Chapter 1

INTRODUCTION

1.1 Problem Description and Scope

Network-structured problems arise directly or indirectly in many practical scenarios where physical flow transfer occurs between a set of origin points (supply nodes) and destination points (demand nodes) in space. This (dynamic) flow transfer could involve inter-nodal movement of any combination of people, raw material, finished products, or electronic data via physical flow channels (links or arcs). A variety of such practical network optimization problems occur in the context of public supply and commercial transportation, emergency response and risk management, and industrial planning (see Francis et al. (1992), and Ahuja et al. (1993)). Frequently, these problems turn out to be nonconvex in nature, which renders standard solution approaches ineffective. On the other hand, such problems possess a network-based structure that enables the development of effective model representations and algorithmic solution procedures. The decisions to be made in these network optimization problems include the location of supply nodes, the routing, allocation, and scheduling of flow between supply and demand nodes, and the design of links in the network. This study is concerned with the analysis and development of discrete and continuous optimization models for solving several such problems.

The first problem addressed in this dissertation is the deterministic time-dependent shortest pair of disjoint paths problem that finds applications in the context of dynamic routing in computer, communication networks, airport airspace and taxiways, and intelligent transportation systems. The time-dependent (single) shortest path problem was first formulated and solved by Cook and Halsey (1958) and has become an active area of research over the last decade. Unlike the standard shortest path problem, this problem assumes the link delays to be a function of the time of arrival at the link.

We examine computational complexity issues, models, and algorithms for the problem of finding a shortest pair of disjoint paths between two nodes of a network such that the total travel delay is minimized, given that the individual arc delays are time-dependent. Such disjoint paths address the issue of network vulnerability by prescribing alternate routes for traffic flows. Applications include the dispatching of duplicate packets of data to improve reliability in communication and computer networks, and the diverting of traffic during congestion to reduce the chances of bottlenecks in transportation networks, in the presence of time-dependent variations in travel delays in the network. It is shown that this problem, and many variations of it, are NP-Hard, and a 0-1 linear programming model that can be used to solve this problem is developed. This model can accommodate various degrees of disjointedness of the pair of paths, from complete to partial with respect to specific arcs. Computational results obtained by solving the above models using CPLEX-MIP are also presented.

The second problem studied is concerned with the development, analysis, and testing of a mathematical model for determining a route that attempts to reduce the risk of low probability-high consequence accidents related with the transportation of hazardous materials (hazmat). The hazmat routing problem was initially solved as a shortest path problem. Since then, this problem had been a subject of intense research due to the complex, multiobjective nature of the problem involving issues such as public concern and social equity, among other considerations. Several models have been proposed based on various objective functions and modeling approaches for solving this problem. In addition, new approaches also address integrated location and routing models.

The hazmat routing model developed in this study considers trade-offs between the conditional expectation of a catastrophic outcome given that an accident has occurred, and more traditional measures of risk dealing with the expected value of the consequence and the accident probability on a selected path. More specifically, the problem addresses in this study involves finding a path that minimizes the conditional expectation objective value, subject to the expected value of the consequence being lesser than or equal to a specified value ν, and the probability of an accident on the path being also constrained to be no more than some value η. Insights into these various modeling constructs are also provided. The values ν and η are user-prescribed and could be prompted by the solution of shortest path problems that minimize the respective corresponding linear risk functions. The proposed model is a discrete, fractional programming problem that is solved using a specialized branch-and-bound approach. A numerical example is used to illustrate the procedure, and some computational experience on randomly generated test cases is provided to study the effort required to solve this problem in different instances. The model is also tested using realistic data associated with a case concerned with routing hazmat through the roadways of Bethlehem, Pennsylvania. Data acquisition as well as algorithmic computational issues are discussed.

The third problem deals with emergency and risk management, and among other features, involves the amalgamation of ideas from the first two problems. A real-time traffic incident response strategy that deals with the determination of optimal resource allocation policies in response to traffic incidents occurring on a network is discussed.

Almost all prior approaches seen in the literature focus on the problem of finding optimal locations of response vehicle depots using location-allocation models. Another approach seen in the literature is the use of simulation models to analyze various incident and response scenarios to determine efficient response strategies. Real-time traffic incident response strategies deal with the determination of optimal resource allocation policies in response to traffic incidents occurring on a network. We develop a discrete optimization model that is equally applicable in concept and implementation to any such generic emergency response problem. We first formulate a mixed-integer programming model MIMR-1 for the multiple incident - multiple response (MIMR) problem using standard modeling techniques. The effect of loss in coverage due to the non-availability of response vehicles that are currently serving certain primary incidents is reflected in a novel manner in the model by including in the objective function a new term related to an opportunity cost for serving future (secondary) incidents that might occur probabilistically on the network. We next present an alternative, equivalent mixed-integer programming model (MIMR-2) that injects a particular structure into the problem. This is shown to yield a dramatic improvement in the performance of the commercial software package CPLEX-MIP when applied to this revised model in comparison with its application to MIMR-1. Furthermore, for certain special cases of the MIMR problem, efficient polynomial-time solution approaches are prescribed that yield a far superior performance over CPLEX-MIP. An algorithmic module composed of the developed specialized solution procedures, model MIMR-2, and a prescribed heuristic scheme, has been incorporated into a real-time area-wide incident management decision support system (WAIMSS).

The next model addressed in this study deals with the development of a global optimization algorithm for solving nonlinear pipe network design problems. Several heuristic and exact solution approaches have been presented in the past to solve this problem. These approaches include a wide range of optimization approaches such as gradient-based search, successive linear programming based approach, global optimization, and stochastic methods such as genetic algorithms and simulated annealing. The proposed methodology improves upon a presently existing global optimization approach to the problem of designing a water distribution network that satisfies specified flow demands at stated pressure head requirements. The nonlinear, nonconvex network problem is transformed into the space of certain design variables. By relaxing the nonlinear constraints in the transformed space via the use of suitable polyhedral outer approximations and applying the Reformulation-Linearization Technique, we derive a tight linear lower bounding problem. This problem provides an enhancement and a more precise representation of previous lower bounding schemes that use similar approximations. Upper bounds are computed by solving a projected linear program which uses the flow conserving solution generated by the lower bounding problem. These bounding strategies are embedded within a branch-and-bound algorithm. Various branching variable selection schemes are explored that reduce the gap from optimality, ensuring the convergence of the procedure to a feasible solution that lies within any prescribed accuracy tolerance of global optimality. A special maximal spanning tree-based procedure is developed to curtail the set of branching variables, and this is shown to reduce computational effort substantially. Computational experience on three standard test problems from the literature is provided to evaluate the relative efficiencies of these approaches. For all these problems, some of which have remained largely unsolved over the past two to three decades, improved and proven global optimal solutions within 10^{-6} % and/or within 1$ of optimality are obtained for the first time in the literature. A new network design test problem based on the Town of Blacksburg Water Distribution System has been created for use by the research community, and computational experience on solving this to global optimality is provided. Transfer of technology to the town engineers in the Blacksburg Water Authority Department is underway.

The last problem addressed in this study is the Euclidean distance multifacility location-allocation problem, as well as its generalization to the case of l_p distances. Continuous location-allocation problems have been studied since Weber posed the first location-allocation problem in 1929. Various extensions of this problems have been investigated since then, and include the rectilinear distance location-allocation problem, the Euclidean distance and squared Euclidean distance location-allocation problems, and the general l_p distance location-allocation problem. Furthermore, the pure location problem, which occurs as a subproblem in this context, has also been a subject of active research. Past studies by researchers have focused on the convergence of Wieszfeld's algorithm (1937) and the development of efficient algorithmic variants of this algorithm. For the capacitated Euclidean and l_p distance location-allocation problems studied in this dissertation, there exists no global optimization algorithm that has been developed and tested, aside from a total enumeration approach. Beginning with the Euclidean distance problem, we design a branch-and-bound algorithm based on a partitioning of the allocation space that finitely converges to a global optimum for this nonconvex problem. For deriving lower bounds at node subproblems in this partial enumeration scheme, we employ two types of procedures. The first approach computes a lower bound via a simple projected location space subproblem. The second approach derives a significantly enhanced lower bound by using the Reformulation Linearization Technique to transform an equivalent representation of the original nonconvex problem into a higher dimensional linear programming relaxation. In addition, certain cut-set inequalities are generated in the allocation space, and objective function based cuts are derived in the location space to further tighten the lower bounding relaxation. The RLT procedure is then extended to the general l_p distance problem, for $1 < p < 2$. A depth-first bases enumeration, as well as a best-first flow interval partitioning based branch-and-bound procedure is developed and tested. Computational experience is provided on a set of difficult test problems to investigate both the projected location space and the RLT-lower bounding schemes. The results indicate that the proposed global optimization approach using a combination of the two bounding schemes offers a promising viable solution procedure. In fact, among the problems solved, for the only two test instances previously available in the literature for the Euclidean distance case, this study reports significantly improved solutions while deriving provable global optimization solutions to these problems for the very first time.

1.2 Research Objectives

The objectives of this study are to develop and analyze optimization models for the continuous and discrete nonconvex network optimization problems introduced in Section 1.1, to design effective exact and/or heuristic algorithms for these problems, and to provide computational experience on implementing such algorithms. The optimization techniques that are to be employed in these algorithms include standard discrete optimization methods available in commercial software packages, specialized branch-and-bound methods that exploit the network structure of the problem, discrete optimization techniques, global optimization procedures, specialized enumerative and graph theoretic methods, as well as classical optimization and calculus methods. It is hoped that the modeling, analysis, insights, and concepts provided for these various network based problems that arise in diverse routing, location, distribution, and design contexts, will provide guidelines for studying many other problems that arise in related situations.