A GENERALIZED SOLUTION FOR THE DESIGN OF ONE-WAY AND TWO-WAY REINFORCED CONCRETE SLABS

by

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I. INTRODUCTION

The design of reinforced concrete slabs has been a topic of continuous study over the past century. Much controversy still exists and improved analytical and experimental studies continue to be prevalent within the profession.

Several techniques for design have been proposed and some of these have found favor as recommended by the American Concrete Institute (1). In their simplest form these recommended methods are tedious to use and involve a trial-and-error type of procedure to converge on an optimum solution.

Any solution for the design of reinforced concrete slabs involves many parameters such as dimensions of length, width and depth, strengths and modular properties of the concrete and steel, percentage of steel and values for dead load and live load as well as conditions of support and constraint.

In this thesis an evaluation is made of the results of approximately 1200 designs for one-way and two-way slabs meeting the requirements of the A.C.I. Code (1). The parameters have been chosen to cover a wide range of practical design conditions. The data resulting from these designs have been plotted using dimensionless parameters evaluated from a dimensional analysis approach. Curves fitting the data proved to be continuous over the working range, so equations for the curves were developed. Data from the original designs were compared with results computed from the developed equations. In a few cases maximum deviations of less than three percent were found. The average deviation for all designs was found to be less than one percent.

The use of the developed equations and the plotted graphs is illustrated. The development of simplified design charts is suggested and discussed.

The techniques for the development of the equations, graphs and charts for the direct design of reinforced concrete slabs presented in this thesis is unique and suggests wider application. The designs included here are for interior panels of continuous one-way and two-way slabs only. Equations, graphs and charts for the design of exterior panels and for the complete design of the various types of flat slabs can be developed in a similar way, and perhaps the techniques can be used also for the design of other types of structures.

II. REVIEW OF LITERATURE

One-way slabs. For many years one of the most common types of fireproof floor construction has been the solid one-way reinforced concrete slab. Under normal conditions it is most suitable and most economical for slabs ranging from six to eight and one-half feet, although for light loads it may be economical up to 12 feet spans. (6)

A one-way slab is designed as a rectangular shallow beam with the main reinforcement extending in the short direction only. At right angles to the main reinforcement, shrinkage and temperature reinforcement is provided (4).

If adjacent continuous spans vary by not more than 20 percent in length and the unit live load does not exceed three times the unit dead load, the bending moment under uniform design load conditions may be computed using coefficients given in the A.C.I. Code (1).

Two-way slabs. One of the popular types of floor systems is that of the two-way slab. This type of slab was introduced around 1900 and design criteria have been in the process of developing over the past fifty years.

This type of slab becomes more efficient as it approaches the shape of a square. As the ratio of sides increases to 2:1, the action of the slab approaches that of a one-way slab. This type of slab is well suited to carry heavy concentrated as well as uniformly distributed loads through plate action (4). Theoretical studies of plate action was initiated in 1766 by Euler (6). Having developed his theory of the flexure of beams and plates, he attempted to explain the tone-producing vibrations of such sound producing plates. It was not until 1872 that interest in the problem changed from the question of vibrations to that of stresses and strength. The years 1912, 1913 and 1916 brought forth a valuable collection of exact or approximately exact studies of rectangular slabs supported on four sides. All of these studies, although concerned with the flat slab, served as a basis for the mathematical theory of load-stress relations for the two-way slab.

The mathematical theory of the two-way slab was developed in 1921 by Westergaard (6) but this precise method was too cumbersome for everyday use by the average designer. The findings, however, have served as a basis from which the more convenient and approximate methods, which are summarized in codes and used for design, have been developed.

There are several methods for designing a two-way slab, three of which are presented in the A.C.I. Code (1). Method 2 of the Code is used for the designs in this investigation.

III. THE ANALYSIS TECHNIQUE

The design of slabs depends upon a large number of variables which makes the application of the usual analytical procedures tedious. Furthermore, analytical procedures have not yet been developed to group all the pertinent variables in one quantitative equation. Because of these complexities, dimensional analysis is used as the theoretical approach to develop a general qualitative expression. The unknown function of this expression is then evaluated using data from typical designs.

The variables affecting the slab design are listed below:

Symbol	Variable	Dimensions
d	effective depth of slab	L
L	long span length of slab	L
S	short span length of slab (two-way slab)	L
f_c	allowable concrete stress	FL^{-2}
f_s	allowable steel stress	FL^{-2}
E_c	modulus of elasticity of concrete	FL^{-2}
E_s	modulus of elasticity of steel	FL^{-2}
p	percentage of tension steel	--
W	total uniform load intensity	FL^{-2}
C	coefficient for determination of moment	--

Some of these variables need to be evaluated before proceeding with dimensional analysis.

In all design equations E_c and E_s can be combined as a fraction E_s/E_c which is equal to the modular ratio n. Since E_s is a constant, then the ratio E_s/E_c can be replaced by the modular ratio, n.

The percentage of steel p can be determined directly if the values of f_c, f_s and n are known. The variable p is therefore redundant and need not appear.

In the design of two-way slabs, S may be defined as the value mL and m = S/L may be used as a variable instead of S.

In design, normally d is unknown and all the other variables are known or may be assumed; therefore, d may be expressed as a function of the other variables and, with the changes suggested above, will appear as:

$$d = f_1(L,\ m,\ f_c,\ f_s,\ n,\ W,\ C) \qquad \ldots (1)$$

Equation 1 contains eight variables expressed in two dimensions, force and length, and therefore can be reduced to an equation with six dimensionless parameters. The variables of equation 1 may be combined in several ways to form many sets of dimensionless parameters. One set of parameters is selected to appear in the functional equation below:

$$L/d = f_2(n,\ f_s/f_c,\ f_s/W,\ m,\ C) \qquad \ldots (2)$$

The variable m can be omitted if the function f_2 is evaluated for each m value separately. Equation 2, therefore, reduces to:

$$L/d = f_3(n,\ f_s/f_c,\ f_s/W,\ C) \qquad . . . (3)$$

For any particular span and for the critical moment within that span the coefficient C is practically constant for the design of one-way slabs. The coefficient C is also constant for any given value of m in the case of two-way slabs. Accordingly, the coefficient C may be omitted from the equation and equation 3 reduces to:

$$L/d = f_4(n,\ f_s/f_c,\ f_s/W) \qquad . . . (4)$$

Equation 4 is indeed general in that it contains all the necessary variables to define the procedure of design for one-way and two-way slabs. Equation 3 may then be written in the following form:

$$\pi_1 = f_4(\pi_2, \pi_3, \pi_4) \qquad . . . (5)$$

where π_1, π_2, π_3, and π_4 represent L/d, n, f_s/f_c and f_s/W, respectively. The variables π_1, π_2, π_3 and π_4 are dimensionless and basically independent in that none of them can be derived from the others. This independence implies that the original variables in equation 1 are all independent and none are redundant. The nature of the function f_4 is determined from the available data.

IV. DESIGN DATA

A. Range of Variables.

The data were obtained by designing one-way and two-way slabs using several values of each variable within reasonable working ranges. The following values for parameters were used:

n = 10, 12, 15

f_s/f_c = 20, 25, 32

f_s/W = 20 to 200

These represented selected values of the independent variables as follows:

n = 10, 12, 15

f_s = 16000, 18000, 20000 psi

f_c = 500 to 1000 psi

W_L = 50, 100, 200, 400 psf

L = 5, 10, 15, 20 feet (for one-way slabs)

L = 10, 15, 20, 35 feet (for two-way slabs)

S/L = 0.50, 0.75, 1.00 (for two-way slabs)

A total of 588 one-way slabs and 612 two-way slabs were designed. Values of f_s/f_c had been chosen to provide a reasonable range of values of f_c. In some cases the selection was very conservative and in a few cases f_c was larger than that permitted by the A.C.I. Code. Values of f_c/f_c' ranged from 0.20 to 0.92 based on the 1963 A.C.I. Code, and from 0.17 to 0.50 based on the 1956 A.C.I. Code.

In a few cases with high strength concrete and short spans, additional values of live load, W_L, of 500 psf and 1000 psf were used.

Values of f_s/W were not selected, but were calculated after depths and total loads had been calculated.

Selected data are tabulated in Tables 1 through 4 in the Appendix. All data have been plotted on Figures 1 through 12 in the Appendix.

B. Design Methods.

The complete design of a concrete slab with given span dimensions to support a given live load requires the determination of the thickness of the slab and the amounts and spacing of reinforcing steel both top and bottom in both directions. Generally the slab thickness is the minimum value that will support the maximum design bending moment, positive or negative, without producing stresses in the concrete or the steel in excess of specified limits.

The solutions here presented are obtained by the elastic theory technique and in accordance with the recommendations of the A.C.I. Code (1), Section 904 for one-way slabs and Section A2002 for two-way slabs.

For the purpose of this study, only partial designs are presented. First, only interior panels of continuous construction are considered. The design of exterior panels would constitute a separate study. Second, only the thickness of slab and amount of tensile reinforcing steel at the section of maximum moment is determined. A complete steel schedule has not been prepared.

The design procedure consists of the following steps:

1. Assume a slab thickness and calculate dead load and total load.
2. Determine the maximum bending on a twelve-inch wide strip using the coefficients given in the A.C.I. Code.
3. Calculate the value of R for the specified materials

$$R = f_c/2\ jk$$

where $k = \dfrac{nf_c}{nf_c + f_s}$ and $j = 1 - \dfrac{k}{3}$

Values of j, k, p, and R have been tabulated (8) for many combinations of n, f_s and f_c and need not be included in the design equations.

4. Calculate d and t.

$$d = \sqrt{\frac{M}{12R}}$$

t = d + 1.12* (for one-way slabs)

t = d + 1.38* (for two-way slabs)

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* Numerical values depend upon the size of reinforcing bars and the concrete cover required.

5. Check the calculated t against the assumed t. If a difference occurs, assume a new thickness and repeat steps 1 through 5.

6. Calculate the maximum shear V and the minimum slab depth d and thickness t to support it.

$$V = \frac{WL}{2} \qquad d = \frac{V}{12v}$$

where $v = 2\sqrt{f_c'}$.

7. Determine the minimum slab thickness as restricted by the A.C.I. Code, Section 909 (b) for one-way slabs, Section 2002 (e) for two-way slabs.

8. Select the largest value for t from steps 4, 6, and 7.

9. Calculate the amount of tensile reinforcing steel, A_s, per foot of width.

$$A_s = \frac{M}{f_s j d}$$

C. Sample Computations.

1. One-way slab panel (interior)

Given: $L = 20'$

$W_L = 100$ psf

$n = 10$

$f_c = 1050$ psi (specified allowable)

$f_s = 16000$ psi

Assume: $t = 8''$

$W_L = 100$ psf

$W = 200$ psf

$M = \frac{1}{11}(200)(20)^2 = 7270$ lb.ft.

$k = 0.396$

$j = 0.868$

$R = 180$ psi

$d = \sqrt{\frac{7270(12)}{12(180)}} = 6.36''$

$t = 6.36 + 1.12 = 7.48''$, say $7\frac{1}{2}''$

$V = \frac{200(20)}{2} = 2000\#$

$d = \frac{2000}{12(100)} = 1.67''$

$t = 1.67 + 1.12 = 2.79''$, say $3''$

$t_{min} = \frac{L}{35} = 6.70''$, say 6-3/4"

Use $t = 7\frac{1}{2}''$, $d = 6.38''$

$$A_s = \frac{7020(12)}{(16000)(.868)(6.38)} = 0.99 \text{ sq.in./ft.}$$

Use #6 bars @ 5" c.c.

2. Two-way slab panel (interior)

Given: $L = 20'$

$S = 15'$

$m = 0.75$

$W_L = 150$ psf

$n = 12$

$f_c = 765$ psi

$f_s = 20{,}000$ psi

Assume: $t = 6"$

$W_D = 75$ psf

$W = 225$ psf

$M_{max} = (0.0515)(225)(15)^2 = 2610$ lb.ft.

$k = 0.314$

$j = 0.895$

$R = 107$ psi

$d = \sqrt{\frac{2610(12)}{12(107)}} = 4.95"$

$t = 4.95 + 1.12 = 6.07"$, say $6\frac{1}{4}"$

$V = \frac{225(15)}{2} = 1690\#$

$d = \frac{1690}{12(83)} = 1.70"$

$t = 1.70 + 1.12 = 2.82"$, say $3"$

$t_{min} = 3.5$

$t_{min} = \frac{2(20 + 15)(12)}{180} = 4.67$

Use $t = 6\frac{1}{4}"$ $d = 5.13$

$A_s = \frac{2610(12)}{(20000)(.895)(5.13)} = 0.342$ sq. in./ft.

Use #5 bars @ 10" c.c.

V. ANALYSIS OF DATA

A. Prediction Equations.

Dimensional analysis of the variables listed in Section III led to the formulation of a functional equation (Equation 5) involving an unknown function f_4. Before a general prediction equation can be formulated, the nature of this unknown function must be investigated. This is accomplished by an analysis of the available data as obtained in Section IV.

If there are n pi-terms ($n \geq 2$) in the functional equation, then n - 1 component equations must exist (see Reference 5). Each of these component equations is a part of the general prediction equation. Component equations are found by expressing the dependent variable, π_1, as a function of all other pi-terms, successively holding constant all terms on the right side except one and observing the effect of this variable on the dependent variable.

The equivalent equation (Equation 5) contains four pi-terms. Therefore there are three component equations. The proper combination of these three equations then forms the general prediction equation.

The data from the several designs of slabs was reduced to the dimensionless form of the parameters expressed as pi-terms, namely: L/d, n, f_s/f_c and f_s/W. All of the data has been plotted on Figures 1 through 12 as derived on the basis of strength. No provision was made to account for control of deflections.

For the solution of the first component equation, π_3 and π_4 remained constant and a relation was developed for π_1 as a function of π_2. This was accomplished, for example, by letting $\pi_3 = f_s/f_c = 25$ and $\pi_4 = f_s/W = 100$. Then, for the one-way slab, L/d = 29.9, 31.4 and 33.3 when n = 10, 12 and 15 respectively, from Figures 1, 2 and 3. This data can be expressed as:

$$\pi_1 = a(\pi_2)^b \qquad (6)$$

$$29.9 = a(10)^b$$

$$31.4 = a(12)^b$$

$$33.3 = a(15)^b$$

Solving for a and b, algebraically or by plotting on log-log graph paper,

$$L/d = 16.21(n)^{0.266} \qquad (7)$$

This process must be repeated several times selecting values for π_3 and π_4 across the range of values that the final equations are expected to cover. A statistical evaluation must be made to select the form of equation and numerical values for coefficients and exponents that will best fit all the data.* Equation (7) was actually developed after such a statistical evaluation.

In a manner similar to that for determining Equation (7), let $\pi_2 = n = 12$ and $\pi_4 = f_s/W = 100$, and using Figure 2, it will be found that

$$L/d = 377(f_s/f_c)^{-0.773} \quad (8)$$

In a similar manner, letting

$$\pi_2 = n = 12 \text{ and } \pi_3 = f_s/f_c = 25$$

and using Figure 2, it will be found that

$$L/d = 3.14(f_s/W)^{0.500} \quad (9)$$

Using the validity test (see Reference 5), the functional equation (Equation 5) was found to exist as a product:

$$\pi_1 = k\,(\pi_2)^a\,(\pi_3)^b\,(\pi_4)^c \quad (10)$$

Properly combining the component equations in multiplication then produces the final equation for the one-way slab:

$$L/d = 19.50\ (n)^{0.266}\ (f_s/f_c)^{-0.773}(f_s/W)^{0.500} \quad (11)$$

In a similar manner, the final equations for the two-way slab are as follows:

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* The reader is referred to Reference 5 for a more comprehensive explanation of the process employed.

when S/L = 0.50

$$L/d = 19.70(n)^{0.266}(f_s/f_c)^{-.773}(f_s/W)^{0.500} \qquad (12)$$

when S/L = 0.75

$$L/d = 24.70(n)^{0.266}(f_s/f_c)^{-.773}(f_s/W)^{0.500} \qquad (13)$$

when S/L = 1.00

$$L/d = 31.35(n)^{0.266}(f_s/f_c)^{-.773}(f_s/W)^{0.500} \qquad (14)$$

Equations 11, 12, 13 and 14 thus constitute final solutions relating the variables in slab design. These equations may be used in the given form or may be expressed in terms of d as a function of L, n, f_s, f_c and W. Equations 11, 12, 13 and 14 thus expressed become equations 15, 16, 17 and 18 respectively.

For a one-way slab:

$$d = \frac{0.05128L\, f_s^{0.273} W^{0.500}}{n^{0.266}\, f_c^{0.773}} \qquad (15)$$

For a two-way slab:

when S/L = 0.50

$$d = \frac{0.05076L\, f_s^{0.273}\, W^{0.500}}{n^{0.266}\, f_c^{0.773}} \qquad (16)$$

when S/L = 0.75

$$d = \frac{0.04049L\, f_s^{0.273}\, W^{0.500}}{n^{0.266}\, f_c^{0.773}} \qquad (17)$$

when S/L = 1.00

$$d = \frac{0.031905\ f_s^{0.273}\ W^{0.500}}{n^{0.266}\ f_c^{0.773}} \qquad (18)$$

The curves drawn through the data on Figures 1 through 12 were computed from these equations and may be used in lieu of the equations. The designer should keep in mind that deflection limitations may control the computation of depth. For one-way slabs, $L/t \leq 35$ is specified for interior continuous panels which would mean that L/d must be less than about 40 to 45. This is called to the reader's attention by the solid horizontal line at L/d = 42 on Figures 1, 2 and 3.

For two-way slabs, $S/t \leq 45$ for square panels to $S/t \leq 60$ for panels where m = 0.50. Solid horizontal lines on Figures 4 through 12 represent approximate upper limits for S/d as controlled by deflection limitations.

B. Illustrated Problems.

In order to illustrate the use of the equations, the problems of Section IV-C will be solved for slab thickness.

1. One-way slab panel (interior).

Given: L = 20' = 240"

W_L = 100 psf

n = 10

f_c = 1050 psi

f_s = 16000 psi

Assume: $t = 8''$

$W_L = 100$ psf

$W = 200$ psf

$$\text{Then } d = \frac{(0.05128)(240)(16000)^{0.273}\ (200)^{0.500}}{(10)^{0.266}\ (1050)^{0.773}}$$

$$= \frac{(0.05128)(240)(14.05)(14.14)}{(1.842)(218)}$$

$$= 6.12''$$

$t = 6.12 + 1.12 = 7.24''$, use $7\frac{1}{4}''$

The difference between the two solutions is less than two percent.

2. Two-way slab panel (interior).

Given: $L = 20'$

$S = 15' = 180$

$m = 0.75$

$W_L = 150$ psf

$n = 12$

$f_c = 765$ psi

$f_s = 20000$ psi

Assume: $t = 6''$

$W_L = 75$ psf

$W = 225$ psf

$$\text{Then } d = \frac{(0.04049)(180)(20000)^{0.273}(225)^{0.500}}{(12)^{0.266}\ (765)^{0.773}}$$

$$= \frac{(0.04049)(180)(14.93)(15.00)}{(1.938)(170)}$$

$$= 4.96''$$

$$t = 4.96 + 1.12 = 6.08'', \text{ say } 6\tfrac{1}{4}''$$

C. Evaluation of Reliability.

For each slab panel designed as in Section IV for data, the depth d was computed again using the equations developed in Section V-A. Corresponding values of L/d for each panel were compared and the deviation of the value computed by the equation from the value computed by conventional elastic design was computed. It was found that the maximum deviation was less than three percent, and there were less than one-fourth of the deviations over two percent, and the average deviation for all designs was less than one percent.

Random samples of the computed values and deviations are shown in tables 1 through 4 in the Appendix.

VI. DISCUSSION OF RESULTS AND CONCLUSIONS

In this investigation, a new approach to the design of slabs is presented. The approach is illustrated for interior panels of continuous construction of one-way and two-way slabs and is limited to the determination of slab thickness when span, live load and material properties are specified.

Design data is obtained by conventional elastic design techniques using the recommended methods and complying with the restrictions of the A.C.I. Code. In the case of one-way slabs, the maximum moment was taken as $WL^2/12$ for spans of 10 feet (A.C.I. Code Section 904). In the case of two-way slabs, the maximum moment was taken as 0.083 WS^2, 0.052 WS^2 and 0.033 WS^2 where m = 0.50, 0.75 and 1.00, respectively (A.C.I. Code Section A2002). The data is plotted and analyzed by dimensional analysis techniques. Component equations are developed and combined to produce effective working equations. The use of the equations is illustrated, and an analysis of the degree of reliability of the equations is made. The equations have been developed to use values of the variables in their usual units: for example, f_s in psi, W in psf, L in feet, etc. If it may be found desirable to use other units, corresponding values for coefficients can be calculated.

When the data was first computed and plotted, it was revealing that the results for each type of slab showed the data restricted to a very narrow band generating a smooth continuous curve. The results appeared to be so good that an "averaging" type curve was used. Subsequent computations showed extremely small deviations, thus justifying the curves that were used. Normally in design it is desirable to select values on the conservative side. Therefore, it would be recommended that in future studies of this nature, consideration should be given to plotting the "curve of fit" to skirt on the conservative side of the data rather than to pass through it.

It is interesting to note that of the 1200 designs used to obtain data, depth was not limited by shear in any of them. Shear is not normally a problem except for very short spans carrying very heavy loads.

The work of this thesis is investigative with the objective to verify that the dimensional analysis approach could be used to develop equations for direct design. It will be observed by the scrutinizing user that many refinements can be made to increase reliability and usefulness of the equations.

It will probably be noted that the equations can be used equally well for analysis as well as design.

It appears that the technique could be used successfully to extend this work to include all types of exterior panels for the one-way and two-way slabs, and perhaps to include the design of flat slabs with and without drop panels and with and without column capitals.

With further study and an evaluation of costs of material and labor, this technique might be used to develop an equation for the determination of total cost. Coupled with the design data, it would provide an excellent tool for comparison of designs.

It is interesting to note that the only difference between any one of the four equations (Equations 15, 16, 17, 18) and another is the value of the coefficient. This suggests that with further data it should be possible to combine these four equations into one. As a preliminary step, the value of the coefficient, C, is plotted against the ratio m in Figure 13 and may be used as a design aid. It is noted also that the coefficient for the two-way slab from m = 1.00 to m = 0.50 becomes progressively smaller and at m = 0.50 nearly equals the coefficient for the one-way slab. The curve becomes asymptotic to the vertical line representing the coefficient for the one-way slab.

As a final step toward making the results more useful, for both design and analysis, the equations might be presented in tabulated or nomographic form. In that case, an additional feature could be incorporated by plotting t rather than d and live load W_L rather than total load W.

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IX. VITA

Abbas N. Al-Khafaji was born in Baghdad, Iraq, on September 12, 1930. He received the B.S. in Civil Engineering from the University of Arizona, Tucson, Arizona, in June, 1953. In December, 1955, he obtained the M.S. in applied hydraulics from Colorado State University, Fort Collins, Colorado. In June, 1961, he obtained his Ph.D. in Hydraulic Engineering from Utah State University, Logan, Utah.

In September, 1961, he joined the faculty of the Civil Engineering Department at Virginia Polytechnic Institute. While working full time he began his graduate study in Structural Engineering.

X. APPENDIX

Table 1. One-Way Slab

L/d from Design vs L/d from Eq. 11

n 1	f_s/f_c 2	L ft. 3	W_L psf 4	f_s ksi 5	$\frac{f_s}{W}$ 6	L/d 7	L/d Eq.11	Dev. % 9
10	20	10	100	20	134	41.4	41.0	1.0
		20	400	16	27.6	18.0	18.6	3.3
		15	50	20	182	46.2	47.8	3.5
	25	10	100	16	98	29.7	29.6	0.3
		15	50	18	139	33.9	35.2	3.8
		20	200	16	43	18.8	19.5	0.4
	32	5	50	18	217	35.9	36.3	1.0
		5	50	16	188	33.3	33.8	1.5
		5	100	18	129	27.6	28.1	1.8
12	20	10	300	18	49	26.2	25.9	1.1
		15	100	16	90	34.2	35.3	3.2
		20	400	18	32	20.3	21.1	1.2
	25	10	300	16	42	20.4	20.2	1.0
		10	100	18	115	33.9	33.7	0.6
		15	50	20	165	39.0	40.3	3.3
	32	10	100	16	93	25.0	24.9	0.4
		15	100	16	73	21.2	22.0	3.8
		20	50	20	113	26.5	27.5	3.8
15	20	5	150	16	88	37.2	37.4	0.5
		10	200	18	70	33.4	33.0	1.2
		15	100	18	106	39.2	40.7	3.8
	25	15	300	16	37.6	19.8	20.4	3.0
		15	100	20	112	34.2	35.2	2.9
		20	100	20	96.7	31.8	32.7	2.8
	32	5	50	20	250	43.8	43.4	0.9
		10	200	18	64.0	22.2	22.0	0.8
		15	100	16	76	23.2	24.0	3.5

Table 2. Two-Way Slab with S/L = 0.5

ℓ/d from Design vs ℓ/d from Eq. 12

n 1	f_s/f_c 2	ℓ ft. 3	W_L psf 4	f_s ksi 5	$\frac{f_s}{W}$ 6	ℓ/d 7	ℓ/d Eq. 12 8	Dev. % 9
10	20	10	400	20	41	23.0	23.0	0.0
		35	400	20	29	19.4	19.3	0.5
		20	200	18	55	26.4	26.6	0.8
	25	10	50	20	196	42.0	42.3	0.7
		20	200	20	57	22.7	22.8	0.4
		35	400	20	26	15.5	15.5	0.3
	32	10	200	20	69	20.3	20.7	1.7
		20	100	20	78	21.6	22.0	1.9
		35	50	20	57.2	18.5	18.8	1.8
12	20	10	100	20	132	43.2	43.2	0.0
		20	50	20	153	46.7	46.5	0.4
		35	200	20	48	26.1	26.1	0.0
	25	10	50	20	200	44.8	44.7	0.2
		20	400	20	35	18.6	18.6	0.0
		35	200	20	43	20.7	20.7	0.0
	32	10	400	20	39	16.3	16.4	0.6
		20	200	20	53.5	19.1	19.2	0.3
		35	100	20	49	18.1	18.2	0.6
15	20	10	100	20	134	46.2	46.2	0.0
		20	50	20	159	50.4	50.3	0.2
		35	100	20	74	34.4	34.3	0.3
	25	10	400	20	41	21.6	21.6	0.0
		20	50	20	140	40.0	39.8	0.5
		35	200	20	44.4	25.5	22.4	0.4
	32	10	50	20	189	38.2	38.1	0.3
		20	100	20	86	25.7	25.6	0.4
		35	400	20	25	13.8	13.8	0.0

Table 3. Two-Way Slab with S/L = 0.75

$\mathcal{S}$/d from Design vs $\mathcal{S}$/d from Eq. 13

n 1	f_s/f_c 2	$\mathcal{S}$ ft. 3	W_L psf 4	f_s ksi 5	$\frac{f_s}{W}$ 6	$\mathcal{S}$/d 7	$\mathcal{S}$/d Eq. 13 8	Dev. % 9
10	20	10	200	20	79	40.4	39.7	1.7
		20	200	18	59	34.7	34.7	0.0
		35	400	20	32.3	25.8	25.6	0.8
	25	10	100	20	132	43.8	43.3	1.1
		20	300	16	36	22.4	22.6	0.9
		35	200	20	48.2	26.4	26.3	0.4
	32	10	200	20	73	26.4	26.7	1.0
		20	400	20	34	18.1	18.3	0.9
		35	200	20	42	20.0	20.1	0.5
12	20	10	100	20	114	51.0	49.7	0.6
		20	200	16	52	34.4	34.4	0.0
		35	400	20	33	27.5	27.1	1.5
	25	10	400	20	42	26.1	25.8	1.2
		20	200	18	57	30.1	29.9	0.7
		35	50	20	102	40.6	41.7	2.7
	32	10	50	20	202	46.9	46.9	0.0
		20	100	20	95	32.3	31.7	1.7
		35	200	20	44	21.9	21.6	1.1
15	20	10	200	20	80	45.3	44.5	1.8
		20	300	18	44	33.3	33.3	0.0
		35	100	20	88	47.5	46.7	1.7
	25	20	200	18	56	31.8	31.6	0.6
		20	300	18	41	27.4	27.2	0.7
		35	400	20	31	24	23.1	2.5
	32	10	200	20	75	30.6	30.1	1.6
		20	400	20	38	21.1	21.4	1.4
		35	400	20	28.7	18.9	18.6	1.6

Table 4. Two-Way Slab with S/L = 1.00

$\mathfrak{S}/d$ from Design vs $\mathfrak{S}/d$ from Eq. 14

n 1	f_s/f_c 2	$\mathfrak{S}$ ft. 3	W_L psf 4	f_s ksi 5	$\frac{f_s}{W}$ 6	$\mathfrak{S}/d$ 7	$\mathfrak{S}/d$ Eq.14 8	Dev. % 9
10	20	10	200	20	72	48.0	48.7	1.5
		15	200	16	59	43.7	43.8	0.2
		35	400	16	27	29.4	29.5	0.2
	25	15	400	20	41	30.4	30.6	0.7
		20	300	16	38	29.4	29.8	1.4
		35	400	20	33	27.3	27.5	0.8
	32	10	100	16	104	39.5	40.4	2.3
		15	200	16	54	28.5	29.1	2.1
		20	300	18	42	28.0	28.0	0.0
12	20	15	200	20	77	52.4	52.5	0.1
		20	300	16	40	38.0	38.2	0.5
		35	200	20	61	46.8	46.8	0.0
	25	10	200	20	80	45.1	45.1	0.0
		20	300	18	44	33.4	33.6	0.6
		35	200	20	56	37.6	37.8	0.5
	32	15	50	20	142	48.9	49.5	1.2
		20	200	16	50	29.1	29.4	0.9
		35	400	20	24	19.9	20.2	1.5
15	20	10	200	16	68	50.8	52.2	2.8
		15	200	16	61	49.4	49.4	0.0
		35	400	16	28	33.3	33.4	0.3
	25	15	100	16	100	53.6	53.5	0.2
		20	200	16	55	39.6	39.5	0.3
		35	100	16	68	43.7	43.9	0.5
	32	10	100	16	107	45.4	45.5	0.2
		20	200	18	59	33.8	33.8	0.0
		35	400	16	24	21.8	21.8	0.0

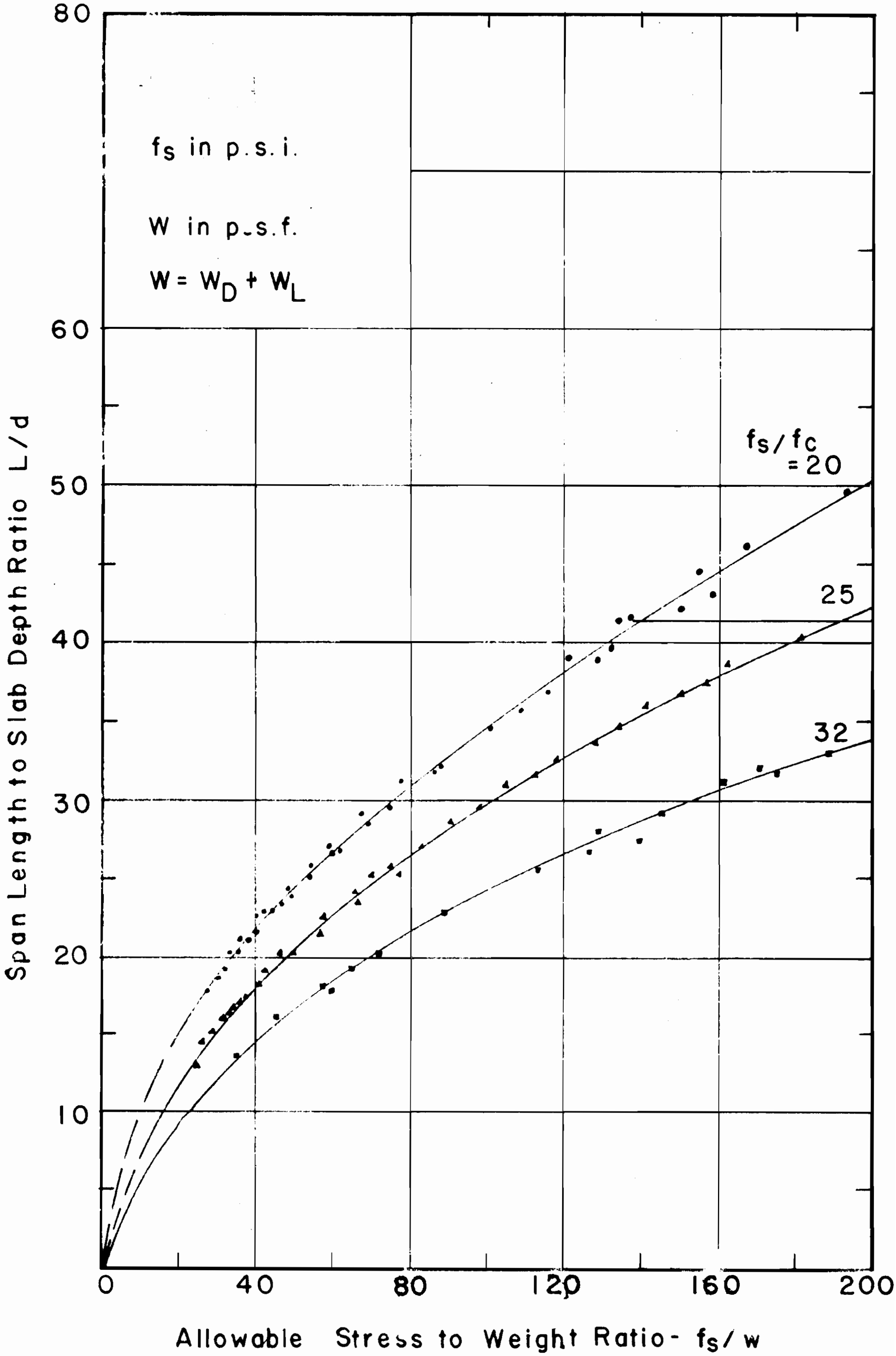

FIG. I L/d AGAINST f_s/w FOR ONE-WAY SLAB

n=10

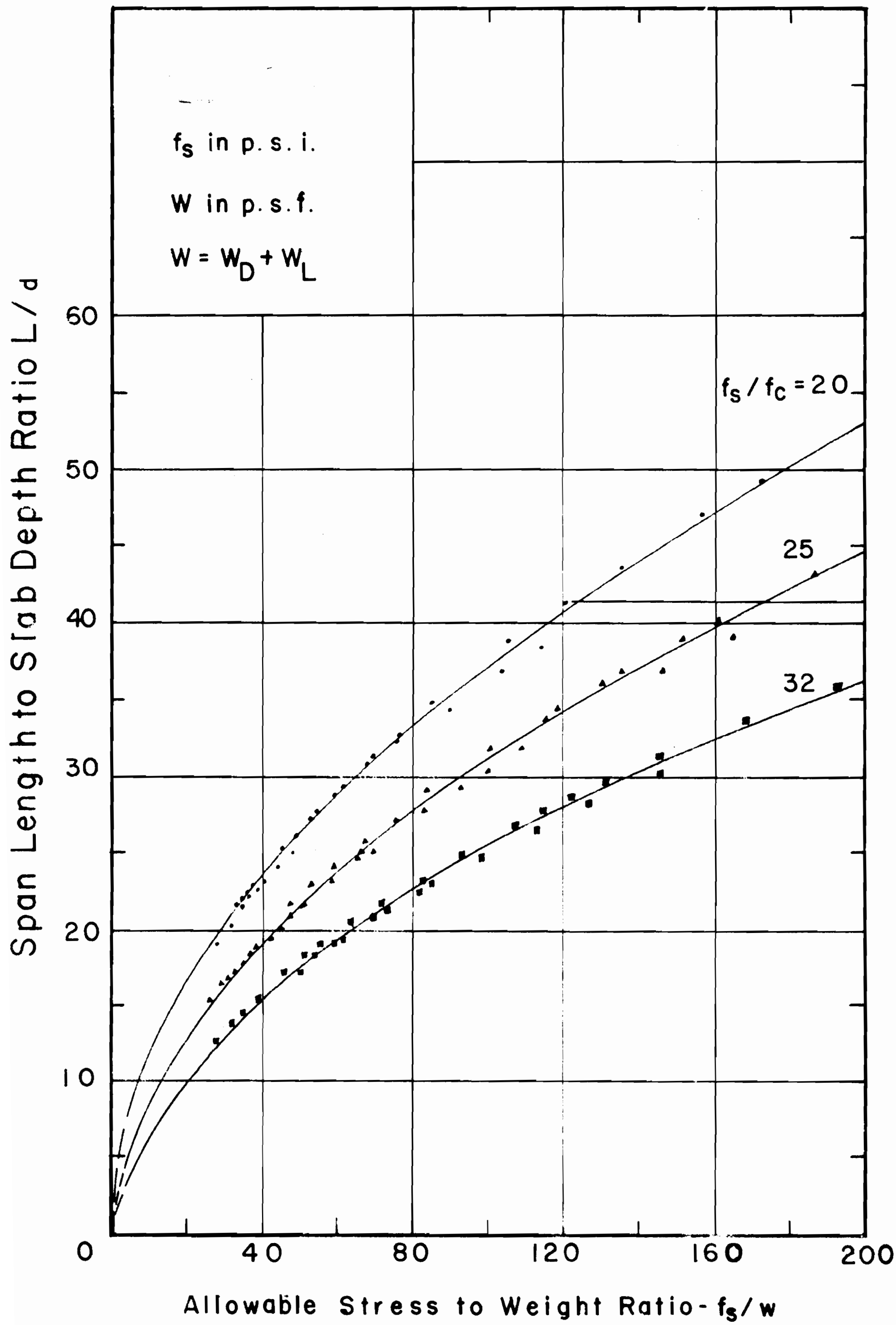

FIG. 2 L/d AGAINST f_s/w FOR ONE-WAY SLAB

n = 12

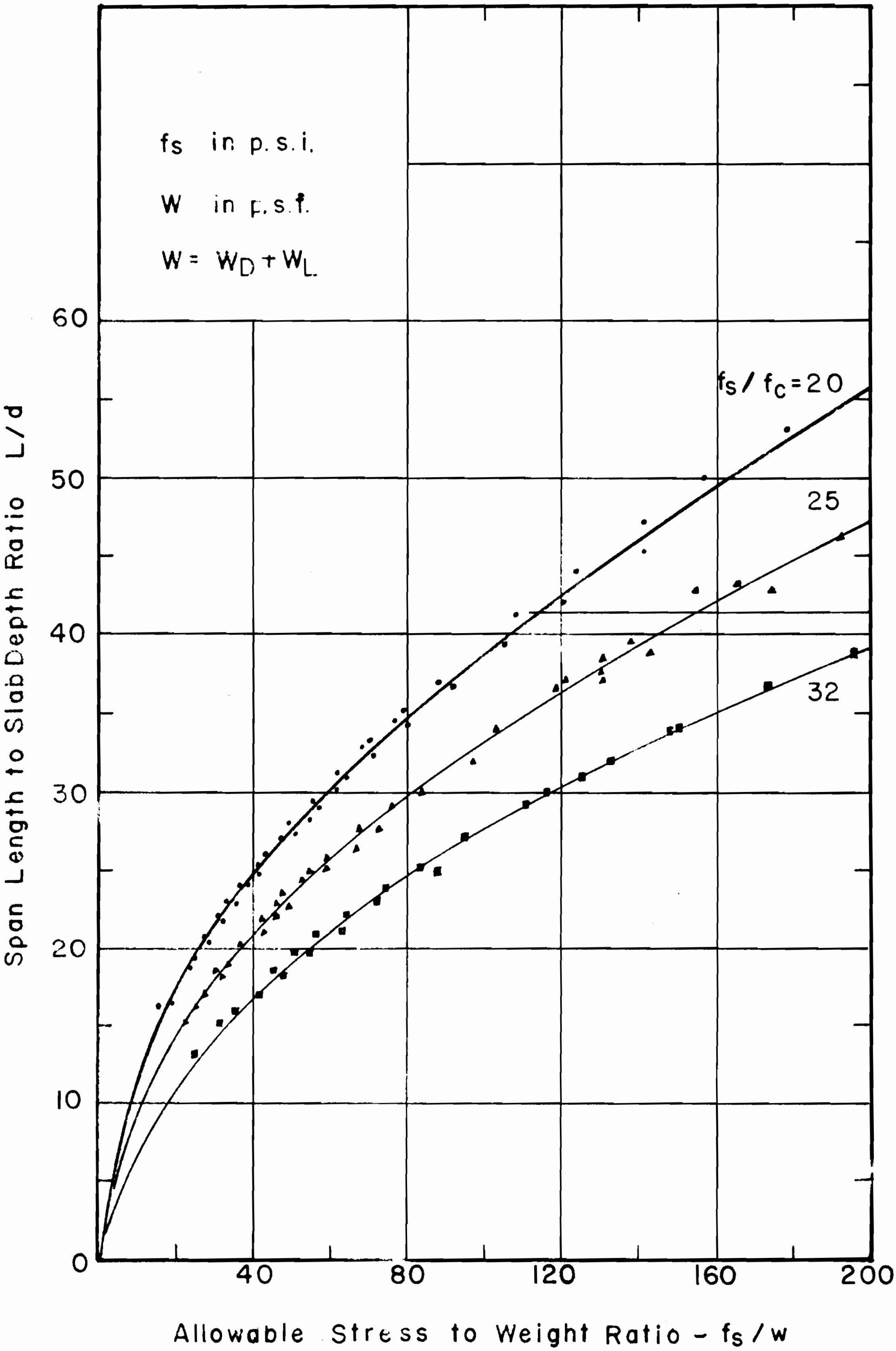

FIG. 3 L/d AGAINST fs/w FOR ONE-WAY SLAB

$n = 15$

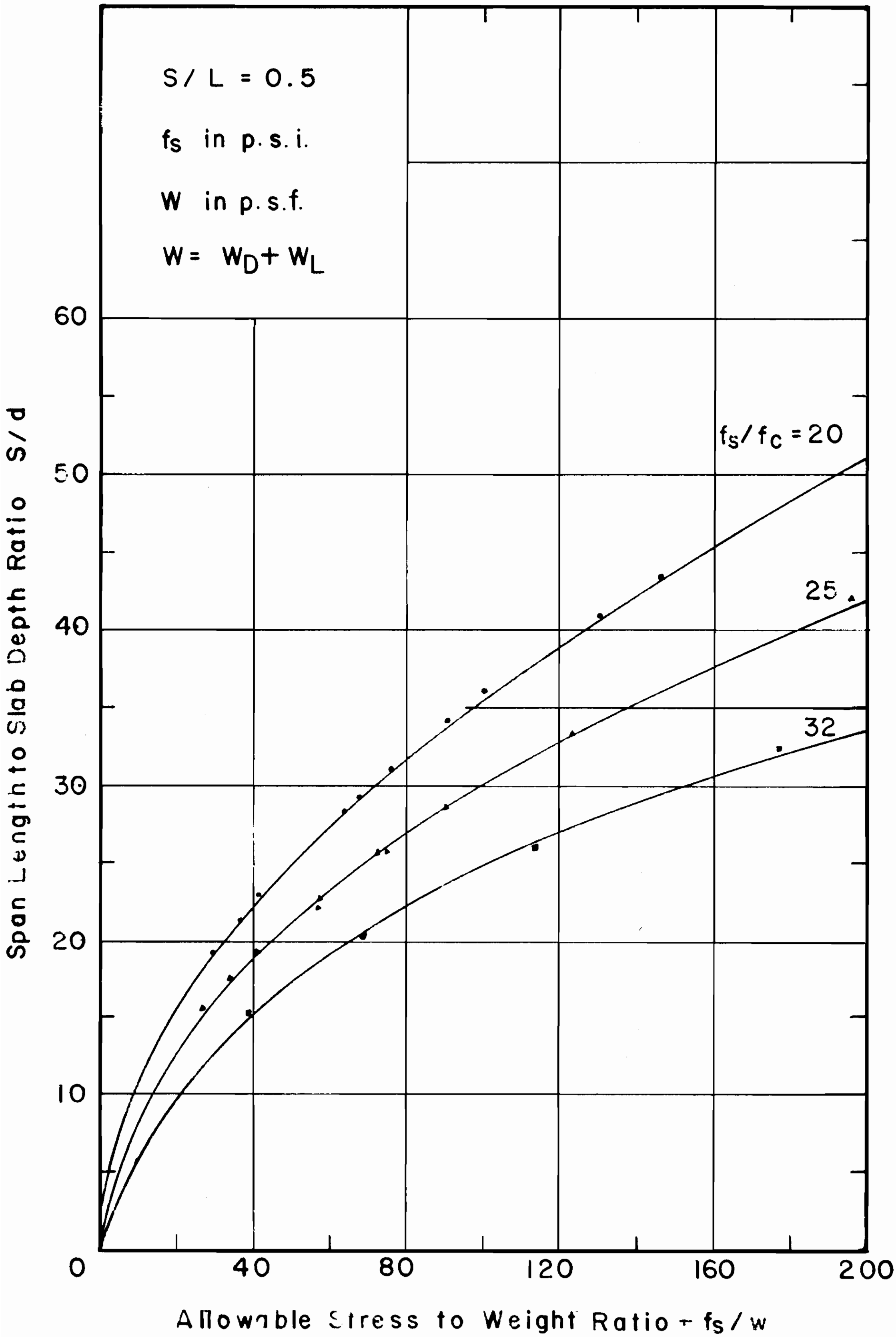

FIG. 4 S/d VS. f_s/w FOR TWO-WAY SLAB

$n = 10$

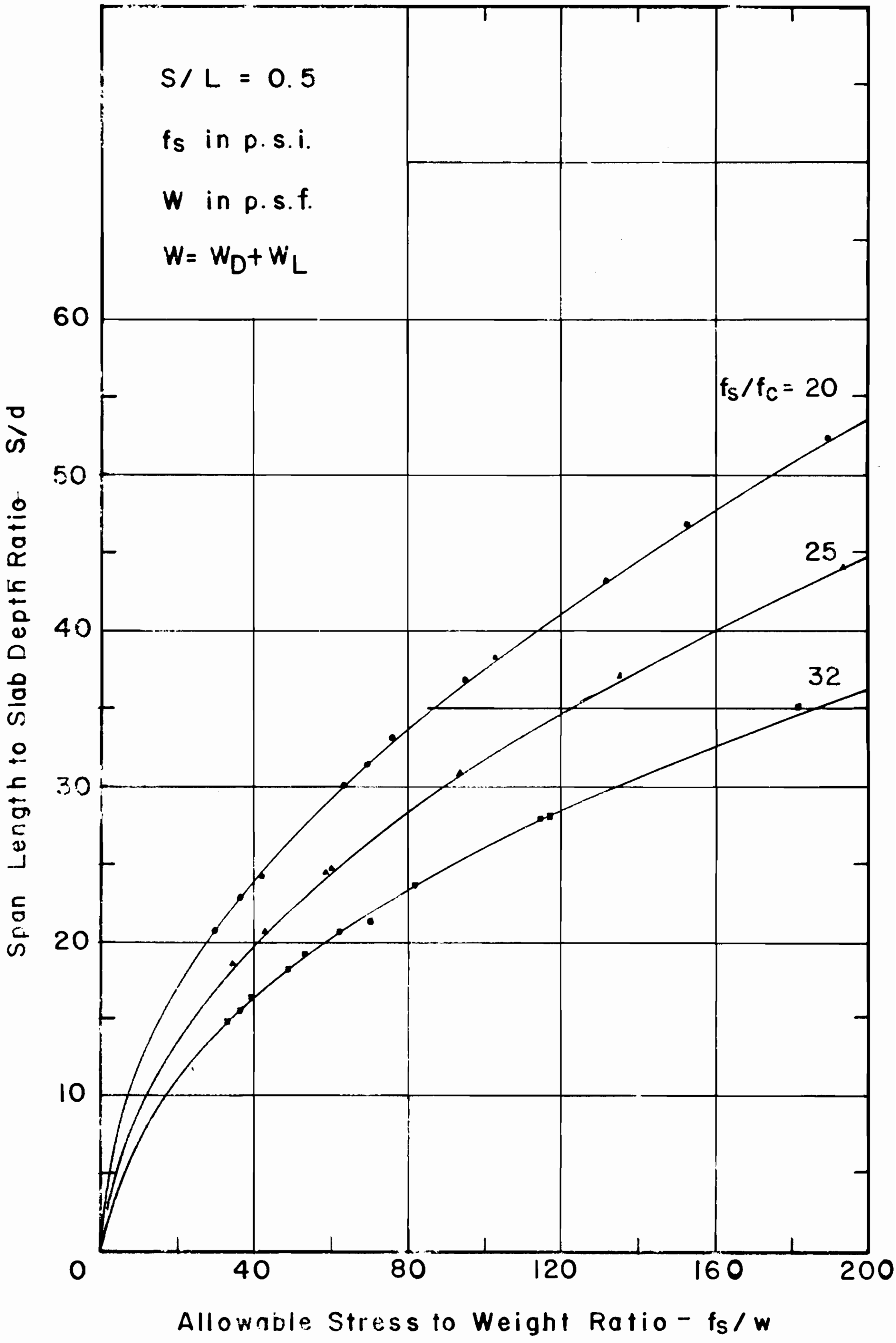

FIG. 5 S/d VS. f_s/w FOR TWO-WAY SLAB

$n = 12$

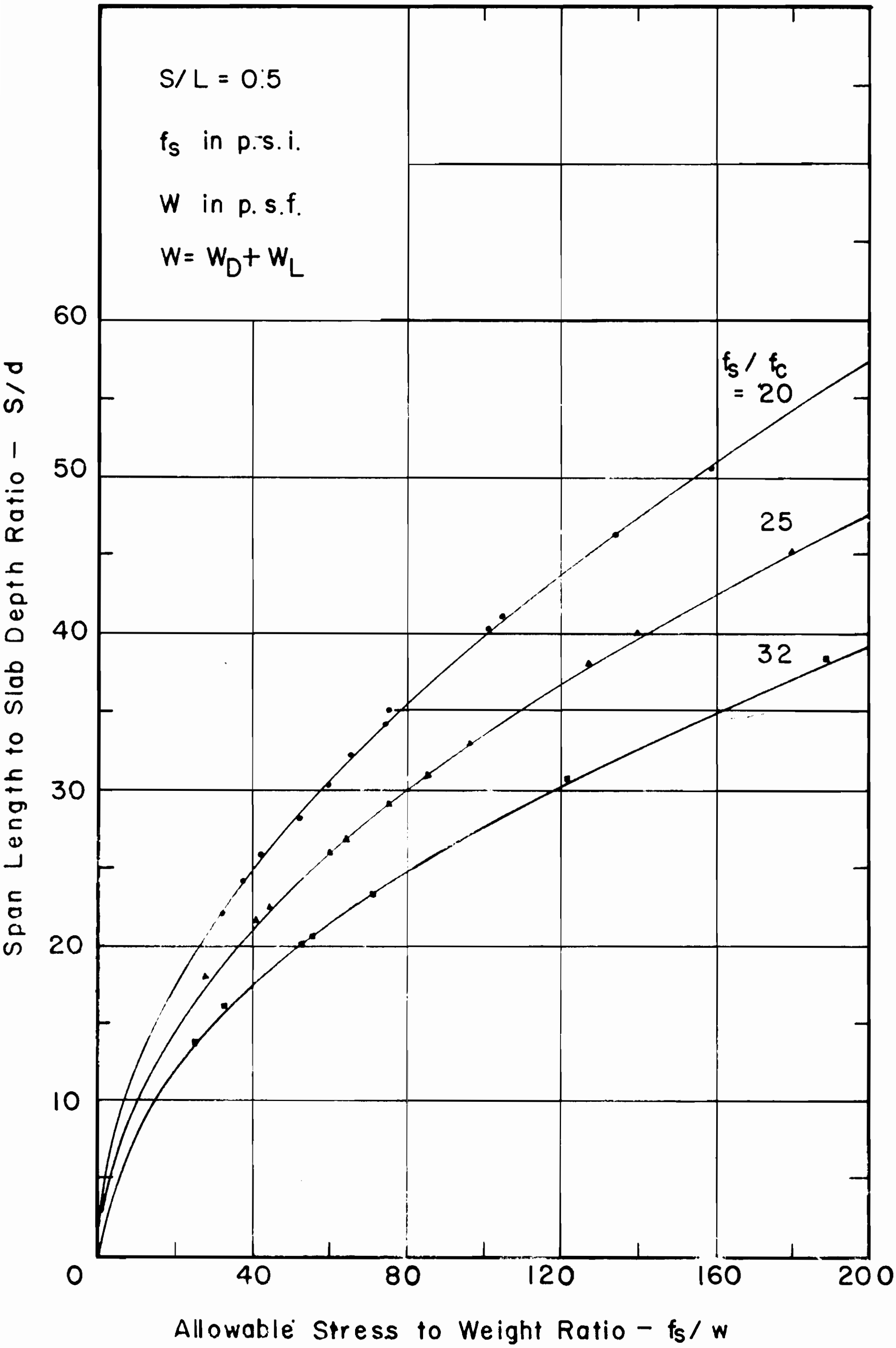

FIG. 6 S/d VS. f_s/w FOR TWO-WAY SLAB

$n = 15$

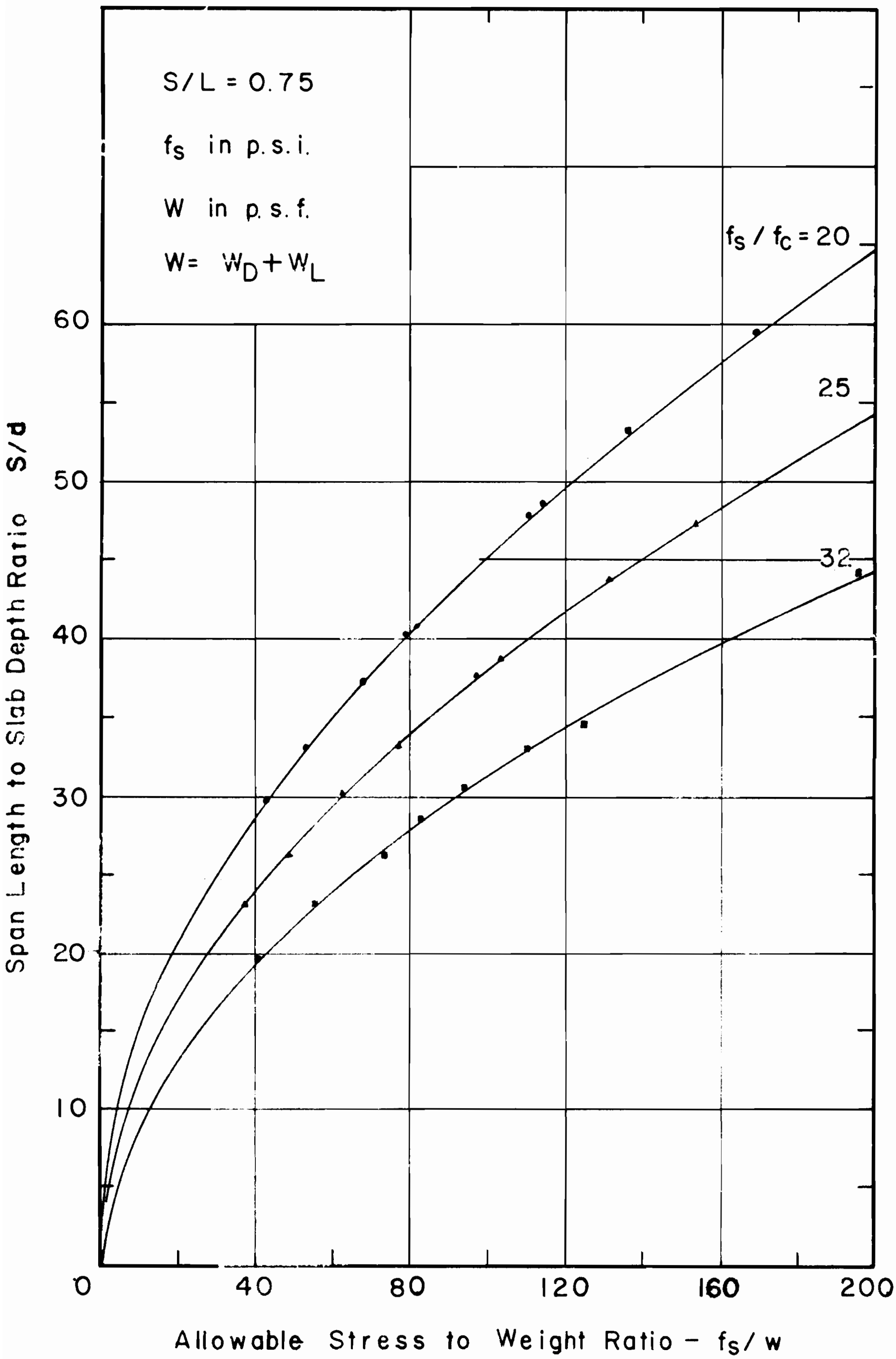

FIG.7 S/d VS. f_s/w FOR TWO-WAY SLAB

n = 10

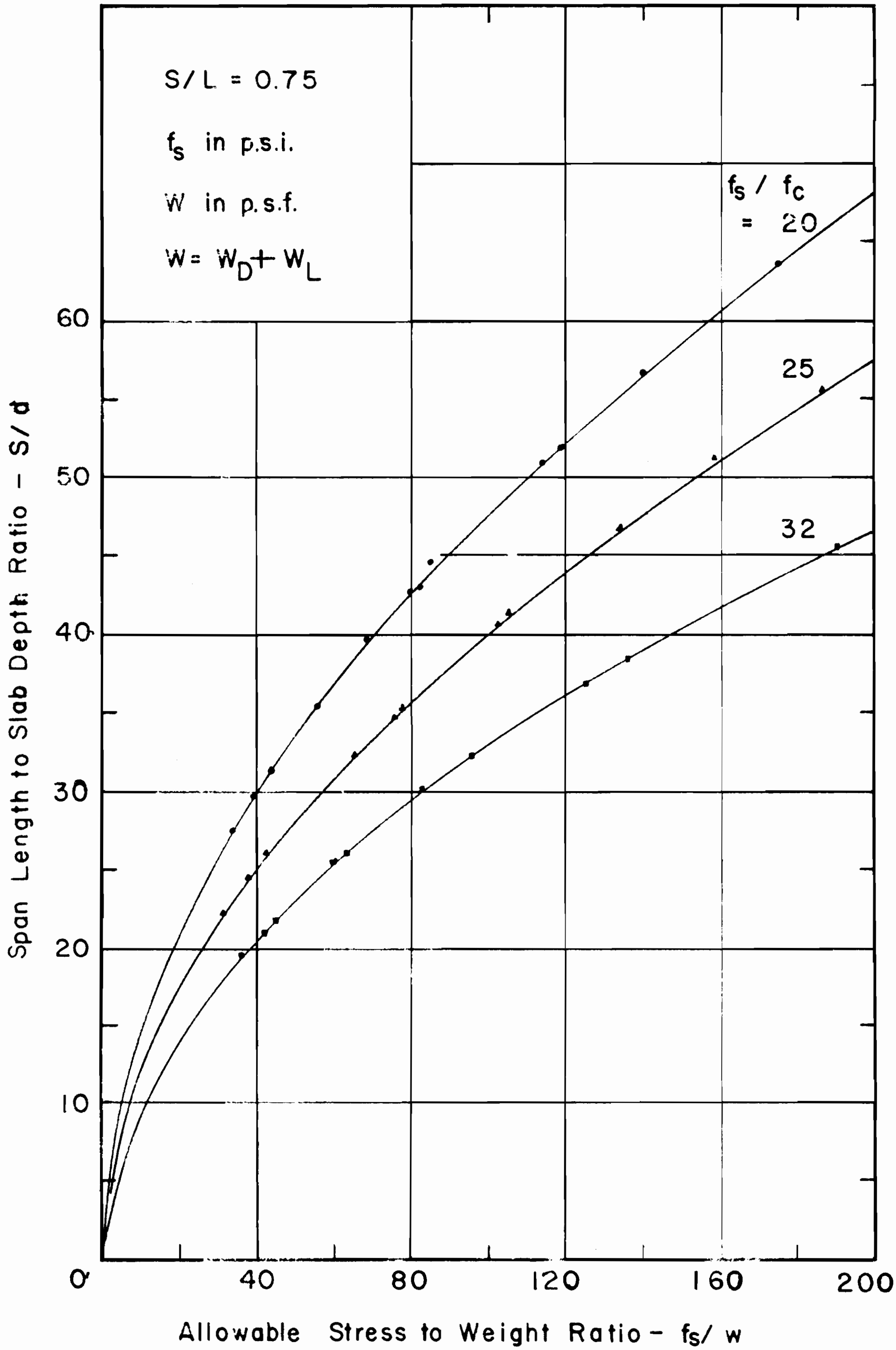

FIG.8 S/d VS. f_s/w FOR TWO-WAY SLAB

n = 12

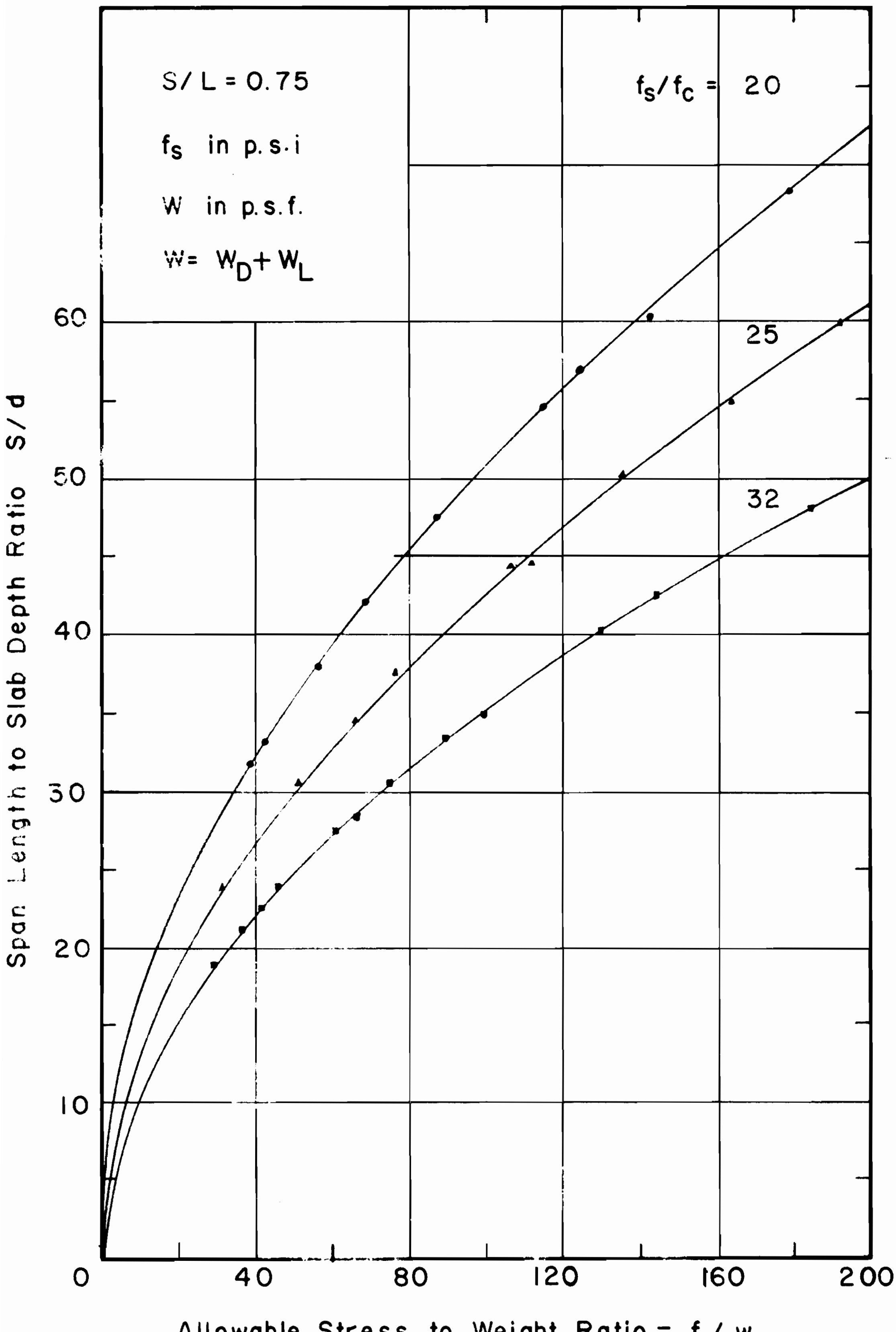

FIG. 9. S/d VS. f_s/w FOR TWO-WAY SLAB

n = 15

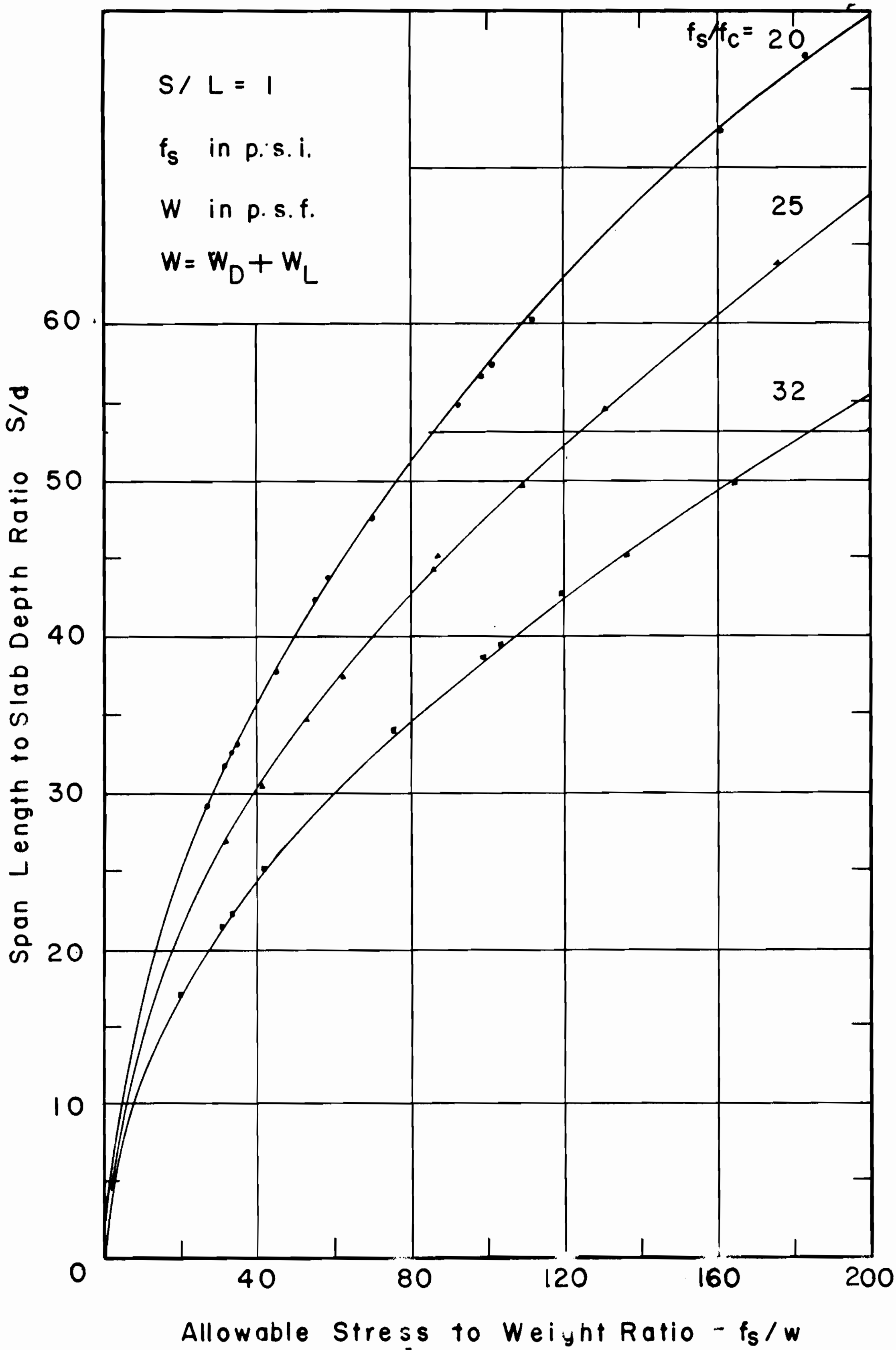

FIG. 10. S/d VS. f_s/w FOR TWO-WAY SLAB

n = 10

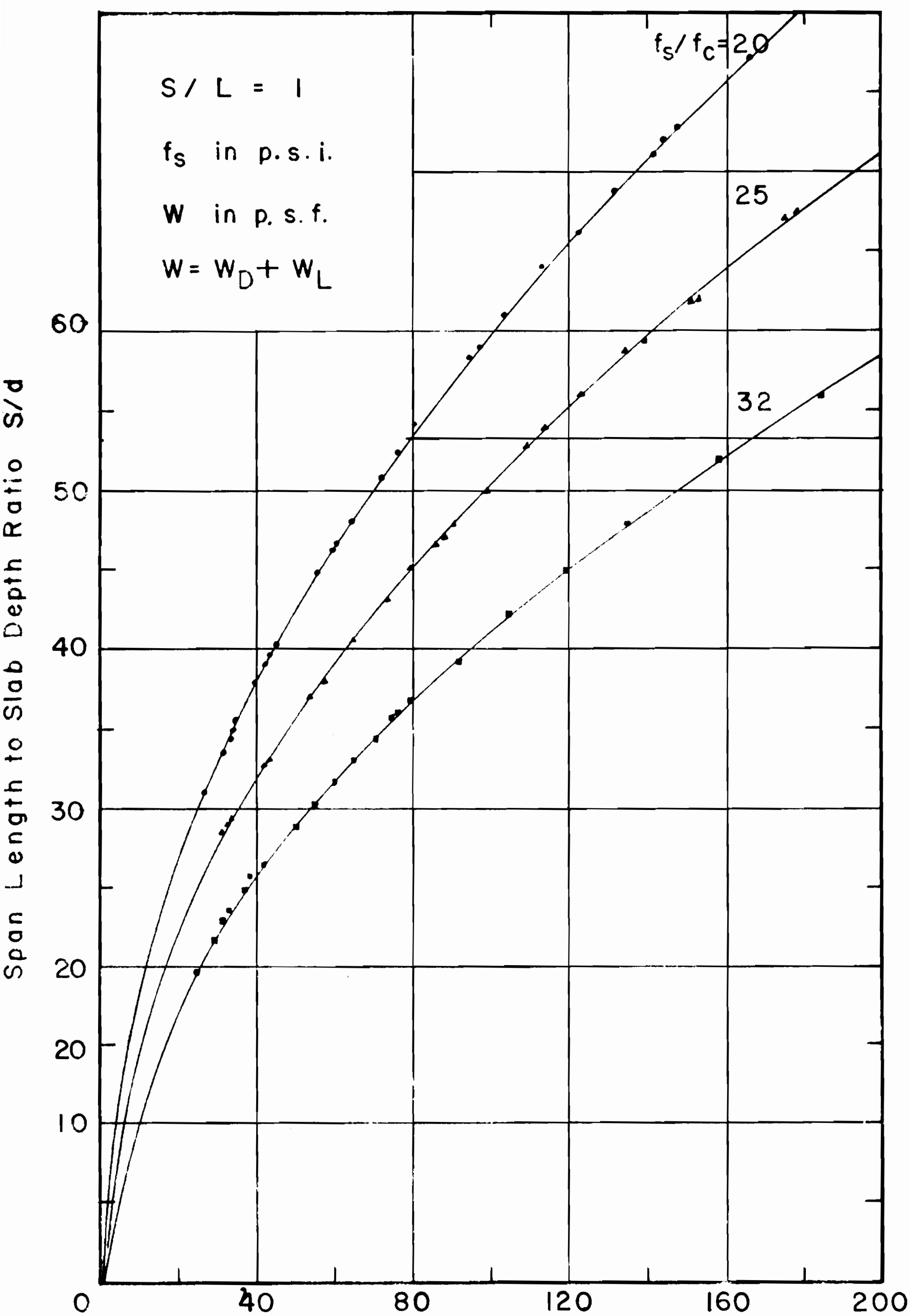

FIG. 11. S/d VS. fs/w FOR TWO-WAY SLAB

n = 12

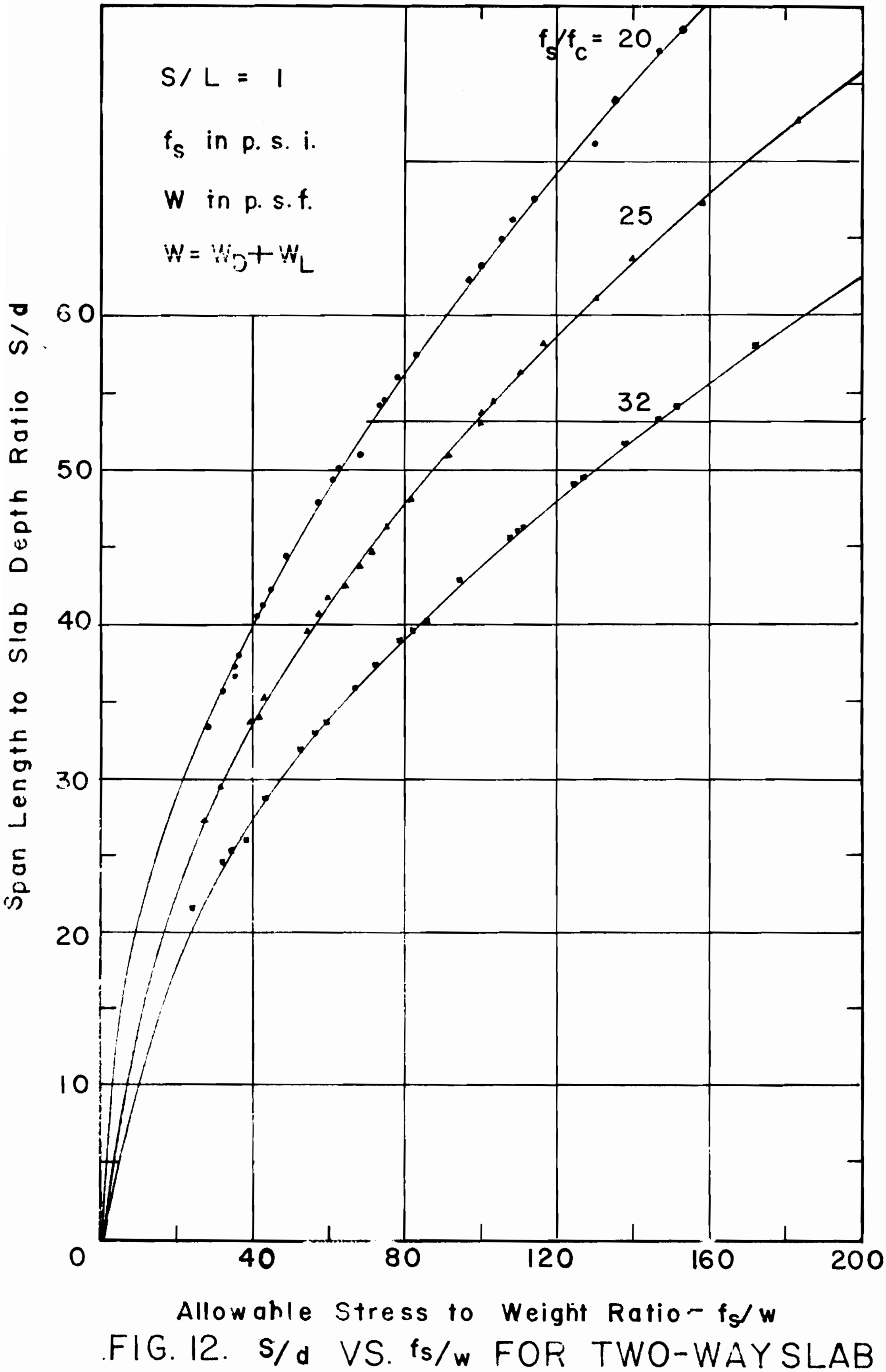

FIG. 12. S/d VS. fs/w FOR TWO-WAY SLAB

n = 15

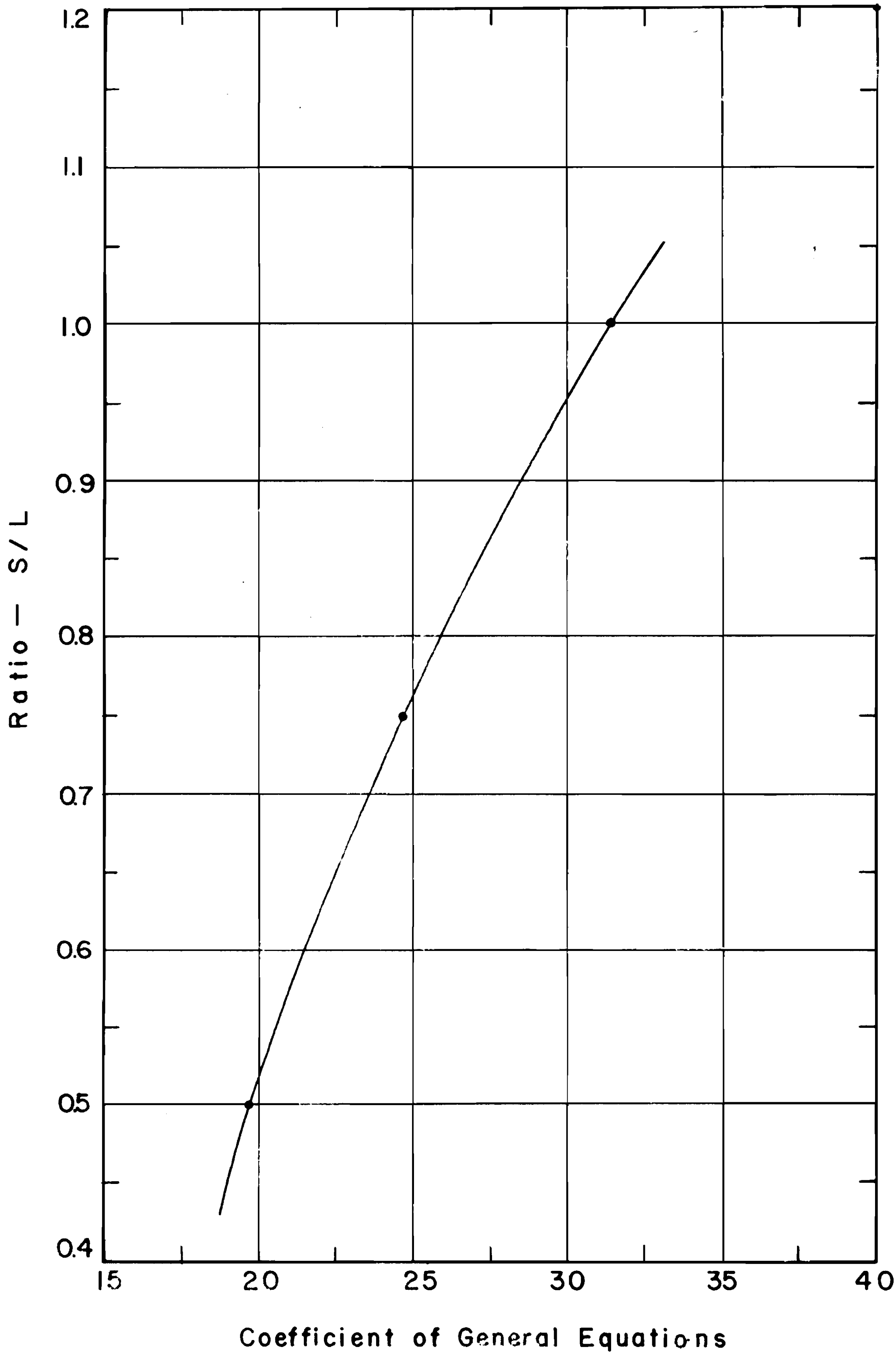

FIG. 13. S/L VS. Coefficient of General Equations

A Generalized Solution for the Design
of One-Way and Two-Way Reinforced Concrete Slabs

by

Abbas N. Al-Khafaji

ABSTRACT

In this investigation a new approach to slab design is presented. This approach is limited to interior panels of continuous one-way and two-way slabs and is limited to the determination of slab thickness when span, live load and material properties are specified.

Data is obtained by conventional elastic design techniques using the recommended methods and complying with the A.C.I. Code.

Based on approximately 1200 designs, general equations are derived and their validity is tested.

The use of the developed equations and the plotted graphs is illustrated.