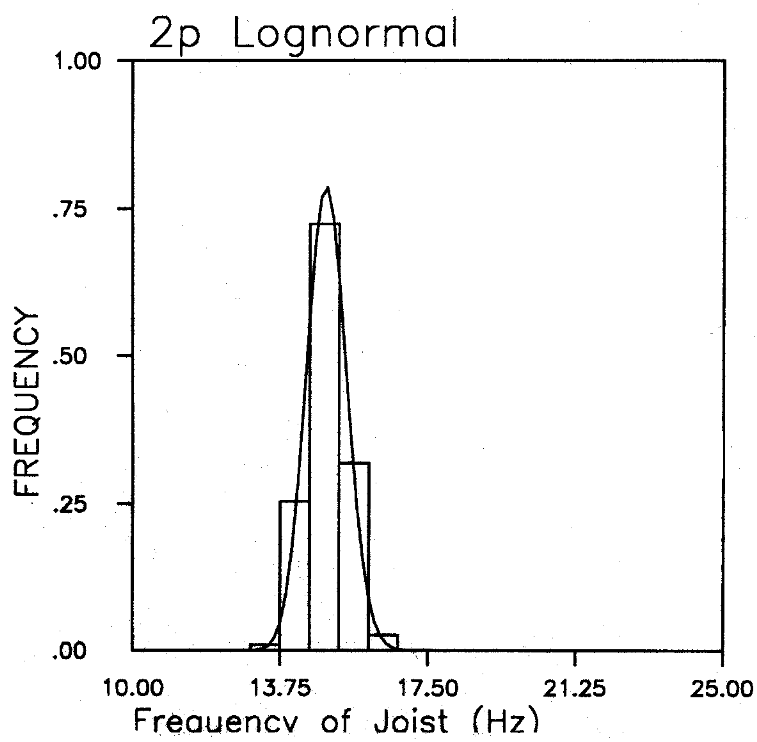

Figure 24 **Five thousand values were generated for the joist frequency (f_{joist}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{joist} for MSR lumber and was determined to be the best-fit distribution for the data.**

to the range of 9 to 23 Hz in Figure 3 for Case 1. For MSR lumber (Figure 24) in Case 5, the range is 13 to 16 Hz compared to the range for MSR lumber (Figure 4) in Case 1 of 12 to 18 Hz.

Case 6 (Load Sharing)

The two-parameter lognormal distribution fit the data in Figures 25 and 26 for the effective E, E_{eff}, of a girder for both VSR and MSR lumber. This distribution was chosen based collectively on the visual test, maximum log-likelihood, Chi-Squared, and Kolmogorov-Smirnov test. The distribution in Figure 26 is less variable than the distribution in Figure 25.

The two-parameter lognormal distribution was also the best fit for the data for girder frequency in Figures 27 and 28. MSR lumber for girder frequency (Figure 28) was also less variable than VSR lumber (Figure 27). Upon inspection of Figures 27 and 28 for VSR and MSR lumber, the probability of f_{girder} being less than 10 Hz is zero.

For Figures 29 and 30, the two-parameter lognormal distribution was the best fit for the system frequency based collectively on the visual test, maximum log-likelihood, and Kolmogorov-Smirnov test. The MSR lumber system frequency (Figure 30) was less variable than the VSR lumber system frequency (Figure 29). The probability that the system frequency is less than 10 Hz for MSR lumber is zero compared to 0.0045 for VSR lumber. The first percentile of MSR system frequency for the distribution in Figure 30 is 10.93 Hz. The first percentile of VSR system frequency for the distribution in Figure 29 is 10.15 Hz. The ratio of the first percentile of the

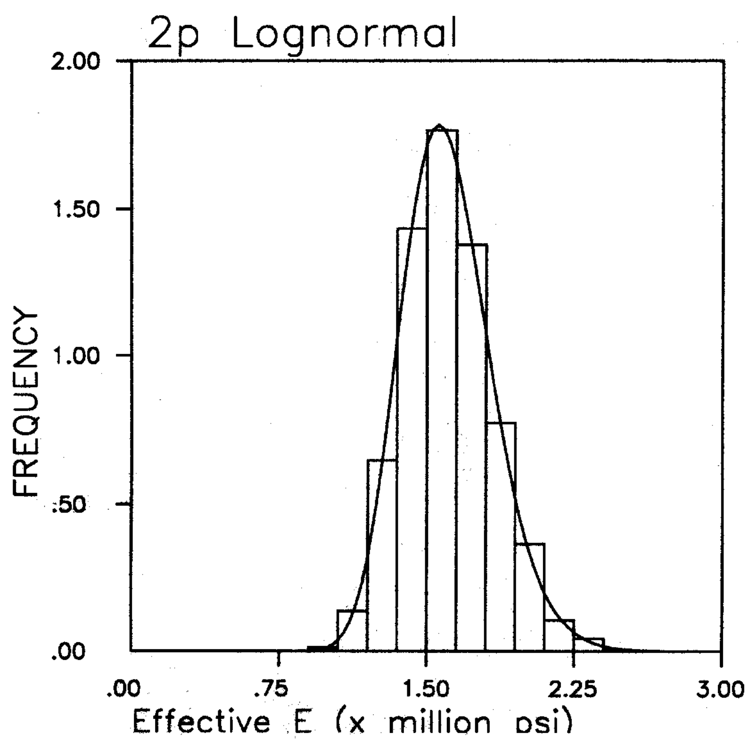

Figure 25 **Five thousand values were generated for the girder effective E (E_{eff}) by Monte Carlo simulation using Equation 14. A two-parameter lognormal distribution was superimposed on the histogram by GDA for the girder (E_{eff}) for VSR lumber and was determined to be the best-fit distribution for the data.**

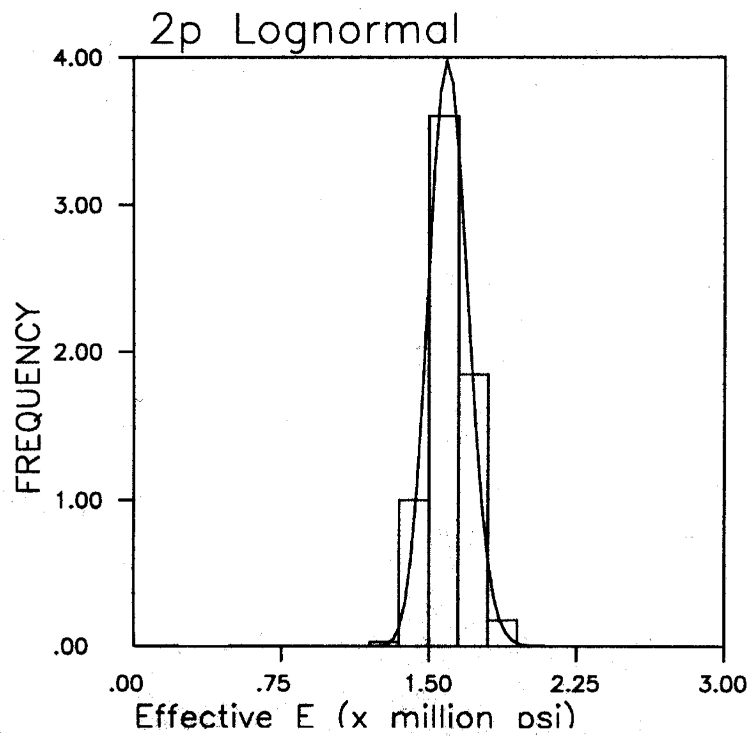

Figure 26 **Five thousand values were generated for the girder effective E (E_{eff}) by Monte Carlo simulation using Equation 14. A two-parameter lognormal distribution was superimposed on the histogram by GDA for the girder (E_{eff}) for MSR lumber and was determined to be the best-fit distribution for the data.**

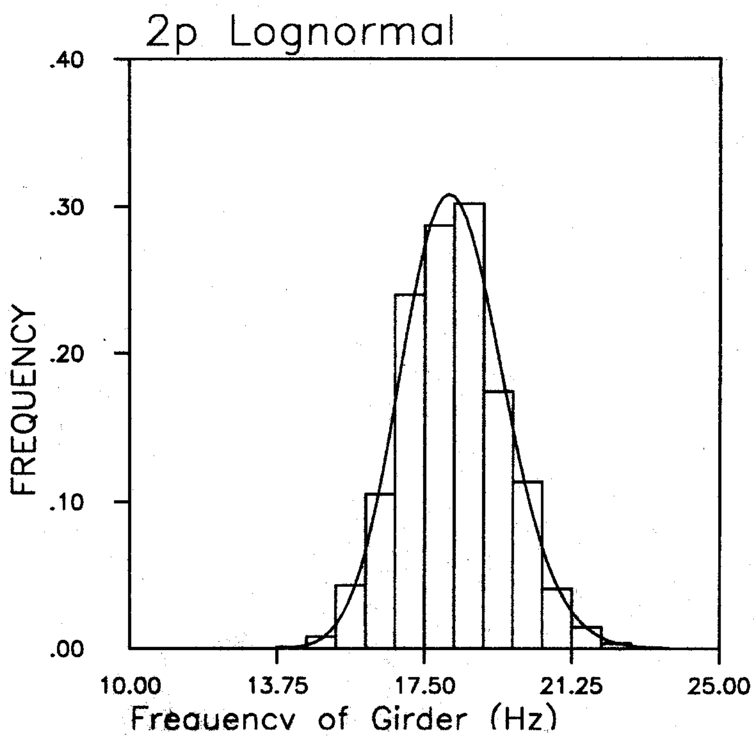

Figure 27 **Five thousand values were generated for the girder frequency (f_{girder}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{girder} for VSR lumber and was determined to be the best-fit distribution for the data.**

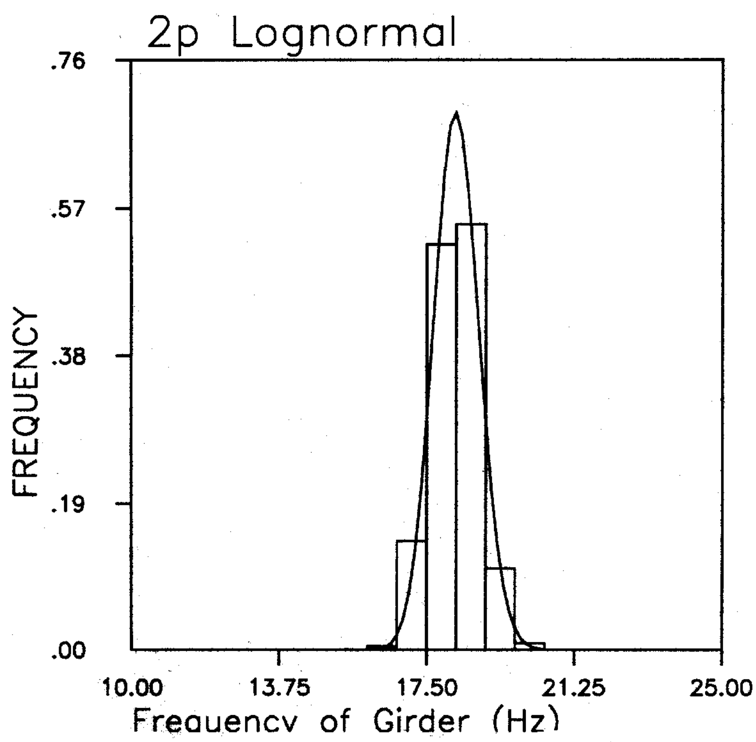

Figure 28 **Five thousand values were generated for the girder frequency (f_{girder}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{girder} for MSR lumber and was determined to be the best-fit distribution for the data.**

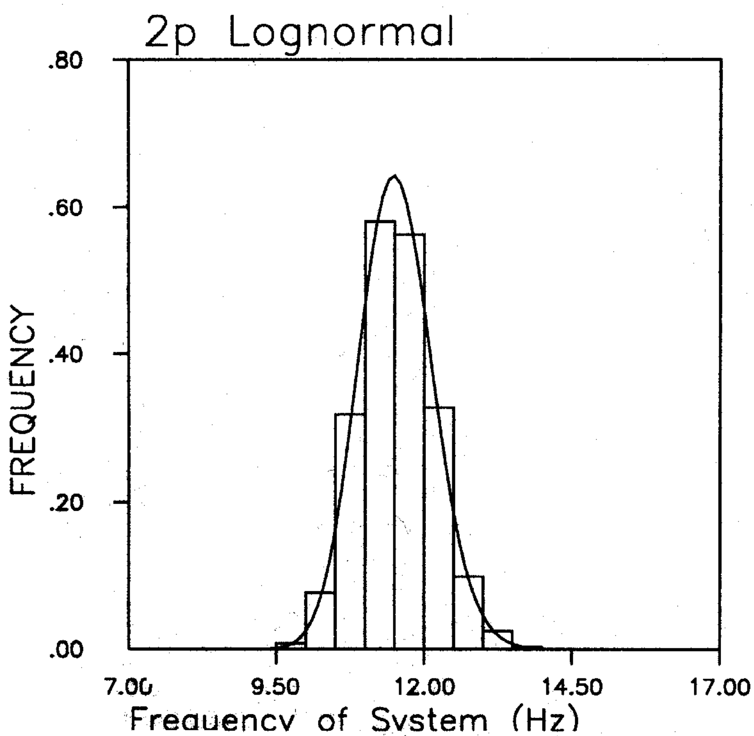

Figure 29 **Five thousand values were generated for the floor system frequency (f_{system}) by Monte Carlo simulation using Equation 3. A two-parameter distribution was superimposed on the histogram by GDA for the f_{system} for VSR lumber and was determined to be the best-fit distribution for the data.**

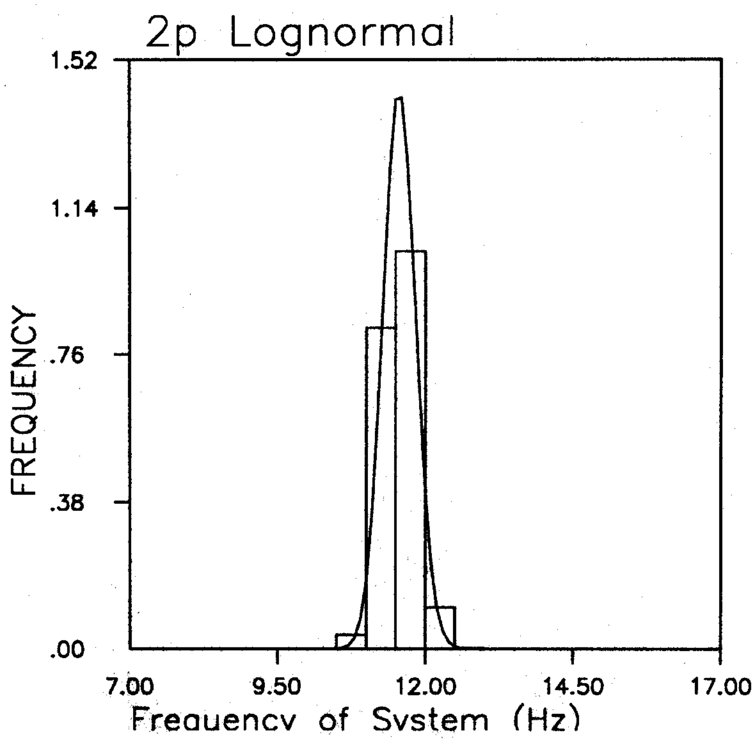

Figure 30 Five thousand values were generated for the floor system frequency (f_{system}) by Monte Carlo simulation using Equation 3. A two-parameter distribution was superimposed on the histogram by GDA for the f_{system} for MSR lumber and was determined to be the best-fit distribution for the data.

predicted system frequency for MSR lumber to the corresponding first percentile of VSR lumber was 1.08 which means that the MSR floor system with load sharing included has an 8 percent better predicted vibrational performance over a VSR floor system.

The f_{system} distributions in Case 6 (Figures 29 and 30) are less variable than the f_{system} distributions (Figures 9 and 10) in Case 2. The first percentile values of f_{system} for MSR and VSR lumber in Case 6 are larger than the first percentile values for MSR and VSR lumber in Case 2.

Case 7 (Load Sharing)

The two-parameter lognormal distribution fit the data well for effective joist E in Figures 31 and 32. The two-parameter lognormal distribution was determined to be the best fit based collectively on the visual test, maximum log-likelihood, Chi-Squared test, and Kolmogorov-Smirnov test. From Figures 31 and 32, it can be seen that the effective joist E for MSR lumber is less variable than VSR lumber.

Based on the visual test, maximum log-likelihood, Chi-Squared test, and Kolmogorov-Smirnov test, the two-parameter lognormal fit the f_{joist} distribution with load sharing for VSR (Figure 33) and MSR (Figure 34) lumber. The distribution for predicted joist frequency in Figure 34 is less variable than the distribution in Figure 33. The probability that f_{joist} is less than 10 Hz for both VSR and MSR lumber is zero. The first percentile of f_{joist} in Figure 34 (MSR lumber) is 16.22 Hz and the first percentile of the comparable distribution in Figure 33 (VSR lumber) is 14.66 Hz.

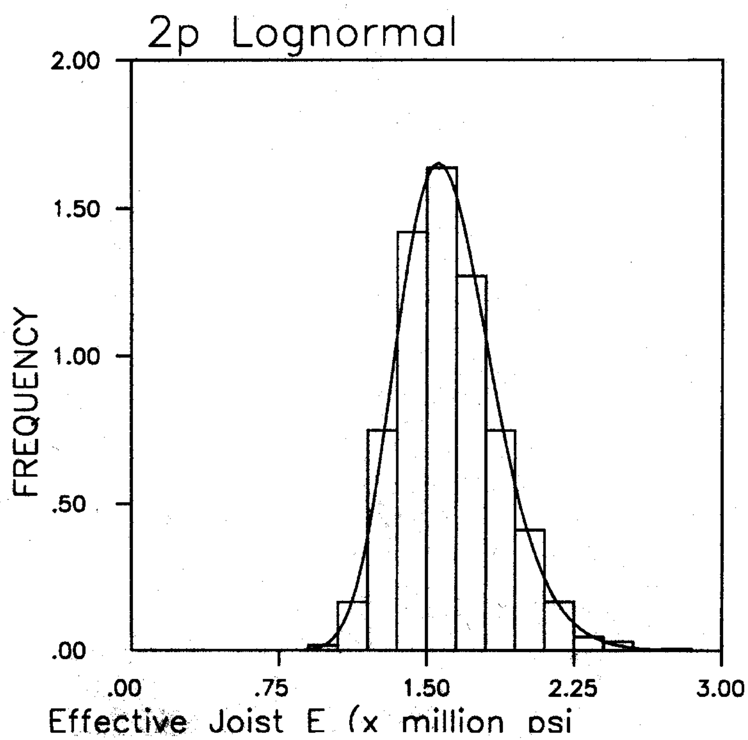

Figure 31 **Five thousand values were generated for the effective joist E (E_j) by Monte Carlo simulation using Equation 15. A two-parameter lognormal distribution was superimposed on the histogram by GDA for E_j for VSR lumber and was determined to be the best-fit distribution for the data.**

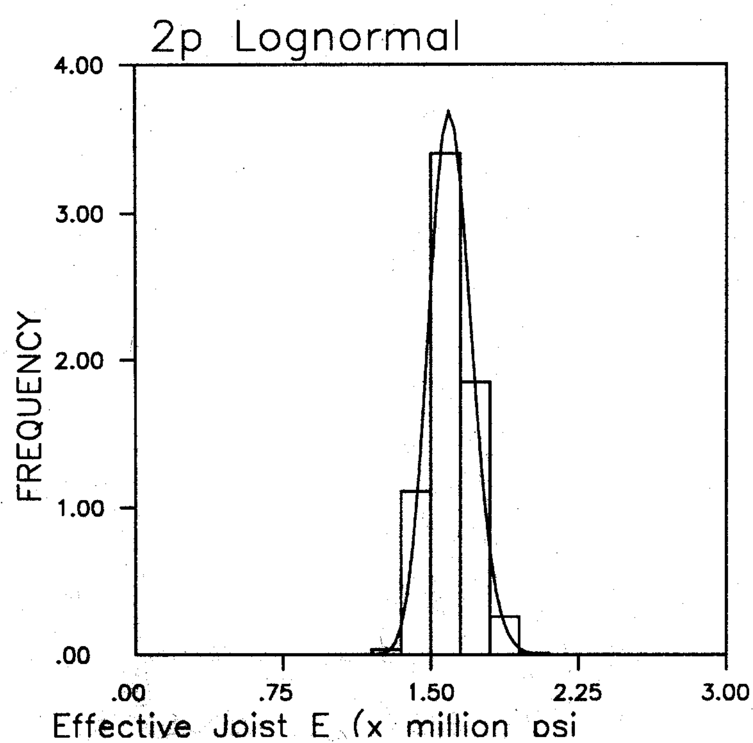

Figure 32 Five thousand values were generated for the effective joist E (E_j) by Monte Carlo simulation using Equation 15. A two-parameter lognormal distribution was superimposed on the histogram by GDA for E_j for MSR lumber and was determined to be the best-fit distribution for the data.

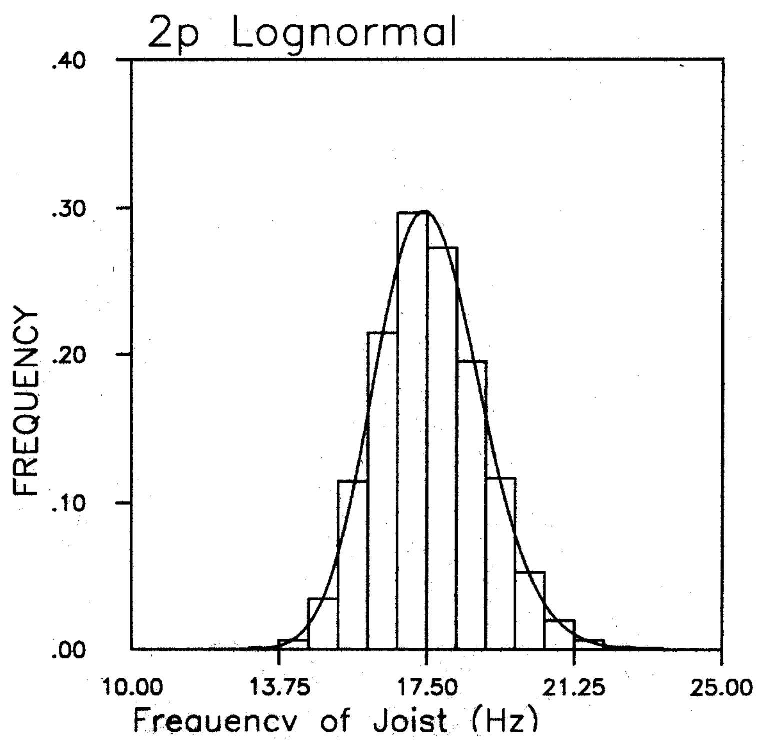

Figure 33 **Five thousand values were generated for the joist frequency (f_{joist}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{joist} for VSR lumber and was determined to be the best-fit distribution for the data.**

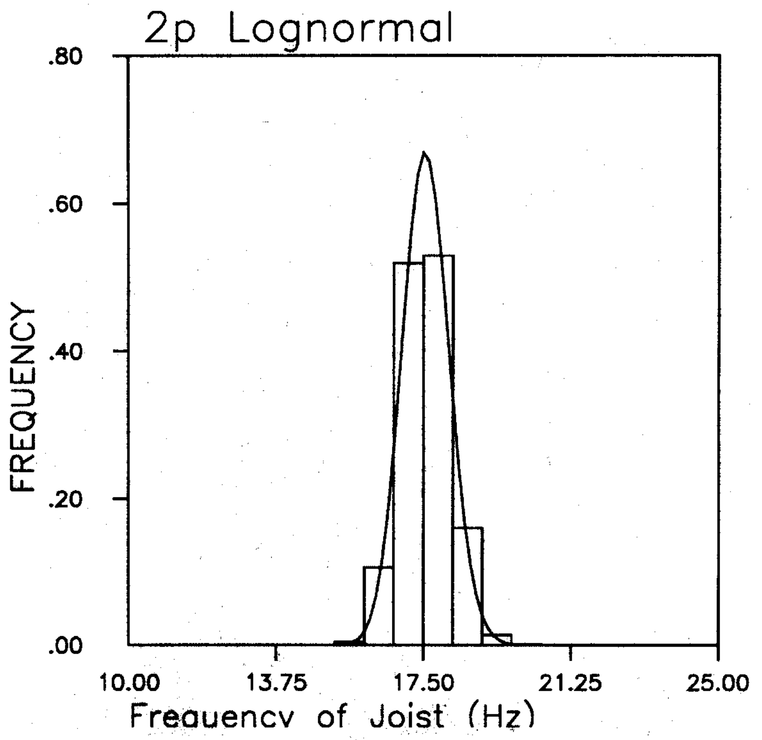

Figure 34 **Five thousand values were generated for the joist frequency (f_{joist}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{joist} for MSR lumber and was determined to be the best-fit distribution for the data.**

The ratio of these two numbers using equation 1 is 1.11 which reveals that MSR lumber has a better predicted vibrational performance than VSR lumber by 11 percent.

The results for f_{joist} in Case 7 are better compared to the results for f_{joist} in Case 5. The f_{joist} distributions in Figures 33 and 34 are shifted to the right more compared to the f_{joist} distributions in Figures 23 and 24.

In comparison with Case 3, Case 7 had better results because the first percentile values of f_{joist} for MSR and VSR lumber are higher than the first percentile values of f_{joist} for MSR and VSR lumber in Case 3. Case 3 did not include a load sharing component, whereas Case 7 included a load sharing model which should predict more realistic floor behavior.

Case 8 (Load Sharing)

The two-parameter lognormal fit the data well for E_{eff} in Figures 35 and 36. This best-fit was based collectively on the visual test, maximum log-likelihood, Chi-Squared test, and Kolmogorov-Smirnov test. The distribution of E_{eff} in Figure 36 (MSR lumber) is less variable than the distribution of E_{eff} in Figure 35 (VSR lumber).

The two-parameter lognormal also fit the data for f_{girder} in Figures 37 and 38 based on the same tests listed above for E_{eff}. The f_{girder} distribution in Figure 38 for MSR lumber is less variable than the f_{girder} distribution in Figure 37 for VSR lumber. Upon inspection of Figures 37 and 38, the probability that f_{girder} is below 10 Hz is zero.

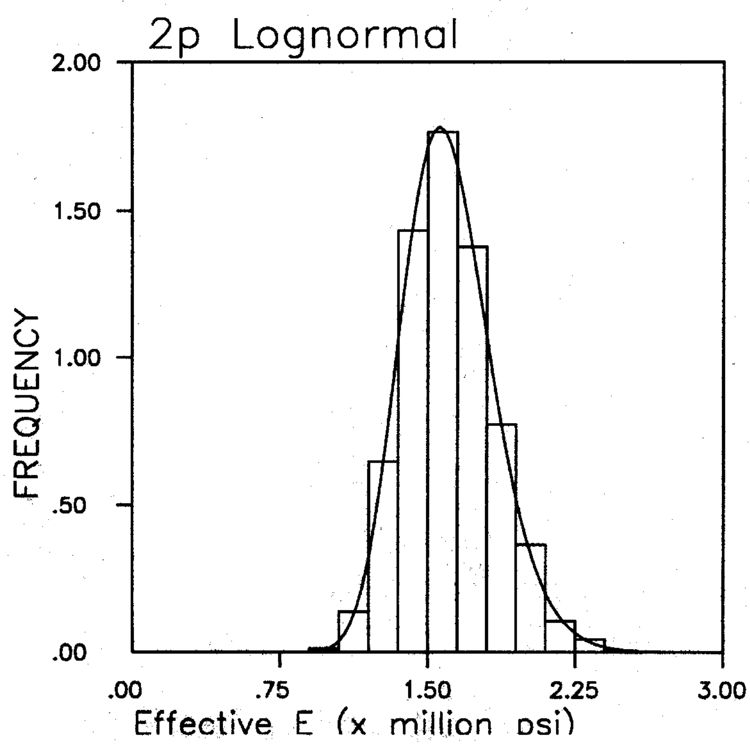

Figure 35 **Five thousand values were generated for the girder effective E (E_{eff}) by Monte Carlo simulation using Equation 14. A two-parameter lognormal distribution was superimposed on the histogram by GDA for the girder E_{eff} for VSR lumber and was determined to be the best-fit distribution for the data.**

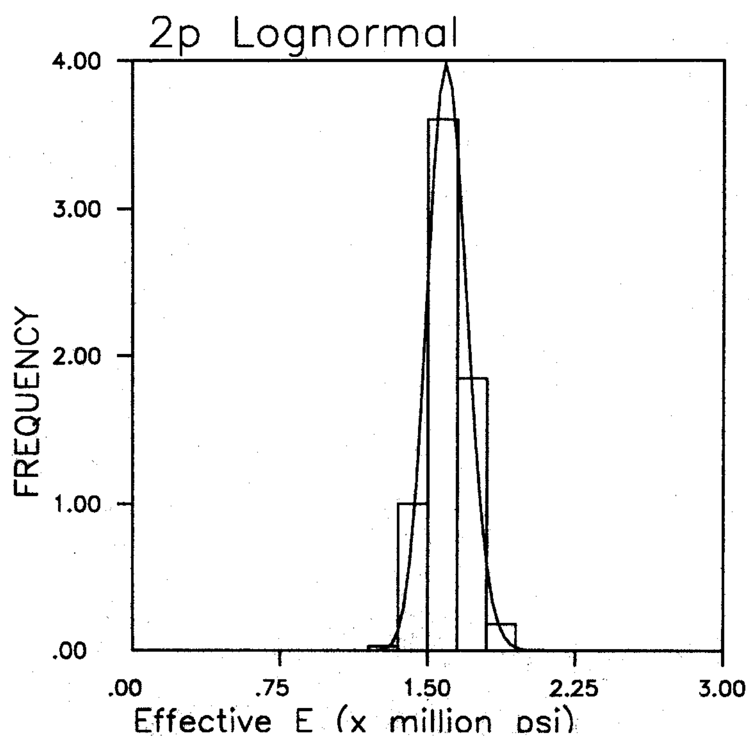

Figure 36 **Five thousand values were generated for the girder effective E (E_{eff}) by Monte Carlo simulation using Equation 14. A two-parameter lognormal distribution was superimposed on the histogram by GDA for the girder E_{eff} for MSR lumber and was determined to be the best-fit distribution for the data.**

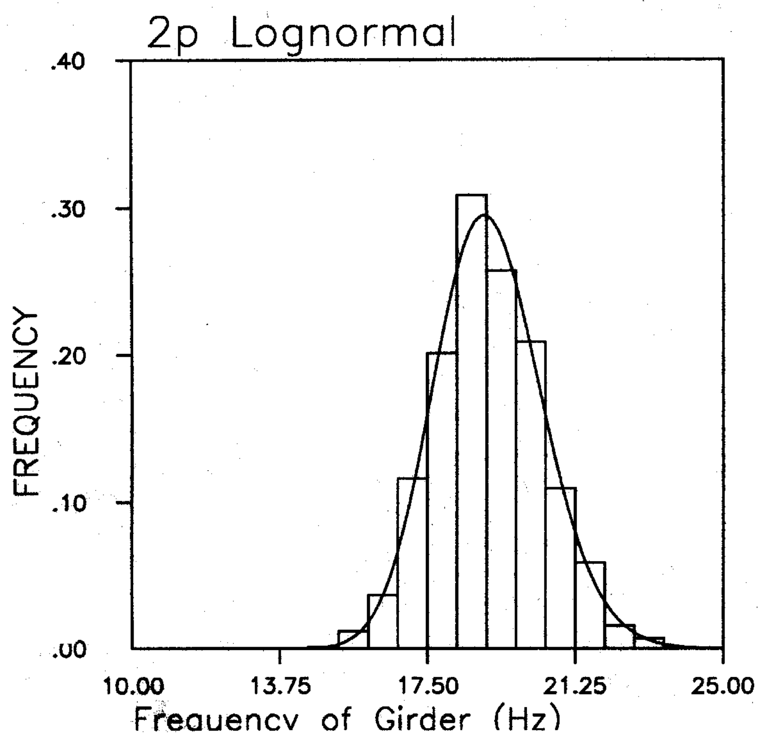

Figure 37 **Five thousand values were generated for the girder frequency (f_{girder}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{girder} for VSR lumber and was determined to be the best-fit distribution for the data.**

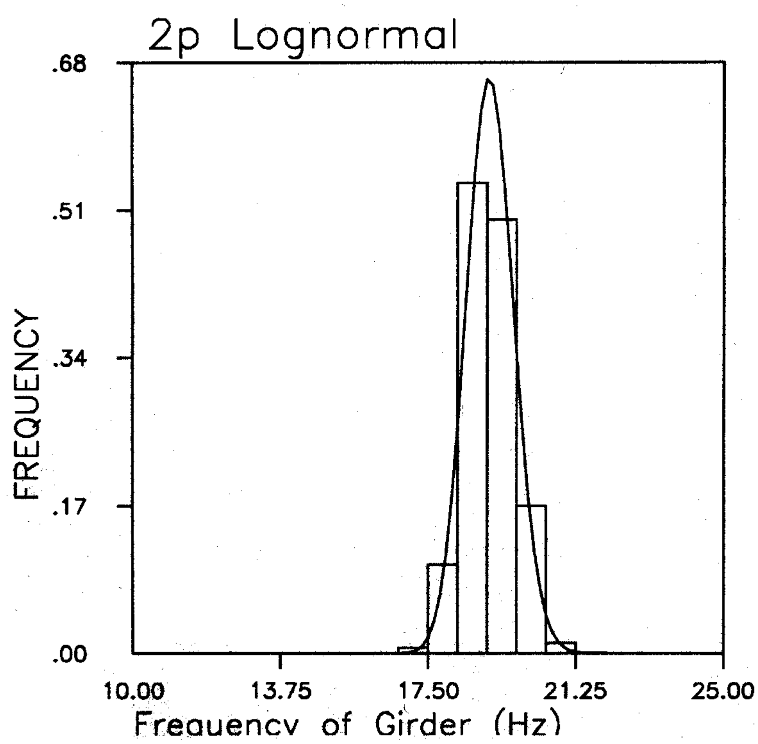

Figure 38 **Five thousand values were generated for the girder frequency (f_{girder}) by Monte Carlo simulation using Equation 2. A two-parameter lognormal distribution was superimposed on the histogram by GDA for f_{girder} for MSR lumber and was determined to be the best-fit distribution for the data.**

The two-parameter lognormal fit the data for f_{system} for both VSR (Figure 39) and MSR (Figure 40) lumber. The f_{system} distribution in Figure 40 is less variable than the f_{system} distribution in Figure 39. The probability of f_{system} less than 10 Hz is zero for both MSR and VSR lumber. The first percentile for Figure 39 (VSR lumber) is 11.38 Hz and the first percentile for Figure 40 (MSR lumber) is 12.24 Hz. The ratio of the first percentile of MSR lumber to the first percentile of VSR lumber is 1.08. The predicted floor vibrational performance of MSR lumber based on the first percentile of frequency is 8 percent better than the predicted floor vibrational performance of VSR lumber.

The floor system frequency in Case 8 for both VSR and MSR lumber is an improvement over the floor system frequency for both VSR and MSR lumber in Case 6 upon comparing Figures 39 and 40 versus Figures 29 and 30. The distributions in Figures 39 and 40 are shifted more toward the right. In addition, the first percentile of f_{system} for both MSR and VSR lumber in case 8 are higher than the first percentile values for MSR and VSR lumber in Case 6.

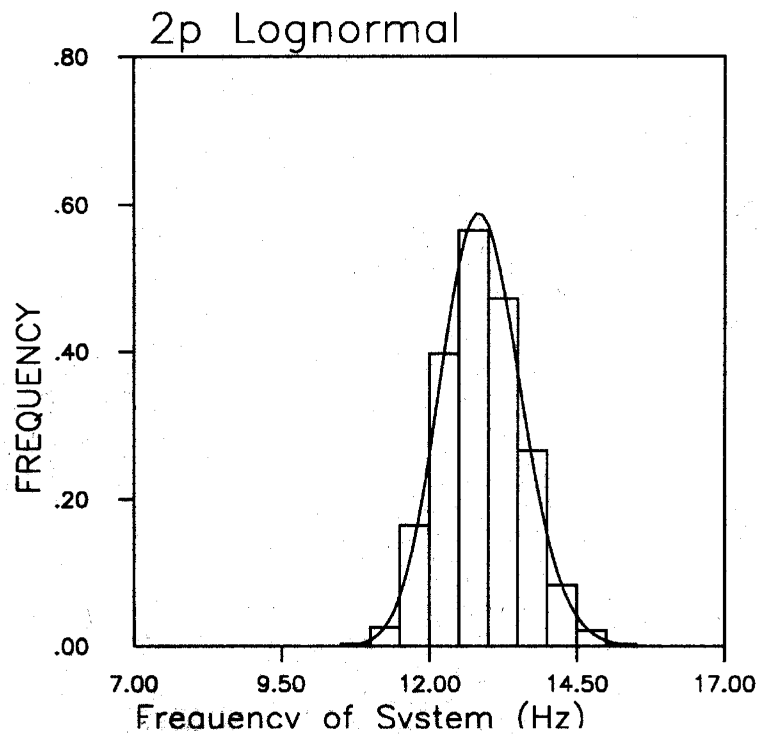

Figure 39 Five thousand values were generated for the floor system frequency (f_{system}) by Monte Carlo simulation using Equation 3. A two-parameter distribution was superimposed on the histogram by GDA for the f_{system} for VSR lumber and was determined to be the best-fit distribution for the data.

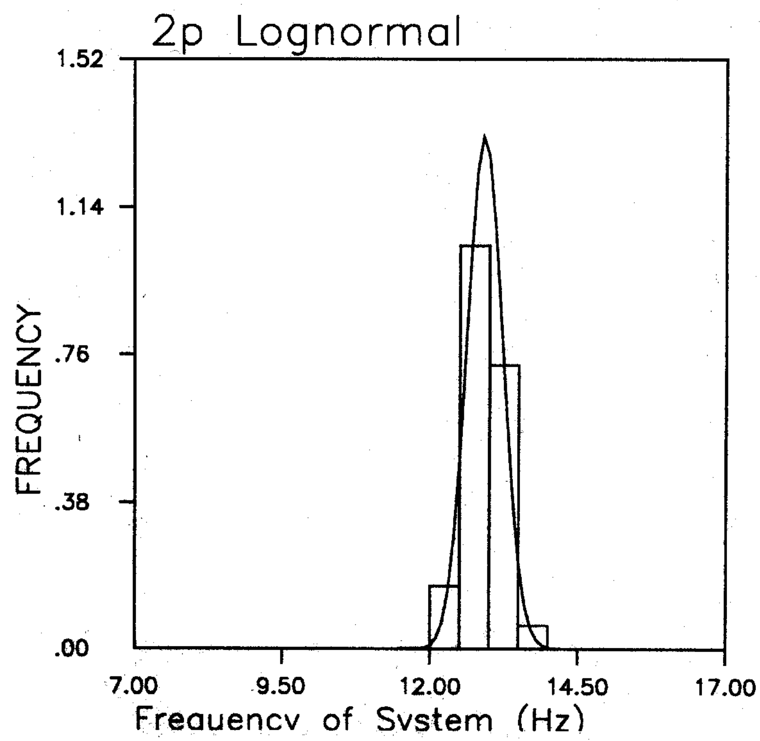

Figure 40 **Five thousand values were generated for the floor system frequency (f_{system}) by Monte Carlo simulation using Equation 3. A two-parameter distribution was superimposed on the histogram by GDA for the f_{system} for MSR lumber and was determined to be the best-fit distribution for the data.**

Summary

An individual joist was studied for Cases 1 through 4 instead of a floor system and thus load sharing was neglected. The joist in Case 1 was rigidly supported and designed for the L/360 live-load deflection limit. MSR lumber in Case 1 had an improved vibrational performance over VSR lumber by 18 percent based on the ratio of the first percentile of the predicted frequency in Equation 1. The probability that f_{joist} is less than 10 Hz was close to zero for both VSR and MSR lumber. However, the MSR lumber floor (Ω_E=0.11) had better predicted floor performance based on the measure of the probability of the joist frequency being less than 10 Hz because its probability was approximately zero.

When the joist was supported by a girder as in Case 2, Ω_E had no detectable positive effect on the predicted vibrational performance of the floor system. The predicted vibrational performance of MSR lumber was 13 percent better than the predicted vibrational performance of VSR lumber. The probability that f_{system} is less than 10 Hz for MSR lumber was zero while the same probability for VSR lumber was 0.0537.

Case 2 provided an example of the potential negative effect of a girder on the vibrational performance of a floor system. Table 2 provides a summary of the vibrational performance measures of both floor cases with respect to the Ω_E level.

Table 2 **Predicted vibrational performance of floor cases 1 and 2 was improved by using lumber having a lower Ω_E.**

		Case 1: Joist (L/360) with Rigid Supports			Case 2: Joist (L/360) Supported by Girder (L/360)		
Lumber E-Grade	Ω_E	1st Percentile of f_{joist}	Probability f_{joist}<10 Hz	$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}}$	1st Percentile of f_{system}	Probability f_{system}<10 Hz	$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}}$
MSR	0.11	13.09	≈ 0.0000	1.18	10.60	0.0000	1.13
VSR	0.25	11.06	0.0008		9.34	0.0537	

The rigidly supported joist in Case 3 was designed for the L/480 live-load deflection limit. Both MSR and VSR lumber had favorable predicted vibrational performance because the probability that f_{joist} is less than 10 Hz was zero for both of them. The floor performance of MSR lumber based on the ratio of the first percentile of the predicted frequency in Equation 1 was 18 percent better than VSR lumber.

The floor system in Case 4 consisted of a joist, designed for the L/480 live-load deflection limit, supported by a girder designed for the L/600 live-load deflection limit. The predicted vibrational performance of MSR lumber based on Equation 1 was 13 percent better than the predicted vibrational performance of VSR lumber. The probability that f_{system} is less than 10 Hz was close to zero for both VSR and MSR lumber.

Case 3 and Case 4 provided examples of the positive effect of designing the joist and girder to a stricter deflection limit on the vibrational performance of a floor system. The distributions shifted to the right and increased the first percentile values in both cases. Table 3 provides a summary of the measures of performance of both MSR and VSR lumber.

Table 3 Predicted vibrational performance of floor case 3 was good regardless of Ω_E due to joist designed for stricter deflection limit (L/480). For floor case 4, the predicted vibrational performance was improved by using lumber having a lower Ω_E.

		Case 3: Joist (L/480) with Rigid Supports			Case 4: Joist (L/480) Supported by Girder (L/600)		
Lumber E-Grade	Ω_E	1st Percentile of f_{joist}	Probability f_{joist}<10 Hz	$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}}$	1st Percentile of f_{system}	Probability f_{system}<10 Hz	$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}}$
MSR	0.11	15.36	0.0000	1.18	11.91	0.0000	1.13
VSR	0.25	12.98	0.0000		10.58	0.0016	

Load sharing was included in the study in Cases 5 through 8. Three joists were studied in Cases 5 through 8 instead of individual joists as in Cases 1 through 4.

Case 5 involved three rigidly supported joists designed for the L/360 live-load deflection limit. The results for the expected vibrational performance measures in Case 5 that included load sharing improved from the results for expected vibrational performance in Case 1. The probability that f_{joist} is less than 10 Hz was zero for both VSR and MSR lumber. However, MSR lumber still had better vibrational performance measures than VSR lumber. According to the ratio of first percentile predicted frequencies given by Equation 1, the predicted performance of the MSR lumber joists is 11 percent over the predicted vibrational performance of VSR lumber joists. This is a decrease in first percentile frequency ratio (Equation 1) by 7 percent from Case 1 which did not include load sharing.

The girder and three joists in Case 6 were both designed for the L/360 live-load deflection limit with the effect of load sharing included. The probability that f_{system} is less than 10 Hz for MSR lumber was zero and the same probability for VSR lumber was 0.0045. The predicted ratio of first percentile frequencies of MSR lumber differed from VSR lumber by 8 percent. This is a reduction from 13 percent in Case 2. The results for case 5 and 6 are summarized in Table 4.

Table 4 Predicted vibrational performance of floor case 5 improved from floor case 1 by including load sharing. The results for floor case 6 were also an improvement over floor case 2 due to load sharing. Predicted vibrational performance for both Case 5 and Case 6 was improved by using lumber having a lower Ω_E.

		Case 5: Joist (L/360) with Rigid Supports; Load Sharing Included			Case 6: Joist (L/360) Supported by Girder (L/360); Load Sharing Included		
Lumber E-Grade	Ω_E	1st Percentile of f_{joist}	Probability f_{joist}<10 Hz	$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}}$	1st Percentile of f_{system}	Probability f_{system}<10 Hz	$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}}$
MSR	0.11	13.79	0.0000	1.11	10.93	0.0000	1.08
VSR	0.25	12.47	0.0000		10.15	0.0045	

Case 7 involved three rigidly supported joists designed for the L/480 live-load deflection limit. The measures of floor performance improved from Case 5 since the joists were designed to a stricter deflection limit and again MSR lumber performed better than VSR lumber. The probability that f_{joist} is less than 10 Hz was zero for both VSR and MSR lumber. The predicted vibrational performance defined by Equation 1 for MSR lumber versus VSR lumber was 1.11, or 11 percent.

In Case 8, the three joists were designed for the L/480 live-load deflection limit. The girder supporting the joists was designed for the L/600 live-load deflection limit. The measures improved from Case 6. The probability that f_{system} is less than 10 Hz was zero for both MSR and VSR lumber. The predicted vibrational performance for MSR lumber differed from VSR lumber by 8 percent. The results for Case 7 and Case 8 are summarized in Table 5.

Table 5 Predicted vibrational performance of floor case 7 was improved from floor case 5 by designing the joists to a stricter deflection limit. For floor case 8, the predicted vibrational performance improved over floor case 6 due to designing the joists and girder to a stricter deflection limit. In addition, these results were also improved by using lumber having a lower Ω_E.

		Case 7: Joist (L/480) with Rigid Supports; Load Sharing Included			Case 8: Joist (L/480) Supported by Girder (L/600);Load Sharing Included		
Lumber E-Grade	Ω_E	1st Percentile of f_{joist}	Probability f_{joist}<10 Hz	$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}}$	1st Percentile of f_{system}	Probability f_{system}<10 Hz	$\frac{f_{0.01}^{MSR}}{f_{0.01}^{VSR}}$
MSR	0.11	16.22	0.0000	1.11	12.24	0.0000	1.08
VSR	0.25	14.66	0.0000		11.38	0.0000	

Conclusions and Recommendations

The variability of E, characterized by Ω_E, did have an effect on the predicted vibrational performance of floor systems. Overall, MSR lumber (Ω_E=0.11) floor systems had improved predicted vibrational performance over VSR floor systems (Ω_E=0.25) based on two measures of floor performance - - fundamental frequency and probability of the fundamental frequency less than 10 Hz. In addition, the lower Ω_E (MSR lumber) helped improve predicted performance of the floor system when a girder was involved.

Based on the two measures studied, the predicted vibrational performance across the floor cases improved as the design deflection limits became more strict, specifically L/360 versus L/480. A recommendation for controlling annoying residential floor vibrations is to design the floor joists to a stricter deflection limit of L/480.

Based on in-situ tests, Shue (1995) calibrated the f_{system} prediction equation by choosing the span between inflection points. The appropriate choice of "span" to be used in the equation for calculating f_{girder} needs to be evaluated for other span conditions including a simple span girder assumed in this study.

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Vita

Ann C. Wilson was born in Rapid City, South Dakota on August 29, 1972. In the spring of 1996 she received her Bachelor of Science degree in Civil Engineering from Virginia Tech. She was employed at Langley AFB until entering the graduate program of Biological Systems Engineering in the spring of 1997. As a graduate student, Ann was invited to join the honor societies of Alpha Epsilon and Gamma Sigma Delta.