How Well Can Two-Wave Models Recover the Three-Wave Second Order Latent Model Parameters?

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Abstract

Although previous studies on structural equation modeling (SEM) have indicated that the second-order latent growth model (SOLGM) is a more appropriate approach to longitudinal intervention effects, its application still requires researchers to collect at least three-wave data (e.g. randomized pretest, posttest, and follow-up design). However, in some circumstances, researchers can only collect two-wave data for resource limitations. With only two-wave data, the SOLGM can not be identified and researchers often choose alternative SEM models to fit two-wave data. Recent studies show that the two-wave longitudinal common factor model (2W-LCFM) and latent change score model (2W-LCSM) can perform well for comparing latent change between groups. However, there still lacks empirical evidence about how accurately these two-wave models can estimate the group effects of latent change obtained by three-wave SOLGM (3W-SOLGM). The main purpose of this dissertation, therefore, is trying to examine to what extent the fixed effects of the tree-wave SOLGM can be recovered from the parameter estimates of the two-wave LCFM and LCSM given different simulation conditions.

Fundamentally, the supplementary study (study 2) using three-wave LCFM was established to help justify the logistics of different model comparisons in our main study (study 1). The data generating model in both studies is 3W-SOLGM and there are in total 5 simulation factors (sample size, group differences in intercept and slope, the covariance between the slope and intercept, size of time-specific residual, change the pattern of

time-specific residual). Three main types of evaluation indices were used to assess the quality of estimation (bias/relative bias, standard error, and power/type I error rate). The results in the supplementary study show that the performance of 3W-LCFM and 3W-LCSM are equivalent, which further justifies the different models' comparison in the main study. The point estimates for the fixed effect parameters obtained from the two-wave models are unbiased or identical to the ones from the three-wave model. However, using two-wave models could reduce the estimation precision and statistical power when the time-specific residual variance is large and changing pattern is heteroscedastic (non-constant). Finally, two real datasets were used to illustrate the simulation results.

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General Audience Abstract

To collect and analyze the longitudinal data is a very important approach to understand the phenomenon of development in the real world. Ideally, researchers who are interested in using a longitudinal framework would prefer collecting data at more than two points in time because it can provide a deeper understanding of the developmental processes. However, in real scenarios, data may only be collected at two-time points. With only two-wave data, the second-order latent growth model (SOLGM) could not be used. The current dissertation compared the performance of two-wave models (longitudinal common factor model and latent change score model) with the three-wave SOLGM in order to better understand how the estimation quality of two-wave models could be comparable to the tree-wave model. The results show that on average, the estimation from two-wave models is identical to the ones from the three-wave model. So in real data analysis with only one sample, the point estimate by two-wave models should be very closed to that of the three-wave model. But this estimation may not be as accurate as it is obtained by the three-wave model when the latent variable has large variability in the first or last time point. This latent variable is more likely to exist as a statelike construct in the real world. Therefore, the current study could provide a reference framework for substantial researchers who could only have access to two-wave data but are still interested in estimating the growth effect that supposed to obtain by three-wave SOLGM.

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As the famous saying “Rome Wasn’t Built in a Day”, completing my dissertations actually costs me more than one and a half years. During this writing process, I have received a great deal of support. I would like to take advantage of this special opportunity to express my gratitude to everyone who keeps supporting me until I finished this “marathon”

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

The study of change has been a pervasive topic in the social sciences for generations. For example, developmental psychologists have proposed many theories to account for the development of cognitive ability such as vocabulary in young children (Brooks & Meltzoff, 2008; Huttenlocher, Haight, Bryk, Seltzer, & Lyons, 1991). Similarly, in criminology, researchers found that antisocial behaviors for adolescents generally have a quadratic shape as a function of their age (Hirschi & Gottfredson, 1983; Tonry, Ohlin, & Farrington, 1991). Unfortunately, to study change is not as easy as many researchers previously thought. Researchers who are interested in studying the change of certain phenomena would like to employ a longitudinal research design for collecting data. Such longitudinal data could often pose challenges to researchers of how to conduct a proper analysis. The main issue is that there is no single statistical procedure for analyzing longitudinal data since different research questions dictate different data collection designs, which further leads to different statistical models or methods (Duncan & Duncan, 2004).

Traditionally, the methods for analyzing longitudinal data could vary from univariate and multivariate analysis of covariance to auto-regressive and cross-lagged multiple regression techniques (Hancock, Kuo, & Lawrence, 2001). Those techniques, if their assumptions are met, can provide useful growth information across time, but each of them has their own limitations. For example, the causal structure in autoregressive models may not be fully reflected in the estimated parameters from the panel data (Hertzog & Nesselroade, 2003), and repeated measures ANOVA can not deal with the unbalanced data frequently seen in longitudinal studies because of the attrition (Raykov & Marcoulides, 2000).

In order to overcome these limitations of traditional analytic approaches, a class of methods emerged from the framework of structural equation modeling (SEM) and that of multilevel modeling (MLM). Under these frameworks, latent curve model from SEM perspective (LCM, Bollen & Curran, 2006), also known as growth curve modeling (GCM) from the MLM perspective (Raudenbush & Bryk, 2002), has gained popularity for evaluating the developmental change of latent construct. Combining the individual and group level of information, the LCM/GCM is not only able to provide hypothesis test on the average initial status and rate of change, but also can detect the variation among individual growth trajectories. The LCM, which inherited the strength of SEM, can test the adequacy of the hypothesized growth from the overall fit indices, and incorporate the assessment of the measurement's property. Because of the flexibility and convenience in testing the different hypotheses for developmental trajectories, many social scientists have argued in favor of LCM's superiority over other analytical approaches (Bollen & Curran, 2006; Curran & Hussong, 2003) for time balanced design that is often employed in psychological research.

One frequent application of LCM in longitudinal studies is to detect the group differences in developmental trajectories (e.g. control vs. treatment group, male vs. female). In the earlier years, many researchers used to run the LCM to compare group trajectories using composite scores as the outcome variable of repeated measure (Curran & Muthén, 1999; Simons-Morton, Chen, Abroms, & Haynie, 2004; Willoughby, Vandergrift, Blair, & Granger, 2007). However, recent studies started to call attention to the drawbacks of using the composite scores, such as the mean of several items, to represent the construct of interest at each measurement occasion. Actually, analyzing the change in this composite score assumes strict invariance for the measurement, which implies that every item assesses the latent construct equally well (Hertzog & Nesselroade, 2003; Isiordia & Ferrer, 2018). In the real world, however, this restrictive assumption is rarely met. When this measurement invariance assumption is violated, LCM could yield bias in the estimates of the model parameters (Leite, 2007; Wirth, 2008), especially for the focal parameter estimates of the group or treatment effects in intervention research (Fraine, Damme, & Onghena, 2007; Kim & Willson, 2014a). With the increasing concerns over the bias obtained from using the composite scores, many researchers proposed an alternative model, the second-order latent growth model (SOLGM, Meredith & Tisak, 1990) or curve-of-factors model (CUFFS, McArdle, 1988), to overcome this limitation.

SOLGM is actually an extension of the LCM described above, which could also be called the first-order latent curve model (FOLCM). The SOLGM incorporates a measurement model to allow an explicit test of measurement property. The first-order factors in the SOLGM are therefore the latent variables measured by multiple-scale items at each measurement occasion. This common factor characterizes how well the latent construct is measured by multiple items. The second-order factors in the SOLGM are level and shape factors, which capture the growth trajectory of the latent construct. One often-cited major advantage of the SOLGM is the opportunity for researchers to test the factorial invariance across occasions of measurement. This invariance guarantees an equivalent definition of a latent construct across measurement occasions. In the early years of the development of LCM, this property has not received much attention among researchers who intended to use LCM for group comparison. However, as evidence was accumulated that violating the measurement invariance could lead to biased estimates of the group effect, many researchers advocated the use of the SOLGM against the FOLCM, especially when their intention was to compare the group difference in the growth trajectories (Ferrer, Balluerka,

& Widaman, 2008; Kim & Willson, 2014a).

Although previous studies have shown that the SOLGM is a more appropriate approach to evaluate interventions or group effects in developmental studies, its application still requires researchers to collect at least three waves of data because it is the necessary condition for the model identification (Bollen & Curran, 2006, p. 23). With only three-wave data, one must fit a simple model with linear growth assumptions (Singer & Willett, 2003, p. 10). While if one could collect data that has more than three waves, one could posit more complex models (Singer & Willett, 2003, p. 10). But in a real-world scenario, it often happens that data is collected only at two-time points because of limited resources. When this happens, pretest-posttest design is still a widely used methodological choice, especially in educational or psychological intervention studies (Alessandri, Zuffianò, & Perinelli, 2017). For example, one may be interested in evaluating the effectiveness of an intervention on a learning task, where participants were randomly assigned to a treatment or control condition and the performance was assessed before and after the experimental manipulation. When examining this kind of two-wave data, the traditional approach researchers would rely on is to use the repeated measures t-test based on the gain/change score. However, this analytical method gradually lost its favor among researchers because the theoretical contributions in the 1980s and 1990s suggested that the observed change score confounds the true change with measurement error (Rogosa, Brandt, & Zimowski, 1982; Willett, 1988). An alternative option of t-test was to use the analysis of covariance (ANCOVA). However, ANCOVA also suffers from the biased treatment effect because of the measurement error in pretest (Cohen, Cohen, West, & Aiken, 2013, p. 351). Because of these shortcomings in the traditional methods, various extensions of SEM have been advocated for the repeated measures data. Many researchers have argued in favor of LCM/LGM's flexibility of testing different research hypotheses related to the developmental trend, such as heteroscedastic residuals, nonlinear growth patterns, and availability of overall model fit (Chan, 1998; Duncan & Duncan, 2004). In the past decade, two models under SEM framework were frequently suggested in the literature for dealing with two-wave data. They are longitudinal common factor model and latent change score model. Both models are particularly useful when modeling interested variables at the latent level because establishing a measurement model can separate true score from error score, leading to a perfectly reliable latent score.

Longitudinal common factor model (LCFM, Grimm, Ram, & Estabrook, 2016) directly conceptualizes the change over time in a latent variable (Finch & Shim, 2018). If two latent variables could be represented with η_t and η_s $(t < s)$ in two different time points respectively in LCFM, their change could be $\Delta\eta$ $(\Delta\eta = \eta_s - \eta_t)$. We therefore could estimate the mean and variance of $\Delta\eta$. When we have only two waves of measurement, a two-wave latent change score model $(2W - LCSM)$ can also be used to directly specify a latent change factor across two-time points (McArdle & Grimm, 2010). One advantage of the 2W-LCSM is that by taking just two measurement occasions, it could directly yield parameters that are isomorphic with change-to-change theories (Henk & Castro-Schilo, 2016). The mean of the change score factor informs whether people are really changing, and the variance of latent change score indicates whether individuals differ in their intraindividual changes.

Some recent studies indicated that both LCFM and LCSM can perform very well for comparing average change between groups, especially in the pretest-posttest intervention research (Mun, von Eye, & White, 2009; Coman et al., 2013). However, there still lacks systematic research about how accurately these models can estimate the group difference in the amount of true change obtained from the real longitudinal data. A recent study demonstrated that when the item-level data is available, the two-wave SOLGM with group covariates can still be identified with certain constraints on the common factor loading (Alessandri et al., 2017). Moreover, their results indicated that the group difference of the growth parameters could be estimated by the latent change model given the use of item-level data. The latent change score model (LCSM) with a group membership covariate is mathematically equivalent to the LCFM with a group membership covariate under SEM framework. With the availability of the multiple indicators at each time point, both LCFM and LCSM can include the measurement-error-free construct, which adds validity and reliability to each assessment compared to the FOLGM, which uses a single measurement as composite scores at each occasion (Newsom, 2015, p. 259). Most importantly, by removing the measurement error, the inflated variance of the composite score could be corrected. With these advantages of using multiple indicators, fitting the two-wave LCFM or LCSM to estimate the fixed effects of the linear growth parameters should be reasonably accurate.

Miyazaki (2017) examined the quality of the estimate of the change in the two-wave LCFM model when item-level information is available. In his Monte Carlo Simulation study, he demonstrated that the two-wave LCFM produced accurate estimates of the fixed effects (average initial status and growth rate) of the three-wave SOLGM. However, the simulation conditions in his study were very limited. For instance, the size of the time-specific error was fixed to be equal across time, which may be not realistic to capture real-world scenarios. In addition, there was no group covariate in the model so the hypothesis that the growth process might systematically differ across groups could not be evaluated. Other simulation parameters such as sample size were also fixed at certain values.

The main purpose of the current study is to examine to what extent the fixed effects parameters of the three-wave model (SOLGM) can be recovered from these two-wave models (LCFM and LCSM) in various realistic settings through Monte Carlo Simulation.

The special focus is given to the estimates of group differences in the growth parameters because examining the group difference in the growth trajectories is a very important topic in longitudinal evaluation studies (Alessandri et al., 2017; Fraine et al., 2007). Therefore, it is important to know how accurately we can estimate this group difference when only two-wave data with item-level information is accessible.

Chapter 2

Literature Review

2.1 Latent Curve Model (LCM)

The LCM can be treated as a special case of Structural Equation Modeling (SEM), where the latent variables are unobserved or unmeasured variables, and are often used as a proxy for hypothetical constructs in social science. A typical SEM consists of the latent variable model, which summarizes the structural relationships between the latent variables, and the measurement model that represents the links between indicators/observed variables and the latent variables (Preacher, Wichman, MacCallum, & Briggs, 2008).

A special case under the SEM system yields the so-called latent curve model (LCM). It has all the advantage of SEM, including the capacity to assess the adequacy of the model fit, the ability to accommodate the measurement errors and ability to handle missing data efficiently (Preacher et al., 2008). Compared with the traditional analytic techniques for the longitudinal data such as repeated measures analysis of variance (ANOVA) and multivariate analysis of covariance (MANCOVA) which focus on the average growth or change, the LCM is able to allow researchers to directly examine both the intraindividual (with-person) changes across time and interindividual (between-person) differences in the changing patterns. Another advantage of LCM is its ability to investigate the antecedents and consequences of the change.

2.1.1 Unconditional Univariate LCM

In the unconditional LCM, the same observed variables are measured repeatedly across time, and the longitudinal change is described by three latent variables: level, shape and error (McArdle & Hamagami, 1992). The level factor represents the initial status of individuals in the process of their development. This reference point is determined by how the loadings of the shape factor are coded. For example, the level can be treated as the intercept if the first measurement time is used as a reference point (Muthen & Khoo, 1998). In this case, we could explain the intercept as the initial status on the trait obtained from the previous experiences. If the reference point is assigned to a measurement time point other than the first (e.g. the second time point), the level represents the individuals' status at that specific measurement time.

The shape factor actually reflects a shape of individual's growth trajectory. If this growth trajectory's shape is linear, the shape factor can also be interpreted as the slope, which indicates the linear rate of change associated with the outcome variable (McArdle & Hamagami, 1992). The score in this slope factor indicates the expected amount of change in the observed variable when the measurement time increases one unit.

The error variable represents the unique portion of an observed variable that can not be explained by the the latent construct being measured. It consists of random measurement errors and specific factors. The measurement errors are assumed to be independent with other variables over time and have a mean of zero (McArdle & Hamagami, 1992). However, the specific factors are allowed to be correlated over time. A general latent growth model with T measurement occasions is represented as:

$$\mathbf{y}_i = \boldsymbol{\Lambda_y \eta}_i + \boldsymbol{\epsilon}_i \tag{2.1}$$

where $\mathbf{y}_i$ is a $T \times 1$ vector of the observed measurement for individual i, and $\boldsymbol{\eta}_i$ is the m×1 vector containing the scores of the common growth factor (α_i, β_i) for individual i, where α_i is referred to as the initial level factor for person i, and β_i is referred to as the shape factor for the same person i . $\boldsymbol{\Lambda}_y$ is a $T \times m$ factor loading matrix containing initial level and shape factors coefficients[1]. $\boldsymbol{\epsilon}_i$ is the $T \times 1$ vector of unique factor score for person i. More explicitly, the above equation can be expressed as:

$$\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ \vdots \\ y_{ti} \\ y_{Ti} \end{bmatrix} = \begin{bmatrix} 1 & \lambda_1 \\ 1 & \lambda_2 \\ 1 & \lambda_3 \\ 1 & \lambda_4 \\ \vdots & \vdots \\ 1 & \lambda_T \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} + \begin{bmatrix} \epsilon_{1i} \\ \epsilon_{2i} \\ \epsilon_{3i} \\ \vdots \\ \epsilon_{ti} \\ \epsilon_{Ti} \end{bmatrix}$$

In the above expression (m=2), the first column of matrix $\boldsymbol{\Lambda}$ is a vector of $\mathbb{1}$s. It corresponds to the loading of level factor (α_i), while the second column consists of the factor loadings of the shape factor (β_i). In order to set up the reference point of the above model, one of the factor loading of shape factor should be fixed to zero.

The means of the observed vector $\mathbf{y}_i$ could be expressed in terms of the mean of the latent variables such as $E(\alpha_i) = \mu_\alpha$ and $E(\beta_i) = \mu_\beta$:

$$E(\mathbf{y}_i) = \boldsymbol{\Lambda_y} E(\boldsymbol{\eta}_i) = \boldsymbol{\Lambda_y \mu_\eta} \tag{2.2}$$

[1]Note that in general it can have m factors, but in this dissertation, m is fixed to 2 because we only study linear growth model.

where $\boldsymbol{\mu_\eta} = (\mu_\alpha, \mu_\beta)^t$ is a 2×1 vector consisting of the mean values of level and shape factors, i.e. $E(\eta_i) = \mu_\eta$. Thus, in the above expression, we formulated a model: $\boldsymbol{\eta_i} = \boldsymbol{\mu_\eta} + \boldsymbol{\zeta}$, where $E(\boldsymbol{\zeta}) = 0$. Though often the values of λ are fitted to the actual spacing time for the measurement occasions by assuming that the shape of growth trajectory is linear. If there is no hypothesis on the growth trajectory, the factor loadings of the slope (shape) factor could also be freely estimated. The difference between the slope factor loadings can be interpreted as the amount of change occurring between two adjacent measurement occasions (Meredith & Tisak, 1990).

If we confine the shape of growth trajectory to be a simple linear form, where the values attached to the factor loading matrix reflect the spacing of time by setting the first time point as the clocking of the time, the model expressed by equation 2.1 could be depicted as in Fig 2.1.

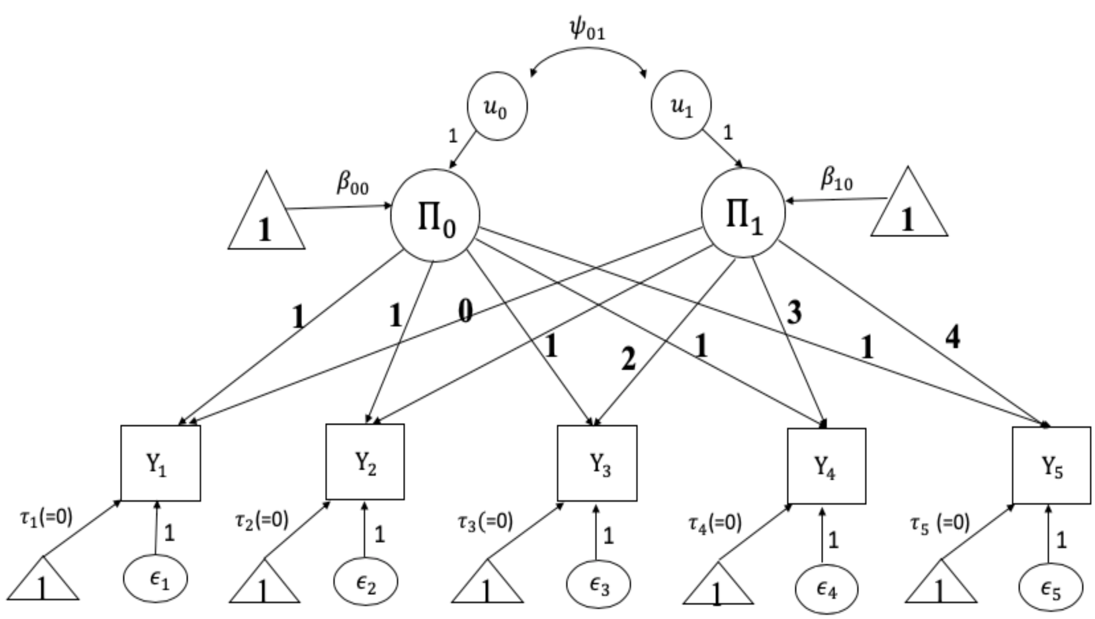

Figure 2.1. Linear Latent Growth Model with Five Waves

It should be Note that the diagram in Figure 2.1 follows the symbols from McArdle-McDonald (RAM) symbolism (Kline, 2015). The triangle represents the mean structure of the model. The following model diagrams all follow RAM symbolism.

An element-wised scalar equation which corresponds to the level 1 model in multilevel modeling (MLM) can be written as:

$$y_{it} = \pi_{0i} + \lambda_t \pi_{1i} + \epsilon_{it} \tag{2.3}$$

where y_{it} is an observed dependent variable for individual i measured at time point t; π_{0i} and π_{1i} are initial status and slope scores for individual i, respectively; ϵ_{it} is a residual term, where $\epsilon_{it} \sim N(0, \sigma^2)$. Since π_{0i} and π_{1i} are also random variables, so they have a joint distribution such as $\boldsymbol{\pi}_i = (\pi_{0i}, \pi_{1i})^t \sim N_2(\boldsymbol{\beta}, \boldsymbol{\Psi})$, where $\boldsymbol{\beta} = (\beta_{00}, \beta_{10})^t$ and $\boldsymbol{\Psi}$ is a 2×2 variance-covariance matrix.

The structure part of LGM, which corresponds to the level 2 in MLM, can be expressed as a function of latent means (β_{00} and β_{10}) and individual deviation from the those means (unconditional model):

$$\pi_{0i} = \beta_{00} + u_{0i} \tag{2.4}$$

$$\pi_{1i} = \beta_{10} + u_{1i} \tag{2.5}$$

where β_{00} is the average initial status of the outcome variable y, while the β_{10} represents an average growth rate (slope) of variable y. $\mathbf{u}_i = (u_{0i}, u_{1i})^t$ are the deviation of the individual scores of growth parameters from their their population means β_{00} and β_{10}, respectively. The covariance between these two disturbances is: $Cov[u_{0i}, u_{1i}] = \psi_{01}$. The corresponding matrix format of the above structural model is:

$$\begin{bmatrix} \pi_{0i} \\ \pi_{1i} \end{bmatrix} = \begin{bmatrix} \beta_{00} \\ \beta_{01} \end{bmatrix} + \begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix}$$

The variance and covariance matrix of the disturbance ($\mathbf{u}_i$) is $\boldsymbol{\Psi}$, which is a 2×2 matrix written as:

$$\boldsymbol{\Psi} = \begin{bmatrix} \psi_{00} & \psi_{01} \\ \psi_{10} & \psi_{11} \end{bmatrix}$$

The variances of the disturbance $\mathbf{u_i}$ are equivalent to the variances of the latent factors (π_0, π_1) because this is an unconditional LGM. If there are any predictors added into this model, the variances of these disturbances become the residual variances of each factor, which indicates the remaining variance after the effects of the initial status and slope factor are removed.

2.1.2 Conditional Univariate LCM

The above section described the model specification and parameter estimates for the unconditional univariate LCM that does not include any covariates in the structure part of the model. However, if there exists individual variability in the initial levels and growth slopes, the covariates could be incorporated to predict this variability. In general, there are two type of variables that can be included as covariates in the model: time-invariant and time-varying variables[2](Willett & Keiley, 2000). Time-invariant variables are measured only at one-time point and thus do not change in the model. However the time-varying variables vary over time and are measured at multiple times. In this subsection, we will

[2]In current dissertation, we only consider the time invariant covariates.

only introduce the models with the time-invariant covariate because it is the main focus in this research. For illustration purpose, all the models introduced here just contain one covariate.

Time-Invariant Covariate

The level 1 equation for the conditional latent growth model is the same as in the unconditional model: $y_{it} = \alpha_i + \lambda_i \beta_i + \epsilon_{it}$ (general LCM) or $y_{it} = \pi_{0i} + \lambda_i \pi_{1i} + \epsilon_{ti}$ (linear LGM). The difference exists in the structure part of the model, which is the level 2 model in multilevel modeling (MLM). In previous section, it was shown that the unconditional LCM does not include any covariates. In contrast, the predictor is included in the conditional LCM because we wish to assess how covariate is related to the initial level and shape factors (See Fig.2.2)

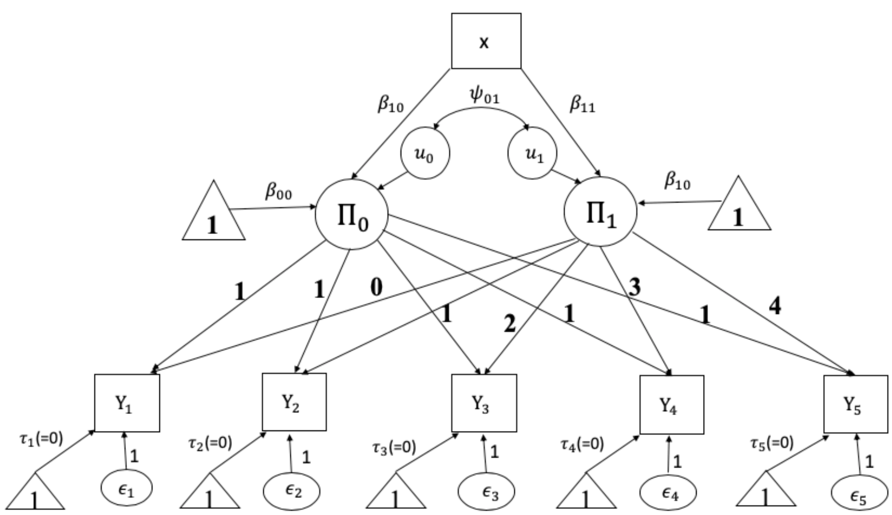

Figure 2.2. Conditional Latent Growth Model with Five Waves

The model specification with the covariates in LCM is very flexible. For example, the covariates may impact the outcome variable through a mediator, or they may have their own measurement model. The structural part of the conditional univariate LCM with a single time-invariant covariate is described below:

$$\pi_{0i} = \beta_{00} + \beta_{01}x_i + u_{0i} \tag{2.6}$$

$$\pi_{1i} = \beta_{10} + \beta_{11}x_i + u_{1i} \tag{2.7}$$

x_i in equation 2.6 and 2.7 is a covariate that can be either discrete or continuous. In social science researches, a discrete covariate is frequently used. For instance, x_i can be a dummy variable representing participants' demographic information such as gender and race ethnicity. x_i can also indicate the treatment/control assignment to the participants. Therefore, x_i is assumed to be a binary variable(i.e.,$x_i \in 0, 1$). For any binary x_i, the corresponding β_{00} and β_{10} represents the average initial status (intercept) and growth rate (slope) in the reference group (x_i=0). Meanwhile, β_{01} and β_{11} represents the effects of being a member of the other group (x_i=1) on the average initial status and growth rate, respectively. Equation 2.6 and 2.7 can be combined together in a matrix form:

$$\begin{bmatrix} \pi_{0i} \\ \pi_{1i} \end{bmatrix} = \begin{bmatrix} \beta_{00} \\ \beta_{10} \end{bmatrix} + \begin{bmatrix} \beta_{01} \\ \beta_{10} \end{bmatrix} [x_i] + \begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix}$$

In the conditional LCM, the interpretation for the disturbance $\mathbf{u}_i$ which consists of u_{0i} and u_{1i} is also different from those in the unconditional model. The disturbance $\mathbf{u}_i$ is actually the residual variability after the effect of the covariate x_i is removed. The intercept β_{00} and β_{10} becomes the adjusted mean when the value of x_i becomes zero.

2.1.3 Applications and Limitations of First-Order Latent Growth Model

The above literature review mainly covered LCM/GCM when the single indicator variable was measured across time. This model is also known as first-order latent growth model (FOLGM) in contrast to SOLGM which will be described in the next section. A majority of educational research involving longitudinal studies have used the first-order LGM (FOLGM, Geiser, Keller, & Lockhart, 2013). This could be seen, for example, in research assessing changes in students' self efficacy (Phan, 2012), ability and skill acquisition (Zyphur, Bradley, Landis, & Thoresen, 2007), problem-based-learning (Wimmers & Lee, 2015), and students' attitude toward science (George, 2000).

One important application of the FOLGM can be found in longitudinal studies trying to detect group differences in the growth patterns (e.g. control vs. treatment groups, male vs. female). Compared to the traditional approaches such as autoregressive model or repeated-measures ANOVA, FOLGM does offer some key advantages. For example, using FOLGM helps examine not only the mean difference between groups, but also the group differences in terms of variance and covariance, which were often under-addressed in more traditional types of fixed effects models (Curran & Muthén, 1999). The latter information may be more important for intervention programs targeting the behavior problems such as alcohol use, reading comprehension or juvenile delinquency because interventions do not work for all people equally well. Therefore, it is important to identify the people who benefit most from the intervention. Another advantage of using FOLGM is that it gives more power to detect treatment or group effects compared to more traditional fixed effect models (Curran & Muthén, 1999). For example, Fan's (2003) simulation study indicated that FOLGM consistently showed a higher statistical power for detecting group differences in the linear growth parameters (initial status and slope) than the repeated-measure

ANOVA. Last but not least, Conceptual match between the model and the substantial theory could be another advantage of FOLGM.

Discrete time-specific measurement, in general, is not consistent with the developmental change of psychological theories; developmental theory typically does not assume the change in terms of time-specific comparisons (Chan, 1998). In contrast, developmental theory tends to posit that change happens in a continuous process over time, and this whole process is depicted in terms of individual difference in initial level, acceleration, plateau and deceleration (Curran & Muthén, 1999). Normally, modeling time-specific comparisons using approaches such as repeated-measures ANOVA could not capture these complicated types of relations across time. Thus, LCM/LGM not only provides aforementioned statistical advantages, but also is more suited to the developmental theory. Though LCM/LGM was quite an improvement as a model for change/growth, recent studies started to identify the drawbacks of using FOLGM in longitudinal studies, especially for assessing the measurement quality. For example, a simulation study (Leite, 2007) found that the FOLGM is able to produce unbiased estimates of the average growth, variance of the initial level and growth rate, and covariance between initial level and growth rate if the indicators are essentially tau-equivalent. However, when such measurement invariance as tau-equivalent measures is not assumed, the FOLGM could yield considerable bias in estimating parameters. Furthermore, the first-order LGM fails to partition the time-specific variance and the measurement residual variance or item-residual variance. This could reduce the estimate of the slope reliability (Newsom, 2015). More importantly, when comparing the group difference in the growth trajectories, Kim and Willson's (2014a) simulation study showed that, noninvariance in factor loadings and intercepts was associated with the Type I error inflation and bias in the parameter estimates of the slope factor (or latent growth) and the intercept factor (or initial status), respectively. As the size of noninvariance increases, the magnitude of Type I error and bias also increases. In general, to analyze change using composite scores implicitly assumes the measurement of a construct has equivalent measurement properties across time, which implies that every item assesses the latent construct equally well (Hertzog & Nesselroade, 2003; Isiordia & Ferrer, 2018).This restrictive assumption is rarely met in real world analysis. When this measurement invariance assumption was violated, LCM could yield bias in the estimates of the model parameters.

In order to overcome the above limitations inherent in the first-order LGM, researchers integrated a common factor measurement model with the LGM, which yields the ”Second-Order Latent Growth Model” (SOLGM) or ”Curve-of-Factors Model” (CUFFS).

2.2 Second-Order Latent Growth Model (SOLGM)

2.2.1 Model Specification and Parameter Interpretation

The second-order latent growth model (SOLGM) or multiple-indicator latent growth model, which was proposed by McArdle (1988) and Meredith and Tisak (1990), can be treated as an extension of the first-order latent growth model (FOLGM). It is actually a second order model where a common factor is specified for each measurement occasion , and a common initial status and growth factor are specified as the second-order factors. (Hancock et al., 2001).

The common factor part of the SOLGM captures how well the indicators measure the latent variables under each time point. While the latent growth part determines the characteristics of the initial status and growth shape of the latent variable across time. The level 1 sub-model of the SOLGM is a measurement model written as :

$$y_{ijt} = \tau_{jt} + \lambda_{jt}\eta_{it} + \epsilon_{ijt} \tag{2.8}$$

where y_{ijt} is the observed value of jth indicator for individual i measured at time point t. τ_{jt} is the intercept of indicator j at time point t. Λ_{jt} is a factor loading of jth indicator at time point t. ϵ_{ijt} is a measurement error for individual i of the jth indicator at time point t. The variance of ϵ_{ijt} is represented as: $\mathrm{Var}[\epsilon_{ijt}] = \sigma^2_{jt}$.

In the measurement part of the SOLGM, the variance of the indicators can be divided into unique variance and common variance. The unique variance consists of error variance and specific variance. The SOLGM can assume either no specific factors influencing the indicators, or allow the measurement errors to be correlated across time to imply the existence of specific factors.

Once the level 1 sub-model/measurement model is established, the latent growth factors (second-order factors) can be incorporated to form the level 2 sub-model of SOLGM. This part can be represented as:

$$\eta_{it} = \pi_{0i} + \pi_{1i} Time_{it} + r_{it} \tag{2.9}$$

where the η_{it} is a true score of person i at time t; π_{0i} and π_{1i} are still initial status and growth rate scores for individual i, respectively; The r_{it} is a time-specific latent residual of person i at wave t, with its variance Var[r_{it}]=θ_t^2.

In fact, this part of the SOLGM is similar to a univariate LGM. The only difference is that the outcome variables in SOLGM are the latent constructs, while the outcome variables in univariate LGM are observed variables. The loadings of the latent growth parameters in the SOLGM can be either fixed to certain values that reflect hypothetical shape of growth, or free to be estimated for the unspecified growth trajectory. Similar to the univariate LGM, fixing one loading in the growth parameter to zero can help define a reference point (initial time point) for the latent growth parameters.

Similar to conditional univariate LCM, we can add a covariate above the level 2 of SOLGM to form a structural part, which is also called the level 3 of SOLGM. The equation for this level is written as:

$$\pi_{0i} = \beta_{00} + \beta_{01} x_i + u_{0i} \tag{2.10}$$

$$\pi_{1i} = \beta_{10} + \beta_{11} x_i + u_{1i} \tag{2.11}$$

The interpretations for the parameters in equation 2.10 and 2.11 are the same as they are in the conditional LCM. The corresponding SEM model is depicted as Fig. 2.3

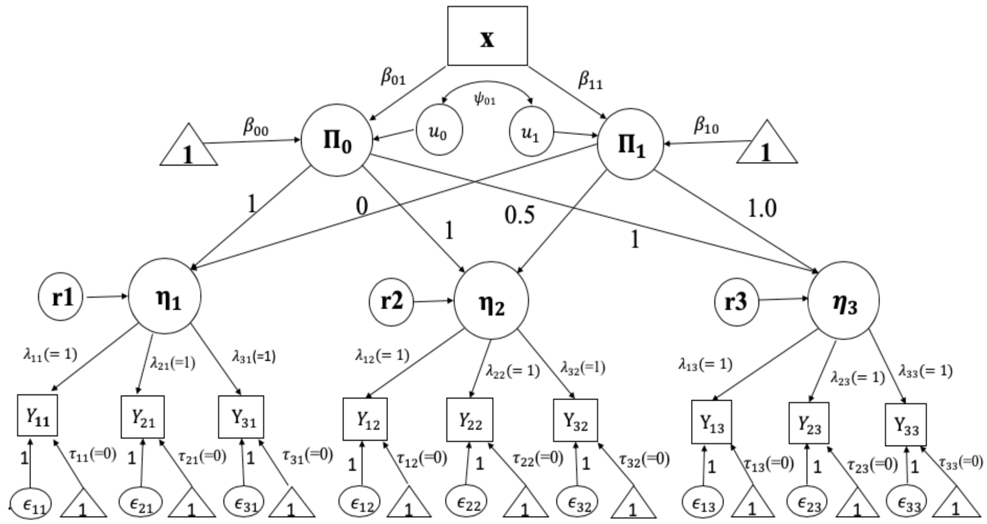

Figure 2.3. Second-Order Linear Latent Growth Model with a Covariate at Leve-3

2.2.2 Longitudinal Measurement Invariance

One of the problems that deserves greater attention for researchers who want to use SOLGM is the longitudinal measurement invariance. In longitudinal studies, the same measurement instruments are repeatedly used because researchers want to ensure that the same construct should be measured over time. However, applying the same instrument does not guarantee that the same construct is being measured across time (Widaman, Ferrer, & Conger, 2010). For example, as time passes, the participants' interpretation for the items of the instrument may change. Therefore, the construct being measured would also be different from the one measured before. As a consequence, the measured change may be partially due to the changes in its factorial structure rather than the pure change in the construct itself. Therefore establishing the longitudinal measurement invariance should be demonstrated prior to employing any SOLGM.

The widely applied taxonomy to evaluate the measurement invariance in longitudinal context is Meredith's (1993) approach. His approach yields four different types of measurement invariance. These types of invariance progress from the least model restriction to the most restriction. The least restrictive invarance, which is also called configural invariance, requires that the pattern of the fixed and free factor loadings are the same across different time points. The second least restrictive invariance is weak factorial invariance. This type of invariance requires that the factor loading of each indicator is identical across time. Next is the strong factorial invariance, which ensures that both the factor loading and intercept of each indicator are the same across time. Finally, the most restrictive type is the strict factorial invariance, which assumes that the intercepts, the factor loadings and error variances are equal over time.

The process of evaluating factorial invariance should start from fitting a model with configural invariance, which means to set the configuration of factor structure, measured by the same set of indicators, the same across time. Then the researchers should fit the model of weak invariance by adding across-time invariance contraints on the parameter of loadings ($\lambda_{21} = \lambda_{22} = \lambda_{23}...;\ \lambda_{31} = \lambda_{32} = \lambda_{33}...$). Third, the strong factorial invariance should be examined, where the across-time contraints on the intercepts are added based on the previous contraints ($\tau_{21} = \tau_{22} = \tau_{23}....;\ \tau_{31} = \tau_{32} = \tau_{33}....$). Finally, the model with strict factorial invariance can be fitted to data by placing the further contraints on the error or unique variances ($\theta_{11} = \theta_{12} = \theta_{13}...;\ \theta_{21} = \theta_{22} = \theta_{23}...$).

The previous sequence of constraining models progressively shows the optimal order of testing the measurement invariance. To compare the fit across models, the likelihood ratio chi-square difference test can be conducted because of the nested structure underlying each successive model (Widaman et al., 2010). However, since the statistic power of the likelihood ratio test will go up with the increase of the sample size, difference in practical fit indices such as the comparative fit index, the Turcker-Lewis index, and the root mean square of error should also be considered when comparing different models (Bentler & Bonett, 1980; Rigdon, 1996).

Horn and Mcardle (1992) and Meredith (1964) argued that in order to meaningfully interpret the factor score, at least the strong invariance should be held across time. In other words, the configural or weak invariance are insufficient to argue that the same latent variable is measured at different time points. As demonstrated by Ferrer et al. (2008), if factorial invariance fails to be held, choice of indicator used to identify the latent variable can have substantial influence on the characterization of patterns of growth. Furthermore, the comparison between groups is also meaningless if the strong invariance can not be reached.

2.2.3 Advantages of SOLGM

There are several advantages of using latent variable to reflect the measured construct. First, the SOLGM should have higher reliability than the first-order LGM, which further lead to greater statistical power. This is because the observed variance at each time is partitioned into measurement residual variance and factor variance: $Var(y_i) = \lambda_{ij}^2\Psi_{jj} + Var(\epsilon_i)$ (von Oertzen, Hertzog, Lindenberger, & Ghisletta, 2010). So the time-specific variance should be reduced. This reduced time-specific variance could improve the reliability estimates. Second, researchers can also model different error structure across time, which can not be done by the first-order LGM. For example, people could include the autocorrelated measurement residuals to account for the stable specific variances (Newsom, 2015). The recent empirical study further emphasized that SOLGM is more preferred when trying to model the different growth trajectories between groups because noninvarince measure could lead to substantial bias in estimating the group difference in the growth parameters (Kim & Willson, 2014a).

One thing that should be noted here is that using the multiple indicators for each occasion does not provide special advantage over the first-order LGM in estimating factor means. The measurement errors actually do not impact the expected measurement mean (Newsom, 2015, p. 7). Meanwhile because the factor (intercept and slope) means are a function of observed means (Newsom, 2015, p. 175), the mean estimate of intercept and slope would also remain unchanged. Although previous studies have shown SOLGM is a more appropriate approach to model intervention or group effects in developmental studies, a standard SOLGM with no special constraints requires at least three time points (Newsom, 2015; Bollen & Curran, 2006, p. 207 and p. 23). In fact, the minimum requirement of three-wave measurement is always a “golden standard” for conducting a longitudinal study, because it provides enough information to fit at least a simple linear model to each individual (Singer & Willett, 2003, p. 11).

However in reality, there are many times that the cost of data collection is high, or staying contact with participants is very challenging, data collection may only be limited with only two waves. The standard SOLGM with group covariate in two-wave data can not be empirically identified without any constrains. Recently, two related SEM models are frequently suggested to deal with two-wave data. They are longitudinal common factor model and latent change score model. For the remaining chapters, we will review these two models.

2.3 Longitudinal Common Factor Model (LCFM)

2.3.1 Model Specification and Application

Longitudinal common factor model (LCFM, Grimm et al., 2016), is a special application of confirmatory factory analysis in the longitudinal study, where the common factor is extracted in each wave. The level 1 sub-model/measurement model for each time can be written as:

$$y_{ijt} = \tau_{jt} + \lambda_{jt}\eta_{it} + \epsilon_{ijt} \tag{2.12}$$

where y_{ijt} represents the observed score of individual i ($i = 1, \ldots, N$) on indicator j ($j = 1 \ldots k$) at measurement time t ($t = 1 \ldots T$). τ_{jt} is the intercept for the indicator j at time t. λ_{jt} is the factor loading for person i at time t. η_{it} represents the latent construct score for person i at measurement time t, ϵ_{ijt} is the unique factor score for person i at time t. The intercepts and factor loadings can be estimated directly, but a person's score on a unique factor can not be fully estimated. Furthermore, the mean and variance of the latent construct can also be estimated. The level 2 of LCFM can be represented as:

$$\eta_{it} = \kappa_t + \gamma_t x_i + \zeta_{it} \tag{2.13}$$

where κ_t is an intercept for η_{jt},γ_{jt} is the regression coefficient of x_i, and ζ_{it} is a residual term.

To date, the LCFM is mainly used to address questions such as ”are the constructs measured equivalently across time?” in longitudinal study (Little, Preacher, Selig, & Card, 2007). In developmental research, the key assumption that the construct is measured in the same metric across time should be first examined before any analysis. If the construct being measured changes over time, the observed growth may be mainly due to the change in the factorial structure of this construct. Therefore rather than assuming it to be true, researchers can formally test this hypothesis using the LCFM.

In addition, LCFM can also be used to address some validity-issues. Because of the repeated measure of the same construct, these validity issues could be assessed more rigorously compared to the cases using cross-sectional data. For example, the content validity of research construct could be assessed by testing the patterns and magnitude of factor loadings and intercepts as well as configural invariance (Little et al., 2007). The criterion of construct validity can be addressed in a number of ways. One way is to examine whether the concurrent pattern of relations among constructs could be replicated across occasions (Little et al., 2007). Another way is to examine the cross-time associations among constructs to confirm if they follow the expected patterns (Little et al., 2007).

2.3.2 Two-Wave LCFM

Beyond the test of measurement invariance, a recent study demonstrated that LCFM could also be applied to model the change across time (Finch & Shim, 2018). For example, if we are modeling a latent variable η, which is measured at two time points (Figure 2.4), the change of this latent variable across time for individual i can be represented as:

$$\triangle \eta_i = \eta_{i2} - \eta_{i1} \tag{2.14}$$

Similarly, we can model the change over time for the indicator y_i, using the difference score:

$$\triangle y_i = y_{i2} - y_{i1} \tag{2.15}$$

Finally, by combining equations 2.14 and 2.15 with equation 2.13, we can construct and estimate the measurement model for the difference factors:

$$\triangle y_i = \tau_{\triangle\eta_i} + \lambda_{\triangle\eta_i} \triangle \eta_i + \epsilon_{\triangle_i} \tag{2.16}$$

The LCFM can incorporate other structural model forms such as the multiple indicators multiple causes (MIMIC) model. Furthermore, it is also very convenient to add covariates above the common latent factors to describe what variables can influence this latent change over time. One normal case is to add a dichotomous variable to predict the latent change score. In this way, one can assess whether two groups experience the similar change in the latent construct. Finally, by testing the measurement invariance, it would be likely to determine whether the relationships between the change of indicators ($\triangle y$) and their latent change factor ($\triangle\eta$) are same across groups.

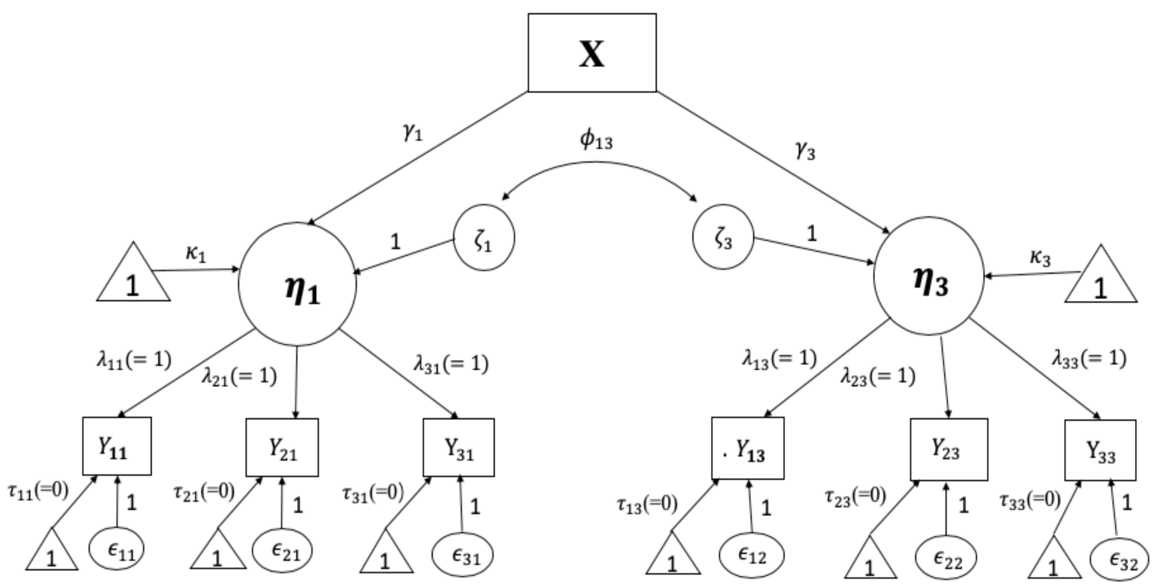

Figure 2.4. 2W-Longitudinal Common Factor Model with a Covariate

2.4 Latent Change Score Model (LCSM)

The latent change score model (LCSM) is a framework of studying longitudinal change which combines the autoregressive cross-lag and latent curve models for the panel data (Grimm, An, McArdle, Zonderman, & Resnick, 2012). Under this framework, the simplest latent difference score model is to explicitly specify a latent difference factor over consecutive waves of data (Newsom, 2015). Recently, LCSM has become a powerful and flexible SEM modeling approach that investigates a wide range of developmental process with relative ease. This section will introduce the basic concepts and specification of LCM .

2.4.1 Univariate Latent Change Score Model

In this section, we started from the simplest cases, the univariate LCSM. The key idea is that we could represent the difference between two adjacent measurement within the autoregressive structure (Newsom, 2015). The residual is equal to the difference score if y_t is regressed on y_{t-1} with the autoregressive slope weight fixed to 1. For a simple example written as a structure model, the equation:

$$y_{it} = \beta_{t,t-1} y_{t-1} + \varsigma_{it} \tag{2.17}$$

When $\beta_{t,t-1}$, the autoregression coefficient, is fixed to 1, the above equation can be rearranged to show that the residual can be expressed as the difference between two observed scores:

$$y_{it} = \beta_{t,t-1} y_{i,t-1} + \varsigma_{it} \tag{2.18}$$

$$y_{it} = (1) y_{i,t-1} + \varsigma_{it} \tag{2.19}$$

$$\varsigma_{it} = y_{it} - y_{i,t-1} \tag{2.20}$$

With the above equations, the average residual is taken by averaging the difference scores across all cases, $E(\varsigma_{it}) = E(y_{it} - y_{i,t-1})$. Based on the linear property of expectation operation, the expectation operation could be taken before the summation or subtract. So $E(\varsigma_{it}) = E(y_{it} - y_{i,t-1}) = E(y_{it}) - E(y_{i,t-1})$. Therefore, the direct test of the significant mean change is equal to a paired t-test or repeated measures ANOVA (Coman et al., 2013)

If we relabel the residual ς_{it} with $\Delta y_{t,t-1}$ in equation (2.20) and fix the $\beta_{t,t-1}$ to 1, the score from the following time point can be defined as the score at the previous time point plus the difference score:

$$\Delta y_{t,t-1} = y_{it} - y_{i,t-1} \tag{2.21}$$

$$y_{it} = y_{i,t-1} + \Delta y_{t,t-1} \tag{2.22}$$

Equation 2.22 set up the foundation of latent change score model (LCSM) because this formulation can be expanded to the model with a series of change. In the SEM context, the simple difference score $\Delta y_{t,t-1}$ can be specified as a latent change score factor $\Delta \eta_t$. This simple change score factor is estimated by second measures, y_{ti} with its loading fixed to 1. y_{ti} is then predicted by measure at the previous time y_{t-1} with the path coefficient set equal to 1. These operations create a latent factor that represents the change between the previous time point and following time point. Finally, a regression coefficient β is added to the path between the previous measure y_{t-1} and the latent change factor $\Delta \eta_t$, which allows us to test how much degree of change is associated with the measures at previous time. With the model formed in the above manner, the latent change factor is now explicitly defined as the part of y_{it} that is not identical to $y_{i,t-1}$ (McArdle, 2009). We therefore can directly estimate and test questions about the change happened in between two time points. The traditional statistical features of the change score are all included as model parameters such as the average of the change ($\mu_{\Delta\eta}$), the variance of the change factor ($\sigma^2_{\Delta\eta}$) and the covariance between the change scores and the initial scores ($\sigma_{\Delta\eta\eta_1}$). For example, we can now test the hypothesis of whether there is a reliable mean change from the two time points by forcing the mean difference to be zero ($\mu_{\Delta\eta}$=0). Under this simple assumptions it fully replicates the paired t-test. Another parameter of considerable interest is the variance of in the latent change factor, $\sigma^2_{\Delta\eta}$. We can use the same model to test whether there is an individual difference in the change score ($\sigma^2_{\Delta\eta}$=0). Finally, we can test the covariance or autoregressive relationship between the score in the initial point and the change score. This captures the extent to which the change is related to or proportional to the scores at the initial time (Kievit et al., 2018).

2.4.2 Multiple Indicator Univariate Latent Change Score Model

The previous section introduced the LCSM based on the observed variables, which assumed there is no measurement error and the observed variables can perfectly reflect the true latent variables. We now expand the previous models by adding the explicit measurement model, where the true latent variable is measured by several indicators. In this section, we will describe the multiple indicator, univariate latent change score model for the purpose of modeling change in terms of the latent variables rather than the observed variables.

To establish this model, we first need to illustrate the association between the latent variable and the indicators under the tradition of confirmatory factor analysis. The level 1 of LCSM/measurement model is expressed as:

$$y_{ijt} = \tau_{jt} + \Lambda_{jt}\eta_{it} + \epsilon_{ijt} \tag{2.23}$$

Since the above measurement model is the same as the LCFM, the interpretations for the parameters will be the same as before.

The Level 2 sub-model/structural part of the mulitple indicator LCSM takes the following form:

$$\eta_{i2} = \eta_{i1} + \Delta\eta_i \tag{2.24}$$

where η_{i1} and η_{i2} are factor scores for the same person i at time 1 and time 2. $\Delta\eta_i$ is the latent change score for person i. As with other models, it is important to establish measurement invariance over time or across groups to improve inference, as was demonstrated in previous section. There are several advantages to use multiple indicators LCSM, but these advantages do not influence the estimate of the average of the difference score factor, because the expected value of any observed variable is not biased by random measurement error (Newsom, 2015).

Although including the multiple indicators does not benefit for estimating the average of the difference scores, there are still some advantages to use multiple indicators at each occasion. First, one can specify the correlated measurement residuals to account for the method variance among repeated measurements of each indicator (McArdle, 2009). This will improve the estimates of the correlations between intercept and slope factors, or between intercept and difference factor (McArdle, 2009). Second, compared to the single measurement at each time point, multiple measures of the same construct at each occasion can add reliability to each assessment. Last but not least, the estimate of variance of latent score is inflated by the existence of measurement error, and this inflation is compounded when the composite is derived from individual measurement, such as difference score. As a consequence, multiple indicators should improve the precision of the difference score, increasing the power of significance tests for the factor means and the effect of covariates

(Newsom, 2015).

2.4.3 Two-Wave Latent Change Score Model

The two-wave LCSM (2W-LCSM) is a type of the multiple indicator univariate latent change score model (Figure 2.5). Its design is well suited to investigate change-to-change hypothesis for the two-wave data. The 2W-LCSM purges latent construct from measurement error by using multiple items (Henk & Castro-Schilo, 2016). At each time of measurement, the model shares the same measurement part:

$$y_{it} = \tau_{jt} + \lambda_{jt}\eta_{it} + \epsilon_{ijt} \quad t = 1, 3 \quad \text{and} \quad i = 1...N \tag{2.25}$$

The Level 2 sub-model is:

$$\eta_{i3} = \eta_{i1} + \triangle\eta_i \tag{2.26}$$

where the subscripts 1 and 3 specify time point, $\triangle\boldsymbol{\eta_i}$ is vector containing latent change score for each person i from time 1 to time 3. One implicit assumption for the above equation is that factor loadings and intercepts are same across time (i.e strong factor in variance) (Henk & Castro-Schilo, 2016). Although the strict factor invariance can be tested by adding equality constraint across time to the unique factors, it is not required for the model.

Recently, the 2W-LCSM has been proposed to estimate the relationships in change over time for latent variable with only two time points (Henk & Castro-Schilo, 2016). The later empirical study indicated that 2W-LCSM is a powerful tool for modeling change with two-wave data (Henk & Castro-Schilo, 2016).

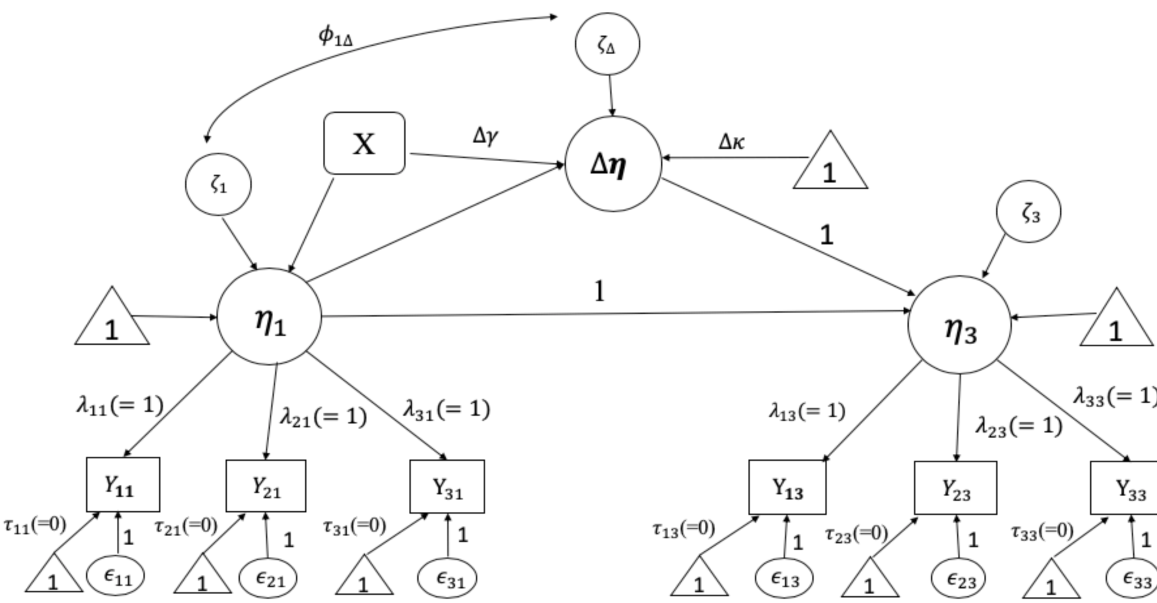

Figure 2.5. 2W-Latent Change Score Model with a Covariate

2.5 Problem Statement and Research Questions

2.5.1 Problem Statement

The previous literature review showed that the second-order latent growth model (SOLGM) is the more appropriate approach to compare the group differences in the linear growth trajectories when multiple items are available at each time of measurement. However, the application of it still requires at minimum three waves of data, which may be difficult to realize in some situations. Actually, we can not neglect the fact that there will inevitably be certain times when only two-wave data has been collected for longitudinal study. At that moment, the SOLGM can not be used because of the identification issue. The choices for researchers are either to wait until the remaining wave of data are collected or to choose alternative models to do analysis.

As demonstrated from the above literature review, two models (LCFM and LCSM) recently have been suggested for dealing with two-wave data. To date, the main application of LCFM was used to address the issue of whether constructs measured are equivalent across time in longitudinal study. This is a starting point for any further model testing in longitudinal study. However, Miyazaki's (2017) recent study showed that two-wave LCFM could also provide accurate estimate of initial status and linear growth rate parameters by taking the mean difference of true scores (i.e., expectation of true difference score). This is a pioneering work shedding light on possible applications of LCFM in estimating linear growth trajectory. Two-wave latent change score model is another model, proposed by Henk and Castro-Schilo (2016), to detect the change-to-change relationships. The characteristic of this model is that multiple items are used to specify a latent variable at each of two occasions. To add this measurement model can separate true score variance from unique variance, leading to a perfectly reliable latent change score. A later simulation study (Finch & Shim, 2018) revealed this model could yield accurate estimates of the correlation between changes in latent constructs. However, no study has been done to examine how well the 2W-LCSM could be used to estimate the group differences in linear growth trajectories.

To sumarize, including the measurement model at each occasion can extract the error-free constructs, which helps attenuate the inflated variance of composite difference score. (Newsom, 2015, . p259). As a consequence, the precision of the estimate and the statistical power of detecting the group effect should be increased (Newsom, 2015, . p260). With these advantages, using the two-wave LCFM and LCSM to estimate the fixed effects of linear growth parameters should be better than the using the composite score at each occasion. However, there is little empirical evidence to show how much this estimation could be improved, especially the precision of estimates.

The current research refers to Miyazaki's (2017) study design, but puts more focus on the the group differences in the linear growth parameters. The reason is that investigating the systemic differences in the growth trajectories between groups is still a very popular topic, especially in the longitudinal intervention studies. A commonly used intervention design for a short time series is randomized pretest, posttest, follow-up or pre–post–post designs, where the long-term intervention effects could be recorded. Based on previous studies, the SOLGM will be the ideal approach to evaluate this long term intervention effect. However in practice, researchers often face different challenges to collect three waves of data. Accordingly, the use of pretest-postest design is widely accepted choice in educational or psychological intervention fields (Finch & Shim, 2018).

The most recent advances in methodological research already shed light on the potential of LCFM and LCSM to estimate the growth trajectory. However, there is still a lack of empirical evidence about the accuracy of these two models when they are being used to estimate the linear growth parameters. Therefore, the main purpose of the current research is trying to explore to what extent the fixed effects of the three-wave model (SOLGM) can be recovered from the parameter estimates of the two-wave models (LCFM and LCSM).

2.5.2 Research Questions

The results in Miyazaki's (2017) study showed that the estimation of the fixed effects via LCFM was sufficiently precise. However, the assumption of homoscedasticity for time-specific variances in his study is often violated in real development study. In addition, researchers are often interested in comparing the group differences (e.g. males vs female or treatment vs control groups) in their growth trajectories,but Miyazaki's findings do not address these issues.

Thus, the present dissertation will develop Miyazaki's research through a number of methods: (1) A binary corariate indicating group membership will be added to the data-generation (SOLGM) and analytical models (LCFM and LCSM); (2)According to the literature review, LCSM wiil be added to pair with LCFM as analytical models. (3) Data-generating conditions will be extended by manipulating (a) sample size, (b) mean group difference in intercept and slope, (c) covariance between intercept and slope, and (d) size and pattern of time-specific residuals. Given the research background above, the main research questions in the current study are:

1. Can the two-wave conditional LCFM and LCSM recover the parameter of the fixed effects of the conditional SOLGM?

2. What factors could influence the accuracy and statistical power in detecting the fixed

effects of the three-wave conditional SOLGM by the two-wave models?

Chapter 3

Methods

In this section, we first used mathematical operations to demonstrate how the fixed effects parameters in the three-wave model (i.e. SOLGM) could be derived from the parameters of the two-wave models (LCFM, LCSM). Then, two simulation studies will be conducted to explore our research questions. For the simulation studies, the item-level data will be first generated from the three-wave SOLGM with a binary covariate that represents group. Six experimental factors will be manipulated for the data-generating conditions: (a) sample size; (b) average group differences in intercept and slope; (c) covariance between the slope and intercept; (d) size and patterns of time-specific residuals. Next, the LCFM, LCSM and SOLGM with the group covariates will be used as the analytic models. They are expected to produce equivalent estimates in terms of the fixed effects. Finally, this study will examine the effects of these experimental factors on the accuracy and statistical power in detecting the fixed effects of the three-wave model by the two-wave models.

3.1 Mathematics Form of Different Models

3.1.1 Second-Order Latent Growth Model (SOLGM)

Level 1 (measurement model):

$$y_{ijt} = \tau_{ju} + \lambda_{jt}\eta_{it} + \epsilon_{ijt} \tag{3.1}$$

y_{ijt} is the observed score for person i of the indicator j at time t, λ_{jt} is the factor loading of the indicator j at time t. η_{it} is the latent factor score for individual i and time measurement t and ϵ_{ijt} is the error score for person i of indicator j at time t.

level 2 (growth model):

$$\eta_{it} = \pi_{0i} + \pi_{1i}Time_{it} + r_{it} \tag{3.2}$$

π_{0i} and π_{1i} are intercept and slope factor for individual i, respectively. They have a joint distribution such as $\pi_{\mathbf{i}}$=$(\pi_{0i}\ \pi_{1i})^t \sim N_2, (\boldsymbol{\beta}, \boldsymbol{\Psi})$ where,
$\beta = (\beta_{00},\ \beta_{10})^t, \boldsymbol{\Psi} = \psi_{kl} \quad (k, l = 0, 1)$. $Time_{it}$ represents the time variable created by researchers. r_{it} is the time specific latent residual for person i at time t, with variance: Var$[r_{it}]$=θ_t^2

Level 3 (structural model)

$$\begin{aligned} \pi_{0i} &= \beta_{00} + \beta_{01}x_i + u_{0i} \\ \pi_{1i} &= \beta_{10} + \beta_{11}x_i + u_{1i} \end{aligned} \tag{3.3}$$

β_{00} and β_{01} are the average initial status for control group and average group difference in the initial status, respectively. β_{01} and β_{11} are the average growth rate for control group, and average group difference in the growth rate. x_i is the binary covariate value for individual i. u_{0i} and u_{1i} are the conditional disturbances for person i, where $(u_{0i}, u_{1i})^t \sim MVN(\mathbf{O}, \boldsymbol{\Psi})$

3.1.2 Two-Wave Longitudinal Common Factor Model (2W-LCFM)

Level 1 (Measurement Model)

$$y_{ijt} = \tau_{ju} + \lambda_{jt}\eta_{it} + \epsilon_{ijt} \qquad \epsilon_{ijt} \sim N(0, \sigma^2) \tag{3.4}$$

the interpretation of this equation is the same as equation 1.

Level 2 (Structure Model)

$$\eta_{it} = \kappa_t + \gamma_t x_i + \varsigma_{it} \tag{3.5}$$

κ_t and γ_t are the regression intercept and regression coefficient for the measurement t. ς_{it} is the residual term for person i at time t, where $COV[\boldsymbol{\varsigma}] = \mathbf{\Phi} = \phi_{kl} \quad (k, l = 0, 1)$.

3.1.3 Two-Wave Latent Change Score Model (2W-LCSM)

$$\begin{aligned} \eta_{i3} &= \eta_{i1} + \triangle\eta_i \\ \triangle\eta_i &= \kappa_\triangle + \gamma_\triangle x_i + \varsigma_{\triangle i} \end{aligned} \tag{3.6}$$

$\triangle\eta_i$ is the latent change score for individual i. $\triangle\eta_i$ and $\gamma_\triangle$ are the regression intercept and coefficient for person i. $\varsigma_{\triangle i}$ is the regression residual.

3.2 Correspondence between Parameters in Different Models

If we substituted 3.3 into 3.2, we could obtain:

$$\begin{aligned} \eta_{it} &= \pi_{0i} + \pi_{1i} Time_{it} + r_{it} \\ &= \beta_{00} + \beta_{01} x_i + u_{0i} + (\beta_{10} + \beta_{11} x_i + u_{1i}) Time_{it} + r_{it} \\ &= (\beta_{00} + \beta_{10} Time_{it}) + (\beta_{01} + \beta_{11} Time_{it}) x_i + u_{0i} + u_{1i} Time_{it} + r_{it} \\ &= \kappa_t + \gamma_t x_i + \varsigma_{it} \end{aligned} \tag{7}$$

where $\beta_{00} + \beta_{10} Time_{it} = \kappa_t$ and $\gamma_t = \beta_{01} + \beta_{11} Time_{it}$. When we analyze a two-wave data (i.e., $t \in \{1, 3\}$), the time variable could be rearranged and coded as $Time_{i1} = 0$ and $Time_{i3} = 1$, which indicate that the duration between t_1 and t_3 becomes unit level. Therefore, by substituting the value 0 or 1 into the time variable from equation 7, we could obtain the following relationships between parameters

$$
\begin{aligned}
\kappa_1 &= \beta_{00} \\
\kappa_3 &= \beta_{00} + \beta_{10} \\
\gamma_1 &= \beta_{01} \\
\gamma_3 &= \beta_{01} + \beta_{11} \\
\varsigma_{i1} &= u_{0i} + r_{0i} \\
\varsigma_{i3} &= u_{0i} + u_{1i} + r_{i3}
\end{aligned}
$$

Furthermore, with the assumption that: $E[u_{0i}] = E[u_{1i}] = E[r_{it}] = 0$, we can derive:

$$
\begin{aligned}
E[\eta_{i1} | x_i = 0] &= \kappa_1 = \beta_{00} \\
E[\eta_{i3} - \eta_{i1} | x_i = 0] &= \kappa_3 - \kappa_1 = \beta_{10} \\
E[\eta_{i1} | x_i = 1] - E[\eta_{i1} | x_i = 0] &= \gamma_1 = \beta_{01} \\
E[\eta_{i3} - \eta_{i1} | x_i = 1] - E[\eta_{i3} - \eta_{i1} | x_i = 0] &= \gamma_3 - \gamma_1 = \beta_{11}
\end{aligned}
$$

Based on the above relationships between different parameters, we could estimate the fixed effects parameters of the SOLGM $(\beta_{00}, \beta_{10}, \beta_{01}, \beta_{11})$ from the parameters of LCFM $(\kappa_1, \kappa_3, \gamma_1, \gamma_3)$ and vice verse. However, In contrast to the fixed effects, variance components of SOLGM cannot be estimated from the parameters of LCFM. This can be demonstrated from the following relationships:

$$Var[\eta_{i1}] = \psi_{00} + \theta_1^2 > \psi_{00} = Var[\pi_{0i}]$$

$$Var[\eta_{i3} - \eta_{i1}] = \psi_{11} + \theta_1^2 + \theta_3^2 > \psi_{11} = Var[\pi_{1i}]$$

$$Cov[\eta_{i1}, \eta_{i3} - \eta_{i1}] = \psi_{01} - \theta_1^2 < \psi_{01} = COV[\pi_{0i}, \pi_{1i}]$$

Similar to the above reasoning process, we could also derive the correspondence between fixed parameters of SOLGM and LCSM. From the equation3.6 , we could obtain:

$$E[\triangle\eta_i|x_i = 0] = \kappa_\triangle = \beta_{10}$$
$$E[\triangle\eta_i|x_i = 1] - E[\triangle\eta_i|x_i = 0] = \gamma_\triangle = \beta_{11}$$

Thus, we can also obtain the estimates of the fixed effects of SOLGM from those of LCSM ($\kappa_\triangle, \gamma_\triangle$).

3.3 Study 1

3.3.1 Data Generation Conditions

The software R (version 3.6.1) with the package lavaan (version 0.6-5) was used to conduct the whole simulation study. All the datasets in study 1 were generated from the SOLGM under different conditions. These conditions were summarized in the following table 3.1

Table 3.1

Summary of the Data Generation Condition

Factors	Levels
Sample Size (n)	50, 200, 600,1000
Average group differences in intercept and slope (β_{01}, β_{11})	(0,0), (3,0), (0,2), (3,2)
Covariance between the slope and intercept (ψ_{01})	0, 1, 1.5
Size of time-specific residual (θ_2^2)	0.25, 1, 4
Patterns of time-specific residuals	Low Edge Pattern$(0.25\theta_2^2, \theta_2^2, 0.25\theta_2^2)$ High Edge Pattern$(4\theta_2^2, \theta_2^2, 4\theta_2^2)$ Moderate Shrinking Pattern$(2\theta_2^2, \theta_2^2, 0.5\theta_2^2)$ Heavy Shrinking Pattern$(4\theta_2^2, \theta_2^2, 0.25\theta_2^2)$ Constant Pattern $(\theta_2^2, \theta_2^2, \theta_2^2)$ Moderate Spreading Pattern$(0.5\theta_2^2, \theta_2^2, 2\theta_2^2)$ Heavy Spreading Pattern $(0.25\theta_2^2, \theta_2^2, 4\theta_2^2)$

First, the binary covariate x_i was generated, which simulated participant's group number ($x_i \in \{0, 1\}$). Since the balanced design was chosen for the current study, x_i were set at x_i=0 for $i = 1, 2, ...\frac{N}{2}$, and $x_i = 1$ for $i = \frac{N}{2} + 1, \frac{N}{2} + 2..., N$, where N is the sample size. The simulated sample sizes were 50, 200, 600, and 1000. With regards to the random effect for each person, the variances of initial status and linear growth rate were set at ψ_{00}=4 and $\psi_{11} = 1$. In terms of the covariance between the initial status and growth rate ψ_{01}, three levels were assumed, namely $\psi_{01} \sim 0, 1, 1.5$. This made the correlation between initial status and growth rate $Cor[\pi_{0i}, \pi_{1i}]$ to at three levels (0, 0.5, 0.75). Then a random effect vector $\mathbf{u}$ was generated from a bivariate normal distribution $N_2(\mathbf{O}, \mathbf{\Psi})$, where

$$\mathbf{\Psi} = \begin{bmatrix} \psi_{00} & \psi_{01} \\ \psi_{01} & \psi_{11} \end{bmatrix}$$

For the fixed effects, the average initial status and growth rate for the control group were fixed at (β_{00}, β_{10})=(10,1). Moreover two cases that whether there exists the group difference in the initial status and growth rate were considered; four sets of (β_{01}, β_{11}) were assumed: (0,0),(3,0),(0,2),(3,2). Given these parameter values, individual intercepts and slopes π_{0i} and π_{1i} were calculated by using the equation 3.3.

Second, true score of the latent construct η_{it} was generated according to equation 3.2, where $Time_{it}$ variable was coded as $Time_{it}$=0, 0.5, 1 for t=1, 2, 3. For the time specific residual variance θ_t^2, the three different sizes were simulated: small size (θ_2^2=0.25), medium (θ_2^2=1), and large size (θ_2^2=4). θ_1^2 and θ_3^2 are determined by relating θ_2^2 in different ways, which resulted into seven different changing patterns. They are: Low Edge Pattern ($0.25\theta_2^2$, θ_2^2, $0.25\theta_2^2$), High Edge Pattern($4\theta_2^2$, θ_2^2, $4\theta_2^2$), Moderate Shrinking Pattern ($2\theta_2^2$, θ_2^2, $0.5\theta_2^2$), Heavy Shrinking Pattern ($4\theta_2^2$, θ_2^2, $0.25\theta_2^2$); Constant Pattern (θ_2^2, θ_2^2, θ_2^2), Moderate Spreading Pattern ($0.5\theta_2^2$, θ_2^2, $2\theta_2^2$), Heavy Spreading Pattern ($0.25\theta_2^2$,θ_2^2, $4\theta_2^2$). Based on these settings, the the time specific residual r_{it} were generated form a normal distribution $N(0, \theta_t^2)$. Note that Cov[r_{it},$r_{it'}$]=0

The settings were determined so that the relative proportion of θ_t^2 varied within a reasonable range. Since the variance of η_{it} is structured by the variance components such that: $Var[\eta_{it}] = \psi_{00} + \psi_{11}Time_{it}^2 + 2\psi_{01}Time_{it} + \theta_t^2$. The quantity $\theta_t^2/Var[\eta_{it}]$ varies as shown from tables 3.2 to 3.4.

Table 3.2

Relative Size of Time-Specific Variance in the Simulation when $\psi_{01} = 0$

Growth-Related Variance/ Covariance				Time-Specific Variance			Relative Size of θ_t^2		
ψ_{00}	ψ_{11}	ψ_{01}	Cor[π_{0i}, π_{1i}]	θ_1^2	θ_2^2	θ_2^3	θ_1^2/ Var[η_{i1}]	θ_2^2/ Var[η_{i2}]	θ_3^2/ Var[η_{i3}]
4	1	0	0	0.0625	0.25	0.0625	1.54%	5.56%	1.23%
4	1	0	0	1	0.25	1	20.00%	5.56%	16.67%
4	1	0	0	0.5	0.25	0.125	11.11%	5.56%	2.44%
4	1	0	0	1	0.25	0.0625	20.00%	5.56%	1.23%
4	1	0	0	0.25	0.25	0.25	5.88%	5.56%	4.76%
4	1	0	0	0.125	0.25	0.5	3.03%	5.56%	9.09%
4	1	0	0	0.0625	0.25	1	1.54%	5.56%	16.67%
4	1	0	0	0.25	1	0.25	5.88%	19.05%	4.76%
4	1	0	0	4	1	4	50%	19.05%	44.44%
4	1	0	0	2	1	0.5	33.33%	19.05%	9.09%
4	1	0	0	4	1	0.25	50%	19.05%	4.76%
4	1	0	0	1	1	1	20%	19.05%	16.67%
4	1	0	0	0.5	1	2	11.11%	19.05%	28.57%
4	1	0	0	0.25	1	4	5.88%	19.05%	44.44%
4	1	0	0	1	4	1	20%	48.48%	16.67%
4	1	0	0	16	4	16	80%	48.48%	76.19%
4	1	0	0	8	4	2	66.67%	48.48%	28.57%
4	1	0	0	16	4	1	80%	48.48%	16.67%
4	1	0	0	4	4	4	50%	48.48%	44.44%
4	1	0	0	2	4	8	33.33%	48.48%	61.54%
4	1	0	0	1	4	16	20%	48.48%	76.19%

Table 3.3

Relative Size of Time-Specific Variance in the Simulation when ψ_{01}=1

Growth-Related Variance/ Covariance				Time-Specific Variance			Relative Size of θ_t^2		
ψ_{00}	ψ_{11}	ψ_{01}	Cor[π_{0i}, π_{1i}]	θ_1^2	θ_2^2	θ_2^3	θ_1^2/ Var[η_{i1}]	θ_2^2/ Var[η_{i2}]	θ_3^2/ Var[η_{i3}]
4	1	1	0.5	0.0625	0.25	0.0625	1.54%	4.55%	0.90%
4	1	1	0.5	1	0.25	1	20.00%	4.55%	12.5%
4	1	1	0.5	0.5	0.25	0.125	11.11%	4.55%	1.75%
4	1	1	0.5	1	0.25	0.0625	20.00%	4.55%	0.90%
4	1	1	0.5	0.25	0.25	0.25	5.88%	4.55%	3.45%
4	1	1	0.5	0.125	0.25	0.5	3.03%	4.55%	6.67%
4	1	1	0.5	0.0625	0.25	1	1.54%	4.54%	12.5%
4	1	1	0.5	0.25	1	0.25	5.88%	16.00%	3.45%
4	1	1	0.5	4	1	4	50%	16.00%	36.36%
4	1	1	0.5	2	1	0.5	33.33%	16.00%	6.67%
4	1	1	0.5	4	1	0.25	50.00%	16.00%	3.45%
4	1	1	0.5	1	1	1	20.00%	16.00%	12.50%
4	1	1	0.5	0.5	1	2	11.11%	16.00%	22.22%
4	1	1	0.5	0.25	1	4	5.88%	16.00%	36.36%
4	1	1	0.5	1	4	1	20%	43.24%	12.50%
4	1	1	0.5	16	4	16	80%	43.24%	69.56%
4	1	1	0.5	8	4	2	66.67%	43.24%	22.2%
4	1	1	0.5	16	4	1	80%	43.24%	12.50%
4	1	1	0.5	4	4	4	50%	43.24%	36.36%
4	1	1	0.5	2	4	8	33.33%	43.24%	53.33%
4	1	1	0.5	1	4	16	20%	43.24%	69.56%

Table 3.4

Relative Size of Time-Specific Variance in the Simulation when $\psi_{01} = 1.5$

Growth-Related Variance/ Covariance				Time-Specific Variance			Relative Size of θ_t^2		
ψ_{00}	ψ_{11}	ψ_{01}	Cor[π_{0i}, π_{1i}]	θ_1^2	θ_2^2	θ_2^3	θ_1^2/ Var[η_{i1}]	θ_2^2/ Var[η_{i2}]	θ_3^2/ Var[η_{i3}]
4	1	1.5	0.75	0.0625	0.25	0.0625	1.54%	4.17%	0.78%
4	1	1.5	0.75	1	0.25	1	20.00%	4.16%	11.11%
4	1	1.5	0.75	0.5	0.25	0.125	11.11%	4.17%	1.54%
4	1	1.5	0.75	1	0.25	0.0625	20.00%	4.17%	0.78%
4	1	1.5	0.75	0.25	0.25	0.25	5.88%	4.17%	3.03%
4	1	1.5	0.75	0.125	0.25	0.5	3.03%	4.16%	5.88%
4	1	1.5	0.75	0.0625	0.25	1	1.54%	4.17%	11.11%
4	1	1.5	0.75	0.25	1	0.25	5.88%	14.81%	3.03%
4	1	1.5	0.75	4	1	4	50%	14.81%	33.33%
4	1	1.5	0.75	2	1	0.5	33.33%	14.81%	5.88%
4	1	1.5	0.75	4	1	0.25	50%	14.81%	3.03%
4	1	1.5	0.75	1	1	1	20%	14.81%	11.11%
4	1	1.5	0.75	0.5	1	2	11.11%	14.81%	20.00%
4	1	1.5	0.75	0.25	1	4	5.88%	14.81%	33.33%
4	1	1.5	0.75	1	4	1	20%	41.03%	11.11%
4	1	1.5	0.75	16	4	16	80.00%	41.03%	66.67%
4	1	1.5	0.75	8	4	2	66.67%	41.03%	20.00%
4	1	1.5	0.75	16	4	1	80%	41.03%	11.11%
4	1	1.5	0.75	4	4	4	50%	41.03%	33.33%
4	1	1.5	0.75	2	4	8	33.33%	41.03%	50.00%
4	1	1.5	0.75	1	4	16	20%	41.03%	66.67%

Finally, five observed variables or indicators were generated for each wave using equation 3.1. This range was used to reflect the scenario where researchers may use several items to measure a single construct from a large survey. In this case, the number of selected items is typically small. For simplicity and to focus on the effects of time-specific variance, measurement invariance over time and parallel measurement were assumed. More specifically, in all waves, $u_{jt} = 0, \lambda_{it} = 1$ and $\sigma^2_{jt} = 4$ were assumed. Note that $\sigma^2_{jt} = \psi_{00} < Var[\eta_{1i}]$, so that communality of each item is greater than 0.50. In sum, six factors: N, (β_{00}, β_{11}), ψ_{01}, θ^2_2, (θ^2_1, θ^2_2,θ^2_3) were fully crossed, resulting into 4*2*2*3*3*7= 1008 total conditions. The replications of each condition were 1000, so there were 1,088,000 data sets in total. Given such a heavy computation load, the parallel computation package "foreach" were used to boost up the computation speed.

3.3.2 Evaluation Procedure

The parallel computation was only applied to the replications part (most inner loop) of the nested loop structure, and the summarized statistics such as "mean" "estimate", "bias", "se" and so on over 1000 replications across models were finally stored into external datasets. When the data were analyzed by the two-wave models, the variables in the second wave were skipped and the third wave was used as the post-test. The analytical models were specified as the same as Figure 2.3, 2.4 and 2.5, where measurement invariance and local independence within a wave was assumed. Meanwhile, no correlation between time-specific residuals was assumed. Furthermore, some parameters of models were fixed at some constants. For example, item intercepts and factor loading of the first item within each wave were set at 0 and 1, respectively $(\mu_{1t} = 1, \lambda_{it} = 1)$. For the SOLGM, the coefficients on the paths from π_{0i} and π_{1i} were set at specific values to draw the individual growth trajectories. For the LCSM, the coefficients of the paths from η_{i1} to η_{i3} and from $\triangle\eta_i$ to η_{i3} were fixed to 1.

3.3.3 Evaluation Indices

Three indices were used to evaluate the quality of estimation. The first one was biased. When the aim of a simulation study is to compare methods for estimating population quantities (also termed as ”estimand”), bias will always be used as the performance measure because it is able to quantify how much an estimator exceeds the true value. (Burton, Altman, Royston, & Holder, 2006).

The bias is defined as the difference between a mean of estimate and its true value:

$$\text{Bias} = \Sigma_{m=1}^{M} \frac{(\hat{\beta}^{(m)} - \beta)}{M}$$

where $\bar{\beta}^{(m)}$ is the estimate of a fixed effect based on the m-th dataset and M is the number of the replications. In our study, M is 1000. Bias can take both positive and negative values, and ideally, it will be zero if the estimate is unbiased. In order to gauge the size of bias, the relative bias (RB) is also calculated along with bias:

$$\frac{1}{M} \Sigma_{m=1}^{M} \frac{(\hat{\beta}^{(m)} - \beta)}{\beta}$$

The most helpful part of relative bias (RB) is that researchers could interpret and quantify the magnitude of the bias under the percentage scale.

The second index is the standard error of an estimate. Its’ definition is:

$$SE = \sqrt{\frac{1}{M-1} \Sigma_{m=1}^{M} \left(\hat{\beta}^{(m)} - \frac{1}{M} \Sigma_{m=1}^{M} \hat{\beta}^{(m)} \right)^2}$$

According to the above definition, the derivation of SE only depends on the estimate $\hat{\beta}$, and does not need to know the true value β. This SE is also called empirical standard error since it estimates the standard deviation of estimate $\hat{\beta}$ over the M replications. In simulation studies, this empirical standard error is often considered as the true standard error for $\hat{\beta}$ because it is obtained from an approximate sampling distribution of $\hat{\beta}$, generated by M replication. In general, a small standard error indicates that most estimates are centered around their mean values and there is little uncertainty in the estimate. The standard error of estimate shares the same property as the standard deviation of statistics. It is supposed to decrease when the sample size increases.

The third index is type I error rate/statistical power, which are essentially the same in the Monte Carlo Simulation. Both indices are calculated as the proportion of cases in which the null hypothesis was rejected ($H_0 : \beta = 0$). If the true value β is equal to 0, this proportion is called Type I error rate, which is also corresponding to the significant level of the test hypothesis (nominal α). However, when the true value of β is not zero (alternative hypothesis is actually true or $H_1 : \beta \neq 0$), this proportion of rate becomes the statistic power which signifies the chance of choosing the alternative hypothesis when the alternative hypothesis is correct.

A desired value for the calculated α is actually the nominal α. In other words, that the difference between the actual α and nominal α is zero should be desired in the simulation study. In contrast, there is no reference level for statistical power. The determinants of power include: sample size, effect size, and significant level (nominal α). Basically, given the same significant level, higher power will be expected with a large sample size and effect size. In simulation studies, power is of particular interest when their purposes are to compare the competing designs (Burton et al., 2006).

In addition to the above main evaluation indices for the fixed effects, goodness-of-fit indices such as CFI, TLI, and RMSEA are also calculated to confirm the appropriateness of the analysis. The Comparative Fit Index (CFI) is an incremental fit index defined as (Chen, 2007):

$$CFI = 1 - \left\{ \frac{\chi_t^2 - df_t}{\chi_n^2 - df_n} \right\}$$

where χ_t^2 represents the chi-square of the proposed model, whereas χ_n^2 is the chi-square of the null model. df_t and df_n represent the degree of freedom for the proposed and null models, respectively. A typical null model is to allow all the variables in the model to have variation but no correlation. CFI ranges from 0 to 1 and is relatively independent of sample size.

The Tucker-Lewis Index (TLI) is another incremental fit index calculated as:

$$TLI = \frac{\frac{\chi_i^2}{v_i} - \frac{\chi_t^2}{v_t}}{\frac{\chi_i^2}{v_i} - 1}$$

where χ_i^2 is the independent model whereas χ_t^2 is the tested model. v_i and v_t are the degree of freedom for each model, respectively. Similar to the CFI, the range of TLI is from 0 to 1, and it is not significantly impacted by the sample size. In sum, the incremental fit index is analogous to R^2, so the value close to 1 indicates a better fit for the model. Normally, a value of CFI or TLI larger than 0.95 is interpreted as an acceptable fit, and 0.97 is treated as an accepted cut-off value in a great deal of research (Cangur & Ercan, 2015).

Root Mean Square Error of Approximation (RMSEA) is an index of the difference between the observed covariance matrix per degree of freedom and the model implied covariance. This absolute fit index is estimated as:

$$RMSEA = \sqrt{\max \left\langle \left\{ \frac{\mathbf{F}(\mathbf{S}, \boldsymbol{\Sigma}\hat{\boldsymbol{\theta}})}{v} - \frac{1}{n-1} \right\}, 0 \right\rangle}$$

where $\mathbf{F}(\mathbf{S}, \boldsymbol{\Sigma}\hat{\boldsymbol{\theta}})$ indicates the fit function is minimized whereas max points to the maximum value of the values given in brackets (Cangur & Ercan, 2015). v represents the value for the degree of freedom, and n is the sample size. According to its formula, RMSEA could produce a better estimation when the sample size is large since the term $\frac{1}{n-1}$ closes to zero asymptotically. The RMSEA values falling between 0.05 and 0.08 are recognized as a good fit.

Finally, the convergence rates and the number of Heywood cases (negative estimates of variances) were also recorded to assess the performance of a method. The maximum likelihood (ML) was used as the estimation method.

3.4 Study 2

In study 1, the direct comparison between the 2-wave LCFM and 3-wave SOLGM has a logic gap as the 2-wave LCFM and 3-wave SOLGM differ in both waves and models. In order to fill this logic gap, study 2 compared the 3-wave LCFM and 2-wave LCFM with the 3-wave SOLGM using the data generated in study 1. Either the performances of 3-wave LCFM or 2-wave LCFM are comparable with 3-wave SOLGM could fill the logic gap in study 1.

3.4.1 Evaluation Procedure

The evaluation procedures in study 2 are similar to study 1. The only difference was that the analytic model used here include the three-wave LCFM. The middle wave was inserted in the 2W-LCFM to form the 3W-LCFM.

3.4.2 Evaluation Indices

All the indices used were same to study 1.

Chapter 4

Results of Study 1

This chapter summarized the results of the previous studies and addressed the research questions in section 2.5.2. The first research question in section 2.5.2 was "Can the two-wave conditional LCFM and LCSM recover the parameters of the fixed effects of the conditional SOLGM?" The fixed effect parameters for 3W-SOLGM include: $\beta_{00}, \beta_{01}, \beta_{10}, \beta_{11}$.

4.1 Heywood Cases

Table 4.1 summarizes the result of Heywood cases (variance estimates). According to this table, n represents the number of conditions that contain at least one Heywood cases, which is a relatively strict criterion. For example, in terms of the ψ_{00} (in the first row), there are 249 conditions over 1008 total conditions, which have at least one Heywood cases. So the corresponding proportion of Heywood cases, which is indicated by p, is 249/1008=0.247. M (5th column) indicates the average Heywood cases per factor. The calculation here includes two steps: the first step is obtaining the number of Heywood cases out of the 1000 replications for each simulation condition. The second step is to obtain the mean of the Heywood cases based on the number of Heywood cases for each simulation condition. That is, we added up the number of Heywood cases across simulation conditions and divided the total number of conditions (1008). "Min" and "Max" in the 6th or 7th column represents the minimum or the maximum number of Heywood cases compared across 1008 simulation conditions. The "Convergence-rate" shows the average converging rate.

First, the variance parameter ψ_{11} from SOLGM has at least one negative estimates under all the simulation conditions. So its corresponding p-value is 1. As for the time-specific residual variance at the first wave (θ_1^2), the likelihood of Heywood cases seems to be highest when its true value equals 1. While the lowest likelihood of Heywood cases occurs when its true value is 16. Second, the average number of Heywood cases per condition (indicated by M) tends to be higher for the parameters with lower true values than those with higher true values. For example, when θ_3=0.063, the average number of Heywood cases per condition is about 454. In contrast, when θ_3=8, the average number of Heywood cases is around 3. Third, the true value of the parameter seems to relate to the minimum and the maximum number of Heywood cases. The smaller the true value, the larger the value of the minimum and the maximum number of Heywood cases. As we can see, the average convergence rate is always 100%, this trend indicates that Heywood cases are more likely to happen when the true parameter is closed to the lower bound of the parameter space (e.g.0) (Kolenikov & Bollen, 2012)

Table 4.1

Summary of Heywood Cases and Convergence Rate

label	True Value	n	p	M	Min	Max	Convergence-rate
ψ_{00}	4	249	0.247	8.441	0	222	1
ψ_{11}	1	1008	1	253.661	10	529	1
θ_1	0.063	96	0.095	443.115	364	520	1
θ_1	0.125	48	0.048	390.896	283	490	1
θ_1	0.25	144	0.143	315.062	133	466	1
θ_1	0.5	96	0.095	187.906	13	409	1
θ_1	1	218	0.216	107.412	0	362	1
θ_1	2	56	0.056	46.99	0	231	1
θ_1	4	43	0.043	9.688	0	74	1
θ_1	8	12	0.012	1.646	0	10	1
θ_1	16	7	0.007	0.115	0	2	1
θ_2	0.25	336	0.333	200.348	14	417	1
θ_2	1	180	0.179	40.182	0	208	1
θ_2	4	76	0.075	1.414	0	22	1
θ_3	0.063	96	0.095	453.875	391	520	1
θ_3	0.125	48	0.048	411.438	312	523	1
θ_3	0.25	144	0.143	345.854	148	474	1
θ_3	0.5	96	0.095	229.979	21	439	1
θ_3	1	232	0.23	137.863	0	400	1
θ_3	2	70	0.069	63.146	0	301	1
θ_3	4	56	0.056	15.472	0	111	1
θ_3	8	12	0.012	2.521	0	21	1
θ_3	16	9	0.009	0.146	0	3	1

4.2 Goodness-of-Fit Indices

Table 4.2-4.4 illustrate the average Goodness-of-Fit indices (CFI, TLI, RMSEA) given different sample sizes. The descriptive statistic in each column represents the corresponding mean value of that Goodness-of-Fit indices. For example, in table 4.2, the columns entitled “M”, “SD” or “Min” demonstrate the marginal average of the mean CFI (a summarized statistics defined by average across 1000 replications), the standard deviation of mean CFI and minimum value of mean CFI across different conditions. As mentioned in the previous section, an acceptable fit requires that the value of CIF or TLI be larger than 0.95, and RMSEA be less than from 0.05 to 0.08. From these results, we could conclude that on average all the models fit the data sufficiently well given different sample sizes because the average CFI and TLI are all above 0.95 and their RMSEA values are less than 0.06.

Table 4.2

Summary of Goodness-of-Fit Indices for CFI

Index	model	N	M	SD	Min	Med	Max
CFI	SOLGM	50	0.966	0.026	0.861	0.969	1.000
CFI	SOLGM	200	0.997	0.004	0.975	0.999	1.000
CFI	SOLGM	600	0.999	0.001	0.992	1.000	1.000
CFI	SOLGM	1000	0.999	0.001	0.995	1.000	1.000
CFI	LCFM	50	0.980	0.022	0.874	0.986	1.000
CFI	LCFM	200	0.997	0.004	0.974	1.000	1.000
CFI	LCFM	600	0.999	0.001	0.992	1.000	1.000
CFI	LCFM	1000	0.999	0.001	0.995	1.000	1.000
CFI	LCSM	50	0.980	0.022	0.874	0.986	1.000
CFI	LCSM	200	0.997	0.004	0.974	1.000	1.000
CFI	LCSM	600	0.999	0.001	0.992	1.000	1.000
CFI	LCSM	1000	0.999	0.001	0.995	1.000	1.000

* Note: *N* is the sample size; *M*: Marginal means of CFI;*SD*: standard deviation of CFI; *Min*: minimum value of CFI; *Med*: median value of CFI;*Max*: maximum value of CFI.

Table 4.3

Summary of Goodness-of-Fit Indices for TLI

Index	model	N	M	SD	Min	Med	Max
TLI	SOLGM	50	0.962	0.033	0.835	0.963	1.065
TLI	SOLGM	200	0.998	0.008	0.970	0.998	1.020
TLI	SOLGM	600	1.000	0.002	0.991	1.000	1.007
TLI	SOLGM	1000	1.000	0.001	0.995	1.000	1.004
TLI	LCFM	50	0.980	0.037	0.835	0.982	1.090
TLI	LCFM	200	0.999	0.008	0.966	0.999	1.021
TLI	LCFM	600	1.000	0.003	0.989	1.000	1.007
TLI	LCFM	1000	1.000	0.002	0.994	1.000	1.004
TLI	LCSM	50	0.980	0.037	0.835	0.982	1.090
TLI	LCSM	200	0.999	0.008	0.966	0.999	1.021
TLI	LCSM	600	1.000	0.003	0.989	1.000	1.007
TLI	LCSM	1000	1.000	0.002	0.994	1.000	1.004

[*] Note: N is the sample size; M: Marginal means of TLI;*SD*: standard deviation of TLI; *Min*: minimum value of TLI; *Med*: median value of TLI;*Max*: maximum value of TLI.

Table 4.4

Summary of Goodness-of-Fit Indices for RMSEA

Index	model	N	M	SD	Min	Med	Max
RMSEA	SOLGM	50	0.056	0.030	0.000	0.061	0.128
RMSEA	SOLGM	200	0.014	0.014	0.000	0.013	0.054
RMSEA	SOLGM	600	0.007	0.008	0.000	0.003	0.030
RMSEA	SOLGM	1000	0.005	0.006	0.000	0.001	0.023
RMSEA	LCFM	50	0.047	0.038	0.000	0.050	0.150
RMSEA	LCFM	200	0.015	0.017	0.000	0.008	0.068
RMSEA	LCFM	600	0.008	0.010	0.000	0.001	0.038
RMSEA	LCFM	1000	0.006	0.007	0.000	0.000	0.030
RMSEA	LCSM	50	0.047	0.038	0.000	0.050	0.150
RMSEA	LCSM	200	0.015	0.017	0.000	0.008	0.068
RMSEA	LCSM	600	0.008	0.010	0.000	0.001	0.038
RMSEA	LCSM	1000	0.006	0.007	0.000	0.000	0.030

[*] Note: N is the sample size; M: Marginal means of RMSEA;*SD*: standard deviation of RMSEA; *Min*: minimum value of RMSEA; *Med*: median value of RMSEA;*Max*: maximum value of RMSEA.

4.3 Overall Summary of Estimates

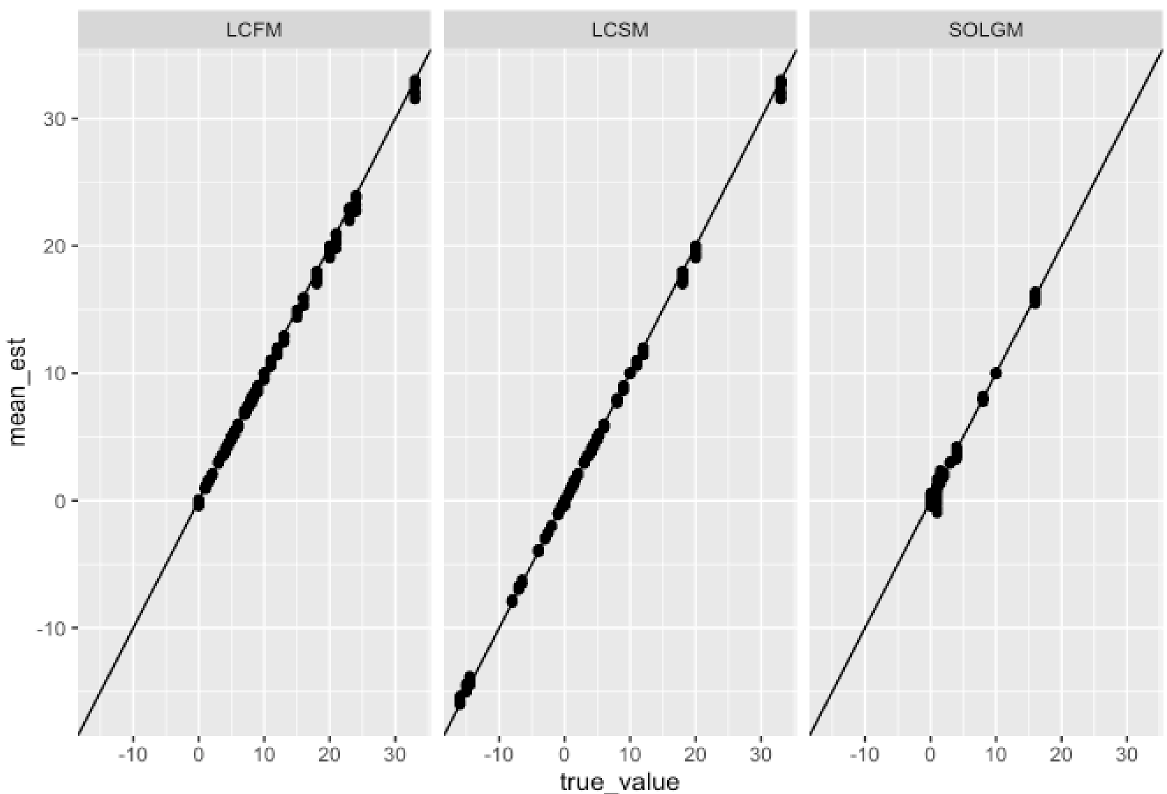

Figure 4.1. Scatter plot of true parameter values and mean estimates per analytic model. SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model. Each dot represents a model parameter. A solid diagonal line is y = x.

Fig 4.1 depicts a scatter plot of the true parameters and their mean estimates. This plot includes all model parameters across 1008 conditions. A point on the solid diagonal line indicates that a mean estimate and its relative true value are identical or the estimate on average is unbiased. Obviously, all the estimates are centered along the diagonal solid line under different analytical models. This indicates that all three analytical models can provide unbiased estimates for their relative true values.

Table 4.5

Mean Initial Status of Reference Group

Model	Label	True Value	N	Bias		Relative Bias		SE		Type I Error/Power	
				Mean	SD	Mean	SD	Mean	SD	Mean	SD
LCFM	κ_1	10	1008	0.001	0.0120	0.000	0.001	0.307	0.201	1	0.000
LCSM	κ_1	10	1008	0.001	0.0120	0.000	0.001	0.307	0.201	1	0.000
SOLGM	β_{00}	10	1008	0.001	0.011	0.000	0.001	0.290	0.185	1	0.000

Table 4.6

Mean Change Rate of Reference Group

Model	Label	True Value	N	Bias		Relative Bias		SE		Type I Error/Power	
				Mean	SD	Mean	SD	Mean	SD	Mean	SD
LCFM	$\Delta\kappa$	1	1008	-0.001	0.015	-0.001	0.015	0.343	0.235	0.790	0.283
LCSM	κ_Δ	1	1008	-0.001	0.015	-0.001	0.015	0.343	0.235	0.790	0.283
SOLGM	β_{10}	1	1008	-0.001	0.014	-0.001	0.014	0.337	0.230	0.795	0.280

Table 4.7

Group Difference in Initial Status

Model	Label	True Value	N	Bias		Relative Bias		SE		Type I Error/Power	
				Mean	SD	Mean	SD	Mean	SD	Mean	SD
LCFM	γ_1	3	504	-0.001	0.017	0.000	0.006	0.415	0.274	0.983	0.062
LCSM	γ_1	3	504	-0.001	0.017	0.000	0.006	0.415	0.274	0.983	0.062
SOLGM	β_{01}	3	504	-0.002	0.016	0.000	0.005	0.392	0.253	0.989	0.041
LCFM	γ_1	0	504	-0.001	0.014	0.000	0.005	0.387	0.262	0.050	0.005
LCSM	γ_1	0	504	-0.001	0.014	0.000	0.005	0.387	0.263	0.050	0.005
SOLGM	β_{01}	0	504	-0.001	0.0145	0.000	0.005	0.368	0.241	0.050	0.004

Table 4.8

Group Difference in Rate of Change

Model	Label	True Value	N	Bias		Relative Bias		SE		Type I Error/Power	
				Mean	SD	Mean	SD	Mean	SD	Mean	SD
LCFM	$\Delta\gamma$	2	504	0.001	0.017	0.000	0.008	0.430	0.311	0.913	0.175
LCSM	$\gamma\Delta$	2	504	0.001	0.017	0.000	0.008	0.430	0.311	0.913	0.175
SOLGM	β_{11}	2	504	0.000	0.015	0.000	0.008	0.419	0.302	0.913	0.168
LCFM	$\Delta\gamma$	0	504	0.001	0.017	0.000	0.009	0.416	0.306	0.050	0.005
LCSM	$\gamma\Delta$	0	504	0.000	0.017	0.000	0.009	0.416	0.306	0.050	0.005
SOLGM	β_{11}	0	504	0.001	0.017	0.000	0.009	0.404	0.294	0.050	0.004

Table 4.5-4.8 show the overall summary statistics for the fixed effect parameters. From the above results, we could see that the average bias or relative bias for $\beta_{00}, \beta_{01}, \beta_{10}$ and β_{11} are all equal to zero (the absolute value is ≤ 0.002), indicating that the fixed effects are, on average, estimated correctly. As for the standard errors, the mean and standard deviation obtained by the second-order latent growth model (SOLGM) are slightly smaller than those obtained by the latent change score model (LCSM) and longitudinal common factor model (LCFM). For example, the average standard error of β_{00} obtained by SOLGM is around 0.290, however, the corresponding standard error (κ_1) in LCFM and LCSM is 0.307. To better interpret this difference, we transform them in the ratio scale. That is, the standard error of β_{00} obtained by SOLGM is about 94.5% of the corresponding κ_1 in LCFM and LCSM. A similar pattern could also be observed in the average change rate of the reference group (β_{10}), where the mean standard error obtained by SOLGM is a little lower than these obtained by the other two models (0.337 vs 0.343) or, is about 98% of the standard error estimated by LCFM and LCSM. Therefore, the estimates of the fixed effect parameters in SOLGM seem to be a little more stable than those estimated by the two-wave models.

In simulation studies, both type I error rate and statistical power are targeted at the null hypothesis, especially for comparing the competing designs or models. In the current study, there seems to be no difference on average type I error rate and statistical power between two-wave models. To be more specific, when the true parameters are 0, the actual α on average is 0.05 for two-wave models , which is equal to the nominal α. These can be observed among $\beta_{01}, \gamma, \beta_{11}$ and $\Delta\gamma$. When the true parameters are not equal to 0, the average power are ranged from 0.790 to 0.983, which are under acceptable levels for all the parameters.

4.4 Results on Main Evaluation Indices

In order to better answer research question one "Can the two-wave conditional LCFM and LCSM recover the parameter of the fixed effects of the conditional SOLGM?", we will dive into the details of the simulation results in this section. The illustration is organized according to the main evaluation indices in which we are interested.

4.4.1 Bias

Bias is one of the main research interests in the current study. Its value quantifies whether the estimate obtained by the prescribed method targets the true parameter on average. Fig A.1-A.39 show the comparison between the average estimate by each model and their parameter values (indicated by the dash line) under different simulation conditions. The results of the mean initial status for the reference group (β_{00}) are illustrated in Fig A.1-A.9; The results of the mean growth rate for the reference group (β_{10}) are shown in Fig A.10-A.18. The results of the group difference in the initial status (β_{01}) are represented in Fig A.19-A.27, and Fig A.28-A.39 show the results of the average group difference in the growth rate (β_{11}). These figures were separately drawn based on the combination of true values of β_{01} and β_{11}.

According to these figures, the major conclusion to be inferred is that the magnitude of the bias may be impacted by the sample size N in all the fixed effect parameters. To be more specific, the average estimates are closed to their true values, or the magnitude of their bias approach to zero with the sample size becoming larger, regardless of the other factors such as ψ_{01}, θ_2^2 and pattern of time-specific residuals. In addition, no discernible patterns regarding the bias could be observed, which implies that the positive and negative bias randomly appears under the combination of different conditions. Therefore, due to the possible cancellation effect, the negative values could underestimate the magnitude of the marginal bias (e.g. table 4.5-4.8) across different conditions. The maximum magnitude of this bias could reach to 0.08, which is not negligible (e.g when the simulation condition is: $\beta_{10}, N = 50, \psi_{01} = 1, \theta_2^2 = 4$ and high_edge)

Relative bias (RB) is another statistic that quantifies the relative deviation of the mean estimate as to its true parameter. In the simulation studies, transforming the statistics of bias into the relative bias will be helpful for interpreting the size of bias. (Harwell, 2019). According to Hoogland and Boomsa (1998), the relative bias (RB)>0.05 indicates that there is a significant bias for the estimate. In other words, if an estimate has more than 5% of bias as to its true parameter, we would flag it as having a significant bias.

Figures 4.2-4.5 depict the box plot of relative bias across different models for each fixed effect parameter. Fig.4.2 shows the distribution of relative bias for the initial status of the reference group (β_{00}). The overall distributions of the two-wave models are the same as the three-wave model in terms of the location of the first quantile, the median, and the third quantile. This indicates 50% of the relative biases are less or equal to 0.001. The red dots represent the potential outliers, which are the largest or smallest values among all the estimates. Compared across different models, the maximum and minimum outliers are far less than 5%. In sum, we could conclude that there is no difference between the two-wave models and the three-level model when estimating the initial status of the reference group (β_{00}). Fig.4.3 is similar to Fig.4.2, except that the inner quantile in Fig. 4.3 is narrower than it is in Fig.4.2. Given their maximum and minimum outliers are still less than 0.05, we could have a similar conclusion that there is no bias for estimating the average group difference in the initial status (β_{01}) by using either the two-wave models or the three-wave model.

Fig.4.4-4.5 show the distribution of the relative bias for estimates of the growth parameters. In both figures, the distribution of the two-wave models are the same as the three-wave model. However, some outliers in both figures are larger or smaller than the absolute value of 0.05. What is worse, a small portion of outliers in Fig.4.4 are even smaller than 0.10. This suggests that there exists some bias under certain conditions even when we use the true model (SOLGM) to estimate the growth parameters β_{10} or β_{11}. In order to further investigate in which conditions the analytical models will generate some biased estimates, we extracted the conditions whose relative bias is either smaller than -0.05 or larger than 0.05, and the results are given in the table 4.9 and 4.10.

Comparing across tables, we could see that there are much more conditions that could generate biased estimates in β_{10} (34 conditions) than they are in the β_{11} (3 conditions). This implies that compared to β_{11}, it is more likely to have biased estimates for β_{10} when we use the three analytical models to estimate the true parameters. In the table 4.9, we could observe that all these biased estimates come from the smallest sample size, $N = 50$. Additionally, the majority of the biased estimates tend to show up when the time-specific residual variance is 4 ($\theta_2^2 = 4$). The patterns under which there exist some biased estimates include high edge, heavy spreading, heavy shrinking, and moderate spreading. Finally, as shown in the column "Relative Bias", there are much more outliers under -0.05 than they are above 0.05. This implies that under those biased conditions, the analytical models are more likely to underestimate the true parameters (β_{10}) than overestimating them.

In the table 4.10, all the biased estimates happened when the sample size is 50 and under the high edge pattern. In contrast with β_{10}, two of three bias estimates exist when the θ_2^2 is 1. Under these biased conditions, we could observe that the two-wave models are more likely to underestimate the true parameters, while the three-wave model tends to overestimate the true parameter.

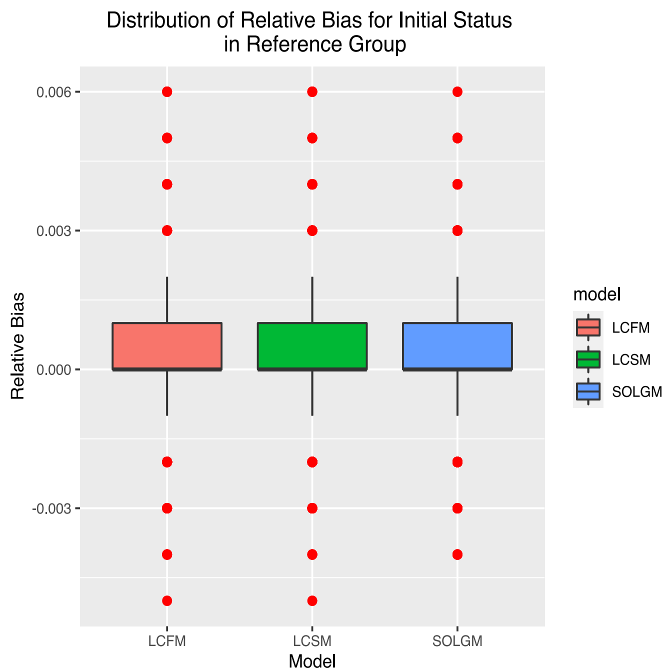

Figure 4.2. Boxplot of Distribution of Relative Bias for β_{00}

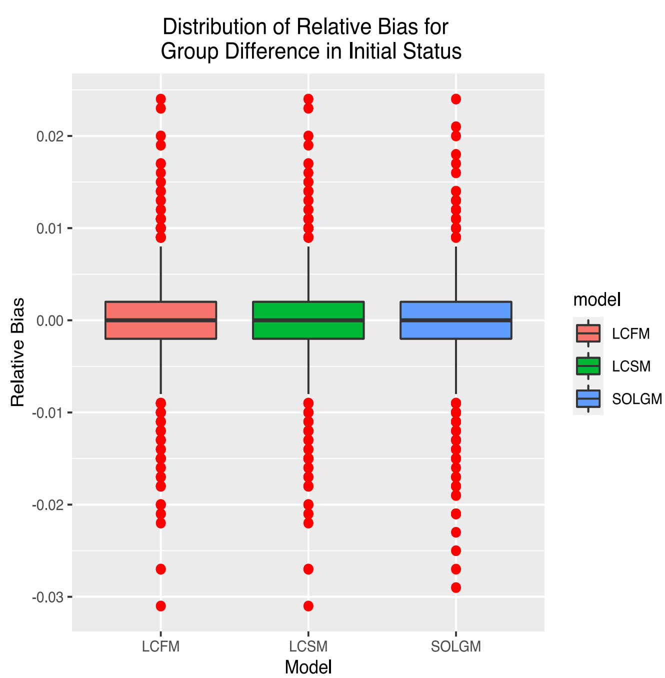

Figure 4.3. Boxplot of Distribution of Relative Bias for β_{01}

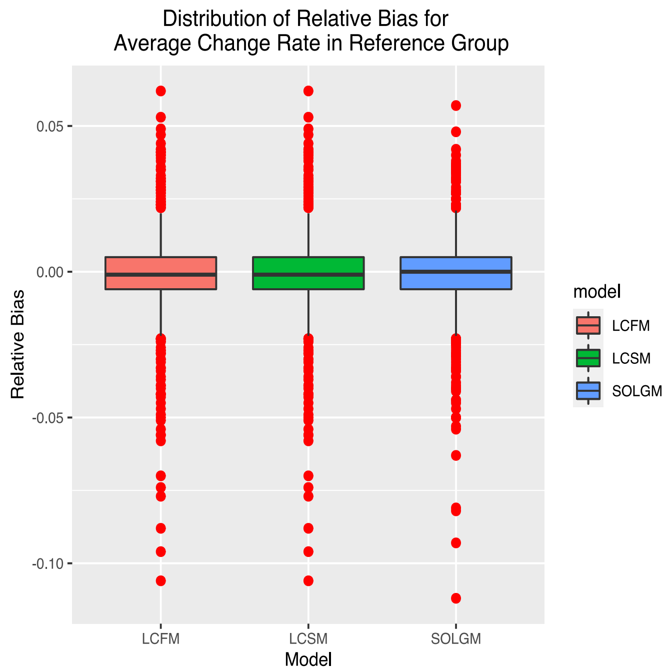

Figure 4.4. Boxplot of Distribution of Relative Bias for β_{10}

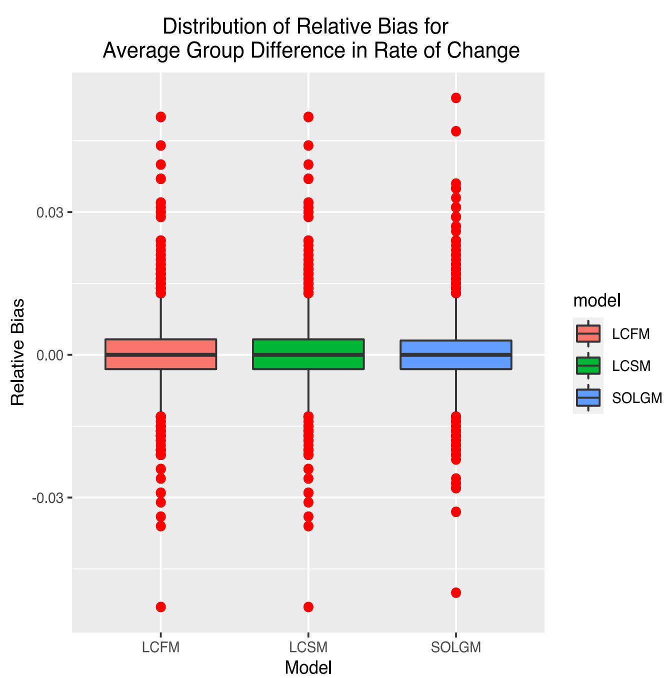

Figure 4.5. Boxplot of Distribution of Relative Bias for β_{11}

Table 4.9

Conditions with significant relative bias in β_{10}

Model	Relative Bias	N	θ_2^2	Pattern
SOLGM	-0.112	50	4	High Edge
LCFM	-0.106	50	4	High Edge
LCSM	-0.106	50	4	High Edge
LCSM	-0.096	50	4	High Edge
LCFM	-0.096	50	4	High Edge
SOLGM	-0.093	50	4	High Edge
LCFM	-0.088	50	4	Heavy Shrinking
LCSM	-0.088	50	4	Heavy Shrinking
SOLGM	-0.082	50	4	Heavy Shrinking
SOLGM	-0.081	50	4	High Edge
LCSM	-0.077	50	4	High Edge
LCFM	-0.077	50	4	High Edge
LCFM	-0.074	50	4	Heavy Spreading
LCSM	-0.074	50	4	Heavy Spreading
LCFM	-0.07	50	4	Heavy Spreading
LCSM	-0.07	50	4	Heavy Spreading
SOLGM	-0.063	50	4	Heavy Spreading
LCSM	-0.058	50	4	Heavy Shrinking
LCFM	-0.058	50	4	Heavy Shrinking
LCFM	-0.056	50	4	Moderate Spreading
LCSM	-0.056	50	4	Moderate Spreading
LCFM	-0.056	50	4	Heavy Spreading
LCSM	-0.056	50	4	Heavy Spreading
SOLGM	-0.054	50	1	High Edge
LCSM	-0.054	50	1	High Edge
LCFM	-0.054	50	1	High Edge
SOLGM	-0.053	50	4	High Edge
LCSM	-0.051	50	4	Heavy Shrinking
LCFM	-0.051	50	4	Heavy Shrinking
LCFM	0.053	50	4	Heavy Spreading
LCSM	0.053	50	4	Heavy Spreading
SOLGM	0.057	50	4	High Edge
LCFM	0.062	50	4	High Edge
LCSM	0.062	50	4	High Edge

Table 4.10

Conditions with significant relative bias in β_{11}

Model	Relative Bias	N	theta_sq	Pattern
SOLGM	0.054	50	4	High Edge
LCSM	-0.053	50	1	High Edge
LCFM	-0.053	50	1	High Edge

In order to further explore what factors could impact the magnitude of the relative bias, four full factorial Analysis of Variance (ANOVA) were conducted for each fixed parameter, with relative bias as the dependent variable and simulation factors as the independent variables. As illustrated above, the reason for using relative bias is that it helps quantify the size of bias relative to its true parameter. The eta-square (η^2), which is measured by the sum of squares for the effect of interests divided by the total sum of squares ($\eta^2 = \frac{SS_{\text{between}}}{SS_{\text{total}}}$), was used as the effect size statistic to determine whether a factor has substantial importance for the dependent variable (Levine & Hullett, 2002). One major advantage of using η^2 as the effect size index is that η^2 could be used on an additive scale, whose sum can never exceed 1. Therefore, it is more intuitive for people to interpret the value as the proportion or percentage of the total variability in the dependent variable that could be explained by the specific term.

Table A.1 shows the results of ANOVA test for the relative bias of β_{00}. The ANOVA model includes all the main effect factors (model, N, $\theta_2^2, \beta_{01}, \beta_{11}, \psi_{01}$ and pattern) and their corresponding interaction terms. The column titled η^2 shows the estimate of effect size for each term. The larger value indicates that more total variance could be explained by the relative term. According to Cohen's rules of thumb (Fritz, Morris, & Richler, 2012), $\eta^2 >= 0.14$ is considered as large effect, $0.06 \geq \eta^2 \leq 0.14$ indicates medium effect and $0.01 \geq \eta^2 \leq 0.06$ shows the small effect. According to the results in the table A.1, the sample size (N) and pattern demonstrate a relatively larger effect compared to other main effects such as the model and β_{01}. However, their η^2 values are still so small (0.014 and 0.012, respectively) that they are not considered as any substantial importance to the relative bias.

The largest effect among all the terms (including interaction terms) in the table A.1 comes from the high-order interaction term $N * \theta_2^2 * pattern * \psi_{01}$, whose η^2 is 0.073. The second largest effect comes from the interaction terms: $N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$ and $N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$ with their η^2 value 0.071. This could be interpreted as about 7% of the total variance in the relative bias could be explained by the above interaction terms. However, according to Cohen's rule, the effect size of these interaction terms could only be treated as medium level. Finally, the substantial contribution by the factor of model and its related interaction terms are negligible since the maximum η^2 among these relative terms are even less than 0.005. In sum, we could conclude that there are no substantially important factors for the variability of the relative bias of β_{00}. In addition, using the two-wave models did not differ from the three-wave model in terms of the magnitude of the bias even when the sample size is as small as 50.

Table A.2 shows the ANOVA test results for the relative bias of the average group difference in the initial status (β_{01}). The ANOVA model include all the main effect factors (model, N, $\theta_2^2, \beta_{01}, \beta_{11}, \psi_{01}$ and pattern) and their corresponding interaction terms. Among the main effect, the largest one for the relative bias is the factor of changing pattern ($\eta^2 = 0.018$). The second largest effect is from the sample size N, and its relative η^2 is 0.014. However, these effect sizes are so small that only 1.8% or 1.4% of the total variance in the relative bias could be accounted for by the factors of changing patterns or sample size. The largest effect among all the terms (including interaction terms) is from the high-order interaction: $N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$, with its η^2 0.094. This indicates that around 9.4% of the total variance in the relative bias could be explained by this interaction term. However, its effect could only be categorized as the median level. Finally, all the model related terms account very small effect ($\eta^2_{\max} \leq 0.02$), indicating that compared with other terms, the model does not have a substantial impact on the size of bias. In sum, we could have similar conclusions as for the β_{00}. That is, no factor has any significant or substantial impact on the total variability of the relative bias. All three analytical models could estimate the true parameter equally well.

Table A.3 illustrates the results of the ANOVA test for the total variability of the relative bias β_{10}. The independent variables included in this model are the same as those in the table A.2. Similarly, according to the size of the η^2, the factor that has the largest effect on the relative bias is the interaction term: $N * \theta_2^2 * pattern * \beta_{01}$ ($\eta^2 = 0.072$). This indicates that nearly 7.2% of the total variance in the relative bias could be explained by this interaction term. Based on Cohen's standard, this effect is just above the threshold of the median size effect (0.06). The second largest effect is from the interaction term: $N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$. However, its effect is still small because the η^2 is 0.056.

Table A.4 demonstrates the ANOVA test results for the relative bias of the average group difference in the growth rate (β_{11}). Similar to the previous results, the interaction term: $N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$ has the largest effect among all the others factors ($\eta^2 = 0.07$). That is to say, about 7% of the total variance in the relative bias was explained by this high-order interaction term. But this term is still considered as the median size of the effect. The second largest effect term, which is also median effect size ($\eta^2 = 0.067$), is the interaction term: $N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$. Finally, the effects from other terms are all very small. Therefore, similar to the findings in the initial status, we could conclude that there is no significant and substantial factor for the variability of the relative bias in terms of the estimates of the growth parameters.

In order to answer research question one, we need to summarize our findings about the estimates for the fixed effect parameters. Given the findings from a series of ANOVA tests that there are no substantially significant main or interaction effects in each parameter, we could conclude that there was no model related difference (three-wave models vs. two-wave models) in the relative bias of the point estimates for each fixed effect parameters across different conditions. In the other words, the two-wave conditional LCFM and LCSM are able to fully recover the parameter estimates of the fixed effects obtained by the three-wave conditional SOLGM. However, it does not guarantee that these models could totally yield unbiased point estimates in every simulation condition. As shown in the table 4.9 and 4.10, the performance of the estimates from the analytical models could be heavily impacted by certain extreme conditions such as the combination of $N = 50$, $\theta_2^2 = 4$ and High Edge Pattern. Under these circumstances, even the true analytical model (3W-SOLGM) could yield a biased point estimate for the growth parameters, especially for the reference group. This implied that the small sample size and large time-specific residual variance could be the main factors to undermine the estimated quality of the 3W-SOLGM.

4.4.2 Standard Error

Empirical Standard Error

In the previous section, we found that using the two-wave model (LCFM and LCSM) to analyze the data generated through the three-wave model (SOLGM) could produce an unbiased estimate for the true parameter of the initial status regardless of the conditions. In contrast, we did observe some biased estimates for the growth parameters β_{10} and β_{11} under certain extreme conditions. This section will further explore how precise or efficient it is to estimate the parameters by using two-wave models compared with the corrected analytical model.

Fig.A.40-A.63 depict the results of empirical standard error under different simulation conditions. In general, as we expected, the standard errors tend to decrease as the sample size increases regardless of the simulation conditions. In terms of the initial status β_{00}, the standard errors of the three-wave model (SOLGM) are smaller than the two-wave models (LCFM and LCSM) when the patterns are shrinking and high-edge. Furthermore, this discrepancy tends to become larger when the time-specific residual variance (θ_2^2) increases. The same pattern could also be observed for the average group difference in the initial status β_{01}. However, the pattern of the standard errors for the growth parameters (β_{10} and β_{11}) seems to be a little different from the parameters of initial status. The cases that the standard errors of the SOLGM are smaller than the LCFM and LCSM mainly occur when the patterns are shrinking and spreading. This discrepancy could be enlarged when the time-specific variance becomes larger.

As we did for the relative bias, in order to systematically explore what are substantially important factors for the variability of the standard error, we conducted four full factorial ANOVA with the standard error being the dependent variable. Table A.5-A.8 show the ANOVA test results for each fixed effect parameter. As for β_{00}, the largest and most dominating effect comes from the sample size, N. It's η^2 is 0.878, meaning 87.8% of the total variance in the standard error could be explained by the main effect of sample size. The effect of other terms is all less than 4%. A similar finding is also true for the average group difference in the initial status, β_{01}. Basically, the sample size N accounts for 84.8% of the total variance in the standard error. The rest of the terms just contribute a very small effect to the total variability of standard error.

Growth rate parameters are often the focal interest for researchers who utilize the growth models in the applied research. Table A.7-A.8 demonstrate the ANOVA test results for β_{10} and β_{11} respectively. The only substantial important factor is the sample size N. To be more detailed, N accounts for 79.6% of the total variance of standard error for β_{10}, and 70.8% of the total variance of standard error for β_{11}. The effect size of all the other factors is less than 0.14. The size of time-specific residual variance θ_2^2 accounted for more variability of standard error than they are in the initial status, however, their corresponding effect is still within the range of median level.

Theoretical standard error

Fig.A.64-Fig.A.87 show the results of the theoretical standard error compared across different models under different conditions. Without any surprise, the theoretical standard errors tend to decrease as the sample size increases regardless of conditions, which is the same as the empirical standard error. The performance of the theoretical standard error for β_{00} and β_{01} are similar to their corresponding empirical standard error. The theoretical standard errors of the three-wave model (SOLGM) are smaller than the two-wave models (LCFM and LCSM) when the patterns are shrinking and high-edge. This discrepancy tends to increase with the time-specific residual variance enlarged. Furthermore, when the sample size is as small as 50, the theoretical standard error obtained from the two-wave models is always larger than the standard error yielded by the three-wave model in almost all the conditions. The performance for the theoretical standard error of growth parameters (β_{10} and β_{11}) is also similar to their corresponding empirical standard error. The two-wave models could give theoretical standard errors comparable to the three-wave model in majority of the cases except when the time-specific variance is large and heteroscedastic. More specifically, the 2W-LCFM and 2W-LCSM yield worse theoretical standard error when θ_2^2 is large and the patterns are "spreading" or "shrinking"

As we did for the empirical standard error, in order to systematically investigate the substantively important factors for the variability of theoretical standard error, we conducted four full factorial ANOVA for each fixed effect parameter. The corresponding results are shown from the A.9-A.12. In general, the conclusion is the same as what has been found for the empirical standard error. That is, the only substantially important factor among all the variability of the tested estimates is the sample size N. The total variability explained by the sample size N could be arranged from 69.5% to 87.2%.

Ratio of Standard Error

The accuracy of the estimated standard error was evaluated by comparing the empirical standard error (SE_E) with the information-based standard error (SE_T), which is theoretically derived from the Fisher information matrix, for each fixed parameter. As mentioned in the method section, SE_E obtained from the simulation study is considered to be a true or approximately true standard error. In contrast, the SE_T is an estimate of the true standard error (SE_E). For each replication, R software generated the SE_T for each parameter, and we averaged these values over 1000 replications to get the average SE_T. If the model is correctly specified with the error term assumption being met, the SE_T should be asymptotically unbiased (Miyazaki, Chungbaek, Shropshire, & Hedeker, 2019). Therefore, the comparison between average SE_T and SE_E is also very informative because it could provide evidence about how well the theoretical standard error (SE_T) could approximate the empirical standard error (SE_E) under different conditions. Therefore, we will first calculate the ratio of SE_T and SE_E ($\frac{SE_T}{SE_E}$) for each fixed parameter. This ratio was then compared to the 1 because for correct analysis, the standard error in the numerator (SE_E) should be very closed or equal to denominator (SE_T). Fig.4.6-4.9 show the histograms of the ratio of SE ($\frac{SE_T}{SE_E}$). Both the distributions of two-wave models and three-wave model for each parameter approximate to the normal distribution, with their mean values closed to 1. Meanwhile, the distributions for the two-wave models seem to be a little narrower than the three-wave model.

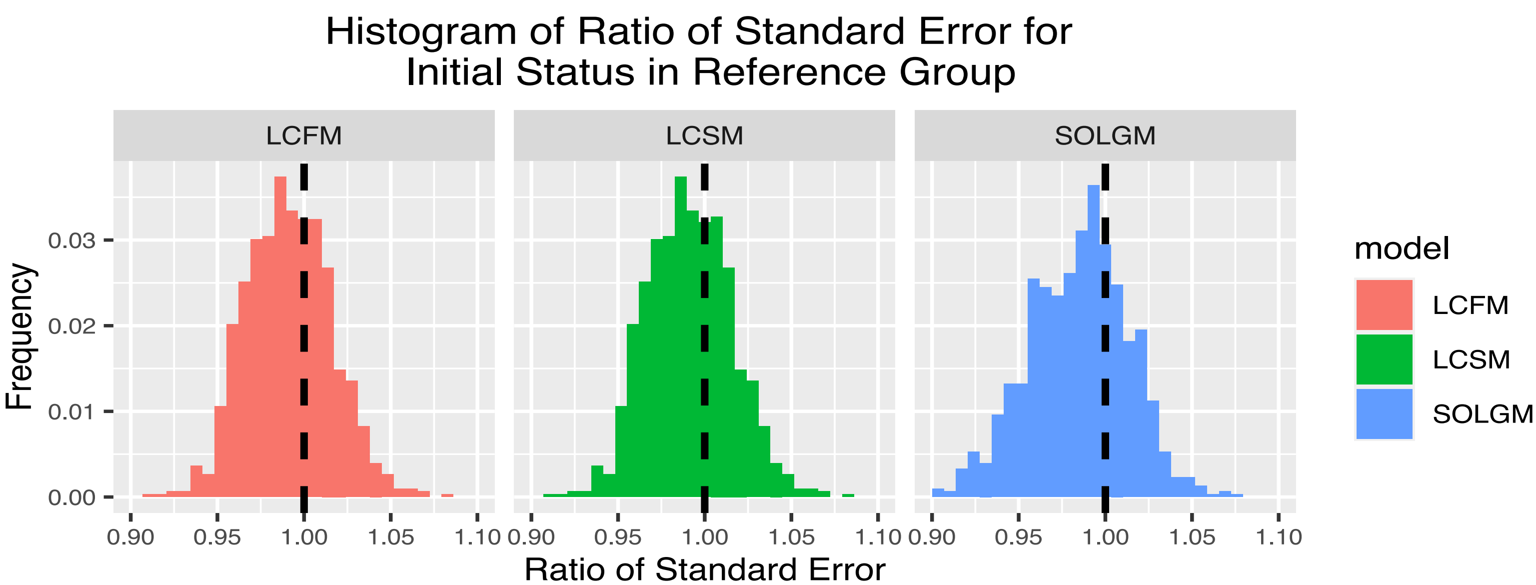

Figure 4.6. Histogram of Ratio of Standard Error for β_{00}

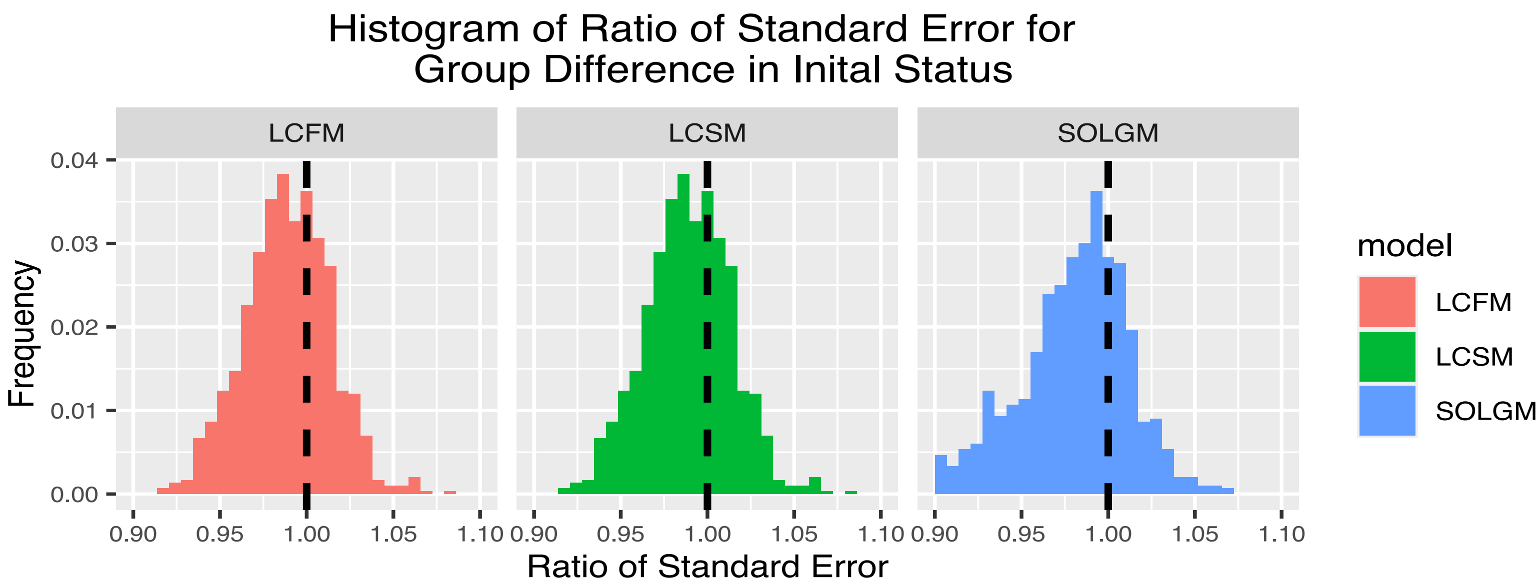

Figure 4.7. Histogram of Ratio of Standard Error for β_{01}

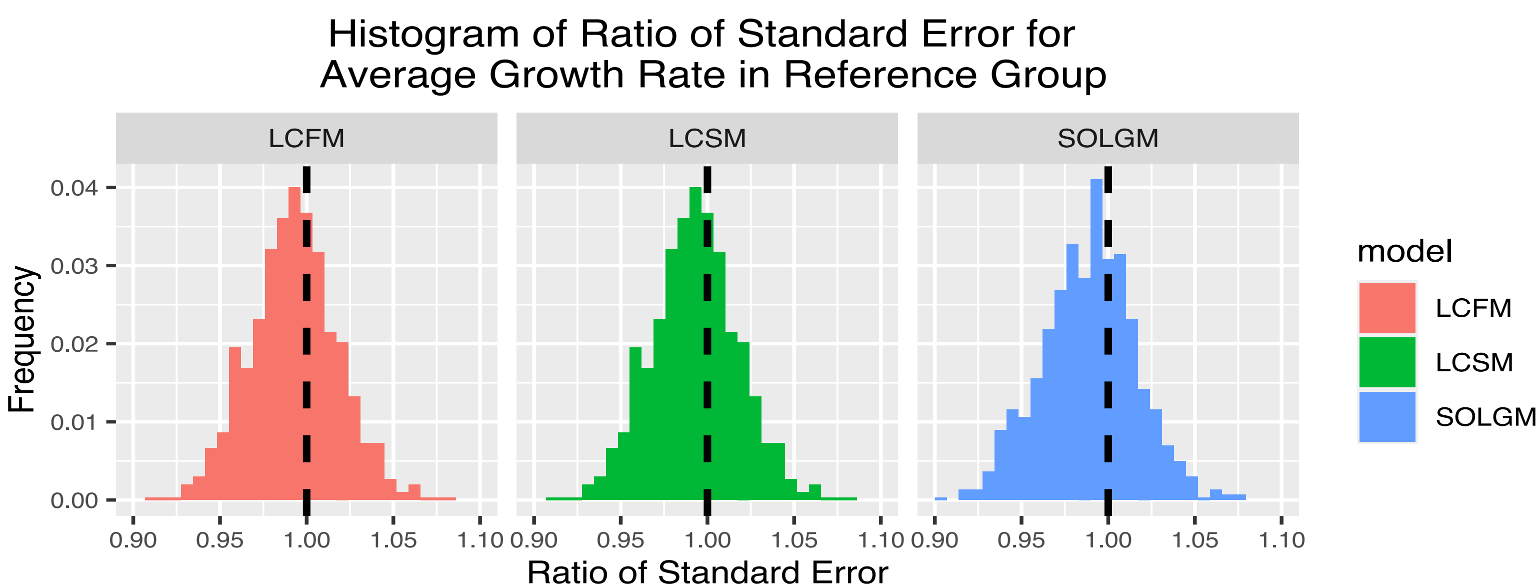

Figure 4.8. Histogram of Ratio of Standard Error for β_{10}

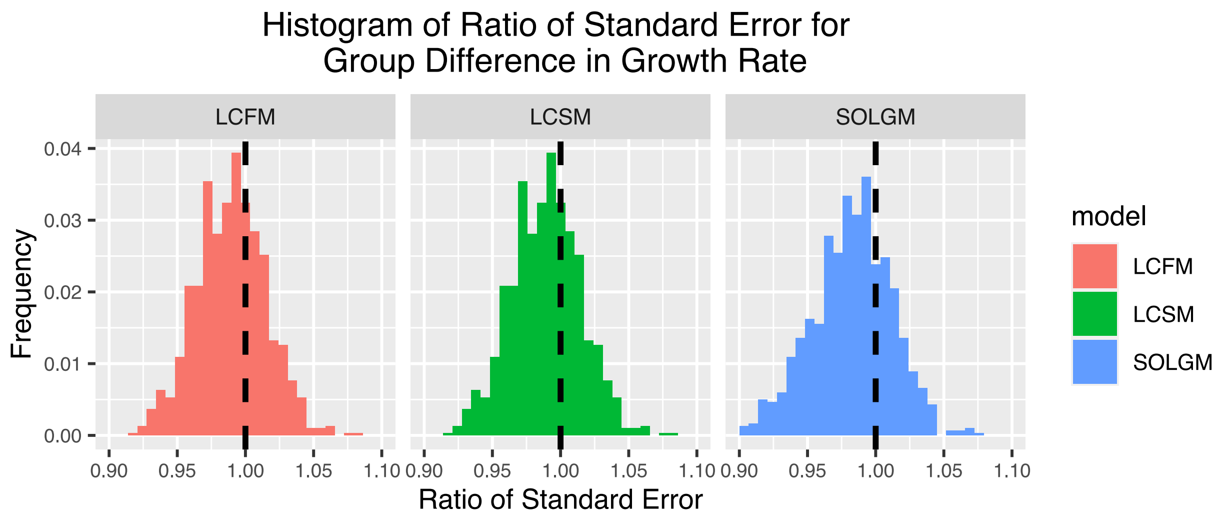

Figure 4.9. Histogram of Ratio of Standard Error for β_{11}

Table 4.11 shows the minimum, maximum, mean, and standard deviation for the ratio of SE in each parameter. As we can see, their mean values are all very closed to 1. The largest value (based on the column "Max") across all the parameters is 1.08. In contrast, the smallest value (based on the column of "Min") is 0.863. These indicate that the theoretical SE_T could underestimate the true SE_E up to 13.7%, and overestimate the SE_E to 8%. Last but not least, we could see that except for the β_{00}, the standard deviation of the ratio SEs given by two-wave models are a little smaller than that of the three-wave model. This is why the distributions of the SE ratio in the two-wave models are a little bit narrower than that of the three-wave model.

Table 4.11

Descriptive Statistics for Ratio of Standard Error

Model	Parameter	N	Min	Mean	Max	SD
LCFM	κ_1	1008	0.912	0.991	1.08	0.025
LCSM	κ_1	1008	0.912	0.991	1.08	0.025
SOLGM	β_{00}	1008	0.889	0.984	1.08	0.023
LCFM	γ_1	1008	0.918	0.989	1.08	0.025
LCSM	γ_1	1008	0.918	0.989	1.08	0.025
SOLGM	β_{01}	1008	0.863	0.979	1.07	0.033
LCFM	Δ_κ	1008	0.911	0.993	1.08	0.025
LCSM	κ_Δ	1008	0.911	0.993	1.08	0.025
SOLGM	β_{10}	1008	0.900	0.989	1.07	0.027
LCFM	Δ_γ	1008	0.917	0.989	1.08	0.024
LCSM	γ_Δ	1008	0.917	0.989	1.08	0.024
SOLGM	β_{11}	1008	0.900	0.983	1.08	0.029

In order to systematically explore what are the substantially important factors for the variability of the ratio of the standard error ($\frac{SE_T}{SE_E}$), we conducted four full factorial ANOVA tests for each fixed effect parameter. Table A.13-A.14 illustrate the ANOVA test results for the parameters of the initial status (β_{00} and β_{01}). The only significant factor that has a large effect is the sample size N. To be more detailed, the value of η^2 is 0.244 for the β_{00} and 0.304 for β_{01}. Therefore, the sample size N could explain 24.4% and 30.4% of total variance of the ratio $\frac{SE_T}{SE_E}$ in β_{00} and β_{01}, respectively. Beyond the sample size, there is no other substantial important factors. Without any surprise, the sample size N is also the only factor that plays a substantially important role in the variance of the ratio $\frac{SE_T}{SE_E}$ among the growth rate parameters. As a matter of fact, it explains 19.9% (η^2=0.199) and 26.9% ($\eta^2 = 0.269$) of total variance among the ratio $\frac{SE_T}{SE_E}$ for β_{10} and β_{11}, respectively.

The above results revealed that the theoretical standard error at maximum could have an 8% deviation from their true standard error in some situations, and the sample size N may be the denominating factor for the variability of this statistics. Therefore, we extracted the conditions that yield the relative large $\frac{SE_T}{SE_E}$ values and summarized their information across tables A.17-A.20. The relative large $\frac{SE_T}{SE_E}$ values here mean the value $\frac{SE_T}{SE_E}$ is either larger than 1.05 or smaller than 0.95. These are actually the cases where their theoretical standard errors are more than 5% off than their empirical/true standard errors. Across different parameters, we could first observe that the majority of the cases with large $\frac{SE_T}{SE_E}$ values occurred when the sample size is 50. Meanwhile, with N=50, the theoretical standard errors under different situations tend to underestimate the empirical standard error because the maximum values given N=50 are all less than 1. At length, it is interesting to observe that, with the sample size increased, the frequency of overestimated cases also increases, resulting in that the average value of $\frac{SE_T}{SE_E}$ approaches to 1. One major concern that we need to clarify is that the relatively large value here is not equivalent to the substantially large value which is treated as being not negligible in SEM. According to Nevitt and Hancock (2001), the cut-off criterion for the substantial large size of $\frac{SE_T}{SE_E}$ should be 10%. That is to say, the absolute value of $\frac{SE_T}{SE_E}$ which is smaller than 0.1 could be considered as negligible in SEM. This result will be further discussed in the discussion section.

In order to directly answer the research question two that how precise it is to use the two-wave models to recover the fixed effect parameters generated by the three-wave model, we would like to directly compare the standard error of estimating the same parameter across different models. Similar to the former operation, this comparison was made on a ratio metric. The theoretical or empirical standard error by the three-wave model (SOLGM) could work as a criterion index with which the standard error of the two-wave models (LCFM and LCSM) could be compared. The most informative point for this comparison is to show how much deviation the standard error of two-wave models can have away from the standard error of a three-wave model when estimating the same parameter.

Therefore, we would like to first describe both the ratio of empirical standard error obtained by the two-wave models to the empirical standard error obtained by the three-wave model ($Ratio - SE_{2E/3E}$), and then the ratio of theoretical standard error obtained by two-wave models to the theoretical standard error obtained by the three-wave model ($Ratio - SE_{2T/3T}$) for each parameter. In real data analysis, we are unable to know any true values for either the empirical standard error of the ratio of the empirical standard error ($Ratio - SE_{2E/3E}$). However, the knowledge about the performance of true standard error can help evaluate how much loss the estimation precision could be when the two-wave models are used to recover the parameters of the three-wave model under the theoretical setting. Last but not the least, we would like to explore what factors could have substantially important effects on the variability of $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$.

Table 4.12 and 4.13 show the descriptive statistics for the $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$, respectively. On average, both the empirical and theoretical SE by the two-wave models are slightly larger than those obtained by the three-wave model in every fixed effect parameter. The minimum values for the ratio $Ratio - SE_{2E/3E}$ across different parameters are all smaller than 1. However, the minimum value for the ratio $Ratio - SE_{2T/3T}$ across different parameters is all larger than 1. This implies that compared to the empirical standard error, the theoretical standard error obtained by the two-wave model tends to be more likely to overestimate the theoretical standard error obtained by the three-wave model in every fixed parameter. The largest maximum values across parameters are 1.301 and 1.269 for $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$ respectively. The standard deviation for the growth parameters seem to be a little smaller than they are in the initial status for both $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$

Table 4.12

Descriptive Statistics for the Ratio of Empirical SE of 2W-Models vs 3W-Model

Parameter	Minimum	Mean	Maximum	Standard Deviation
beta_00	0.995	1.056	1.243	0.047
beta_01	0.968	1.049	1.301	0.057
beta_10	0.979	1.016	1.169	0.034
beta_11	0.977	1.025	1.244	0.049

Table 4.13

Descriptive Statistics for the Ratio of Theoretical SE of 2W-Models vs 3W-Model

Parameter	Minimum	Mean	Maximum	Standard Deviation
beta_00	1.013	1.064	1.227	0.046
beta_01	1.010	1.060	1.269	0.055
beta_10	1.000	1.020	1.139	0.034
beta_11	1.000	1.032	1.212	0.049

Given there is still some variability among these ratio statistics, we conducted eight full factorial ANOVAs to further examine what factors are significantly impacting the variability of the $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$. Table A.21-A.24 show the results of ANOVA tests for each parameter. In terms of the initial status for the reference group (β_{00}), we can see that the changing pattern, θ_2^2 and $pattern * \theta_2^2$ have large effect on the variance of both $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$. The η^2 value are 0.487, 0.275 and 0.165 for the pattern, $pattern * \theta_2^2$ and θ_2^2, respectively in $Ratio - SE_{2E/3E}$. While the η^2 value are 0.523, 0.297 and 0.162 for the pattern, $pattern * \theta_2^2$ and θ_2^2, respectively in $Ratio - SE_{2T/3T}$. Since the interaction term between pattern and θ_2^2 is significant, we create the interaction effect plot to further examine how the two terms jointly work together to impact the ratio of standard error.

Fig.4.10 depicts the interaction effect between pattern and θ_2^2 on the $Ratio - SE_{2E/3E}$ for β_{00}. As the θ_2^2 increases from 0.25 to 4, the average $Ratio - SE_{2E/3E}$ also increases. However, this increasing trend is not applicable to every pattern. To be more specific, when the η_2^2 increases, the $Ratio - SE_{2E/3E}$ increases only at shrinking, constant, and high edge patterns. Meanwhile, the $Ratio - SE_{2E/3E}$ at the low edge and spreading patterns tend to slightly decrease. Finally, this increasing rate tends to become larger as the θ_2^2 becomes larger and the shrinking pattern gets worse. This could be directly observed from the heavy shrinking pattern, where the top red line suddenly rises up after the median size of θ_2^2 (where θ_2^2=1). Ideally, we expect the $Ratio - SE_{2E/3E}$ to be 1 if there is no real loss of precision to use two-wave models to estimate the true parameters in comparison with the three-wave model. However, in the current study, the loss of precision could up to 20% when the changing pattern is Heavy Shrinking and the θ_2^2 is 4. On the other side, the smallest loss is around 5% which happens when the θ_2^2 is 0.25. A similar pattern could also be observed for the $Ratio - SE_{2T/3T}$, which we could actually obtain by real data analysis. (See Fig 4.11) However, the maximum loss in precision is a little bigger than the $Ratio - SE_{2E/3E}$. It could up to 23% when the θ_2^2 is 4 and the changing pattern is heavy shrinking.

As for the average group difference in the initial status (β_{01}), we could find the same set of terms (pattern, $\theta^2 * pattern$ and θ^2) that have large effects on the variability of both the $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$. The η^2 values are 0.483, 0.253 and 0.189 for the pattern, $pattern * \theta_2^2$ and θ_2^2, respectively in $Ratio - SE_{2E/3E}$. In contrast, the η^2 values are 0.517, 0.253 and 0.163 for the pattern, $pattern * \theta_2^2$ and θ_2^2, respectively in $Ratio - SE_{2T/3T}$. Given the interaction term between pattern and θ_2^2 is significant, we created the interaction effect plots for $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$, respectively. Figure 4.12 and 4.13 show that as the size of θ_2^2 increases , both $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$ increases only at shrinking, constant and high edge pattern. Meanwhile, both the $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$ at the low edge and spreading patterns tend to be slightly decreased. This increasing rate tends to become larger as the θ_2^2 becomes larger and the shrinking pattern becomes worse. As for the average loss of precision, the minimum level is still around 5%. However, the maximum loss level could reach 24% when the θ_2^2 is 4 and the changing pattern is shrinking or having an edge.

Since the above results for β_{00} and β_{01} are highly similar, we, therefore, could summarize them together in this subsection. That is, these results implied that in the real data analysis, using the two-wave models may fall into the risk of losing a maximum 24% precision on average for estimating the initial status parameters (β_{00} and β_{01}). In fact, this big loss could occur even when we have a large sample size such as 600 or 1000, as long as the time-specific residual variance θ_2^2 is large enough (e.g. $\theta_2^2 = 4$) under the shrinking or edge pattern.

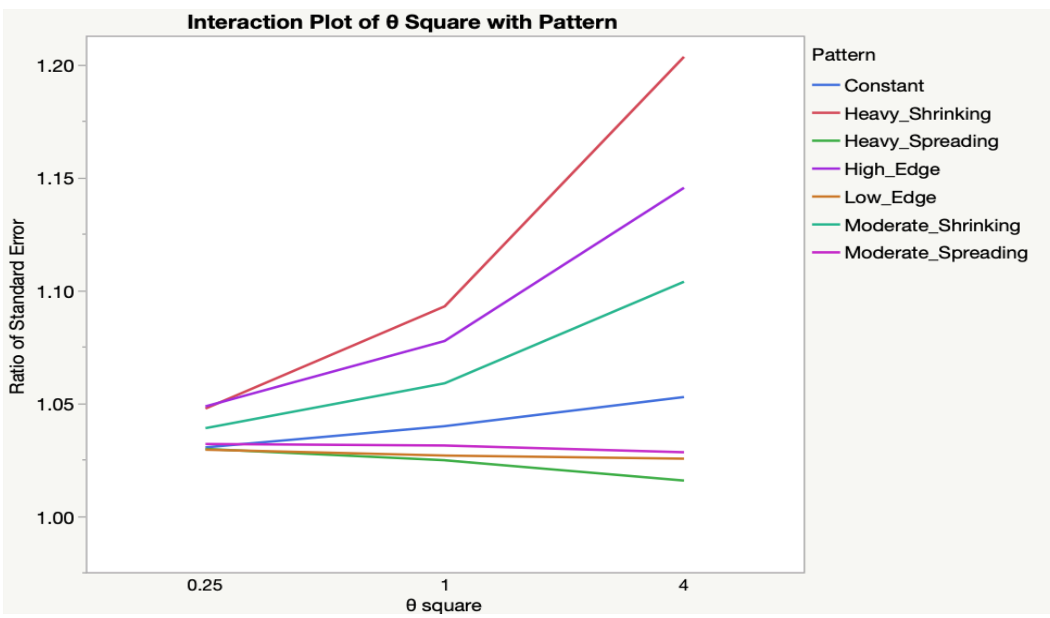

Figure 4.10. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of empirical standard error $(Ratio - SE_{2E/3E})$ for $\boldsymbol{\beta_{00}}$

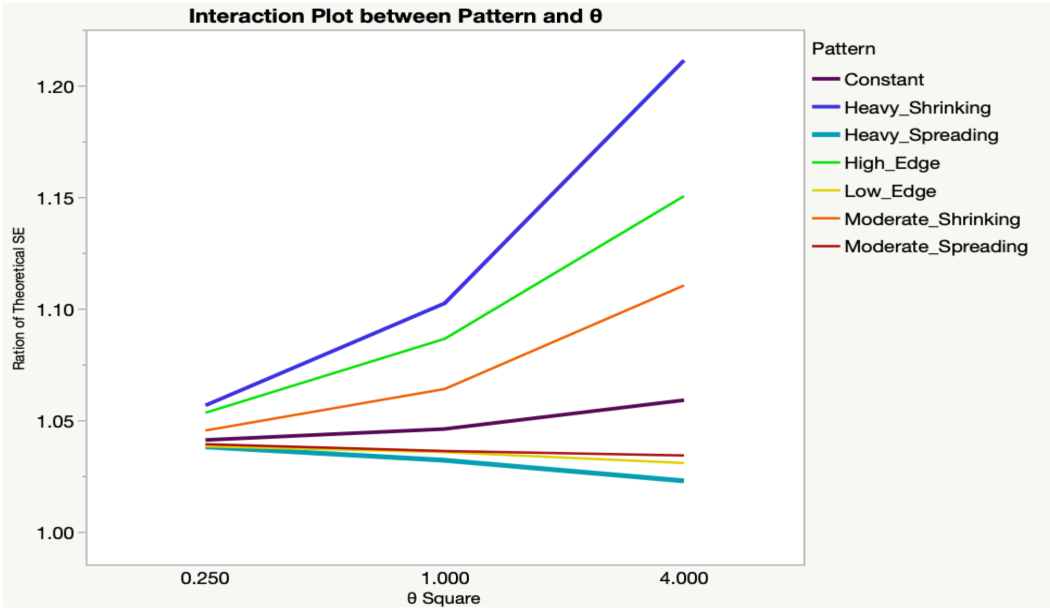

Figure 4.11. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error $(Ratio - SE_{2T/3T})$ for $\boldsymbol{\beta_{00}}$

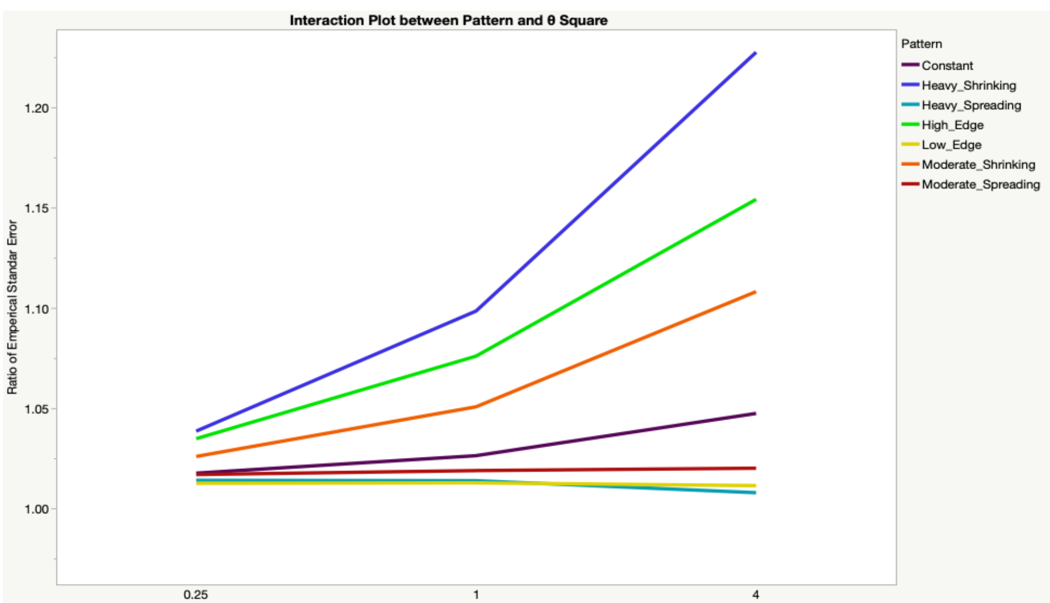

Figure 4.12. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of empirical standard error $(Ratio - SE_{2E/3E})$ for $\boldsymbol{\beta_{01}}$

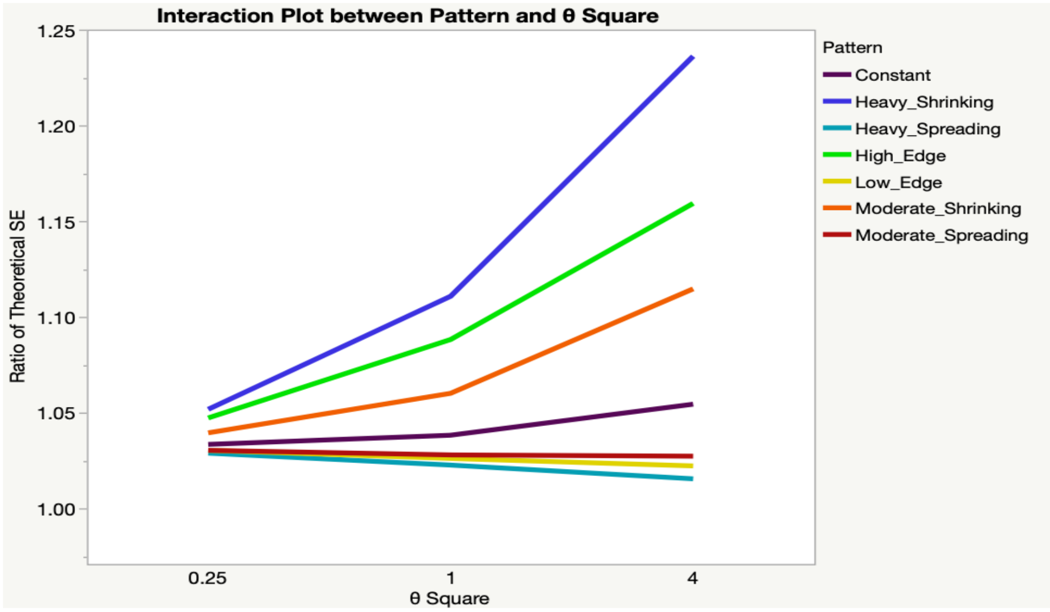

Figure 4.13. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error ($Ratio - SE_{2T/3T}$) for $\boldsymbol{\beta_{01}}$

Table A.23 and A.24 demonstrate the ANOVA test results for the growth parameters (β_{10} and β_{11}). For both parameters, the pattern, $pattern * \theta_2^2$ and θ_2^2 are still playing a significant impact on the $Ratio - SE_{2E/3E}$ or $Ratio - SE_{2T/3T}$. For example, corresponding to β_{01}, the values of η^2 are 0.384, 0.339 and 0.219 for terms of pattern, $pattern * \theta_2^2$ and θ_2^2, respectively in testing $Ratio - SE_{2E/3E}$. While the values of η^2 are 0.382,0.345 and 0.249 for terms of pattern, $pattern * \theta_2^2$ and θ_2^2, respectively in $Ratio - SE_{2T/3T}$. Similar to the previous section, we further created the plots of interaction effect between pattern and θ_2^2 to further check this joint influence.

Fig.4.14-4.17 depict the interaction effect between pattern and θ_2^2 on the $Ratio - SE_{2E/3E}$ or $Ratio - SE_{2T/3T}$ for β_{10} and β_{11}, respectively. A little different from the trend found in the initial status, as the size of θ_2^2 increases, both $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$ increases only at the spreading and shrinking patterns. Under the other patterns, the average value for both ratios tends to remain stable regardless of how large the θ_2^2 is. In addition, this increase tends to be larger when the spreading and shrinking becomes heavier, which can be easily observed among the top two lines and two lines in the middle position. The range of precision loss is also different from the initial status. In terms of β_{10}, the average loss of precision could up to 12% for both $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$ when the θ_2^2 is 4 and the changing patterns are either spreading or shrinking. The minimum average loss of precision is around 2% when the θ_2^2 is 0.25. The maximum loss in the β_{11} is a little bit larger. They could reach 18% when the changing patterns are either heavy spreading or shrinking. Thus in the real data analysis, using the two-wave models may fall into the risk of losing a maximum 18% precision on average for estimating the growth-related parameters (β_{00} and β_{01}). Regardless of the sample size, this big loss could occur as long as the time-specific residual variance θ_2^2 is large enough (e.g. $\theta_2^2 = 4$) under the shrinking or spreading pattern.

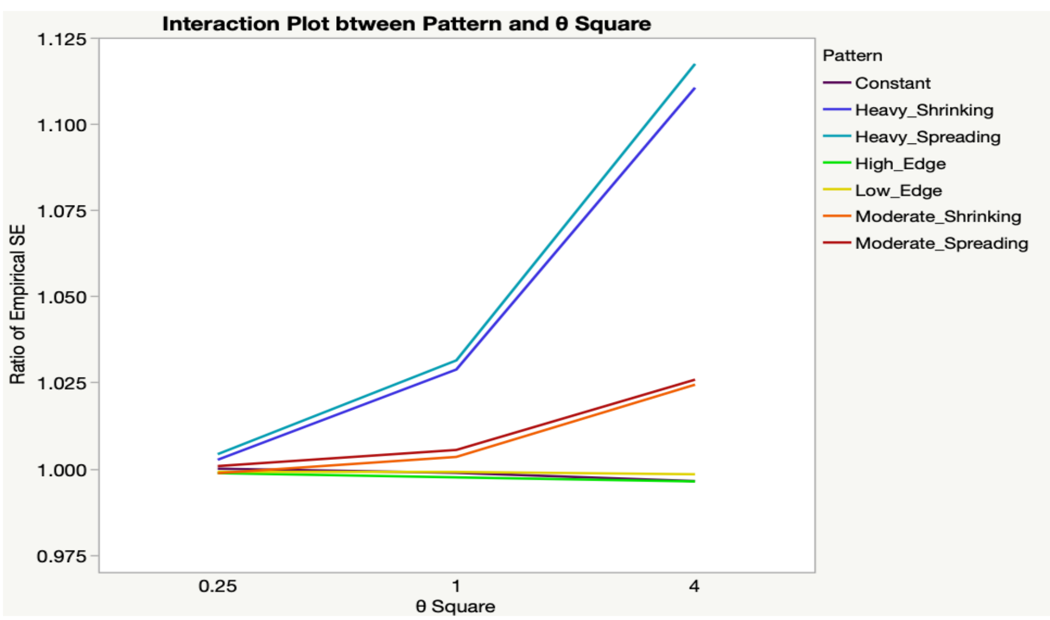

Figure 4.14. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error ($Ratio - SE_{2E/3E}$) for $\boldsymbol{\beta_{10}}$

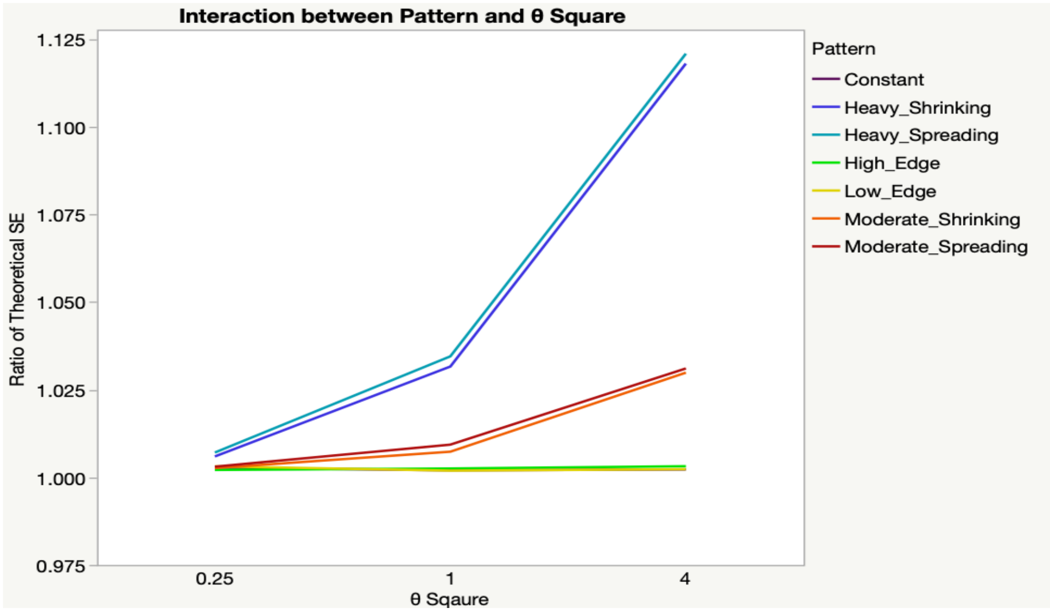

Figure 4.15. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error ($Ratio - SE_{2T/3T}$) for $\boldsymbol{\beta_{10}}$

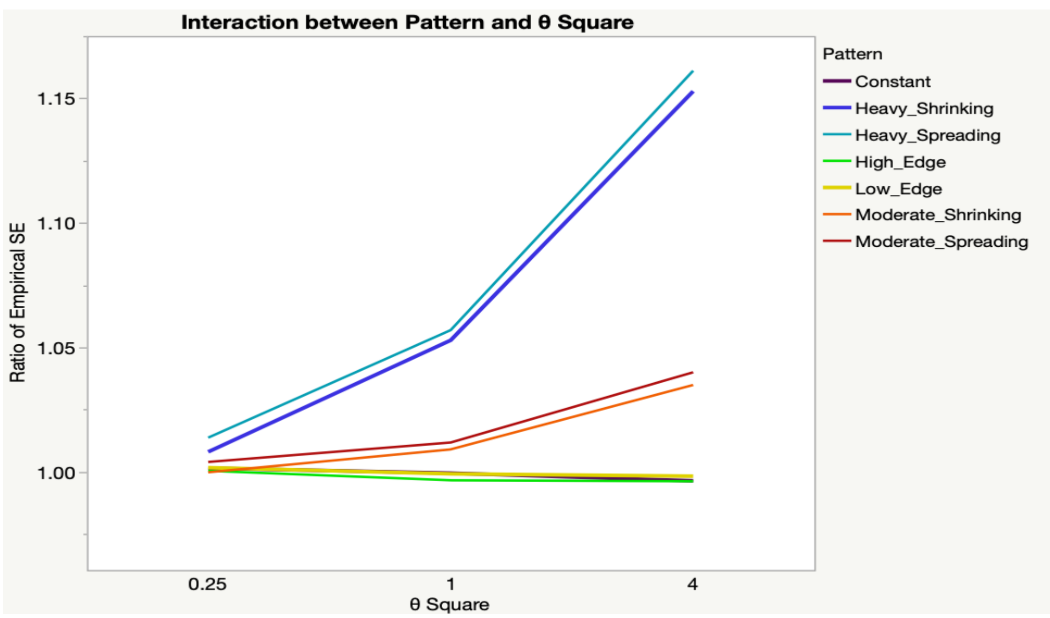

Figure 4.16. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of empirical standard error ($Ratio - SE_{2E/3E}$) for $\boldsymbol{\beta_{11}}$

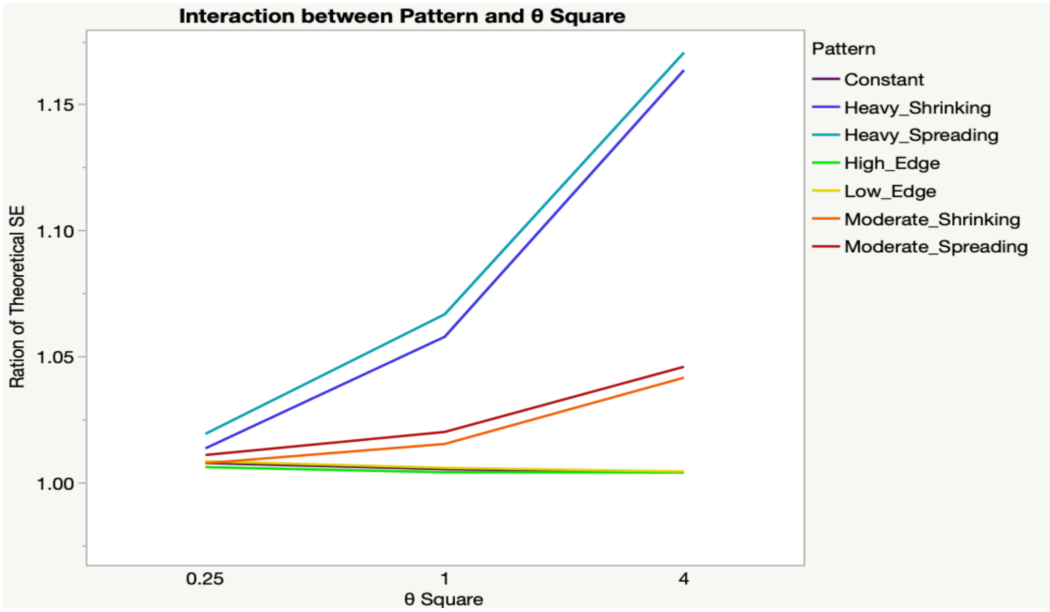

Figure 4.17. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error $(Ratio - SE_{2T/3T})$ for $\boldsymbol{\beta_{11}}$

In all, the summary of the above results could help answer part of the research question two "What factors could influence the accuracy of estimating the fixed effect parameters by using the two-wave models?" As the results shown in tables A.21-A.24, both the changing pattern and θ_2^2 are jointly influencing the variability of the $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$. When $\theta_2^2 = 0.25$, regardless of any changing pattern, both the empirical standard error and the theoretical standard error of the point estimate obtained by two-wave models are almost equivalent to that of the three-wave model since the average ratio is so closed to 1. Therefore, we could conclude that on average there is no loss of efficiency or precision when using the two-wave models to estimate the true parameters compared with the three-wave model when the θ_2^2 is very small with the constant pattern. However, the average difference between the SE of the two-wave models and SE of the three-wave model could be increased as the θ_2^2 becomes larger under patterns where the

the-time specific variance differ between the first and the last time point. That is why we could observe the largest discrepancy between the two-wave and three-wave models when the θ_2^2 is 4 at certain patterns. Finally, we could conclude that the maximum average loss of precision or efficiency by using two-wave models could reach 24% in estimating the initial status and 18% in estimating the growth parameters.

4.4.3 Type I Error Rate and Statistical Power

Fig.A.88-Fig.A.108 demonstrate the statistical power and type I error rate for the fixed effect parameters under different conditions. More specifically, Fig.A.88-Fig.A.99 show the results for statistical power. For the average initial status of the reference group (β_{00}), the two-wave models (LCFM and LCSM) have the same level of statistical power as the three-wave model (SOLGM) in terms of detecting the true parameter value across all the conditions. This power maintains around 1 across different conditions, which indicates that regardless of conditions, both the two-wave and three-wave models could be almost 100% correct at choosing the alternative hypothesis when it is true. However, the power pattern of β_{01}, the average group difference in the initial status, seems a little different from the pattern in β_{00}. To be more detailed, when the sample size N decreases from 200, the power of analysis could go down slightly no matter what model is used. This is why the lowest level of statistical power could always be observed when the sample size N is 50 in every condition. Other factors that seem to influence the power of each model are time-specific residual variance and pattern. Under shrinking and high edge patterns, the three-wave model (SOLGM) tends to have higher power than the two-wave models when the sample size is less than 200. This discrepancy could become larger when the time-specific variance increases. At length, it should be noted that when the sample size is greater than 200 and approaching 1000, the level of power gets converged so that no obvious difference could be observed among different models.

The slope parameters (β_{01} and β_{11}) are the main focal points for researchers who are interested in the growth phenomenon. Fig.A.91 - A.93 show the power level of β_{10} across conditions. Compared to the parameters of initial status in the reference group, a larger sample size could yield a higher level of statistical power such as 0.9. For example in the β_{00} and β_{01}, with sample size just being 200, the power by different models could approximate to 1 in almost every simulation condition. However, with the same sample size, the maximum statistical power in β_{10} is around 0.9. What is worse, the power of testing β_{10} under the sample size N=50 is much smaller than that of testing the initial status in the reference group. Other factors that could influence the power level are the size of time-specific residual variance and the changing pattern. From these figures, we could see that the three-wave model has a higher statistical power than the two-wave models when the pattern are shrinking and spreading. This difference could be boosted when the size of time-specific residual variance gets larger. A similar pattern could also be observed for the average group difference of the growth rate parameter β_{11}. The power level reaches the lowest when the sample size is 50 compared to other sample sizes. In addition, The factors (time-specific residual variance and changing pattern) play a similar effect on the power of β_{11} as they did for the β_{10}.

The final result to check is the type I error rate for β_{01} and β_{11} in our study. From Fig. A.100-A.108, we could see that in majority of the cases, the actual α approaches the nominal α=0.05 as the sample size increases from 50. In addition, the changing pattern and θ_2^2 seem to have no effect on the type I error. Table 4.14 and 4.15 describe the type I error rate across different models for β_{01} and β_{11}, respectively. On average, we could see that given the same sample size, different analytical models yield the same level of the type I error. In other words, given the same sample size, all the analytical models have the same

Table 4.14

Descriptive statistics of Type I error rate across sample size for β_{01}

N	Model	Count	Min	Mean	Max	SD
50.000	LCFM	126.000	0.042	0.051	0.064	0.004
50.000	LCSM	126.000	0.042	0.051	0.064	0.004
50.000	SOLGM	126.000	0.039	0.051	0.064	0.004
200.000	LCFM	126.000	0.040	0.050	0.063	0.005
200.000	LCSM	126.000	0.040	0.050	0.063	0.005
200.000	SOLGM	126.000	0.036	0.049	0.060	0.004
600.000	LCFM	126.000	0.037	0.050	0.062	0.005
600.000	LCSM	126.000	0.037	0.050	0.062	0.005
600.000	SOLGM	126.000	0.040	0.050	0.066	0.005
1000.000	LCFM	126.000	0.037	0.050	0.063	0.005
1000.000	LCSM	126.000	0.037	0.050	0.063	0.005
1000.000	SOLGM	126.000	0.036	0.050	0.061	0.004

level of chance to reject our null hypothesis when the alternative hypothesis is true. This level of chance is all-around 5%, which is actually our nominal α level, regardless of the sample sizes. This pattern could be observed through Fig.A.109 and Fig. A.110, where the marginal type I error rate was given under different sample sizes and time-specific residual variances.

In order to systematically investigate the substantially important factors for the statistical power, several full factorial Analysis of Variance (ANOVA) were conducted for each fixed parameter, with statistical power as the dependent variable and all the simulation factors as independent variables. In addition, the η^2 was used to quantify the magnitude of effect size. Tables A.25-A.27 show the results of ANOVA tests for the statistical power of each fixed effect parameter. Since the power of correctly detecting the alternative hypothesis is always 1 in β_{00} regardless of the different conditions, there is no need to conduct the ANOVA test. As for β_{01}, the substantially significant terms given by the table A.25 are: $N, N * \theta_2^2, N * Pattern\ and\ N * Pattern * \theta_2^2$, with their corresponding η^2 values 0.211,0.194,0.164 and 0.201. Since the interaction term "$N * Pattern * \theta_2^2$" is significant, we

Table 4.15

Descriptive statistics of Type I error rate across sample size for β_{11}

N	Model	Count	Min	Mean	Max	SD
50.000	LCFM	126.000	0.038	0.050	0.063	0.005
50.000	LCSM	126.000	0.038	0.050	0.063	0.005
50.000	SOLGM	126.000	0.040	0.050	0.062	0.004
200.000	LCFM	126.000	0.040	0.051	0.063	0.004
200.000	LCSM	126.000	0.040	0.051	0.063	0.004
200.000	SOLGM	126.000	0.041	0.051	0.061	0.004
600.000	LCFM	126.000	0.037	0.050	0.063	0.005
600.000	LCSM	126.000	0.037	0.050	0.063	0.005
600.000	SOLGM	126.000	0.039	0.050	0.062	0.004
1000.000	LCFM	126.000	0.034	0.050	0.061	0.004
1000.000	LCSM	126.000	0.034	0.050	0.061	0.004
1000.000	SOLGM	126.000	0.034	0.050	0.061	0.005

further created the plot 4.18 to check how they jointly influence the power. That is, when the sample size is as small as 50, the pattern that the statistical power goes down with the increase of the θ_2^2 could be easily observed among spreading and high edge pattern. However, this pattern is not that clear under other changing patterns such as constant and shrinking. The most dominating factor that impacts the total variability of the statistical power for β_{10} is the sample size N ($\eta^2 = 0.87$). In another word, 87% of the total variance in statistical power could be explained by the sample size N. Fig 4.19 shows that as the sample size increase from 50 to 1000, the statistical power is also increased with different rate. Apparently, the statistical power increases much faster as the N moves from 50 to 200. According to the table A.27, we could see that N and interaction term $N * \theta_2^2$ have a substantially important influence on the variability of statistical power in terms of β_{11}. They actually explain 57.7% and 15.6% total variance of the statistical power, respectively. Fig.4.20 illustrates how the effect of θ_2^2 on statistical power relies on the sample size N. When the sample size is as small as 50, we could observe that the smaller the θ_2^2 is, the larger the power is. However, this pattern no longer exists when the sample size approach

1000. Basically, the discrepancy of the power levels obtained by different sizes of θ_2^2 is gradually shrunken to 0 as the N becomes larger and larger. Therefore, we could conclude none of the model-related factors have a substantially large effect on the total variance of the statistical power.

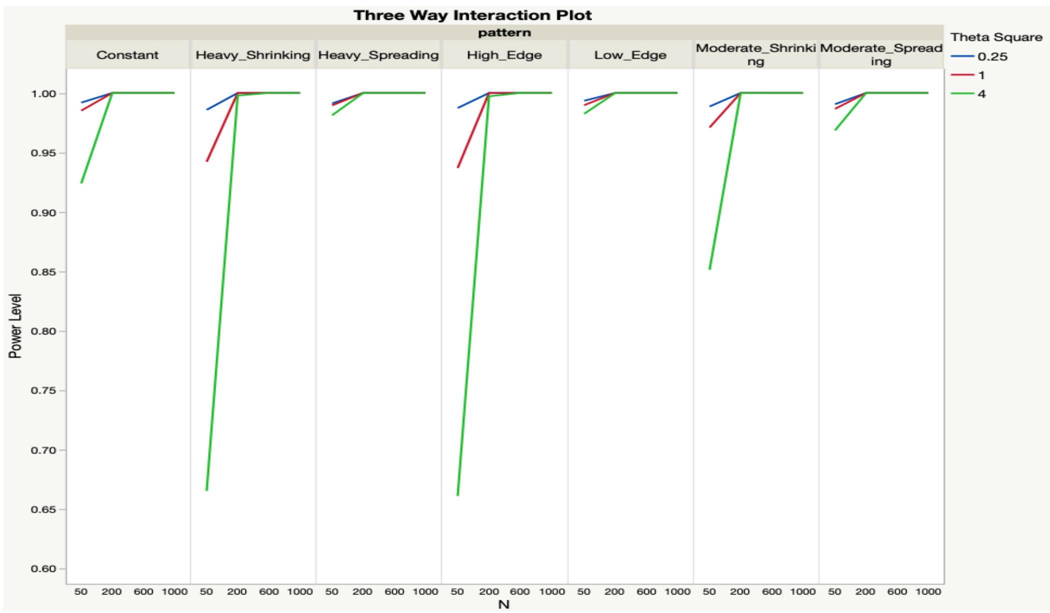

Figure 4.18. Power Plot of Three-Way Interaction among $\boldsymbol{\theta_2^2}$, pattern and $\boldsymbol{N}$ for $\boldsymbol{\beta_{01}}$

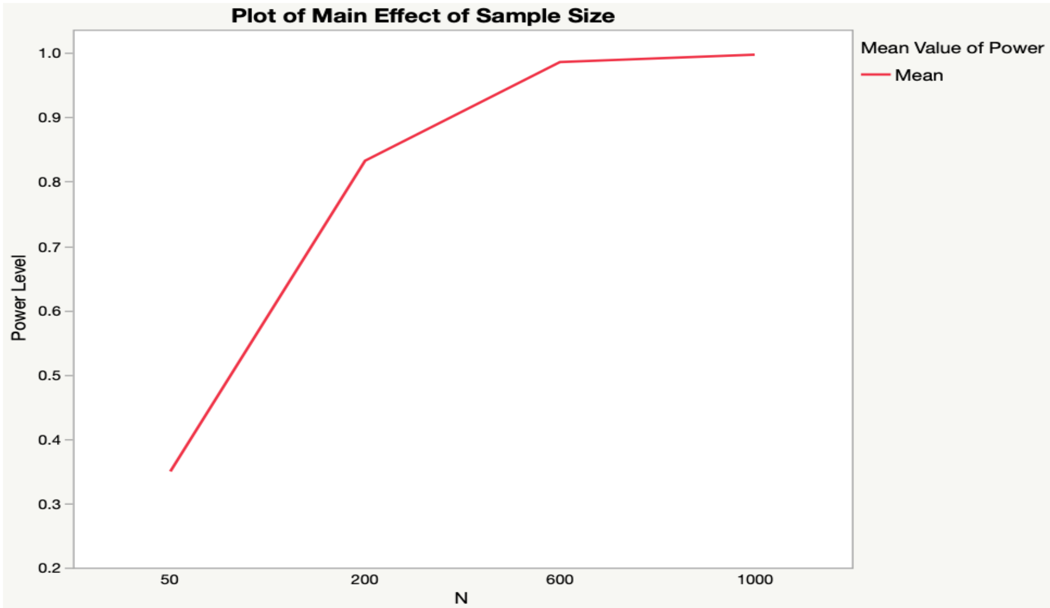

Figure 4.19. Power Plot of main effect for β_{10}

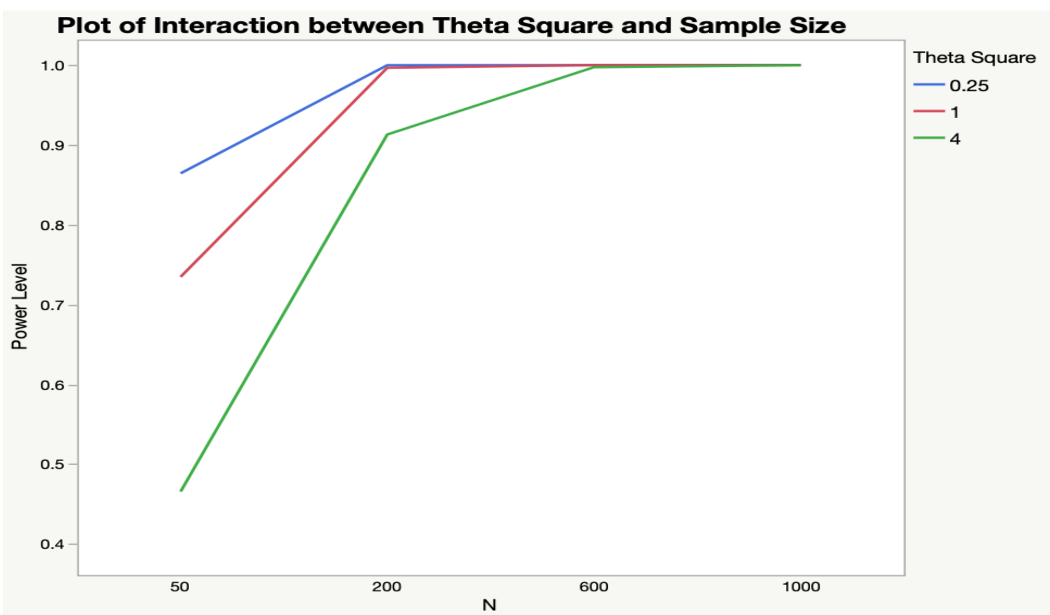

Figure 4.20. Power Plot of Interaction between N and θ_2^2 for β_{11}

Chapter 5

Results of Study 2

5.1 Overall Summary of Estimates

The results in the table 5.1 are very similar to the table 4.1 in study 1. First, the variance parameter ψ_{11} from SOLGM has at least one negative estimate under all the simulation conditions. So its corresponding p-value is 1. As for the time-specific residual variance at the first wave (θ_1^2) , the likelihood of Heywood cases seems to be highest when its true value equals 1. While the lowest likelihood of Heywood cases occurs when its true value is 16. Second, the average number of Heywood cases per condition (indicated by M) tends to be higher for the parameters with lower values than those with higher values. For example, when θ_3=0.063, the average number of Heywood cases per condition is about 454. In contrast, when the θ_3=8, the average number of Heywood cases is around 3. Third, the values of the parameters seem to relate to the minimum and the maximum number of Heywood cases. The smaller the parameters are, the larger the value of the minimum and the maximum number of Heywood cases are. Finally, as we can see, the average convergence rate is always 100%. This trend indicates that Heywood cases are more likely to happen when the values of the parameters are closed to the lower bound of the parameter space (e.g.0)

Table 5.2-5.4 summarized the average Goodness-of-Fit indices (CFI, TLI, RMSEA) given different sample sizes. The descriptive statistic in each column represents the corresponding

Table 5.1

Summary of Heywood Cases and Convergence Rate

Label	True Value	N	P	Mean	Minimum	Maximum	Mean Convergence
ψ_{00}	4.000	249	0.247	8.441	0	222	1
ψ_{11}	1.000	1008	1.000	253.661	10	529	1
θ_1	0.063	96	0.095	443.115	364	520	1
θ_1	0.125	48	0.048	390.896	283	490	1
θ_1	0.250	144	0.143	315.063	133	466	1
θ_1	0.500	96	0.095	187.906	13	409	1
θ_1	1.000	218	0.216	107.413	0	362	1
θ_1	2.000	56	0.056	46.990	0	231	1
θ_1	4.000	43	0.043	9.688	0	74	1
θ_1	8.000	12	0.012	1.646	0	10	1
θ_1	16.000	7	0.007	0.115	0	2	1
θ_2	0.250	336	0.333	200.348	14	417	1
θ_2	1.000	180	0.179	40.182	0	208	1
θ_2	4.000	76	0.075	1.414	0	22	1
θ_3	0.063	96	0.095	453.875	391	520	1
θ_3	0.125	48	0.048	411.438	312	523	1
θ_3	0.250	144	0.143	345.854	148	474	1
θ_3	0.500	96	0.095	229.979	21	439	1
θ_3	1.000	232	0.230	137.863	0	400	1
θ_3	2.000	70	0.069	63.146	0	301	1
θ_3	4.000	56	0.056	15.472	0	111	1
θ_3	8.000	12	0.012	2.521	0	21	1
θ_3	16.000	9	0.009	0.146	0	3	1

statistics based on the mean of Goodness-of-Fit indices. For example, in the table 5.2, the columns titled “M” and “SD” in each table demonstrate the marginal average of the mean-CFI (a summarized statistics that is averaged across 1000 replications) and standard deviation of mean-CFI across the different conditions. As mentioned in the previous section, an acceptable fit requires that the value of CIF or TLI be larger than 0.95, and RMSEA be less than from 0.05 to 0.08. From these results, we could conclude that on average all the models fit the data sufficiently well given different sample sizes because their average CFI and TLI values are all above 0.95 and their RMSEA values are less than 0.06.

Table 5.2

Summary of Goodness-of-Fit Indices for CFI

Model	N	Mean	SD	Min	Median	Max
LCFM	50	0.980	0.005	0.980	0.981	0.989
LCFM	200	0.997	0.001	0.997	0.997	0.998
LCFM	600	0.999	0.000	0.999	0.999	1.000
LCFM	1000	0.999	0.000	0.999	1.000	1.000
LCFM-3W	50	0.966	0.007	0.966	0.968	0.979
LCFM-3W	200	0.997	0.001	0.997	0.997	0.998
LCFM-3W	600	0.999	0.000	0.999	0.999	0.999
LCFM-3W	1000	0.999	0.000	0.999	1.000	1.000
SOLGM	50	0.966	0.007	0.966	0.968	0.979
SOLGM	200	0.997	0.001	0.997	0.997	0.998
SOLGM	600	0.999	0.000	0.999	0.999	0.999
SOLGM	1000	0.999	0.000	0.999	1.000	1.000

* Note: *N* is the sample size; *M*: Marginal means of CFI;*SD*: standard deviation of CFI; *Min*: minimum value of CFI; *Med*: median value of CFI; *Max*: maximum value of CFI.

Table 5.3

Summary of Goodness-of-Fit Indices for RMSEA

Model	N	Mean	SD	Min	Median	Max
LCFM	50	0.047	0.001	0.044	0.047	0.051
LCFM	200	0.015	0.001	0.014	0.015	0.017
LCFM	600	0.008	0.000	0.007	0.008	0.009
LCFM	1000	0.006	0.000	0.006	0.006	0.007
LCFM-3W	50	0.056	0.001	0.053	0.056	0.059
LCFM-3W	200	0.014	0.000	0.013	0.014	0.015
LCFM-3W	600	0.007	0.000	0.006	0.007	0.007
LCFM-3W	1000	0.005	0.000	0.005	0.005	0.006
SOLGM	50	0.056	0.001	0.053	0.056	0.059
SOLGM	200	0.014	0.000	0.013	0.014	0.015
SOLGM	600	0.007	0.000	0.006	0.007	0.007
SOLGM	1000	0.005	0.000	0.005	0.005	0.006

* Note: *N* is the sample size; *M*: Marginal means of RMSEA;*SD*: standard deviation of RMSEA; *Min*: minimum value of RMSEA; *Med*: median value of RMSEA;*Max*: maximum value of RMSEA.

Table 5.4

Summary of Goodness-of-Fit Indices for TLI

Model	N	Mean	SD	Min	Median	Max
LCFM	50	0.980	0.005	0.965	0.980	0.988
LCFM	200	0.999	0.000	0.998	0.999	1.000
LCFM	600	1.000	0.000	1.000	1.000	1.000
LCFM	1000	1.000	0.000	1.000	1.000	1.000
LCFM-3W	50	0.962	0.008	0.941	0.963	0.976
LCFM-3W	200	0.998	0.000	0.997	0.998	0.999
LCFM-3W	600	1.000	0.000	1.000	1.000	1.000
LCFM-3W	1000	1.000	0.000	1.000	1.000	1.000
SOLGM	50	0.962	0.008	0.941	0.963	0.976
SOLGM	200	0.998	0.000	0.997	0.998	0.999
SOLGM	600	1.000	0.000	1.000	1.000	1.000
SOLGM	1000	1.000	0.000	1.000	1.000	1.000

* Note: N is the sample size; M: Marginal means of CFI;SD: standard deviation of CFI; *Min*: minimum value of CFI; *Med*: median value of CFI;*Max*: maximum value of CFI.

As shown in study 1, Fig. 5.1 depicts the relationship between the true parameter values and their mean estimates. This plot includes all model parameters across 1008 conditions. A point on the solid diagonal line indicates that a mean estimate and its true value are identical or the estimate on average is unbiased. Obviously, all the estimates are centered along the diagonal solid line under different analytical models. This indicates that all three analytical models can provide unbiased estimates for their relative true values.

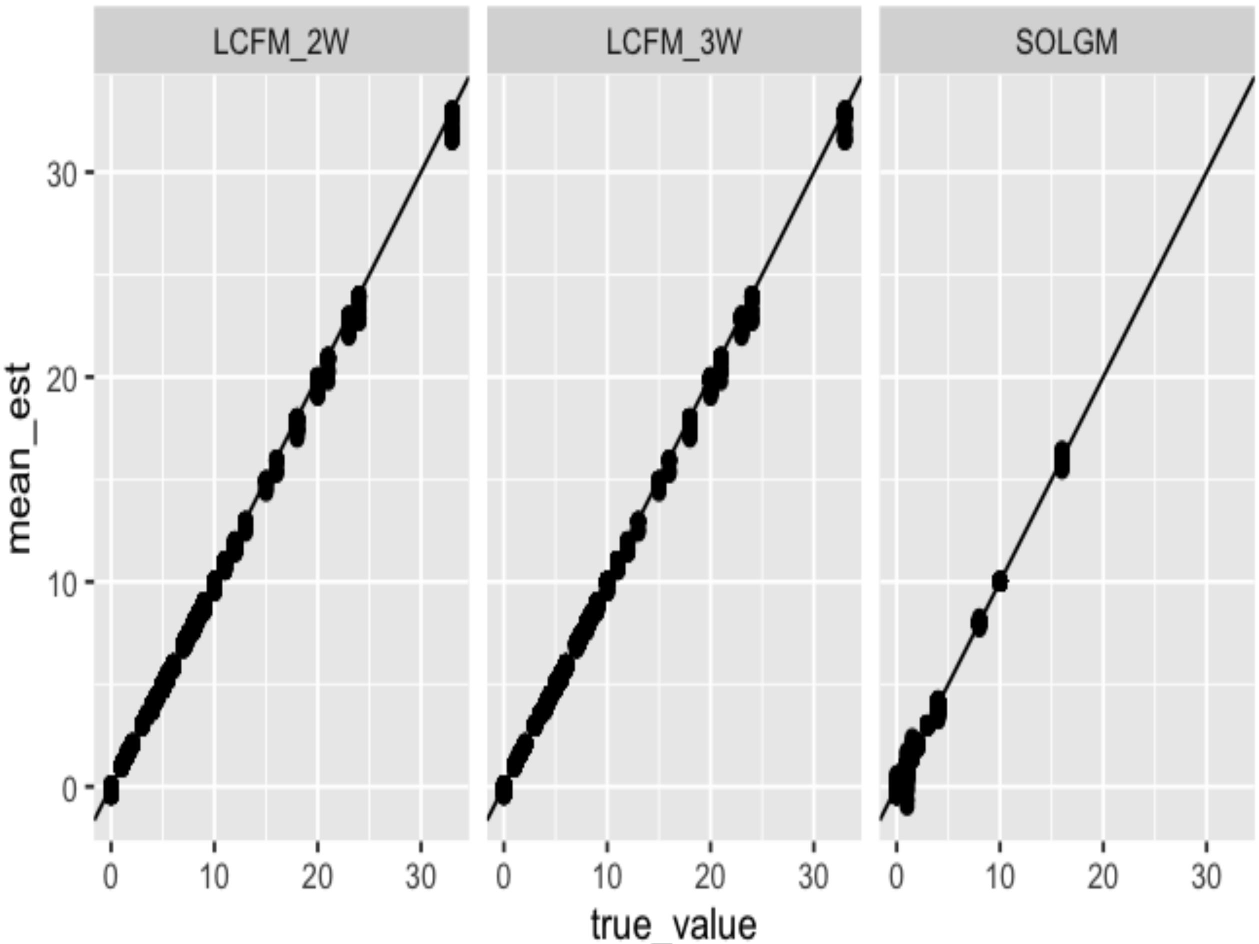

Figure 5.1. Scatter plot of true parameter values and mean estimates per analytic model. SOLGM = 3-wave second-order latent growth model second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model. Each dot represent a model parameter. A solid diagonal line is y = x.

Table 5.5-5.8 are overall summary statistics for the fixed effect parameters. Looking across these tables, we could find that both the bias and the relative bias (Relative_Bias_m) for $\beta_{00}, \beta_{01}, \beta_{10}$ and β_{11} are almost all equal to zero (the absolute value is no larger than ≤ 0.002) no matter what model is used for analysis. This indicates the fixed effects are, on average, estimated correctly. As for the standard deviation of the relative bias (Relative_Bias_sd), three models are equal in most cases except for β_{10}

SE_m and SE_sd are the averages of the empirical standard errors of the estimates and the standard deviation of the empirical standard errors of the estimates. For all the fixed effect parameters, the SE_m obtained from the second-order latent growth model (SOLGM) and 3W-LCFM are slightly smaller than those obtained by the two waves latent change score model (LCSM). For example, the average standard error of β_{00} obtained by SOLGM and 3W-LCFM is around 0.290, however, the corresponding average standard error of κ_1 in the 2-wave LCFM is 0.307. To better interpret this difference, we transform them into a ratio scale. That is, the standard error of β_{00} obtained by SOLGM or 3W-LCFM is about 94.5% of the corresponding κ_1 in the 2W-LCFM. A similar pattern could also be observed in the average growth rate of the reference group (β_{10}), where the mean standard errors obtained by SOLGM and 3W-LCFM are a little lower than the one obtained from the two models (0.337 vs 0.343). In other words, the average standard errors obtained by SOLGM or 3W-LCFM is about 98% of the standard error estimated by the 2-wave LCFM. Therefore, the estimates of the fixed effect parameters using SOLGM/3W-LFCM seem to be relatively more stable than that of the two-wave models.

In simulation studies, type I error rate and statistical power are targeted at the null hypothesis, especially for comparing the competing designs or models. In the current study, there seems to be no difference in the average type I error rate and statistical power (Type I Error Rate/Power M) across three-wave models. This indicates that on average the three-wave models are equivalent in terms of power or type I error rate.

Table 5.5

Mean Change Rate of Reference Group

Model	Label	True Value	N	Bias		Relative Bias		SE		Type I Error/Power	
				Mean	SD	Mean	SD	Mean	SD	Mean	SD
LCFM_2W	κ_1	10	1008	0.001	0.011	0.000	0.001	0.307	0.201	1	0
LCFM_3W	κ_1_3W	10	1008	0.001	0.011	0.000	0.001	0.290	0.185	1	0
SOLGM	β_{00}	10	1008	0.001	0.011	0.000	0.001	0.290	0.185	1	0

Table 5.6

Average Growth Rate in Reference Group

Model	Label	True Value	N	Bias		Relative Bias		SE		Type I Error/Power	
				Mean	SD	Mean	SD	Mean	SD	Mean	SD
LCFM_2W	$\Delta\kappa$	1	1008	-0.001	0.015	-0.001	0.015	0.343	0.235	0.790	0.004
LCFM_3W	$\Delta\kappa\_3W$	1	1008	-0.001	0.014	-0.001	0.014	0.337	0.230	0.795	0.004
SOLGM	β_{10}	1	1008	-0.001	0.014	-0.001	0.014	0.337	0.230	0.794	0.005

Table 5.7

Group Difference in Initial Status

Model	Label	True Value	N	Bias		Relative Bias		SE		Type I Error/Power	
				Mean	SD	Mean	SD	Mean	SD	Mean	SD
LCFM_2W	γ_1	3	504	-0.001	0.017	0.000	0.006	0.415	0.274	0.983	0.061
LCFM_3W	γ_1_3W	3	504	-0.002	0.016	0.001	0.005	0.392	0.253	0.989	0.004
SOLGM	β_{01}	3	504	-0.002	0.016	-0.001	0.005	0.392	0.253	0.989	0.004
LCFM_2W	γ_1	0	504	-0.001	0.014	0.000	0.005	0.387	0.262	0.050	0.005
LCFM_3W	γ_1_3W	0	504	-0.001	0.014	0.000	0.005	0.368	0.241	0.050	0.004
SOLGM	β_{01}	0	504	-0.001	0.014	0.000	0.005	0.368	0.241	0.050	0.004

Table 5.8

Group Difference in Rate of Change

Model	Label	True Value	N	Bias		Relative Bias		SE		Type I Error/Power	
				Mean	SD	Mean	SD	Mean	SD	Mean	SD
LCFM_2W	$\Delta\gamma$	2	504	0.001	0.017	0.000	0.008	0.430	0.311	0.913	0.175
LCFM_3W	$\Delta\gamma\_3W$	2	504	0.000	0.015	0.000	0.008	0.419	0.302	0.917	0.168
SOLGM	β_{11}	2	504	0.000	0.015	0.000	0.008	0.419	0.302	0.917	0.168
LCFM_2W	$\Delta\gamma$	0	504	0.001	0.017	0.000	0.009	0.416	0.306	0.050	0.005
LCFM_3W	$\Delta\gamma\_3W$	0	504	0.001	0.017	0.000	0.009	0.404	0.294	0.050	0.004
SOLGM	β_{11}	0	504	0.001	0.017	0.000	0.009	0.403	0.294	0.050	0.004

5.1.1 Bias

Fig.B.1-Fig. B.12 exemplify the bias of fixed effect parameters under some conditions. The average estimates of fixed parameters are the same for both SOLGM and three-wave LCFM regardless of different simulation conditions. That is why the green line (for three-wave LCFM) is completely overlaid by the blue line (for SOLGM) in every panel. Therefore, we could conclude that three-wave LCFM and the three-wave SOLGM would, on average, produce the equivalent estimates for the fixed effect parameters. In other words, given the same number of waves, the two models, on average, (3W-LCFM vs. 3W-SOLGM) do not differ in terms of the estimation ability.

In contrast, the average estimates of the fixed parameters obtained by 2W-LCFM (for red line) have a slight deviation from the 3W-LCFM/SOLGM, especially when the sample size is 50. Furthermore, the comparison between the results of 3W-LCFM and 2W-LCFM seems to imply that given the same model (LCFM), the number of waves may have a little impact on the magnitude of the bias, especially when the sample size is very small. However, we still do not know whether this visualized difference could really reach a statistically significant level. One more thing to be noted here is that no discernible patterns of the direction in the bias could be observed. This provides evidence that positive and negative biases could randomly appear under the combinations of different conditions.

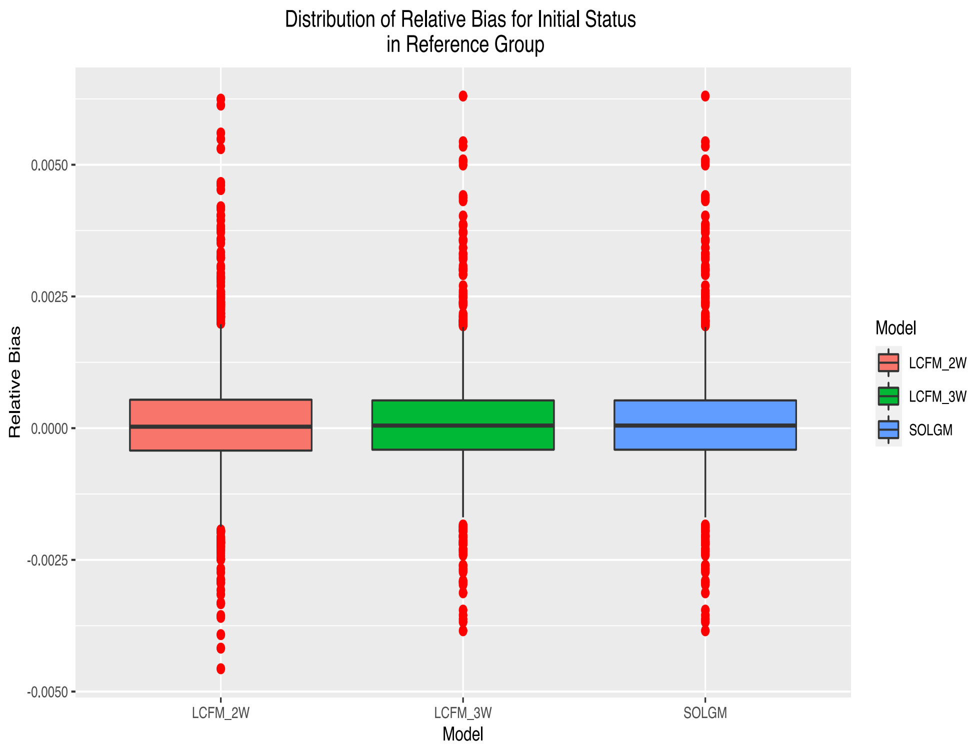

Figure 5.2. Boxplot of Distribution of Relative Bias for β_{00}

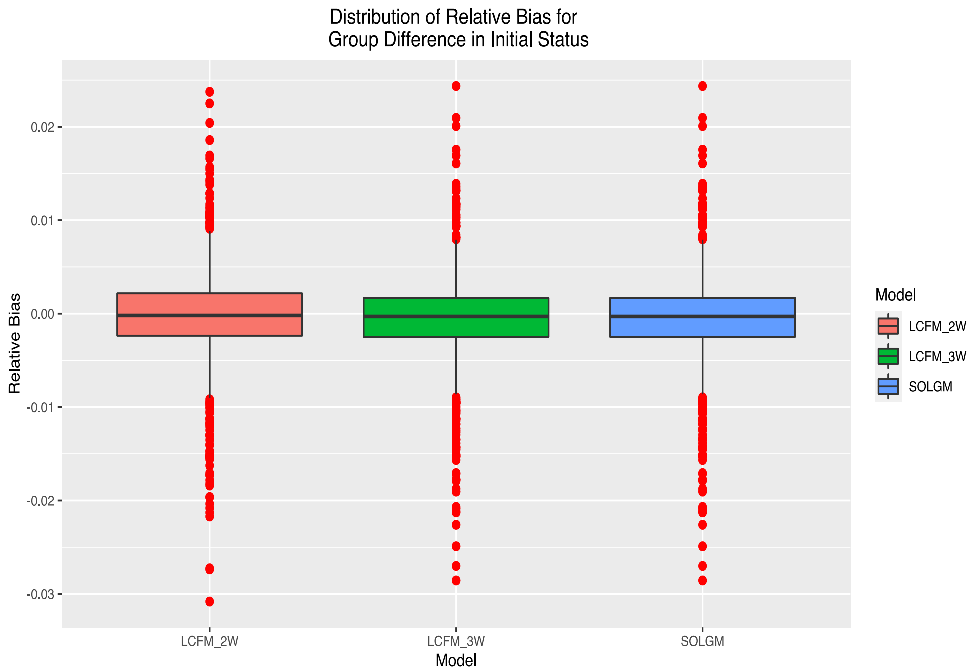

Figure 5.3. Boxplot of Distribution of Relative Bias for β_{01}

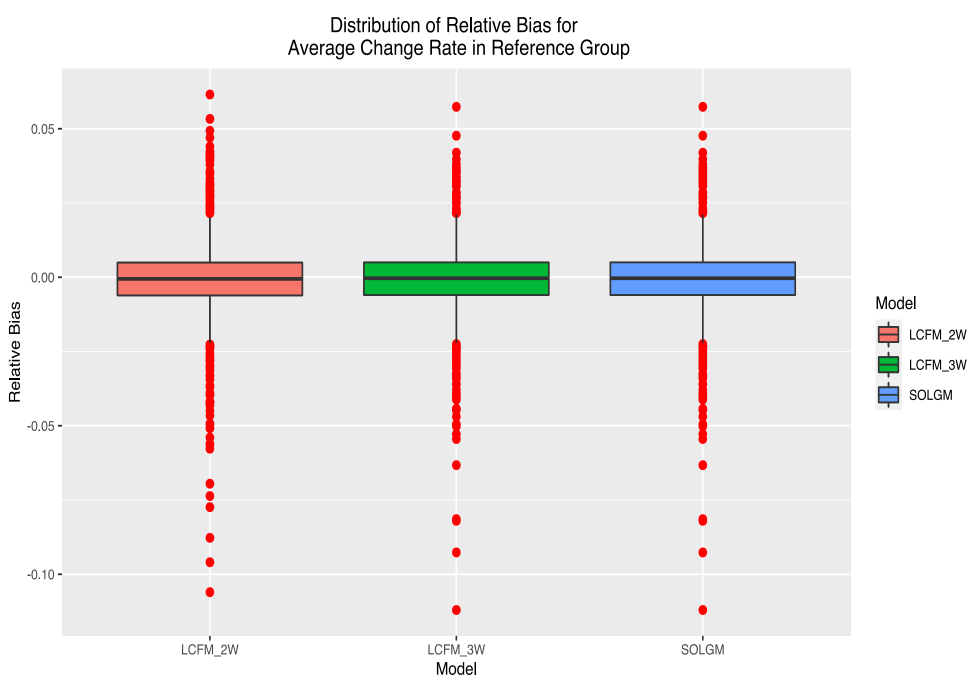

Figure 5.4. Boxplot of Distribution of Relative Bias for β_{10}

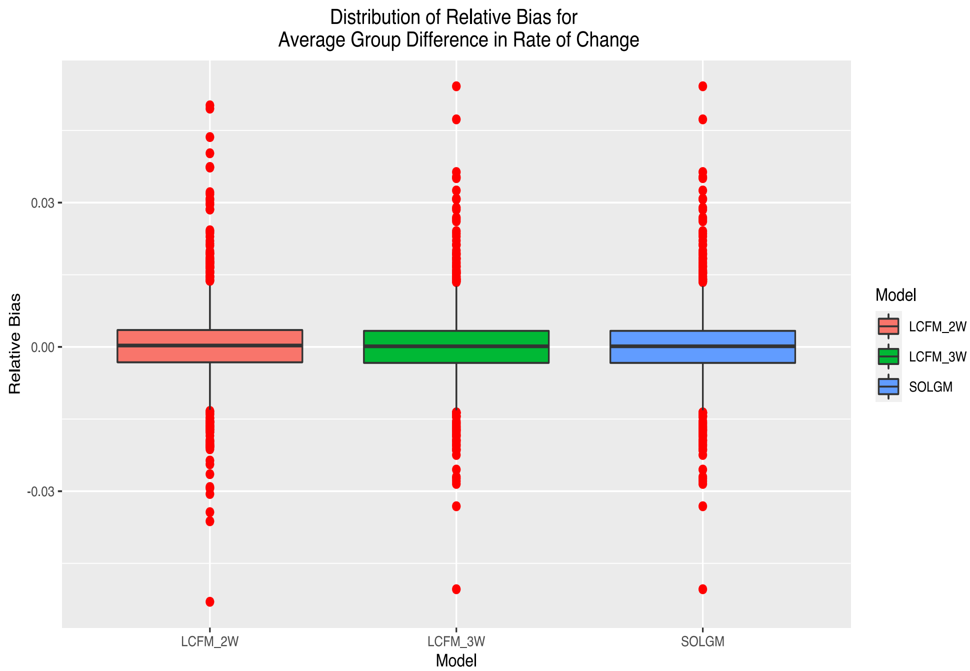

Figure 5.5. Boxplot of Distribution of Relative Bias for β_{11}

Figure 5.2-5.5 are the box plots of relative bias across different models for each fixed effect parameter. Fig 5.2 depicts the distribution of relative bias for the initial status of the reference group (β_{00}). The distributions of the 3W-LCFM are almost the same as the three-wave SOLGM. This could be easily observed for the location of different quantiles and the span of outliers (red dots). It seems that given the same wave of data, the performance of bias is not impacted by different models in the current study. In other words, the point estimate abilities are equivalent for both models given the same number of waves. However, the distributions of relative bias between the 2W-LCFM and 3W-LCFM seem a little different in the span of the outliers. This difference only comes from the outliers attached to the bottom whisker. That is, the span of outliers in 2W-LCFM is slightly wider than they are in the 3W-LCFM. Therefore, we could infer that given the same longitudinal common factor model (LCFM), the performance of the point estimate by the two-wave model almost fully recovers its performance in the three-wave model except for a few extreme conditions. In summary, the minimum and maximum outliers in the three models are still far less than 5%, and we, therefore, could conclude that there is no bias when using either different models (SOLGM vs LCFM) or different waves (2W vs. 3W) to estimate the initial status of the reference group (β_{00}).

Fig 5.3 depicts the box-plot of relative bias for the average group difference in the initial status (β_{01}). As expected, the pattern of the distribution is almost the same as the Fig.5.2. The performance of the relative bias given by 3W-LCFM is the same as the SOLGM, but slightly differs from the 2W-LCFM. This difference is still from the outliers attached to the bottom whisker, where the span of 2W-LCFM is slightly wider than the 3W-LCFM/SOLGM. However, the absolute values of the outliers in every model are less than the 5% criterion. Therefore, given the above evidence, we could conclude that there is still no bias when using either different models (SOLGM vs LCFM) or different waves (2W vs. 3W) to estimate the average group difference in the initial status (β_{01}).

Fig 5.4-5.5 are the distributions of relative bias in the growth rate parameters (β_{10} and β_{11}). In both plots, the distribution of 3W-LCFM performs the same as the SOGLM, which could be observed in terms of the location of quantiles and the span of outliers. This implies that given the same wave of the data, the longitudinal common factor model could show the same point estimate ability as the SOLGM for the growth parameters. However, some absolute values of outliers in both plots are larger than 5%, even for the data generation model (SOLMG). What is worse, there is a small portion of outliers in Fig.5.4 whose absolute values are even larger than 10%. This suggests that under some extreme conditions, there may exist some degree of bias even when the true model (SOLGM) was used to estimate the growth parameters β_{10} and β_{11}. Since there are more extreme outliers attached to the bottom whisker than they are along with the top whiskers, the likelihood of underestimating the true value of (β_{10}) may be slightly higher than the likelihood of overestimating the true (β_{10}). In contrast, the magnitude of estimating the bias in β_{11} is not as large as they are in the β_{10} since no outliers' absolute values are larger than 0.1. In addition, the chances of overestimation and underestimation to the true values seem to be more balanced in the β_{11}.

In order to further explore in which conditions the three analytical models will generate biased estimates for the β_{01} and β_{11}, we extracted the conditions whose relative biases are either smaller than -0.05 or larger than 0.05, and the results are given in tables 5.9 and 5.10. Similar to the results in study 1, we could observe that there are much more simulation conditions that generated the bias estimates in β_{10} (31 conditions) than they are in the β_{11} (6 conditions). Therefore, compared to β_{11}, it is more likely to have biased estimates for β_{10}. For both β_{10} and β_{11}, all these biased estimates come from $N = 50$. In addition, the majority of the biased estimates tend to come from the $\theta_2^2 = 4$ with regards to β_{10}. The patterns under which there exist some biased estimates include High Edge, Heavy spreading, Heavy shrinking, and moderate spreading.

Table 5.9

Conditions with significant relative bias in β_{10}

Model	Relative Bias	N	θ_2^2	Pattern
LCFM	0.062	50	4	High_Edge
SOLGM	0.057	50	4	High_Edge
LCFM_3W	0.057	50	4	High_Edge
LCFM	0.053	50	4	Heavy_Spreading
LCFM_3W	-0.050	50	4	High_Edge
SOLGM	-0.050	50	4	High_Edge
LCFM	-0.051	50	4	Heavy_Shrinking
SOLGM	-0.053	50	4	High_Edge
LCFM_3W	-0.053	50	4	High_Edge
LCFM	-0.054	50	1	High_Edge
SOLGM	-0.054	50	1	High_Edge
LCFM_3W	-0.054	50	1	High_Edge
LCFM	-0.056	50	4	Heavy_Spreading
LCFM	-0.056	50	4	Moderate_Spreading
LCFM	-0.058	50	4	Heavy_Shrinking
LCFM_3W	-0.063	50	4	Heavy_Spreading
SOLGM	-0.063	50	4	Heavy_Spreading
LCFM	-0.070	50	4	Heavy_Spreading
LCFM	-0.074	50	4	Heavy_Spreading
LCFM	-0.077	50	4	High_Edge
SOLGM	-0.081	50	4	High_Edge
LCFM_3W	-0.081	50	4	High_Edge
SOLGM	-0.082	50	4	Heavy_Shrinking
LCFM_3W	-0.082	50	4	Heavy_Shrinking
LCFM	-0.088	50	4	Heavy_Shrinking
LCFM_3W	-0.093	50	4	High_Edge
SOLGM	-0.093	50	4	High_Edge
LCFM	-0.096	50	4	High_Edge
LCFM	-0.106	50	4	High_Edge
LCFM_3W	-0.112	50	4	High_Edge
SOLGM	-0.112	50	4	High_Edge

Table 5.10

Conditions with significant relative bias in β_{11}

Model	Relative Bias	N	θ_2^2	Pattern
SOLGM	0.054	50	4	High_Edge
LCFM_3W	0.054	50	4	High_Edge
LCFM	0.050	50	4	High_Edge
LCFM_3W	-0.050	50	1	High_Edge
SOLGM	-0.050	50	1	High_Edge
LCFM	-0.053	50	1	High_Edge

In order to further explore the possible factors that could impact the magnitude of the relative bias, four full factorial Analysis of Variance(ANOVA) were conducted for each fixed parameter, with relative bias as the dependent variable and simulation factors as independent variables. Tables B.1-B.4 show the ANOVA test results for the relative bias of each fixed effect parameters, respectively (β_{00}, β_{01},β_{10} and β_{11}). Across all four ANOVA results, we could conclude that there is no substantially important factor for the variability of relative bias in each fixed effect parameter because the largest value of η^2 is from 0.068 to 0.087, which means that less than 9% of the total variability could be explained by the most influential terms in each table.

5.1.2 Standard Error

Empirical Standard Error

Fig.B.13-B.24 exemplify the results of empirical standard error across different fixed effect parameters. In general, the empirical standard error tends to decrease as the sample size increases regardless of simulation conditions. That is why all the line plots in each panel show a similar down-slope pattern starting from the top left and reach down to the bottom right. As for the average initial status β_{00}, the blue and green lines that represent the standard errors of the SOLGM and 3W-LCFM respectively, are totally intertwined in every panel. However, they are all a little lower than the red line that represents the 2W-LCFM, when the changing patterns are shrinking and high-edge. Meanwhile, we can observe that this discrepancy tends to become larger when the time-specific residual variance becomes larger (θ^2). A similar pattern also exists for the average group difference in the initial status β_{01}. This indicates that given the same number of waves, LCFM and SOLGM seem to have the same level of efficiency when estimating the parameter of the initial status. However, given the same model (LCFM), increasing the number of waves seems to improve the efficiency of the estimate when the changing pattern is at the shrinking and high-edge. This improvement in estimating efficiency seems more salient when the θ^2 becomes larger.

In contrast, the pattern of empirical standard errors in growth parameters (β_{10} and β_{11}) seems a little different from the parameters of the initial status. Basically, the blue (SOLGM) and green (3W-LCFM) lines are still intertwined together to show that there is no difference in terms of estimation efficiency. This is also true when the sample size is 50. However, the discrepancy happens between 2W-LCFM and 3W-LCFM/SOLGM. Different from the patterns shown in the initial status, The empirical standard errors of 2W-LCFM (red line) tend to be a little higher than that of three-wave models when the changing patterns are at shrinking and spreading. In addition and similar to the pattern in the initial status, this discrepancy is enlarged when the time-specific residual variance θ_2^2 gets larger.

In order to systematically explore the factors that are substantially important for the standard error of estimating the fixed effect parameters, we conducted four full factorial ANOVA with the standard error being the dependent variable. Table B.5-B.8 show the ANOVA test results for the fixed effects parameters. As expected, the sample size N is the only substantially important factor for the variability of standard error in each estimate of the fixed effect parameter. The corresponding values of η^2 are 0.895, 0.871, 0.802 and 0.715

for β_{00}, β_{01}, β_{10} and β_{11}, respectively. That means the total variance of standard error explained by the sample size N could be ranged from 71% to 90%. The rest of the terms, especially those related to the models, do not have any substantial contribution to the variance of the standard error of the estimate. The major conclusion inferred from this evidence is that there is actually no loss of efficiency when we use the two-wave LCFM to estimate the fixed effect parameters compared to the three-wave SOLGM.

Theoretical Standard Error

Fig.B.25-Fig.B.36 depict some examples of theoretical standard error across different models under different conditions. The theoretical standard errors tend to decrease as the sample size increases regardless of condition, which is the same as the pattern observed for the empirical standard error. The performance of the theoretical standard error for β_{00} and β_{01} are also similar to their corresponding empirical standard error. The theoretical standard errors of the three-wave models (3W-SOLGM and 3W-LCFM) are smaller than the two-wave models (2W-LCFM) when the patterns are shrinking and high-edge. This discrepancy tends to increase with the time-specific residual variance enlarged. Furthermore, when the sample size is as small as 50, the theoretical standard error obtained from the two-wave models is always larger than the standard error yielded by the three-wave models in almost all the conditions. In addition, the performance for the theoretical standard error of growth parameters (β_{10} and β_{11}) indicated that two-wave models could give theoretical standard errors comparable to the three-wave models in the majority of cases except when the time-specific variance are large and heteroscedastic. More specifically, the 2W-LFCM yields worse theoretical standard error compared to the three-wave models when θ_2^2 is large and the patterns are spreading or shrinking.

In order to systematically explore the factors that are substantially important for the theoretical standard error of estimating the fixed effect parameters, we conducted four full factorial ANOVA with the standard error being the dependent variable. The corresponding results are listed across tables B.9-B.12. Again the sample size N is the only substantially important factor for the variability of theoretical standard error in each estimate of the fixed effect parameter. The corresponding values of η^2 are 0.889, 0.859, 0.796 and 0.701 for β_{00}, β_{01}, β_{10} and β_{11}, respectively. That means the total variance of standard error explained by the sample size N could be ranged from 70% to 89%. The rest of the terms, especially those related to the models, do not have any substantial contribution to the variance of the standard error of the estimate. The major conclusion inferred from this evidence is that there is actually no loss of efficiency when we use the two-wave LCFM to estimate the fixed effect parameters compared to the three-wave SOLGM.

Ratio of Theoretical standard error

Table 5.11 shows the descriptive statistics for the $Ratio - SE_{2T/3T}$. On average, the theoretical SE by the two-wave models is slightly larger than the theoretical SE obtained from the three-wave model in every fixed effect parameter. The minimum values of the ratio $Ratio - SE_{2T/3T}$ for all fixed effect parameters are larger or equal to 1. This implies that on average, the two-wave models tend to be larger than the theoretical SE of the three-wave model, especially for estimating the initial status parameters. The largest maximum values across fixed effect parameters are ranged from 1.135 to 1.269. This indicates in certain situations, the theoretical SE by the two-wave models could be 26% higher than the corresponding one by the three-wave model.

Table 5.11

Descriptive Statistics for the Ratio of Theoretical SE of 2W-Models vs 3W-Model

Parameters	Minimum	Median	Maximum	Mean	Standard Deviation
β_{00}	1.018	1.046	1.228	1.064	0.045
β_{01}	1.008	1.040	1.269	1.060	0.054
β_{10}	1.000	1.006	1.137	1.020	0.034
β_{11}	1.000	1.011	1.212	1.032	0.049

In order to examine what factors have substantial importance on the variability of $Ratio - SE_{2T/3T}$, we conducted four full factorial ANOVA for each fixed effect parameter. The results are shown from tables B.13 to B.16. Similar as the results in study 1, the term pattern, θ_2^2 and pattern*θ_2^2 have the largest effect on the variability of $Ratio - SE_{2T/3T}$ for all the fixed effect parameters. Given the interaction term between pattern and θ_2^2 are significantly important, we first created the interaction plots for both β_{00} and β_{01}. Fig.5.6 and 5.7 depict the interaction effect between pattern and θ_2^2 on the $Ratio - SE_{2T/3T}$ for β_{00} and β_{01}. The results are similar to the findings in the study. In general, as the θ_2^2 increases from 0.25 to 4, the average value of $Ratio - SE_{2T/3T}$ also increases. However, this increasing trend can only be observed between shrinking and edge patterns. Meanwhile, the $Ratio - SE_{2T/3T}$ at the low edge and spreading patterns tend to slightly decrease. Finally, this increasing rate tends to become larger as the θ_2^2 becomes larger with the shrinking pattern become worse. Similar to study 1, the average maximum loss of using the two-wave models could reach 24% for both β_{00} and β_{01}. Fig.5.8-5.9 depict the interaction effect between pattern and θ_2^2 on the $Ratio - SE_{2T/3T}$ for β_{10} and β_{11}, respectively. In general, as the size of θ_2^2 increases, The average $Ratio - SE_{2T/3T}$ increases only at the spreading and shrinking patterns. Under the other patterns, the average value for both ratios tends to remain stable regardless of how large the θ_2^2 is.

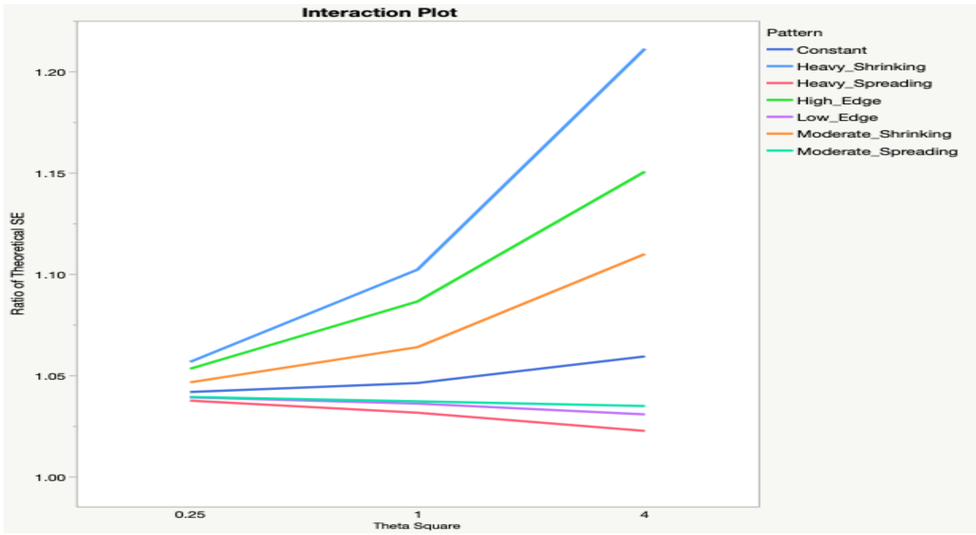

Figure 5.6. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error $(Ratio - SE_{2T/3T})$ for $\boldsymbol{\beta_{00}}$

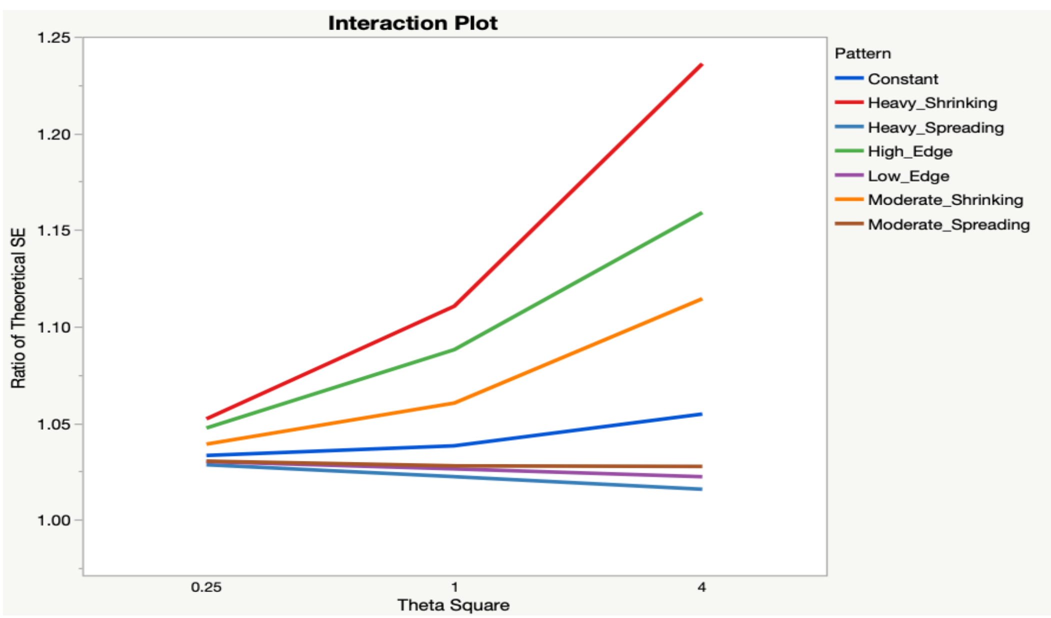

Figure 5.7. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error $(Ratio - SE_{2T/3T})$ for $\boldsymbol{\beta_{01}}$

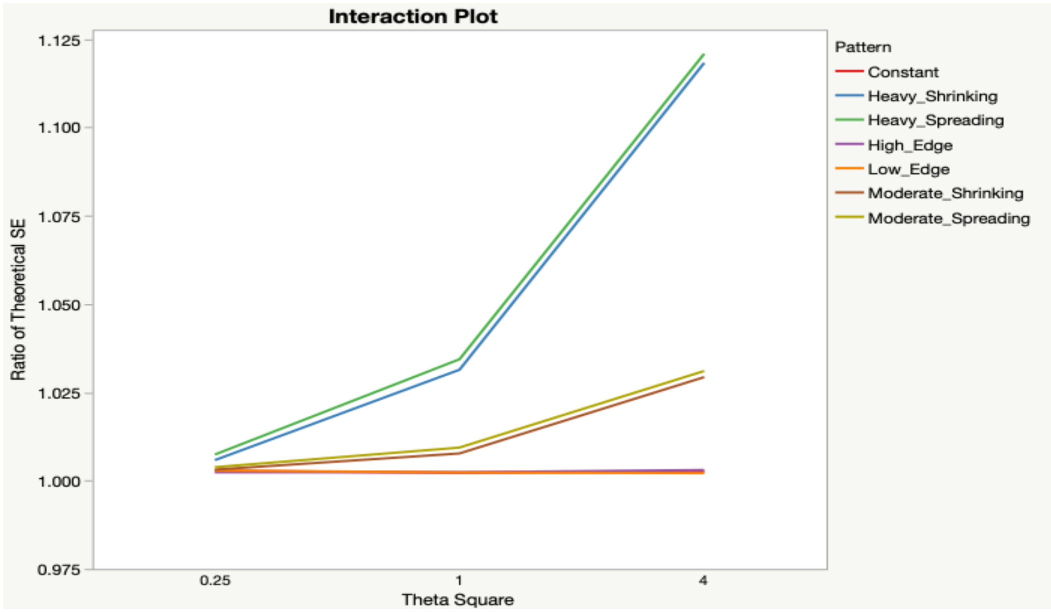

Figure 5.8. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error $(Ratio - SE_{2T/3T})$ for $\boldsymbol{\beta_{10}}$

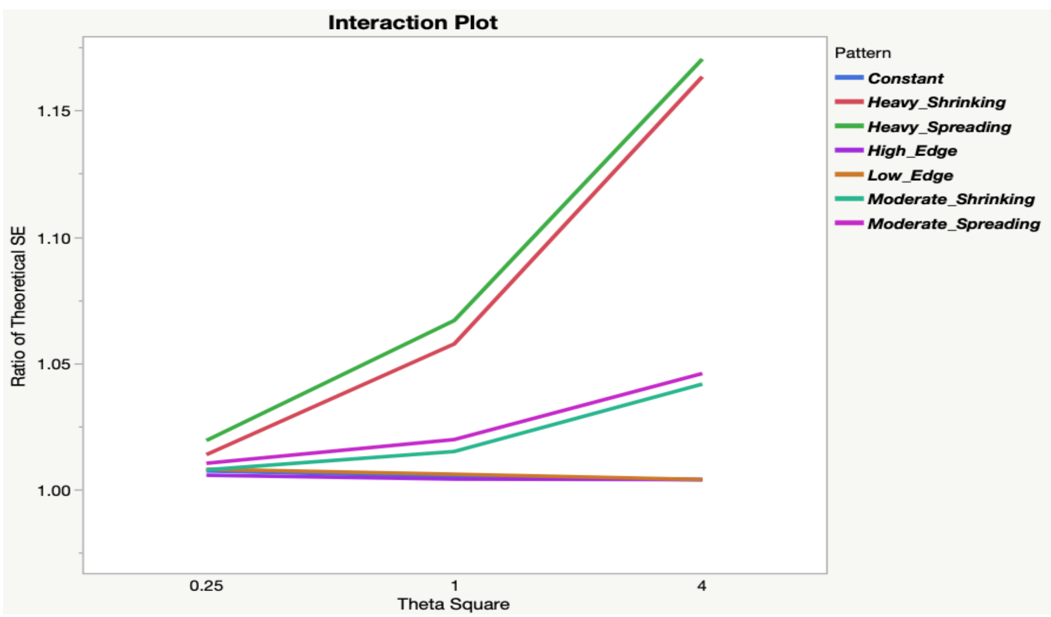

Figure 5.9. Interaction between the pattern and $\boldsymbol{\theta_2^2}$ on the ratio of theoretical standard error $(Ratio - SE_{2T/3T})$ for $\boldsymbol{\beta_{11}}$

5.1.3 Type I Error Rate and Statistical Power

Fig B.37-B.45 list some examples of plots for the statistical power and type I error rate in different parameters. On the whole, we could easily observe that the three-wave models (3W-LCFM vs. SOLGM) are exactly the same in terms of statistical power in every panel. That explains why the green line and blue line again come together. In addition, the sample size N has a dominating effect on the statistical power except for the β_{00}. To be more specific, when the statistical inference is for the average initial status in the reference group β_{00}, our statistical power always reaches 100% regardless of analytical models. When the inference is for other fixed effect parameters, we could see that the statistical power goes up as the sample size increases from 50 to 1000. When the sample size N is 50, the statistical power in most cases is less than 0.5. Then it goes up gradually with the increase of sample size. After the sample size is larger than 600 or 700, the statistical power in most cases starts to approach 1. Other possible factors that may influence the statistical power are time-specific residual variance θ_2^2 and changing patterns of the time-specific residual variance. the power of 3-wave SOLGM/LCFM seems to be slightly higher than the 2-wave LCFM when the patterns are at spreading and shrinking. Furthermore, increasing the time-specific residual variance θ_2^2 could slightly enlarge this disparity.

In order to systematically investigate the substantially important factors for the statistical power, we conducted three full factorial Analysis of Variance (ANOVA) for the fixed parameter $\beta_{00}, \beta_{10}\ and\ \beta_{11}$, with statistical power as dependent variable and simulation factors as independent variables. Similar to the reason as in study 1, we neglect the ANOVA analysis for β_{00} because all the power value is 1. As shown in Table B.17, the substantially significant terms for the statistical power of β_{01} are:
$N, N * \theta_2^2, N * Pattern\ and\ N * Pattern * \theta_2^2$ with their corresponding η^2 value 0.214, 0.194,

0.157 and 0.198. Further three-way interaction plot 5.10 shows almost the same pattern observed in study 1. That is, when the sample size is as small as 50, the discrepancy between the power level given by the different sizes of θ_2^2 could be observed among most changing patterns except for the heavy spreading and low edge, where to increase θ_2^2 from 0.25 to 1 does not reduce the statistical power.

Table B.18 shows the results of the ANOVA test for β_{10}, whose value is 1. Consistent with the results in study 1, N is the only substantially important factor for the total variability of the statistical power. Around 88% of the total variance in statistical power could be explained by N. The increasing rate is the same as in study 1 (Fig 5.11). Finally, in the table B.19, we could conclude that N and interaction term $N * \theta_2^2$ are still the substantially important factors on the variability of statistical power for β_{11}, with their corresponding η^2 0.581 and 0.154. The interpretations for Fig 5.12 are the same as they are in study 1. Therefore we could conclude that the findings in study 1 are all consistent with the findings in study 2.

Finally, we would like to check the type I error rate. Table 5.12 and 5.13 describe the type I error rate across different models for β_{01} and β_{11}, respectively. On average, we could see that given the same sample size, different analytical models yield the same level of the type I error. In other words, given the same sample size, all the analytical models have the same level of chance to reject our null hypothesis when the alternative hypothesis is true. This level of chance is all-around 5%, which is actually our nominal α level, regardless of the sample sizes.

Table 5.12

Descriptive statistics of Type I error rate across sample size for β_{01}

N	Model	Count	Min	Mean	Max	SD
50.000	LCFM	126.000	0.042	0.051	0.064	0.004
50.000	LCFM_3W	126.000	0.039	0.051	0.064	0.004
50.000	SOLGM	126.000	0.039	0.051	0.064	0.004
200.000	LCFM	126.000	0.040	0.050	0.063	0.005
200.000	LCFM_3W	126.000	0.036	0.049	0.060	0.004
200.000	SOLGM	126.000	0.036	0.049	0.060	0.004
600.000	LCFM	126.000	0.037	0.050	0.062	0.005
600.000	LCFM_3W	126.000	0.040	0.050	0.066	0.005
600.000	SOLGM	126.000	0.040	0.050	0.066	0.005
1000.000	LCFM	126.000	0.037	0.050	0.063	0.005
1000.000	LCFM_3W	126.000	0.036	0.050	0.061	0.004
1000.000	SOLGM	126.000	0.036	0.050	0.061	0.004

Table 5.13

Descriptive statistics of Type I error rate across sample size for β_{11}

N	Model	Count	Min	Mean	Max	SD
50.000	LCFM	126.000	0.038	0.050	0.063	0.005
50.000	LCFM_3W	126.000	0.040	0.050	0.062	0.004
50.000	SOLGM	126.000	0.040	0.050	0.062	0.004
200.000	LCFM	126.000	0.040	0.051	0.063	0.004
200.000	LCFM_3W	126.000	0.041	0.051	0.061	0.004
200.000	SOLGM	126.000	0.041	0.051	0.061	0.004
600.000	LCFM	126.000	0.037	0.050	0.063	0.005
600.000	LCFM_3W	126.000	0.039	0.050	0.062	0.004
600.000	SOLGM	126.000	0.039	0.050	0.062	0.004
1000.000	LCFM	126.000	0.034	0.050	0.061	0.004
1000.000	LCFM_3W	126.000	0.034	0.050	0.061	0.005
1000.000	SOLGM	126.000	0.034	0.050	0.061	0.005

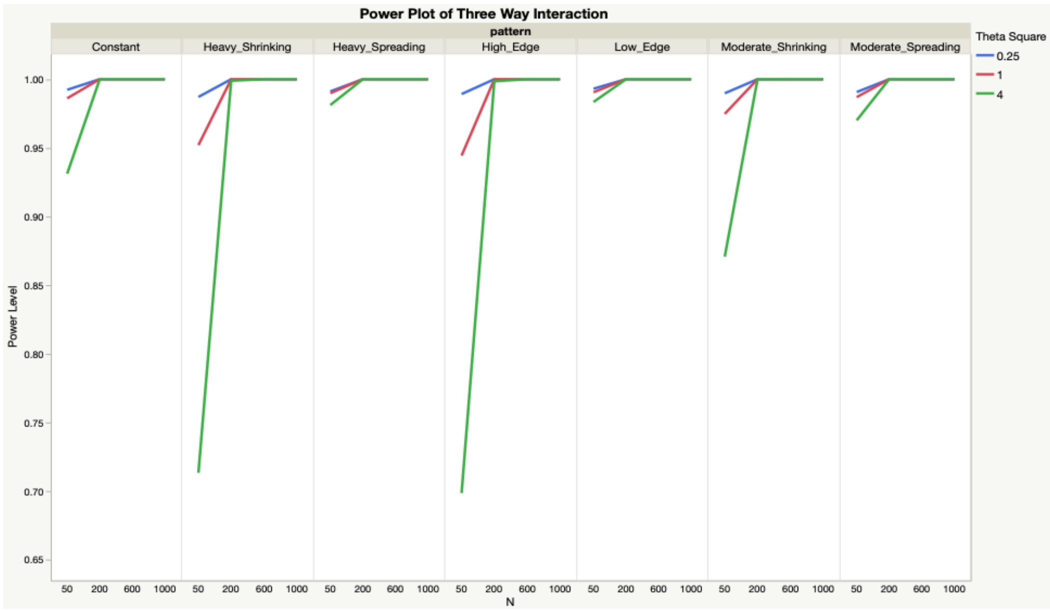

Figure 5.10. Power Plot of Three-Way Interaction among $\boldsymbol{\theta_2^2}$, pattern and $\boldsymbol{N}$ for $\boldsymbol{\beta_{01}}$

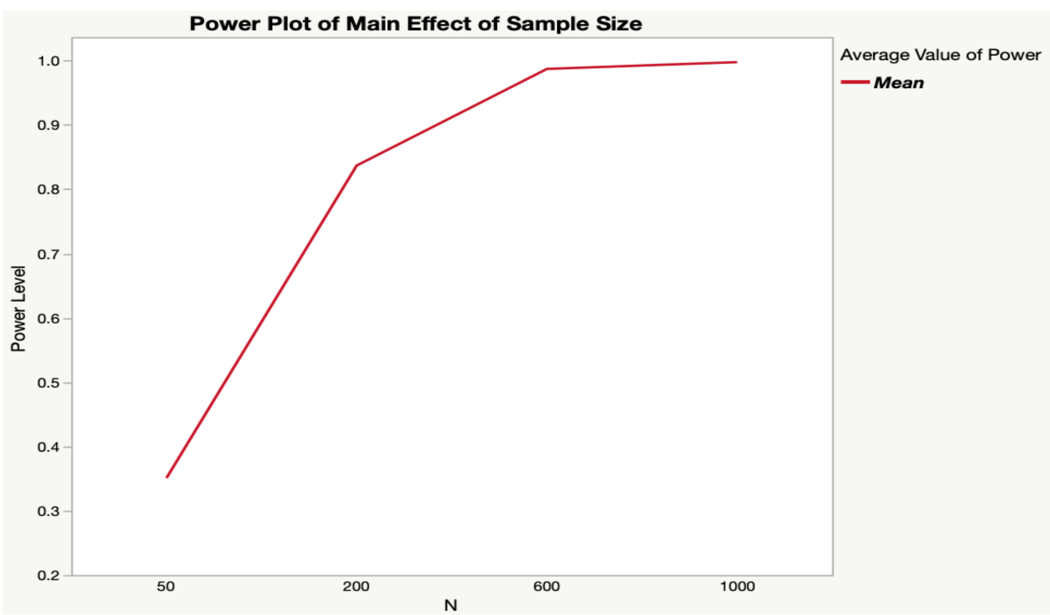

Figure 5.11. Power Plot of main effect for $\boldsymbol{\beta_{10}}$

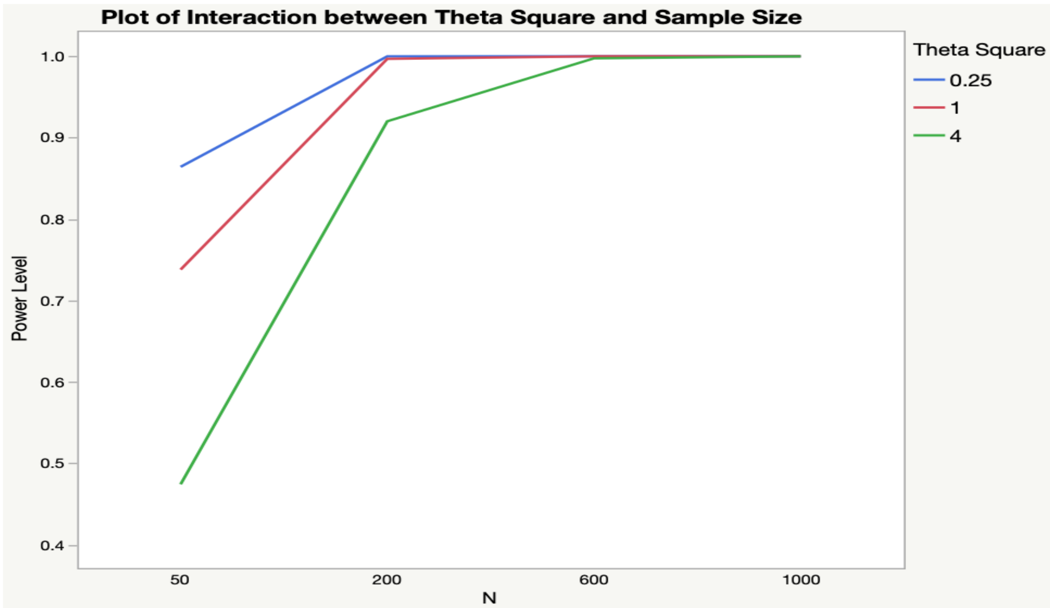

Figure 5.12. Power Plot of Interaction between N and $\boldsymbol{\theta_2^2}$ for $\boldsymbol{\beta_{11}}$

Chapter 6

Discussion

The major purpose of this dissertation was to determine to what extent the fixed effect parameters of the 3W-SOLGM can be recovered from the 2W-LCFM and 2W-LCSM in various settings and how precise the two-wave models could approach when they are used to estimate those fixed effect parameters. In social science research, we sometimes have to face the dilemma that researchers can only collect two-wave data, but still expect to estimate the growth effect under the longitudinal framework when the item-level information is available. Therefore, it is very important to know the estimation quality by adopting the two-wave models (LCFM and LCSM) given different settings.

In the supplementary study, we did not find any factors that have a substantially important impact on the variability of bias for β_{00} and β_{01}. This provided direct evidence that the fixed effect parameters related to the initial status could be equally estimated by both the two-wave and three-wave LCFM/SOLGM without any substantial bias. The corresponding results in our main study (study 1) further confirmed that this equivalent point estimate could also be observed between the two-wave LCSM and three-wave SOLGM in terms of the β_{00} and β_{01}. Therefore, from the above chain reasoning in terms of the equal bias: 3W-SOLGM $\longleftrightarrow$ 3W-LCFM $\longleftrightarrow$ 2W-LCFM / 2W-LCSM, we could finally conclude that the average initial status in the reference group (β_{00}) and their average group difference (β_{01}) in the 3W-SOLGM could be accurately recovered by the proposed two-wave models (LCFM and LCSM) in the current study.

As for the growth-related parameters, the findings seem a little more complicated. In both studies, the ANOVA tests did not detect any factors that had substantially large effect on the variability of the relative bias. Following the same reasoning process, as we did for the initial status, we could derive that there are no significant differences among these models in terms of the point estimates, and the parameters related to the growth rate estimated by the 3W-SOLGM could also be accurately recovered by the two-wave lCFM and LCSM. However, the box-plots for the distribution of relative bias in both studies (e.g. Fig.4.4 and Fig.4.5) indicated that the true model itself (3W-SOLGM) is able to provide biased estimates under certain scenarios. As shown from the tables 4.9-5.10, it is more likely to produce biased estimates for the average growth rate in the reference group (β_{01}) than the average group difference of the growth rate (β_{11}). Furthermore, the majority of the biased estimates occurred when the sample size is as small as 50 and the time-specific residual variance is large ($\theta_2^2 = 4$).

In order to further investigate the distribution of the biased estimates under different changing patterns, the information from tables 4.9 and 5.9 was reorganized, based on the changing patterns, into table 6.1 and 6.2. Both tables provide the summary statistics for the biased cases across their relative changing patterns. For example, in table 6.1, the first row starting with the “heavy shrinking” summarized (from the left and based on the absolute relative bias) the “number of biased estimates”, “minimum value of the relative bias”, “median of the relative bias”, “mean of the relative bias”, “maximum of the relative bias” and the “standard deviation of the relative bias” under the “heavy shrinking” pattern. In general, we could conclude that the “High Edge” pattern in both studies is actually the worst scenario in that the analytical models may fall into a higher risk of producing a larger bias for estimating the average growth rate for the reference group (β_{01}) since the total number of biased cases and the average size of the bias are larger compared to most other patterns.

In addition, it is worth noting that under each biased pattern, the alternative analytic models tend to have more biased cases than the true analytical model (3W-SOLGM) in both studies. An extreme case occurred that no biased cases were detected by analyzing with the true analytical model compared with the alternative models under the "Moderate Spreading" pattern. Finally, it is interesting to find that the major difference between table 6.1 and 6.2 is that the models with the same number of waves have exactly the same performance in terms of bias. To be more specific, the two alternative models in study 1 (2W-LCFM and 2W-LCSM) are exactly the same in all the descriptive statistics for the biased cases. Similarly, the 3W-LCFM and 3W-SOLGM models are also fully equal in their descriptive statistics. The reason for this equivalence in study 1 may come from the fact that 2W-LCMF and 2W-LCSM are equivalent models (Hershberger & Marcoulides, 2013, p. 11). By definition, when two equivalent models are used to fit the same dataset, they should finally yield the identical fit functions and goodness-of-fit indices (Hershberger & Marcoulides, 2013; Lai et al., 2016, p. 15). This explains why all average point estimates and the descriptive statistics for the goodness-of-fit indices between 2W-LCSM and 2W-LCFM are identical among tables 4.2 and 4.4. This same phenomenon could also be observed between 3W-LCFM and 3W-SOLGM, which implies that they are also equivalent.

We should note that it is not the focus of the current dissertation to approve whether the models are mathematically close or equivalent. The main purpose is to see whether the alternative models could provide the same point estimates as our true model. Based on the ANOVA tests in study 1 and 2, we, therefore, could conclude that the alternative models could on average recover the point estimates of true analytical model (3W-SOLGM) under all conditions even when the growth estimates given by the true analytical model (3W-SOLGM) was biased when the sample size is small (50) and time-specific residual variance (θ_2^2) is large (1 or 4). A previous simulation study by Leite (2007) demonstrated that the three-wave SOLGM was able to provide unbiased estimates under different conditions when the sample size was as small as 100. However, so far there is no empirical evidence to show how the 3W-SOLGM could performance when the sample size is as small as 50.

So far, what is the practical implication we could get from the current empirical results? The major implication of the current findings is that this empirical evidence could help justify the use of two-wave models to evaluate the long-term strength (T=3) of intervention effects. Although researchers should be certainly preferable to gather data at more than two-time points, it is not always feasible to do so. For example, some large-scale testing programs such as those conducted by the Northwest Evaluation Association are carried out in a biannual fashion. (Finch & Shim, 2018). Given this limitation, the proposed two-wave models still could successfully estimate the average longitudinal treatment effect designed for the three-wave of measurement in most of the conditions.

In addition, the finding that 3W-SOLGM itself could yield a biased estimate when the sample size is limited to around 50 and the time-specific residual variance is large should also bring enough cautions for the researchers who are interested in the application of the 3W-SOLGM. Although it is well known that obtaining more than three measures of outcome permit a more detailed understanding of individual differences in the growth trajectory, real-world constraints may often impose practical limitations on the number of waves the data could be collected. For example, in the clinical research study related to the saliva data among young children, applied researchers routinely adopted the pretest, posttest, follow-up (PPF) design (Rausch, Maxwell, & Kelly, 2003) to collect three (and only three) samples of cortisol (Willoughby et al., 2007). If researchers are really interested in applying the 3W-SOLGM to analyze such a dataset, it may be more advisable to make the sample size no less than 100 to avoid the risk of getting a biased estimates.

The other important points to be discussed in current study are how accurate or efficient when these alternative models are used to recover the fixed effect parameters of 3W-SOLGM and what factors are related to the precision of the estimation obtained by two-wave models. Before starting a formal discussion about research question two, we would like to first put our lens on the justification of the logic to compare different models (LCFM vs. SOLGM) in terms of their standard error. As the figures listed across B.13-B.45 in the supplementary study, the performance of both empirical standard error and theoretical standard error between the 3W-SOLGM and 3W-LCFM are exactly the same in every graph. This equality, therefore, justifies that we can directly compare the standard error of LCFM or LCSM with that of the SOLGM in study 1 since the 3W-LCFM and 3W-SOLGM are equivalent in terms of standard error.

Another important point to check, before targeting research question two, is to examine how accurate the theoretical standard error could reach for each model parameter. This information was provided by the results of $\frac{SE_T}{SE_E}$ in study 1. In general, the SE_T could underestimate the true SE (SE_E) by 13.87% and overestimate by 8% across parameters. However, as we mentioned in results 1, the criterion recommended by the Nevitt and Hancock (2001) would more favor the conclusion that the theoretical standard errors tend to accurately estimate the true stand error in majority of the conditions because neither their underestimation or overestimation rates are larger than 10%. With a further look at the table A.17-A.20, we can find that all the biased estimate for the true standard error (SE_E) occur when sample size is 50, and the parameter estimates are for β_{00} and β_{01}. More interestingly, they all occurred when the true model (3W-SOLGM) is applied. These findings should give gentle cautions again to researchers who try to apply the three-wave second-order latent growth model. When the sample size is as small as 50 and the time-specific is large enough, the model may not only generate biased estimates for the initial status, but also downplay its estimated standard error compared with two-wave models.

To answer part of the main research question two "How accurate or efficient when these alternative models are used to recover the fixed effect parameters of 3W-SOLGM", we need to turn back our discussion to the findings of the $Ratio - SE_{2E/3E}$ and $Ratio - SE_{2T/3T}$ in study 1. First, the comparison between the tables 4.12 and 4.13 may imply that the performance of the $Ratio - SE_{2T/3T}$ is highly similar to the performance of the $Ratio - SE_{2E/3E}$. The average values of both ratios are around 1 across each parameters. In addition, both the empirical and theoretical SEs in the two-wave models tend to overestimate their counterparts obtained by the three-wave model. The maximum overestimation rate could be as large as 30%. Except for this risk of precision loss by using alternative two-wave models, the findings from the ANOVA tests on both $Ratio - SE_{2T/3T}$ and $Ratio - SE_{2E/3E}$ also revealed that the substantively important sources of this precision loss for both standard errors are originated from two factors, i.e., the time-specific residual variance (θ_2^2) and changing pattern. Clearly shown by figures 4.10-4.17, the patterns starting with higher variability in the initial measurement have a relatively larger impact on both ratios of initial status compared to other patterns. Under these patterns, the average loss of precision for both ratios could be increased up to 23% as the θ_2^2 increase to 4. In contrast, the heterogeneity patterns, where the residual variance for the true latent construct shows differ in the first and third-time point, tends to have a larger impact on both ratios of growth parameters compared to other patterns. Given these patterns, the average loss of precision for both ratios could be increased up to 12% as the θ_2^2 increase to 4.

The empirical standard error estimates the long-run standard deviation of parameter estimates over repeated samplings, which is inaccessible in real data analysis since we only collect one dataset (Burton et al., 2006). Therefore, a more informative interpretation for the substantial researchers should be based on the ratio of theoretical standard error. As for the theoretical standard error, the overestimation rate by using the two-wave models to estimate the parameters of the three-wave model could range from 3% to 23% for the initial status (β_{00} and β_{01}) or from 1% to 13% for the growth rate parameters (β_{10} and β_{11}), respectively. Given that the theoretical standard errors of 3W-LCFM are equivalent to the 3W-SOLGM in the preliminary results, this overestimation is more likely attributed to the lack of the middle-wave (2nd wave) measurement for the two-wave models. In addition, it is very interesting to observe that the overestimation rate by two-wave models for the initial status (β_{00} and β_{01}) could be as twice as the overestimation rate by two-wave models for the growth parameters (β_{10} and β_{11}) when the time-specific variances are as large as 4. This may imply that compared with estimating the initial status parameters (β_{00} and β_{01}), the two-wave models are more robust to the loss of precision when estimating the growth related parameters (β_{10} and β_{11}).

As mentioned before, when the time-specific variance is very large (e.g. θ^2=4), the average maximum loss of precision could reach to 13% or 23% under certain patterns. In consequence, this loss would have a negative impact on the statistical power of detecting the true growth effects (Miyazaki et al., 2019). For example, as shown from figures A.88-A.99, the power loss of using the two-wave models compared with the three-wave model is consistently observed when the time-specific residual variance is large (e.g. 4). Moreover, this loss could be gradually shrunk as the sample size keep growing. Specifically, this shrinkage will not totally disappear until the sample size is close to 1000. This indicates when sample size is small and time-specific variance is large, the two-wave models may be correct on average, but tend to have smaller power to detect the group difference in the initial status and growth rate compared with the three-wave model. This result may not be surprising because SOLGM utilizes the data from the second wave, which gains more information than the two-wave models. One last notice should be given to the statistical power for detecting the initial status for the reference group (β_{00}) since its power level is always 1 regardless of conditions. This consistent high power should be mainly due to the setting for the initial status in the reference group (β_{00}). In the current study, the true value of β_{00} is set to be 10, which is relatively easier to be detected in any situations (Diallo, Morin, & Parker, 2014).

As for the substantial important factors influencing the empirical power, the findings in the current study show that the factors may vary across parameters. The common factor is the sample size N. That is, the power rate to detect the true parameter is related to sample size. The power rate increases with the increase in the sample size, controlling for other conditions. This finding is consistent with the results in other power analysis using the univariate LCM (Hertzog, Lindenberger, Ghisletta, & von Oertzen, 2006; Hertzog, von Oertzen, Ghisletta, & Lindenberger, 2008; Diallo et al., 2014), and as expected, also consistent with the statistical theory stating that statistical power in any context depends on sample size, effect size, and the chosen Type I error rate (Cohen, 1998). Another important factor is time-specific residual variance (θ_2^2). Expect for the average initial status and growth rate for the reference group (β_{00} and β_{10}), the time-specific variance is found to be related to the power rate among (β_{01} and β_{11}). On average, given the smaller sample size (e.g. 50), the statistical power will be lower than 75% when the θ_2^2 is closed to 4. As the θ_2^2 drops down to 0.25, the average power rate could be significantly boosted especially when the sample size is very small.

6.1 Implications for Practice

The randomized pretest, posttest, follow-up, (Rausch et al., 2003) or pre-post-post design (Willoughby et al., 2007) is a commonly used design for testing hypotheses about intervention effects in clinical child and adolescent research (e.g. cortisol data) or long-term strength of intervention effects. Of course, if applied researchers are able to collect three waves of data, it is more preferable to use second-order latent growth model (SOLGM) to evaluate the longitudinal intervention effect. However, the real world constraint often imposes practical limitation on collecting one of the post-test/follow-up data. For example, some large-scale testing programs such as these conducted by the Northwest Evaluation Association are carried out in a biannual fashion (Finch & Shim, 2018) or some clinical studies collecting the saliva samples among the young or infant children (Willoughby et al., 2007). With only two waves of data, the use of SOLGM is impossible because there are not sufficient degrees of freedom with which to fit the model (Finch & Shim, 2018).

The current study extended Miyazaki (Miyazaki, 2017)'s recent study that provided a promising solution for using the two-wave LFCM to recovery the fixed effects of three-wave SOLGM. When considering the implication of these results for practice, the researchers should take several considerations into account. First, with only two-wave data collected, researcher could still use LCFM or LCSM to estimate the fixed effects of the three-wave SOLGM. When researchers' original pre-post-post design was interrupted by the lack of sustained funding or maintained participants, they could still try to collect two waves of data to estimate the true growth effect under the pre-post-post design. Another important implication is for the situation when the data of a certain wave is missing in a nonrandom way. Although full information maximum likelihood estimation has been consistently suggested to deal with missing data in SEM (Enders & Bandalos, 2001; Raykov, 2005), this is only suitable for the cases when missing are random. However, one can sometimes run into cases where this assumption is not valid. For example, studies of infant cortisol response following immunization shots will invariably have some missing data due to children falling asleep as a regulation strategy (Willoughby et al., 2007). If the missing response rate is very high in the post-test (2nd-wave), the researcher could drop the middle wave and use the alternative two-wave models to estimate the average growth effects.

The second one is related to the three-wave SOLGM itself. The recent evidence accumulated from methodological research has consistently demonstrated the advantage of using SOLGM over the first-order LGM when multiple-scale items are available at each measurement time. In the social science field where the latent constructs are primary research interest, the application of SOLGM is very important because the measurement quality for the construct could be fully assessed. The findings in our study that the 3W-SOLGM may yield biased estimates for the growth parameters when the sample size is 50 and time-specific residual variance is large actually extends our knowledge about the property of 3-wave SOLGM. Especially for the substantial researchers who are interested in using the SOLGM with the pre-post-post/follow-up design, a warning should be flagged with the collection of a small sample size such as 50. Although the findings in study 1 indicated that sample size 50 does not necessarily give rise to the biased estimates because time-specific residual and changing patterns should be also considered simultaneously. But these two factors may not be easily sensible before the data is collected. So we would like to recommend applied researchers to collect a sample larger than 100 to ward off falling the risk of biased estimation.

Last but not the least, compared with other empirical studies, the most salient contribution of the current work is the emphasis on how important it is to consider the size and pattern of the time specific residual variance when SOLGM is implemented. According to the literature review, to date, rarely have any empirical studies of SOLGM discussed the structure and size of the time-specific residual variance and how they could influence the estimation bias. Thus, there is little empirical knowledge about to what extent the size and structure of the residual variance could have an impact on the estimation performance of the SOLGM. Let alone the knowledge about how the time-specific residual could influence the recovery of 3W-SOLGM by the two-wave models. In the second-order latent growth model, the measurement error has been removed from these composites by means of the first-order part of the model (Sayer & Cumsille, 2001). Thus θ_2^2 here represents random variance attributable to fluctuations unique to each time of measurement. In the substantial theory such as Latent State-Trait Theory (LST), the time-specific residual, also called occasion-specific residual is the portion of a latent state variable that does not rely on the person's trait, but on the effect of the situations (e.g. psycho-social and psycho-biological factors). The person's trait, within the LST, corresponds to the growth factors in the SOLGM and represents the person's characteristic independent of the situational effect (Steyer, Schmitt, & Eid, 1999; Kenny & Zautra, 2001). Therefore, people's trait should not be primary reasons for intraindividual state variability. Rather, intraindividual state variability will, in general, be a consequence of the fact that the person is in different situations on different occasions of measurement (Steyer et al., 1999).

In the substantial research, should the use of SOLGM be framed under the LST, the researcher needs clearly understand what the test really measures. This is because the traitlike construct and statelike normally differ as for the proportion of latent state residual. If the measurement is for the traitlike construct (e.g intelligence score), the proportion of variance of the latent state that is determined by the latent trait should be higher enough so the whole measurement is less dependent on the measurement occasion compared to the statelike construct (e.g depression), which constitutes a high level of occasion-specific residual variance. In latter case, the second-order latent growth should (LGM) be preferred than the first-order LGM because the LGM tend to underestimate the indicator's reliability (Geiser et al., 2013). From the theoretical perspective, the SOLGM is often preferred in measuring the psychological construct because it is more appropriate to conceive the psychological construct contains both trait and state component(Hertzog & Nesselroade, 1987).

Although the purpose of this study is not to show how well the SOLGM could align with the LST in real application (see Geiser et al. (2013) for the detailed discussion), the outline of the time-specific residual under the LST should be very beneficial for substantial researchers to understand to what extent and in what situations the two-wave longitudinal common factor model or two-wave latent change score model could recover the fixed effects of 3W-SOLGM. Based on the current results, the two-wave models could accurately estimate the fixed effects parameters of SOLGM in various conditions. However, the precision and statistical power of this estimation (standard error) by two-wave models could only be as comparable as the 3W-SOLGM when the time-specific residual is small and stable across time (constant pattern). As discussed in the previous paragraph, this is more likely to happen when the measurement is about the cognitive construct. For example the mathematical ability should be more stable across time (the relative ranking order on the construct being measured is highly similar across time (Kenny & Zautra, 2001) is, even it could change slowly across time. Even the sample size is as small as 50, the researchers should feel confident enough to use the alternative two-wave models if they are only interested in the fixed effects such as the gender difference or treatment group effect.

Things become more complicated when the interested construct is more statelike. Many studies showed that the psychological construct such as depressed mood or anxiety (Dumenci & Windle, 1996; Cole & Martin, 2005; Olatunji & Cole, 2009) has a substantial amount of occasion-specific variance. In other words, their corresponding score is more prone to fluctuate dependent on the measurement occasion. If we are interested in anxiety, for instance, one person may just have been told by the doctor that she/he needed surgery to remove cancerous skin tissue while another person may just have been offered a job which means the end of worries about unemployment. These two individuals will, at the occasion of measurement (time point), differ in their anxiety state (Geiser et al., 2013). In these cases, the analysis of changes should take more occasion-specific effects into account. With reference to our results, using two-wave models to estimate the fixed effect of 3W-SOLGM may be still correct on average, but will substantially lose the precision and statistical power when the size of the time-specific residual variance is ranged from medium to large and unequal or not constant over time (pattern). Therefore, with psychological constructs be measured, a forewarning should be given to those who want to take advantage of the 2W-LCFM or 2W-LCSM to recover the estimate of these fixed effect. Therefore, the best remedy for researchers who are interested in the change of the stateliked construct is to collect at least three-wave data. So they could properly estimate the measurement error, time-specific residual variance, and growth related variance, which are corresponding to the property of the measurement, property of the construct and property of the sample or population.

Table 6.1

Study 1: Summary of the Descriptive Statistics for Biased Cases with Relative Bias for β_{10}

Pattern	Model	N	Min	Median	Mean	Max	SD
Heavy Shrinking	LCFM	3	0.051	0.058	0.066	0.088	0.020
Heavy Shrinking	LCSM	3	0.051	0.058	0.066	0.088	0.020
Heavy Shrinking	SOLGM	1	0.082	0.082	0.082	0.082	NA
Heavy Spreading	LCFM	4	0.053	0.063	0.063	0.074	0.010
Heavy Spreading	LCSM	4	0.053	0.063	0.063	0.074	0.010
Heavy Spreading	SOLGM	1	0.063	0.063	0.063	0.063	NA
High Edge	LCFM	5	0.054	0.077	0.079	0.106	0.022
High Edge	LCSM	5	0.054	0.077	0.079	0.106	0.022
High Edge	SOLGM	6	0.053	0.069	0.075	0.112	0.024
Moderate Spreading	LCFM	1	0.056	0.056	0.056	0.056	NA
Moderate Spreading	LCSM	1	0.056	0.056	0.056	0.056	NA

Table 6.2

Supplementary Study: Summary of the Descriptive Statistics for Biased Cases with Relative Bias for β_{10}

Pattern	Model	N	Min	Median	Mean	Max	SD
Heavy Shrinking	LCFM	3	0.051	0.058	0.065	0.088	0.020
Heavy Shrinking	LCFM-3W	1	0.082	0.082	0.082	0.082	NA
Heavy Shrinking	SOLGM	1	0.082	0.082	0.082	0.082	NA
Heavy Spreading	LCFM	4	0.053	0.063	0.063	0.074	0.010
Heavy Spreading	LCFM-3W	1	0.063	0.063	0.063	0.063	NA
Heavy Spreading	SOLGM	1	0.063	0.063	0.063	0.063	NA
High Edge	LCFM	5	0.054	0.077	0.079	0.106	0.022
High Edge	LCFM-3W	7	0.050	0.057	0.072	0.112	0.024
High Edge	SOLGM	7	0.050	0.057	0.072	0.112	0.024
Moderate Spreading	LCFM	1	0.056	0.056	0.056	0.056	NA

6.2 Limitations and Future Orientations

First, it is a sad reality that 2nd-order LGM has been much less frequently used in the empirical studies compared with the 1st-order LGM. As the recent review by (Yang, Luo, & Zhang, 2020), among 300 studies using the LGMs as analytical method, only 11 papers adopted the 2nd-order LGMs. The major issue that discourages researchers from using this model may be because the requirement of the longitudinal invariance and lack of well understanding the identification and scaling for the latent variables in this type of model. For current study, we intentionally simulated all the test items as parallel measurement, where all the items intercept and loading parameters and error variance are equal across waves because they are not our research focus. However, in real application, it is more common that this strong assumption could not be met. Depending on where the invariance exist (e.g. intercept vs. factor loading) and the magnitude of this invariance, the bias may happen to different fixed parameters (Kim & Willson, 2014a) , and the the resulting growth parameter estimates may not be meaningful. Another issue that may be faced by substantial researchers is the identification and scaling for the latent variables in the 2nd-order LGM. In practise, there are mainly three identification methods for SEM analysis: the marker variable identification method, the effect-coding identification method, and the latent-standardization identification method. Yang et al. (2020)'s study shows that only the latent-standardization method could yield unique parameter estimates in the SOLGM. The other two methods will give varying parameters estimations dependent on the selection of the marker variables or how to constrain the loading and intercept. However, in the current study, by strong constraint, we only compared models based on the simple scenario where the measurement is totally parallel. In future researchers, it may be interesting to expand the current study by considering how measurement non-invariance and different identification methods could impact the current results.

Second, the measurement design of the current study is only to mimic the scenario where researchers may use several items to measure a single construct from a large survey(e.g. the NELS is a large survey containing a few items measuring different latent constructs, such as motivation and parental involvement). Therefore, only five observed variables were generated for each easement occasions. However, in other longitudinal studies and, especially for educational and psychological scales, it is more common for one factor to have 10 or 15 items (Mason, 2001; Li et al., 2001). With a large number of items per factor, it seems that the SOLGM needs to have a large sample size to compensate for the decrease of assessment of global fit under Leite (2007)'s study. The increase of the item number tends to increase the relative bias of covariance between initial status and growth factors, the chi-square test when sample size is small (e.g.N=100). In more general scenarios, the number of items per factor could affect the degrees of freedom, the power to detect model misspecification, and the number of iterations necessary for convergence. in the SEM studies (Leite, 2007). However, whether the number of items per factor could impact the parameter recovery by two-wave models may still deserve a further examination for the substantial researchers.

Finally, the setting of the time-specific residual variance in the current study is based on the proportion of the time-specific residual variance to the true score variance $\frac{Var[\theta_t^2]}{Var[\eta_{it}]}$. The size of large time-specific variance which is equal to 4 can make it account for 20% to 80% of total variance in the true latent score. According to the LST, the corresponding construct is more likely to be a statelike construct or even a measurement of a certain mood (e.g. depression mood). This construct may be also very rare in the real world. Geiser et al. (2013) demonstrated a similar statistic "occasion-specific (OS) coefficient" to reflection of the proportion of observed variable that is due to occasion- or time-specific effect: $\frac{\lambda_{it}^2 Var(SR_t)}{Var(Y_{it}}$. The only difference is just the denominator, where our justification criterion $\frac{Var[\theta_t^2]}{Var[\eta_{it}]}$ uses true score variance as the denominator. The main point here is the current settings for the occasion or time-specific residual may not be sufficient to capture all the real world situations. Different research field and studies may have their own unique consideration in terms of the size of this occasional effect.

Chapter 7

Illustration Examples

In this section, two different applied research datasets will be used to demonstrate the results of simulation studies. The first one was used to measure the students' cognitive ability while the second one intended to measure students' psychological construct.

7.1 Illustrative Example 1

This dataset is from the Early Childhood Longitudinal Study-Kindergarten Cohort (ECLS-K). The Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K) is a longitudinal study that followed the same children from kindergarten through the 8th grade. It focuses on children's early school experiences starting from kindergarten until middle school. The data provides descriptive information on children's status at entry to school, their transition into school, and their progression through 8th grade. Reading, Mathematics, and Science Tests were administered in 3rd, 5th, and 8th grades, and are treated as indicators of an academic achievement common factor. The achievement test in the ECLS-K was adaptive, and the scores used in the analysis are the estimated number of correct responses if the entire test was administrated (Grimm et al., 2016). The dataset can be accessed from url:

`https://www.guilford.com/companion-site/Growth-Modeling/9781462526062`

7.1.1 Descriptive information

Table 7.1 demonstrates the estimated statistics for each indicator variable/test (Science, Reading, and Math) across grades (3rd, 5th, and 8th) and their correlation matrix. The first two rows give the mean and standard deviation for each indicator variable/test across time. The mean performance of three tests (Science, Reading, and Math) actually is improved across the grade. For example, the mean sample score of the science test is 51.01 in the 3rd grade, 64.81 in the 5th grade, and 83.35 in the 8th grade. This indicates there is an average increasing trend for each test. The rest of the rows show the correlation between each indicator variable/test with the same grade and also across different grades. In general, all the sample correlations are relatively high. However, the correlations within the same grade tend to be stronger than the correlation across grades, except for the correlations involving the same test over grade (e.g. math in grade 3 and math in grade 5. $r = 0.69$)

Table 7.1

Descriptive Statistics of Interested Variables

	1	2	3	4	5	6	7	8	9
Mean	51.00	127.66	99.72	65.25	151.09	124.35	84.89	172.05	142.47
Variance	15.62	29.22	25.54	16.18	27.31	25.17	16.71	27.73	22.50
3th Science Grade	1								
3th Reading Grade	0.76	1							
3th Math Grade	0.71	0.75	1						
5th Science Grade	0.85	0.73	0.70	1					
5th Reading Grade	0.73	0.85	0.72	0.77	1				
5th Math Grade	0.69	0.70	0.88	0.74	0.75	1			
8th Science Grade	0.75	0.70	0.71	0.81	0.74	0.74	1		
8th Reading Grade	0.68	0.76	0.66	0.73	0.80	0.68	0.78	1	
8th Math Grade	0.66	0.69	0.81	0.70	0.71	0.86	0.79	0.75	1

7.1.2 Longitudinal Measurement Invariance

As discussed in the literature review section, the first step to use SOLGM is always to check the measurement invariance across time. By following Meredith's (1993) approach, the four consecutive models that differ in terms of the model constraint (configural invariance, week factorial invariance, strong factorial invariance and strict factorial invariance) will be fitted into the data. The results are shown in the following table 7.2. Starting from the 2nd column, which gives the different model fit statistics, we could conclude that the configural invariance model fit the data very well with TLI of 0.996, CFI of 0.998, and RMSEA of 0.03. This supports the notion to have a common factor to represent students' ability across grades. After the configural invariance, we fitted a week invariance model by constraining the loading parameters to be equal across grades. There is a significant jump for the χ^2 test from the configural invariance to the week invariacne ($\Delta\chi^2(4) = 81.31, p < 0.001$), which indicates the existence of non-invariance. However, it is consistently agreed by researchers Chen (2007) and Cheung and Rensvold (2002) that χ^2 difference test depends on the sample size. So having a large sample size will boost our power to detect even very small non-invariance, and we should not rely on it given the sample size is as large as 2108. According to Cheung and Rensvold (2002), the main goodness of fit statistics that shows good property in determining the non-invariance decision is the change of CFI (ΔCFI). As long as the $|\Delta CFI| \leq 0.01$, the invariance hypothesis should not be rejected. Therefore, the week invariance model should be maintained. However, from the week invarince model to the strict invariance model, the $|\Delta CFI|$ is always larger than 0.01. In addition, the RMSEA is always above 0.08. All these evidence indicates that even the strong invariance assumption could not be satisfied. This actually happens a lot in real world, which is one of the main reasons to obstruct the utility of SOLGM. Here, for the purpose of our final illustration, we will still treat the strict invariance assumption is met.

Table 7.2

Longitudinal Measurement Invariance Test

Fit Statistics	Configural Invariance	Week Invariance	Strong Invariance	Strict Invariance
AIC	83,914	83,987	84,403	84,451
BIC	84,120	84,172	84,567	84,584
χ^2(df)	35.52(15)	116.83(19)	540.75(23)	600.61(29)
RMSEA	0.030	0.059	0.123	0.115
TLI	0.996	0.984	0.930	0.939
CFI	0.998	0.992	0.955	0.951

As the strict invariance assumption is met in the current dataset, we then moved to examine the parameter estimates of this model and the overall development of the academic ability across time. Table 7.3 shows the unstandardized parameter estimates given by the strict invariance model. Their intercept and residual variance are set equal across time. So, the corresponding estimates in the table are all equal across time. The standardized estimates for the factor loadings indicated that the science test is relatively weighed more for representing the academic ability than the other two tests (indicators) because its factor loadings are always larger than the other two (e.g. 0.890 in grade 3) at any time. The mean of the latent variable, academic ability (η_t), was 1.02 in the 5th grade and 1.95 in the 8th grade. Since η_1 was standardized for scaling and identification, we could interpret the factor in a standardized metrics such as $\eta_1 \sim N(0, 1)$. Thus, from 3rd to 5th grade, the academic ability (latent variable) actually changed a little more than one standard deviation of the 3rd-grade distribution of academic ability. In contrast, the academic ability changed by a little less than one standard deviation from the 5th grade to 8th grade. Another important statistic is the variance and covariance ($Var[\eta_t]$ and $Cov[\eta_i, \eta_j]$) of the latent variable. The variance of the latent variable in 5th and 8th grade was 1.01 and 0.97, indicating that the magnitude of interindividual difference in academic ability did not change much from the third to eighth grades. The correlations between factors across time were also significant ($p < 0.01$), meaning that factors are strongly correlated across time.

Table 7.3

Parameter Estimate by Strict Invariance Model

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Factor loading					Mean				
η_1					η_1	0.000	0.000	999.000	999.000
Science_G3	15.176 (0.890)	0.319	47.527	0.000	η_2	1.020 (1.015)	0.025	41.016	0.000
Reading_G3	22.982 (0.847)	0.520	44.171	0.000	η_3	1.947 (1.976)	0.045	43.248	0.000
Math_G3	21.236 (0.850)	0.473	44.917	0.000	Intercept				
η_2					Science_G3	51.433	0.448	114.744	0.000
Science_G5	15.176 (0.891)	0.139	47.527	0.000	Reading_G3	126.363	0.702	179.898	0.000
Reading_G5	22.982 (0.848)	0.520	44.171	0.000	Math_G3	100.193	0.651	153.912	0.000
Math_G5	21.236 (0.851)	0.473	44.917	0.000	Science_G5	51.433	0.448	114.744	0.000
η_3					Reading_G5	126.363	0.702	179.898	0.000
Science_G8	15.176 (0.887)	0.139	47.527	0.000	Math_G5	100.193	0.651	153.912	0.000
Reading_G8	22.982 (0.843)	0.520	44.171	0.000	Science_G8	51.433	0.448	114.744	0.000
Math_G8	21.236 (0.846)	0.473	44.917	0.000	Reading_G8	126.363	0.702	179.898	0.000
Covariance					Math_G8	100.193	0.651	153.912	0.000
η_1 & η_2	0.968	0.013	72.280	0.000	Variance				
η_1 & η_3	0.904	0.019	48.240	0.000	η_1	1.000	0.000	999.000	999.000
η_2 & η_3	0.949	0.027	35.095	0.000	η_2	1.011	0.026	38.332	0.000
Science_G3 & Science_G5	28.853	3.118	9.254	0.000	η_3	0.971	0.036	27.168	0.000
Science_G3 & Science_G8	6.616	3.3031	2.004	0.045	Residual Variance				
Science_G5 & Science_G8	5.353	3.311	1.617	0.106	Science_G3	60.274	2.939	20.506	0.000
Reading_G3 & Reading_G5	112.069	8.673	12.921	0.000	Reading_G3	208.700	8.172	25.537	0.000
Reading_G3 & Reading_G8	69.528	9.345	7.440	0.000	Math_G3	173.322	7.236	23.953	0.000
Reading_G5 & Reading_G8	79.187	9.640	8.214	0.000	Science_G5	60.274	2.939	20.506	0.000
Math_G3 & Math_G5	112.591	7.308	15.047	0.000	Reading_G5	208.700	8.172	25.537	0.000
Math_G3 & Math_G8	93.469	7.990	11.698	0.000	Math_G5	173.322	7.236	23.953	0.000
Math_G5 & Math_G8	102.540	8.014	12.795	0.000	Science_G8	60.274	2.939	20.506	0.000
					Reading_G8	208.700	8.172	25.537	0.000
					Math_G8	173.322	7.236	23.952	0.000

* Note: value in the () is the standardized estimates

7.1.3 Three-Wave Model vs. Two-wave Models

After the description of the measurement scale, we fitted a three-wave second-order latent growth model, two-wave longitudinal common factor model, two-wave latent change score model, and three-wave longitudinal common factor model to the data. The purpose of this section is to demonstrate how to use the two-wave models to recover the fixed effect of the three-wave model when substantial data was collected. As for factor loadings, we followed the Grimm et al.'s (2016) method by fixing the factor loadings to the science variables to 15.176. As shown in the table 7.4, starting from the left, we consecutively fitted 3W-SOLGM, 2W-LCFM, and 2W-LCSM with participants' gender as the between person's covariate. The first four rows indicate the estimates for our fixed effect parameters in each model. For the identification and interpretation purpose, the (β_{00}) was fixed at 51 in each model. β_{00} represents the average academic ability score for the female students in the 3rd grade and 51 actually is their estimated mean score. β_{01} represents the average group difference of the academic ability between male and female students. Its value is -0.228, which indicates that on average female students are 0.228 points higher than their male counterparts in the academic ability scale. If we use 2W-LCFM or 2W-LCSM to estimate β_{01}, the corresponding values are -0.235 or -0.237. So the difference between the estimates from the two-wave models and the three-wave model is just around 0.009, which is only 3% off from -0.228. These results are consistent with the conclusions in study 1, which shows that on average there is no bias for point estimate. Similar results could be observed for β_{10}, which is the average growth rate of the academic ability among female students. In 3W-SOLGM, this estimate is 1.94, meaning that among female students, moving up one grade will on average give rise to a 1.94 point increase in their academic ability scale. The corresponding estimates from two-wave models is 1.933 or 1.986, which has less than a 3% deviation from the 1.94. The most deviated estimates occur at the β_{11}, which is the average group difference between female and male students in terms of the growth rate. Two-wave models' estimates (0.093 and 0.098) could have 36% off from the three-wave model. Finally, the tree-wave LCFM appears to have the lowest fit to the data since all the GFIs (goodness of fit indices) such as TLI and CFL are much smaller than the two-wave models (table 7.5). All the other point estimates for fixed effect parameters using 3W-LCFM are also far off the point estimates by 3W-SOGLM compared with the two-wave models. For example, for the group difference in the initial status, the estimated β_{01} is -0.270, which is 15.56% deviated off from the point estimate of 3W-SOLGM. These results are not surprising because of the fact that the measurement invariance does not hold. So from the findings in other simulation studies, the relative point estimate will definitely be impacted by this non-invariance (Kim & Willson, 2014a, 2014b). We expected that once the measurement invariance truly holds, all the point estimates of the two-wave models should approach the point estimate of the the-wave models.

Looking across models, we could also find that the theoretical standard error of the fixed effect estimates in two-wave models is always very closed to their standard error in the three-wave models. In fact, this difference is only about 1%, where the two-wave models tend to be a little higher. This result is consistent with our simulation results as well. In our simulation part, we already demonstrated that when the sample size is as large as 1000, the loss of accuracy will tend to be minimized when using the two-wave models to recover the estimates of the three-wave model. Given the current sample size is much over 1000, this performance is easily observed. Given the time-specific residual variance (θ_t^2), the current data seems to mimic the middle-level edge pattern because the time-specific residual variance of the academic ability for students in 3th grade (θ_1^2)and 8th grade (θ_3^2) is as 2.5 times as they are in the 5th grade ($\theta_1^2 = 0.045$, $\theta_2^2 = 0.018$,$\theta_3^2 = 0.043$). However, the largest residual variance here only accounts for 4.5% of the total true score variance. In other words, more than 95% variability in academic ability is captured by its growth trajectory. This is sufficient to state that academic ability is still a stable construct. Finally, it is not surprising to see that except for CFI, all the other fit indices are below the accepted level in any model. This is so different from our simulation results. The main reason is that we constrain that all the items are parallel measurement. This is a very strict constraint, which rarely happens in the reality. If the current measurement could reach the strict invariance, we would anticipate seeing much improvement in the goodness of fit indices.

Table 7.4

Model Comparison for the Parameter Estimate

	Model: 3W-SOLGM			Model: 2W-LCFM			Model: 2W-LCSM		
Fixed Effect Parameters	Estimates (std)	S.E (std)	Two-Tailed P-Value (std)	Estimates (std)	S.E (std)	Two-Tailed P-Value (std)	Estimates (std)	S.E (std)	Two-Tailed P-Value (std)
β_{00}	51			51			51		
β_{01}	-0.228 (-0.234)	0.055 (0.056)	0.000	-0.235 (-0.116)	0.056 (0.055)	0.00	-0.237 (-0.119)	0.055 (0.028)	0.000
β_{10}	1.940 (7.632)	0.028 (1.697)	0.000	1.993	0.030	0.000	1.986 (5.100)	0.029 (0.267)	0.00
β_{11}	0.072 (0.282)	0.034 (0.144)	0.036	0.093	0.037	0.012	0.098 (0.126)	0.036 (0.046)	0.007
Growth Factor Variance Covariance									
ψ_{00}	0.937 (0.986)	0.039 (0.007)	0.000						
ψ_{11}	0.063 (0.980)	0.029 (0.02)	0.026						
ψ_{01}	0								
Time-Specific Residual Variance									
θ_1^2	0.045 (0.045)	0.013 (0.013)	0.001	1.010 (0.987)	0.042 (0.006)	0.000	0.978 (0.986)	0.041(0.007)	0.000
θ_2^2	0.018 (0.019)	0.008 (0.008)	0.024				0.149^a (0.984^a)	0.015 (0.012)	0.000
$\theta 2_3$	0.043 (0.041)	0.017 (0.017)	0.014	0.998 (0.995)	0.047 (0.004)	0.000	0		
Model Fit Indices									
$\chi^2(df)$	840.280 (38)		0.000	492.237 (16)		0.000	514.342 (17)		0.000
TLI	0.920			0.899			0.901		
CFI	0.932			0.923			0.920		
RMSEA (95% CI)	0.120 (0.113, 0.127)		0.000	0.142(0.131, 0.153)			0.141 (0.130, 0.151)		0.000
SRMR	0.111			0.134			0.160		

* Note: (std) indicates the value in the parenthesis is the standardized estimates.

Table 7.5

Three-Wave Longitudinal Common Factor Model

Model: 3W-LCFM			
Fixed Effect Parameters	Estimate (std)	S.E. (std)	Two-Tailed P-Value (std)
β_{00}	51		
β_{01}	-0.270 (-0.269)	0.044 (0.043)	0.000
β_{10}	0.1869	0.037	0.000
β_{11}	0.148	0.047	0.001
Growth Factor Variance Covariance			
ψ_{00}			
ψ_{11}			
ψ_{01}			
Time Specific Residual Invariance			
θ_1^2	0.992 (0.082)	0.043 (0.006)	0.000
θ_2^2	1.007 (0.991)	0.048 (0.004)	0.000
θ_3^2	0.750 (0.995)	0.050 (0.004)	0.000
Model Fit Indices			
$\chi^2(df)$	3791.302 (38)		
TLI	0.625		
CFI	0.683		
RMSEA(95% CI)	0.259 (0.252 0.266)		
SRMR	0.396		

* Note: (std) indicates the value in the parenthesis is the standardized estimates.

7.2 Illustrative Example 2

The data illustrated by this example comes from the Geiser's (2012) textbook. This dataset is based on the self-reported anxiety of children, which is measured on four (equally spaced) time points T1–T4. There is a time interval of approximately 6 months between the measurement occasions. For each measurement occasion, there are two items to measure the construct of anxiety. So there are in total 8 variables in this dataset. In order to fit our illustration purpose, we only used the first three waves as our analytic dataset.

Table 7.6 shows the estimated statistics for each indicator across time and their correlation coefficient. Each indicator was symbolised by a_{ij}, where the first subscript i ($i = 1, 2$) indicates the test item and the second subscript j ($j = 1, 2, 3$) indicates the measurement occasion. The first and the second row of the table show the mean and standard deviation of each indicator. For example, the mean of the first indicator in the first measurement occasion a_{11} is 0.72 and its standard deviation is 0.446. Similarly, the second indicator in the first measurement occasion a_{21} has a mean score of 0.736 and a standard deviation of 0.441. In general, we could observe that a declining mean score. This implied that the children's anxiety level is going down across the time in the current dataset. In contrast, the standard deviation is ranged from 0.431 to 0.446 across indicators. So the variability of indicators is small in this dataset. Finally, the indicator correlations given the same measurement occasion are all above 0.86, which is much larger than the correlation across time. This indicates a common underlying construct is measured by the anxiety scale.

Table 7.6

Descriptive Statistics of Interested Variables

	1	2	3	4	5	6
Standard Deviation	0.720	0.736	0.614	0.630	0.563	0.587
Variance	0.446	0.441	0.449	0.443	0.431	0.444
a_{11}	1					
a_{21}	0.863	1				
a_{12}	0.695	0.662	1			
a_{22}	0.644	0.657	0.870	1		
a_{13}	0.624	0.590	0.756	0.701	1	
a_{23}	0.597	0.609	0.734	0.739	0.878	1

7.2.1 longitudinal Measurement Invariance

Table 7.7 demonstrates the step-by-step measurement invariance test following the Meredith's (1993) approach. First, the configural invariance model fits the data very well since all the GFIs values are under their cut-off values: $CFI = 1.003$, $TLI = 1.000$, $RMSEA = 0.00$ and $\chi^2 = 4.062$ ($df = 5$, $p = 0.841$). Next, the factor loadings of the configural invariance model were fixed across time to obtain the week invariance model. Compared with configural invariance model, the χ^2 was increased to 7.580. The relative χ^2 difference test ($\Delta\chi^2 = 5.518, \Delta df = 2, p > 0.05$) is not statistically significant. RMSEA is 0.013 , which is less 0.08. Both TLI and CFI stay the same as they are in the configural invariance model. So we could conclude that the week invariance model fits the data as well as the configural invariance model. Then the measurement intercepts were fixed to be equal across time to form the strong invariance model. Similarly, the χ^2 difference test is still not significant ($\Delta\chi^2 = 0.205, \Delta df = 2, p > 0.05$). RMSEA, TLI, and CFI almost stay unchanged compared to the week invariance model. So the strong invariance model should be maintained. Finally, following the same logic, we could conclude that the strick invariance model fits the data as well as the strong invariance model ($\Delta\chi^2 = 3.09, \Delta df = 4, RMESA = 0.00, TLI = 1.001$ and $CFI = 1.000$). In all, the assumption of longitudinal measurement invariance is met for the current dataset.

Table 7.7

Test of Longitudinal Measurement Invariance

Fit Statistics	Configural Invariance	Week Invariance	Strong Invariance	Strick Invariance
AIC	604.647	606.165	602.370	597.460
BIC	696.652	689.806	677.647	656.009
χ^2 (df)	2.062(5)	7.580(7)	7.785(9)	10.875(13)
RMSEA	0.000	0.013	0.000	0.000
TLI	1.000	1.000	1.001	1.001
CFI	1.003	1.000	1.001	1.000

7.2.2 Three-Wave Model vs. Two-wave Models

After the measurement invariance assumption is met, we fitted the same data with 3-wave SOLGM, 2-Wave LCSM, 2-Wave LCFM, and 3-wave LCFM. The corresponding comparison based on the estimates of the fixed effect parameters and the goodness of fit indices is given by the table 7.8 and 7.9. Since the current dataset does not contain covariate variables, we only can show the initial status (β_{00}) and overall growth rate (β_{10}) without referring to any single group. The far left panel in the table 7.8 shows the estimate for the growth effects of the three-wave model (true model). The average initial status is 0.712 with a standard error of 0.02. The values in the parenthesis are their corresponding standardized coefficients. The average growth rate is -0.151, which is statistically significant ($p < 0.05$) from 0. The substantial meaning of this pattern is: The average anxiety level of these children tends to decrease with time. To be more specific, every 6 months, the average anxiety level of these children decreases by 0.151 points in terms of the anxiety scale. Furthermore, as shown in the covariance/variance section, both variances of initial status (ψ_{00}=0.152) and growth rate (ψ_{11}=0.083) are statistically significant. This indicates that the interindividual difference of people's growth trajectory does exist.

The covariance between initial status and growth rate (ψ_{01}=-0.031) is also negatively and significantly related. This means the people with higher anxiety levels at the starting point tend to have a relatively lower decreasing trend compared with the person whose initial anxiety level is lower. The time-specific residuals are 0.029, 0.042, and 0.002 across the first three measurements. In fact, the residual variance approach 0 at the 3rd time point. This indicates that people with higher anxiety levels or lower anxiety levels will eventually get more converged at the third measurement occasion. It is interesting to see the occasion-specific variance at the 2nd measurement occasion suddenly increased compared with the first measurement occasion, and then quickly reduced to a zero level. This pattern is different from any of our simulation patterns. The possible reasons may be some unexpected event happen just before the 2nd time measurement, and people's relative anxiety become more diluted, with some people become more anxious than others. Without this event, a more expected changing pattern could be a shrinking trend. Finally, we could see that all the GIFs indicate the model fit the data very well.

Next to the left panel was the 2-wave latent change score model. We could see the estimated average initial status β_{10} is 0.669, which is very close to 0.712. The deviation between the two models is only 0.043, which 0.6% off from the true analytical model. The average growth rate estimated by 2W-LCSM is almost the same as the 3W-SOLGM (-0.151 vs. -0.154). Compared to the former illustrative example, this better recovery should be mostly attributed to the existence of the measurement invariance. These results are consistent with our simulation findings even the changing pattern is not captured by our simulation conditions. As for the standard error, we could see that the 2W-LCSM tends to overestimate it. However, the size of this estimate is so small to be noticed. This is also demonstrated by our simulation result, where the discrepancy of standard error will approach zero as the sample size become large. Finally, the majority of the GFIs statistics show our 2W-LCSM has a least an acceptable fit.

In the far right panel of table 7.8, it is no surprising to see that 2W-LCFM exactly recovers the estimate of growth rate given by the 3W-SOLGM. The standard error for both estimates is almost the same as the 3-wave model. The GIF also confirms that this model could fit the data as well as other models. It may be not advisable to make the conclusion that 2W-LCFM is a little better than the 2W-LCSM just based on a single sample here. But at least, by using this measurement invariance data, we could partially confirm our simulation results. That is, given no measurement issue, we could just use two-wave data to estimate the growth rate obtained from the three-wave data. Finally, from the table 7.9, the average initial status is estimated as 0.720, which is far more deviated from the estimates by 3W-SOLGM compared with the two-wave models. However, this deviation is about only 1.11%. Similarly, the average growth rate estimated by 3W-LCFM is -0.155, which is 2.58% deviated from the estimated by 3W-SOLGM. All the GFIs become a little worse compared with the two-wave models. But they are still in the acceptable range.

Table 7.8

Model Comparison for the Parameter Estimate

	Model: 3W-SOLGM			Model: 2W-LSCM			Model: 2W-LCFM		
Fixed Effect	Estimates (std)	S.E. (std)	Two-Tailed P-Value (std)	Estimates (std)	S.E. (std)	Two-Tailed P-Value (std)	Estimates (std)	S.E.(std)	Two-Tailed P-Value (std)
Parameters									
β_{00}	0.712 (1.827)	0.020 (0.096)	0	0.699 (1.742)	0.022 (0.087)	0	0.718 (1.758)	0.02 (0.082)	0
β_{01}									
β_{10}	-0.151 (-0.523)	0.017 (0.086)	0	-0.154 (-1.175)	0.016 (0.220)	0	-0.151	0.016	0
β_{11}									
Growth Variance / Covariance									
ψ_{00}	0.152	0.014	0						
ψ_{11}	0.083	0.02	0						
ψ_{01}	-0.031 (-0.274)	0.012	0.009 (0)						
Time-Specific Residual Variance									
θ_1^2	0.029 (0.160)	0.01(0.055)	0.005(0.004)	0.161	0.014	0.002	0.167		0
θ_2^2	0.042 (0.227)	0.006 (0.025)	0						
θ_3^2	0.002 (0.009)	0.01 (0.057)	0.878 (0.878)	0.084	0.007	0	0.163		0
Model Fit Indices									
χ^2 (df)	17.822 (14)				7.646 (2)		5.407 (4)		
TLI	0.999				0.99		0.999		
CFI	0.999				0.997		0.999		
RMSEA	0.024				0.076		0.027		
(95% CI)	(0.000, 0.053)				(0.025, 0.137)		(0.000, 0.078)		
SRMR	0.02				0.055		0.013		

* Note: (std) indicates the value in the parenthesis is the standardized estimates.

Table 7.9

Fixed Effect Estimated by the Three-Wave Longitudinal Common Factor Model

Model: 3W-LCFM			
Fixed Effect Parameters	Estimate (std)	S.E. (std)	Two-Tailed P-Value (std)
β_{00}	0.720 (1.722)	0.02 (0.077)	0.000
β_{01}			
β_{10}	-0.155	0.016	0.000
β_{11}			
Growth Factor Variance Covariance			
ψ_{00}			
ψ_{11}			
ψ_{01}			
Time Specific Residual Invariance			
θ_1^2	0.175	0.012	0.000
θ_2^2	0.178	0.013	0.000
θ_3^2	0.170	0.021	0.000
Model Fit Indices			
$\chi^2(df)$	59.326 (14)		
TLI	0.983		
CFI	0.985		
RMSEA(95% CI)	0.082 (0.061, 0.104)		
SRMR	0.015		

* Note: (std) indicates the value in the parenthesis is the standardized estimates.
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Appendix A

Results of Study 1

A.0.1 List of Tables for Relative Bias

Table A.1

ANOVA Test Results for Relative Bias of β_{00}

Source	**DF**	**Sum of Squares**	η^2
$model$	2.000	0.000	0.000
N	3.000	0.000	0.014
θ_2^2	2.000	0.000	0.004
$pattern$	6.000	0.000	0.012
β_{01}	1.000	0.000	0.000
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.002
$model * N$	6.000	0.000	0.000
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.000	0.001
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.000	0.002
$N * pattern$	18.000	0.000	0.030

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Table A.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \beta_{01}$	3.000	0.000	0.000
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.000	0.004
$\theta_2^2 * pattern$	12.000	0.000	0.008
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.001
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.001
$\theta_2^2 * \psi_{01}$	4.000	0.000	0.001
$pattern * \beta_{01}$	6.000	0.000	0.004
$pattern * \beta_{11}$	6.000	0.000	0.006
$pattern * \psi_{01}$	12.000	0.000	0.004
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.001
$\beta_{11} * \psi_{01}$	2.000	0.000	0.003
$model * N * \theta_2^2$	12.000	0.000	0.000
$model * N * pattern$	36.000	0.000	0.001
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.000	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000

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Table A.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern$	36.000	0.000	0.039
$N * \theta_2^2 * \beta_{01}$	6.000	0.000	0.013
$N * \theta_2^2 * \beta_{11}$	6.000	0.000	0.004
$N * \theta_2^2 * \psi_{01}$	12.000	0.000	0.004
$N * pattern * \beta_{01}$	18.000	0.000	0.014
$N * pattern * \beta_{11}$	18.000	0.000	0.012
$N * pattern * \psi_{01}$	36.000	0.000	0.034
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.001
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.002
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.000	0.009
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.000	0.009
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.000	0.035
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.002
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.000	0.006
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.006

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Table A.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$pattern * \beta_{01} * \beta_{11}$	6.000	0.000	0.004
$pattern * \beta_{01} * \psi_{01}$	12.000	0.000	0.016
$pattern * \beta_{11} * \psi_{01}$	12.000	0.000	0.013
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.004
$model * N * \theta_2^2 * pattern$	72.000	0.000	0.003
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.001
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.001
$model * N * pattern * \beta_{11}$	36.000	0.000	0.001
$model * N * pattern * \psi_{01}$	72.000	0.000	0.001
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.001
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.001

Continued on next page

Table A.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.000	0.044
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.000	0.025
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.000	0.073
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.000	0.017
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.000	0.005
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.000	0.018
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.000	0.013
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.000	0.023
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.000	0.037
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.000	0.012
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.011
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.013
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.035
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.002
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.007
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.001
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.000	0.005
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000

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Table A.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.001
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.002
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.001
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.001
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.042
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.071
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.055
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.005
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.000	0.027
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.020
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.000	0.003
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.000	0.003
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.002
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.001

Continued on next page

Table A.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.071
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.000	0.003

Table A.2

ANOVA Test Results for Relative Bias of β_{01}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.000	0.000
N	3.000	0.001	0.007
θ_2^2	2.000	0.000	0.000
$pattern$	6.000	0.002	0.018
β_{01}	1.000	0.000	0.000
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.005
$model * N$	6.000	0.000	0.001
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.000	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.000	0.001
$N * pattern$	18.000	0.003	0.036
$N * \beta_{01}$	3.000	0.000	0.000
$N * \beta_{11}$	3.000	0.000	0.000

Continued on next page

Table A.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \psi_{01}$	6.000	0.001	0.007
$\theta_2^2 * pattern$	12.000	0.001	0.016
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.004
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.002
$\theta_2^2 * \psi_{01}$	4.000	0.000	0.002
$pattern * \beta_{01}$	6.000	0.000	0.003
$pattern * \beta_{11}$	6.000	0.001	0.009
$pattern * \psi_{01}$	12.000	0.001	0.017
$\beta_{01} * \beta_{11}$	1.000	0.000	0.002
$\beta_{01} * \psi_{01}$	2.000	0.000	0.002
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.000	0.000
$model * N * pattern$	36.000	0.000	0.001
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.000	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000

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Table A.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern$	36.000	0.004	0.046
$N * \theta_2^2 * \beta_{01}$	6.000	0.001	0.014
$N * \theta_2^2 * \beta_{11}$	6.000	0.001	0.008
$N * \theta_2^2 * \psi_{01}$	12.000	0.000	0.006
$N * pattern * \beta_{01}$	18.000	0.000	0.005
$N * pattern * \beta_{11}$	18.000	0.001	0.017
$N * pattern * \psi_{01}$	36.000	0.003	0.037
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.005
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.003
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.003
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.000	0.005
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.001	0.013
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.002	0.026
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.000	0.005
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.001	0.007
$pattern * \beta_{01} * \beta_{11}$	6.000	0.000	0.004
$pattern * \beta_{01} * \psi_{01}$	12.000	0.001	0.006

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Table A.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$pattern * \beta_{11} * \psi_{01}$	12.000	0.001	0.014
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.005
$model * N * \theta_2^2 * pattern$	72.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.001
$model * N * pattern * \psi_{01}$	72.000	0.000	0.001
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.001
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table A.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.002	0.020
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.002	0.025
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.005	0.059
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.008
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.001	0.007
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.001	0.017
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.001	0.015
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.002	0.021
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.003	0.040
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.001	0.018
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.001	0.009
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.001	0.016
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.002	0.026
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.002
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.008
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.000	0.002
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.001

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Table A.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.002
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.001
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.003	0.038
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.005	0.058
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.005	0.061
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.007
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.003	0.031
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.002	0.025
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.001
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.000	0.002
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.001
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.008	0.094
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.000	0.002

Table A.3

ANOVA Test Results for Relative Bias of β_{10}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.000	0.000
N	3.000	0.004	0.006
θ_2^2	2.000	0.003	0.004
$pattern$	6.000	0.003	0.005
β_{01}	1.000	0.003	0.005
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.001	0.002
$model * N$	6.000	0.000	0.000
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.000	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.005	0.008
$N * pattern$	18.000	0.011	0.017
$N * \beta_{01}$	3.000	0.005	0.008
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.002	0.003
$\theta_2^2 * pattern$	12.000	0.007	0.011
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.001	0.001

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Table A.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.003	0.005
$pattern * \beta_{01}$	6.000	0.004	0.006
$pattern * \beta_{11}$	6.000	0.004	0.006
$pattern * \psi_{01}$	12.000	0.007	0.011
$\beta_{01} * \beta_{11}$	1.000	0.001	0.002
$\beta_{01} * \psi_{01}$	2.000	0.004	0.006
$\beta_{11} * \psi_{01}$	2.000	0.007	0.012
$model * N * \theta_2^2$	12.000	0.000	0.000
$model * N * pattern$	36.000	0.000	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.000	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table A.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	0.019	0.031
$N * \theta_2^2 * \beta_{01}$	6.000	0.002	0.004
$N * \theta_2^2 * \beta_{11}$	6.000	0.008	0.012
$N * \theta_2^2 * \psi_{01}$	12.000	0.007	0.012
$N * pattern * \beta_{01}$	18.000	0.018	0.030
$N * pattern * \beta_{11}$	18.000	0.015	0.024
$N * pattern * \psi_{01}$	36.000	0.019	0.030
$N * \beta_{01} * \beta_{11}$	3.000	0.004	0.006
$N * \beta_{01} * \psi_{01}$	6.000	0.006	0.010
$N * \beta_{11} * \psi_{01}$	6.000	0.008	0.012
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.009	0.015
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.006	0.009
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.007	0.011
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.003	0.004
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.002	0.004
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.005	0.008
$pattern * \beta_{01} * \beta_{11}$	6.000	0.004	0.006
$pattern * \beta_{01} * \psi_{01}$	12.000	0.010	0.017
$pattern * \beta_{11} * \psi_{01}$	12.000	0.014	0.022
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table A.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.045	0.072
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.021	0.034
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.025	0.040
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.012	0.020

Continued on next page

Table A.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.005	0.007
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.007	0.011
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.010	0.016
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.023	0.036
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.023	0.036
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.002	0.003
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.008	0.013
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.011	0.018
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.017	0.027
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.001	0.001
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.003	0.005
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.001	0.001
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

Continued on next page

Table A.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.028	0.045
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.035	0.056
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.028	0.044
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.003	0.005
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.025	0.039
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.009	0.015
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.001	0.002
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.001	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.032	0.051
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.001

Table A.4

ANOVA Test Results for Relative Bias of β_{11}

Source	**DF**	**Sum of Squares**	η^2
$model$	2.000	0.000	0.000
N	3.000	0.000	0.002
θ_2^2	2.000	0.001	0.004
$pattern$	6.000	0.001	0.006
β_{01}	1.000	0.000	0.002
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.002	0.007
$model * N$	6.000	0.000	0.000
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.000	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.001	0.005
$N * pattern$	18.000	0.007	0.032
$N * \beta_{01}$	3.000	0.001	0.004
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.002	0.010
$\theta_2^2 * pattern$	12.000	0.005	0.024
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.000

Continued on next page

Table A.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.002	0.008
$pattern * \beta_{01}$	6.000	0.002	0.007
$pattern * \beta_{11}$	6.000	0.001	0.004
$pattern * \psi_{01}$	12.000	0.003	0.016
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.001	0.003
$\beta_{11} * \psi_{01}$	2.000	0.001	0.003
$model * N * \theta_2^2$	12.000	0.000	0.000
$model * N * pattern$	36.000	0.000	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.000	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

Continued on next page

Table A.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	0.011	0.053
$N * \theta_2^2 * \beta_{01}$	6.000	0.001	0.006
$N * \theta_2^2 * \beta_{11}$	6.000	0.002	0.008
$N * \theta_2^2 * \psi_{01}$	12.000	0.004	0.017
$N * pattern * \beta_{01}$	18.000	0.005	0.023
$N * pattern * \beta_{11}$	18.000	0.004	0.018
$N * pattern * \psi_{01}$	36.000	0.010	0.045
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.001
$N * \beta_{01} * \psi_{01}$	6.000	0.002	0.010
$N * \beta_{11} * \psi_{01}$	6.000	0.001	0.006
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.001	0.002
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.001	0.003
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.002	0.009
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.001
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.001	0.003
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.002
$pattern * \beta_{01} * \beta_{11}$	6.000	0.001	0.007
$pattern * \beta_{01} * \psi_{01}$	12.000	0.002	0.009
$pattern * \beta_{11} * \psi_{01}$	12.000	0.003	0.014
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

Continued on next page

Table A.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.000	0.001
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.007	0.033
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.004	0.017
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.013	0.062
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.004

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Table A.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.002	0.008
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.004	0.017
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.003	0.015
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.006	0.028
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.008	0.039
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.000	0.002
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.002	0.008
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.005	0.022
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.006	0.027
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.001	0.004
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.009
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.001
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.001
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.001
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.001
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table A.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.008	0.039
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.014	0.067
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.013	0.062
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.011
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.008	0.040
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.004	0.019
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.001	0.003
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.015	0.070
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.000	0.001

A.0.2 List of Tables for Standard Error

Table A.5

ANOVA Test Results for Empirical Standard Error of β_{00}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.203	0.002
N	3.000	101.694	0.878
θ_2^2	2.000	3.910	0.034
$pattern$	6.000	3.427	0.030
β_{01}	1.000	0.054	0.000
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.000
$model * N$	6.000	0.058	0.000
$model * \theta_2^2$	4.000	0.054	0.000
$model * pattern$	12.000	0.104	0.001
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	1.451	0.013
$N * pattern$	18.000	1.312	0.011
$N * \beta_{01}$	3.000	0.018	0.000
$N * \beta_{11}$	3.000	0.001	0.000
$N * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern$	12.000	2.353	0.020
$\theta_2^2 * \beta_{01}$	2.000	0.004	0.000

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Table A.5 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01}$	6.000	0.004	0.000
$pattern * \beta_{11}$	6.000	0.000	0.000
$pattern * \psi_{01}$	12.000	0.001	0.000
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.020	0.000
$model * N * pattern$	36.000	0.035	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.079	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000

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Table A.5 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern$	36.000	0.895	0.008
$N * \theta_2^2 * \beta_{01}$	6.000	0.003	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.002	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.003	0.000
$N * pattern * \beta_{01}$	18.000	0.004	0.000
$N * pattern * \beta_{11}$	18.000	0.002	0.000
$N * pattern * \psi_{01}$	36.000	0.004	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.001	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.002	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.002	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.001	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.002	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.003	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.027	0.000

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Table A.5 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.005	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.004	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.013	0.000

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Table A.5 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.002	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.006	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.007	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.007	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.005	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.004	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.001	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000

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Table A.5 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.006	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.013	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.015	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.008	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.005	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.012	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.000

Table A.6

ANOVA Test Results for Empirical Standard Error of β_{01}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.288	0.001
N	3.000	175.230	0.848
θ_2^2	2.000	8.648	0.042
$pattern$	6.000	7.408	0.036
β_{01}	1.000	0.523	0.003
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.000
$model * N$	6.000	0.070	0.000
$model * \theta_2^2$	4.000	0.141	0.001
$model * pattern$	12.000	0.252	0.001
$model * \beta_{01}$	2.000	0.002	0.000
$model * \beta_{11}$	2.000	0.001	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	3.275	0.016
$N * pattern$	18.000	2.705	0.013
$N * \beta_{01}$	3.000	0.181	0.001
$N * \beta_{11}$	3.000	0.002	0.000
$N * \psi_{01}$	6.000	0.003	0.000
$\theta_2^2 * pattern$	12.000	5.130	0.025
$\theta_2^2 * \beta_{01}$	2.000	0.020	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.000

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Table A.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01}$	6.000	0.021	0.000
$pattern * \beta_{11}$	6.000	0.003	0.000
$pattern * \psi_{01}$	12.000	0.002	0.000
$\beta_{01} * \beta_{11}$	1.000	0.001	0.000
$\beta_{01} * \psi_{01}$	2.000	0.001	0.000
$\beta_{11} * \psi_{01}$	2.000	0.001	0.000
$model * N * \theta_2^2$	12.000	0.052	0.000
$model * N * pattern$	36.000	0.085	0.000
$model * N * \beta_{01}$	6.000	0.001	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.185	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.003	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.002	0.000
$model * pattern * \beta_{11}$	12.000	0.001	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table A.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	2.013	0.010
$N * \theta_2^2 * \beta_{01}$	6.000	0.004	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.001	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.004	0.000
$N * pattern * \beta_{01}$	18.000	0.013	0.000
$N * pattern * \beta_{11}$	18.000	0.004	0.000
$N * pattern * \psi_{01}$	36.000	0.011	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.002	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.009	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.004	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.009	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.002	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.002	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.008	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.003	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.062	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.002	0.000

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Table A.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.001	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.017	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.019	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.030	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.000

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Table A.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.009	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.004	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.008	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.009	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.005	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.011	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.006	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.006	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.001	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table A.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.010	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.021	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.021	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.018	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.010	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.021	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.000

Table A.7

ANOVA Test Results for Empirical Standard Error of β_{10}

Source	**DF**	**Sum of Squares**	η^2
$model$	2.000	0.024	0.000
N	3.000	130.788	0.796
θ_2^2	2.000	14.039	0.085
$pattern$	6.000	6.247	0.038
β_{01}	1.000	0.189	0.001
β_{11}	1.000	0.016	0.000
ψ_{01}	2.000	0.001	0.000
$model * N$	6.000	0.005	0.000
$model * \theta_2^2$	4.000	0.031	0.000
$model * pattern$	12.000	0.051	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	5.082	0.031
$N * pattern$	18.000	2.261	0.014
$N * \beta_{01}$	3.000	0.059	0.000
$N * \beta_{11}$	3.000	0.006	0.000
$N * \psi_{01}$	6.000	0.002	0.000
$\theta_2^2 * pattern$	12.000	3.767	0.023
$\theta_2^2 * \beta_{01}$	2.000	0.014	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.001	0.000

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Table A.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01}$	6.000	0.013	0.000
$pattern * \beta_{11}$	6.000	0.002	0.000
$pattern * \psi_{01}$	12.000	0.004	0.000
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.009	0.000
$model * N * pattern$	36.000	0.019	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.056	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table A.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	1.354	0.008
$N * \theta_2^2 * \beta_{01}$	6.000	0.010	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.001	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01}$	18.000	0.010	0.000
$N * pattern * \beta_{11}$	18.000	0.007	0.000
$N * pattern * \psi_{01}$	36.000	0.006	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.002	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.006	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.003	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.007	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.002	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.003	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.001	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.005	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.001	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.021	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table A.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.017	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.008	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.023	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.000

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Table A.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.003	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.005	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.008	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.014	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.005	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.006	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table A.7 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.007	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.016	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.022	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.011	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.005	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.017	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.000

Table A.8

ANOVA Test Results for Empirical Standard Error of β_{11}

Source	**DF**	**Sum of Squares**	η^2
$model$	2.000	0.088	0.000
N	3.000	199.121	0.708
θ_2^2	2.000	34.005	0.121
$pattern$	6.000	15.042	0.054
β_{01}	1.000	2.112	0.008
β_{11}	1.000	0.149	0.001
ψ_{01}	2.000	0.010	0.000
$model * N$	6.000	0.018	0.000
$model * \theta_2^2$	4.000	0.094	0.000
$model * pattern$	12.000	0.166	0.001
$model * \beta_{01}$	2.000	0.002	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	12.062	0.043
$N * pattern$	18.000	5.350	0.019
$N * \beta_{01}$	3.000	0.671	0.002
$N * \beta_{11}$	3.000	0.048	0.000
$N * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern$	12.000	8.267	0.029
$\theta_2^2 * \beta_{01}$	2.000	0.187	0.001
$\theta_2^2 * \beta_{11}$	2.000	0.007	0.000

Continued on next page

Table A.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.004	0.000
$pattern * \beta_{01}$	6.000	0.072	0.000
$pattern * \beta_{11}$	6.000	0.017	0.000
$pattern * \psi_{01}$	12.000	0.007	0.000
$\beta_{01} * \beta_{11}$	1.000	0.006	0.000
$\beta_{01} * \psi_{01}$	2.000	0.002	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.027	0.000
$model * N * pattern$	36.000	0.057	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.160	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.002	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.004	0.000
$model * pattern * \beta_{11}$	12.000	0.002	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table A.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	2.861	0.010
$N * \theta_2^2 * \beta_{01}$	6.000	0.059	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.003	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.003	0.000
$N * pattern * \beta_{01}$	18.000	0.034	0.000
$N * pattern * \beta_{11}$	18.000	0.013	0.000
$N * pattern * \psi_{01}$	36.000	0.012	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.001	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.002	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.004	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.019	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.005	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.009	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.001	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.003	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.001	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.006	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.005	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.002	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.055	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table A.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.002	0.000
$model * N * pattern * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.002	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.001	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.020	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.014	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.027	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.003	0.000

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Table A.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.005	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.003	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.002	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.008	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.014	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.005	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.006	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.007	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.003	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.001	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table A.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.013	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.018	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.035	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.013	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.010	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.025	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.000

Table A.9

ANOVA test for the Theoretical Standard Error of β_{00}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.280	0.003
N	3	93.024	0.872
θ_2^2	2	3.806	0.036
$pattern$	6	3.350	0.031
β_{01}	1	0.061	0.001
β_{11}	1	0.000	0.000
ψ_{01}	2	0.000	0.000
$model * N$	6	0.122	0.001
$model * \theta_2^2$	4	0.052	0.000
$model * pattern$	12	0.107	0.001
$model * \beta_{01}$	2	0.000	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	1.338	0.013
$N * pattern$	18	1.177	0.011
$N * \beta_{01}$	3	0.024	0.000
$N * \beta_{11}$	3	0.000	0.000
$N * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern$	12	2.303	0.022
$\theta_2^2 * \beta_{01}$	2	0.003	0.000
$\theta_2^2 * \beta_{11}$	2	0.000	0.000

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Table A.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.002	0.000
$pattern * \beta_{11}$	6	0.000	0.000
$pattern * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11}$	1	0.000	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2$	12	0.018	0.000
$model * N * pattern$	36	0.038	0.000
$model * N * \beta_{01}$	6	0.000	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.081	0.001
$model * \theta_2^2 * \beta_{01}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

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Table A.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	0.813	0.008
$N * \theta_2^2 * \beta_{01}$	6	0.001	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.001	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.001	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2 * pattern$	72	0.030	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

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Table A.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.000	0.000
$model * N * pattern * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000

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Table A.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

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Table A.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.000	0.000

Table A.10

ANOVA test for the Theoretical Standard Error of β_{01}

Source	**DF**	**Sum of Squares**	η^2
$model$	2.000	0.477	0.003
N	3.000	157.105	0.838
θ_2^2	2.000	8.345	0.045
$pattern$	6.000	7.304	0.039
β_{01}	1.000	0.592	0.003
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.000
$model * N$	6.000	0.234	0.001
$model * \theta_2^2$	4.000	0.122	0.001
$model * pattern$	12.000	0.255	0.001
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.001	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	2.956	0.016
$N * pattern$	18.000	2.571	0.014
$N * \beta_{01}$	3.000	0.237	0.001
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern$	12.000	4.953	0.026
$\theta_2^2 * \beta_{01}$	2.000	0.032	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.000

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Table A.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01}$	6.000	0.026	0.000
$pattern * \beta_{11}$	6.000	0.000	0.000
$pattern * \psi_{01}$	12.000	0.000	0.000
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.040	0.000
$model * N * pattern$	36.000	0.092	0.000
$model * N * \beta_{01}$	6.000	0.002	0.000
$model * N * \beta_{11}$	6.000	0.001	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.191	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.002	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table A.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	1.744	0.009
$N * \theta_2^2 * \beta_{01}$	6.000	0.014	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.000	0.000
$N * pattern * \beta_{01}$	18.000	0.009	0.000
$N * pattern * \beta_{11}$	18.000	0.000	0.000
$N * pattern * \psi_{01}$	36.000	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.006	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.069	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table A.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.001	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.002	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.000	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.001	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.000	0.000

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Table A.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table A.10 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.000	0.000

Table A.11

ANOVA test for the Theoretical Standard Error of β_{10}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.045	0.000
N	3	121.107	0.790
θ_2^2	2	13.678	0.089
$pattern$	6	6.111	0.040
β_{01}	1	0.177	0.001
β_{11}	1	0.013	0.000
ψ_{01}	2	0.002	0.000
$model * N$	6	0.022	0.000
$model * \theta_2^2$	4	0.041	0.000
$model * pattern$	12	0.048	0.000
$model * \beta_{01}$	2	0.000	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	4.758	0.031
$N * pattern$	18	2.125	0.014
$N * \beta_{01}$	3	0.058	0.000
$N * \beta_{11}$	3	0.004	0.000
$N * \psi_{01}$	6	0.001	0.000
$\theta_2^2 * pattern$	12	3.687	0.024
$\theta_2^2 * \beta_{01}$	2	0.011	0.000
$\theta_2^2 * \beta_{11}$	2	0.001	0.000

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Table A.11 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.005	0.000
$pattern * \beta_{11}$	6	0.000	0.000
$pattern * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11}$	1	0.001	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2$	12	0.016	0.000
$model * N * pattern$	36	0.017	0.000
$model * N * \beta_{01}$	6	0.000	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.054	0.000
$model * \theta_2^2 * \beta_{01}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

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Table A.11 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	1.283	0.008
$N * \theta_2^2 * \beta_{01}$	6	0.003	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.001	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.001	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.001	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2 * pattern$	72	0.019	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

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Table A.11 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.000	0.000
$model * N * pattern * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000

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Table A.11 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

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Table A.11 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.000	0.000

Table A.12

ANOVA test for the Theoretical Standard Error of β_{11}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.166	0.001
N	3	181.440	0.695
θ_2^2	2	33.226	0.127
$pattern$	6	14.666	0.056
β_{01}	1	2.084	0.008
β_{11}	1	0.157	0.001
ψ_{01}	2	0.016	0.000
$model * N$	6	0.081	0.000
$model * \theta_2^2$	4	0.115	0.000
$model * pattern$	12	0.164	0.001
$model * \beta_{01}$	2	0.005	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	11.512	0.044
$N * pattern$	18	5.081	0.019
$N * \beta_{01}$	3	0.704	0.003
$N * \beta_{11}$	3	0.055	0.000
$N * \psi_{01}$	6	0.007	0.000
$\theta_2^2 * pattern$	12	8.110	0.031
$\theta_2^2 * \beta_{01}$	2	0.182	0.001
$\theta_2^2 * \beta_{11}$	2	0.013	0.000

Continued on next page

Table A.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.002	0.000
$pattern * \beta_{01}$	6	0.074	0.000
$pattern * \beta_{11}$	6	0.007	0.000
$pattern * \psi_{01}$	12	0.001	0.000
$\beta_{01} * \beta_{11}$	1	0.006	0.000
$\beta_{01} * \psi_{01}$	2	0.004	0.000
$\beta_{11} * \psi_{01}$	2	0.001	0.000
$model * N * \theta_2^2$	12	0.045	0.000
$model * N * pattern$	36	0.056	0.000
$model * N * \beta_{01}$	6	0.003	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.160	0.001
$model * \theta_2^2 * \beta_{01}$	4	0.002	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.005	0.000
$model * pattern * \beta_{11}$	12	0.002	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

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Table A.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	2.826	0.011
$N * \theta_2^2 * \beta_{01}$	6	0.057	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.005	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.001	0.000
$N * pattern * \beta_{01}$	18	0.022	0.000
$N * pattern * \beta_{11}$	18	0.002	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.002	0.000
$N * \beta_{01} * \psi_{01}$	6	0.002	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.011	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.002	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2 * pattern$	72	0.055	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

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Table A.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.002	0.000
$model * N * pattern * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.002	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.003	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.001	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.001	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000

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Table A.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

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Table A.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.000	0.000

Table A.13 1

ANOVA Test Results for the Ratio of SE_T to SE_E in β_{00}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.031	0.015
N	3.000	0.507	0.244
θ_2^2	2.000	0.002	0.001
$pattern$	6.000	0.016	0.008
β_{01}	1.000	0.001	0.001
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.000
$model * N$	6.000	0.041	0.020
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.001	0.000
$model * \beta_{01}$	2.000	0.003	0.001
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.004	0.002
$N * pattern$	18.000	0.029	0.014
$N * \beta_{01}$	3.000	0.003	0.001
$N * \beta_{11}$	3.000	0.008	0.004
$N * \psi_{01}$	6.000	0.004	0.002
$\theta_2^2 * pattern$	12.000	0.017	0.008
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.001	0.000

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Table A.13 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.008	0.004
$pattern * \beta_{01}$	6.000	0.003	0.002
$pattern * \beta_{11}$	6.000	0.005	0.002
$pattern * \psi_{01}$	12.000	0.017	0.008
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.003	0.002
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.002	0.001
$model * N * pattern$	36.000	0.002	0.001
$model * N * \beta_{01}$	6.000	0.003	0.001
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.001	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.002	0.001
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.001	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.001	0.000

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Table A.13 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	0.039	0.019
$N * \theta_2^2 * \beta_{01}$	6.000	0.008	0.004
$N * \theta_2^2 * \beta_{11}$	6.000	0.010	0.005
$N * \theta_2^2 * \psi_{01}$	12.000	0.021	0.010
$N * pattern * \beta_{01}$	18.000	0.019	0.009
$N * pattern * \beta_{11}$	18.000	0.020	0.010
$N * pattern * \psi_{01}$	36.000	0.046	0.022
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.011	0.005
$N * \beta_{11} * \psi_{01}$	6.000	0.002	0.001
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.012	0.006
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.022	0.010
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.037	0.018
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.003	0.001
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.002	0.001
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.006	0.003
$pattern * \beta_{01} * \beta_{11}$	6.000	0.012	0.006
$pattern * \beta_{01} * \psi_{01}$	12.000	0.027	0.013
$pattern * \beta_{11} * \psi_{01}$	12.000	0.007	0.003
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.002	0.001
$model * N * \theta_2^2 * pattern$	72.000	0.003	0.001
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table A.13 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.001	0.001
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01}$	36.000	0.002	0.001
$model * N * pattern * \beta_{11}$	36.000	0.001	0.001
$model * N * pattern * \psi_{01}$	72.000	0.004	0.002
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.002	0.001
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.001	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.001	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.045	0.022
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.036	0.017
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.122	0.059
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.010	0.005

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Table A.13 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.014	0.007
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.010	0.005
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.019	0.009
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.064	0.031
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.053	0.026
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.008	0.004
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.020	0.010
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.052	0.025
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.038	0.019
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.004	0.002
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.014	0.007
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.002	0.001
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.003	0.002
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.007	0.003
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.001	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.002	0.001
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.006	0.003
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.002	0.001
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.001	0.000

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Table A.13 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.001	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.001	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.038	0.018
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.125	0.060
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.116	0.056
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.014	0.007
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.049	0.024
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.033	0.016
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.004	0.002
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.006	0.003
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.005	0.003
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.004	0.002
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.001	0.001
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.094	0.045
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.005	0.003

[1] SE_E: empirical standard error; SE_T: Theoretical standard error.

Table A.14 2

ANOVA Test Results for the Ratio of SE_T to SE_E in β_{01}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.067	0.028
N	3.000	0.739	0.304
θ_2^2	2.000	0.002	0.001
$pattern$	6.000	0.007	0.003
β_{01}	1.000	0.014	0.006
β_{11}	1.000	0.002	0.001
ψ_{01}	2.000	0.001	0.000
$model * N$	6.000	0.093	0.038
$model * \theta_2^2$	4.000	0.005	0.002
$model * pattern$	12.000	0.001	0.001
$model * \beta_{01}$	2.000	0.011	0.005
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.010	0.004
$N * pattern$	18.000	0.016	0.007
$N * \beta_{01}$	3.000	0.014	0.006
$N * \beta_{11}$	3.000	0.003	0.001
$N * \psi_{01}$	6.000	0.013	0.005
$\theta_2^2 * pattern$	12.000	0.018	0.007
$\theta_2^2 * \beta_{01}$	2.000	0.005	0.002
$\theta_2^2 * \beta_{11}$	2.000	0.003	0.001

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Table A.14 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.001	0.001
$pattern * \beta_{01}$	6.000	0.011	0.005
$pattern * \beta_{11}$	6.000	0.015	0.006
$pattern * \psi_{01}$	12.000	0.011	0.004
$\beta_{01} * \beta_{11}$	1.000	0.009	0.004
$\beta_{01} * \psi_{01}$	2.000	0.001	0.001
$\beta_{11} * \psi_{01}$	2.000	0.005	0.002
$model * N * \theta_2^2$	12.000	0.004	0.002
$model * N * pattern$	36.000	0.002	0.001
$model * N * \beta_{01}$	6.000	0.013	0.005
$model * N * \beta_{11}$	6.000	0.001	0.000
$model * N * \psi_{01}$	12.000	0.001	0.000
$model * \theta_2^2 * pattern$	24.000	0.001	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.007	0.003
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.001	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table A.14 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	0.060	0.025
$N * \theta_2^2 * \beta_{01}$	6.000	0.015	0.006
$N * \theta_2^2 * \beta_{11}$	6.000	0.010	0.004
$N * \theta_2^2 * \psi_{01}$	12.000	0.018	0.007
$N * pattern * \beta_{01}$	18.000	0.017	0.007
$N * pattern * \beta_{11}$	18.000	0.018	0.007
$N * pattern * \psi_{01}$	36.000	0.060	0.025
$N * \beta_{01} * \beta_{11}$	3.000	0.002	0.001
$N * \beta_{01} * \psi_{01}$	6.000	0.008	0.003
$N * \beta_{11} * \psi_{01}$	6.000	0.003	0.001
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.009	0.003
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.008	0.003
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.038	0.015
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.001	0.001
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.005	0.002
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.004	0.002
$pattern * \beta_{01} * \beta_{11}$	6.000	0.005	0.002
$pattern * \beta_{01} * \psi_{01}$	12.000	0.039	0.016
$pattern * \beta_{11} * \psi_{01}$	12.000	0.006	0.002
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.003	0.001
$model * N * \theta_2^2 * pattern$	72.000	0.002	0.001
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.005	0.002

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Table A.14 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01}$	36.000	0.001	0.000
$model * N * pattern * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \psi_{01}$	72.000	0.003	0.001
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.001	0.001
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.001	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.001	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.071	0.029
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.055	0.023
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.124	0.051
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.005	0.002

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Table A.14 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.032	0.013
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.018	0.007
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.009	0.004
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.032	0.013
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.026	0.011
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.004	0.002
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.021	0.008
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.057	0.023
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.032	0.013
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.019	0.008
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.003	0.001
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.004	0.001
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.007	0.003
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.001	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.001	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.002	0.001
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.003	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.003	0.001
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.001	0.000

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Table A.14 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.003	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.001	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.056	0.023
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.071	0.029
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.105	0.043
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.019	0.008
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.046	0.019
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.034	0.014
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.002	0.001
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.006	0.002
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.006	0.003
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.003	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.001	0.001
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.086	0.036
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.004	0.002

[2] SE_E: empirical standard error; SE_T: Theoretical standard error.

Table A.15 3

ANOVA Test Results for the Ratio of SE_T to SE_E in β_{10}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.011	0.005
N	3.000	0.403	0.199
θ_2^2	2.000	0.007	0.003
$pattern$	6.000	0.007	0.004
β_{01}	1.000	0.004	0.002
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.008	0.004
$model * N$	6.000	0.013	0.006
$model * \theta_2^2$	4.000	0.001	0.000
$model * pattern$	12.000	0.000	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.006	0.003
$N * pattern$	18.000	0.019	0.009
$N * \beta_{01}$	3.000	0.006	0.003
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.009	0.004
$\theta_2^2 * pattern$	12.000	0.027	0.013
$\theta_2^2 * \beta_{01}$	2.000	0.001	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.001	0.001

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Table A.15 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.004	0.002
$pattern * \beta_{01}$	6.000	0.012	0.006
$pattern * \beta_{11}$	6.000	0.006	0.003
$pattern * \psi_{01}$	12.000	0.016	0.008
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.002	0.001
$model * N * \theta_2^2$	12.000	0.001	0.000
$model * N * pattern$	36.000	0.001	0.000
$model * N * \beta_{01}$	6.000	0.001	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.000	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table A.15 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	0.059	0.029
$N * \theta_2^2 * \beta_{01}$	6.000	0.010	0.005
$N * \theta_2^2 * \beta_{11}$	6.000	0.003	0.002
$N * \theta_2^2 * \psi_{01}$	12.000	0.018	0.009
$N * pattern * \beta_{01}$	18.000	0.041	0.020
$N * pattern * \beta_{11}$	18.000	0.035	0.017
$N * pattern * \psi_{01}$	36.000	0.055	0.027
$N * \beta_{01} * \beta_{11}$	3.000	0.014	0.007
$N * \beta_{01} * \psi_{01}$	6.000	0.002	0.001
$N * \beta_{11} * \psi_{01}$	6.000	0.011	0.005
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.022	0.011
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.019	0.010
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.030	0.015
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.008	0.004
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.003	0.002
$pattern * \beta_{01} * \beta_{11}$	6.000	0.011	0.005
$pattern * \beta_{01} * \psi_{01}$	12.000	0.008	0.004
$pattern * \beta_{11} * \psi_{01}$	12.000	0.028	0.014
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.003	0.001
$model * N * \theta_2^2 * pattern$	72.000	0.001	0.001
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table A.15 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \psi_{01}$	72.000	0.001	0.001
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.064	0.032
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.053	0.026
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.123	0.061
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.011	0.005

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Table A.15 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.018	0.009
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.020	0.010
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.028	0.014
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.077	0.038
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.091	0.045
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.004	0.002
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.009	0.005
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.039	0.019
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.028	0.014
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.006	0.003
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.012	0.006
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.001	0.001
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.001	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.002	0.001
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table A.15 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.031	0.015
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.096	0.048
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.114	0.056
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.016	0.008
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.065	0.032
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.041	0.020
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.003	0.001
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.002	0.001
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.001	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.101	0.050
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.002	0.001

[3] SE_E: empirical standard error; SE_T: Theoretical standard error.

Table A.16 4

ANOVA Test Results for the Ratio of SE_T to SE_E in β_{11}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.029	0.013
N	3.000	0.584	0.269
θ_2^2	2.000	0.004	0.002
$pattern$	6.000	0.017	0.008
β_{01}	1.000	0.000	0.000
β_{11}	1.000	0.004	0.002
ψ_{01}	2.000	0.000	0.000
$model * N$	6.000	0.032	0.015
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.000	0.000
$model * \beta_{01}$	2.000	0.004	0.002
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.019	0.009
$N * pattern$	18.000	0.019	0.009
$N * \beta_{01}$	3.000	0.023	0.011
$N * \beta_{11}$	3.000	0.008	0.004
$N * \psi_{01}$	6.000	0.011	0.005
$\theta_2^2 * pattern$	12.000	0.022	0.010
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.006	0.003

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Table A.16 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.006	0.003
$pattern * \beta_{01}$	6.000	0.009	0.004
$pattern * \beta_{11}$	6.000	0.020	0.009
$pattern * \psi_{01}$	12.000	0.013	0.006
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.000	0.000
$model * N * pattern$	36.000	0.001	0.000
$model * N * \beta_{01}$	6.000	0.003	0.001
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.001	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

Continued on next page

Table A.16 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	0.059	0.027
$N * \theta_2^2 * \beta_{01}$	6.000	0.017	0.008
$N * \theta_2^2 * \beta_{11}$	6.000	0.004	0.002
$N * \theta_2^2 * \psi_{01}$	12.000	0.008	0.004
$N * pattern * \beta_{01}$	18.000	0.026	0.012
$N * pattern * \beta_{11}$	18.000	0.024	0.011
$N * pattern * \psi_{01}$	36.000	0.064	0.029
$N * \beta_{01} * \beta_{11}$	3.000	0.001	0.001
$N * \beta_{01} * \psi_{01}$	6.000	0.008	0.004
$N * \beta_{11} * \psi_{01}$	6.000	0.021	0.010
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.024	0.011
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.023	0.011
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.021	0.010
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.003	0.001
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.009	0.004
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.003	0.002
$pattern * \beta_{01} * \beta_{11}$	6.000	0.007	0.003
$pattern * \beta_{01} * \psi_{01}$	12.000	0.037	0.017
$pattern * \beta_{11} * \psi_{01}$	12.000	0.016	0.007
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.009	0.004
$model * N * \theta_2^2 * pattern$	72.000	0.001	0.001
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table A.16 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \psi_{01}$	72.000	0.002	0.001
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.002	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.001	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.001	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.037	0.017
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.059	0.027
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.106	0.049
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.017	0.008

Continued on next page

Table A.16 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.021	0.009
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.022	0.010
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.011	0.005
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.053	0.024
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.067	0.031
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.008	0.004
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.011	0.005
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.026	0.012
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.033	0.015
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.005	0.002
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.019	0.009
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.001	0.001
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.005	0.002
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.002	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.002	0.001
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table A.16 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.002	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.035	0.016
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.072	0.033
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.098	0.045
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.017	0.008
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.057	0.026
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.043	0.020
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.002	0.001
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.003	0.001
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.004	0.002
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.001	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.090	0.042
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.003	0.001

[4] SE_E: empirical standard error; SE_T: Theoretical standard error.

Table A.17

The Ratio of Theoretical SE and Empirical SE for β_{00} on Extreme Cases

N	Model	Pattern	Count	Minimum	Mean	Maximum
50	LCFM	Constant	4	0.923	0.937	0.949
50	LCFM	Heavy_Shrinking	3	0.935	0.941	0.949
50	LCFM	Heavy_Spreading	5	0.912	0.934	0.948
50	LCFM	High_Edge	3	0.936	0.945	0.949
50	LCFM	Low_Edge	4	0.919	0.936	0.949
50	LCFM	Moderate_Shrinking	5	0.928	0.937	0.948
50	LCFM	Moderate_Spreading	4	0.936	0.942	0.950
50	LCSM	Constant	4	0.923	0.937	0.949
50	LCSM	Heavy_Shrinking	3	0.935	0.941	0.949
50	LCSM	Heavy_Spreading	5	0.912	0.934	0.948
50	LCSM	High_Edge	3	0.936	0.945	0.949
50	LCSM	Low_Edge	4	0.919	0.936	0.949
50	LCSM	Moderate_Shrinking	5	0.928	0.937	0.948
50	LCSM	Moderate_Spreading	4	0.936	0.942	0.950
50	SOLGM	Constant	13	0.903	0.934	0.948
50	SOLGM	Heavy_Shrinking	18	0.901	0.932	0.949
50	SOLGM	Heavy_Spreading	15	0.901	0.932	0.949
50	SOLGM	High_Edge	18	0.911	0.938	0.950
50	SOLGM	Low_Edge	17	0.889	0.933	0.947
50	SOLGM	Moderate_Shrinking	15	0.916	0.937	0.947
50	SOLGM	Moderate_Spreading	17	0.922	0.935	0.944

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Table A.17 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
200	LCFM	Heavy_Spreading	1	1.061	1.061	1.061
200	LCFM	High_Edge	1	0.946	0.946	0.946
200	LCFM	Moderate_Spreading	2	0.934	0.940	0.947
200	LCSM	Heavy_Spreading	1	1.061	1.061	1.061
200	LCSM	High_Edge	1	0.946	0.946	0.946
200	LCSM	Moderate_Spreading	2	0.934	0.940	0.947
200	SOLGM	Heavy_Spreading	2	0.945	1.000	1.054
200	SOLGM	Moderate_Shrinking	1	0.946	0.946	0.946
200	SOLGM	Moderate_Spreading	2	0.917	0.931	0.944
600	LCFM	Constant	2	0.945	0.998	1.051
600	LCFM	Heavy_Shrinking	1	1.071	1.071	1.071
600	LCFM	High_Edge	1	0.948	0.948	0.948
600	LCFM	Moderate_Shrinking	1	1.052	1.052	1.052
600	LCFM	Moderate_Spreading	3	0.950	1.017	1.052
600	LCSM	Constant	2	0.945	0.998	1.051
600	LCSM	Heavy_Shrinking	1	1.071	1.071	1.071
600	LCSM	High_Edge	1	0.948	0.948	0.948
600	LCSM	Moderate_Shrinking	1	1.052	1.052	1.052
600	LCSM	Moderate_Spreading	3	0.950	1.017	1.052
600	SOLGM	Constant	1	0.932	0.932	0.932
600	SOLGM	Moderate_Shrinking	1	1.054	1.054	1.054
600	SOLGM	Moderate_Spreading	1	1.057	1.057	1.057

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Table A.17 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
1000	LCFM	Heavy_Shrinking	2	1.058	1.060	1.063
1000	LCFM	Heavy_Spreading	1	0.950	0.950	0.950
1000	LCFM	High_Edge	1	1.081	1.081	1.081
1000	LCFM	Low_Edge	2	0.950	1.005	1.061
1000	LCFM	Moderate_Spreading	2	1.051	1.059	1.067
1000	LCSM	Heavy_Shrinking	2	1.058	1.060	1.063
1000	LCSM	Heavy_Spreading	1	0.950	0.950	0.950
1000	LCSM	High_Edge	1	1.081	1.081	1.081
1000	LCSM	Low_Edge	2	0.950	1.005	1.061
1000	LCSM	Moderate_Spreading	2	1.051	1.059	1.067
1000	SOLGM	Heavy_Shrinking	2	1.060	1.063	1.066
1000	SOLGM	High_Edge	2	1.053	1.061	1.068
1000	SOLGM	Low_Edge	1	0.950	0.950	0.950
1000	SOLGM	Moderate_Shrinking	2	0.950	1.000	1.050
1000	SOLGM	Moderate_Spreading	2	1.051	1.062	1.073

Table A.18

The Ratio of Theoretical SE and Empirical SE for β_{01} on Extreme Cases

N	Model	Pattern	Count	Minimum	Mean	Maximum
50	LCFM	Constant	8	0.926	0.939	0.950
50	LCFM	Heavy_Shrinking	9	0.931	0.944	0.949
50	LCFM	Heavy_Spreading	8	0.919	0.934	0.942
50	LCFM	High_Edge	5	0.929	0.943	0.948
50	LCFM	Low_Edge	9	0.935	0.943	0.950
50	LCFM	Moderate_Shrinking	6	0.917	0.938	0.948
50	LCFM	Moderate_Spreading	11	0.924	0.938	0.948
50	LCSM	Constant	8	0.926	0.939	0.950
50	LCSM	Heavy_Shrinking	9	0.931	0.944	0.949
50	LCSM	Heavy_Spreading	8	0.919	0.934	0.942
50	LCSM	High_Edge	5	0.929	0.943	0.948
50	LCSM	Low_Edge	9	0.935	0.943	0.950
50	LCSM	Moderate_Shrinking	6	0.917	0.938	0.948
50	LCSM	Moderate_Spreading	11	0.924	0.938	0.948
50	SOLGM	Constant	24	0.882	0.925	0.944
50	SOLGM	Heavy_Shrinking	25	0.869	0.922	0.948
50	SOLGM	Heavy_Spreading	22	0.880	0.920	0.945
50	SOLGM	High_Edge	17	0.883	0.923	0.947
50	SOLGM	Low_Edge	25	0.874	0.918	0.944
50	SOLGM	Moderate_Shrinking	22	0.897	0.926	0.949
50	SOLGM	Moderate_Spreading	24	0.862	0.919	0.947

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Table A.18 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
200	LCFM	Heavy_Shrinking	1	1.064	1.064	1.064
200	LCFM	Heavy_Spreading	3	0.941	0.984	1.068
200	LCFM	Low_Edge	1	1.057	1.057	1.057
200	LCFM	Moderate_Shrinking	1	0.931	0.931	0.931
200	LCFM	Moderate_Spreading	3	0.939	0.942	0.945
200	LCSM	Heavy_Shrinking	1	1.064	1.064	1.064
200	LCSM	Heavy_Spreading	3	0.941	0.984	1.068
200	LCSM	Low_Edge	1	1.057	1.057	1.057
200	LCSM	Moderate_Shrinking	1	0.931	0.931	0.931
200	LCSM	Moderate_Spreading	3	0.939	0.942	0.945
200	SOLGM	Constant	3	0.938	0.943	0.949
200	SOLGM	Heavy_Shrinking	3	0.943	0.945	0.948
200	SOLGM	Heavy_Spreading	6	0.936	0.960	1.057
200	SOLGM	High_Edge	2	0.936	0.937	0.939
200	SOLGM	Low_Edge	3	0.946	0.987	1.064
200	SOLGM	Moderate_Spreading	4	0.933	0.942	0.950
600	LCFM	Constant	1	0.946	0.946	0.946
600	LCFM	Heavy_Spreading	1	1.060	1.060	1.060
600	LCFM	Low_Edge	1	1.078	1.078	1.078
600	LCFM	Moderate_Shrinking	2	1.054	1.057	1.061
600	LCFM	Moderate_Spreading	1	0.948	0.948	0.948
600	LCSM	Constant	1	0.946	0.946	0.946

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Table A.18 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
600	LCSM	Heavy_Spreading	1	1.060	1.060	1.060
600	LCSM	Low_Edge	1	1.078	1.078	1.078
600	LCSM	Moderate_Shrinking	2	1.054	1.057	1.061
600	LCSM	Moderate_Spreading	1	0.948	0.948	0.948
600	SOLGM	Constant	1	0.944	0.944	0.944
600	SOLGM	Heavy_Shrinking	1	1.053	1.053	1.053
600	SOLGM	Heavy_Spreading	2	0.949	1.003	1.057
600	SOLGM	Low_Edge	1	1.070	1.070	1.070
600	SOLGM	Moderate_Shrinking	1	1.067	1.067	1.067
600	SOLGM	Moderate_Spreading	1	0.944	0.944	0.944
1000	LCFM	Heavy_Shrinking	1	0.944	0.944	0.944
1000	LCFM	Heavy_Spreading	1	1.060	1.060	1.060
1000	LCFM	High_Edge	1	1.063	1.063	1.063
1000	LCFM	Low_Edge	1	0.945	0.945	0.945
1000	LCFM	Moderate_Shrinking	1	1.056	1.056	1.056
1000	LCFM	Moderate_Spreading	1	1.057	1.057	1.057
1000	LCSM	Heavy_Shrinking	1	0.944	0.944	0.944
1000	LCSM	Heavy_Spreading	1	1.060	1.060	1.060
1000	LCSM	High_Edge	1	1.063	1.063	1.063
1000	LCSM	Low_Edge	1	0.945	0.945	0.945
1000	LCSM	Moderate_Shrinking	1	1.056	1.056	1.056
1000	LCSM	Moderate_Spreading	1	1.057	1.057	1.057

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Table A.18 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
1000	SOLGM	Heavy_Spreading	1	1.061	1.061	1.061
1000	SOLGM	High_Edge	1	1.064	1.064	1.064

Table A.19

The Ratio of Theoretical SE and Empirical SE for β_{10} on Extreme Cases

N	Model	Pattern	Count	Minimum	Mean	Maximum
50	LCFM	Constant	8	0.915	0.939	0.950
50	LCFM	Heavy_Shrinking	6	0.934	0.944	0.950
50	LCFM	Heavy_Spreading	3	0.932	0.940	0.950
50	LCFM	High_Edge	3	0.943	0.944	0.947
50	LCFM	Low_Edge	2	0.928	0.939	0.950
50	LCFM	Moderate_Shrinking	7	0.912	0.936	0.946
50	LCFM	Moderate_Spreading	2	0.944	0.945	0.946
50	LCSM	Constant	8	0.915	0.939	0.950
50	LCSM	Heavy_Shrinking	6	0.934	0.944	0.950
50	LCSM	Heavy_Spreading	3	0.932	0.940	0.950
50	LCSM	High_Edge	3	0.943	0.944	0.947
50	LCSM	Low_Edge	2	0.928	0.939	0.950
50	LCSM	Moderate_Shrinking	7	0.912	0.936	0.946
50	LCSM	Moderate_Spreading	2	0.944	0.945	0.946
50	SOLGM	Constant	14	0.900	0.935	0.949
50	SOLGM	Heavy_Shrinking	13	0.926	0.940	0.946
50	SOLGM	Heavy_Spreading	8	0.921	0.938	0.947
50	SOLGM	High_Edge	12	0.930	0.941	0.950
50	SOLGM	Low_Edge	9	0.920	0.942	0.950
50	SOLGM	Moderate_Shrinking	14	0.903	0.935	0.949
50	SOLGM	Moderate_Spreading	8	0.925	0.938	0.949

Continued on next page

Table A.19 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
200	LCFM	Constant	1	0.950	0.950	0.950
200	LCFM	Heavy_Shrinking	1	1.055	1.055	1.055
200	LCFM	Heavy_Spreading	2	0.935	0.938	0.941
200	LCFM	High_Edge	2	0.941	0.943	0.944
200	LCFM	Low_Edge	1	1.050	1.050	1.050
200	LCFM	Moderate_Shrinking	2	0.949	1.000	1.052
200	LCFM	Moderate_Spreading	1	0.938	0.938	0.938
200	LCSM	Constant	1	0.950	0.950	0.950
200	LCSM	Heavy_Shrinking	1	1.055	1.055	1.055
200	LCSM	Heavy_Spreading	2	0.935	0.938	0.941
200	LCSM	High_Edge	2	0.941	0.943	0.944
200	LCSM	Low_Edge	1	1.050	1.050	1.050
200	LCSM	Moderate_Shrinking	2	0.949	1.000	1.052
200	LCSM	Moderate_Spreading	1	0.938	0.938	0.938
200	SOLGM	Heavy_Shrinking	1	1.051	1.051	1.051
200	SOLGM	Heavy_Spreading	2	0.936	0.941	0.946
200	SOLGM	High_Edge	2	0.937	0.938	0.940
200	SOLGM	Moderate_Shrinking	1	0.944	0.944	0.944
200	SOLGM	Moderate_Spreading	1	0.935	0.935	0.935
600	LCFM	Constant	1	0.941	0.941	0.941
600	LCFM	Heavy_Shrinking	2	1.062	1.074	1.086
600	LCFM	High_Edge	1	0.945	0.945	0.945

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Table A.19 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
600	LCFM	Moderate_Shrinking	3	1.063	1.067	1.075
600	LCSM	Constant	1	0.941	0.941	0.941
600	LCSM	Heavy_Shrinking	2	1.062	1.074	1.086
600	LCSM	High_Edge	1	0.945	0.945	0.945
600	LCSM	Moderate_Shrinking	3	1.063	1.067	1.075
600	SOLGM	Constant	1	0.941	0.941	0.941
600	SOLGM	Heavy_Shrinking	2	1.062	1.066	1.071
600	SOLGM	Heavy_Spreading	1	1.054	1.054	1.054
600	SOLGM	High_Edge	1	0.945	0.945	0.945
600	SOLGM	Moderate_Shrinking	3	1.065	1.068	1.071
1000	LCFM	Constant	2	0.941	0.996	1.050
1000	LCFM	Heavy_Shrinking	2	1.066	1.066	1.066
1000	LCFM	High_Edge	2	1.053	1.057	1.061
1000	LCFM	Moderate_Shrinking	2	0.947	1.007	1.067
1000	LCFM	Moderate_Spreading	1	0.934	0.934	0.934
1000	LCSM	Constant	2	0.941	0.996	1.050
1000	LCSM	Heavy_Shrinking	2	1.066	1.066	1.066
1000	LCSM	High_Edge	2	1.053	1.057	1.061
1000	LCSM	Moderate_Shrinking	2	0.947	1.007	1.067
1000	LCSM	Moderate_Spreading	1	0.934	0.934	0.934
1000	SOLGM	Constant	1	0.941	0.941	0.941
1000	SOLGM	Heavy_Shrinking	2	0.944	1.002	1.061

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Table A.19 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
1000	SOLGM	High_Edge	2	1.051	1.056	1.061
1000	SOLGM	Moderate_Shrinking	2	0.949	1.007	1.066
1000	SOLGM	Moderate_Spreading	1	0.934	0.934	0.934

Table A.20

The Ratio of Theoretical SE and Empirical SE for β_{11} on Extreme Cases

N	Model	Pattern	Count	Minimum	Mean	Maximum
50	LCFM	Constant	10	0.917	0.936	0.949
50	LCFM	Heavy_Shrinking	4	0.927	0.936	0.947
50	LCFM	Heavy_Spreading	10	0.924	0.941	0.950
50	LCFM	High_Edge	5	0.929	0.940	0.948
50	LCFM	Low_Edge	6	0.927	0.935	0.949
50	LCFM	Moderate_Shrinking	6	0.935	0.942	0.949
50	LCFM	Moderate_Spreading	5	0.926	0.938	0.946
50	LCSM	Constant	10	0.917	0.936	0.949
50	LCSM	Heavy_Shrinking	4	0.927	0.936	0.947
50	LCSM	Heavy_Spreading	10	0.924	0.941	0.950
50	LCSM	High_Edge	5	0.929	0.940	0.948
50	LCSM	Low_Edge	6	0.927	0.935	0.949
50	LCSM	Moderate_Shrinking	6	0.935	0.942	0.949
50	LCSM	Moderate_Spreading	5	0.926	0.938	0.946
50	SOLGM	Constant	21	0.904	0.932	0.949
50	SOLGM	Heavy_Shrinking	15	0.920	0.937	0.950

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Table A.20 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
50	SOLGM	Heavy_Spreading	22	0.907	0.933	0.949
50	SOLGM	High_Edge	15	0.912	0.932	0.947
50	SOLGM	Low_Edge	15	0.907	0.931	0.950
50	SOLGM	Moderate_Shrinking	18	0.909	0.934	0.950
50	SOLGM	Moderate_Spreading	16	0.900	0.934	0.950
200	LCFM	Constant	1	0.939	0.939	0.939
200	LCFM	Heavy_Spreading	4	0.948	1.008	1.084
200	LCFM	Low_Edge	1	0.940	0.940	0.940
200	LCFM	Moderate_Shrinking	1	0.939	0.939	0.939
200	LCSM	Constant	1	0.939	0.939	0.939
200	LCSM	Heavy_Spreading	4	0.948	1.008	1.084
200	LCSM	Low_Edge	1	0.940	0.940	0.940
200	LCSM	Moderate_Shrinking	1	0.939	0.939	0.939
200	SOLGM	Constant	2	0.931	0.939	0.946
200	SOLGM	Heavy_Shrinking	1	0.945	0.945	0.945
200	SOLGM	Heavy_Spreading	3	0.944	0.945	0.946
200	SOLGM	Low_Edge	1	0.931	0.931	0.931
200	SOLGM	Moderate_Shrinking	1	0.930	0.930	0.930
200	SOLGM	Moderate_Spreading	2	0.947	0.948	0.950
600	LCFM	Constant	1	0.938	0.938	0.938
600	LCFM	High_Edge	1	1.059	1.059	1.059
600	LCFM	Low_Edge	1	0.946	0.946	0.946

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Table A.20 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
600	LCFM	Moderate_Shrinking	1	0.945	0.945	0.945
600	LCFM	Moderate_Spreading	1	1.073	1.073	1.073
600	LCSM	Constant	1	0.938	0.938	0.938
600	LCSM	High_Edge	1	1.059	1.059	1.059
600	LCSM	Low_Edge	1	0.946	0.946	0.946
600	LCSM	Moderate_Shrinking	1	0.945	0.945	0.945
600	LCSM	Moderate_Spreading	1	1.073	1.073	1.073
600	SOLGM	Constant	1	0.941	0.941	0.941
600	SOLGM	High_Edge	1	1.059	1.059	1.059
600	SOLGM	Low_Edge	1	0.945	0.945	0.945
600	SOLGM	Moderate_Shrinking	1	0.942	0.942	0.942
600	SOLGM	Moderate_Spreading	1	1.075	1.075	1.075
1000	LCFM	Heavy_Shrinking	3	0.935	0.976	1.055
1000	LCFM	High_Edge	4	0.944	1.034	1.065
1000	LCFM	Low_Edge	1	1.059	1.059	1.059
1000	LCFM	Moderate_Spreading	2	0.940	1.000	1.060
1000	LCSM	Heavy_Shrinking	3	0.935	0.976	1.055
1000	LCSM	High_Edge	4	0.944	1.034	1.065
1000	LCSM	Low_Edge	1	1.059	1.059	1.059
1000	LCSM	Moderate_Spreading	2	0.940	1.000	1.060
1000	SOLGM	Constant	1	0.950	0.950	0.950
1000	SOLGM	Heavy_Spreading	1	0.947	0.947	0.947

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Table A.20 – continued from previous page

N	Model	Pattern	Count	Minimum	Mean	Maximum
1000	SOLGM	High_Edge	5	0.944	1.018	1.068
1000	SOLGM	Low_Edge	1	1.055	1.055	1.055
1000	SOLGM	Moderate_Spreading	2	0.939	1.003	1.067

Table A.21 5

ANOVA Test Results for the Ratio of SE_{2T} to SE_{3T} and SE_{2E} to SE_{3E} in β_{00}

Source	**DF**	η^2_E	η^2_T
N	3.000	0.008	0.009
θ^2_2	2.000	0.165	0.162
$pattern$	6.000	0.488	0.523
β_{01}	1.000	0.001	0.000
ψ_{01}	2.000	0.000	0.000
β_{11}	1.000	0.000	0.001
$N * \theta^2_2$	6.000	0.000	0.000
$N * pattern$	18.000	0.001	0.000
$N * \beta_{01}$	3.000	0.000	0.002
$N * \psi_{01}$	6.000	0.000	0.000
$N * \beta_{11}$	3.000	0.000	0.000
$\theta^2_2 * pattern$	12.000	0.275	0.297
$\theta^2_2 * \beta_{01}$	2.000	0.002	0.000
$\theta^2_2 * \psi_{01}$	4.000	0.000	0.000
$\theta^2_2 * \beta_{11}$	2.000	0.000	0.000
$pattern * \beta_{01}$	6.000	0.002	0.002
$pattern * \psi_{01}$	12.000	0.000	0.000
$pattern * \beta_{11}$	6.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\psi_{01} * \beta_{11}$	2.000	0.000	0.000

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Table A.21 – continued from previous page

Source	**DF**	η^2_E	η^2_T
$N * \theta^2_2 * pattern$	36.000	0.002	0.000
$N * \theta^2_2 * \beta_{01}$	6.000	0.000	0.001
$N * \theta^2_2 * \psi_{01}$	12.000	0.001	0.000
$N * \theta^2_2 * \beta_{11}$	6.000	0.001	0.000
$N * pattern * \beta_{01}$	18.000	0.001	0.000
$N * pattern * \psi_{01}$	36.000	0.003	0.000
$N * pattern * \beta_{11}$	18.000	0.001	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01} * \beta_{11}$	6.000	0.000	0.000
$\theta^2_2 * pattern * \beta_{01}$	12.000	0.002	0.001
$\theta^2_2 * pattern * \psi_{01}$	24.000	0.001	0.000
$\theta^2_2 * pattern * \beta_{11}$	12.000	0.000	0.000
$\theta^2_2 * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$\theta^2_2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta^2_2 * \psi_{01} * \beta_{11}$	4.000	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.001	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$pattern * \psi_{01} * \beta_{11}$	12.000	0.001	0.000
$\beta_{01} * \psi_{01} * \beta_{11}$	2.000	0.000	0.000
$N * \theta^2_2 * pattern * \beta_{01}$	36.000	0.002	0.000
$N * \theta^2_2 * pattern * \psi_{01}$	72.000	0.005	0.000

Continued on next page

Table A.21 – continued from previous page

Source	DF	η_E^2	η_T^2
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.002	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$N * \theta_2^2 * \psi_{01} * \beta_{11}$	12.000	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.005	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.001	0.000
$N * pattern * \psi_{01} * \beta_{11}$	36.000	0.002	0.000
$N * \beta_{01} * \psi_{01} * \beta_{11}$	6.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$\theta_2^2 * pattern * \psi_{01} * \beta_{11}$	24.000	0.001	0.000
$\theta_2^2 * \beta_{01} * \psi_{01} * \beta_{11}$	4.000	0.000	0.000
$pattern * \beta_{01} * \psi_{01} * \beta_{11}$	12.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.004	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.003	0.000
$N * \theta_2^2 * pattern * \psi_{01} * \beta_{11}$	72.000	0.004	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01} * \beta_{11}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \psi_{01} * \beta_{11}$	36.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01} * \beta_{11}$	24.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01} * \beta_{11}$	72.000	0.004	0.000

[5] SE_{2T}:Theoretical Standard Error of two-wave LCFM;SE_{3T}: Theoretical Standard Error of Three-Wave SOLGM;SE_{2E}:Empirical Standard Error of Two-Wave LCFM; SE_{3E}: Empirical Standard Error of Three-Wave SOLGM; η_E^2: Effect size for the ratio of $\frac{SE_{2E}}{SE_{3E}}$;η_T^2:Effect size for the ratio of $\frac{SE_{2T}}{SE_{3T}}$.

Table A.22 6

ANOVA Test Results for the Ratio of SE_{2T} to SE_{3T} and SE_{2E} to SE_{3E} in β_{01}

Source	**DF**	η_E^2	η_T^2
N	3.000	0.010	0.018
θ_2^2	2.000	0.189	0.163
$pattern$	6.000	0.483	0.517
β_{01}	1.000	0.005	0.000
β_{11}	1.000	0.002	0.003
ψ_{01}	2.000	0.000	0.000
$N * \theta_2^2$	6.000	0.000	0.001
$N * pattern$	18.000	0.001	0.000
$N * \beta_{01}$	3.000	0.000	0.005
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern$	12.000	0.253	0.278
$\theta_2^2 * \beta_{01}$	2.000	0.007	0.001
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01}$	6.000	0.008	0.008
$pattern * \beta_{11}$	6.000	0.001	0.001
$pattern * \psi_{01}$	12.000	0.000	0.000
$\beta_{01} * \beta_{11}$	1.000	0.001	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000

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Table A.22 – continued from previous page

Source	DF	η^2_E	η^2_T
$N * \theta_2^2 * pattern$	36.000	0.001	0.000
$N * \theta_2^2 * \beta_{01}$	6.000	0.000	0.001
$N * \theta_2^2 * \beta_{11}$	6.000	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.000	0.000
$N * pattern * \beta_{01}$	18.000	0.000	0.000
$N * pattern * \beta_{11}$	18.000	0.000	0.000
$N * pattern * \psi_{01}$	36.000	0.002	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.002	0.001
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.001	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.002	0.000

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Table A.22 – continued from previous page

Source	DF	η_E^2	η_T^2
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.004	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.002	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.002	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.003	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.004	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.002	0.000

[6] SE_{2T}:Theoretical Standard Error of two-wave LCFM;SE_{3T}: Theoretical Standard Error of Three-Wave SOLGM;SE_{2E}:Empirical Standard Error of Two-Wave LCFM; SE_{3E}: Empirical Standard Error of Three-Wave SOLGM; η_E^2: Effect size for the ratio of $\frac{SE_{2E}}{SE_{3E}}$;η_T^2:Effect size for the ratio of $\frac{SE_{2T}}{SE_{3T}}$.

Table A.23 7

ANOVA Test Results for the Ratio of SE_{2T} to SE_{3T} and SE_{2E} to SE_{3E} in β_{10}

Source	**DF**	η^2_E	η^2_T
N	3.000	0.005	0.003
θ_2^2	2.000	0.219	0.249
$pattern$	6.000	0.385	0.383
β_{01}	1.000	0.002	0.005
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.000
$N * \theta_2^2$	6.000	0.001	0.000
$N * pattern$	18.000	0.000	0.000
$N * \beta_{01}$	3.000	0.000	0.000
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern$	12.000	0.339	0.345
$\theta_2^2 * \beta_{01}$	2.000	0.002	0.001
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01}$	6.000	0.005	0.006
$pattern * \beta_{11}$	6.000	0.001	0.002
$pattern * \psi_{01}$	12.000	0.001	0.000
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000

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Table A.23 – continued from previous page

Source	**DF**	η^2_E	η^2_T
$N * \theta^2_2 * pattern$	36.000	0.001	0.000
$N * \theta^2_2 * \beta_{01}$	6.000	0.000	0.000
$N * \theta^2_2 * \beta_{11}$	6.000	0.001	0.000
$N * \theta^2_2 * \psi_{01}$	12.000	0.000	0.000
$N * pattern * \beta_{01}$	18.000	0.000	0.000
$N * pattern * \beta_{11}$	18.000	0.001	0.000
$N * pattern * \psi_{01}$	36.000	0.002	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta^2_2 * pattern * \beta_{01}$	12.000	0.002	0.002
$\theta^2_2 * pattern * \beta_{11}$	12.000	0.001	0.000
$\theta^2_2 * pattern * \psi_{01}$	24.000	0.001	0.000
$\theta^2_2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta^2_2 * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$\theta^2_2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.001	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$N * \theta^2_2 * pattern * \beta_{01}$	36.000	0.001	0.000
$N * \theta^2_2 * pattern * \beta_{11}$	36.000	0.002	0.000

Continued on next page

Table A.23 – continued from previous page

Source	DF	η^2_E	η^2_T
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.002	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.001	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.002	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.002	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.004	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.003	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.003	0.000

[7] SE_{2T}:Theoretical Standard Error of two-wave LCFM;SE_{3T}: Theoretical Standard Error of Three-Wave SOLGM;SE_{2E}:Empirical Standard Error of Two-Wave LCFM; SE_{3E}: Empirical Standard Error of Three-Wave SOLGM; η^2_E: Effect size for the ratio of $\frac{SE_{2E}}{SE_{3E}}$;η^2_T:Effect size for the ratio of $\frac{SE_{2T}}{SE_{3T}}$.

Table A.24 8

ANOVA Test Results for the Ratio of SE_{2T} to SE_{3T} and SE_{2E} to SE_{3E} in β_{11}

Source	**DF**	η^2_E	η^2_T
N	3.000	0.006	0.005
θ^2_2	2.000	0.187	0.193
$pattern$	6.000	0.424	0.427
β_{01}	1.000	0.011	0.023
β_{11}	1.000	0.001	0.001
ψ_{01}	2.000	0.000	0.000
$N * \theta^2_2$	6.000	0.000	0.000
$N * pattern$	18.000	0.001	0.000
$N * \beta_{01}$	3.000	0.000	0.001
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.000	0.000
$\theta^2_2 * pattern$	12.000	0.290	0.300
$\theta^2_2 * \beta_{01}$	2.000	0.006	0.005
$\theta^2_2 * \beta_{11}$	2.000	0.001	0.001
$\theta^2_2 * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01}$	6.000	0.024	0.026
$pattern * \beta_{11}$	6.000	0.007	0.008
$pattern * \psi_{01}$	12.000	0.001	0.001
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000

Continued on next page

Table A.24 – continued from previous page

Source	DF	η_E^2	η_T^2
$N * \theta_2^2 * pattern$	36.000	0.001	0.000
$N * \theta_2^2 * \beta_{01}$	6.000	0.000	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01}$	18.000	0.000	0.000
$N * pattern * \beta_{11}$	18.000	0.001	0.000
$N * pattern * \psi_{01}$	36.000	0.001	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.006	0.007
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.002	0.001
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.002	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.001	0.000

Continued on next page

Table A.24 – continued from previous page

Source	**DF**	η_E^2	η_T^2
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.004	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.001	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.002	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.001	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.002	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.003	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.002	0.000

[8] SE_{2T}:Theoretical Standard Error of two-wave LCFM;SE_{3T}: Theoretical Standard Error of Three-Wave SOLGM;SE_{2E}:Empirical Standard Error of Two-Wave LCFM; SE_{3E}: Empirical Standard Error of Three-Wave SOLGM; η_E^2: Effect size for the ratio of $\frac{SE_{2E}}{SE_{3E}}$;η_T^2:Effect size for the ratio of $\frac{SE_{2T}}{SE_{3T}}$.

A.0.3 List of Tables for Statistical Power

Table A.25

ANOVA Test Results for Statistical Power of β_{01}

Source	DF	Sum of Squares	η^2
$model$	2	0.009	0.002
N	3	0.982	0.211
θ_2^2	2	0.306	0.066
$pattern$	6	0.259	0.056
β_{11}	1	0.000	0.000
ψ_{01}	2	0.000	0.000
$model * N$	6	0.026	0.006
$model * \theta_2^2$	4	0.010	0.002
$model * pattern$	12	0.011	0.002
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	0.903	0.194
$N * pattern$	18	0.764	0.164
$N * \beta_{11}$	3	0.000	0.000
$N * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern$	12	0.319	0.068
$\theta_2^2 * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{11}$	6	0.001	0.000
$pattern * \psi_{01}$	12	0.000	0.000

Continued on next page

Table A.25 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$\beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2$	12	0.027	0.006
$model * N * pattern$	36	0.031	0.007
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.012	0.002
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern$	36	0.935	0.201
$N * \theta_2^2 * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{11}$	18	0.001	0.000
$N * pattern * \psi_{01}$	36	0.001	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.001	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.001	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern$	72	0.032	0.007

Continued on next page

Table A.25 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{11}$	36	0.001	0.000
$model * N * pattern * \psi_{01}$	72	0.001	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.002	0.001
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.002	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.001	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.001	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.002	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.001	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.007	0.001
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.001	0.000

Table A.26

ANOVA Test Results for Statistical Power of β_{10}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.016	0.000
N	3	209.451	0.870
θ_2^2	2	10.688	0.044
$pattern$	6	4.552	0.019
β_{01}	1	0.132	0.001
β_{11}	1	0.012	0.000
ψ_{01}	2	0.001	0.000
$model * N$	6	0.014	0.000
$model * \theta_2^2$	4	0.021	0.000
$model * pattern$	12	0.034	0.000
$model * \beta_{01}$	2	0.000	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	7.821	0.032
$N * pattern$	18	2.765	0.011
$N * \beta_{01}$	3	0.111	0.000
$N * \beta_{11}$	3	0.011	0.000
$N * \psi_{01}$	6	0.011	0.000
$\theta_2^2 * pattern$	12	2.561	0.011
$\theta_2^2 * \beta_{01}$	2	0.014	0.000
$\theta_2^2 * \beta_{11}$	2	0.002	0.000

Continued on next page

Table A.26 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.005	0.000
$pattern * \beta_{01}$	6	0.018	0.000
$pattern * \beta_{11}$	6	0.003	0.000
$pattern * \psi_{01}$	12	0.008	0.000
$\beta_{01} * \beta_{11}$	1	0.001	0.000
$\beta_{01} * \psi_{01}$	2	0.003	0.000
$\beta_{11} * \psi_{01}$	2	0.002	0.000
$model * N * \theta_2^2$	12	0.016	0.000
$model * N * pattern$	36	0.024	0.000
$model * N * \beta_{01}$	6	0.000	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.032	0.000
$model * \theta_2^2 * \beta_{01}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

Continued on next page

Table A.26 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$N * \theta_2^2 * pattern$	36	1.799	0.007
$N * \theta_2^2 * \beta_{01}$	6	0.043	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.006	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.007	0.000
$N * pattern * \beta_{01}$	18	0.048	0.000
$N * pattern * \beta_{11}$	18	0.025	0.000
$N * pattern * \psi_{01}$	36	0.017	0.000
$N * \beta_{01} * \beta_{11}$	3	0.002	0.000
$N * \beta_{01} * \psi_{01}$	6	0.004	0.000
$N * \beta_{11} * \psi_{01}$	6	0.006	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.013	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.005	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.019	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.001	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.003	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.001	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.002	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.007	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.008	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.003	0.000
$model * N * \theta_2^2 * pattern$	72	0.026	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

Continued on next page

Table A.26 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.001	0.000
$model * N * pattern * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \psi_{01}$	72	0.001	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.027	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.016	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.054	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.006	0.000

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Table A.26 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.004	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.006	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.008	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.016	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.020	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.008	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.014	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.001	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.004	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.001	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.001	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

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Table A.26 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.012	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.033	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.045	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.007	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.016	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.008	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.002	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.001	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.033	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.001	0.000

Table A.27

ANOVA Test Results for Statistical Power of β_{11}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.008	0.000
N	3.000	25.966	0.577
θ_2^2	2.000	4.012	0.089
$pattern$	6.000	1.737	0.039
β_{01}	1.000	0.281	0.006
ψ_{01}	2.000	0.001	0.000
$model * N$	6.000	0.008	0.000
$model * \theta_2^2$	4.000	0.009	0.000
$model * pattern$	12.000	0.016	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	7.019	0.156
$N * pattern$	18.000	2.569	0.057
$N * \beta_{01}$	3.000	0.714	0.016
$N * \psi_{01}$	6.000	0.003	0.000
$\theta_2^2 * pattern$	12.000	0.889	0.020
$\theta_2^2 * \beta_{01}$	2.000	0.018	0.000
$\theta_2^2 * \psi_{01}$	4.000	0.002	0.000
$pattern * \beta_{01}$	6.000	0.006	0.000
$pattern * \psi_{01}$	12.000	0.007	0.000
$\beta_{01} * \psi_{01}$	2.000	0.001	0.000

Continued on next page

Table A.27 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2$	12.000	0.009	0.000
$model * N * pattern$	36.000	0.020	0.000
$model * N * \beta_{01}$	6.000	0.001	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.014	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.001	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern$	36.000	1.352	0.030
$N * \theta_2^2 * \beta_{01}$	6.000	0.101	0.002
$N * \theta_2^2 * \psi_{01}$	12.000	0.005	0.000
$N * pattern * \beta_{01}$	18.000	0.035	0.001
$N * pattern * \psi_{01}$	36.000	0.015	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.007	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.008	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.004	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.016	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.001	0.000

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Table A.27 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.001	0.000
$model * N * pattern * \beta_{01}$	36.000	0.002	0.000
$model * N * pattern * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.033	0.001
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.027	0.001
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.011	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.009	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.031	0.001
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.002	0.000

A.0.4 List of Figures in Bias

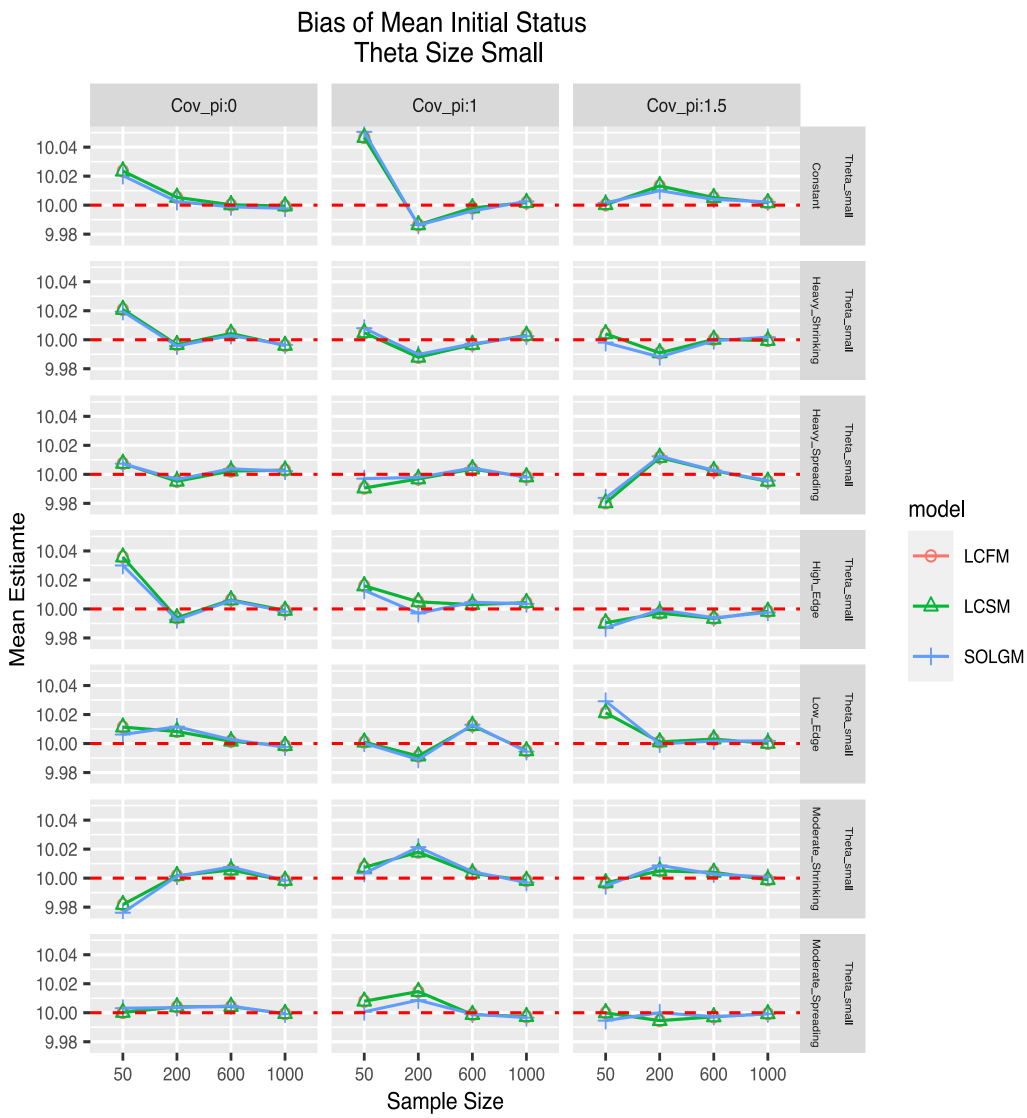

Figure A.1. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

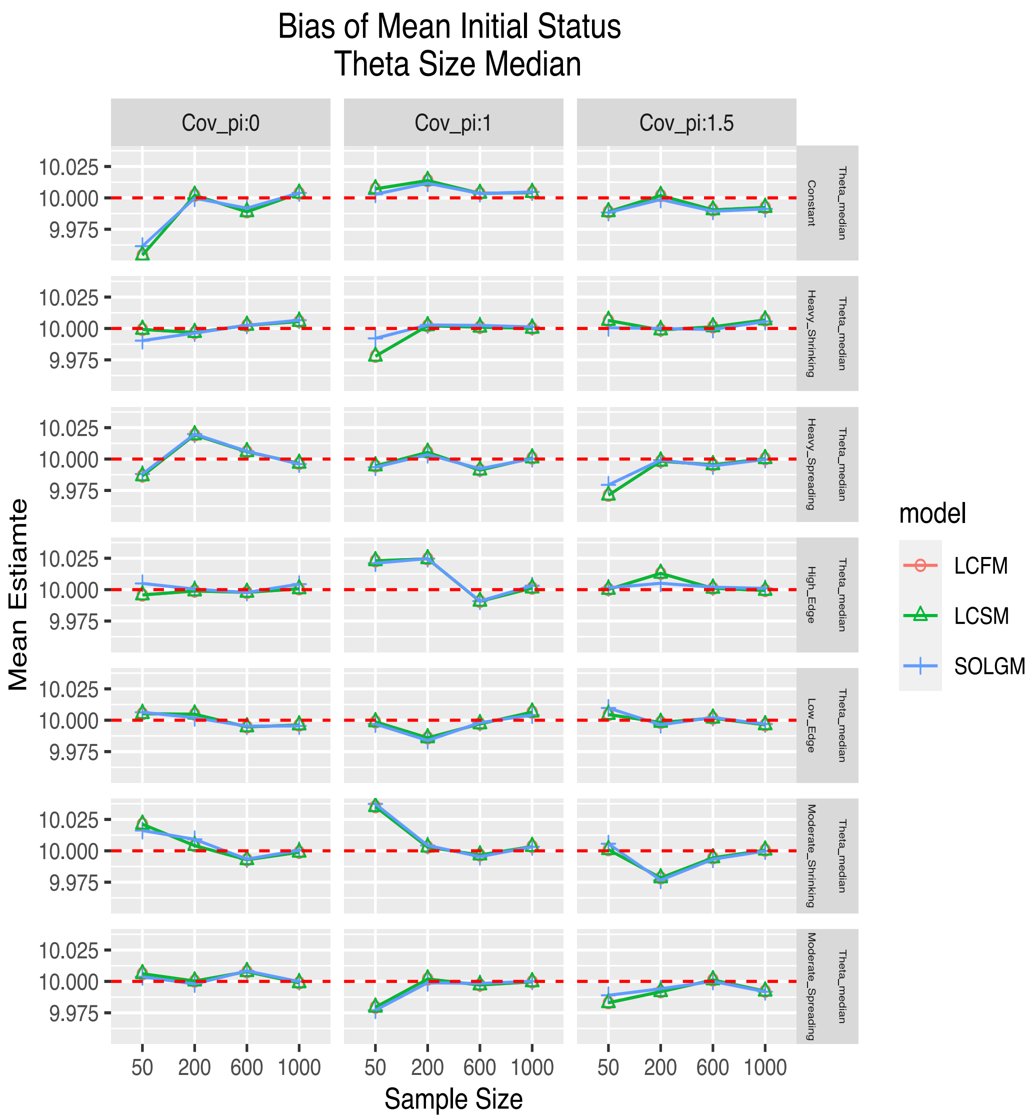

Figure A.2. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

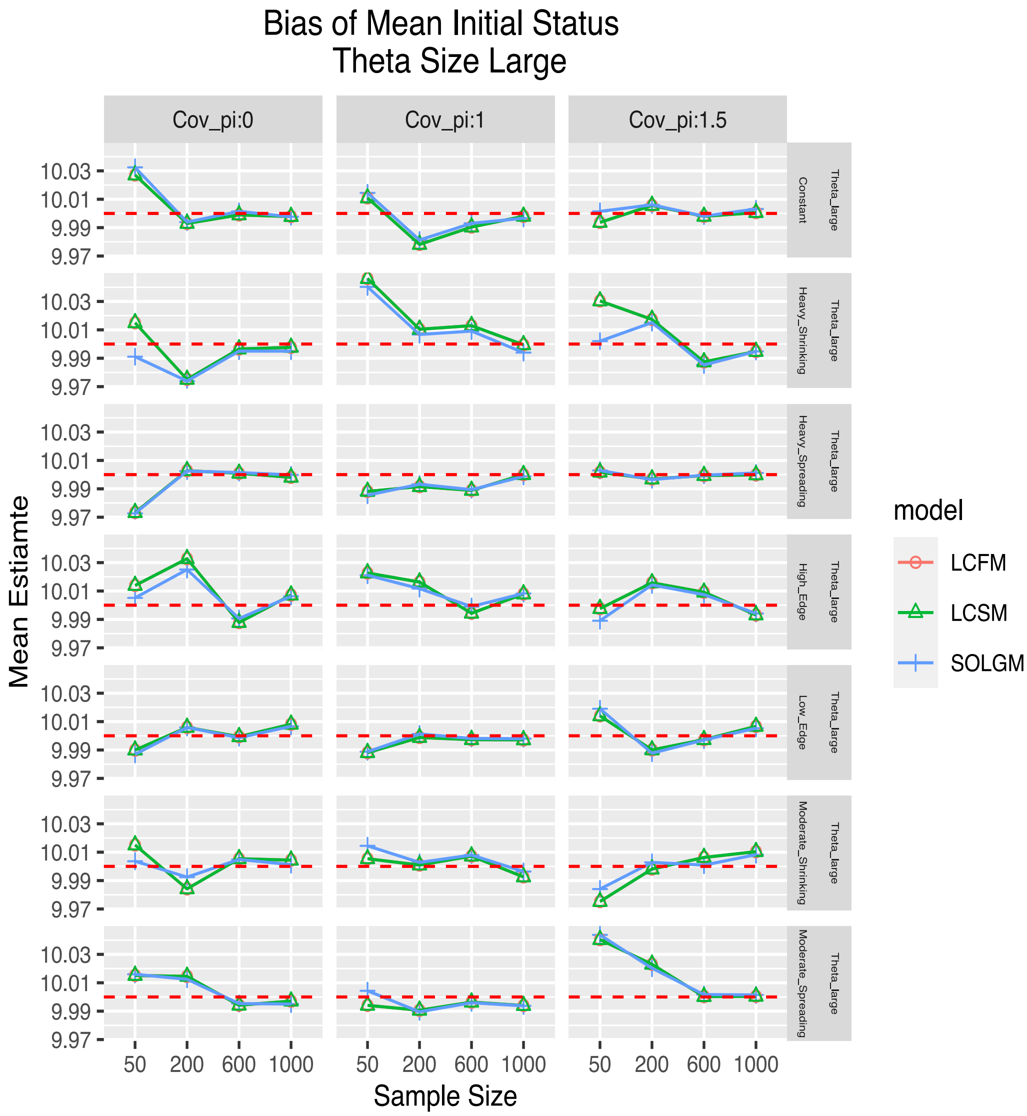

Figure A.3. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

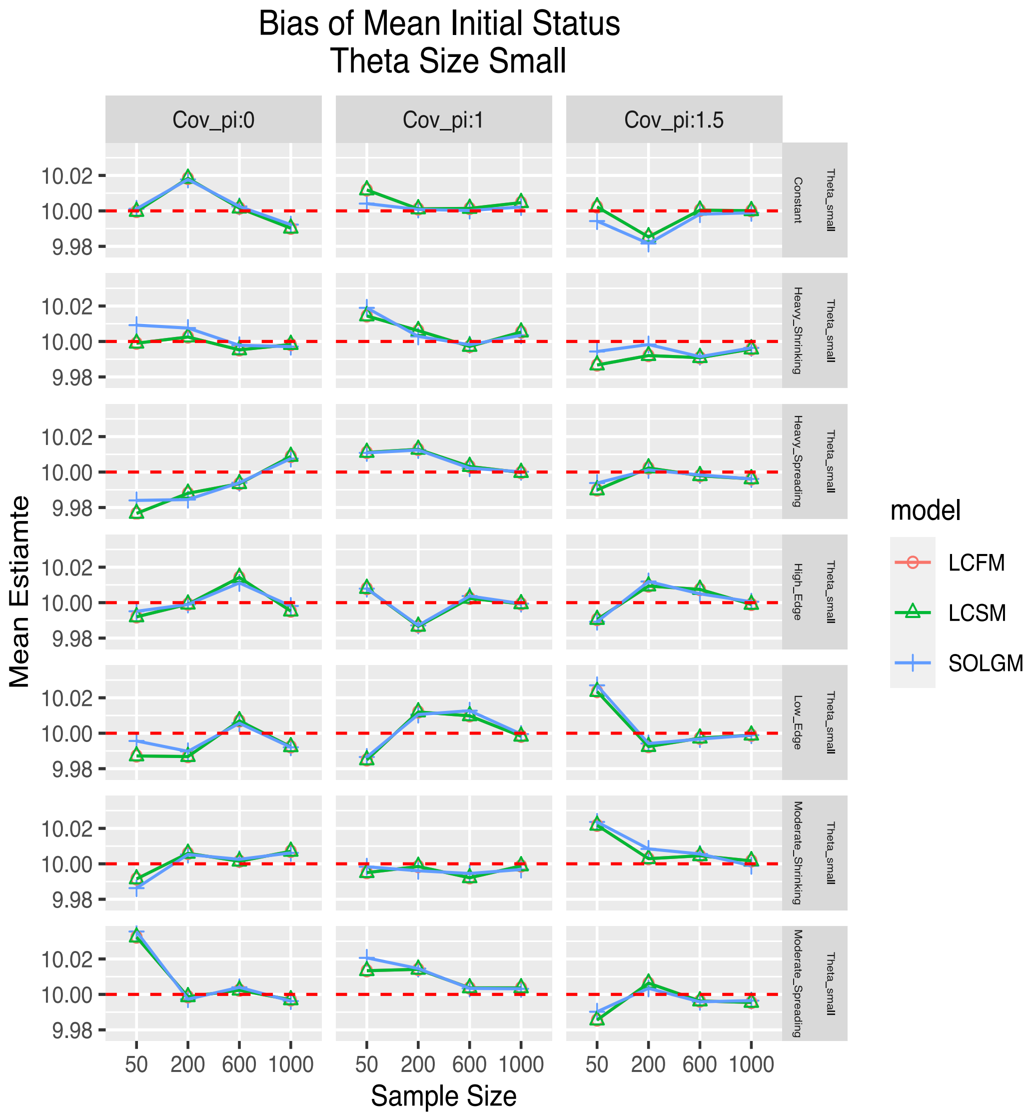

Figure A.4. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

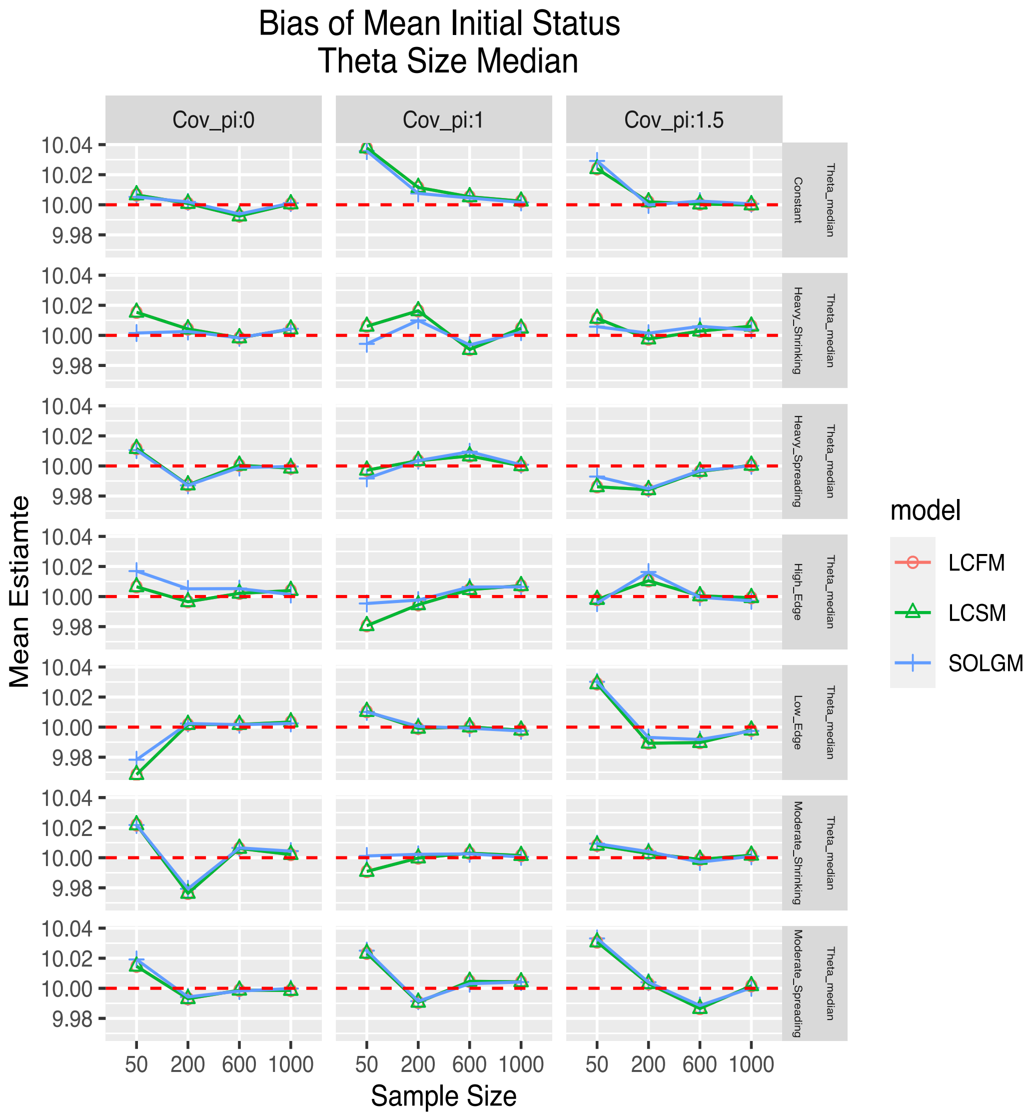

Figure A.5. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00}} = 10$, by time-specific error size (**median**) and correlation between intercept and slope ($Cor_p i$), when $\boldsymbol{\beta_{01}} = 3$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

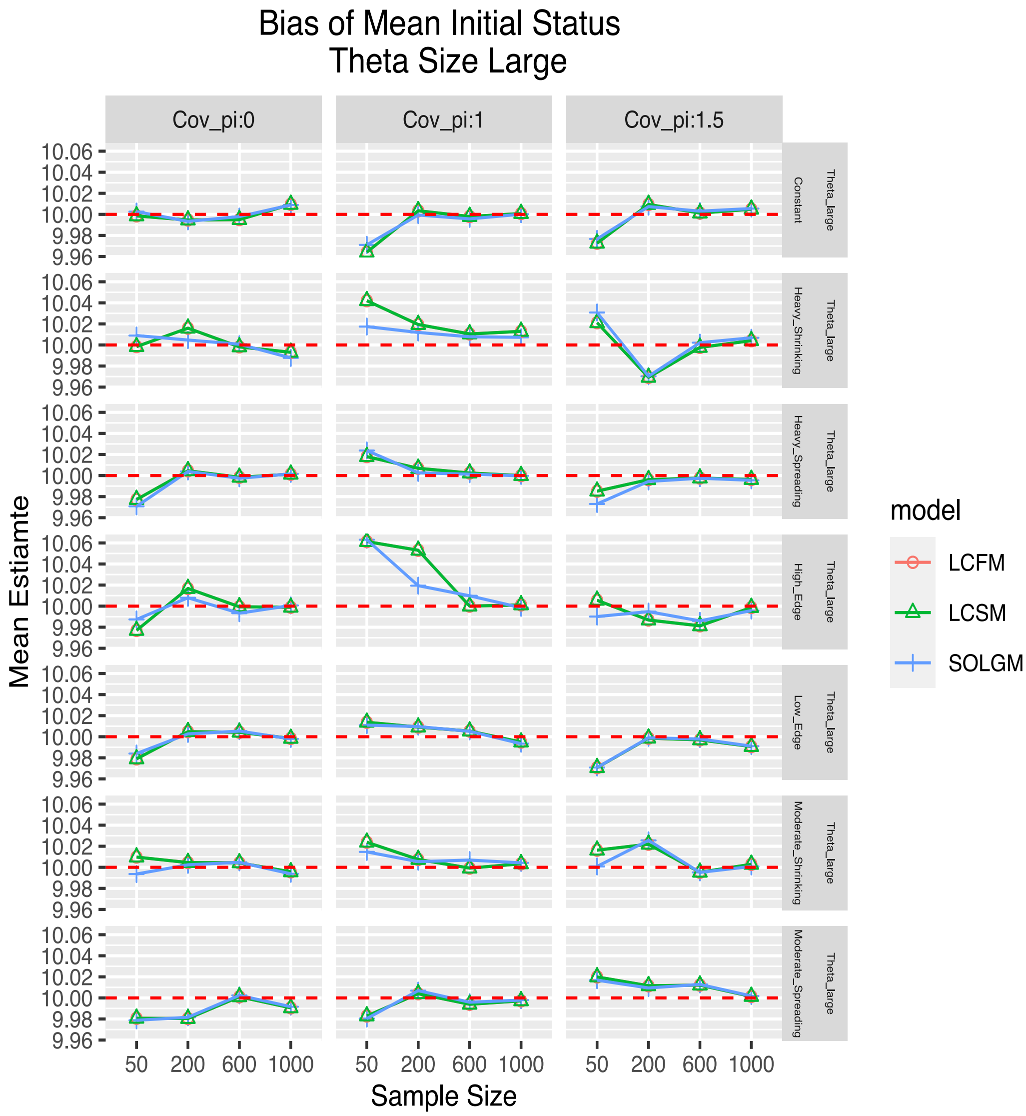

Figure A.6. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00}} = 10$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01}} = 3$ and $\boldsymbol{\beta_{11}} = 0$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

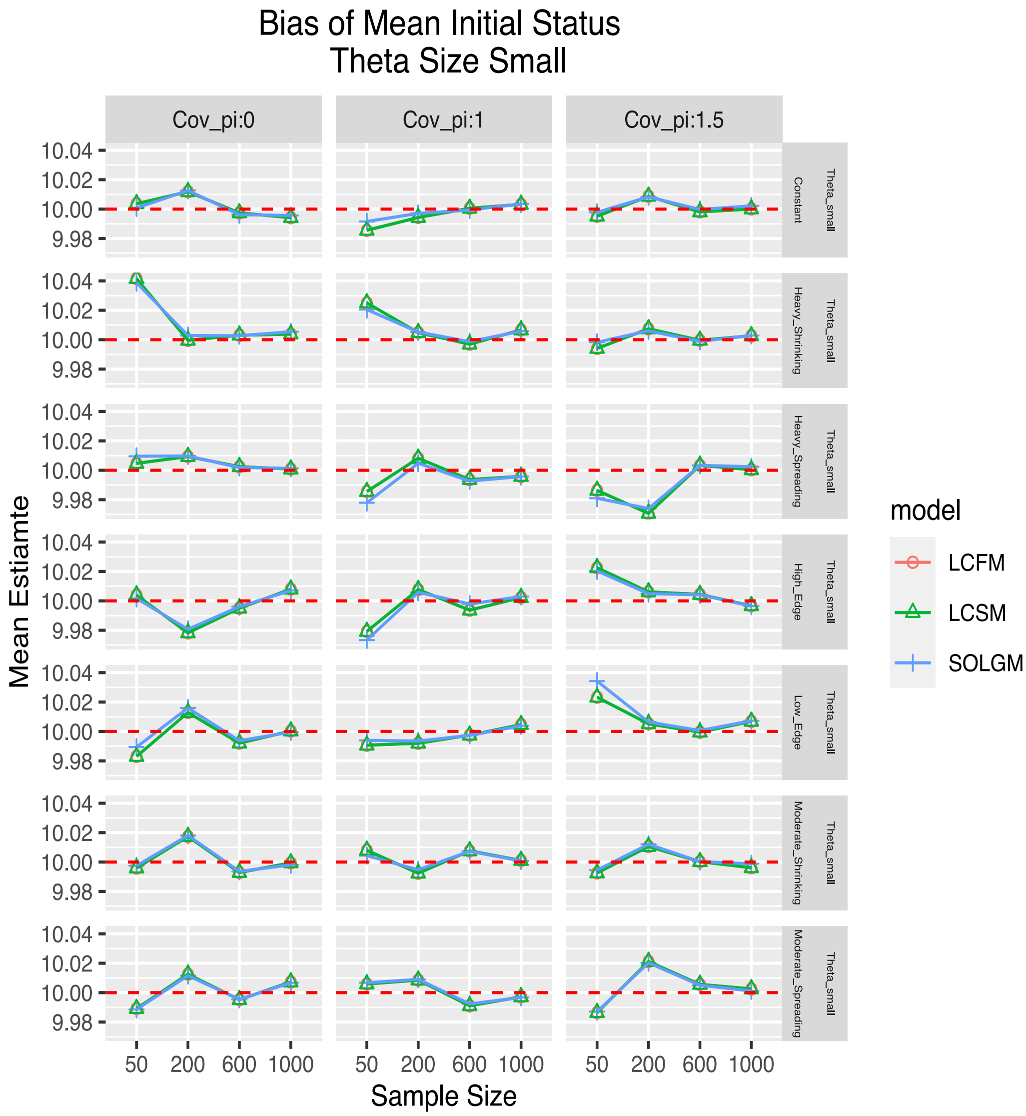

Figure A.7. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

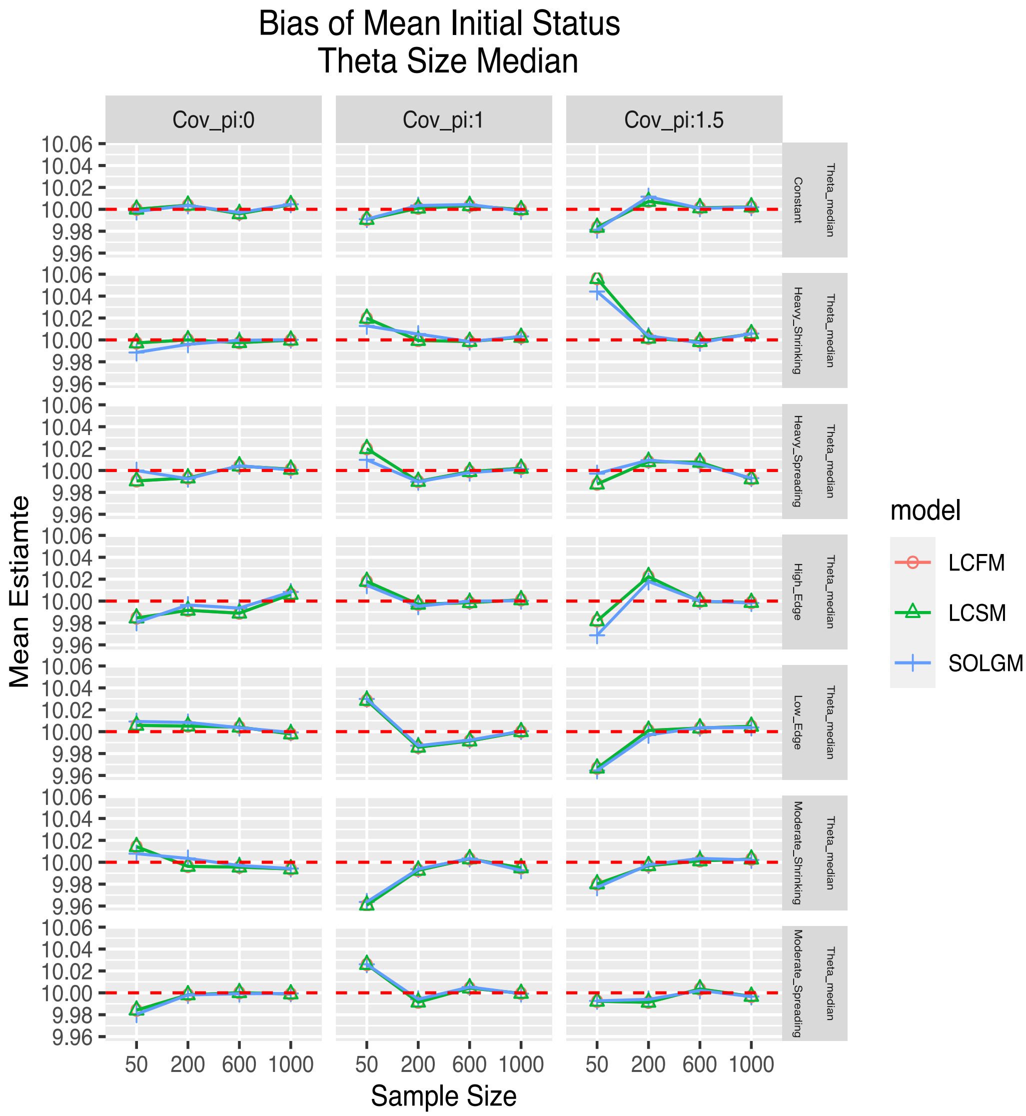

Figure A.8. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00}} = 10$, by time-specific error size (**median**) and correlation between intercept and slope ($Cor_p i$), when $\boldsymbol{\beta_{01}} = 3$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

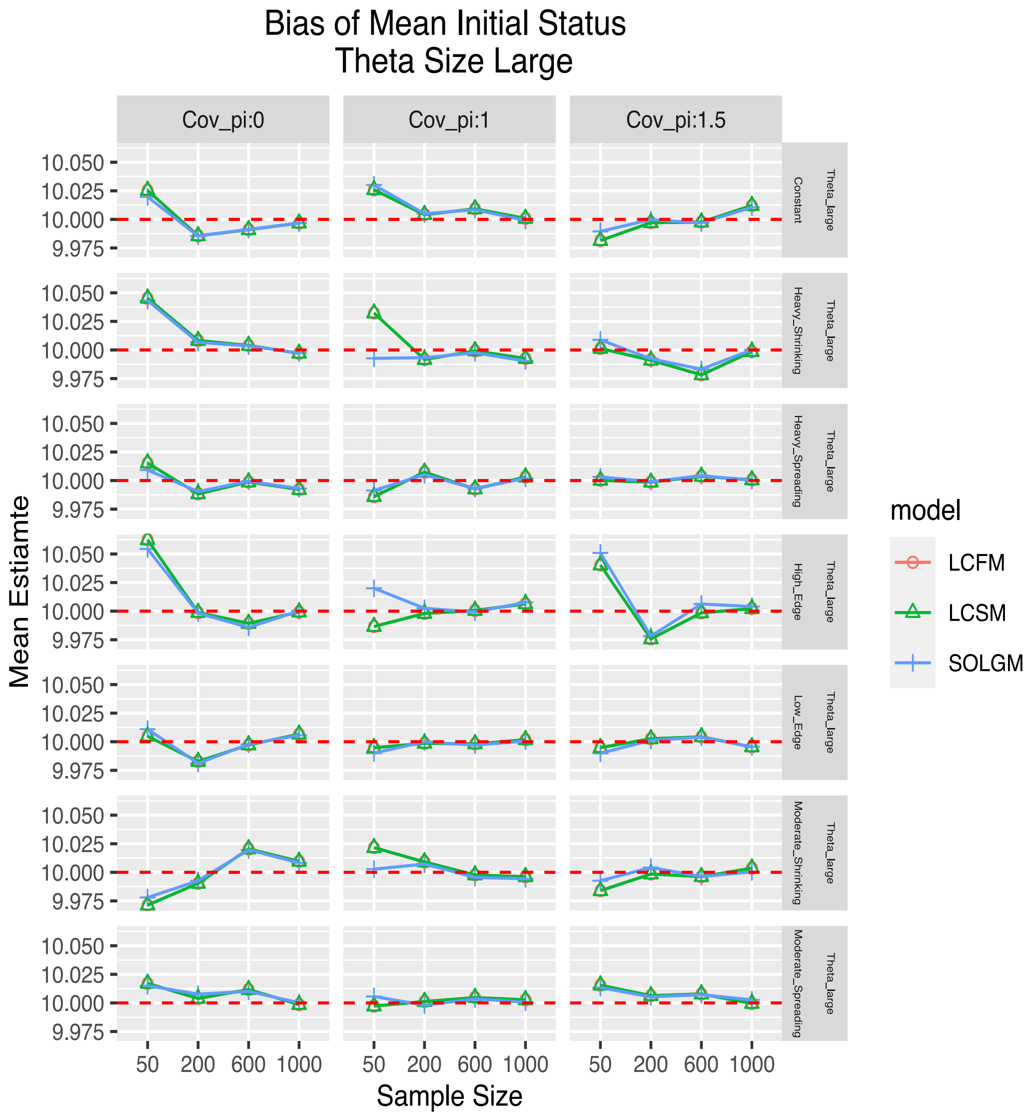

Figure A.9. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00}} = 10$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01}} = 3$ and $\boldsymbol{\beta_{11}} = 2$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

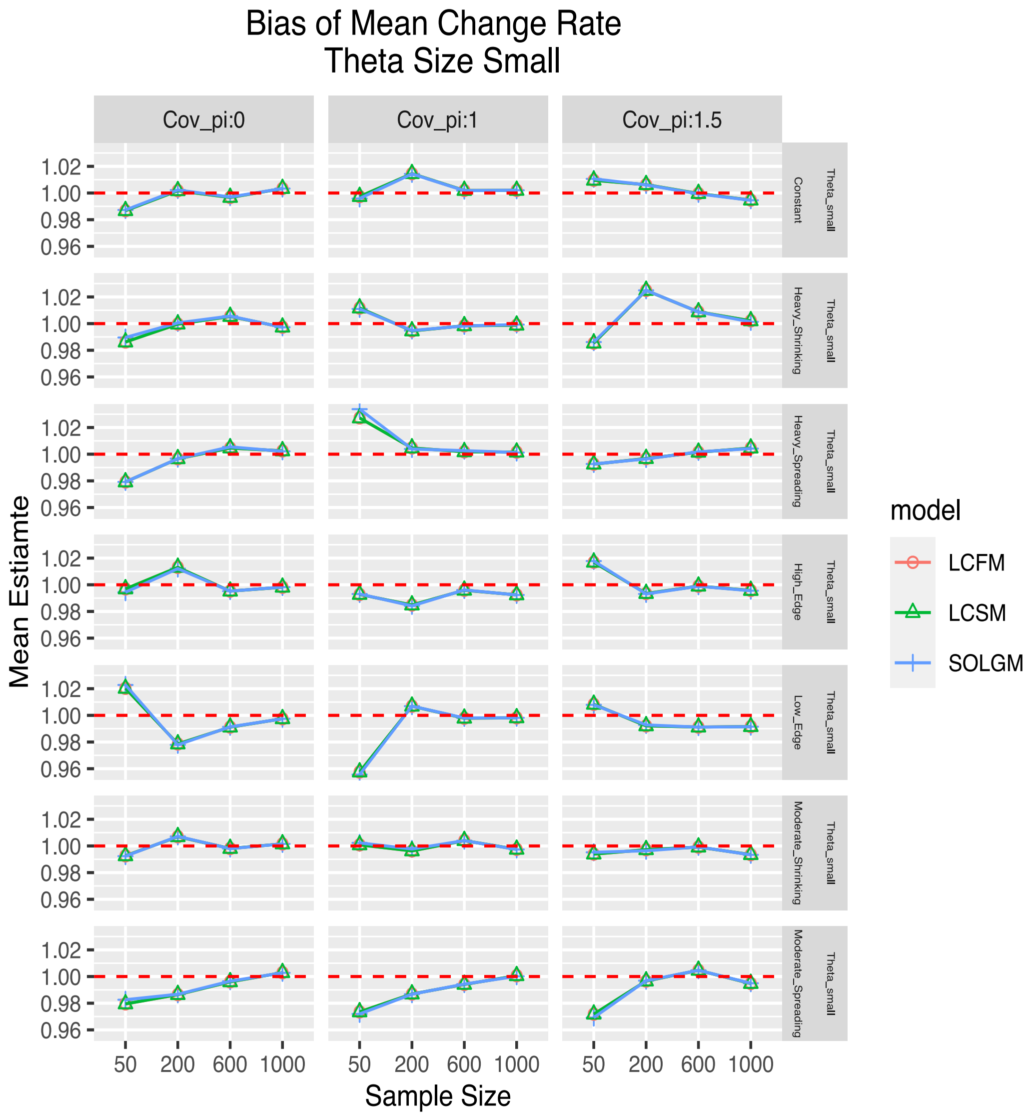

Figure A.10. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

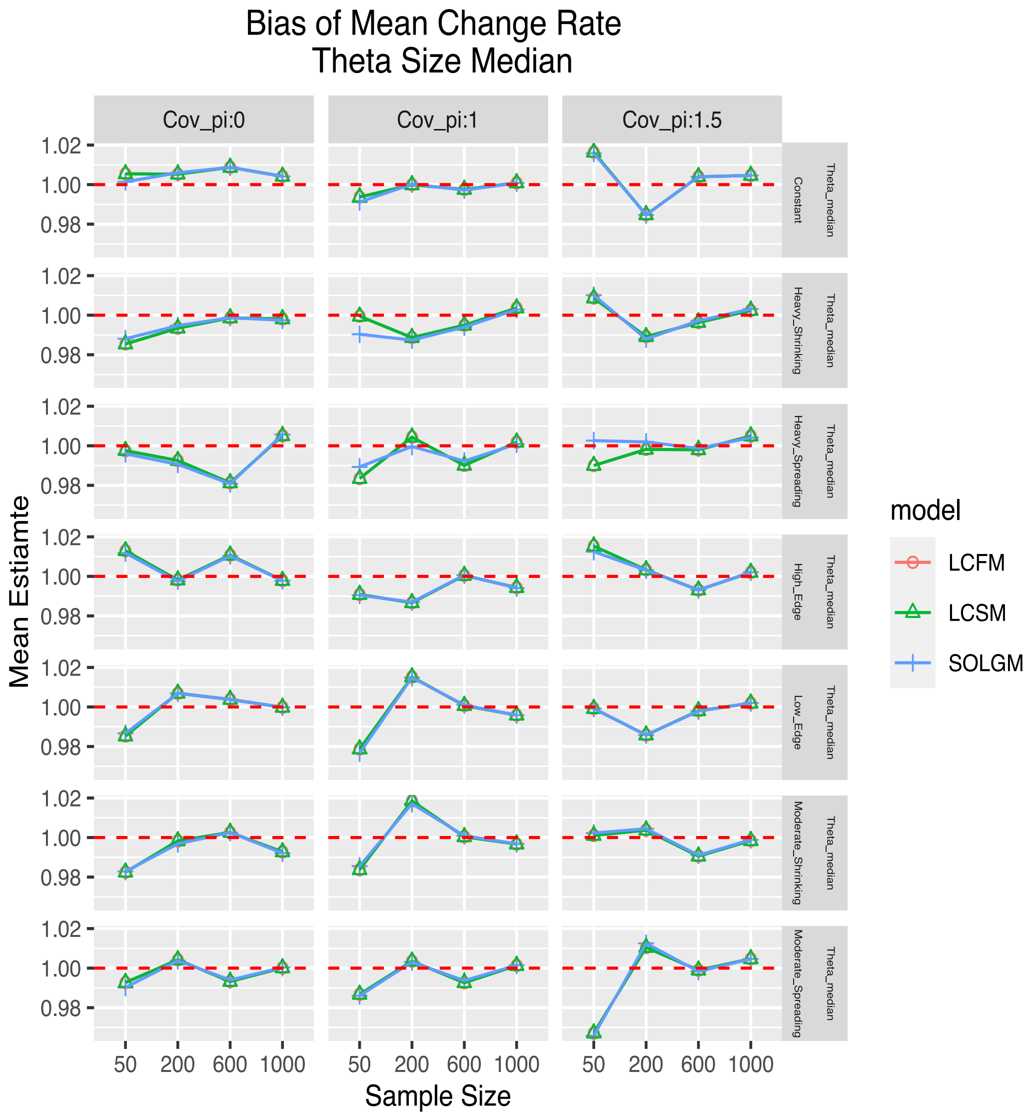

Figure A.11. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

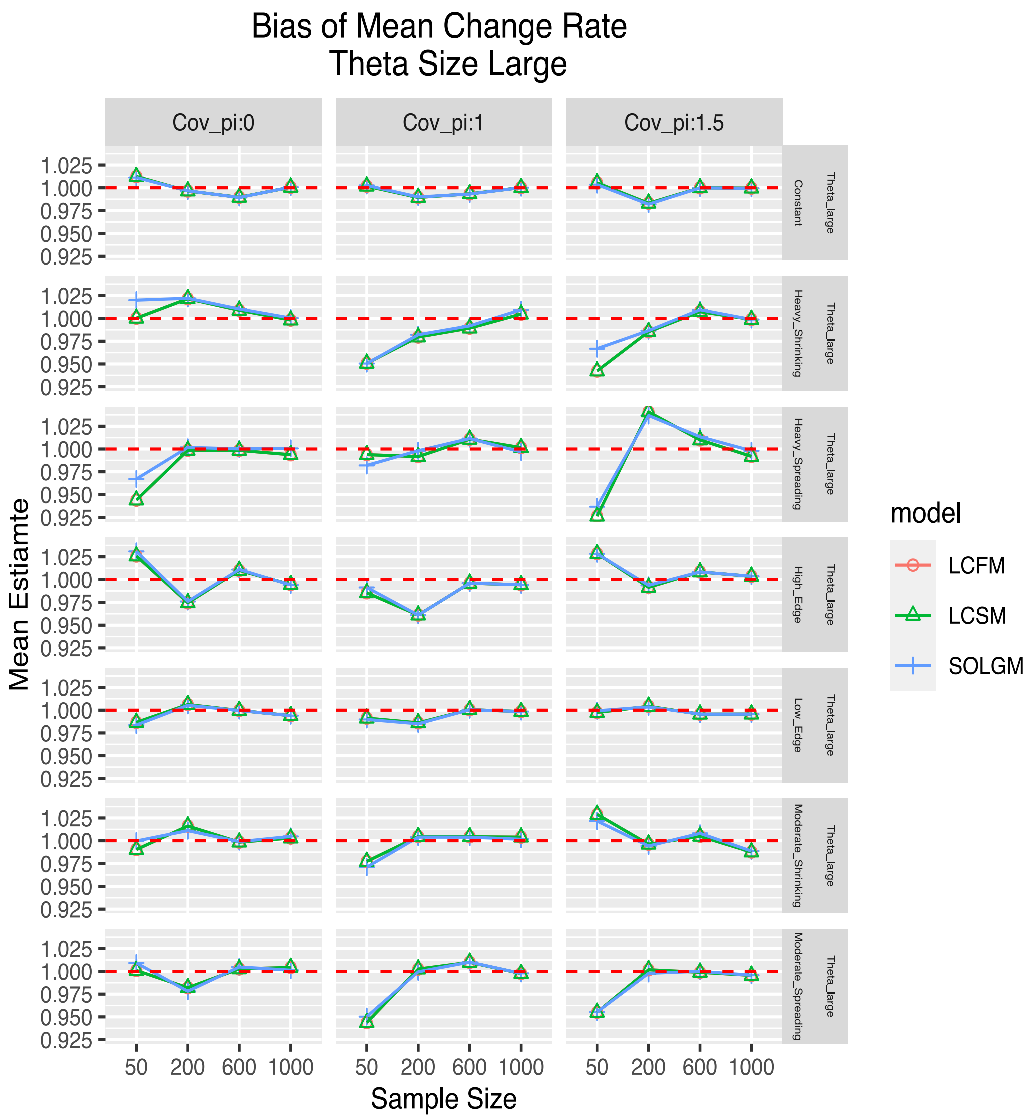

Figure A.12. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

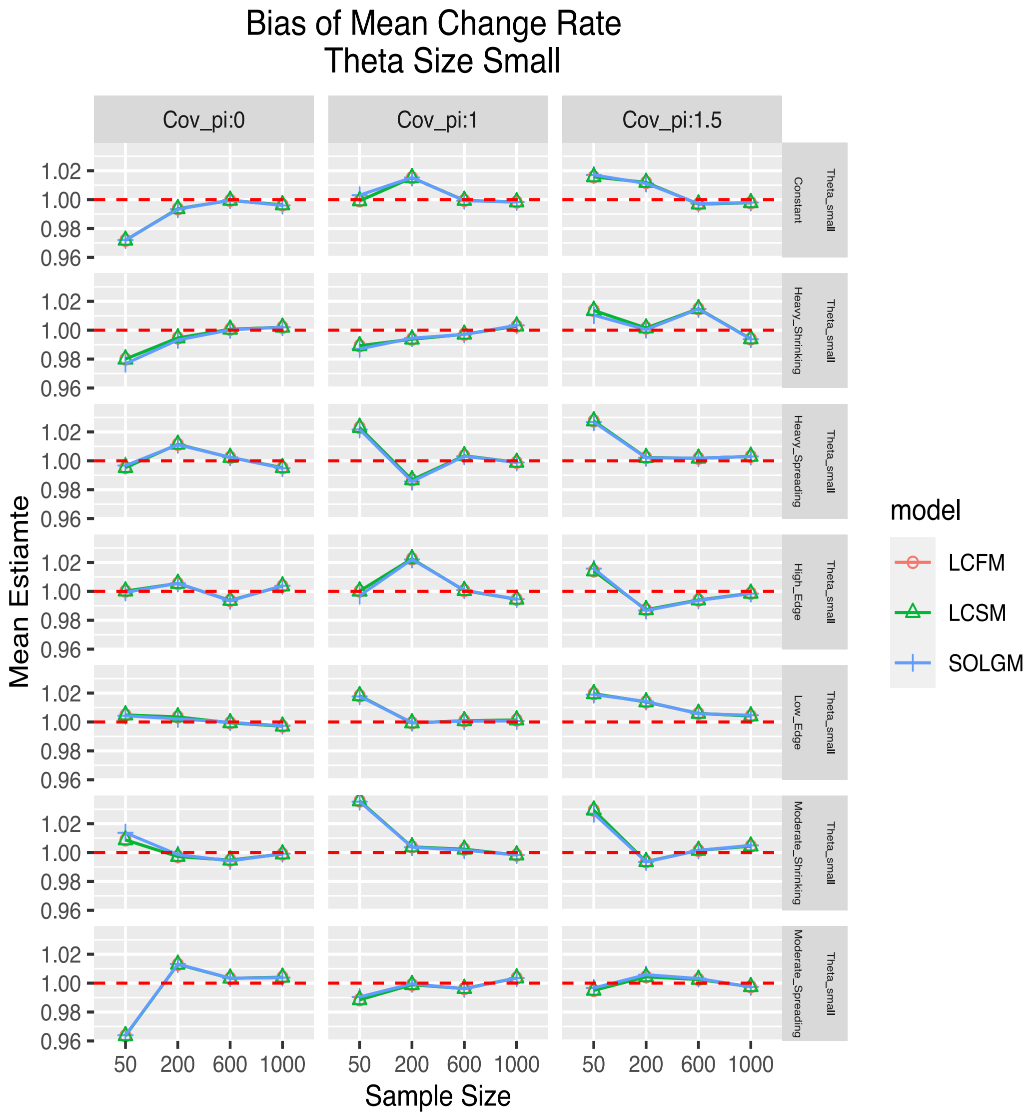

Figure A.13. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

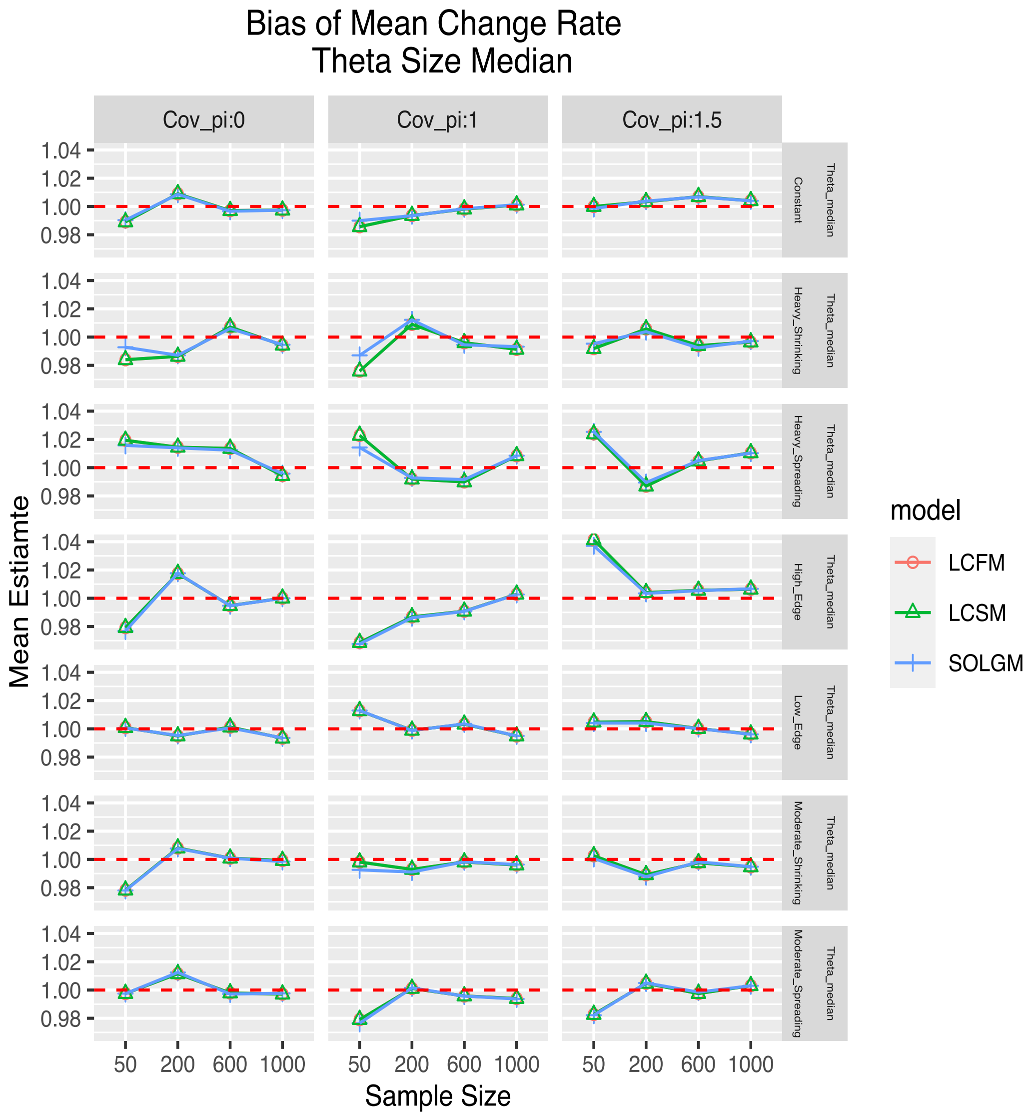

Figure A.14. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

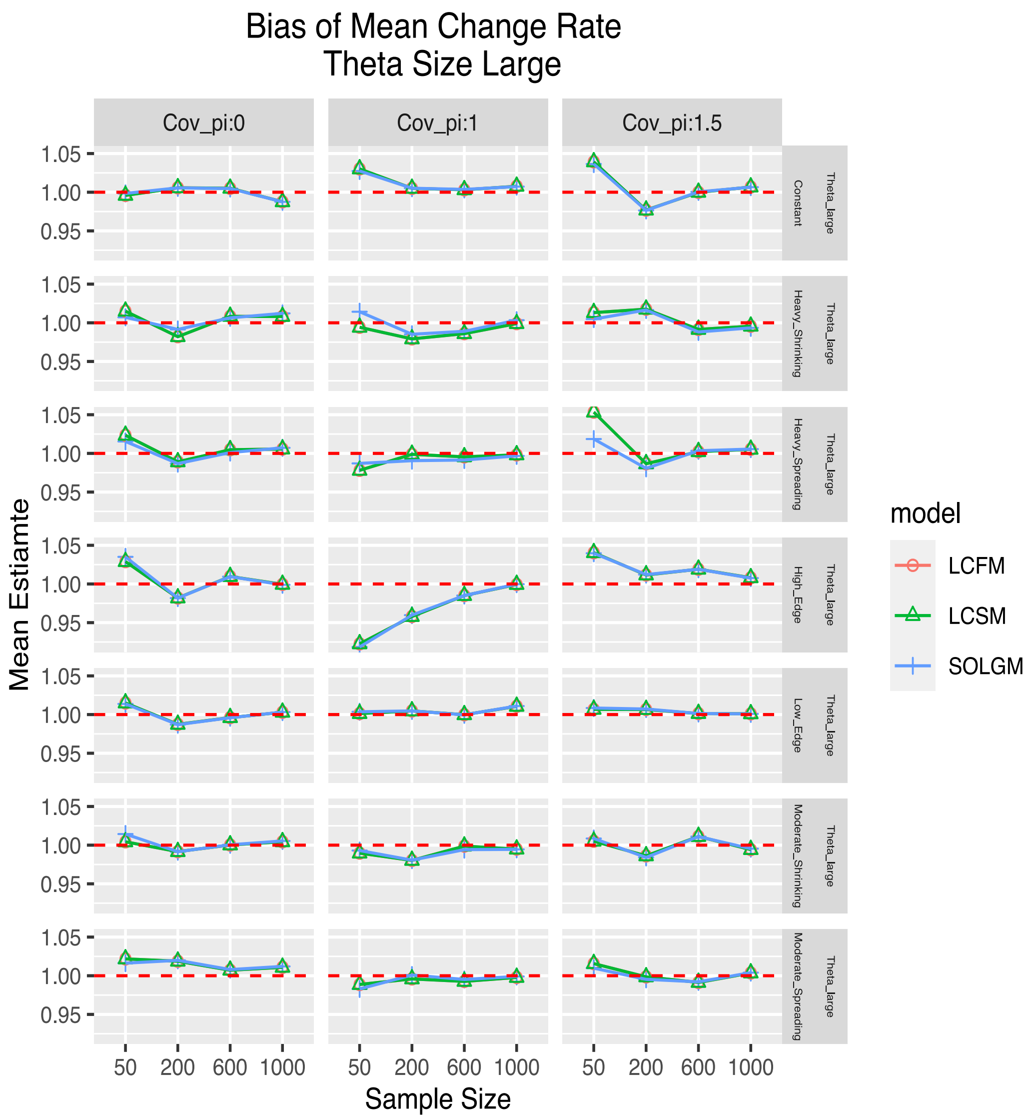

Figure A.15. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

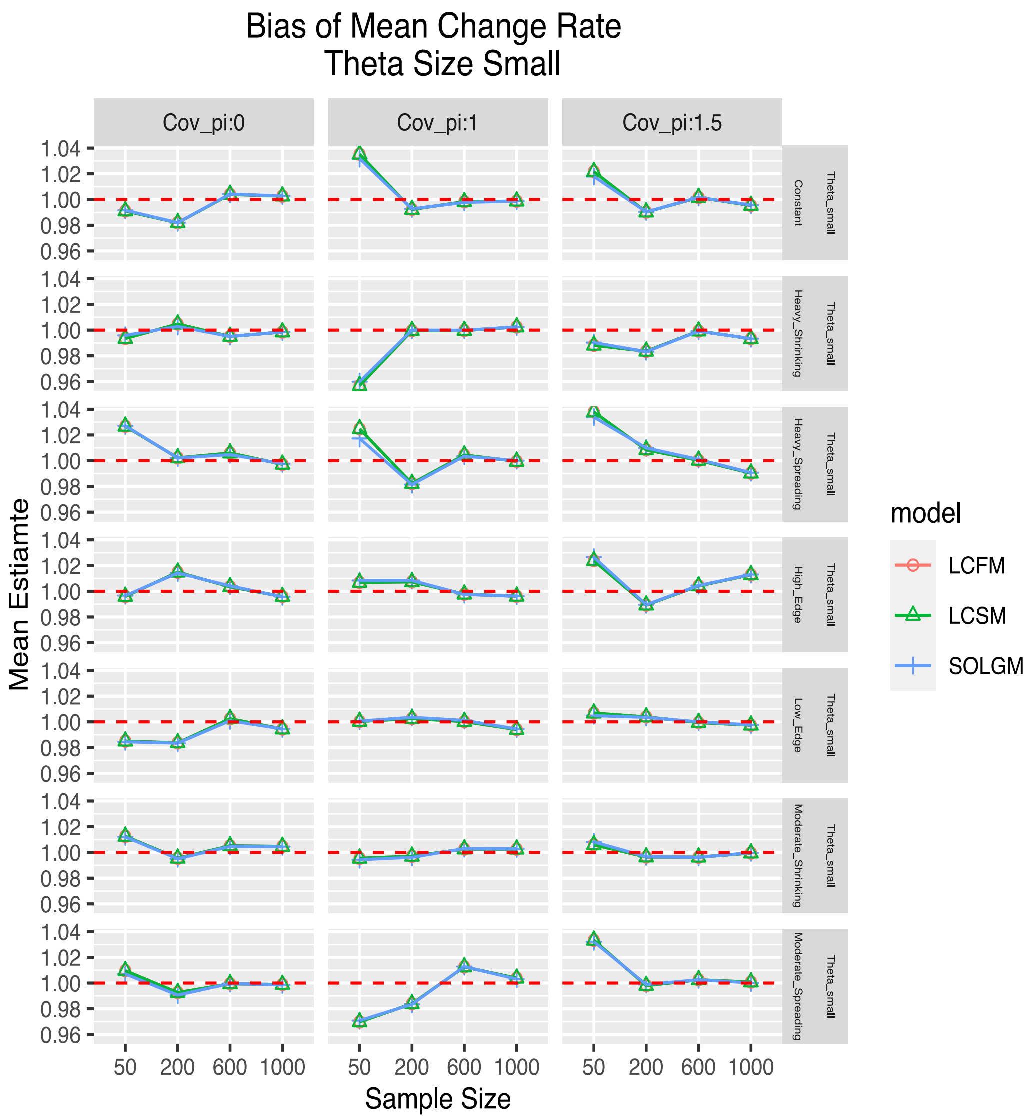

Figure A.16. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

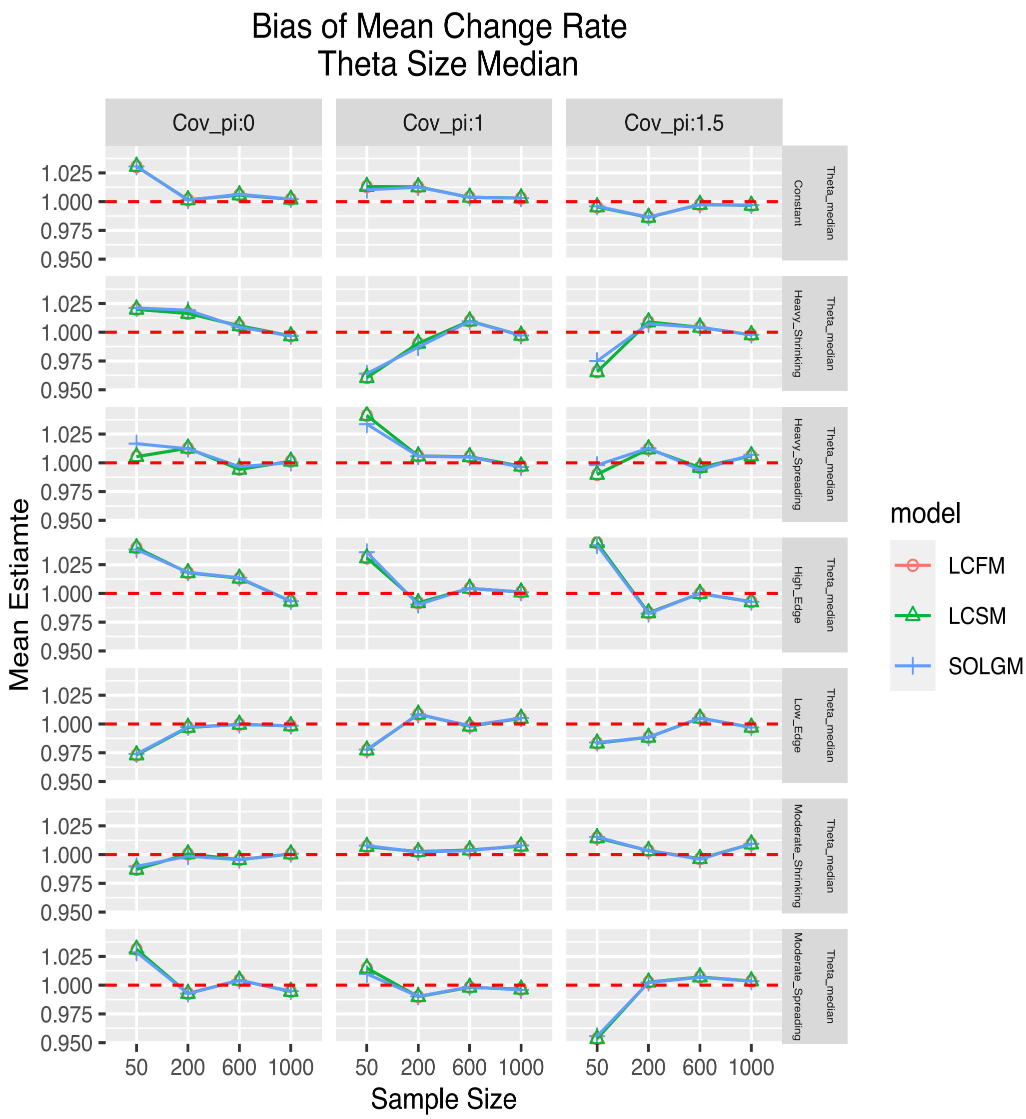

Figure A.17. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

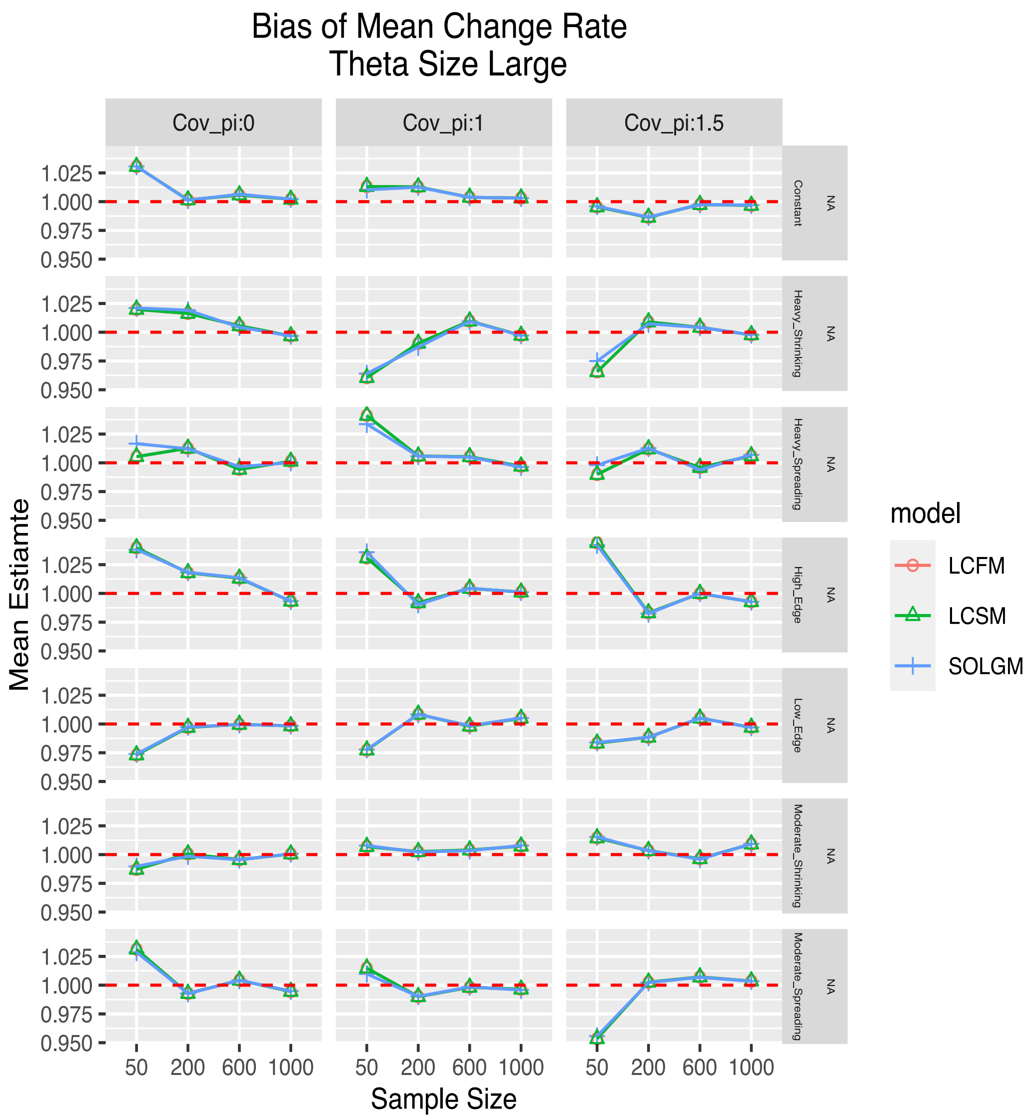

Figure A.18. Bias of the Average Growth Rate for a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

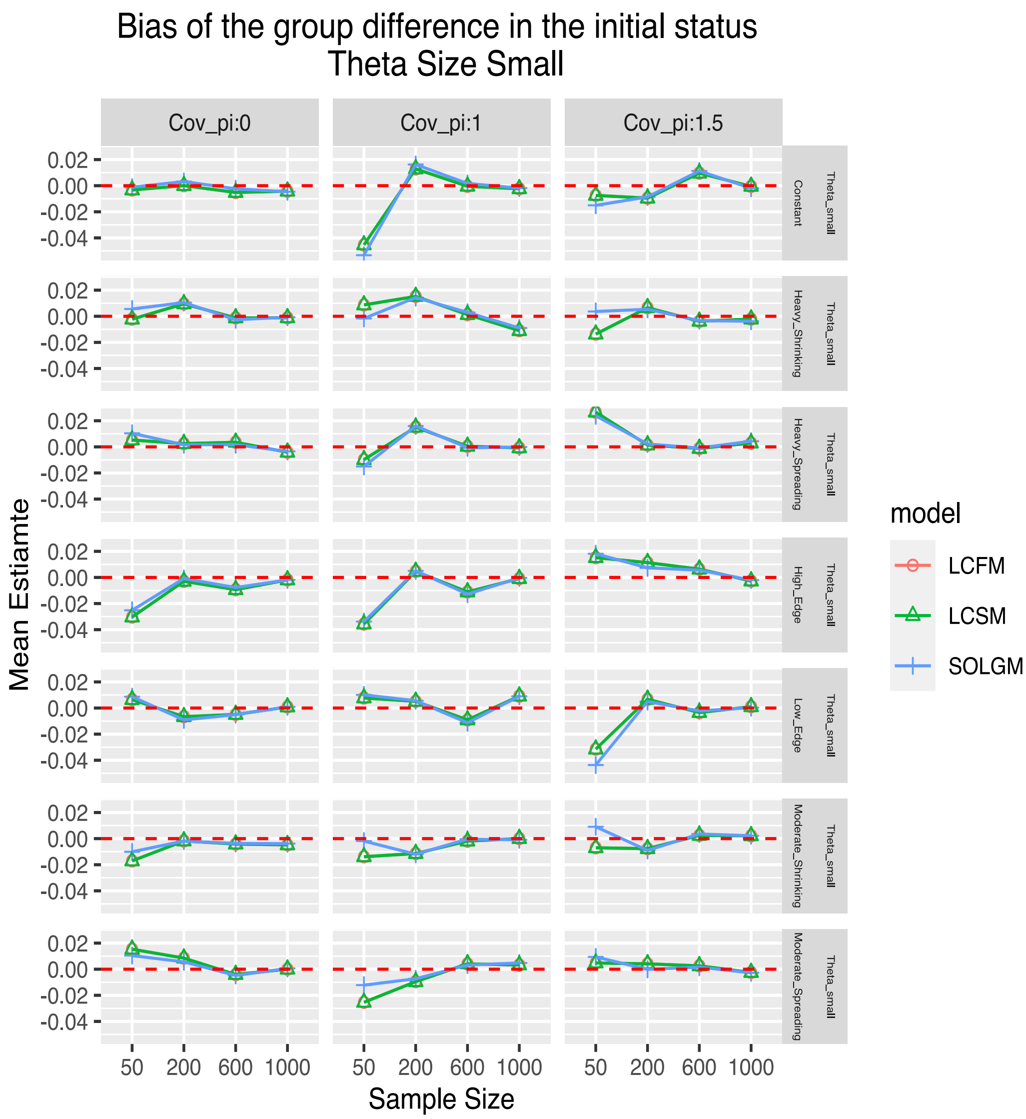

Figure A.19. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

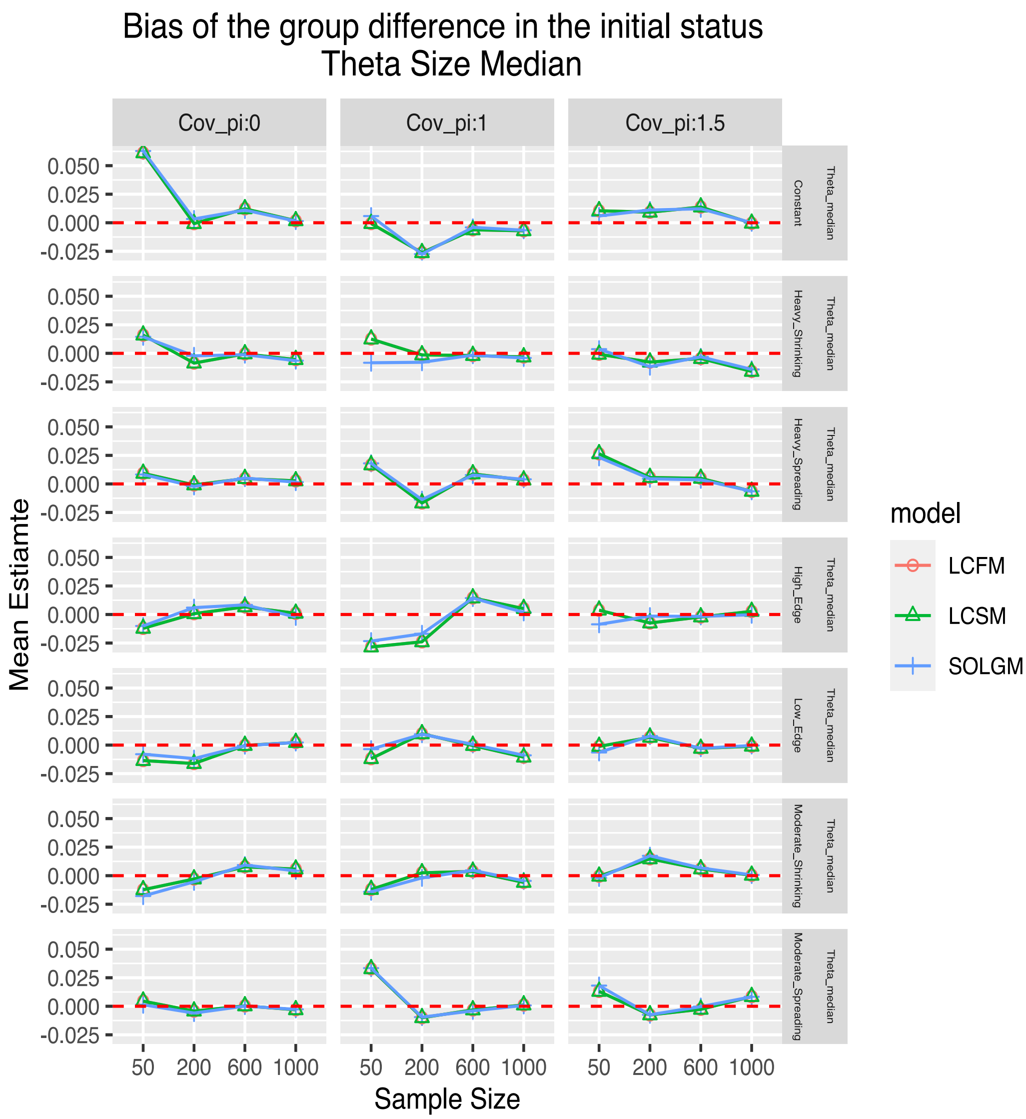

Figure A.20. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

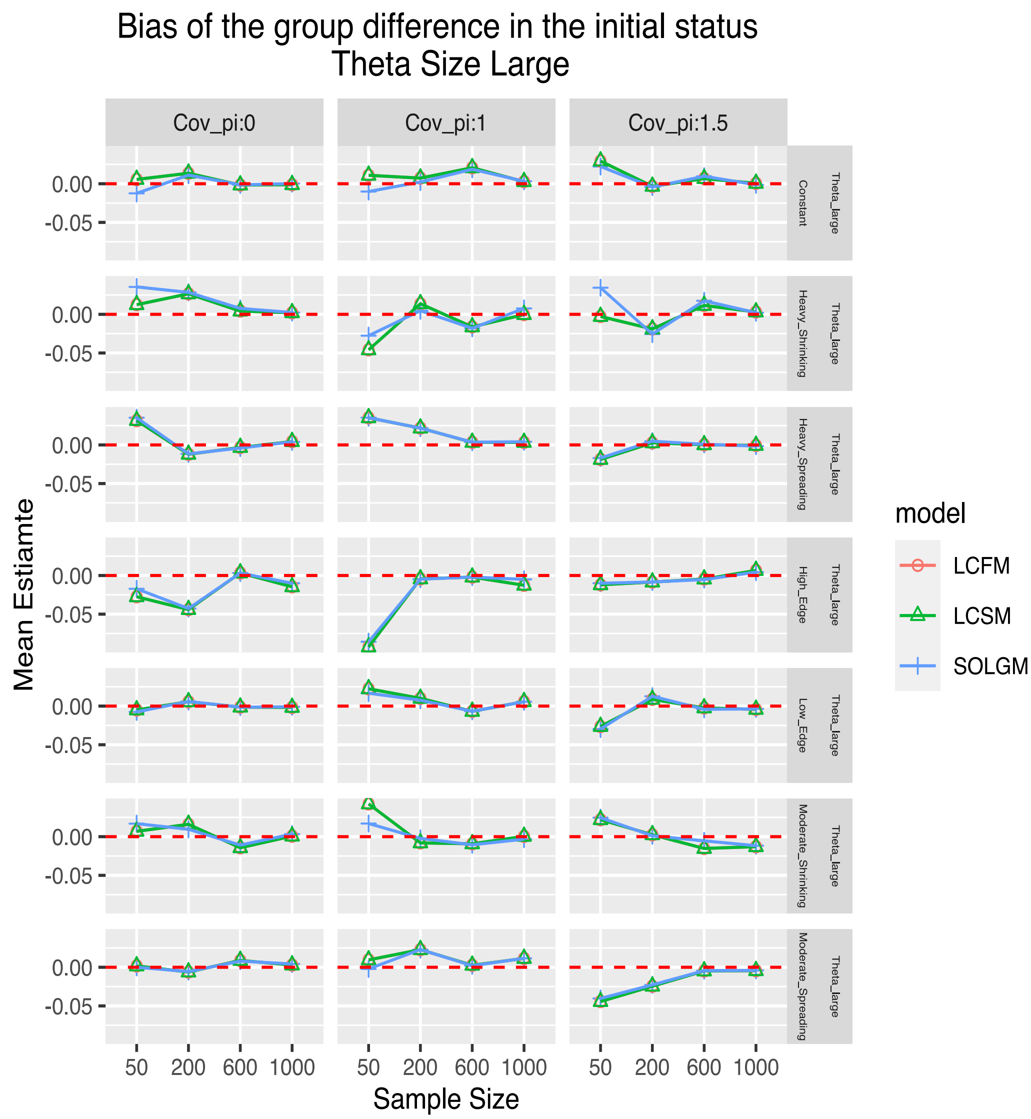

Figure A.21. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

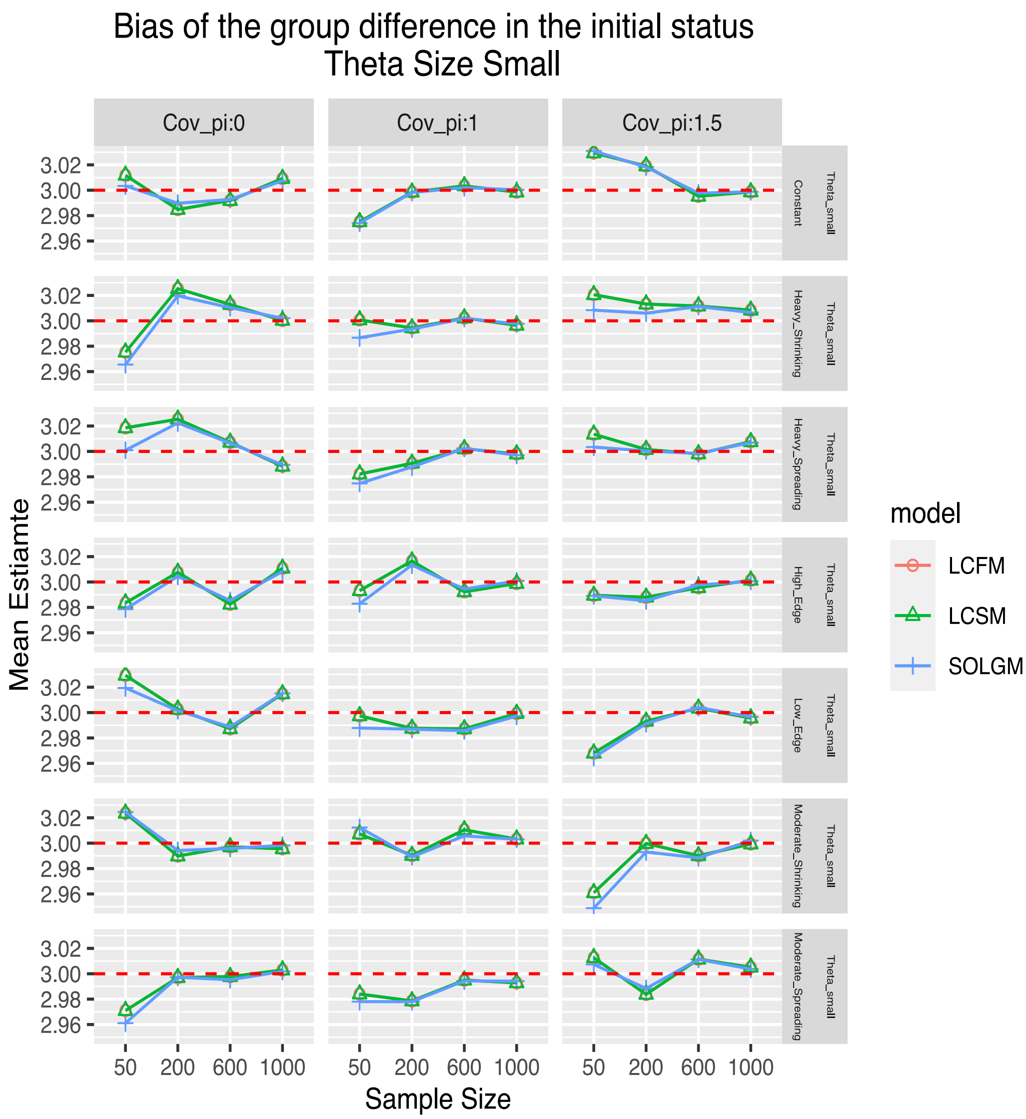

Figure A.22. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

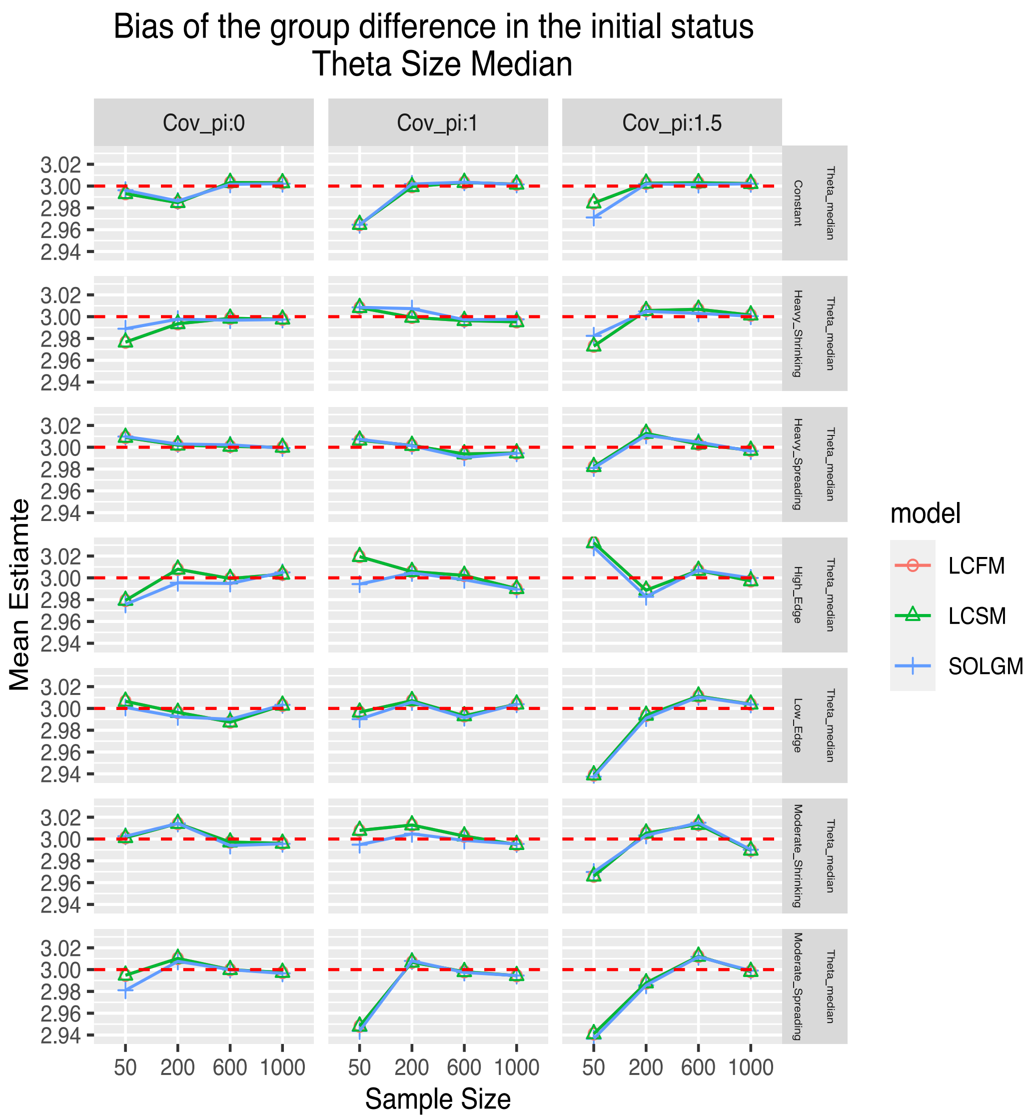

Figure A.23. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

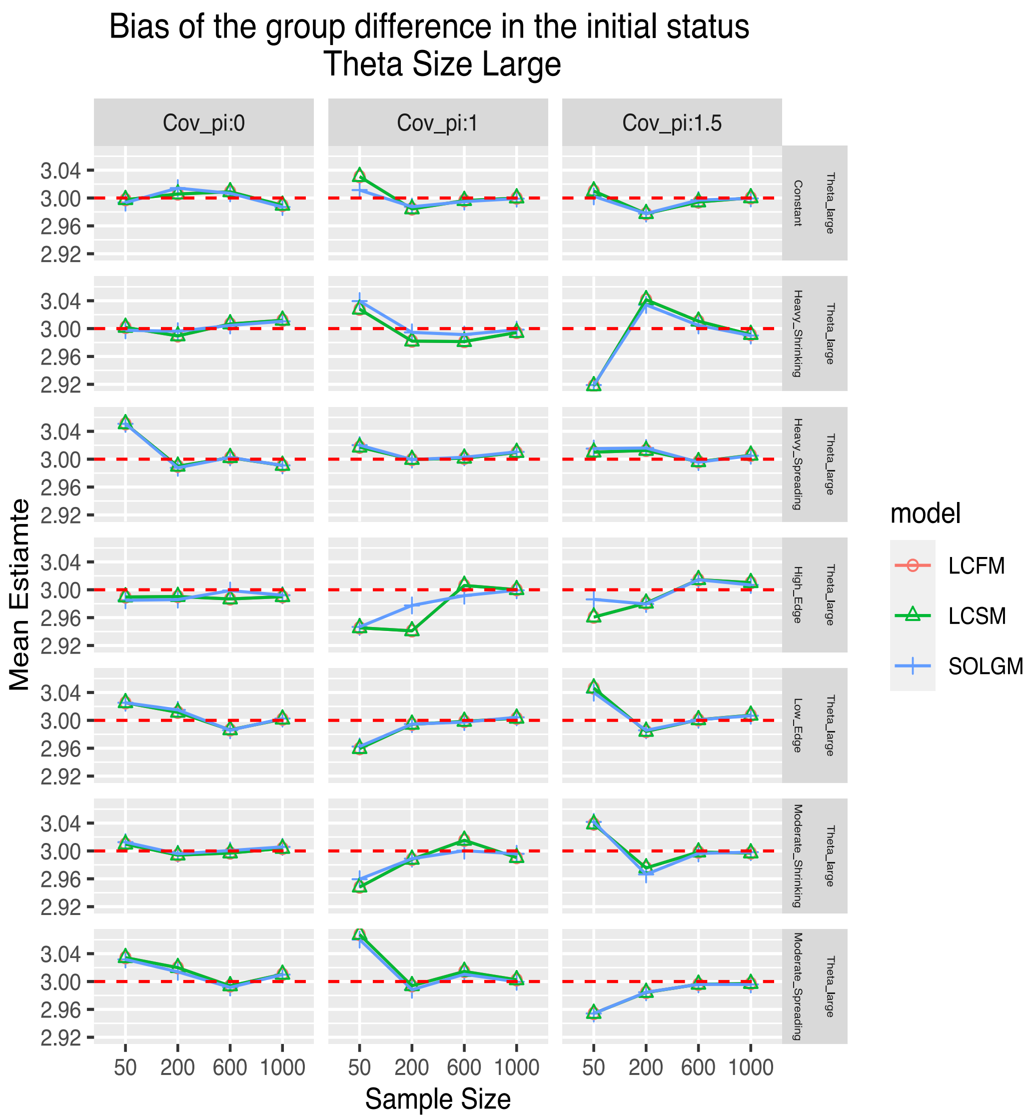

Figure A.24. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

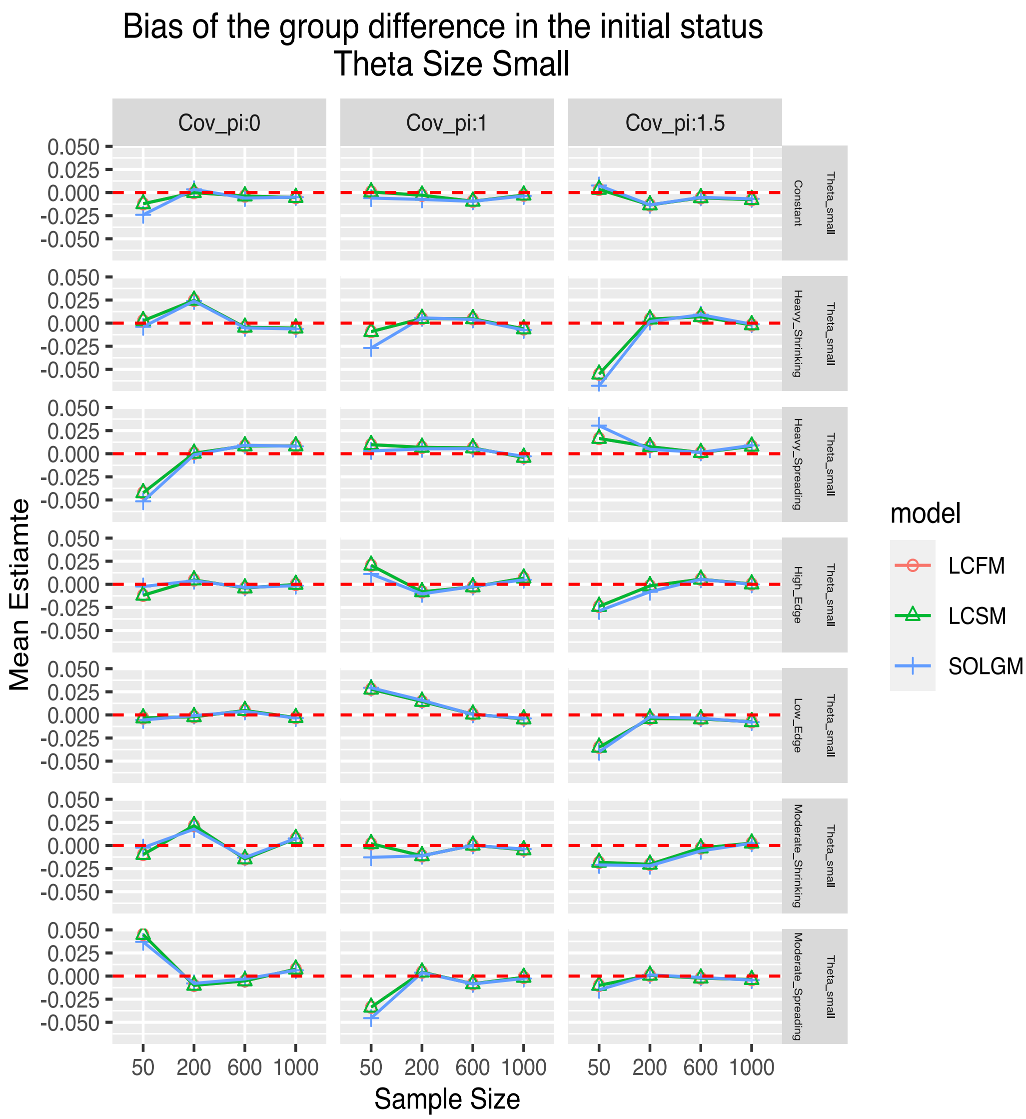

Figure A.25. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

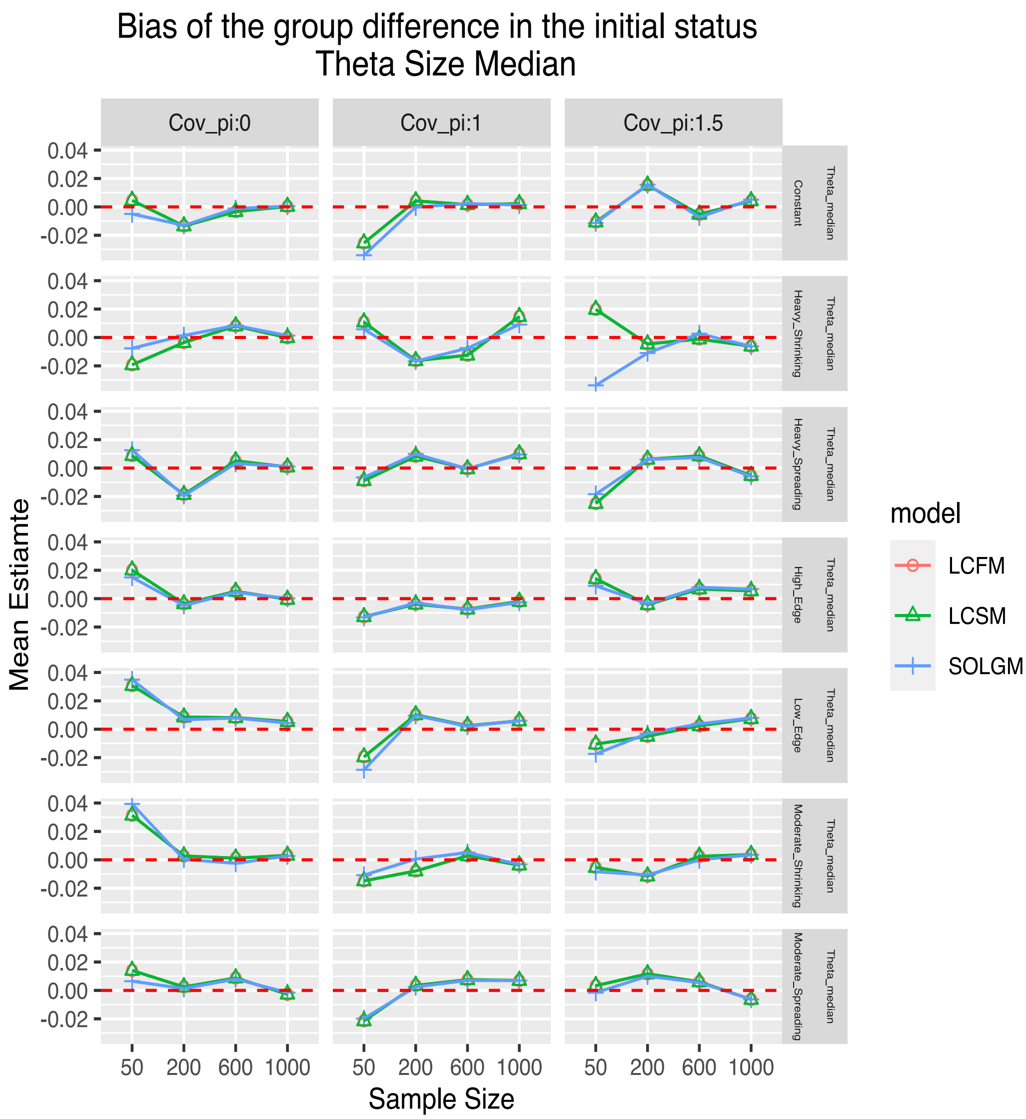

Figure A.26. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

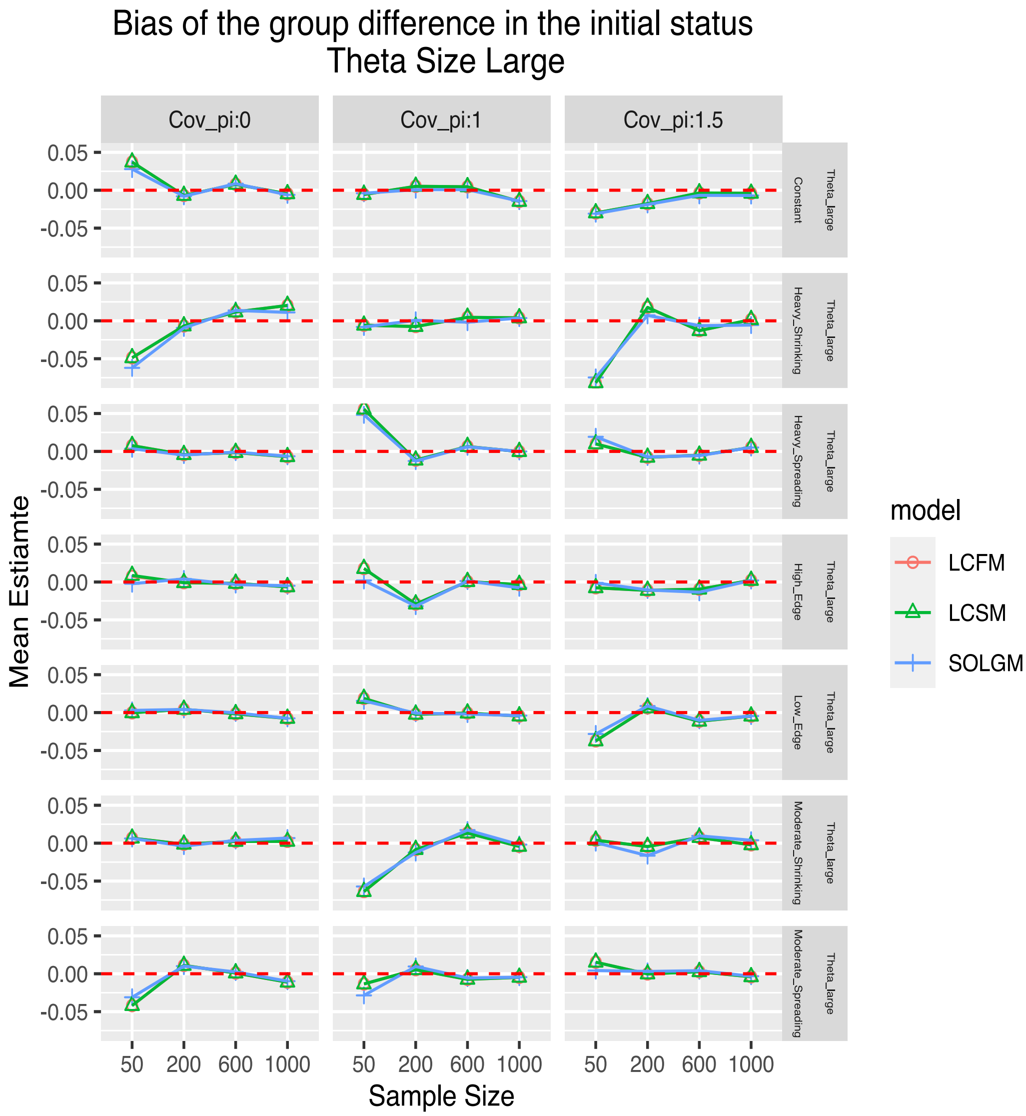

Figure A.27. Bias of the Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

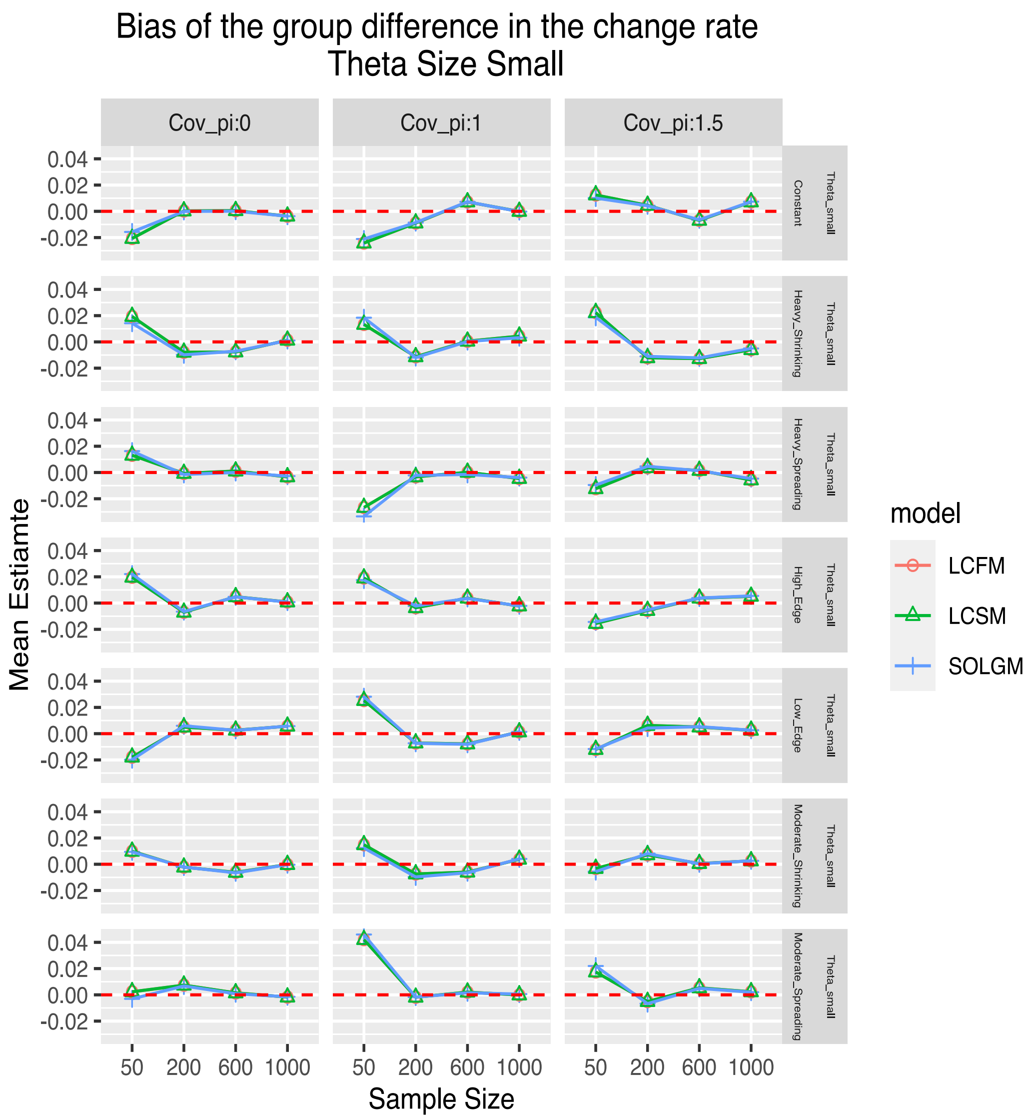

Figure A.28. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

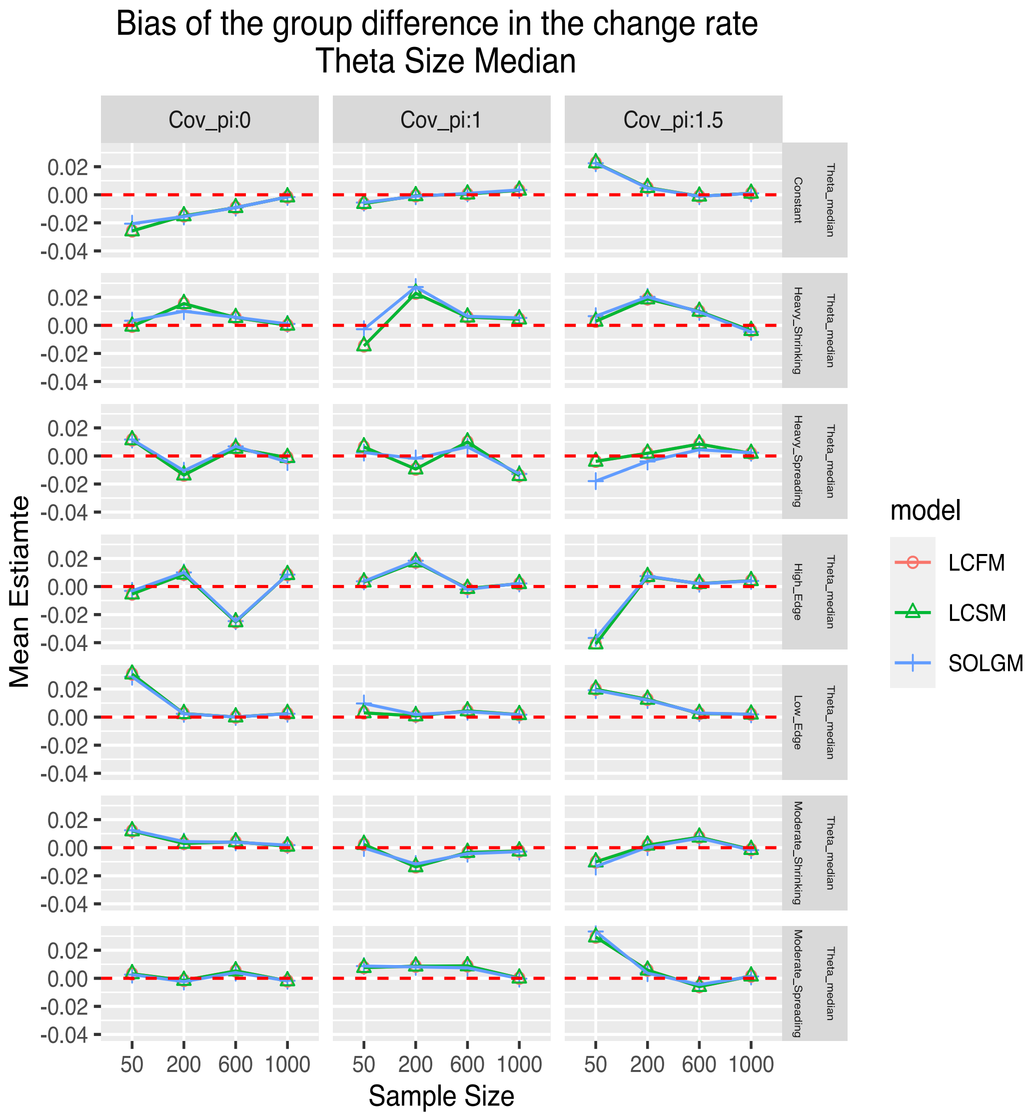

Figure A.29. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

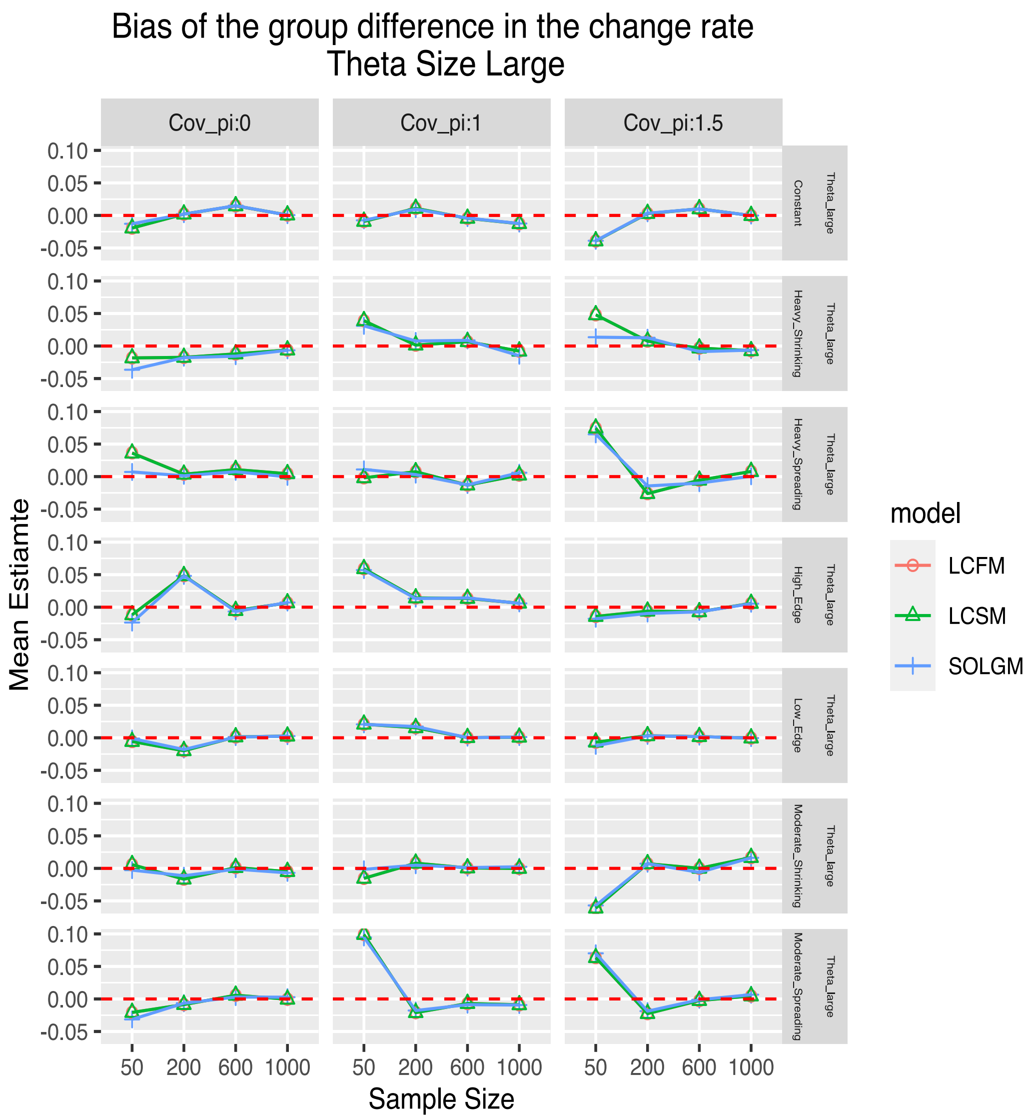

Figure A.30. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

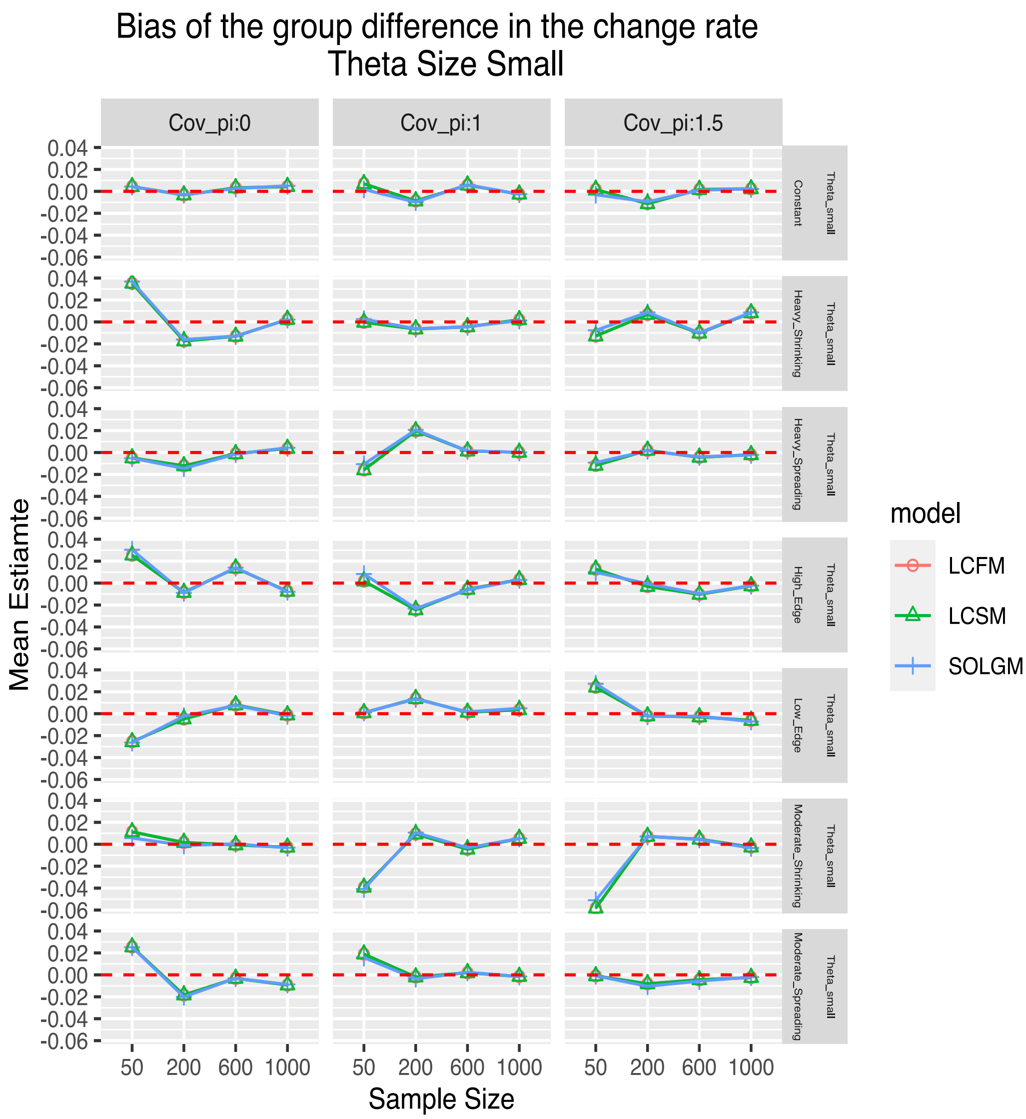

Figure A.31. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

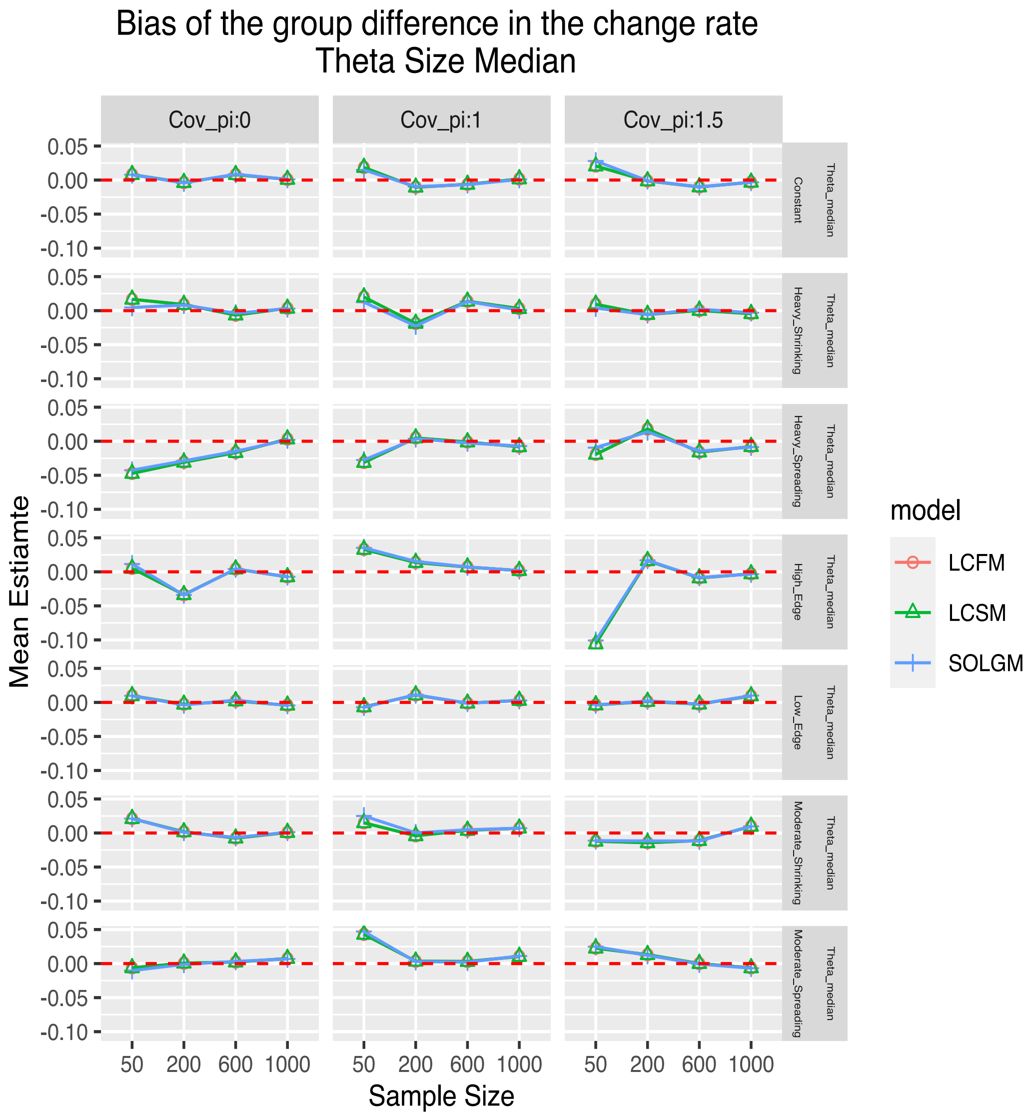

Figure A.32. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

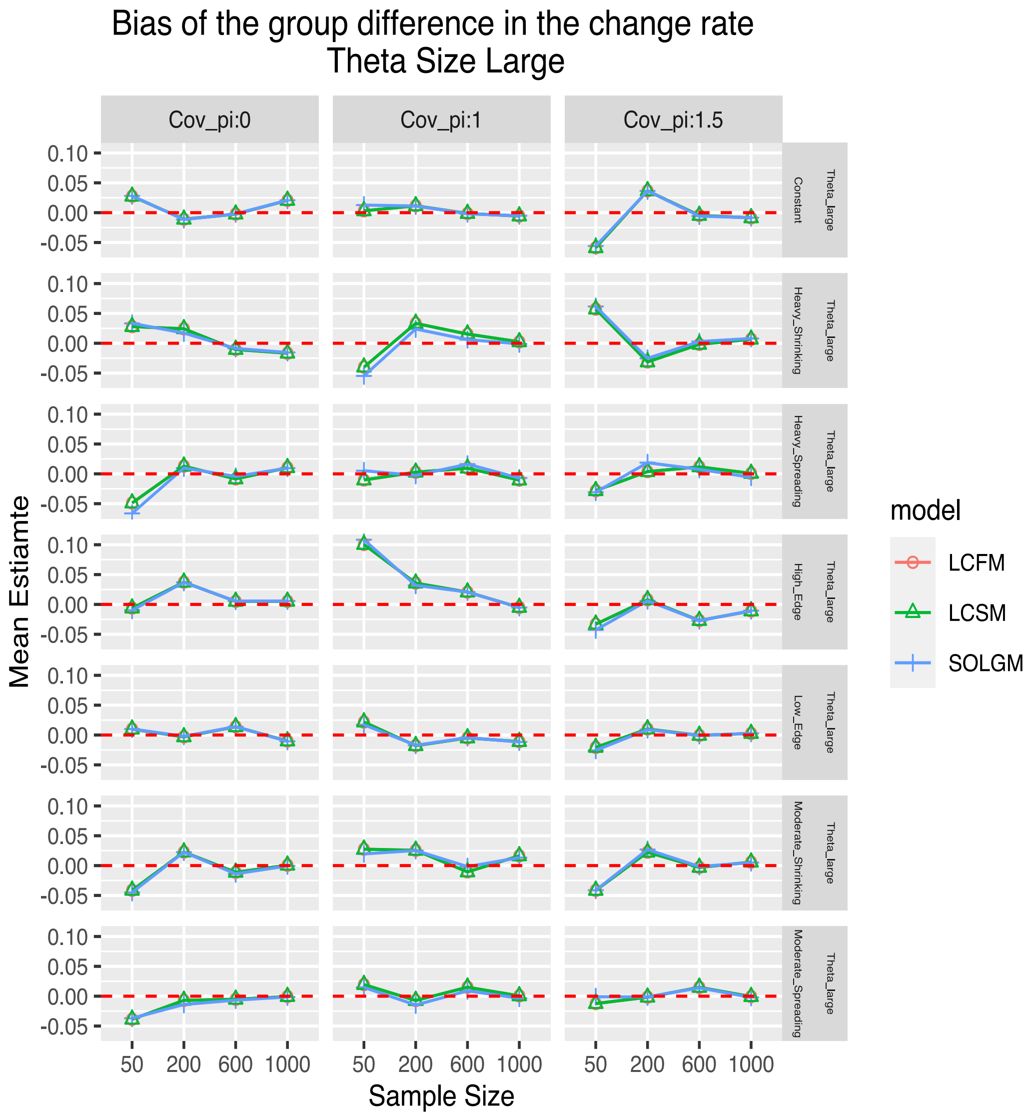

Figure A.33. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

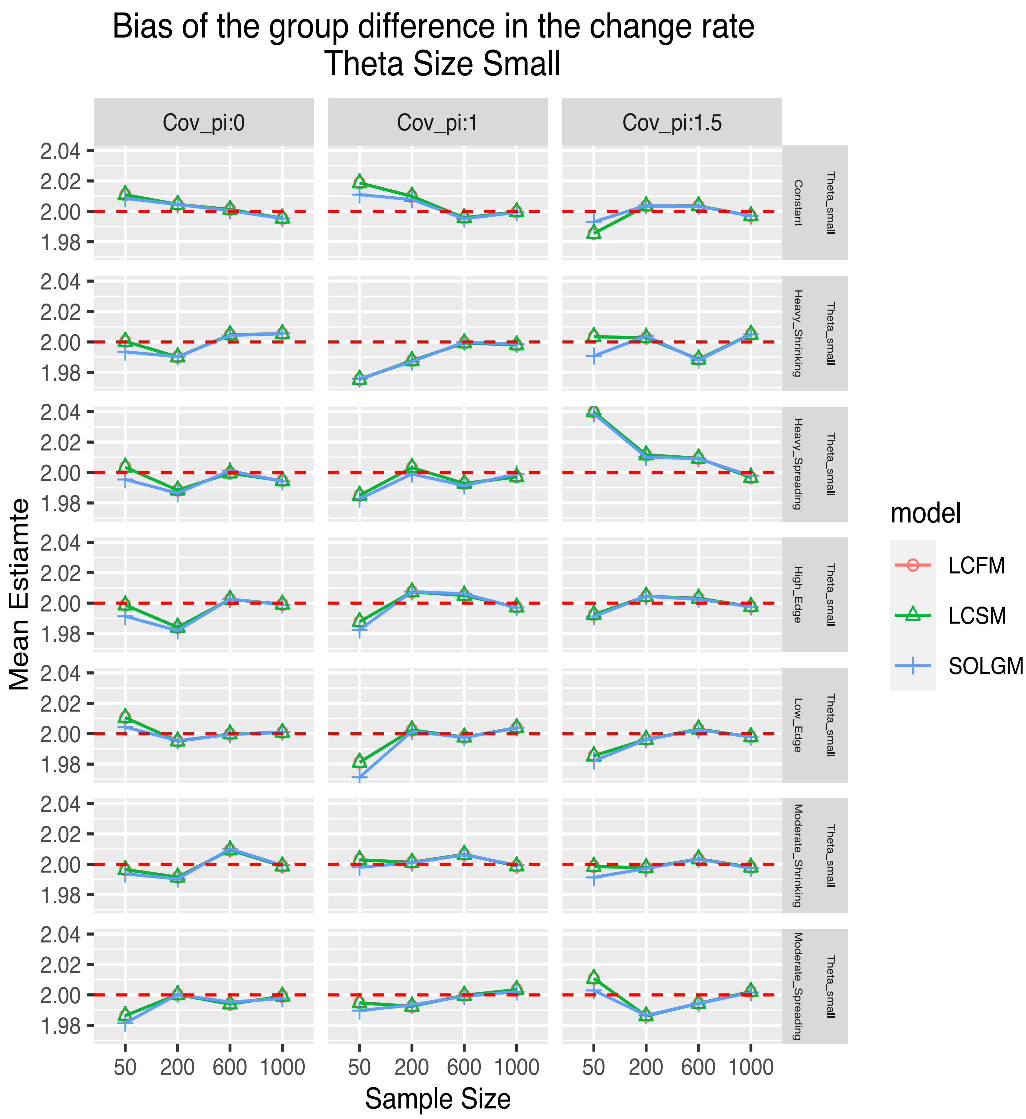

Figure A.34. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

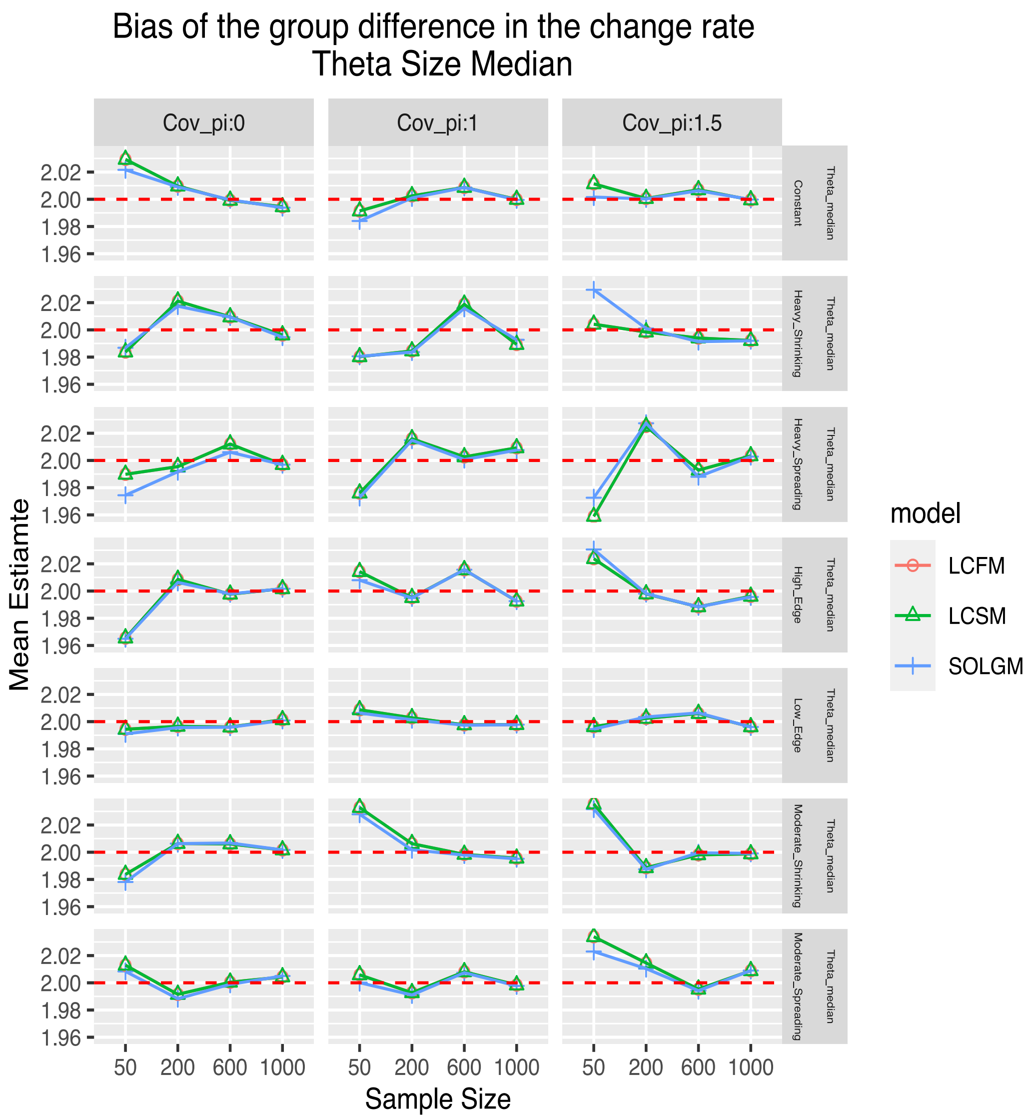

Figure A.35. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

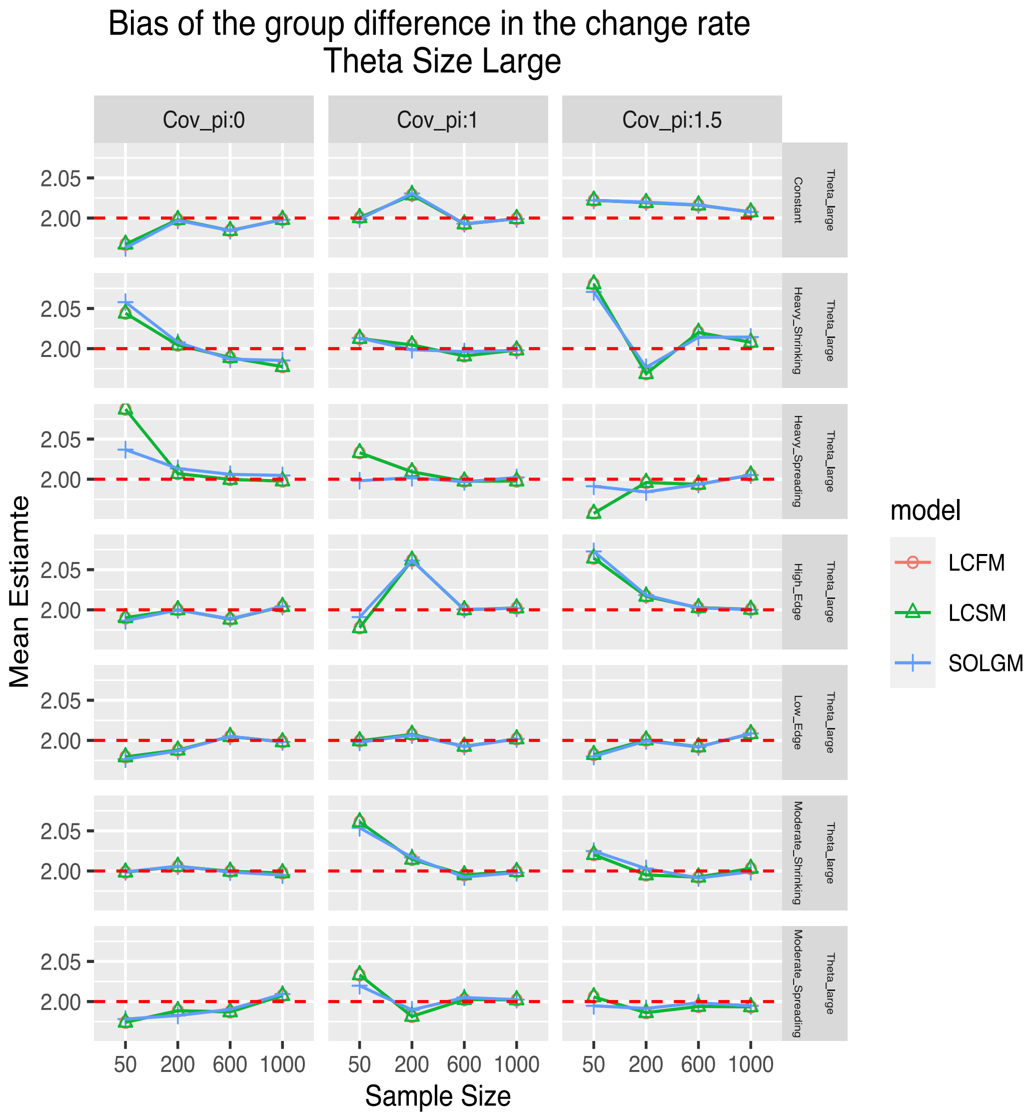

Figure A.36. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

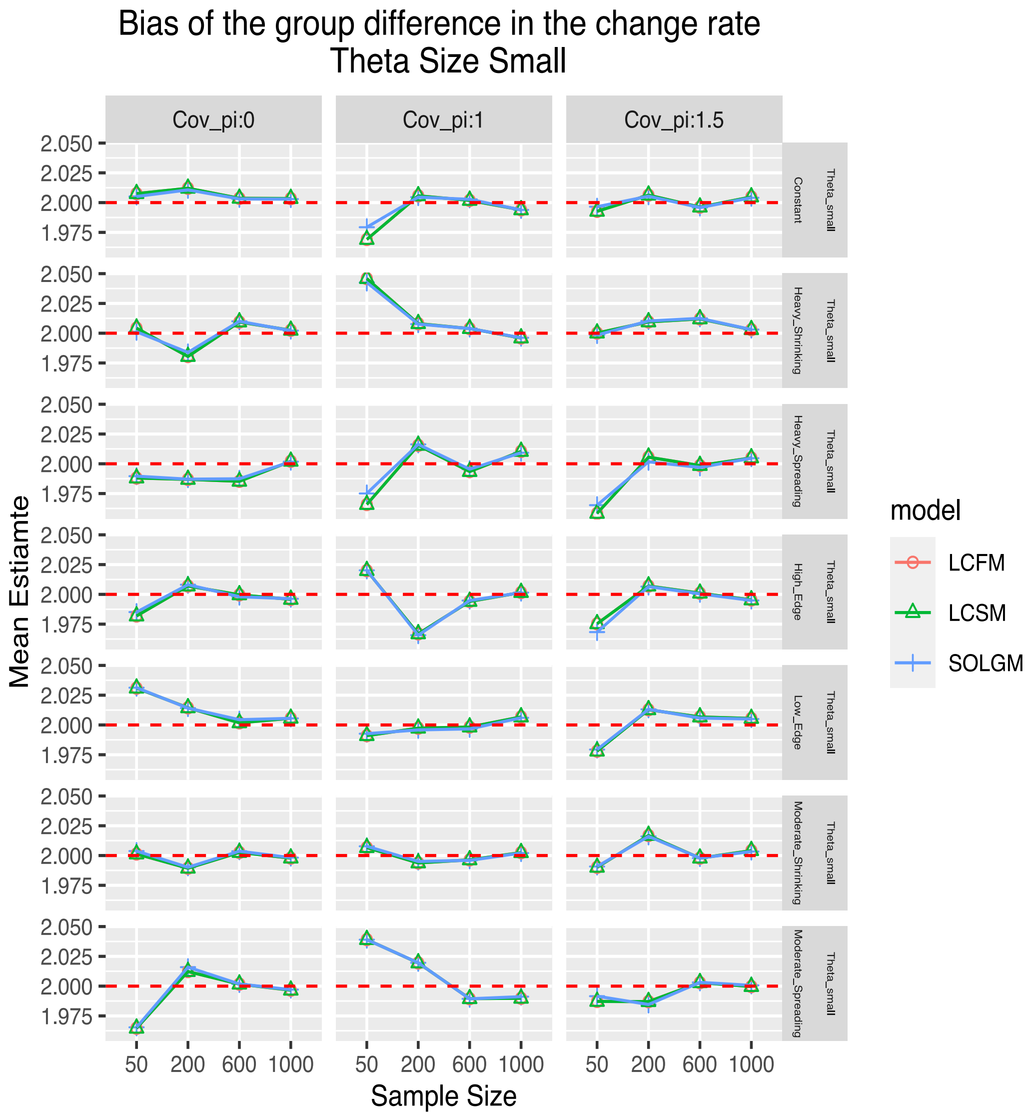

Figure A.37. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

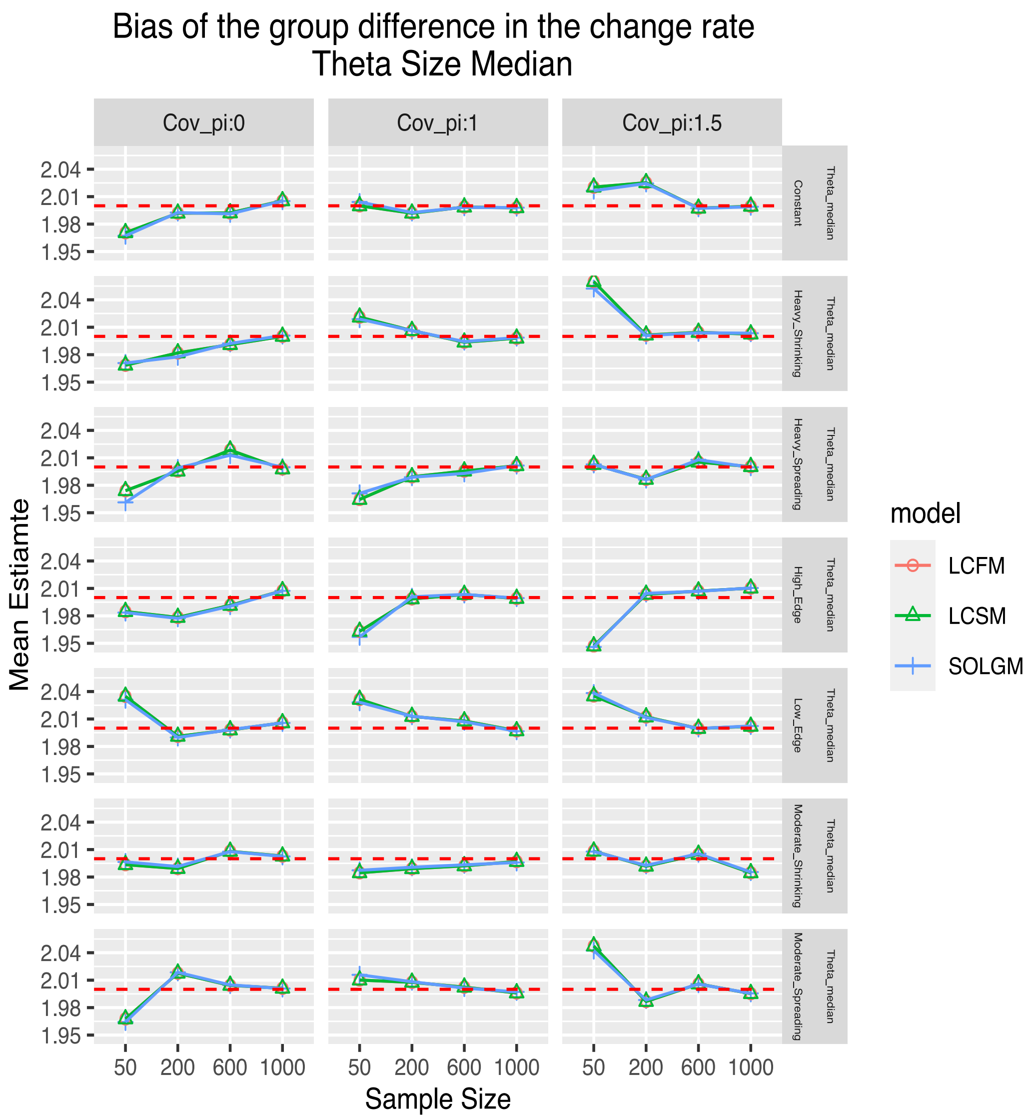

Figure A.38. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

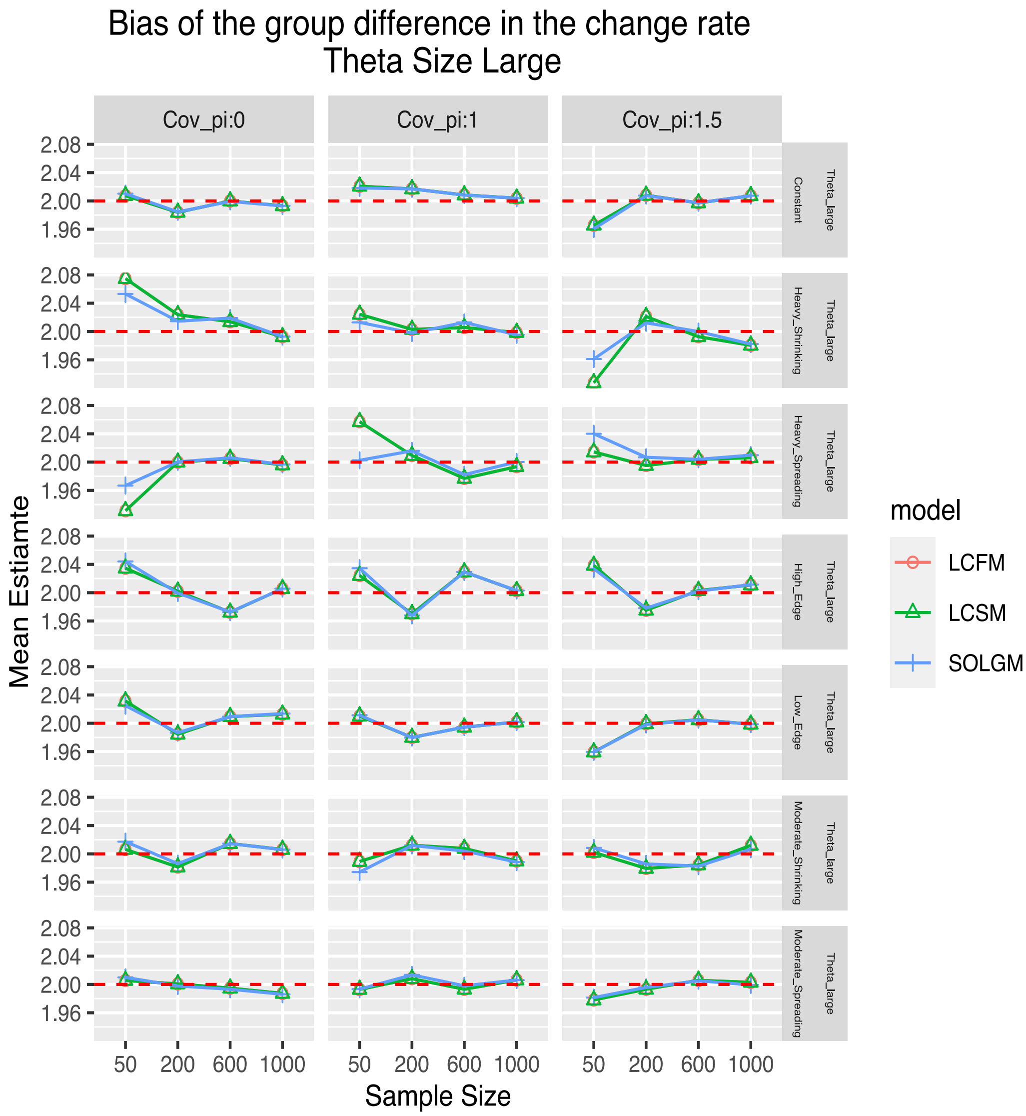

Figure A.39. Bias of the Average Group Difference in the Rate of Change, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

A.0.5 List of Figures in Empirical Standard Error

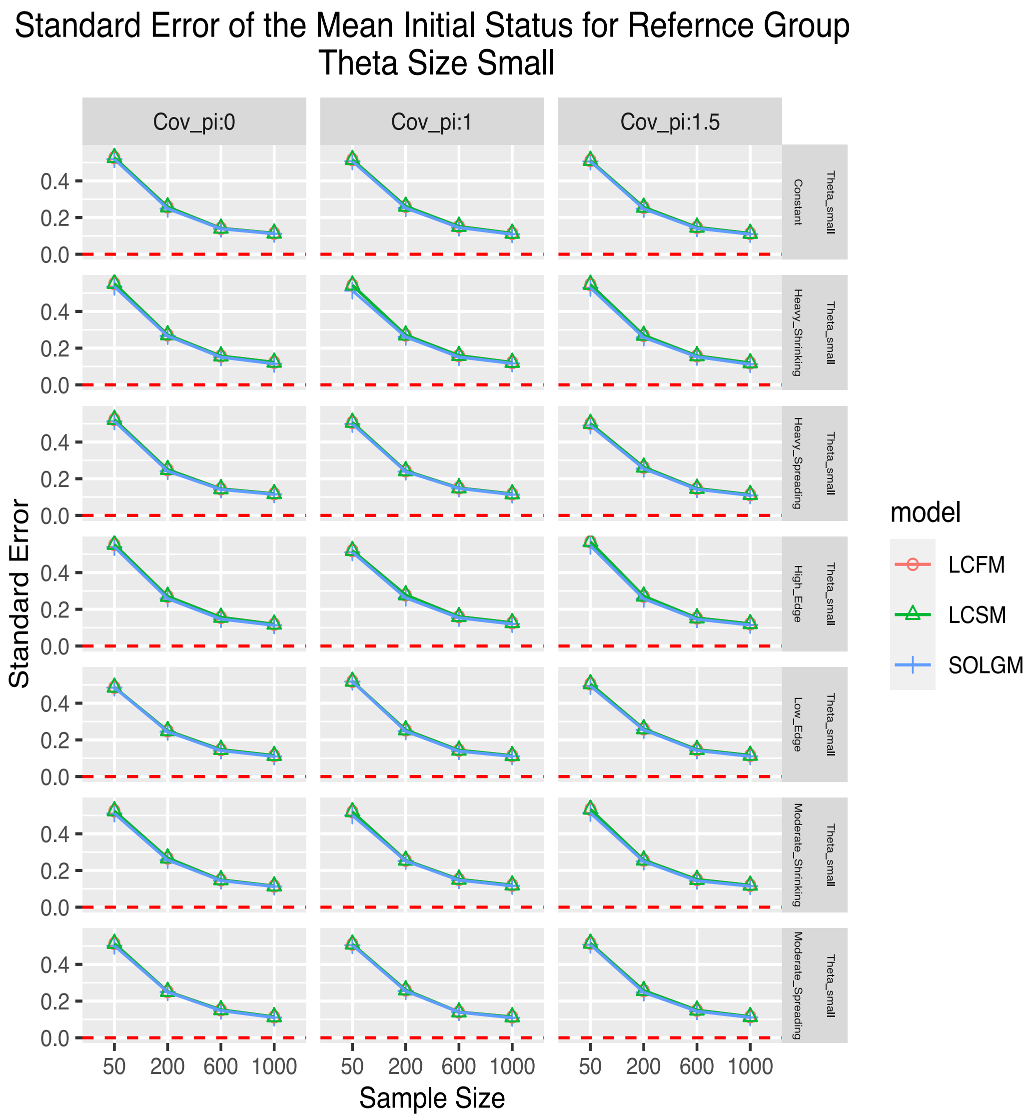

Figure A.40. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

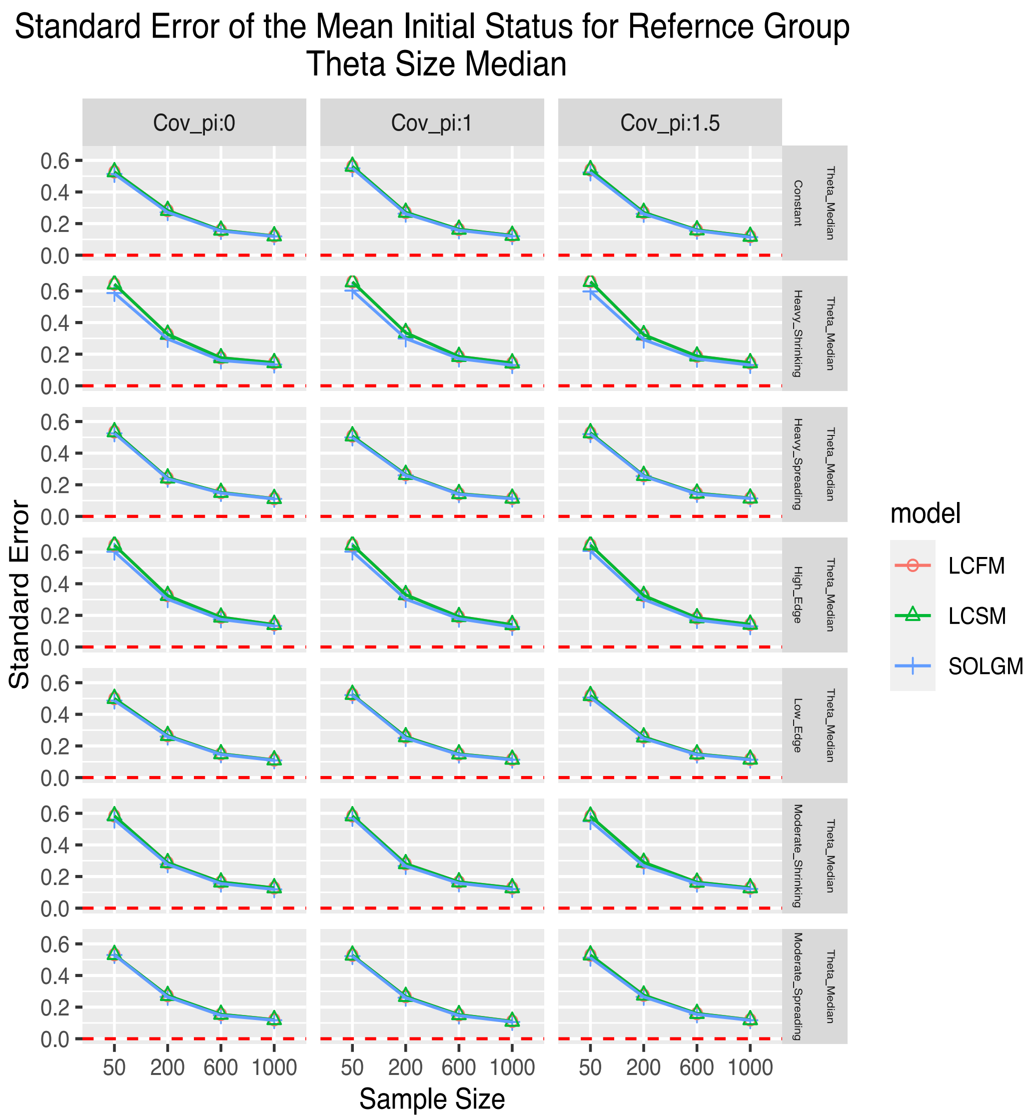

Figure A.41. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

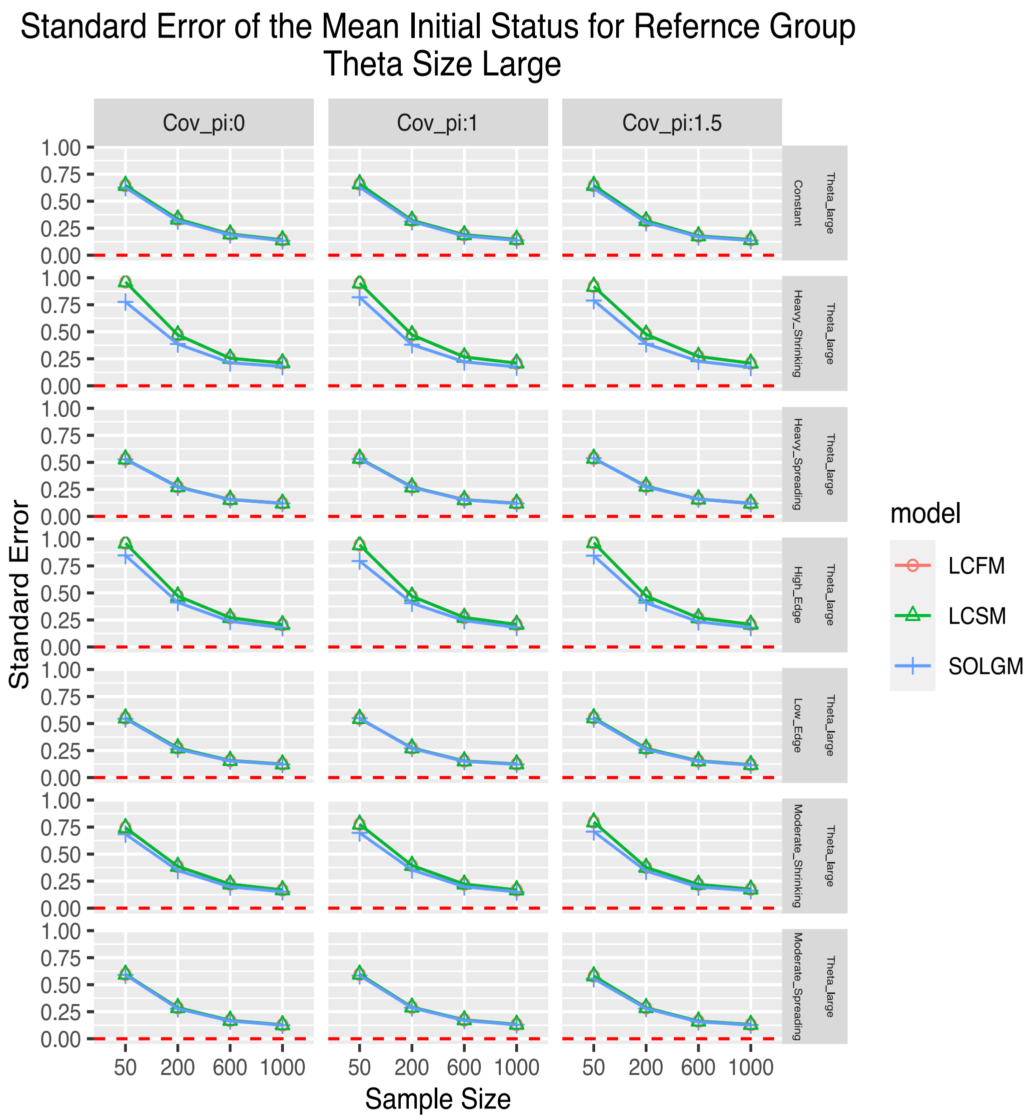

Figure A.42. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

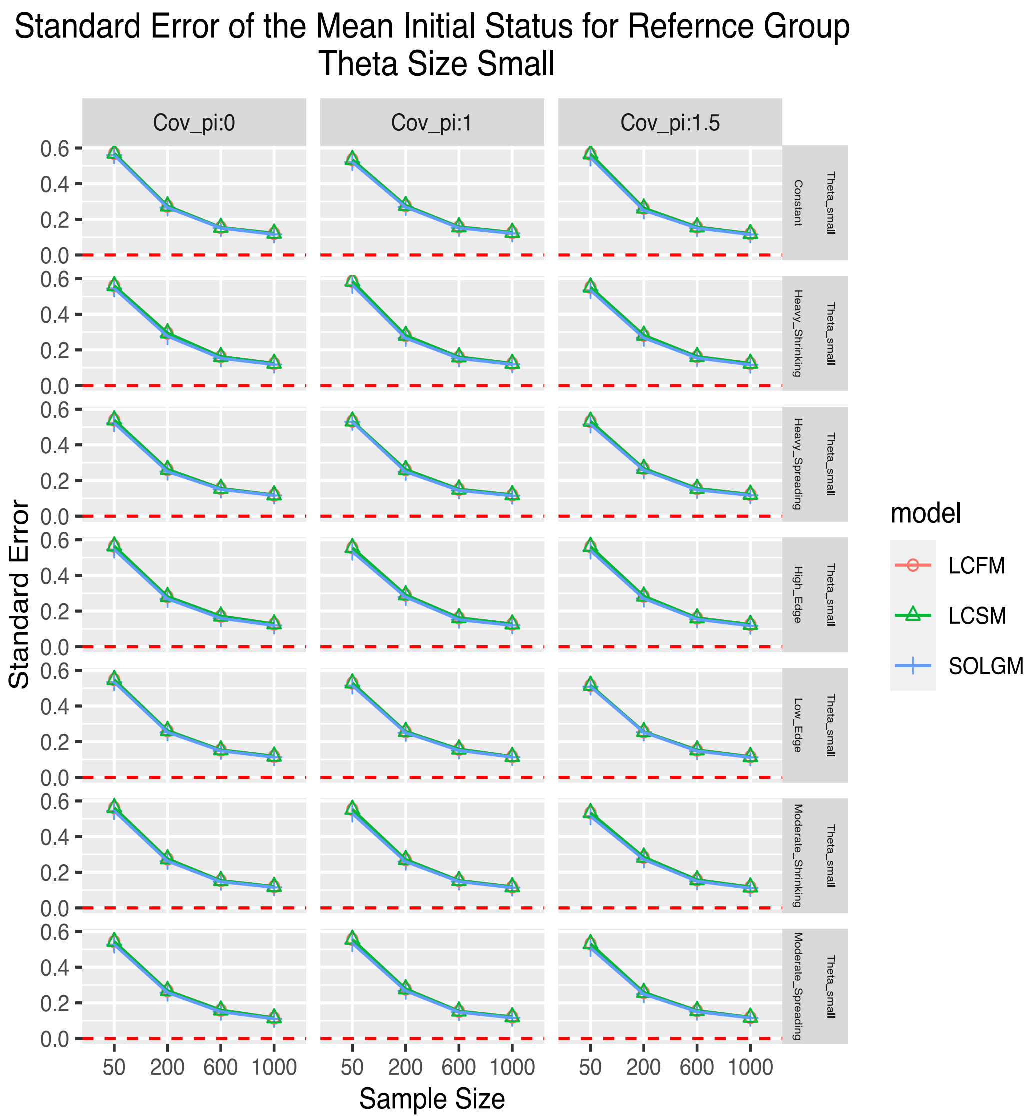

Figure A.43. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

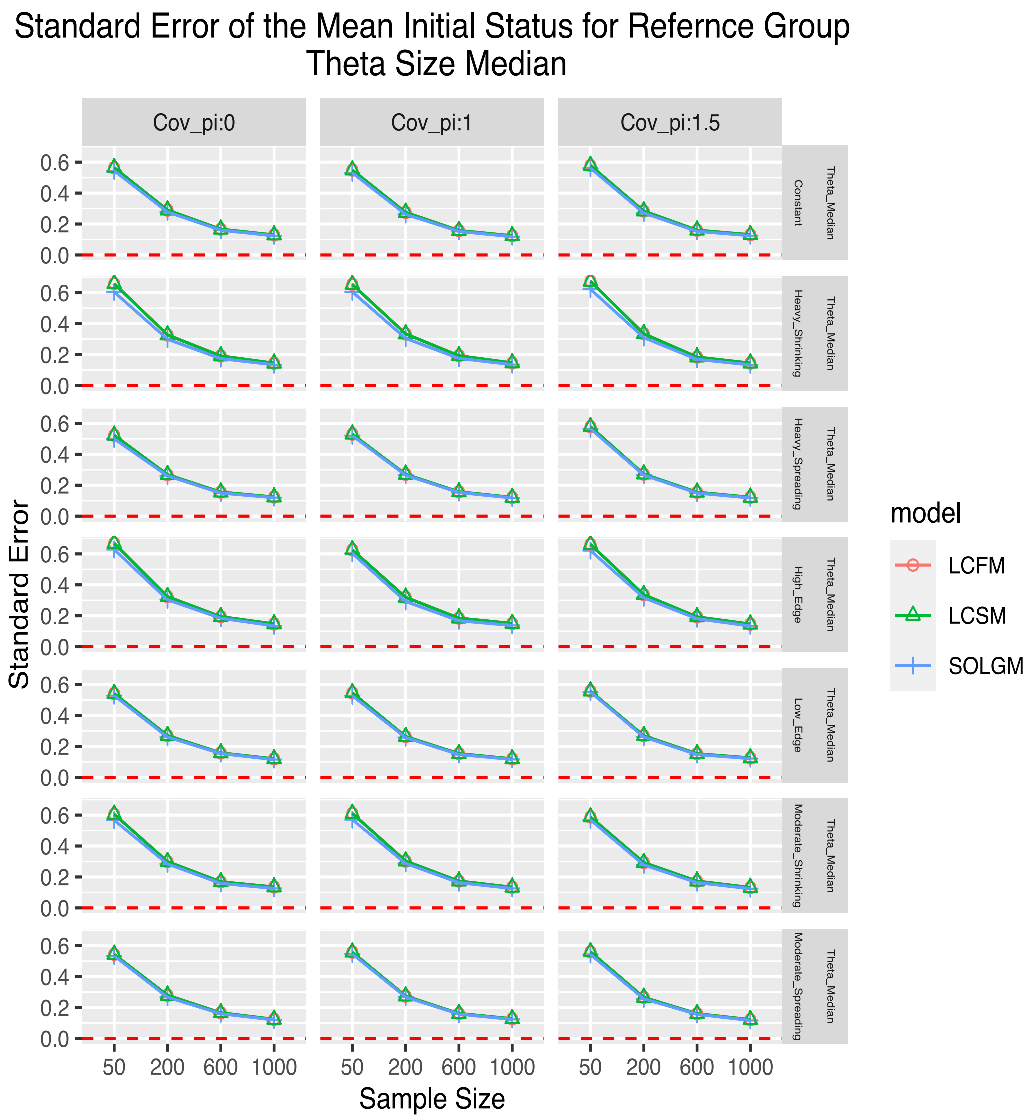

Figure A.44. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

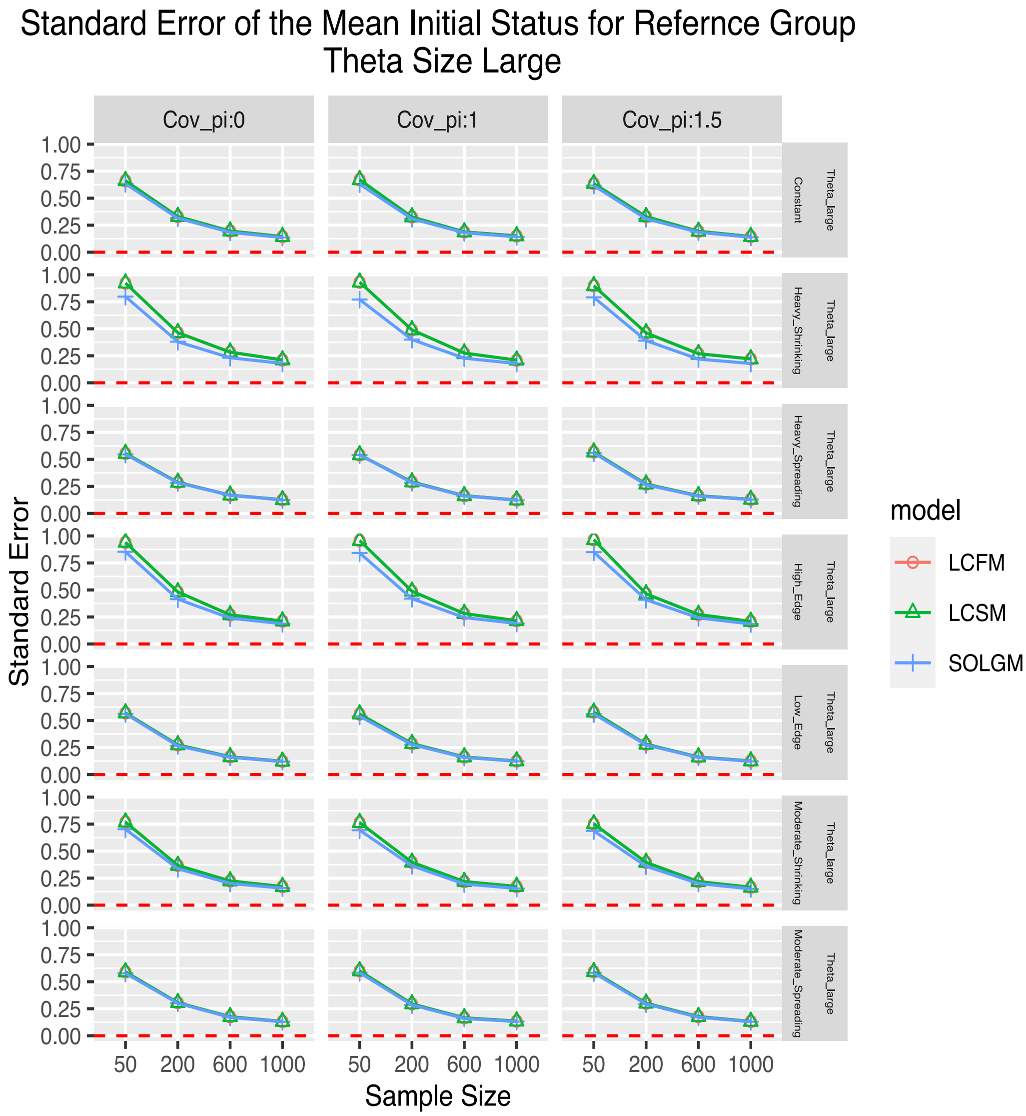

Figure A.45. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

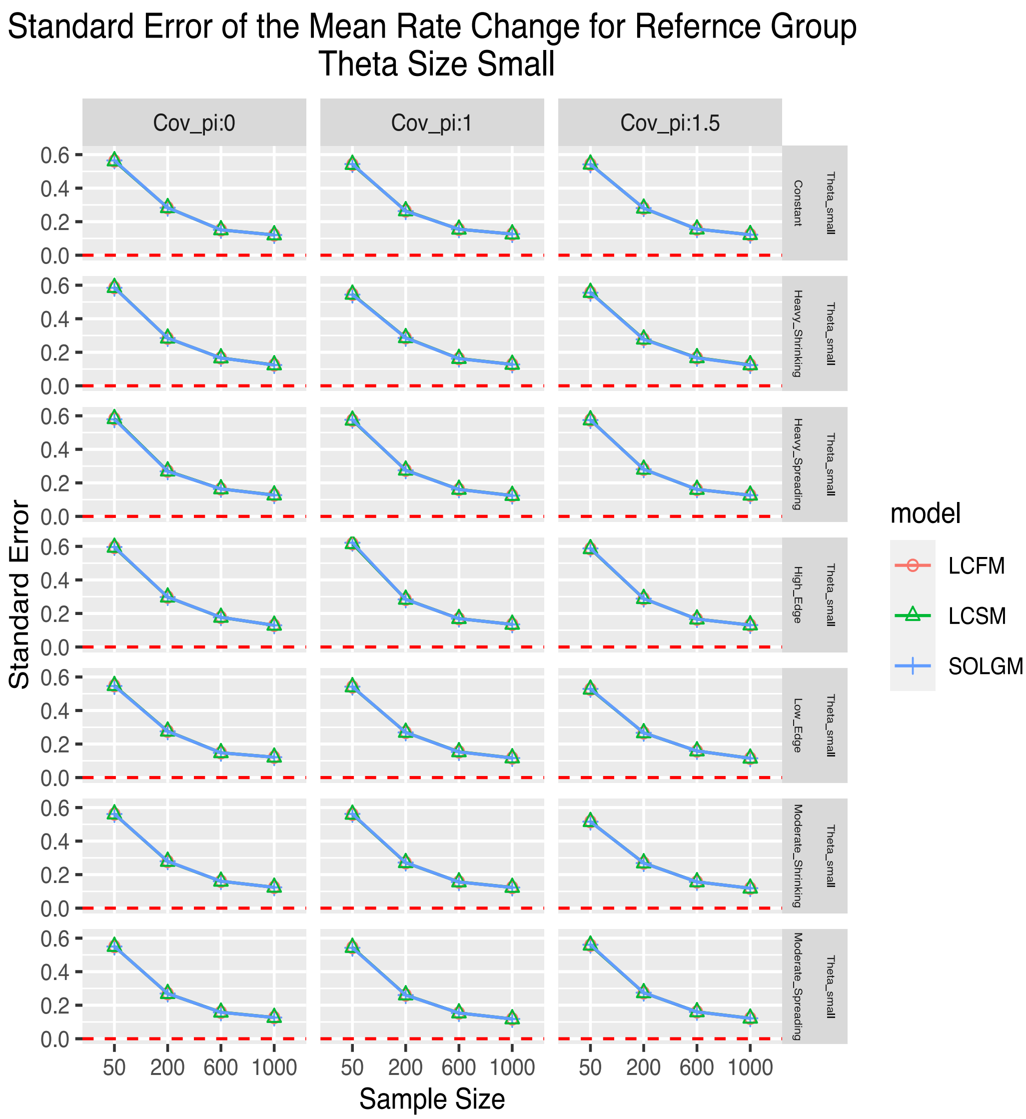

Figure A.46. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

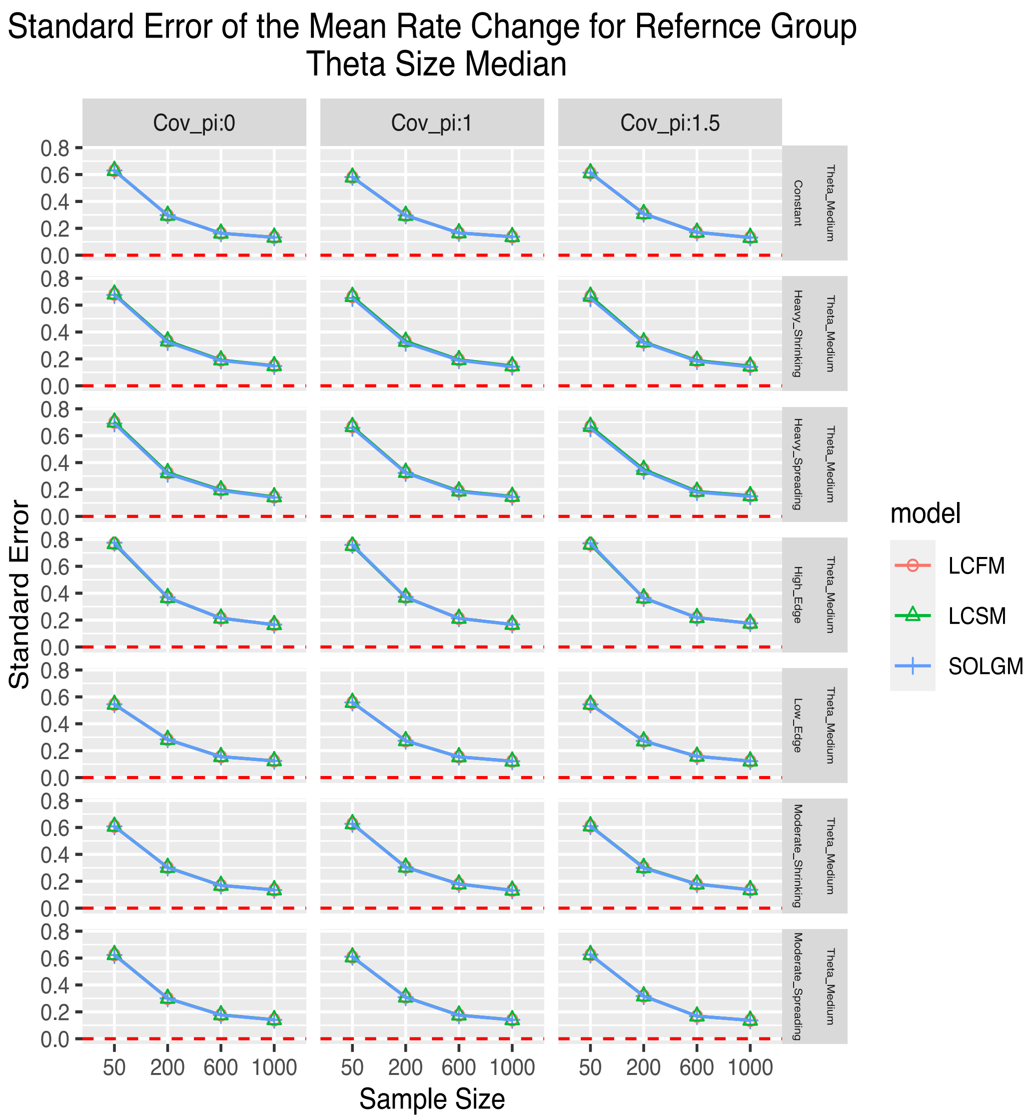

Figure A.47. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

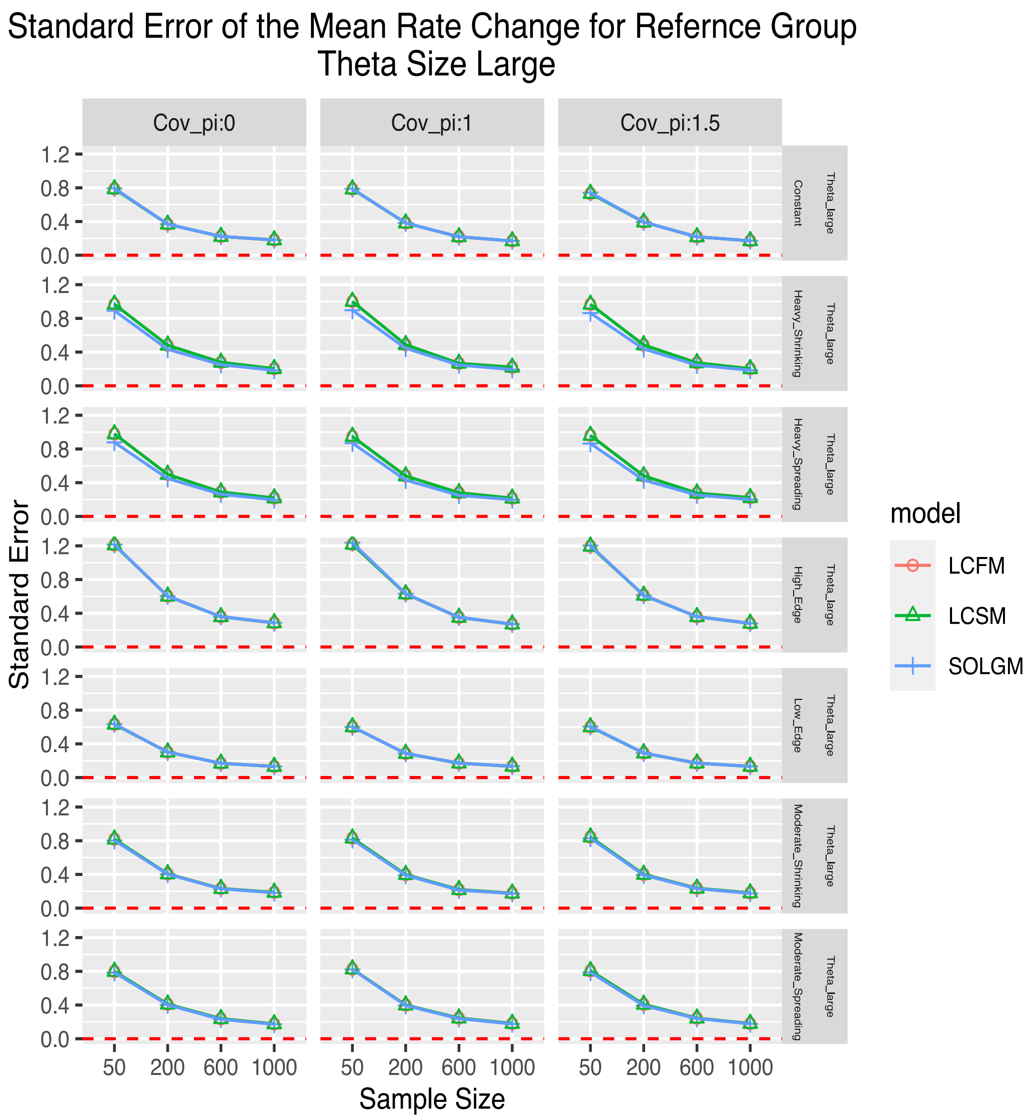

Figure A.48. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

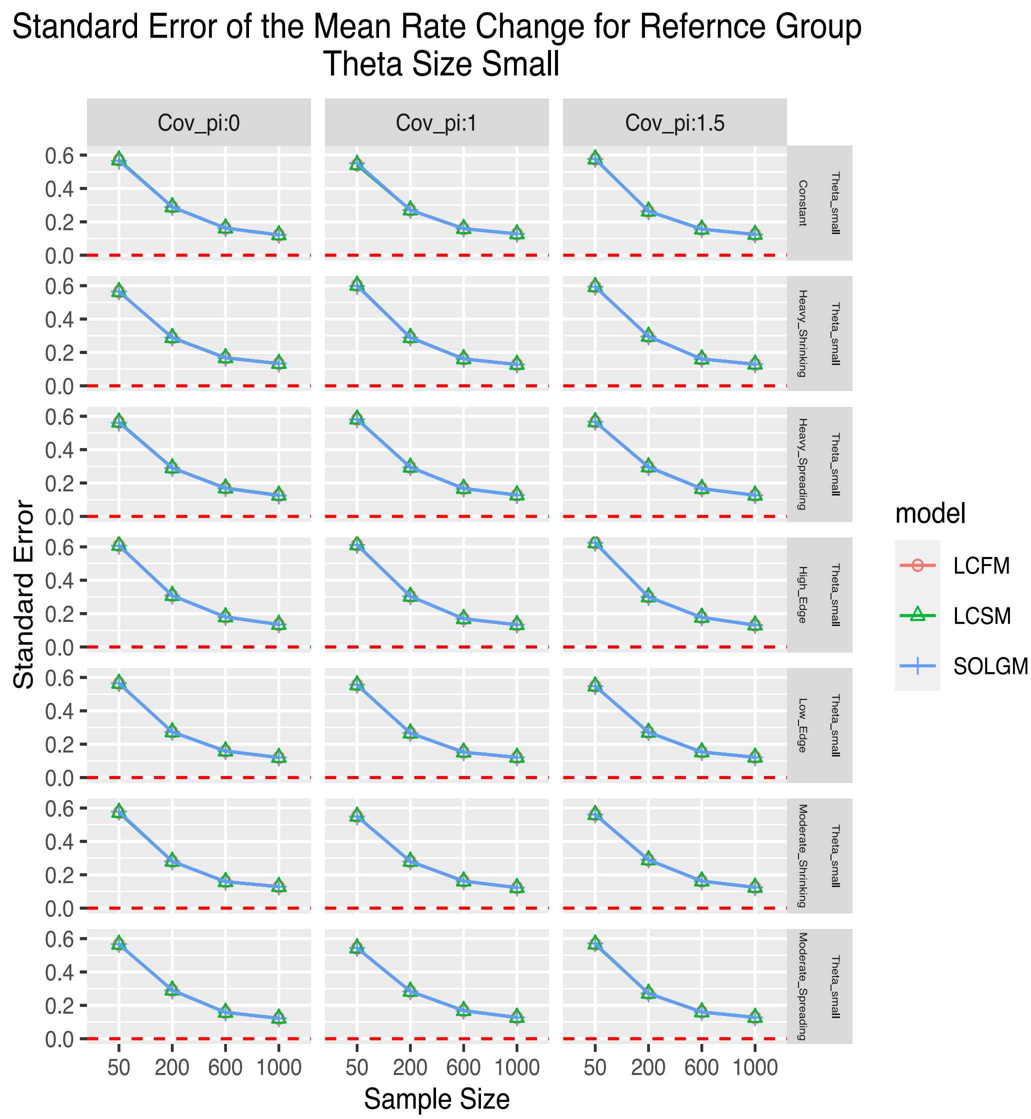

Figure A.49. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

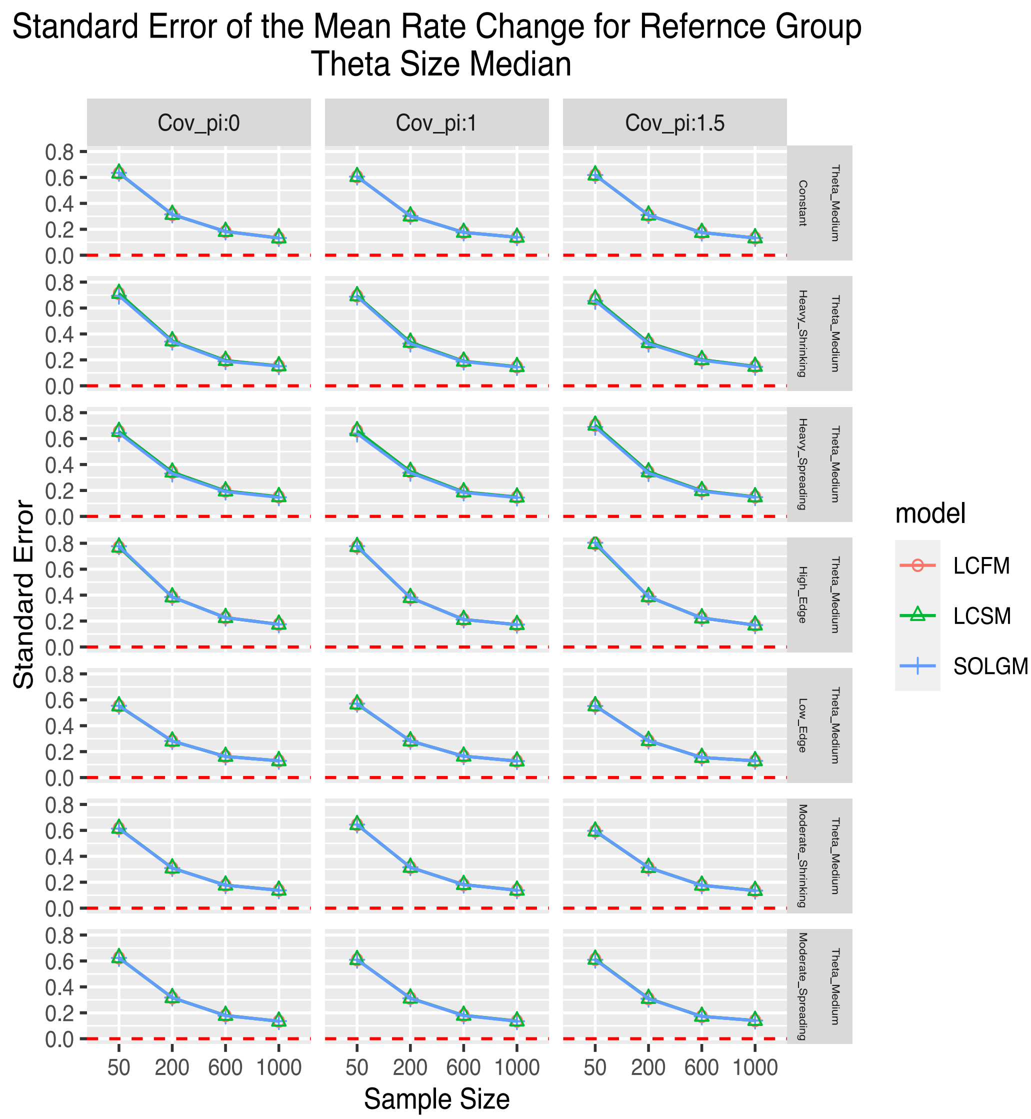

Figure A.50. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

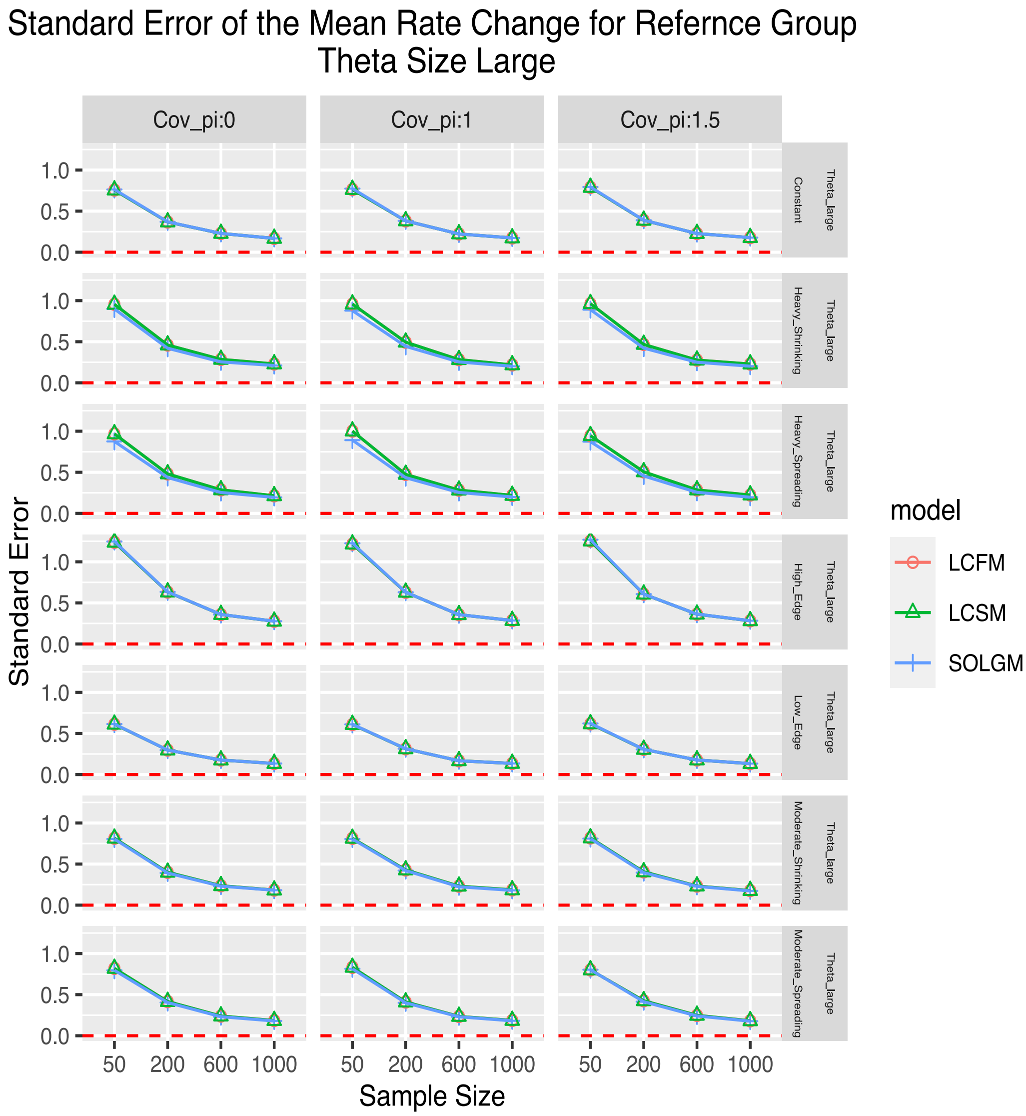

Figure A.51. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

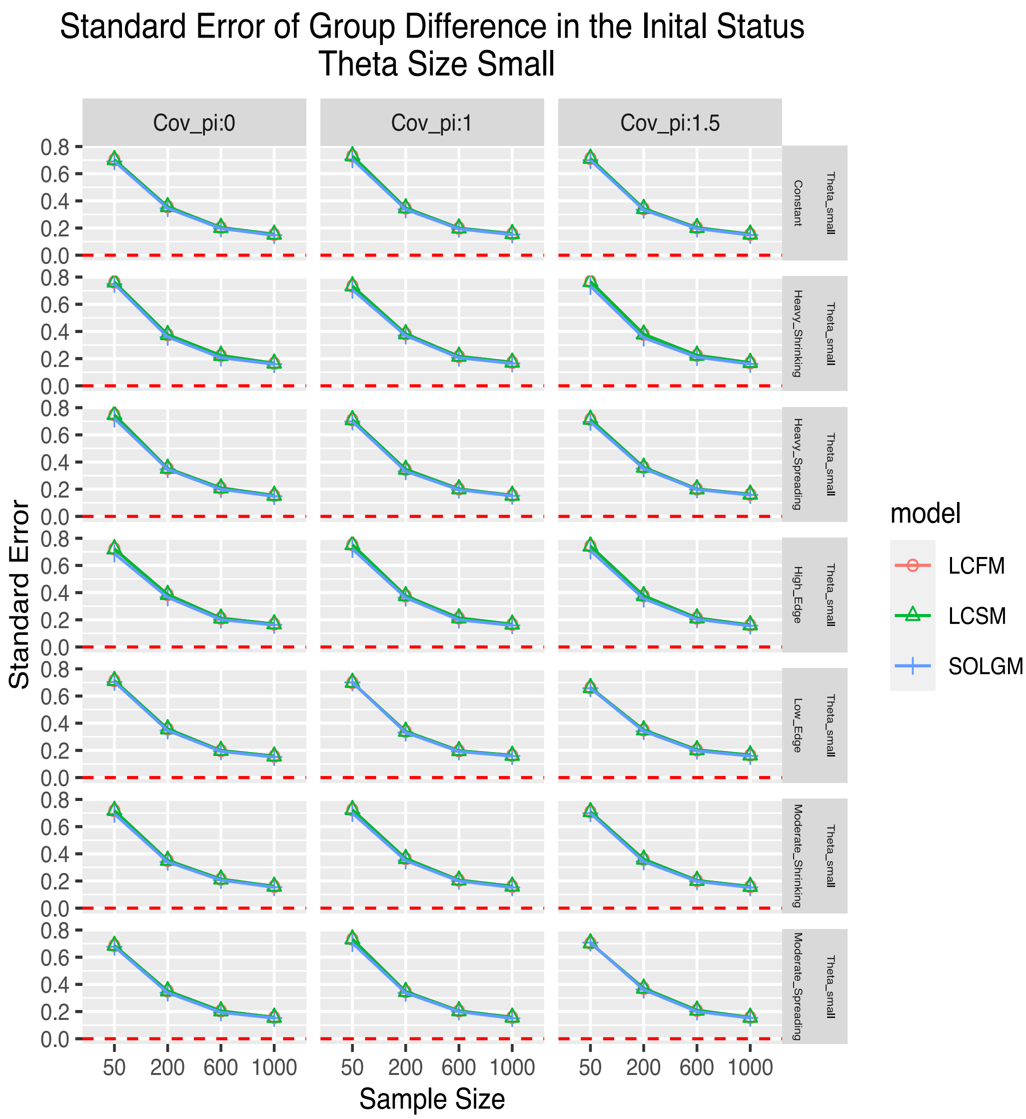

Figure A.52. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

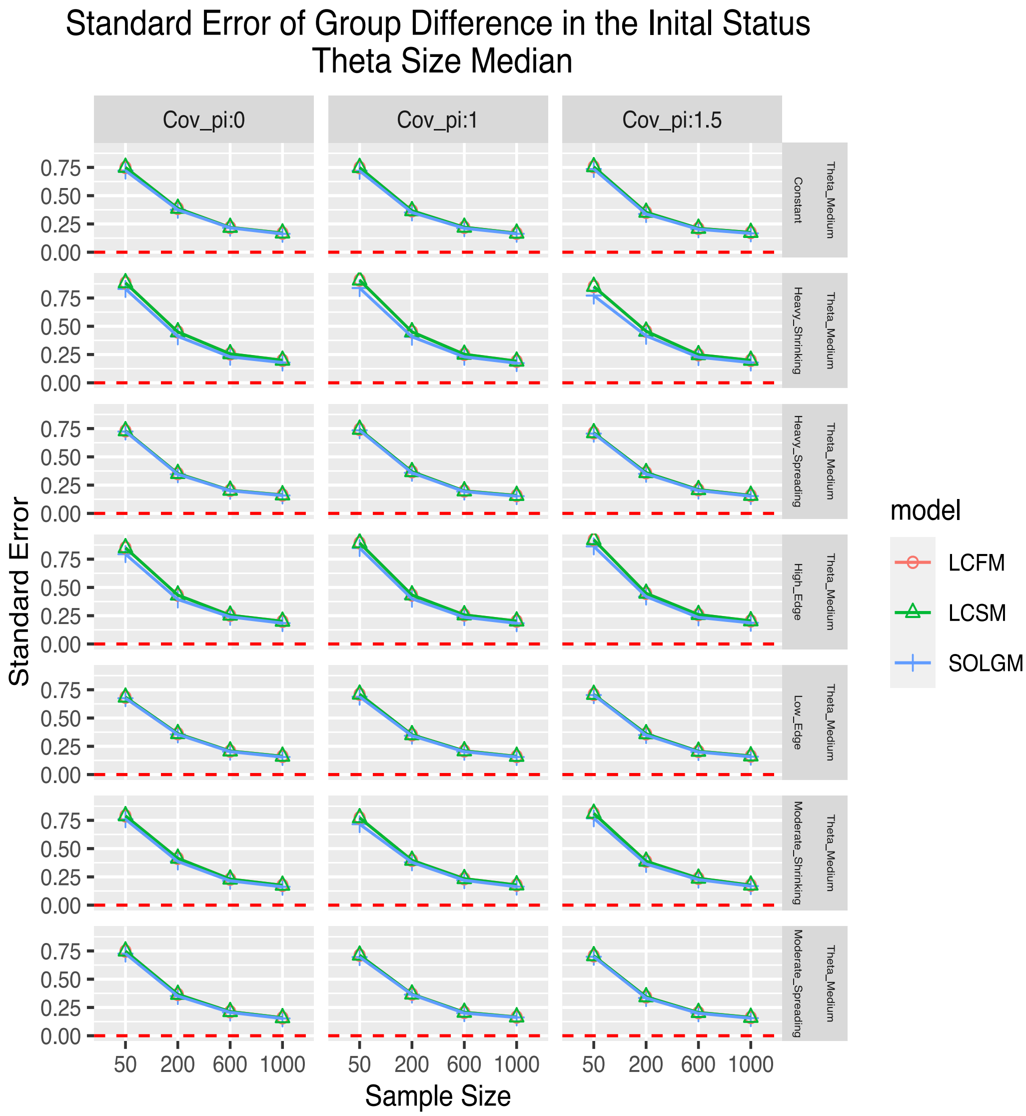

Figure A.53. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

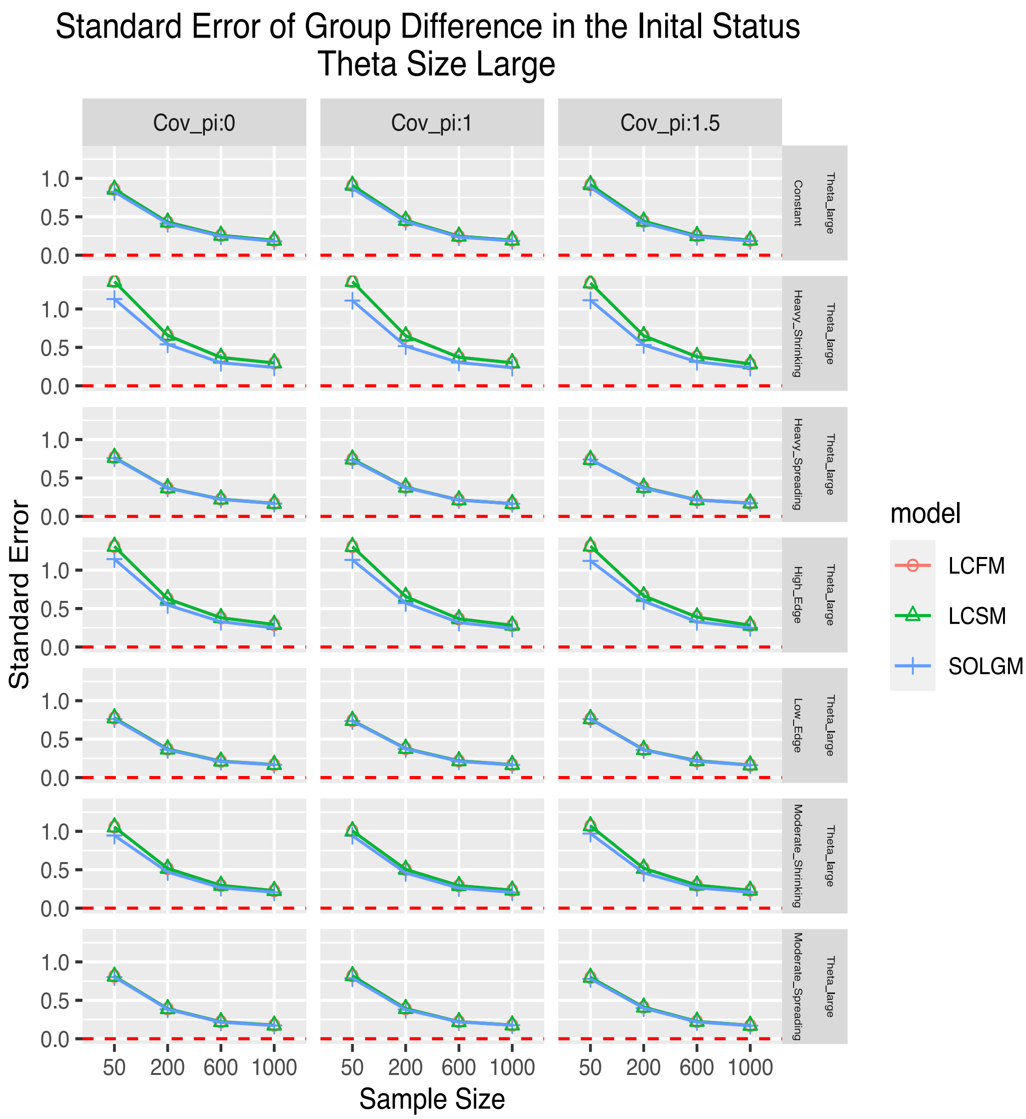

Figure A.54. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

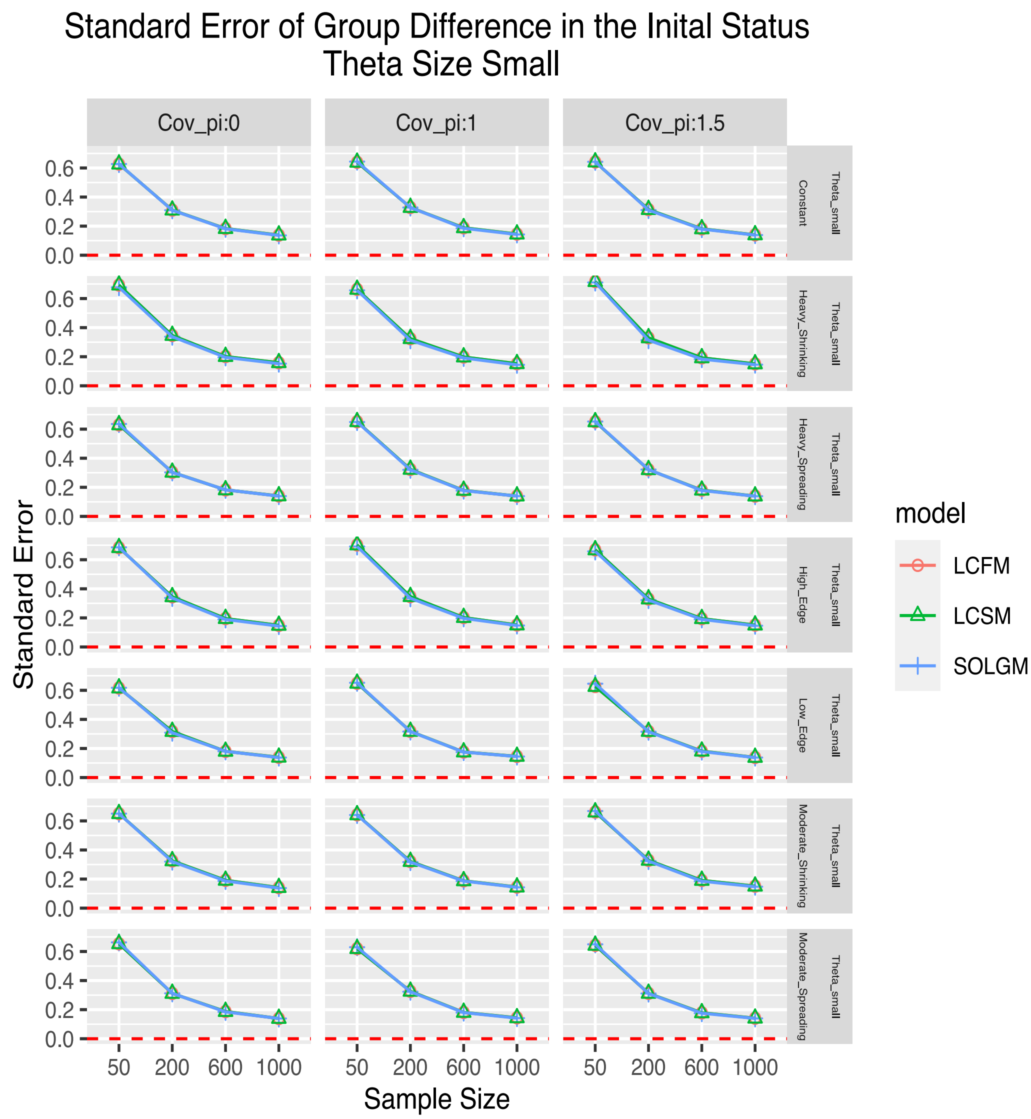

Figure A.55. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

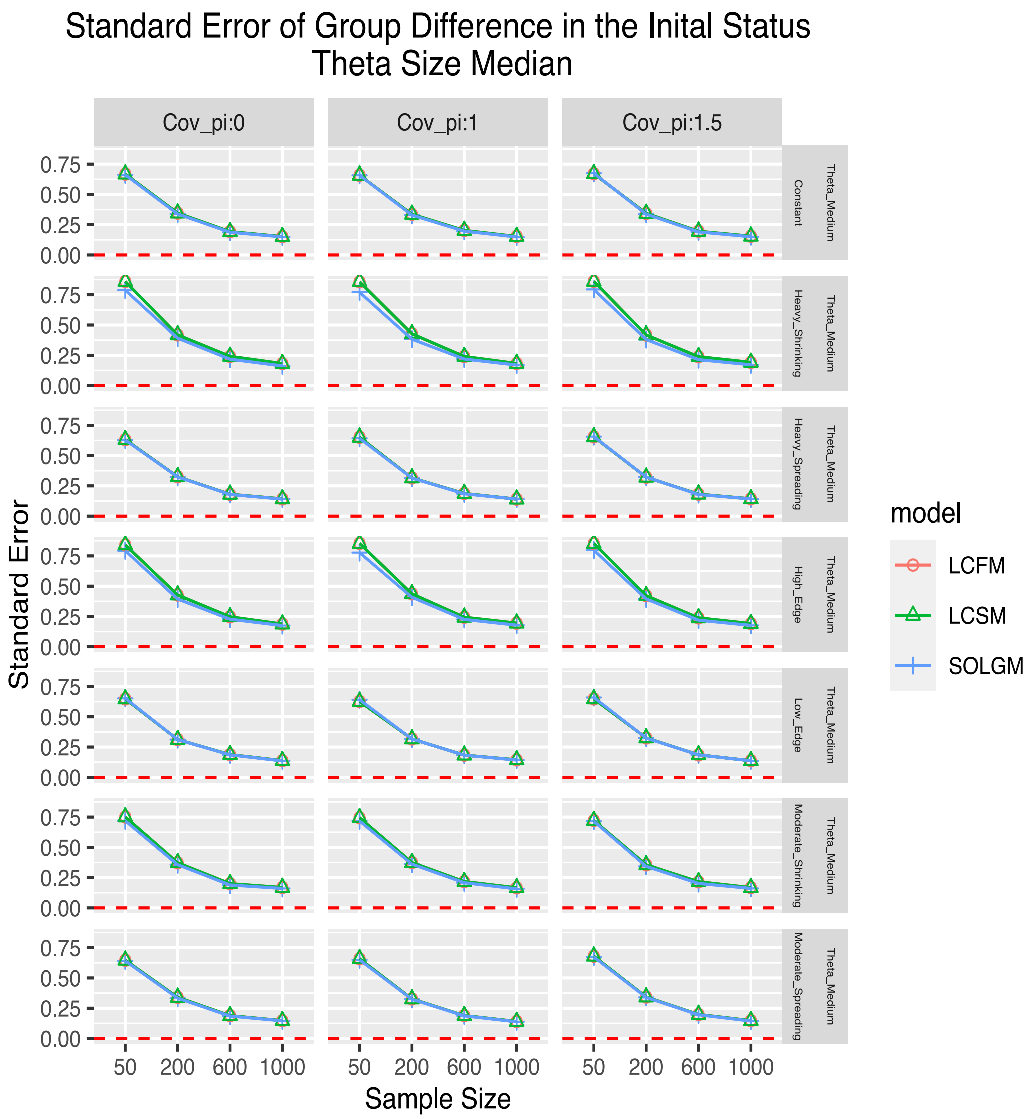

Figure A.56. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

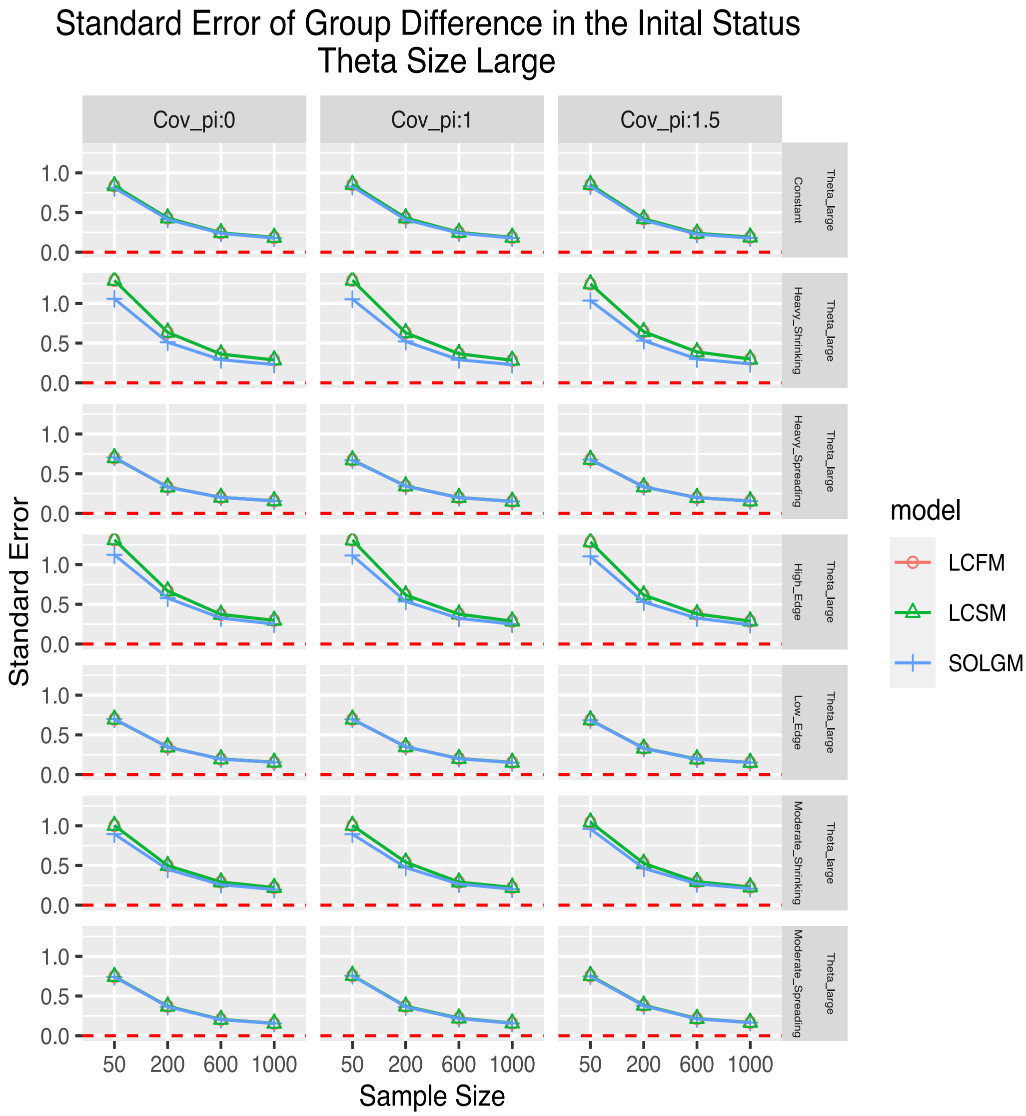

Figure A.57. Standard Error of Group Difference in Initial Status , $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

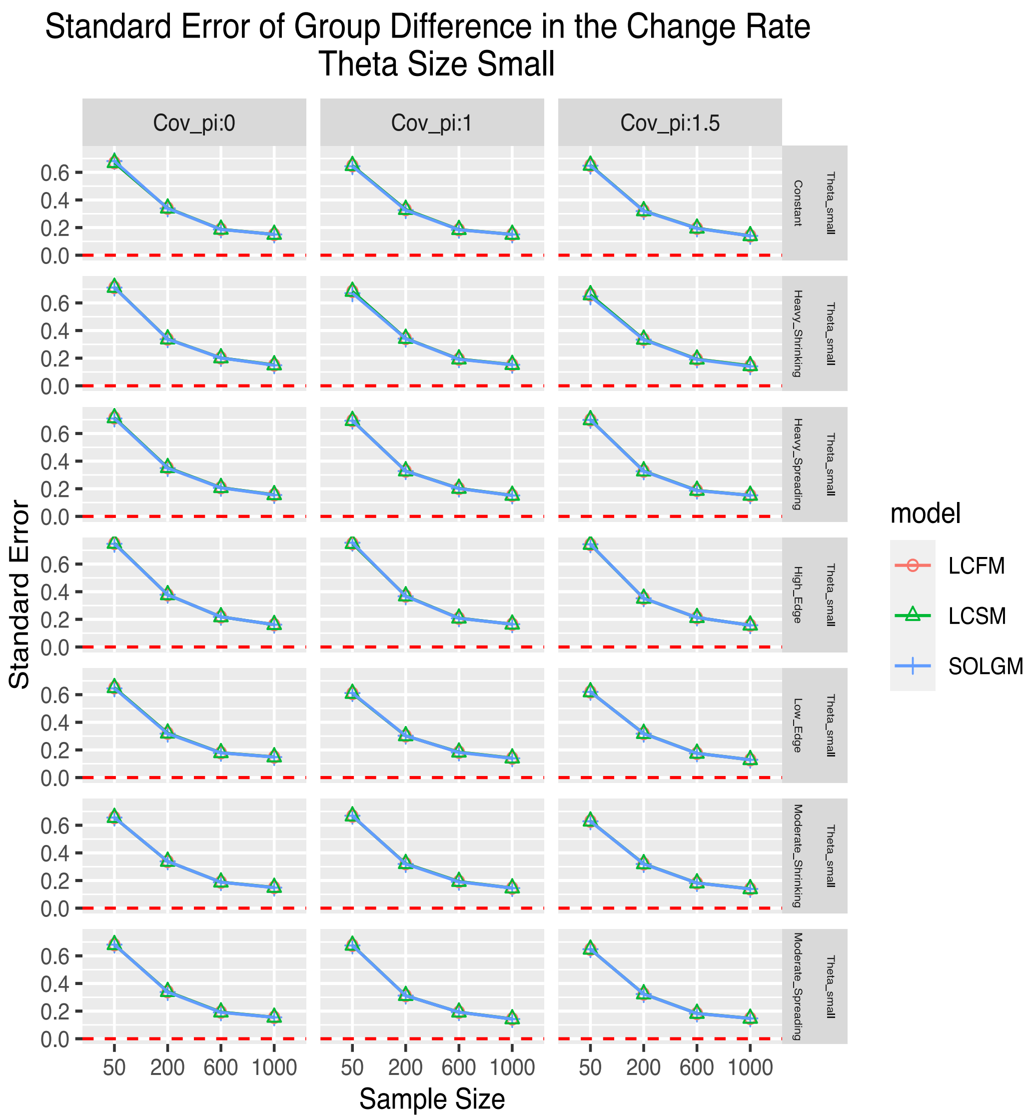

Figure A.58. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

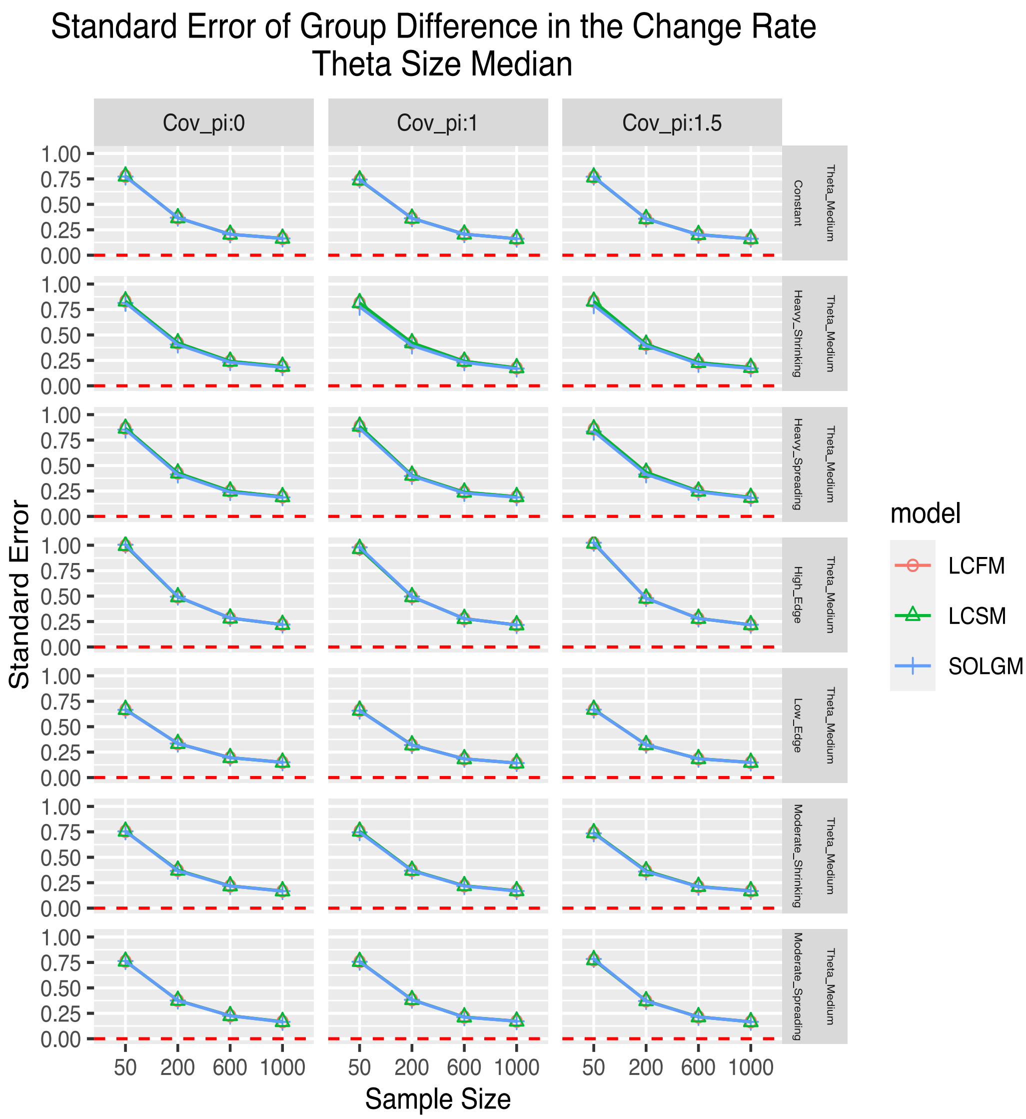

Figure A.59. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

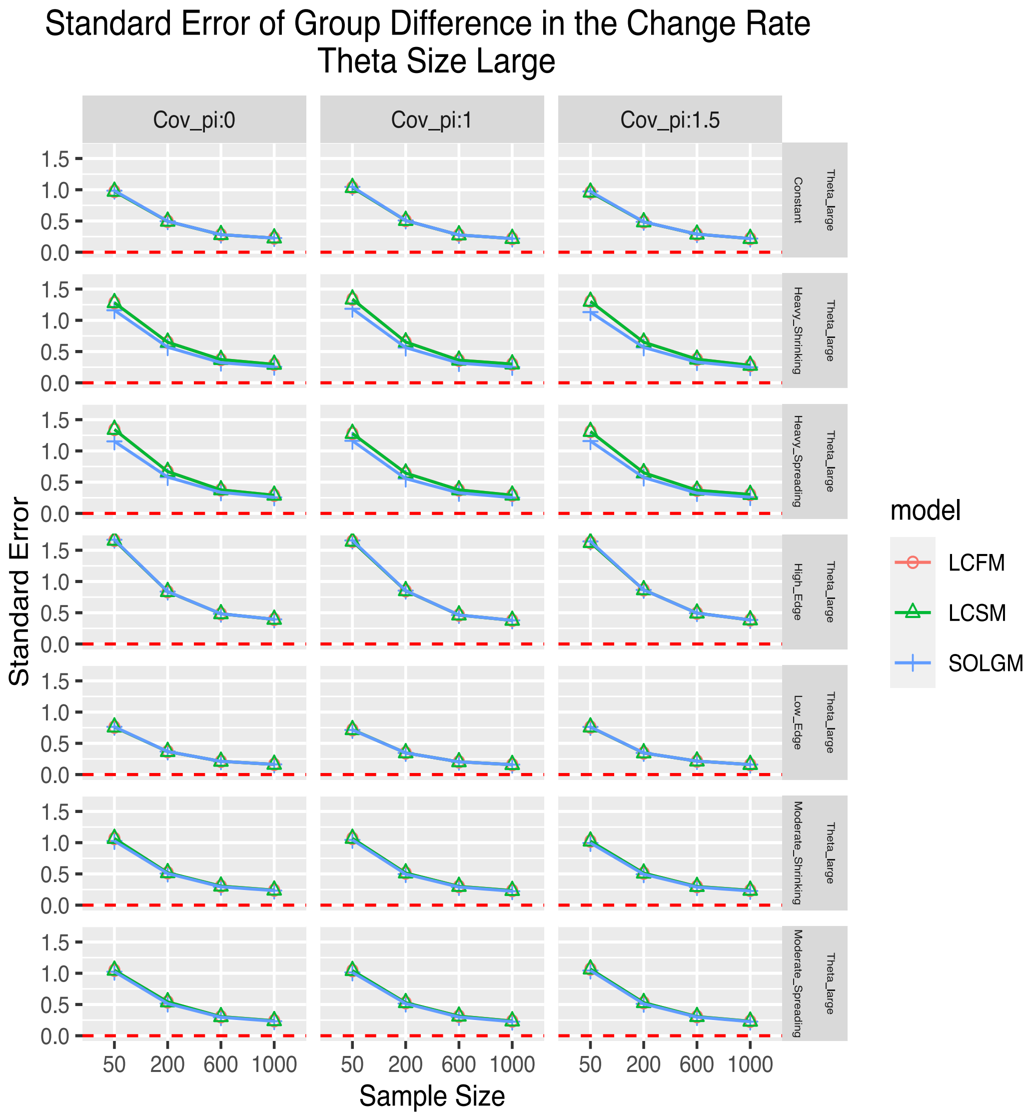

Figure A.60. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

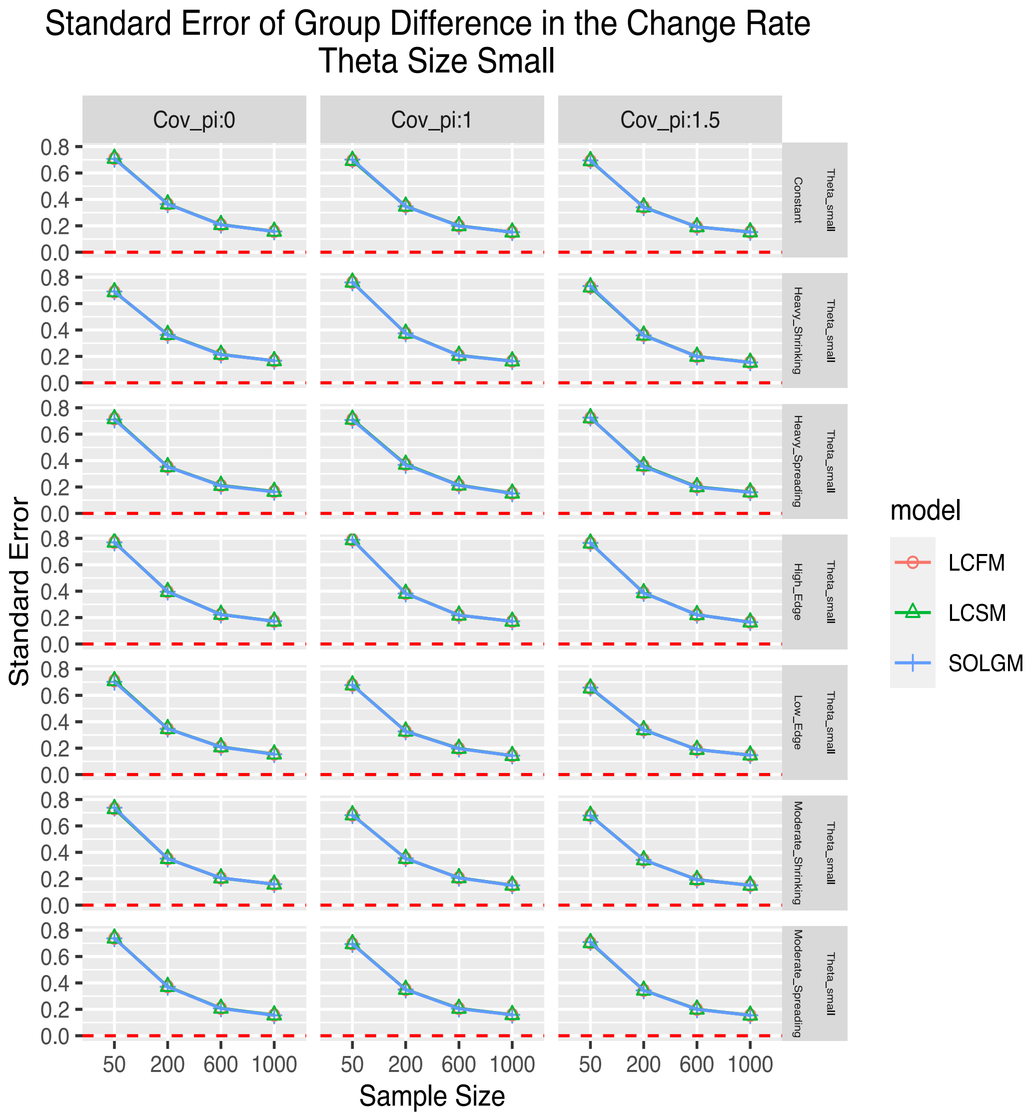

Figure A.61. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

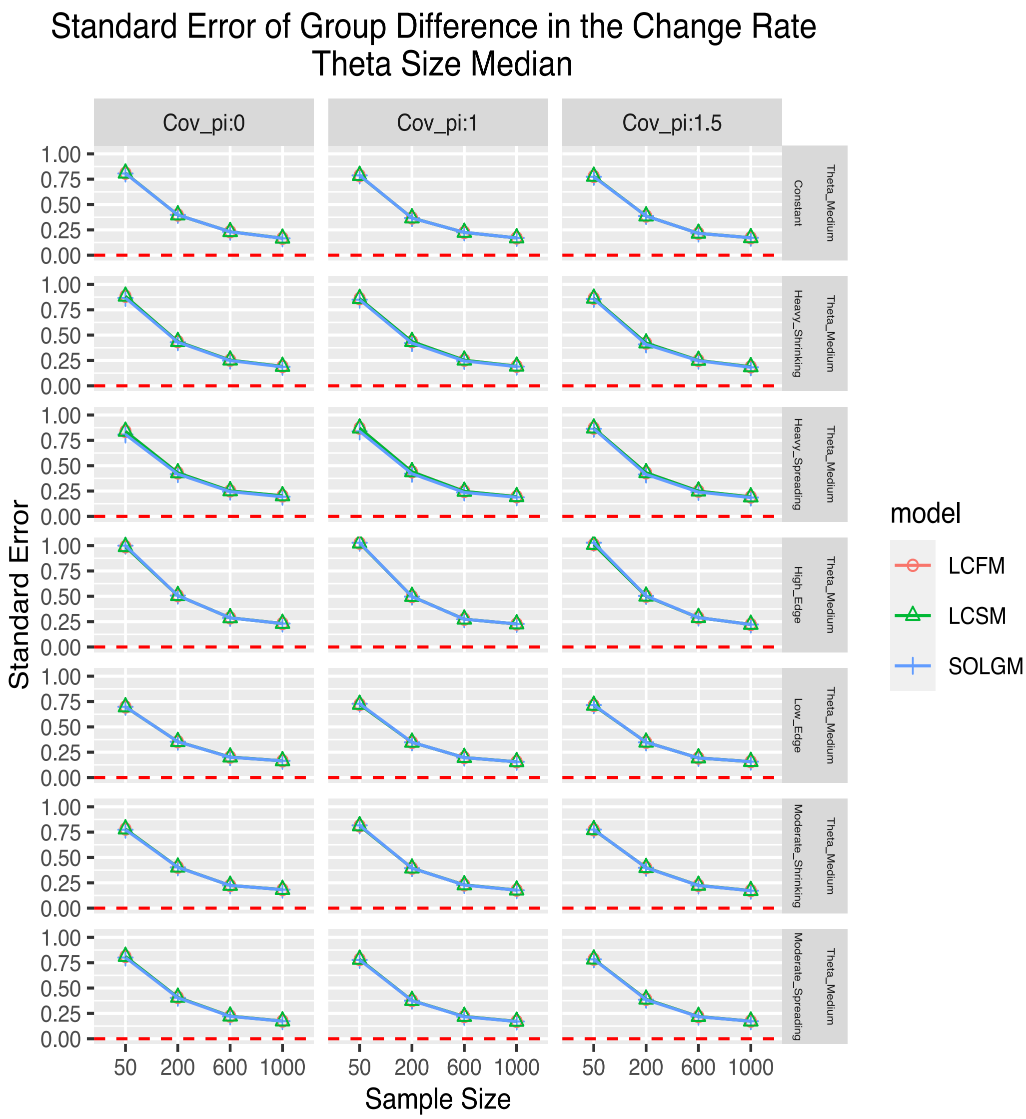

Figure A.62. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

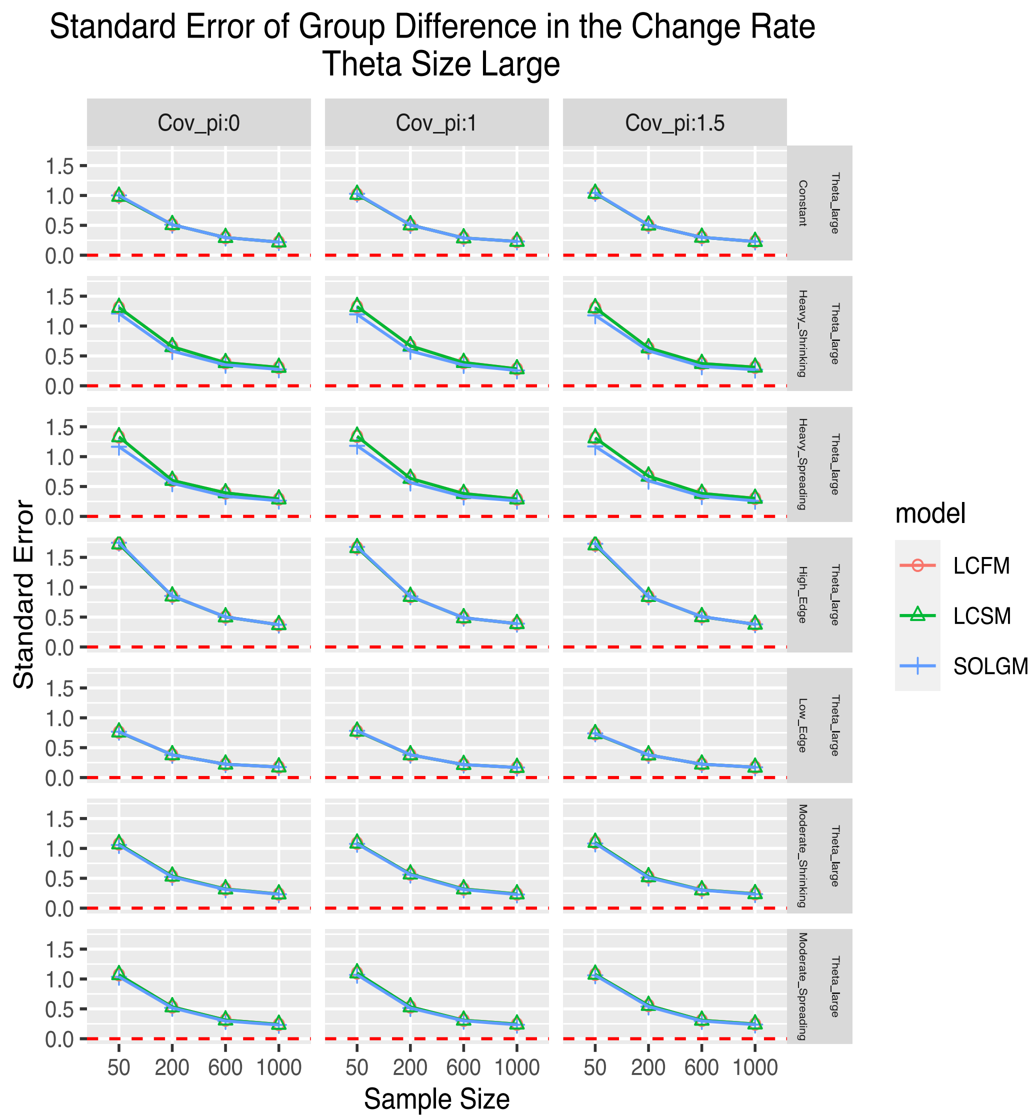

Figure A.63. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

A.0.6 List of Figures in Theoretical Standard Error

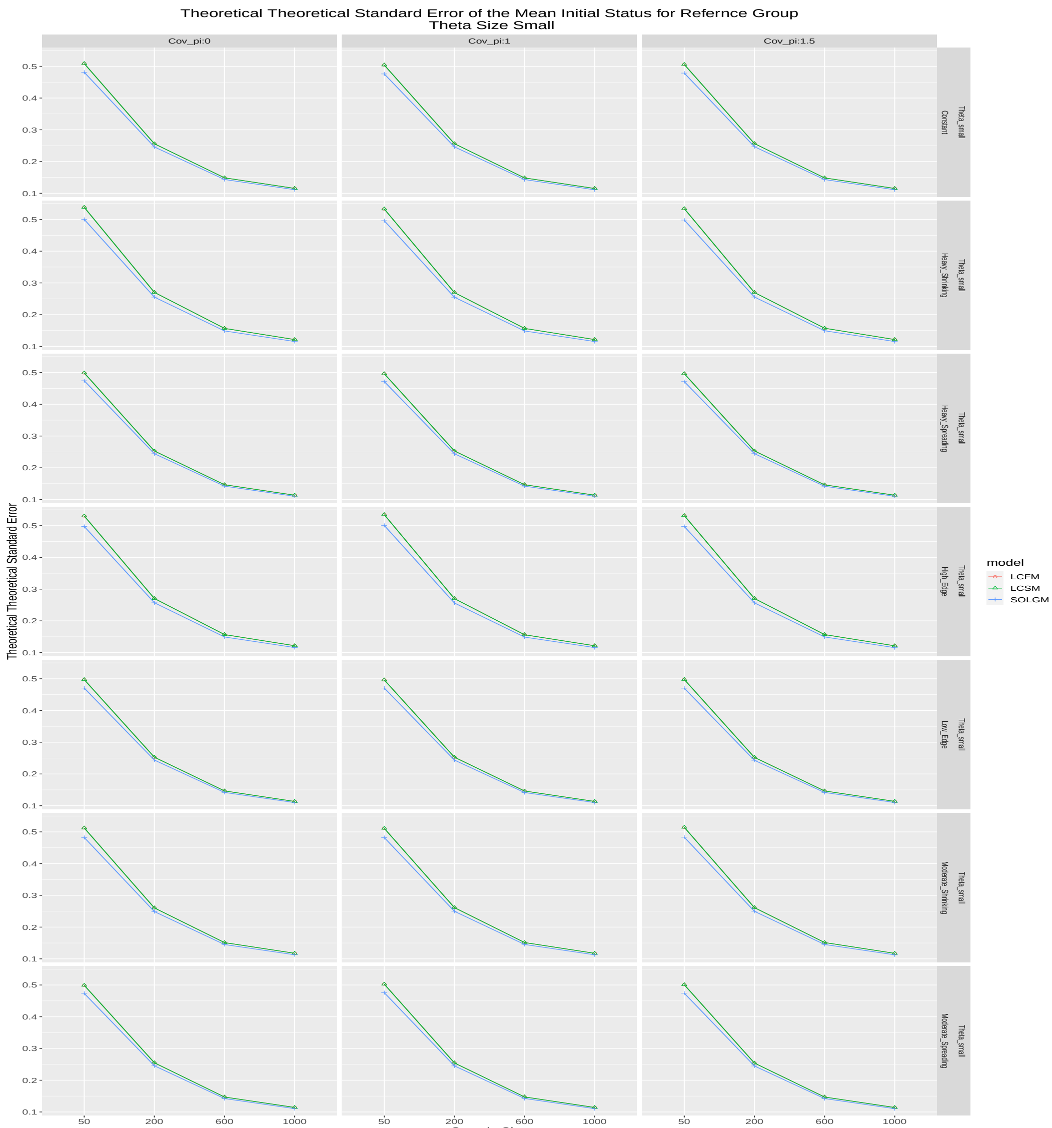

Figure A.64. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

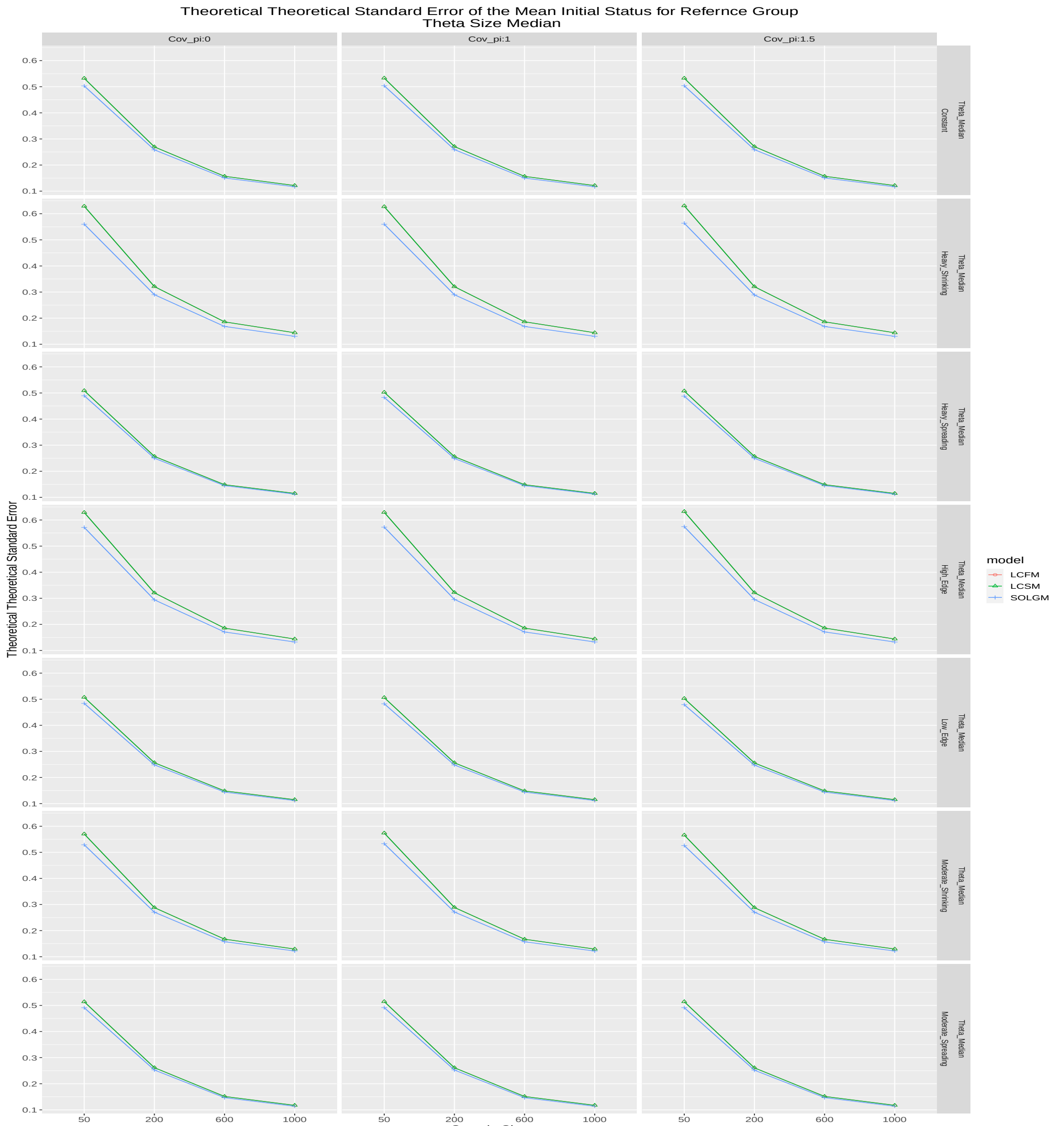

Figure A.65. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

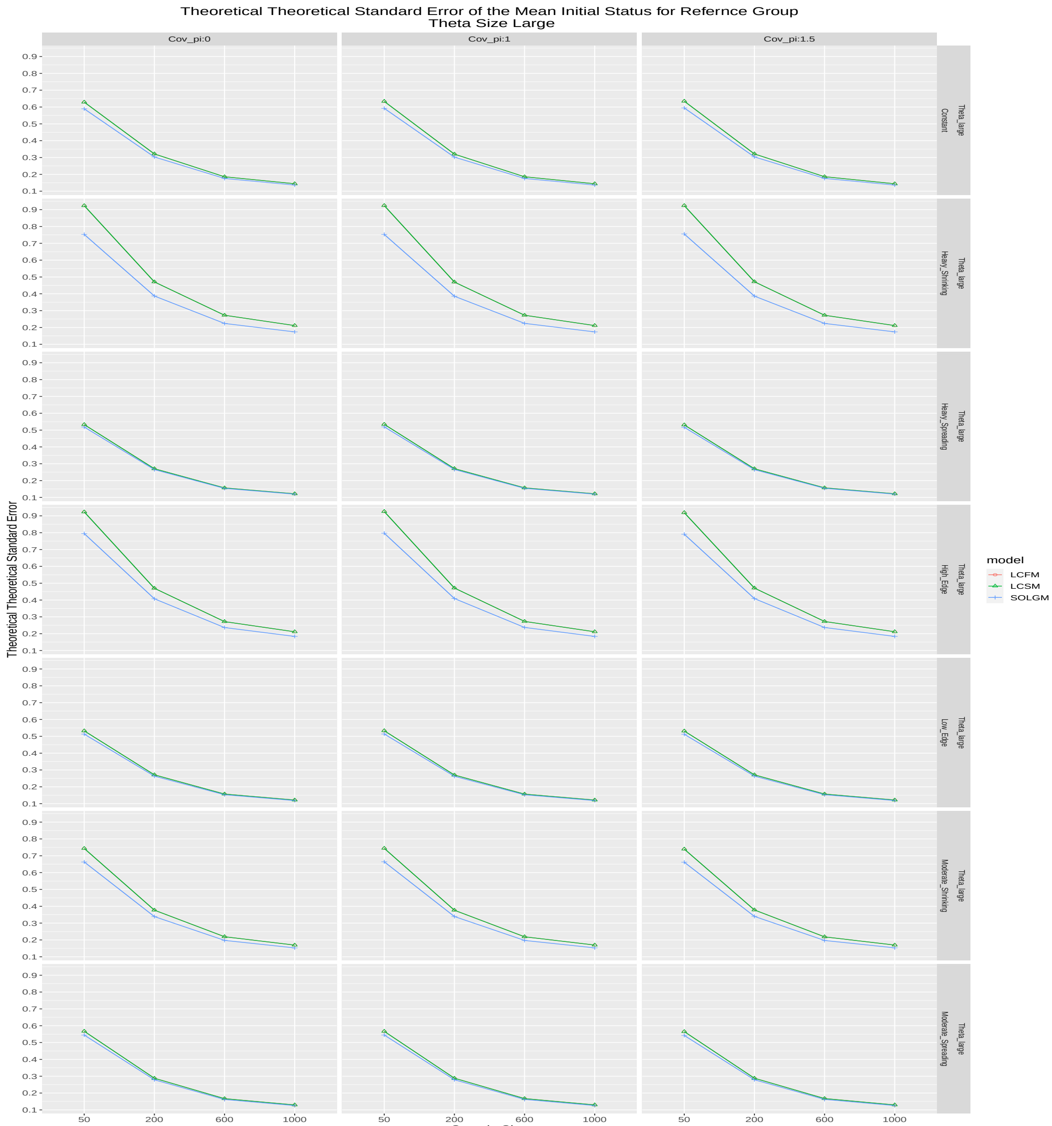

Figure A.66. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

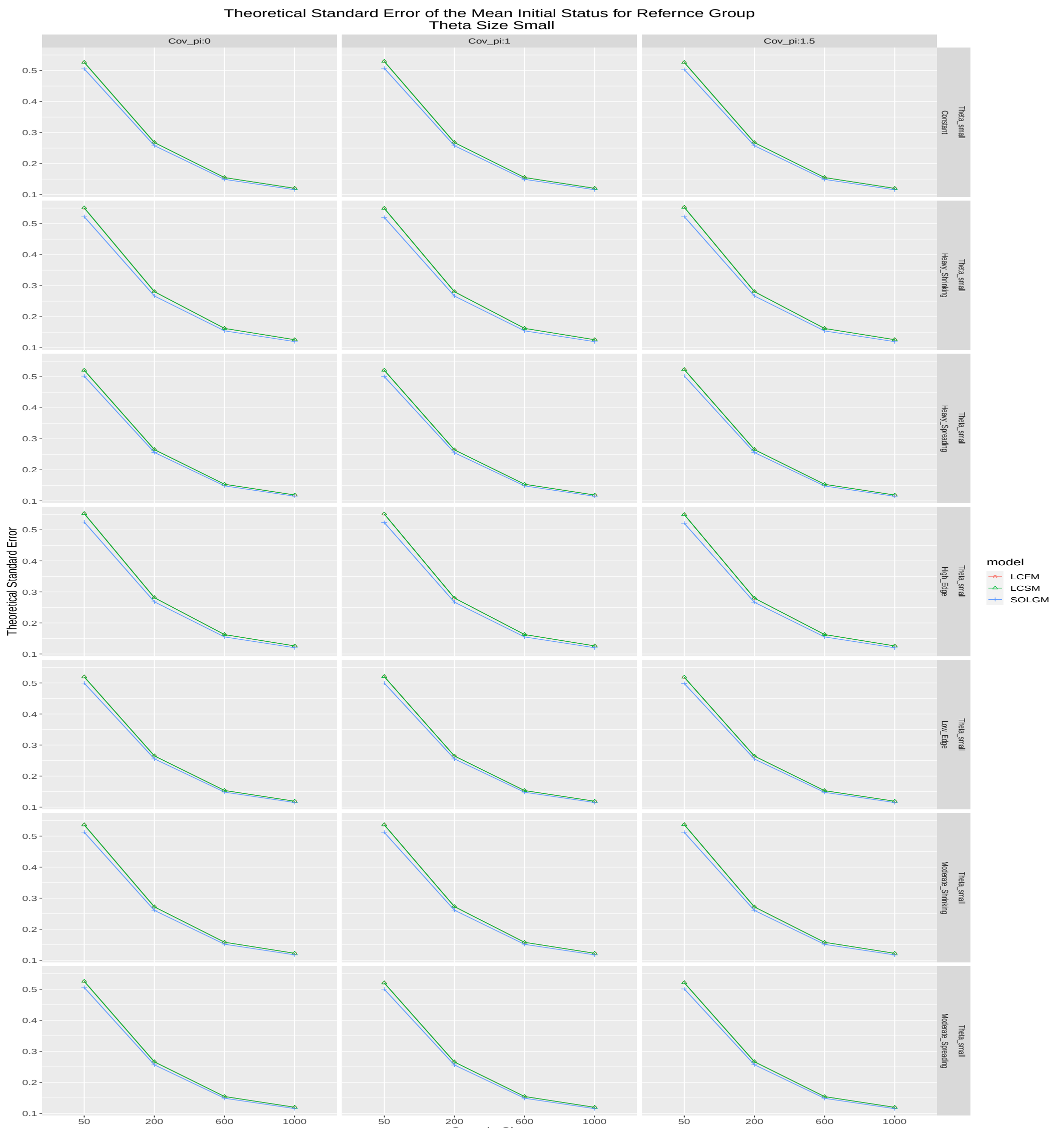

Figure A.67. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

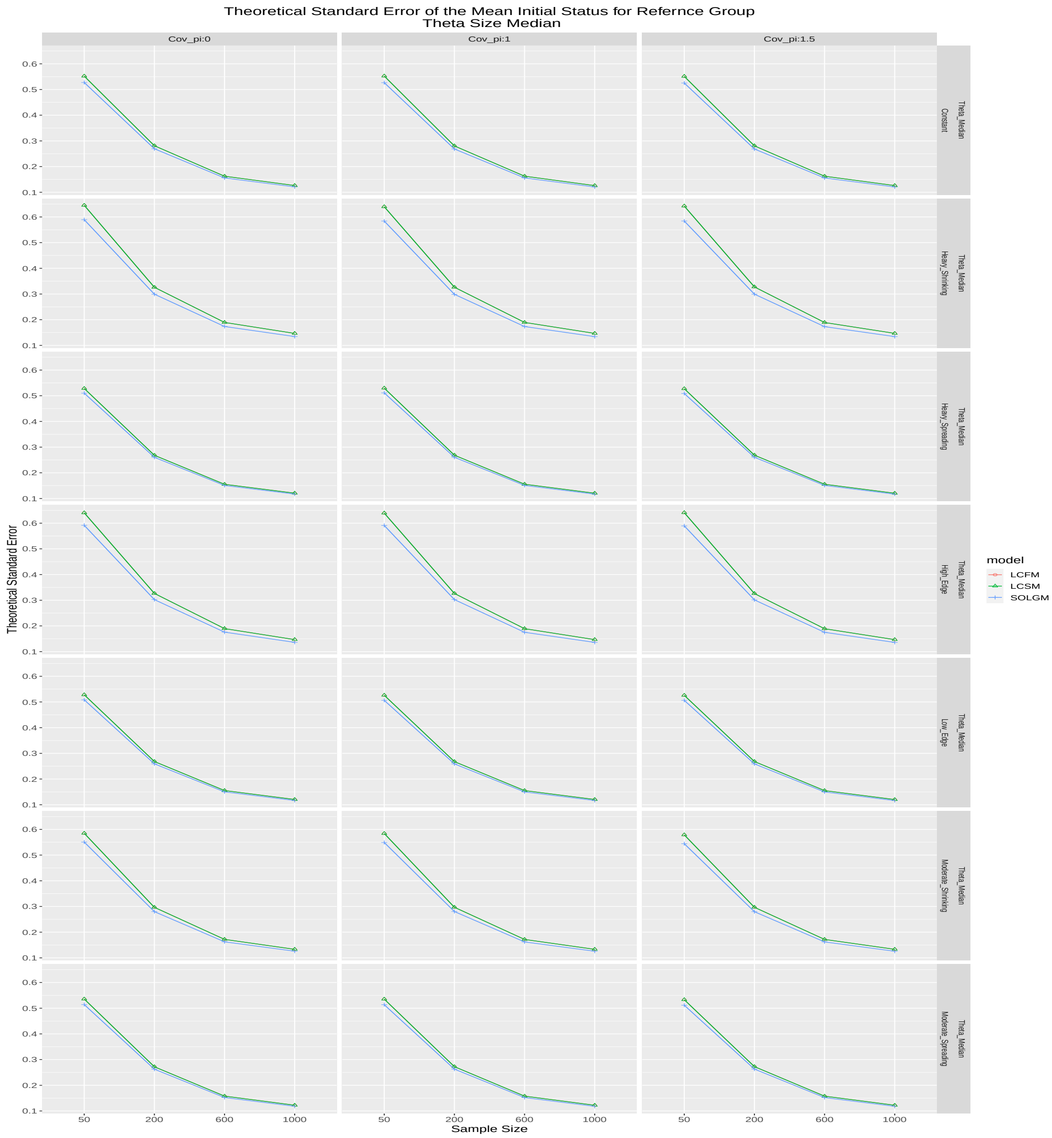

Figure A.68. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

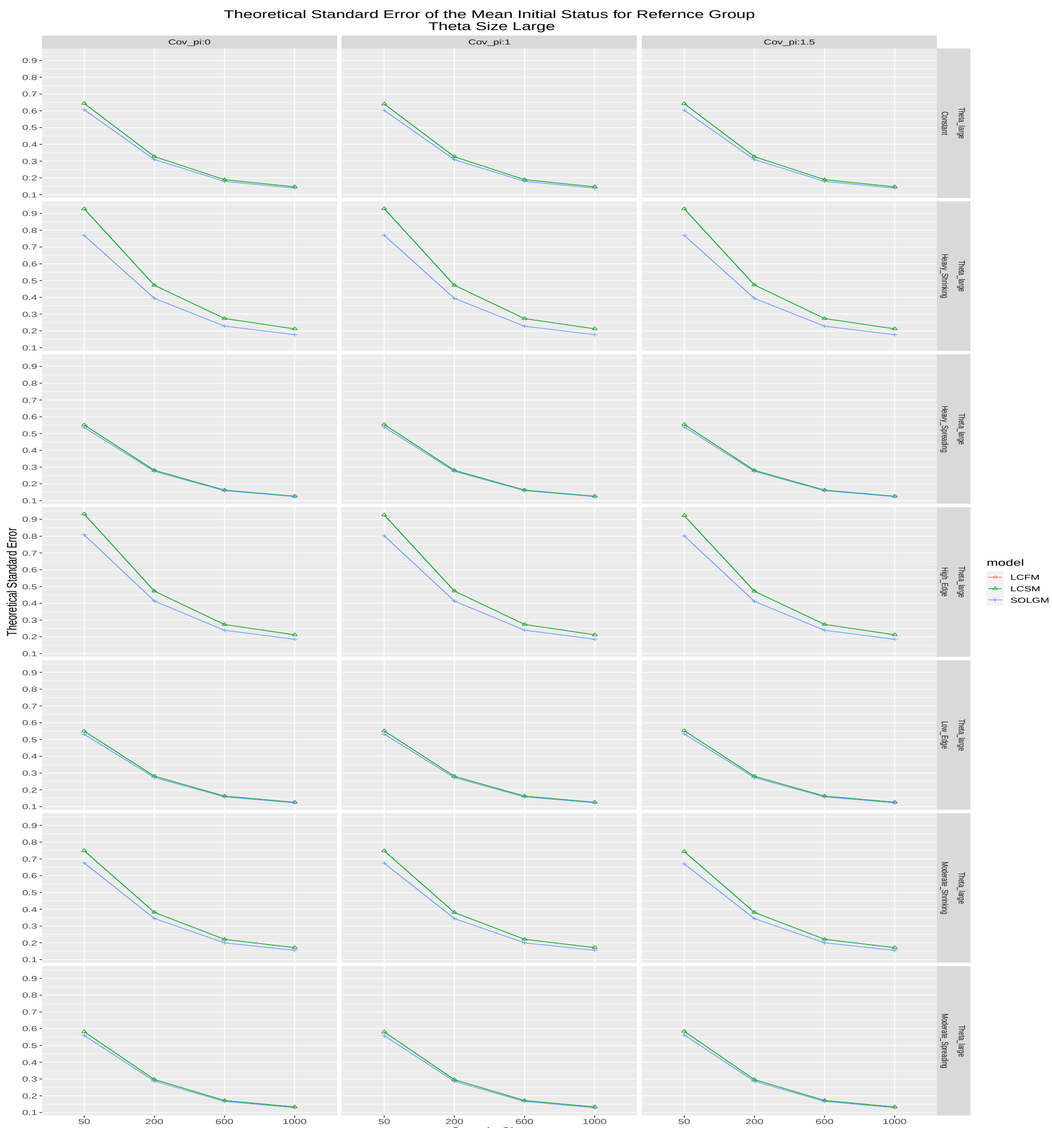

Figure A.69. Standard Error of the Mean Initial Status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

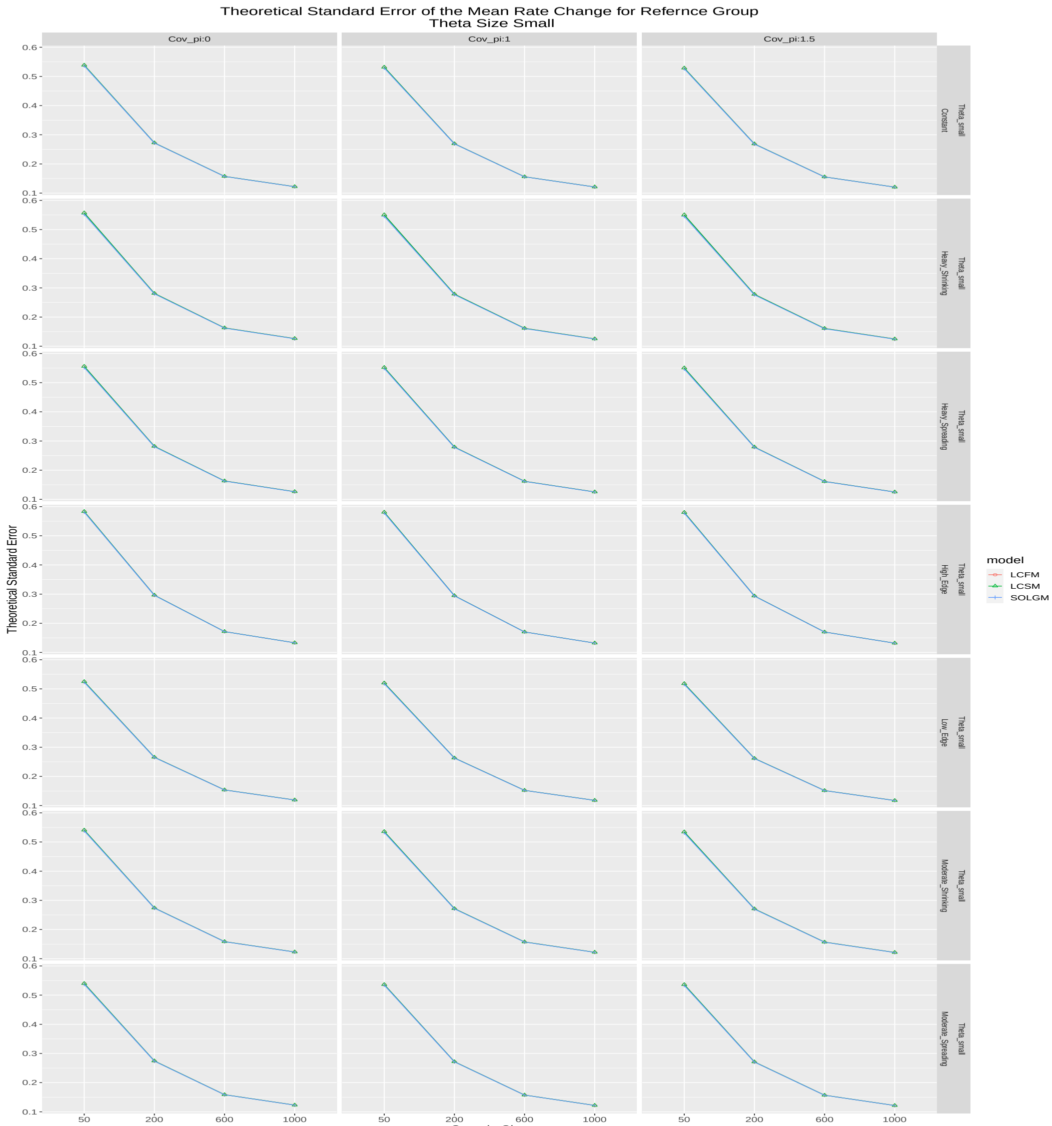

Figure A.70. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

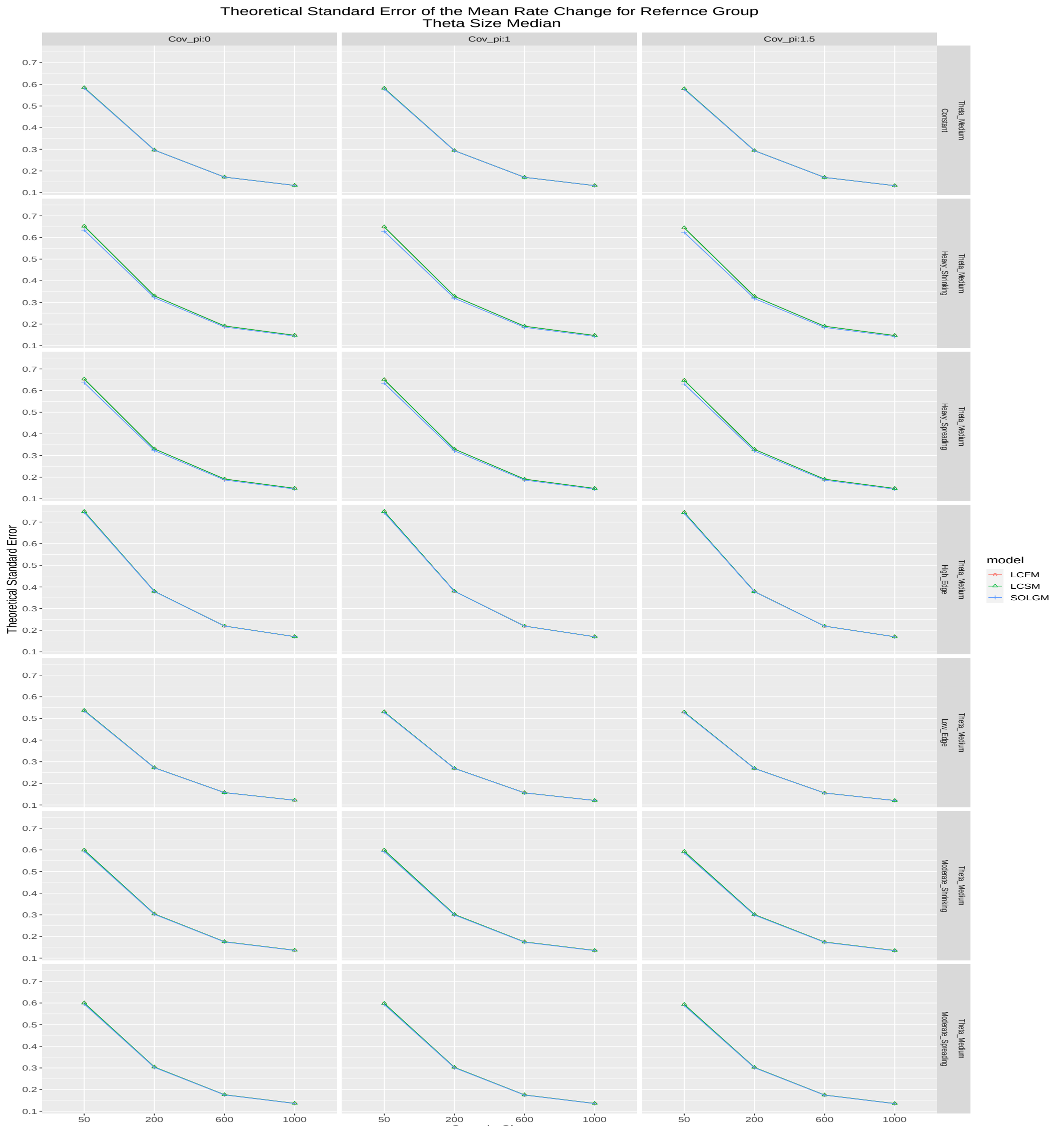

Figure A.71. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

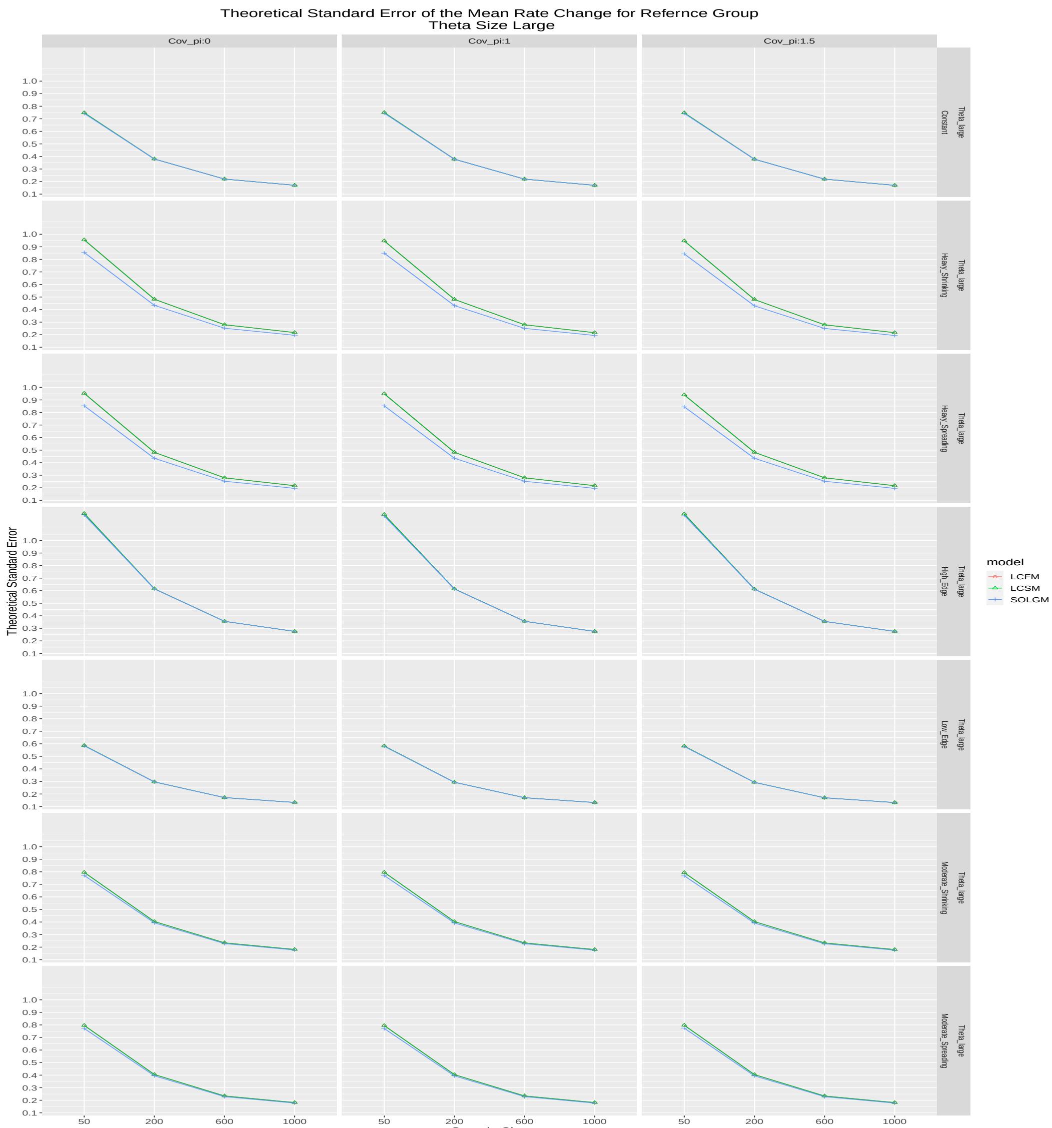

Figure A.72. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

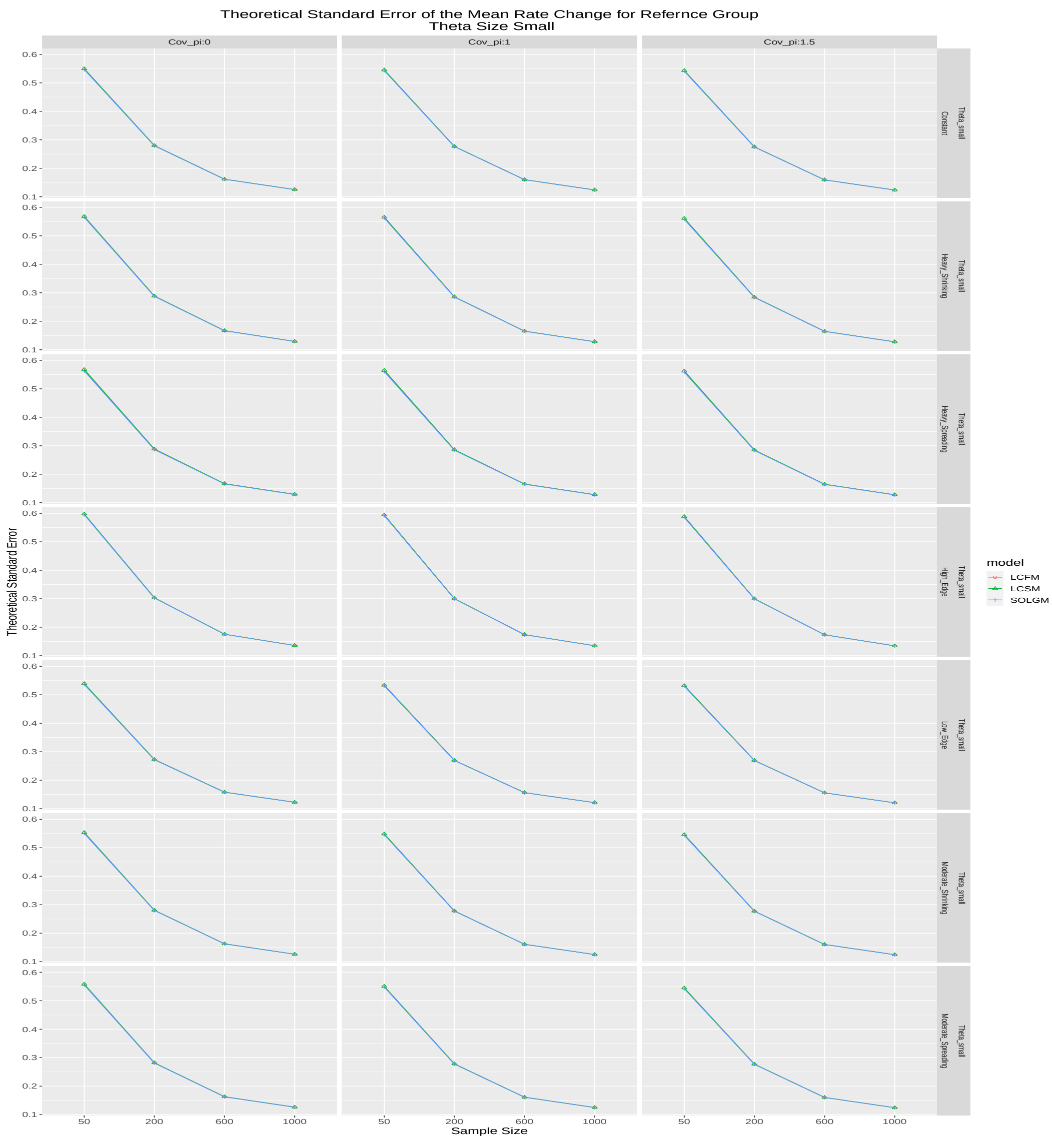

Figure A.73. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

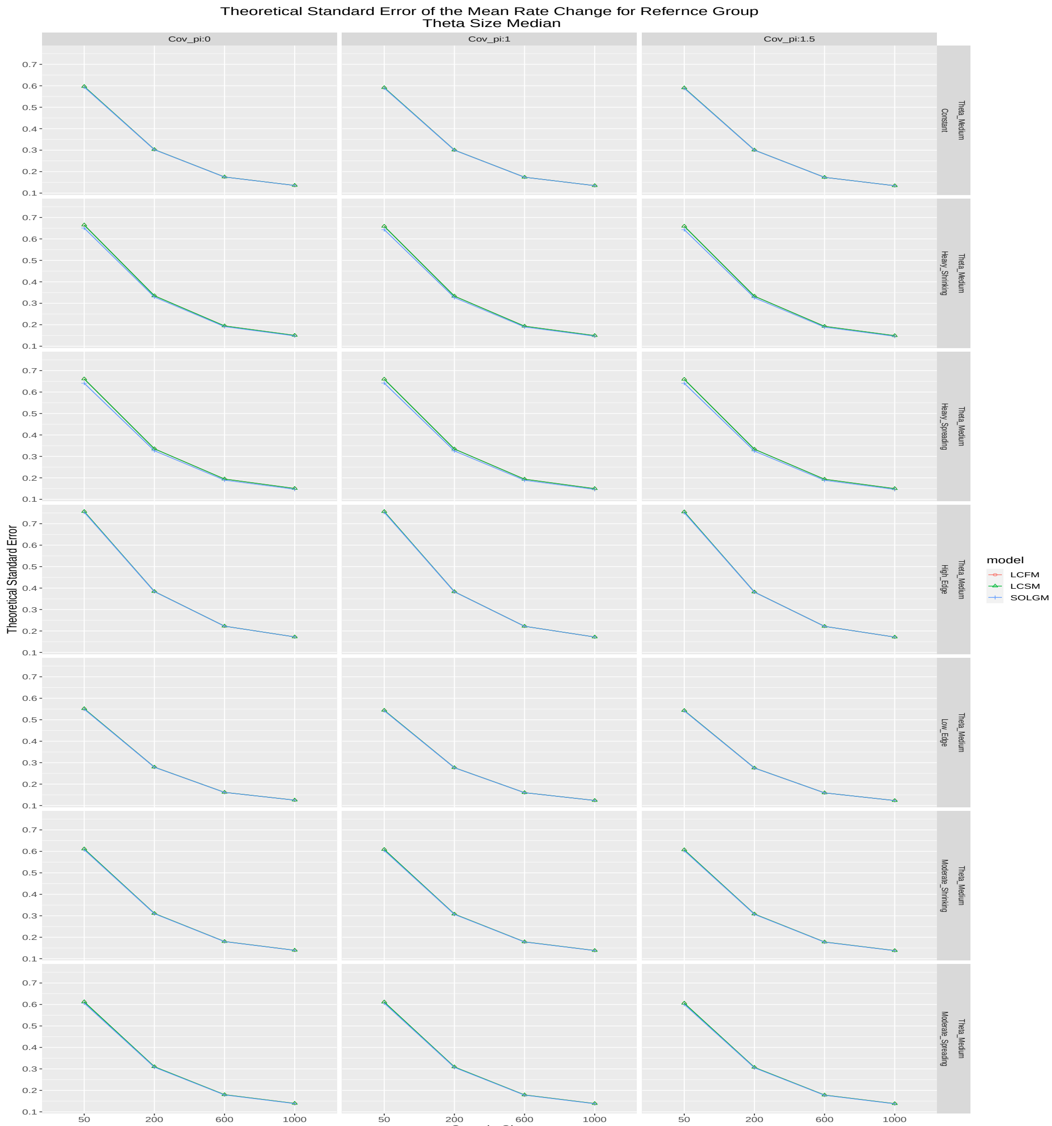

Figure A.74. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

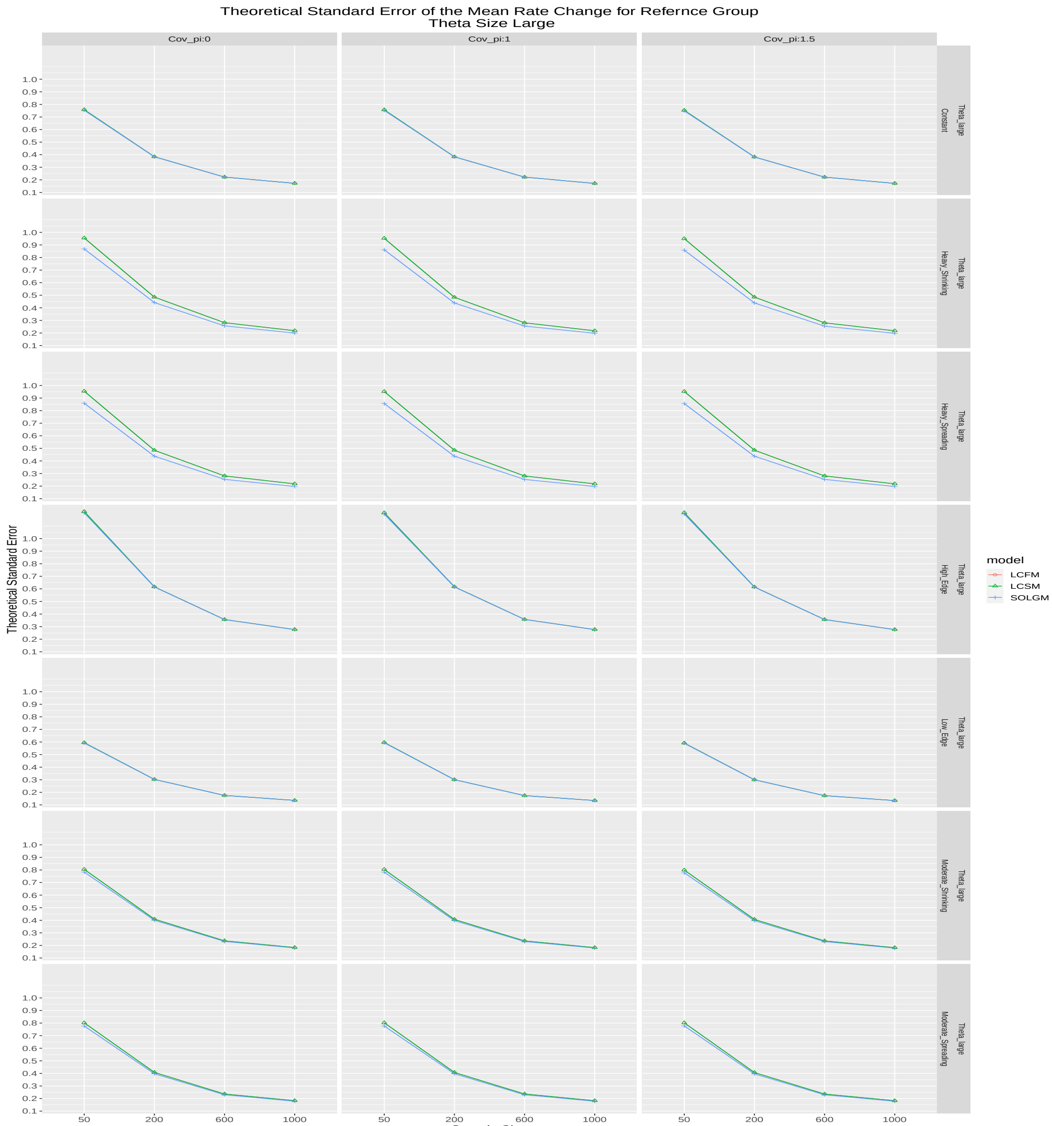

Figure A.75. Standard Error of Average Change Rate of Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

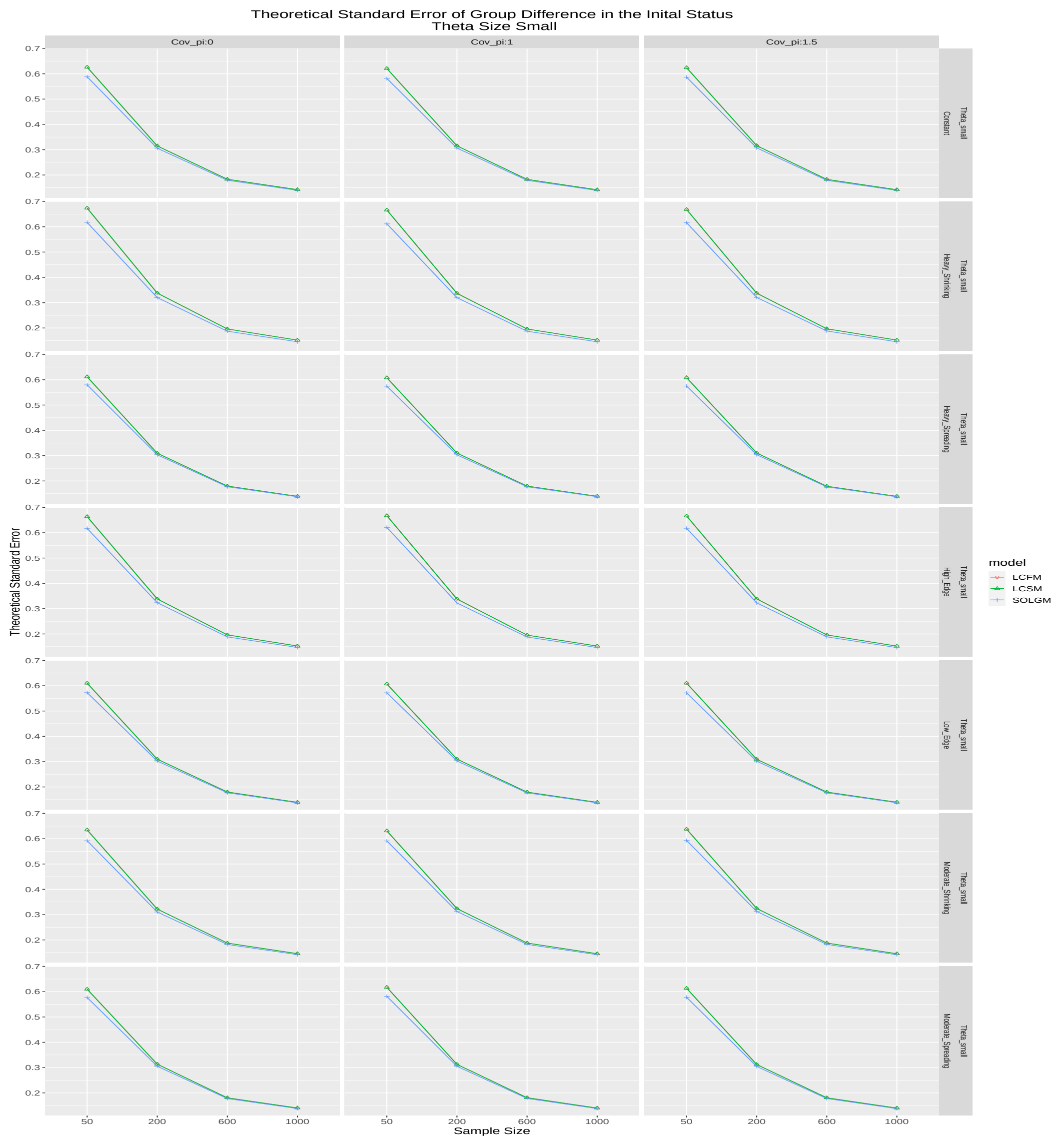

Figure A.76. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

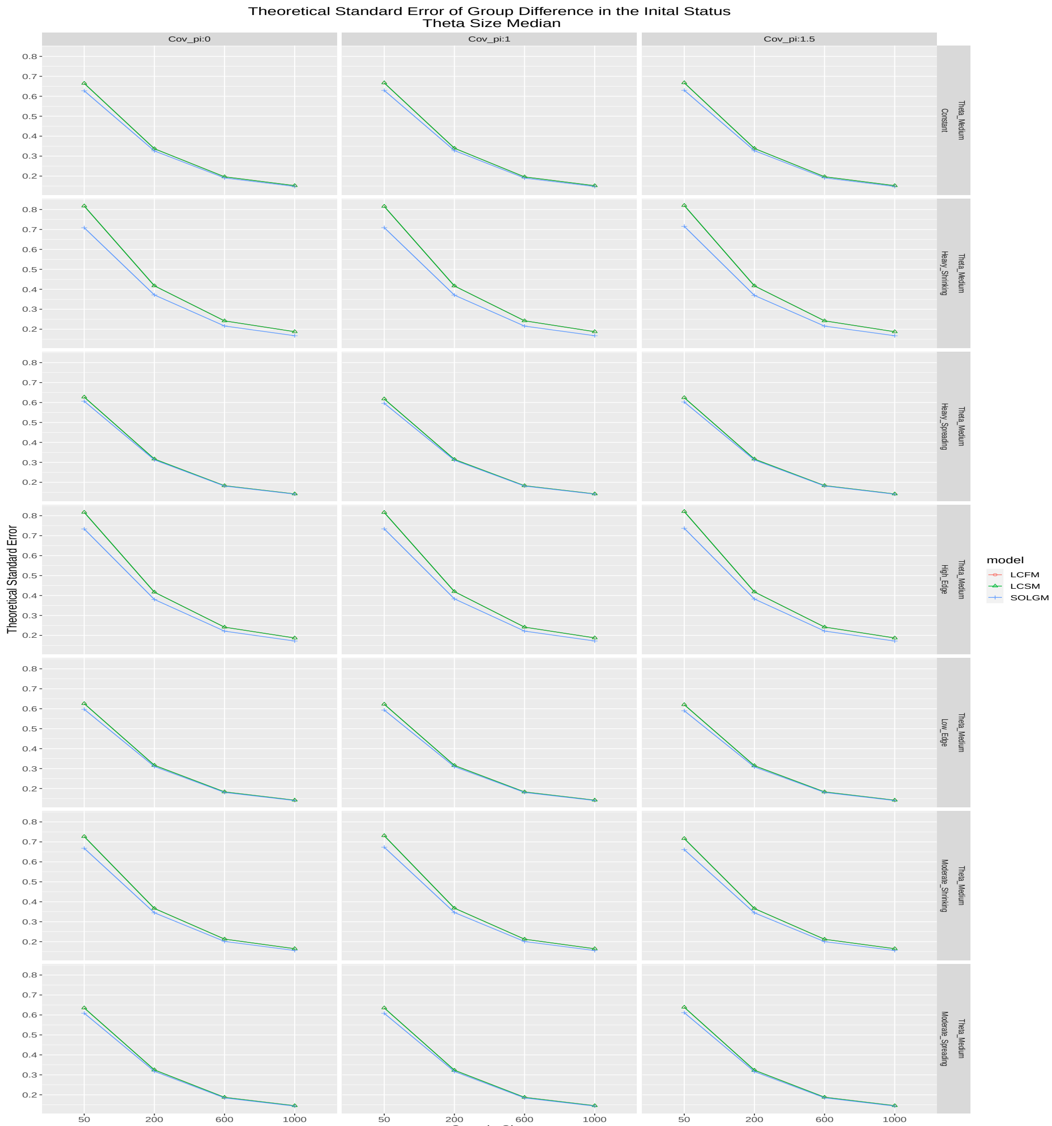

Figure A.77. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

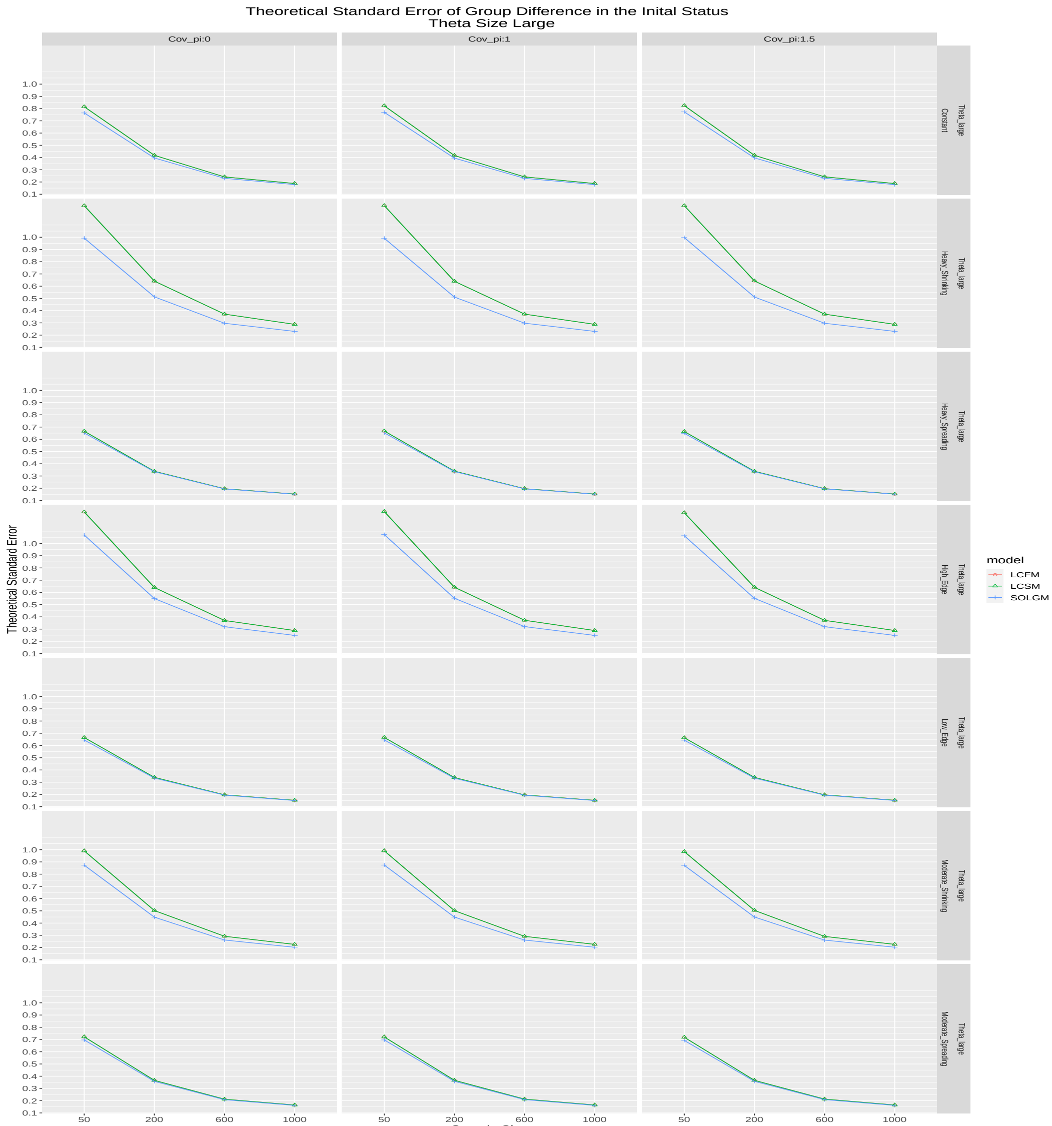

Figure A.78. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

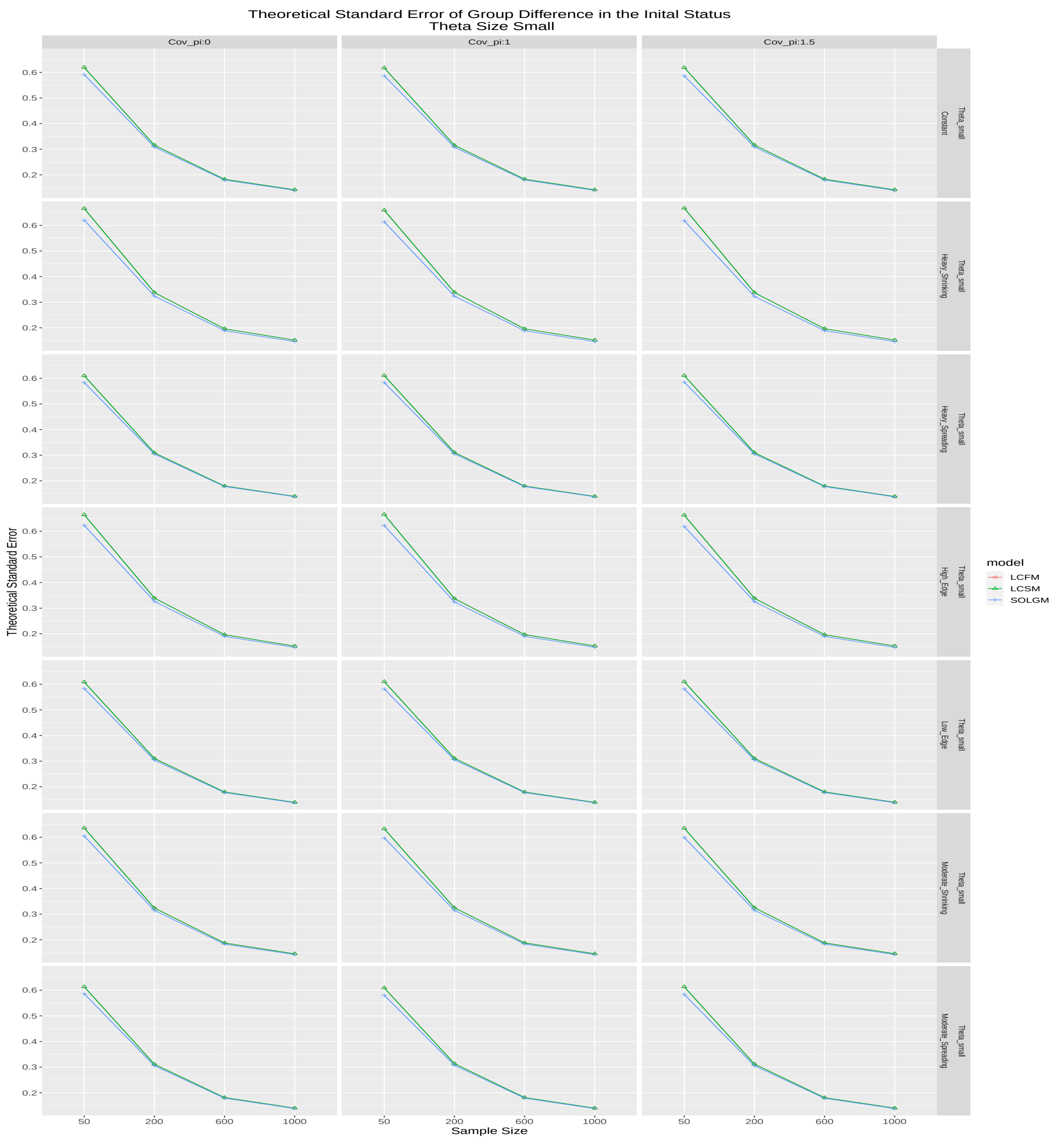

Figure A.79. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

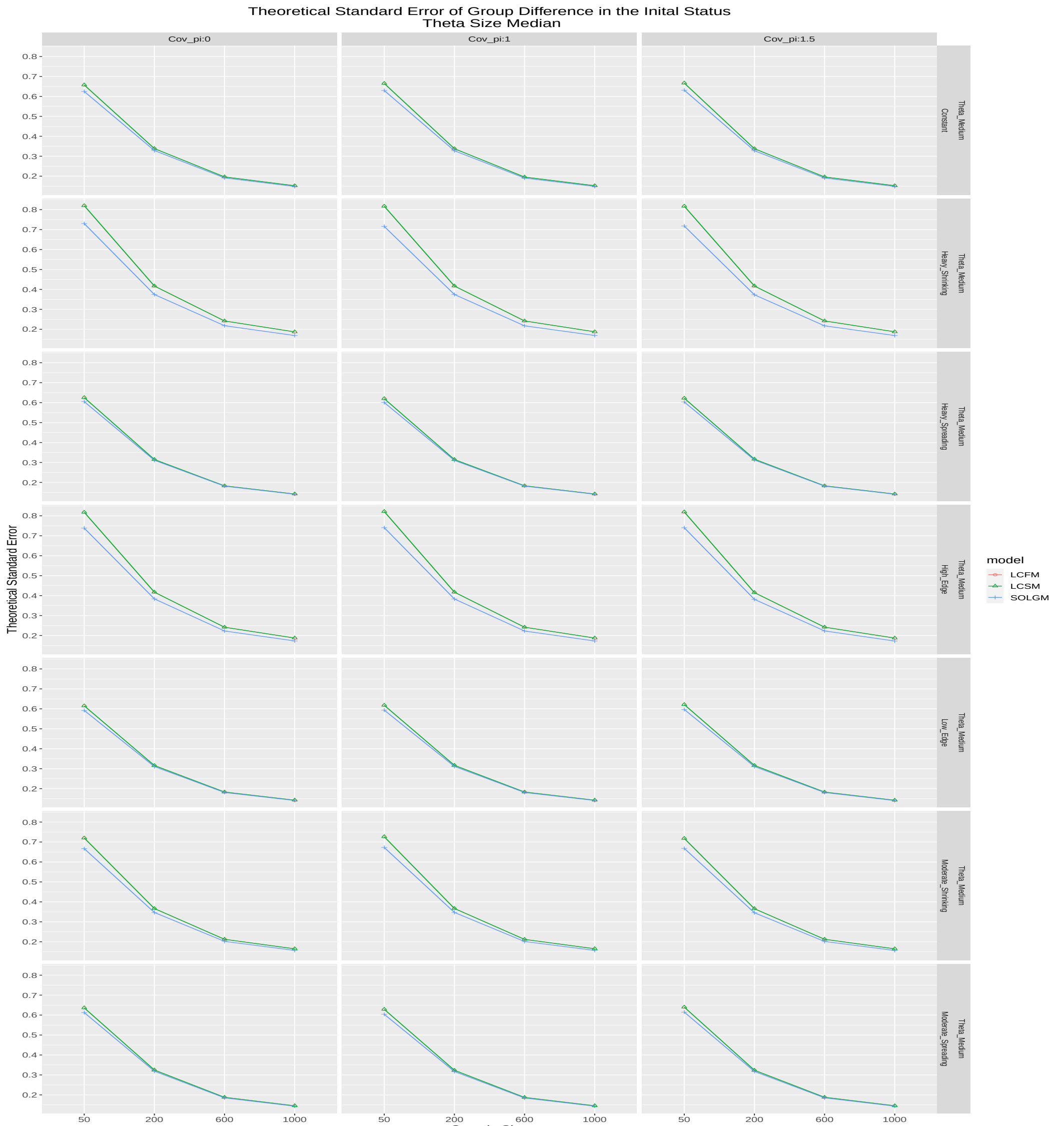

Figure A.80. Standard Error of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

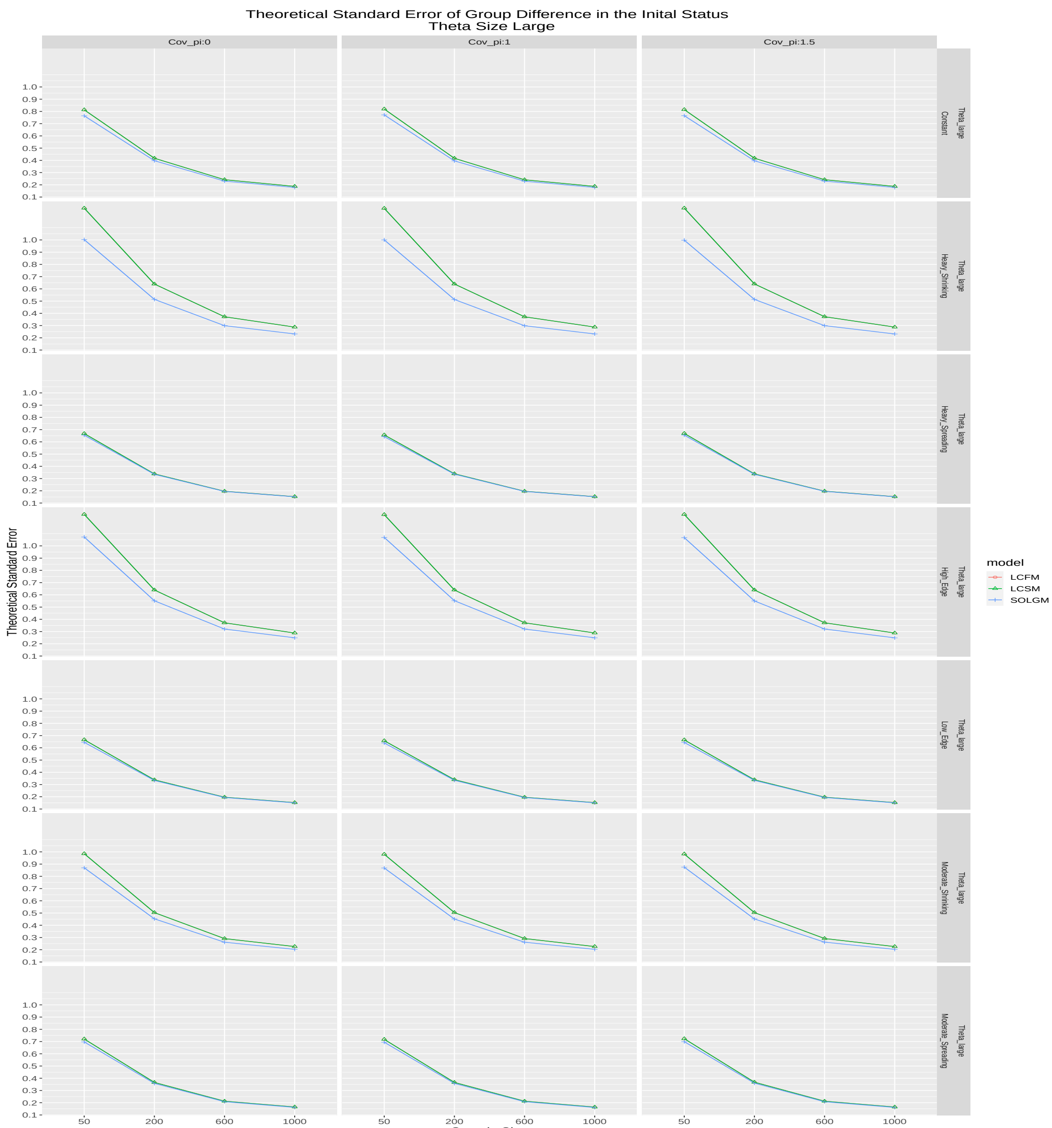

Figure A.81. Standard Error of Group Difference in Initial Status , $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

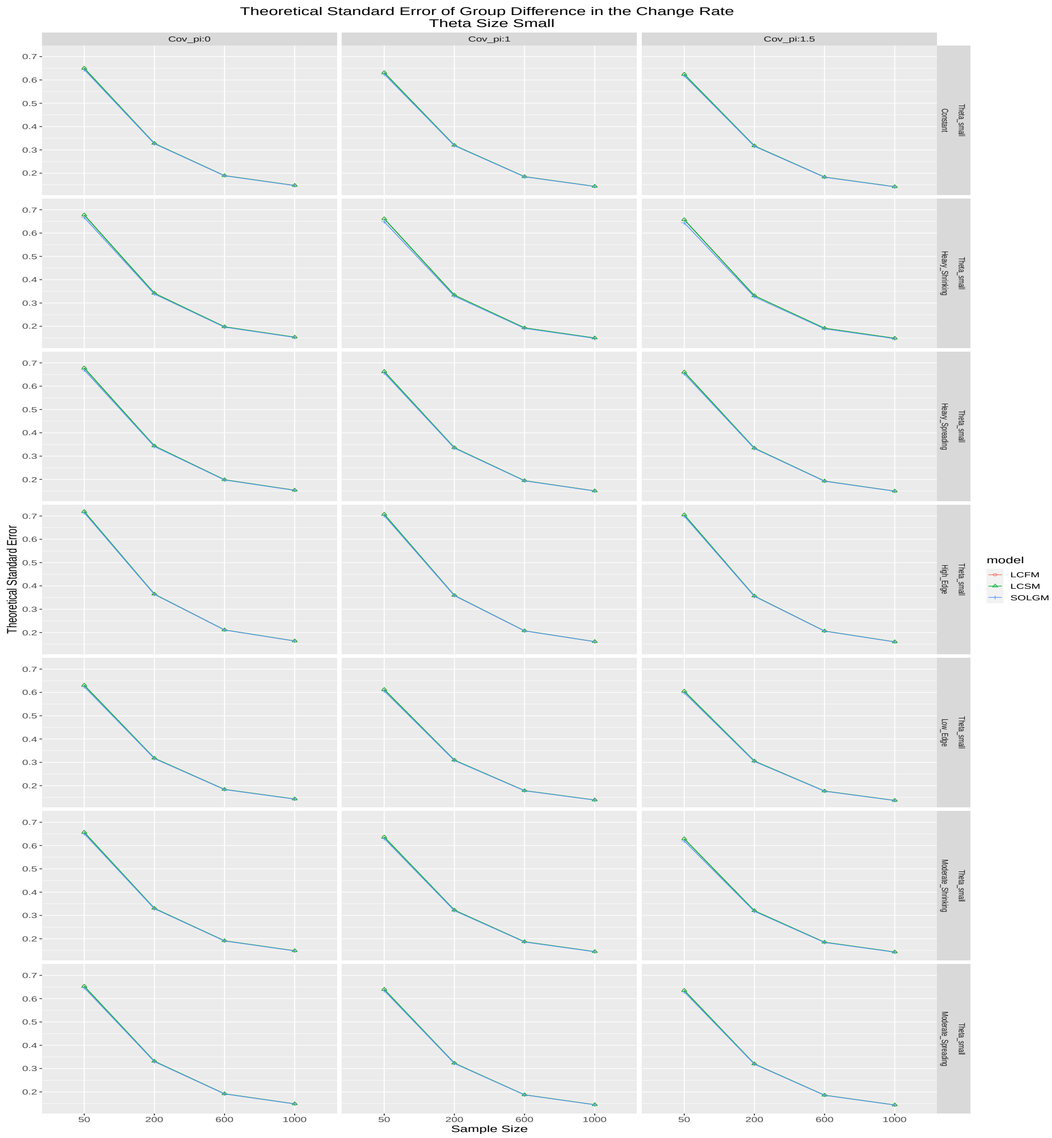

Figure A.82. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

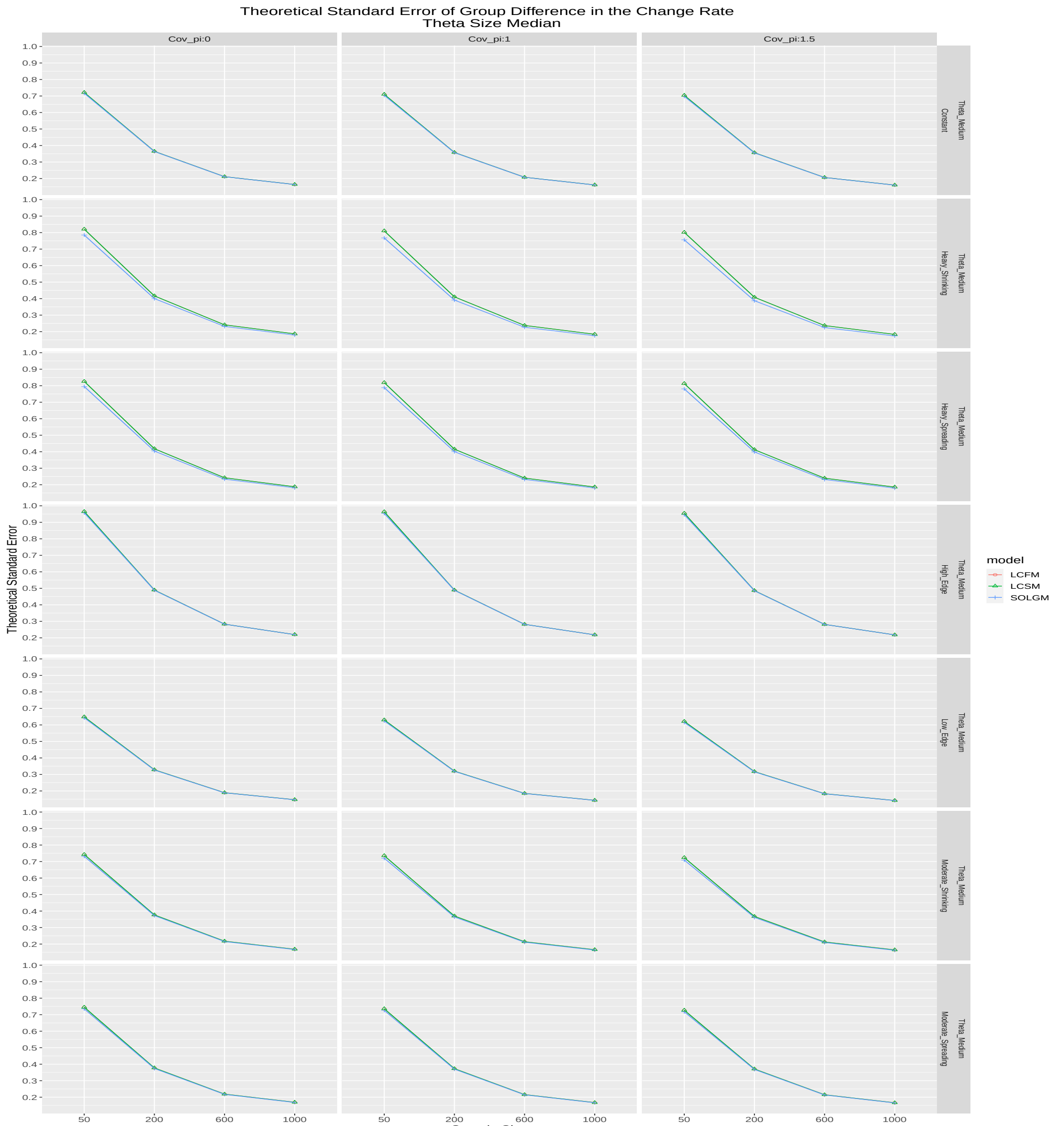

Figure A.83. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

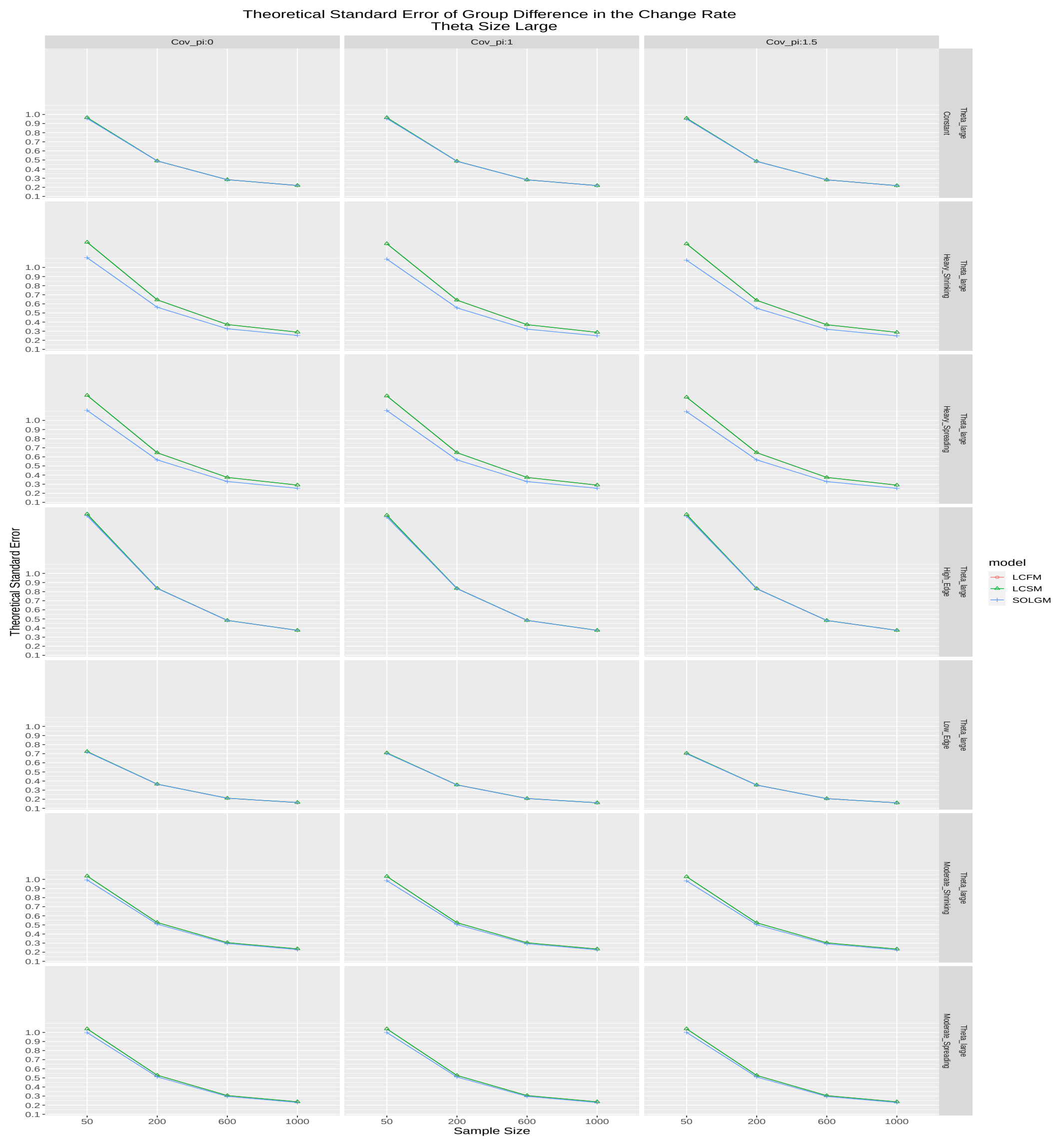

Figure A.84. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

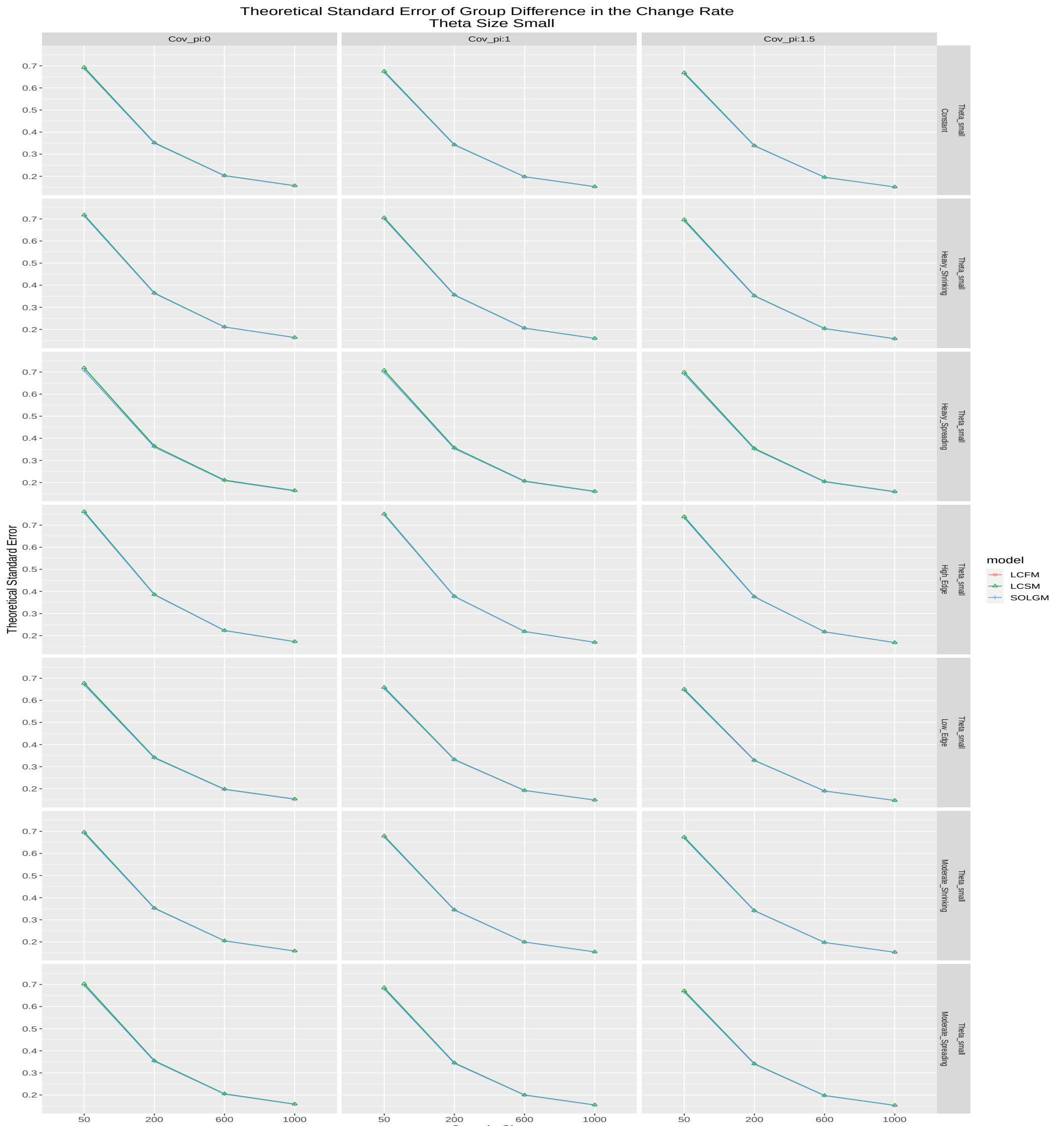

Figure A.85. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

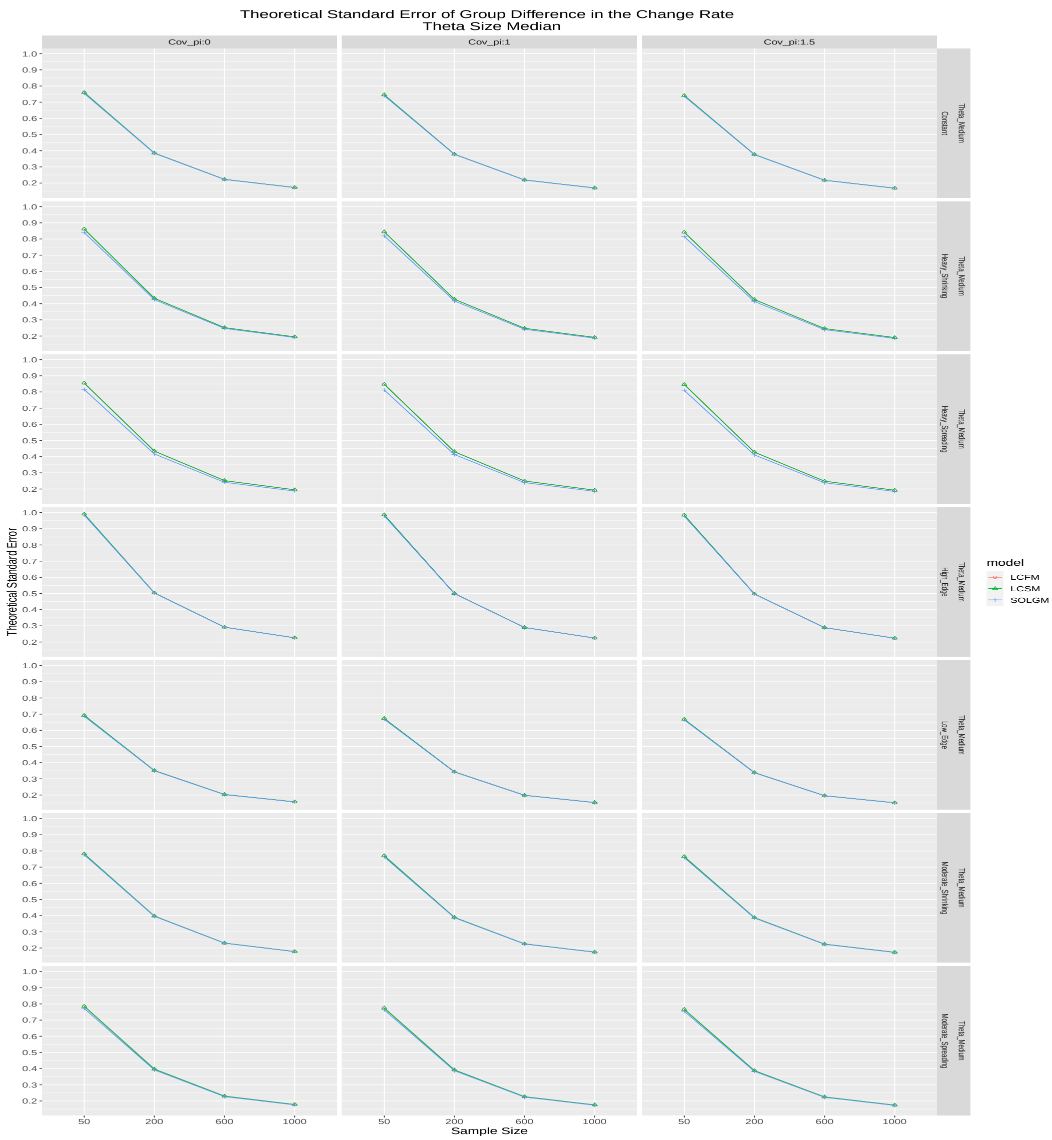

Figure A.86. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

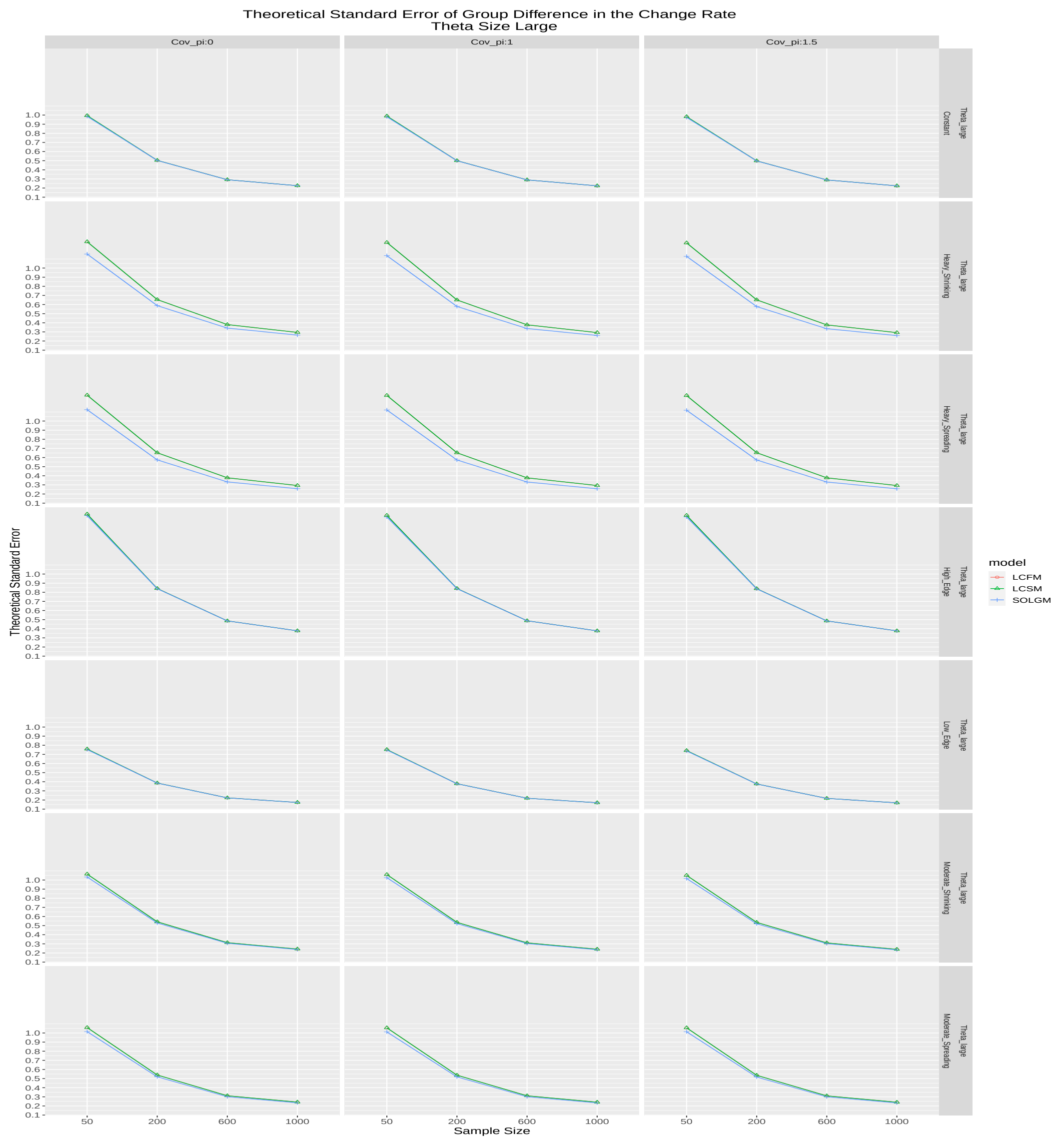

Figure A.87. Standard Error of the Mean Group Difference in the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

A.0.7 List of Figures in Type I Error/Statistical Power

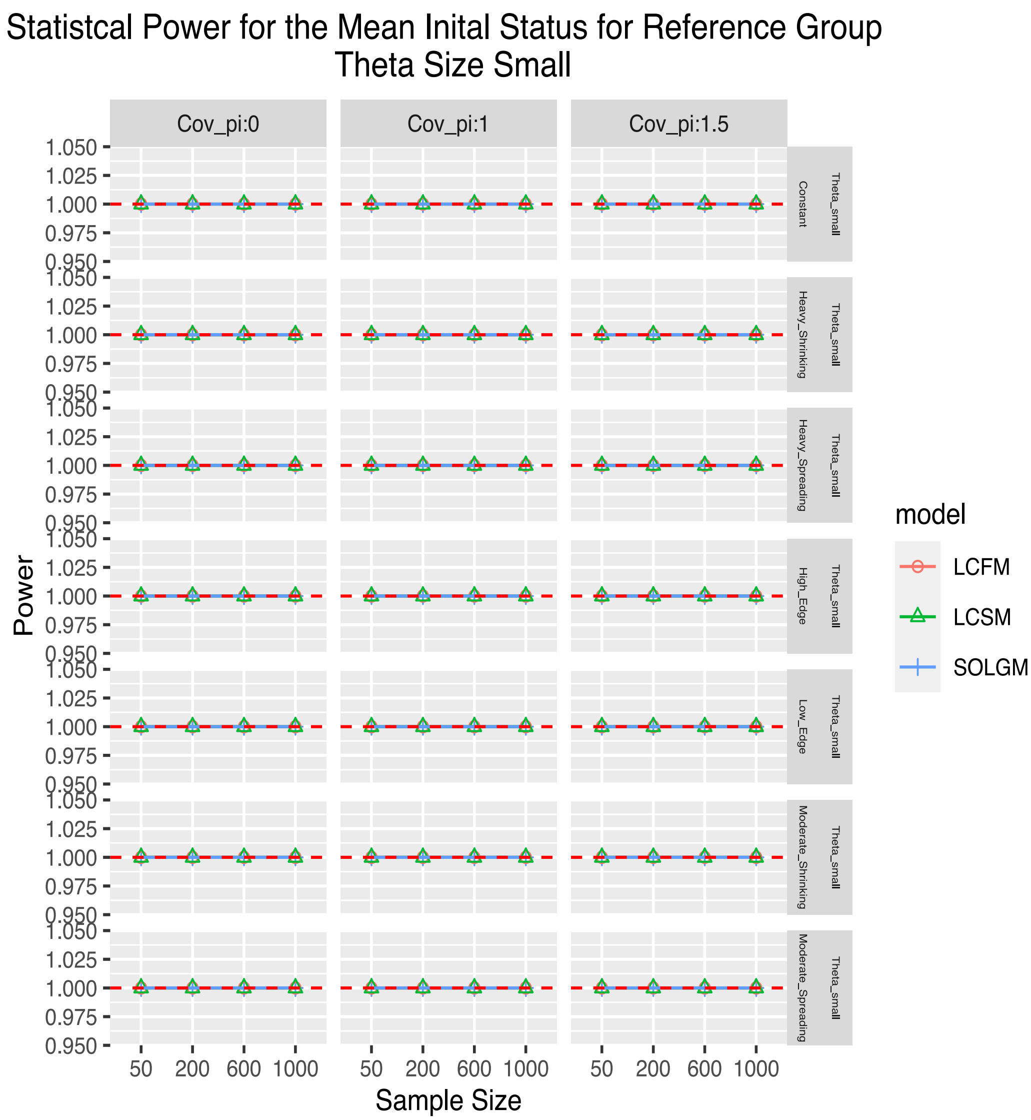

Figure A.88. Statistical Power of the mean initial status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**Small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

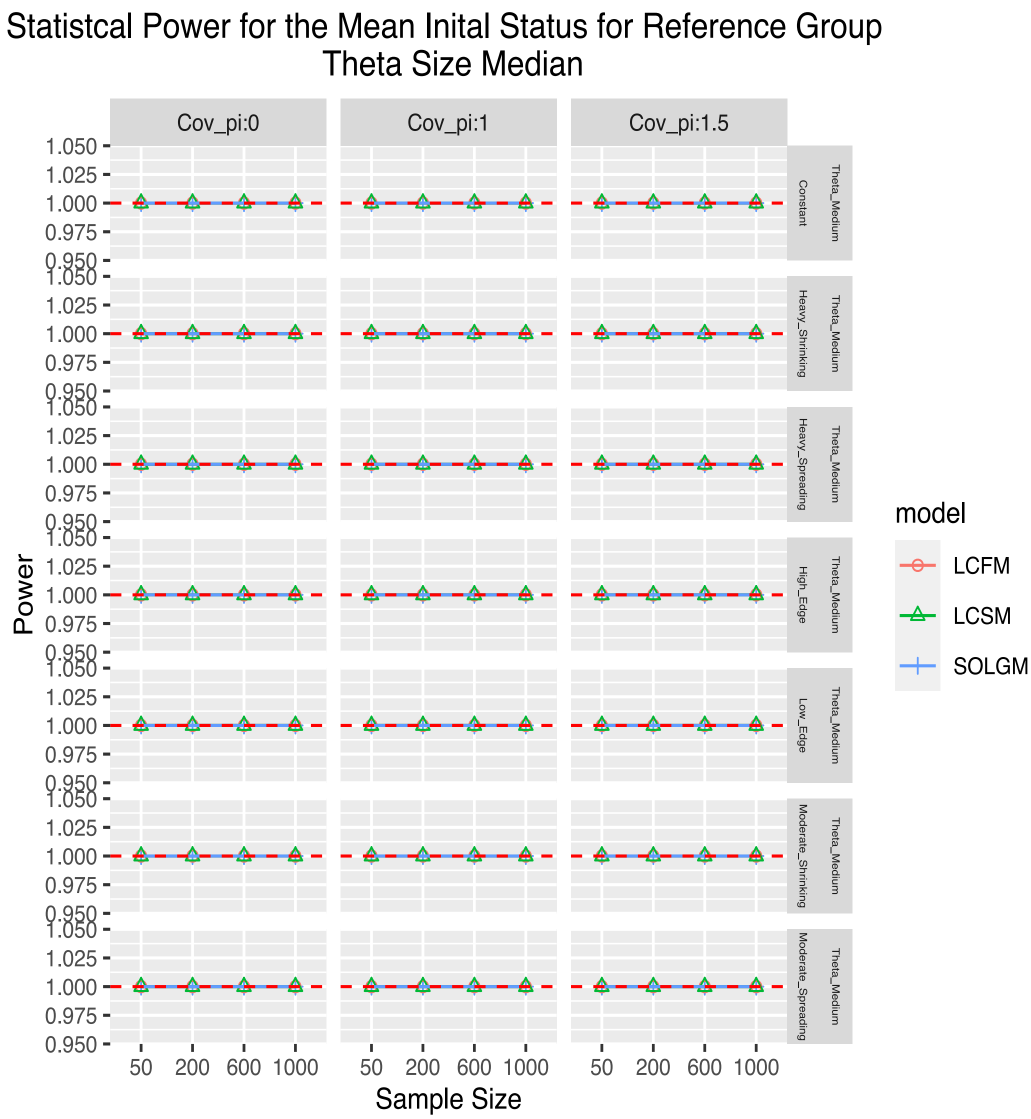

Figure A.89. Statistical Power of the mean initial status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**Median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

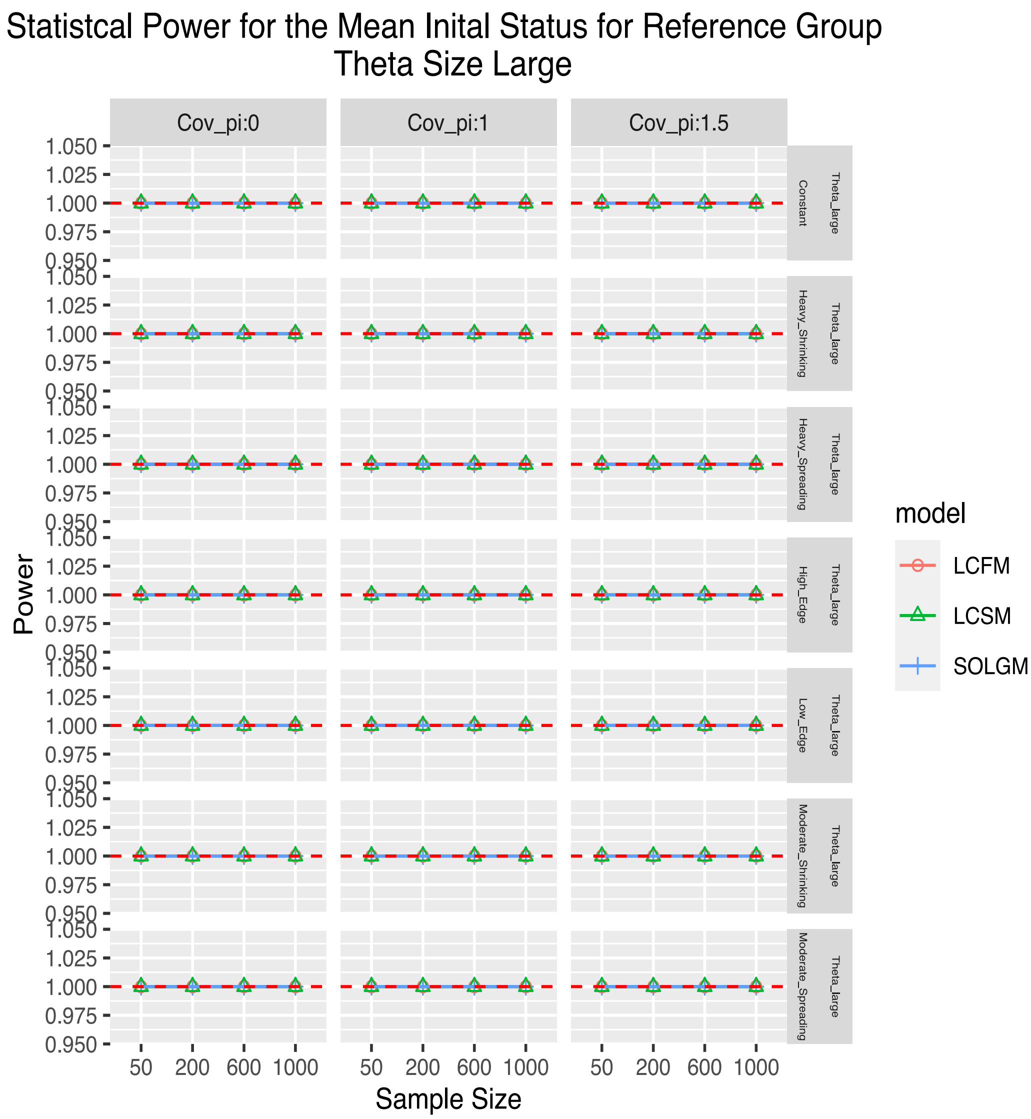

Figure A.90. Statistical Power of the mean initial status for the Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**Large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

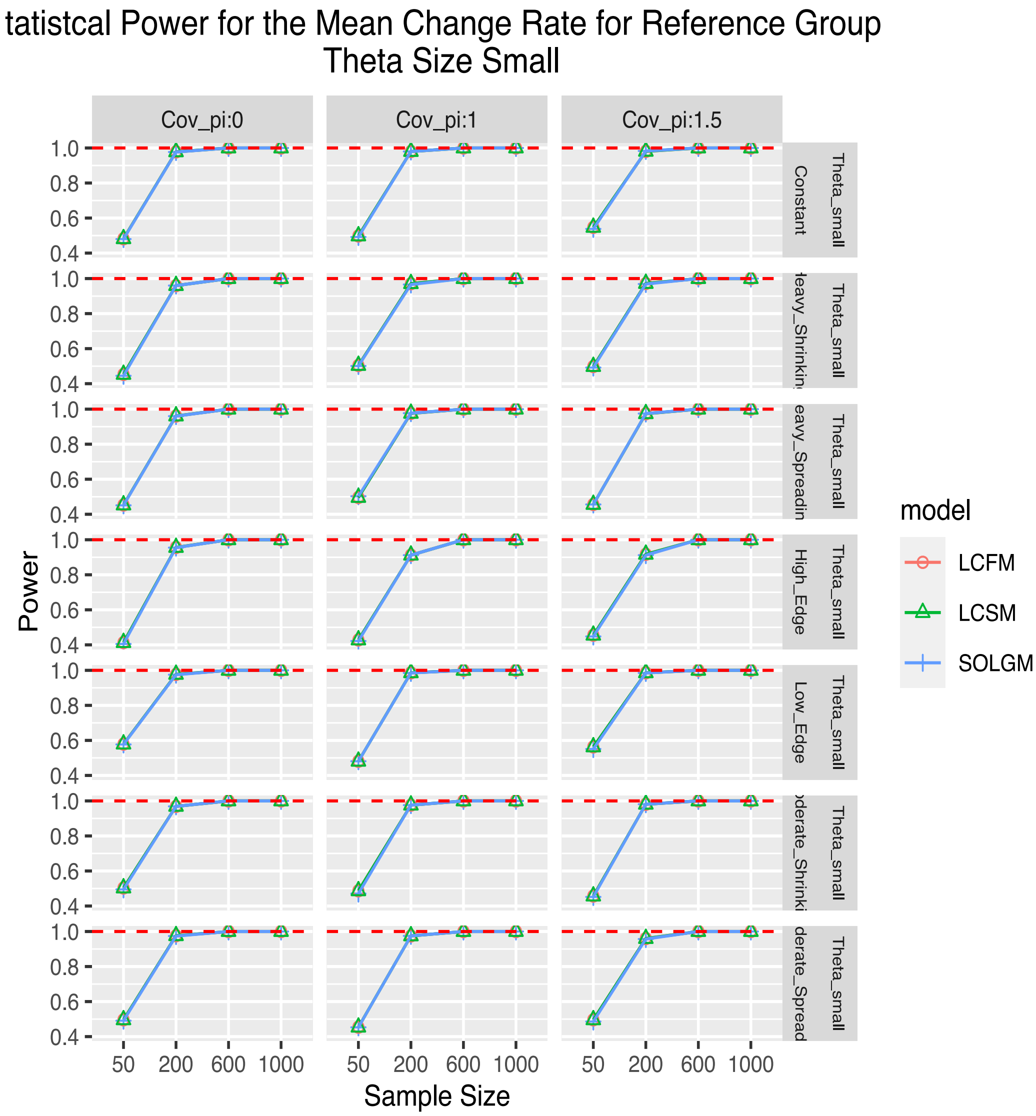

Figure A.91. Statistical Power of Mean Change Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**Small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

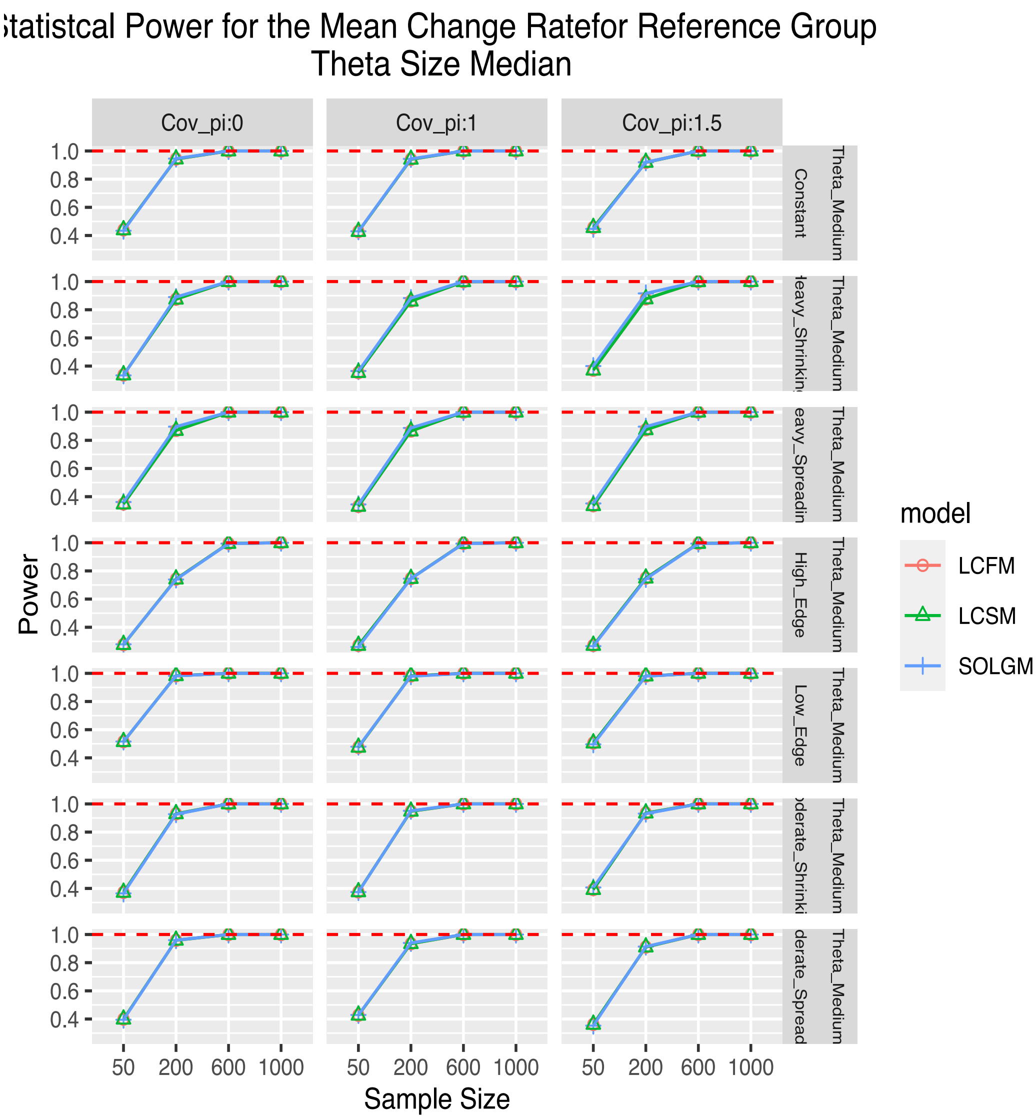

Figure A.92. Statistical Power of Mean Change Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**Median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

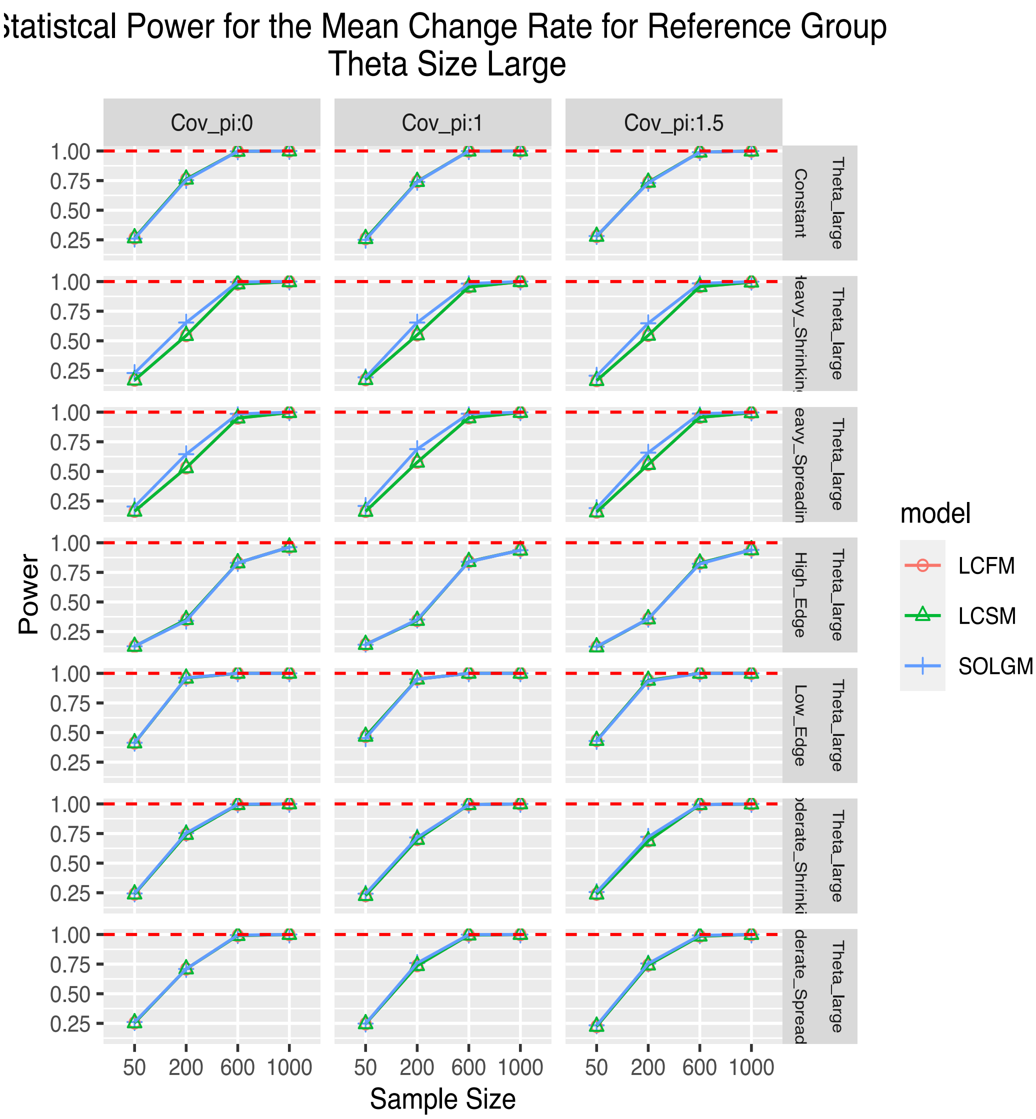

Figure A.93. Statistical Power of Mean Change Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**Large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

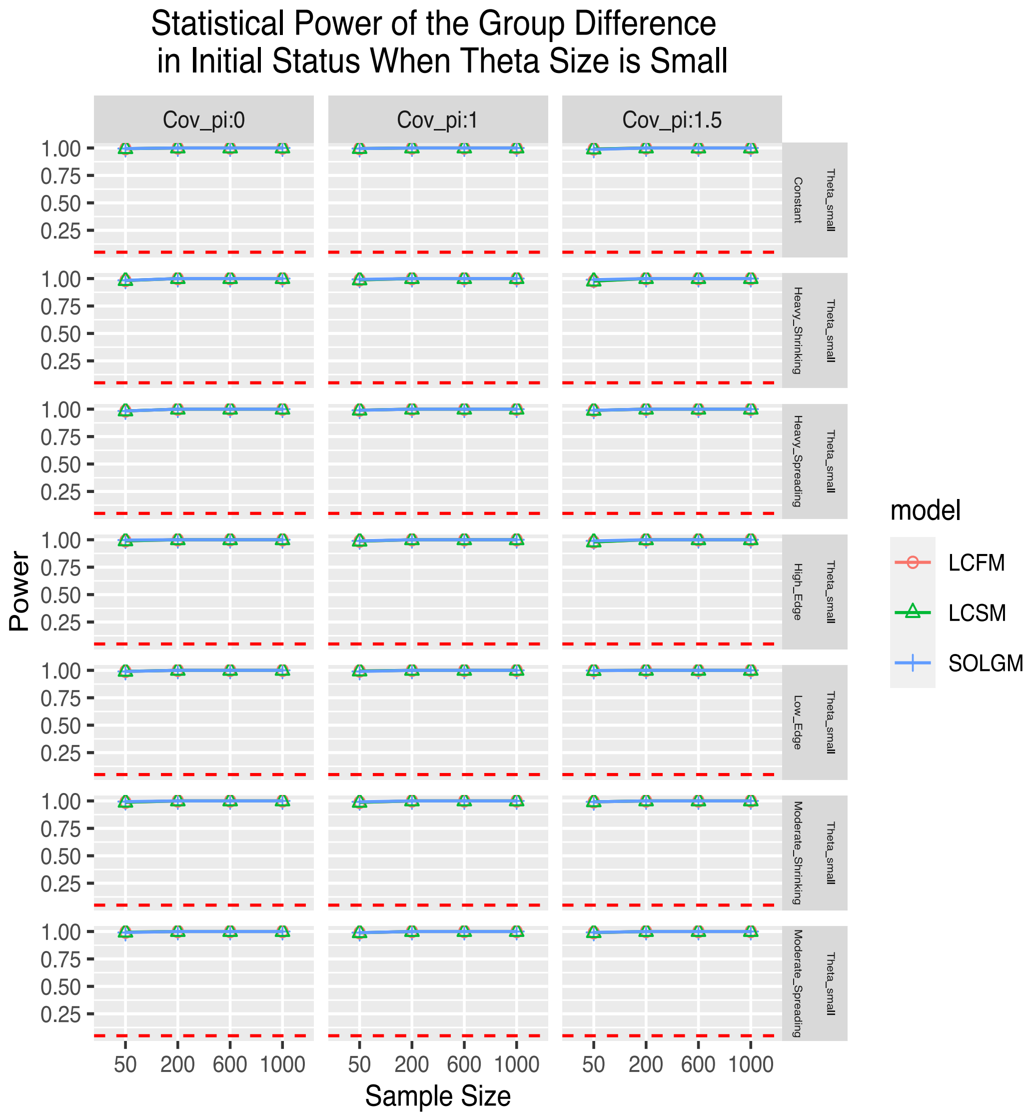

Figure A.94. Statistical Power of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**Small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

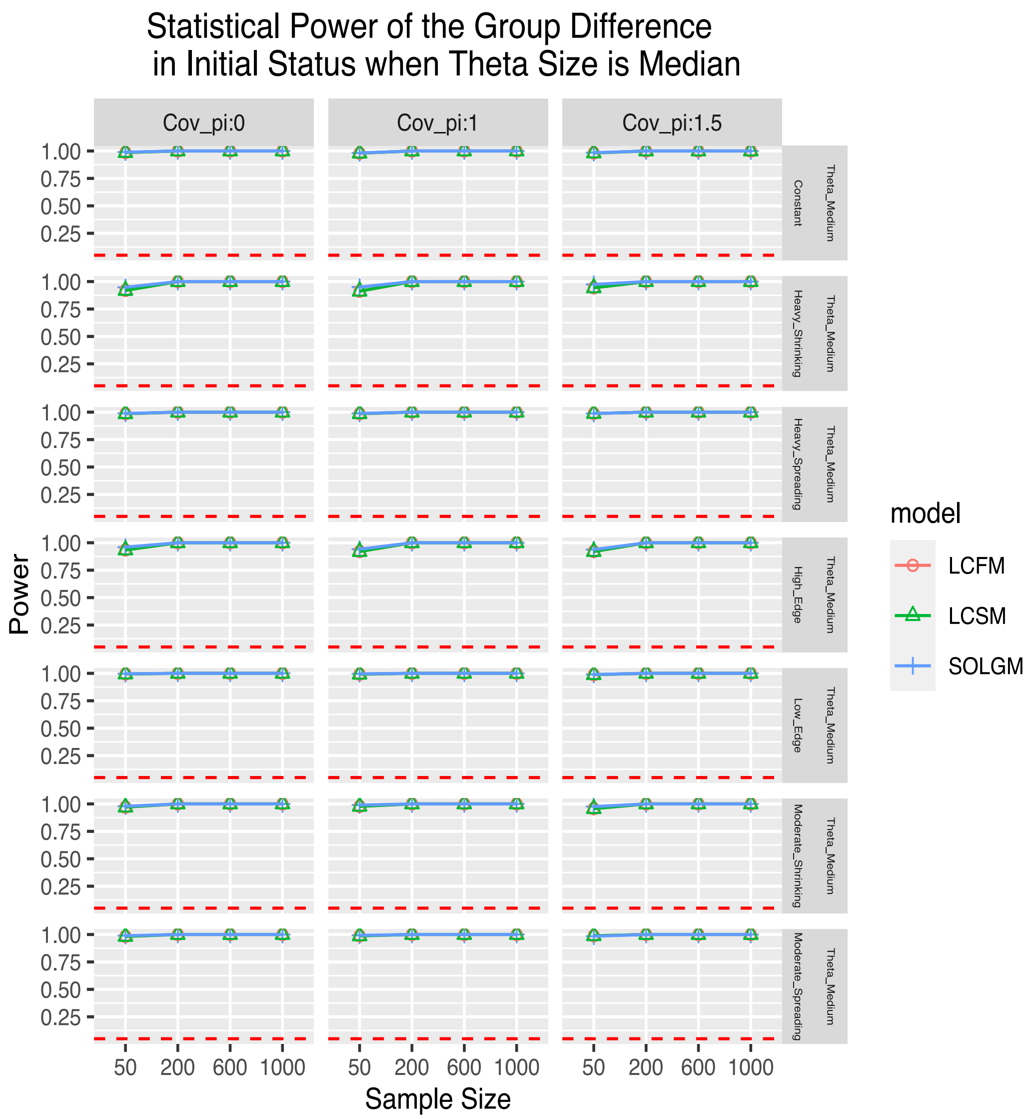

Figure A.95. Statistical Power of Group Difference In Initial Status, $\boldsymbol{\beta_{01}=3}$, by time-specific error size (**Median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01}=3}$ and $\boldsymbol{\beta_{11}=0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

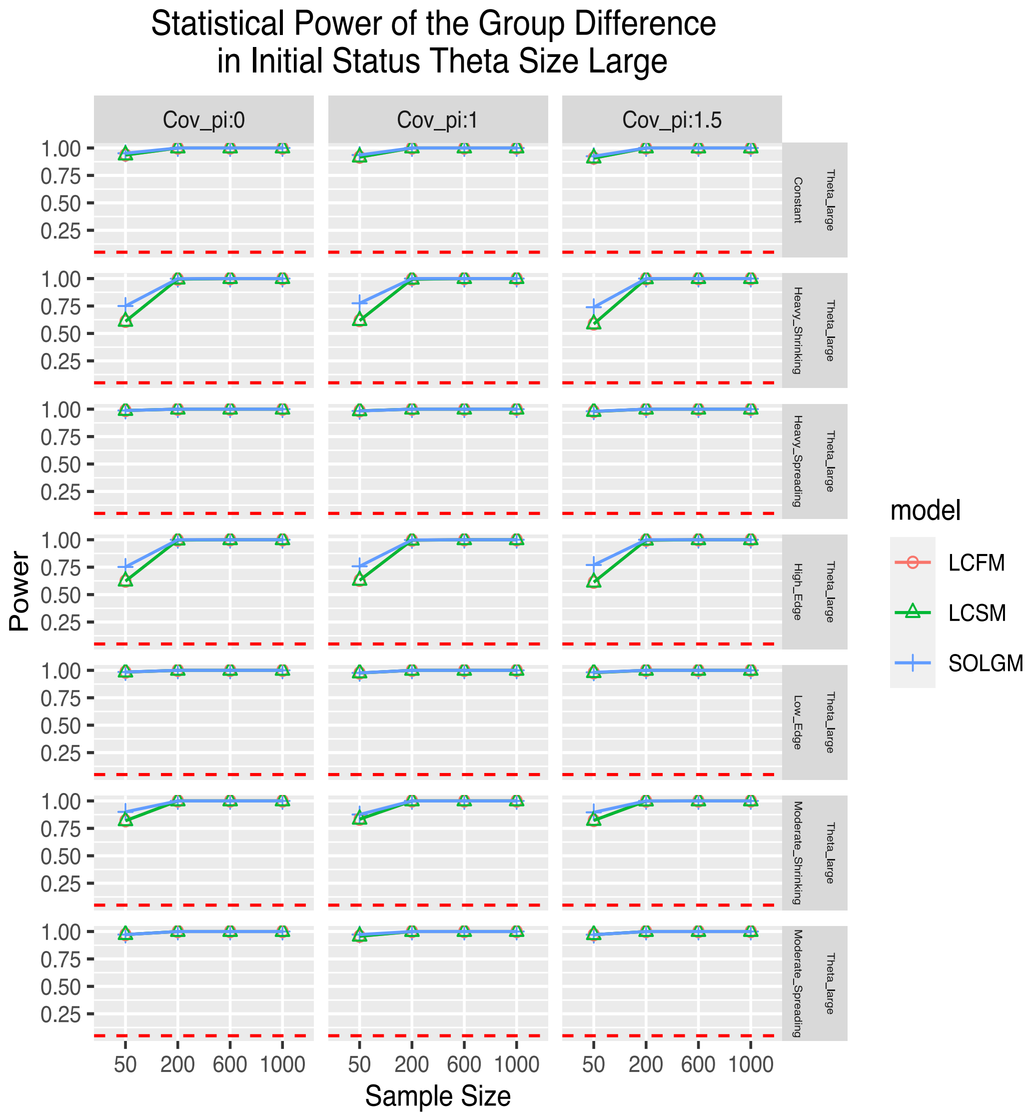

Figure A.96. Statistical Power of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**Large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

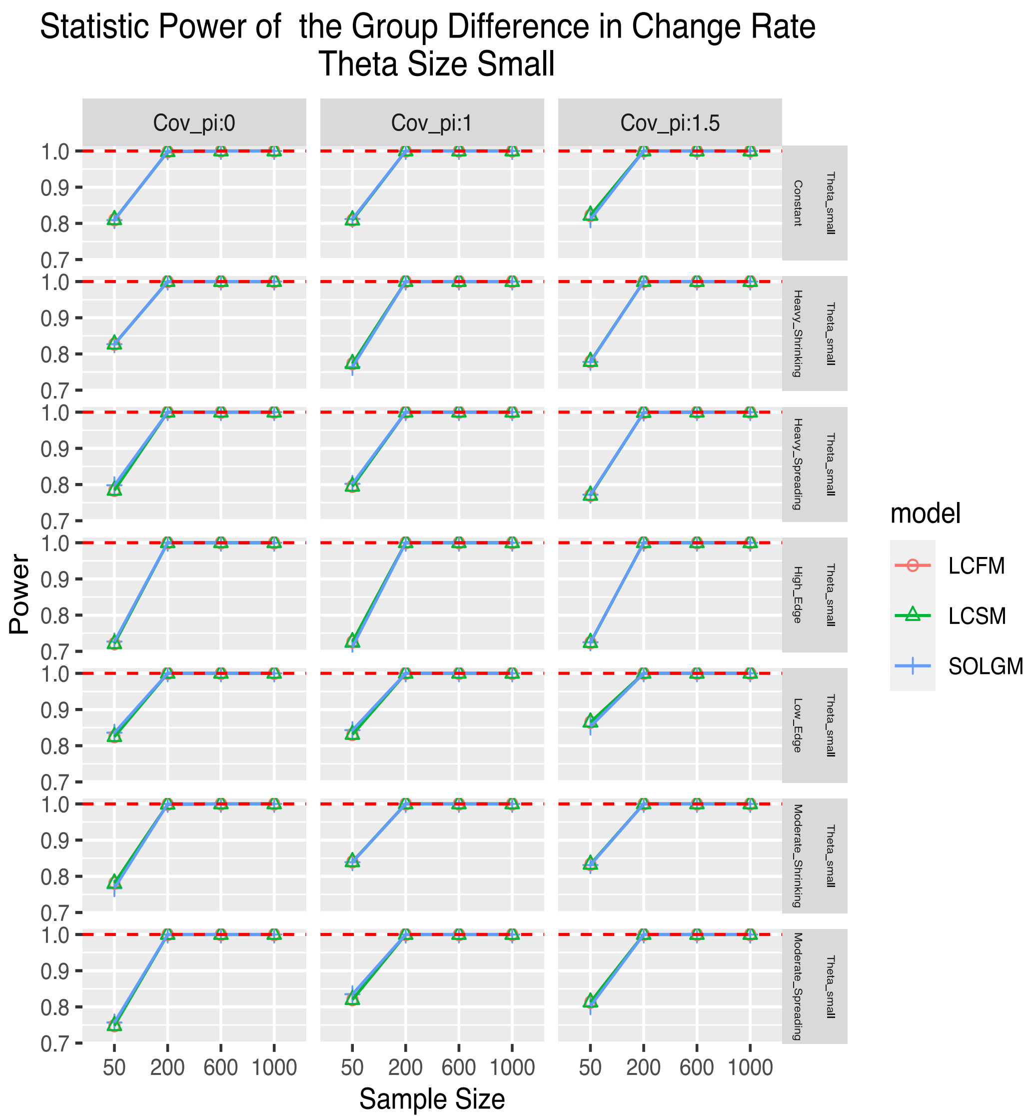

Figure A.97. Statistical Power of Group Difference in Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**Small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

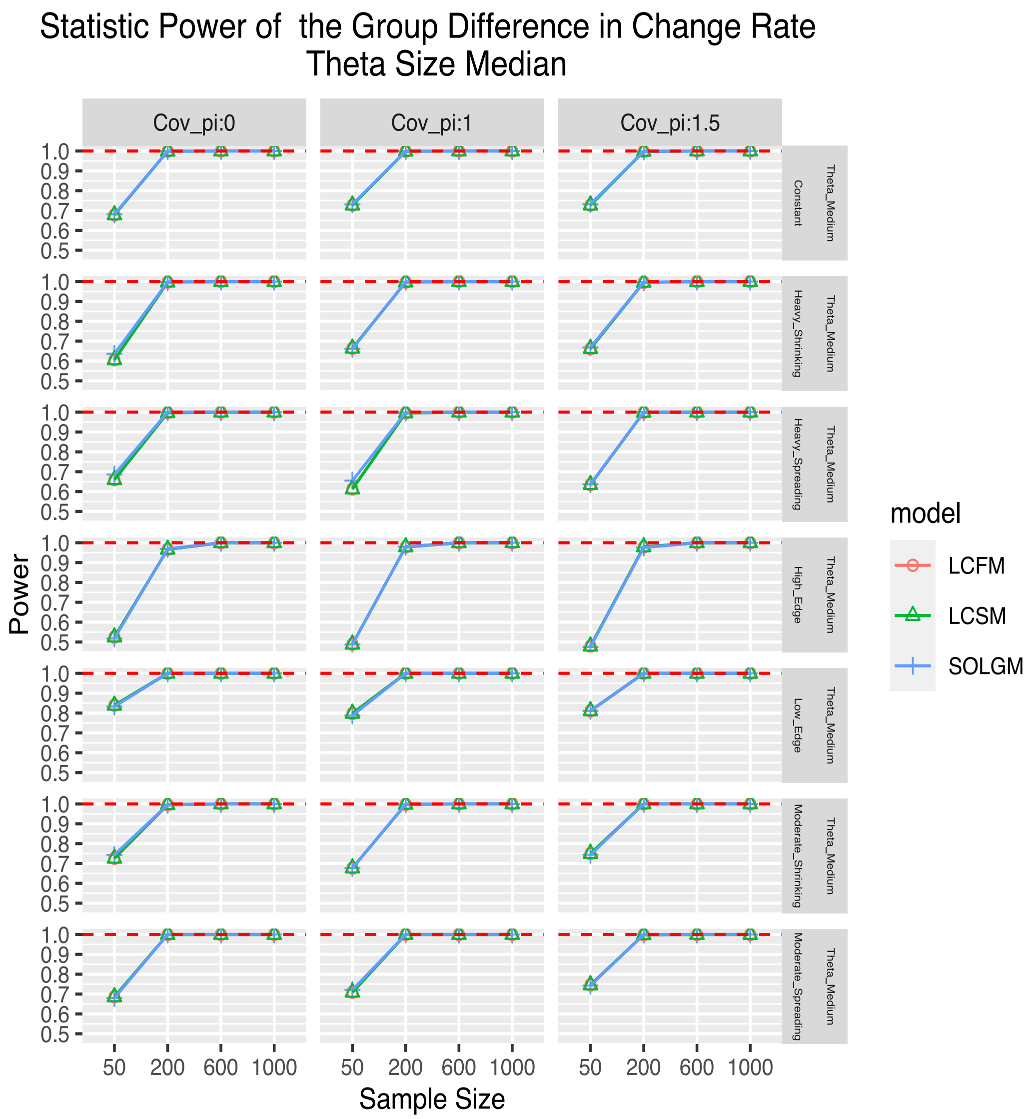

Figure A.98. Statistical Power of Group Difference In Growth Rate, $\boldsymbol{\beta_{11} = 3}$, by time-specific error size (**Median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 2}$ and $\boldsymbol{\beta_{11} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

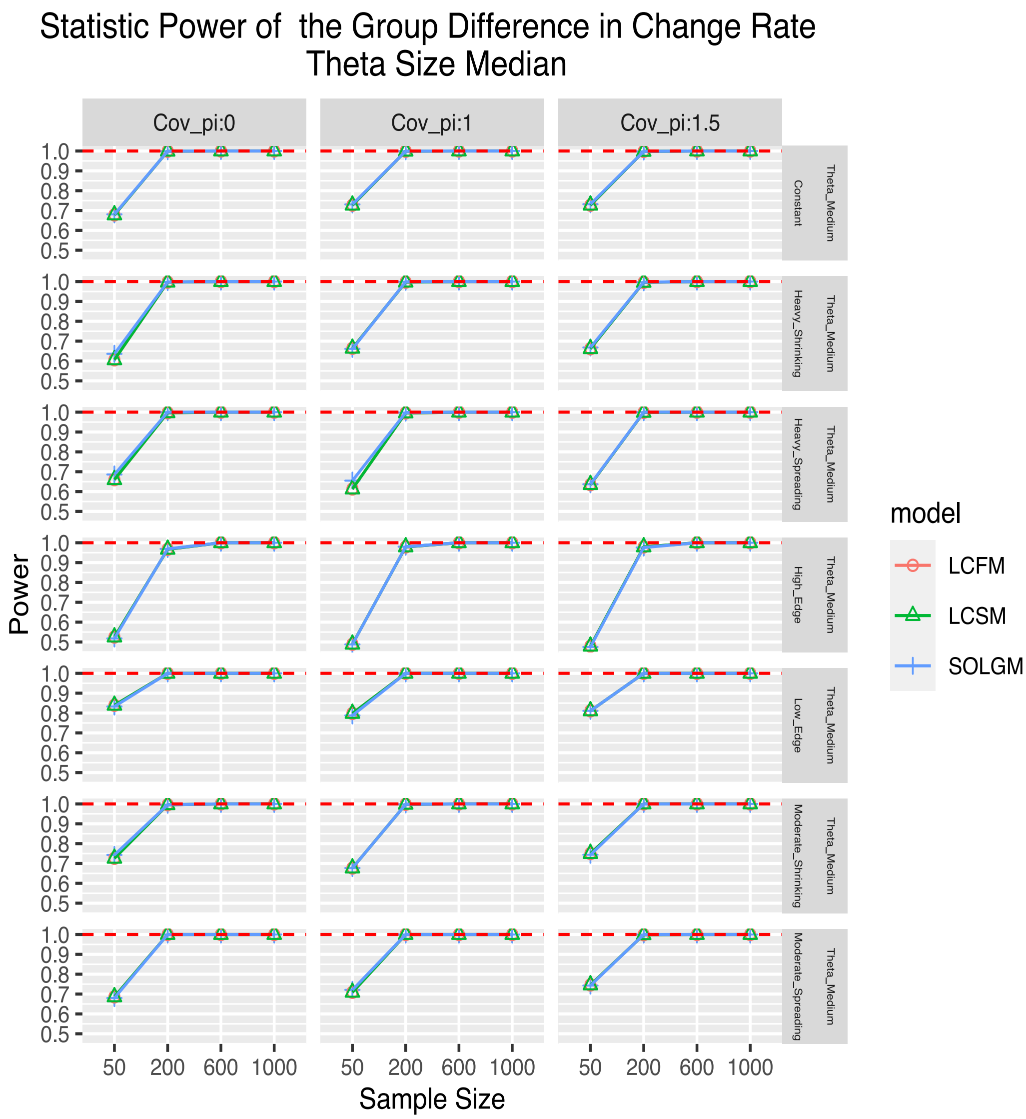

Figure A.99. Statistical Power of Group Difference in Growth Rate, $\boldsymbol{\beta_{11} = 3}$, by time-specific error size (**Large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 2}$ and $\boldsymbol{\beta_{11} = 3}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

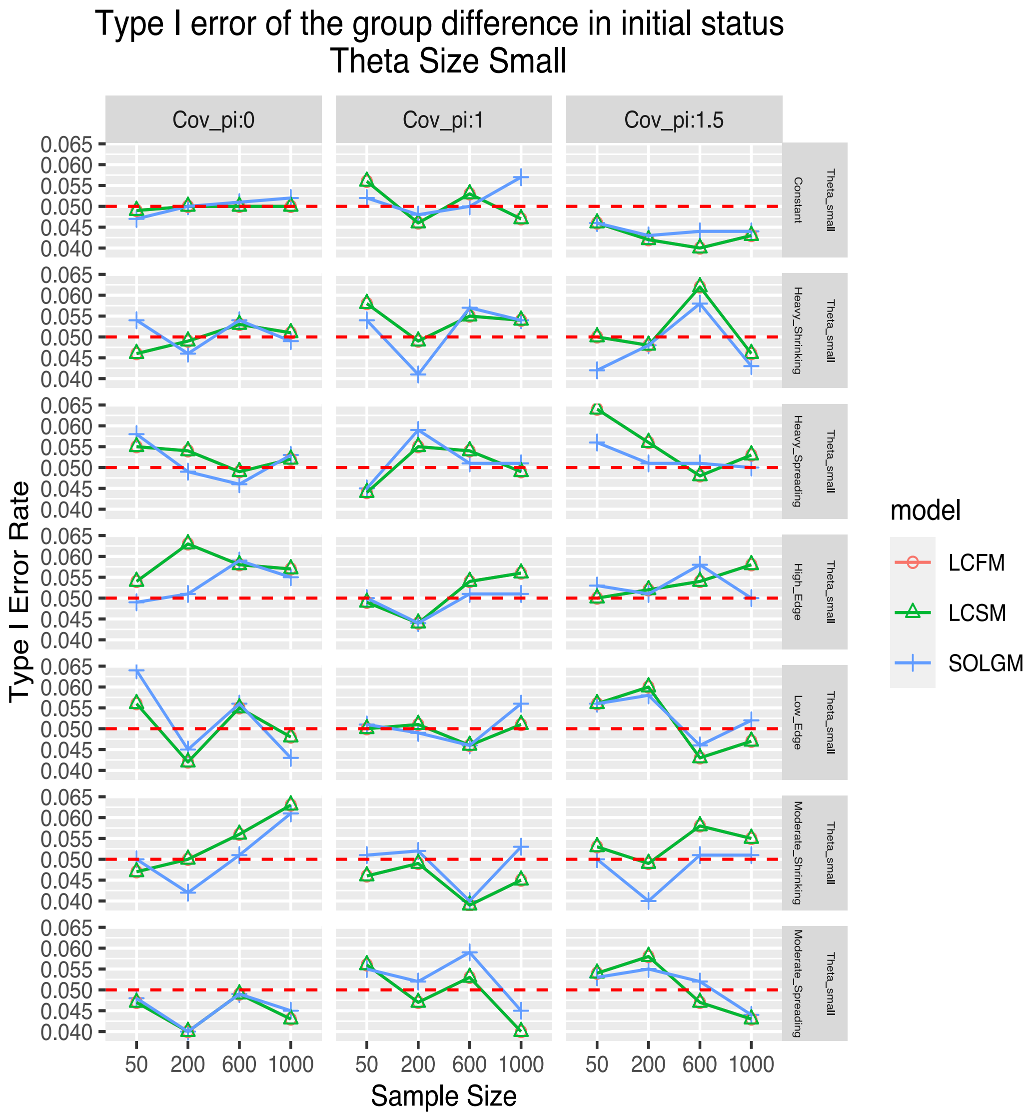

Figure A.100. Type I Error Rate of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**Small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

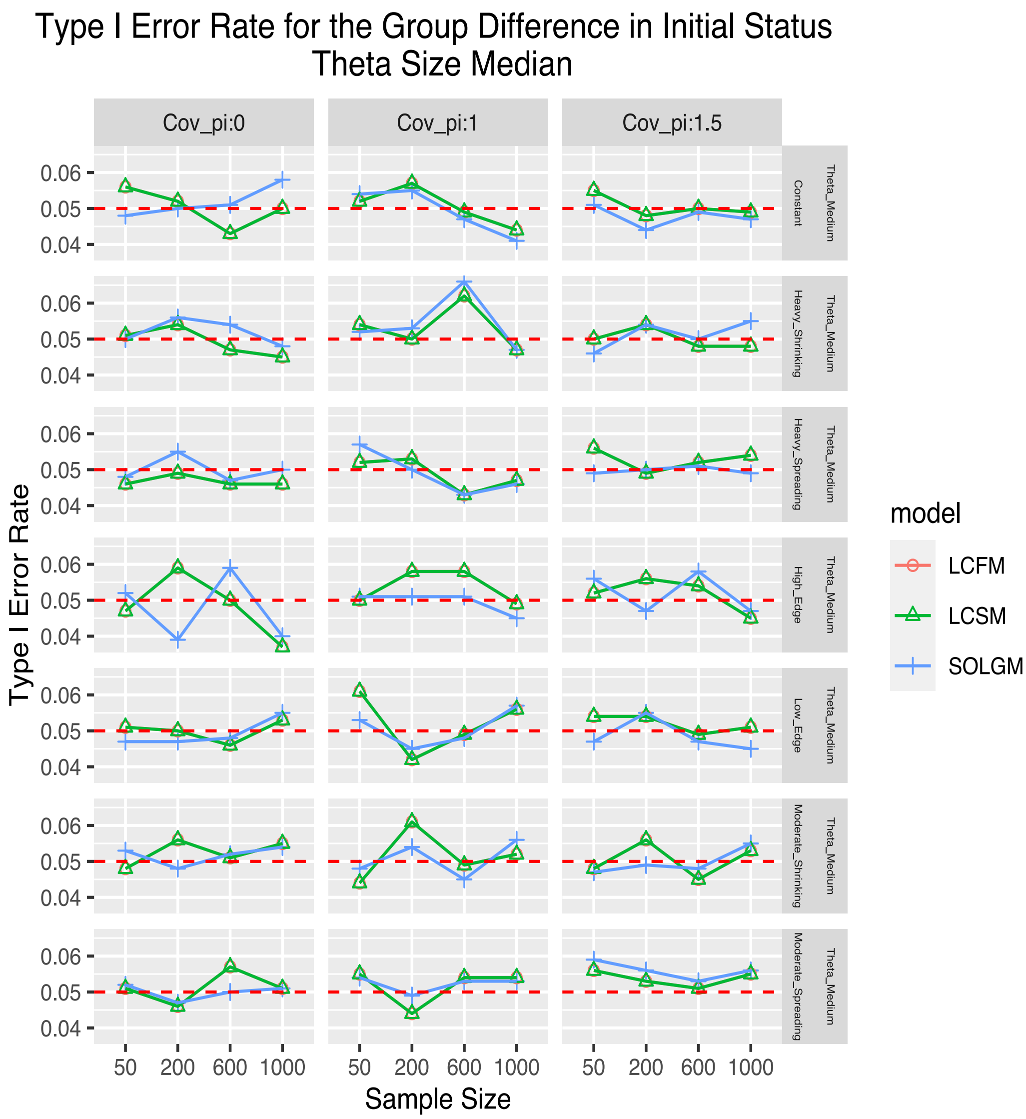

Figure A.101. Type I Error Rate of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**Median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

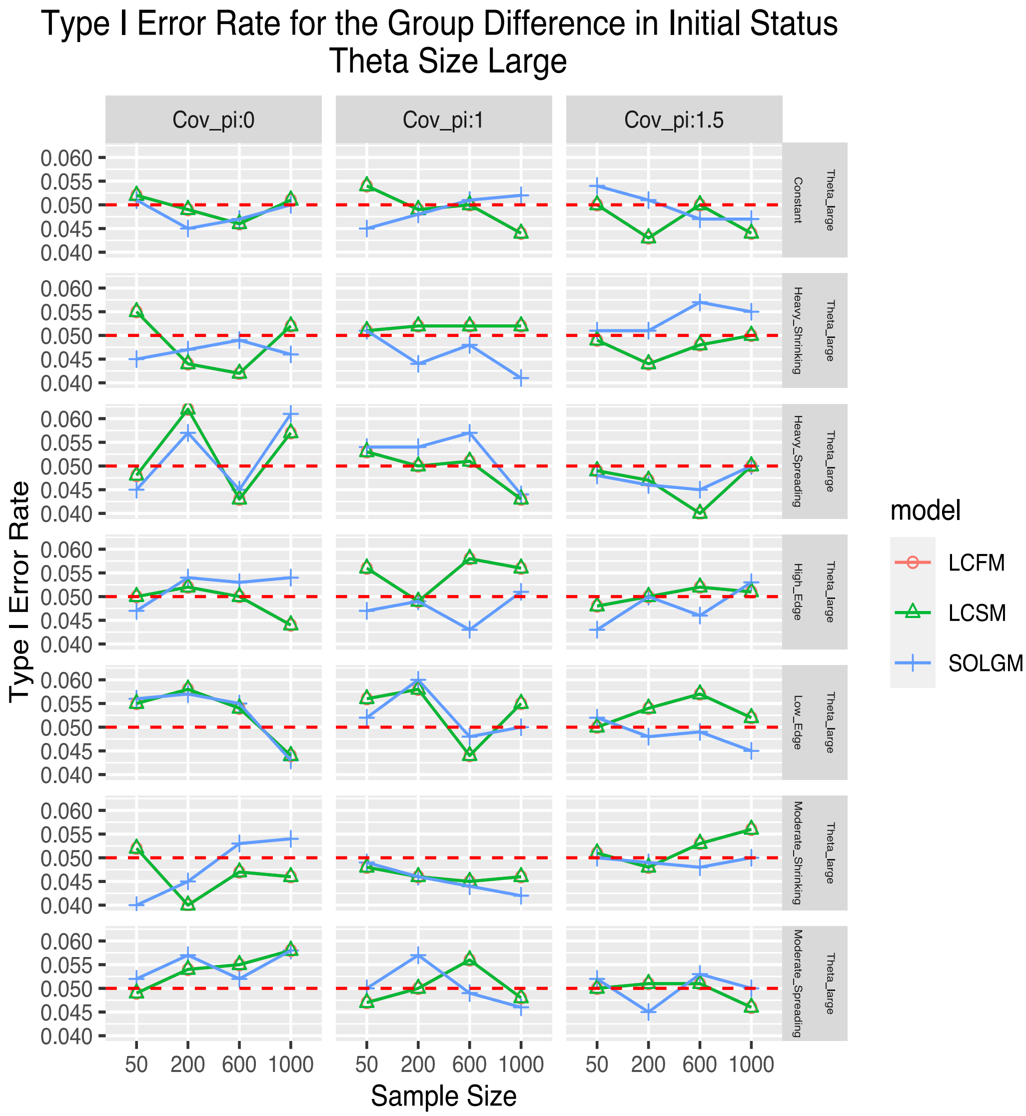

Figure A.102. Type I Error Rate of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**Large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

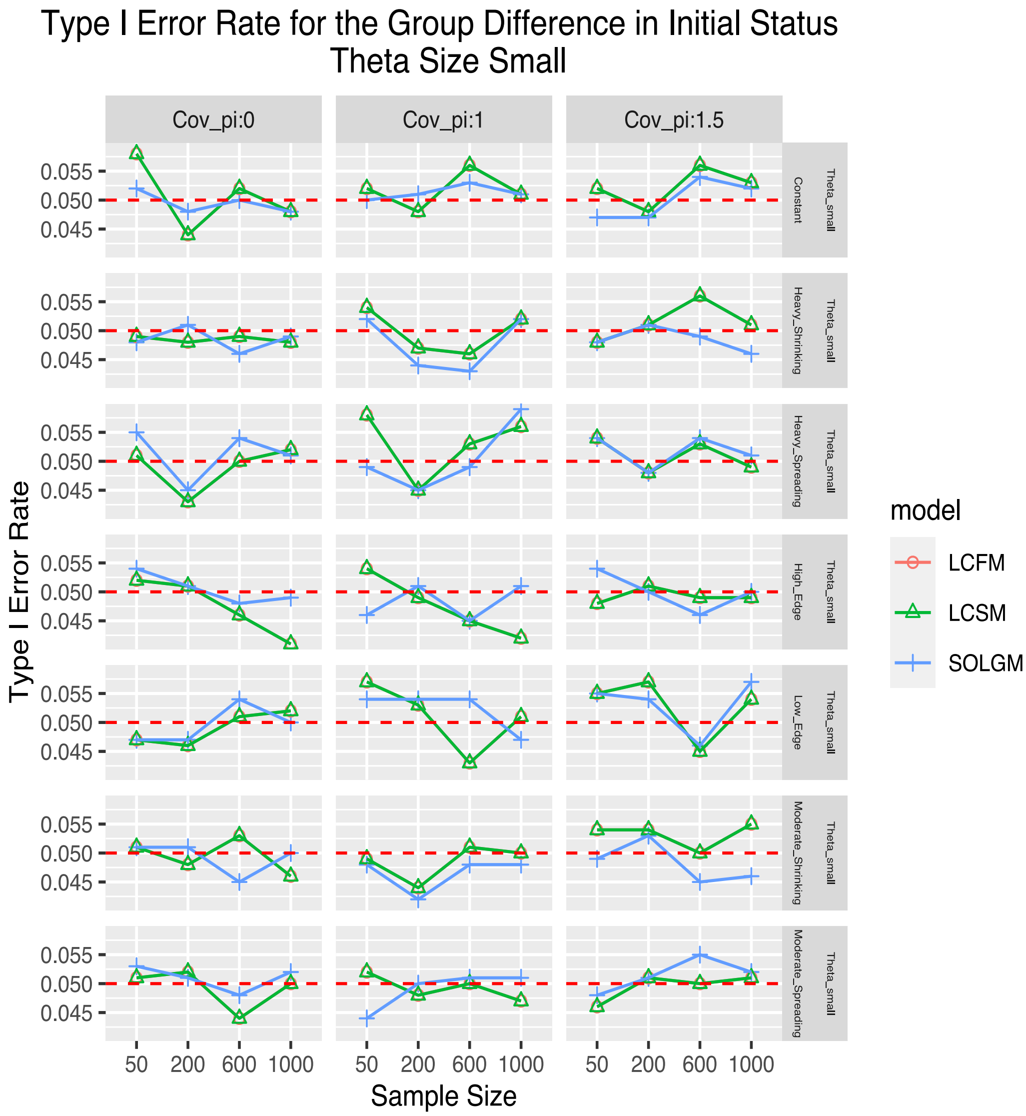

Figure A.103. Type I Error Rate of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**Small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

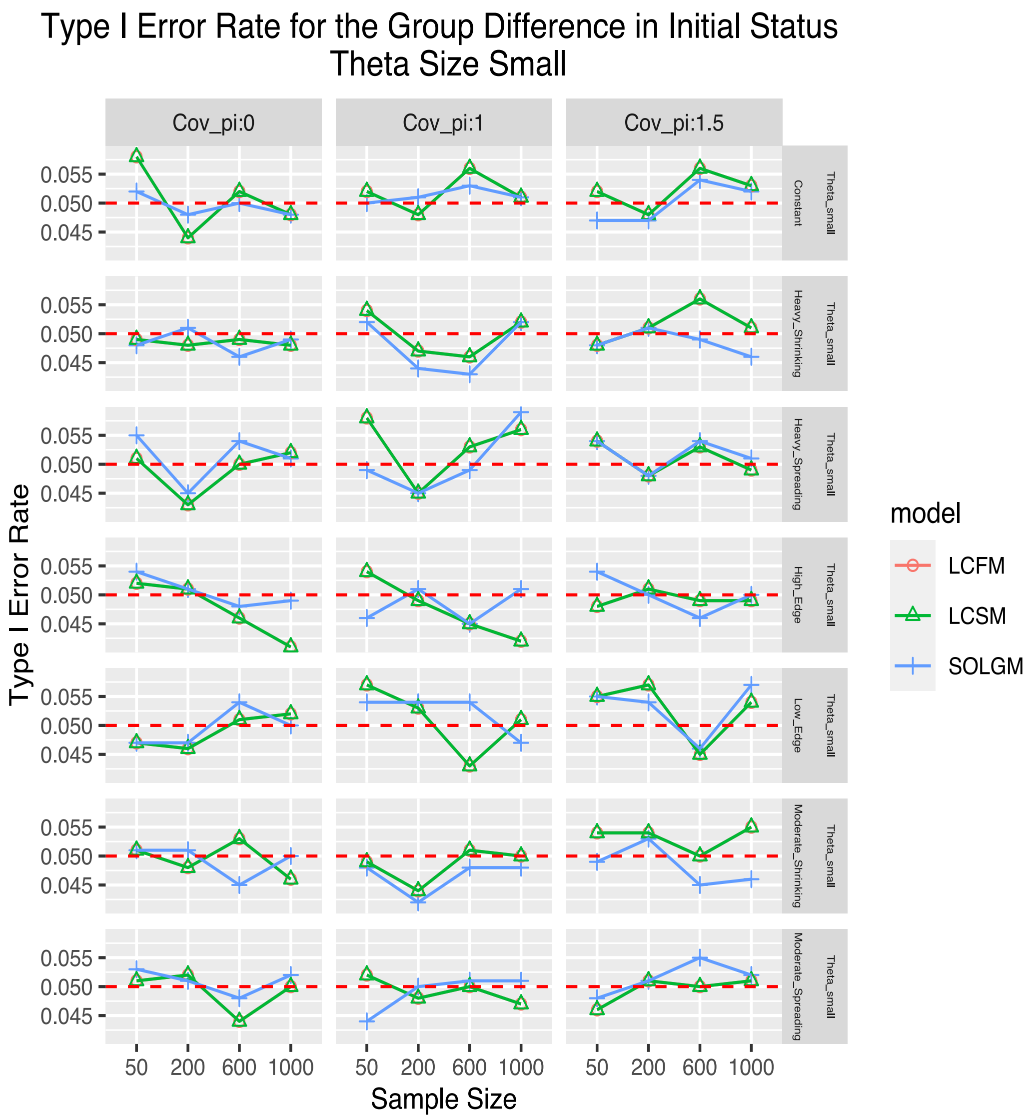

Figure A.104. Type I Error Rate of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**Median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

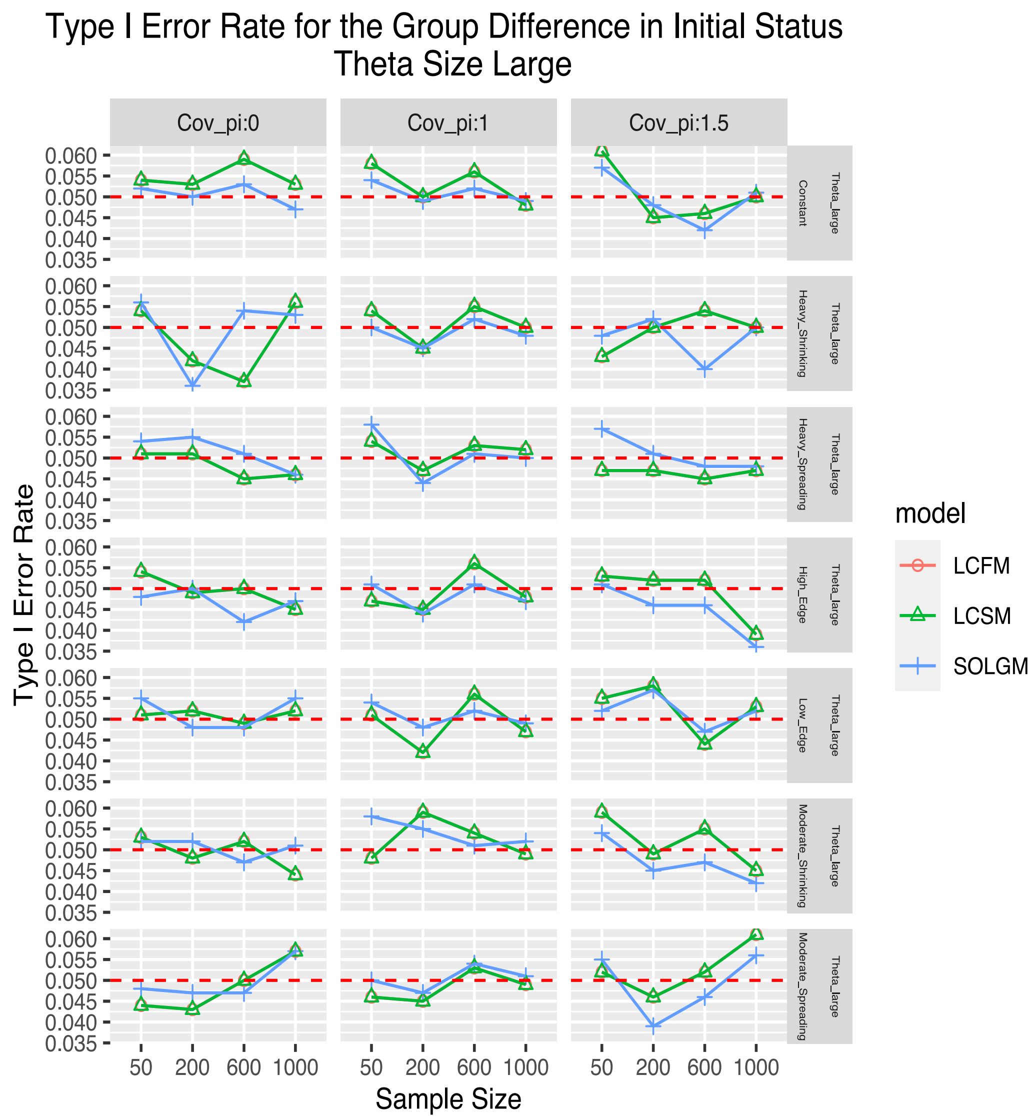

Figure A.105. Type I Error Rate of Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**Large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

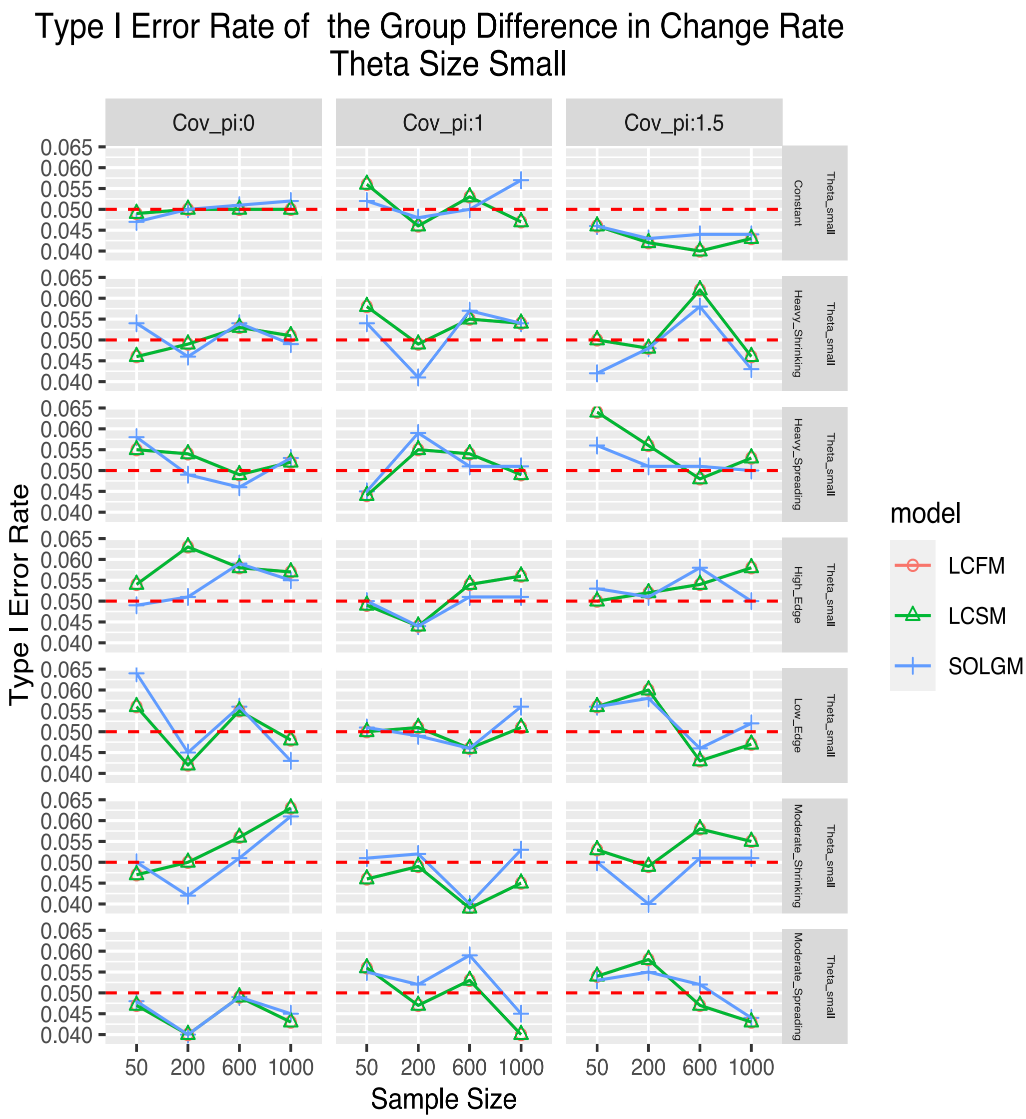

Figure A.106. Type I Error Rate of Group Difference in Growth Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**Small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

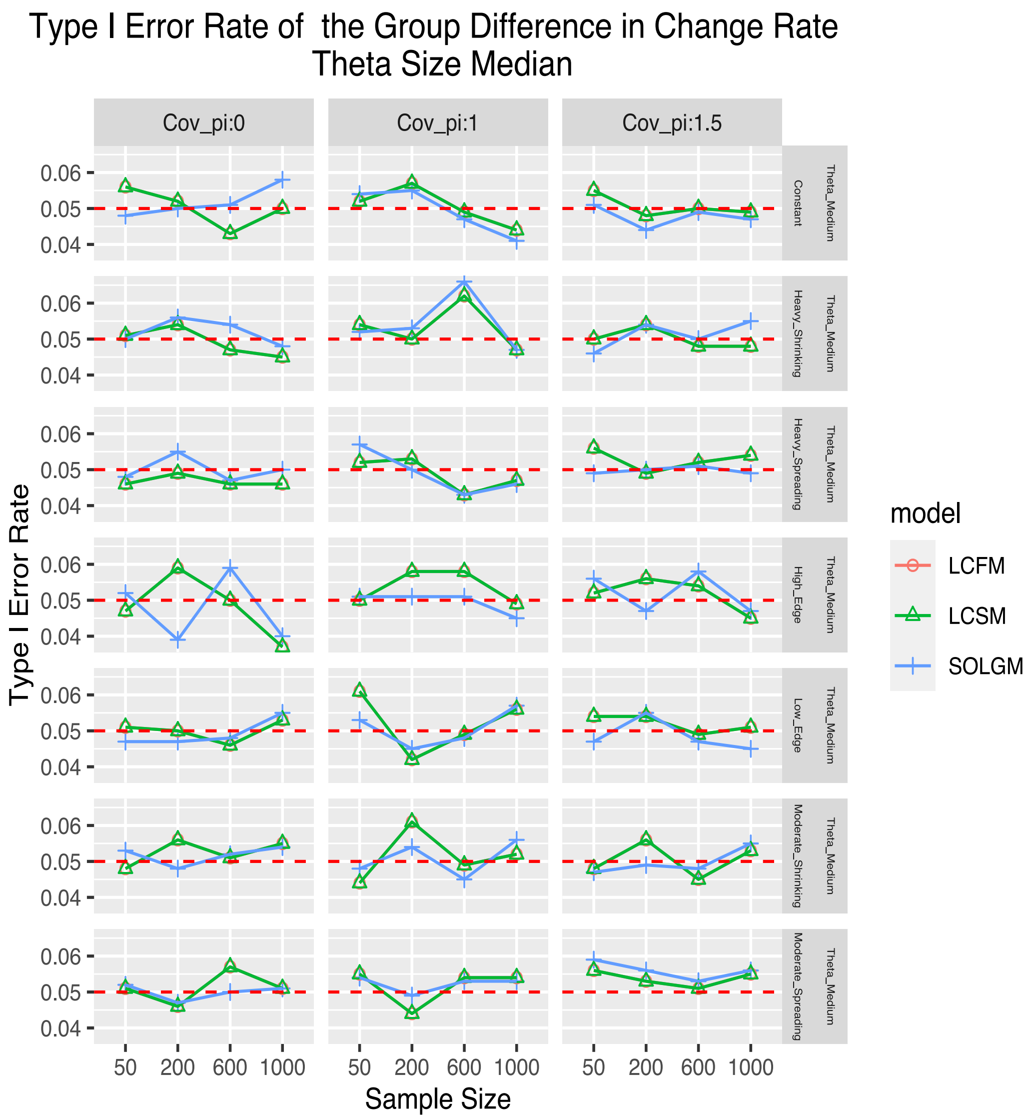

Figure A.107. Type I Error Rate of Group Difference in Group Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**Median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

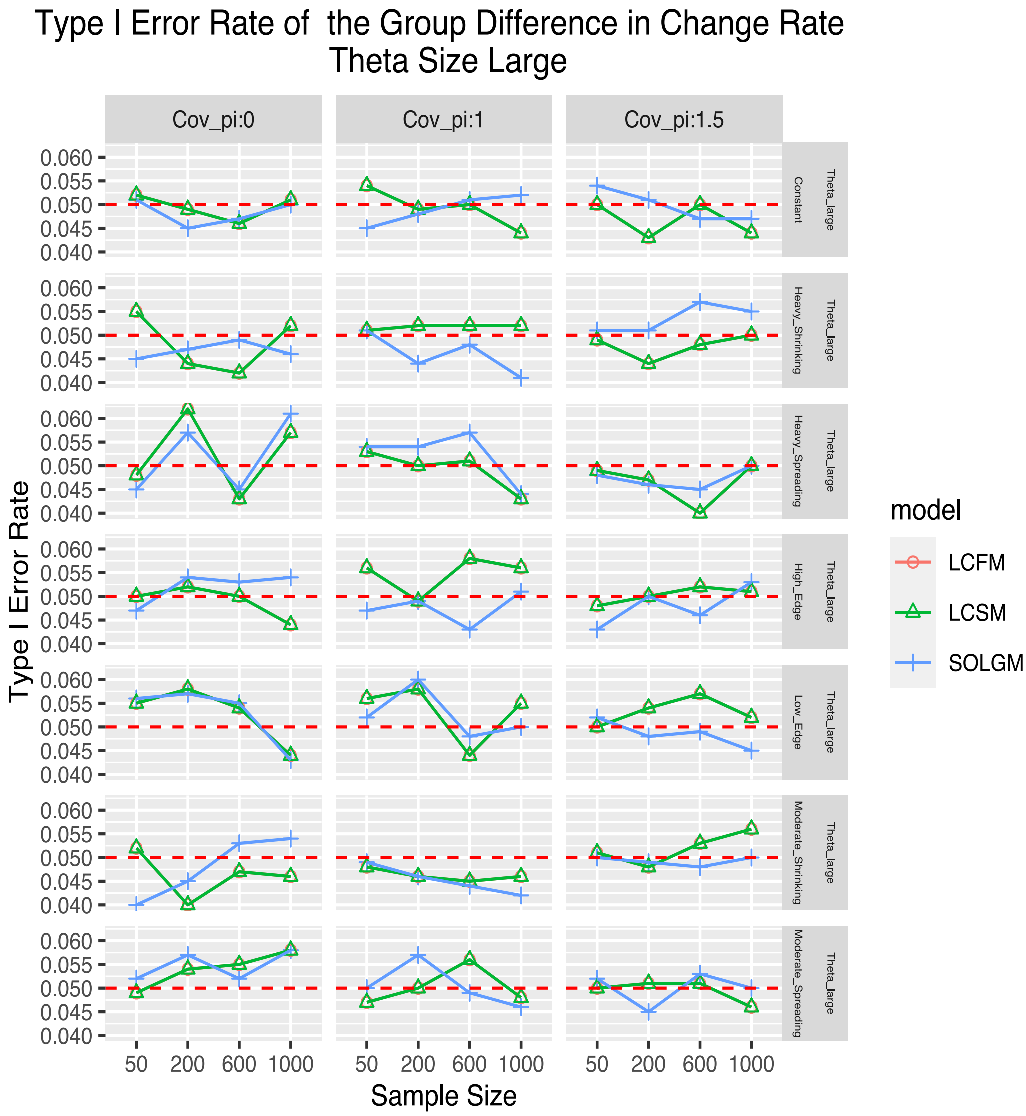

Figure A.108. Type I Error Rate of Group Difference in Growth Rate, $\boldsymbol{\beta_{11} = 0}$, by time-specific error size (**Large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

Figure A.109. Marginal mean of type I error rate for the group differences in initial status $\boldsymbol{\beta_{01} = 0}$, The horizontal dahsed line displays the nominal $\boldsymbol{\alpha = 0.05}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

Figure A.110. Marginal mean of type I error rate for the group differences in Growth Rate $\boldsymbol{\beta_{11} = 0}$, The horizontal dahsed line displays the nominal $\boldsymbol{\alpha = 0.05}$, SOLGM = second-order latent growth model, LCFM = longitudinal common factor model, LCSM = latent change score model.

Appendix B

Results of Study 2

B.0.1 List of Tables For Relative Bias

Table B.1

ANOVA Test Results for Relative Bias of β_{00}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0	0.000
N	3	0	0.016
θ_2^2	2	0	0.003
$pattern$	6	0	0.012
β_{01}	1	0	0.000
β_{11}	1	0	0.000
ψ_{01}	2	0	0.003
$model * N$	6	0	0.000
$model * \theta_2^2$	4	0	0.000
$model * pattern$	12	0	0.000
$model * \beta_{01}$	2	0	0.000
$model * \beta_{11}$	2	0	0.000
$model * \psi_{01}$	4	0	0.000
$N * \theta_2^2$	6	0	0.003
$N * pattern$	18	0	0.031

Continued on next page

Table B.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \beta_{01}$	3	0	0.000
$N * \beta_{11}$	3	0	0.001
$N * \psi_{01}$	6	0	0.003
$\theta_2^2 * pattern$	12	0	0.008
$\theta_2^2 * \beta_{01}$	2	0	0.002
$\theta_2^2 * \beta_{11}$	2	0	0.002
$\theta_2^2 * \psi_{01}$	4	0	0.001
$pattern * \beta_{01}$	6	0	0.003
$pattern * \beta_{11}$	6	0	0.006
$pattern * \psi_{01}$	12	0	0.006
$\beta_{01} * \beta_{11}$	1	0	0.000
$\beta_{01} * \psi_{01}$	2	0	0.001
$\beta_{11} * \psi_{01}$	2	0	0.005
$model * N * \theta_2^2$	12	0	0.000
$model * N * pattern$	36	0	0.001
$model * N * \beta_{01}$	6	0	0.000
$model * N * \beta_{11}$	6	0	0.000
$model * N * \psi_{01}$	12	0	0.000
$model * \theta_2^2 * pattern$	24	0	0.001
$model * \theta_2^2 * \beta_{01}$	4	0	0.000
$model * \theta_2^2 * \beta_{11}$	4	0	0.000
$model * \theta_2^2 * \psi_{01}$	8	0	0.000

Continued on next page

Table B.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * pattern * \beta_{01}$	12	0	0.000
$model * pattern * \beta_{11}$	12	0	0.000
$model * pattern * \psi_{01}$	24	0	0.000
$model * \beta_{01} * \beta_{11}$	2	0	0.000
$model * \beta_{01} * \psi_{01}$	4	0	0.000
$model * \beta_{11} * \psi_{01}$	4	0	0.000
$N * \theta_2^2 * pattern$	36	0	0.039
$N * \theta_2^2 * \beta_{01}$	6	0	0.011
$N * \theta_2^2 * \beta_{11}$	6	0	0.005
$N * \theta_2^2 * \psi_{01}$	12	0	0.003
$N * pattern * \beta_{01}$	18	0	0.013
$N * pattern * \beta_{11}$	18	0	0.012
$N * pattern * \psi_{01}$	36	0	0.037
$N * \beta_{01} * \beta_{11}$	3	0	0.000
$N * \beta_{01} * \psi_{01}$	6	0	0.001
$N * \beta_{11} * \psi_{01}$	6	0	0.003
$\theta_2^2 * pattern * \beta_{01}$	12	0	0.010
$\theta_2^2 * pattern * \beta_{11}$	12	0	0.008
$\theta_2^2 * pattern * \psi_{01}$	24	0	0.033
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0	0.002
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0	0.006
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0	0.004

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Table B.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$pattern * \beta_{01} * \beta_{11}$	6	0	0.003
$pattern * \beta_{01} * \psi_{01}$	12	0	0.014
$pattern * \beta_{11} * \psi_{01}$	12	0	0.015
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0	0.004
$model * N * \theta_2^2 * pattern$	72	0	0.002
$model * N * \theta_2^2 * \beta_{01}$	12	0	0.000
$model * N * \theta_2^2 * \beta_{11}$	12	0	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0	0.000
$model * N * pattern * \beta_{01}$	36	0	0.000
$model * N * pattern * \beta_{11}$	36	0	0.000
$model * N * pattern * \psi_{01}$	72	0	0.001
$model * N * \beta_{01} * \beta_{11}$	6	0	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0	0.001
$model * \theta_2^2 * pattern * \psi_{01}$	48	0	0.002
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0	0.001

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Table B.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * pattern * \beta_{11} * \psi_{01}$	24	0	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0	0.047
$N * \theta_2^2 * pattern * \beta_{11}$	36	0	0.023
$N * \theta_2^2 * pattern * \psi_{01}$	72	0	0.076
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0	0.018
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0	0.006
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0	0.020
$N * pattern * \beta_{01} * \beta_{11}$	18	0	0.014
$N * pattern * \beta_{01} * \psi_{01}$	36	0	0.022
$N * pattern * \beta_{11} * \psi_{01}$	36	0	0.043
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0	0.009
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0	0.012
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0	0.012
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0	0.037
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0	0.002
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0	0.009
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0	0.001
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0	0.001
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0	0.003
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0	0.000

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Table B.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0	0.001
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0	0.002
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0	0.001
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0	0.001
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0	0.042
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0	0.067
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0	0.056
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0	0.006
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0	0.029
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0	0.020
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0	0.001
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0	0.003
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0	0.002
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0	0.001

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Table B.1 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0	0.067
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0	0.002

Table B.2

ANOVA Test Results for Relative Bias of β_{01}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.000	0.000
N	3.000	0.001	0.010
θ_2^2	2.000	0.000	0.000
$pattern$	6.000	0.001	0.018
β_{01}	1.000	0.000	0.000
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.005
$model * N$	6.000	0.000	0.001
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.000	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.000	0.001
$N * pattern$	18.000	0.003	0.036
$N * \beta_{01}$	3.000	0.000	0.000
$N * \beta_{11}$	3.000	0.000	0.000

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Table B.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \psi_{01}$	6.000	0.001	0.009
$\theta_2^2 * pattern$	12.000	0.001	0.016
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.004
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.002
$\theta_2^2 * \psi_{01}$	4.000	0.000	0.001
$pattern * \beta_{01}$	6.000	0.000	0.003
$pattern * \beta_{11}$	6.000	0.001	0.010
$pattern * \psi_{01}$	12.000	0.002	0.018
$\beta_{01} * \beta_{11}$	1.000	0.000	0.003
$\beta_{01} * \psi_{01}$	2.000	0.000	0.002
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.000	0.000
$model * N * pattern$	36.000	0.000	0.001
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.000	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000

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Table B.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern$	36.000	0.003	0.042
$N * \theta_2^2 * \beta_{01}$	6.000	0.001	0.012
$N * \theta_2^2 * \beta_{11}$	6.000	0.001	0.009
$N * \theta_2^2 * \psi_{01}$	12.000	0.000	0.006
$N * pattern * \beta_{01}$	18.000	0.000	0.006
$N * pattern * \beta_{11}$	18.000	0.002	0.020
$N * pattern * \psi_{01}$	36.000	0.003	0.038
$N * \beta_{01} * \beta_{11}$	3.000	0.001	0.007
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.002
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.003
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.000	0.006
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.001	0.011
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.002	0.026
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.000	0.004
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.001	0.007
$pattern * \beta_{01} * \beta_{11}$	6.000	0.000	0.003
$pattern * \beta_{01} * \psi_{01}$	12.000	0.000	0.006

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Table B.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$pattern * \beta_{11} * \psi_{01}$	12.000	0.001	0.014
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.005
$model * N * \theta_2^2 * pattern$	72.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.001
$model * N * pattern * \psi_{01}$	72.000	0.000	0.001
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.001
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table B.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.002	0.022
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.002	0.023
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.005	0.060
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.010
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.000	0.006
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.001	0.016
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.001	0.017
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.002	0.022
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.003	0.041
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.001	0.016
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.001	0.008
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.001	0.015
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.002	0.027
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.002
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.010
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.000	0.002
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.001

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Table B.2 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.002
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.001
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.003	0.039
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.005	0.057
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.005	0.061
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.007
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.003	0.032
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.002	0.025
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.001
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.000	0.002
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.001
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.007	0.087
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.000	0.002

Table B.3

ANOVA Test Results for Relative Bias of β_{10}

Source	**DF**	**Sum of Squares**	η^2
$model$	2.000	0.000	0.000
N	3.000	0.003	0.005
θ_2^2	2.000	0.003	0.004
$pattern$	6.000	0.003	0.004
β_{01}	1.000	0.003	0.005
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.001	0.002
$model * N$	6.000	0.000	0.000
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.000	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.005	0.009
$N * pattern$	18.000	0.011	0.017
$N * \beta_{01}$	3.000	0.004	0.007
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.002	0.003
$\theta_2^2 * pattern$	12.000	0.007	0.011
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.001	0.002

Continued on next page

Table B.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.003	0.006
$pattern * \beta_{01}$	6.000	0.003	0.005
$pattern * \beta_{11}$	6.000	0.003	0.006
$pattern * \psi_{01}$	12.000	0.007	0.012
$\beta_{01} * \beta_{11}$	1.000	0.001	0.002
$\beta_{01} * \psi_{01}$	2.000	0.003	0.005
$\beta_{11} * \psi_{01}$	2.000	0.008	0.013
$model * N * \theta_2^2$	12.000	0.000	0.000
$model * N * pattern$	36.000	0.000	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.000	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

Continued on next page

Table B.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	0.019	0.032
$N * \theta_2^2 * \beta_{01}$	6.000	0.002	0.004
$N * \theta_2^2 * \beta_{11}$	6.000	0.008	0.014
$N * \theta_2^2 * \psi_{01}$	12.000	0.007	0.012
$N * pattern * \beta_{01}$	18.000	0.017	0.028
$N * pattern * \beta_{11}$	18.000	0.015	0.025
$N * pattern * \psi_{01}$	36.000	0.018	0.030
$N * \beta_{01} * \beta_{11}$	3.000	0.004	0.006
$N * \beta_{01} * \psi_{01}$	6.000	0.006	0.010
$N * \beta_{11} * \psi_{01}$	6.000	0.008	0.014
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.010	0.016
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.006	0.010
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.007	0.012
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.002	0.004
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.002	0.003
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.005	0.009
$pattern * \beta_{01} * \beta_{11}$	6.000	0.004	0.007
$pattern * \beta_{01} * \psi_{01}$	12.000	0.010	0.016
$pattern * \beta_{11} * \psi_{01}$	12.000	0.014	0.023
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

Continued on next page

Table B.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.044	0.073
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.021	0.035
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.024	0.040
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.011	0.019

Continued on next page

Table B.3 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.004	0.007
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.007	0.012
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.009	0.016
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.021	0.034
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.022	0.037
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.002	0.003
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.008	0.013
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.010	0.016
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.017	0.029
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.001	0.001
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.004	0.006
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.001	0.001
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

Continued on next page

Table B.3 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.028	0.046
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.031	0.051
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.028	0.046
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.003	0.006
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.024	0.039
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.009	0.014
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.001	0.002
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.001	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.029	0.048
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.001

Table B.4

ANOVA Test Results for Relative Bias of β_{11}

Source	**DF**	**Sum of Squares**	η^2
$model$	2.000	0.000	0.000
N	3.000	0.000	0.002
θ_2^2	2.000	0.001	0.004
$pattern$	6.000	0.001	0.007
β_{01}	1.000	0.000	0.001
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.001	0.007
$model * N$	6.000	0.000	0.000
$model * \theta_2^2$	4.000	0.000	0.000
$model * pattern$	12.000	0.000	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	0.001	0.005
$N * pattern$	18.000	0.007	0.032
$N * \beta_{01}$	3.000	0.001	0.003
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.002	0.010
$\theta_2^2 * pattern$	12.000	0.005	0.024
$\theta_2^2 * \beta_{01}$	2.000	0.000	0.001
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.001

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Table B.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.002	0.008
$pattern * \beta_{01}$	6.000	0.001	0.006
$pattern * \beta_{11}$	6.000	0.001	0.004
$pattern * \psi_{01}$	12.000	0.004	0.017
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.001	0.003
$\beta_{11} * \psi_{01}$	2.000	0.001	0.004
$model * N * \theta_2^2$	12.000	0.000	0.000
$model * N * pattern$	36.000	0.000	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.000	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table B.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	0.011	0.051
$N * \theta_2^2 * \beta_{01}$	6.000	0.001	0.006
$N * \theta_2^2 * \beta_{11}$	6.000	0.002	0.009
$N * \theta_2^2 * \psi_{01}$	12.000	0.003	0.016
$N * pattern * \beta_{01}$	18.000	0.004	0.021
$N * pattern * \beta_{11}$	18.000	0.004	0.020
$N * pattern * \psi_{01}$	36.000	0.010	0.048
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.001
$N * \beta_{01} * \psi_{01}$	6.000	0.002	0.010
$N * \beta_{11} * \psi_{01}$	6.000	0.002	0.008
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.001	0.003
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.001	0.003
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.002	0.010
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.001
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.001	0.004
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.002
$pattern * \beta_{01} * \beta_{11}$	6.000	0.001	0.007
$pattern * \beta_{01} * \psi_{01}$	12.000	0.002	0.009
$pattern * \beta_{11} * \psi_{01}$	12.000	0.003	0.014
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table B.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.000	0.001
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.006	0.030
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.004	0.019
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.013	0.064
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.004

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Table B.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.002	0.009
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.004	0.019
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.003	0.015
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.006	0.028
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.007	0.036
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.000	0.002
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.002	0.009
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.004	0.021
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.005	0.026
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.001	0.003
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.011
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.001
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.001
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.000	0.001
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.001
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.001
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.001
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table B.4 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.001
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.008	0.040
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.013	0.063
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.012	0.059
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.011
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.008	0.041
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.004	0.018
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.000	0.002
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.001	0.003
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.001
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.014	0.068
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.000	0.001

B.0.2 List of Tables For Standard Error

Table B.5

ANOVA test for the Empirical Standard Error of β_{00}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.203	0.002
N	3.000	98.318	0.895
θ_2^2	2.000	3.288	0.030
$pattern$	6.000	2.651	0.024
β_{01}	1.000	0.049	0.000
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.000
$model * N$	6.000	0.057	0.001
$model * \theta_2^2$	4.000	0.054	0.000
$model * pattern$	12.000	0.104	0.001
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	1.221	0.011
$N * pattern$	18.000	1.030	0.009
$N * \beta_{01}$	3.000	0.016	0.000
$N * \beta_{11}$	3.000	0.000	0.000
$N * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern$	12.000	1.804	0.016
$\theta_2^2 * \beta_{01}$	2.000	0.003	0.000

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Table B.5 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01}$	6.000	0.002	0.000
$pattern * \beta_{11}$	6.000	0.000	0.000
$pattern * \psi_{01}$	12.000	0.002	0.000
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.020	0.000
$model * N * pattern$	36.000	0.035	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.079	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000

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Table B.5 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern$	36.000	0.693	0.006
$N * \theta_2^2 * \beta_{01}$	6.000	0.003	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.001	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.002	0.000
$N * pattern * \beta_{01}$	18.000	0.003	0.000
$N * pattern * \beta_{11}$	18.000	0.001	0.000
$N * pattern * \psi_{01}$	36.000	0.004	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.001	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.002	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.002	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.001	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.002	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.002	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.027	0.000

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Table B.5 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.006	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.004	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.010	0.000

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Table B.5 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.003	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.006	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.007	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.006	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.006	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.004	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.001	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000

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Table B.5 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.006	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.013	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.015	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.008	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.004	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.012	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.000

Table B.6

ANOVA test for the Empirical Standard Error of β_{01}

Source	**DF**	**Sum of Squares**	η^2
$model$	2.000	0.288	0.001
N	3.000	170.316	0.871
θ_2^2	2.000	7.157	0.037
$pattern$	6.000	5.646	0.029
β_{01}	1.000	0.478	0.002
β_{11}	1.000	0.000	0.000
ψ_{01}	2.000	0.000	0.000
$model * N$	6.000	0.071	0.000
$model * \theta_2^2$	4.000	0.140	0.001
$model * pattern$	12.000	0.252	0.001
$model * \beta_{01}$	2.000	0.002	0.000
$model * \beta_{11}$	2.000	0.001	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	2.716	0.014
$N * pattern$	18.000	2.075	0.011
$N * \beta_{01}$	3.000	0.161	0.001
$N * \beta_{11}$	3.000	0.001	0.000
$N * \psi_{01}$	6.000	0.003	0.000
$\theta_2^2 * pattern$	12.000	3.896	0.020
$\theta_2^2 * \beta_{01}$	2.000	0.011	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.000	0.000

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Table B.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01}$	6.000	0.014	0.000
$pattern * \beta_{11}$	6.000	0.003	0.000
$pattern * \psi_{01}$	12.000	0.001	0.000
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.053	0.000
$model * N * pattern$	36.000	0.085	0.000
$model * N * \beta_{01}$	6.000	0.001	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.185	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.003	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.002	0.000
$model * pattern * \beta_{11}$	12.000	0.001	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table B.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	1.559	0.008
$N * \theta_2^2 * \beta_{01}$	6.000	0.002	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.001	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.004	0.000
$N * pattern * \beta_{01}$	18.000	0.009	0.000
$N * pattern * \beta_{11}$	18.000	0.003	0.000
$N * pattern * \psi_{01}$	36.000	0.010	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.002	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.006	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.004	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.008	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.002	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.002	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.001	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.007	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.003	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.001	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.063	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.002	0.000

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Table B.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.001	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.016	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.020	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.026	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.000

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Table B.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.009	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.003	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.007	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.008	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.005	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.010	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.006	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.006	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.001	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table B.6 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.011	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.021	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.020	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.018	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.009	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.020	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.000

Table B.7

ANOVA test for the Empirical Standard Error of β_{10}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.024	0.000
N	3.000	129.720	0.802
θ_2^2	2.000	13.118	0.081
$pattern$	6.000	6.150	0.038
β_{01}	1.000	0.196	0.001
β_{11}	1.000	0.017	0.000
ψ_{01}	2.000	0.001	0.000
$model * N$	6.000	0.004	0.000
$model * \theta_2^2$	4.000	0.031	0.000
$model * pattern$	12.000	0.051	0.000
$model * \beta_{01}$	2.000	0.000	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	4.789	0.030
$N * pattern$	18.000	2.242	0.014
$N * \beta_{01}$	3.000	0.060	0.000
$N * \beta_{11}$	3.000	0.007	0.000
$N * \psi_{01}$	6.000	0.002	0.000
$\theta_2^2 * pattern$	12.000	3.684	0.023
$\theta_2^2 * \beta_{01}$	2.000	0.012	0.000
$\theta_2^2 * \beta_{11}$	2.000	0.001	0.000

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Table B.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01}$	6.000	0.012	0.000
$pattern * \beta_{11}$	6.000	0.002	0.000
$pattern * \psi_{01}$	12.000	0.004	0.000
$\beta_{01} * \beta_{11}$	1.000	0.000	0.000
$\beta_{01} * \psi_{01}$	2.000	0.000	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.009	0.000
$model * N * pattern$	36.000	0.019	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.056	0.000
$model * \theta_2^2 * \beta_{01}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.000	0.000
$model * pattern * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table B.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	1.332	0.008
$N * \theta_2^2 * \beta_{01}$	6.000	0.008	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01}$	18.000	0.009	0.000
$N * pattern * \beta_{11}$	18.000	0.007	0.000
$N * pattern * \psi_{01}$	36.000	0.007	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.002	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.006	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.003	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.007	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.002	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.001	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.003	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.001	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.001	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.021	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table B.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.000	0.000
$model * N * pattern * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.018	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.007	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.023	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.001	0.000

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Table B.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.003	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.002	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.005	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.008	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.013	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.001	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.005	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.007	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table B.7 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.006	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.017	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.022	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.001	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.011	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.005	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.017	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.000	0.000

Table B.8

ANOVA test for the Empirical Standard Error of β_{11}

Source	DF	Sum of Squares	η^2
$model$	2.000	0.088	0.000
N	3.000	196.496	0.715
θ_2^2	2.000	31.540	0.115
$pattern$	6.000	14.753	0.054
β_{01}	1.000	2.200	0.008
β_{11}	1.000	0.159	0.001
ψ_{01}	2.000	0.009	0.000
$model * N$	6.000	0.018	0.000
$model * \theta_2^2$	4.000	0.094	0.000
$model * pattern$	12.000	0.166	0.001
$model * \beta_{01}$	2.000	0.002	0.000
$model * \beta_{11}$	2.000	0.000	0.000
$model * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2$	6.000	11.275	0.041
$N * pattern$	18.000	5.291	0.019
$N * \beta_{01}$	3.000	0.694	0.003
$N * \beta_{11}$	3.000	0.053	0.000
$N * \psi_{01}$	6.000	0.001	0.000
$\theta_2^2 * pattern$	12.000	8.036	0.029
$\theta_2^2 * \beta_{01}$	2.000	0.164	0.001
$\theta_2^2 * \beta_{11}$	2.000	0.005	0.000

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Table B.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4.000	0.005	0.000
$pattern * \beta_{01}$	6.000	0.070	0.000
$pattern * \beta_{11}$	6.000	0.021	0.000
$pattern * \psi_{01}$	12.000	0.008	0.000
$\beta_{01} * \beta_{11}$	1.000	0.007	0.000
$\beta_{01} * \psi_{01}$	2.000	0.002	0.000
$\beta_{11} * \psi_{01}$	2.000	0.000	0.000
$model * N * \theta_2^2$	12.000	0.027	0.000
$model * N * pattern$	36.000	0.057	0.000
$model * N * \beta_{01}$	6.000	0.000	0.000
$model * N * \beta_{11}$	6.000	0.000	0.000
$model * N * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern$	24.000	0.160	0.001
$model * \theta_2^2 * \beta_{01}$	4.000	0.002	0.000
$model * \theta_2^2 * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01}$	12.000	0.005	0.000
$model * pattern * \beta_{11}$	12.000	0.002	0.000
$model * pattern * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2.000	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4.000	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4.000	0.000	0.000

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Table B.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36.000	2.792	0.010
$N * \theta_2^2 * \beta_{01}$	6.000	0.052	0.000
$N * \theta_2^2 * \beta_{11}$	6.000	0.002	0.000
$N * \theta_2^2 * \psi_{01}$	12.000	0.003	0.000
$N * pattern * \beta_{01}$	18.000	0.032	0.000
$N * pattern * \beta_{11}$	18.000	0.013	0.000
$N * pattern * \psi_{01}$	36.000	0.014	0.000
$N * \beta_{01} * \beta_{11}$	3.000	0.001	0.000
$N * \beta_{01} * \psi_{01}$	6.000	0.001	0.000
$N * \beta_{11} * \psi_{01}$	6.000	0.004	0.000
$\theta_2^2 * pattern * \beta_{01}$	12.000	0.018	0.000
$\theta_2^2 * pattern * \beta_{11}$	12.000	0.005	0.000
$\theta_2^2 * pattern * \psi_{01}$	24.000	0.009	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2.000	0.001	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4.000	0.002	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6.000	0.002	0.000
$pattern * \beta_{01} * \psi_{01}$	12.000	0.006	0.000
$pattern * \beta_{11} * \psi_{01}$	12.000	0.005	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2.000	0.002	0.000
$model * N * \theta_2^2 * pattern$	72.000	0.055	0.000
$model * N * \theta_2^2 * \beta_{01}$	12.000	0.000	0.000

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Table B.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01}$	36.000	0.002	0.000
$model * N * pattern * \beta_{11}$	36.000	0.001	0.000
$model * N * pattern * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11}$	6.000	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12.000	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24.000	0.002	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24.000	0.001	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8.000	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36.000	0.019	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36.000	0.013	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72.000	0.025	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6.000	0.002	0.000

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Table B.8 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12.000	0.005	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12.000	0.003	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18.000	0.002	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36.000	0.008	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36.000	0.013	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6.000	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12.000	0.005	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24.000	0.007	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24.000	0.006	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4.000	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144.000	0.003	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24.000	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36.000	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72.000	0.001	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24.000	0.000	0.000

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Table B.8 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8.000	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36.000	0.012	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72.000	0.019	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72.000	0.033	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12.000	0.004	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36.000	0.013	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.010	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144.000	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144.000	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24.000	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48.000	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72.000	0.024	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144.000	0.001	0.000

Table B.9

ANOVA test for the Theoretical Standard Error of β_{00}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.281	0.003
N	3	88.341	0.889
θ_2^2	2	3.204	0.032
$pattern$	6	2.576	0.026
β_{01}	1	0.063	0.001
β_{11}	1	0.000	0.000
ψ_{01}	2	0.000	0.000
$model * N$	6	0.122	0.001
$model * \theta_2^2$	4	0.052	0.001
$model * pattern$	12	0.107	0.001
$model * \beta_{01}$	2	0.000	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	1.127	0.011
$N * pattern$	18	0.902	0.009
$N * \beta_{01}$	3	0.028	0.000
$N * \beta_{11}$	3	0.000	0.000
$N * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern$	12	1.752	0.018
$\theta_2^2 * \beta_{01}$	2	0.003	0.000
$\theta_2^2 * \beta_{11}$	2	0.000	0.000

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Table B.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.002	0.000
$pattern * \beta_{11}$	6	0.000	0.000
$pattern * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11}$	1	0.000	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2$	12	0.018	0.000
$model * N * pattern$	36	0.038	0.000
$model * N * \beta_{01}$	6	0.000	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.081	0.001
$model * \theta_2^2 * \beta_{01}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

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Table B.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	0.614	0.006
$N * \theta_2^2 * \beta_{01}$	6	0.001	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.000	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2 * pattern$	72	0.030	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

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Table B.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.000	0.000
$model * N * pattern * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000

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Table B.9 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

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Table B.9 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.000	0.000

Table B.10

ANOVA test for the Theoretical Standard Error of β_{01}

Source	DF	Sum of Squares	η^2
model	2	0.045	0.000
N	3	118.829	0.796
θ_2^2	2	12.649	0.085
pattern	6	5.958	0.040
β_{01}	1	0.189	0.001
β_{11}	1	0.014	0.000
ψ_{01}	2	0.002	0.000
$model * N$	6	0.022	0.000
$model * \theta_2^2$	4	0.040	0.000
$model * pattern$	12	0.048	0.000
$model * \beta_{01}$	2	0.000	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	4.375	0.029
$N * pattern$	18	2.060	0.014
$N * \beta_{01}$	3	0.063	0.000
$N * \beta_{11}$	3	0.005	0.000
$N * \psi_{01}$	6	0.001	0.000
$\theta_2^2 * pattern$	12	3.567	0.024
$\theta_2^2 * \beta_{01}$	2	0.009	0.000
$\theta_2^2 * \beta_{11}$	2	0.001	0.000

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Table B.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.005	0.000
$pattern * \beta_{11}$	6	0.001	0.000
$pattern * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11}$	1	0.001	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2$	12	0.016	0.000
$model * N * pattern$	36	0.017	0.000
$model * N * \beta_{01}$	6	0.000	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.054	0.000
$model * \theta_2^2 * \beta_{01}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

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Table B.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	1.234	0.008
$N * \theta_2^2 * \beta_{01}$	6	0.003	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.001	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.001	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.001	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2 * pattern$	72	0.019	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

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Table B.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.000	0.000
$model * N * pattern * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000

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Table B.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

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Table B.10 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.000	0.000

Table B.11

ANOVA test for the Theoretical Standard Error of β_{10}

Source	DF	Sum of Squares	η^2
$model$	2	0.045	0.000
N	3	121.107	0.790
θ_2^2	2	13.678	0.089
$pattern$	6	6.111	0.040
β_{01}	1	0.177	0.001
β_{11}	1	0.013	0.000
ψ_{01}	2	0.002	0.000
$model * N$	6	0.022	0.000
$model * \theta_2^2$	4	0.041	0.000
$model * pattern$	12	0.048	0.000
$model * \beta_{01}$	2	0.000	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	4.758	0.031
$N * pattern$	18	2.125	0.014
$N * \beta_{01}$	3	0.058	0.000
$N * \beta_{11}$	3	0.004	0.000
$N * \psi_{01}$	6	0.001	0.000
$\theta_2^2 * pattern$	12	3.687	0.024
$\theta_2^2 * \beta_{01}$	2	0.011	0.000
$\theta_2^2 * \beta_{11}$	2	0.001	0.000

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Table B.11 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.005	0.000
$pattern * \beta_{11}$	6	0.000	0.000
$pattern * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11}$	1	0.001	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2$	12	0.016	0.000
$model * N * pattern$	36	0.017	0.000
$model * N * \beta_{01}$	6	0.000	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.054	0.000
$model * \theta_2^2 * \beta_{01}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

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Table B.11 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	1.283	0.008
$N * \theta_2^2 * \beta_{01}$	6	0.003	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.001	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.001	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.001	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2 * pattern$	72	0.019	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

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Table B.11 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.000	0.000
$model * N * pattern * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000

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Table B.11 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

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Table B.11 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.000	0.000

Table B.12

ANOVA test for the Theoretical Standard Error of β_{11}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.166	0.001
N	3	176.074	0.701
θ_2^2	2	30.528	0.121
$pattern$	6	14.256	0.057
β_{01}	1	2.229	0.009
β_{11}	1	0.166	0.001
ψ_{01}	2	0.017	0.000
$model * N$	6	0.081	0.000
$model * \theta_2^2$	4	0.115	0.000
$model * pattern$	12	0.164	0.001
$model * \beta_{01}$	2	0.005	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	10.515	0.042
$N * pattern$	18	4.907	0.020
$N * \beta_{01}$	3	0.768	0.003
$N * \beta_{11}$	3	0.059	0.000
$N * \psi_{01}$	6	0.007	0.000
$\theta_2^2 * pattern$	12	7.820	0.031
$\theta_2^2 * \beta_{01}$	2	0.159	0.001
$\theta_2^2 * \beta_{11}$	2	0.011	0.000

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Table B.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.001	0.000
$pattern * \beta_{01}$	6	0.072	0.000
$pattern * \beta_{11}$	6	0.010	0.000
$pattern * \psi_{01}$	12	0.001	0.000
$\beta_{01} * \beta_{11}$	1	0.007	0.000
$\beta_{01} * \psi_{01}$	2	0.004	0.000
$\beta_{11} * \psi_{01}$	2	0.001	0.000
$model * N * \theta_2^2$	12	0.045	0.000
$model * N * pattern$	36	0.056	0.000
$model * N * \beta_{01}$	6	0.003	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.160	0.001
$model * \theta_2^2 * \beta_{01}$	4	0.002	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.005	0.000
$model * pattern * \beta_{11}$	12	0.002	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

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Table B.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	2.706	0.011
$N * \theta_2^2 * \beta_{01}$	6	0.050	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.005	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.022	0.000
$N * pattern * \beta_{11}$	18	0.003	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.003	0.000
$N * \beta_{01} * \psi_{01}$	6	0.002	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.011	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.002	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.001	0.000
$model * N * \theta_2^2 * pattern$	72	0.055	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

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Table B.12 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.002	0.000
$model * N * pattern * \beta_{11}$	36	0.001	0.000
$model * N * pattern * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.002	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.003	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.001	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.001	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000

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Table B.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

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Table B.12 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.000	0.000

Table B.13

ANOVA test for the Ratio of Theoretical Standard Error for β_{00}

Source	**DF**	**Sum of Squares**	η^2
N	3	0.019	0.016
θ_2^2	2	0.337	0.283
$pattern$	6	1.090	0.915
β_{01}	1	0.001	0.000
β_{11}	1	0.002	0.001
ψ_{01}	2	0.000	0.000
$N * \theta_2^2$	6	0.000	0.000
$N * pattern$	18	0.000	0.000
$N * \beta_{01}$	3	0.003	0.003
$N * \beta_{11}$	3	0.000	0.000
$N * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern$	12	0.622	0.523
$\theta_2^2 * \beta_{01}$	2	0.000	0.000
$\theta_2^2 * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.004	0.003
$pattern * \beta_{11}$	6	0.000	0.000
$pattern * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11}$	1	0.000	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000

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Table B.13 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	0.000	0.000
$N * \theta_2^2 * \beta_{01}$	6	0.001	0.001
$N * \theta_2^2 * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.000	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.001	0.001
$\theta_2^2 * pattern * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000

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Table B.13 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000

Table B.14

ANOVA test for the Ratio of Theoretical Standard Error for β_{01}

Source	**DF**	**Sum of Squares**	η^2
N	3	0.053	0.018
θ_2^2	2	0.490	0.164
$pattern$	6	1.552	0.519
β_{01}	1	0.000	0.000
β_{11}	1	0.009	0.003
ψ_{01}	2	0.001	0.000
$N * \theta_2^2$	6	0.003	0.001
$N * pattern$	18	0.000	0.000
$N * \beta_{01}$	3	0.015	0.005
$N * \beta_{11}$	3	0.001	0.000
$N * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern$	12	0.825	0.276
$\theta_2^2 * \beta_{01}$	2	0.002	0.001
$\theta_2^2 * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.024	0.008
$pattern * \beta_{11}$	6	0.002	0.001
$pattern * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11}$	1	0.000	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000

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Table B.14 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	0.000	0.000
$N * \theta_2^2 * \beta_{01}$	6	0.005	0.002
$N * \theta_2^2 * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.000	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.004	0.001
$\theta_2^2 * pattern * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000

Continued on next page

Table B.14 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000

Table B.15

ANOVA test for the Ratio of Theoretical Standard Error for β_{10}

Source	**DF**	**Sum of Squares**	η^2
N	3	0.004	0.003
θ_2^2	2	0.295	0.247
$pattern$	6	0.457	0.383
β_{01}	1	0.006	0.005
β_{11}	1	0.000	0.000
ψ_{01}	2	0.000	0.000
$N * \theta_2^2$	6	0.000	0.000
$N * pattern$	18	0.000	0.000
$N * \beta_{01}$	3	0.000	0.000
$N * \beta_{11}$	3	0.000	0.000
$N * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern$	12	0.415	0.348
$\theta_2^2 * \beta_{01}$	2	0.002	0.002
$\theta_2^2 * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.007	0.006
$pattern * \beta_{11}$	6	0.002	0.002
$pattern * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11}$	1	0.000	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000

Continued on next page

Table B.15 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	0.000	0.000
$N * \theta_2^2 * \beta_{01}$	6	0.000	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.000	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.002	0.002
$\theta_2^2 * pattern * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000

Continued on next page

Table B.15 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000

Table B.16

ANOVA test for the Ratio of Theoretical Standard Error for β_{11}

Source	**DF**	**Sum of Squares**	η^2
N	3	0.013	0.005
θ_2^2	2	0.474	0.194
$pattern$	6	1.050	0.429
β_{01}	1	0.057	0.023
β_{11}	1	0.002	0.001
ψ_{01}	2	0.000	0.000
$N * \theta_2^2$	6	0.000	0.000
$N * pattern$	18	0.000	0.000
$N * \beta_{01}$	3	0.002	0.001
$N * \beta_{11}$	3	0.000	0.000
$N * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern$	12	0.729	0.298
$\theta_2^2 * \beta_{01}$	2	0.012	0.005
$\theta_2^2 * \beta_{11}$	2	0.002	0.001
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01}$	6	0.064	0.026
$pattern * \beta_{11}$	6	0.018	0.008
$pattern * \psi_{01}$	12	0.001	0.000
$\beta_{01} * \beta_{11}$	1	0.000	0.000
$\beta_{01} * \psi_{01}$	2	0.000	0.000
$\beta_{11} * \psi_{01}$	2	0.000	0.000

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Table B.16 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern$	36	0.000	0.000
$N * \theta_2^2 * \beta_{01}$	6	0.000	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01}$	18	0.000	0.000
$N * pattern * \beta_{11}$	18	0.000	0.000
$N * pattern * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11}$	3	0.000	0.000
$N * \beta_{01} * \psi_{01}$	6	0.000	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.017	0.007
$\theta_2^2 * pattern * \beta_{11}$	12	0.002	0.001
$\theta_2^2 * pattern * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.000	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.000	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.000	0.000

Continued on next page

Table B.16 – continued from previous page

Source	DF	Sum of Squares	η^2
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.000	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.000	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.000	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.000	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.000	0.000

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B.0.3 list of Tables For Statistical Power

Table B.17

ANOVA test for the statistical power of β_{01}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.009	0.002
N	3	0.982	0.211
θ_2^2	2	0.306	0.066
$pattern$	6	0.259	0.056
β_{11}	1	0.000	0.000
ψ_{01}	2	0.000	0.000
$model * N$	6	0.026	0.006
$model * \theta_2^2$	4	0.010	0.002
$model * pattern$	12	0.011	0.002
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	0.903	0.194
$N * pattern$	18	0.764	0.164
$N * \beta_{11}$	3	0.000	0.000
$N * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern$	12	0.319	0.068
$\theta_2^2 * \beta_{11}$	2	0.000	0.000
$\theta_2^2 * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{11}$	6	0.001	0.000
$pattern * \psi_{01}$	12	0.000	0.000

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Table B.17 – continued from previous page

Source	DF	Sum of Squares	η^2
$\beta_{11} * \psi_{01}$	2	0.000	0.000
$model * N * \theta_2^2$	12	0.027	0.006
$model * N * pattern$	36	0.031	0.007
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.012	0.002
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern$	36	0.935	0.201
$N * \theta_2^2 * \beta_{11}$	6	0.000	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.000	0.000
$N * pattern * \beta_{11}$	18	0.001	0.000
$N * pattern * \psi_{01}$	36	0.001	0.000
$N * \beta_{11} * \psi_{01}$	6	0.000	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.001	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.001	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.000	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * N * \theta_2^2 * pattern$	72	0.032	0.007

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Table B.17 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{11}$	36	0.001	0.000
$model * N * pattern * \psi_{01}$	72	0.001	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.002	0.001
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.002	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.001	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.001	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.002	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.001	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.007	0.001
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.001	0.000

Table B.18

ANOVA test for the statistical power of β_{10}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.016	0.000
N	3	209.451	0.870
θ_2^2	2	10.688	0.044
$pattern$	6	4.552	0.019
β_{01}	1	0.132	0.001
β_{11}	1	0.012	0.000
ψ_{01}	2	0.001	0.000
$model * N$	6	0.014	0.000
$model * \theta_2^2$	4	0.021	0.000
$model * pattern$	12	0.034	0.000
$model * \beta_{01}$	2	0.000	0.000
$model * \beta_{11}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	7.821	0.032
$N * pattern$	18	2.765	0.011
$N * \beta_{01}$	3	0.111	0.000
$N * \beta_{11}$	3	0.011	0.000
$N * \psi_{01}$	6	0.011	0.000
$\theta_2^2 * pattern$	12	2.561	0.011
$\theta_2^2 * \beta_{01}$	2	0.014	0.000
$\theta_2^2 * \beta_{11}$	2	0.002	0.000

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Table B.18 – continued from previous page

Source	DF	Sum of Squares	η^2
$\theta_2^2 * \psi_{01}$	4	0.005	0.000
$pattern * \beta_{01}$	6	0.018	0.000
$pattern * \beta_{11}$	6	0.003	0.000
$pattern * \psi_{01}$	12	0.008	0.000
$\beta_{01} * \beta_{11}$	1	0.001	0.000
$\beta_{01} * \psi_{01}$	2	0.003	0.000
$\beta_{11} * \psi_{01}$	2	0.002	0.000
$model * N * \theta_2^2$	12	0.016	0.000
$model * N * pattern$	36	0.024	0.000
$model * N * \beta_{01}$	6	0.000	0.000
$model * N * \beta_{11}$	6	0.000	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.032	0.000
$model * \theta_2^2 * \beta_{01}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.000	0.000
$model * pattern * \beta_{11}$	12	0.000	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11}$	2	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$model * \beta_{11} * \psi_{01}$	4	0.000	0.000

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Table B.18 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$N * \theta_2^2 * pattern$	36	1.799	0.007
$N * \theta_2^2 * \beta_{01}$	6	0.043	0.000
$N * \theta_2^2 * \beta_{11}$	6	0.006	0.000
$N * \theta_2^2 * \psi_{01}$	12	0.007	0.000
$N * pattern * \beta_{01}$	18	0.048	0.000
$N * pattern * \beta_{11}$	18	0.025	0.000
$N * pattern * \psi_{01}$	36	0.017	0.000
$N * \beta_{01} * \beta_{11}$	3	0.002	0.000
$N * \beta_{01} * \psi_{01}$	6	0.004	0.000
$N * \beta_{11} * \psi_{01}$	6	0.006	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.013	0.000
$\theta_2^2 * pattern * \beta_{11}$	12	0.005	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.019	0.000
$\theta_2^2 * \beta_{01} * \beta_{11}$	2	0.001	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.003	0.000
$\theta_2^2 * \beta_{11} * \psi_{01}$	4	0.001	0.000
$pattern * \beta_{01} * \beta_{11}$	6	0.002	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.007	0.000
$pattern * \beta_{11} * \psi_{01}$	12	0.008	0.000
$\beta_{01} * \beta_{11} * \psi_{01}$	2	0.003	0.000
$model * N * \theta_2^2 * pattern$	72	0.026	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.000	0.000

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Table B.18 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01}$	36	0.001	0.000
$model * N * pattern * \beta_{11}$	36	0.000	0.000
$model * N * pattern * \psi_{01}$	72	0.001	0.000
$model * N * \beta_{01} * \beta_{11}$	6	0.000	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * N * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11}$	4	0.000	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * \theta_2^2 * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * pattern * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.027	0.000
$N * \theta_2^2 * pattern * \beta_{11}$	36	0.016	0.000
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.054	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11}$	6	0.006	0.000

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Table B.18 – continued from previous page

Source	**DF**	**Sum of Squares**	η^2
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.004	0.000
$N * \theta_2^2 * \beta_{11} * \psi_{01}$	12	0.006	0.000
$N * pattern * \beta_{01} * \beta_{11}$	18	0.008	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.016	0.000
$N * pattern * \beta_{11} * \psi_{01}$	36	0.020	0.000
$N * \beta_{01} * \beta_{11} * \psi_{01}$	6	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11}$	12	0.002	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.008	0.000
$\theta_2^2 * pattern * \beta_{11} * \psi_{01}$	24	0.014	0.000
$\theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	4	0.001	0.000
$pattern * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.004	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{11}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11}$	12	0.000	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * \theta_2^2 * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11}$	36	0.001	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.000	0.000
$model * N * pattern * \beta_{11} * \psi_{01}$	72	0.001	0.000
$model * N * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	24	0.000	0.000

Continued on next page

Table B.18 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	48	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	36	0.012	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.033	0.000
$N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	72	0.045	0.000
$N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	12	0.007	0.000
$N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	36	0.016	0.000
$\theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.008	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.002	0.000
$model * N * \theta_2^2 * pattern * \beta_{11} * \psi_{01}$	144	0.001	0.000
$model * N * \theta_2^2 * \beta_{01} * \beta_{11} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	48	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	72	0.033	0.000
$model * N * \theta_2^2 * pattern * \beta_{01} * \beta_{11} * \psi_{01}$	144	0.001	0.000

Table B.19

ANOVA test for the statistical power of β_{11}

Source	**DF**	**Sum of Squares**	η^2
$model$	2	0.008	0.000
N	3	25.966	0.577
θ_2^2	2	4.012	0.089
$pattern$	6	1.737	0.039
β_{01}	1	0.281	0.006
ψ_{01}	2	0.001	0.000
$model * N$	6	0.008	0.000
$model * \theta_2^2$	4	0.009	0.000
$model * pattern$	12	0.016	0.000
$model * \beta_{01}$	2	0.000	0.000
$model * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2$	6	7.019	0.156
$N * pattern$	18	2.569	0.057
$N * \beta_{01}$	3	0.714	0.016
$N * \psi_{01}$	6	0.003	0.000
$\theta_2^2 * pattern$	12	0.889	0.020
$\theta_2^2 * \beta_{01}$	2	0.018	0.000
$\theta_2^2 * \psi_{01}$	4	0.002	0.000
$pattern * \beta_{01}$	6	0.006	0.000
$pattern * \psi_{01}$	12	0.007	0.000
$\beta_{01} * \psi_{01}$	2	0.001	0.000

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Table B.19 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2$	12	0.009	0.000
$model * N * pattern$	36	0.020	0.000
$model * N * \beta_{01}$	6	0.001	0.000
$model * N * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern$	24	0.014	0.000
$model * \theta_2^2 * \beta_{01}$	4	0.000	0.000
$model * \theta_2^2 * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01}$	12	0.001	0.000
$model * pattern * \psi_{01}$	24	0.000	0.000
$model * \beta_{01} * \psi_{01}$	4	0.000	0.000
$N * \theta_2^2 * pattern$	36	1.352	0.030
$N * \theta_2^2 * \beta_{01}$	6	0.101	0.002
$N * \theta_2^2 * \psi_{01}$	12	0.005	0.000
$N * pattern * \beta_{01}$	18	0.035	0.001
$N * pattern * \psi_{01}$	36	0.015	0.000
$N * \beta_{01} * \psi_{01}$	6	0.001	0.000
$\theta_2^2 * pattern * \beta_{01}$	12	0.007	0.000
$\theta_2^2 * pattern * \psi_{01}$	24	0.008	0.000
$\theta_2^2 * \beta_{01} * \psi_{01}$	4	0.001	0.000
$pattern * \beta_{01} * \psi_{01}$	12	0.004	0.000
$model * N * \theta_2^2 * pattern$	72	0.016	0.000
$model * N * \theta_2^2 * \beta_{01}$	12	0.001	0.000

Continued on next page

Table B.19 – continued from previous page

Source	DF	Sum of Squares	η^2
$model * N * \theta_2^2 * \psi_{01}$	24	0.001	0.000
$model * N * pattern * \beta_{01}$	36	0.002	0.000
$model * N * pattern * \psi_{01}$	72	0.001	0.000
$model * N * \beta_{01} * \psi_{01}$	12	0.000	0.000
$model * \theta_2^2 * pattern * \beta_{01}$	24	0.000	0.000
$model * \theta_2^2 * pattern * \psi_{01}$	48	0.001	0.000
$model * \theta_2^2 * \beta_{01} * \psi_{01}$	8	0.000	0.000
$model * pattern * \beta_{01} * \psi_{01}$	24	0.000	0.000
$N * \theta_2^2 * pattern * \beta_{01}$	36	0.033	0.001
$N * \theta_2^2 * pattern * \psi_{01}$	72	0.027	0.001
$N * \theta_2^2 * \beta_{01} * \psi_{01}$	12	0.001	0.000
$N * pattern * \beta_{01} * \psi_{01}$	36	0.011	0.000
$\theta_2^2 * pattern * \beta_{01} * \psi_{01}$	24	0.009	0.000
$model * N * \theta_2^2 * pattern * \beta_{01}$	72	0.001	0.000
$model * N * \theta_2^2 * pattern * \psi_{01}$	144	0.002	0.000
$model * N * \theta_2^2 * \beta_{01} * \psi_{01}$	24	0.000	0.000
$model * N * pattern * \beta_{01} * \psi_{01}$	72	0.001	0.000
$model * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	48	0.001	0.000
$N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	72	0.031	0.001
$model * N * \theta_2^2 * pattern * \beta_{01} * \psi_{01}$	144	0.002	0.000

B.0.4 List of Figures in Bias

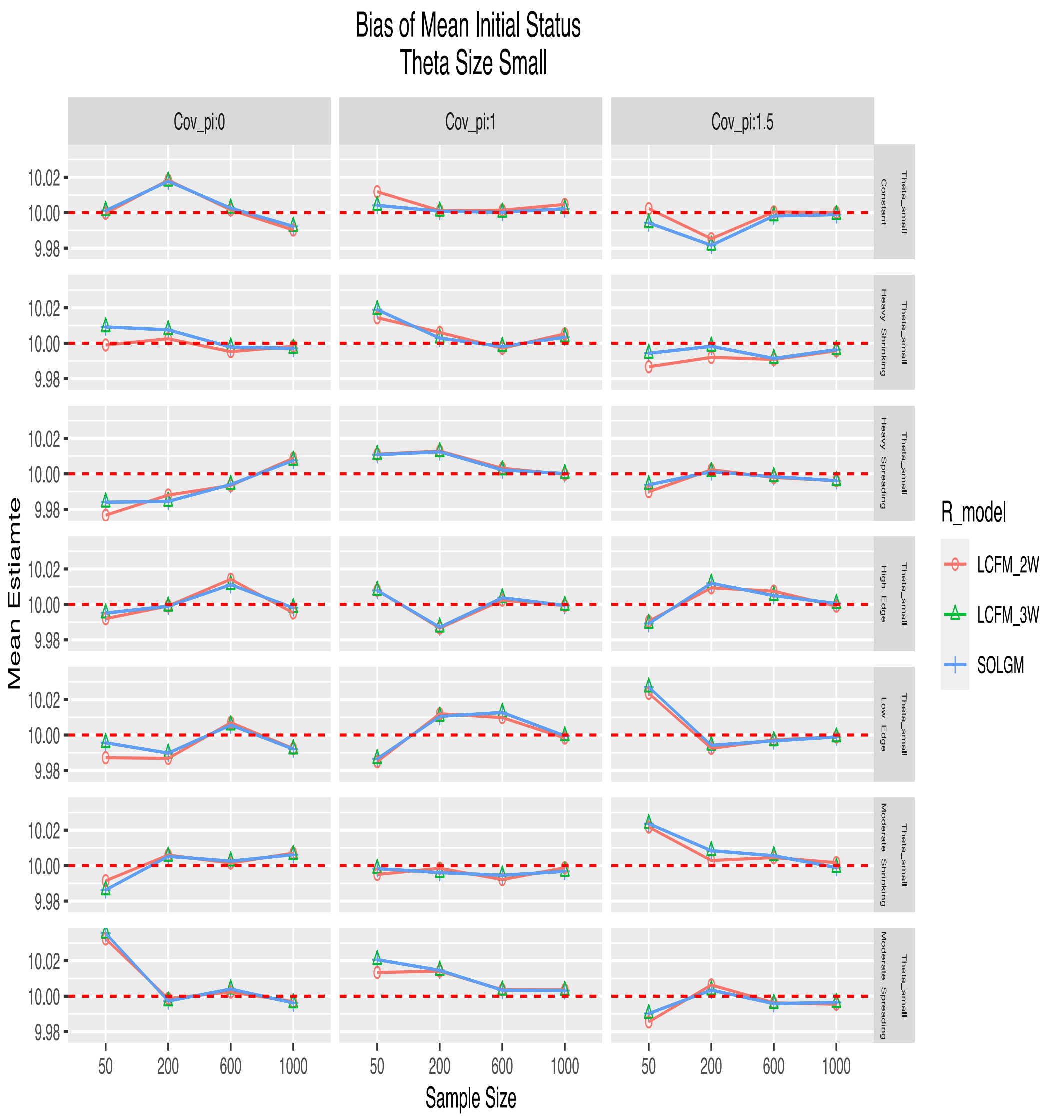

Figure B.1. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

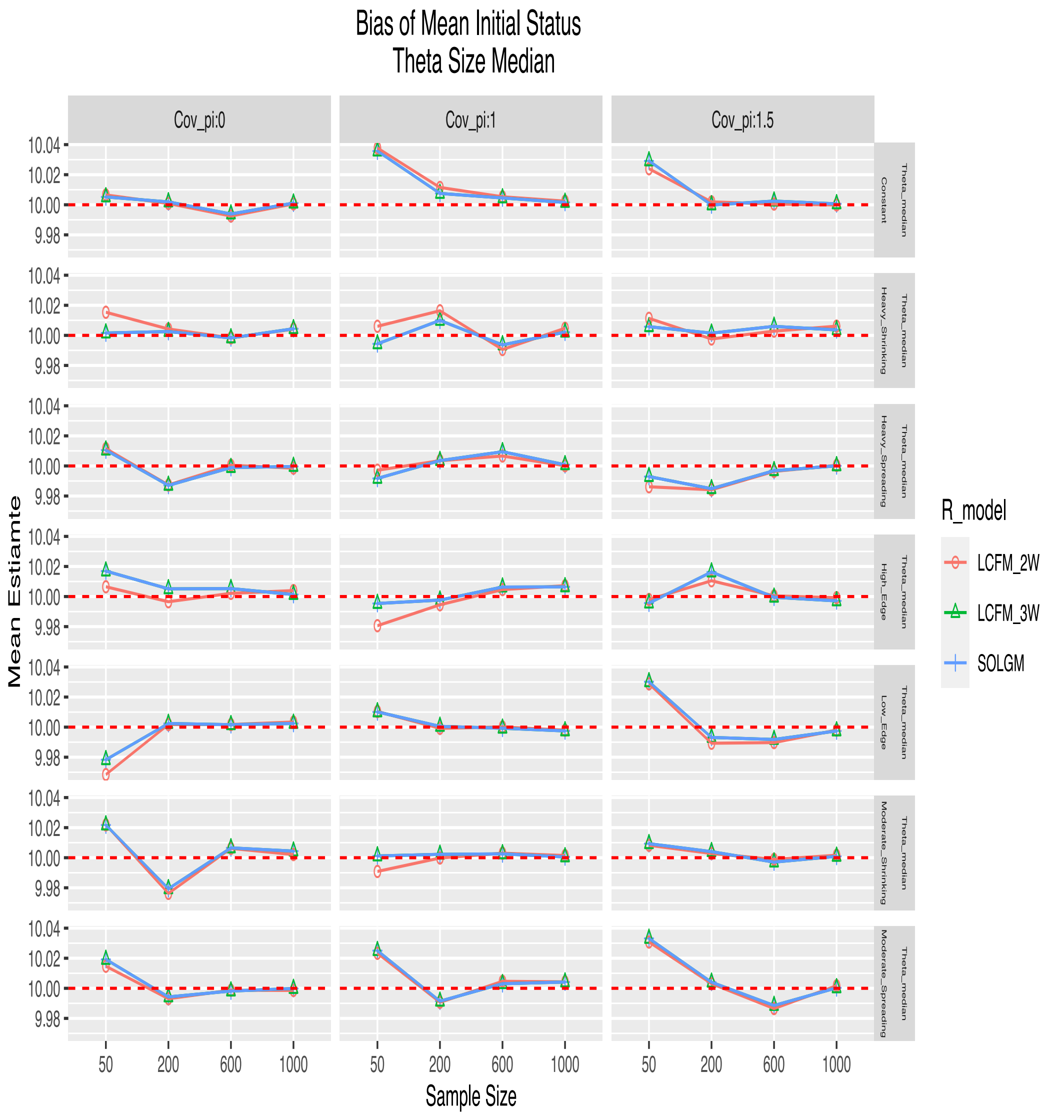

Figure B.2. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

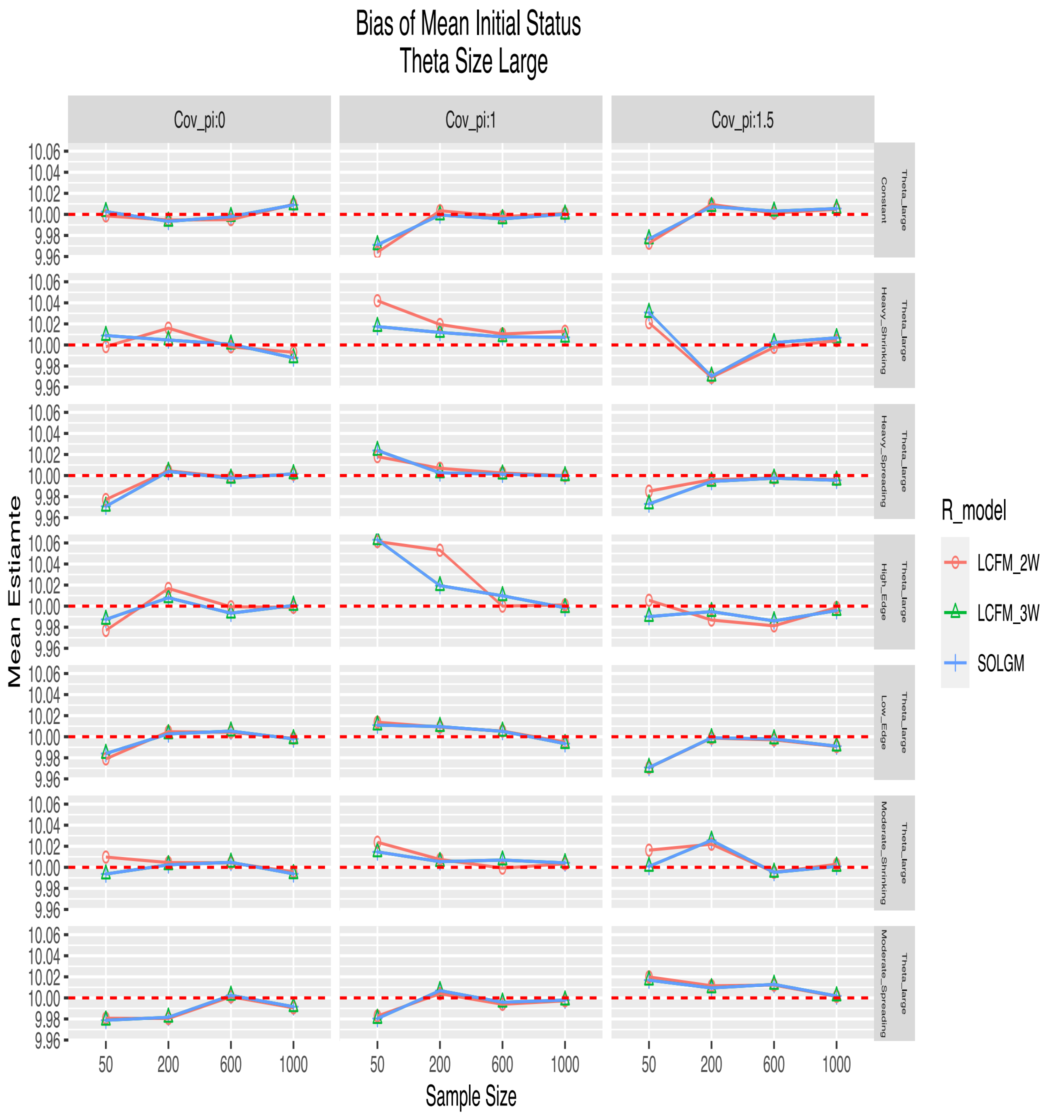

Figure B.3. Bias of the Mean Initial Status for a Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

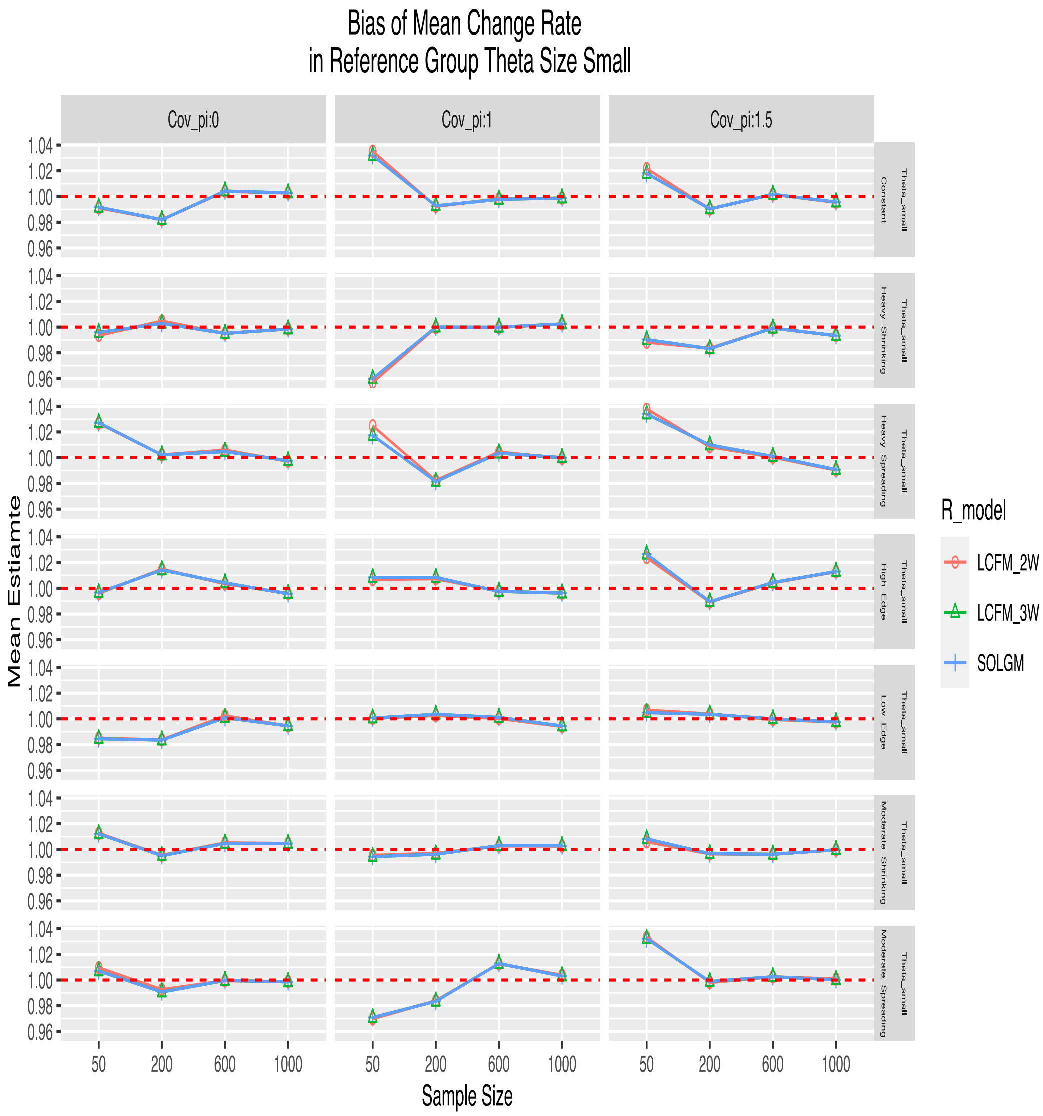

Figure B.4. Bias of the Average Growth Rate in a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

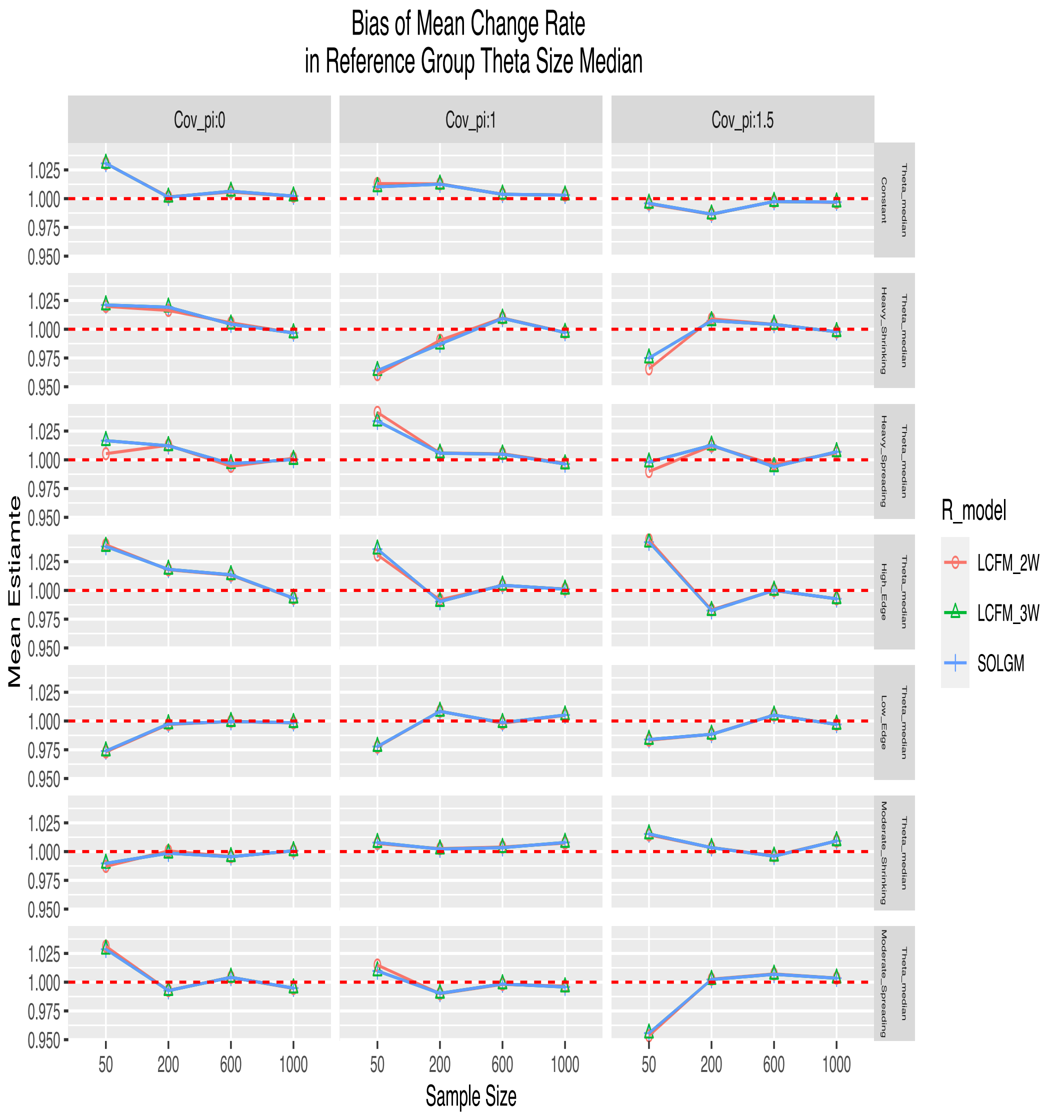

Figure B.5. Bias of the Average Growth Rate in a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

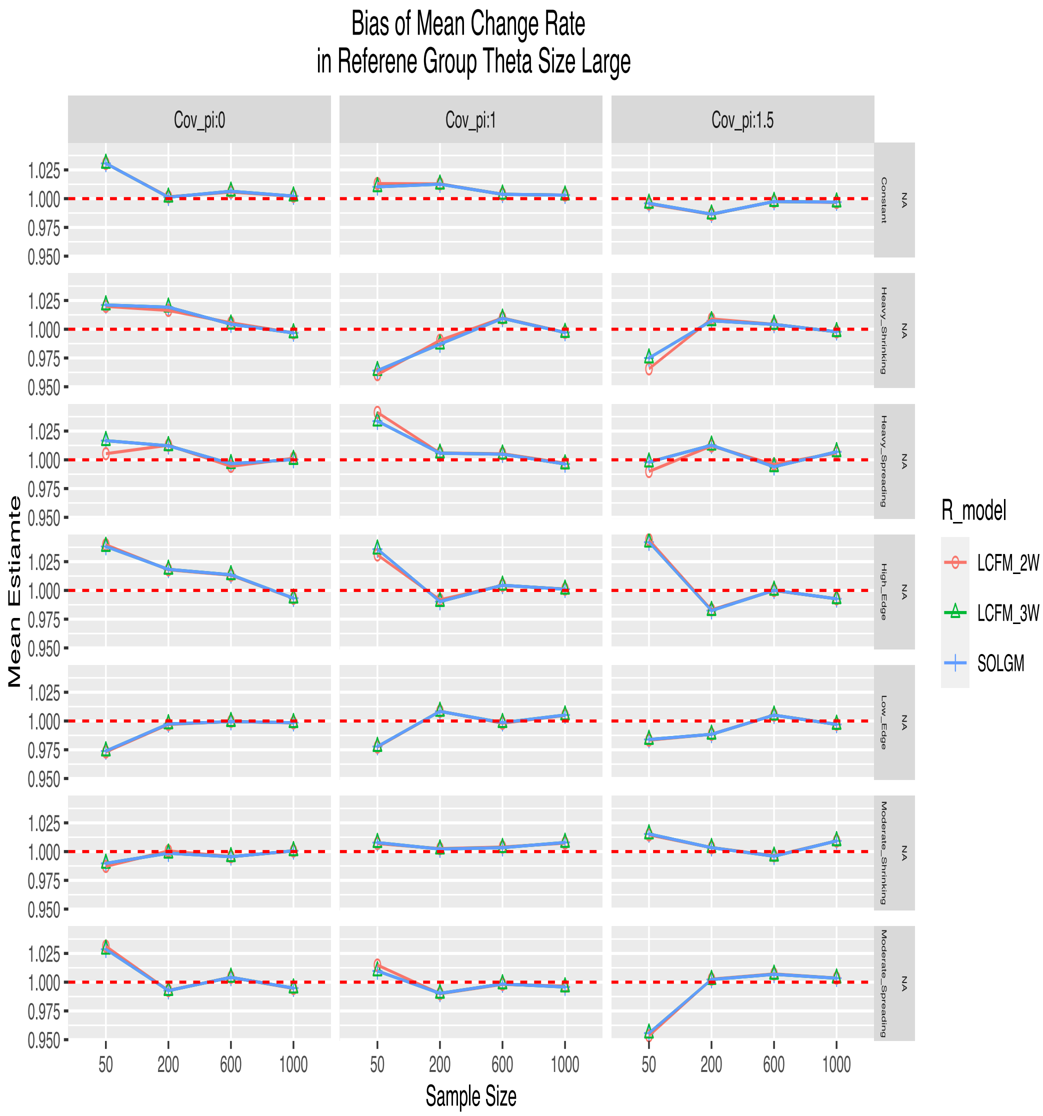

Figure B.6. Bias of the Average Growth Rate in a Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

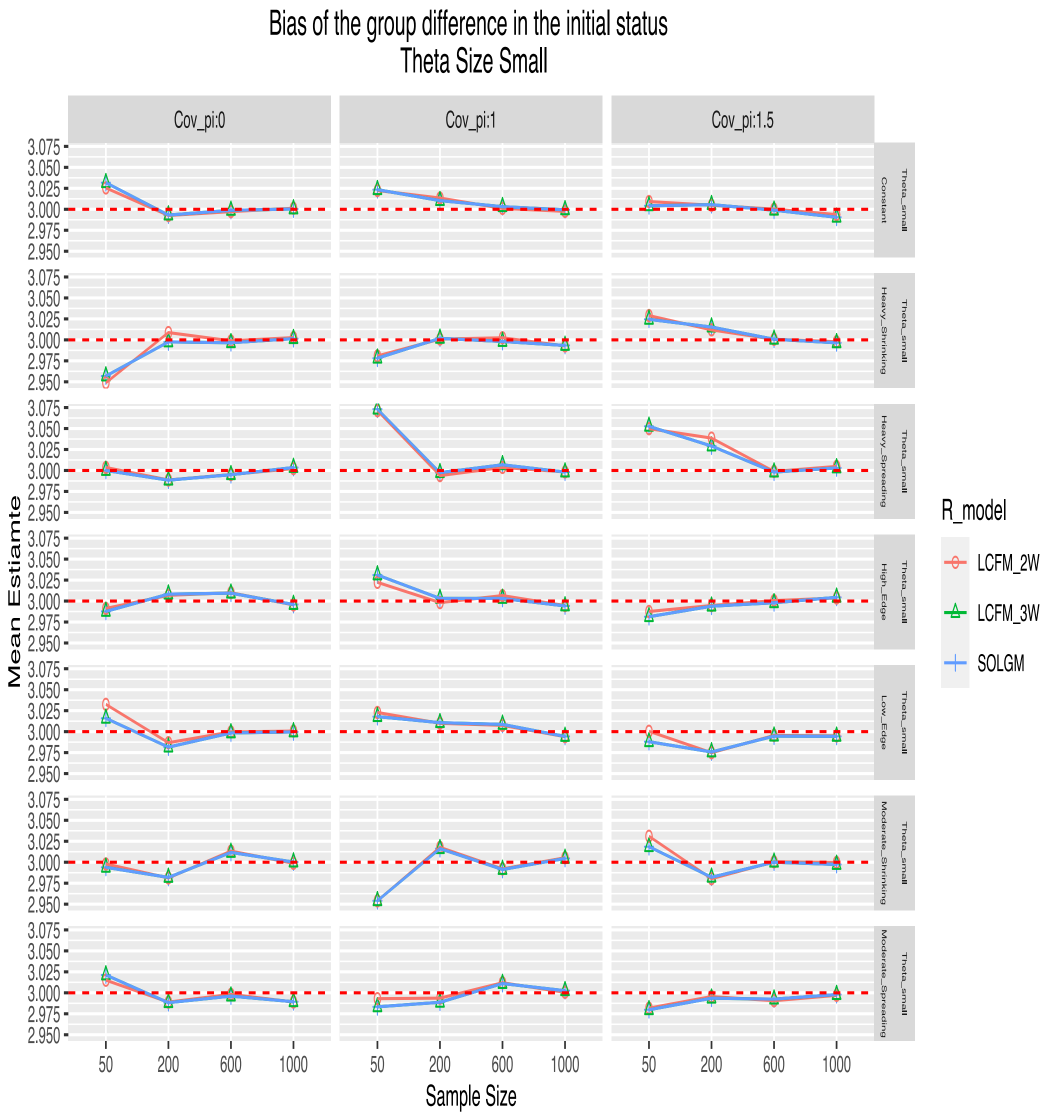

Figure B.7. Bias of the Average Group Difference in the initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

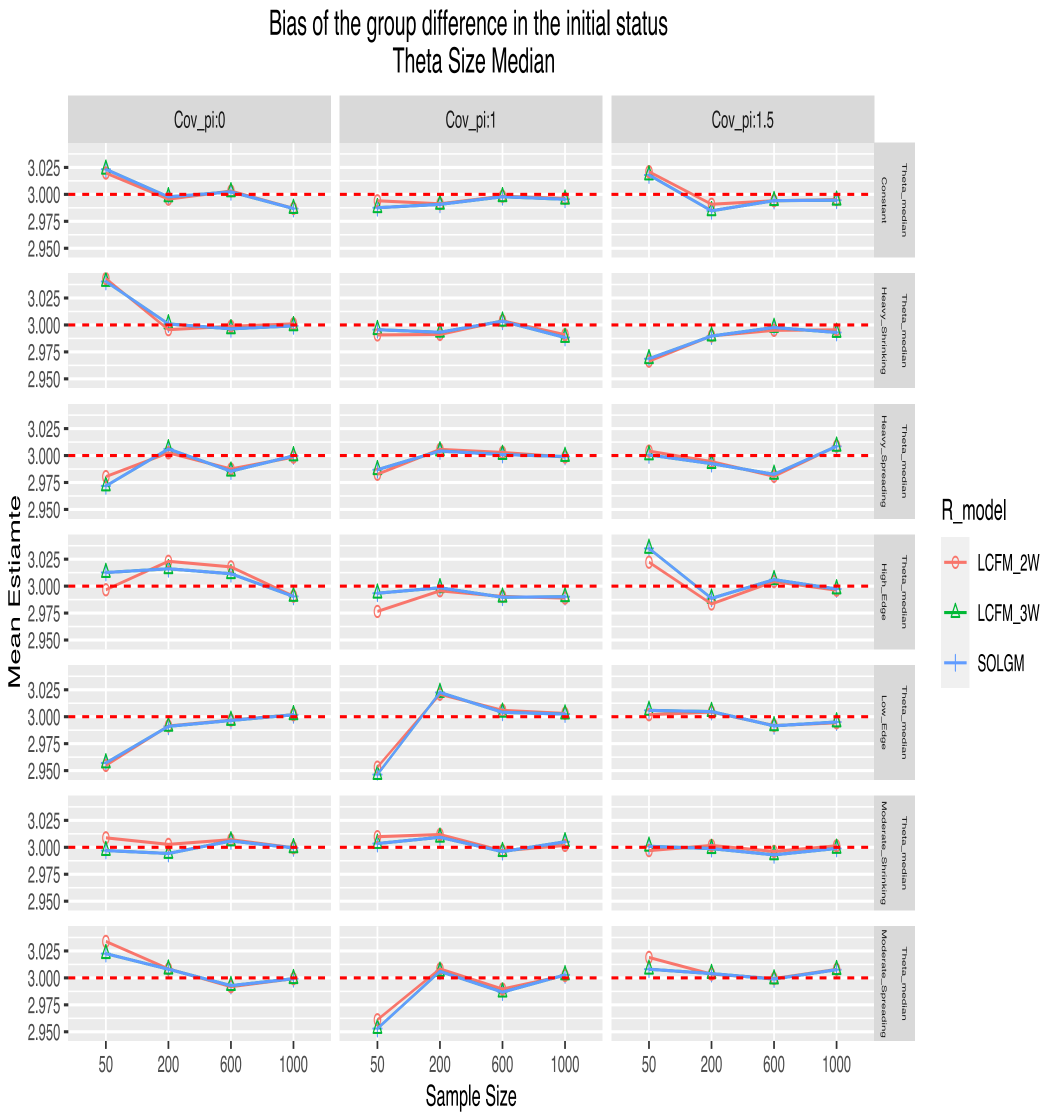

Figure B.8. Bias of the Average Group Difference in the initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

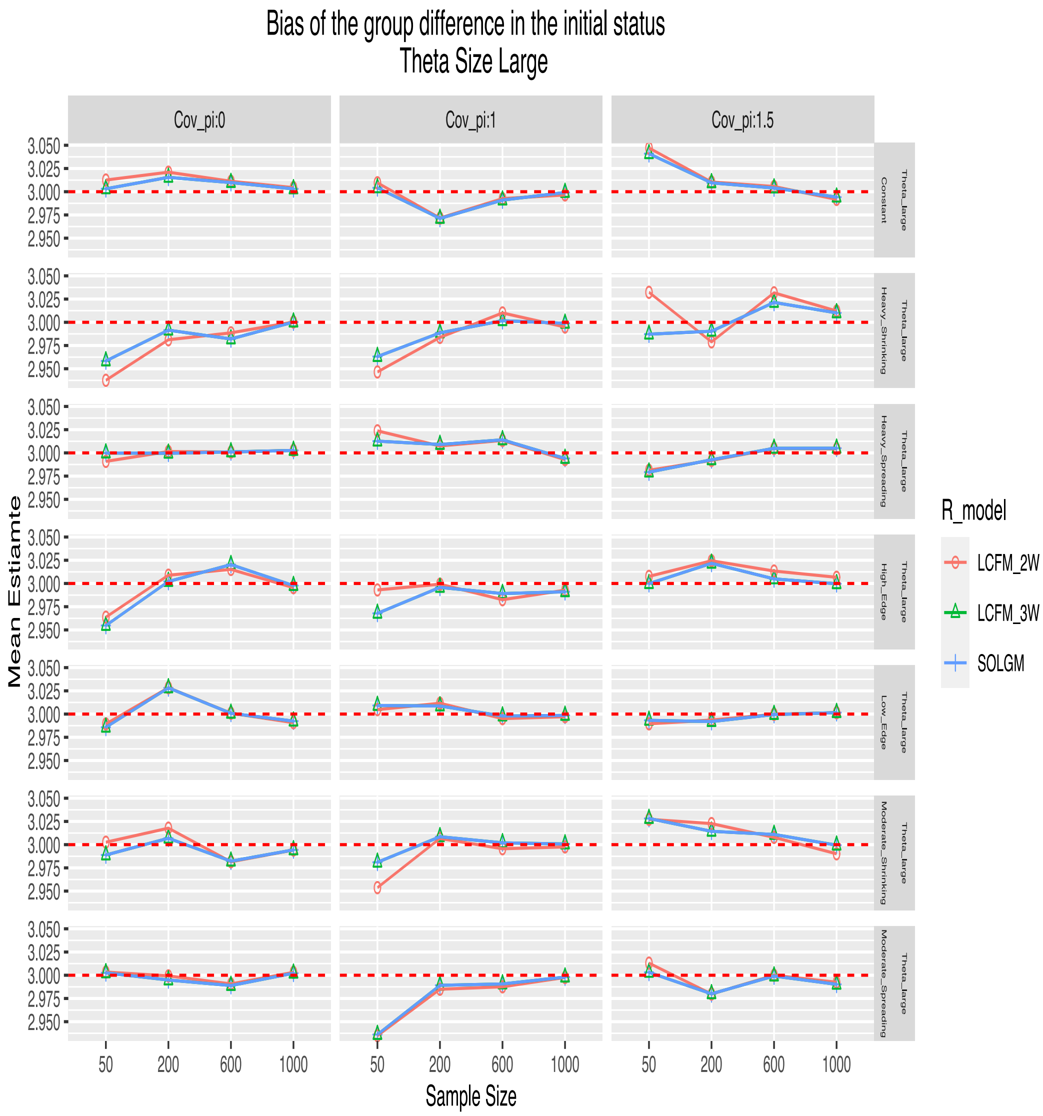

Figure B.9. Bias of the Average Group Difference in the initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

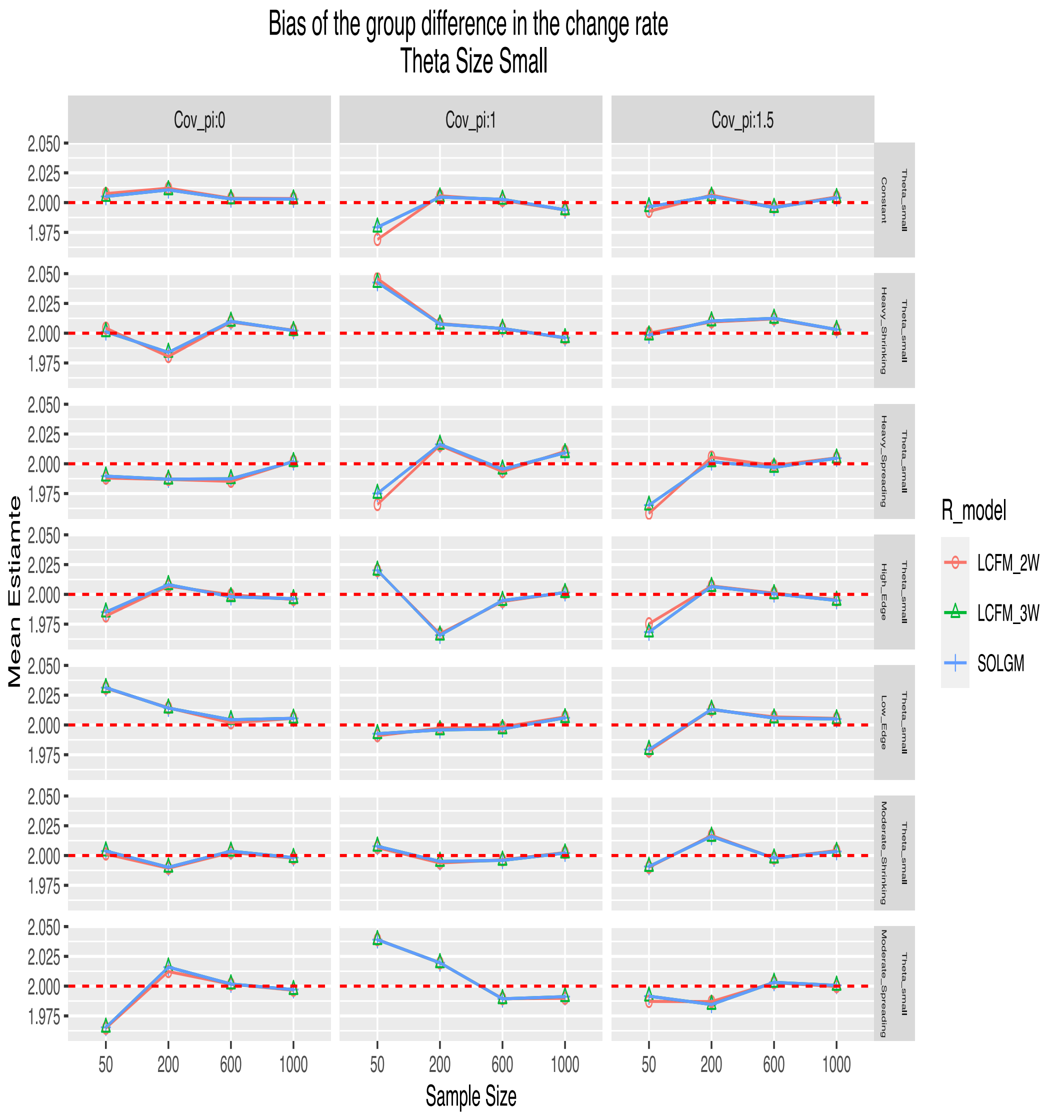

Figure B.10. Bias of Mean Group Difference in Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

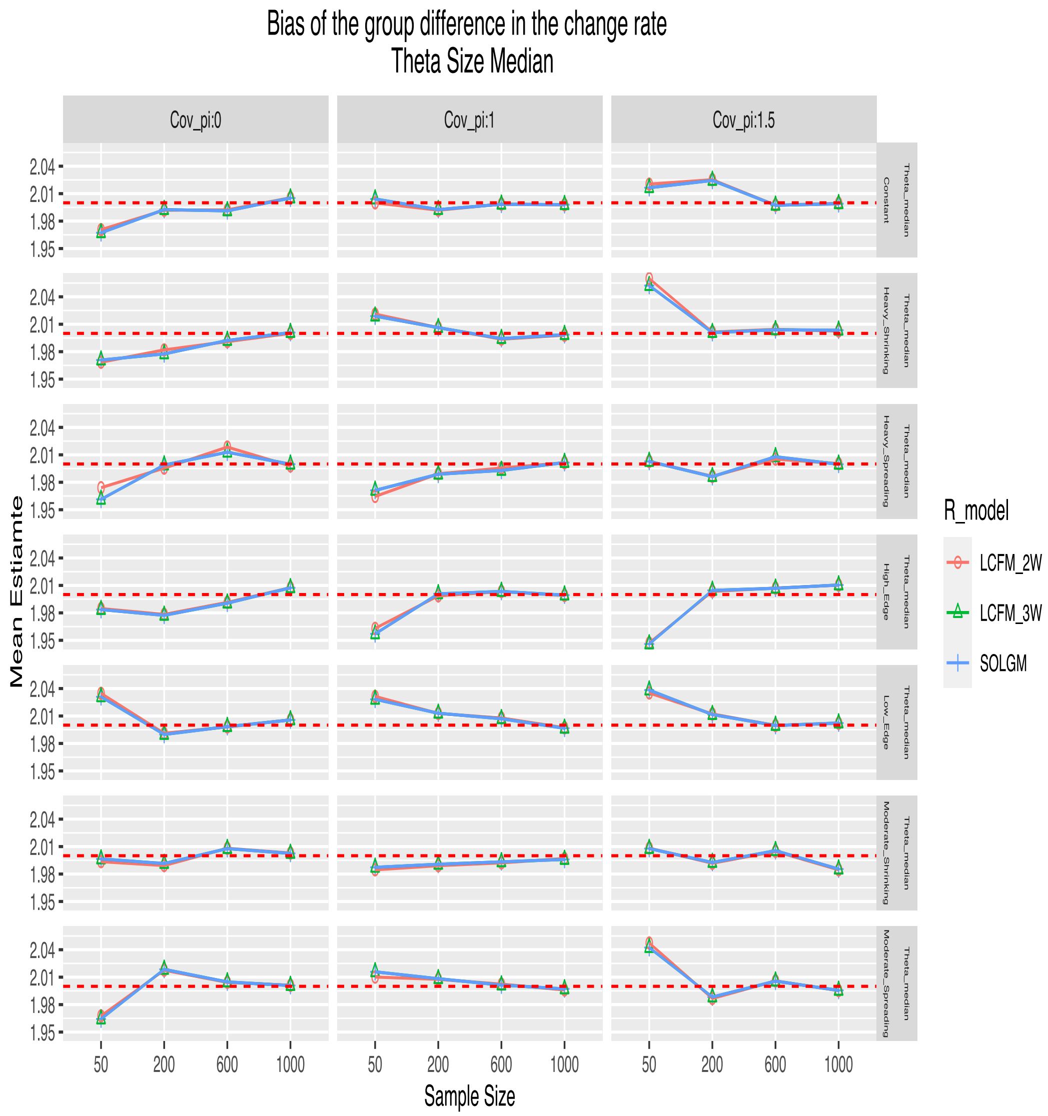

Figure B.11. Bias of Mean Group Difference in Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

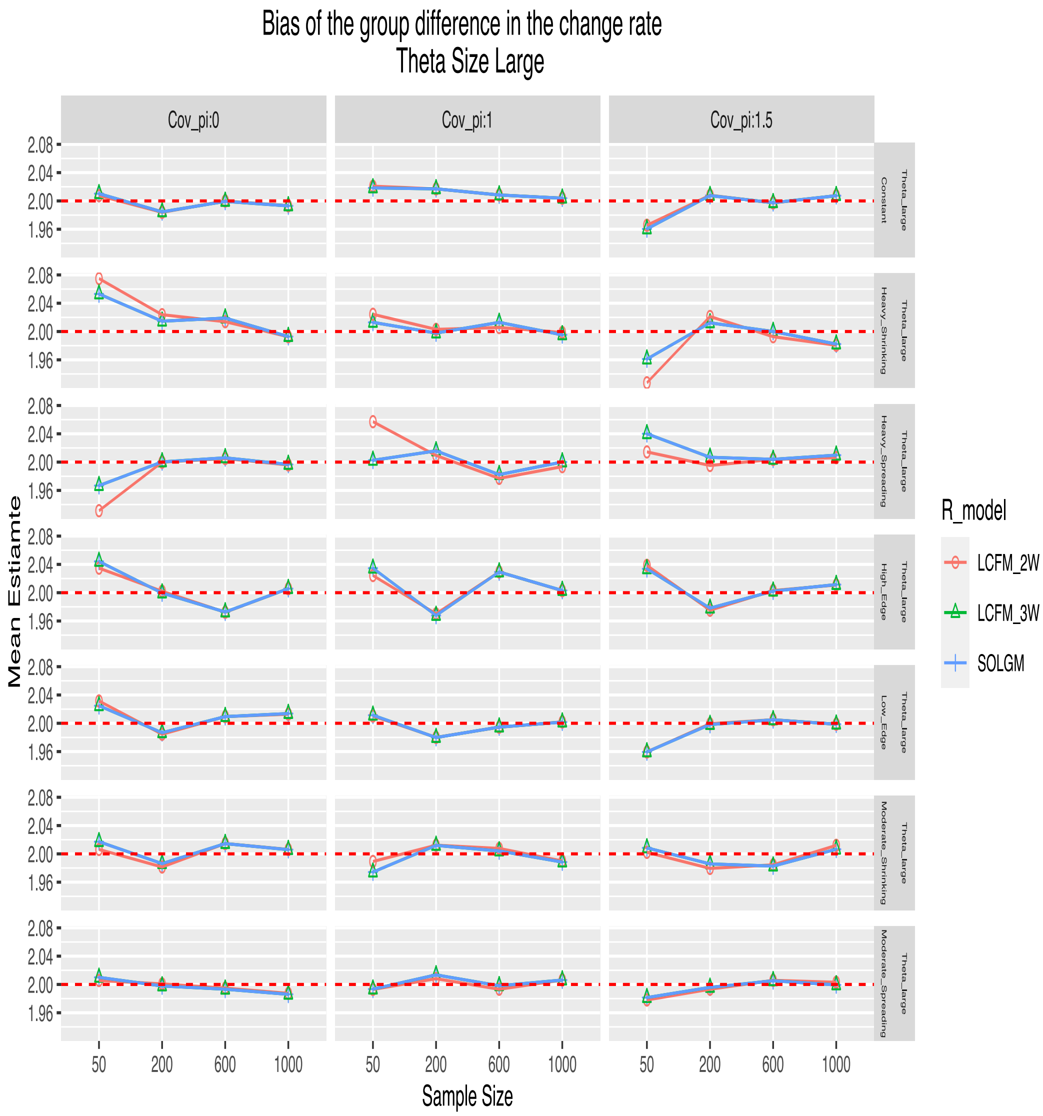

Figure B.12. Bias of Mean Group Difference in Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

B.0.5 List of Figures in Empirical Standard Error

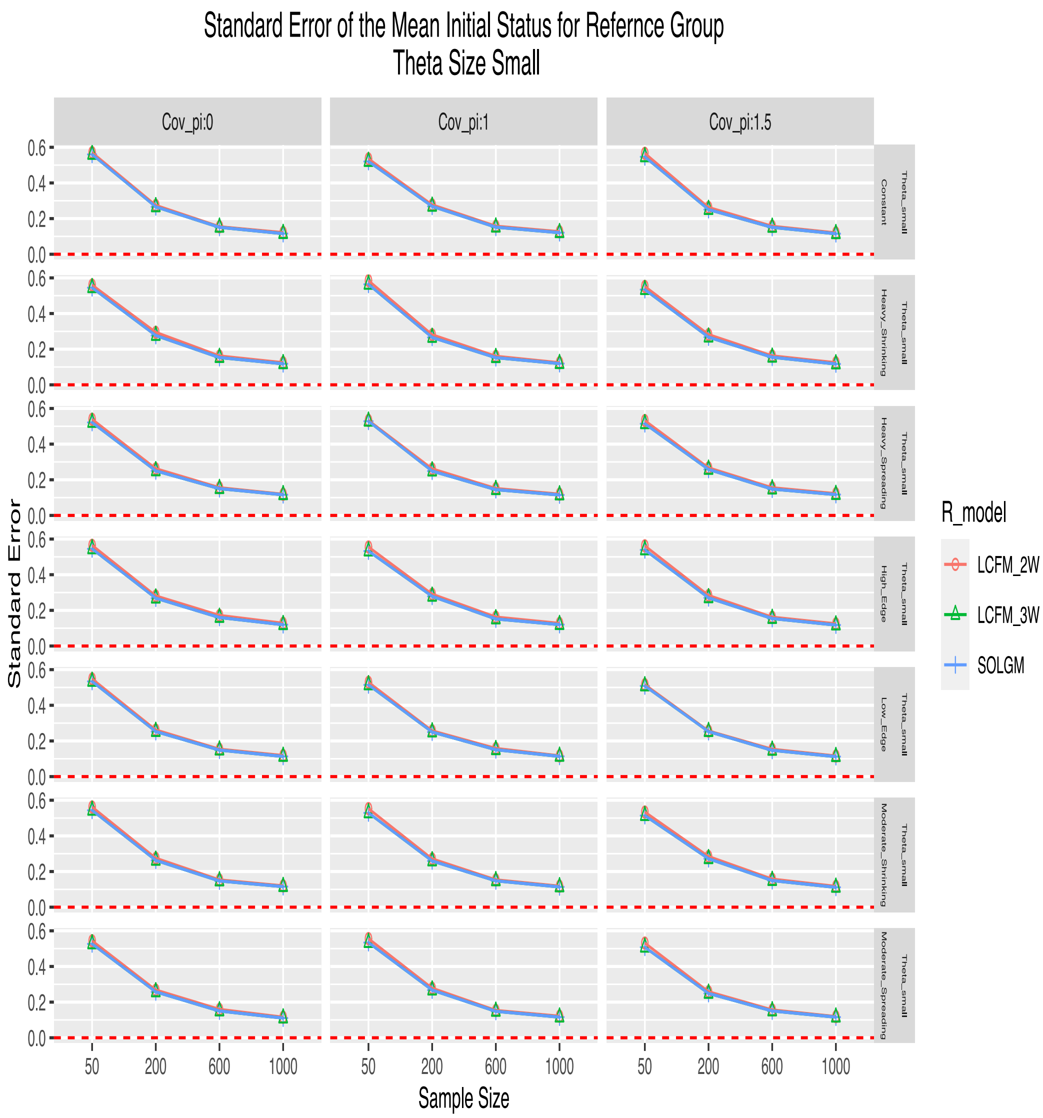

Figure B.13. Standard Error of the Average Initial Status in Refrence Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

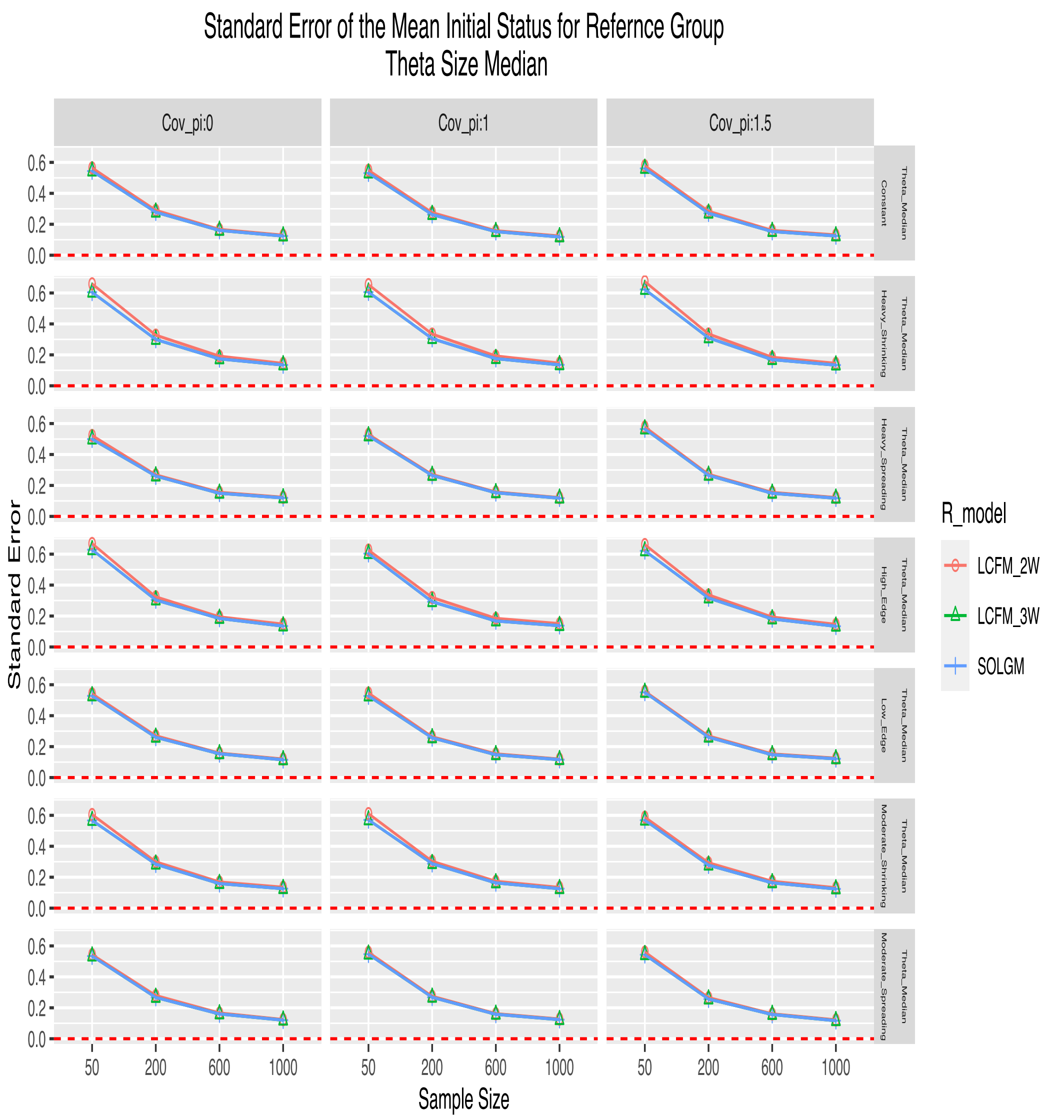

Figure B.14. Standard Error of the Average Initial Status in Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

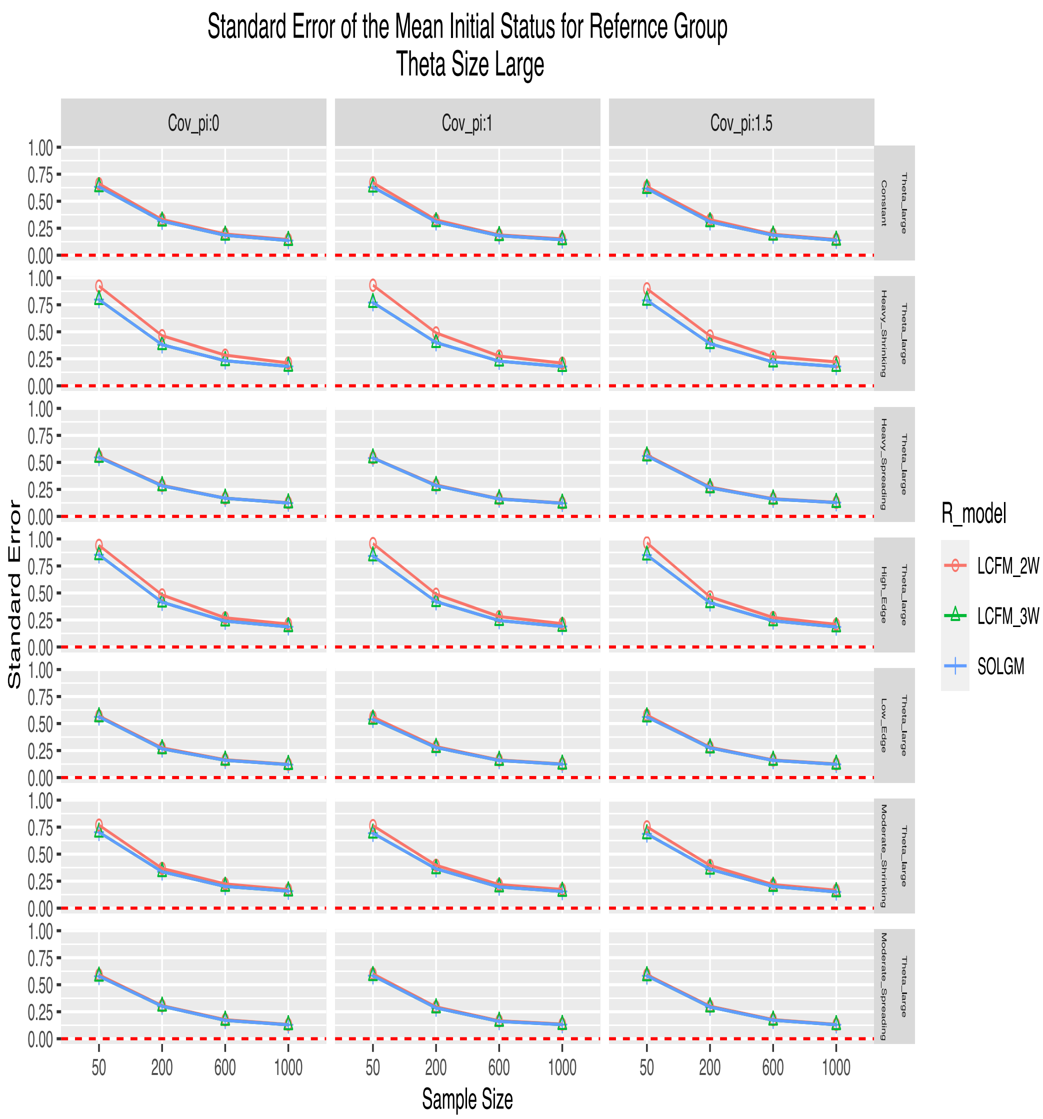

Figure B.15. Standard Error of the Average Initial Status in Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

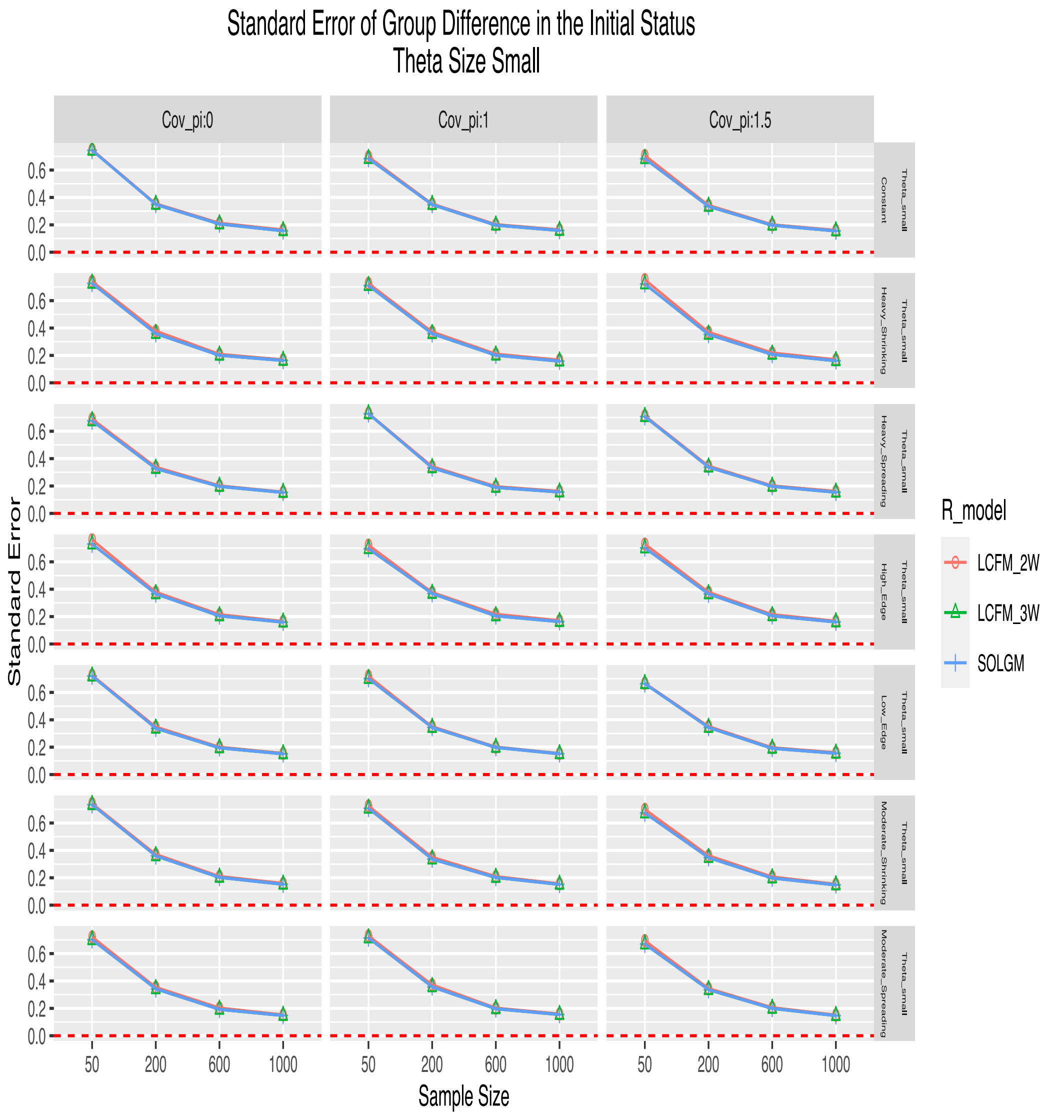

Figure B.16. Standard Error of the Average Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

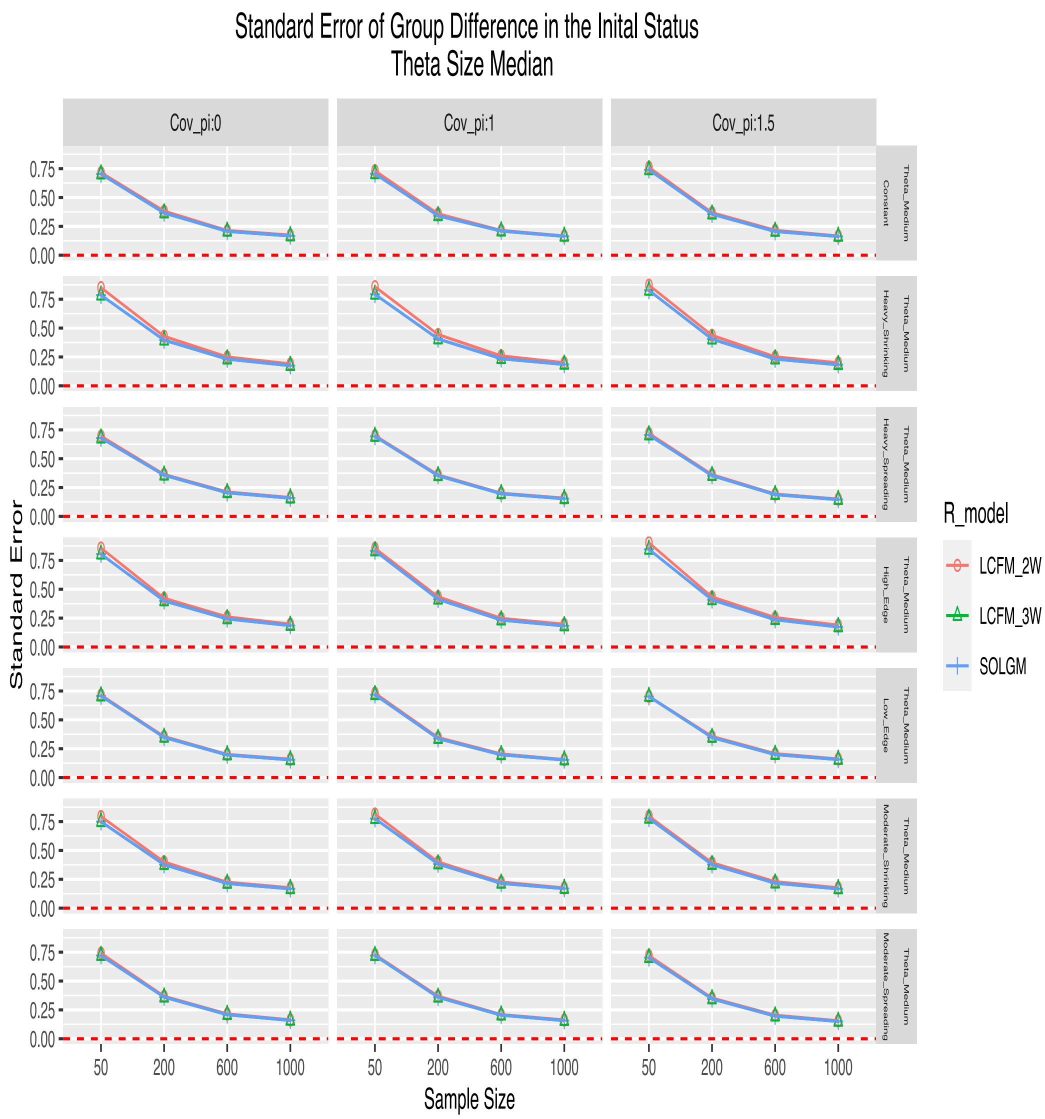

Figure B.17. Standard Error of the Average Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

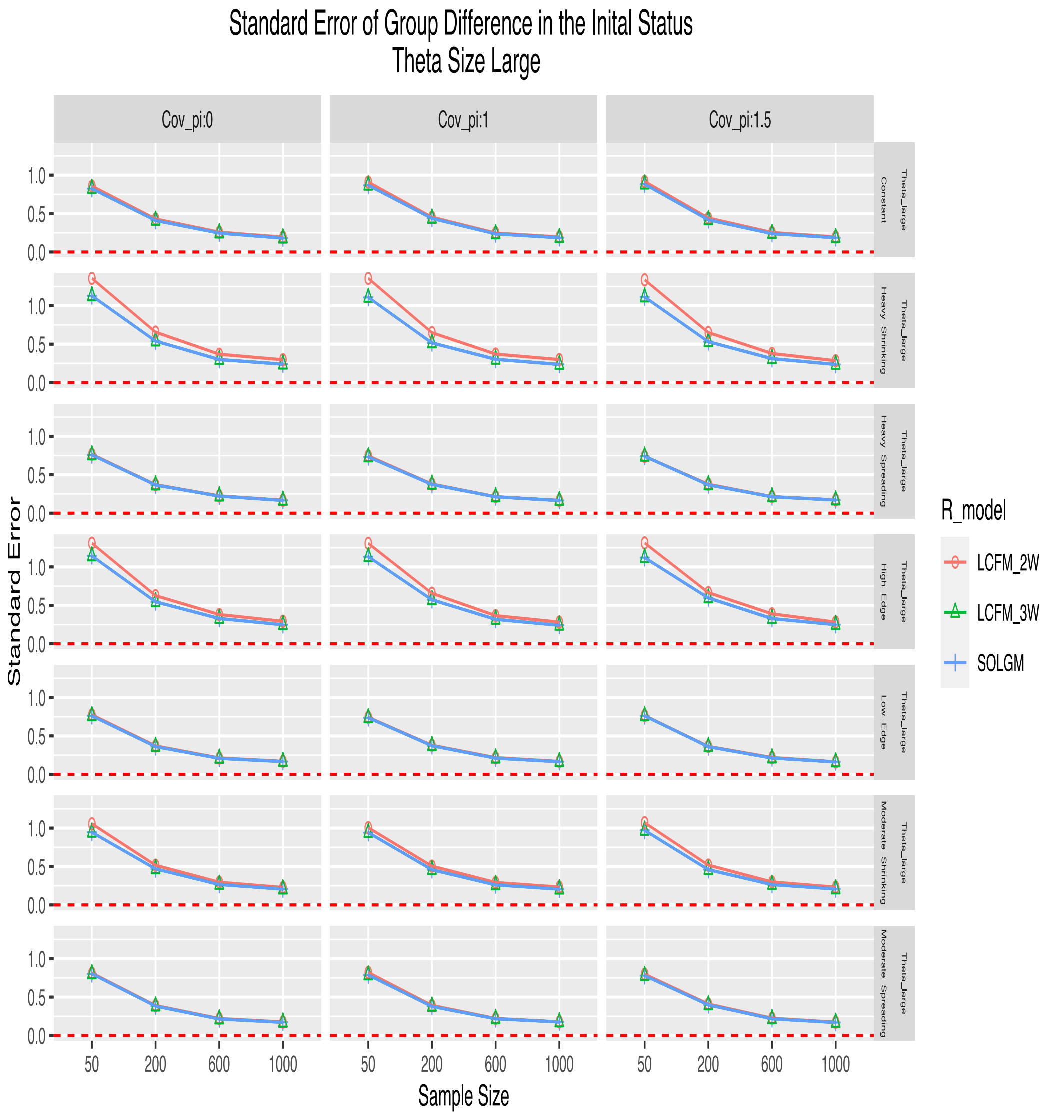

Figure B.18. Standard Error of the Average Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

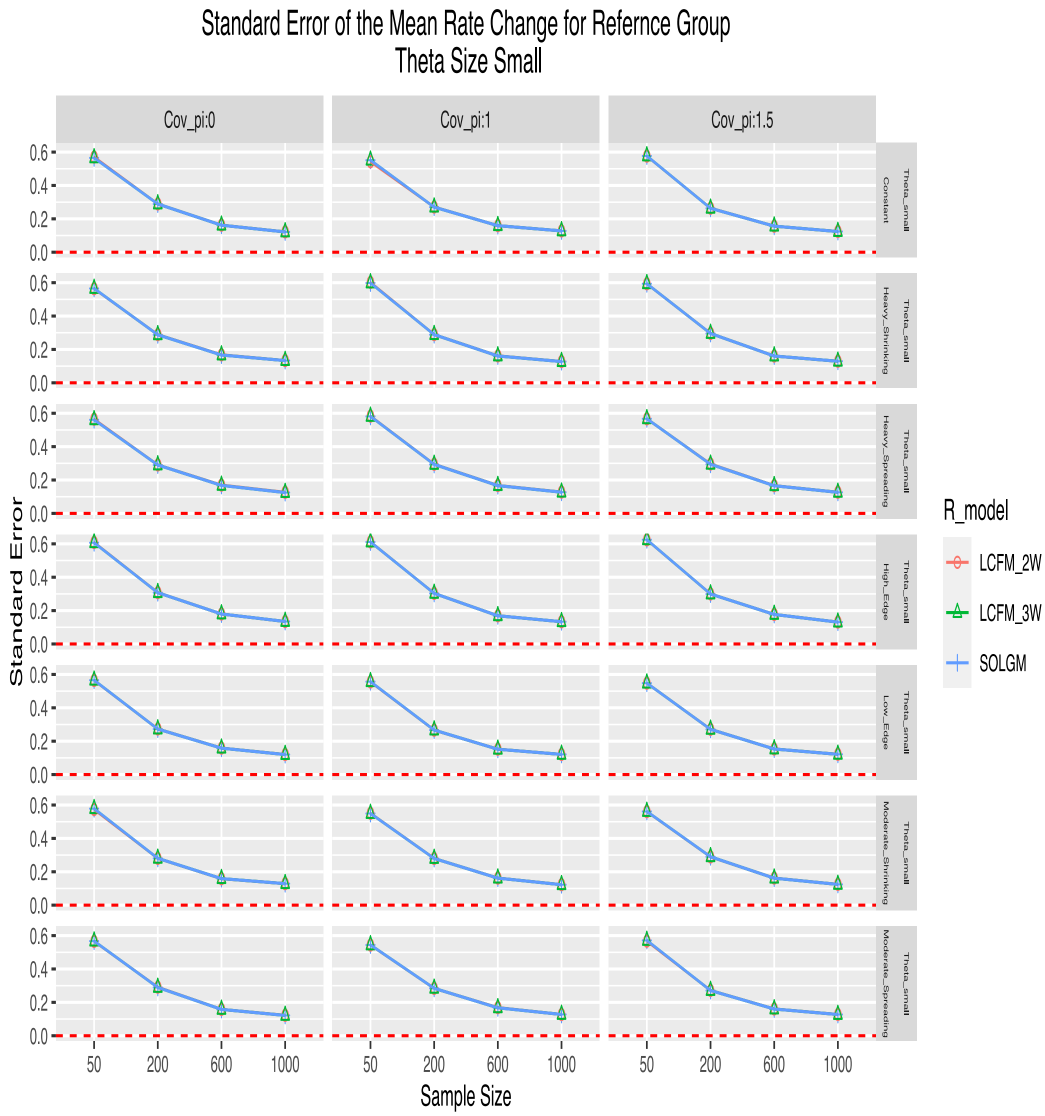

Figure B.19. Standard Error of the Average Growth Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

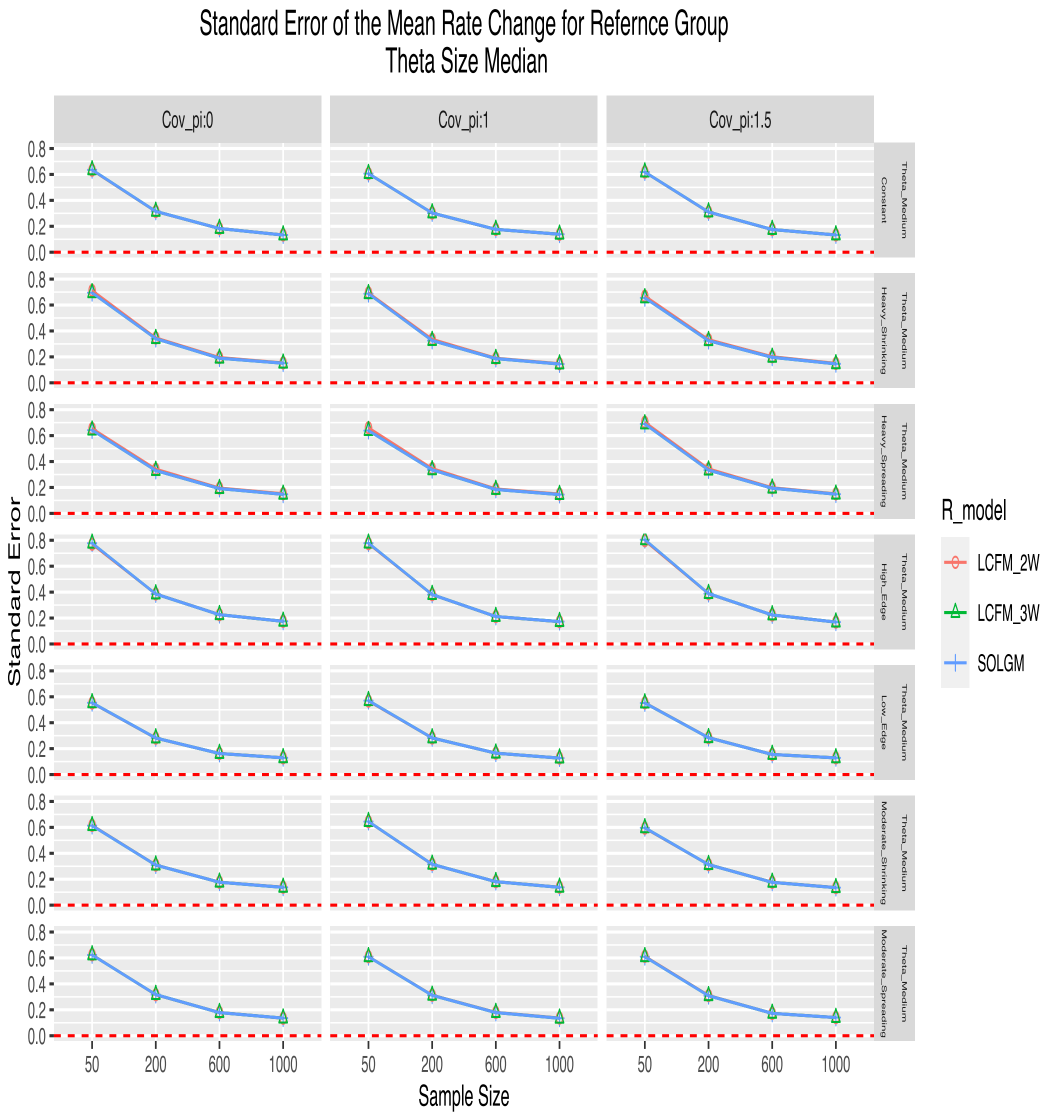

Figure B.20. Standard Error of the Average Growth Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

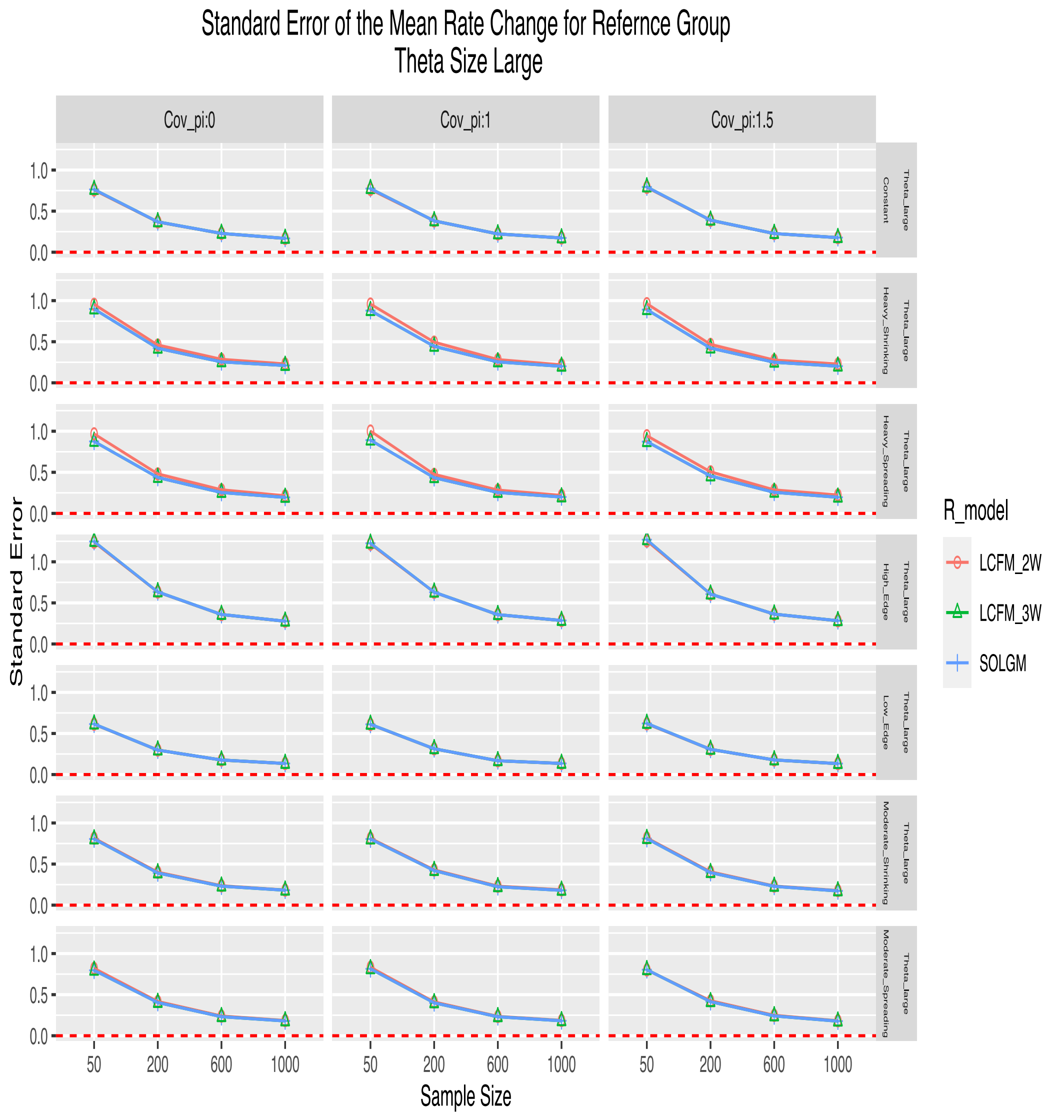

Figure B.21. Standard Error of the Average Growth Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

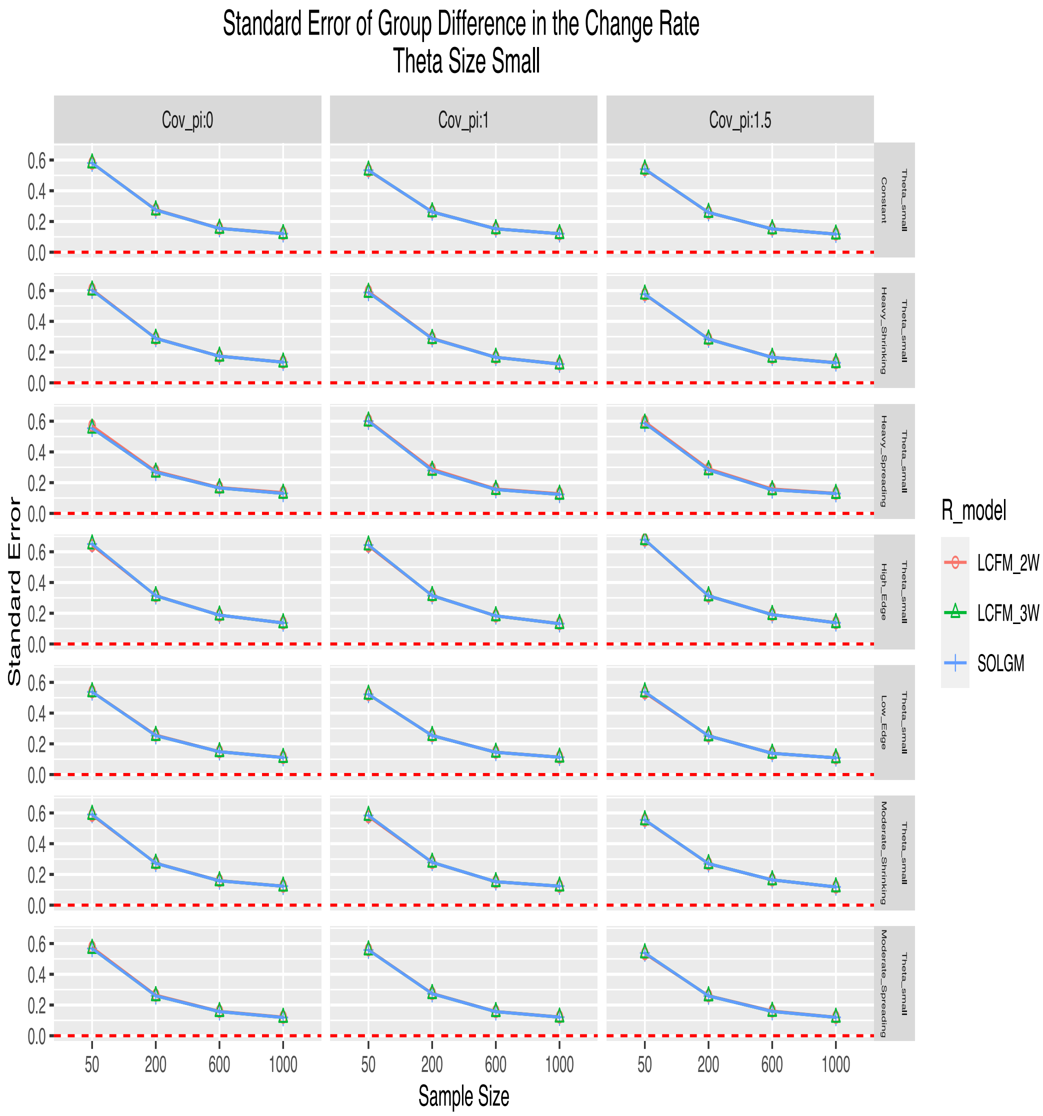

Figure B.22. Standard Error of Average Group Difference for the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

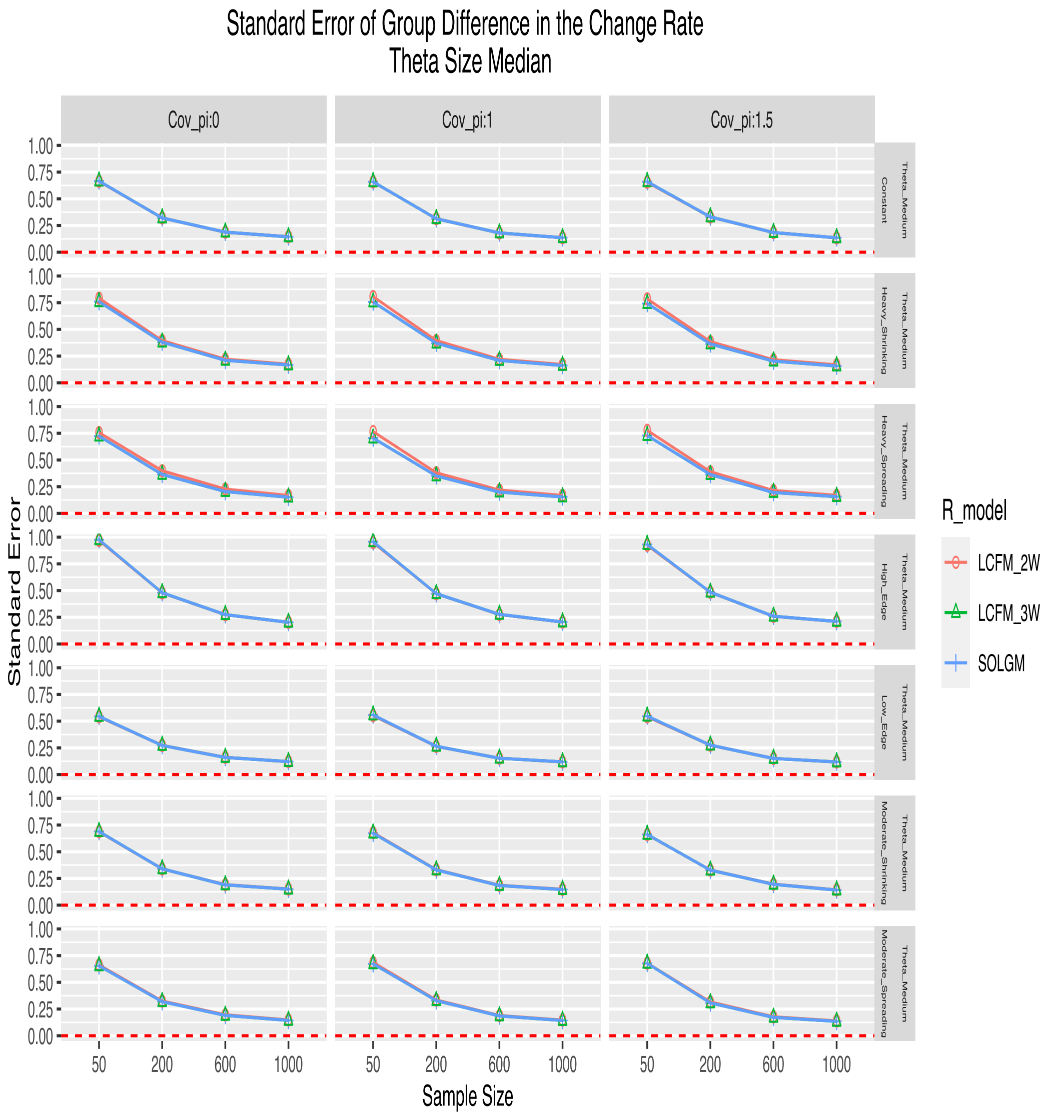

Figure B.23. Standard Error of Average Group Difference for the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

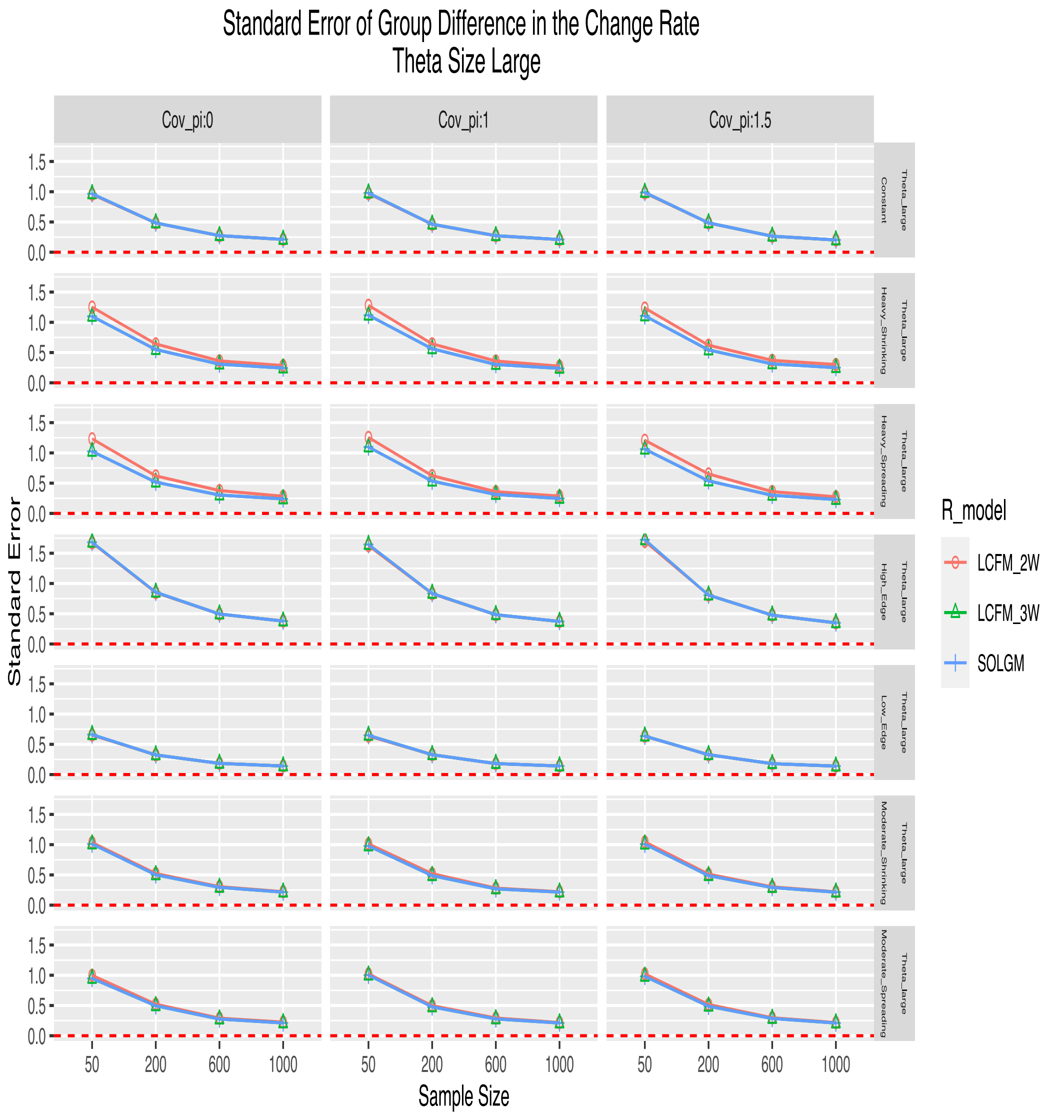

Figure B.24. Standard Error of Average Group Difference for the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

B.0.6 List of figured in Theoretical Standard Error

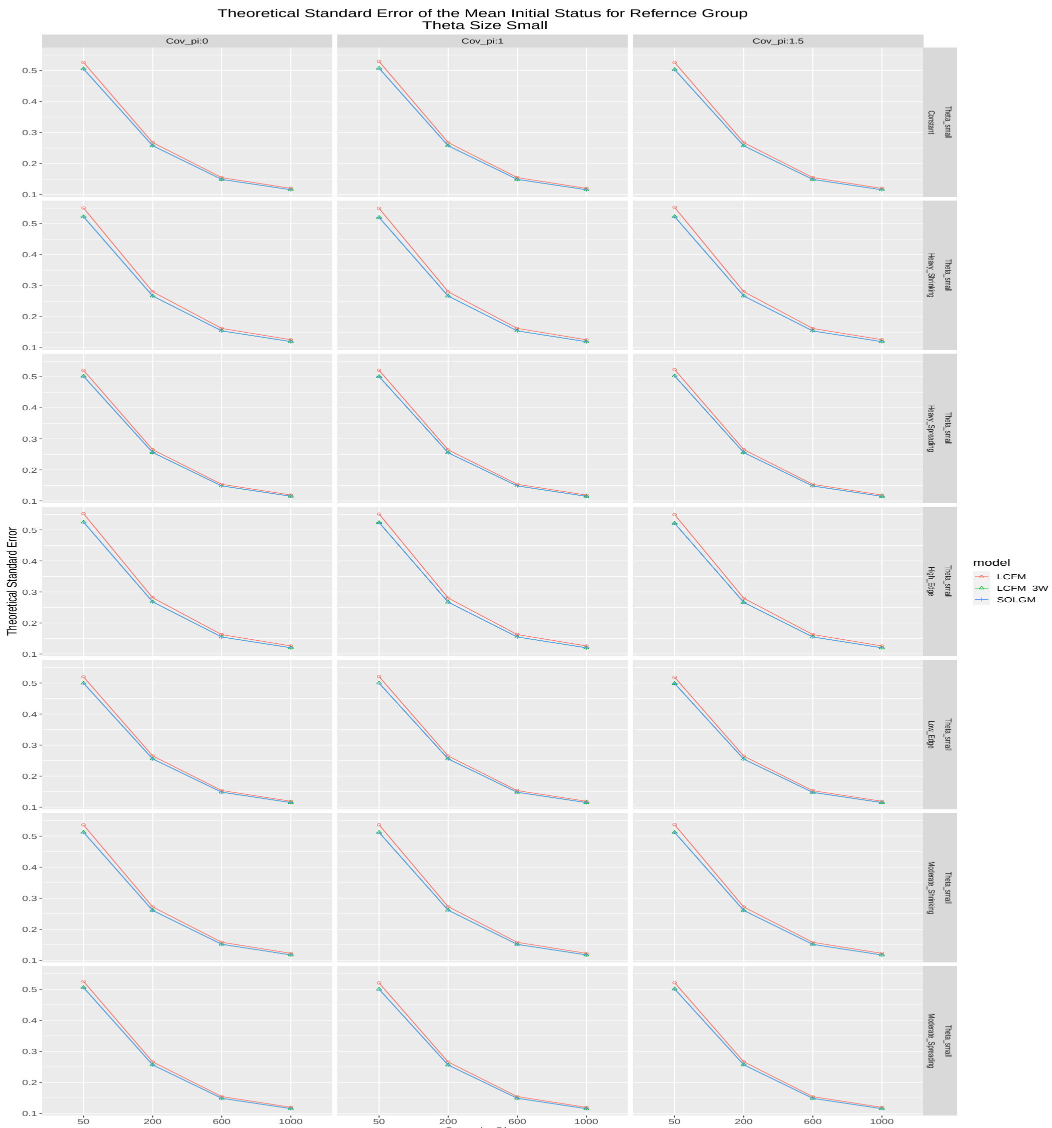

Figure B.25. Theoretical Standard Error of the Average Initial Status in Refrence Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

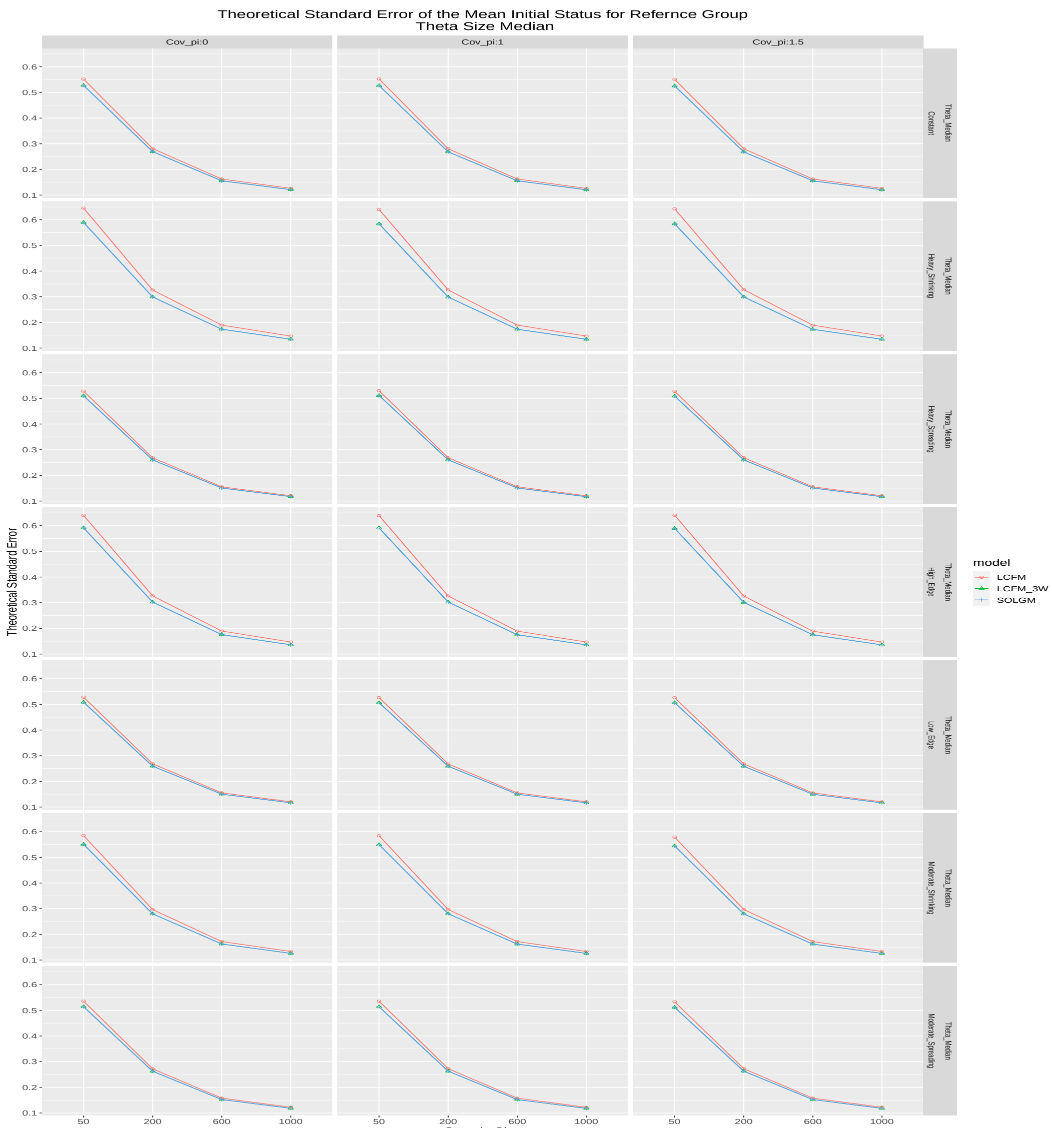

Figure B.26. Theoretical Standard Error of the Average Initial Status in Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

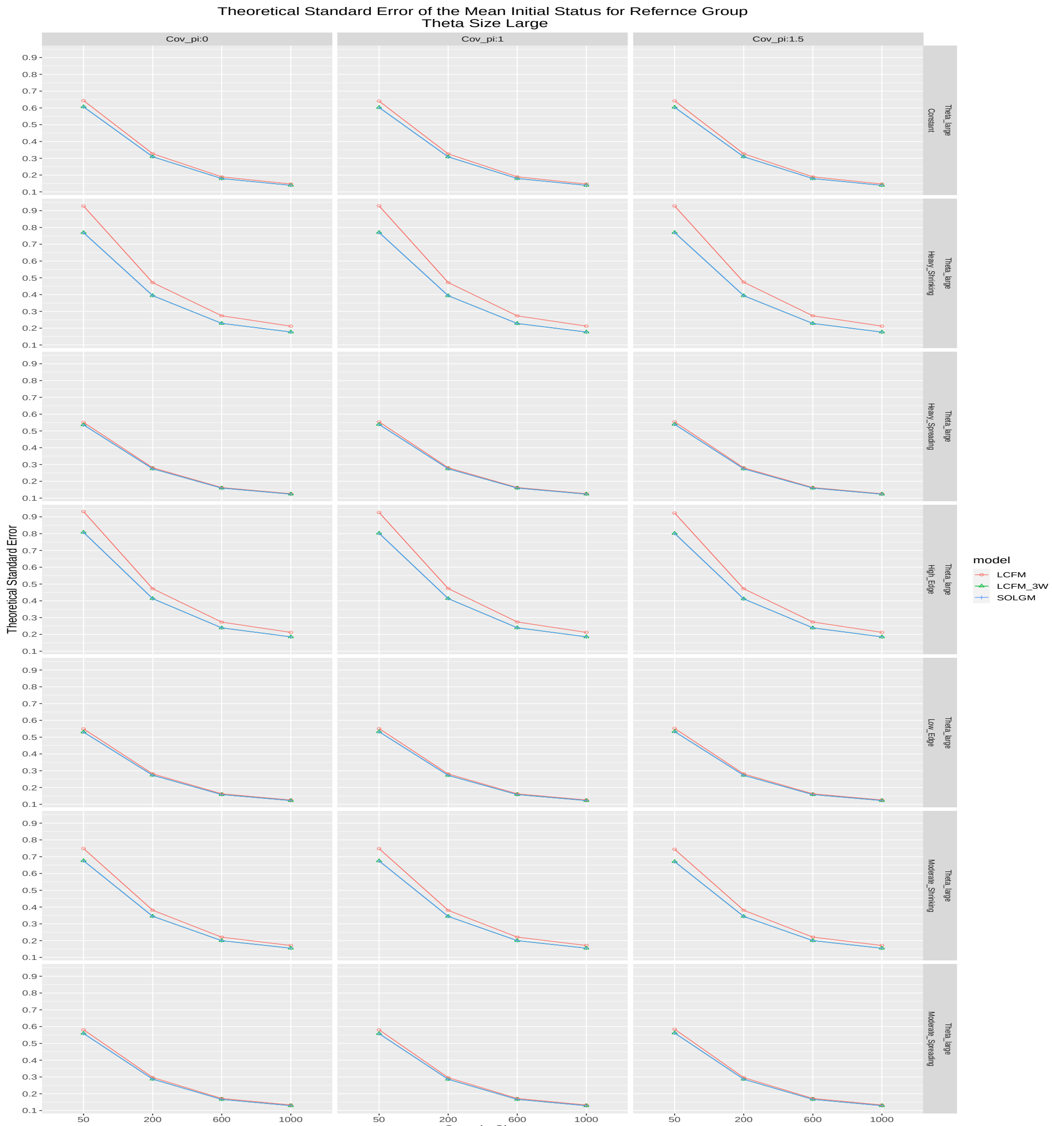

Figure B.27. Theoretical Standard Error of the Average Initial Status in Reference Group, $\boldsymbol{\beta_{00} = 10}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

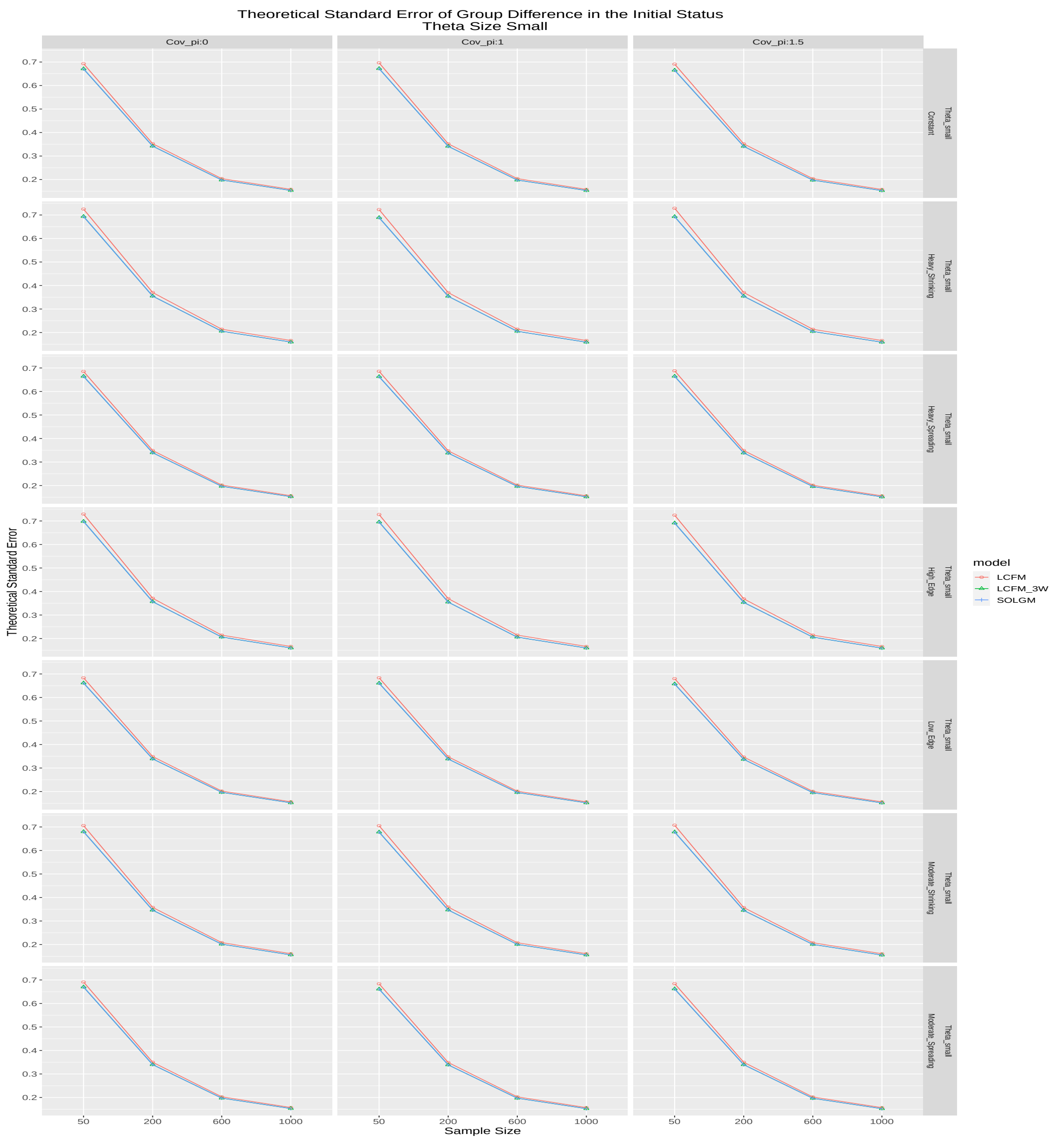

Figure B.28. Theoretical Standard Error of the Average Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

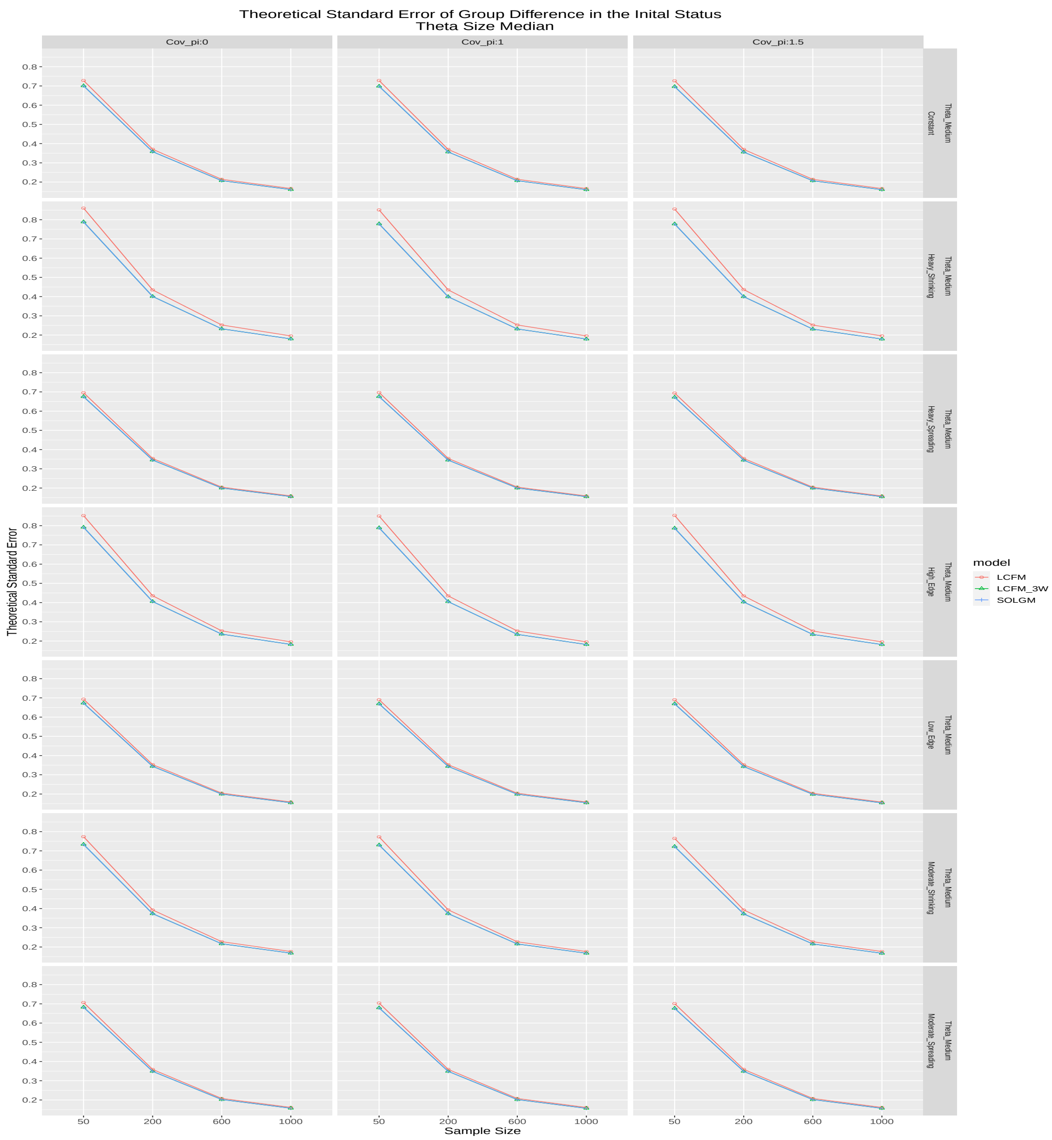

Figure B.29. Theoretical Standard Error of the Average Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

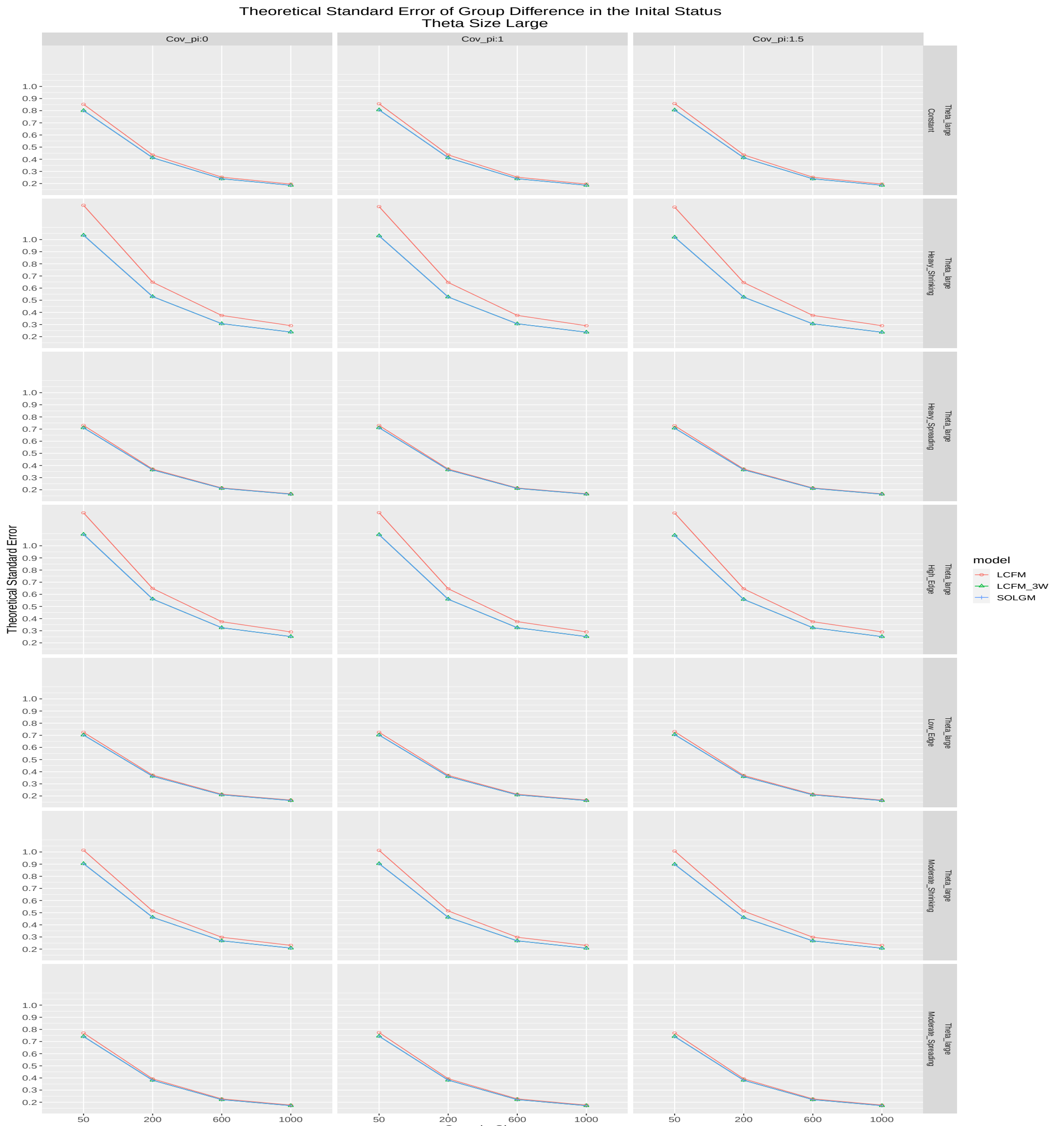

Figure B.30. Theoretical Standard Error of the Average Group Difference in Initial Status, $\boldsymbol{\beta_{01} = 3}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

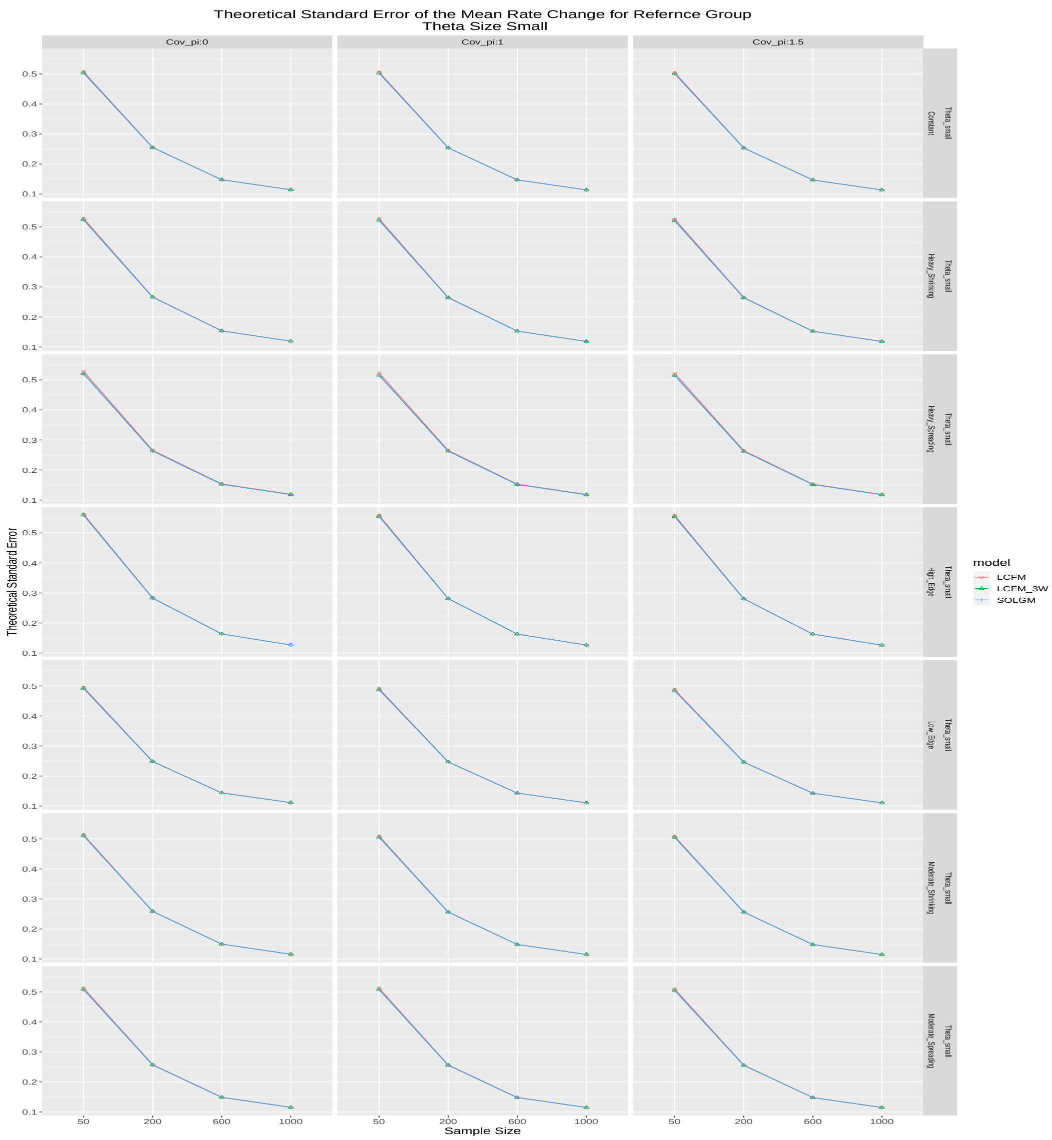

Figure B.31. Theoretical Standard Error of the Average Growth Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

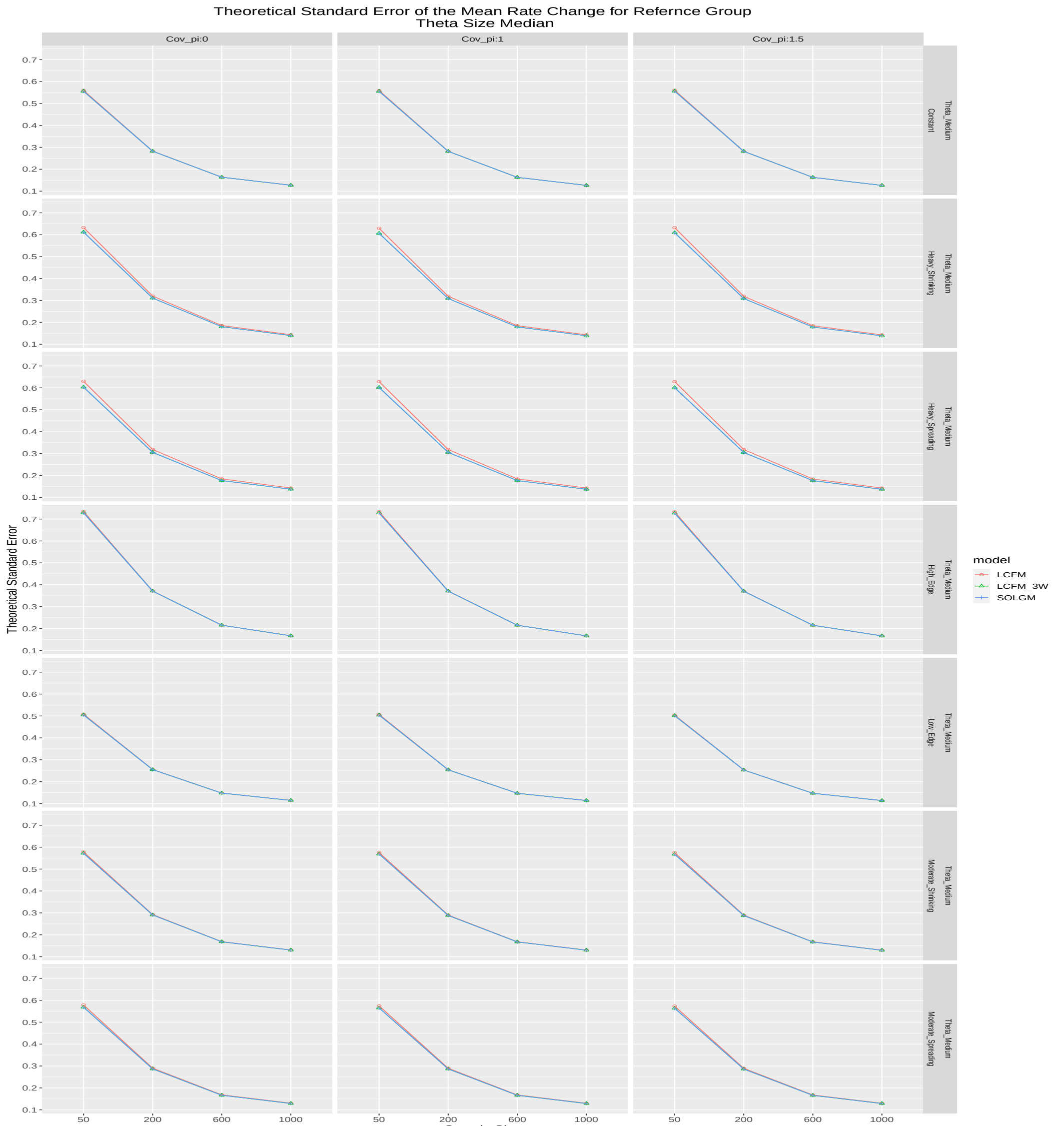

Figure B.32. Theoretical Standard Error of the Average Growth Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

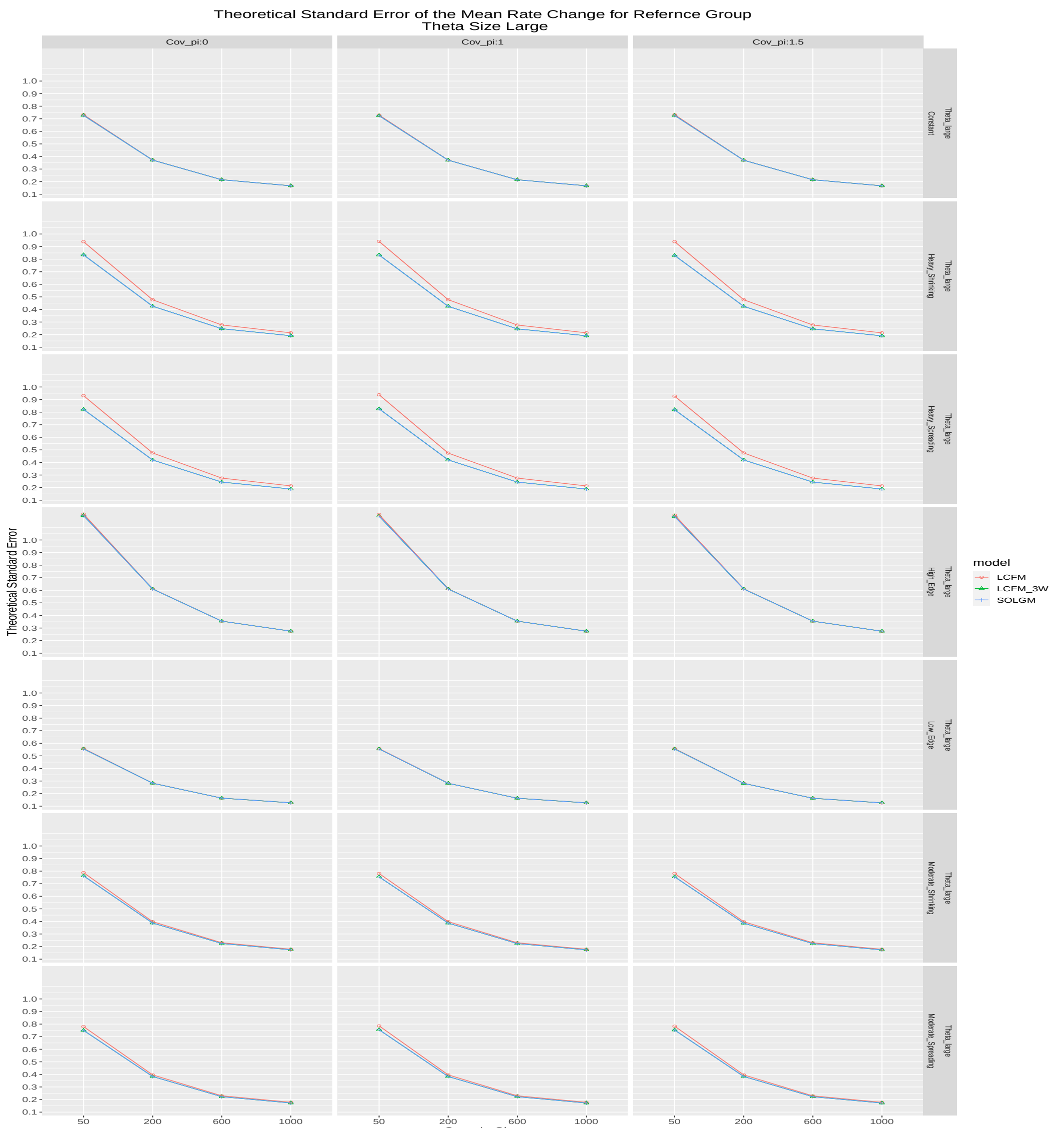

Figure B.33. Theoretical Standard Error of the Average Growth Rate for the Reference Group, $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

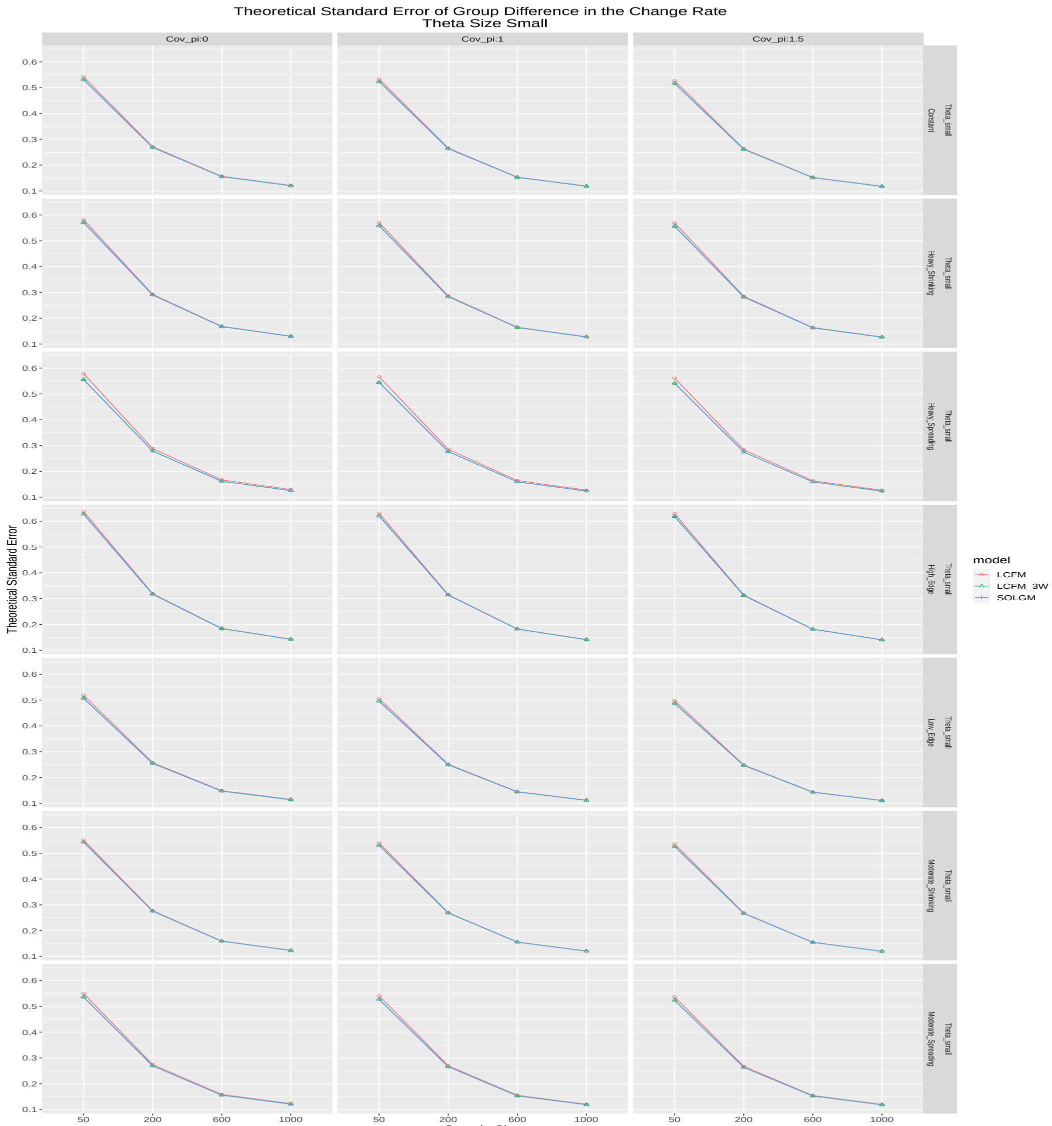

Figure B.34. Theoretical Standard Error of Average Group Difference for the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

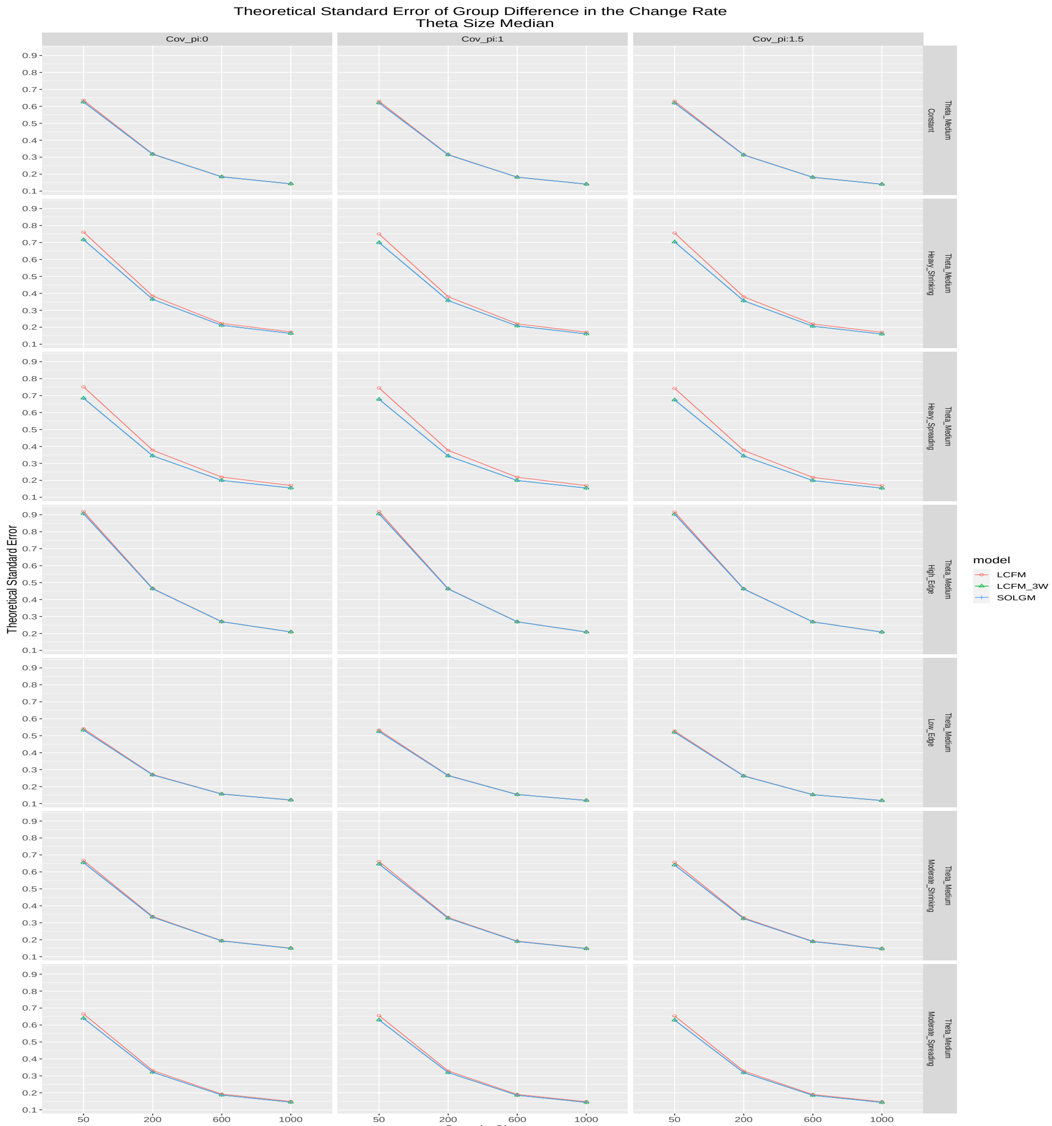

Figure B.35. Theoretical Standard Error of Average Group Difference for the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

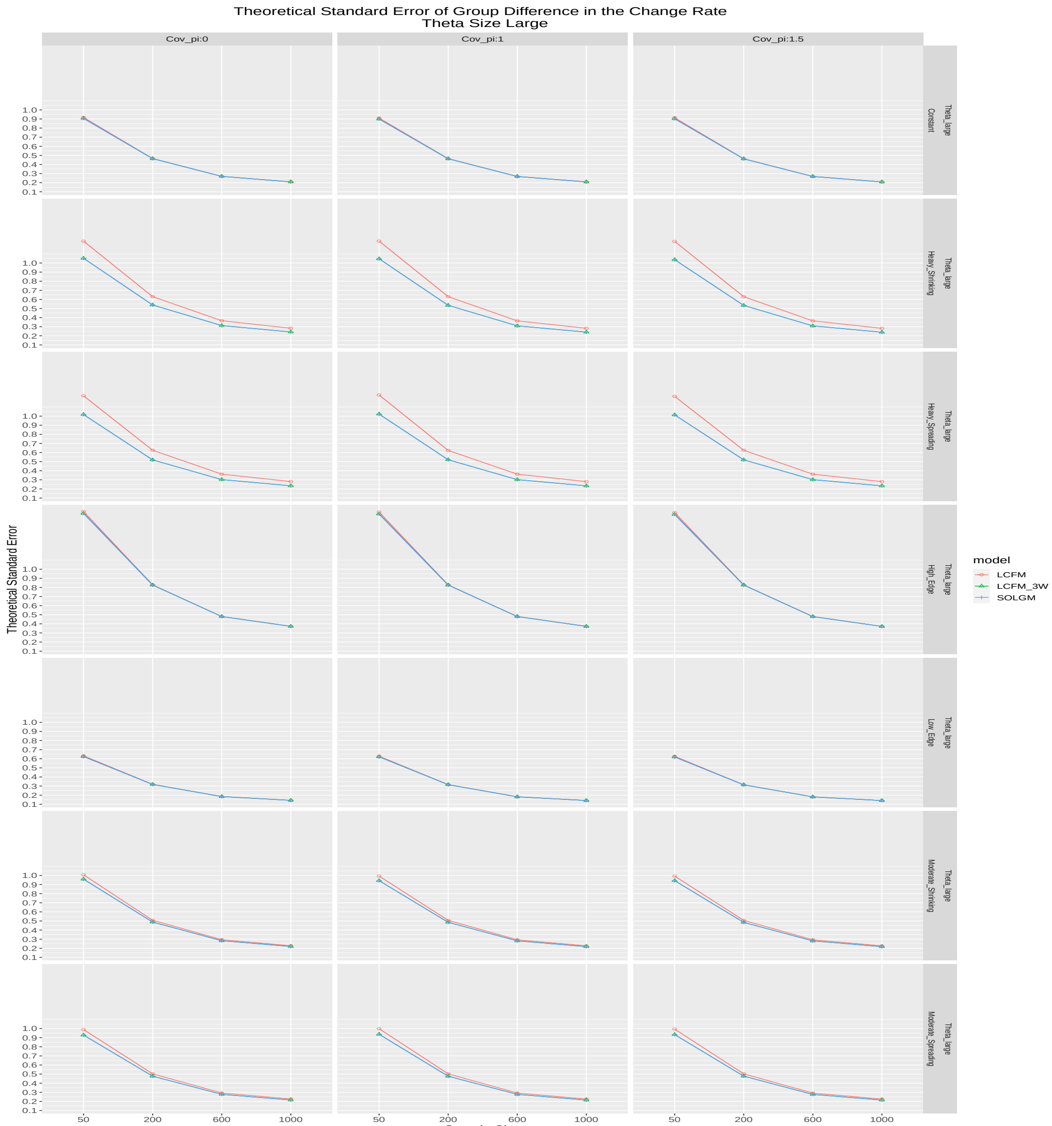

Figure B.36. Theoretical Standard Error of Average Group Difference for the Growth Rate, $\boldsymbol{\beta_{11} = 2}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

B.0.7 List of Figures in Type I Error/Statistical Power

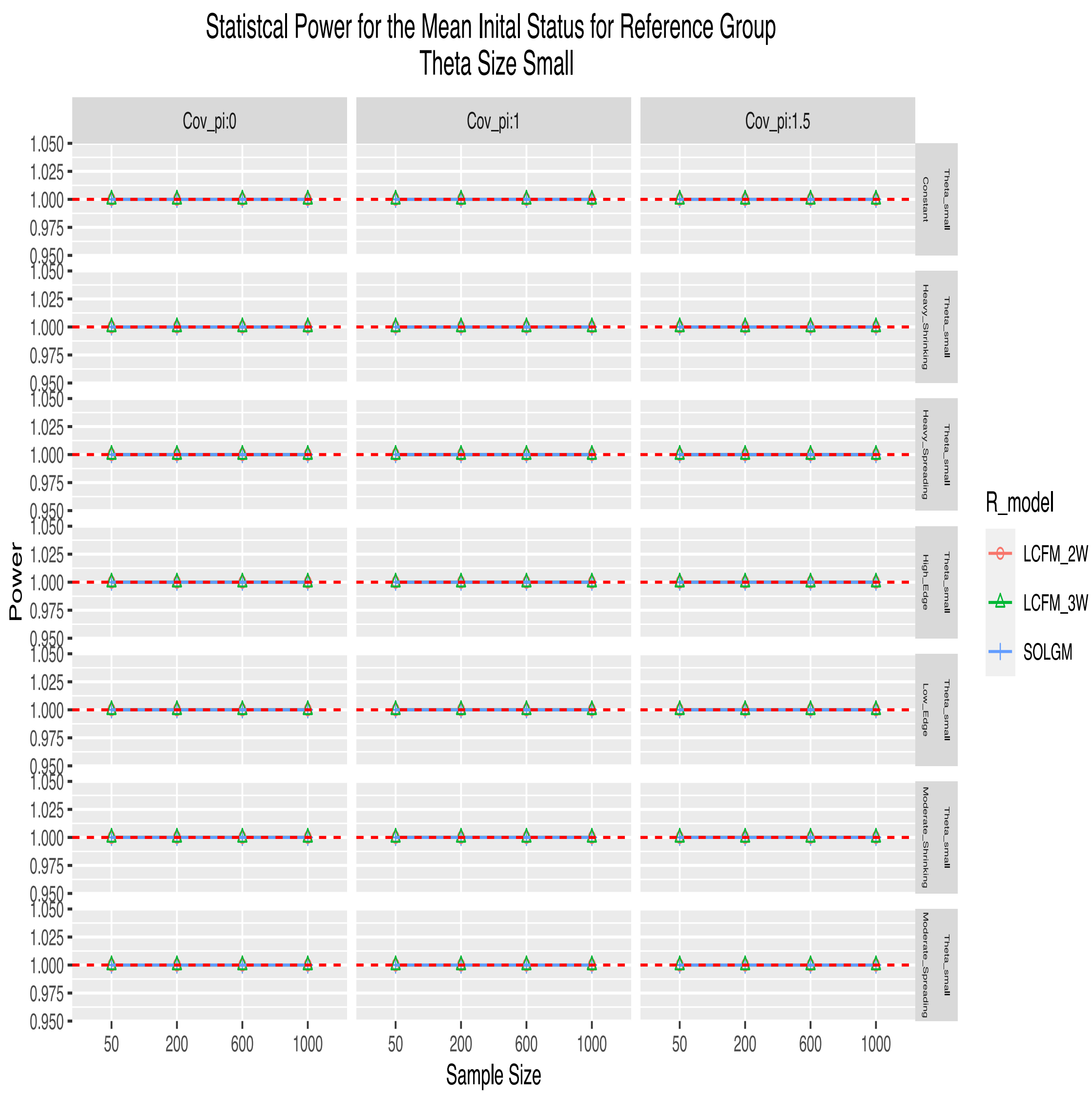

Figure B.37. Statistical Power of Mean Initial Status for the Reference Group $\boldsymbol{\beta_{00} = 2}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

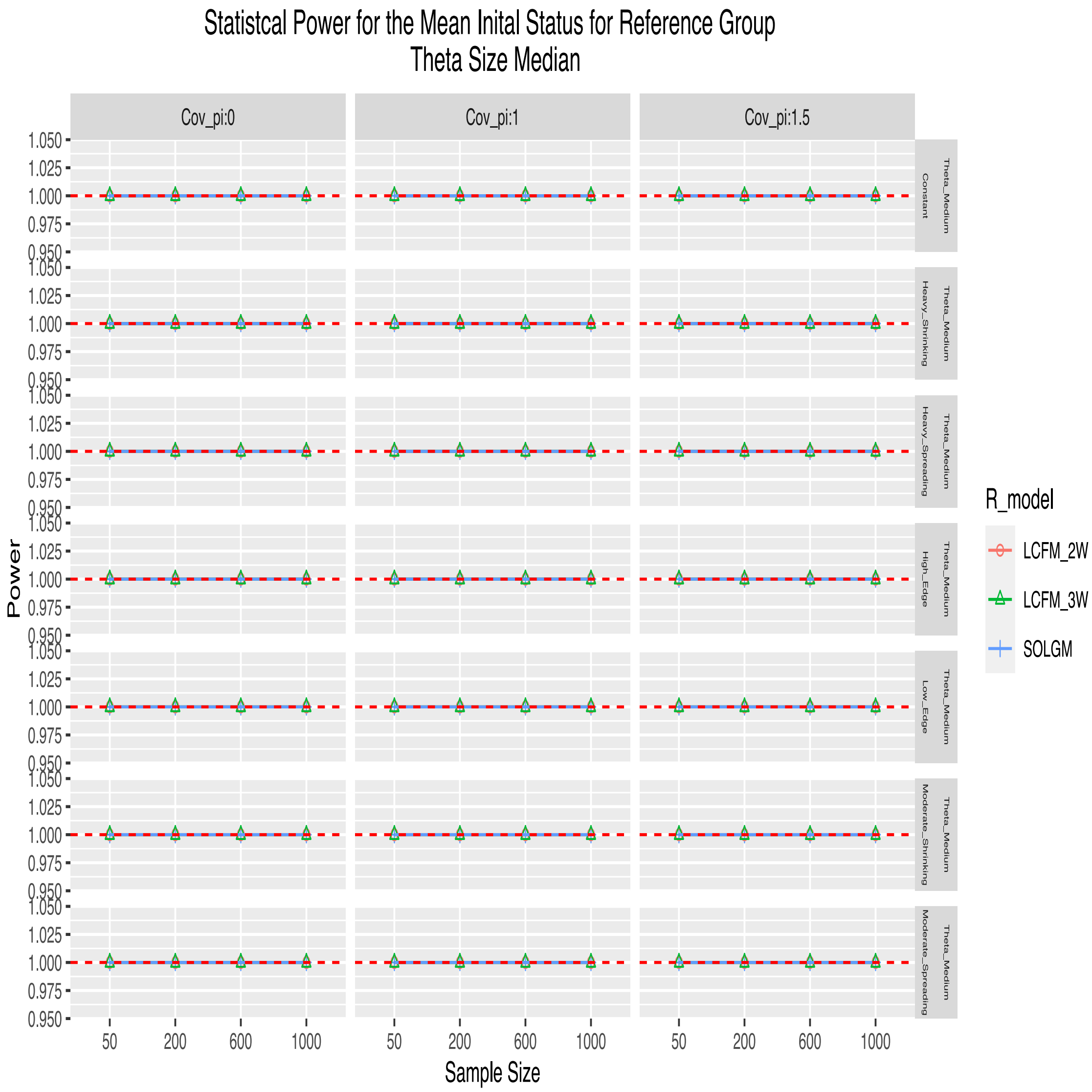

Figure B.38. Statistical Power of Mean Initial Status for the Reference Group $\boldsymbol{\beta_{00} = 2}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

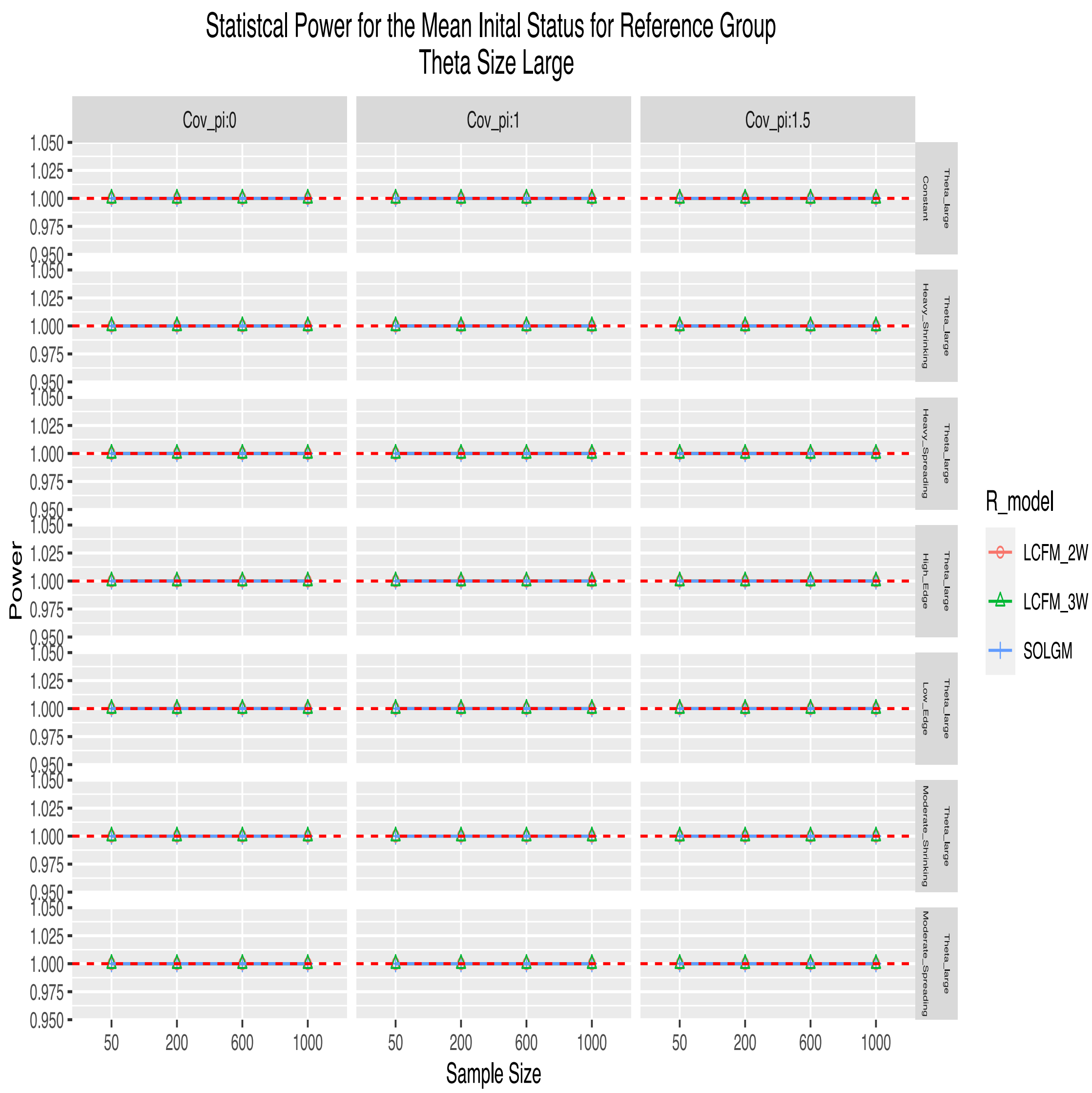

Figure B.39. Statistical Power of Mean Initial Status for the Reference Group $\boldsymbol{\beta_{00} = 2}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 3}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

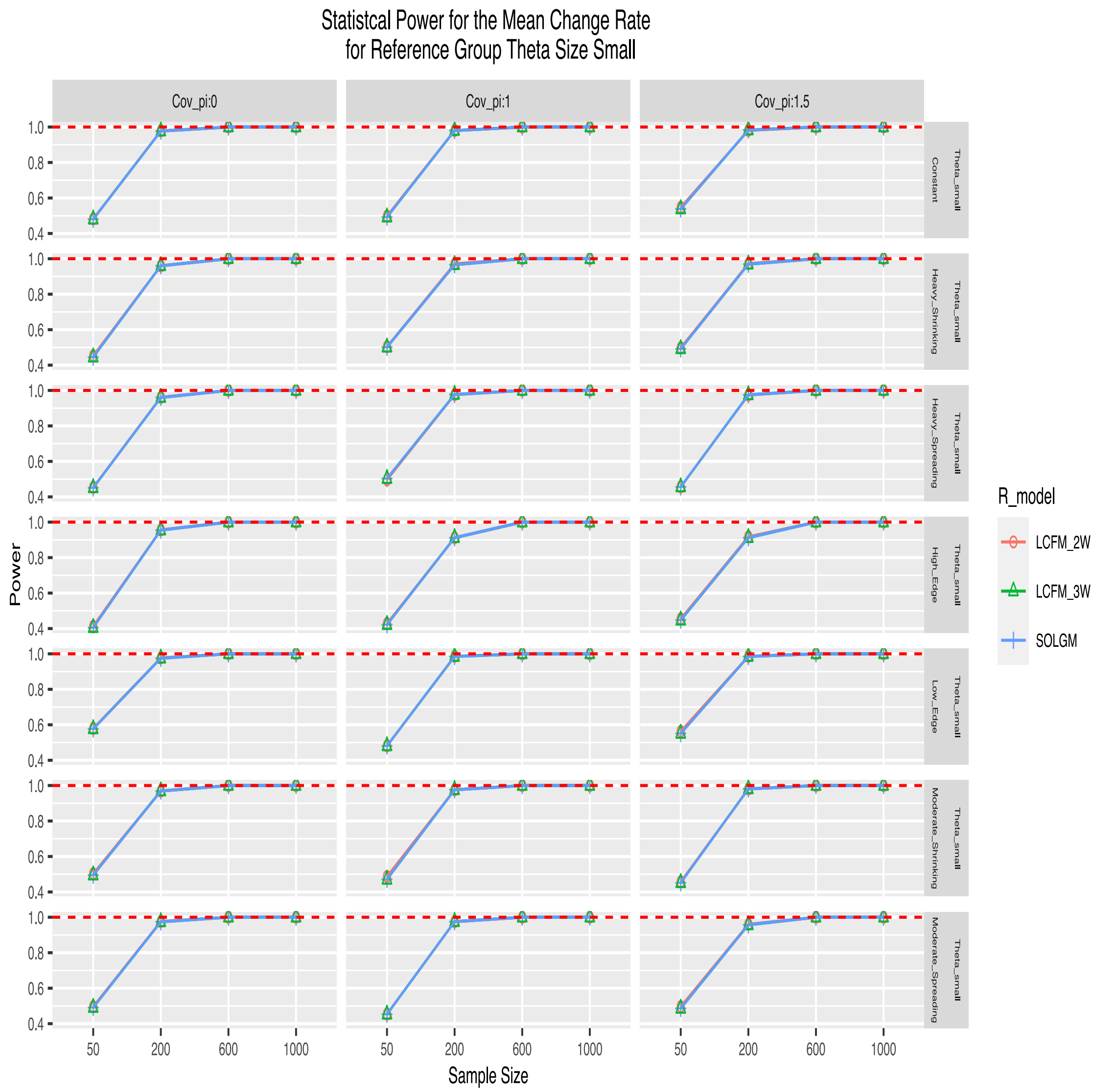

Figure B.40. Statistical Power of the Average Growth Rate in Reference Group $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

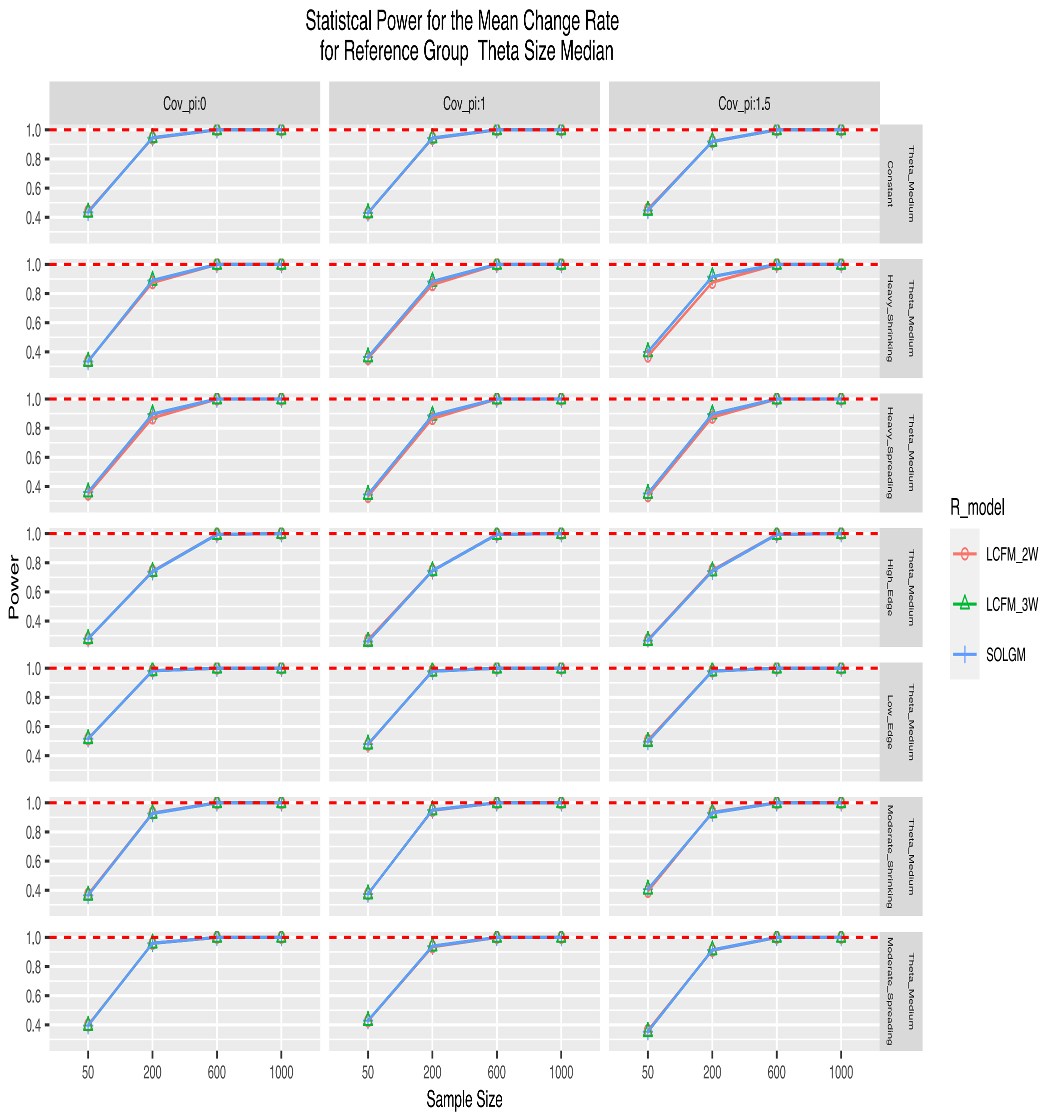

Figure B.41. Statistical Power of the Average Growth Rate in Reference Group $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

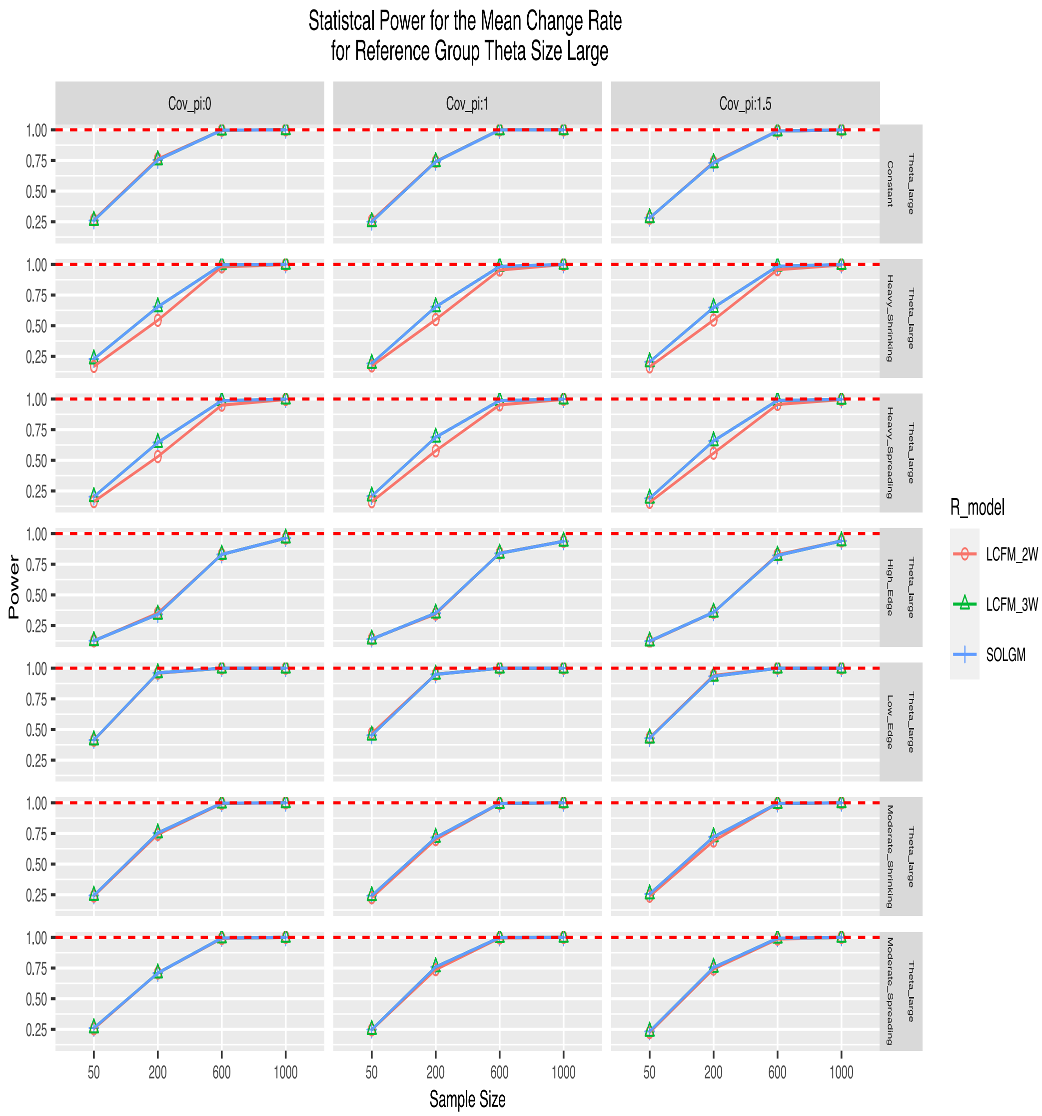

Figure B.42. Statistical Power of the Average Growth Rate in Reference Group $\boldsymbol{\beta_{10} = 1}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 0}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

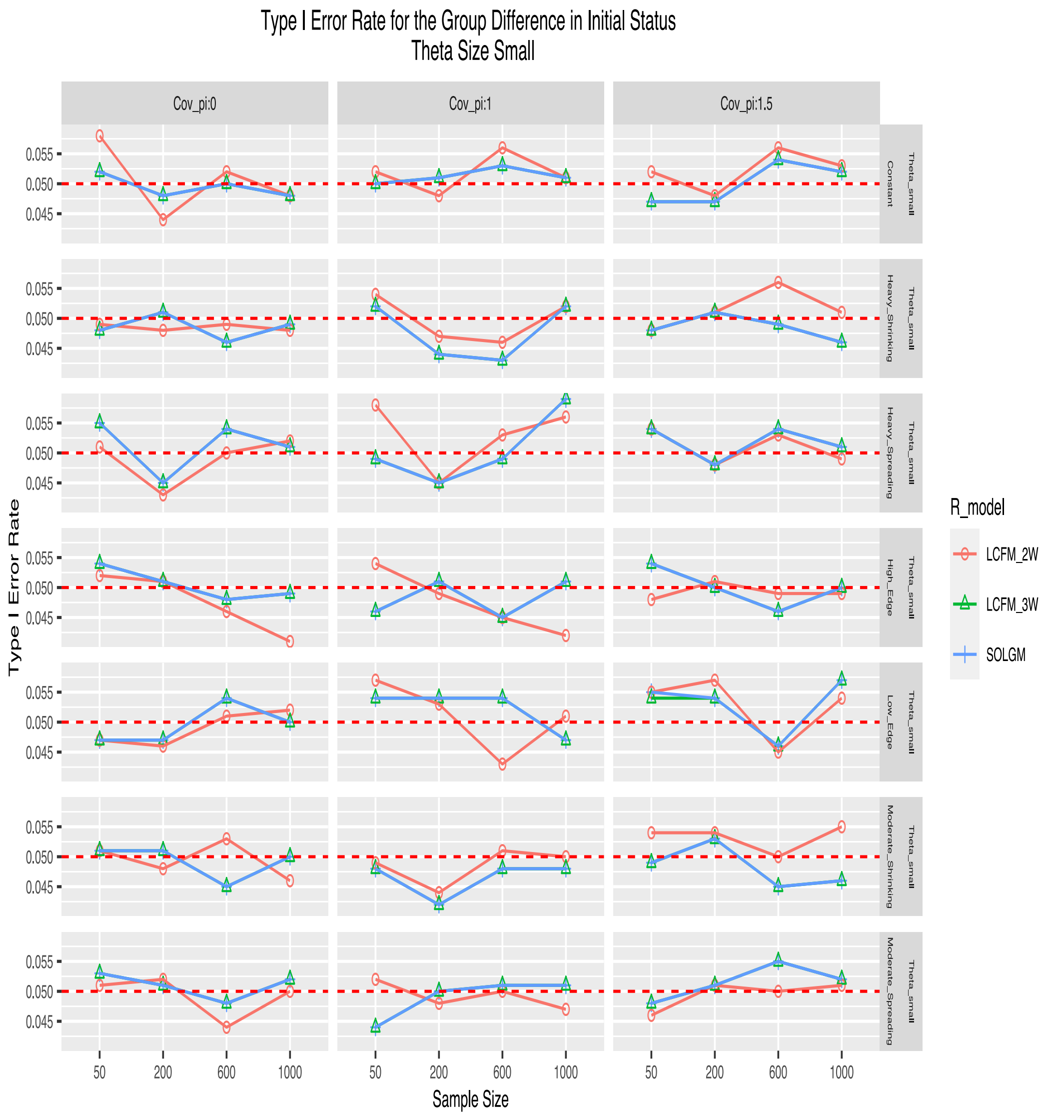

Figure B.43. Type I Error Rate of the Average Growth Difference in Initial Status$\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**small**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

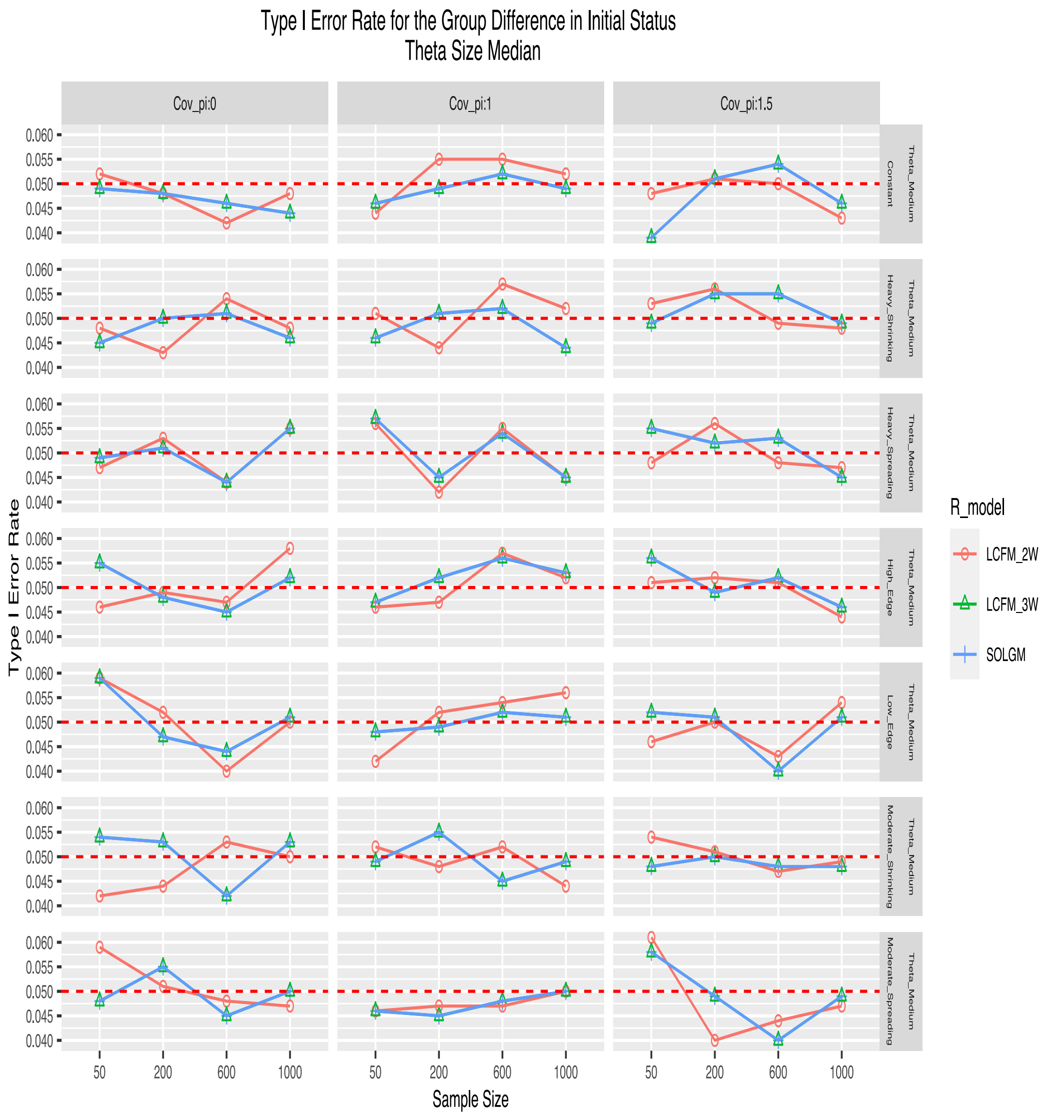

Figure B.44. Type I Error Rate of the Average Growth Difference in Initial Status$\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**median**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.

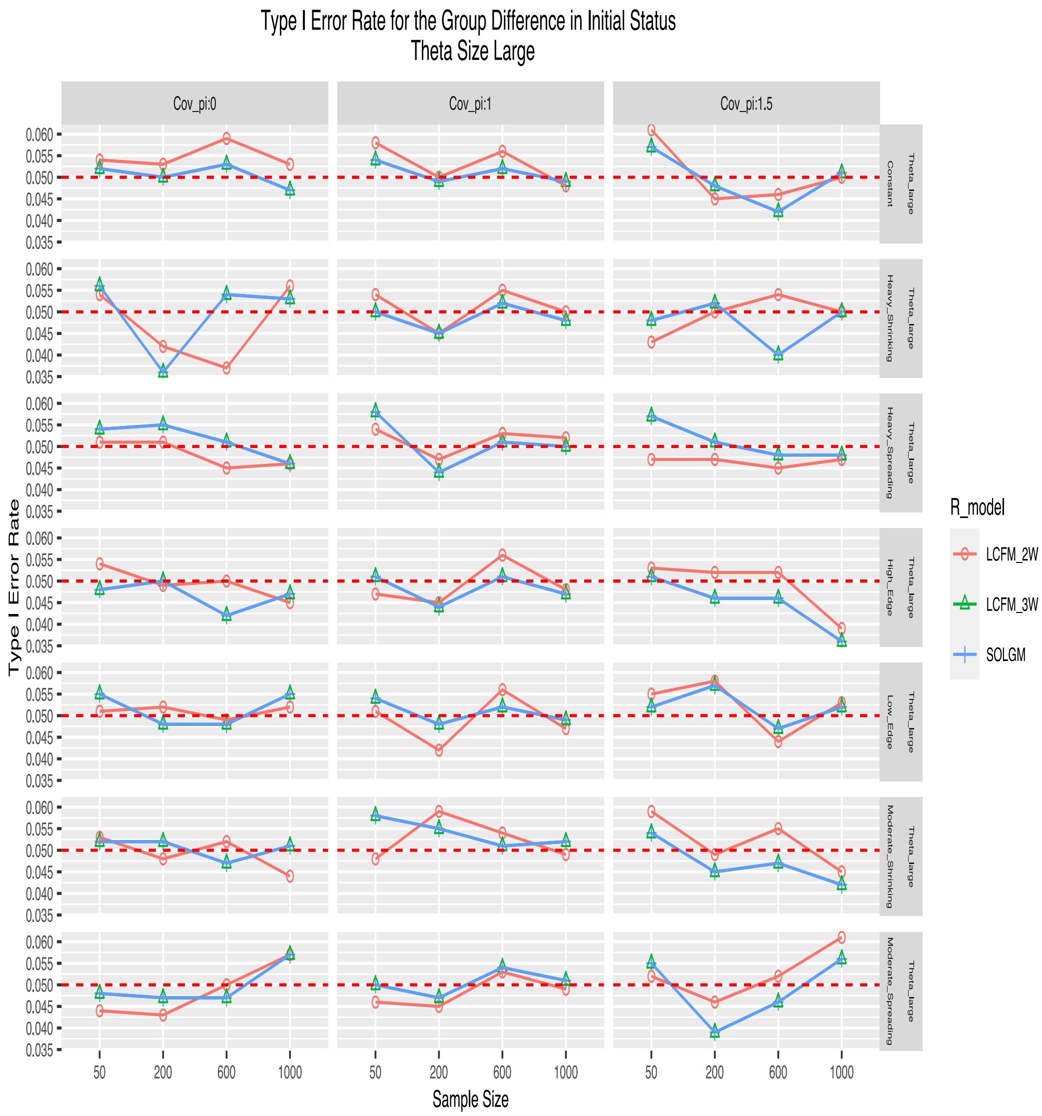

Figure B.45. Type I Error Rate of the Average Growth Difference in Initial Status$\boldsymbol{\beta_{01} = 0}$, by time-specific error size (**large**) and correlation between intercept and slope (Cor_{pi}), when $\boldsymbol{\beta_{01} = 0}$ and $\boldsymbol{\beta_{11} = 2}$, SOLGM = second-order latent growth model, LCFM_2W = 2-wave longitudinal common factor model, LCFM_3W = 3-wave longitudinal common factor model.