

EMPIRICAL DIAMETER DISTRIBUTIONS AND
PREDICTED YIELDS OF THINNED LOBLOLLY PINE PLANTATIONS,

by

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TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS	ii
TABLE OF CONTENTS	iii
LIST OF TABLES	v
LIST OF FIGURES	vi
INTRODUCTION	1
LITERATURE REVIEW	3
Approaches to Growth and Yield Modeling	3
Whole stand models	3
Diameter distribution models	4
Relationship between whole stand models and diameter distribution models	7
Modeling Thinned Loblolly Pine Stands	8
MATERIALS AND METHODS	9
Segmented Cumulative Distribution Function	9
Evaluation of the Segmented CDF	12
Data Used for Model Construction	16
Model for Thinned Loblolly Pine Plantations	18
Stand-level model	23
Deriving diameter distributions from stand attributes	29
RESULTS AND DISCUSSION	32
Equations for Thinned Loblolly Pine Plantations	32
Program THINNED	32
Prediction of the present stand	32

	Page
Thinning	38
Projection	40
Diameter distribution of a previously low-thinned stand	41
Yield Tables	41
A Numerical Example	45
Effect of Thinning Regimes on Yield	52
Comparison with Published Information on Thinning	58
Coile and Schumacher's (1964) model	58
Yields reported by Goebel <u>et al.</u> (1974)	60
Possible Modifications and Refinements	62
LITERATURE CITED	64
APPENDICES	70
VITA	100
ABSTRACT	

LIST OF TABLES

<u>Table</u>	<u>Page</u>
1 The Kolmogorov-Smirnov statistics and p-levels computed from different data sets for the Weibull and segmented distributions	15
2 Description of plots immediately before and after first thinning and amount of first thinning	19
3 Description of plots immediately before and after second and third thinnings and amount of second and third thinnings	20
4 Distribution of all observations by site index (base age 25), age, basal area, and number of trees per acre	21
5 Equation forms that were considered in the development of a whole stand model for thinned loblolly pine plantations	24
6 Performance of the candidate models based on three criteria, computed separately from the fit and test data sets	33
7 Regression equations that form a whole stand model for thinned loblolly pine plantations	34
8 Total cubic-foot volume outside bark per acre for thinned loblolly pine plantations	43
9 Yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option	54
10 Comparison of predicted yields of Coile and Schumacher (1964) and those from program THINNED on a per acre basis for thinned loblolly pine plantations	59
11 Comparison of observed yields of Goebel <u>et al.</u> (1974) and predicted yields from program THINNED on a per acre basis for thinned loblolly pine plantations	61

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	A segmented cumulative distribution function and its corresponding probability density function	11
2	Plots of the Weibull and segmented distributions that approximate observed diameter distributions . . .	17
3	Generalized flowchart of program THINNED	37
4	Generalized flowchart of subprogram YIELD	39
5	Generalized flowchart of subprogram GROW	42
6	Example output from program THINNED	
6a	Step 1: Yield prediction of the stand before thinning	46
6b	Step 2: Row thinning at age 20	47
6c	Step 3: Low thinning at age 20	48
6d	Step 4: Project to age 40	49

INTRODUCTION

Growth and yield predictions are essential to forest management planning. Reliable growth and yield models assist managers in analyzing alternative management strategies. For loblolly pine (Pinus taeda L.), a myriad of yield information for unmanaged stands has accumulated over the years. On the other hand, yield models for thinned loblolly pine plantations still seem inadequate, and flexible models that supply information about diameter distributions are needed.

Different probability density functions (pdf's) have been used to characterize diameter distributions; most recently the beta, Weibull, and Johnson's S_B distributions have been employed to develop yield estimates. The so-called probability density function approach to yield modeling involves prediction of the pdf parameters from stand variables (age, site, and density) using regression techniques, and then calculation of the number of trees and yield per acre in each dbh class. The drawbacks of this approach are (a) that these pdf's can assume only a limited variety of shapes, and (b) that the regression models for predicting the pdf parameters usually account for only a small percentage of the variation (i.e. low R^2 values). Recently, research has been conducted to develop methods for approximating the parameters in a theoretical diameter distribution (e.g. the beta or Weibull) from overall stand values such as total basal area and mean diameter.

The objectives of this study were: (1) to investigate a more flexible probability density function approach for diameter distribution modeling, (2) to develop a whole stand model for thinned loblolly pine plantations using regression techniques, and (3) to derive diameter distributions from predicted stand attributes.

LITERATURE REVIEW

The following review of literature is limited to past work that is germane to the methodology used in this study. Emphasis is given to southern pines and to thinned stands since this study dealt with thinned loblolly pine plantations.

Approaches to Growth and Yield Modeling

Whole Stand Models

Graphic techniques were used to construct yield tables in the early years of yield prediction. These tables presented volumes of fully-stocked stands of "normal" density and were thus called normal yield tables. A well-known normal yield table for southern pines, "Miscellaneous Publication 50" (Anon. 1929), is still being used to some extent in the South.

Multiple regression techniques began to replace graphic methods in constructing variable-density yield tables when MacKinney et al. (1937) used a form of the Pearl-Reed growth function for nonnormal loblolly pine stands. Modifying an equation form suggested by Schumacher (1939), MacKinney and Chaiken (1939) predicted the logarithm of yield as a function of stand variables (age, site, density, and composition). This approach, with certain modifications, has been employed in recent models for loblolly pine (such as Goebel and Warner 1969, Burkhart et al. 1972a, 1972b).

Yield models for natural stands of four major southern pine

species were published by Schumacher and Coile (1960). Coile and Schumacher (1964) presented a prediction system for old-field and non-old-field slash and loblolly pine plantations. Both thinned and unthinned stands can be modeled by this system.

Growth and yield are not two separate attributes but closely related to one another. Buckman (1962) used a yield model for red pine where yield is obtained by mathematically integrating the growth equation over time. The concept of compatibility between growth and yield prediction was discussed in detail by Clutter (1963), who introduced a compatible growth and yield model for loblolly pine natural stands. Sullivan and Clutter (1972) later refined Clutter's equations and developed a model that produced not only analytically but also numerically compatible growth and yield predictions. A similar approach has been used by several other researchers including Brender and Clutter (1970), Bennett (1970), Beck and Della-Bianca (1972), Murphy and Sternitzke (1979).

Diameter Distribution Models

Diameter distributions in unevenaged stands were thought of as following the pattern of a geometric series by de Liocourt in 1898 (Meyer and Stevenson 1943). This general model, having a reversed J shape, was employed by Meyer and Stevenson (1943), Meyer (1952), Schmelz and Lindsey (1965) and was later formalized as the exponential distribution by Leak (1965).

Other distributions have been used to characterize diameter

distributions in evenaged stands, among them, the Gram-Charlier series (Meyer 1928, 1930; Schumacher 1928, 1930; Schnur 1934), the modified Pearl-Reed growth curve (Osborne and Schumacher 1935, Nelson 1964), Pearsonnian curves (Schnur 1934), and the log-normal distribution (Bliss and Reinker 1964).

Clutter and Bennett (1965) applied the beta pdf to approximate diameter distributions in slash pine stands. Bennett and Clutter (1968) later developed a yield model to estimate multiple-product yields for slash pine plantations by using the stand table generated via the Clutter and Bennett (1965) diameter distribution model. In this model, the parameters of the beta pdf that approximated the diameter distribution were predicted from stand variables (age, site, and density). The number of trees and volume per acre in each diameter class were calculated and per acre yield estimates were obtained by summing over diameter classes of interest. A similar approach was applied to loblolly pine plantations by Lenhart and Clutter (1971), Lenhart (1972), and Burkhart and Strub (1974).

The main drawback of using the beta distribution is that its cumulative distribution function (cdf) does not exist in closed form. As a result, the proportion of trees in each diameter class has to be solved by using numerical integration techniques. Bailey and Dell (1973) discovered that the Weibull distribution fit diameter data reasonably well and was easy to use. The cumulative distribution function of the Weibull distribution exists in closed form and this is a major advantage over the beta function. Schreuder and

Swank (1974) used the Weibull distribution to fit diameter, basal area, surface area, biomass, and crown profile data of coniferous species. The Weibull function was applied in plantation models for loblolly pine (Smalley and Bailey 1974a, Feduccia et al. 1979), slash pine (Clutter and Belcher 1978, Dell et al. 1979), shortleaf pine (Smalley and Bailey 1974b), and longleaf pine (Lohrey and Bailey 1976).

Hafley and Schreuder (1977) evaluated six pdf's (normal, log-normal, gamma, Weibull, beta, S_B) and concluded that the S_B distribution (Johnson 1949a) was consistently better than the others in fitting diameter and height data. A bivariate distribution, S_{BB} (Johnson 1949b), was suggested by Schreuder and Hafley (1977) to quantify simultaneously diameter and height distributions. The S_B is a transformation of the standard normal; thus in spite of the lack of a closed form expression for its cdf, the proportion of trees in a specified diameter class can be easily obtained from a standard normal table.

Using the same pdf to model the diameter distributions of a stand at two different times implies that the diameter growth follows certain rules. Bailey (1980) showed that for the normal, exponential, beta, and S_B distributions, the relationship is linear:

$$X_2 = b_0 + b_1 (X_1 - b_2)$$

where X_i is tree diameter at breast height (dbh) at time i and the b_j 's are coefficients. On the other hand, the following relationship

is implied if the generalized gamma, Weibull, and log-normal distribution are used:

$$X_2 = b_0 + b_1 (X_1 - b_2)^{b_3}$$

Relationship between Whole Stand Models and Diameter Distribution Models

Strub and Burkhart (1975) presented a class-interval-free method for predicting whole stand yield per unit area from diameter distribution models:

$$TV = N \int_L^U g(D) f(D) dD$$

where TV = expected stand volume per unit area,

N = number of trees per unit area,

D = diameter at breast height,

g(D) = individual tree volume equation,

f(D) = pdf for D, and

(L,U) = merchantability limits for the product described by g(D).

Using this relationship, Hyink (1980) introduced a method of solving for the parameters of the pdf approximating the diameter distribution, using attributes predicted from a whole stand model. The same concept was employed by Matney and Sullivan (1981) in their model for thinned loblolly pine plantations. In the first phase of Matney and Sullivan's study, stand volume and basal area were predicted using compatible growth and yield equations. The second phase involved solving for two parameters of the Weibull pdf which

characterized the diameter distribution such that the resulting stand volume and basal area per acre would be identical to those predicted in the first phase.

Modeling Thinned Loblolly Pine Stands

Coile and Schumacher (1964) included amount of thinning as input in their model. Different types of thinning (thinning by rows, from below, or by a combination of both) can be specified in Daniel and Burkhardt's (1975) and Daniels et al.'s (1979) individual tree models. Other models based on data from thinned loblolly pine stands include Clutter (1963), Brender and Clutter (1970), Sullivan and Clutter (1972).

The Weibull function was used by Bailey et al. (1981) to describe diameter distribution of slash pine plantations before and after thinning. Matney and Sullivan (1981) also used the Weibull distribution to produce compatible stand and stock tables for thinned loblolly pine plantations. In addition to the models mentioned above, growth and yield of thinned loblolly pine stands were reported by many researchers (such as Bassett 1966, Bruner and Goebel 1968, Andrulot et al. 1972, Shepard 1974, Goebel et al. 1974, Mann and Feduccia 1977, Sullivan and Williston 1977, Burton 1980).

MATERIALS AND METHODS

Segmented Cumulative Distribution Function

A simple cumulative distribution function may not be flexible enough to model certain data. An alternative approach is to use different forms of the cdf's to characterize different portions of the range of the random variable and then join them together.

Let X be a continuous random variable defined over the interval $[x_{\min}, x_{\max}]$. Consider n points

$$x_1 < x_2 < \dots < x_n,$$

where $x_1 = x_{\min}$ and $x_n = x_{\max}$. The x_j 's will be referred to as either percentile points or join points.

Let $p_j = \Pr(X \leq x_j)$, $j = 1, 2, \dots, n$; thus $p_1 = 0$ and $p_n = 1$.

A segmented cdf $F(x)$ is defined here as

$$F(x) = \begin{cases} 0, & x < x_1, \\ F_1(x), & x_1 \leq x < x_2, \\ F_2(x), & x_2 \leq x < x_3, \\ \vdots & \\ F_j(x), & x_j \leq x < x_{j+1}, \\ \vdots & \\ F_{n-1}(x), & x_{n-1} \leq x \leq x_n, \\ 1, & x_n \leq x. \end{cases}$$

where the segment $F_j(x)$ has to satisfy the following conditions:

- (1) $F_j(x)$ must be monotonic nondecreasing.
- (2) $F_j(x)$ must be continuous.
- (3) $F(x)$ must be continuous at the join points, i.e.

$$F_j(x_{j+1}) = F_{j+1}(x_{j+1}) = p_{j+1}, \quad j = 1, 2, \dots, n-2.$$

- (4) $f_j(x)$, which is the derivative of $F_j(x)$ with respect to x , must be continuous at the join points, i.e.

$$f_j(x_{j+1}) = f_{j+1}(x_{j+1}), \quad j = 1, 2, \dots, n-2.$$

Conditions (1, 2, and 3) ensure that $F(x)$ is a continuous cdf. Condition (4) requires that $F(x)$ be smooth and the corresponding pdf, $f(x)$, continuous at the join points. A segmented cdf and its corresponding probability density function (pdf) are shown in Figure 1.

In this study, a modified form of the Weibull cdf is used for each segment as follows:

$$F_j(x) = e_j \left\{ 1 - d_j \exp \left[- \left(\frac{x-a_j}{b_j} \right)^{c_j} \right] \right\} \quad \text{for } x_j \leq x \leq x_{j+1}, \\ j = 1, 2, \dots, n-1.$$

A method for computing a_j , b_j , c_j , d_j , and e_j is described in Appendix 1. If the p_j 's are specified, the x_j 's define a segmented cdf. The parameters of this cdf are actually the x_j 's since a_j , b_j , c_j , d_j , and e_j can be computed from the x_j 's and p_j 's. Thus a

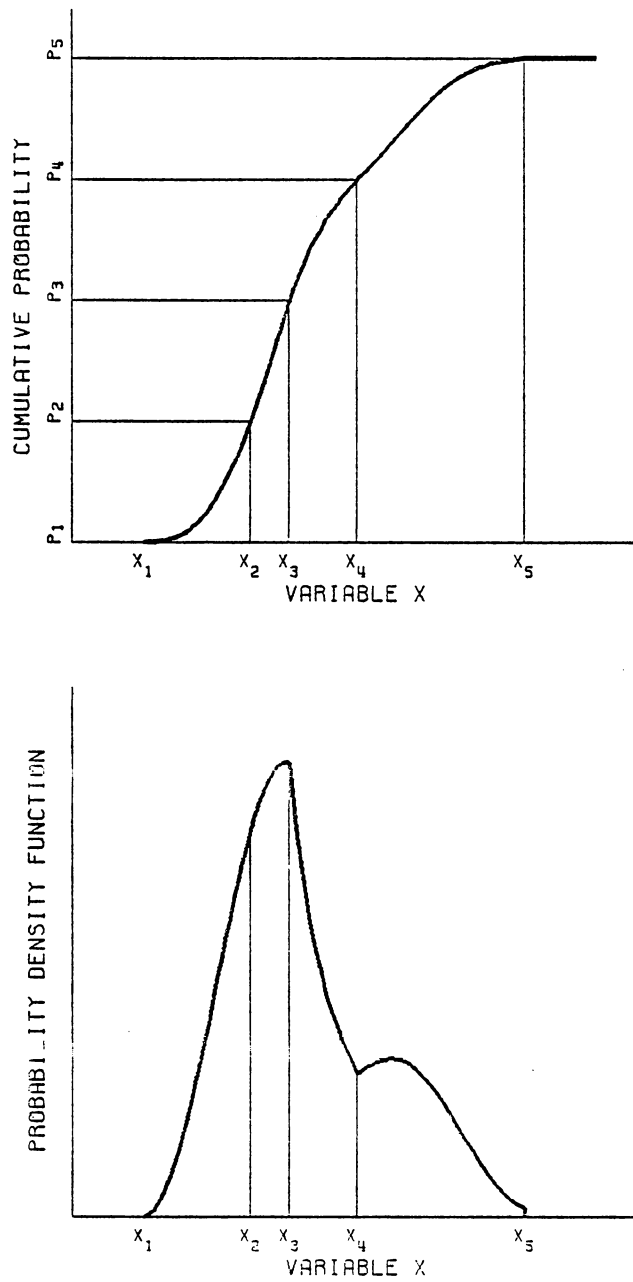


Figure 1. A segmented cumulative distribution function and its corresponding probability density function.

segmented cdf defined by n percentile points has n parameters.

If $a_j = a_{j'}$, $b_j = b_{j'}$, $c_j = c_{j'}$, $d_j = d_{j'}$, $e_j = e_{j'}$, $= 1$
 $(j, j' = 1, 2, \dots, n-1)$, then the segmented cdf is a Weibull cdf.

Evaluation of the Segmented CDF

The segmented cdf was evaluated against the 3-parameter Weibull distribution using data from old-field loblolly pine plantations that underwent different cultural treatments. The first data set was the region-wide series from the North Carolina State Forest Fertilization Cooperative (NCSFFC) study. Permanent plot data were obtained from eleven installations located in the Piedmont and Coastal Plain regions of Maryland, Virginia, Georgia, North and South Carolina. Plot size ranges from 1/30 to 1/5 acre. Two treatments were applied: control and thinning. There were between 3 and 5 replications for each treatment at each installation. Two remeasurements were taken 2 and 4 years after plot establishment. There were a total of 126 plot measurements for each treatment.

Measurements from a mutual competition study at the North Louisiana Hill Farm Experiment Station form the second data set. The natural pines and hardwoods on an abandoned crop field were cut and the stumps burned. Bare-rooted seedlings were planted in 1958 at 6 by 6 foot spacing. Stepwise thinning was initiated in 1962 and 4 replications of 5 levels of treatment (1000, 600, 300, 200, and 100 remaining trees per acre) were reached by 1965. Twenty 0.25-acre plots were measured repeatedly since 1962; up to 8 remeasurements

were taken for each plot, totaling 170 plot measurements.

Diameter at breast height (dbh) were measured to the nearest 0.01 inch in both data sets. The diameter distribution from plot measurement was fitted with a 3-parameter Weibull function and then a segmented cdf. The method of maximum likelihood was employed to estimate the Weibull parameters $\frac{1}{\lambda}$, and the segmented cdf was determined from five observed percentile points (the 0th, 25th, 50th, 75th, and 100th percentiles).

The range of the diameter variable was defined by the 0th and 100th percentiles; whereas the 25th, 50th, and 75th percentiles dictated the shape of the segmented cdf. The choice of the number of percentiles was entirely arbitrary. Preliminary investigations (e.g. plotting of diameter distribution histograms) indicated that five points should be satisfactory for describing the range of shapes commonly found in pine plantation data. The 0th, 50th, and 100th percentile points have meaningful interpretations as the minimum, median, and maximum stand diameters. Points midway between the 0th and 50th, and between the 50th and 100th percentiles (i.e. the 25th and 75th percentiles) were chosen for use in this investigation. Although many other choices of percentile points could be made, equally-spaced points (every 25%) were selected as a logical set of values.

^{1/} The Weibull fitting program developed by Zutter and Oderwald (1981) was used for this purpose.

One-sample Kolmogorov-Smirnov (K-S) statistics were then computed for both the Weibull and segmented cdf as a means of comparison. For each plot measurement, a rank of 1 was given to that distribution with a smaller value of the K-S statistic whereas the other distribution received a rank of 0. The sum of these ranks for one distribution denotes simply the number of plot measurements where this distribution was better than the other. Table 1 presents the K-S statistics, p-levels, and the sum of the ranks for the Weibull and segmented distributions by type of treatment.

Past work on the Weibull function showed that, for unthinned stands, this pdf was suitable for quantifying diameter distributions which are often unimodal. The Weibull distribution performed slightly better (by a margin of 65 to 61) than the segmented cdf on the NCSFFC unthinned data, based on the one-sample K-S statistic.

In the Hill Farm mutual competition study, trees were removed with the emphasis on maintaining a relatively uniform spatial pattern. As a result, the diameter distributions of these plots were similar to those of unthinned stands, i.e., unimodal for the most part. The Weibull function also out-performed the segmented cdf by a margin of 91 to 79.

The segmented cdf was clearly superior to the Weibull cdf for the NCSFFC thinned data (81 to 45). This is due largely to the fact that the greater flexibility of the segmented cdf allows it to handle the irregularities of the diameter distributions in thinned stands.

Table 1. The Kolmogorov-Smirnov statistics and p-levels computed from different data sets for the Weibull and segmented distributions.

Data	K-S statistic			Sum of the ranks $\frac{1}{2}$	p-level			Number of rejections at .05-level
	Minimum	Mean	Maximum		Minimum	Mean	Maximum	
<u>NCSFFC $\frac{2}{2}$ Unthinned</u>								
Segmented CDF	.0547	.1042	.1865	61	.0329	.7734	.9999	2
Weibull	.0571	.1042	.2057	65	.2918	.7766	.9995	0
				126				
<u>NCSFFC Thinned</u>								
Segmented CDF	.0684	.1273	.2341	81	.3064	.8236	.9997	0
Weibull	.0622	.1428	.2944	45	.0817	.7429	.9997	0
				126				
<u>Hill Farm Mutual Competition Study</u>								
Segmented CDF	.0206	.0848	.1988	79	.0153	.6574	.9999	5
Weibull	.0244	.0853	.2691	91	.0545	.6824	.9985	0
				170				

1/ For each plot measurement, a rank of 1 was given to the distribution with a smaller value of the K-S statistic, and the other distribution received a rank of 0.

2/ North Carolina State Forest Fertilization Cooperative.

Plots of the Weibull and segmented pdf's that approximate different types of observed diameter distributions are shown in Figure 2.

Based on this evaluation, the segmented cdf was selected to approximate diameter distributions in the model for thinned loblolly pine plantations developed in this study.

Data Used for Model Construction

The growth and yield model for thinned loblolly pine plantations developed in this study was based on data from the Virginia Division of Forestry (VDF). This data set consists of 128 0.2-acre permanent plots from old-field plantations in the Virginia Piedmont and Coastal Plain. Number of remeasurements varied from plot to plot, ranging from 1 to 7. There were a total of 490 plot measurements.

Diameter at breast height (dbh) was recorded to the nearest inch and total height was measured to the nearest foot. Trees in the 1- and 2-inch classes were not tallied separately but combined to form one class whose midpoint was arbitrarily set at 1.5 inches. In each plot, measurements of dbh of all trees were taken but only some tree heights were measured. Height corresponding to each dbh class was predicted for each plot measurement using a regression equation of the form

$$\log_e(H) = b_0 + b_1/D,$$

where H = total tree height in feet,

D = diameter at breast height in inches,

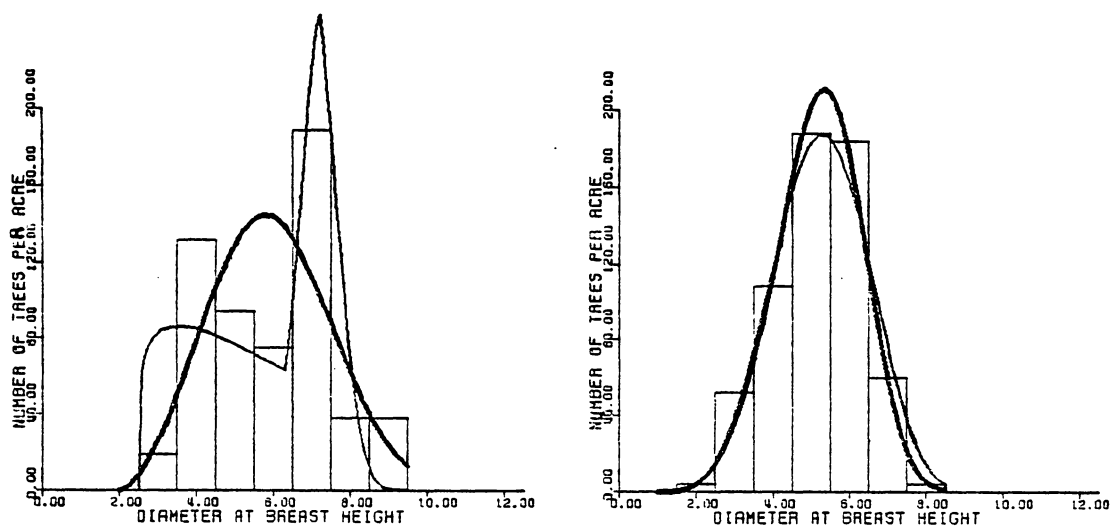
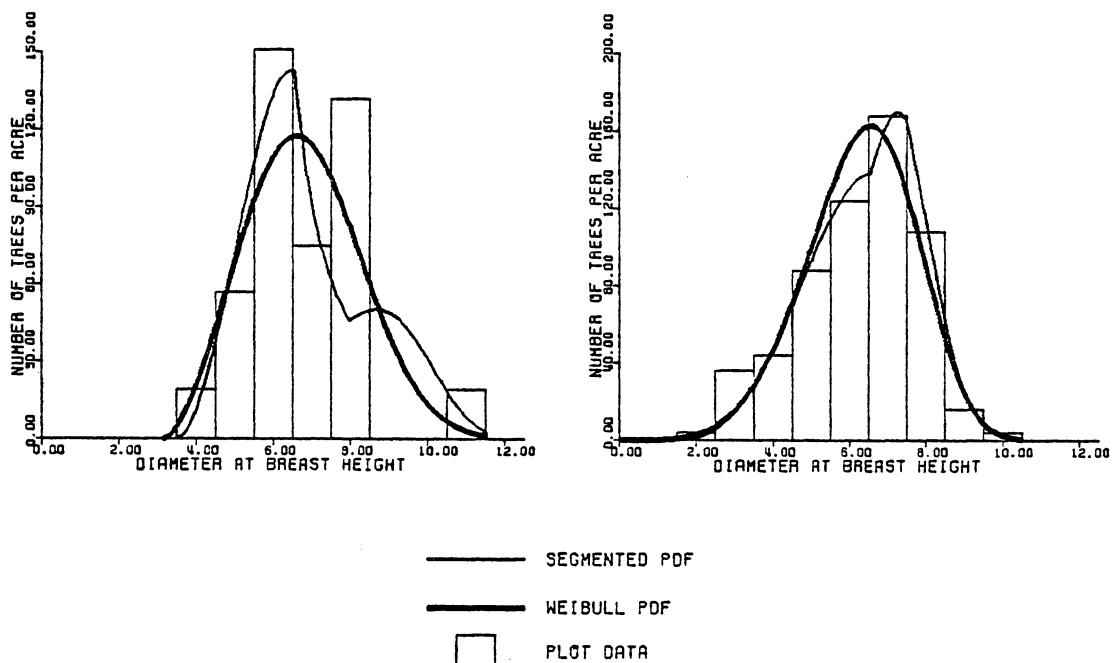


Figure 2. Plots of the Weibull and segmented distributions that approximate observed diameter distributions.

b_0, b_1 = regression equations.

Site index was determined from the average height of the dominants and codominants in the plot, using a site index equation developed by Devan (1979). Total cubic-foot volume outside bark per acre was computed using Burkhart et al.'s (1972b) individual tree volume equation.

The stands were thinned up to 3 times and, for the most part, thinnings were from below. Table 2 presents a description of plots in this data set immediately before and after first thinning and also amount of first thinning. A similar table was constructed for subsequent thinnings (Table 3). The distribution of all observations by site index, age, basal area, and number of trees per acre is presented in Table 4.

Ninety-one plot measurements were selected at random and referred to as the test data set. This subset, considered representative of the original data, was withheld for testing purposes. The remaining 399 plot measurements, which form the fit data set, were used for initial fitting of the equations.

Model for Thinned Loblolly Pine Plantations

The model for thinned loblolly pine plantations developed in this study consisted of two stages. In the first stage, stand-level attributes were predicted using regression techniques. The second stage involved determining the parameters of the segmented cdf so

Table 2. Description of plots immediately before and after first thinning and amount of first thinning. 1/

Variable	Before first thinning	Amount of first thinning	After first thinning
<u>Number of trees/acre</u>			
Minimum	355	165	160
Mean	774	459	339
Maximum	1305	770	1040
<u>Basal area (sq.ft./acre)</u>			
Minimum	107	29	50
Mean	174	87	90
Maximum	227	148	145
<u>Total outside-bark volume (cu.ft./acre)</u>			
Minimum	1700	475	1080
Mean	3839	1910	1975
Maximum	6235	3705	3885
<u>Average DBH (inches)</u>			
Minimum	4.5		4.0
Mean	6.4		7.1
Maximum	9.5		10.1
<u>Age (years)</u>			
Minimum	12		12
Mean	21		21
Maximum	30		30

1/ Discrepancies in the plot description (e.g. the means of a stand attribute after thinning and amount of thinning do not sum to the mean of that attribute before thinning as expected) are due to missing observations either before or after thinning.

Table 3. Description of plots immediately before and after second and third thinnings and amount of second and third thinnings. 1/

Variable	Before thinning	Amount of thinning	After thinning
<u>Number of trees/acre</u>			
Minimum	120	25	115
Mean	322	126	205
Maximum	925	435	410
<u>Basal area (sq.ft./acre)</u>			
Minimum	87	12	58
Mean	131	38	92
Maximum	185	77	137
<u>Total outside-bark volume (cu.ft./acre)</u>			
Minimum	2305	295	1335
Mean	3538	944	2466
Maximum	5935	1625	4330
<u>Average DBH (inches)</u>			
Minimum	6.0		6.3
Mean	8.9		9.2
Maximum	12.8		12.3
<u>Age (years)</u>			
Minimum	18		18
Mean	28		28
Maximum	39		39

1/ Discrepancies in the plot description (e.g. the means of a stand attribute after thinning and amount of thinning do not sum to the mean of that attribute before thinning as expected) are due to missing observations either before or after thinning.

Table 4. Distribution of all observations by site index (base age 25), age, basal area, and number of trees per acre.

Site Index (feet)	Age (years)	Basal Area (sq.ft. /acre)	Number of trees per acre						Total
			< 300	301-500	501-700	701-900	901-1100	> 1100	
50	20	50	3	2					5
		100	1	13					14
		150		2	1	6			9
		200				1	2		3
			—	—	—	—	—		—
			4	17	1	7	2		31
	30	50	5	2					7
		100	33	11					44
		150		11	2	2			15
		200			2	1			3
			—	—	—	—			—
			38	24	4	3			69
	40	50	1						1
		100	22						22
		150	5						5
	50		—						—
			28						28
		100	2						2
		150	1						1
			—						—
			3						3
60	10	50		1					1
		100					1		1
				—			—		—
				1			1		2
	20	50	4	3					7
		100	21	32					53
		150	1	8	3	3	6		21
		200		1	7	8	2		18
			—	—	—	—	—		—
			26	44	10	11	8		99

Table 4. Distribution of all observations by site index (base age 25), age, basal area, and number of trees per acre (cont.).

Site Index (feet)	Age (years)	Basal Area (sq.ft. /acre)	Number of trees per acre						Total
			< 300	301- 500	501- 700	701- 900	901- 1100	> 1100	
600	30	50	6						6
		100	88	11					99
		150	19	20	2				41
		200			1	1			2
			113	31	3	1			148
	40	100	23						23
		150	20						20
			43						43
	50	100	2						2
		150	2						2
		200	3						3
			7						7
70	10	50	2	2	2				6
		100		4	2	1			7
		150				4	4	2	10
			2	6	4	5	4	2	23
	20	100	7	11	3				21
		150	1	6	1				8
		200			2	2			4
			8	17	6	2			33
	30	100	1						1
		150	3						3
			4						4
TOTAL			276	140	28	29	15	2	490

that the resulting diameter distribution would produce stand basal area and average dbh estimates identical to those predicted from regression equations in the first stage. By linking these two stages, the size-class distribution information produced is conditioned to provide aggregate values that are consistent with the predicted overall stand attributes.

Stand-Level Model

The stand-level model consisted of regression equations that predict (1) stand attributes (such as number of trees, basal area, minimum, median, maximum, and average diameters), and (2) density of a stand in the future (age A_2) based on stand information at present (age A_1). Also needed was a mean height equation that predicts total height corresponding to a given dbh.

Different equation forms were developed through a series of plotting dependent variables against various independent variables and their combinations. The equation forms that were considered and included in the evaluation are shown in Table 5. Equations (1a to 1d) and (2a to 2d) were constrained such that projected and current values of basal area and number of trees per acre were identical when $A_2 = A_1$.

When both measures of density (number of trees and basal area per acre) are either known or predicted, quadratic mean diameter (D_q) of the stand can be computed. Since quadratic mean dbh is always greater than or equal to arithmetic mean dbh ($\bar{D}$), $\log_e(D_q - \bar{D})$

Table 5. Equation forms that were considered in the development of a whole stand model for thinned loblolly pine plantations.

Equation Number	Equation form $\frac{1}{}$
1a	$\ln(B_2/B_1) = b_1 \ln(HD_2/HD_1) + b_2 (1/A_2 - 1/A_1)$
1b	$\ln(B_2/B_1) = b_1 \ln(HD_2/HD_1) + b_2 (1/A_2 - 1/A_1) + b_3 \left[1/(A_2 HD_2) - 1/(A_1 HD_1) \right]$
1c	$\ln(B_2/B_1) = (1/A_2 - 1/A_1) \left[b_1 \ln(HD_1) + b_2/A_1 + b_3 \ln(N_1) \right]$
1d	$\ln(B_2/B_1) = (1/A_2 - 1/A_1) \left[b_1 \ln(HD_1) + b_2/A_1 + b_3 \ln(N_1) + b_4/(A_1 HD_1) \right]$
2a	$\ln(N_2/N_1) = b_1 \ln(HD_2/HD_1) + b_2 (1/A_2 - 1/A_1)$
2b	$\ln(N_2/N_1) = b_1 \ln(HD_2/HD_1) + b_2 (1/A_2 - 1/A_1) + b_3 \left[1/(A_2 HD_2) - 1/(A_1 HD_1) \right]$
2c	$\ln(N_2/N_1) = (1/A_2 - 1/A_1) \left[b_1 \ln(HD_1) + b_2/A_1 + b_3 \ln(N_1) \right]$
2d	$\ln(N_2/N_1) = (1/A_2 - 1/A_1) \left[b_1 \ln(HD_1) + b_2/A_1 + b_3 \ln(N_1) + b_4/(A_1 HD_1) \right]$

Table 5. Equation forms that were considered in the development of a whole stand model for thinned loblolly pine plantations (cont.).

Equation Number	Equation form
3a	$\ln(B) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(N)$
3b	$\ln(B) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(N) + b_4/(A \cdot HD)$
4a	$\ln(N) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B)$
4b	$\ln(N) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4/(A \cdot HD)$
5a	$\ln(H) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N) + b_5/D$
5b	$\ln(H) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N) + b_5/D + b_6/(A \cdot D)$
5c	$\ln(H) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N) + b_5/D + b_6/(A \cdot HD)$
5d	$\ln(H) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N) + b_5/D + b_6 (A \cdot D)$
6a	$\ln(Dmin) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N)$

Table 5. Equation forms that were considered in the development of a whole stand model for thinned loblolly pine plantations (cont.).

Equation Number	Equation form
6b	$\ln(D_{\min}) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N) + b_5/(A \cdot HD)$
6c	$\ln(D_{\min}) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N) + b_5/(B \cdot HD)$
7a	$\ln(D_{\max}) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N)$
7b	$\ln(D_{\max}) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N) + b_5/(A \cdot HD)$
7c	$\ln(D_{\max}) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N) + b_5/(B \cdot HD)$
8a	$\ln(D_{\text{med}}) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N)$
8b	$\ln(D_{\text{med}}/D_q) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N)$
8c	$\ln(D_{\text{med}}/D_q) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(N)$
8d	$\ln(D_{\text{med}}/D_q) = b_0 + b_1 \ln(HD) + b_2 \ln(N)$

Table 5. Equation forms that were considered in the development of a whole stand model for thinned loblolly pine plantations (cont.).

Equation Number	Equation form
9a	$\ln(Dq-\bar{D}) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(B) + b_4 \ln(N)$
9b	$\ln(Dq-\bar{D}) = b_0 + b_1/A + b_2 \ln(HD) + b_3 \ln(N)$
9c	$\ln(Dq-\bar{D}) = b_0 + b_1 \ln(HD) + b_2 \ln(N)$

1/ Notation:

$\ln(x)$ = Natural logarithm of x,

A = Stand age in years,

B = Basal area in square feet per acre,

D = Tree diameter at breast height (dbh) in inches,

$\bar{D}$ = Arithmetic mean dbh in inches,

Dmax = Maximum dbh in inches,

Dmed = Median dbh in inches,

Dmin = Minimum dbh in inches,

Dq = Quadratic mean dbh in inches,

H = Total height in feet of a tree having dbh D,

HD = Average height in feet of the dominants and codominants,

N = Number of surviving trees per acre.

Subscript i denotes that the measurement is taken at time i.

was selected as the dependent variable in Equations (9a to 9c) that predict $\bar{D}$ in order to guarantee the above relationship between D_q and $\bar{D}$.

Regression coefficients for the equations presented in Table 5 were estimated from the fit data set. In order to determine the "best" equation form for each stand attribute, candidate models were compared based on the following criteria computed separately from the fit and test data sets.

<u>Criterion</u>	<u>Fit Data Set</u>	<u>Test Data Set</u>
$\overline{\text{DIFF}}$	$\sum_i (y_i - \hat{y}_i) / n_{\text{fit}}$	$\sum_j (y_j - \hat{y}_j) / n_{\text{test}}$
$\overline{ \text{DIFF} }$	$\sum_i y_i - \hat{y}_i / n_{\text{fit}}$	$\sum_j y_j - \hat{y}_j / n_{\text{test}}$
% Variation explained	$\frac{\sum_i (y_i - \bar{y}_1)^2 - \sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y}_1)^2}$	$\frac{\sum_j (y_j - \bar{y}_2)^2 - \sum_j (y_j - \hat{y}_j)^2}{\sum_j (y_j - \bar{y}_2)^2}$

where y_i or y_j = the i th or j th observed stand attribute,

$\hat{y}_i$ or $\hat{y}_j$ = the i th or j th predicted stand attribute,

$i = 1, 2, \dots, n_{\text{fit}}; j = 1, 2, \dots, n_{\text{test}},$

n_{fit} = number of observations in the fit data set,

n_{test} = number of observations in the test data set,

$\sum_i$ = sum over i where $i = 1, 2, \dots, n_{\text{fit}},$

$\sum_j$ = sum over j where $j = 1, 2, \dots, n_{\text{test}},$

$\bar{y}_1 = \sum_i y_i / n_{\text{fit}},$

$$\bar{y}_2 = \sum_j y_j / n_{\text{test}}.$$

$\overline{\text{DIFF}}$ and $|\overline{\text{DIFF}}|$ are measures of bias and precision of a model, respectively, whereas percent variation explained by a model is a measure of fit, similar to R^2 .

Published equations used in this study include Burkhardt et al.'s (1972b) individual tree volume equations, volume ratio models developed by Burkhardt (1977) and Cao and Burkhardt (1980) for estimating merchantable volumes, and the site index equation for old-field loblolly pine plantations prepared by Devan (1979). The site index equation was used to predict the average height of the dominants and codominants (HD) from site index and stand age, or to estimate site index from HD and stand age.

Deriving Diameter Distributions from Stand Attributes

The segmented cdf was employed here to approximate diameter distributions. A total of five percentile points were used to determine the cdf:

$$x_1 = D_{\min}, \quad x_2 = D_{.25}, \quad x_3 = D_{\text{med}}, \quad x_4 = D_{.75}, \quad x_5 = D_{\max},$$

where $D_{\min}$ = minimum dbh in inches,

D_{med} = median dbh in inches,

$D_{\max}$ = maximum dbh in inches,

$D_{.25}$ = the 25th percentile for dbh where $\text{Pr}(\text{dbh} \leq D_{.25}) = .25$,

$D_{.75}$ = the 75th percentile for dbh.

Dmin, Dmax, and Dmed were predicted from regression equations. To completely specify a segmented cdf to quantify the diameter distribution, D_{.25} and D_{.75} had to be determined. The 25th and 75th percentile points were searched for such that the resulting segmented cdf would produce stand basal area and arithmetic mean dbh estimates identical to those predicted from regression equations. In other words, D_{.25} and D_{.75} were solutions of the following system of two equations:

$$\hat{B} = 0.005454 N \int_{D_{\min}}^{D_{\max}} x^2 f(x) dx \quad (10)$$

$$\hat{D} = \int_{D_{\min}}^{D_{\max}} x f(x) dx \quad (11)$$

where $\hat{B}$ = predicted basal area in square feet per acre,

$\hat{D}$ = predicted arithmetic mean dbh in inches,

N = number of surviving trees per acre,

$f(x) = dF(x)/dx$ = diameter pdf, determined by Dmin, D_{.25}, Dmed, D_{.75}, and Dmax.

In most diameter distribution models, stand volume and basal area are often obtained by first computing these attributes for each dbh class and then summing over diameter classes of interest. Equations (10) and (11) can be approximated in a similar manner by replacing the integral sign with a summation sign. The equations become:

$$B = 0.005454 N \sum_{x_i=Dmin}^{Dmax} x_i^2 f_i \quad (12)$$

$$D = \sum_{x_i=Dmin}^{Dmax} x_i f_i \quad (13)$$

where x_i = midpoint of the i th dbh class,

f_i = proportion of trees in the i th dbh class, computed
from the segmented cdf which were specified by $Dmin$,

$D_{.25}$, D_{med} , $D_{.75}$, and $Dmax$.

Equations (12, 13) were used in place of equations (10, 11)
to form a system of two nonlinear equations and two unknowns,
namely, $D_{.25}$ and $D_{.75}$.

RESULTS AND DISCUSSION

Equations for Thinned Loblolly Pine Plantations

In general, the candidate models for each stand attribute performed almost equally well based on the three criteria computed from the fit and test data sets (Table 6). Simpler models were chosen since they behaved better in the case of extrapolation and there were fewer parameters to estimate.

After the "best" equation forms were selected, coefficients of these regression equations were estimated from all data; the results are presented in Table 7.

Program THINNED

All of the techniques described earlier were incorporated into program THINNED, which was written in standard FORTRAN. This program can generate stand and stock tables for different combinations of site, stand age, and density. It is also able to simulate a loblolly pine stand for a specified period during which thinning options are available at any point in time (Figure 3).

Prediction of the Present Stand

The inputs needed are:

- (1) age of the present stand,
- (2) site index (or average height of the current dominants

Table 6. Performance of the candidate models based on three criteria, computed separately from the fit and test data sets.

Equation Number ^{1/}	Fit data set			Test data set		
	$\overline{\text{DIFF}}$	$ \overline{\text{DIFF}} $	% Variation explained	$\overline{\text{DIFF}}$	$ \overline{\text{DIFF}} $	% Variation explained
1a	0.6675	7.2813	80.85	1.0687	7.1581	76.93
1b	0.4190	7.2319	80.77	0.6567	6.9404	76.42
1c	-0.4439	7.9881	75.64	-1.4729	6.8427	70.27
1d	-0.4082	7.1094	79.73	-1.3855	6.7992	71.08
2a	-0.8179	14.2019	96.63	-4.1747	12.9764	91.64
2b	-0.5149	13.4681	97.14	-2.5347	13.2317	92.52
2c	0.4938	13.6452	97.20	-1.3066	12.7931	93.28
2d	0.3743	13.7791	97.20	-1.2863	12.8062	93.28
3a	1.0640	12.0161	77.25	2.3746	12.4210	75.55
3b	1.0564	11.9255	77.63	2.2320	12.1435	75.97
4a	6.7692	52.8856	85.24	1.1996	42.9483	88.57
4b	7.3819	50.8495	86.01	0.4791	41.6650	88.93
5a	0.1580	1.6887	96.60	0.1796	1.7286	96.00
5b	0.0844	1.4168	97.34	-0.0883	1.4153	97.04
5c	0.1501	1.6579	96.68	0.1422	1.6804	96.15
5d	0.0978	1.5433	97.05	-0.0482	1.5464	96.73
6a	-0.4485	1.5787	45.13	0.1027	1.2135	51.92
6b	-0.4422	1.5703	45.55	0.1062	1.2267	51.24
6c	-0.4494	1.5793	45.00	0.1006	1.2170	51.84
7a	0.0281	0.6081	87.19	0.2577	0.7704	80.28
7b	0.0276	0.6044	87.23	0.2544	0.7627	80.59
7c	0.0278	0.6061	87.21	0.2578	0.7641	80.48
8a	0.0060	0.2561	97.33	-0.0152	0.2778	96.70
8b	0.0060	0.2561	97.33	-0.0152	0.2778	96.70
8c	0.0057	0.2561	97.33	-0.0159	0.2773	96.72
8d	0.0056	0.2559	97.33	-0.0140	0.2579	96.74
9a	-0.0217	0.0592	99.79	-0.0217	0.0573	99.84
9b	-0.0218	0.0597	99.79	-0.0215	0.0570	99.84
9c	-0.0220	0.0597	99.79	-0.0228	0.0564	99.84

^{1/} Equation forms were presented in Table 5.

Table 7. Regression equations that form a whole stand model for thinned loblolly pine plantations.

Equation Number	Equation $\frac{1}{2}$
1	$\ln(B_2/B_1) = 0.66153 \ln(HD_2/HD_1) - 10.36715 (1/A_2 - 1/A_1)$ $n=234 \frac{2}{2}; \quad \overline{\ln(B_2/B_1)}=0.1846; \quad s_{y.x}=0.0932$ $R^2=81.98\%; \quad R^2(B_2)=78.63\%$
2	$\ln(N_2/N_1) = -0.36449 \ln(HD_2/HD_1) - 0.12207 (1/A_2 - 1/A_1)$ $n=234 \frac{2}{2}; \quad \overline{\ln(N_2/N_1)}=-0.0596; \quad s_{y.x}=0.0861$ $R^2=31.02\%; \quad R^2(N_2)=96.03\%$
3	$\ln(B) = -4.39181 + 0.19054 /A + 1.34753 \ln(HD)$ $+ 0.63902 \ln(N)$ $n=490 \frac{3}{3}; \quad \overline{\ln(B)}=4.7149; \quad s_{y.x}=0.1407$ $R^2=75.48\%; \quad R^2(B)=77.01\%$
4	$\ln(N) = 7.79805 + 2.10495 /A - 1.89908 \ln(HD)$ $+ 1.16744 \ln(B)$ $n=490 \frac{3}{3}; \quad \overline{\ln(N)}=5.6732; \quad s_{y.x}=0.1902$ $R^2=87.19\%; \quad R^2(N)=85.78\%$
5	$\ln(H) = 0.46152 + 0.43275 /A + 0.93333 \ln(HD)$ $- 0.08583 \ln(B) + 0.07596 \ln(N)$ $- 2.15312 /D$ $n=3559 \frac{4}{4}; \quad \overline{\ln(H)}=4.0404; \quad s_{y.x}=0.0422$ $R^2=96.76\%; \quad R^2(H)=97.62\%$

Table 7. Regression equations that form a whole stand model for thinned loblolly pine plantations (cont.).

Equation Number	Equation
6	$\ln(D_{\min}) = 1.10835 + 5.10755 /A + 0.50531 \ln(HD)$ $+ 0.28544 \ln(B) - 0.57131 \ln(N)$ $n=427 \frac{5/}{\quad}; \quad \overline{\ln(D_{\min})}=1.52530; \quad s_{y.x}=0.2972$ $R^2=46.84\%; \quad R^2(D_{\min})=51.02\%$
7	$\ln(D_{\max}) = 2.20990 - 2.14356 /A + 0.04418 \ln(HD)$ $+ 0.41893 \ln(B) - 0.31663 \ln(N)$ $n=490 \frac{3/}{\quad}; \quad \overline{\ln(D_{\max})}=2.4818; \quad s_{y.x}=0.0699$ $R^2=86.77\%; \quad R^2(D_{\max})=86.43\%$
8	$\ln(D_{\text{med}}/D_q) = 0.15560 - 0.02965 \ln(HD) - 0.00828 \ln(N)$ $n=490 \frac{3/}{\quad}; \quad \overline{\ln(D_{\text{med}}/D_q)}=-0.0118; \quad s_{y.x}=0.0389$ $R^2=1.25\%; \quad R^2(D_{\text{med}})=97.26\%$
9	$\ln(D_q - \overline{D}) = -9.05733 + 0.89274 \ln(HD) + 0.58151 \ln(N)$ $n=489 \frac{6/}{\quad}; \quad \overline{\ln(D_q - \overline{D})}=-2.1316; \quad s_{y.x}=0.6206$ $R^2=11.50\%; \quad R^2(\overline{D})=99.80\%$

Table 7. Regression equations that form a whole stand model for thinned loblolly pine plantations (cont.).

1/ Notation:

$\ln(x)$ = Natural logarithm of x ,
 $R^2(x)$ = Percent variation in x explained by the model,
 A = Stand age in years,
 B = Basal area in square feet per acre,
 D = Tree diameter at breast height (dbh) in inches,
 $\bar{D}$ = Arithmetic mean dbh in inches,
 $D_{\max}$ = Maximum dbh in inches,
 D_{med} = Median dbh in inches,
 $D_{\min}$ = Minimum dbh in inches,
 D_q = Quadratic mean dbh in inches,
 H = Total height in feet of a tree having dbh D ,
 HD = Average height in feet of the dominants and codominants,
 N = Number of surviving trees per acre,
 Subscript i denotes that the measurement is taken at time i .

2/ Each observation consisted of two successive plot measurements.

3/ Each observation consisted of one plot measurement.

4/ Each observation was a height-diameter pair. In each plot, there was one observation for each diameter class in which some trees were tallied.

5/ Each observation consisted of one plot measurement; observations with $D_{\min}=0$ were deleted since $\ln(0)$ is undefined.

6/ Each observation consisted of one plot measurement; one observation with $\bar{D} > D_q$ was deleted since logarithm of a negative number is undefined. Numerical round-off errors resulted in this inconsistency.

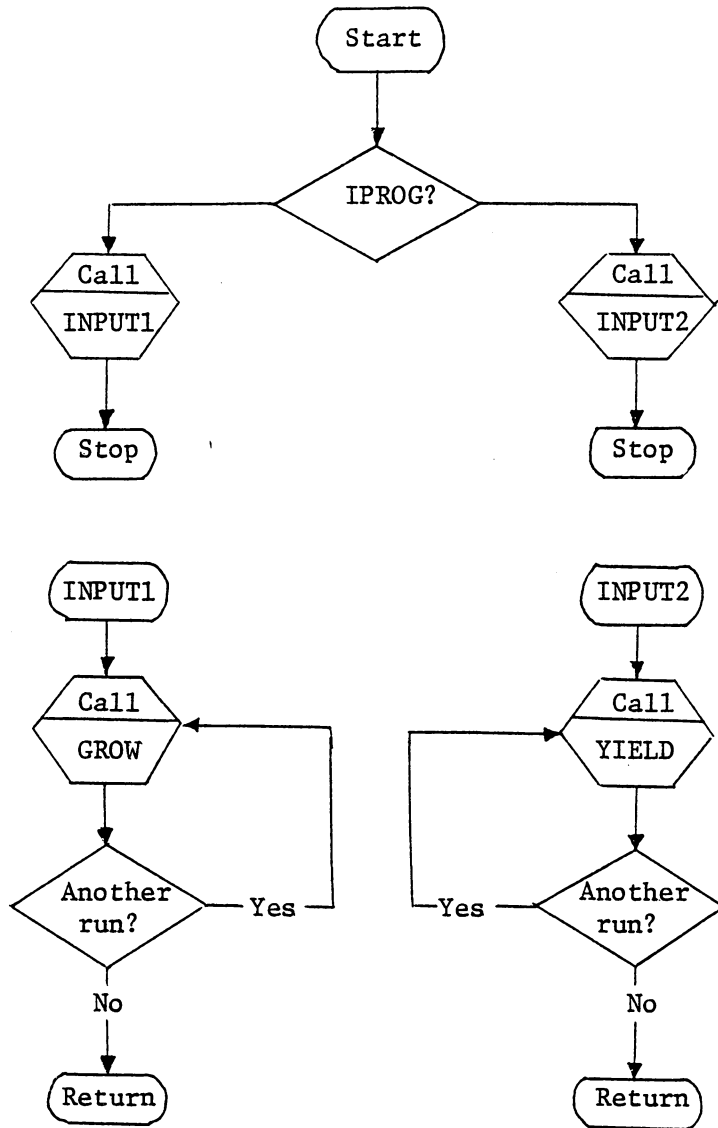


Figure 3. Generalized flowchart of program THINNED.

and codominants),

- (3) two measures of density (total basal area and number of trees per acre).

If only one measure of density is available, the other can be estimated by employing the appropriate equation (3 or 4) of Table 7. The procedures for predicting the present stand are outlined in Figure 4.

Equations (6, 7, 8, 9) of Table 7 predict the minimum, maximum, median, and arithmetic mean dbh of the stand. The 25th and 75th percentiles for dbh are obtained by solving equations (12) and (13) simultaneously. The five percentile points were used to determine a segmented cdf that approximates the diameter distribution of the stand. As a result, number of trees and basal area per acre for each dbh class can be computed. The mean height equation (equation 5 of Table 7) predicts total height corresponding to the midpoint of each dbh class. Total volumes outside and inside bark can be obtained from the individual tree volume equations published by Burkhardt et al. (1972b). Merchantable volumes can also be calculated using the volume ratio methods developed by Burkhardt (1977) and Cao and Burkhardt (1980).

Thinning

Inputs for the thinning option include age of the stand when thinning occurs and type of thinning. Thinning can be carried out

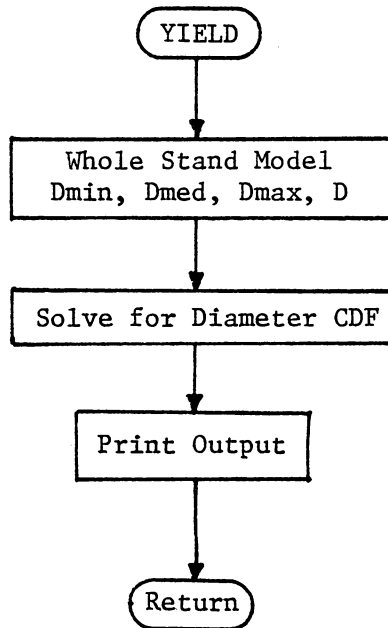


Figure 4. Generalized flowchart of subprogram YIELD.

by rows, from below, or a combination of both.

It is assumed that the diameter distribution does not change due to row thinning. Thus the number of trees, basal area, and volume per acre in each dbh class are reduced by the proportion of trees removed in thinning.

Thinning from below is defined here as removing all trees with dbh values less than a specified diameter. Input for this type of thinning can be either this diameter limit or a residual basal area. A combination of row and low thinnings involves first a row thinning followed by thinning from below.

Projection

Basal area and number of trees per acre at some age in the future can be projected using equations (1) and (2) of Table 7 for thinned stands, or the following equations from Coile and Schumacher (1964) for unthinned loblolly pine plantations:

$$\log_{10}(N) = \log_{10}(N_0) + \left[2.1346 - 1.1103 \log_{10}(N_0) + 0.1384 (OF) \right] A/100$$

$$\log_{10}(B) = 1.4366 \log_{10}(S) - 0.7084 (10/A) + 0.4888 \log_{10}(N) + 0.0585 (OF) - 1.4436$$

where A = age in years,

B = stand basal area in square feet per acre at age A,

N = number of surviving trees per acre at age A,

N_0 = number of trees planted per acre,

OF = +1 if old-field origin, and -1 otherwise,

S = site index in feet (base age 25 years).

Procedures similar to those for predicting the present stand are then employed to produce stand and stock tables for the future stand (Figure 5).

Diameter Distribution of a Previously Low-Thinned Stand

Suppose the stand was previously thinned from below where all trees having dbh below D_{thin} were cut. If predicted D_{min} of the present stand is greater than or equal to D_{thin} , then the segmented cdf is used to characterize the current diameter distribution. On the other hand, when D_{min} is less than D_{thin} , a left-truncated segmented cdf is more appropriate where D_{thin} is the truncation point. The proportion of trees in the i th dbh class is given by:

$$f_i = \frac{F(i+0.5) - F(i-0.5)}{1 - F(D_{thin})}$$

where $F(x) = \Pr(X \leq x)$ from the segmented cdf.

Yield Tables

Stand and stock tables of thinned loblolly pine plantations were generated for various combinations of site, stand age, and density. Only total cubic-foot volumes outside bark per acre for these combinations are shown in Table 8.

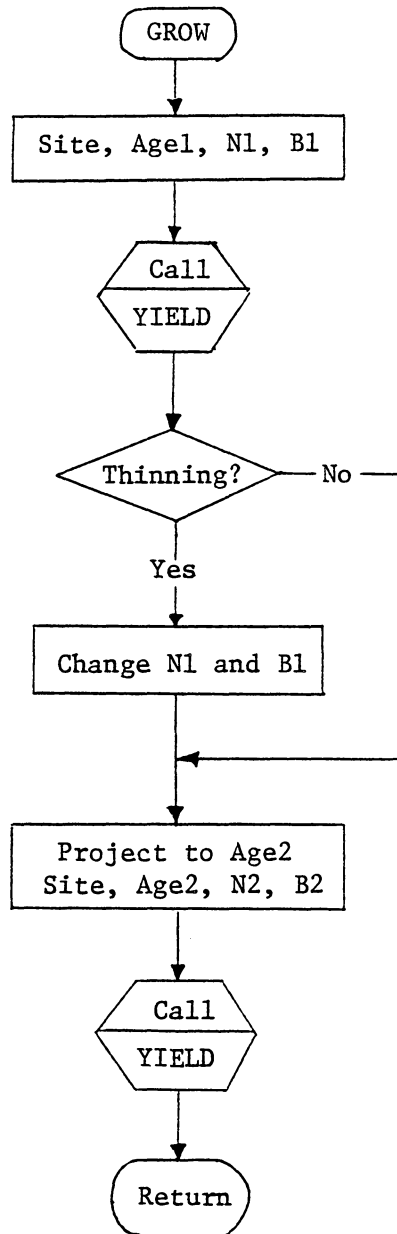


Figure 5. Generalized flowchart of subprogram GROW.

Table 8. Total cubic-foot volume outside bark per acre for thinned loblolly pine plantations. 1/

Site <u>2/</u> Index (feet)	Age (years)	Basal area (sq.ft. per acre)	Number of trees per acre				
			200	400	600	800	1000
50	10	50	424	470	517	570	626
		100	801	843	887	933	982
		150	1173	1219	1262	1312	1351
		200	1541	1592	1636	1678	1722
	20	50	923	936	962	999	1040
		100	1833	1840	1854	1872	1900
		150	2728	2745	2755	2767	2787
		200	3614	3644	3656	3666	3681
	30	50	1249	1244	1260	1286	1323
		100	2499	2488	2482	2490	2507
		150	3735	3734	3724	3725	3730
		200	4954	4977	4965	4961	4963
	40	50	1464	1447	1456	1477	1515
		100	2941	2918	2903	2903	2917
		150	4402	4388	4367	4359	4359
		200	5839	5850	5838	5822	5819
	50	50	1613	1590	1593	1616	1653
		100	3249	3218	3199	3196	3207
		150	4867	4849	4822	4810	4804
		200	6461	6470	6446	6430	6418
60	10	50	544	580	623	672	723
		100	1048	1088	1117	1156	1200
		150	1549	1584	1618	1653	1693
		200	2038	2084	2120	2162	2190
	20	50	1099	1102	1123	1152	1192
		100	2195	2189	2193	2207	2227
		150	3275	3281	3279	3285	3297
		200	4341	4366	4366	4369	4373
	30	50	1454	1438	1446	1470	1507
		100	2923	2900	2886	2886	2899
		150	4373	4362	4342	4335	4333
		200	5803	5815	5799	5787	5781

Table 8. Total cubic-foot volume outside bark per acre for thinned loblolly pine plantations (cont.).

Site Index (feet)	Age (years)	Basal area (sq.ft. per acre)	Number of trees per acre				
			200	400	600	800	1000
60	40	50	1690	1660	1663	1684	1721
		100	3404	3366	3342	3336	3347
		150	5096	5076	5046	5030	5027
		200	6767	6773	6748	6727	6714
	50	50	1849	1820	1816	1836	1877
		100	3738	3695	3664	3658	3667
		150	5602	5574	5539	5516	5510
		200	7438	7440	7413	7386	7370
	70	50	676	704	742	784	832
		100	1322	1346	1373	1405	1443
		150	1960	1990	2014	2041	2072
		200	2589	2629	2655	2680	2709
	20	50	1276	1270	1282	1310	1347
		100	2556	2541	2535	2543	2560
		150	3821	3818	3810	3806	3811
		200	5070	5087	5079	5072	5072
	30	50	1659	1633	1636	1655	1692
		100	3337	3305	3281	3286	3286
		150	4999	4980	4951	4933	4931
		200	6638	6646	6621	6598	6588
	40	50	1907	1868	1869	1887	1928
		100	3853	3804	3773	3765	3768
		150	5776	5741	5706	5680	5674
		200	7665	7666	7636	7606	7590
	50	50	2081	2038	2035	2052	2092
		100	4211	4155	4121	4110	4120
		150	6313	6275	6231	6206	6196
		200	8381	8379	8344	8312	8293

$\frac{1}{2}$ The boxes indicate the range of the data.
 $\frac{2}{2}$ Site index at base age 25.

A Numerical Example

The following example is chosen to illustrate the techniques employed in program THINNED. Consider a loblolly pine plantation on soil of site index 60 feet (base age 25 years), with 600 trees and 150 sq.ft. of basal area per acre at age 20. The stand is thinned to 100 sq.ft. per acre at age 20; the thinning method is a combination of 25% row thinning and 75% low thinning (i.e. a row thinning removes 25% of the total basal area scheduled to be thinned, and then a thinning from below removes the remaining 75%). No minimum diameter for removal in the low thinning is specified in this example. The stand is then left to grow until it is harvested at age 40. The card input needed by program THINNED to simulate this particular stand is presented in Appendix 3a. Figures 6a to 6d show the outputs of this simulation from program THINNED. The computational steps (on a per acre basis) are outlined as follows.

Step 1: Yield prediction of the stand before thinning.

Stand variables: Site index = 60 feet, A = 20 years, N = 600 trees, B = 150 sq.ft. (variable names were defined in Table 7).

From Devan's (1979) site index equation, average height of the dominants and codominants at age 20 is 49.55 feet. Substituting the values into the appropriate stand variables in equations (6, 7, 8, 9) of Table 7 gives: $D_{min} = 3.04$, $D_{max} = 10.47$, $D_{med} = 6.68$, $\bar{D} = 6.61$ (all in inches).

25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

INPUTS				PREDICTED			
-----				-----			
SITE = 60.00				HD = 49.55			
AGE = 20.00				AVERAGE DBH = 6.61			
NUMBER OF TREES = 600.00				MINIMUM DBH = 3.04			
BASAL AREA = 150.00				MEDIAN DBH = 6.68			
				MAXIMUM DBH = 10.47			

DBH CLASS	NUMBER OF TREES	AVERAGE HEIGHT	BASAL AREA	TOTAL VOLUME O.B.	TOTAL VOLUME I.B.	VOLUME O.B. TO 4. IN	VOLUME I.B. TO 4. IN
2	0.7	22.3	0.0	0.4	0.2	0.0	0.0
3	9.2	31.9	0.5	9.4	6.0	0.0	0.0
4	37.1	38.2	3.2	65.6	46.3	0.0	0.0
5	86.5	42.6	11.8	243.7	180.4	163.8	117.3
6	138.1	45.7	27.1	575.7	436.8	460.5	342.7
7	157.8	48.1	42.2	918.8	707.2	797.8	606.0
8	118.1	50.0	41.2	918.5	713.4	834.3	641.8
9	44.8	51.6	19.8	450.0	351.6	420.0	325.8
10	7.7	52.8	4.2	96.8	76.0	92.0	71.8
11	0.0	53.8	0.0	0.0	0.0	0.0	0.0
-----				-----	-----	-----	-----
	600.0		150.0	3278.9	2518.0	2768.3	2105.3

AVERAGE DBH = 6.61
CORD VOLUME TO 4. IN = 31.42

CONVERGENCE ATTAINED

Figure 6a. Example output from program THINNED --
Step 1: Yield prediction of the stand
before thinning.

25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

ROW THINNING AT AGE 20.

8.33% OF TREES IN ALL DIAMETER CLASSES ARE CUT

BEFORE ROW THINNING

SITE = 60.00
AGE = 20.00
NUMBER OF TREES = 600.00
BASAL AREA = 150.01
AVERAGE DBH = 6.61

AFTER ROW THINNING

DBH CLASS	NUMBER OF TREES	AVERAGE HEIGHT	BASAL AREA	TOTAL VOLUME O.B.	TOTAL VOLUME I.B.	VOLUME O.B. TO 4.1N	VOLUME I.B. TO 4.1N
2	0.6	22.3	0.0	0.3	0.2	0.0	0.0
3	8.5	31.9	0.4	8.6	5.5	0.0	0.0
4	34.0	38.2	3.0	60.1	42.5	0.0	0.0
5	79.3	42.6	10.8	223.4	165.4	150.1	107.5
6	126.6	45.7	24.9	527.7	400.4	422.1	314.1
7	144.7	48.1	38.7	842.2	648.3	731.3	555.5
8	108.3	50.0	37.8	842.0	654.0	764.7	588.3
9	41.1	51.6	18.2	412.5	322.3	385.0	298.6
10	7.0	52.8	3.8	88.8	69.7	84.3	65.8
11	0.0	53.8	0.0	0.0	0.0	0.0	0.0
-----			-----	-----	-----	-----	-----
550.0			137.5	3005.6	2308.1	2537.6	1929.8

SITE = 60.00
AGE = 20.00
NUMBER OF TREES = 549.99
BASAL AREA = 137.51
AVERAGE DBH = 6.61

AMOUNT REMOVED IN THINNING

NUMBER OF TREES = 50.01
BASAL AREA = 12.50
TOTAL CU.FT. VOLUME O.B. = 273.27
CU.FT. VOLUME O.B. TO 4.1N = 230.72
CORD VOLUME TO 4.1N = 2.62

Figure 6b. Example output from program THINNED --
Step 2: Row thinning at age 20.

25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

LOW THINNING AT AGE 20.

THIN TO 100.00 SQ.FT. RESIDUAL BASAL AREA

BEFORE LOW THINNING

SITE = 60.00
AGE = 20.00
NUMBER OF TREES = 549.99
BASAL AREA = 137.51
AVERAGE DBH = 6.61

AFTER LOW THINNING

DBH CLASS	NUMBER OF TREES	AVERAGE HEIGHT	BASAL AREA	TOTAL VOLUME O.B.	TOTAL VOLUME I.B.	VOLUME O.B. TO 4. IN	VOLUME I.B. TO 4. IN
6	7.9	45.7	1.6	33.0	25.0	26.4	19.6
7	144.7	48.1	38.7	842.2	648.3	731.3	555.5
8	108.3	50.0	37.8	842.0	654.0	764.7	588.3
9	41.1	51.6	18.2	412.5	322.3	385.0	298.6
10	7.0	52.8	3.8	88.8	69.7	84.3	65.8
11	0.0	53.8	0.0	0.0	0.0	0.0	0.0
	-----		-----	-----	-----	-----	-----
	309.0		100.0	2218.4	1719.2	1991.7	1527.8

SITE = 60.00
AGE = 20.00
NUMBER OF TREES = 308.97
BASAL AREA = 100.00
AVERAGE DBH = 7.66

AMOUNT REMOVED IN THINNING

NUMBER OF TREES = 241.03
BASAL AREA = 37.51
TOTAL CU.FT. VOLUME O.B. = 787.18
CU.FT. VOLUME O.B. TO 4. IN = 545.85
CORD VOLUME TO 4. IN = 6.44

Figure 6c. Example output from program THINNED --
Step 3: Low thinning at age 20.

25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

INPUTS	PREDICTED
-----	-----
SITE = 60.00	HD = 81.14
AGE = 40.00	AVERAGE DBH = 11.13
NUMBER OF TREES = 258.93	MINIMUM DBH = 5.84
BASAL AREA = 179.56	MEDIAN DBH = 11.04
	MAXIMUM DBH = 15.89

THIS STAND WAS PREVIOUSLY THINNED FROM BELOW
ALL TREES UNDER 5.5 INCHES IN DBH WERE CUT

DBH CLASS	NUMBER OF TREES	AVERAGE HEIGHT	BASAL AREA	TOTAL VOLUME O.B.	TOTAL VOLUME I.B.	VOLUME O.B. TO 4. IN	VOLUME I.B. TO 4. IN
6	0.8	66.2	0.2	4.7	3.6	3.8	2.9
7	4.4	69.7	1.2	36.3	28.2	31.5	24.2
8	13.5	72.4	4.7	149.5	117.0	135.8	105.3
9	29.4	74.6	13.0	422.6	332.3	394.4	307.8
10	48.7	76.5	26.6	881.6	695.1	837.5	656.5
11	59.2	78.0	39.1	1316.7	1040.4	1265.8	995.5
12	45.2	79.2	35.5	1212.4	959.5	1175.4	926.6
13	30.3	80.3	28.0	966.4	765.7	942.6	744.5
14	17.1	81.3	18.3	639.8	507.5	627.0	495.9
15	7.4	82.1	9.1	321.4	255.1	316.0	250.3
16	2.4	82.9	3.4	119.0	94.5	117.4	93.0
17	0.4	83.5	0.6	22.6	18.0	22.4	17.7
-----				-----	-----	-----	-----
	258.9		179.6	6093.0	4816.9	5869.4	4620.3

AVERAGE DBH = 11.12
CORD VOLUME TO 4. IN = 62.79

CONVERGENCE ATTAINED

Figure 6d. Example output from program THINNED --
Step 4: Project to age 40.

Dmin and Dmax are then adjusted as follows:

$$D_{\min} = \text{FLOOR} (D_{\min} - 0.5) - 0.49 = 1.51 \text{ inches,}$$

$$D_{\max} = \text{FLOOR} (D_{\max} + 1.5) + 0.49 = 11.49 \text{ inches,}$$

where $\text{FLOOR}(x)$ = integer portion of x .

These adjustments simply set Dmin and Dmax at the lower and upper ends of their 1-inch dbh class, respectively, and then decrease Dmin and increase Dmax by 1 inch.

The solution of the system of two nonlinear equations:

$$150 = 0.005454 (600) \sum_{x_i=2}^{11} x_i^2 f_i$$

$$6.61 = \sum_{x_i=2}^{11} x_i f_i$$

is found to be $D_{.25} = 5.64$ inches and $D_{.75} = 7.63$ inches.

The segmented cdf that approximates the diameter distribution of this stand is now defined on the following 5 percentile points:

$$D_{\min} = 1.51, \quad D_{.25} = 5.64, \quad D_{\text{med}} = 6.68, \quad D_{.75} = 7.63,$$

$D_{\max} = 11.49$ (all in inches).

Per acre number of trees, basal area, and volume for each dbh class can be computed. For example, number of trees in the 6-inch class is $600 [F(6.5) - F(5.5)] = 138.1$ trees, where $F(x)$ is the segmented cdf evaluated at x . Basal area in the 6-inch class:

$$138.1 (0.005454) (6)^2 = 27.1 \text{ sq.ft.}$$

Average height of a tree with a 6-inch dbh in this plantation is calculated to be 45.7 feet from equation (5) of Table 7. Burkhardt et al.'s (1972b) tree volume equation is applied on 138.1 trees of dbh 6 inches and total height 45.7 feet, resulting in a total of 575.7 cu.ft. of outside bark volume in the 6-inch dbh class. Summing volume estimates over dbh classes gives a stand volume value of 3279 cu.ft./acre, which is one entry in Table 8 for this combination of stand variables.

Step 2: Row thinning at age 20.

In this example, 25% of the basal area removed is due to row thinning. Total basal area removed in two thinnings = $150 - 100 = 50$ sq.ft. Residual basal area after row thinning:

$$150 - 0.25(50) = 137.5 \text{ sq.ft.}$$

Let Q be the ratio of basal area after row thinning and basal area before thinning, $Q = 137.5 / 150 = 0.9167$. The stand and stock table after row thinning is constructed by multiplying the residual ratio Q by the entries in the stand and stock table before row thinning.

Number of trees in the 6-inch class: $0.9167 (138.1) = 126.6$ trees.

Basal area in the 6-inch class: $0.9167 (27.1) = 24.86$ sq.ft.

Volume in the 6-inch class: $0.9167 (575.7) = 527.7$ cu.ft.

Step 3: Low thinning at age 20.

Basal area removed in low thinning: $0.75 (50) = 37.5$ sq.ft.

The diameter limit (D_{thin}) is searched for by summing basal area in each dbh class, starting from the lowest class, until the total is closest to but not greater than 37.5 sq.ft. Basal area of cut trees having dbh's of 5.5 inches and below:

$$0.4 + 3.0 + 10.8 = 14.2 \text{ sq.ft.}$$

Basal area of trees in the 6-inch class that are removed in low thinning: $37.5 - 14.2 = 23.3$ sq.ft., which corresponds to:

$$126.6 (23.3) / 24.857 = 118.7 \text{ trees.}$$

Residual number of trees in the 6-inch class: $126.6 - 118.7 = 7.9$ trees/acre. Trees in the 7-inch class and above are left in this low thinning.

Step 4: Project to age 40.

Stand attributes at age 40 are predicted from those at age 20 after thinning. The procedures for constructing the stand and stock table are similar to those described earlier in Step 1, except that a segmented cdf left-truncated at diameter of 5.5 inches is used in this case.

Effect of Thinning Regimes on Yield

In order to demonstrate the effect of thinning types and inten-

sities on yield, different thinning options were applied to loblolly pine plantations on site index 60 soil. These hypothetical stands had 800 trees and 150 sq.ft. per acre of basal area at age 15, and would be harvested at age 30. Option A was the control where no thinning was applied. In the rest of the thinning options, the stands were thinned repeatedly at ages 15, 20, and 25 to a specified residual basal area. Residual basal areas were arbitrarily set at 80, 95, and 110 sq.ft. per acre for options B, C, and D, respectively. Three types of thinning were considered for each residual density: (1) row thinning, (2) low thinning, and (3) a combination of row and low thinnings, where 25% of the basal area removed was first cut in a row thinning and then the remainder in a thinning from below. Option C2, for example, means low thinning to 95 sq.ft./acre of residual basal area.

Yields of these stands under different regimes are presented in Table 9. Total cubic-foot volume production (amount removed in thinnings plus final harvest volume) did not differ much from row to low thinning for a given thinning level. Stand average diameter, however, was lowest in row thinning, highest in low thinning, and somewhere between these two extremes in the combination of row and low thinnings, as expected. As found by other researchers (such as Feduccia and Mann 1976, Sullivan and Williston 1977), cubic-foot volume production increased with higher residual basal area. On the other hand, average dbh increased as the thinnings were more severe, which implies an increase in board-foot volume production. Yield

Table 9. Yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option.

Age (years)	Before thinning				After thinning				Total	
	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume ob (cu.ft.)	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume ob (cu.ft.)	Volume removed (cu.ft.)	Volume Production (cu.ft.)
<u>OPTION A: No thinning.</u>										
15	800	130	5.3	2225						2225
30	540	186	7.8	5387						5387
<u>OPTION B1: Row thinning -- Residual basal area = 80 sq.ft./acre.</u>										
15	800	130	5.3	2225	493	80	5.3	1369	856	2225
20	442	116	6.8	2542	305	80	6.8	1751	791	3398
25	285	101	7.9	2611	226	80	7.9	2074	537	4258
30	216	93	8.8	2730						4914
<u>OPTION B2: Low thinning -- Residual basal area = 80 sq.ft./acre.</u>										
15	800	130	5.3	2225	353	80	6.4	1380	845	2225
20	318	120	8.2	2630	172	80	9.2	1781	849	3475
25	161	101	10.6	2618	114	80	11.3	2095	523	4312
30	109	94	12.5	2749						4966

Table 9. Yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (cont.).

Age (years)	Before thinning				After thinning				Total	
	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume ob (cu.ft.)	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume ob (cu.ft.)	Volume removed (cu.ft.)	Volume Production (cu.ft.)
<u>OPTION B3: 25% row thinning and 75% low thinning -- Residual basal area = 80 sq.ft./acre.</u>										
15	800	130	5.3	2225	365	80	6.3	1379	846	2225
20	328	116	8.0	2546	188	80	8.8	1773	773	3392
25	176	102	10.2	2669	124	80	10.8	2097	572	4288
30	118	94	12.0	2730						4921
<u>OPTION C1: Row thinning -- Residual basal area = 95 sq.ft./acre.</u>										
15	800	130	5.3	2225	585	95	5.3	1626	599	2225
20	525	138	6.8	3016	362	95	6.8	2077	939	3615
25	338	120	7.9	3097	269	95	7.9	2460	637	4635
30	256	111	8.8	3242						5417
<u>OPTION C2: Low thinning -- Residual basal area = 95 sq.ft./acre.</u>										
15	800	130	5.3	2225	451	95	6.2	1633	592	2225
20	405	138	7.8	3016	222	95	8.8	2108	908	3608
25	208	120	10.2	3130	145	95	10.9	2487	643	4630
30	138	111	12.0	3245						5388

Table 9. Yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (cont.).

Age (years)	Before thinning				After thinning				Total	
	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume ob (cu.ft.)	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume ob (cu.ft.)	Volume removed (cu.ft.)	Production (cu.ft.)
<u>OPTION C3: 25% row thinning and 75% low thinning -- Residual basal area = 95 sq.ft./acre.</u>										
15	800	130	5.3	2225	468	95	6.1	1632	593	2225
20	420	138	7.6	3021	235	95	8.6	2107	914	3614
25	220	120	9.9	3107	156	95	10.5	2484	623	4614
30	149	111	11.6	3246						5376
<u>OPTION D1: Row thinning -- Residual basal area = 110 sq.ft./acre.</u>										
15	800	130	5.3	2225	677	110	5.3	1883	342	2225
20	609	160	6.8	3492	419	110	6.8	2404	1088	3834
25	391	138	7.9	3585	311	110	7.9	2846	739	5015
30	296	129	8.8	3748						5917
<u>OPTION D2: Low thinning -- Residual basal area = 110 sq.ft./acre.</u>										
15	800	130	5.3	2225	561	110	5.9	1885	340	2225
20	503	167	7.6	3673	249	110	8.9	2457	1216	4013
25	233	142	10.5	3678	175	110	10.6	2866	812	5234
30	167	129	11.8	3749						6117

Table 9. Yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (cont.).

Age (years)	Before thinning				After thinning				Total	
	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume ob (cu.ft.)	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume ob (cu.ft.)	Volume removed (cu.ft.)	Volume Production (cu.ft.)
<u>OPTION D3: 25% row thinning and 75% low thinning -- Residual basal area = 110 sq.ft./acre.</u>										
15	800	130	5.3	2225	571	110	5.9	1886	339	2225
20	512	169	7.6	3710	257	110	8.8	2455	1255	4049
25	240	138	10.2	3590	171	110	10.8	2874	716	5184
30	163	129	11.9	3754						6064

of the control stand (no thinning) was equivalent to total yield of the stands with residual basal area of 95 sq.ft./acre. Residual basal areas after thinning of less than 95 sq.ft./acre resulted in less total volume production than an unthinned stand, whereas residual basal areas greater than 95 sq.ft./acre resulted in increased total volume production.

Comparison with Published Information on Thinning

Coile and Schumacher's (1964) Model

Program THINNED was compared with the model for thinned loblolly pine plantations developed by Coile and Schumacher (1964); results are presented in Table 10. Both row and low thinning options were tried, for the thinning in practice would likely be somewhere between these two extremes. Care was taken such that cord volume removed in each thinning was identical to that specified by Coile and Schumacher. Examination of the residual stands at age 30 revealed that the number of surviving trees from Coile and Schumacher's model was between the predicted values from two types of thinning of program THINNED. Residual basal area, quadratic mean dbh, and volume from Coile and Schumacher's predictions were consistently higher than those from THINNED.

Coile and Schumacher's predicted total volume production of thinned stands far exceeded that of unthinned counterparts. On the other hand, total volume predictions (i.e. volume removed in thinnings

Table 10. Comparison of predicted yields of Coile and Schumacher (1964) and those from program THINNED on a per acre basis for thinned loblolly pine plantations.

Source	Site ^{1/} Index (feet)	Number of trees at age 5	Basal Area (sq.ft.) at age 5	Age when thinned (years)	Amount of thinning		Residual stand at age 30				Total Volume Production (cords)
					Basal area (sq.ft.)	Volume ^{2/} (cords)	Quadratic mean DBH (inches)	Number of trees	Basal area (sq.ft.)	Volume (cords)	
C&S ^{3/}	50	600	9.9	20	68	10	13.3 (7.7) ^{6/}	140 (365)	135 (118)	28.7 (26.7)	38.7 (26.7)
Row ^{4/}					58		8.4	163	62	15.7	25.7
Low ^{5/}					61		9.9	106	57	14.8	24.8
C&S	50	800	11.4	20	82	12	13.4 (7.3)	146 (448)	142 (130)	30.3 (29.1)	42.3 (29.1)
Row					72		7.8	176	59	14.8	26.8
Low					77		9.7	100	52	13.3	25.3
C&S	60	600	12.9	17,22	45,36	7,7	13.6 (8.8)	168 (365)	170 (153)	43.7 (42.9)	47.7 (42.9)
Row					38,29		9.8	192	100	30.0	44.0
Low					43,30		12.1	112	89	27.0	41.0
C&S	60	800	14.8	17,22	58,47	9,9	14.6 (8.3)	159 (448)	185 (169)	47.1 (47.2)	65.1 (47.2)
Row					51,38		9.2	198	91	27.2	45.1
Low					59,38		12.2	99	80	24.3	42.0
C&S	70	600	16.1	15,20,25	37,37,39	6,8,10	15.1 (9.8)	158 (365)	196 (191)	60.6 (63.4)	84.6 (63.4)
Row					31,31,39		11.0	178	118	40.7	64.7
Low					36,31,33		14.2	98	108	37.2	61.2
C&S	70	800	18.5	15,20,25	43,47,51	7,10,13	14.7 (9.3)	189 (448)	222 (211)	68.2 (70.0)	98.2 (70.0)
Row					37,39,43		10.3	202	118	40.5	70.6
Low					46,40,43		14.2	89	98	33.7	63.7

^{1/} Site index at base age 25.

^{2/} Cord volume to a 4-inch top, converted from cubic-foot volume outside bark to a 4-inch top, using ratios from Burkhart *et al.* (1972).

^{3/} Coile and Schumacher (1964).

^{4/} Row thinning, program THINNED.

^{5/} Low thinning, program THINNED.

^{6/} Numbers in parentheses are for unthinned stands.

plus residual volume) of thinned stands at age 30 from program THINNED were close to volumes of unthinned stands at age 30 from Coile and Schumacher's model. This agrees well with what other investigators have found, namely, that total cubic-foot volume production is generally little affected by thinning (Smith 1962, Andrulot et al. 1972, Goebel et al. 1974).

Yields Reported by Goebel et al. (1974)

Goebel et al. (1974) reported yields of old-field loblolly pine plantations which were thinned 4 to 5 times to specified residual basal area per acre. Site indices were determined from curves developed by Goebel and Shipman (1964). Goebel and Warner (1969) recognized a significant site-age bias in these site index curves and revised their yield model using Clutter and Lenhart's (1968) polymorphic site index curves. Devan's (1979) site index equation was replaced with that of Clutter and Lenhart (1968) in program THINNED when simulating the stands based on the guidelines set forth by Goebel et al. (1974). The observed number of trees per acre and average dbh in each plot fell between values predicted from THINNED using the row and low thinning options (Table 11).

Comparison of total volume production in these 9 stands showed that the predicted yields (averages of yields from the low and row thinning options) were within 4% of the observed yields in 7 stands. The relative differences between observed and predicted yields in the other 2 stands were -7.35% and 19.82%, respectively. The mean

Table 11. Comparison of observed yields of Goebel *et al.* (1974) and predicted yields from program THINNED on a per acre basis for thinned loblolly pine plantations.

Source	Site Index (feet)	Before first thinning					After periodic thinnings					Age when thinned (years)	Basal Area Limit (sq.ft.)	Volume removed in thinning (cu.ft.)	Total Volume Production (cu.ft.)
		Age (years)	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Total Volume 1b (cu.ft.)	Age (years)	Number of trees	Basal Area (sq.ft.)	Average DBH (inches)	Residual Volume (cu.ft.)				
Observed	51 ^{1/}	13	790	121	5.3	1476	34	140	75	9.9	1870	13,21,	75	2325	4195
Row ^{3/}	60 ^{2/}				5.2	1492		147	75	9.6	1784	27,34		2674	4458
Low ^{4/}	60 ^{2/}				5.2	1492		70	75	14.0	1804			2254	4088
Observed	51	13	800	116	5.0	2116	34	160	84	9.8	2075	13,21,	85	2188	4263
Row	60				5.0	1428		177	85	9.3	2018	27,34		2358	4376
Low	60				5.0	1428		84	85	13.5	2042			2391	4433
Observed	51	13	780	129	5.3	1579	34	160	94	10.4	2349	13,21,	95	2189	4538
Row	60				5.4	1595		173	95	9.9	2260	27,34		2640	4900
Low	60				5.4	1595		86	95	14.2	2279			2563	4843
Observed	51	13	1016	124	4.6	1409	34	132	80	10.5	2065	13,18,20,	80	2261	4326
Row	60				4.6	1518		196	80	8.5	1892	25,34		2198	4090
Low	60				4.6	1518		88	80	12.8	1929			2303	4232
Observed	51	13	1004	122	4.6	1350	34	148	89	10.5	2436	13,18,20,	90	2431	4867
Row	60				4.6	1493		221	90	8.5	2126	25,34		2257	4383
Low	60				4.6	1493		89	90	13.6	2163		100	2707	5641
Observed	51	13	924	105	4.5	1133	34	176	103	10.4	2934	13,18,20,	100	2707	5641
Row	60				4.4	1282		261	100	8.2	2357	25,34		2128	4485
Low	60				4.4	1282		115	100	12.6	2407			2153	4561
Observed	55	17	1180	196	5.3	2784	30	252	85	7.8	2107	17,20,	85	2401	4508
Row	61				5.3	3044		263	85	7.6	1911	24,30		2681	4593
Low	61				5.3	3044		105	85	12.2	1959			2693	4652
Observed	55	17	1220	187	5.4	3054	30	280	111	8.6	2854	17,20,	110	2192	5046
Row	61				5.1	2898		365	110	7.3	2462			2430	4893
Low	61				5.1	2898		165	110	11.0	2534			2663	5197
Observed	55	17	1212	180	5.3	2884	30	372	129	8.0	3232	17,20,	135	1896	5128
Row	61				5.0	2785		516	135	6.7	3005	24,30		1906	4910
Low	61				5.0	2785		235	135	10.2	3092			2072	5164

^{1/} Site Index (base age 25) from Goebel and Shipman (1964).
^{2/} Site Index (base age 25) from Clutter and Lenhart (1968).

^{3/} Row thinning, program THINNED.
^{4/} Low thinning, program THINNED.

relative difference for all 9 stands was 0.82%.

Possible Modifications and Refinements

In this study, a flexible method -- the segmented cdf -- was introduced to characterize diameter distributions in forest stands. A growth and yield model for thinned loblolly pine plantations was developed in which the segmented cdf was searched for such that the resulting stand basal area and average dbh estimates were identical to those predicted from stand variables using regression techniques. This study is merely a start in an effort to search for a method of modeling thinned stands using the flexible cdf approach. Although the segmented cdf seemed to be able to approximate diameter distributions in thinned stands where unimodal distributions, such as the Weibull, sometimes failed, there is still room for improvement in the segmented cdf approach and also in yield model for thinned stands.

Some specific areas for further investigation are:

(1) Improvement in the flexibility of the segmented cdf approach can be achieved by requiring more percentile points or join points. This is accompanied by a burden, however, of estimating these new parameters.

(2) Further investigations are also needed to search for an appropriate function form (other than the modified form of the Weibull cdf used in this study) that is both simple and flexible.

(3) Various methods for deriving a distribution in general, and a segmented cdf in particular, from stand attributes for thinned stands need to be more fully evaluated.

(4) More realistic removal patterns for thinning from below should be developed. One possibility is to establish stochastic models in which trees in each dbh class are assigned probabilities of being removed, and are cut or left in each thinning operation depending on values of the random numbers generated.

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APPENDICES

Appendix 1. Method for computing the parameters of the segmented cumulative density function.

The segmented cdf is of the form:

$$F_j(x) = e_j \left\{ 1 - d_j \exp \left[- \left(\frac{x-a_j}{b_j} \right)^{c_j} \right] \right\}, \quad x_j \leq x \leq x_{j+1},$$

$$j = 1, 2, \dots, n-1,$$

where the x_j 's are join points.

If $d_j = e_j = 1$, $F_j(x)$ is a segment of the Weibull cdf.

Computation of the parameters of the segmented cdf is carried out in the following steps.

Step 1: The first two segments; $j = 1, 2$.

One function is used for both of these segments, i.e. $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, and $e_1 = e_2$.

Set $d_1 = e_1$ and $a_1 = x_1$.

From $F_2(x_2) = p_2$ and $F_2(x_3) = p_3$, the remaining parameters can be derived as

$$b_2 = \frac{x_2 - x_1}{\left[-\ln(1-p_2) \right]^{1/c_2}}$$

$$c_2 = \frac{\ln \left[\frac{\ln(1-p_3)}{\ln(1-p_2)} \right]}{\ln \left[\frac{x_3 - x_1}{x_2 - x_1} \right]}$$

and

where $\ln(x)$ denotes natural logarithm of x .

Step 2: Subsequent segments; $j = 3, 4, \dots, n-2$.

Set $d_j = e_j = 1$. For the interval $[x_j, x_{j+1}]$, $F_j(x)$ has to satisfy

- (1) $F_j(x_j) = p_j$,
- (2) $f_j(x_j) = f_{j-1}(x_j)$, and
- (3) $F_j(x_{j+1}) = p_{j+1}$,

where $f_j(x) = dF_j(x)/dx$.

These conditions result in the following relationship between the parameters:

$$b_j = \frac{x_j - a_j}{\left[-\ln \left(\frac{e_j - p_j}{d_j e_j} \right) \right]^{1/c_j}}$$

$$c_j = \frac{(x_j - a_j) \left[\frac{f_{j-1}(x_j)}{e_j - p_j} \right]}{\left[-\ln \left(\frac{e_j - p_j}{d_j e_j} \right) \right]}$$

The location parameter, a_j , is the solution of

$$g(a_j) = F_j(x_{j+1}) - p_{j+1} = 0$$

Newton-Raphson method is used to search for a_j :

$$a_j^{(2)} = a_j^{(1)} - \frac{g(a_j)}{g'(a_j)}$$

where $a_j^{(2)}$ = improved estimate of a_j based on initial guess $a_j^{(1)}$,

$$g'(a_j) = dg(a_j) / da_j \doteq \left[g(a_j+h) - g(a_j) \right] / h ,$$

h = a small positive value.

The search is terminated when $g(a_j)$ is very close to 0. Then the current a_j will be the solution. If convergence does not occur, d_j or e_j has to be adjusted depending on which of the following two cases happens to take place.

Case 1: $g(a_j) < 0$.

The above-mentioned procedures are repeated with

$$d_j = \left[1 - p_j/2 \right] , \left[1 - 3p_j/4 \right] , \left[1 - 7p_j/8 \right] , \dots$$

until convergence is obtained.

Case 2: $g(a_j) > 0$.

Parameter e_j is adjusted:

$$e_j = \left[p_{j+1} + (1-p_{j+1})/2 \right] , \left[p_{j+1} + (1-p_{j+1})/4 \right] , \\ \left[p_{j+1} + (1-p_{j+1})/8 \right] , \dots$$

until convergence is reached.

Step 3: Last segment; $j = n-1$.

Procedures similar to step 2 are applied to the last segment of the cdf, except that e_{n-1} is initially set at 1.001. If convergence does not occur and $g(a_{n-1}) > 0$ (Case 2), e_{n-1} is adjusted as follows:

$$e_{n-1} = \left[1 + .001/2 \right], \left[1 + .001/4 \right], \left[1 + .001/8 \right], \dots$$

Appendix 2a. Input variable formats and description for program
 THINNED -- Subprogram identification card (first card).

Column	Format	Variable	Description
1	I1	I PROG	= 1 = Call INPUT1: project a stand through time. = 2 = Call INPUT2: stand and stock tables for specified combinations of age, site, and density.

Appendix 2b. Input variable formats and descriptions for program
THINNED -- Subprogram INPUT1.

Card Type	Column	Format	Variable	Description
1				STAND DESCRIPTION CARD.
	1-3	F3.0	SI1	Site index in feet (base age 25).
	4-6	F3.0	AGE1	Age in years of the present stand.
	7-10	F4.0	XN1	Number of trees per acre at AGE1.
	11-16	F6.2	BA1	Basal area in square feet per acre at AGE1.
				(Either XN1 or BA1 has to be specified).
	17-18	I2	INDEX	= 1 = XN1 and BA1 are both inputs. = 2 = Only XN1 is input for density. = 3 = Only BA1 is input for density.
	19-23	F5.2	DTHIN1	= 0 = This stand has never been thinned from below. 0 = All trees having dbh below DTHIN1 were cut in a previous low thinning.
	24-26	F3.0	AGE2	Age at the next input or decision period.
	27-28	I2	NDEC	Number of decision cards, each card describes management routine (thinning or not) at a specified age.
	29-30	I2	IOPT	= 0 = No title card for this stand. = 1 = Title card immediately follows this card.
	31-32	I2	MORE	= 0 = No other stand. Stop when this stand is finished. = 1 = Another stand follows.

Appendix 2b. Input variable formats and descriptions for program
THINNED -- Subprogram INPUT1 (cont.).

Card Type	Column	Format	Variable	Description
2				DECISION CARD.
	1-3	F3.0	AGE1	Current stand age, equal to AGE2 specified in the previous card.
	4-6	F3.0	AGE2	Age at the next input or decision period (harvest age if this is the last decision card of this stand).
	7-8	I2	ITHIN	= 1 = No thinning at AGE1. = 2 = Row thinning at AGE1. = 3 = Low thinning at AGE1. = 4 = Row thinning followed by low thinning at AGE1.
	9-10	I2	JOPT	(Needed only when IROW=2 or ILOW=2) = 1 = BTHIN is specified. = 2 = BRESR or BRES is specified.
	11-12	I2	IROW	(Needed only when ITHIN=2 or 4) = 1 = Specify residual ratio (Q). = 2 = Residual ratio not specified.
	13-17	F5.2	Q	= Residual ratio (after/before thinning), when ITHIN=2 and IROW=1. = Ratio of basal area removed in row thinning and total basal area removed, when ITHIN=4 and IROW=2.
	18-23	F6.2	BRESR	(Needed only when JOPT=2 and IROW=2) Residual basal area per acre after row thinning.
	24-29	F6.2	BTHINR	(Needed only when JOPT=1 and IROW=2) Basal area per acre removed in row thinning.
	30-31	I2	ILOW	(Needed only when ITHIN=3 or 4) = 1 = All trees below a specified diameter limit (DTHIN) are cut. = 2 = Thin to a specified residual basal area (BRES).

Appendix 2b. Input variable formats and descriptions for program
THINNED -- Subprogram INPUT1 (cont.).

Card Type	Column	Format	Variable	Description
2	32-36	F5.2	DTHIN	(Needed only when ILOW=1) All trees having dbh below DTHIN are cut.
	37-42	F6.2	BRES	(Needed only when JOPT=2 and ILOW=2) Residual basal area per acre after low thinning.
	43-48	F6.2	BTHIN	(Needed only when JOPT=1 and ILOW=2) Basal area per acre removed in low thinning.

Appendix 2c. Input variable formats and descriptions for program
THINNED -- Subprogram INPUT2.

Column	Format	Variable	Description
1-4	I4	ISB	Site index: Begin
5-8	I4	ISE	End
9-12	I4	ISI	Increment
13-16	I4	IAB	Stand age: Begin
17-20	I4	IAE	End
21-24	I4	IAI	Increment
25-28	I4	INB	Trees/acre: Begin
29-32	I4	INE	End
33-36	I4	INI	Increment
37-40	I4	IBB	Basal area: Begin
41-44	I4	IBE	End
45-48	I4	IBI	Increment
49-52	I4	INDEX	= 1 = Number of trees (IN) and basal area (IB) per acre are both inputs. = 2 = Only IN is input for density. = 3 = Only IB is input for density.
53-56	I4	IOPT	= 0 = No title card. = 1 = Title card immediately follows this card.

Appendix 3a. Input example for program THINNED -- simulate a stand through time.

Stand 1:

Site index = 60 feet (base age 25 years).

Density at age 5 = 600 trees/acre.

Thinning: Age = 17. Amount = 38 sq.ft./acre. Type = ROW.

Age = 22. Amount = 29 sq.ft./acre. Type = ROW.

Harvest age = 30 years.

Title: COILE AND SCHUMACHER (1964)

Stand 2:

Site index = 60 feet (base age 25 years).

Density at age 20 = 600 trees and 150 sq.ft./acre.

Thinning: Age = 20. Thin to 100 sq.ft./acre. Type = 25% ROW,
75% LOW.

Harvest age = 40 years.

Card input:

	1	2	3	4
Column:	1234567890....5....0....5....0....5....0			
1	60	5	600	
		2	17	2 1 1
			COILE AND SCHUMACHER (1964)	
	17	22	2 1 2	38.00
	22	30	2 1 2	29.00
	60	20	600150.00	1 20 1 1 0
			25% ROW, 75% LOW THINNING	
	20	40	4 2 2 0.25	100.00 2 100.00

Appendix 3b. Input example for program THINNED -- stand and stock tables for specified combinations of site index, age, and density.

Combinations:

Site index = 50 feet (base age 25 years).

Stand age = 10, 15, 20, 25, 30 years.

Number of trees = 200, 400, 600, 800 trees/acre

Basal area = 50, 100, 150, 200 sq.ft./acre.

No title wanted.

Card input:

[illegible]

```
C
C *****
C *
C * PROGRAM THINNED PRODUCES STAND AND STOCK *
C * TABLES FOR THINNED LOBLOLLY PINE PLANTATIONS. *
C *
C * DEVELOPED BY QUANG V. CAO *
C * VPI & SU. AUGUST 1, 1981 *
C *
C *****
C CALL ERRSET(208,256,-1,1)
C CALL ERRSET(207,256,-1,1)
C CALL ERRSET(209,256,-1,1)
C CALL ERRSET(262,256,-1,1)
C CALL ERRSET(263,256,-1,1)
500 READ(5,500) IPROG
      FORMAT(I1)
      IF(IPROG.EQ.1) CALL INPUT1
      IF(IPROG.EQ.2) CALL INPUT2
      RETURN
      END
      SUBROUTINE INPUT1
C
C *****
C *
C * SUBROUTINE INPUT1 READS THE NECESSARY INPUTS *
C * FOR SUBROUTINE GROW. *
C *
C *****
C IMPLICIT REAL*8 (A-H,O-Z)
C COMMON /ONE/ C(2), S1, AGE1, XN1, BA1, HD, DMIN, DMED, DMAX, DBAR, IMAX, IMIN
C COMMON /TWO/ S11, AGE1, XN1, BA1, DTHIN1, AGE2, Q, DTHIN, BRES, BRESR, QTHIN
: INDEX, ITHIN, ILOW, IROW
COMMON /THREE/ ITITLE(20), AINV, XNLOG, BLOG, HDLOG, TVOB1, TVOB41
: CVOB41, IOPT, JJJ
COMMON /SPLIN1/ S(4,5), X(5), Y(5), YP, N, N1, J, J1
DATA IBLANK/' '/
C----- READ STAND DESCRIPTION CARD.
C
1 READ(5,500,END=999) S11, AGE1, XN1, BA1, INDEX, DTHIN1, AGE2, NDEC, IOPT
: , MORE
500 FORMAT(2F3.0, F4.0, F6.2, I2, F5.2, F3.0, 3I2)
      ITHIN=1
      JJJ=0
C
C----- READ TITLE CARD IF ANY.
C
DO 2 II=1,20
2 ITITLE(II)=IBLANK
IF(IOPT.EQ.1) READ(5,501) (ITITLE(II), II=1,20)
501 FORMAT(20A4)
      CALL GROW
```

Appendix 4. Source listing of program THINNED (cont.).

```

C----- READ DECISION CARDS.
C
      IF(MORE.EQ.1.AND.NDEC.EQ.0) GO TO 1
      IF(MORE.NE.1.AND.NDEC.EQ.0) RETURN
      DO 3 I=1,NDEC
      READ(5,502) AGE2, ITHIN, JOPT, IROW, Q, BRESR, BTHINR, ILOW, DTHIN, BRES
      :      , BTHIN
502  FORMAT(3X, F3.0, 2I2, 2(12, F5.2, 2F6.2))
      IF(ITHIN.NE.1) JJJ=1
      AGE1=AGE
      XN1=XN
      BA1=BA
      IF(JOPT.EQ.1.AND.IROW.EQ.2) BRESR=BA1-BTHINR
      IF(JOPT.EQ.1.AND.ILOW.EQ.2) BRES=BA1-BTHIN
      IF(JOPT.EQ.1.AND.ILOW.EQ.2.AND.ITHIN.EQ.4) BRES=BRESR-BTHIN
      INDEX=1
      3  CALL GROW
      IF(MORE.EQ.1) GO TO 1
999  RETURN
      END
      SUBROUTINE INPUT2
C
C
C      *****
C      *
C      * SUBROUTINE INPUT2 READS THE NECESSARY INPUTS *
C      * FOR SUBROUTINE YIELD. *
C      *
C      *****
C
      IMPLICIT REAL*8 (A-H, O-Z)
      COMMON /ONE/ C(2), SI, AGE, XN, BA, HD, DMIN, DMED, DMAX, DBAR, IMAX, IMIN
      COMMON /TWO/ SI1, AGE1, XN1, BA1, DTHIN1, AGE2, Q, DTHIN, BRES, BRESR, QTHIN
      :      , INDEX, ITHIN, ILOW, IROW
      COMMON /THREE/ ITITLE(20), AINV, XNLOG, BLOG, HDLOG, TVOB1, TVOB41
      :      , CVOB41, IOPT, JJJ
      COMMON /SPLIN1/ S(4,5), X(5), Y(5), YP, N, N1, J, J1
      DATA IBLANK/' /
      1  READ(5,500,END=999) ISB, ISE, ISI, IAB, IAE, IAI, INB, INE, INI, IBB
      :      , IBE, IBI, INDEX, IOPT
500  FORMAT(14I4)
      DO 2 II=1,20
      2  ITITLE(II)=IBLANK
      IF(IOPT.EQ.1) READ(5,501) (ITITLE(II), II=1,20)
501  FORMAT(20A4)
C
C----- DO LOOPS. CHECK INDEX FOR INPUTS FOR STAND DENSITY.
C

```

```

THI00600
THI00610
THI00620
THI00630
THI00640
THI00650
THI00660
THI00670
THI00680
THI00690
THI00700
THI00710
THI00720
THI00730
THI00740
THI00750
THI00760
THI00770
THI00780
THI00790
THI00800
THI00810
THI00820
THI00830
THI00840
THI00850
THI00860
THI00870
THI00880
THI00890
THI00900
THI00910
THI00920
THI00930
THI00940
THI00950
THI00960
THI00970
THI00980
THI00990
THI01000
THI01010
THI01020
THI01030
THI01040
THI01050
THI01060
THI01070
THI01080
THI01090

```

Appendix 4. Source listing of program THINNED (cont.).

DO 40 IS=ISB,ISE,ISI	THI01100
SI=DFLOAT(IS)	THI01110
DO 30 IA=IAB,IAE,IAI	THI01120
AGE=DFLOAT(IA)	THI01130
AINV=1.DO/AGE	THI01140
CALL HEIGHT	THI01150
AHDI=AINV/HD	THI01160
GO TO (13,11,12), INDEX	THI01170
11 IBB=100	THI01180
IBE=IBB	THI01190
IBI=50	THI01200
GO TO 13	THI01210
12 INB=100	THI01220
INE=INB	THI01230
INI=50	THI01240
13 DO 20 IN=INB,INE,INI	THI01250
GO TO (21,22,23), INDEX	THI01260
21 XN=DFLOAT(IN)	THI01270
XNLOG=DLOG(XN)	THI01280
GO TO 23	THI01290
22 XN=DFLOAT(IN)	THI01300
XNLOG=DLOG(XN)	THI01310
BLOG=-4.39180687D0 + 0.19054366D0*AINV	THI01320
: + 1.34753473D0*HDLOG + 0.63902092D0*XNLOG	THI01330
BA=DEXP(BLOG)	THI01340
DO 10 IB=IBB,IBE,IBI	THI01350
GO TO (31,33,32), INDEX	THI01360
31 BA=DFLOAT(IB)	THI01370
BLOG=DLOG(BA)	THI01380
GO TO 33	THI01390
32 BA=DFLOAT(IB)	THI01400
BLOG=DLOG(BA)	THI01410
XNLOG=7.79805237D0 + 2.10495039D0*AINV	THI01420
: - 1.89908311D0*HDLOG + 1.16743646D0*BLOG	THI01430
XN=DEXP(XNLOG)	THI01440
33 CONTINUE	THI01450
C	THI01460
C----- SOLVE FOR DIAMETER CDF.	THI01470
C	THI01480
CALL YIELD	THI01490
10 CONTINUE	THI01500
20 CONTINUE	THI01510
30 CONTINUE	THI01520
40 CONTINUE	THI01530
GO TO 1	THI01540
999 RETURN	THI01550
END	THI01560

Appendix 4. Source listing of program THINNED (cont.).

```

C      SUBROUTINE GROW
C
C      *****
C      *
C      *   SUBROUTINE GROW PRODUCES A STAND AND STOCK
C      *   TABLE AT AGE1. THE STAND IS THEN SUBJECT TO
C      *   THINNING (OR NO THINNING), AND THEN PROJECTED
C      *   TO AGE2.
C      *
C      *****
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      COMMON /ONE/ C(2), SI, AGE, XN, BA, HD, DMIN, DMED, DMAX, DBAR, IMAX, IMIN
C      COMMON /TWO/ SI1, AGE1, XN1, BA1, DTHIN1, AGE2, Q, DTHIN, BRES, BRESR, QTHIN
C      :      , INDEX, ITHIN, ILOW, IROW
C      COMMON /THREE/ ITITLE(20), AINV, XNLOG, BLOG, HDLOG, TVOB1, TVOB41
C      :      , CVOB41, IOPT, JJJ
C      COMMON /SPLIN1/ S(4,5), X(5), Y(5), YP, N, N1, J, J1
C      DATA B1/0.02273D0/, B2/-0.011103D0/
C      IF(AGE1.EQ.AGE.AND.XN1.EQ.XN.AND.BA1.EQ.BA) GO TO 5
C      IDTHIN=DTHIN1+0.5D0
C      DTHIN1=DFLOAT(IDTHIN)-0.5D0
C      SI=SI1
C      AGE=AGE1
C      AINV=1.D0/AGE
C      CALL HEIGHT
C      GO TO (1,2,3), INDEX
C
C----- INDEX = 1 = BOTH XN1 AND BA1 ARE INPUTS FOR STAND DENSITY.
C
C      1  XNLOG=DLOG(XN1)
C        BLOG=DLOG(BA1)
C        GO TO 4
C
C----- INDEX = 2 = ONLY XN1 IS INPUT FOR STAND DENSITY.
C
C      2  XNLOG=DLOG(XN1)
C        IF(JJJ.EQ.0) BLOG=DLOG(10.D0)*(1.4366D0*DLOG10(SI)-7.084D0*AINV
C      :      +0.4888D0*DLOG10(XN1)-1.3851D0)
C        IF(JJJ.EQ.1) BLOG=-4.39180687D0 + 0.19054366D0*AINV
C      :      + 1.34753473D0*HDLOG + 0.63902092D0*XNLOG
C        BA1=DEXP(BLOG)
C        GO TO 4
C
C----- INDEX = 3 = ONLY BA1 IS INPUT FOR STAND DENSITY.
C
C      3  BLOG=DLOG(BA1)
C        IF(JJJ.EQ.0) XNLOG=DLOG(10.D0)*(1.4366D0*DLOG10(SI)-7.084D0*AINV
C      :      -DLOG10(BA1)-1.3851D0)/(-0.4888D0)
C        IF(JJJ.EQ.1) XNLOG=7.79805237D0 + 2.10495039D0*AINV
C      :      - 1.89908311D0*HDLOG + 1.16743646D0*BLOG
C        XN1=DEXP(XNLOG)
C
C----- SOLVE FOR DIAMETER CDF.
C
C      4  BA=BA1
C        XN=XN1
C        CALL YIELD

```

```

THI01570
THI01580
THI01590
THI01600
THI01610
THI01620
THI01630
THI01640
THI01650
THI01660
THI01670
THI01680
THI01690
THI01700
THI01710
THI01720
THI01730
THI01740
THI01750
THI01760
THI01770
THI01780
THI01790
THI01800
THI01810
THI01820
THI01830
THI01840
THI01850
THI01860
THI01870
THI01880
THI01890
THI01900
THI01910
THI01920
THI01930
THI01940
THI01950
THI01960
THI01970
THI01980
THI01990
THI02000
THI02010
THI02020
THI02030
THI02040
THI02050
THI02060
THI02070
THI02080
THI02090
THI02100
THI02110
THI02120
THI02130
THI02140
THI02150
THI02160

```

Appendix 4. Source listing of program THINNED (cont.).

C		TH102170
C-----	THINNING AT AGE1.	TH102180
C		TH102190
5	CALL THIN	TH102200
	IF(AGE.EQ.AGE2) RETURN	TH102210
C		TH102220
C-----	PROJECT TO AGE2.	TH102230
C		TH102240
	HD1LOG=HDLOG	TH102250
	AINV1=AINV	TH102260
	XN1LOG=XNLOG	TH102270
	AGE=AGE2	TH102280
	AINV=1.D0/AGE2	TH102290
	CALL HEIGHT	TH102300
	C1=HDLOG-HD1LOG	TH102310
	C2=AINV-AINV1	TH102320
	XNPLOG=DLOG10(XN1)	TH102330
	XNPLOG=(XNPLOG-B1*AGE1)/(1.+B2*AGE1)	TH102340
	IF(JJJ.EQ.0) XNLOG=DLOG(10.D0)*(XNPLOG+AGE*(B1+B2*XNPLOG))	TH102350
	IF(JJJ.EQ.1) XNLOG=XNLOG-0.36449027D0*C1-0.12207393D0*C2	TH102360
	XN=DEXP(XNLOG)	TH102370
	IF(JJJ.EQ.0) BLOG=DLOG(10.D0)*(1.4366D0*DLOG10(S1)-7.084D0*AINV	TH102380
:	+0.4888D0*DLOG10(XN)-1.3851D0)	TH102390
	IF(JJJ.EQ.1) BLOG=BLOG+0.66152665D0*C1-10.36715220*C2	TH102400
	BA=DEXP(BLOG)	TH102410
C		TH102420
C-----	SOLVE FOR DIAMETER CDF.	TH102430
C		TH102440
	CALL YIELD	TH102450
	RETURN	TH102460
	END	TH102470
	SUBROUTINE YIELD	TH102480
C		TH102490
C		TH102500
C	*****	TH102510
C	*	TH102520
C	* SUBROUTINE YIELD PRODUCES A STAND AND STOCK *	TH102530
C	* TABLE FOR A SPECIFIED COMBINATION OF AGE, *	TH102540
C	* SITE, AND DENSITY. *	TH102550
C	*	TH102560
C	*****	TH102570
C		TH102580
C		TH102590
	CALL MODEL	TH102600
	CALL DIST	TH102610
	CALL OUTPUT(1)	TH102620
	RETURN	TH102630
	END	TH102640

Appendix 4. Source listing of program THINNED (cont.).

```

SUBROUTINE HEIGHT
*****
*
* SUBROUTINE HEIGHT COMPUTES HEIGHT OF THE
* DOMINANTS AND CODOMINANTS OF A STAND, GIVEN
* SITE INDEX AND AGE.
* FROM JIM DEVAN'S THESIS (1979).
*
*****

IMPLICIT REAL*8 (A-H,O-Z)
COMMON /ONE/ C(2), SI, AGE, XN, BA, HD, DMIN, DMED, DMAX, DBAR, IMAX, IMIN
COMMON /THREE/ ITITLE(20), AINV, XNLOG, BLOG, HDLOG, TVOB1, TVOB41
: , CVOB41, IOPT, JJJ
DATA X0/0.04D0/, XL/0.2D0/, A/8.96178D0/,
: B1/-5.27794D0/, B2/19.90047D0/, B3/-58.76122D0/
X=AINV
Z=DEXP(A*(X-X0))
X0Z=X0*Z
Y0=DLOG(SI)
HDLOG=Y0*Z + B1*(Z-1.D0) + B2*(X0Z-X) + B3*(X0Z*X0-X*X)
HD=DEXP(HDLOG)
RETURN
END
SUBROUTINE MODEL

*****
*
* SUBROUTINE MODEL PREDICTS FROM THE STAND
* CHARACTERISTICS DMIN, DMED, DMAX, AND DBAR.
*
*****

IMPLICIT REAL*8 (A-H,O-Y)
COMMON /ONE/ C(2), SI, AGE, XN, BA, HD, DMIN, DMED, DMAX, DBAR, IMAX, IMIN
COMMON /THREE/ ITITLE(20), AINV, XNLOG, BLOG, HDLOG, TVOB1, TVOB41
: , CVOB41, IOPT, JJJ
DQ=(BA/((0.545415D-2*XN))**0.5D0
DMIN=DEXP(1.10834919D0 + 5.10754613D0*AINV + 0.50530582*HDLOG
: + 0.28543547D0*BLOG - 0.57131133D0*XNLOG)
DMAX=DEXP(2.20990158D0 - 2.14356249D0*AINV + 0.04417647D0*HDLOG
: + 0.41893213D0*BLOG - 0.31662682D0*XNLOG)
DMED=DQ * DEXP(0.15559652D0 - 0.02965339D0*HDLOG
: - 0.00827993*XNLOG)
DBAR=DQ - DEXP(-9.05733308D0 + 0.89273788D0*HDLOG
: + 0.58151144*XNLOG)
IF((DMAX-DMED).GT.(DQ+2.D0)) DMAX=DQ+DQ
1 IF(DMIN.LT.DMED) GO TO 2
DMIN=DMIN-1.D0
GO TO 1
2 IF(DMAX.GT.DMED) RETURN
DMAX=DMAX+1.D0
GO TO 2
END

```

Appendix 4. Source listing of program THINNED (cont).

```

C          SUBROUTINE DIST
C
C          *****
C          *
C          * SUBROUTINE DIST SOLVES FOR CDF OF DBH, GIVEN *
C          * DMIN, DMAX, MEDIAN DBH, AVERAGE DBH, BA, N. *
C          *
C          *****
C
C          IMPLICIT REAL*8 (A-H,O-Y)
C          DIMENSION WA(20),FF(2),W(2)
C          COMMON /ONE/ C(2),SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
C          COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41
C          :          ,CVOB41,IOP,T,JJJ
C          COMMON /SPLIN1/ S(4,5),X(5),Y(5),YP,N,N1,J,J1
C          EXTERNAL FCN
C          DATA NPAR/2/,LWA/20/
C
C-----  INITIALIZE VARIABLES.
C
C          C(1)=-BA
C          C(2)=-DBAR
C          Y(1)=0.00
C          Y(2)=0.2500
C          Y(3)=0.500
C          Y(4)=0.7500
C          Y(5)=1.00
C          I=DMIN-0.500
C          X(1)=I-0.4900
C          I=DMAX+1.500
C          X(5)=I+0.4900
C          IF(X(1).LT.0.00) X(1)=0.00
C          X(3)=DMED
C          W(1)=0.500
C          W(2)=-0.500
C          IMAX=0.500+X(5)
C          IMIN=0.500+X(1)
C          IF(IMIN.LE.0) IMIN=1
C
C-----  SOLVE A SYSTEM OF 2 NONLINEAR EQUATIONS
C          AND 2 UNKNOWN, W(1) AND W(2).
C
C          CALL SOLVE(FCN,W,FF,0.50-2,0.50-4,IER)
C          X(2)=X(1)+0.500*(X(3)-X(1))*(1.00+DERF(W(1)))
C          X(4)=X(3)+0.500*(X(5)-X(3))*(1.00+DERF(W(2)))
C          RETURN
C          END

```

TH103240
 TH103250
 TH103260
 TH103270
 TH103280
 TH103290
 TH103300
 TH103310
 TH103320
 TH103330
 TH103340
 TH103350
 TH103360
 TH103370
 TH103380
 TH103390
 TH103400
 TH103410
 TH103420
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 TH103560
 TH103570
 TH103580
 TH103590
 TH103600
 TH103610
 TH103620
 TH103630
 TH103640
 TH103650
 TH103660
 TH103670
 TH103680
 TH103690
 TH103700
 TH103710
 TH103720

Appendix 4. Source listing of program THINNED (cont.).

```

C          SUBROUTINE SOLVE(FCN,W1,FF,ERROR1,ERROR2,IER)
C
C          *****
C          *
C          *   SUBROUTINE SOLVE SOLVES A SYSTEM OF TWO NON-
C          *   LINEAR EQUATIONS AND TWO UNKNOWNNS.
C          *
C          *****
C
C          IMPLICIT REAL*8 (A-H,O-Y)
C          DIMENSION FFW(2,2),FF1(2),FF2(2),FF3(2),FF(2),W1(2),W2(2),WINC(2)
C          COMMON /SPLIN1/ S(4,5),X(5),Y(5),YP,N,N1,J,J1
C          DATA H/1.D-2/
C          ITER=-1
C          IER=1
C          ICOUNT=0
C
C-----  COMPUTE FF AND P1=(FF)'(FF).
C
C          CALL FCN(W1(1),W1(2),FF)
C          P1=FF(1)*FF(1)+FF(2)*FF(2)
C
C-----  COMPUTE JACOBIAN MATRIX (FFW) AND WINC.
C
C          1  ITER=ITER+1
C             FF3(1)=FF(1)
C             FF3(2)=FF(2)
C             CALL FCN(W1(1)+H,W1(2),FF1)
C             CALL FCN(W1(1),W1(2)+H,FF2)
C             DO 2 I=1,2
C                FFW(I,1)=(-FF1(I)+FF(1))/H
C                FFW(I,2)=(-FF2(I)+FF(1))/H
C          2  DET=FFW(1,1)*FFW(2,2)-FFW(1,2)*FFW(2,1)
C             WINC(1)=(FFW(2,2)*FF(1)-FFW(1,2)*FF(2))/DET
C             WINC(2)=(FFW(1,1)*FF(2)-FFW(2,1)*FF(1))/DET
C             ALPHA=1.D0
C             ICOUNT=0
C
C-----  COMPUTE NEW W AND CHECK WMIN AND WMAX.
C
C          3  W2(1)=W1(1)+ALPHA*WINC(1)
C             W2(2)=W1(2)+ALPHA*WINC(2)
C             GO TO 5
C
C-----  HALF THE STEP SIZE.
C
C          4  ICOUNT=ICOUNT+1
C             IF(ICOUNT.GE.16) RETURN
C             ALPHA=0.5D0*ALPHA
C             GO TO 3
C
C-----  COMPUTE NEW FF AND P2.
C
C          5  CALL FCN(W2(1),W2(2),FF)
C             P2=FF(1)*FF(1)+FF(2)*FF(2)

```

Appendix 4. Source listing of program THINNED (cont.).

```

C
C----- CHECK IF THE PERFORMANCE INDEX IS IMPROVED
C          (P2 < P1) AND CHECK FOR CONVERGENCE.
C
      IF(P2.LT.ERROR1) GO TO 6
      IF(P2.GE.P1) GO TO 4
      IF(P1-P2.LT.ERROR2) RETURN
C
C----- GOOD MOVE : RESET THE VARIABLES.
C
      W1(1)=W2(1)
      W1(2)=W2(2)
      P1=P2
      GO TO 1
6     CONTINUE
      W1(1)=W2(1)
      W1(2)=W2(2)
      IER=0
      RETURN
      END
      SUBROUTINE FCN(W1,W2,FF)
C
C
C          *****
C          *
C          * SUBROUTINE FCN IS CALLED BY SUBROUTINE SOLVE *
C          * TO EVALUATE THE VECTOR FUNCTION FF.          *
C          *
C          *****
C
      IMPLICIT REAL*8 (A-H,O-Y)
      DIMENSION FF(2),W(2)
      COMMON /ONE/ C(2),SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
      COMMON /TWO/ S11,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
      :      ,INDEX,ITHIN,ILOW,IROW
      COMMON /SPLIN1/ S(4,5),X(5),Y(5),YP,N,N1,J,J1
C
C----- FIT THE SEGMENTED CDF.
C
      X(2)=X(1)+0.5D0*(X(3)-X(1))*(1.D0+DERF(W1))
      X(4)=X(3)+0.5D0*(X(5)-X(3))*(1.D0+DERF(W2))
      CALL SPLINE
C
C----- INITIALIZATION.
C
      FF(1)=0.D0
      FF(2)=C(2)
      IF(X(1).LT.DTHIN1) GO TO 2
      F1=0.D0
C
C----- COMPUTE FF.
C
      DO 1 I=IMIN,IMAX
      XI=DFLOAT(I)
      F2=CDF(XI+0.5D0)
      F=F2-F1
      IF(F.LT.0.D0) F=0.D0
      F1=F2
      FF(1)=FF(1)+XI*XI*F
1     FF(2)=FF(2)+F*XI
      FF(1)=C(1)+FF(1)*0.545415D-2*XN
      RETURN

```

Appendix 4. Source listing of program THINNED (cont.).

```

C
C----- WHEN DMIN IS LOWER THAN DTHIN1.
C
2 F1=CDF(DTHIN1)
  FRES=1.D0-F1
  IMIN1=DTHIN1+0.51D0
  DO 3 I=IMIN1,IMAX
    XI=DFLOAT(I)
    F2=CDF(XI+0.5D0)
    F=(F2-F1)/FRES
    IF(F.LT.0.D0) F=0.D0
    F1=F2
    FF(1)=FF(1)+F*XI*XI
3  FF(2)=FF(2)+F*XI
  FF(1)=C(1)+FF(1)*0.54545D-2*NXN
  RETURN
  END
  DOUBLE PRECISION FUNCTION CDF(XX)
C
C
C *****
C *
C * FUNCTION CDF EVALUATES THE SEGMENTED CDF.
C *
C *****
C
  IMPLICIT REAL*8 (A-H,O-Y)
  COMMON /SPLIN1/ S(4,5),X(5),Y(5),YP,N,N1,J,J1
  CDF=0.D0
  IF(XX.LE.X(1)) RETURN
  IF(XX.GE.X(N)) GO TO 2
  K=N1
  DO 1 I=1,N1
1  IF(XX.GT.X(I)) K=I
    C1=S(K,3)*DLOG((XX-S(K,1))/S(K,2))
    C2=0.D0
    IF(C1.GT.-50.D0.AND.C1.LT.50.D0) C2=-DEXP(C1)
    IF(C1.GT.50.D0) C2=-1.D8
    C3=0.D0
    IF(C2.GT.-50.D0) C3=DEXP(C2)
    CDF=S(K,5)*(1.D0-S(K,4)*C3)
    IF(CDF.GT.1.D0) CDF=1.D0
  RETURN
2  CDF=1.D0
  RETURN
  END

```

TH104930
 TH104940
 TH104950
 TH104960
 TH104970
 TH104980
 TH104990
 TH105000
 TH105010
 TH105020
 TH105030
 TH105040
 TH105050
 TH105060
 TH105070
 TH105080
 TH105090
 TH105100
 TH105110
 TH105120
 TH105130
 TH105140
 TH105150
 TH105160
 TH105170
 TH105180
 TH105190
 TH105200
 TH105210
 TH105220
 TH105230
 TH105240
 TH105250
 TH105260
 TH105270
 TH105280
 TH105290
 TH105300
 TH105310
 TH105320
 TH105330
 TH105340
 TH105350
 TH105360
 TH105370
 TH105380
 TH105390

Appendix 4. Source listing of program THINNED (cont.).

```

C      SUBROUTINE SPLINE
C
C      *****
C      *
C      *   SUBROUTINE SPLINE FITS A SEGMENTED CDF TO
C      *   N PAIRS OF DATA POINTS.
C      *
C      *****
C
C      IMPLICIT REAL*8 (A-H,O-Y)
C      DIMENSION W(4)
C      COMMON /ONE/ C(2), SI, AGE, XN, BA, HD, DMIN, DMED, DMAX, DBAR, IMAX, IMIN
C      COMMON /SPLIN1/ S(4,5), X(5), Y(5), YP, N, N1, J, J1
C      DATA H/1.D-2/
C
C----- FIT THE FIRST THREE POINTS.
C
C      J=1
C      S(1,4)=1.D0
C      S(1,5)=1.D0
C      S(1,1)=X(1)
C      C1=DLOG(1.D0-Y(2))
C      C2=1.D0-Y(3)
C      C3=DLOG(DLOG(C2)/C1)
C      C4=X(2)-X(1)
C      C5=X(3)-X(1)
C      S(1,3)=C3/DLOG(C5/C4)
C      C6=DLOG(-C1)/S(1,3)
C      S(1,2)=1.D8
C      IF(C6.GT.-50.D0) S(1,2)=C4/DEXP(C6)
C      DO 1 I=1,5
1  S(2,I)=S(1,I)
C      C6=(S(2,3)-1.D0)*DLOG(C5/S(2,2))
C      YP=0.D0
C      IF(C6.GT.-50.D0) YP=C2*DEXP(C6)*S(2,3)/S(2,2)
C
C----- SUBSEQUENT SPLINES.
C
C      DO 7 J=3,N1
C      J1=J+1
C      C1=X(J)-H
C      AA=X(1)
C      K3=0
20  K=0
C      K1=0
C      K2=0
C      A=AA
C      S(J,4)=1.D0
C      S(J,5)=1.D0
C      IF(J.EQ.N1) S(J,5)=1.001D0
C      FA1=0.D0

```

```

TH105400
TH105410
TH105420
TH105430
TH105440
TH105450
TH105460
TH105470
TH105480
TH105490
TH105500
TH105510
TH105520
TH105530
TH105540
TH105550
TH105560
TH105570
TH105580
TH105590
TH105600
TH105610
TH105620
TH105630
TH105640
TH105650
TH105660
TH105670
TH105680
TH105690
TH105700
TH105710
TH105720
TH105730
TH105740
TH105750
TH105760
TH105770
TH105780
TH105790
TH105800
TH105810
TH105820
TH105830
TH105840
TH105850
TH105860
TH105870
TH105880
TH105890
TH105900
TH105910
TH105920

```

Appendix 4. Source listing of program THINNED (cont.).

C		TH105930
C-----	SEARCH FOR THE LOCATION PARAMETER S(J,1).	TH105940
C		TH105950
2	S(J,1)=A	TH105960
	K=K+1	TH105970
	FAH=F(S(J,1)+H)	TH105980
	FA=F(S(J,1))	TH105990
	FPA=(FAH-FA)/H	TH106000
	DFA=DABS(FA)	TH106010
	IF(DFA.LE.1.D-3) GO TO 6	TH106020
	IF(DABS(FA-FA1).LE.1.D-5) GO TO 4	TH106030
	IF((DFA.GE.1.D-2).AND.(K.GE.25)) GO TO 4	TH106040
	IF(FPA.EQ.0.D0) GO TO 4	TH106050
	FA1=FA	TH106060
	AINC=FA/FPA	TH106070
3	A=S(J,1)-AINC	TH106080
	IF(A.LT.C1) GO TO 2	TH106090
	AINC=AINC*0.5D0	TH106100
	GO TO 3	TH106110
C		TH106120
C-----	NEED TO ADJUST VALUES OF S(J,4) OR S(J,5).	TH106130
C		TH106140
4	K=0	TH106150
	IF(FA.GT.0.D0) GO TO 5	TH106160
	K1=K1+1	TH106170
	S(J,4)=1.D0-Y(J)+Y(J)/(2.D0**K1)	TH106180
	IF(K1.GT.15) GO TO 60	TH106190
	A=AA	TH106200
	GO TO 2	TH106210
5	K2=K2+1	TH106220
	IF(K2.GE.6) GO TO 60	TH106230
	S(J,5)=Y(J1)+(1.D0-Y(J1))/(2.D0**K2)	TH106240
	IF(J.EQ.N1) S(J,5)=1.D0+1.D-3/(2.D0**K2)	TH106250
	A=AA	TH106260
	GO TO 2	TH106270
60	K3=K3+1	TH106280
	IF(K3.GE.10) GO TO 6	TH106290
	IF(K3.LT.4) AA=X(1)+(X(J)-X(1))*0.1D0*DFLOAT(K3)	TH106300
	IF(K3.GE.4) AA=X(J)-(X(J)-X(1))/(2.D0**(K3-3))	TH106310
	GO TO 20	TH106320
C		TH106330
C-----	CONVERGENCE ATTAINED. COMPUTE YP, WHICH IS THE PDF	TH106340
C	EVALUATED AT X(J+1).	TH106350
C		TH106360
6	C1=(X(J1)-S(J,1))/S(J,2)	TH106370
	C2=(S(J,3)-1.D0)*DLOG(C1)	TH106380
	C3=0.D0	TH106390
	IF(C2.GT.-50.D0) C3=DEXP(C2)	TH106400
	C4=C3*C1	TH106410
	C5=0.D0	TH106420
	IF(C4.LT.50.D0) C5=DEXP(-C4)	TH106430
	YP=S(J,5)*S(J,4)*S(J,3)*C3*C5/S(J,2)	TH106440
7	CONTINUE	TH106450
	RETURN	TH106460
	END	TH106470

Appendix 4. Source listing of program THINNED (cont.).

```

C      DOUBLE PRECISION FUNCTION F(A)
C
C      *****
C      *
C      * FUNCTION F IS USED TO SEARCH FOR THE LOCATION *
C      * PARAMETER, S(J,1). IT ALSO COMPUTES THE OTHER *
C      * TWO PARAMETERS, S(J,2) AND S(J,3). *
C      *
C      *****
C
C      IMPLICIT REAL*8 (A-H,O-Y)
C      COMMON /SPLIN1/ S(4,5),X(5),Y(5),YP,N,N1,J,J1
C      C1=X(J)-A
C      1 C2=S(J,5)-Y(J)
C      C3=-DLOG(C2/(S(J,4)*S(J,5)))
C      IF(C3.GT.0.D0) GO TO 2
C      F=1.D8
C      RETURN
C
C----- COMPUTE S(J,2) AND S(J,3).
C
C      2 C4=C2*C3
C      S(J,3)=1.D8
C      IF(C4.GT.1.D-20) S(J,3)=C1*YP/C4
C      C5=0.D0
C      IF(S(J,3).GT.1.D-20) C5=DLOG(C3)/S(J,3)
C      S(J,2)=1.D8
C      IF(C5.GT.50.D0) S(J,2)=1.D-8
C      IF(C5.GT.-50.D0.AND.C5.LT.50.D0) S(J,2)=C1/DEXP(C5)
C
C----- COMPUTE F = F(J)(X(J+1)) - Y(J+1)
C
C      C1=DLOG((X(J1)-A)/S(J,2))
C      C2=S(J,3)*C1
C      C3=0.D0
C      IF(C2.GT.-50.D0.AND.C2.LT.50.D0) C3=DEXP(C2)
C      IF(C2.GT.50.D0) C3=1.D8
C      C4=0.D0
C      IF(C3.LT.50.D0) C4=DEXP(-C3)
C      F=S(J,5)*(1.D0-S(J,4)*C4)-Y(J1)
C      RETURN
C      END
C      SUBROUTINE OUTPUT(III)
C
C      *****
C      *
C      * SUBROUTINE OUTPUT PRINTS THE STAND AND STOCK *
C      * TABLE. *
C      *
C      *****
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      DIMENSION CF(20),ROB(3),RIB(3),BOB(2),BIB(2),BH(2)
C      COMMON /ONE/ C(2),SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
C      COMMON /TWO/ S11,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
C      : INDEX,ITHIN,ILOW,IROW
C      COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41
C      : CVOB41,IOP,JJJ
C      COMMON /SPLIN1/ S(4,5),X(5),Y(5),YP,N,N1,J,J1

```

Appendix 4. Source listing of program THINNED (cont.).

```

DATA ROB/-0.3235400, 3.157900, -2.711500/
: ,RIB/-0.3520600, 3.076300, -2.654000/
: ,BOB/ 0.3486400, 0.0023200/
: ,BIB/ 0.1169100, 0.0018500/
: ,TOP/4.00/,KROW/'ROW '/,KLOW/'LOW '/,KTYPE/' '/
DATA CF/0.,0.,0.,0.,84.,85.,87.,90.,91.,92.,93.,94.,95.,95.,95.
: ,95.,95.,95.,95.,95./
BH(1)=0.4615154000 + 0.4327452100*AINV + 0.9333308100*HDLOG
: - 0.0858328800*BLOG + 0.07596439*XNLOG
BH(2)=-2.1531226400
TOPOB=TOP**ROB(2)
TOPIB=TOP**RIB(2)
C
C----- WRITE HEADINGS.
C
IF(III.EQ.2) GO TO 11
WRITE(6,666) (ITITLE(II),II=1,20)
666 FORMAT('1'//10X,20A4)
WRITE(6,599) SI,HD,AGE,DBAR,XN,DMIN,BA,DMED,DMAX
599 FORMAT(/33X,'INPUTS',22X,'PREDICTED'/33X,6(' - '),22X,9(' - '))
: /31X,'SITE =' ,F7.2,18X,'HD =' ,F6.2
: /32X,'AGE =' ,F7.2,9X,'AVERAGE DBH =' ,F6.2
: /20X,'NUMBER OF TREES =' ,F7.2,9X,'MINIMUM DBH =' ,F6.2
: /25X,'BASAL AREA =' ,F7.2,10X,'MEDIAN DBH =' ,F6.2
: /53X,'MAXIMUM DBH =' ,F6.2)
GO TO 12
11 KTYPE=KLOW
IF(ITHIN.NE.3) KTYPE=KROW
WRITE(6,600) KTYPE
600 FORMAT(/15X,'BEFORE ' ,A4,'THINNING'/15X,6(' - '), ' --- ' ,8(' - '))
WRITE(6,601) SI,AGE1,XN,BA,DBAR
601 FORMAT(/42X,'SITE =' ,F7.2/43X,'AGE =' ,F7.2/31X,'NUMBER OF '
: 'TREES =' ,F7.2/36X,'BASAL AREA =' ,F7.2
: /35X,'AVERAGE DBH =' ,F7.2)
12 IF(A.LT.DTHIN1) WRITE(6,602) DTHIN1
602 FORMAT(/26X,'THIS STAND WAS PREVIOUSLY THINNED FROM BELOW'
: /26X,'ALL TREES UNDER',F5.1,' INCHES IN DBH WERE CUT')
IF(III.EQ.2) WRITE(6,603) KTYPE
603 FORMAT(/15X,'AFTER ' ,A4,'THINNING'/15X,5(' - '), ' --- ' ,8(' - '))
WRITE(6,604) TOP,TOP
604 FORMAT(/50X,'TOTAL' ,6X,'TOTAL' ,5X,'VOLUME' ,5X,'VOLUME'
: /9X,'DBH' ,5X,'NUMBER' ,4X,'AVERAGE' ,5X,'BASAL' ,5X
: /VOLUME' ,5X,'VOLUME' ,7X,'O.B.' ,7X,'I.B.'
: /8X,'CLASS' ,3X,'OF TREES' ,4X,'HEIGHT' ,6X,'AREA' ,7X
: /O.B.' ,7X,'I.B.' ,4X,'TO' ,F3.0,' IN' ,4X,'TO' ,F3.0,' IN'//)
C
C----- INITIALIZATION.
C
F1=0.00
DAVG=0.00
BB=0.00
XNRES=0.00
TVOB=0.00
TVIB=0.00
TVOB4=0.00
TVIB4=0.00
CVOB4=0.00
XNT=XN
IMIN1=IMIN
IF(III.EQ.2) GO TO 13

```

Appendix 4. Source listing of program THINNED (cont.).

```

      IF(A.GE.DTHIN1) GO TO 3
      F1=CDF(DTHIN1)
      IMIN1=DTHIN1+0.51D0
      XNT=XN/(1.D0-F1)
      GO TO 3
13  IF(ITHIN.EQ.1) GO TO 3
      IF(ITHIN.EQ.3) GO TO 2
      IF(A.LT.DTHIN1) GO TO 1
C
C----- ROW THINNING. NO PREVIOUS LOW THINNING.
C
      XNT=XN*Q
      GO TO 3
C
C----- ROW THINNING. PREVIOUS LOW THINNING.
C
      1 F1=CDF(DTHIN1)
        XNT=XN*Q/(1.D0-F1)
        IMIN1=DTHIN1+0.51D0
        GO TO 3
C
C----- LOW THINNING.
C
      2 F1=CDF(DTHIN)
        IF(A.LT.DTHIN1) XNT=XN/(1.D0-CDF(DTHIN1))
        IMIN1=DTHIN+0.51D0
C
C----- LOOP OVER DBH CLASSES.
C
      3 CONTINUE
        DO 5 I=IMIN1,IMAX
          XI=DFLOAT(I)
          F2=CDF(XI+0.5D0)
          F=XNT*(F2-F1)
          IF(1.EQ.IMIN1.AND.111.EQ.2) F=F*QTHIN
          IF(F.LT.0.D0) F=0.D0
          F1=F2
          XI2=XI*XI
          BASAL=0.545415D-2*XI2*F
          H=DEXP(BH(1)+BH(2)/XI)
          D2H=XI2*H
          VOB=F*(BOB(1)+BOB(2)*D2H)
          VIB=F*(BIB(1)+BIB(2)*D2H)
          VOB4=0.D0
          VIB4=0.D0
          IF(1.LT.5) GO TO 4
          VOB4=VOB*(1.D0+ROB(1)*TOPOB*XI**ROB(3))
          VIB4=VIB*(1.D0+RIB(1)*TOPIB*XI**RIB(3))
          IF(1.LE.20) CVOB4=CVOB4+VOB4/CF(1)

```

```

TH107710
TH107720
TH107730
TH107740
TH107750
TH107760
TH107770
TH107780
TH107790
TH107800
TH107810
TH107820
TH107830
TH107840
TH107850
TH107860
TH107870
TH107880
TH107890
TH107900
TH107910
TH107920
TH107930
TH107940
TH107950
TH107960
TH107970
TH107980
TH107990
TH108000
TH108010
TH108020
TH108030
TH108040
TH108050
TH108060
TH108070
TH108080
TH108090
TH108100
TH108110
TH108120
TH108130
TH108140
TH108150
TH108160
TH108170
TH108180
TH108190

```

Appendix 4. Source listing of program THINNED (cont.).

```

4  DAVG=DAVG+F*XI
   BB=BB+BASAL
   XNRES=XNRES+F
   TVOB=TVOB+VOB
   TVIB=TVIB+VIB
   TVOB4=TVOB4+VOB4
   TVIB4=TVIB4+VIB4
5  WRITE(6,605) I,F,H,BASAL,VOB,VIB,VOB4,VIB4
605  FORMAT(111,7F11.1)
C
C-----  END LOOP.
C
   DAVG=DAVG/XNRES
   IF(111.EQ.2) GO TO 7
   WRITE(6,608) XNRES,BB,TVOB,TVIB,TVOB4,TVIB4,DAVG,
608  TOP,CVOB4
   FORMAT(16X,6(' '),11X,5(5X,6(' '))/11X,F11.1,11X,5F11.1
:  //35X,'AVERAGE DBH =',F7.2
:  /27X,'CORD VOLUME TO',F3.0,'IN =',F7.2)
   C1=DABS(BA-BB)
   C2=DABS(DBAR-DAVG)
   IER=1
   IF(C1.LT.0.07.AND.C2.LT.0.07) IER=0
   IF(1.EQ.0) WRITE(6,609)
609  FORMAT(/35X,'CONVERGENCE ATTAINED')
   IF(1.EQ.0) WRITE(6,610)
610  FORMAT(/39X,'NO CONVERGENCE')
   WRITE(6,611)
611  FORMAT(////4X,'K',6X,'X(K)',8X,'X(K+1)',16X,'S(K,1)',
:  11X,'S(K,2)',11X,'S(K,3)',11X,'S(K,4)',11X,'S(K,5)'////)
   DO 6 K=1,4
6  WRITE(6,612) K,X(K),X(K+1),(S(K,I),I=1,5)
612  FORMAT(/15,2F13.8,5X,5F17.8)
   GO TO 8
7  WRITE(6,606) XNRES,BB,TVOB,TVIB,TVOB4,TVIB4
606  FORMAT(16X,6(' '),11X,5(5X,6(' '))/11X,F11.1,11X,5F11.1)
   WRITE(6,601) SI,AGE1,XNRES,BB,DAVG
   XNTHIN=XN-XNRES
   BATHIN=BA-BB
   TVTHIN=TVOB1-TVOB
   TV4T=TVOB41-TVOB4
   CV4T=CVOB41-CVOB4
   WRITE(6,607) XNTHIN,BATHIN,TVTHIN,
607  TOP,TV4T,
   TOP,CV4T
   FORMAT(/15X,'AMOUNT REMOVED IN THINNING'
:  /15X,6(' '),1X,7(' '),1X,'--',1X,8(' ')
:  //31X,'NUMBER OF TREES =',F7.2
:  /36X,'BASAL AREA =',F7.2
:  /22X,'TOTAL CU.FT. VOLUME O.B. =',F7.2
:  /20X,'CU.FT. VOLUME O.B. TO',F3.0,'IN =',F7.2
:  /27X,'CORD VOLUME TO',F3.0,'IN =',F7.2)
   XN=XNRES
8  BA=BB
   DAVG=DBAR
   TVOB1=TVOB
   TVOB41=TVOB4
   CVOB41=CVOB4
   RETURN
   END

```

Appendix 4. Source listing of program THINNED (cont.).

```
C SUBROUTINE THIN C TH108770  
C C TH108780  
C C TH108790  
C ***** TH108800  
C * TH108810  
C * SUBROUTINE THIN TAKES CARE OF THE THINNING * TH108820  
C * OPTIONS AT AGE1. * TH108830  
C * * TH108840  
C ***** TH108850  
C TH108860  
C TH108870  
C IMPLICIT REAL*8 (A-H,O-Z) TH108880  
COMMON /ONE/ C(2), S1,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN TH108890  
COMMON /TWO/ S11,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN TH108900  
: INDEX,IHIN,ILOW,IROW TH108910  
COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDLG,TVOB1,TVOB41 TH108920  
: CVOB41,IOP,JJJ TH108930  
COMMON /SPLIN1/ S(4,5),X(5),Y(5),YP,N,N1,J,J1 TH108940  
QTHIN=1.DO TH108950  
GO TO (1,2,3,2), IHIN TH108960  
C----- IHIN = 1 = NO THINNING AT AGE1. TH108970  
C TH108980  
C 1 RETURN TH108990  
C TH109000  
C TH109010  
C----- IHIN = 2 = ROW THINNING AT AGE1. EVERYTHING IS REDUCED TH109020  
C BY A FACTOR Q. TH109030  
C TH109040  
C 2 IF(IHIN.EQ.2.AND.IROW.EQ.2) Q=BRESR/BA TH109050  
IF(IHIN.EQ.4.AND.IROW.EQ.2) Q=1.DO-Q*(1.DO-BRESR/BA) TH109060  
Q1=100.DO-Q*100.DO TH109070  
WRITE(6,666) (ITITLE(I),I=1,20) TH109080  
666 FORMAT('1'//1X,20A4) TH109090  
WRITE(6,600) AGE1,Q1 TH109100  
600 FORMAT('//32X,'ROW THINNING AT AGE',F4.0 TH109110  
: //F26.2,'% OF TREES IN ALL DIAMETER CLASSES ARE CUT') TH109120  
IF(Q1.GE.100.DO) RETURN TH109130  
CALL OUTPUT(2) TH109140  
IF(IHIN.EQ.2) GO TO 10 TH109150  
IHIN=3 TH109160  
C----- IHIN = 3 = LOW THINNING AT AGE1. TH109170  
C TH109180  
C 3 GO TO (4,5), ILOW TH109190  
C TH109200  
C----- ILOW = 1 = ALL TREES HAVING DBH LESS THAN DTHIN ARE CUT. TH109210  
C TH109220  
C TH109230  
C 4 IDTHIN=DTHIN+0.5DO TH109240  
DTHIN=DFLOAT(IDTHIN)-0.5DO TH109250  
IF(DTHIN.LT.A.OR.DTHIN.LT.DTHIN1) RETURN TH109260  
WRITE(6,666) (ITITLE(I),I=1,20) TH109270  
WRITE(6,601) AGE1,DTHIN TH109280  
601 FORMAT('//32X,'LOW THINNING AT AGE',F4.0 TH109290  
: //23X,'ALL TREES UNDER',F5.1,' INCHES DBH ARE CUT') TH109300  
CALL OUTPUT(2) TH109310  
DTHIN=DTHIN TH109320  
GO TO 10 TH109330
```

Appendix 4. Source listing of program THINNED (cont.).

```

C----- ILOW = 2 = THIN TO A SPECIFIED RESIDUAL BASAL AREA (BRES).
C
5 BTHIN=BA-BRES
  BB=0.D0
  IF(A.LT.DTHIN1) GO TO 6
  F1=0.D0
  XNT=XN
  IMIN1=IMIN
  GO TO 7
6 F1=CDF(DTHIN1)
  XNT=XN/(1.D0-F1)
  IMIN1=DTHIN1+0.51D0
C
C----- FIND DTHIN CORRESPONDING TO BRES.
C
7 DO 8 I=IMIN1,IMAX
  X1=DFLOAT(I)
  F2=CDF(X1+0.5D0)
  F=XNT*(F2-F1)
  IF(F.LT.0.D0) F=0.D0
  F1=F2
  BASAL=0.545415D-2*F*X1*X1
  BB=BB+BASAL
  IF(BB.GT.BTHIN) GO TO 9
8 CONTINUE
C
C----- QTHIN IS THE RESIDUAL PROPORTION (AFTER / BEFORE THINNING)
C          OF THE DBH CLASS WHOSE LOWER LIMIT IS DTHIN.
C
9 QTHIN=(BB-BTHIN)/BASAL
  DTHIN=X1-0.5D0
  WRITE(6,666) (ITITLE(I),I=1,20)
  WRITE(6,602) AGE1,BRES
602 FORMAT(/32X,'LOW THINNING AT AGE',F4.0
:      //23X,'THIN TO',F7.2,' SQ.FT. RESIDUAL BASAL AREA')
  IF(BRES.LE.0.D0) RETURN
  CALL OUTPUT(2)
  DTHIN1=DTHIN
10 XNLOG=DLOG(XN)
  BLOG=DLOG(BA)
  RETURN
  END
  BLOCK DATA
  IMPLICIT REAL*8 (A-H,O-Z)
  COMMON /ONE/ C(2),SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
  COMMON /TWO/ SI1,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
:      ,INDEX,ITHIN,ILOW,IROW
  COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41
:      ,CVOB41,IOP,JJJ
  COMMON /SPLIN1/ S(4,5),X(5),Y(5),YP,N,N1,J,J1
  DATA AGE/0.D0/,XN/0.D0/,BA/0.D0/,DTHIN1/0.D0/,N/5/,N1/4/,ITHIN/1/
  END

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TH109340
 TH109350
 TH109360
 TH109370
 TH109380
 TH109390
 TH109400
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 TH109440
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EMPIRICAL DIAMETER DISTRIBUTIONS AND
PREDICTED YIELDS OF THINNED LOBLOLLY PINE PLANTATIONS

by

Quang Van Cao

(ABSTRACT)

A flexible method to approximate diameter distributions in forest stands was developed. Different functions, each in the form of a modified Weibull cumulative distribution function (cdf), were joined together to form a segmented cdf. Five percentile points (the 0th, 25th, 50th, 75th, and 100th percentiles) were used to determine a segmented cdf. Based on the one-sample Kolmogorov-Smirnov goodness-of-fit statistic, the segmented cdf was found to perform almost as well as the Weibull distribution for unthinned data and to be superior in case of thinned stands, especially when the data were irregular.

A growth and yield model for thinned loblolly pine plantations was developed based on data from 128 0.2-acre permanent plots in the Virginia Piedmont and Coastal Plain. The segmented cdf which characterized the diameter distribution of a stand was searched for such that the resulting total basal area and average dbh estimates were identical to those predicted from stand variables using regression equations. Program THINNED was written in standard FORTRAN to provide stand and stock tables for thinned old-field loblolly pine plantations.

Trials with several thinning intensities at ages 15, 20, and 25 and a final harvest at age 30 showed that, for average sites (site index 60 feet at base age 25 years), residual basal areas of 95 square feet per acre resulted in approximately the same total volume (volume removed in thinnings plus final harvest volume) production as unthinned stands. Heavier thinnings (i.e. residual basal areas of less than 95 square feet) resulted in a slight decrease in total volume production, whereas residual basal areas of more than 95 square feet showed a slight increase in total volume production.

Estimates from the new model was compared with those from Coile and Schumacher (1964). Total volume production predicted by program THINNED was little affected by thinning, as confirmed by many researchers. The Coile and Schumacher model, however, predicts higher stand response to thinning and consequently its predicted total volumes were consistently higher than those from program THINNED. On the other hand, total volumes resulting from periodic thinnings predicted from the new model agreed closely with observed yields reported by Goebel et al. (1974).