DO UNIQUE CONCEPTIONS OF CONSTRUCTS OVERTLY INFLUENCE STUDENTS' USE OF THE REPRESENTATIVENESS HEURISTIC? NOT QUITE!

Modiu Olaguro
Virginia Tech
olaguro@vt.edu

Jesse L. M. Wilkins
Virginia Tech
wilkins@vt.edu

The predictive basis of the constructs associated with the use of the representativeness heuristic has rarely been investigated. To address this gap, we obtained, via written responses, qualitative and quantitative data on high school students' (n=70) conceptions of randomness, fairness, and independence, parsing them into categories to allow us to infer if they influence students' use of the heuristic. Amidst variations within each construct, we found no relationship between the unique ways randomness and independence are conceived and subsequent use of heuristic. Our study not only provides quantitative support for the naiveness of certain conceptions of constructs but their limited potential to serve predictive purposes. Implications for research and instruction follow.

Keywords: Cognition, High School Education, Probability

Findings from probabilistic conceptions research suggest that although the use of the representativeness heuristic is one of the most researched, its understanding remains at the descriptive level (see Tenenbaum & Griffiths, 2001). In their explication, Kahneman and Tversky (1972) described the heuristic as a mental shortcut taken by individuals in their dealings with uncertainty. To these authors, the use of the representativeness heuristic fits into rendering verdicts on a chance event by assuming that "it is: (i) similar in essential properties to its parent population; and (ii) reflects the salient features of the process by which it is generated" (p. 431).

Incorporated in this is the view that these assumptions owe to certain conceptions of uncertainty that inform the heuristic use. They include the fairness of a chance object, perceptions of independence of trials, or randomness of a process. But despite grounding the heuristic within these constructs, researchers continue to background their presumed effects on its use. However, evidence suggesting that probabilistic constructs are conceived in a plurality of ways (Ingram, 2024) might be a reason to infer that the use of the heuristic might be directly influenced by the unique ways students conceive of these constructs. To this end, we report on the findings of a study that sought to answer the following questions: (1) How do students conceive of fairness, randomness, and independence? (2) How do students' unique conceptions of fairness, randomness, and independence influence their use of the representativeness heuristic?

Theoretical Framework

Conceptions of Probabilistic Constructs

The broad nature of the probability register implies that reasoning under uncertainty might depend on an understanding of multiple foundational constructs. Hence, a grasp of any of the approaches to probability requires facility with constructs such as random, likely, possible, or certain (Batanero & Diaz, 2007), motivating researchers to delve into the extent students' conceptions of these constructs align with the views of experts. In one of them, Pratt (1998) engaged 16 10-11 years-olds in a computer microworld designed so the sampling distribution may vary as to cause the students to infer randomness from the experimental outcomes. Pratt realized that children's attempt at ensuring that the devices were random followed from their view that random generators are fair, their outcomes should be unpredictable and irregular, and its process usually lacking in control.

Similarly, Kazima (2007) elicited first year middle schoolers' conceptions of likely, unlikely, certain, and impossible. She assumed that since the students had not yet taken formal probability, their conceptions of these constructs may fit outside of school probability. Furthermore, by seeking responses via the local language (Chichewa), she expected conceptions to be mediated by the language structure. Her analysis not only aligned with her assumptions but revealed an interplay of both. Specifically, she found that among some students that held conceptions that aligned with formal probability, the structural differences between English and Chichewa caused them to confuse constructs and their complements (e.g., certain and impossible). Across these studies, it is apparent that the devotion of researchers to the ways chance constructs are conceived suggests the existence of a probability register that shares affinities with others but is nevertheless unique in its formulation.

Constructs: A Window into the use of the Representativeness Heuristic

Investigations into the use of the representativeness heuristic has often involved students' judgment on the relative likelihood of sequences of binary outcomes, outcomes on the next trial after a sequence of runs, or number sequences that differ by order (Kahneman & Tversky, 1972; Olaguro, 2024; Rubel, 2007). Since the choice of these tasks follows from the various manifestations of representativeness, the extent to which students can accommodate seemingly deterministic sequences as chance events determine their proclivity to adopt the heuristic.

Within this understanding, Rubel (2007) engaged 173 middle and high schoolers in a series of tasks. In one of them, the *Four Heads Item,* students were asked to state the outcome on the fifth throw of a coin that had landed heads on the previous trials. In the *Coin Sequences Item,* the expectation was to judge the likelihood of a sequence of outcomes in six tosses of a fair coin. Informing of the relevance of the tasks, the researcher drew inferences on students' use of the heuristic from their responses. Results show that 26% and 33% of the students used the heuristic in each of the items, respectively. In their justifications of the heuristic use, the students expressed a recognition of the randomness of the process within the expectation that randomness should guarantee fairness. Consequently, they ignored the independence of trials, causing one of them to argue that the next outcome on the *Four Heads item* should be "Tails, because the coin has two sides with equal chances. If you flip it four times and get heads all four times on one side, the probability has to even out, to equal it out" (p. 545).

In a recent study, Olaguro (2024) engaged 71 PSTs on two tasks that might elicit the use of the heuristic. Like Rubel, he found that the PSTs use of the heuristic owe to their implicit assumptions of the associated constructs. In the first item involving the expected number of heads in the next 90 flips that had returned all heads in the first 10 trials, 82% failed to account for the base-rate frequency. To a respondent, the fairness of the coin would nullify the apparent unfairness that had produced all heads in the first 10 flips. A similar justification was made in the second item involving the likelihood of sequences of outcomes when a fair coin is thrown six times. Although only 37% used the heuristic via the selection of one among four equally likely sequences, most of them thought the sequence, *HHHTTT* was least likely. To these respondents, this sequence depicts the fairness of the coin but fails the most basic test of randomness which is an absence of pattern that manifests via absolute runs or slow alternations (see Falk, 1981; Pratt, 1998). As these studies reveal, students' use of the heuristic does not occur in isolation. Instead, it manifests in the yearning to ensure that their judgment reflects an understanding of the core constructs undergirding the chance process.

Methods

Seventy high schoolers (ages 14-17) in southwestern Nigeria provided responses to an instrument containing two sets of items (see Table 1). Item 1 contains an adaptation of Fischbein and Schnarch's (1997) task for eliciting the use of the representativeness heuristic. In Item 2, students were asked to define fairness, independence, and randomness. Their responses were subjected to both quantitative and qualitative analyses. For Item 1, responses were coded 1 (normative; option *b*) or 0 (use of the heuristic; options *a* or *c*). For the qualitative analysis, each definition was read and interpreted via open and axial coding based on the core idea a student communicated (see Marshall & Rossman, 2014). The data was subsequently analyzed via descriptive statistics and a Chi-square Test of Independence. The choice of chi square was due to the categorical nature of the variables whose association with the use of the heuristic is to be ascertained.

Table 1: Tasks

Task	Task Statements		
1	When tossing a coin, there are two possible outcomes: either heads or tails. Opebe flipped a coin three times and in all cases, heads came up. Opebe intends to flip the coin again. What is the chance of getting heads the fourth time? (a) Smaller than the chance of getting tails (b) Equal to the chance of getting tails (c) Greater than the chance of getting tails.		
2	What do you understand by the word *Fairness*?	When are two events said to be *Independent*?	What do you understand by the word *Random*?

Results

Research Question 1

We found that at 35.6%, equiprobability is the most common way students conceive of fairness. Others include a lack of bias (30.5%), absence of cheating (27.1%), or the openness of the process (6.8%). Similarly, independence is most associated with the absence of a causation (55.8%), lack of influence (26.9%), or temporal simultaneity (17.3%). Random conceptions recorded the most plurality of meanings. To these students, randomness could mean unpredictability (35.3%), lack of pattern (23.5%) or plan (15.7%). It may also mean rare events (9.8%), equiprobability (7.8%), absence of bias (3.9%) or connection (3.9%).

The constructs' conceptions. Although the various meanings students have for these constructs seem similar, subtle differences exist. Detailed descriptions of the meanings are given below.

Fairness. It is unsurprising that the notion of fairness as equality was most prominent among the four meanings. The understanding of fairness as equality maps into fair sharing, equiprobable outcomes, or turn-taking in game contexts (e.g., Jones et al., 1997). For openness, fairness is clear elucidation and transparency of rules. This meaning fits within gaming situations where ignorance of rules might skew the chances of players.

Independence. As stated, students conceive of independence in three ways viz: (1) influence, (2) causality, and (3) simultaneity. *Independence as lack of influence* expresses an absence of relational interactions between two events. They are oftentimes characterized by outcomes of compound experiments involving a pair of same or different random generators. In the second category, independence was described with respect to causation. Amir and Williams (1999) described this meaning as "causal orientation…a very manipulative, suspicious and non-random view of situations…" (p. 93) that makes students attribute chance outcomes to God, skill, or superstition. Finally, regarding *independence as simultaneity* characterizes temporal invariance. Although this meaning might align within formal probability, it better describes mutually exclusive events (Kelly & Zwiers, 1986).

Randomness. The plurality of meanings students had for randomness is not unexpected. This is because it remains one of the most elusive constructs in probability (Tsonis, 2008). To this day, little success has been recorded in creating an objective measure of randomness (cf. Falk & Konold, 1998), leaving it to be conceived in mostly subjective ways (Falk & Konold, 1998; Pratt, 1998). Randomness is perceived relative to bias if its process is outside of human control, what Pratt (1998) termed as unsteerability. Students who perceive it as an absence of connection believe that random outcomes lack initial conditions that might lend it to an input-output relationship. Randomness as absence of pattern or prediction aligns with the recognition that random sequences or outcomes should not share deterministic properties. The perception of randomness as an unplanned event draws on its haphazard perception. For students who conceive it as rarity, events are random when the likelihood of their occurrence is very low. Finally, conceived with respect to speed, randomness is events that take place in a flash. The examples these respondents provided include earthquake and tsunami.

Research Question 2

To answer the second research question on the predictive possibility of the unique conceptions of each construct on the use of the representativeness heuristic, we report on the results of three Chi-Square Tests of independence. Except for fairness, we found that the unique ways constructs are conceived do not overtly influence students' use of the heuristic. Specifically, the test statistic for fairness returned, $\chi^2[3, 59] = 10.143$, p=.017; for independence χ^2 [2, 52] = .334, p=.846; and for randomness, χ^2 [6, 51] = 11.490, p=.074. At an alpha level of .05, these three results show evidence that, for this particular task (Table 1), the meaning a student has for fairness plays a role in attendant use of the heuristic, however, such does not exist for randomness and independence.

Conclusions

We investigated how students' conceptions of randomness, fairness, and independence relate to the use of the representativeness heuristic. These constructs are often assumed to be understood, but our findings suggest otherwise. Like prior studies (e.g., Pratt, 1998), we found that randomness is viewed in multiple, sometimes conflicting ways: as a presence (e.g., equality, rarity) or absence (e.g., bias, pattern, plan) of a property. All seven identified meanings, though diverse, describe necessary but insufficient aspects of randomness. Students also held varied conceptions of fairness and independence. For instance, some viewed independence as simultaneity—confusing it with mutual exclusivity—while others viewed fairness as bias. These conceptions highlight the need for explicit instruction, particularly in contexts where language or curriculum (e.g., Nigeria) may not foreground conceptual clarity (see Lagos State Ministry of Education, 2021). Interestingly, these diverse meanings did not predict use of the heuristic, suggesting that the unique ways these constructs are conceived does not necessarily influence heuristic use. However, fairness, unlike the others, may be multidimensional—incorporating structural, procedural, and ethical judgments—which could explain its partial influence on the use of the heuristic. Future research should explore this further and emphasize the importance of addressing students' interpretations in probability instruction.

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