Appendix A

Uncertainty Analysis for the Skin-friction Measurements

A.1 Classification of the Uncertainties

The uncertainty in the skin-friction measurements should be examined in two levels. The first part corresponds to the uncertainty in the calibration of the hot-film sensors and can be thought as a bias source of uncertainty. This uncertainty directly affects the skin-friction magnitudes, however it does not have an effect on the flow separation locations. The second one is the random relative uncertainty between the measurements made by one sensor at different circumferential locations on the model in different flow conditions. In other words, this uncertainty determines how accurately one sensor can measure the same wall shear stress τ_w value in a given circumferential location at successive measurements. As also described in [2], this is the uncertainty that determines the accuracy of the flow separation locations.

A.2 Uncertainty Calculations

The bias (calibration) uncertainty is mainly related to the accuracy of the wall shear stress values τ_w used in the calibration process. As described in chapter 4, these shear stress values were calculated by using the boundary layer properties that were obtained from the boundary layer velocity profile measurements. The previous experiments that used

the same technique approximates an uncertainty of ±5% in the skin friction coefficient C_f (Simpson [36]). Since the uncertainty in the determination of the free-stream velocity δU_∞ was less than 1% during the measurements, the uncertainty of the wall shear stress $\delta\tau_w$ can also be thought as ±5%. Thus, this value may be taken as the bias uncertainty in the calibration process.

In order to calculate the relative uncertainty in the actual measurements, we should again consider the King's equation written for the constant-temperature hot-film anemometers (CTA):

$$\tau_w^{1/3} = \bar{A}\frac{E^2}{\theta} + \bar{B} \tag{A.1}$$

where θ is the difference between the sensor operating temperature (which is constant for CTA) and the free-stream temperature:

$$\theta = T_w - T_\infty \tag{A.2}$$

The relative uncertainty of τ_w measured by each sensor will be due to the random uncertainties of the measured sensor voltage δE and the temperature difference $\delta\theta$. Note that the calibration constants $\bar{A}$ and $\bar{B}$ are not considered here, since they don't have a random variation. These coefficients have the same value between each calibration run as described in chapter 4. The overall relative uncertainty in τ_w can be determined by using the *Kline and McClintock method* given in Doebelin [37]:

$$\delta W = \sqrt{\left(\frac{\partial W}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial W}{\partial x_2}\delta x_2\right)^2 + \ldots + \left(\frac{\partial W}{\partial x_n}\delta x_n\right)^2} \tag{A.3}$$

In the above equation, $W = W(x_1, x_2, \ldots, x_n)$ represents the measured variable; δW, the uncertainty of the measured variable; x_i, each variable that effect the measured quantity; and δx_i, the uncertainty in each of these variables. The partial derivatives are the sensitivities of W with respect to each x_i. In order to use this method, each δx_i should be random and uncorrelated and have the same confidence level (e.g. 20:1 odds). Then the overall random uncertainty in W will also have the same odds.

By using equation A.3, $\delta\tau_w^{1/3}$ can be approximated by:

$$\frac{\delta\tau_w^{1/3}}{\tau_w^{1/3}} = \frac{\tau_w^{1/3} - \bar{B}}{\tau_w^{1/3}}\sqrt{\left(\frac{\delta\theta}{\theta}\right)^2 + \left(\frac{\delta E^2}{E^2}\right)^2} \tag{A.4}$$

Since the sensor temperature T_w is constant, temperature difference uncertainty will be equal to the uncertainty of the free-stream temperature which was at most ±0.5 °C

between each calibration run in the experiments. By using the nominal 1.1 value of the overheat ratio,

$$\frac{\delta\theta}{\theta} = \pm 0.013 \tag{A.5}$$

can be obtained. In order to get statistically meaningful uncertainty results, equation A.4 has been calculated for each steady and unsteady skin-friction value in the corresponding steady and the unsteady data reduction programs. The value of each δE^2 has been obtained by using the voltage standard deviation info calculated in the same programs. At the last step, the overall random uncertainty of τ_w was obtained by considering:

$$\frac{\delta\tau_w}{\tau_w} = 3.0\frac{\delta\tau_w^{1/3}}{\tau_w^{1/3}} \tag{A.6}$$

Note that the uncertainty in the skin-friction δC_f is equal to the wall shear stress uncertainty $\delta\tau_w$, since the uncertainty of the free-stream velocity is negligible. As the final results, the random uncertainties in C_f with 20:1 odds were obtained as:

for the steady measurements	$\delta C_f = 6\%$ of C_f
for the unsteady measurements	$\delta C_f = 8\%$ of C_f

The higher uncertainty in the unsteady measurements come from the relatively higher unsteady voltage uncertainties. Above values lead to an uncertainty of $\pm 2\,^\circ$ in the determination of the separation locations.

Appendix B

Robust Locally Weighted Regression and Smoothing *LOESS*

Robust locally weighted regression technique (Loess) is a smoothing method which uses the pre-determined windows of the original data in the regression process. Within each individual window, the data points near to the location where the smoothed value of the dependent variable is sought are given higher weights. Outliers in the data set are detected during the fitting process in an iterative manner and not used in the regression. These features of Loess technique enable to smooth the data without changing the original pattern.

B.1 Mathematical Description

Mathematical details of the Loess method are described in Cleveland [29] and [30]. The theory behind the smoothing process can be summarized as follows: Let (x, y) be the point where we seek for the smoothed value $\hat{g}(x)$. Here x corresponds to the independent and y to the dependent variable in our data set. Two parameters are needed to be chosen to fit a loess curve. The first parameter, α, is a smoothing parameter; it can be any positive number but typical values are 0.25 to 1.0. As α increases, the curve becomes smoother. The second parameter, λ, is the degree of the certain polynomials that are fitted by the method; λ can be 1 or 2. Suppose first that $\alpha \leq 1.0$. Let q be αn truncated to an integer where n is the total number of the data points. We assume that α is large enough so that q is at least 1, although q is much larger than 1 in most of the

applications.

Let $\Delta_i(x) = |x_i - x|$ be the distance from x to x_i, and $\Delta_{(i)}(x)$ be these distances ordered from the smallest to the largest. $T(u)$, the *tricube weight function* is defined as:

$$T(u) = \left\{ \begin{array}{cc} (1-|u|^3)^3 & for \quad |u| \leq 1.0 \\ 0.0 & otherwise \end{array} \right\} \tag{B.1}$$

Then the neighborhood weight given to the observation (x_i, y_i) for the fit at x is

$$w_i(x) = T\left(\frac{\Delta_i(x)}{\Delta_{(q)}(x)}\right) \tag{B.2}$$

For x_i such that $\Delta_i(x) < \Delta_{(q)}(x)$, the weights are positive and decrease as $\Delta_i(x)$ increases. For $\Delta_i(x) > \Delta_{(q)}(x)$, the weights are zero. If $\alpha > 1$, the $w_i(x)$ are defined in the same manner, but $\Delta_{(q)}(x)$ is replaced by $\Delta_{(n)}(x)\alpha$.

If $\lambda = 1$, a line is fitted to the local data using weighted least squares with weight $w_i(x)$ at (x_i, y_i); and the values of a and b are found that minimize:

$$\sum_{i=1}^{n} w_i(x) \left(y_i - a - bx_i\right)^2 \tag{B.3}$$

Let $\hat{a}$ and $\hat{b}$ the minimizing values, then the fit at x is

$$\hat{g}(x) = \hat{a} + \hat{b}x \tag{B.4}$$

For $\lambda = 2$, a quadratic polynomial is fitted to the local data using weighted least squares; values of a, b, and, c are found that minimize

$$\sum_{i=1}^{n} w_i(x) \left(y_i - a - bx_i - cx_i^2\right)^2 \tag{B.5}$$

If $\hat{a}$, $\hat{b}$, and $\hat{c}$ are the minimizing values, then the fit at x is

$$\hat{g}(x) = \hat{a} + \hat{b}x + \hat{c}x^2 \tag{B.6}$$

B.2 Selection of the Loess Parameters

In the previous section, two important parameters α and λ that must be chosen in the smoothing process are defined. As described in Cleveland [29], in any specific application

of the loess method, the choice of these two parameters must be based on a combination of judgment and of trial and error. As α increases, the loess fit becomes smoother. However after a certain value of α, a lack of fit may be observed as the peaks and valleys in the data are missed because of the large value of α. For small values of α, the underlying pattern is tracked, but the local wiggles do not appear to be supported by the data. *The goal in choosing α is to produce a fit as smooth as possible without unduly distorting the underlying pattern in the data.* Cleveland [29] also shows that a locally quadratic fit ($\lambda = 2$) may be superior to the linear fit ($\lambda = 1$) in terms of picking the local minima and maxima in certain data sets.

In this study, the loess method is used in smoothing some C_f vs ϕ distributions, and the optimum combination of the parameters for each configuration and test condition have been found to be as follows:

Unsteady, barebody	$\alpha = 0.25$ and $\lambda = 1$
Steady, sail-on-side	$\alpha = 0.10$ and $\lambda = 1$
Unsteady, sail-on-side (sail region)	$\alpha = 0.20$ and $\lambda = 1$
Unsteady, sail-on-side (non-sail region)	$\alpha = 0.25$ and $\lambda = 1$

Figures B.1 and B.2 show the loess fits to the barebody unsteady C_f vs. ϕ data at $\alpha(t') = 11.3\,^\circ$ and $x/L = 0.819$, and at $\alpha(t') = 21.5\,^\circ$ and $x/L = 0.638$ respectively. As can be seen from these figures, by using the loess parameters $\lambda = 1$ and $\alpha = 0.25$, a desirable level of smoothing is achieved without changing the general trend and the local minima of the data.

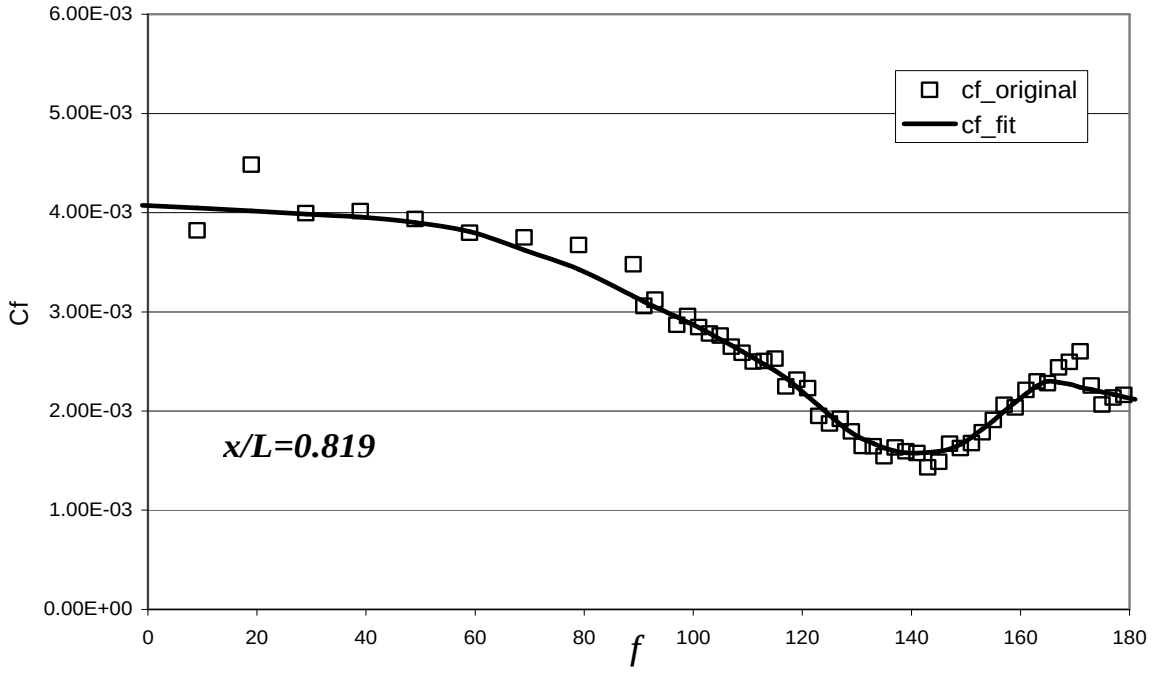

Figure B.1: Loess smoothing for unsteady barebody data at $\alpha(t') = 11.3^\circ$ and $x/L = 0.819$.

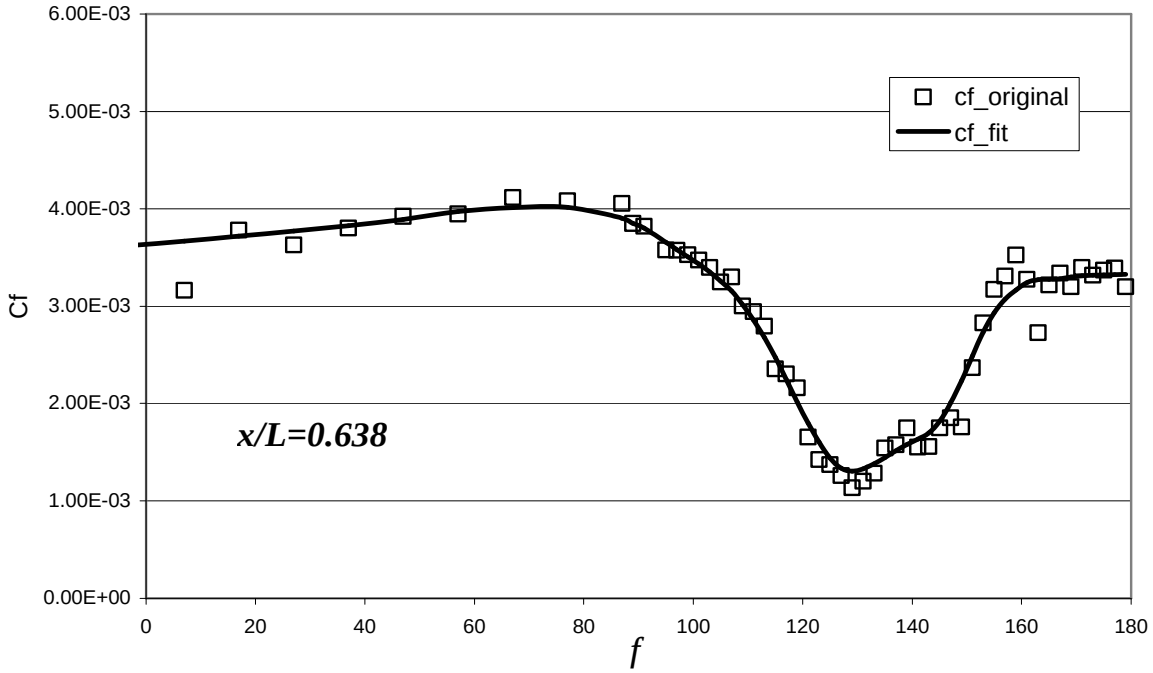

Figure B.2: Loess smoothing for unsteady barebody data at $\alpha(t') = 21.5^\circ$ and $x/L = 0.638$.

Vita

Serhat Hosder was born on November 4, 1975 in Ankara, the capital of Turkey. He continued his high school education in different places due to his father's job. In 1993, he graduated from Bornova Anatolian High School (BAL), Izmir. In the same year, he began his undergraduate study in Aeronautics and Astronautics Department of Istanbul Technical University (ITU), Istanbul. In 1997, he graduated from the same school with a Bachelor degree in Aeronautical Engineering. He came to Virginia Tech in the Fall of 1998 to get his M.S. degree in Aerospace Engineering.