

A Computational Fluid Dynamics Investigation of Thermoacoustic Instabilities in Premixed Laminar and Turbulent Combustion Systems

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Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Mechanical Engineering

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July 9, 2004
Blacksburg, Virginia

Keywords: flame dynamics, vortex-flame interaction, thermoacoustic instability, CFD,
frequency response function

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(ABSTRACT)

Lean premixed combustors have been designed to lower NO_x and other pollutant levels in land based gas turbines. These combustors are often susceptible to thermo-acoustic instabilities, which manifest as pressure and heat release oscillations in the combustor. To be able to predict and control these instabilities, it is required that both the acoustics of the system, and a frequency-resolved response of the combustion process to incoming perturbations be understood.

Currently, a system-level approach is being used widely to predict the thermoacoustic instabilities. This approach requires simple, yet accurate models which would describe the behavior of each dynamic block within the loop. The present study is directed toward using computational fluid dynamics (CFD) as a tool in developing reduced order models for the dynamics of laminar flat flames and swirl stabilized turbulent flames. A finite-volume based approach is being used to simulate reacting flows in both laminar and turbulent combustors. The study has been divided into three parts – the first part involves the modeling of a self-excited combustor (the acoustics of the combustor are coupled with the unsteady heat release); the second part of the research aims to study the effect of velocity perturbations on the unsteady heat release rate from a burner stabilized laminar flat flame; the third and final part of work involves an extension of the laminar flat flame study to turbulent reacting flows in a swirl stabilized combustor, and study the effects on the turbulent heat release due to the velocity perturbations.

A Rijke tube combustor was selected to study self-excited combustion phenomenon. A laminar premixed methane-air flat flame was stabilized on a honeycomb flame-stabilizer. The flame stabilizer was placed at the center of the 5 *ft* vertical tube. The position of the

flame at the center of the tube leads to a thermoacoustic instability of the 2nd acoustic mode. The fundamental thermoacoustic frequency was predicted accurately by the CFD model and the amplitude was reasonably matched (for a flow rate of $Q = 120 \text{ cc/s}$ and equivalence ratio $\phi = 1.0$). Other characteristics of the pressure power spectrum were captured to a good degree of accuracy. This included the amplitude modulation of the fundamental and the harmonics due to a subsonic pulsating instability.

The flat flame study has been being conducted for $Q = 200 \text{ cc/s}$ and equivalence ratio $\phi = 0.75$. The objective has been to obtain a frequency response function (FRF) of the unsteady heat release rate (output) due to incoming velocity perturbations (input). A range of frequencies (15 Hz - 500 Hz) have been selected for generating the FRF. The aim of this part of the study has been to validate the computational model against the experimental results and propose a physics based interpretation of the flame response. Detailed heat transfer modeling (including radiation heat transfer) and two-step chemistry models have been implemented in the model. The FRF generated has been able to reproduce the experimentally observed phenomena, like the low frequency pulsating instability occurring at 30 Hz . A heat transfer study has been conducted to explain the pulsating instability and a fuel variability study has been performed. Both the heat transfer study and the fuel variability study proved the role of heat transfer in creating the pulsating instability.

The final part of the study involves simulation of reacting flow in a turbulent swirl stabilized combustor. The effect of velocity perturbations on the unsteady heat release has been studied by creating an FRF between the unsteady velocity and the unsteady heat release rate. A Large Eddy Simulation (LES) approach has been selected. A swirl number of $S = 1.19$ corresponding to a flow rate of $Q = 20 \text{ SCFM}$ with an equivalence ratio of $\phi = 0.75$ have been implemented. Reduced reaction chemistry modeling, turbulence-chemistry interaction and heat transfer modeling have been incorporated in the model. The LES of reacting flow has shown vortex-flame interaction occurring inside the combustor. This interaction has been shown to occur at 255 Hz . The FRF obtained between unsteady velocity and unsteady heat release rate shows good comparison with the experimentally obtained FRF.

To my parents, Mrs. Mala Chatterjee and Dr. P. K. Chatterjee

Acknowledgments

First and foremost, I would like to offer my deepest gratitude to my committee chair, Dr. Uri Vandsburger. Throughout the course of my graduate studies at Virginia Tech, he provided encouragement and valuable technical knowledge and helped me maintain my focus on the research. I appreciate the technical suggestions he has constantly offered and thank him for being patient with me during my stay at Virginia Tech. I have come to appreciate the emphasis he places on the understanding of fundamental aspects of combustion research while applying them to numerical simulations of complex combustion systems.

I would like to thank Dr. William R. Saunders for helping me understand the basics of thermoacoustics and for the numerous occasions he spent helping me decipher the intricacies of combustion control research. In particular, I would like to acknowledge his help in furthering my understanding of the Matlab code, especially the spectrum analysis portion. I am grateful to Dr. William T. Baumann, who made my life easier by frequently helping me analyze my data. Without his help I would not have been able to figure out several controls related fundamentals.

I would also like to thank Dr. Danesh K. Tafti for guiding me on the numerical issues encountered in the research. I would like to acknowledge his generosity in offering to let me work with his LES code. Dr. Robert L. West deserves my thanks for understanding the endless problems I encountered with accessing faster computers for my research. He was kind enough to provide me access to a fast computer for performing turbulent flow simulations. I am also thankful to Dr. Andrew G. Godfrey for his valuable suggestions. I have enjoyed

interacting with him while working on an STTR project together.

There are several students in the department who have been extremely helpful to me during my stay at Virginia Tech. Stephen D. LePera deserves a generous thanks for always helping me sort out computer related issues. I would also like to thank him for sharing with me his vast knowledge of computers, experimental methods in fluid flows and his understanding of combustion. Thanks are due to Wajid A. Chishty for his help in conducting several combustion related experiments which produced valuable data for boundary conditions and validations for my simulations. Both Steve and Wajid are good friends of mine and have helped me outside the workplace on several occasions. I would like to thank Christopher J. Wieczorek for his friendship and for throwing those wonderful parties over the years. Vivek K. Khanna mentored me during the first part of my stay at Virginia Tech. I am thankful to him for sharing his knowledge of combustion and thermal radiation. Ludwig C. Haber, Adam Hendricks, Denzil John and Salah Basaran have extended their help to me on numerous occasions and I am grateful to them.

I owe my thanks to several people in the Mechanical engineering department. In particular, thanks are due to Mike Harness, Cathy Hill, Eloise McCoy, Kathy Taszarek, Lisa Stables, Ben Poe and Jamie Archual.

My friends in Blacksburg have contributed considerably by supporting me and helping me survive graduate school. I would like to extend my thanks particularly to Dhaval Makhecha, Konda Reddy, Suhas Subramanya, Navin Verma, Ramya Ramanath, Nina Sengupta and Alok Mullick, Edwin and Valarie Robinson and Helen Castaneda amongst several others.

My parents have given me infinite support and have kept their faith in me during these grueling graduate school years. I thank them for always being there to listen, for all the advise they gave me from time to time and for praying for me constantly. Last but not the least, I am indebted to my wife Sucharita for her continuous support. She gets my gratitude for her never-ending patience and for believing in me.

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Nomenclature

$\dot{\omega}$	chemical reaction rate
j_j^h	laminar diffusion flux
μ_t	turbulent viscosity
ω	frequency (<i>rad/s</i>)
\overline{var}	average value of variable <i>var</i>
Φ	wave energy dissipation
ϕ	equivalence ratio
ρ	density
σ	Stephan-Boltzmann constant
τ	period of oscillation
τ_c	chemical time scale
τ_t	turbulent time scale
\widetilde{var}	filtered variable <i>var</i>
a	radiation absorption coefficient

c	reaction progress variable
G	irradiation
$G(s)$	open loop transfer function of a system
h_t	total enthalpy
Ka	Karlovitz number
l	integral length scale
l_F	flame thickness
Ma	Mach number
P	probability density function
p	pressure
Q	flow rate
q	heat release
R	universal gas constant
Re	Reynolds number
RR	reaction rate
S	Swirl number
S_g	geometrical Swirl number
S_L	laminar flame speed
Sc_{kt}	turbulent Schmidt number
T	temperature

t time

t_d time delay

u_i i th component of velocity

V combustor volume

var' fluctuating component of variable var

Y_k k th specie mass fraction

Da Damkohler number