### 2 Research Review

One of the major additions to *Tmin* was the inclusion of analysis of a 2-Dimensional vertical piping span. The original plan from Dupont was to include several types of 2-D and 3-D vertical spans. However, due to time constraints only this piping span was implemented into *Tmin*. This 2-D vertical piping span is only in the x and y-axis and is thus called the 2-D vertical piping span. In order for the analysis of a 2-D vertical piping span to be added to *Tmin*, certain known mechanical methods had to be applied. In this chapter, these methods will be described and explained. The purpose of this chapter is to identify the known mechanical methods that are used for evaluation of stresses, which could lead to pipe fatigue. Some fundamental problems behind the shortened pipe life are internal corrosion and erosion of the internal pipe, which increase pipe stresses. Corrosion fatigue can result from a combination of the corrosive chemicals inside the piping and cyclic stress. An example of cyclic stress would be the on—off pressurization of piping by pumps.

The primary purpose of this chapter is to discuss the analysis of piping stress and fatigue. In Section 2.1, the background of piping stresses and fatigue will be identified. The information in this section will be used throughout this thesis. Since piping stress can be derived mathematically using several different methods, the primary goals in this chapter will be to detail the assumptions and mechanical methods followed. These methods follow certain standards practiced, national standards, and national codes for beam and piping analysis.

The American Society of Mechanical Engineers (ASME) has followed the engineering and mechanical methods for the analysis of piping stresses and fatigue in their *Unfired Piping and Pressure Vessel* standards and codes [3]. They have compiled their findings in piping standards and codes that will be described in Section 2.2. The primary purpose of the standards and codes followed in the *Unfired Piping and Pressure Vessel* standards and codes are the safety of personnel.

# 2.1 Background of Piping Stresses

Jack A. Collins [5] in his book Failure of Materials in Mechanical Design says, "In designing a machine part, it is necessary to conceive a combination of material and geometry that will fulfill the design objective without failure. To do this a designer must have at his or her disposal a calculable mechanical modulus that is physically related to the governing failure mode in such a way that failure is accurately predictable when the mechanical modulus reaches a determinable critical value. For most failure modes it is found that which may utilize stress as a modulus failure can be predicted and, therefore, averted by a proper design configuration. The concepts of states of stress at a point...become topics of great importance to the designer."

Den Hartog [6] explains the stresses that occur in engineering materials in his book, *Strength of Materials* by defining the primary causes of stress. A normal stress is defined as the force per unit cross-sectional area where the direction of the force is normal to the plane of the cross section. There are two principals that can be followed to get the materials performance. The first principle is the stress analysis of existing structures, in order to predict their behavior under specified loading conditions. The second principle is in the design of a new structure that will safely perform a specified function [7].

# 2.1.1 Differential Stress Element Analysis

This project will be using thin-pipe wall theory, described in Section 2.1.4, and only 2-dimensional (2-axis) states of stress will be used [5]. Besides the use of 2-axis states of stress, it can be seen that many triaxial (3-dimensional) shear stresses are identical in numerical value. As a result, the outer pipe surface is the critical position with a differential stress element, and can be broken down to a 2-axis state of stress, called a differential stress element, is seen in Figure 2-1. The differential stress element seen in this figure will be used throughout this thesis for analysis of pipe stress. As seen in this figure, shear elements follow the face of the cube, while normal stresses are seen normal to the element edges. Tensile normal stresses are seen as outward normals, while compressive stresses are seen as

inward normal to the differential stress element. This normal stress and shear analysis will be derived in detail in Chapter 4 using various element areas on the piping span.

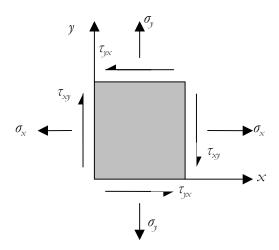


Figure 2-1. Differential Stress Element with Normal and Shear Stresses

In order to analyze the differential stress element properly, the design engineer must follow certain criteria. Structural elements made of ductile materials must be designed so the material will not yield under expected loading conditions [7]. Once the engineer has decided on the task at hand, whether it is upgrading an existing structure, or the replacement of fatigued pipes, stress states must be analyzed for safety of people. To do this, the engineer would need to know the ultimate tensile strength, or another name is tensile strength,  $S_{UT}$ , the yield strength,  $S_Y$ , and the fatigue strength,  $S_P$ , of the material. Upon knowing these values, the engineer can then begin the design of the project in which they are involved. Next, a graphical representation is needed to show the state of stress that a piping span may experience is detailed.

# 2.1.2 Mohr Circle Analysis

Otto Mohr developed a graphical representation of the equations developed through the Maximum-Shear-Stress Theory (detailed in Section 2.1.3) in the 1900's, and is an extension of the maximum shear stress theory of failure, which is based on a 3-Dimensional graphical circle [5]. Using coordinates of normal stress,  $\sigma$ , and shear,  $\tau$ , he had shown that these

equations can be represented graphically in a circle, resulting in Mohr's Circle. Mohr's circle is also dependent on differential stress elements. Figure 2-2 shows the graphical creation of a 3-D Mohr's circle. The use of a 3-D Mohr's circle will enable the engineer to visually see the largest stress, or failure envelope, that are found in a piping system. The failure envelope is defined as the largest Mohr's circle with a maximum shear of  $\tau_{Max}$ .

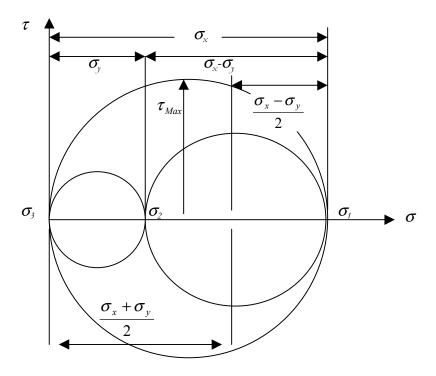


Figure 2-2. Mohr's Circle Representation of Principle Stresses

As a result of the 3-D Mohr's circle, each circle will have a representative equation either in shear or in stress. The centers of the circles, smallest through the largest are defined as Equations (2.1) through (2.3).

$$C_1 = \frac{\sigma_2 - \sigma_3}{2} \tag{2.1}$$

$$C_2 = \frac{\sigma_1 - \sigma_3}{2} \tag{2.2}$$

$$C_3 = \frac{\sigma_1 - \sigma_2}{2} \tag{2.3}$$

The radius of each circle, smallest through the largest, are seen in Equations (2.4) - (2.6).

$$\tau_1 = \frac{\sigma_2 - \sigma_3}{2} \tag{2.4}$$

$$\tau_{Max} = \tau_2 = \frac{\sigma_1 - \sigma_3}{2} \tag{2.5}$$

$$\tau_3 = \frac{\sigma_1 - \sigma_2}{2} \tag{2.6}$$

These equations will be used for analysis of piping stresses, which are discussed in Section 2.1.4. In Figure 2-2, the 3-D Mohr's circle shows stress-states that are positive in sign are in tension. When all stress-states are in tension, none of the circles will cross the  $\sigma$  = 0.0 axis. However, as seen in Chapter 4, upon evaluation of compressive piping stresses some stress states will be negative, causing a shift of the Mohr's circle over the  $\sigma$  = 0.0 axis. Using the graphical representation of the stress-states, the American Society of Mechanical Engineers has chosen to use the Maximum-Shear-Stress Theory for failure analysis of piping spans [8]. As a result of the ASME, the Maximum-Shear-Stress Theory was used as the criteria theory in this thesis.

# 2.1.3 Maximum-Shear-Stress Theory

According to Shigley and Mishke [9], "The Maximum-Shear-Stress Theory states that yielding begins when the maximum shear stress in any element becomes equal to the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield." In contrast to other failure theories, the Maximum-Shear-Stress Theory has been found to be more successful in the prediction of failure of a ductile material under hydrostatic states of stress [5]. The Maximum-Shear-Stress Theory relies on the use of a tensile-test specimen. Tensile tests and the resultant stresses observed are described in Section 3.1.

The American Society of Mechanical Engineer dictates in the *Unfired Piping and Pressure Vessel* standards and code, that the Maximum-Shear-Stress Theory is to be used for calculation of stresses [8]. The Maximum-Shear-Stress Theory results in several equations

that are formed upon analysis of the differential element discussed previously. The criterion for this theory states that a structural component is safe as long as the maximum value of shear stress,  $\tau_{Max}$ , of a specimen in tensile remains smaller than the one-half of the yield strength,  $S_Y$ , seen as Equation (2.7). As a result of Equations (2.1) through (2.3), the Maximum-Shear-Stress Theory predicts that failure will occur when Equations (2.7) or (2.8) are satisfied. As seen in Equation (2.8), the difference between the principle stresses must be greater than or equal to the yield strength, which is also equal to the equivalent stress. The equivalent stress will be defined in more detail in Chapter 4.

$$\tau_{Max} \ge \frac{S_{Y}}{2} \tag{2.7}$$

$$\sigma_{eq} = (\sigma_1 - \sigma_3) \ge S_Y \tag{2.8}$$

Through observation of the two previous equations, it was found that the yield strength is always less than or equal to two times the maximum shear. Using the Maximum-Shear-Stress Theory, these two equations will be used as limits observed in a piping system. To find what stresses are observed in a piping system, or pressure vessel, pipe-wall theory must be understood.

# 2.1.4 Pipe-Wall Theory

There are two different types of pipe-wall theories that are known to the engineering community, thin-wall and thick-wall theory. Both of these theories have been shown through experimental analysis in early engineering years as valid for use. Thick-wall piping theory will not be used in this thesis. Thick-wall piping theory relies on the assumption of a gradient radial normal stress through the pipe thickness that results in a multi-axial stress state being present n a 3-D differential stress element [10]. Since this thesis will be using only 2-D differential stress elements, thin-wall piping theory will be used on the pressure vessel when the piping thickness, t, to the radius, r, of the pipe is less than 1/20 [7]. When an internal pressure is placed inside the pressure vessel, stresses occur along the length of the pipe and around its radius.

The stress in the circumferential direction on the pressure vessel is called the hoop stress,  $\sigma_H$ , and is seen as Equation (2.9). It is called the hoop stress because it is the type of stress that is found in hoops to hold together the slats of a wooden barrel [7]. The stress in the longitudinal direction of the pressure vessel is defined as the longitudinal stress,  $\sigma_L$ , seen as Equation (2.10). In a closed cylinder the longitudinal stress exists because of pressure on the ends of the pressure vessel [9].

$$\sigma_H = \frac{P(D_o - 2t)}{2t} \tag{2.9}$$

$$\sigma_L = \frac{P(D_o - 2t)}{4t} \tag{2.10}$$

It can be observed that the longitudinal stress is half of the hoop stress. This explains why piping systems fail along the longitudinal line and rarely show a tear along the radius of the circle [6]. Figure 2-3 shows a differential stress element with the effect of hoop and longitudinal stresses acting on a pressure vessel. As seen in this figure, the pressure P, is the gage pressure of the internal fluid,  $D_{\sigma}$  is the outside diameter of the pipe, and t, is the pipewall thickness.

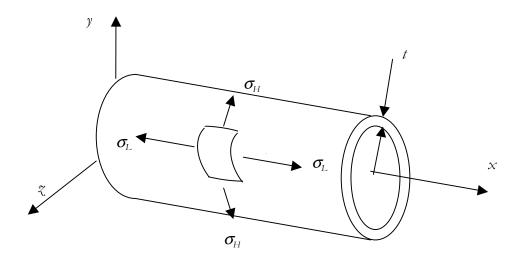
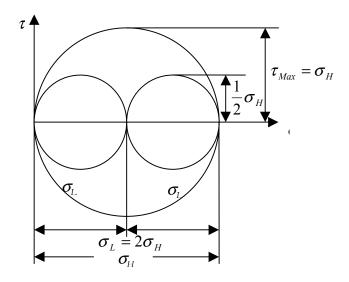


Figure 2-3. Hoop and Longitudinal Stresses acting on a Differential Element

The next Figure 2-4 shows the relationship between 3-D Mohr's circle and the aforementioned hoop and longitudinal stresses.



**Figure 2-4.** Relationship between use of Mohr's Circle with Hoop and Longitudinal Stresses

The relationship between hoop and longitudinal stresses are more obvious upon drawing the Mohr's circle. In addition to pressurization stresses one has additional stresses by body force loading and external force loading. Thus, shear and moment diagrams must be analyzed for each piping system and are described in more detail in Chapter 4.

Shear is defined as the shear force, V, over the cross-sectional area of the pipe as seen in Equation (2.11) [6]. This shear equation will be used in Chapter 4 when the analysis of shear and moment diagrams of the vertical piping span is detailed.

$$\tau_{Shear} = \frac{2V}{A} \tag{2.11}$$

A stress distribution created by torsion is seen in Figure 2-5. They are at a maximum at the outer edge of the rod, while tending toward zero in the center [6]. A pipe or cylinder has a similar distribution, but cannot tend toward zero at the center since there is no mass in the center of a pipe.

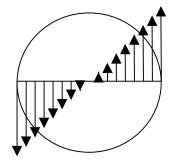
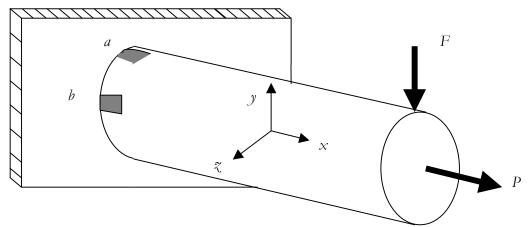


Figure 2-5. Stress Distribution Created by Torsion Acting on a Cross-Section of a Bar

There are even more stresses for consideration. To see the effects of these stresses acting on a point, Figure 2-6 shows two differential stress elements, seen as a and b, on a bar, being affected by transverse and axial forces. A transverse force is seen as the downward force F. This downward force is multiplied by the distance of the bar to cause bending stresses through summation of moments. The pulling force, P, acting over the area of the bar causes an axial stress.



**Figure 2-6.** A Cantilever Bar Subjected to Downward and Axial Forces. Differential Stress Elements *a* and *b* are Shown

As a result of the effect of the downward force, F, a bending moment occurs. The bending moment at any point on a beam or bar is positive when the external forces bend or squeeze the fibers at this section of the beam at the point being analyzed [7]. The effect is seen in Figure 2-7, where the moment, M, is seen to bend the beam at a given point. In this

figure, the top of the beam experiences a compressive moment, while the bottom of the beam fibers experiences a stretching, or a tensile moment.

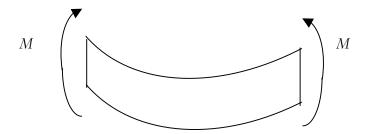


Figure 2-7. Effect of External Force Results in a Bending Moment

The stress that occurs because of the bending moment is seen as Equation (2.12). The bending stress sign is either positive (tensile moment) or negative (compressive moment) depending on the direction of this stress in relation to the differential stress element [7]. An additional stress seen on the bar using differential stress elements is axial stress, given in Equation (2.13).

$$\sigma_B = \frac{M}{Z} \tag{2.12}$$

$$\sigma_A = \frac{P}{A} \tag{2.13}$$

Analysis of the differential elements a and b in Figure 2-6 yield stress equations and shear in the x, y, and z-axis. In general, the summation of stresses at point a is seen as Equations (2.14) through (2.16). More specifically, the shear stress at point a is zero because shear stress passes through the center of the differential stress element. As a result, the shear is zero on the top and bottom of the bar. However, looking at the previous Figure 2-6 the shear created by the downward force F, at point b passes parallel to the face of the differential stress element and is not zero.

$$\sigma_{x} = \sigma_{B} + \sigma_{A} \tag{2.14}$$

$$\sigma_z = 0 \tag{2.15}$$

$$\tau_{xv} = \tau_{Shear} = 0 \tag{2.16}$$

To continue the evaluation of Figure 2-6, the summation of stresses and shear at point b as seen as Equations (2.17) through (2.19).

$$\sigma_{r} = \sigma_{R} + \sigma_{A} \tag{2.17}$$

$$\sigma_{y} = 0 \tag{2.18}$$

$$\tau_{xy} = \tau_{Shear} \tag{2.19}$$

In Chapter 6, a detailed numerical analysis of the stresses that occur to the piping system will be shown in more detail for the 2-D vertical piping span. In this major section, the definitions of the mechanical methods and criteria used for the development of the *Tmin* 2-D vertical piping span analysis was shown. In order to justify these theories for use in piping spans and to make this a trustworthy computer program, certain criteria must be followed using standardized codes developed by ASME [3]. These standardized codes were put in effect to create a safer working environment for personnel working around these piping systems. The next major section documents the standards and codes that were followed during the development of this computer program.

# 2.2 Piping Codes and Standards Requirements

The purpose of this section is to identify the known mechanical methods that are used for evaluation of piping stresses. The methodology used for the calculation these stresses have been documented in a National Code written by the American Society of Mechanical Engineers [11]. The evaluation of piping used in the industry for pressure vessels was written by the ASME to ensure that a unified code structure exists. As a result, the codes and standards written by ASME must be followed and properly applied.

### 2.2.1 Unfired Piping and Pressure Vessel Code

One primary addition to *Tmin* was the inclusion of fatigue curves (S-N) in its internal Microsoft Access database. To allow for cyclic operations that may occur in the piping system, Appendix 5 of Section VIII in the ASME Unfired Piping and Pressure Vessel standards and codes must be followed [11]. The title of Appendix 5 of Section VIII of the 1999 standards and codes is called "Mandatory Design Based on Fatigue Analysis." From this title alone, it can be seen that this appendix must be followed for fatigue analysis. A quote taken from paragraph 5-100 in this section states, "The stability of a [piping] vessel component for a specified operating conditions involving cyclic application of loads and thermal conditions shall be determined by the methods herein" [12].

In the Unfired Piping and Pressure Vessel standards and codes Section VIII, paragraph 5-101 explains that conditions and procedures in AD-160 and 5-110 are based on the comparison of stresses with fatigue data [11]. The aforementioned paragraph states that peak stresses are based on a comparison of peak stresses using strain-based fatigue data [10]. In Appendix 5 of Section VIII of the Unfired Piping and Pressure Vessel standards and codes, the ASME provides several S-N curves in this appendix and explains that the stress amplitude,  $\sigma_a$ , is calculated under the assumption of fully-elastic behavior [12]. However, the ASME explains that this type of strain-based S-N fatigue curve does not represent a real stress when the elastic range is exceeded [12].

The S-N fatigue curves in Appendix 5 of Section VIII of the Unfired Piping and Pressure Vessel standards and code were obtained through fully reversed uniaxial strain cycling as well as compression stress-based fatigue curves [12]. A fully reversed uniaxial strain cycling S-N curve is defined as an R-ratio equal to minus one (R = -1). R-ratio and fully reversed fatigue curves will be explained in more detail in Chapter 3. To create the S-N curves in this section, ASME has multiplied the imposed strains by the elastic modulus and then has taken half of the product to reach a pseudo elastic alternating strength. Doing this creates a design margin to make the calculated stress intensity,  $S_B$ , amplitude and the allowable stress amplitude directly comparable [12]. The stress intensity is the equivalent intensity of combined stress, or is twice the maximum shear stress; which is algebraically the difference between the largest principal stress and the smallest principal stress at a given

point [10]. Many of the concepts that ASME uses are described in Chapter 3, such as how strain-based and stress-based fatigue data are obtained.

In paragraph AD-160 of Appendix 6 in the Unfired Piping and Pressure Vessel standards and codes, explains whether or not fatigue analysis will be required [14]. Paragraph AD-160 states that pressure vessels must follow conditions of the next two sections, AD-160.1 and AD-160.2. AD-160.1 states that piping supports used shall be opposed to the integral strength of the pressure vessel. In paragraph, AD-160.2, conditions must be followed as rules to determine the need for fatigue analysis. The following list documents Condition A from paragraph AD-160.2 [14]:

- Condition A: Fatigue analysis is not necessary for materials whose yield strength exceeds 80,000 psi when the total number of cycles of types (a) plus (b) plus (c), defined below, does not exceed 1000 cycles, in addition to the following sub-conditions.
  - a) Is the expected (design) number of full-range pressure cycles including startup and shutdown
  - b) Is the expected number of operating pressure cycles in with the range of pressure variation exceeds 20% of the design pressure. (Cycles in which the pressure variation does not exceed 20% of the design pressure are not limited in number. Pressure cycles caused by fluctuations in atmospheric conditions need not to be considered.)
  - c) Is the effective number of changes in metal temperature between any two adjacent points in the pressure vessel.

According to the 1969 ASME Criteria Document, described in Section 2.2.2, the fatigue curves that are used must be adjusted for two items [8]. An adjusted S-N fatigue curve can be seen in Appendix B. This figure was adapted from the ASME Unfired Piping and Pressure Vessel standards and codes Appendix 5, Section VIII, Figure 5-110.2.1 [12]. In this figure, the ASME has adjusted the strain-based data. The fatigue curve has an adjustment for maximum possible mean stress under a state of perfect plasticity, and independent adjustment for factors of 2 on stress, and 20 on number of cycles; again, these concepts are described in more detail in the next section.

If only stress based fatigue curve data are available, the data should also be adjusted for the maximum effects of mean stress. In Figure 2-8, a reproduction of Figure 5-110.2.2 is shown [12]. In this figure, curve B is stress based, and C has been adjusted for the maximum effects of mean stress. Mean stress adjustment will be discussed in Section 2.2 and mean stress effects will be documented in Chapter 3. Because the fatigue curves have been mean-stress adjusted, ASME requires that paragraphs 4-135 and 4-137 of Appendix 4, Section VIII in the Unfired Piping and Pressure Vessel standards and code be satisfied [15].

The first paragraph, 4-135, states that the stress intensity, derived as the highest value at any point across the thickness of a section, is to be compared to 2 times the value of the allowable values taken from an S-N fatigue curve [15]. The criteria of paragraph 4-137 states that the mathematical sum of the three principal stresses must not exceed four times the tabulated value of mean stress,  $\sigma_M$  [15].

Reproduction of Curve B from Figure 5-1100.2.2, Appendix 5.

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Curve B from Appendix 5, figure 5-110.2.2

Fatigue Curve B, Adjusted for Mean Stress

To fully understand the adjustment to the SN curves by the ASME for factors of safety, the ASME *Criteria Document* must be followed [8].

#### 2.2.2 Criteria Document

The *Criteria Document* of the 1969 ASME Unfired Boiler and Pressure Vessel standards and codes was developed from a footnote that appears in Division 1, Section VIII on page 9. The footnote gives the following statement, "the maximum hoop stress will not exceed the allowable stress" [8]. This means that Section III and VIII of the ASME Unfired Boiler and Pressure Vessel standards and codes does not call for a detailed stress analysis of a piping system, but just will require the minimum pipe-wall thickness needed to sustain the hoop stress. The criteria document was developed in 1969 and has since been abolished with its some of its contents added to Section III and Section VIII of the ASME Unfired Boiler and Pressure Vessel standards and codes 1999 version [31]. However, DuPont has requested the criteria document format of the adjustment of the S-N fatigue curves is to be followed. In the criteria document, an adjustment of the S-N fatigue curve for the

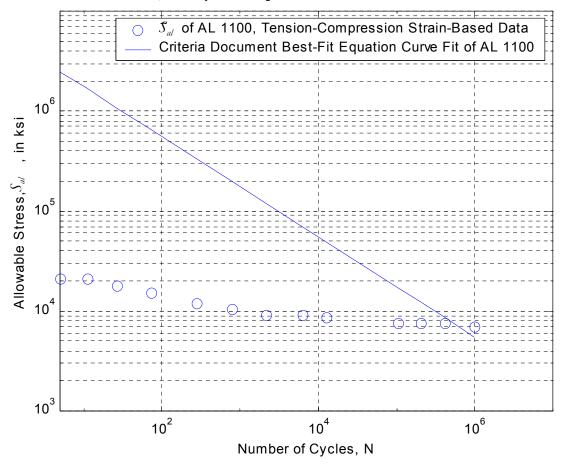
maximum mean stress, a factor of 2 on allowable stress, and a factor of 20 on the number of cycles are proposed [8]. The following section will detail these adjustments.

Most S-N curves supplied by the industry are in various formats, from load-based to strain-based. The ASME allows both load-based to strain-based S-N fatigue curves, as detailed in the previous sections. In the criteria document they developed a general equation to fit this data. This equation is termed the "best-fit" equation, seen as Equation (2.20) [8]. The "best-fit" equation incorporates the reduction area of the material, "A," in percent as well as the elastic modulus, "E," the endurance limit,  $S_o$ , of the material, which they define as "B." An example of the best-fit equation of Aluminum 1100 is seen in Figure 2-9 using a value of  $10x10^6$  PSI for the elastic modulus, 45% for reduction area percent, A, and an endurance limit of 7000 PSI for B. As seen in this figure, the "best-fit" equation is a poor fit for this data.

$$S = \frac{E}{4\sqrt{N}} Log_{10} \left(\frac{100}{100 - A}\right) + B \tag{2.20}$$

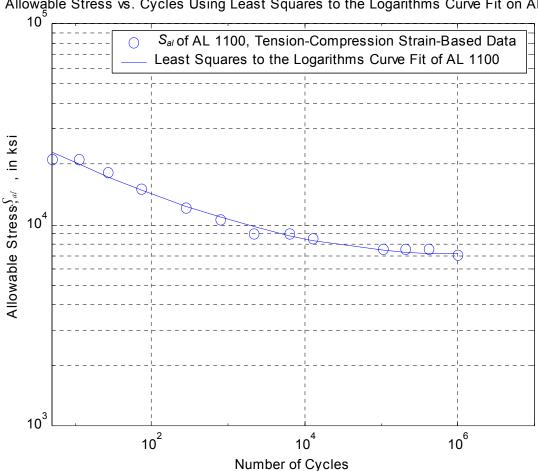
However, since this criteria document has been abolished, a new best-fit curve to the experimental data has been adapted. The new curve fit format is found in Article III-2200 of Section III in the Unfired Boiler and Pressure Vessel standards and codes, and is obtained by applying the method of least squares to the logarithms of the stress values [16]. One possible reason for the abolishment of the criteria document "best-fit" Equation (2.20) is that this equation will not create an accurate reproduction of the stress values when applied on some materials such as Aluminum, which does not have a true endurance limit.





**Figure 2-9.** Example of Criteria Document "Best-Fit" Equation to Aluminum 1100 (stress data from NASA, Technical Note -157 [17])

Upon application of the least squares to the logarithms of the stress values, a more accurate representation of the stress values can be obtained. In contrast to the "best-fit" equation, an example S-N curve created by the least squares to the logarithms shows that the curve does indeed follow the supplied data and is seen as Figure 2-10. As seen in this figure, the least squares to the logarithms curve fit is a better fit to the stress data. The least square to the logarithms is explained more in Chapter 3.



Allowable Stress vs. Cycles Using Least Squares to the Logarithms Curve Fit on AL 1100

Figure 2-10. Example of Least Squares to the Logarithms Best-Fit Equation to Aluminum 1100 (stress data from NASA, Technical Note-1574 [17])

Since Dupont required an inclusion of four additional S-N fatigue curves, they needed to be ASME compliant. This meant that the least squares to the logarithms of the stress values must be applied to the stress data. In this next Figure 2-10, the least squares to the logarithms curve fit shows a better fit to the stress data in comparison to the use of the criteria document best-fit curve fit equation.

In the ASME 1999 version of the Unfired Boiler and Pressure Vessel standards and codes, Section III, Article III-2200, which is a revision of a section in the criteria document, they propose to use a conversion factor using the engineering materials ultimate and yield

strengths [8, 16]. However, in order to use these two strengths, the ASME proposes the use of the Modified-Goodman criteria [8].

The Modified-Goodman criterion uses a diagram as a graphical representation of the multiple stress combinations that will act as a failure criterion. The Modified-Goodman diagram has been termed conservative for use by the ASME [5]. All of the stress-states used in the following pages are defined in Chapter 3. The yield strength in the Modified-Goodman diagram comes from the assumption of perfect plasticity of the material. Perfect plasticity of a material limits the stress in a stress-strain diagram. This relation can be seen in Figure 2-11.

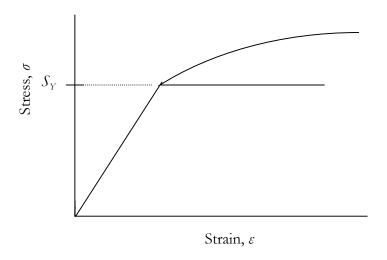
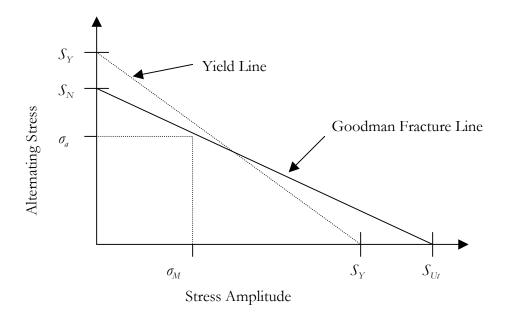


Figure 2-11. Yield Stress Limitation on a Stress-Strain Diagram

As a result of the perfect plasticity assumption, a relation between the mean stress, and the stress amplitude, seen as Equation (2.21). In this equation, these stresses are equated to the yield strength.

$$\sigma_M + \sigma_a = S_V \tag{2.21}$$

An adaptation of the Modified-Goodman diagram from Shigley and Mishke [9] is seen in Figure 2-12. On the vertical axis is the stress amplitude  $\sigma_a$ , the yield strength, and the stress at any life,  $S_N$ . On the horizontal axis, is the mean stress,  $\sigma_M$ , the yield strength, and the ultimate strength of the material are seen.



**Figure 2-12.** Modified Goodman Diagram (adapted from Shigley and Mishke [4])

To create the Goodman fracture line, the stress amplitude and mean stress, the endurance and the ultimate strengths are put into a relation, seen as Equation (2-22) [8, 9].

$$\frac{\sigma_a}{S_N} + \frac{\sigma_M}{S_{Ut}} = 1 \tag{2-22}$$

Equation (2.22) is the Goodman line. ASME created a new relation based upon the maximization of maximum stress at the yield strength of the material. This attempts to correct the allowed stress at any cyclic life for the maximum means stress that could exist based upon a perfectly plastic material model where the total stress is limited to the yield strength of the material. Using the new relation based on the maximum stress at the yield strength of the material, Equation (2.21) is solved for mean stress, and then substituted back into Equation (2.22). The new variable seen in this equation is  $S'_N$ .

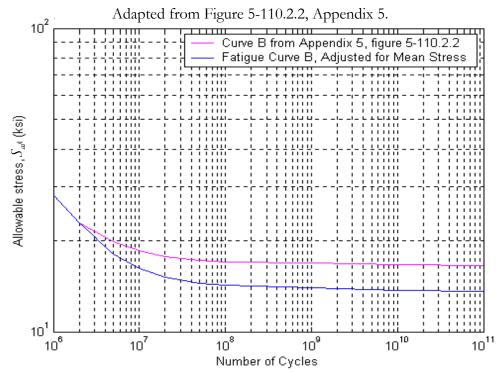
Using this new relation, the ASME had derived a way to create an S-N curve that can be mean stress adjusted. The following Equation (2.23) shows the conversion factor used for creating an S-N curve that incorporates mean stress [8]. However, upon analysis of this equation in the Criteria document, a new variable  $S_n$  was found [8]. This variable has similar

to the  $S_N$  found in the Modified Goodman equation, but had a lower case "n" instead of an upper case "N."

$$S_{N}^{'} = S_{n} \left[ \frac{S_{UT} - S_{y}}{S_{UT} - S_{N}} \right]$$
 (2.23)

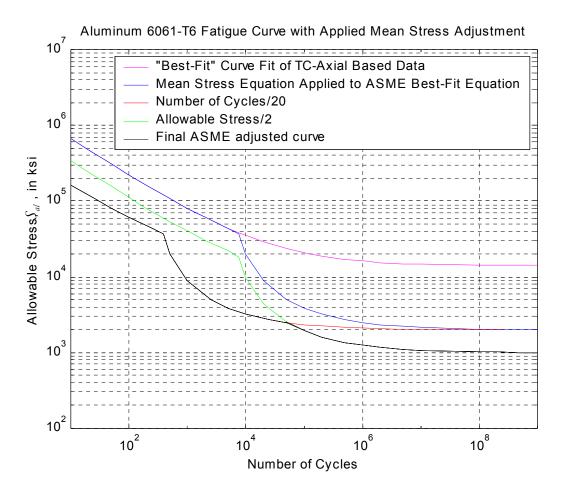
This discrepancy in equations could have arisen from the mislabeling of variables, and lack of error checking by ASME. However, upon application of Equation (2.23), it was found that to use this conversion factor, the factor needed to be multiplied by the stress at any life,  $S_N$ , in order for the mean stress adjustment to be applied to the S-N curve. As a result, Equation (2.23) was used, but with the variable  $S_n$  replaced with the variable  $S_N$ . Finally, the mean stress adjustment to an S-N curve could be implemented to the fatigue data that was input into Tmin.

By creating Equation (2.23), and using the ultimate and yield strengths, the mean-stress adjusted S-N curve is forced to deviate from the original S-N curve when  $S_N$  equals  $S_Y$ . When the yield strength equals the stress at any life, N, the mean-stress adjusted curve falls below the original S-N curve for life cycles greater N. This is shown as Curve B in Figure 2-13. The mean-stress adjustment prepares the S-N curve for the further adjustment according to ASME standards and codes.



**Figure 2-13.** Design Fatigue Curve of Series 3XX Alloy Steel. Curve A is a Load-Based Curve, and Curve B is the Mean Adjusted Curve (adapted from Appendix 5, Section VIII, Figure 5-110.2.2 [12])

The previous mean stress adjusted S-N curve can be seen to lower the endurance limit of the material, thus creating a conservative fatigue curve. Next, the design stress values were obtained from the best-fit curve by dividing a factor of two on stress or a factor of twenty on cycle. This second procedure implemented by ASME into their codes and standards for S-N fatigue curves. This has been dubbed the 2/20 format. Figure 2-14 shows an S-N curve that has been adjusted for mean-stress, and also has been adjusted according to the 2/20 format. These newly adjusted S-N curves are used to develop a master curve, which takes more conservative point at each number of cycles [8].



**Figure 2-14.** Fatigue Curve of Aluminum 6061-T6 with Mean Stress Adjustment, and  $S_{al}$  Divided by a Factor of 2, and Number of Cycles Divided by a Factor of 20

Some discrepancy was found in the *Criteria Document* as to whether or not the 2/20 format was a mandatory ASME design. As discussed previously in Article III-2000 of Section III of the ASME Unfired Boiler and Pressure Vessel standards and codes, this section documented this procedure [16]. The summary of this article is as follows, the data from the S-N fatigue curves are obtained, then the least squares to the logarithm curve fit of the fatigue data was then performed. The curves were then adjusted for mean stress. Once these operations were complete, the 2/20 format was applied. Finally, an order of operations can be created from the known ASME standards and codes on how to create a "safe" S-N curve. This order of operations is seen as follows:

- 1. Obtain tension compression S-N fatigue source data
- 2. Use least squares to the logarithms curve fit to create a curve of the data

- 3. Apply the mean-stress adjustment from Equation 2.23 to the least squares to the logarithms curve fit
- 4. Reduce the mean stress adjusted curve fit, curve by a factor of 2
- 5. Divide the number of cycles, N, of the mean-stress adjusted curve fit curve by a factor of 20
- 6. Retrieve the allowable stress values from the adjusted ASME curves, that are more conservative of the two new 2/20 curves

The above ASME procedure can be applied to piping materials that are not documented within Appendix 5, in Section VIII of the Unfired Boiler and Pressure Vessel standards and codes [12]. Use of this procedure will produce new ASME curves for use in the ASME fatigue design process. The end result is an S-N curve that will be more conservative than the original recorded data. In the next few the creation of S-N curves and the implementation of other topics covered in this chapter will be presented.