THE APPLICATION OF IMPLICIT METHOD TO OPEN-CHANNEL SURGES

by

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I. INTRODUCTION

Water resource engineers have recently recognized that it is more advantageous to mathematically model a river system than to simulate it by means of a scaled-down physical model. Thus for normal floods, the equations of continuity and momentum for gradually varied flow in open channels can be solved numerically by the implicit method. However, special care has to be exercised when a surge is likely to develop in an open channel. For such a situation, the method of characteristics can be used to model the surge. Explicit schemes are also available that model surges accurately.

The need to find a numerical scheme that would handle the fast-rising type of flows in natural open-channels led Vasiliev to come up with the two-stage implicit scheme. This scheme was certainly a step in the right direction. However, this scheme did not give results as good as the explicit schemes.

There is a tendency on the part of engineers to try and apply the implicit method to open-channels for its obvious merits. The implicit method, however, can not be applied to open-channel surges without some modification.

It was toward this end that this exploratory work was undertaken in the hope that we might come up with a better

numerical scheme that would tackle surges occurring in open-channels.

II. LITERATURE REVIEW

For the computation of shocks in rectangular open-channels and other simple cases the method of characteristics has been applied successfully [6,9]. The explicit scheme [7] is an improvement over the method of characteristics in that it is easier to program for a computer. The only known implicit scheme that can handle surges in open-channels is the two-stage method of Vasiliev [15]. This thesis explores the potentialities of a single-stage implicit method for computing open channel surges.

The equations that describe unsteady one-dimensional, gradually-varied flow in open-channels have been solved by three basic procedures:

1. Explicit finite difference methods applied directly to the partial differential equations;
2. Implicit finite difference methods applied directly to the equations, and
3. The method of characteristics.

The method of characteristics is particularly suitable for the fast-rising type of disturbance. This method is also applicable to natural flood waves. However, it is difficult to apply the method of characteristics to natural channels with varying geometry.

In the method of characteristics the limitation that must be imposed on the ratio of the time increment to the space increment is desirable not only from the standpoint of convergence of the numerical scheme but also from the standpoint of the physical property of the flow.

For example, a small disturbance in the flow created over a limited portion of the river--an influx of water from a tributary, say--will create a disturbance the front of which propagates both upstream and downstream at a rate, relative to the stream flow, which depends on the local value of the depth y. The propagation of the speed is given by $\sqrt{gy}$, where y is the average depth.

Thus, a disturbance created in a certain region at a given time spreads out in a definite fashion and affects the flow only in definite regions of the x-t plane.

The magnitude of the depth y and velocity V at any particular time and space, that is, at any given point of the x-t plane, are determined from the values of these quantities at an earlier time only on a definite and uniquely determined segment along the river, and are totally uninfluenced by what happens at any other points along the river. This has a most important bearing on the problem of calculating a wave motion.

The explicit formulation is the simplest method to program. It provides the most direct solution and the

fastest solution for short reach lengths. In cases in which the dependent variables are changing rapidly, short reach lengths must be used in order to maintain accuracy. This is necessary because of the poor representation of the non-linear terms in the equations. If the dependent variables are changing slowly, the reach lengths are increased in order to take advantage of larger time steps. If the time steps become too large the explicit formulation becomes unstable. The well-known Courant Condition is: $\Delta t \leq \frac{\Delta x}{|v + c|}$.

Although it is commonly believed that high friction may tend to stabilize a numerical computation, yet this factor can lead to instabilities in the solution of problems when the explicit formulation is used.

The implicit schemes are considerably more complicated to program. However, they are inherently stable even though the Courant Condition is violated [1].

The implicit method gives unstable results for abruptly rising hydrographs.

According to Amien and Fang [2], the implicit method, besides its inherent stability, can handle varying channel geometry even where the changes from section to section and in the bottom slope are quite significant. Thus the

implicit method plays an important role in flood routing. Collins and Fersht used the "mixed technique" for computing surges in channels [5]. In this method, they use the finite difference form combined with a fourth-order Runge-Kutta technique. The space variations in discharge and height are evaluated by a finite difference scheme and the integrations in time are performed using a fourth order Runge-Kutta technique.

$$Q_{m,n+1} = Q_{m,n} + \frac{1}{6}(k_m^1 + 2k_m^2 + 2k_m^3 + k_m^4) \text{ and}$$

$$H_{m,n+1} = H_{m,n} + \frac{1}{6}(\ell_m^1 + 2\ell_m^2 + 2\ell_m^3 + \ell_m^4)$$

where k_m^1, k_m^2, k_m^3, k_m^4, and ℓ_m^1, ℓ_m^2, ℓ_m^3, ℓ_m^4 are the successive approximations of the changes in "Q" and "H" over the time interval $n\Delta t$ to $(n+1)\Delta t$. The initial discharge is taken as constant for all x. The input surge is prescribed as a hydrograph. The downstream boundary condition is established by the stage discharge relationship.

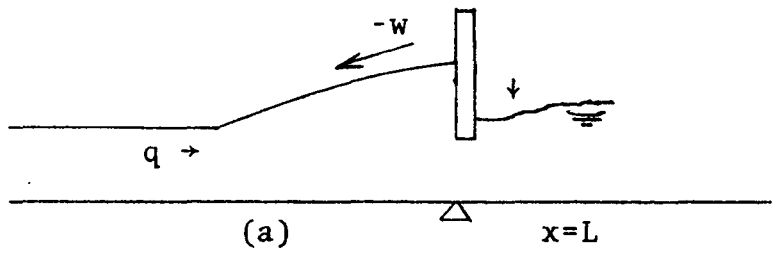
Several conditions were tried for the upstream boundary condition. Each one produced some distortion of the surge as it approached the final station. Such distortion was minimal upto values of x equal to about 80% of $M \cdot \Delta x$. The upstream boundary condition introduced a dissipative mechanism into the simple wave. As the wave

approached the upstream boundary its height decreased suddenly in the proximity of the boundary. This dissipation is most severe from quiescent starting conditions with no friction. Computations for about 70-80% of the total x-t grid were used.

$$\Delta t/\Delta x \quad < \quad 1/\sqrt{gA/B}$$

The time step satisfying the above Courant Condition gave stable results.

A profile propagating with velocity w, in a flow where fluid velocity is u, is called a positive wave if the pressure intensity experienced by individual fluid particles increases with time as the wave passes over [7].



The definition sketch explains the formation of a wave. As a positive wave steepens, the internal streamlines eventually become highly curvilinear and the pressure distribution in the wave differs significantly from hydrostatic. For small changes in depth the profile is undular. With large changes in depth, the frontal wave curls over and breaks, forming a turbulent, steep-fronted roller. As far as the calculation of water-surface profiles

incorporating such shocks is concerned the following essential features should be considered:

1. The entire phenomenon involving depth change occurs in a relatively short distance, about 5 to 10 times the larger depth, ie., it represents a short zone of rapidly varied flow propagating in a long field of gradually varied flow. Thus it can be thought of as bounded by two cross-sections of essentially hydrostatic pressure distribution and nearly uniform velocity distribution.

2. Once the long wave has broken into a bore, it proceeds with little further change in shape.

3. The energy losses within the shock are greater than those associated with boundary shear.

In view of the properties (1) and (2) it is possible to find an algebraic relationship between shock speed and depth and velocity changes across the shock that stems from volume-integrated one-dimensional equations of continuity and momentum. Δx should be large enough to encompass the entire rapidly varied shock zone.

III. DEVELOPMENT OF THE COMPUTER MODEL

The one-dimensional, unsteady equations of continuity and momentum, including lateral flow, are:

$$\frac{\partial y}{\partial t} + v\frac{\partial y}{\partial x} + \frac{A}{T}\frac{\partial v}{\partial x} + \frac{v}{T}\frac{\partial A}{\partial x}\Big|_y - \frac{q}{T} = 0 \qquad (1)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = g(S_o - S_f) - g\frac{\partial y}{\partial x} - \frac{qv}{A} \qquad (2)$$

where v = average velocity

A = area of flow, T = top width of channel

q = lateral inflow/ft. length of channel

S_o = bed slope

S_f = friction slope = $\frac{n^2v^2}{2.22R}$ 4/3, where n is Manning roughness coefficient and R is the hydraulic radius.

y = depth of flow

x = distance along the channel

t = time

$\frac{\partial A}{\partial x}\Big|_y$ = Rate of change of area of channel with distance x for constant depth.

In this study a rectangular channel 10 miles long and 100 feet wide, with no lateral inflow, has been used. Therefore, the terms $\frac{V}{T} \cdot \frac{\partial A}{\partial x}\Big|_y$, $\frac{q}{T}$ and $\frac{qV}{A}$ in equations (1) and (2) are equal to zero. Thus, we have equation of

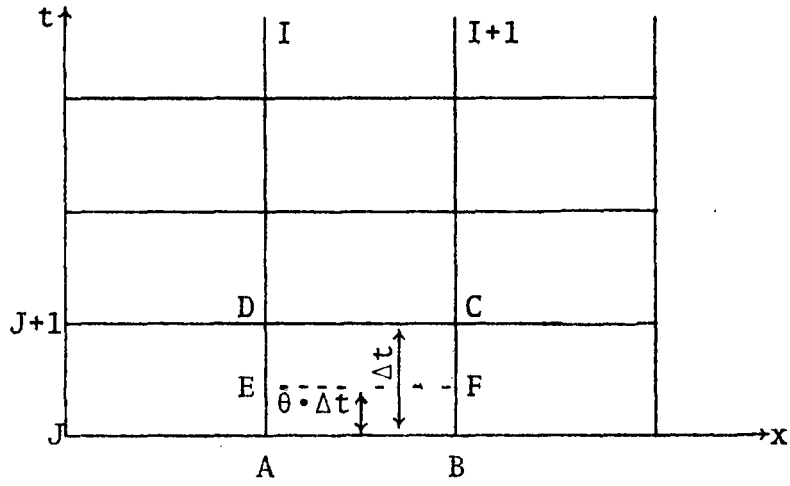
continuity:

$$\frac{\partial y}{\partial t} + v\,\frac{\partial y}{\partial x} + \frac{A}{T}\,\frac{\partial v}{\partial x} = 0$$

Equation of momentum:

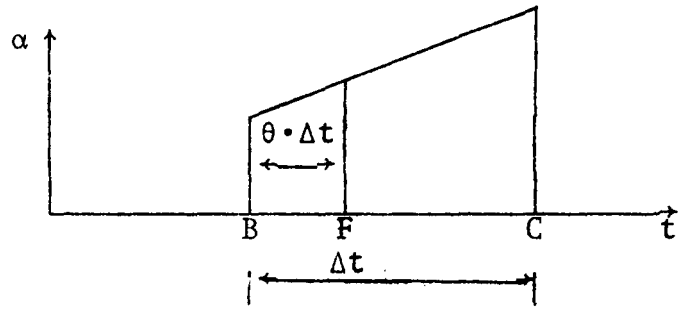
$$\frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial x} = g(S_o - S_f) - g\,\frac{\partial y}{\partial x}$$

These equations are solved by numerical means, using the implicit method. In this particular method a grid is established in the x-t plane



The x-t plane used in the implicit method.

Any property of flow α_F at point F can be expressed in terms of that particular property at points C and B.



θ is a constant factor that varies from 0 to 1.0.

$$\alpha_F = \alpha_B + (\alpha_c - \alpha_B)\,\frac{\theta \cdot \Delta t}{\Delta t}$$

or $\alpha_F = \alpha_B(1 - \theta) + \theta\alpha_c$

$$= \alpha\Big|_{I+1,J}\,(1 - \theta) + \theta \cdot \alpha\Big|_{I+1,J+1}$$

Similarly for point E:

$$\alpha_E = \alpha_A + (\alpha_D - \alpha_A)\,\theta \cdot \frac{\Delta t}{\Delta t}$$

$$= \alpha_A\,(1 - \theta) + \theta \cdot \alpha_D$$

$$= \alpha\Big|_{I,J}\,(1 - \theta) + \theta \cdot \alpha\Big|_{I,J+1}$$

$$\therefore\ \frac{\partial \alpha}{\partial x} = \frac{\alpha_F - \alpha_E}{\Delta x}$$

$$= \frac{\left\{\alpha\Big|_{I+1,J}(1-\theta) + \theta \cdot \alpha\Big|_{I+1,J+1}\right\} - \left\{\alpha\Big|_{I,J}\,(1-\theta) + \theta \cdot \alpha\Big|_{I,J+1}\right\}}{\Delta x}$$

Therefore:

$$V(E) = V\Big|_{I,J}(1-\theta) + \theta \cdot V\Big|_{I,J+1}$$

$$V(F) = V\Big|_{I+1,J}\,(1-\theta) + \theta \cdot V\Big|_{I+1,J+1}$$

$$Y(E) = Y\Big|_{I,J}\,(1-\theta) + \theta \cdot Y\Big|_{I,J+1}$$

$$Y(F) = Y\Big|_{I+1,J}\,(1-\theta) + \theta \cdot Y\Big|_{I+1,J+1}$$

Now the equations of continuity and momentum can be written in finite difference form.

The equation of continuity takes the form:

$$F = \frac{[(Y(I+1,J+1) + Y(I,J+1)) - (Y(I,J) + Y(I+1,J))]}{2\ \Delta t}$$

$$+ \frac{[V(E) + V(F)]}{2} * \frac{[Y(F) - Y(E)]}{\Delta x}$$

$$+ \frac{[Y(F) + Y(E)]}{2} * \frac{[V(F) - V(E)]}{\Delta x} \qquad (3)$$

[Note: $\frac{A}{T} = Y$; the variables at "E" and "F" have been differentiated with respect to x only].

The momentum equation is:

$$G = \frac{[(V(I,J+1) + V(I+1,J+1))}{2} - \frac{(V(I,J) + V(I+1,J))]}{2}$$

$$* \ 1/\Delta t$$

$$+ \left[\frac{V(F) + V(E)}{2}\right] * \left[\frac{V(F) - V(E)}{\Delta x}\right]$$

$$- g\ \frac{(So(I) + So(I+1))}{2} + g\ \frac{(S_f(E) + S_f(F))}{2}$$

$$+ g\ \frac{[Y(F) - Y(E)]}{\Delta x} \qquad (4)$$

With conditions known at time J for all I, equations (3) and (4) contain four unknowns,

V(I,J+1), V(I+1,J+1), Y(I,J+1) and Y(I+1,J+1).

The equations can be rewritten in a form in which an algebraic function of the unknowns is equal to zero, thus

$$F_I(V_{I,J+1}, V_{I+1,J+1}, Y_{I,J+1}, Y_{I+1,J+1}) = 0$$

$$G_I(V_{I,J+1}, V_{I+1,J+1}, Y_{I,J+1}, Y_{I+1,J+1}) = 0$$

The equations can be written for all interior reaches of the distance counter I. Since there are 2*N unknowns and 2*(N-1) equations for all the interior reaches, there being two equations for each reach. We need two additional equations from the boundary conditions.

N is the number of stations along the reach. At the upstream end the discharge hydrograph $Q_{I,J}$ for all J's is given. Hence

$$F_{US_{1,J+1}} = (VA)_{1,J+1} - Q_{1,J+1} = 0 \qquad (5)$$

At the downstream end (I = N), the stage discharge relationship is given.

$$F_{DS_{N,J+1}} = Q_{N,J+1} - (AV)_{N,J+1} = 0 \qquad (6)$$

where $Q_{N,J+1}$ is obtained from the stage discharge relationship at $Y_{N,J+1}$. Thus the equations (3) and (4) written for each reach and the two boundary conditions constitute a system of 2*N non-linear equations to be solved by the iterative method called the Newton Method. In the Newton Method, the values of Y and V at the unknown time step J+1 are assumed for all I steps, and constitute the first trial values. However, in general, the values of the functions F, G, F_{US} and F_{DS} thus calculated will not be

equal to zero, but will have a finite value called the residue (R). New values of Y and V at time J+1 are to be calculated so that the residues become zero; or in other words, ΔY and ΔV--- changes in Y and V between iterations --- are to be chosen so that the total differentials of the functions F, G, F_{US} and F_{DS} are equal to the negative of the calculated residues.

In order to find the values of constants (k's) to be used in the banded matrix we differentiate the equations (3) and (4) with respect to $Y_{I,J+1}$, $Y_{I+1,J+1}$, $V_{I,J+1}$, $V_{I+1,J+1}$.

Therefore,

$$\frac{\partial F}{\partial Y_{I,J+1}} = 1/2\Delta t - \frac{\theta}{\Delta x} * V(E).$$

$$\frac{\partial F}{\partial V_{I,J+1}} = - \frac{\theta}{\Delta x} \cdot Y(E)$$

$$\frac{\partial F}{Y_{I+1,J+1}} = \frac{1}{2\Delta t} + \frac{\theta}{\Delta x} \cdot V(F)$$

$$\frac{\partial F}{\partial V_{I+1,J+1}} = \frac{\theta}{\Delta x} * Y(F)$$

$$\frac{\partial G}{Y_{I,J+1}} = - \frac{g}{2} \cdot \frac{4}{3} \cdot S_f(E) \cdot \frac{\theta}{Y(E)} - \frac{g\theta}{\Delta x}$$

$$\frac{\partial G}{V_{I,J+1}} = \frac{1}{2\Delta t} - \frac{\theta}{\Delta x} \cdot V(E) + g \frac{S_f(E)}{V(E)} \cdot \theta$$

$$\frac{\partial G}{Y_{I+1,J+1}} = -\frac{g}{2} \cdot \frac{4}{3} \cdot S_f(F) \quad \frac{\theta}{Y(F)} + \frac{g\theta}{\Delta x}$$

$$\frac{\partial G}{V_{I+1,J+1}} = \frac{1}{2\Delta t} + \frac{\theta}{\Delta x} \cdot V(F) + g \frac{S_f(F)}{V(F)} \cdot \theta$$

In general, the results, in the form of total differentials, are:

$$D(F_{US}) = K^{US}_{Y,1} \; dY_1 + K^{US}_{V,1} \; dV_1 = - R^{US}_1 \tag{7}$$

$$D(F) = K^F_{Y,I} \, dY_I + K^F_{Y,I+1} \, dY_{I+1} + K^F_{V,I} \, dV_I$$

$$+ K^F_{V,I+1} \, dV_{I+1} = - R^F_I \tag{8}$$

$$D(G) = K^G_{Y,I} \, dY_I + K^G_{Y,I+1} \, dY_{I+1} + K^G_{V,I} \, dV_I$$

$$+ K^G_{V,I+1} \, dV_{I+1} = - R^G_I \tag{9}$$

and

$$D(F_{DS}) = K^{DS}_{Y,N} \, dY_N + K^{DS}_{V,N} \, dV_N = - R^{DS}_N \tag{10}$$

where $K^{\psi}_{W,\xi} = \left.\frac{\partial \psi}{\partial w}\right|_{\xi}$

and R values are the residues.

The equations (7), (8), (9) and (10) form a set of 2*N linear differential equations with 2*N unknowns (dY_1, dV_1, ----, dY_N, dV_N). The coefficients of the equations form a banded matrix, and a program in the IBM Scientific Subroutine Package can be used to solve this system of equations. The particular program chosen was GELB, which stands for Gauss Elimination Banded. The solution of these equations provides increments of Y_1, V_1, ----, Y_N, V_N, which will give new values of those variables which in turn will make the residues approach zero. An appropriate tolerance can be put on the relative changes in Y and V, stopping the iterative procedure when the specified tolerance is met.

The initial conditions at J = 1 were obtained by calculating the normal depth at that section for the discharge at that section.

IV. METHODOLOGY

A channel 10 miles long and 100 feet wide was chosen. Inflow hydrographs (including abrupt rise and fall in discharge) were specified at the upstream end. The downstream condition was specified with stage-discharge curve calculated from Manning formula:

$$Q = \frac{1.49}{n} \frac{A^{5/3}}{p^{2/3}} \sqrt{S_o}$$

Lateral inflow was neglected.

The channel was divided into ten reaches, each one being one mile long. The value of Δt was made equal to 0.25 HRS. The value of t_{max} was 12 HRS. A bed slope of 5 ft/mile was used. The Manning's roughness co-efficient "n" was chosen to be 0.03, to start with. Later this value of "n" was varied since the results were very sensitive to the value of "n".

Three types of hydrographs were used at the upstream end, both rising and receding.

Type 1 (rising and receding): The discharge changed linearly from 1000 CFS to 8000 CFS in one time interval and vice versa.

Type 2 (rising and receding): The discharge changed linearly from 1000 to 8000 with increments of 1700 CFS in each time interval and vice-versa.

Type 3 (rising and receding): The discharge changed linearly from 1000-8000 CFS with increments of 700 CFS in each time interval and vice versa.

The graphic representation of type 1, 2, and 3 hydrographs is shown in Figures 1 through 6. Also, two other inflow hydrographs that changed linearly from 1000 to 2000 CFS and from 1000 to 4000 CFS in one time interval were used.

The values of the variable theta were changed from 0.5 to 1.0.

The hydrographs were plotted for stations 1, 5 and 11.

In the case of type 1 rising hydrographs the time of arrival of the shock at stations 5 and 11 was calculated and compared with the theoretical time of arrival.

The formula for a "frictionless surge wave in a rectangular channel" [8] was used.

$$(V_1 + C)\ Y_1 = (V_2 + C)\ Y_2$$

and

$$\frac{\gamma}{2}\ (Y_1{}^2 - Y_2{}^2) = \frac{\gamma}{g}\ Y_1\ (V_1 + C)\ (V_2 + C - V_1 - C)$$

After finding the wave speed "C", the time of arrival of the wave was calculated thus: $t = \frac{X}{C}$.

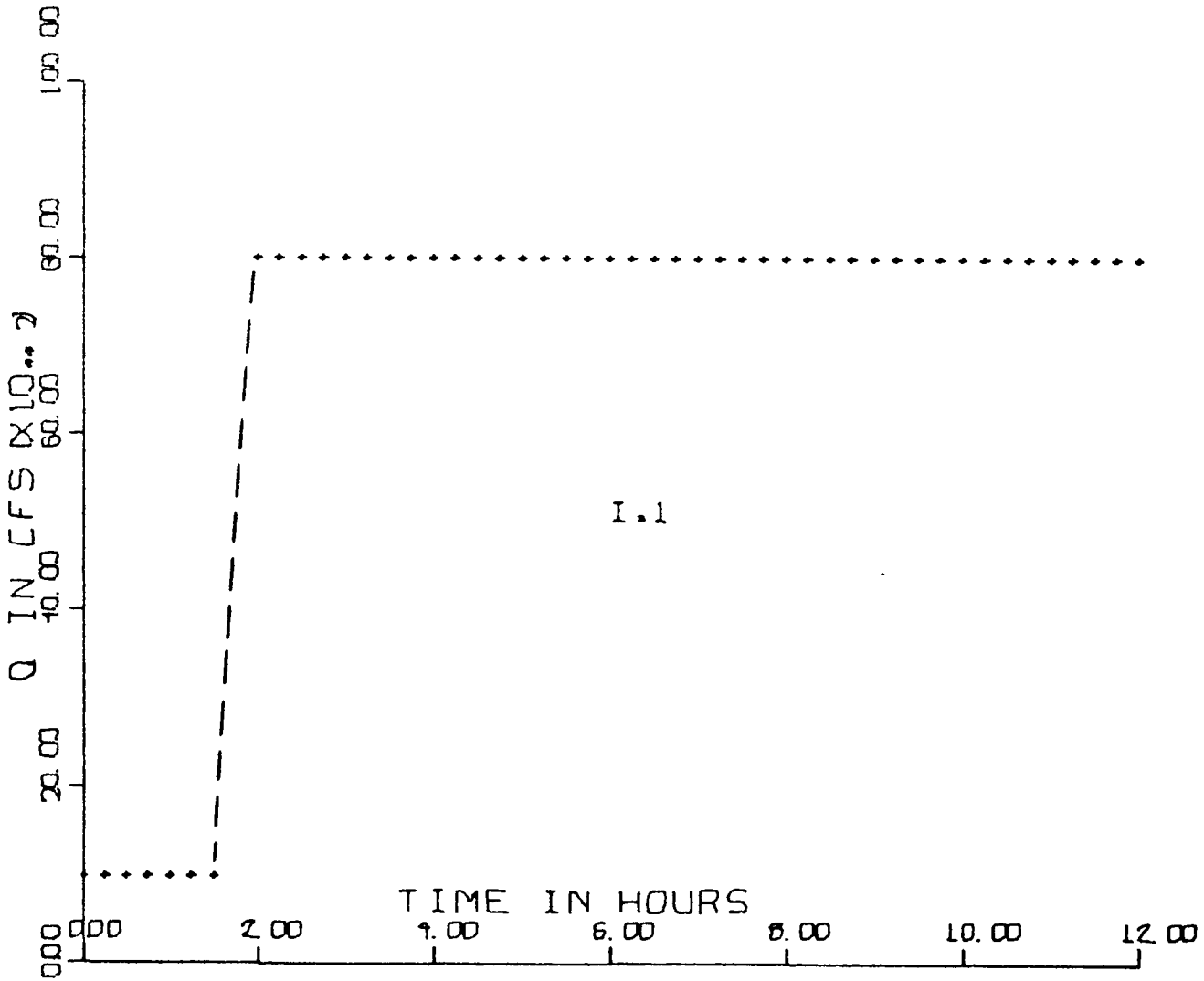


FIG.1. HYDROGRAPHS TYPE 1. RISING.

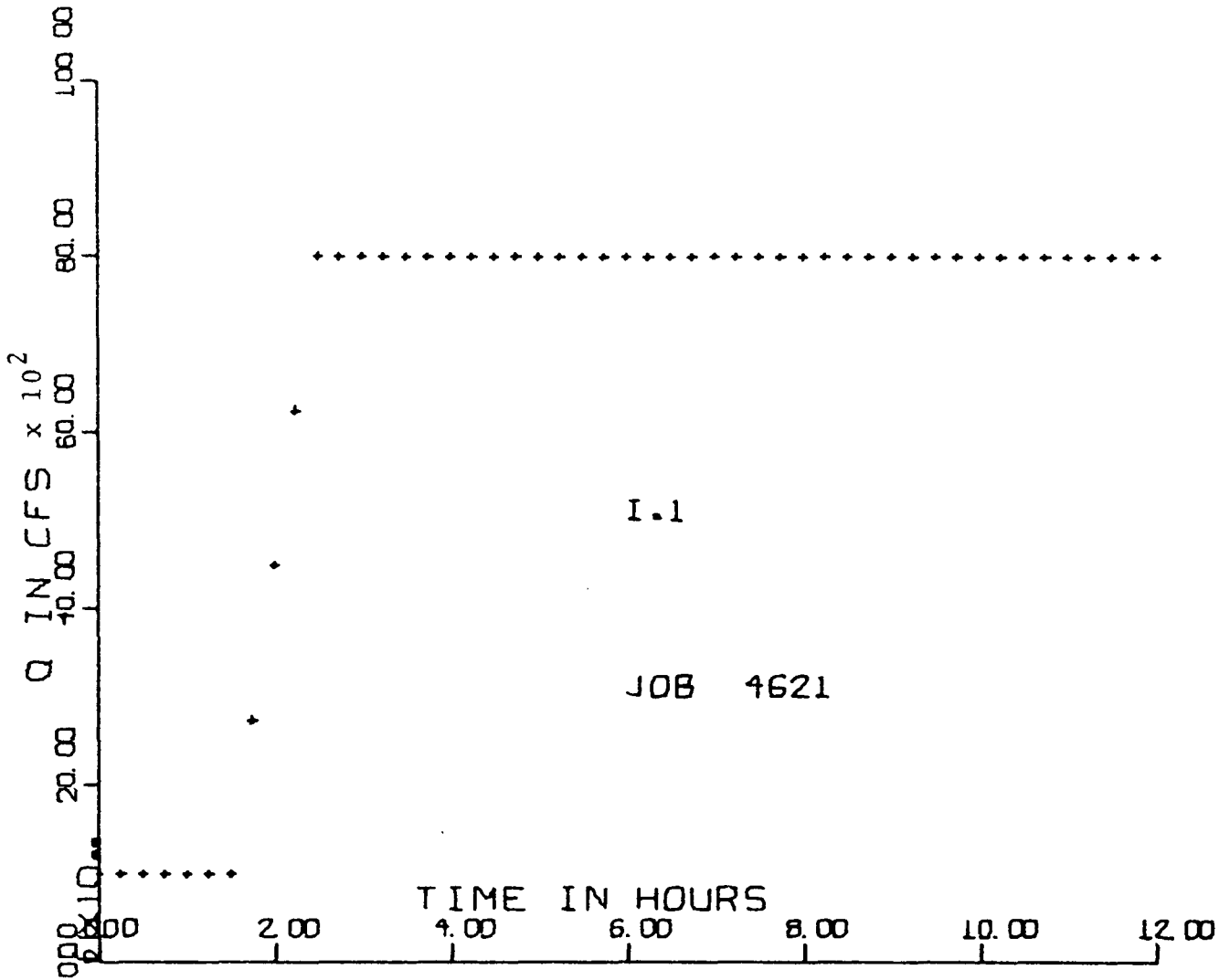


FIG.2. HYDROGRAPHS TYPE 2 . RISING .

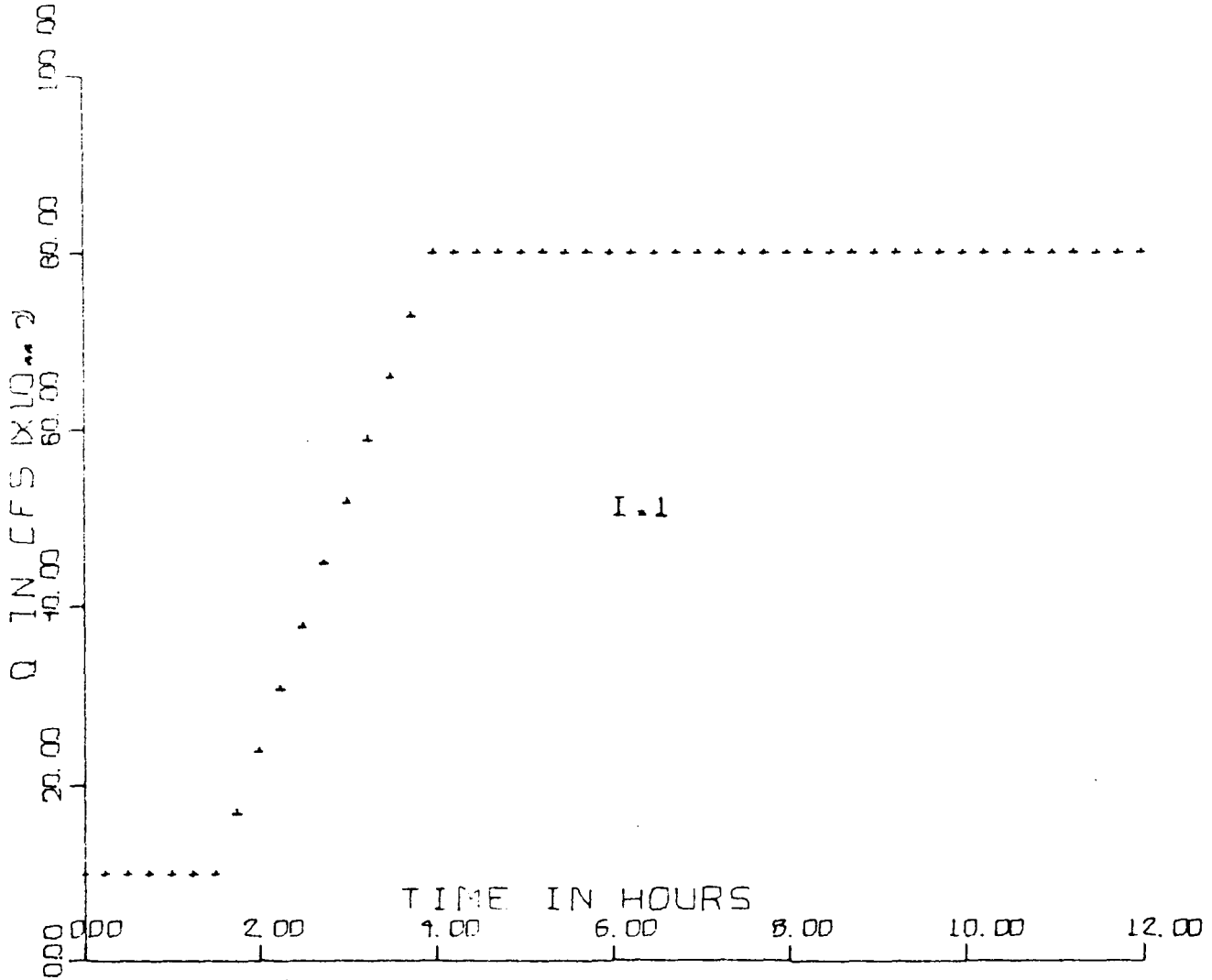


FIG.3.HYDROGRAPHS TYPE 3 . RISING .

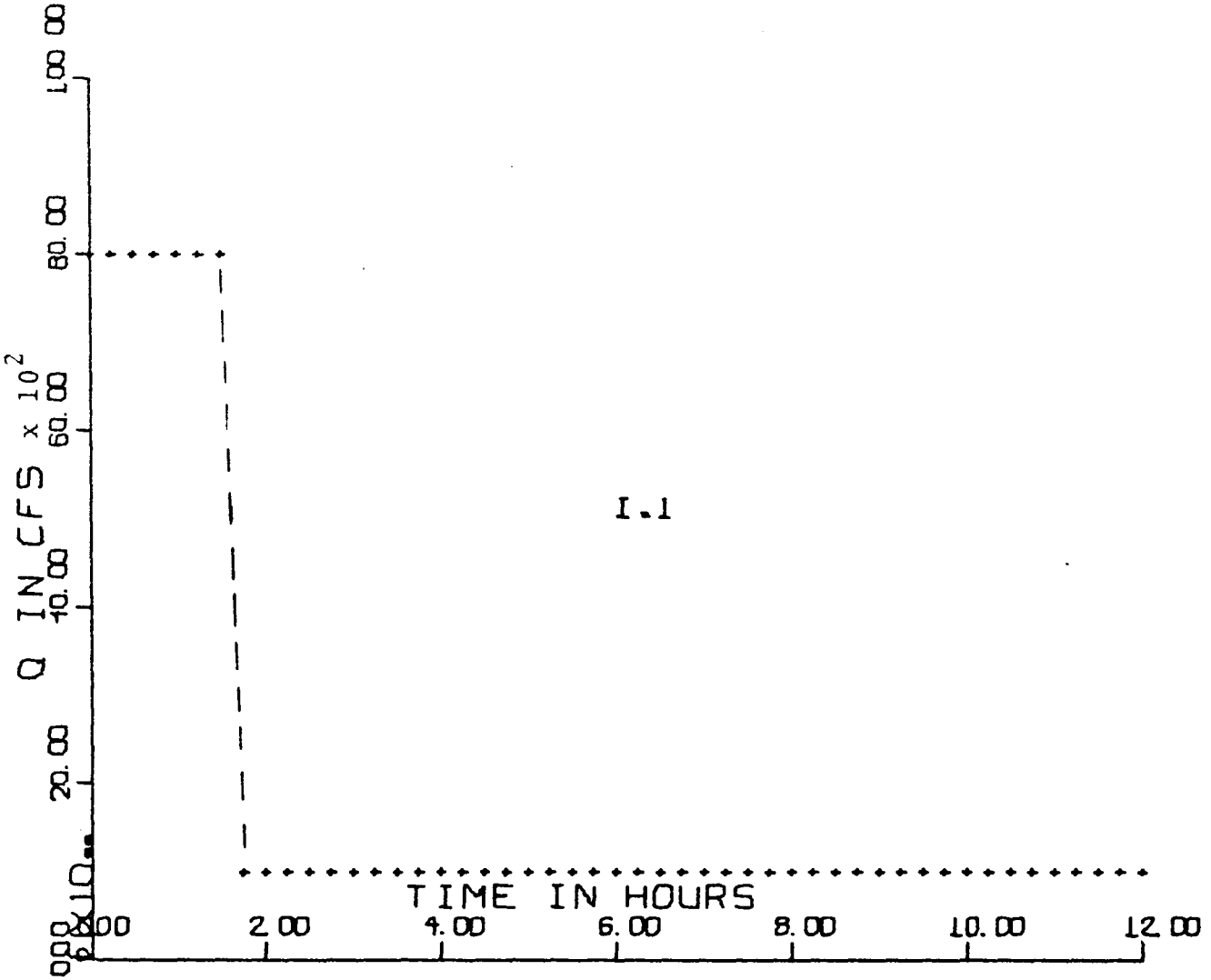


FIG.4.HYDROGRAPHS TYPE 1 . RECEDING .

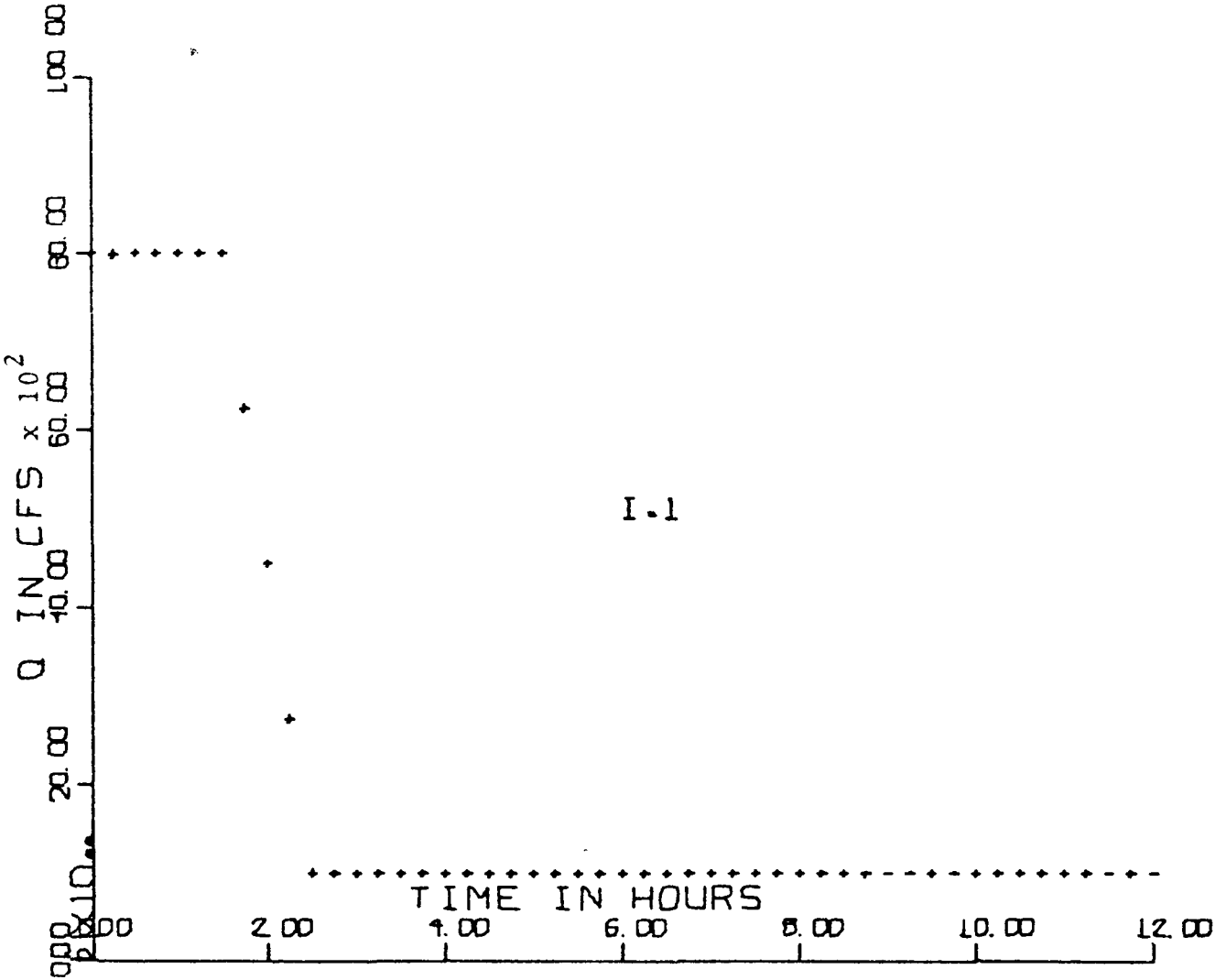


FIG.5.HYDROGRAPHS TYPE 2 RECEDING .

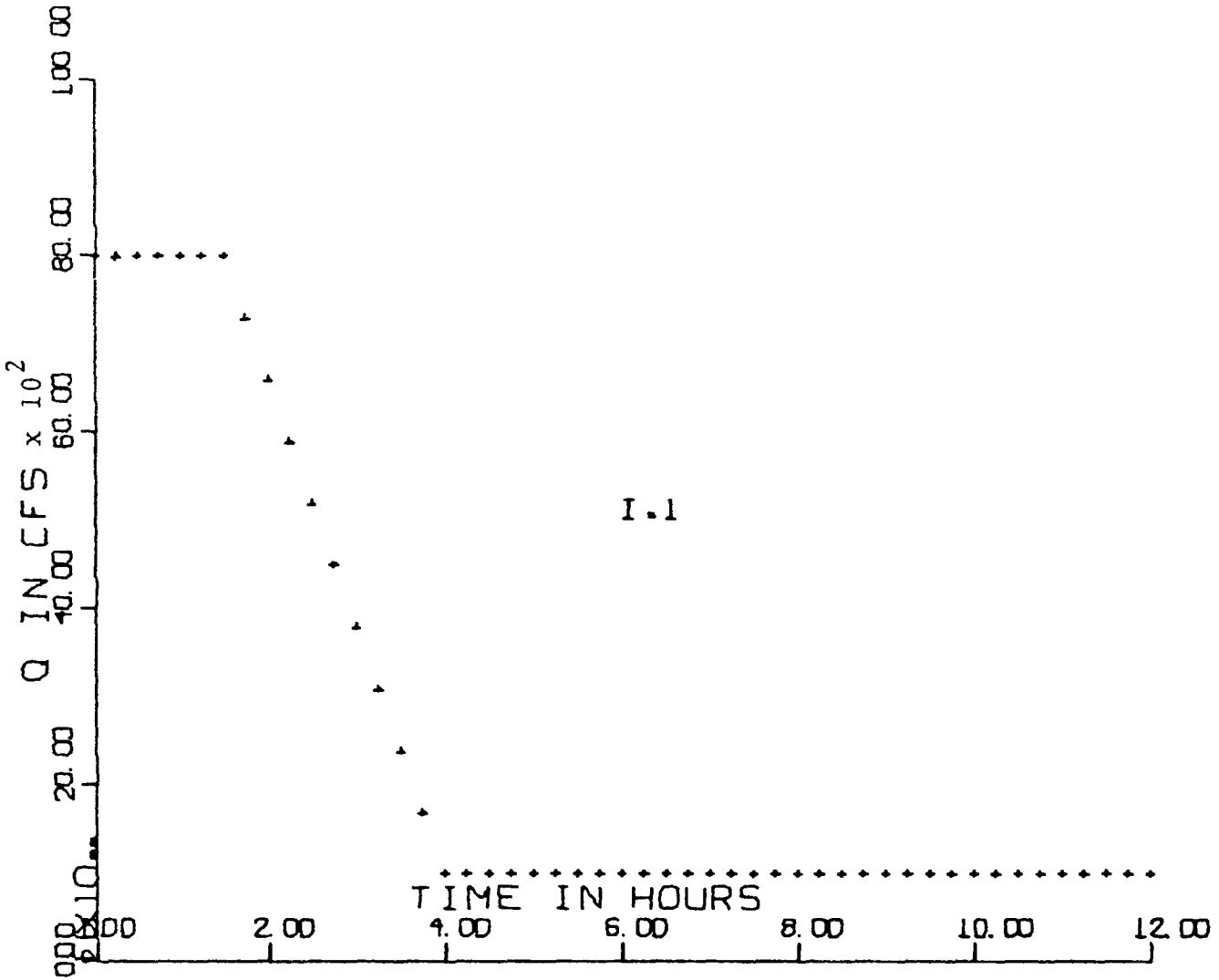


FIG.6.HYDROGRAPHS TYPE 3 . RECEDING .

This program was also applied to Faure's experiment 7, Series III [7]. The horizontal rectangular channel is 75 m long and 0.42 m wide. The Manning roughness co-efficient equals 0.01. Initially water stands at a depth of 0.205 m. Then at the upper boundary the discharge is increased from 0ℓ/sec to 28ℓ/sec in 0.2 sec. The flow is subcritical To make the solution converge, an initial discharge of 0.1 CFS was assumed. Also the outflow was made equal to 0.1 CFS. The flow profiles at 20 and 40 sec. were plotted. The calculated values and measured values are fairly satisfactory.

V. RESULTS AND DISCUSSION

Type 1 Rising Hydrographs

These hydrographs represent the abruptly rising hydrographs. To start with a Manning's roughness co-efficient "N" equal to 0.03 was used. The values of theta were varied from 0.5 to 1.0, see Figs. 7-15. The surge corresponding to $\theta = 1.0$ arrives earlier than the theoretical time of arrival. The surge corresponding to $\theta = 0.5$ arrives later, at stations 5 and 11, than the theoretical time of arrival. A value of theta equal to 0.9 gave comparatively better results, though not quite satisfactory.

The Manning roughness co-efficient "N", was changed to 0.02. This brought the hydrographs for $\theta = 1.0$ and $\theta = 0.5$ fairly close to each other, see Figs. 16-20. Next the Manning roughness coefficient was changed to 0.012. In this case the hydrograph for $\theta = 0.5$ matched the theoretical shape of the shock, see Figs. 21-22. However the value of θ equal to 0.5 gave some fluctuations in the discharge. Therefore a value of θ equal to 0.6 was chosen. This value gave satisfactory results.

In Figs. 23-26, the discharge changes linearly from 1000 CFS to 4000 CFS and from 1000 CFS to 2000 CFS in one time interval. In these two cases, too, the value of theta equal to 0.6 gave better results. However type 5

hydrograph, Fig. 26, shows greater spreading.

The values of θ from 0.0 to 0.4 caused numerical instabilities in the computer program.

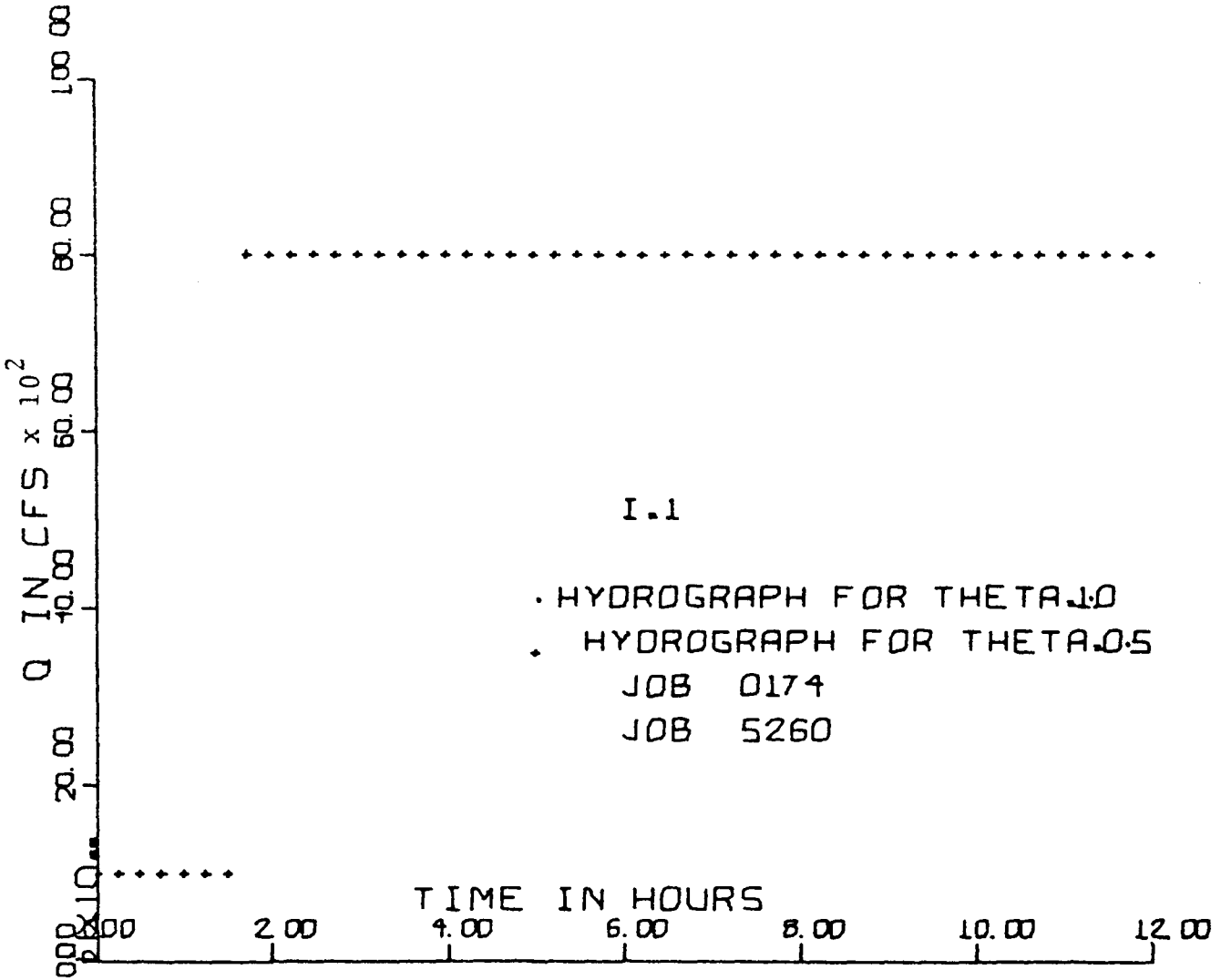


FIG.7.HYDROGRAPHS TYPE 1 . RISING .

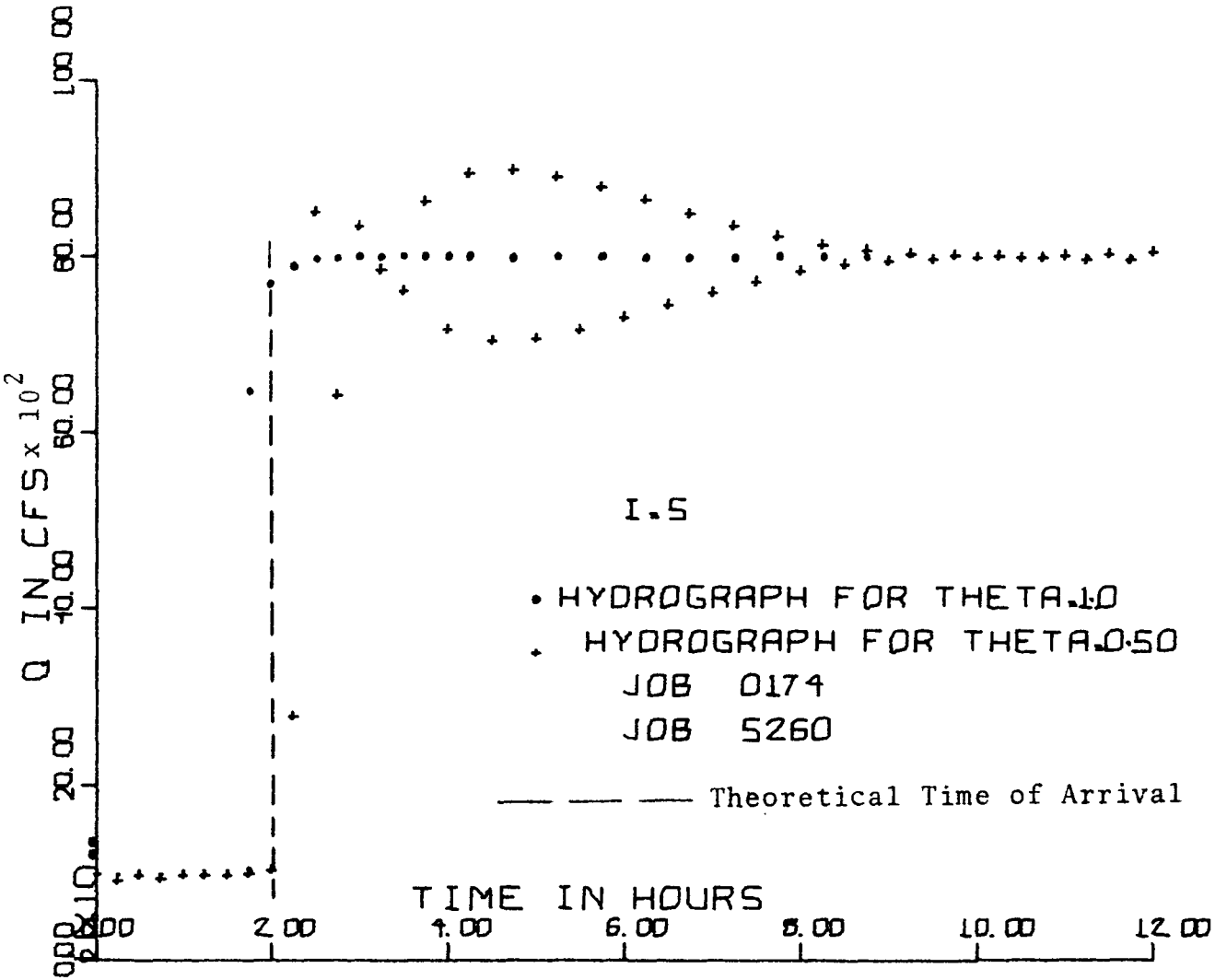


FIG.8.HYDROGRAPHS TYPE 1 . RISING .

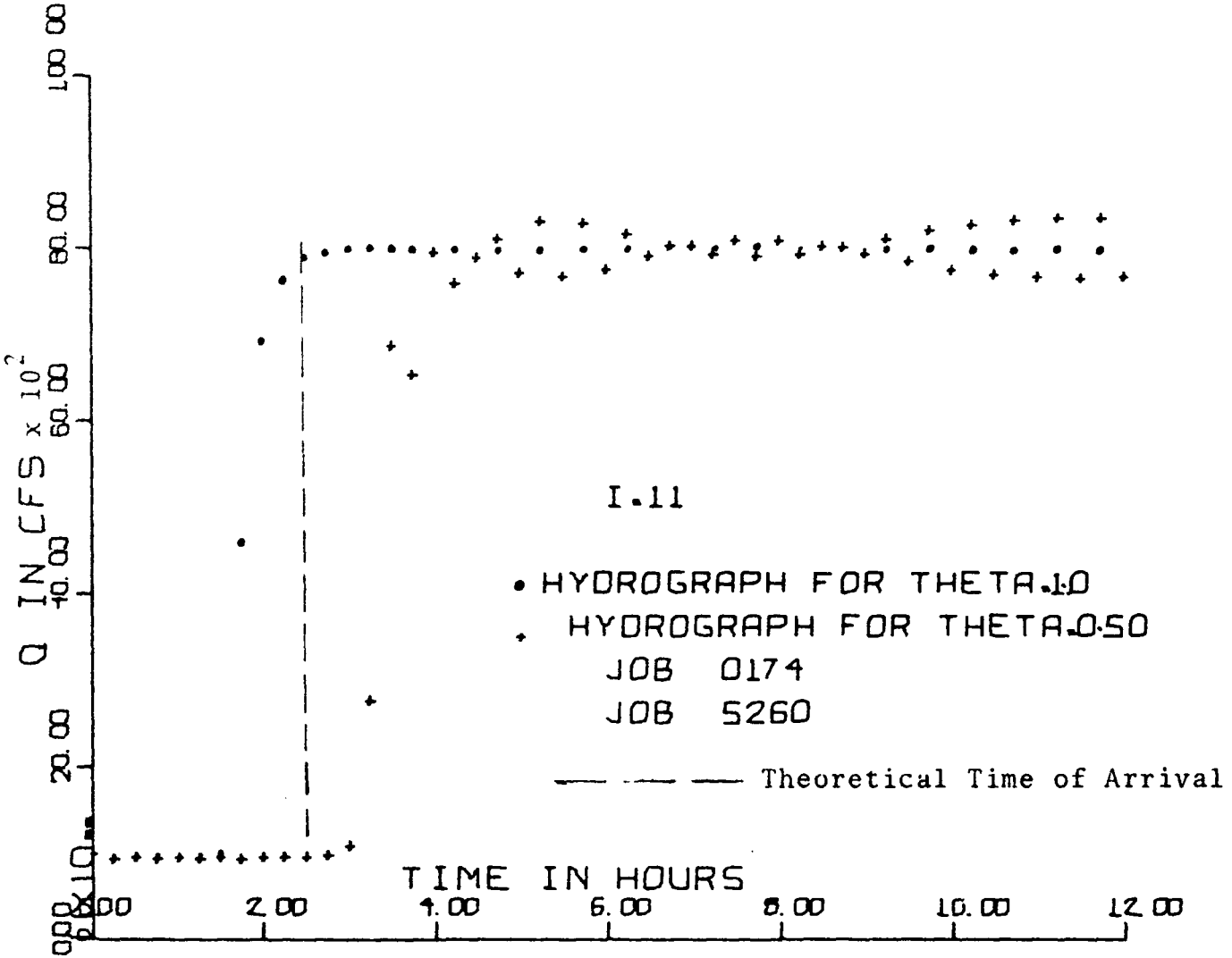


FIG.9.HYDROGRAPHS TYPE 1 . RISING .

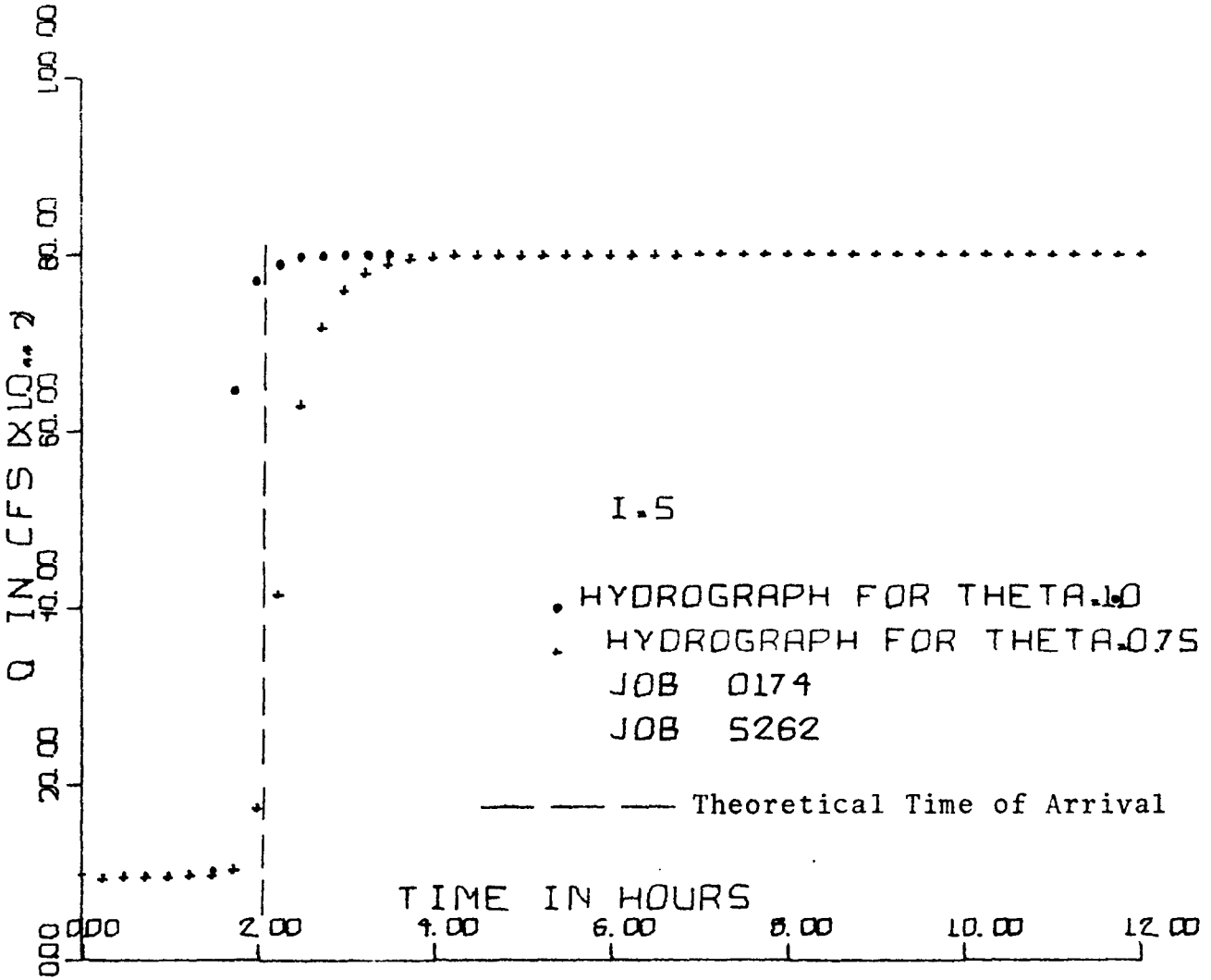


FIG.10.HYDROGRAPHS TYPE 1. RISING.

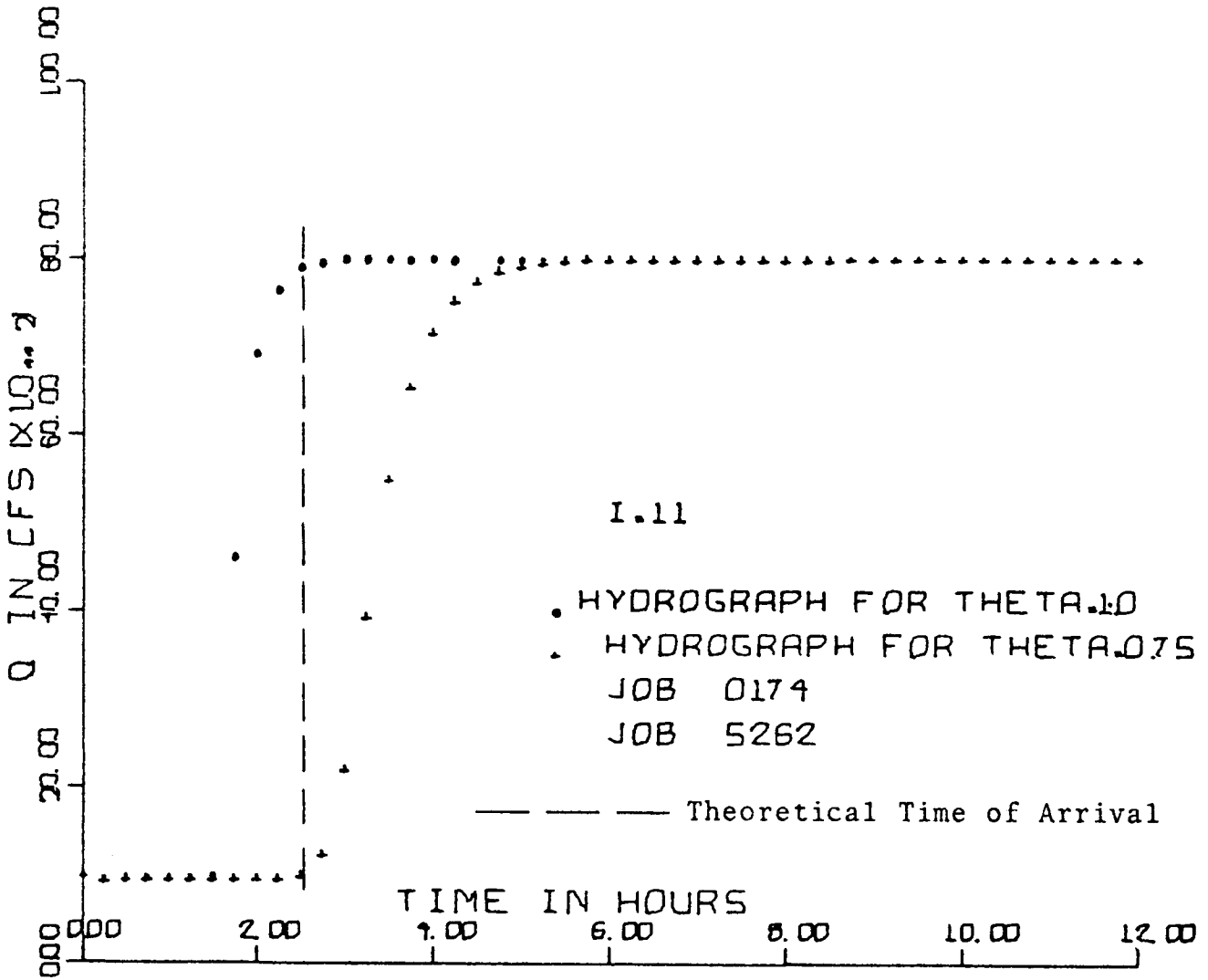


FIG.11 HYDROGRAPHS TYPE 1. RISING.

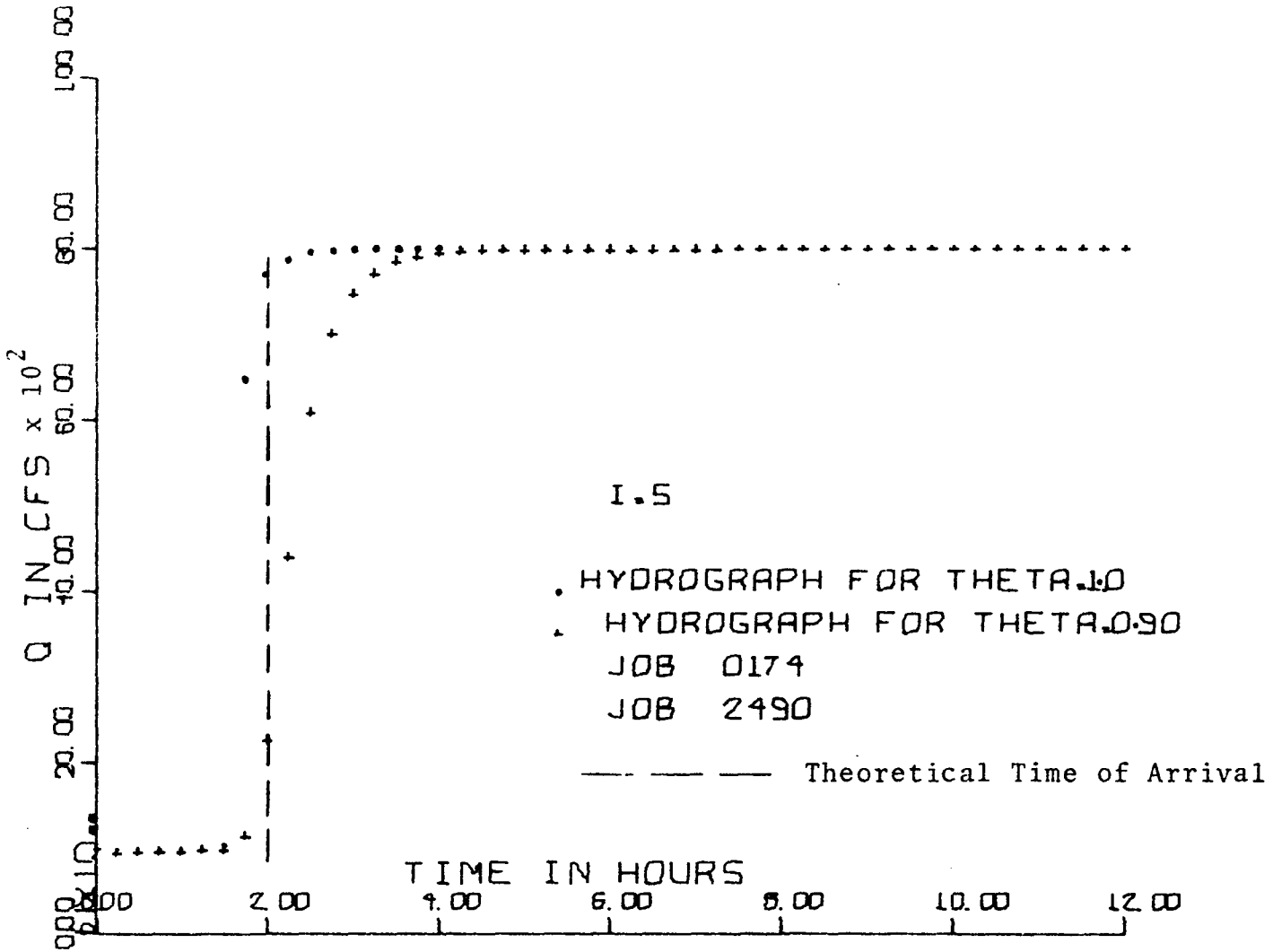


FIG12.HYDROGRAPHS TYPE 1. RISING.

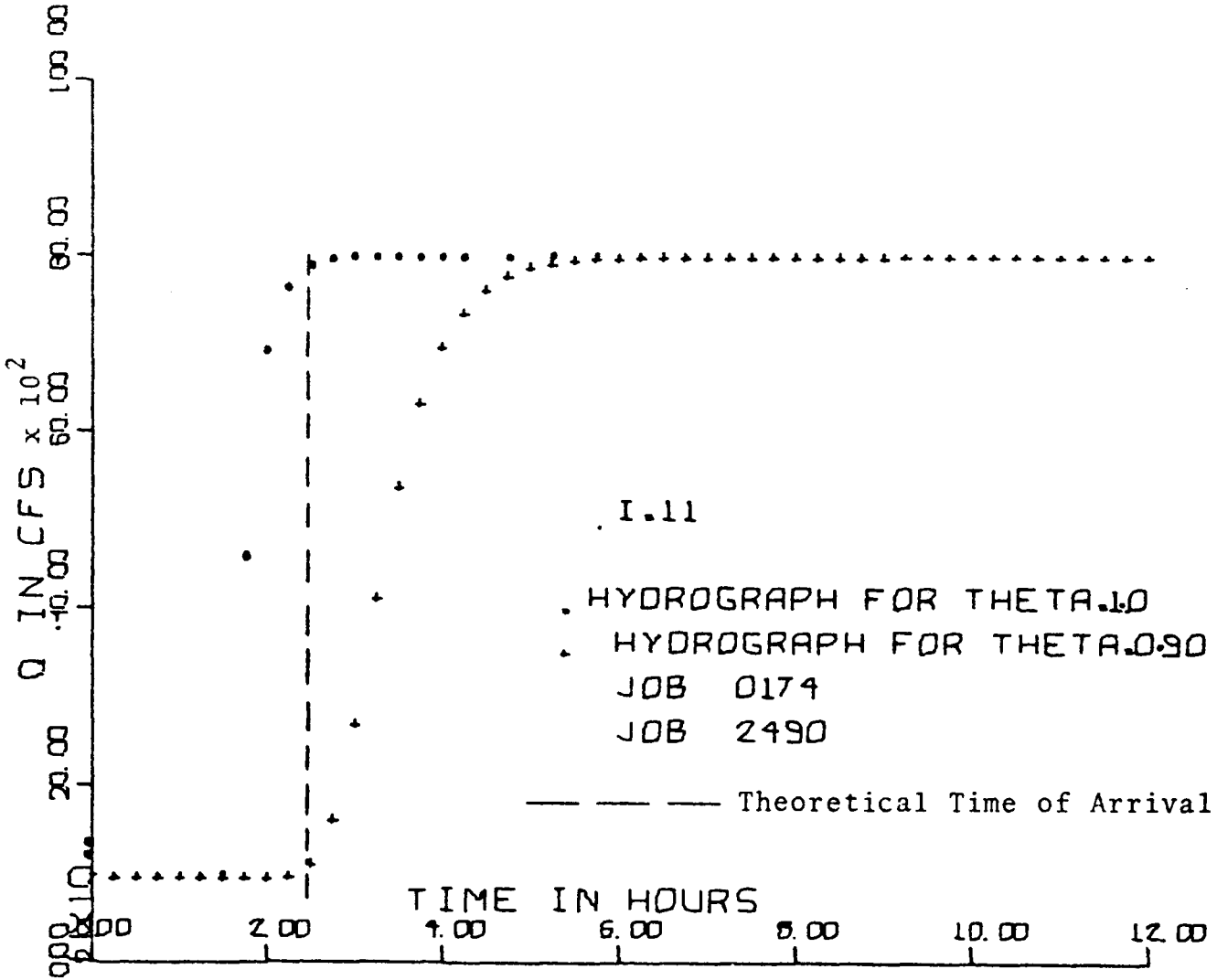


FIG.13.HYDROGRAPHS TYPE 1. RISING.

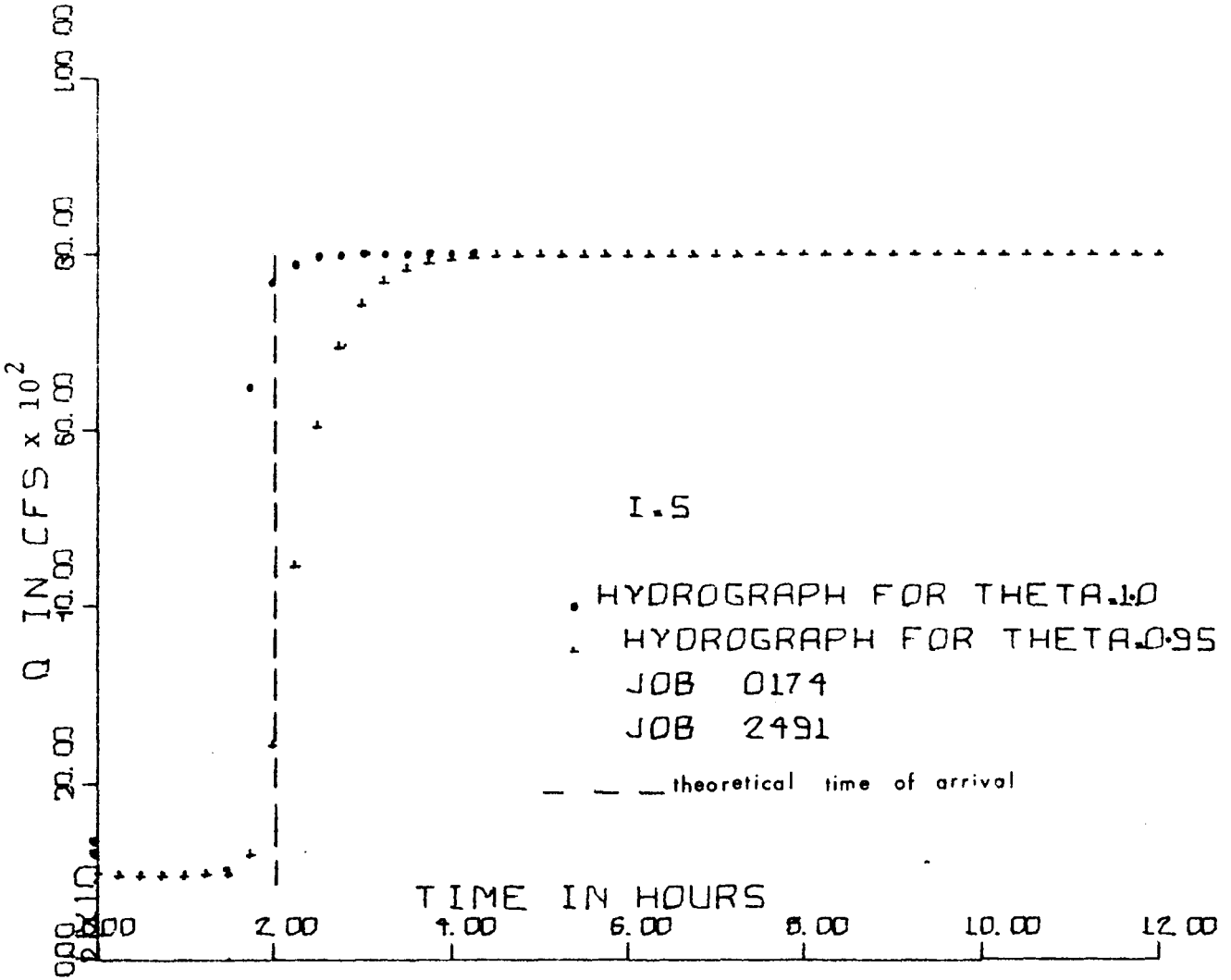


FIG.14.HYDROGRAPHS TYPE 1. RISING.

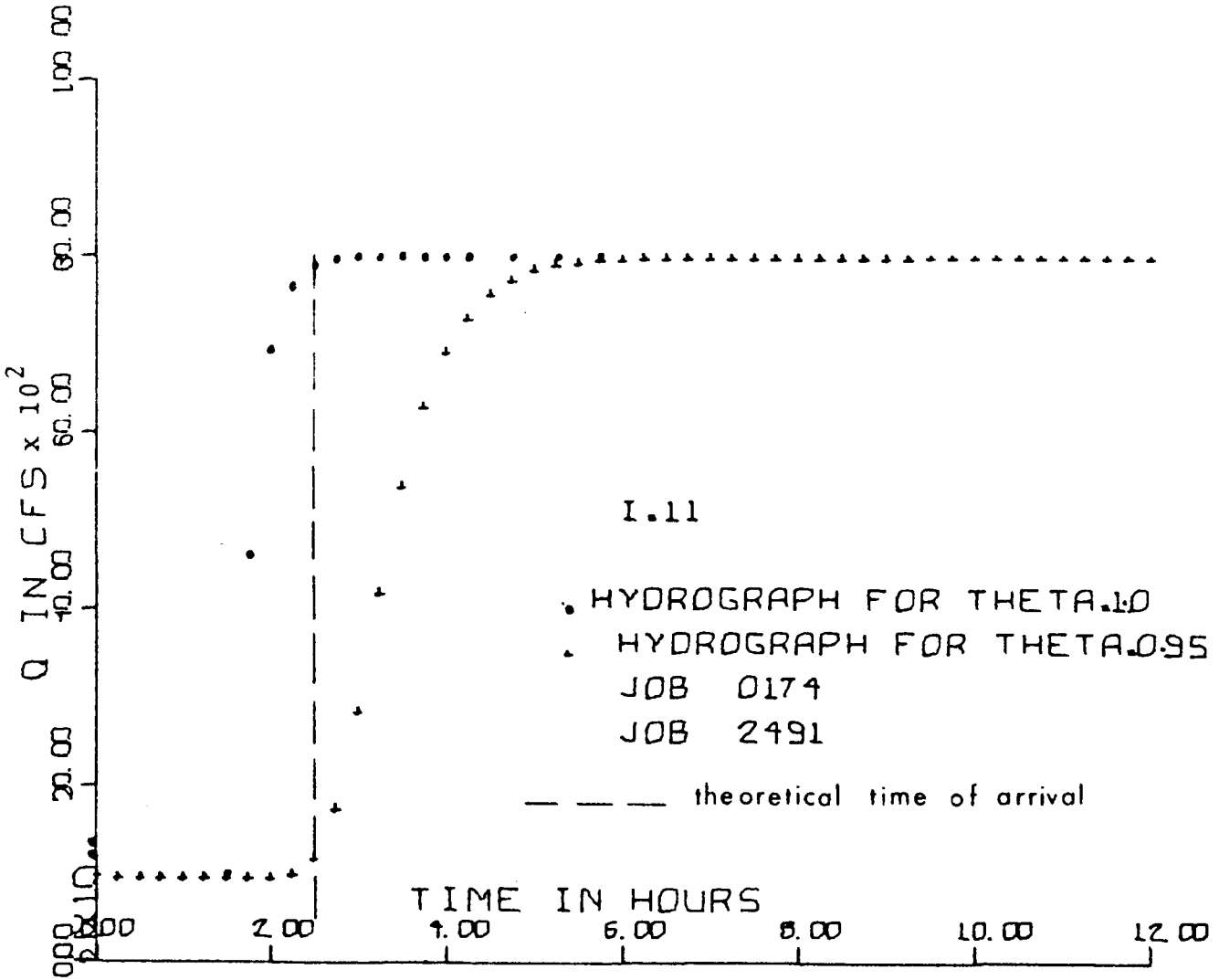


FIG.15. HYDROGRAPHS TYPE 1. RISING.

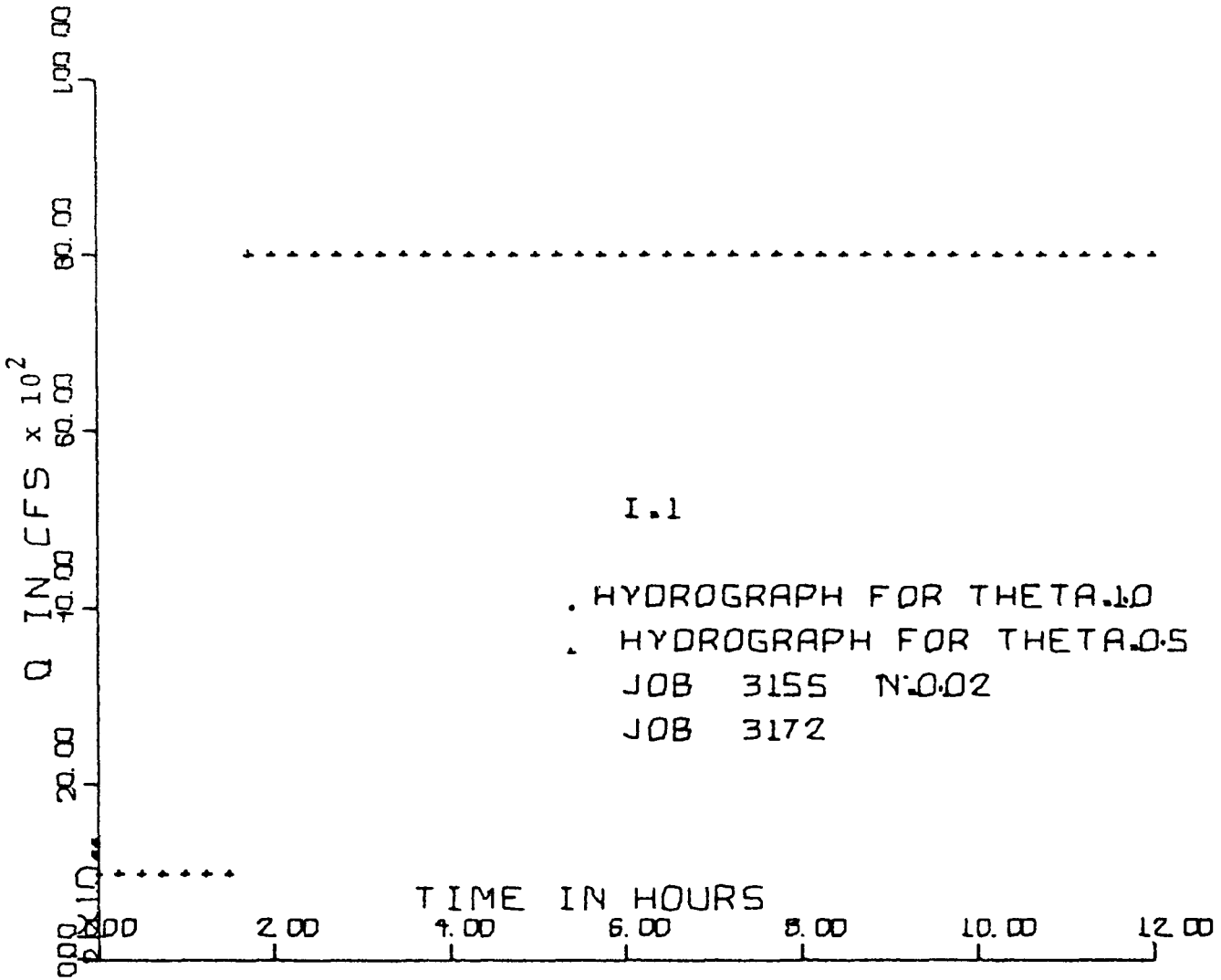


FIG.16.HYDROGRAPHS TYPE 1 . RISING .

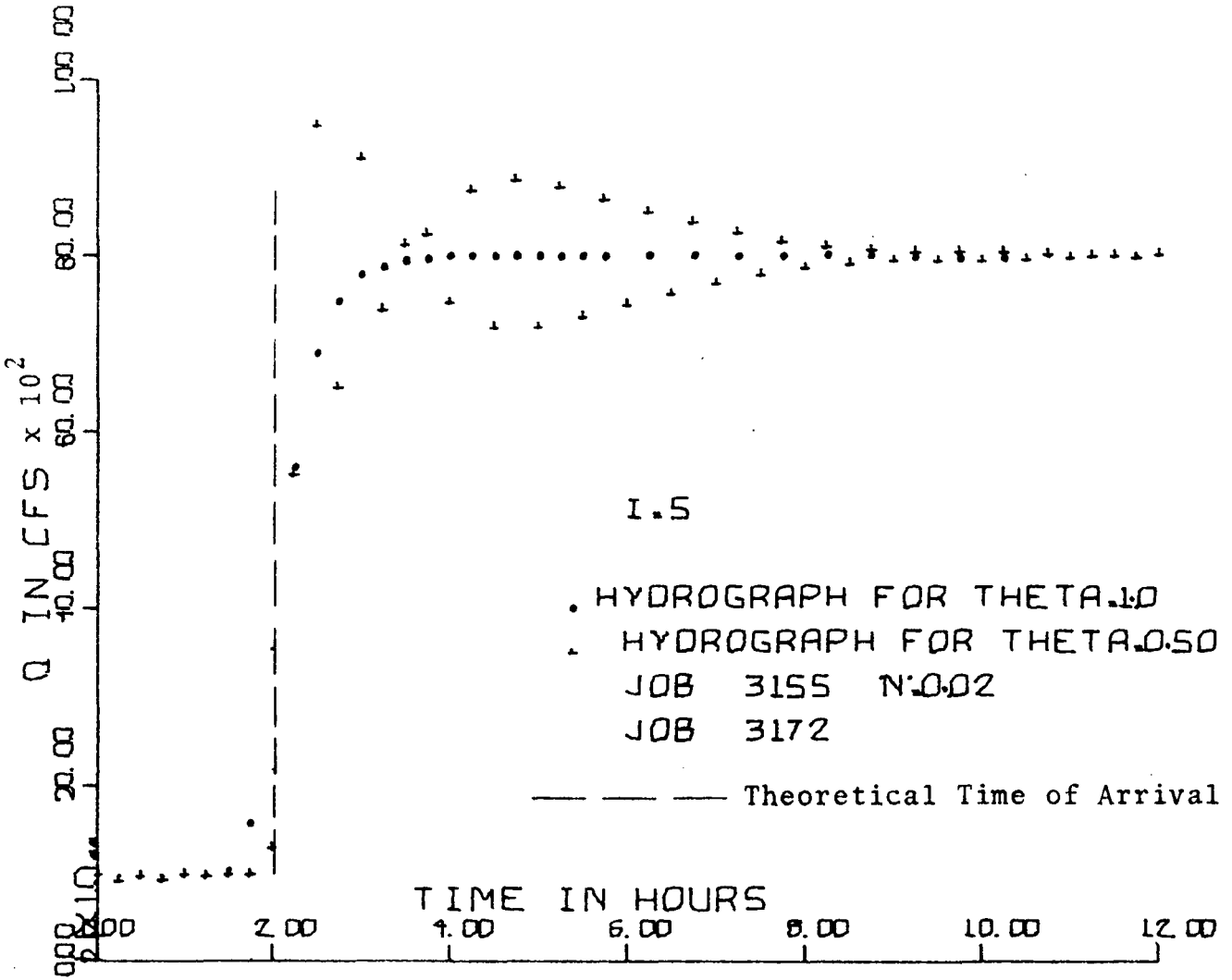


FIG.17.HYDROGRAPHS TYPE 1 . RISING .

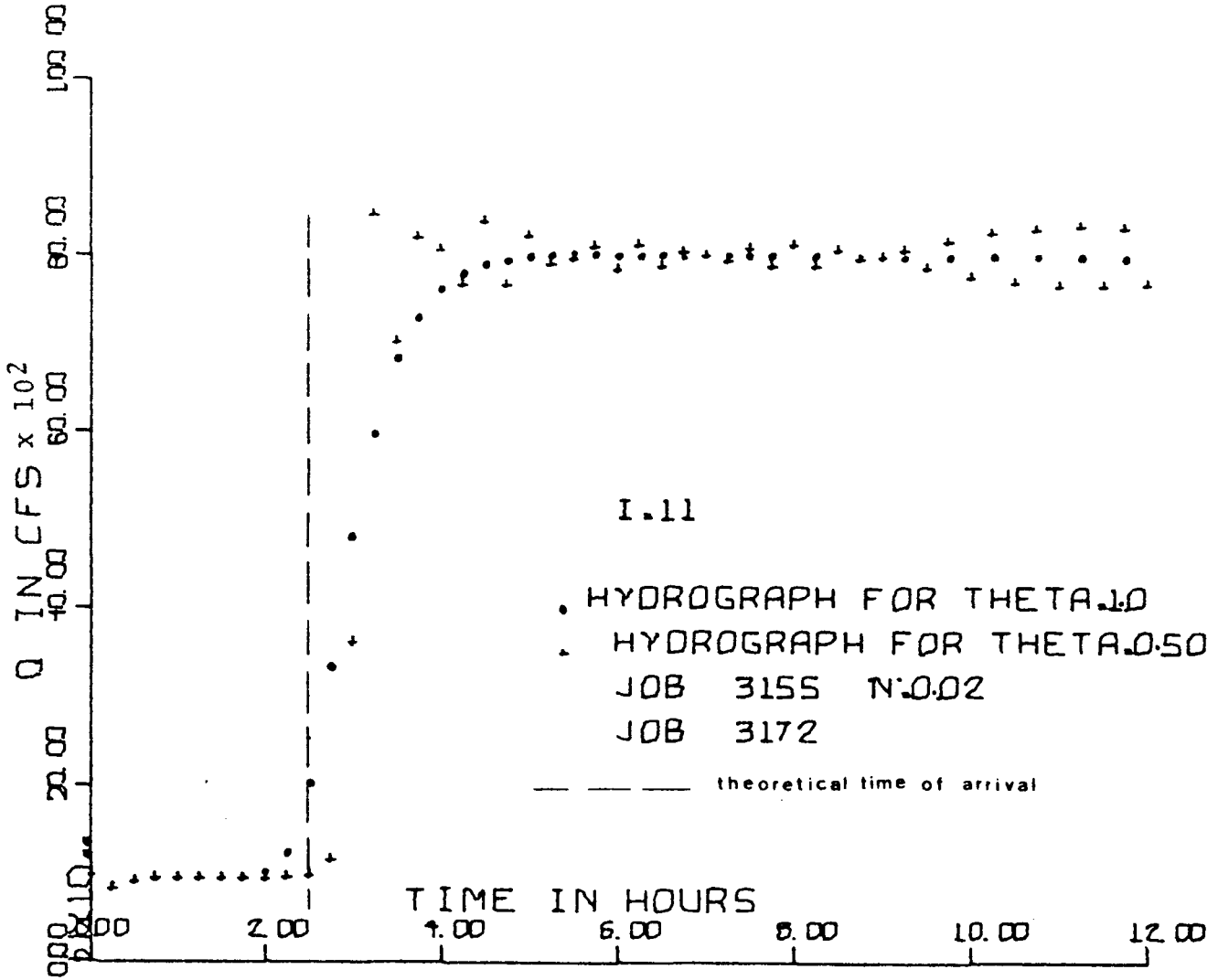


FIG.18 HYDROGRAPHS TYPE 1. RISING.

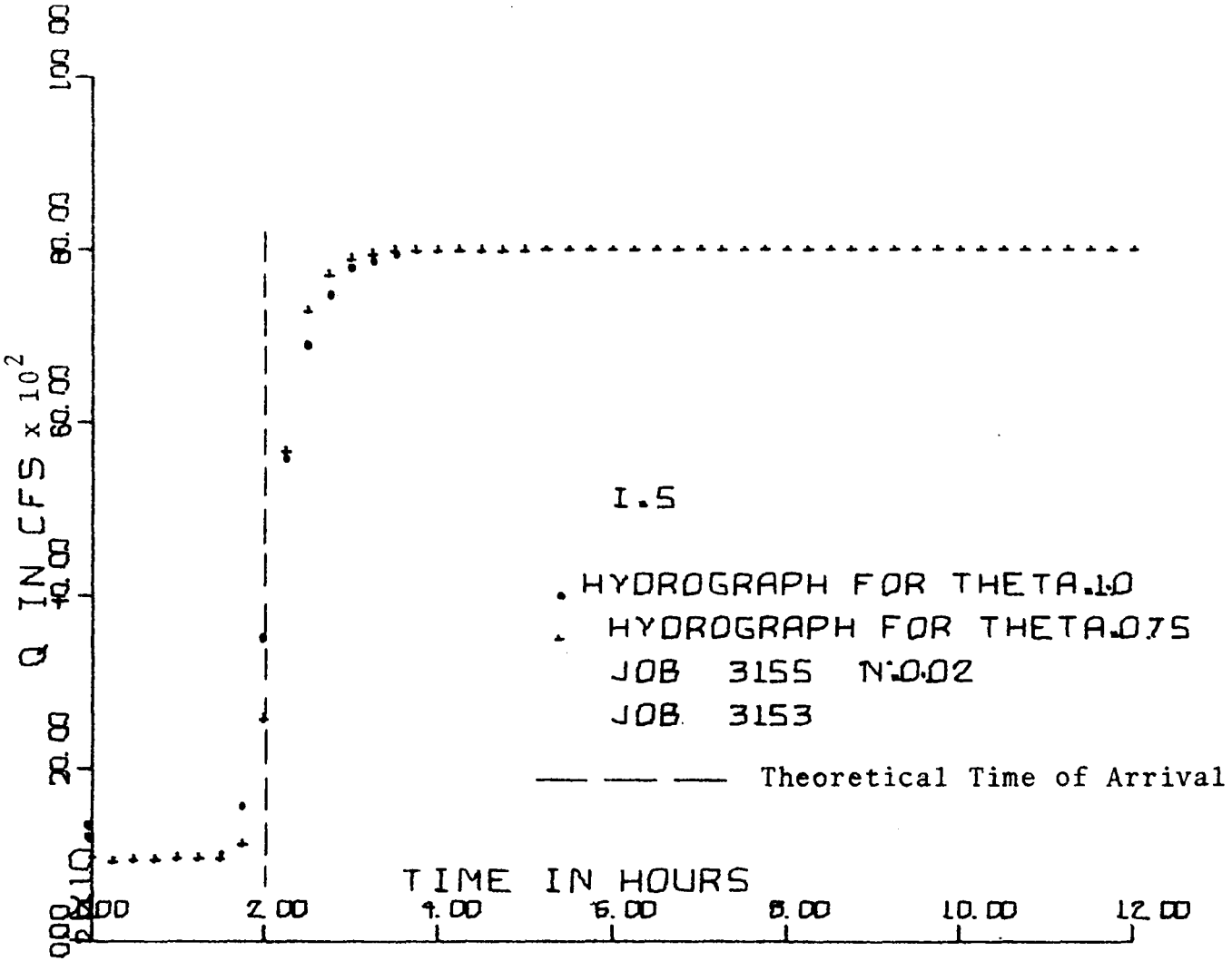


FIG.19.HYDROGRAPHS TYPE 1 . RISING .

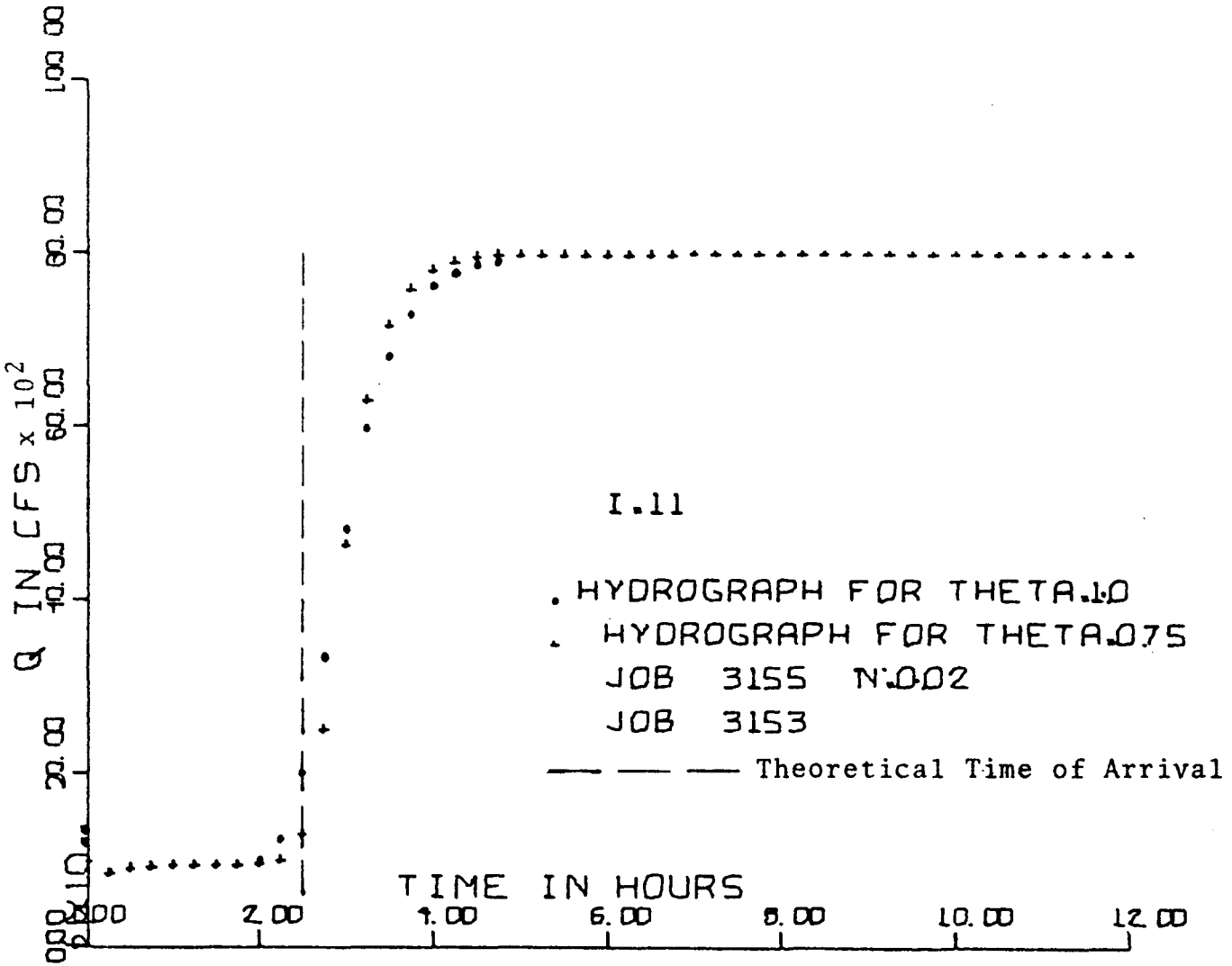


FIG.20 HYDROGRAPHS TYPE I . RISING .

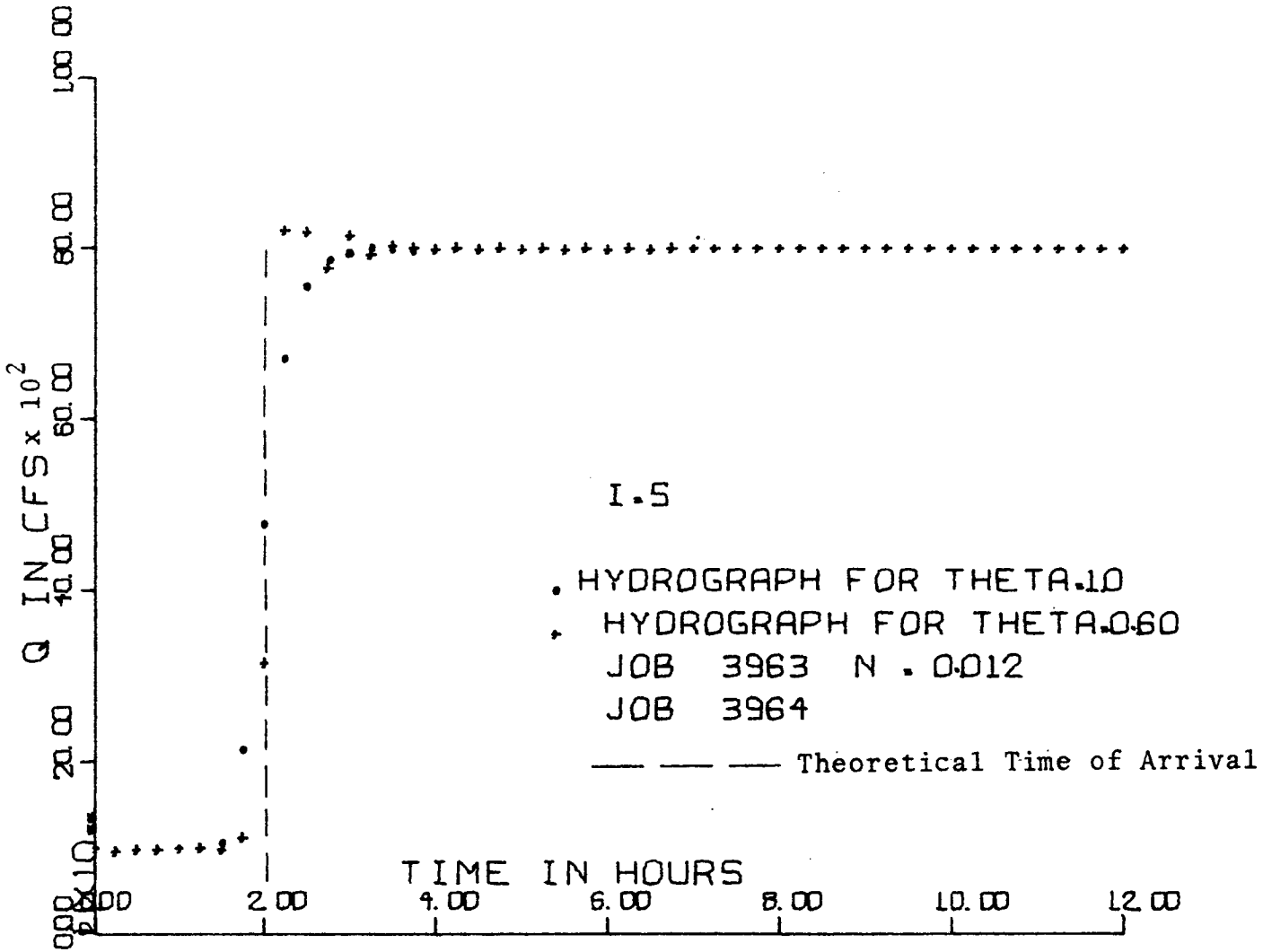


FIG.21 HYDROGRAPHS TYPE 1 . RISING .

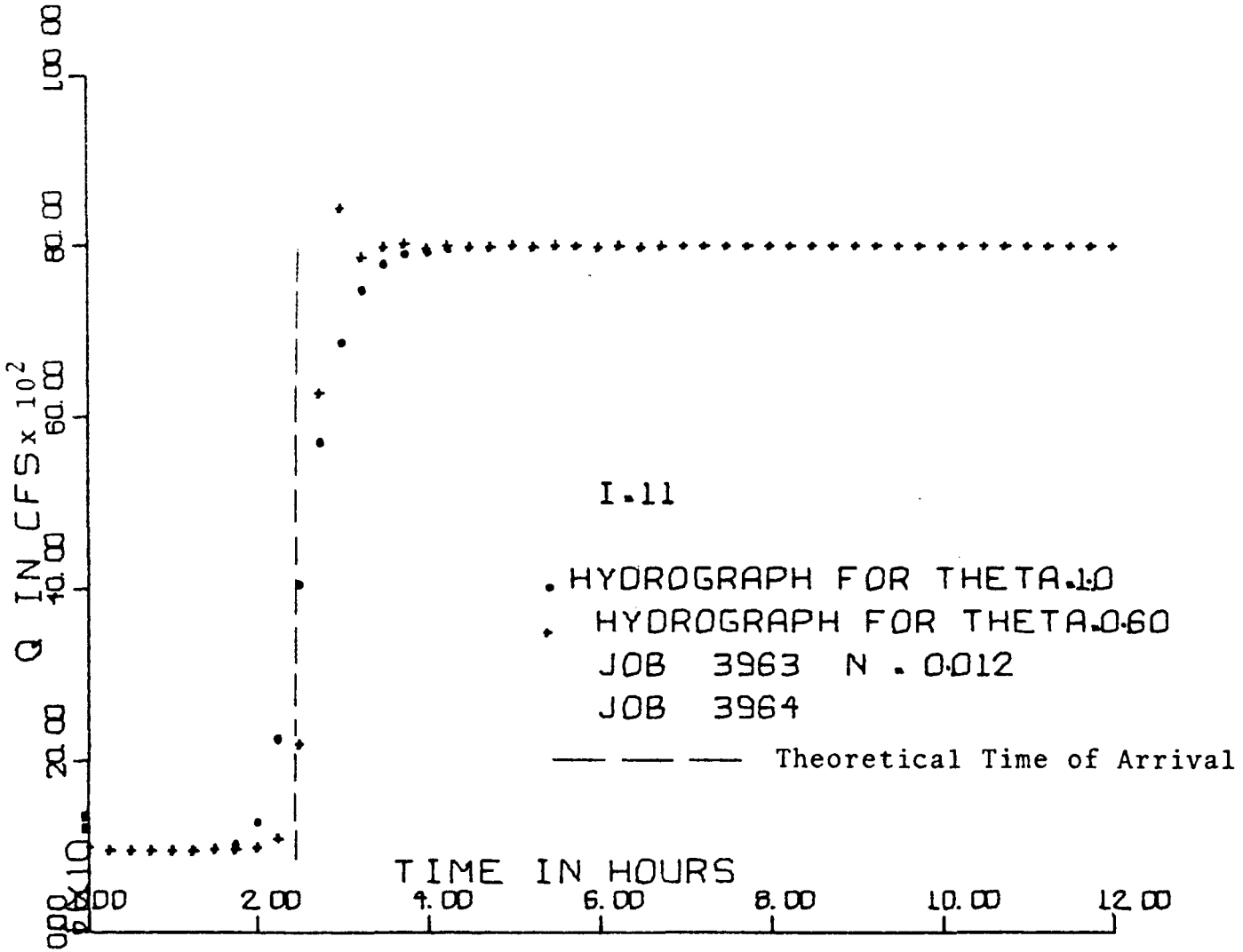


FIG.22 HYDROGRAPHS TYPE 1 . RISING .

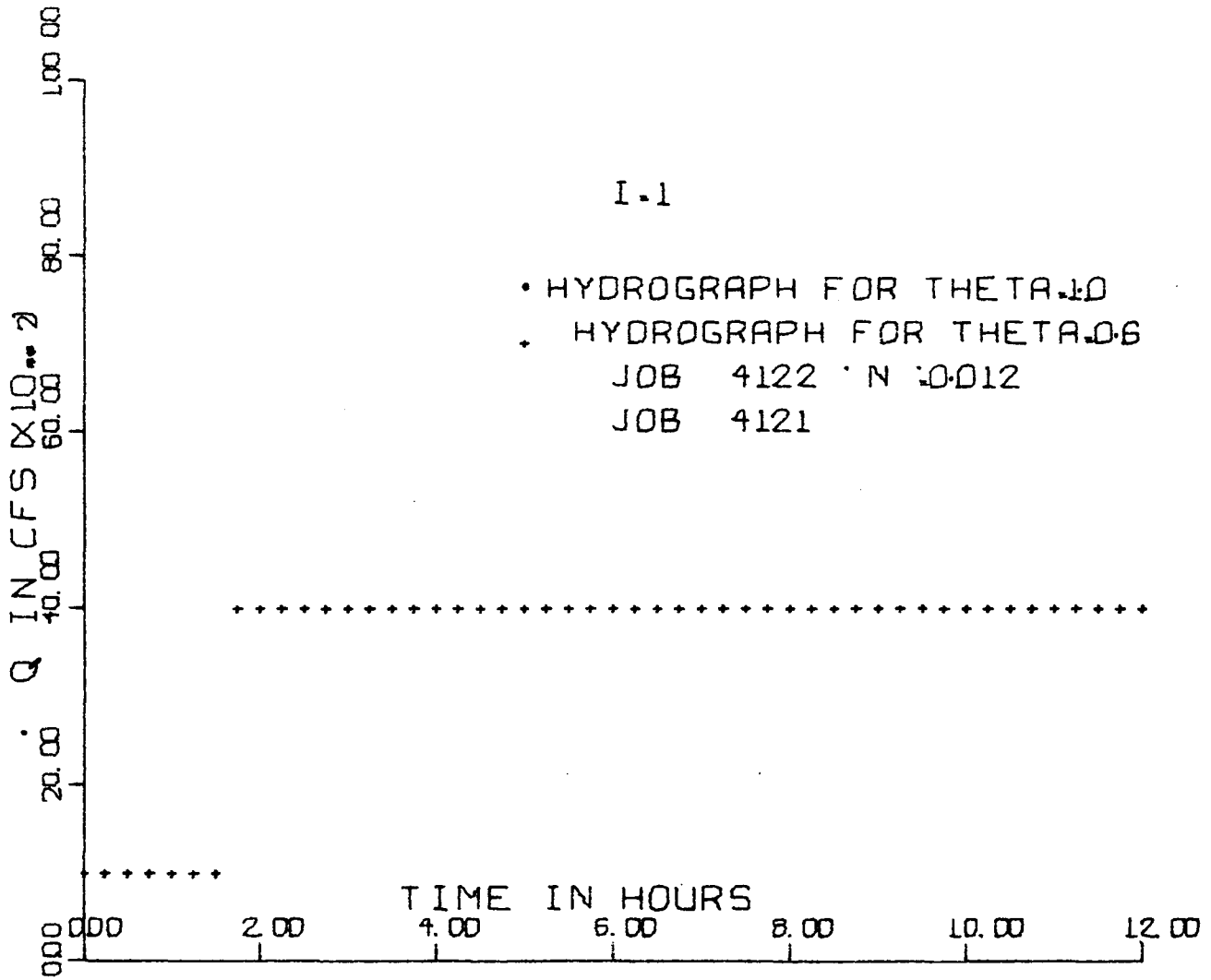


FIG.23.HYDROGRAPHS TYPE 4 . RISING .

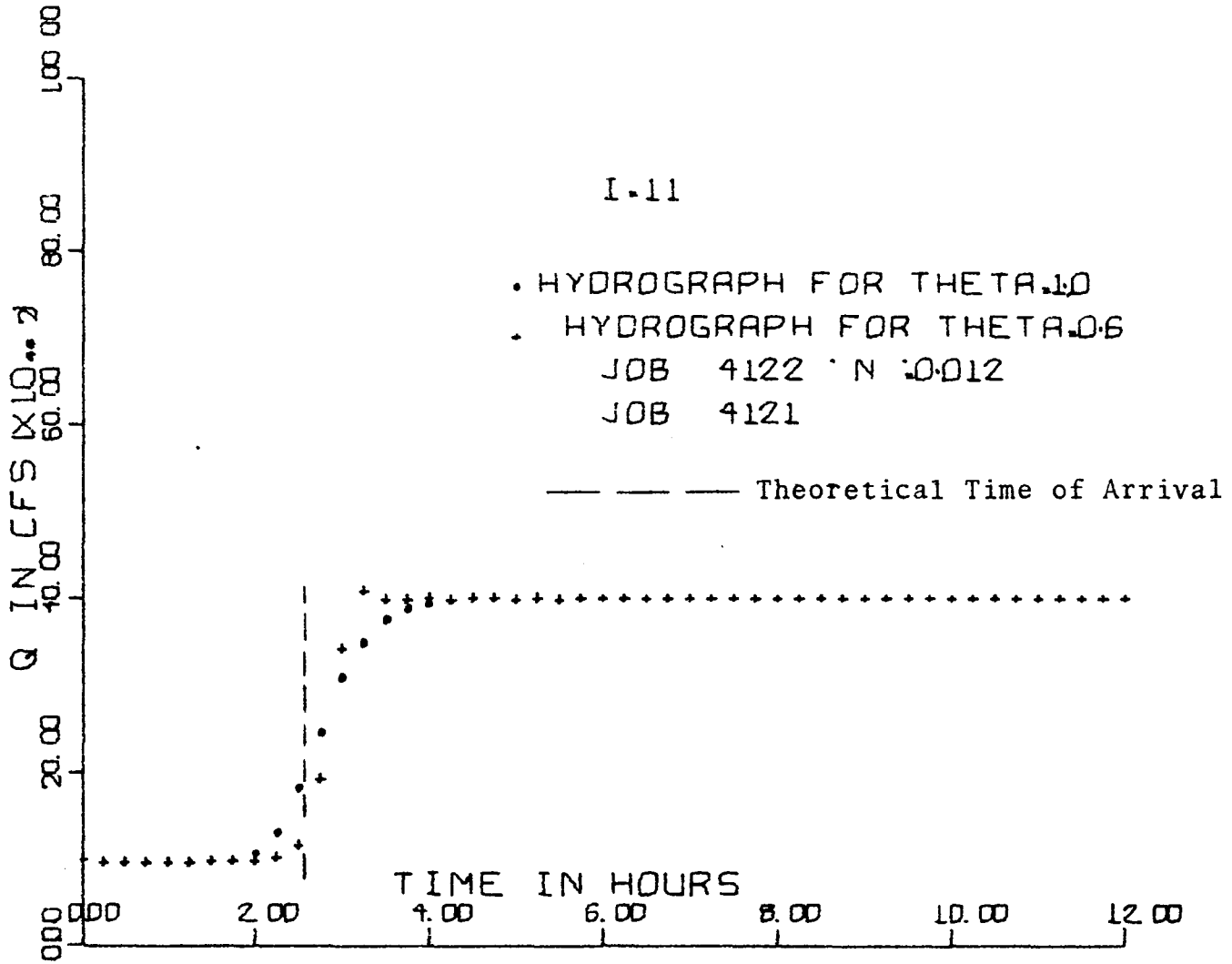


FIG.24 HYDROGRAPHS TYPE 4 . RISING .

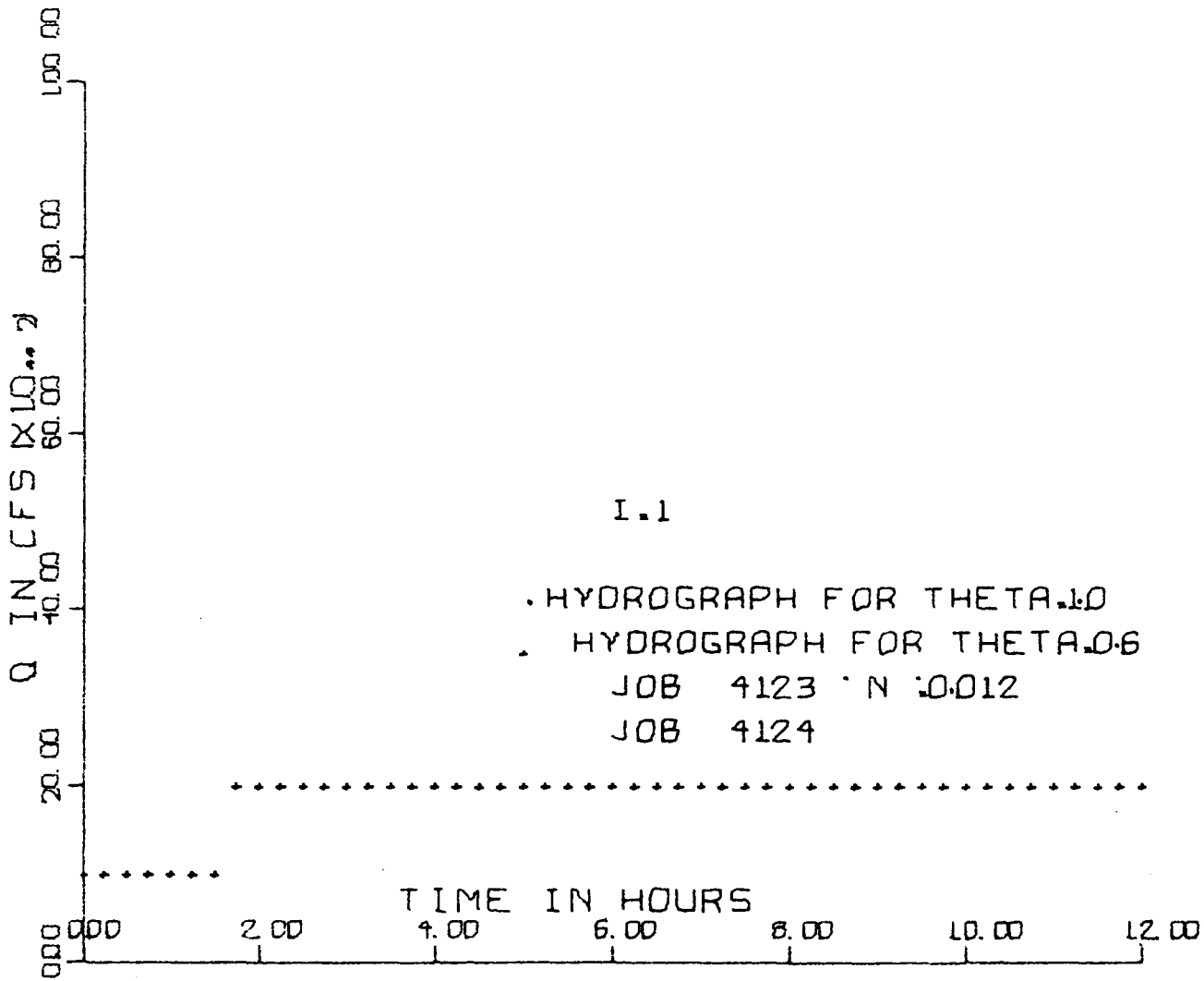


FIG.25 HYDROGRAPHS TYPE 5 . RISING .

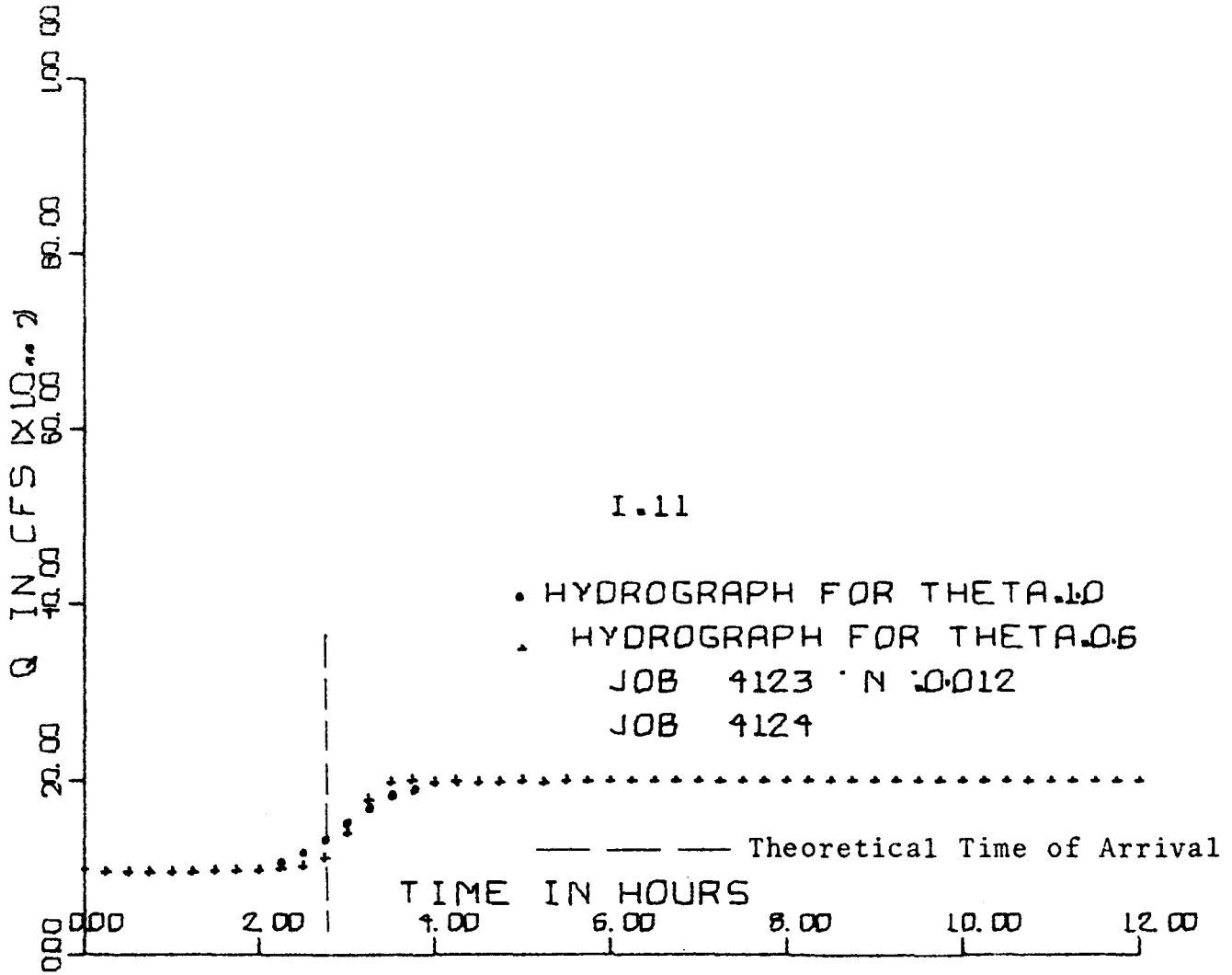


FIG.26.HYDROGRAPHS TYPE 5 . RISING .

Type 2 Rising Hydrographs

In this case, for N = 0.03, the hydrographs for θ = 0.5 and 1.0 indicated different wave speeds. At station 11 the hydrograph for θ = 0.5 started rising at 3.25 HRS whereas the hydrographs for θ = 1.0 started rising at 1.75 HRS, see Fig. 29. The hydrographs for θ = 1.0 and 0.75 were also drawn, see Fig. 31. These hydrographs showed practically the same results. In this case, as well as in all the subsequent cases, the theoretical time of arrival of the wave can be determined by the method of characteristics.

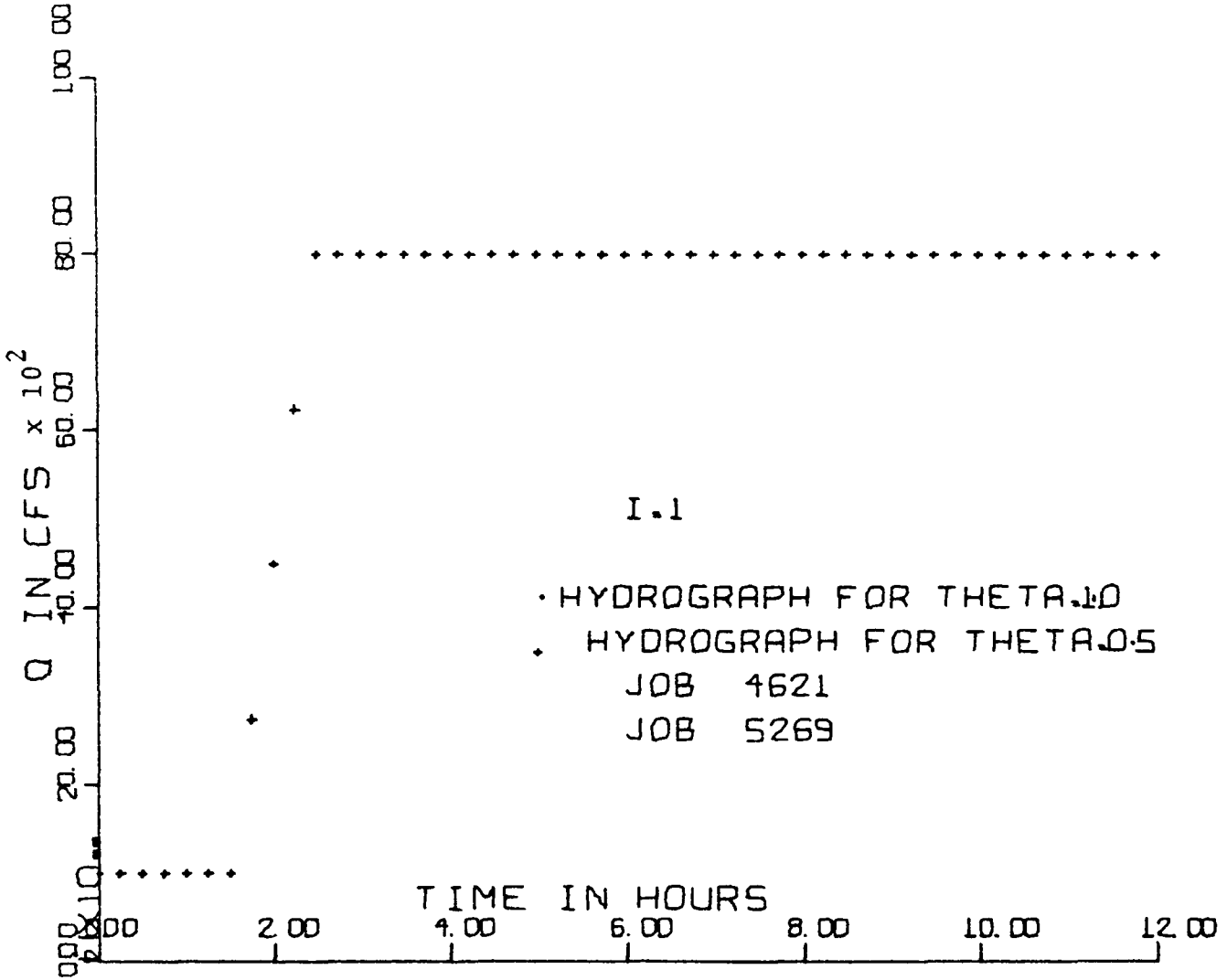


FIG.27.HYDROGRAPHS TYPE 2 . RISING .

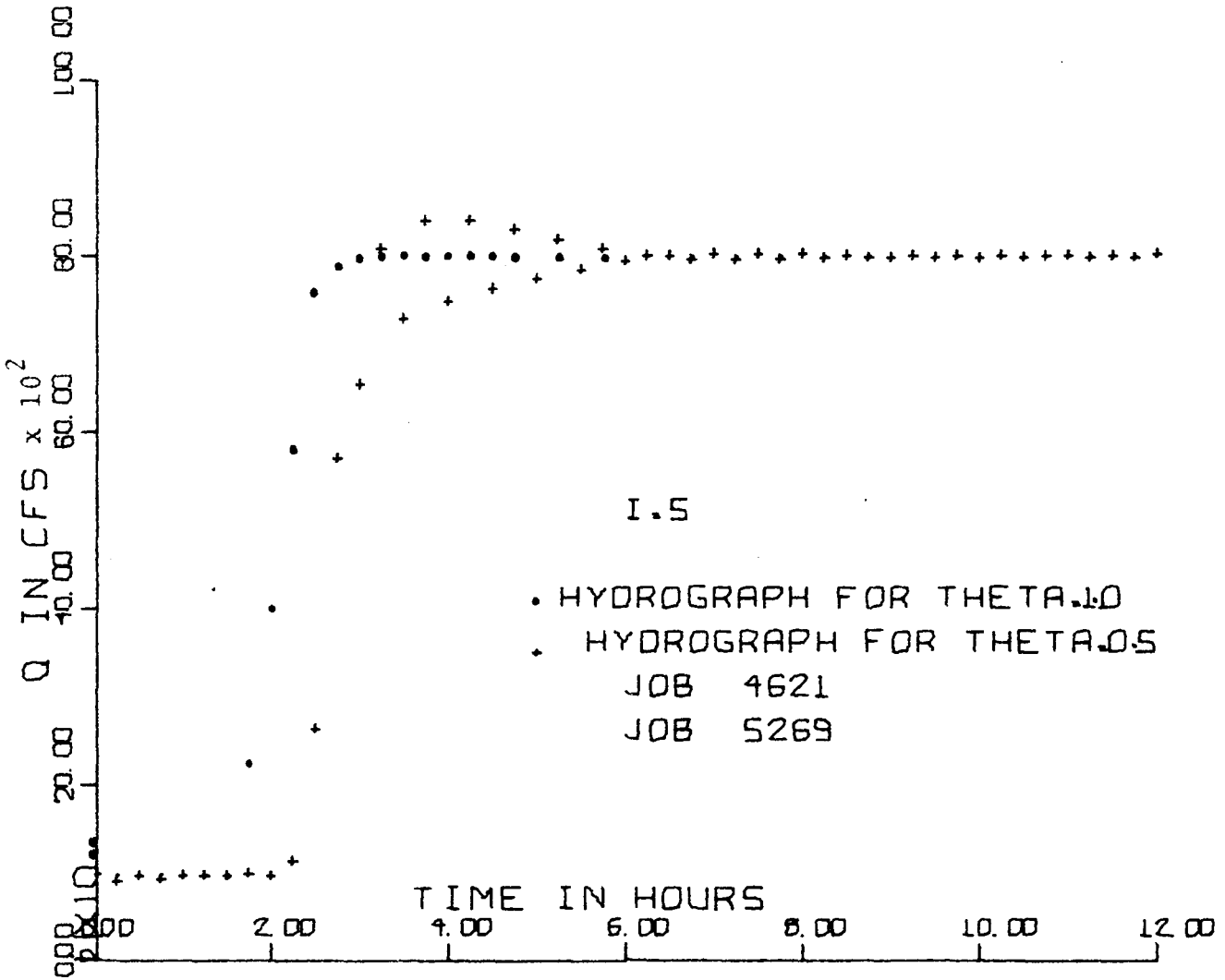


FIG.28 HYDROGRAPHS TYPE 2 . RISING .

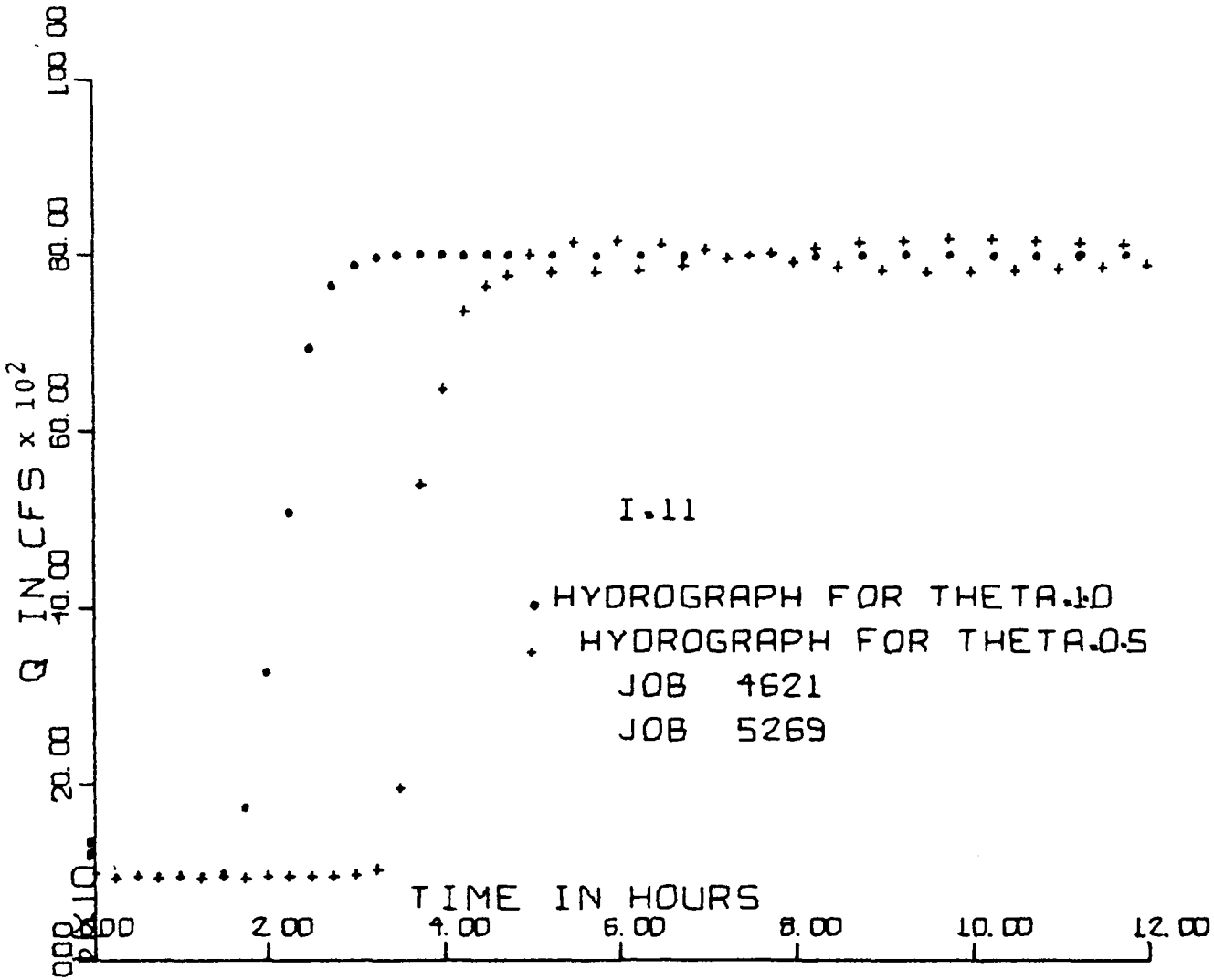


FIG.29. HYDROGRAPHS TYPE 2 . RISING .

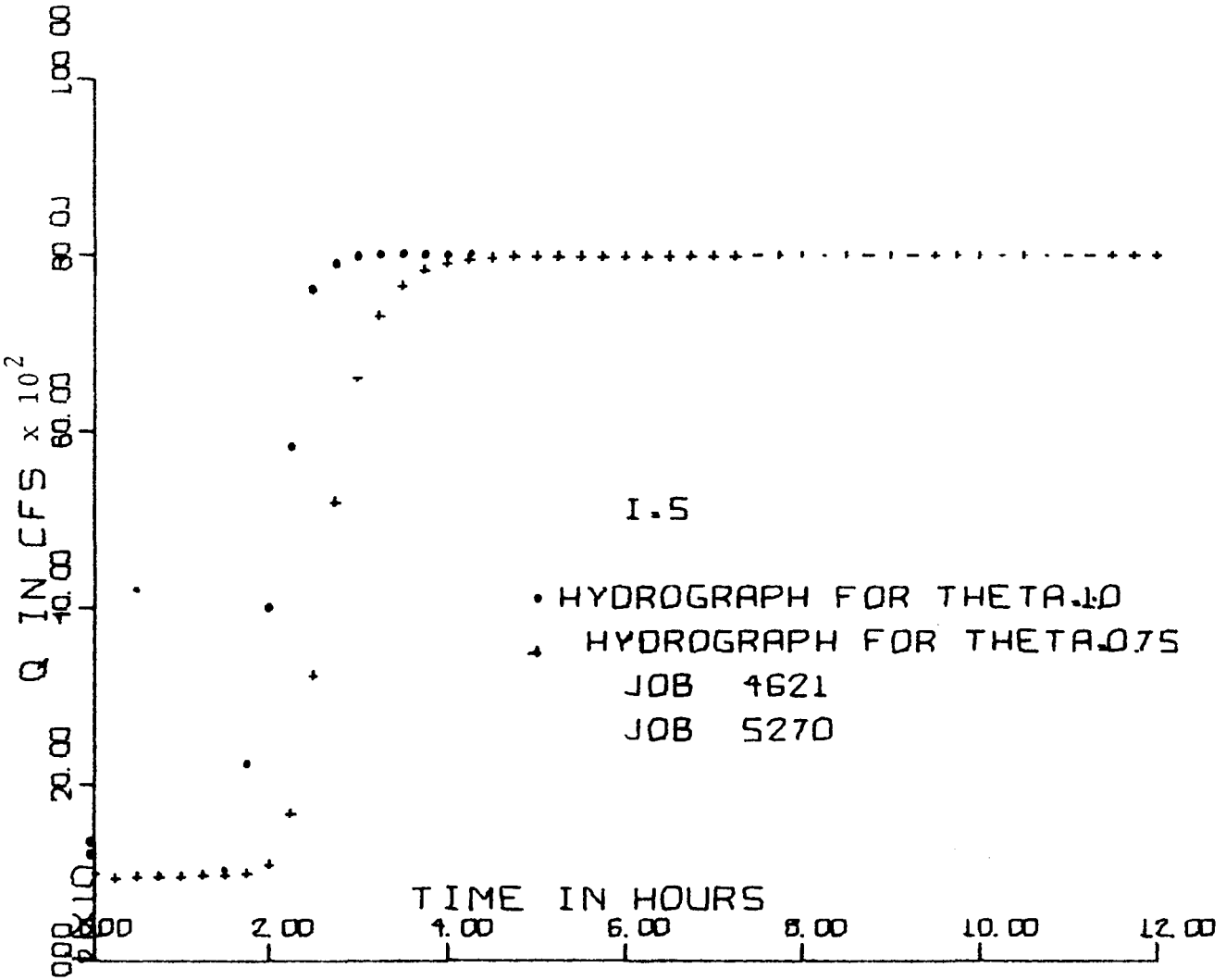


FIG.30 HYDROGRAPHS TYPE 2 . RISING .

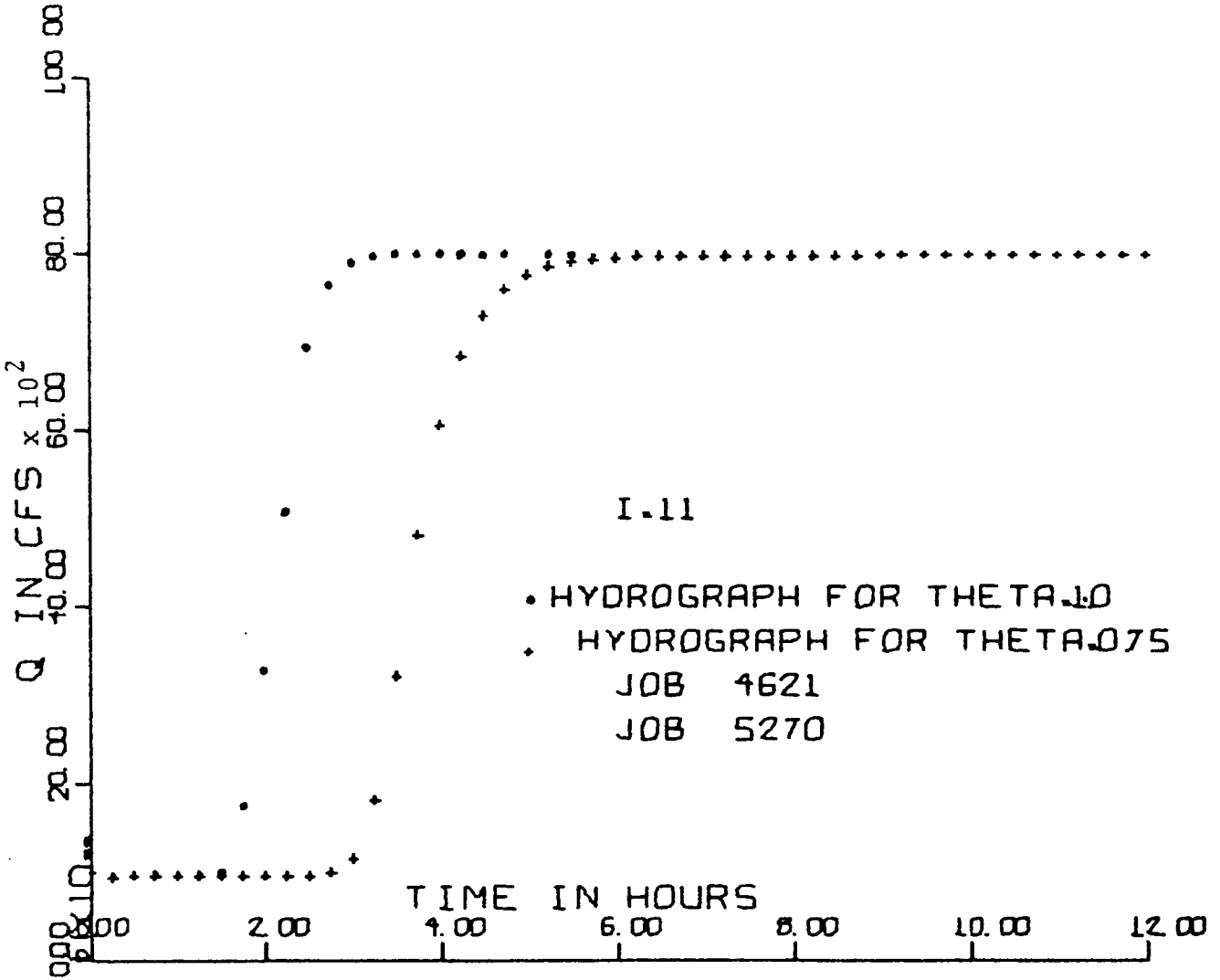


FIG.31 HYDROGRAPHS TYPE 2 . RISING .

Type 3 Rising Hydrographs

A value of "N" equal to 0.03 was used. The different values of theta did not make any significant difference in the hydrographs, as is evident from Figures 32-36.

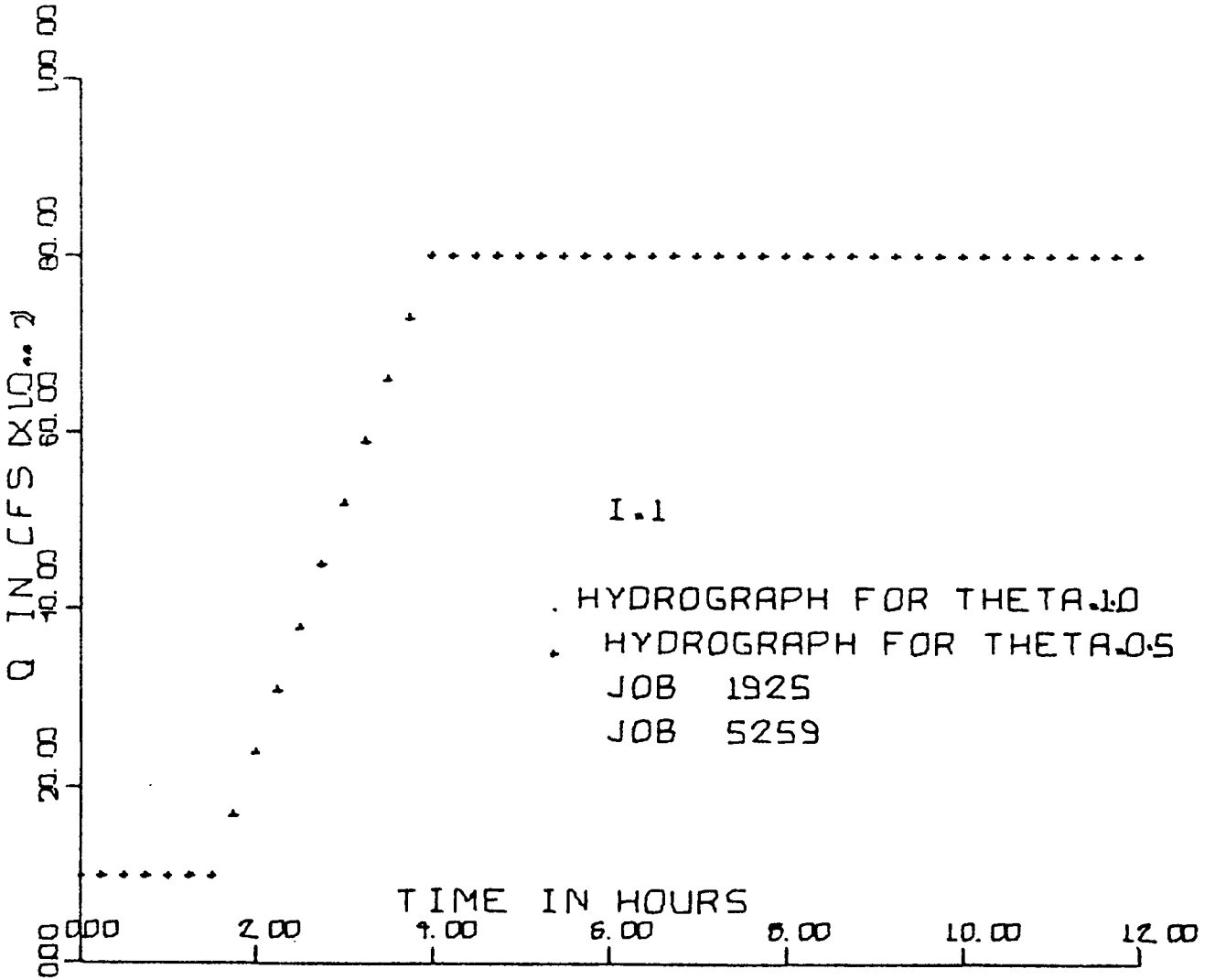


FIG.32 HYDROGRAPHS TYPE 3 . RISING .

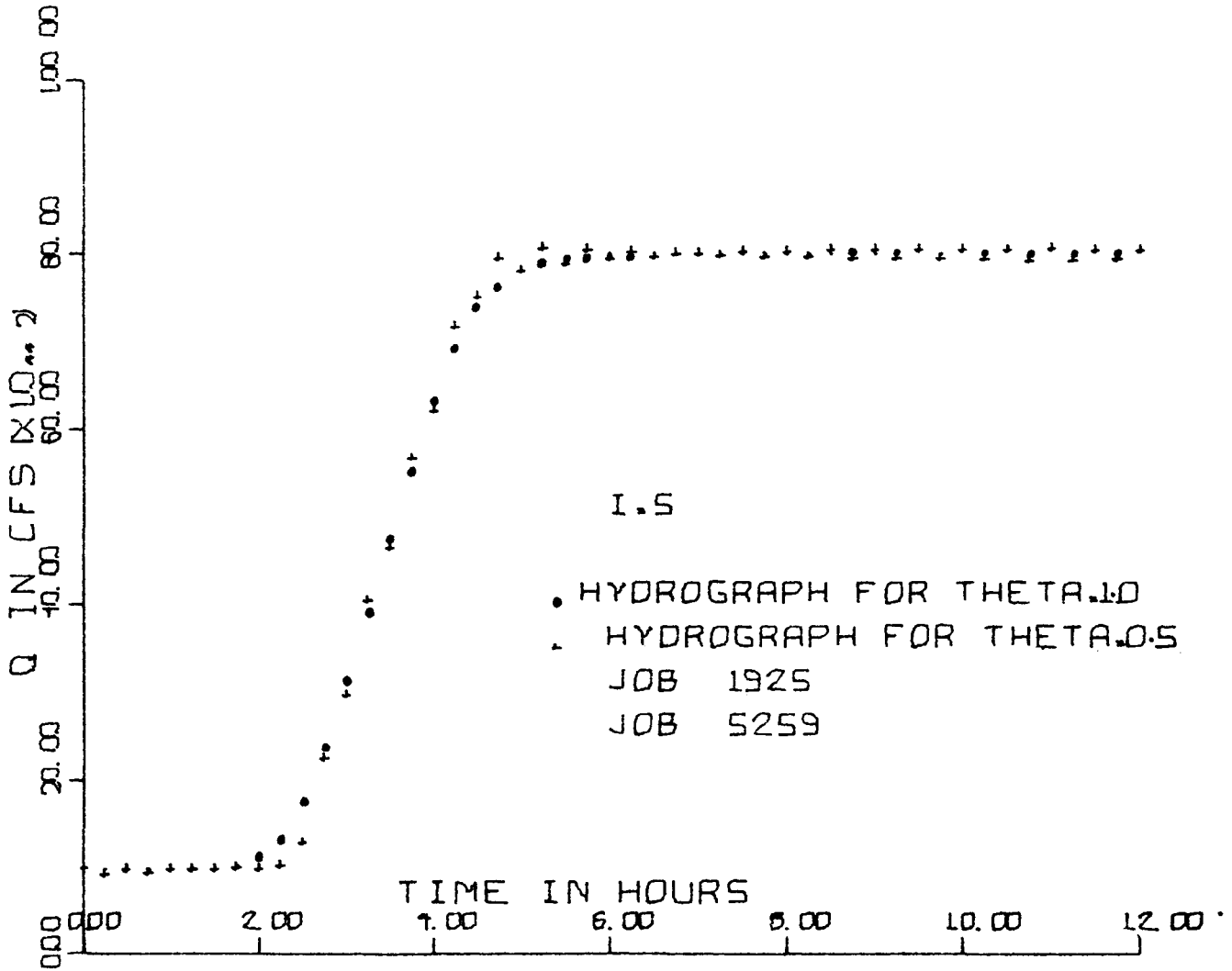


FIG.33.HYDROGRAPHS TYPE 3 . RISING .

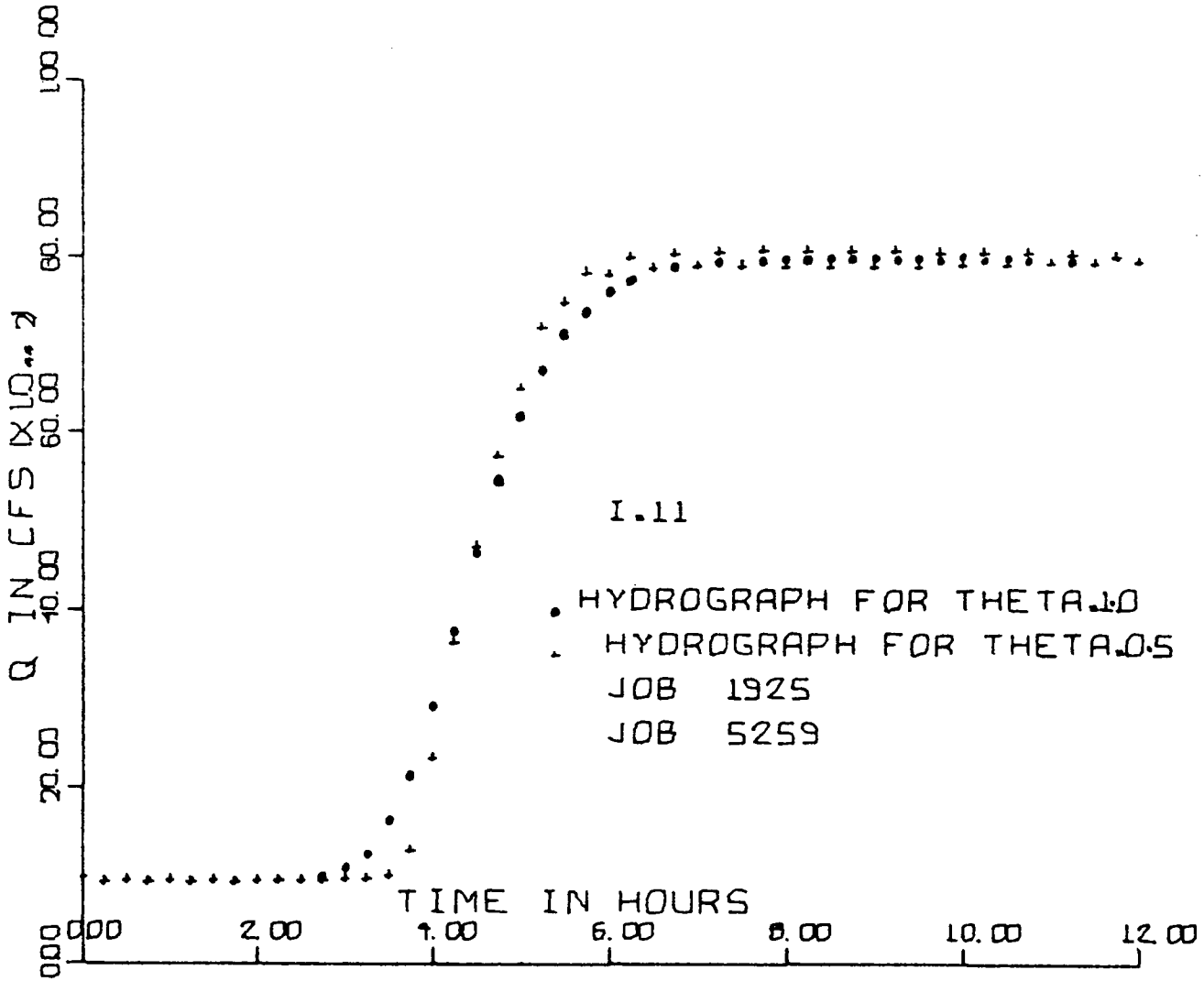


FIG.34. HYDROGRAPHS TYPE 3 . RISING .

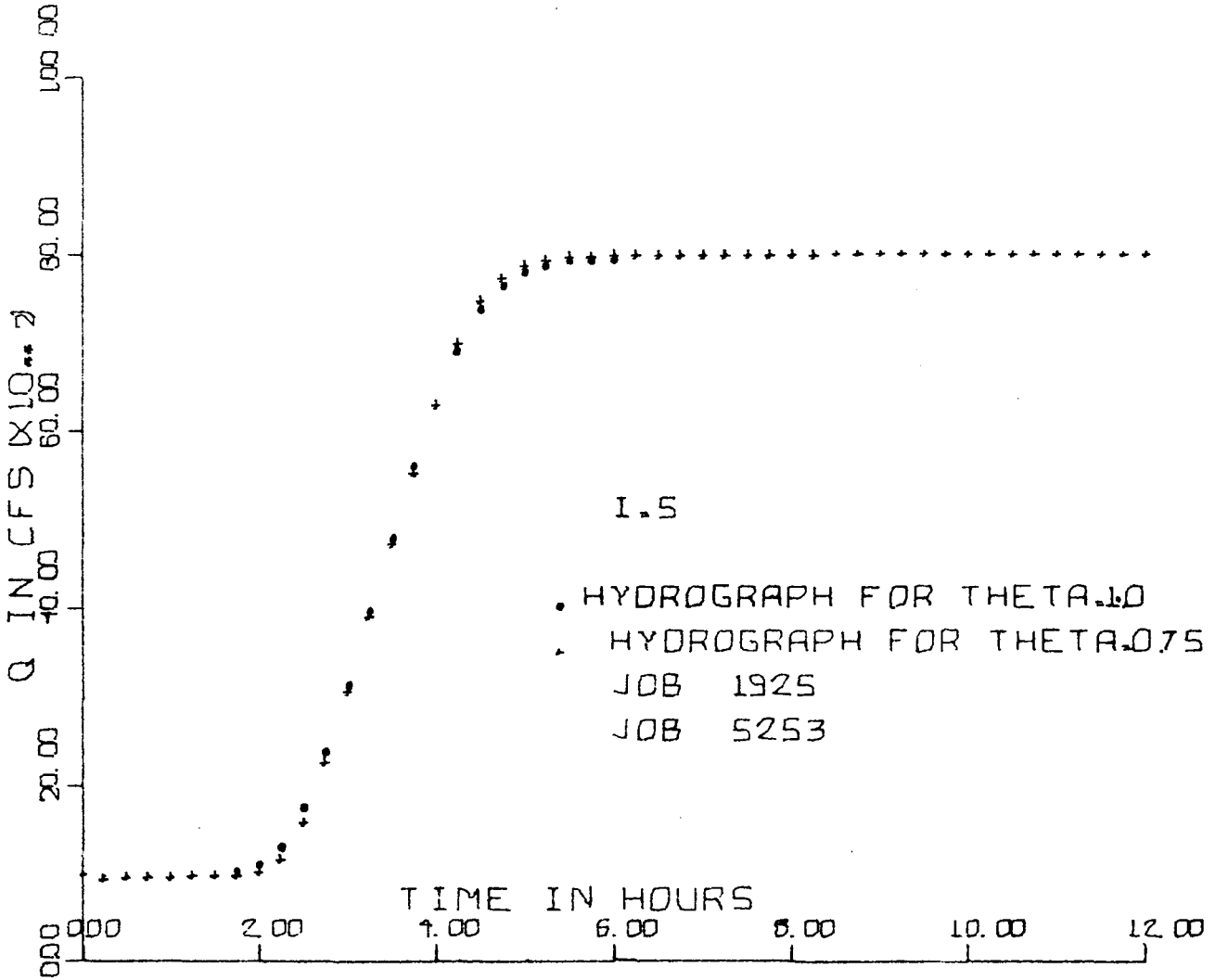


FIG.35HYDROGRAPHS TYPE 3 . RISING .

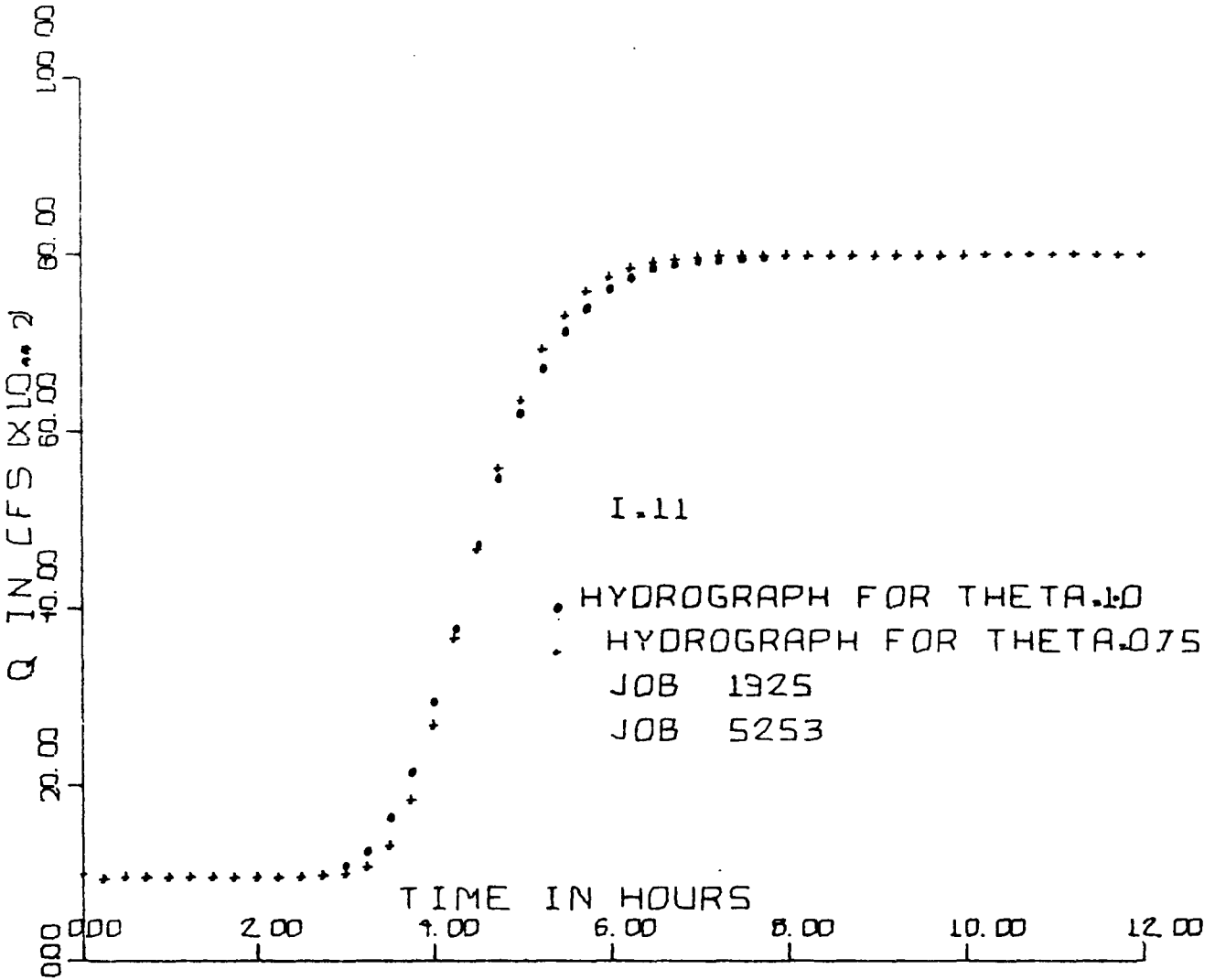


FIG.36 HYDROGRAPHS TYPE 3 . RISING .

Type 1 Receding Hydrographs

In this case, too, the Manning roughness coefficient was 0.03. The hydrographs for values of theta equal to 0.5, 0.75 and 1.0 were drawn. The different values of θ did not affect the hydrographs to any significant degree, see Figs. 37-39.

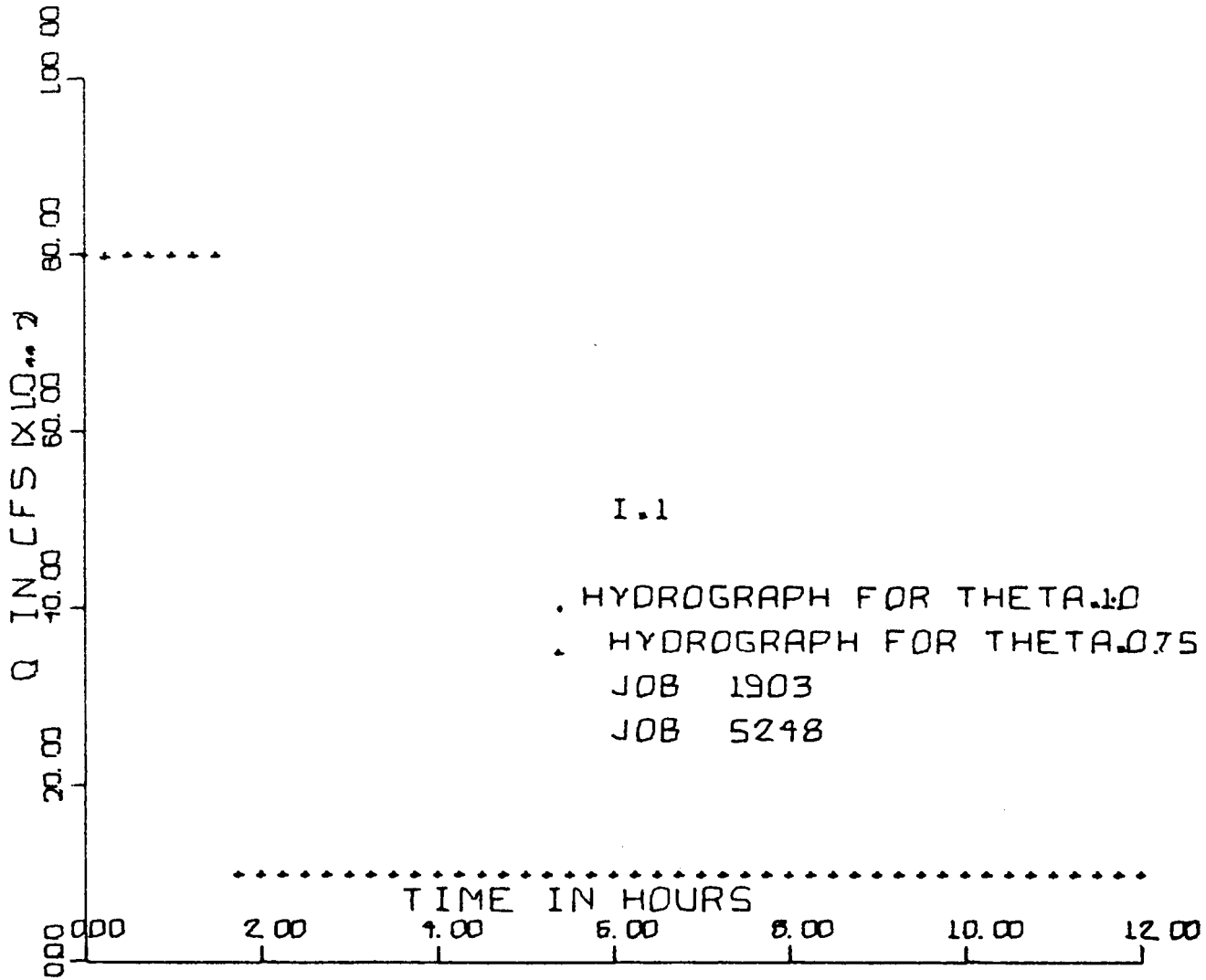


FIG.37.HYDROGRAPHS TYPE 1. RECEDING.

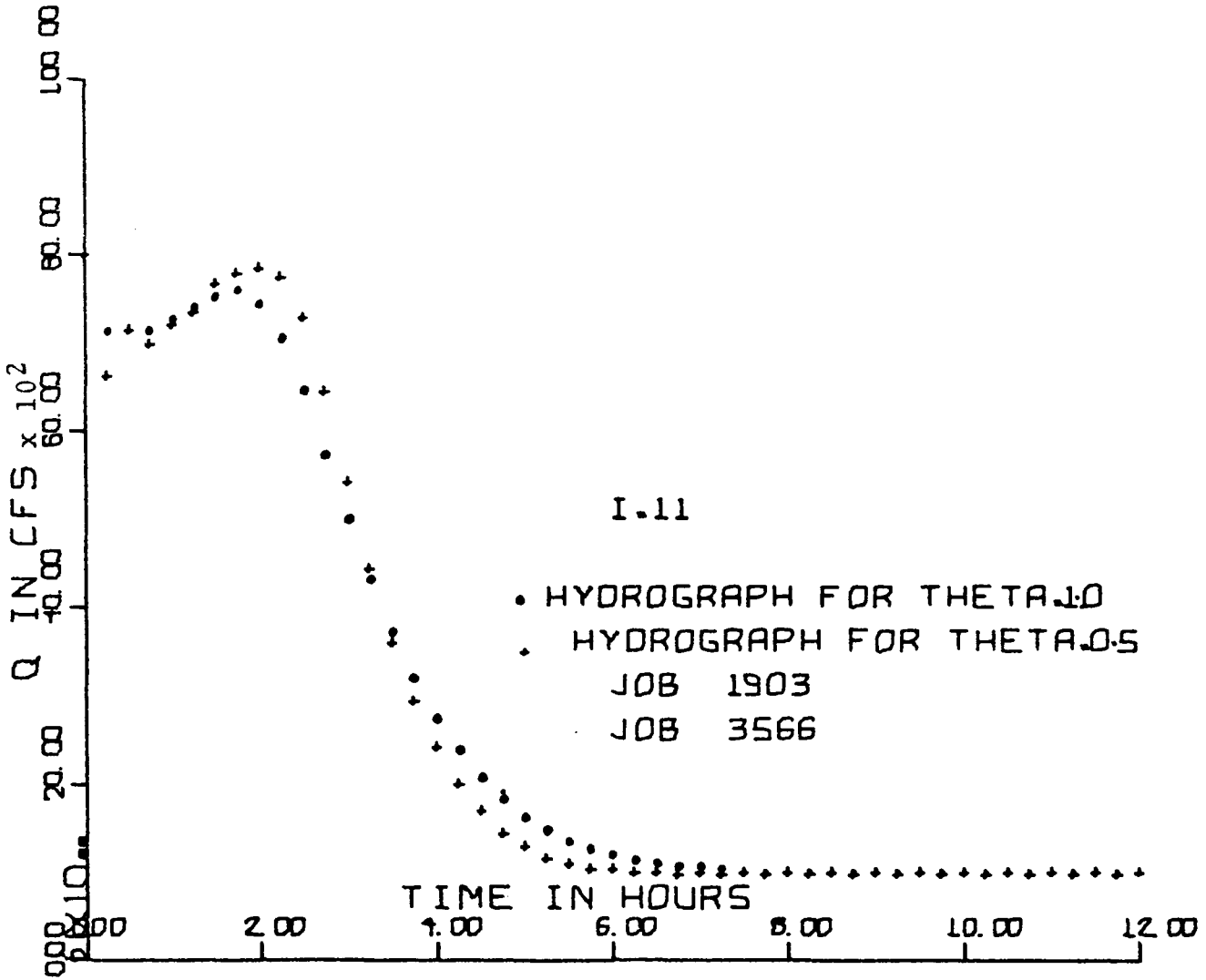


FIG.38. HYDROGRAPHS TYPE 1 . RECEDING .

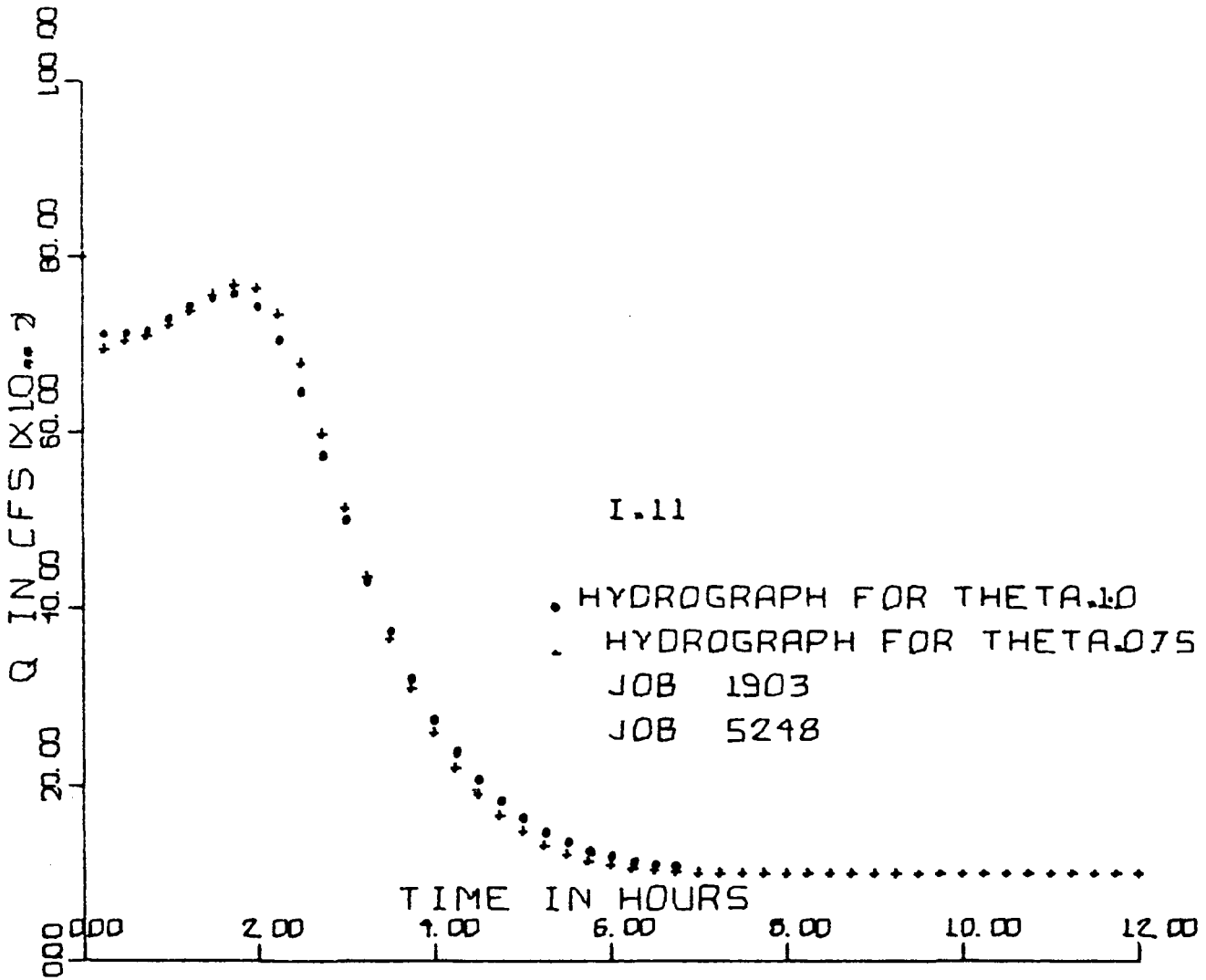


FIG39.HYDROGRAPHS TYPE 1. RECEDING.

Type 2 and Type 3 Receding Hydrographs

For N = 0.03, different values of θ gave markedly different results. The values of θ = 0.5 gave a greater spreading of the hydrograph as compared to the hydrograph for θ = 1.0. Theoretically there should have been some spreading but exactly how much can not be evaluated. Hydrographs for θ = 1.0 remain undeformed even at station 11. However, the hydrographs for smaller values of θ become deformed at station 11, which seems to be in conformity with theory, see Figs. 40-47.

The smaller values of theta gave some initial fluctuations.

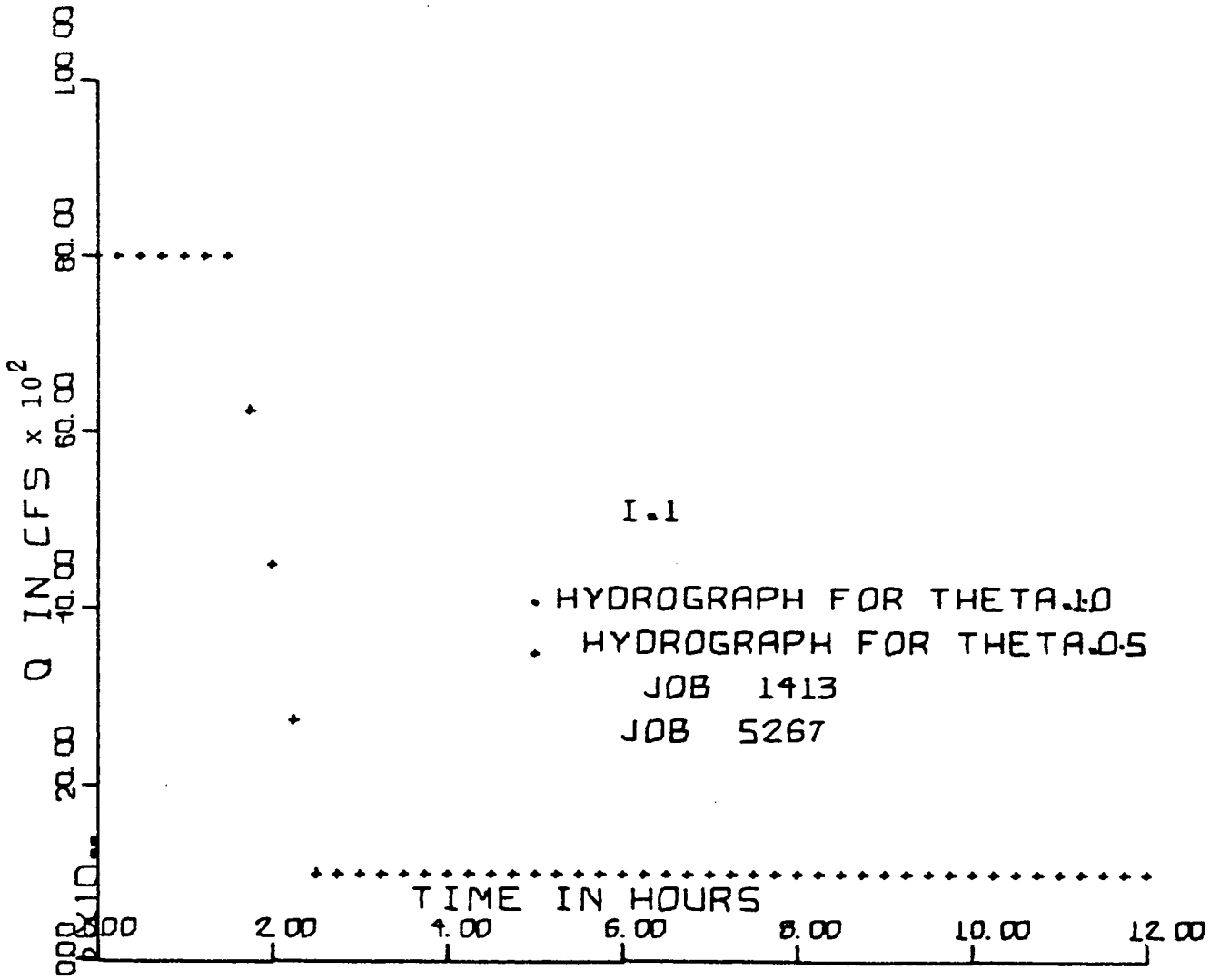


FIG.40. HYDROGRAPHS TYPE 2 RECEDING .

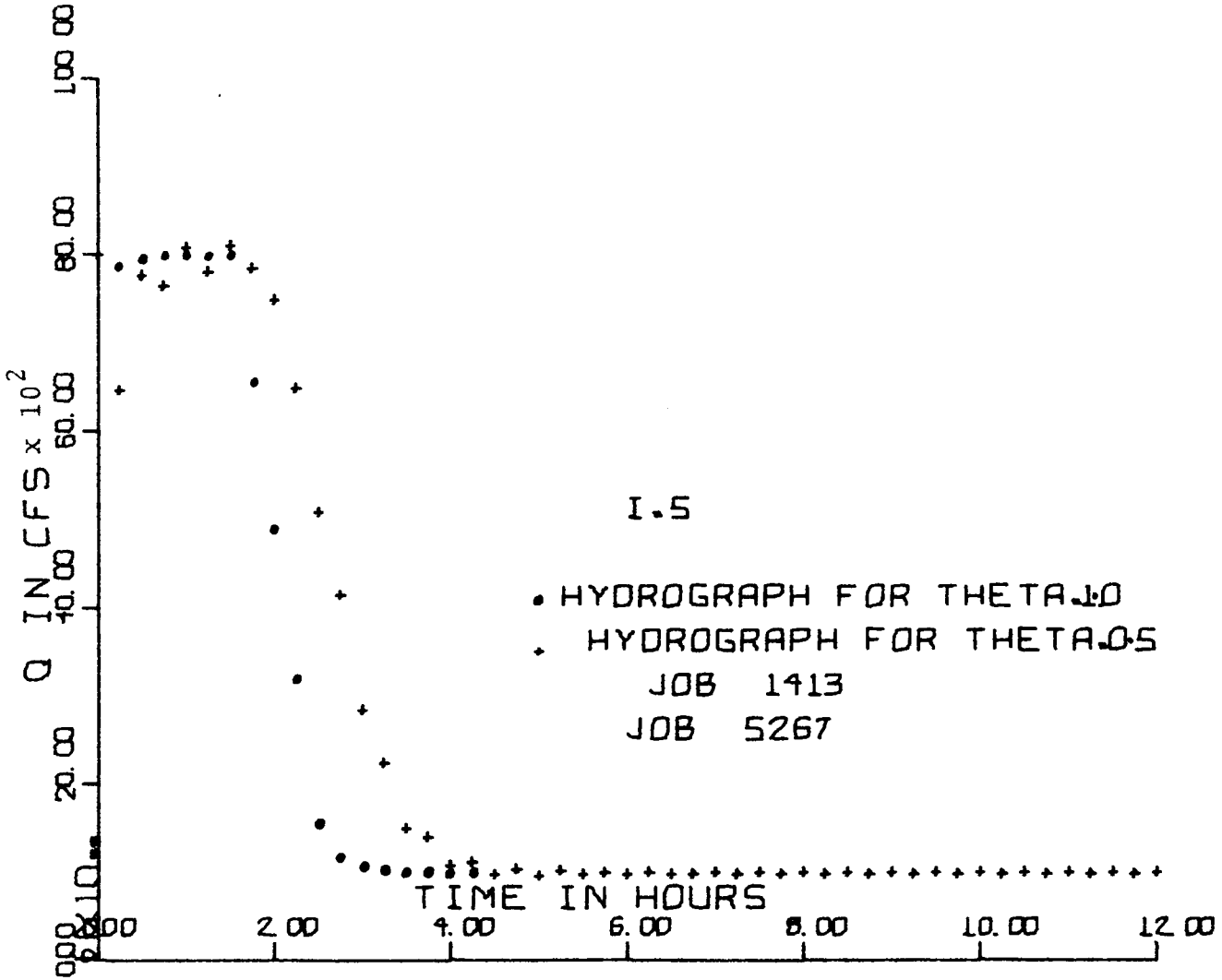


FIG.41. HYDROGRAPHS TYPE 2 RECEDING .

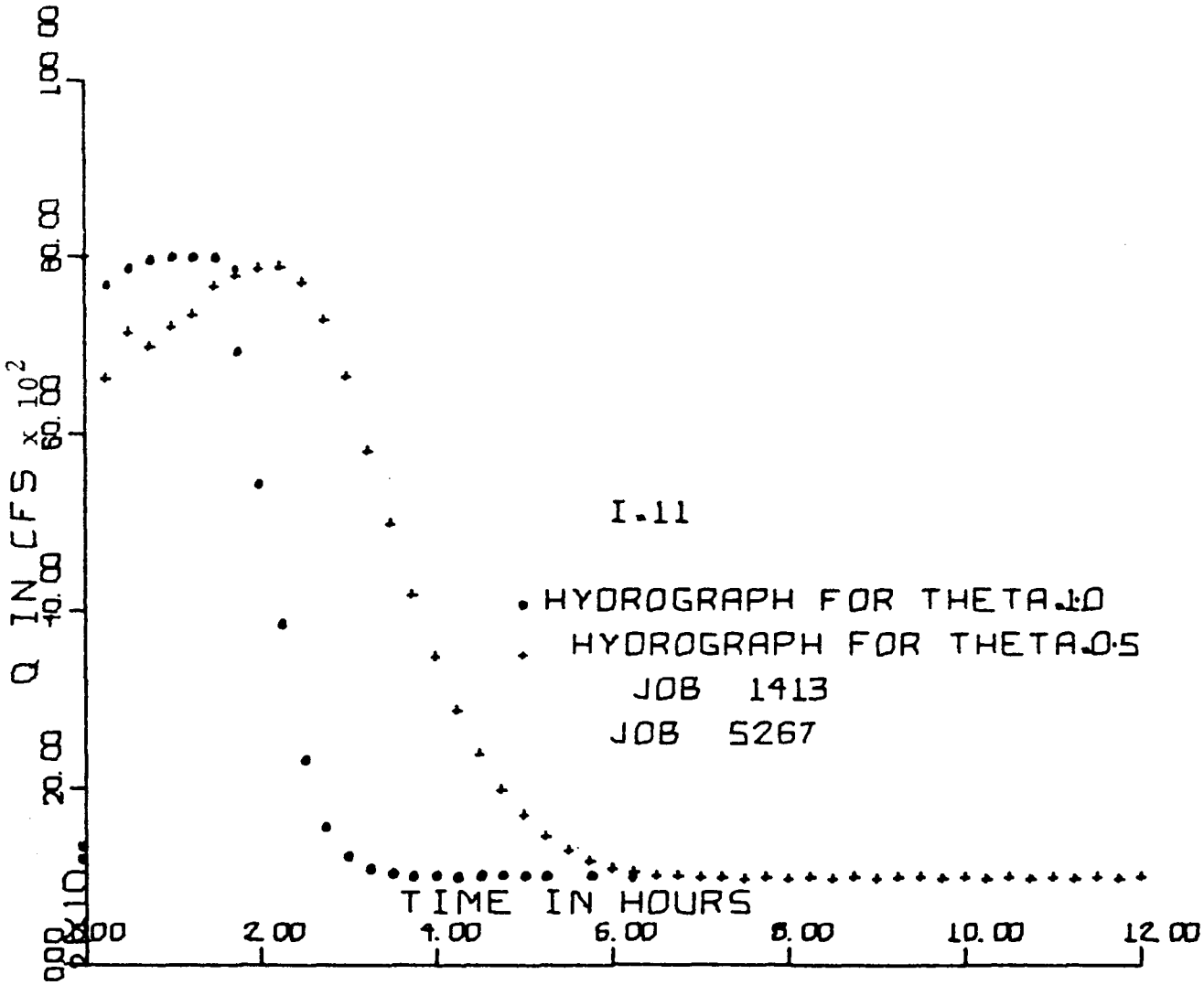


FIG.42. HYDROGRAPHS TYPE 2 RECEDING .

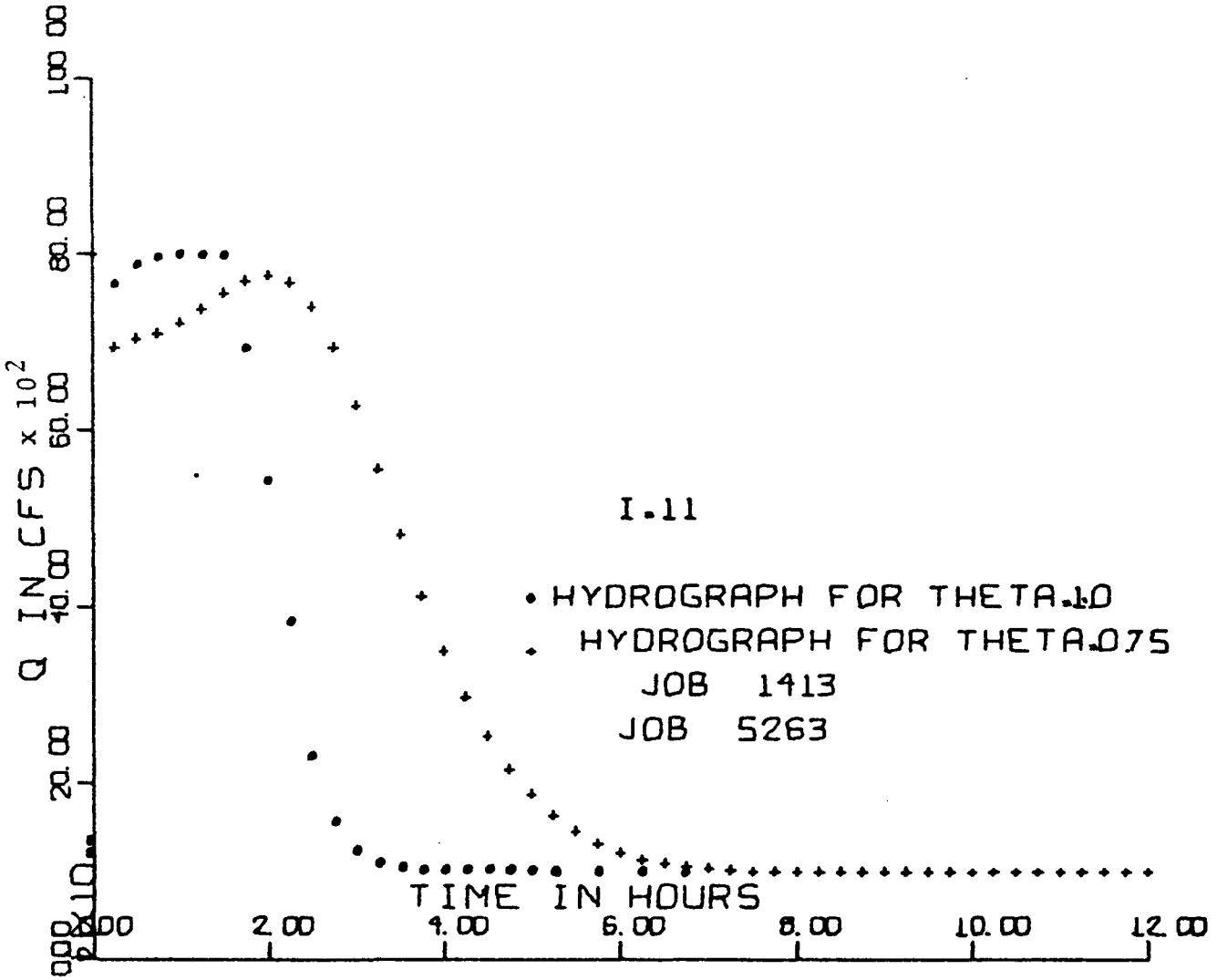


FIG.43.HYDROGRAPHS TYPE 2 RECEDING .

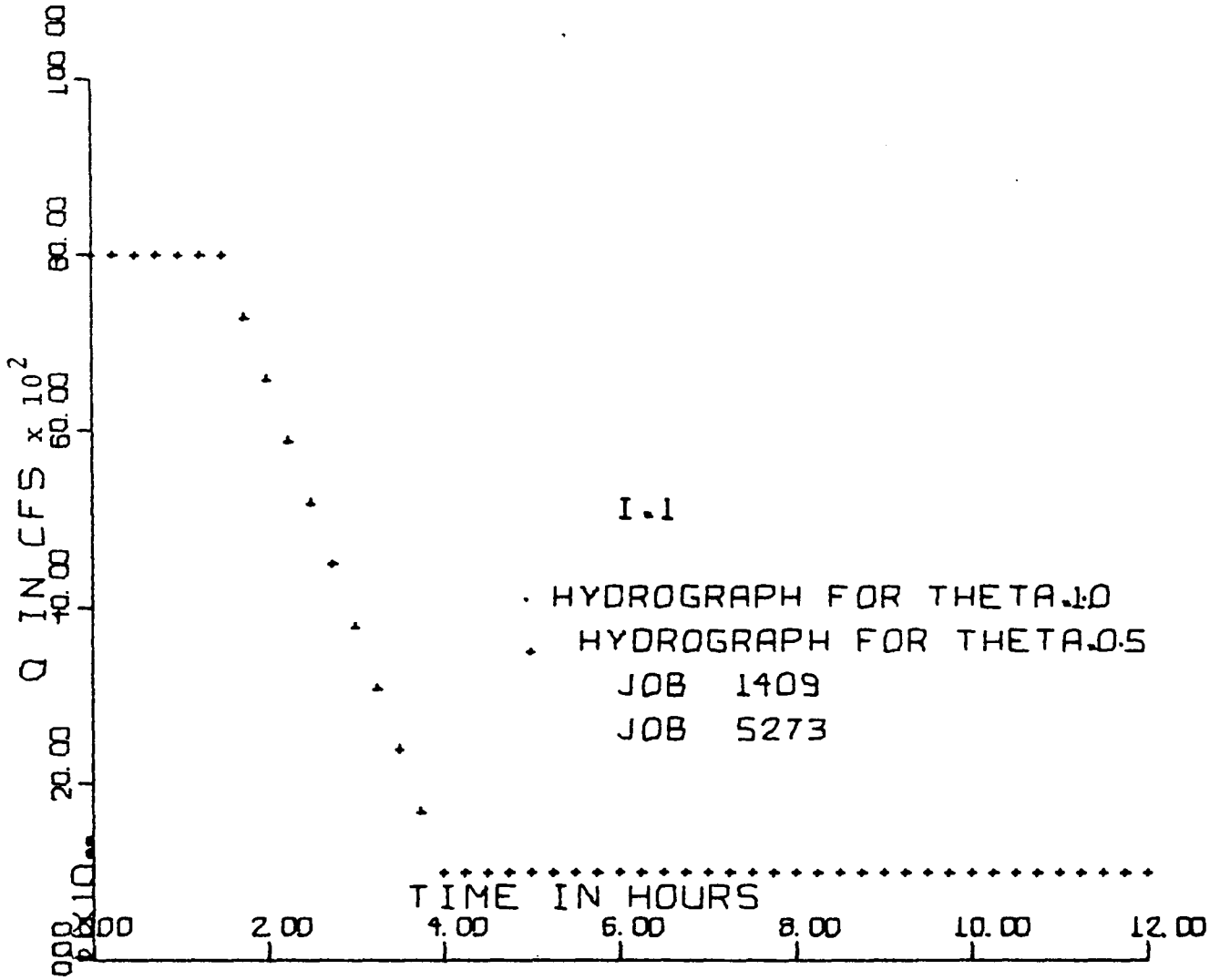


FIG.44 HYDROGRAPHS TYPE 3 . RECEDING .

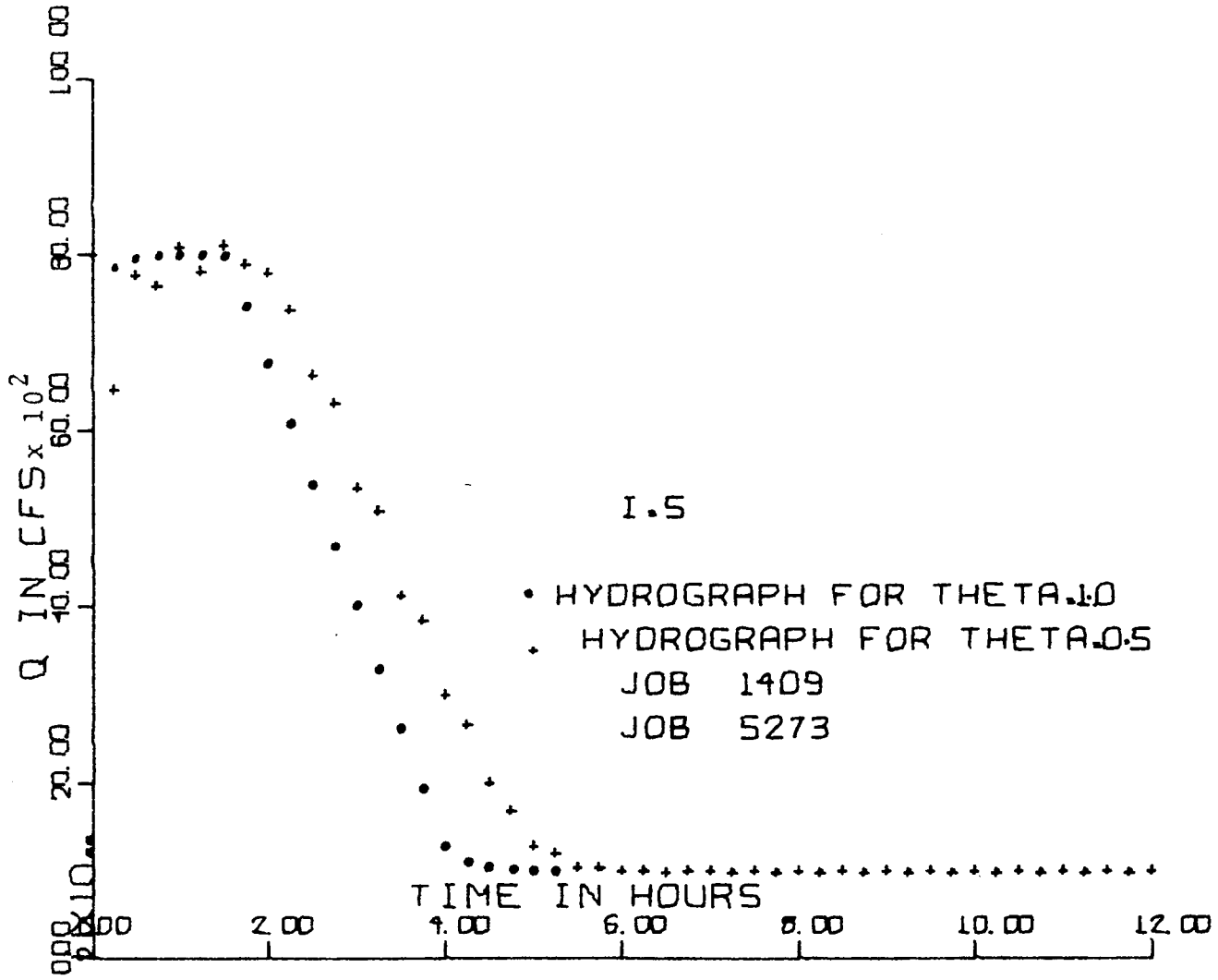


FIG.45.HYDROGRAPHS TYPE 3 . RECEDING .

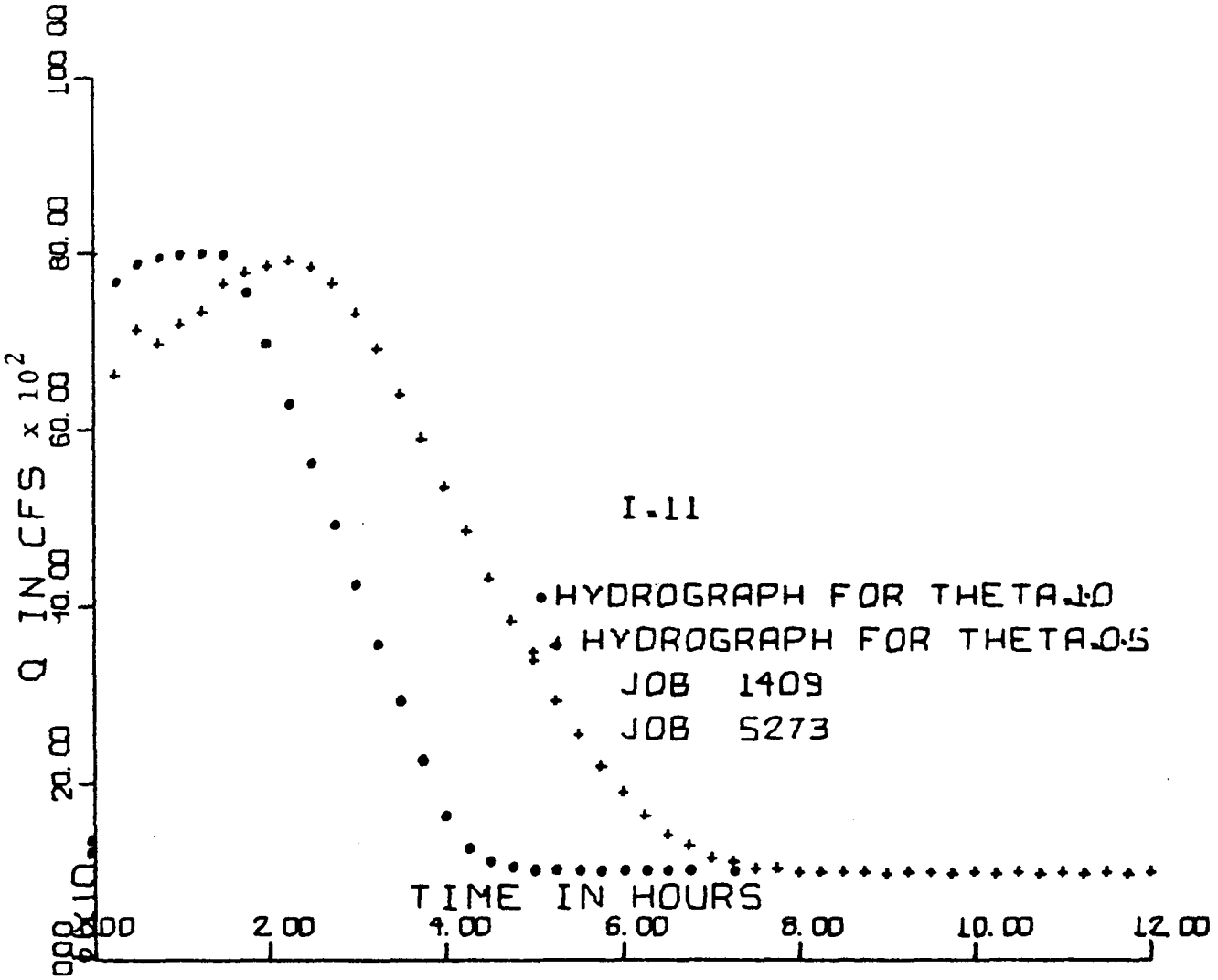


FIG.46 HYDROGRAPHS TYPE 3 . RECEDING .

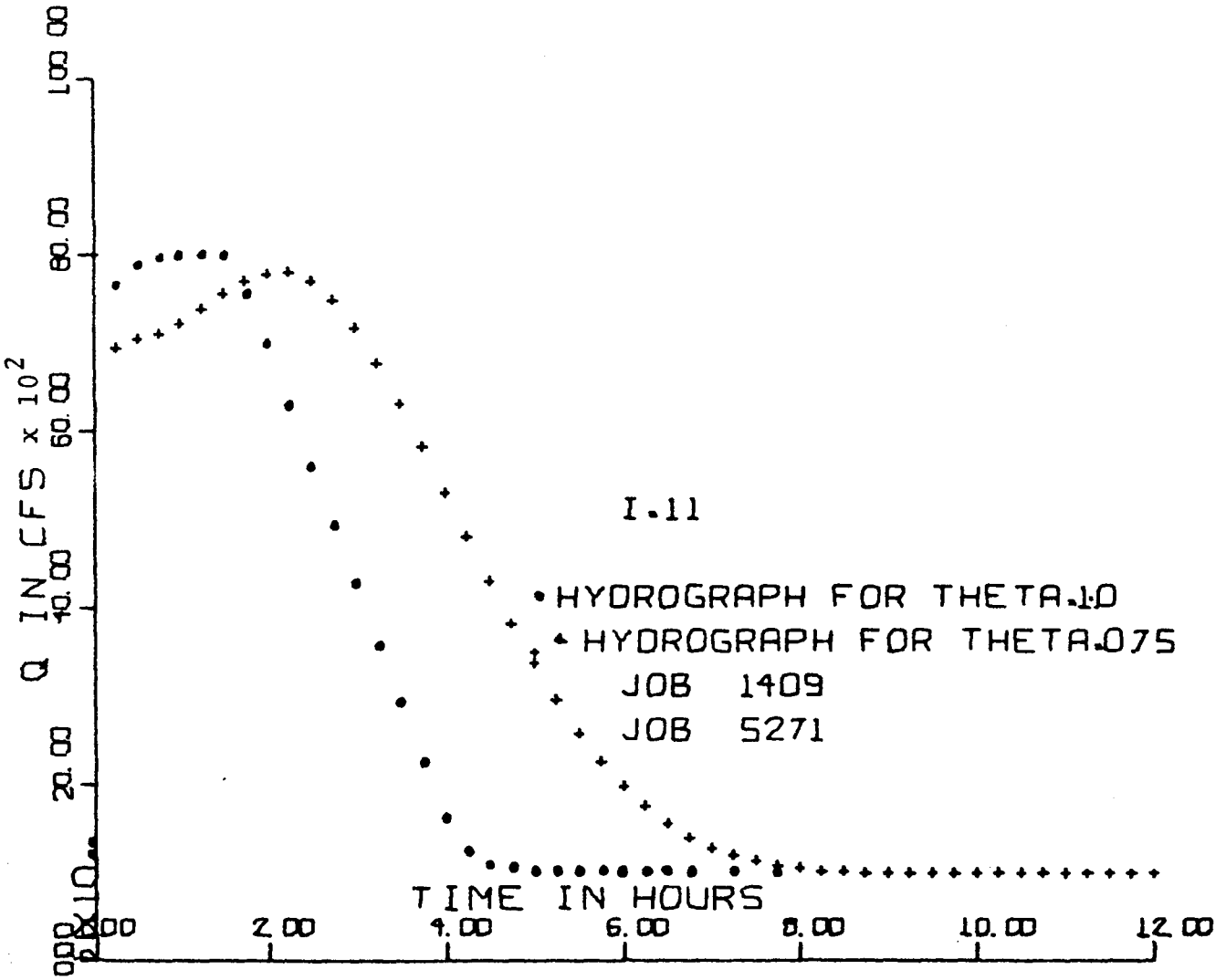


FIG.47. HYDROGRAPHS TYPE 3 . RECEDING .

Comparison with Faure's experiments.

This experiment is described in the section entitled "Methodology." This is a case of unsteady subcritical flow with friction in a horizontal rectangular channel. Δt and Δx were made equal to 1.0 sec. and 2 m respectively. The graphs of depth versus time were plotted at various locations in the channel, as shown in Fig. 48. Solution by the single-stage implicit scheme (θ = 0.6) appears better than solution of the same scheme for theta equal to 1.0 and solution by the two-stage implicit scheme. Solution by the single-stage implicit scheme is almost comparable to solution by the explicit scheme. In fact the solution by the single-stage implicit scheme for theta equal to 1.0 is the worst of all solutions.

Figure 49 shows the flow profiles at 20 sec. and 40 sec. Solution by the single-stage implicit scheme (θ = 0.6) again appears better than solution of the same scheme for theta equal to 1.0. The solutions by the single-stage implicit scheme and the explicit scheme are practically the same. The calculated depths differed slightly from the measured depths because of the initial assumed conditions (initial discharge = 0.1 CFS and outflow = 0.1 CFS). These conditions were assumed in order to make the solution converge. When the initial conditions specified by Faure were used in the program the solution

did not converge.

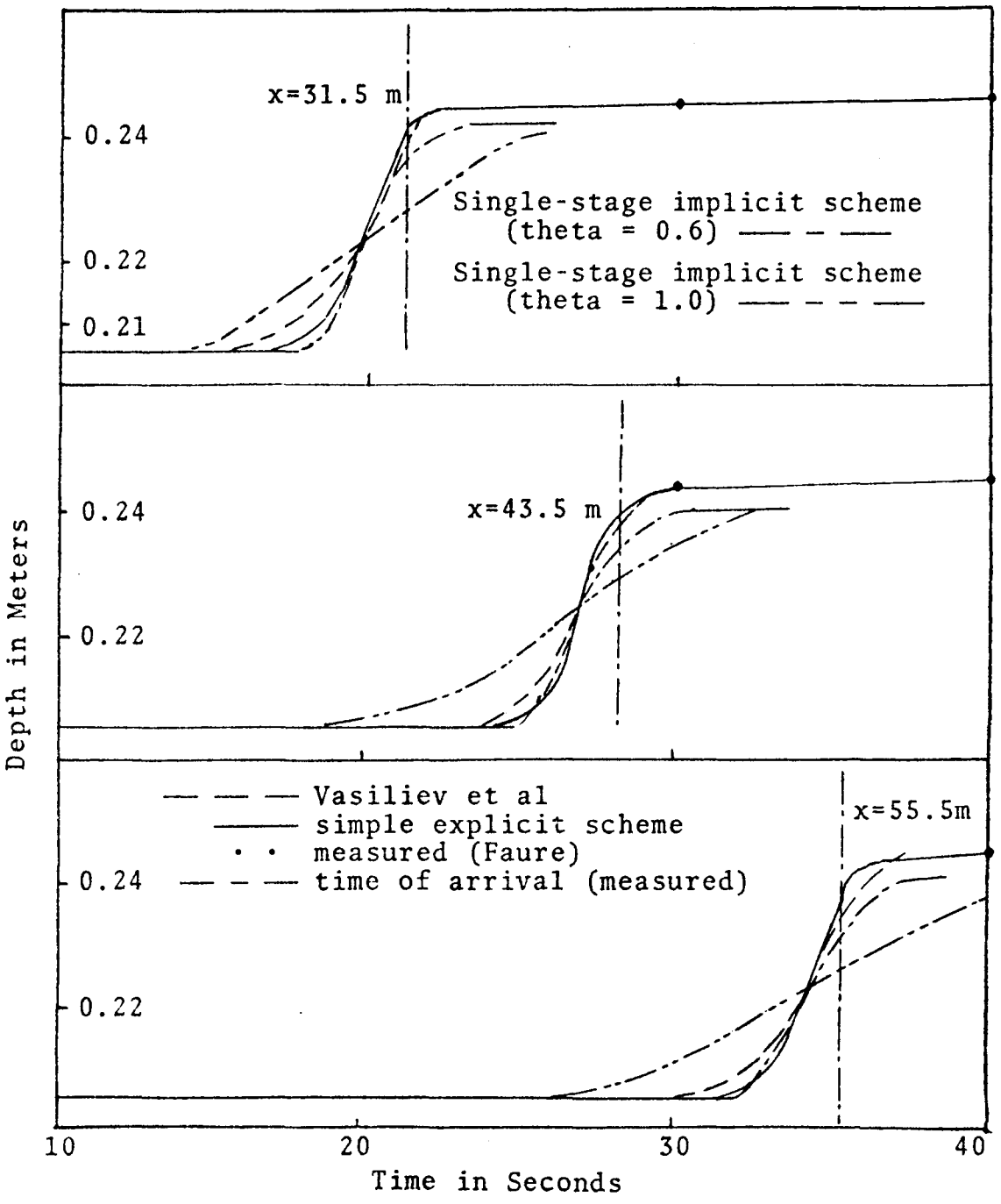


Fig. 48. Propagation of Shock Wave in Still Liquid with Depth as Function of Time at Various Locations in Channel.

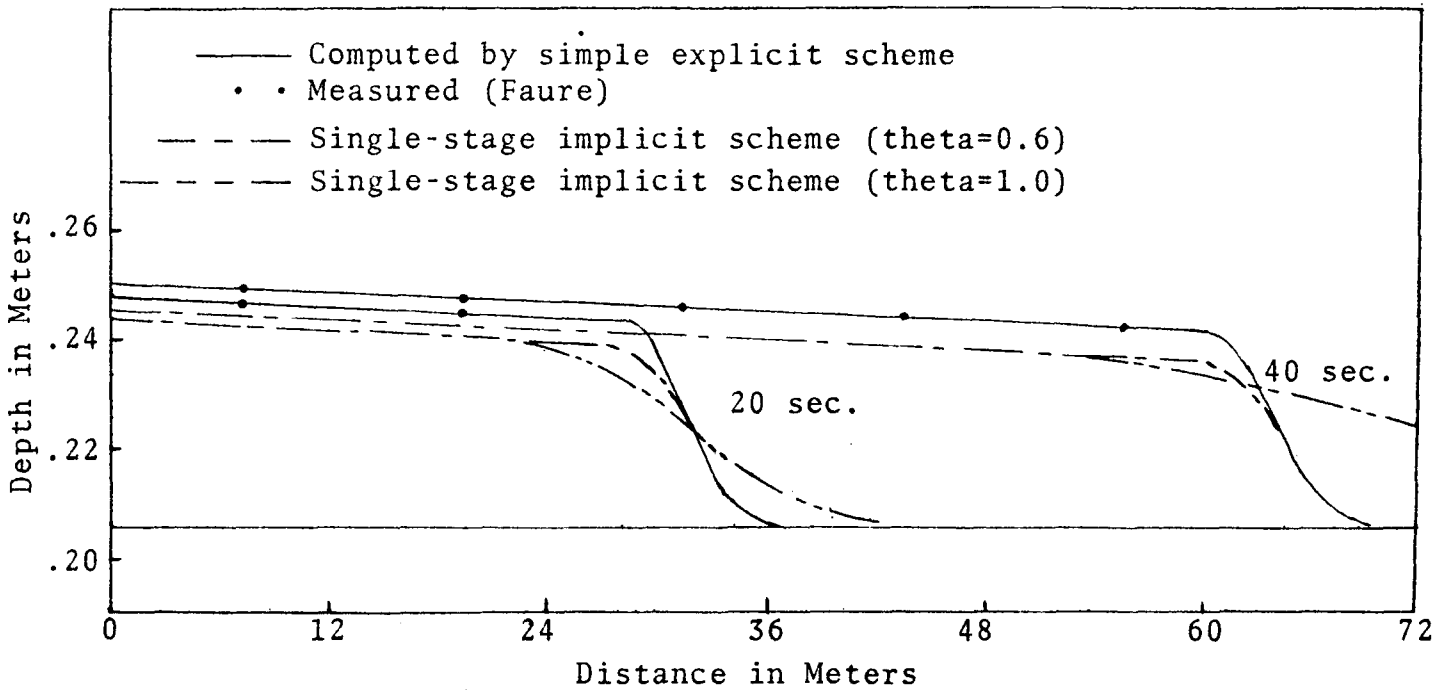


Fig. 49. Propagation of Shock Wave, Successive Flow Profiles.

CONCLUSIONS

1. The wave speed was found to be very sensitive to Manning's roughness coefficient "N". The values of "N" equal to 0.03 gave significantly different wave speeds for values of theta ranging from 0.6 to 1.0 in the case of type 1 rising hydrographs. There was a considerable departure from the actual wave speed. The values of "N" = 0.012 caused the wave speed to approach the theoretical wave speed.
2. The values of θ from 0.0-0.4 caused numerical instabilities in the computer program.
3. The value of theta equal to 0.5 gave better results than the one for θ = 1.0 for the fast-rising type of hydrographs.
4. The value of theta equal to 0.5 caused some fluctuations in the computed discharge values. Therefore a value of θ equal to 0.6 was chosen.
5. For types 2 & 3 receding hydrographs, the values of theta equal to 0.5 gave some spreading of the shock. But for values of theta equal to 1.0, the wave traveled almost undeformed, which is contrary to observation and theory.
6. In the case of gradually varied flow, the different values of theta gave identical results.

7. The values of theta have a more pronounced effect on the hydrographs that change linearly from 1000 CFS to 2000 CFS in one time interval as compared to their effect on type 1 rising hydrographs.
8. In Faure's experiment, Series III [7], the value of theta equal to 0.6 gave better results than that obtained by the two-stage implicit scheme and practically the same as the one obtained by the explicit scheme.

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THE APPLICATION OF IMPLICIT METHOD TO

OPEN-CHANNEL SURGES

by

Yusuf M. Chaudhry

(ABSTRACT)

The implicit method of flood routing has been modified so as to make it applicable to open-channel shocks.

The output of the computer program with $\theta = 0.6$ compared favorably with the experimental result. A comparison was also made with other explicit schemes and a two step implicit scheme proposed by other investigators. The implicit method with $\theta = 0.6$ gave results as good as the explicit scheme and much better than the two step implicit scheme.

In this method the equations of continuity and momentum for gradually varied flow have been written in finite difference form for a point in the (x,t) plane specified by the co-ordinates $(x+dx/2,\ t+\theta dt)$. The variable θ was made to take on values from 0 to 1.0. A computer program was written to obtain the output hydrograph at the downstream end of a rectangular channel for any type of inflow hydrograph at the upstream end of the channel. Three types of inflow hydrographs were chosen and the outflow hydrographs were obtained for different

values of θ. For that type of inflow hydrograph that caused a surge to occur in the channel, it was found that $\theta = 0.6$ gave the most satisfactory results.

Experimental results of a surge in a channel were obtained from published literature.