

Disaggregating Within-Person and Between-Person Effects in the Presence of Linear Time
Trends in Time-Varying Predictors: Structural Equation Modeling Approach

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Abstract

Educational researchers are often interested in phenomena that unfold over time within a person and at the same time, relationships between their characteristics that are stable over time. Since variables in a longitudinal study reflect both within- and between-person effects, researchers need to disaggregate them to understand the phenomenon of interest correctly. Although the person-mean centering technique has been believed as the gold standard of the disaggregation method, recent studies found that the centering did not work when there was a trend in the predictor. Hence, they proposed some detrending techniques to remove the systematic change; however, they were only applicable to multilevel models. Therefore, this dissertation develops novel detrending methods based on structural equation modeling (SEM). It also establishes the links between centering and detrending by reviewing a broad range of literature. The proposed SEM-based detrending methods are compared to the existing centering and detrending methods through a series of Monte Carlo simulations. The results indicate that (a) model misspecification for the time-varying predictors or outcomes leads to large bias of and standard error, (b) statistical properties of estimates of the within- and between-person effects are mostly determined by the type of between-person predictors (i.e., observed or latent), and (c) for unbiased estimation of the effects, models with latent between-person predictors require nonzero growth factor variances, while those with observed predictors at the between level need either nonzero or zero variance, depending on the parameter. As concluding remarks, some practical recommendations are provided based on the findings of the present study.

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General Audience Abstract

Educational researchers are often interested in longitudinal phenomena within a person and relations between the person's characteristics. Since repeatedly measured variables reflect their within- and between-person aspects, researchers need to disaggregate them statistically to understand the phenomenon of interest. Recent studies found that the traditional centering method, where the individual's average of a predictor was subtracted from the original predictor value, could not correctly disentangle the within- and between-person effects when the predictor showed a systematic change over time (i.e., trend). They proposed some techniques to remove the trend; however, the detrending methods were only applicable to multilevel models. Therefore, the present study develops novel detrending methods using structural equation modeling. The proposed models are compared to the existing methods through a series of Monte Carlo simulations, where we can manipulate a data-generating model and its parameter values. The results indicate that (a) model misspecification for the time-varying predictor or outcome leads to systematic deviation of the estimates from their true values, (b) statistical properties of estimates of the effects are mostly determined by the type of between-person predictors (i.e., observed or latent), and (c) the latent predictor models require nonzero growth factor variances for unbiased estimation, while the observed predictor models need either nonzero or zero variance, depending on the parameter. As concluding remarks, some recommendations for the practitioners are provided.

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Contents

List of Figures	xii
List of Tables	xxxvi
List of Abbreviations	xxxviii
1 Introduction	1
1.1 Background	1
1.1.1 Within-Person Process and Between-Person Relation	1
1.1.2 Time-Varying Predictor and Centering	3
1.1.3 Detrending	5
1.2 Purpose of the Study	7
2 Literature Review	9
2.1 Regression Models with Time-Varying Predictors	10
2.1.1 Fixed Effects Model	10
2.1.2 Random Effects Model	13
2.1.3 Correlated Random Effects Model	15
2.1.4 Within-Between Model	17
2.1.5 Adaptive Centering	20
2.1.6 Latent Variable Centering	22
2.1.7 Centering and Interpretation of the Coefficients	26
2.2 Latent Curve Models with Time-Varying Predictors	28
2.2.1 Latent Curve Model with Cross-Level Covariances	28

2.2.2	Latent Curve Model with a Random Intercept	33
2.3	Detrending Methods	36
2.3.1	Two-Step Detrending	36
2.3.2	Time-as-Covariate Detrending	40
2.3.3	Multivariate Multilevel Model	42
2.3.4	Relationship between Detrending Methods	46
2.3.5	Relationship between Detrending and Centering	50
2.4	Comparison of Structural Equation Modeling and Multilevel Modeling . . .	51
2.5	Summary of the Literature Review and Its Implications for the Present Research	56
3	Proposed Methods	60
3.1	Latent Curve Model with Residualized Predictors and Outcomes	60
3.2	Latent Curve Model with Residualized Predictors	63
3.3	Latent Curve Model for Time-Varying Predictors with a Random Intercept for Outcomes	64
3.4	Latent Curve Model for Time-Varying Predictors with Cross-Level Covariances	66
3.5	Summary and Discussion on the Proposed Models	70
3.6	Motivation and Research Questions for the Simulation Studies	72
4	Simulation Designs	76
4.1	Data-Generating Models	76
4.2	Experimental Factors	78
4.3	Settings for Other Parameters	80
4.4	Analytic Models	86
4.5	Evaluation Indices	87
4.6	Analysis	90
4.7	Implementation	91

5	Results	92
5.1	Simulation 1	92
5.1.1	Auxiliary Indices	93
5.1.2	Within-Person Effect	95
5.1.3	Between-Person Effect of Intercept on Intercept	97
5.1.4	Between-Person Effect of Intercept on Slope	100
5.1.5	Between-Person Effect of Slope on Intercept	102
5.1.6	Between-Person Effect of Slope on Slope	104
5.2	Simulation 2	107
5.2.1	Auxiliary Indices	107
5.2.2	Within-Person Effect	109
5.2.3	Between-Person Effect of Intercept on Intercept	112
5.2.4	Between-Person Effect of Intercept on Slope	115
5.2.5	Between-Person Effect of Slope on Intercept	119
5.2.6	Between-Person Effect of Slope on Slope	121
5.3	Simulation 3	123
5.3.1	Auxiliary Indices	124
5.3.2	Within-Person Effect	126
5.3.3	Between-Person Effect of Intercept on Intercept	128
5.3.4	Between-Person Effect of Intercept on Slope	132
5.3.5	Between-Person Effect of Slope on Intercept	135
5.3.6	Between-Person Effect of Slope on Slope	138
5.4	Simulation 4	140
5.4.1	Auxiliary Indices	141
5.4.2	Within-Person Effect	143
5.4.3	Between-Person Effect of Intercept on Intercept	147

5.4.4	Between-Person Effect of Intercept on Slope	152
5.4.5	Between-Person Effect of Slope on Intercept	156
5.4.6	Between-Person Effect of Slope on Slope	159
5.5	Simulation 5	163
5.5.1	Auxiliary Indices	164
5.5.2	Within-Person Effect	166
5.5.3	Between-Person Effect of Intercept on Intercept	169
5.5.4	Between-Person Effect of Intercept on Slope	173
5.5.5	Between-Person Effect of Slope on Intercept	178
5.5.6	Between-Person Effect of Slope on Slope	182
5.6	Summary of the Simulation Results	185
5.6.1	Bias and Empirical SE	185
5.6.2	SE Ratio, Coverage Rate, and Power	190
6	Discussion	194
6.1	Summary of the Main Findings	194
6.2	Answers to the Research Questions	198
6.3	Other Findings	201
6.3.1	Standard Error of the Models with Latent Between-Person Predictors	201
6.3.2	Properties of LCMMy-CLC	203
6.4	Limitations and Future Directions	208
6.5	Recommendations for Practitioners	211
	References	215
	Appendices	229

Appendix A	Figures in Simulation 1	230
A.1	Auxiliary Indices	230
A.2	Within-Person Effect	236
A.3	Between-Person Effect of Intercept on Intercept	243
A.4	Between-Person Effect of Intercept on Slope	251
A.5	Between-Person Effect of Slope on Intercept	258
A.6	Between-Person Effect of Slope on Slope	268
Appendix B	Figures in Simulation 2	279
B.1	Auxiliary Indices	279
B.2	Within-Person Effect	285
B.3	Between-Person Effect of Intercept on Intercept	296
B.4	Between-Person Effect of Intercept on Slope	303
B.5	Between-Person Effect of Slope on Intercept	313
B.6	Between-Person Effect of Slope on Slope	319
Appendix C	Figures in Simulation 3	328
C.1	Auxiliary Indices	328
C.2	Within-Person Effect	334
C.3	Between-Person Effect of Intercept on Intercept	341
C.4	Between-Person Effect of Intercept on Slope	349
C.5	Between-Person Effect of Slope on Intercept	355
C.6	Between-Person Effect of Slope on Slope	362
Appendix D	Figures in Simulation 4	369
D.1	Auxiliary Indices	369
D.2	Within-Person Effect	392

D.3	Between-Person Effect of Intercept on Intercept	421
D.4	Between-Person Effect of Intercept on Slope	453
D.5	Between-Person Effect of Slope on Intercept	480
D.6	Between-Person Effect of Slope on Slope	498
Appendix E Figures in Simulation 5		514
E.1	Auxiliary Indices	514
E.2	Within-Person Effect	521
E.3	Between-Person Effect of Intercept on Intercept	537
E.4	Between-Person Effect of Intercept on Slope	567
E.5	Between-Person Effect of Slope on Intercept	590
E.6	Between-Person Effect of Slope on Slope	607
Appendix F R Script for the Simulations		619
F.1	simulation.R	619
F.2	checkers.R	636

List of Figures

1.1	An Example of a Within-Person Effect and a Between-Person Effect That Differ in Their Sign (Within-Person Effect = Negative, Between-Person Effect = Positive).	4
2.1	Latent Curve Model for variable Y (LCMy for short)	29
2.2	Latent Curve Model for Y with Cross-Level Covariances (LCMy-CLC for short)	31
2.3	Fixed Effects Model as an SEM Model (Allison, 2005, 2009)	34
2.4	Latent Curve Model for Y with Random Intercept for X (LCMy-RIx)	35
2.5	Centering and Detrending under the Linear Trend over Time	38
3.1	Latent Curve Model with Residualized Time-Varying Predictors and Outcomes (LCMxy-RPO)	62
3.2	Latent Curve Model with Residualized Time-Varying Predictors (LCMxy-RP)	63
3.3	Latent Curve Model for X with Random Intercept for Y (LCMx-RIy)	65
3.4	Latent Curve Model for X with Cross-Level Covariances (LCMx-CLC)	67
4.1	Data-Generating Models (DGMs; Population Models)	77
4.2	Expected Growth Curve for the Time-Varying Predictors in Simulation 4	83
4.3	Expected Growth Curve for the Outcomes in Simulation 4	84
5.1	Data-Generating Model in Simulation 1	93
5.2	Data-Generating Model in Simulation 2	107
5.3	Data-Generating Model in Simulation 3	124
5.4	Data-Generating Model in Simulation 4	141
5.5	Data-Generating Model in Simulation 5	164

6.1	Sampling Distribution of $\hat{\tau}_S^Y$ of LCM _y -CLC in Simulation 3	204
A.1	Proportion of Converged Cases in Simulation 1	230
A.2	Proportion of Cases with Problems in Estimating SE in Simulation 1	231
A.3	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 1	232
A.4	CFI in Simulation 1	233
A.5	RMSEA in Simulation 1	234
A.6	SRMR in Simulation 1	235
A.7	Bias for the Within-Person Effect γ in Simulation 1	236
A.8	Empirical SE of the Within-Person Effect γ in Simulation 1	237
A.9	Theoretical SE for the Within-Person Effect γ in Simulation 1	238
A.10	SE Ratio for the Within-Person Effect γ in Simulation 1	239
A.11	RMSE for the Within-Person Effect γ in Simulation 1	240
A.12	Coverage Rate for the Within-Person Effect γ in Simulation 1	241
A.13	Power for Detecting the Within-Person Effect γ in Simulation 1	242
A.14	Bias for the Between-Person Effect β_{II} in Simulation 1	243
A.15	Empirical SE for the Between-Person Effect β_{II} in Simulation 1	244
A.16	Empirical SE for the Between-Person Effect β_{II} in Simulation 1 except LCM _x -CLC	245
A.17	Theoretical SE for the Between-Person Effect β_{II} in Simulation 1 except LCM _x -CLC	246
A.18	SE Ratio for the Between-Person Effect β_{II} in Simulation 1 except LCM _x -CLC	247
A.19	RMSE for the Between-Person Effect β_{II} in Simulation 1 except LCM _x -CLC	248
A.20	Coverage Rate for the Between-Person Effect β_{II} in Simulation 1	249
A.21	Power of the Between-Person Effect β_{II} in Simulation 1	250

A.22	Bias for the Between-Person Effect β_{SI} in Simulation 1	251
A.23	Bias for the Between-Person Effect β_{SI} in Simulation 1 except LCMx-CLC . .	252
A.24	Empirical SE for the Between-Person Effect β_{SI} in Simulation 1 except LCMx- CLC	253
A.25	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 1 except LCMx- CLC	254
A.26	SE Ratio for the Between-Person Effect β_{SI} in Simulation 1	255
A.27	RMSE for the Between-Person Effect β_{SI} in Simulation 1 except LCMx-CLC .	256
A.28	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 1	257
A.29	Bias for the Between-Person Effect β_{IS} in Simulation 1	258
A.30	Empirical SE for the Between-Person Effect β_{IS} in Simulation 1	259
A.31	Empirical SE for the Between-Person Effect β_{IS} in Simulation 1 except LCMx- CLC	260
A.32	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 1	261
A.33	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 1 except LCMx- CLC	262
A.34	SE Ratio for the Between-Person Effect β_{IS} in Simulation 1	263
A.35	SE Ratio for the Between-Person Effect β_{IS} in Simulation 1 except LCMx-CLC	264
A.36	RMSE for the Between-Person Effect β_{IS} in Simulation 1	265
A.37	RMSE Ratio for the Between-Person Effect β_{IS} in Simulation 1 except LCMx- CLC	266
A.38	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 1	267
A.39	Bias for the Between-Person Effect β_{SS} in Simulation 1	268
A.40	Empirical SE for the Between-Person Effect β_{SS} in Simulation 1	269
A.41	Empirical SE for the Between-Person Effect β_{SS} in Simulation 1 except LCMx- CLC	270

A.42	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 1	271
A.43	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 1 except LCMx- CLC	272
A.44	SE Ratio for the Between-Person Effect β_{SS} in Simulation 1	273
A.45	SE Ratio for the Between-Person Effect β_{SS} in Simulation 1 except LCMx-CLC	274
A.46	SE Ratio for the Between-Person Effect β_{SS} in Simulation 1	275
A.47	RMSE for the Between-Person Effect β_{SS} in Simulation 1 except LCMx-CLC .	276
A.48	Coverage Rate for the Between-Person Effect β_{SS} in Simulation 1	277
A.49	Scatter Plot of $\hat{\gamma}$ and $\hat{\beta}_{SS}$ in Simulation 1 (10 samples per condition)	278
B.1	Proportion of Converged Cases in Simulation 2	279
B.2	Proportion of Cases with Problems in Estimating SE in Simulation 2	280
B.3	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simula- tion 2	281
B.4	CFI in Simulation 2	282
B.5	RMSEA in Simulation 2	283
B.6	SRMR in Simulation 2	284
B.7	Bias for the Within-Person Effect γ in Simulation 2	285
B.8	Bias for the Within-Person Effect γ in Simulation 2 except LCMx-RIy	286
B.9	Empirical SE for the Within-Person Effect γ in Simulation 2	287
B.10	Empirical SE for the Within-Person Effect γ in Simulation 2 except LCMx-RIy	288
B.11	Theoretical SE for the Within-Person Effect γ in Simulation 2	289
B.12	Theoretical SE for the Within-Person Effect γ in Simulation 2 except LCMx-RIy	290
B.13	SE Ratio for the Within-Person Effect γ in Simulation 2	291
B.14	RMSE for the Within-Person Effect γ in Simulation 2	292
B.15	RMSE for the Within-Person Effect γ in Simulation 2 except LCMx-RIy	293

B.16	Coverage Rate for the Within-Person Effect γ in Simulation 2	294
B.17	Power for the Within-Person Effect γ in Simulation 2	295
B.18	Bias for the Between-Person Effect β_{II} in Simulation 2	296
B.19	Empirical SE for the Between-Person Effect β_{II} in Simulation 2	297
B.20	Theoretical SE for the Between-Person Effect β_{II} in Simulation 2	298
B.21	SE Ratio for the Between-Person Effect β_{II} in Simulation 2	299
B.22	RMSE for the Between-Person Effect β_{II} in Simulation 2	300
B.23	Coverage Rate for the Between-Person Effect β_{II} in Simulation 2	301
B.24	Power for the Between-Person Effect β_{II} in Simulation 2 (by Model)	302
B.25	Bias for the Between-Person Effect β_{SI} in Simulation 2	303
B.26	Empirical SE for the Between-Person Effect β_{SI} in Simulation 2	304
B.27	Empirical SE for the Between-Person Effect β_{SI} in Simulation 2 except LCMx- CLC	305
B.28	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 2	306
B.29	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 2 except LCMx- CLC	307
B.30	SE Ratio for the Between-Person Effect β_{SI} in Simulation 2	308
B.31	RMSE for the Between-Person Effect β_{SI} in Simulation 2	309
B.32	RMSE for the Between-Person Effect β_{SI} in Simulation 2 except LCMx-CLC .	310
B.33	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 2	311
B.34	Power for the Between-Person Effect β_{SI} in Simulation 2	312
B.35	Bias for the Between-Person Effect β_{IS} in Simulation 2	313
B.36	Empirical SE for the Between-Person Effect β_{IS} in Simulation 2	314
B.37	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 2	315
B.38	SE Ratio for the Between-Person Effect β_{IS} in Simulation 2	316
B.39	RMSE for the Between-Person Effect β_{IS} in Simulation 2	317

B.40	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 2	318
B.41	Bias for the Between-Person Effect β_{SS} in Simulation 2	319
B.42	Scatter Plot of $\hat{\gamma}$ and $\hat{\beta}_{SS}$ in Simulation 2 (10 samples per condition)	320
B.43	Empirical SE for the Between-Person Effect β_{SS} in Simulation 2	321
B.44	Empirical SE for the Between-Person Effect β_{SS} in Simulation 2 except LCMx- CLC	322
B.45	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 2	323
B.46	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 2 except LCMx- CLC	324
B.47	SE Ratio for the Between-Person Effect β_{SS} in Simulation 2	325
B.48	RMSE for the Between-Person Effect β_{SS} in Simulation 2	326
B.49	Coverage Rate for the Between-Person Effect β_{SS} in Simulation 2	327
C.1	Proportion of Converged Cases in Simulation 3	328
C.2	Proportion of Cases with Problems in Estimating SE in Simulation 3	329
C.3	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simula- tion 3	330
C.4	CFI in Simulation 3	331
C.5	RMSEA in Simulation 3	332
C.6	SRMR in Simulation 3	333
C.7	Bias for the Within-Person Effect γ in Simulation 3	334
C.8	Empirical SE for the Within-Person Effect γ in Simulation 3	335
C.9	Theoretical SE for the Within-Person Effect γ in Simulation 3	336
C.10	SE Ratio for the Within-Person Effect γ in Simulation 3	337
C.11	RMSE for the Within-Person Effect γ in Simulation 3	338
C.12	Coverage Rate for the Within-Person Effect γ in Simulation 3	339

C.13	Power for the Within-Person Effect γ in Simulation 3	340
C.14	Bias for the Between-Person Effect β_{II} in Simulation 3	341
C.15	Empirical SE for the Between-Person Effect β_{II} in Simulation 3	342
C.16	Theoretical SE for the Between-Person Effect β_{II} in Simulation 3	343
C.17	SE Ratio for the Between-Person Effect β_{II} in Simulation 3	344
C.18	RMSE for the Between-Person Effect β_{II} in Simulation 3	345
C.19	RMSE for the Between-Person Effect β_{II} in Simulation 3 except LCM _y -RI _x and LCM _x -CLC	346
C.20	Coverage Rate for the Between-Person Effect β_{II} in Simulation 3	347
C.21	Power for the Between-Person Effect β_{II} in Simulation 3	348
C.22	Bias for the Between-Person Effect β_{SI} in Simulation 3	349
C.23	Empirical SE for the Between-Person Effect β_{SI} in Simulation 3	350
C.24	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 3	351
C.25	SE Ratio for the Between-Person Effect β_{SI} in Simulation 3	352
C.26	RMSE for the Between-Person Effect β_{SI} in Simulation 3	353
C.27	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 3	354
C.28	Bias for the Between-Person Effect β_{IS} in Simulation 3	355
C.29	Empirical SE for the Between-Person Effect β_{IS} in Simulation 3	356
C.30	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 3	357
C.31	SE Ratio for the Between-Person Effect β_{IS} in Simulation 3	358
C.32	RMSE for the Between-Person Effect β_{IS} in Simulation 3	359
C.33	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 3	360
C.34	Power for the Between-Person Effect β_{IS} in Simulation 3	361
C.35	Bias for the Between-Person Effect β_{SS} in Simulation 3	362
C.36	Empirical SE for the Between-Person Effect β_{SS} in Simulation 3	363
C.37	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 3	364

C.38	SE Ratio for the Between-Person Effect β_{SS} in Simulation 3	365
C.39	RMSE for the Between-Person Effect β_{SS} in Simulation 3	366
C.40	RMSE for the Between-Person Effect β_{SS} in Simulation 3	367
C.41	Coverage Rate for the Between-Person Effect β_{SS} in Simulation 3	368
D.1	Proportion of Converged Cases in Simulation 4 except LCMx-RIx ($\gamma = -0.6$) .	369
D.2	Proportion of Converged Cases in Simulation 4 except LCMx-RIx ($\gamma = 0$) . . .	370
D.3	Proportion of Converged Cases in Simulation 4 except LCMx-RIx ($\gamma = 0.6$) . .	371
D.4	Proportion of Converged Cases in Simulation 4 (LCMx-RIx)	372
D.5	Proportion of Cases with Problems in Estimating SE in Simulation 4 except LCMx-RIy ($\gamma = -0.6$)	373
D.6	Proportion of Cases with Problems in Estimating SE in Simulation 4 except LCMx-RIy ($\gamma = 0$)	374
D.7	Proportion of Cases with Problems in Estimating SE in Simulation 4 except LCMx-RIy ($\gamma = 0.6$)	375
D.8	Proportion of Cases with Problems in Estimating SE in Simulation 4 (LCMx-RIy)	376
D.9	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simula- tion 4 except LCMy-RIx and LCMy-RIy ($\gamma = -0.6$)	377
D.10	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simula- tion 4 except LCMy-RIx and LCMy-RIy ($\gamma = 0$)	378
D.11	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simula- tion 4 except LCMy-RIx and LCMy-RIy ($\gamma = 0.6$)	379
D.12	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simula- tion 4 (LCMy-RIy)	380
D.13	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simula- tion 4 (LCMx-RIy)	380

D.14	CFI in Simulation 4 ($\gamma = -0.6$)	381
D.15	CFI in Simulation 4 ($\gamma = 0$)	382
D.16	CFI in Simulation 4 ($\gamma = 0.6$)	383
D.17	RMSEA in Simulation 4 ($\gamma = -0.6$)	384
D.18	RMSEA in Simulation 4 ($\gamma = 0$)	385
D.19	RMSEA in Simulation 4 ($\gamma = 0.6$)	386
D.20	RMSEA in Simulation 4 (LCMy-RIx)	387
D.21	RMSEA in Simulation 4 (LCMx-RIy)	387
D.22	SRMR in Simulation 4 ($\gamma = -0.6$)	388
D.23	SRMR in Simulation 4 ($\gamma = 0$)	389
D.24	SRMR in Simulation 4 ($\gamma = 0.6$)	390
D.25	SRMR in Simulation 4 (LCMy-RIx)	391
D.26	SRMR in Simulation 4 (LCMx-RIy)	391
D.27	Bias for the Within-Person Effect γ in Simulation 4 (MLM-WB)	392
D.28	Bias for the Within-Person Effect γ in Simulation 4 (MLM-E2S)	392
D.29	Bias for the Within-Person Effect γ in Simulation 4 (MLM-TAC)	393
D.30	Bias for the Within-Person Effect γ in Simulation 4 (MLM-MV)	393
D.31	Bias for the Within-Person Effect γ in Simulation 4 (LCMy-RIx)	394
D.32	Bias for the Within-Person Effect γ in Simulation 4 (LCMy-CLC)	394
D.33	Bias for the Within-Person Effect γ in Simulation 4 (LCMxy-RP)	395
D.34	Bias for the Within-Person Effect γ in Simulation 4 (LCMx-RIy)	395
D.35	Bias for the Within-Person Effect γ in Simulation 4 (LCMx-CLC)	396
D.36	Empirical SE for the Within-Person Effect γ in Simulation 4 (LCMx-RIy) . . .	397
D.37	Empirical SE for the Within-Person Effect γ in Simulation 4 Except for LCMx- RIy ($\gamma = -0.6$)	398

D.38	Empirical SE for the Within-Person Effect γ in Simulation 4 for LCMx-RIy ($\gamma = 0$)	399
D.39	Empirical SE for the Within-Person Effect γ in Simulation 4 for LCMx-RIy ($\gamma = 0.6$)	400
D.40	Theoretical SE for the Within-Person Effect γ in Simulation 4 (LCMx-RIy) . .	401
D.41	Theoretical SE for the Within-Person Effect γ in Simulation 4 for LCMx-RIy ($\gamma = -0.6$)	402
D.42	Theoretical SE for the Within-Person Effect γ in Simulation 4 for LCMx-RIy ($\gamma = 0$)	403
D.43	Theoretical SE for the Within-Person Effect γ in Simulation 4 for LCMx-RIy ($\gamma = 0.6$)	404
D.44	SE Ratio for the Within-Person Effect γ in Simulation 4 (MLM-WB)	405
D.45	SE Ratio for the Within-Person Effect γ in Simulation 4 (MLM-E2S)	405
D.46	SE Ratio for the Within-Person Effect γ in Simulation 4 (MLM-TAC)	406
D.47	SE Ratio for the Within-Person Effect γ in Simulation 4 (MLM-MV)	406
D.48	SE Ratio for the Within-Person Effect γ in Simulation 4 (LCMy-RIx)	407
D.49	SE Ratio for the Within-Person Effect γ in Simulation 4 (LCMy-CLC)	407
D.50	SE Ratio for the Within-Person Effect γ in Simulation 4 (LCMxy-RP)	408
D.51	SE Ratio for the Within-Person Effect γ in Simulation 4 (LCMx-RIy)	408
D.52	SE Ratio for the Within-Person Effect γ in Simulation 4 (LCMx-CLC)	409
D.53	RMSE for the Within-Person Effect γ in Simulation 4 (MLM-WB)	409
D.54	RMSE for the Within-Person Effect γ in Simulation 4 (LCMx-RIy)	410
D.55	RMSE for the Within-Person Effect γ in Simulation 4 ($\gamma = -0.6$)	411
D.56	RMSE for the Within-Person Effect γ in Simulation 4 ($\gamma = 0$)	412
D.57	RMSE for the Within-Person Effect γ in Simulation 4 ($\gamma = 0.6$)	413
D.58	Coverage Rate for the Within-Person Effect γ in Simulation 4 (MLM-WB) . .	414

D.59	Coverage Rate for the Within-Person Effect γ in Simulation 4 (MLM-E2S)	. . . 414
D.60	Coverage Rate for the Within-Person Effect γ in Simulation 4 (MLM-TAC)	. . . 415
D.61	Coverage Rate for the Within-Person Effect γ in Simulation 4 (MLM-MV)	. . . 415
D.62	Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMy-RIx)	. . . 416
D.63	Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMy-CLC)	. . . 416
D.64	Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMxy-RP)	. . . 417
D.65	Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMx-RIy)	. . . 417
D.66	Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMx-CLC)	. . . 418
D.67	Power for the Within-Person Effect γ in Simulation 4 ($\gamma = -0.6$)	 419
D.68	Power for the Within-Person Effect γ in Simulation 4 ($\gamma = 0.6$)	 420
D.69	Bias for the Between-Person Effect β_{II} in Simulation 4 (MLM-WB)	 421
D.70	Bias for the Between-Person Effect β_{II} in Simulation 4 (MLM-E2S)	 421
D.71	Bias for the Between-Person Effect β_{II} in Simulation 4 (MLM-TAC)	 422
D.72	Bias for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)	 422
D.73	Bias for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)	 423
D.74	Bias for the Between-Person Effect β_{II} in Simulation 4 (LCMy-CLC)	 423
D.75	Bias for the Between-Person Effect β_{II} in Simulation 4 (LCMxy-RP)	 424
D.76	Bias for the Between-Person Effect β_{II} in Simulation 4 (LCMx-RIy)	 424
D.77	Bias for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)	 425
D.78	Empirical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)	 425
D.79	Empirical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-RIy)	 426
D.80	Empirical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)	 426
D.81	Empirical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = -0.6$)	 427
D.82	Empirical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0$)	 428
D.83	Empirical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)	 429
D.84	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)	 430

D.85	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)	430
D.86	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMxy-RP)	431
D.87	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-RIy)	431
D.88	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)	432
D.89	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = -0.6$)	433
D.90	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0$)	434
D.91	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)	435
D.92	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (MLM-WB)	436
D.93	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (MLM-E2S)	436
D.94	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (MLM-TAC)	437
D.95	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)	437
D.96	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)	438
D.97	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCMy-CLC)	438
D.98	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCMxy-RP)	439
D.99	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCMx-RIy)	439
D.100	SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)	440
D.101	RMSE for the Between-Person Effect β_{II} in Simulation 4 (MLM-WB)	440
D.102	RMSE for the Between-Person Effect β_{II} in Simulation 4 (MLM-E2S)	441
D.103	RMSE for the Between-Person Effect β_{II} in Simulation 4 (MLM-TAC)	441
D.104	RMSE for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)	442
D.105	RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)	442
D.106	RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMy-CLC)	443
D.107	RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMxy-RP)	443
D.108	RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-RIy)	444
D.109	RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)	444
D.110	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (MLM-WB)	445

D.111	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (MLM-E2S)	445
D.112	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (MLM-TAC)	446
D.113	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)	446
D.114	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)	447
D.115	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMy-CLC)	447
D.116	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMxy-RP)	448
D.117	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMx-RIy)	448
D.118	Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)	449
D.119	Power for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = -0.6$)	450
D.120	Power for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0$)	451
D.121	Power for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)	452
D.122	Bias for the Between-Person Effect β_{SI} in Simulation 4 (MLM-E2S)	453
D.123	Bias for the Between-Person Effect β_{SI} in Simulation 4 (MLM-TAC)	453
D.124	Bias for the Between-Person Effect β_{SI} in Simulation 4 (MLM-MV)	454
D.125	Bias for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)	454
D.126	Bias for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-CLC)	455
D.127	Bias for the Between-Person Effect β_{SI} in Simulation 4 (LCMxy-RP)	455
D.128	Bias for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)	456
D.129	Empirical SE for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx) .	456
D.130	Empirical SE for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)	457
D.131	Empirical SE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = -0.6$) . .	458
D.132	Empirical SE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0$)	459
D.133	Empirical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$) . . .	460
D.134	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)	461
D.135	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)	461
D.136	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = -0.6$) .	462

D.137	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0$) . . .	463
D.138	Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$) . .	464
D.139	SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (MLM-E2S) . . .	465
D.140	SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (MLM-TAC) . . .	465
D.141	SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (MLM-MV) . . .	466
D.142	SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx) . . .	466
D.143	SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-CLC) . .	467
D.144	SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (LCMxy-RP) . . .	467
D.145	SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC) . .	468
D.146	RMSE for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)	468
D.147	RMSE for the Between-Person Effect β_{SI} in Simulation 4 (LCMxy-RP)	469
D.148	RMSE for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)	469
D.149	RMSE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = -0.6$)	470
D.150	RMSE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0$)	471
D.151	RMSE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)	472
D.152	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (MLM-E2S)	473
D.153	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (MLM-TAC)	473
D.154	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (MLM-MV)	474
D.155	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)	474
D.156	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-CLC)	475
D.157	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (LCMxy-RP)	475
D.158	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)	476
D.159	Power for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = -0.6$)	477
D.160	Power for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0$)	478
D.161	Power for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0.6$)	479
D.162	Bias for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)	480

D.163	Bias for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)	480
D.164	Bias for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)	481
D.165	Bias for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-RIy)	481
D.166	Bias for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)	482
D.167	Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)	482
D.168	Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)	483
D.169	Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)	483
D.170	Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-RIy)	484
D.171	Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)	484
D.172	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)	485
D.173	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)	485
D.174	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)	486
D.175	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-RIy)	486
D.176	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)	487
D.177	SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)	487
D.178	SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)	488
D.179	SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)	488
D.180	SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-RIy)	489
D.181	SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)	489
D.182	RMSE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)	490
D.183	RMSE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)	490
D.184	RMSE for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)	491
D.185	RMSE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-RIy)	491
D.186	RMSE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)	492
D.187	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)	492
D.188	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)	493

D.189	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)	493
D.190	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-RIy)	494
D.191	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)	494
D.192	Power for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)	495
D.193	Power for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)	495
D.194	Power for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)	496
D.195	Power for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-RIy)	496
D.196	Power for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)	497
D.197	Bias for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)	498
D.198	Bias for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)	498
D.199	Bias for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)	499
D.200	Bias for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)	499
D.201	Empirical SE for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)	500
D.202	Empirical SE for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)	500
D.203	Empirical SE for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)	501
D.204	Empirical SE for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)	501
D.205	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)	502
D.206	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)	502
D.207	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)	503
D.208	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)	503
D.209	SE Ratio for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)	504
D.210	SE Ratio for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)	504
D.211	SE Ratio for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)	505
D.212	SE Ratio for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)	505
D.213	RMSE for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)	506
D.214	RMSE for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)	506

D.215	RMSE for the Between-Person Effect β_{SS} in Simulation 4 ($\gamma = -0.6$)	507
D.216	RMSE for the Between-Person Effect β_{SS} in Simulation 4 ($\gamma = 0$)	508
D.217	RMSE for the Between-Person Effect β_{SS} in Simulation 4 ($\gamma = 0.6$)	509
D.218	Coverage Rate for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)	510
D.219	Coverage Rate for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)	510
D.220	Coverage Rate for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)	511
D.221	Coverage Rate for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)	511
D.222	Power for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)	512
D.223	Power for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)	512
D.224	Power for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)	513
D.225	Power for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)	513
E.1	Proportion of Converged Cases in Simulation 5	514
E.2	Proportion of Cases with Problems in Estimating SE in Simulation 5	515
E.3	Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 5	516
E.4	CFI in Simulation 5	517
E.5	RMSEA in Simulation 5	518
E.6	SRMR in Simulation 5	519
E.7	SRMR in Simulation 5 (LCMy-RIx)	520
E.8	Bias for the Within-Person Effect γ in Simulation 5 (MLM-WB)	521
E.9	Bias for the Within-Person Effect γ in Simulation 5 (MLM-E2S)	521
E.10	Bias for the Within-Person Effect γ in Simulation 5 (MLM-TAC)	522
E.11	Bias for the Within-Person Effect γ in Simulation 5 (MLM-MV)	522
E.12	Bias for the Within-Person Effect γ in Simulation 5 (LCMy-RIx)	523
E.13	Bias for the Within-Person Effect γ in Simulation 5 (LCMy-CLC)	523

E.14	Bias for the Within-Person Effect γ in Simulation 5 (LCMxy-RP)	524
E.15	Bias for the Within-Person Effect γ in Simulation 5 (LCMx-RIy)	524
E.16	Bias for the Within-Person Effect γ in Simulation 5 (LCMx-CLC)	525
E.17	Empirical SE for the Within-Person Effect γ in Simulation 5	526
E.18	Theoretical SE for the Within-Person Effect γ in Simulation 5	527
E.19	SE Ratio for the Within-Person Effect γ in Simulation 5 (MLM-WB)	528
E.20	SE Ratio for the Within-Person Effect γ in Simulation 5 (MLM-E2S)	528
E.21	SE Ratio for the Within-Person Effect γ in Simulation 5 (MLM-TAC)	529
E.22	SE Ratio for the Within-Person Effect γ in Simulation 5 (MLM-MV)	529
E.23	SE Ratio for the Within-Person Effect γ in Simulation 5 (LCMy-RIx)	530
E.24	SE Ratio for the Within-Person Effect γ in Simulation 5 (LCMy-CLC)	530
E.25	SE Ratio for the Within-Person Effect γ in Simulation 5 (LCMxy-RP)	531
E.26	SE Ratio for the Within-Person Effect γ in Simulation 5 (LCMx-RIy)	531
E.27	SE Ratio for the Within-Person Effect γ in Simulation 5 (LCMx-CLC)	532
E.28	RMSE for the Within-Person Effect γ in Simulation 5 (MLM-WB)	532
E.29	RMSE for the Within-Person Effect γ in Simulation 5 (LCMx-RIy)	533
E.30	RMSE for the Within-Person Effect γ in Simulation 5 except MLM-WB and LCMx-RIy	534
E.31	Coverage Rate for the Within-Person Effect γ in Simulation 5	535
E.32	Power for the Within-Person Effect γ in Simulation 5	536
E.33	Bias for the Between-Person Effect β_{II} in Simulation 5 (MLM-WB)	537
E.34	Bias for the Between-Person Effect β_{II} in Simulation 5 (MLM-E2S)	537
E.35	Bias for the Between-Person Effect β_{II} in Simulation 5 (MLM-TAC)	538
E.36	Bias for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)	538
E.37	Bias for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)	539
E.38	Bias for the Between-Person Effect β_{II} in Simulation 5 (LCMy-CLC)	539

E.39	Bias for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)	540
E.40	Bias for the Between-Person Effect β_{II} in Simulation 5 (LCMx-RIy)	540
E.41	Bias for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)	541
E.42	Empirical SE for the Between-Person Effect β_{II} in Simulation 5	542
E.43	Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)	543
E.44	Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)	543
E.45	Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)	544
E.46	Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-RIy)	544
E.47	Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)	545
E.48	Theoretical SE for the Between-Person Effect β_{II} in Simulation 5	546
E.49	Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)	547
E.50	Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)	547
E.51	Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)	548
E.52	Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-RIy)	548
E.53	Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)	549
E.54	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (MLM-WB)	549
E.55	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (MLM-E2S)	550
E.56	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (MLM-TAC)	550
E.57	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)	551
E.58	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)	551
E.59	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMy-CLC)	552
E.60	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)	552
E.61	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMx-RIy)	553
E.62	SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)	553
E.63	RMSE for the Between-Person Effect β_{II} in Simulation 5	554
E.64	RMSE for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)	555

E.65	RMSE for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)	555
E.66	RMSE for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)	556
E.67	RMSE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-RIy)	556
E.68	RMSE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)	557
E.69	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (MLM-WB)	557
E.70	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (MLM-E2S)	558
E.71	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (MLM-TAC)	558
E.72	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)	559
E.73	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)	559
E.74	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMy-CLC)	560
E.75	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)	560
E.76	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMx-RIy)	561
E.77	Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)	561
E.78	Power for the Between-Person Effect β_{II} in Simulation 5 (MLM-WB)	562
E.79	Power for the Between-Person Effect β_{II} in Simulation 5 (MLM-E2S)	562
E.80	Power for the Between-Person Effect β_{II} in Simulation 5 (MLM-TAC)	563
E.81	Power for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)	563
E.82	Power for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)	564
E.83	Power for the Between-Person Effect β_{II} in Simulation 5 (LCMy-CLC)	564
E.84	Power for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)	565
E.85	Power for the Between-Person Effect β_{II} in Simulation 5 (LCMx-RIy)	565
E.86	Power for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)	566
E.87	Bias for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S)	567
E.88	Bias for the Between-Person Effect β_{SI} in Simulation 5 (MLM-TAC)	567
E.89	Bias for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV)	568
E.90	Bias for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx)	568

E.91	Bias for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-CLC)	569
E.92	Bias for the Between-Person Effect β_{SI} in Simulation 5 (LCMxy-RP)	569
E.93	Bias for the Between-Person Effect β_{SI} in Simulation 5 (LCMx-CLC)	570
E.94	Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM-TAC, LCMy-CLC)	571
E.95	Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV) .	572
E.96	Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx) .	572
E.97	Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMxy-RP)	573
E.98	Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMx-CLC)	573
E.99	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM-TAC, LCMy-CLC)	574
E.100	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV)	575
E.101	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx)	575
E.102	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMxy-RP)	576
E.103	Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMx-CLC)	576
E.104	SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S) . . .	577
E.105	SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (MLM-TAC) . . .	577
E.106	SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV) . . .	578
E.107	SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx) . . .	578
E.108	SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-CLC) . .	579
E.109	SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (LCMxy-RP) . . .	579
E.110	SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (LCMx-CLC) . .	580
E.111	RMSE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM- TAC, LCMy-CLC)	581
E.112	RMSE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV)	582
E.113	RMSE for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx)	582

E.114	RMSE for the Between-Person Effect β_{SI} in Simulation 5 (LCMxy-RP)	583
E.115	RMSE for the Between-Person Effect β_{SI} in Simulation 5 (LCMx-CLC)	583
E.116	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM-TAC, LCMMy-CLC)	584
E.117	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV, LCMxy-RP, LCMx-CLC)	585
E.118	Coverage Rate for the Between-Person Effect β_{SI} in Simulation 5 (LCMMy-RIx)	586
E.119	Power for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM- TAC, LCMMy-CLC)	587
E.120	Power for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV, LCMxy- RP, LCMx-CLC)	588
E.121	Power for the Between-Person Effect β_{SI} in Simulation 5 (LCMMy-RIx)	589
E.122	Bias for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)	590
E.123	Bias for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)	590
E.124	Bias for the Between-Person Effect β_{IS} in Simulation 5 (LCMxy-RP)	591
E.125	Bias for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-RIy)	591
E.126	Bias for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-CLC)	592
E.127	Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S) .	592
E.128	Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV) .	593
E.129	Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMxy-RP)	593
E.130	Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-RIy) .	594
E.131	Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-CLC)	594
E.132	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)	595
E.133	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)	595
E.134	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMxy-RP)	596
E.135	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-RIy)	596

E.136	Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-CLC)	597
E.137	SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)	597
E.138	SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)	598
E.139	SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (LCMxy-RP)	598
E.140	SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-RIy)	599
E.141	SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-CLC)	599
E.142	RMSE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)	600
E.143	RMSE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)	600
E.144	RMSE for the Between-Person Effect β_{IS} in Simulation 5 (LCMxy-RP)	601
E.145	RMSE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-RIy)	601
E.146	RMSE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-CLC)	602
E.147	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV, LCMxy-RP, LCMx-CLC)	603
E.148	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)	604
E.149	Coverage Rate for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-RIy)	604
E.150	Power for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S, MLM- MV, LCMxy-RP, LCMx-CLC)	605
E.151	Power for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-RIy)	606
E.152	Bias for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)	607
E.153	Bias for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)	607
E.154	Bias for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)	608
E.155	Bias for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)	608
E.156	Empirical SE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)	609
E.157	Empirical SE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)	609
E.158	Empirical SE for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)	610
E.159	Empirical SE for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)	610

E.160	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)	611
E.161	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)	611
E.162	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)	612
E.163	Theoretical SE for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)	612
E.164	SE Ratio for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)	613
E.165	SE Ratio for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)	613
E.166	SE Ratio for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)	614
E.167	SE Ratio for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)	614
E.168	RMSE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)	615
E.169	RMSE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)	615
E.170	RMSE for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)	616
E.171	RMSE for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)	616
E.172	Coverage Rate for the Between-Person Effect β_{SS} in Simulation 5	617
E.173	Power for the Between-Person Effect β_{SS} in Simulation 5	618

List of Tables

2.1	Summary of Regression Models with Time-Varying Predictor	28
2.2	Example of Singly Stacked and Doubly Stacked Data (Three-Wave Data with a Single Time-Varying Predictor)	46
2.3	Wang & Maxwell's (2015) Analytical Results on the Relationship between the Estimated Individual Within-Person Effects and Detrending Methods	47
2.4	Summary of Within-Person and Between-Person Predictors in Detrending Meth- ods	52
2.5	Summary of the Reviewed Methods and Proposed Methods	59
4.1	Simulation Conditions	79
4.2	Other Model Parameters	81
4.3	Proportion of Within-Person and Between-Person Components in the Variance of the Outcome	85
4.4	Summary of the Nine Analytic Models in the Simulations	86
4.5	Target Parameters of Analytic Models for Evaluating Simulation Results . . .	90
5.1	Marginal Means of Absolute Bias (Empirical SE) in Simulation 1	94
5.2	Marginal Means of Absolute Bias (Empirical SE) in Simulation 2	108
5.3	Marginal Means of Absolute Bias (Empirical SE) in Simulation 3	125
5.4	Marginal Means of Absolute Bias (Empirical SE) in Simulation 4	142
5.5	Marginal Means of Absolute Bias (Empirical SE) in Simulation 5	165
5.6	Mean Absolute Bias (Empirical SE) of $\gamma \in \{-0.6, 0, 0.6\}$ across Simulations . .	188
5.7	Mean Absolute Bias (Mean Empirical SE) of $\beta_{II} \in \{0, 0.6\}$ across the Simulations	188

5.8	Mean Absolute Bias (Mean Empirical SE) of $\beta_{SI} \in \{0, 0.3\}$ across the Simulations	189
5.9	Mean Absolute Bias (Mean Empirical SE) of $\beta_{IS} \in \{0, 0.3\}$ across the Simulations	189
5.10	Mean Absolute Bias (Mean Empirical SE) of $\beta_{SS} \in \{0, 0.6\}$ across the Simulations	189
5.11	Average SE Ratio/Coverage Rate/Power of $\gamma \in \{-0.6, 0, 0.6\}$ across Simulations	192
5.12	Average SE Ratio/Coverage Rate/Power of $\beta_{II} \in \{0, 0.6\}$ across Simulations .	192
5.13	Average SE Ratio/Coverage Rate/Power of $\beta_{SI} \in \{0, 0.3\}$ across Simulations .	192
5.14	Average SE Ratio/Coverage Rate/Power of $\beta_{IS} \in \{0, 0.3\}$ across Simulations .	193
5.15	Average SE Ratio/Coverage Rate/Power of $\beta_{SS} \in \{0, 0.6\}$ across Simulations .	193
6.1	Model Groups That Show a Similar Pattern in the Simulation Results	196
6.2	Variance Component Conditions for Unbiased Estimation of the Within-Person and Between-Person Effects in Simulation 5	198
6.3	Estimates (SEs) from An Ad-Hoc Supplemental Analysis	203
6.4	Marginal Mean, Standard Error, and p -Value of Parameter Estimates Related to S_i^Y Based on LCMxy-RP in Simulation 3	206

List of Abbreviations

2S	2-Step (detrending)
AC	Adaptive Centering
CFA	Confirmatory Factor Analysis
CFI	Comparative Fit Index
CLC	Cross-Level Covariances
CR	Coverage Rate
CRE	Correlated Random Effect (model)
DGM	Data-Generating Model
E2S	Extended 2-Step (detrending)
FEM	Fixed Effects Model
FGLS	Feasible Generalized Least Squares
FWL	Frisch-Waugh-Lovell (theorem)
GLS	Generalized Least Squares
LCM	Latent Curve Model
LSDV	Least Squares Dummy Variable
ML	Maximum Likelihood

MLM Multilevel Model/Multilevel Modeling

MSE Mean Square Error

NPD Non-Positive Definite

OLS Ordinary Least Squares

REM Random Effects Model

REML Restricted Maximum Likelihood

RMSE Root Mean Square Error

RMSEA Root Mean Square Error of Approximation

RP Residualized Predictors

RPO Residualized Predictors and Outcomes

RQ Research Question

SE Standard Error

SEM Structural Equation Model/Structural Equation Modeling

SR Significance Rate

SRMR Standardized Root Mean Square Residual

TAC Time-As-Covariate (detrending)

TLI Tucker Lewis Index

WB Within-Between (model)

Chapter 1

Introduction

1.1 Background

1.1.1 Within-Person Process and Between-Person Relation

Longitudinal data analysis is attracting considerable attention in psychology and educational research. Researchers in these fields adopt longitudinal designs for several reasons. For instance, longitudinal designs enable us to enhance the validity of causal inference by establishing temporal precedence between variables and by increasing statistical power (Curran & Bauer, 2011). Another advantage of longitudinal studies is that one can simultaneously test their hypotheses on within-person processes and between-person relations between variables (Curran & Bauer, 2011; Hoffman, 2015). A *between-person* or *interindividual* relation is the relationship between time-invariant variables such as sex assigned at birth, year of birth, and assignment of treatment in an experiment. Examples of the between-person, time-invariant variables include psychological traits that are stable at least during a study (e.g., personality, intelligence, socioeconomic status). Also, a time-varying variable may be addressed as time-invariant when it is measured once in a study (e.g., at the beginning of the study, at the end of the study). Examples of between-person relations include the one between treatment assignment and endpoint measurement, gender and extraversion, and socioeconomic status and test score on a standardized test in a specific grade.

In contrast, a *within-person* or *intraindividual* process refers to change and fluctuation in variables over time within a person. Examples of within-person, time-varying variables include anxiety level, stress level, motivation level, change in blood pressure, and change in heart rate. Age and other time-related variables such as time elapsed since the beginning of the study may also be relevant to a time-varying, within-person process. For instance, natural history or growth can be seen as an effect of time passage. Within-person change in these time-varying variables is not an absolute measure but relative to their time-invariant references such as baseline measurement, individual's average, and endpoint measurement. In other words, within-person change represents an increase or decrease of a variable after controlling for time-invariant between-person differences.

Within-person processes are one of the popular research topics in psychology, education, and developmental sciences. In fact, after Molenaar's seminal works (Molenaar, 2004; Molenaar & Campbell, 2009), many studies that focus on the within-person process have been published. Beal and Gabriel (2019) and Podsakoff et al. (2019) reported in their comprehensive literature review that around 300 articles had been published using the experience sampling method, which is a technique to collect intensive longitudinal data to examine within-person processes. Moreover, more than half of the studies were published within the last several years. One reason for the surge of popularity is that many psychological and educational theories address within-person phenomena that unfold over time. In addition to the within-person nature of the theories, the recent development of information and communication technology (ICT) enables researchers to collect (intensive) longitudinal data with less difficulty (e.g., experience sampling, ecological momentary assessment, and ambulatory assessment, which are implemented with ICT devices such as smartphones and tablets as well as online survey platforms; Beal, 2015; Beal & Gabriel, 2019; Mehl & Conner, 2012).

On the other hand, between-person relations have been a central issue of scientific

inquiry in a variety of research fields. Especially in longitudinal studies, between-person differences in within-person change are often of interest (Fitzmaurice et al., 2011; Singer & Willett, 2003). For example, researchers may be interested in the effect of an experimental intervention on anxiety levels. In this case, parameters related to the within-person change may be compared between the treatment group and control group after controlling for individual-level, time-invariant covariates such as biological sex and anxiety level before the treatment.

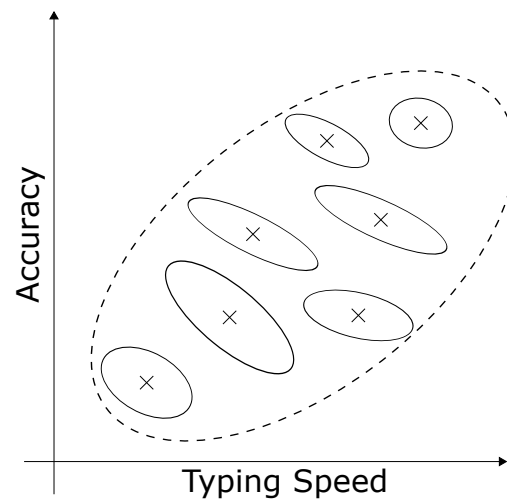
1.1.2 Time-Varying Predictor and Centering

Examining within-person and between-person relations requires not only longitudinal data but also an appropriate statistical analysis. Analytic methods are important especially when one wants to include time-varying predictors in their analysis. A *time-varying predictor* is a variable that can take different values over time and is used to explain variation in an outcome variable. A well-known issue in addressing time-varying predictors is that they contain both within-person and between-person components simultaneously. Because of this dual nature of time-varying predictors, simply regressing an outcome on a time-varying predictor (i.e., pooled regression) does not provide an answer to a research question about a within-person process or a between-person relation. Instead, as stated in Cronbach (1976) and Raudenbush and Bryk (2002), the resulting regression coefficients represent an “uninterpretable blend” of the within-person effect and the between-person effect. What is more problematic is that the two effects can differ not only in their size but also in their direction. Therefore, researchers may reach an incorrect conclusion if they do not properly address the time-varying variables.

Figure 1.1 illustrates this case. The figure shows a hypothetical relationship between typing speed (e.g., the number of words typed per minute; x -axis) and accuracy

Figure 1.1

An Example of a Within-Person Effect and a Between-Person Effect That Differ in Their Sign (Within-Person Effect = Negative, Between-Person Effect = Positive).



Note. Solid ellipses represent a distribution of data points for a single person. A cross mark shows the averages of typing speed and accuracy for that person. The large dashed ellipse represents the entire distribution of the observations.

(inverse of the number of typos per minute; y -axis). Each solid ellipse represents a distribution of data points for an individual, and a cross mark indicates the average point of that individual. The large dashed ellipse means the entire distribution of the observations. In this example, the correlation between typing speed and accuracy is negative for a given person—the faster a person types, the more typos they get. On the other hand, the correlation between the two variables is positive as a whole. If we do not distinguish the within-person correlation and the between-person correlation, we may draw a conclusion that people have fewer typos when they type faster, which is obviously incorrect. To avoid such fallacious inference, researchers need to conceptually and statistically distinguish the within-person and between-person relations between variables (Curran & Bauer, 2011; Hamaker, 2012; Murayama et al., 2017).

In order to disaggregate the within-person effect and between-person effect, methodologists have recommended using the random effects model with person-mean centering (Allison, 2005, 2009; Hamaker & Muthén, 2020; Raudenbush, 2009). Let Y_{ti} and X_{ti} be an outcome and a time-varying predictor of person i measured at time t , respectively. Then, a multilevel model with person-mean centering can be written as

$$\begin{aligned} Y_{ti} &= \beta_{0i} + \beta_{1i}(X_{ti} - \bar{X}_{\cdot i}) + r_{ti}, \\ \beta_{0i} &= \gamma_{00} + \gamma_{01}\bar{X}_{\cdot i} + u_{0i}, \\ \beta_{1i} &= \gamma_{10}, \end{aligned}$$

or equivalently,

$$Y_{ti} = \gamma_{00} + \gamma_{01}\bar{X}_{\cdot i} + \gamma_{10}(X_{ti} - \bar{X}_{\cdot i}) + u_{0i} + r_{ti},$$

where $\bar{X}_{\cdot i}$ is person i 's average of the time-varying predictor; u_{0i} is a random intercept, which is distributed as $u_{0i} \sim N(0, \tau_{00})$; and r_{ti} is an error term that has a distribution of $N(0, \sigma^2)$. This model is called within-between model (Bell & Jones, 2015; Bell et al., 2018, 2019) and hybrid model (Allison, 2005, 2009) in social sciences. The model can appropriately capture both types of effect by decomposing the time-varying predictor into two orthogonal components, namely the person mean $\bar{X}_{\cdot i}$ and the person-mean centered predictor $X_{ti} - \bar{X}_{\cdot i}$. Because the person mean does not change over time and the person-mean centered predictor represent person i 's deviation from their average at time t , the corresponding coefficients, γ_{01} and γ_{10} , can be interpreted as between-person effect and within-person effect, respectively.

1.1.3 Detrending

The person-mean centering or group-mean centering was originally developed in the context of organizational research that uses multilevel models (e.g., Bell et al., 2018, 2019;

Cronbach, 1976; Enders & Tofighi, 2007; Hofmann & Gavin, 1998; Kreft et al., 1995; Paccagnella, 2006; Rights et al., 2019). In this field, the group-mean centering works successfully, and it is considered the gold standard to disentangle the within-group effect and between-group effect. In fact, the person-mean centering is ubiquitous in popular textbooks of multilevel analysis (e.g., Hoffman, 2015; Hox et al., 2018; Raudenbush & Bryk, 2002; Singer & Willett, 2003; Snijders & Bosker, 2012).

Although organizational data and longitudinal data have a similar data property (i.e., dependency among observations within a unit at a higher level; nested data structure) and they share analytic methods (multilevel model; MLM), there is a critical and underrepresented difference between the two types of data. Specifically, longitudinal studies concern the temporal order of the observations, whereas organizational studies do not (Fitzmaurice et al., 2011). In longitudinal studies, the temporal order of the observations determines the direction of change. Suppose that we have two longitudinal observations and the measured values are $(Y_1, Y_2) = (1, 2)$. The change score in this case is $2 - 1 = 1$. However, if the order is reversed, that is, if $(Y_1, Y_2) = (2, 1)$, the change score becomes negative ($1 - 2 = -1$). In contrast, observations in organizational studies are considered random samples from a given higher-level unit. Thus, the order of observations does not matter (i.e., lower-level units are exchangeable for a given higher-level unit; Curran, 2003; Mehta & Neale, 2005; Lüdtke et al., 2008).

Recent studies revealed that the person-mean centering would not perform well in longitudinal studies when a time-varying predictor exhibited systematic change over time (Curran & Bauer, 2011; Curran et al., 2012; Hoffman, 2015; Wang & Maxwell, 2015). More specifically, when a time-varying predictor shows a trend over time, the within-between model produces a biased estimate of the within-person effect regardless of the data collection design (i.e., balanced or unbalanced). Besides, the studies showed that the between-person effect in the within-between model could also be estimated with bias when

the time design was unbalanced. This is because the trend in the time-varying predictor is not taken into account, and the centered predictors and person means fail to capture the true within-person and between-person components of the data-generating mechanism.

To obtain unbiased estimates of the within-person effect and between-person effect in the presence of a time trend in a time-varying predictor, the researchers suggested removing the trend prior to or during the analysis. This procedure is called *detrending* and can be done either by transforming the data or by explicitly modeling the trend.

Detrending has been well studied in time series analysis (Frisch & Waugh, 1933; Lovell, 1963, 2008) and is a popular practice to make a time series stationary (i.e., having a time-invariant distribution, at least time-invariant mean, variance, and covariance).

Recently, psychologists and psychometricians started using time series models for intensive longitudinal data analysis, and some articles that illustrated detrending were published (e.g., Hamaker et al., 2018; Falkenström et al., 2017; Helm et al., 2018; Jebb et al., 2015).

Detrending is also studied in the context of longitudinal data analysis (Curran & Bauer, 2011; Hoffman, 2015; Wang & Maxwell, 2015).

1.2 Purpose of the Study

Most of the previous research on centering and detrending used MLM as their population and analytic models. However, structural equation modeling (SEM) can also be used for the same purpose. Specifically, the latent curve model (LCM) can be extended in a way that it can incorporate time-varying predictors and estimate the within-person effects and between-person effects (Bollen & Curran, 2006; Curran et al., 2012, 2014). Unfortunately, only a few studies compared the performance of MLM-based and SEM-based detrending methods. For example, Curran et al. (2012) proposed a centering approach based on SEM, which is an SEM counterpart of the within-between model. However, they only addressed

centering and did a small simulation under a specific condition. Hamaker and Muthén (2020) compared a broad range of MLM and SEM models, but they also focused on centering and did not compare detrending methods. Wang and Maxwell (2015) addressed both centering and detrending, but they only examined MLM and did not consider any SEM models. Plewis (2001) compared MLM and SEM models with time-varying predictors; however, Plewis lacked attention to detrending and within-person effects.

Therefore, in this dissertation, a variety of novel SEM models are proposed, which can detrend time-varying predictors and properly disaggregate the within-person and between-person effects. The performance of the proposed methods is compared to each other as well as MLM-based methods through a series of Monte Carlo simulations. The focus of the simulations is on whether the proposed models can correctly disentangle the within-person and between-person effects when a data-generating model does and does not have a slope factor for time-varying predictors and/or outcome variables. The use of SEM as data-generating models enable us to implement such simulations. Also, the current research provides a framework that characterizes centering and detrending methods and classifies them from a unified viewpoint.

The remaining chapters of this dissertation are organized as follows. In Chapter 2, a comprehensive literature review is provided, describing the existing models with time-varying predictors in detail. Chapter 2 also presents a framework to classify centering and detrending methods from the unified viewpoint. Chapter 3 develops new SEM-based detrending methods based on the framework. Also, research questions are explicitly stated at the end of this chapter. Chapter 4 gives the details of simulations, including settings of data-generating models, analytic models, evaluation indices, and technical aspects of the simulations. Results from the simulations are presented in Chapter 5. Finally, Chapter 6 summarizes and discusses the results from the simulations and provides answers to the research questions as well as some recommendations for practitioners.

Chapter 2

Literature Review

Chapter 2 provides a comprehensive literature review on longitudinal models with a time-varying predictor. Although various models are addressed in this chapter, the review is limited to a certain class of models for simplicity. First, I only consider models with one time-varying predictor. Also, no person-level covariate is considered except for variables derived from the time-varying predictor (e.g., person means). However, the models can easily be extended so that multiple time-varying predictors and multiple person-level covariates are incorporated. Second, in order to compare MLM and SEM, the within-person effect is constrained to be fixed (i.e., the average within-person effect), though some MLM models can make it random. In other words, the ability to incorporate individual differences in the within-person effect is an advantage of MLM over SEM. Third, lagged effects such as autoregressive effects (e.g., effects of X_{t-1} on X_t , effects of Y_t on Y_{t+2}) and cross-lagged effects (effects of X_{t-1} on Y_t , effects of Y_{t-1} on X_{t+1}) are not considered in this dissertation since they pose considerable difficulty in parameter estimation and interpretation. Finally, data is assumed to be *time-balanced* or *time-structured*, which means that all persons have the same measurement schedule (i.e., the same number of measurements at the same time points).

Chapter 2 is organized as follows. Section 2.1 is devoted to explaining regression and multilevel models with a time-varying predictor. The focus of this section is on centering the time-varying predictor and its interpretation. Section 2.2 describes SEM-based methods to handle a time-varying predictor, that is, latent curve model and its

extensions. Section 2.3 overviews existing detrend methods that are based on MLM. In this section, I discuss the relationship between MLM-based detrending methods as well as the similarity of detrending and centering. Then, in Section 2.4, SEM and MLM are compared in terms of centering and detrending.. Finally, the summary of the literature review and its implications for the current research are presented in Section 2.5.

2.1 Regression Models with Time-Varying Predictors

2.1.1 Fixed Effects Model

*Fixed effects model*¹ or *fixed effects regression* is a popular regression model for analyzing longitudinal and panel data with time-varying predictors. It is widely used in social science fields that are methodologically influenced by econometrics such as sociology and political science (Allison, 2005, 2009; Bell & Jones, 2015; Bell et al., 2018, 2019; Bollen & Brand, 2010; Clark & Linzer, 2015; Firebaugh et al., 2013; Imai & Kim, 2019, 2020; Greene, 2018; Wooldridge, 2010, 2016). Fixed effects model is also becoming popular in psychology (Hamaker & Muthén, 2020; Kim & Steiner, 2020; McNeish & Kelley, 2019) and public health fields (Dieleman & Templin, 2014; Hirai & Kaufman, 2017).

Fixed effects model is expressed as follows. Let Y_{ti} and X_{ti} be an outcome variable and a time-varying predictor of person i measured at time t , respectively ($i = 1, \dots, N$; $t = 1, \dots, T$). Then, one-way or unit fixed effects regression can be written as

$$Y_{ti} = \beta_{0i} + \beta_1 X_{ti} + r_{ti} \quad (2.1)$$

where β_{0i} is a random variable, which represents a person-specific intercept, β_1 is a

¹The term “fixed effects” is ambiguous and can have different meanings, depending on the tradition of the research field. Here, it does not mean constant parameters in the sense of mixed effects model. It means unit-specific effects that are not explicitly modeled. See a footnote of Gelman and Hill (2007, p. 245) for various definitions of fixed and random effects.

common regression coefficient, and r_{ti} is an error term, which is assumed to be distributed as $r_{ti} \sim N(0, \sigma^2)$. When we want to control the effects of measurement timing, we can add time-specific intercepts μ_t to the model. In this case, the model is called the two-way fixed effects model (Imai & Kim, 2020). The person-specific intercept β_{0i} and the time-varying predictor X_{ti} can be correlated, whereas the error r_{ti} is assumed to be uncorrelated with X_{ti} and β_{0i} . That is, $\text{COV}[\beta_{0i}, X_{ti}] \neq 0$, $\text{COV}[\beta_{0i}, r_{ti}] = 0$, and $\text{COV}[X_{ti}, r_{ti}] = 0$. Because β_{0i} does not assume a specific distributional form, it can absorb any unobserved individual difference, meaning that the fixed effects model can control any between-person difference, even if it is unobserved. Consequently, the coefficient of the time-varying predictor β_1 can be interpreted as a within-person effect. This ability to control for individual differences is one reason why econometricians, who often pursue rigorous causal inference, prefer fixed effects model to random or mixed effects model.

There are several approaches to estimate the within-person effect β_1 . Among them, the most popular estimator is *fixed effects estimator* or *within-group estimator* (Bell & Jones, 2015; Allison, 2005; Greene, 2018; Wooldridge, 2010, 2016)². The fixed effects estimator is the ordinary least squares (OLS) estimator using time-demeaned data. *Time-demeaning* refers to centering both dependent and independent variables around person means. Specifically, the fixed effects estimator is obtained from OLS regression with the following centered model:

$$\begin{aligned}
 Y_{ti} - \bar{Y}_{\cdot i} &= (\beta_{0i} + \beta_1 X_{ti} + r_{ti}) - \frac{1}{T} \sum_{t=1}^T (\beta_{0i} + \beta_1 X_{ti} + r_{ti}) \\
 &= (\beta_{0i} + \beta_1 X_{ti} + r_{ti}) - (\beta_{0i} + \beta_1 \bar{X}_{\cdot i} + \bar{r}_{ti}) \\
 &= \beta_1 (X_{ti} - \bar{X}_{\cdot i}) + (r_{ti} - \bar{r}_{ti})
 \end{aligned} \tag{2.2}$$

²For other estimators such as pooled OLS and first differencing, see Wooldridge (2010).

$$\Rightarrow \hat{\beta}_1 = \frac{\sum_{i=1}^N \sum_{t=1}^T (Y_{ti} - \bar{Y}_{.i})(X_{ti} - \bar{X}_{.i})}{\sum_{i=1}^N \sum_{t=1}^T (X_{ti} - \bar{X}_{.i})^2} \quad (2.3)$$

As in Equation 2.2, time-demeaning eliminates the person-specific intercepts β_{0i} from the estimation. Hence, we do not need to make any distributional assumption on β_{0i} , including correlations with the other random variables. The removal of the individual intercepts implies that the fixed effects model cannot have any person-level, time-invariant covariates since they are always canceled out by demeaning. This inability to incorporate person-level covariates is one reason why psychologists and educational researchers use random effects model or mixed effects model rather than fixed effects model; they are often interested in examining both within-person and between-person relationships as discussed in Chapter 1.

Note that the time-demeaned error $r_{ti} - \bar{r}_{.i}$ in Equation 2.2 is no longer uncorrelated over time (i.e., serially correlated), thus it violates the i.i.d. assumption. Hence, we need to use generalized least squares (GLS) estimation to address the violation. However, fortunately and interestingly, the GLS estimator coincides with the OLS estimator in Equation 2.3 (Fitzmaurice et al., 2011). Note also that the fixed effects estimator requires correcting the degree of freedom. The correct degree of freedom is $NT - N - 1$, not $NT - 1$, since the individual intercepts β_{0i} are treated as N distinct parameters (Wooldridge, 2016)³.

³When conducting multiple regression with p predictors, the adjusted degree of freedom is $NT - p - 1$.

2.1.2 Random Effects Model

Another traditional econometric model to deal with a time-varying predictor is *random effects model*. Random effects model can be written as

$$Y_{ti} = \beta_{0i} + \beta_{1i}X_{ti} + r_{ti}, \quad (2.4)$$

$$\beta_{0i} = \gamma_{00} + u_{0i}, \quad (2.5)$$

$$\beta_{1i} = \gamma_{10}, \quad (2.6)$$

or equivalently,

$$Y_{ti} = \gamma_{00} + \gamma_{10}X_{ti} + u_{0i} + r_{ti}, \quad (2.7)$$

where γ_{00} and γ_{10} are coefficients to be estimated, and u_{0i} and r_{ti} are random components at level 2 (i.e., person level) and level 1 (measurement occasion level), respectively. In the multilevel literature, this model is also known as random intercept model, or more generally, random coefficients model (Rabe-Hesketh & Skrondal, 2012; Raudenbush & Bryk, 2002; Singer & Willett, 2003; Snijders & Bosker, 2012), linear mixed effects model (McNeish & Kelley, 2019; Laird & Ware, 1982), and varying intercept, varying slope model (Gelman & Hill, 2007).

An important difference between fixed effects model and random effects model is that the latter assumes a normal distribution for the unobserved individual heterogeneity u_{0i} ; that is, $u_{0i} \sim N(0, \tau_{00})$. Another key assumption is the *exogeneity*, which means that the unobserved heterogeneity u_{0i} , thus the individual intercepts β_{0i} , are uncorrelated with the time-varying predictor X_{ti} (i.e., $\text{COV}[X_{ti}, u_{0i}] = \text{COV}[X_{ti}, \beta_{0i}] = 0$)⁴. These two assumptions enable the random effects model to incorporate person-level covariates (Allison, 2009; Hamaker & Muthén, 2020). Unlike the fixed effects model, the random

⁴In econometric textbooks (e.g., Wooldridge, 2010, p. 907), more strict assumption, $\mathbb{E}[u_{0i}|X_{i1}, \dots, X_{ti}] = 0$, is often made. This form of assumption is particularly called the *mean independence* and implies the exogeneity (i.e., uncorrelatedness). Uncorrelatedness, however, does not mean the mean independence.

effects model does not eliminate all of the level-2 variability since u_{0i} is constrained to be distributed as $u_{0i} \sim N(0, \tau_{00})$ and $\text{COV}[X_{ti}, u_{0i}] = 0$. Thus, there is room for additional level-2 covariates to account for the remaining variability of the dependent variable. However, if the model fails to fully explain the level-2 variability, that is, if there is an omitted variable at level 2, the covariance between the unobserved heterogeneity and the time-varying predictor becomes nonzero, leading to the omitted variable bias in the within-person effect γ_{10} (Dieleman & Templin, 2014; Rabe-Hesketh & Skrondal, 2012; Raudenbush & Bryk, 2002). The bias introduced by the violation of the exogeneity is called *endogeneity bias* in econometrics (Greene, 2018; Wooldridge, 2010, 2016).

There are several methods to estimate the parameters of random effects model. Econometricians prefer the feasible generalized least squares (FGLS), while others prefer the maximum likelihood (ML) or restricted maximum likelihood (REML). FGLS and GLS transform data by premultiplying it by $\mathbf{V}^{-\frac{1}{2}}$, where $\mathbf{V}$ is the covariance matrix of the composite error (i.e., $v_{ti} = u_{0i} + r_{ti}$) to make the nonstandard error structure homoscedastic. For the random effects model in Equation 2.7, a factor λ defined in the following way is estimated using the variance estimates from the pooled OLS regression:

$$\lambda = 1 - \left(\frac{\hat{\sigma}^2}{\hat{\sigma}^2 + T\hat{\tau}_{00}} \right)^{\frac{1}{2}}. \quad (2.8)$$

Then, the coefficients γ_{00} and γ_{10} are estimated through a pooled OLS regression with the following transformed model:

$$Y_{ti} - \lambda \bar{Y}_{\cdot i} = \gamma_{00}(1 - \lambda) + \gamma_{10}(X_{ti} - \lambda \bar{X}_{\cdot i}) + (r_{ti} - \lambda \bar{r}_{\cdot i}). \quad (2.9)$$

Wooldridge (2010, 2016) called this transformation *quasi-time-demeaning* and the resulting FGLS estimator *random effect estimator*. Similar to the time-demeaning in the fixed effects estimator, we can see the quasi-time-demeaning as a variation of centering. A difference is that the quasi-time-demeaning subtracts a fraction of person mean rather than full person

mean from a raw variable. As λ approaches the unity, the two estimates become closer to each other. Put another way, when λ deviates from one, the $\hat{\gamma}_{10}$ in the random effects model becomes dissimilar to the within-person effect $\hat{\beta}_1$ in the fixed effects model, implying that the two coefficients have different interpretations. Details are discussed later in this section.

2.1.3 Correlated Random Effects Model

Correlated random effects model (Chamberlain, 1982; Mundlak, 1978; Wooldridge, 2010, 2016, 2019) regards the person-specific intercept β_{0i} in the random effects model as a function of the time-varying predictors. As a result, the person-specific intercept can be correlated with the time-varying predictors. In organizational studies, a linear function of a person mean is often used as the modeling function (Mundlak, 1978). In the simplest case, the correlated random effects model can be written as

$$Y_{ti} = \beta_{0i} + \beta_{1i}X_{ti} + r_{ti}, \quad (2.10)$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01}\bar{X}_{\cdot i} + u_{0i}, \quad (2.11)$$

$$\beta_{1i} = \gamma_{10}, \quad (2.12)$$

or equivalently,

$$Y_{ti} = \gamma_{00} + \gamma_{01}\bar{X}_{\cdot i} + \gamma_{10}X_{ti} + u_{0i} + r_{ti}. \quad (2.13)$$

Note that $\text{COV}[X_{ti}, u_{0i}] = \text{COV}[\bar{X}_{\cdot i}, u_{0i}] = 0$ by construction. In multilevel literature, Model 2.13 is also called (*multilevel*) *contextual effects model* or *compositional effects model* (Burstein, 1980; Castellano et al., 2014; Raudenbush, 1989a; Raudenbush & Bryk, 2002; Raudenbush & Willms, 1995; Willms, 1986)⁵.

⁵Literature before the advent of multilevel models called Model 2.13 without the random intercept a contextual model.

A difference between the correlated and standard random effects model is that the former has a person mean $\bar{X}_{.i}$ as a level-2 predictor in addition to a time-varying predictor X_{ti} at level 1. Since X_{ti} and $\bar{X}_{.i}$ are simultaneously included in the model and $\bar{X}_{.i}$ only contains the between-person variation, the between-person part of X_{ti} is partialled out and only the within-person effect is reflected in its coefficient γ_{10} . Conversely, the coefficient of the person mean γ_{01} reflects the difference between the between-person effect and the within-person effect because of the shared variance between X_{ti} and $\bar{X}_{.i}$. In other words, γ_{01} represents an increment of the outcome by a unit increase of the person mean after controlling for the individual level predictor X_{ti} . In organizational research, this increment is known as *contextual effect* or *compositional effect* (Blalock, 1984; Erbring & Young, 1979; Firebaugh, 1978; Raudenbush, 1989a; Raudenbush & Bryk, 2002). In educational settings, it is also known as *big-fish-little-pond effect* (Marsh, 1987; Marsh & Parker, 1984) or *frog-pond effect* (Burstein, 1980).

One practical issue in using the correlated random effects model is that the standard error of the coefficients may become extremely large when the correlation between X_{ti} and $\bar{X}_{.i}$ is high. Specifically, the correlated random effects model may suffer from multicollinearity (Blalock, 1984; Plewis, 1989; Raudenbush, 1989a, 1989b). In such a case, the group-mean centering of the raw predictor can alleviate the multicollinearity (Raudenbush & Bryk, 2002). Another practical issue is that the contextual effects may be difficult to interpret in longitudinal studies. The contextual effects in longitudinal studies represent a sort of individual differences; however, it is unclear what is left in the effects after controlling for the temporal level of the time-varying predictors. For this reason, the correlated random effects model may not be popular among researchers who are interested in longitudinal studies.

2.1.4 Within-Between Model

Within-between model (Bell & Jones, 2015; Bell et al., 2018, 2019; Dieleman & Templin, 2014; Greene, 2018), which is also known as *hybrid model* (Allison, 2005, 2009; Hirai & Kaufman, 2017) and *including-the-group-means approach* (Castellano et al., 2014), is an extension of the standard random effect model. The within-between model centers a time-varying predictor around a person mean and the person mean is included in the model as a level-2 covariate of the individual intercept. More specifically, the within-between model can be expressed as

$$Y_{ti} = \beta_{0i} + \beta_{1i}(X_{ti} - \bar{X}_{\cdot i}) + r_{ti}, \quad (2.14)$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01}\bar{X}_{\cdot i} + u_{0i}, \quad (2.15)$$

$$\beta_{1i} = \gamma_{10}. \quad (2.16)$$

In a combined form, we have

$$Y_{ti} = \gamma_{00} + \gamma_{01}\bar{X}_{\cdot i} + \gamma_{10}(X_{ti} - \bar{X}_{\cdot i}) + u_{0i} + r_{ti}. \quad (2.17)$$

In multilevel literature, this model does not have a specific name. Instead, it is often introduced as a variant of the random intercept model and addressed as one of the centering options for level-1 predictors (Fitzmaurice et al., 2011; Hox et al., 2018; Rabe-Hesketh & Skrondal, 2012; Raudenbush & Bryk, 2002; Snijders & Bosker, 2012).

One of the important features of the within-between model is that it has separate coefficients that directly represent the within-person effect and between-person effects. This direct capture of both effects can be achieved by decomposing the level-1 predictor into two orthogonal (i.e., uncorrelated) components, namely the person-mean centered predictor $X_{ti} - \bar{X}_{\cdot i}$ and the person mean $\bar{X}_{\cdot i}$. As seen in its subscript, the person mean $\bar{X}_{\cdot i}$ does not change over time and only includes the between-person variation. Furthermore,

since the person mean is subtracted from the raw predictor, the mean deviation score $X_{ti} - \bar{X}_{\cdot i}$ only contains the within-level variation and no between-person component. As such, the corresponding coefficients, γ_{01} and γ_{10} , can be interpreted as the between-person effect and the within-person effect, respectively.

Although Paul Allison, one of the significant contributors of the within-between model, named the model a hybrid model (i.e., a hybrid of fixed effects model and random effects model), the model is considered as an extension of the random effects model. Allison (2005) actually illustrated the within-between model using the MIXED procedure, which is a SAS command for the mixed effects model. Also, Allison (2009) noted in describing the within-between model that “instead of doing OLS regression, we estimate a random effects model to ensure that the standard errors reflect the dependence among the multiple observations for each person” (p. 24). Andrew Bell and his colleagues emphasized this point, arguing that the fixed effects model is also a submodel of the within-between and random effects model because they can incorporate person-level covariates, while the fixed effect model cannot (Bell & Jones, 2015; Bell et al., 2019). Bell’s argument is partly correct but partly incorrect because the random effects model and within-between model can incorporate level-2 covariates by constraining the covariance between the time-varying predictor and the random intercept to be zero (i.e., the exogeneity assumption), whereas the fixed effects model does not have such a constraint. Thus, we can also say that the fixed effects model is a generalization, rather than a special case, of the random effects model in terms of the model constraint.

The within-between model has a close relationship with the standard and correlated random effects model. The connection between them can be found when the within-between model is reparametrized as follows:

$$Y_{ti} = \gamma_{00}^{WB} + \gamma_{01}^{WB} \bar{X}_{\cdot i} + \gamma_{10}^{WB} (X_{ti} - \bar{X}_{\cdot i}) + u_{0i} + r_{ti}$$

$$\begin{aligned}
&= \gamma_{00}^{WB} + (\gamma_{01}^{WB} - \gamma_{10}^{WB})\bar{X}_{.i} + \gamma_{10}^{WB}X_{ti} + u_{0i} + r_{ti}, \\
&= \gamma_{00}^{CR} + \gamma_{01}^{CR}\bar{X}_{.i} + \gamma_{10}^{CR}X_{ti} + u_{0i} + r_{ti},
\end{aligned} \tag{2.18}$$

and if $\gamma_{01}^{CR} = 0$, the model becomes

$$Y_{ti} = \gamma_{00}^{RE} + \gamma_{10}^{RE}X_{ti} + u_{0i} + r_{ti}. \tag{2.19}$$

Here, γ_{pq}^{WB} , γ_{pq}^{CR} , and γ_{pq}^{RE} are the coefficients of the within-between model (WB), the correlated random effects model (CR), and the standard random effects model (RE), respectively ($p, q \in \{0, 1\}$). Equations 2.18 and 2.19 mean that (a) the within-between model is mathematically equivalent to the correlated random effects model, (b) both γ_{10}^{WB} and γ_{10}^{CR} can be interpreted as the within-person effect (i.e., $\gamma_{10}^{WB} = \gamma_{10}^{CR}$), and (c) the contextual effect is equal to a difference of between-person effect and within-person effect from within-between model ($\gamma_{01}^{CR} = \gamma_{01}^{WB} - \gamma_{10}^{WB}$). In other words, the between-person effect is a sum of the within-person effect and the contextual effect ($\gamma_{01}^{WB} = \gamma_{10}^{CR} + \gamma_{01}^{CR}$).

Furthermore, as described before, the standard random effects model can be seen as a constrained correlated random effects model with $\gamma_{01}^{CR} = 0$ (i.e., no contextual effect). This relation implies that the standard random effects model assumes the equality of the within-person effect and the between-person effect ($\gamma_{10}^{WB} = \gamma_{01}^{WB}$). If the equality assumption is violated, the estimate of γ_{10}^{RE} becomes different from the within-person effect γ_{10}^{WB} , resulting in an optimally weighted average of the within-person effect and the between-person effect (Fitzmaurice et al., 2011; Greene, 2018; Hamaker & Muthén, 2020; Raudenbush & Bryk, 2002; Wooldridge, 2010). More specifically,

$$\gamma_{10}^{RE} = \eta\gamma_{01}^{WB} + (1 - \eta)\gamma_{10}^{WB}, \tag{2.20}$$

or

$$\beta_T = \eta\beta_B + (1 - \eta)\beta_W, \tag{2.21}$$

where

$$\eta = \frac{(1 - \rho_y)\rho_x}{(1 - \rho_y) + T\rho_y(1 - \rho_x)}, \quad (2.22)$$

$\beta_T = \gamma_{10}^{RE}$, $\beta_B = \gamma_{01}^{WB}$, and $\beta_W = \gamma_{10}^{WB}$. ρ_y and ρ_x are the proportion of between-person variability to the total variation in the outcome and predictor, respectively. Some researchers call β_T *conflated effect* (Preacher et al., 2010; Rights et al., 2019) or *smushed effect* (Hoffman, 2015, 2019) and others see it as a biased estimate of the within-person effect (Castellano et al., 2014). The formula for the weight η tells us conditions under which β_T approximates the within-person effect β_W . That is, for the coefficient β_T to be close to the within-person effect β_W , the weight η needs to be as small as possible. This occurs when (a) ρ_y is large (i.e., between-level variation is relatively large in the outcome), (b) ρ_x is small (between-level variation is relatively small), and/or (c) T is large (more measurements). Fitzmaurice et al. (2011) provided possible scenarios in which these could occur. For example, ρ_x could be small when variation in the covariate is constrained or fixed by design (e.g., time elapsed, covariate under crossover designs).

2.1.5 Adaptive Centering

Raudenbush (2009) proposed *adaptive centering* as an alternative to the fixed effects model. Adaptive centering model is a random effects model that only has within-person predictors (i.e., person-mean centered time-varying predictors). The basic model equation of the adaptive centering is

$$Y_{ti} = \gamma_{00} + \gamma_{10}(X_{ti} - \bar{X}_{\cdot i}) + u_{0i} + r_{ti}. \quad (2.23)$$

This model is also called *within model* (Bell et al., 2018), *group-mean centering method* (Castellano et al., 2014), and *hybrid fixed effects model with cluster-mean centering* (Hirai & Kaufman, 2017).

In addition to the basic model, Raudenbush (2009) also provided a general form of adaptive centering. Let $\mathbf{y}$ be a vector of outcomes and $\mathbf{X}^*$ be a matrix of adaptively centered predictors. Then, the general model can be written as

$$\mathbf{y} = \mathbf{1}\theta + \mathbf{X}^*\boldsymbol{\beta} + \mathbf{A}\mathbf{b} + \mathbf{e}, \quad (2.24)$$

where θ is an intercept parameter; $\mathbf{1}$ is a vector of ones; $\boldsymbol{\beta}$ is a vector of the fixed effects; $\mathbf{A}$ is a design matrix for the random effects; $\mathbf{b}$ is a vector of the random effects, which is distributed as $\mathbf{b} \sim N(\mathbf{0}, \boldsymbol{\Omega})$; and $\mathbf{e}$ is an error term, which is distributed as $\mathbf{e} \sim N(\mathbf{0}, \boldsymbol{\Sigma})$. Note that $\text{VAR}[\mathbf{y}] = \mathbf{V} = \mathbf{A}\boldsymbol{\Omega}\mathbf{A}' + \boldsymbol{\Sigma}$. Then, the adaptive centering of the raw predictors $\mathbf{X}$ is defined as

$$\begin{aligned} \mathbf{X}^* &= \mathbf{X} - \mathbf{A}(\mathbf{A}'\boldsymbol{\Sigma}^{-1}\mathbf{A})^{-1}\mathbf{A}\boldsymbol{\Sigma}^{-1}\mathbf{X} \\ &= [\mathbf{I} - \mathbf{A}(\mathbf{A}'\boldsymbol{\Sigma}^{-1}\mathbf{A})^{-1}\mathbf{A}'\boldsymbol{\Sigma}^{-1}]\mathbf{X} \\ &= (\mathbf{I} - \mathbf{P}_{\mathbf{A}/\boldsymbol{\Sigma}^{-1}})\mathbf{X}, \end{aligned} \quad (2.25)$$

where $\mathbf{P}_{\mathbf{A}/\boldsymbol{\Sigma}^{-1}} = \mathbf{A}(\mathbf{A}'\boldsymbol{\Sigma}^{-1}\mathbf{A})^{-1}\mathbf{A}'\boldsymbol{\Sigma}^{-1}$. This general form reduces to Equation 2.23 when $\mathbf{A} = \mathbf{1}$ and $\boldsymbol{\Sigma} = \sigma^2\mathbf{I}$. In this case, the projection matrix $\mathbf{P}_{\mathbf{A}/\boldsymbol{\Sigma}^{-1}}$ is equal to

$$\mathbf{P}_{\mathbf{A}/\boldsymbol{\Sigma}^{-1}} = \mathbf{1}[\mathbf{1}'(\sigma^{-2}\mathbf{I})\mathbf{1}]^{-1}\mathbf{1}'(\sigma^{-2}\mathbf{I}) = \frac{1}{N}\mathbf{J},$$

where N is the total number of observations and $\mathbf{J} = \mathbf{1}\mathbf{1}'$ is an $N \times N$ matrix of ones.

Therefore, $\mathbf{X}^* = (\mathbf{I} - \mathbf{P}_{\mathbf{A}/\boldsymbol{\Sigma}^{-1}})\mathbf{X} = (\mathbf{I} - \frac{1}{N}\mathbf{J})\mathbf{X}$, representing person-mean centered predictors. More generally, $\mathbf{X}^* = (\mathbf{I} - \mathbf{P}_{\mathbf{A}/\boldsymbol{\Sigma}^{-1}})\mathbf{X}$ can be seen as a projection of $\mathbf{X}$ onto the orthogonal complement of the column space spanned by $\mathbf{A}$ with the metric matrix $\boldsymbol{\Sigma}^{-1}$ (Yanai et al., 2011). As a result, $\mathbf{X}^*$ is always uncorrelated with the random effects $\mathbf{A}\mathbf{b}$, which contain all the between-person variability in the model. This in turn means that $\mathbf{X}^*$ only includes the within-person variability.

Raudenbush (2009) pointed out some advantages of the adaptive centering approach

over the fixed effects model. First, the adaptive centering can produce the same estimate for the within-person effect as that of the fixed effects model. Especially in the case of balanced data, the adaptive centering and the fixed effects model yield the identical point estimate and standard error⁶ for the within-person effect. Second, the adaptive centering approach can be extended to incorporate multiple levels, including cross-classified cases. For example, to illustrate adaptive centering, Raudenbush used a “simple” example, in which students are longitudinally assessed, and between-teacher and between-school variations are considered (i.e., a cross-classified, four-level model). In contrast, the fixed effects model cannot cope with multiple levels, though it can accommodate a certain type of cross-classified membership (e.g., two-way fixed effects; Bell & Jones, 2015; McNeish & Kelley, 2019)⁷. Third, adaptive centering can have random effects for the treatment effects at each level. This is one of the most important advantages of the adaptive centering over the fixed effects model and standard (i.e., non-multilevel) SEM models. Finally, the computation is relatively less burdensome and more straightforward when the model includes many level-1 predictors. This is so because centering does not increase the number of independent variables in a model, thus reducing the risk of multicollinearity.

2.1.6 Latent Variable Centering

The models reviewed so far use an observed person mean $\bar{X}_{.i}$ to disentangle the within-person effect and between-person effect. However, it is well known in organizational studies that the observed group mean can be unreliable when the cluster size is small. In longitudinal studies, the cluster size in calculating $\bar{X}_{.i}$ is the number of measurements T , which is typically small, compared to the group size in organizational studies. We do not

⁶In the case of unbalanced data, the point estimate is still the same. The standard error is, however, slightly different when the sample size is small.

⁷McNeish and Kelley (2019) mentioned to the possibility of including the fixed effects for the highest-level units and random effects for lower-level units.

usually expect hundreds of measurements in a longitudinal study. Instead, only several measurements are available in most cases. Therefore, the low reliability issue seems inevitable in longitudinal studies as long as we use observed means as a means of the within-between disaggregation.

Some researchers recently developed latent variable approaches to the low reliability issue, in which a person mean is replaced by its true score. The replacement is achieved by modeling a time-varying predictor and an outcome in the following manner:

$$X_{ti} = \mu^X + u_i^X + r_{ti}^X = X_i^B + X_{ti}^W, \quad (2.26)$$

$$Y_{ti} = \mu^Y + u_i^Y + r_{ti}^Y = Y_i^B + Y_{ti}^W, \quad (2.27)$$

where $X_i^B = \mu^X + u_i^X$ and $Y_i^B = \mu^Y + u_i^Y$ are the between-level components of the time-varying predictor and outcome, respectively; $X_{ti}^W = r_{ti}^X$ and $Y_{ti}^W = r_{ti}^Y$ are the within-person components; μ^X and μ^Y are the grand means; u_i^X and u_i^Y are random variables representing how much the person mean deviates from the grand mean; and r_{ti}^X and r_{ti}^Y are random variables, which are the composites of time-specific deviations from the person mean and random error. These random variables are assumed to be distributed as $\mathbf{u}_i = (u_i^X, u_i^Y)' \sim N(\mathbf{0}, \mathbf{T})$ and $\mathbf{r}_{ti} = (r_{ti}^X, r_{ti}^Y)' \sim N(\mathbf{0}, \mathbf{\Sigma})$, where

$$\mathbf{T} = \text{VAR}[\mathbf{u}_i] = \begin{pmatrix} \tau_{XX} & \tau_{XY} \\ \tau_{XY} & \tau_{YY} \end{pmatrix}, \quad (2.28)$$

$$\mathbf{\Sigma} = \text{VAR}[\mathbf{r}_{ti}] = \begin{pmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{pmatrix}. \quad (2.29)$$

Then, a within-level regression and a between-level regression can be written as

$$Y_{ti}^W = \beta_W X_{ti}^W + \varepsilon_{ti}^W, \quad (2.30)$$

$$Y_i^B = \mu^Y + \beta_B X_i^B + \varepsilon_i^B. \quad (2.31)$$

In a combined form, we have

$$\begin{aligned}
Y_{ti} &= Y_i^B + Y_{ti}^W \\
&= \mu^Y + \beta_B X_i^B + \beta_W X_{ti}^W + \varepsilon_i^B + \varepsilon_{ti}^W \\
&= \mu^Y + \beta_B(\mu^X + u_i^X) + \beta_W r_{ti}^X + \varepsilon_i^B + \varepsilon_{ti}^W \\
&= \beta_0 + \beta_B u_i^X + \beta_W r_{ti}^X + \varepsilon_i^B + \varepsilon_{ti}^W,
\end{aligned} \tag{2.32}$$

where $\beta_0 = \mu^Y + \beta_B \mu^X$. This model is called *latent variable centering model*, *latent cluster mean model*, and *latent covariate model* (Asparouhov & Muthén, 2019; Croon & van Veldhoven, 2007; Gottfredson, 2019; Hoffman, 2019; Lüdtke et al., 2008, 2011; Marsh et al., 2009; Shin & Raudenbush, 2010; Yang & Cai, 2014; Zitzmann et al., 2015, 2016; Zitzmann, 2018). Similar to the relationship between the within-between model and the correlated random model, the latent variable centering model can also be rewritten as

$$\begin{aligned}
Y_{ti} &= \mu^Y + \beta_B X_i^B + \beta_W X_{ti}^W + \varepsilon_i^B + \varepsilon_{ti}^W \\
&= \mu^Y + (\beta_B - \beta_W) X_i^B + \beta_W (X_i^B + X_{ti}^W) + \varepsilon_i^B + \varepsilon_{ti}^W \\
&= \mu^Y + \beta_C X_i^B + \beta_W X_{ti} + \varepsilon_i^B + \varepsilon_{ti}^W,
\end{aligned} \tag{2.33}$$

where $\beta_C = \beta_B - \beta_W$. Since X_i^B is a latent variable, this model is called *latent contextual effects model*.⁸

Although those studies addressed the same single model, they differed in their estimation methods, which can be classified into two categories; two-step ad-hoc adjustment approaches and one-step model-based approaches. Croon and van Veldhoven (2007) developed ANOVA type estimation (i.e., the method of moments). This is a two-step procedure, where the person mean is adjusted for its unreliability in the first step and OLS regression with the adjusted group mean is conducted in the second step.

⁸The conventional context effects model is sometimes called the *constant centering model*, *manifest covariate model*, or *sample cluster mean model* since it includes observed person mean.

Zitzmann (2018) extended Croon and van Veldhoven’s approach such that the estimation of the level-2 covariance matrix becomes stabilized when the cluster size is small. Zitzmann recognized the ANOVA technique as a special case of the Bayesian expected a posteriori (EAP) estimator and proposed using an EAP-based estimator for the between-level covariance matrix. Although these adjustment methods can provide an unbiased estimate of the contextual effects, Croon and van Veldhoven’s approach and Zitzmann’s approach are considered less efficient than one-step methods because they only use the limited information (i.e., the first and second moments). Grilli and Rampichini (2011) and Gottfredson (2019) proposed another type of two-step adjustment. They adjusted the estimates of the coefficients rather than the group mean by dividing the estimates by the reliability of the group mean. All of the two-step approaches can give us an unbiased estimate of the within-person and between-person effect; however, they require the computation of the standard error of the corrected estimates by some means (e.g., bootstrap, delta method).

Oliver Lüdtke and their colleagues derived a formula for the bias in the between-person effect and contextual effect due to small sample size at level 1, which Asparouhov and Muthén (2019) called *Lüdtke’s bias*:

$$\mathbb{E}[\hat{\beta}_B] - \beta_B = \mathbb{E}[\hat{\beta}_C] - \beta_C = (\beta_W - \beta_B) \frac{(1 - ICC)/T}{ICC + (1 - ICC)/T}, \quad (2.34)$$

where ICC is the intraclass correlation of the time-varying predictor. In order to eliminate the bias, they developed the full information maximum likelihood estimation for the latent centering model (Lüdtke et al., 2008, 2011). Unlike Croon and van Veldhoven’s (2007) approach, Lüdtke et al.’s (2008) approach was a one-step estimation of the latent contextual effect model, which utilized the full information in the data rather than the limited information. Shin and Raudenbush (2010) and Yang and Cai (2014) also adopted the full information maximum likelihood estimation. In their approach, the latent group

mean was seen as missing data, and the EM algorithm and Metropolis-Hastings Robbins-Monro algorithm were used in the parameter estimation. As a result, their methods outperformed the traditional observed-mean centering in their simulation studies, especially when the level-1 and level-2 sample size and ICC are small. In addition, the one-step methods have an advantage in computing standard error; they do not need additional computation, thus standard statistical software can produce the standard error for the between-person or contextual effect. However, the latent centering methods resulted in a larger root mean square error (RMSE) than the two-step adjustment methods when either the level-1 or level-2 sample size is small or when ICC is small (Lüdtke et al., 2008; Zitzmann et al., 2015). This is because the latent variable centering model has more parameters, thus requires a larger sample size than the step-wise adjustment approaches. Zitzmann et al. (2015) and Zitzmann et al. (2016) tackled the large variability issue from the Bayesian perspective. They investigated how different prior distributions such as inverse gamma, half-Cauchy, and uniform distribution could stabilize the estimation of variance components and the resulting contextual effects. Similar to regularization techniques (e.g., LASSO, ridge regression, elastic net), by introducing a small amount of bias, the Bayesian approach successfully reduced RMSE and produced more precise estimates than the maximum likelihood approach (i.e., Lüdtke's approach) even though the priors were noninformative. The effect of introducing a prior distribution was greater when ICC was small.

2.1.7 Centering and Interpretation of the Coefficients

Table 2.1 summarizes the models reviewed in this section from the viewpoints of the level-1 and level-2 predictors and their interpretation. For the level-1 predictor, there are two types of models; one has person-mean centered predictors (i.e., fixed effects,

within-between, adaptive centering, latent variable centering) and the other has raw predictors (random effects, correlated random effects). Except for the random effects model, the coefficient of the level-1 predictor can be interpreted as a within-person effect. The coefficient of the random effect model cannot generally be interpreted as a within-person effect because the model does not have any term to control for the between-person differences (i.e., no level-2 predictor). The coefficient can be interpreted as a within-person effect only when there is no contextual effect, or equivalently, when the size and direction of the within-person effect are the same as those of the between-person effect. When none of them does hold, the coefficient becomes biased (conflated effect).

For level-2 predictors, there are also two types of models. That is, models with person mean (within-between, correlated random effects, latent variable centering) and models without person mean (fixed effects, random effects). The interpretation of the coefficient of the person mean depends on what type of level-1 predictors the model has. When the model has a person-mean centered predictor at level 1, the level-2 coefficient can be interpreted as the between-person effect since the person-mean centered predictor does not contain the between-person component and the between-person component in the level-2 predictor is fully reflected in the coefficient. In contrast, when the model has a raw level-1 predictor, the level-2 coefficient represents the contextual effect, which is the difference of the between-person effect and within-person effect. This change in the interpretation occurs because the raw level-1 predictor and the level-2 predictor (i.e., person mean) are correlated, thus the shared variance between level-1 and level-2 predictors is partialled out.

In sum, we have two ways to obtain the within-person effect; one is removing the between-person component from the time-varying predictor by centering, and the other is partialling out the between-person component by including a person mean. Depending on which strategy we take, the interpretation of the coefficient of the level-2 predictor changes.

Table 2.1*Summary of Regression Models with Time-Varying Predictor*

Model	Level 1		Level 2	
	Predictor	Coefficient	Predictor	Coefficient
Fixed Effects	$X_{ti} - \bar{X}_{.i}$	within-person effect	—	—
Random Effects	X_{ti}	conflated effect	—	—
Within-Between	$X_{ti} - \bar{X}_{.i}$	within-person effect	$\bar{X}_{.i}$	between-person effect
Correlated Random Effects	X_{ti}	within-person effect	$\bar{X}_{.i}$	contextual effect
Adaptive Centering	$X_{ti} - \bar{X}_{.i}$	within-person effect	—	—
Latent Variable Centering	r_{ti}^X	within-person effect	u_i^X	between-person effect

Note. All models except the fixed effects model have an random intercept u_i^Y and can incorporate covariates at level 2. Fixed effects model centers Y and eventually centers X . The level-1 coefficient of random effects model can be interpreted as the within-person effect if there is no contextual effect. The level-1 and level-2 predictors in latent variable centering are latent variables.

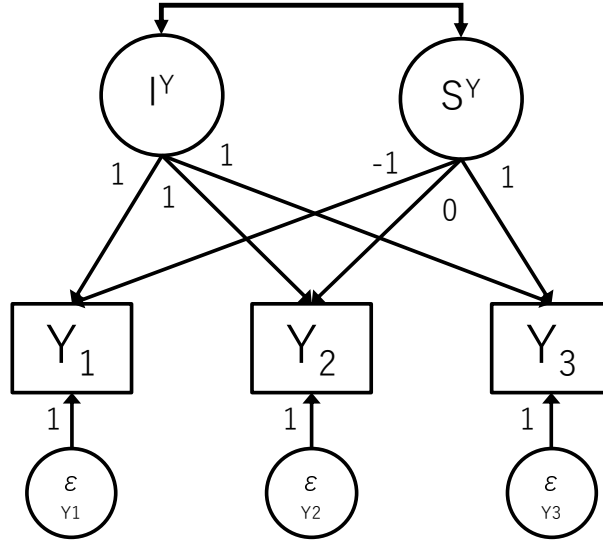
2.2 Latent Curve Models with Time-Varying Predictors

2.2.1 Latent Curve Model with Cross-Level Covariances

The models described in Section 2.1 are variations of the mixed effects model, more specifically, the random intercept model. In this section, the latent curve model (LCM) is introduced, which is another class of longitudinal models that can handle time-varying predictors.

LCM is one of the popular models in structural equation modeling (SEM) and it is tailored for analyzing change and growth over time. The basic idea of LCM is that we can draw a growth trajectory for each person as a function of time. When the functional form is linear, the standard LCM can be written as

$$Y_{ti} = \pi_{0i} + \pi_{1i}\text{Time}_{ti} + \varepsilon_{ti}. \quad (2.35)$$

Figure 2.1*Latent Curve Model for variable Y (LCMy for short)*

In this model equation, the regression coefficients π_{0i} and π_{1i} have an index i , indicating they can vary across persons ($i = 1, \dots, N$). These coefficients can be seen as random variables (i.e., latent factors) that are distributed as $(\pi_{0i}, \pi_{1i})' \sim N(\mathbf{0}, \Psi)$. Time_{ti} is a time variable, which represents, for example, the time elapsed from a reference time point ($t = 1, \dots, T$). ε_{ti} is an error term that is assumed to be distributed as $\varepsilon_{ti} \sim N(0, \sigma^2)$. Note that the time variable Time_{ti} is centered around person mean such that $\sum_{t=1}^T \text{Time}_{ti} = 0$ for all i . Model 2.35 is also called *growth curve model* and can be implemented within the mixed effects model framework (Laird & Ware, 1982; Bryk & Raudenbush, 1987; Raudenbush & Bryk, 2002) and the SEM framework (Meredith & Tisak, 1990; Duncan et al., 1998; Bollen & Curran, 2006). In particular, when the growth curve model is implemented as an SEM model, it is called the latent curve model or *latent growth curve model*.

Figure 2.1 is a path diagram of an LCM with three waves. Y_1 , Y_2 , and Y_3 are observed outcomes at time 1, 2, and 3, and latent factors denoted by I^Y and S^Y are an

intercept factor and a slope factor, respectively. Unlike ordinary confirmatory factor analysis (CFA), LCM has fixed factor loadings to draw growth trajectories. Specifically, the loadings of the intercept factor are fixed at one and those of the slope factor are fixed at the time elapsed from a reference time point (i.e., Time_{ti}). Together with the error term ε_{ti}^Y , the LCM depicted in Figure 2.1 has a model equation of

$$Y_{ti} = I_i^Y + \lambda_t S_i^Y + \varepsilon_{ti}^Y, \quad (2.36)$$

where I_i^Y , S_i^Y , and ε_{ti}^Y are assumed to be distributed as

$$I_i^Y \sim N(\tau_I^Y, \psi_{II}^Y), \quad (2.37)$$

$$S_i^Y \sim N(\tau_S^Y, \psi_{SS}^Y), \quad (2.38)$$

$$\varepsilon_{ti}^Y \sim N(0, \sigma_Y^2). \quad (2.39)$$

It is also assumed that $\text{COV}[I_i^Y, S_i^Y] = \psi_{IS}^Y$, $\text{COV}[\varepsilon_{ti}^Y, \varepsilon_{it'}^Y] = 0$, $\text{COV}[I_i^Y, \varepsilon_{ti}^Y] = 0$, and $\text{COV}[S_i^Y, \varepsilon_{ti}^Y] = 0$. λ_t is a factor loading at time t , representing time elapsed since a reference time point. In Figure 2.1, the reference time point is set at the second wave ($t = 2$), thus λ_t 's are coded as $\lambda_1 = -1$, $\lambda_2 = 0$, and $\lambda_3 = 1$ (i.e., λ_t 's are centered). As implied by its subscript, the factor loadings at time t take the same value across individuals, meaning that LCM assumes a time-balanced design.

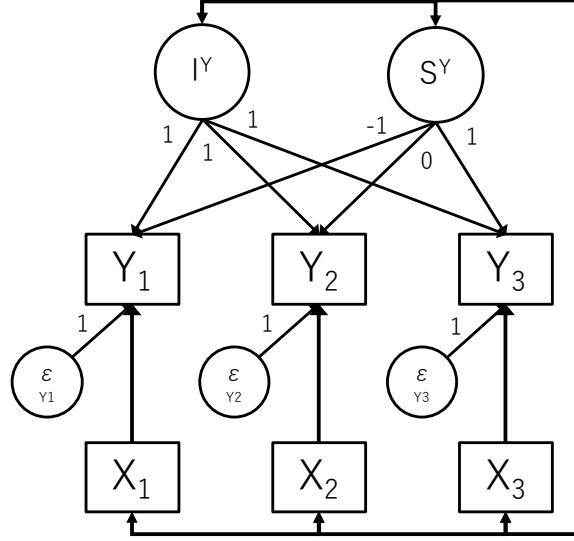
Time-varying predictors can be added to Model 2.36 in various ways. For instance, the simplest way is adding a time-varying predictor to Equation 2.36 as a simultaneous predictor:

$$Y_{ti} = I_i^Y + \lambda_t S_i^Y + \gamma X_{ti} + \varepsilon_{ti}^Y. \quad (2.40)$$

This model can be seen as a restricted multivariate, multiple regression model in which I_i^Y , S_i^Y , and X_{ti} are independent variables and X_{ti} only affects the outcome that is measured at the same time point. In other words, this model does not consider the lagged effects (i.e.,

Figure 2.2

Latent Curve Model for Y with Cross-Level Covariances (LCMy-CLC for short)



autoregressive effect and cross-lagged effect). Also, as in the usual multiple regression model, all independent variables are allowed to be correlated with each other. Hence, the model has the following covariances as model parameters:

$$\text{COV}[X_{ti}, I_i^Y] = \phi_{tI}^{XY}, \quad (2.41)$$

$$\text{COV}[X_{ti}, S_i^Y] = \phi_{tS}^{XY}, \quad (2.42)$$

$$\text{COV}[X_{ti}, X_{t'i}] = \phi_{tt'}^{XX}. \quad (2.43)$$

In Figure 2.2, these covariances are depicted as double-headed arrows. The covariances can be called *cross-level covariances* since the time-varying predictors and the growth factors are at different levels (i.e., X_{ti} is at level 1 [occasion level], while I_i^Y and S_i^Y are at level 2 [person level]). I refer to this model as a *latent curve model for Y with cross-level covariances* or LCMY-CLC for short.

As with the contextual effects model, the between-person effects in the time-varying predictor are partialled out since I_i^Y and S_i^Y are also in the model (recall that I_i^Y and S_i^Y

only have the subscript i , indicating that they are time-invariant, between-person components). Consequently, the regression coefficient γ can be interpreted as the within-person effect of the time-varying predictors (Bollen & Curran, 2006; Curran et al., 2012; Holt, 2008; Hox et al., 2018; B. Muthén & Curran, 1997). In addition, by explicitly correlating the time-varying predictors with the person-level growth factors, the between-person effects of the time-varying predictors, more precisely, the contextual effects, are absorbed in the cross-level covariances. Curran et al. (2012) showed that the cross-level covariances contain information enough to estimate the contextual effects. They derived formulae to estimate the contextual effects from the available quantities:

$$\hat{\delta}_0 = \frac{\frac{1}{T} \sum_{t=1}^T \text{COV}[X_{ti}, I_i^Y]}{\frac{1}{T^2} \left\{ \sum_{t=1}^T \text{VAR}[X_{ti}] + \sum_{t \neq t'}^T \text{COV}[X_{ti}, X_{it'}] \right\}}, \quad (2.44)$$

$$\hat{\delta}_1 = \frac{\frac{1}{T} \sum_{t=1}^T \text{COV}[X_{ti}, S_i^Y]}{\frac{1}{T^2} \left\{ \sum_{t=1}^T \text{VAR}[X_{ti}] + \sum_{t \neq t'}^T \text{COV}[X_{ti}, X_{it'}] \right\}}. \quad (2.45)$$

Here, $\hat{\delta}_0$ and $\hat{\delta}_1$ are the estimates of the contextual effect regarding intercept and slope, respectively. Consequently, the between-person effects can be estimated as a sum of the contextual effects and the within-person effect ($\hat{\delta}_0 + \hat{\gamma}$ and $\hat{\delta}_1 + \hat{\gamma}$; Curran et al., 2012).⁹

Unfortunately, the derivation of Equation 2.45 by Curran et al. is partially incorrect. Specifically, the right-hand side of the Equation 2.45 estimates $\gamma + \delta_1$ (i.e., between-person effect) rather than δ_1 (contextual effect). This fact can be confirmed in the following way. From Equations 9.A1–9.A3 of Curran et al. (p. 247), with our notation, we have

$$\begin{aligned} Y_{ti} &= \tau_I^Y + (\gamma + \delta_0)\bar{X}_{.i} + \zeta_i^{I^Y} + [\tau_S^Y + (\gamma + \delta_1)\bar{X}_{.i}]\text{Time}_{ti} + \gamma(X_{ti} - \bar{X}_{.i}) + \varepsilon_{ti} \\ &= \tau_I^Y + \tau^{SY}\text{Time}_{ti} + \delta_0\bar{X}_{.i} + (\gamma + \delta_1)\bar{X}_{.i}\text{Time}_{ti} + \gamma X_{ti} + \zeta_i^{I^Y} + \zeta_i^{SY}\text{Time}_{ti} + \varepsilon_{ti} \end{aligned}$$

⁹Although we can easily compute the point estimates of the between-person effects, we need a special functionality of software to compute the standard errors and confidence intervals. Examples are ESTIMATE statement in the MIXED procedure in SAS and NEW option under MODELS CONSTRAINT command in Mplus.

$$= \tau^{I^Y} + \tau^{S^Y} \text{Time}_{ti} + \zeta_i^{*I^Y} + \zeta_i^{*S^Y} \text{Time}_{ti} + \varepsilon_{ti}, \quad (2.46)$$

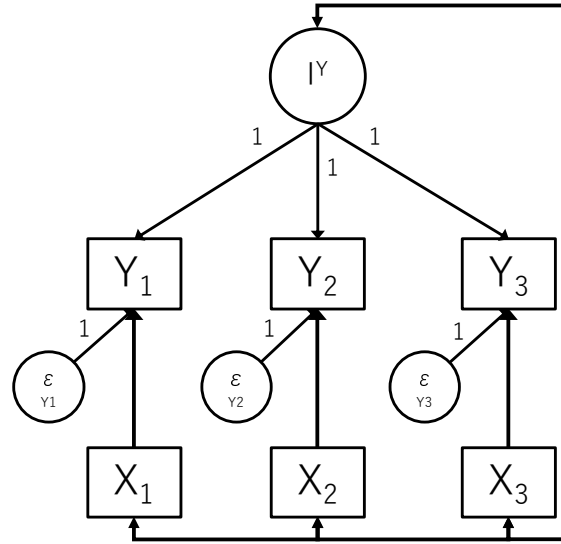
where $\zeta_i^{*I^Y} = \delta_0 \bar{X}_{.i} + \zeta_i^{IS}$ and $\zeta_i^{*S^Y} = (\gamma + \delta_1) \bar{X}_{.i} + \zeta^{SY}$. Therefore, based on the same reasoning as Curran et al.,

$$\widehat{\gamma + \delta_1} = \frac{\text{COV}[\bar{X}_{.i}, \zeta_i^{*S^Y}]}{\text{VAR}[\bar{X}_{.i}]} = \frac{\frac{1}{T} \sum_{t=1}^T \text{COV}[X_{ti}, S_i^Y]}{\frac{1}{T^2} \{ \sum_{t=1}^T \text{VAR}[X_{ti}] + \sum_{t \neq t'}^T \text{COV}[X_{ti}, X_{it'}] \}}. \quad (2.47)$$

Interestingly, LCMMy-CLC has a close relationship with the fixed effects model. As explained in the previous section, the fixed effects model is traditionally estimated by OLS with demeaning. However, Allison (2005, 2009) introduced the maximum likelihood estimation of the fixed effects model using SEM (see also Bollen & Brand, 2010; Allison et al., 2017; Moral-Benito et al., 2019). Figure 2.3 is a path diagram illustrating Allison's SEM approach. Both Allison's fixed effects model and LCMMy-CLC have an intercept factor, simultaneous within-person paths, and double-headed arrows between the random intercept and the time-varying predictors. The difference between the two models is a slope factor; LCMMy-CLC has a slope factor, while the SEM-based fixed effects model does not. Hence, the fixed effects model can be seen as a special case of LCMMy-CLC under the SEM framework, or conversely, LCMMy-CLC can be seen as an extended fixed effects model that takes a trend in the outcome into consideration. Also, no slope factor in the fixed effects model implies that it does not assume a systematic change over time in the outcome.

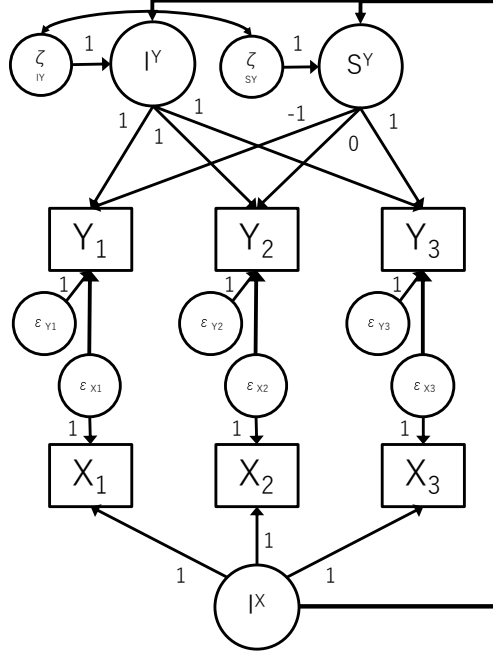
2.2.2 Latent Curve Model with a Random Intercept

One concern of LCM in addressing time-varying predictors is that it cannot include person means and time-varying predictors simultaneously in the model. This is because the standard SEM requires so-called wide-form data and a person mean in the wide-form data is a linear combination of the columns (i.e., raw time-varying predictors). Thus, the data matrix with person means cannot have a full column rank. This rank deficiency prohibits

Figure 2.3*Fixed Effects Model as an SEM Model (Allison, 2005, 2009)*

the maximum likelihood estimation of model parameters (Curran et al., 2012). Also, time-varying predictors centered around a person mean cannot be included in the model since the centered variables are ipsative or compositional (i.e., all individuals have the same mean value of zero). Consequently, the centered data has a rank of (full rank) – 1, which again makes the maximum likelihood estimation impossible (Curran et al., 2012). Note that this is not the case for multilevel models because they use so-called long-form or tall data (i.e., person-mean centered variables at different time points are stacked thus not in different columns).

Curran et al. (2012) developed an SEM model that can estimate both within-person effect and between-person effect without person-mean centering. They introduced a random intercept for the time-varying predictors (i.e., a latent variable with loadings fixed at one) and used the residuals as predictors of the outcomes. In addition, the between-person components of the outcome (the growth factors; intercept and slope factors) are regressed on the random intercept of the time-varying predictor (see Figure

Figure 2.4*Latent Curve Model for Y with Random Intercept for X (LCMy-Rlx)*

2.4). Eventually, Curran et al.'s model, which I call *latent curve model for Y with a random intercept for X* or LCMY-Rlx for short, has the following measurement and structural equations:

$$X_{ti} = I_i^X + \varepsilon_{ti}^X, \quad (2.48)$$

$$Y_{ti} = I_i^Y + \lambda_t S_i^Y + \gamma \varepsilon_{ti}^X + \varepsilon_{ti}^Y, \quad (2.49)$$

$$I_i^Y = \delta^{I^Y} + \beta_{II} I_i^X + \zeta_i^{I^Y}, \quad (2.50)$$

$$S_i^Y = \delta^{S^Y} + \beta_{SI} I_i^X + \zeta_i^{S^Y}, \quad (2.51)$$

where $I_i^X \sim N(\tau_I^X, \psi_{II}^X)$ and $\varepsilon_{ti}^X \sim N(0, \sigma_X^2)$; $\zeta_i^{I^Y}$ and $\zeta_i^{S^Y}$ are person-level residuals with distributions $\zeta_i^{I^Y} \sim N(0, \theta_{II}^Y)$ and $\zeta_i^{S^Y} \sim N(0, \theta_{SS}^Y)$. The covariance between $\zeta_i^{I^Y}$ and $\zeta_i^{S^Y}$ is denoted by θ_{IS}^Y . Also, the error term for the Y-model ε_{ti}^Y is assumed to be distributed as $\varepsilon_{ti}^Y \sim N(0, \sigma_Y^2)$ and to have no correlation with any random variables.

The core idea of LCMy-RIx is the residualized predictor ε_{ti}^X in Equation 2.49. For a given person i , the expectation of X_{ti} is equal to I_i^X . Thus, the residual $\varepsilon_{ti}^X = X_{ti} - I_i^X$ can be seen as a time-varying predictor centered around latent person mean. The corresponding coefficient γ then represents the within-person effect as in the within-between model and adaptive centering (i.e., models with time-varying predictor centered around observed person mean). On the other hand, the regression coefficients related to the growth factors, β_{II} and β_{SI} , can be interpreted as the between-person effects (recall that all terms in Equations 2.50 and 2.51 do not include the time index t).

LCMy-RIx is similar to the latent mean centering (i.e., Model 2.26–2.32) in that both methods employ the true person mean in centering, or equivalently, in residualizing the time-varying predictors. Hamaker and Muthén (2020) pointed out this similarity and empirically showed that the two models could yield unbiased estimates for the within-person and between-person effect (see also Curran et al., 2012). One of the practical differences between the two models is that the latent mean centering requires long-form data, while LCMy-RIx requires a wide-form data (Hamaker & Muthén, 2020).

2.3 Detrending Methods

2.3.1 Two-Step Detrending

As briefly described in Chapter 1, some recent studies revealed that centering could fail to disaggregate the within-person effect and the between-person effect when a time-varying predictor showed a systematic change over time. Curran and Bauer (2011) theoretically and empirically illustrated the impact of the trend in time-varying predictors. According to their results, a person mean could correctly catch the between-person variability in the time-varying predictors and produce unbiased estimates of the between-person effects as

long as the design was balanced with respect to time. Contrastingly, person-mean centering could not fully isolate the within-person component, thus yielded a biased estimate for the within-person effect when there was a trend in time-varying predictors. This occurred even when the data was time-structured. Besides, the centering approach could not produce unbiased estimates for either within-person effect or between-person effect when the design was time-unbalanced.

The primary reason why the centering approach did not perform well under a trend is that centering does not take into account the systematic change in time-varying predictors. As a result, the relationship between the predictors and outcomes is distorted in the estimation and the estimates become bias. Curran and Bauer (2011) proposed a simple solution to this issue. They suggested removing the person mean conditional on time, rather than the marginal mean, before an analysis using OLS regression analyses (see Figure 2.5). Specifically, Curran and Bauer proposed the following *two-step detrending* procedure:

1. For each person, run a simple OLS regression analysis in which the time-varying predictor is a dependent variable and time is an independent variable.
2. Run a regression analysis with a random effects model where the outcome variable is a dependent variable and the residual and the estimate of intercept from Step 1 are independent variables.

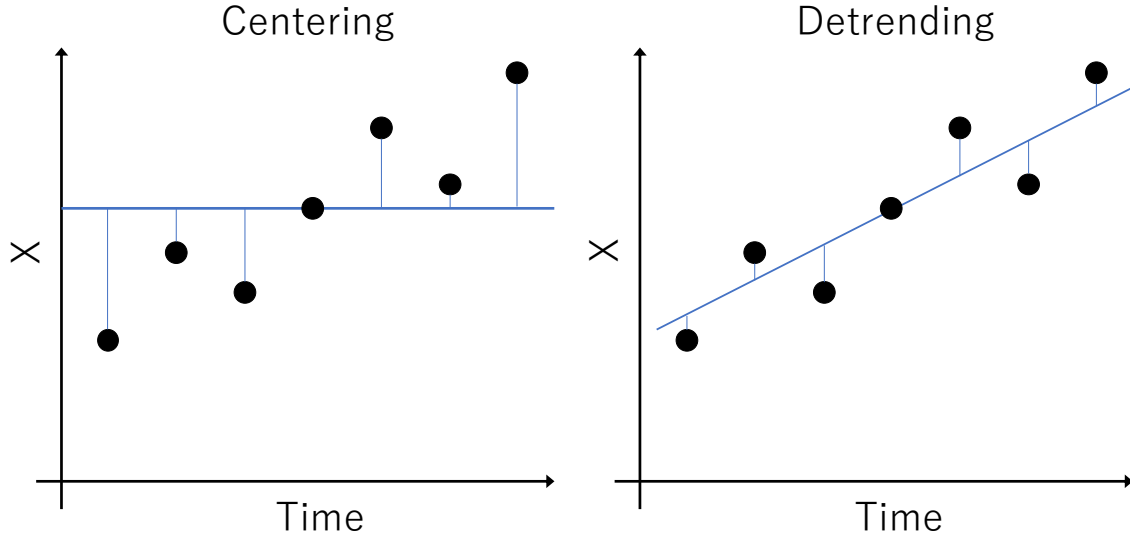
This two-step procedure can be written as the following set of equations:

$$X_{ti} = \beta_{0i}^X + \beta_{1i}^X \text{Time}_{ti} + r_{ti}^X, \quad (2.52)$$

$$Y_{ti} = \beta_{0i}^Y + \beta_{1i}^Y \hat{r}_{ti}^X + r_{ti}^Y, \quad (2.53)$$

$$\beta_{0i}^Y = \gamma_{00}^Y + \gamma_{01}^Y \hat{\beta}_{0i}^X + u_{0i}, \quad (2.54)$$

$$\beta_{1i}^Y = \gamma_{10}^Y, \quad (2.55)$$

Figure 2.5*Centering and Detrending under the Linear Trend over Time*

or equivalently,

$$Y_{ti} = \gamma_{00}^Y + \gamma_{01}^Y \hat{\beta}_{0i}^X + \gamma_{10}^Y \hat{r}_{ti}^X + u_{0i} + r_{ti}^Y. \quad (2.56)$$

Equation 2.52 corresponds to the first step of the individual OLS regressions, from which we obtain the estimates of the individual intercept and residuals ($\hat{\beta}_{0i}^X$ and $\hat{r}_{ti}^X$). Equations 2.53 through 2.55 correspond to the second step, in which the random effects model is estimated using $\hat{\beta}_{0i}^X$ and $\hat{r}_{ti}^X$ as between-person predictors and a within-person predictor, respectively.

The two-step detrending is a simple and straightforward solution to the trend problem. As Wang and Maxwell (2015) pointed out, however, this kind of tandem analysis is in general not as precise and efficient as single-step approaches. Actually, Curran and Bauer (2011) acknowledged that their proposed two-step approach did not consider the uncertainty in the estimation of β_{0i}^X and r_{ti}^X . Furthermore, the individual OLS regressions in the first step require N runs of estimation. Thus, the computation becomes more demanding as the sample size increases. Curran and Bauer (2011) suggested solutions to

the computation issue, including the use of empirical Bayes estimator of the random effects (Raudenbush & Bryk, 2002) and factor score estimates (Bollen & Curran, 2006). However, to the present, no study has investigated the performance of these approaches. In addition, researchers need to modify the degrees of freedom and related indices by themselves since the results from standard software often ignore the fact that two degrees of freedom per person are already consumed in the first step (i.e., one for the intercept and one for the slope).

Hoffman (2015) presented another procedure to conduct two-step detrending. Instead of the individual OLS regression, Hoffman used the least squares dummy variable (LSDV) approach, which is an estimation method for the fixed effects model. Specifically, in the first step of detrending, dummy variables are added in Equation 2.52 so that the coefficients can take different values for different persons. That is,

$$X_{ti} = \left(\kappa_{01}^X + \sum_{i=2}^N \kappa_{0i}^X D_{0i}^X \right) + \left(\kappa_{11}^X + \sum_{i=2}^N \kappa_{1i}^X D_{1i}^X \right) \text{Time}_{ti} + r_{ti}^X,$$

where D_{0i}^X and D_{1i}^X are dummy variables indicating person i ($i = 2, 3, \dots, N$), and κ_{0i}^X and κ_{1i}^X are the coefficients corresponding to D_{0i}^X and D_{1i}^X , respectively. The coefficients represent the differences in intercept and slope between person i and the reference person, who is person 1 in this case. Using Hoffman's approach, we can estimate the parameters in a single run. Thus, the estimation is more computationally efficient especially when N is large. This computational efficiency is accomplished at the cost of many degrees of freedom.

Hoffman (2015) further extended the two-step detrending by including the individual slope estimates in addition to the intercept and residual estimates in the Y -model (i.e., Equations 2.54 and 2.55). In cases of linear trends, the inclusion of the slope estimates enables us to catch all sources of the between-person variability in the time-varying predictor. Hoffman also incorporated a time variable as a covariate. As a

result, we have the following *extended two-step detrending* model:

$$\begin{aligned} X_{ti} &= \left(\kappa_{01}^X + \sum_{i=2}^N \kappa_{0i}^X D_{0i}^X \right) + \left(\kappa_{11}^X + \sum_{i=2}^N \kappa_{1i}^X D_{1i}^X \right) \text{Time}_{ti} + r_{ti}^X \\ &= \beta_{0i}^X + \beta_{1i}^X \text{Time}_{ti} + r_{ti}^Y, \end{aligned} \quad (2.57)$$

$$Y_{ti} = \beta_{0i}^Y + \beta_{1i}^Y \text{Time}_{ti} + \beta_{2i}^Y \hat{r}_{ti}^X + r_{ti}^Y, \quad (2.58)$$

$$\beta_{0i}^Y = \gamma_{00}^Y + \gamma_{01}^Y \hat{\beta}_{0i}^X + \gamma_{02}^Y \hat{\beta}_{1i}^X + u_{0i}^Y, \quad (2.59)$$

$$\beta_{1i}^Y = \gamma_{10}^Y + \gamma_{11}^Y \hat{\beta}_{0i}^X + \gamma_{12}^Y \hat{\beta}_{1i}^X + u_{1i}^Y, \quad (2.60)$$

$$\beta_{2i}^Y = \gamma_{20}^Y, \quad (2.61)$$

where $\beta_{iq}^X = \kappa_{q1}^X + \sum_{i=2}^N \kappa_{qi}^X D_{qi}^X$ ($q = 0, 1$ and $i = 2, \dots, N$). In the combined form, we have

$$Y_{ti} = (\gamma_{00}^Y + \gamma_{01}^Y \hat{\beta}_{0i}^X + \gamma_{02}^Y \hat{\beta}_{1i}^X) + (\gamma_{10}^Y + \gamma_{11}^Y \hat{\beta}_{0i}^X + \gamma_{12}^Y \hat{\beta}_{1i}^X) \text{Time}_{ti} + \gamma_{20}^Y \hat{r}_{ti}^X + u_{0i}^Y + u_{1i}^Y \text{Time}_{ti} + r_{ti}^Y. \quad (2.62)$$

2.3.2 Time-as-Covariate Detrending

As with the latent curve models, multilevel models can also incorporate a time variable as a covariate. For example, when we use the within-between model as a base model, adding a time variable with a random slope leads to the following model:

$$Y_{ti} = \beta_{0i} + \beta_{1i}(X_{ti} - \bar{X}_{\cdot i}) + \beta_{2i} \text{Time}_{ti} + r_{ti}, \quad (2.63)$$

$$\beta_{0i} = \gamma_{00} + \gamma_{01} \bar{X}_{\cdot i} + u_{0i}, \quad (2.64)$$

$$\beta_{1i} = \gamma_{10}, \quad (2.65)$$

$$\beta_{2i} = \gamma_{20} + \gamma_{21} \bar{X}_{\cdot i} + u_{2i}, \quad (2.66)$$

or equivalently,

$$Y_{ti} = \gamma_{00} + \gamma_{01} \bar{X}_{\cdot i} + \gamma_{10}(X_{ti} - \bar{X}_{\cdot i}) + \gamma_{20} \text{Time}_{ti} + \gamma_{21} \bar{X}_{\cdot i} \text{Time}_{ti} + u_{2i} \text{Time}_{ti} + r_{ti}. \quad (2.67)$$

Wang and Maxwell (2015) called this approach *time-as-covariate* detrending and compared it to other detrending methods, including Curran and Bauer’s (2011) two-step detrending¹⁰. They analytically and empirically showed the time-as-covariate approach could produce the same point estimate of the within- and between-person effects with smaller standard errors as those based on the two-step detrending. According to these results, they recommended using the time-as-covariate method when detrending is needed.

Hoffman and Stawski (2009) also used a similar model to that of Wang and Maxwell (2015). They introduced a time variable to control for the effect of time (e.g., natural history, spontaneous recovery) and to catch all other within-person variations in the data. Hoffman and Stawski also pointed out that under time-balanced design, time is a “pure” within-person variable, meaning that it contains only within-person variation. Although their argument is true only for the time-structured design, it is essentially identical to Curran and Bauer’s (2011) discussion, maintaining that whether the data collection design is time-balanced influences the unbiasedness of estimates of within-person and between-person effect.

In contrast to Wang and Maxwell (2015) and Hoffman and Stawski (2009), Curran et al. (2012) introduced the time-as-covariate model as an MLM-based growth curve model rather than a detrending method. They assumed no trend in time-varying predictors and focused on sampling errors in the person means and the resulting bias in the between-person effects (i.e., Lüdtke’s bias). One important contribution of Curran et al.’s study is that they proposed LCMMy-RIx as a counterpart of the time-as-covariate model. We can see the correspondence of the two models by equating the model equations, namely Equations 2.48–2.51 and Equations 2.63–2.66. More specifically,

¹⁰Wang and Maxwell (2015) did not include a person mean in Equation 2.66.

$$\begin{aligned}
Y_{ti} &= I_i^Y + \lambda_t S_i^Y + \gamma \varepsilon_{ti}^X + \varepsilon_{ti}^Y \\
&= (\delta^{I^Y} + \beta_{II} I_i^X + \zeta_i^{I^Y}) + \lambda_t (\delta^{S^Y} + \beta_{SI} I_i^X + \zeta_i^{S^Y}) + \gamma (X_{ti} - I_i^X) + \varepsilon_{ti}^Y \\
&= \underbrace{\delta^{I^Y}}_{\gamma_{00}} + \underbrace{\beta_{II} I_i^X}_{\gamma_{01} \bar{X}_{\cdot i}} + \underbrace{\gamma (X_{ti} - I_i^X)}_{\gamma_{10} (X_{ti} - \bar{X}_{\cdot i})} + \underbrace{\delta^{S^Y} \lambda_t}_{\gamma_{20} \text{Time}_{ti}} + \underbrace{\lambda_t \beta_{SI} I_i^X}_{\gamma_{21} \bar{X}_{\cdot i} \text{Time}_{ti}} + \underbrace{\zeta_i^{I^Y}}_{u_{0i}} + \underbrace{\lambda_t \zeta_i^{S^Y}}_{u_{1i} \text{Time}_{ti}} + \underbrace{\varepsilon_{ti}^Y}_{r_{ti}}. \quad (2.68)
\end{aligned}$$

Note that this correspondence of LCM-RI to time-as-covariate model relies on the time-balanced design and centering of the time variable, or more precisely, assumptions that $\lambda_t = \text{Time}_{ti}$ and $\sum_{t=1}^T \text{Time}_{ti} = 0$.

2.3.3 Multivariate Multilevel Model

Hoffman's (2015) extended two-step detrending successfully incorporates all sources of the between-person variability in time-varying predictors. However, as Hoffman noted, this strategy is still tandem, thus statistically inefficient. Therefore, Hoffman proposed a multivariate multilevel modeling approach to the trend issue. Specifically, they suggested using two level-1 models, in which both X and Y are modeled with linear growth curves:

$$X_{ti} = \beta_{0i}^X + \beta_{1i}^X \text{Time}_{ti} + r_{ti}^X, \quad (2.69)$$

$$Y_{ti} = \beta_{0i}^Y + \beta_{1i}^Y \text{Time}_{ti} + r_{ti}^Y, \quad (2.70)$$

$$(r_{ti}^X, r_{ti}^Y)' \sim N(\mathbf{0}, \mathbf{R}), \quad (2.71)$$

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_{XX} & \mathbf{R}_{XY} \\ \mathbf{R}_{XY}' & \mathbf{R}_{YY} \end{pmatrix} = \left(\begin{array}{c|c} \sigma_X^2 & \sigma_{XY} \\ \hline \sigma_{XY} & \sigma_Y^2 \end{array} \right). \quad (2.72)$$

Then, each coefficient is modeled as in the random coefficients model:

$$\beta_{0i}^X = \gamma_{00}^X + u_{0i}^X, \quad (2.73)$$

$$\beta_{1i}^X = \gamma_{10}^X + u_{1i}^X, \quad (2.74)$$

$$\beta_{0i}^Y = \gamma_{00}^Y + u_{0i}^Y, \quad (2.75)$$

$$\beta_{1i}^Y = \gamma_{10}^Y + u_{1i}^Y, \quad (2.76)$$

$$(u_{0i}^X, u_{1i}^X, u_{0i}^Y, u_{1i}^Y)' \sim N(\mathbf{0}, \mathbf{G}), \quad (2.77)$$

$$\mathbf{G} = \begin{pmatrix} \mathbf{G}_{XX} & \mathbf{G}_{XY} \\ \mathbf{G}'_{XY} & \mathbf{G}_{YY} \end{pmatrix} = \left(\begin{array}{cc|cc} \tau_{00}^{XX} & \tau_{01}^{XX} & \tau_{00}^{XY} & \tau_{01}^{XY} \\ \tau_{01}^{XX} & \tau_{11}^{XX} & \tau_{10}^{XY} & \tau_{11}^{XY} \\ \hline \tau_{00}^{XY} & \tau_{10}^{XY} & \tau_{00}^{YY} & \tau_{01}^{YY} \\ \tau_{01}^{XY} & \tau_{11}^{XY} & \tau_{01}^{YY} & \tau_{11}^{YY} \end{array} \right). \quad (2.78)$$

Although Hoffman (2015) used the within-person and between-person correlations as indicators of the effects, we can transform them into the regression coefficients as follows¹¹:

$$\mathbf{B}_W = \beta_W = \mathbf{R}_{XX}^{-1} \mathbf{R}_{XY}, \quad (2.79)$$

$$\mathbf{B}'_B = \begin{pmatrix} \beta_{II} & \beta_{SI} \\ \beta_{IS} & \beta_{SS} \end{pmatrix} = \mathbf{G}_{XX}^{-1} \mathbf{G}_{XY}. \quad (2.80)$$

In the scalar form,

$$\beta_W = \frac{\sigma_{XY}}{\sigma_X^2}, \quad (2.81)$$

$$\beta_{II} = \frac{\tau_{11}^{XX} \tau_{00}^{XY} - \tau_{01}^{XX} \tau_{10}^{XY}}{\tau_{00}^{XX} \tau_{11}^{XX} - (\tau_{01}^{XX})^2}, \quad (2.82)$$

$$\beta_{IS} = \frac{\tau_{00}^{XX} \tau_{10}^{XY} - \tau_{01}^{XX} \tau_{00}^{XY}}{\tau_{00}^{XX} \tau_{11}^{XX} - (\tau_{01}^{XX})^2}, \quad (2.83)$$

$$\beta_{SI} = \frac{\tau_{11}^{XX} \tau_{01}^{XY} - \tau_{01}^{XX} \tau_{11}^{XY}}{\tau_{00}^{XX} \tau_{11}^{XX} - (\tau_{01}^{XX})^2}, \quad (2.84)$$

¹¹Again, we need a special functionality of software in order to obtain the standard errors, p -values, and confidence intervals of within- and between-person effects.

$$\beta_{SS} = \frac{\tau_{00}^{XX} \tau_{11}^{XY} - \tau_{11}^{XX} \tau_{01}^{XY}}{\tau_{00}^{XX} \tau_{11}^{XX} - (\tau_{01}^{XX})^2}. \quad (2.85)$$

Hoffman (2015) introduced two ways to implement the multivariate multilevel approach. One is to use a “truly” multivariate software and the other is to use univariate multilevel software with double-stacked data and dummy variables. The truly multivariate approach uses SEM software such as Mplus and implements a multivariate multilevel model as a multilevel SEM model. The univariate approach uses dummy variables to switch the dependent variable and the corresponding submodel (Goldstein, 2011; Hox et al., 2018; Raudenbush & Bryk, 2002; Singer & Willett, 2003; Snijders & Bosker, 2012). With this trick, we can use popular statistical software that can handle multilevel models such as SPSS, SAS, HLM, and nlme package in R to analyze data with the multivariate multilevel models.

Table 2.2 shows an example of a single-stacked data (the left table) and a double-stacked data (the right table). In the single-stacked format, which is also known as a long-form format, an outcome variable Y and a time-varying predictor X have their own column. A row represents a measurement record of a single person at a single time point. On the other hand, the double-stacked format lays out all measurements in a single column (i.e., in the Value column) and has two additional dummy variables (dX and dY). The two dummy variables indicate from what variable the values came (X or Y). With the double-stacked format, the multivariate multilevel model can be reformulated as the following three-level model without the level-1 intercept and residual:

$$Z_{itj} = \alpha_{ti}^X dX_{itj} + \alpha_{ti}^Y dY_{itj} \quad (2.86)$$

$$\alpha_{ti}^X = \beta_{0i}^X + \beta_{1i}^X \text{Time}_{ti}^X + r_{ti}^X, \quad (2.87)$$

$$\alpha_{ti}^Y = \beta_{0i}^Y + \beta_{1i}^Y \text{Time}_{ti}^Y + r_{ti}^Y, \quad (2.88)$$

$$\beta_{0i}^X = \gamma_{00}^X + u_{0i}^X, \quad (2.89)$$

$$\beta_{1i}^X = \gamma_{10}^X + u_{1i}^X, \quad (2.90)$$

$$\beta_{0i}^Y = \gamma_{00}^Y + u_{0i}^Y, \quad (2.91)$$

$$\beta_{1i}^Y = \gamma_{10}^Y + u_{1i}^Y, \quad (2.92)$$

where $Z_{itj} = X_{ti}$ for $j = 1$ and $Z_{itj} = Y_{ti}$ for $j = 2$; dX_{itj} and dY_{itj} are the level-1 (variable level) dummy variables taking the unity when the outcome variable Z_{itj} is equal to X_{ti} and Y_{ti} , respectively; α_{ti}^X and α_{ti}^Y are the level-1 random coefficients; and β_{0i}^X , β_{1i}^X , β_{0i}^Y , and β_{1i}^Y are the level-2 (occasion level) random coefficients. Note that the level-2 residuals r_{ti}^X and r_{ti}^Y and the level-3 (person level) residuals u_{qi}^X and u_{qi}^Y have the same distributional assumptions as Equations 2.71 and 2.78. Note also that the within-person effect and the between-person effects can be calculated based on Equations 2.79 and 2.80 or Equations 2.81–2.85. One caution for using the double-stacked technique is that there is no level-1 error term (i.e., variable-level error). Hence, we need to restrict the level-1 error variance at zero, or in practice, at a small value. According to Hoffman's illustrative analysis, the truly multivariate approach and the univariate approach resulted in the same estimates and log-likelihood.

Hoffman (2015) also compared the multivariate multilevel approach to the two-step detrending with persons as fixed effects (i.e., LSDV approach) and as random effects (empirical Bayes estimates). As anticipated, the three methods resulted in similar estimates for the fixed effects (i.e., the average effects) but different for the variance components. The estimated variance components using the empirical Bayes method were smaller than those using the multivariate multilevel model. This was due to the shrinkage of the estimates to the overall means (Raudenbush & Bryk, 2002). On the other hand, the estimated variances based on LSDV were larger than those of the empirical Bayes approach and multivariate multilevel approach. Consequently, the three methods had different (quasi-) standardized estimates for both within-person and between-person effects.

Table 2.2

Example of Singly Stacked and Doubly Stacked Data (Three-Wave Data with a Single Time-Varying Predictor)

ID	Time	X	Y	ID	Variable	Time	Value	dX	dY
1	1	X_{11}	Y_{11}	1	X	1	X_{11}	1	0
1	2	X_{12}	Y_{12}	1	X	2	X_{12}	1	0
1	3	X_{13}	Y_{13}	1	X	3	X_{13}	1	0
2	1	X_{21}	Y_{21}	1	Y	1	Y_{11}	0	1
2	2	X_{22}	Y_{22}	1	Y	2	Y_{12}	0	1
2	3	X_{23}	Y_{23}	1	Y	3	Y_{13}	0	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	2	X	1	X_{21}	1	0
				2	X	2	X_{22}	1	0
				2	X	3	X_{23}	1	0
				2	Y	1	Y_{21}	0	1
				2	Y	2	Y_{22}	0	1
				2	Y	3	Y_{23}	0	1
				$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

2.3.4 Relationship between Detrending Methods

Wang and Maxwell (2015) compared the two-step detrending and the time-as-covariate detrending. For the two-step detrending, they distinguished detrending only X , detrending only Y , and detrending both X and Y . According to their analytical results, the two-step detrending of X , both X and Y , and the time-as-covariate detrending produce the identical point estimates of the within-person effect for a given person. The other methods (i.e., no detrending and two-step detrending of Y) may or may not be equal to the three equivalent methods, depending on the existence of the trend as well as the size and sign of correlations between X , Y , and the time variable. Table 2.3 summarizes these scenarios. Wang and Maxwell also noted that the equivalence they found for a given person does not hold at the aggregated level (i.e., the fixed effect of the within-person effect).

Table 2.3

Wang & Maxwell's (2015) Analytical Results on the Relationship between the Estimated Individual Within-Person Effects and Detrending Methods

Trend	Relationship between Detrending Methods
no trend	$a = b = c = d = e$
Y has a trend	$a = b = c = d = e$
X has a trend	$a = e < b = c = d$
X and Y have a trend	
in different directions	$a < e < b = c = d$
both positive and $r_{XY} \cdot r_{X,\text{Time}} > r_{Y,\text{Time}}$	$e < a < b = c = d$
both negative and $r_{XY} \cdot r_{X,\text{Time}} < r_{Y,\text{Time}}$	$e < a < b = c = d$
both positive and $r_{XY} \cdot r_{X,\text{Time}} < r_{Y,\text{Time}}$	$e < b = c = d < a$
both negative and $r_{XY} \cdot r_{X,\text{Time}} > r_{Y,\text{Time}}$	$e < b = c = d < a$

Note. a = no detrending, b = two-step method for X , c = time-as-covariate method, d = two-step method for X and Y , e = two-step method for Y . These relationships hold for the estimates of within-person effects for a given person. X is centered with person means when X is not detrended.

Econometricians have known Wang and Maxwell's (2015) findings as *Frisch-Waugh theorem* or *Frisch-Waugh-Lovell theorem* (FWL theorem for short; Frisch & Waugh, 1933; Lovell, 1963). In econometric textbooks (e.g., Davidson & MacKinnon, 2004; Greene, 2018), the FWL theorem is introduced as one regarding the following *partitioned regression*:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} = \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon}, \quad (2.93)$$

where $\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2)$ and $\boldsymbol{\beta} = (\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2)'$ with appropriate sizes. Here, we are interested in estimating $\boldsymbol{\beta}_2$ but not $\boldsymbol{\beta}_1$. The FWL theorem states that the estimates of $\boldsymbol{\beta}_2$ and the residuals $\boldsymbol{\varepsilon}$ are identical to $\hat{\boldsymbol{\beta}}_2^*$ and $\hat{\boldsymbol{\varepsilon}}^*$ from the following regression with transformed data:

$$\mathbf{Q}_1\mathbf{y} = \mathbf{Q}_1\mathbf{X}_2\boldsymbol{\beta}_2^* + \boldsymbol{\varepsilon}^* \quad (2.94)$$

where $\mathbf{Q}_1 = \mathbf{I} - \mathbf{P}_1 = \mathbf{I} - \mathbf{X}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{X}'_1$. The matrix $\mathbf{P}_1$ is a projection matrix onto the

column space of $\mathbf{X}_1$ and $\mathbf{Q}_1$ is its orthogonal complement. Premultiplying a vector by $\mathbf{Q}_1$ produces a residual vector that we obtain when regressing the vector on $\mathbf{X}_1$. Thus, the FWL theorem says that we can obtain the estimates of β_2 in the partitioned regression as the estimates of β_2^* in the regression of the residualized dependent variable $\mathbf{Q}_1\mathbf{y}$ on the residualized predictors $\mathbf{Q}_1\mathbf{X}_2$.

Frisch and Waugh (1933) and Lovell (1963) applied the theorem to remove a trend and seasonality in time-series data. In particular, Lovell (1963) showed the relations between the following seven seasonality adjustment (or *deseasoning*) methods:

1. $\mathbf{y} = \mathbf{X}\beta_1 + \mathbf{D}\alpha_1 + \varepsilon_1$, (adding an adjustment term $\mathbf{D}\alpha_1$)
2. $\mathbf{y}_\alpha = \mathbf{X}_\alpha\beta_2 + \varepsilon_2$, (deseasoning both $\mathbf{y}$ and $\mathbf{X}$)
3. $\mathbf{y} = \mathbf{X}\beta_3 + \varepsilon_3$, (no adjustment)
4. $\mathbf{y} = \mathbf{X}_\alpha\beta_4 + \varepsilon_4$, (deseasoning only $\mathbf{X}$)
5. $\mathbf{y}_\alpha = \mathbf{X}\beta_5 + \varepsilon_5$, (deseasoning only $\mathbf{y}$)
6. $\mathbf{y} = \mathbf{X}_\alpha\beta_6 + \mathbf{D}\alpha_6 + \varepsilon_6$, (deseasoning only $\mathbf{X}$ plus an adjustment term)
7. $\mathbf{y}_\alpha = \mathbf{X}_\alpha\beta_7 + \mathbf{D}\alpha_7 + \varepsilon_7$, (deseasoning both $\mathbf{y}$ and $\mathbf{X}$ plus an adjustment term)

where $\mathbf{D}$ is a set of dummy variables indicating seasons to be adjusted for, and α 's are vectors of the seasonality effects. The vectors and matrices with a subscript α indicate that they are seasonally adjusted. That is,

$$\mathbf{y}_\alpha = \mathbf{Q}_D\mathbf{y} = \mathbf{y} - \mathbf{D}(\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'\mathbf{y}, \quad (2.95)$$

$$\mathbf{X}_\alpha = \mathbf{Q}_D\mathbf{X} = \mathbf{X} - \mathbf{D}(\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'\mathbf{X}. \quad (2.96)$$

The error terms ε 's are assumed to have a normal distribution of $N(\mathbf{0}, \sigma^2\mathbf{I})$. Then, Lovell established as Theorem 4.1 (p. 1001) that

$$\beta_1 = \beta_2 = \beta_4 = \beta_6 = \beta_7 \quad (2.97)$$

$$= \beta_3 + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'[(\mathbf{X} - \mathbf{X}_\alpha)\beta_1 - (\mathbf{y} - \mathbf{y}_\alpha)] \quad (2.98)$$

$$= \beta_5 - (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'(\mathbf{X}_\alpha - \mathbf{X})\beta_1, \quad (2.99)$$

and

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_4 - (\mathbf{y} - \mathbf{y}_\alpha) = \varepsilon_6 = \varepsilon_7. \quad (2.100)$$

Lovell (1963) summarized the findings as follows. First, the identity $\beta_1 = \beta_2$ means the equivalence of the two-step least squares adjustment and the single-step adjustment by adding seasonality dummy variables. Second, the identity $\beta_2 = \beta_4$ implies that the adjustment of the dependent variable does not matter as long as the independent variables are adjusted. These findings coincide with those by Wang and Maxwell (2015). Lovell further noted that the adjustment of the dependent variable in the second and seventh method could worsen R^2 even though they led to the identical estimates and residuals to those of the first and sixth method. This occurs because adjusting the dependent variable reduces its variance but does not affect the error variance.

Lovell (1963) mentioned cases in which the error variance is not homogeneous (i.e., $\text{VAR}[\varepsilon] \neq \sigma^2\mathbf{I}$). Lovell discussed the GLS approach, which transforms $\mathbf{y}$ and $\mathbf{X}$ by premultiplying them by the inverse of the square root of the error covariance matrix ($\mathbf{V}^{-\frac{1}{2}}$, where $\mathbf{V} = \text{VAR}[\varepsilon]$). Krishnakumar (2006) also discussed the same issue and provided the generalized Frisch-Waugh theorem, or generalized FWL theorem for short, in which the error covariance has a general structure. Specifically, consider Model 2.93 with a general error structure $\text{VAR}[\varepsilon] = \mathbf{V}$. Then, the generalized FWL theorem tells us that the estimates of β_2 in Equation 2.93 can be obtained by the following two-step adjustment:

$$\hat{\beta}_2 = (\mathbf{R}_2'\mathbf{V}^{-1}\mathbf{R}_2)^{-1}\mathbf{R}_2'\mathbf{V}^{-1}\mathbf{R}_1, \quad (2.101)$$

where

$$\mathbf{R}_1 = \mathbf{y} - \mathbf{X}_1(\mathbf{X}_1'\mathbf{V}^{-1}\mathbf{X}_1)^{-1}\mathbf{X}_1\mathbf{V}^{-1}\mathbf{y} = \mathbf{Q}_{1/\mathbf{V}^{-1}}\mathbf{y}, \quad (2.102)$$

$$\mathbf{R}_2 = \mathbf{X}_2 - (\mathbf{X}_1\mathbf{V}^{-1}\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{V}^{-1}\mathbf{X}_2 = \mathbf{Q}_{1/\mathbf{V}^{-1}}\mathbf{X}_2. \quad (2.103)$$

Here, $\mathbf{Q}_{1/\mathbf{V}^{-1}} = \mathbf{I} - \mathbf{P}_{1/\mathbf{V}^{-1}} = \mathbf{I} - \mathbf{X}_1(\mathbf{X}_1'\mathbf{V}^{-1}\mathbf{X}_1)^{-1}\mathbf{X}_1\mathbf{V}^{-1}$, that is, $\mathbf{Q}_{1/\mathbf{V}^{-1}}$ is a projector onto the orthogonal complement of the column space of $\mathbf{X}_1$ with the metric matrix $\mathbf{V}^{-1}$ (Yanai et al., 2011). Note that as in the standard FWL theorem, we can replace $\mathbf{Q}_{1/\mathbf{V}^{-1}}\mathbf{y}$ with $\mathbf{y}$ (i.e., the adjustment of $\mathbf{y}$ does not affect $\hat{\beta}_2$).

2.3.5 Relationship between Detrending and Centering

Centering and detrending are often recognized as unrelated transformations. In fact, Wang and Maxwell (2015) addressed them as separate problems. However, these two data transformation methods are closely related to each other. We can find the connection when we compare the general form of the adaptive centering and the generalized FWL theorem. That is, as in Equations 2.25 (i.e., adaptive centering) and 2.103 (generalized FWL theorem), both centering and detrending can be written in the same form:

$$\mathbf{X}^* = \mathbf{X} - \mathbf{Z}(\mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\boldsymbol{\Omega}^{-1}\mathbf{X} = (\mathbf{I} - \mathbf{P}_{\mathbf{Z}/\boldsymbol{\Omega}^{-1}})\mathbf{X} = \mathbf{Q}_{\mathbf{Z}/\boldsymbol{\Omega}^{-1}}\mathbf{X}, \quad (2.104)$$

where $\mathbf{Z}$ is a design matrix for the random effects in the growth curve model and $\boldsymbol{\Omega}$ is a positive definite matrix. In person-mean centering, we have $\mathbf{Z} = \mathbf{1}_N$ and $\boldsymbol{\Omega} = \sigma^2\mathbf{I}$, while in detrending, the columns of $\mathbf{Z}$ consist of $\mathbf{1}_N$ and the functions of time. In other words, centering can be seen as a special case of detrending, in which the time-varying predictors are simply modeled by person means. Conversely, detrending can be regarded as a generalization of centering, in which the time-varying predictors are modeled with functions of time. Both procedures use the residuals as independent variables to obtain the within-person effect.

Projecting or residualizing the time-varying predictors is one of the two key components in disaggregating the within-person and between-person effects. That is, residualization is a way of obtaining within-person components. The other piece is the estimation of the between-person effects. As we saw in Table 2.1, the between-person effects can be captured by a person means in multilevel centering (i.e., within-between model, contextual model, latent centering model). The between-person effects can also be caught by the latent factors in LCM (intercept and slope factors of LCM_y-RI_x; cross-level covariances of LCM_y-CLC).

Similar to the centering methods, the detrending methods take various strategies to estimate the between-person effects. Table 2.4 summarizes the within-person and between-person predictors in each detrending method. The detrending methods mainly differ in their between-person predictors. For example, Curran and Bauer's (2011) two-step detrending and the time-as-covariate detrending only use mean-related predictors (intercept estimates and person means), while the other methods use both intercept and slope. Among the four methods in the table, only the multivariate multilevel model uses latent predictors at both levels and the other methods use some sort of estimates. Hence, the multivariate multilevel model is considered the most general and accurate of the four methods.

2.4 Comparison of Structural Equation Modeling and Multilevel Modeling

As we saw in the previous sections, multilevel models have their corresponding SEM models in many cases. This parallelism of SEM and MLM has extensively been studied and illustrated in many articles and textbooks. For example, Bauer (2003), Curran (2003), and

Table 2.4*Summary of Within-Person and Between-Person Predictors in Detrending Methods*

Detrending Methods	Within-Person Predictor	Between-Person Predictor
CB's two-step	OLS residuals	OLS intercept
H's two-step	L1 EB residuals	LSDV intercept/slope
		L2 EB intercept/slope
time-as covariate	person-mean-centered TVP	person mean
multivariate multilevel	L1 latent residual	L2 latent residuals
		(intercept/slope)

Note. CB = Curran and Bauer (2011), H = Hoffman (2015), OLS = ordinary least squares, L1 = level-1, L2 = level-2, LSDV = least squares dummy variable, EB = empirical Bayes, TVP = time-varying predictor.

Mehta and Neale (2005) provided general comparisons of the two frameworks. They showed how we could analyze nested data, including longitudinal data, with multilevel models. They also illustrated procedures to do the same analysis with standard structural equation modeling (i.e., single-level SEM). These studies especially focused on how the unbalanced data can be addressed in the SEM framework since the standard SEM requires balanced data in analyzing nested data.

Other studies compared SEM and MLM in the context of longitudinal studies, in particular, growth modeling. McNeish and Matta (2018) compared the mixed effects model and the latent curve model from the practical viewpoints. According to their discussion, the practical differences lie in how the two approaches deal with (a) small samples, (b) time-unstructured data, (c) nonlinear growth, (d) multiple-group models, (e) model fit, (f) residual structure, (g) three or higher-level models, (h) growth models in a larger complex model, (i) missing data, and (j) time-varying predictors. McNeish and Matta summarized these differences by saying that MLM is useful for simple models with complex data structure, whereas LCM is suited to complex models with a straightforward data structure.

For example, if the data is time-unbalanced but growth is assumed to be linear, then MLM may be preferred since it can incorporate the unbalanced data without difficulty. In contrast, when the growth model is a part of a larger model, as is in multivariate growth models (Bollen & Curran, 2006; MacCallum et al., 1997) and when the data is time-structured, then LCM may be preferred.

McNeish and Matta (2018) further examined strategies for incorporating time-varying predictors under time-unbalanced designs. They proposed a dummy variable technique, which allows MLM to have time-varying effects of time-varying predictors. They also introduced *definition variables* (Curran, 2003; Mehta & West, 2000). Definition variables allow different persons to have different fixed parameter values, so that LCM can accommodate time-unbalanced data¹².

Plewis (2001) showed how we could model longitudinal data with an outcome and at least one time-varying explanatory variable. Plewis provided applications of multilevel models, for example, a three-level random intercept model (occasion, pupil, and school level); *distributed lag model* with and without lagged dependent variable; within-between model; and fixed effects regression. Plewis called these models the conditional approach. Plewis also introduced a multivariate multilevel model as an unconditional model, which examines the between-person relationship (i.e., models for the intercept, linear, and quadratic coefficients of the dependent and independent variables). Plewis further illustrated the *bivariate latent curve model* to conduct the same analyses as those with conditional and unconditional approaches. Although Plewis examined diverse and sophisticated models, the study only provided illustrations using real data. Moreover, neither are they concerned with a trend in the exploratory variables (i.e., time-varying predictors) nor the within-person effect of them. The same tendency can also be found in multilevel textbooks (e.g., Holt, 2008; Hox et al., 2018). This lack of attention to the

¹²Currently, the definition variable technique is only available in Mplus and OpenMx.

lower-level components may be due to their emphasis on examining causal relations at the person level or the group level.

Curran et al. (2012) evaluated the performance of the multilevel within-between model (MLM-WB), LCMMy-CLC, LCMMy with zero cross-level covariances, and LCMMy-RIx based on simulations. The result showed that MLM-WB, LCMMy-CLC, and LCMMy-RIx produced accurate estimates of the within-person effect. However, LCMMy with zero cross-level covariances resulted in attenuated estimates in a similar way to the random effects model. For the between-person effects, LCMMy-RIx yielded accurate estimates, whereas MLM-WB suffered from Lüdtke's bias (the bias due to the small sample size within a cluster). On the other hand, LCMMy-CLC produced accurate contextual effects related to the random intercept and random slope for the outcomes, leading to accurate estimates of the between-person effects.

Hamaker and Muthén (2020) conducted two simulation studies, comparing MLM-based and SEM-based centering methods. In their first simulation, nine models were compared, five of which are univariate multilevel models with long-form data and the other four models are multivariate models (SEM) with wide-form data. The nine models were random effects model, which they called Model L1 (L1), adaptive centering model (L2), within-between model (L3a), latent variable centering model (L3b), SEM-based random effects model (i.e., without cross-level covariances; W1), SEM-based fixed effects model (W2), LCMMy-RIx (W3), and latent contextual model (W4). Using the latent variable centering model as the data-generating model, they simulated a data set with 5,000 persons and four time points. The simulation revealed that (1) the models without centering or between-person predictor (i.e., L1 and W1) failed to obtain an accurate estimate for the within-person effect, (2) the models using observed person-mean as a between-person predictor (L3a and W3) suffered from Lüdtke's bias, and (3) the estimates of the contextual effects model from L4 and W4 were equal to the difference of the

between-person effect and within-person effect from L3a and W3, respectively. This means that the contextual effect obtained from L4 was biased, while that from W4 was unbiased.

In the second simulation of Hamaker and Muthén (2020), they changed the settings; the data set included 100 persons with four time points (i.e., $T = 4$; panel data) or 40 time points ($T = 40$; intensive longitudinal data). In this simulation, they simulated 1,000 data sets for each condition, and only used a univariate, long-form data approach when $T = 40$. According to their results, all models except for L1 and W2 could obtain unbiased estimates for the within-person effect when $T = 4$. However, all models resulted in empirical standard errors (the standard deviation of the estimates) that were smaller than the theoretical standard errors (the averages of standard errors from software output). As a result, the coverage rates (i.e., the proportion that confidence intervals caught the true values across replications) were overestimated. For the between-person effect, the univariate models (L3a, L3b, and L4) underestimated the coverage rates, while the multivariate models (W3 and W4) could obtain accurate ones. This was due to the Lüdtke's bias in the between-person effects.

In contrast to the cases of $T = 4$, the univariate models resulted in slightly lower theoretical standard errors than the empirical standard errors for the within-person effect when $T = 40$. This led to slightly underestimated coverage rates. Also, the models with the between-person predictor no longer suffered from Lüdtke's bias, though the standard errors and the coverage rates were still underestimated.

2.5 Summary of the Literature Review and Its Implications for the Present Research

Chapter 2 has reviewed the literature on disaggregation of the within-person and between-person effects with a focus on centering and detrending. From the review, we can find some essential points of the disaggregation. First, in order to isolate the within-person component, we need to control for the between-person components in time-varying predictors. The between-person components can be controlled by residualizing a time-varying predictor using a between-person model (i.e., centering and detrending) or by including in a model between-person predictors that are derived from the time-varying predictor. Cross-level covariances also work for the control of the between-person components in an indirect way.

Second, we can use both MLM and SEM to disentangle the within-person and the between-person effects and can convert one model to the other (Bauer, 2003; Curran, 2003). Good examples of this are the comparability of the time-as-covariate model and LCMMy-RIx shown in Equation 2.68; Allison's SEM-based fixed effects regression in Figure 2.3; and the latent version of the contextual effects model, which was referred to as Model W4 in Hamaker and Muthén (2020).

Third, we can assume different models for different series of variables. For example, LCMMy-RIx has a linear growth model for the Y -series, while only a random intercept is assumed for the X -series. We can also assume no specific model behind the X -series, as in LCMMy-CLC and SEM-based fixed effects model. Moreover, even when two models have an identical between-person model, they may assume different between-person relations. In LCMMy-RIx, for example, we may constrain the path coefficient from I^X to S^Y to be zero, while the path from I_i^X to I_i^Y is left free to be estimated.

Based on these observations, I developed a framework to characterize the reviewed

models (see Table 2.5). The framework analyzes the models from the following viewpoints. First, the models can be characterized by the submodels behind the time-varying predictors (i.e., X -model) and outcomes (Y -model). Each series can have different submodels that govern the process. These include, but are not limited to, intercept-only models, linear growth models, quadratic curve models, and piecewise regression models. Among them, this study particularly focuses on whether a series has a slope factor (i.e., linear growth models).

Second, the type of the dependent and independent variables in the within-person regression features the centering/detrending methods. In the within-person regression, the dependent variables are regressed on the residualized time-varying predictors or raw predictors. The residualized predictors may be latent as in LCM_{My}-RI_x; otherwise, the estimated residuals are used (e.g., two-step detrending, person-mean centering). The same classification rule can be applied to the dependent variables, though residualizing outcomes is not necessary to extract the within-person effect. The latent/observed rule also holds for the between-level predictors. That is, the between-person predictors can be latent factors (LCM_{My}-RI_x) or the ones derived from the observed regressors (person means, estimated regression coefficients).

Finally, the between-person models can differ in the patterns of between-person effects. Some models may constrain part of the between-person effects to be zero in the population. Alternatively, models may have cross-level covariances to address the between-level relationship.

Based on this framework, the within-between model (MLM-WB in Table 2.5), for example, can be viewed as a model that has an intercept-only model for both X and Y , uses the raw observed outcomes and residualized observed predictors for the within-person regressions, and only assumes an observed intercept-intercept relationship at the between-person level. Also, LCM_{My}-CLC is a model that assumes a linear growth model for

Y but no particular model behind X , accompanied by the cross-level covariances with the growth factors for Y . It also has raw observed within-person predictors and outcomes rather than residualized ones.

Based on the classification table, we can further find the relations between the existing models. For instance, one may see LCM_y-RI_x as a latent variable centering model with time as a covariate since the differences between these two models are the slope factor of the time-varying predictors (S_i^X) and its relevant between-person effect (β_{SI}). However, this view of LCM_y-RI_x may be insufficient because of the nature of adding a time variable as a predictor rather than a fixed parameter as in SEM. Specifically, a time variable can capture the trend in the time-varying predictors, but at the same time, it also captures the trend in the outcome by the same time variable. I will discuss this point later in detail.

In addition to the classification of the existing models, careful examination of Table 2.5 brings us an insight into developing novel models that can disentangle the within-person and between-person effects. Specifically, no LCM model has been developed that assumes a random slope for the time-varying predictors. In other words, LCM has never been used as a means of detrending. Therefore, I propose four SEM models that can disaggregate the within-person and between-person effects by detrending time-varying predictors. The next chapter illustrates the details of the new models.

Table 2.5

Summary of the Reviewed Methods and Proposed Methods

Model	X-model		Y-model		WP Outcomes			WP Predictor			BP Predictor			BP Effect		
	I	S	I	S	Residualized	Observed	Latent	Residualized	Observed	Latent	Observed	Latent	β_{II}	β_{SI}	β_{SS}	CLC
FEM			✓		✓ ^a	✓		✓ ^a	✓							✓
REM			✓		✓ ^a	✓		✓ ^a	✓							
MLM-CRE			✓		✓ ^a	✓		✓ ^a	✓		✓		✓ ^b			
MLM-AC	✓		✓			✓		✓	✓							
MLM-WB	✓		✓			✓		✓	✓		✓					
MLM-LVC	✓		✓			✓		✓	✓	✓						
MLM-TAC	✓	✓ ^c	✓			✓		✓	✓		✓					
MLM-2S	✓	✓	✓	✓ ^d	✓ ^d	✓ ^d		✓	✓		✓					
MLM-E2S	✓	✓	✓	✓	✓ ^d	✓ ^d		✓	✓		✓		✓	✓	✓	
MLM-MV	✓	✓	✓	✓	✓ ^e	✓ ^e	✓ ^e	✓ ^e	✓ ^e	✓ ^e	✓ ^e	✓ ^e	✓ ^e	✓ ^e	✓ ^e	✓ ^e
LCMy-Rlx	✓		✓	✓		✓		✓								
LCMy-CLC			✓	✓					✓							✓
LCMxy-RPO*	✓	✓	✓	✓	✓		✓	✓		✓		✓	✓	✓	✓	
LCMxy-RP*	✓	✓	✓	✓		✓		✓		✓		✓	✓	✓	✓	
LCMx-Rly*	✓	✓	✓	✓		✓		✓		✓		✓	✓	✓	✓	
LCMx-CLC*	✓	✓				✓		✓		✓		✓	✓	✓		✓

Note. BP = between-person, WP = within-person, MLM = multilevel model, I = intercept, S = slope, CLC = cross-level covariances, FEM = fixed effects model, REM = random effects model, CRE = correlated random effects, AC = adaptive centering, WB = within-between model, LVC = latent variable centering, TAC = time-as-covariate, 2S = Curran and Bauer's (2011) two-step detrending, E2S = Hoffman's (2015) extended two-step detrending, MV = multivariate (multilevel model), RI = random intercept, RPO = residualized time-varying predictors and outcomes, RP = residualized time-varying predictors, * = proposed model. All models can estimate the within-person effect. ^a (quasi-) demeaning in the estimation, ^b contextual effect, ^c a time covariate catches the trends in both X and Y (composite trend), ^d optional residualization, ^e no explicit predictors, within-/between-person correlations.

Chapter 3

Proposed Methods

In Chapter 3, four novel SEM-based detrending models are proposed. These models are derived from the classification table shown in Table 2.5. The relations among the proposed models as well as between the proposed models and existing models are also discussed. In addition, motivations and research questions for the simulations in the subsequent chapter are described in the last section of this chapter.

3.1 Latent Curve Model with Residualized Predictors and Outcomes

The first proposed model is a variant of the latent curve model with structured residuals (LCM-SR), which was first developed by Curran et al. (2014). LCM-SR was developed for panel data analysis, in which researchers are interested in the lagged effects (i.e., autoregressive effects and cross-lagged effects) and the unobserved heterogeneity (individual differences). LCM-SR decomposes the variation of outcomes and predictors into the between-person variation and within-person variation using LCMs. Then, regression models are set up at the within-person and between-person levels. In the present research, no lagged effects are assumed and the within-person relation is restricted to the concurrent effects. Specifically, the proposed model is

$$X_{ti} = I_i^X + \lambda_t S_i^X + \varepsilon_{ti}^X, \quad (3.1)$$

$$Y_{ti} = I_i^Y + \lambda_t S_i^Y + \varepsilon_{ti}^Y, \quad (3.2)$$

$$\varepsilon_{ti}^Y = \gamma \varepsilon_{ti}^X + \nu_{ti}^Y, \quad (3.3)$$

$$I_i^Y = \delta^{I^Y} + \beta_{II} I_i^X + \beta_{IS} S_i^X + \zeta_i^{I^Y}, \quad (3.4)$$

$$S_i^Y = \delta^{S^Y} + \beta_{SI} I_i^X + \beta_{SS} S_i^X + \zeta_i^{S^Y}, \quad (3.5)$$

where $(I_i^X, S_i^X)' \sim N(\mathbf{0}, \mathbf{\Psi})$, $(\zeta_i^{I^Y}, \zeta_i^{S^Y})' \sim N(\mathbf{0}, \mathbf{\Theta})$, $\varepsilon_{ti}^X \sim N(0, \sigma_X^2)$, $\nu_{ti}^Y \sim N(0, \omega_Y^2)$, and

$$\mathbf{\Psi} = \begin{pmatrix} \psi_{II}^X & \psi_{IS}^X \\ \psi_{IS}^X & \psi_{SS}^X \end{pmatrix}, \quad (3.6)$$

$$\mathbf{\Theta} = \begin{pmatrix} \theta_{II}^Y & \theta_{IS}^Y \\ \theta_{IS}^Y & \theta_{SS}^Y \end{pmatrix}. \quad (3.7)$$

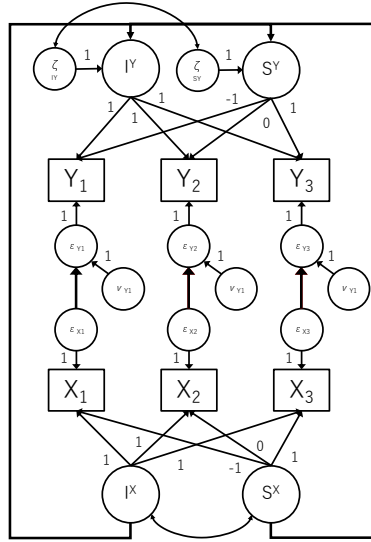
Among these equations, Equation 3.3 represents the within-level regression, and Equations 3.4 and 3.5 show the between-level regressions. In this sense, the present model is considered as an SEM-counterpart of the two-step detrending in the (generalized) FWL theorem. It differs from the two-step detrending in that the outcomes and predictors at a within-person level and between-person level are all latent variables. Also, unlike the two-step detrending, the current model only requires a single run to obtain parameter estimates. I refer to this model as *latent curve model with residualized predictors and outcomes* (LCMxy-RPO)¹.

Figure 3.1 is a path diagram of LCMxy-RPO. As the figure shows, LCMxy-RPO is similar to a standard bivariate LCM (Bollen & Curran, 2006). Differences between LCMxy-RPO and the standard bivariate LCM are that LCMxy-RPO uses regression models at level 2 (i.e., between-person regression) whereas the bivariate LCM uses

¹Subscripts are added to a model acronym to indicate for what variables are modeled by LCMs. For example, LCMxy means that both X and Y are modeled by LCM (i.e., the model has intercept and slope factors for both X and Y)

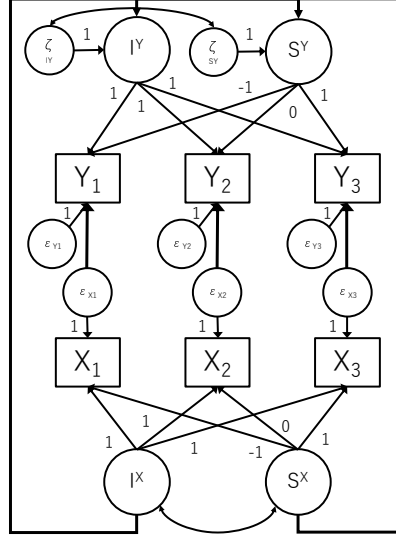
Figure 3.1

Latent Curve Model with Residualized Time-Varying Predictors and Outcomes (LCM_{xy}-RPO)



correlations and that LCM_{xy}-RPO regresses residuals of the time-varying predictors on those of the outcomes, whereas the bivariate LCM assumes no relation between the residuals.

Some may think that LCM_{xy}-RPO is more similar to the multivariate multilevel model than the two-step detrending and the bivariate LCM. In fact, multivariate multilevel models also decompose the outcomes and predictors into latent within-person and between-person components (e.g., X_{ti}^W and X_i^B). However, multivariate multilevel models use correlations rather than regressions to model the within-person and between-person relations. Thus, the multivariate multilevel models need an additional step to transform the correlations to the regression coefficients (i.e., the within-person effect and between-person effects; see Equations 2.82–2.85).

Figure 3.2*Latent Curve Model with Residualized Time-Varying Predictors (LCMxy-RP)*

3.2 Latent Curve Model with Residualized Predictors

The second proposed model is the *latent curve model with residualized time-varying predictors*, abbreviated to LCMxy-RP. As with LCMxy-RPO, this model has two growth curve models, one for each series (see Figure 3.2). Thus, it can capture the trends in X and Y separately. For the within-level regression, the residuals ε_{ti}^X serves as a predictor of Y_{ti} . Since the growth factors isolate the between-person variation related to the trend, the residualized time-varying predictors ε_{ti}^X only contain the within-person variation and measurement error. Therefore, the resulting coefficient is considered to represent the within-person effect.

The model equations of LCMxy-RP are

$$X_{ti} = I_i^X + \lambda_t S_i^X + \varepsilon_{ti}^X, \quad (3.8)$$

$$Y_{ti} = I_i^Y + \lambda_t S_i^Y + \gamma \varepsilon_{ti}^X + \varepsilon_{ti}^Y, \quad (3.9)$$

$$I_i^Y = \delta^{I^Y} + \beta_{II} I_i^X + \beta_{IS} S_i^X + \zeta_i^{I^Y}, \quad (3.10)$$

$$S_i^Y = \delta^{S^Y} + \beta_{SI} I_i^X + \beta_{SS} S_i^X + \zeta_i^{S^Y}. \quad (3.11)$$

As found in Figure 3.2 and Equation 3.9, LCMxy-RP directly predicts the raw outcomes Y_{ti} rather than the residuals ε_{ti}^Y . Although LCMxy-RP and LCMxy-RPO have different dependent variables in the within-person regression, both models are expected to have the same estimates for the within-person effects. This is so because the residualization of the time-varying predictors isolates the within-person variation from the between-person variation, and the within-level variation is correlated only with that in the outcomes, regardless of whether the outcomes are residualized. The validity of this argument can also be confirmed by equating the two models. That is, by substituting Equation 3.3 to Equation 3.2, we have

$$Y_{ti} = I_i^Y + \lambda_t S_i^Y + \varepsilon_{ti}^Y = I_i^Y + \lambda_t S_i^Y + \gamma \varepsilon_{ti}^X + \nu_{ti}^Y, \quad (3.12)$$

which is the exact same form as Equation 3.9 (i.e., ν_{ti}^Y in LCMxy-RPO is equal to ε_{ti}^Y in LCMxy-RP). Note that these arguments agree with the generalized FWL theorem, which says that the adjustment of dependent variables does not affect the estimation of within-level coefficients.

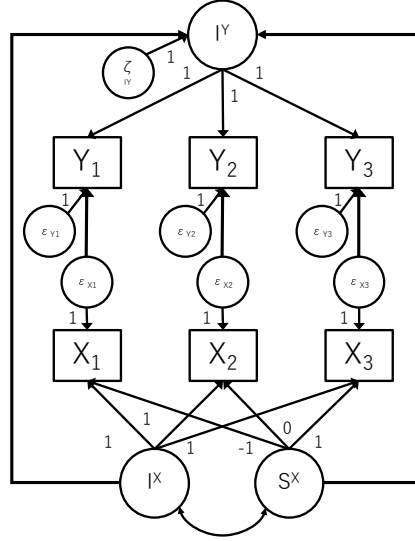
3.3 Latent Curve Model for Time-Varying Predictors with a Random Intercept for Outcomes

The third proposed model is obtained by “flipping over” LCMxy-RP (cf. Figures 2.4 and 3.3). Specifically, in the third model, an LCM is assumed for the time-varying predictors and a random intercept model for the outcomes. In other words, LCMxy-RP reduced to this model by assuming no slope factor for the outcomes.

This model, named *latent curve model for time-varying predictors with a random*

Figure 3.3

Latent Curve Model for X with Random Intercept for Y (LCMx-Rly)



intercept for outcomes (LCMx-RIy), can also be seen as a detrended fixed effects model.

We can find the similarity of LCMx-RIy and SEM-based fixed effects model when comparing Figures 2.3 and 3.3. One difference between the two models lies in their between-person models. In the fixed effects model, there is no explicit model at the between-person level and the between-person relations are absorbed in the cross-level covariances. In contrast, the cross-level covariances are replaced with the regression of the random intercept on the growth factors. Then, the model equations of LCMx-RIy can be written as:

$$X_{ti} = I_i^X + \lambda_t S_i^X + \epsilon_{ti}^X, \quad (3.13)$$

$$Y_{ti} = I_i^Y + \gamma \epsilon_{ti}^X + \epsilon_{ti}^Y, \quad (3.14)$$

$$I_i^Y = \delta^{I^Y} + \beta_{II} I_i^X + \beta_{IS} S_i^X + \zeta_i^{I^Y}. \quad (3.15)$$

These model equations imply the similarity of LCMx-RIy to Curran and Bauer's (2011) and Hoffman's (2015) two-step detrending. More precisely, LCMx-RIy is in between

the two detrending methods. Both Curran and Bauer's method and LCMx-RIy assume a linear growth model for X and a random intercept model for Y (see Equations 2.52 and 3.13); however, Curran and Bauer only uses the intercept of X for the between-person regression, whereas LCMx-RIy and Hoffman's extended two-step method uses both intercept and slope as between-level regressors (Equations 2.56, 2.62, and 3.15). A difference between LCMx-RIy and Hoffman's method can also be seen in their between-level models. That is, LCMx-RIy only has a between-level regression for the intercept of Y , while Hoffman's method has two between-person regressions; it regresses the intercept and slope of Y on the intercept and slope of X (Equations 2.59–2.60 and 3.15).

3.4 Latent Curve Model for Time-Varying Predictors with Cross-Level Covariances

The fourth proposed model, the *latent curve model for time-varying predictors with cross-level covariances* (LCMx-CLC), is obtained by replacing the between-level models in the other three proposed models with cross-level covariances, keeping the growth model of the time-varying predictors. The resulting path diagram is presented in Figure 3.4, and the model equations of this model are

$$X_{ti} = I_i^X + \lambda_t S_i^X + \varepsilon_{ti}^X, \quad (3.16)$$

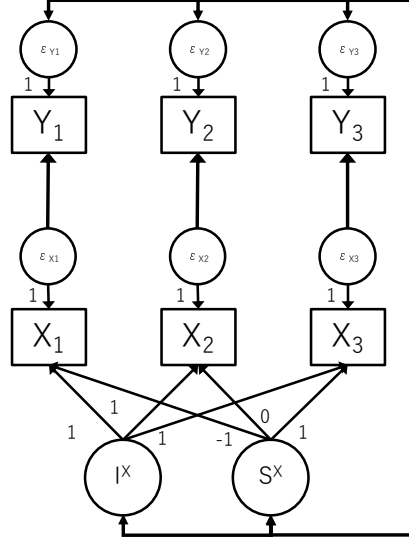
$$Y_{ti} = \mu_t^Y + \gamma \varepsilon_{ti}^X + \varepsilon_{ti}^Y, \quad (3.17)$$

$$\text{COV}[I_i^X, Y_{ti}] = \phi_{It}^{XY}, \quad (3.18)$$

$$\text{COV}[S_i^X, Y_{ti}] = \phi_{St}^{XY}, \quad (3.19)$$

$$\text{COV}[Y_{ti}, Y_{t'i}] = \phi_{tt'}^{YY}. \quad (3.20)$$

As with the other proposed models, LCMx-CLC has residualized predictors. Thus,

Figure 3.4*Latent Curve Model for X with Cross-Level Covariances (LCMx-CLC)*

if the trends in the predictors are linear in the population, the corresponding coefficient γ represents the within-person effects. On the other hand, the between-person relationship between X and Y is modeled simply by cross-level covariances, and the mean changes in Y are captured by the time-specific item intercepts μ_t^Y . Therefore, LCMx-CLC is expected to capture a trend in Y in a flexible manner.

Although LCMx-CLC does not have an explicit between-level model, one can recover the between-person effects from the available estimates in a similar way to LCM_y-CLC. Suppose that LCM_{xy}-RPO or LCM_{xy}-RP holds in the population. Then, the between-person effect β_{II} can be written as

$$\beta_{II} = \frac{\psi_{SS}^X \text{COV}[I_i^X, I_i^Y] - \psi_{IS}^X \text{COV}[S_i^X, I_i^Y]}{\psi_{II}^X \psi_{SS}^X - (\psi_{IS}^X)^2}. \quad (3.21)$$

Similarly, the other three between-person effects can be written as follows:

$$\beta_{SI} = \frac{\psi_{SS}^X \text{COV}[I_i^X, S_i^Y] - \psi_{IS}^X \text{COV}[S_i^X, S_i^Y]}{\psi_{II}^X \psi_{SS}^X - (\psi_{IS}^X)^2}, \quad (3.22)$$

$$\beta_{IS} = \frac{\psi_{II}^X \text{COV}[S_i^X, I_i^Y] - \psi_{IS}^X \text{COV}[I_i^X, S_i^Y]}{\psi_{II}^X \psi_{SS}^X - (\psi_{IS}^X)^2}, \quad (3.23)$$

$$\beta_{SS} = \frac{\psi_{II}^X \text{COV}[S_i^X, S_i^Y] - \psi_{IS}^X \text{COV}[I_i^X, S_i^Y]}{\psi_{II}^X \psi_{SS}^X - (\psi_{IS}^X)^2}. \quad (3.24)$$

Thus, we need to estimate the cross-variable covariances in the numerators, namely $\text{COV}[I_i^X, I_i^Y]$, $\text{COV}[I_i^X, I_i^Y]$, $\text{COV}[S_i^X, I_i^Y]$, and $\text{COV}[S_i^X, S_i^Y]$. To estimate these covariances, consider the averages of cross-level covariances. Using the model equation of LCMxy-RPO (Equation 3.2), we have

$$\begin{aligned} \frac{1}{T} \sum_{t=1}^T \text{COV}[I_i^X, Y_{ti}] &= \frac{1}{T} \sum_{t=1}^T \text{COV}[I_i^X, I_i^Y + \lambda_t S_i^Y + \varepsilon_{ti}^Y] \\ &= \frac{1}{T} \sum_{t=1}^T (\text{COV}[I_i^X, I_i^Y] + \lambda_t \text{COV}[I_i^X, S_i^Y]) \\ &= \frac{1}{T} \sum_{t=1}^T \text{COV}[I_i^X, I_i^Y] \\ &= \text{COV}[I_i^X, I_i^Y], \end{aligned} \quad (3.25)$$

$$\begin{aligned} \frac{1}{T} \sum_{t=1}^T \text{COV}[S_i^X, Y_{ti}] &= \frac{1}{T} \sum_{t=1}^T \text{COV}[S_i^X, I_i^Y + \lambda_t S_i^Y + \varepsilon_{ti}^Y] \\ &= \frac{1}{T} \sum_{t=1}^T (\text{COV}[S_i^X, I_i^Y] + \lambda_t \text{COV}[S_i^X, S_i^Y]) \\ &= \frac{1}{T} \sum_{t=1}^T \text{COV}[S_i^X, I_i^Y] \\ &= \text{COV}[S_i^X, I_i^Y]. \end{aligned} \quad (3.26)$$

Furthermore, consider the difference between the cross-level covariances at two different time points, t and t' ($t > t'$). Then, we have

$$\begin{aligned} &\text{COV}[I_i^X, Y_{ti}] - \text{COV}[I_i^X, Y_{t'i}] \\ &= \text{COV}[I_i^X, I_i^Y + \lambda_t S_i^Y + \varepsilon_{ti}^Y] - \text{COV}[I_i^X, I_i^Y + \lambda_{t'} S_i^Y + \varepsilon_{t'i}^Y] \end{aligned}$$

$$\begin{aligned}
&= (\text{COV}[I_i^X, I_i^Y] + \lambda_t \text{COV}[I_i^X, S_i^Y]) - (\text{COV}[I_i^X, I_i^Y] + \lambda_{t'} \text{COV}[I_i^X, S_i^Y]) \\
&= (\lambda_t - \lambda_{t'}) \text{COV}[I_i^X, S_i^Y] \\
&\implies \text{COV}[I_i^X, S_i^Y] = \frac{\text{COV}[I_i^X, Y_{ti}] - \text{COV}[I_i^X, Y_{t'i}]}{\lambda_t - \lambda_{t'}}, \tag{3.27}
\end{aligned}$$

$$\begin{aligned}
&\text{COV}[S_i^X, Y_{ti}] - \text{COV}[S_i^X, Y_{t'i}] \\
&= \text{COV}[S_i^X, I_i^Y + \lambda_t S_i^Y + \varepsilon_{ti}^Y] - \text{COV}[S_i^X, I_i^Y + \lambda_{t'} S_i^Y + \varepsilon_{t'i}^Y] \\
&= (\text{COV}[S_i^X, I_i^Y] + \lambda_t \text{COV}[S_i^X, S_i^Y]) - (\text{COV}[S_i^X, I_i^Y] + \lambda_{t'} \text{COV}[S_i^X, S_i^Y]) \\
&= (\lambda_t - \lambda_{t'}) \text{COV}[S_i^X, S_i^Y] \\
&\implies \text{COV}[S_i^X, S_i^Y] = \frac{\text{COV}[I_i^X, Y_{ti}] - \text{COV}[I_i^X, Y_{t'i}]}{\lambda_t - \lambda_{t'}}. \tag{3.28}
\end{aligned}$$

Therefore, by the method of moments, we have the following estimators for the between-person effects:

$$\hat{\beta}_{II} = \frac{\hat{\psi}_{SS}^X \hat{\phi}_{I\cdot}^{XY} - \hat{\psi}_{IS}^X \hat{\phi}_{S\cdot}^{XY}}{\hat{\psi}_{II}^X \hat{\psi}_{SS}^X - (\hat{\psi}_{IS}^X)^2}, \tag{3.29}$$

$$\hat{\beta}_{SI} = \frac{\hat{\psi}_{SS}^X \hat{\Delta}_{I\cdot}^{XY} - \hat{\psi}_{IS}^X \hat{\Delta}_{S\cdot}^{XY}}{\hat{\psi}_{II}^X \hat{\psi}_{SS}^X - (\hat{\psi}_{IS}^X)^2}, \tag{3.30}$$

$$\hat{\beta}_{IS} = \frac{\hat{\psi}_{II}^X \hat{\phi}_{S\cdot}^{XY} - \hat{\psi}_{IS}^X \hat{\phi}_{I\cdot}^{XY}}{\hat{\psi}_{II}^X \hat{\psi}_{SS}^X - (\hat{\psi}_{IS}^X)^2}, \tag{3.31}$$

$$\hat{\beta}_{SS} = \frac{\hat{\psi}_{II}^X \hat{\Delta}_{S\cdot}^{XY} - \hat{\psi}_{IS}^X \hat{\Delta}_{I\cdot}^{XY}}{\hat{\psi}_{II}^X \hat{\psi}_{SS}^X - (\hat{\psi}_{IS}^X)^2}, \tag{3.32}$$

where

$$\bar{\phi}_{I\cdot}^{XY} \equiv \frac{1}{T} \sum_{t=1}^T \text{COV}[I_i^X, Y_{ti}], \tag{3.33}$$

$$\bar{\phi}_{S\cdot}^{XY} \equiv \frac{1}{T} \sum_{t=1}^T \text{COV}[S_i^X, Y_{ti}], \tag{3.34}$$

$$\Delta_{I\cdot}^{XY} \equiv \frac{1}{T(T-1)} \sum_{t>t'} \frac{\phi_{It}^{XY} - \phi_{It'}^{XY}}{\lambda_t - \lambda_{t'}}, \tag{3.35}$$

$$\Delta_{S.}^{XY} \equiv \frac{1}{T(T-1)} \sum_{t>t'} \frac{\phi_{St}^{XY} - \phi_{St'}^{XY}}{\lambda_t - \lambda_{t'}}, \quad (3.36)$$

Note that the data collection design is assumed to be time-balanced and the time variable is centered. As such, LCMx-CLC can provide estimates of the between-person effects. However, they are secondary concerns; LCMx-CLC mainly focuses on estimating the within-person effect. In this sense, LCMx-CLC is similar to Raudenbush's (2009) adaptive centering although they differ in that LCMx-CLC does not have a random intercept for the outcomes (see Equation 2.23).

3.5 Summary and Discussion on the Proposed Models

In this chapter, I have proposed four SEM models that can disaggregate the within-person effect and between-person effects by detrending the time-varying predictors. These models were induced from the classification framework that is shown in Table 2.5. More specifically, the proposed models can be derived by identifying the patterns in Table 2.5 that the previous studies have not examined yet.

We can further develop some variations of the proposed models. For example, if we assume random intercepts for X and Y and regress I^Y on I^X , and if we use raw observed within-person predictors and outcomes, then we have a latent contextual effect model, which is an SEM-counterpart of Mundlak's model². Also, the residualized predictors in the proposed models can be replaced with the raw predictors X_{ti} . In this case, the between-level coefficients β_{II} , β_{IS} , β_{SI} , and β_{SS} are interpreted as the contextual effects (i.e., the differences between the between-person effects and the within-person effect).

Table 2.5 also provides us with a different perspective from the literature on what variables are detrended by each model. Based on their analytic result, Wang and Maxwell

²This model is also called Model W4 in (Hamaker & Muthén, 2020).

(2015) concluded that the time-as-covariate method detrends the time-varying covariate rather than the outcomes, while Hoffman and Stawski (2009) stated that it is a detrending of the outcomes. However, as mentioned in the note of Table 2.5, the time-as-covariate method simultaneously detrends the time-varying predictors and the outcomes with a single time variable. Suppose that both time-varying predictor and outcome show a linear trend in the population and that data from the population is analyzed by MLM-TAC. Specifically, assume that the population model can be written as

$$\begin{aligned} X_{ti} &= \beta_{0i}^X + \beta_{1i}^X \text{Time}_{ti} + \varepsilon_{ti}^X \\ &= (\gamma_{00}^X + u_{0i}^X) + (\gamma_{10}^X + u_{1i}^X) \text{Time}_{ti} + \varepsilon_{ti}^X, \end{aligned}$$

$$\begin{aligned} Y_{ti} &= \beta_{0i}^Y + \beta_{1i}^Y (X_{ti} - \bar{X}_{\cdot i}) + \beta_{2i}^Y \text{Time}_{ti} + \varepsilon_{ti}^Y \\ &= (\gamma_{00}^Y + \gamma_{01}^Y \bar{X}_{\cdot i} + u_{0i}^Y) + \gamma_{10}^Y (X_{ti} - \bar{X}_{\cdot i}) + (\gamma_{20}^Y + \gamma_{21}^Y \bar{X}_{\cdot i} + u_{2i}^Y) \text{Time}_{ti} + \varepsilon_{ti}^Y. \end{aligned}$$

Since the time variable is centered and balanced, we have

$$\bar{X}_{\cdot i} = \beta_{0i}^X.$$

Then, the fixed part (i.e., non-random part) of the time-as-covariate model can be rewritten as

$$\begin{aligned} &\gamma_{00}^Y + \gamma_{01}^Y \bar{X}_{\cdot i} + \gamma_{10}^Y (X_{ti} - \bar{X}_{\cdot i}) + (\gamma_{20}^Y + \gamma_{21}^Y \bar{X}_{\cdot i}) \text{Time}_{ti} \\ &= \gamma_{00}^Y + \gamma_{01}^Y \beta_{0i}^X + \gamma_{10}^Y (\beta_{1i}^X \text{Time}_{ti} + \varepsilon_{ti}^X) + (\gamma_{20}^Y + \gamma_{21}^Y \beta_{0i}^X) \text{Time}_{ti} \\ &= \underbrace{\gamma_{00}^Y}_{\text{intercept}} + \underbrace{\gamma_{01}^Y \beta_{0i}^X}_{\text{between-person predictor}} + \underbrace{\gamma_{10}^Y \varepsilon_{ti}^X}_{\text{within-person predictor}} + \underbrace{(\gamma_{20}^Y + \gamma_{10}^Y \beta_{1i}^X + \gamma_{21}^Y \beta_{0i}^X) \text{Time}_{ti}}_{\text{time as covariate}} \end{aligned}$$

Thus, the coefficient of the time variable in the time-as-covariate model represents a *composite trend*, which is a sum of the trend related to X (i.e., $\gamma_{10}^Y \beta_{1i}^X + \gamma_{21}^Y \beta_{0i}^X$) and the one inherent in Y (γ_{20}^Y). Accordingly, the time-as-covariate method cannot differentiate the trend that is attributable to the systematic change of the time-varying predictor from the

trend that is intrinsic to the outcomes. This inability of the time-as-covariate method to distinguish the source of trends is considered as one of the major differences between the MLM approach and the SEM approach to detrending. Note that Frisch and Waugh (1933) referred to the entire coefficient of the time variable (i.e., $\gamma_{20}^Y + \gamma_{10}^Y \hat{\beta}_{1i}^X + \gamma_{21}^Y \hat{\beta}_{0i}^X$) as *total trend*, the trend attributable to the time-varying predictors ($\gamma_{10}^Y \hat{\beta}_{1i}^X + \gamma_{21}^Y \hat{\beta}_{0i}^X$) as *indirect trend*, and the trend inherent in Y (γ_{20}^Y) as *partial trend* or *direct trend*.

3.6 Motivation and Research Questions for the Simulation Studies

In the previous sections of Chapter 3, I described the proposed models and compared them with the existing models from the theoretical viewpoint. The remaining parts of this dissertation are then devoted to empirical comparisons of the proposed and existing models. Monte Carlo simulation rather than real data analysis is considered appropriate for such comparisons because Monte Carlo simulation, where we know the exact data-generating mechanism and true parameter values, allows for not only the comparisons of the models but also the performance evaluation of the proposed models.

As discussed in Section 3.5, one of the advantages of the SEM-based detrending is that it can specify series-specific slope factors that capture the trends in X and Y separately. Using this property, we can differentiate various data-generating models (i.e., population models) that we cannot with MLM. For example, when we use LCMx-RIy as a population model, we can simulate data sets where the trend in Y is completely determined by the trend in X . Such a simulation can complement the previous studies, for example, Wang and Maxwell (2015), where the time-as-covariate model, which cannot differentiate the trends in time-varying predictors and the ones in outcomes, was used as a

data-generating model.

Additionally, SEM-based population models are more realistic in that they take into consideration the sampling variability of the person means of the time-varying predictors. When one uses MLM as a population model, person means are often incorporated directly as part of a data-generating mechanism. This means that they ignore the sampling variability in the person means and regard the person means as true scores. On the other hand, SEM-based population models properly consider the sampling variability by assuming latent factors behind the observed variables. This difference can be crucial for the estimation of the between-person effects especially when the number of measurements is small (i.e., when “cluster” size is small) since the sampling error can be large in such a situation.

SEM-based detrending approaches can also complement previous studies when they are used as an analytic model. For instance, Wang and Maxwell (2015) investigated when and how to detrend through real data analyses and simulations, but they only examined multilevel models. Hamaker and Muthén (2020) only used centering models, though their analytic models covered a variety of SEM and MLM models. Curran et al. (2012) also compared SEM- and MLM-based growth models (i.e., detrending methods) through a simulation. Unfortunately, they just consider a single data-generating condition, thus the performance of the detrending methods in various situations is still unclear. Plewis (2001) also examined both MLM and SEM models; however, Plewis was only concerned with the between-person effects and lacked attention to the within-person effect. Moreover, Plewis did not introduce the concept of centering/detrending.

Another important viewpoint for the disaggregation of the within-person effect and between-person effects is variance partitioning of time-varying predictors. Hoffman (2015) illustrated why the centering approach does not work when there is a trend in time-varying predictors from this viewpoint. Hoffman pointed out that since the person-mean centered

level-1 predictors and the person means cannot fully capture the slope variation in the time-varying predictors, it is absorbed in residuals at each level, resulting in the biased effect estimates. This argument implies that if the variation in time-varying predictors and outcomes is fully explained, it is anticipated that the within-person effect and between-person effects can be estimated without bias even though there is a trend in time-varying predictors. Such a situation may occur when there is a nonzero average slope with zero variance for the time-varying predictors. Or more generally, the size of between-person variances for time-varying predictors may affect the estimation of within-person and between-person effects. With SEM-based population models, we can manipulate the size of variance for the time-varying predictors separately from that inherent in the outcomes. Therefore, using an SEM model as a data-generating mechanism can extend Hoffman (2015).

Given these considerations, this dissertation extends the previous research and evaluates the performance of MLM- and SEM-based disaggregation methods including both the existing and proposed methods. The performance of the disaggregation methods is evaluated through a series of Monte Carlo simulations, where the population models implemented by SEM have one of the following characteristics regarding trends over time:

1. Neither time-varying predictors nor outcomes has a trend.
2. Outcomes have a linear trend but time-varying predictors do not.
3. Time-varying predictors have a linear trend but outcomes do not.
4. Both time-varying predictors and outcomes have linear trends
5. Both time-varying predictors and outcomes show trends and the slope variance of time-varying predictors varies, including no individual difference (i.e., zero variance).

Furthermore, the current study poses the following research questions for the simulations.

These research questions are explicitly answered in the Discussion chapter:

- [RQ1] Can the proposed and existing methods correctly disentangle the within-person effect and between-person effects under various population models with and without trends in time-varying predictors and outcomes?
- [RQ2] Can the proposed and existing methods accurately estimate the within-person and between-person effects when the time-varying predictors show a trend with no or large individual differences?
- [RQ3] Are the proposed methods comparable to the existing methods in terms of the quality of estimation and statistical inference regarding the within-person effect and the between-person effects?

Chapter 4

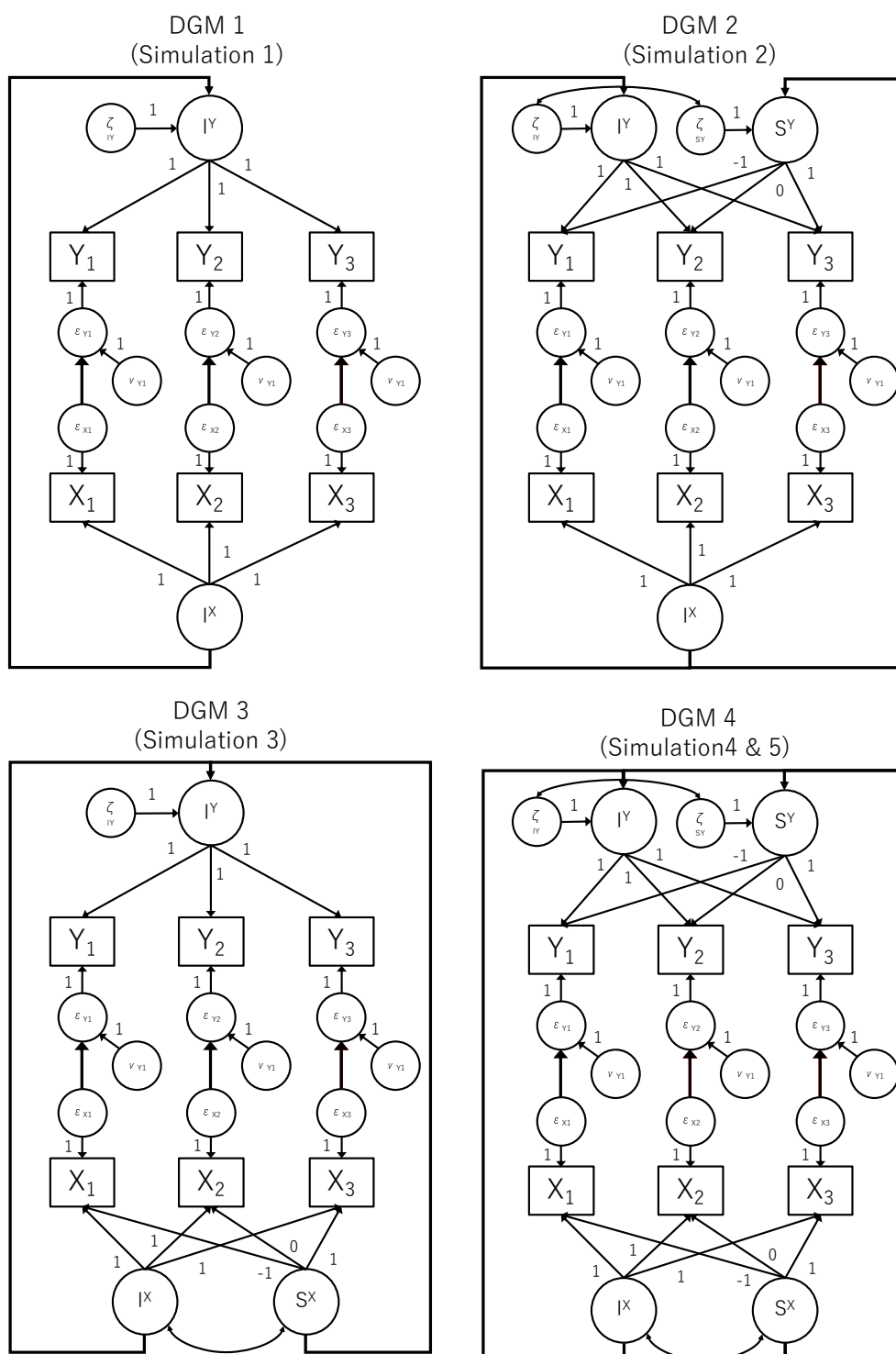
Simulation Designs

4.1 Data-Generating Models

The present study conducts five simulations to investigate the performance of SEM-based and MLM-based centering and detrending. Among the reviewed and proposed models, the latent curve model with residualized predictors and outcomes (LCMxy-RPO) and its submodels are used as the data-generating models (DGMs). Figure 4.1 illustrates the four DGMs that are used in the five simulations. DGM 4 is the full LCMxy-RPO, which has an intercept factor and a slope factor for the time-varying predictors (I_i^X and S_i^X) and the outcomes (I_i^Y and S_i^Y). This model is used in Simulations 4 and 5. Although Simulation 4 and 5 use the same DGM, they have different sets of true values for the within-person and between-person effects and the variance components (see the next section for details of settings).

DGMs in Simulations 2 and 3 are similar to the latent curve model with a random intercept for X (LCMy-RIx) and latent curve model for X with a random intercept for Y (LCMx-RIy), respectively. However, the DGMs differ from LCMy-RIx and LCMx-RIy in that the DGMs have residualized outcomes. Since the residualization of the outcomes is considered optional, LCMy-RIx and LCMx-RIy are expected to perform well in Simulation 2 and 3, respectively.

Simulation 1 uses the most restricted model as a DGM, in which no slope factor is assumed for the time-varying predictors and outcomes. In other words, Simulation 1

Figure 4.1*Data-Generating Models (DGMs; Population Models)*

assumes stationary predictors and outcomes. Accordingly, analytic models do not need to have a slope factor for X and Y , and centering is sufficient to disaggregate the within-person and between-person effects. Also, DGM 1 can be regarded as a special case of random intercept cross-lagged panel model (Hamaker et al., 2015; Mulder & Hamaker, 2020; Usami, Murayama, & Hamaker, 2019), where the cross-lagged effects and the autoregressive effects are constrained to be zero, and the concurrent effects are modeled by regression rather than correlation. Additionally, DGM 1 is the same as the Model W3 in Hamaker and Muthén (2020) except that DGM 1 uses the residualized outcomes, while Model W3 uses the observed outcomes.

4.2 Experimental Factors

Among the five simulations, Simulation 5 has a slightly different purpose from the other simulations. While Simulations 1–4 manipulate the presence and absence of the between-person effects with the variance components fixed, Simulation 5 concerns the variance components of the time-varying predictors, setting the within-person and between-person effects at constants.

Table 4.1 summarizes the experimental factors and their levels in each simulation. The sample size N has four levels ($N = 100, 300, 500, 1000$), simulating small, medium, medium-large, and large samples. The number of measurements has three levels ($T = 3, 5, 7$). Note that three-wave data ($T = 3$) is the minimum required to conduct a linear growth curve analysis (Bollen & Curran, 2006; Singer & Willett, 2003). These two factors are always manipulated in all five simulations.

Table 4.1

Simulation Conditions

Experimental Factor	Parameter	Simulation 1 (No Slope)	Simulation 2 (Slope for Y)	Simulation 3 (Slope for X)	Simulation 4 (Slope for X & Y)	Simulation 5 (Slope for X & Y)
Sample Size	N	{100, 300, 500, 1000}	{100, 300, 500, 1000}	{100, 300, 500, 1000}	{100, 300, 500, 1000}	{100, 300, 500, 1000}
Measurements	T	{3, 5, 7}	{3, 5, 7}	{3, 5, 7}	{3, 5, 7}	{3, 5, 7}
WPE	γ	{-0.6, 0.0, 0.6}	{-0.6, 0.0, 0.6}	{-0.6, 0.0, 0.6}	{-0.6, 0.0, 0.6}	0.6
BPE (intercept-intercept)	β_{II}	{0.0, 0.6}	{0.0, 0.6}	{0.0, 0.6}	{0.0, 0.6}	0.6
BPE (intercept-slope)	β_{SI}	—	{0.0, 0.3}	—	{0.0, 0.3}	0.3
BPE (slope-intercept)	β_{IS}	—	—	{0.0, 0.3}	{0.0, 0.3}	0.3
BPE (slope-slope)	β_{SS}	—	—	—	{0.0, 0.6}	0.6
Variance of I_i^X	ψ_{II}^X	16	16	16	16	{0, 16, 64}
Variance of S_i^X	ψ_{SS}^X	—	—	1	1	{0, 1, 4}
Correlation of I_i^X and S_i^X	ρ_{IS}^X	—	—	0.3	0.3	{0.0, 0.3, 0.6}
Total Number of Conditions		72	144	144	576	324

Note. WPE = within-person effect, BPE = between-person effect.

In Simulations 1–4, three levels of within-person effects and two levels of between-person effects are considered. The levels of the within-person effect include both positive and negative values as well as the null effect (i.e., $\gamma \in \{-0.6, 0.0, 0.6\}$). As for the between-person effects, one level is set at zero and the other level is set at a positive value. Specifically, for the path from an intercept factor, the population coefficient takes 0.0 and either 0.6 or 0.3 (i.e., $\beta_{II} \in \{0.0, 0.6\}$ and $\beta_{SI} \in \{0.0, 0.3\}$). For the path from a slope factor, the population coefficient takes 0.0 and either 0.3 or 0.6 ($\beta_{IS} \in \{0.0, 0.3\}$ and $\beta_{SS} \in \{0.0, 0.6\}$). As a result, the total numbers of the conditions amount to 72 (Simulation 1), 144 (Simulation 2 and 3), and 576 (Simulation 4).

In contrast to Simulation 1–4, Simulation 5 has fixed, nonzero population values for the within-person and between-person effects. These values are taken from the population settings for Simulation 4. Instead of manipulating the within-person and between-person effects, Simulation 5 examines the impact of the variance components of the time-varying predictors. The levels of the variances are set at zero, the same value as Simulation 4, and larger value than Simulation 4. For example, the variance of the intercept factor of X , ψ_{II}^X , takes 0 (no variation), 16 (the same as Simulation 4), or 64 (larger than Simulation 4). See Table 4.1 for the settings of ψ_{SS}^X and ψ_{IS}^X . In the end, Simulation 5 has 324 conditions with five experimental factors.

4.3 Settings for Other Parameters

Other model parameters are unchanged across the simulations. Table 4.2 shows these parameter values. Using the parameter values in Tables 4.1 and 4.2, we can compute the moments (i.e., expectations, variances, and covariances) of the endogenous variables under each condition. For example, the expectations and variances of the time-varying predictor X_{ti} and the outcomes Y_{ti} can be expressed as

Table 4.2*Other Model Parameters*

Parameter		Value
Mean of I_i^X	τ_I^X	50
Mean of S_i^X	τ_S^X	5
Variance of ε_{ti}^X	σ_X^2	16
Intercept of I_i^Y	δ^{I^Y}	100
Intercept of S_i^Y	δ^{S^Y}	10
Variance of $\zeta_i^{I^Y}$	θ_{II}^Y	4
Variance of $\zeta_i^{S^Y}$	θ_{SS}^Y	1
Covariance of ζ^{I^Y} and ζ^{S^Y}	θ_{IS}^Y	0.8
(Correlation of ζ^{I^Y} and ζ^{S^Y})	θ_{IS}^Y	0.2
Variance of ν_{ti}^Y	ω_Y^2	4

$$\mathbb{E}[X_{ti}] = \mathbb{E}[I_i^X + \lambda_t S_i^X + \varepsilon_{ti}^X] = \tau_I^X + \lambda_t \tau_S^X \equiv \mu_t^X, \quad (4.1)$$

$$\text{VAR}[X_{ti}] = \text{VAR}[I_i^X + \lambda_t S_i^X + \varepsilon_{ti}^X] = \psi_{II}^X + \lambda_t^2 \psi_{SS}^Y + 2\lambda \psi_{IS}^X \equiv \phi_t^{XX}, \quad (4.2)$$

$$\begin{aligned} \mathbb{E}[Y_{ti}] &= \mathbb{E}[I_i^Y + \lambda_t S_i^Y + \gamma \varepsilon_{ti}^X + \nu_{ti}^Y] \\ &= \mathbb{E}[(\beta_{II} + \lambda_t \beta_{IS})I_i^X + (\beta_{SI} + \lambda_t \beta_{SS})S_i^X + \delta^{I^Y} + \lambda_t \delta^{S^Y}] \\ &= (\beta_{II} + \lambda_t \beta_{IS})\tau_I^X + (\beta_{SI} + \lambda_t \beta_{SS})\tau_S^X + \delta^{I^Y} + \lambda_t \delta^{S^Y} \equiv \mu_t^Y, \end{aligned} \quad (4.3)$$

$$\begin{aligned} \text{VAR}[Y_{ti}] &= \text{VAR}[I_i^Y + \lambda_t S_i^Y + \gamma \varepsilon_{ti}^X + \nu_{ti}^Y] \\ &= \text{VAR}[(\beta_{II} + \lambda_t \beta_{IS})I_i^X + (\beta_{SI} + \lambda_t \beta_{SS})S_i^X + (\zeta_i^{I^Y} + \lambda_t \zeta_i^{S^Y}) + \gamma \varepsilon_{ti}^X + \nu_{ti}^Y] \\ &= (\beta_{II} + \lambda_t \beta_{IS})^2 \psi_{II}^X + (\beta_{SI} + \lambda_t \beta_{SS})^2 \psi_{SS}^Y + (\beta_{II} + \lambda_t \beta_{IS})(\beta_{SI} + \lambda_t \beta_{SS}) \psi_{IS}^X \\ &\quad + (\theta_{II}^Y + \lambda_t^2 \theta_{SS}^Y + 2\lambda_t \theta_{IS}^Y) + \gamma^2 \sigma_X^2 + \omega_Y^2 \equiv \phi_t^{YY}. \end{aligned} \quad (4.4)$$

Using these equations, we can monitor the data-generating process from different viewpoints. For example, we can draw the expected growth curves using Equation 4.1 or 4.3. Figures 4.2 and 4.3 depict the expected growth curves in Simulation 4. As can be seen in the figures, the averages of X_{ti} range between 35 and 65, while the averages of Y_{ti} lie in between 16 and 215.5. Since the time variable is centered, these ranges become broader when the number of measurements is large.

Also, we can compute the proportion of the within-person components and the between-person components to the observed score variance. For example, we can decompose Equation 4.4 in the following way:

$$\begin{aligned} \text{VAR}[Y_{ti}] = & \underbrace{(\beta_{II} + \lambda_t \beta_{IS})^2 \psi_{II}^X + (\beta_{SI} + \lambda_t \beta_{SS})^2 \psi_{SS}^X + (\beta_{II} + \lambda_t \beta_{IS})(\beta_{SI} + \lambda_t \beta_{SS}) \psi_{IS}^X}_{\text{between-person effects}} \\ & + \underbrace{(\theta_{II}^Y + \lambda_t^2 \theta_{SS}^Y + 2\lambda_t \theta_{IS}^Y)}_{\text{between-person residuals}} + \underbrace{\gamma^2 \sigma_X^2}_{\text{within-person effect}} + \underbrace{\omega_Y^2}_{\text{within-person residual}}. \end{aligned}$$

Based on this decomposition, we can check the proportion of each component to the observed score variance. Table 4.3 summarizes the proportion of the four components. Under the conditions and settings explained above, each component accounts for roughly 15–40% of the outcome variance. Under some conditions, the proportion can be extreme (i.e., zero and one). These are expected to happen frequently in Simulation 5, where the variance components can be zero.

All parameter settings above (i.e., the choices of the levels of true parameter values), including both experimental and non-experimental factors, were determined based on these structural values. Specifically, the true values were chosen so that expected growth curves takes positive values to simulate the reality (e.g., standardized test scores, psychological scale scores) and that the performance of the models can be examined under various settings, while the average proportions of the within-person and between-person components are bounded between 15% and 40% to avoid too extreme case in which one of

Figure 4.2

Expected Growth Curve for the Time-Varying Predictors in Simulation 4

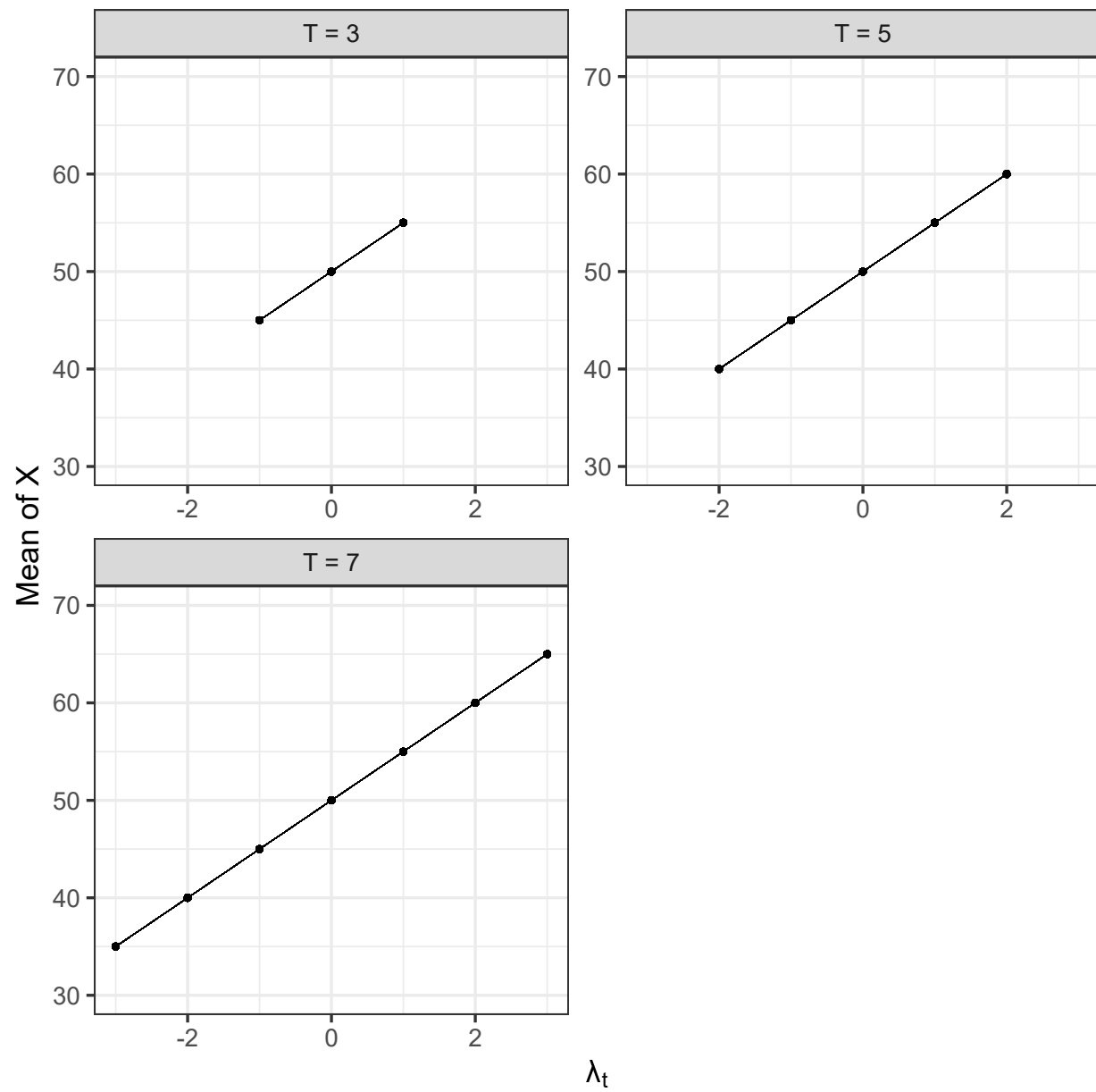


Figure 4.3

Expected Growth Curve for the Outcomes in Simulation 4

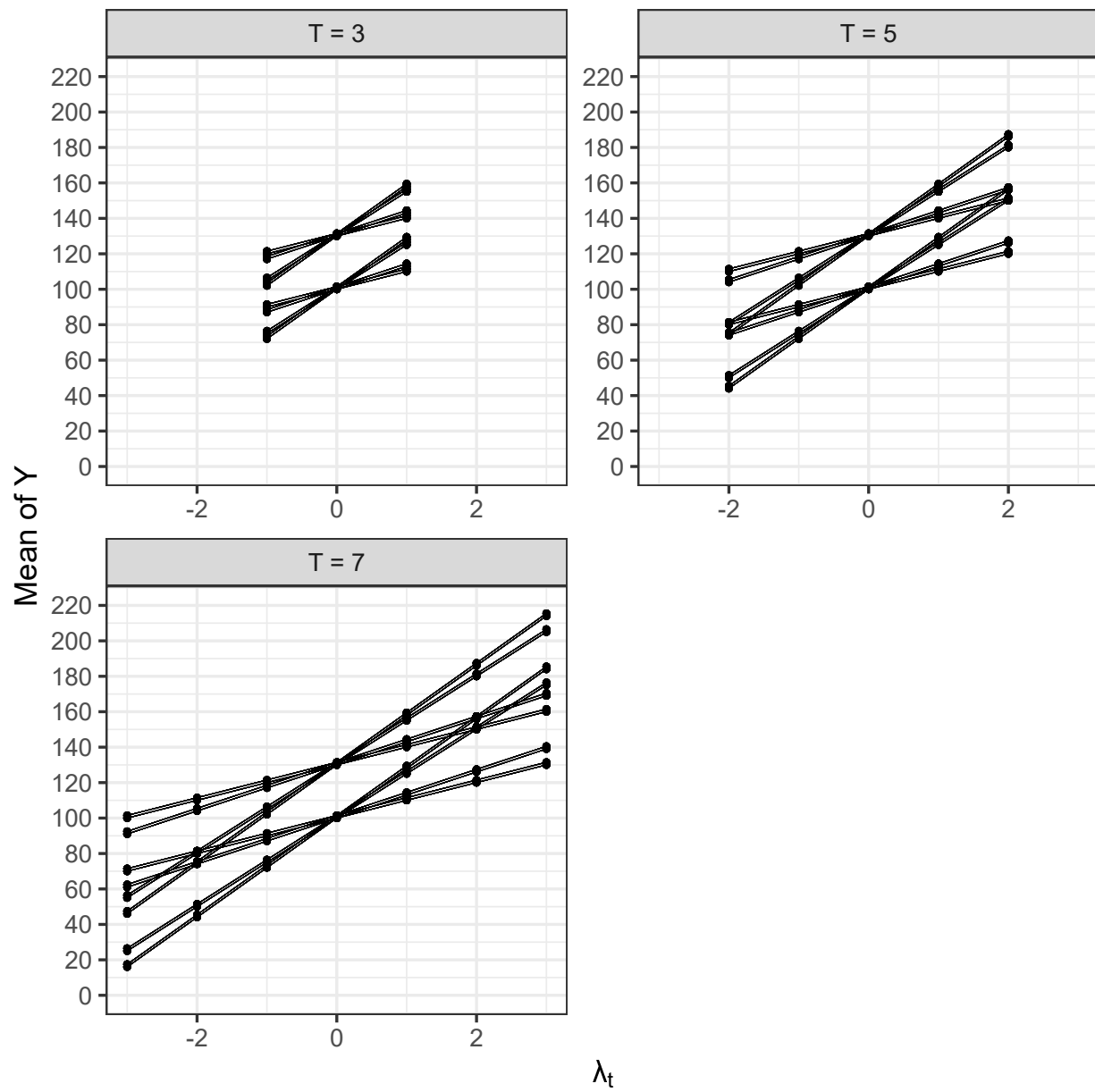


Table 4.3*Proportion of Within-Person and Between-Person Components in the Variance of the Outcome*

Component	n	M	SD	Min	Mdn	Max
Simulation 1						
BPE	270	0.148	0.155	0.000	0.093	0.419
BPR	270	0.389	0.140	0.205	0.373	0.794
WPE	270	0.209	0.158	0.000	0.270	0.419
WPR	270	0.254	0.088	0.129	0.248	0.500
Simulation 2						
BPE	540	0.193	0.182	0.000	0.168	0.650
BPR	540	0.363	0.127	0.203	0.318	0.794
WPE	540	0.201	0.157	0.000	0.246	0.419
WPR	540	0.243	0.095	0.065	0.235	0.500
Simulation 3						
BPE	540	0.152	0.155	0.000	0.099	0.425
BPR	540	0.387	0.140	0.200	0.373	0.794
WPE	540	0.207	0.157	0.000	0.264	0.419
WPR	540	0.252	0.088	0.127	0.246	0.500
Simulation 4						
BPE	2160	0.213	0.178	0.000	0.203	0.656
BPR	2160	0.349	0.119	0.185	0.312	0.794
WPE	2160	0.195	0.155	0.000	0.229	0.419
WPR	2160	0.236	0.096	0.055	0.226	0.500
Simulation 5						
BPE	1215	0.325	0.287	0.000	0.291	0.881
BPR	1215	0.293	0.153	0.062	0.283	0.794
WPE	1215	0.169	0.154	0.000	0.155	0.419
WPR	1215	0.203	0.116	0.016	0.206	0.500

Note. n = the number of data points, WPE = within-person effect, WPR = within-person residual, BPE = between-person effect, BPR = between-person residual.

Table 4.4*Summary of the Nine Analytic Models in the Simulations*

		centering	detrending
Existing Models	MLM	MLM-WB	MLM-E2S, MLM-TAC, MLM-MV
	SEM	LCMy-RIx	LCMy-CLC
Proposed Models	SEM	—	LCMxy-RP, LCMx-RIy, LCMx-CLC

the components dominates the others.

Another possible approach to determining the simulation conditions is to set the true values based on the parameter estimates from existing applied research. However, the literature review showed that the size of the within-person effect and between-person effect in reality varies across studies, and it seems that there is no typical situation. Moreover, a broad range of simulation conditions may be preferred to specific conditions even though they are realistic since the performance of the proposed models has never been examined. For these reasons, this study employed the systematic settings of the simulations rather than the applied settings.

All random variables are assumed to be normally distributed. Under each data generating model and condition, 1,000 data sets are generated to evaluate the accuracy and precision of the estimation as well as the inferential properties.

4.4 Analytic Models

The present study uses four MLM models and five SEM models as analytic models. Specifically,

1. multilevel within-between model (MLM-WB; Equation 2.17)
2. multilevel model with extended two-step detrending using individual OLS estimates

(MLM-E2S; Equation 2.62)

3. multilevel model with time as covariate (MLM-TAC; Equation 2.67)
4. multivariate multilevel model (MLM-MV; Equations 2.69–2.78)
5. latent curve model for Y with cross-level covariances (LCMy-CLC; Equations 2.40–2.43)
6. latent curve model for Y with a random intercept for X (LCMy-RIx; Equations 2.49–2.51)
7. latent curve model for X and Y with residualized predictors (LCMxy-RP, Equations 3.8–3.11)
8. latent curve model for X with a random intercept Y (LCMx-RIy; Equations 3.13–3.15)
9. latent curve model for X with cross-level covariances (LCMx-CLC; Equations 3.16–3.20)

Two of the nine models above are centering approaches (i.e., MLM-WB, LCMy-RIx), whereas the other seven models are detrending models. Also, the four models described in the Chapter 3 are the proposed models, while the other five models are the existing models. See Table 4.4 for the classification of the analytic models

4.5 Evaluation Indices

The simulations in this study evaluate the estimation and inference quality of the within-person and between-person effects, denoted by $\theta \in \{\gamma, \beta_{II}, \beta_{SI}, \beta_{IS}, \beta_{SS}\}$, by means of bias, standard error, and root mean squared error, coverage rate, and significance rate.

Bias is defined as a difference between the average estimate and its true value.

Specifically,

$$Bias = \frac{1}{R} \sum_{r=1}^R \hat{\theta}^{(r)} - \theta = \bar{\theta} - \theta, \quad (4.5)$$

where $\hat{\theta}^{(r)}$ is the r -th estimate out of the R replications and $\bar{\theta}$ is the average of the estimates over the replications. Bias measures the accuracy of estimation (i.e., how the estimates are, on average, close to its true value). It can take both positive and negative values, and zero is the desired value.

Standard error (SE) is defined as the standard deviation of estimates over replications:

$$SE = \sqrt{\frac{1}{R} \sum_{r=1}^R (\hat{\theta}^{(r)} - \bar{\theta})^2}. \quad (4.6)$$

SE represents the precision of estimation (how much the estimates spread over the replications). SE only takes non-negative values, and smaller SE is preferable. The standard error measured as the standard deviation of estimates is sometimes called empirical standard error, while the SE derived from statistical theory is called theoretical SE. These two SEs can differ when the number of observations (i.e., sample size and/or the number of measurements) is small. Therefore, the ratio of the two SEs (empirical SE over theoretical SE) is also monitored.

Root mean squared error (RMSE) measures the estimation quality, which takes both of the systematic error (i.e., bias) and random variability (SE) into account. It is defined as the square root of mean square error (MSE):

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{R} \sum_{r=1}^R (\hat{\theta}^{(r)} - \theta)^2}. \quad (4.7)$$

RMSE also takes non-negative values and smaller values are preferable. It is well known that the sum of the squared bias and squared SE (i.e., the variance of the estimates) is

equal to MSE ($MSE = Bias^2 + SE^2$).

Coverage rate (CR) indicates the estimation quality as an interval estimator. It is defined as the proportion of confidence intervals that capture the true value over replications. That is,

$$CR = \frac{1}{R} \sum_{r=1}^R I \left(CI_L^{(r)} < \theta < CI_U^{(r)} \right), \quad (4.8)$$

where $CI_L^{(r)}$ and $CI_U^{(r)}$ are the lower and upper bound of the r -th confidence interval and $I(\cdot)$ is an indicator function that takes one when its argument (statement) is true; otherwise, $I(\cdot)$ returns zero. CR takes values between zero and one, and the desired value (i.e., nominal rate) is $1 - \alpha$. In the present simulations, α is set at $\alpha = 0.05$, thus the desired value of CR is 0.95. The coverage rate is computed based on the confidence intervals that are returned by software rather than the empirical standard errors.

As for the inference quality, this study uses the significance rate as a measure. *Significance rate* (SR) is the proportion of the replications in which an estimate $\hat{\theta}^{(r)}$ is declared statistically significant. When the true value of the parameter is zero, the significant rate is called *Type I error rate*. On the other hand, it is called *statistical power* or simply *power* when the true value is not zero. Both indices can be written in the same form (i.e., the significance rate):

$$SR = \frac{1}{R} \sum_{r=1}^R I \left(CI_L^{(r)} < 0 < CI_U^{(r)} \right). \quad (4.9)$$

When the true value is zero, the coverage rate and the type I error rate becomes equivalent. In addition, the simulation conditions of this study include true value of zero for all target parameters. Thus, there is an overlap of information between the two indices. In order to avoid the redundancy, this study only reports the power and omit the type I error rate.

In addition to these indices, several auxiliary indices are also monitored. The auxiliary indices include global model fit indices such as CFI, RMSEA, SRMR (only for

Table 4.5*Target Parameters of Analytic Models for Evaluating Simulation Results*

Model	Equations	WPE	BPE (Intercept-Intercept)	BPE (Intercept-Slope)	BPE (Slope-Intercept)	BPE (Slope-Slope)
MLM-WB	2.17	γ_{10}	γ_{01}	—	—	—
MLM-E2S	2.62	γ_{20}^Y	γ_{01}^Y	γ_{11}^Y	γ_{02}^Y	γ_{12}^Y
MLM-TAC	2.67	γ_{10}	γ_{01}	γ_{21}	—	—
MLM-MV	2.81–2.85	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}
LCMy-RIx	2.48–2.51	γ	β_{II}	β_{SI}	—	—
LCMy-CLC	2.40–2.45	γ	$\delta_0 + \gamma$	δ_1	—	—
LCMxy-RP	3.8–3.11	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}
LCMx-RIy	3.13–3.15	γ	β_{II}	—	β_{IS}	—
LCMx-CLC	3.16–3.20	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}

Note. WPE = within-person effect, BPE = between-person effect

LCMs), and the frequency of converged estimation under each condition, and the frequency of the nonpositive definite model-implied covariance matrices, which may imply the negative variance estimates.

Table 4.5 summarizes the target parameters of each analytic model. As can be seen in the table, all models have a parameter to represent the within-person effect, while different models have different sets of parameters for the between-person effect. Some of the models have the full set of parameters (i.e., MLM-E2S, MLM-MV, LCMxy-RP, LCMx-CLC). In contrast, the others only have one or two between-person effects (MLM-WB, MLM-TAC, LCMy-RIx, LCMy-CLC, LCMx-RIy).

4.6 Analysis

Since nonconverged cases and cases without standard error estimates are highly likely to produce invalid estimates, these cases are eliminated from the following analyses. Due to the deletion of these invalid cases, the quality of the evaluation indices varies across the simulation conditions and analytic models.

In order to identify influential experimental factors, the analysis of variance (ANOVA) with Type III sum of squares is applied to each evaluation index in the five simulations. In ANOVA, all main effects and interactions of the experimental factors as well as the analytic models are included as independent variables. For each evaluation index, the effect size is measured by the eta-square η^2 , which is defined as the proportion of the total variance to the variance of each source. Note that R replications are used when calculating evaluation indices, thus there is no degree of freedom for the error term.

4.7 Implementation

All data sets are generated with R (R Core Team, 2021). For the analyses with SEM models and MLM-MV, Mplus (L. K. Muthén & Muthén, 1998–2017) is used, while the lme4 package (Bates et al., 2015) and the lmerTest package (Kuznetsova et al., 2017) are used for multilevel models except for MLM-MV. In addition to these analysis packages, some utility packages are used, for example, the doParallel package (Microsoft Corporation & Weston, 2019) and the foreach package (Microsoft & Weston, 2020) for parallel computing, and the MplusAutomation package (Hallquist & Wiley, 2018) for creating input files, calling Mplus from R, and reading output files into R. ANOVA is conducted with the GLM procedure in SAS (SAS Institute Inc., 2018) or with SAS JMP[®] Pro. The R codes for the five simulations are provided in Appendix F.

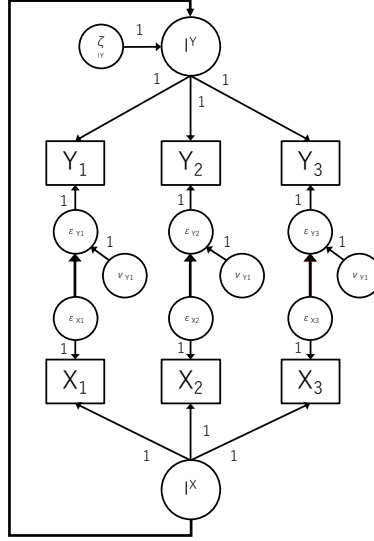
Chapter 5

Results

In Chapter 5, results from the five simulations are provided. Each section presents results from a simulation, in which subsections are organized based on the target parameters. In other words, each section consists of six subsections, namely Auxiliary Indices, Within-Person Effect, Between-Person Effect of Intercept on Intercept, Between-Person Effect of Intercept on Slope, Between-Person Effect of Slope on Intercept, and Between-Person Effect of Slope on Slope. The results are shown as figures, which are compiled per simulation in Appendices A–E.

5.1 Simulation 1

In Simulation 1, the model shown in Figure 5.1 is used as a data-generating model (see also Equations 3.1–3.7). This DGM only assumes intercepts for both time-varying predictors and outcomes. In addition to the sample size $N = \{100, 300, 500, 1000\}$ and the number of measurements $T = \{3, 5, 7\}$, there are two experimental conditions: $\gamma = \{-0.6, 0, 0.6\}$ and $\beta_{II} = \{0, 0.6\}$. Accordingly, 72 conditions are considered in Simulation 1. A brief summary of the results is provided in Table 5.1. All figures in Simulation 1 are provided in Appendix A.

Figure 5.1*Data-Generating Model in Simulation 1*

5.1.1 Auxiliary Indices

Convergence The analytic models other than LCMxy-RP and LCMx-RIy converged in all analyses. For LCMxy-RP, 46.6% of the 72,000 analyses converged. For LCMx-RIy, 84.9% of the analyses converged. Figure A.1 shows the convergence rate under each condition. According to the figure, the sample size N does not affect the convergence, while the number of measurement T does. In particular, LCMxy-RP is less likely to converge as T grows when $\beta_{II} = 0.6$.

Problems in SE Estimation Figure A.2 depicts the proportion of cases with problems in SE estimation. Similar to the convergence rate, LCMxy-RP and LCMx-RIy have no standard errors in the output from the software. In the worst case, 73.3% of the analyses under a single condition have a problem in estimating SE (LCMxy-RP). Note that most of the no SE cases coincide with the nonconvergence.

Table 5.1*Marginal Means of Absolute Bias (Empirical SE) in Simulation 1*

Model	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}
MLM-WB	0.000 (0.016)	0.091 (0.033)	—	—	—
MLM-E2S	0.001 (0.020)	0.091 (0.033)	0.000 (0.011)	0.003 (0.117)	0.400 (0.030)
MLM-TAC	0.000 (0.016)	0.091 (0.033)	0.000 (0.011)	—	—
MLM-MV	0.000 (0.017)	0.003 (0.068)	0.000 (0.015)	0.065 (2.552)	0.399 (0.281)
LCMy-RIx	0.000 (0.016)	0.002 (0.043)	0.000 (0.014)	—	—
LCMy-CLC	0.001 (0.020)	0.091 (0.033)	0.000 (0.011)	—	—
LCMxy-RP	0.001 (0.020)	0.010 (0.083)	0.001 (0.033)	0.130 (3.382)	0.452 (1.075)
LCMx-RIy	0.001 (0.017)	0.016 (0.114)	—	0.171 (6.060)	—
LCMx-CLC	0.001 (0.020)	0.022 (0.702)	0.010 (0.326)	0.378 (15.82)	0.459 (6.332)

Nonpositive-Definite Model-Implied Covariance Matrix Nonpositive-definite model-implied covariance matrices were observed for seven models (i.e., all models other than MLM-WB and MLM-MV; see Figure A.3). Among 593,675 analyses that converged and returned SEs, 262,517 cases (44.2%) had a model-implied covariance matrix that was diagnosed as non-positive definite. For most of the models, the percentage does not greatly change across the conditions; however, LCMxy-RP improves the rate as T becomes large.

Goodness of Fit Figures A.4–A.6 present the goodness-of-fit indices under each condition. These indices show that model-data fit is good for all models, especially when more data points are available (i.e., either N or T is large). In particular, all models achieve the desired level of CFI and RMSEA ($CFI > 0.95$, $RMSEA < 0.05$). SRMR is slightly conservative and requires more participants rather than measurements for the models to reach its criterion of 0.08.

5.1.2 Within-Person Effect

Bias Figure A.7 shows the bias in the estimates of within-person effect, $\hat{\gamma}$, under each simulation condition. The bias ranges from -0.007 to 0.004 , which is at most 1% of the true value in absolute value. The bias is relatively large when the number of data points is small (i.e., when either N or T is small). In fact, the interaction terms including N and T , such as $T \times \gamma \times \beta_{II}$ and $N \times T \times \gamma \times \beta_{II}$, have the eta-square larger than 0.1^1 and these two interactions explain 22.7% of the total variability of the bias (i.e., the total sum of squares; SST for short). However, since the mean absolute bias is 0.0005 and SST is 0.0004 , the estimates of the within-person effect are considered unbiased for all models (see also Table 5.1).

Empirical SE Figure A.8 shows the empirical standard error of the within-person effect, that is, the standard deviation of $\hat{\gamma}$. Although visual inspection of the graph may suggest differences between the models when T is small, ANOVA does not support it. According to the results from ANOVA, the most influential factor is the sample size N , which explains 59.8% of SST . This is followed by the number of measurements T with $\eta^2 = 0.273$, and no other factor has an eta-square larger than 0.1 .

Theoretical SE and SE Ratio The mean theoretical standard error shown in Figure A.9 has a similar pattern to the empirical SE. However, their magnitude is systematically different. Thus, the ratio of the theoretical SE over the empirical SE varies across the models. The main effect of Model has the largest eta-square ($\eta^2 = 0.561$), followed by an interaction $\text{Model} \times T$ ($\eta^2 = 0.140$). Figure A.10 illustrates that MLM-MV overestimates SE by 11.8% on average, while LCMx-CLC underestimates by 2.2%. The overestimation by MLM-MV is large especially when T is small (the ratios were 1.20 when $T = 3$; 1.09

¹Hereafter, only the main effects and interactions with $\eta^2 > 0.1$ are mentioned as important factors in the text.

when $T = 5$; and 1.06 when $T = 7$, on average). In contrast, the underestimation by LCMx-CLC is not affected by T (the ratio was 0.982 when $T = 3$; 0.975 when $T = 5$; and 0.977 when $T = 7$).

RMSE Figure A.11 presents the RMSE of $\hat{\gamma}$. The results are almost equivalent to those for the empirical SE. This approximate equivalence is due to the unbiasedness of $\hat{\gamma}$ and the well-known relation between bias and SE (i.e., $RMSE^2 = Bias^2 + SE^2$). More than 85% of the total variability in RMSE is explained by the sample size ($\eta^2 = 0.598$) and the number of measurements ($\eta^2 = 0.273$).

Coverage Rate The results of the coverage rate are aligned with those of the empirical SE. Since MLM-MV overestimates SE, the resulting confidence intervals (CIs) are wider than they should be and the CIs capture the true value more frequently than the nominal rate of 0.95. As such, the coverage rate of MLM-MV becomes higher than 0.95 especially when T is small (see Figure A.12). Note, however, that ANOVA does not find T important but Model with $\eta^2 = 0.356$, which is the only term that has an eta-square larger than 0.1.

Power For the statistical power, all models successfully detect nonzero γ under all conditions (Figure A.13). This means that the magnitude of the within-person effect in the current conditions (i.e., -0.6 and 0.6) is large enough for the models to detect even when $N = 100$ and $T = 3$.

Summary All analytic models can unbiasedly estimate the within-person effect. Differences due to the models can be found in the theoretical standard errors and the resulting SE ratios. In particular, MLM-MV overestimated SE, leading to too high coverage rates. However, the overestimation did not lead to a decrease in the power under the current conditions.

5.1.3 Between-Person Effect of Intercept on Intercept

Bias As reviewed in Chapter 1 and 2, the models that use an observed mean as a between-person predictor suffer from the bias due to sampling error, which is known as Lüdtke's bias. Equation 2.34 indicates that the bias is a function of $\gamma - \beta_{II}$ and T ,² meaning that the bias becomes zero when γ is equal to β_{II} or when T goes to infinity. MLM-WB, MLM-E2S, MLM-TAC, and LCM_y-CLC in Figure A.14 are obviously subject to this rule. That is, the bias disappears when $\gamma = \beta_{II} = 0.6$ or when $\gamma = \beta_{II} = 0$. Also, the bias appears to be a decreasing function of T . In contrast, the models that have latent between-person predictors can unbiasedly estimate the between-person effect, though the estimates from LCM_x-CLC can be somewhat biased when the number of data points is small (i.e., when either N or T is small). This bias may be due to unstable estimates of the variance components of the latent growth factors. The result of ANOVA coincides with these observations. The main effects of Model and γ as well as the interactions such as Model $\times$ γ and Model $\times$ β_{II} significantly influence the bias. These four terms explain 79.7% of SST ($\eta^2 = 0.313$ for Model $\times$ γ , $\eta^2 = 0.229$ for γ , $\eta^2 = 0.147$ for Model, and $\eta^2 = 0.107$ for Model $\times$ β_{II}).

Empirical SE Figure A.15 illustrates the empirical standard errors of β_{II} . As can be seen in the figure, the SE of LCM_x-CLC considerably fluctuates when N is small. This fluctuation results in the inflation of the variability in the SE, which makes it difficult to appropriately assess the influence of the experimental factors. Actually, when LCM_x-CLC is excluded from ANOVA, SST reduces by 98.4% (from 64.1 to 1.06). Hence, I decided to remove the extreme models from ANOVA so that we can make a fair comparison of the other models. Figure A.16 shows the empirical SE after LCM_x-CLC is removed from the analysis. We can see from the figure the effect of Model, N , and T . ANOVA indicates the

² ICC is a constant under the current settings; thus it does not affect the bias.

main effects of Model and N ($\eta^2 = 0.444$ and 0.292 , respectively) and the interaction between Model and T ($\eta^2 = 0.104$) have an impact on the empirical SE. These three terms explain 84.0% of SST .

Theoretical SE and SE Ratio As with the empirical SE, the theoretical SE of LCMx-CLC is much larger than the other models. The mean theoretical SE of LCMx-CLC is 26.2, while that of the other models are all less than 0.4. As a result, LCMx-CLC inflates SST and distorts the result of ANOVA. In fact, after excluding LCMx-CLC from ANOVA, SST drops from 141821.7 to 16.7 (i.e., reduction by 99.99%). Also, the pattern in the theoretical SE is similar to that of the empirical SE after removing LCMx-CLC (see Figure A.17). Specifically, the main effects of Model and N and the interactions Model $\times$ T and Model $\times$ N have the eta-square that is larger than 0.1 ($\eta^2 = 0.498$ for Model, 0.171 for Model $\times$ T , 0.109 for Model $\times$ N , and 0.107 for N). These four terms explain 88.5% of the total variability in the theoretical SE.

Although the patterns in the empirical and theoretical SEs are similar, their size is systematically different across the models; thus, the ratio of SE for some models is substantially away from the unity. Figure A.18 illustrates the SE ratio after removing LCMx-CLC. Among the eight analytic models, MLM-MV, LCMxy-RP, and LCMx-RIy show SE ratios substantially higher than one. The ratios of LCMx-RIy seem to decrease as T gets large; however, the main effect of Model is the only factor that has an eta-square larger than 0.1 ($\eta^2 = 0.833$).

RMSE RMSE shows mixed results of the bias and the empirical standard error. In Figure A.19, where the RMSE under each condition is shown, we can see the effect of Lüdtke's bias for MLM-WB, MLM-E2S, and LCMy-CLC. These models are greatly influenced by a combination of γ and β_{II} (i.e., clearly distinct lines in a subplot and the

left column and right column obviously exhibit different patterns). Also, the three models are negatively influenced by the number of measurements T . On the other hand, lines that represent MLM-MV, LCMMy-RIx, LCMxy-RP, and LCMx-RIy are close to each other, and their trajectories in the left and right columns are similar to each other. This similarity means that the models with latent between-person predictors are less affected by γ and β_{II} (i.e., being free from Lüdtke's bias). ANOVA indicates that γ , $\gamma \times \beta_{II}$, Model, and T are the influential factors, of which the eta-squares are 0.162, 0.140, 0.124, and 0.110, respectively.

Coverage Rate Coverage rate also reflects Lüdtke's bias (see Figure A.20). When Lüdtke's bias exists, the coverage rate deviates from the proper rate of 0.95. In particular, the coverage rates drop to 0.05 when the bias is large (i.e., when the point estimates are systematically off from the true value) and the sample size is large (when the confidence intervals become narrower). Conversely, CIs of MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC capture the true value too frequently than they are supposed. As a result, the main effect of Model and the interactions $\text{Model} \times \gamma \times \beta_{II}$ and $\gamma \times \beta_{II}$ have a large eta-square in ANOVA ($\eta^2 = 0.504$, 0.160, and 0.129, respectively).

Power In the current settings, models such as MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC resulted in imperfect detection of the nonzero β_{II} (see Figure A.21). The level of the power is mainly determined by Model ($\eta^2 = 0.664$), $\text{Model} \times N$ ($\eta^2 = 0.183$), and N ($\eta^2 = 0.107$). Some may see the impact of T , for example, the main effect of T or the interaction between Model and T . However, the eta-square for these terms are actually small ($\eta^2 = 0.019$ for Model; $\eta^2 = 0.007$ for $\text{Model} \times T$). On the contrary, MLM-WB, MLM-E2S, MLM-TAC, and LCMMy-CLC achieved the perfect detection of nonzero β_{II} even though they suffered from Lüdtke's bias. This means that the biased estimates are distant

from zero enough, and the standard error is small enough for those models to detect the effect even when N and T are small.

Summary The estimates of β_{II} can be biased if a model uses an observed between-person predictor. This bias is known as Lüdtke's bias and is a function of γ , β_{II} , and T as well as ICC . The bias affects the coverage rate but not the power in the present settings. Both empirical and theoretical SEs of LCMx-CLC tend to be extremely large although its estimates are unbiased. Therefore, one needs to be cautious in evaluating the uncertainty in $\hat{\beta}_{II}$ when using LCMx-CLC.

5.1.4 Between-Person Effect of Intercept on Slope

Not all analytic models have a term estimating β_{SI} . Specifically, MLM-WB and LCMx-RIy do not have a term corresponding to β_{SI} . Hence, all results in this subsection are about analytic models other than MLM-WB and LCMx-RIy. Note also that β_{SI} in this simulation is always fixed at zero, thus the power is not reported for this parameter.

Bias Figure A.22 shows the bias in $\hat{\beta}_{SI}$. As can be seen in the figure, all models but LCMx-CLC yield unbiased estimates. LCMx-CLC, on the other hand, produces a biased estimate especially when either N or T is small. Because of the instability of LCMx-CLC, ANOVA was conducted after excluding LCMx-CLC. This exclusion reduced SST from 0.0236 to 0.0004, which amounts to a 98.5% reduction. Figure A.23 is a similar figure to Table A.22 but without LCMx-CLC. It shows small biases of LCMxy-RP in cases where T is small. ANOVA without LCMx-CLC also indicates that interaction terms such as $N \times T \times \gamma$ ($\eta^2 = 0.150$) and $\text{Model} \times N \times T \times \gamma$ ($\eta^2 = 0.122$) are influential, though the proportion of variability explained by these terms is relatively small.

Empirical SE Figure A.24 depicts the empirical SE of $\hat{\beta}_{SI}$, excluding LCMx-CLC. LCMx-CLC was removed from the analysis since it produced extreme SEs when T is small. In fact, removing LCMx-CLC decreases SST by 99.3% (from 18.9 to 0.125). After excluding LCMx-CLC, ANOVA finds four important factors. That is, T ($\eta^2 = 0.296$), Model ($\eta^2 = 0.223$), N ($\eta^2 = 0.186$), and Model $\times T$ ($\eta^2 = 0.117$). These four terms explain 82.1% of SST .

Theoretical SE and SE Ratio Results of theoretical SE show similar patterns to the empirical SE with a slightly different set of influential factors (see Figure A.25). In addition to the main effects of Model ($\eta^2 = 0.322$) and T ($\eta^2 = 0.142$) as well as the interaction Model $\times T$ ($\eta^2 = 0.209$), ANOVA finds that instead of the main effect of N , the interaction between Model and N is an important factor.³ Also, as with the results of empirical SE, LCMxy-RP has relatively large theoretical SEs especially when T is small.

Although the empirical and theoretical SEs exhibit a similar pattern, their size is substantially different for some analytic models including LCMx-CLC. Figure A.26 presents the ratios of the theoretical SE over the empirical SE under each condition. Models such as MLM-MV, LCMxy-RP, and LCMx-CLC show the SE ratio larger than one, implying that these models overestimate the SE. According to the figure, the size of overestimation is at least 50%. One may see that N and T affect the ratio; however, ANOVA indicates that the main effect of Model is the sole and dominant factor with $\eta^2 = 0.845$.

RMSE Reflecting the fact that the models except LCMx-CLC can produce unbiased estimates, RMSE has a similar pattern to the empirical SE (see Figure A.27). Dominant factors suggested by ANOVA are the same as those of the two SEs. Specifically, the influential factors identified by ANOVA are T with $\eta^2 = 0.296$, Model with $\eta^2 = 0.223$, N

³The main effect of N is marginally influential with $\eta^2 = 0.092$.

with $\eta^2 = 0.186$, and $\text{Model} \times T$ with $\eta^2 = 0.117$, which together account for 82.1% of SST .

Coverage Rate Figure A.28 shows the coverage rate of all models under each condition. In this graph, we can see three groups of the models: (a) over-covering models (the average coverage rate is higher than 0.98; MLM-MV, LCMxy-RP, and LCMx-CLC), (b) just-covering models (the average coverage is around 0.95; MLM-E2S and MLM-TAC), and (c) under-covering models (the average coverage is 0.94; LCMy-RIx and LCMy-CLC). The model difference is so clear that ANOVA indicates the main effect of Model is the sole and dominant factor in terms of the eta-square ($\eta^2 = 0.867$).

Summary All models except LCMx-CLC can unbiasedly estimate β_{SI} when the population model does not have a slope factor behind predictors and outcomes. In contrast, LCMx-CLC produces biased estimates especially when T is small. This bias comes with an extremely large SE. LCMxy-RP can also have a large standard error, which is more evident when T is small. As for the theoretical SE, more general models such as LCMxy-RP and MLM-MV tend to overestimate it. This overestimation leads to a too high coverage rate of those models.

5.1.5 Between-Person Effect of Slope on Intercept

Some of the analytic models can estimate β_{IS} , but the others cannot. The models that can estimate β_{IS} include MLM-E2S, MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC. Therefore, the following results are about these five models. Note also that β_{IS} is always fixed at zero in Simulation 1, thus the power is not reported in this subsection.

Bias Figure A.29 depicts the bias in $\hat{\beta}_{IS}$. We can see in this figure that MLM-E2S can accurately estimate β_{IS} under all conditions. In contrast, the other four models produce

substantially biased estimates. In particular, LCMx-CLC and LCMx-RIy yield bias of which absolute size is larger by 100% of the true value. The direction of bias can be both positive and negative and it is difficult to find a systematic pattern in the bias. In ANOVA including LCMx-CLC, only higher order interactions are significant ($\text{Model} \times N \times T \times \gamma \times \beta_{II}$ with $\eta^2 = 0.185$; $\text{Model} \times N \times T \times \gamma$ with $\eta^2 = 0.184$). ANOVA without LCMx-CLC has a similar set of significant terms, but they are also higher order interactions ($\text{Model} \times N \times \gamma \times \beta_{II}$ with $\eta^2 = 0.122$; $\text{Model} \times N \times T \times \gamma$ with $\eta^2 = 0.122$; $\text{Model} \times N \times T \times \gamma \times \beta_{II}$ with $\eta^2 = 0.116$; $\text{Model} \times N \times T$). In addition, significant terms that have $\eta^2 > 0.1$ in ANOVA can only explain 36.9% of SST when including LCMx-CLC and 46.2% when excluding LCMx-CLC).

Empirical SE Figures A.30 and A.31 show the empirical standard errors with and without LCMx-CLC, respectively. The figures clearly shows systematic differences in the SEs due to model (i.e., the main effect of Model ; $\eta^2 = 0.693$ with LCMx-CLC and $\eta^2 = 0.776$ without LCMx-CLC). Also, all models except MLM-E2S increase SE as T increases (the main effect of T , $\eta^2 = 0.084$ with LCMx-CLC and $\eta^2 = 0.103$ without LCMx-CLC; the interaction of Model and T , $\eta^2 = 0.122$ with LCMx-CLC and $\eta^2 = 0.034$). In addition to these systematic changes, the size of empirical SE is considerably large for all models but MLM-E2S.

Theoretical SE and SE Ratio Figures A.32 and A.33 show theoretical SE including and excluding LCMx-CLC, respectively. One may think that T still has an impact on the SE; however, Model is the sole factor that has $\eta^2 > 0.1$ even after excluding LCMx-CLC ($\eta^2 = 0.708$ and 0.850 before and after the exclusion, respectively). The ratio of the two types of SE approaches one as T or N increases (see Figures A.34 and A.35). However, the ratios are still substantially higher than one except for MLM-E2S.

RMSE Since the magnitude of SE is much larger than that of bias, the resulting RMSE reflects the empirical SE more than the bias. Thus, most models increase RMSE as T gets large (see Figures A.36 and A.37). As a result, ANOVA identifies the following terms as influential factors: Model ($\eta^2 = 0.693$) and Model $\times T$ ($\eta^2 = 0.122$) when including LCMx-CLC, and Model ($\eta^2 = 0.776$) and T ($\eta^2 = 0.102$) when excluding LCMx-CLC. Among the five analytic models, MLM-E2S is the one that has the smallest RMSE for β_{IS} . MLM-E2S is the only model with an average RMSE less than one (i.e., 0.117), while the second-best model, MLM-MV, has a much larger average RMSE (2.55).

Coverage Rate Figure A.38 presents the coverage rate of all models. As clearly shown in the figure, all models other than MLM-E2S capture the true value too many times than they are supposed. This may be due to the theoretical standard errors that are too high. As a result, these models almost always capture the true value. In contrast, the coverage rate of MLM-E2S is at the proper level in all conditions.

Summary In brief, MLM-E2S can estimate β_{IS} without bias and with a reasonable standard error, while the other models produce biased estimates and/or excessively large standard error. In particular, larger empirical and theoretical standard errors are obtained as we collect data from an individual more frequently (i.e., with larger T). Also, all models but MLM-E2E estimate the standard error, on average, two and half times as large as it should be. Accordingly, these models have too high coverage rates across the conditions, which is almost 100%.

5.1.6 Between-Person Effect of Slope on Slope

There are only four models that can estimate β_{SS} . These include MLM-E2S, MLM-MV, LCMxy-RP, and LCMx-CLC. Therefore, the results reported in the following are only

regarding these four models. Note that since there is no condition that has nonzero β_{SS} , the power is not reported in this subsection.

Bias Figure A.39 shows the bias in $\hat{\beta}_{SS}$. The size and direction of bias are almost equal to those of γ . This pattern is more evident for multilevel models (i.e., MLM-E2S and MLM-MV). The pattern is equivocal for LCM-based methods (LCMxy-RP and LCMx-CLC), but we can still see the same pattern. Accordingly, γ is the only factor that has an eta-square greater than 0.1, and it can explain 95% of the variability in the bias.

Empirical SE Figures A.40 and A.41 present the empirical SE with and without LCMx-CLC. The average SE for LCMx-CLC is larger than the other models, and it greatly fluctuates. The SE of LCMxy-RP decreases as T increases, while MLM-E2S and MLM-MV do not exhibit this pattern. Also, the level of SE for LCMxy-RP is substantially higher than the multilevel models (Figure A.41). Both before and after the removal of LCMx-CLC, the main effect of Model is the sole influential factor in ANOVA, which has an eta-square higher than 0.85 ($\eta^2 = 0.870$ before the removal; $\eta^2 = 0.892$ after the removal).

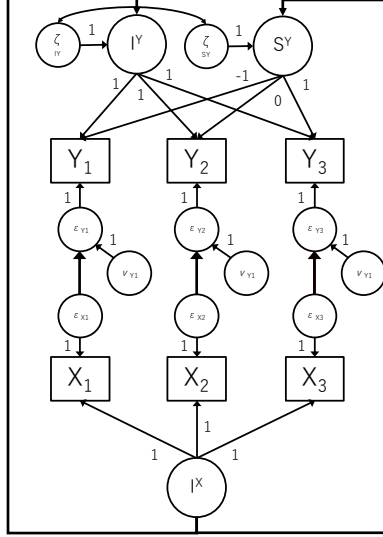
Theoretical SE and SE Ratio As with the other evaluated parameters, the theoretical SE exhibits a similar pattern to the empirical SE, but its level is more extreme (Figures A.42 and A.43). In addition, we can see in Figures A.44 and A.45 that the SE ratio for LCMx-CLC can exceed 30. For the other models, the ratios are much more moderate—around 4 for MLM-MV, 3 for LCMxy-RP, 1 for MLM-E2S.

RMSE Since the magnitude of the empirical SE is larger than the bias, RMSE shows a similar pattern to the empirical SE (see Figure A.46). Thus, the estimation error of LCMx-CLC is substantially larger than the other models. LCMxy-RP seems to decrease RMSE as T gets larger; however, ANOVA does not identify T as a significant factor.

MLM-E2S has a very small estimation error when $\gamma = 0$ (i.e., when the bias is zero), whereas the level of RMSE is similar to that of MLM-MV when $\gamma \neq 0$ (Figure A.47). As a result, the main effect of Model and γ account for 84.0% of SST ($\eta^2 = 0.668$ for Model and 0.171 for γ).

Coverage Rate Figure A.48 presents the coverage rate for $\hat{\beta}_{SS}$. As we can clearly see in the figure, the coverage rate of MLM-E2S greatly depends on γ . This dependence may be due to the bias, which systematically diverts the point estimates from their true value. On the other hand, MLM-MV, LCMxy-RP, and LCMx-CLC suffer from both large bias and large standard error. Specifically, when γ is not zero and the bias is not zero, the point estimates of those models are systematically off from their true values, leading to the under-coverage around 0.8. When γ is zero and the estimates are unbiased, the three models can capture the true value too frequently. This is likely due to the too wide CIs, which are based on the theoretical SE.

Summary The results show that γ is a significant factor that determines the distribution of $\hat{\beta}_{SS}$. The bias seems to be a function of γ . In fact, the main effect of γ can explain 95% of the variability in the bias. Figure A.49 is a supplemental scatter plot, where the estimates of γ and β_{SS} are jointly plotted by model (the red lines represent $\hat{\beta}_{SS} = \hat{\gamma}$). For presentation purpose, 10 pairs of $(\hat{\gamma}, \hat{\beta}_{SS})$ are randomly sampled under each condition. According to this scatter plot, $\hat{\beta}_{SS}$ seems to be estimated independently of $\hat{\gamma}$ with the expectation of $\mathbb{E}[\hat{\beta}_{SS}] = \gamma$. This relation is evident for MLM-E2S, which produced more stable estimates. For the other three models, the means of the estimates lie around $\hat{\beta}_{SS} = \gamma$, though the estimates are highly variable.

Figure 5.2*Data-Generating Model in Simulation 2*

5.2 Simulation 2

In Simulation 2, the data-generating model depicted in Figure 5.2 is used (see also Equation 3.1–3.7). There is a new experimental factor $\beta_{SI} = \{0, 0.3\}$ in addition to those in Simulation 1. Therefore, the total number of conditions is $72 \times 2 = 144$. A brief summary of the results from Simulation 2 is presented in Table 5.2. All figures in Simulation 2 are provided in Appendix B.

5.2.1 Auxiliary Indices

Convergence In Simulation 2, LCMxy-RP and LCMx-RIy have nonconverged cases (see Figure B.1). Although the average convergence rate is around 0.5 for both models, the pattern of convergence is different. Specifically, while LCMxy-RP fluctuates around 0.5, LCMx-RIy increases or decreases as T grows, or stays at a certain level, depending on the conditions. For example, the convergence rate rapidly rises as T gets large when $(\gamma, \beta_{SI}) = (-0.6, 0)$, but it sharply drops when $(\gamma, \beta_{SI}) = (0.6, 0)$, $(-0.6, 0.3)$, and $(0.6, 0.3)$.

Table 5.2*Marginal Means of Absolute Bias (Empirical SE) in Simulation 2*

model	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}
MLM-WB	0.006 (0.205)	0.091 (0.033)	—	—	—
MLM-E2S	0.000 (0.020)	0.091 (0.033)	0.027 (0.018)	0.003 (0.116)	0.401 (0.061)
MLM-TAC	0.000 (0.017)	0.091 (0.033)	0.027 (0.018)	—	—
MLM-MV	0.001 (0.019)	0.006 (0.098)	0.002 (0.048)	0.133 (4.622)	0.386 (2.323)
LCMy-RIx	0.000 (0.017)	0.002 (0.043)	0.001 (0.023)	—	—
LCMy-CLC	0.000 (0.020)	0.091 (0.033)	0.027 (0.018)	—	—
LCMxy-RP	0.001 (0.020)	0.010 (0.086)	0.002 (0.051)	0.347 (3.464)	0.527 (1.879)
LCMx-RIy	0.386 (0.606)	1.129 (0.415)	—	1.449 (2.411)	—
LCMx-CLC	0.000 (0.020)	0.020 (0.653)	0.011 (0.415)	0.409 (16.04)	0.536 (10.20)

Also, the changes in the convergence rate are relatively small when $\gamma = 0$.

Problems in SE Estimation Figure B.2 depicts the proportion of cases that had problems in SE estimation. Since the SE estimation problems mostly coincide with nonconvergence, the patterns in the occurrence of the SE problems are opposite to those of the convergence rate. That is, LCMxy-RP has a proportion around 0.5 across the conditions, while LCMx-RIy has a proportion that steeply increases and decreases as T becomes large.

Nonpositive-Definite Model-Implied Covariance Matrix Figure B.3 shows obvious differences between the models in the proportion of cases with nonpositive-definite (NPD) model-implied covariance matrix.⁴ While the average proportion was less than 0.01 for the existing models except for MLM-WB, the proposed models produced more NPD covariances. Among the proposed models, LCMx-RIy is strongly affected by T . The impact of T can be both positive and negative, depending on the other conditions.

⁴A model may not have all possible points in a panel when it always failed to converge.

Goodness of Fit Figures B.4–B.6 show the average goodness-of-fit indices under each condition. In these figures, all indices consistently indicate that among the four SEM models, LCMx-RIy does not fit to the data well. This misfit coincides with the results from the nonconvergence and the SE estimation problems. Also, LCMx-RIy constantly exhibits ill behaviors for all evaluation indices.

5.2.2 Within-Person Effect

Bias Figure B.7 shows the bias in $\hat{\gamma}$ under each condition. In this figure, we can see the positive and negative bias for LCMx-RIy, depending on the true value of γ . When $\gamma = -0.6$, LCMx-RIy constantly underestimates γ regardless of T and N . The opposite pattern can be found when $\gamma = 0.6$ (i.e., overestimation). When $\gamma = 0$, the bias is dependent on β_{SI} . Specifically, LCMx-RIy can unbiasedly estimate γ when $\beta_{SI} = 0$, while it produces biased estimates when $\beta_{SI} = 0.3$. These observations agree with the result of ANOVA, where the interaction of Model and γ and the main effect of γ account for 93.5% of the variability in the bias ($\eta^2 = 0.831$ for Model $\times \gamma$ and $\eta^2 = 0.104$ for γ).

Figure B.8 is the one that omits LCMx-RIy. We can see in the graph that MLM-WB yields a slightly biased estimate of γ especially when $\beta_{SI} = 0.3$. However, the pattern is not as simple as LCMx-RIy. In fact, the result from ANOVA shows that higher-order interaction terms explain a small portion of the variability. For example, the highest interaction term (Model $\times N \times T \times \gamma \times \beta_{II} \times \beta_{SI}$), which has the largest eta-square, can only explain 10.8% of SST . Note that omitting LCMx-RIy from ANOVA reduced SST from 27.6 to 0.01, which amounts to a 96.3% reduction.

Empirical SE Figure B.9 presents the empirical SE of $\hat{\gamma}$ under each condition. It shows that when $\gamma = 0$, LCMx-RIy increases the SE as T gets large and decreases as N gets large. The impact of T seems larger when $\beta_{SI} = 0.3$ than when $\beta_{SI} = 0$. ANOVA supports

part of these observations; 36.8% of SST is explained by the interaction of Model and γ , 19.8% by the main effect of Model, 14.7% by the interaction of Model, γ , and β_{SI} . However, ANOVA does not support the effect of N and T .

Figure B.10 shows the results after LCMx-RIy is removed from the analysis. In this figure, MLM-WB constantly exhibits a larger SE than the other models. Similar to LCMx-RIy, the SE of MLM-WB are heavily affected by β_{SI} . However, the size of SEs is smaller than that of LCMx-RIy. The other models estimate γ very stably (the average SE over the conditions is less than 0.02). Overall, 87.7% of the total variability in the empirical SE, after removing LCMx-RIy, can be explained by the main effect of Model ($\eta^2 = 0.620$), the interaction between Model and β_{SI} ($\eta^2 = 0.135$), and the interaction between Model and N ($\eta^2 = 0.122$).

Theoretical SE and SE Ratio As shown in Figures B.11 and B.12, the theoretical SE exhibits the same pattern as the empirical SE. That is, γ and β_{SI} strongly affect the size of the estimated SE of LCMx-RIy. The level of the theoretical and empirical SEs are similar to each other except for MLM-WB; it underestimates SE in all settings (see Figure B.13), though it approaches one as T grows. The SE ratio of MLM-MV is higher than one when $T = 3$, but it also improves as T becomes large. As a result, the main effect of Model and the interaction between Model and T explain 67.5% of SST , each of which has $\eta^2 = 0.562$ and 0.113, respectively.

RMSE Since the analytic models except LCMx-RIy and MLM-WB can produce unbiased estimates, the RMSE mostly represents the empirical SE. Thus, Figures B.14 and B.15, which depict RMSE with and without LCMx-RIy, are similar to those of the empirical SE. The influential factors that can account for the variability in RMSE are also similar to those of the empirical SE. That is, when LCMx-RIy is included, 74.5% of SST is

explained by Model ($\eta^2 = 0.434$), Model $\times \gamma$ ($\eta^2 = 0.178$), and Model $\times \gamma \times \beta_{SI}$ ($\eta^2 = 0.133$). When LCMx-RIy is excluded, 87.7% of *SST* can be explained by Model ($\eta^2 = 0.620$), Model $\times \beta_{SI}$ ($\eta^2 = 0.135$), and Model $\times N$ ($\eta^2 = 0.122$).

Coverage Rate Results of the coverage rate follow those of the bias and the theoretical SE (see Figure B.16). Specifically, the coverage rate of LCMx-RIy is zero when the estimates are severely biased and have relatively small SE (i.e., when $\gamma \neq 0$). Also, MLM-WB has a slightly lower coverage rate than it should because of the underestimation of the SE across the conditions (Figure B.13). ANOVA indicates that Model is a dominant factor ($\eta^2 = 0.715$ with LCMx-RIy and $\eta^2 = 0.554$ without LCMx-RIy) and the interaction Model $\times \gamma$ (with LCMx-RIy) and Model $\times T$ (without LCMx-RIy) are secondary factors with $\eta^2 = 0.247$ and $\eta^2 = 0.119$, respectively.

Power Figure B.17 presents the power to detect nonzero γ . As found in the figure, MLM-WB is the only model that fails to achieve perfect detection. It decreases the power as T becomes large and increases as N gets large. Also, the power seems dependent on the size of β_{SI} ; MLM-WB can achieve higher power when $\beta_{SI} = 0$, while the average level becomes lower when $\beta_{SI} = 0.3$. In ANOVA, Model, Model $\times N$, Model $\times \beta_{SI}$, and Model $\times N \times \beta_{SI}$ are the four terms that have the eta-square higher than 0.1 ($\eta^2 = 0.320$, 0.238, 0.217, 0.115, respectively).

Summary The data-generating model in Simulation 2 assumes a slope factor behind outcomes but not time-varying predictors. As a result, models that do not have a slope factor for the outcomes, such as LCMx-RIy and MLM-WB, suffer from bias and large SE as well as underestimation of the SE. LCMx-RIy can produce unbiased estimates of γ when $\gamma = 0$ and $\beta_{SI} = 0$, while MLM-WB always provides biased estimates under the current conditions. The underestimation of the SE by MLM-WB led to relevant issues, including

the under-coverage and lower power.

5.2.3 Between-Person Effect of Intercept on Intercept

Bias As can be seen in Figure B.18, LCMx-RIy produces extremely large, positive and negative biases. This pattern is akin to Lüdtke's bias; however, unlike Lüdtke's bias, the bias here seems affected not only by T , γ , and β_{II} but also β_{SI} . More specifically, it seems that β_{SI} makes the bias moderate, compared to that with the same β_{II} and γ , and that β_{SI} suppresses the effect of T . Also, the bias of LCMx-RIy is different from Lüdtke's bias in that LCMx-RIy does not have an observed between-person predictor (e.g., observed person mean). For MLM-WB, MLM-E2S, MLM-TAC, and LCMy-CLC, Figure B.18 presents a pattern analogous to Figure A.14. Those models are considered to show Lüdtke's bias even with the current data-generating model and simulation conditions. Unlike the case of LCMx-RIy, β_{SI} does not influence the size and direction of the bias. ANOVA indicates that Model $\times \gamma$ ($\eta^2 = 0.296$), γ ($\eta^2 = 0.272$), model ($\eta^2 = 0.118$), Model $\times \beta_{II}$ ($\eta^2 = 0.112$), and β_{II} ($\eta^2 = 0.100$) account for 89.8% of the total variability in the bias.

Empirical SE Figure B.19 shows the empirical SE of $\hat{\beta}_{II}$. In this figure, we can see that LCMx-RIy and LCMx-CLC, and MLM-MV take an extreme SE in some cases. While LCMx-RIy exhibits a systematic pattern, LCMx-CLC and MLM-MV unstably move upward and downward across the conditions. Figure B.19 also shows that the remaining models have reasonable changes in the SE. In ANOVA without MLM-MV, LCMx-RIy, and LCMx-CLC, the main effects of Model and N explain 79.2% of SST , each of which explains 40.0% and 39.2%, respectively.

Theoretical SE and SE Ratio Figure B.20 presents the theoretical SE under each condition. The pattern shown in this graph is akin to the empirical SE. One difference is

that the theoretical SE of MLM-WB seems more affected by T . The theoretical SE rises along with T , especially when $\beta_{SI} = 0.3$, while the empirical SE does not change or decrease as T increases. Another difference between the theoretical and empirical SEs is that their sizes. We can see in Figure B.20 that MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC have a wide range of the theoretical SE. In particular, LCMx-CLC shows the maximum close to 200. On the other hand, the maximum of the empirical SE is much smaller, which is about 2.5.

Figure B.21, which illustrates the SE ratio, is another representation of this observation. In this figure, MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC show the SE ratios larger than one in most cases. LCMx-CLC particularly suffers from an extreme SE ratio, which can sometimes be higher than 100. ANOVA of the SE ratio after excluding LCMx-CLC finds that the main effect of Model and the interaction between Model and β_{SI} are influential, which have $\eta^2 = 0.480$ and 0.115, respectively.

RMSE Figure B.22 illustrates RMSE for each model. This figure clarifies that there are two groups of models. The first group consists of MLM-WB, MLM-E2S, MLM-TAC, LCMxy-CLC, and LCMx-RIy, and the second group includes MLM-MV, LCMxy-RIx, LCMxy-RP, and LCMx-CLC. The first group is strongly affected by γ , while the second group is not. This pattern stems from the bias (see Figure B.18), thus we can say that the estimation error of the first group is mainly due to bias. In contrast, the RMSE of the second group is more likely to reflect SE because the models in the second group can produce unbiased estimates. Among the seven models, LCMx-RIy and LCMx-CLC show relatively large estimation errors. On average, the RMSEs of these models are, respectively, 1.4 and 0.65, while the RMSEs of the other models are around 0.1. Hence, LCMx-RIy and LCMx-CLC are removed from ANOVA so that we can avoid the inflation of variation and the effect masking due to the inflation. After excluding LCMx-RIy and LCMx-CLC,

ANOVA identifies three terms as significant factors: γ ($\eta^2 = 0.160$), $\gamma \times \beta_{II}$ ($\eta^2 = 0.144$), and $\text{Model} \times \gamma$ ($\eta^2 = 0.101$). In addition, there are two marginally important factors (i.e., Model with $\eta^2 = 0.093$ and $\text{Model} \times \gamma \times \beta_{SI}$ with $\eta^2 = 0.087$). However, these terms can only explain 58.4% of SST even with the marginal factors.

Coverage Rate Figure B.23 shows the coverage rate of $\hat{\beta}_{SI}$ under each condition. In this figure, we can find some model groups that share the pattern in changes of the coverage rate. For example, MLM-E2S, MLM-TAC, and LCM_y-CLC show a similar pattern, where they behave well with $(\gamma, \beta_{II}) = (0, 0), (0.6, 0.6)$; otherwise, they result in too low coverage rate. MLM-MV, LCM_{xy}-RP, and LCM_x-CLC is another group of models, which exhibit too high coverage rates under all conditions. MLM-WB is less likely to capture the true value when T is small; otherwise, it captures the true value too frequently. LCM_x-RI_y also misses the proper rate of 0.95 under all conditions; however, unlike MLM-MV, LCM_{xy}-RP, and LCM_x-CLC, its coverage rate fluctuates across the conditions. Finally, LCM_y-RI_x is the only model that could achieve the appropriate rate on average (i.e., the overall average is 0.947). ANOVA tells us that the main effect of Model as well as the interactions such as $\text{Model} \times \gamma$ and $\text{Model} \times \gamma \times \beta_{II}$ can explain 72.9% of the variability in the coverage rate, each of which explain 45.4%, 15.0%, and 12.5%, respectively.

Power Figure B.24 presents the power to detect nonzero β_{II} . It shows that MLM-MV, LCM_{xy}-RP, and LCM_x-CLC do not achieve perfect detection under any condition, while the other models do under most of the conditions. Some models can result in very low power in a specific setting. For example, the power of LCM_x-RI_y is less than 0.25 when $(\gamma, \beta_{SI}) = (0, 0.3)$. As a result, Model -related terms such as the main effect of Model , the interaction $\text{Model} \times \gamma$, $\text{Model} \times \gamma \times \beta_{SI}$, and $\text{Model} \times N$ are identified as influential factors by ANOVA.

Summary The results of this subsection clarify four points. First, LCMx-RIy cannot provide valid estimates of β_{II} . The estimates from LCMx-RIy are often biased and their SEs are highly likely to take an extreme value. In addition, goodness-of-fit indices show that LCMx-RIy does not fit data well under the present conditions. Second, Lüdtke's bias can occur in the existence of a slope factor for the outcomes. Lüdtke's bias is observed in the same way as Simulation 1 even though the current data generating model assumes a slope factor for the outcomes. This means that additional slope factor for the outcomes and the relevant between-person effect β_{SI} do not exert an impact on the occurrence of Lüdtke's bias. Third, the models with latent between-person predictors can produce unbiased estimates for β_{II} , but they are less efficient (i.e., larger standard errors), compared to those with observed between-person predictors. In particular, LCMx-CLC is highly likely to suffer from an extreme SE, which can undermine the unbiasedness of its estimates. Forth, some models have an SE ratio that is substantially higher than one. This overestimation of SE can occur not only for the models with a latent between-person predictor but also for MLM-WB.

5.2.4 Between-Person Effect of Intercept on Slope

Among the nine analytic models, MLM-WB and LCMx-RIy do not have a term estimating β_{SI} . Thus, results are reported with regard to the other seven models (i.e., MLM-E2S, MLM-TAC, MLM-MV, LCMy-RIx, LCMy-CLC, LCMxy-RP, and LCMx-CLC).

Bias Figure B.25 clearly shows that there are two types of models: those with biased and unbiased $\hat{\beta}_{SI}$. The two groups again correspond to models that have observed between-person predictors and models that have latent between-person predictors. The bias is not always present but only in cases of $\beta_{SI} = 0.3$. Also, similar to Lüdtke's bias, it seems to be a function of T . However, the current bias differs from Lüdtke's bias in that it is not a

function of the within-person effect γ . In fact, ANOVA does not identify terms including γ as an important term. Instead, the main effect of Model and β_{SI} as well as the interaction of them are identified as influential terms, which can account for 84.8% of SST in ANOVA. The eta-square of these terms are 0.324 (Model), 0.301 (Model $\times$ β_{SI}), and 0.224 (β_{SI}).

Empirical SE Figures B.26 and B.27 present the empirical SEs including and excluding LCMx-CLC. Figure B.26 shows that LCMx-CLC has a larger SE than the other models. There seems no pattern in the SE of LCMx-CLC and it appears to fluctuate randomly. For the other models, more general models such as MLM-MV and LCMxy-RP result in a larger SE than the other models (Figure B.27). The differences between the models are more evident when N and T are small. Also, the SE of MLM-MV seems to be a function of N and T with some exceptions of “spikes”. In fact, after removing LCMx-CLC from the analysis, Model, N , and T consist of the top five terms with large eta-square in ANOVA; however, only the main effects of Model and N have the eta-square larger than 0.1 ($\eta^2 = 0.383$ for Model; $\eta^2 = 0.254$ for N).

Theoretical SE and SE Ratio Figures B.28 and B.29 represent the mean theoretical SE with and without LCMx-CLC, respectively. As with the case of empirical SE, LCMx-CLC has a large theoretical SE, but its size is more extreme. MLM-MV also exhibits the spikes in theoretical SE, but they are also more extreme than those in empirical SE.

These differences between the theoretical and empirical SEs can also be confirmed in Figure B.30, where the ratios of them are provided. This figure shows that the models with latent between-person predictors have SE ratios larger than one (i.e., overestimating the SEs). In contrast, the SE ratios for the models with observed between-person predictors are fluctuating around one. Since the models with latent between-person predictors take extreme SE ratios, only the main effect of Model is identified as an

important factor when including them. When excluding those models, on the other hand, the highest order interaction without Model ($N \times T \times \gamma \times \beta_{II} \times \beta_{SI}$) is the sole important term with $\eta^2 = 0.125$.

RMSE Figure B.31 shows that LCMx-CLC has the worst estimation error indexed by RMSE. Since the size and variability across the conditions are very large, LCMx-CLC inflates the total variability in RMSE and may mask the effects of factors other than Model. In fact, removing LCMx-CLC from ANOVA decrease SST from 36.9 to 0.569, which amounts to a 98.5% reduction. Figure B.32 shows RMSE after removing LCMx-CLC from the analysis. In this figure, we can see that MLM-MV and LCMxy-RP have a larger RMSE than MLM-E2S, MLM-TAC, LCMMy-RIx, and LCMMy-CLC. This means that the impact of SE is relatively larger than the bias (recall that MLM-MV and LCMxy-RP can yield unbiased estimates for β_{SI}). Figure B.32 also shows that the RMSE of LCMMy-RIx becomes the smallest among the models when $\beta_{SI} = 0.3$. In addition, LCMMy-RIx is the best model in terms of the average RMSE across models; the overall average RMSE of LCMMy-RIx is 0.023, while that of the other models is at least 0.037. Reflecting these facts, ANOVA indicates that the main effects of β_{SI} , N , T , and Model, as well as the interaction between Model and β_{IS} , are the influential terms regarding RMSE. Their eta-squares are 0.185 (β_{SI}), 0.162 (N), 0.140 (T), 0.125 (Model $\times \beta_{IS}$), and 0.122 (Model).

Coverage Rate Figure B.33 presents the coverage rate of each model. Again, we can see the three types of models. The first group of models, including MLM-E2S, MLM-TAC, and LCMMy-CLC, achieve the proper coverage rate of 0.95 when $\beta_{SI} = 0$, but the rate rapidly drops as N becomes large. In particular, the coverage rate is almost zero when $N = 1,000$ regardless of T . This is because the size of bias does not change along with N but the SE becomes smaller, leading to narrower CIs. The second group is the models with latent

between-level predictors. They capture the true value too many times across 1,000 samples. This over-coverage is due to the overestimation of SE, which leads to unduly wide CIs. The third group only consists of LCMMy-RIx. It can produce unbiased point estimates and its SEs are relatively small and free from overestimation and underestimation. As a result, the CIs of LCMMy-RIx captured the true value at the proper rate. Since the deviation of the first model group from the proper rate is evident, ANOVA identifies Model, Model $\times$ β_{SI} , and β_{SI} as influential terms. The eta-squares of these terms are 0.352 (Model), 0.302 (Model $\times$ β_{SI}), and 0.226 (β_{SI}), and these three terms explain 87.9% of SST in total.

Power Figure B.34 is the result of power under each condition. The first group (i.e., MLM-E2S, MLM-TAC, LCMMy-RIx, and LCMMy-CLC) can always detect the nonzero β_{SI} , while the second group (MLM-MV, LCMxy-RP, and LCMx-CLC) is strongly affected by N and T . Recall that the first model group except LCMMy-RIx suffers from bias when true β_{SI} is not zero. This may indicate that their SEs are sufficiently small to detect β_{SI} under the current conditions. As a result, ANOVA suggests the main effect of Model and the interaction between Model and N are significant factors, which have $\eta^2 = 0.649$ and 0.172, respectively.

Summary In this subsection, we repeatedly saw the three model groups: the models with observed between-person predictors (i.e., MLM-E2S, MLM-TAC, LCMMy-CLC), the models with latent between-person predictors (MLM-MV, LCMxy-RP, and LCMx-CLC), and LCMMy-RIx as a singleton. While the first group has biased point estimates with relatively small SE, the second group produced unbiased estimates with large and possibly extreme SE. LCMMy-RIx can take a balance between the bias and SE, leading to the smallest RMSE. For the statistical inference (i.e., coverage rate and power), the SE ratio plays an important role in addition to the bias. Since the second type of models have an

overestimated SE, the resulting coverage rate and power are undermined.

5.2.5 Between-Person Effect of Slope on Intercept

There are only five models that can estimate β_{IS} (i.e., MLM-E2S, MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC). Hence, the following reports are with regard to these five analytic models. Note that under the current data-generating model, the target parameter β_{IS} is always zero, thus the power is not reported for this parameter.

Bias Figure B.35 illustrates the bias in $\hat{\beta}_{IS}$. The figure clearly shows that LCMx-RIy exhibits systematic changes in the bias, which appear to be a function of the experimental factors other than N . Also, the size of bias is larger than the other models. In contrast, the other four models attain unbiasedness on average, though individual conditions can result in a biased estimate. ANOVA indicates that three interaction terms (i.e., Model $\times$ γ , Model $\times$ $\gamma \times \beta_{SI}$, and Model $\times$ β_{SI}) are influential with the eta-square of 0.285, 0.126, and 0.113, when all models are analyzed. However, neither do these terms nor the other terms show a large eta-square after removing LCMx-CLC.

Empirical SE Figure B.36 depicts the empirical SE by model. In this figure, it is obvious that the SE increases in T in most cases. An exception is LCMx-CLC when $\gamma = -0.6$ or 0.6 . In these cases, the SE is decreasing in both N and T . In general, the size of SE is considerable except MLM-E2S. This is true even when N is large, implying that N does not contribute to the change in the SE. As a result, the main effect of Model and the interaction of Model with T explain 79.3% of SST ($\eta^2 = 0.665$ for Model; $\eta^2 = 0.128$ for Model $\times$ T).

Theoretical SE and SE Ratio Figure B.37 presents the theoretical SE of $\hat{\beta}_{IS}$ under each condition. The results resemble those of the empirical SE, but the level of theoretical SE is more severe for all models other than MLM-E2S. For example, the SEs estimated by MLM-MV and LCMx-CLC can be higher than 100, and those estimated by LCMxy-RP and LCMx-RIy can be higher than 10. As such, all models other than MLM-E2S have SE ratios higher than one in most settings (see Figure B.38). On the other hand, MLM-E2S can keep the ratio around one, meaning that the SEs returned by the software are basically reliable. ANOVA reflects these differences between the models, resulting in a large eta-square ($\eta^2 = 0.785$ for Model).

RMSE Since the size of SE is larger than the bias, the patterns of RMSE, which are shown in Figure B.39, are similar to those in Figure B.36. Accordingly, the influential factors identified by ANOVA are identical to those for the empirical SE. That is, the main effect of Model and the interaction Model $\times$ T account for 66.1% and 12.9% of the total variability in the RMSE, respectively.

Coverage Rate As can be seen in Figure B.40, MLM-E2S is the only model that can achieve the proper level of coverage rate. For MLM-MV, LCMxy-RP, and LCMx-CLC, CIs capture the true value too frequently in all settings. The coverage rate of LCMx-RIy exceeds the proper rate when $\gamma = 0$, while it falls below 0.95 when $\gamma \neq 0$. As a result, the main effect of Model and the interactions of Model with γ and β_{SI} have large eta-squares ($\eta^2 = 0.348$ for Model, 0.204 for Model $\times$ β_{SI} , 0.185 for Model $\times$ γ , and 0.104 for Model $\times$ $\gamma \times \beta_{SI}$).

Summary Overall, MLM-E2S performs better than the other analytic models. MLM-E2S can produce unbiased point estimates and the estimates are relatively stable and reliable since the estimated SE agrees with that based on the Monte Carlo samples. As

a result, MLM-E2S achieves the appropriate coverage rate. In contrast, the other models suffer from bias and/or large SE. LCMx-RIy can produce a large bias, depending on γ and β_{SI} . All models but MLM-E2S also suffer from the overestimation of SE. As a result, the coverage rates of these models are too high than they are supposed.

5.2.6 Between-Person Effect of Slope on Slope

Among the analytic models, only four models can estimate the between-person effect of slope on slope β_{SS} (i.e., MLM-E2S, MLM-MV, LCMxy-RP, and LCMx-CLC). Thus, the results reported in this subsection are only relevant to these four models. Note also that the true value of β_{SS} in Simulation 2 is always fixed at zero, thus the power is not reported for this parameter.

Bias Figure B.41 exhibits a very similar pattern to Figure A.39 in Simulation 1.

Specifically, the bias appears to be a function of γ . Since the true value of β_{SS} is always zero in the current conditions, this pattern means that the distribution of the estimates has the expectation of γ . Figure B.42 shows this distribution. Each circle represents a set of estimates $(\hat{\gamma}, \hat{\beta}_{SS})$ and the red lines in the subplots represent the points of $\hat{\beta}_{SS} = \hat{\gamma}$.

Although the different models spread to a different extent, the centers are on the red line, implying that the expectations of $\hat{\gamma}$ and $\hat{\beta}_{SS}$ are equal for all models. ANOVA supports this observation by the eta-square of 0.803 for the main effect of γ and no other term has the eta-square larger than 0.1.

Empirical SE Figures B.43 and B.44 illustrate the empirical SE including and excluding LCMx-CLC, respectively. The figures show the effect of T , which increases the SE. The extent of the effect varies across the models, implying the interaction of Model and T . However, since the impact of the main effects of Model and T are large, the effect of

Model $\times T$ remains marginal ($\eta^2 = 0.069$ for Model $\times T$, 0.763 for Model when including LCMx-CLC; $\eta^2 = 0.086$ for Model $\times T$, 0.479 for Model, and 0.103 for T when excluding LCMx-CLC).

Theoretical SE and SE Ratio Figures B.45 and B.46 present the theoretical SE with and without LCMx-CLC, respectively. Compared to the empirical SE, the theoretical SE takes larger values for MLM-MV, LCMxy-RP, and LCMx-CLC. The ratios of the two types of SE also indicate the overestimation of SE across the conditions (Figure B.47). We can also see in Figure B.47 that MLM-E2S can keep the SE ratio at around one. As such, the eta-squares from ANOVA suggest that Model is the only influential factor for the theoretical SE and SE ratio even after excluding LCMx-CLC from the analyses ($\eta^2 = 0.721$ for the theoretical SE with LCMx-CLC; $\eta^2 = 0.303$ for the theoretical SE without LCMx-CLC; $\eta^2 = 0.812$ for the SE ratio with LCMx-CLC; $\eta^2 = 0.426$ for the SE ratio without LCMx-CLC).

RMSE Figure B.48 shows RMSE observed in Simulation 2. It is obvious that LCMx-CLC is the worst model in terms of the estimation error measured by RMSE. LCMx-CLC is followed by MLM-MV and LCMxy-RP, and MLM-E2S is the best model of the four. In addition to the model difference, T also affects the RMSE in an increasing manner. The result of ANOVA aligns with these observations. When ANOVA includes LCMx-CLC, the main effect of Model is the sole factor that has the eta-square larger than 0.1 ($\eta^2 = 0.755$). Without LCMx-CLC, the main effects of Model and T are identified as influential factors with $\eta^2 = 0.405$ and 0.112, respectively.

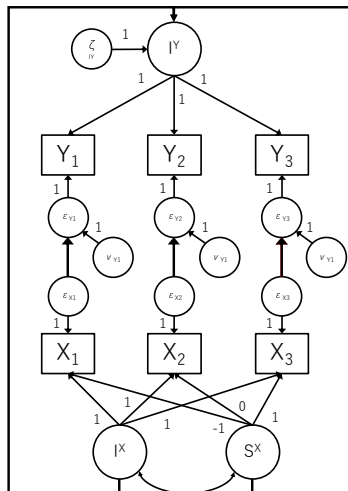
Coverage Rate Figure B.49 shows the coverage rate of each model under each condition. As with the other indices, MLM-E2S and the remaining models exhibit different patterns in the coverage rate across conditions. Both types of models are heavily affected

by the true value of γ . MLM-E2S can achieve the proper coverage rate when $\gamma = 0$, while the rate is almost always zero when $\gamma \neq 0$. In contrast, the coverage rates of MLM-MV, LCMxy-RP, and LCMx-CLC are almost always one when $\gamma = 0$. When $\gamma \neq 0$, the coverage rates of these models seem to be an increasing function of T . As a result, the main effects and interaction between Model and γ explain 98.7% of SST . More specifically, the main effect of Model has $\eta^2 = 0.588$, the interaction between Model and γ has $\eta^2 = 0.249$, and the main effect of γ has $\eta^2 = 0.150$.

Summary The estimation of β_{SS} under the current data-generating model and the simulation conditions can be considered considerably unstable. The point estimates are biased regardless of the models. The pattern in the bias is very akin to those found in Simulation 1—it seems that $\mathbb{E}[\hat{\beta}_{SS}] = \gamma$ regardless of the simulation conditions. In addition, the SE of $\hat{\beta}_{SS}$ can be extremely large, depending on an analytic model. That is, the empirical and theoretical SEs based on MLM-E2S are stable and they are close to each other. However, the SEs based on the other models are unstable and can be seriously large. As a result, the statistical inference regarding this parameter is compromised.

5.3 Simulation 3

Simulation 3 uses a data-generating model shown in Figure 5.3. Unlike Simulations 1 and 2, the data-generating model of Simulation 3 assumes a linear trend for the time-varying predictors, but not for the outcomes. Hence, the data-generating model does not have β_{SI} , but does have β_{IS} . Eventually, Simulation 3 examines the performance of the nine analytic models across 144 conditions. A brief summary of the results from Simulation 3 is presented in Table 5.3. All figures related to Simulation 3 are provided in Appendix C.

Figure 5.3*Data-Generating Model in Simulation 3*

5.3.1 Auxiliary Indices

Convergence Figure C.1 shows the proportion of converged cases across the conditions. The figure indicates that three models (i.e., LCM γ -RI x , LCM $x\gamma$ -RP, and LCM x -RI y) experienced nonconvergence during the simulation. While the LCM $x\gamma$ -RP and LCM x -RI y improve the convergence rate as T increases, LCM γ -RI x results in a lower rate when T is large. Also, γ seems to affect the convergence rate. Specifically, for LCM $x\gamma$ -RP and LCM x -RI y , a small γ leads to a better convergence rate. For LCM γ -RI x , the extent to which γ affects the convergence rate also depends on β_{II} . In the worst case, the convergence rate can be lower than 25%.

Problems in SE Estimation Figure C.2 shows the proportion of cases with problems in estimating SE. Since most cases with SE problems coincide with nonconvergence, the pattern in this figure resembles Figure C.1. One difference between them is that LCM x -CLC has a few cases with SE problems (i.e., 67 cases out of 144,000). These cases occur when T is small ($T = 3$).

Table 5.3*Marginal Means of Absolute Bias (Empirical SE) in Simulation 3*

Model	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}
MLM-WB	0.307 (0.009)	0.090 (0.033)	—	—	—
MLM-E2S	0.000 (0.020)	0.092 (0.033)	0.015 (0.012)	0.103 (0.085)	0.255 (0.027)
MLM-TAC	0.031 (0.016)	0.090 (0.033)	0.023 (0.012)	—	—
MLM-MV	0.004 (0.018)	0.003 (0.060)	0.005 (0.020)	0.015 (0.396)	0.068 (0.120)
LCMy-RIx	0.031 (0.016)	3.287 (2.190)	0.184 (0.196)	—	—
LCMy-CLC	0.000 (0.020)	0.090 (0.033)	0.025 (0.013)	—	—
LCMxy-RP	0.001 (0.020)	0.008 (0.084)	0.015 (0.062)	0.059 (0.716)	0.173 (0.628)
LCMx-RIy	0.001 (0.018)	0.011 (0.082)	—	0.065 (0.726)	—
LCMx-CLC	0.000 (0.020)	0.013 (0.433)	0.015 (0.406)	0.076 (2.458)	0.099 (2.396)

Nonpositive-Definite Model-Implied Covariance Matrix As shown in Figure C.3, different analytic models exhibit different patterns in the proportion of analyses with a nonpositive-definite (NPD) model-implied covariance matrix. For example, MLM-WB and MLM-MV achieve zero NPD covariance across the conditions. LCMx-RIy and LCMx-CLC yield NPD covariance matrices especially when N and/or T is small. MLM-E2S, MLM-TAC, and LCMy-CLC have NPD covariance matrices mostly when $\gamma = 0$. LCMxy-RP shows a pattern in which the proportion increases in T across conditions. LCMy-RIx exhibits a mixed pattern; the proportion increases in T when $\gamma = 0$ and $\beta_{II} = 0$, whereas it decreases in T when $\gamma \neq 0$ and $\beta_{II} = 0$. The pattern for LCMy-RIx changes when β_{II} is not zero; the proportion decreases in T when $\gamma = 0, -0.6$, while the proportion increases in T when $\gamma = 0.6$.

Goodness of Fit Figures C.4–C.6 constantly show that LCMy-RIx is ill-fitted to the data sets that are generated in the current settings. In particular, the model-data fit becomes worse as T increases. It is worth noting that SRMR indicates most models do not

fit the data across conditions. LCMx-CLC is an exception, which satisfies the criterion of $SRMR = 0.08$. This occurs when $\gamma = 0$.

5.3.2 Within-Person Effect

Bias Figure C.7 displays the bias in $\hat{\gamma}$. Some models show systematic patterns, which are a function of T and γ . MLM-WB, MLM-TAC, and LCMy-CLC can produce unbiased estimates when $\gamma = 0$, while they yield estimates with positive and negative biases when $\gamma \neq 0$. The magnitude of the bias becomes large when T is large. The bias size of MLM-WB is substantial when $T = 7$, while the bias of MLM-TAC and LCMy-CLC are moderate. MLM-MV also produces unbiased estimates when $\gamma = 0$ but biased estimates when $\gamma \neq 0$. However, T affects the bias in a reverse direction to MLM-WB, MLM-TAC, and LCMy-CLC. That is, the bias approaches zero as N and/or T increases. Also, the bias size of MLM-MV is not substantial. In fact, the maximum size of the bias is around 0.01, whereas the size of the true value is 0.6. Although we can see the effect of T , it is not identified as an important term by ANOVA. Instead, ANOVA finds the main effects of Model and γ important, which account for 97.9% of SST in ANOVA. Most of SST is explained by by Model ($\eta^2 = 0.822$).

Empirical SE Figure C.8 plots the empirical SE under each condition. The figure evidently shows that the SE is a decreasing function of N and T across the conditions ($\eta^2 = 0.533$ for N and 0.267 for T). In all subplots, the SE of MLM-WB is lower than the other models. Also, the model difference in the SE is more obvious when N and T are small. However, the model difference is not identified as a marginally influential factor with $\eta^2 = 0.093$.

Theoretical SE and SE Ratio Figure C.9, where the theoretical SE is depicted, exhibits the same pattern as Figure C.8. The SE decrease as a function of N and T , and MLM-WB shows slightly better results than the other models. Figure C.10 presents the ratio of the theoretical SE over the empirical SE. Most models have the SE ratio of one, implying that their theoretical SE is close to the Monte Carlo-based SE (i.e., empirical SE). MLM-WB and MLM-MV overestimate the SE by 5% on average and the ratios approach one as T grows. In contrast, MLM-TAC underestimates the SE by 5% on average and the ratio deviates from one as T gets large. The ratio of LCMx-CLC is slightly affected by N rather than T . It underestimates the SE when N is small, but the magnitude of underestimation is not substantial (around 2%). ANOVA suggests the main effect of Model and the interaction between Model and T as important factors with $\eta^2 = 0.375$ and 0.112, respectively.

RMSE Figure C.11 displays RMSE of each model across the conditions. The figure shows a clear pattern in RMSE. First, MLM-WB has a relatively large RMSE when $\gamma \neq 0$. In this case, the RMSE increases as T gets large. MLM-TAC and LCMy-RIx present a similar pattern, but their RMSE is much smaller than that of MLM-WB. Second, MLM-E2S, MLM-MV, LCMy-CLC, LCMxy-RP, LCMx-RIy, and LCMx-CLC exhibit a similar pattern to the SE. This is because these models can produce unbiased estimates, thus the RMSE reflects the SE more strongly. Even though we can see a clear effect of N and T , ANOVA identifies only the main effect of Model and the interaction between Model and γ as influential factors. These two terms explain 91.3% of SST (59.4% by Model and 32.0% by Model $\times$ γ).

Coverage Rate Figure C.12 depicts the coverage rate under each condition. The figure shows that when $\gamma = 0$, all models have the proper coverage rate of 0.95 and that when

$\gamma \neq 0$, models can be classified into three categories. The first category includes MLM-E2S, MLM-MV, LCM_y-CLC, LCM_{xy}-RP, LCM_x-RI_y, and LCM_x-CLC. These models can reach the proper coverage rate. The second category consists of MLM-TAC and LCM_y-RI_x. The coverage rate of these models sharply drops as N and/or T get large. In particular, when $N \geq 300$ and $T \geq 5$, the coverage rate is approximately zero. The third category only consists of MLM-WB, which results in zero coverage rate across the conditions. The result of ANOVA reflects these facts. Specifically, the main effects of Model and γ , as well as the interaction between them, explain 89.4% of the total variation in the coverage rate. The eta-square of these terms are 0.512 (Model), 0.257 (Model $\times$ γ), and 0.125 (γ).

Power Figure C.13 presents the statistical power for detecting γ under each condition. The figure obviously shows that all models always reject the null hypothesis $H_0 : \gamma = 0$ when γ is actually nonzero. Note that this result is specific to the current settings.

Summary In Simulation 3, the estimation of γ is relatively stable. This is supported by the fact that RMSE is relatively small except for MLM-WB. Although the small amount of bias and/or large standard error may be obtained under specific conditions such as the one with small N and/or T , we can use, with caution, the point estimates and results from the statistical inference that are based on the analytic models other than MLM-WB.

5.3.3 Between-Person Effect of Intercept on Intercept

Bias Figure C.14 shows three groups of models based on patterns in the bias of $\hat{\beta}_{II}$. The first group consists of MLM-WB, MLM-E2S, MLM-TAC, and LCM_y-CLC. The models in this group show Lüdke's bias. Hence, the size and direction of bias are a function of T , γ , and β_{II} . The second group includes MLM-MV, LCM_{xy}-RP, LCM_x-RI_y, and LCM_x-CLC. These models can unbiasedly estimate β_{II} when N and/or T is large. When either N or T

is small, MLM-MV, LCMxy-RP, and LCMx-RIy produce positive bias, while LCMx-CLC can yield positive and negative bias. The third group consists of LCMy-RIx only. This model exhibits a distinctive pattern in which the bias sharply rises as T increases, especially when $\gamma = -0.6$. In the worst case, the bias comes to 25, which amounts to more than 4,000% of the true value. Thus, I decided to exclude LCMy-RIx from ANOVA to avoid the inflation of SST by a single model. Actually, removing LCMy-RIx successfully reduce SST from 4369.5 to 7.867 (i.e., 99.8% reduction). ANOVA summarizes the observed patterns and indicates that $\text{Model} \times \gamma$, γ , β_{II} , and $\text{Model} \times \beta_{II}$ as important factors. These four terms have $\eta^2 = 0.300$ ($\text{Model} \times \gamma$), 0.279 (γ), 0.110 (β_{II}), and 0.108 ($\text{Model} \times \beta_{II}$). ANOVA also finds the main effect of Model marginally significant ($\eta^2 = 0.095$).

Empirical SE The same model classification as the bias can be applied to the result of empirical SE. We can see in Figure C.15 that the first group, that is, MLM-WB, MLM-E2S, MLM-TAC, and LCMy-CLC are less affected by T but mostly affected by N . The size of SE is relatively small, compared to the other groups. The second group, which includes MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC, shows a higher SE than the first group. Unlike the first group, the magnitude of the SE is strongly affected by T —the more measurements we have, the less varying the estimates become. This tendency is more evident for LCMx-CLC. The third group, LCMy-RIx, increases the SE as a function of T . However, the effect of T becomes much smaller when $\gamma = \beta_{II}$. In this case, the size of SE becomes large, which is comparable with or higher than LCMx-CLC. Overall, ANOVA finds the main effect of Model and the interaction between Model and T significant and their eta-squares are 0.419 for Model and 0.248 for $\text{Model} \times T$.

Theoretical SE and SE Ratio We can further apply the same model classification to the results of theoretical SE since the pattern in the theoretical SE is the same as the

empirical SE. However, the magnitude of the SE is larger for the second and third groups. We can see this difference in Figure C.17, where the ratio of the theoretical SE over the empirical SE is provided. The first model group keeps the SE ratio around one, but the second and third groups have the SE ratio higher than one (i.e., overestimation). While the ratio of the second group approaches to one as T increases, the third group (i.e., LCM_y-RI_x) increases the ratio as T grows. As a result, the interaction between Model and T and the main effect of Model are identified as important factors in ANOVA. Their eta-squares are 0.371 for Model $\times$ T and 0.197 for Model.

RMSE Figure C.18 displays RMSE of each model. The model classification still holds in this figure. The RMSE of the second group is affected by T . The first and third groups are affected by T as well. In addition, they are also influenced by γ and β_{II} . The first and second group reduces the RMSE as a function of T , while the third group increases. Among the nine analytic models, the estimation error indexed by RMSE is much larger for LMC_y-RI_x and LCM_x-CLC. Figure C.19 focuses on the other seven models so that we can examine differences between the remaining models. In Figure C.19, we can find that LCM_x_y-RP and LCM_x-RI_y tend to have higher RMSE when $T = 3$, but once we have more measurement occasions, the RMSE drops to the same level as or lower level than the other models, depending on the within-person and between-person effects. MLM-WB, MLM-E2S, MLM-TAC, and LCM_y-CLC have a small to moderate level of RMSE, depending on whether they suffer from Lüdke's bias. In other words, these models have a smaller RMSE than the other models when $\gamma = \beta_{II}$; otherwise, their RMSE can be larger than MLM-MV, LCM_x_y-RP, and LCM_x-RI_y. Even though we can see these model differences, ANOVA indicates that Model does not impact on the RMSE after excluding LCM_y-RI_x and LCM_x-CLC. Instead, the main effects of T and γ and the interaction between γ and β_{II} are indicated as influential factors with $\eta^2 = 0.212$ (T), 0.151 ($\gamma \times \beta_{II}$), and 0.130 (γ).

Coverage Rate Figure C.20 depicts the coverage rate of each model. Again, we can find the three types of models. The first group (i.e., MLM-WB, MLM-E2S, MLM-TAC, and LCM_y-CLC) shows a highly variable coverage rate, ranging from 0 to 0.95. The patterns seem dependent on γ and β_{II} as well as T . The second group of models, that is, MLM-MV, LCM_{xy}-RP, LCM_x-RI_y, and LCM_x-CLC have coverage rates slightly higher than 0.95. This over-coverage is improved as T increases. The third group of the model (i.e., LCM_y-RI_x) is also affected by γ , β_{II} and T . LCM_y-RI_x is more likely to achieve the proper rate of 0.95 when $T = 7$. Although we can see the effect of T in the figure, ANOVA does not support it. Instead, ANOVA specifies the main effect of Model, the interaction $\gamma \times \beta_{II}$, and the interaction Model $\times \gamma \times \beta_{II}$ as important factors. Their eta-squares are 0.427 (Model), 0.148 (Model $\times \beta_{II}$), and 0.133 (Model $\times \gamma \beta_{II}$).

Power In Figure C.21, LCM_y-RI_x exhibits a different pattern in the statistical power from the other analytic models. It is the only model that losses power when T increases. Models such as MLM-WB, MLM-E2S, MLM-TAC, and LCM_y-CLC achieve almost perfect detection of nonzero β_{II} , while MLM-MV, LCM_{xy}-RP, LCM_x-RI_y, and LCM_x-CLC have lower power, which is around 0.75 when $T = 3$. Overall, ANOVA identifies the main effects of Model and T as influential factors with $\eta^2 = 0.487$ and 0.396, respectively.

Summary We repeatedly saw in this subsection that there were three types of model and the quality of estimation and inference heavily depend on the model type. The first group is the models that have observed between-person predictors (i.e., MLM-WB, MLM-E2S, MLM-TAC, and LCM_y-CLC). This type of model produces biased estimates with relatively small SE. The second type of models, which possess latent between-person predictors (i.e., MLM-MV, LCM_{xy}-RP, LCM_x-RI_y, and LCM_x-CLC), suffer from large SE, though they can estimate β_{II} unbiasedly. The quality of estimation and inference for this

model group can be improved by increasing T . The third type of model, that is LCMMy-RIx, is the most problematic model in the current settings. This model produces extreme bias and SE that are an increasing function of T . As such, LCMMy-RIx shows RMSE that is almost 100 times as large as the other models. Fortunately, researchers may be able to notice the ill-behavior of LCMMy-RIx by the auxiliary indices such as nonconvergence, no SE estimates, NPD model-implied covariance matrix, and/or poor goodness of fit.

5.3.4 Between-Person Effect of Intercept on Slope

MLM-WB and LCMx-RIy do not have a term that corresponds to β_{SI} . Hence, these two models are excluded from the analyses in this subsection. Also, the power is not reported for this parameter since it is always fixed at zero in the population model.

Bias Figure C.22 shows the bias in $\hat{\beta}_{SI}$ by model. As can be seen in the figure, the analytic models exhibit some patterns in the bias. First, MLM-E2S can estimate β_{SI} unbiasedly when $\gamma = 0$ regardless of the settings of the other parameters. When $\gamma \neq 0$, however, it produces slightly biased estimates, of which the size becomes smaller as T increases. MLM-TAC and LCMMy-CLC show a similar pattern to MLM-E2S but in the opposite direction. That is, when $\gamma \neq 0$, the size of their bias increases along with T . Second, MLM-MV, LCMxy-RP, and LCMx-CLC yield biased estimates when T is small. However, this bias disappears as T grows. Note that N and the other factors seem not to affect the results. Third, LCMMy-RIx produces unbiased estimates when $\gamma = 0$, but biased estimates when $\gamma \neq 0$. The pattern that LCMMy-RIx shows is more complex than the other models. When $\gamma = -0.6$, the bias increases as an increasing function of T . However, when $\gamma = 0.6$, LCMMy-RIx exhibits a V-shaped pattern, in which the size of bias for $T = 5$ is larger than that for $T = 3$ and 7. We can see this pattern across the conditions regardless of the true value of the between-person effects (i.e., β_{II} and β_{SI}). Note that the absolute

size of the bias is much larger for LCM_y-R_Ix than the other models. As a result, ANOVA identifies the terms related to Model, T , and γ as important factors. Their eta-squares are 0.239 (Model $\times$ γ), 0.229 (Model $\times$ T), 0.151 (Model $\times$ $T \times \gamma$), 0.123 (Model), and 0.113 (γ).

Empirical SE Figure C.23 depicts the empirical SE of each model across the conditions. In this figure, LCM_y-R_Iy shows a distinctive pattern of SE. Unlike the other models, the SE of LCM_y-R_Ix grows as T increases. The size of SE is relatively large, but smaller than that of LCM_x-CLC. In contrast, the models other than LCM_x-CLC reduce the SE along with T . The magnitude of the SE for MLM-E2S, MLM-TAC, and LCM_y-CLC tend to be smaller than MLM-MV, LCM_{xy}-RP, and LCM_x-CLC. This is true especially when T is small. In other words, SEs of MLM-MV, LCM_{xy}-RP, and LCM_x-CLC are likely to be high when T is small. This model difference disappears as T gets large. Since the magnitude of SE for LCM_x-CLC is much larger than the other models, ANOVA is conducted without LCM_x-CLC so that we can avoid the inflated variation masking a meaningful pattern. According to the result from ANOVA, the main effect of Model and the interaction Model $\times$ T are identified as important determinants of the SE with $\eta^2 = 0.443$ and 0.223, respectively.

Theoretical SE and SE Ratio Figure C.24 presents the theoretical SE in the same way as the empirical SE. Figure C.25 shows the ratio of theoretical SE over empirical SE. The theoretical SE replicates the same pattern as the empirical SE, but MLM-MV, LCM_{xy}-RP, and LCM_x-CLC have theoretical SE that is higher than their empirical counterpart. This overestimation is more evident when $T = 3$. In contrast, MLM-E2S, MLM-TAC, and LCM_y-CLC keep the two types of SEs at a similar level. As such, the set of the influential factors found by ANOVA is identical to that for the empirical SE. More specifically, the main effect of Model and the interaction Model $\times$ T explain SST in the

theoretical SE by 33.7% and 32.4%, respectively. Also, the same set of terms account for 39.6% (Model) and 21.1% (Model $\times$ T) of SST of the SE ratio.

RMSE Since RMSE is a function of bias and SE, Figure C.26 shows the impacts of Model, T , and γ , which are the significant determinants of the bias and SE. MLM-E2S decreases the RMSE as a function of T . Its average level, on the other hand, depends on γ . More specifically, the RMSE with $\gamma = 0$ is smaller than that with $\gamma \neq 0$. MLM-TAC and LCM_y-CLC exhibit a pattern in which the RMSE decreases as T grows when $\gamma = 0$, but does not change so much when $\gamma \neq 0$. MLM-MV, LCM_{xy}-RP, and LCM_x-CLC also show a decreasing pattern; however, the effect of T seems greater than the previous two types of models. This may be because the SE of MLM-MV, LCM_{xy}-RP, and LCM_x-CLC are greatly affected by T . Finally, LCM_y-RI_x presents the opposite pattern to the other models. It increases the RMSE as T becomes large when $\gamma \neq 0$, while the RMSE does not change over T when $\gamma = 0$. Following the analyses of empirical and theoretical SE, LCM_x-CLC is removed from ANOVA. The result indicates three influential factors, namely Model ($\eta^2 = 0.464$), Model $\times$ T ($\eta^2 = 0.221$), and Model $\times$ γ ($\eta^2 = 0.115$).

Coverage Rate Figure C.27 displays the coverage rate of each model across the conditions. The figure shows that all models result in the proper coverage rate when $\gamma = 0$ (i.e., red lines), while some models fail to keep the rate at the proper level when $\gamma \neq 0$ (black and blue lines). In cases of $\gamma \neq 0$, MLM-E2S, MLM-TAC, and LCM_y-CLC decrease the coverage rate as either N or T increases. LCM_y-RI_x also decreases the coverage rate when $\gamma \neq 0$ as N increases, but increases as T increases, or more precisely, it shows a V-shaped pattern. For MLM-MV, LCM_{xy}-RP, and LCM_x-CLC, the average rates are approximately 0.95. However, individual coverage rates can deviate from 0.95. In particular, it seems that the coverage rate of those models decreases as T increases when

$\gamma = 0$ (red lines). Despite these observations, ANOVA does not support the effects relevant to T and N . Instead of them, the main effects of Model ($\eta^2 = 0.361$) and γ ($\eta^2 = 0.205$) as well as the interaction of them ($\eta^2 = 0.170$) explain most part of SST (73.6%).

Summary Across the figures presented in this subsection, the effects of Model and γ are persistently observed. In particular, the models obtain desirable results (i.e., unbiased estimates, small SE, proper coverage rate) when $\gamma = 0$. In addition, the effect of T is also seen in most evaluation indices, where increasing T leads to improving the qualities of estimation and inference. One exception to this tendency is LCMMy-RIx, For LCMMy-RIx, increasing T deteriorates the performance. It is worthwhile noting that MLM-TAC, which is an MLM counterpart of LCMMy-RIx, does not show similar results to LCMMy-RIx. Rather, the results of MLM-TAC are very close to those of LCMMy-CLC.

5.3.5 Between-Person Effect of Slope on Intercept

Models that can estimate β_{IS} include MLM-E2S, MLM-MV, LCMxy-RP, LCMx-Rly, and LCMx-CLC, and this subsection only provides the results regarding these five models.

Bias Figure C.28 presents the bias in $\hat{\beta}_{IS}$. It clearly shows a systematic bias of MLM-E2S. The bias is dependent on T and γ (i.e., within a subplot) as well as β_{II} and β_{IS} (across the subplots). The impact of T varies along with β_{IS} . We can also see that the average level of bias decrease in β_{IS} and increases in β_{II} . For the other models, the estimates can be biased when $T = 3$ and the bias gradually becomes small as T gets large. In spite of these observations, the interaction between Model and β_{IS} is the sole influential factor found by ANOVA with $\eta^2 = 0.108$. Thus, most of the variation in the bias left unexplained.

Empirical SE Figure C.29 depicts the empirical SE of $\hat{\beta}_{IS}$ under each condition. MLM-E2S again shows a different pattern from the other models. Specifically, the SE of MLM-E2S increases along with T , while the other models decrease. In addition, the SE of MLM-E2S is much smaller than the other four models when $T = 3$. The result from ANOVA aligns with these findings. The interaction of Model with T as well as the main effects of Model and T are specified as influential terms, and their eta-squares are 0.293 (Model $\times T$), 0.221 (T), and 0.207 (Model).

Theoretical SE and SE Ratio Figure C.30 presents the theoretical SE and Figure C.31 shows the ratio of the theoretical SE over the empirical counterpart. The theoretical SE exhibits the same pattern as the empirical SE, but the theoretical SE is higher than the empirical SE for all models but MLM-E2S. In particular, these models tend to have an extreme theoretical SE when $T = 3$. Accordingly, they have SE ratios that are much higher than one when $T = 3$. ANOVA also derives a similar set of important terms for the theoretical SE and SE ratio. For the theoretical SE, the interaction between Model and T ($\eta^2 = 0.287$) and the main effect of Model ($\eta^2 = 0.159$) are found influential. For the SE ratio, the interaction between Model and T ($\eta^2 = 0.350$) and the main effects of T ($\eta^2 = 0.200$) and Model ($\eta^2 = 0.196$) are identified as important factors.

RMSE Figure C.32 shows RMSE of the five analytic models across the conditions. The models except MLM-E2S present a decreasing pattern in T , while MLM-E2S shows both increasing and decreasing patterns when $\beta_{IS} = 0$ and $\beta_{IS} = 0.3$, respectively. MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC have much larger RMSE than MLM-E2S when $T = 3$. However, when $T = 7$, there is little difference among the models (the average RMSE is 0.140 for MLM-E2S, and around 0.190 for the other models). ANOVA indicates the interaction between Model and T ($\eta^2 = 0.291$) and the main effects of T ($\eta^2 = 0.227$)

and Model ($\eta^2 = 0.202$) as influential factors.

Coverage Rate Figure C.33 depicts the coverage rate of each model. This figure again shows two types of models, namely MLM-E2S and the other models. The coverage rate of MLM-E2S is greatly affected by β_{IS} . The deviation of the coverage rate from 0.95 is larger when $\beta_{IS} = 0.3$ than when $\beta_{IS} = 0$. In particular, when $T = 3$, the coverage rate of MLM-E2S can drop to zero. In contrast, the coverage rate of MLM-E2S does not radically change across the conditions when $\beta_{IS} = 0$. In this case, lower coverage rates are observed when N is large. For the other models, too high coverage rates are observed when $T = 3$, and as T increases, the rate gradually approaches to the proper rate of 0.95. This may be due to the overestimation of SE, which is shown in Figure C.31. The result of ANOVA is greatly influenced by the dramatic change of MLM-E2S along with β_{IS} . ANOVA then found that the main effect of Model and the interaction of Model and β_{IS} are influential. Their eta-square are 0.355 (Model) and 0.216 (Model $\times \beta_{IS}$). Note that the interaction between Model and T and the one between Model, T , and β_{IS} are at the marginal level ($\eta^2 = 0.097$ and 0.090, respectively).

Power Figure C.34 presents the statistical power for detecting β_{IS} under each condition. In Figure C.34, we can see that the power of MLM-E2S can be larger than the other models when $(\gamma, \beta_{II}) = (-0.6, 0)$, $(-0.6, 0.6)$, or $(0, 0.6)$. However, when $(\gamma, \beta_{II}) = (0, 0)$ or $(0.6, 0.6)$, the power of MLM-E2S is as high as the other models. Furthermore, it can be lower when $(\gamma, \beta_{II}) = (0.6, 0)$. LCMx-RIy shows higher power when $\gamma \neq 0$, while the power is almost the same across the four models when $\gamma = 0$. Even though we visually observed the model differences, ANOVA does not support them. Instead, ANOVA identifies the main effect of T ($\eta^2 = 0.442$), N ($\eta^2 = 0.332$), and the interaction between them ($\eta^2 = 0.109$) as influential factors, which together account for 88.3% of SST .

Summary To summarize, MLM-E2S behaves differently from the other analytic models. On the other hand, MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC show results that are very similar to each other across the evaluation indices. In general, the results of MLM-E2S are better than the other models. This is true especially when T is small (i.e., $T = 3$). However, under some conditions, the inferential properties of MLM-E2S such as the coverage rate and statistical power can be worse than the other models, and sometimes it can be the worst model.

5.3.6 Between-Person Effect of Slope on Slope

Since models that can estimate β_{SS} are limited to MLM-E2S, MLM-MV, LCMxy-RP, and LCMx-CLC, this subsection describes results regarding β_{SS} for these four models.

Bias Figure C.35 shows the bias in $\hat{\beta}_{SS}$. In this figure, MLM-E2S and MLM-MV clearly show a specific pattern, in which the bias appears to be a function of T and γ . When $\gamma = 0$, the bias is zero across the conditions, while the bias is decreasing in T when $\gamma \neq 0$. Although the effect of T is linear for MLM-E2S, it seems nonlinear for MLM-MV. Also, MLM-MV may be negatively influenced by the sample size N ; the larger N becomes, the smaller the bias becomes. For the other two models (i.e., LCMxy-RP and LCMx-CLC), the pattern is equivocal, but it seems that a large bias occurs when T is small. ANOVA indicates by the eta-square that the main effect and interactions related to Model, T , and γ are influential. The term that has the largest eta-square is the main effect of γ ($\eta^2 = 0.287$). This is followed by the interaction $T \times \gamma$ ($\eta^2 = 0.221$), Model $\times \gamma$ ($\eta^2 = 0.179$), and Model $\times T \times \gamma$ ($\eta^2 = 0.102$).

Empirical SE Results related to the empirical SE are provided in Figure C.36. We can see in the figure that all models exhibit a decreasing pattern in T . However, the size of the

largest SE, which occurs $T = 3$, varies across the models. LCMx-CLC produces an extreme SE, which is the largest of the four models. Note that the models converge to a similar level of SE when $T = 7$ (about 0.01 for MLM-E2S and 0.02 for the other three models). As such, ANOVA indicates that the interaction of Model and T and the main effects of Model and T are significant factors. The eta-squares are 0.311 for Model $\times$ T , 0.220 for Model, and 0.195 for T .

Theoretical SE and SE Ratio Figures C.37 and C.38 present the mean theoretical SE and the SE ratio, respectively. The theoretical SE has the same pattern as the empirical SE; however, its level is much larger than the empirical SE, which is also indicated by the SE ratios higher than one. In particular, LCMx-CLC greatly overestimates the SE when $T = 3$. The ratio reaches 30 in the worst case. In contrast, MLM-E2S keeps the ratio at around one across the conditions. Eventually, both theoretical SE and SE ratio have the same set of important determinants as the empirical SE. Specifically, for the theoretical SE, 55.0% of SST is explained by Model $\times$ T ($\eta^2 = 0.282$), Model ($\eta^2 = 0.158$), and T ($\eta^2 = 0.110$). On the other hand, Model $\times$ T ($\eta^2 = 0.316$), T ($\eta^2 = 0.215$), and Model ($\eta^2 = 0.170$) account for 70.1% of the variation in the SE ratio.

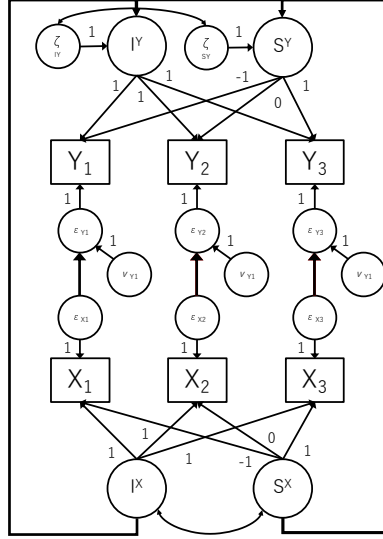
RMSE Each model's RMSE of $\hat{\beta}_{SS}$ is shown in Figure C.39. All models exhibit a decreasing pattern in T , but LCMx-CLC and LCMxy-RP estimate β_{SS} with a much larger error than MLM-E2S and MLM-MV. Figure C.40 plots the RMSE of MLM-E2S and MLM-MV in the same subplot to facilitate the model comparison. We can see in the figure that MLM-E2S has more estimation error when $\gamma \neq 0$, while it becomes very small and is smaller than that of MLM-MV when $\gamma = 0$. As a result, ANOVA finds Model $\times$ T , T , and Model influential, with eta-square of 0.304, 0.214, and 0.201, respectively.

Coverage Rate Figure C.41 displays the coverage rate under each condition. In Figure C.41, MLM-E2S presents bipolar results. That is, the coverage rate of MLM-E2S is, on average, 0.952 when $\gamma = 0$, while it is almost zero when $\gamma = -0.6$ and 0.6 . In contrast, MLM-MV, LCMxy-RP, and LCMx-CLC show a pattern in which the rate converges to 0.95 as T gets large. When $T = 3$, however, the confidence intervals of the three models capture the true value too many times when $\gamma = 0$ and too few times when $\gamma \neq 0$. Because of the distinctive results of MLM-E2S, ANOVA mainly finds Model-relevant factors important. In fact, the main effect of Model accounts for more than half of the variation in the coverage rate ($\eta^2 = 0.582$). In addition, the interaction of Model with γ and the main effect of γ have a large eta-square ($\eta^2 = 0.273$ for Model $\times \gamma$; $\eta^2 = 0.136$ for γ). As a result, these three terms together explain more than 99% of SST .

Summary The results in this subsection repeatedly indicate the model differences, which mainly contrast MLM-E2S with the other three models. The general patterns are similar to each other; for example, they are decreasing in T for most indices. Overall, we can say that all models can produce biased estimates for β_{SS} when $T = 3$, but the SE of MLM-E2S is much smaller and its theoretical SE is closer to the truth. However, we need to be cautious about the inference results based on MLM-E2S because it is likely to reject the null hypothesis of $H_0 : \beta_{SS} = 0$ even if there is no such an effect in the population model.

5.4 Simulation 4

Simulation 4 assumes the data-generating model shown in Figure 5.4. The data-generating model has all possible between-person effects (i.e., β_{II} , β_{SI} , β_{IS} , and β_{SS}), thus the number of simulation conditions is larger than the previous simulations (576 conditions). In order to grasp a big picture of the result, a brief summary averaged over the conditions is

Figure 5.4*Data-Generating Model in Simulation 4*

provided in Table 5.5. Also, all figures from Simulation 4 are provided in Appendix D.

5.4.1 Auxiliary Indices

Convergence Figures D.1–D.4 show the proportion of converged cases. According to these figures, LCM y -RI x , LCM xy -RP, and LCM x -RI y have at least one nonconverged case during the simulation. Their average convergence rates are 0.896 (LCM y -RI x), 0.521 (LCM xy -RP), and 0.605 (LCM x -RI y). The patterns in the proportions are not uniform but unique to each model. For example, LCM y -RI x decreases the convergence rate as T increases and the rate of decrease seems to depend on γ , β_{II} , and β_{SI} . In contrast, LCM xy -RP improves the convergence rate as T goes higher. The pattern is more complex for LCM x -RI y (see Figure D.4); the proportion fluctuates, depending on γ , β_{SI} , and β_{SS} .

Problems in SE Estimation Similar to the previous simulations, most of the problems in SE estimation coincide with nonconvergence. In fact, the percentage of cases that

Table 5.4*Marginal Means of Absolute Bias (Empirical SE) in Simulation 4*

model	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}
MLM-WB	2.823 (0.046)	0.090 (0.033)	—	—	—
MLM-E2S	0.000 (0.020)	0.093 (0.033)	0.032 (0.020)	0.103 (0.085)	0.319 (0.048)
MLM-TAC	0.017 (0.017)	0.090 (0.033)	0.037 (0.020)	—	—
MLM-MV	0.001 (0.019)	0.004 (0.064)	0.004 (0.040)	0.024 (0.455)	0.048 (0.299)
LCMy-RIx	0.017 (0.017)	3.299 (1.932)	1.070 (0.873)	—	—
LCMy-CLC	0.000 (0.020)	0.090 (0.033)	0.037 (0.020)	—	—
LCMxy-RP	0.001 (0.020)	0.007 (0.082)	0.012 (0.079)	0.057 (0.693)	0.144 (0.772)
LCMx-RIy	0.956 (0.474)	1.660 (0.848)	—	1.418 (1.286)	—
LCMx-CLC	0.001 (0.020)	0.012 (0.366)	0.014 (0.449)	0.075 (2.620)	0.120 (3.090)

converged but do not have an SE is less than 1% for all models. MLM-MV and LCMx-CLC do not have a nonconverged case but do have cases without SE estimates (1 case for MLM-MV and 231 cases for LCMx-CLC). All of these cases were obtained when $T = 3$. Therefore, the pattern in the proportion of no-SE cases is almost the same as that in nonconverged cases; it is increasing in T for LCMy-RI but decreasing for LCMxy-RP (Figures D.5–D.7), and complex for LCMx-RIy (Figure D.8).

Nonpositive-Definite Model-Implied Covariance Matrix The proportion of nonpositive-definite (NPD) model-implied covariance matrices exhibits different patterns from that of the convergence rate and SE problems (see Figures D.9–D.13). First, MLM-WB almost always results in an NPD covariance matrix. Only 216 cases out of 576,000 had a positive-definite covariance matrix. In contrast, MLM-MV achieved 100% positive definiteness of a model-implied covariance matrix. Also, more than 90% of the cases of MLM-E2S, MLM-TAC, LCMy-CLC, LCMxy-RP, and LCMx-CLC result in a positive-definite covariance.

On the other hand, the proportion of the NPD covariance matrices for LCMy-RIx and LCMx-RIy largely vary, depending on the simulation conditions (Figure D.12 and Figure D.13). For these models, the proportion can take extreme values (i.e., one and zero) or values close to them when $T = 7$. However, this is not always the case. For example, when $(\beta_{SI}, \beta_{SS}) = (0, 0)$, the proportion for LCMx-RIy does not drop to zero, whereas it never goes up to one when $(\beta_{SI}, \beta_{SS}) = (0.3, 0.6)$, for example.

Goodness-of-Fit Figures D.14–D.26 are the plots displaying averages of CFI, RMSEA, and SRMR under each condition. These model-fit indices consistently show that (a) LCMy-CLC, LCMxy-RP, and LCMx-CLC achieve the sufficient level of model fit, (b) LCMx-RIy exhibits the worst model fit among the five SEM models, and (c) LCMy-RIx positions in between them, but never achieves the sufficient levels of goodness-of-fit.

5.4.2 Within-Person Effect

Bias Figures D.27–D.35 show the bias in $\hat{\gamma}$ for each analytic model. Figure D.27 clearly shows the large positive bias of MLM-WB across the conditions. The bias increases as T gets large, and the average level becomes higher when $\beta_{SI} = 0.3$ and/or $\gamma = -0.6$.

LCMx-RIy also produces a large bias, which appears to be a function of the model parameters (see Figure D.34). Specifically, when $\beta_{SI} = \beta_{SS} = 0$ (i.e., in the first and third column), the bias is determined by γ and not by N and T . However, when $\beta_{SS} = 0.6$ and $\gamma = 0$ (the red lines in the second and fourth row), the bias increases along with T . A similar pattern can be found when $\beta_{SI} = 0.3$ (in the second and fourth column) except when $(\beta_{SI}, \beta_{IS}, \beta_{SS}) = (0.3, 0.3, 0)$. MLM-TAC (Figure D.29) and LCMy-RIx (Figure D.31) also show a systematic pattern in the bias. Each row in these figures exhibits the same pattern, meaning that β_{II} and β_{SI} do not affect the bias. Also, the first and third rows have a similar pattern, while the second and fourth rows have another pattern. This

means that β_{SS} affects the pattern in the bias but β_{IS} does not. In addition to the between-person effects, γ also seems to play an important role in determining the bias of MLM-TAC and LCM_y-RI_x. In particular, the estimates are unbiased when $\gamma = \beta_{SS} = 0$ and when $\gamma = \beta_{SS} = 0.6$. The remaining models (i.e., MLM-E2S, MLM-MV, LCM_y-CLC, LCM_x-RP, and LCM_x-CLC) are considered to yield unbiased estimates of the within-person effect. In fact, their mean absolute bias is almost zero, as shown in Table 5.4.

ANOVA supports these observations. Specifically, when MLM-WB and LCM_x-RI_y are included in an analysis, the main effect of Model ($\eta^2 = 0.748$) and the interaction between Model and β_{SI} ($\eta^2 = 0.117$) account for 86.4% of *SST*. On the other hand, once the two models are excluded from the analysis, the *SST* reduces to 0.009% (from 6204.5 to 0.464), and γ plays an important role to determine the bias. That is, the interaction between Model and γ has the largest eta-square ($\eta^2 = 0.403$), followed by the main effect of γ ($\eta^2 = 0.164$). These terms are further followed by the main effect of Model ($\eta^2 = 0.161$) and the interaction of Model with β_{SS} ($\eta^2 = 0.142$). In the end, these four terms together explain 87.0% of *SST* after removing MLM-WB and LCM_x-RI_y.

Empirical SE Figure D.36 depicts the empirical standard error of LCM_x-RI_y. From this figure, it is obvious that $\hat{\gamma}$ based on LCM_x-RI_y is highly unstable when the true value is zero. The SE can even increase as T increases when β_{SS} is zero. Figures D.37–D.39 also show the empirical SE excluding LCM_x-RI_y. In these figures, we can see that LCM_x-RI_y has the same level of SE as the other models when $\gamma \neq 0$ and that MLM-WB can produce three times as large SE as the other models when $\beta_{SI} = 0.3$ (in the second and fourth column). ANOVA including all models indicates that an interaction term $\text{Model} \times N \times \gamma$ is the sole factor with eta-square higher than 0.1 and explains more than half of the variation in the empirical SE ($\eta^2 = 0.575$). On the other hand, after excluding LCM_x-RI_y, the main effect of Model ($\eta^2 = 0.574$) and the interactions $\text{Model} \times \beta_{IS}$ ($\eta^2 = 0.218$) and $\text{Model} \times N$

($\eta^2 = 0.111$) account for 90.4% of the variability.

Theoretical SE and SE Ratio Figures D.40–D.43 display the theoretical SE in the same ways as Figures D.36–D.39. The patterns of the theoretical SE are akin to the empirical SE. The level of the theoretical SE is also similar to the empirical SE except for some conditions (see Figures D.44–D.52 for SE ratio). For example, LCMx-RIy sometimes overestimates the SE, as shown in Figure D.51. MLM-MV shows a systematic pattern in the SE ratio. It underestimates and overestimates SE when $T = 3$ and $T = 7$, respectively (Figure D.44). ANOVA identifies the interaction term $\text{Model} \times \gamma$ and $\text{Model} \times \gamma \times \beta_{SI}$ as important factors for the theoretical SE, and $\text{Model} \times T$ for the SE Ratio. The eta-squares are 0.461 for $\text{Model} \times \gamma$, 0.159 for $\text{Model} \times \gamma \times \beta_{SI}$, and 0.229 for $\text{Model} \times T$.

RMSE Figure D.53 and Figure D.54 are the plots showing the RMSE of MLM-WB and LCMx-RIy, respectively. These two models are displayed separately from the other models since their RMSE is much larger than those of the other models. Although both MLM-WB and LCMx-RIy show a large RMSE, there are some differences between them. First, while the RMSE of MLM-WB is large across the conditions, LCMx-RIy takes large RMSE only when $\gamma = 0$. Second, the large RMSE of MLM-WB stems mainly from bias, whereas that of LCMx-RIy is the result of bias and SE.

Figures D.55–D.57 present RMSE of the remaining seven models. The level of the RMSE for these models is much smaller than MLM-WB and LCMx-RIy, and their pattern is similar to each other. One may find two model groups in the figures (i.e., MLM-TAC and LCMy-RIx versus the others); however, this notion is not supported by ANOVA. ANOVA without MLM-WB and LCMx-RIy finds that the main effects of N and T are the two terms that have an impact on the RMSE ($\eta^2 = 0.453$ and 0.297 , respectively), implying that there is no significant model difference.

Coverage Rate Figures D.58 through D.66 show the coverage rate of each model.

Figure D.58 is the result of MLM-WB and shows that the confidence intervals of MLM-WB never capture the true value of γ under all conditions. This zero coverage is due to the large bias and small SE, which we saw above. The coverage rates of MLM-TAC and LCM_y-RI_x deviate from the proper rate of 0.95 under some conditions (Figure D.60 and Figure D.62). These patterns in the coverage rate correspond to the ones in bias (Figure D.29 and Figure D.31). That is, the confidence intervals fail to capture the true value as bias increases, and the coverage rates become close to 0.95 when the estimates are unbiased. Because of these systematic patterns, the main effect of Model is the sole factor with a large eta-square in ANOVA including all models. In this analysis, the Model term explains more than 80% of SST ($\eta^2 = 0.812$). Even when MLM-WB and LCM_x-RI_y are excluded from ANOVA, Model remains important ($\eta^2 = 0.371$) in addition to Model $\times$ N ($\eta^2 = 0.136$) and Model $\times$ $\gamma \times \beta_{SS}$ ($\eta^2 = 0.112$).

Power Figures D.67 and D.68 present the power to detect γ under each condition. These figures show the almost perfect detection of the within-person effect. The only exception occurs when $(\beta_{II}, \beta_{SI}, \beta_{IS}, \beta_{SS}) = (0.6, 0.3, 0, 0)$ and $T = 3$ for MLM-MV. In all cases but this single case, the models can detect a statistically significant within-person effect.

Summary Figures presented in this subsection consistently indicate that the quality of estimation and inference based on MLM-WB and LCM_x-RI_y is poor. MLM-WB mainly suffers from its large bias, while LCM_x-RI_y from both bias and SE. Compared to these two models, MLM-TAC and LCM_y-RI_x performed better; however, they do not show satisfactory performance especially for the coverage rate. The other models can achieve unbiased estimation with reasonably small SE, leading to the proper inference (the coverage rate of 0.95 and power of 1).

5.4.3 Between-Person Effect of Intercept on Intercept

Bias Figures D.69–D.77 are plots that show the bias in $\hat{\beta}_{II}$. Among the nine analytic models, MLM-WB, MLM-E2S, MLM-TAC, and LCMMy-CLC exhibit Lüdtke’s bias. The size and direction of the bias are affected by $\gamma - \beta_{II}$ and T . In particular, the estimates become unbiased when $\gamma = \beta_{II} = 0$ (i.e., the red lines in the first and second column) and $\gamma = \beta_{II} = 0.6$ (the blue lines in the third and fourth column). LCMx-RIy has a similar pattern, but the bias size is much larger than Lüdtke’s bias. This is true especially when $(\gamma, \beta_{SI}, \beta_{SS}) = (0, 0.3, 0.6)$. LCMMy-RIx is another model that shows extremely large bias. The bias pattern of LCMMy-RIx looks different from Lüdtke’s bias, though the estimates become unbiased under the same condition as Lüdtke’s bias (i.e., $\gamma = \beta_{II}$). The bias of LCMMy-RIx is basically positive and increasing in T , and can be extreme under some conditions (e.g., $N = 1000$, $T = 7$, $\gamma = -0.6$). The remaining models (MLM-MV, LCMxy-RP, and LCMx-CLC) can basically yield unbiased estimates. They may have a small amount of bias under certain conditions (e.g., $T = 3$), but the magnitude of the bias is at most 10% of the biased models.

ANOVA including all models indicates that the main effect of Model ($\eta^2 = 0.232$) and the interactions Model $\times \gamma$ ($\eta^2 = 0.229$) and Model $\times T$ ($\eta^2 = 0.194$) are influential, while ANOVA without the severely biased models (i.e., LCMMy-RIx and LCMx-RIy) finds important the main effects of γ ($\eta^2 = 0.325$) and β_{II} ($\eta^2 = 0.271$) and the interactions Model $\times \gamma$ ($\eta^2 = 0.122$) and Model $\times \beta_{II}$ ($\eta^2 = 0.102$).

Empirical SE Figures D.78–D.80 are plots of the empirical SE, in which models with extreme SE are separately shown (i.e., LCMMy-RIy, LCMx-RIy, and LCMx-CLC). In contrast, Figures D.81–D.83 display the other models in the same subplots to facilitate model comparisons. The empirical SE of LCMMy-RIx, which is shown in Figure D.78, increases along with T . The increment becomes large as γ and β_{II} get distant. LCMx-RIy

also exhibits an increasing pattern in T when $(\beta_{SI}, \beta_{SS}) = (0.3, 0.6)$ and $\gamma = 0$ (Figure D.79). The large average SE of LCMx-RIy, which we can see in Table 5.4, may be due to these extreme cases. In contrast to LCMy-RIx and LCMx-RIy, LCMx-CLC demonstrates a decreasing pattern in T (Figure D.80). Thus, most cases with extreme SE occurred when $T = 3$. The magnitude of the extreme SE is relatively moderate, compared to LCMy-RIx and LCMx-RIy.

The remaining models can be examined by visually inspecting Figures D.81–D.83. MLM-MV and LCMxy-RP have a higher SE than the other models when T is small. Because of the decreasing pattern of MLM-MV and LCMxy-RP, this gap gradually disappears as T grows. Note that MLM-WB, MLM-E2S, MLM-TAC, and LCMy-CLC do not show covariation with T . Also, there is no influence of the within-person and between-person effects.

When all models are included in an analysis, ANOVA indicates that the main effect of Model and the interactions $\text{Model} \times T$ and $\text{Model} \times \gamma \times \beta_{II}$ are influential factors that have a large eta-square. These three terms account for 80.4% of the variation in the empirical SE. More specifically, Model, $\text{Model} \times T$, and $\text{Model} \times \gamma \times \beta_{II}$ explain 44.8%, 23.3%, and 12.4% of SST , respectively. Once LCMy-RIx and LCMx-RIy are excluded, SST reduces from 6122.8 to 244.3 (96% reduction), and only Model ($\eta^2 = 0.311$) and $\text{Model} \times T$ ($\eta^2 = 0.212$) remain influential after the exclusion.

Theoretical SE and SE Ratio Figures D.84–D.88 individually present the theoretical SE of five analytic models (i.e., MLM-MV, LCMy-RIx, LCMx-RIy, LCMxy-RP, and LCMx-CLC). The patterns of these five models are the same as the corresponding empirical SE; however, the magnitude becomes larger. In fact, the marginal means of SE ratio exceed one for the five models (i.e., 1.36 for MLM-MV, 1.82 for LCMy-RIx, 1.45 for LCMxy-RP, 1.21 for LCMx-RIy, and 7.76 for LCMx-CLC). On the contrary, the models

shown in Figure D.89–D.91 correctly estimate the SE, except for MLM-WB (the marginal means of SE ratio are 2.39 for MLM-WB, while 0.998 for MLM-E2S, 0.994 for MLM-TAC, and 0.994 for LCMY-CLC). MLM-WB overestimates the SE especially when $\beta_{SI} = 0.3$ (see Figure D.92). This tendency affects the result of ANOVA for the SE ratio when MLM-WB, MLM-E2S, MLM-TAC, and LCMY-CLC are only included in the analysis. In this analysis, the main effect of Model and the interaction between Model and β_{SI} are identified as important factors with eta-square of 0.634 (Model) and 0.247 (Model $\times$ β_{SI}). On the other hand, ANOVA including all models identifies Model ($\eta^2 = 0.381$) and Model $\times$ T ($\eta^2 = 0.188$) as influential terms. This is so because the theoretical SE of MLM-MV, LCMY-RIx, LCMx-RIy, LCMxy-RP, and LCMx-CLC tends to be extremely large when T is small.

RMSE Figures D.101–D.109 present RMSE of the nine analytic models. We can see some patterns in the RMSE across the models. First, MLM-WB, MLM-E2S, MLM-TAC, and LCMY-CLC exhibit a similar pattern, which is mainly due to the bias (i.e., Lüdtke’s bias). Because Lüdtke’s bias is increasing in $\gamma - \beta_{II}$ and decreasing in T , the RMSE becomes largest when $(\gamma, \beta_{II}) = (0.6, -0.6)$ and $T = 3$. Also, there is a small impact of N , which stems from the SE. Second, MLM-MV, LCMxy-RP, and LCMx-CLC show a common pattern, which is decreasing in T and N , though they have different maxima at $(N, T) = (100, 300)$. Also, there seems no clear influence of γ , which we saw for the first group of models. The remaining two models, that is, LCMY-RIx and LCMx-RIy show respective patterns, both of which produce extremely large RMSE under certain conditions. LCMY-RIx shows an increasing pattern in N and T , which is amplified by β_{II} . LCMx-RIy, in contrast, shows mixed patterns. While it exhibits a decreasing pattern when $\beta_{SI} = 0$, the model also shows an increasing RMSE when $(\beta_{SI}, \gamma) = (0.3, 0)$.

Reflecting these patterns, ANOVA with all models indicate that Model, Model $\times$ T ,

and $\text{Model} \times \gamma$ are influential factors with a large eta-square (0.339 for Model, 0.201 for $\text{Model} \times T$, and 0.113 for $\text{Model} \times \gamma$), whereas ANOVA without LCM_y-RI_x and LCM_x-RI_y identifies $\text{Model} \times T$ ($\eta^2 = 0.222$) and model ($\eta^2 = 0.118$) as important factors. In other words, the within-person effect influences the patterns of LCM_y-RI_x and LCM_x-RI_y but not the other models.

Coverage Rate Figures D.110–D.118 show the coverage rate of each model. They can be classified into similar groups to those for RMSE, but we can also see some differences within a group. For the first group (i.e., MLM-WB, MLM-E2S, MLM-TAC, LCM_y-RI_x, and LMC_y-CLC), MLM-E2S, MLM-TAC, and LCM_y-CLC have almost the same coverage rates across the conditions. They achieve the proper coverage rate of 0.95 when their estimates are unbiased (i.e., when $\gamma = \beta_{II}$). Since the bias becomes small as T increases, the corresponding coverage rate approaches to 0.95 as T grows. However, as N increases, the SE becomes small and the resulting confidence interval becomes narrower, leading to a low coverage rate. MLM-WB (Figure D.110) and LCM_y-RI_x (Figure D.114) have a similar pattern; however, MLM-WB has a higher average coverage rate, and LCM_y-RI_x shows a less clear effect of γ .

MLM-MV, LCM_{xy}-RP, and LCM_x-CLC, which constitute the second model group, can capture the true value at the proper rate on average. However, their confidence intervals capture the true value too frequently when $T = 3$. In fact, the average coverage rate is decreasing in T (i.e., 0.975 when $T = 3$, 0.952 when $T = 5$, and 0.948 when $T = 7$). Also, there is no clear effect of γ .

In contrast, the coverage rate of LCM_x-RI_y is heavily affected by γ (see Figure D.117). This model can have the proper coverage rate only when $\gamma = 0$, though the coverage rate can deviate from 0.95 even when $\gamma = 0$ if $(\beta_{SI}, \beta_{SS}) = (0, 0.6)$, $N \geq 300$, and $T \geq 5$.

According to the result from ANOVA, these patterns in the coverage rate can be explained by the following four terms: Model ($\eta^2 = 0.375$), $\gamma \times \beta_{II}$ ($\eta^2 = 0.143$), Model $\times \gamma \times \beta_{II}$ ($\eta^2 = 0.139$), and Model $\times \gamma$ ($\eta^2 = 0.129$).

Power Figures for the statistical power evidently show four types of models (see Figures D.119-D.121). The first group is the one that achieved almost perfect detection of β_{II} (i.e., MLM-E2S, MLM-TAC, and LCMMy-CLC). The second group has moderate level of power when $T = 3$ and almost 100% power when $T = 5$ and $T = 7$. This group consists of MLM-MV, LCMxy-RP, and LCMx-CLC. One may add MLM-WB to this group, but its power can drop to 0.25 (see Figure D.119). The third and fourth group consists only of a single model. LCMMy-RIx is the only model that shows a decreasing pattern in T . This pattern holds even when $N = 1000$. LCMx-RIy takes zero or one when $\beta_{SI} = 0.3$, while it can take moderate values when $\beta_{SI} = 0$ and $\gamma = 0$ (Figure D.120). Note that LCMx-RIy almost always takes 100% power when $\gamma \neq 0$. The result from ANOVA shows that the main effect of Model and the interactions of Model with T and γ have an impact on the power. Their eta-squares are 0.357 (Model), 0.277 (Model $\times T$), and 0.184 (Model $\times \gamma$).

Summary We again observed Lüdtke's bias in $\hat{\beta}_{II}$ for the models that have observed between-person predictors. Those models tend to have small, stable standard errors across the conditions. In contrast, the models with latent between-person predictors resulted in unbiased estimates with relatively large standard error. LCMMy-RIx and LCMx-RIx suffered from large bias and large SE. This may be due to their inability to capture the time trend behind either outcomes or time-varying predictors.

5.4.4 Between-Person Effect of Intercept on Slope

Since MLM-WB and LCMx-RIy do not possess a term for β_{SI} , the following results are with regard to the other seven analytic models (i.e., MLM-E2S, MLM-TAC, MLM-MV, LCMMy-RIx, LCMMy-CLC, LCMxy-RP, LCMx-CLC).

Bias Figures D.122–D.128 show the bias in $\hat{\beta}_{SI}$ for each model. Among the seven models, MLM-E2S, MLM-TAC, and LCMMy-CLC produce a systematic bias. The bias size of MLM-E2S is basically decreasing (i.e., approaching zero), whereas that of MLM-TAC and LCMMy-CLC is increasing in T (the bias keeps increasing). These three models can yield unbiased estimates when $\beta_{SI} = 0$ and $\gamma = \beta_{II}$. The estimates from MLM-MV, LCMxy-RP, and LCMx-CLC are unbiased except when $T = 3$. The bias tends to be positive, but this is not always the case. In addition, the estimates can be biased even when $N = 1000$. LCMMy-RIx can produce an extremely large bias when $\beta_{SI} = 0.3$ and $T = 7$. Furthermore, the bias becomes larger as the N gets large.

ANOVA was conducted, including and excluding LCMMy-RIx. For both analyses, the interaction between Model and β_{SI} are detected as an important factor. Moreover, ANOVA with all models indicates that the main effect of Model and the interaction term $\text{Model} \times T \times \gamma$ are also influential. ANOVA excluding LCMMy-RIx identified as important factors the main effects of γ and β_{SI} as well as the interaction of Model and γ . For ANOVA with LCMMy-RIx, the eta-squares of the significant factors are 0.187 (Model), 0.111 (Model $\times \beta_{SI}$), and 0.111 (Model $\times T \times \gamma$). For ANOVA without LCMMy-RIx, the eta-squares are 0.186 (γ), 0.155 (Model $\times \beta_{SI}$), 0.140 (β_{SI}), and 0.109 (Model $\times \gamma$).

Empirical SE Figure D.129 and Figure D.130 show the empirical standard error of LCMMy-RIx and LCMx-CLC, respectively. These two models are displayed individually because their SE is much larger, on average, than the other models. LCMMy-RIx increases

the SE as T increases. The increment is extreme when $\beta_{SI} = 0.3$. In contrast, the empirical SE of LCMx-CLC is decreasing in T , and it gets close to the other models when $T = 5$ and $T = 7$.

In Figures D.131–D.133, the remaining models (i.e., MLM-E2S, MLM-TAC, MLM-MV, LCMy-CLC, and LCMxy-RP) are plotted together to facilitate the model comparisons. These three figures constantly show that the estimates of MLM-MV and LCMxy-RP are unstable when T is small. In particular, when $T = 3$, the SE of LCMxy-RP is 2.59 times and 7.36 times larger than MLM-MV and MLM-E2S, respectively. This discrepancy reduces to 1.22 times and 2.08 times when $T = 5$ and 1.00 times and 1.28 times when $T = 7$.

As with the bias, ANOVA was conducted, including and excluding LCMy-RIx and LCMx-CLC. The main effect of Model has a large eta-square in both analyses ($\eta^2 = 0.187$ when including the two models and 0.239 when excluding them). When LCMy-RIx and LCMx-CLC are included, ANOVA indicates that the interactions $\text{Model} \times \beta_{SI}$ ($\eta^2 = 0.111$) and $\text{Model} \times T \times \gamma$ ($\eta^2 = 0.111$) have an impact on the empirical SE. When the two models are excluded, ANOVA also suggests T and $\text{Model} \times T$ as important factors ($\eta^2 = 0.181$ and $\eta^2 = 0.307$, respectively).

Theoretical SE and SE Ratio Figures D.134–D.138 present the theoretical SE in the same way as the empirical SE. The patterns in these figures are very similar to those for the empirical SE, but some of the models show the theoretical SE higher than the empirical SE. For example, for LCMxy-RP, the SE ratio is 3.04 when $N = 100$ and $T = 3$ (see Figure D.144). Furthermore, MLM-MV, LCMy-RIx, and LCMx-CLC severely overestimate the SE (Figures D.141, D.142, and D.145). ANOVA indicates the significant effects of Model and T for the theoretical SE and SE ratio. More specifically, the interaction between Model and T and the main effect of Model are found important for the theoretical SE and

SE ratio, including all models. Their eta-squares are 0.247 ($\text{Model} \times T$) and 0.120 (Model) for theoretical SE, and .0341 ($\text{Model} \times T$) and 0.174 (Model) for SE Ratio.

RMSE In Figures D.146–D.148, LCM_y-RI_x, LCM_{xy}-RP, and LCM_x-CLC are individually plotted since they take large RMSE. While LCM_{xy}-RP and LCM_x-CLC exhibit a common pattern in which RMSE is decreasing as N or T increases, LCM_y-RI_x shows an increasing pattern. This increasing pattern is due to both bias and SE, as shown in Figures D.125 and D.129.

Figures D.149–D.151 jointly plot the remaining models. While MLM-MV constantly exhibits the same decreasing pattern across the conditions, the other models show some variation in the magnitude and pattern. Depending on the conditions, their RMSE can be higher and lower than that of MLM-MV. For example, when $\gamma = 0.6$ (i.e., Figure D.149), MLM-TAC and LCM_y-CLC show a flat or slightly increasing pattern if $\beta_{SS} = 0.6$ or $(\beta_{SS}, \beta_{SI}) = (0, 0)$. Since their RMSE does not decrease so much along with N , MLM-MV can have a lower estimation error than MLM-TAC and LCM_y-CLC, especially when both N and T are large. MLM-MV can also have smaller RMSE than MLM-E2S when $(\beta_{SS}, \beta_{SI}) = (0, 0.3)$ regardless of N and T . For other conditions such as $T = 3$, MLM-MV can be the worst model.

Although we can see the model difference among MLM-E2S, MLM-TAC, MLM-MV, and LCM_y-CLC, it is not detected by ANOVA. Specifically, when excluding LCM_y-RI_x, LCM_{xy}-RP, and LCM_x-CLC from the analysis, ANOVA only finds influential the main effect of T and the interaction between γ and β_{SI} ($\eta^2 = 0.177$ and 0.166 , respectively) as well as N ($\eta^2 = 0.097$).

Coverage Rate Figures D.152–D.158 show the coverage rate of each model. The figures indicate that the coverage rate of some models can greatly deviate from the proper

coverage rate of 0.95. These models include MLM-E2S, MLM-TAC, LCMMy-RIx, and LCMMy-CLC. Among them, MLM-E2S and LCMMy-RIx improve the rate as T increases, while MLM-TAC and LCMMy-CLC show mixed results, depending on the conditions. For example, when $(\beta_{SI}, \beta_{SS}) = (0.3, 0.6)$, the coverage rate of MLM-TAC and LCMMy-CLC deviates from 0.95 as T increases if $\gamma = -0.6$, but improves when $\gamma = 0$ or 0.6 . On the other hand, the coverage rates of MLM-MV, LCMxy-RP, and LCMx-CLC are, on average, at the proper level, though they may be off from 0.95 when the number of measurements is small. The model differences are so clear that the main effect of Model is the sole term with $\eta^2 > 0.1$ in ANOVA ($\eta^2 = 0.346$).

Power Figure D.159–D.161 present the statistical power across the conditions. As can be seen in the figures, MLM-E2S, MLM-TAC, LCMMy-CLC achieved approximately perfect detection, while the other models can have power less than one under some conditions. The power of MLM-MV, LCMxy-RP, and LCMx-CLC seems to be an increasing function of T . However, the power of LCMMy-RIx declines as T increases regardless of the settings of the other parameters. ANOVA supports these observations by indicating Model and Model $\times$ T as significant factors. Their eta-squares are 0.443 for Model and 0.424 for Model $\times$ T .

Summary To summarize, there are two types of models. One has biased point estimates with small SE (i.e., MLM-E2S, MLM-TAC, LCMMy-CLC) and the other produces unbiased but unstable estimates (MLM-MV, LCMxy-RP, and LCMx-CLC). LCMMy-RIx does not belong to either of them, yielding large RMSE due to the bias and SE. As a result, the former model group can have a coverage rate that is too low, whereas the latter model group can keep the proper coverage rate across the conditions. However, the models in the second group may fail to detect nonzero β_{SI} when only a few observations are available (i.e., either T or N is small).

5.4.5 Between-Person Effect of Slope on Intercept

Among the nine analytic models, MLM-E2S, MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC can estimate the between-person effect of slope on intercept β_{IS} . Hence, the following reports are about these five models.

Bias Figures D.162–D.166 present the bias in $\hat{\beta}_{IS}$ for each model. We can clearly see that MLM-E2S and LCMx-RIy show systematic biases. For MLM-E2S, the bias seems to be an increasing function of T , and the increment of the bias seems governed by γ , β_{II} , and β_{IS} . This is not the case for LCMx-RIy; for example, the bias approaches zero along with T when $(\beta_{SI}, \beta_{SS}) = (0, 0)$. In addition, the bias of LCMx-RIy can be extreme when $(\gamma, \beta_{SI}, \beta_{SS}) = (0, 0.3, 0.6)$. The remaining models can yield unbiased estimates except when $T = 3$. When $T = 3$, $\hat{\beta}_{IS}$ from MLM-MV, LCMxy-RP, and LCMx-CLC tends to be biased. For example, LCMxy-RP often has positively biased estimates when $T = 3$, but the bias of MLM-MV and LCMx-CLC can be both positive and negative. Unfortunately, it is difficult to predict the direction of the bias.

ANOVA produces different sets of significant variables, depending on whether or not LCMx-RIy is included. When LCMx-RIy is included in the analysis, the Model-related interactions such as $\text{Model} \times \gamma$ ($\eta^2 = 0.151$), $\text{Model} \times \gamma \times \beta_{SS}$ ($\eta^2 = 0.149$), $\text{Model} \times T \times \gamma \times \beta_{SS}$ ($\eta^2 = 0.126$), and $\text{Model} \times T \times \gamma$ ($\eta^2 = 0.118$) are identified as important factors. On the other hand, when LCMx-RIy is excluded, only two terms are detected: $\text{Model} \times \beta_{IS}$ with $\eta^2 = 0.128$ and Model with $\eta^2 = 0.102$.

Empirical SE Figures D.167–D.171 plot the empirical SE by model. In general, the empirical SE is a decreasing function of N and T . However, MLM-E2S exhibits an increasing pattern in T , though the bias is still decreasing in N (see Figure D.167). It is also worth noting that the bias size of MLM-E2S is relatively small, compared to the other

models. Even the largest bias of MLM-E2S is smaller than the other models. For the remaining models, SE can be greatly large. While the largest SE is 0.232 for MLM-E2S, that for the other models ranges from 1.80 (MLM-MV) to 14.2 (LCMx-CLC). In addition, these largest SEs occur with $T = 3$ or 5, when the SE of MLM-E2S is relatively small. Thus, we can say that the models other than MLM-E2S are quite unstable in terms of the estimation of β_{IS} . Overall, ANOVA indicates that the main effect of Model and the interaction of Model with T are significant factors with $\eta^2 = 0.451$ and 0.325, respectively.

Theoretical SE and SE Ratio Theoretical SE shows the same pattern as the empirical SE although its magnitude is inflated (see Figures D.172–D.176). In particular, the maximum of theoretical SE reaches 315 for LCMx-CLC and the ratio of theoretical SE over the empirical SE can be higher than 30 (Figure D.181). MLM-E2S is an exception to this inflation tendency. It can keep the SE ratio around the unity on average, implying that MLM-E2S can correctly estimate the SE across the conditions (Figure D.177).

Although the magnitude of SE is inflated, the pattern remains the same, leading to the same set of significant factors in ANOVA. Specifically, the main effect of Model and the interaction between Model and T can account for a substantial amount of the variation in the theoretical SE and SE ratio. The eta-squares are 0.430 (Model $\times$ T) and 0.238 (Model) for the theoretical SE, and 0.502 (Model $\times$ T) and 0.285 (Model) for the SE ratio.

RMSE Figures D.182–D.186 display RMSE of each model. As can be seen in the figures, there are three types of models that have different patterns in the RMSE. A dominant pattern is the one where RMSE is decreasing in T . Models such as MLM-MV, LCMxy-RP, and LCMx-CLC show this pattern. MLM-E2S exhibits a mixed pattern; that is, the RMSE is increasing in T when $\beta_{IS} = 0$ but decreasing when $\beta_{IS} = 0.3$. Finally, LCMx-RIy has a peculiar pattern, in which the RMSE becomes large when $\gamma = 0$. This pattern seems to

stem from the SE (see Figure D.170).

Because of these model differences in RMSE, ANOVA specifies the main effect of Model and the interaction between Model and T as influential factors. Their eta-squares are 0.430 for the interaction and 0.238 for the main effect, meaning that these two terms explain two thirds of the total variation in the RMSE of $\hat{\beta}_{IS}$.

Coverage Rate Again, three types of models can be observed in the results of the coverage rate (Figures D.187–D.191). The confidence intervals of MLM-MV, LCMxy-RP, and LCMx-CLC capture the true value too frequently (i.e., too high coverage rate) when $T = 3$. The coverage rates gradually approach 0.95 and on average, the coverage rates of these models are approximately 0.95. MLM-E2S shows increasing and decreasing patterns, depending on β_{IS} (Figure D.187). While MLM-E2S can keep relatively high coverage rate when $\beta_{IS} = 0$, the rate stays at a very low level when $\beta_{IS} = 0.3$ and $T = 3$, and then soars as T increases. LCMx-RIy tends to take an extreme rate (i.e., zero or one) and can take values between zero and one when $\beta_{SI} = 0$. In this case, the coverage rate decreases along with T and N (see Figure D.190), and the decrement seems dependent on γ . ANOVA supports these observations and finds influential the interactions $\text{Model} \times \gamma$ and $\text{Model} \times \beta_{IS}$ as well as the main effect of Model. The eta-squares of these terms are 0.258 for $\text{Model} \times \gamma$, 0.247 for Model, and 0.149 for $\text{Model} \times \beta_{IS}$.

Power Figures D.192 through D.196 are plots in which the statistical power of each model is shown. All models appear to be an increasing function T and N , meaning that we can enhance the ability of a model to detect β_{IS} by collecting more observations. Model differences can be seen in how γ affects the power. For example, it seems that MLM-MV, LCMxy-RP, and LCMx-CLC are not affected by γ , while the increment of power along with T and N seems dependent on γ for MLM-E2S and LCMx-RIy. The power of

MLM-E2S becomes large as γ becomes small with a fixed T and N ; however, this is not the case for LCMx-RIy. For example, the power of LCMx-RIy with $\gamma = 0$ is lower than that with $\gamma = -0.6$ or 0.6 (see Figure D.195). As a result, ANOVA indicates that the interaction between Model and γ is the most influential factor with $\eta^2 = 0.359$. It is followed by the main effect of T , which accounts for 24.2% of the variation in the power.

Summary To conclude, MLM-MV, LCMxy-RP, and LCMx-CLC have similar results across the evaluation indices. They can produce unbiased estimates unless the number of measurements is small. If T is small (e.g., $T = 3$), then the estimates may be biased and extremely unstable (i.e., have huge standard error). MLM-E2S is better than the other models in terms of total estimation error (i.e., RMSE); however, its inferential performance, measured by coverage rate, type I error rate, and statistical power, is inferior to MLM-MV, LCMxy-RP, and LCMx-CLC. On the other hand, LCMx-RIy suffers from a large estimation error, which is due to both large bias and large SE. Thus, neither can point estimates nor hypothesis testing be trusted.

5.4.6 Between-Person Effect of Slope on Slope

The following four models can estimate the between-person effect of slope on slope β_{SS} : MLM-E2S, MLM-MV, LCMxy-RP, and LCMx-CLC. In this subsection, the results are reported only for those models.

Bias Figures D.197–D.200 display the bias in $\hat{\beta}_{SS}$ for each model. Figure D.197 evidently shows a systematic bias of MLM-E2S. Similar to Lüdtke’s bias in $\hat{\beta}_{II}$, the bias in $\hat{\beta}_{SS}$ appears to be a function of $\gamma - \beta_{SS}$ and T . On the other hand, the other three models produced unbiased estimates unless $T = 3$. This bias may have a deterministic relation with the parameters since the models exhibit a pattern somehow similar to each other. However,

it is difficult to identify the relationship based only on the visual inspection of the figures.

Overall, ANOVA identifies the interactions $\text{Model} \times \gamma$ and $T \times \gamma$ as well as the main effect of γ as important factors that determine the behavior of the bias. However, the proportions of variance explained by these terms are not so high; 0.193 for $\text{Model} \times \gamma$, 0.152 for γ , and 0.109 for $T \times \gamma$.

Empirical SE Figures D.201–D.204 display the empirical SE of each model. As shown in Figure D.201, the empirical SE of MLM-E2S is much smaller than the other models. The number of measurements T increases the empirical SE of MLM-E2S, while the sample size N decreases the SE. The other analytic models show a decreasing pattern in which the SE becomes very large when $T = 3$. In particular, the empirical SE of LCMx-CLC can be higher than 20, which is extremely large for the true value of the target parameter ($\beta_{SS} = 0.6$).

ANOVA detects such heterogeneous effect of T by Model (i.e., the interaction of T with Model). The interaction between Model and T can solely explain 32.1% of the total variation in the empirical SE. This percentage is the highest of the three terms that have an eta-square larger than 0.1 (0.226 for Model and 0.204 for T).

Theoretical SE and SE Ratio Figures D.205–D.208 present the mean theoretical SE of each model. The patterns of the theoretical SE are similar to those of the empirical SE—MLM-E2S shows an increasing pattern in T , while the other models show a decreasing one. One difference between the two types of SE is their levels; all models but MLM-E2S overestimate the SE. This overestimation is severe particularly for LCMx-CLC, of which the SE ratio exceeds 20 in the worst case (see Figure D.212). The overestimation of SE is most likely to happen when $T = 3$.

Because of the overestimation, the interaction between Model and T has the largest

eta-square of $\eta^2 = 0.502$ in ANOVA of the SE ratio. This is followed by the main effect of Model, which has $\eta^2 = 0.285$. In the end, these two terms account for 78.7% of the variation in the SE ratio.

RMSE Figures D.213 and D.214 show RMSE of LCMxy-RP and LCMx-CLC, respectively. These models are individually plotted since their RMSE is very large. As shown in the figures, both models have a decreasing pattern in T . That is, the RMSE can be extremely high when $T = 3$, but it sharply drops as T grows.

Figures D.215–D.217 compare the RMSE of MLM-E2S and MLM-MV. These models are basically comparable, but under some conditions, they show clear differences. For example, when $\gamma = -0.6$ (i.e., Figure D.215), MLM-E2S tends to have a larger RMSE than MLM-MV, while MLM-MV can have much higher RMSE when $(\gamma, \beta_{SS}) = (0, 0.6)$ and $(0.6, 0.6)$. Note that MLM-MV takes similar RMSE values across the conditions, thus the model differences between MLM-E2S and MLM-MV are attributable to the changes of MLM-E2S.

As with the empirical and theoretical SE, ANOVA indicates that most of the variation in RMSE can be explained by the main effects of Model and T as well as the interaction between them. Their eta-squares of these terms are 0.321 (Model $\times$ γ), 0.226 (Model), and 0.204 (T).

Coverage Rate Figures D.218–D.221 depict the coverage rate of each model. The figures clearly show the similarity of MLM-MV, LCMxy-RP, and LCMx-CLC. Their coverage rate is deviated from 0.95 when $T = 3$, but it converges to 0.95 as T increases. The direction of the deviation seems dependent on the combination of γ and β_{SS} . That is, when $\beta_{SS} = 0$ (and $T = 3$), then the coverage rate is too high if $\gamma = 0$ and too low if $\gamma \neq 0$. In contrast, when $\beta_{SS} = 0.6$, then the coverage rate is too high if $\gamma = 0.6$ and too low if

$\gamma = -0.6$ or 0 . In other words, when $\gamma = \beta_{SS}$, the coverage rate tends to be too high and when $\gamma \neq \beta_{SS}$, it is likely to be too low. However, this deviation from the proper rate disappears as we increase the number of measurements.

Figure D.218 shows the coverage of MLM-E2S. Obviously, MLM-E2S exhibits a different pattern from the other three models. This model takes one of two extreme values, namely either 0.95 or zero. It seems that which of the two values MLM-E2S takes depends on the true value of γ and β_{SS} . More specifically, MLM-E2S takes the coverage rate of 0.95 when $\gamma = \beta_{SS}$ and 0 when $\gamma \neq \beta_{SS}$. Note that the coverage rate becomes less extreme when T is large.

ANOVA supports these observations. ANOVA tells us that 59.4% of the variation in the coverage rate can be explained by the main effect of Model and 19.5% by the interaction $\text{Model} \times \gamma \times \beta_{SS}$. Also, the interaction between γ and β_{SS} is marginally influential ($\eta^2 = 0.090$).

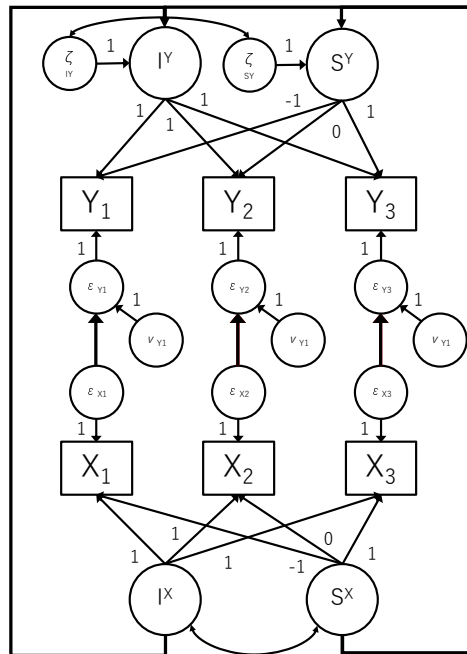
Power Figures D.222–D.225 display the power to detect nonzero β_{SS} . While MLM-MV, LCMxy-RP, and LCMx-CLC show a simple pattern in which the power is increasing in T and N , MLM-E2S exhibits a more complex pattern (Figure D.222). First, the nonzero β_{SS} is always detected when $\gamma = 0.6$. In the other cases (i.e., $\gamma = 0$ and -0.6), the power is dependent on T and N . When $\gamma = 0$, the power is simply increasing in T and N . However, when $\gamma = -0.6$, MLM-E2S shows a decreasing pattern, and under some conditions, it also shows a small “rebound” of the power when T increases from 5 to 7.

In ANOVA, the main effect of T is identified as the most influential factor with $\eta^2 = 0.509$. This is followed by the interaction between Model and T , which has $\eta^2 = 0.144$. Other factors such as the main effects of Model and N do not have the eta-square larger than 0.1 and remain marginal.

Summary In sum, there are two types of models regarding the estimation of β_{SS} . The first type of models, MLM-MV, LCMxy-RP, and LCMx-CLC, produce unbiased estimates with relatively large and possibly extreme standard error. The second type (i.e., MLM-E2S) produces substantially biased point estimates with a small standard error. MLM-E2S can yield unbiased estimates, but the unbiased estimation is possible only when $\gamma = \beta_{SS}$. As for the inference properties, the first type of models performed well unless $T = 3$, while MLM-E2S performed well only when $\gamma = 0$.

5.5 Simulation 5

In Simulation 5, data sets are generated from the same model as Simulation 4 (Figure 5.5). However, unlike Simulation 4, where the within-person and between-person effect take different values across the conditions, Simulation 5 fixes these effect parameters at a certain level and changes the levels of variance components of the growth factors for the time-varying predictors (i.e., ψ_{II}^X , ψ_{SS}^X , and ρ_{IS}^X). Specifically, the variance of I^X takes $\psi_{II}^X = \{0, 16, 64\}$, the variance of S^X takes $\psi_{SS}^X = \{0, 1, 4\}$, and the correlation between I^X and S^X takes $\rho_{IS}^X = \{0, 0.3, 0.6\}$, while the within-person and between-person effects are set at $(\gamma, \beta_{II}, \beta_{SI}, \beta_{IS}, \beta_{SS}) = (0.6, 0.6, 0.3, 0.3, 0.6)$. Thus, the results in Simulation 4 with $(\gamma, \beta_{II}, \beta_{SI}, \beta_{IS}, \beta_{SS}) = (0.6, 0.6, 0.3, 0.3, 0.6)$ correspond to those in Simulation 5 with $(\psi_{II}^X, \psi_{SS}^X, \rho_{IS}^X) = (16, 1, 0.3)$. As such, Simulation 5 has 324 conditions. A brief summary regarding the bias and SE is presented in Table 5.5. Figures from Simulation 5 are compiled in Appendix E. Note that there exists a linear trend behind the time-varying predictors even when a variance is zero. Zero variance means that the intercept and/or slope are identical for all individuals (i.e., there is no individual differences in the growth curves).

Figure 5.5*Data-Generating Model in Simulation 5*

5.5.1 Auxiliary Indices

Convergence Figure E.1 shows the proportion of converged cases among 1,000 replications. As shown in the figure, LCM_Y-R_{I_X}, LCM_{XY}-R_P, and LCM_X-R_{I_Y} experience nonconvergence. LCM_Y-R_{I_X} has its nonconverged cases only when $\psi_{II}^X = 16$ and the proportion appears to be a decreasing function of T . On the other hand, LCM_Y-R_{I_X} and LCM_{XY}-R_P have their nonconverged cases across the conditions. While LCM_{XY}-R_P basically shows an increasing pattern in T , LCM_X-R_{I_X} exhibits a decreasing pattern when $\psi_{II}^X \neq 0$ and $\psi_{SS}^X \neq 0$.

Problems in SE Estimation Figure E.2 presents the proportion of cases with problems in SE estimation. In addition to the models with nonconvergence, LCM_X-CLC also has a small number of cases where SE is not returned by software. These cases are very rare (440

Table 5.5*Marginal Means of Absolute Bias (Empirical SE) in Simulation 5*

Model	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}
MLM-WB	3.642 (0.063)	0.011 (0.040)	—	—	—
MLM-E2S	0.001 (0.020)	0.006 (0.040)	0.124 (0.024)	0.202 (0.079)	0.009 (0.047)
MLM-TAC	0.002 (0.017)	0.011 (0.040)	0.123 (0.024)	—	—
MLM-MV	0.001 (0.020)	0.054 (1.241)	0.103 (0.638)	0.131 (1.583)	0.067 (0.881)
LCMy-RIx	0.002 (0.017)	0.042 (0.127)	0.552 (0.581)	—	—
LCMy-CLC	0.001 (0.020)	0.011 (0.040)	0.123 (0.024)	—	—
LCMxy-RP	0.001 (0.020)	0.125 (0.515)	0.082 (0.306)	0.184 (1.236)	0.099 (0.779)
LCMx-RIy	0.494 (0.033)	0.513 (0.036)	—	0.804 (0.283)	—
LCMx-CLC	0.001 (0.020)	0.089 (4.018)	0.117 (2.543)	0.188 (6.231)	0.122 (4.422)

cases out of 326,000; 0.14%) and most of them occur when either ψ_{II}^X or ψ_{SS}^X is zero. For the other models with nonconvergence, no SE cases mostly coincide with the nonconverged cases. For example, LCMxy-RP has 15,066 cases (4.7%) where its estimation converged but no SE estimate was obtained. And this is the highest percentage of converged cases with no SE. Therefore, we can consider that Figure E.2 contains almost the same information as Figure E.1.

Nonpositive-Definite Model-Implied Covariance Matrix Figure E.3 depicts the proportion of cases that produced a nonpositive-definitive (NPD) model-implied covariance matrix, which may imply a negative variance estimate. The figure indicates that MLM-WB always results in an NPD covariance matrix whereas MLM-MV never had it. The other models lie in between MLM-WB and MLM-MV. While MLM-E2S, MLM-TAC, and LCMy-CLC mostly keep a low NPD rate, LCMy-RIx, LCMxy-RP, LCMx-RIy, and LCMx-CLC take a range of rates across the conditions, which may be increasing or decreasing in T , depending on the condition.

Goodness-of-Fit Figures E.4–E.7 are plots showing goodness-of-fit indices. The plots constantly indicate that LCMy-CLC, LCMxy-RP, and LCMx-CLC fit data sufficiently well and that LCMx-RIy is the worst model, followed by LCMy-RIx. We can also find that increasing N or T does not necessarily improve the model fit, but rather it can deteriorate the fit. Note that LCMx-RIy does not provide a goodness-of-fit index under some conditions because it never converged in these cases.

5.5.2 Within-Person Effect

Bias Figures E.8–E.16 plot the bias in $\hat{\gamma}$ under each condition. The figures show that MLM-WB and LCMx-RIy produce a substantial amount of bias across the conditions. In particular, the bias of MLM-WB is extremely large and it increases along with T . Also, some other models exhibit systematic patterns. For example, the bias of MLM-TAC and LCMy-RIx is affected by T and ρ_{IS}^X when $\psi_{II}^X \neq 0$ and $\psi_{SS}^X \neq 0$. MLM-MV also seems influenced by T , but the absolute size of bias is small. Accordingly, Model is a dominant factor in ANOVA with all models, which accounts for 97.5% of the variation in the bias of $\hat{\gamma}$. It is followed by the interaction of Model with T , but the eta-square is only 0.02. When MLM-WB and LCMx-RIy are excluded from ANOVA, only the interaction between Model and T has an eta-square larger than 0.1 ($\eta^2 = 0.115$).

Empirical SE Figure E.17 displays the empirical SE of each model across the conditions. We can see two notable points in this figure. First, all models decrease the SE as N or T increases. Second, the rank order among the models is stable. That is, MLM-WB is the model with the largest SE, followed by LCMx-RIy. Furthermore, the other seven models stay at similar levels of SE over the conditions. ANOVA supports these observations by showing the significant main effects of Model, N , and T and by showing no

large eta-square for interaction terms relevant to Model⁵. The eta-squares of these three main effects are 0.452 (Model), 0.296 (N), and 0.101 (T), meaning that these three terms can explain 84.9% of SST in the empirical SE.

Theoretical SE and SE Ratio Theoretical SE shown in Figure E.18 exhibits the same pattern as the empirical SE. In fact, a set of terms with large eta-square is identical (i.e., Model, N , and T) and their size is very close to each other ($\eta^2 = 0.452$ for Model; $\eta^2 = 0.296$ for N ; $\eta^2 = 0.101$ for T).

The ratio of the theoretical SE over the empirical SE is approximately one for all models, as shown in Figures E.19–E.27. There seems no pattern except for MLM-WB; the ratio of MLM-WB appears to be a decreasing function of T . Also, LCMx-RIy has some outliers when $\psi_{II}^X \neq 0$, $\psi_{SS}^X \neq 0$, and $T = 7$. As a result, Model $\times$ T is the sole term with eta-square larger than 0.1 ($\eta^2 = 0.127$).

RMSE Figures E.28 and E.29 show RMSE of MLM-WB and LCMx-RIy, respectively, and Figure E.30 shows RMSE of the remaining models. The first two graphs indicate that MLM-WB and LCMx-RIy have different patterns from the other models in that MLM-WB exhibits an increasing pattern in T , while LCMx-RIy shows a flat pattern. They are also different in their size of RMSE. The overall average RMSE of MLM-WB is much larger than that of LCMx-RIy (3.64 for MLM-WB and 0.459 for LCMx-RIy), which is in turn larger than the average of the other models (around 0.02). The third graph displays the remaining models in the same subplot so that we can compare the models easily. In Figure E.30, we may see small model differences when $T = 3$; however, they are not substantial.

ANOVA with all models finds that Model is the only important term that can explain much variation in RMSE. The eta-squares show that the main effect of Model is

⁵An interaction between Model and T is marginally influential with $\eta^2 = 0.090$.

dominant, which explains more than 95% of SST in RMSE ($\eta^2 = 0.974$). In contrast, ANOVA excluding MLM-WB and LCMx-RIy reduces the total variability by 99.99%, and the main effects of N and T are the only terms that has a large eta-square ($\eta^2 = 0.582$ for N and $\eta^2 = 0.320$ for T).

Coverage Rate Figure E.31 shows the coverage rate across the conditions. The figure clearly shows that the confidence intervals of MLM-WB and LCMx-RIy cannot capture the true value of γ . This is mainly because of the large bias we saw in Figure E.8 and E.15. Also, Figure E.31 indicates that the other models can capture the true value at the proper rate with some exceptions. These exceptions occur when $\psi_{II}^X = 16$ and $\psi_{SS}^X \neq 0$ for MLM-TAC and LCMy-RIx. Under this setting, the coverage rate drops below 0.90 especially when $\rho_{IS}^X = 0.6$. ANOVA is heavily affected by the large RMSE of MLM-WB and LCMx-RIy, resulting in the main effect of Model with an extreme eta-square higher than 0.99 ($\eta^2 = 0.993$). However, once MLM-WB and LCMx-RIy are excluded from the analysis, there is no term with an eta-square larger than 0.1, implying that all models, on average, achieved the proper coverage rate across the conditions.

Power Figure E.32 presents the power across the conditions. The result is obvious; all models achieved the perfect detection of nonzero within-person effect under all conditions. Since there is no variation in the power, ANOVA was not conducted.

Summary We observe model differences in all evaluation indices other than power. These differences can be attributed to the large bias of MLM-WB and LCMx-RIy. In fact, when the two models are excluded from the analyses, the model differences become less influential.

5.5.3 Between-Person Effect of Intercept on Intercept

Bias Figures E.33–E.41 are plots that individually show the bias of $\hat{\beta}_{II}$. As found in the figures, there are some types of models that have different patterns of bias. First, MLM-WB, MLM-E2S, MLM-TAC, and LCMMy-CLC show small positive bias when $\psi_{II}^X \neq 0$, $\psi_{SS}^X \neq 0$, and $\rho_{IS}^X \neq 0$. When these models produce bias, its magnitude appears to be an increasing function of ρ_{IS}^X . Second, the estimates based on MLM-MV, LCMxy-RP, and LCMx-CLC are moderately biased when $\psi_{II}^X = 0$. The bias size seems to increase in T for MLM-MV and LCMxy-RP, but it may not be the case for LCMx-CLC. Third, LCMMy-RIx produces biased estimates when $\psi_{II}^X = 16$ but not in the other cases. Although there seems no pattern in the bias when $\psi_{SS}^X = 0$, the bias size tends to become large along with ρ_{IS}^X when $\psi_{SS}^X \neq 0$. In particular, LCMMy-RIx can produce unbiased estimates when $\rho_{IS}^X = 0$. Finally, LCMx-RIy always produces biased estimates whatever value ψ_{II}^X , ψ_{SS}^X , and ρ_{IS}^X take—it produces bias of which the magnitude is constantly around 0.5.

Including all models, the main effect of Model is identified as an important factor by ANOVA with an eta-square of 0.676. This is followed by the interaction between Model and ψ_{II}^X with an eta-square of 0.083. Therefore, the main effect of Model is the sole influential factor from the viewpoint of eta-square.

Empirical SE Figure E.42 shows the empirical SE of four models (i.e., MLM-WB, MLM-E2S, MLM-TAC, and LCMMy-CLC) in the same subplots. As we can see in the figure, the empirical SE of these models is almost identical. It is mainly affected by N and ψ_{II}^X , though it is also slightly influenced by T when $\psi_{II}^X = 0$.

Figures E.43–E.47 individually show the empirical SE for MLM-MV, LCMMy-RIx, LCMxy-RP, LCMx-RIy, and LCMx-CLC. These models are displayed separately from those in Figure E.42 because they take much larger values or exhibit a different pattern. Obviously, those five models present a large SE when they produce biased estimates. For

example, MLM-MV, LCMxy-RP, and LCMx-CLC take large empirical SE only when $\psi_{II}^X = 0$, and LCMy-RIx changes its pattern only when $\psi_{II}^X = 16$. Also, LCMx-RIy shows the empirical SE of which the size is similar to the models in Figure E.42, but its pattern is decreasing in T .

As a result, ANOVA including all models reveals that the interaction term $\text{Model} \times \psi_{II}^X$ and the main effects of Model and ψ_{II}^X can account for most variation in the empirical SE. More specifically, 48.5% of the variation is explained by $\text{Model} \times \psi_{II}^X$, 27.7% by Model, and 12.7% by ψ_{II}^X . Accordingly, these three terms explain 88.9% of SST .

Theoretical SE and SE Ratio Results regarding the theoretical SE are presented in the same manner as the empirical SE. Figure E.48 collectively shows the theoretical SE of MLM-WB, MLM-E2S, MLM-TAC, and LCMy-CLC. This figure shows the overestimation of the standard error by MLM-WB. That is, MLM-WB deviates from the other models in all panels although they are close to each other in the figure for the empirical SE (i.e., Figure E.42). This overestimation can also be confirmed in the plots for the ratio of the SEs. For example, Figure E.54 presents the SE ratio of MLM-WB. The figure shows ratios higher than one for MLM-WB, implying that it overestimates SE. In contrast, MLM-E2S, MLM-TAC, and LCMy-CLC have ratios that are, on average, one (see Figures E.55, E.56, and E.59). This means that these three models can estimate the standard error correctly across the conditions.

The theoretical SE of MLM-MV, LCMy-RIx, LCMxy-RP, LCMx-RIy, and LCMx-CLC are individually provided in Figures E.49 through E.53. Figures E.57, E.58, E.60, E.61, and E.62 are the corresponding graphs of the SE ratio. These figures indicate that most of the five models overestimate the standard error. In particular, those which can when $T = 3$ and when either ψ_{II}^X or ψ_{SS}^X is zero. The most severe overestimation can be found in Figure E.62 (i.e., LCMx-CLC). In this worst case, the size of overestimation

reaches 15,000%. On the other hand, the overestimation of LCMy-RIx is more moderate, which is at most 200%. Therefore, we can say that the magnitude of the overestimation highly varies across the models.

Overall, the main effect of Model and the interaction between Model and ψ_{SS}^X are indicated as important factors by ANOVA. Their eta-squares are 0.321 for Model and 0.134 for Model $\times$ ψ_{SS}^X .

RMSE RMSE for $\hat{\beta}_{II}$ is presented in the same way as the empirical and theoretical SE, that is, collectively (Figure E.63) and individually (Figures E.64–E.68). Figure E.63 plots the RMSE of MLM-WB, MLM-E2S, MLM-TAC, and LCMy-CLC in the same subplots. We can see in this plot that MLM-E2S has a lower RMSE than the other three models under some conditions. In particular, the nonzero variance components seem to amplify the gap between MLM-E2S and the other three models. In contrast, the model difference is unlikely to occur when either ψ_{II}^X , ψ_{SS}^X , or ρ_{IS}^X is zero.

Figures E.64–E.68 show that MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC can have estimates with larger estimation errors. For most of these models, such large estimation error occurs under specific conditions (e.g., $\psi_{II}^X = 0$); however, the estimates of LCMx-RIy constantly contain a moderate or large estimation error (Figure E.67).

ANOVA mainly reflects the results from the latter group of models; thus, the interaction between Model and ψ_{II}^X and the main effects of Model and ψ_{II}^X are found influential. These three terms can explain 49.0% (Model $\times$ ψ_{II}^X), 27.0% (Model), and 12.8% (ψ_{II}^X) of the total variation in the RMSE.

Coverage Rate Figures E.69–E.77 show the coverage rate of each model. We can find some patterns in these figures. First, MLM-E2S, MLM-TAC, LCMy-RIx, and LCMy-CLC achieve the proper coverage rate of 0.95 when $\psi_{II}^X = 0$ or $\psi_{SS}^X = 0$. When both ψ_{II}^X and ψ_{SS}^X

are not zero, these models require ρ_{IS}^X to be zero so that they can have the appropriate level of coverage rate. If ρ_{IS}^X is not zero, then the coverage rate diverts from 0.95.

Second, in contrast to the models mentioned above, MLM-MV, LCMxy-RP, and LCMx-CLC require that both ψ_{II}^X and ψ_{SS}^X are not zero so that they can have the coverage rate of 0.95. Otherwise, their coverage rate becomes higher than it should be. Note that the size of the correlation between growth factors for time-varying predictors seems not to affect the coverage rate for these three models.

Finally, MLM-WB and LCMx-RIy take extreme coverage rates, that is, namely one or zero. In other words, a confidence interval of MLM-WB almost always captures the true value, while that of LCMx-RIy never does. These extreme coverage rates may be due to the overestimation of SE (for MLM-WB) or the constant bias across the conditions (for LCMx-RIy).

The model differences are so obvious that the main effect of Model is identified as the only influential factor by ANOVA. In fact, it can solely explain 83.0% of the variation in the coverage rate.

Power Figures E.78–E.86 present the statistical power to detect nonzero β_{II} . These figures indicate that (a) MLM-E2S, MLM-TAC, LCMMy-CLC, and LCMx-RIy achieve approximately perfect detection (i.e., a power higher than 0.99) across the conditions, (b) MLM-MV, LCMxy-RP, and LCMx-CLC need a nonzero variance of the intercept factor (ψ_{II}^X) to obtain high power, and (c) MLM-WB and LCMMy-RIx lose substantial amount of power as T increases for a fixed N . As a result, ANOVA detected the interaction between Model and ψ_{II}^X as well as the main effects of Model and ψ_{II}^X as influential factors with eta-squares of 0.428 (Model $\times$ ψ_{II}^X), 0.247 (Model), and 0.222 (ψ_{II}^X).

Summary To summarize, we can classify the nine analytic models into three types of models based on the results from this subsection. The first group (MLM-WB, MLM-E2S, MLM-TAC, LCMMy-CLC) can unbiasedly estimate β_{II} with a small SE when one of the three variance components is zero, but once the population model has variation in growth curves, then the estimates from the first group of models become biased. The second group (MLM-MV, LCMxy-RP, and LCMx-CLC) needs nonzero variance of the intercept factor to obtain unbiased estimates. If the intercept factor has no variation, then the models in the second group cannot produce an unbiased estimate and suffer from a large SE. Models in the third group (LCMMy-RIx and LCMx-RIy) have a unique pattern in the bias and standard error. While LCMMy-RIx seems to perform well unless $\psi_{II}^X = 16$, LCMx-RIy suffers from the bias that constantly appears across the conditions, which leads to its zero coverage rate.

5.5.4 Between-Person Effect of Intercept on Slope

Since MLM-WB and LCMx-RIy do not have a term corresponding to β_{SI} , the results reported in the following only address the other seven analytic models (i.e., MLM-E2S, MLM-TAC, MLM-MV, LCMMy-RIx, LCMMy-CLC, LCMxy-RP, and LCMx-CLC).

Bias Figures E.87–E.93 show the bias of each model. Overall, ψ_{II}^X strongly affects the pattern of bias. In particular, all models have the bias around -0.3 , on average, when $\psi_{II}^X = 0$. The bias becomes small as ψ_{II}^X becomes large for MLM-E2S, MLM-TAC, and LCMMy-CLC, while the bias becomes approximately zero for MLM-MV, LCMxy-RP, and LCMx-CLC when $\psi_{II}^X \neq 0$. Note that there seems no effect of ρ_{IS}^X .

ANOVA finds that ψ_{II}^X , $\text{Model} \times \psi_{II}^X$, and $\text{Model} \times T \times \psi_{II}^X \times \psi_{SS}^X$ are important. They have eta-squares of 0.199 (ψ_{II}^X), 0.138 ($\text{Model} \times \psi_{II}^X$), 0.126 ($\text{Model} \times T \times \psi_{II}^X \times \psi_{SS}^X$). Also, $\text{Model} \times T \times \psi_{II}^X$ and Model are identified as marginally important factors, of which

eta-squares are $\eta^2 = 0.094$ (Model $\times T \times \psi_{II}^X$) and $\eta^2 = 0.090$ (Model).

Empirical SE Figure E.94 collectively shows the empirical SE of MLM-E2S, MLM-TAC, and LCM_y-CLC. As this figure shows, the three models take almost identical SE values across the conditions. The SE seems to be a decreasing function of T and N and also weakly dependent on ψ_{II}^X . The empirical SE of the remaining models (MLM-MV, LCM_y-RI_x, LCM_{xy}-RP, and LCM_x-CLC) are individually provided in Figures E.95–E.98. These models appear influenced more strongly by ψ_{II}^X . For LCM_y-RI_x, the SE becomes very large when $\psi_{II}^X = 16$, while MLM-MV, LCM_{xy}-RP, and LCM_x-CLC have large SE when $\psi_{II}^X = 0$. This impact of ψ_{II}^X seems much greater than T and N .

The result of ANOVA supports these findings. The interaction between Model and ψ_{II}^X has the largest eta-square ($\eta^2 = 0.481$), followed by the main effects of Model ($\eta^2 = 0.248$) and ψ_{II}^X ($\eta^2 = 0.127$). These three terms can together explain 85.5% of the total variation in the empirical SE of $\hat{\beta}_{SI}$.

Theoretical SE and SE Ratio Figures E.99–E.103 present the theoretical SE of each model. The results are provided in the same way as the empirical SE so as to compare the two types of SE. The overall pattern of the theoretical SE is akin to the empirical SE. That is, the theoretical SE is considerably influenced by the size of ψ_{II}^X . However, some models have a theoretical SE that is higher than the empirical SE. This overestimation can be confirmed by the ratio of the theoretical SE over the empirical SE, which is shown in Figures E.104–E.110. According to the figures, MLM-MV, LCM_{xy}-RP, and LCM_x-CLC have an SE ratio larger than one, implying that the theoretical SE gets larger than the empirical SE. This happens when either ψ_{II}^X or ψ_{SS}^X is zero or when both ψ_{II}^X and ψ_{SS}^X are nonzero and T is small. LCM_y-RI_x can also have an SE ratio larger than one. But unlike MLM-MV, LCM_{xy}-RP, and LCM_x-CLC, it has a large theoretical SE only when $\psi_{II}^X = 16$.

ANOVA indicates that the pattern of the SE ratio is more complex. In addition to the main effect of Model ($\eta^2 = 0.251$) and the interaction between Model and ψ_{SS}^X ($\eta^2 = 0.117$), ANOVA finds important the highest order interaction, that is, the interaction between all factors ($\text{Model} \times N \times T \times \psi_{II}^X \times \psi_{SS}^X \times \rho_{IS}^X$; $\eta^2 = 0.101$). Also, ANOVA does not indicate ψ_{II}^X and its relevant interactions other than the highest interaction as an important factor.

RMSE Figures E.111–E.115 show RMSE in the same way as the empirical and theoretical SE. Figure E.111 jointly presents the RMSE of MLM-E2S, MLM-TAC, and LCMMy-CLC. As the figure shows, the pattern of the RMSE for these models clearly depends on ψ_{II}^X . When $\psi_{II}^X = 0$ (i.e., the first three rows), they have a constant RMSE, which is around 0.3, while the RMSE seems dependent on T when $\psi_{II}^X \neq 0$. Although the pattern looks similar when $\psi_{II}^X \neq 0$, the magnitude is larger when $\psi_{II}^X = 16$ than when $\psi_{II}^X = 64$.

The RMSE of the remaining models (i.e., MLM-MV, LCMMy-RIx, LCMxy-RP, and LCMx-CLC) are separately presented in Figures E.112–E.115. For these models, ψ_{II}^X seems a deterministic factor of their RMSE. When $\psi_{II}^X = 0$, MLM-MV, LCMxy-RP, and LCMx-CLC tend to have a high RMSE, but when $\psi_{II}^X = 16$ and 64, their RMSE is very small. LCMMy-RIx, on the other hand, have a large RMSE when $\psi_{II}^X = 16$; otherwise, the RMSE is quite small.

The result of ANOVA reflects these facts, identifying the interaction between Model and ψ_{II}^X and the main effects of Model and ψ_{II}^X are important factors. These three terms have the eta-square of 0.474 ($\text{Model} \times \psi_{II}^X$), 0.231 (Model), and 0.151 (ψ_{II}^X). As a result, the three terms account for 85.6% of the total variation in the RMSE.

Coverage Rate As Figure E.116 shows, the coverage rate of MLM-E2S, MLM-TAC, and LCMMy-CLC depend mainly on ψ_{II}^X , T , and N . Specifically, when $\psi_{II}^X = 0$, the confidence

intervals of these three models almost never capture the true value of β_{SI} . On the other hand, the average level of coverage rate seems higher when $\psi_{II}^X = 64$ than when $\psi_{II}^X = 16$. Also, within a condition defined by ψ_{II}^X and ψ_{SS}^X , the coverage rate becomes higher as T increases and lower as N increases.

Figure E.117 shows the coverage rate of MLM-MV, LCMxy-RP, and LCMx-CLC, and Figure E.118 individually shows the coverage rate of LCMy-RIx. Among these models, MLM-MV, LCMxy-RP, and LCMx-CLC have a similar pattern, where the proper coverage rate of 0.95 is achieved when $\psi_{II}^X \neq 0$, $\psi_{SS}^X \neq 0$, and T is not small. When any of them is not satisfied, then the coverage rate tends to be higher than 0.95. On the other hand, LCMy-RIx has a distinctive pattern. The coverage rate tends to be low when $\psi_{II}^X \neq 16$. In particular, the rate is almost always zero when $\psi_{II}^X = 0$. In cases of $\psi_{II}^X = 16$, LCMy-RIx decreases the coverage rate as N increases. It also increases the coverage rate along with T with a fixed sample size. However, this is not the case when $\psi_{SS}^X = 4$. In this case, the coverage rate exhibits a spike when $T = 5$.

Overall, the main effect of Model and the interaction between Model and ψ_{II}^X are identified as important factors by ANOVA. These two terms have the eta-squares of 0.701 (Model) and 0.102 (Model $\times \psi_{II}^X$). In other words, these two factors can explain 80.3% of the variability in the coverage rate.

Power The statistical power to detect nonzero β_{SI} is summarized in three figures (Figures E.119–E.121). Figure E.119 is for MLM-E2S, MLM-TAC, and LCMy-CLC. The result from this figure is dichotomous. That is, the power is almost always zero when $\psi_{II}^X = 0$, while the power is approximately one when $\psi_{II}^X \neq 0$. The other experimental factors such as N and T appear to have no effect in the current settings.

Figure E.120 provides the power for MLM-MV, LCMxy-RP, and LCMx-CLC. Similar to the previous models, these three models cannot detect the nonzero β_{SI} when

$\psi_{II}^X = 0$. In contrast, these models achieve very high power when $\psi_{II}^X \neq 0$. On the other hand, when $\psi_{SS}^X = 0$, the average power becomes lower than when $\psi_{SS}^X \neq 0$, though the power is still at a moderate to high level. In other words, MLM-MV, LCMxy-RP, and LCMx-CLC need nonzero factor variances, both ψ_{II}^X and ψ_{SS}^X , to achieve high power to detect β_{SI} .

Finally, Figure E.121 presents the power of LCMMy-RIx. As with the other models, LCMMy-RIx also has very low, or almost zero power when $\psi_{II}^X = 0$. In addition, LCMMy-RIx achieved the perfect detection of β_{SI} when $\psi_{II}^X = 64$. When ψ_{II}^X is in between them, that is, when $\psi_{II}^X = 16$, the power seems a decreasing function of T . However, it exhibits a V-shaped pattern when $\psi_{SS}^X = 4$, so the power of LCMMy-RIx may not be a simple linear function of T .

As we see above, ψ_{II}^X has a great impact on the statistical power to detect β_{SI} . Actually, ψ_{II}^X is the only factor that is recognized as influential by ANOVA. ψ_{II}^X can solely explain 91.5% of the variability in the power, and moreover, the term with the second largest eta-square still includes ψ_{II}^X , though its eta-square is very small ($\eta^2 = 0.023$ for Model $\times \psi_{II}^X$).

Summary The seven analytic models can be categorized into three types of models based on the results presented in this subsection. In terms of bias, all models but LCMMy-RIx need nonzero ψ_{II}^X for unbiased estimation. In terms of SE, the first model group (i.e., MLM-E2S, MLM-TAC, and LCMMy-CLC) does not necessarily require nonzero ψ_{II}^X , but larger ψ_{II}^X can contribute to obtaining a small SE. In contrast, the second type of models (MLM-MV, LCMxy-RP, and LCMx-CLC) does need nonzero ψ_{II}^X ; otherwise, the SE, especially the theoretical SE, can take an extremely high value. This difference between the model groups in how ψ_{II}^X affects SE leads to the differences in the inferential properties (i.e., the coverage rate and power).

5.5.5 Between-Person Effect of Slope on Intercept

The following results are reported only for MLM-E2S, MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC since only these models have a term that corresponds to β_{IS} .

Bias Figures E.122–E.126 are plots in which the bias in $\hat{\beta}_{IS}$ is shown by each model. MLM-E2S shown in Figure E.122 underestimates β_{IS} across the conditions. When $\psi_{II}^X = 0$, it results in the bias of -0.3 regardless of the other settings. On the other hand, the bias appears to be an increasing function of T when $\psi_{II}^X \neq 0$ (i.e., the size of bias becomes small along with T). Interestingly, the average bias size with $\psi_{SS}^X = 4$ is smaller than that with $\psi_{SS}^X = 1$.

LCMx-RIy also produces biased estimates across the conditions. Unlike MLM-E2S, the bias of LCMx-RIy seems not affected by T or N , but by ψ_{II}^X and ψ_{SS}^X . ψ_{II}^X increases the level of bias, not the absolute magnitude, while ψ_{SS}^X decreases it. As a result, the bias of LCMx-RIy can be both positive and negative. This pattern contrasts with MLM-E2S, which has a negative bias across the conditions. Also, the bias size of LCMx-RIy is larger than that of MLM-E2S and the other models.

MLM-MV, LCMxy-RP, and LCMx-CLC are mainly affected by ψ_{SS}^X . When $\psi_{SS}^X = 0$ (i.e., in the first row of a graph), these models have biased estimates. For MLM-MV and LCMx-CLC, the sign of bias is negative, but for LCMxy-RP, it depends on ψ_{II}^X (i.e., positively biased when $\psi_{II}^X = 64$; otherwise negatively biased). Moreover, when $(\psi_{II}^X, \psi_{SS}^X) = (64, 0)$, then the bias seems to be an increasing function of T .

These findings can be confirmed by ANOVA if LCMx-CLC is excluded from the analysis. ANOVA without LCMx-RIy finds $\text{Mode} \times \psi_{SS}^X$, Model , and $\text{Model} \times \psi_{II}^X \times \psi_{SS}^X$ influential with eta-squares of 0.132 ($\text{Mode} \times \psi_{SS}^X$), 0.112 (Model), and 0.101 ($\text{Model} \times \psi_{II}^X \times \psi_{SS}^X$)⁶.

⁶When including LCMx-RIy to the analysis, ANOVA is strongly affected and detects $\text{Model} \times \psi_{II}^X$,

Empirical SE Figures E.127–E.131 show the empirical SE of each model. These figures can be classified into three groups in terms of the pattern of SE; increasing in T (i.e., MLM-E2S), decreasing in T (LCMx-RIy), and a mix of the two (MLM-MV, LCMxy-RP, LCMx-CLC). The mix type shows an increasing pattern when $\psi_{SS}^X = 0$, while it shows a decreasing pattern when $\psi_{SS}^X = 1$ and 4. Also, on average, the mix type tends to have a higher average SE than the other models. Note that there seems no effect of ρ_{IS}^X .

ANOVA identifies ψ_{SS}^X and its relevant terms as influential factors. It suggests that the interaction of Model with ψ_{SS}^X and the main effects of them as important terms. Their eta-squares are 0.290 for Model $\times \psi_{SS}^X$, 0.278 for Model, and 0.184 for ψ_{SS}^X . Also, the interaction Model $\times T \times \psi_{SS}^X$ has an eta-square that is slightly smaller than 0.1. (i.e., $\eta^2 = 0.099$). These four terms including Model $\times T \times \psi_{SS}^X$ account for 85.1% of SST .

Theoretical SE and SE Ratio Figures E.132–E.136 present the theoretical SE of each model. These figures show that the patterns in the theoretical SE are the same as the empirical SE, but that its magnitude tends to be larger than the empirical SE. This overestimation of SE can also be confirmed by Figures E.137–E.141, where the ratio of the theoretical SE over the empirical SE are plotted. For example, MLM-MV (Figure E.138) overestimates SE when either ψ_{II}^X or ψ_{SS}^X is zero or when $T = 3$ and $(\psi_{II}^X, \psi_{SS}^X) \neq (0, 0)$. This pattern can also be seen for LCMxy-RP (Figure E.139) and LCMx-CLC (Figure E.141). On the contrary, MLM-E2S can keep the ratio around one, meaning that it can correctly estimate the SE even when the variance parameters are zero and when T is small.

As a result, ANOVA for the theoretical SE results in the same set of influential terms with slightly different eta-squares (0.340 for Model, 0.317 for Model $\times \psi_{SS}^X$, and 0.105 for ψ_{SS}^X). Also, ANOVA for the SE ratio indicates that Model is the most influential factor with $\eta^2 = 0.431$, followed by Model $\times \psi_{II}^X$ with $\eta^2 = 0.163$.

($\eta^2 = 0.363$) Model ($\eta^2 = 0.238$), and ψ_{II}^X ($\eta^2 = 0.136$).

RMSE Figures E.142–E.146 are plots that show the RMSE of each model. In these figures, we can see a clear order of these models in terms of RMSE. MLM-E2S has the smallest average RMSE (0.224), which means that its estimate contains the least estimation error, on average. This is followed by LCMx-RIy (0.906), LCMxy-RP (1.25), and MLM-MV (1.59). LCMx-CLC is the worst model and its average RMSE is much higher than the other models (6.23). This is so because LCMx-CLC takes extreme RMSEs when $\psi_{SS}^X = 0$.

ANOVA identifies four influential terms. The interaction between Model and ψ_{SS}^X has the largest eta-square ($\eta^2 = 0.290$) and it is followed by the main effects of Model ($\eta^2 = 0.261$) and ψ_{SS}^X ($\eta^2 = 0.199$) as well as the interaction between Model, T , and ψ_{SS}^X ($\eta^2 = 0.100$).

Coverage Rate Figure E.147 jointly displays the coverage rate of MLM-MV, LCMxy-RP, and LCMx-CLC. These three models show approximately the same pattern of the coverage rate across the conditions. They mostly take similar convergence rates, though LCMxy-RP can deviate from the other two models under some conditions, for example, when $(\psi_{II}^X, \psi_{SS}^X) = (64, 4)$. The figure also shows that the coverage rate is evidently higher than 0.95 when $\psi_{II}^X = 0$ (the first three rows) and when $\psi_{SS}^X = 0$ (the first, fourth, and seventh row). In other words, these three models can achieve the proper coverage rate when both ψ_{II}^X and ψ_{SS}^X are not zero. Figure E.148 and Figure E.149 present the coverage rate of MLM-E2S and LCMx-RIy, respectively. The figures show that the coverage rate of these models is heavily influenced by T and N as well as ψ_{II}^X . In particular, LCMx-RIy almost never capture the true value of β_{IS} when $\psi_{II}^X = 64$.

Although we can see different patterns that are more than a simple model difference, ANOVA indicates the main effect of Model is the only influential factor. It can solely explain 74.0% of the variation in the coverage rate. Model $\times T$ is a marginal factor

with $\eta^2 = 0.092$, and 83.2% of SST can be explained by Model and Model $\times T$.

Power The statistical power to detect β_{IS} is summarized in Figure E.150 and Figure E.151. In Figure E.150, all models but LCMx-RIy are plotted collectively. MLM-E2S can have higher power under some conditions (e.g., when $\psi_{II}^X = 0$), but generally speaking, the four models exhibit a similar pattern of power. The figure clearly shows that these models cannot find nonzero β_{IS} when ψ_{SS}^X is zero. Also, the power tends to be low in general when fewer observations are available (i.e., when either T or N is small). In contrast to the four models, LCMx-RIy frequently rejects the null hypothesis of $H_0 : \beta_{IS} = 0$ even when $\psi_{SS}^X = 0$ (recall that LCMx-RIy produces biased estimates across the conditions). It is interesting that ψ_{II}^X rather than ψ_{SS}^X lowers the power to detect β_{IS} . In particular, when $(\psi_{II}^X, \psi_{SS}^X) = (0, 4)$, LCMx-RIy cannot achieve high power and the power in this case stays at a low level around 0.1.

As such, ANOVA indicates Model and ψ_{SS}^X are the key factors that determine the level of power. That is, the main effect of ψ_{SS}^X has the largest eta-square of 0.282, followed by the main effect of Model with $\eta^2 = 0.173$. The interaction between these two key factors is also identified as an important term, which has the eta-square of 0.118. In total, the top three terms can explain 57.3% of the variability in the statistical power.

Summary We repeatedly saw in this subsection that there are three types of models. That is, (1) MLM-E2S, (2) MLM-MV, LCMxy-RP, and LCMx-CLC, and (3) LCMx-RIy. The first group can provide an estimate β_{IS} that is biased but stable over samples, while the second group can offer an estimate that is unbiased but unstable. As far as this simulation considers, it seems that ψ_{SS}^X has a great impact on the properties of estimation and inference regarding β_{IS} . In particular, all models may have some problems in the parameter estimation and/or the statistical inference when $\psi_{SS}^X = 0$.

5.5.6 Between-Person Effect of Slope on Slope

Since only MLM-E2S, MLM-MV, LCMxy-RP, and LCM-CLC can estimate β_{SS} , the following reports are limited to those four models.

Bias Figures E.152–E.155 are plots that show the bias in $\hat{\beta}_{SS}$. In these figures, different models have different conditions in which the estimates are unbiased. For example, MLM-E2S can produce unbiased estimates when one of the variance components is zero (i.e., either when $\psi_{II}^X = 0$, $\psi_{SS}^X = 0$, or $\rho_{IS}^X = 0$). In contrast, the other three models yield unbiased estimates of β_{SS} when T is not small. LCMxy-RP may need another condition of $\psi_{SS}^X \neq 0$. In cases where one of the conditions is violated, MLM-MV underestimates β_{SS} , while LCMxy-RP overestimates it. LCMx-CLC can overestimate and underestimate β_{SS} ; however, it is difficult to predict the size or direction of the bias from the current result.

Although we can see such patterns in the bias, ANOVA does not support them. It actually indicates the main effect of Model as a sole influential factor. However, the eta-square of the main effect of Model is not so large ($\eta^2 = 0.104$). Therefore, one needs to be cautious when interpreting the results.

Empirical SE Figures E.156–E.159 display empirical SE of each model. The empirical SE of MLM-E2S is increasing in T and at the same time, decreasing in N (Figure E.156). The size of SE is relatively small, compared to the other models. MLM-MV also shows an increasing pattern in T ; however, this is the case only when $\psi_{SS}^X = 0$. When $\psi_{SS}^X > 0$, the pattern reverses and the magnitude becomes smaller than that with $\psi_{SS}^X = 0$. The other models also exhibit a similar pattern to MLM-MV with different magnitude. LCMxy-RP is more stable (i.e., smaller SE) than MLM-MV, and LCMx-CLC can have an extreme standard error, particularly when $\psi_{SS}^X = 0$.

As such, ANOVA finds Model and ψ_{SS}^X important. More specifically, the main effect

of Model has the largest eta-square of 0.274. This is followed by the interaction between Model and ψ_{SS}^X with $\eta^2 = 0.272$ and the main effect of ψ_{SS}^X with $\eta^2 = 0.213$.

Theoretical SE and SE Ratio The theoretical SE shows a similar pattern to the empirical SE with more extreme values (see Figures E.160–E.163). As a result, the ratio of theoretical SE over the empirical SE is higher than one for all models but MLM-E2S (Figures E.164–E.167). In the worst case (LCMx-CLC in Figure E.167), the ratio reaches 100, meaning that SE is overestimated by 10,000%. Although this is the most extreme case, the other models also overestimate SE by 200–1,000%.

Since these extreme cases happen when either ψ_{II}^X or ψ_{SS}^X is zero, ANOVA indicates that the main effect of Model and the interaction between Model and ψ_{II}^X are influential factors. In fact, they have the eta-square of 0.385 for Model and 0.150 for Model $\times$ ψ_{II}^X .

RMSE Figures E.168–E.171 present RMSE of each model. Most figures show similar patterns to SE since it is dominant in the composition of RMSE. Then, MLM-E2S is considered as the best model of the four analytical models in terms of the RMSE, though it can produce larger RMSE than the other models under certain conditions. Although MLM-E2S show an increasing or flat pattern across the conditions, MLM-MV, LCMxy-RP, and LCMx-CLC have a mixed pattern. Specifically, the RMSE is increasing in T when $\psi_{SS}^X = 0$. In contrast, the models show a decreasing pattern in T when $\psi_{SS}^X > 0$. In addition, the RMSE seems to decrease as ψ_{SS}^X becomes large.

ANOVA for the RMSE results in the same set of important factors as empirical and theoretical SE. The main effect of Model is the most influential factor with $\eta^2 = 0.273$, followed by Model $\times$ ψ_{SS}^X ($\eta^2 = 0.272$) and ψ_{SS}^X ($\eta^2 = 0.213$).

Coverage Rate Figure E.172 summarizes all models' coverage rates in a single plot. The figure clearly shows that (a) when ψ_{II}^X or ψ_{SS}^X is zero, MLM-MV, LCMxy-RP, and

LCMx-CLC take too high a coverage rate, while MLM-E2S still keeps the proper rate of 0.95, and (b) when both ψ_{II}^X and ψ_{SS}^X is not zero and ρ_{IS}^X is high, MLM-E2S has a decreasing pattern in T and N , while the other models can and cannot keep the proper rate, depending on the setting of $(\psi_{II}^X, \psi_{SS}^X)$. As a result, only the main effect of Model is identified as an important factor with $\eta^2 = 0.230$.

Power The power to detect β_{SS} is presented in Figure E.173. The figure clearly shows that ψ_{SS}^X has a large impact on the power of MLM-MV, LCMxy-RP, and LCMx-CLC, but not on MLM-E2S. Specifically, when $\psi_{SS}^X = 0$ (i.e., in the first, fourth, and seventh row), MLM-MV, LCMxy-RP, and LCMx-CLC have low power (less than 25%). In the other cases, the power of these models appears to be an increasing function of T and N , and it changes over the whole range of $[0, 1]$.

As such, 82.0% of the variation in the power can be explained by the main effects of ψ_{SS}^X and Model as well as the interaction $\text{Model} \times \psi_{SS}^X$. Their eta-squares are 0.435 for ψ_{SS}^X , 0.241 for Model, and 0.144 for $\text{Model} \times \psi_{SS}^X$.

Summary The results presented in this subsection repeatedly show MLM-E2S has different characteristics from MLM-MV, LCMxy-RP, LCMx-CLC regarding estimation and inference of β_{SS} . In general, MLM-E2S show better performance than the other models—smaller bias and SE, accurate estimation of SE, the proper level of coverage, and higher power to detect nonzero β_{SS} . However, under some circumstances, MLM-MV, LCMxy-RP, and LCMx-CLC can have better results than MLM-E2S for example, in terms of the coverage rate when $\psi_{II}^X \neq 0$, $\psi_{SS}^X \neq 0$, and $\rho_{IS}^X = 0.6$.

5.6 Summary of the Simulation Results

5.6.1 Bias and Empirical SE

In this chapter, the results from the five simulations are reported in detail by simulation. This section summarizes and reviews those results from a different perspective than the previous sections, that is, in terms of the average absolute bias and the average empirical SE across the simulations. Note that the magnitude of bias may be offset because the average is taken over biased and unbiased cases. In other words, when the estimates are biased, the bias can be more severe than those shown in the summary tables (Tables 5.6–5.10).

Tables 5.6–5.10 present the same information as Tables 5.1–5.5; however the current tables show the information in a model-by-simulation form for each target parameter so that we can compare the models, focusing on a single target parameter across the simulations. Table 5.6 summarizes the absolute bias and the empirical SE for the within-person effect γ . The table indicates that the models can estimate γ unbiasedly in most cases, but there are some cases in which a small to large bias is observed. For example, MLM-WB produces a moderate size of bias in Simulation 3, of which the size is around 50% of the nonzero true value ($\gamma \in \{-0.6, 0, 0.6\}$), and a large bias in Simulations 4 and 5, whose magnitude is larger than the true value. LCMx-RIy also has a moderate bias in Simulations 2 and 5, and a large bias in Simulation 4. MLM-TAC and LCMy-RIx show a small bias in Simulations 3 and 4, which amounts to 3–5% of the true value. These observations imply that underspecification (i.e., insufficient modeling) of time-varying predictors and/or outcomes can introduce a bias in estimates of the within-person effect, while overspecification does not. An exception to this is MLM-WB in Simulation 2. MLM-WB does not assume a slope factor for the outcomes, which is actually assumed in the data-generating model (i.e., underspecification). Nevertheless, MLM-WB can produce

relatively accurate estimates, though the estimates come with relatively large SE.

Table 5.7 provides a summary of the absolute bias and the empirical SE for β_{II} . In this table, we can find two model groups, one of which consists of MLM-WB, MLM-E2S, MLM-TAC, LCM_y-CLC, the other includes MLM-MV, LCM_y-RI_x, LCM_{xy}-RP, LCM_x-RI_y, and LCM_x-CLC. The former group has observed between-person predictors (e.g., observed person means, individual OLS estimates of intercept and slope) and constantly shows a moderate size of bias, which is about 15% of the true value ($\beta_{II} \in \{0, 0.6\}$). This is what is called Lüdtke's bias (Asparouhov & Muthén, 2019; Lüdtke et al., 2008) and it is a function of $\gamma - \beta_{II}$, T , and ICC (see Equation 2.34). On the other hand, the latter model group has latent between-person predictors (i.e., latent growth factors) and can produce unbiased estimates with a relatively large SE. Unlike the first group, which shows the bias regardless of the model specification and the data-generating mechanisms, the models in the second group can have an extremely large bias if they fail to appropriately model the time-varying predictors and/or outcomes (e.g., LCM_y-RI_x in Simulations 3 and 4, LCM_x-RI_y in Simulations 2 and 4).

Table 5.8 shows a summary for the between-person effect of intercept on slope β_{SI} . This parameter has a similar pattern to β_{II} . Specifically, the models with the observed between-person predictors yield slightly biased estimates across the simulations except for Simulation 1. The average bias size is around 10% of the true value ($\beta_{SI} \in \{0, 0.3\}$). The second model group with the latent between-person predictors can estimate β_{SI} almost unbiasedly, and at worst, produces estimates with a comparable size of bias to the first group if the modeling of the time-varying predictors and/or outcomes is sufficient.

For the estimation of β_{IS} summarized in Table 5.9, MLM-E2S delivers unbiased estimates when a data-generating model does not have a slope factor for the time-varying predictor (i.e., in Simulation 1s and 2), but has biased estimates for the remaining simulations. The other models exhibit the opposite pattern to MLM-E2S. Specifically,

MLM-MV, LCMxy-RP, LCMx-RIy, and LCMx-CLC produce biased estimates in Simulations 1 and 2; however, when a data-generating model assumes a slope factor for the time-varying predictors, they can unbiasedly estimate β_{IS} , though the standard error is relatively large. In particular, LCMx-CLC shows an extremely large SE in Simulations 1 and 2 (15.82 and 16.04).

In Simulations 1 and 2, where the data-generating models do not have a slope factor for the time-varying predictors, all models suffer from a substantial amount of bias in $\hat{\beta}_{SS}$ (see Table 5.10). This bias shows a peculiar pattern in which the bias size appears to be equal to the true value of the within-person effect γ . We can confirm the unique distribution of $\hat{\beta}_{SS}$ in Figure A.49 from Simulation 1 and Figure B.42 from Simulation 2, where $\hat{\beta}_{SS}$ shows a sampling distribution seemingly independent of $\hat{\gamma}$ with an expectation of $\mathbb{E}[\hat{\beta}_{SS}] = \gamma$. In addition, the models cannot estimate β_{SS} without bias in the other simulations. MLM-MV shows relatively small bias, but it is still around 10% of the true value ($\beta_{SS} \in \{0, 0.3\}$). Note that LCMx-CLC again has an extremely large average SE in Simulation 2 (10.20).

Finally, as we saw in Section 5.5, the models with observed between-person predictors estimate the between-person effects with bias when both the intercept and slope factors have a nonzero variance, while they can estimate the β 's without bias or with only a small bias when either of the growth factors has zero variance. In contrast, the models with latent between-person predictors need nonzero variance for the corresponding between-person predictor; otherwise, their estimates of β 's become biased.

Table 5.6*Mean Absolute Bias (Empirical SE) of $\gamma \in \{-0.6, 0, 0.6\}$ across Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
1.MLM-WB	0.000 (0.016)	0.006 (0.205)	0.307 (0.009)	2.823 (0.046)	3.642 (0.063)
2.MLM-E2S	0.001 (0.020)	0.000 (0.020)	0.000 (0.020)	0.000 (0.020)	0.001 (0.020)
3.MLM-TAC	0.000 (0.016)	0.000 (0.017)	0.031 (0.016)	0.017 (0.017)	0.002 (0.017)
4.MLM-MV	0.000 (0.017)	0.001 (0.019)	0.004 (0.018)	0.001 (0.019)	0.001 (0.020)
5.LCM _y -RI _x	0.000 (0.016)	0.000 (0.017)	0.031 (0.016)	0.017 (0.017)	0.002 (0.017)
6.LCM _y -CLC	0.001 (0.020)	0.000 (0.020)	0.000 (0.020)	0.000 (0.020)	0.001 (0.020)
7.LCM _{xy} -RP	0.001 (0.020)	0.001 (0.020)	0.001 (0.020)	0.001 (0.020)	0.001 (0.020)
8.LCM _x -RI _y	0.001 (0.017)	0.386 (0.606)	0.001 (0.018)	0.956 (0.474)	0.494 (0.033)
9.LCM _x -CLC	0.001 (0.020)	0.000 (0.020)	0.000 (0.020)	0.001 (0.020)	0.001 (0.020)

Table 5.7*Mean Absolute Bias (Mean Empirical SE) of $\beta_{II} \in \{0, 0.6\}$ across the Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
1.MLM-WB	0.091 (0.033)	0.091 (0.033)	0.090 (0.033)	0.090 (0.033)	0.011 (0.040)
2.MLM-E2S	0.091 (0.033)	0.091 (0.033)	0.092 (0.033)	0.093 (0.033)	0.006 (0.040)
3.MLM-TAC	0.091 (0.033)	0.091 (0.033)	0.090 (0.033)	0.090 (0.033)	0.011 (0.040)
4.MLM-MV	0.003 (0.068)	0.006 (0.098)	0.003 (0.060)	0.004 (0.064)	0.054 (1.241)
5.LCM _y -RI _x	0.002 (0.043)	0.002 (0.043)	3.287 (2.190)	3.299 (1.932)	0.042 (0.127)
6.LCM _y -CLC	0.091 (0.033)	0.091 (0.033)	0.090 (0.033)	0.090 (0.033)	0.011 (0.040)
7.LCM _{xy} -RP	0.010 (0.083)	0.010 (0.086)	0.008 (0.084)	0.007 (0.082)	0.125 (0.515)
8.LCM _x -RI _y	0.016 (0.114)	1.129 (0.415)	0.011 (0.082)	1.660 (0.848)	0.513 (0.036)
9.LCM _x -CLC	0.022 (0.702)	0.020 (0.653)	0.013 (0.433)	0.012 (0.366)	0.089 (4.018)

Table 5.8*Mean Absolute Bias (Mean Empirical SE) of $\beta_{SI} \in \{0, 0.3\}$ across the Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
2.MLM-E2S	0.000 (0.011)	0.027 (0.018)	0.015 (0.012)	0.032 (0.020)	0.124 (0.024)
3.MLM-TAC	0.000 (0.011)	0.027 (0.018)	0.023 (0.012)	0.037 (0.020)	0.123 (0.024)
4.MLM-MV	0.000 (0.015)	0.002 (0.048)	0.005 (0.020)	0.004 (0.040)	0.103 (0.638)
5.LCMy-RIx	0.000 (0.014)	0.001 (0.023)	0.184 (0.196)	1.070 (0.873)	0.552 (0.581)
6.LCMy-CLC	0.000 (0.011)	0.027 (0.018)	0.025 (0.013)	0.037 (0.020)	0.123 (0.024)
7.LCMxy-RP	0.001 (0.033)	0.002 (0.051)	0.015 (0.062)	0.012 (0.079)	0.082 (0.306)
9.LCMx-CLC	0.010 (0.326)	0.011 (0.415)	0.015 (0.406)	0.014 (0.449)	0.117 (2.543)

Table 5.9*Mean Absolute Bias (Mean Empirical SE) of $\beta_{IS} \in \{0, 0.3\}$ across the Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
2.MLM-E2S	0.003 (0.117)	0.003 (0.116)	0.103 (0.085)	0.103 (0.085)	0.202 (0.079)
4.MLM-MV	0.065 (2.552)	0.133 (4.622)	0.015 (0.396)	0.024 (0.455)	0.131 (1.583)
7.LCMxy-RP	0.130 (3.382)	0.347 (3.464)	0.059 (0.716)	0.057 (0.693)	0.184 (1.236)
8.LCMx-RIy	0.171 (6.060)	1.449 (2.411)	0.065 (0.726)	1.418 (1.286)	0.804 (0.283)
9.LCMx-CLC	0.378 (15.82)	0.409 (16.04)	0.076 (2.458)	0.075 (2.620)	0.188 (6.231)

Table 5.10*Mean Absolute Bias (Mean Empirical SE) of $\beta_{SS} \in \{0, 0.6\}$ across the Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
2.MLM-E2S	0.400 (0.030)	0.401 (0.061)	0.255 (0.027)	0.319 (0.048)	0.009 (0.047)
4.MLM-MV	0.399 (0.281)	0.386 (2.323)	0.068 (0.120)	0.048 (0.299)	0.067 (0.881)
7.LCMxy-RP	0.452 (1.075)	0.527 (1.879)	0.173 (0.628)	0.144 (0.772)	0.099 (0.779)
9.LCMx-CLC	0.459 (6.332)	0.536 (10.20)	0.099 (2.396)	0.120 (3.090)	0.122 (4.422)

5.6.2 SE Ratio, Coverage Rate, and Power

Tables 5.11–5.15 summarize the averages of SE ratio (i.e., theoretical SE divided by empirical SE), the coverage rate, and the power for a single target parameter. In Table 5.11, these three indices are presented for the within-person effect γ . In most cases, the ratio of the standard errors is around one, leading to the desired coverage rate of 0.95. An exception is MLM-WB in Simulation 2, which underestimates the standard error, thus resulting in a lower coverage rate than it should be (recall that γ is estimated without bias in this case; see Table 5.6). Another case in which the SE ratio deviates from one is MLM-MV in Simulation 1. MLM-MV overestimates the standard error by 11.8% and eventually has a coverage rate slightly higher than 0.95 ($CR = 0.966$).

The coverage rate is determined not only by the accuracy of standard error (i.e., SE ratio being close to one) but also by the accuracy of the point estimate (bias being close to zero). Hence, the models are likely to have a lower coverage rate when the point estimate is biased. For example, the average coverage rate of MLM-WB is zero in Simulations 4 and 5 since it has a large bias in these simulations (2.823 and 3.642, respectively, for $\gamma = \{-0.6, 0, 0.6\}$). Likewise, the following cases are considered to have a low coverage rate due to their bias: LCMx-RIy in Simulation 2; MLM-WB, MLM-TAC, and LCMy-RIx in Simulation 3; MLM-TAC, LCMy-RIx, and LCMx-RIy in Simulation 4; and LCMx-RIy in Simulation 5.

In spite of the systematic bias and the erroneous SE, all models but MLM-WB in Simulation 2 achieve perfect detection of nonzero within-person effect. Since MLM-WB can unbiasedly estimate γ in this case and the SE is underestimated rather than overestimated, this imperfect power may be due to the inherently large SE of MLM-WB.

The results for β_{II} in Table 5.12 can be seen from the viewpoint of the model groups that are introduced in the previous subsections. The models with observed

between-person predictors tend to have an SE ratio around one, a low coverage rate around 0.4, and a power that is approximately one. These tendencies can be considered as a result of biased point estimates and correctly estimated small SE. MLM-WB can exceptionally have a large SE ratio, a higher coverage rate, and lower power even though it has an observed between-person predictor (i.e., person mean). This is so because MLM-WB does not have a correct model for the time-varying predictor (in Simulations 3, 4, and 5) and the outcomes (in Simulations 2, 4, and 5).

In contrast, the second group of models, that is, the models with latent between-person predictors tend to have a large SE ratio, thus leading to too high coverage rate and slightly lower power. LCMMy-RIx performed well in Simulations 1 and 2, where LCMMy-RIx correctly models the time-varying predictors. However, LCMx-RIy did not perform so well in Simulation 3 as LCMMy-RIx does in Simulation 2. Note that LCMx-CLC constantly shows an extreme SE ratio across the simulation. This extreme overestimation is likely to occur in simulations where the time-varying predictor does not systematically evolve over time (i.e., when no slope factor is assumed).

Similar two-group patterns can be found for β_{SI} , β_{IS} , and β_{SS} (see Tables 5.13, 5.14, and 5.15, respectively). One difference between (β_{IS}, β_{SS}) and (β_{II}, β_{SI}) is that the power to detect β_{IS} and β_{SS} is low across the simulations, compared to those for β_{II} and β_{SI} . Specifically, the overall averages of the power for β_{IS} are 0.299, 0.355, and 0.408 in Simulations 3, 4, and 5, and those for β_{SS} are 0.707 in Simulation 4 and 0.659 in Simulation 5. These averages are smaller than those for β_{II} and β_{SI} , which are greater than 0.85 except for β_{SI} in Simulation 5. This lower power for β_{IS} and β_{SS} may imply that the true values of them may be set too close to zero for the analytic models to detect them.

Table 5.11*Average SE Ratio/Coverage Rate/Power of $\gamma \in \{-0.6, 0, 0.6\}$ across Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
1.MLM-WB	1.001/0.950/1.000	0.889/0.917/0.829	1.050/0.317/1.000	0.989/0.000/1.000	0.968/0.000/1.000
2.MLM-E2S	0.992/0.948/1.000	0.999/0.950/1.000	0.997/0.949/1.000	0.999/0.950/1.000	1.000/0.950/1.000
3.MLM-TAC	0.998/0.949/1.000	0.997/0.950/1.000	0.959/0.486/1.000	0.989/0.710/1.000	0.994/0.943/1.000
4.MLM-MV	1.118/0.966/1.000	1.016/0.949/1.000	1.058/0.952/1.000	1.004/0.947/1.000	1.005/0.947/1.000
5.LCM _y -RI _x	0.995/0.945/1.000	1.000/0.946/1.000	0.980/0.486/1.000	1.010/0.704/1.000	1.002/0.939/1.000
6.LCM _y -CLC	0.995/0.945/1.000	0.997/0.946/1.000	0.997/0.945/1.000	0.996/0.945/1.000	0.997/0.945/1.000
7.LCM _{xy} -RP	0.998/0.944/1.000	0.998/0.946/1.000	0.989/0.942/1.000	0.993/0.944/1.000	0.993/0.943/1.000
8.LCM _x -RI _y	0.995/0.943/1.000	0.978/0.277/1.000	0.999/0.943/1.000	1.062/0.166/1.000	1.023/0.000/1.000
9.LCM _x -CLC	0.978/0.940/1.000	0.980/0.941/1.000	0.980/0.941/1.000	0.980/0.941/1.000	0.981/0.941/1.000

Table 5.12*Average SE Ratio/Coverage Rate/Power of $\beta_{II} \in \{0, 0.6\}$ across Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
1.MLM-WB	0.995/0.410/1.000	4.575/0.842/0.854	0.998/0.413/1.000	2.385/0.698/0.977	3.467/1.000/0.909
2.MLM-E2S	1.000/0.412/1.000	0.997/0.412/0.999	1.002/0.411/0.999	0.998/0.410/0.999	0.997/0.916/0.999
3.MLM-TAC	0.995/0.410/1.000	0.993/0.411/0.999	0.998/0.413/1.000	0.994/0.412/1.000	0.993/0.845/0.999
4.MLM-MV	2.056/0.983/0.908	3.634/0.986/0.880	1.265/0.957/0.975	1.359/0.958/0.964	2.690/0.977/0.628
5.LCM _y -RI _x	0.993/0.948/1.000	0.992/0.947/1.000	1.763/0.688/0.540	1.819/0.706/0.559	1.086/0.892/0.889
6.LCM _y -CLC	0.996/0.408/1.000	0.993/0.409/0.999	0.998/0.410/1.000	0.994/0.410/1.000	0.993/0.841/0.999
7.LCM _{xy} -RP	2.265/0.980/0.907	2.392/0.980/0.901	1.474/0.958/0.951	1.450/0.958/0.955	1.839/0.973/0.660
8.LCM _x -RI _y	3.086/0.983/0.869	1.230/0.343/0.791	1.390/0.955/0.974	1.205/0.299/0.746	1.038/0.000/1.000
9.LCM _x -CLC	30.890/0.983/0.836	31.076/0.983/0.837	7.770/0.959/0.945	7.764/0.959/0.944	15.356/0.976/0.637

Table 5.13*Average SE Ratio/Coverage Rate/Power of $\beta_{SI} \in \{0, 0.3\}$ across Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
2.MLM-E2S	1.013/0.953/NA	1.000/0.584/1.000	1.003/0.590/NA	0.999/0.564/1.000	0.998/0.250/0.683
3.MLM-TAC	1.010/0.952/NA	0.995/0.581/1.000	0.999/0.486/NA	0.995/0.504/1.000	0.993/0.257/0.683
4.MLM-MV	2.023/0.983/NA	3.607/0.987/0.860	1.397/0.952/NA	1.428/0.953/0.924	2.732/0.974/0.615
5.LCM _y -RI _x	0.987/0.941/NA	0.994/0.945/1.000	1.564/0.778/NA	1.715/0.687/0.541	1.126/0.208/0.535
6.LCM _y -CLC	1.003/0.939/NA	0.996/0.577/1.000	0.995/0.466/NA	0.995/0.496/1.000	0.994/0.251/0.684
7.LCM _{xy} -RP	2.329/0.984/NA	2.400/0.982/0.883	1.493/0.949/NA	1.461/0.951/0.912	1.852/0.964/0.629
9.LCM _x -CLC	37.31/0.985/NA	35.24/0.985/0.817	7.320/0.952/NA	7.909/0.954/0.904	17.75/0.969/0.614

Table 5.14*Average SE Ratio/Coverage Rate/Power of $\beta_{IS} \in \{0, 0.3\}$ across Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
2.MLM-E2S	0.995/0.948/NA	1.001/0.950/NA	1.001/0.666/0.393	0.999/0.665/0.392	1.000/0.346/0.409
4.MLM-MV	2.629/0.995/NA	4.592/0.996/NA	1.376/0.966/0.262	1.477/0.965/0.264	2.859/0.978/0.286
7.LCM _{xy} -RP	2.816/1.000/NA	3.020/1.000/NA	1.553/0.967/0.260	1.544/0.966/0.258	1.860/0.971/0.310
8.LCM _x -RI _y	3.832/1.000/NA	1.699/0.628/NA	1.442/0.960/0.320	1.240/0.408/0.612	1.093/0.194/0.737
9.LCM _x -CLC	13.32/1.000/NA	13.93/1.000/NA	4.689/0.968/0.262	5.191/0.967/0.261	11.60/0.974/0.298

Table 5.15*Average SE Ratio/Coverage Rate/Power of $\beta_{SS} \in \{0, 0.6\}$ across Simulations*

Model	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5
2.MLM-E2S	1.014/0.319/NA	0.998/0.317/NA	1.001/0.317/NA	1.000/0.327/0.882	0.998/0.917/0.999
4.MLM-MV	4.036/0.915/NA	4.482/0.973/NA	1.850/0.941/NA	1.640/0.946/0.643	2.867/0.982/0.512
7.LCM _{xy} -RP	2.811/0.826/NA	2.998/0.929/NA	1.578/0.933/NA	1.554/0.938/0.656	1.906/0.981/0.583
9.LCM _x -CLC	17.85/0.861/NA	15.42/0.943/NA	4.746/0.941/NA	5.064/0.942/0.646	13.32/0.982/0.544

Chapter 6

Discussion

This dissertation reviewed a range of literature on longitudinal models with time-varying predictors and their strategies to disentangle the within-person and between-person effects (i.e., centering and detrending). Based on the literature review, a framework for model classification was developed and four SEM-based detrending methods were derived from the framework. In order to evaluate the performance of the proposed models and compare them to the existing models, a series of Monte Carlo simulations were conducted, where the population models may or may not have a slope factor unique to the time-varying predictors and/or the outcome variables. In this chapter, the findings from the simulation studies are first summarized and the answers to the Research Questions are provided. Then, after discussing other findings of the results, limitations of the current study, directions for future research, and recommendations for practitioners are discussed.

6.1 Summary of the Main Findings

The first main finding is that insufficient modeling of the time-varying predictors and/or the outcomes leads to bias and large SE. In the case of a linear trend, insufficient modeling means that an analytic model does not have a slope factor or the corresponding observed between-person variable. For example, MLM-WB only has a person mean of time-varying predictors but no slope-related variables. Eventually, it had a bias for the within-person effect in Simulations 3, 4, and 5, where the data-generating models actually have a slope

factor for the time-varying predictors and/or the outcomes. Another example is LCMMy-RIx and LCMx-RIy. LCMMy-RIx has no slope factor for X and failed to unbiasedly estimate the parameters in Simulations 3, while LCMx-RIy has no slope factor for Y and produced biased estimates in Simulation 2. Furthermore, both models produced biased estimates in Simulation 4 and 5, where all possible between-person effects are assumed.

One interesting result regarding the first finding is that LCMMy-CLC successfully estimated the within-person effect even under the presence of a time trend in time-varying predictors. Instead of specifying a between-level model for time-varying predictors, LCMMy-CLC has the cross-level covariances between the observed time-varying predictors and the latent growth factors of the outcomes. Judging from the unbiasedness of $\hat{\gamma}$, the cross-level covariances seems to successfully capture the between-person relationships at least in the current settings. This contrasts with LCMMy-RIx and LCMx-RIy, which explicitly model the time-varying predictors with the growth factors but can fail to unbiasedly estimate γ when they specify an incorrect model for the time-varying predictors.

The second main finding is the model groups that we repeatedly observed in the Results section. Table 6.1 describes these model groups. The first model group (Group 1) consists of MLM-WB, MLM-E2S, MLM-TAC, and LCMMy-CLC. These models are characterized by their observed between-person predictors. On the other hand, the second model group (Group 2) includes MLM-MV, LCMMy-RIx, LCMxy-RP, LCMx-RIy, and LCMx-CLC. These models have latent between-person predictors. Group 1 constantly shows the bias in the between-person effects because of sampling error in the observed between-person predictors. This bias can be seen as a variation or generalization of Lüdtke's bias, which is decreasing in T , not in N , and determined by γ and β 's. In contrast, Group 2 is basically free from bias due to the sampling error because it uses latent variables for the between-level model. Thus, Group 2 can produce unbiased estimates of the between-person effects. However, this unbiasedness comes at the cost of

Table 6.1*Model Groups That Show a Similar Pattern in the Simulation Results*

Group	Between-Person Predictors	Model
1	observed (e.g., person mean, individual OLS estimates)	MLM-WB, MLM-E2S, MLM-TAC, LCMMy-CLC
2	latent (e.g., intercept factor, slope factor)	MLM-MV, LCMMy-RIx, LCMxy-RP, LCMx-RIy, LCMx-CLC

large SE. In general, Group 1 tends to produce biased estimates of the between-person effects with relatively small SE, while Group 2 can unbiasedly estimate the between-person effects but its standard error is likely to be large.

The third main finding is also related to the model groups. That is, the models in Group 2 are likely to overestimate SE of between-person effects, while the models in Group 1 can accurately estimate it in most cases. The overestimation becomes severe especially when the number of measurements T is small (e.g., $T = 3$). Increasing sample size N can mitigate the overestimation to some extent, but severe overestimation can still be observed even when $N = 1,000$. The severity of the overestimation differs depending on the models. For instance, LCMx-CLC is the worst model in terms of the overestimation of SE, which sometimes has 100 or more times as large standard errors as the true standard error (i.e., the standard deviation of the estimates in the simulations). In contrast, the other models in Group 2 show the overall average SE ratio smaller than 2.0, and more complex models tend to have a larger average SE ratio.

Due to the overestimation of SE, the models in Group 2 tend to have a larger coverage rate than it is supposed when the point estimates are unbiased. When the estimates are biased, on the other hand, the coverage rate becomes lower than the proper rate of 0.95 regardless of the overestimation of SE. This is true not only for the model in Group 2 but also for those in Group 1. As mentioned above, the models in Group 1 tend to have biased estimates due to the sampling error and the SE ratio is around one in most

cases. Hence, the coverage rate of Group 1 is often lower than 0.95. Moreover, since SE and the width of confidence intervals are decreasing in N in general, the coverage rate is also decreasing in N when the point estimates are biased. As such, the uncertainty of $\hat{\beta}$'s, which is quantified by the confidence interval, may not be reliable when one uses a model in Group 1 with a large sample. In other words, a confidence interval based on a model in Group 1 may be unduly narrow and may not capture the true value at the anticipated rate over samples.

Finally, the fourth main finding is derived from Simulation 5. In Simulation 5, the models systematically change their patterns of unbiasedness, depending on whether a variance component is zero or not. For example, the models in Group 1 produce unbiased estimates for β_{II} when either ψ_{II}^X or ψ_{SS}^X is zero, while they yield biased estimates when both ψ_{II}^X and ψ_{SS}^X are not zero. On the other hand, the models in Group2 except LCMY-RIX and LCMX-Riy have biased estimates for β_{II} when $\psi_{II}^X = 0$, and unbiased estimates when $\psi_{II}^X \neq 0$. As for β_{SI} , all models but LCMY-RIX have a bias around -0.3 when $\psi_{II}^X = 0$. In contrast, when $\psi_{II}^X \neq 0$, the bias becomes very small (Group 1) or almost zero (Group 2). The result for β_{IS} is similar to those for β_{SI} but with a different variance component. Specifically, when $\psi_{SS}^X = 0$, the bias is large for all models, while the bias becomes small when $\psi_{SS}^X \neq 0$ (Group 1) or almost unbiased unless $T = 3$ (Group 2). The result for β_{SS} has a similar pattern to β_{II} , where the model in Group 1 (i.e., MLM-E2S) produces unbiased estimates when either ψ_{II}^X or ψ_{SS}^X is zero but biased estimates when both ψ_{II}^X and ψ_{SS}^X are not zero. The models in Group 2 have biased estimates for β_{SS} when $\psi_{SS}^X = 0$; otherwise, they produce unbiased estimates. Note that the estimation of γ in Simulation 5 is not affected by the variance components. The estimates are unbiased insofar as a model successfully captures all significant sources of individual differences (e.g., latent slope factor for the time-varying predictors and/or outcomes). These observations are summarized in Table 6.2. In sum, Group 1 and Group 2 have the opposite conditions

Table 6.2

Variance Component Conditions for Unbiased Estimation of the Within-Person and Between-Person Effects in Simulation 5

Model	γ	β_{II}	β_{SI}	β_{IS}	β_{SS}
Group 1	correct growth models	$\psi_{II}^X = 0$ or $\psi_{SS}^X = 0$	large ψ_{II}^X and T (still biased)	large ψ_{SS}^X and T (still biased)	$\psi_{II}^X = 0$ or $\psi_{SS}^X = 0$
Group 2	correct growth models	$\psi_{II}^X \neq 0$	$\psi_{II}^X \neq 0$	$\psi_{SS}^X \neq 0$ and large T	$\psi_{SS}^X \neq 0$ and large T

(zero variance and nonzero variance) for unbiased estimation of β_{II} and β_{SS} , while they have similar conditions for β_{SI} and β_{IS} .

6.2 Answers to the Research Questions

Given the four main findings described in the previous section, Section 6.2 provides answers to the research questions that were developed in Section 3.6. The first research question concerns the accurate estimation of within-person and between-person effects when a data-generating model does and does not have a trend in its time-varying predictors and/or outcomes:

[RQ1] Can the proposed and existing methods correctly disentangle the within-person effect and between-person effects under various population models with and without trends in time-varying predictors and outcomes?

The first and second main findings address this research question. Specifically, the answers to RQ1 are:

[A1.1] Yes, if the time-varying predictors and outcomes are sufficiently modeled. However, model misspecification, especially underparameterization (i.e., failing to assume a slope factor when the population model truly has it), can result in a large bias and a large standard error.

[A1.2] Even when the time-varying predictors and outcomes are correctly modeled, the models with observed between-person predictors generally suffer from the bias of between-person effects unless the number of measurements is large.

The second research question deals with the effect of variance components of the time-varying predictors on the estimation accuracy. Specifically,

[RQ2] Can the proposed and existing methods accurately estimate the within-person and between-person effects when the time-varying predictors show a trend with no or large individual differences?

We can answer this research question based on the results from Simulation 5 and the fourth main finding:

[A2.1] Yes, if the variance of a growth factor is large. This means that large factor variance does not cause a problem in the parameter estimation and the results are similar to those with the moderate level of variance components.

[A2.2] In cases where a variance component is zero, whether the estimates become biased or not depends on the type of between-person predictors (observed or latent) as well as the parameter of interest, as summarized in Table 6.2.

Finally, the third research question compares the proposed models (LCMxy-RP, LCMx-RIy, and LCMx-CLC) to the existing models (MLM-WB, MLM-E2S, MLM-TAC, MLM-MV, LCMy-RIy, and LCMy-CLC) in terms of the estimation of and the statistical inference on the within-person and between-person effects:

[RQ3] Are the proposed methods comparable to the existing methods in terms of the quality of estimation and statistical inference regarding the within-person effect and the between-person effects?

The second and third main findings address this research question. In particular, the second finding is concerned with parameter estimation (the bias and SE), while the third finding relates to statistical inference (the coverage rate and power). The following are the answers for each of the four evaluation indices:

- [A3.1] In terms of the bias, the answer to RQ3 is “Yes” insofar as time-varying variables are sufficiently modeled. The proposed methods can perform better than the existing methods other than MLM-MV and LCMY-RIx because the existing methods are likely to suffer from bias due to sampling error.

- [A3.2] In terms of the SE, the answer to RQ3 is “No”; the performance of the proposed models is lower than the existing models. The SE of the proposed methods is generally larger than that of the existing methods. Furthermore, the proposed methods overestimate the SE especially when the number of measurements T is small.

- [A3.3] In terms of the coverage rate, the answer to RQ3 is “Yes and No”; the simulations show mixed results. When the proposed models can unbiasedly estimate a parameter, they tend to have a coverage rate that is higher than the proper rate of 0.95. This overcoverage is due to the overestimation of the SE, which often occurs when T is small. In contrast, the existing models can keep the proper coverage rate as long as they produce unbiased estimates.

- [A3.4] In terms of the statistical power, the answer to the RQ3 is “No”; the proposed methods generally show lower power than the existing models. This relatively low performance of the proposed models can be attributed to the overestimation of SE.

6.3 Other Findings

In addition to the main findings, there are some findings from the simulations. Section 6.3 discusses these miscellaneous findings.

6.3.1 Standard Error of the Models with Latent Between-Person Predictors

In the Results chapter, we repeatedly saw that the models with latent between-person predictors had a larger SE for the between-person effects than those with observed between-person predictors, especially when the number of measurements is small. In particular, LCMx-CLC often resulted in an extreme SE, which undermines the practicality of the model. This unstableness issue seems to have a close relation to the studies by Steffen Zitzmann and his colleagues (Zitzmann et al., 2015, 2016; Zitzmann, 2018). Zitzmann et al. (2015) focused on the multilevel latent contextual effect model, which assumes that individuals (level-1 units) are nested within a group (level-2 unit). As with the latent between-person effect models, the multilevel latent contextual effect model suffers from a large SE of the level-2 effect (i.e., the contextual effect) due to the small sample size at level 1 (in this case, the number of individuals rather than measurements). Zitzmann and his colleagues proposed a Bayesian approach to stabilize the estimation of the level-2 covariance matrix. The Bayesian approach introduces a prior distribution for each parameter whereby one can add a small amount of information to the likelihood, resulting in a shrinkage of estimates to the prior mean. Since Zitzmann's studies and the current study have many points in common (e.g., data structure, model characteristics, large SE issue due to small sample size at level 1), the Bayesian approach may be also effective in reducing the variability of the between-person effects of the proposed models.

In order to illustrate the effectiveness of the Bayesian approach, an ad-hoc

supplemental analysis was conducted using a simulated data set based on the data-generating model in Simulation 4. In this analysis, the parameters were set at $N = 100$, $T = 3$, $\gamma = 0.6$, $\beta_{II} = 0.6$, $\beta_{SI} = 0.3$, $\beta_{IS} = 0.3$, and $\beta_{SS} = 0.6$, and the generated data set was analyzed by LCMx-CLC using the maximum likelihood estimation and the Bayesian estimation. In the Bayesian estimation, the Gibbs sampling, which is an algorithm of Markov Chain Monte Carlo (MCMC) methods, was used with a 10,000 burn-in period. Two chains were independently run. The convergence of the chains was checked by visually inspecting the trace plots of each parameter and by monitoring the potential scale reduction (PSR; Gelman & Rubin, 1992). After the burn-in period, every fifty samples was taken and stored to reduce the autocorrelation between the samples (i.e., thinning). Point estimates were computed as a multivariate mode of the posterior distribution (the maximum a posteriori [MAP] estimates). For the prior distributions, the default settings in Mplus were used. More specifically, the normal distribution with a mean of zero and variance of infinity, that is, $N(0, \infty)$ was used for the non-variance component parameters. For the inverse of the covariance matrix of the latent variables (i.e., Ψ^{-1} ; see Equation 3.6), the inverse Wishart distribution with scale matrix of $\mathbf{O}$ and the degree of freedom of -3 , that is, $IW(\mathbf{O}, -3)$ was used. Note that these prior distributions are noninformative and improper.

Table 6.3 summarizes the result of the supplemental analysis. For this specific data set, the Bayesian method performs very well. In particular, the point estimates of β_{SI} , β_{IS} , and β_{SS} get close to their true value and the standard errors¹ are successfully reduced from those of the maximum likelihood estimation. In contrast, the other parameters, γ and β_{II} , remains approximately the same for both the point estimate and the standard error. Accordingly, although this is just a single case analysis and does not provide sufficient evidence, the Bayesian estimation may be preferred than the maximum likelihood

¹More precisely, this is the posterior standard deviation.

Table 6.3*Estimates (SEs) from An Ad-Hoc Supplemental Analysis*

Parameter	True Value	ML	Bayes
γ	0.6	0.667 (0.046)	0.676 (0.048)
β_{II}	0.6	0.529 (0.090)	0.520 (0.081)
β_{IS}	0.3	-0.393 (1.315)	-0.241 (0.434)
β_{SI}	0.3	0.416 (0.190)	0.337 (0.082)
β_{SS}	0.6	-0.510 (3.060)	-0.077 (0.430)

Note. ML = maximum likelihood estimator, Bayes = MAP estimator.

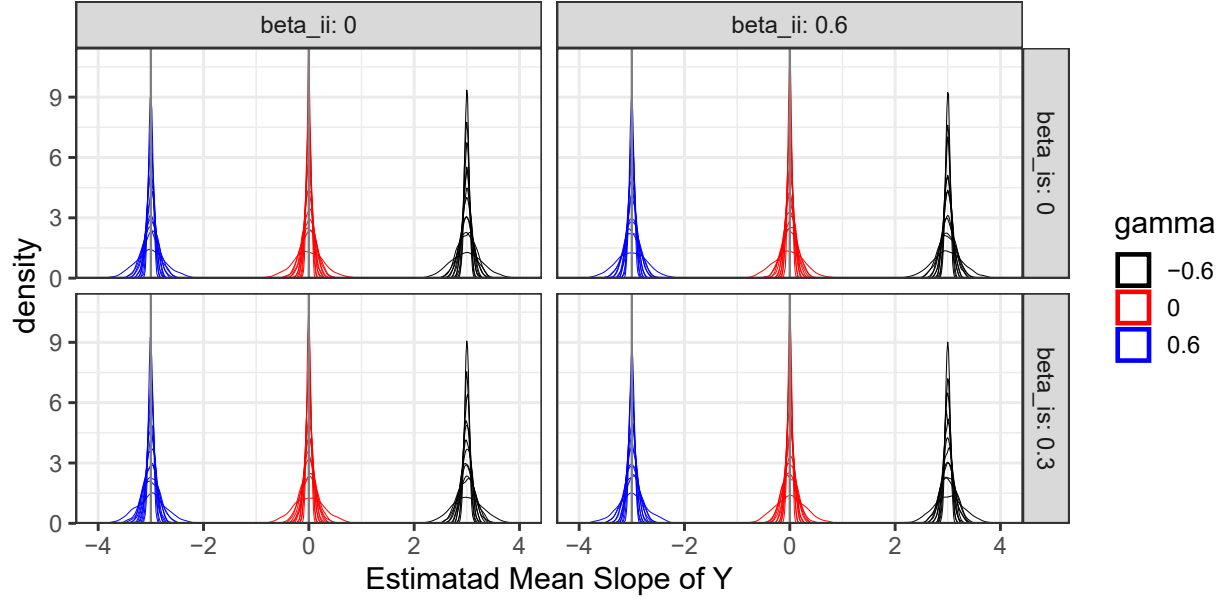
estimation when one wants to use a model with latent between-person predictors such as the proposed models and the number of measurements is small.

6.3.2 Properties of LCMY-CLC

Relationship with Other Models LCMY-CLC performed very well in Simulation 3, where the data-generating model does not have a slope factor for the outcomes (see Figure 5.3). The goodness-of-fit indices of LCMY-CLC indicate that it fits the data very well. An interesting result is that the estimates of the slope mean, τ_S^Y , are statistically significant when $\gamma \neq 0$. Figure 6.1 shows the distribution of $\hat{\tau}_S^Y$ across the conditions. Each line represents a sample distribution of $\hat{\tau}_S^Y$ from a single condition. As evidently shown in the figure, the sampling distributions deviate from zero when $\gamma \neq 0$ although the population model has no slope factor. On the other hand, LCMY-CLC appears to have an expectation of zero for S_i^Y when $\gamma = 0$. These facts imply that the systematic change captured as S_i^Y includes a change in the time-varying predictors, which is transmitted through the within-person effect. Thus, S_i^Y does not only reflect the systematic change inherent in the outcome. In fact, LCMY-CLC can be expressed in terms of the parameters of the

Figure 6.1

Sampling Distribution of $\hat{\tau}_S^Y$ of LCMy-CLC in Simulation 3



data-generating model in Simulation 3, which is a special case of LCMxy-RPO.

Suppose that LCMxy-RPO holds in the population. Then, we can rewrite the population model in the following way:

$$\begin{aligned}
 X_{ti} &= I_i^X + \lambda_t S_i^X + \varepsilon_{ti}^X, \\
 Y_{ti} &= I_i^Y + \varepsilon_{ti}^Y, \\
 \varepsilon_{ti}^Y &= \gamma \varepsilon_{ti}^X + \nu_{ti}^Y, \\
 \implies Y_{ti} &= I_i^Y + \lambda_t S_i^Y + \gamma \varepsilon_{ti}^X + \nu_{ti}^Y \\
 &= (\delta^{I^Y} + \beta_{II} I_i^X + \beta_{IS} S_i^X + \zeta_i^{I^Y}) \\
 &\quad + \lambda_t (\delta^{S^Y} + \beta_{SI} I_i^X + \beta_{SS} S_i^X + \zeta_i^{S^Y}) \\
 &\quad + \gamma (X_{ti} - I_i^X - \lambda_t S_i^X) + \nu_{ti}^Y
 \end{aligned}$$

$$\begin{aligned}
&= [\delta^{I^Y} + (\beta_{II} - \gamma)I_i^X + \beta_{IS}S_i^X + \zeta_i^{I^Y}] \\
&\quad + \lambda_t[\delta^{S^Y} + \beta_{SI}I_i^X + (\beta_{SS} - \gamma)S_i^X + \zeta_i^{S^Y}] + \gamma X_{ti} + \nu_{ti}^X \\
&= \tilde{I}_i^Y + \lambda_t \tilde{S}_i^Y + \gamma X_{ti} + \tilde{\varepsilon}_{ti}^Y,
\end{aligned} \tag{6.1}$$

where $\tilde{I}_i^Y = \delta^{I^Y} + (\beta_{II} - \gamma)I_i^X + \beta_{IS}S_i^X + \zeta_i^{I^Y}$ and $\tilde{S}_i^Y = \delta^{S^Y} + \beta_{SI}I_i^X + (\beta_{SS} - \gamma)S_i^X + \zeta_i^{S^Y}$.

Equation 6.1 has the same form as Equation 2.40. Thus, we can obtain the parameters of LCM_Y-CLC from those of LCM_{xy}-RPO. Specifically,

$$\tau_{\tilde{I}}^Y = \mathbb{E}[\tilde{I}_i^Y] = \delta^{I^Y} + (\beta_{II} - \gamma)\tau_I^X + \beta_{IS}\tau_S^X, \tag{6.2}$$

$$\tau_{\tilde{S}}^Y = \mathbb{E}[\tilde{S}_i^Y] = \delta^{S^Y} + \beta_{SI}\tau_I^X + (\beta_{SS} - \gamma)\tau_S^X, \tag{6.3}$$

$$\psi_{\tilde{I}\tilde{I}}^Y = \text{VAR}[\tilde{I}_i^Y] = (\beta_{II} - \gamma)^2\psi_{II}^X + \beta_{IS}^2\psi_{SS}^X + 2(\beta_{II} - \gamma)\beta_{IS}\psi_{IS}^X + \theta_{II}^Y, \tag{6.4}$$

$$\psi_{\tilde{S}\tilde{S}}^Y = \text{VAR}[\tilde{S}_i^Y] = \beta_{SI}^2\psi_{II}^X + (\beta_{SS} - \gamma)^2\psi_{SS}^X + 2\beta_{SI}(\beta_{SS} - \gamma)\psi_{IS}^X + \theta_{SS}^Y, \tag{6.5}$$

$$\begin{aligned}
\psi_{\tilde{I}\tilde{S}}^Y &= \text{COV}[\tilde{I}_i^Y, \tilde{S}_i^Y] \\
&= (\beta_{II} - \gamma)\beta_{SI}\psi_{II}^Y + \beta_{IS}(\beta_{SS} - \gamma)\psi_{SS}^Y \\
&\quad + [(\beta_{II} - \gamma)(\beta_{SS} - \gamma) + \beta_{IS}\beta_{SI}]\psi_{IS}^X + \theta_{IS}^Y.
\end{aligned} \tag{6.6}$$

In Simulation 3, no Y -slope assumption implies that $\delta^{S^Y} = 0$, $\beta_{SI} = 0$, $\beta_{SS} = 0$, and $\zeta_i^{S^Y} = 0$. Thus, $\tilde{S}_i^Y = -\gamma S_i^X$ and $\tau_{\tilde{S}}^Y = -\gamma\tau_S^X$. Since τ_X^Y was set at 5 across the simulations, we have $\tau_{\tilde{S}}^Y = 3$ when $\gamma = -0.6$; $\tau_{\tilde{S}}^Y = 0$ when $\gamma = 0$; and $\tau_{\tilde{S}}^Y = -3$ when $\gamma = 0.6$. These numbers agree with the location of the sampling distributions in Figure 6.1. In words, LCM_Y-CLC does not distinguish spontaneous change of the outcomes (i.e., direct trend; $\delta^{S^Y} + \zeta_i^{S^Y}$) from the one that is attributable to the change in the time-varying predictors (indirect trend; $\beta_{SI}I_i^X + (\beta_{SS} - \gamma)S_i^X$). This contrasts with models that have a separate model for the time-varying model and such as LCM_{xy}-RP. Table 6.4 summarizes the point estimate, standard error, and p -value of the parameters of LCM_{xy}-RP in Simulation 3,

Table 6.4

Marginal Mean, Standard Error, and p -Value of Parameter Estimates Related to S_i^Y Based on LCMxy-RP in Simulation 3

Parameter	M	SE	p
β_{SI}	-0.001	0.073	0.045
β_{SS}	0.017	0.742	0.056
δ^{SY}	-0.013	2.639	0.042
θ_{SS}^Y	0.007	0.651	0.044

which are related to the slope of outcomes:

$$S_i^Y = \delta^{SY} + \beta_{SI}I_i^X + \beta_{SS}S_i^X + \zeta_i^{SY}.$$

That is, the intercept for the outcomes, δ^{SY} ; the between-person effects on the slope, β_{SI} and β_{SS} ; and the residual variance of the slope factor for the outcomes θ_{SS}^Y . All of these parameters are constrained to be zero in the population. As can be seen in the table, LCMxy-RP correctly estimated the slope-related parameters as zero, on average. In other words, in contrast to LCMY-CLC, LCMxy-RP could find the absence of the linear change in the outcomes.

Estimation of Between-Person Effects According to the results from the simulations, a major limitation of LCMY-CLC is the bias of the between-person effect due to the small sample size at level-1. However, recall that LCMY-CLC does not have a specific model for the time-varying predictors, but the between-person level relation is freely estimated by introducing the cross-level covariances. The estimation of the between-person effects in the simulations of this study draws upon the modified formulae of Curran et al. (2012, see Equations 2.44 and 2.47). The estimation formulae were derived from a model that is called MLM-TAC in this study, meaning that a time trend in time-varying predictors was

not assumed in the derivation of the formulae. In fact, Curran et al. (2012, p. 246) acknowledged this limitation and the need for further research to accommodate the systematic growth over time.

Suppose that LCMxy-RPO holds in the population and LCMy-CLC is used to analyze the data from the population model. In terms of the parameters of LCMxy-RPO, the between-person effects can be written as:

$$\beta_{II} = \frac{\psi_{SS}^X \text{COV}[I_i^X, I_i^Y] - \psi_{IS}^X \text{COV}[S_i^X, I_i^Y]}{\psi_{II}^X \psi_{SS}^X - (\psi_{IS}^X)^2}. \quad (6.7)$$

$$\beta_{SI} = \frac{\psi_{SS}^X \text{COV}[I_i^X, S_i^Y] - \psi_{IS}^X \text{COV}[S_i^X, S_i^Y]}{\psi_{II}^X \psi_{SS}^X - (\psi_{IS}^X)^2}, \quad (6.8)$$

$$\beta_{IS} = \frac{\psi_{II}^X \text{COV}[S_i^X, I_i^Y] - \psi_{IS}^X \text{COV}[I_i^X, S_i^Y]}{\psi_{II}^X \psi_{SS}^X - (\psi_{IS}^X)^2}, \quad (6.9)$$

$$\beta_{SS} = \frac{\psi_{II}^X \text{COV}[S_i^X, S_i^Y] - \psi_{IS}^X \text{COV}[I_i^X, S_i^Y]}{\psi_{II}^X \psi_{SS}^X - (\psi_{IS}^X)^2}. \quad (6.10)$$

Rewriting the cross-level covariances, we have

$$\begin{aligned} \text{COV}[X_{ti}, I_i^Y] &= \text{COV}[I_i^X + \lambda_t S_i^X + \varepsilon_{ti}^X, I_i^Y] = \text{COV}[I_i^X, I_i^Y] + \lambda_t \text{COV}[S_i^X, I_i^Y] \\ \implies \text{COV}[I_i^X, I_i^Y] &= \frac{1}{T} \sum_{t=1}^T \text{COV}[X_{ti}, I_i^Y], \end{aligned} \quad (6.11)$$

$$\begin{aligned} \text{COV}[X_{ti}, S_i^Y] &= \text{COV}[I_i^X + \lambda_t S_i^X + \varepsilon_{ti}^X, S_i^Y] = \text{COV}[S_i^Y, I_i^Y] + \lambda_t \text{COV}[S_i^X, S_i^Y] \\ \implies \text{COV}[I_i^X, S_i^Y] &= \frac{1}{T} \sum_{t=1}^T \text{COV}[X_{ti}, S_i^Y], \end{aligned} \quad (6.12)$$

$$\begin{aligned} \text{COV}[X_{ti}, I_i^Y] - \text{COV}[X_{t'i}, I_i^Y] &= (\lambda_t - \lambda_{t'}) \text{COV}[S_i^X, I_i^Y] \\ \implies \text{COV}[S_i^X, I_i^Y] &= \frac{1}{T(T-1)} \sum_{t > t'} \frac{\text{COV}[X_{ti}, I_i^Y] - \text{COV}[X_{t'i}, I_i^Y]}{\lambda_t - \lambda_{t'}}, \end{aligned} \quad (6.13)$$

$$\begin{aligned} \text{COV}[X_{ti}, S_i^Y] - \text{COV}[X_{t'i}, S_i^Y] &= (\lambda_t - \lambda_{t'}) \text{COV}[S_i^X, S_i^Y] \\ \Rightarrow \text{COV}[S_i^X, S_i^Y] &= \frac{1}{T(T-1)} \sum_{t > t'} \frac{\text{COV}[X_{ti}, S_i^Y] - \text{COV}[X_{t'i}, S_i^Y]}{\lambda_t - \lambda_{t'}}. \end{aligned} \quad (6.14)$$

Thus, the four covariances in the numerators of Equations 6.7–6.10 can be estimated from the parameter estimates of LCMMy-CLC. Unfortunately, neither does LCMMy-CLC have the variance components of the time-varying predictors as parameters (i.e., ψ_{II}^X , ψ_{SS}^X , and ψ_{IS}^X) nor can they be estimated from the available parameter estimates. This is a key difference between the models with cross-level covariances, that is, LCMMy-CLC and LCMx-CLC. It is because of the variance component parameters that LCMx-CLC can estimate the between-person effects. This availability of the estimates of between-person effects is considered as an important advantage of LCMx-CLC over LCMMy-CLC.

6.4 Limitations and Future Directions

As with any other studies, this thesis has several limitations which may weaken the generalizability of the current study. First, from the methodological viewpoint, the present study completely relies on the Monte Carlo simulation and does not show the effectiveness of the proposed models in a real context. Unlike the simulated data sets, real data is not necessarily structured in the way that the current study supposes. In particular, data from education and social sciences typically contains a variety of noises and relationships between variables are highly complex. Hence, it is crucially important to show the practicality of the proposed models through real data analyses. Future studies must include some applications of the proposed models to the data collected from educational settings.

Second, most of the model properties shown in this dissertation are based on the time-balanced design. More specifically, many derivations rely on the assumption that all

individuals have the same measurement schedule and the time variable is centered (i.e., $\bar{\lambda}_i = \frac{1}{T} \sum_{t=1}^T \lambda_{it} = 0$ for all i). In practice, it may be plausible to assume the time balancedness if the study conducts experiments with not so many participants. However, if the study is observational and utilizes survey data with hundreds of respondents, it is almost impossible for the respondents to have the same measurement schedule. In such a case, some properties presented in the current study, for example, the estimation of the between-person effects based on LCM_y-CLC and LCM_x-CLC, may no longer hold since $\bar{\lambda}_i$ is not zero or constant. Curran and Bauer (2011) discussed the decomposition of the within-person and between-person effects, and centering and detrending methods under the time unbalanced designs. According to their simulation result, the person-mean centering approach could produce highly biased estimates for both within-person and between-person effects. The two-step detrending approach also resulted in biased estimates though the magnitude of bias was moderate. The issues of the time unbalancedness may have a more crucial impact on the proposed models since SEM-based growth models in general requires the time-balanced design (McNeish & Matta, 2020). Thus, examining the impact of time-unbalancedness on SEM-based detrending can be a topic in future research.

Third, the present study focuses on linear time trends in a time-varying variable. The linear trend is the simplest form of time trends and the linear approximation of the systematic change over time may be justifiable when the number of measurements is not large. However, in order to obtain reliable estimates, one may need to increase the number of measurements especially when an SEM-based method is used (recall the results of the simulations). In cases with large T , a more complex pattern may be observed in reality. Therefore, research on accommodating a nonlinear trend is necessary particularly for the SEM-based detrending methods. Polynomial models and piecewise linear models are relatively easy to implement. In contrast, nonparametric or semiparametric detrending using generalized estimating equation (GEE), spline smoothing, and other nonlinear

regression techniques would be difficult to develop within the SEM framework.

Another possible choice is the detrending by differencing. Differencing is a popular detrending method in econometrics, especially in time-series analysis (Shumway & Stoffer, 2017). Differencing is also known as an estimation method for the fixed effects regression (Wooldridge, 2010, 2016), which is reviewed in Chapter 2. For the first-differencing, a measurement at time $t - 1$ is subtracted from the one at time t (i.e., $X_t - X_{t-1}$) and the differenced values are used in the analysis in a usual manner. It is not necessary to specify a specific functional form of trend, thus differencing can be used for both linear and nonlinear trends. Also, differencing may correspond to an SEM model for studying change. Specifically, the latent change score model (Hamagami & McArdle, 2001) considers difference scores (also called change scores and gain scores) for latent variables, namely $f_t - f_{t-1}$, instead of the difference scores for the observed scores, $X_t - X_{t-1}$. Since the link between differencing and latent change score model has not been studied, it would be a promising research topic.

Last but not least, detrending models can be extended such that dynamic relationships are incorporated. Examples of the dynamic relationship include autoregression effects (e.g., the effect of X_{t-1} on X_t , the effect of Y_{t-1} on Y_t) and cross-lagged effects (the effect of X_{t-1} on Y_t , the effect of Y_{t-1} on X_t). Research on latent variable models with the dynamic relationships is getting increasingly active these days (Asparouhov et al., 2018; Asparouhov & Muthén, 2019, 2020; Bainter & Howard, 2016; Bianconcini & Bollen, 2018; Falkenström et al., 2017; Grimm & Ram, 2018; Hamaker et al., 2015; Moral-Benito et al., 2019; Mulder & Hamaker, 2020; McNeish & Hamaker, 2020; Shamsollahi et al., 2021; Usami, Murayama, & Hamaker, 2019; Usami, Todo, & Murayama, 2019; Usami, 2020; Zyphur, Allison, et al., 2020; Zyphur, Voelkle, et al., 2020; Zyphur et al., 2021). Among such models, the latent curve model with structured residuals (LCM-SR; Curran et al., 2014) is an immediate extension of the proposed models to the dynamic

models. LCM-SR residualizes the outcomes and predictors by means of LCMs and regresses the residualized variables on themselves at past time points (i.e., autoregression effects) and each other (cross-lagged effects). These dynamic effects are considered as a sort of the within-person effect because both are based on the residualization. Another line of extension is the dynamic structural equation modeling (DSEM; Asparouhov et al., 2018; Asparouhov & Muthén, 2020; McNeish & Hamaker, 2020). DSEM is an implementation of time series analysis within the SEM framework and has been developed to analyze intensive longitudinal data, which can include more than a hundred measurements. DSEM can address not only typical longitudinal data with many N and small T but also data with $N = 1$ and large T and data with many N and large T . DSEM also incorporates a multilevel framework (i.e., random coefficients); thus, it can disentangle the within-person effect and between-person effects in the way of multilevel SEM. A variant of DSEM is the residual DSEM (RDSEM), where the autoregression part is absorbed in the residual model and isolated from the between-level model. In this sense, RDSEM can be seen as a direct extension of the proposed models. As such, DSEM and RDSEM open up the possibility of modeling more complex patterns of within-person and between-person effects in large and long longitudinal data. One practical limitation is software—DSEM and RDSEM have been implemented only in Mplus 8.1 or later versions. Therefore, the access to DSEM and RDSEM may be limited to applied researchers at this time.

6.5 Recommendations for Practitioners

Given the results and discussions in the previous and current chapters, some recommendations can be provided for practitioners who are interested in examining the relationship between variables over time. First, analyze time-varying outcomes and time-varying predictors separately in a preliminary analysis and check whether there is a

systematic change over time in each variable. According to the results of the simulations in this study, misspecification of a growth model for either outcome or predictor can result in seriously large bias and standard error of the parameter of interest. The separate preliminary analysis can help us avoid missing important components of the change in the following steps (e.g., linear and/or quadratic growth factor). Some may think this advice too basic; however, especially when one wants to use a regression-based model, predictors may be included in a model without sufficient examination of their nature just because substantial theory suggests that the predictors be included in the model. In this sense, SEM models may be advantageous since model building in the SEM framework requires us to separately specify a model for the outcomes and predictors.

Second, in designing a study, maximize the number of measurements when practical conditions allow. As the simulation results repeatedly showed, the number of measurements T greatly affects the bias and standard error of the within-person and between-person effects. This is true for both models with observed between-level predictors and those with latent between-level predictors. Since the models with observed between-person predictors typically have a bias in the between-person effects due to sampling error, increasing T is a direct and simple solution to the bias issue. For the latent between-person predictor models, the estimates are basically unbiased, but the standard error can be extremely large when T is small. Hence, increasing T can reduce the standard error and stabilize the estimates of the within-person and between-person effects. Increasing the measurement occasions has another benefit for practitioners. It is well known that we need at least three measurements per person to use a linear growth curve model to estimate the residual variance. Similarly, we need at least $p + 2$ measurements per person to use a polynomial growth curve model of order p (Bollen & Curran, 2006, p. 7). Thus, the more measurement occasions we have, the more choices we have for the functional form of growth. Of course, conducting a longitudinal study with many waves

requires more resources and time. Therefore, researchers need to contemplate the study design in terms of measurement schedule, taking the balance between the practicality and statistical soundness as well as the research interests. One can increase T by either increasing measurement frequency with a fixed study duration or expanding the study duration with a fixed measurement frequency. Development scientists, for example, may prefer increasing frequency rather than duration because they are often interested in an event in a specific time period (e.g., infancy, childhood, adolescence). In contrast, researchers who are interested in a long-term effect of educational policy change may choose increasing duration rather than the frequency. See Raudenbush and Liu (2001) for the technical details on the power to detect group differences in polynomial growth curves in terms of study duration, frequency, and sample size.

Third, select an appropriate analytic model based on the study design and research questions. Based on the results from the simulations, the within-person effect can be unbiasedly estimated as long as the growth models are correctly specified or the cross-level covariances are used. If researchers are interested in the change intrinsic to the outcome independently of the change attributable to the time-varying predictors, then they should not use MLM-WB, MLM-TAC, and LCMMy-CLC because these models cannot distinguish the direct or partial trend from the indirect trend (see Section 3.5 and Subsection 6.3.2). In this case, the models that have a separate submodel for the predictors are appropriate (e.g., the proposed models, MLM-E2S, MLM-MV). If the data is collected many times, for example, more than five times, then the models with latent between-person predictors may be preferable since they are free from the bias and the large SE issue in this case. However, when T is much larger (e.g., $T \geq 10$), the analysis by SEM-based models with wide-form data may not be practical since the model specification in software (e.g., writing a syntax file) gets more burdensome as T increases. In this case, MLM-MV may be a good choice since it has latent between-person predictors but uses long-form data, which can reduce the

workload of model specification in software. One practical issue of MLM-MV is that software which can run MLM-MV is very limited (e.g., SAS PROC MIXED, Mplus). Alternative software such as HLM, SPSS, and nlme package in R can be used with double-stacked data and variable-level dummies, but a deeper understanding of the model and model specification in the software is needed (see Section 2.3.3 for the details; see also Chapter 9 of Hoffman, 2019). In this sense, the proposed models can provide an excellent compromise between the statistical soundness (i.e., the unbiased estimation of the effects, the separability of the direct and indirect trends) and the practicality (easy implementation in software, easy access to the software).

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Appendices

Appendix A

Figures in Simulation 1

A.1 Auxiliary Indices

Figure A.1

Proportion of Converged Cases in Simulation 1

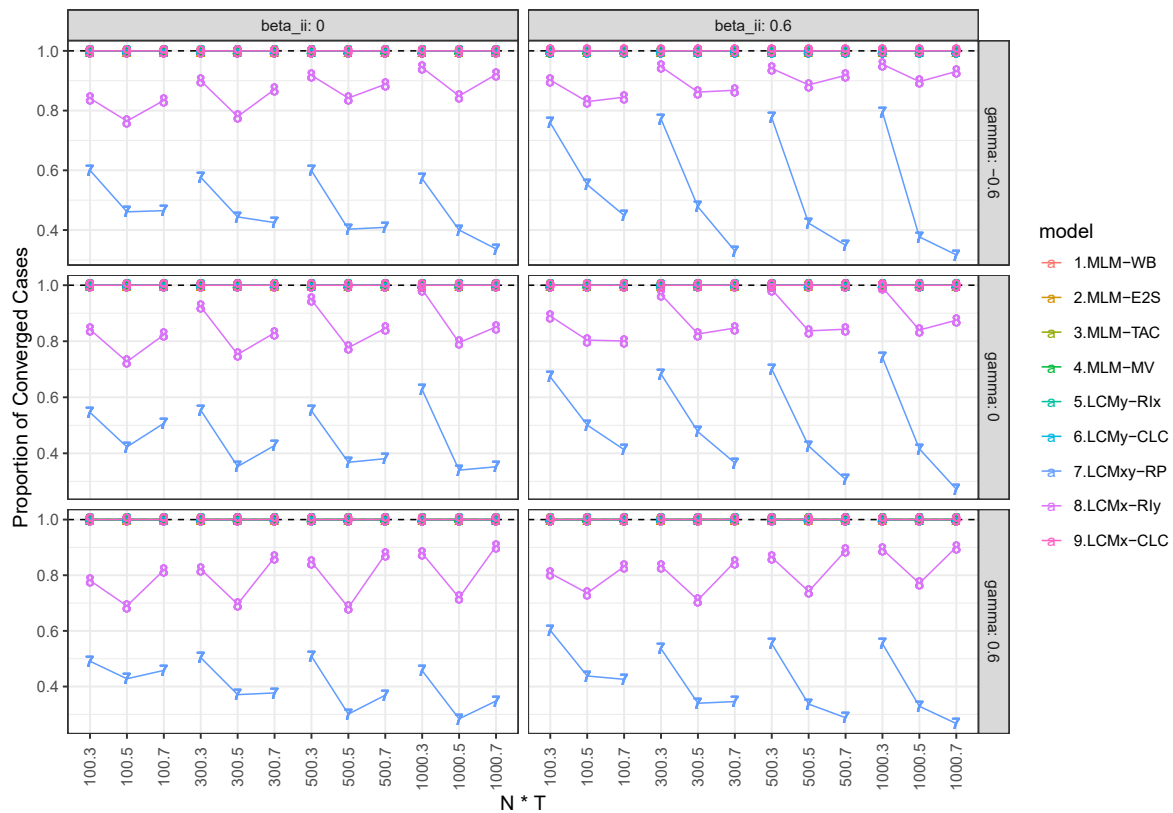


Figure A.2

Proportion of Cases with Problems in Estimating SE in Simulation 1

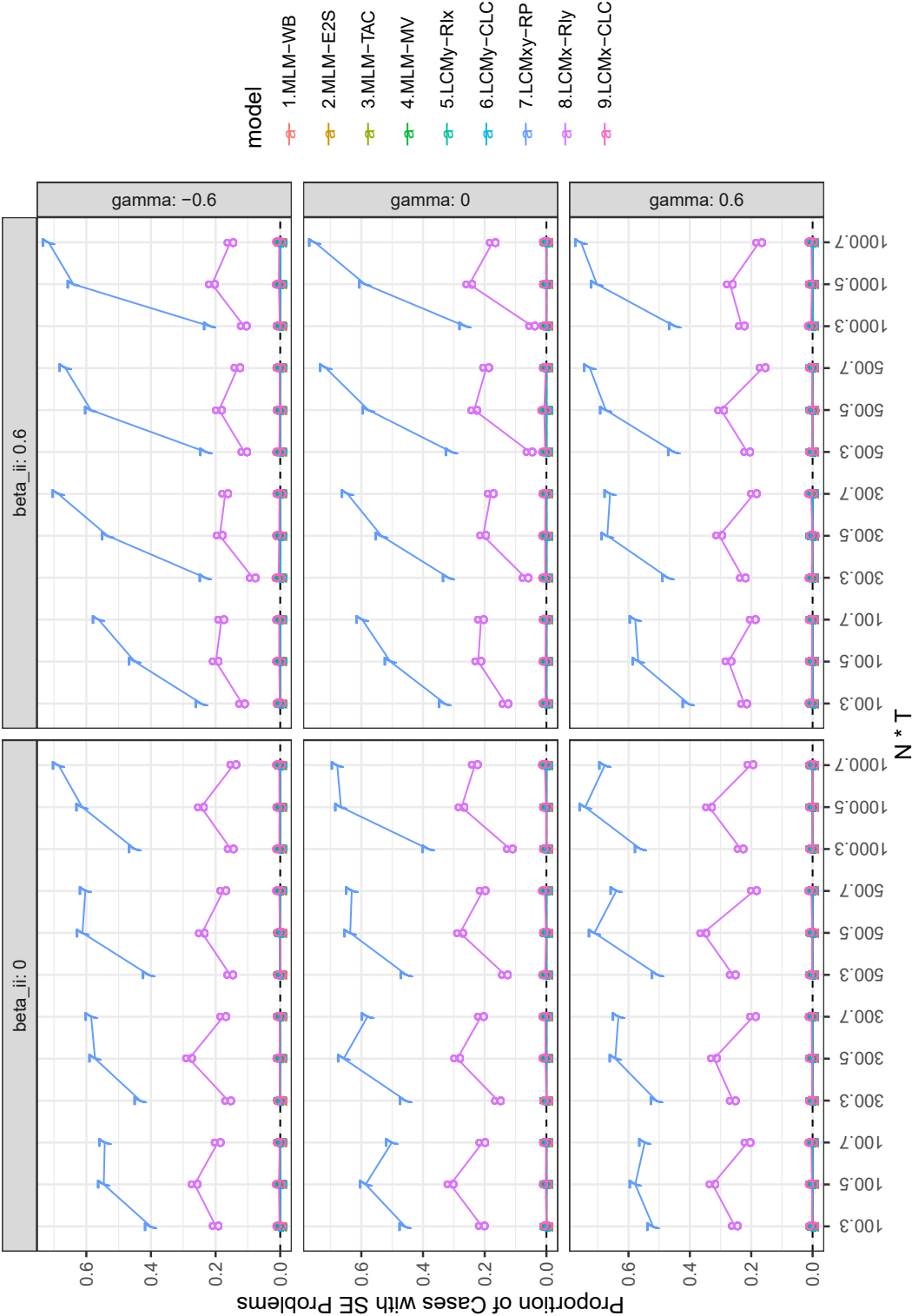


Figure A.3

Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 1

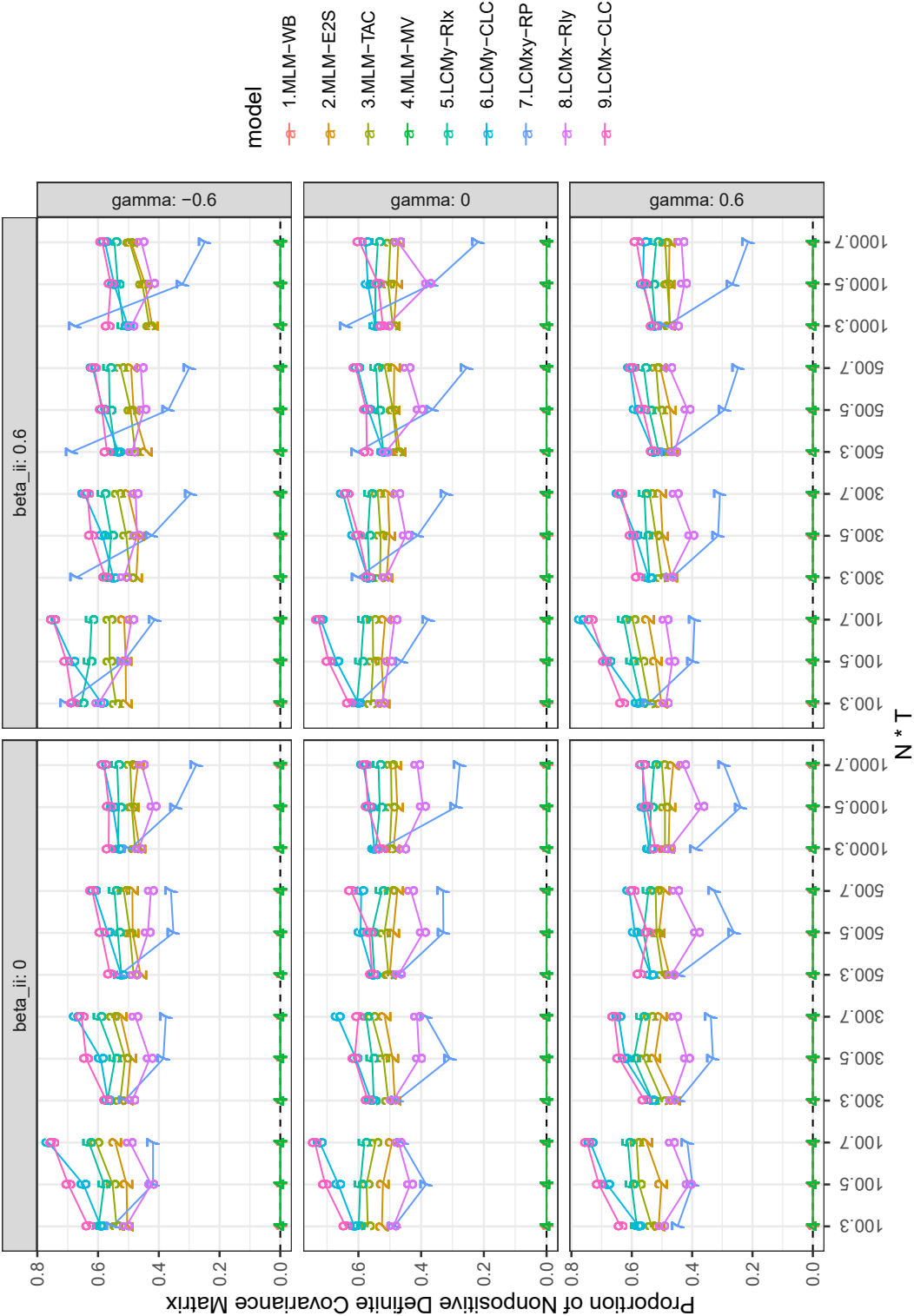


Figure A.4

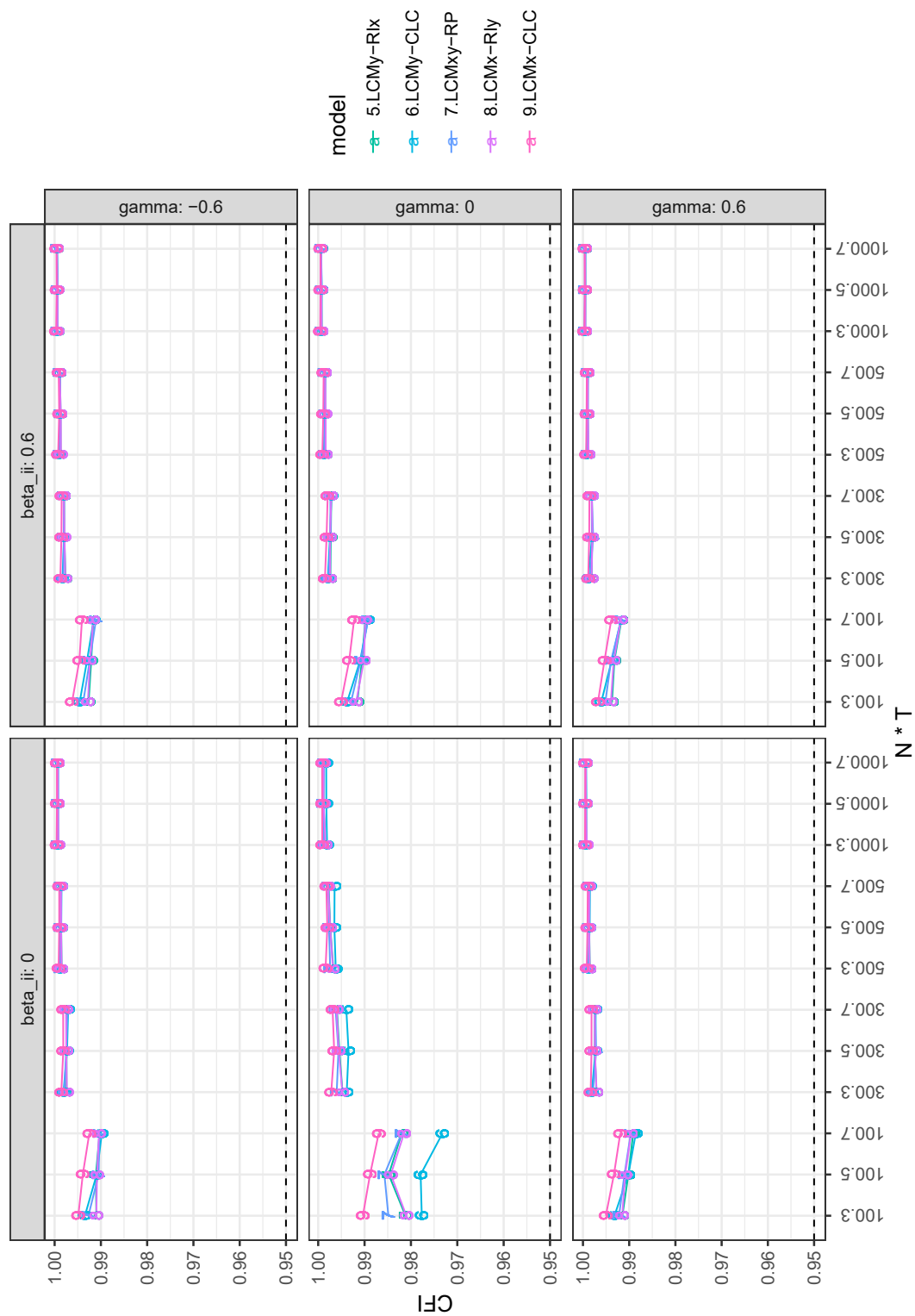


Figure A.5

RMSEA in Simulation 1

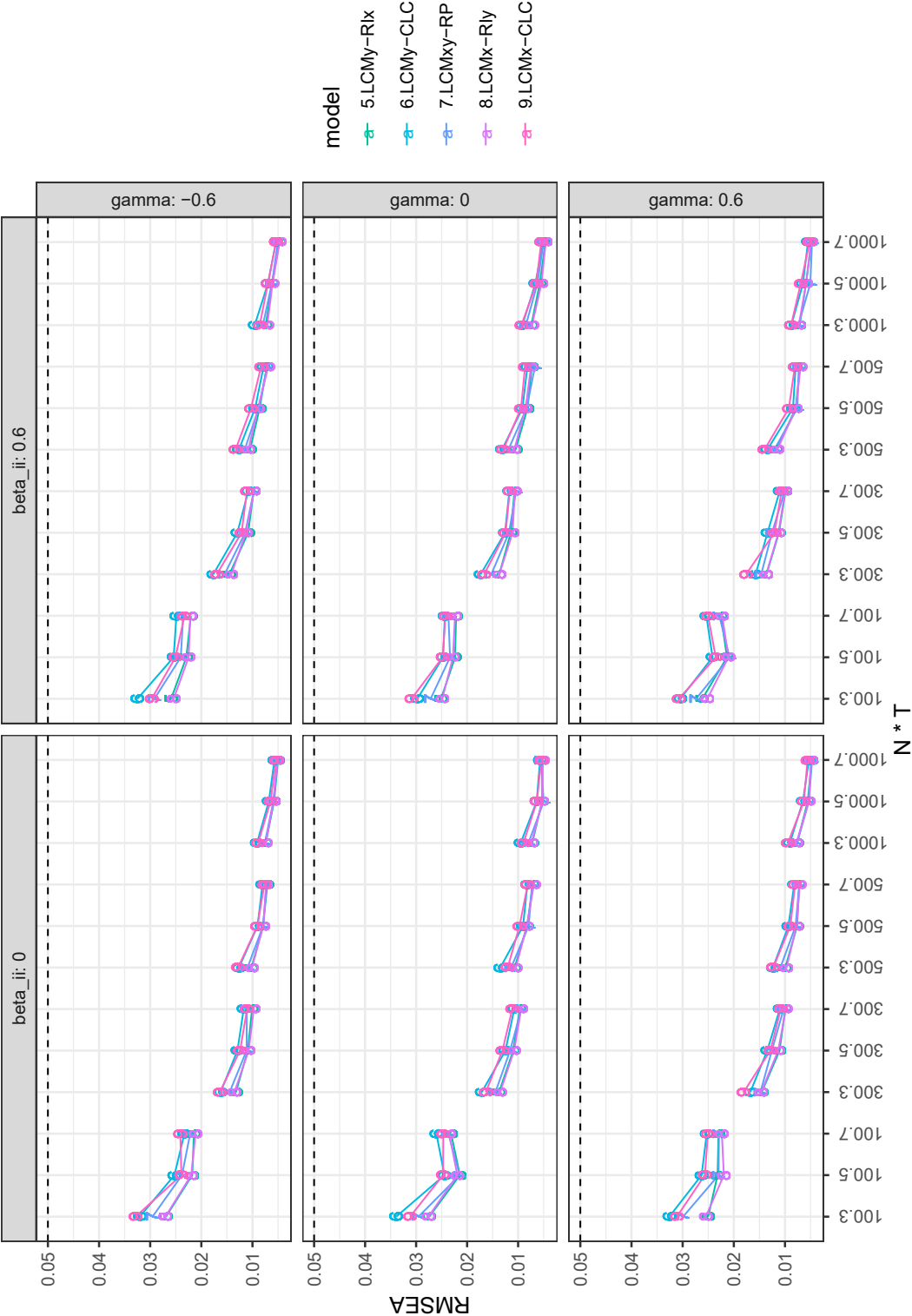
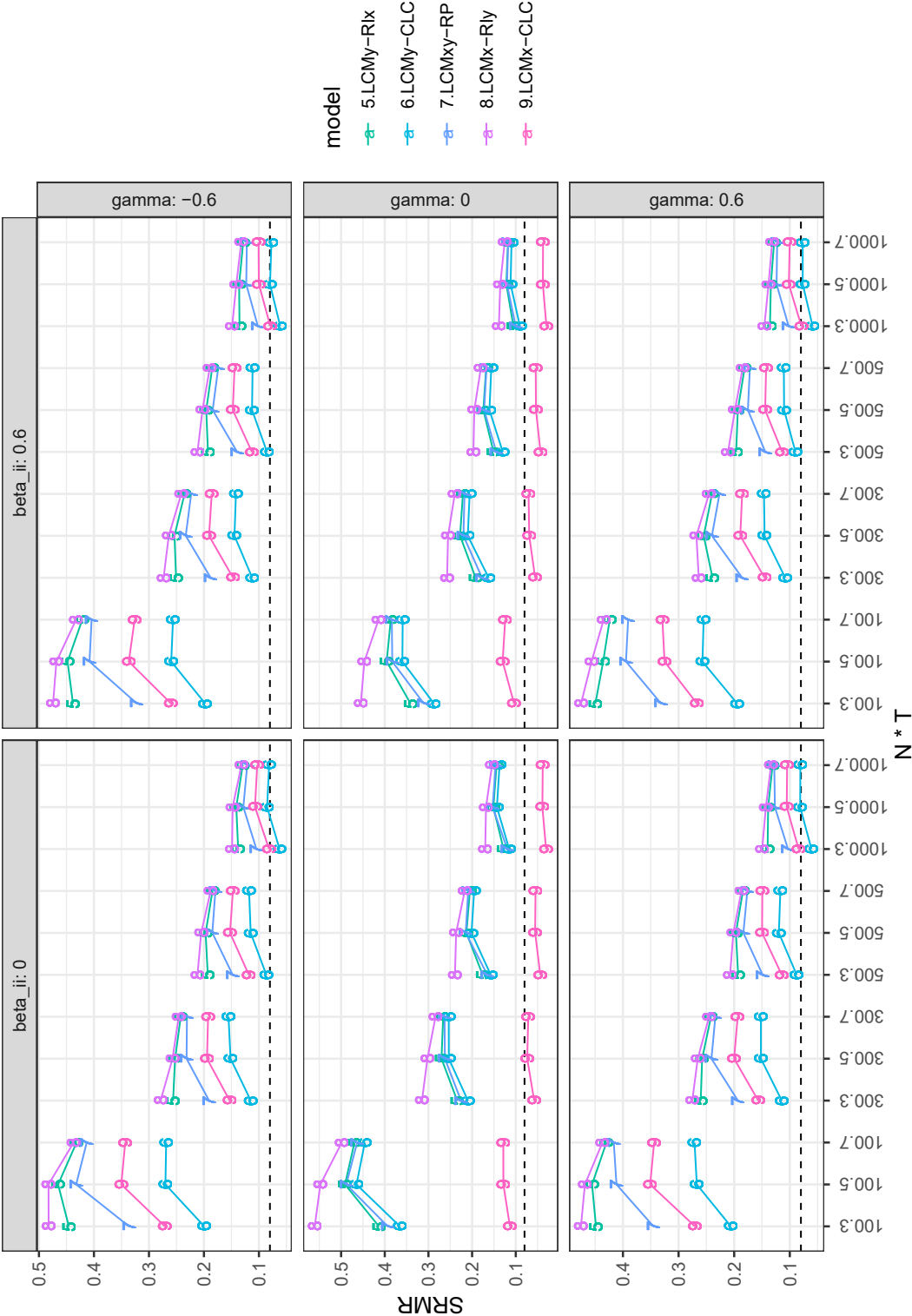


Figure A.6

SRMR in Simulation 1



A.2 Within-Person Effect

Figure A.7

Bias for the Within-Person Effect γ in Simulation 1

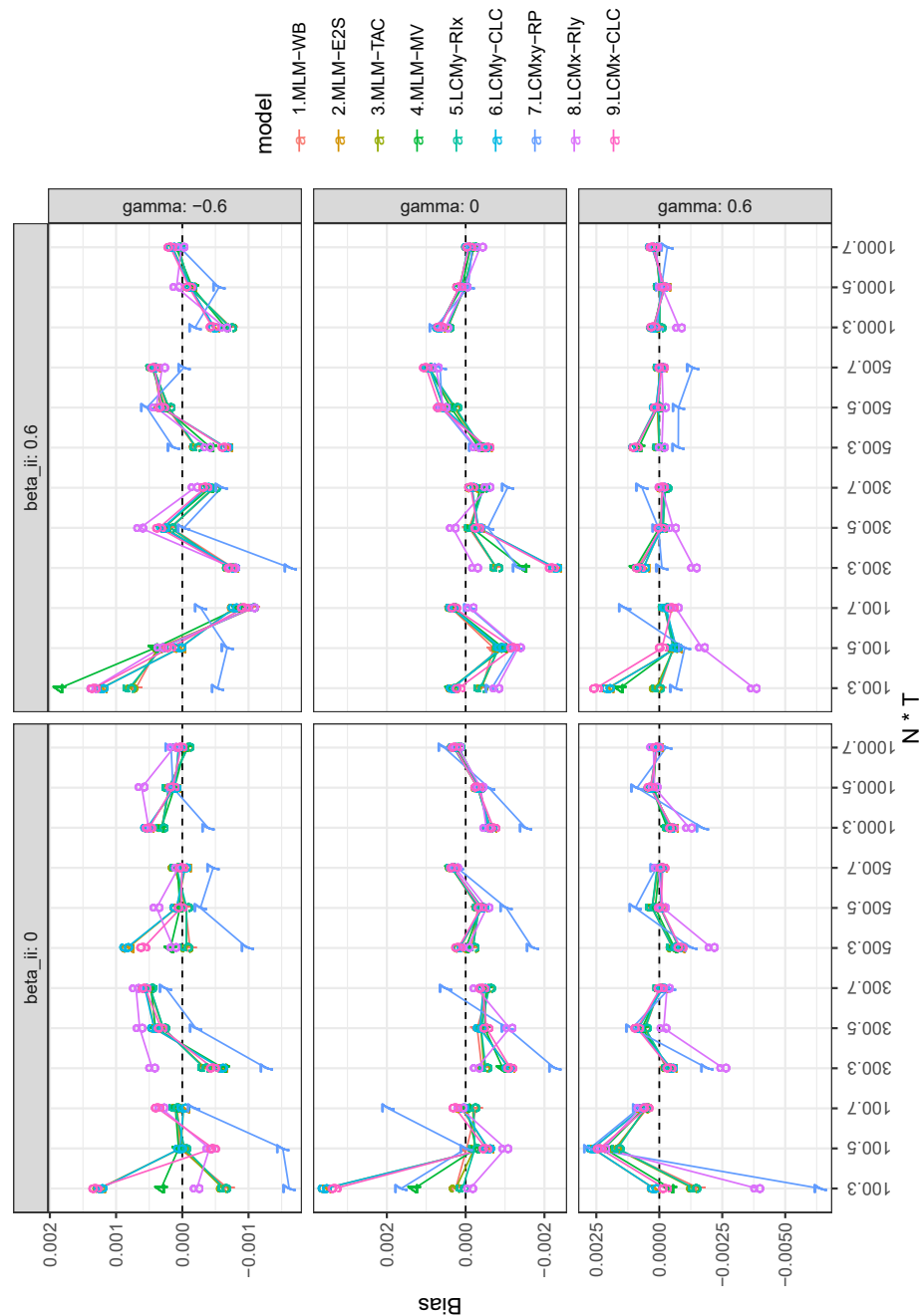


Figure A.8

Empirical SE of the Within-Person Effect γ in Simulation 1

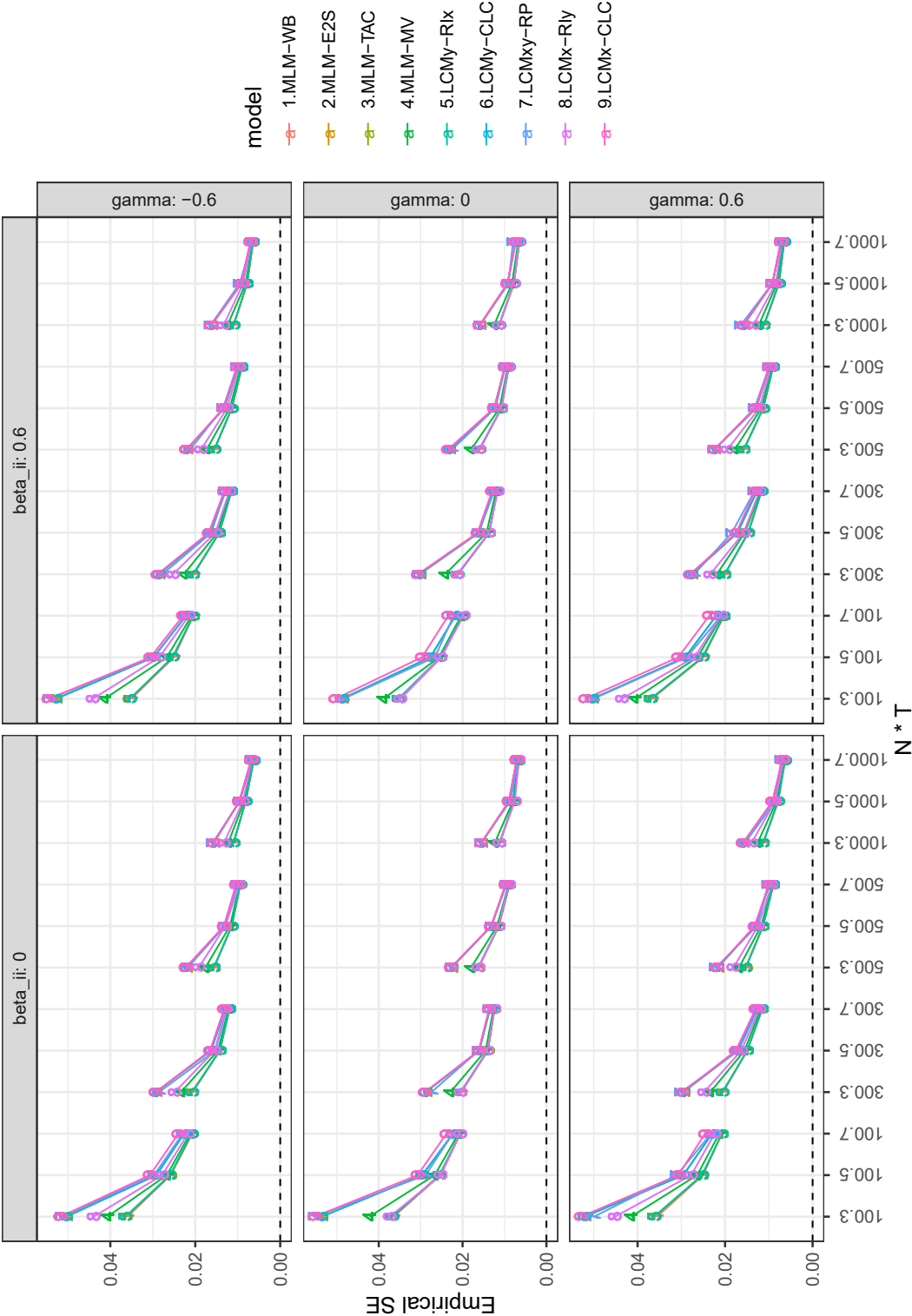


Figure A.9

Theoretical SE for the Within-Person Effect γ in Simulation 1

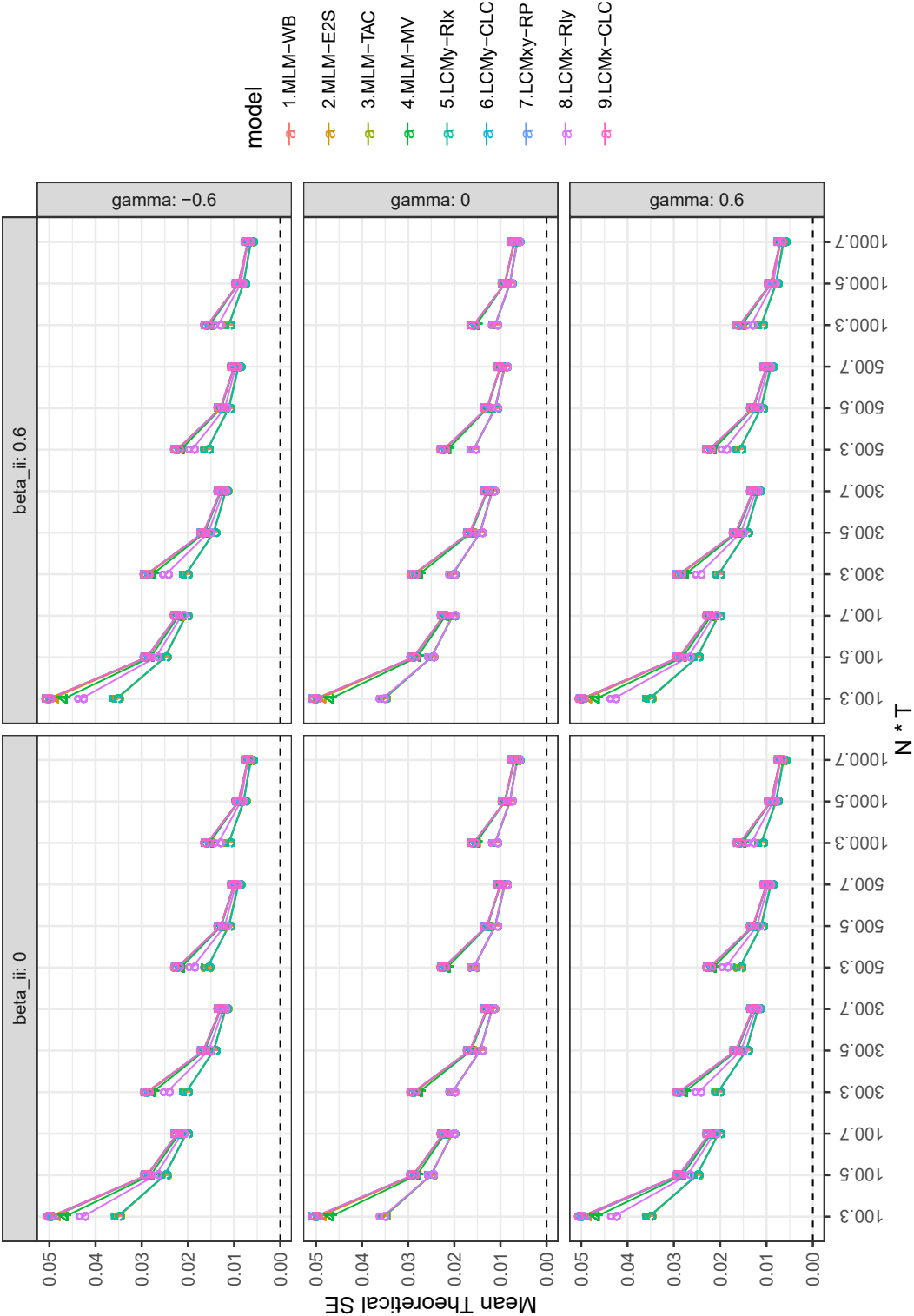


Figure A.10

SE Ratio for the Within-Person Effect γ in Simulation 1

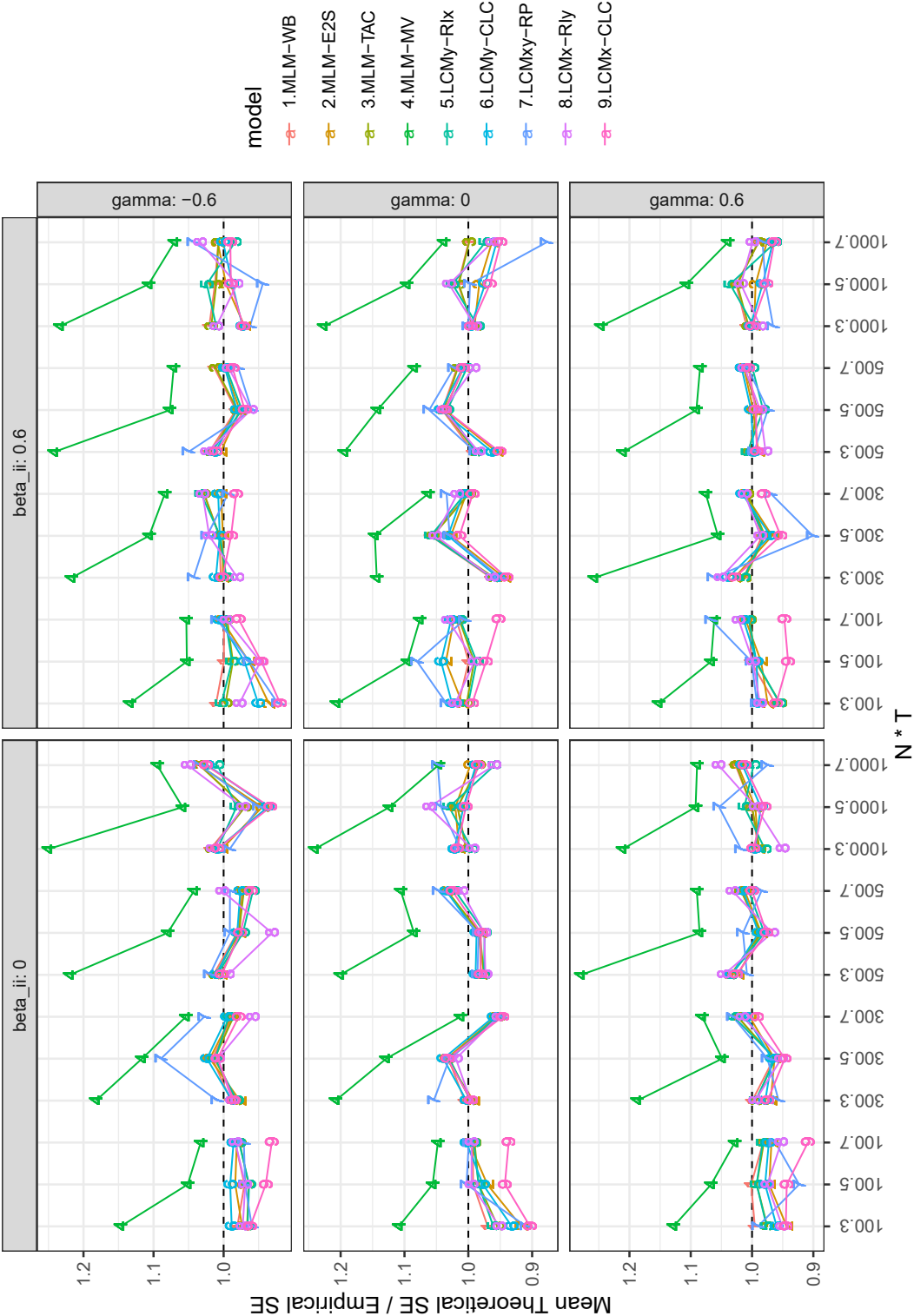


Figure A.11

RMSE for the Within-Person Effect γ in Simulation 1

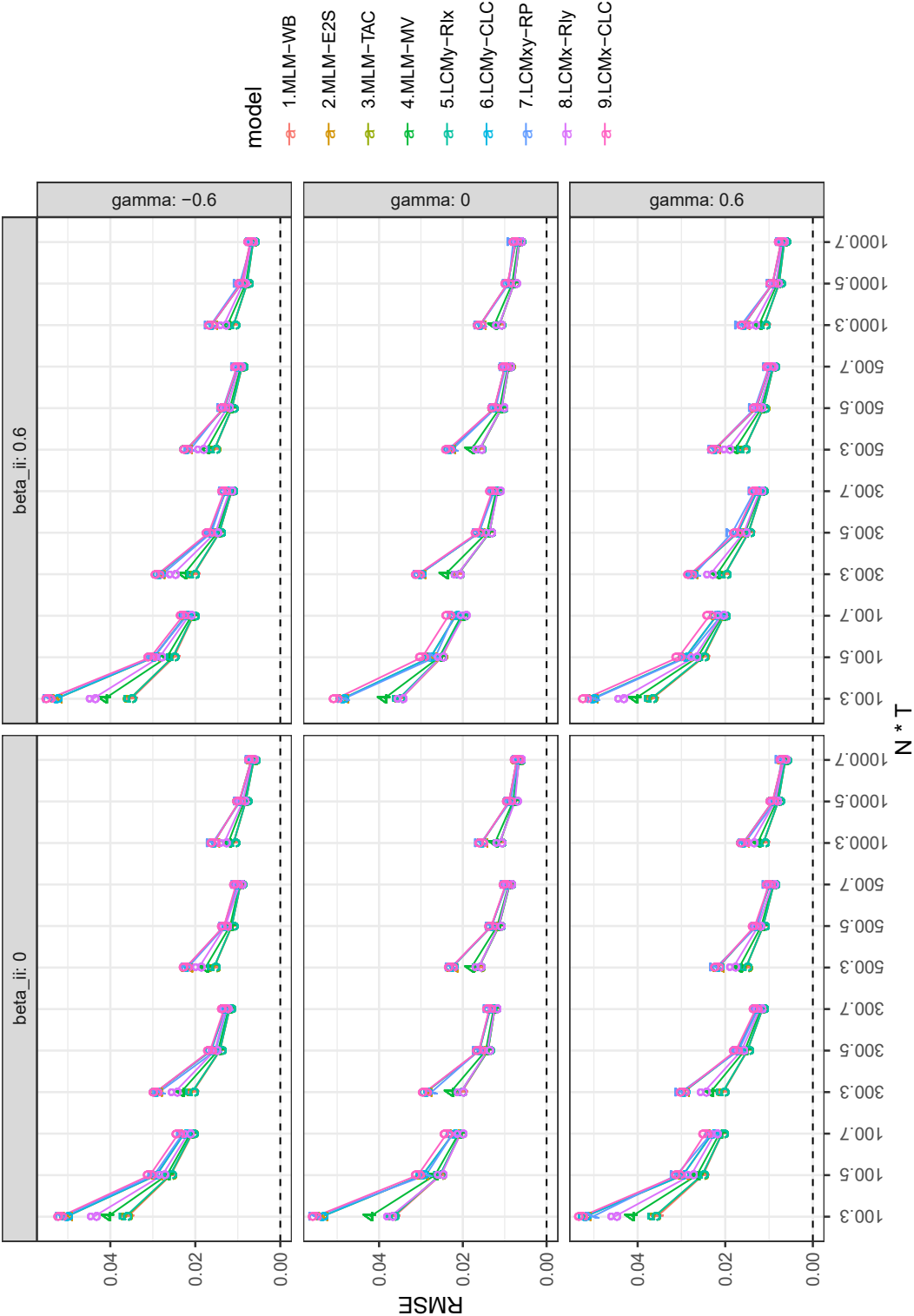


Figure A.12

Coverage Rate for the Within-Person Effect γ in Simulation 1

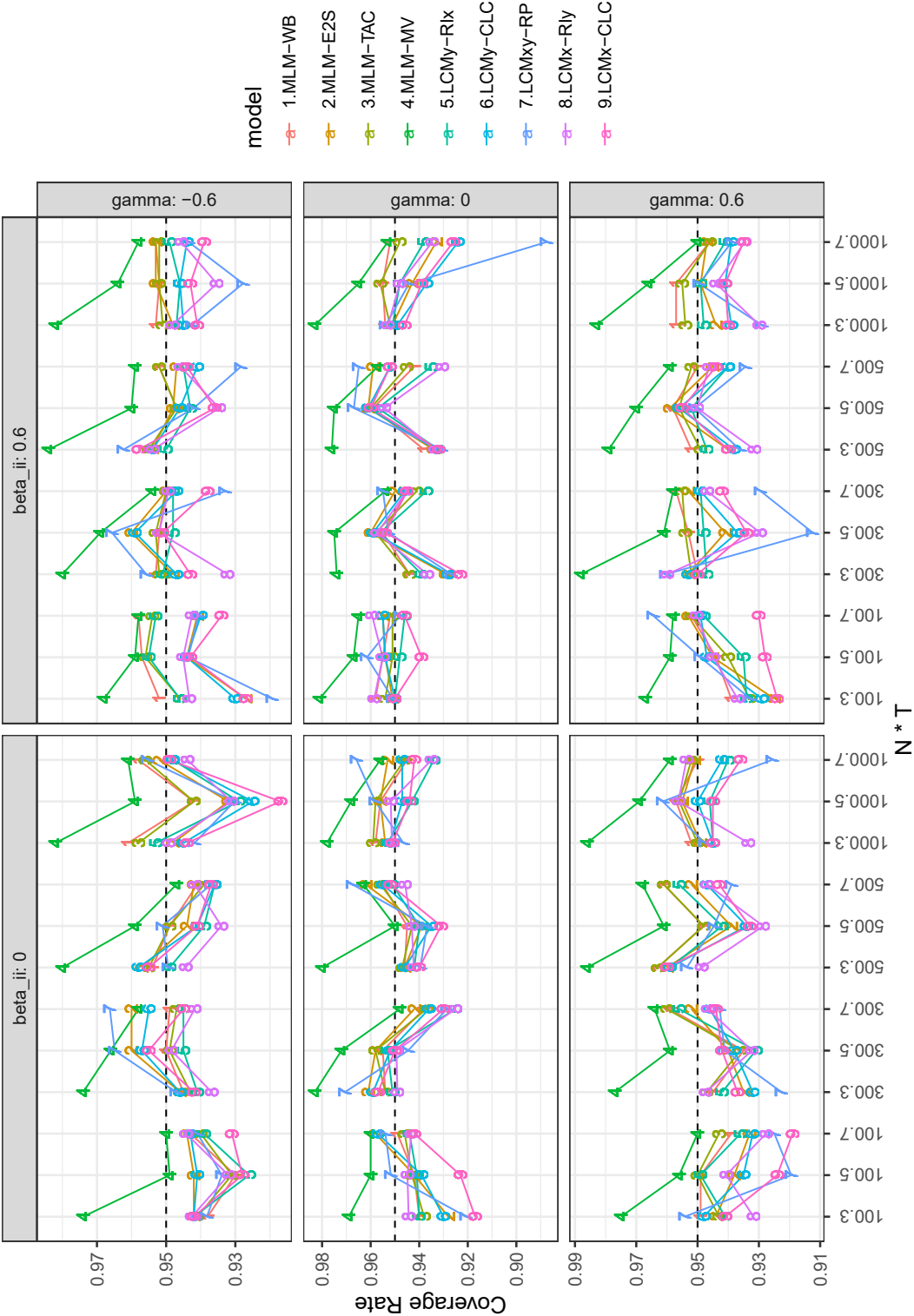
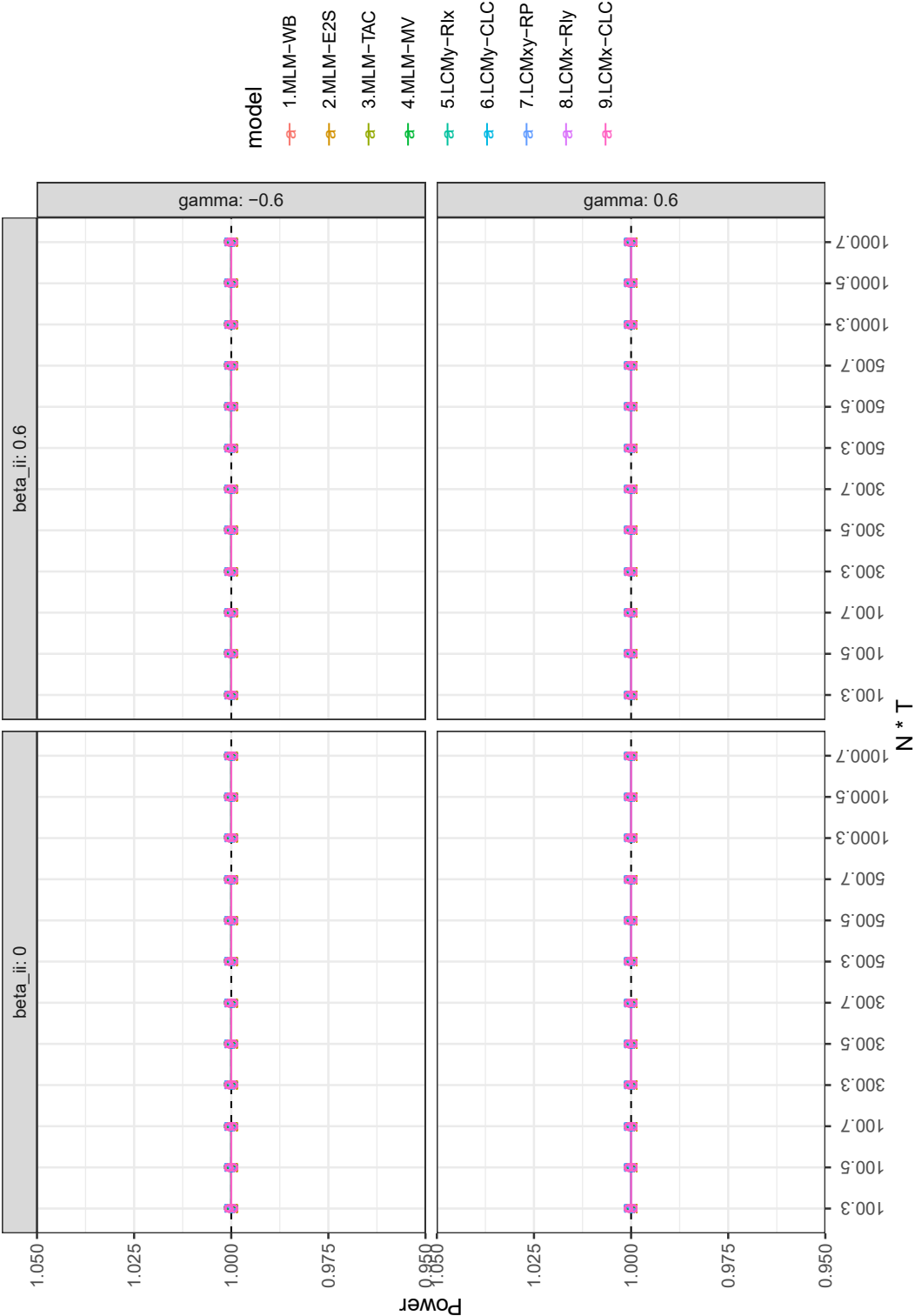


Figure A.13

Power for Detecting the Within-Person Effect γ in Simulation 1



A.3 Between-Person Effect of Intercept on Intercept

Figure A.14

Bias for the Between-Person Effect β_{II} in Simulation 1

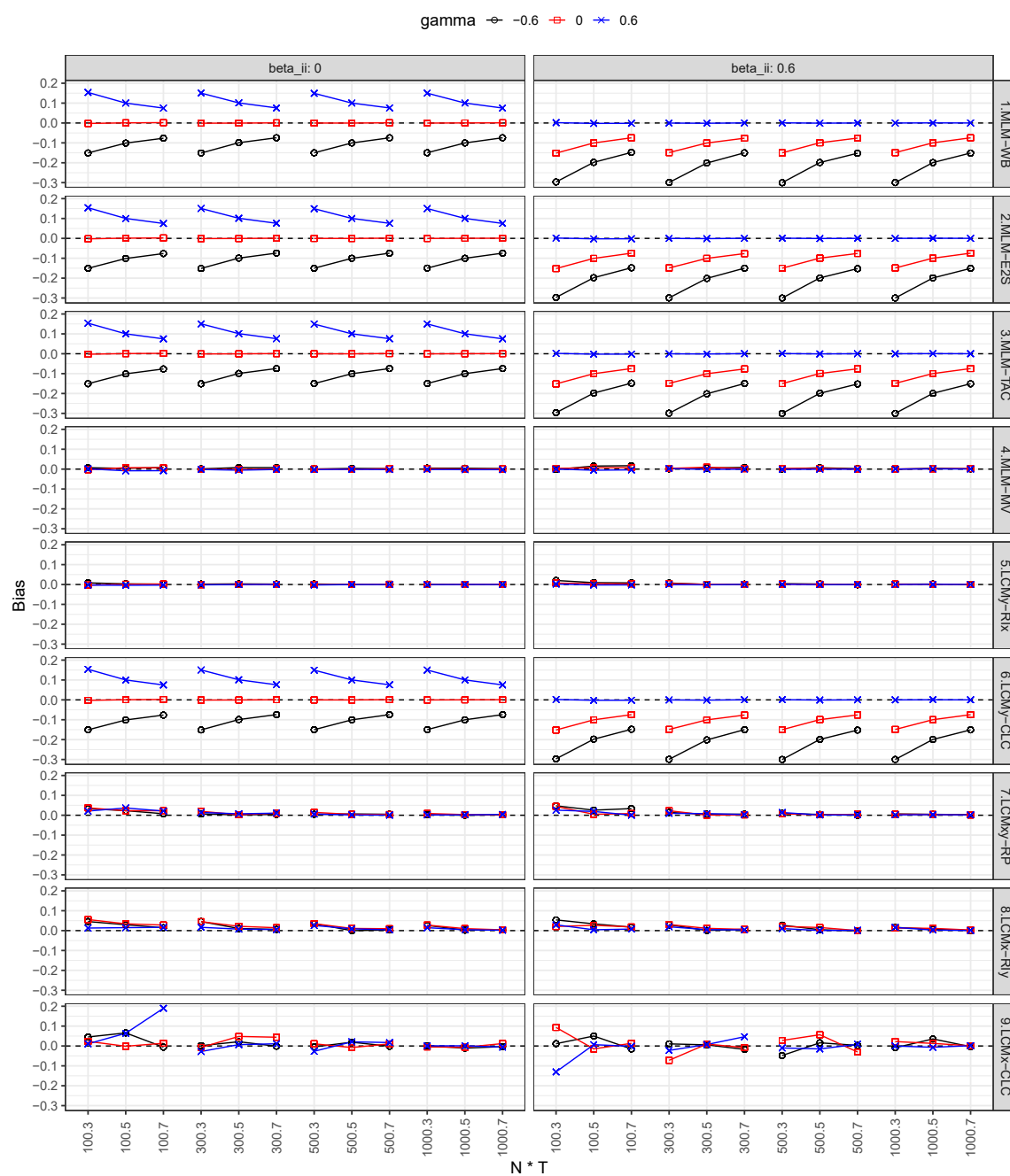


Figure A.15

Empirical SE for the Between-Person Effect β_{II} in Simulation 1

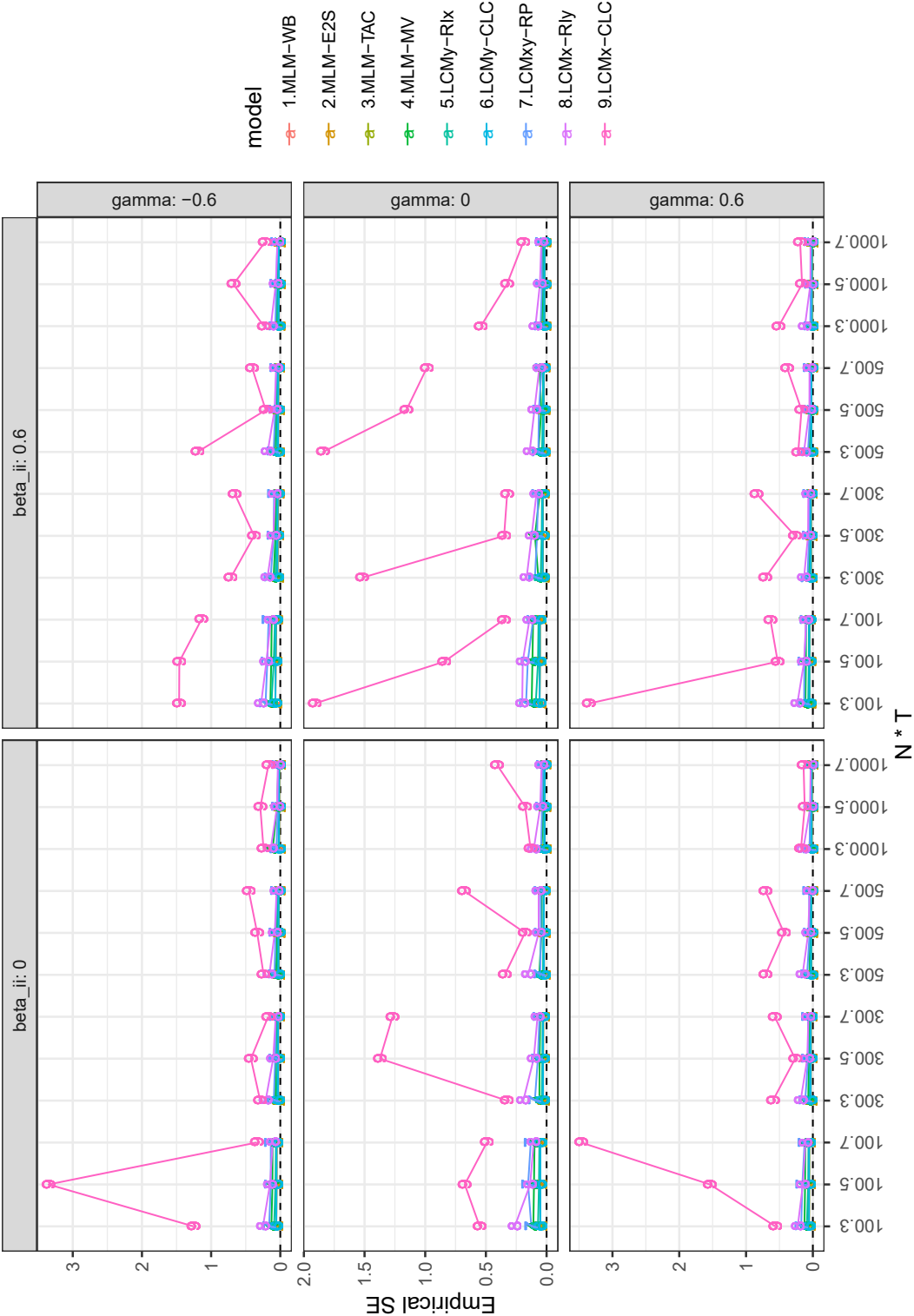


Figure A.16

Empirical SE for the Between-Person Effect β_{II} in Simulation 1 except LCMx-CLC

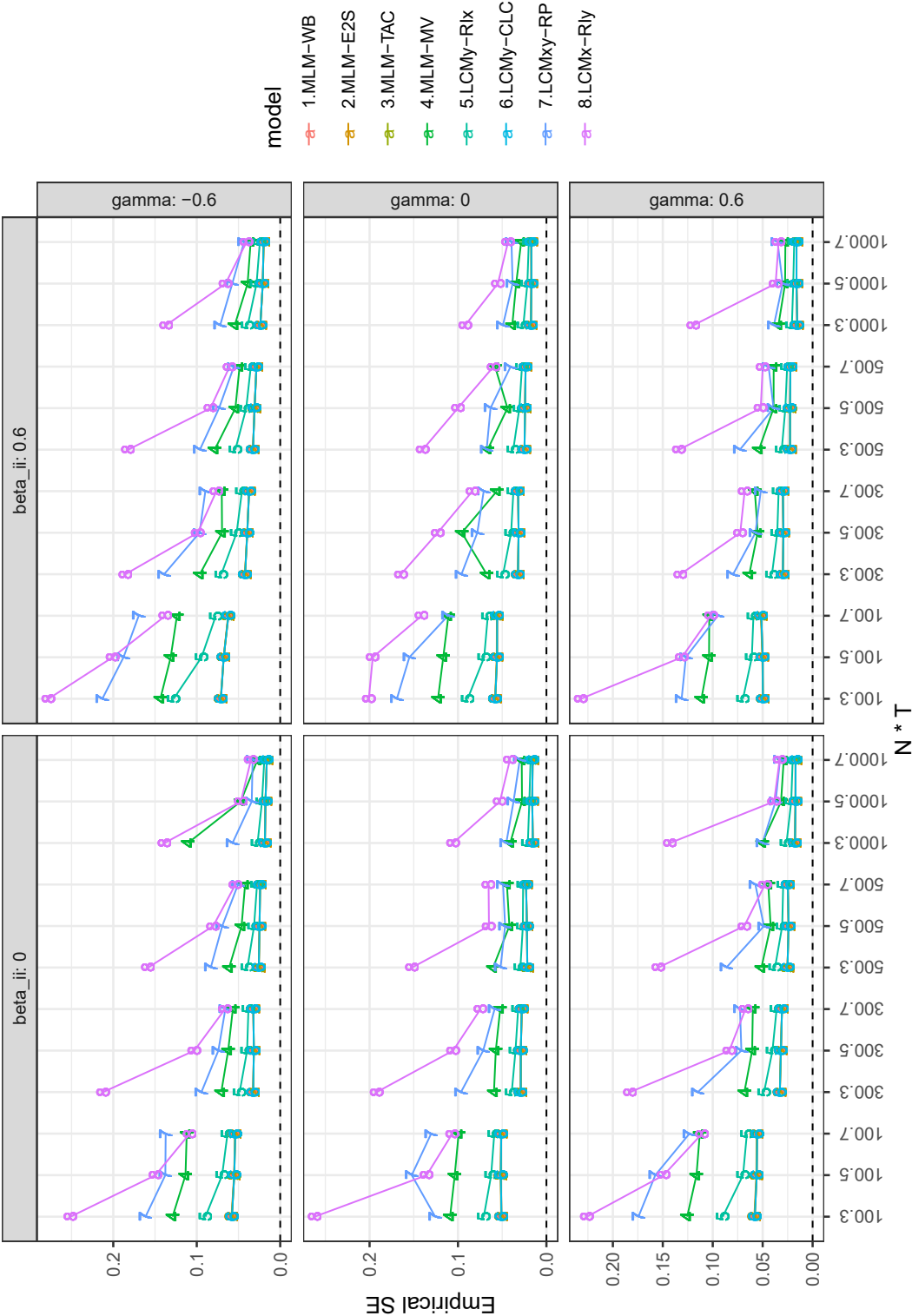


Figure A.17

Theoretical SE for the Between-Person Effect β_{II} in Simulation 1 except LCMx-CLC

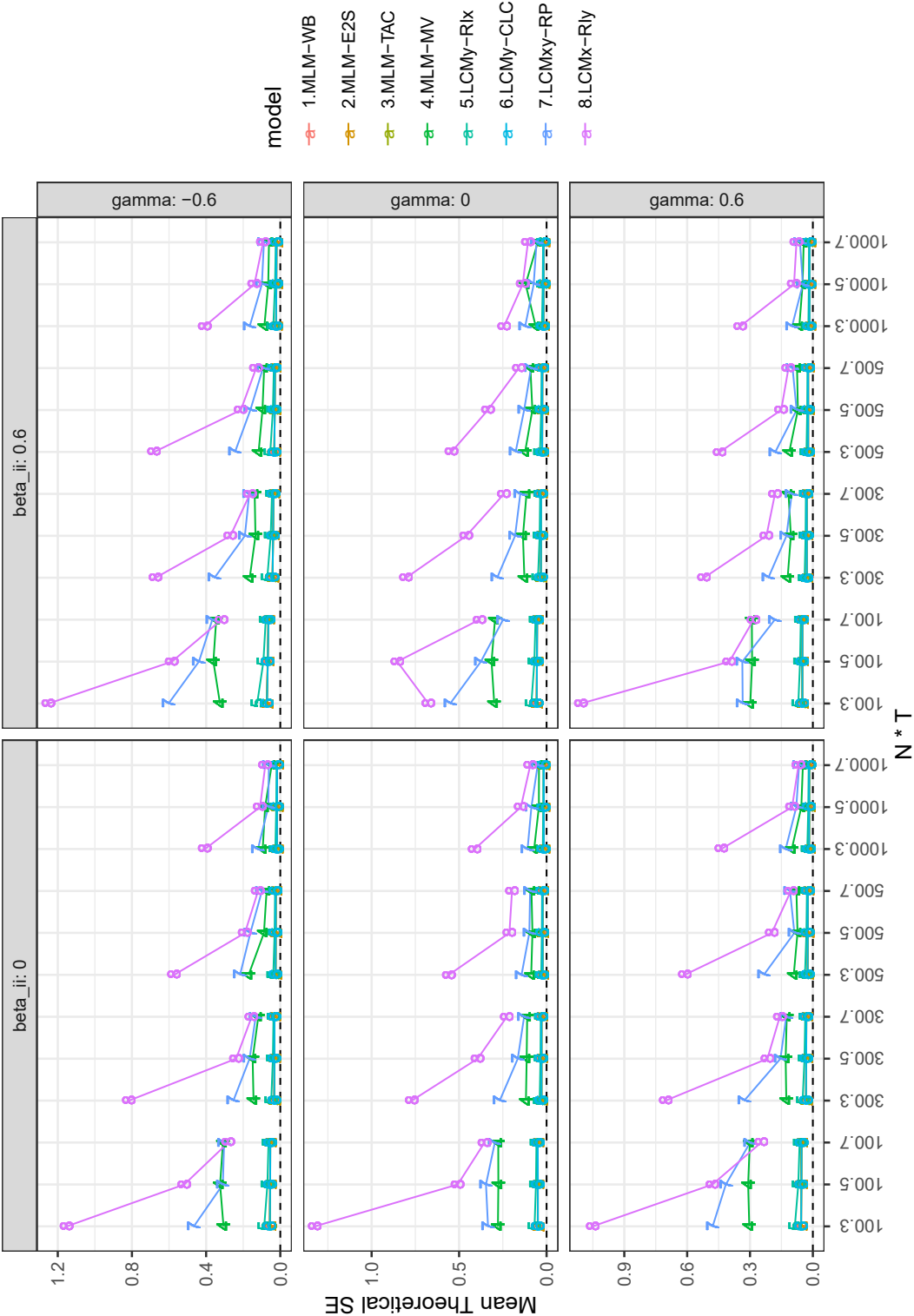


Figure A.18

SE Ratio for the Between-Person Effect β_{II} in Simulation 1 except LCMx-CLC

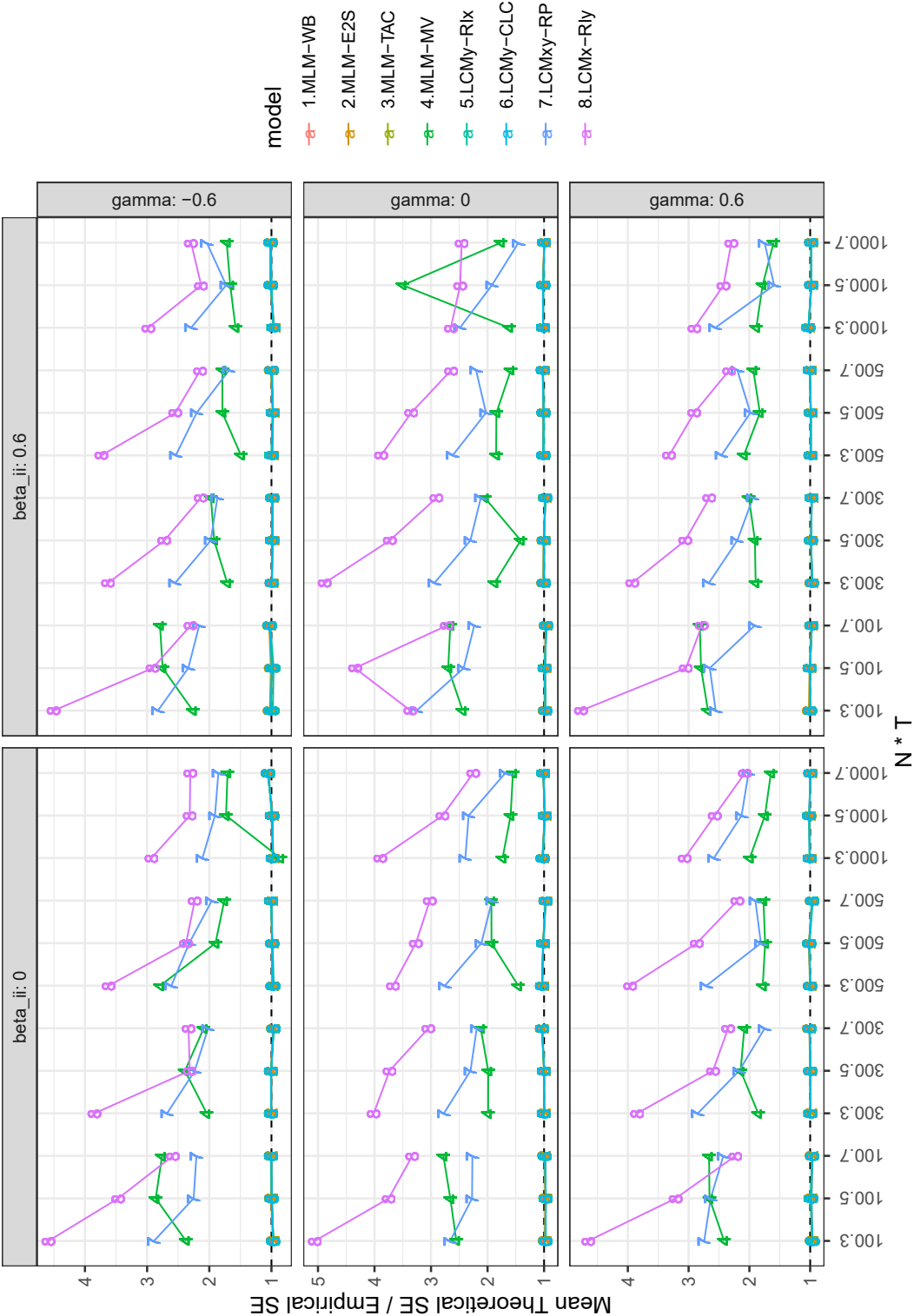


Figure A.19

RMSE for the Between-Person Effect β_{II} in Simulation 1 except LCMx-CLC

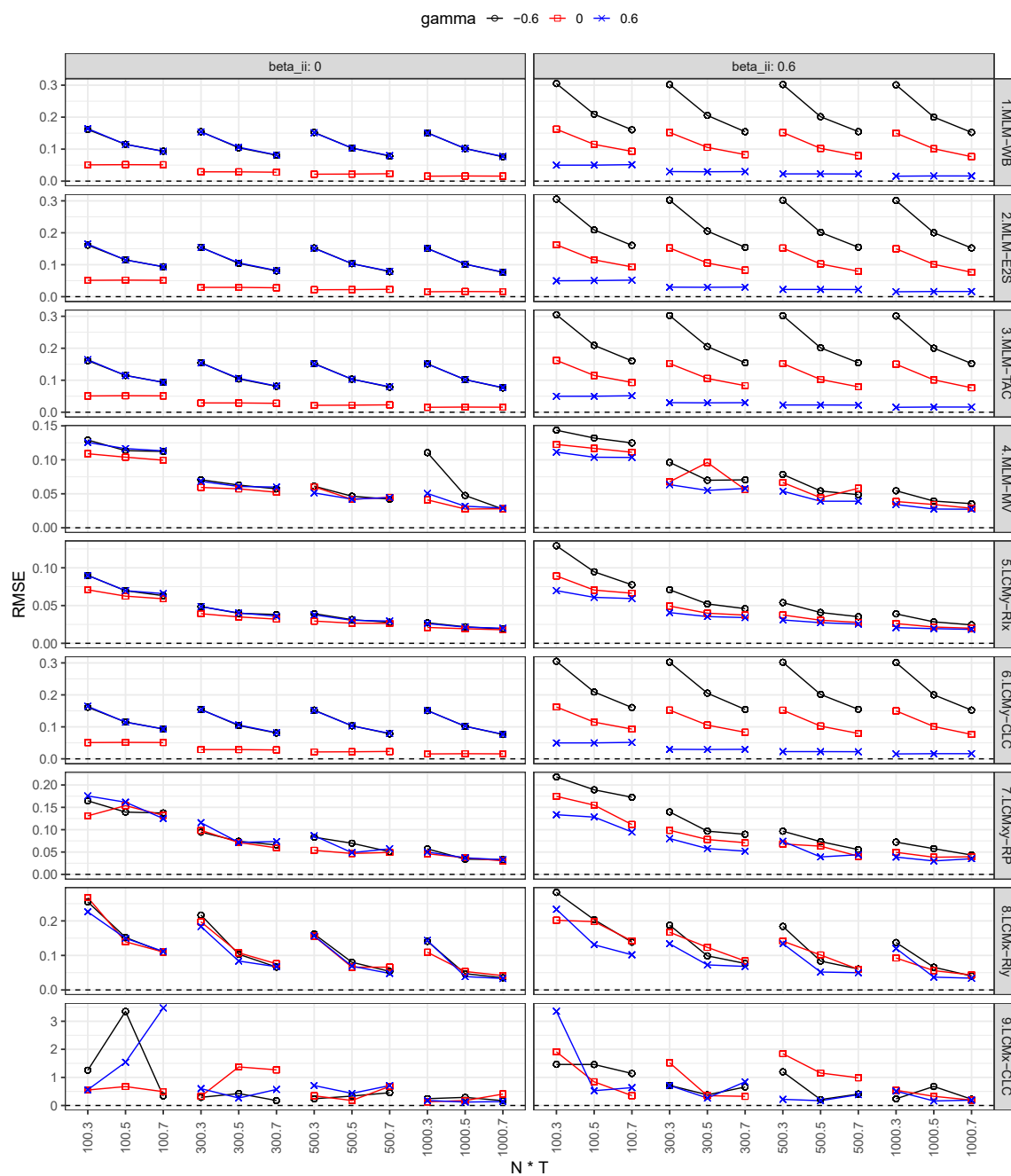


Figure A.20

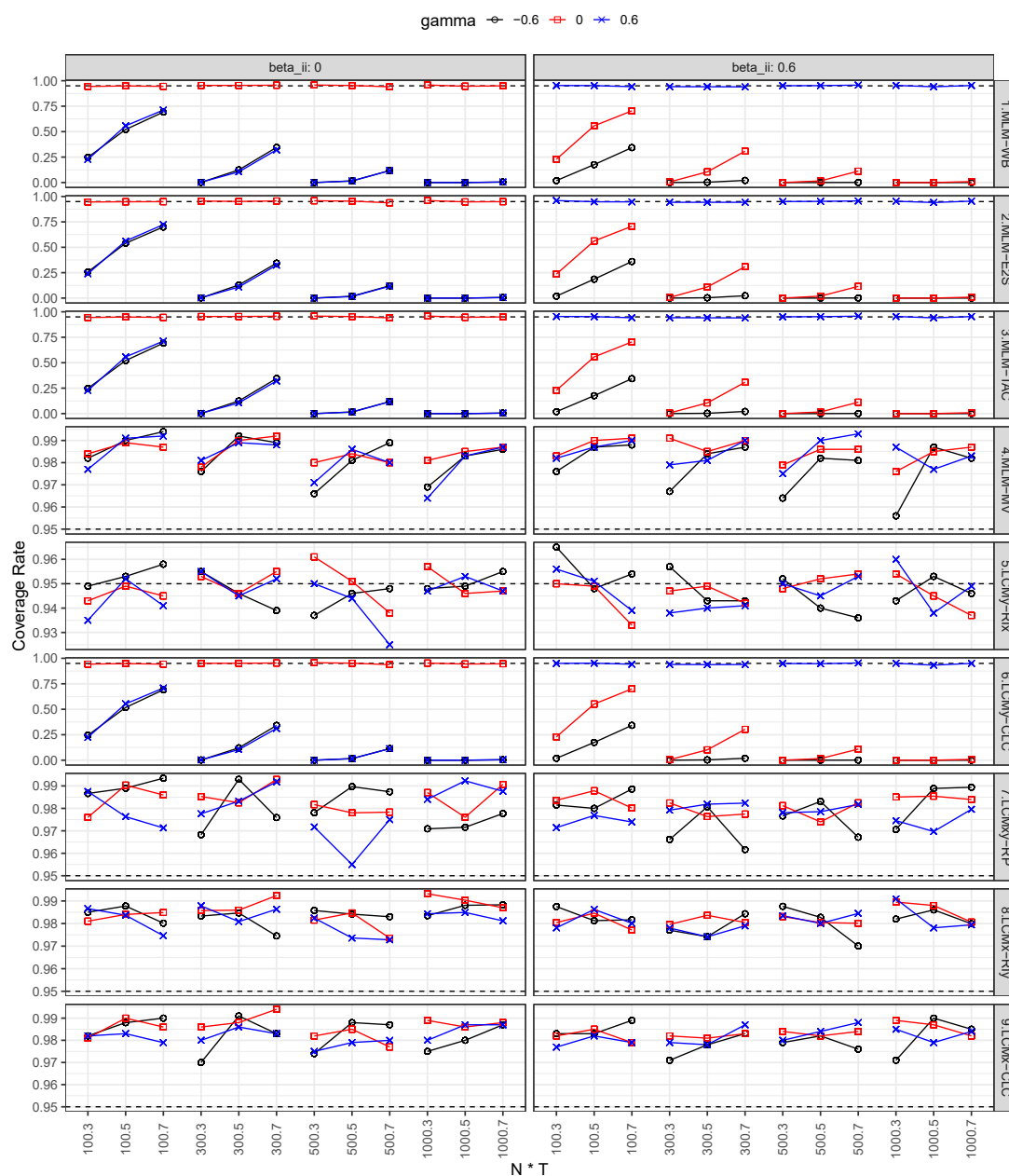
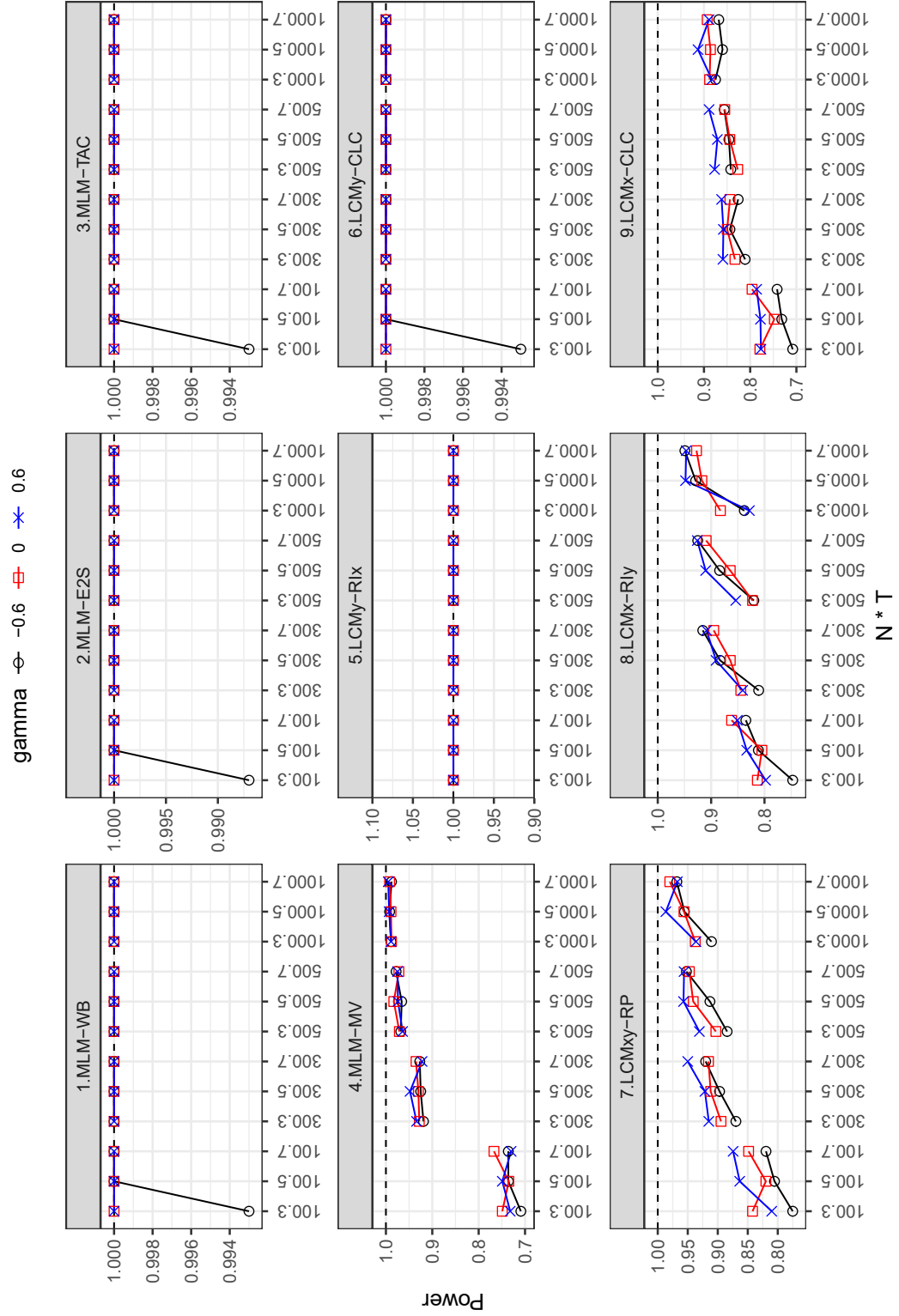
Coverage Rate for the Between-Person Effect β_{II} in Simulation 1

Figure A.21

Power of the Between-Person Effect β_{II} in Simulation 1

A.4 Between-Person Effect of Intercept on Slope

Figure A.22

Bias for the Between-Person Effect β_{SI} in Simulation 1

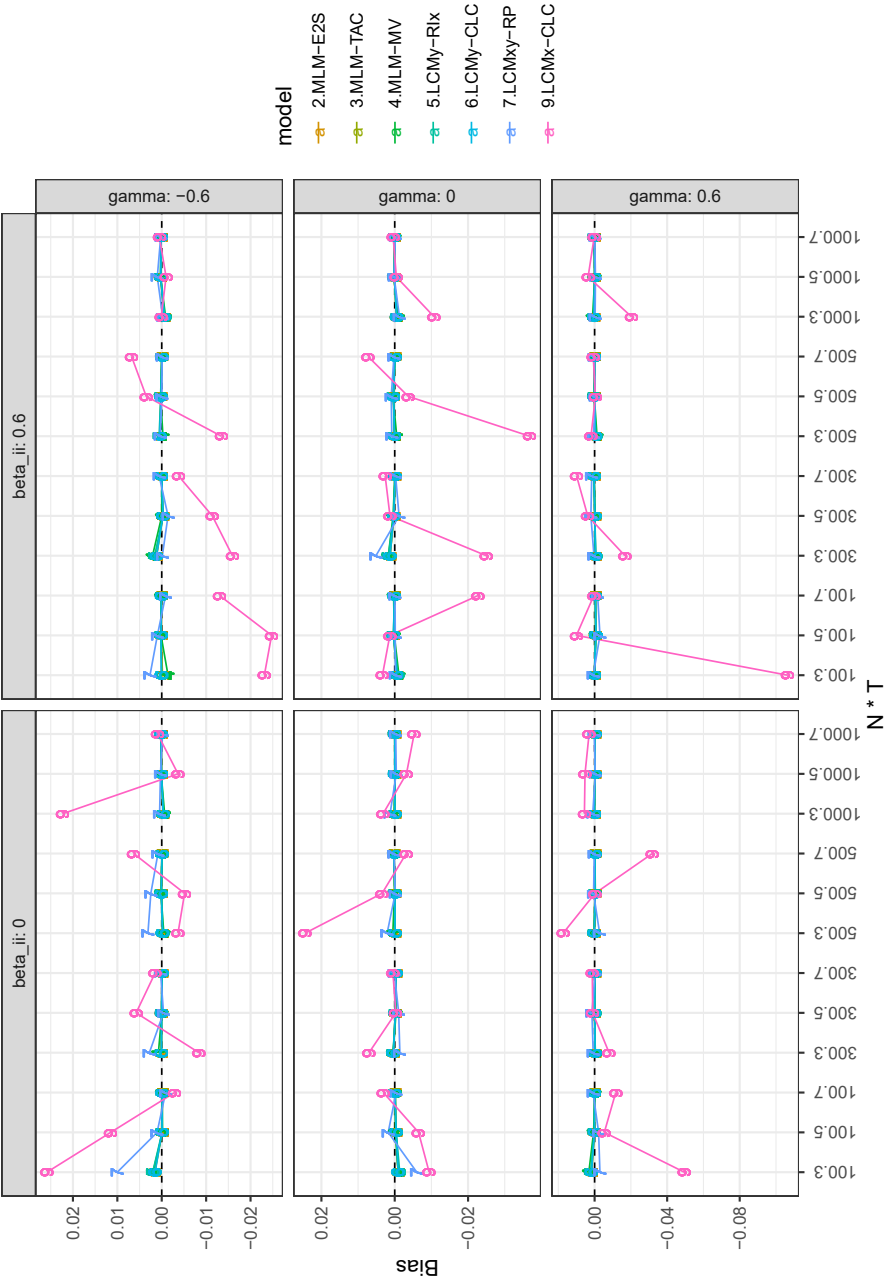


Figure A.23

Bias for the Between-Person Effect β_{SI} in Simulation 1 except LCMx-CLC

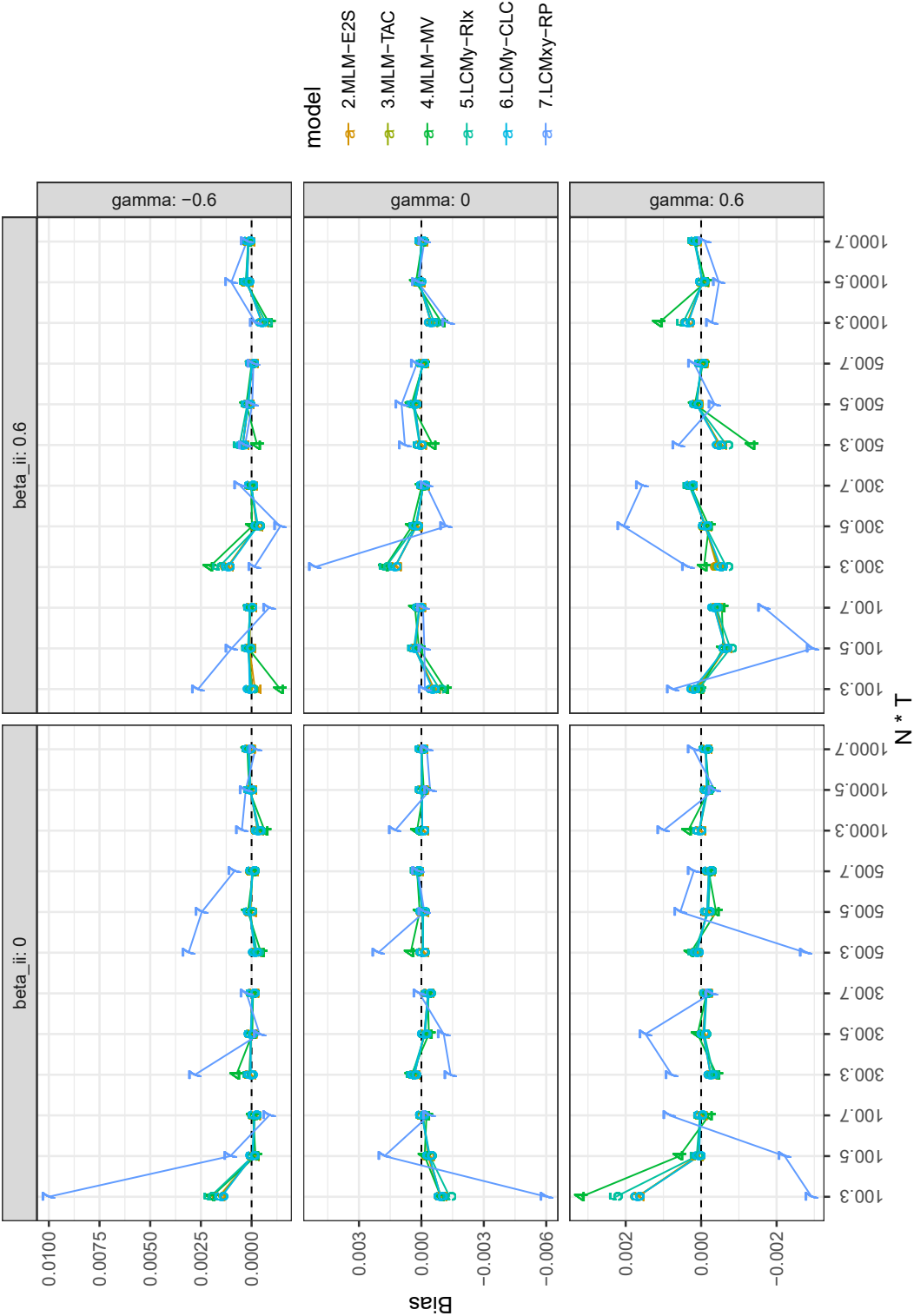


Figure A.24

Empirical SE for the Between-Person Effect β_{SI} in Simulation 1 except LCMx-CLC

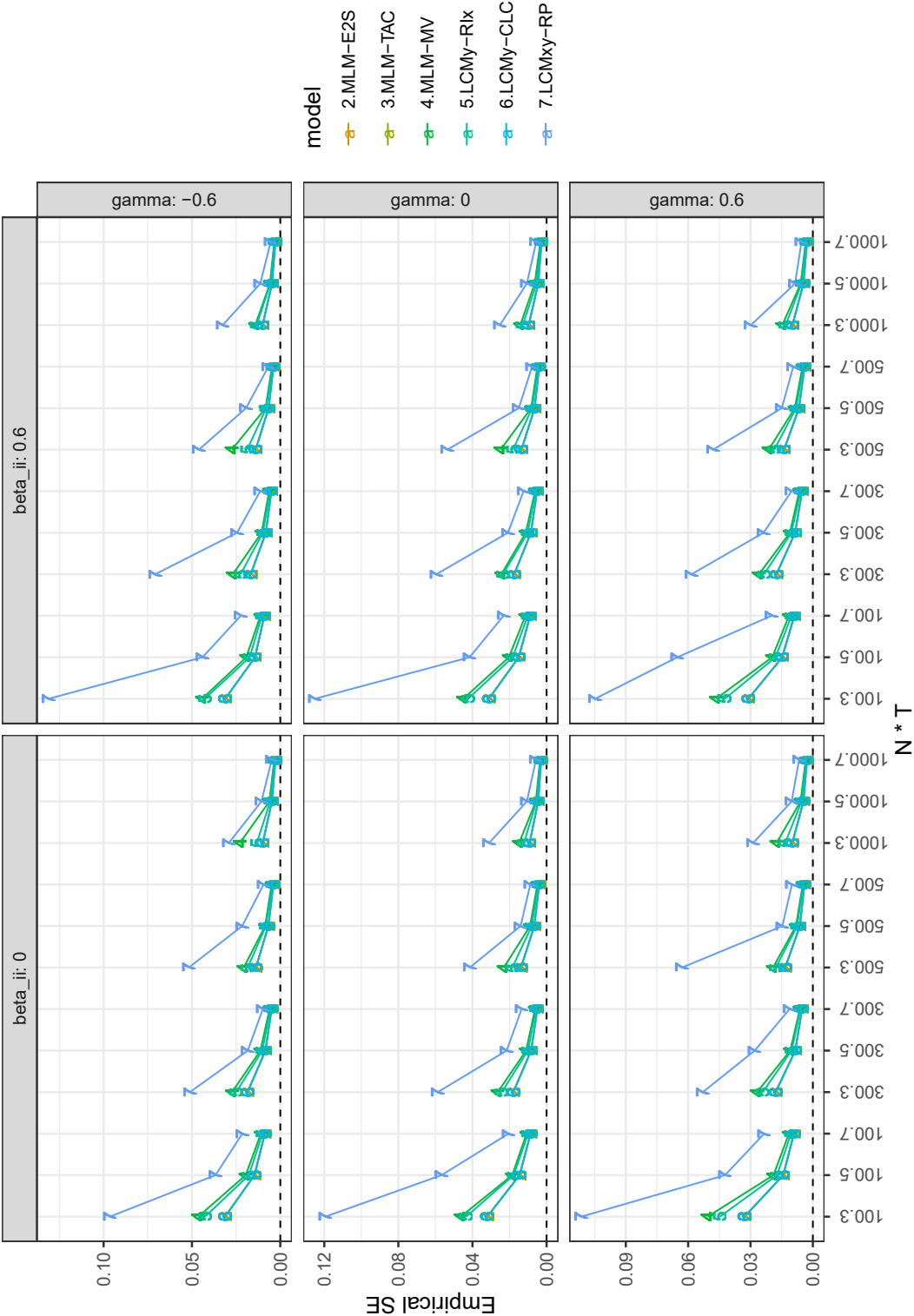


Figure A.25

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 1 except LCMx-CLC

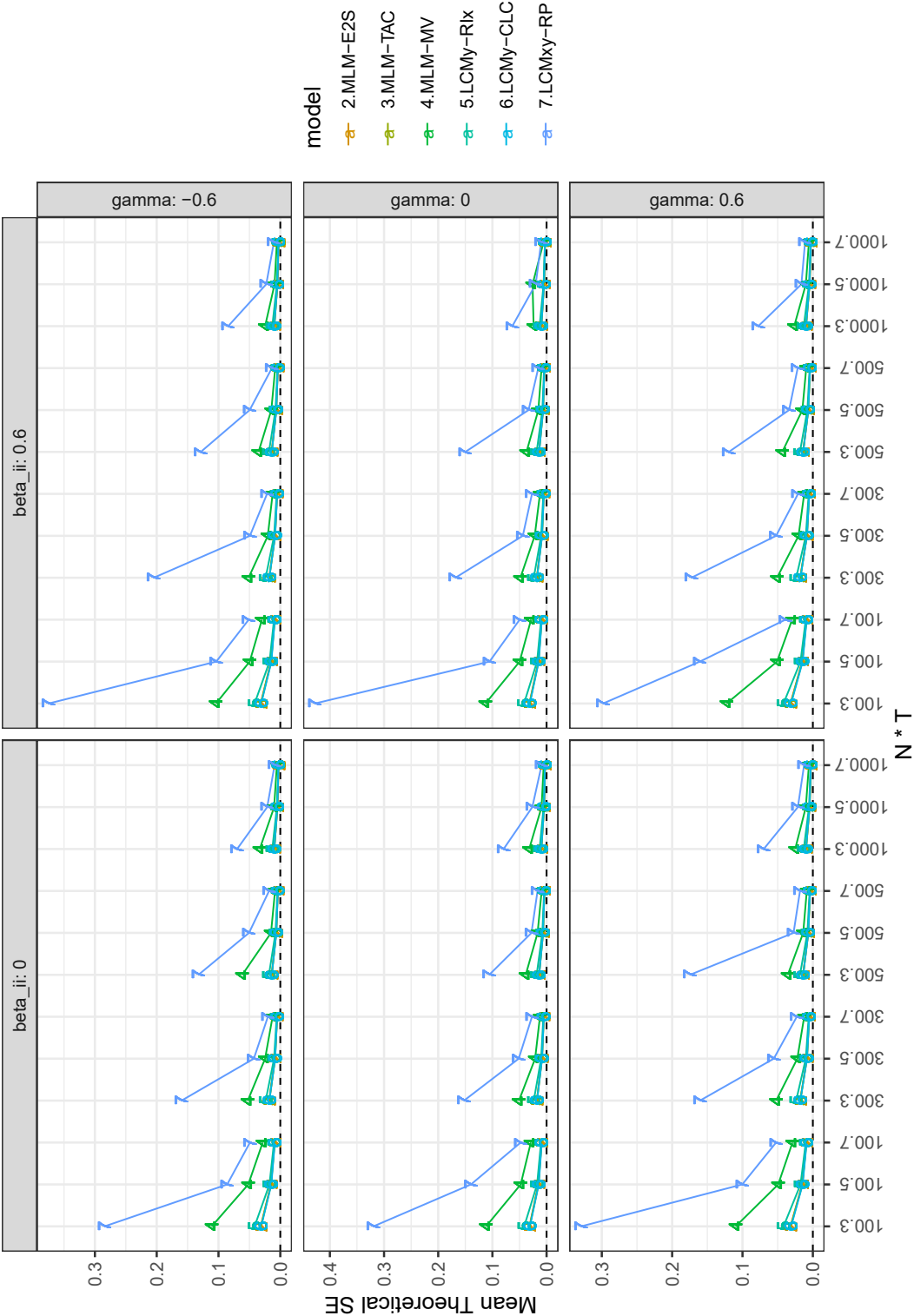


Figure A.26

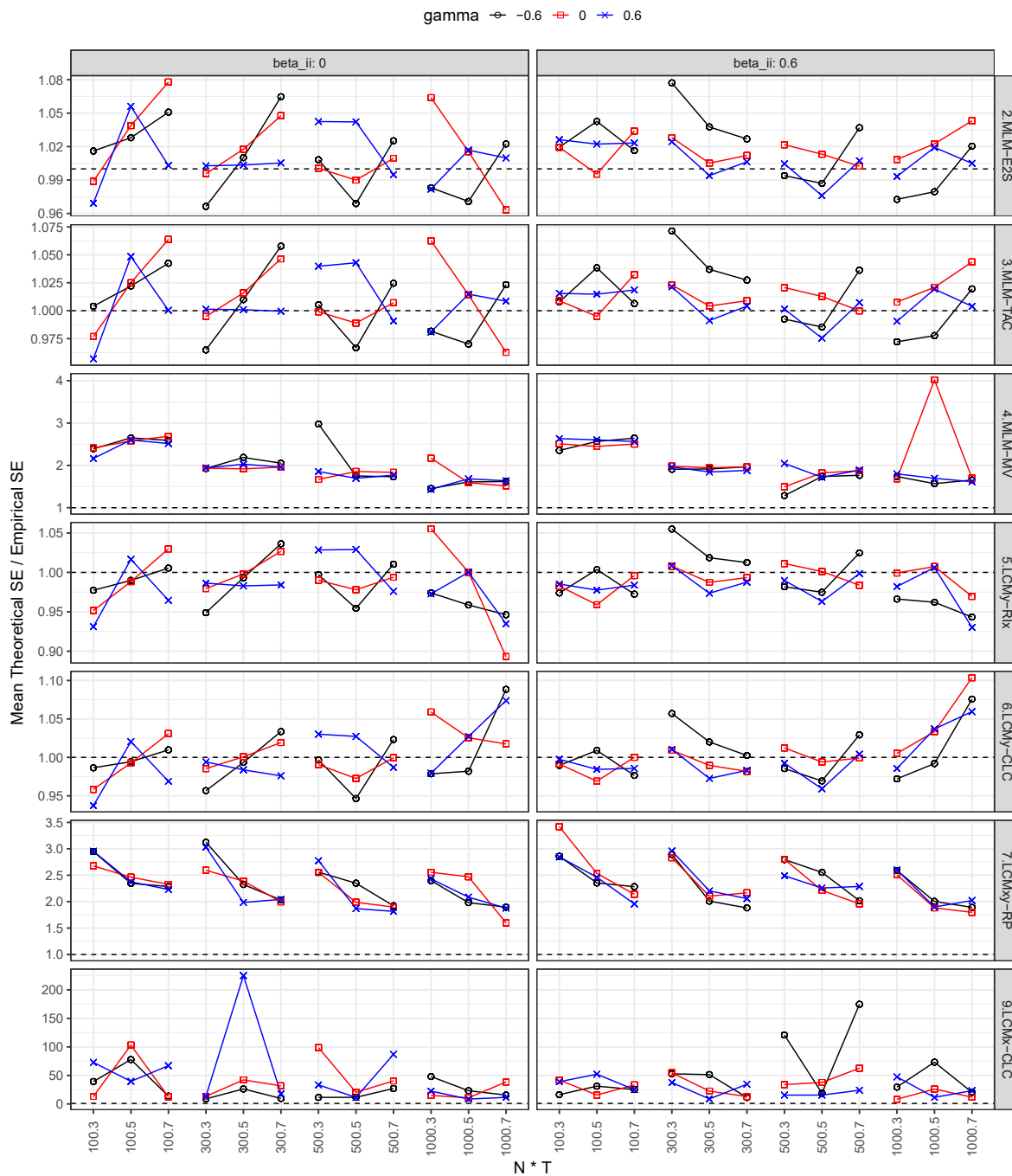
SE Ratio for the Between-Person Effect β_{SI} in Simulation 1

Figure A.27

RMSE for the Between-Person Effect β_{SI} in Simulation 1 except LCMx-CLC

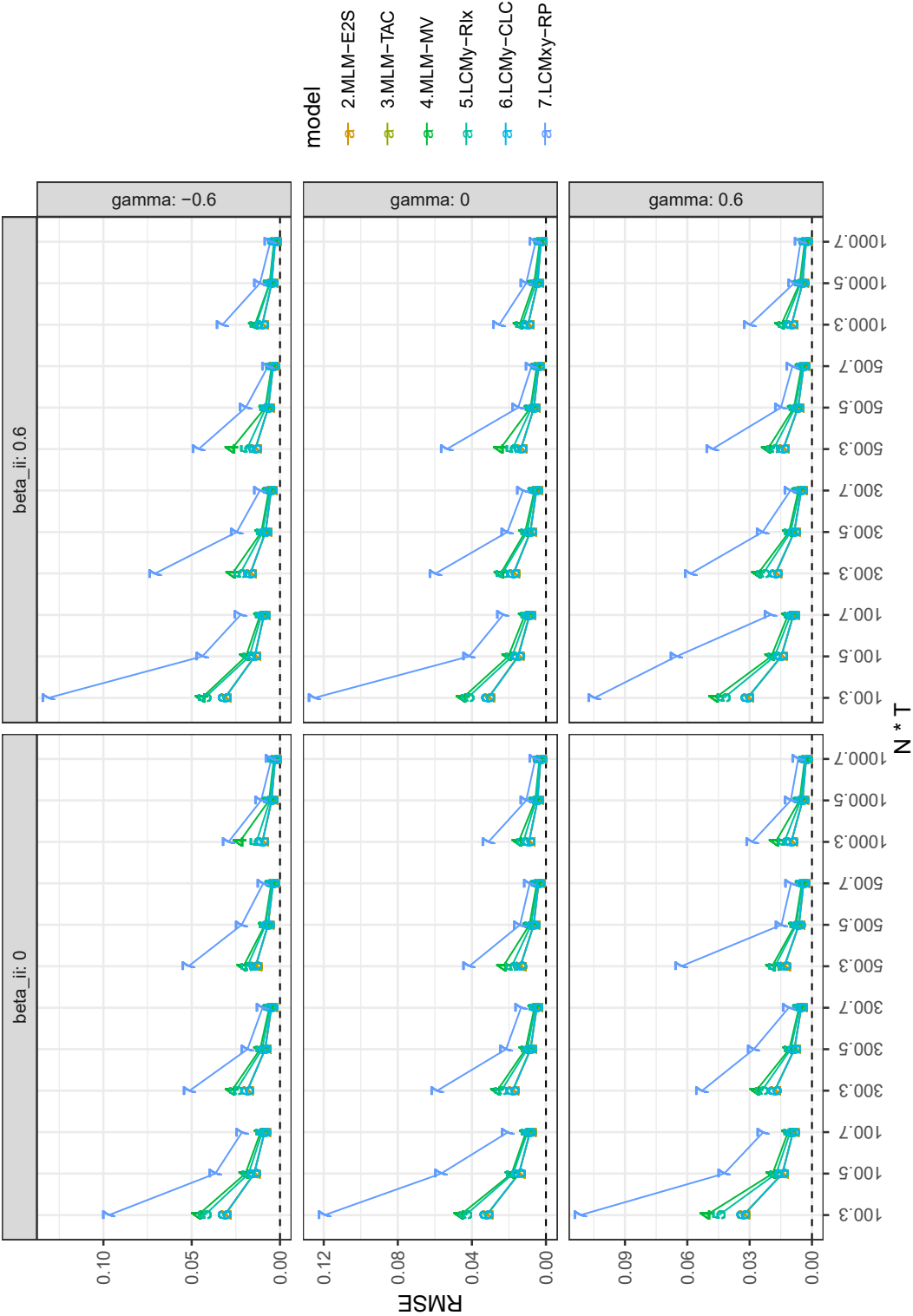


Figure A.28

Coverage Rate for the Between-Person Effect β_{SI} in Simulation 1

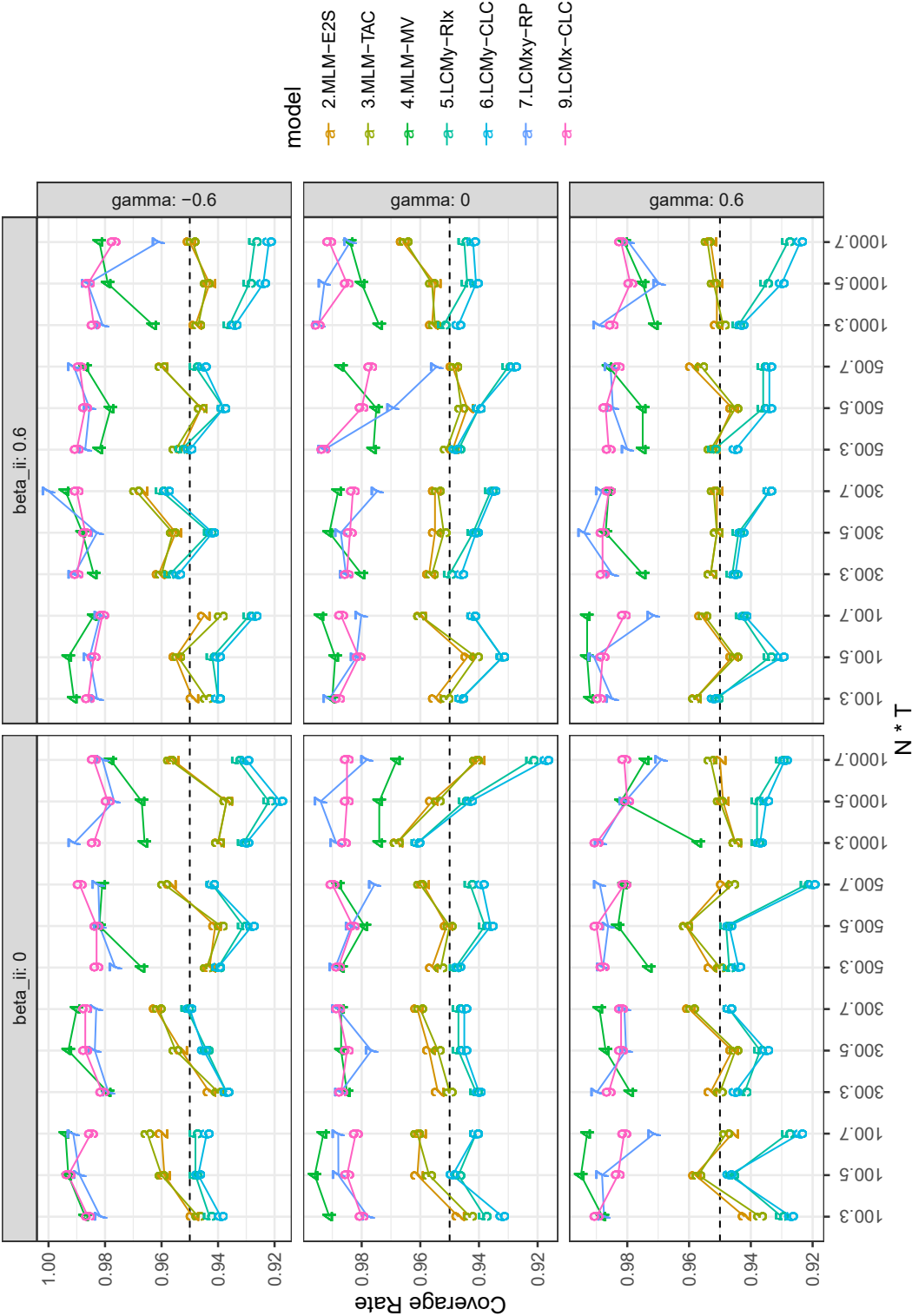


Figure A.29

Bias for the Between-Person Effect β_{IS} in Simulation 1

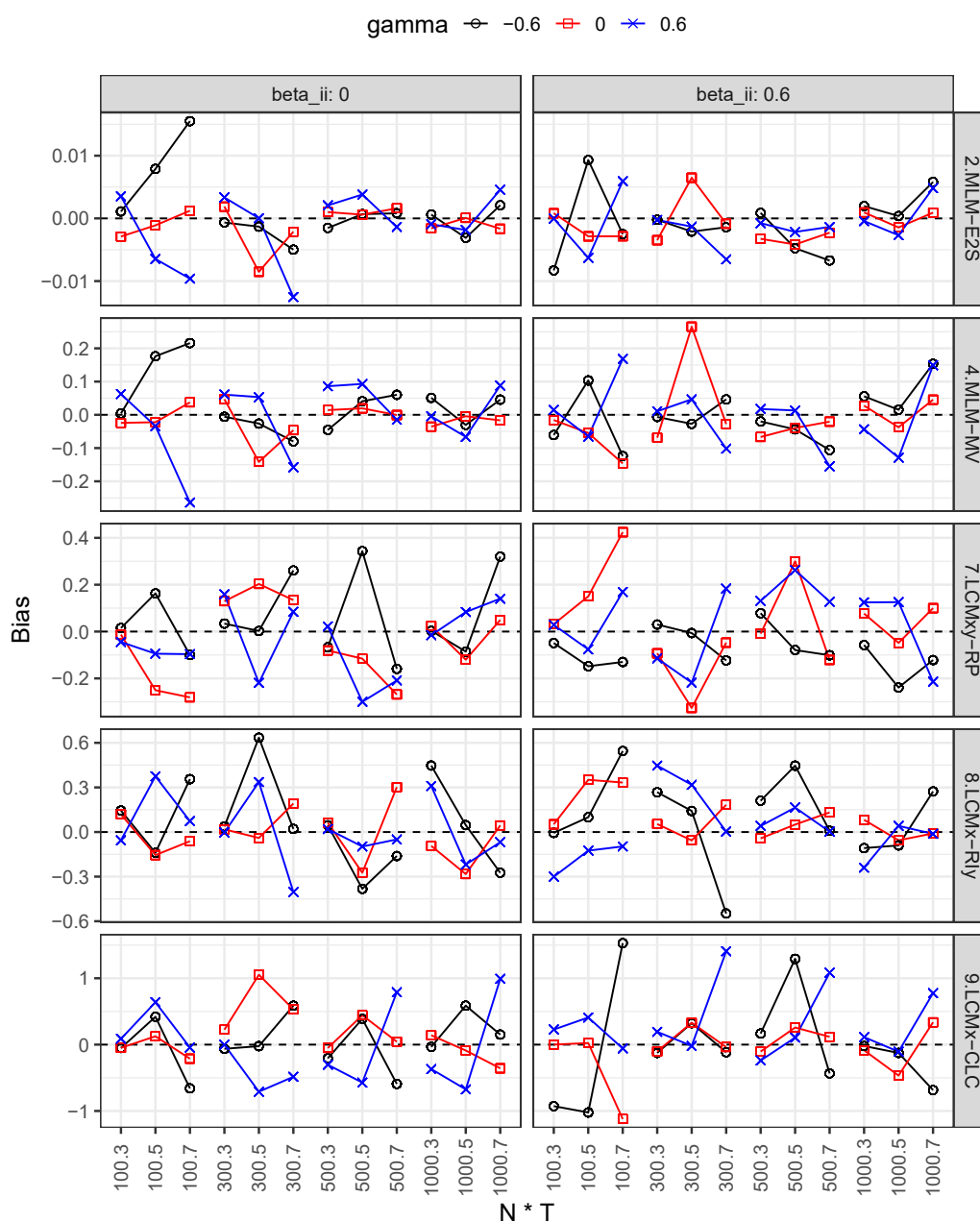


Figure A.30

Empirical SE for the Between-Person Effect β_{IS} in Simulation 1

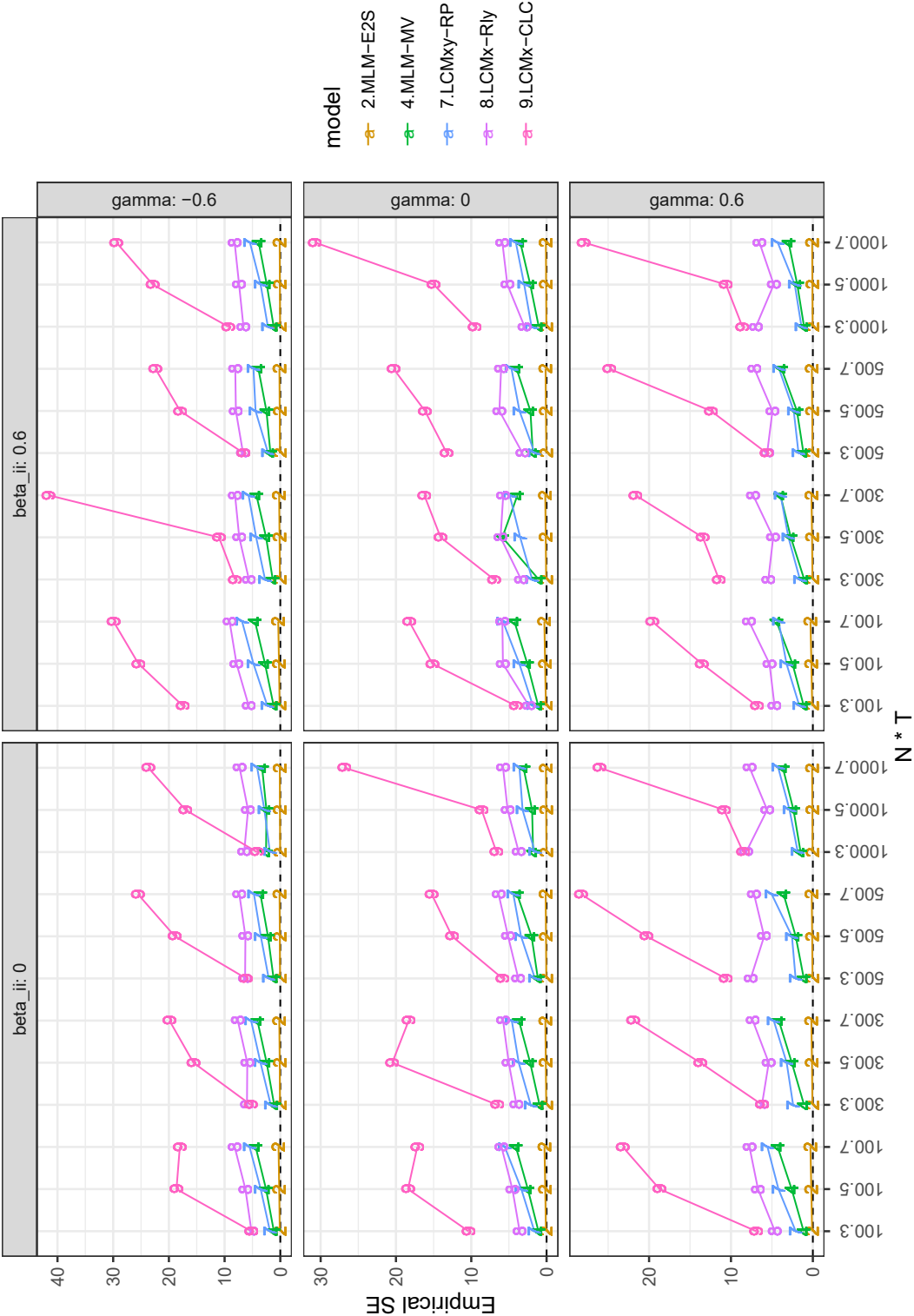


Figure A.31

Empirical SE for the Between-Person Effect β_{1S} in Simulation 1 except LCMx-CLC

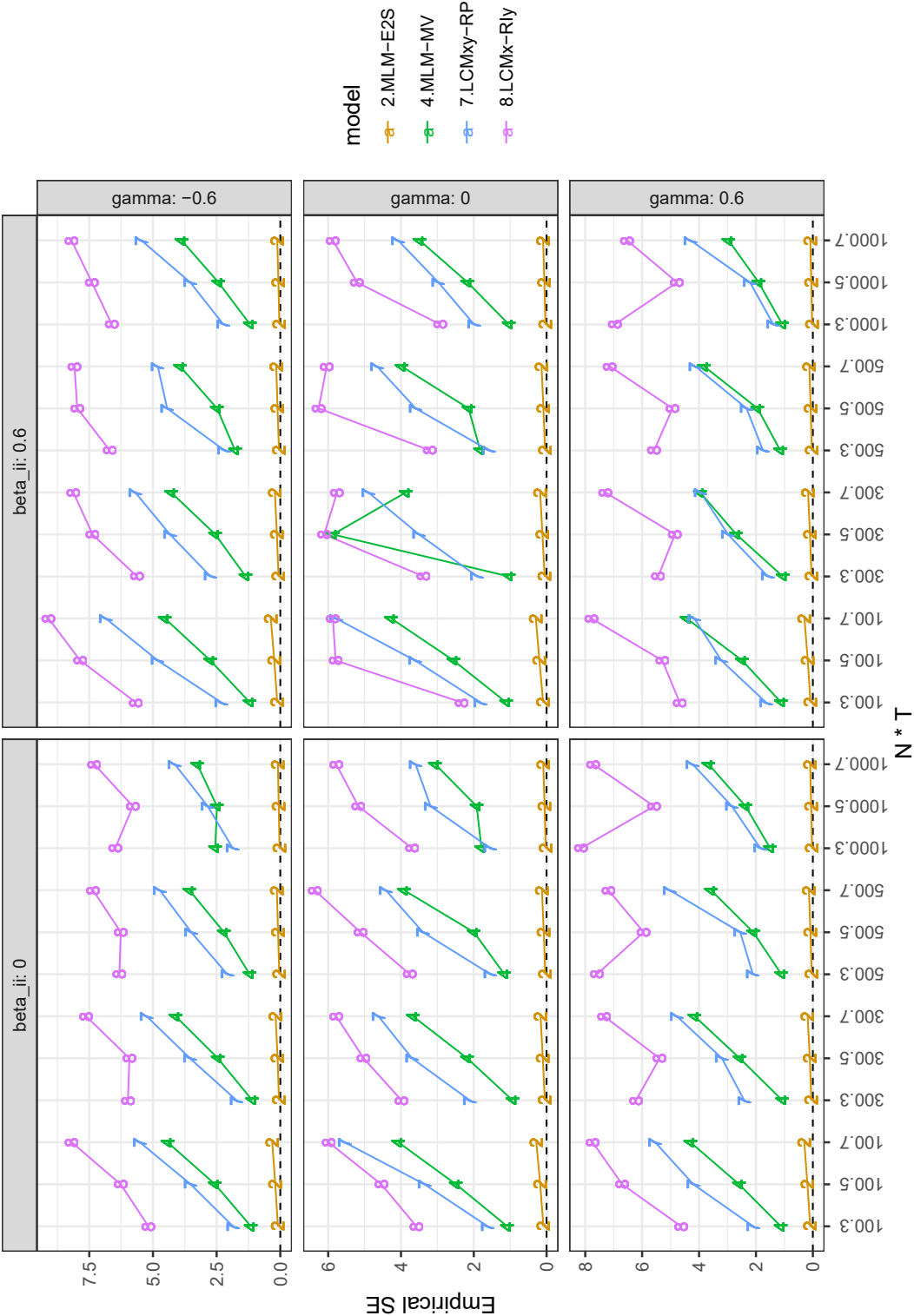


Figure A.32

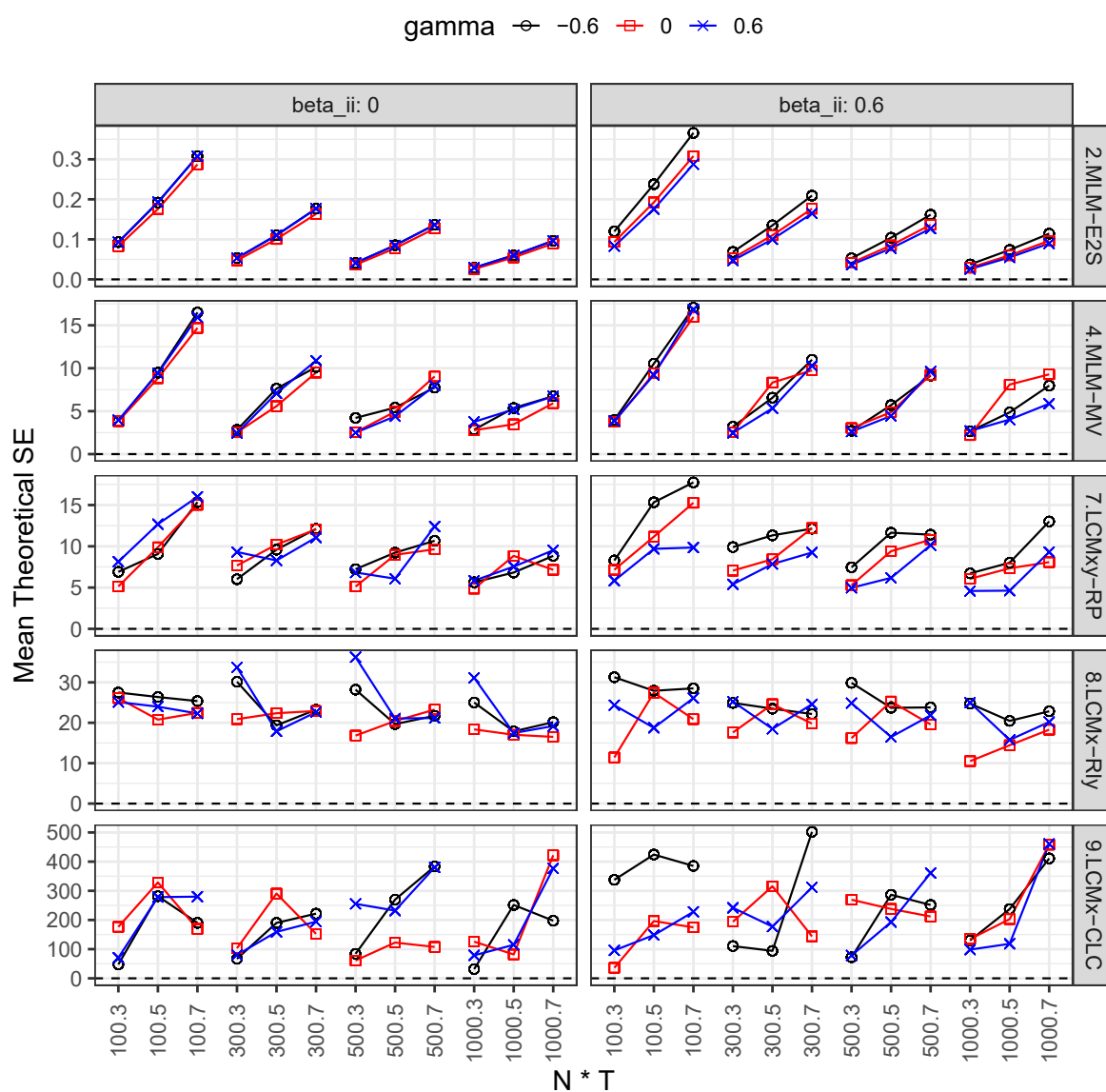
Theoretical SE for the Between-Person Effect β_{IS} in Simulation 1

Figure A.33

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 1 except LCMx-CLC

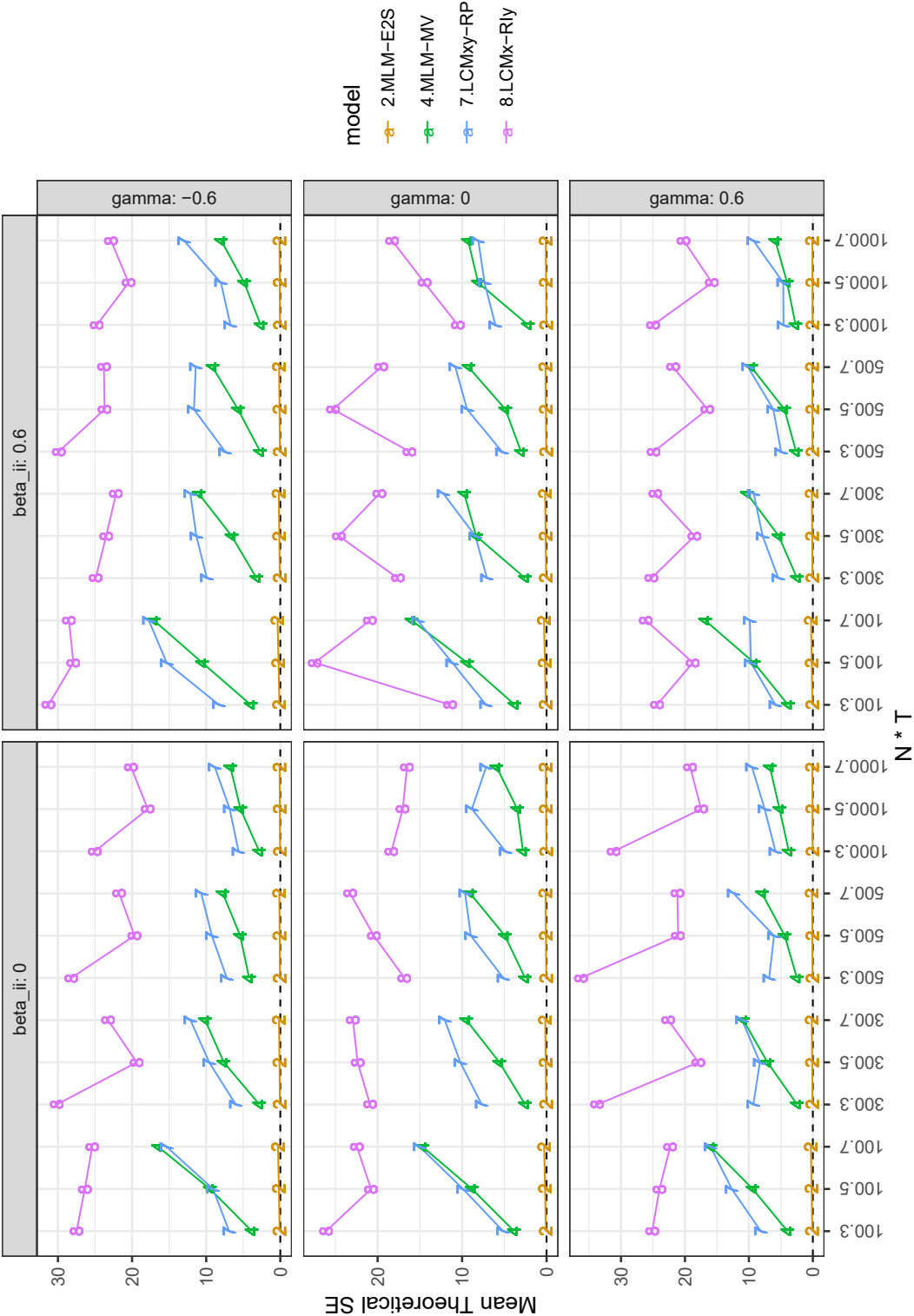


Figure A.34

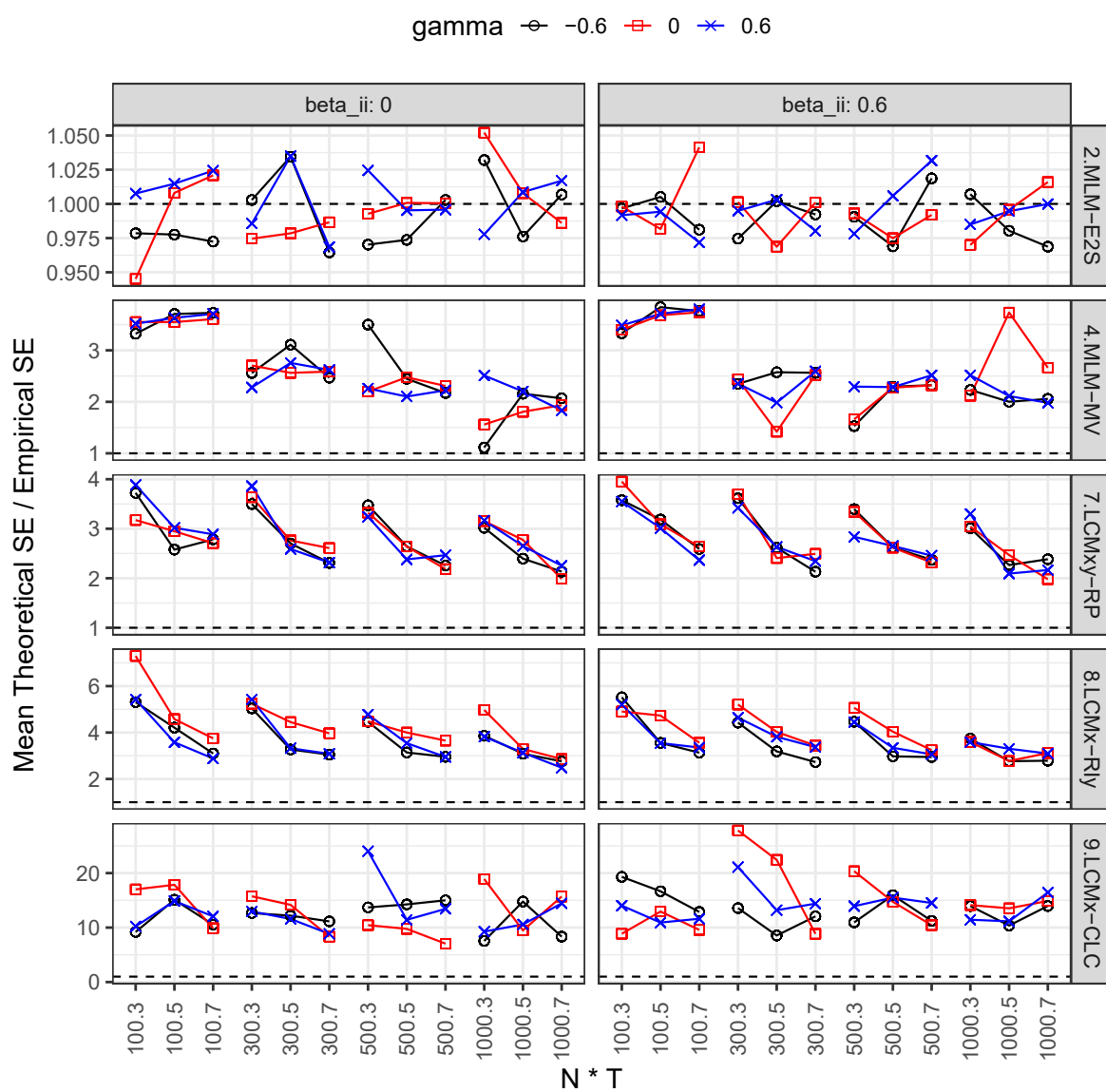
SE Ratio for the Between-Person Effect β_{IS} in Simulation 1

Figure A.35

SE Ratio for the Between-Person Effect β_{IS} in Simulation 1 except LCMx-CLC

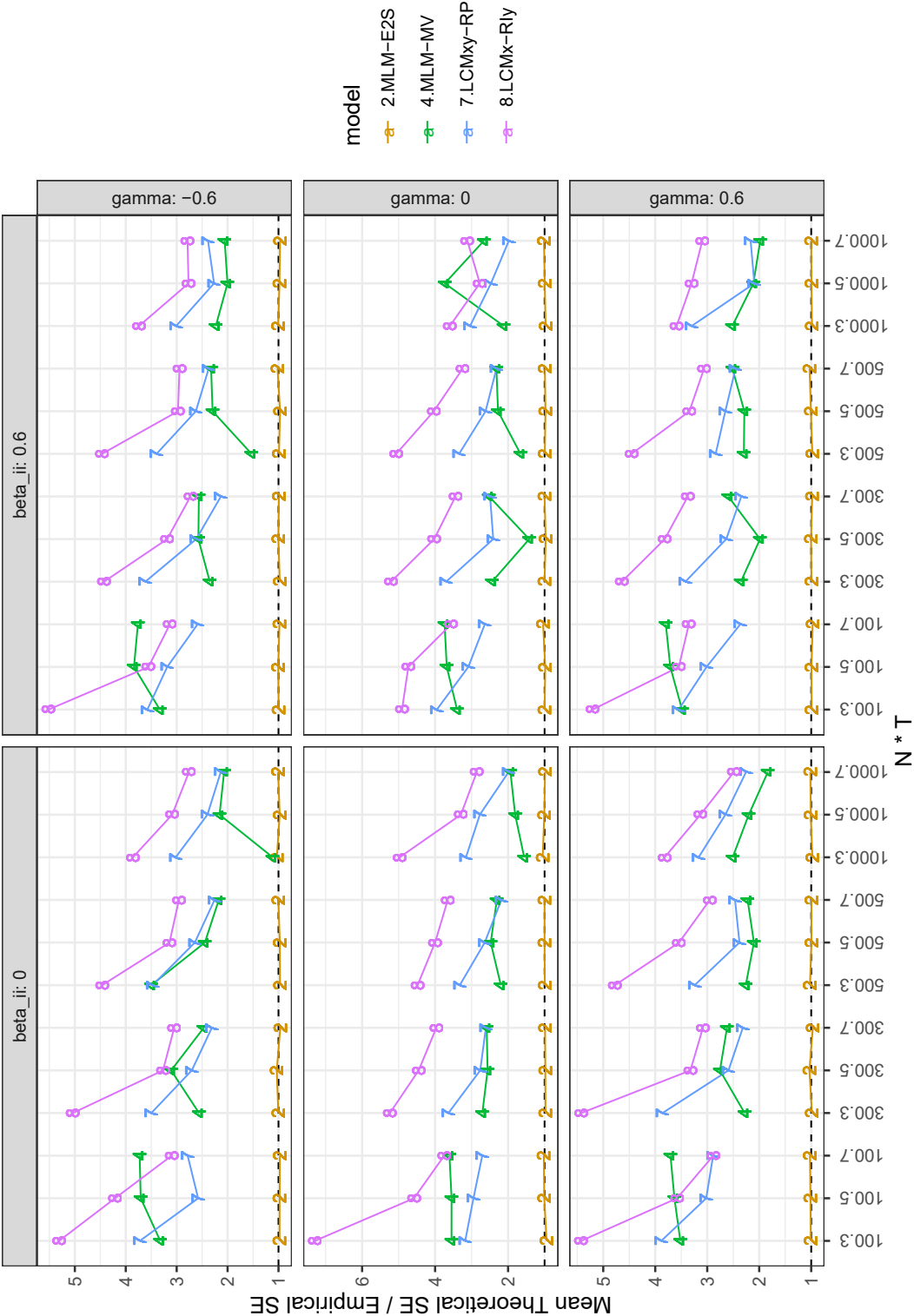
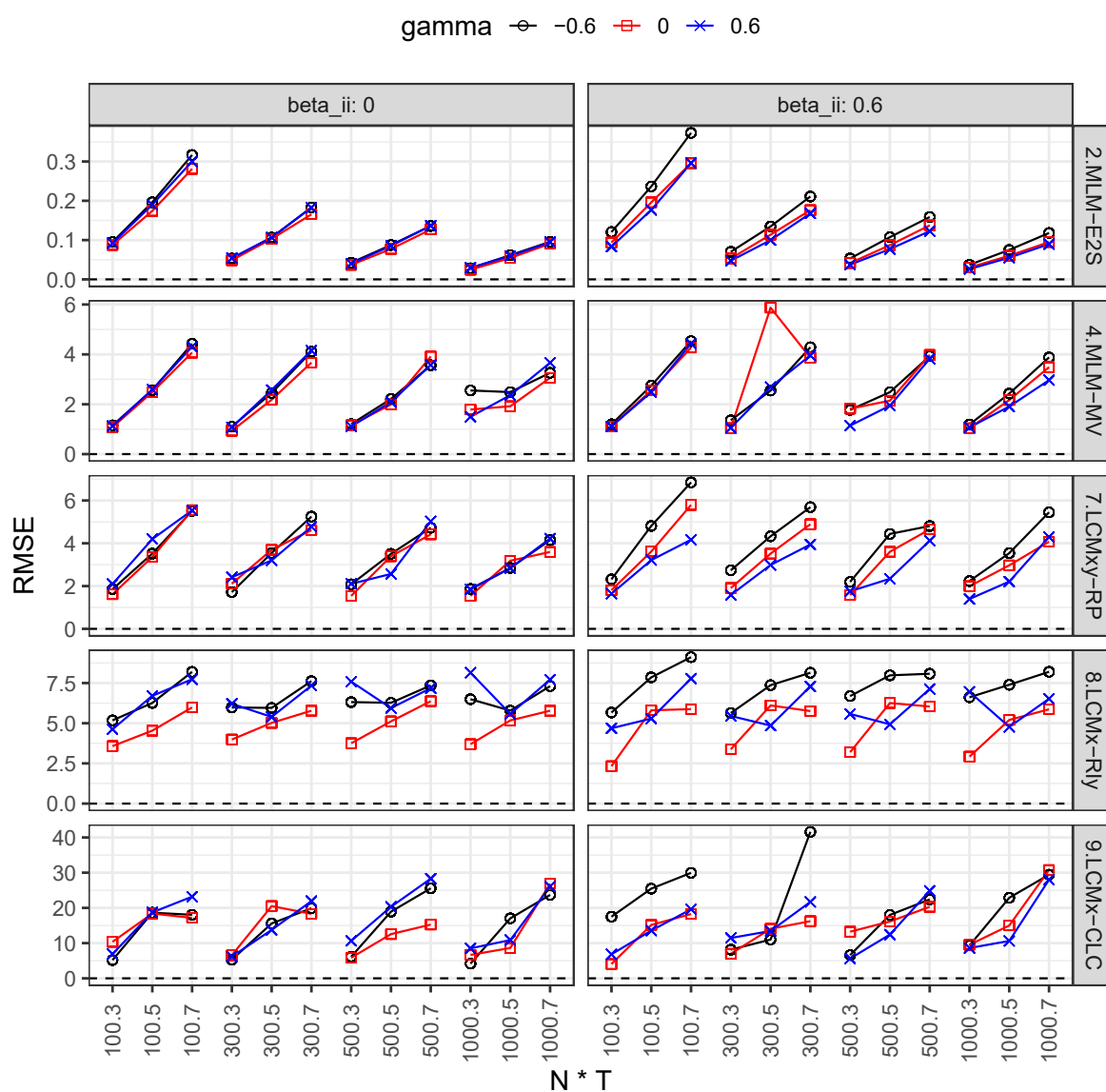


Figure A.36*RMSE for the Between-Person Effect β_{IS} in Simulation 1*

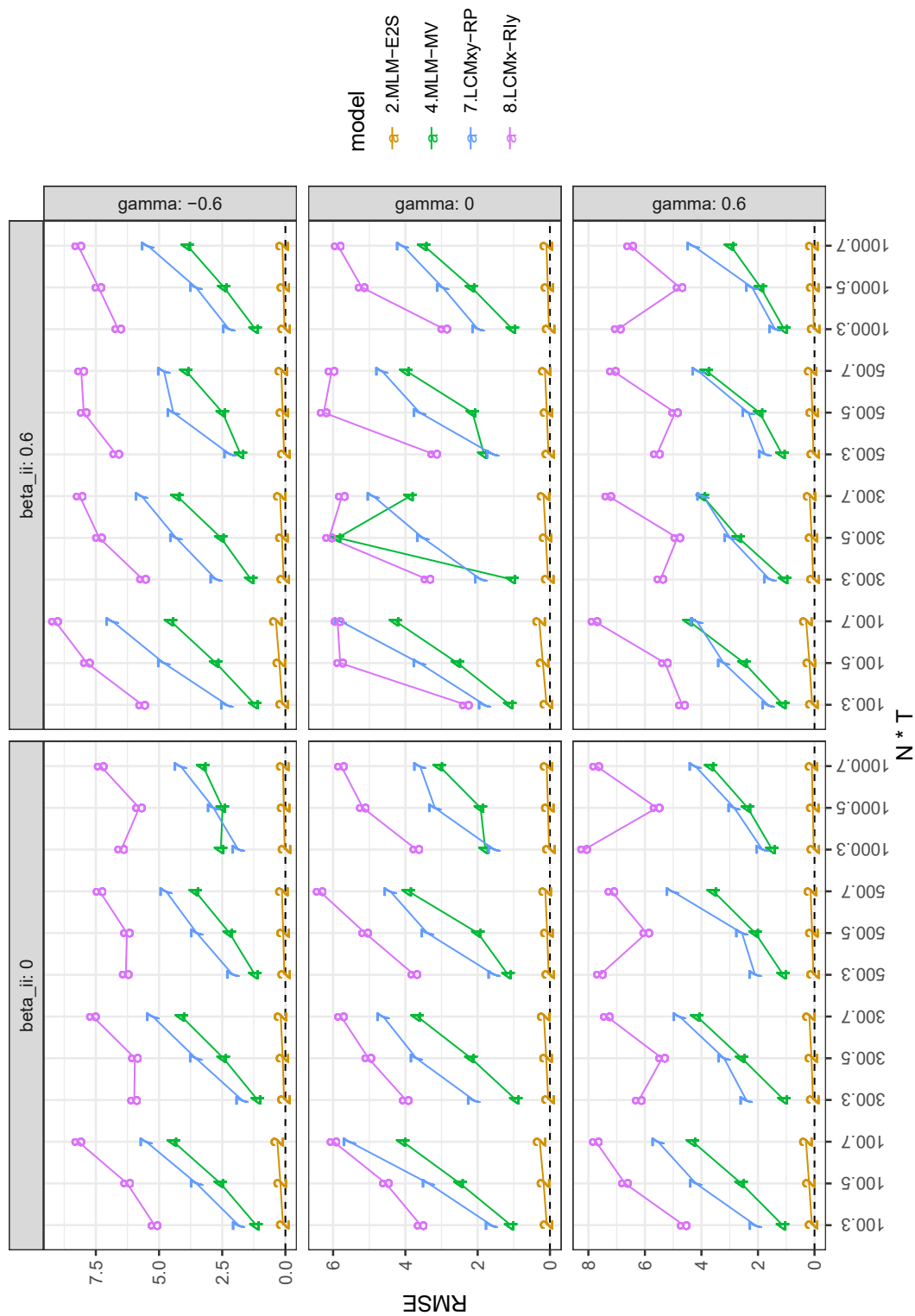
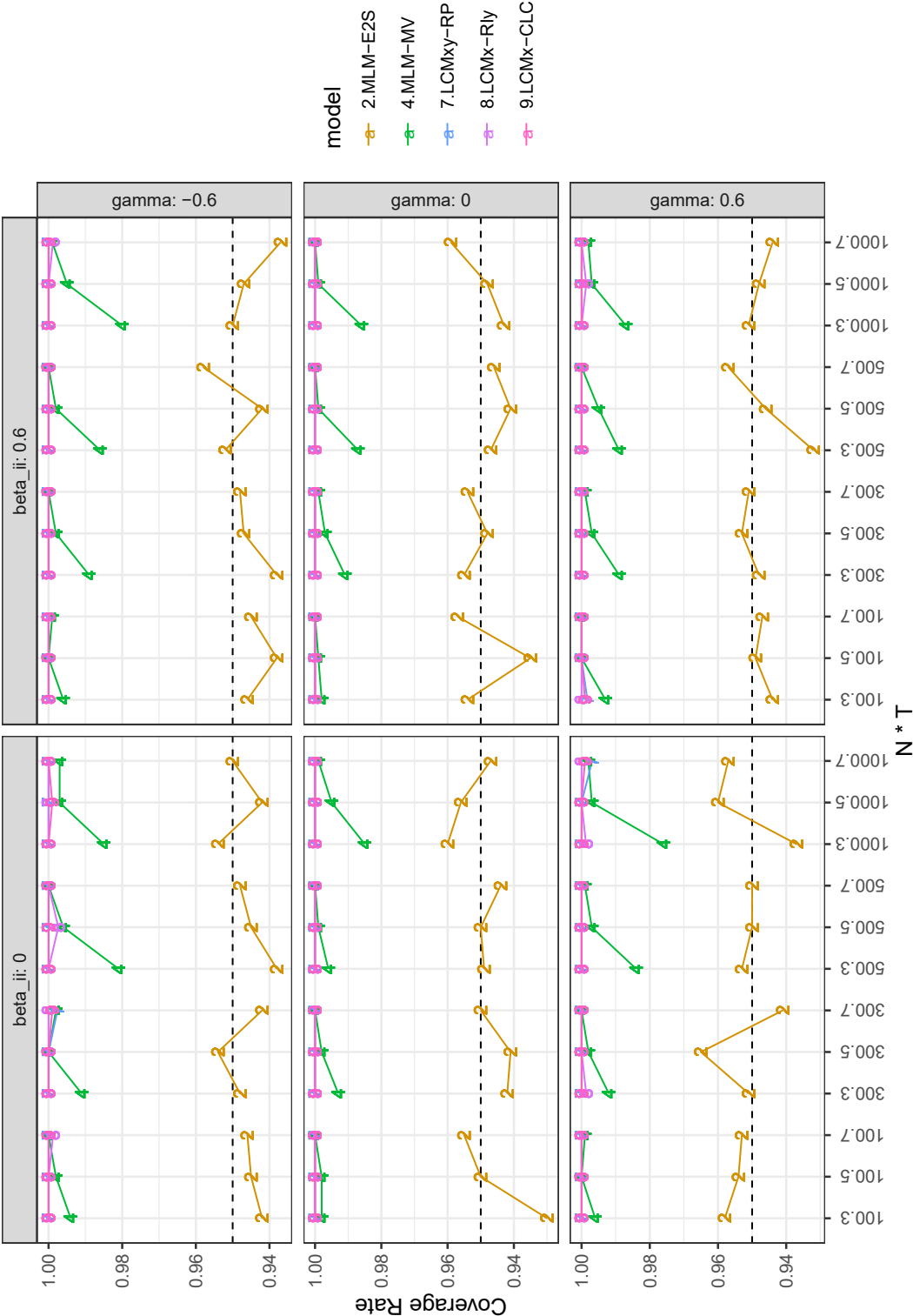
RMSE Ratio for the Between-Person Effect β_{IS} in Simulation 1 except LCMx-CLC

Figure A.38

Coverage Rate for the Between-Person Effect β_{IS} in Simulation 1



A.6 Between-Person Effect of Slope on Slope

Figure A.39

Bias for the Between-Person Effect β_{SS} in Simulation 1

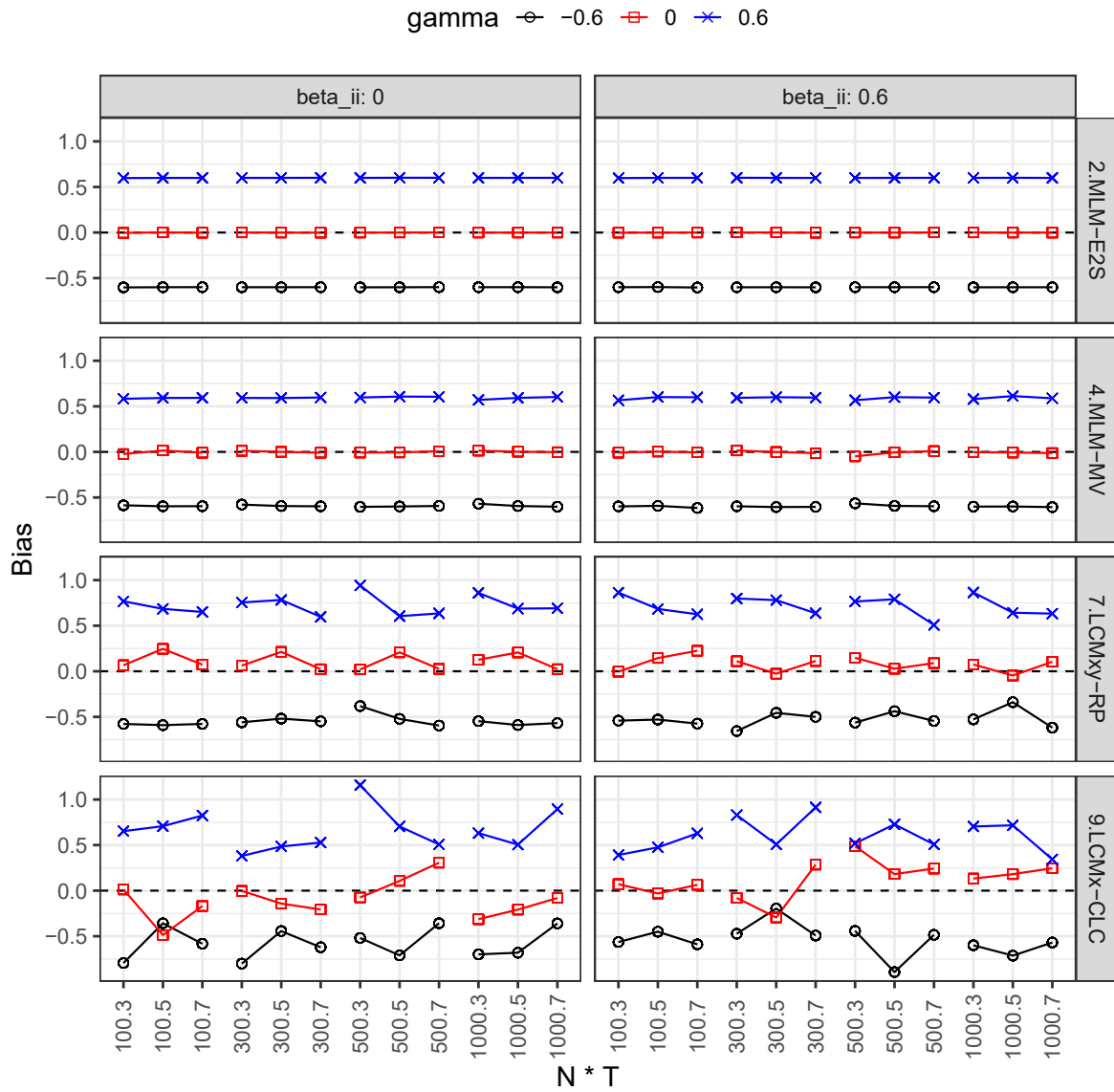
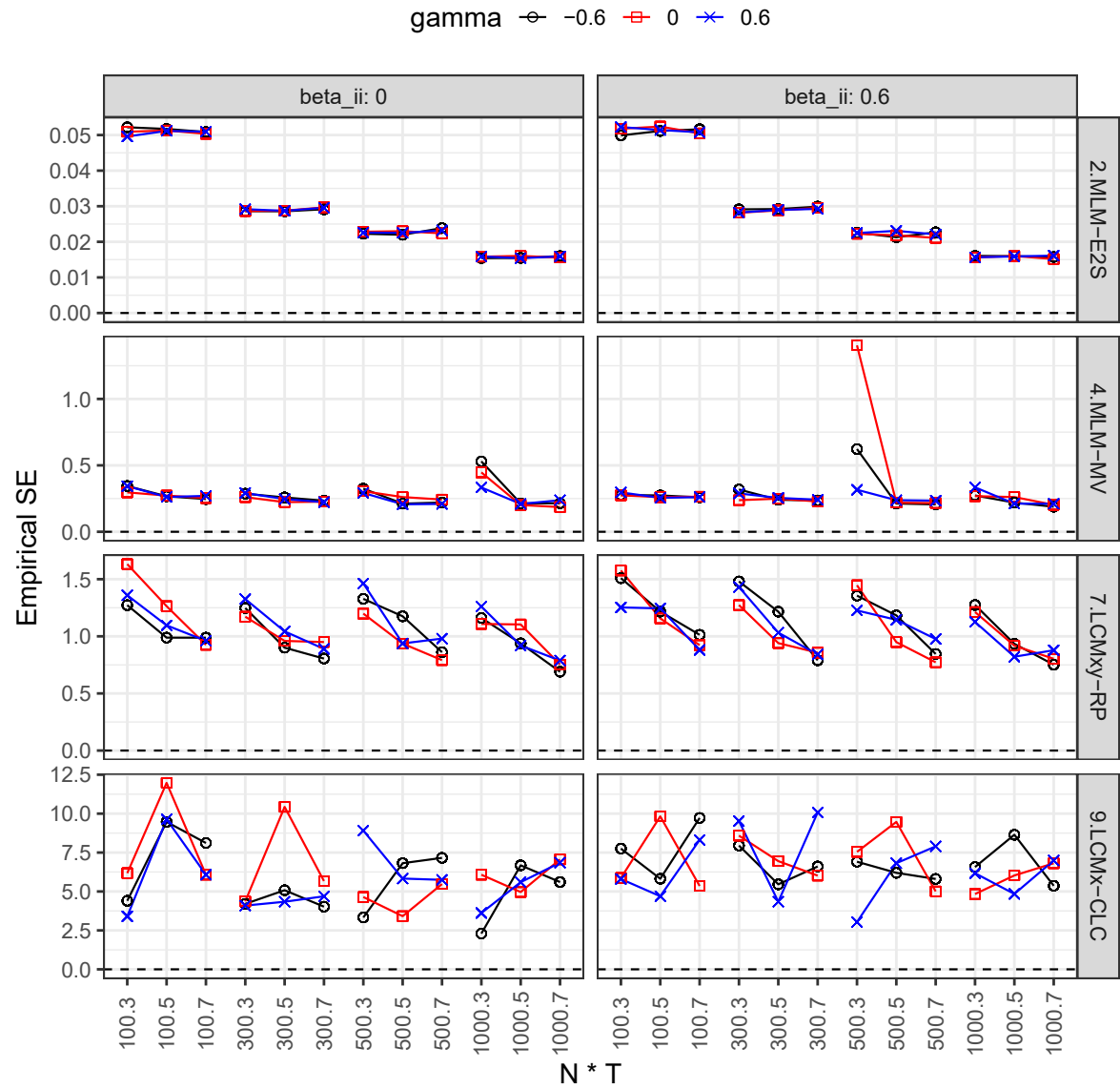


Figure A.40

Empirical SE for the Between-Person Effect β_{SS} in Simulation 1



Empirical SE for the Between-Person Effect β_{SS} in Simulation 1 except LCMx-CLC

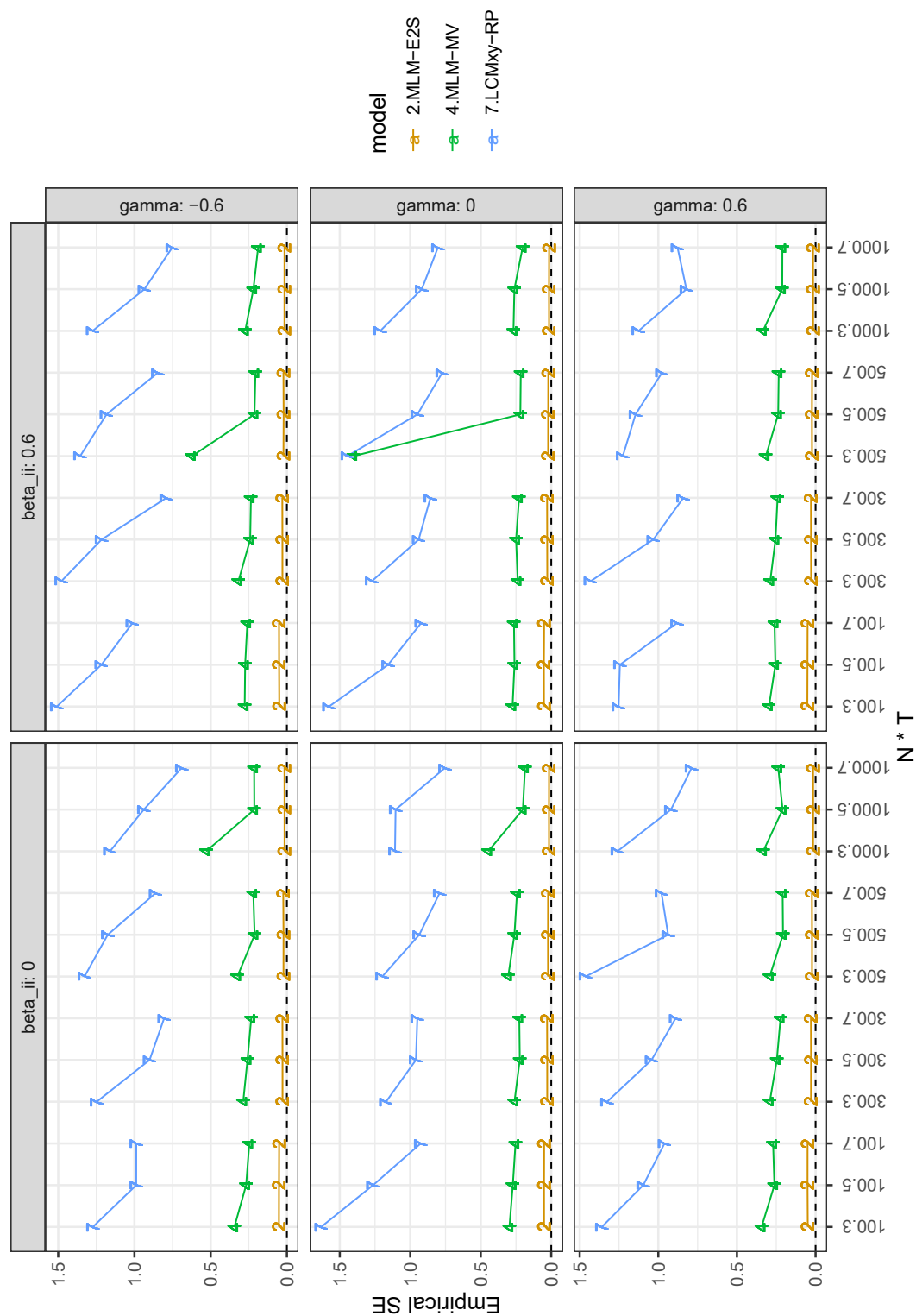


Figure A.42

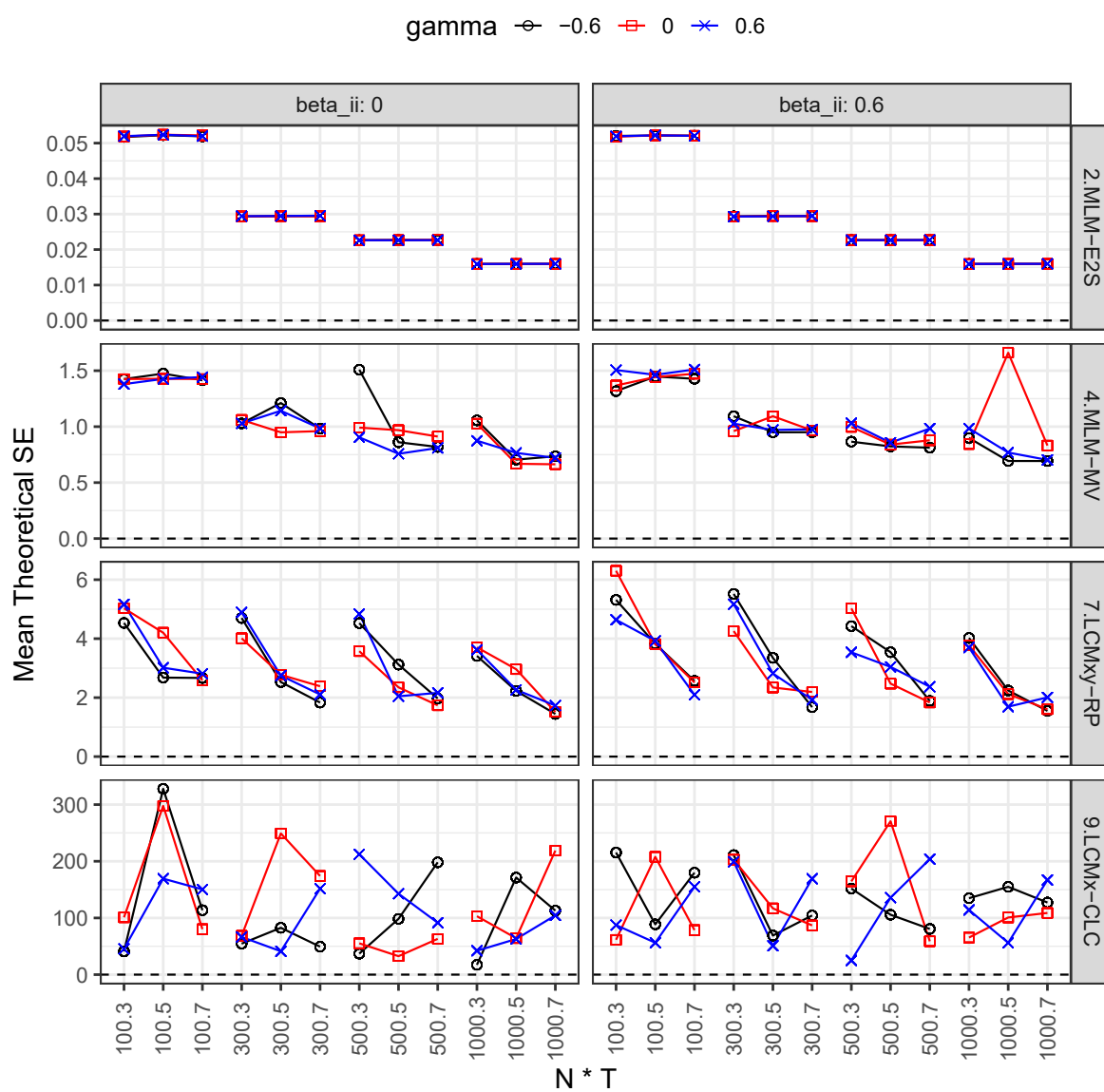
Theoretical SE for the Between-Person Effect β_{SS} in Simulation 1

Figure A.43

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 1 except LCMx-CLC

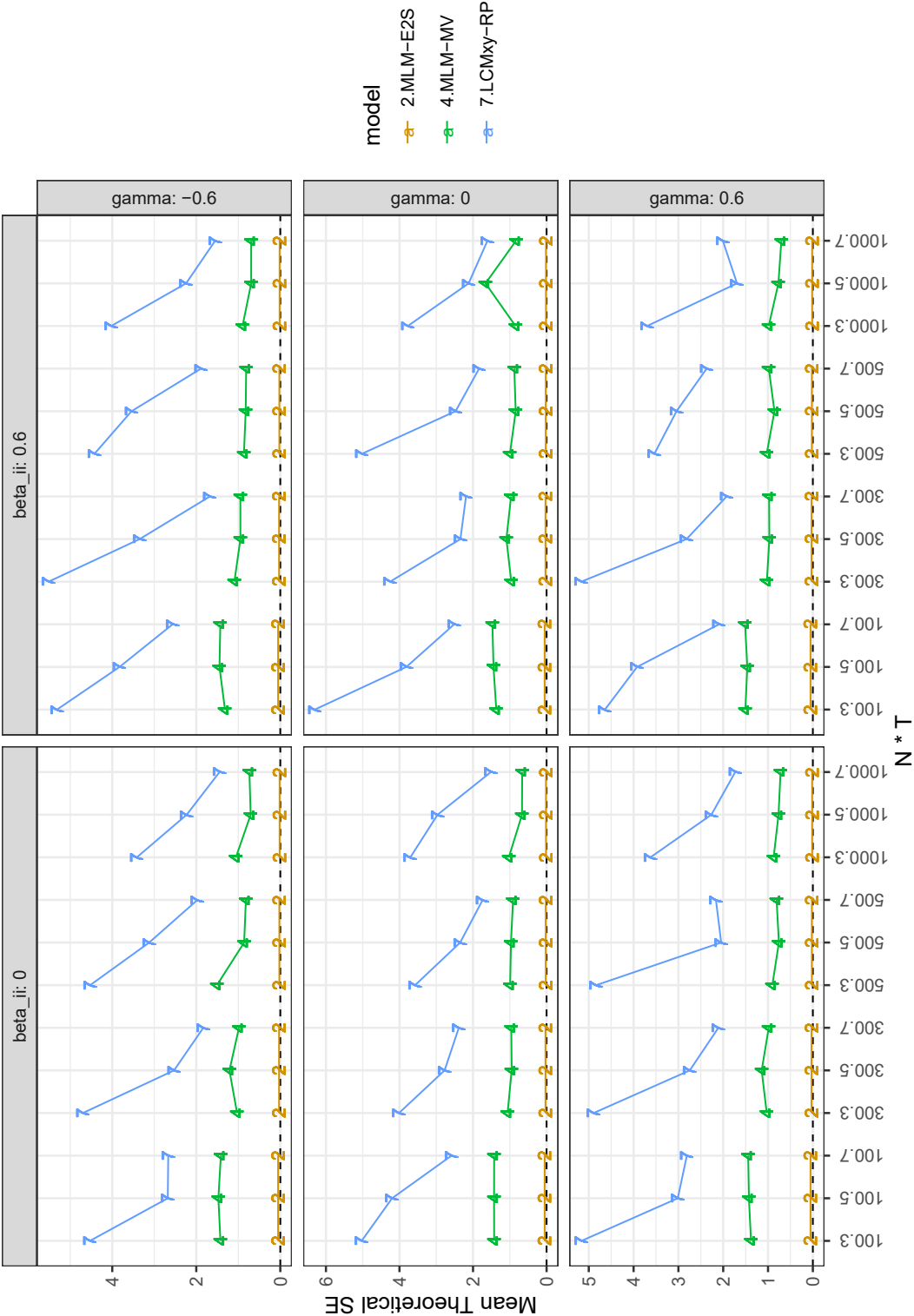


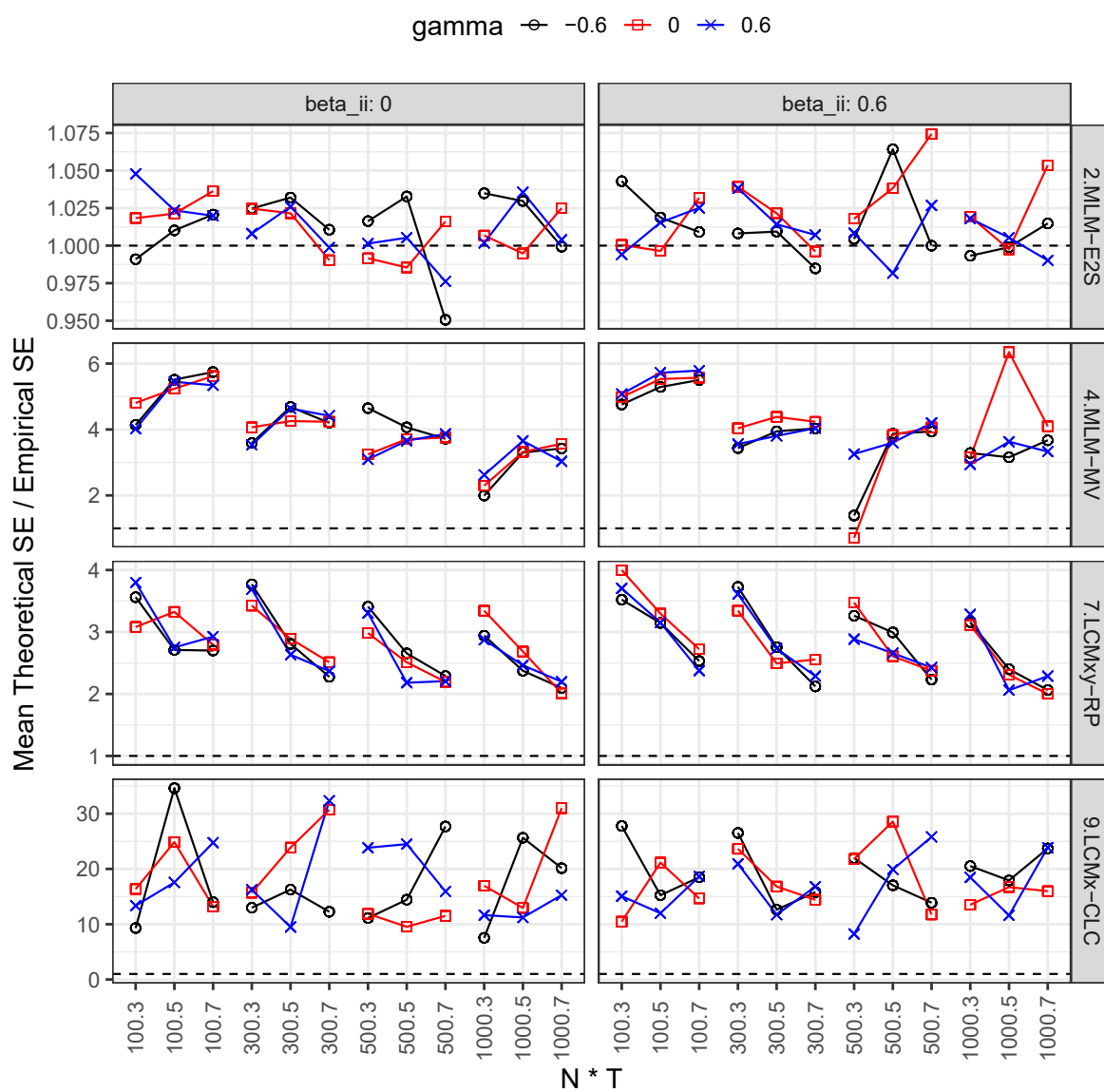
Figure A.44*SE Ratio for the Between-Person Effect β_{SS} in Simulation 1*

Figure A.45

SE Ratio for the Between-Person Effect β_{SS} in Simulation 1 except LCMx-CLC

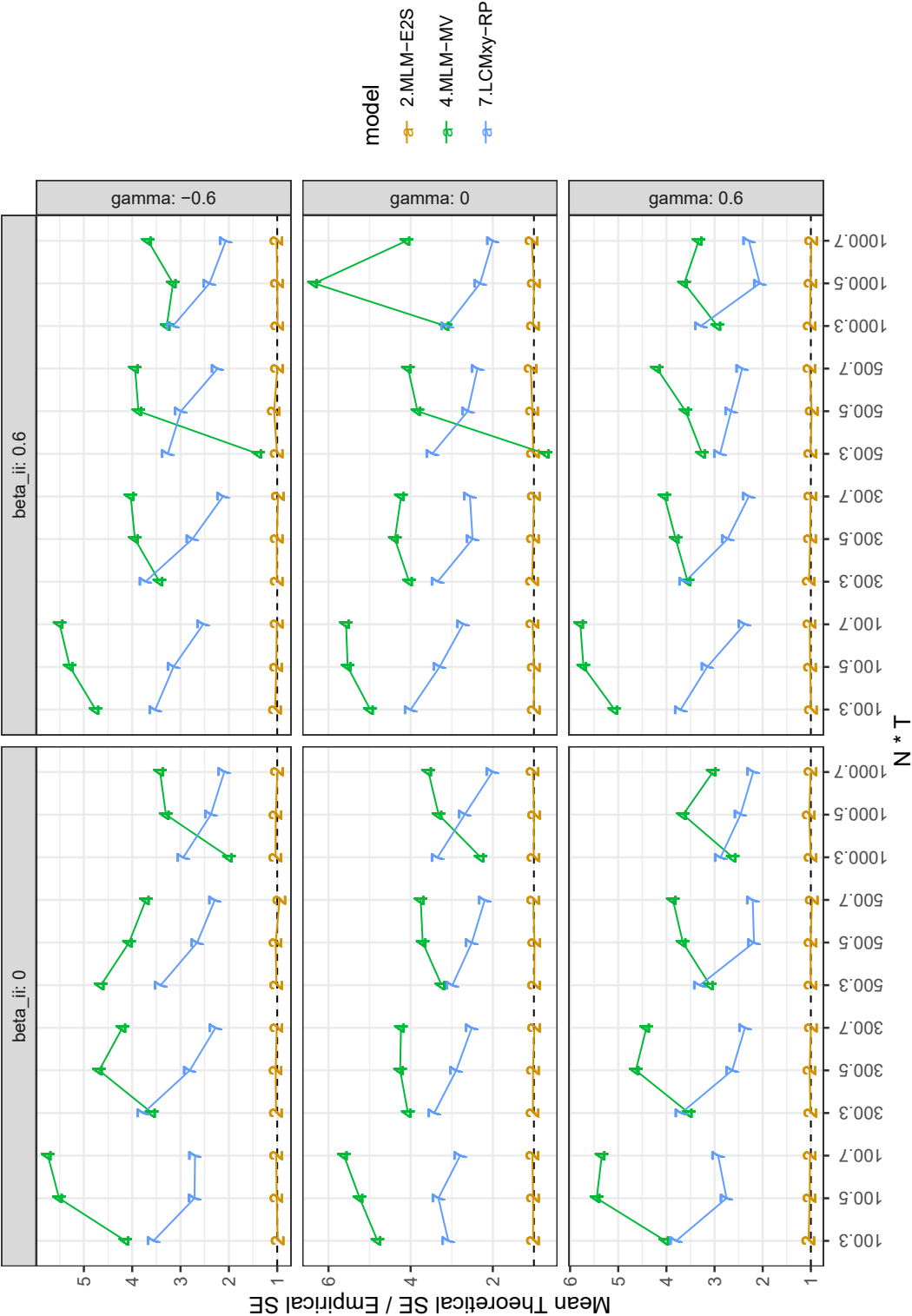


Figure A.46

SE Ratio for the Between-Person Effect β_{SS} in Simulation 1

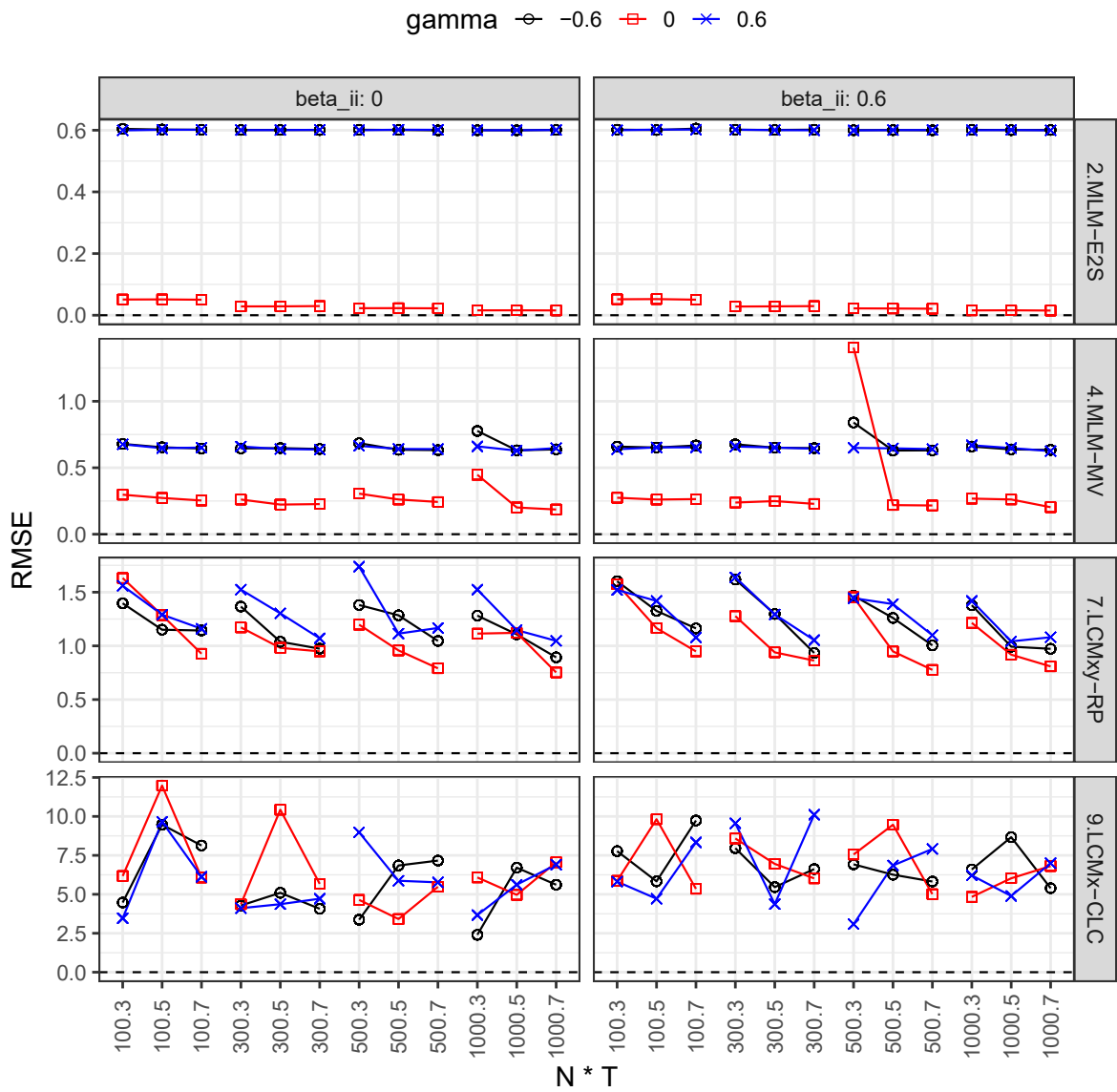


Figure A.47

RMSE for the Between-Person Effect β_{SS} in Simulation 1 except LCMx-CLC

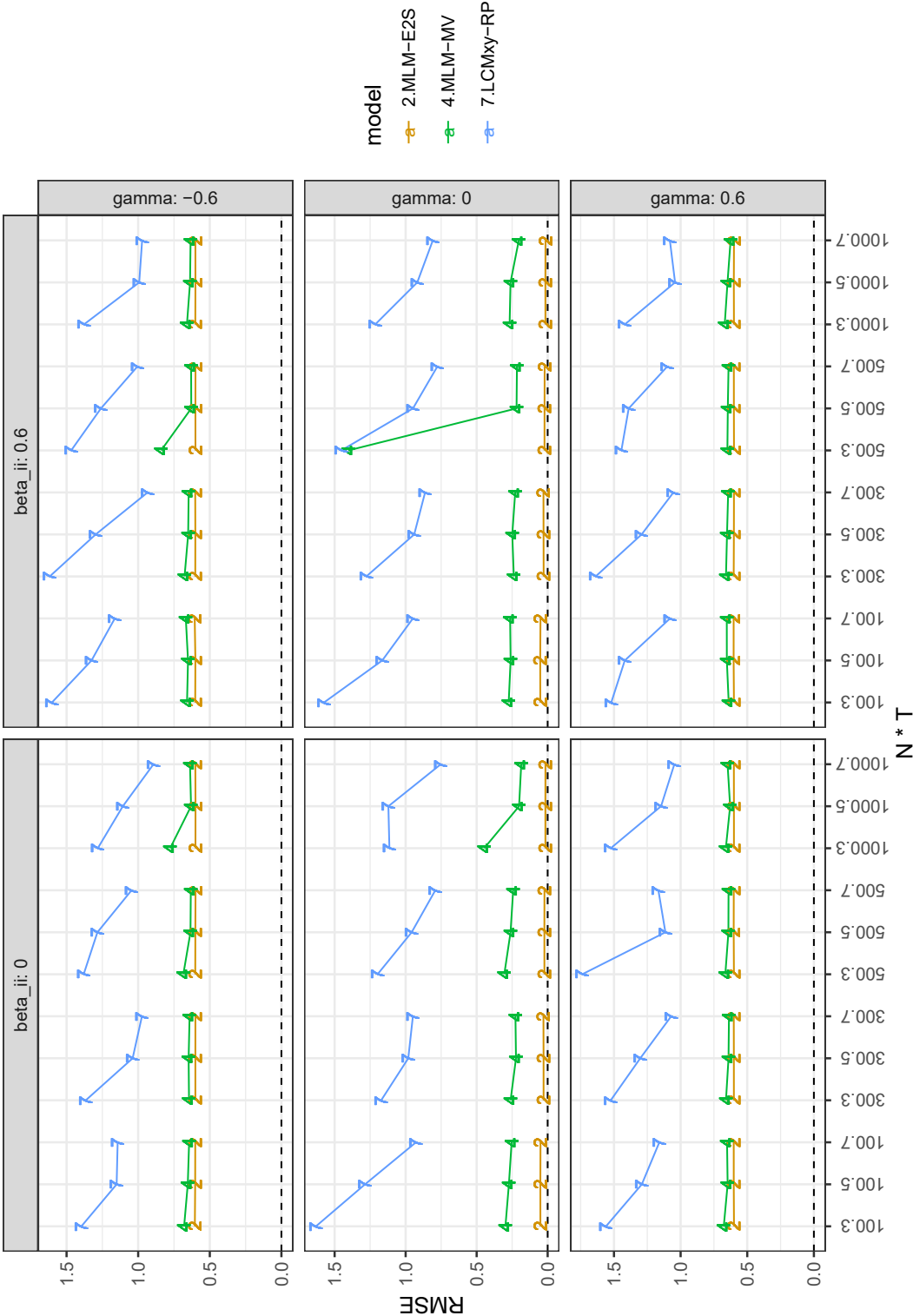


Figure A.48

Coverage Rate for the Between-Person Effect β_{SS} in Simulation 1

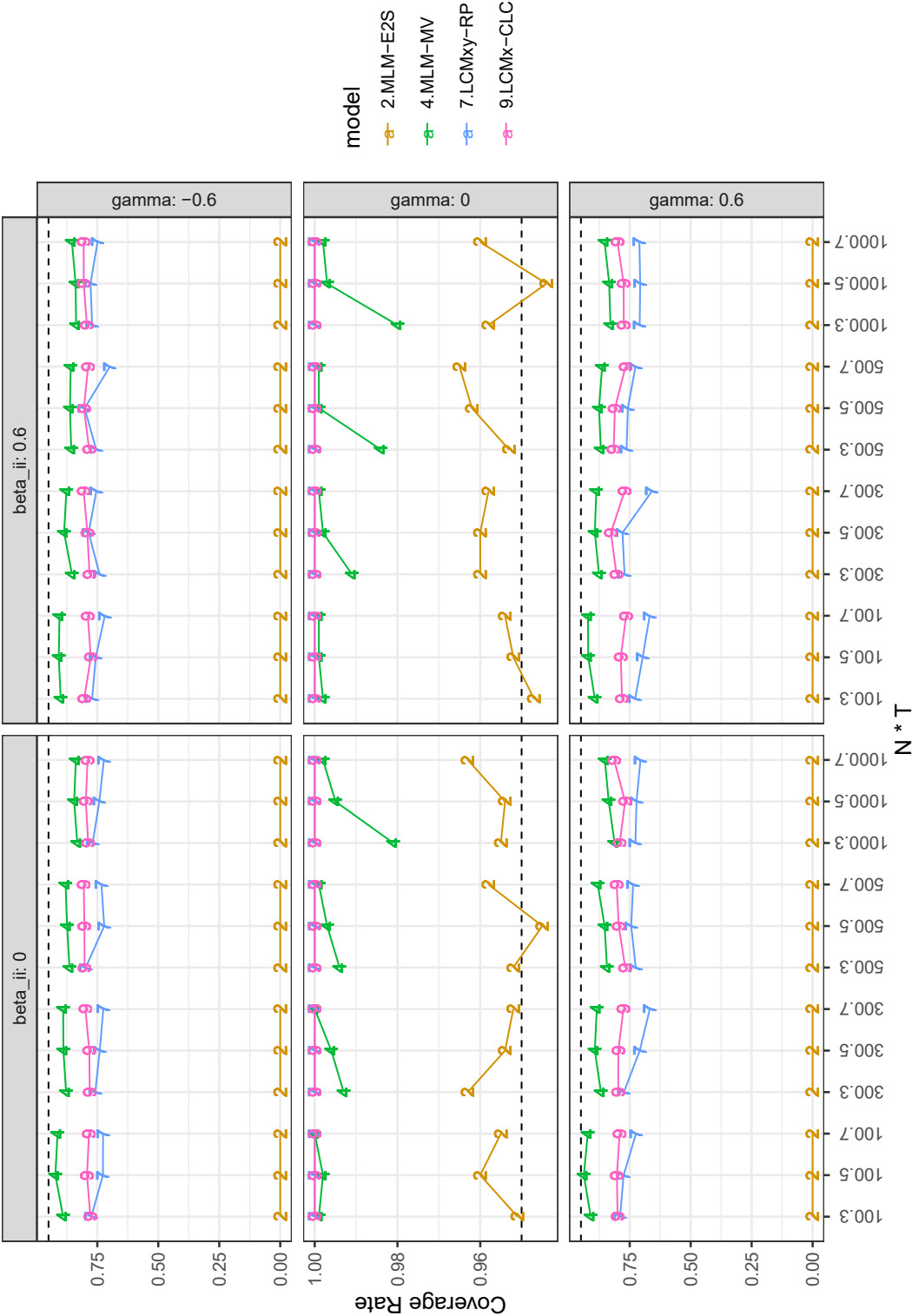
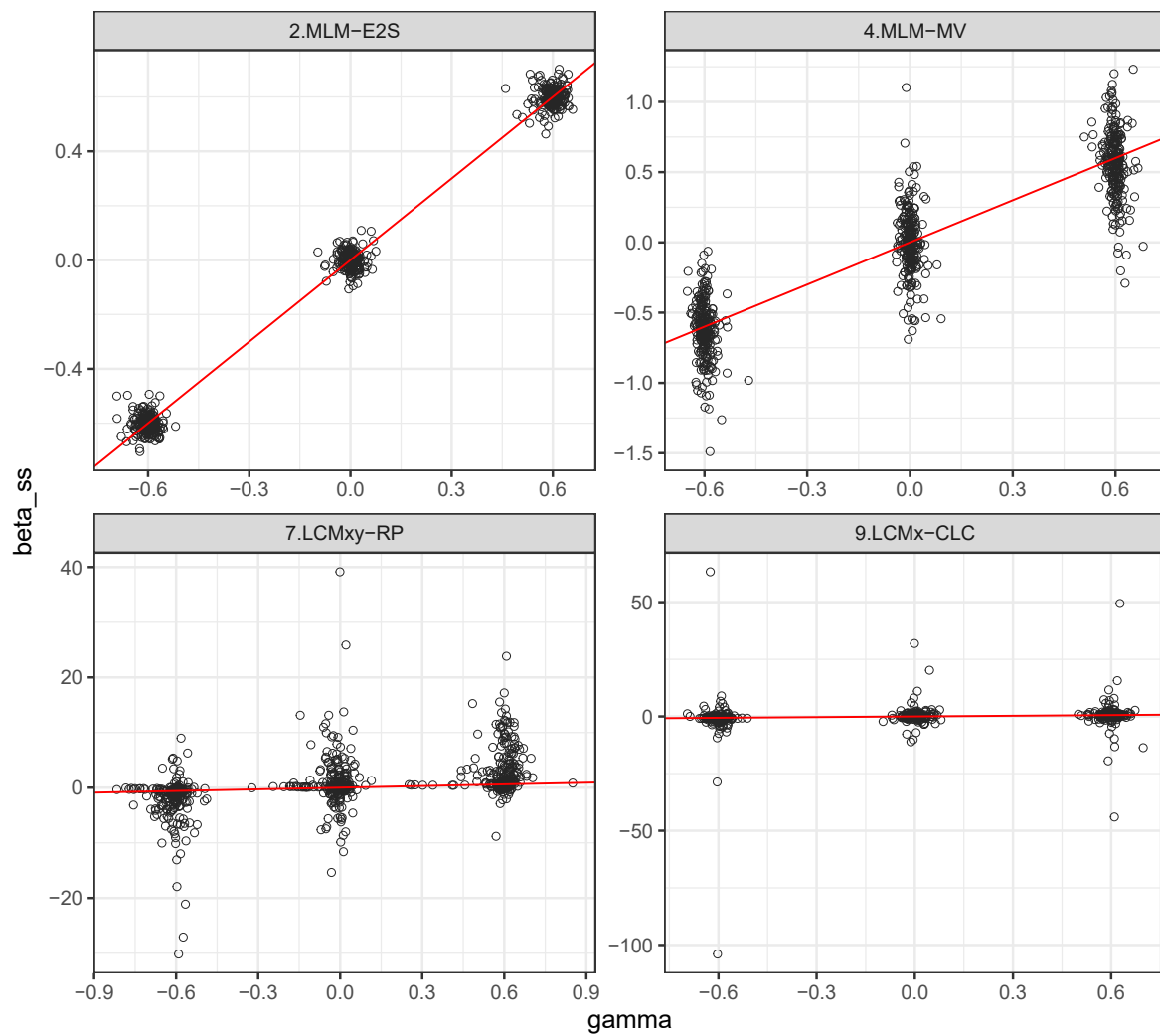


Figure A.49

Scatter Plot of $\hat{\gamma}$ and $\hat{\beta}_{SS}$ in Simulation 1 (10 samples per condition)



Appendix B

Figures in Simulation 2

B.1 Auxiliary Indices

Figure B.1

Proportion of Converged Cases in Simulation 2

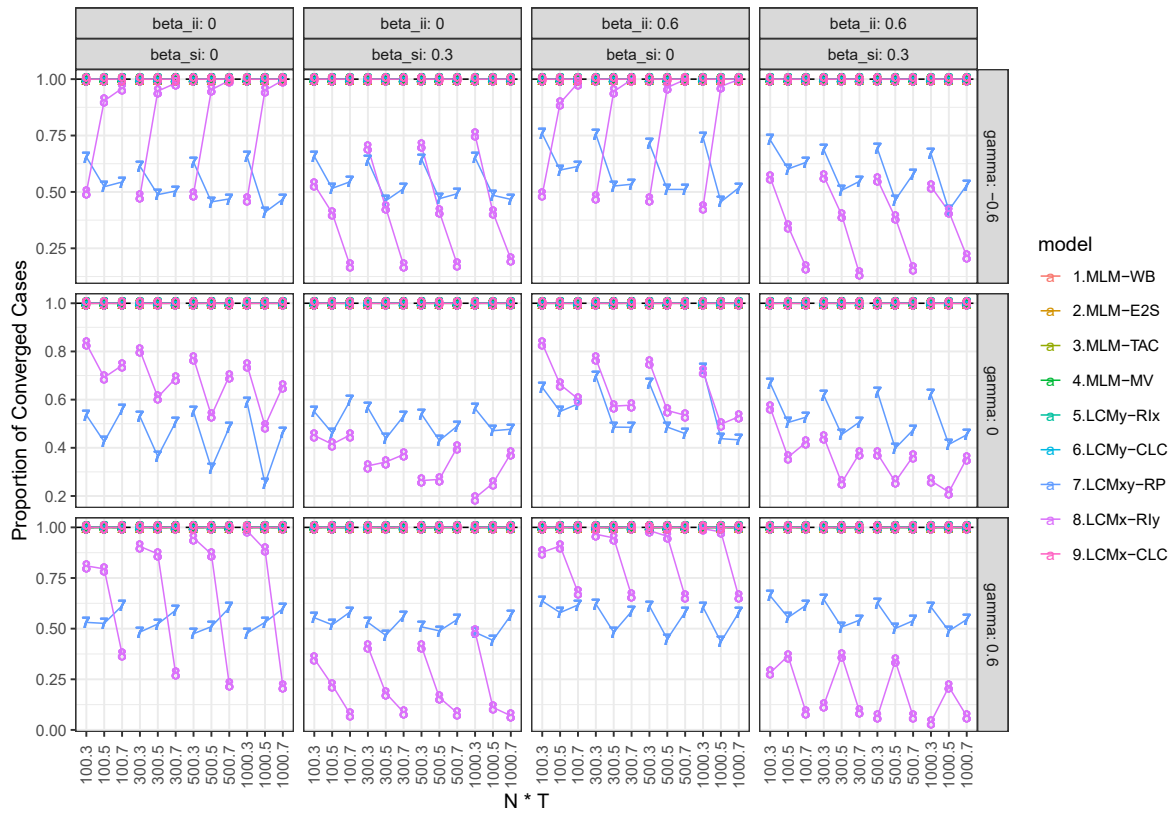
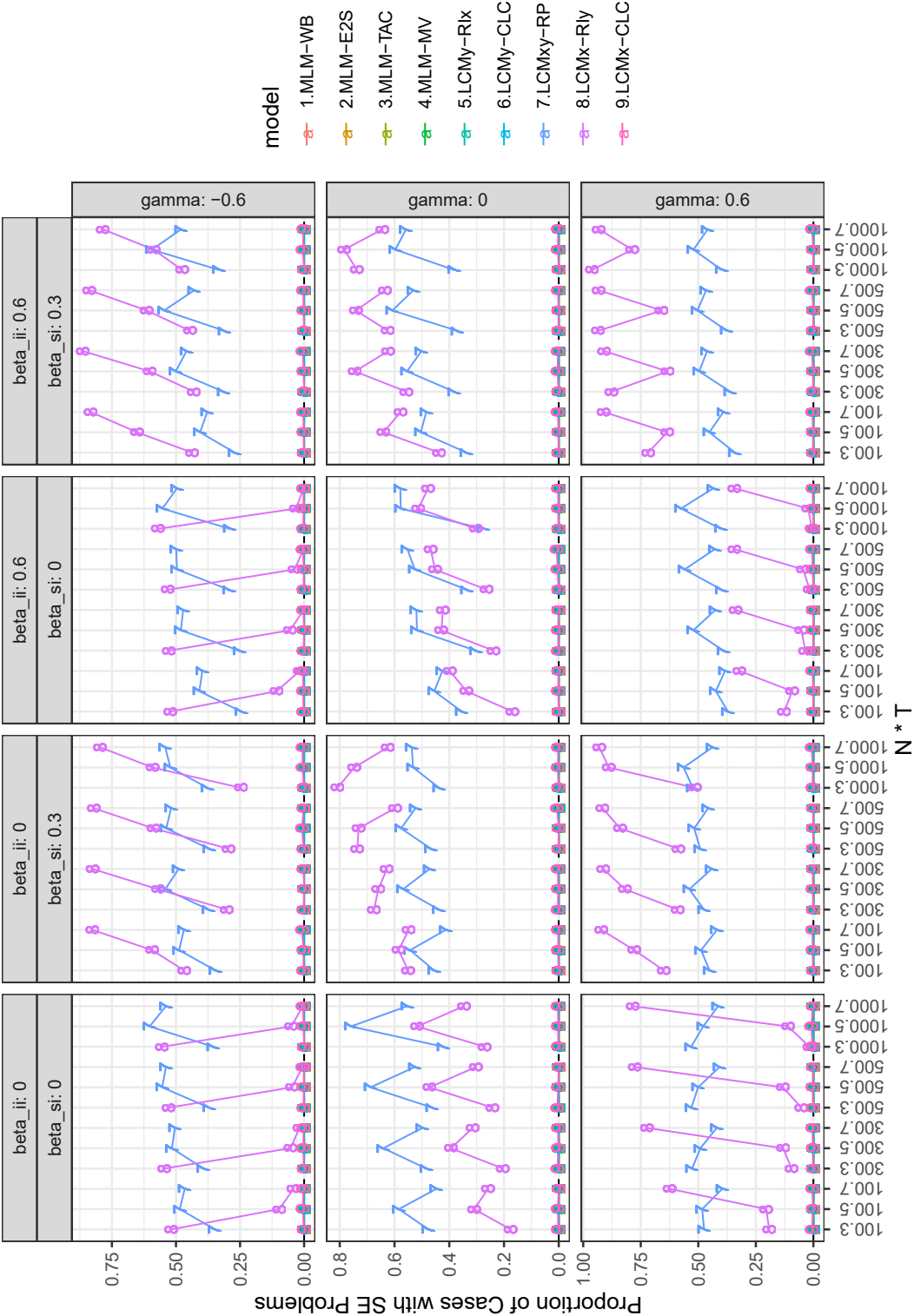


Figure B.2

Proportion of Cases with Problems in Estimating SE in Simulation 2



Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 2

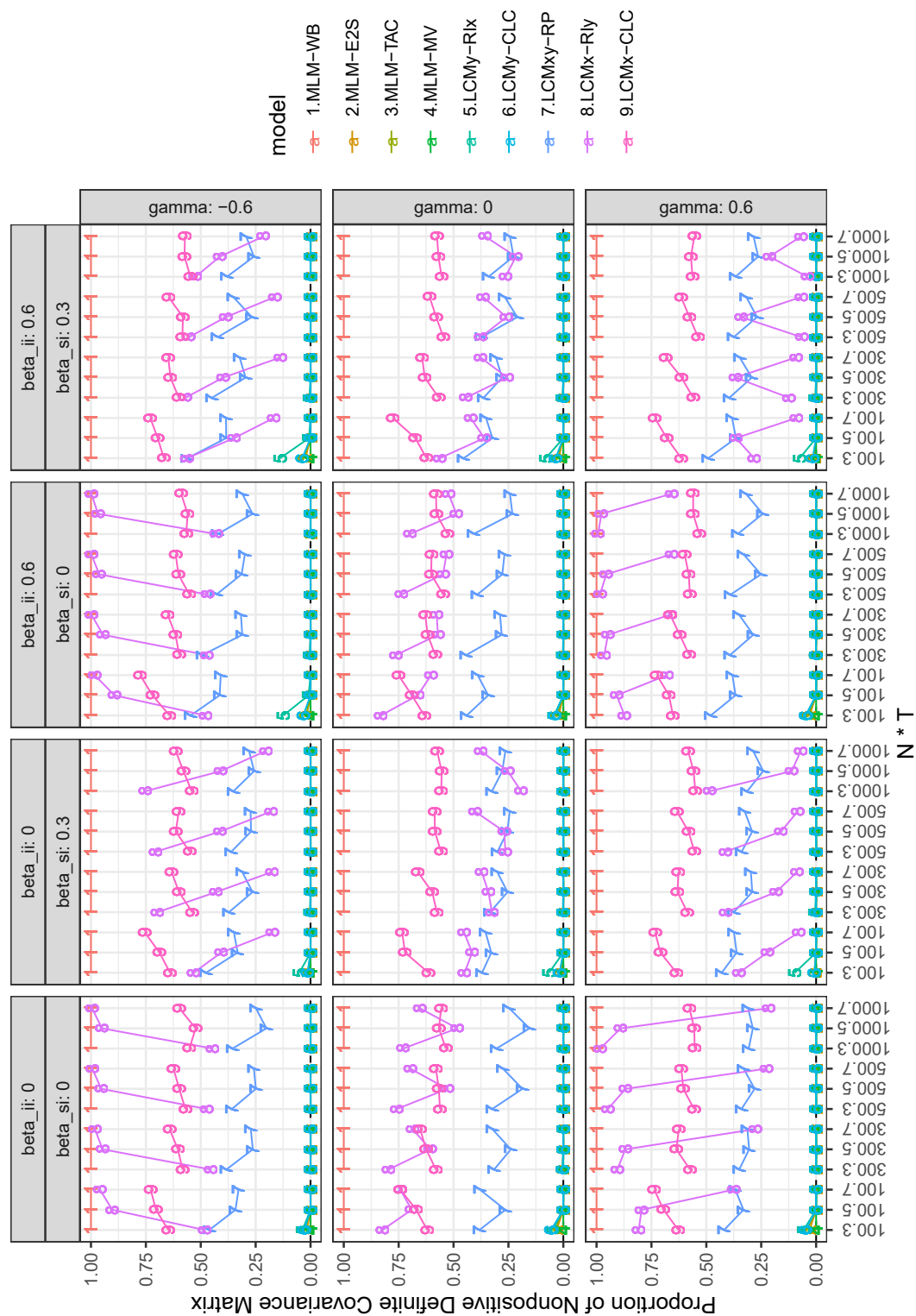


Figure B.4

CFI in Simulation 2

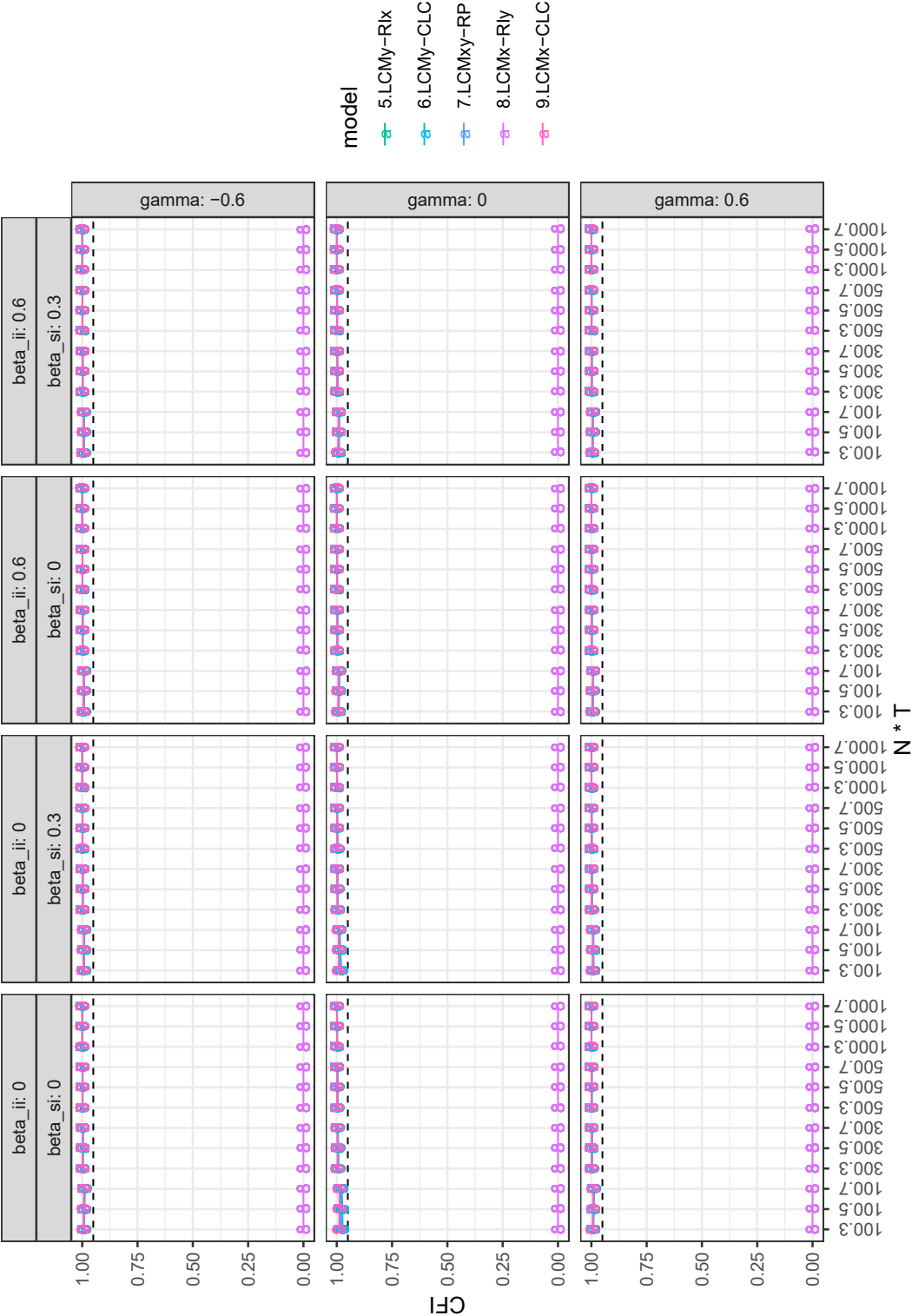


Figure B.5
RMSEA in Simulation 2

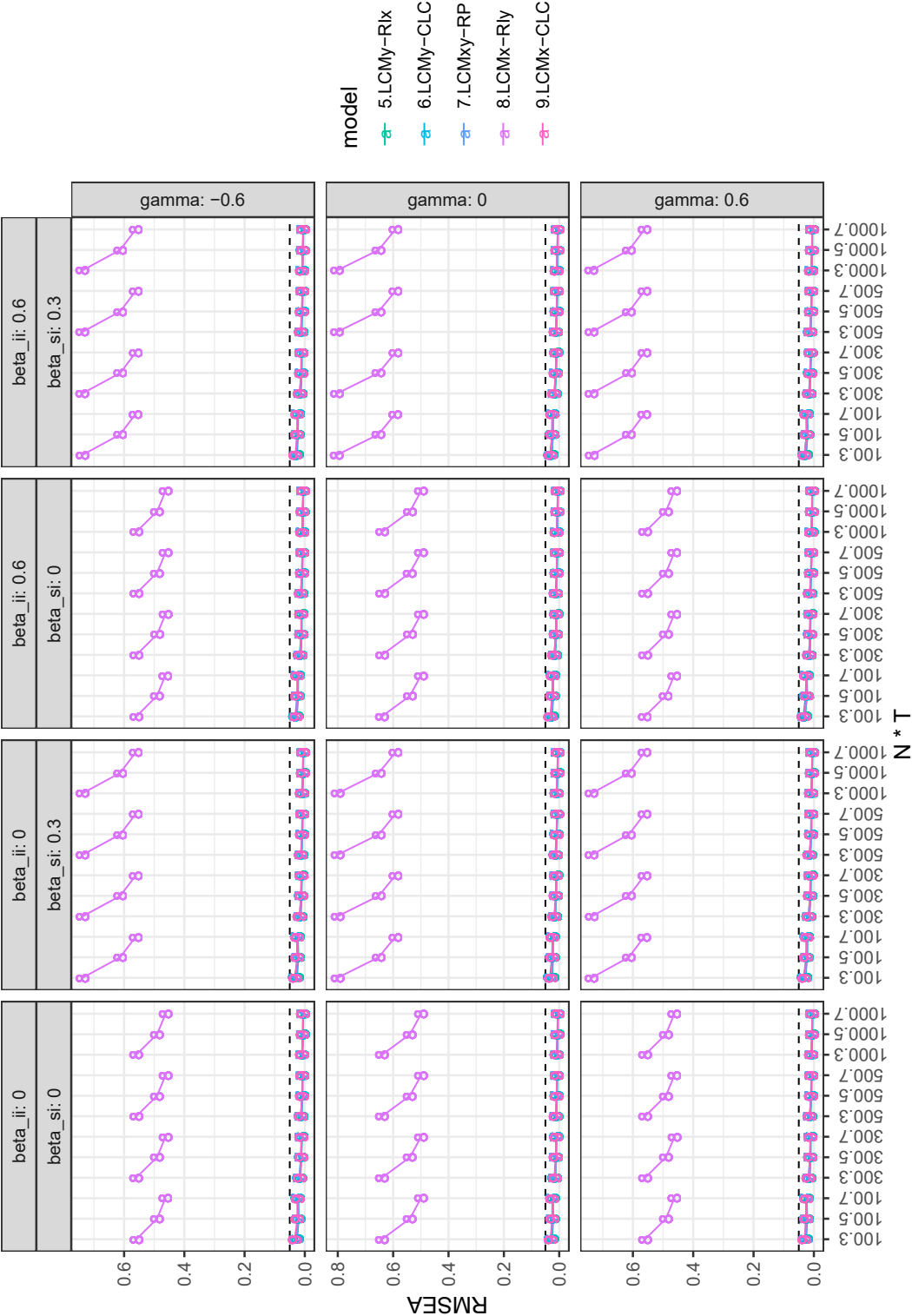
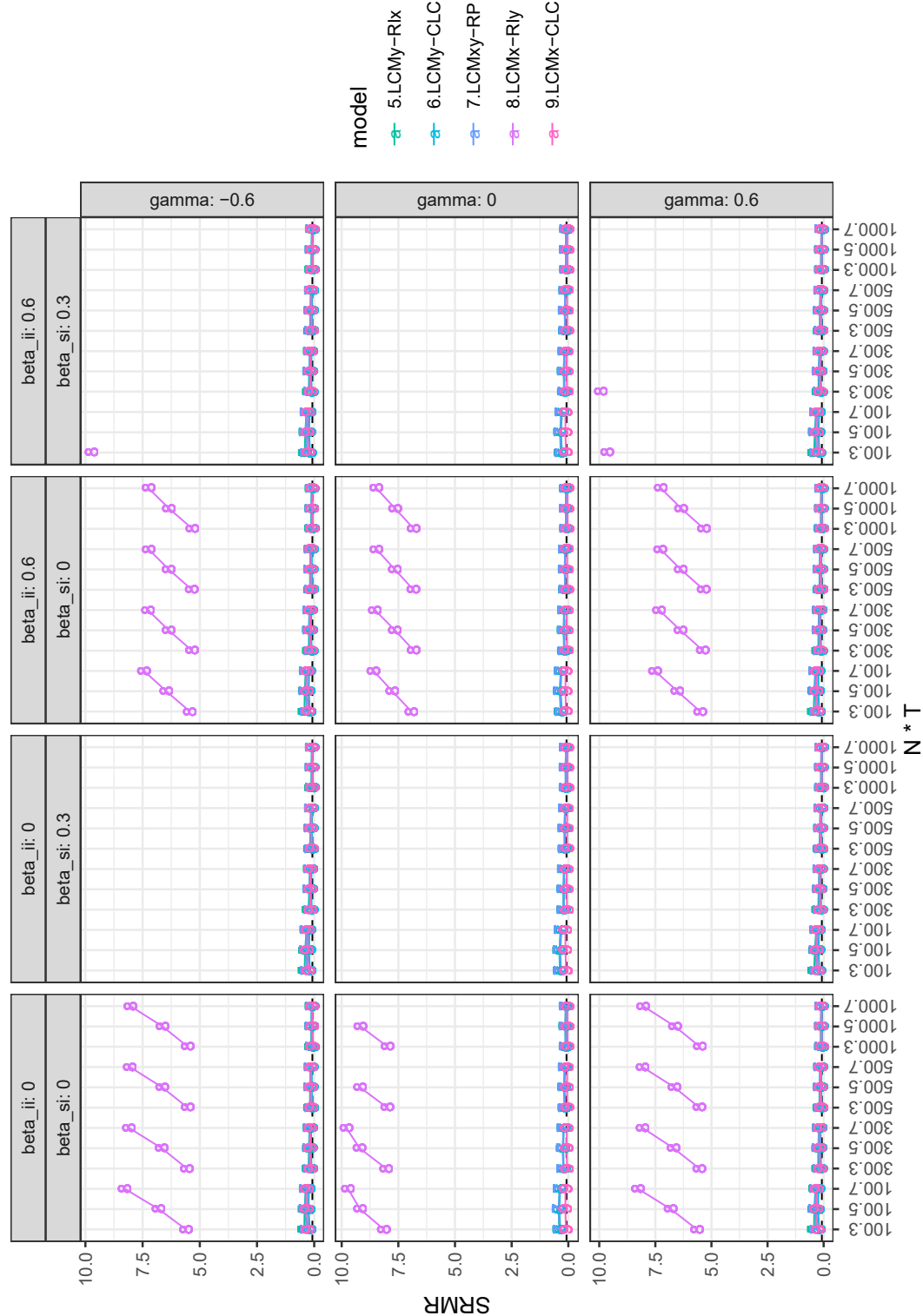


Figure B.6

SRMR in Simulation 2



B.2 Within-Person Effect

Figure B.7

Bias for the Within-Person Effect γ in Simulation 2

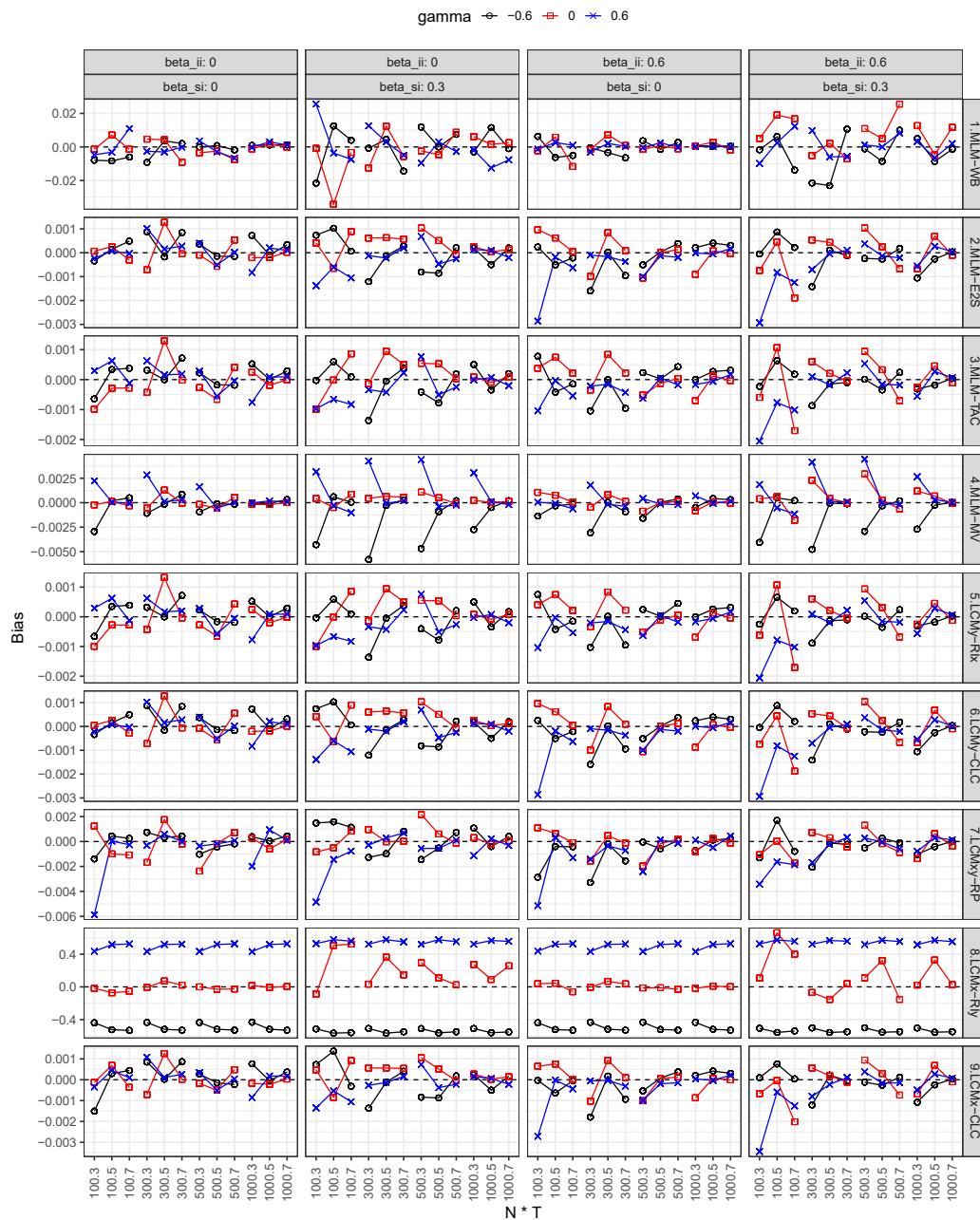


Figure B.8

Bias for the Within-Person Effect γ in Simulation 2 except LCMx-Rly

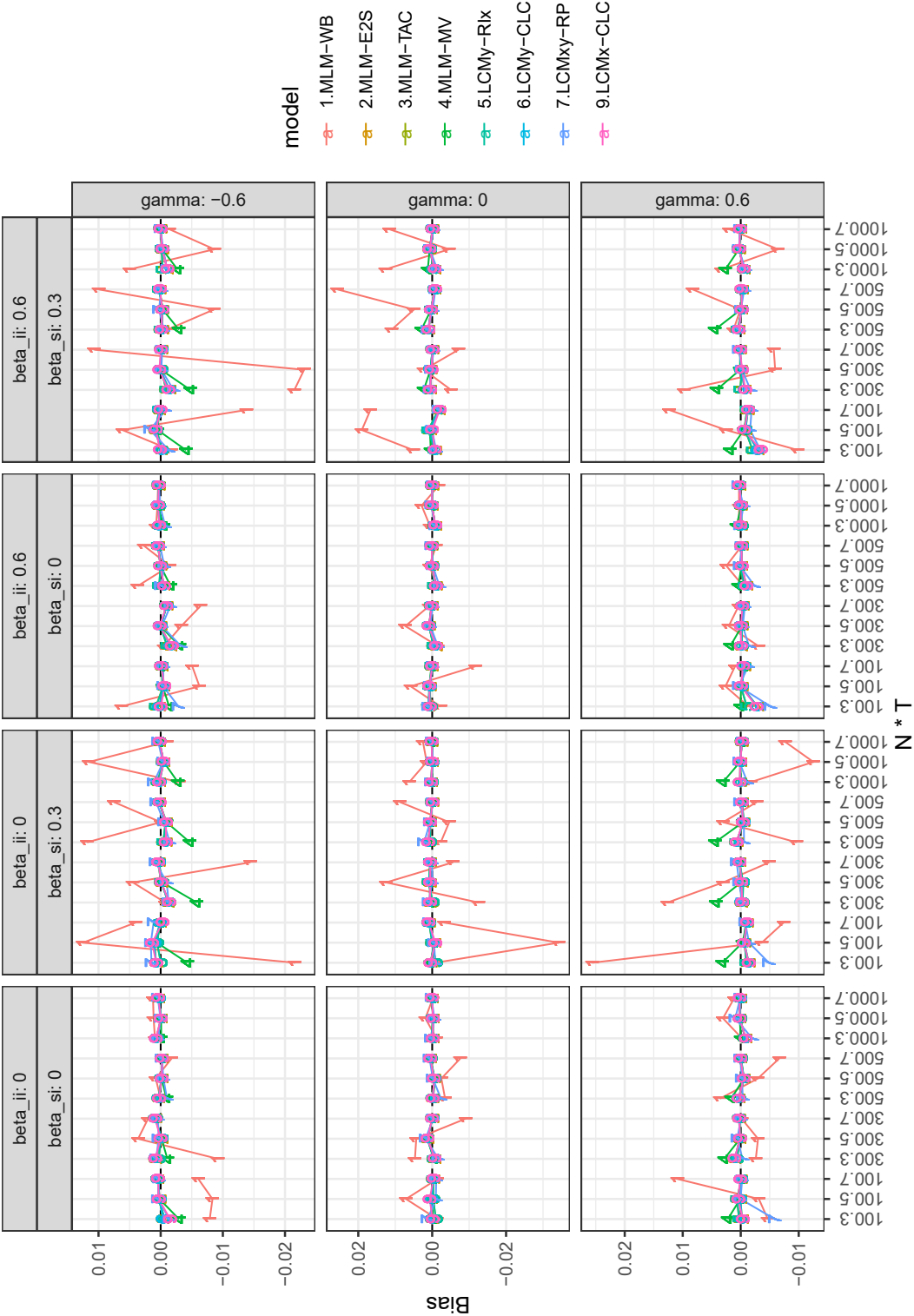


Figure B.9

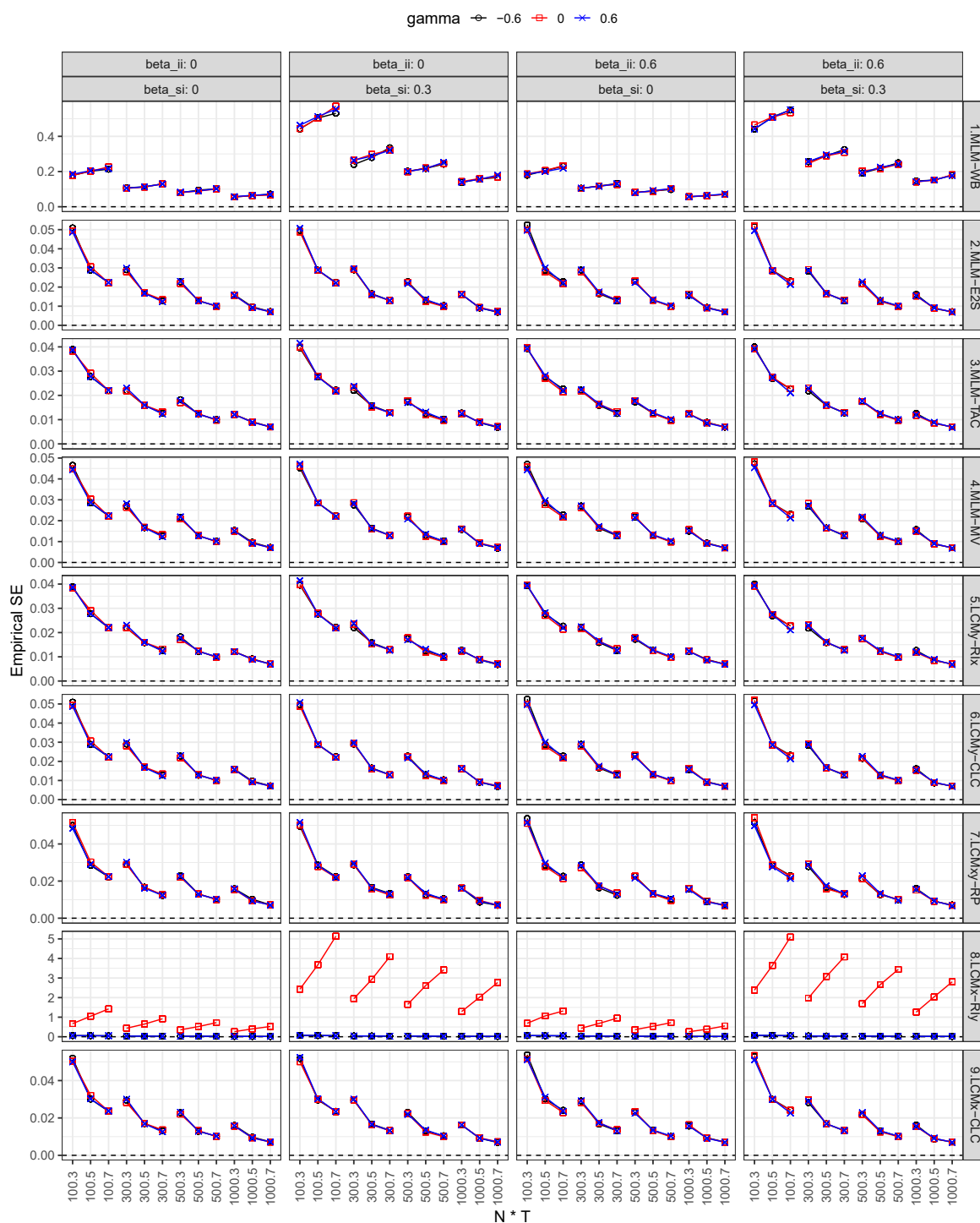
Empirical SE for the Within-Person Effect γ in Simulation 2

Figure B.10

Empirical SE for the Within-Person Effect γ in Simulation 2 except LCMx-Rly

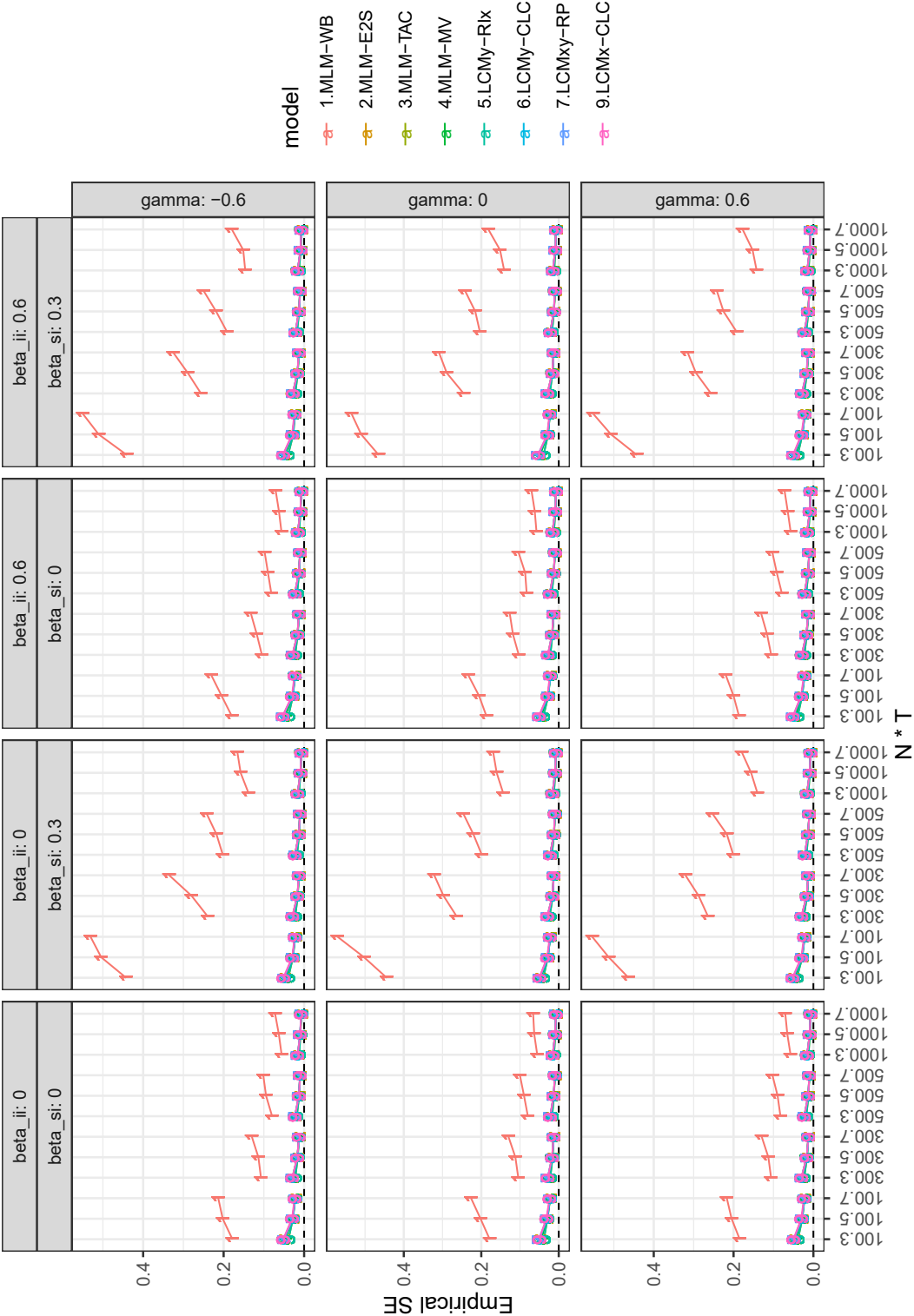


Figure B.11

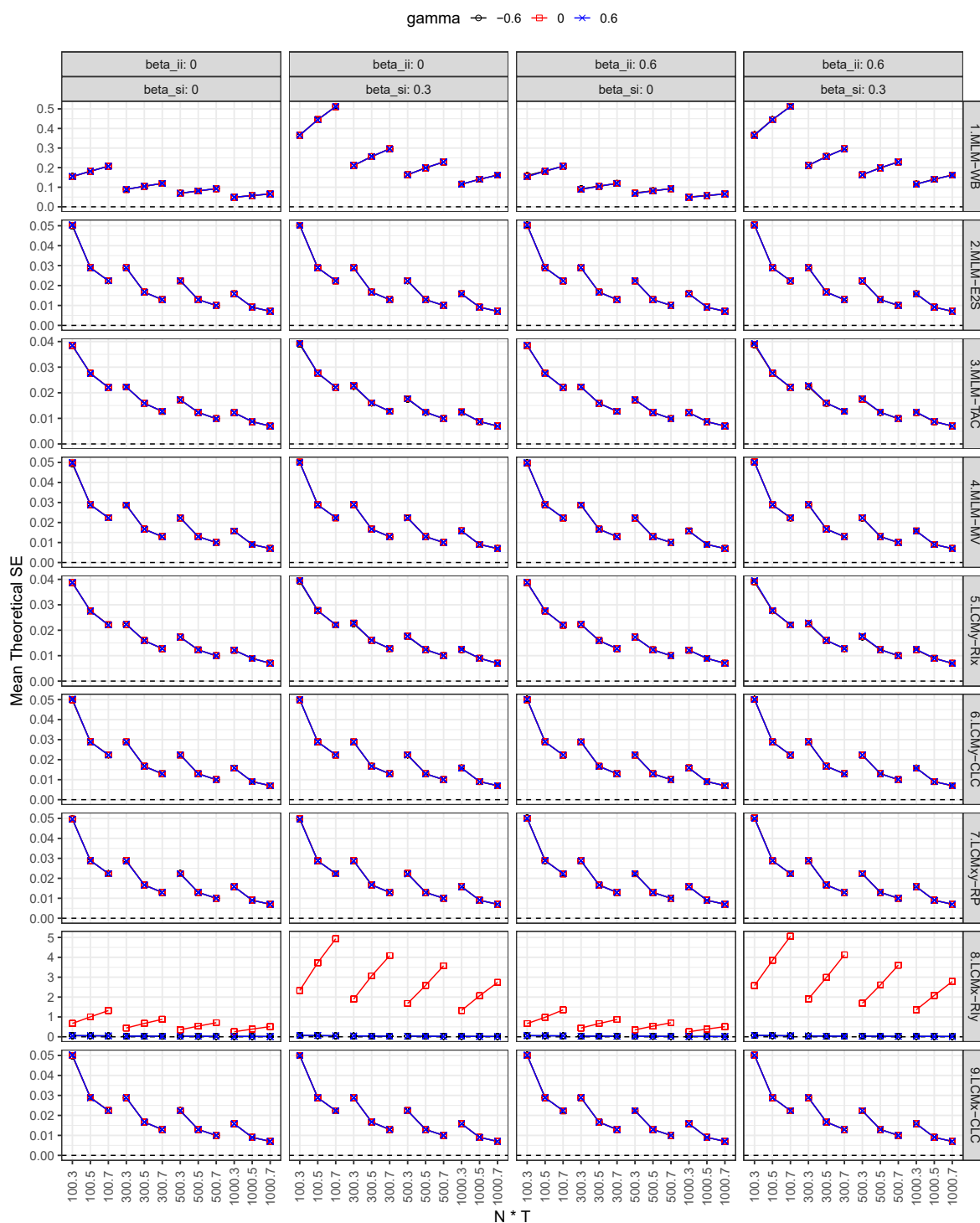
Theoretical SE for the Within-Person Effect γ in Simulation 2

Figure B.12

Theoretical SE for the Within-Person Effect γ in Simulation 2 except LCMx-Rly

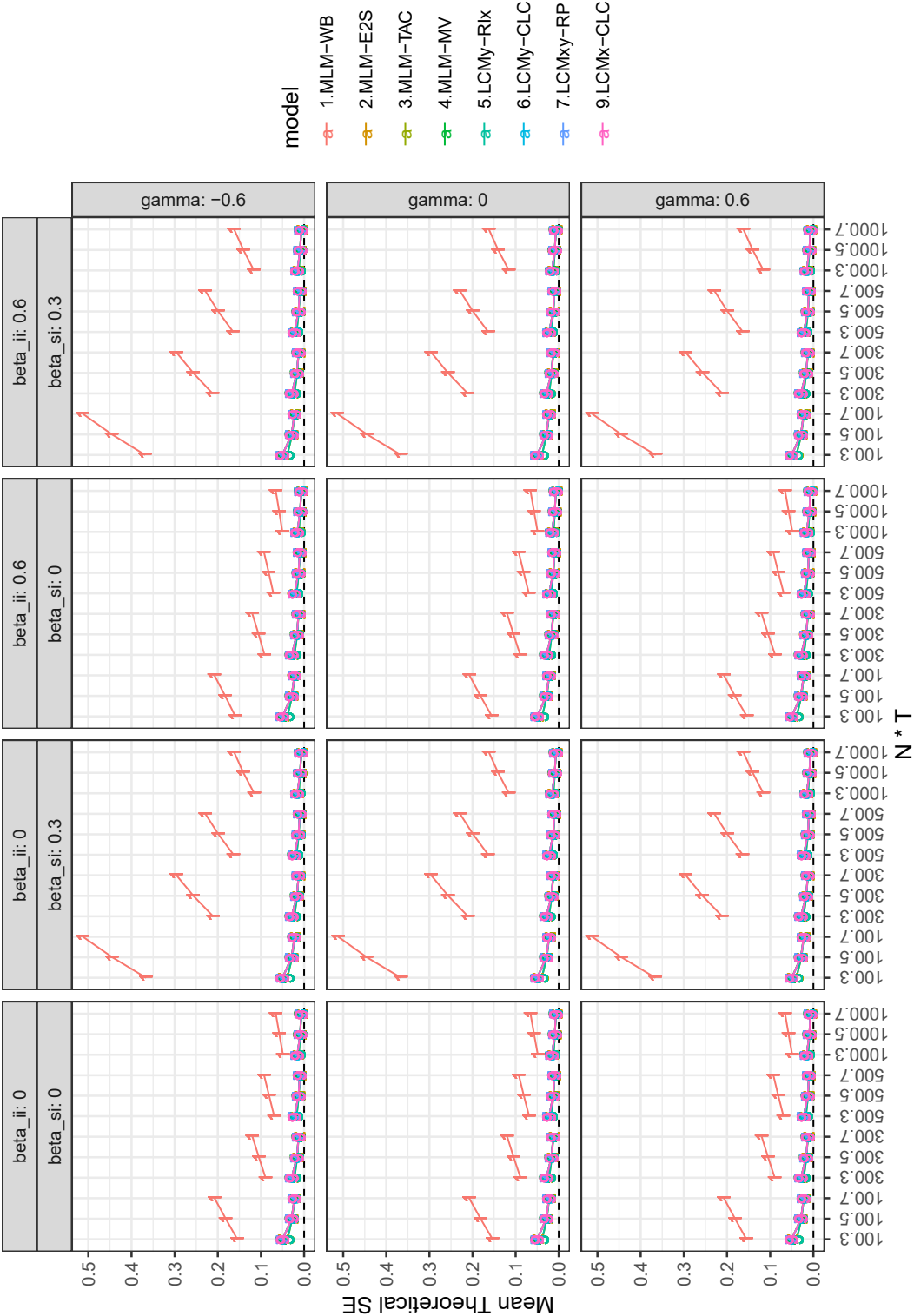


Figure B.13

SE Ratio for the Within-Person Effect γ in Simulation 2

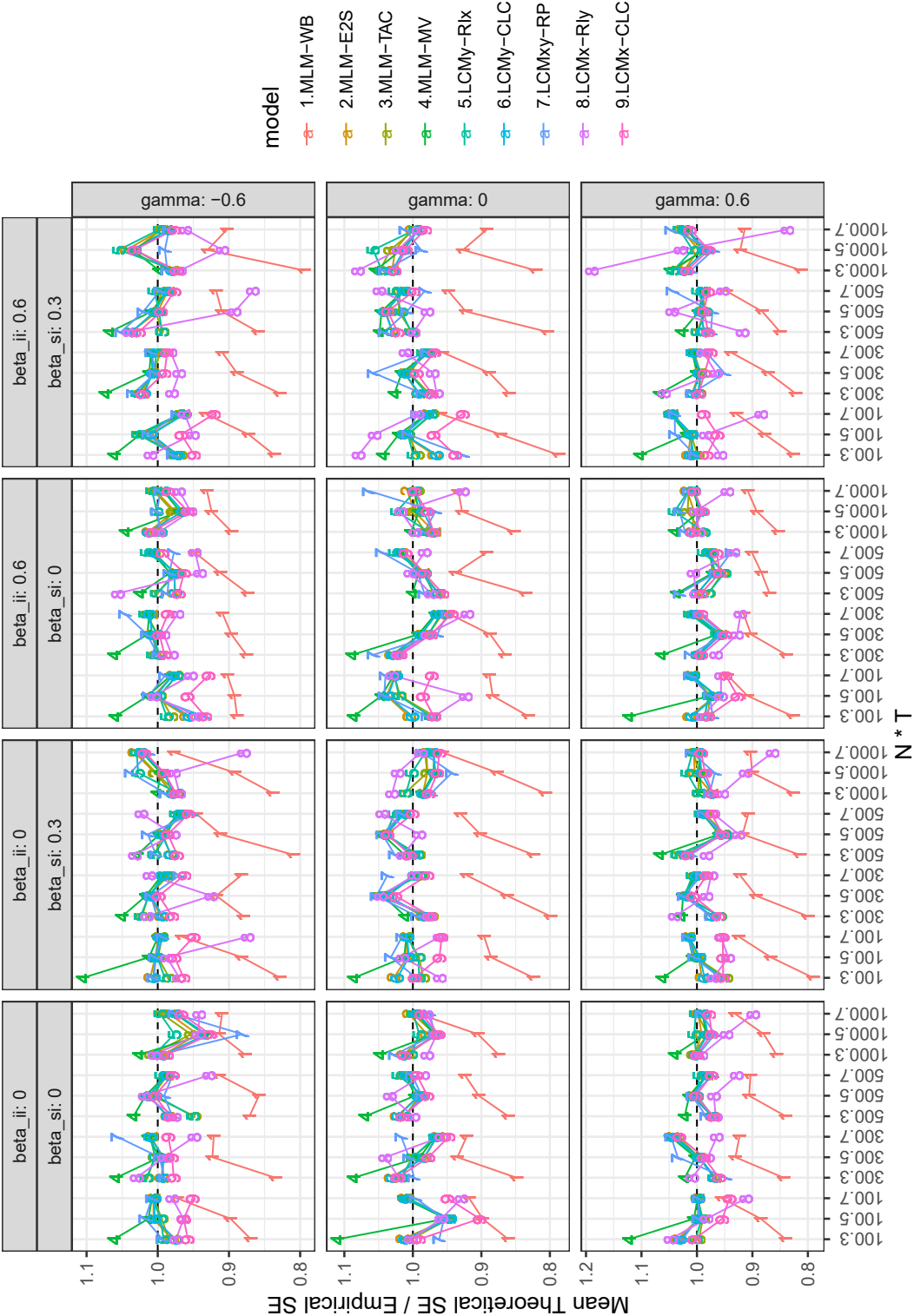


Figure B.14

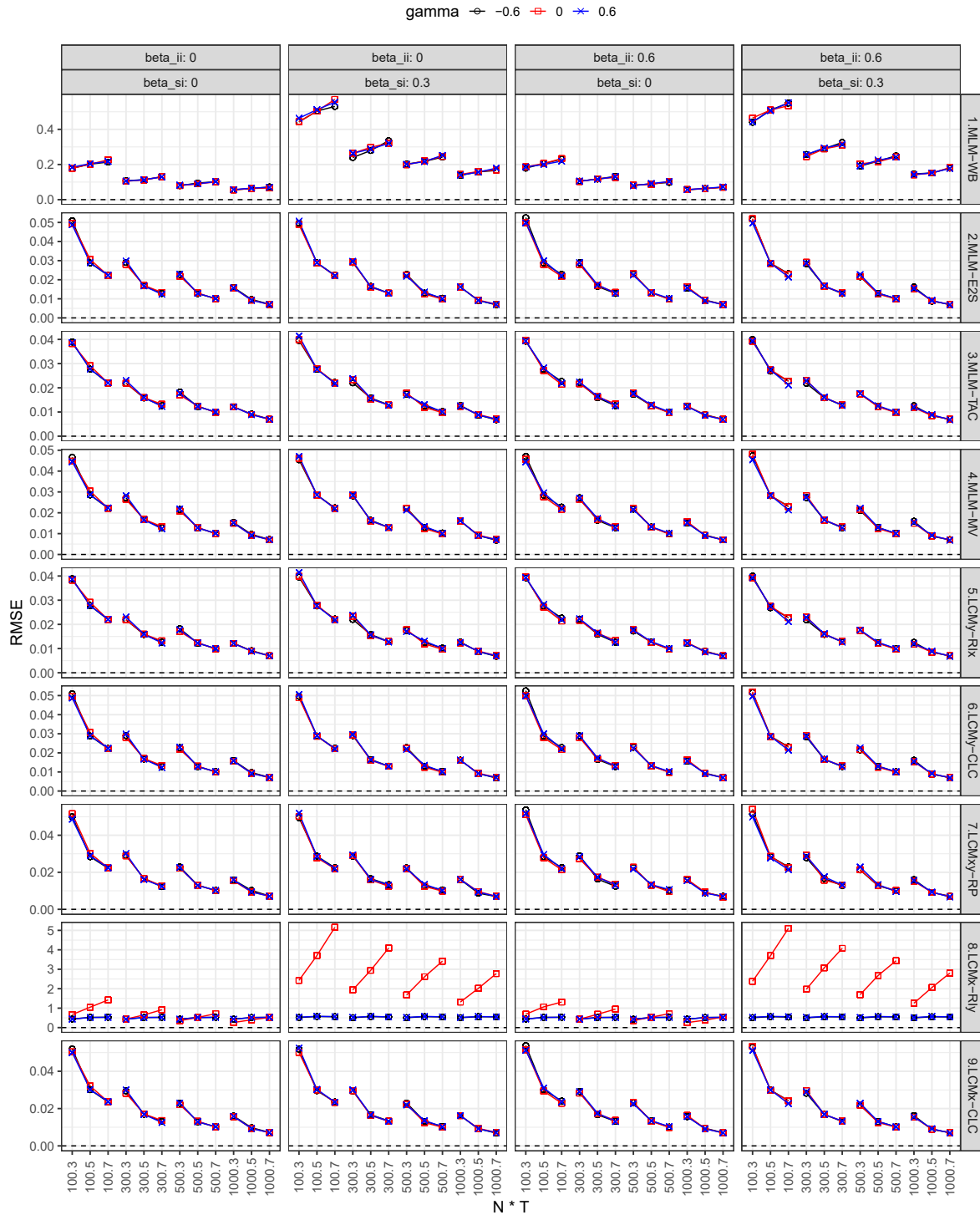
RMSE for the Within-Person Effect γ in Simulation 2

Figure B.15

RMSE for the Within-Person Effect γ in Simulation 2 except LCMx-Rly

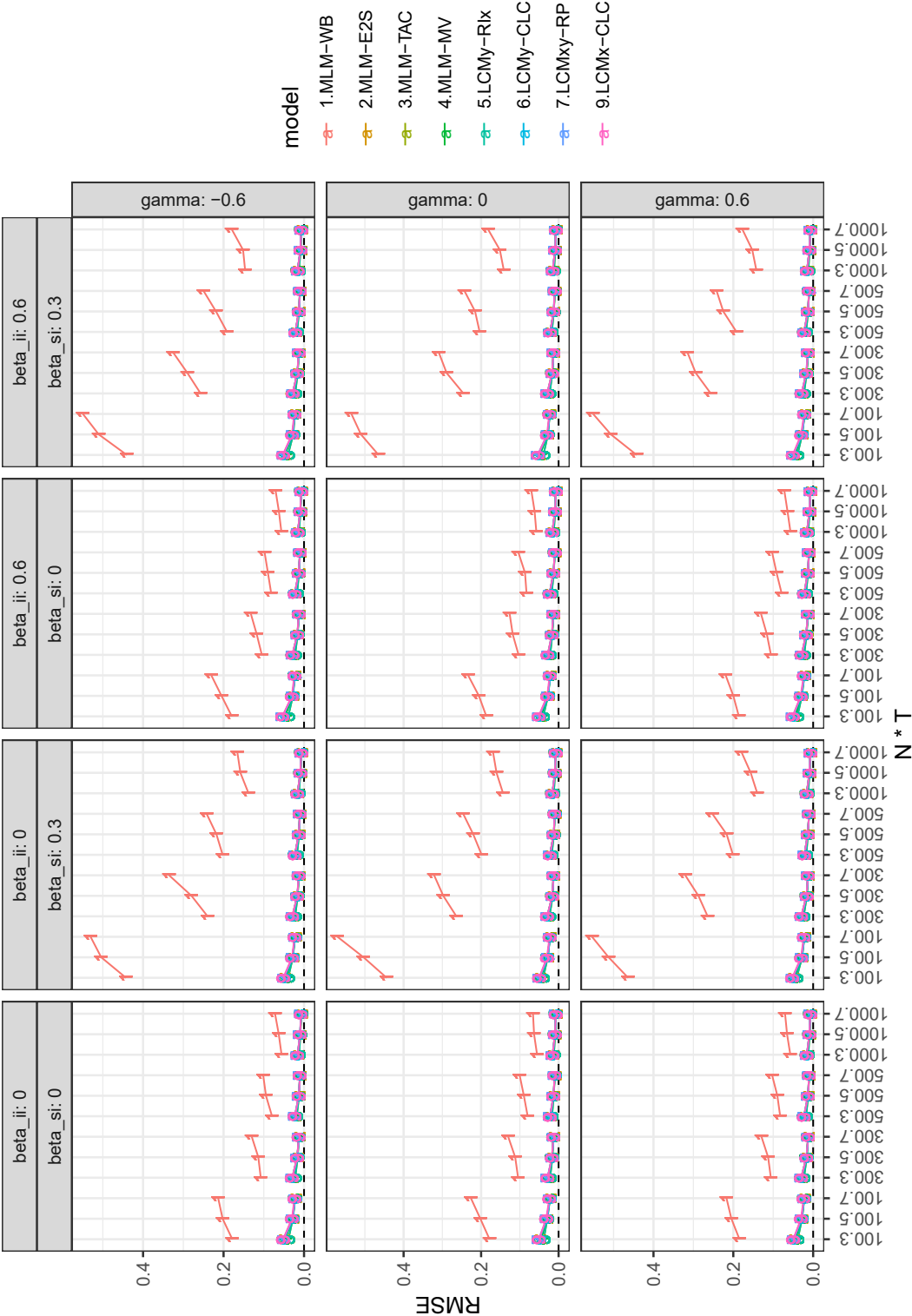


Figure B.16

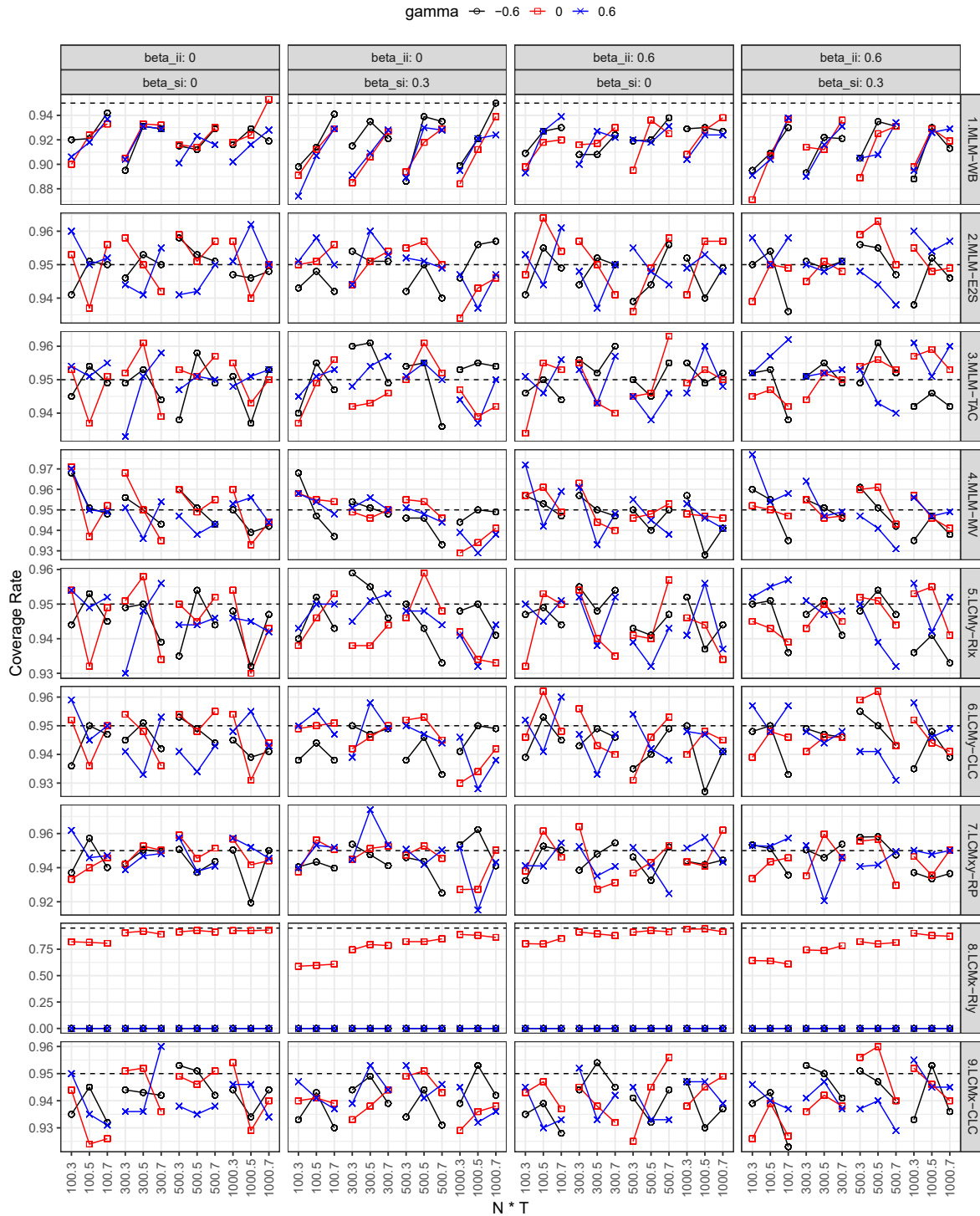
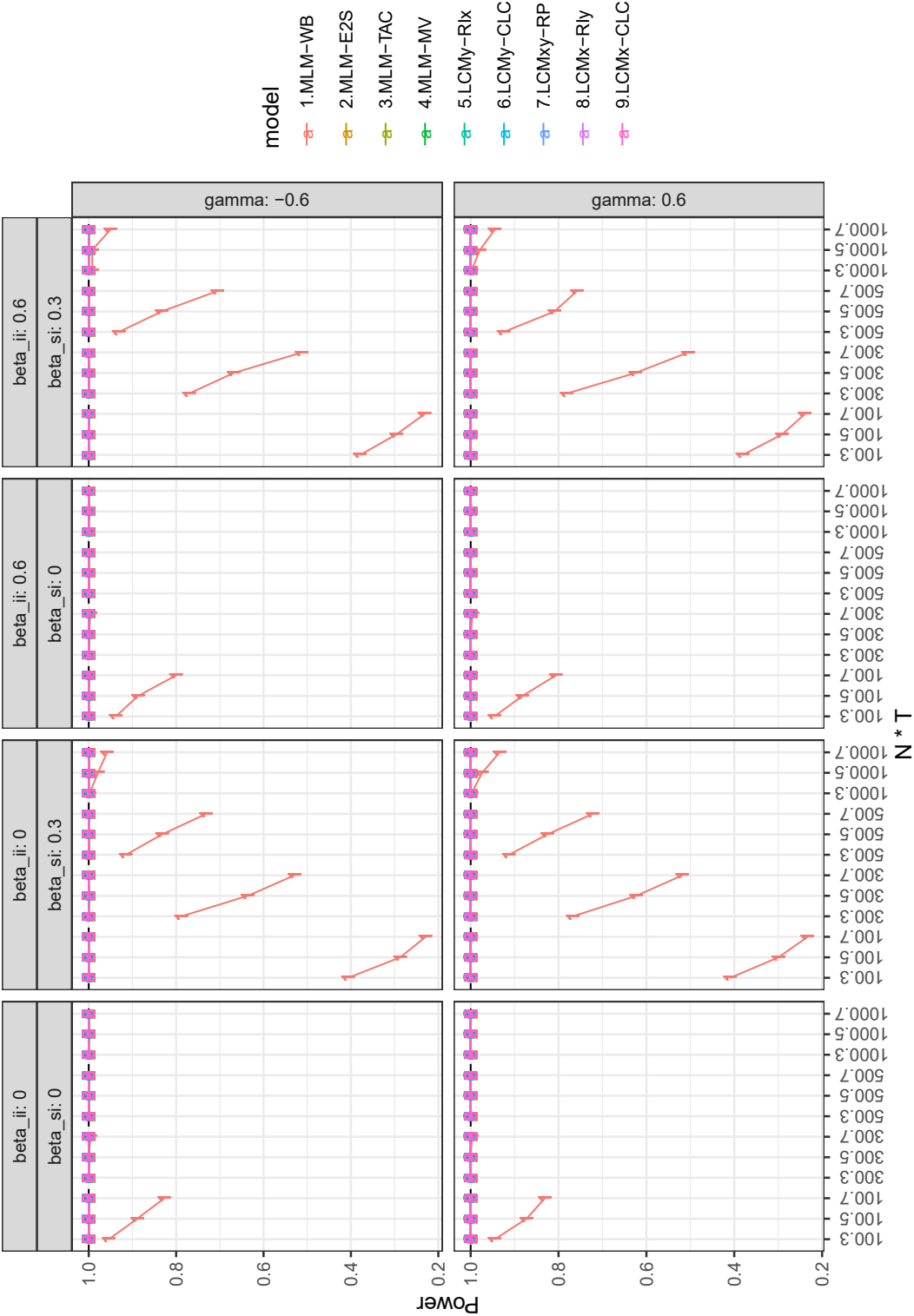
Coverage Rate for the Within-Person Effect γ in Simulation 2

Figure B.17

Power for the Within-Person Effect γ in Simulation 2



B.3 Between-Person Effect of Intercept on Intercept

Figure B.18

Bias for the Between-Person Effect β_{II} in Simulation 2

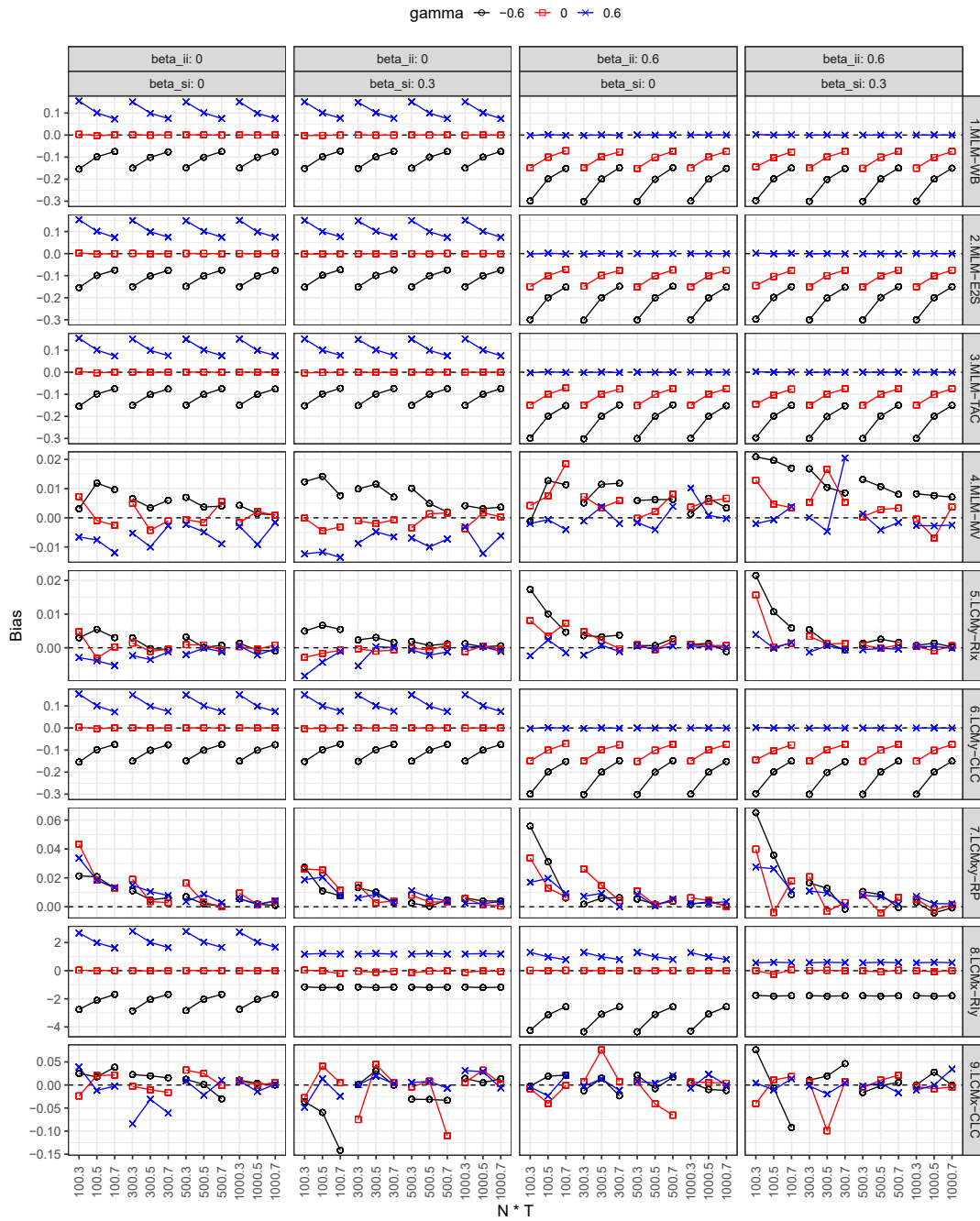


Figure B.19

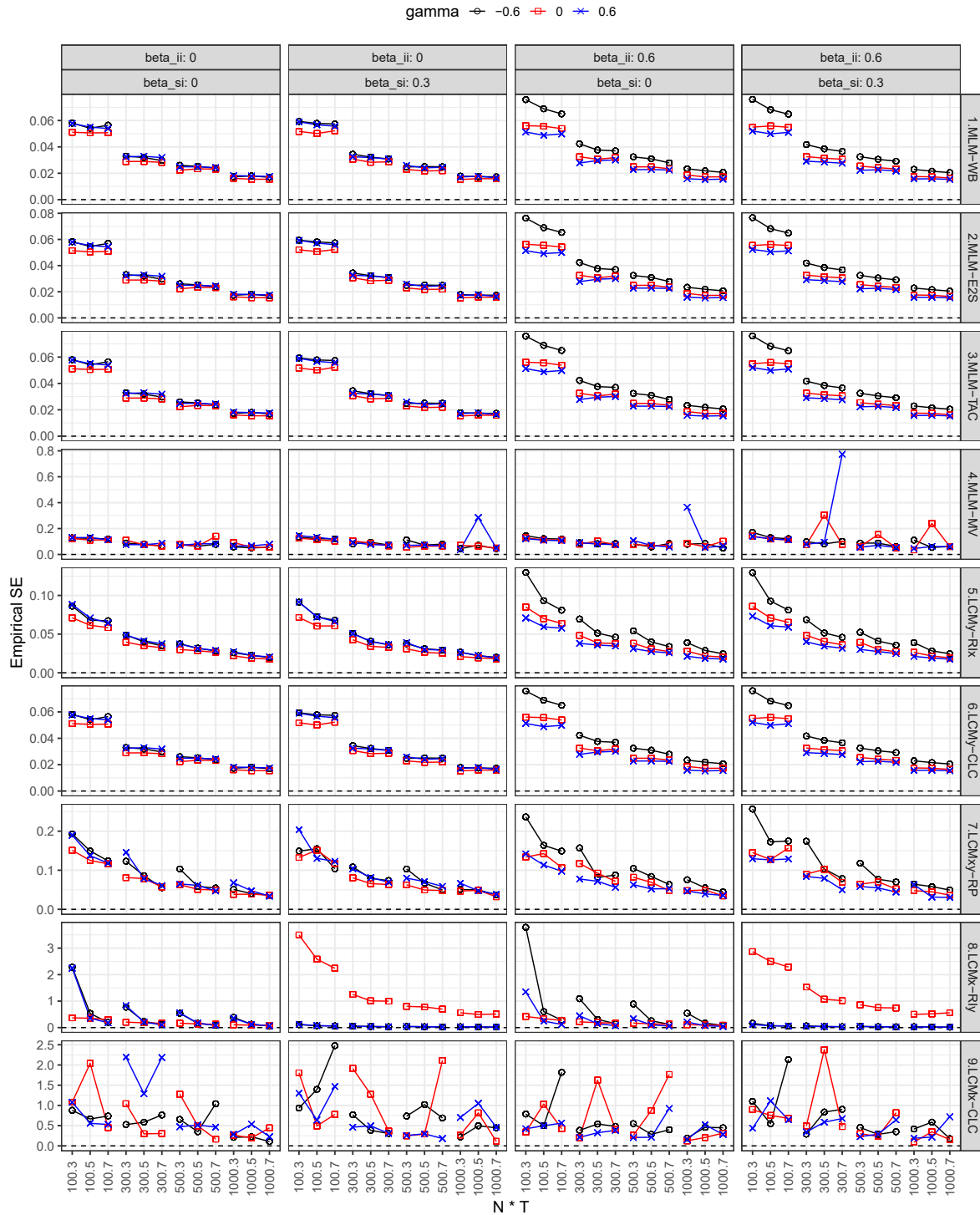
Empirical SE for the Between-Person Effect β_{II} in Simulation 2

Figure B.20

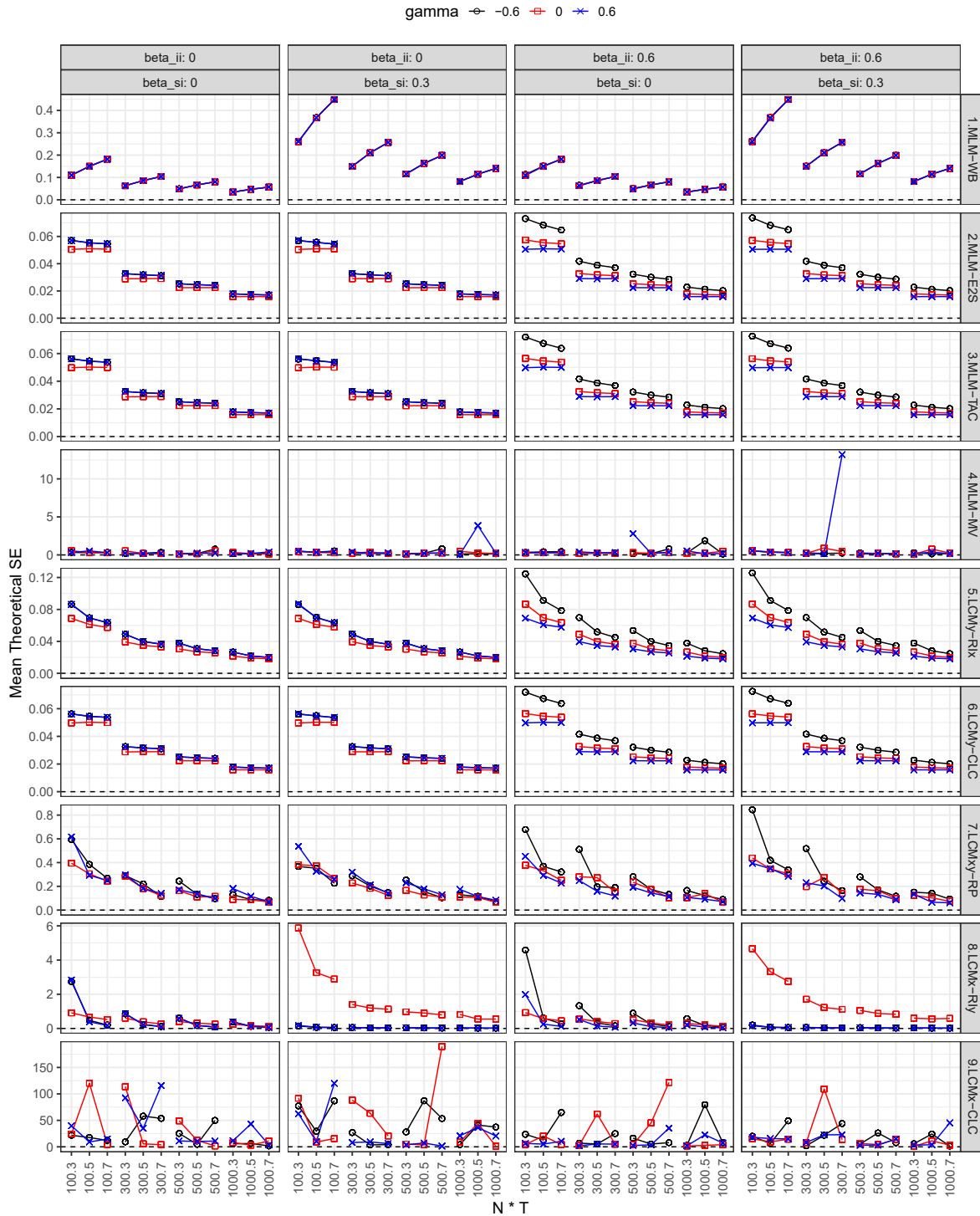
Theoretical SE for the Between-Person Effect β_{II} in Simulation 2

Figure B.21

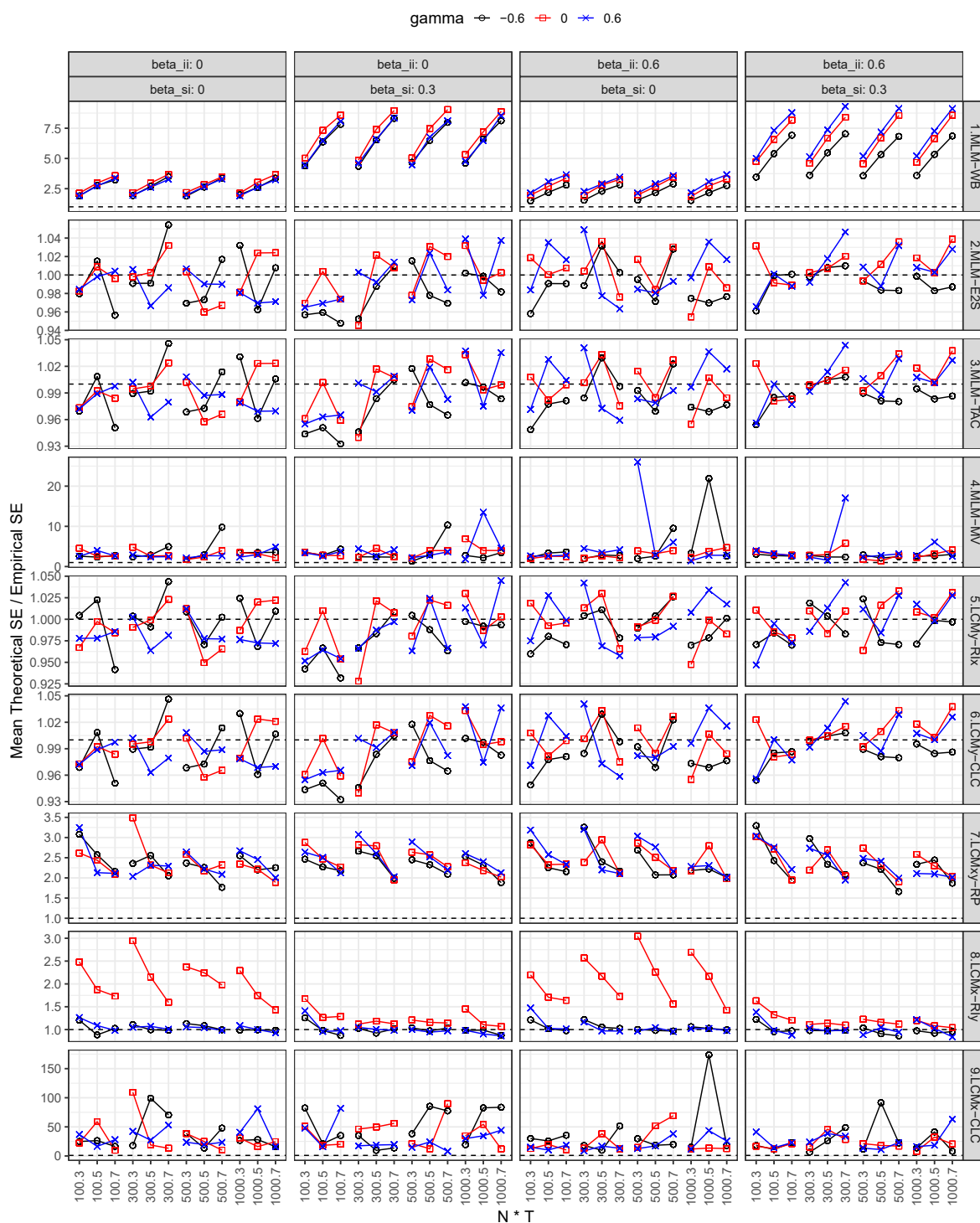
SE Ratio for the Between-Person Effect β_{II} in Simulation 2

Figure B.22

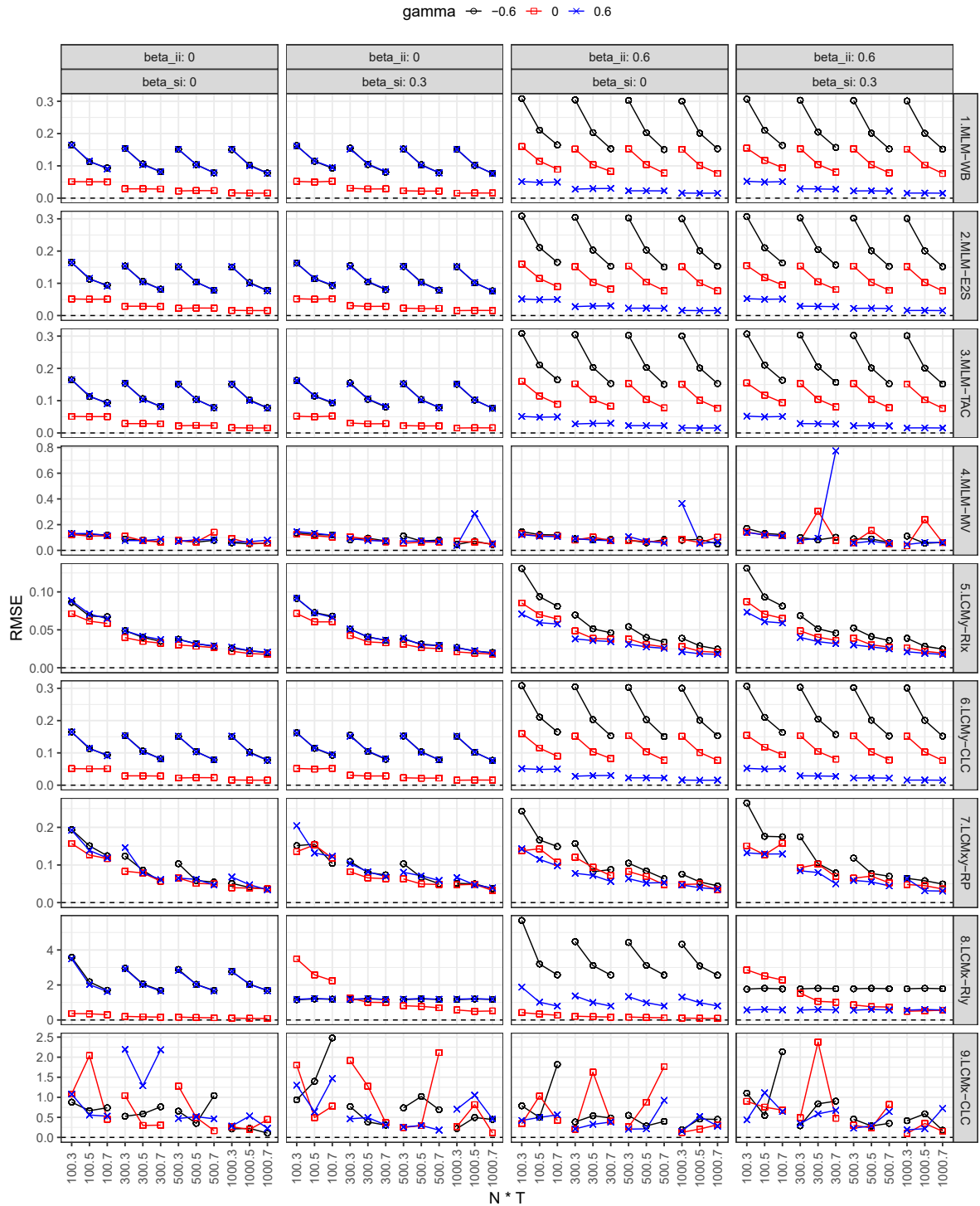
RMSE for the Between-Person Effect β_{II} in Simulation 2

Figure B.23

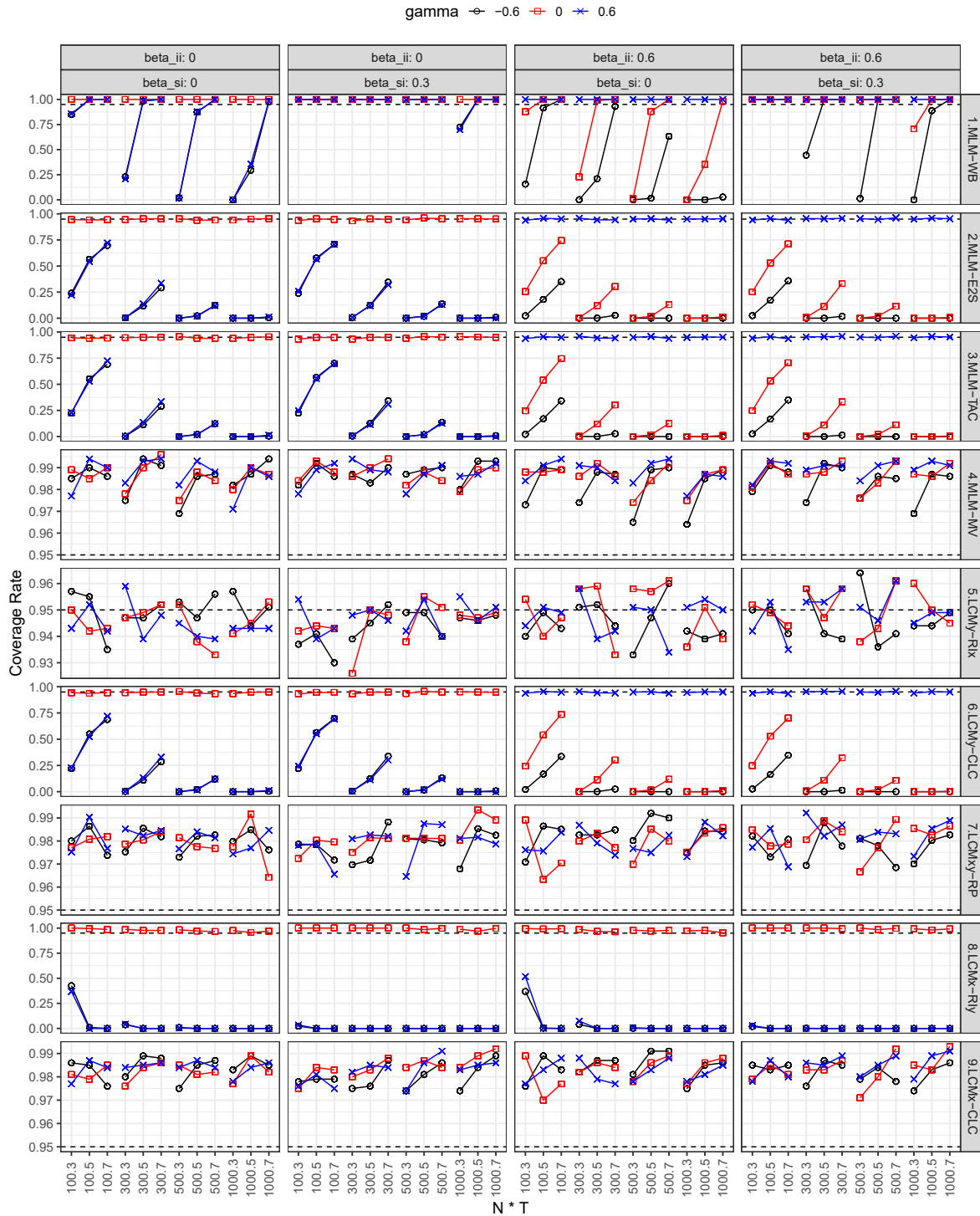
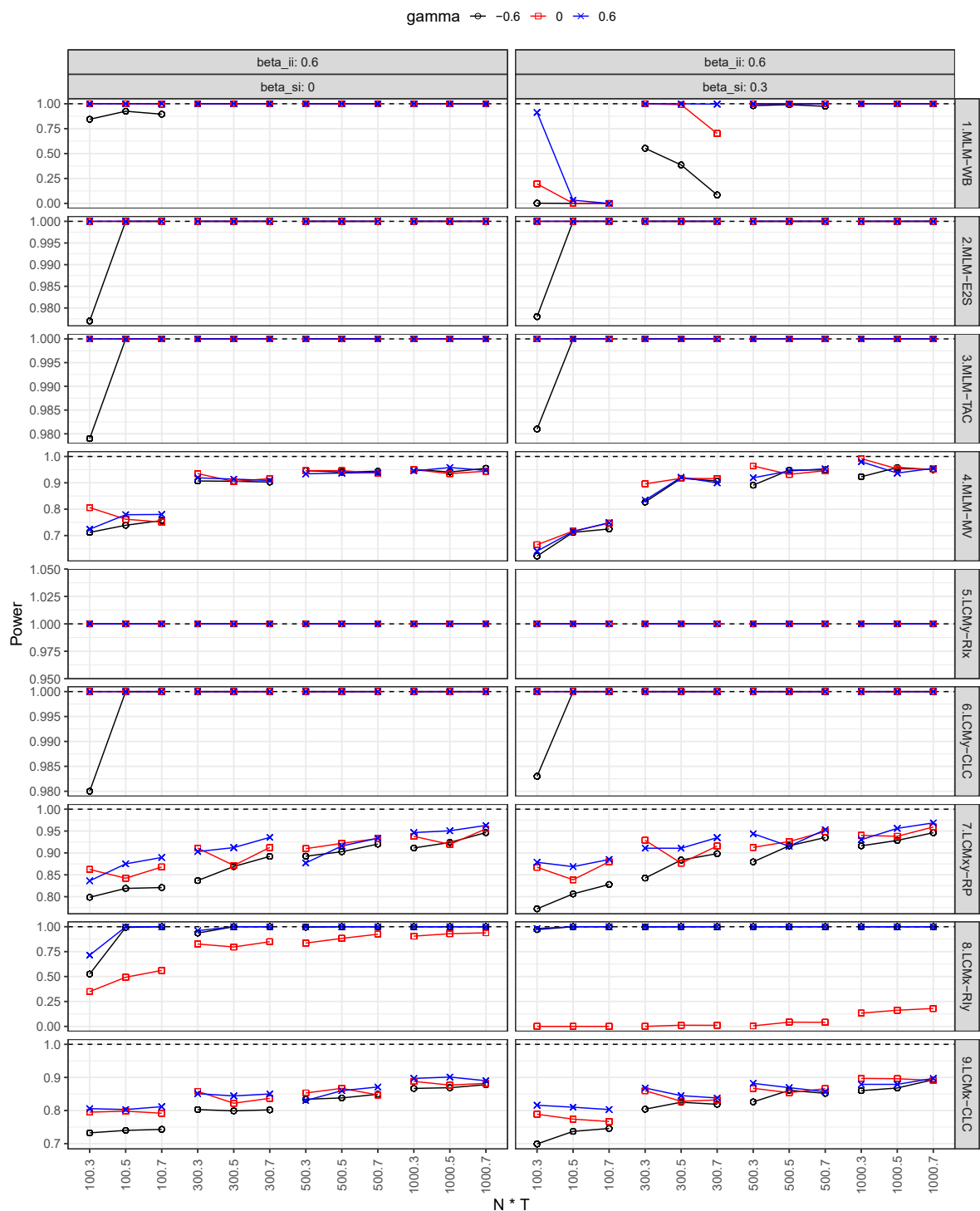
Coverage Rate for the Between-Person Effect β_{II} in Simulation 2

Figure B.24
Power for the Between-Person Effect β_{II} in Simulation 2 (by Model)



B.4 Between-Person Effect of Intercept on Slope

Figure B.25

Bias for the Between-Person Effect β_{SI} in Simulation 2

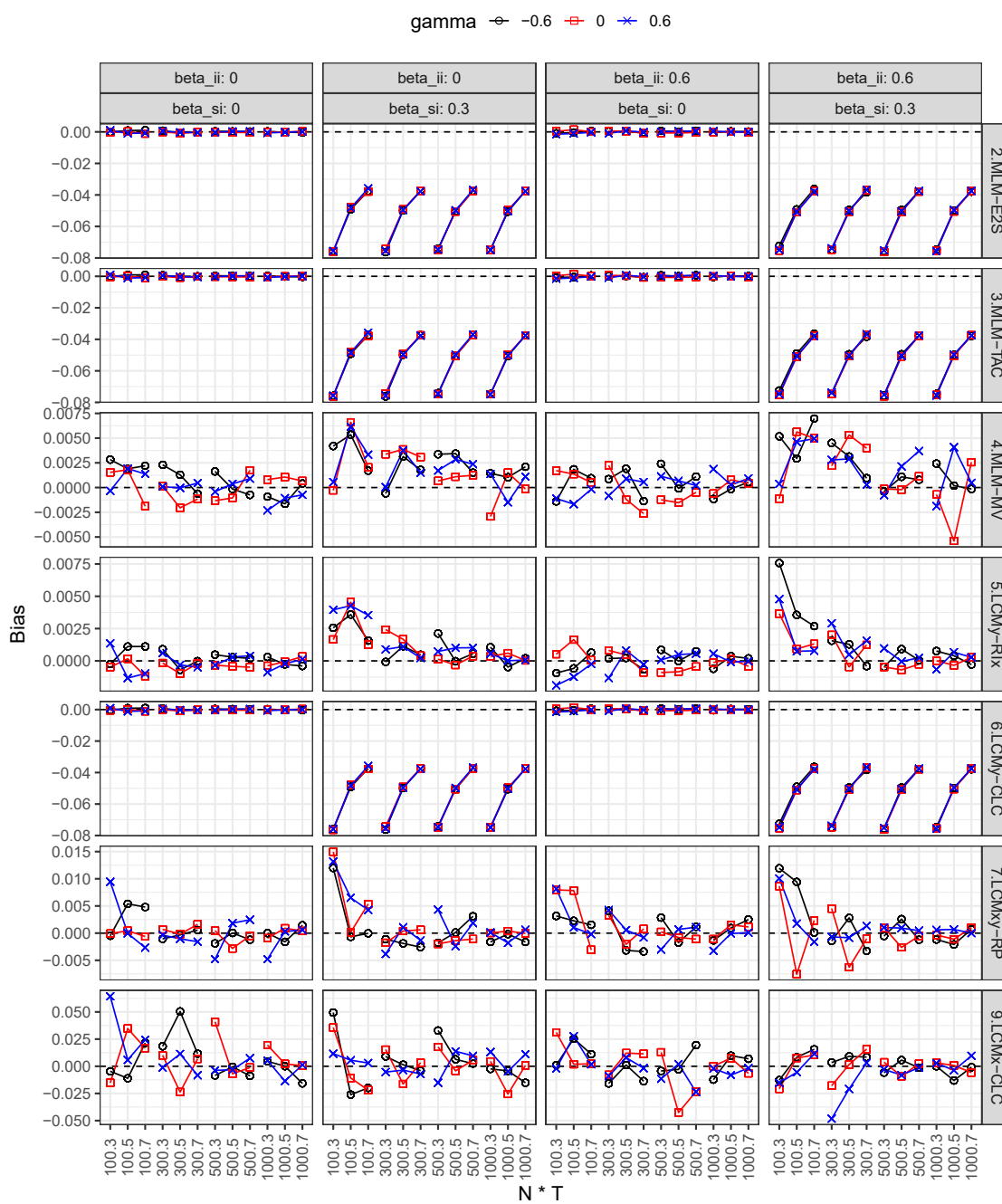


Figure B.26

Empirical SE for the Between-Person Effect β_{SI} in Simulation 2

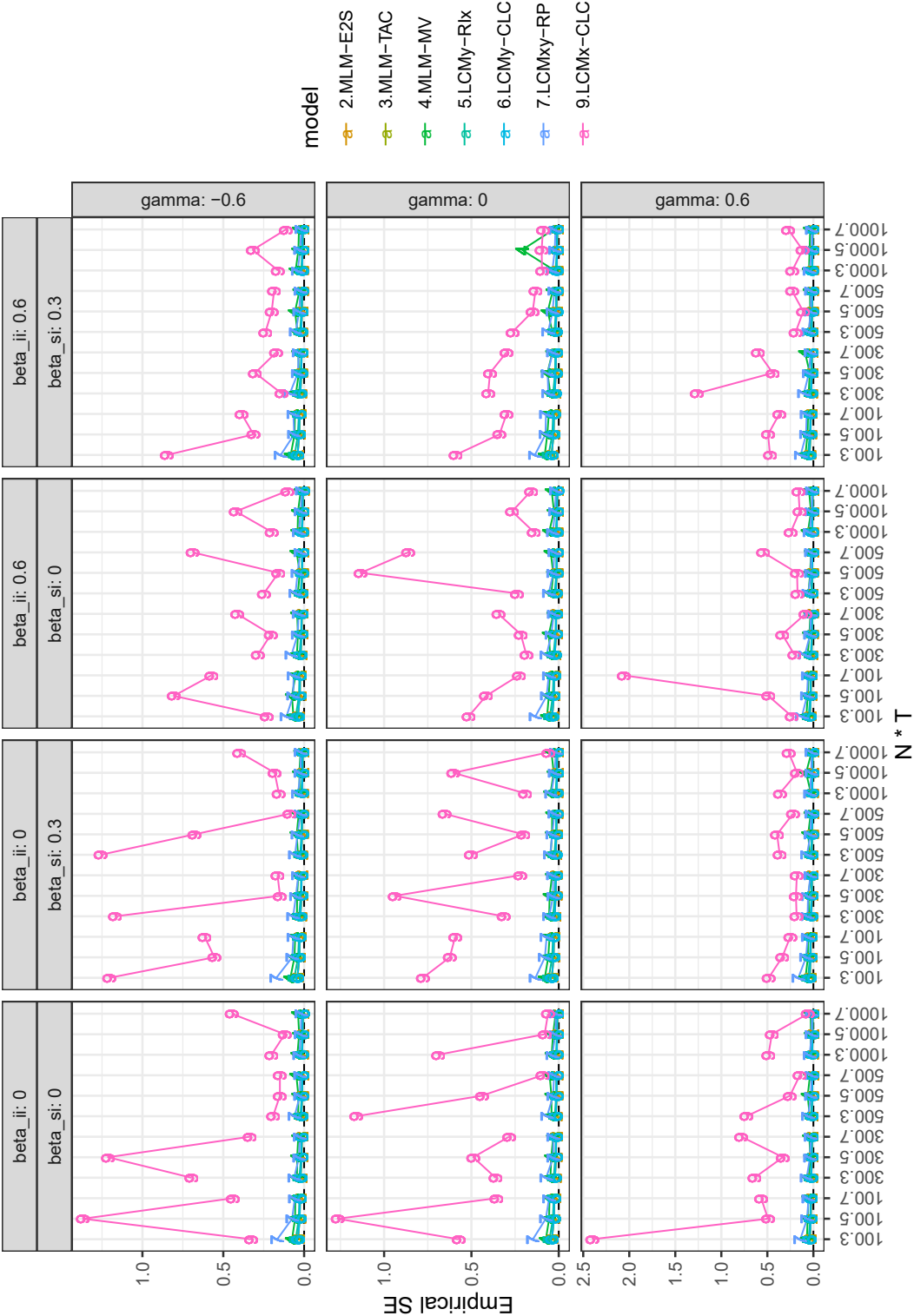


Figure B.27

Empirical SE for the Between-Person Effect β_{SI} in Simulation 2 except LCMx-CLC

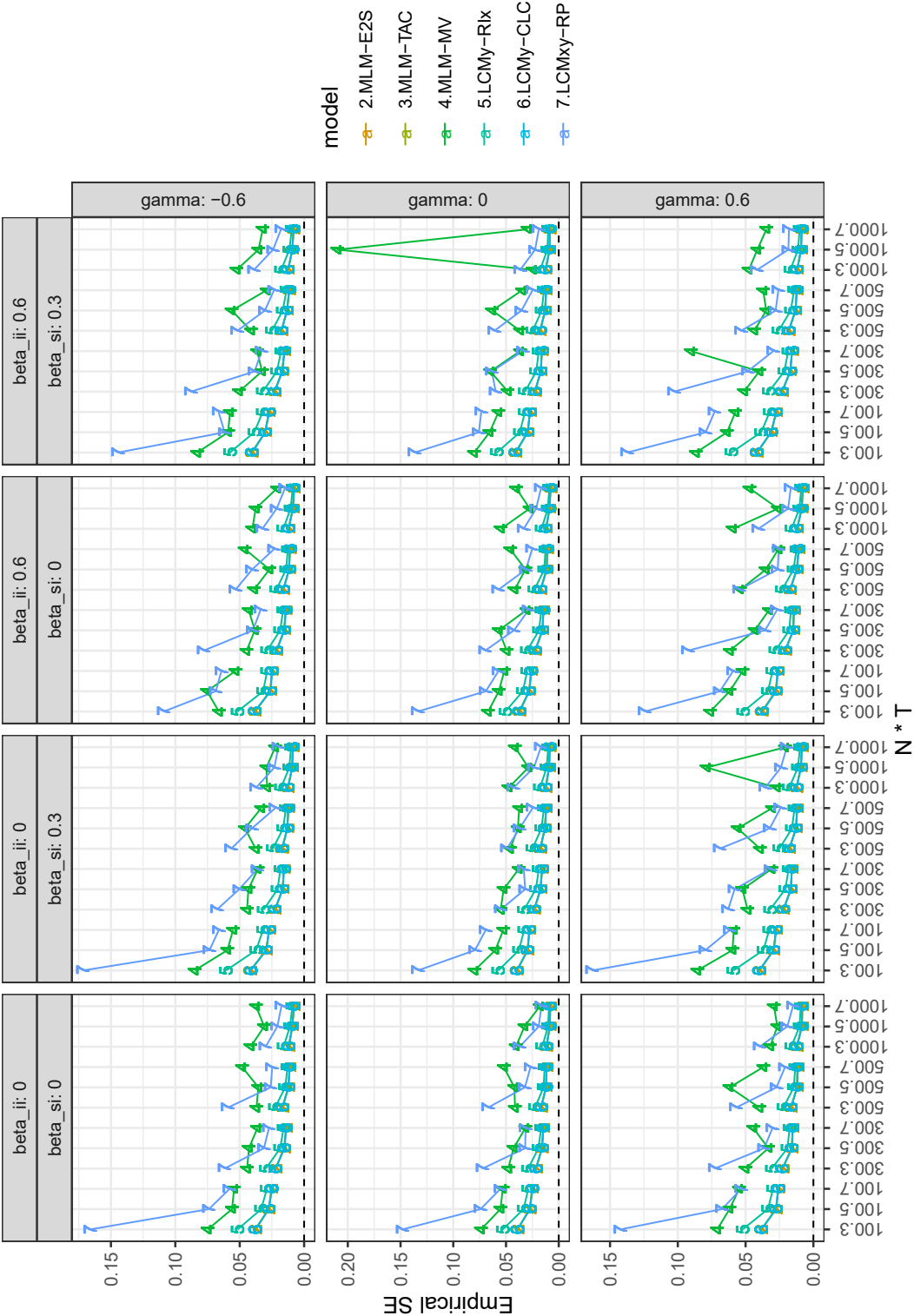


Figure B.28

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 2

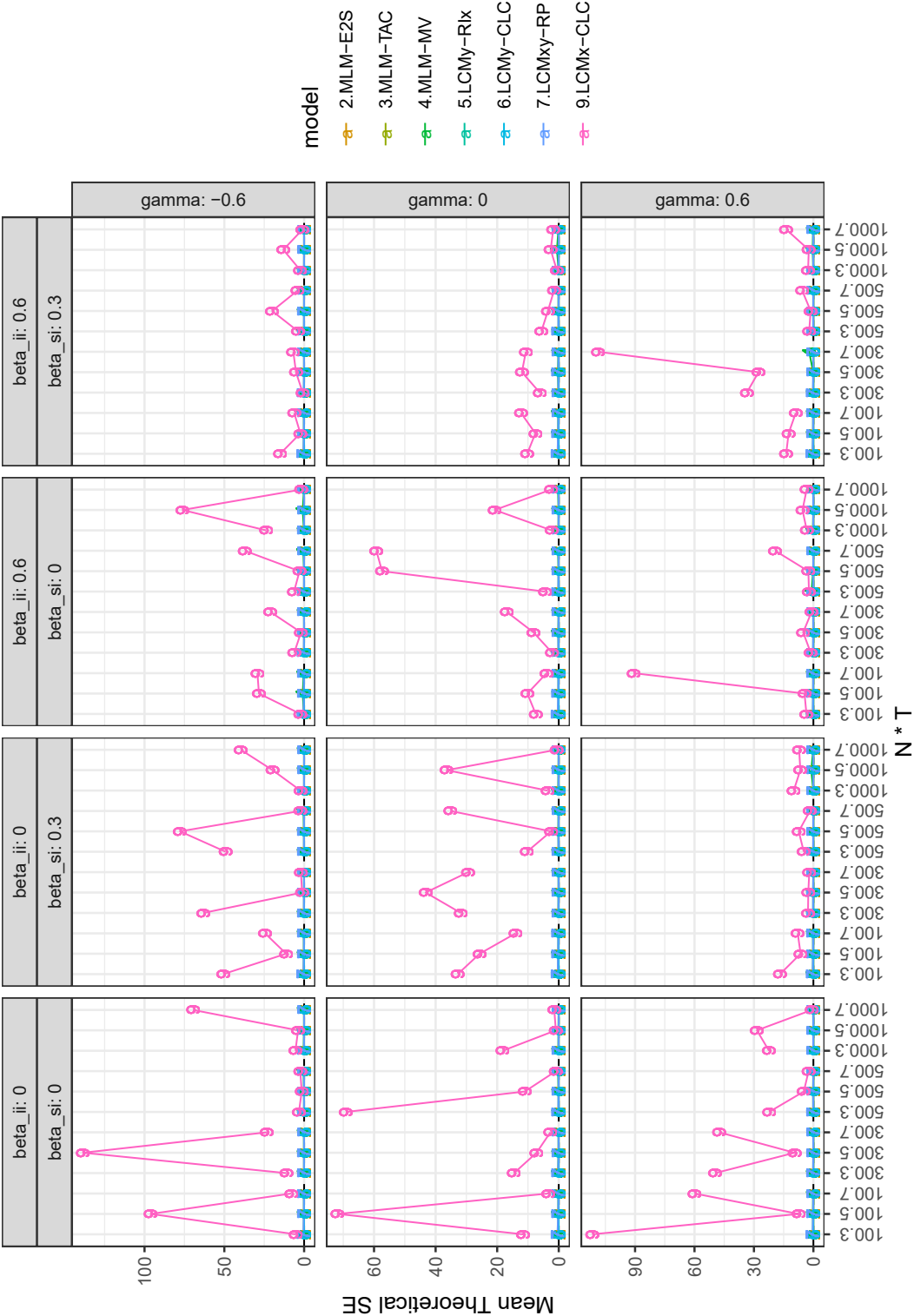


Figure B.29

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 2 except LCMx-CLC

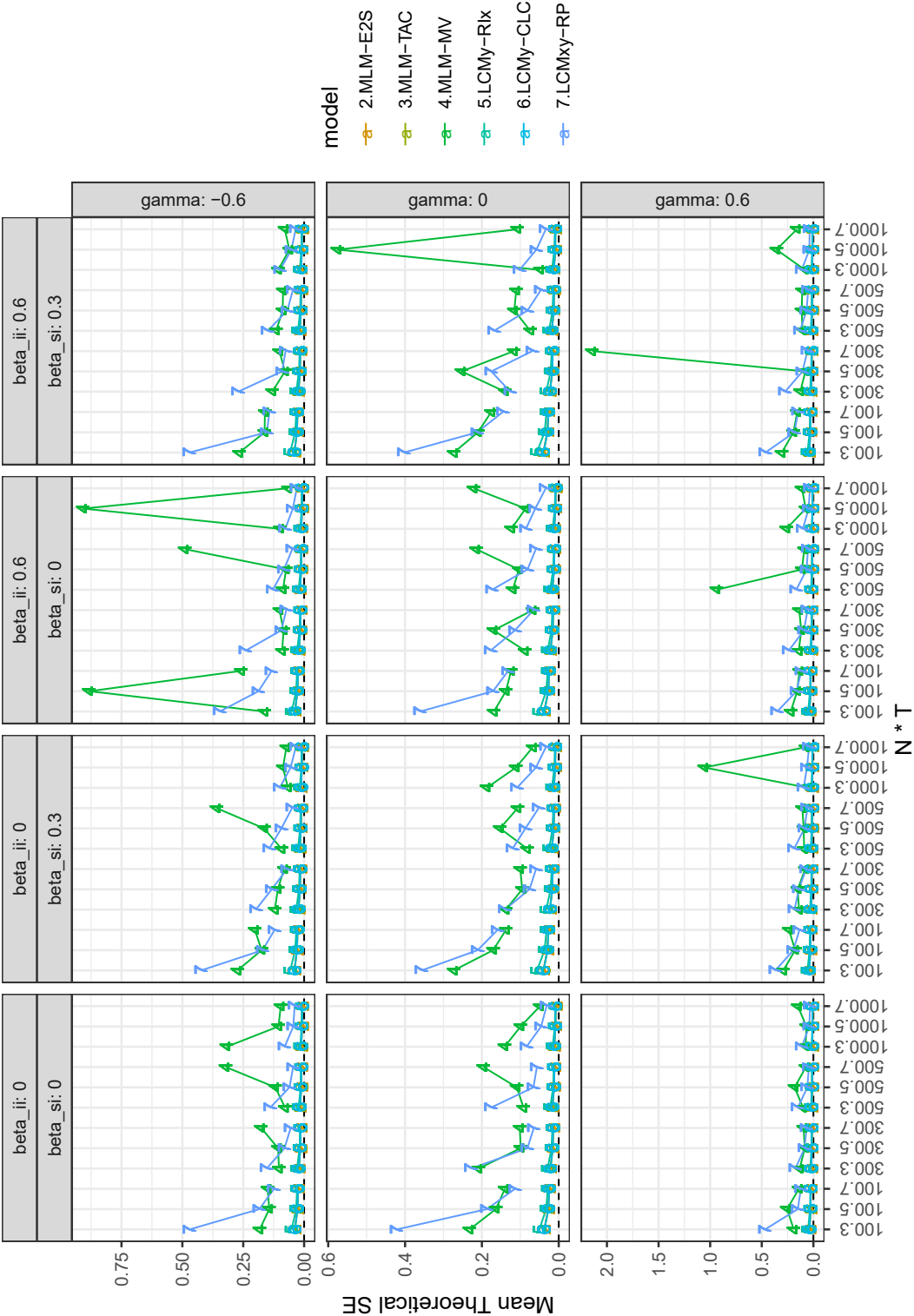


Figure B.30

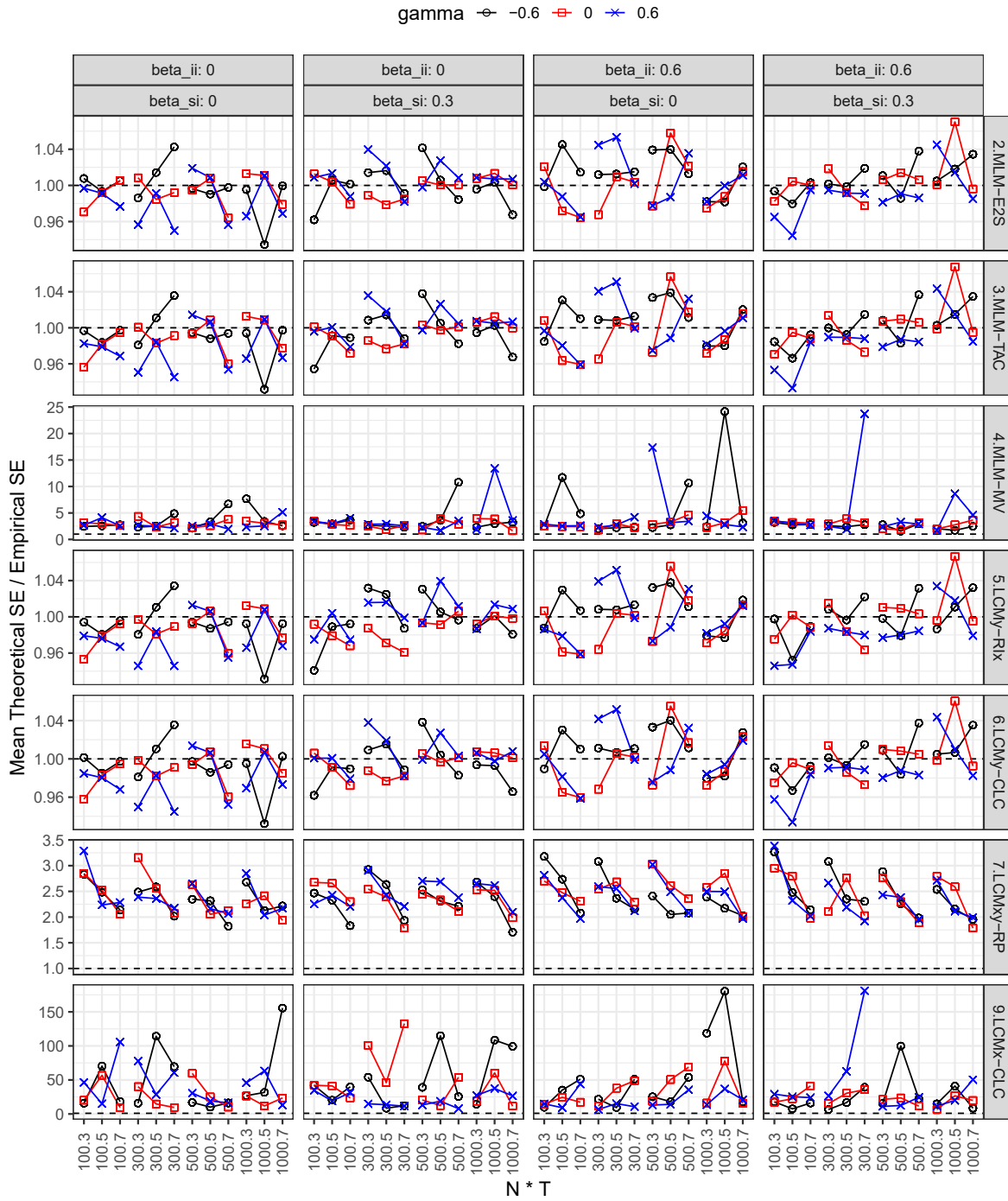
SE Ratio for the Between-Person Effect β_{SI} in Simulation 2

Figure B.31

RMSE for the Between-Person Effect β_{SI} in Simulation 2

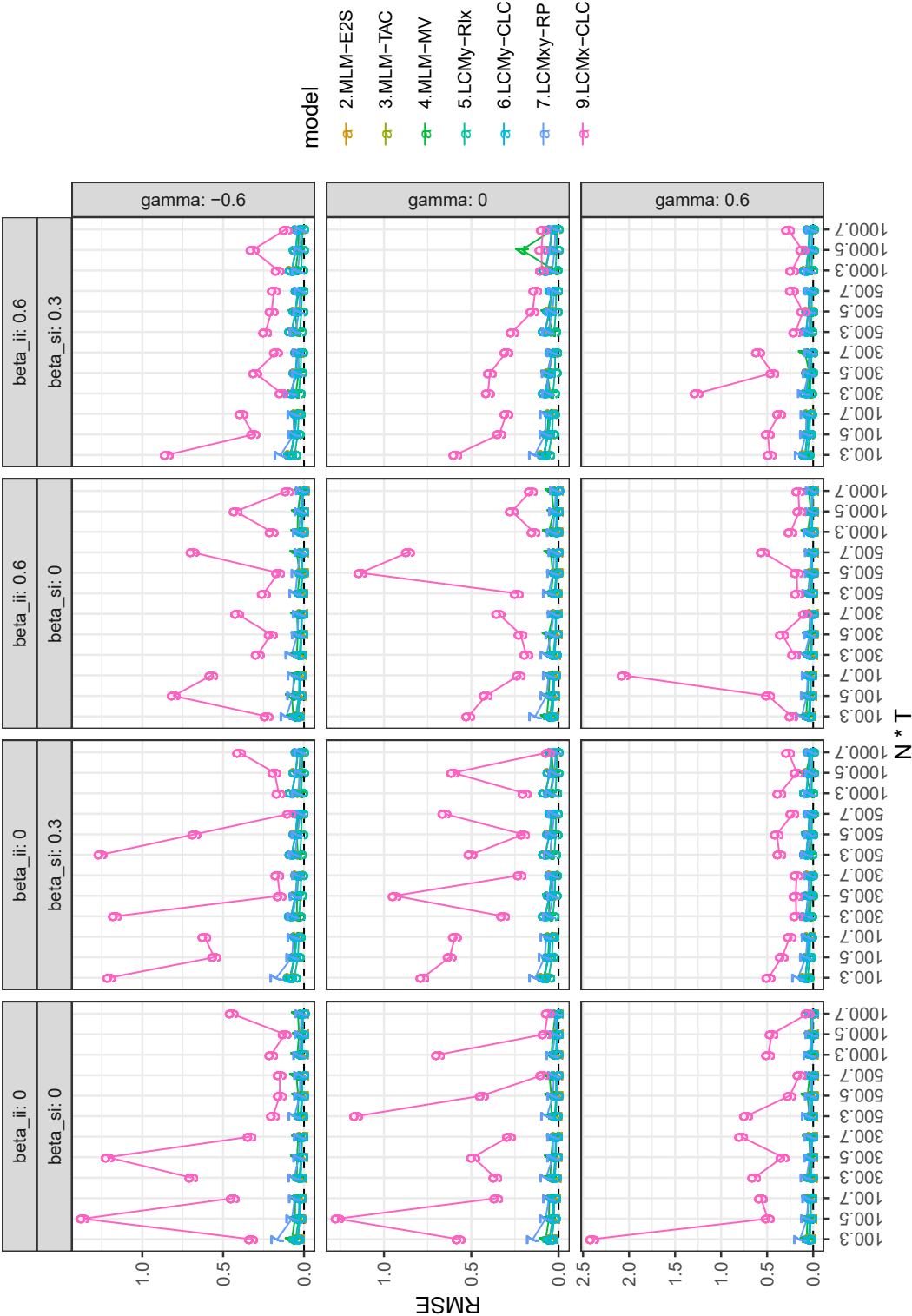


Figure B.32

RMSE for the Between-Person Effect β_{SI} in Simulation 2 except LCMx-CLC

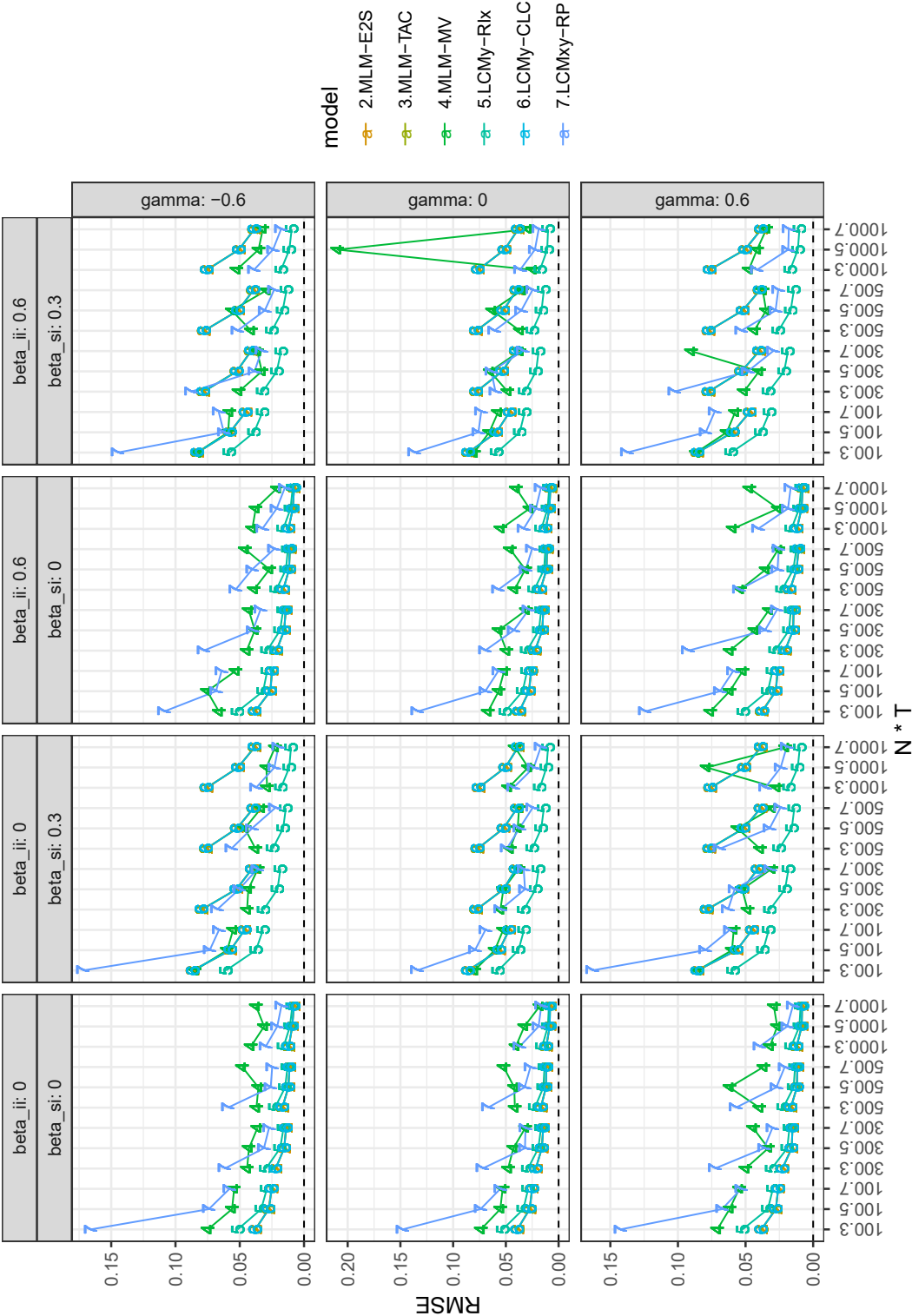


Figure B.33

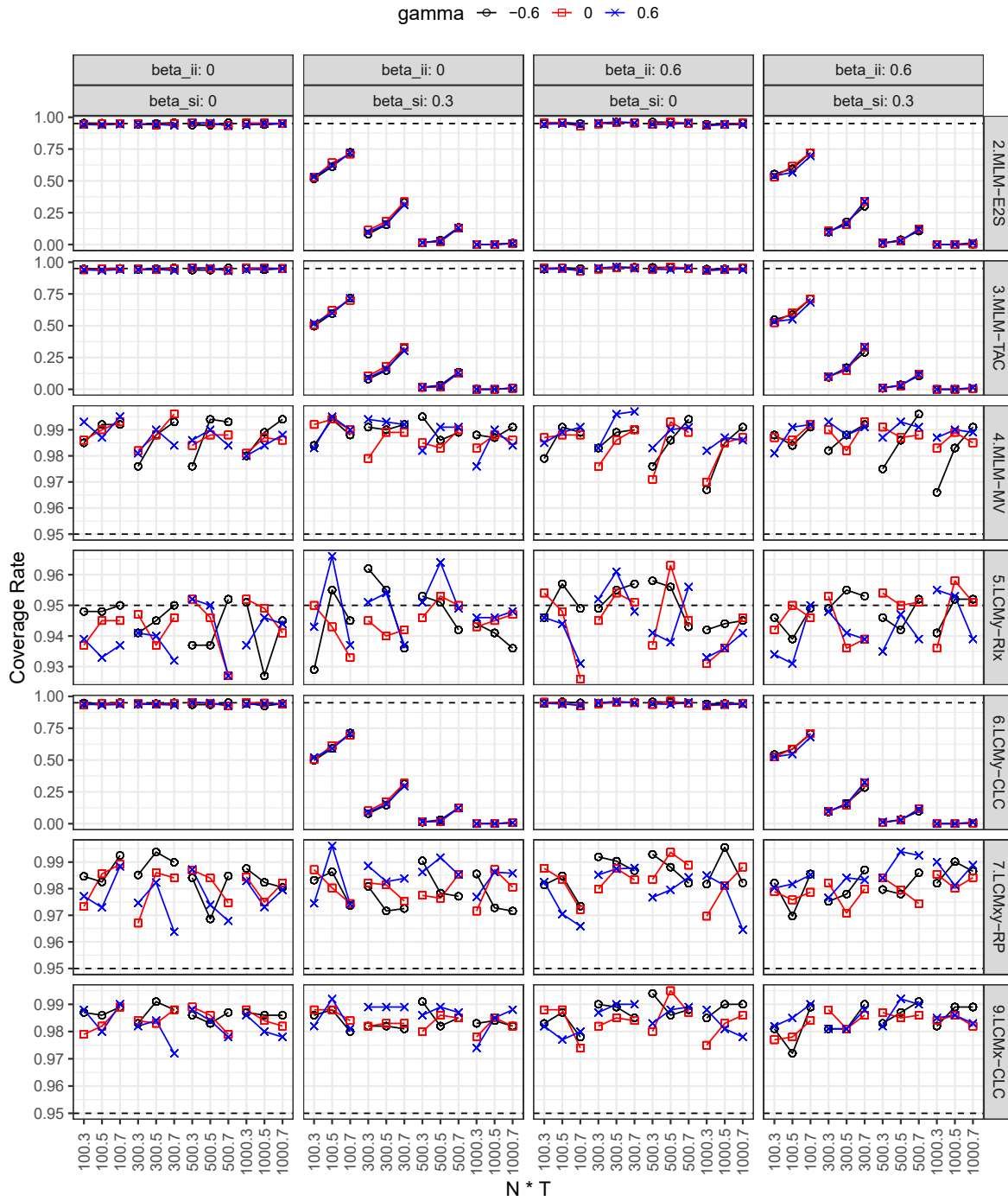
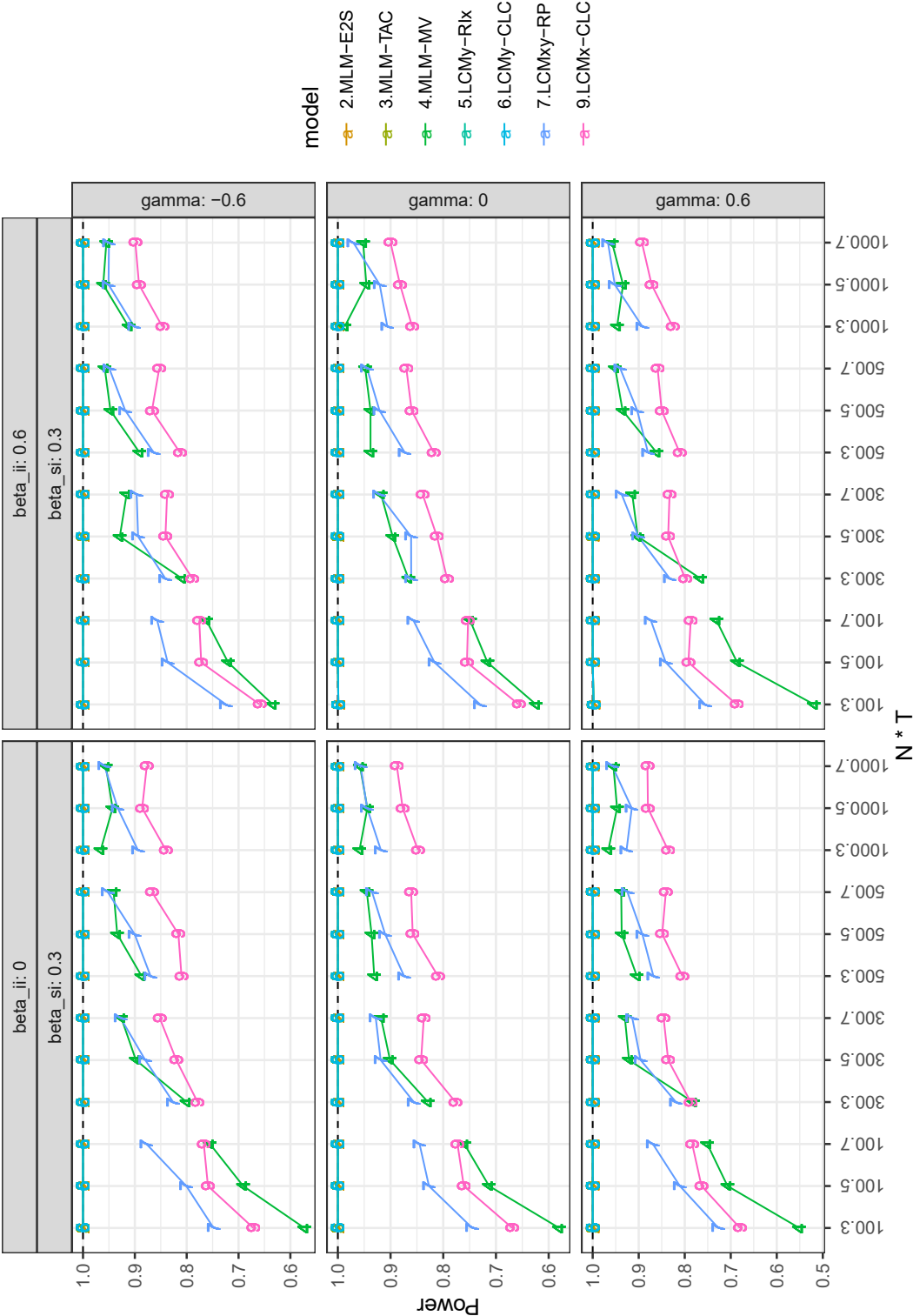
Coverage Rate for the Between-Person Effect β_{SI} in Simulation 2

Figure B.34

Power for the Between-Person Effect β_{SI} in Simulation 2



B.5 Between-Person Effect of Slope on Intercept

Figure B.35

Bias for the Between-Person Effect β_{IS} in Simulation 2

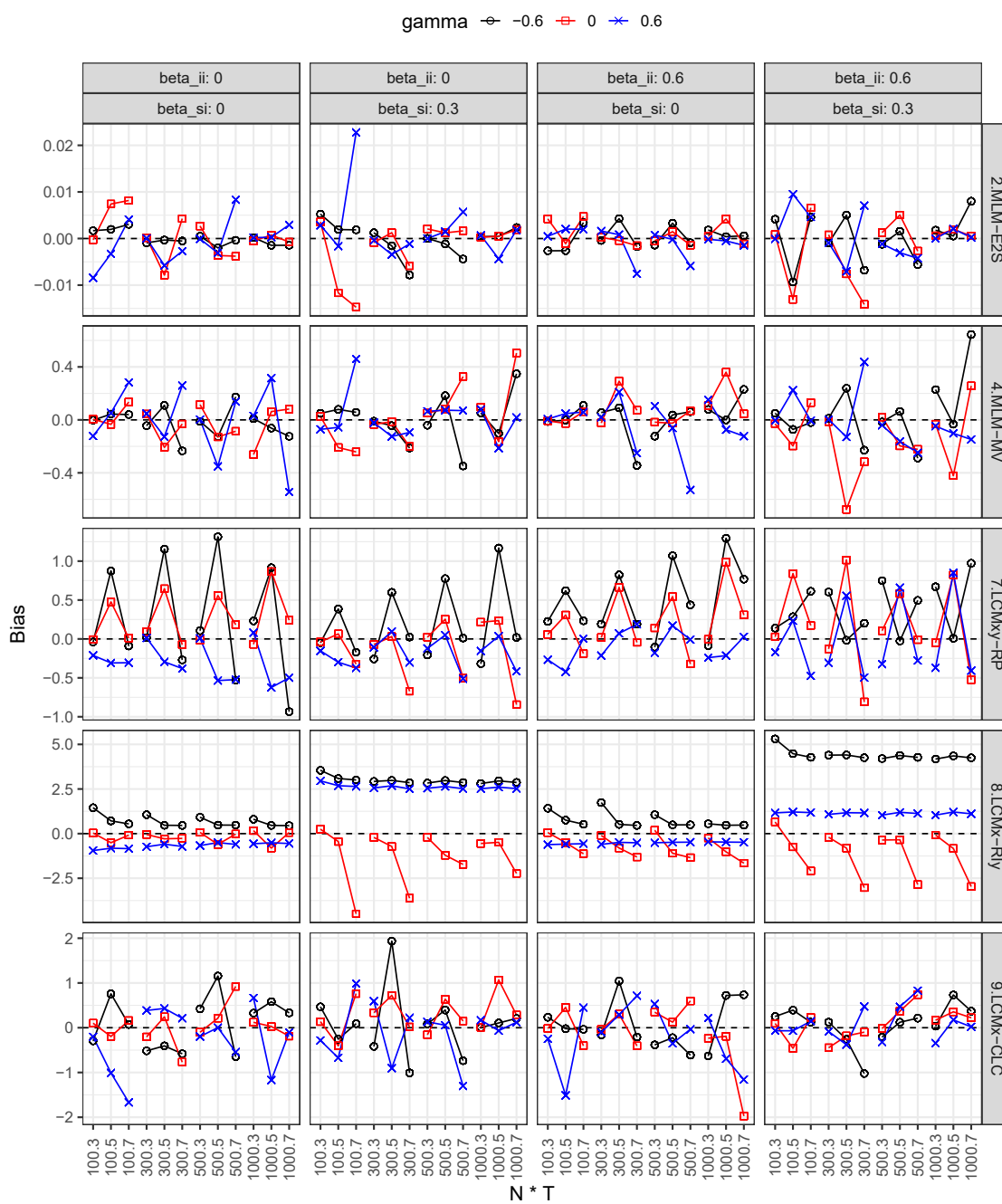


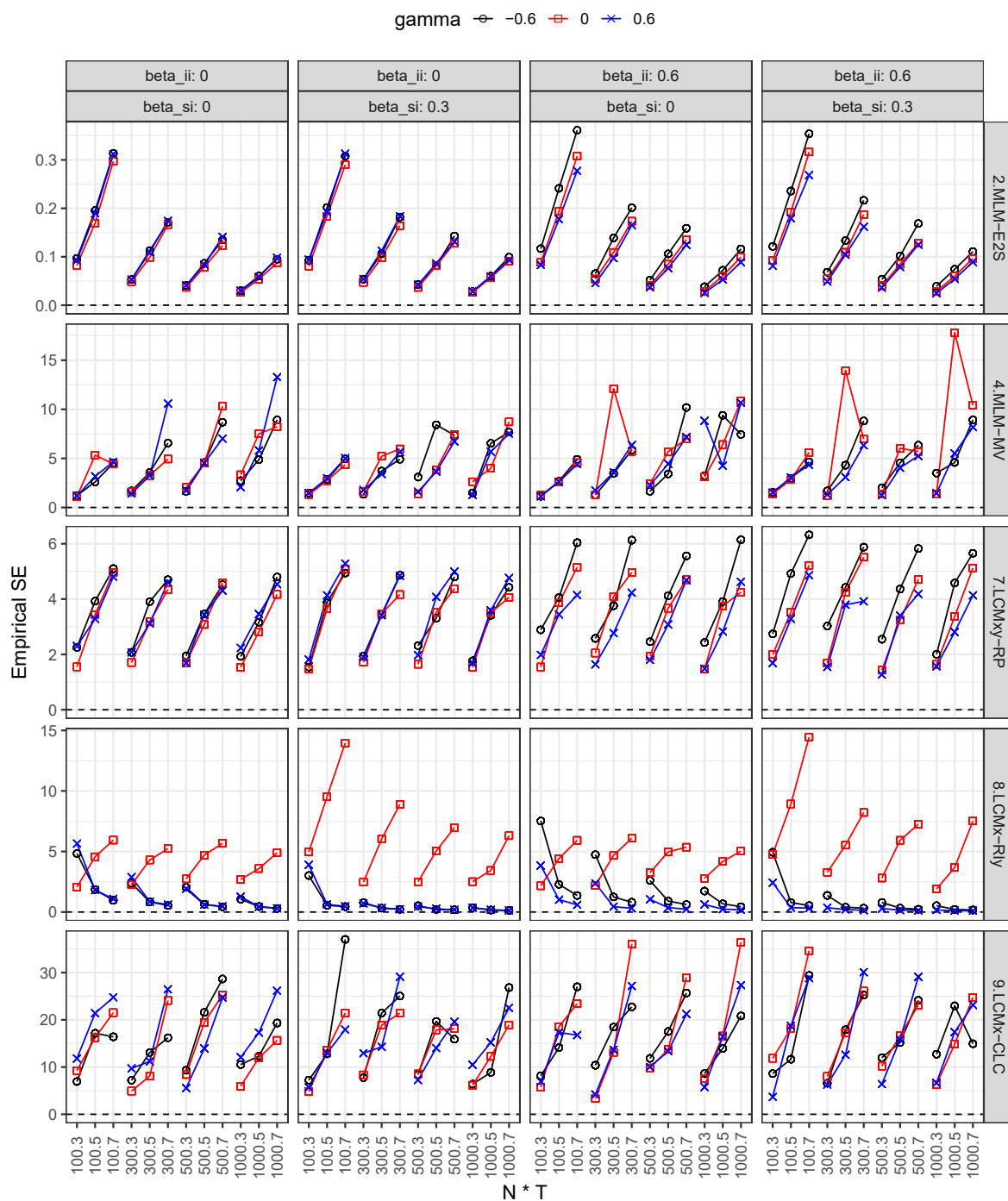
Figure B.36*Empirical SE for the Between-Person Effect β_{IS} in Simulation 2*

Figure B.37

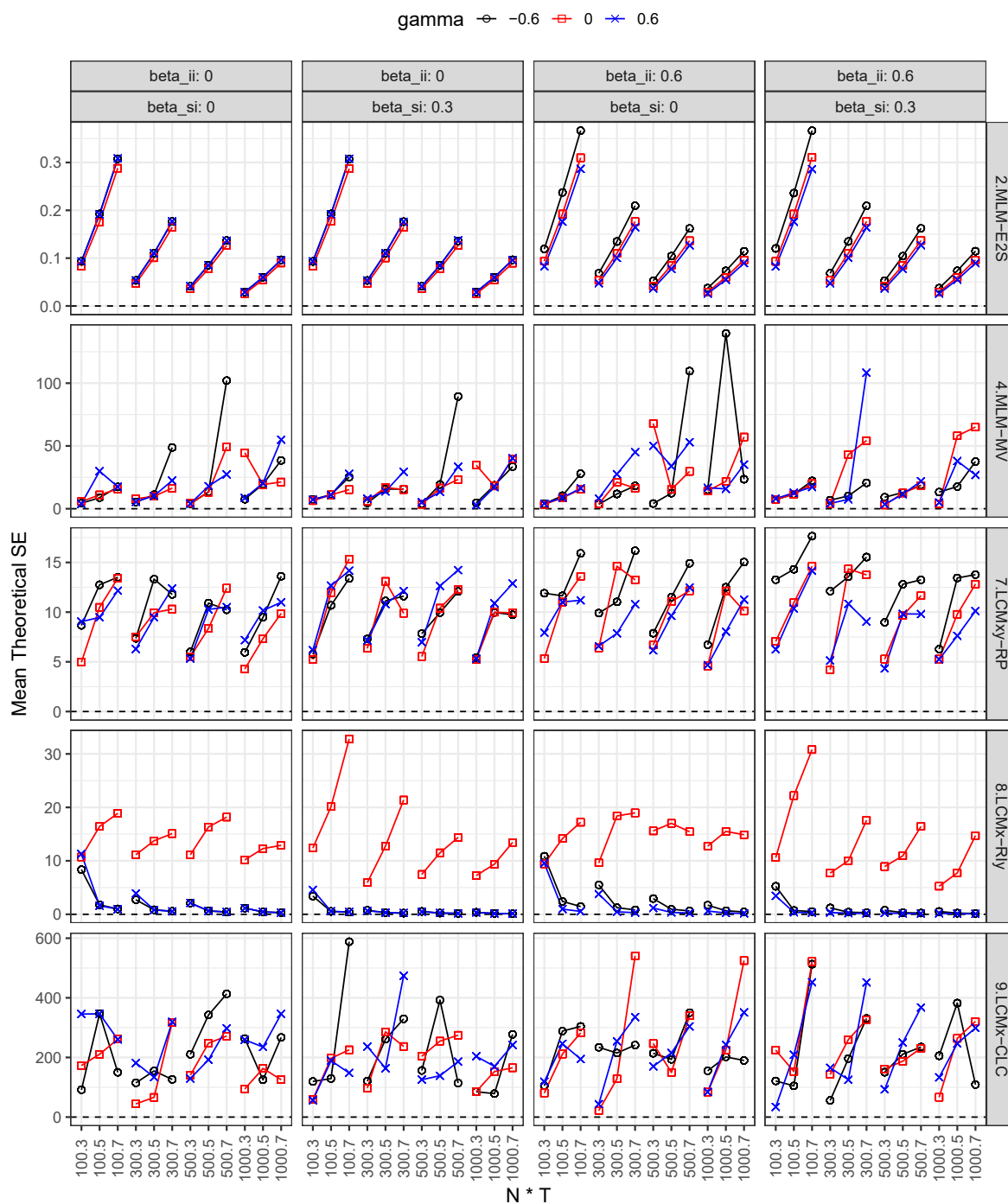
Theoretical SE for the Between-Person Effect β_{IS} in Simulation 2

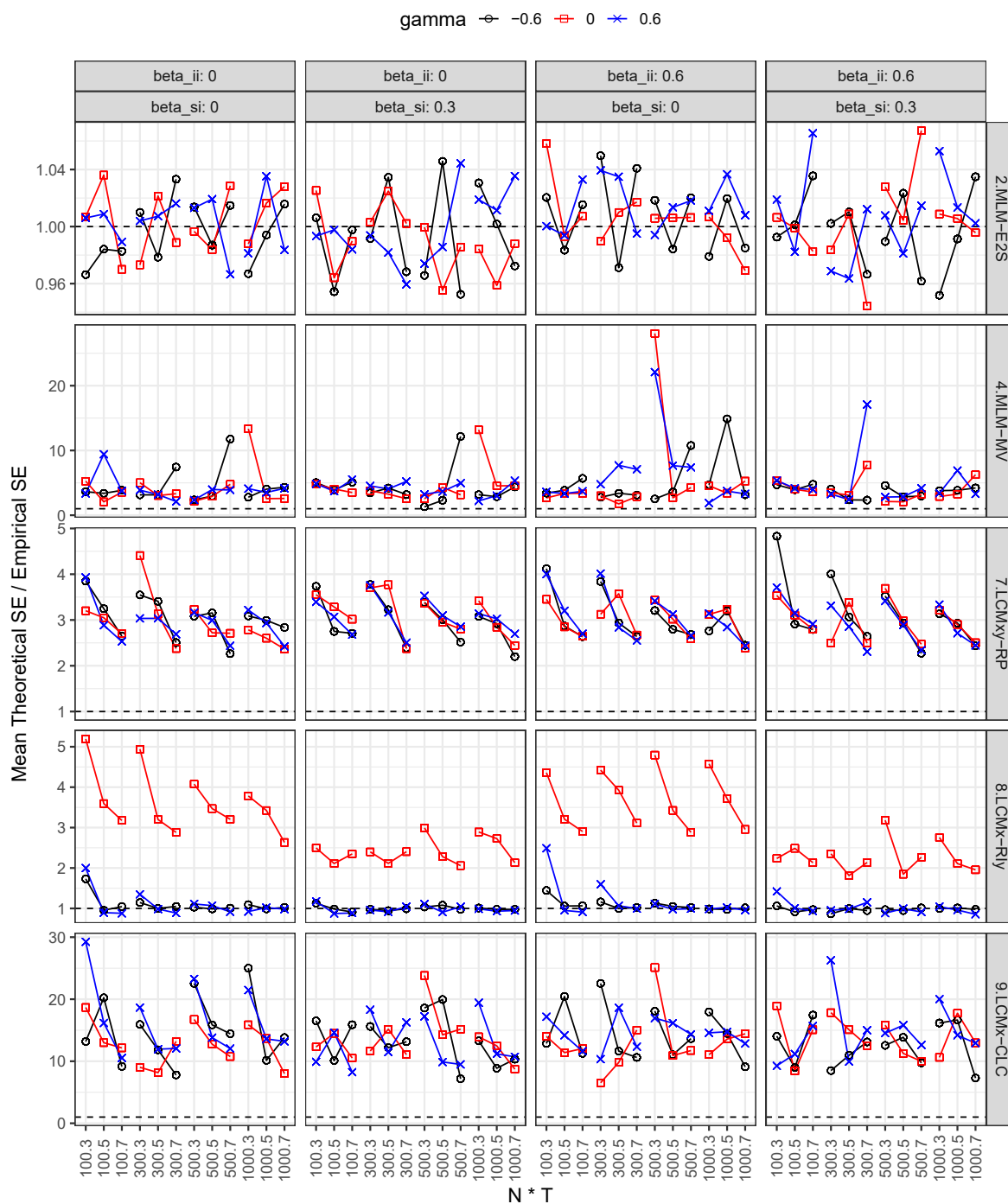
Figure B.38*SE Ratio for the Between-Person Effect β_{IS} in Simulation 2*

Figure B.39

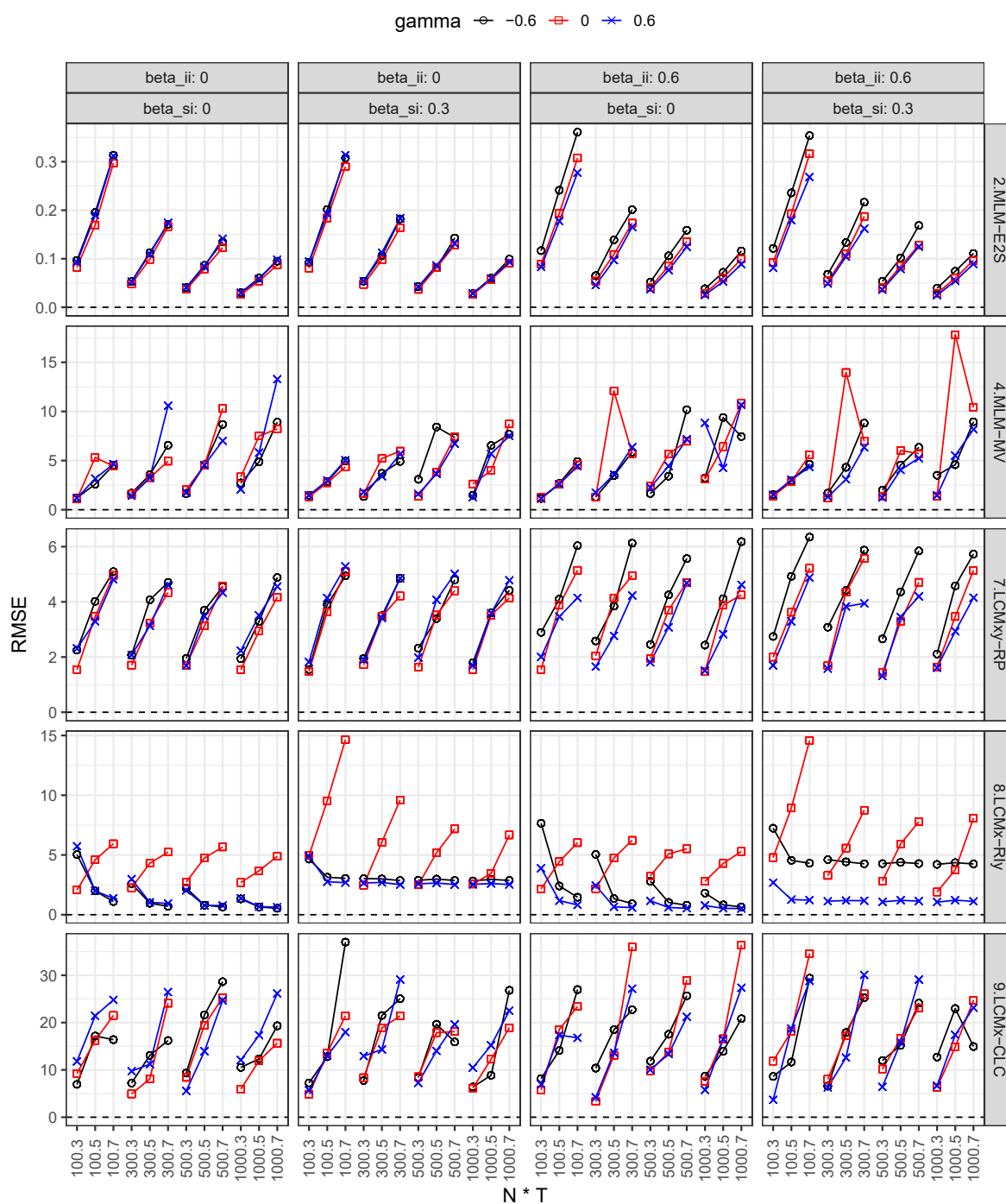
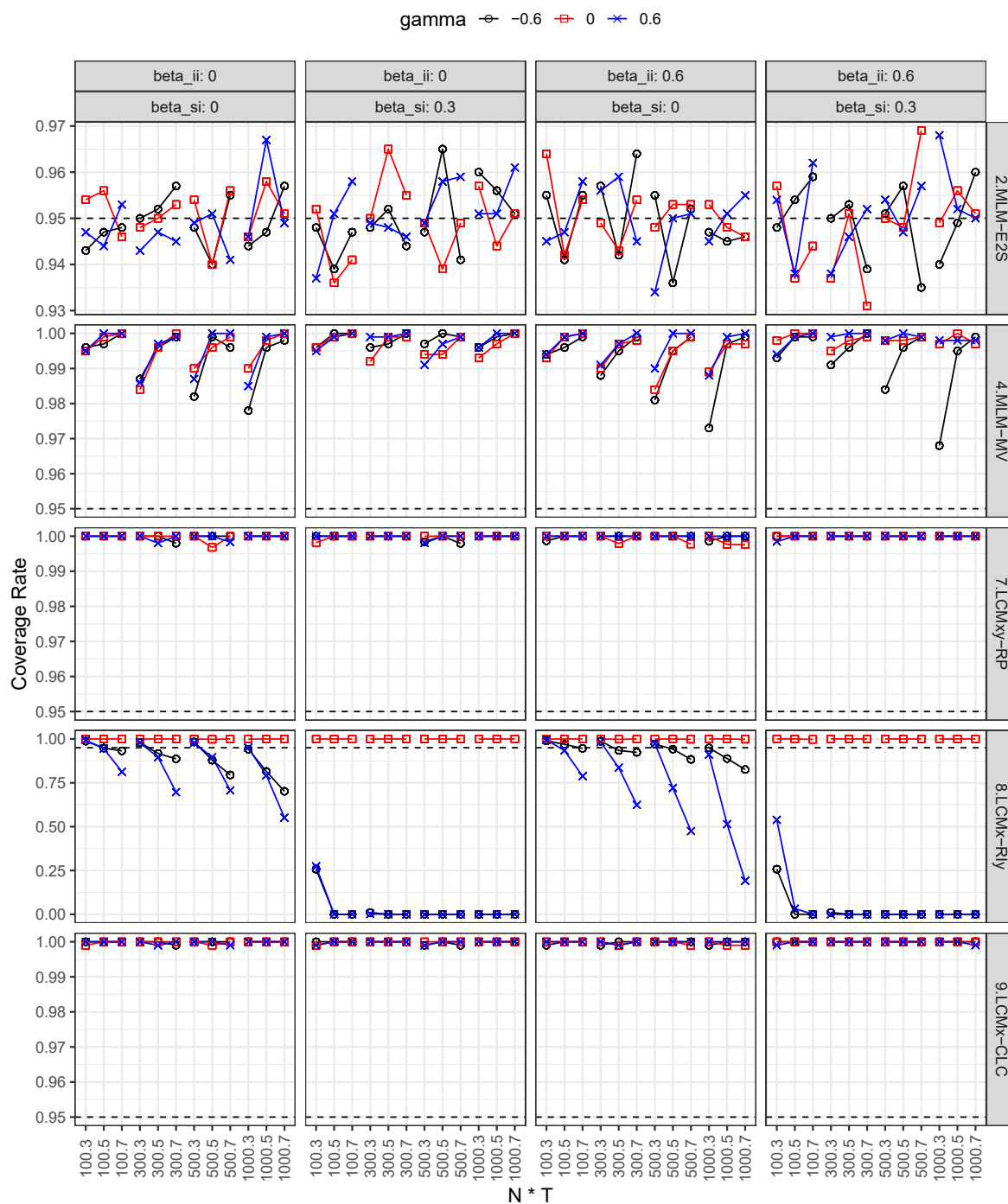
RMSE for the Between-Person Effect β_{IS} in Simulation 2

Figure B.40*Coverage Rate for the Between-Person Effect β_{IS} in Simulation 2*

B.6 Between-Person Effect of Slope on Slope

Figure B.41

Bias for the Between-Person Effect β_{SS} in Simulation 2

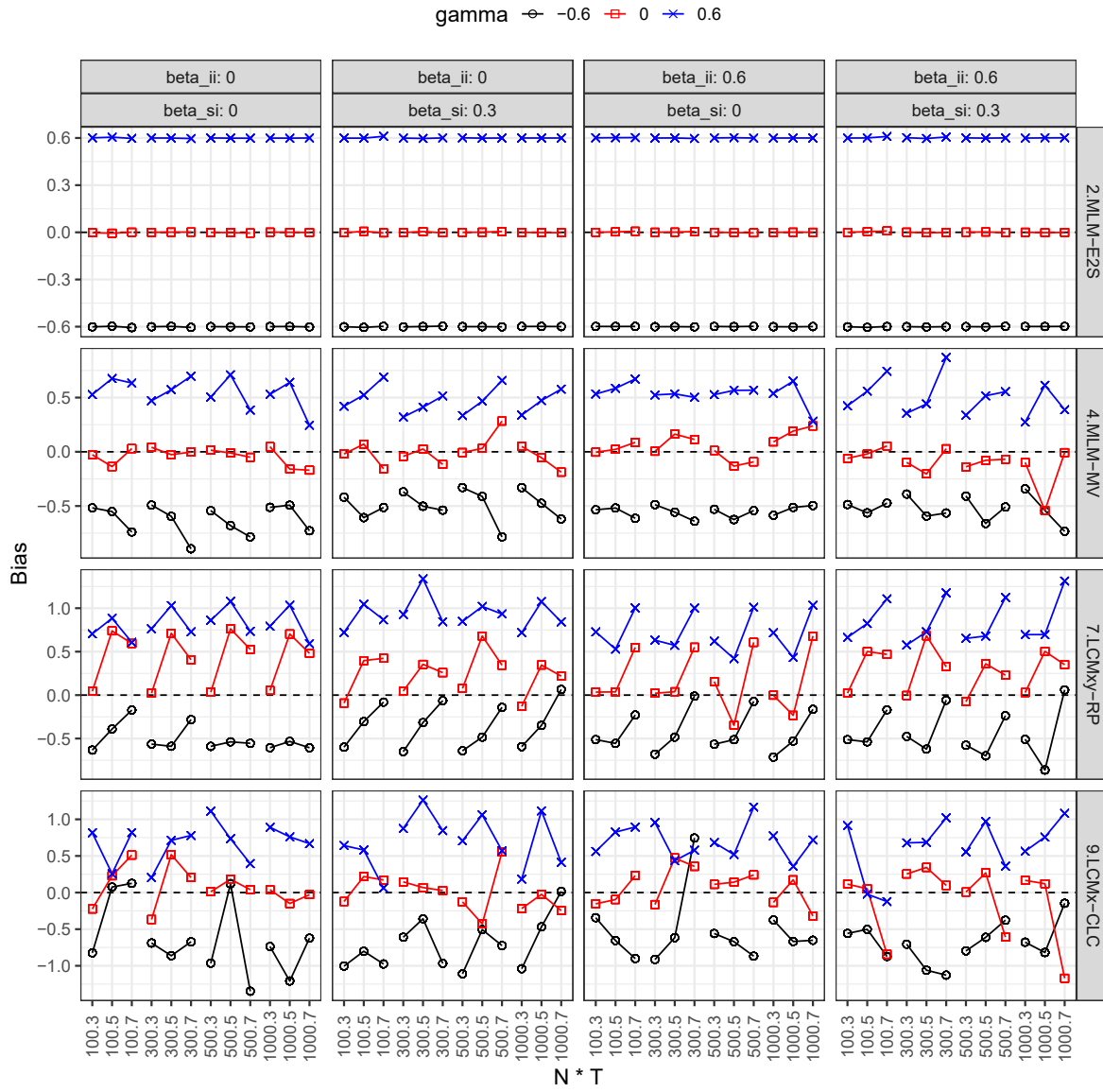


Figure B.42

Scatter Plot of $\hat{\gamma}$ and $\hat{\beta}_{SS}$ in Simulation 2 (10 samples per condition)

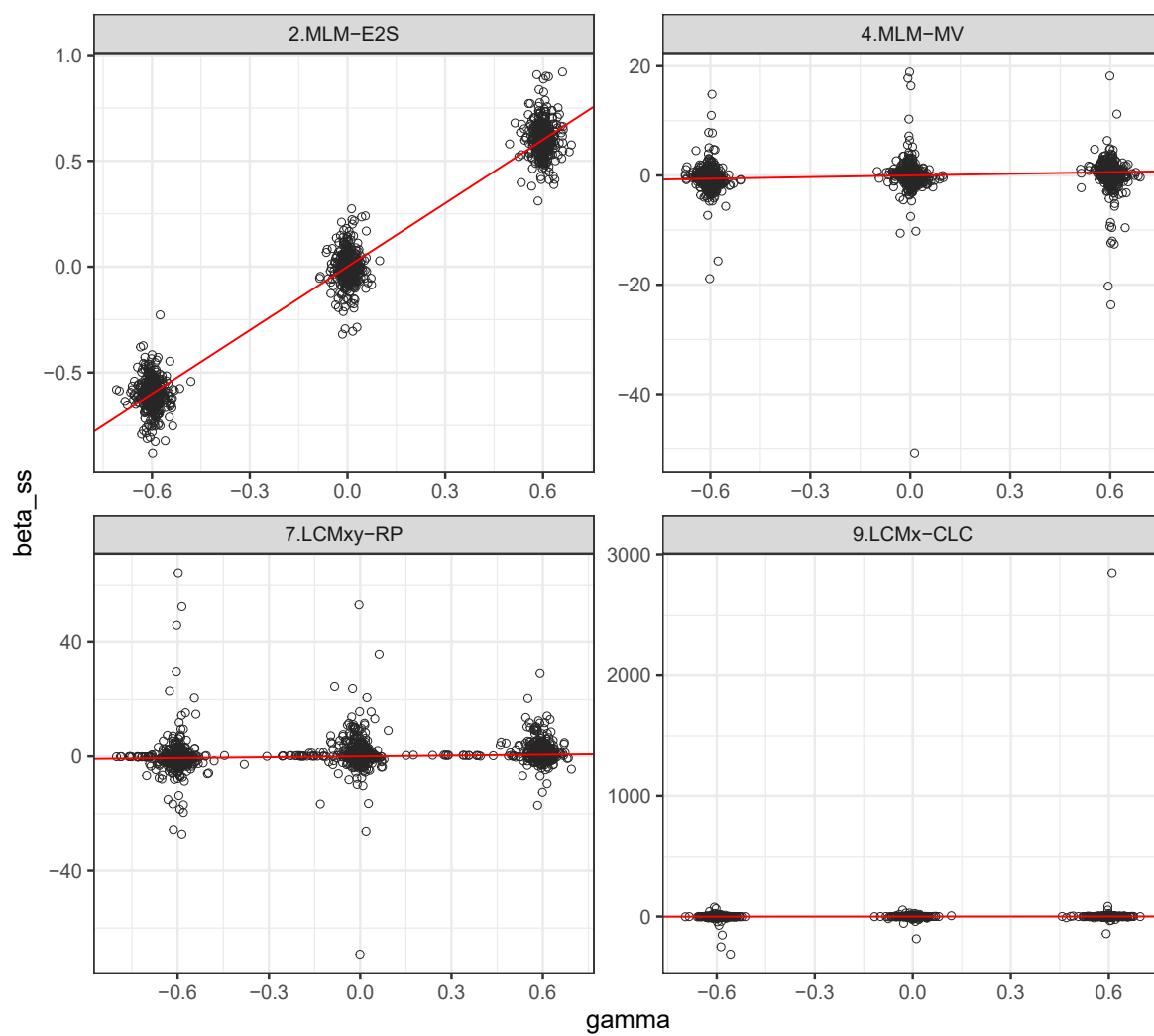


Figure B.43

Empirical SE for the Between-Person Effect β_{SS} in Simulation 2

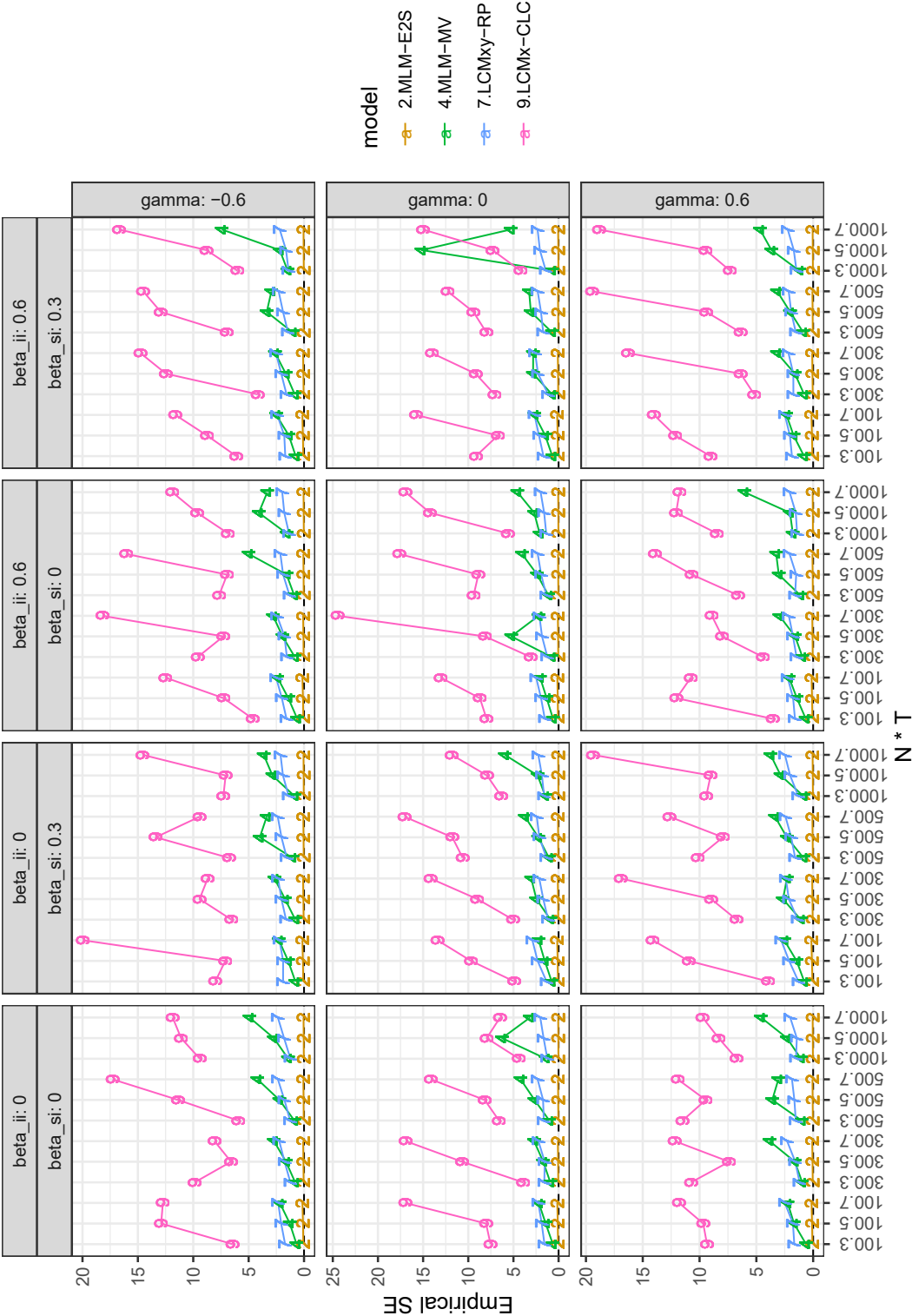


Figure B.44

Empirical SE for the Between-Person Effect β_{SS} in Simulation 2 except LCMx-CLC

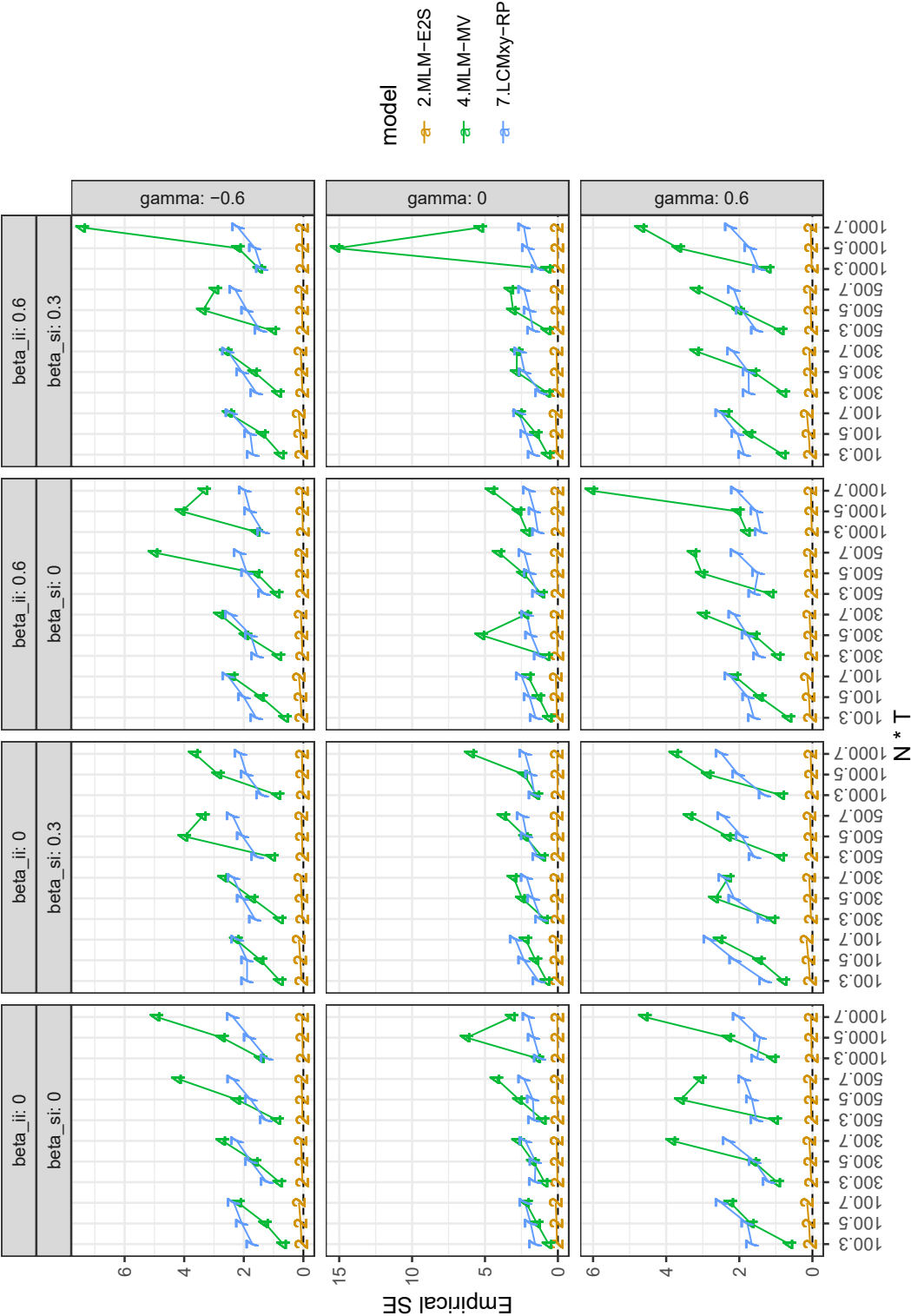


Figure B.45

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 2

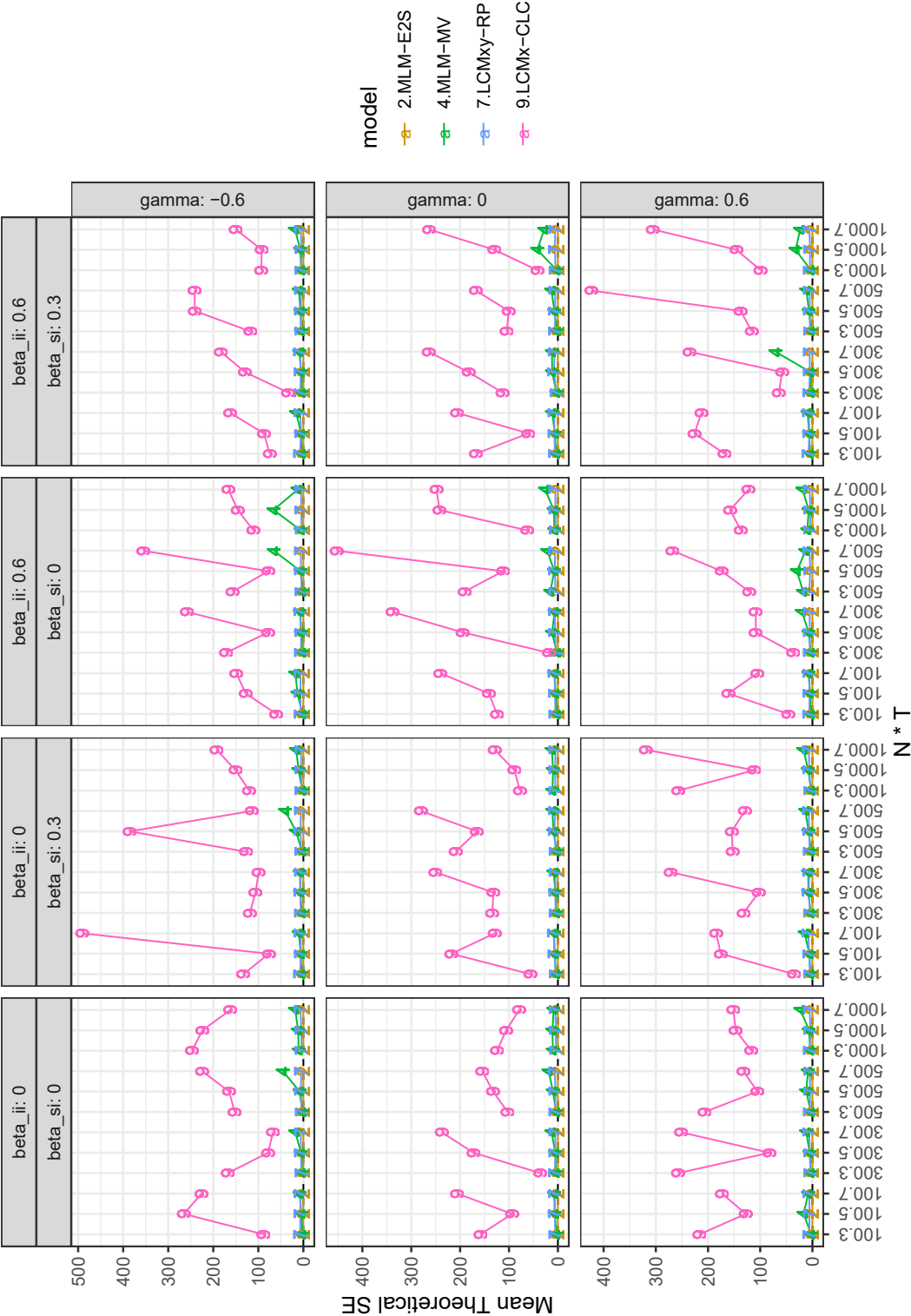


Figure B.46

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 2 except LCMx-CLC

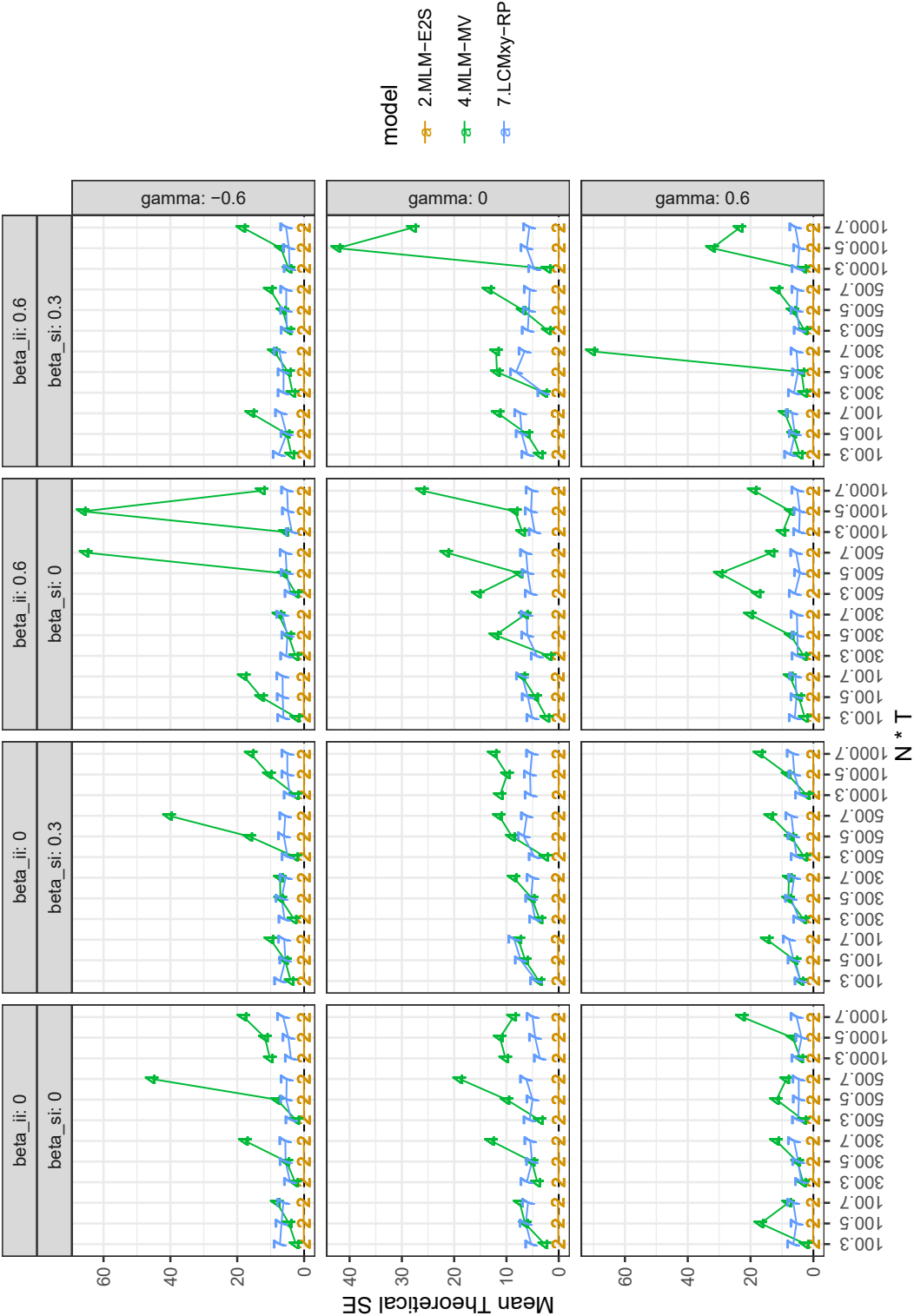


Figure B.47

SE Ratio for the Between-Person Effect β_{SS} in Simulation 2

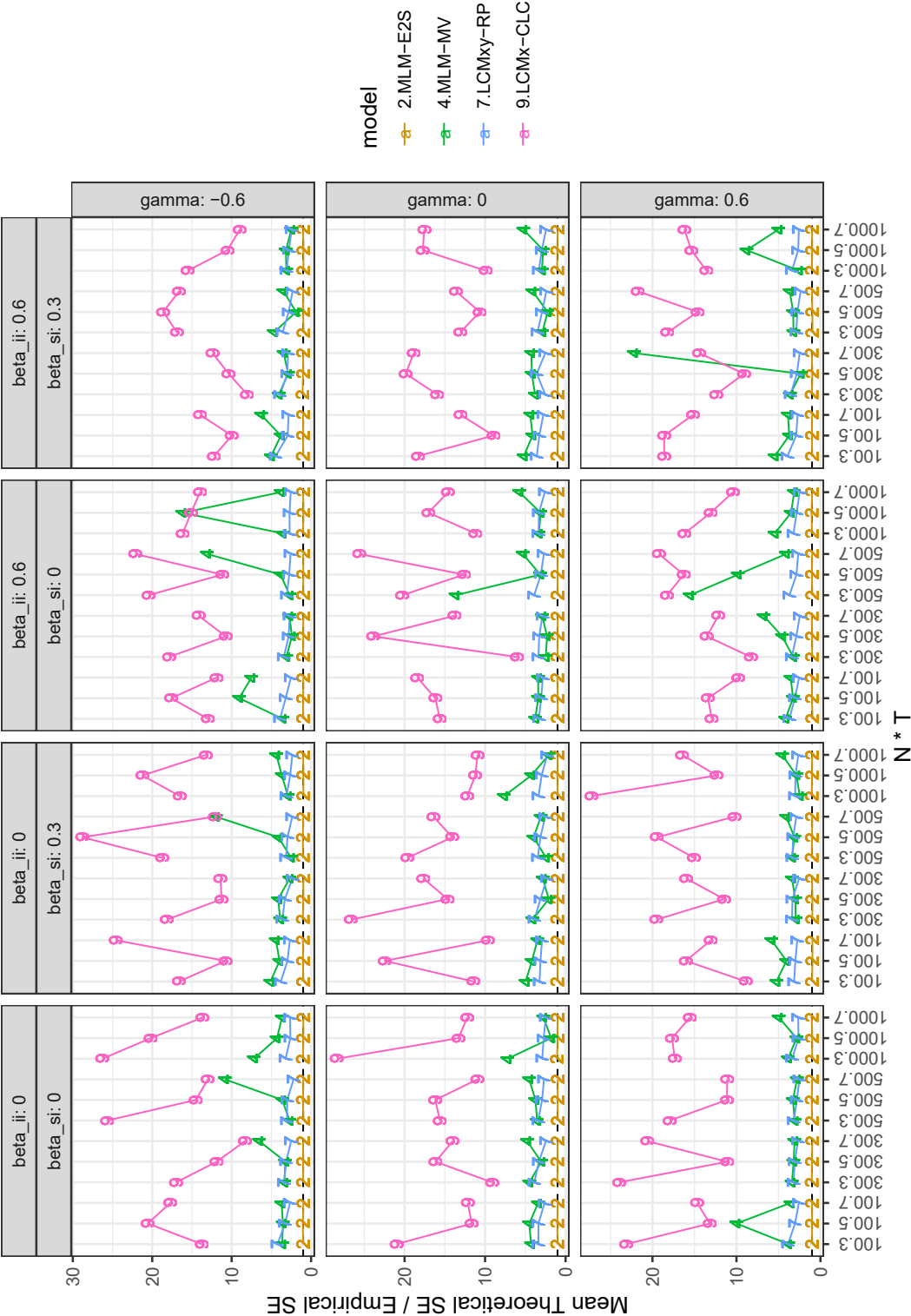


Figure B.48

RMSE for the Between-Person Effect β_{SS} in Simulation 2

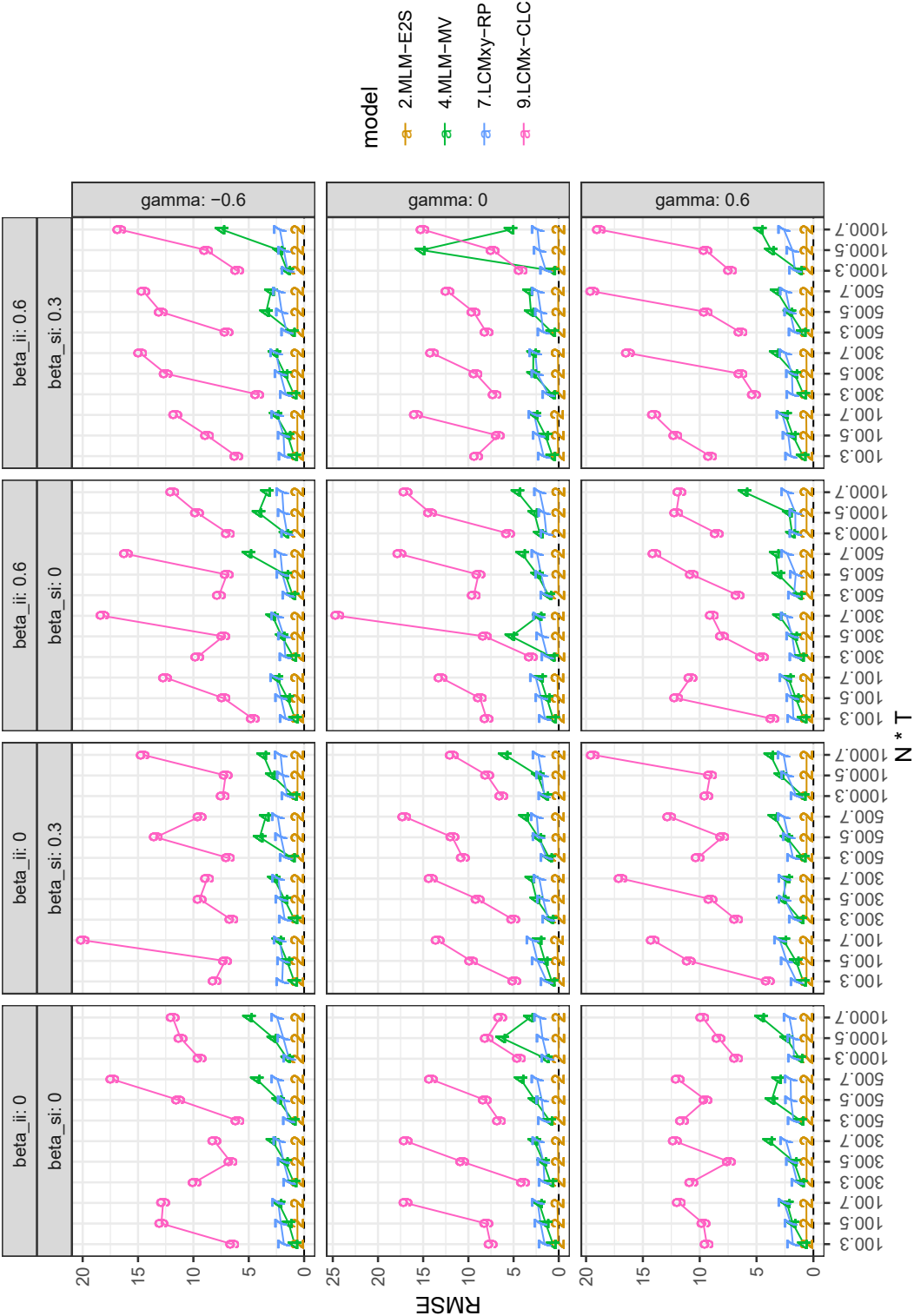
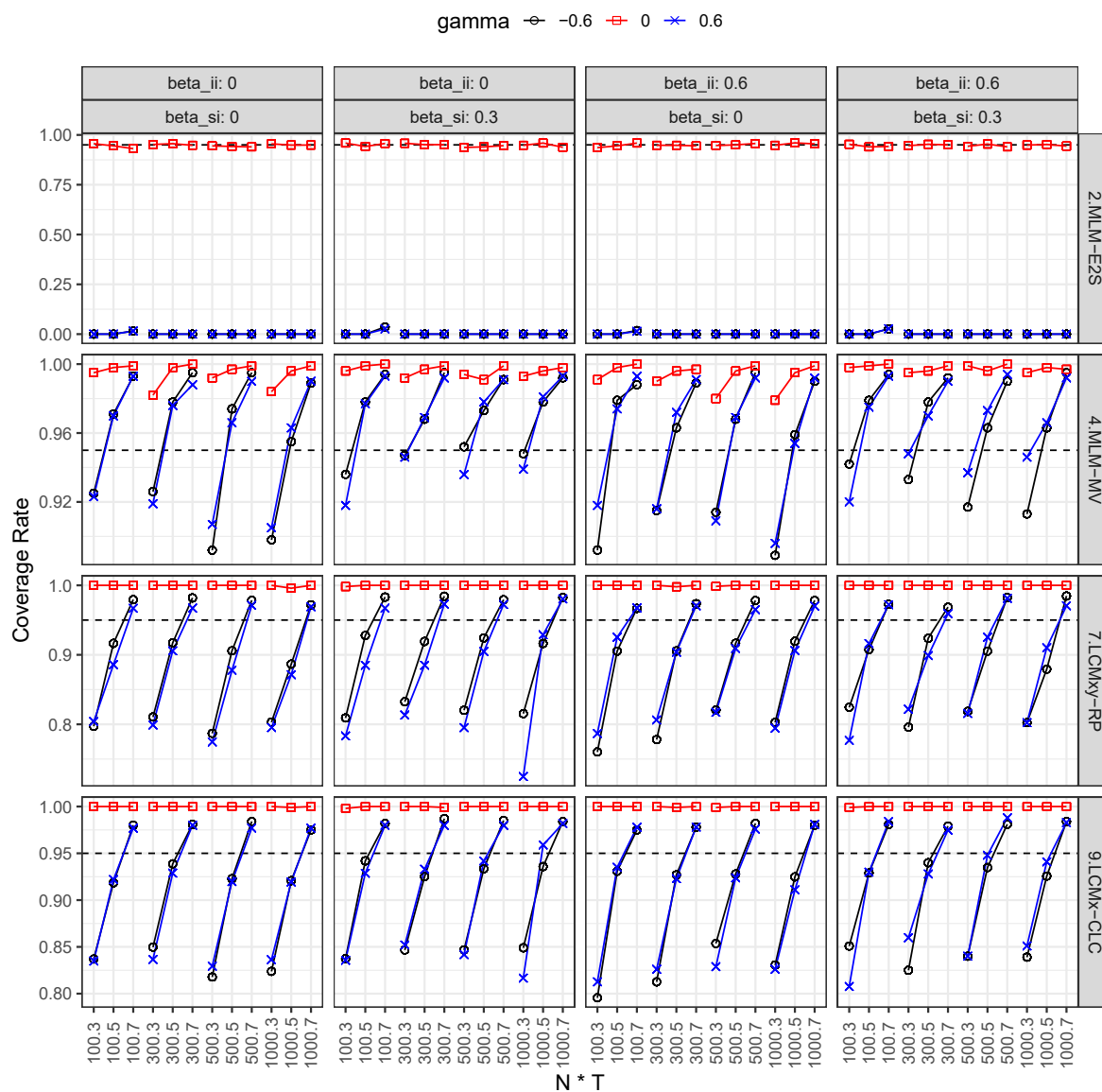


Figure B.49

Coverage Rate for the Between-Person Effect β_{SS} in Simulation 2

Appendix C

Figures in Simulation 3

C.1 Auxiliary Indices

Figure C.1

Proportion of Converged Cases in Simulation 3

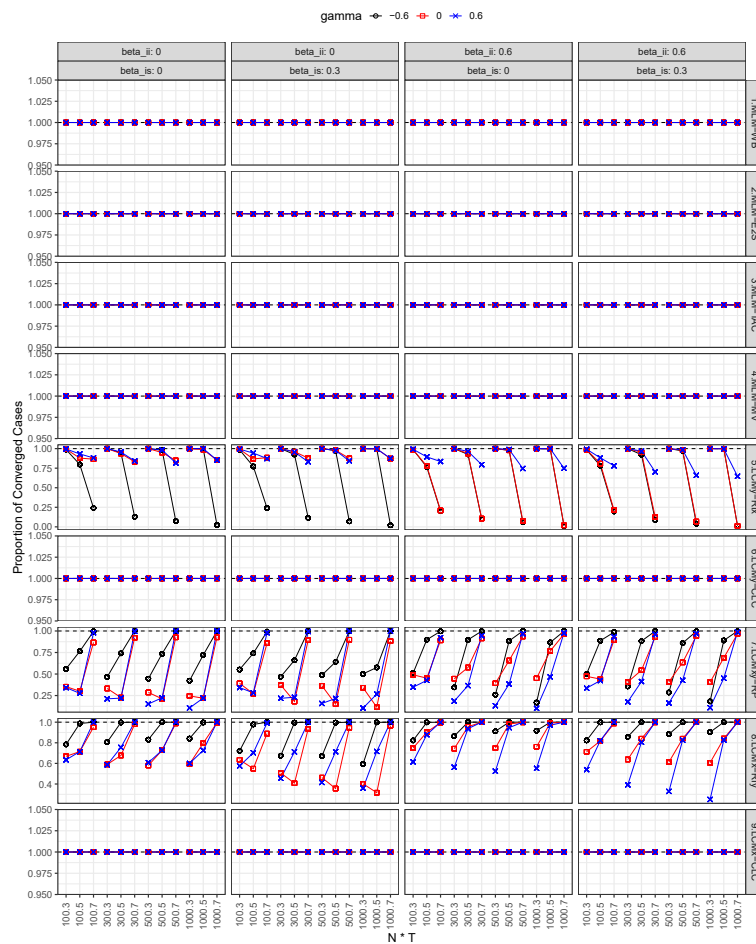


Figure C.2

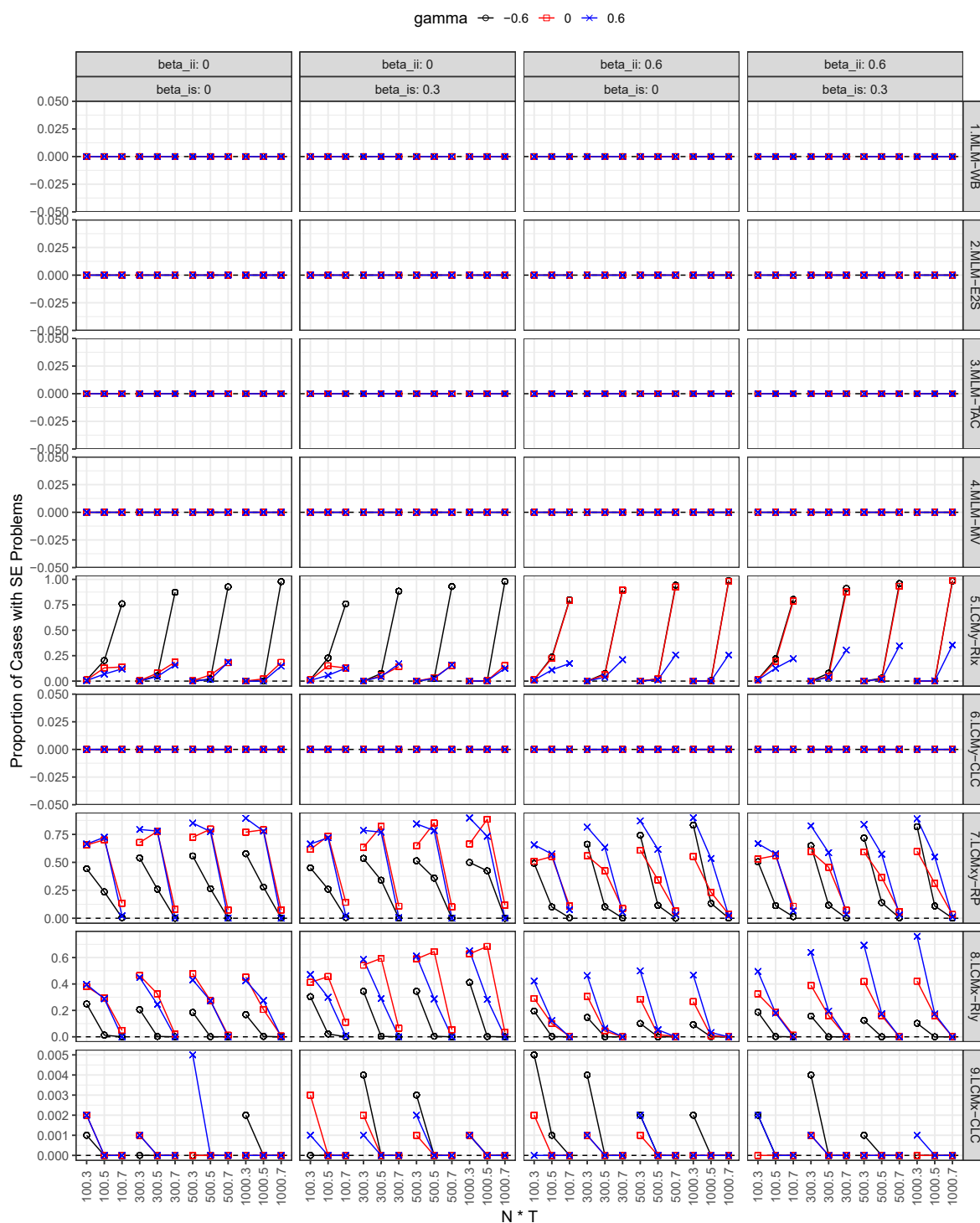
Proportion of Cases with Problems in Estimating SE in Simulation 3

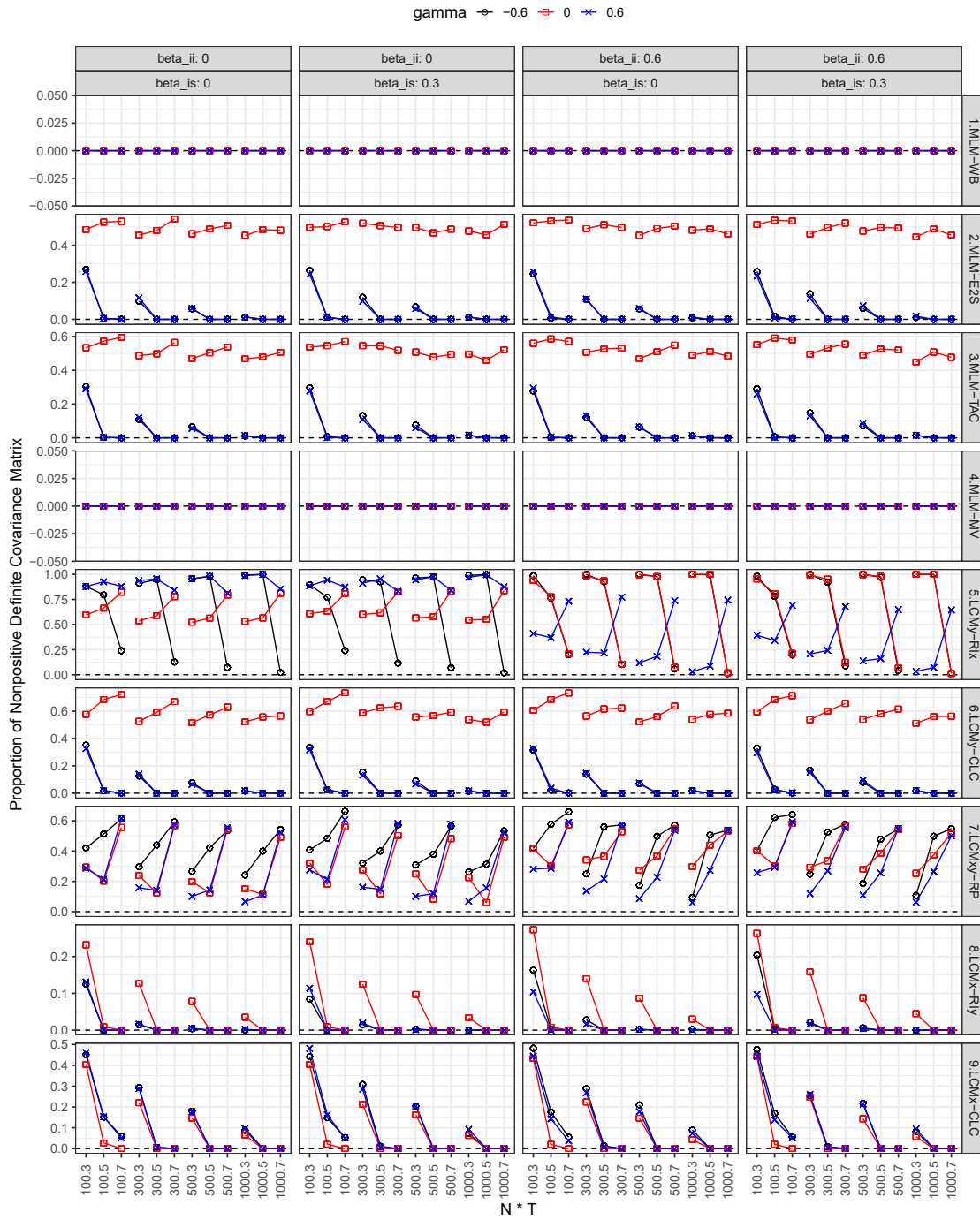
Figure C.3*Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 3*

Figure C.4

CFI in Simulation 3

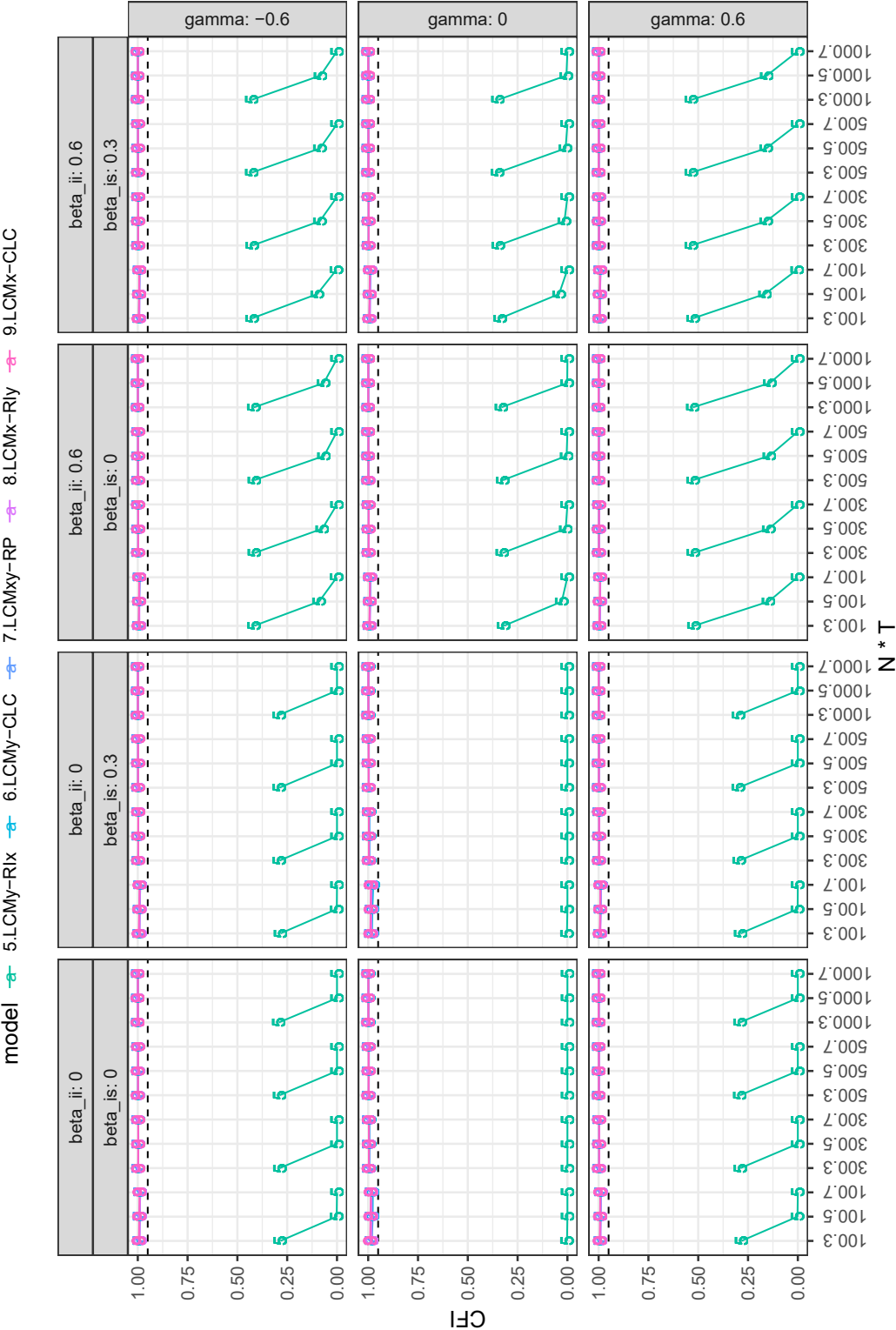


Figure C.5
RMSEA in Simulation 3

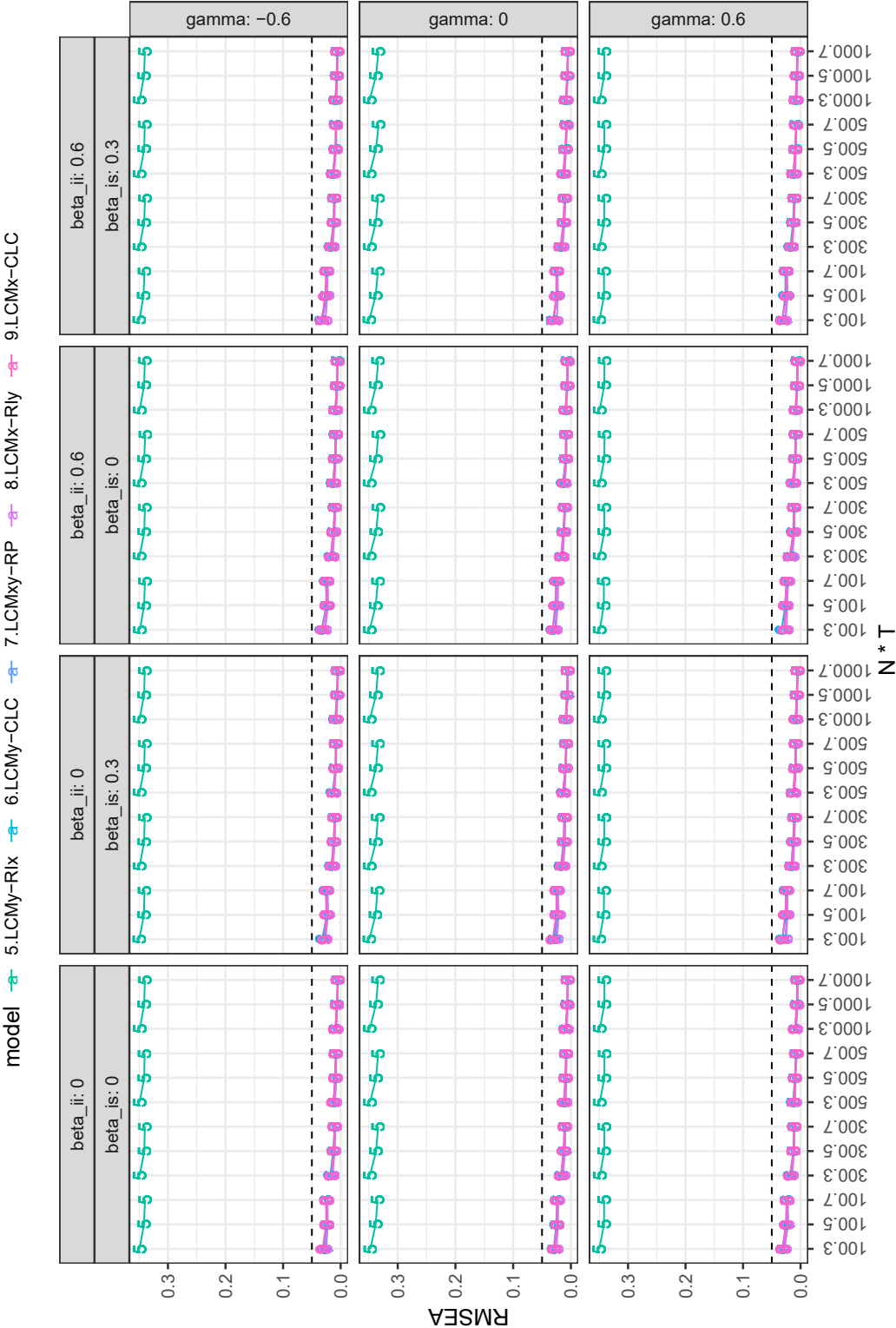
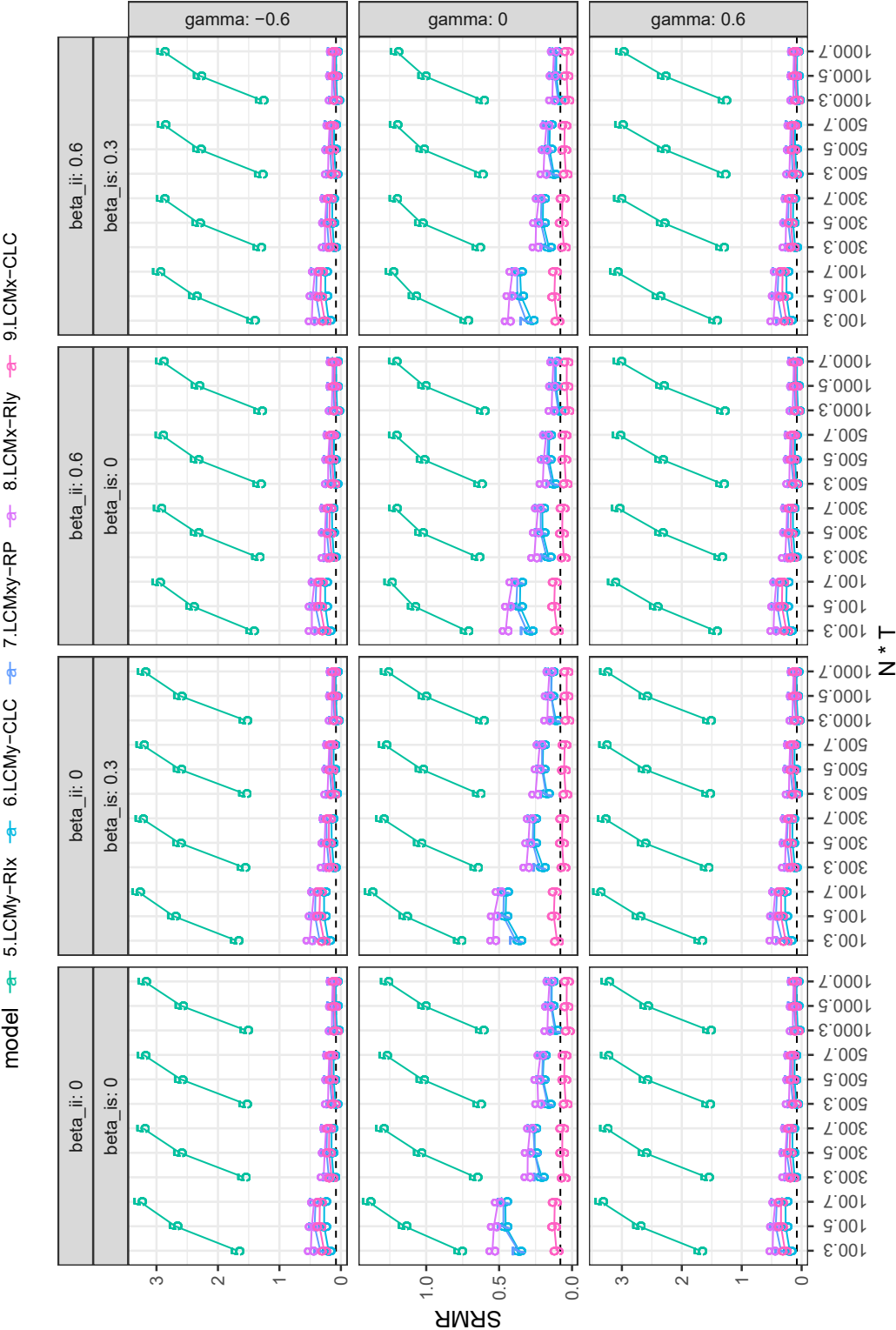


Figure C.6
SRMR in Simulation 3



C.2 Within-Person Effect

Figure C.7

Bias for the Within-Person Effect γ in Simulation 3

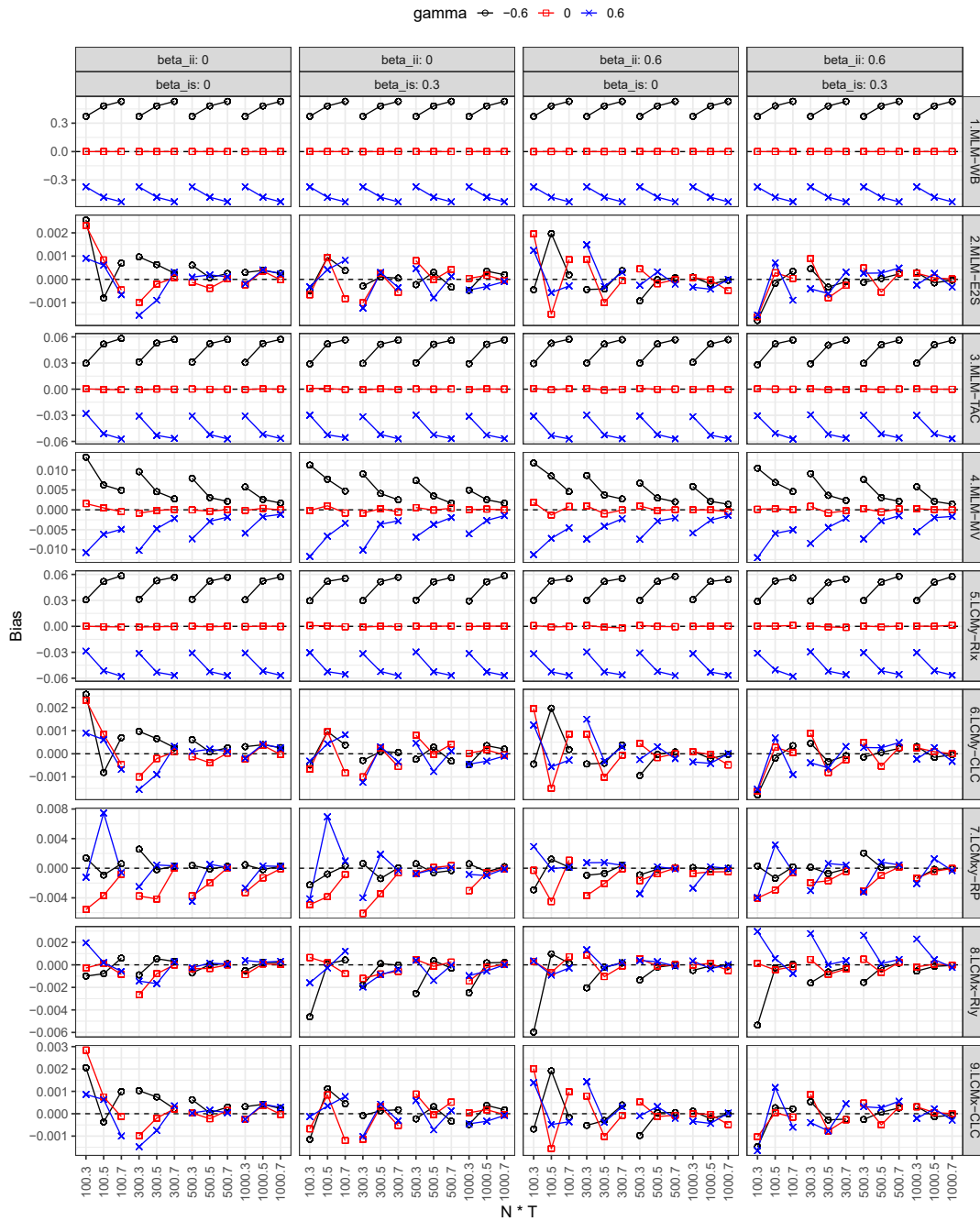


Figure C.8

Empirical SE for the Within-Person Effect γ in Simulation 3

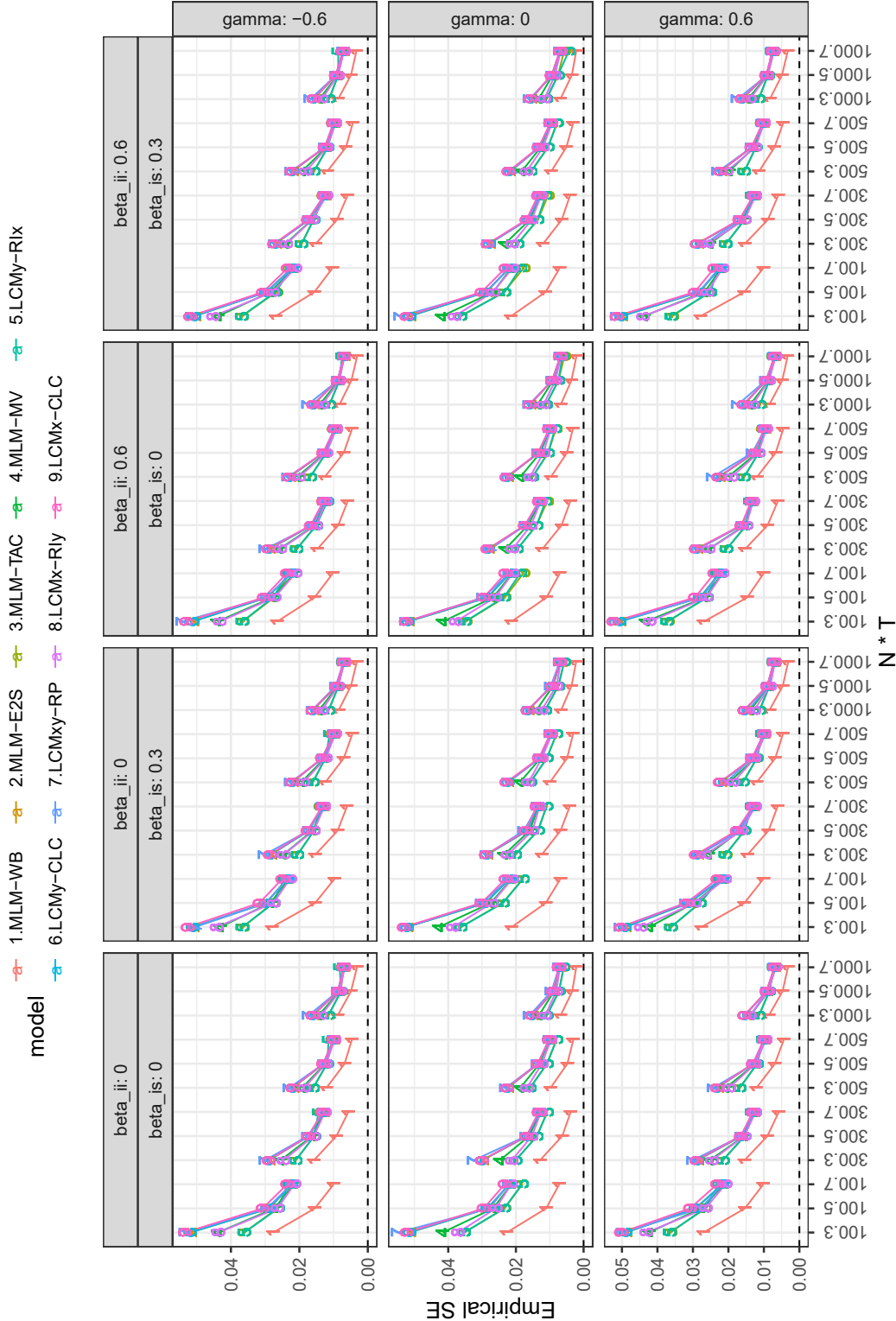


Figure C.9

Theoretical SE for the Within-Person Effect γ in Simulation 3

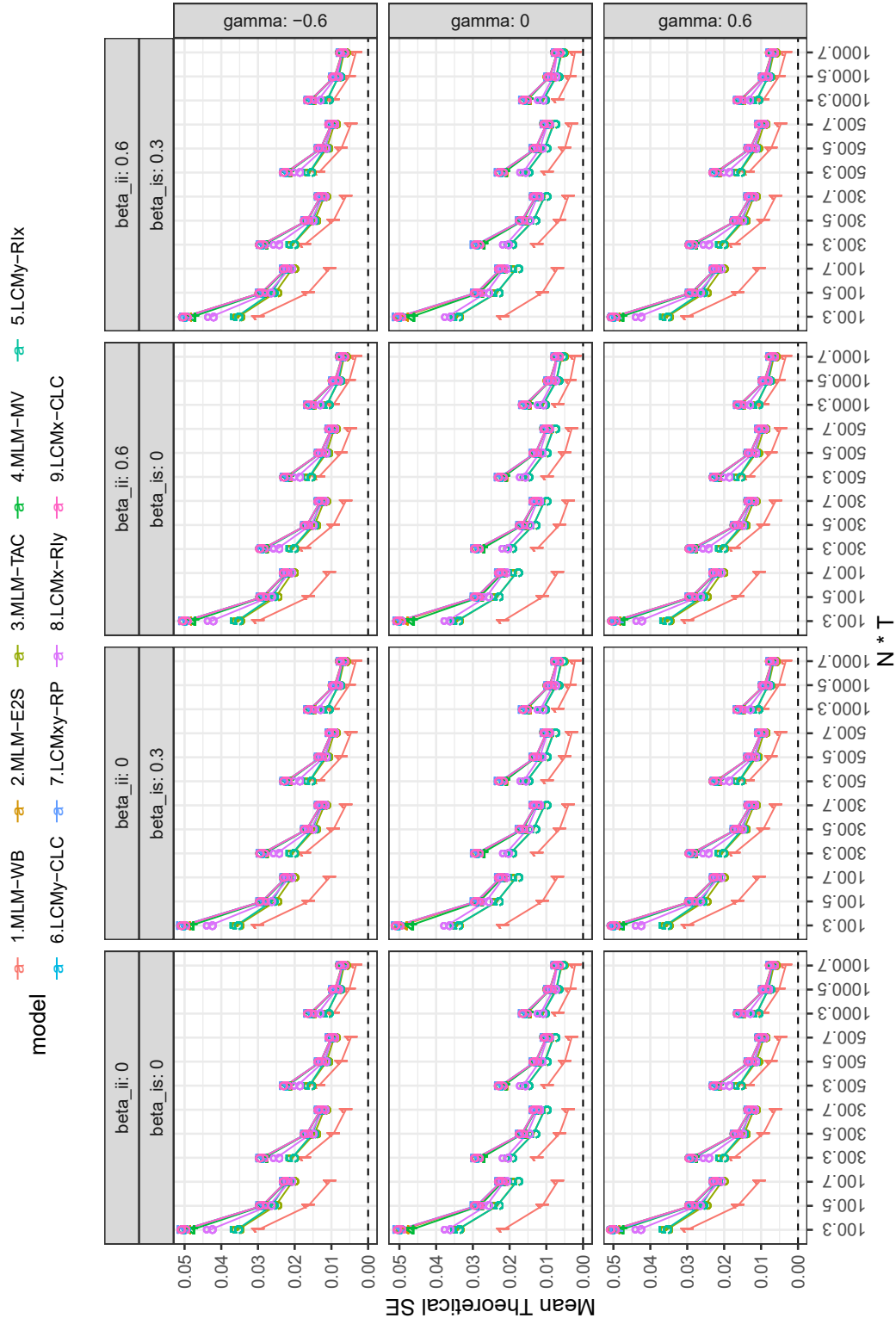


Figure C.10

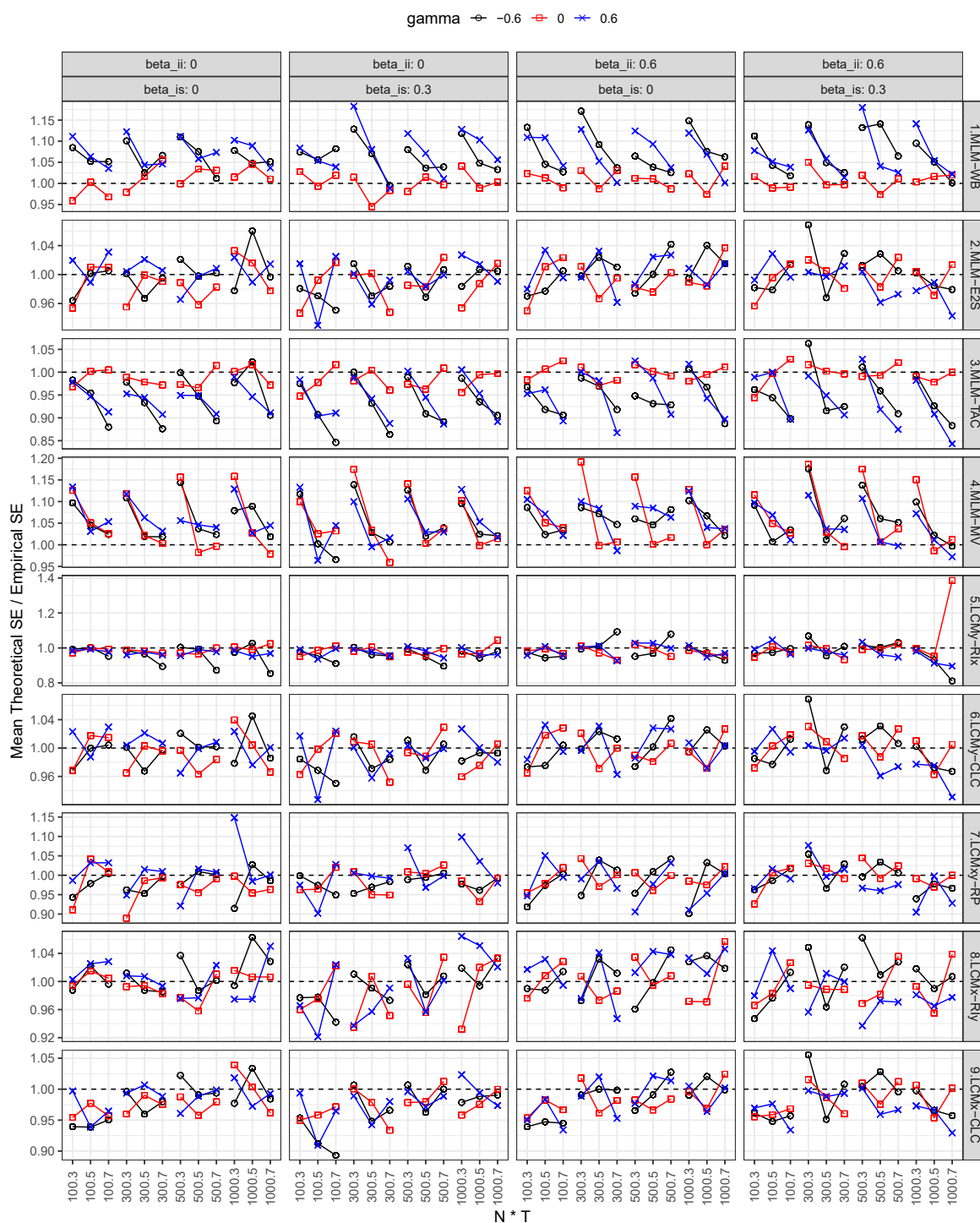
SE Ratio for the Within-Person Effect γ in Simulation 3

Figure C.11

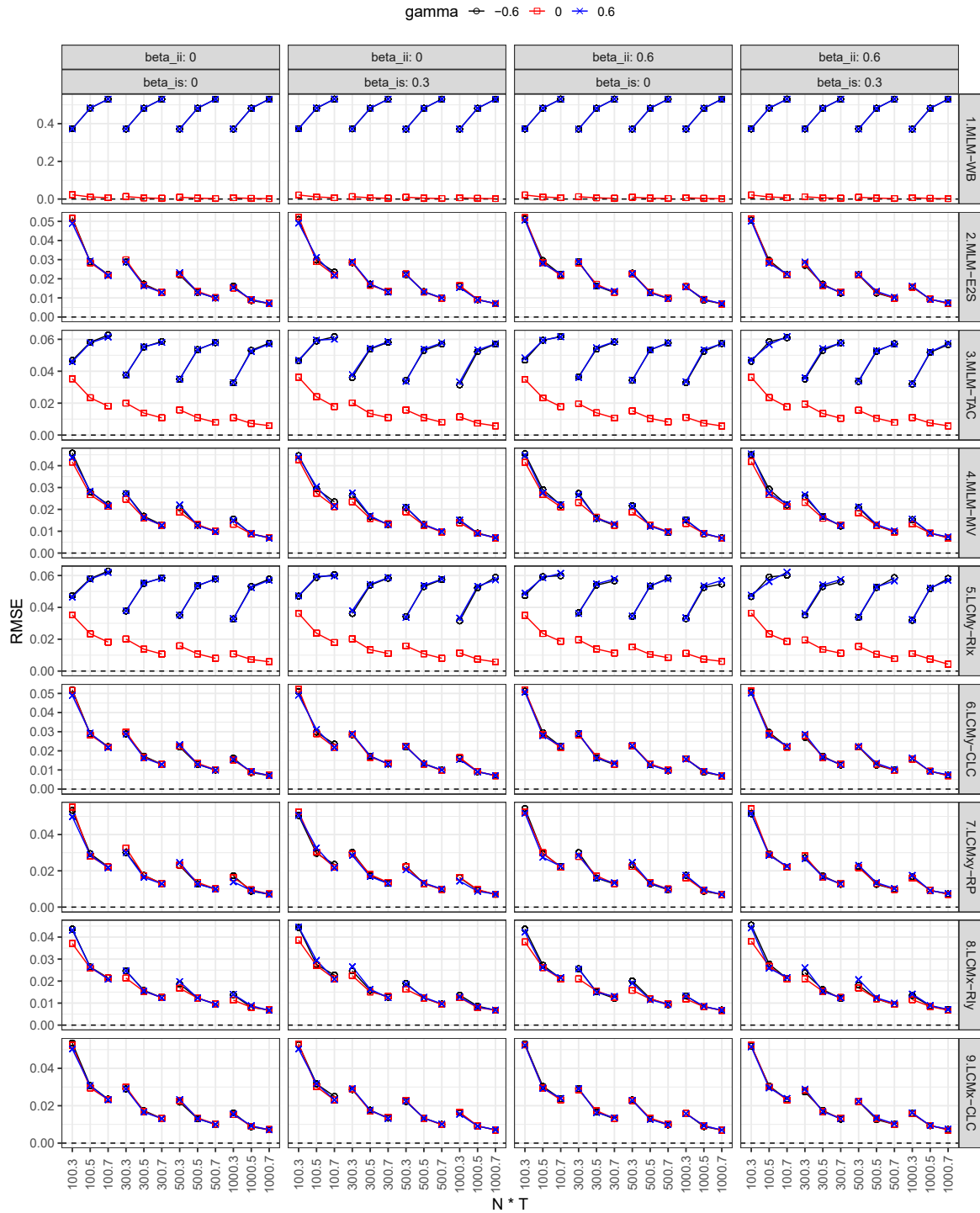
RMSE for the Within-Person Effect γ in Simulation 3

Figure C.12

Coverage Rate for the Within-Person Effect γ in Simulation 3

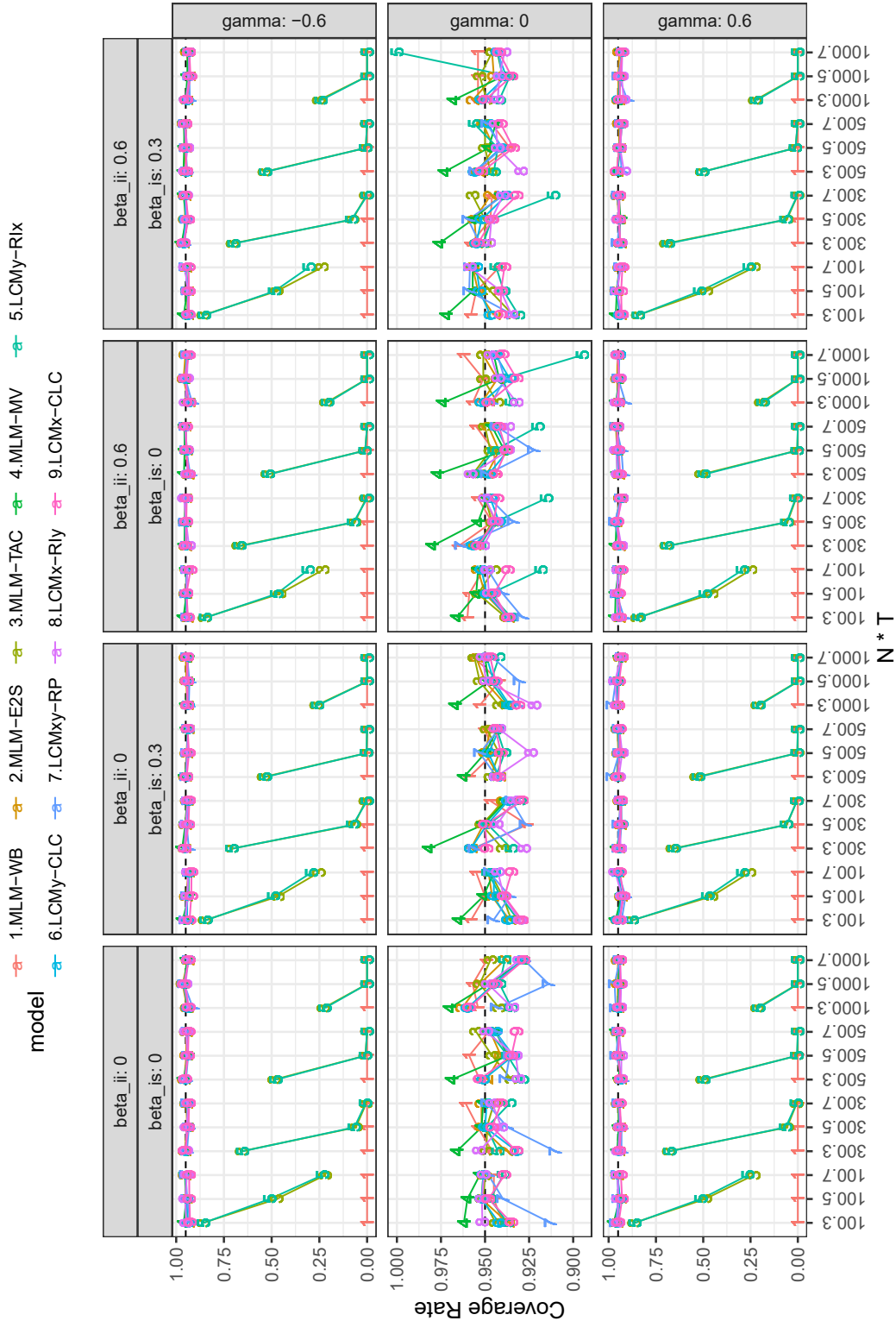
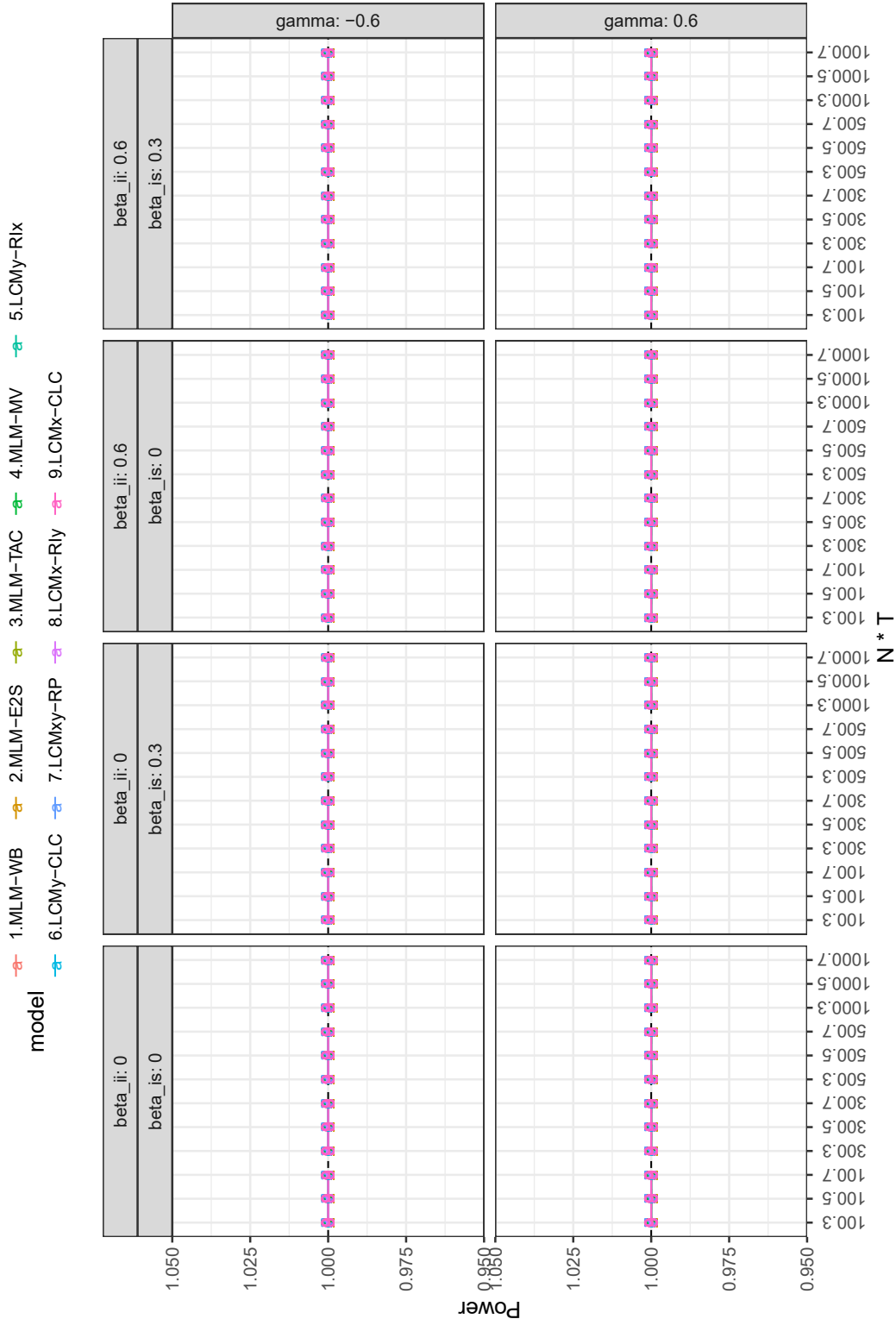


Figure C.13

Power for the Within-Person Effect γ in Simulation 3



C.3 Between-Person Effect of Intercept on Intercept

Figure C.14

Bias for the Between-Person Effect β_{II} in Simulation 3

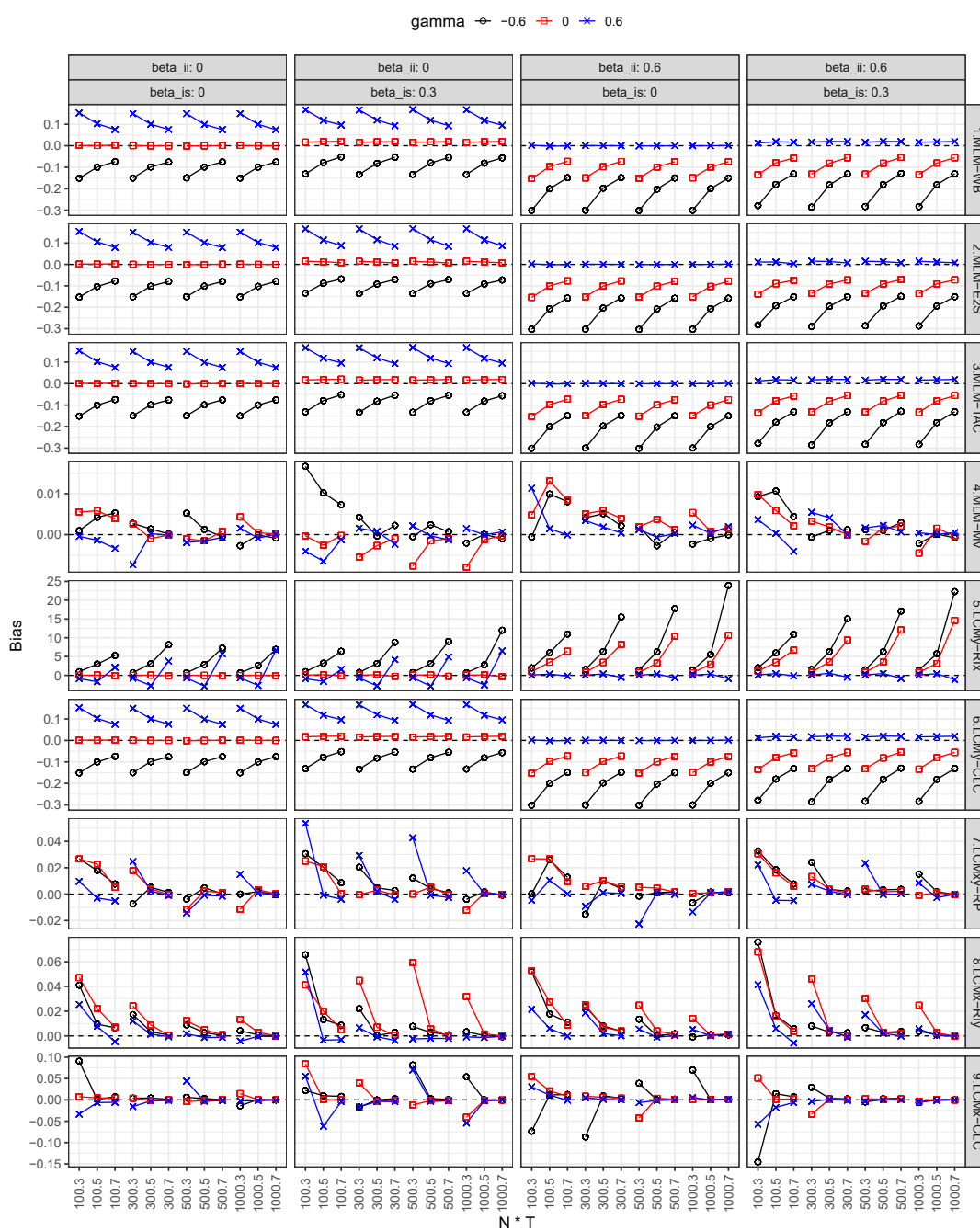


Figure C.15

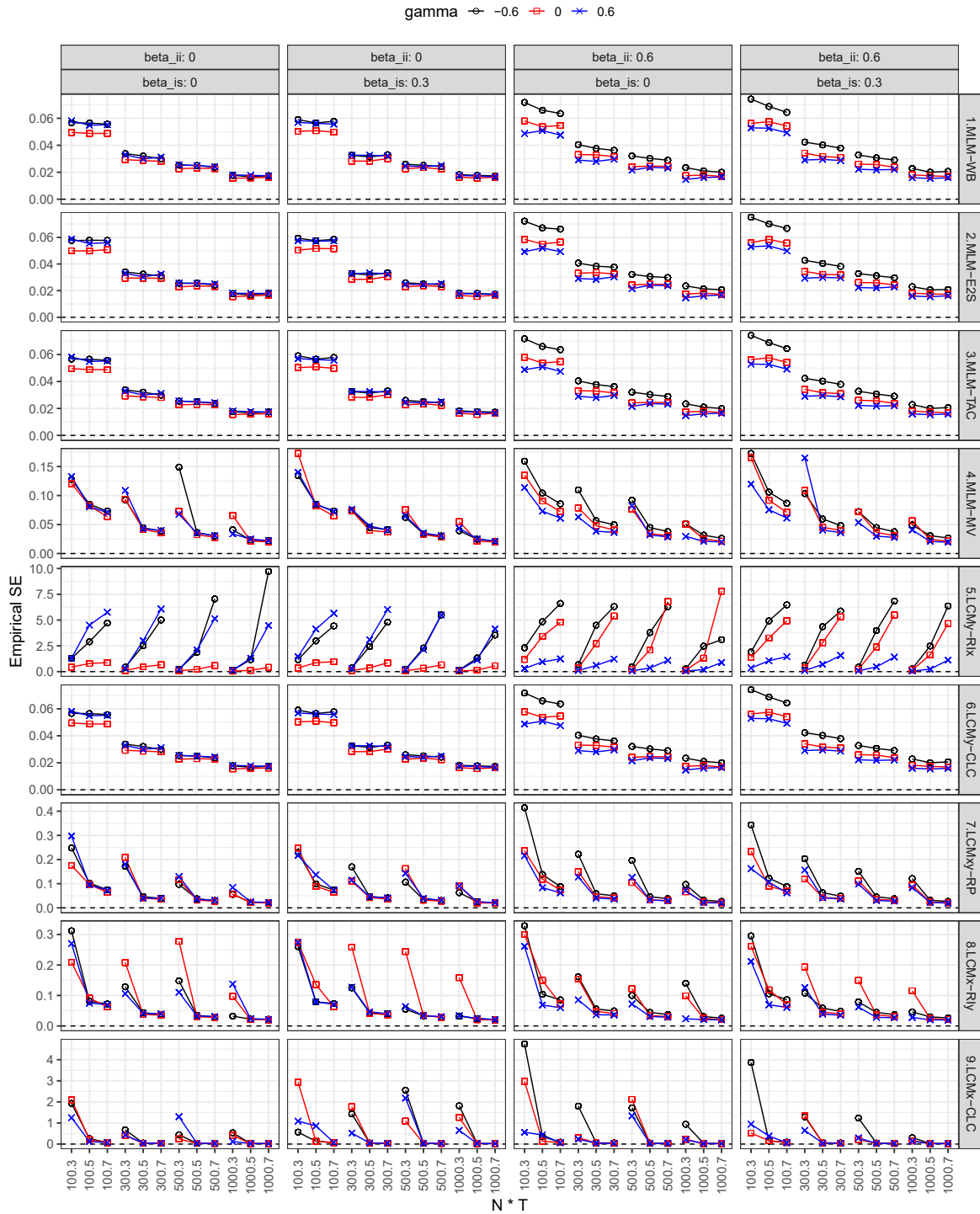
Empirical SE for the Between-Person Effect β_{II} in Simulation 3

Figure C.16

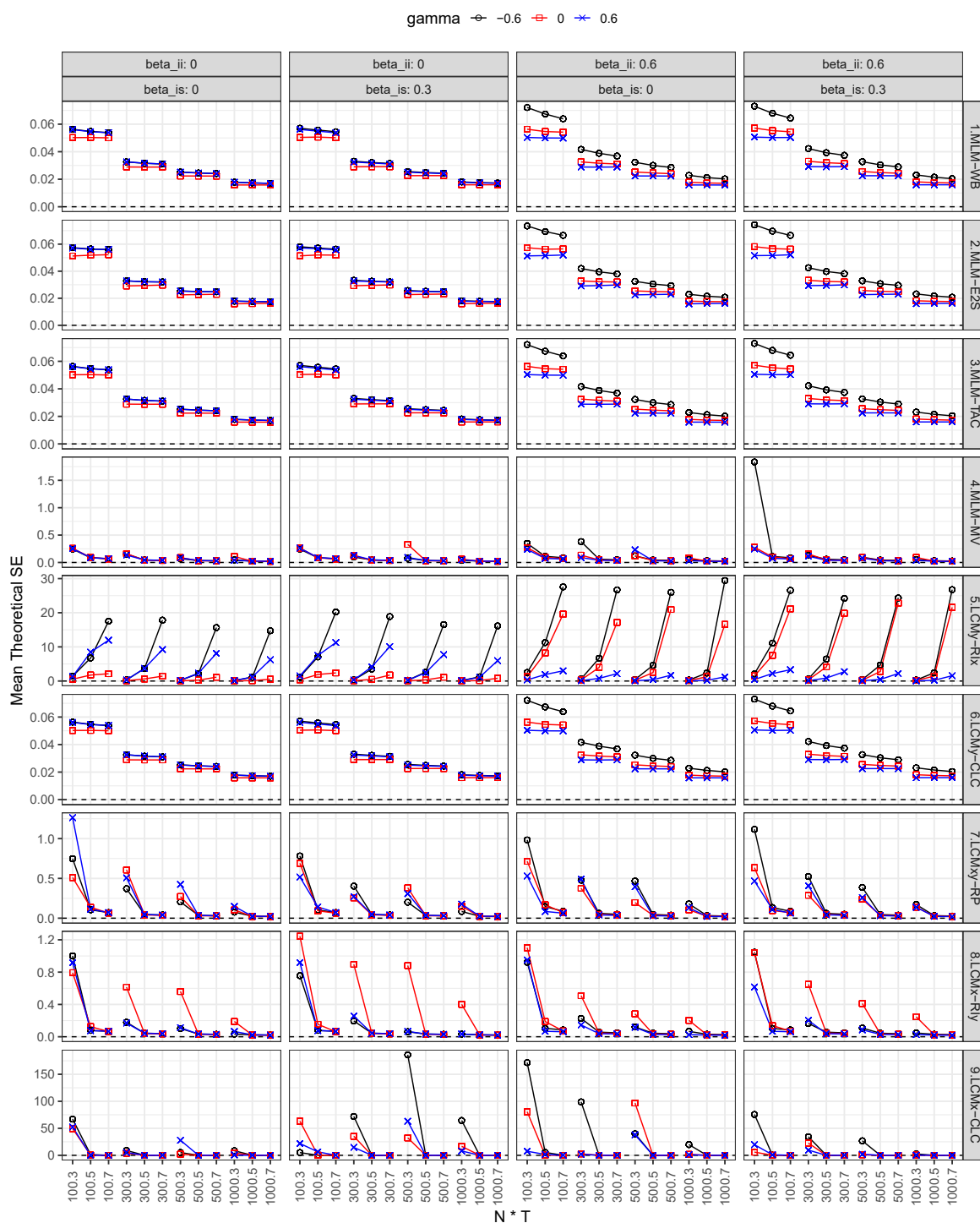
Theoretical SE for the Between-Person Effect β_{II} in Simulation 3

Figure C.17

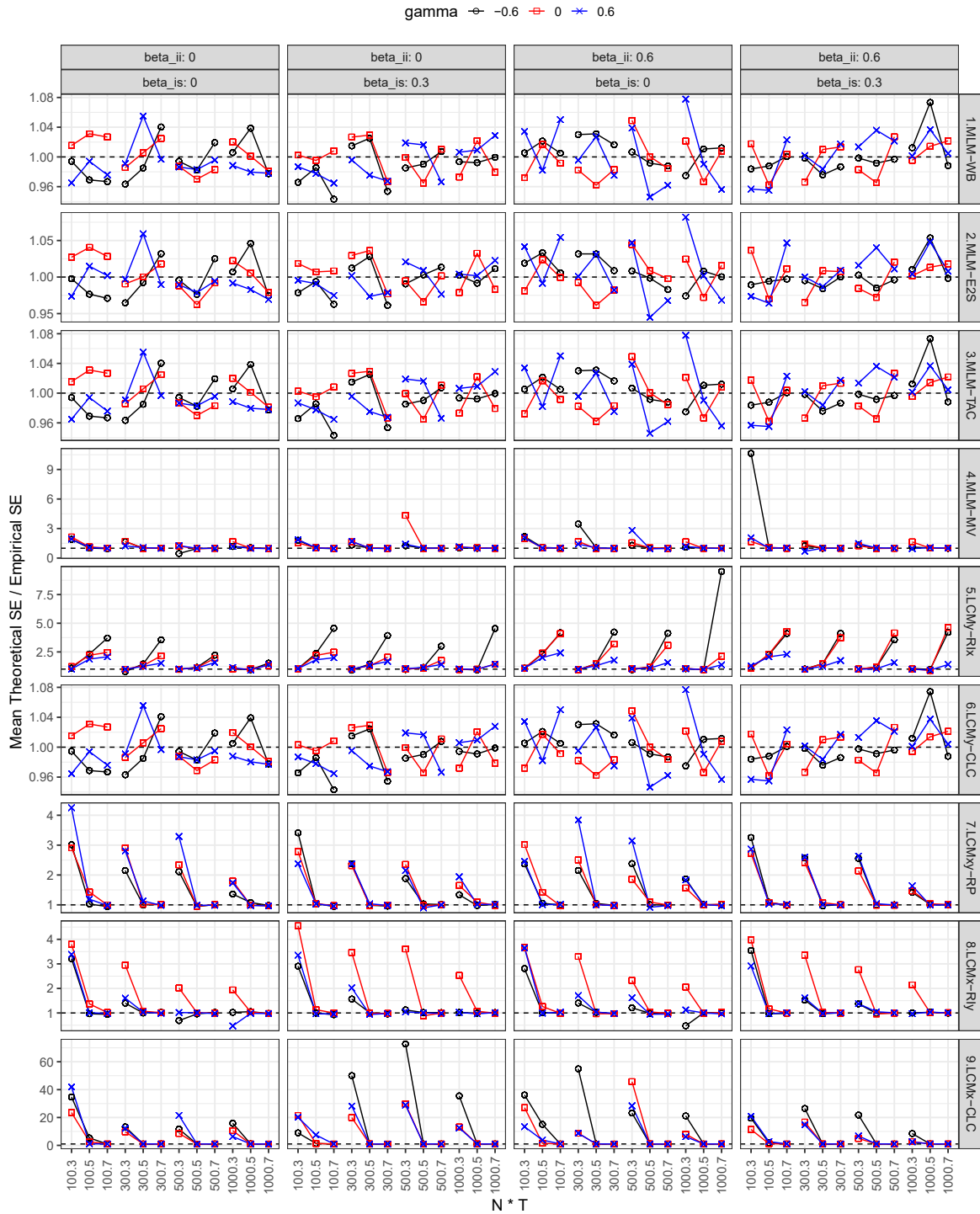
SE Ratio for the Between-Person Effect β_{II} in Simulation 3

Figure C.18

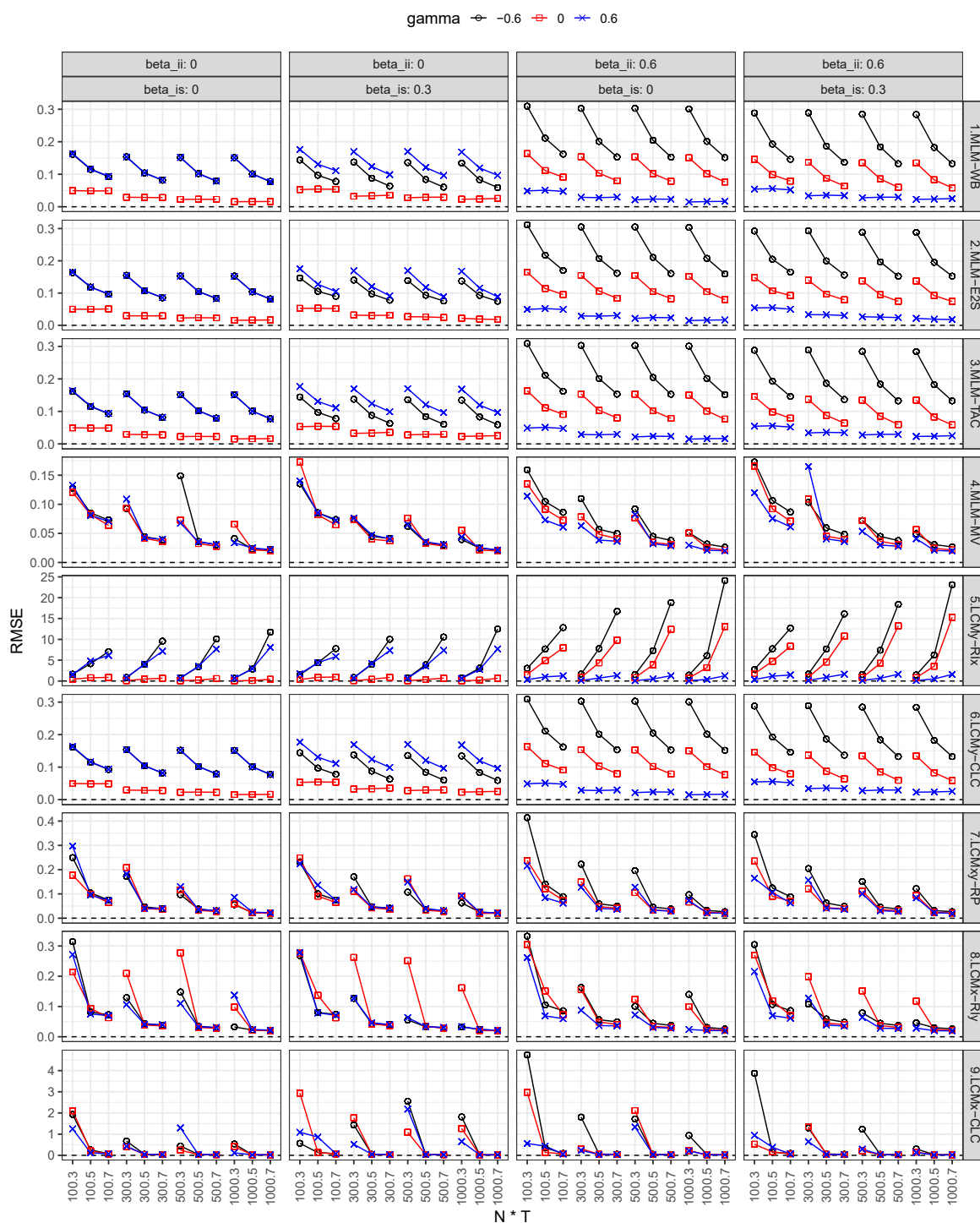
RMSE for the Between-Person Effect β_{II} in Simulation 3

Figure C.19

RMSE for the Between-Person Effect β_{II} in Simulation 3 except LCMy-Rlx and LCMx-CLC

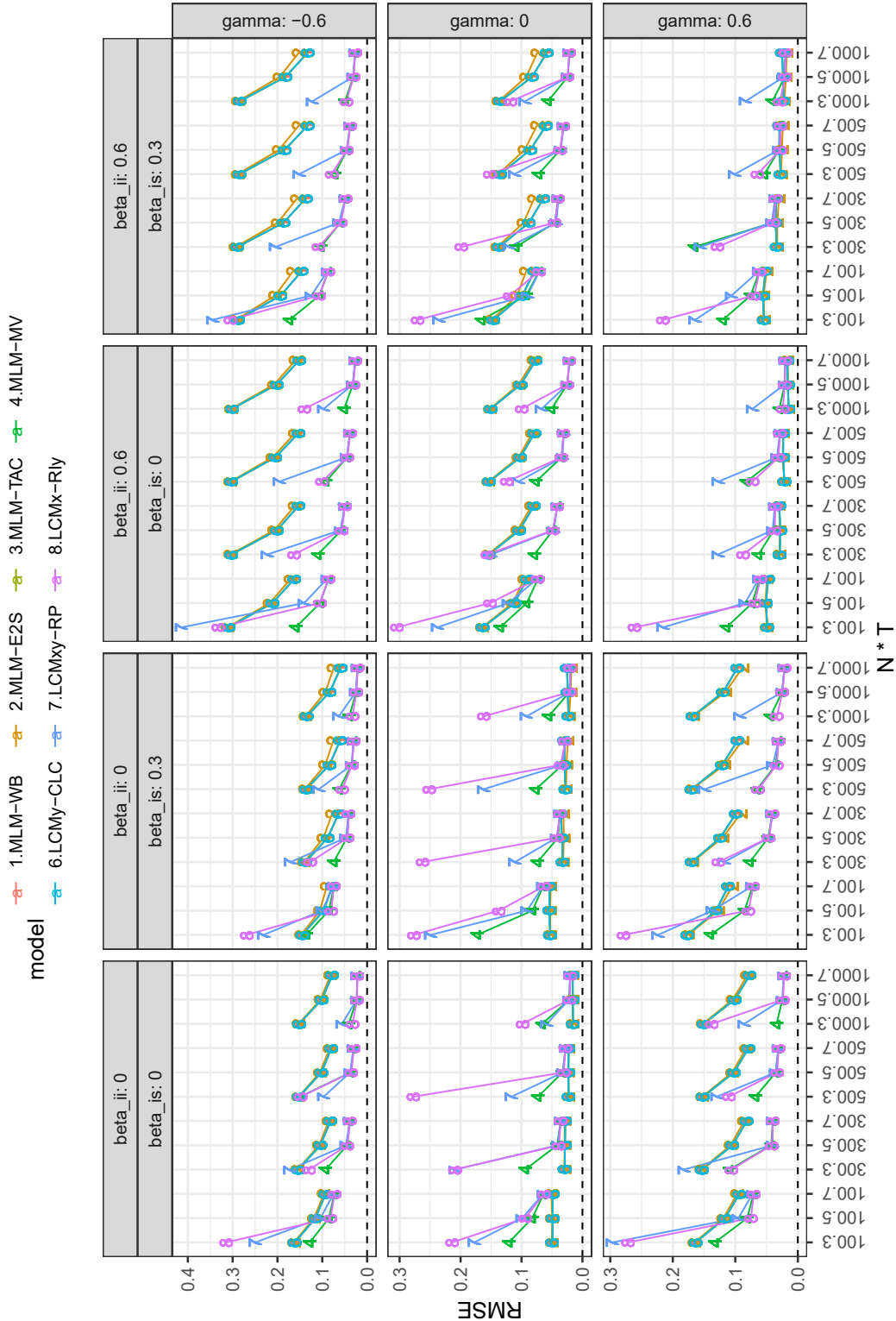


Figure C.20

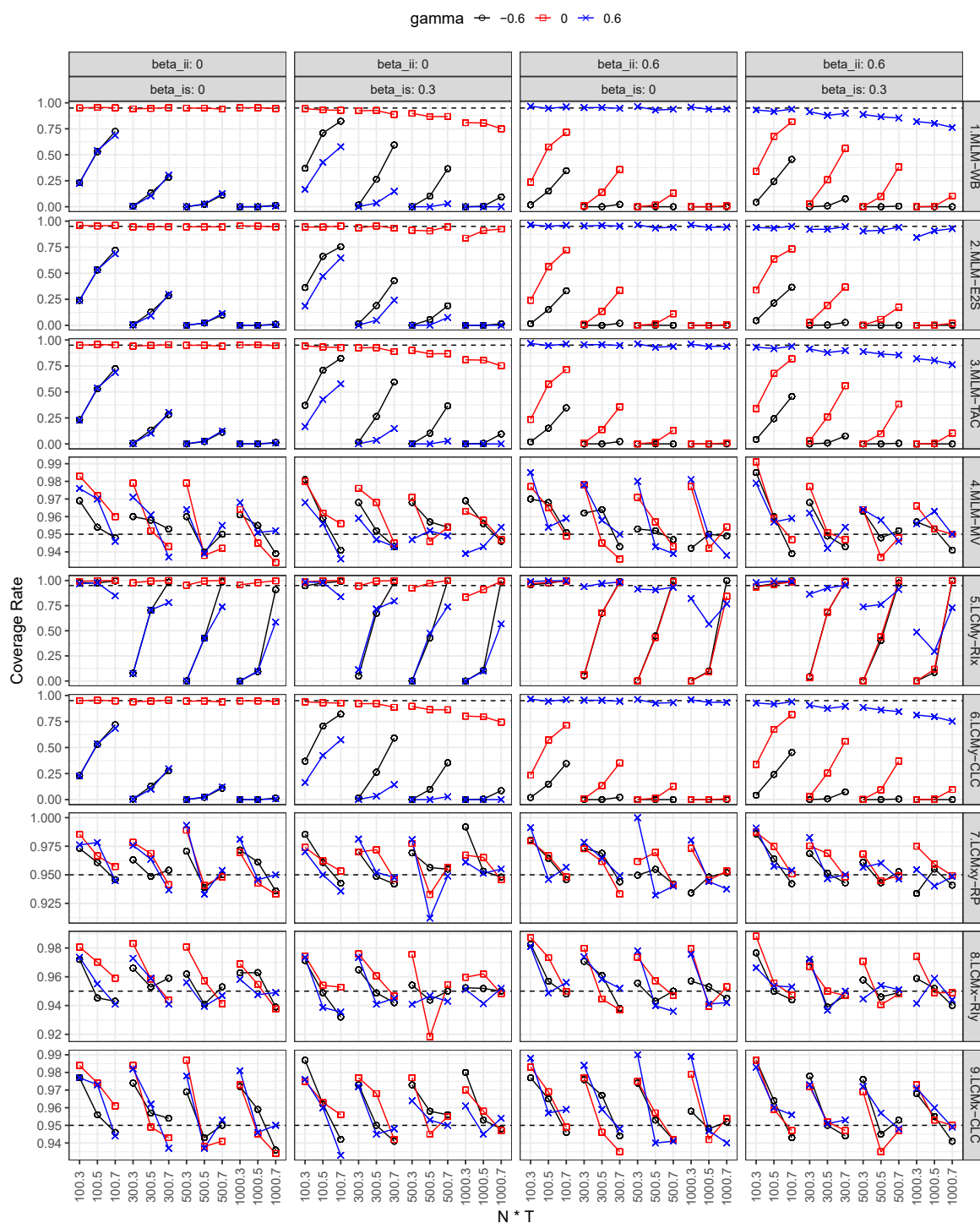
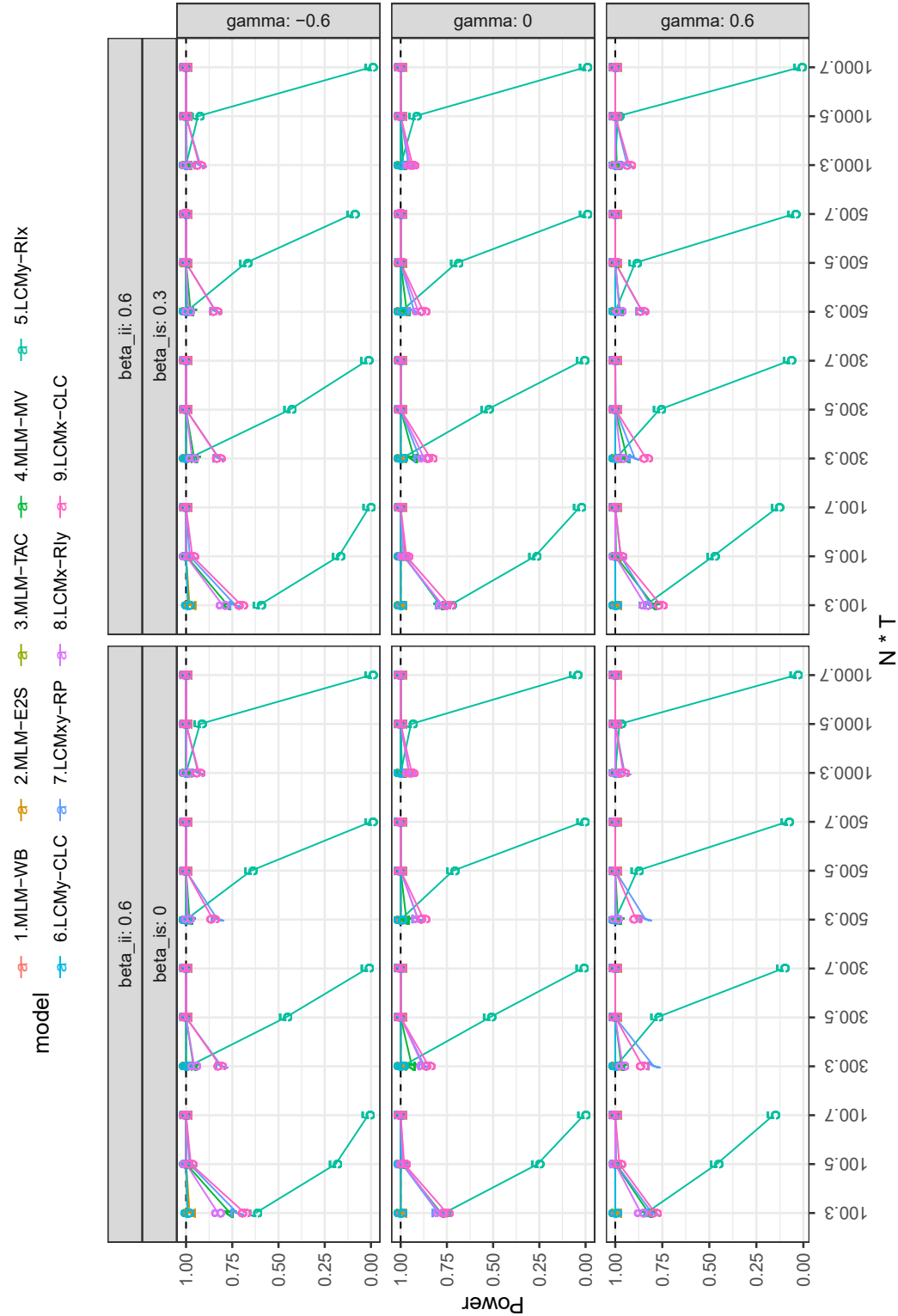
Coverage Rate for the Between-Person Effect β_{II} in Simulation 3

Figure C.21
Power for the Between-Person Effect β_{II} in Simulation 3



C.4 Between-Person Effect of Intercept on Slope

Figure C.22

Bias for the Between-Person Effect β_{SI} in Simulation 3

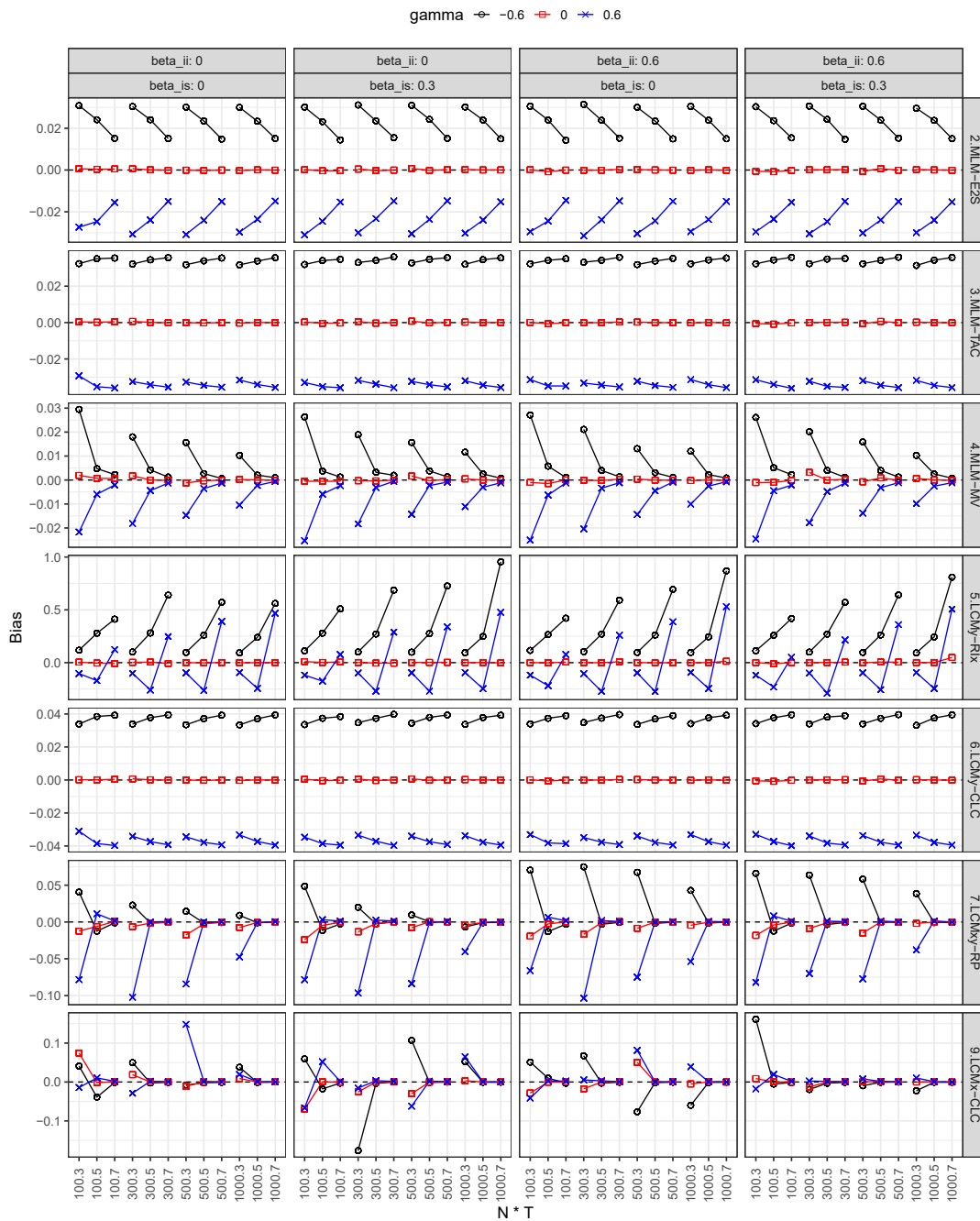


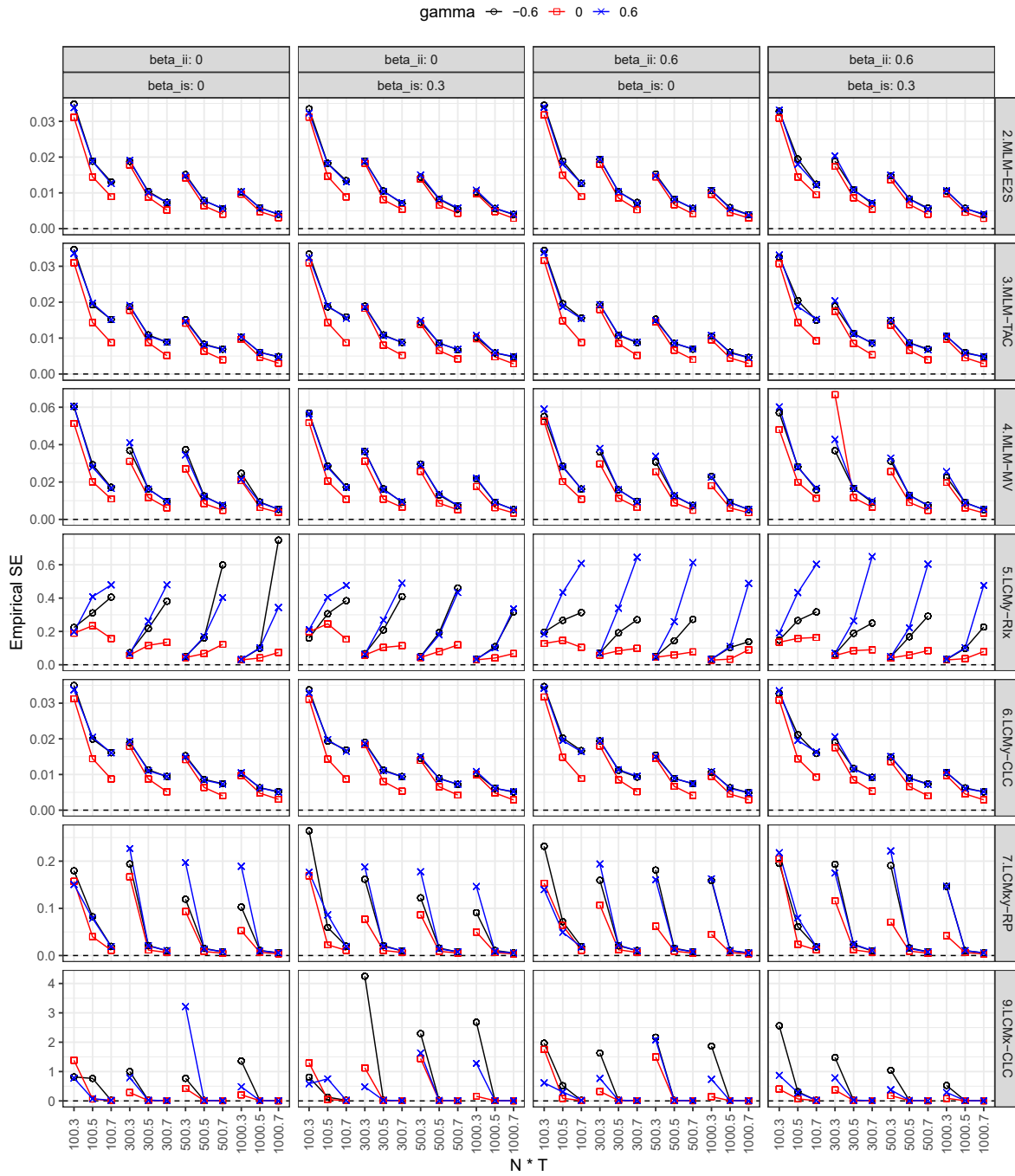
Figure C.23*Empirical SE for the Between-Person Effect β_{SI} in Simulation 3*

Figure C.24

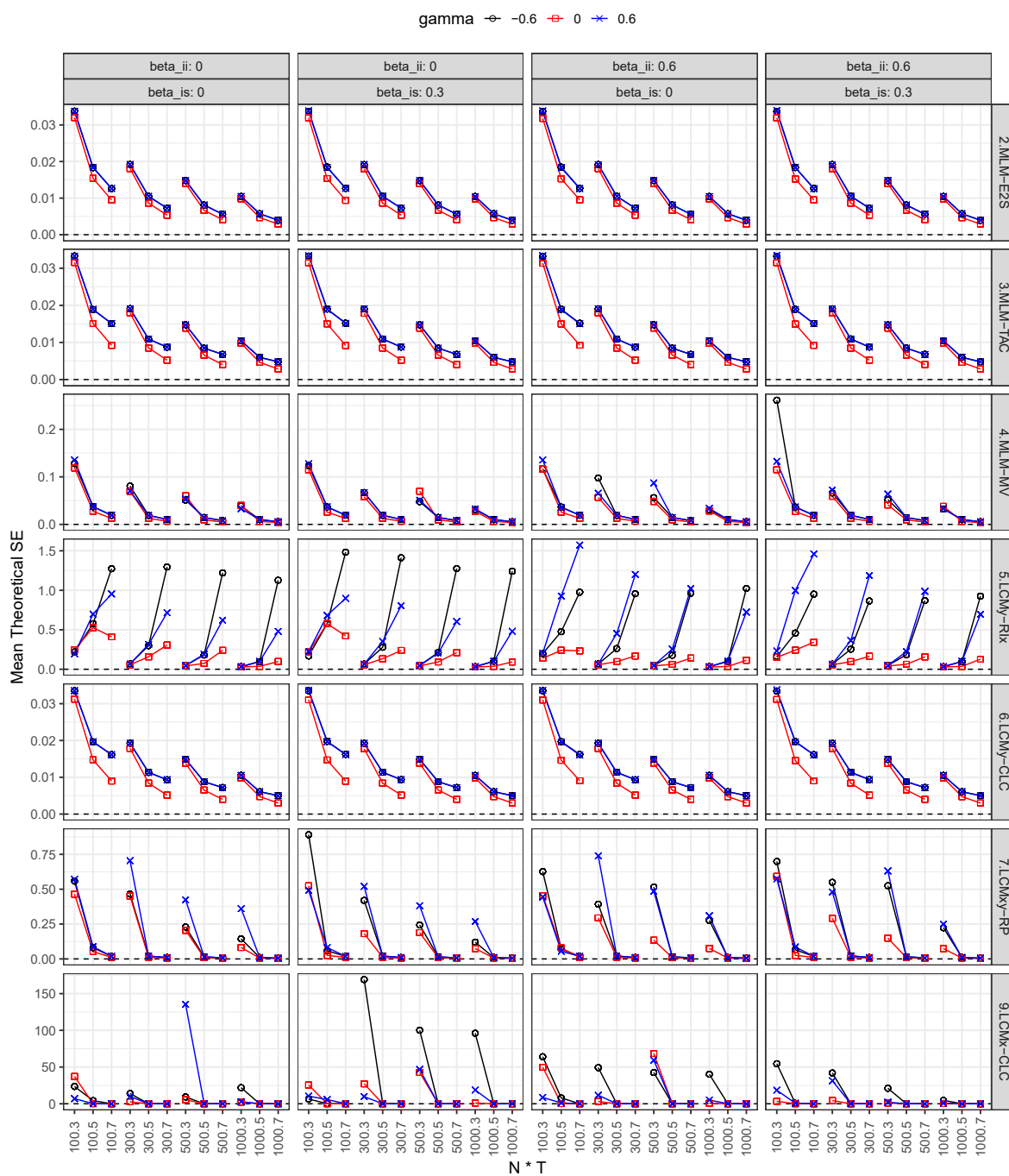
Theoretical SE for the Between-Person Effect β_{SI} in Simulation 3

Figure C.25

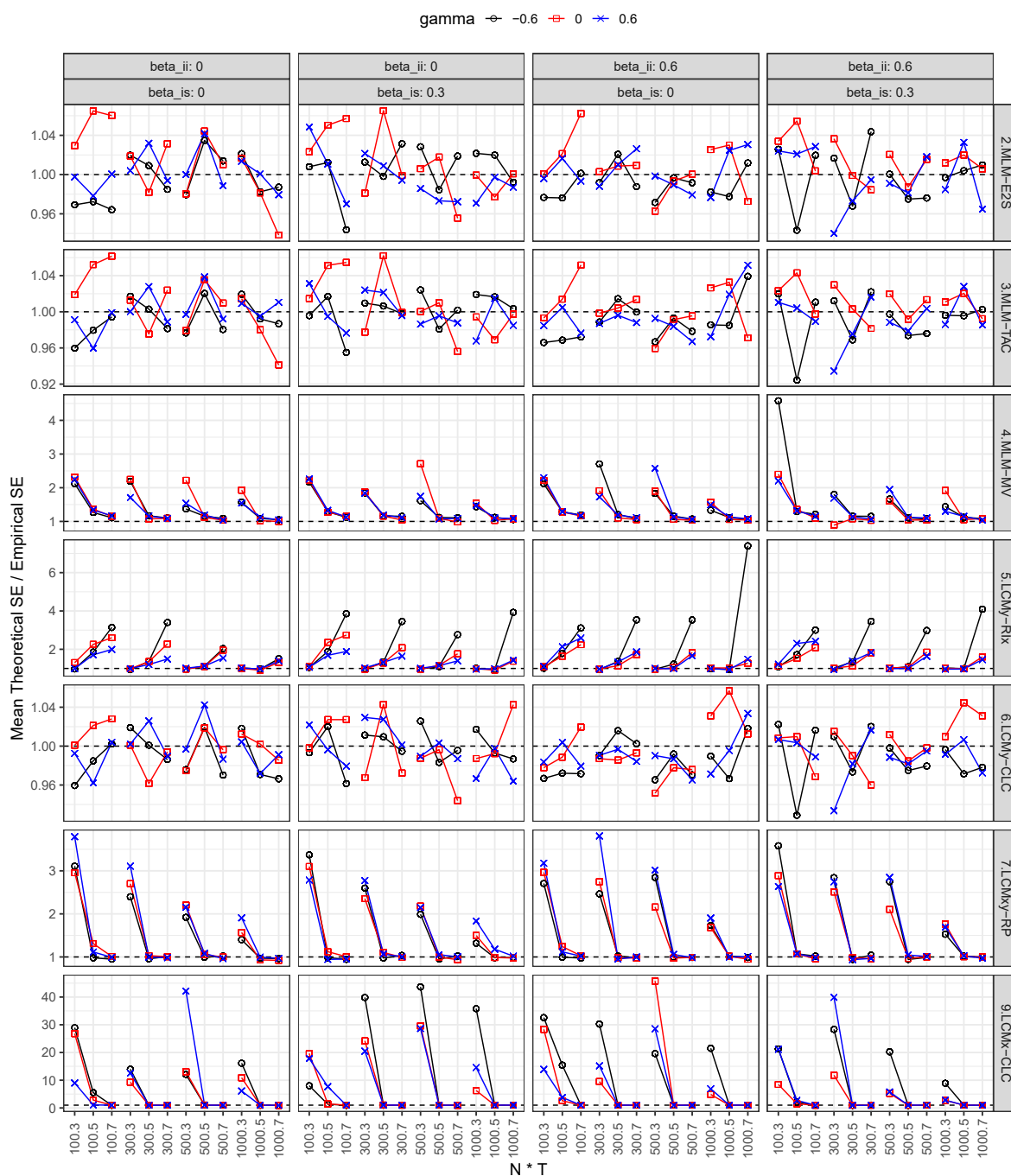
SE Ratio for the Between-Person Effect β_{SI} in Simulation 3

Figure C.26

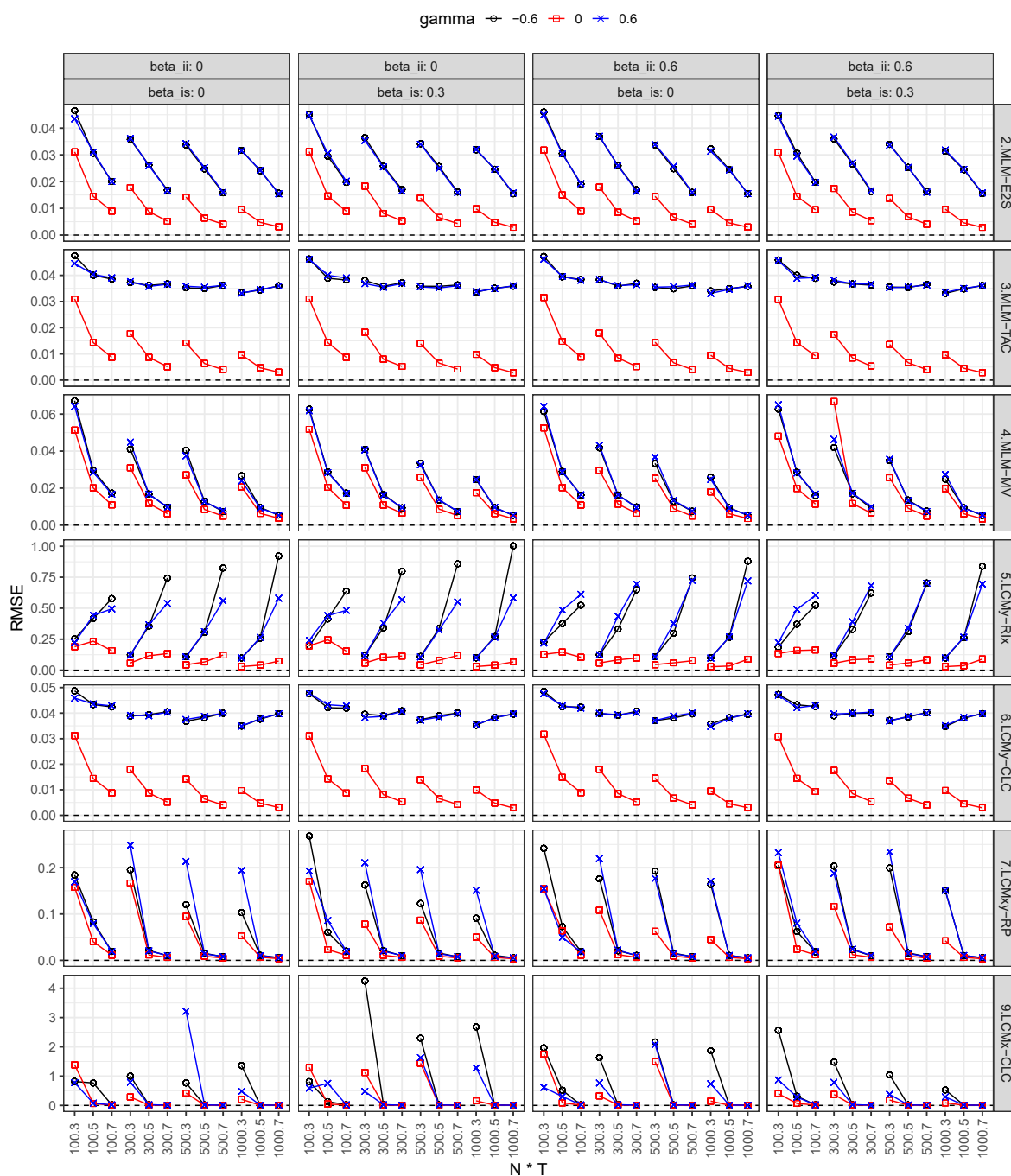
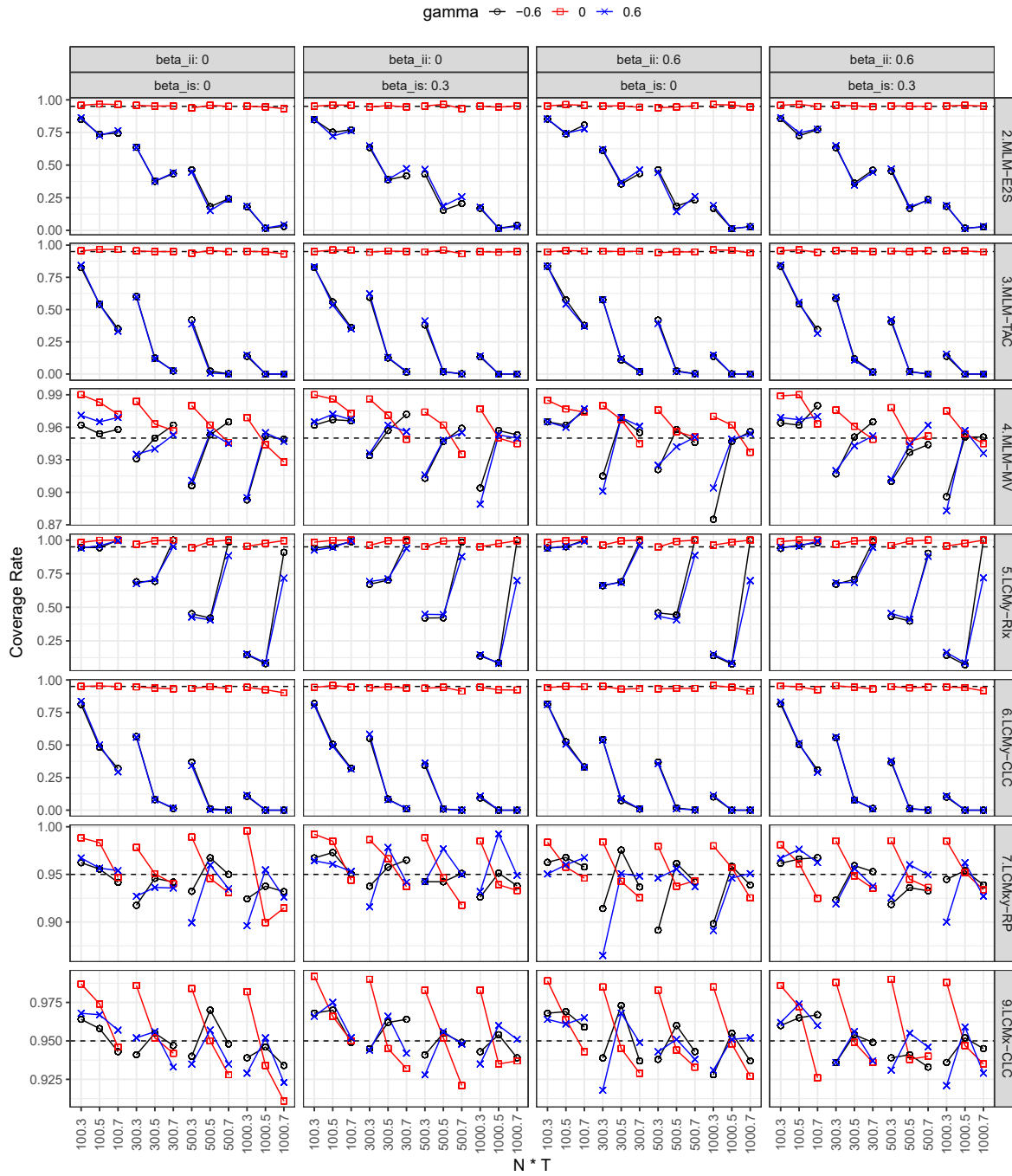
RMSE for the Between-Person Effect β_{SI} in Simulation 3

Figure C.27

Coverage Rate for the Between-Person Effect β_{SI} in Simulation 3

C.5 Between-Person Effect of Slope on Intercept

Figure C.28

Bias for the Between-Person Effect β_{IS} in Simulation 3

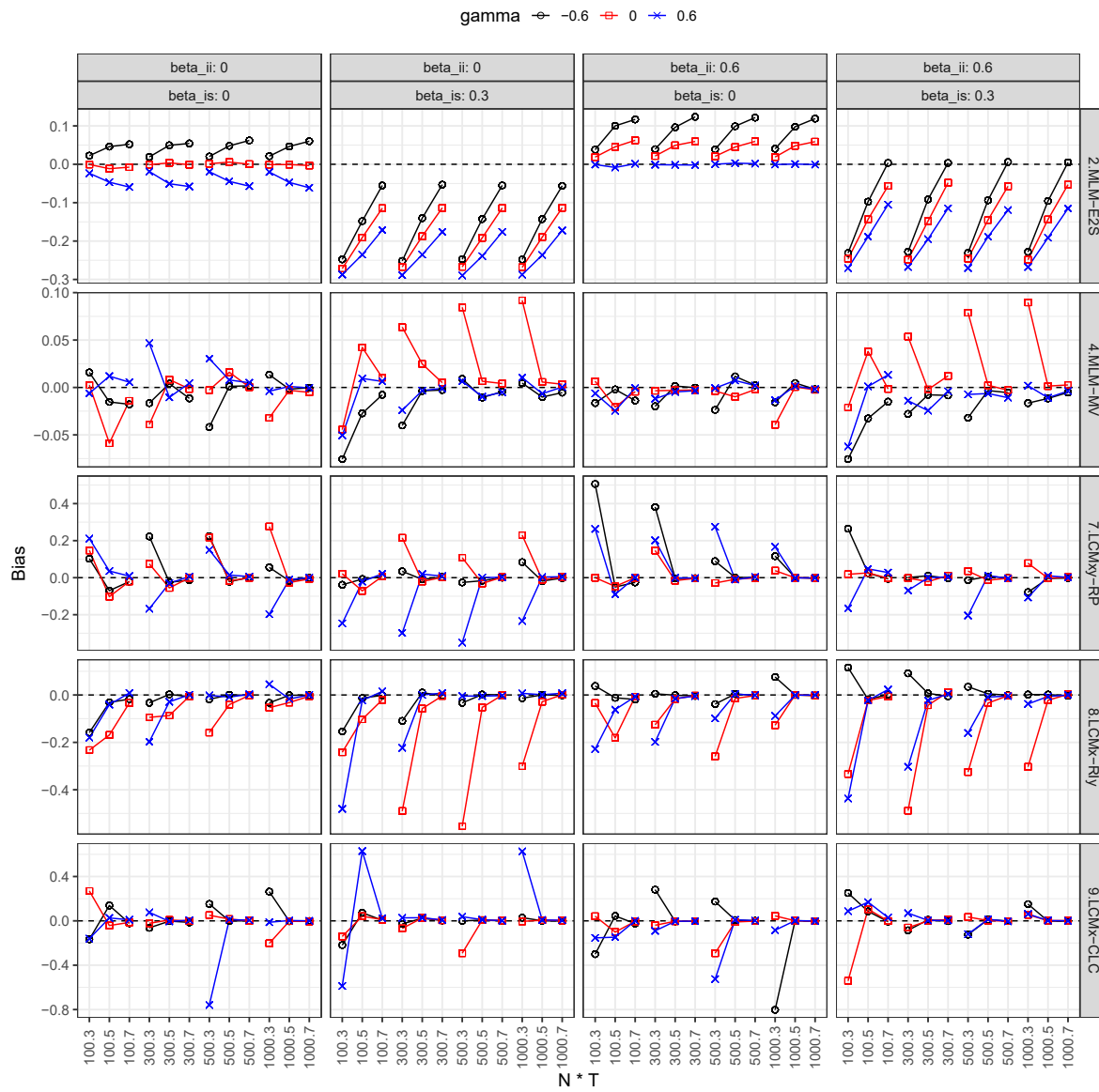


Figure C.29

Empirical SE for the Between-Person Effect β_{IS} in Simulation 3

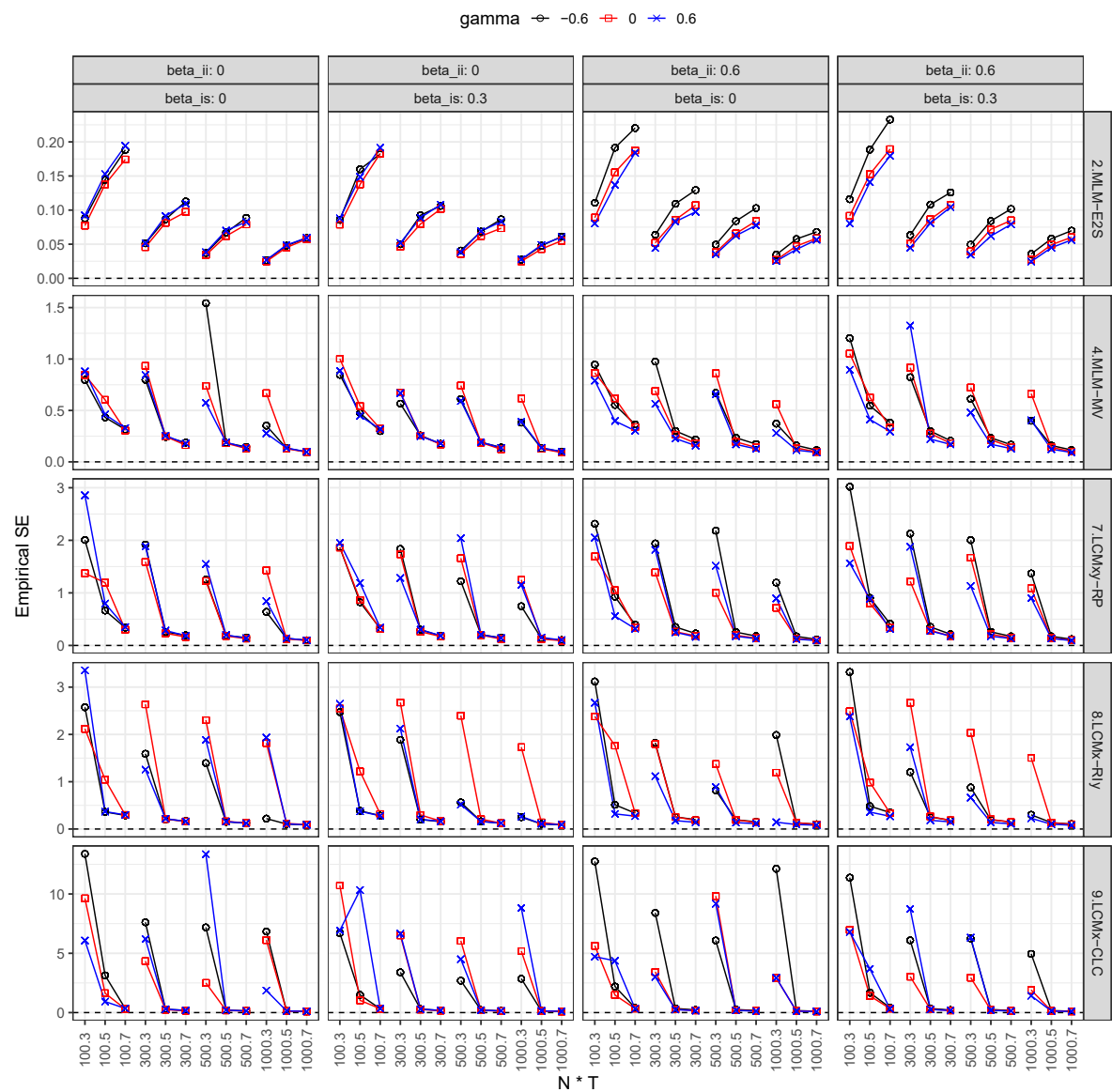


Figure C.30

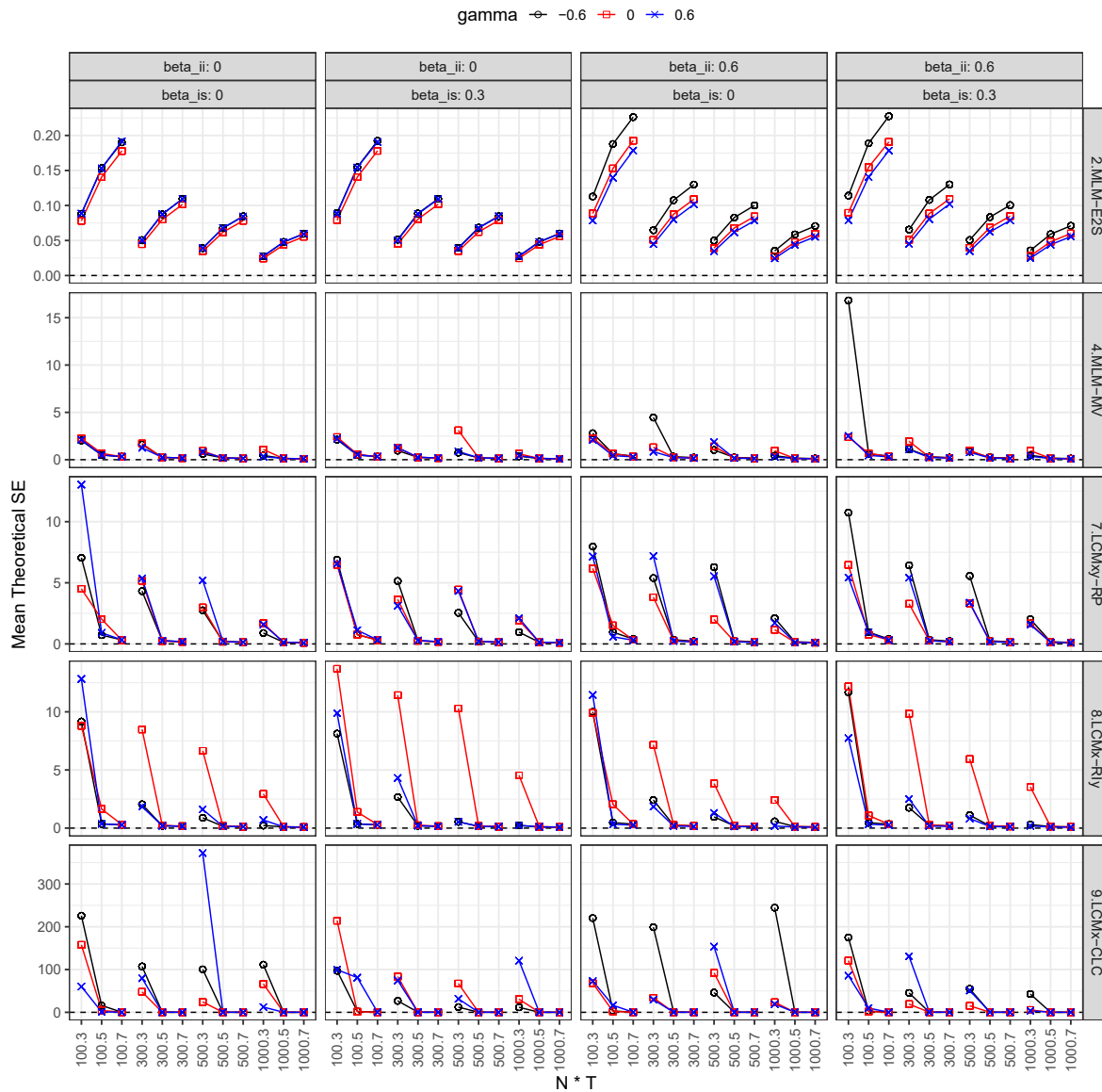
Theoretical SE for the Between-Person Effect β_{IS} in Simulation 3

Figure C.31

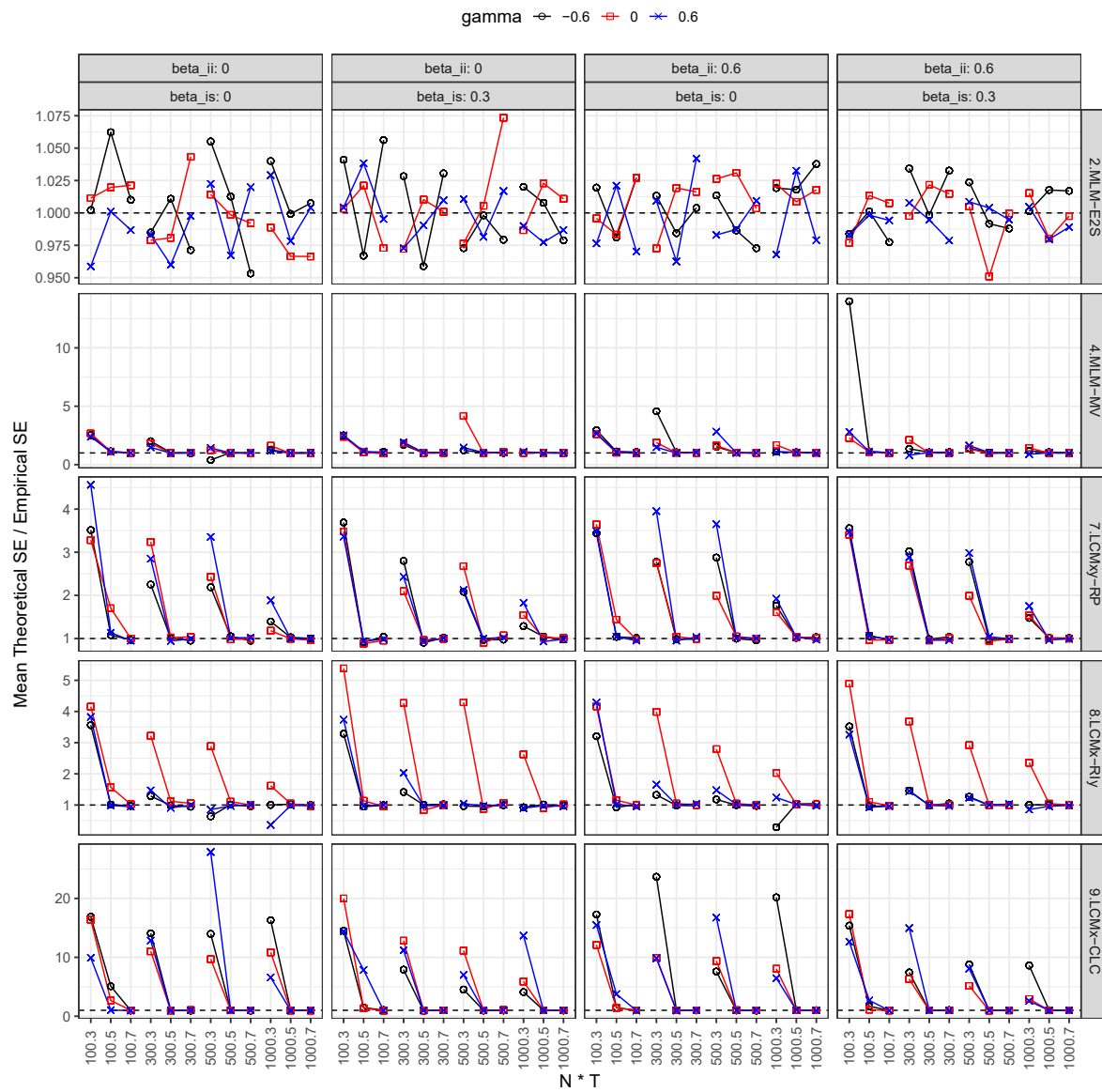
SE Ratio for the Between-Person Effect β_{IS} in Simulation 3

Figure C.32

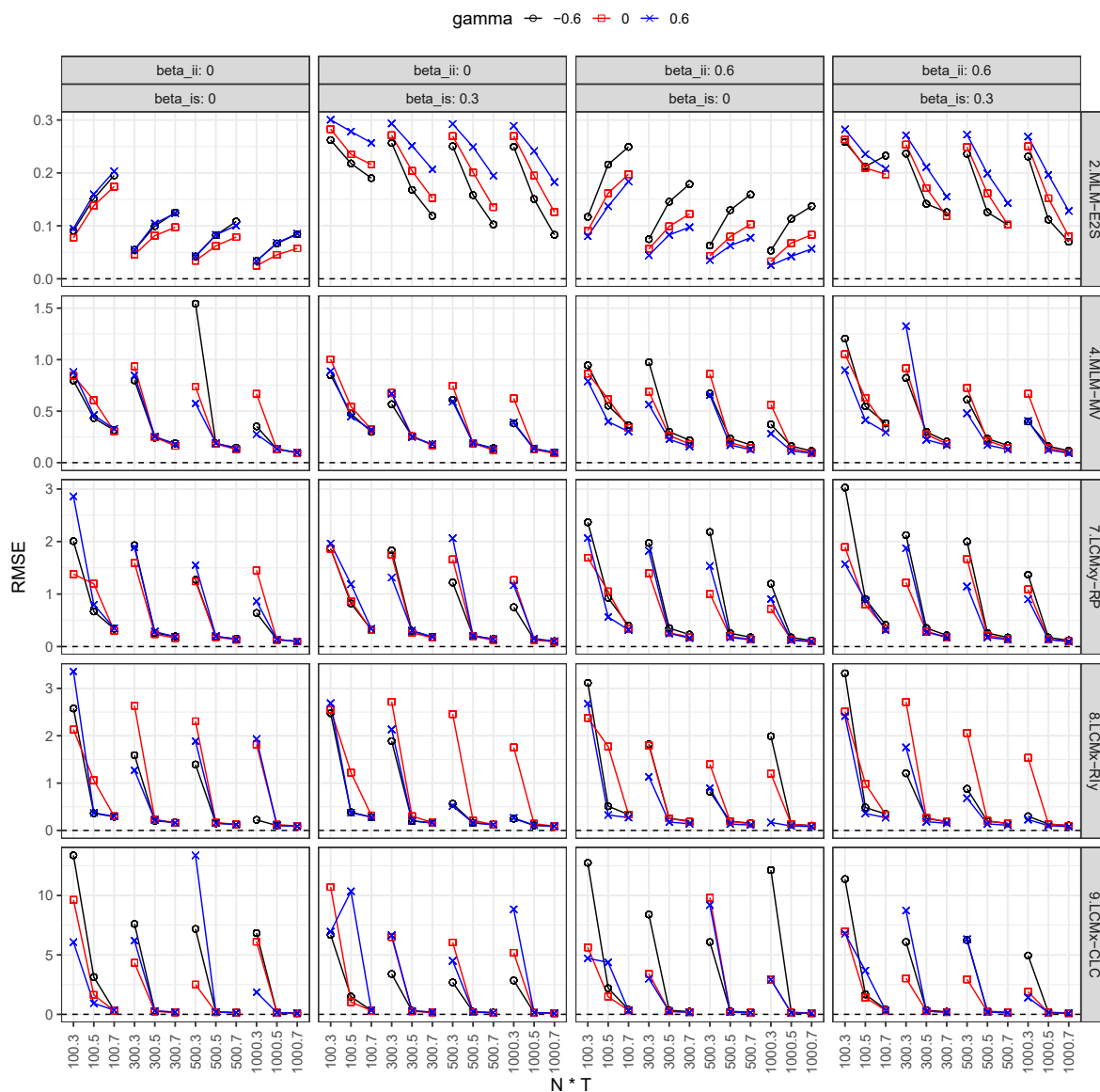
RMSE for the Between-Person Effect β_{IS} in Simulation 3

Figure C.33

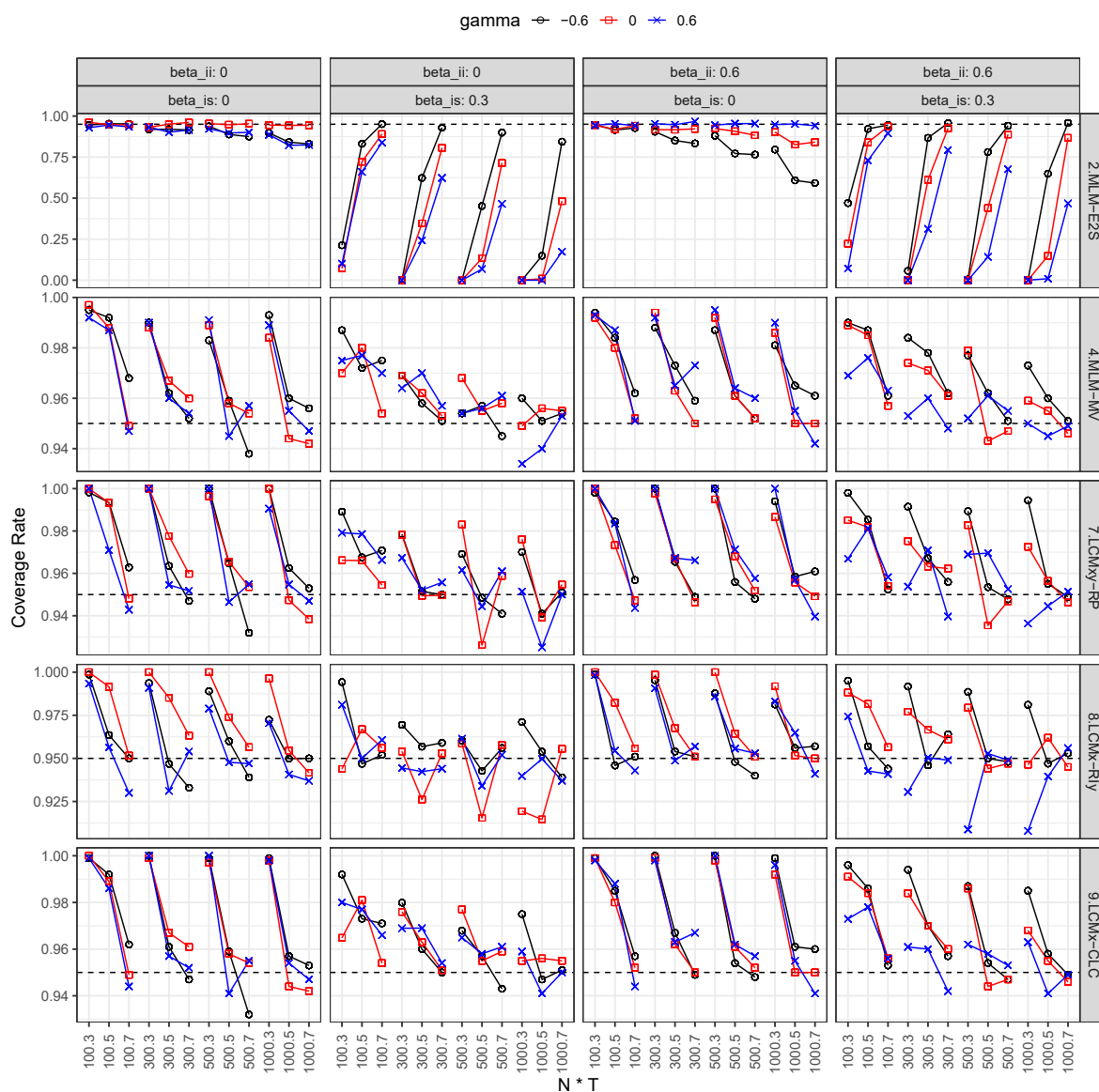
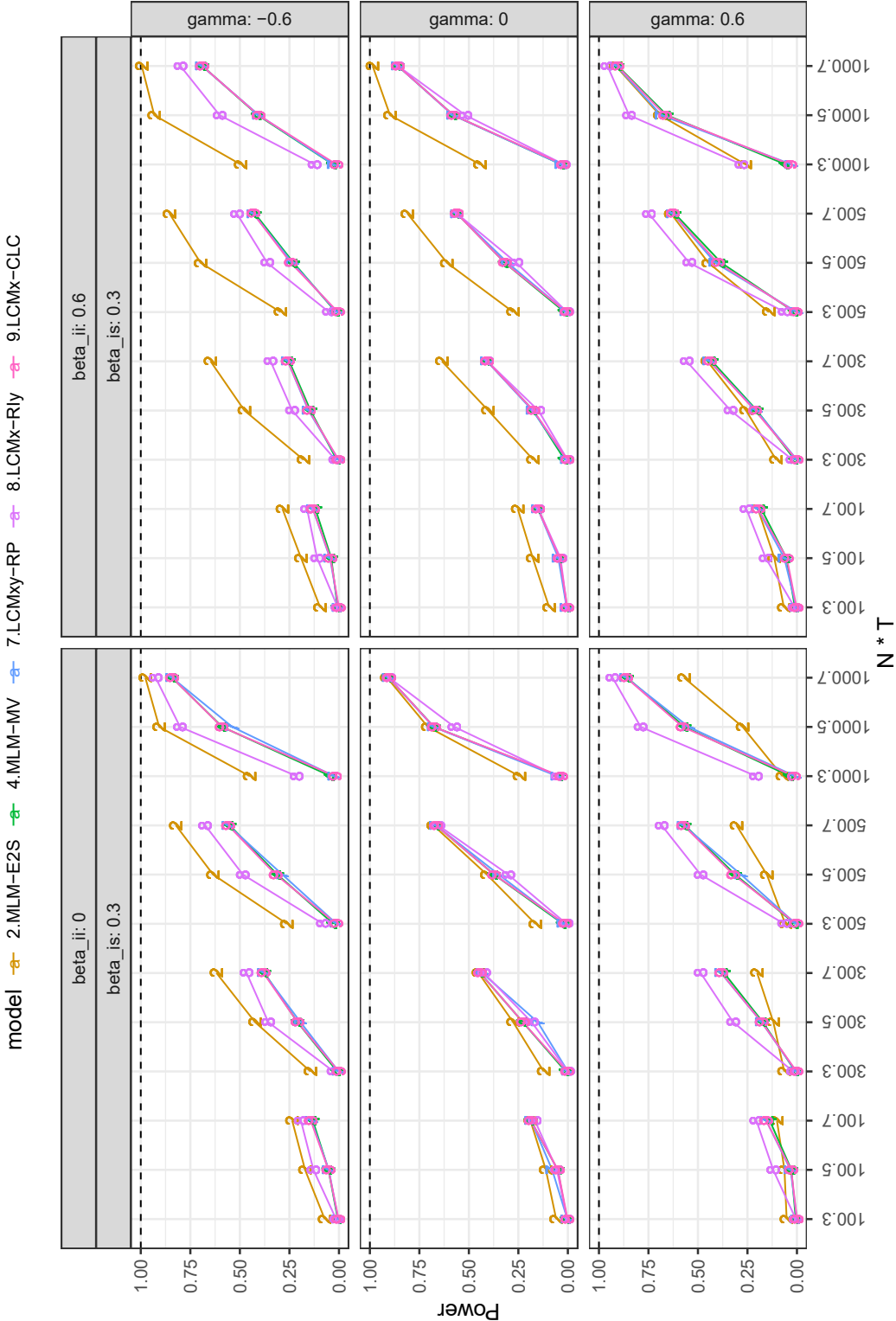
Coverage Rate for the Between-Person Effect β_{IS} in Simulation 3

Figure C.34

Power for the Between-Person Effect β_{IS} in Simulation 3



C.6 Between-Person Effect of Slope on Slope

Figure C.35

Bias for the Between-Person Effect β_{SS} in Simulation 3

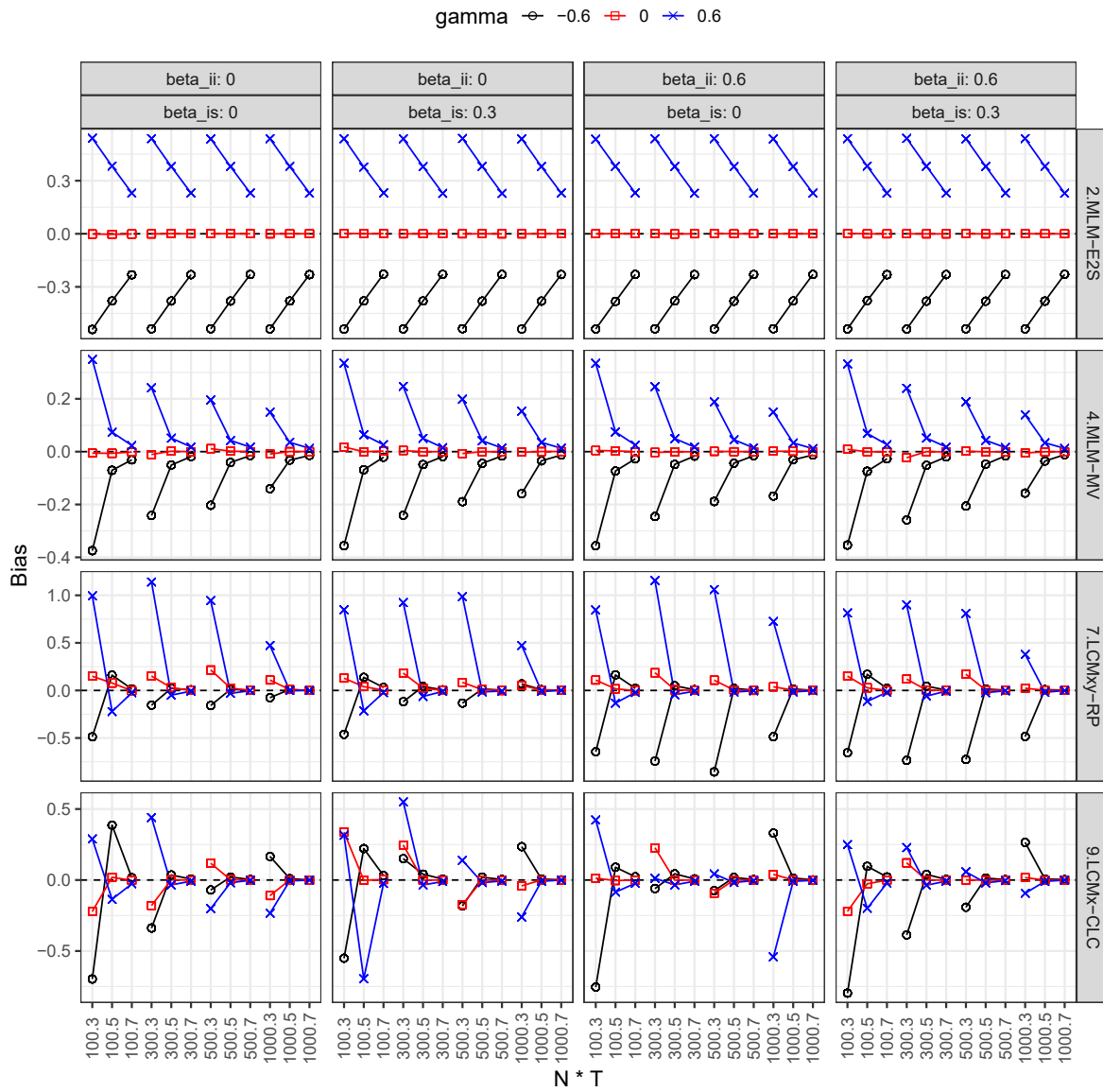


Figure C.36

Empirical SE for the Between-Person Effect β_{SS} in Simulation 3

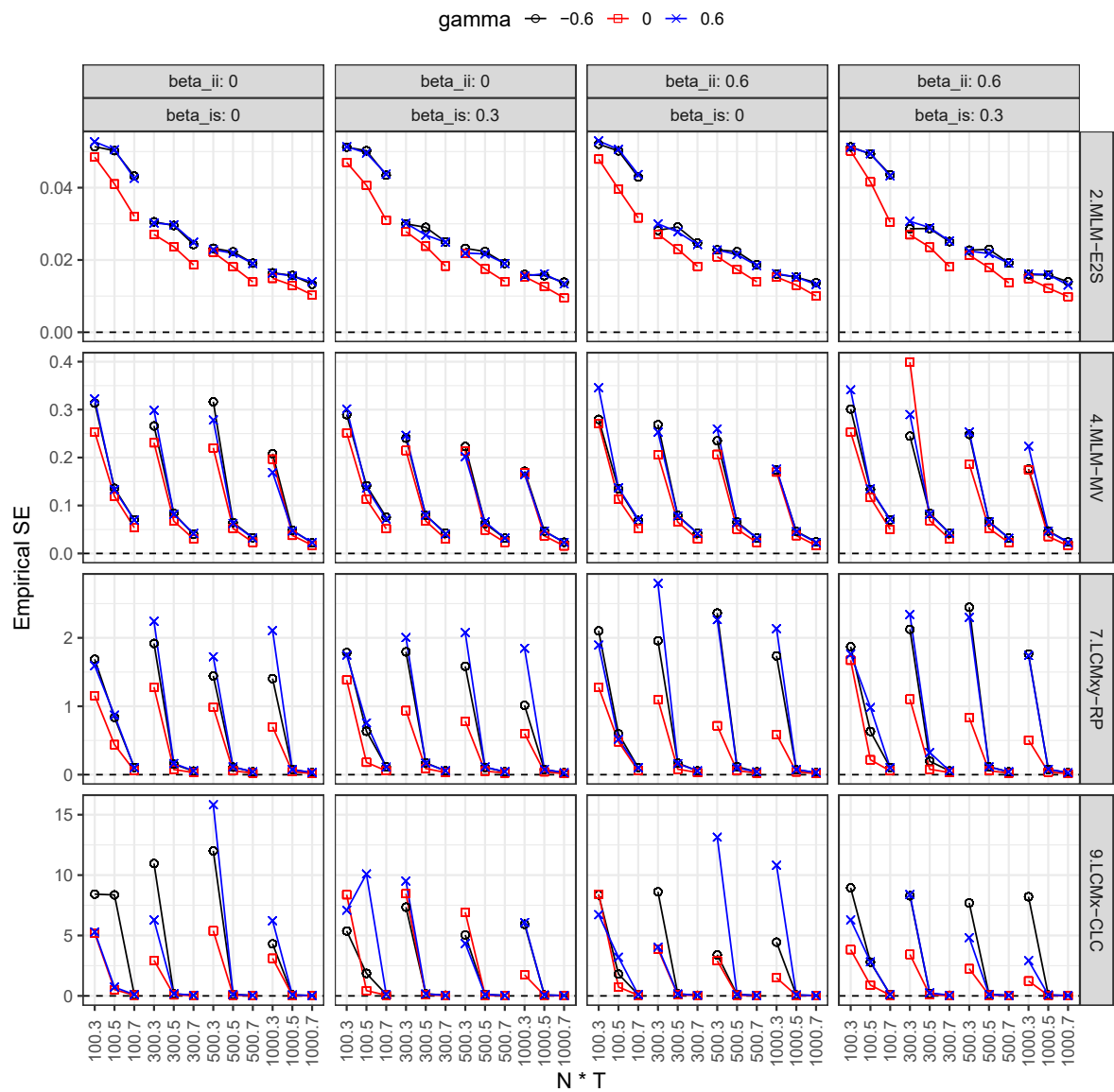


Figure C.37

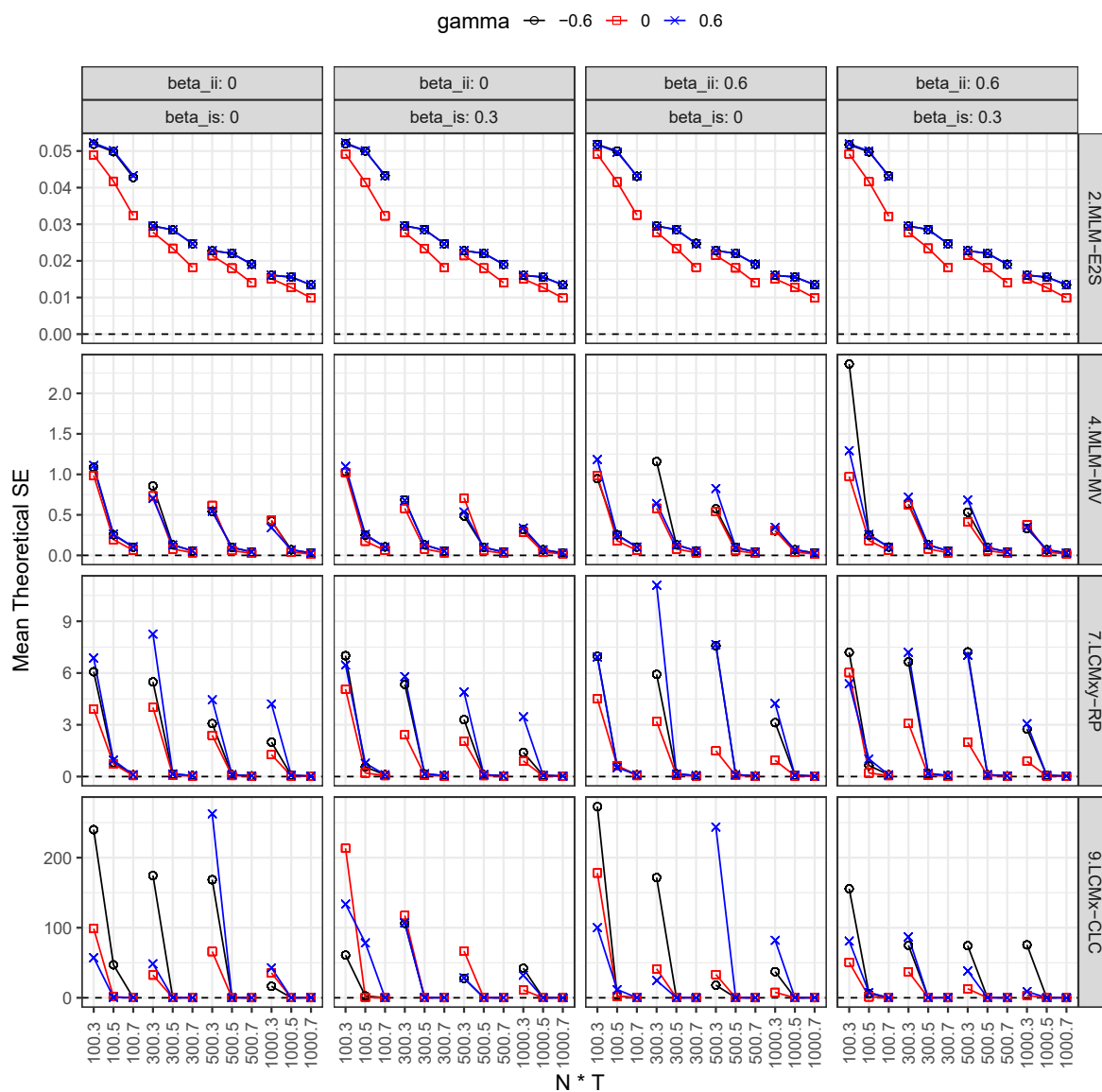
Theoretical SE for the Between-Person Effect β_{SS} in Simulation 3

Figure C.38

SE Ratio for the Between-Person Effect β_{SS} in Simulation 3

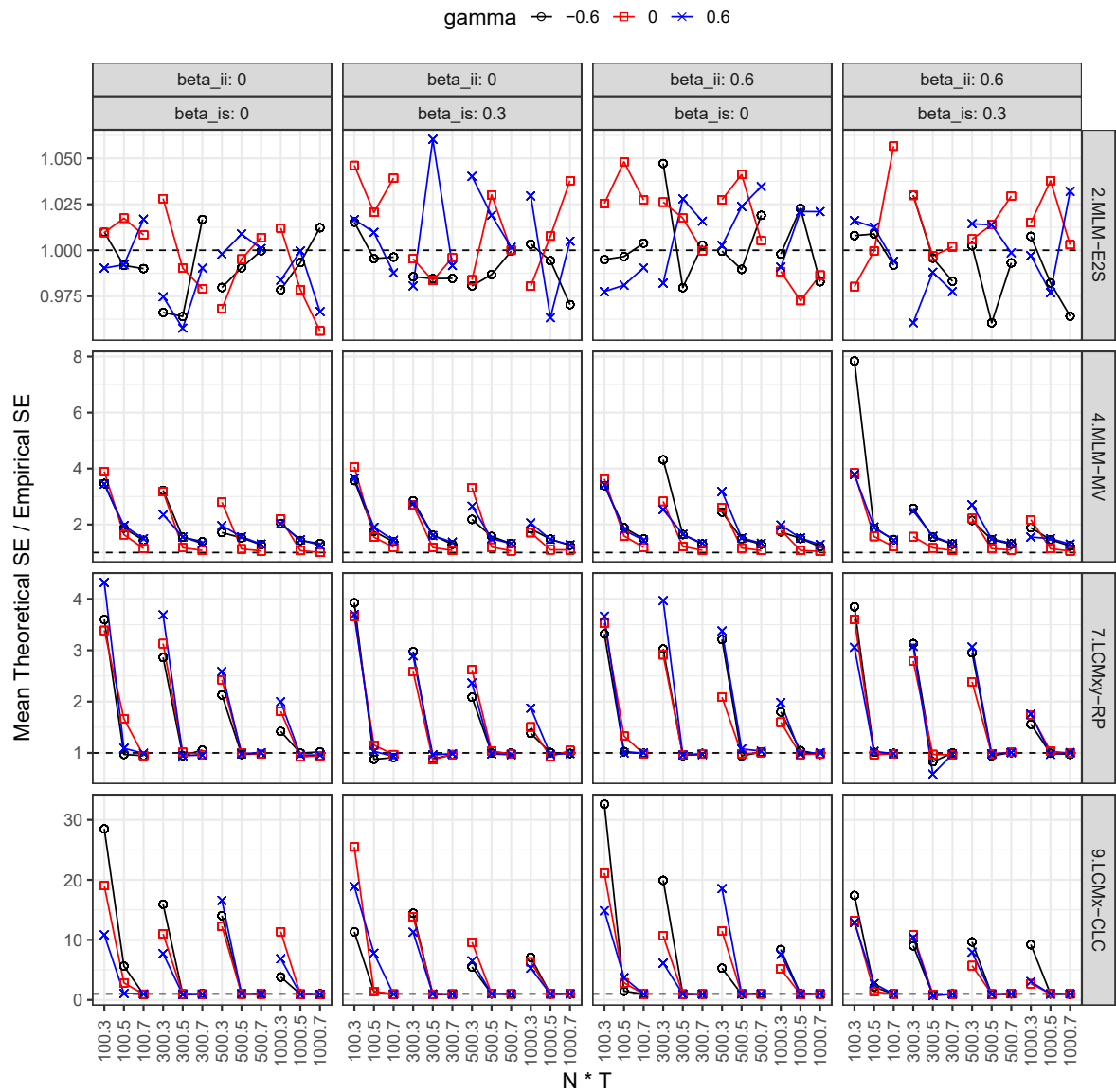


Figure C.39

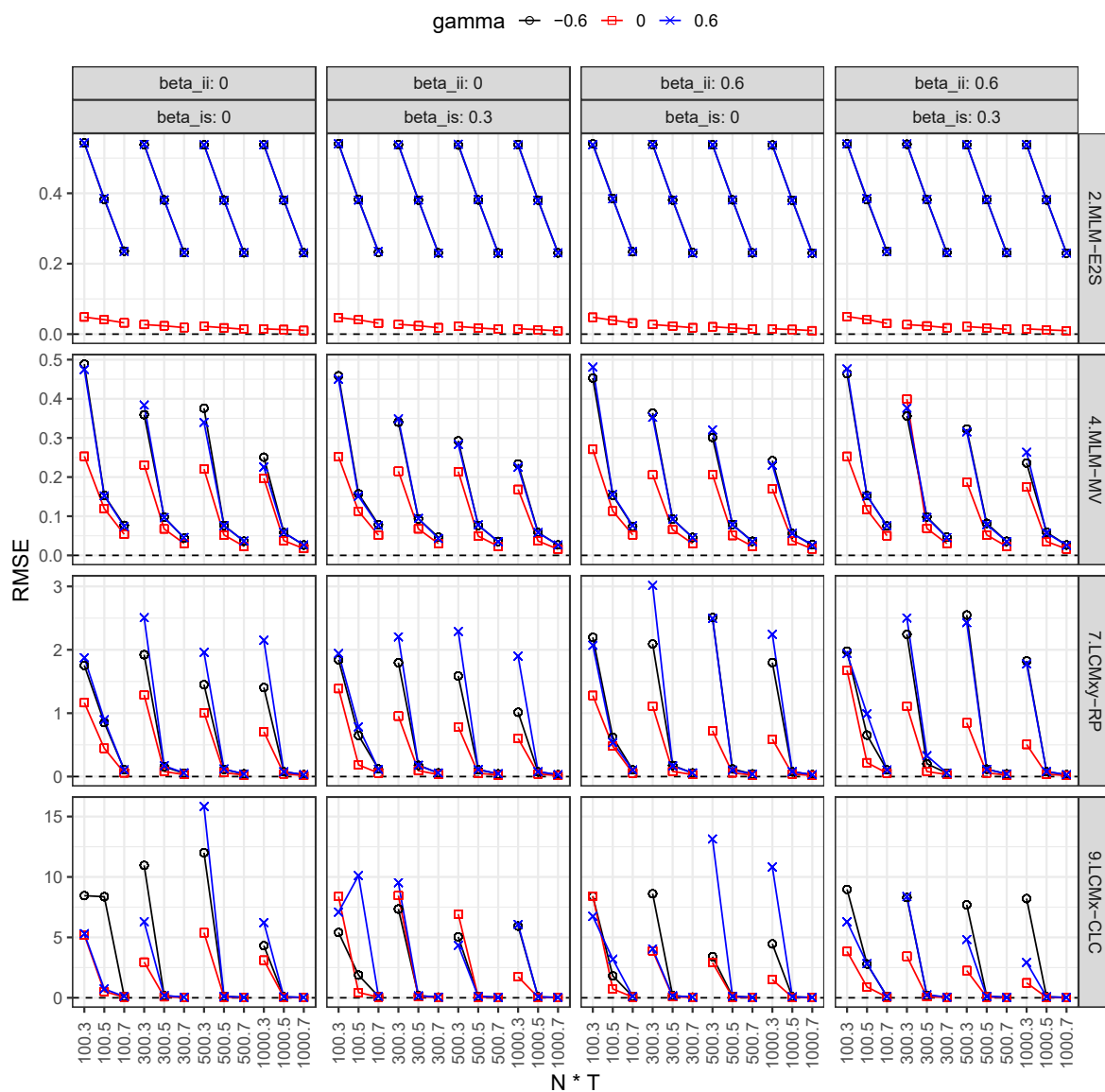
RMSE for the Between-Person Effect β_{SS} in Simulation 3

Figure C.40

RMSE for the Between-Person Effect β_{SS} in Simulation 3

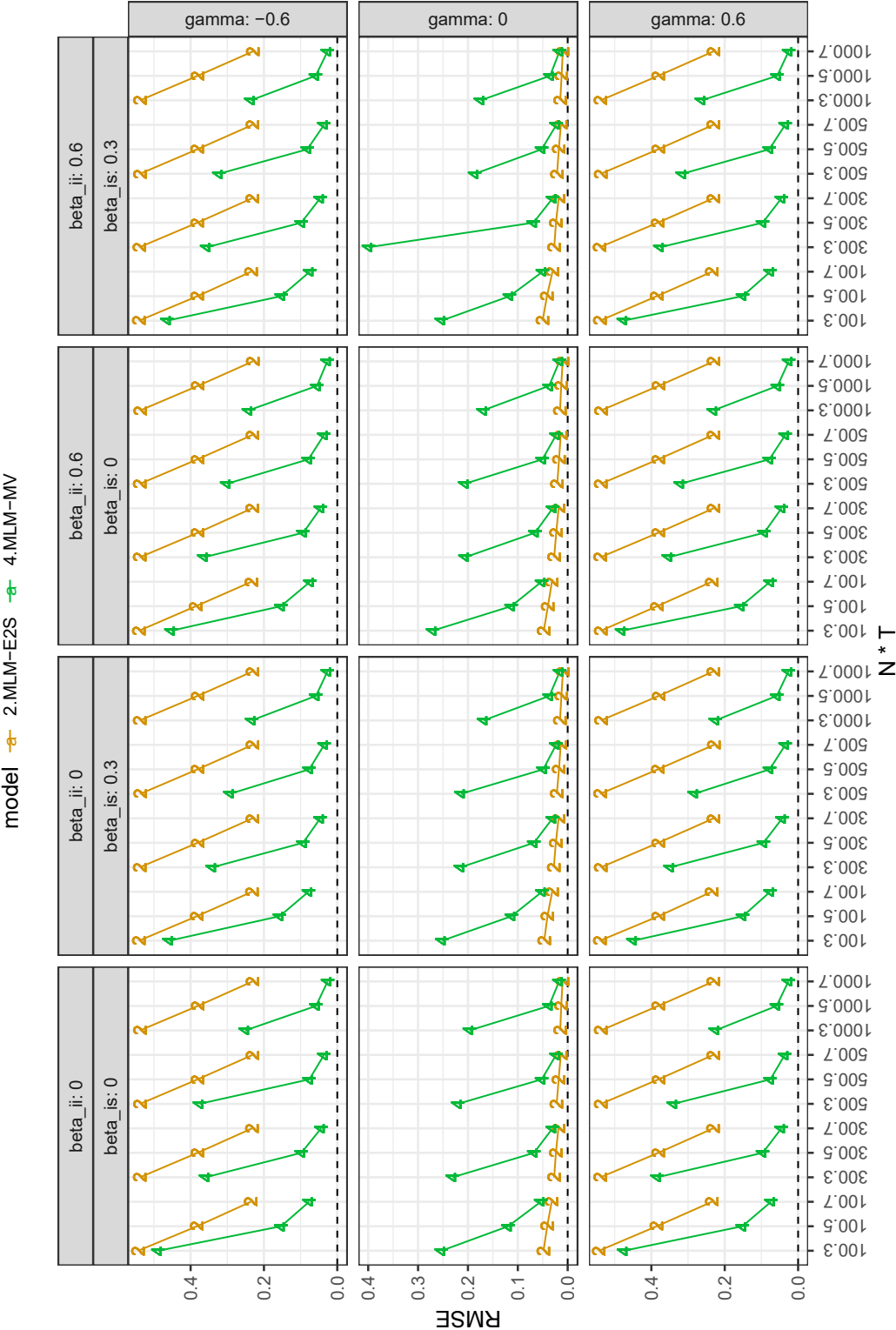
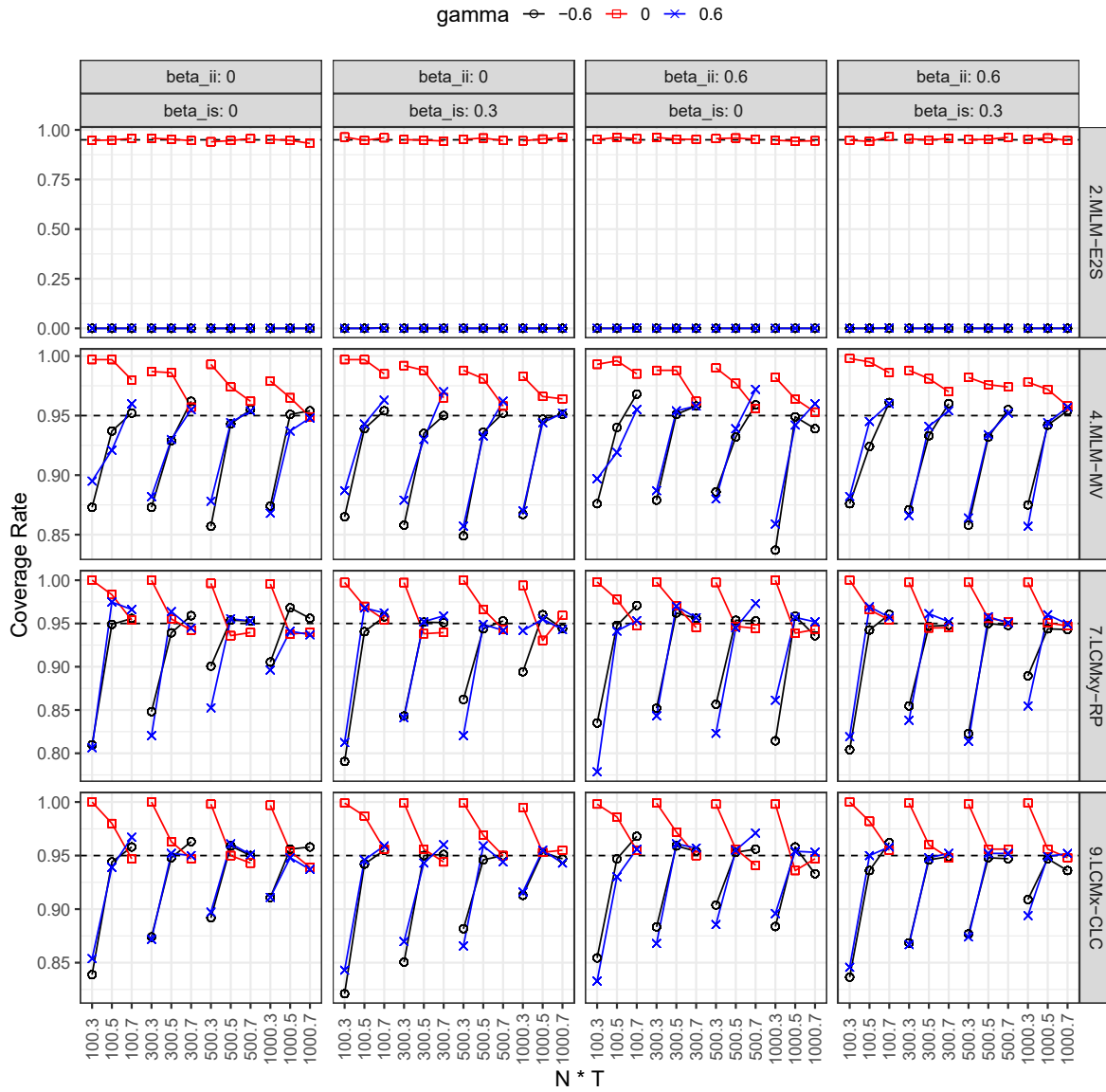


Figure C.41

Coverage Rate for the Between-Person Effect β_{SS} in Simulation 3

Appendix D

Figures in Simulation 4

D.1 Auxiliary Indices

Figure D.1

Proportion of Converged Cases in Simulation 4 except LCMx-Rlx ($\gamma = -0.6$)

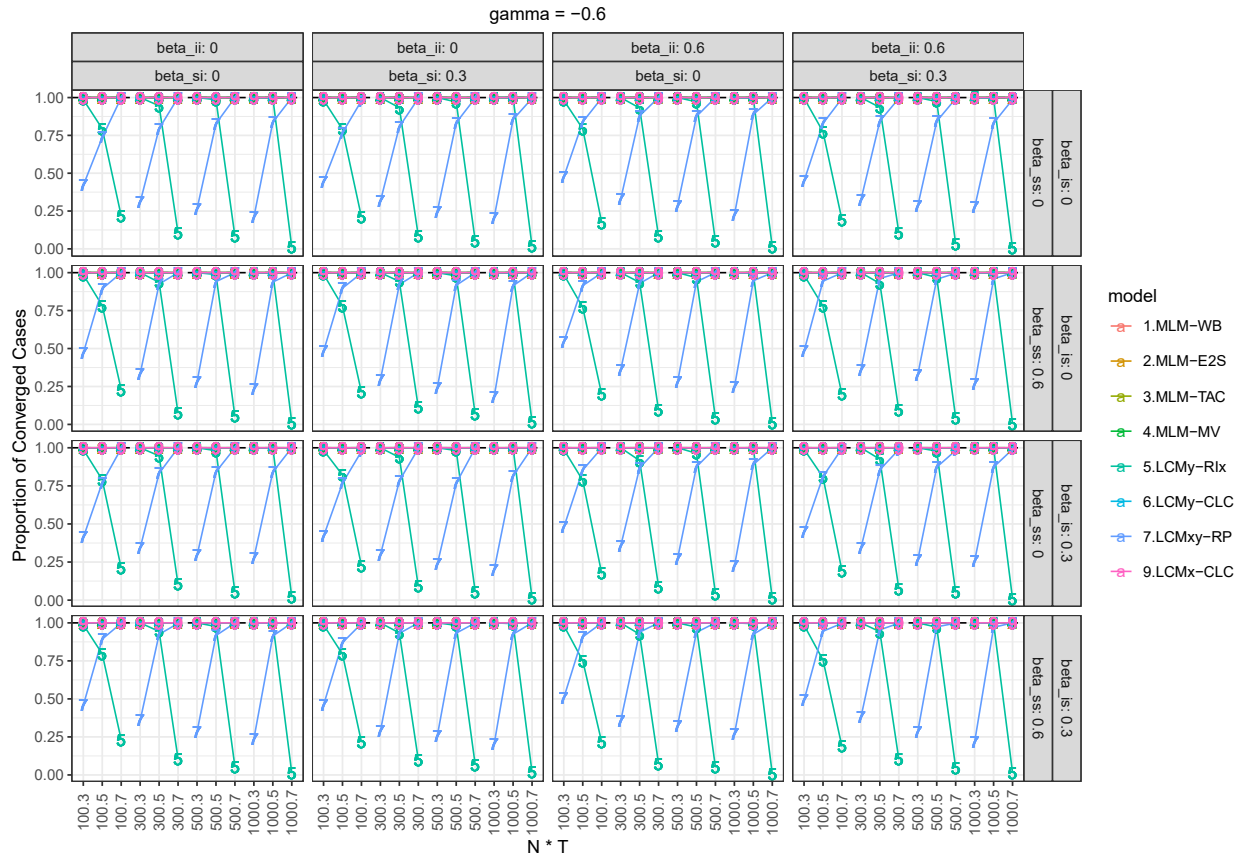


Figure D.2
Proportion of Converged Cases in Simulation 4 except LCMx-RLx ($\gamma = 0$)

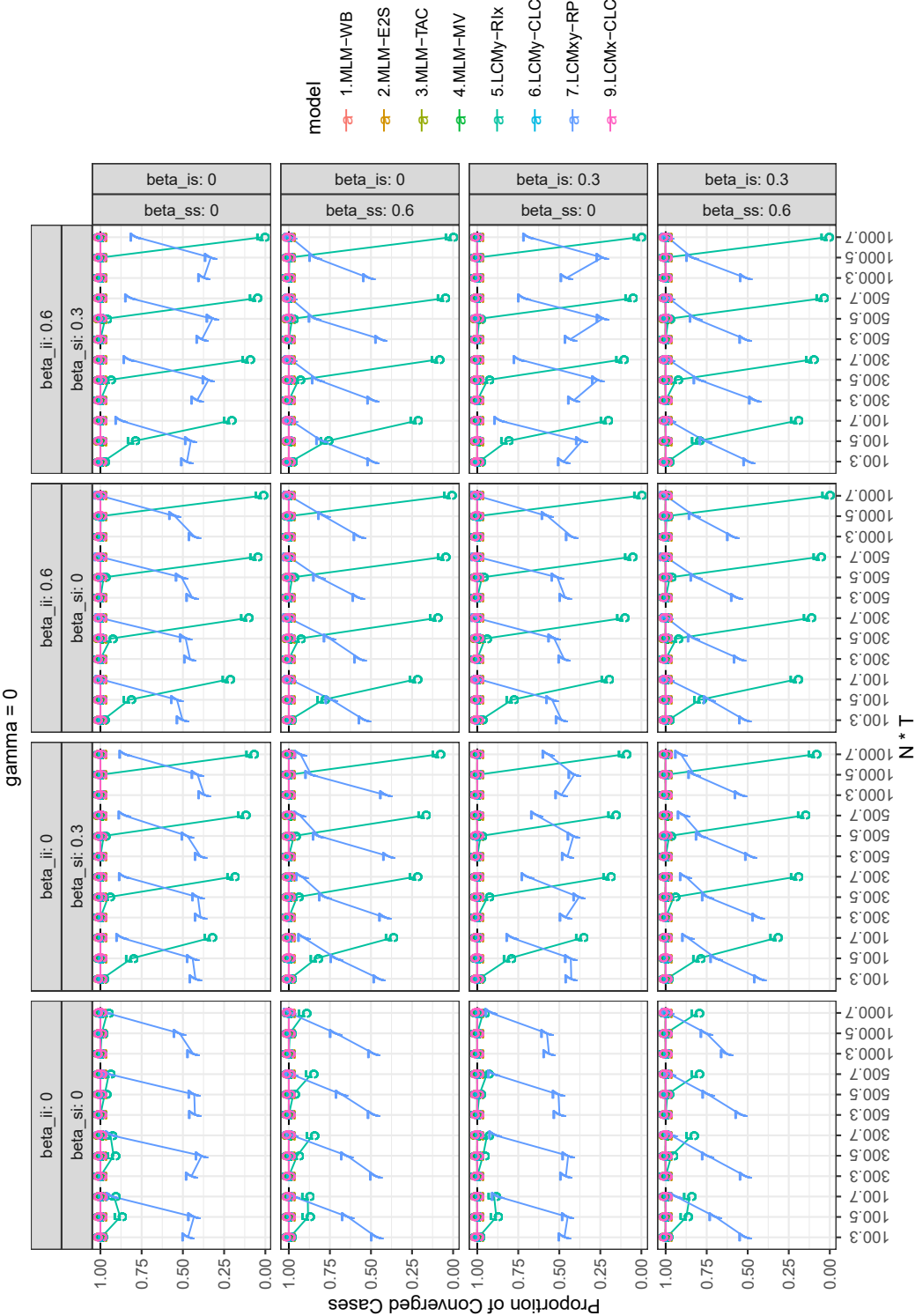


Figure D.3

Proportion of Converged Cases in Simulation 4 except LCMx-RLx ($\gamma = 0.6$)

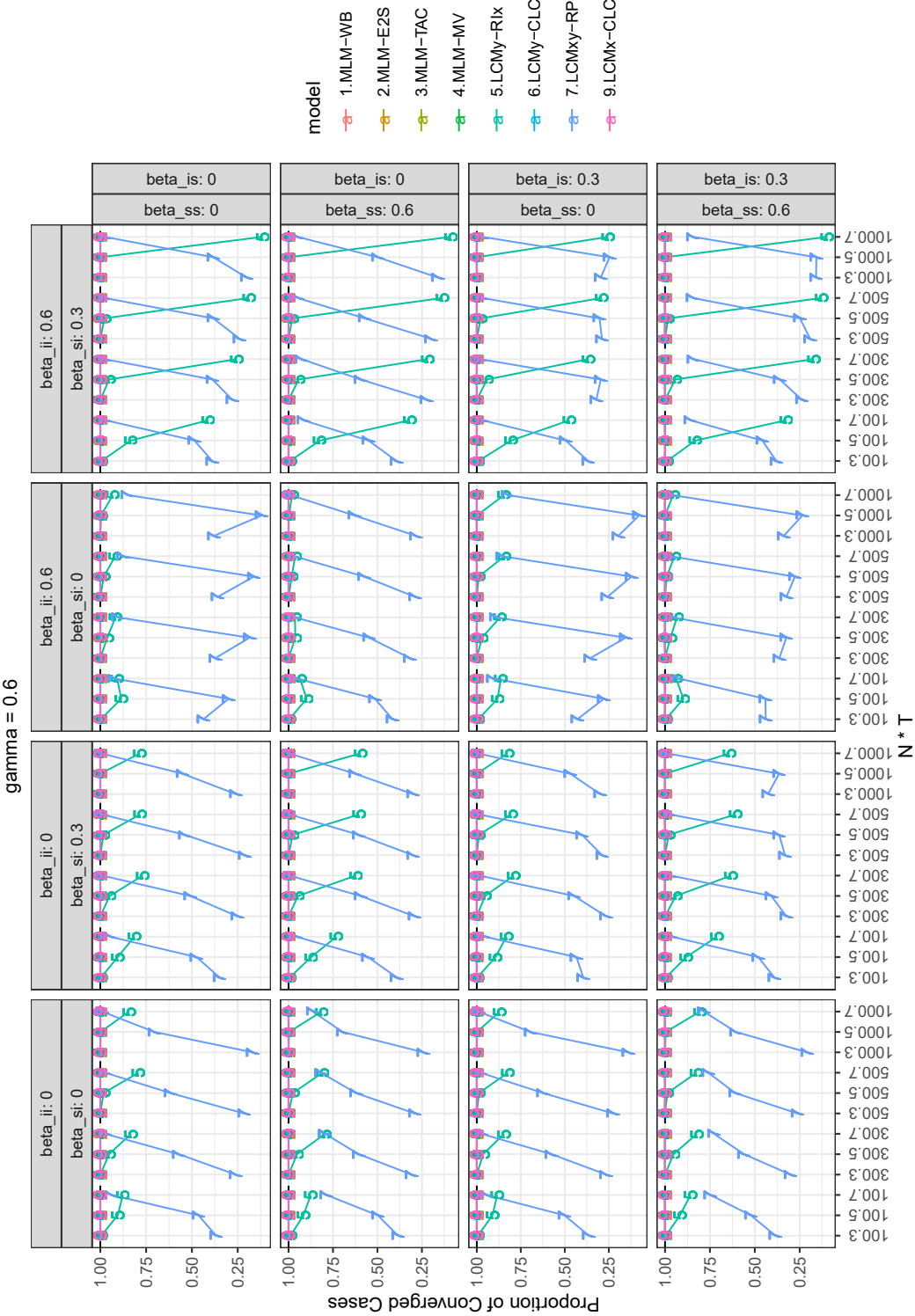


Figure D.4
Proportion of Converged Cases in Simulation 4 (LCMx-Rlx)

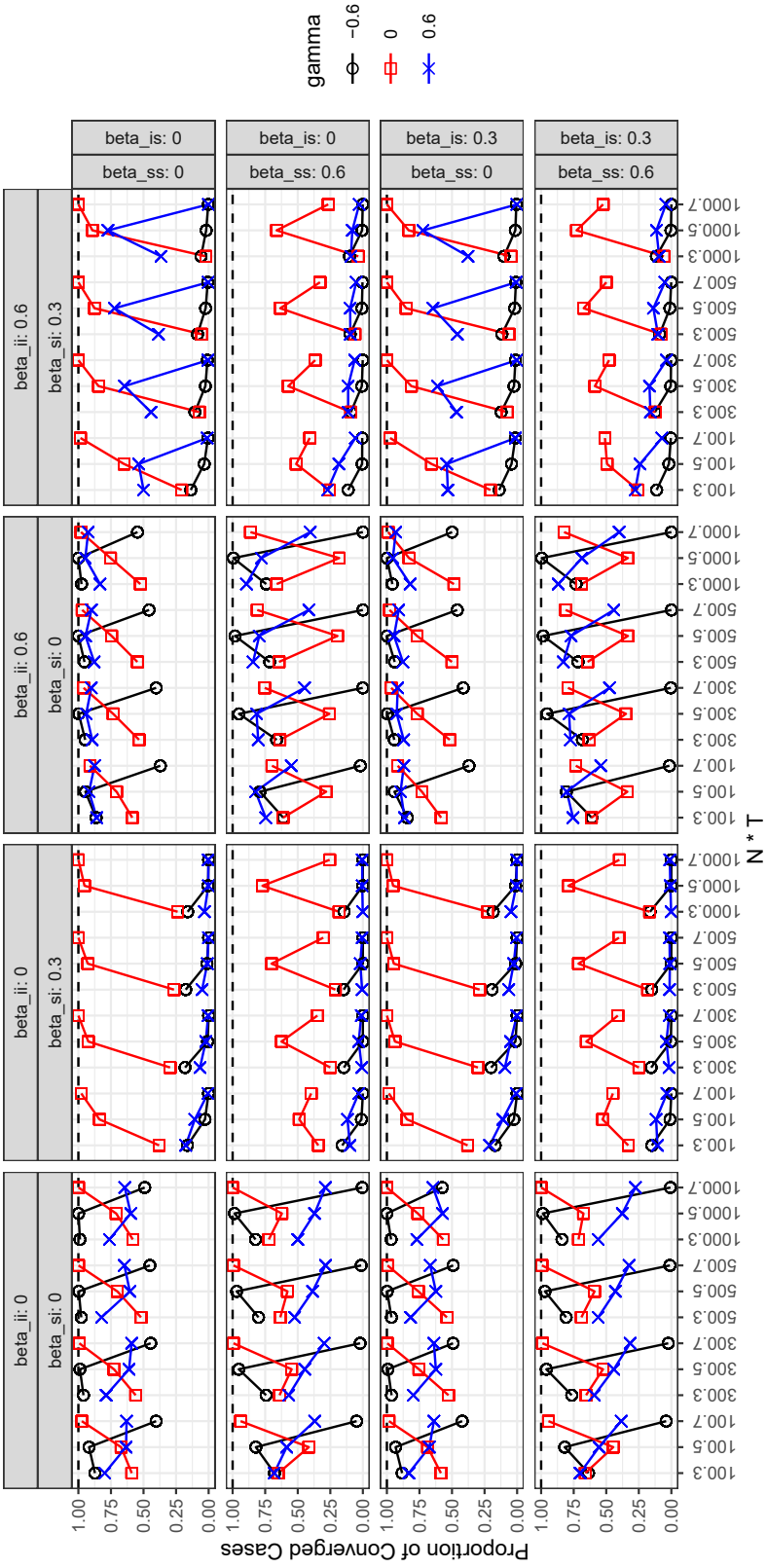


Figure D.5

Proportion of Cases with Problems in Estimating SE in Simulation 4 except LCMx-Rly ($\gamma = -0.6$)

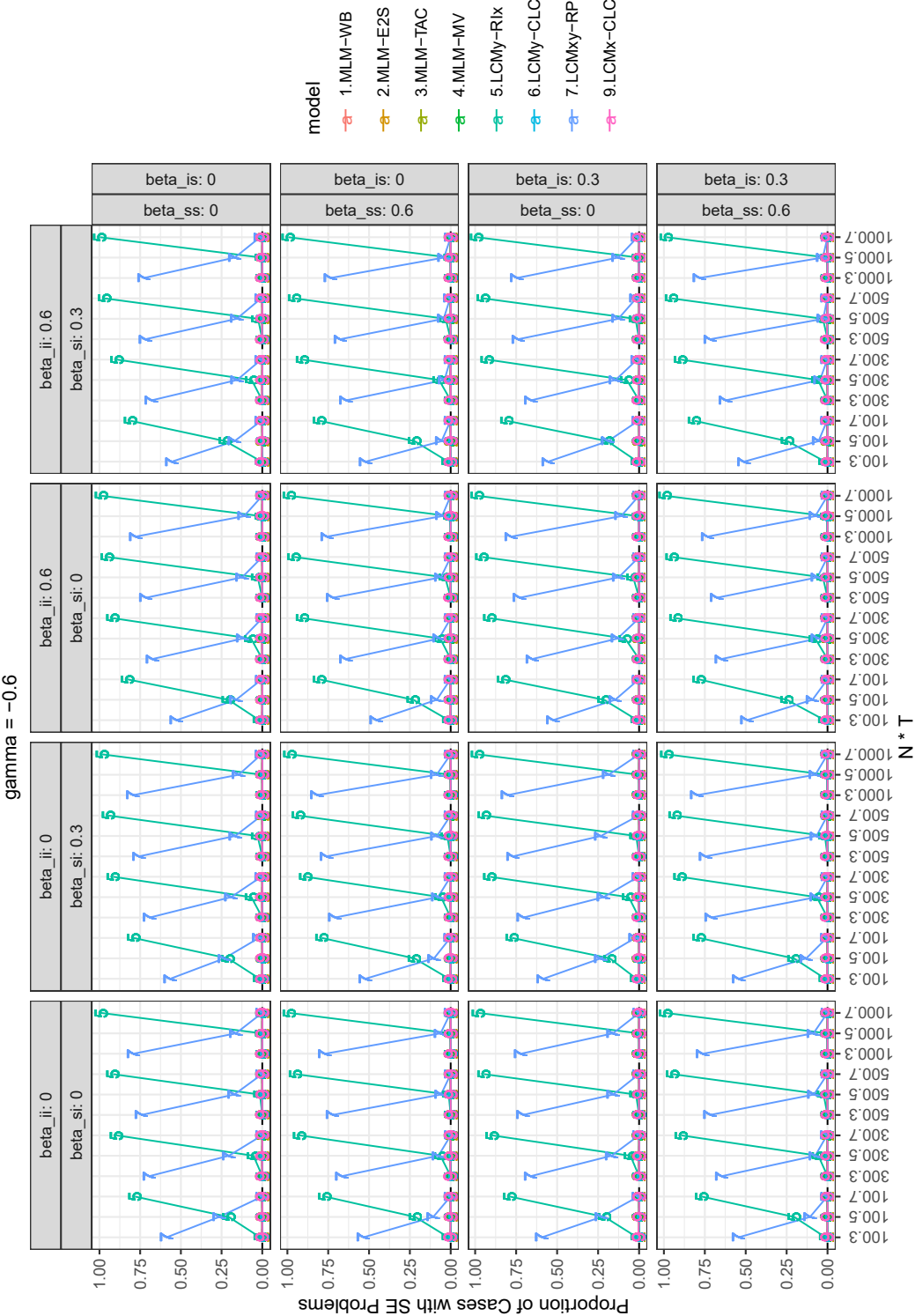


Figure D.6

Proportion of Cases with Problems in Estimating SE in Simulation 4 except LCMx-Rly ($\gamma = 0$)

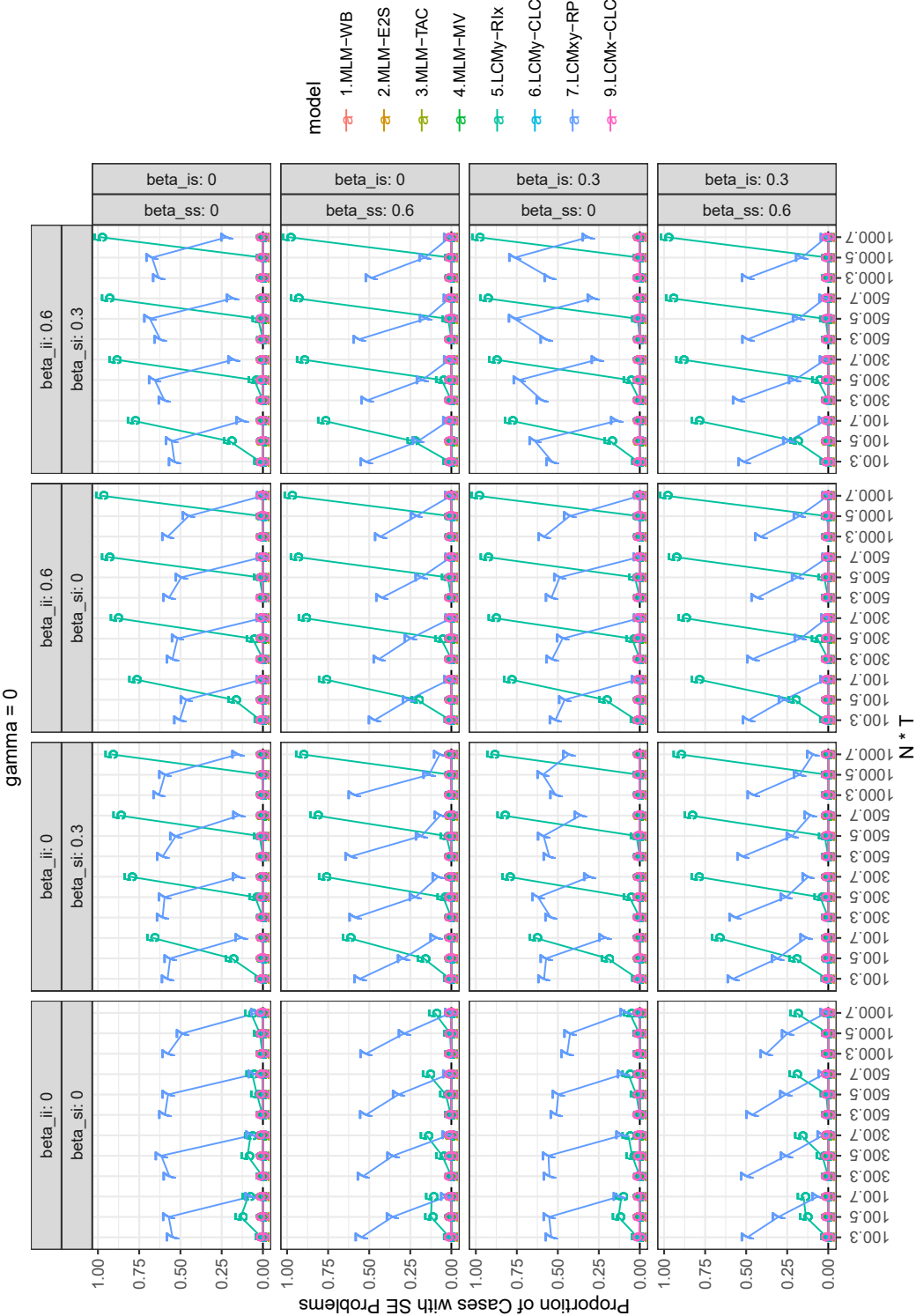


Figure D.7

Proportion of Cases with Problems in Estimating SE in Simulation 4 except LCMx-Rly ($\gamma = 0.6$)

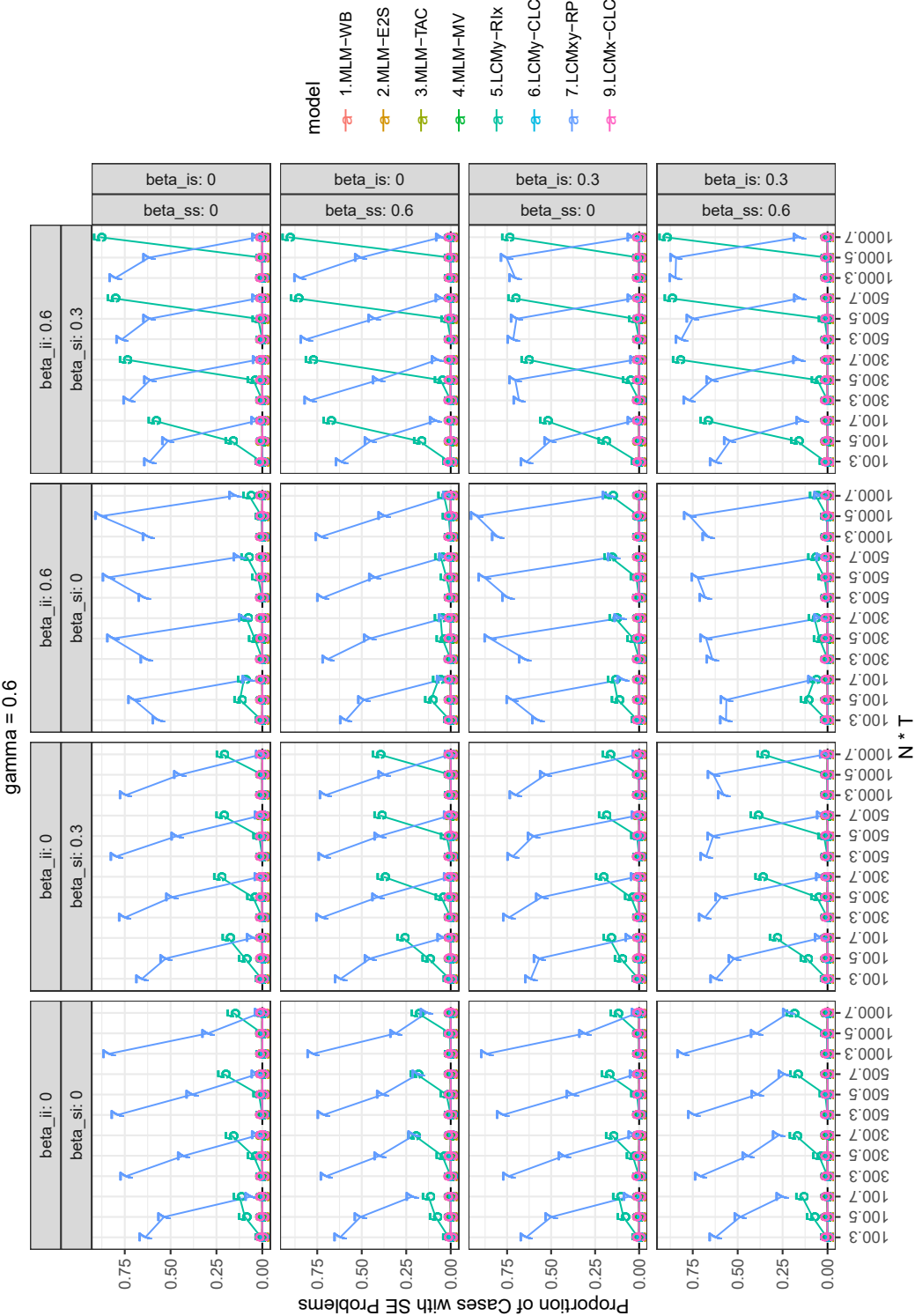


Figure D.8
Proportion of Cases with Problems in Estimating SE in Simulation 4 (LCMx-Rly)

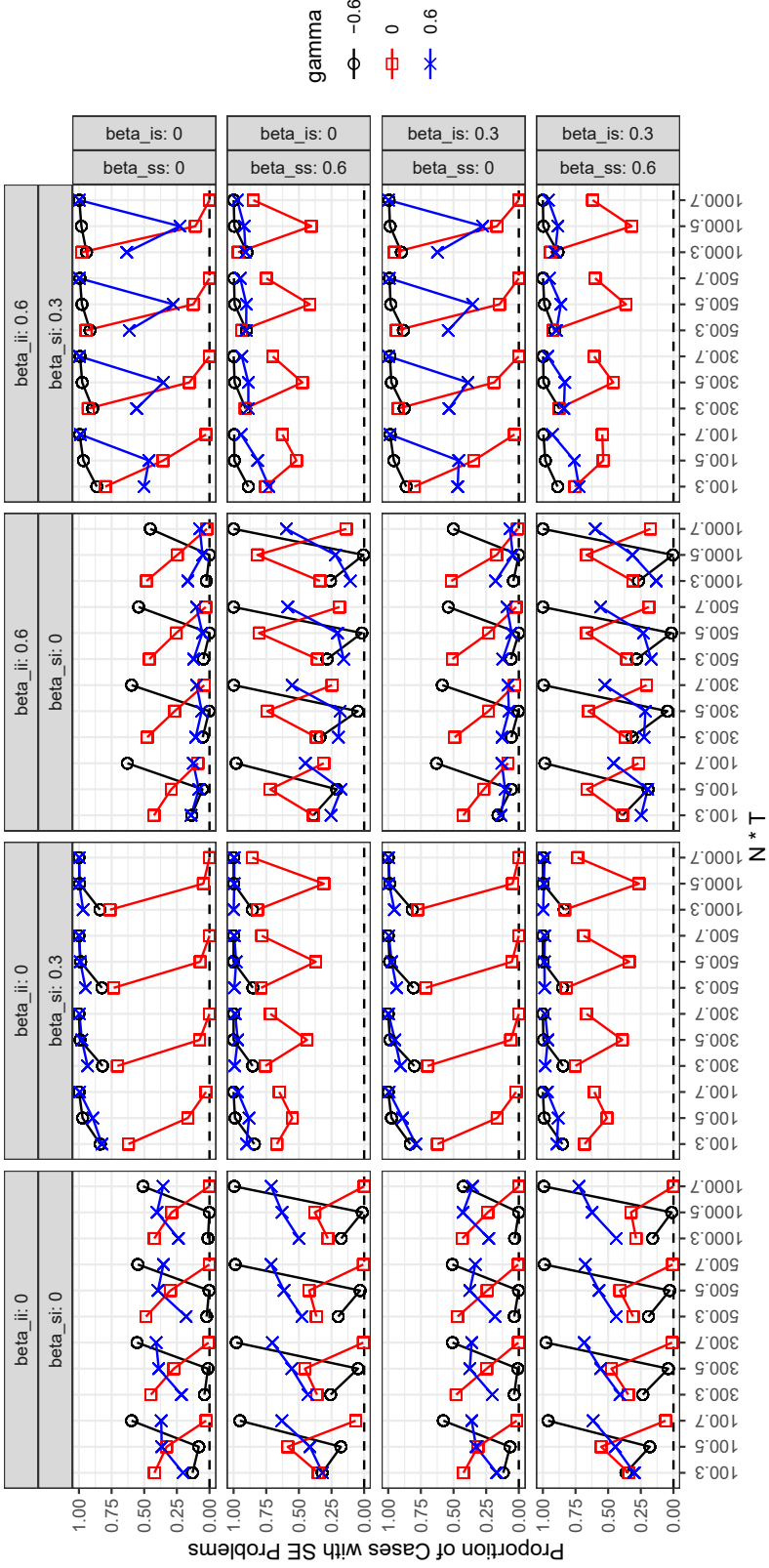


Figure D.9

Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 4 except LCMy-Rlx and LCMy-Rly
($\gamma = -0.6$)

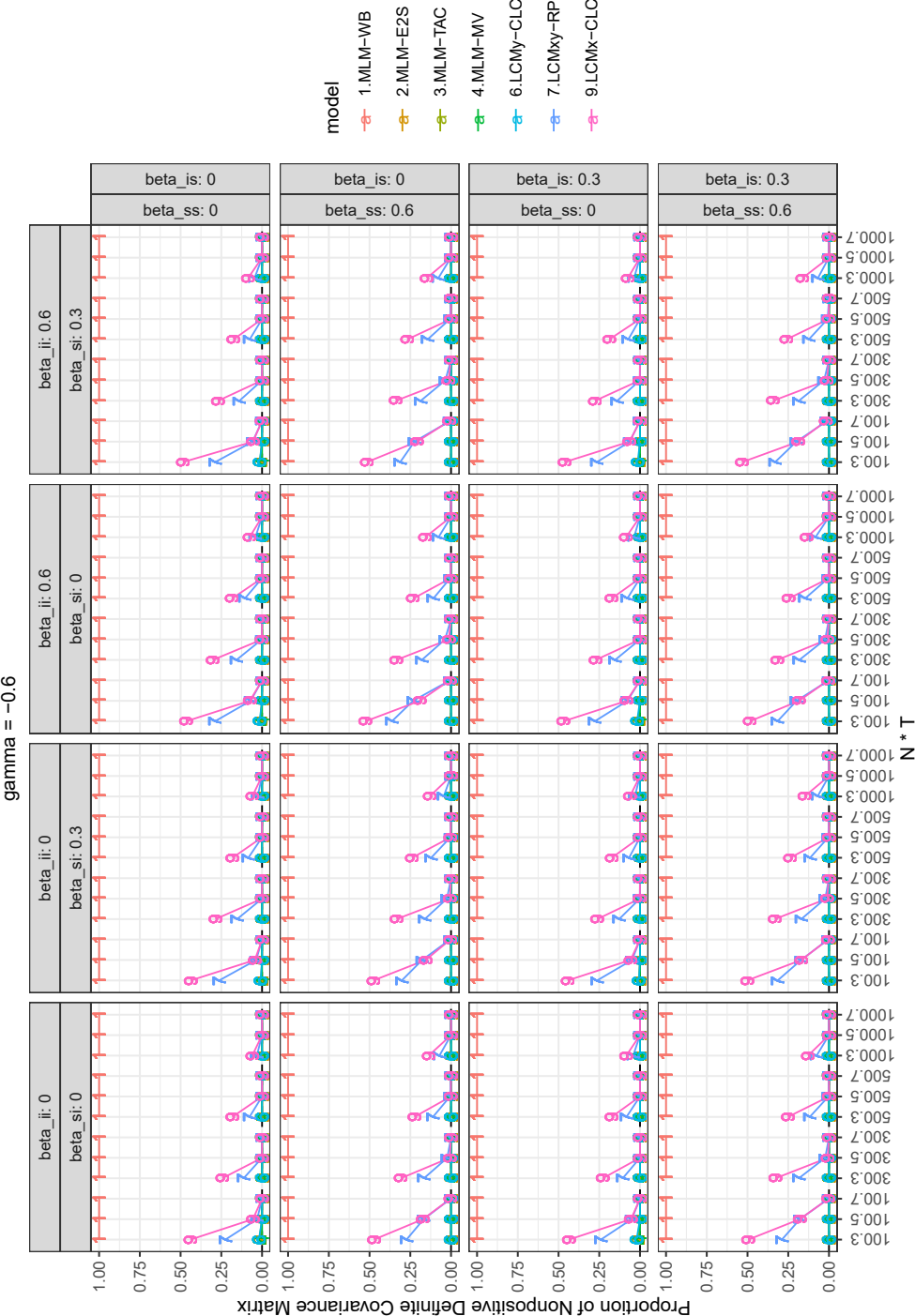


Figure D.10

Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 4 except LCMy-Rlx and LCMy-Rly ($\gamma = 0$)

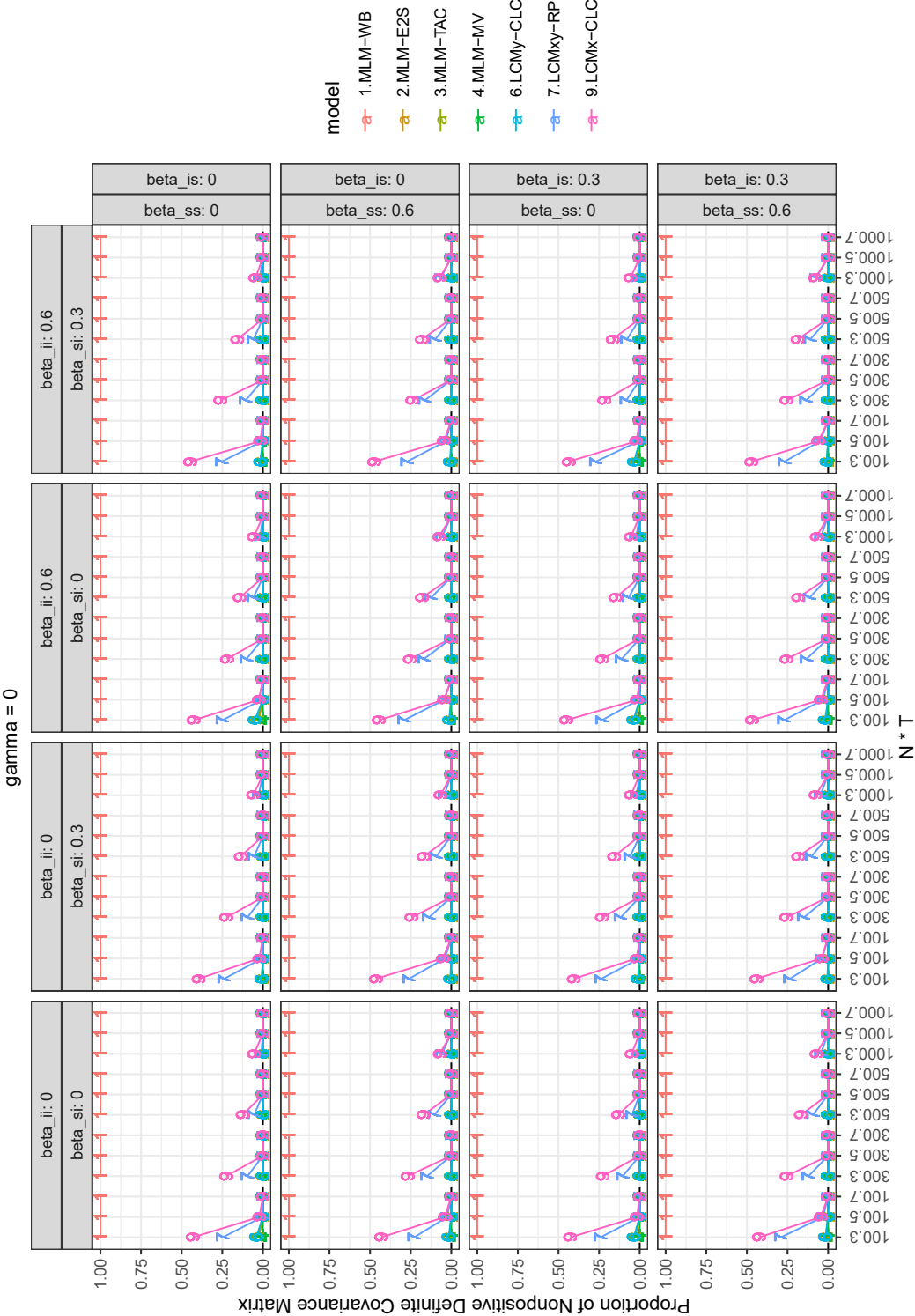


Figure D.11

Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 4 except LCMy-Rlx ($\gamma = 0.6$)

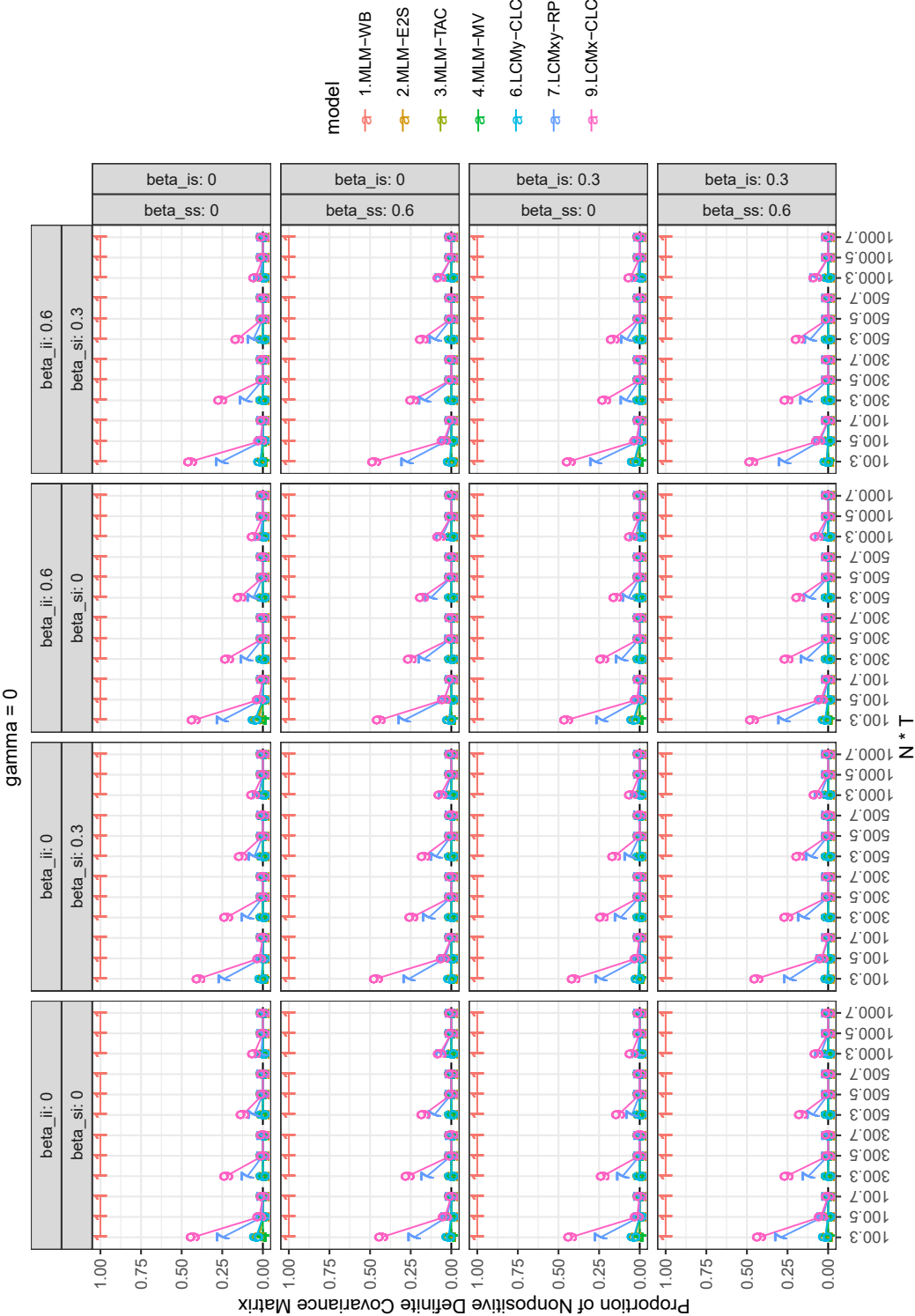


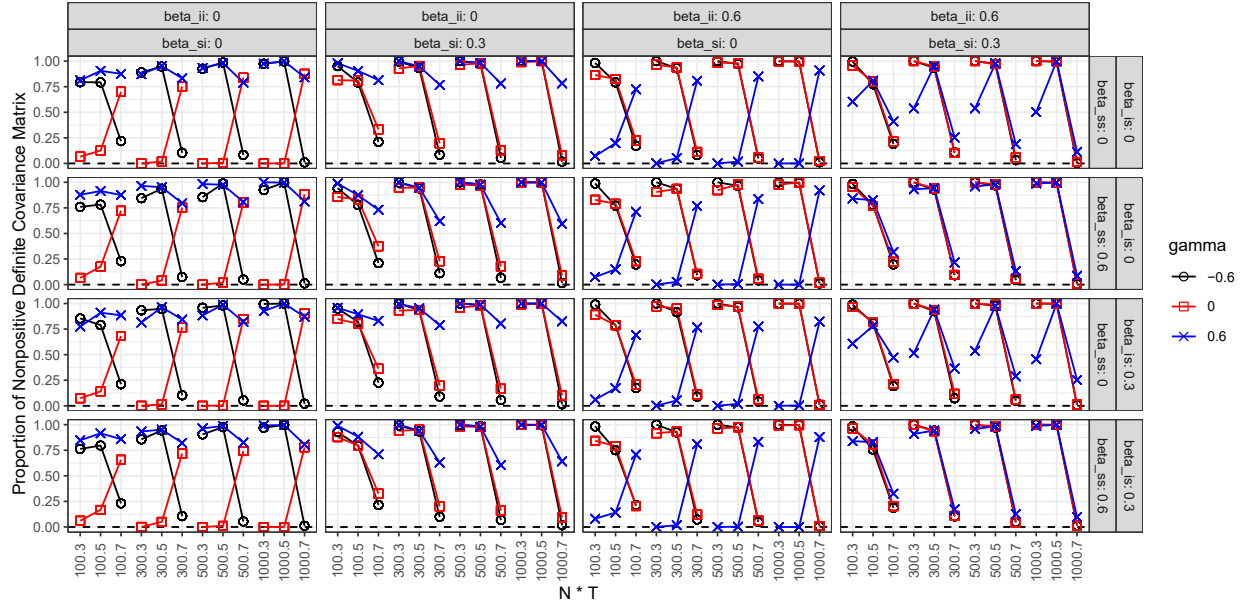
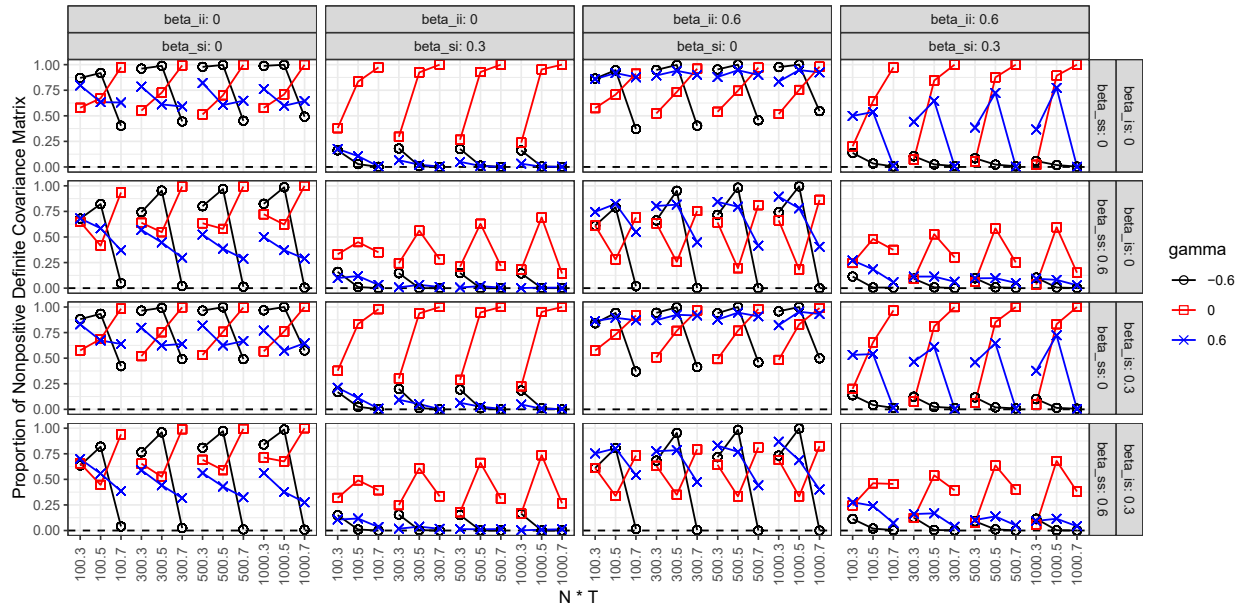
Figure D.12*Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 4 (LCMy-Rly)***Figure D.13***Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 4 (LCMx-Rly)*

Figure D.14

CFI in Simulation 4 ($\gamma = -0.6$)

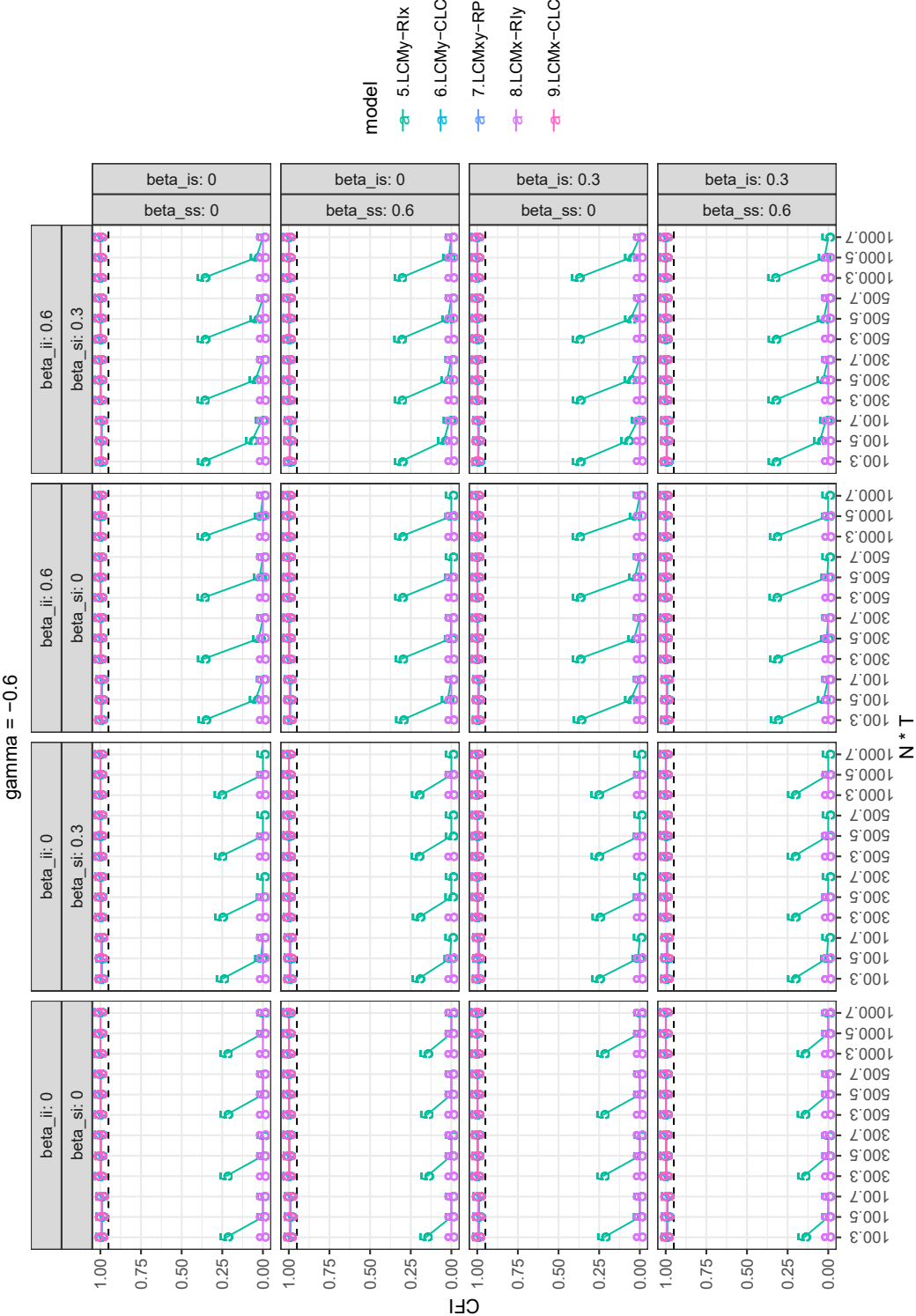


Figure D.15

CFI in Simulation 4 ($\gamma = 0$)

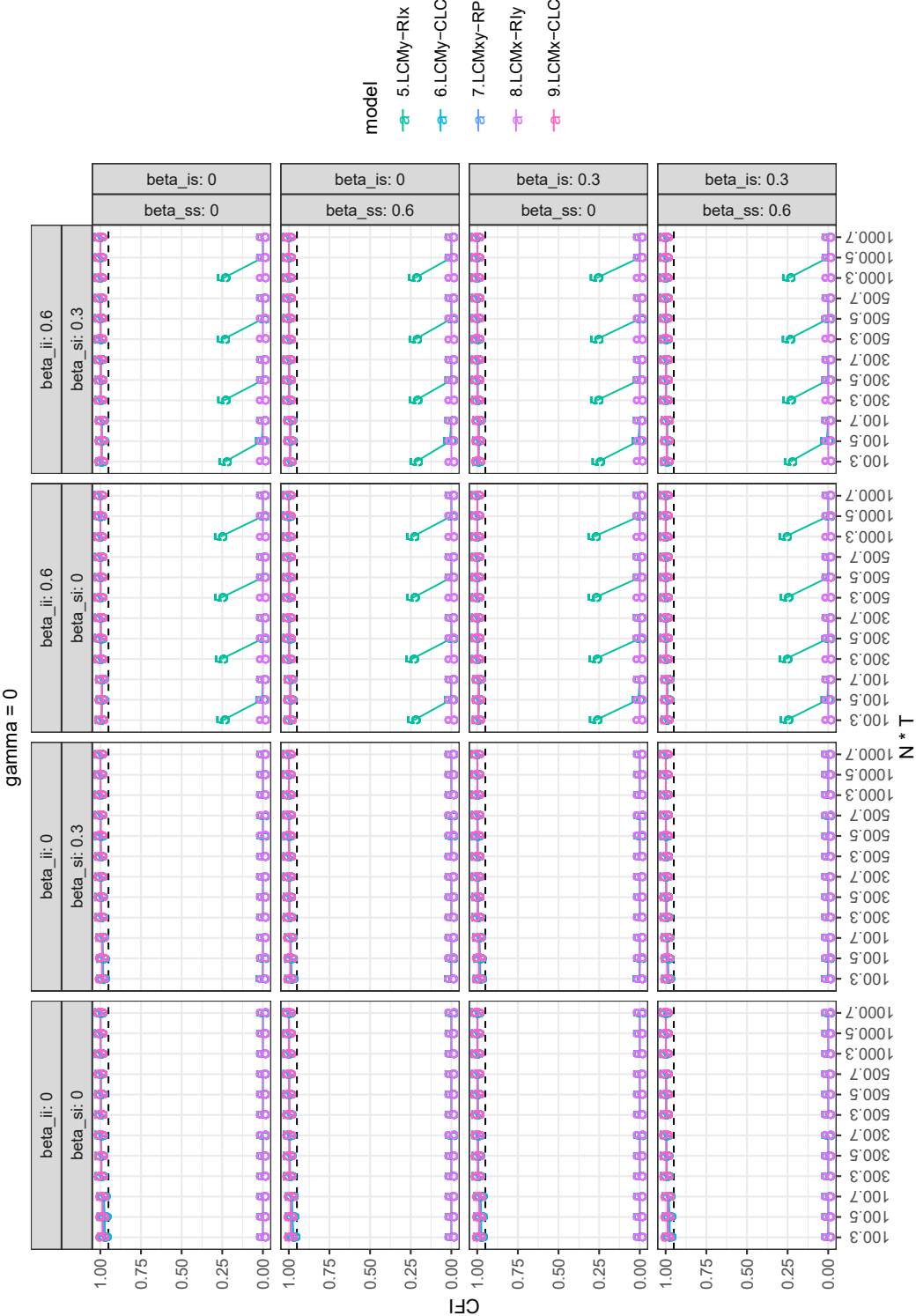


Figure D.16
CFI in Simulation 4 ($\gamma = 0.6$)

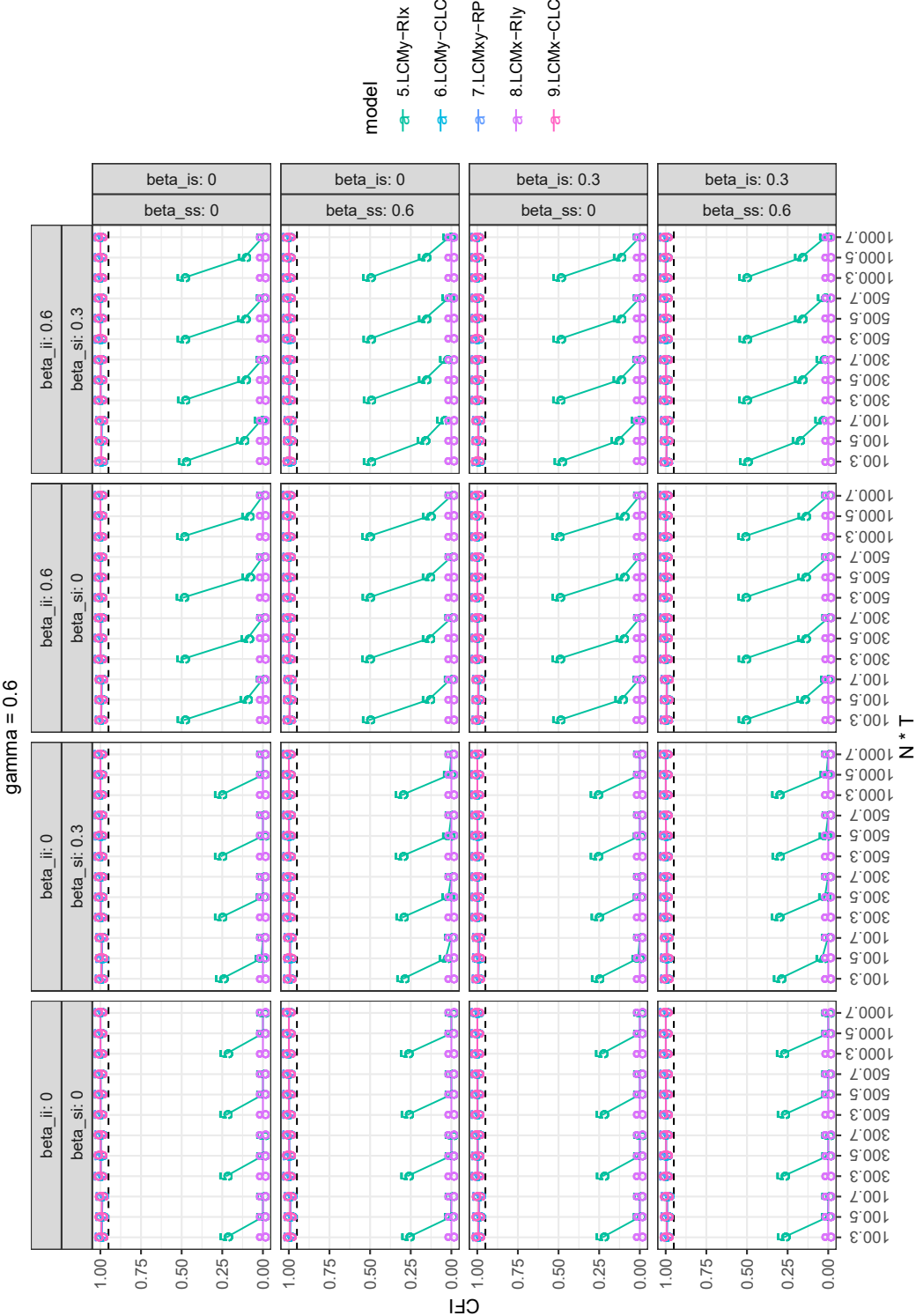


Figure D.17

RMSEA in Simulation 4 ($\gamma = -0.6$)

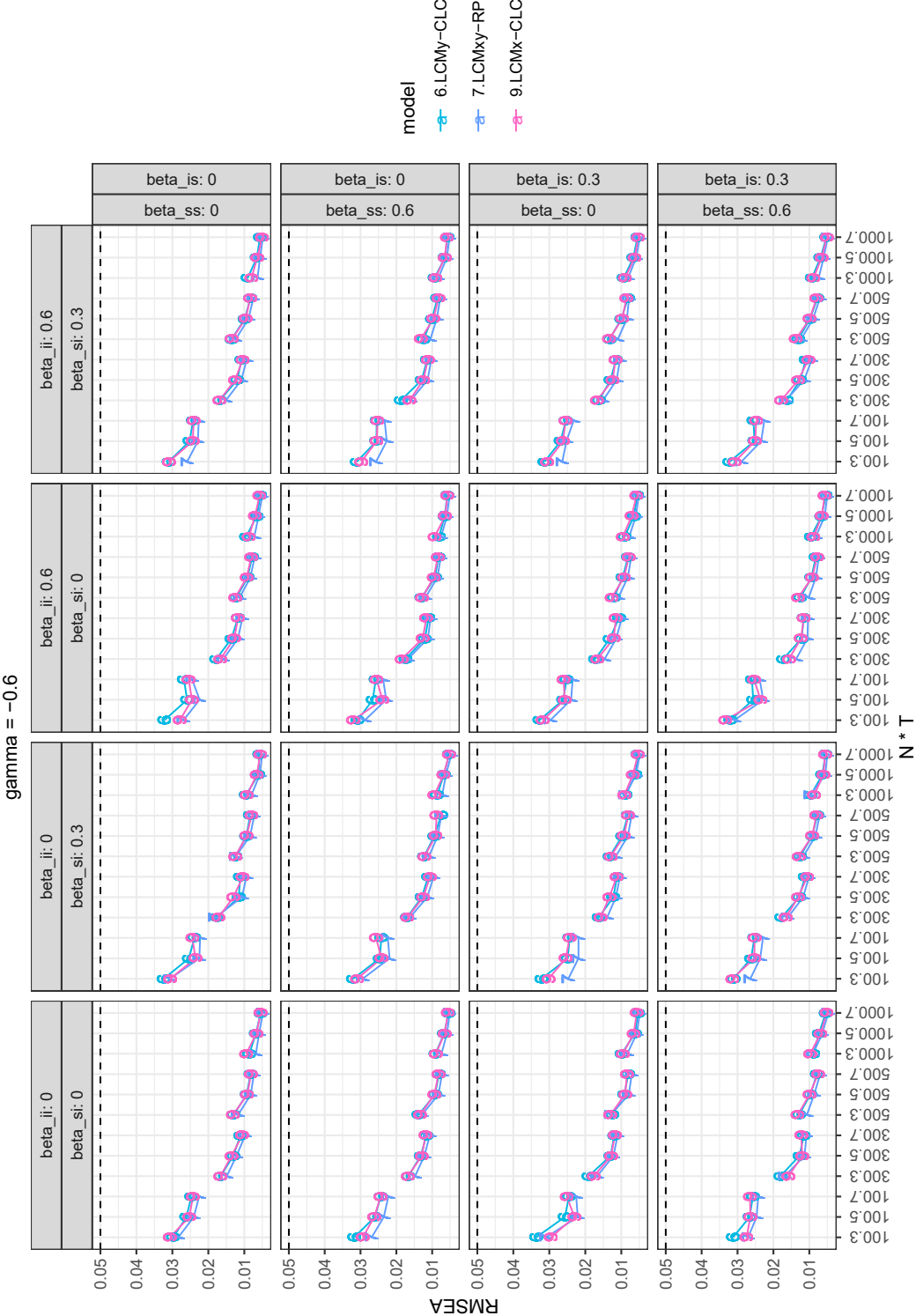


Figure D.18
RMSEA in Simulation 4 ($\gamma = 0$)

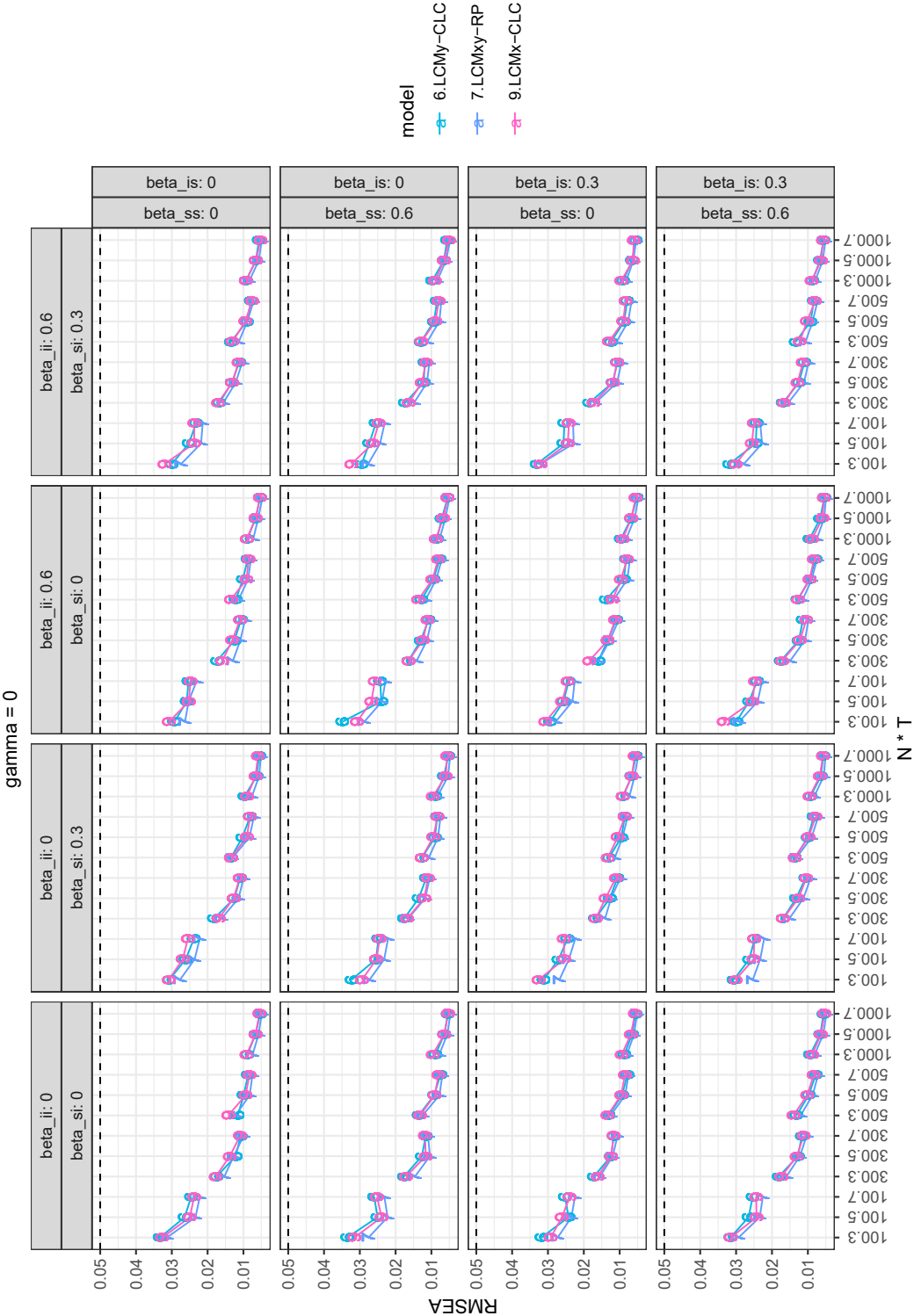


Figure D.19

RMSEA in Simulation 4 ($\gamma = 0.6$)

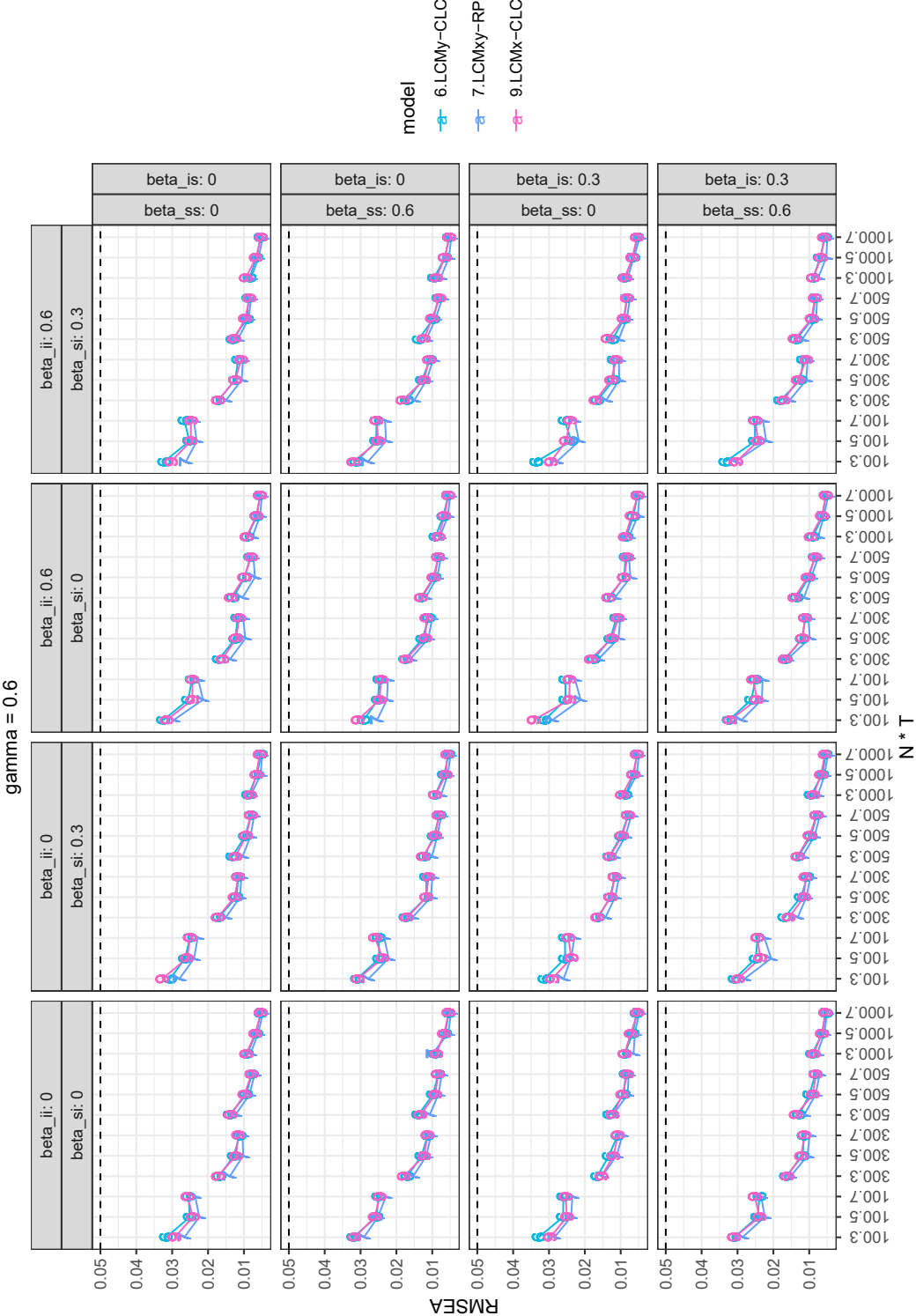


Figure D.20
RMSEA in Simulation 4 (LCMy-Rlx)



Figure D.21
RMSEA in Simulation 4 (LCMx-Rly)

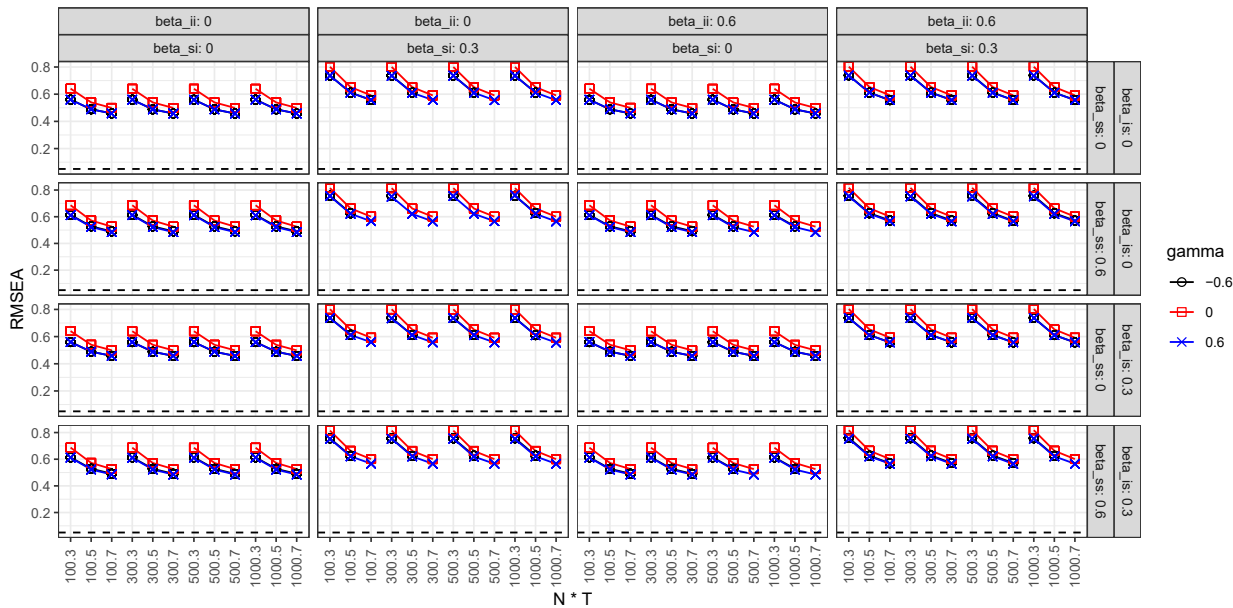


Figure D.22

SRMR in Simulation 4 ($\gamma = -0.6$)

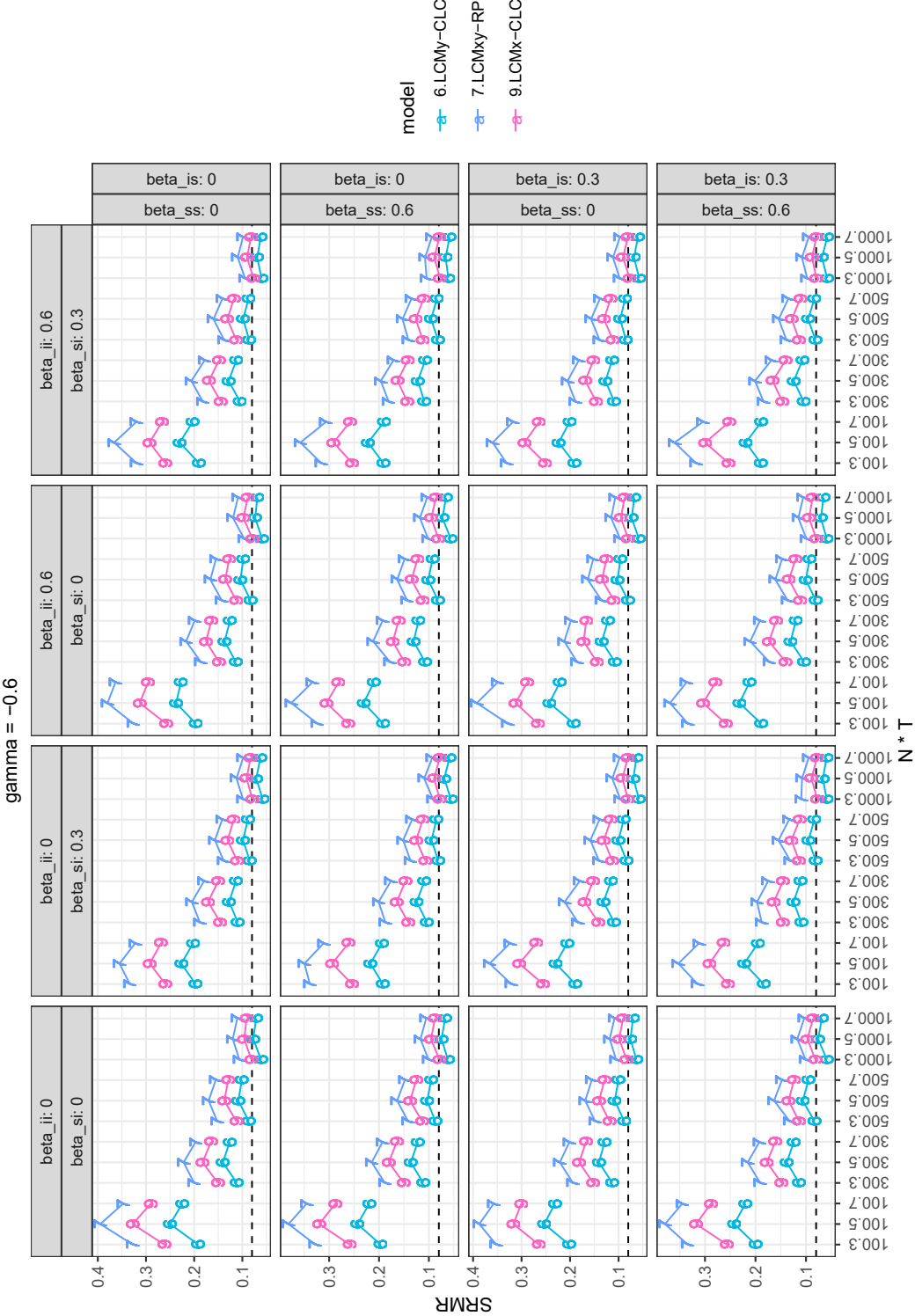


Figure D.23

SRMR in Simulation 4 ($\gamma = 0$)

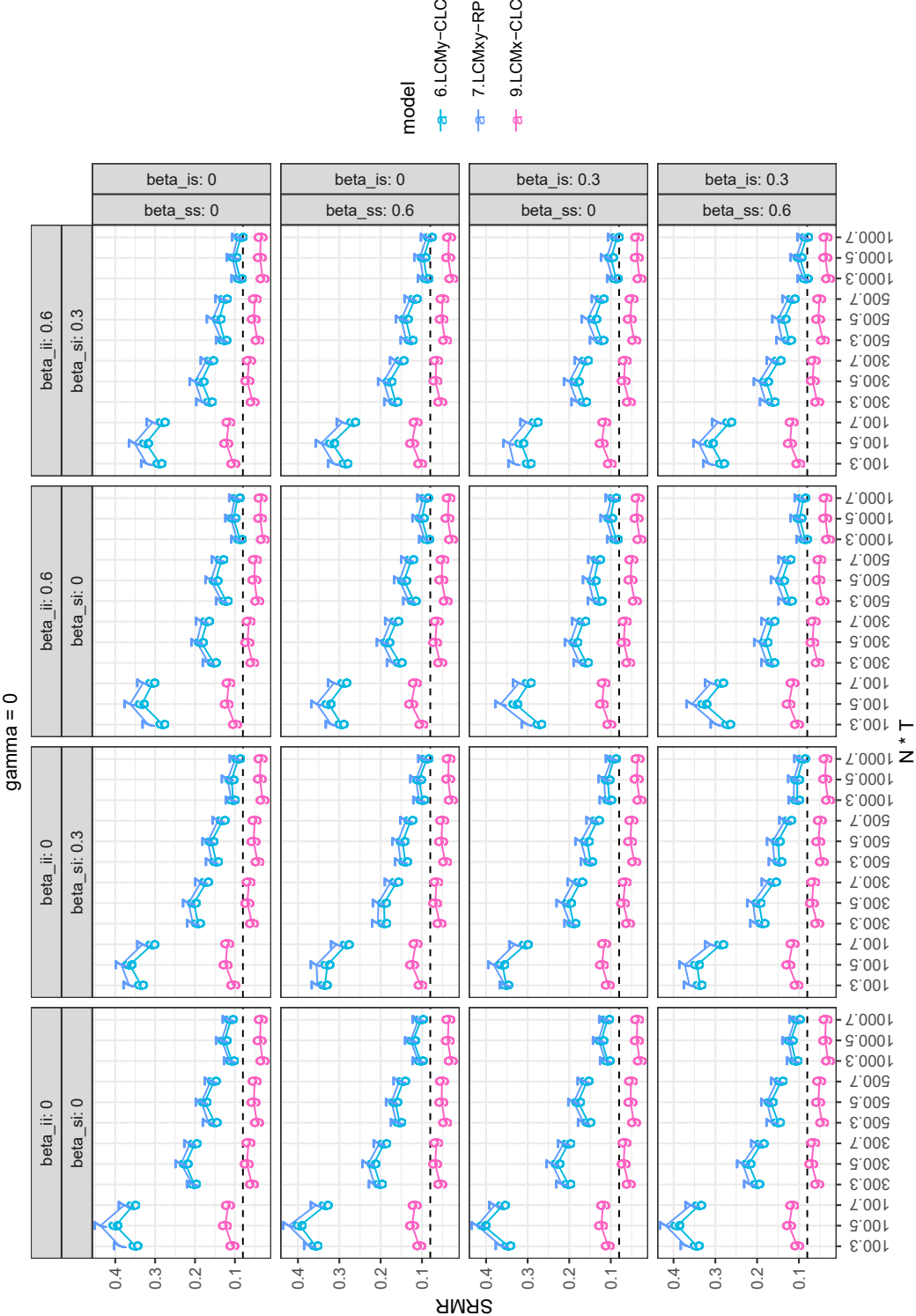


Figure D.24

SRMR in Simulation 4 ($\gamma = 0.6$)

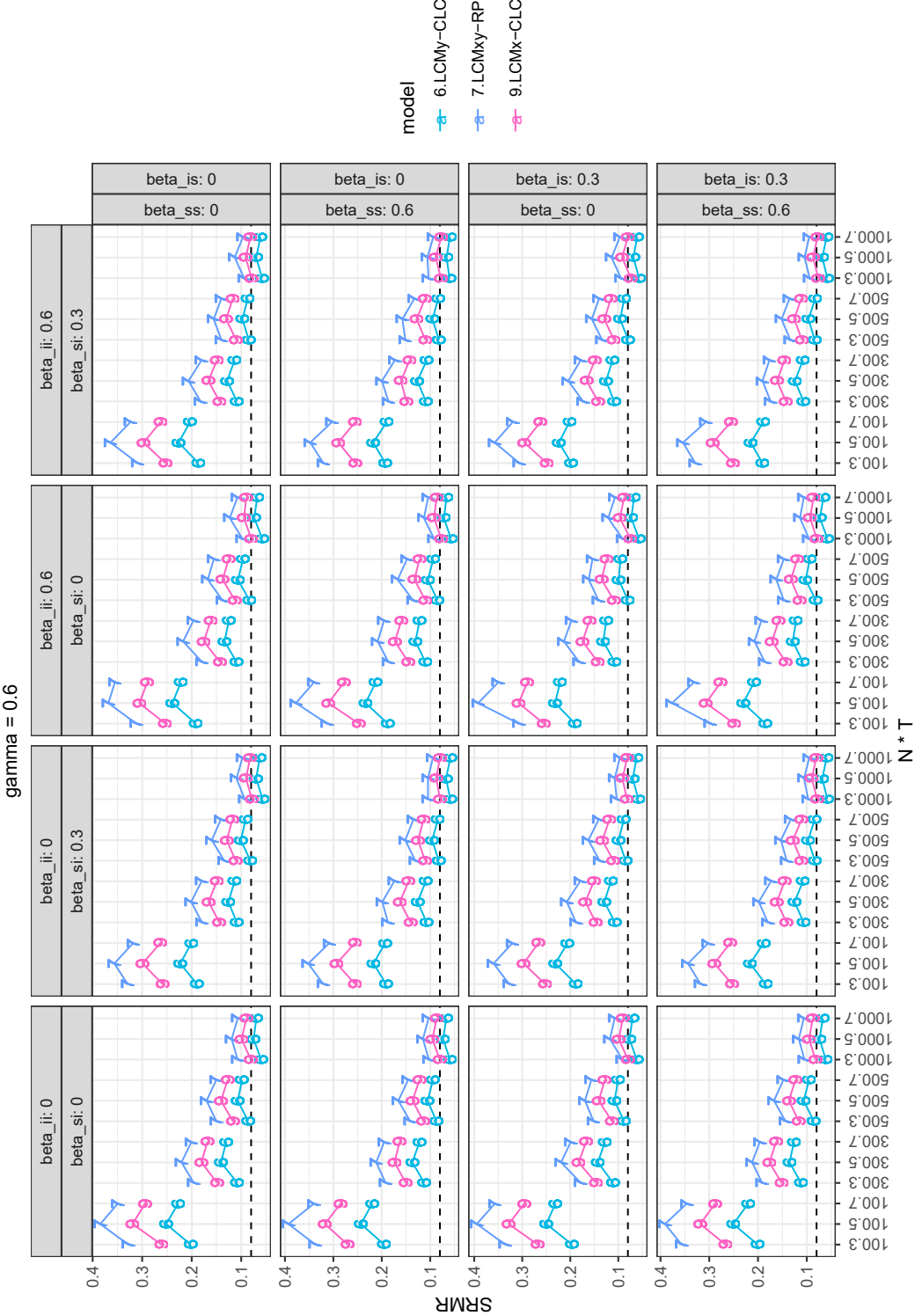


Figure D.25

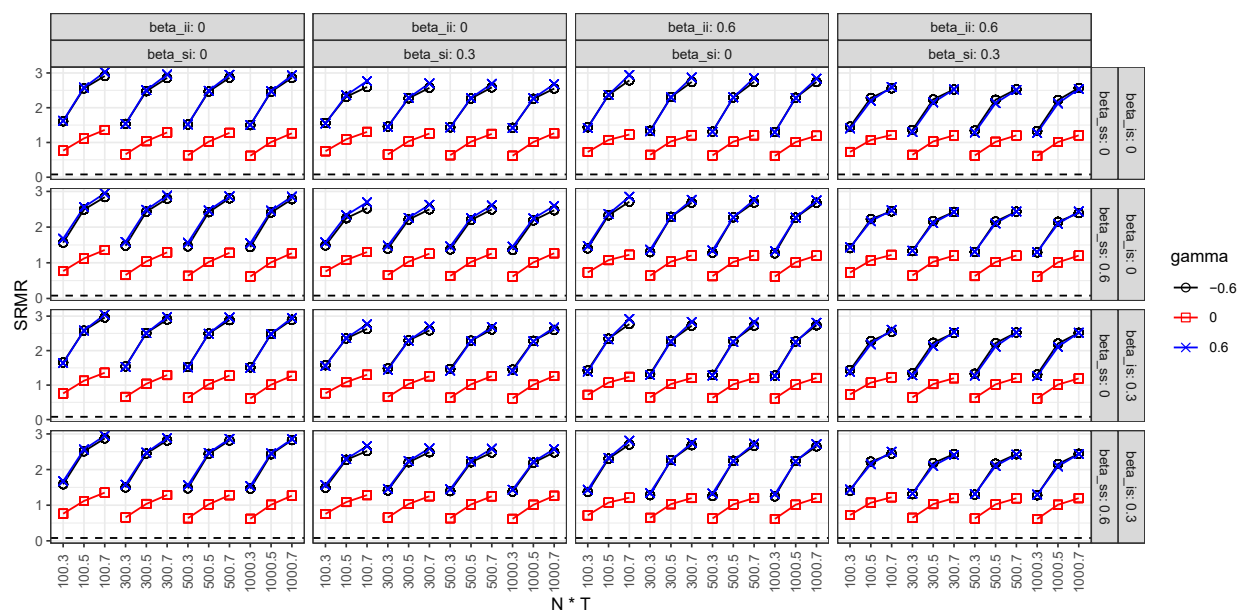
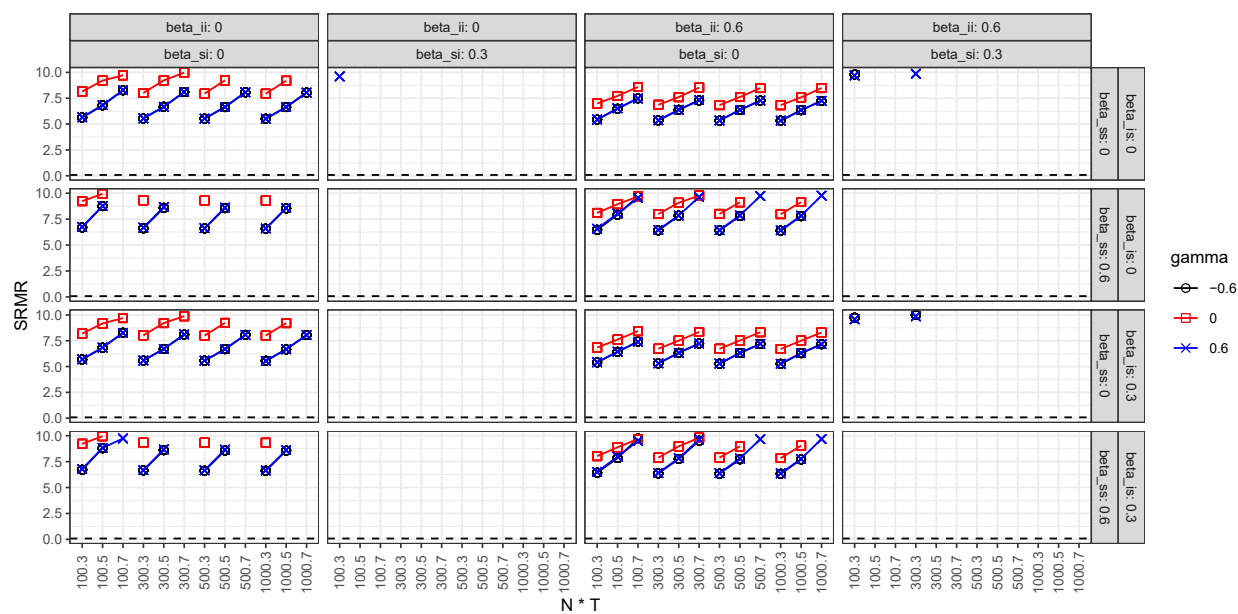
SRMR in Simulation 4 (LCMy-Rlx)

Figure D.26

SRMR in Simulation 4 (LCMx-Rly)

D.2 Within-Person Effect

Figure D.27

Bias for the Within-Person Effect γ in Simulation 4 (MLM-WB)

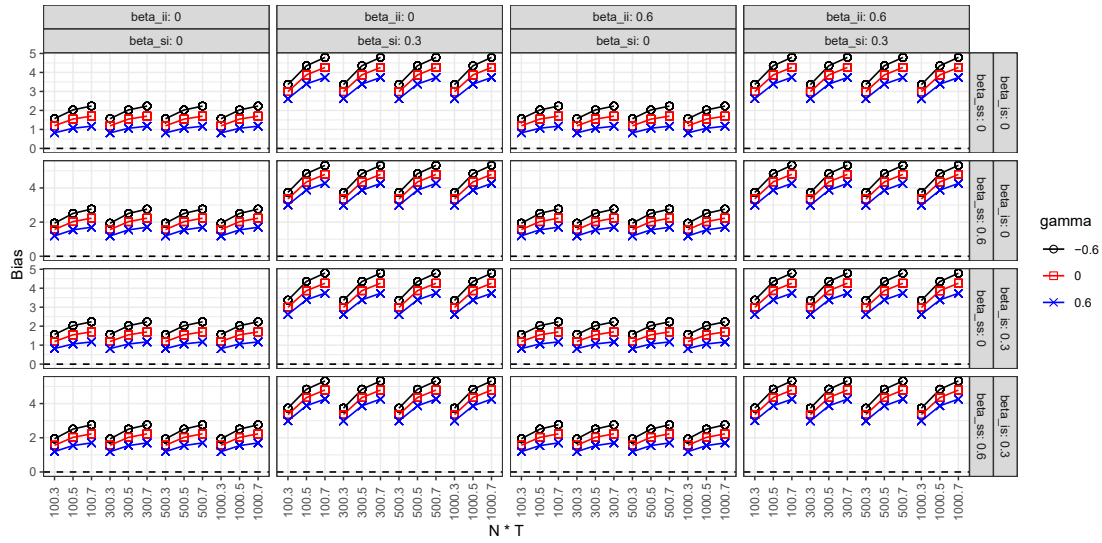


Figure D.28

Bias for the Within-Person Effect γ in Simulation 4 (MLM-E2S)

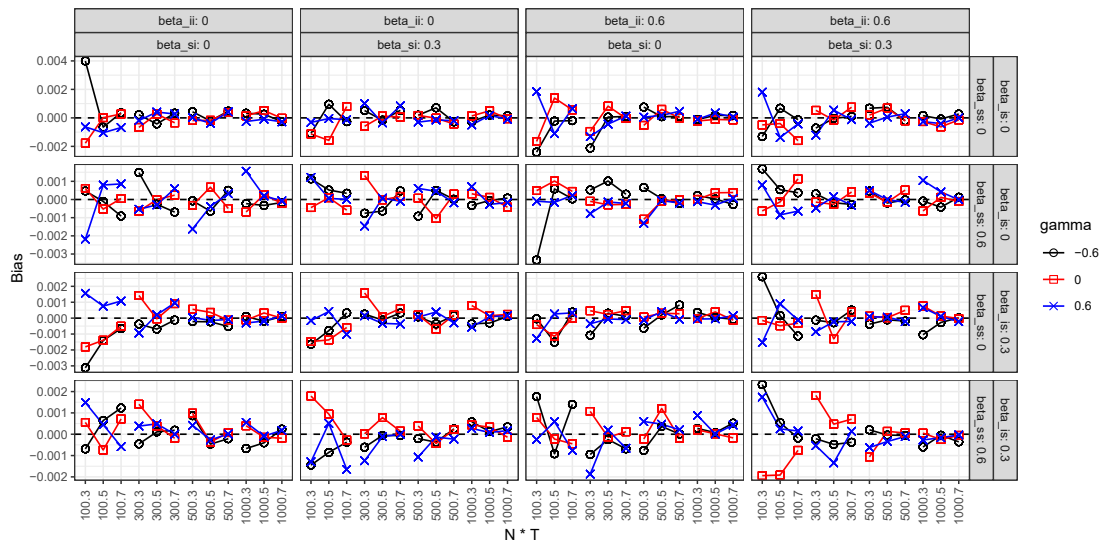


Figure D.29

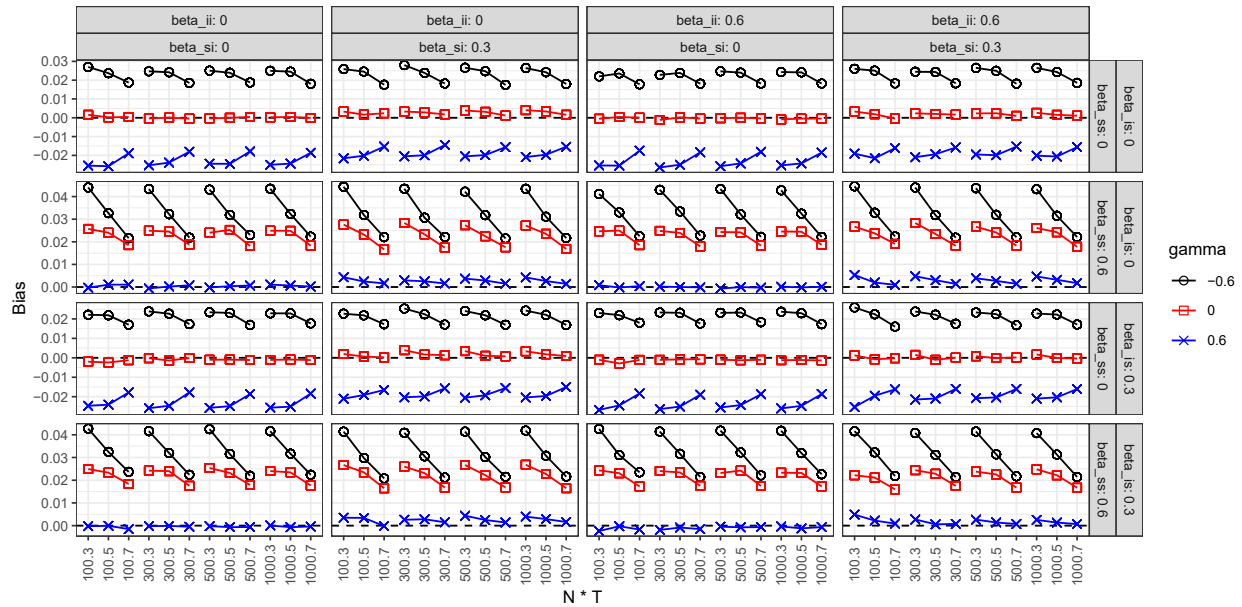
Bias for the Within-Person Effect γ in Simulation 4 (MLM-TAC)

Figure D.30

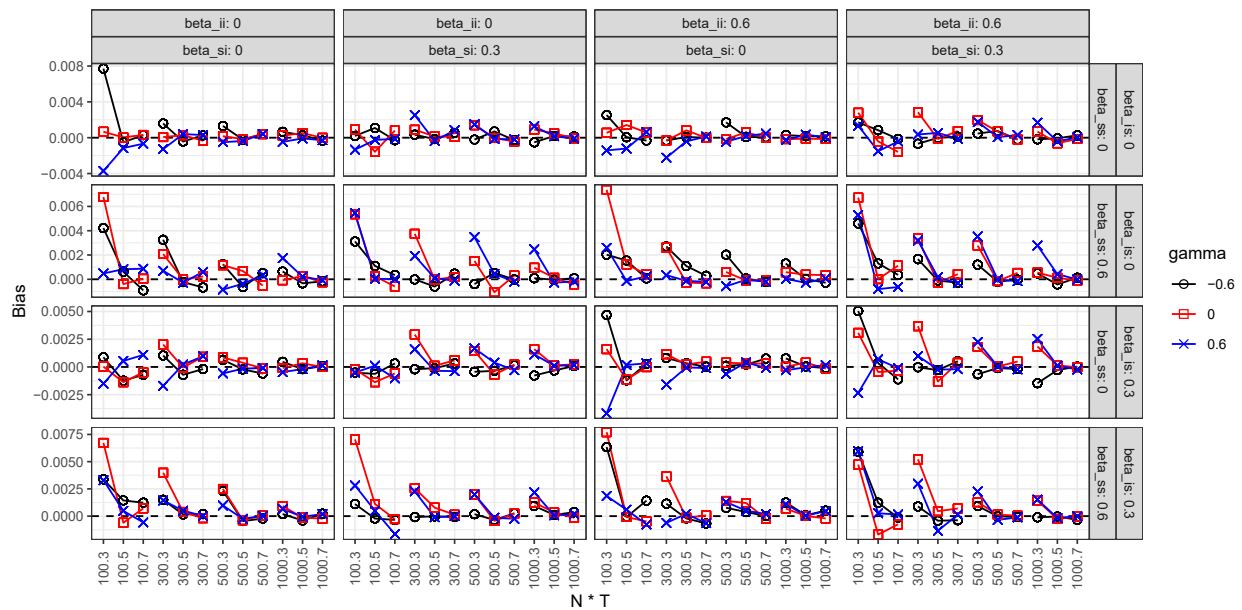
Bias for the Within-Person Effect γ in Simulation 4 (MLM-MV)

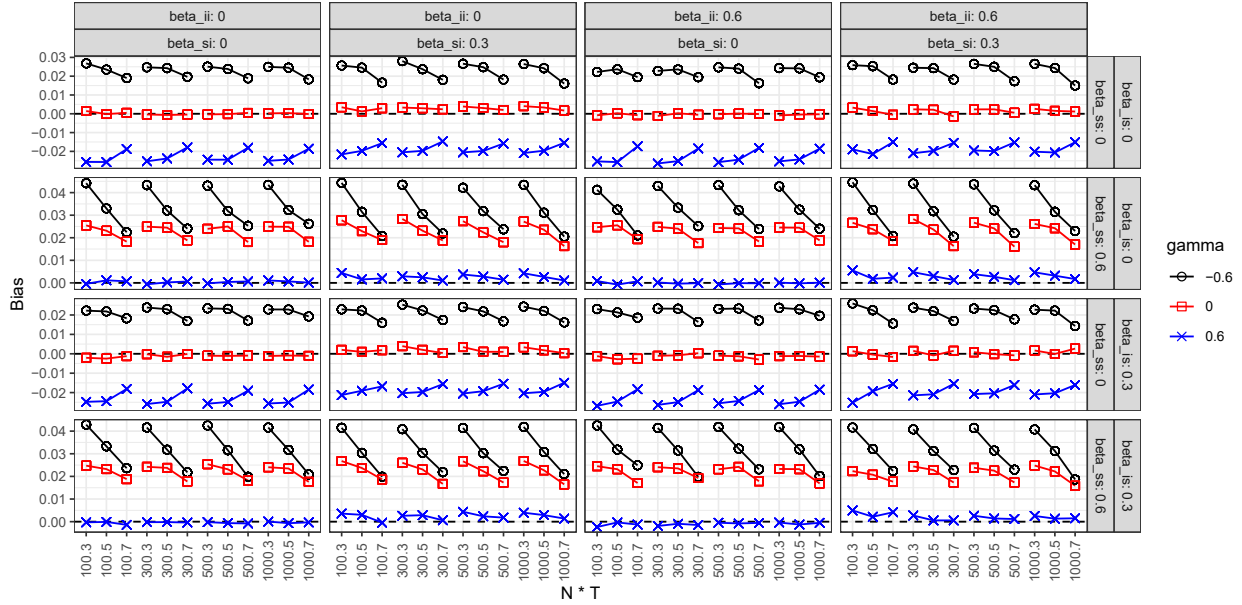
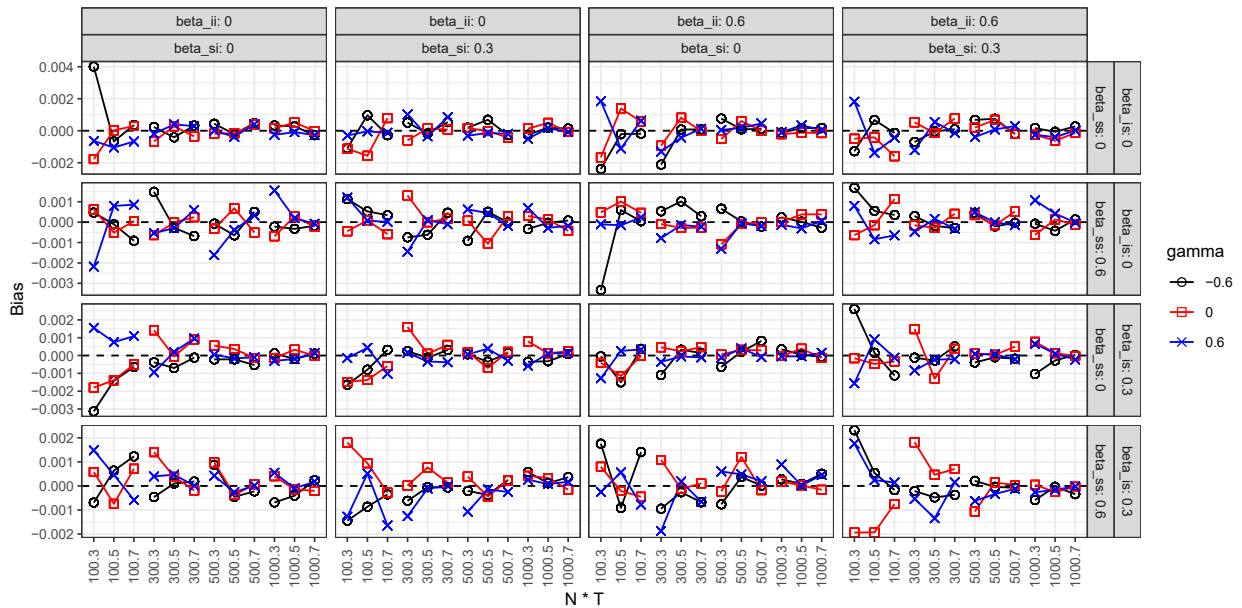
Figure D.31*Bias for the Within-Person Effect γ in Simulation 4 (LCMy-RLx)***Figure D.32***Bias for the Within-Person Effect γ in Simulation 4 (LCMy-CLC)*

Figure D.33

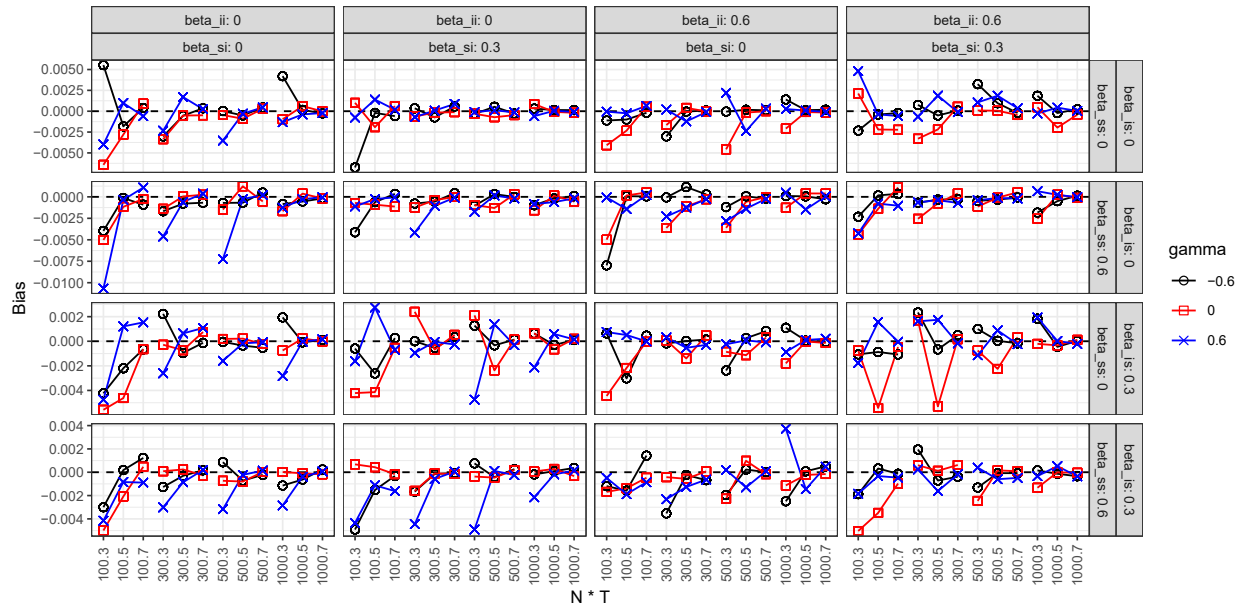
Bias for the Within-Person Effect γ in Simulation 4 (LCMxy-RP)

Figure D.34

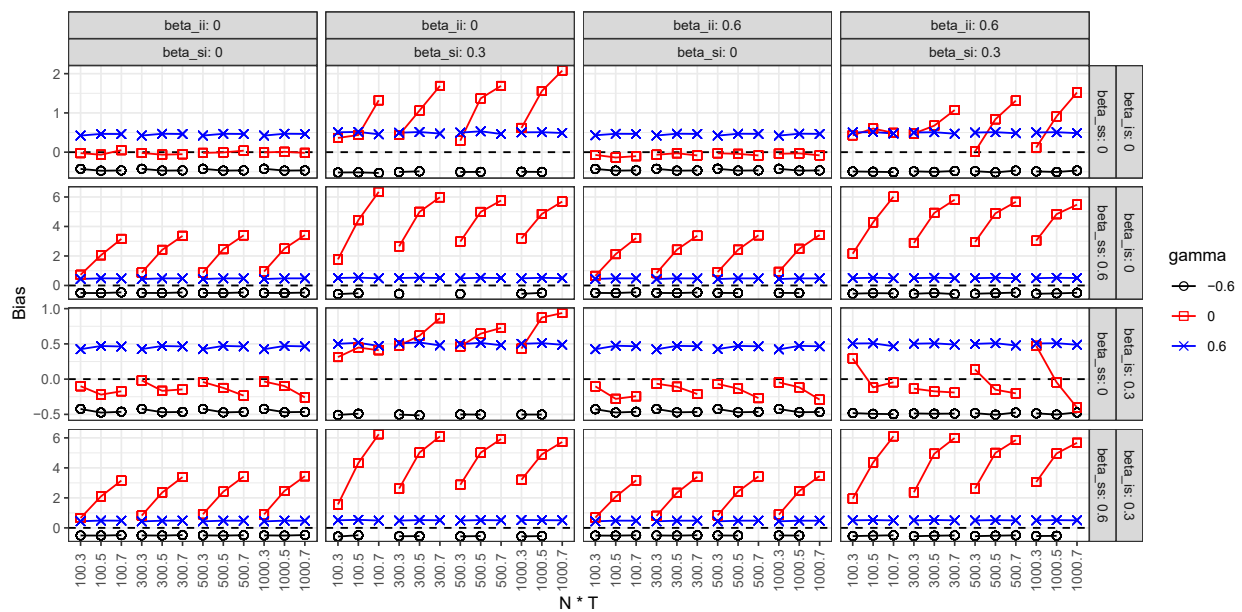
Bias for the Within-Person Effect γ in Simulation 4 (LCMx-Rly)

Figure D.35

Bias for the Within-Person Effect γ in Simulation 4 (LCMx-CLC)

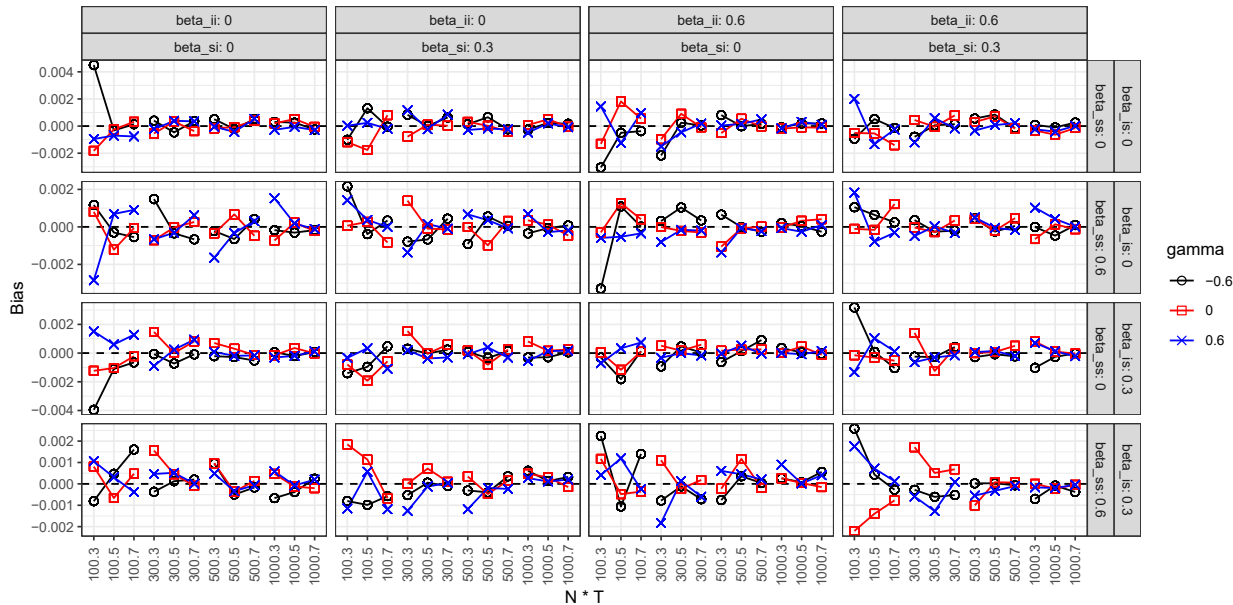


Figure D.36
Empirical SE for the Within-Person Effect γ in Simulation 4 (LCMx-Rly)

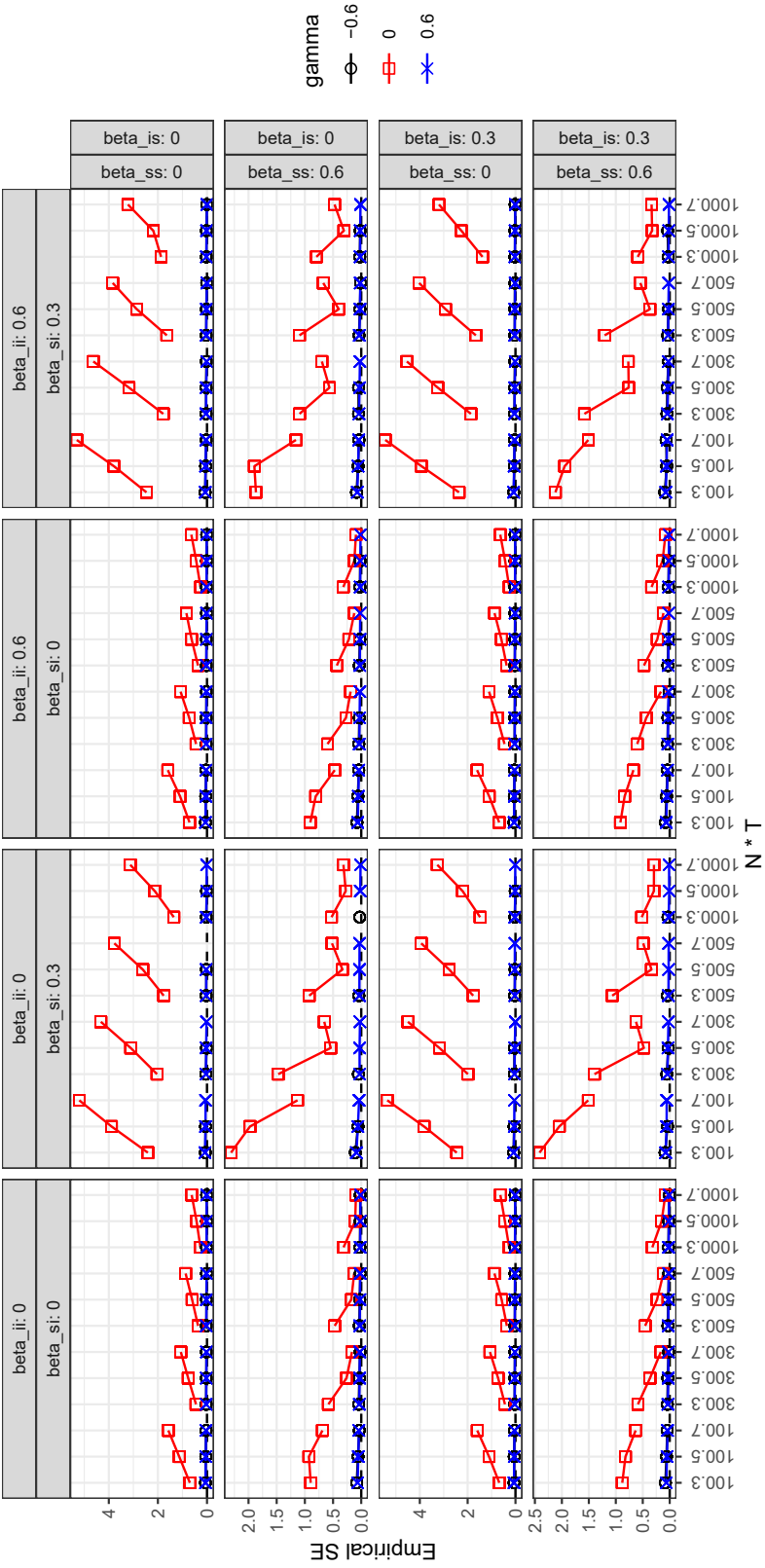


Figure D.37

Empirical SE for the Within-Person Effect γ in Simulation 4 Except for LCMx-Rly ($\gamma = -0.6$)

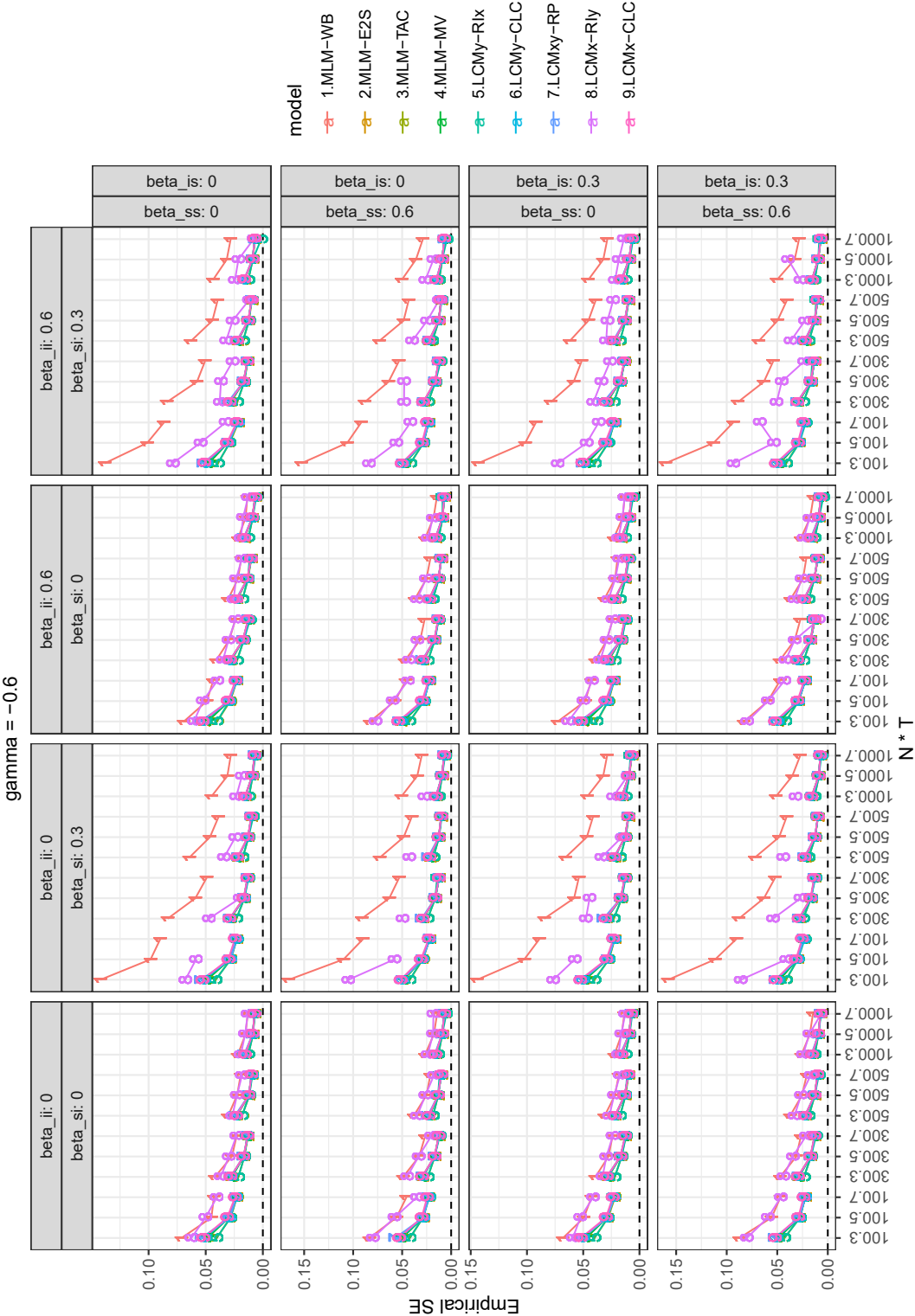


Figure D.38

Empirical SE for the Within-Person Effect γ in Simulation 4 for LCMx-Rly ($\gamma = 0$)

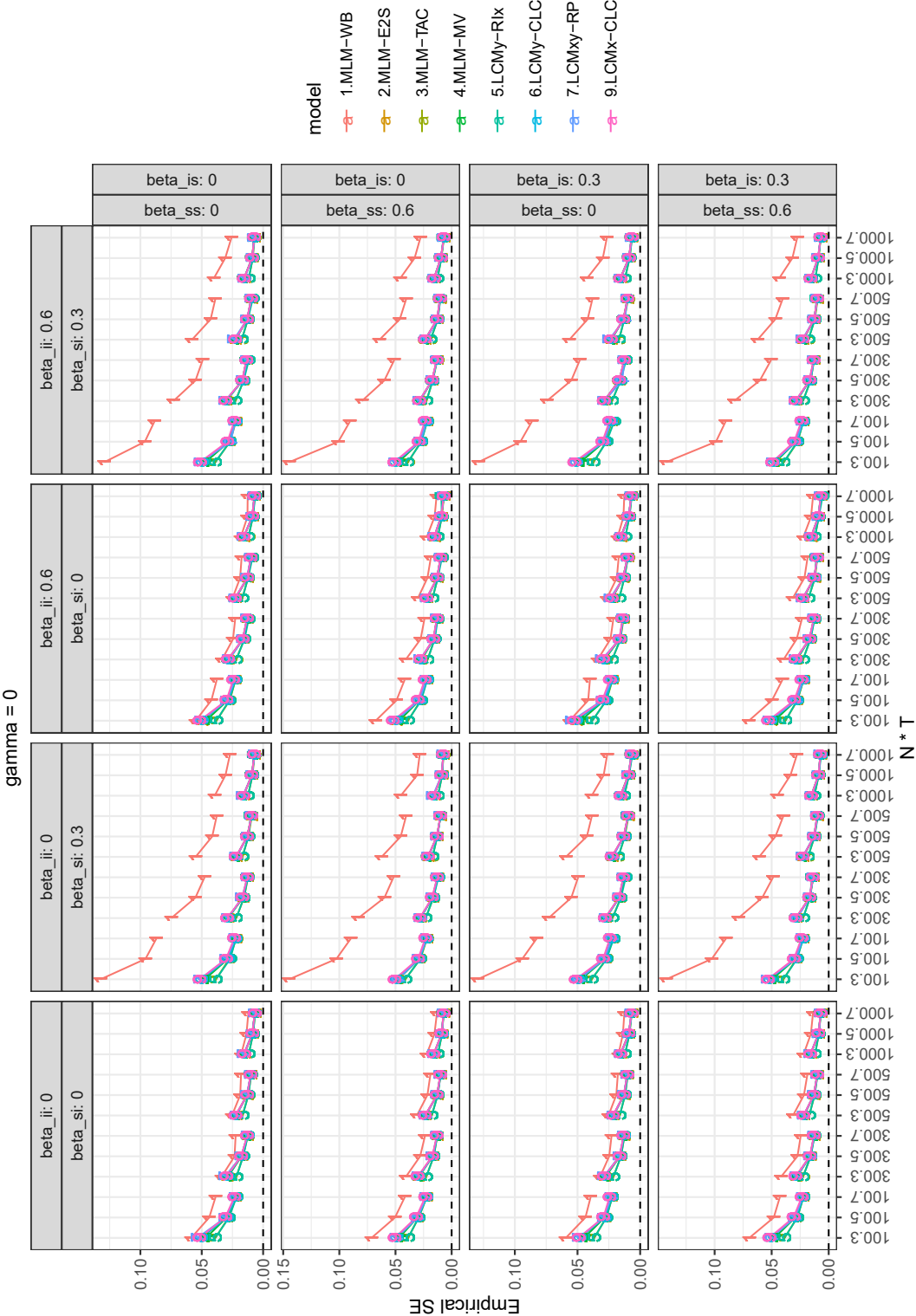


Figure D.39

Empirical SE for the Within-Person Effect γ in Simulation 4 for LCMx-Rly ($\gamma = 0.6$)

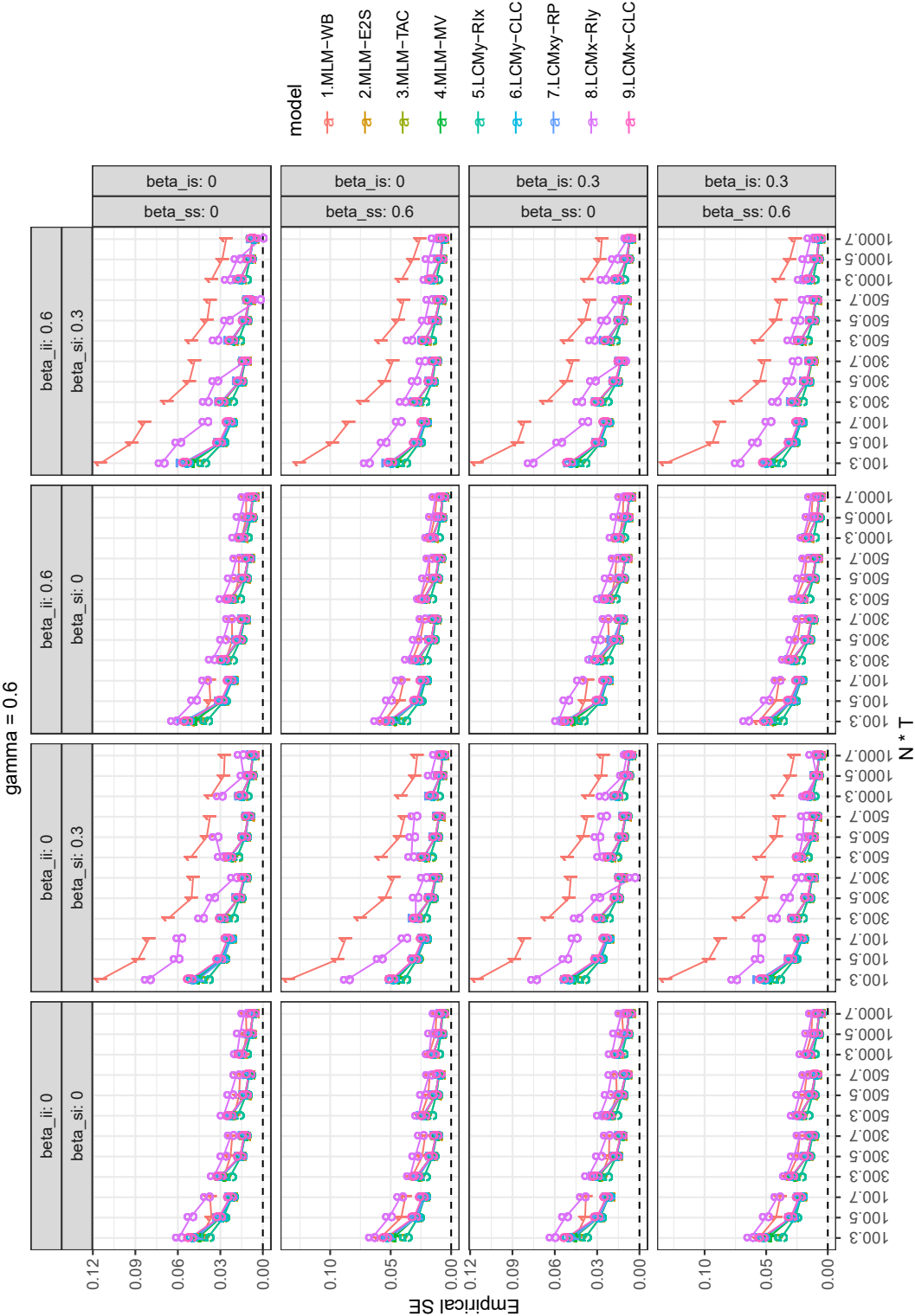


Figure D.40
Theoretical SE for the Within-Person Effect γ in Simulation 4 (LCMx-Rly)

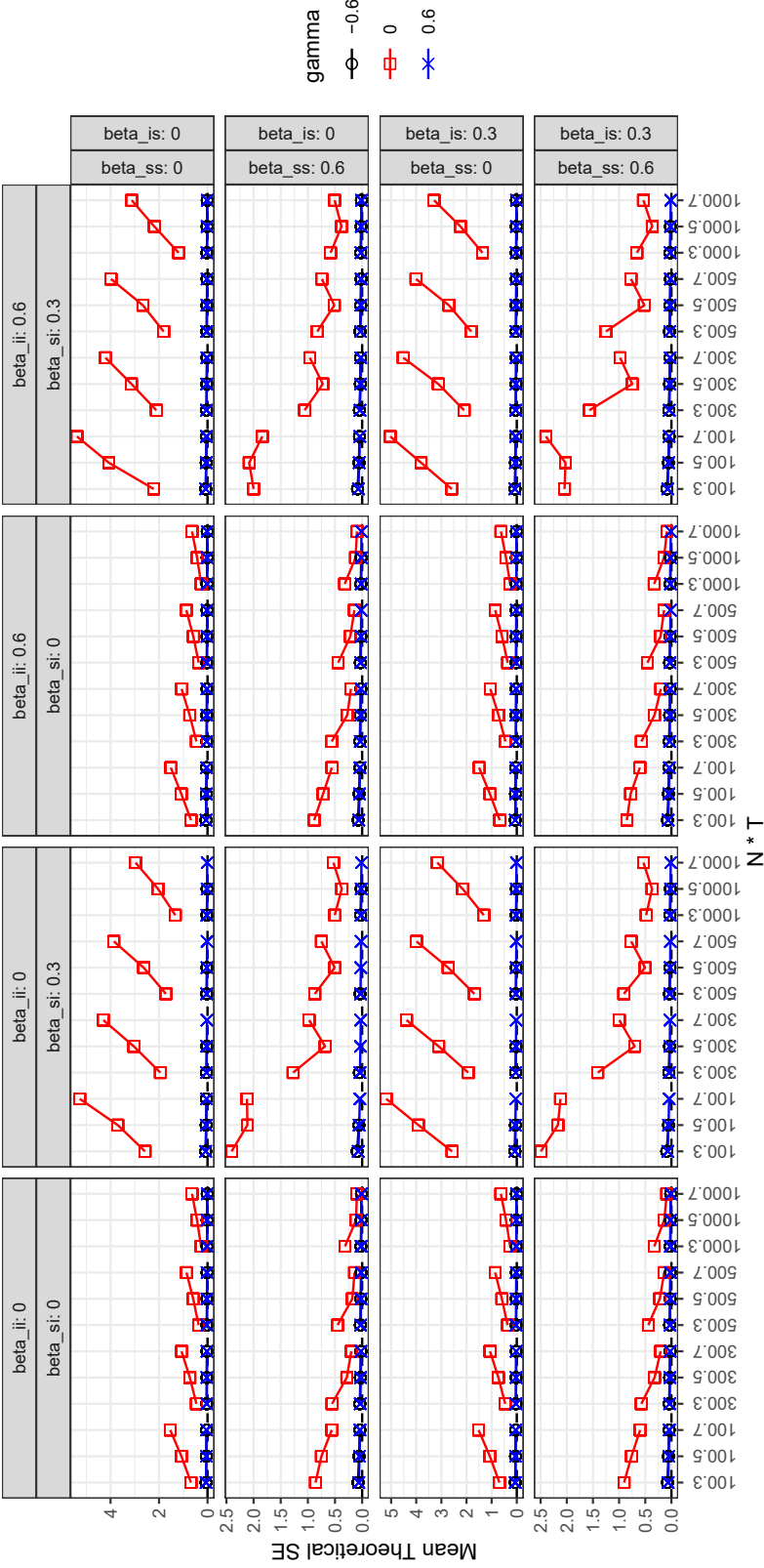


Figure D.41

Theoretical SE for the Within-Person Effect γ in Simulation 4 for LCMx-Rly ($\gamma = -0.6$)

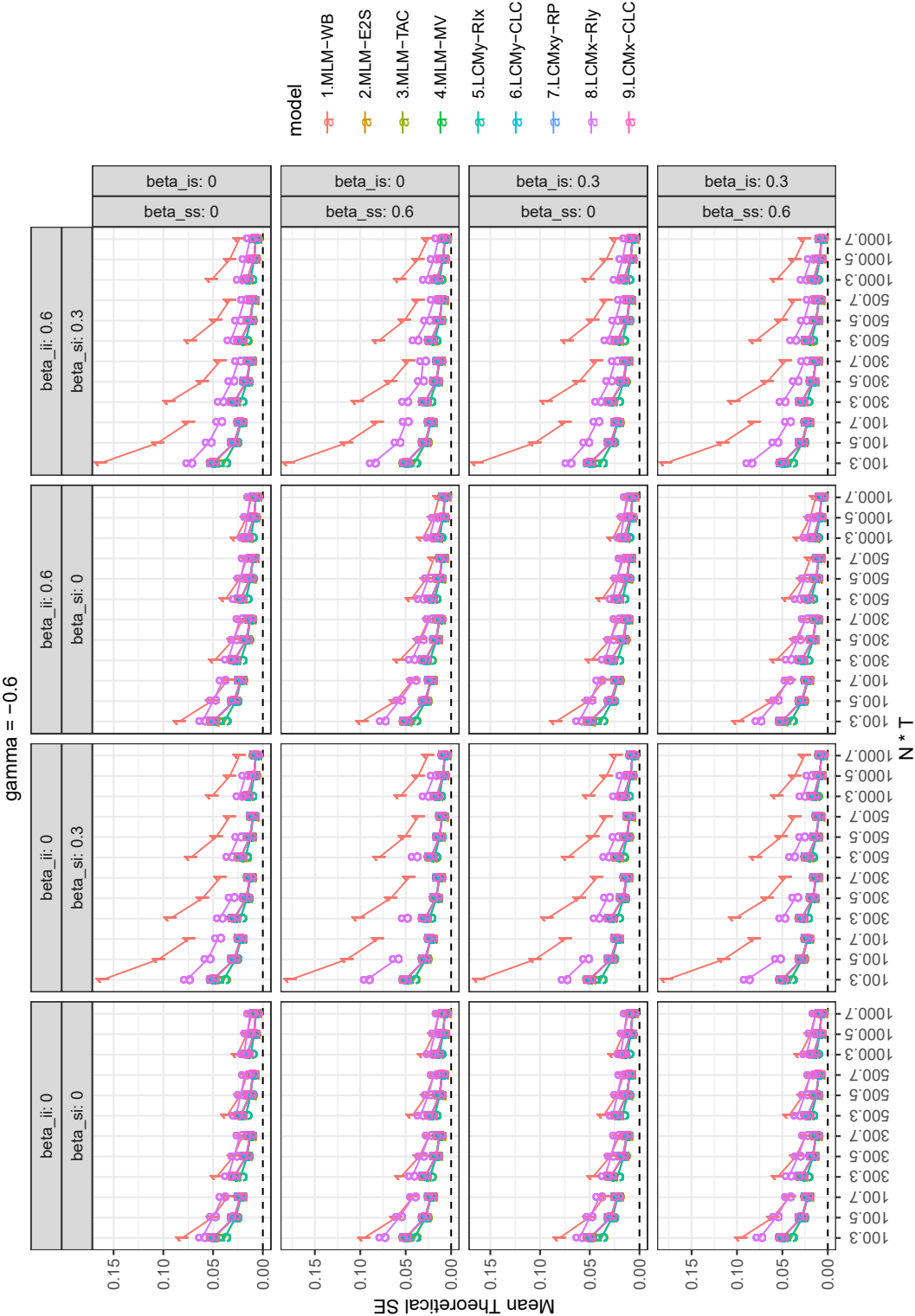
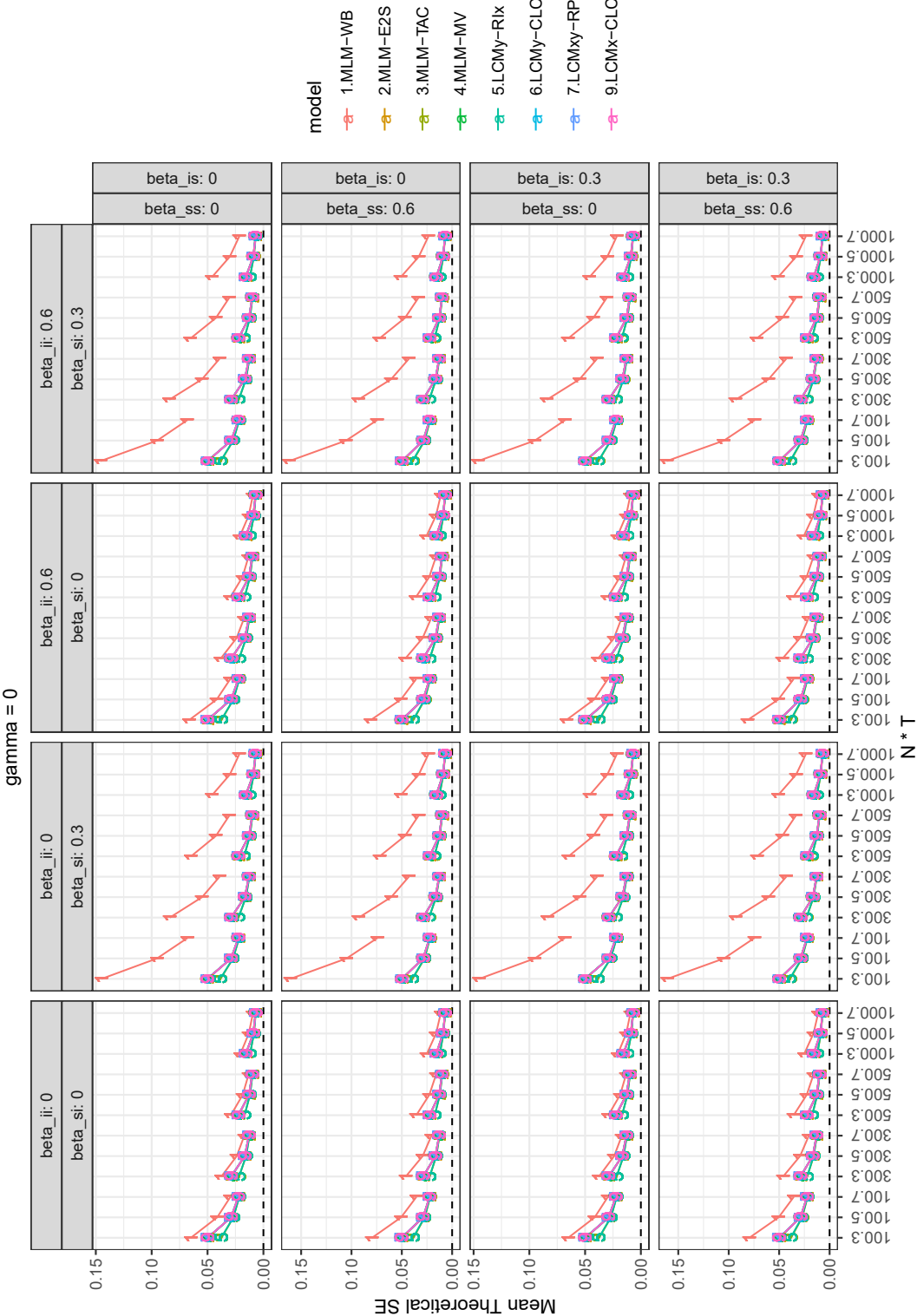


Figure D.42

Theoretical SE for the Within-Person Effect γ in Simulation 4 for LCMx-Rly ($\gamma = 0$)



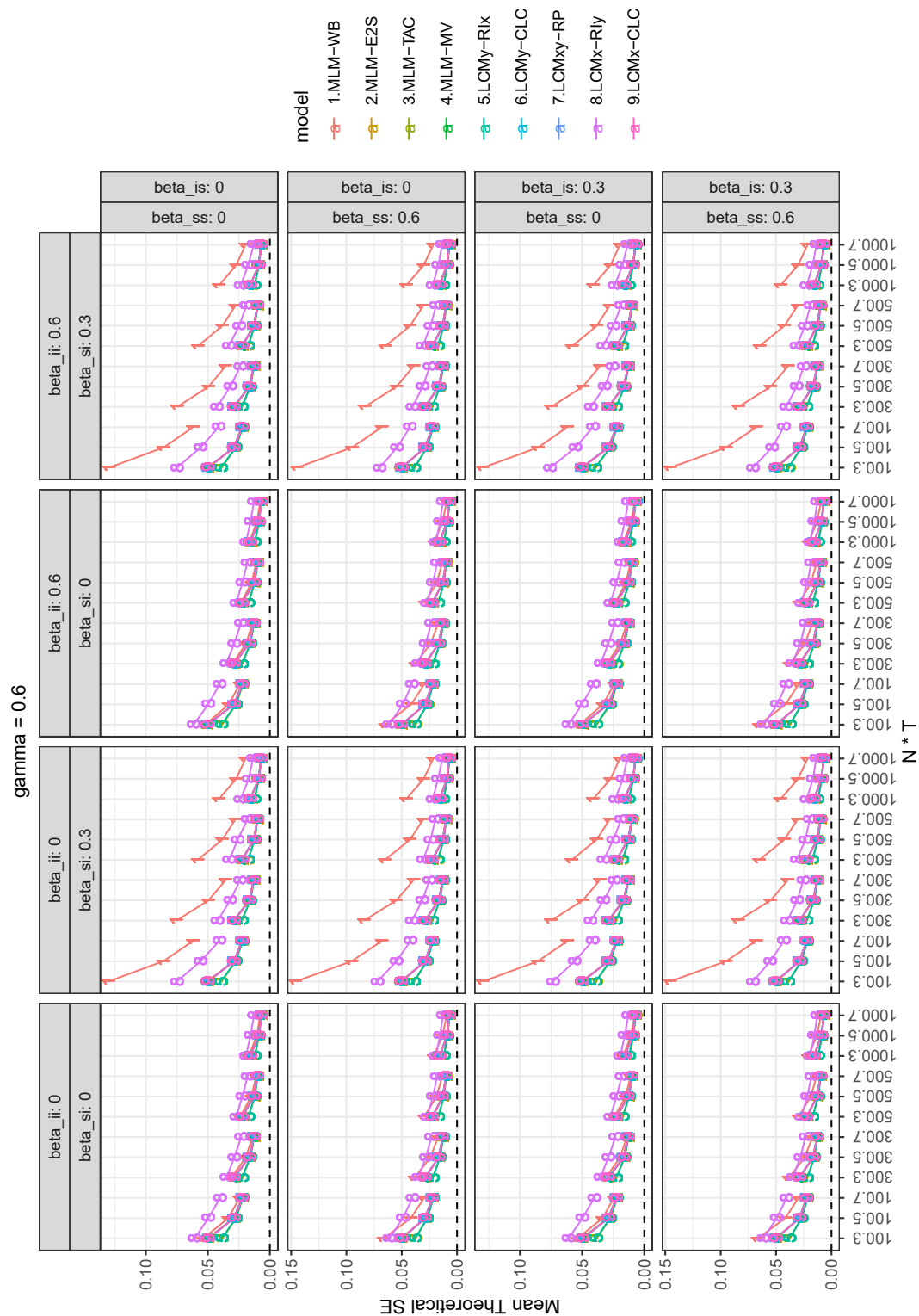
Theoretical SE for the Within-Person Effect γ in Simulation 4 for LCMx-Rly ($\gamma = 0.6$)

Figure D.44

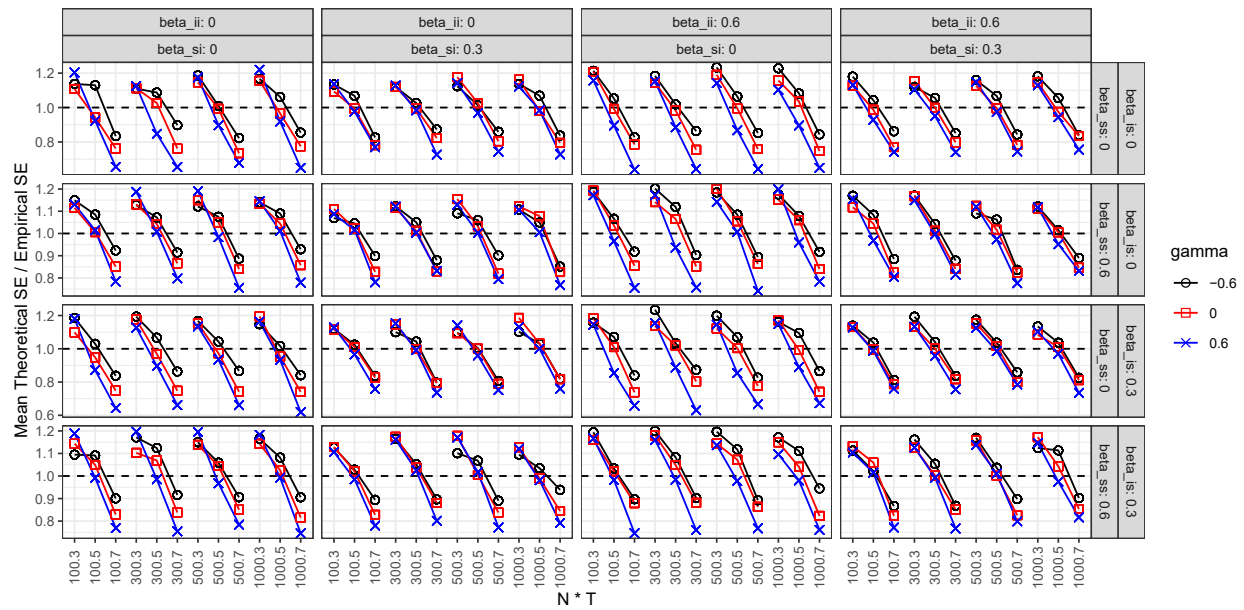
SE Ratio for the Within-Person Effect γ in Simulation 4 (MLM-WB)

Figure D.45

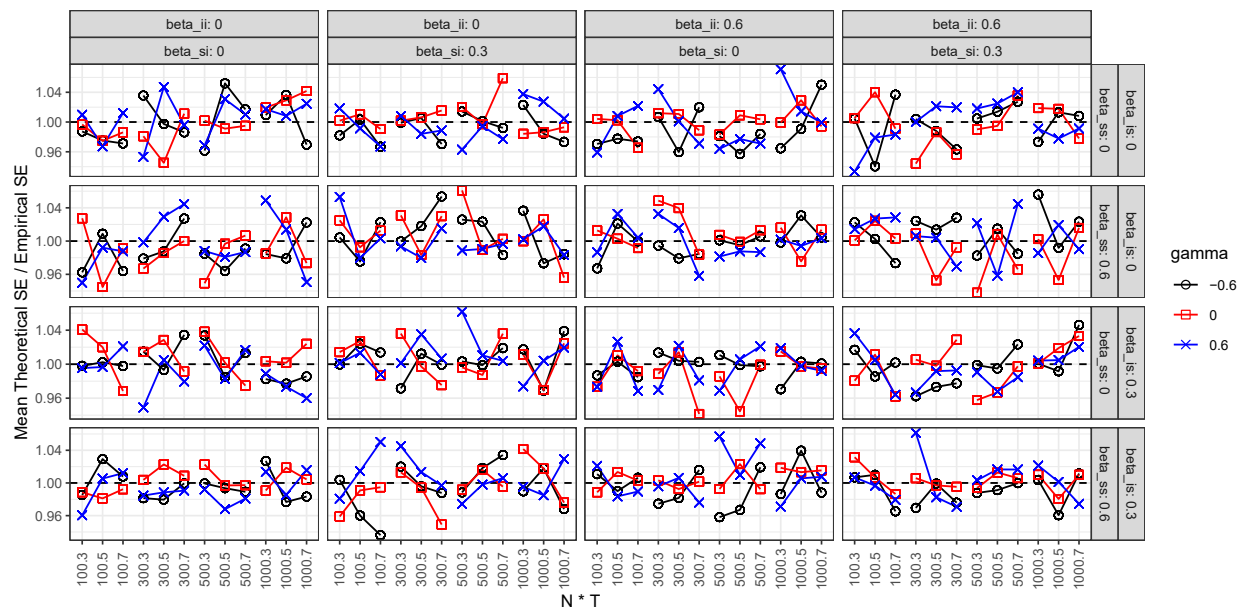
SE Ratio for the Within-Person Effect γ in Simulation 4 (MLM-E2S)

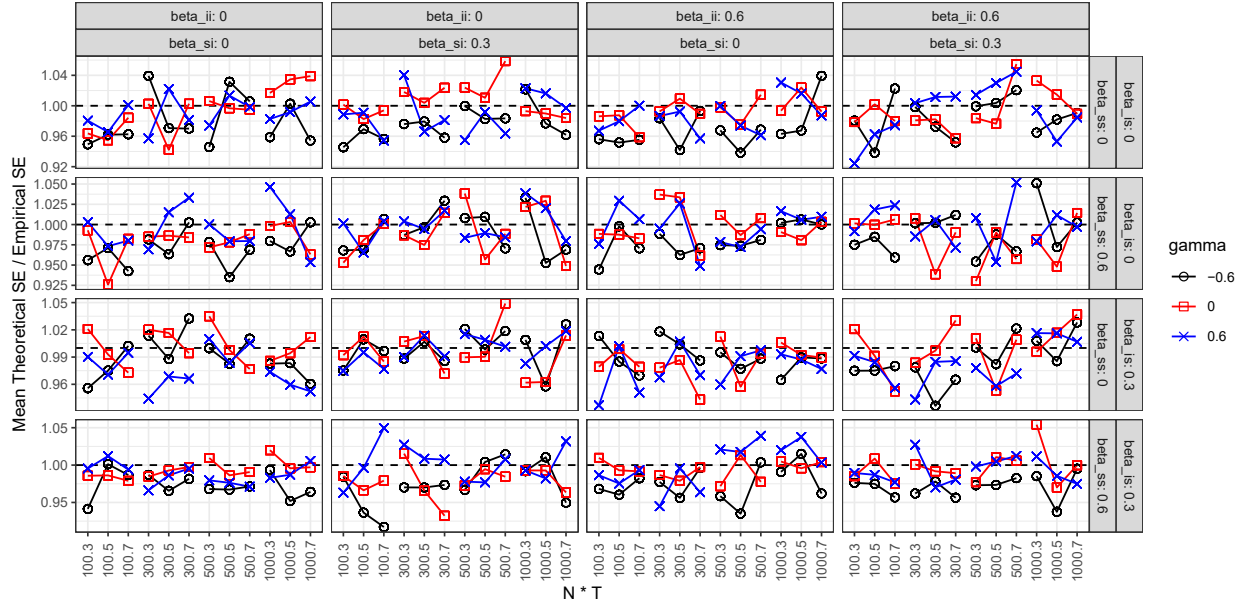
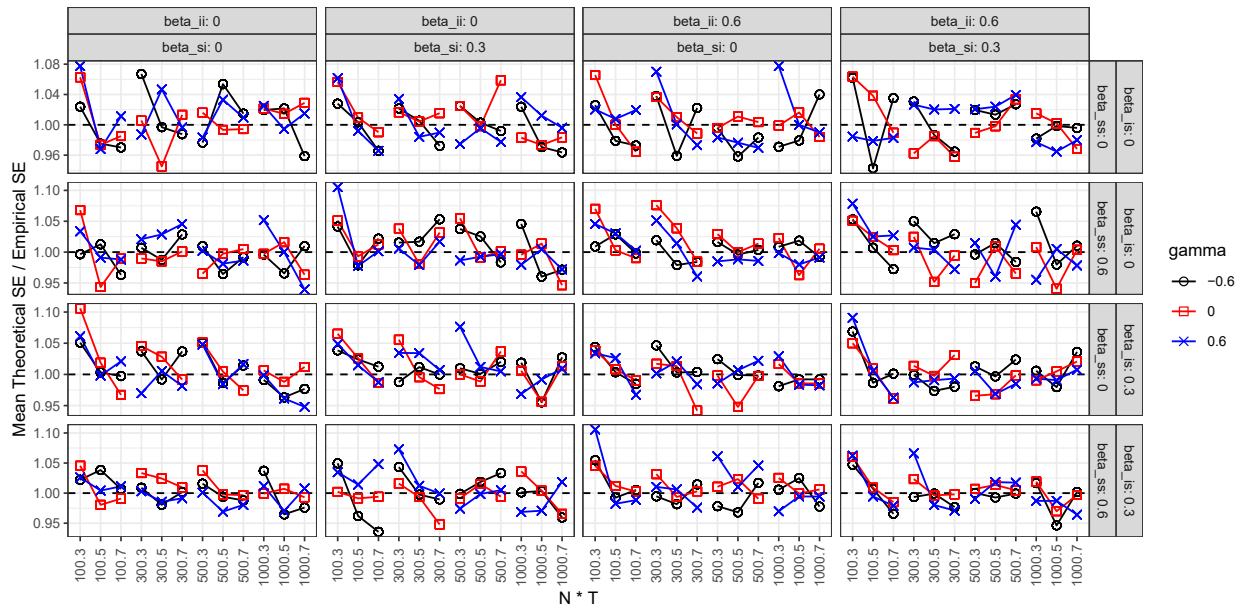
Figure D.46*SE Ratio for the Within-Person Effect γ in Simulation 4 (MLM-TAC)***Figure D.47***SE Ratio for the Within-Person Effect γ in Simulation 4 (MLM-MV)*

Figure D.48

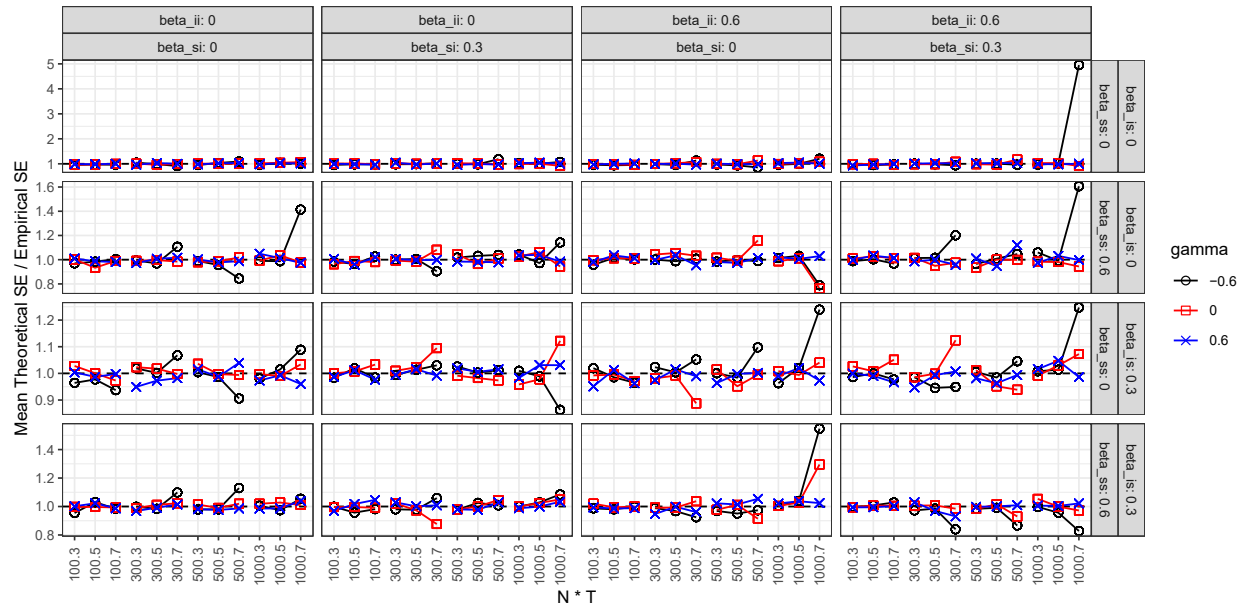
SE Ratio for the Within-Person Effect γ in Simulation 4 (LCMy-RIx)

Figure D.49

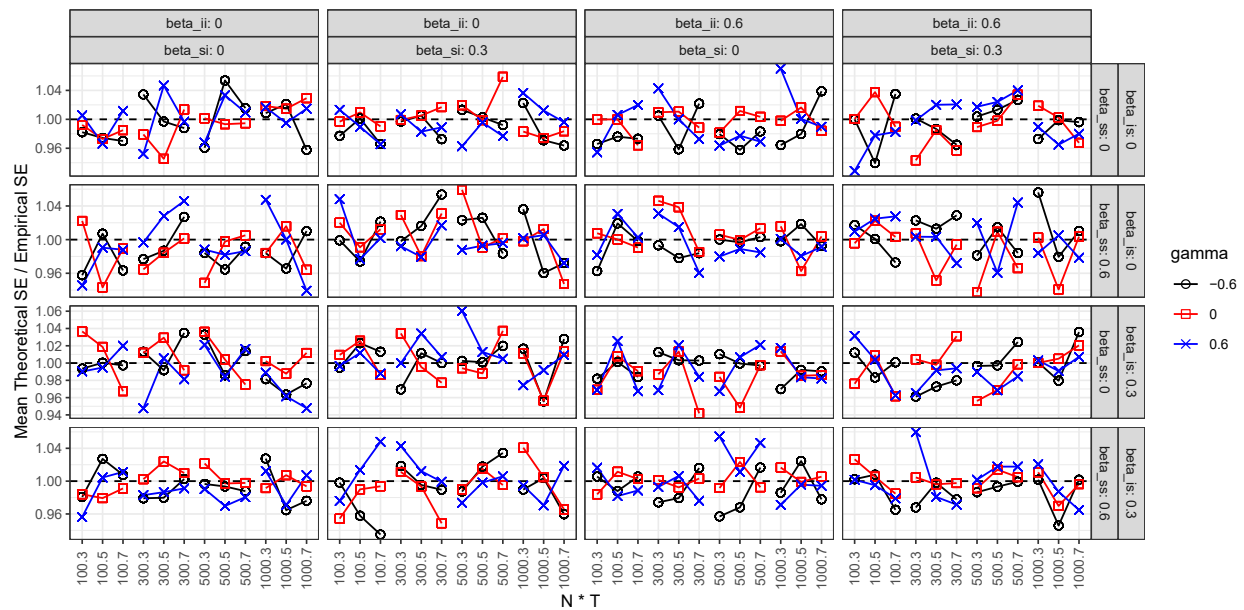
SE Ratio for the Within-Person Effect γ in Simulation 4 (LCMy-CLC)

Figure D.50

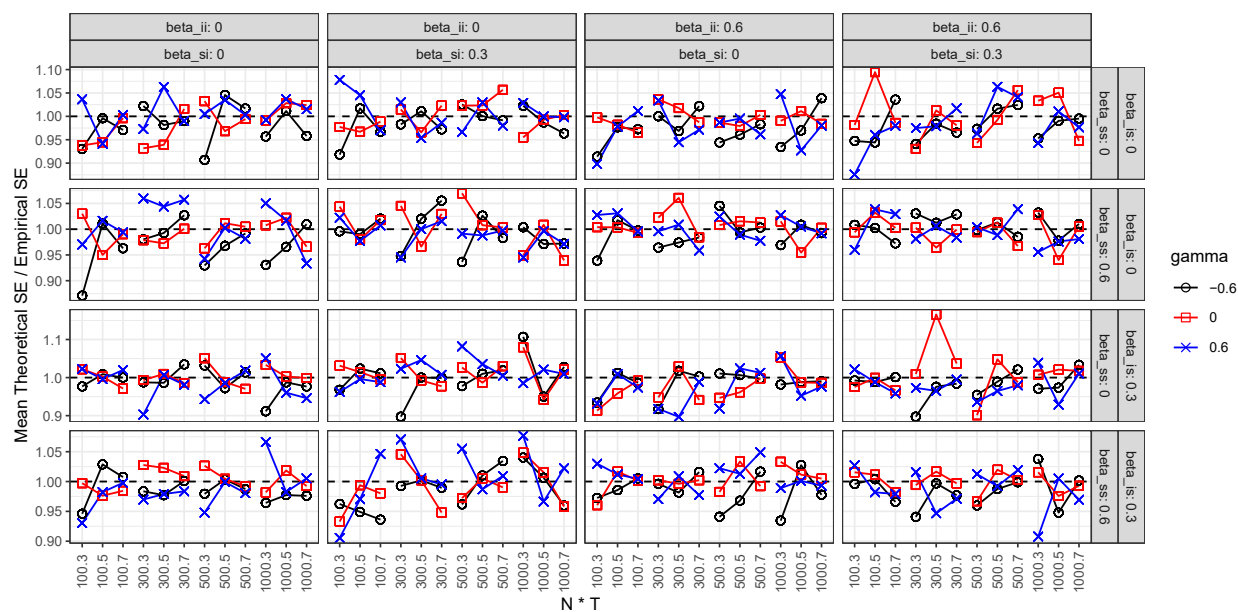
SE Ratio for the Within-Person Effect γ in Simulation 4 (LCM_{xy}-RP)

Figure D.51

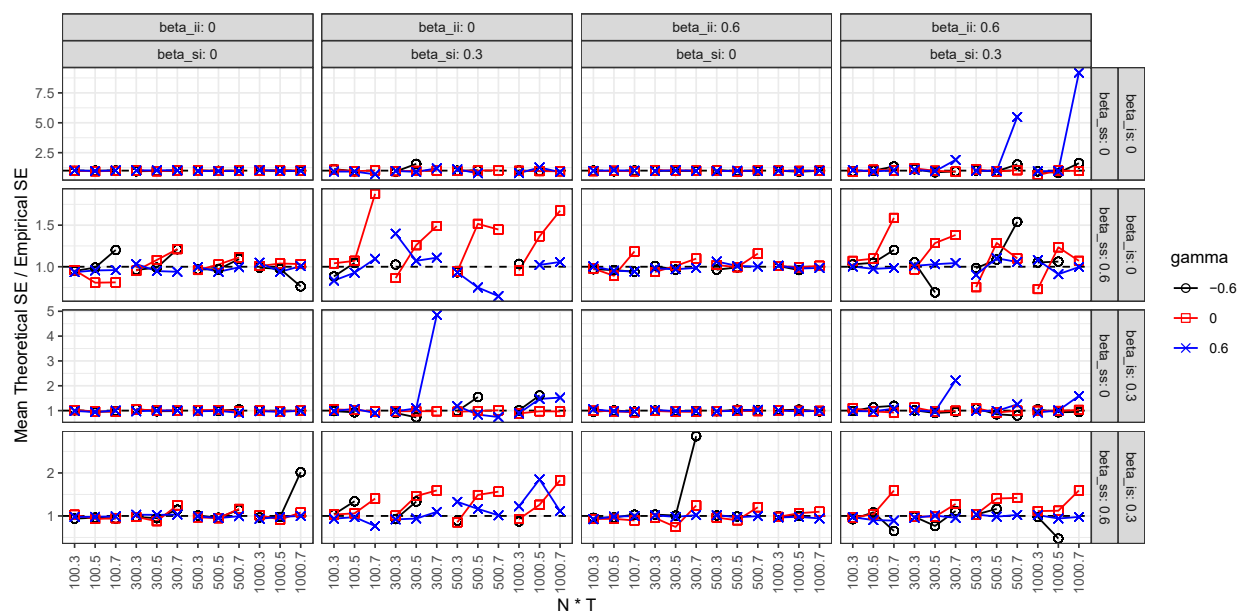
SE Ratio for the Within-Person Effect γ in Simulation 4 (LCM_x-RIy)

Figure D.52

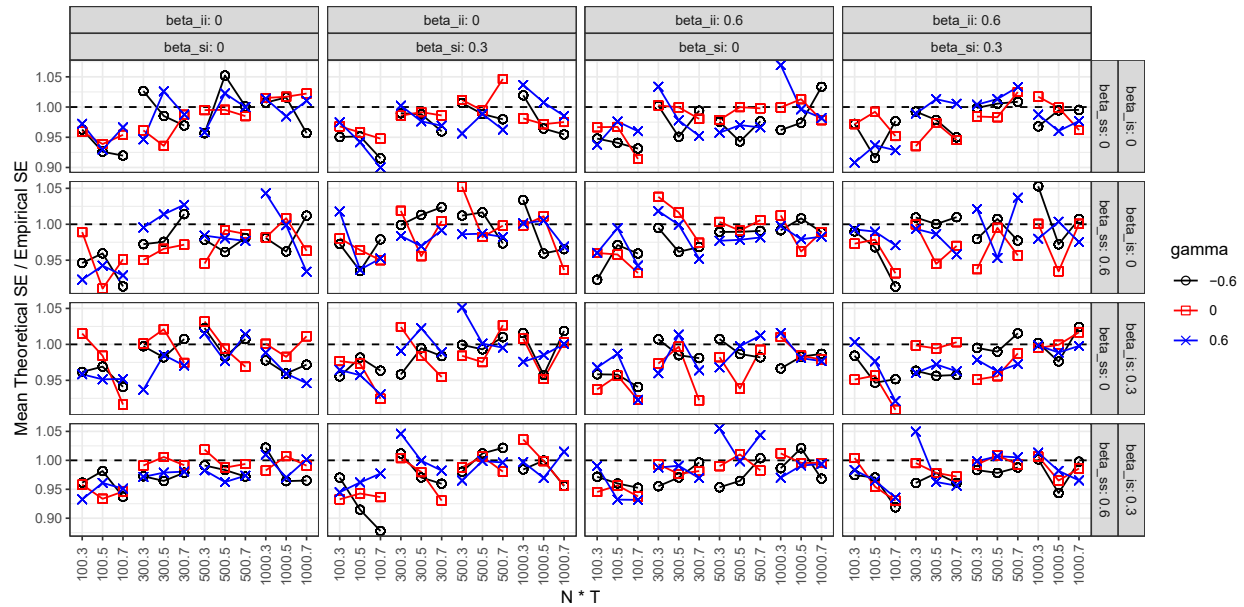
SE Ratio for the Within-Person Effect γ in Simulation 4 (LCMx-CLC)

Figure D.53

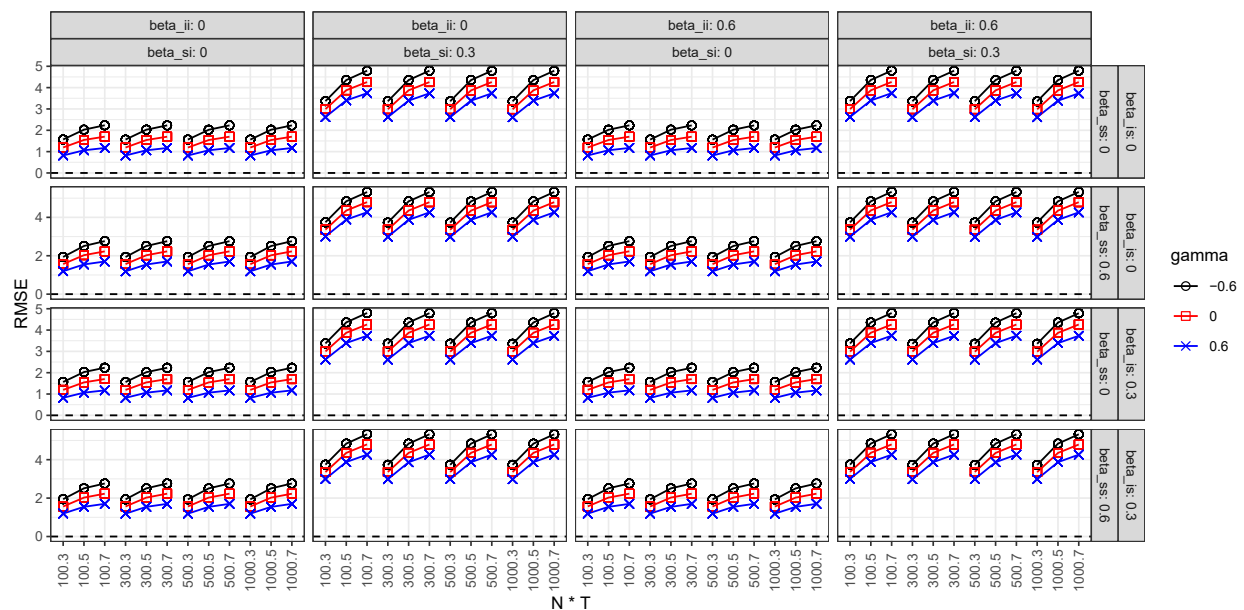
RMSE for the Within-Person Effect γ in Simulation 4 (MLM-WB)

Figure D.54

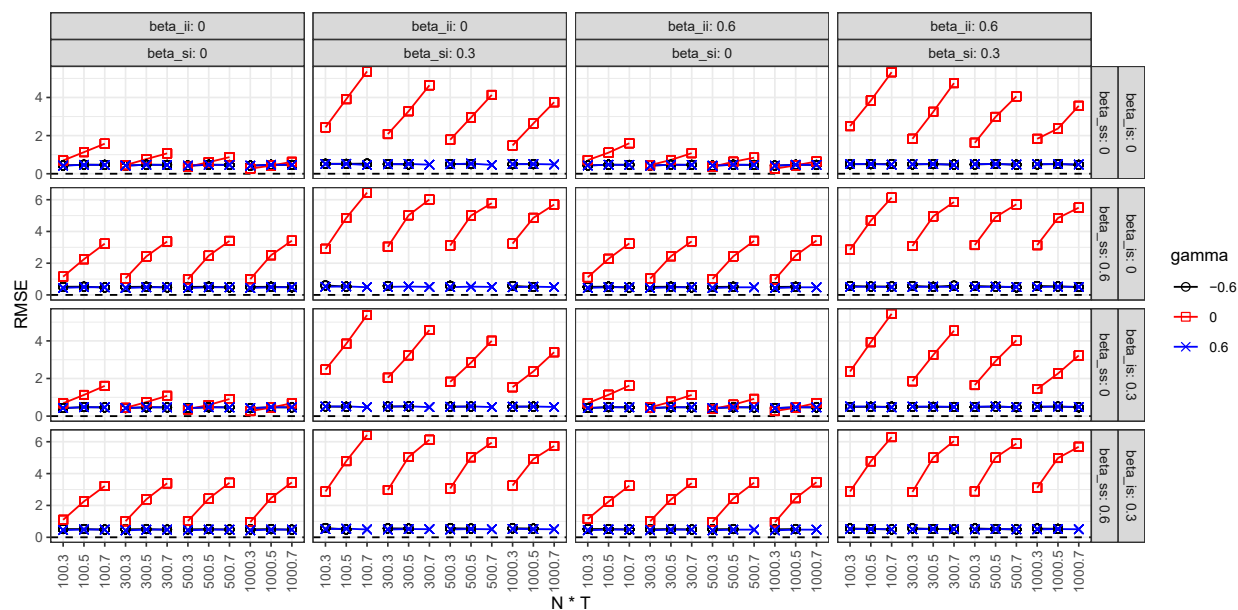
RMSE for the Within-Person Effect γ in Simulation 4 (LCM_x-Rly)

Figure D.55

RMSE for the Within-Person Effect γ in Simulation 4 ($\gamma = -0.6$)

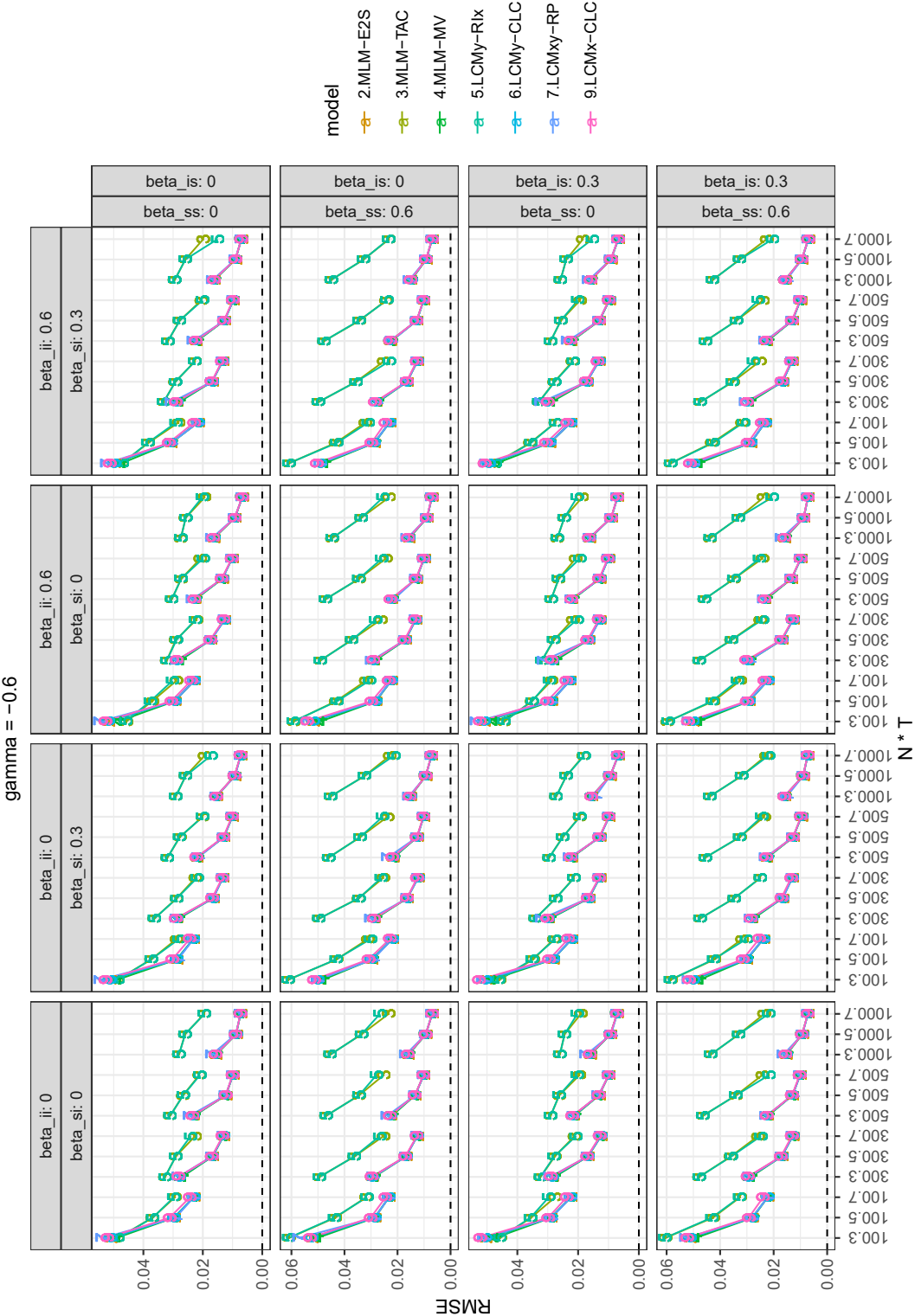


Figure D.56

RMSE for the Within-Person Effect γ in Simulation 4 ($\gamma = 0$)

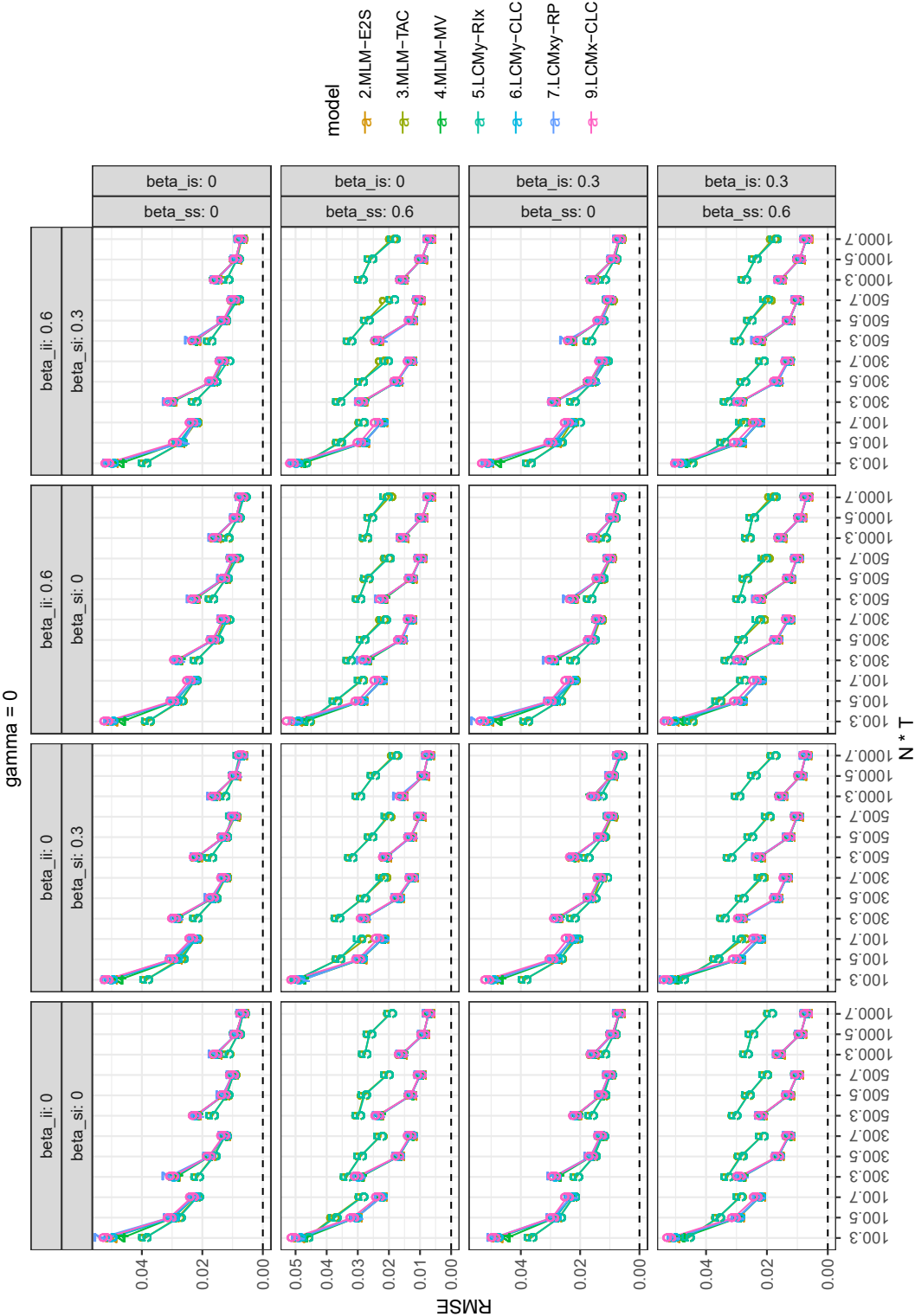


Figure D.57

RMSE for the Within-Person Effect γ in Simulation 4 ($\gamma = 0.6$)

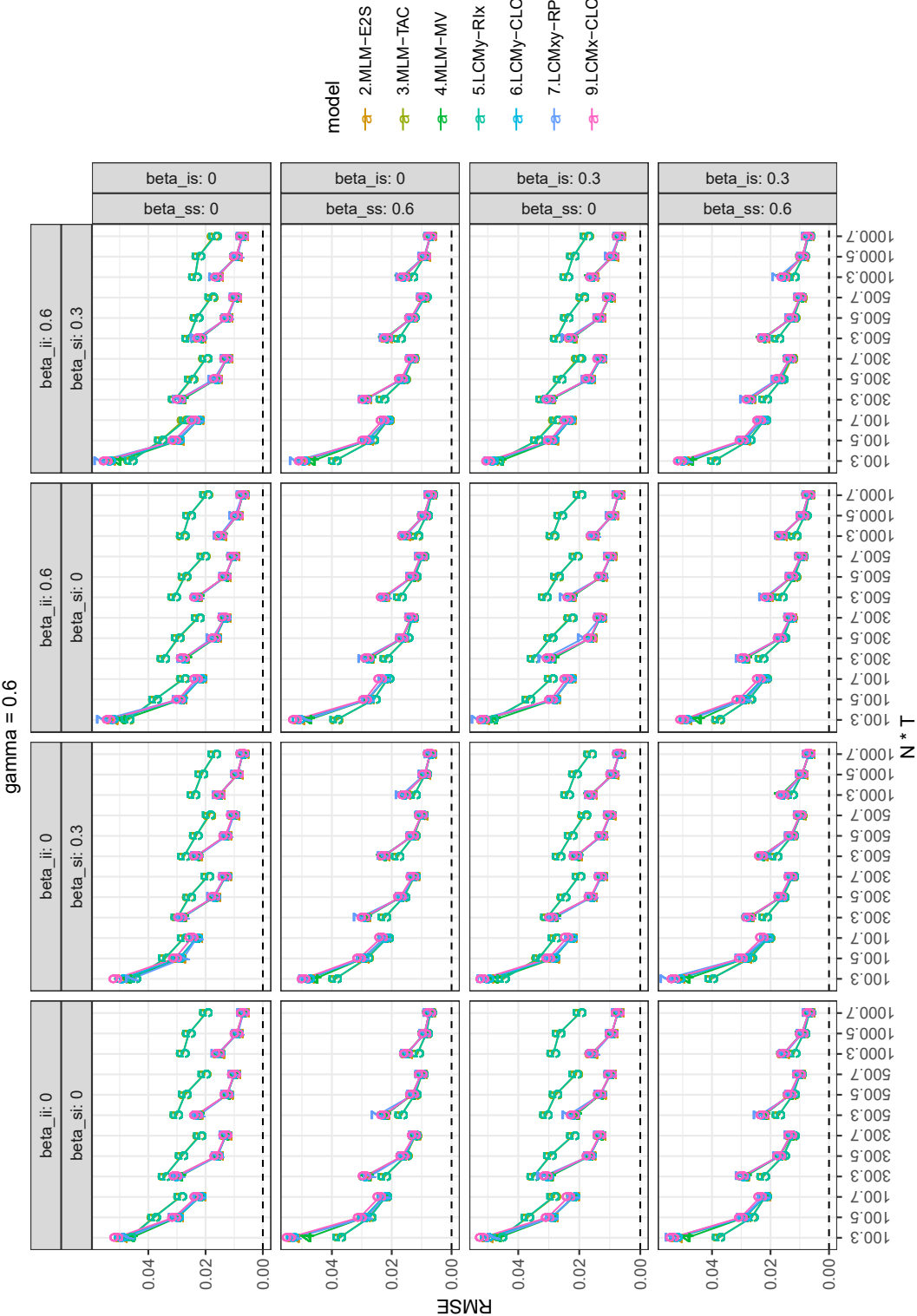


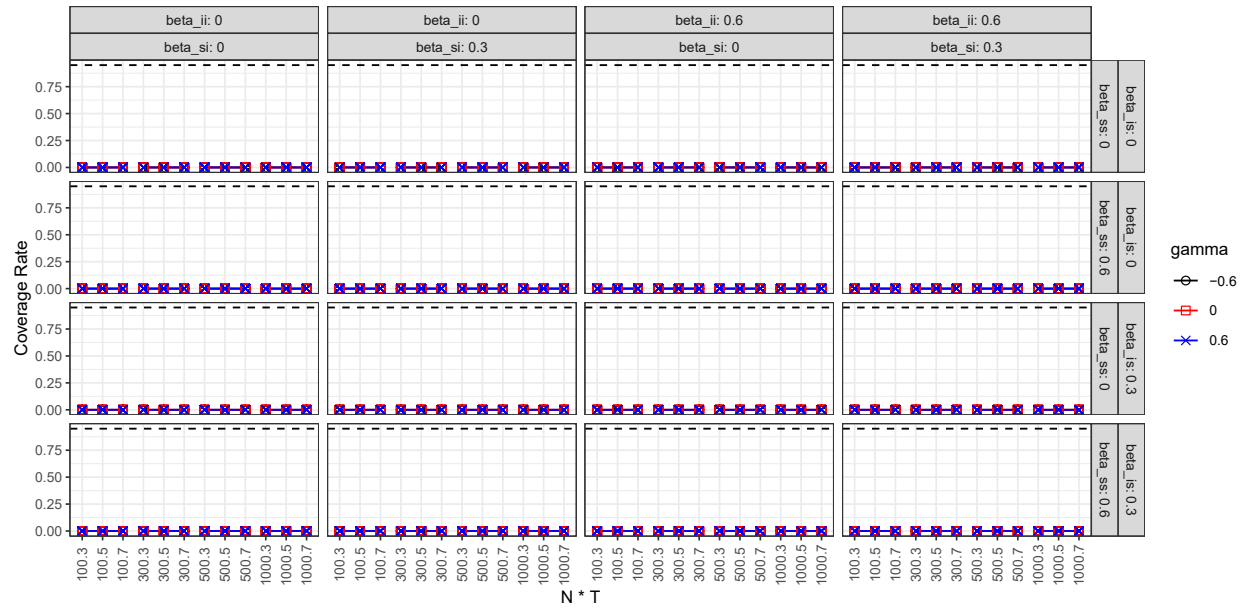
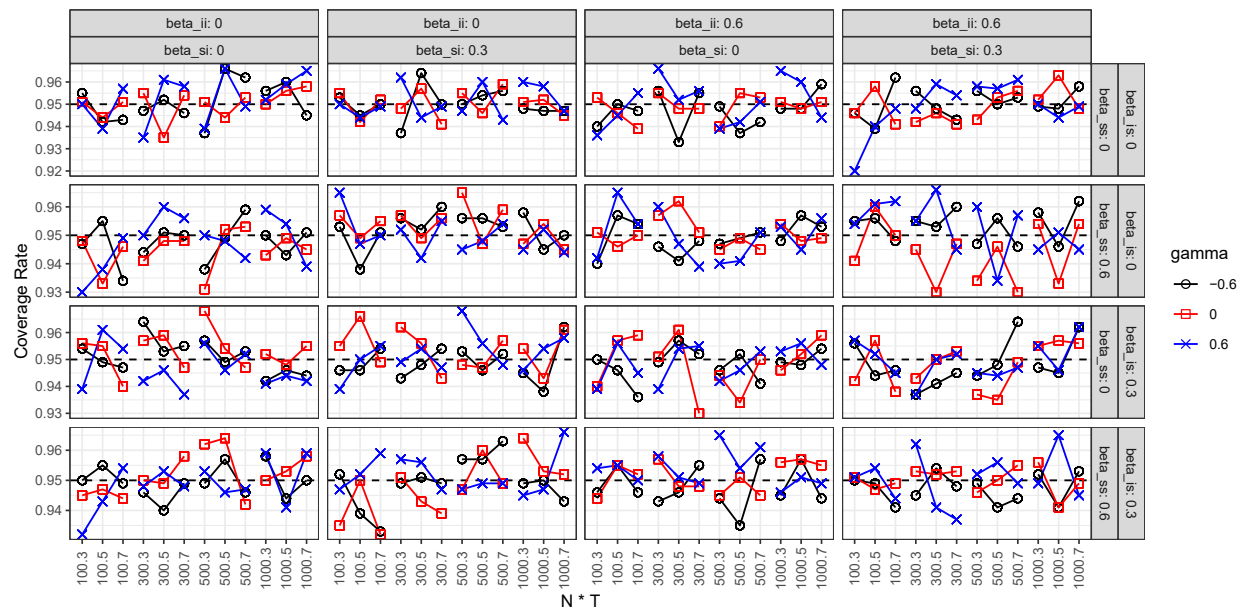
Figure D.58*Coverage Rate for the Within-Person Effect γ in Simulation 4 (MLM-WB)***Figure D.59***Coverage Rate for the Within-Person Effect γ in Simulation 4 (MLM-E2S)*

Figure D.60

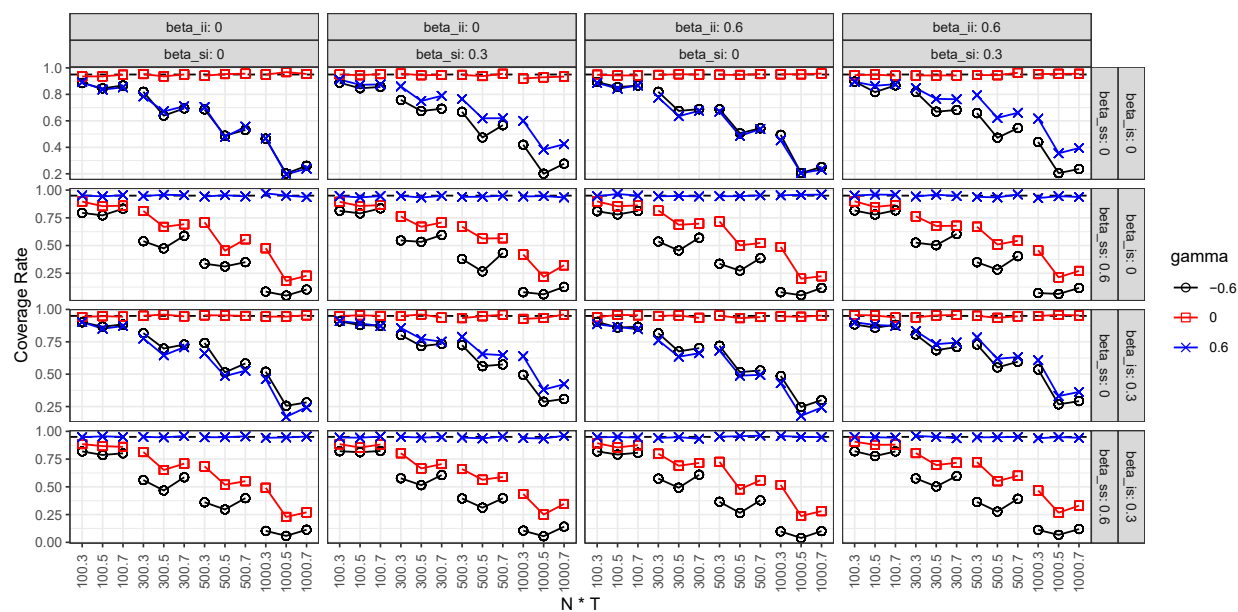
Coverage Rate for the Within-Person Effect γ in Simulation 4 (MLM-TAC)

Figure D.61

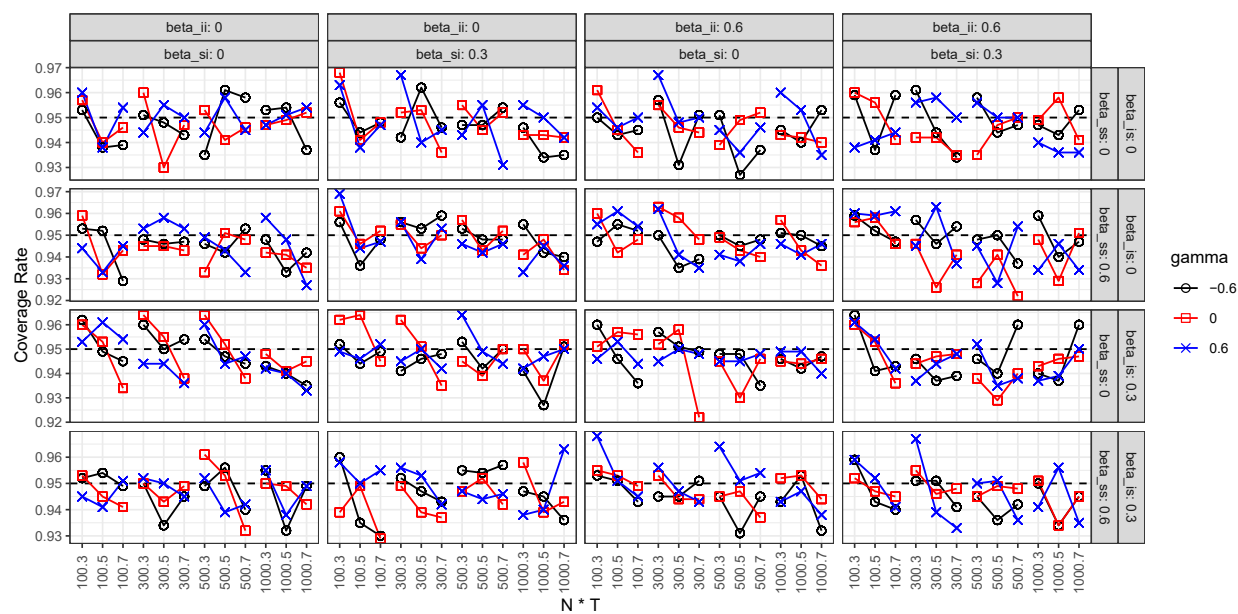
Coverage Rate for the Within-Person Effect γ in Simulation 4 (MLM-MV)

Figure D.62

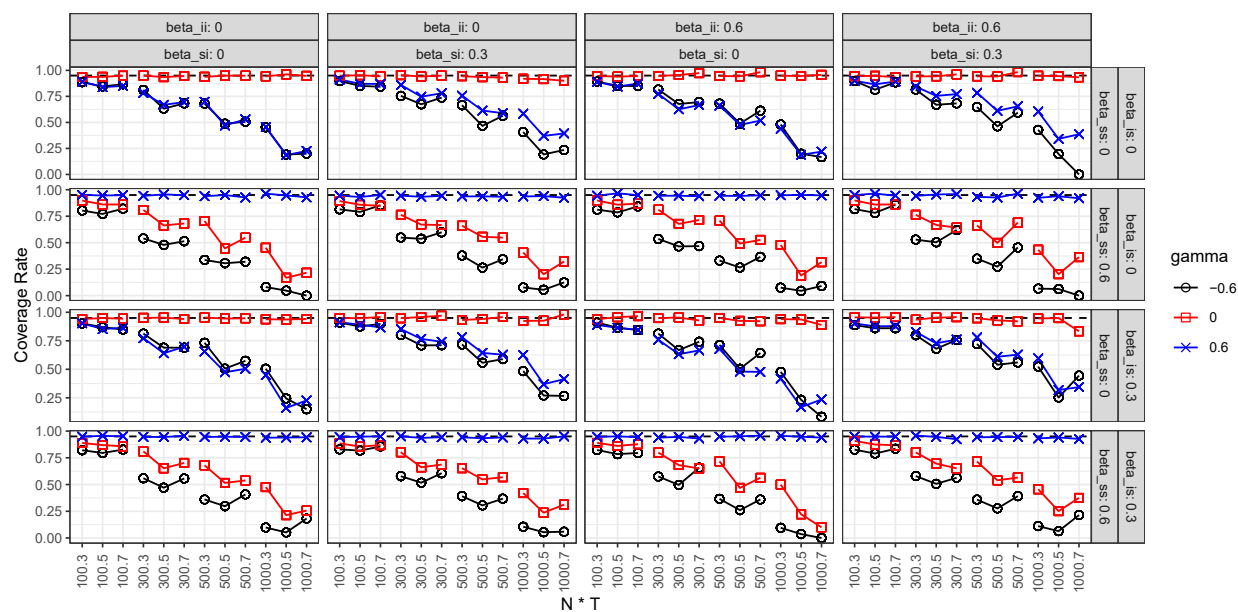
Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMy-RLx)

Figure D.63

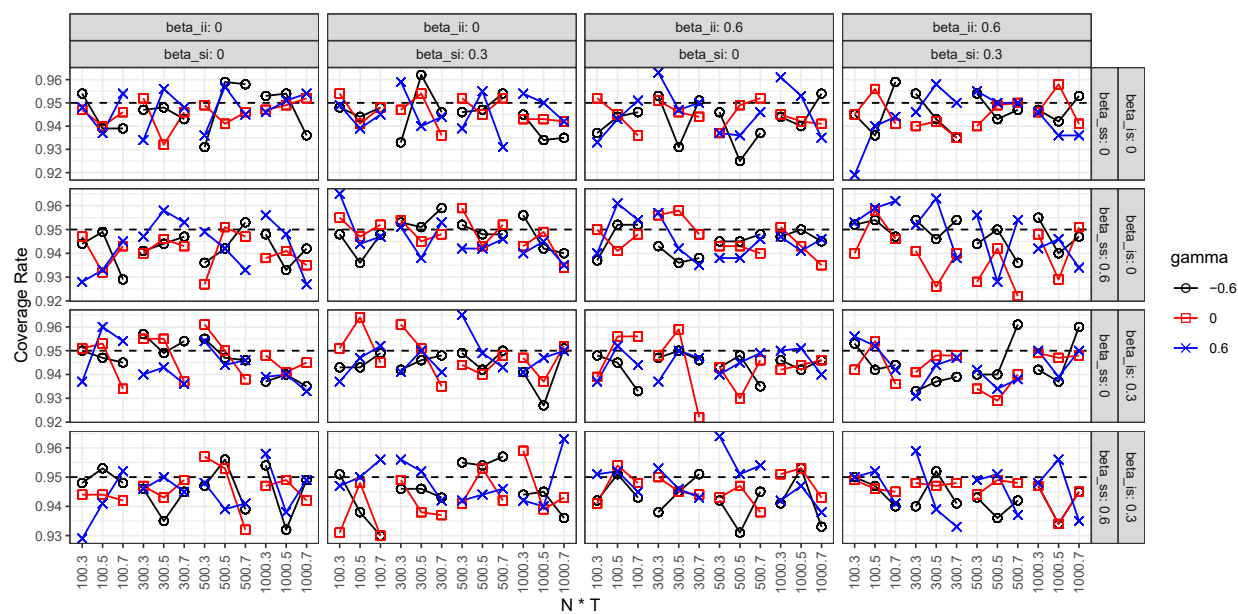
Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMy-CLC)

Figure D.64

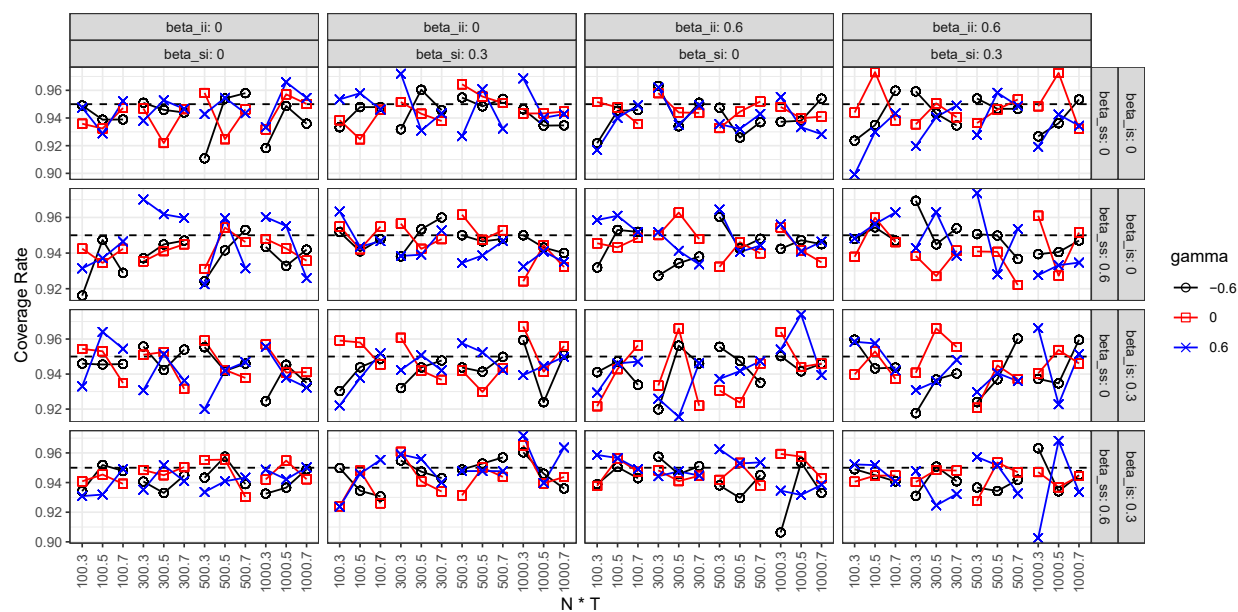
Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMxy-RP)

Figure D.65

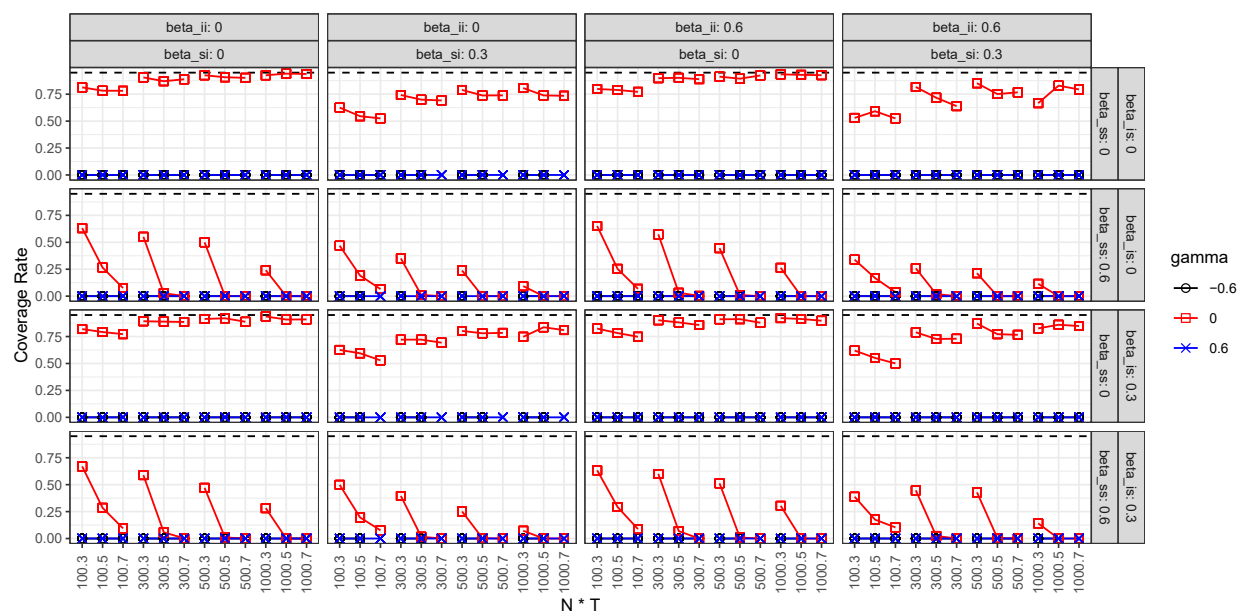
Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMx-Rly)

Figure D.66

Coverage Rate for the Within-Person Effect γ in Simulation 4 (LCMx-CLC)

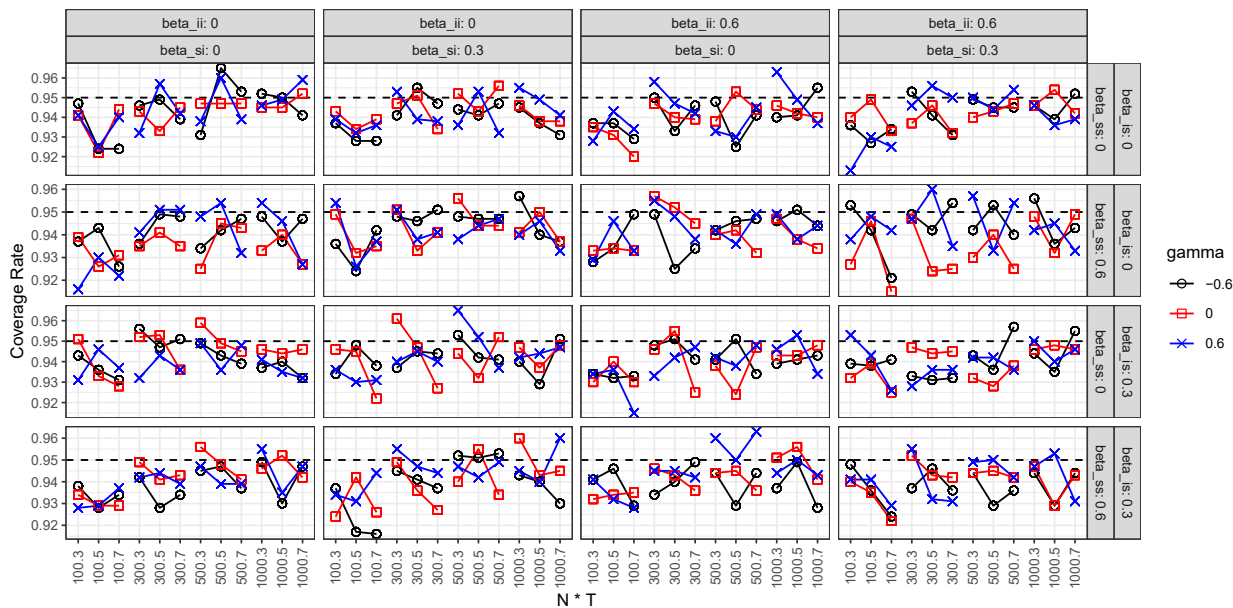


Figure D.67

Power for the Within-Person Effect γ in Simulation 4 ($\gamma = -0.6$)

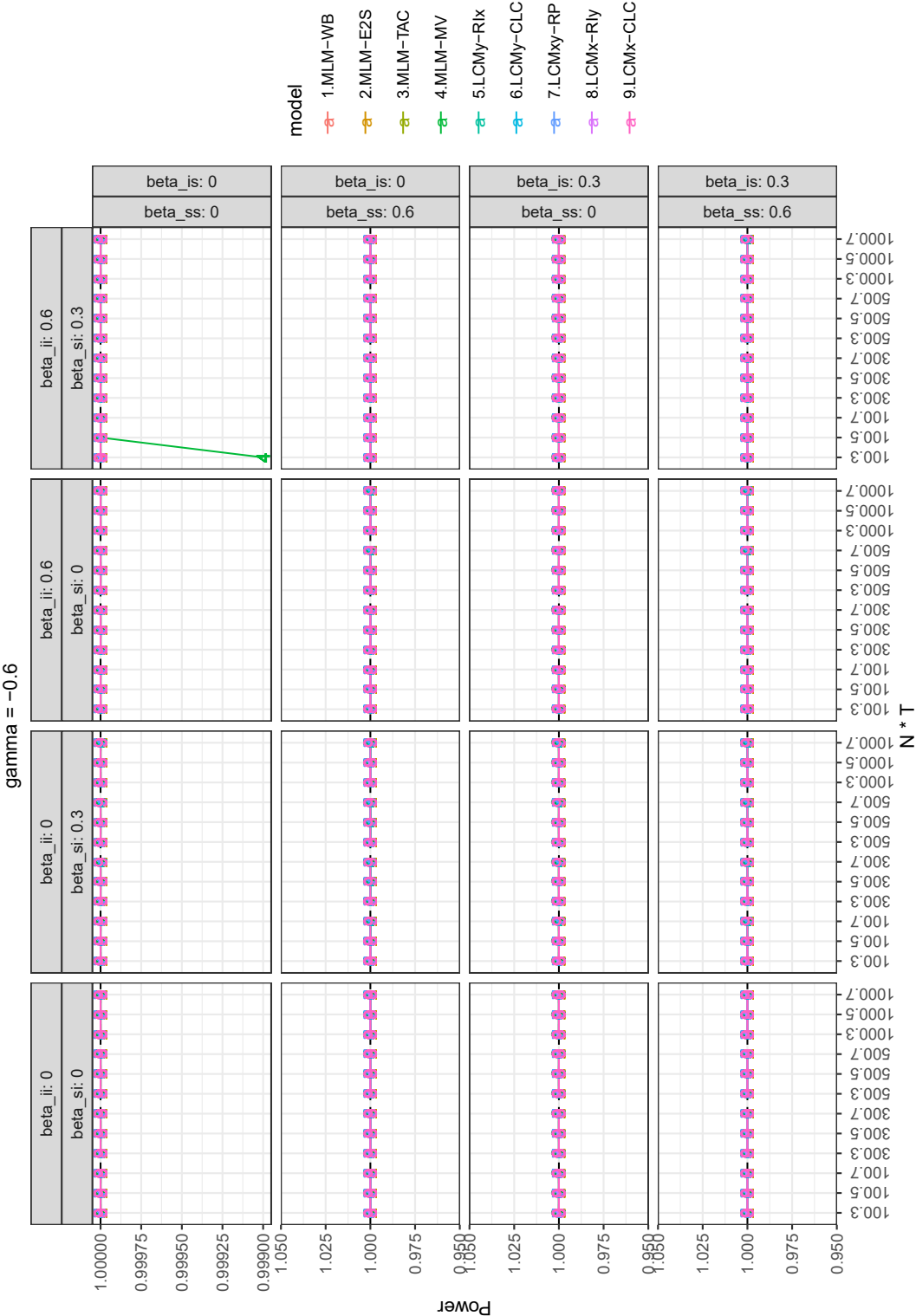
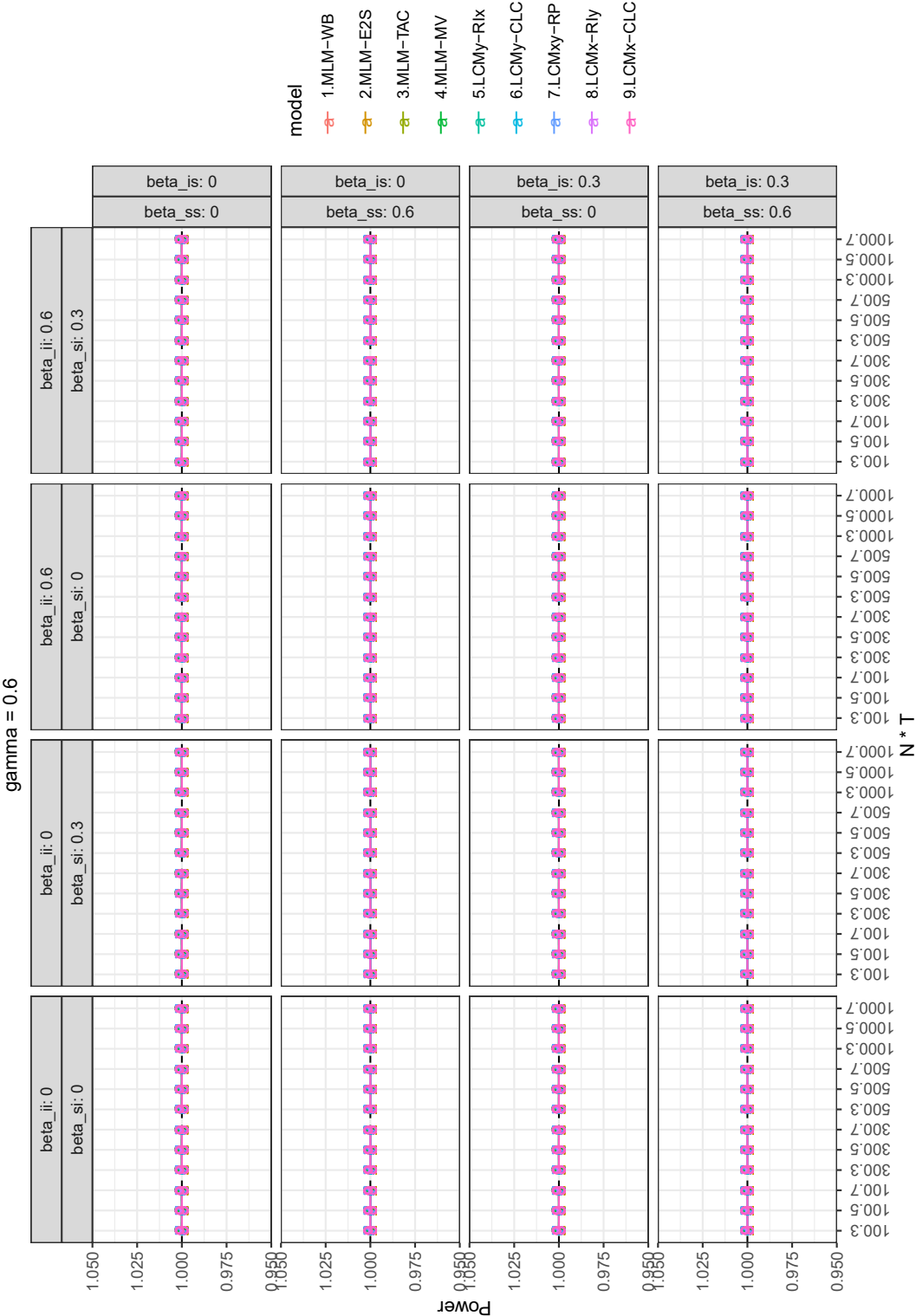


Figure D.68

Power for the Within-Person Effect γ in Simulation 4 ($\gamma = 0.6$)



D.3 Between-Person Effect of Intercept on Intercept

Figure D.69

Bias for the Between-Person Effect β_{II} in Simulation 4 (MLM-WB)

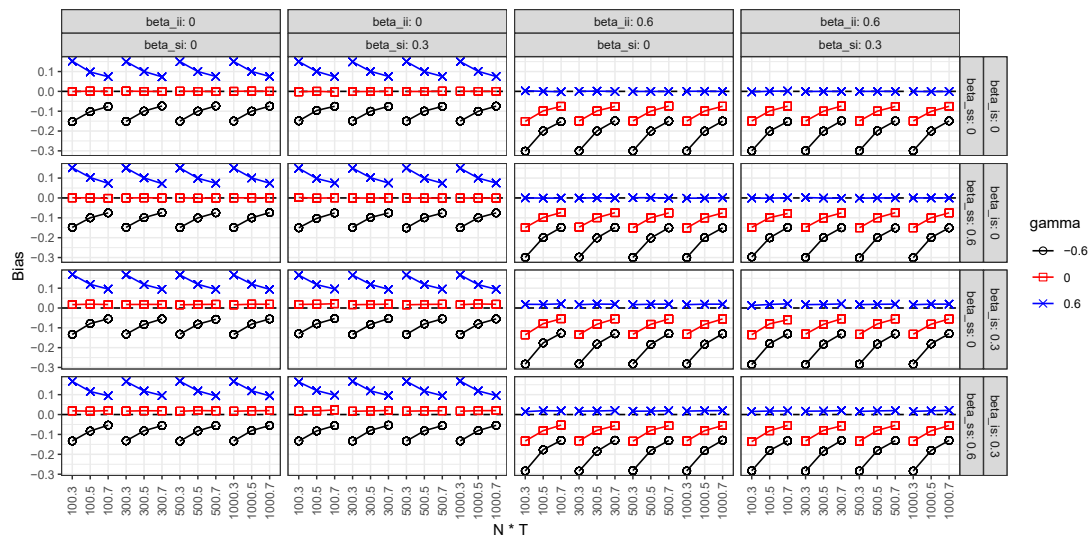


Figure D.70

Bias for the Between-Person Effect β_{II} in Simulation 4 (MLM-E2S)

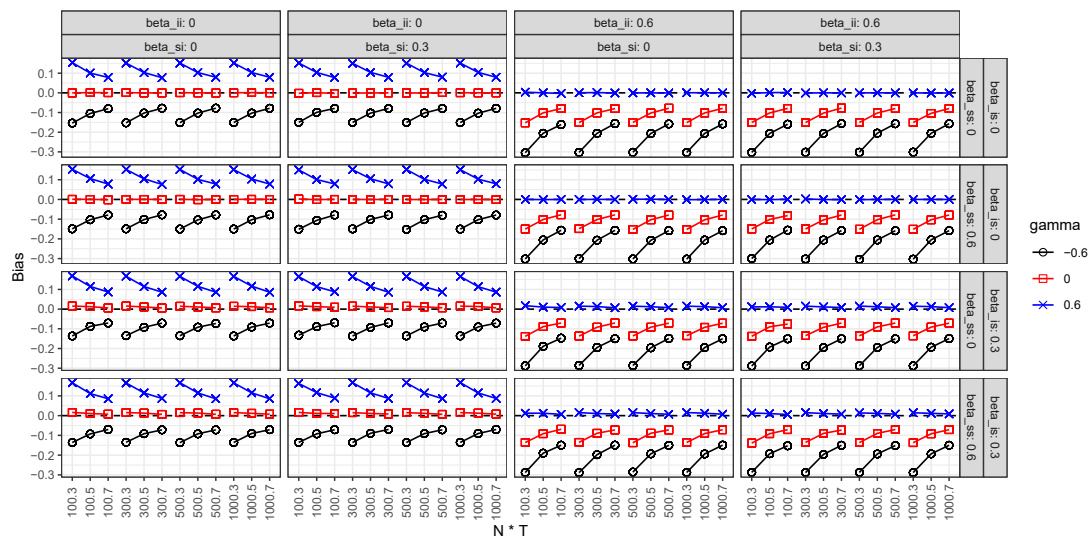


Figure D.71

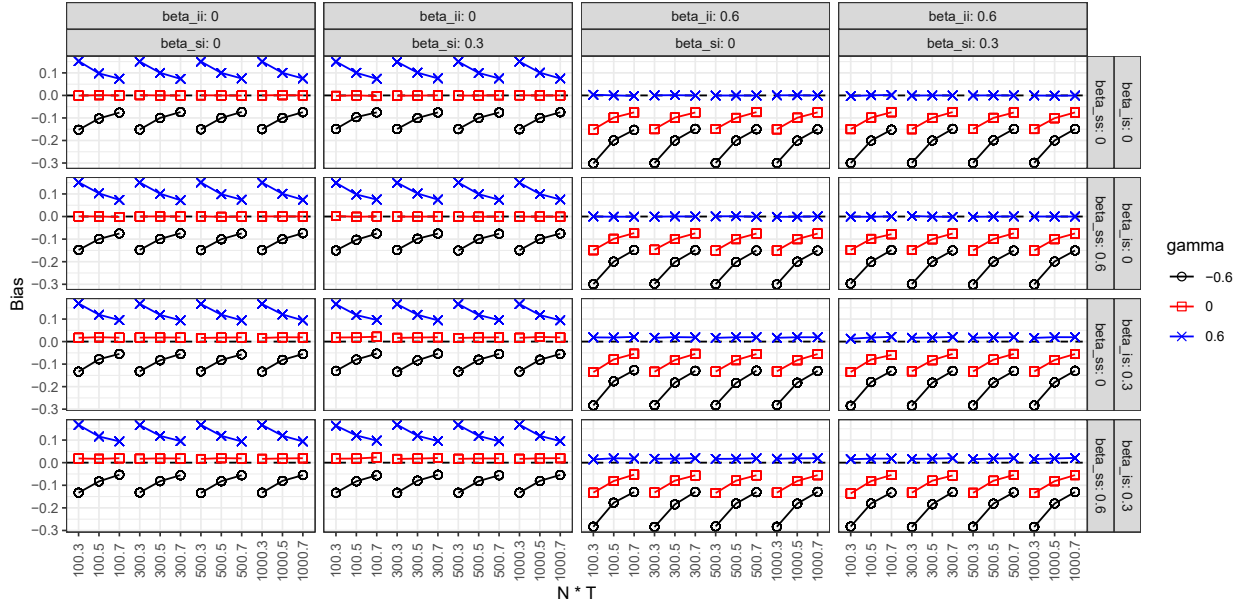
Bias for the Between-Person Effect β_{II} in Simulation 4 (MLM-TAC)

Figure D.72

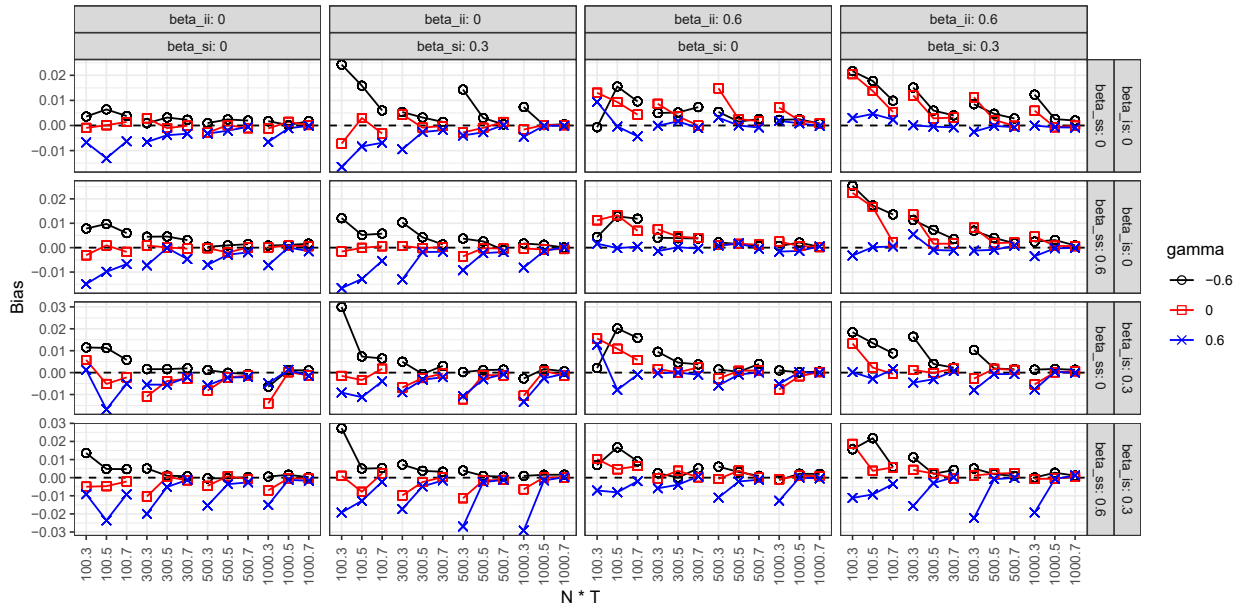
Bias for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)

Figure D.73

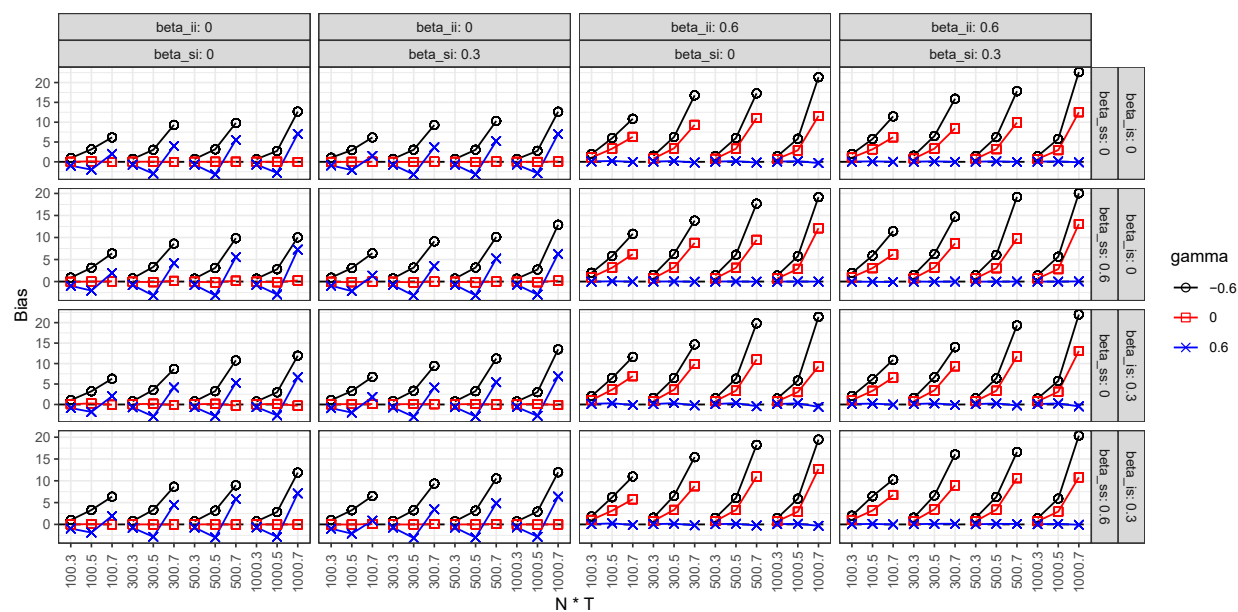
Bias for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)

Figure D.74

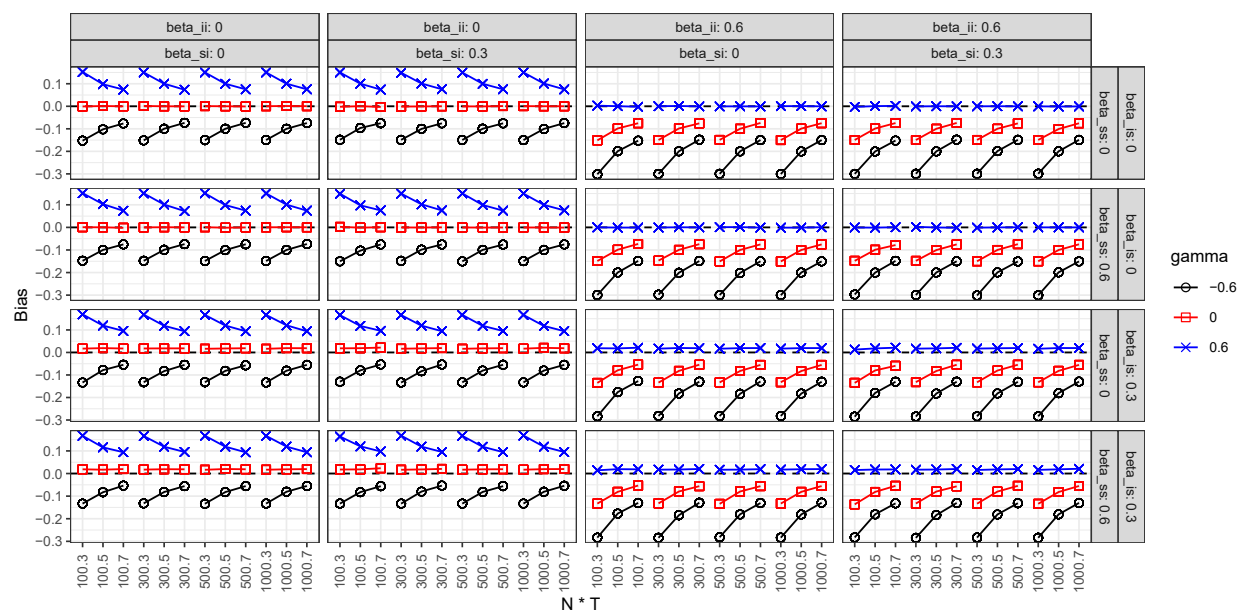
Bias for the Between-Person Effect β_{II} in Simulation 4 (LCMy-CLC)

Figure D.75

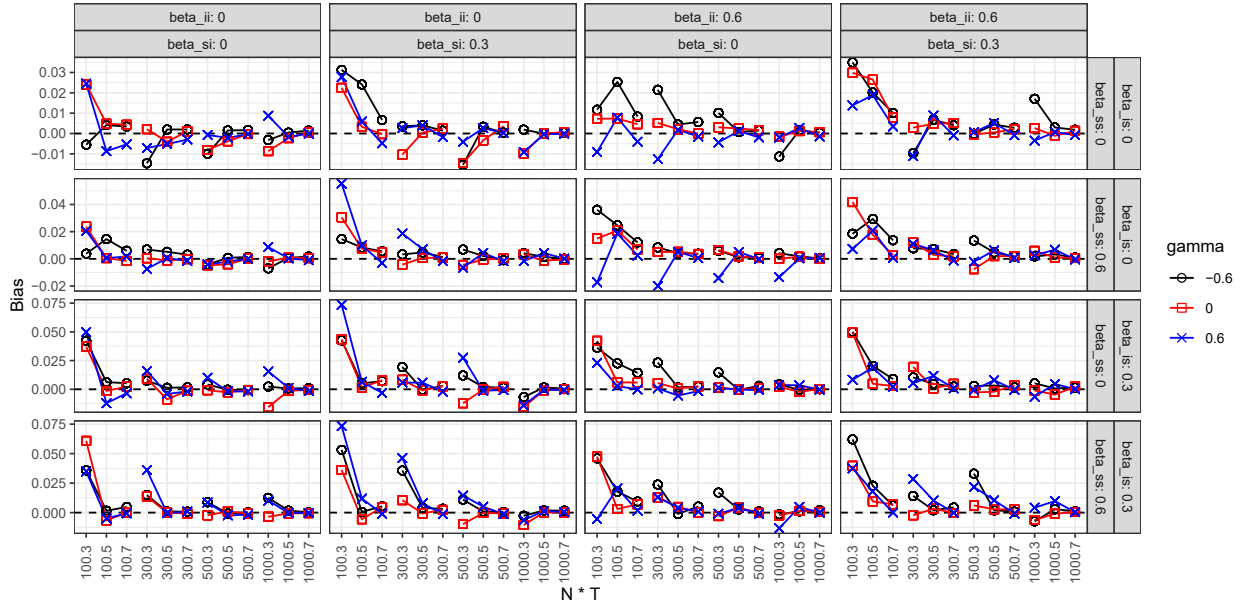
Bias for the Between-Person Effect β_{II} in Simulation 4 (LCM_{xy}-RP)

Figure D.76

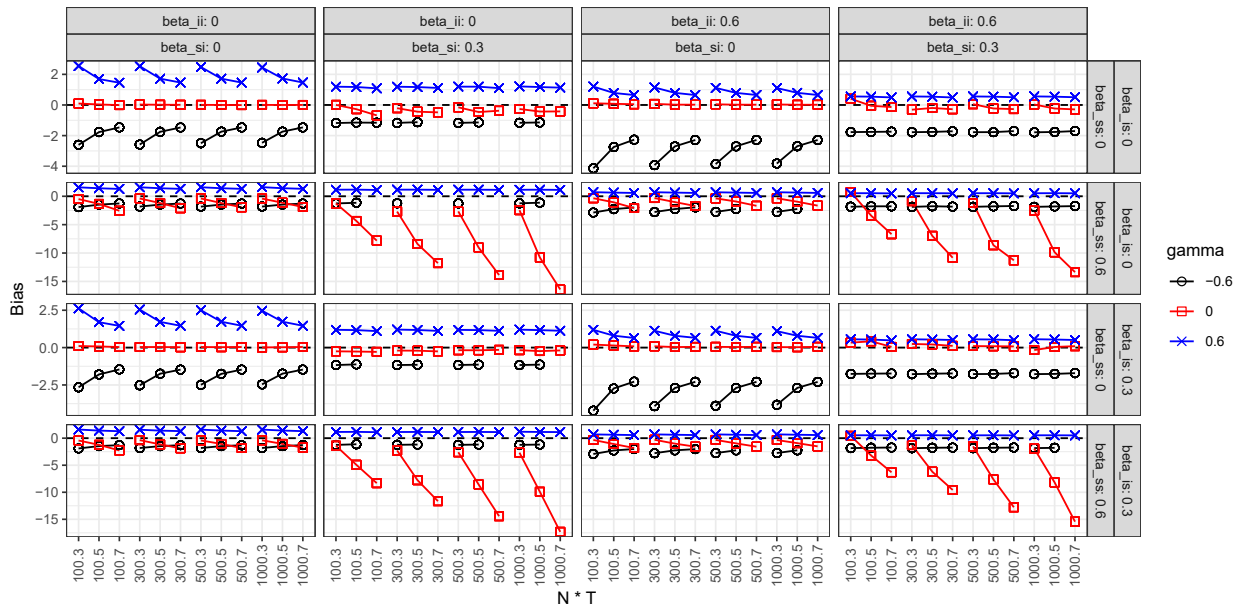
Bias for the Between-Person Effect β_{II} in Simulation 4 (LCM_x-Rly)

Figure D.77

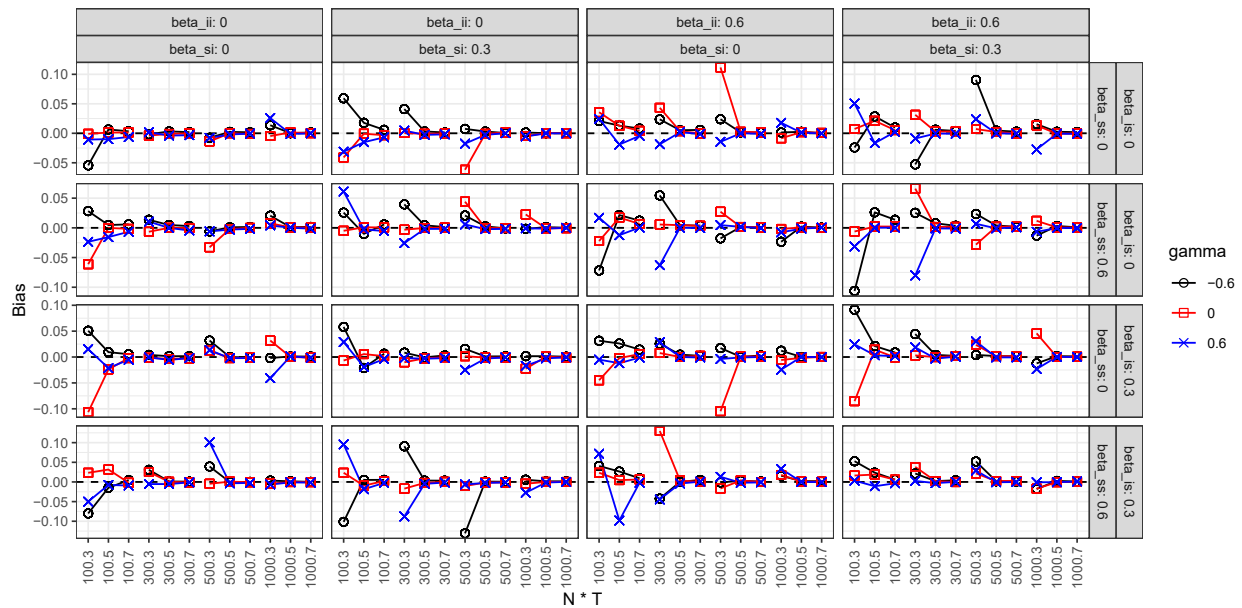
Bias for the Between-Person Effect β_{II} in Simulation 4 (LCM_x-CLC)

Figure D.78

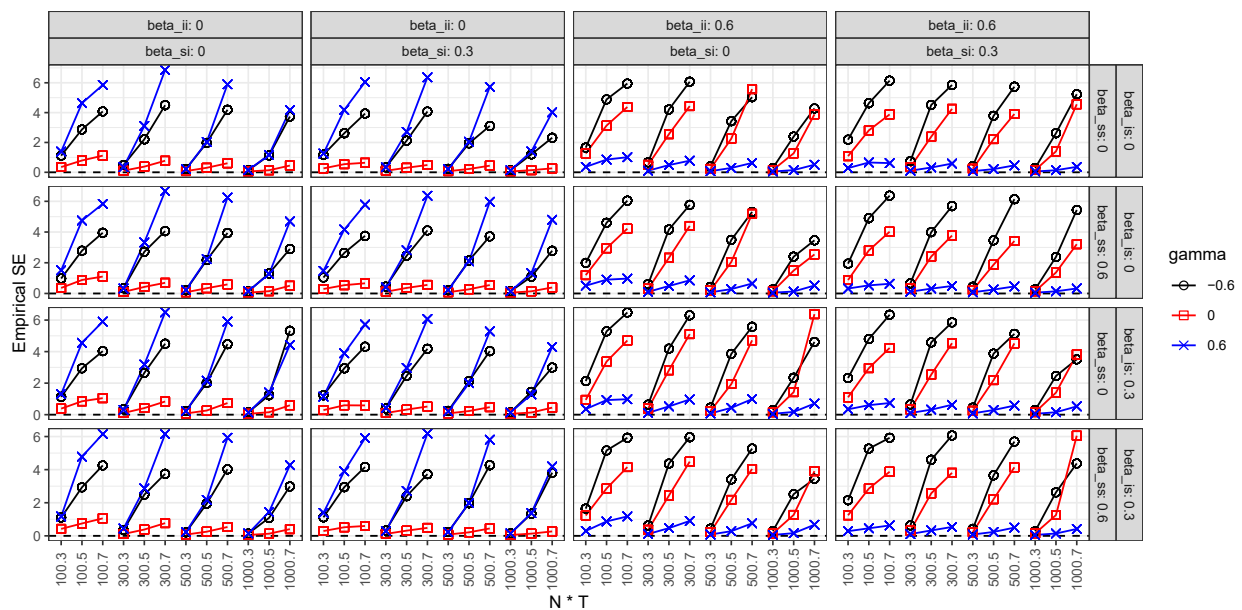
Empirical SE for the Between-Person Effect β_{II} in Simulation 4 (LCM_y-RI_x)

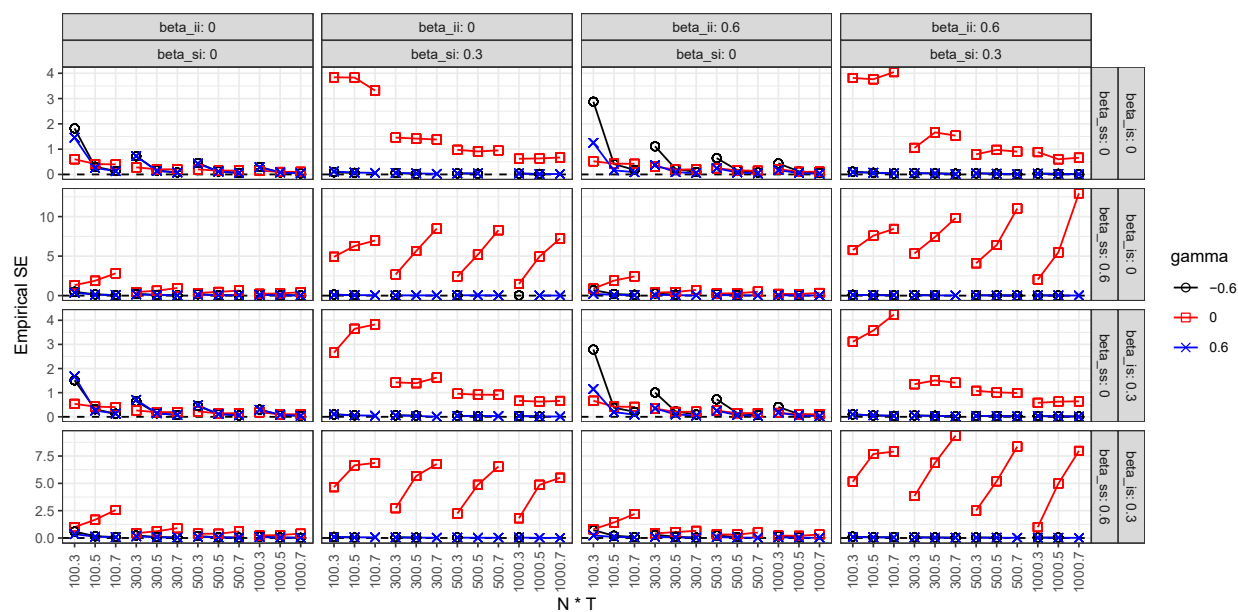
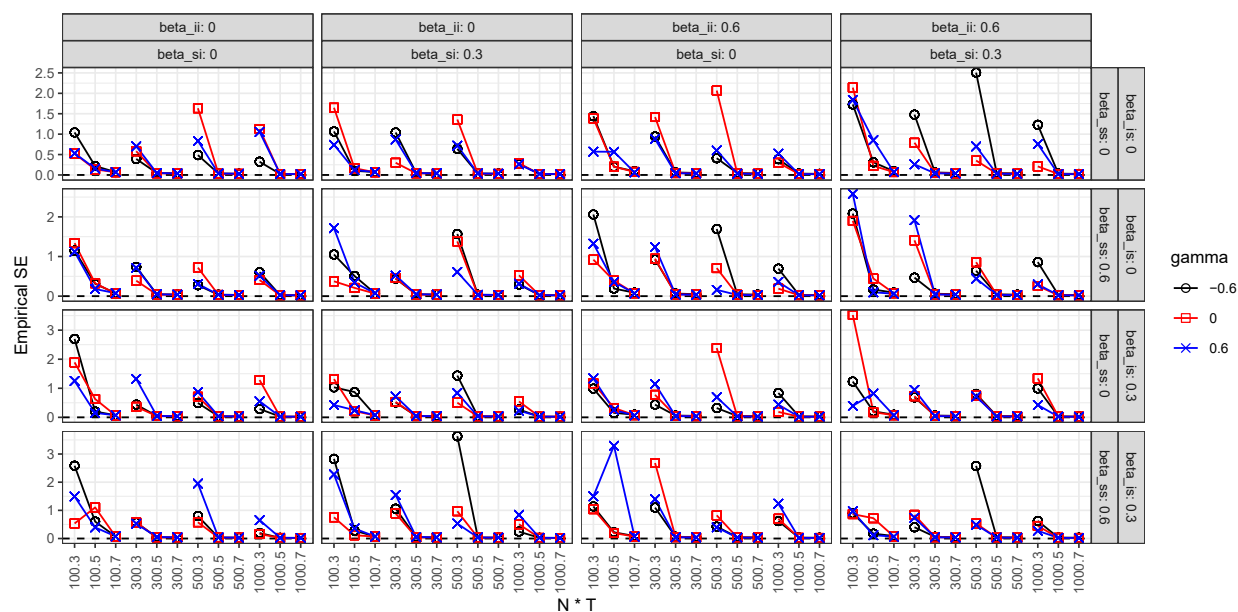
Figure D.79*Empirical SE for the Between-Person Effect β_{II} in Simulation 4 (LCM_X-RIy)***Figure D.80***Empirical SE for the Between-Person Effect β_{II} in Simulation 4 (LCM_X-CLC)*

Figure D.81

Empirical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = -0.6$)

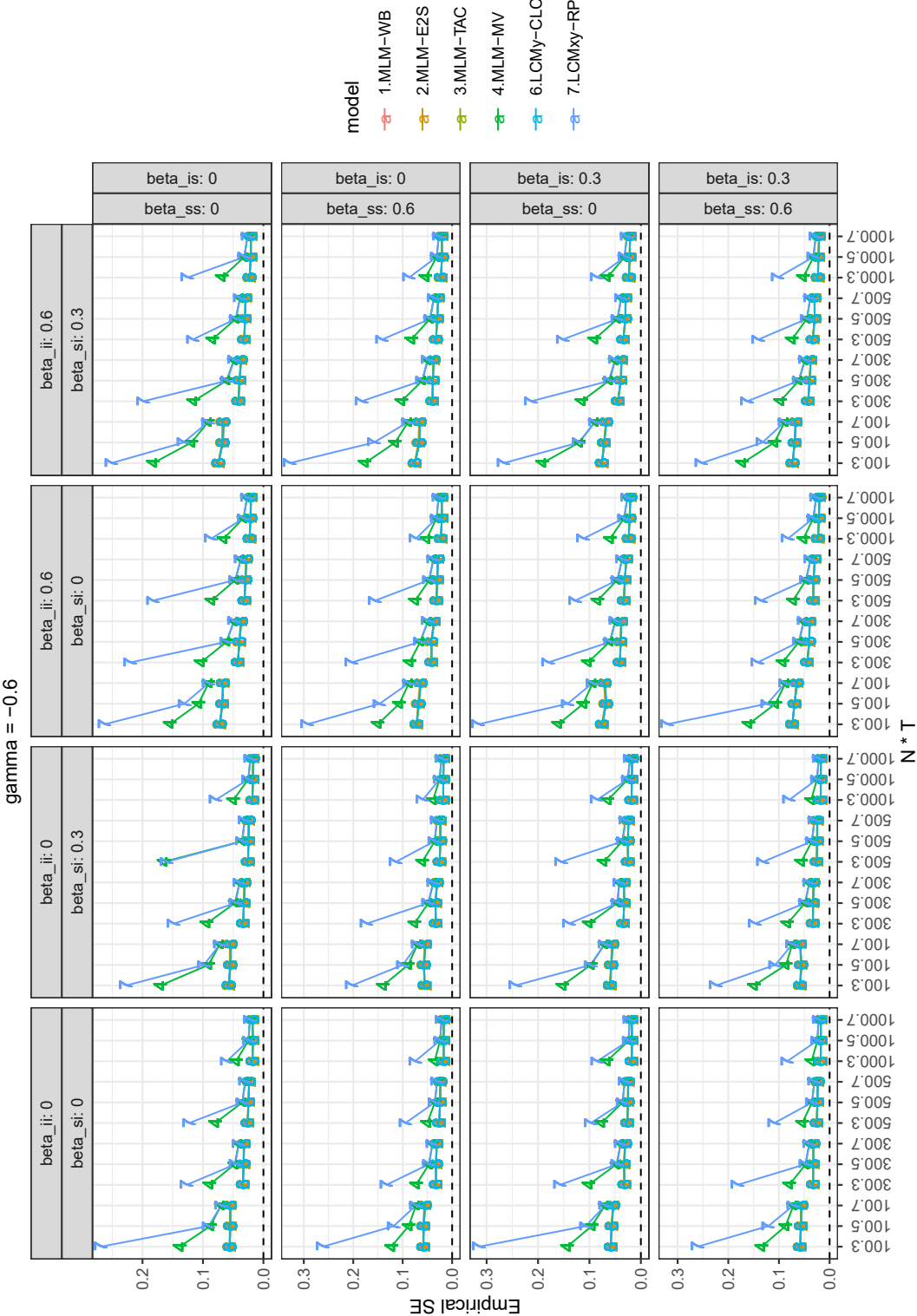


Figure D.82

Empirical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0$)

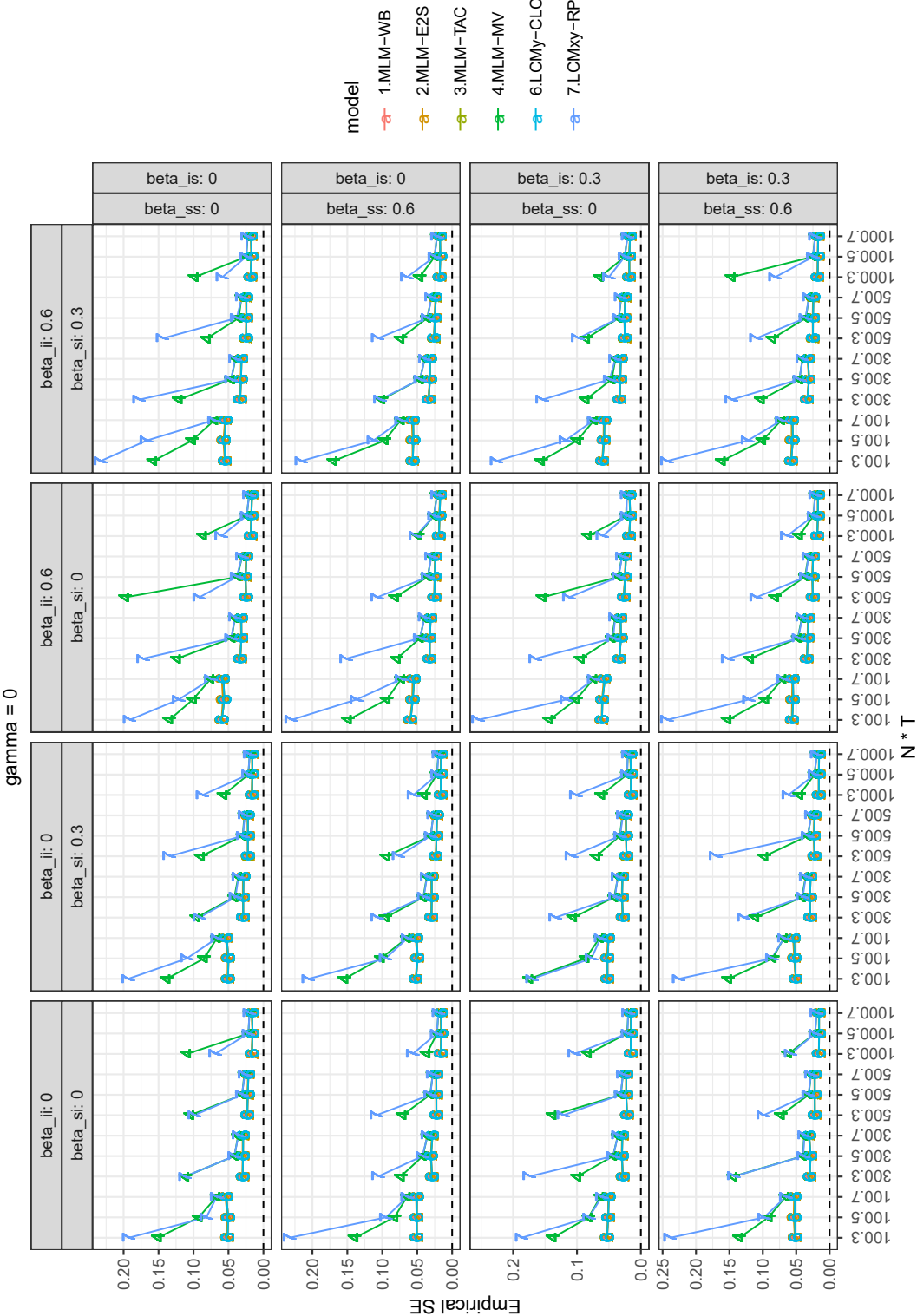


Figure D.83

Empirical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)

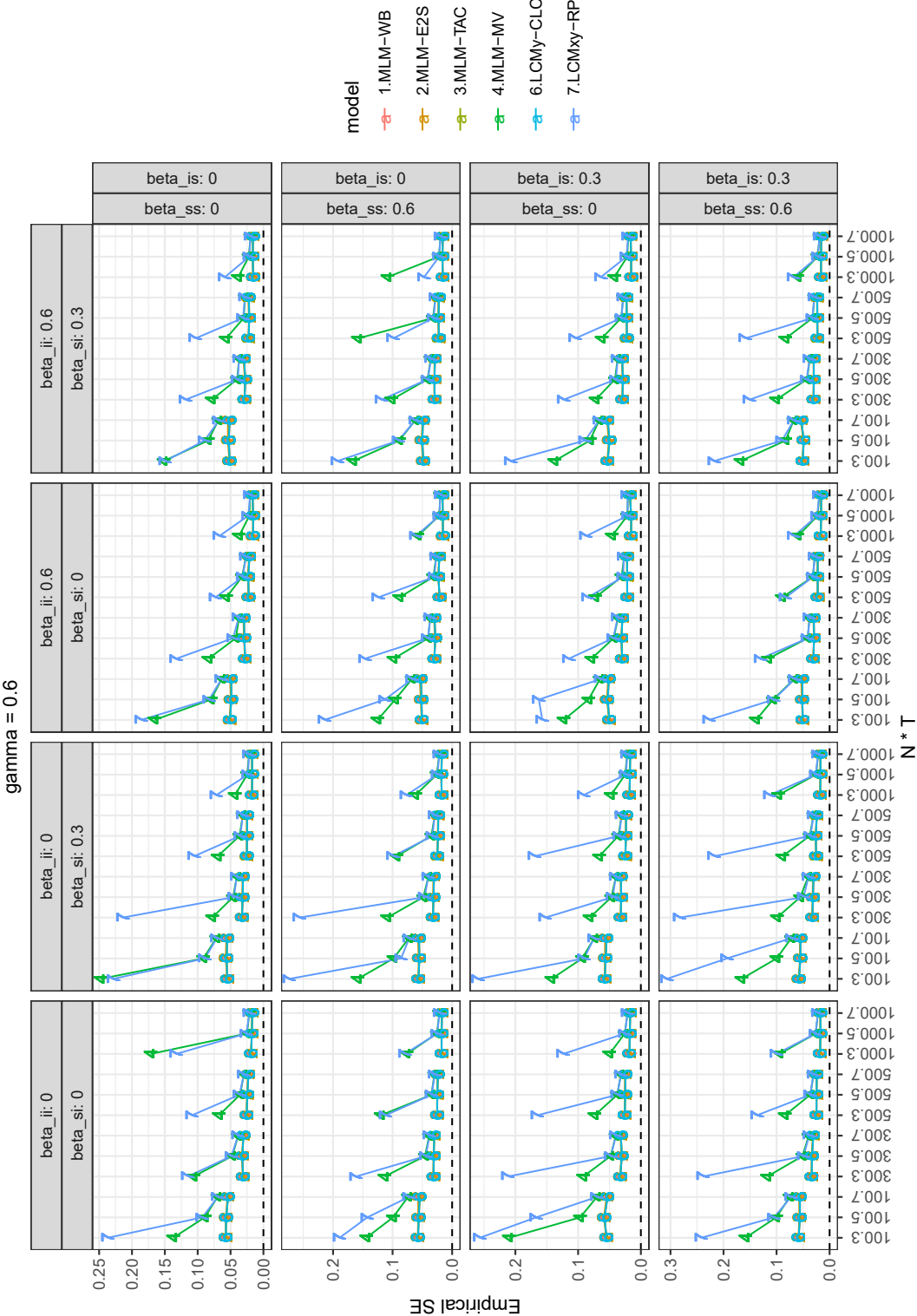


Figure D.84

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)

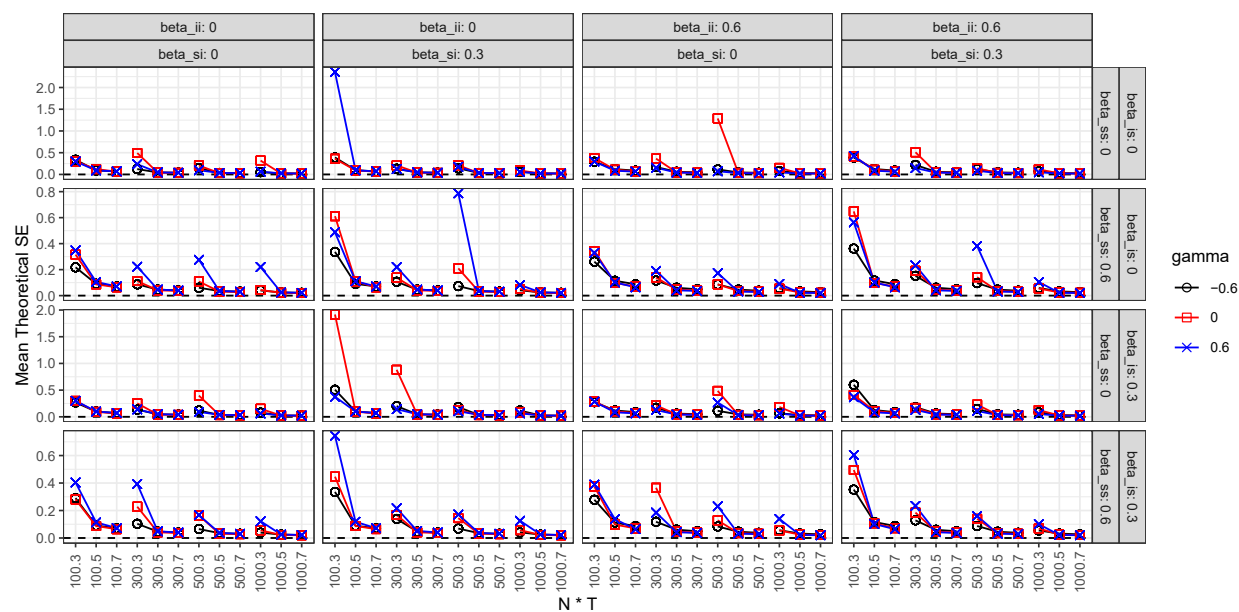


Figure D.85

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)

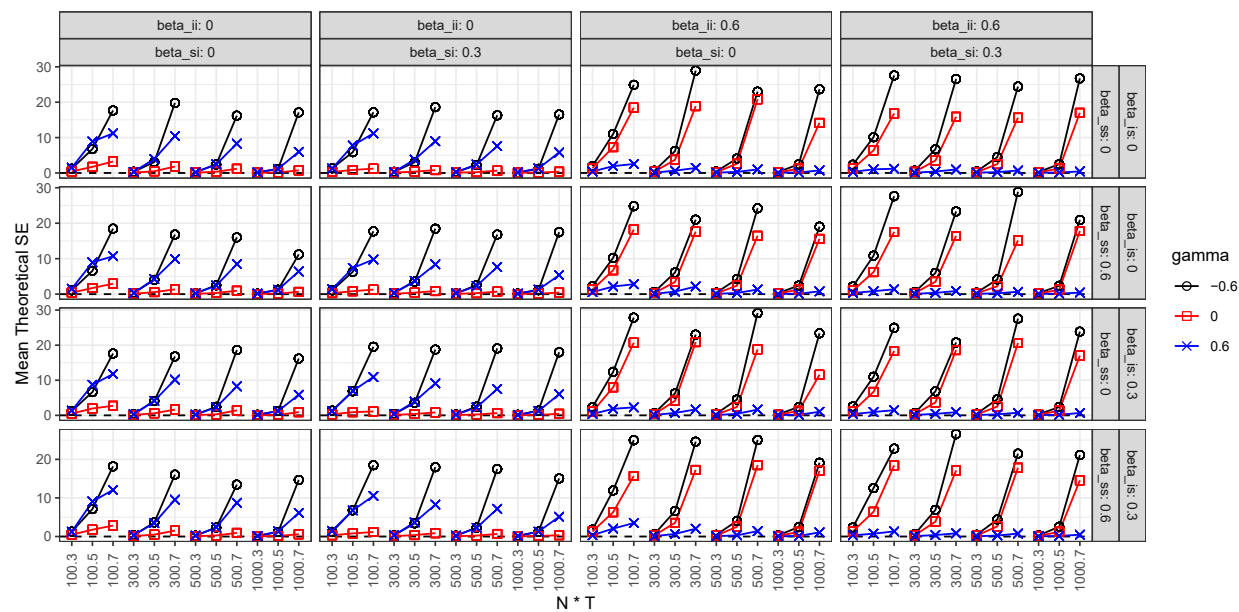


Figure D.86

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMxy-RP)

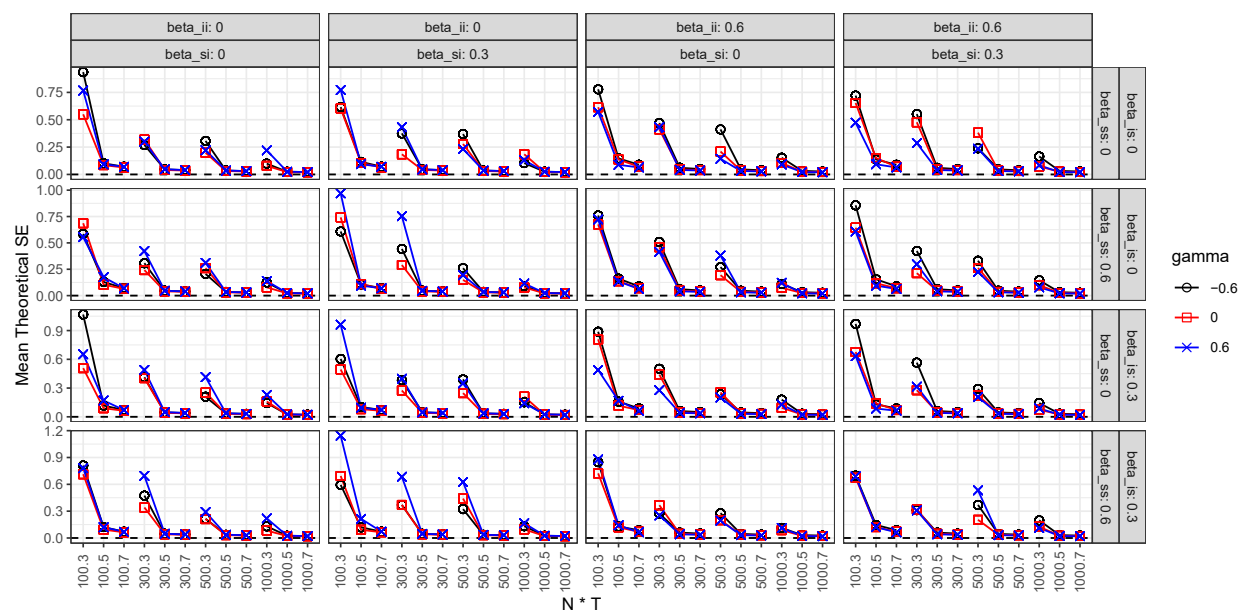


Figure D.87

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-Rly)

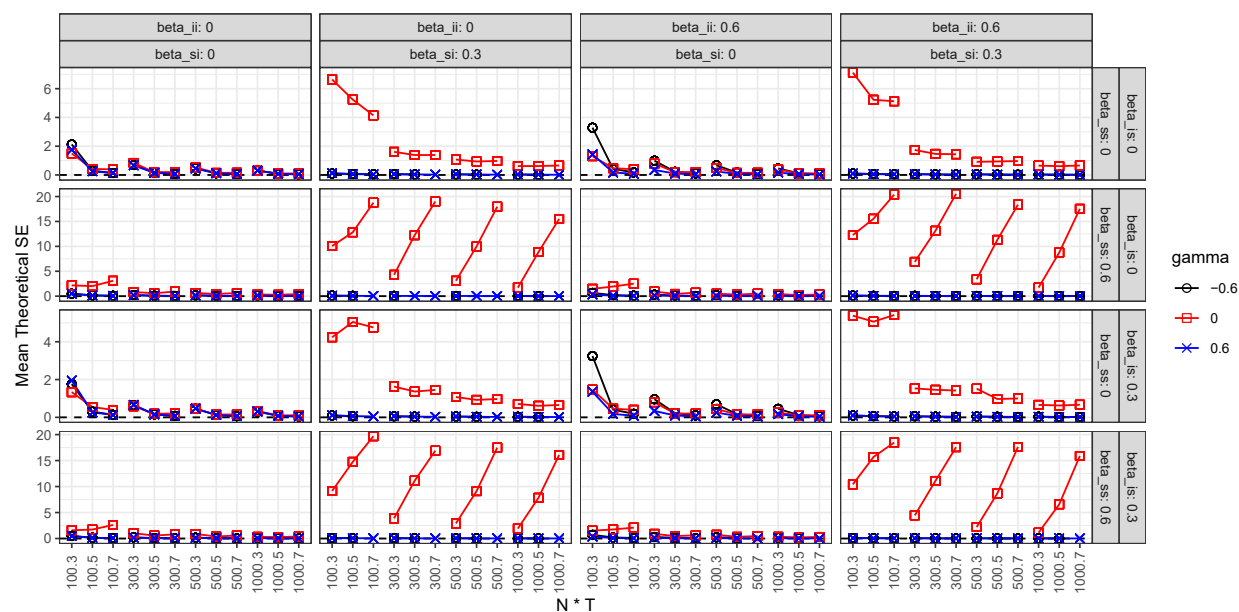


Figure D.88

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)

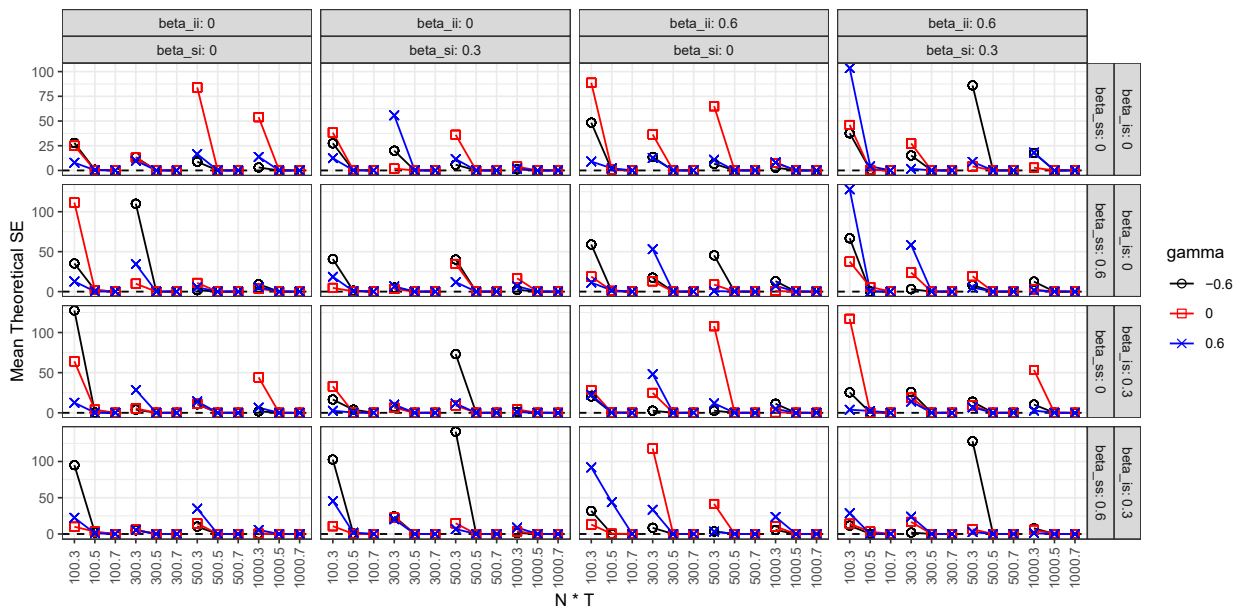


Figure D.89

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = -0.6$)

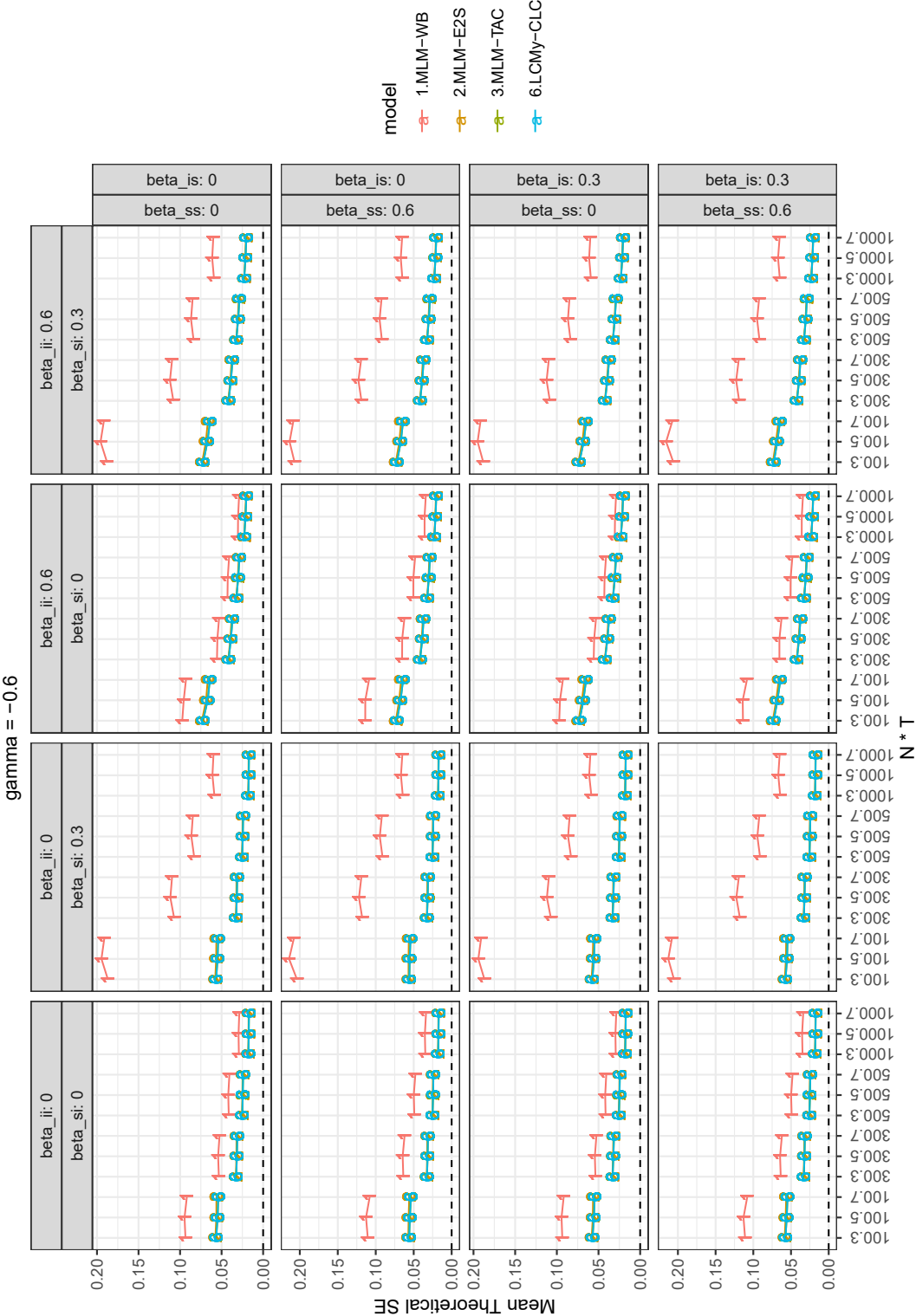


Figure D.90

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0$)

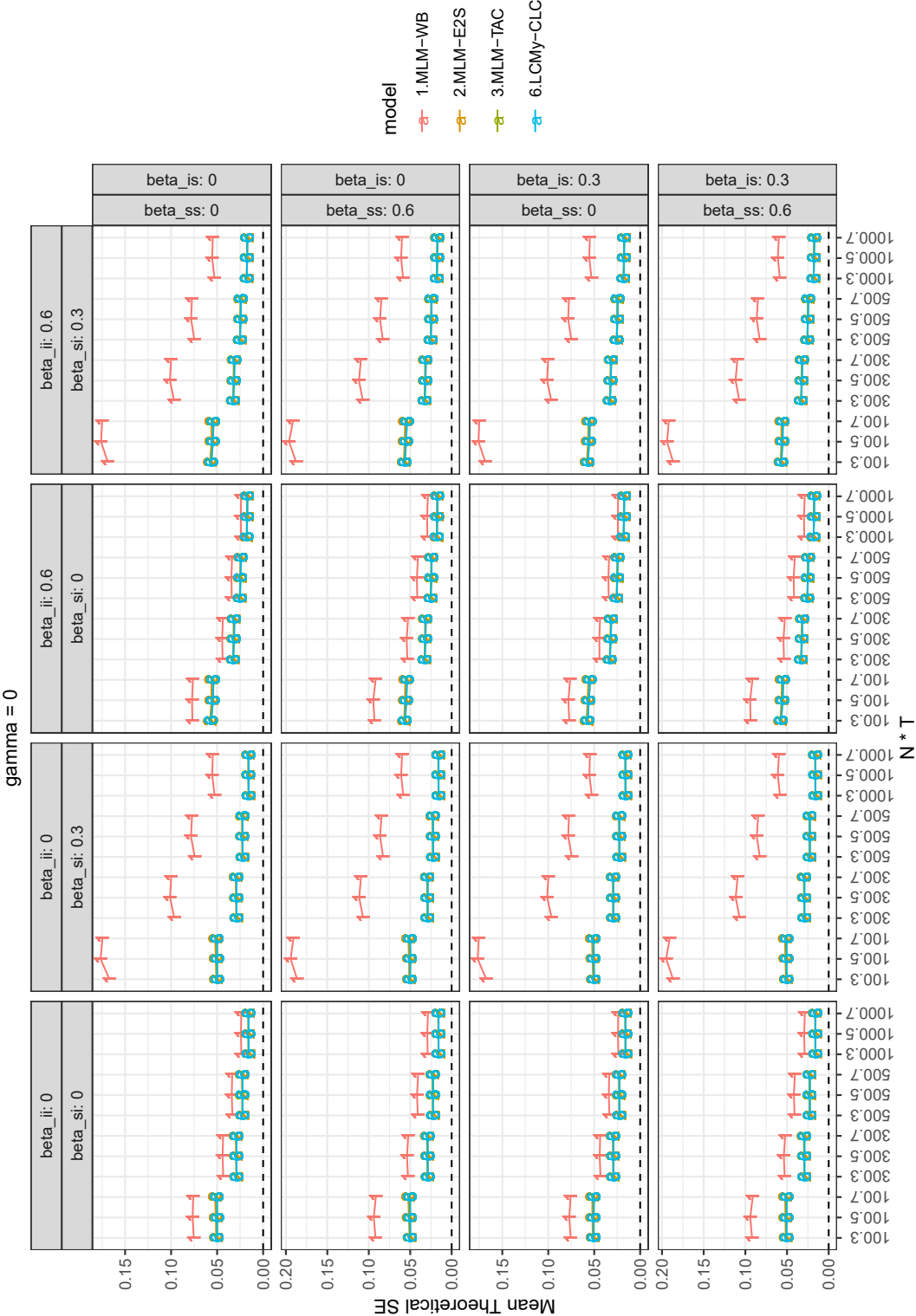


Figure D.91

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)

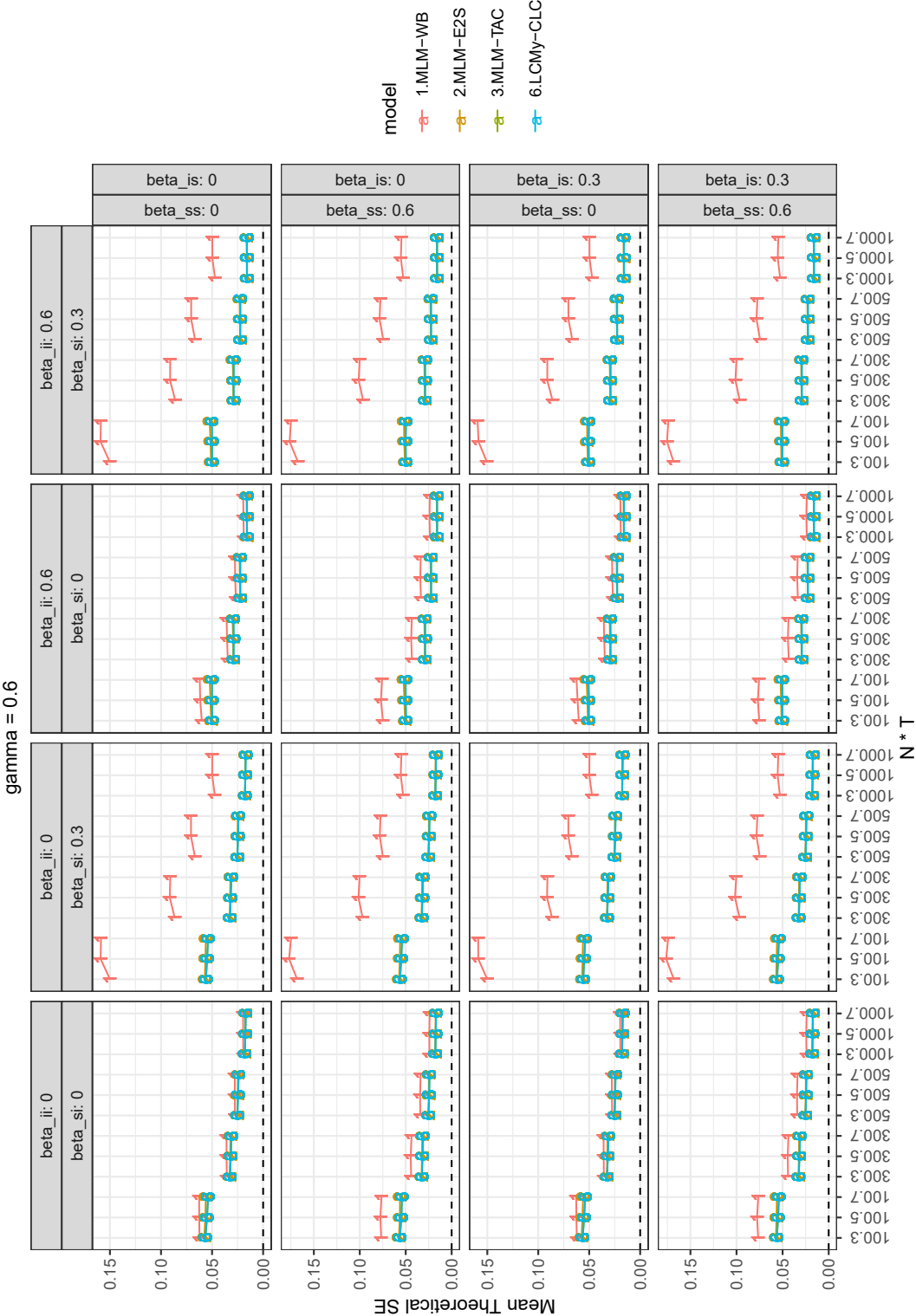


Figure D.92

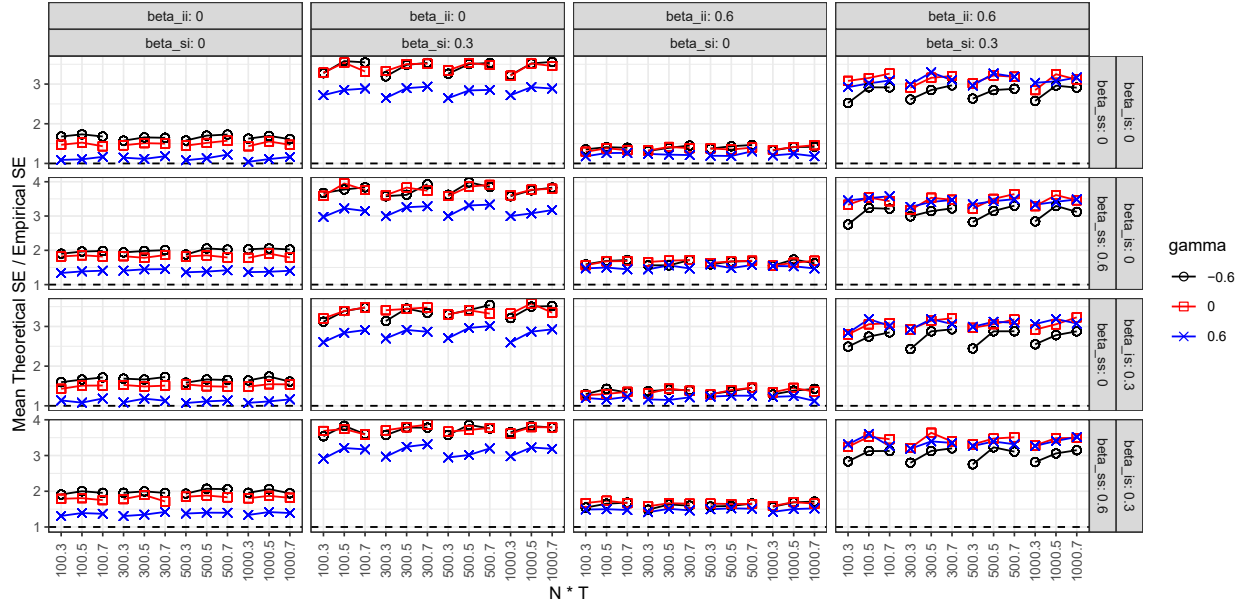
SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (MLM-WB)

Figure D.93

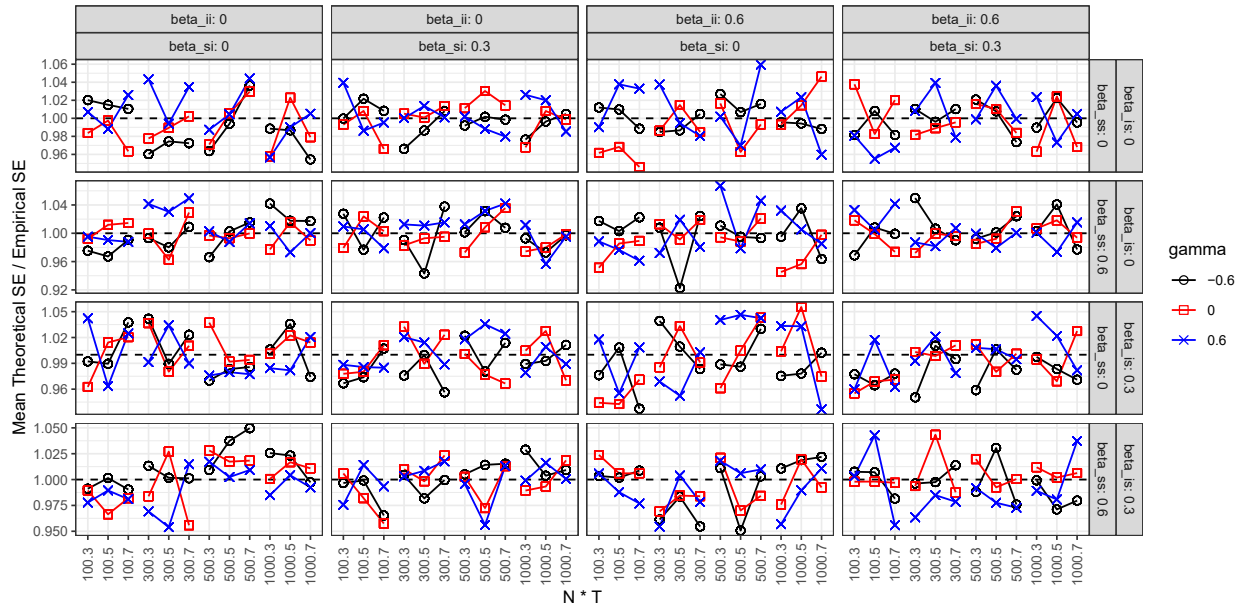
SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (MLM-E2S)

Figure D.94

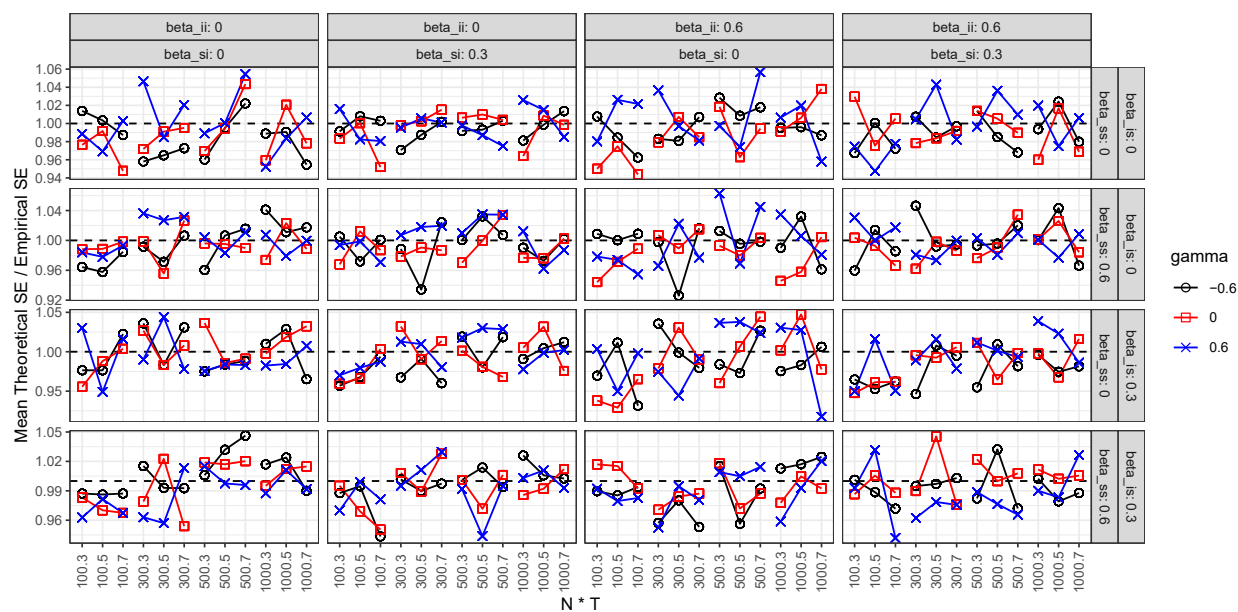
SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (MLM-TAC)

Figure D.95

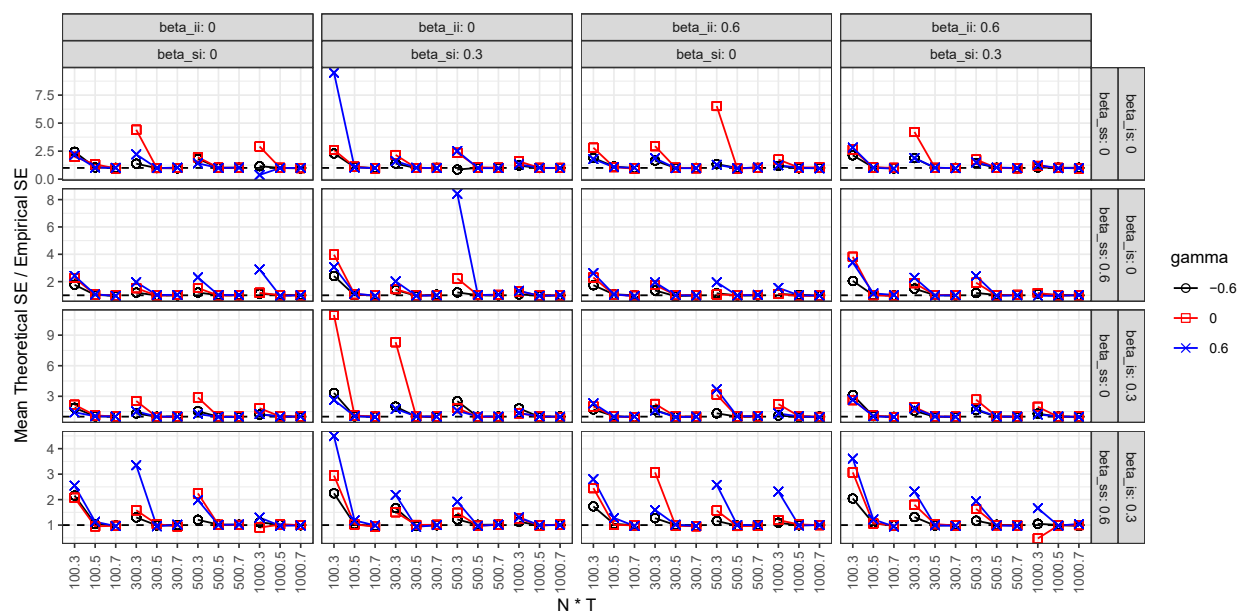
SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)

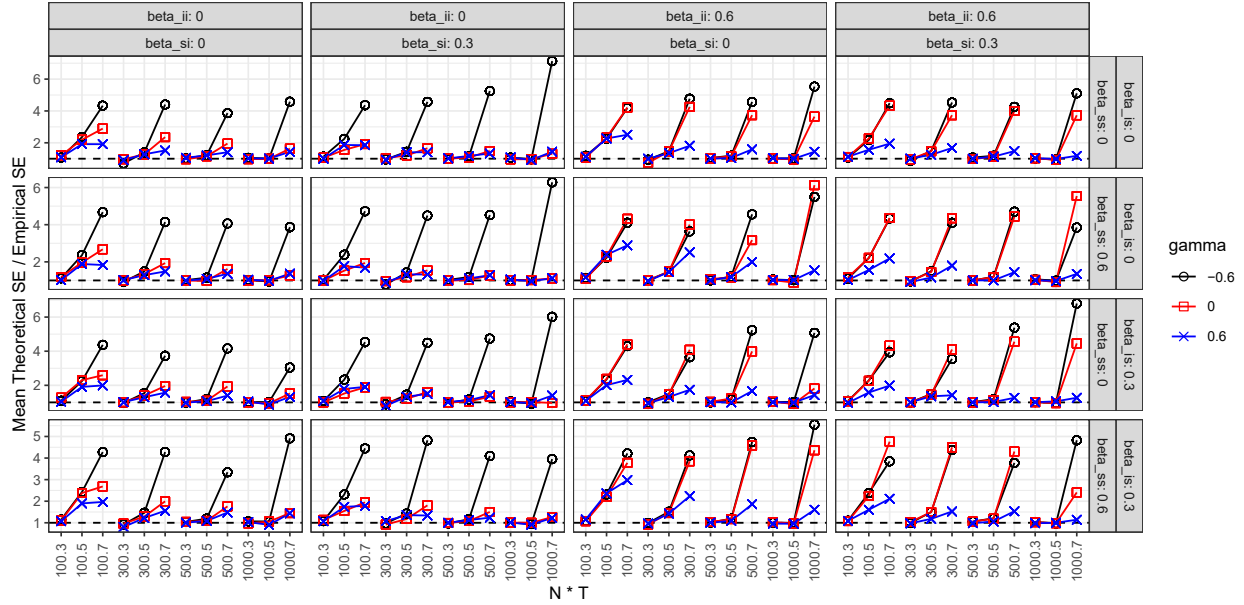
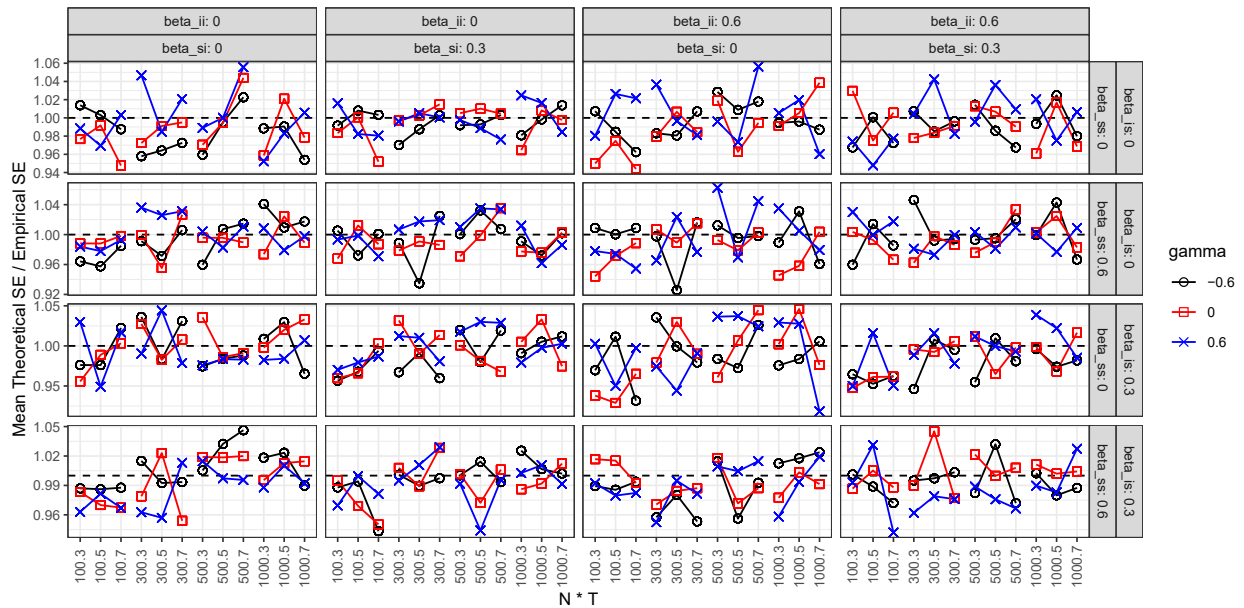
Figure D.96*SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)***Figure D.97***SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCMy-CLC)*

Figure D.98

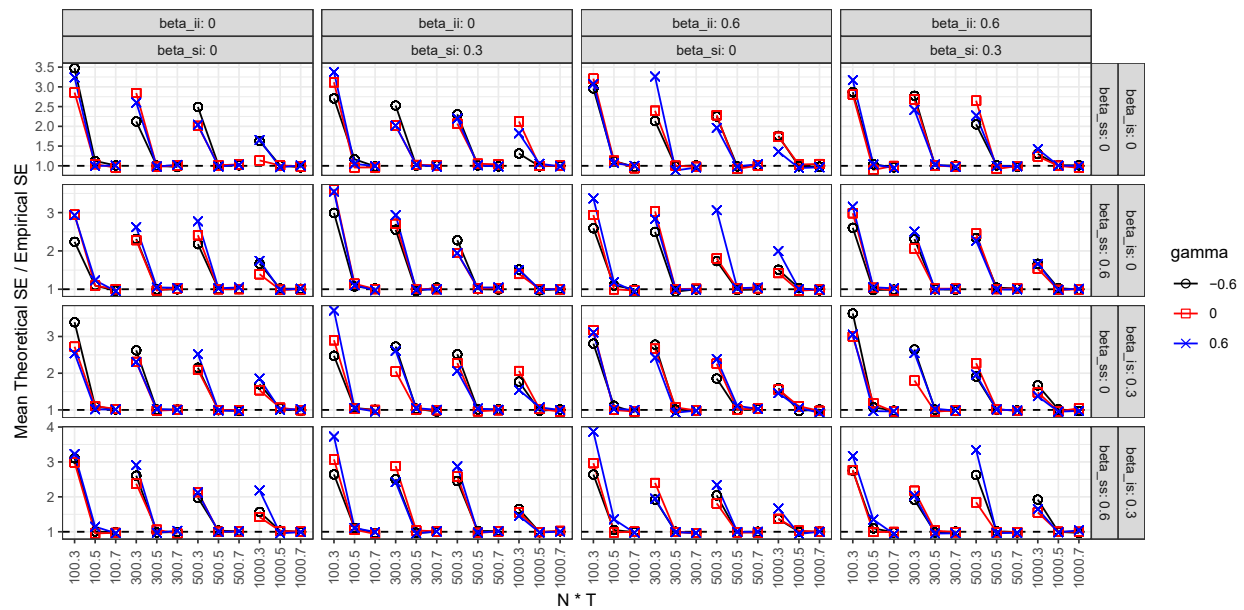
SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCM_{xy}-RP)

Figure D.99

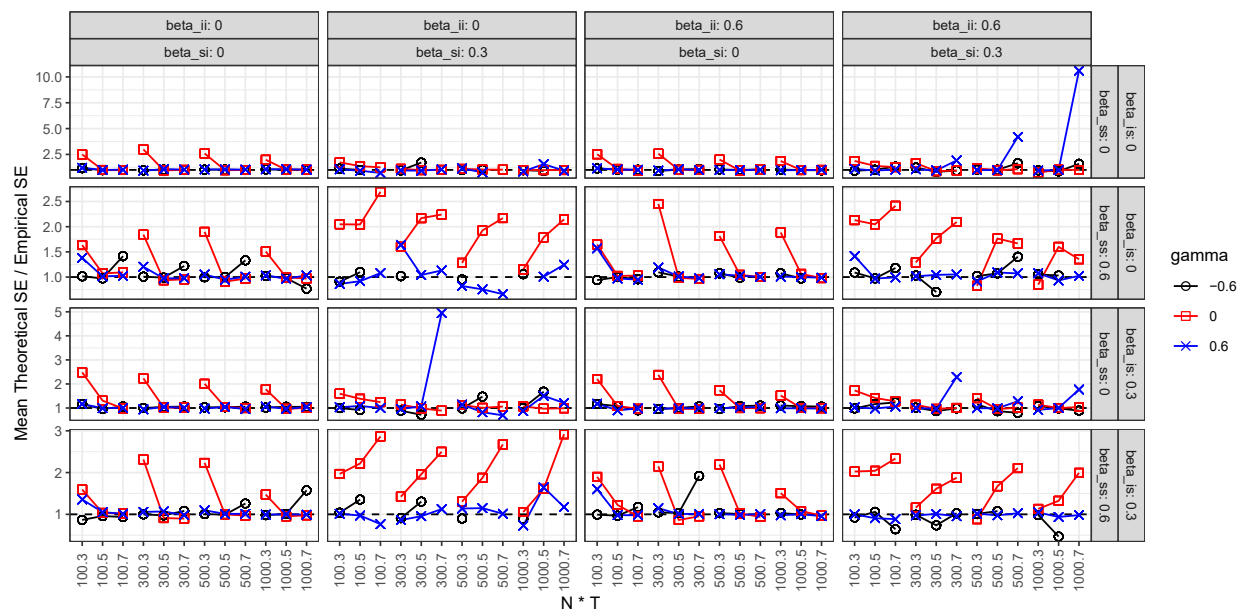
SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCM_x-Rly)

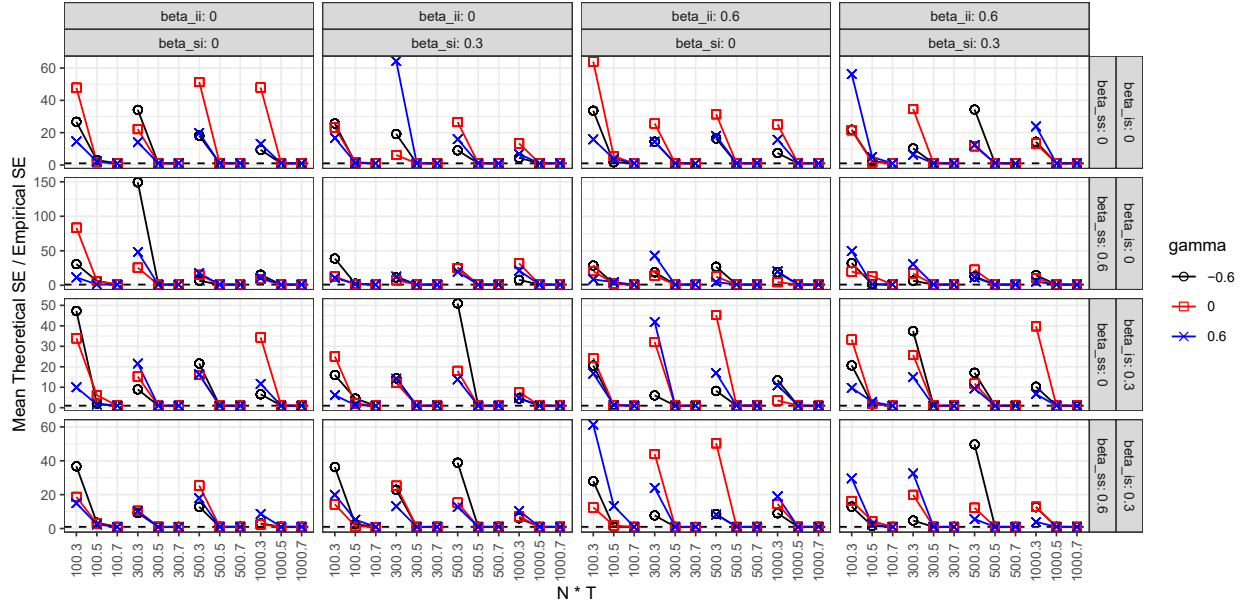
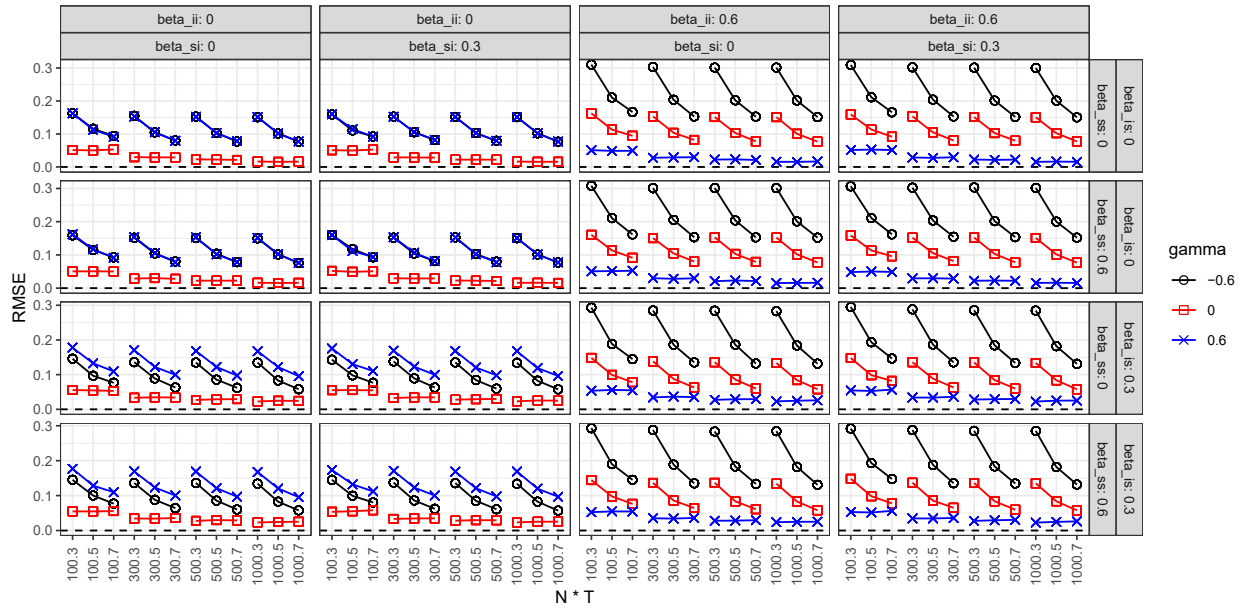
Figure D.100*SE Ratio for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)***Figure D.101***RMSE for the Between-Person Effect β_{II} in Simulation 4 (MLM-WB)*

Figure D.102

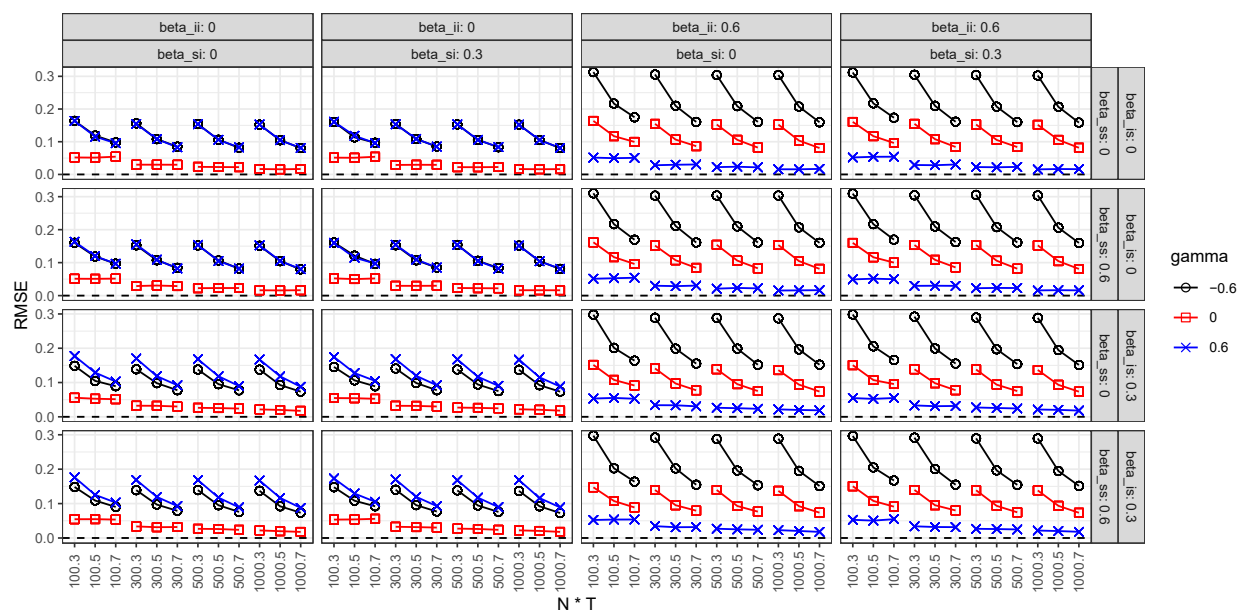
RMSE for the Between-Person Effect β_{II} in Simulation 4 (MLM-E2S)

Figure D.103

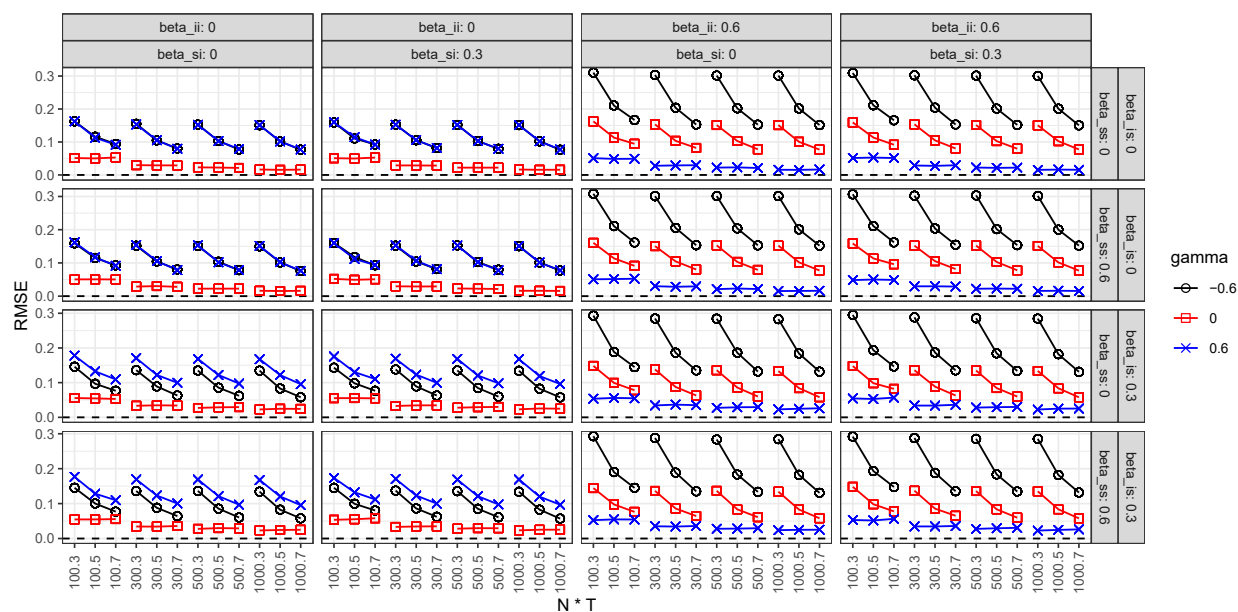
RMSE for the Between-Person Effect β_{II} in Simulation 4 (MLM-TAC)

Figure D.104

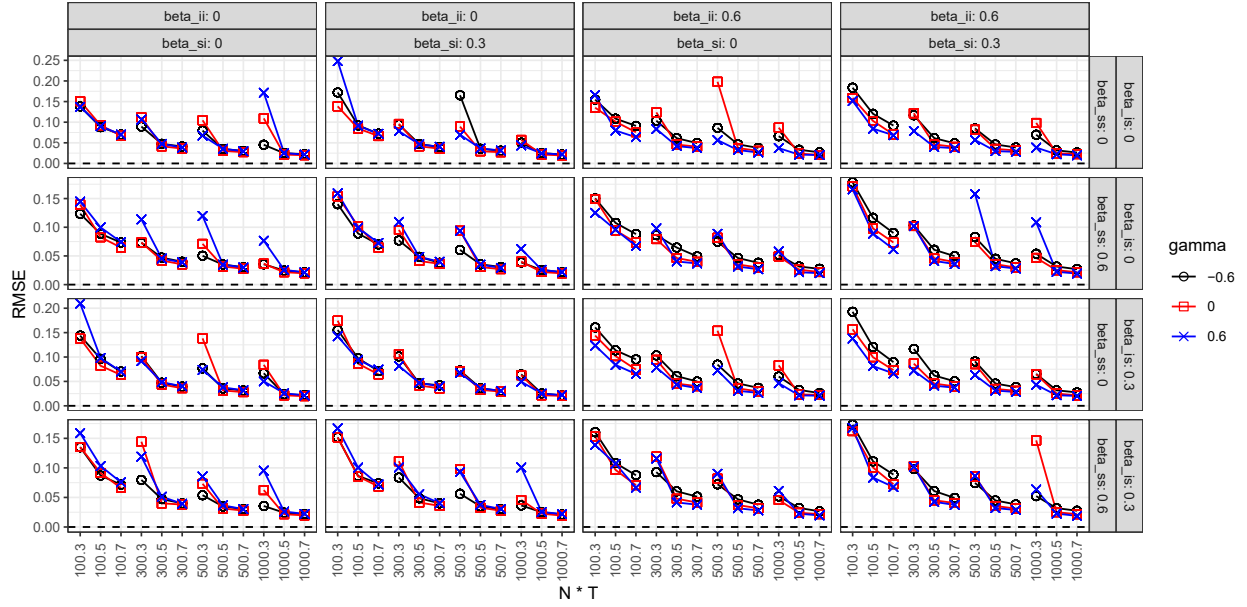
RMSE for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)

Figure D.105

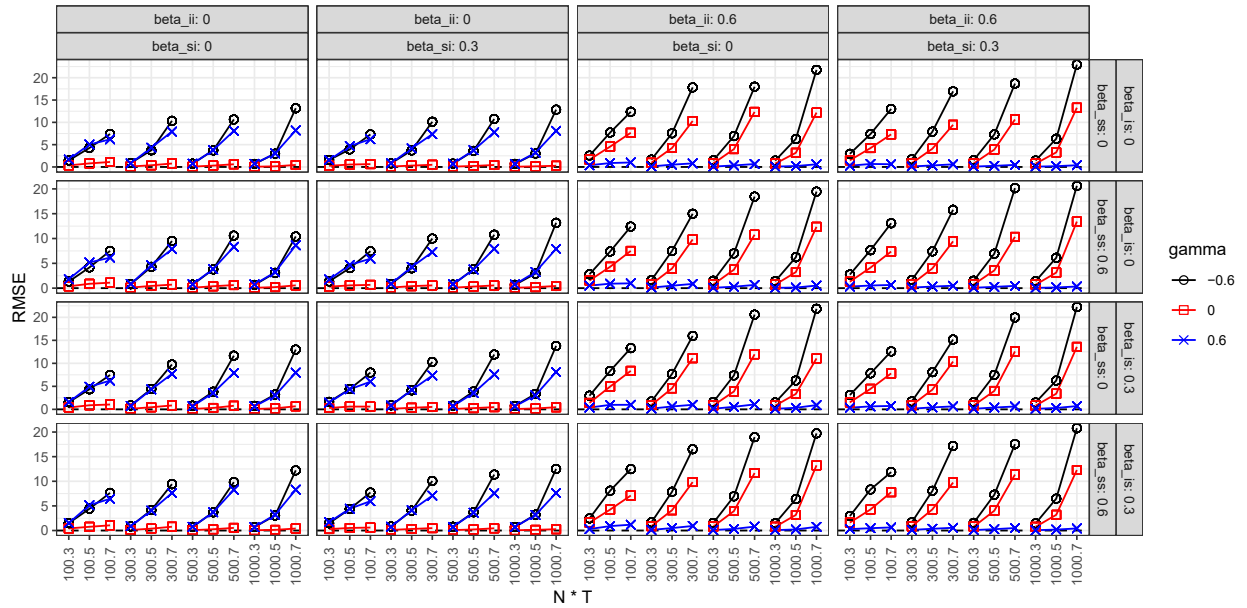
RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)

Figure D.106

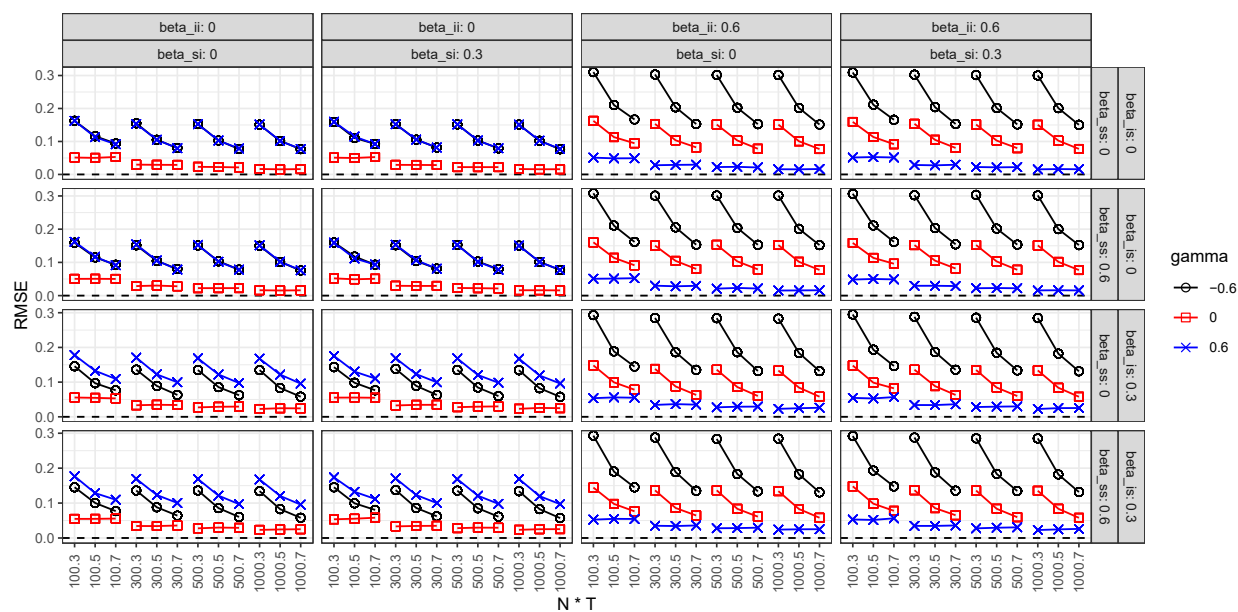
RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMy-CLC)

Figure D.107

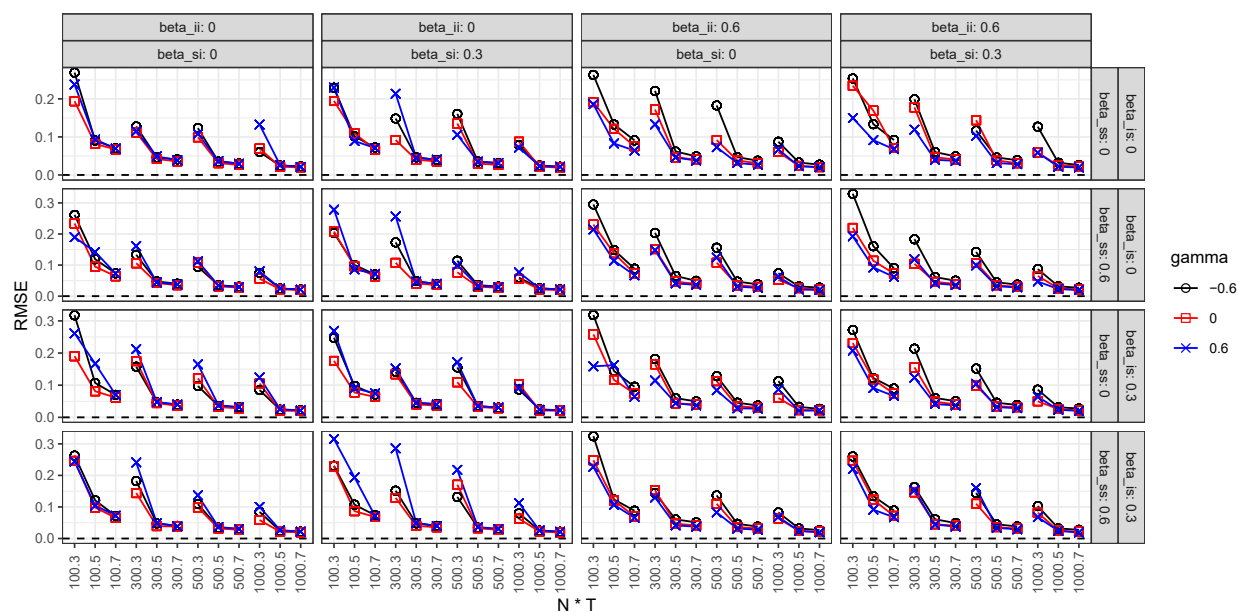
RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMxy-RP)

Figure D.108

RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-Rly)

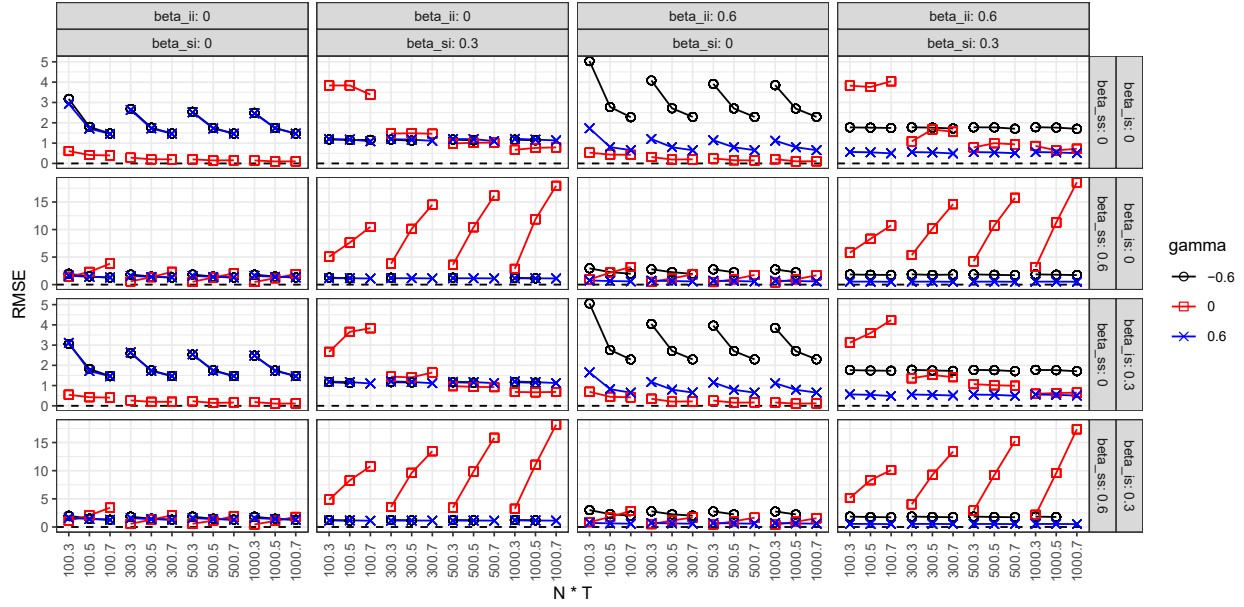


Figure D.109

RMSE for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)

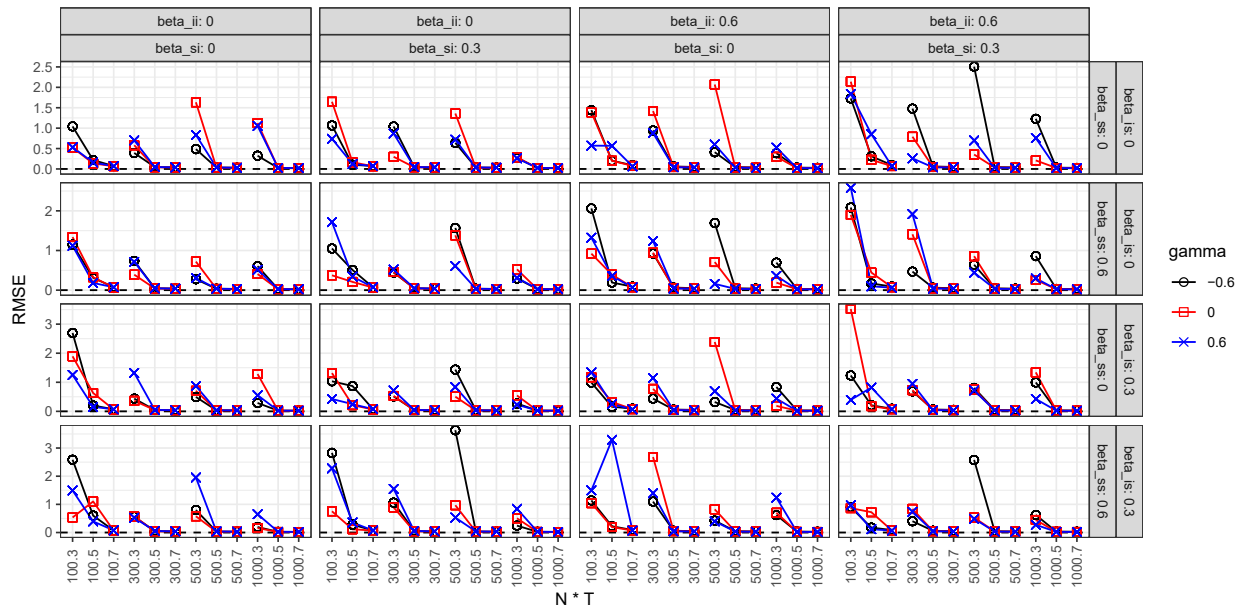


Figure D.110

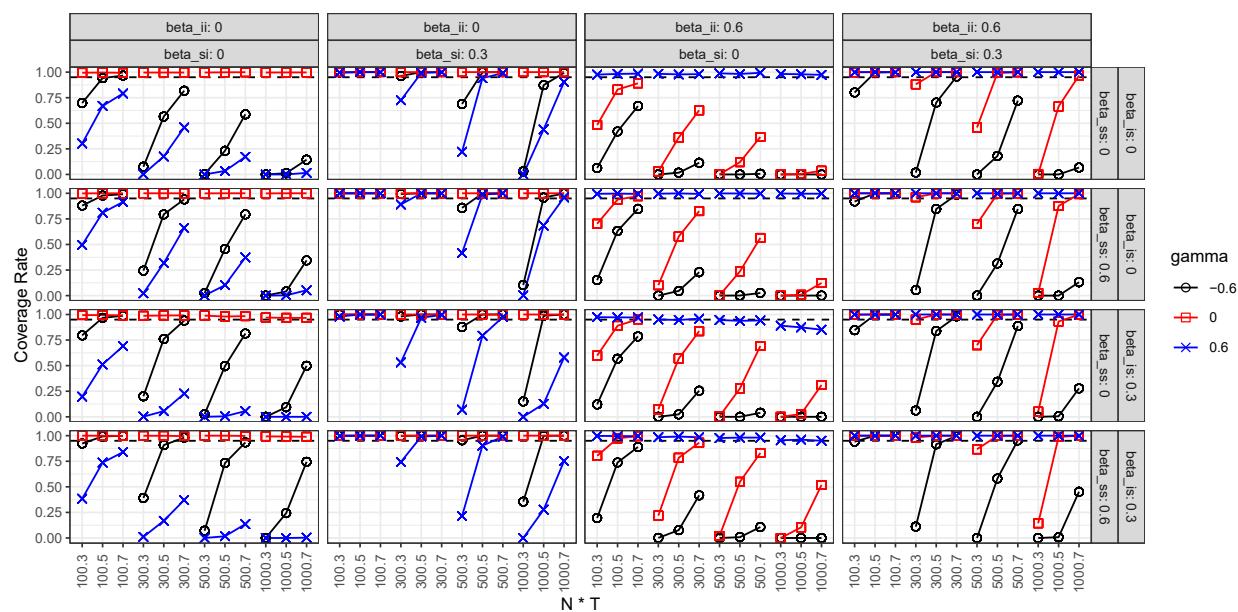
Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (MLM-WB)

Figure D.111

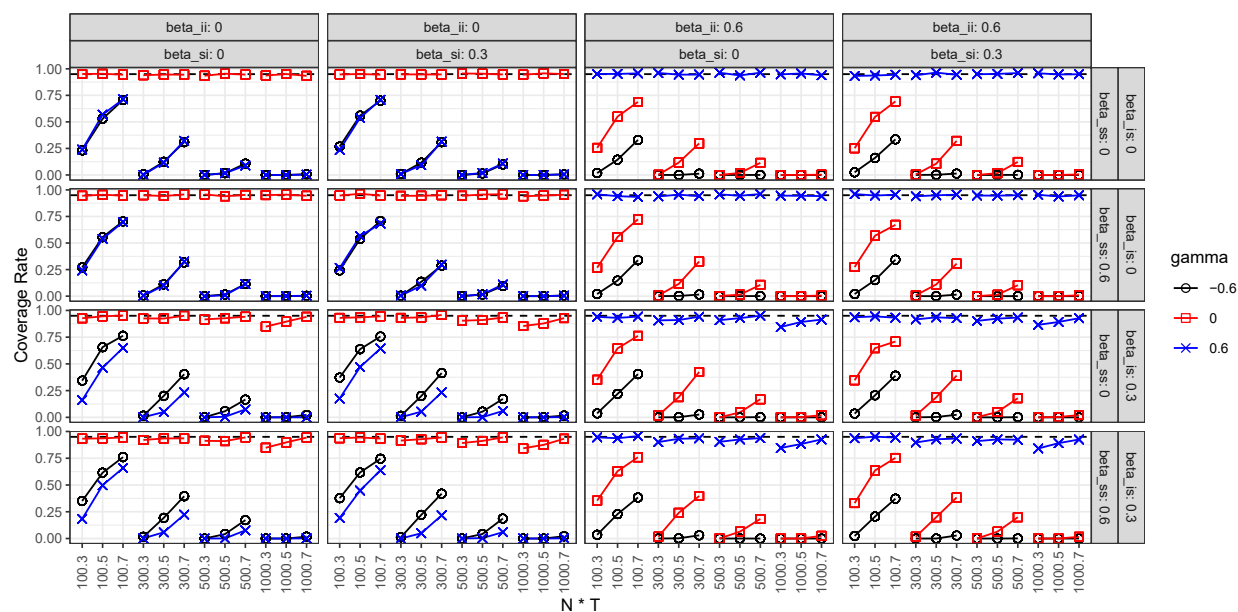
Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (MLM-E2S)

Figure D.112

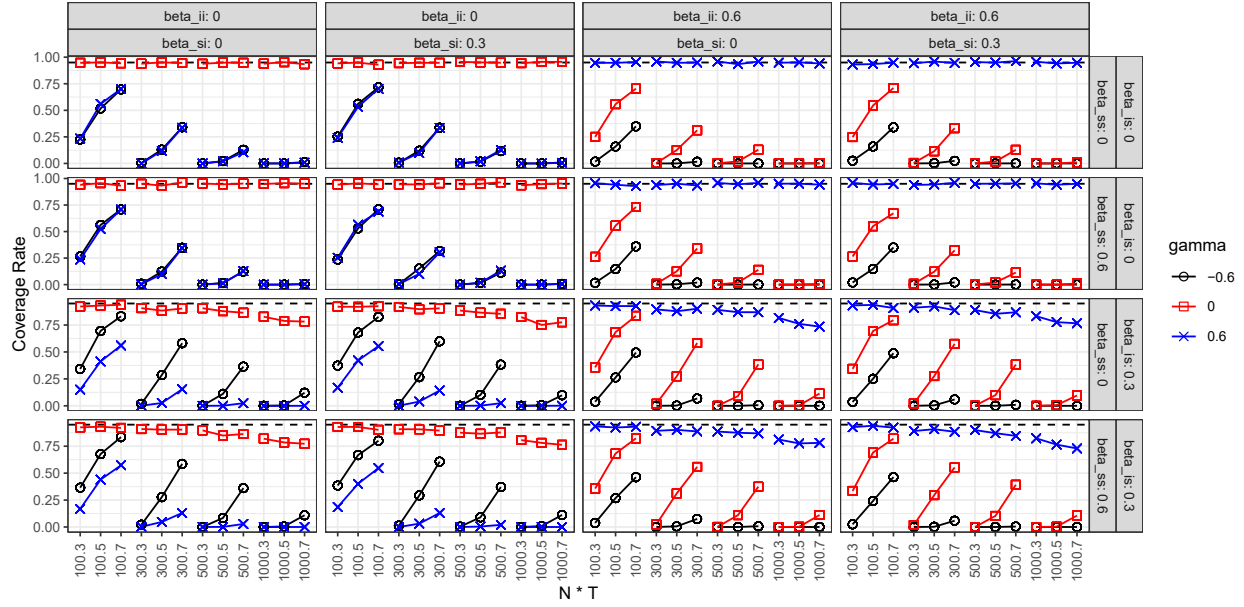
Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (MLM-TAC)

Figure D.113

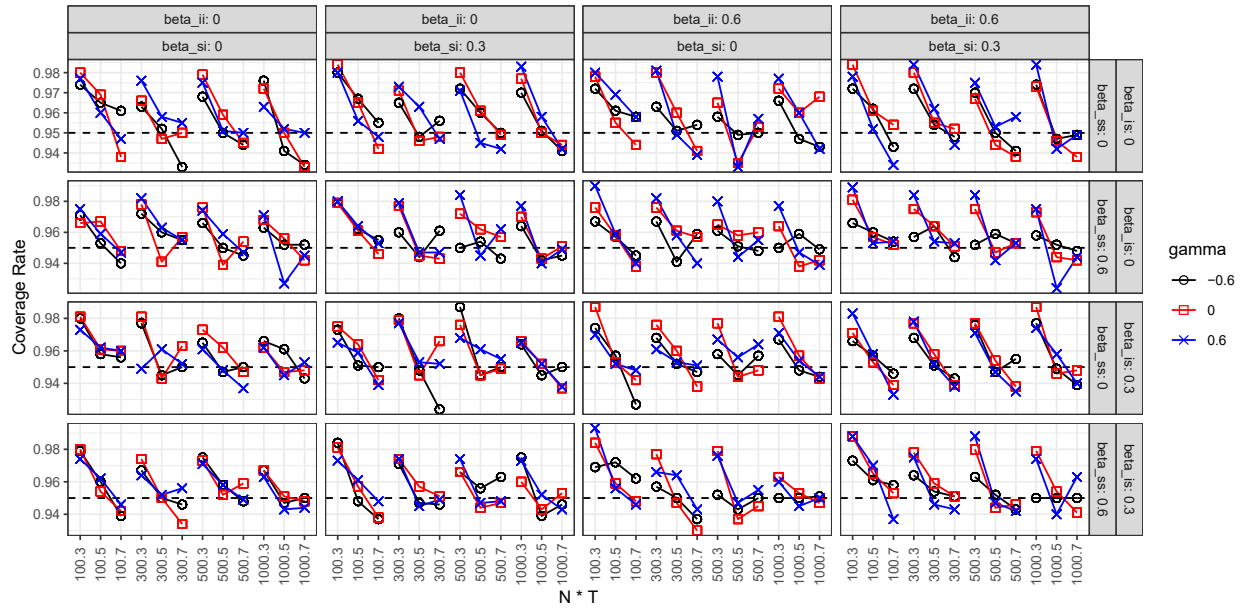
Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (MLM-MV)

Figure D.114

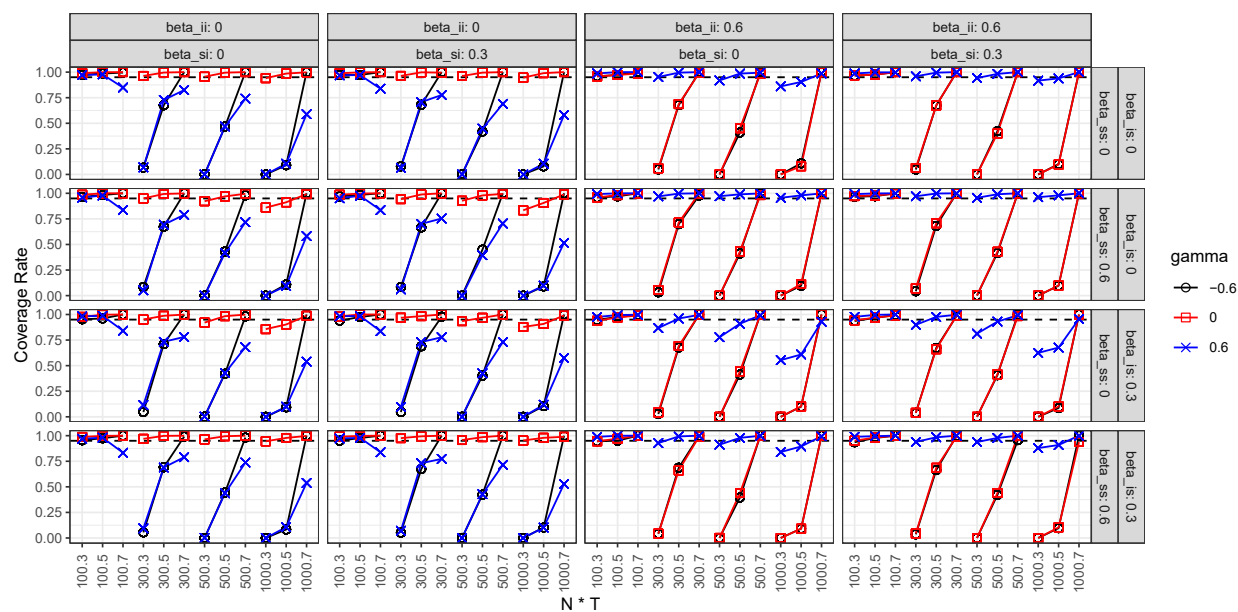
Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMy-RIx)

Figure D.115

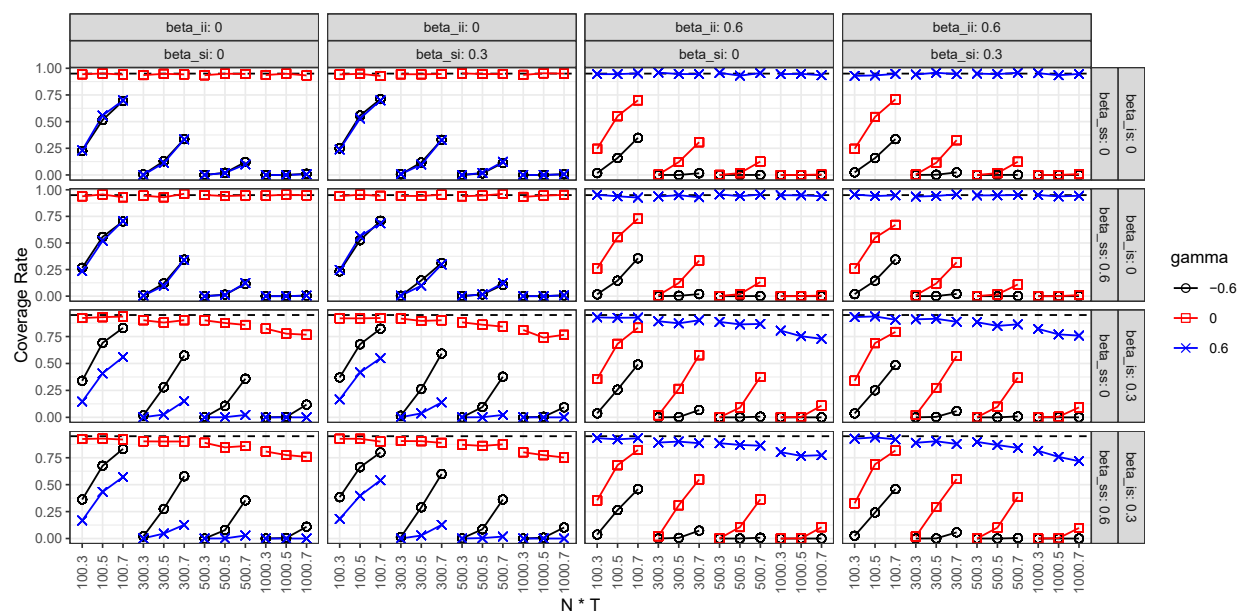
Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMy-CLC)

Figure D.116

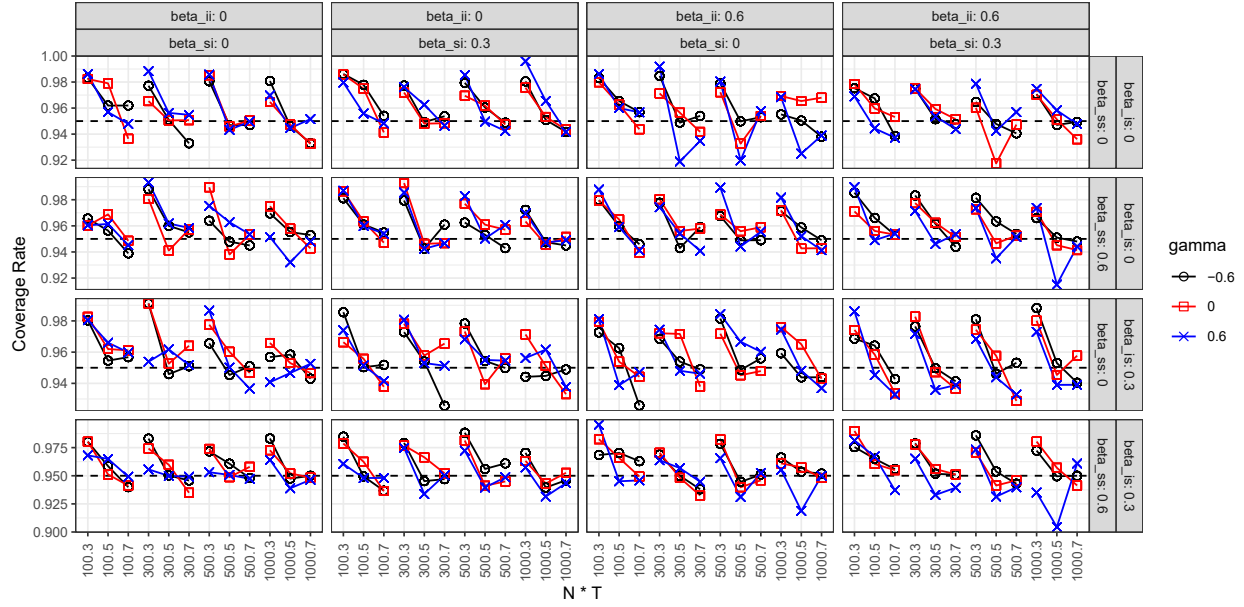
Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMxy-RP)

Figure D.117

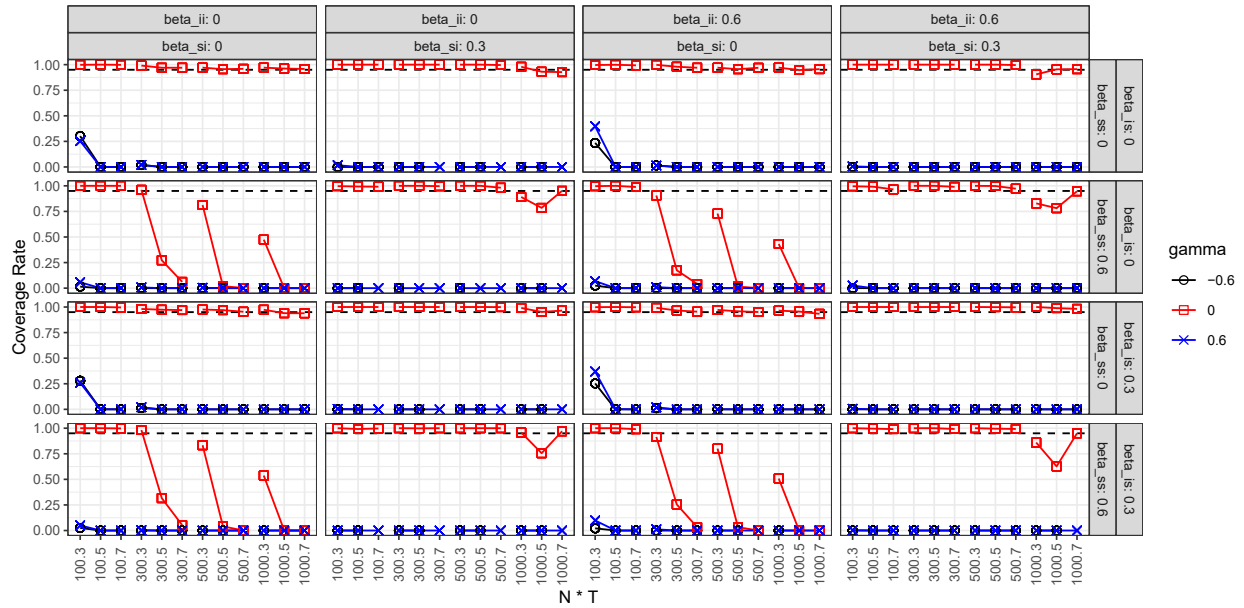
Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMx-Rly)

Figure D.118

Coverage Rate for the Between-Person Effect β_{II} in Simulation 4 (LCMx-CLC)

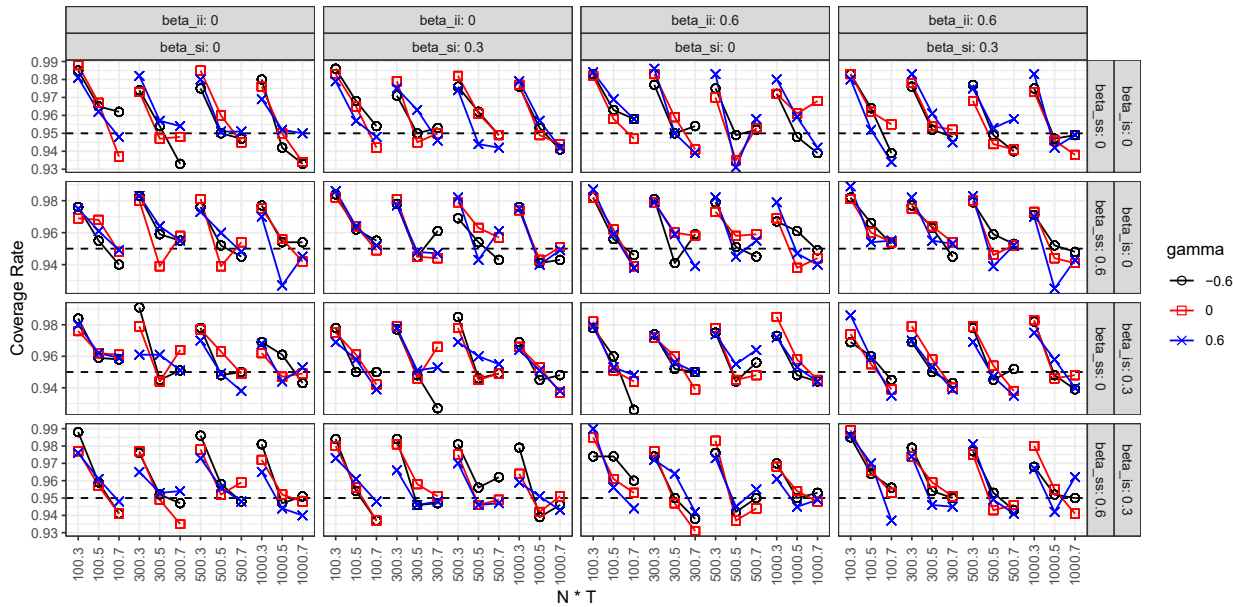


Figure D.119

Power for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = -0.6$)

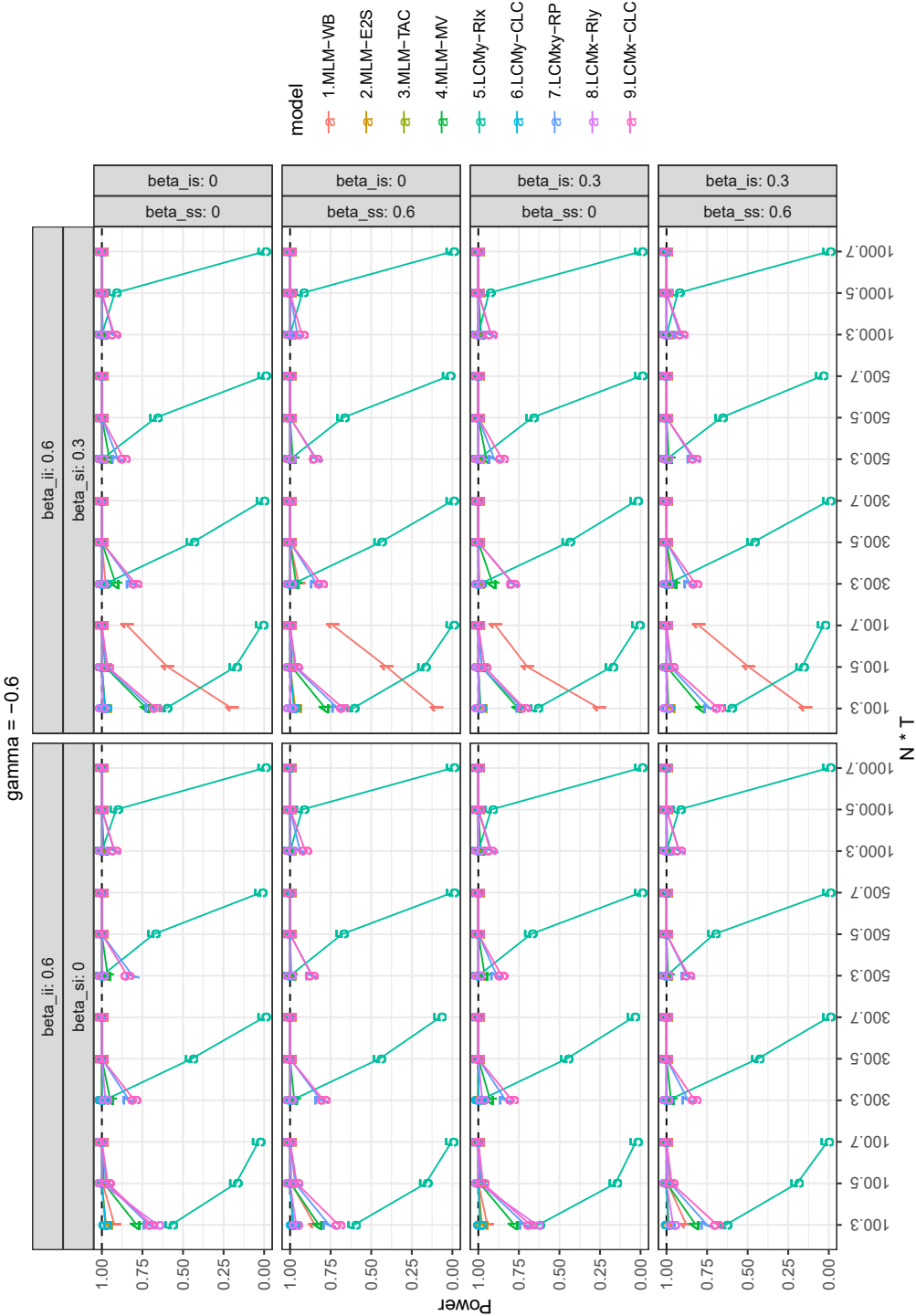


Figure D.120

Power for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0$)

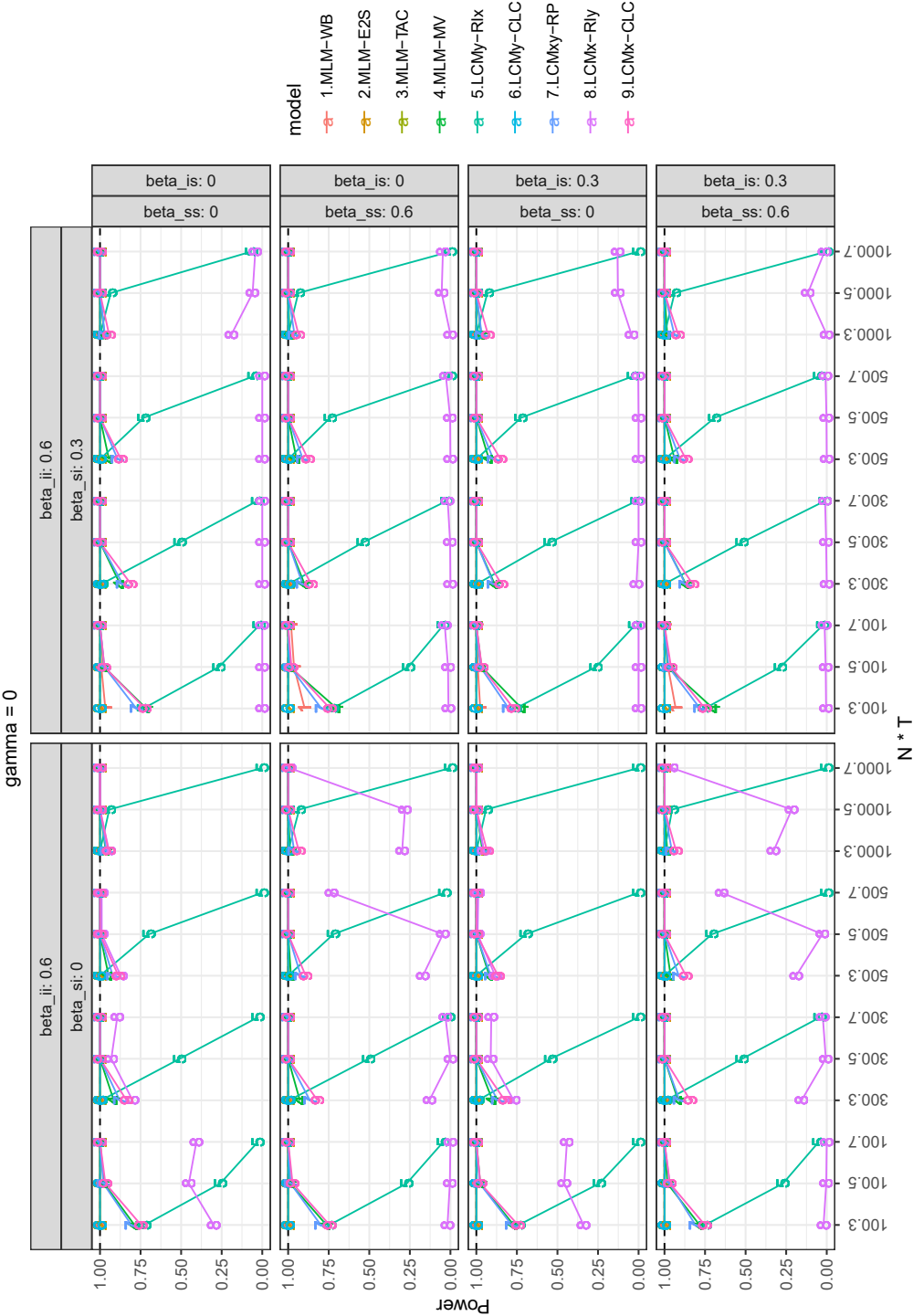
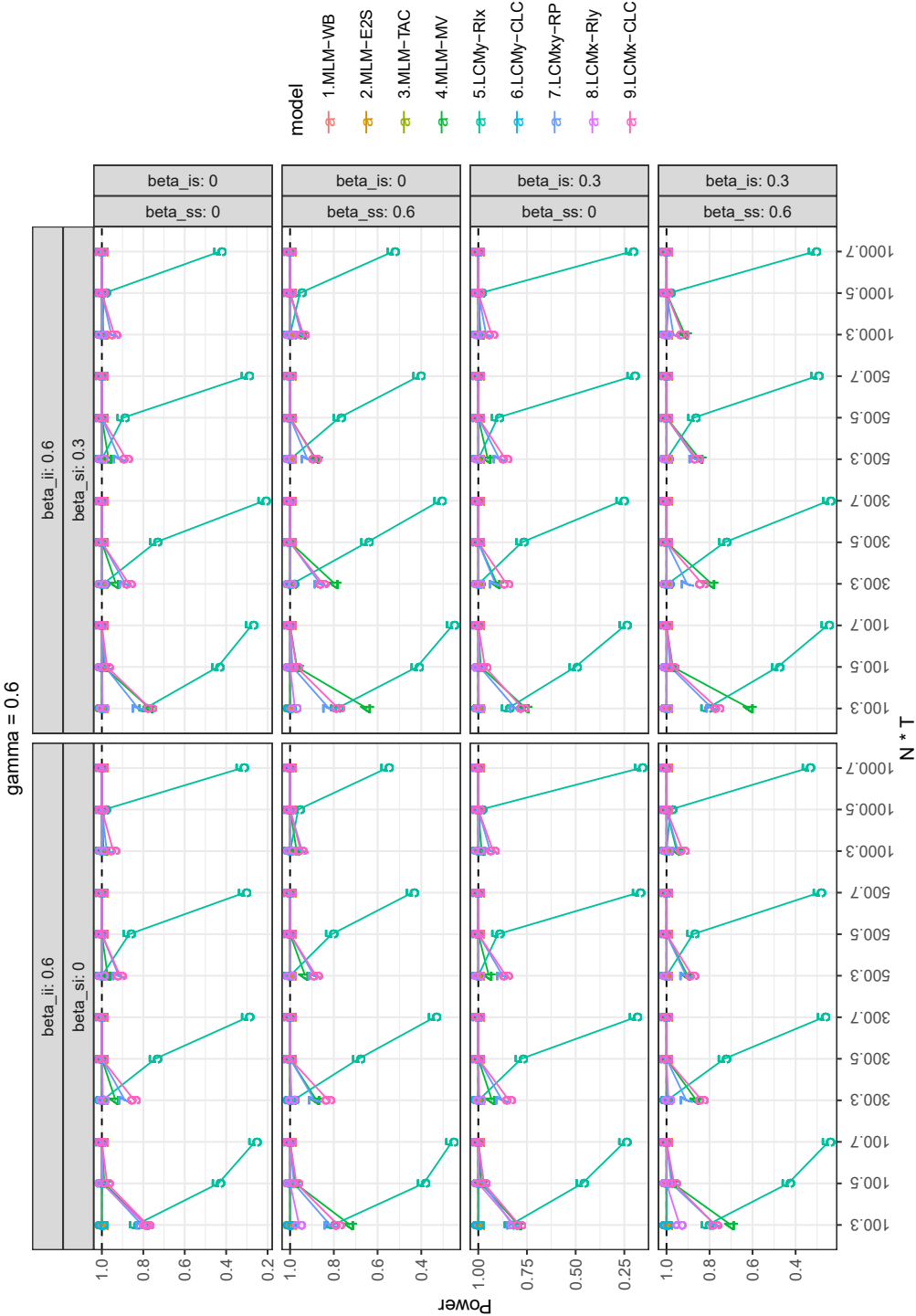


Figure D.121

Power for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)



D.4 Between-Person Effect of Intercept on Slope

Figure D.122

Bias for the Between-Person Effect β_{SI} in Simulation 4 (MLM-E2S)

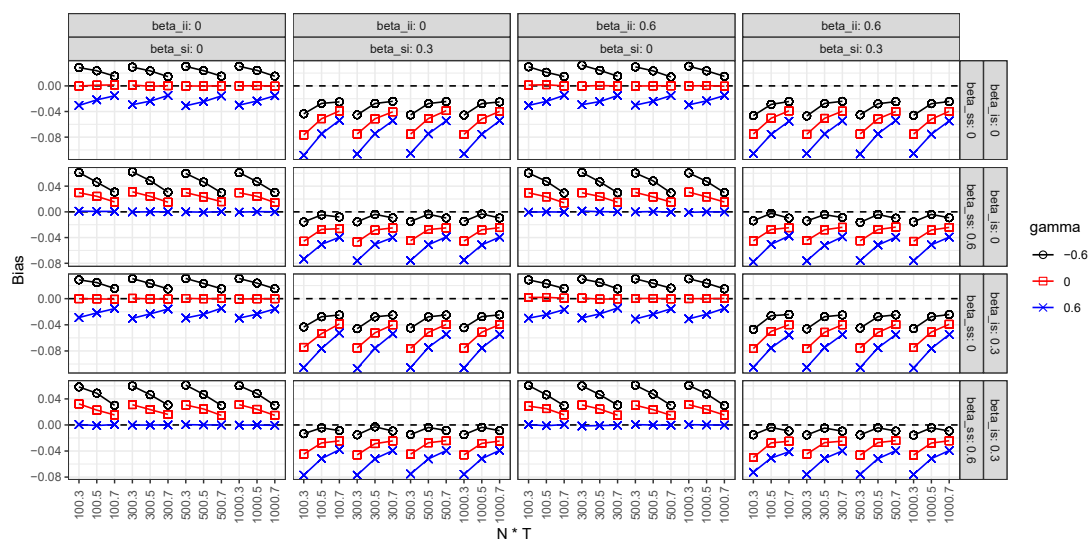


Figure D.123

Bias for the Between-Person Effect β_{SI} in Simulation 4 (MLM-TAC)

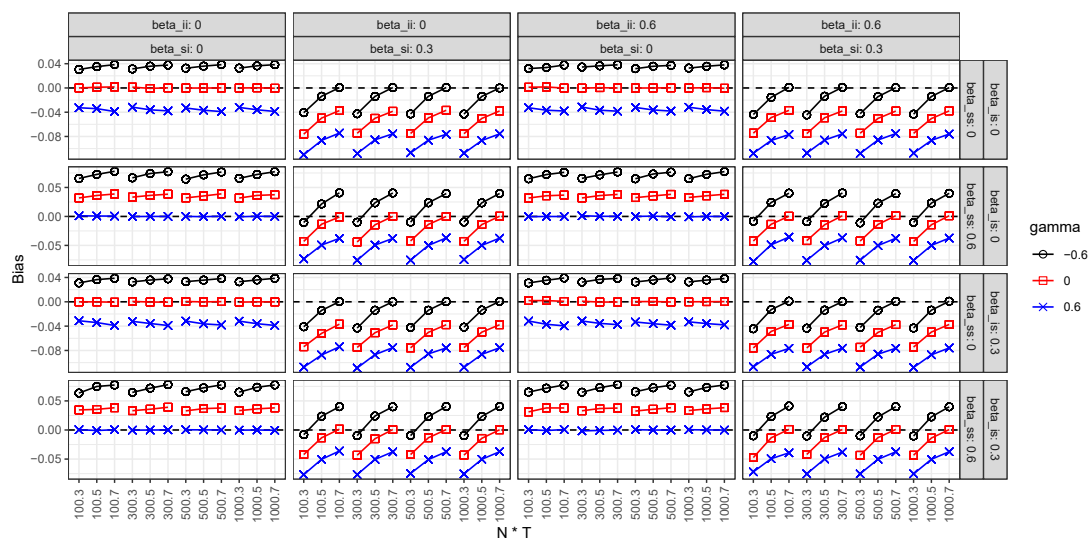


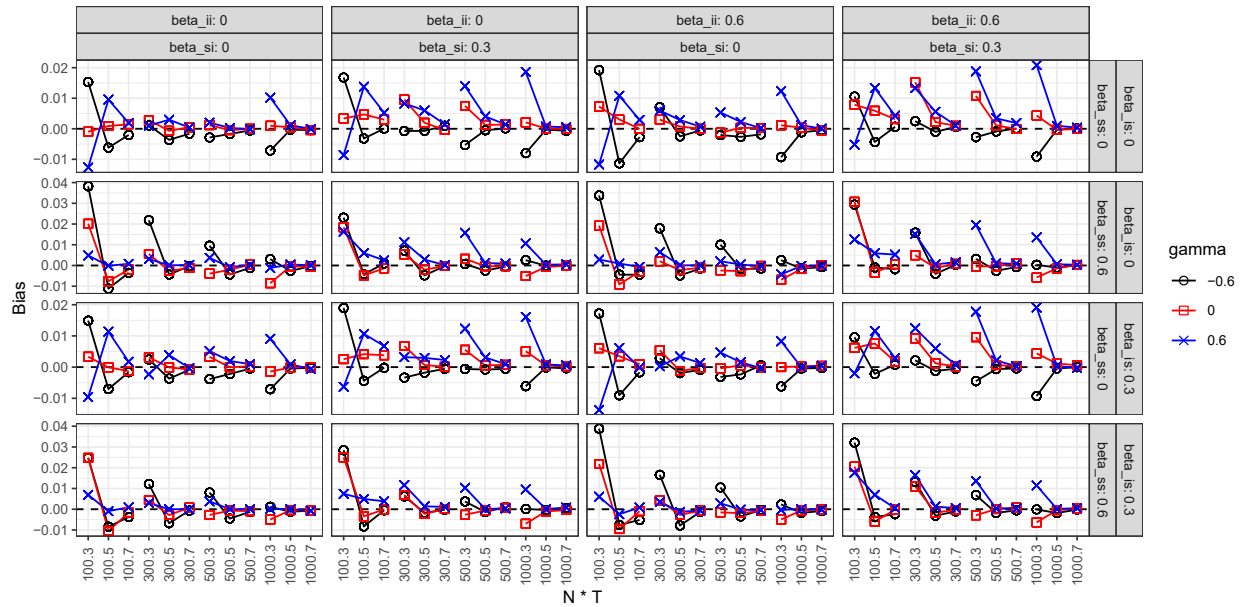
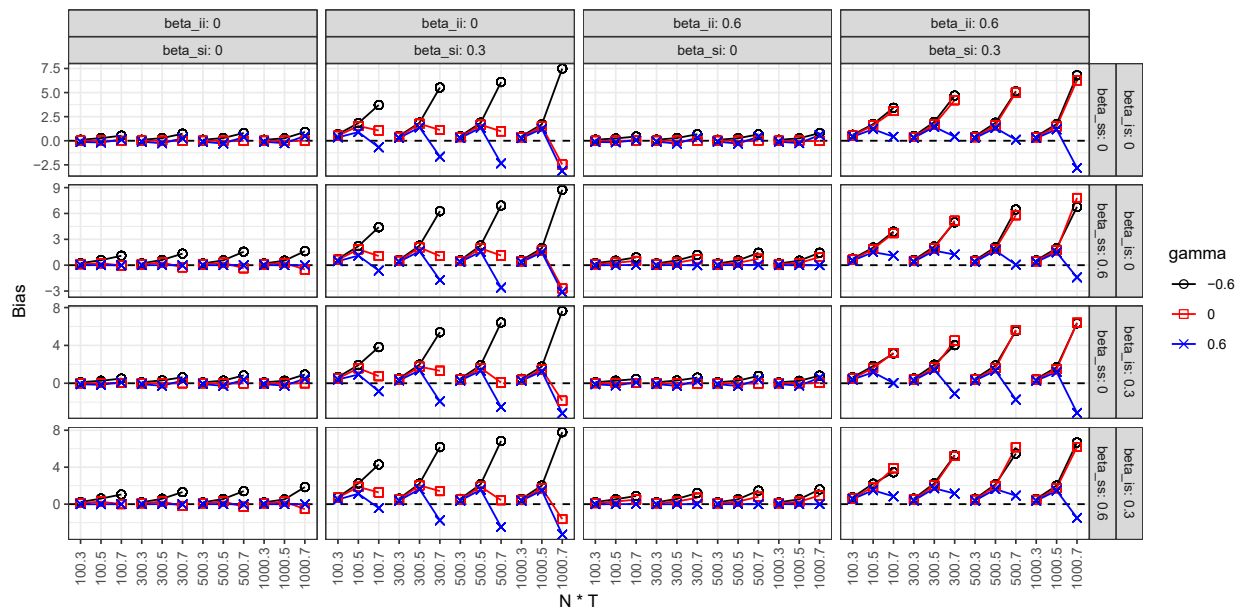
Figure D.124*Bias for the Between-Person Effect β_{SI} in Simulation 4 (MLM-MV)***Figure D.125***Bias for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)*

Figure D.126

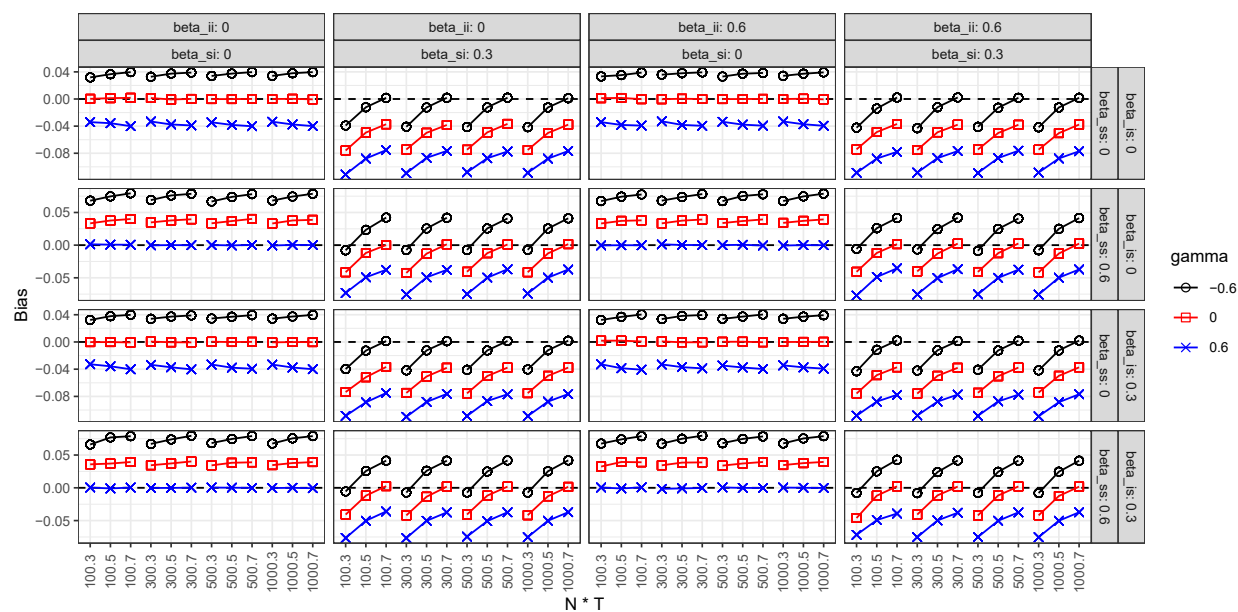
Bias for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-CLC)

Figure D.127

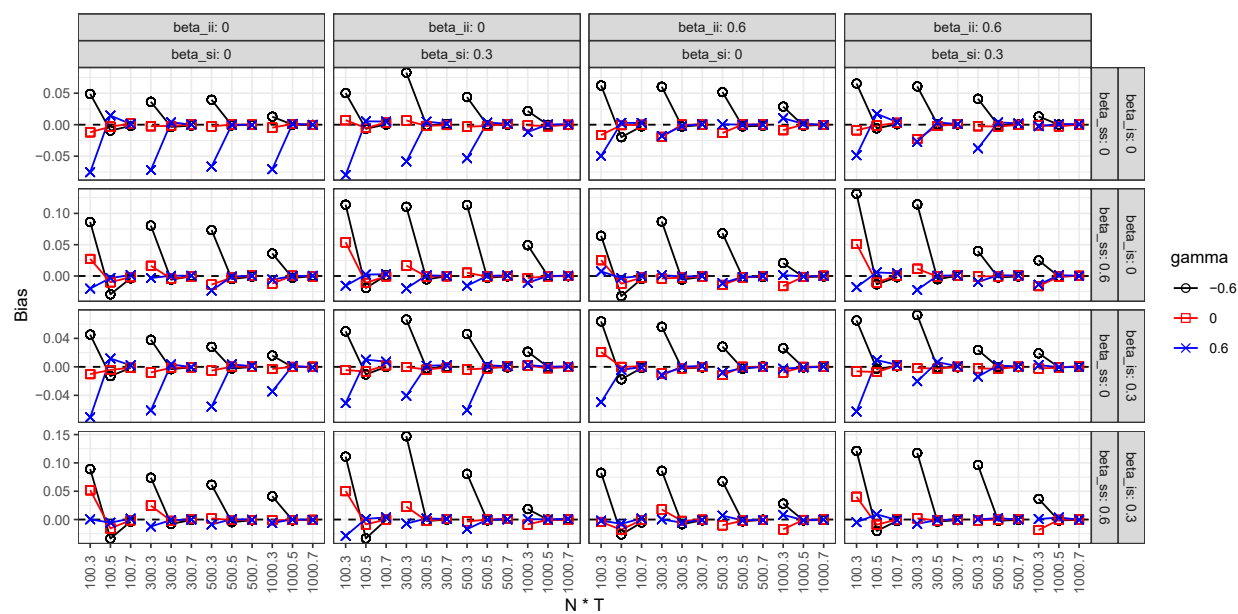
Bias for the Between-Person Effect β_{SI} in Simulation 4 (LCMxy-RP)

Figure D.128

Bias for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)

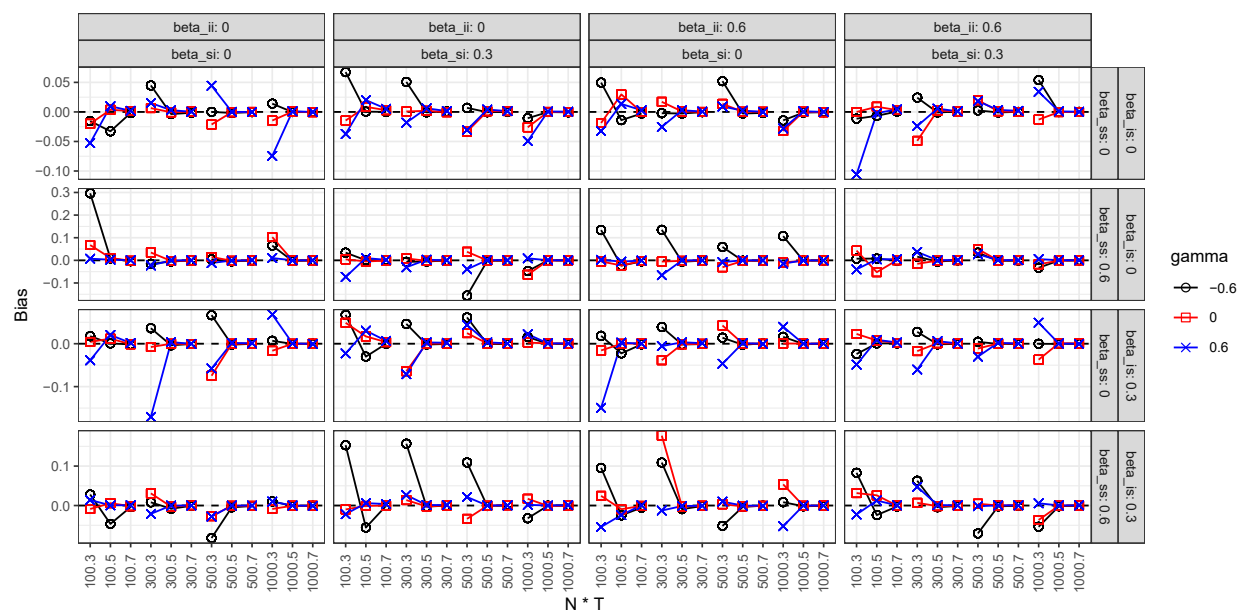


Figure D.129

Empirical SE for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)

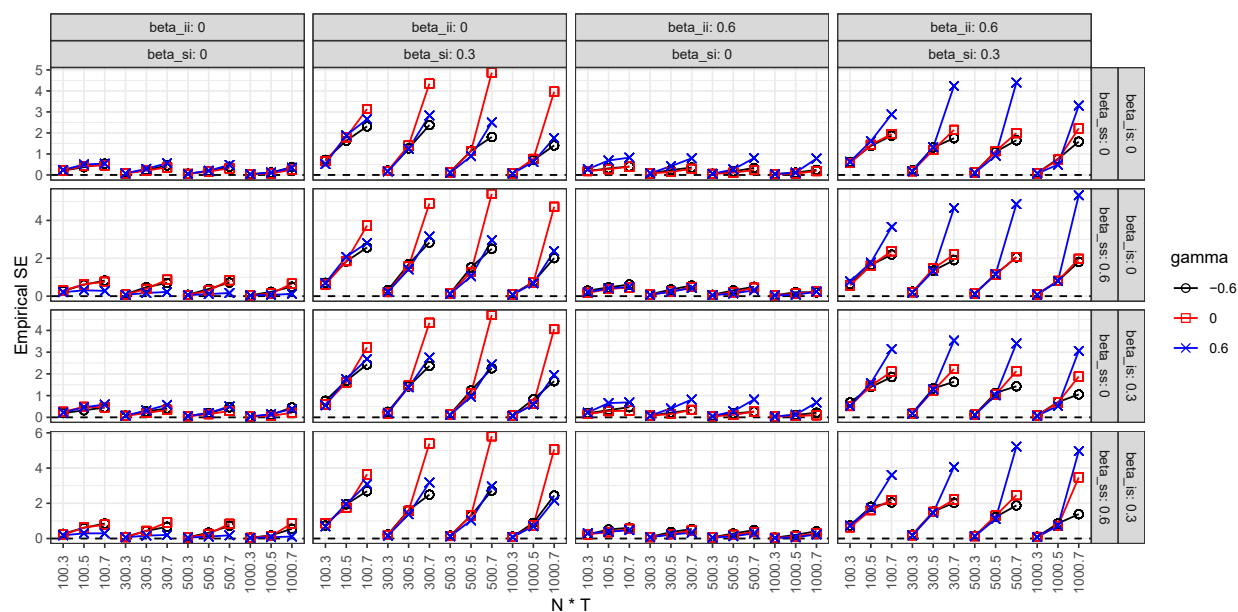


Figure D.130

Empirical SE for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)

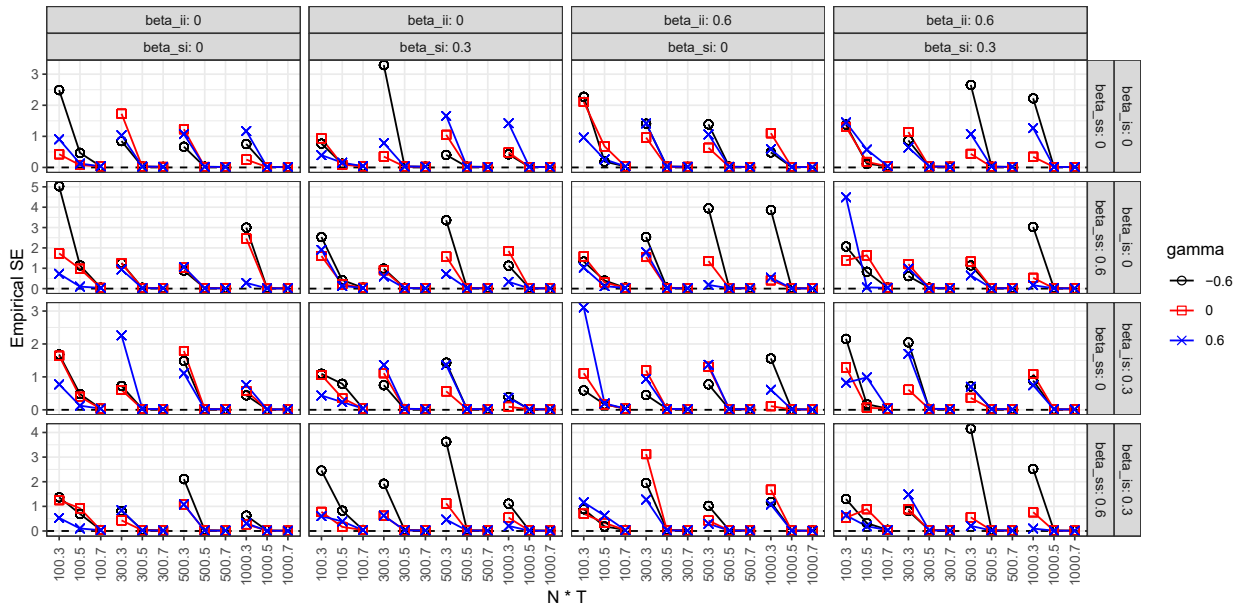


Figure D.131

Empirical SE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = -0.6$)

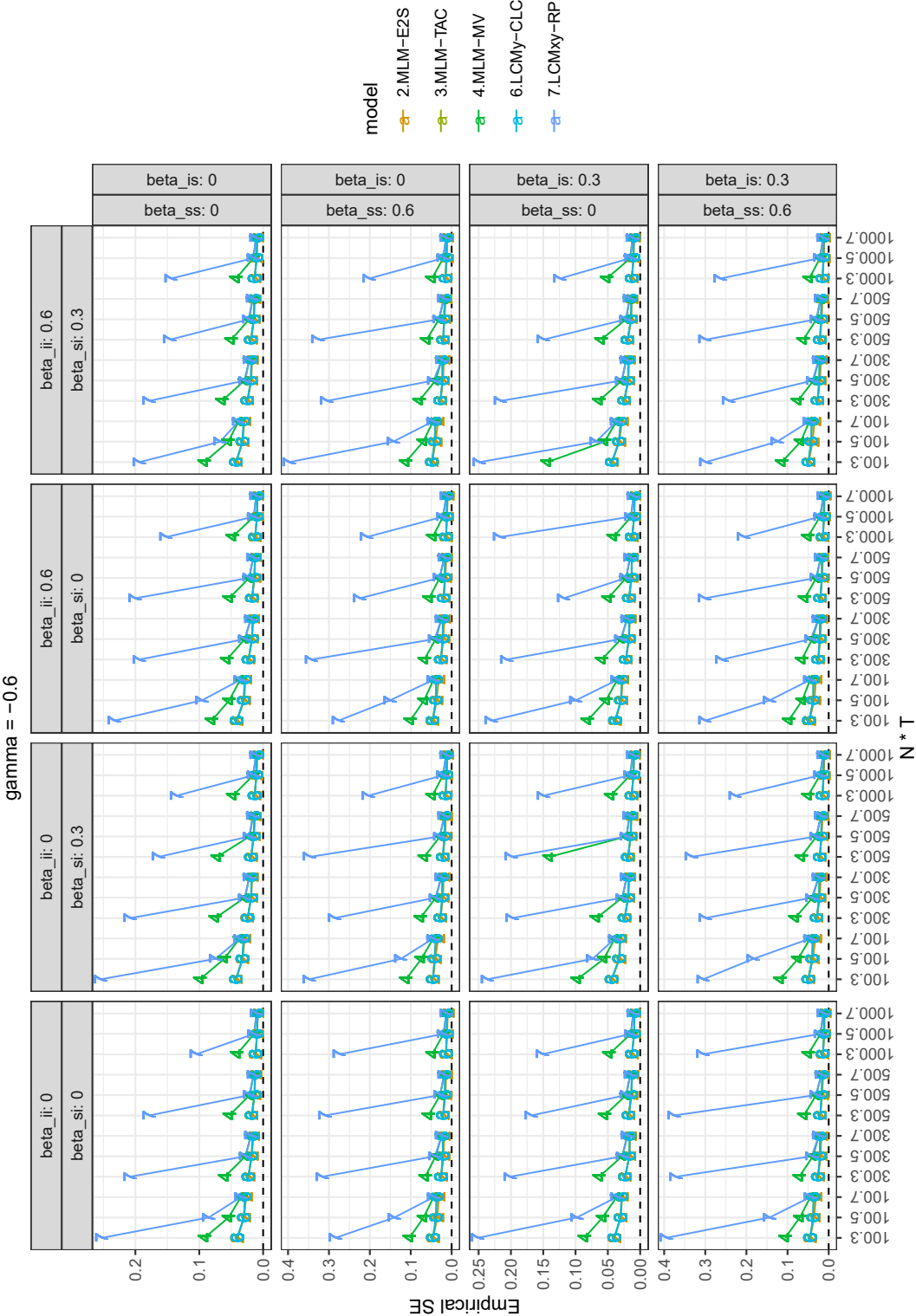


Figure D.132

Empirical SE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0$)

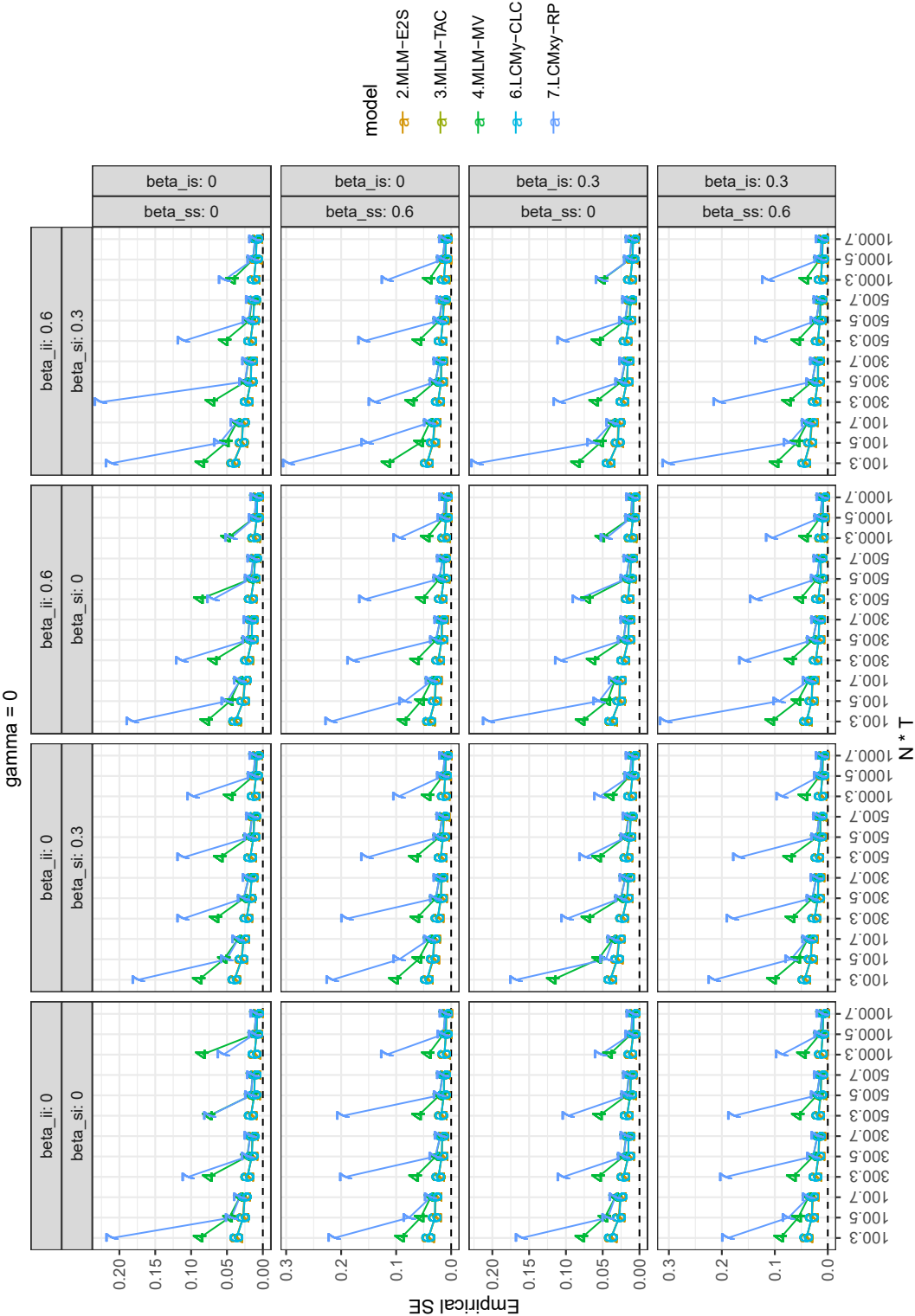


Figure D.133

Empirical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)

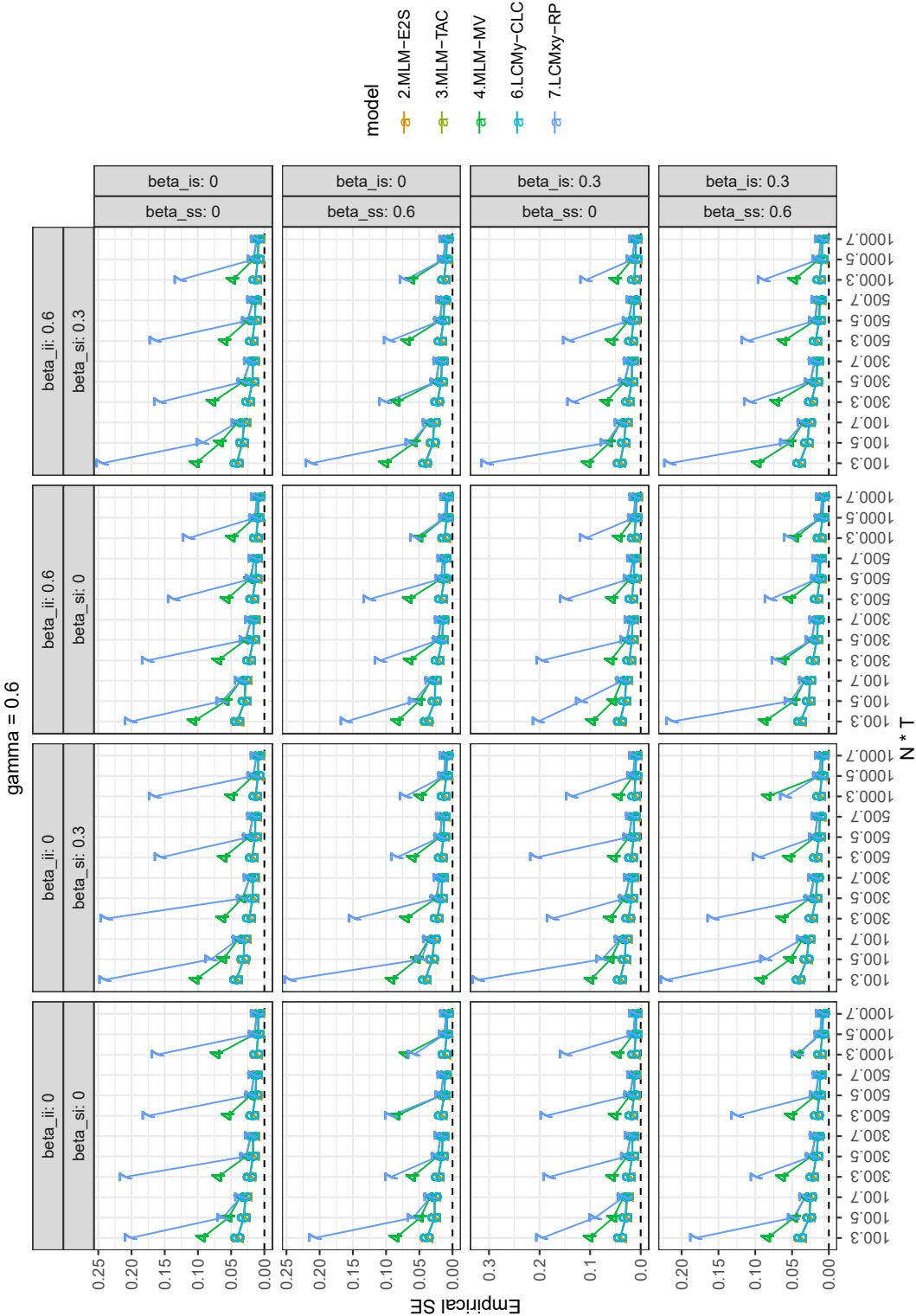


Figure D.134

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)

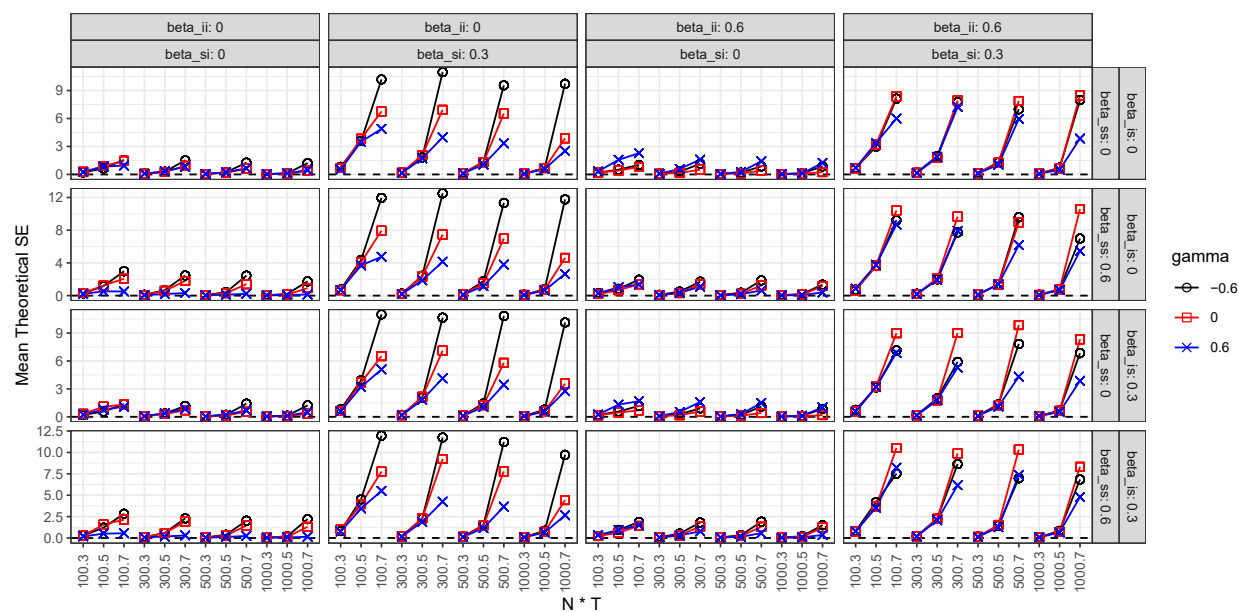


Figure D.135

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)

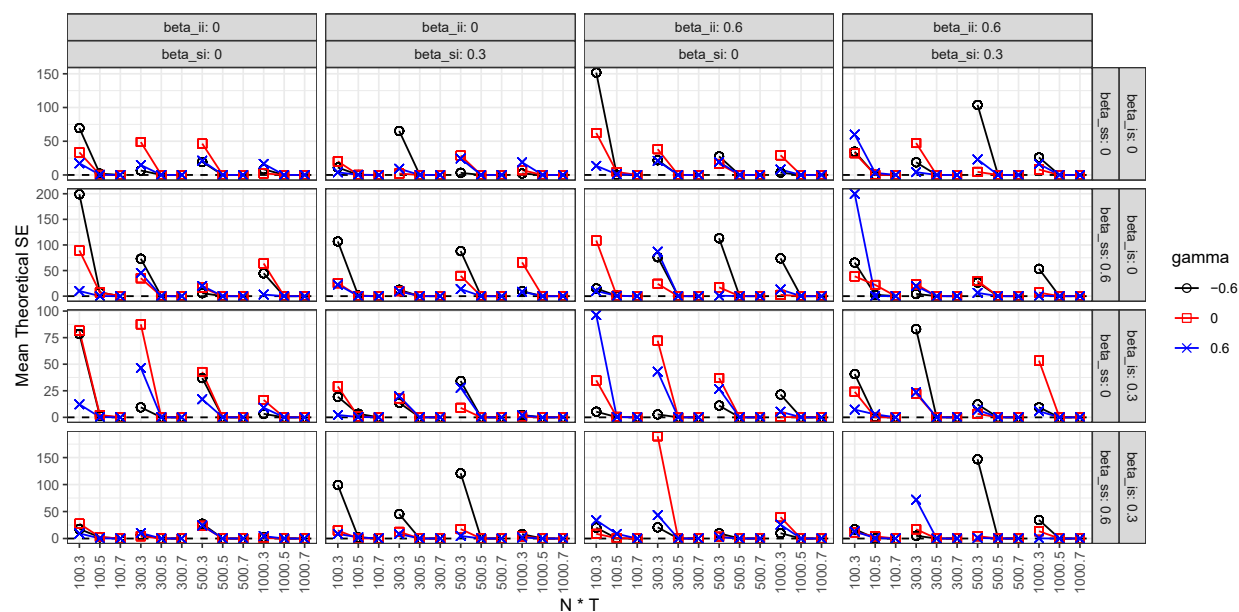


Figure D.136

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = -0.6$)

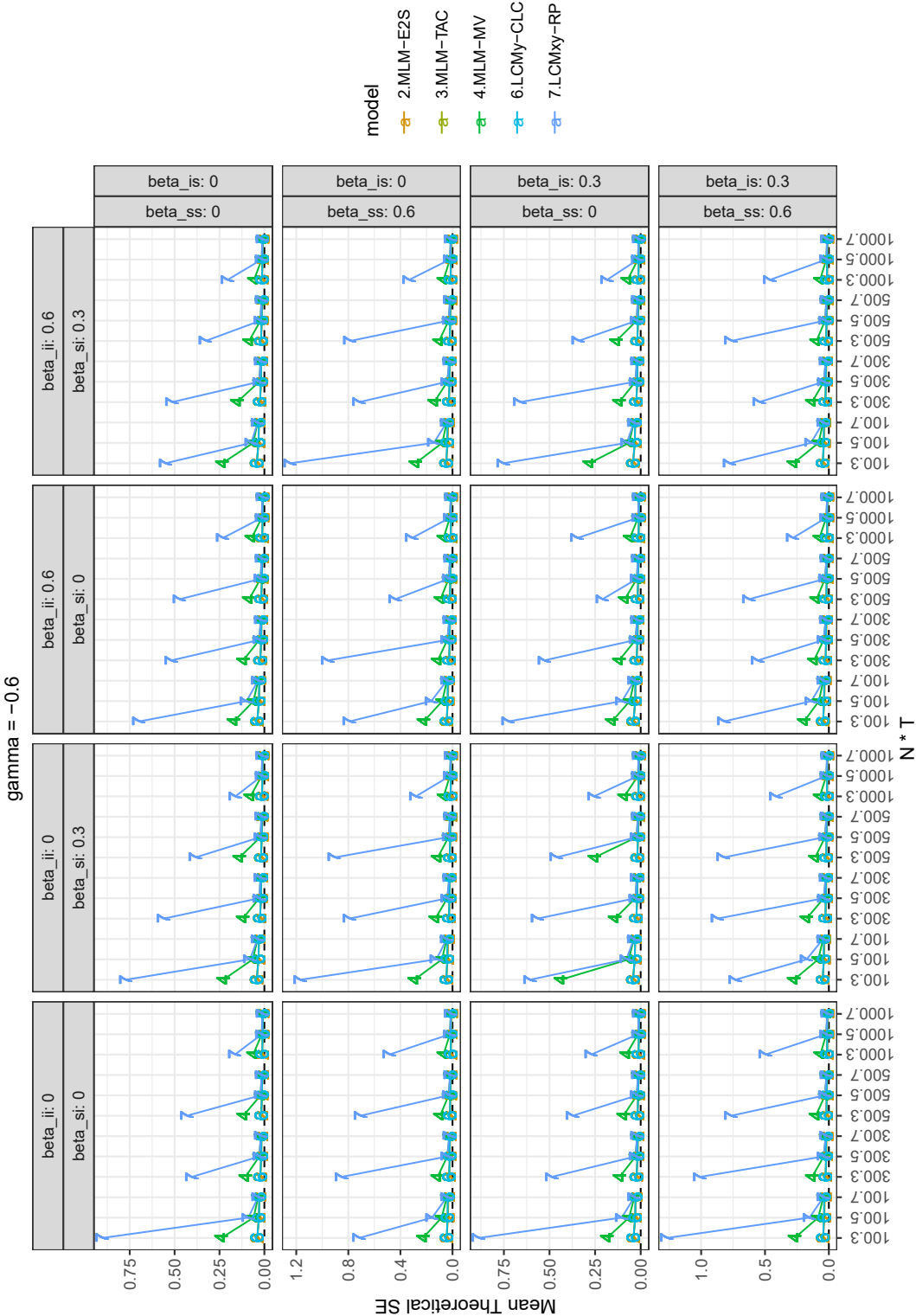


Figure D.137

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0$)

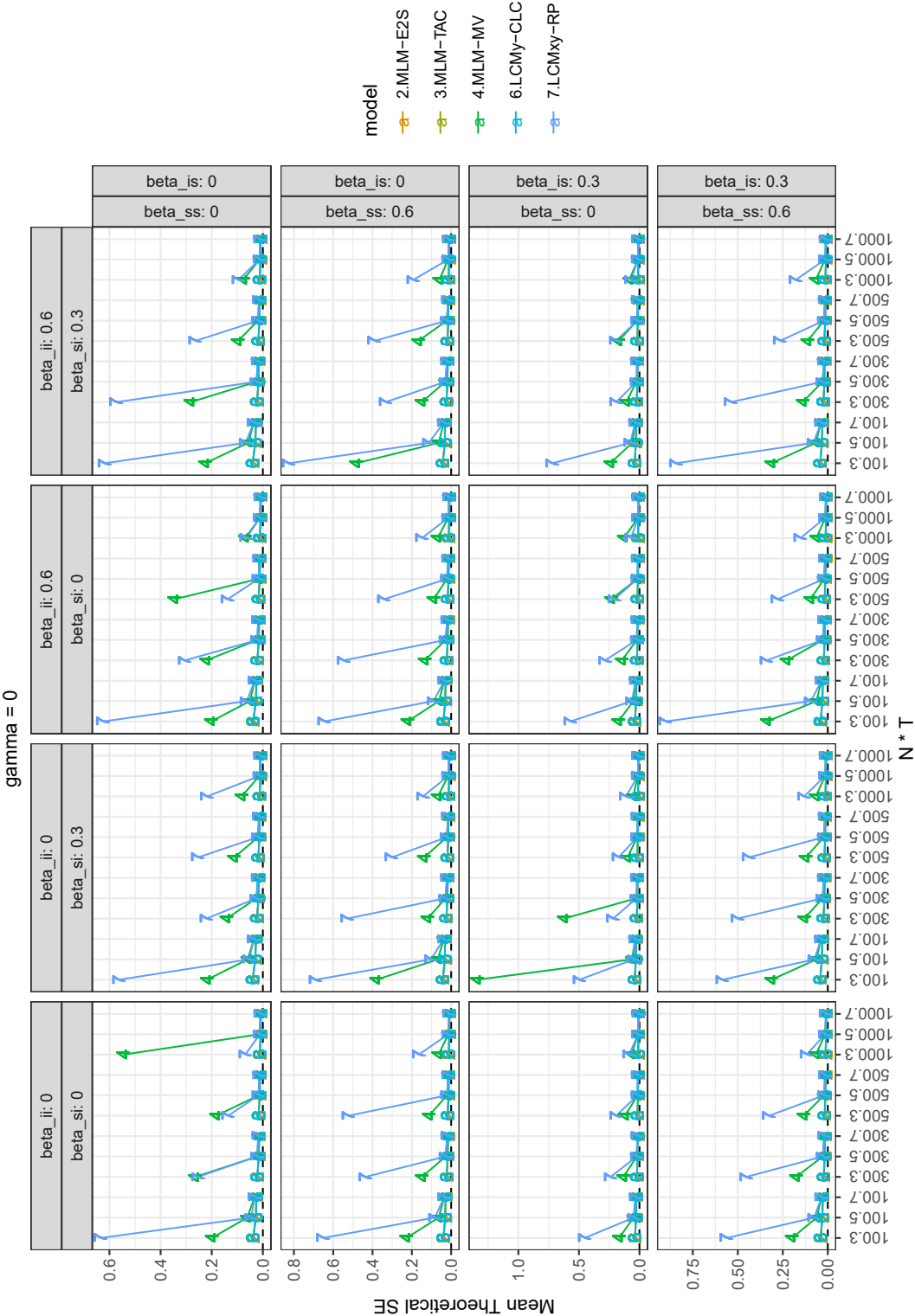


Figure D.138

Theoretical SE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)

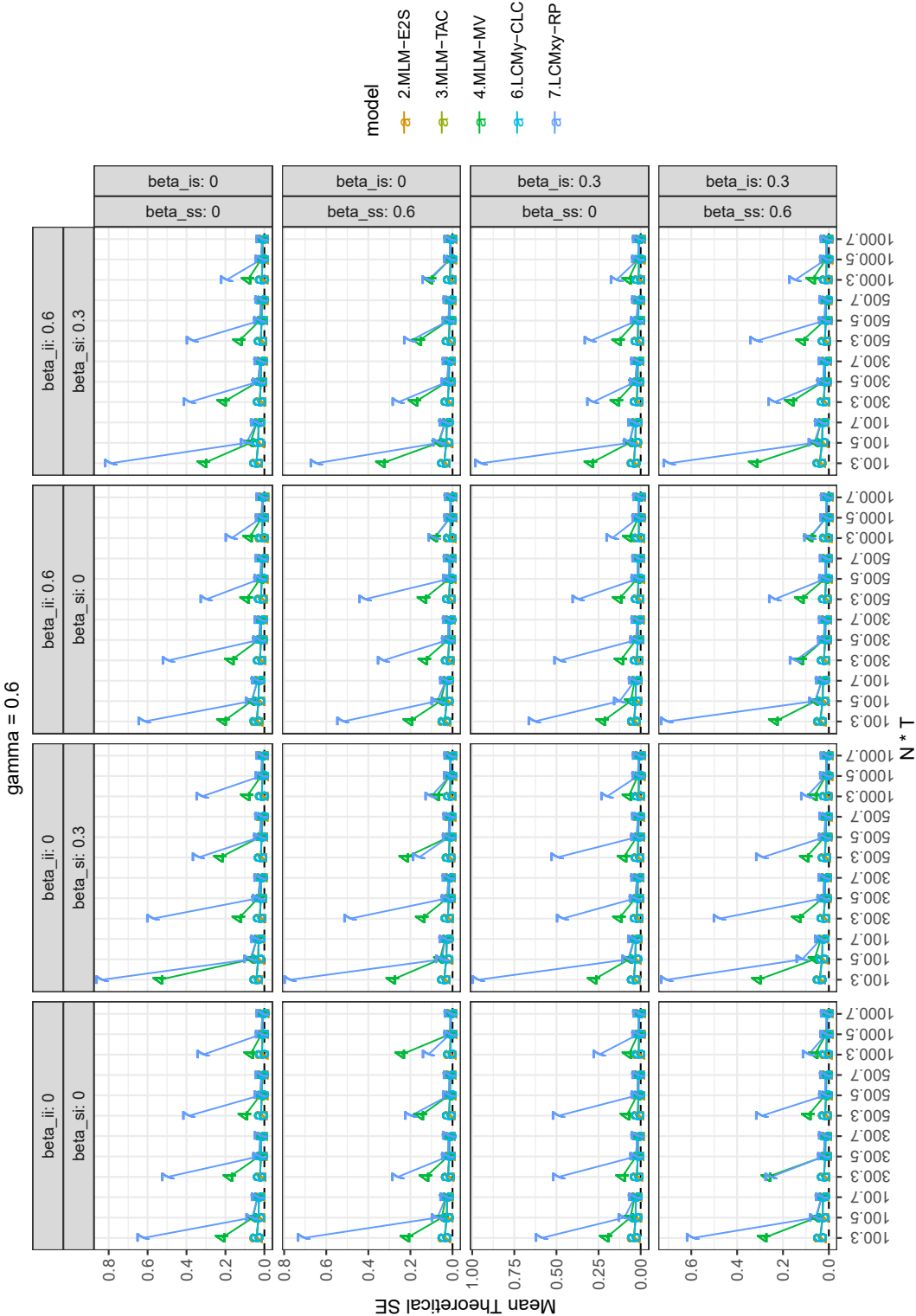


Figure D.139

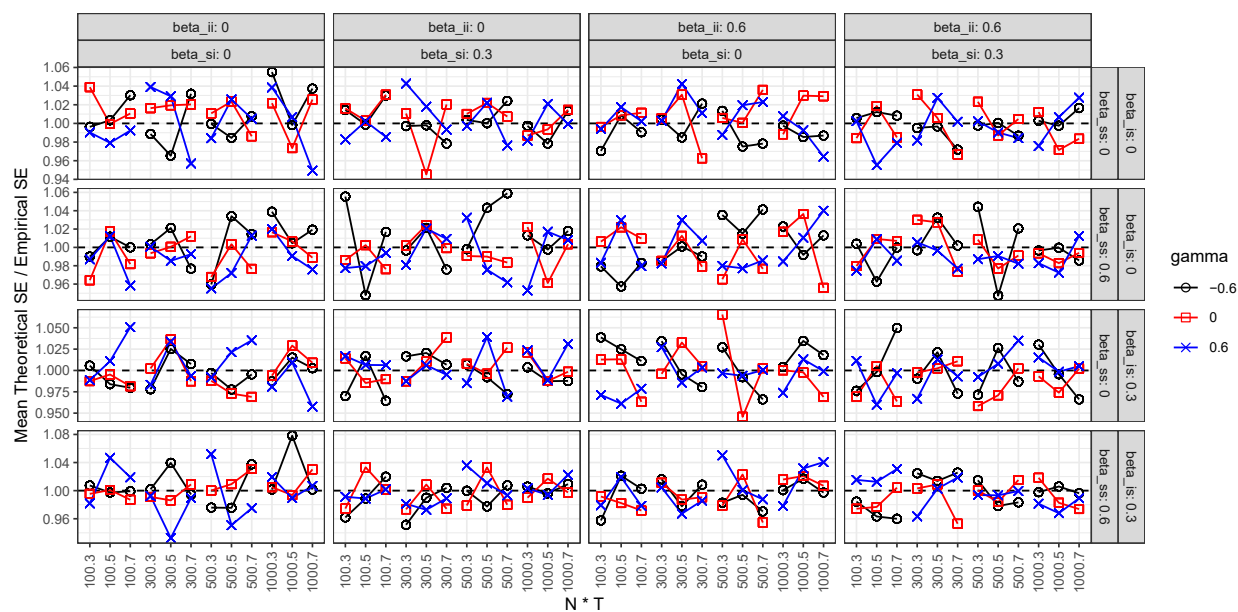
SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (MLM-E2S)

Figure D.140

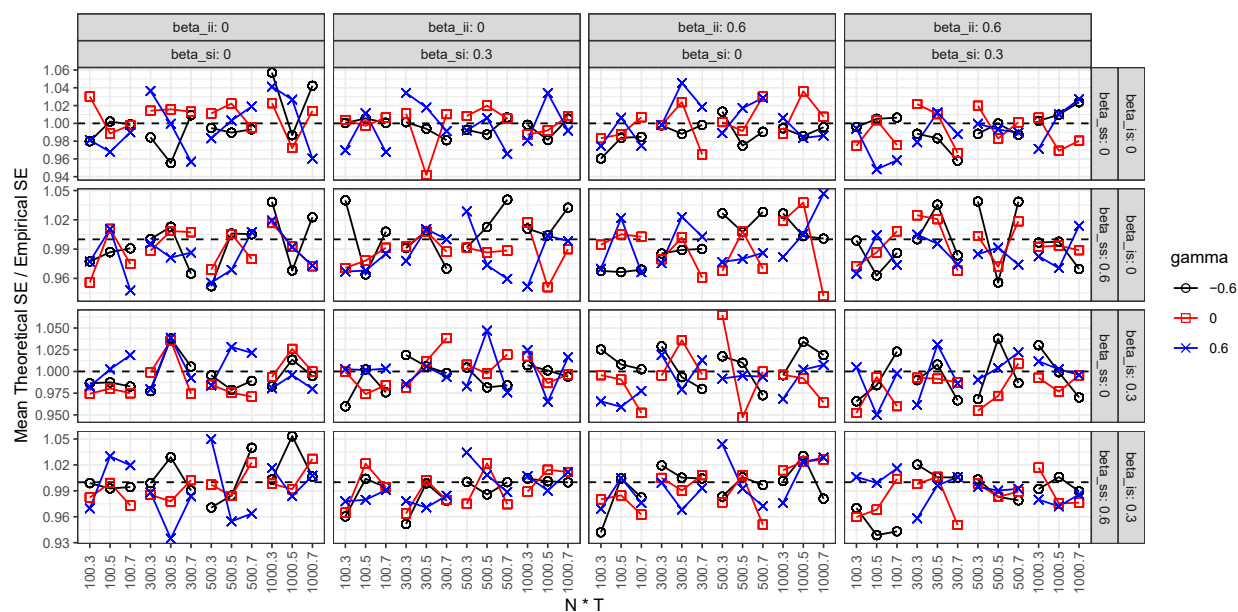
SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (MLM-TAC)

Figure D.141

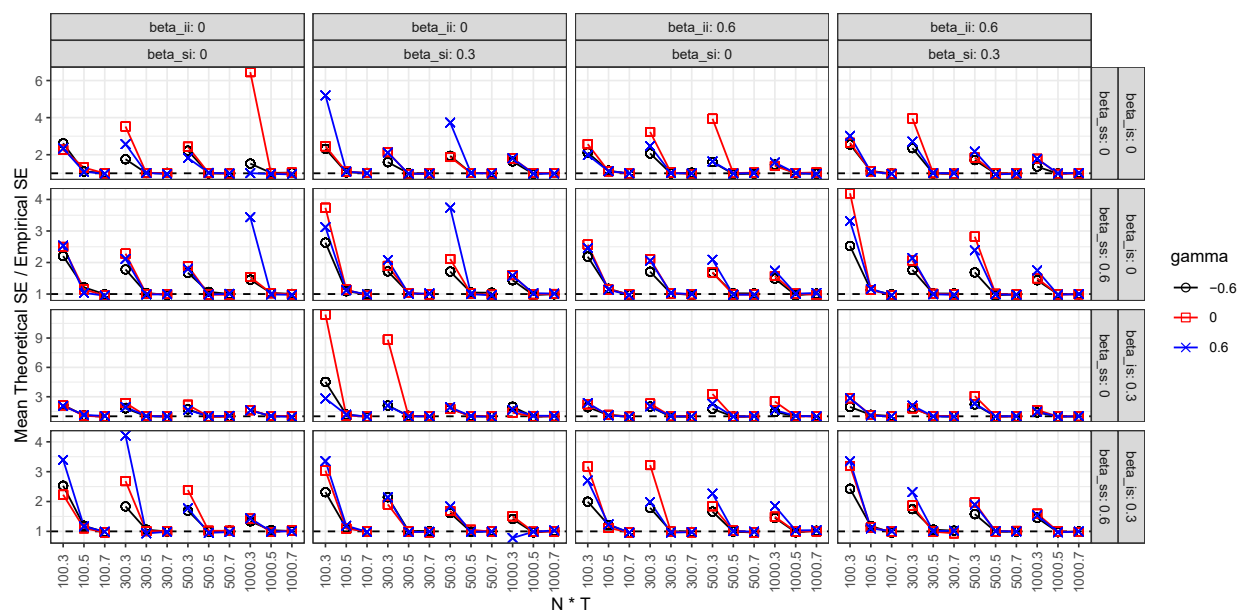
SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (MLM-MV)

Figure D.142

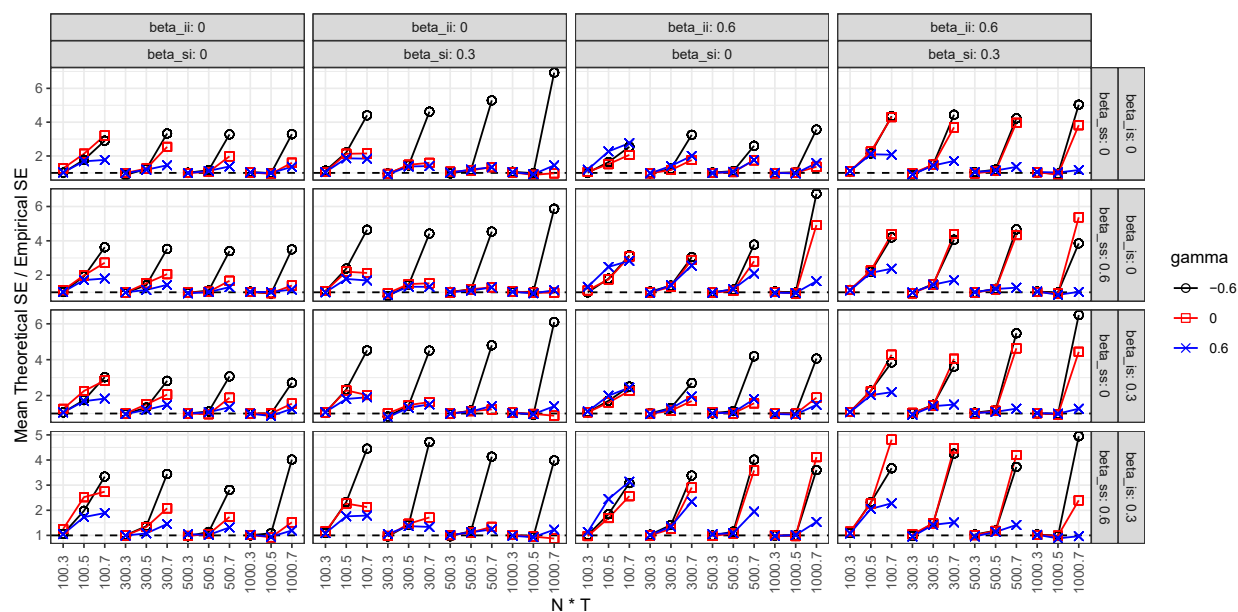
SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)

Figure D.143

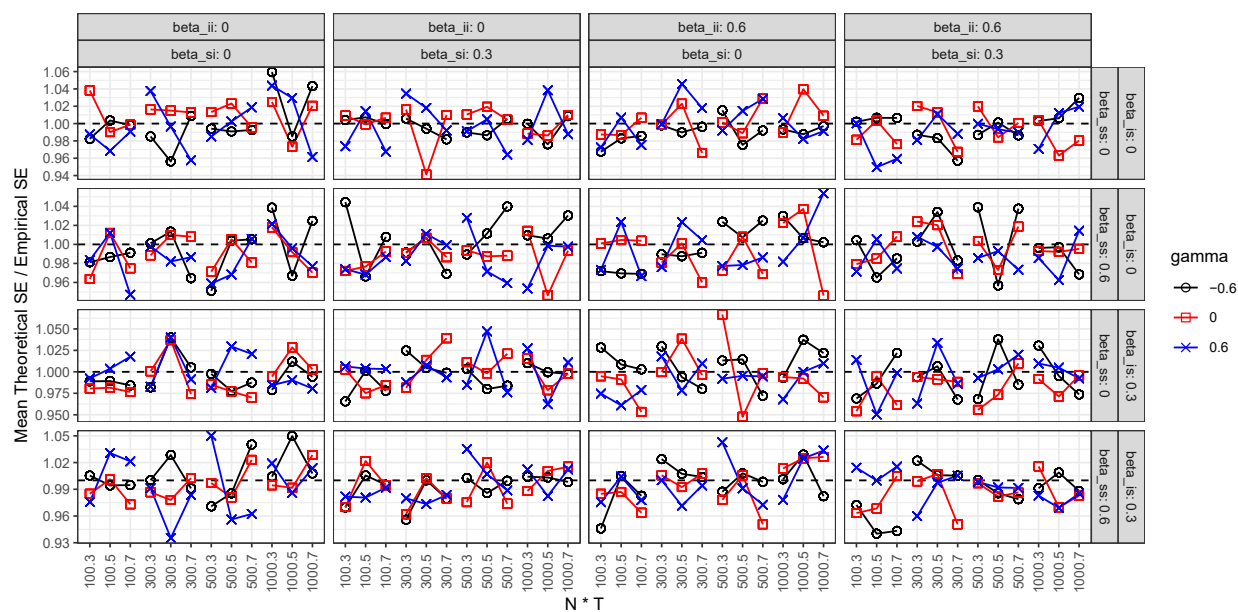
SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-CLC)

Figure D.144

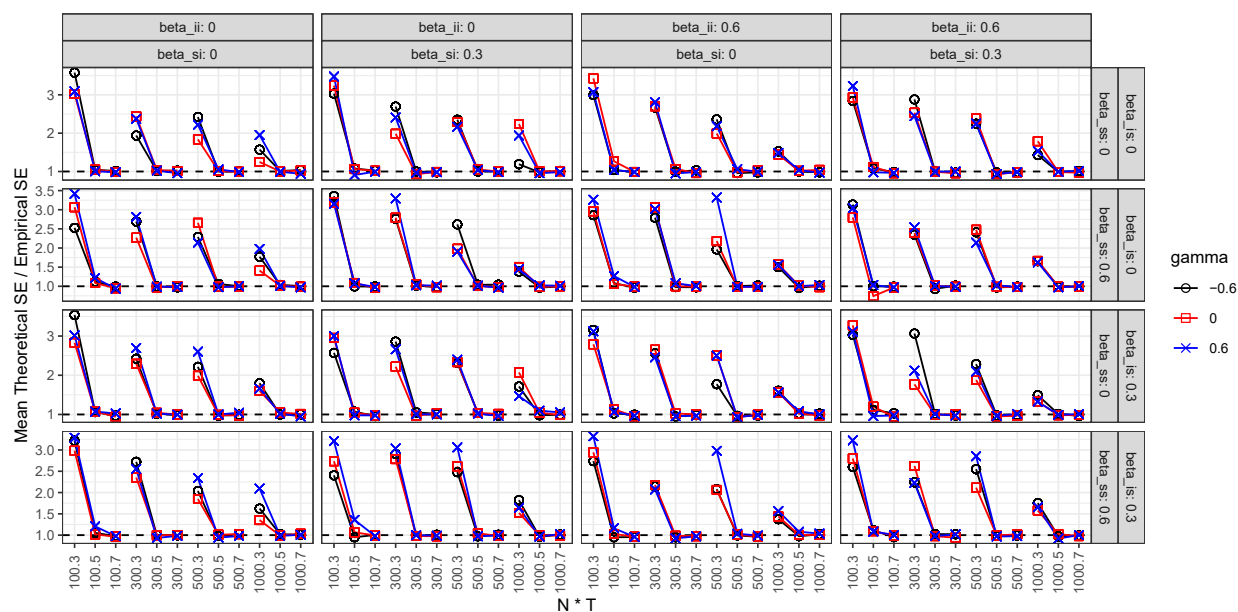
SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (LCMxy-RP)

Figure D.145

SE Ratio for the Between-Person Effect β_{SI} in Simulation 4 (LCM_x-CLC)

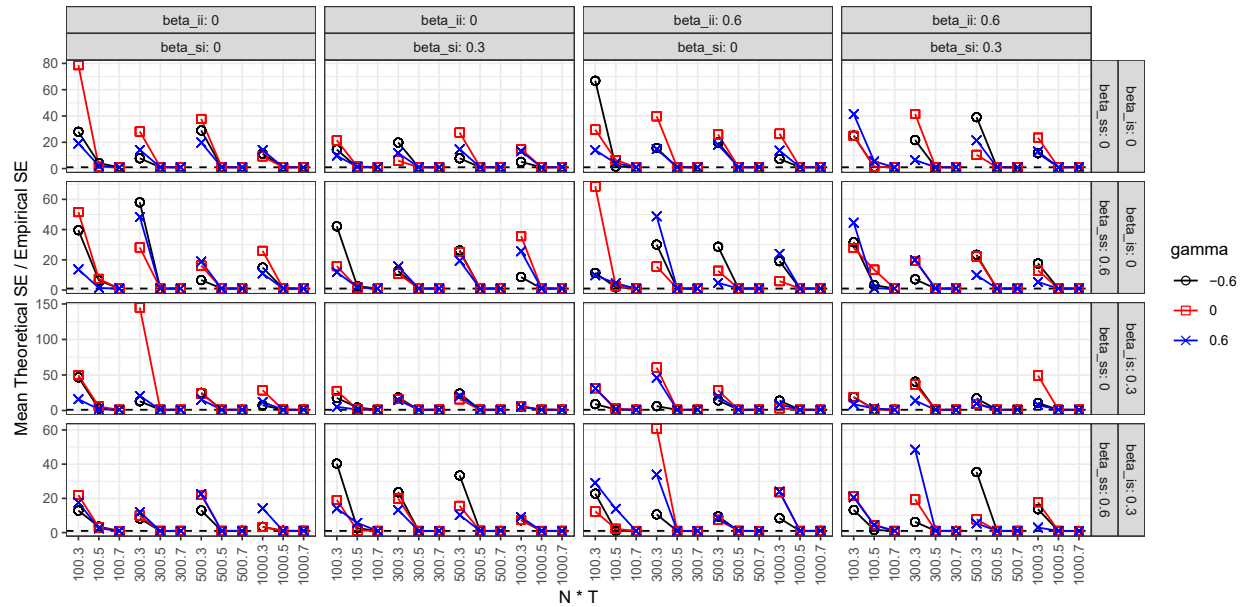


Figure D.146

RMSE for the Between-Person Effect β_{SI} in Simulation 4 (LCM_y-RIx)

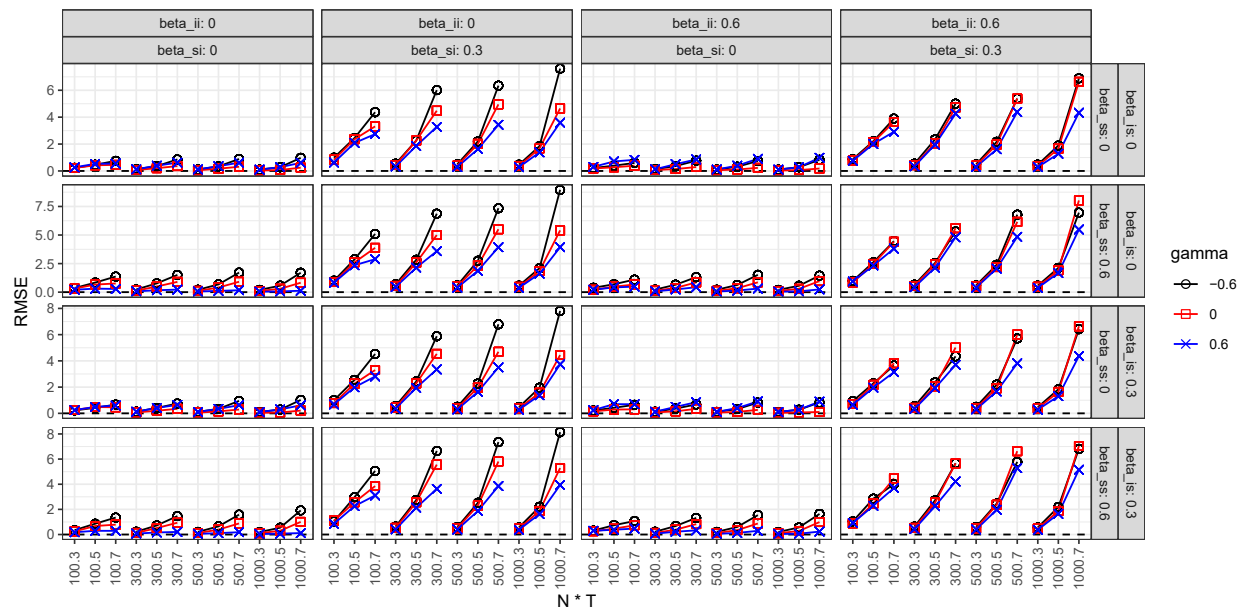


Figure D.147

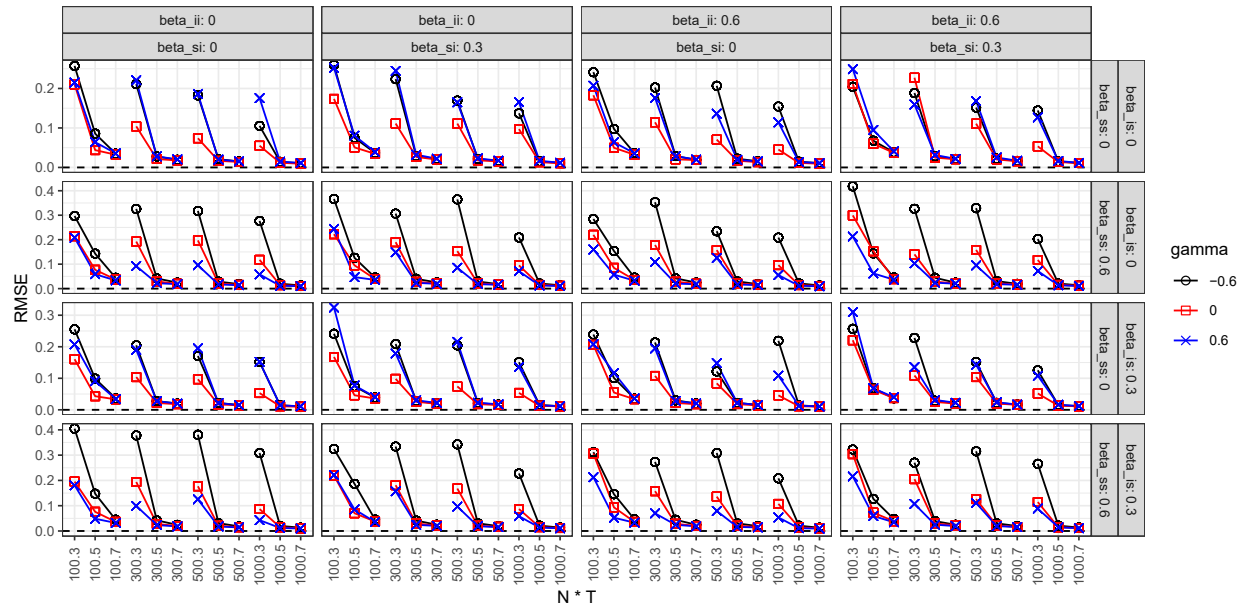
RMSE for the Between-Person Effect β_{SI} in Simulation 4 (LCM_{xy}-RP)

Figure D.148

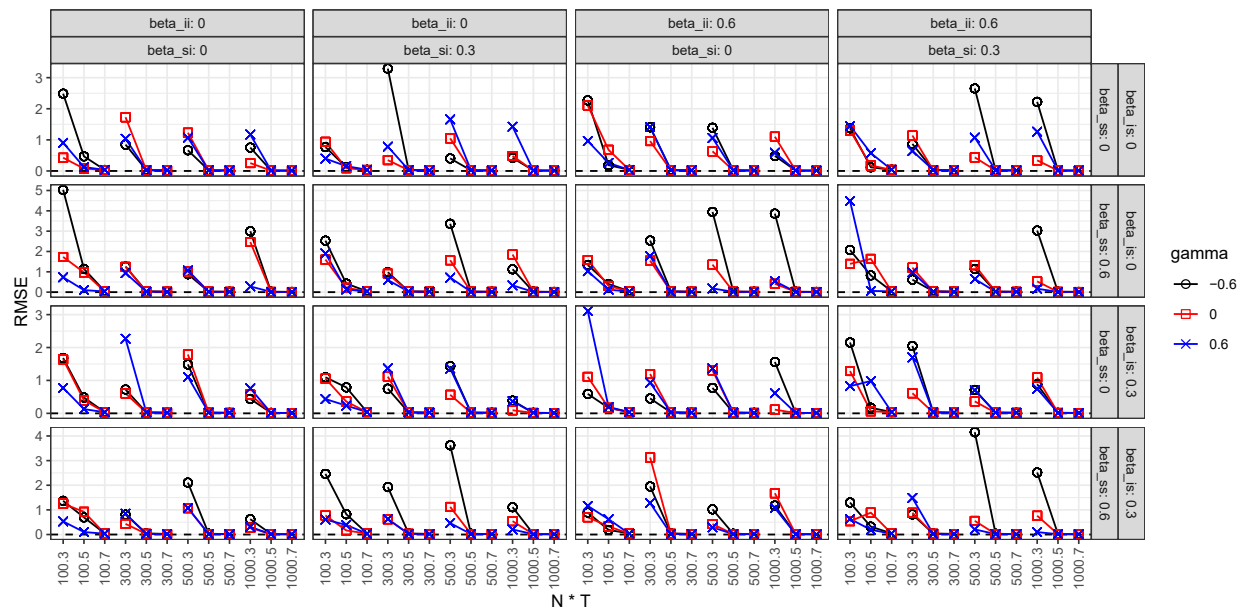
RMSE for the Between-Person Effect β_{SI} in Simulation 4 (LCM_x-CLC)

Figure D.149

RMSE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = -0.6$)

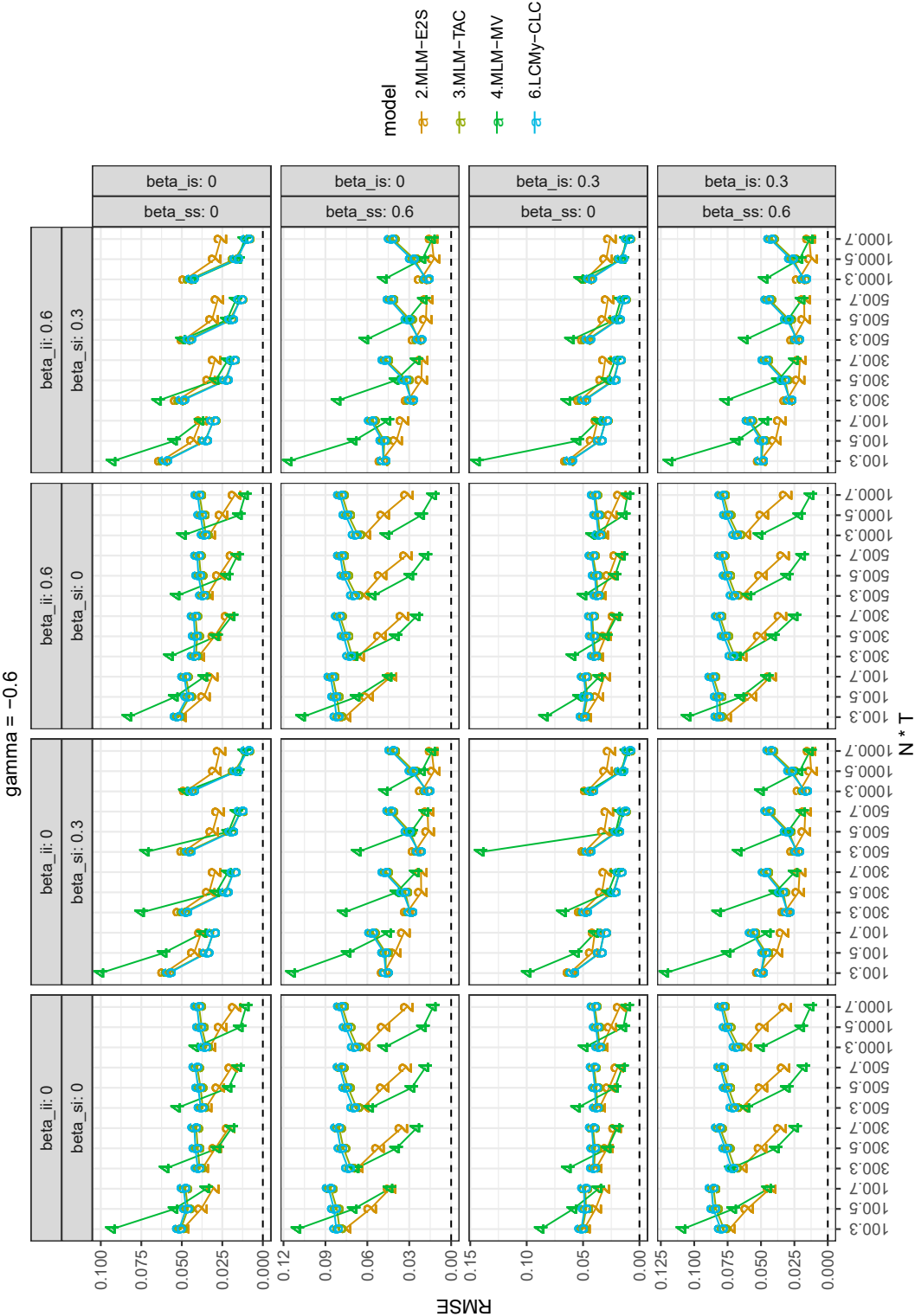


Figure D.150

RMSE for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0$)

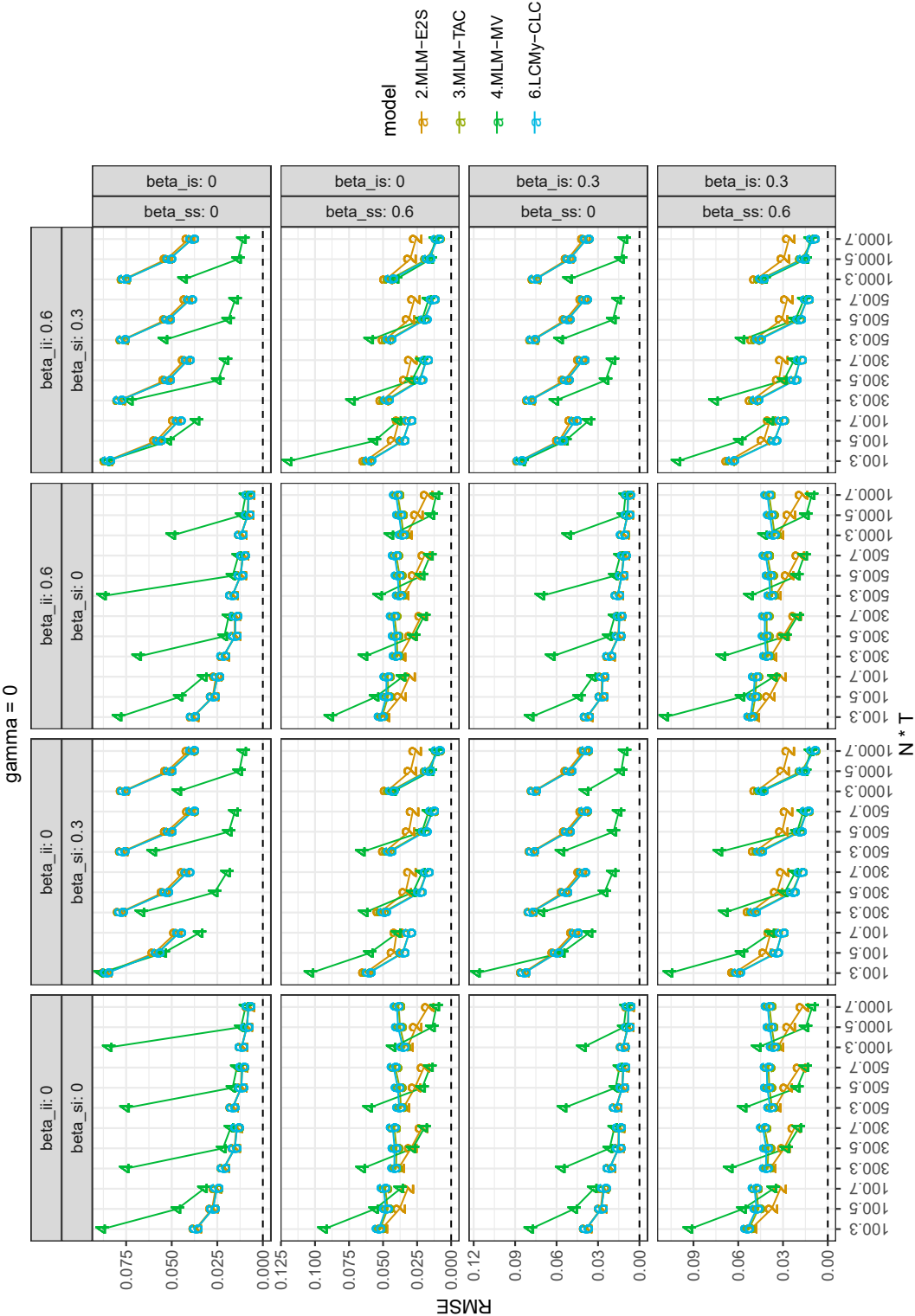


Figure D.151

RMSE for the Between-Person Effect β_{II} in Simulation 4 ($\gamma = 0.6$)

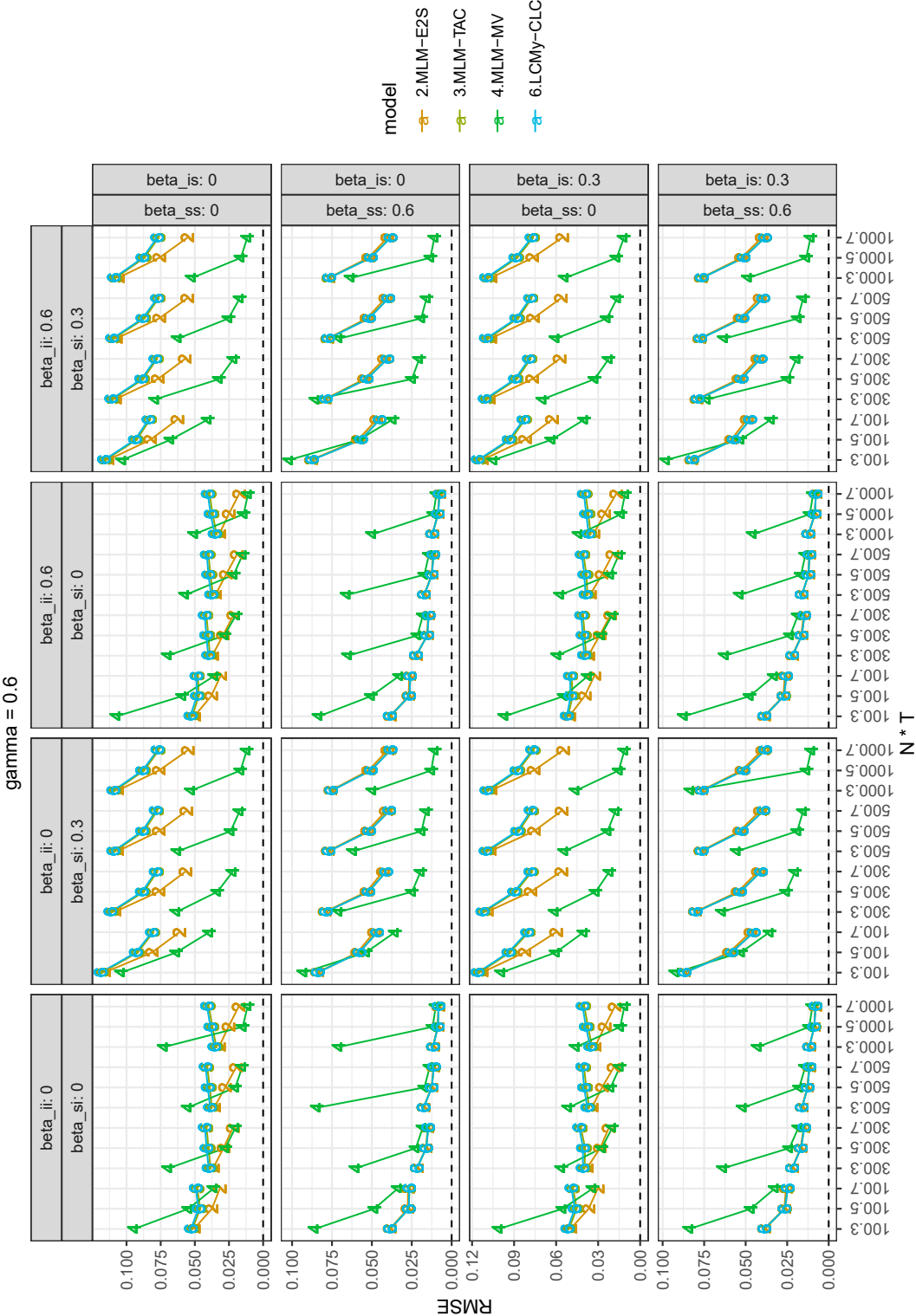


Figure D.152

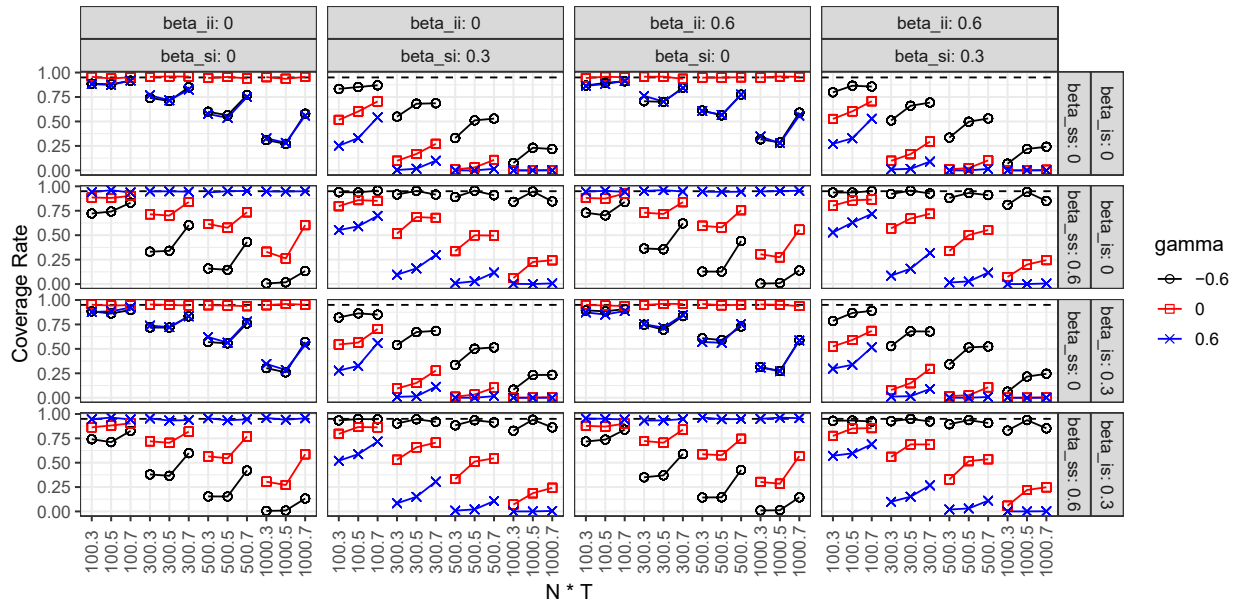
Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (MLM-E2S)

Figure D.153

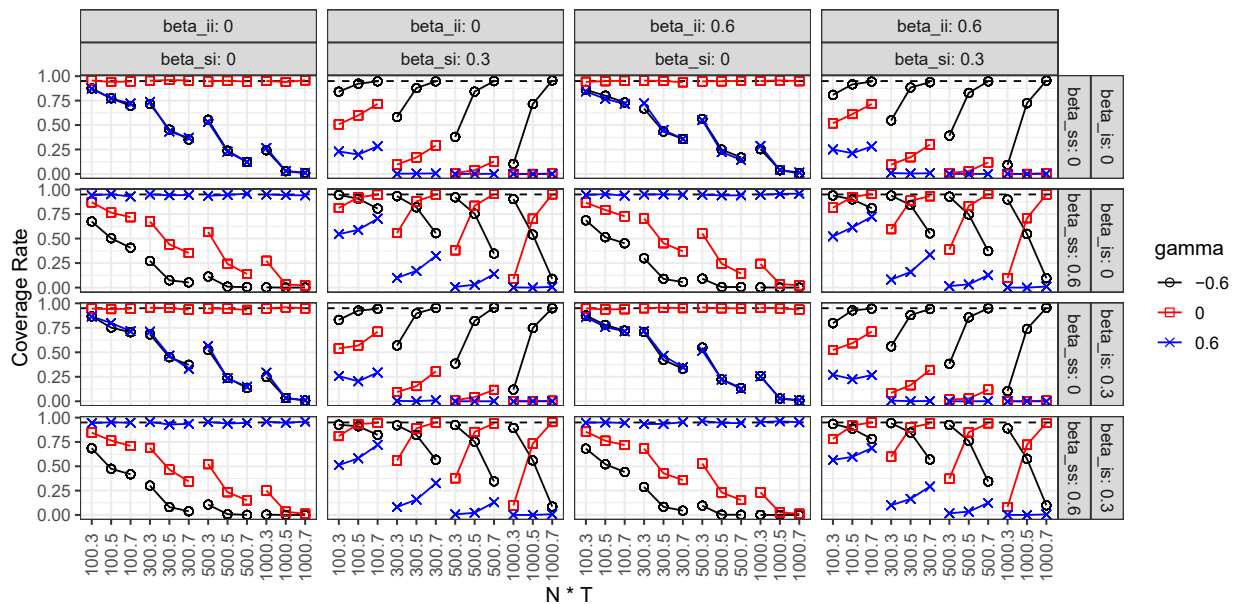
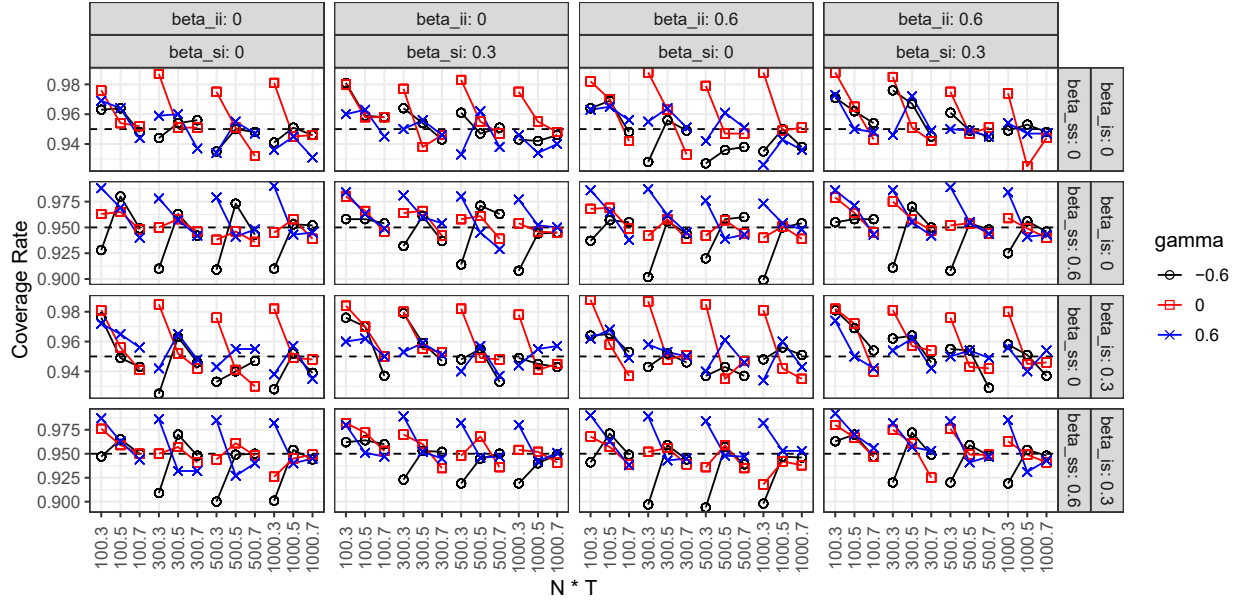
Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (MLM-TAC)

Figure D.154

Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (MLM-MV)

**Figure D.155**

Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-RIx)

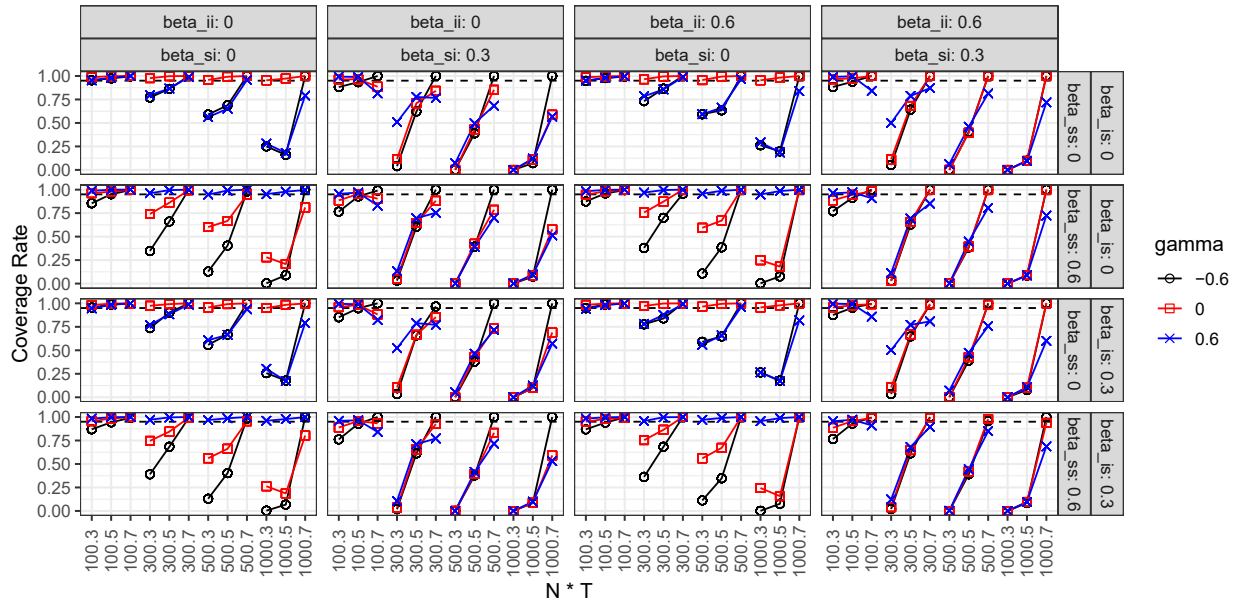


Figure D.156

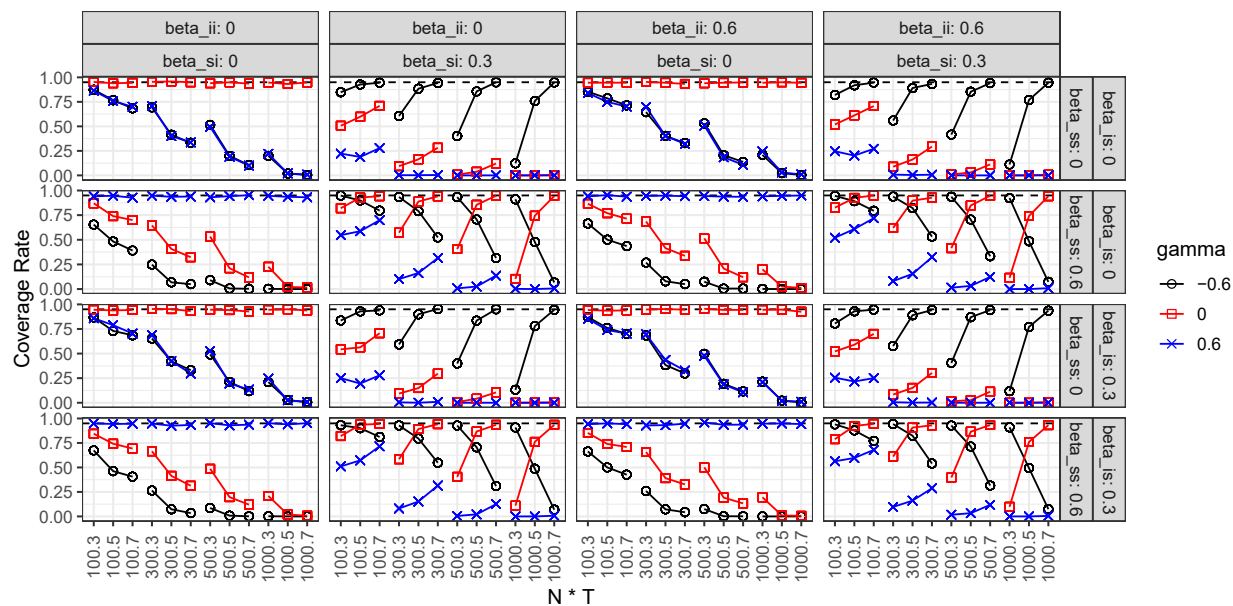
Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (LCMy-CLC)

Figure D.157

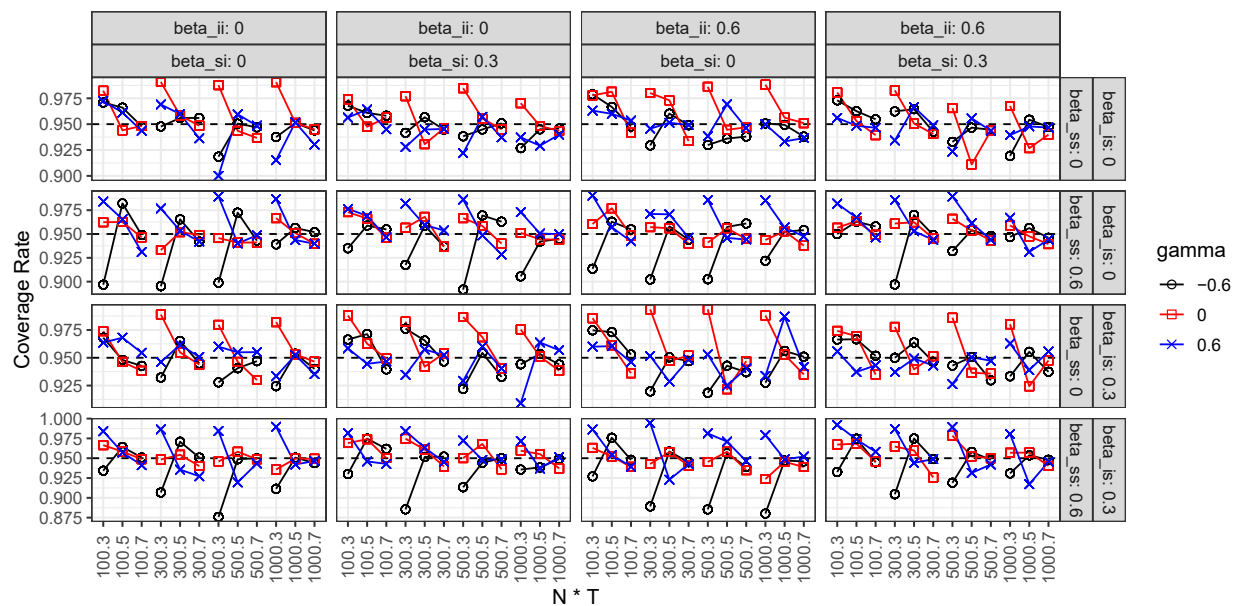
Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (LCMxy-RP)

Figure D.158

Coverage Rate for the Between-Person Effect β_{SI} in Simulation 4 (LCMx-CLC)

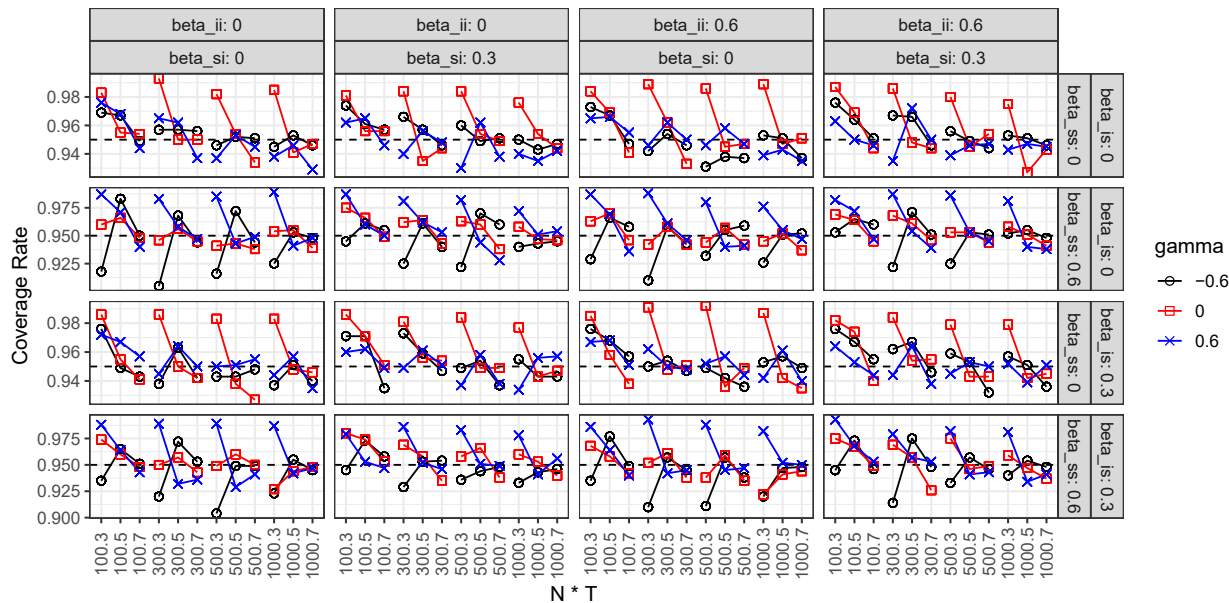


Figure D.159

Power for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = -0.6$)

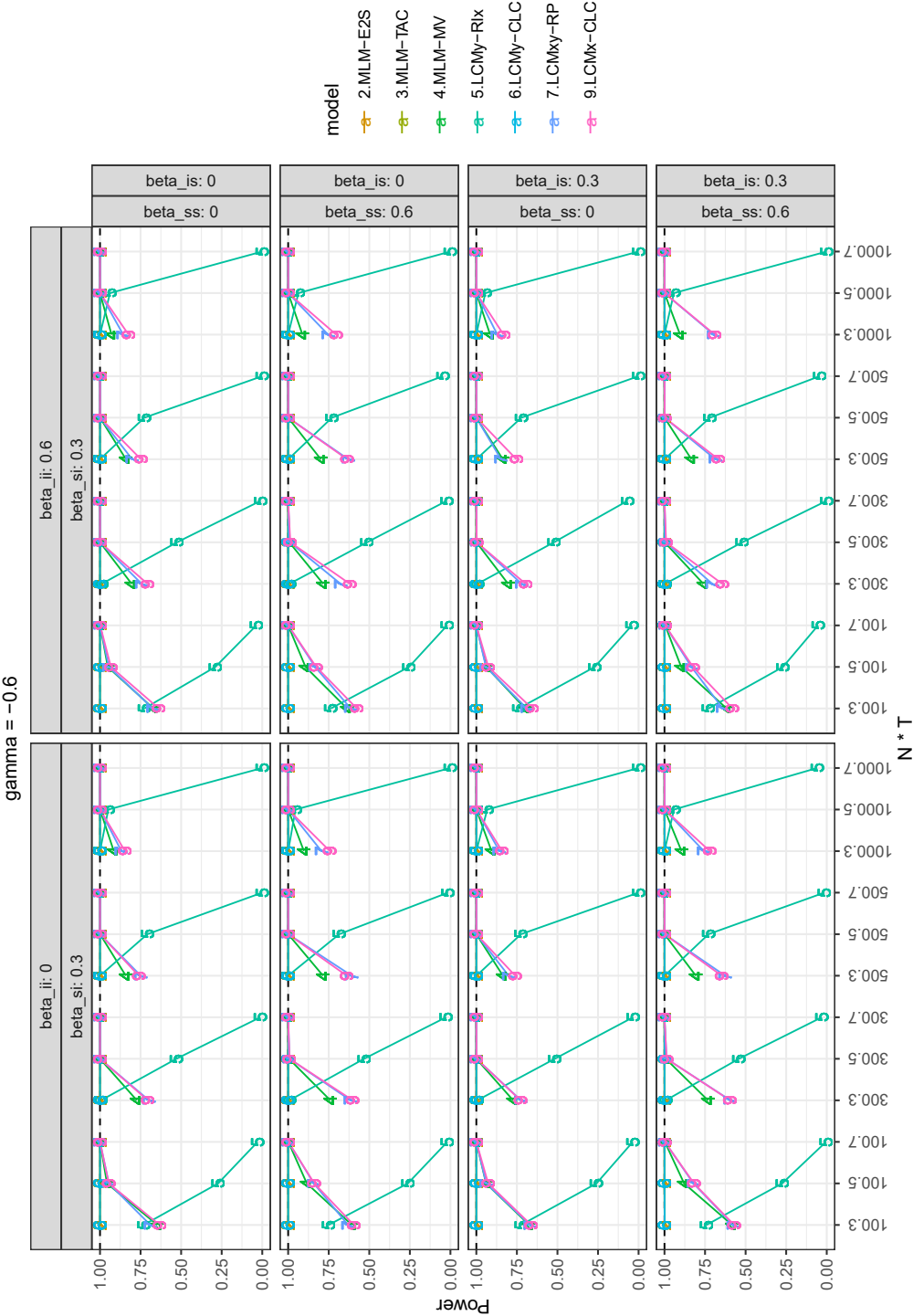


Figure D.160

Power for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0$)

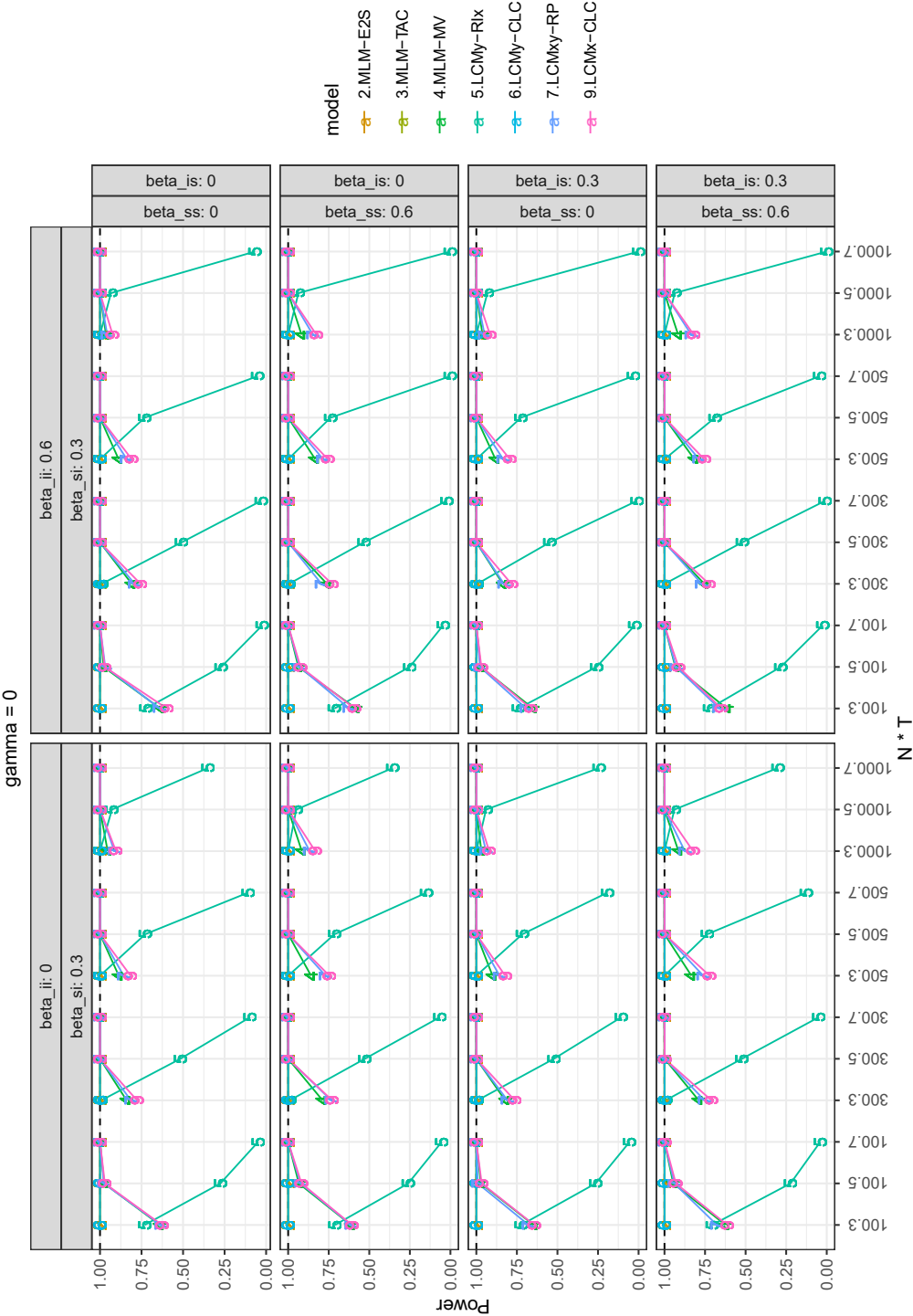
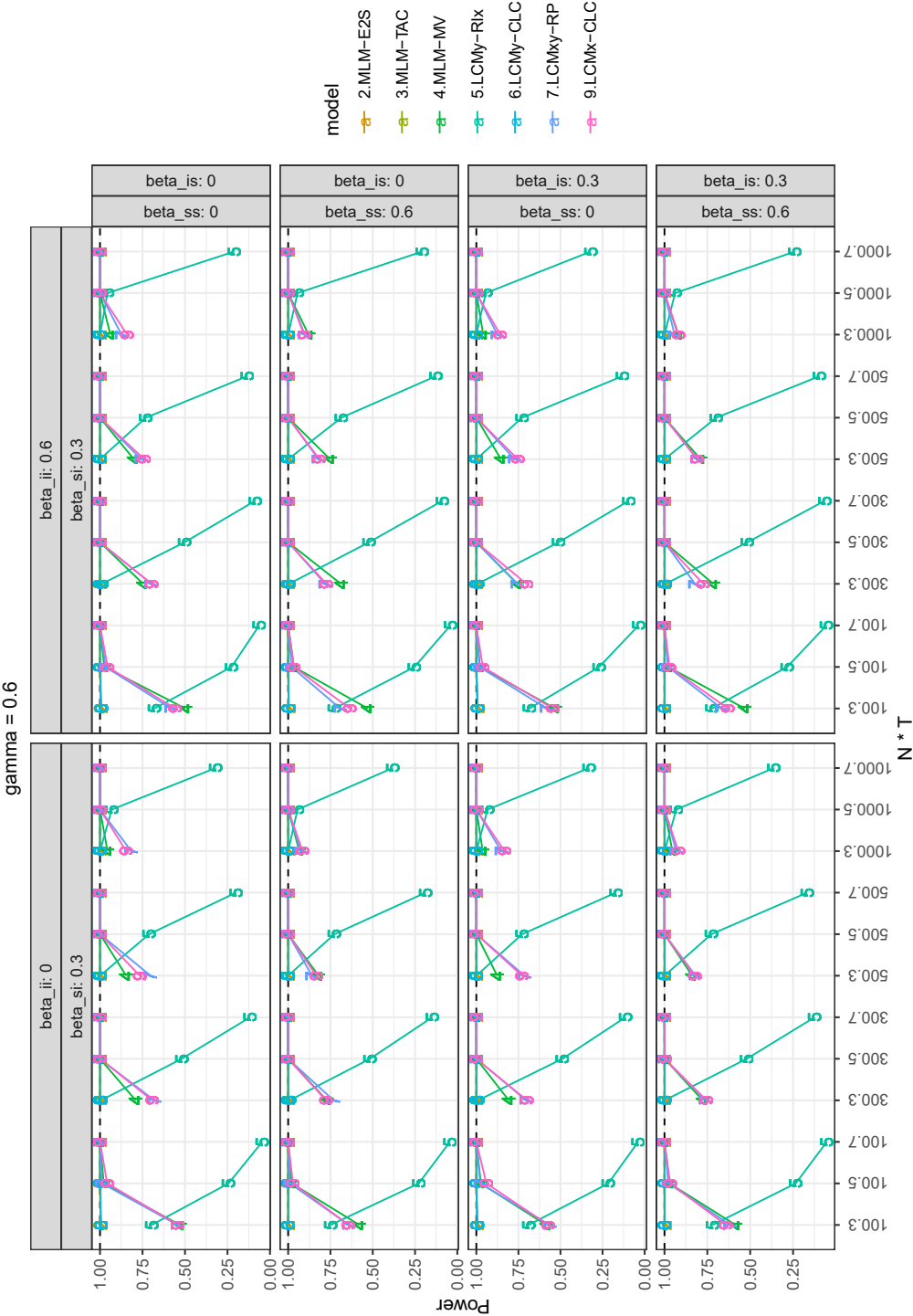


Figure D.161

Power for the Between-Person Effect β_{SI} in Simulation 4 ($\gamma = 0.6$)



D.5 Between-Person Effect of Slope on Intercept

Figure D.162

Bias for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)

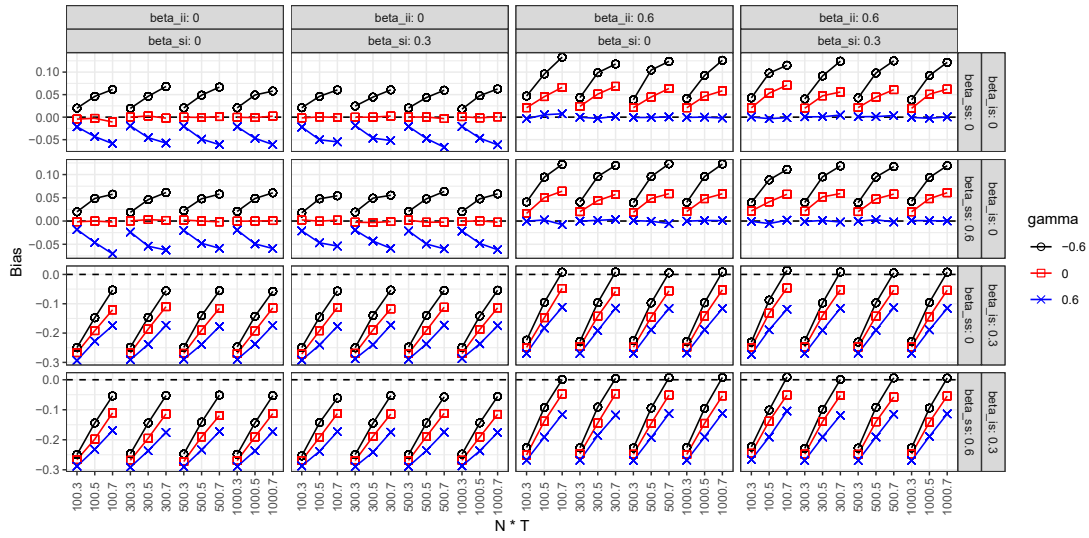


Figure D.163

Bias for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)

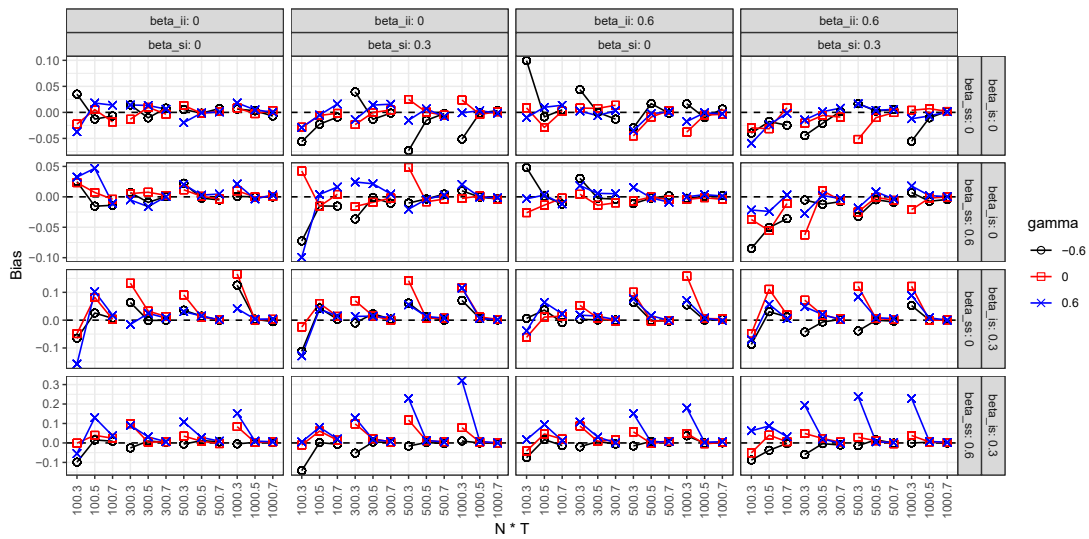


Figure D.164

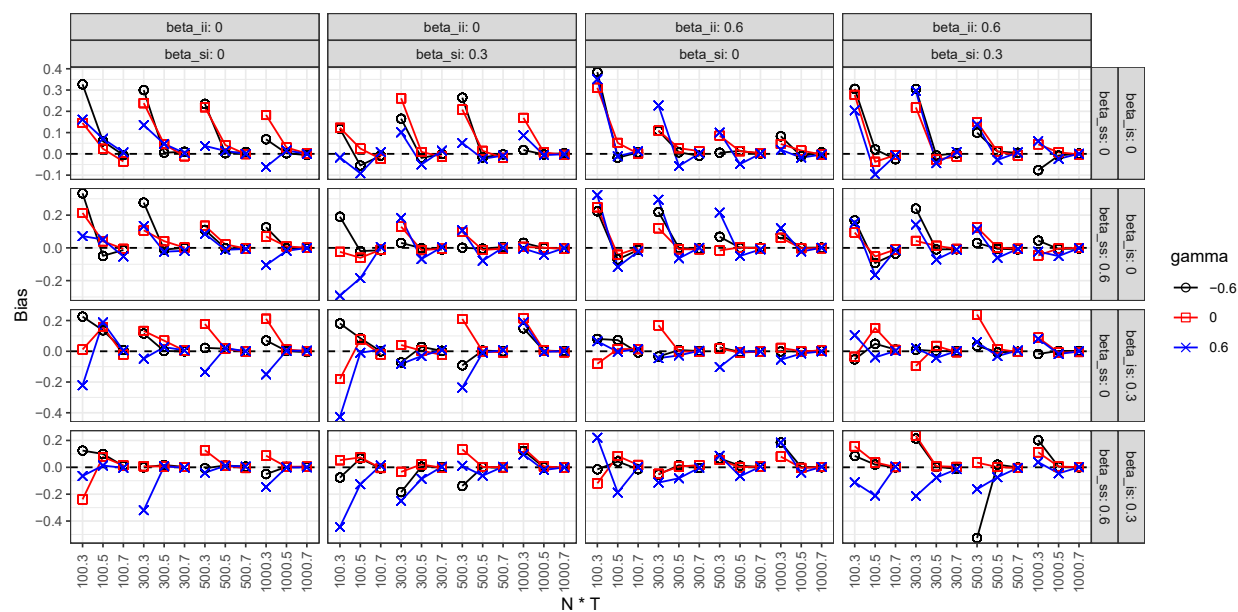
Bias for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)

Figure D.165

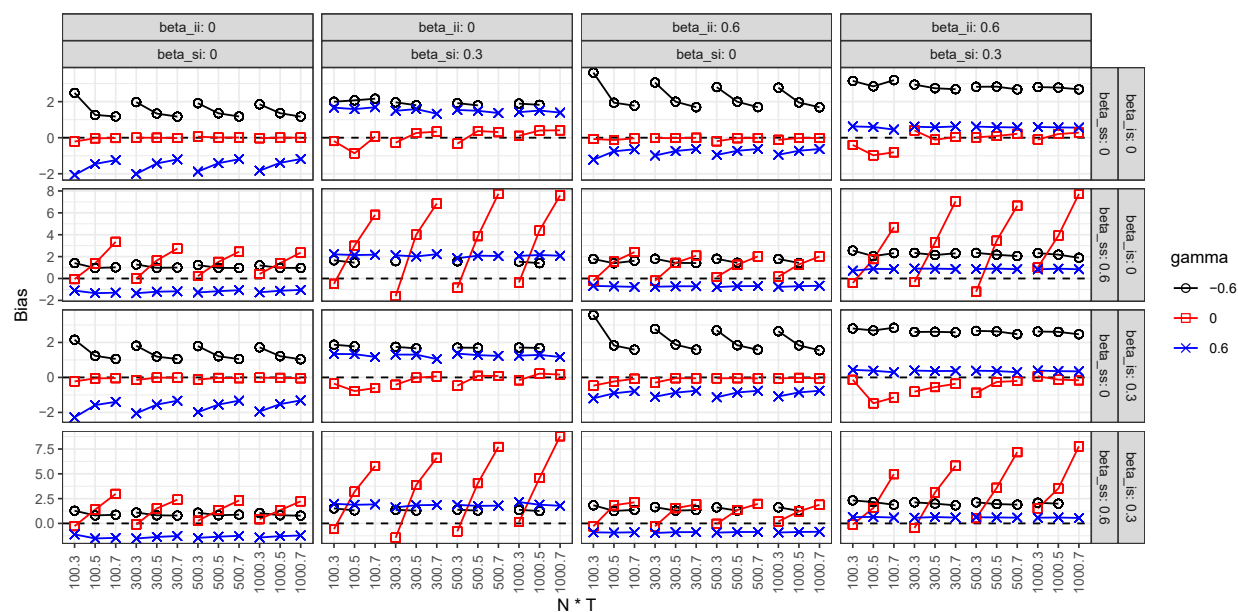
Bias for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-Rly)

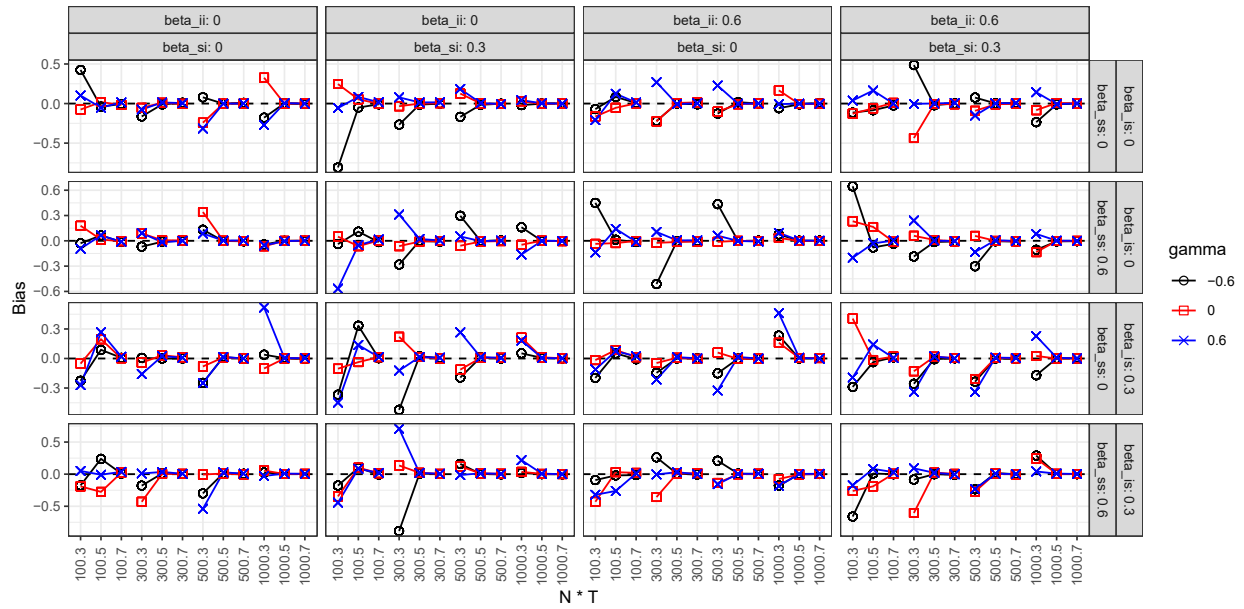
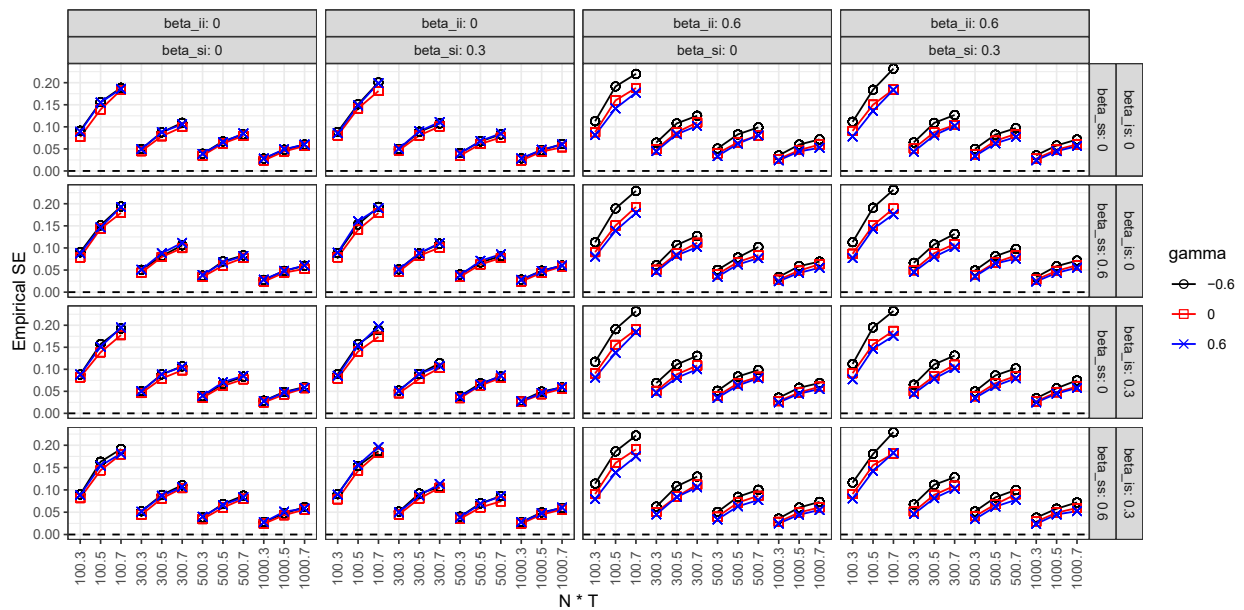
Figure D.166*Bias for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)***Figure D.167***Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)*

Figure D.168

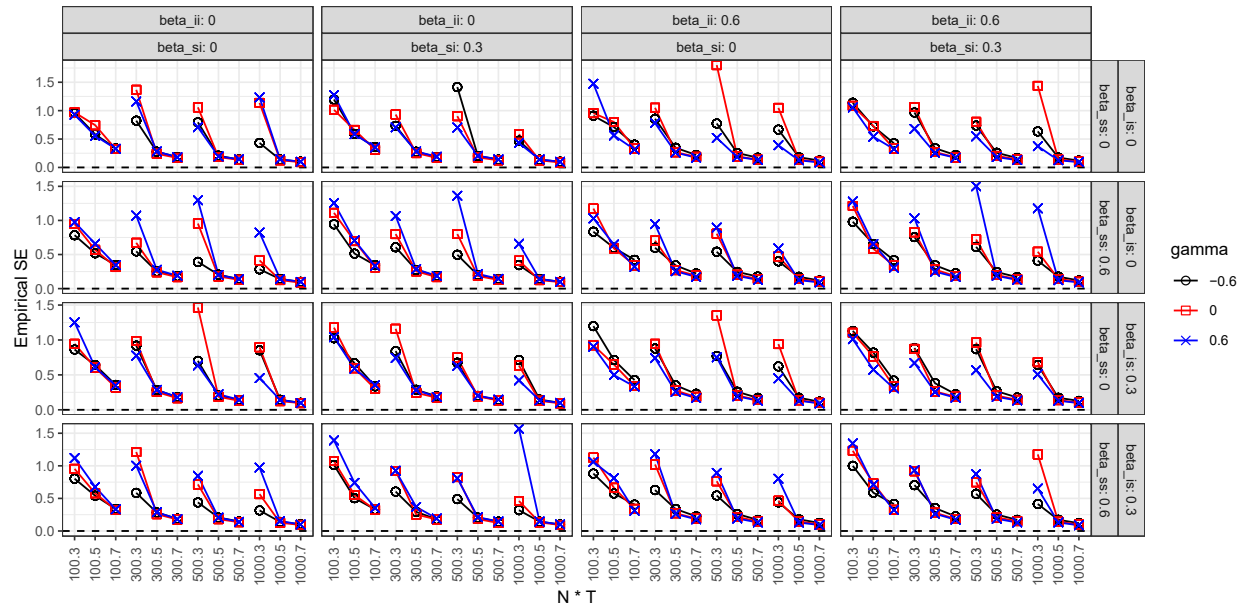
Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)

Figure D.169

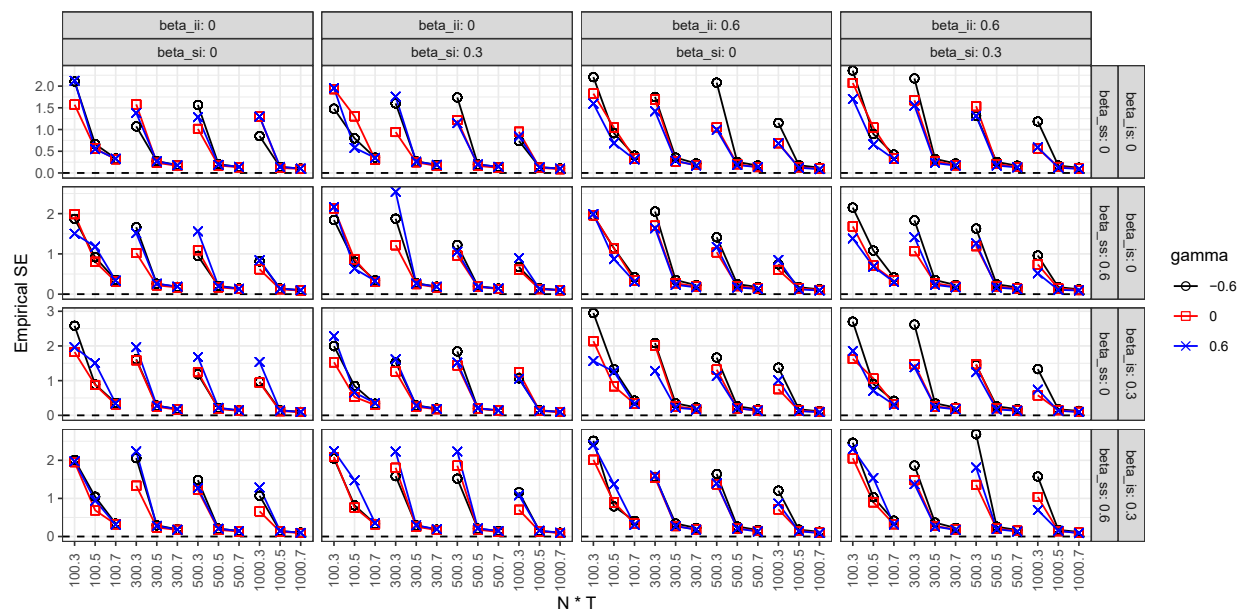
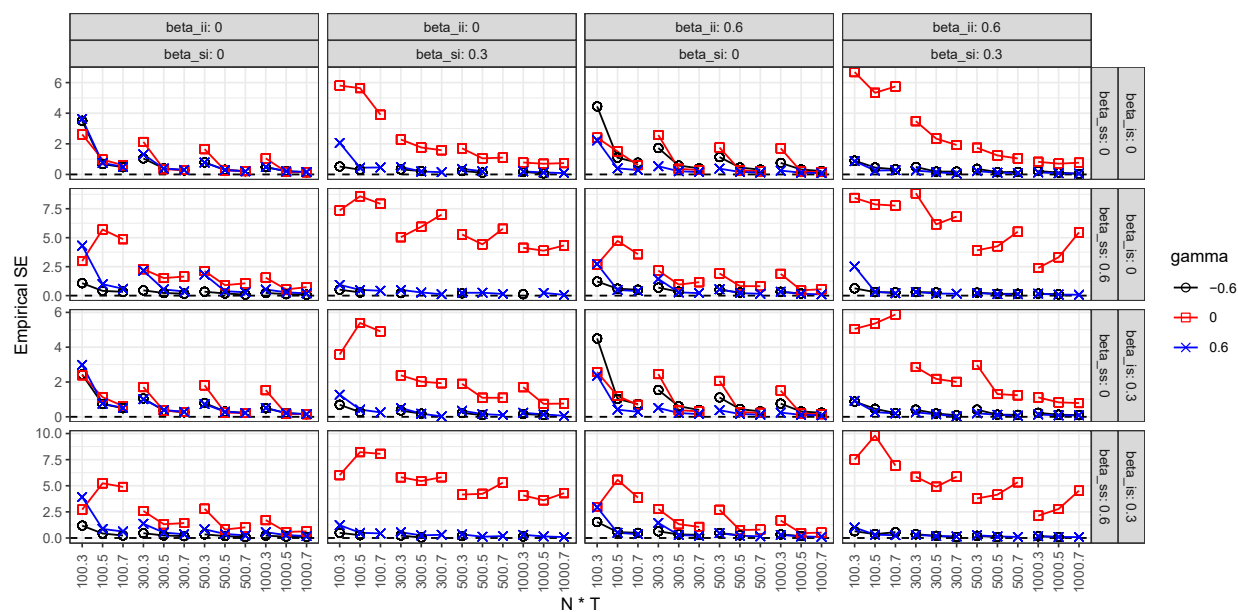
Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)

Figure D.170

Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-Rly)

**Figure D.171**

Empirical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)

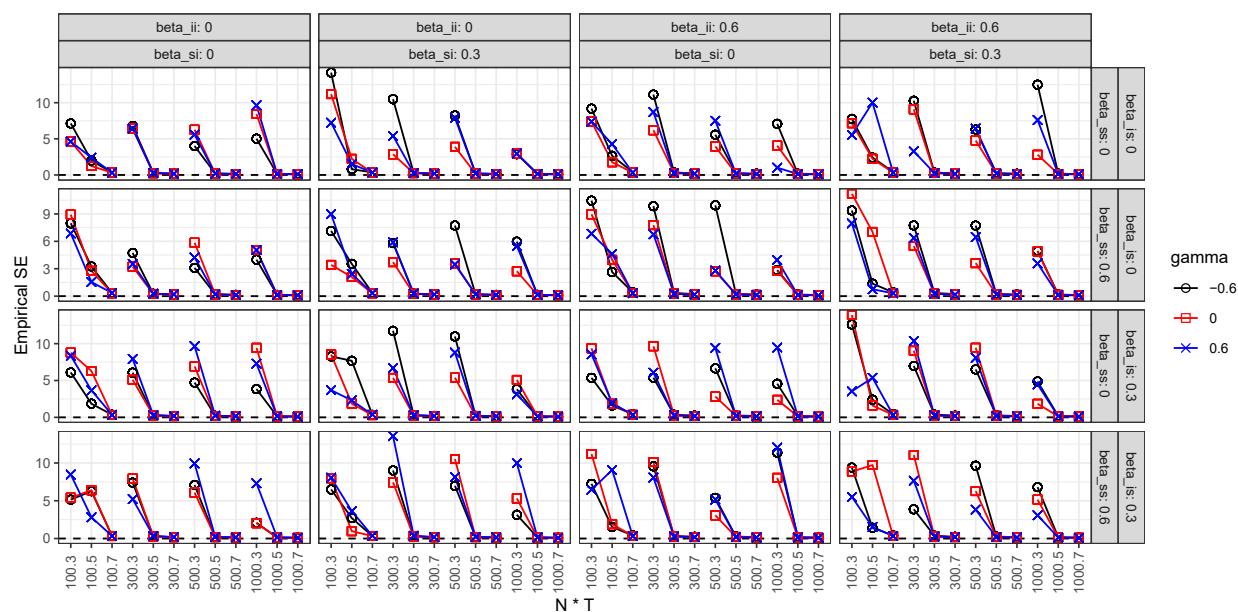


Figure D.172

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)

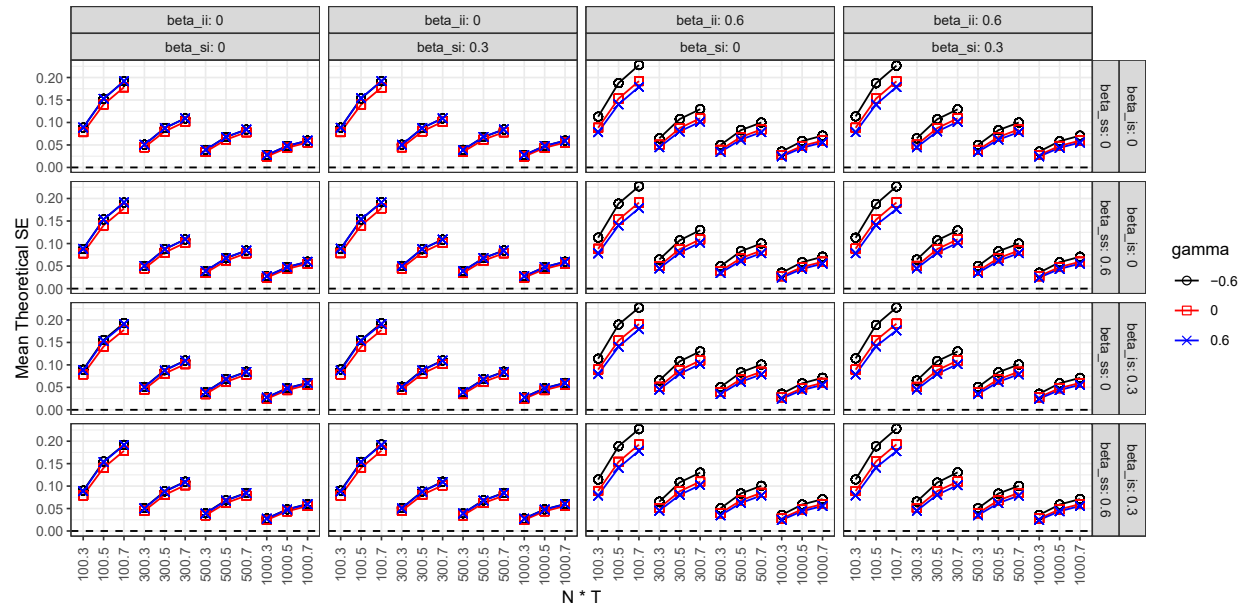


Figure D.173

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)

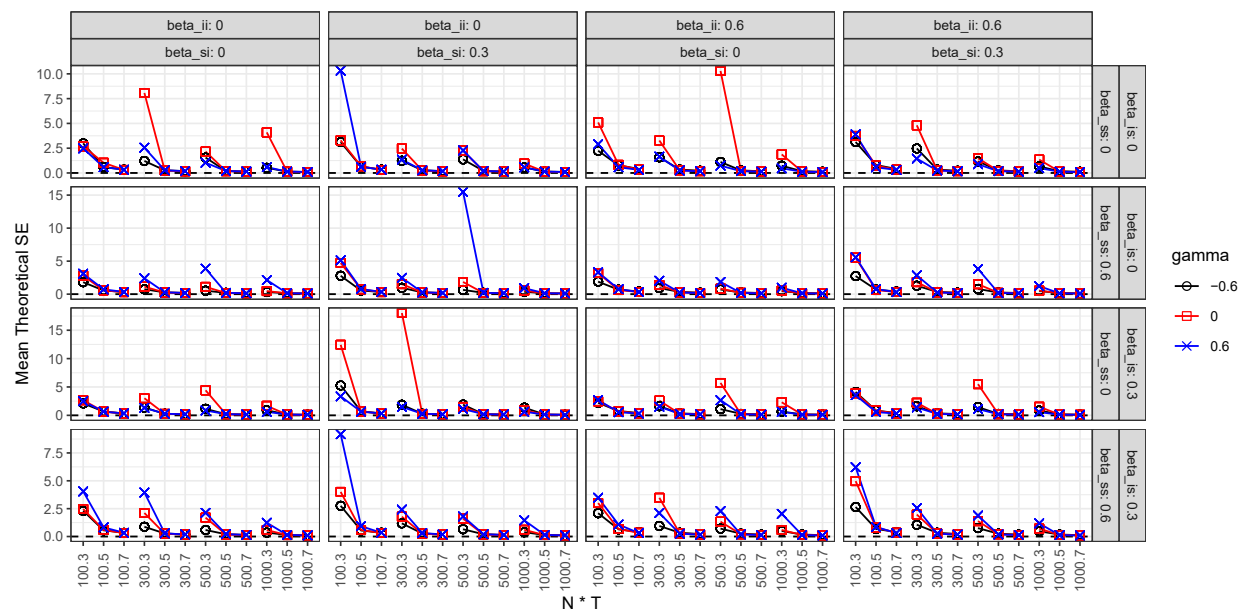


Figure D.174

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCM_{xy}-RP)

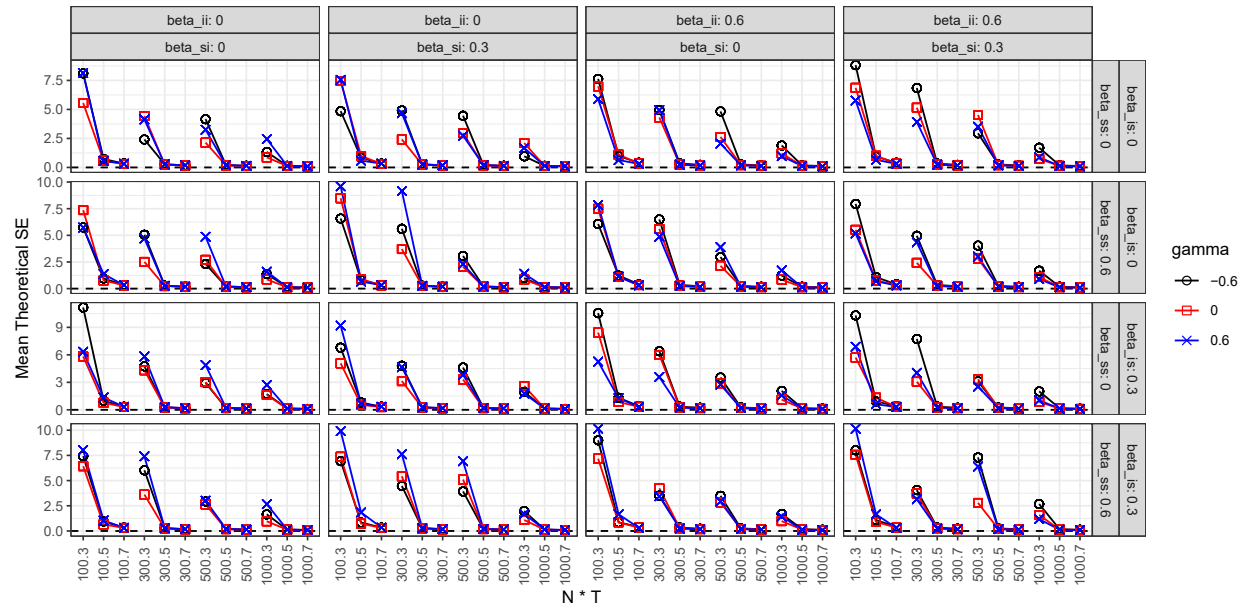


Figure D.175

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCM_x-Rly)

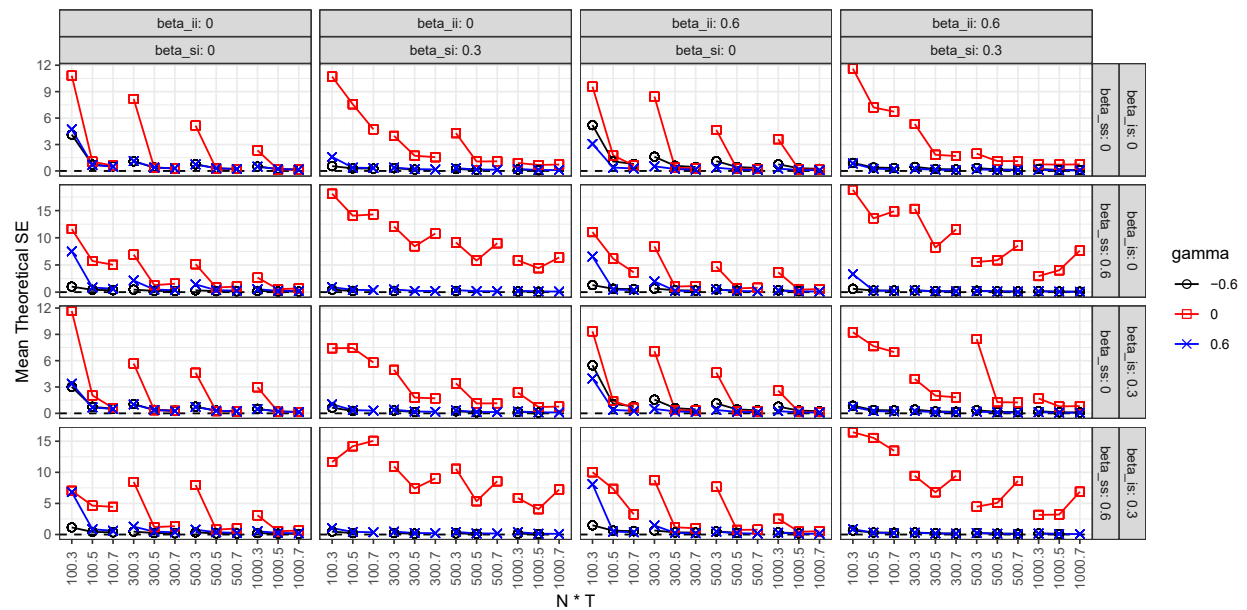


Figure D.176

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)

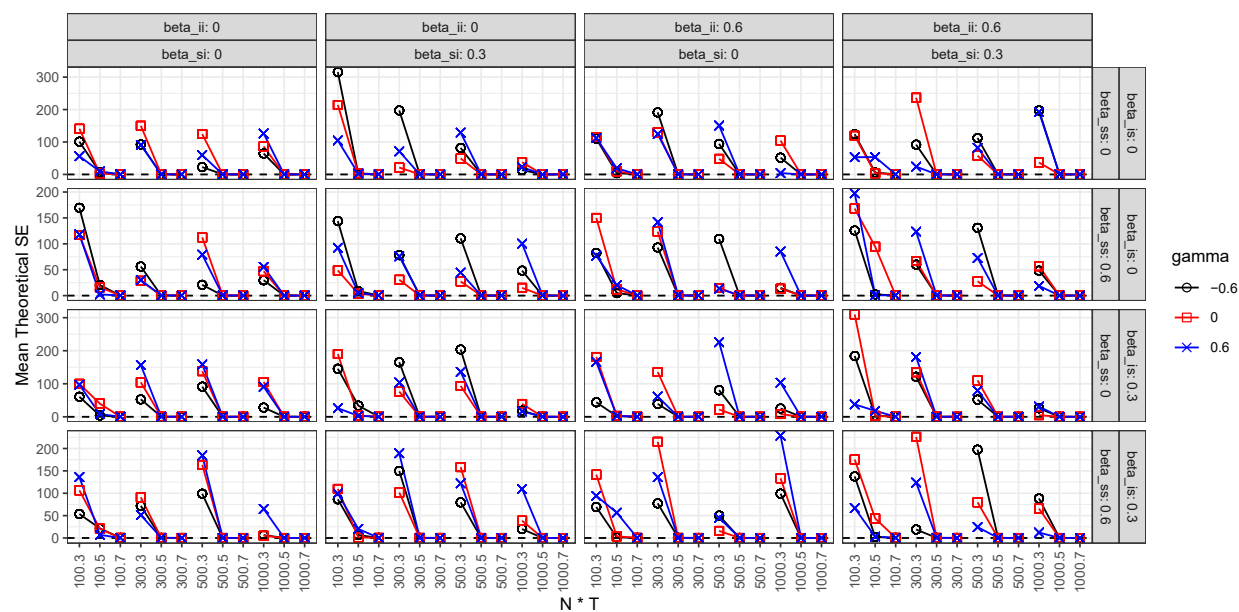


Figure D.177

SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)

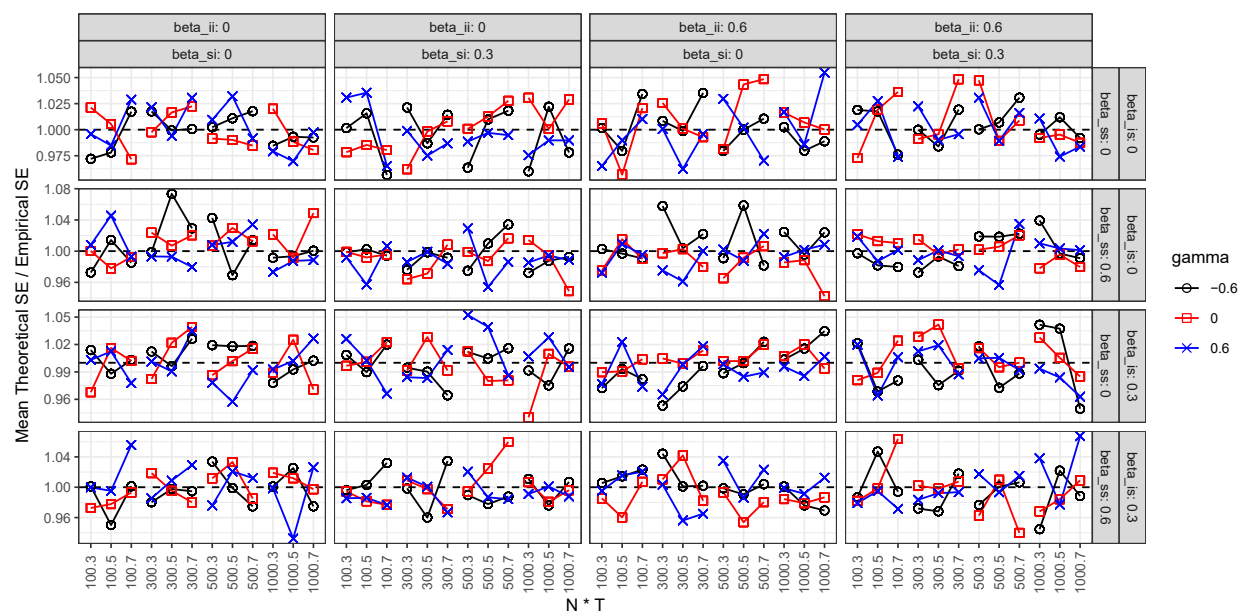


Figure D.178

SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)

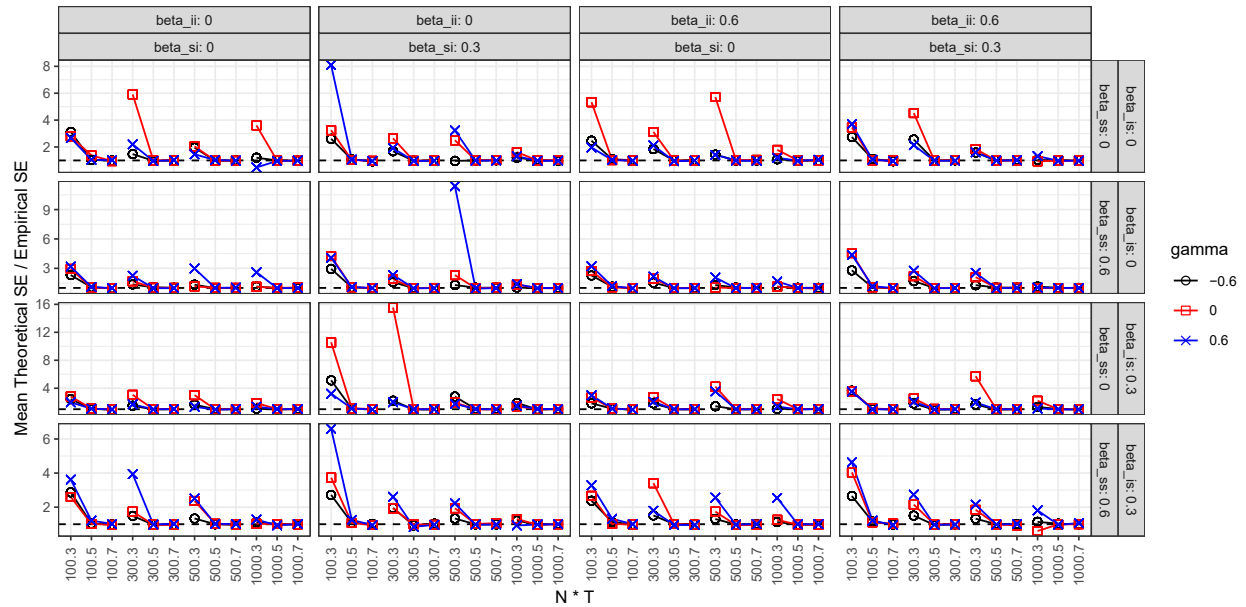


Figure D.179

SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (LCM_{xy}-RP)

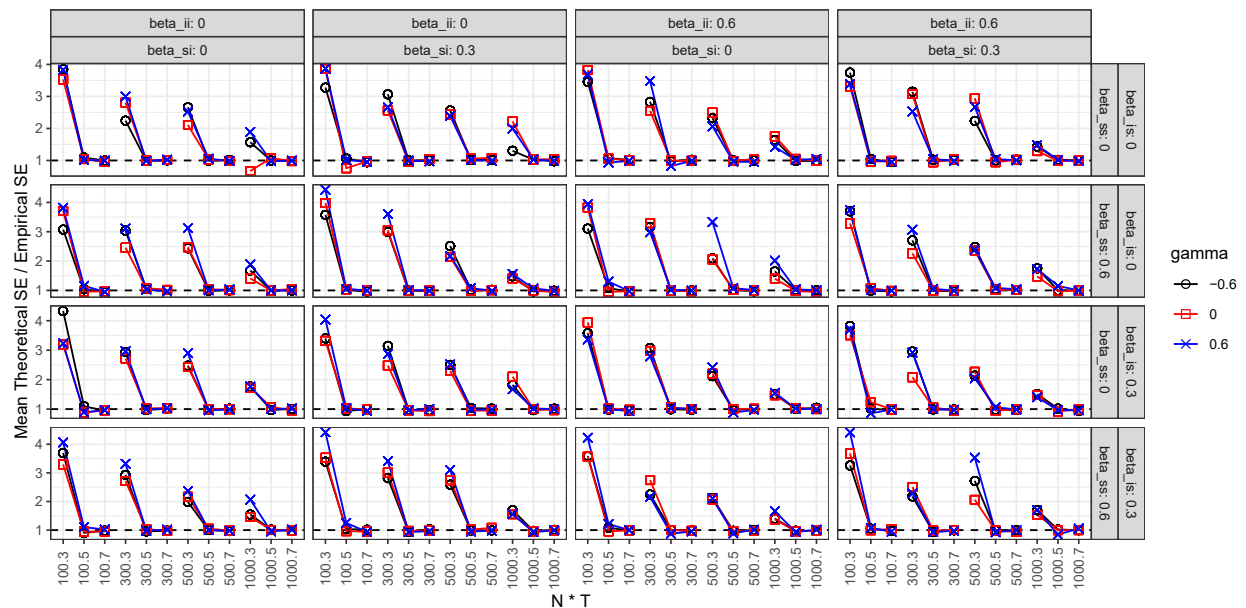


Figure D.180

SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (LCM_x-RLy)

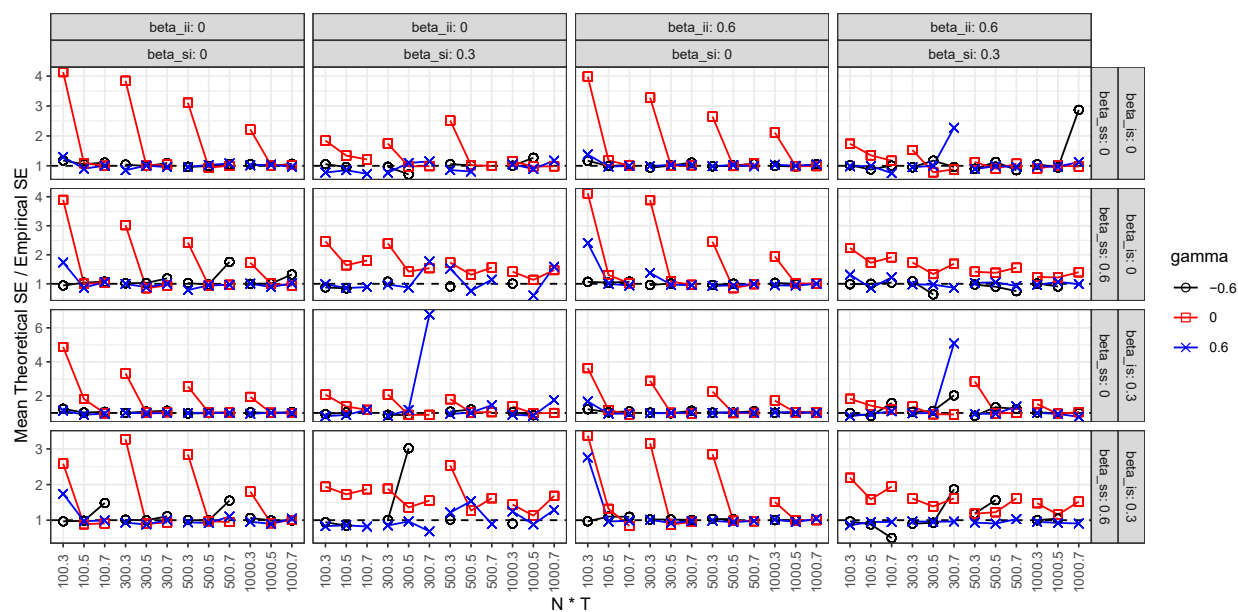


Figure D.181

SE Ratio for the Between-Person Effect β_{IS} in Simulation 4 (LCM_x-CLC)

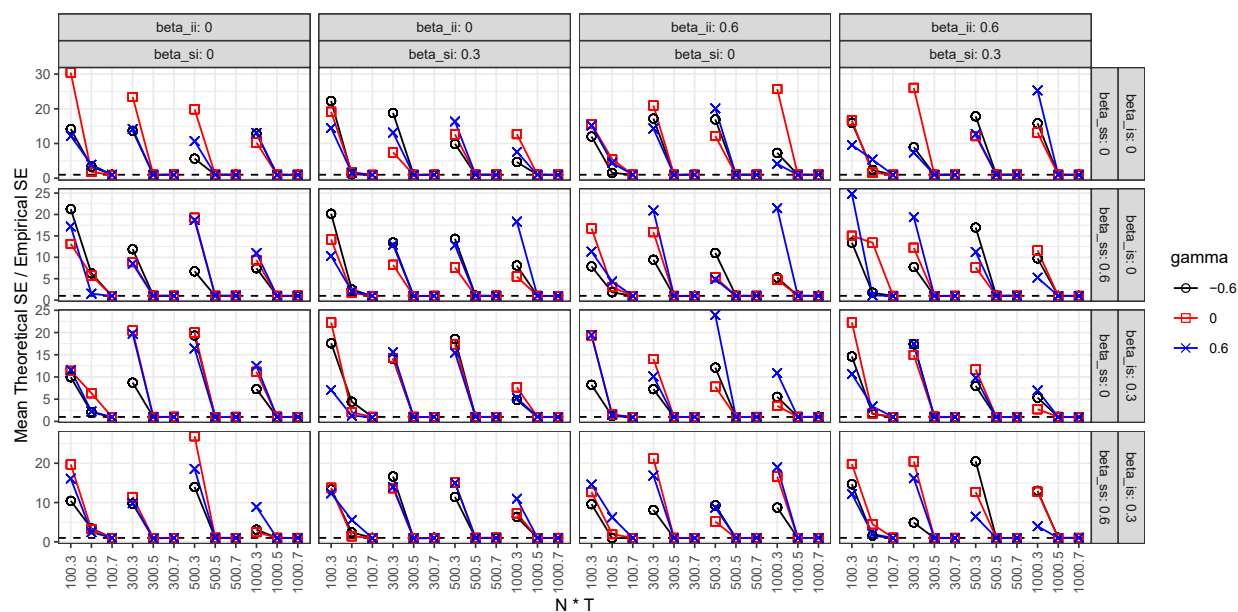


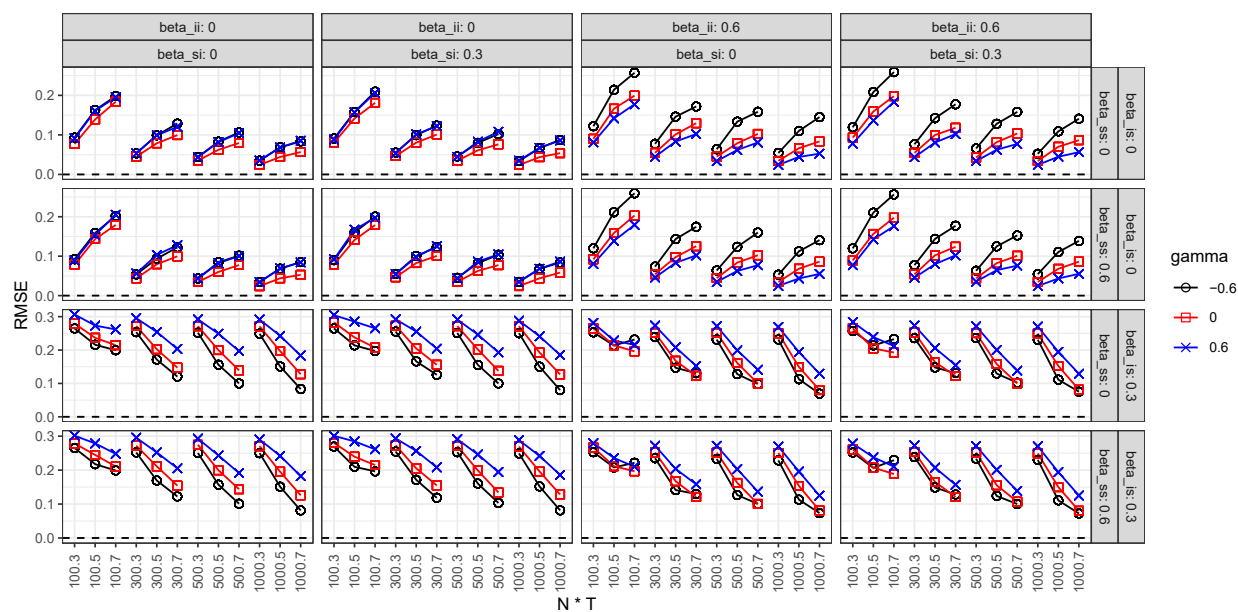
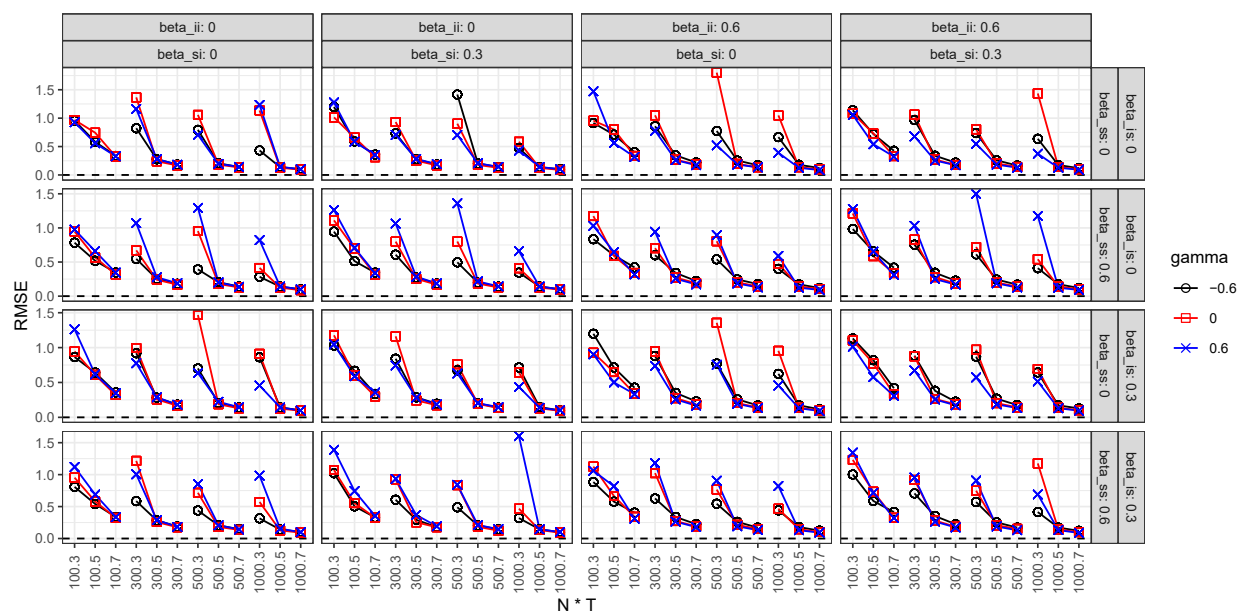
Figure D.182*RMSE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)***Figure D.183***RMSE for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)*

Figure D.184

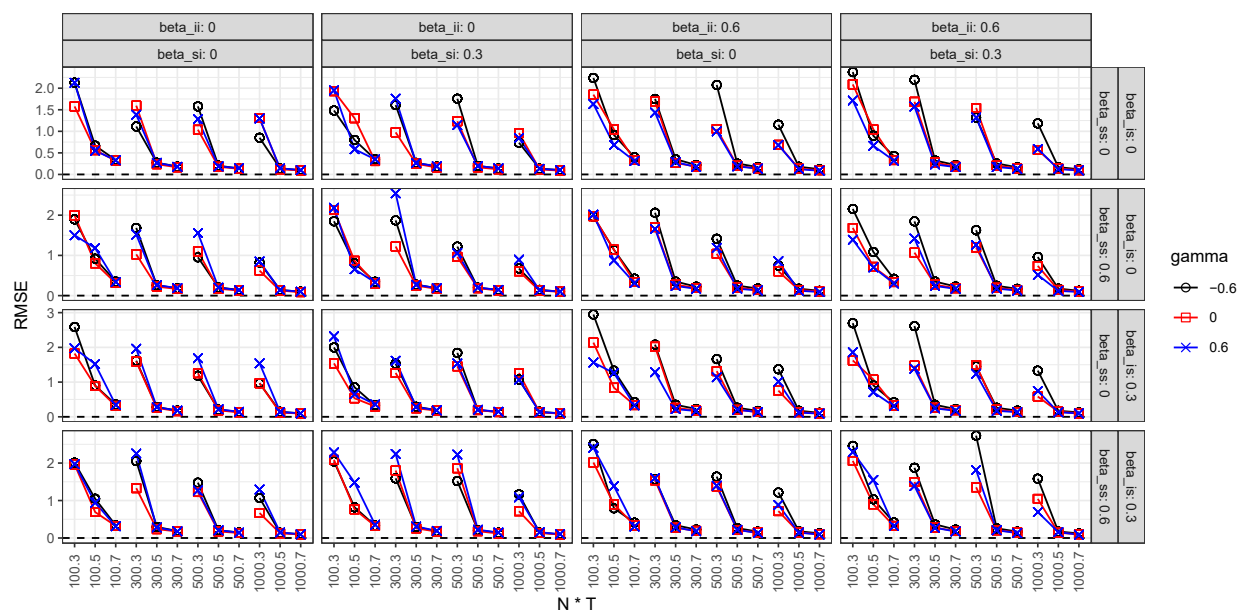
RMSE for the Between-Person Effect β_{IS} in Simulation 4 (LCM_{xy}-RP)

Figure D.185

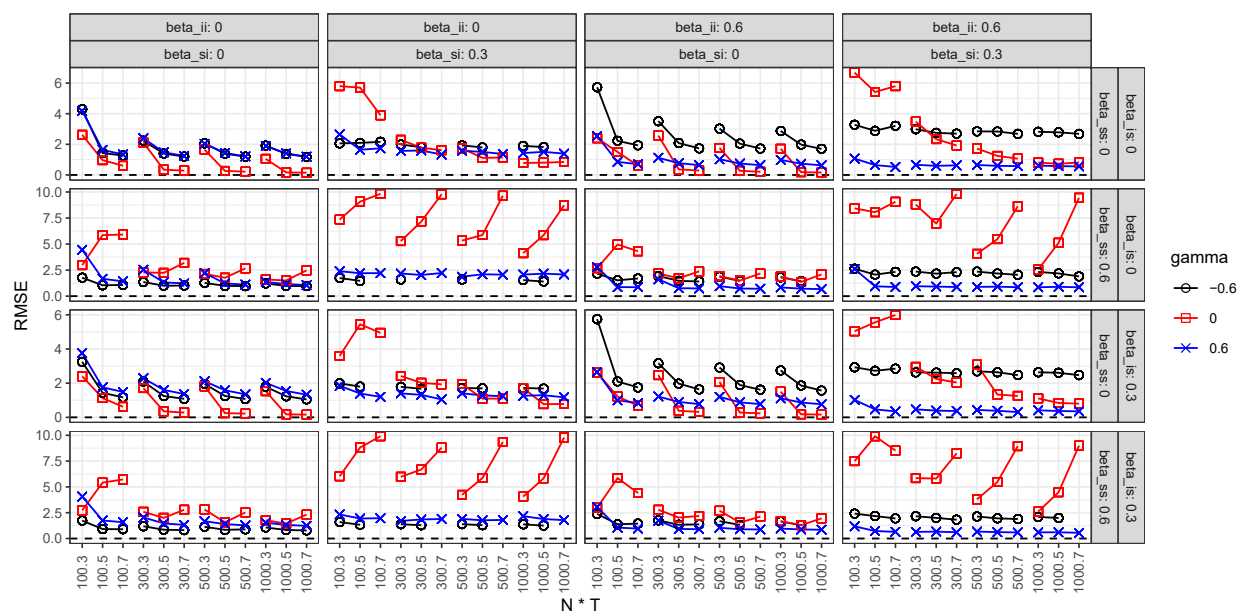
RMSE for the Between-Person Effect β_{IS} in Simulation 4 (LCM_x-Rly)

Figure D.186

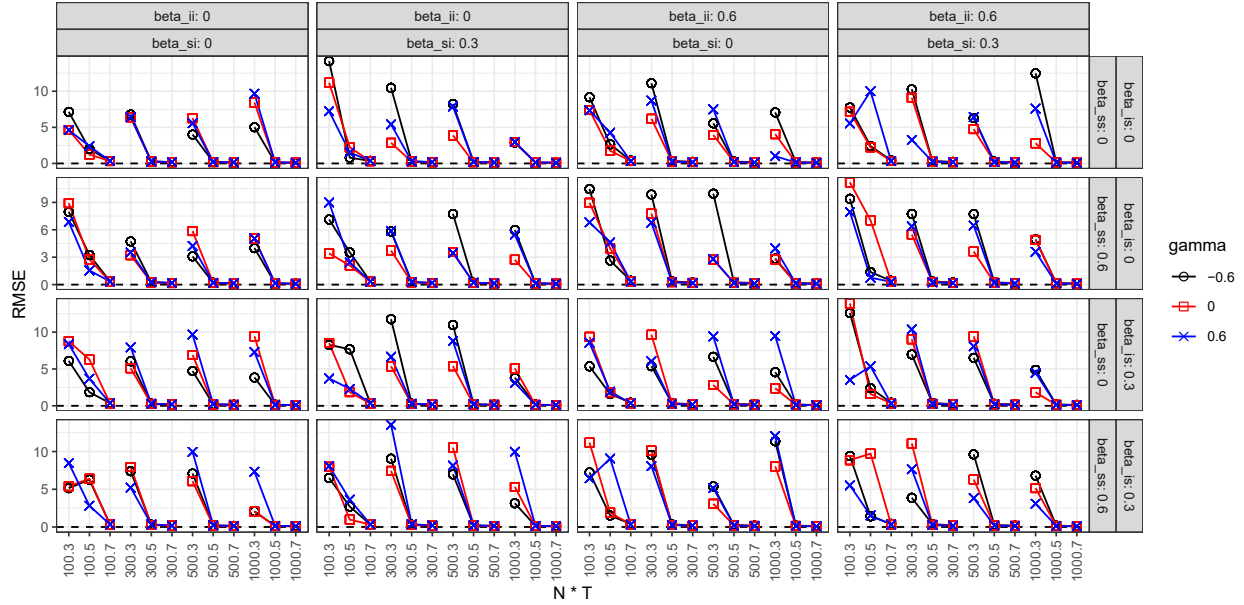
RMSE for the Between-Person Effect β_{IS} in Simulation 4 (LCM_x-CLC)

Figure D.187

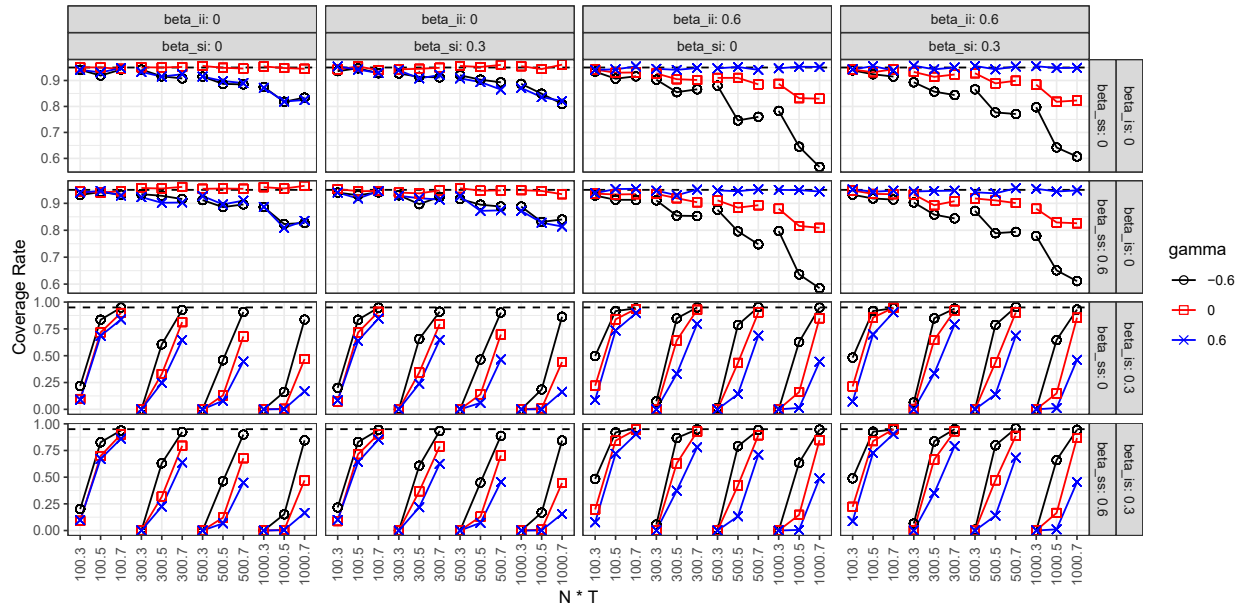
Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)

Figure D.188

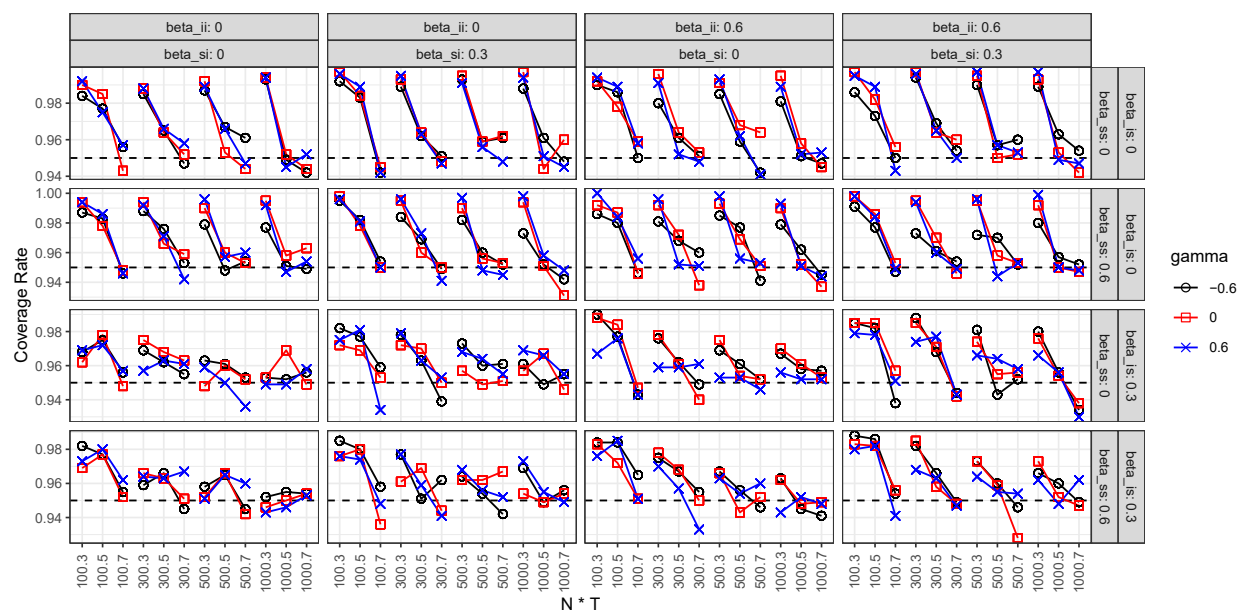
Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)

Figure D.189

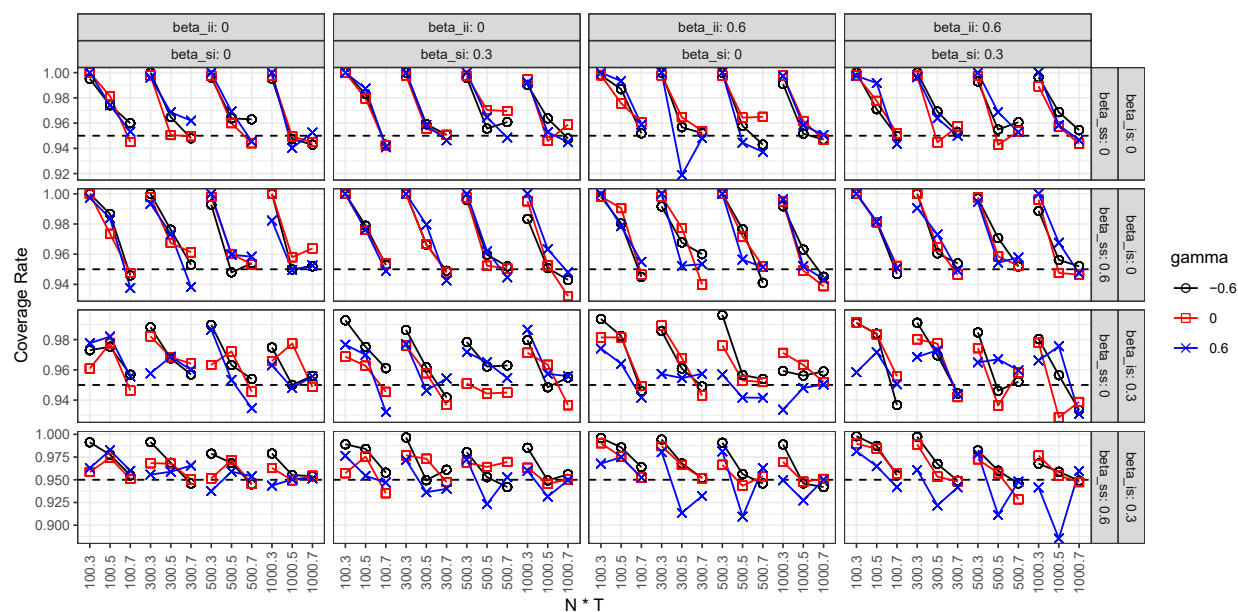
Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (LCMxy-RP)

Figure D.190

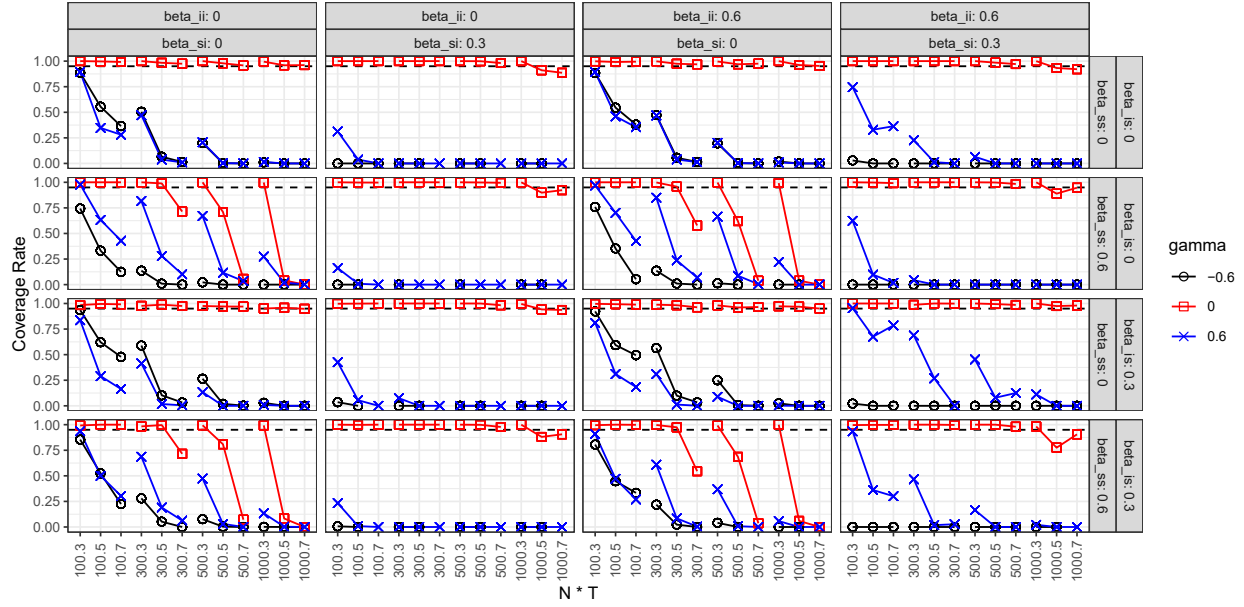
Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-Rly)

Figure D.191

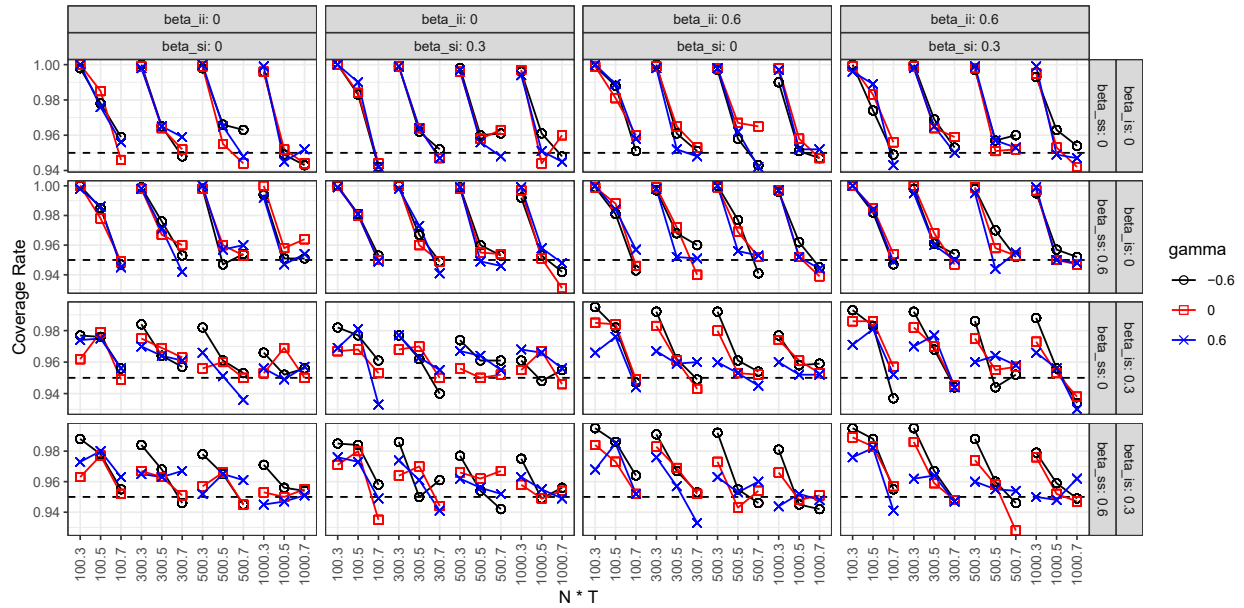
Coverage Rate for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)

Figure D.192

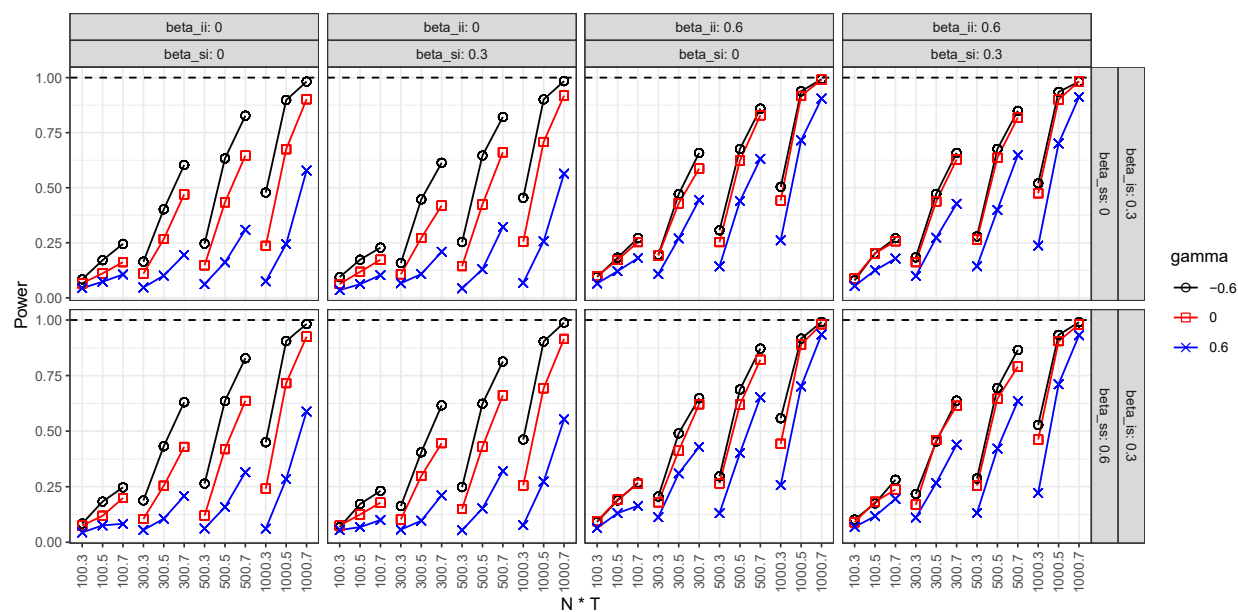
Power for the Between-Person Effect β_{IS} in Simulation 4 (MLM-E2S)

Figure D.193

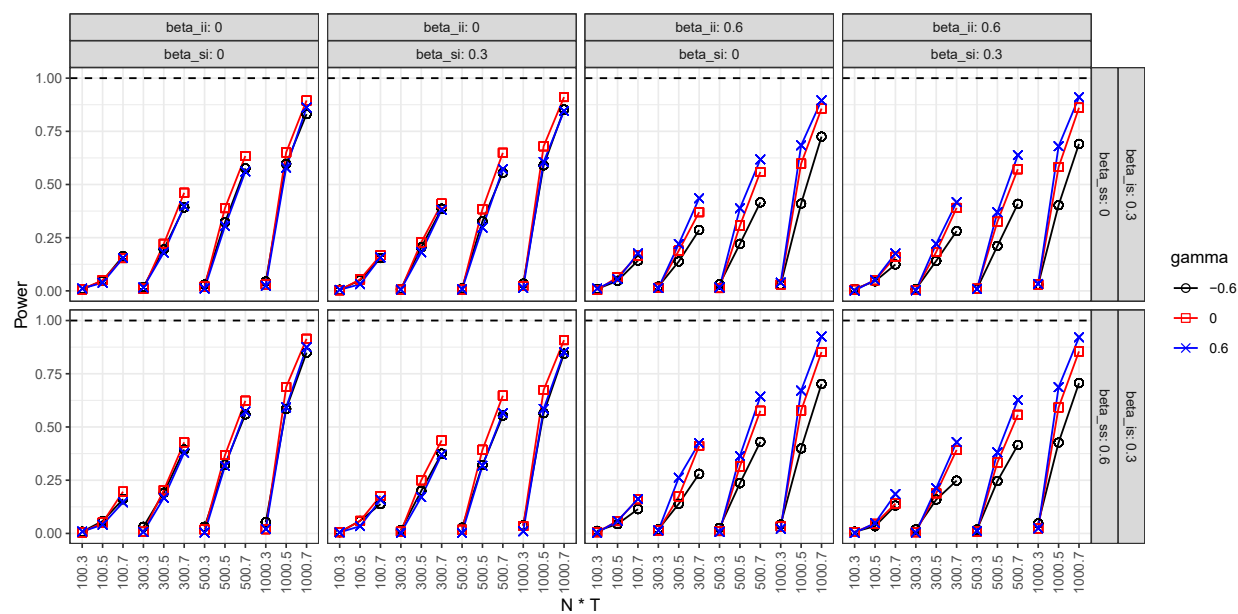
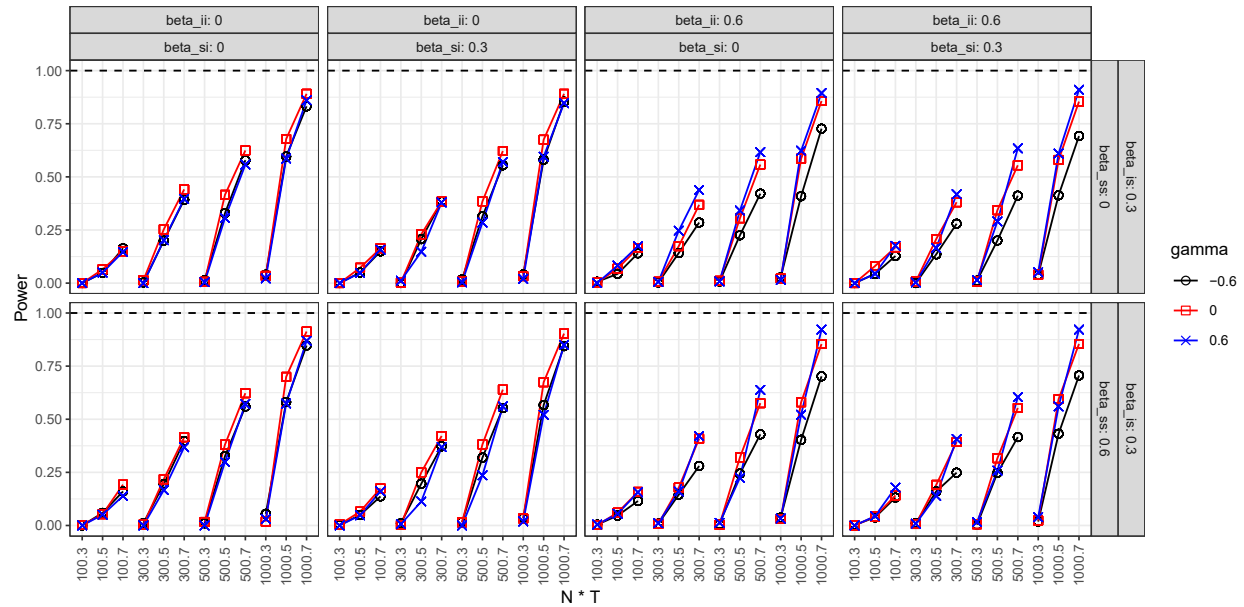
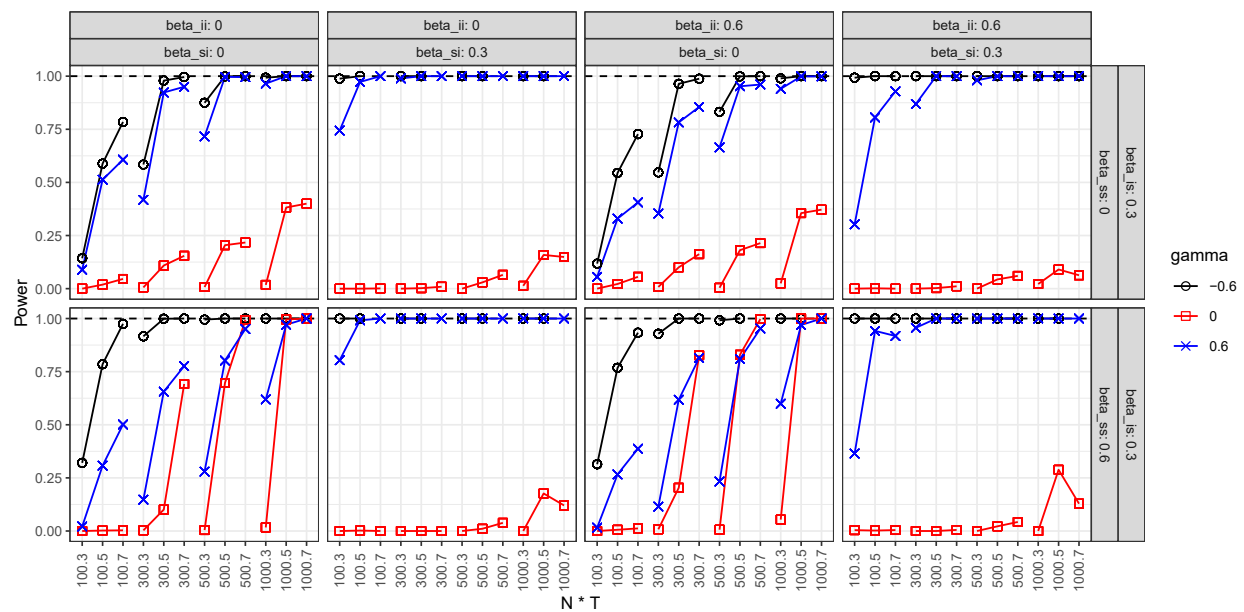
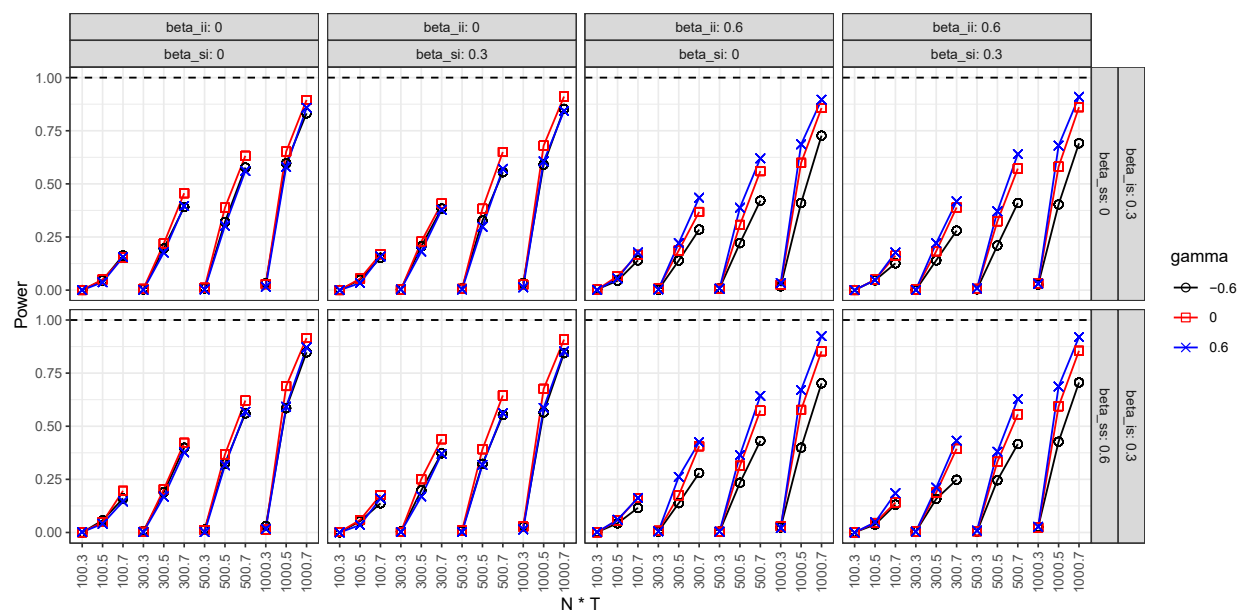
Power for the Between-Person Effect β_{IS} in Simulation 4 (MLM-MV)

Figure D.194*Power for the Between-Person Effect β_{IS} in Simulation 4 (LCM_{xy}-RP)***Figure D.195***Power for the Between-Person Effect β_{IS} in Simulation 4 (LCM_x-Rly)*

Power for the Between-Person Effect β_{IS} in Simulation 4 (LCMx-CLC)



D.6 Between-Person Effect of Slope on Slope

Figure D.197

Bias for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)

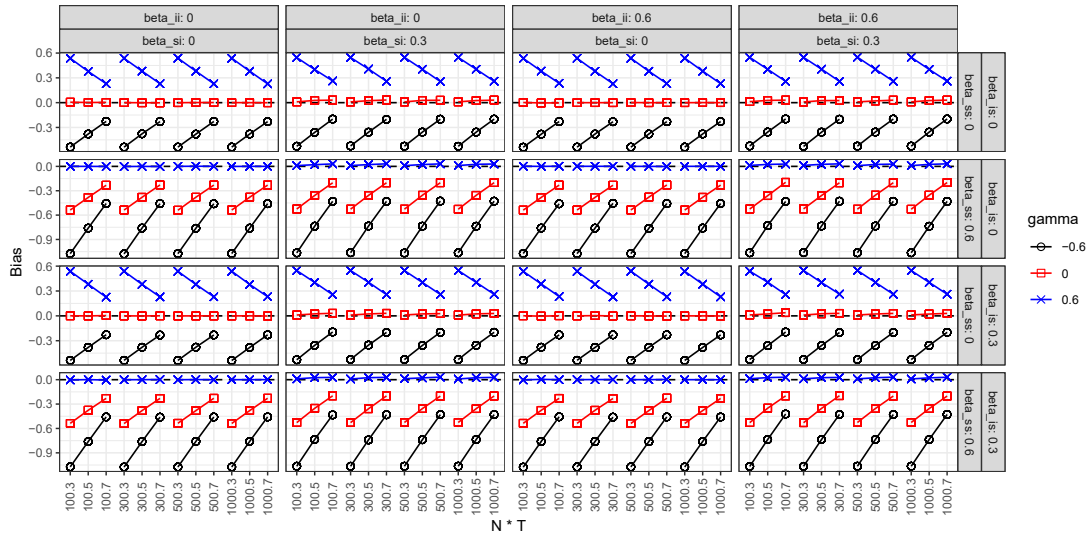


Figure D.198

Bias for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)

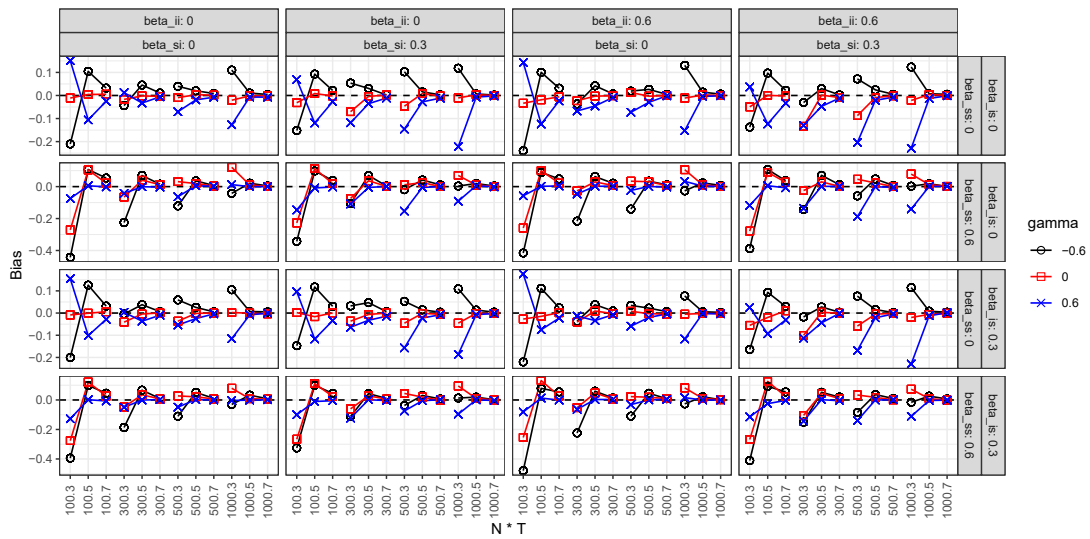


Figure D.199

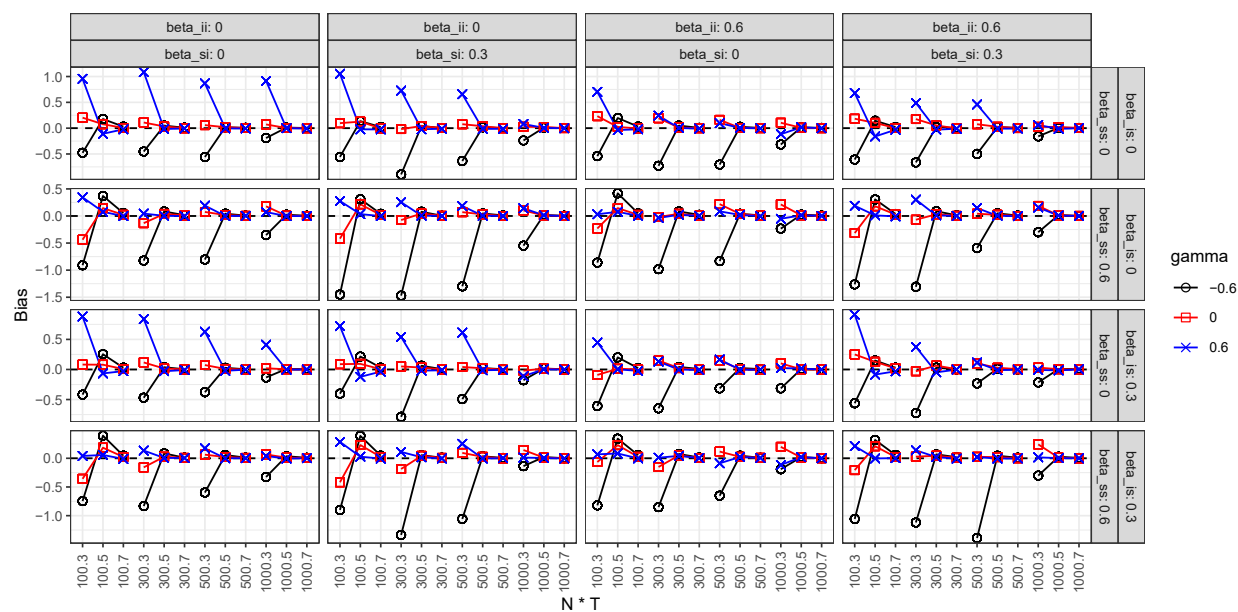
Bias for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)

Figure D.200

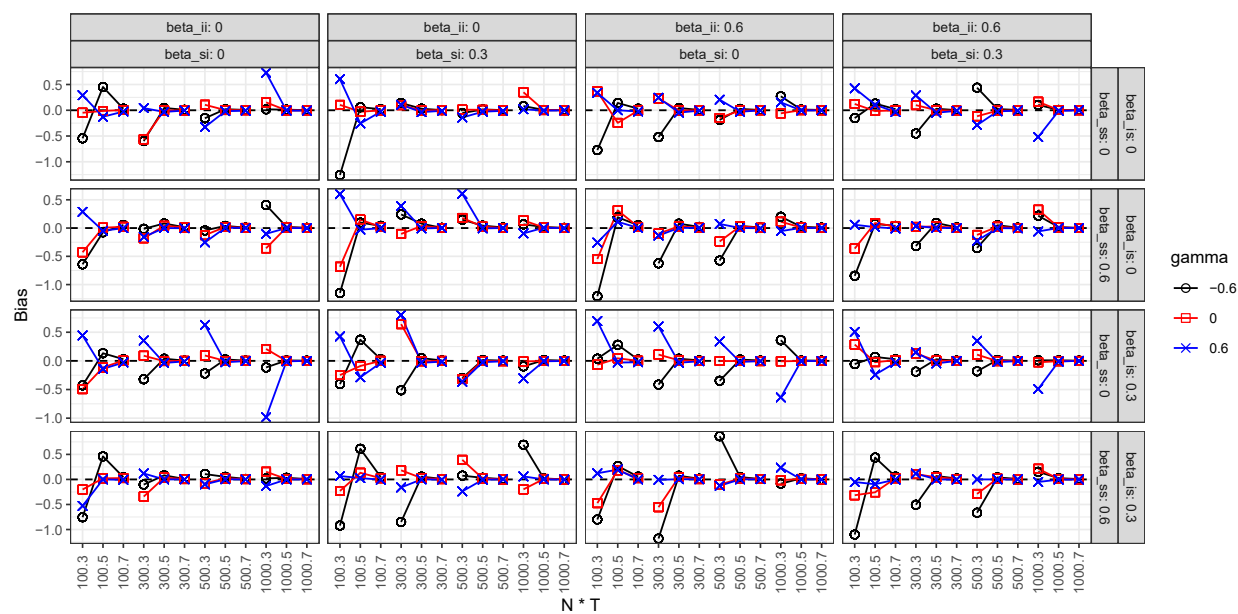
Bias for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)

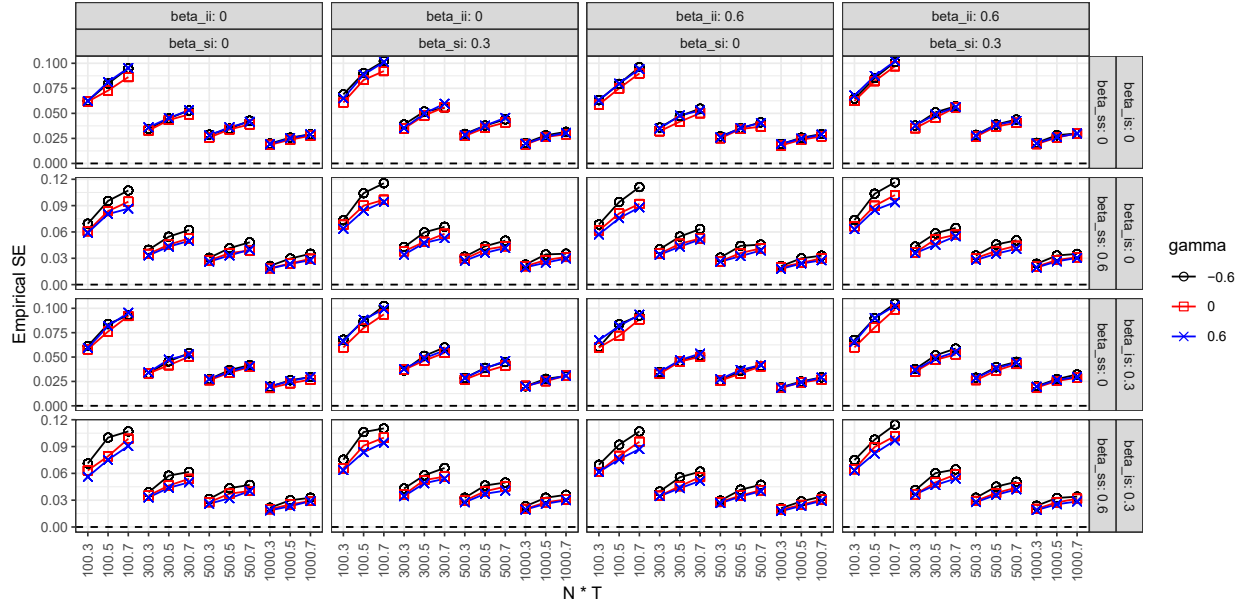
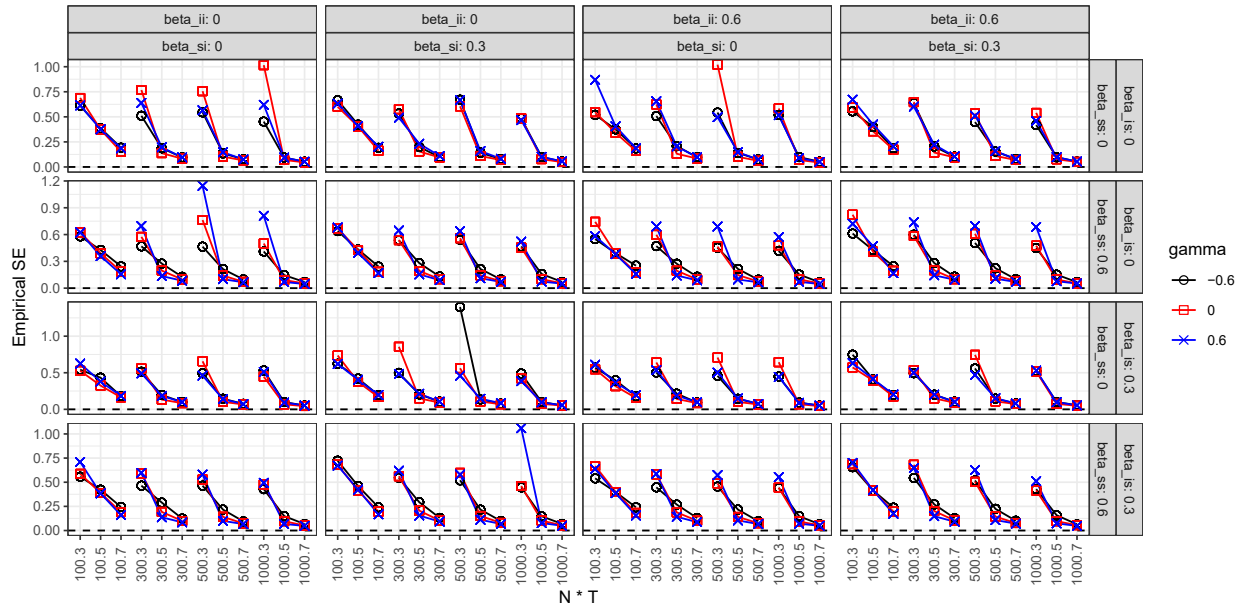
Figure D.201*Empirical SE for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)***Figure D.202***Empirical SE for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)*

Figure D.203

Empirical SE for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)

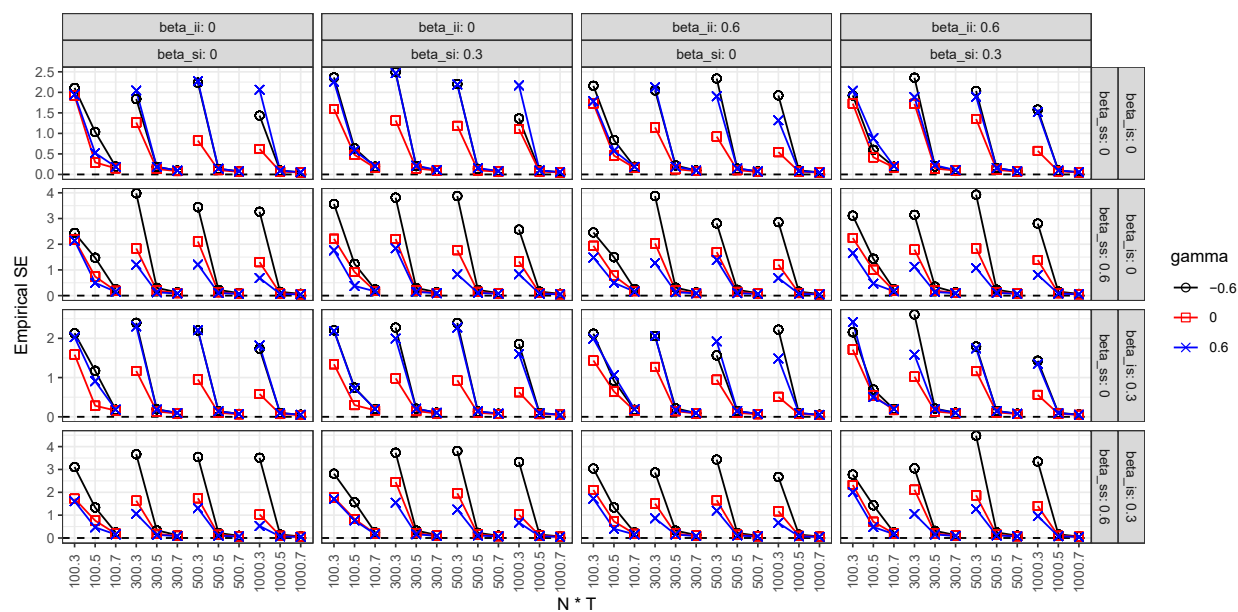


Figure D.204

Empirical SE for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)

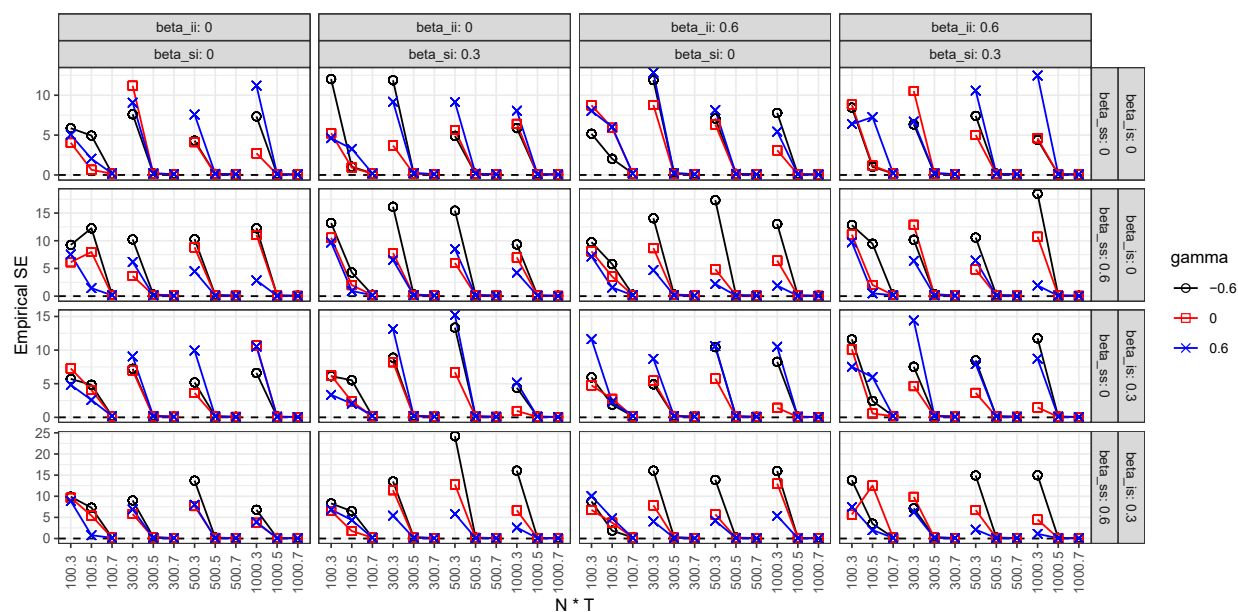


Figure D.205

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)

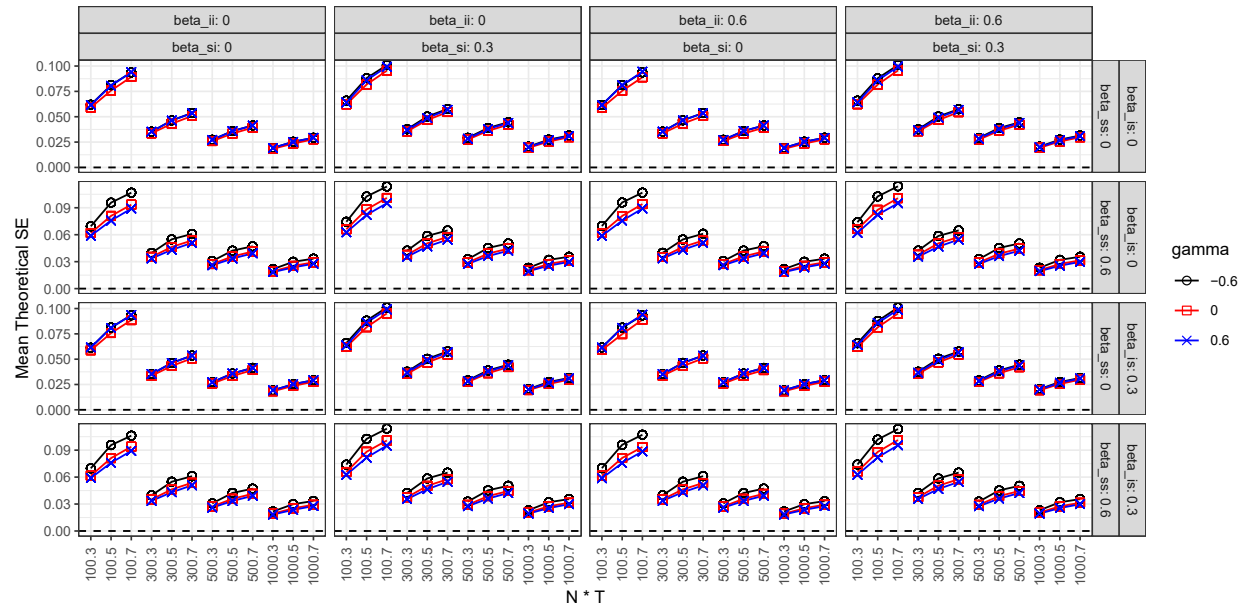


Figure D.206

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)

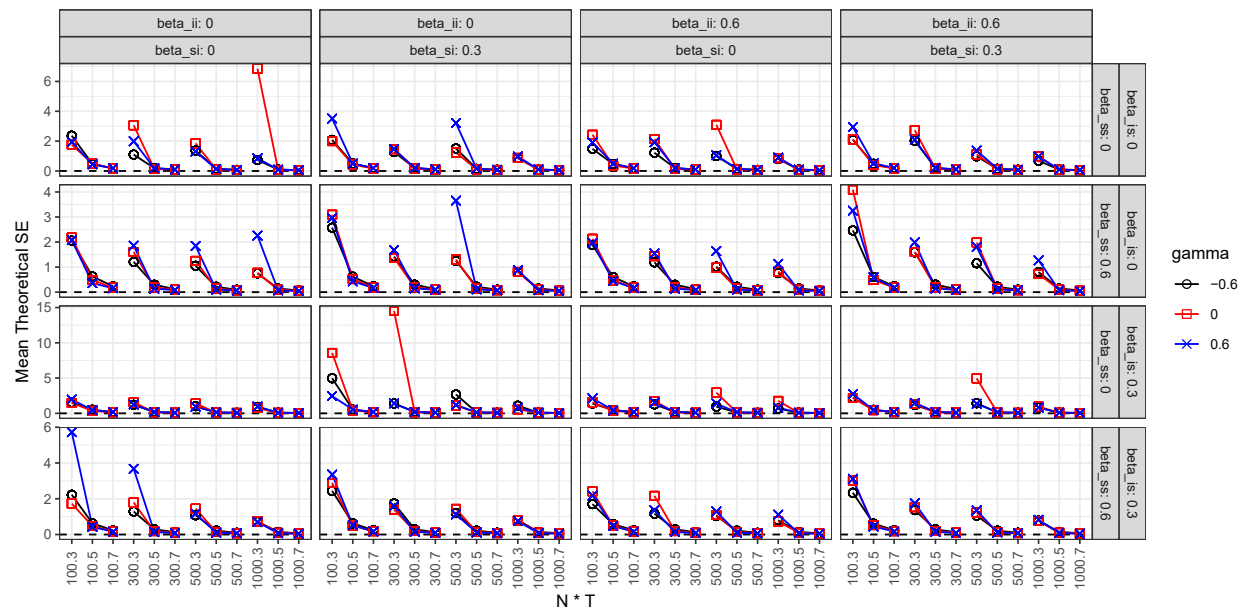


Figure D.207

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 4 (LCM_{xy}-RP)

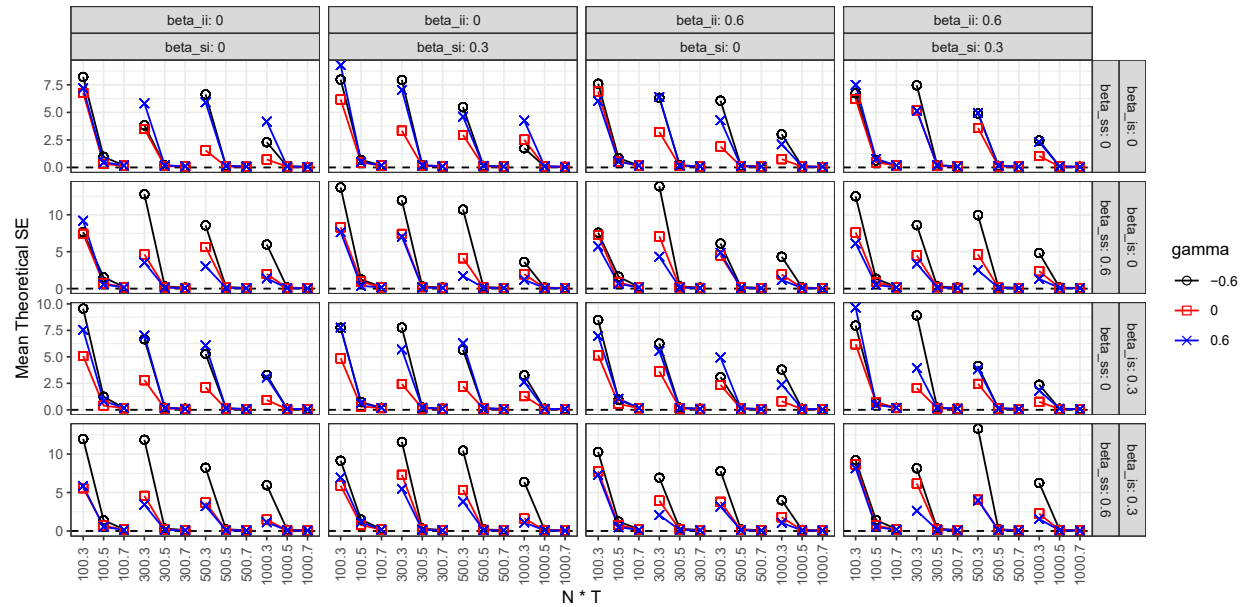


Figure D.208

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 4 (LCM_x-CLC)

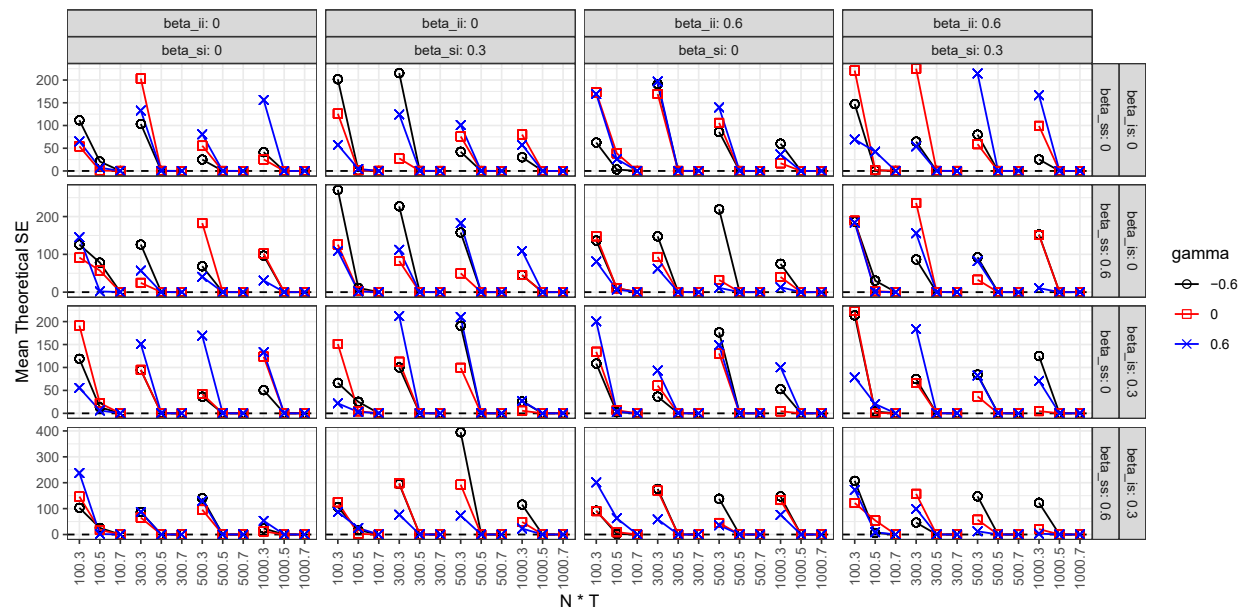


Figure D.209

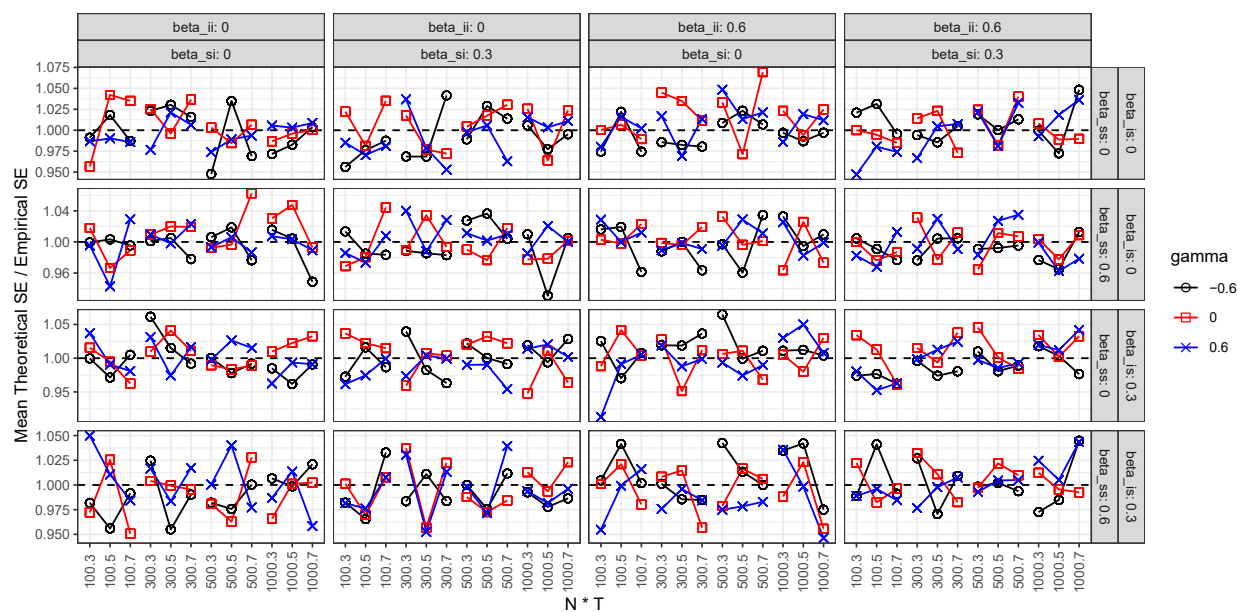
SE Ratio for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)

Figure D.210

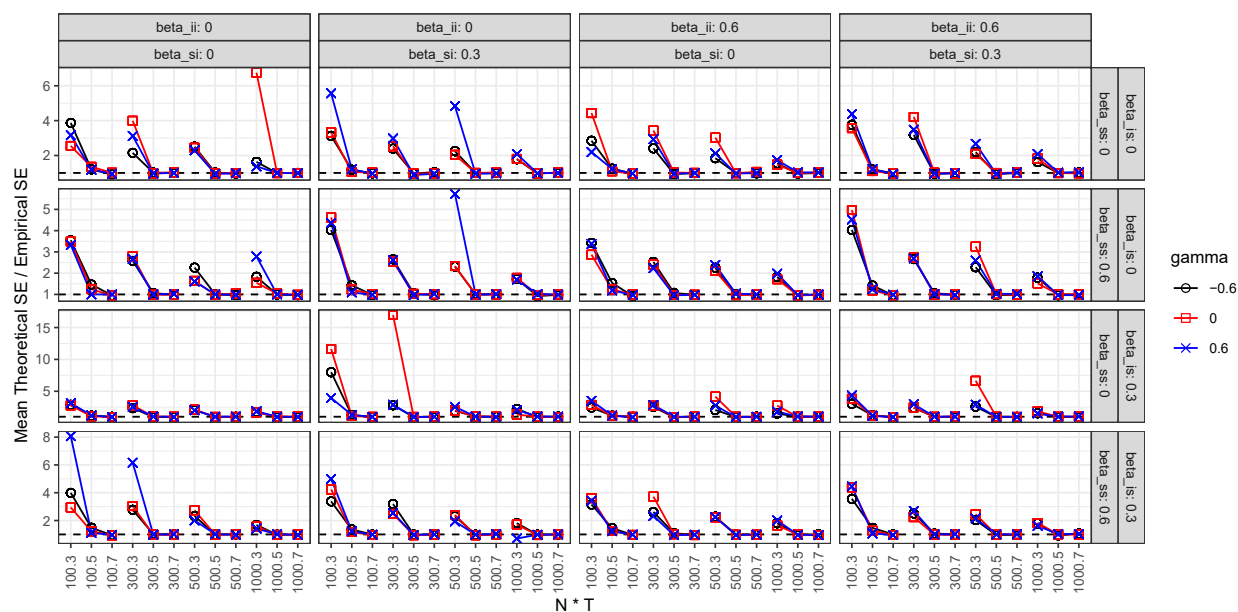
SE Ratio for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)

Figure D.211

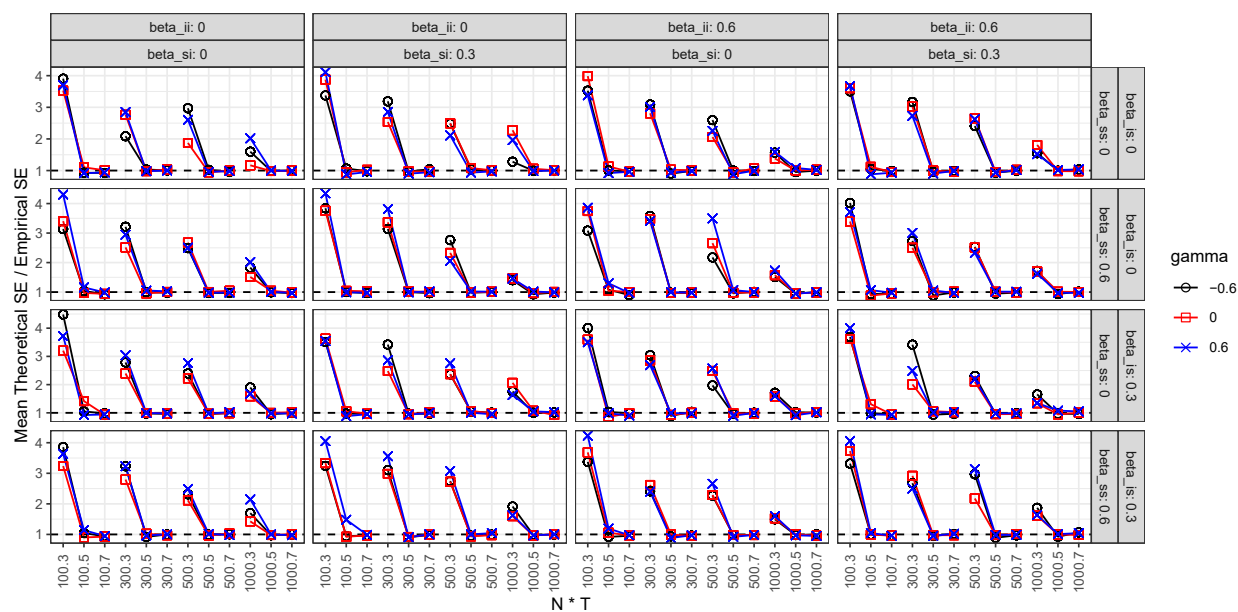
SE Ratio for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)

Figure D.212

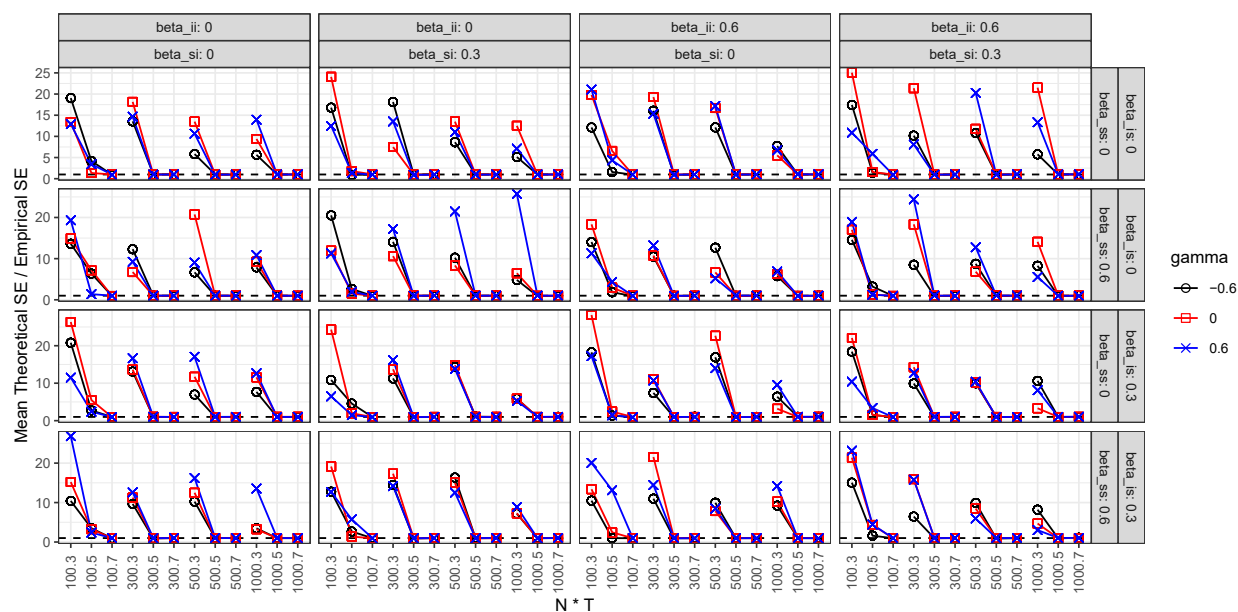
SE Ratio for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)

Figure D.213

RMSE for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)

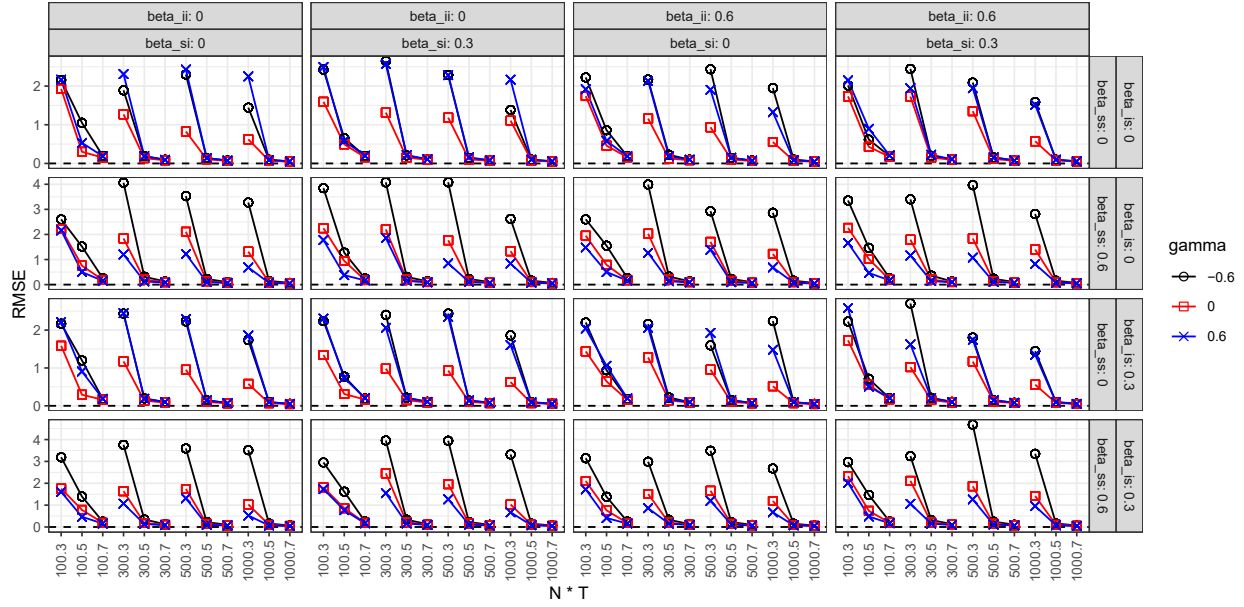


Figure D.214

RMSE for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)

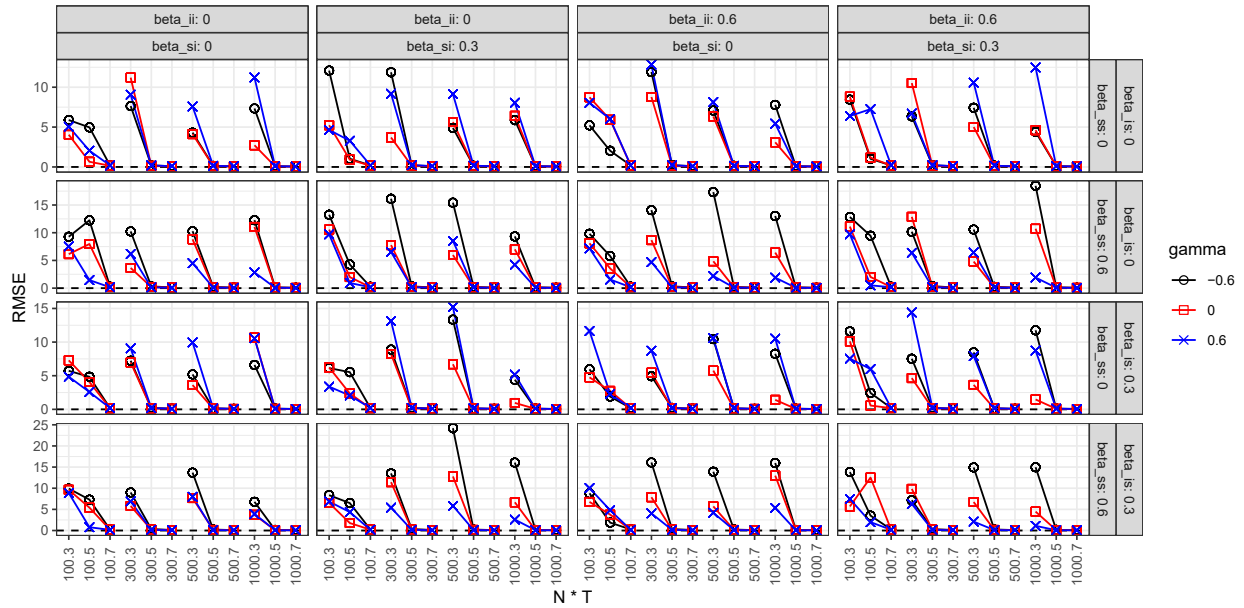


Figure D.215

RMSE for the Between-Person Effect β_{SS} in Simulation 4 ($\gamma = -0.6$)

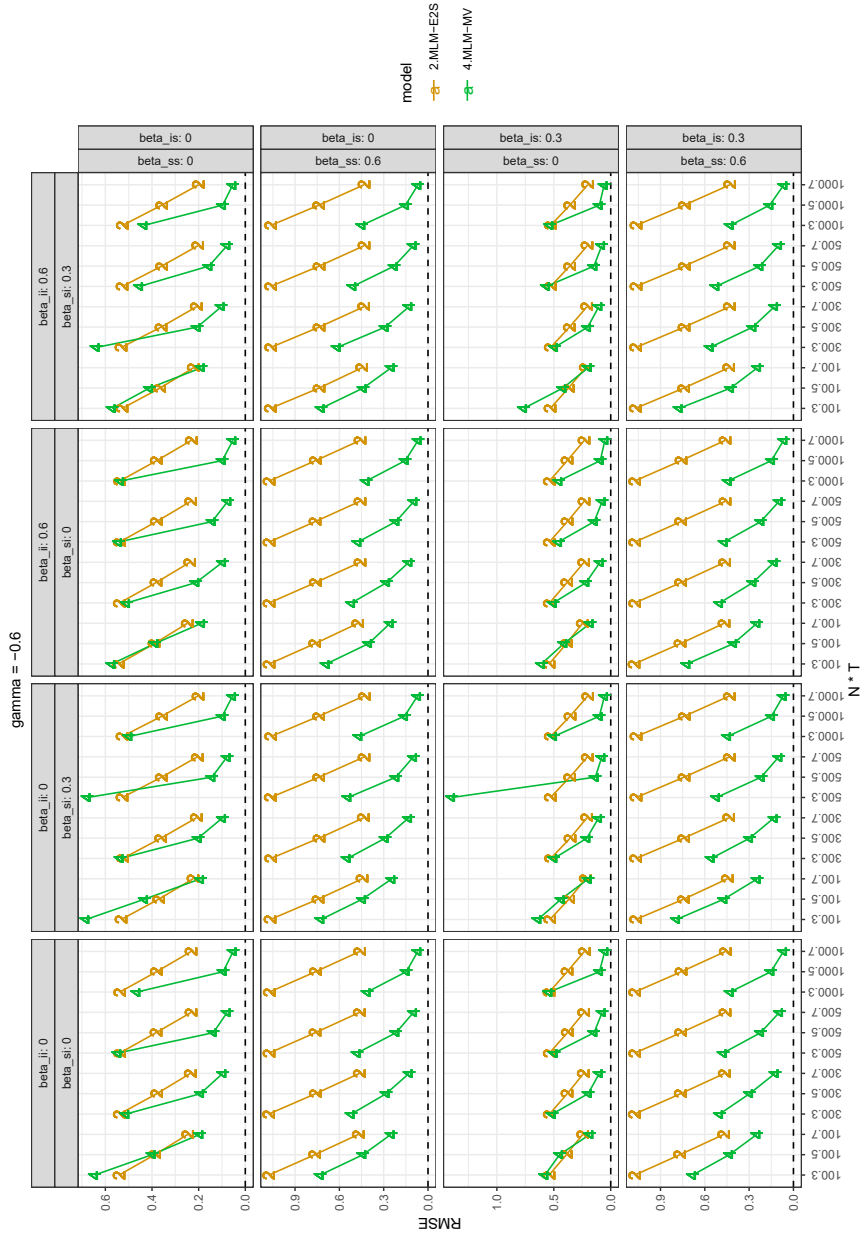


Figure D.216

RMSE for the Between-Person Effect β_{SS} in Simulation 4 ($\gamma = 0$)

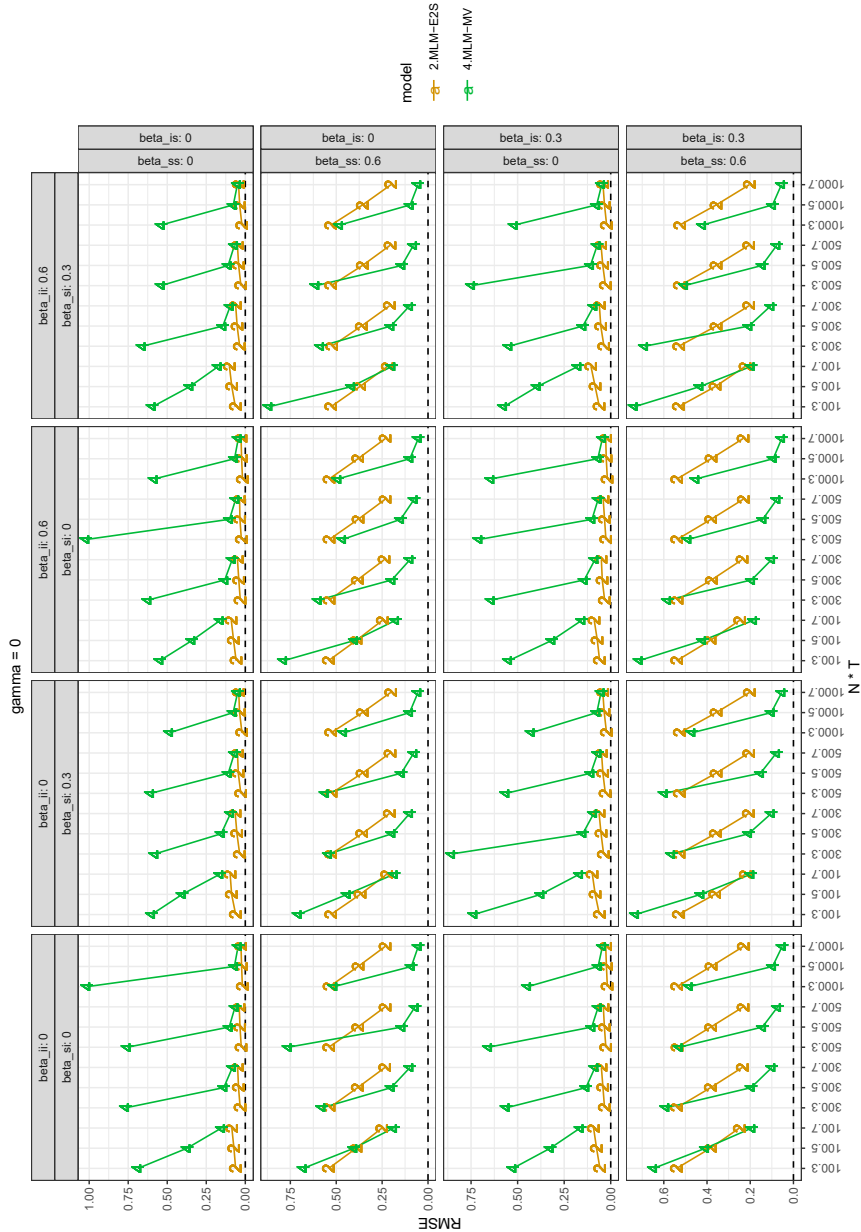


Figure D.217

RMSE for the Between-Person Effect β_{SS} in Simulation 4 ($\gamma = 0.6$)

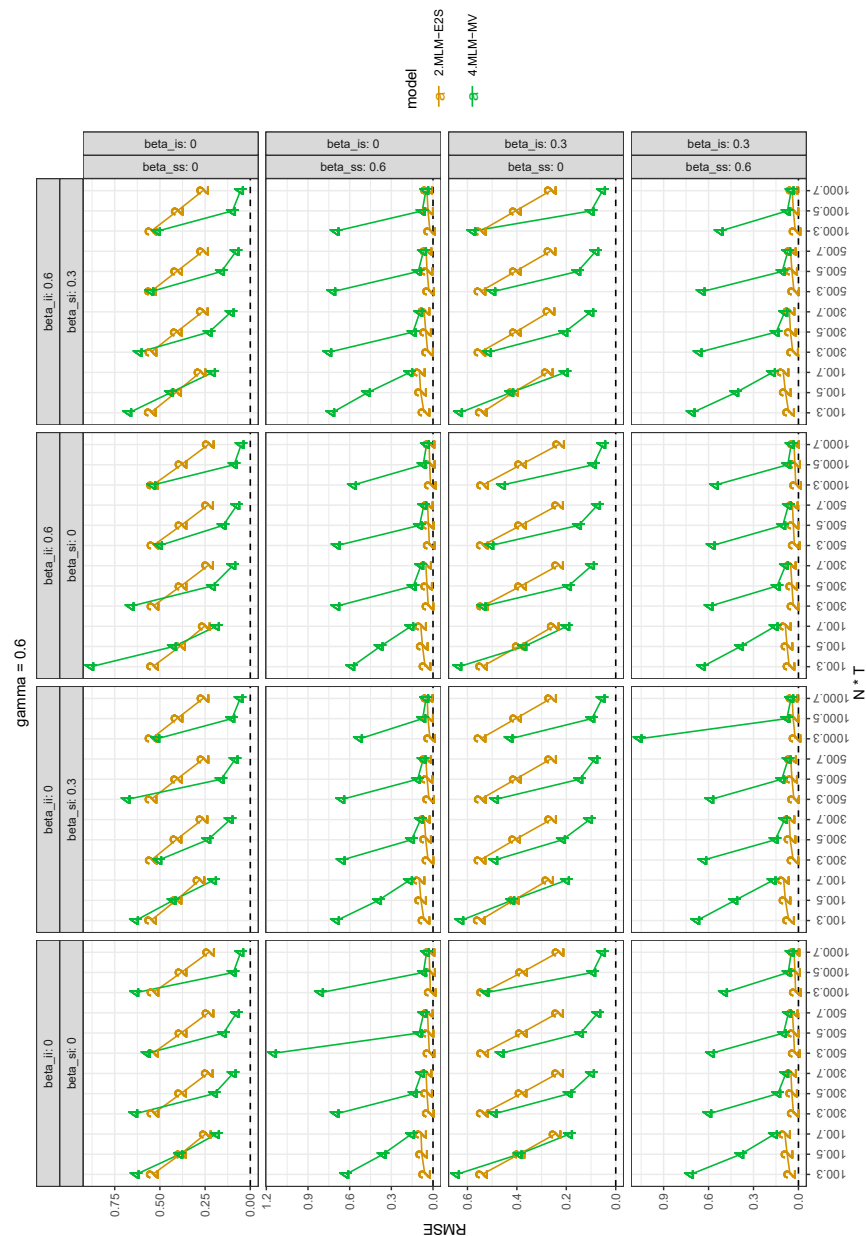


Figure D.218

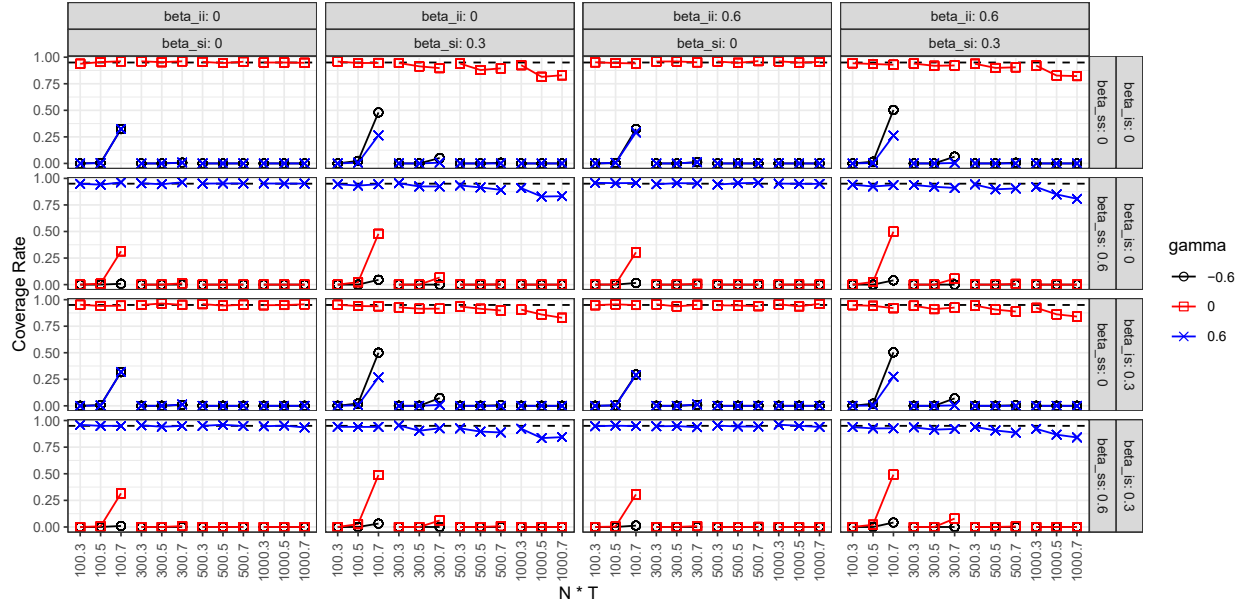
Coverage Rate for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)

Figure D.219

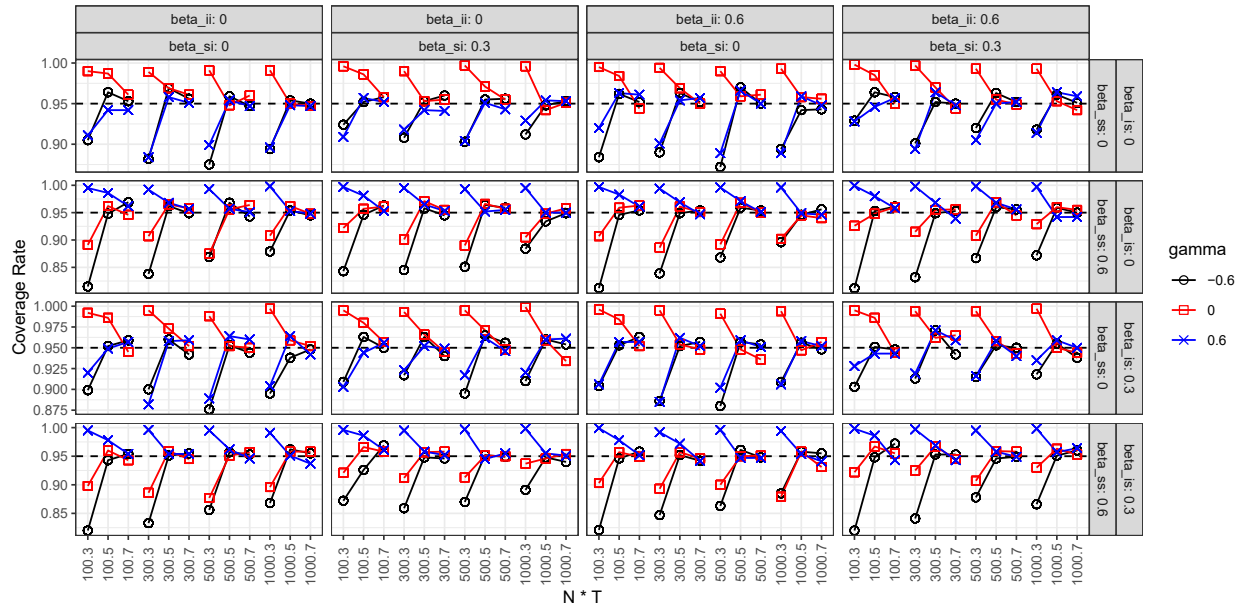
Coverage Rate for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)

Figure D.220

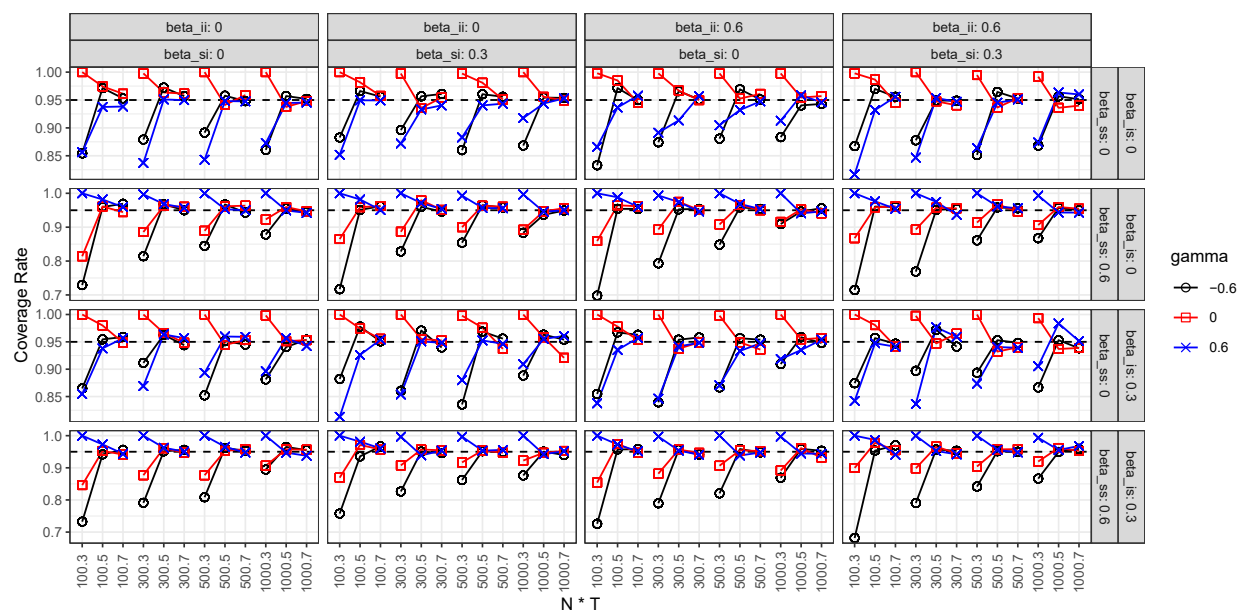
Coverage Rate for the Between-Person Effect β_{SS} in Simulation 4 (LCMxy-RP)

Figure D.221

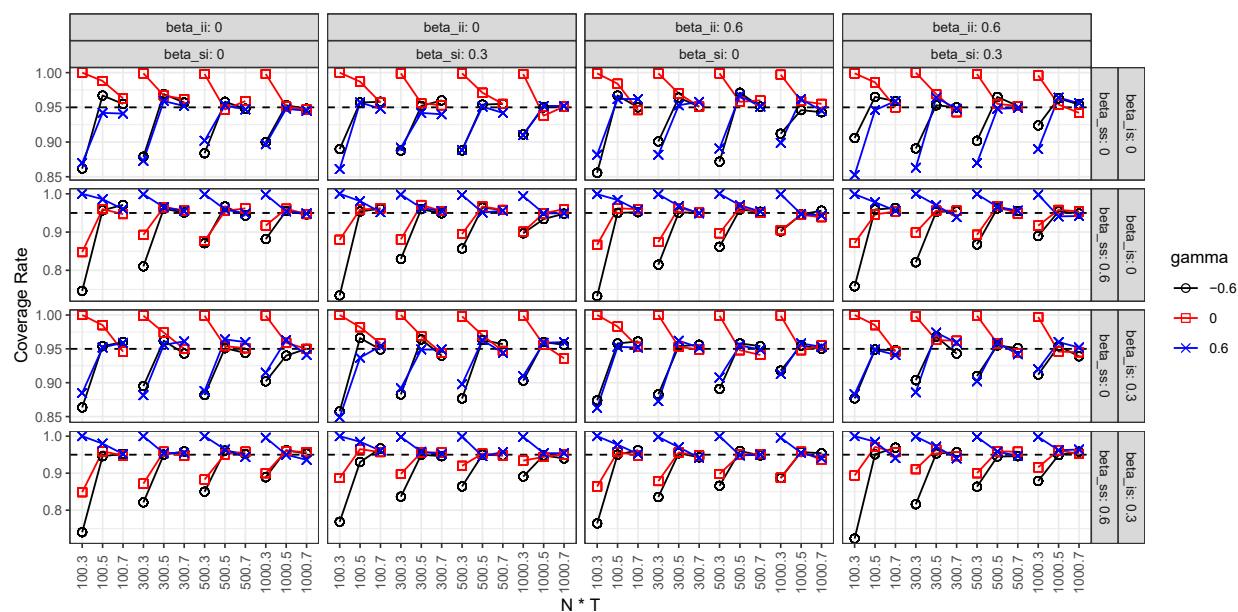
Coverage Rate for the Between-Person Effect β_{SS} in Simulation 4 (LCMx-CLC)

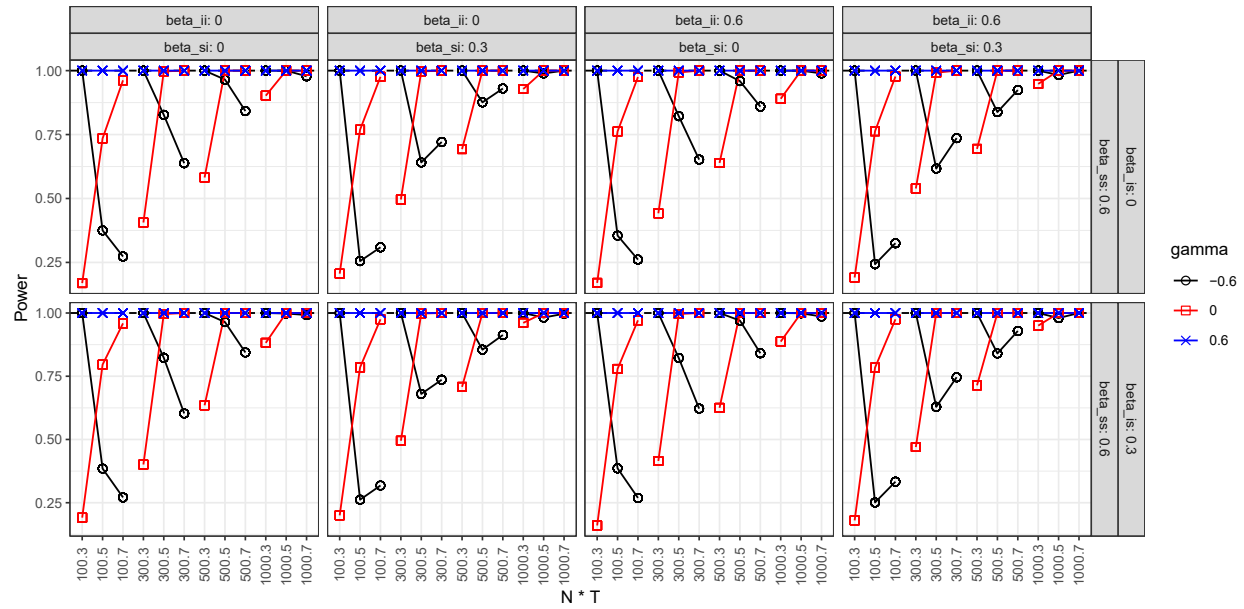
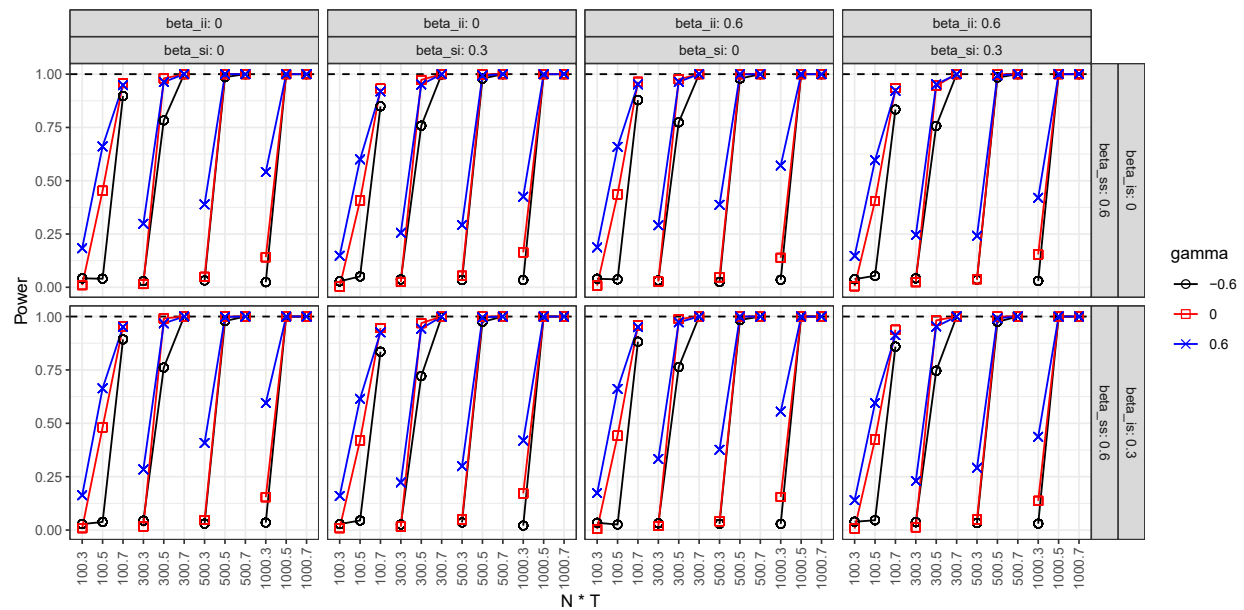
Figure D.222*Power for the Between-Person Effect β_{SS} in Simulation 4 (MLM-E2S)***Figure D.223***Power for the Between-Person Effect β_{SS} in Simulation 4 (MLM-MV)*

Figure D.224

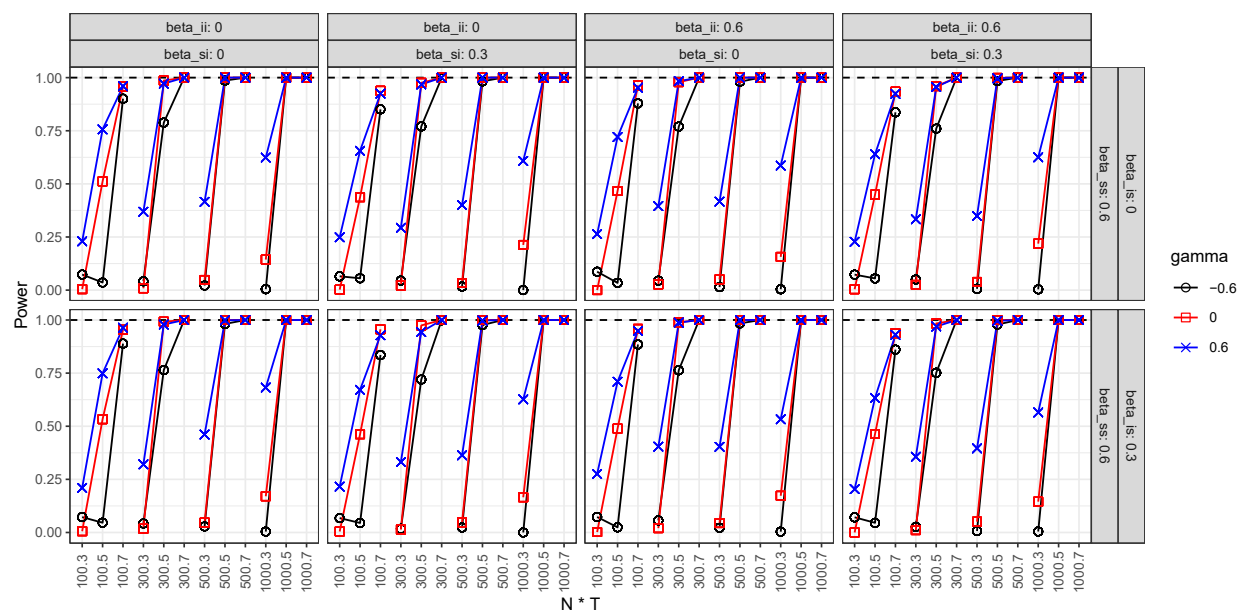
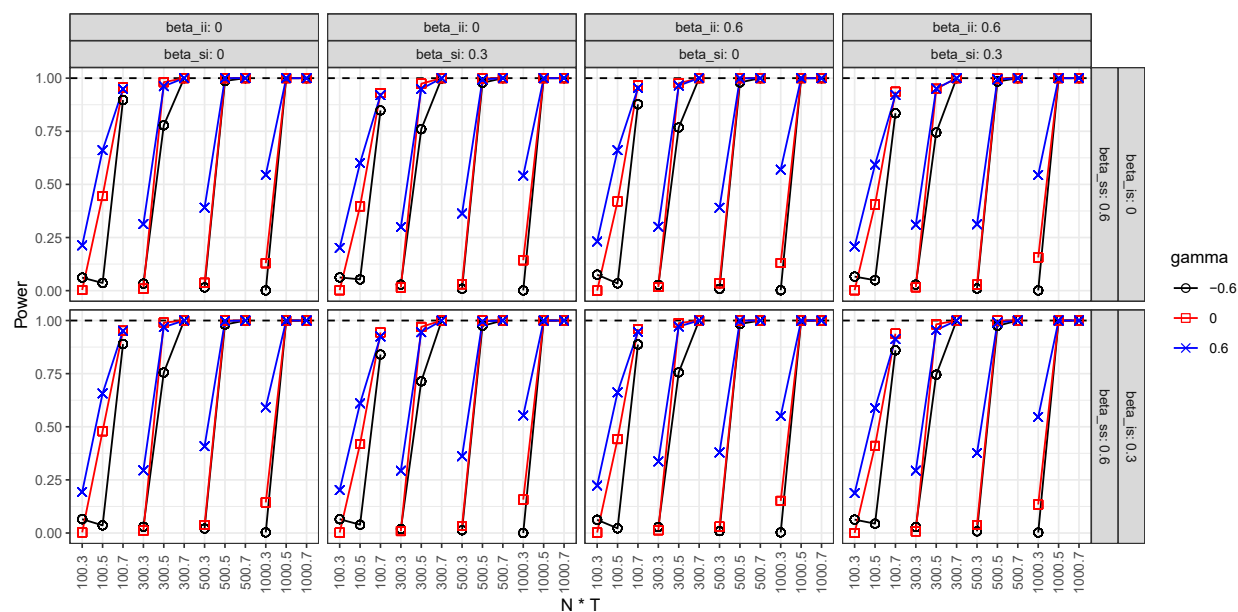
Power for the Between-Person Effect β_{SS} in Simulation 4 (LCM_{xy}-RP)

Figure D.225

Power for the Between-Person Effect β_{SS} in Simulation 4 (LCM_x-CLC)

Appendix E

Figures in Simulation 5

E.1 Auxiliary Indices

Figure E.1

Proportion of Converged Cases in Simulation 5

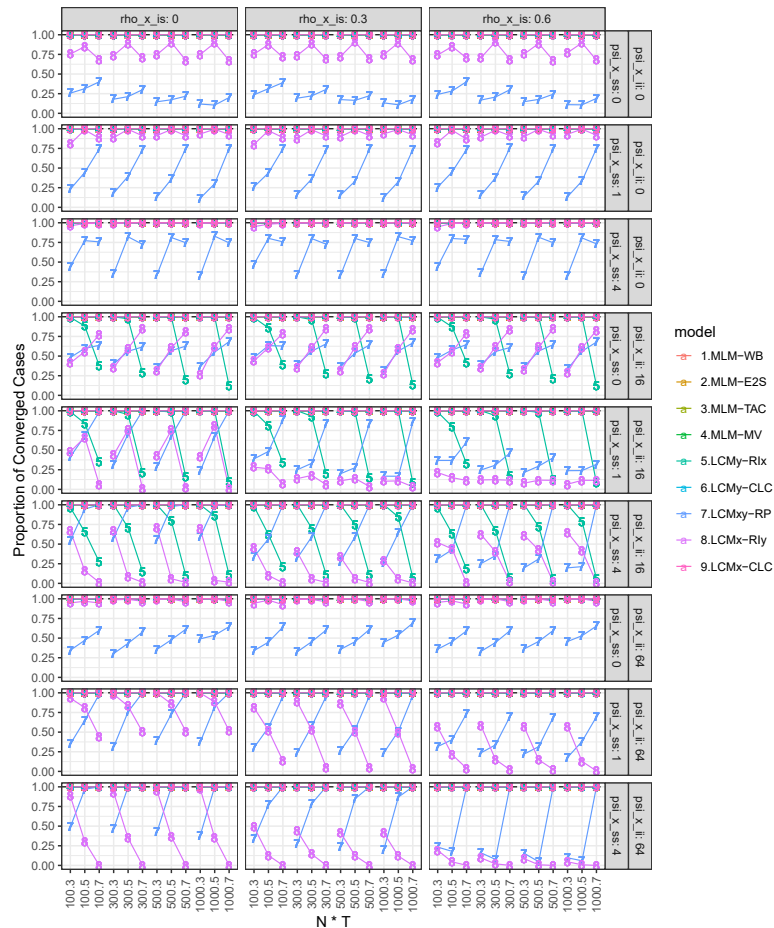


Figure E.2

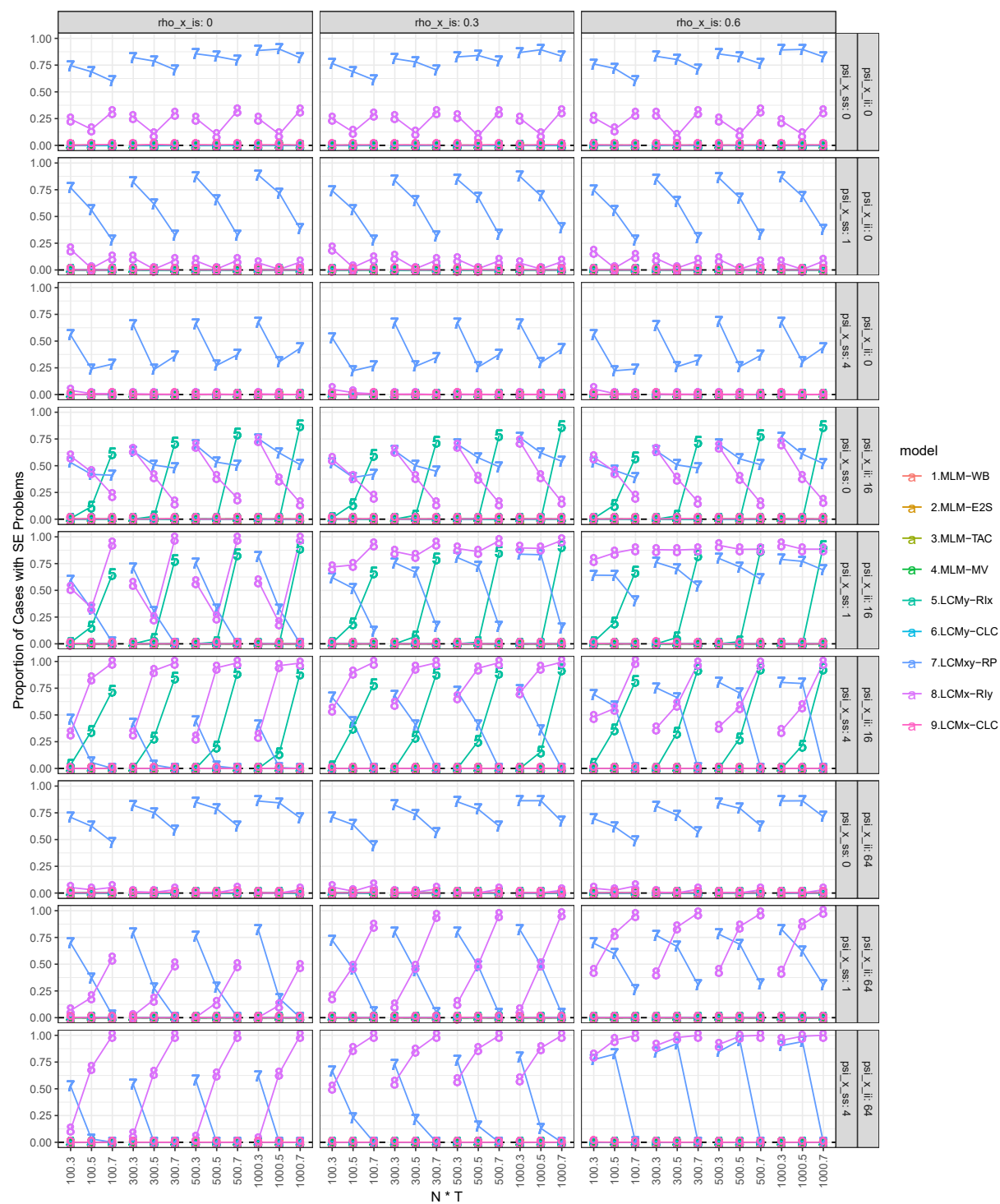
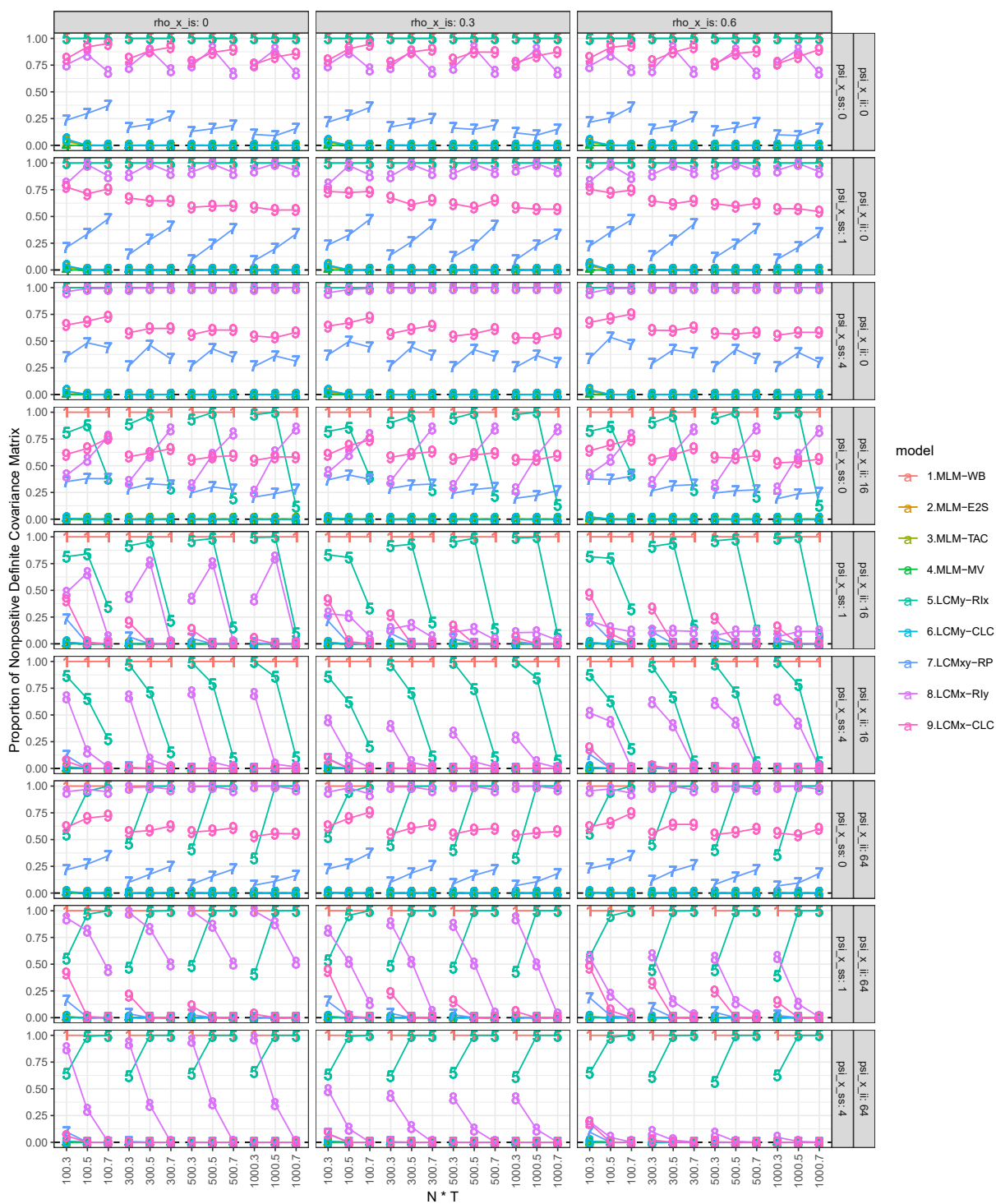
Proportion of Cases with Problems in Estimating SE in Simulation 5

Figure E.3

Proportion of Cases with NPD Model-Implied Covariance Matrices in Simulation 5



CFI in Simulation 5

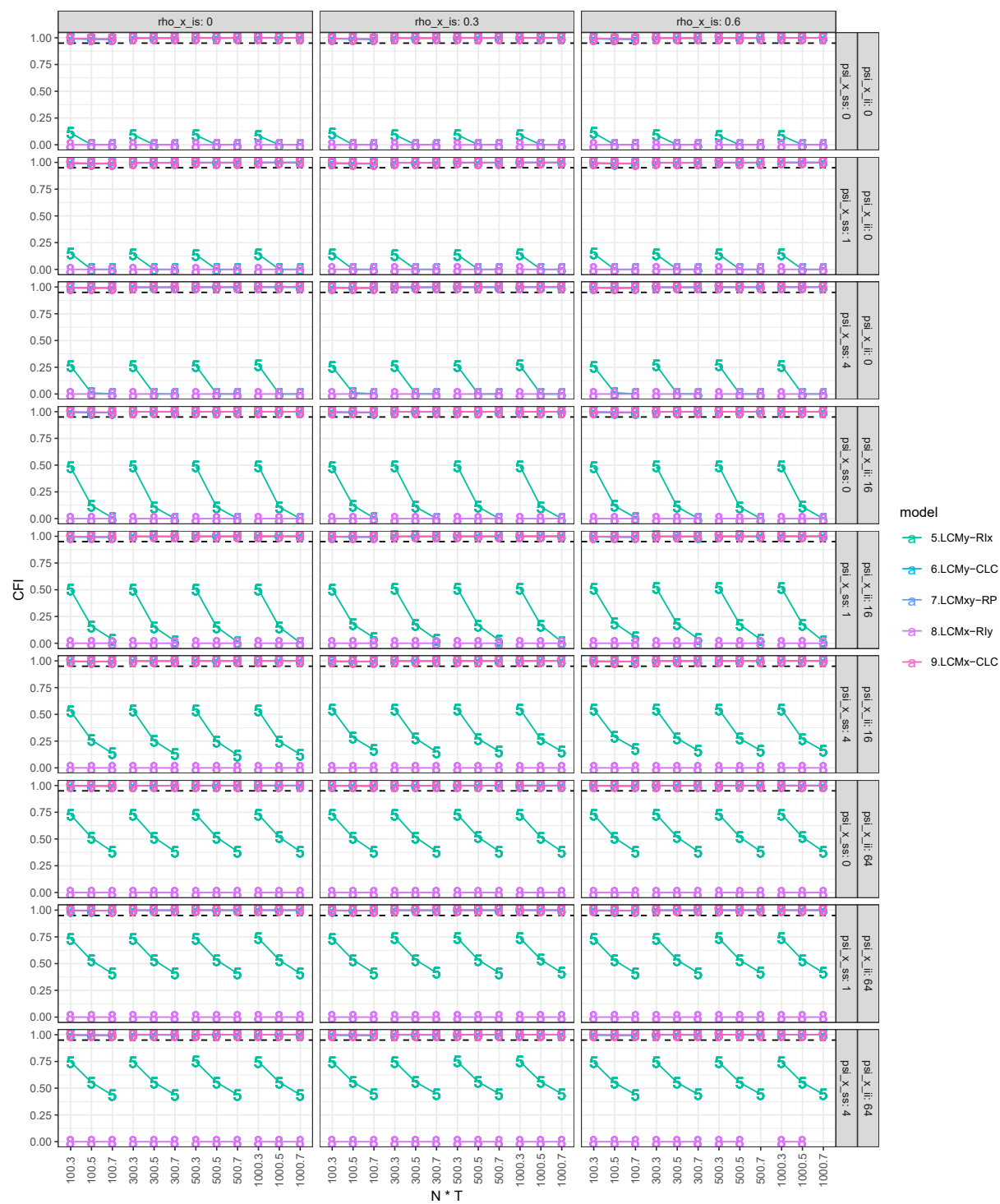


Figure E.5

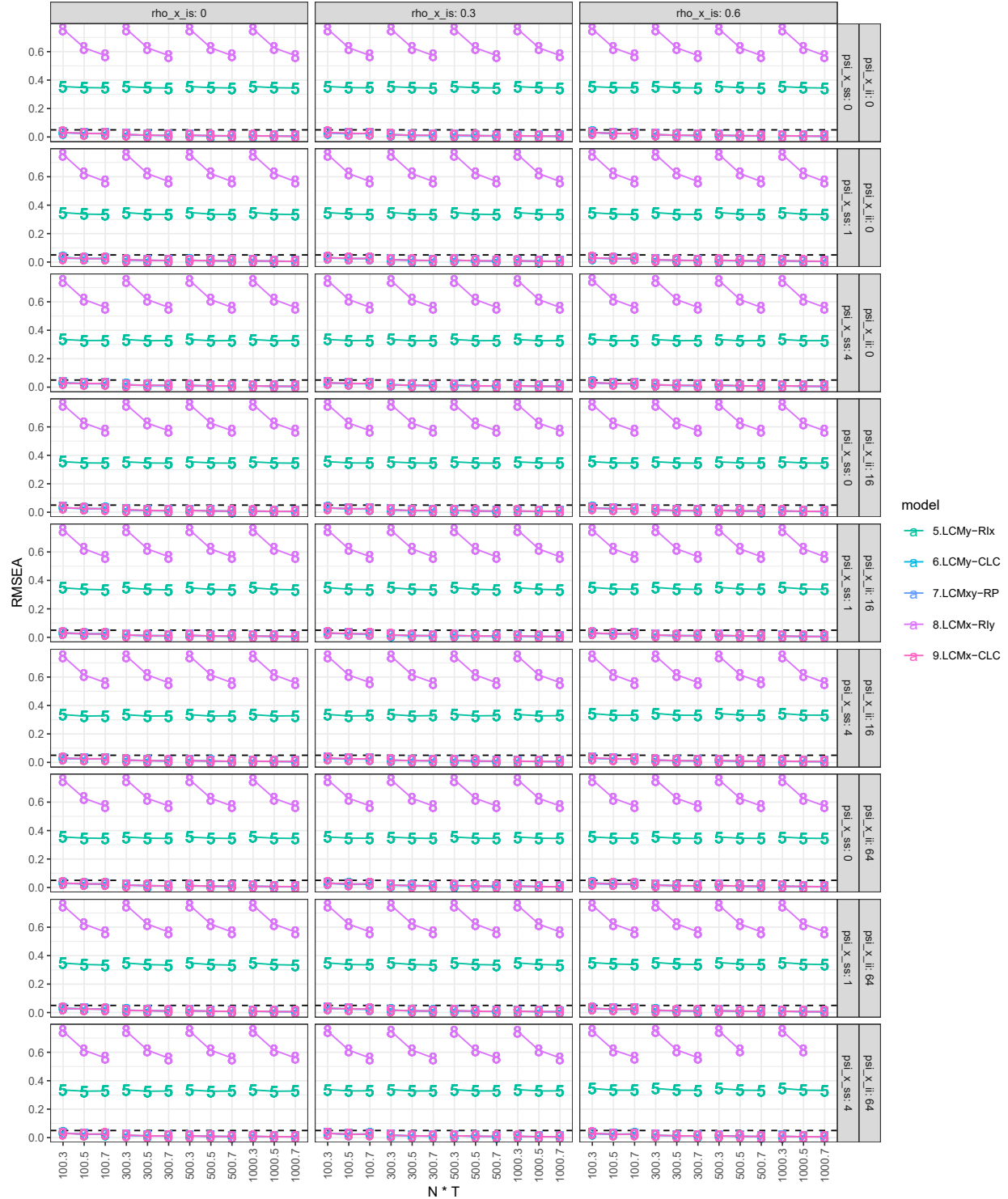
RMSEA in Simulation 5

Figure E.6

SRMR in Simulation 5

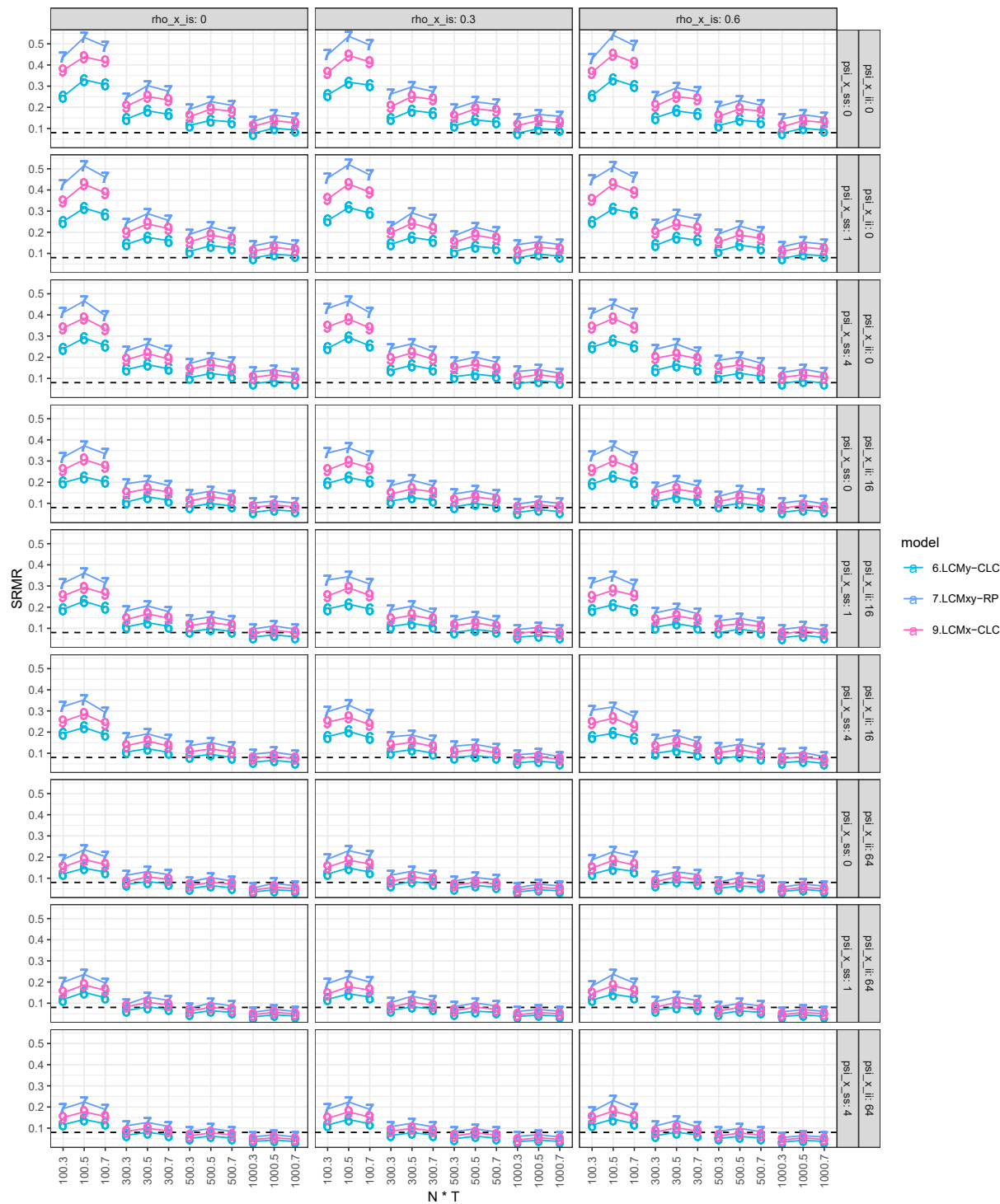
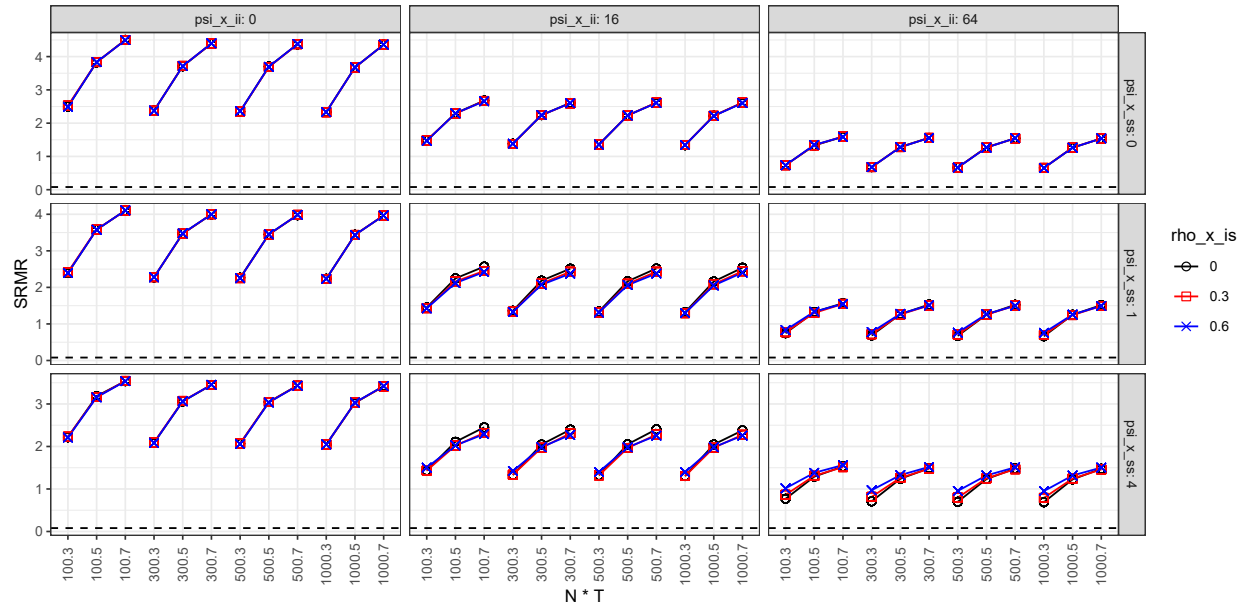


Figure E.7*SRMR in Simulation 5 (LCMy-RIx)*

E.2 Within-Person Effect

Figure E.8

Bias for the Within-Person Effect γ in Simulation 5 (MLM-WB)

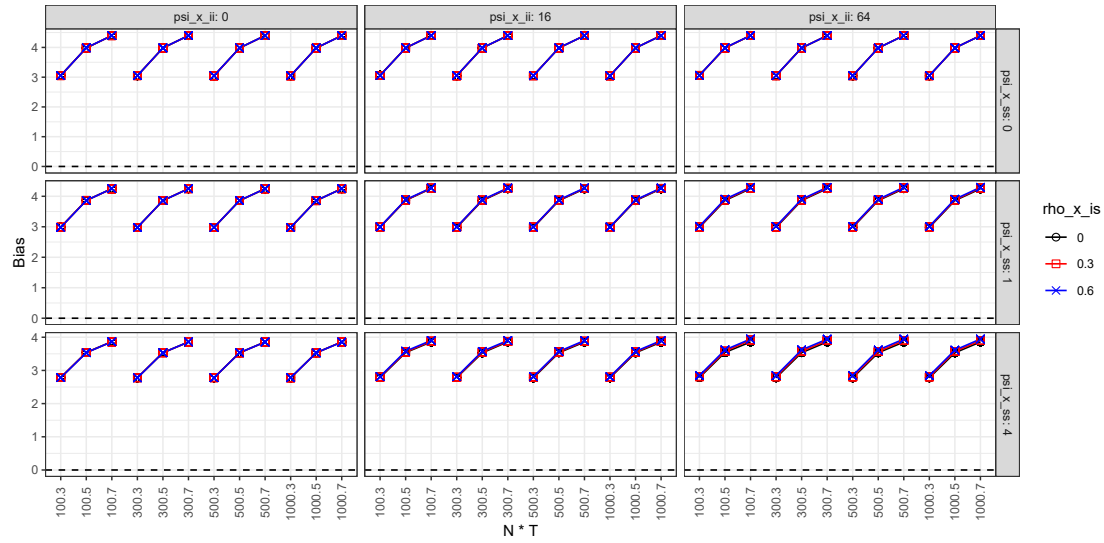


Figure E.9

Bias for the Within-Person Effect γ in Simulation 5 (MLM-E2S)

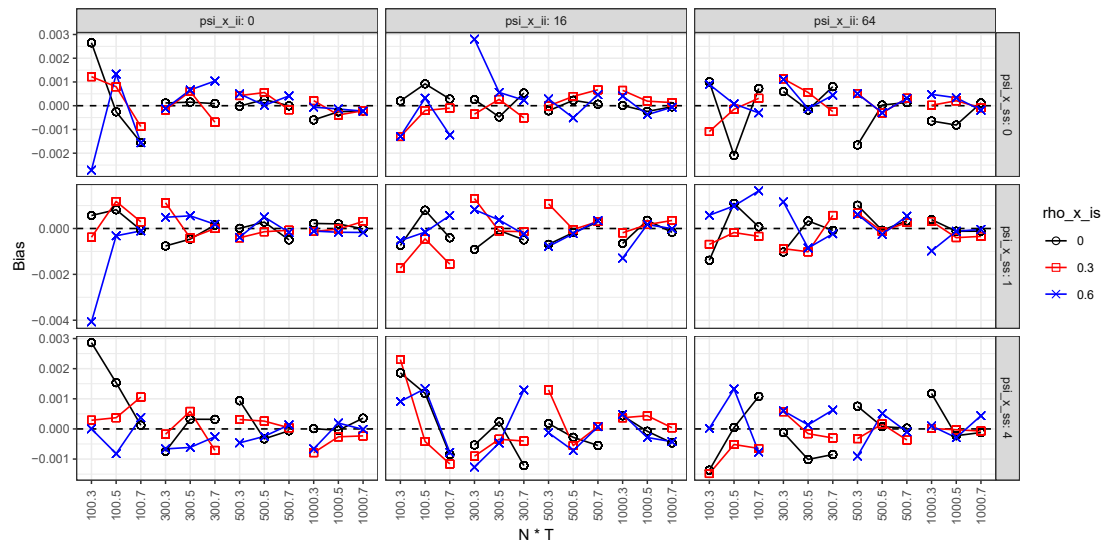


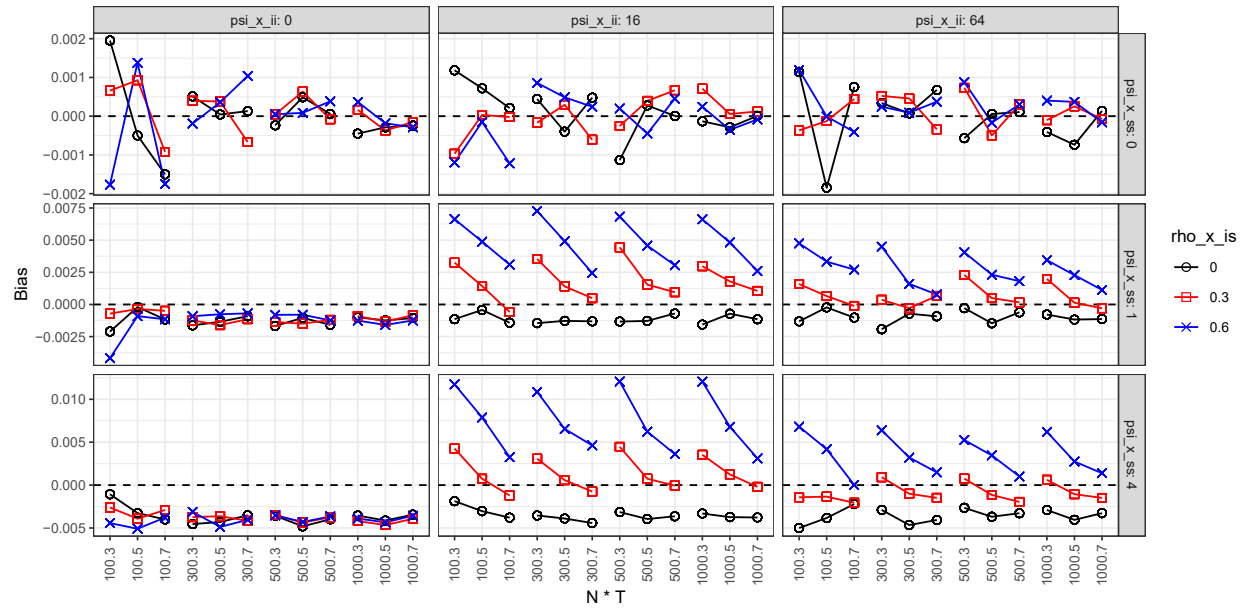
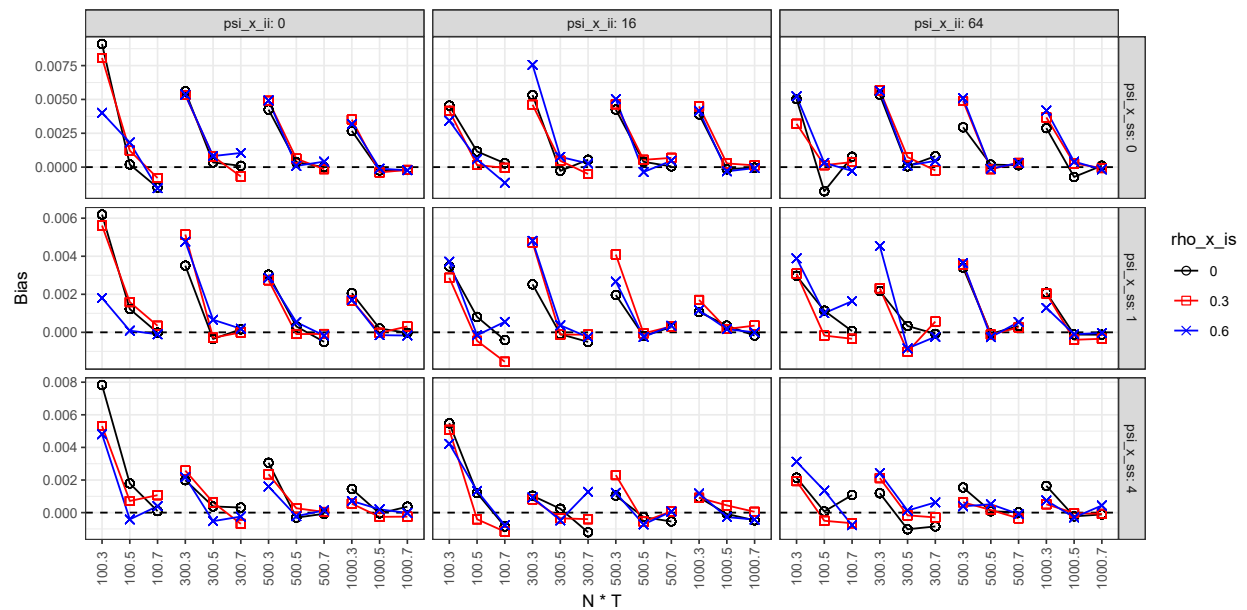
Figure E.10*Bias for the Within-Person Effect γ in Simulation 5 (MLM-TAC)***Figure E.11***Bias for the Within-Person Effect γ in Simulation 5 (MLM-MV)*

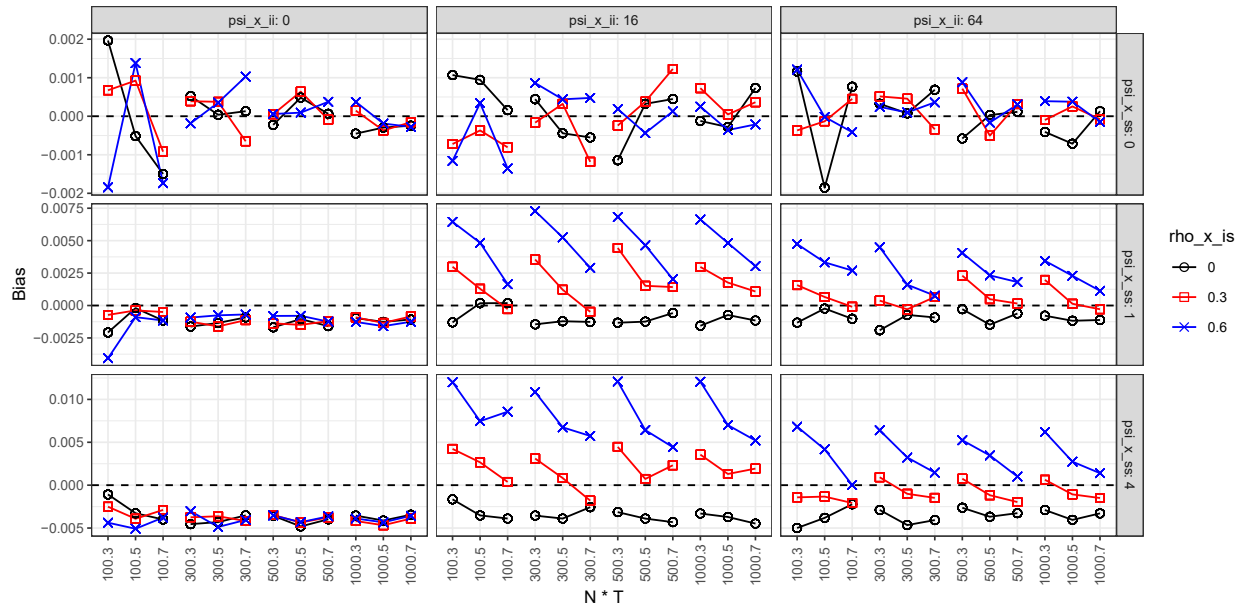
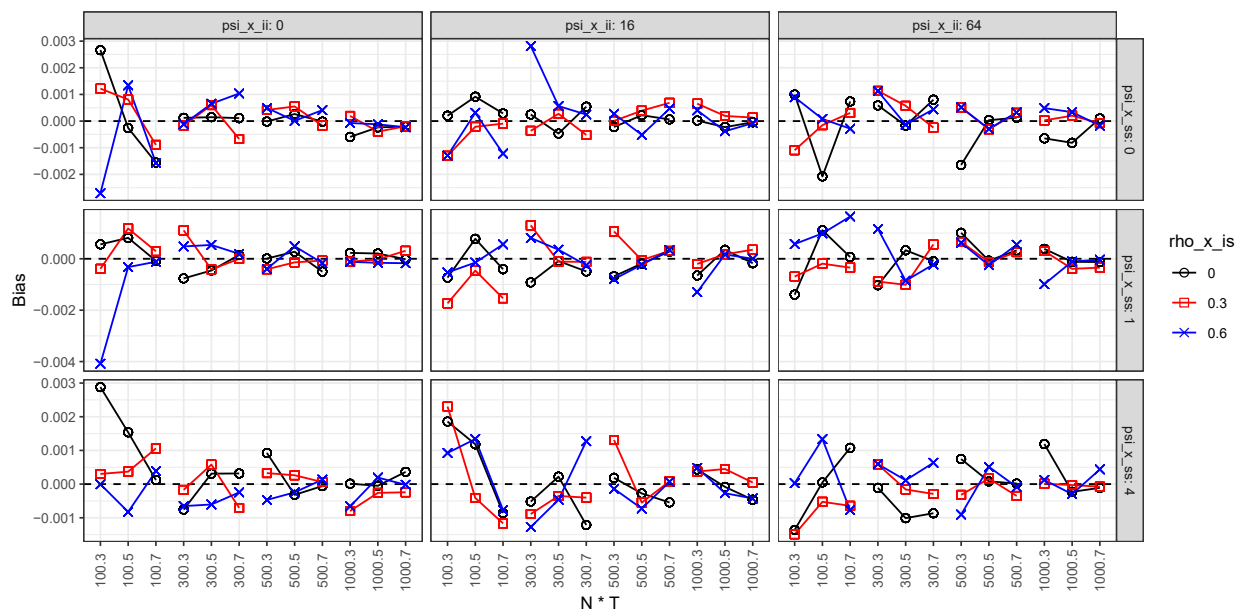
Figure E.12*Bias for the Within-Person Effect γ in Simulation 5 (LCMy-RLx)***Figure E.13***Bias for the Within-Person Effect γ in Simulation 5 (LCMy-CLC)*

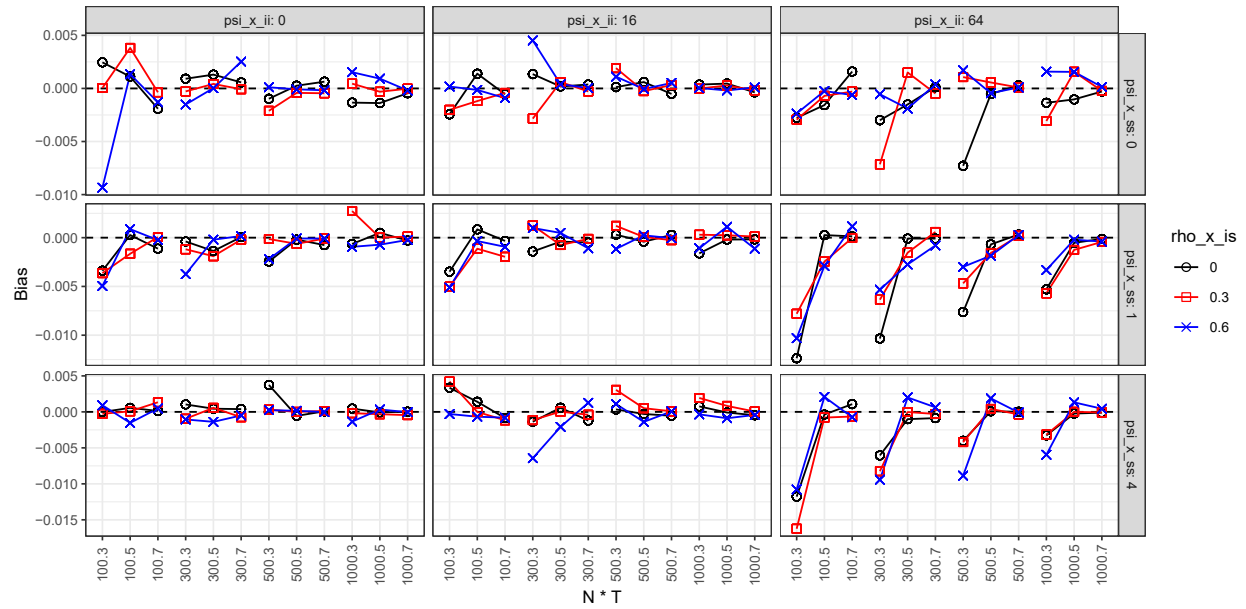
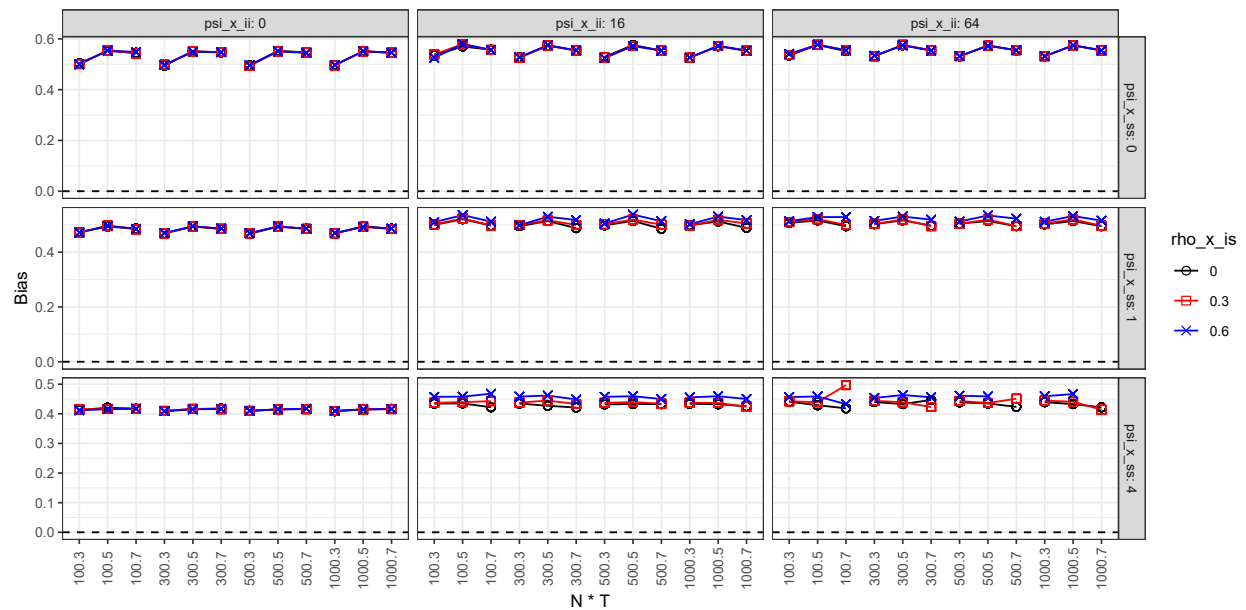
Figure E.14*Bias for the Within-Person Effect γ in Simulation 5 (LCMxy-RP)***Figure E.15***Bias for the Within-Person Effect γ in Simulation 5 (LCMx-Rly)*

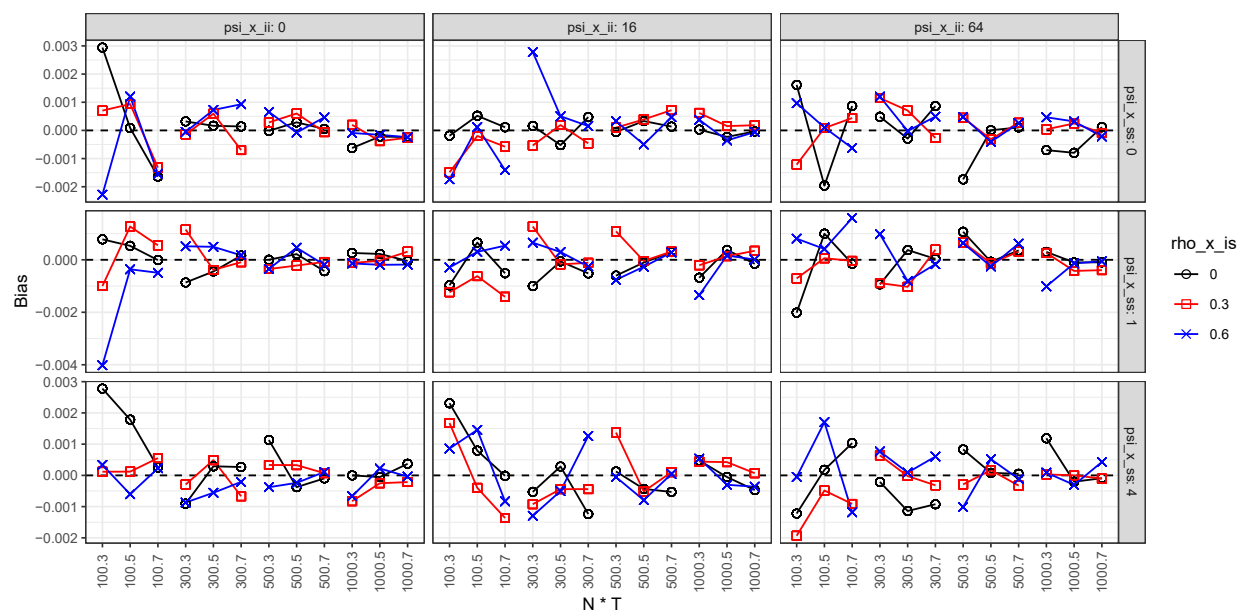
Figure E.16*Bias for the Within-Person Effect γ in Simulation 5 (LCMx-CLC)*

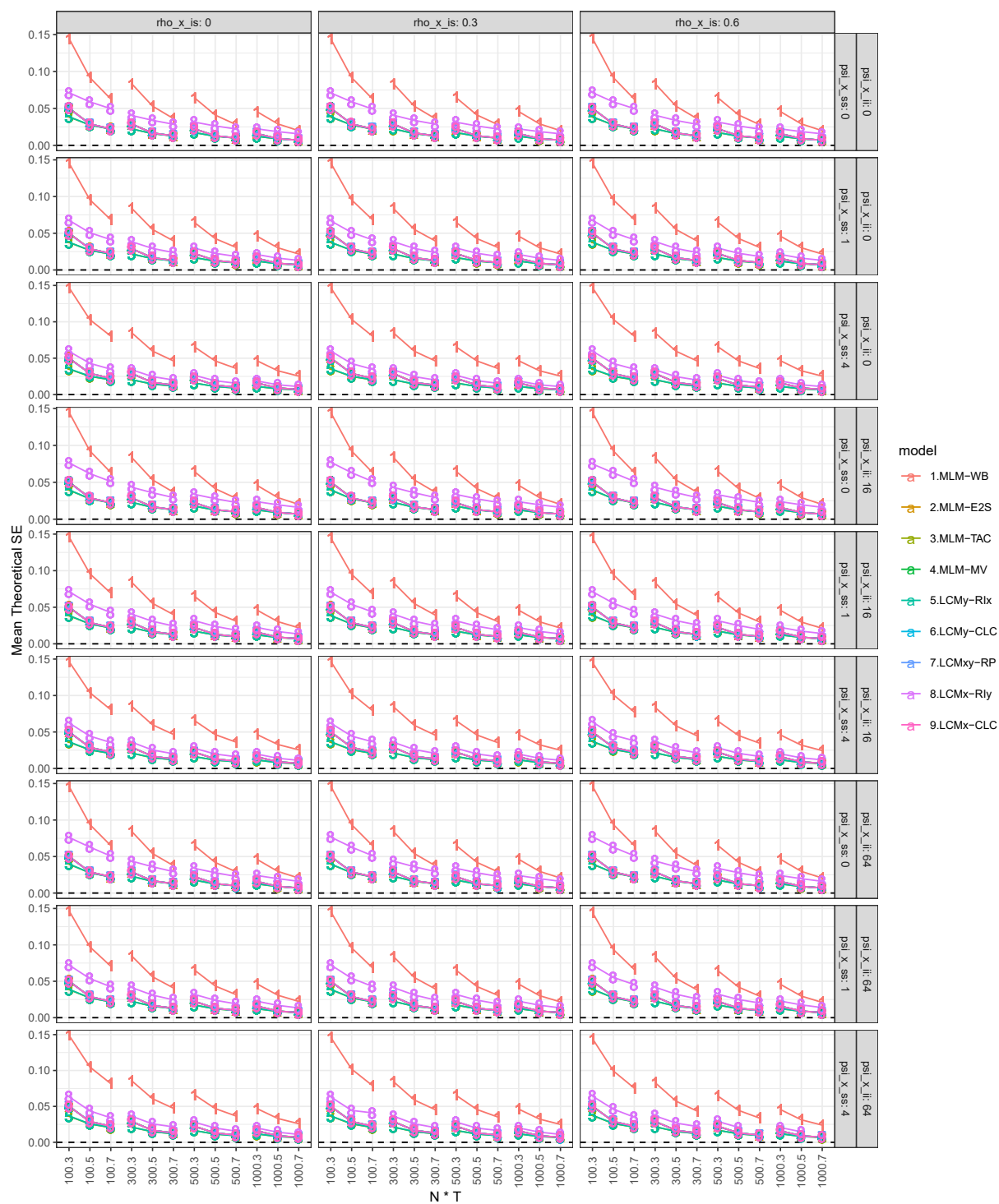
Figure E.17*Empirical SE for the Within-Person Effect γ in Simulation 5*

Figure E.18

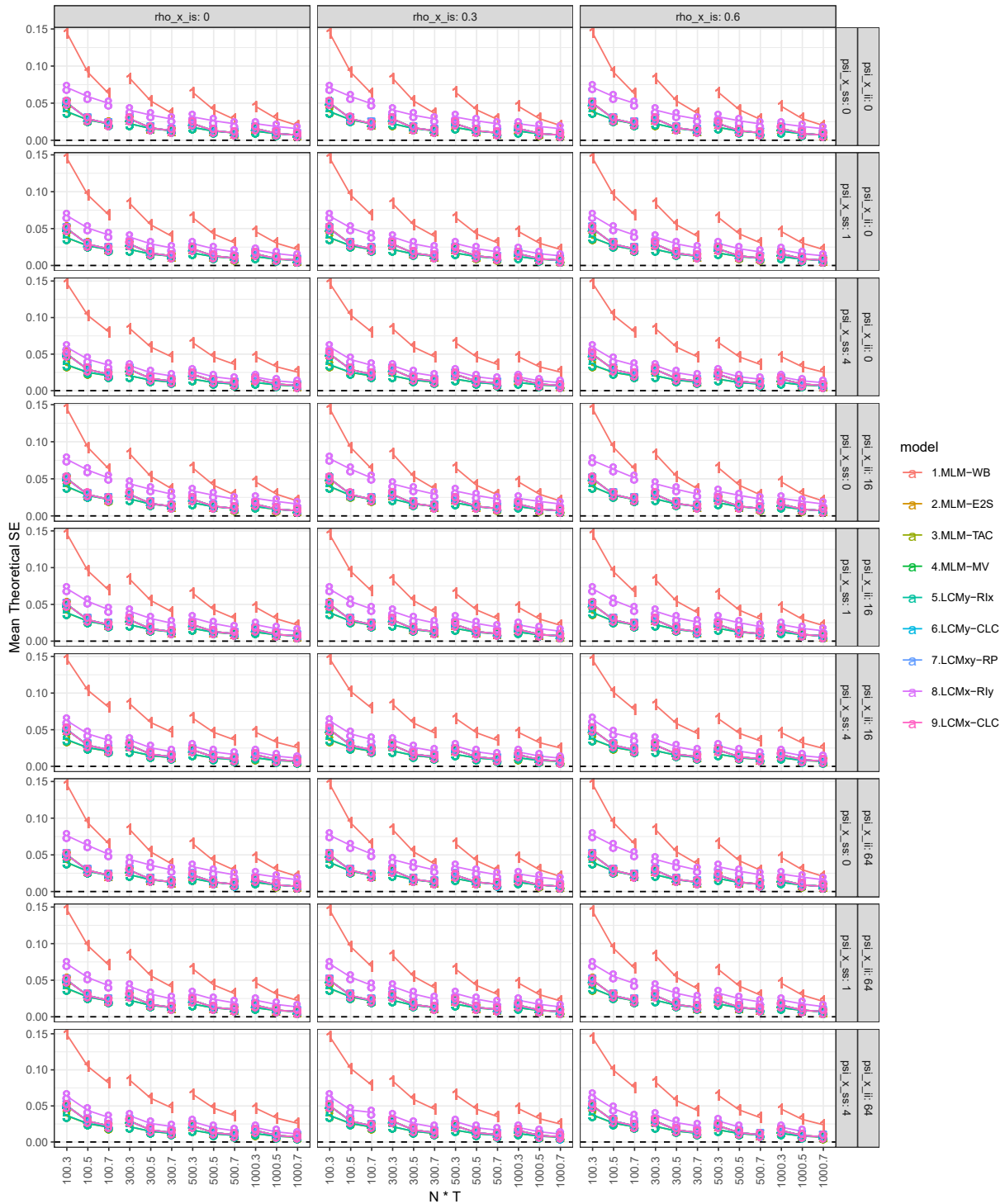
Theoretical SE for the Within-Person Effect γ in Simulation 5

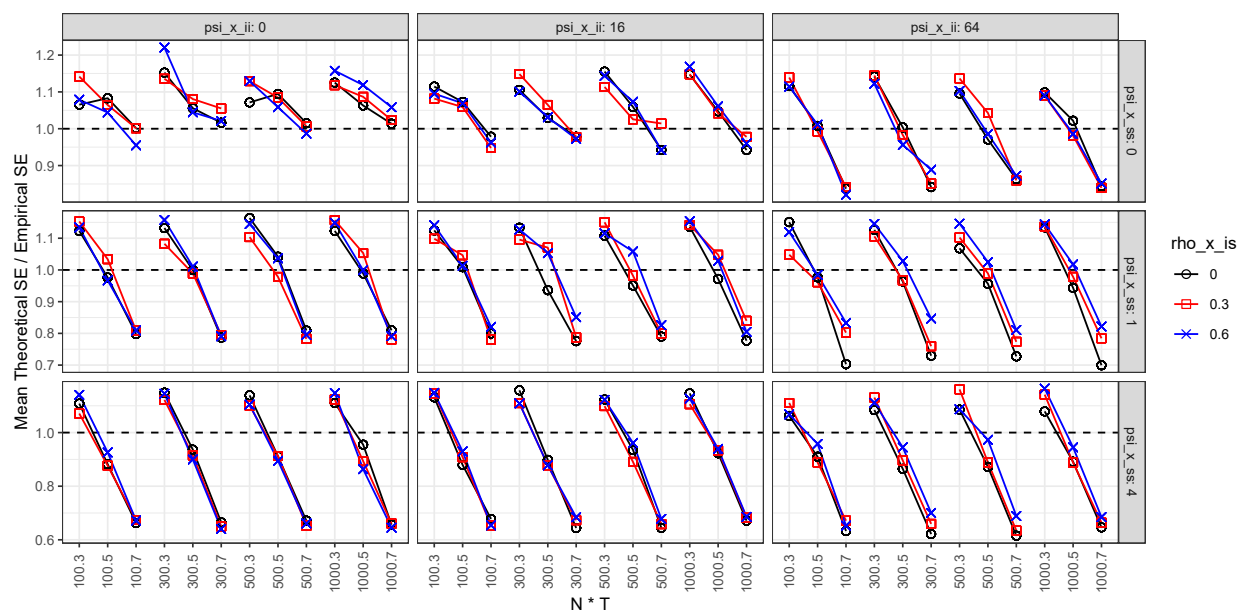
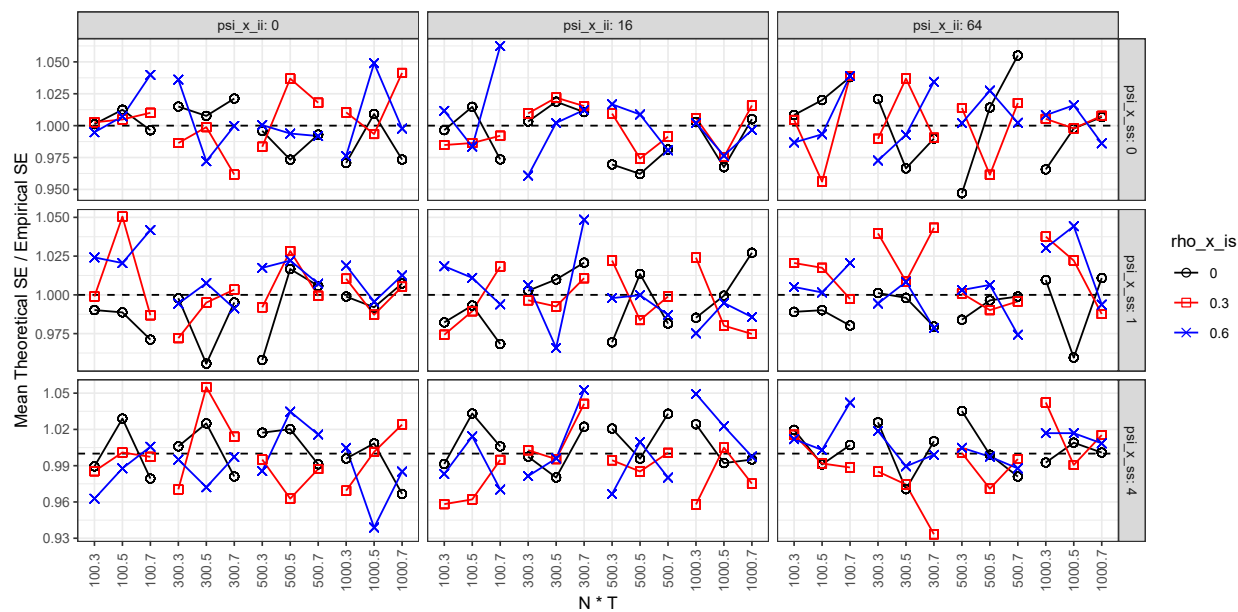
Figure E.19*SE Ratio for the Within-Person Effect γ in Simulation 5 (MLM-WB)***Figure E.20***SE Ratio for the Within-Person Effect γ in Simulation 5 (MLM-E2S)*

Figure E.21

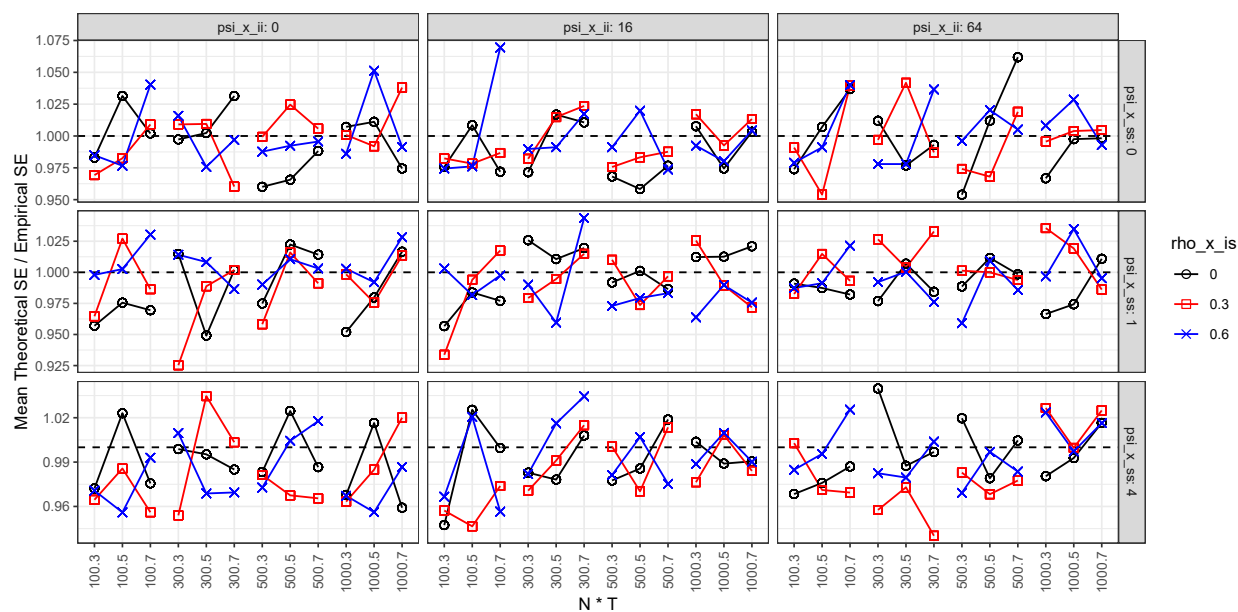
SE Ratio for the Within-Person Effect γ in Simulation 5 (MLM-TAC)

Figure E.22

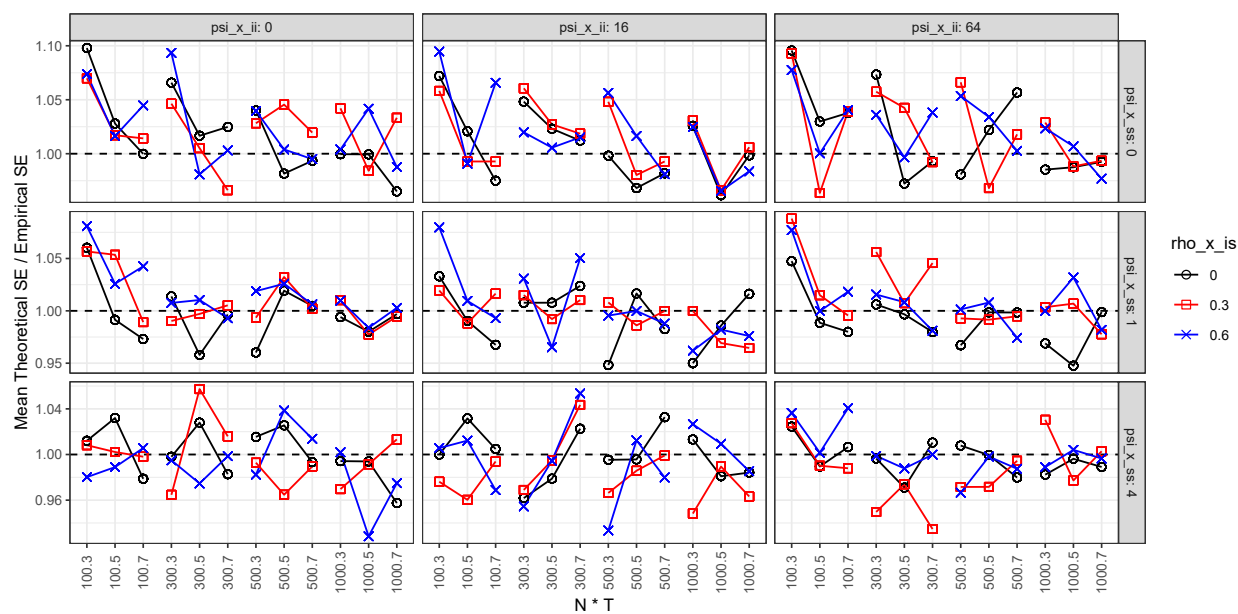
SE Ratio for the Within-Person Effect γ in Simulation 5 (MLM-MV)

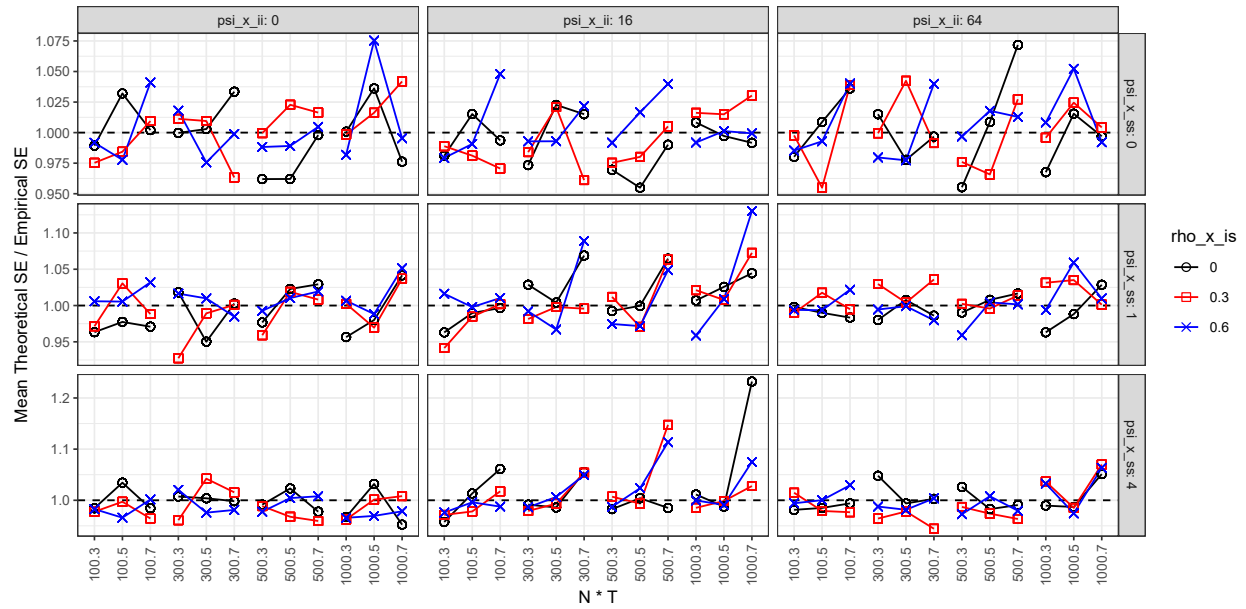
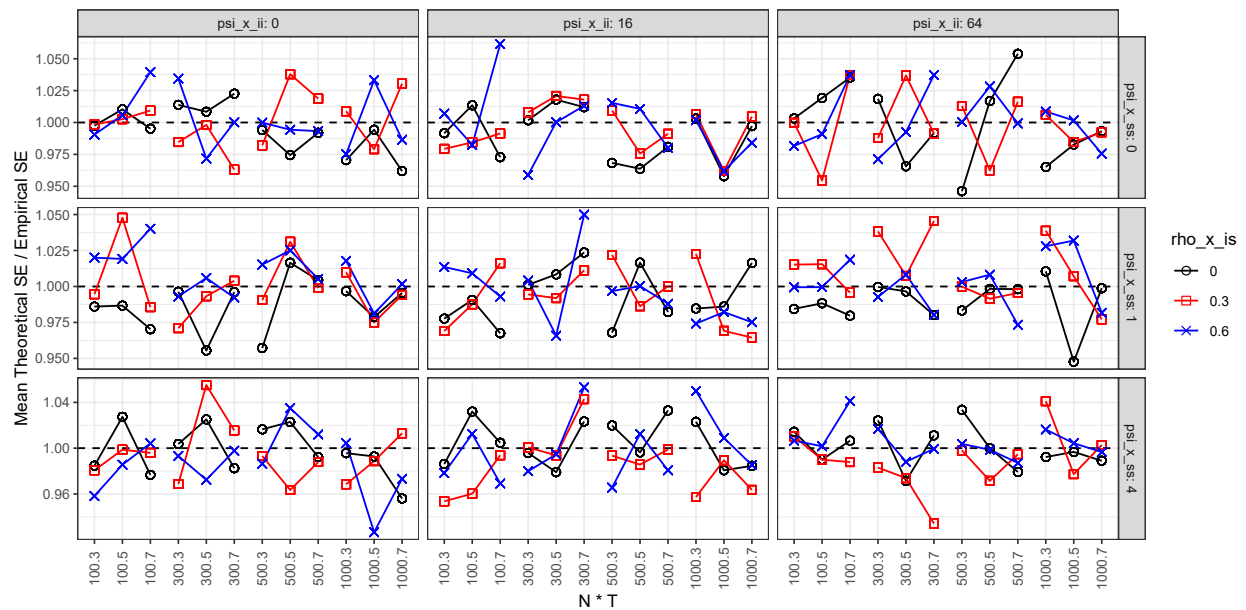
Figure E.23*SE Ratio for the Within-Person Effect γ in Simulation 5 (LCMy-RIx)***Figure E.24***SE Ratio for the Within-Person Effect γ in Simulation 5 (LCMy-CLC)*

Figure E.25

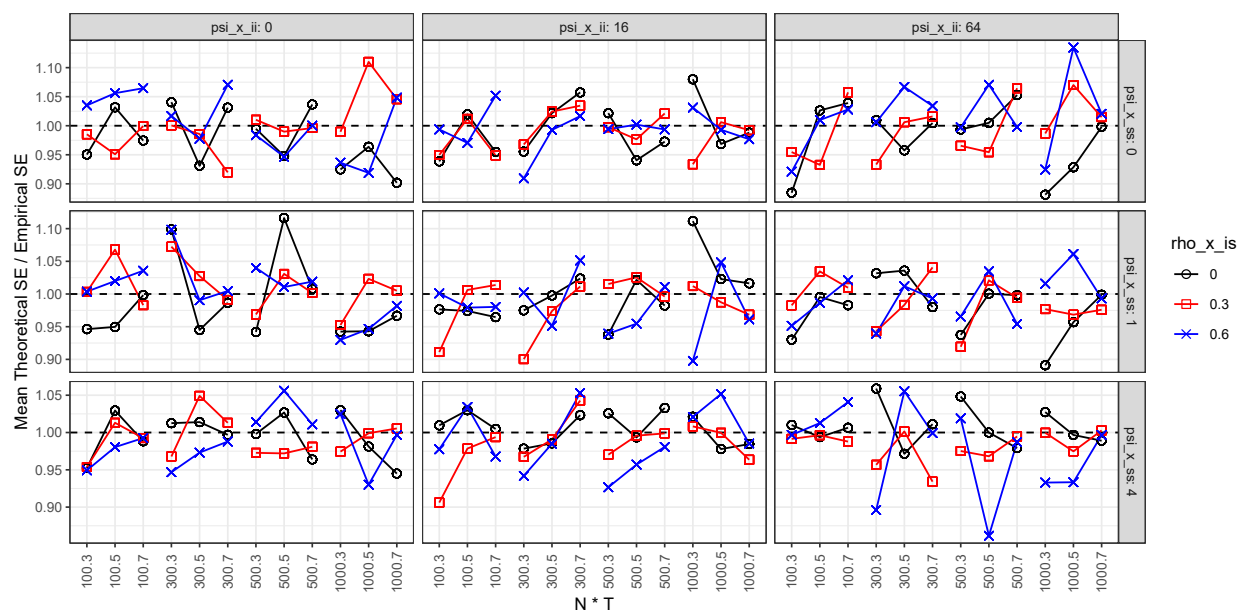
SE Ratio for the Within-Person Effect γ in Simulation 5 (LCM_{xy}-RP)

Figure E.26

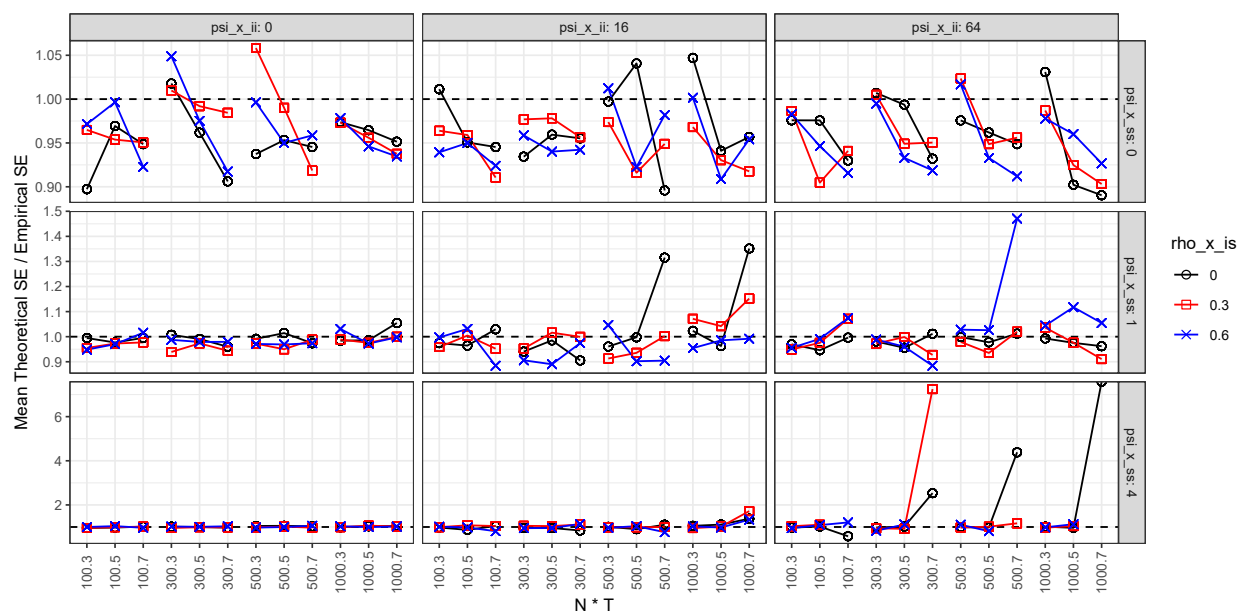
SE Ratio for the Within-Person Effect γ in Simulation 5 (LCM_x-RI_y)

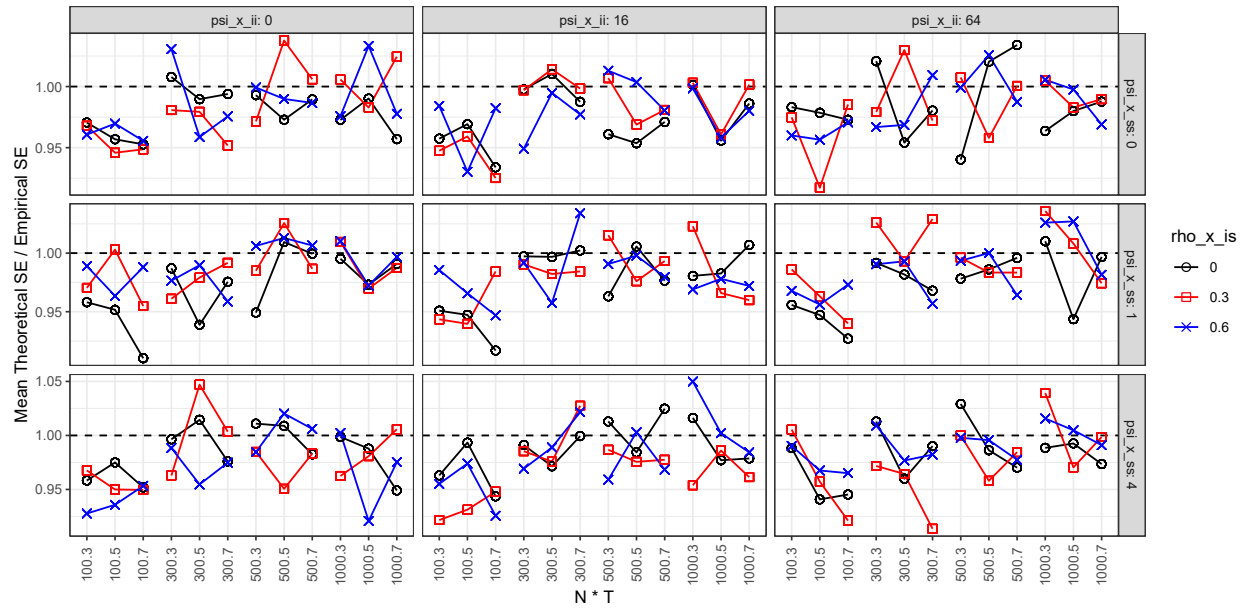
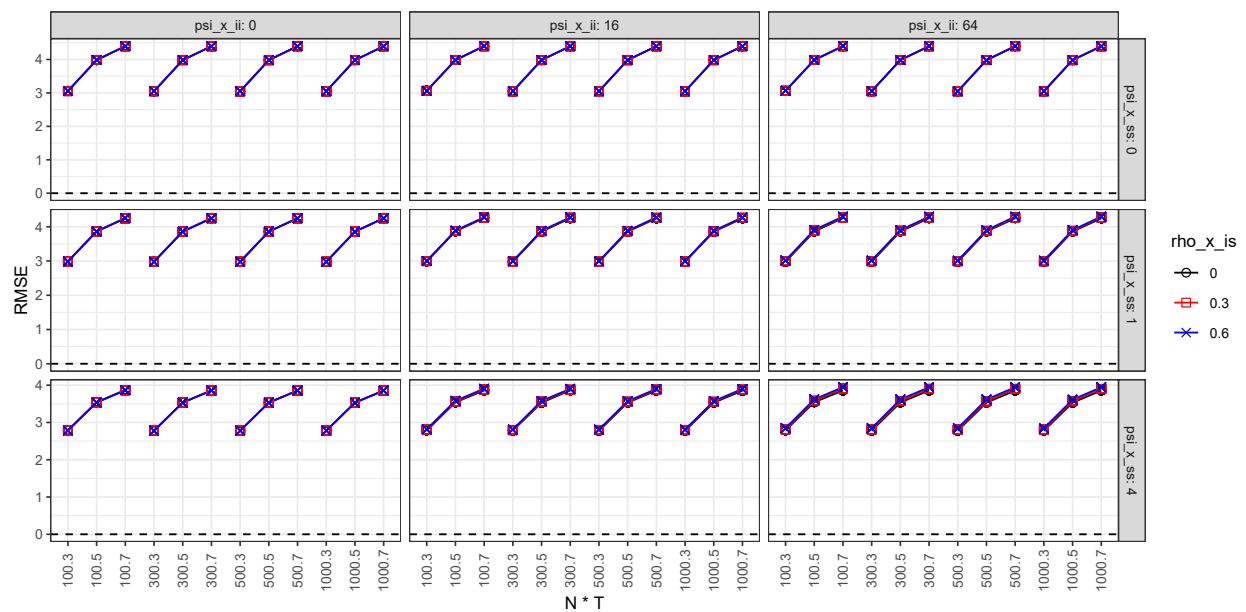
Figure E.27*SE Ratio for the Within-Person Effect γ in Simulation 5 (LCM_x-CLC)***Figure E.28***RMSE for the Within-Person Effect γ in Simulation 5 (MLM-WB)*

Figure E.29

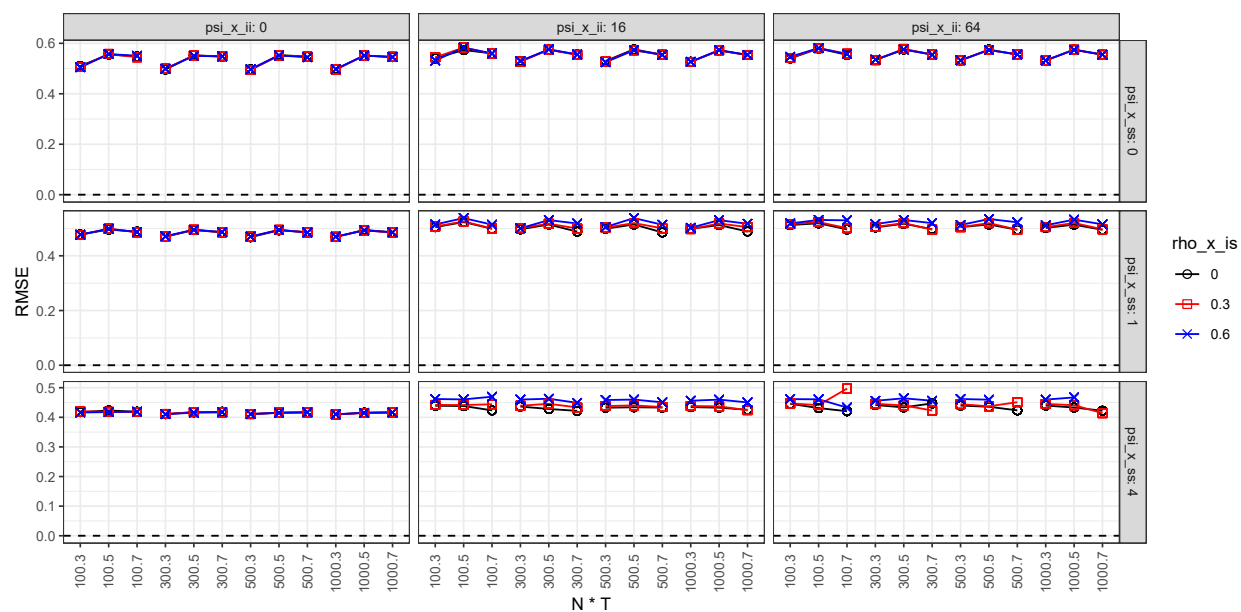
RMSE for the Within-Person Effect γ in Simulation 5 (LCMx-Rly)

Figure E.30

RMSE for the Within-Person Effect γ in Simulation 5 except MLM-WB and LCMx-Rly

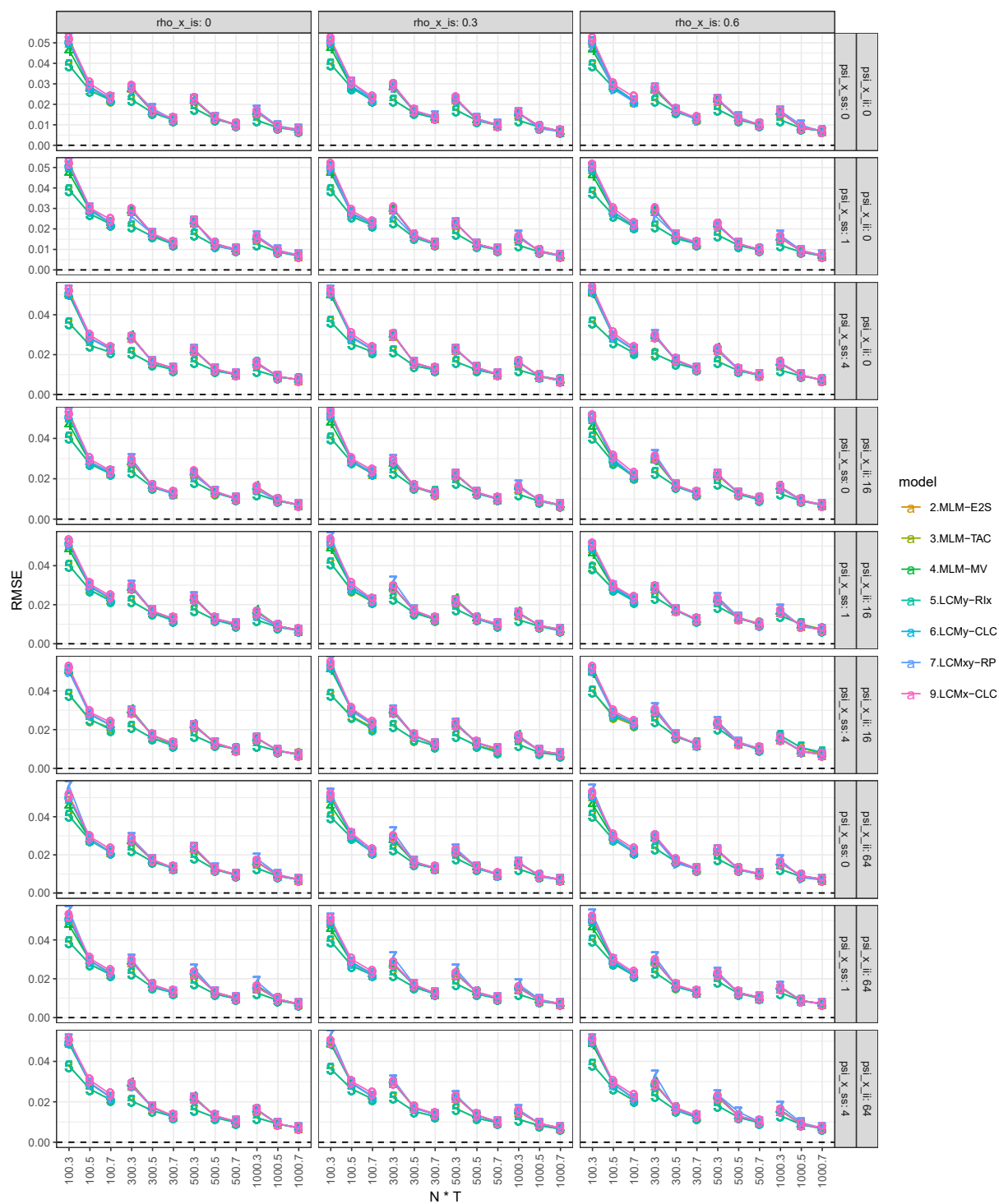
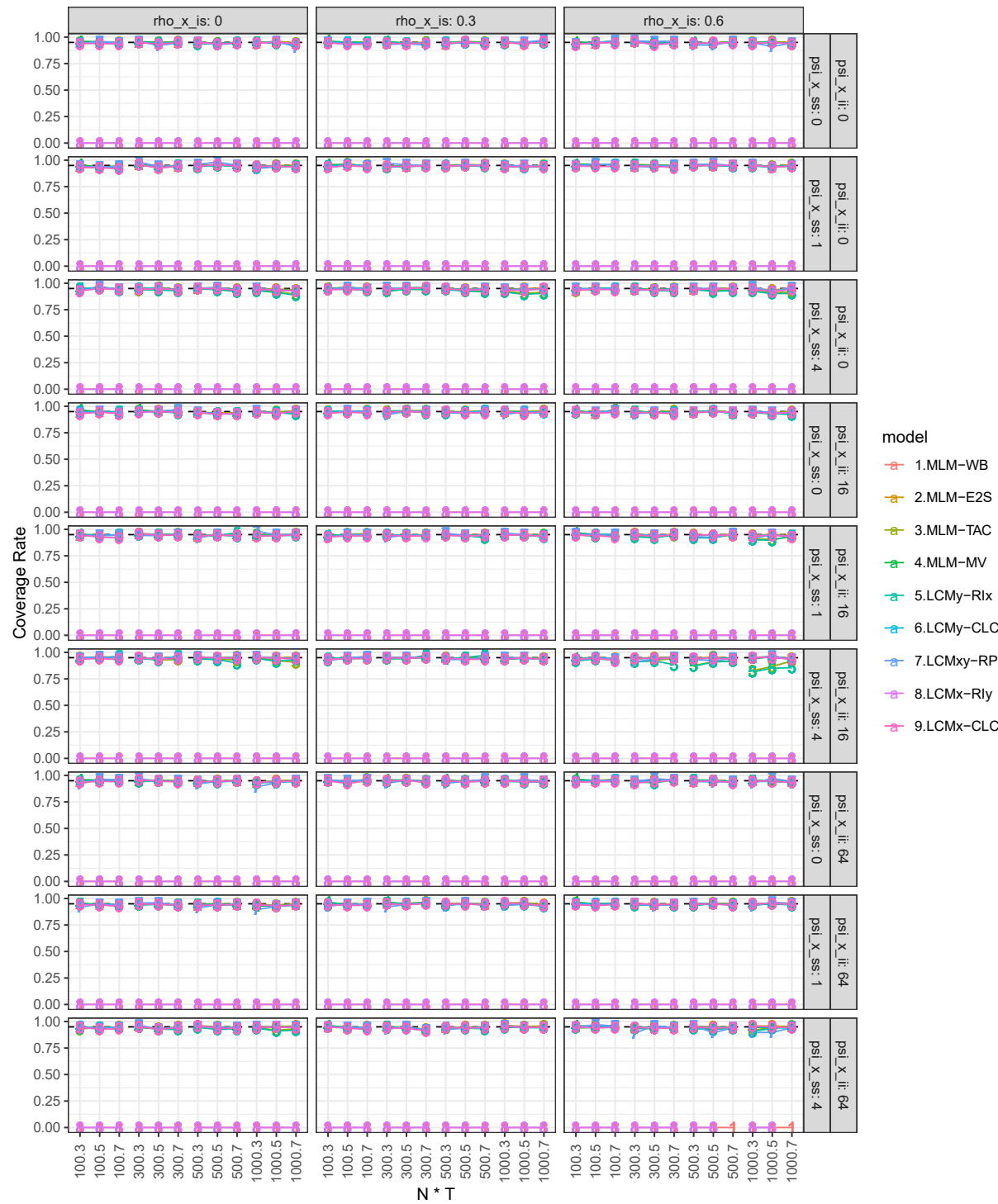
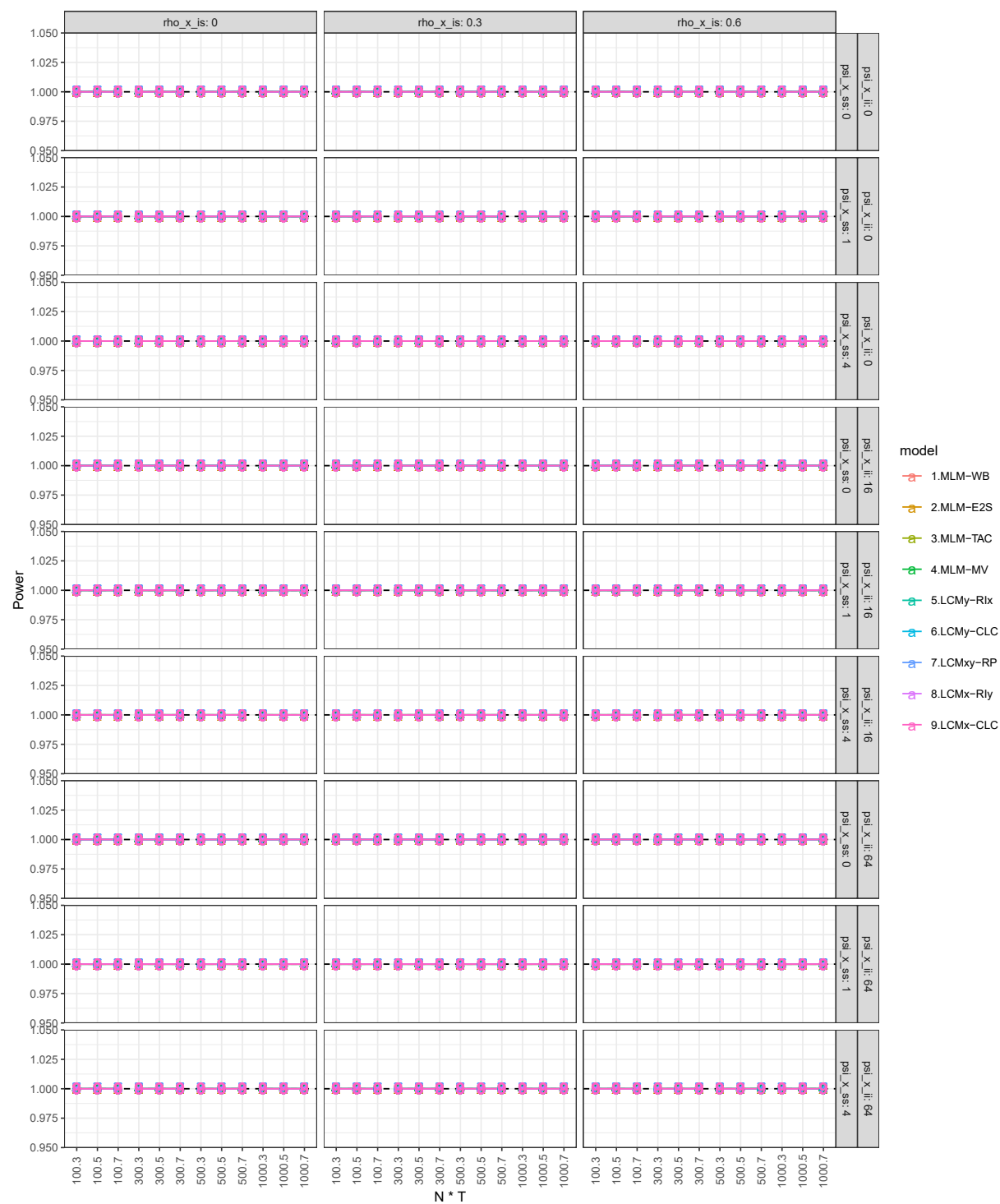


Figure E.31
Coverage Rate for the Within-Person Effect γ in Simulation 5



Power for the Within-Person Effect γ in Simulation 5



E.3 Between-Person Effect of Intercept on Intercept

Figure E.33

Bias for the Between-Person Effect β_{II} in Simulation 5 (MLM-WB)

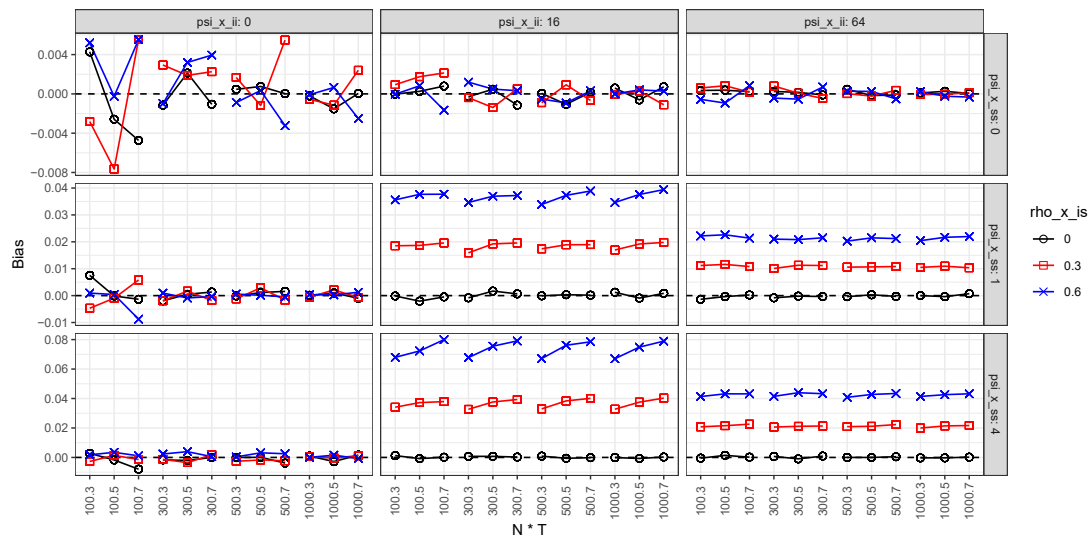


Figure E.34

Bias for the Between-Person Effect β_{II} in Simulation 5 (MLM-E2S)

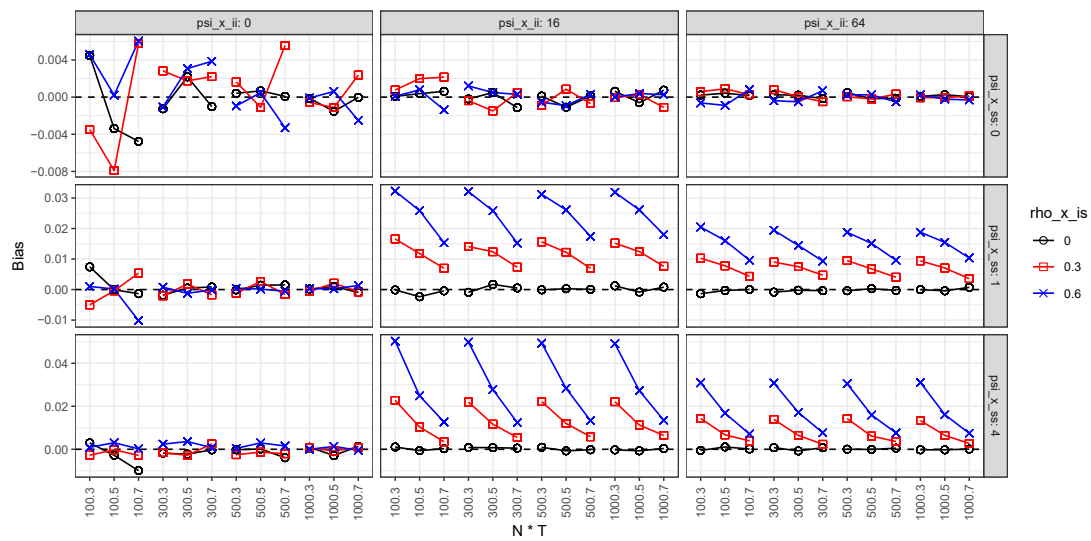


Figure E.35

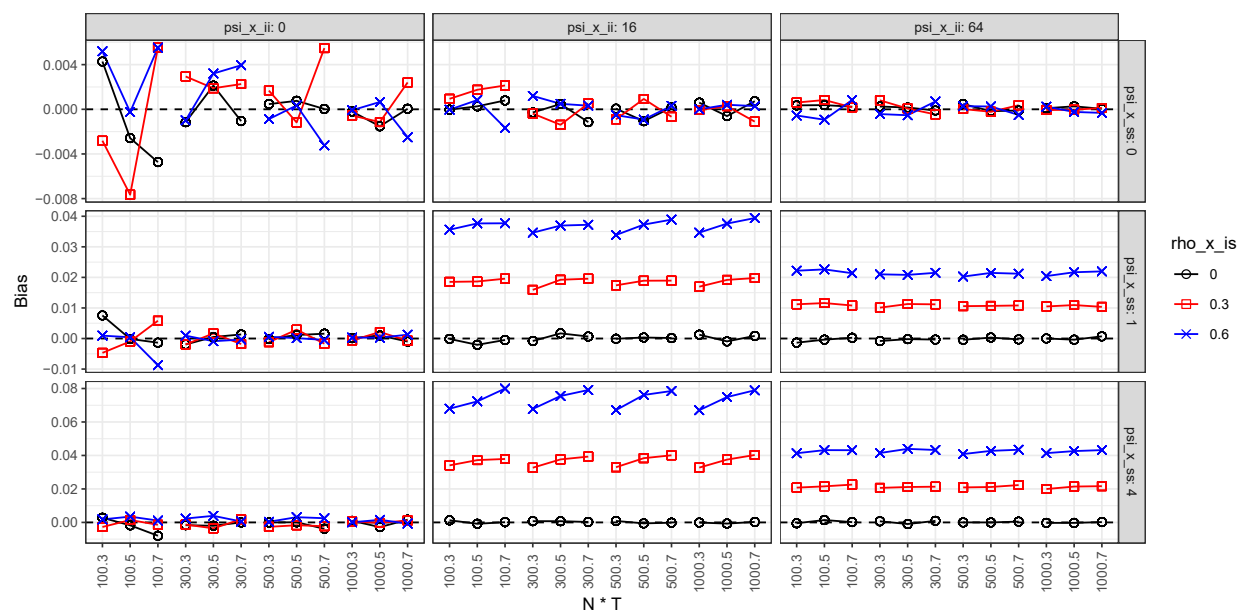
Bias for the Between-Person Effect β_{II} in Simulation 5 (MLM-TAC)

Figure E.36

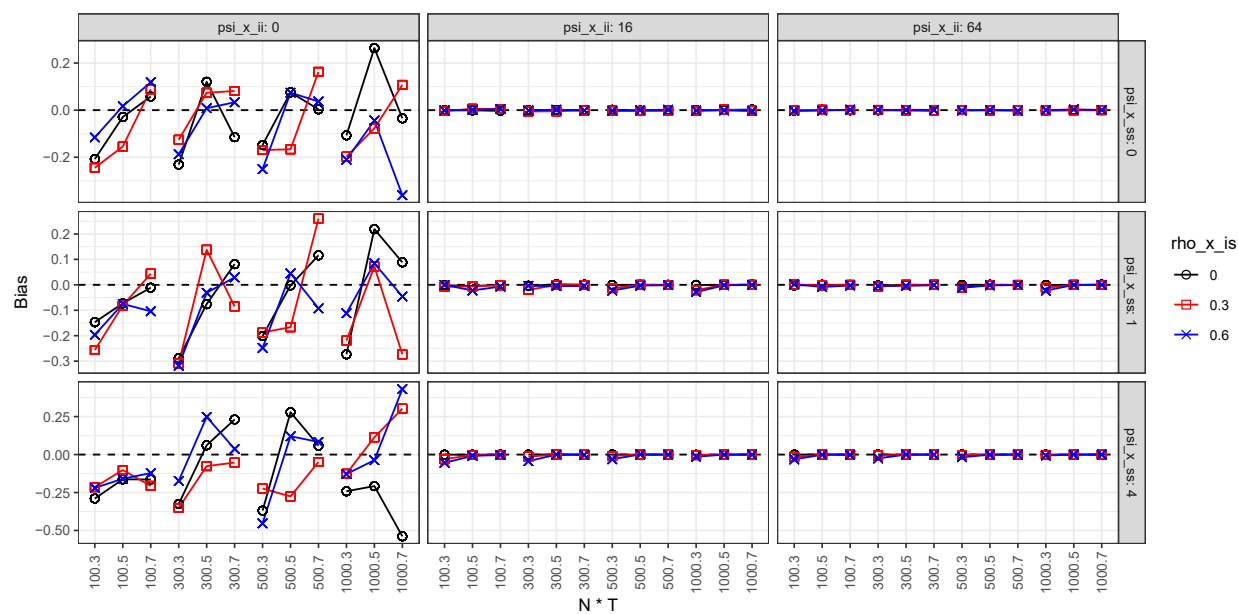
Bias for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)

Figure E.37

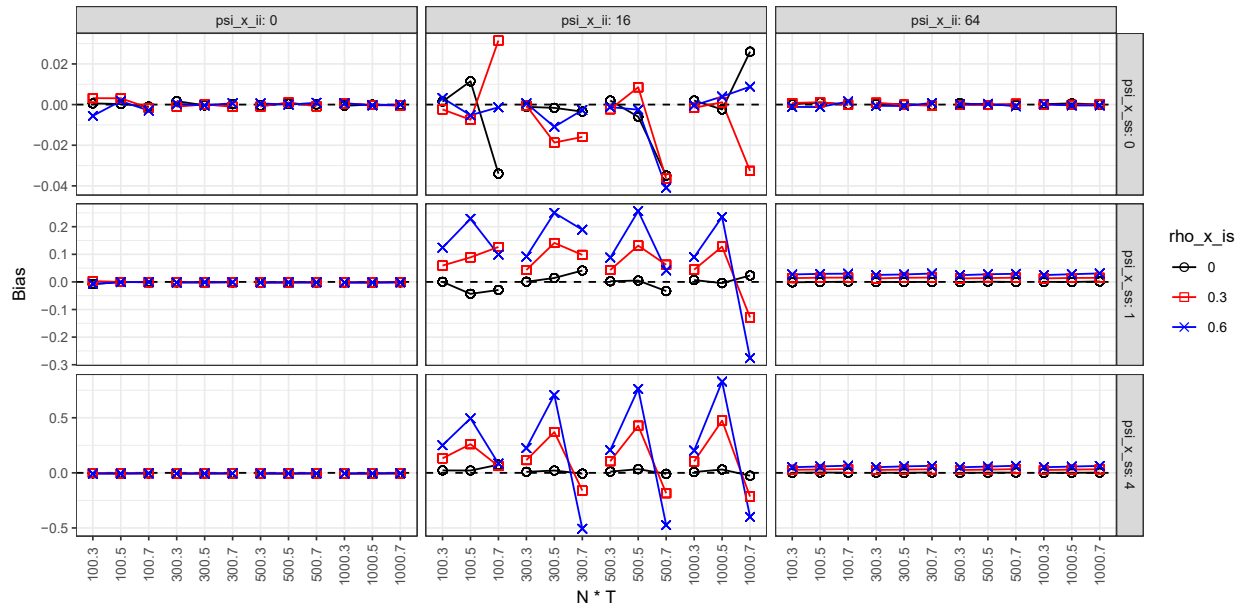
Bias for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)

Figure E.38

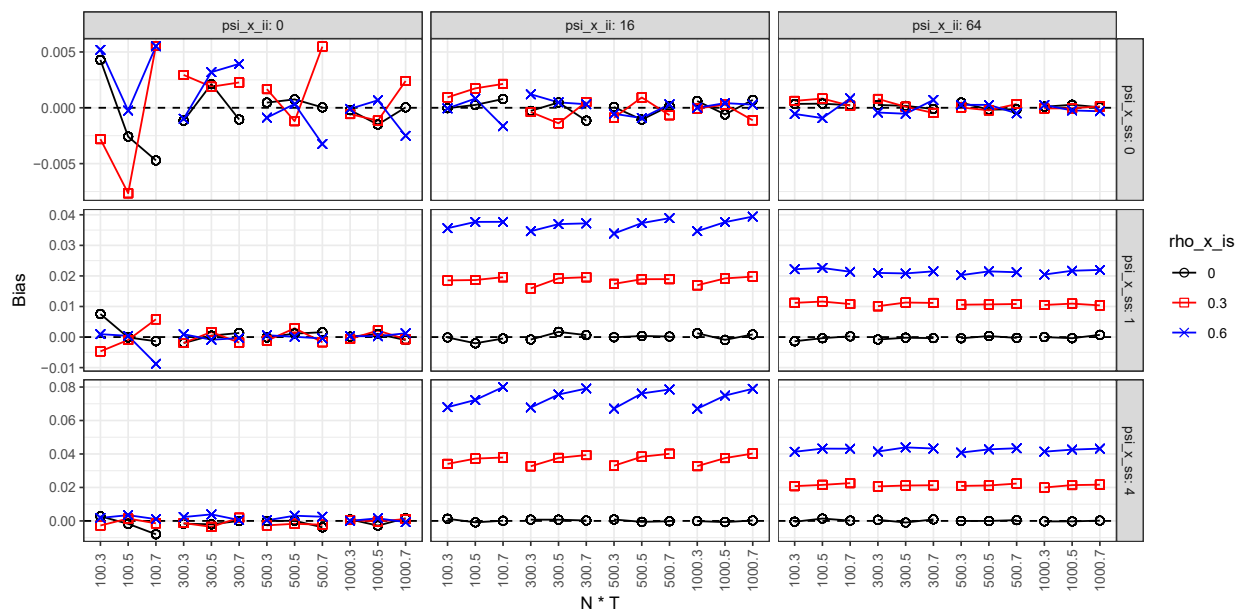
Bias for the Between-Person Effect β_{II} in Simulation 5 (LCMy-CLC)

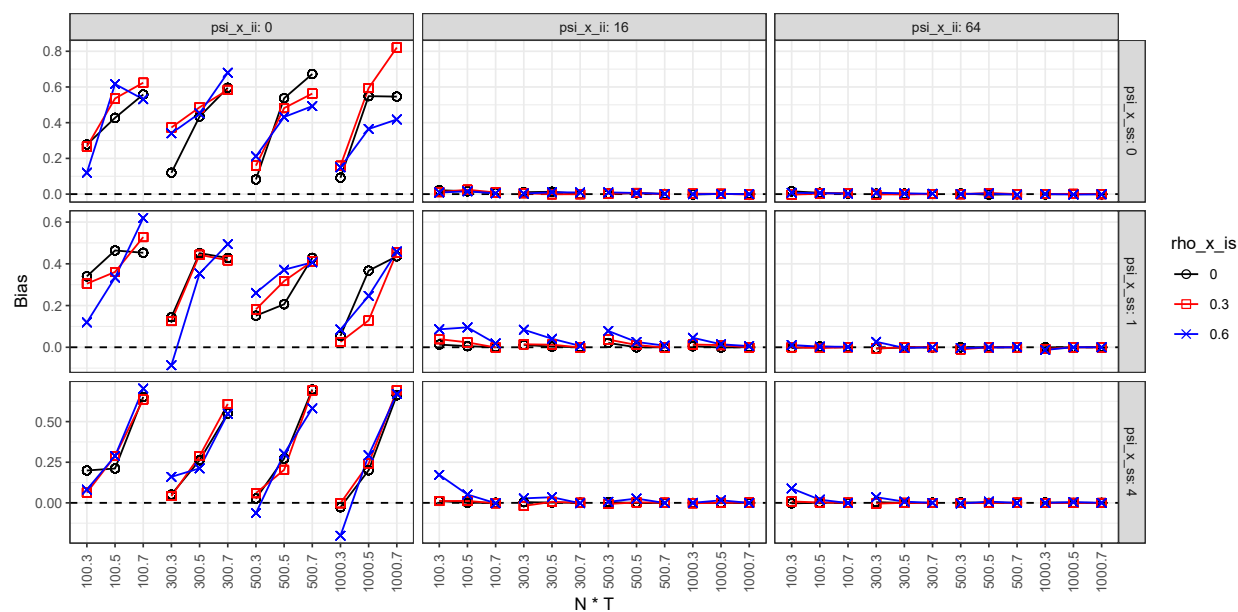
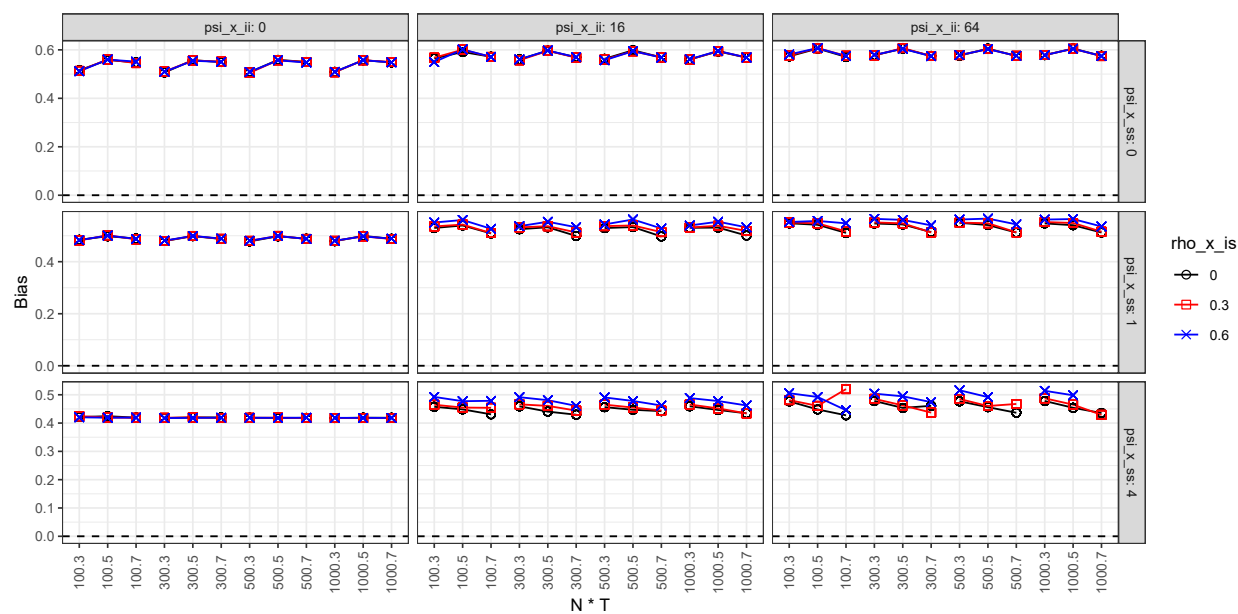
Figure E.39*Bias for the Between-Person Effect β_{II} in Simulation 5 (LCM_{xy}-RP)***Figure E.40***Bias for the Between-Person Effect β_{II} in Simulation 5 (LCM_x-Rly)*

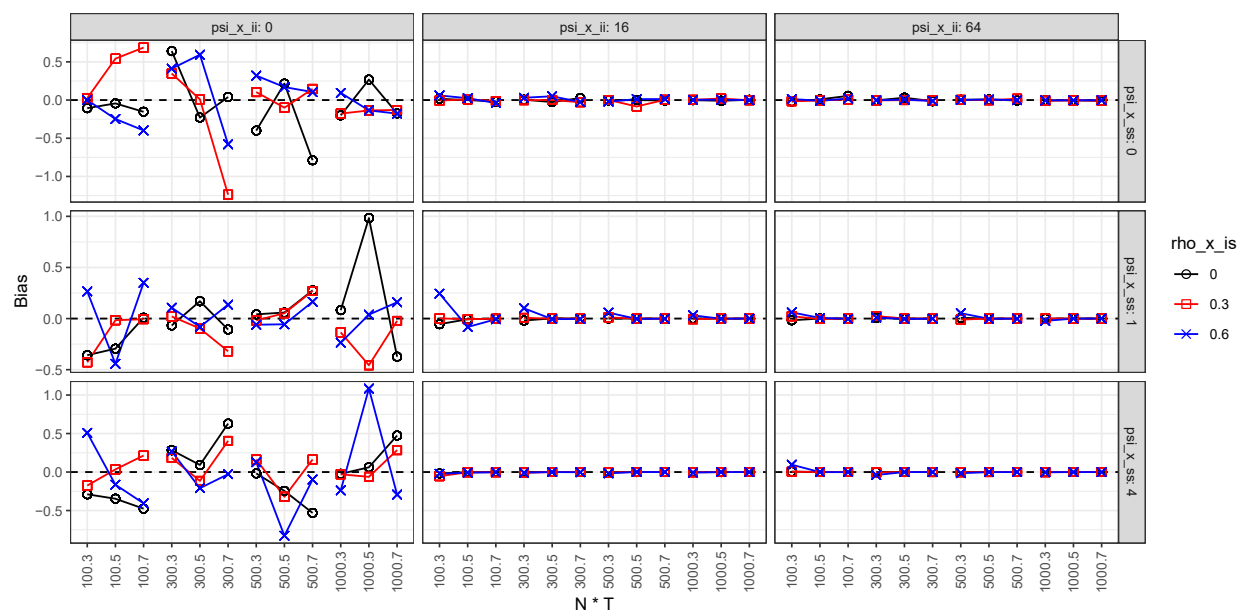
Figure E.41*Bias for the Between-Person Effect β_{II} in Simulation 5 (LCM_X-CLC)*

Figure E.42

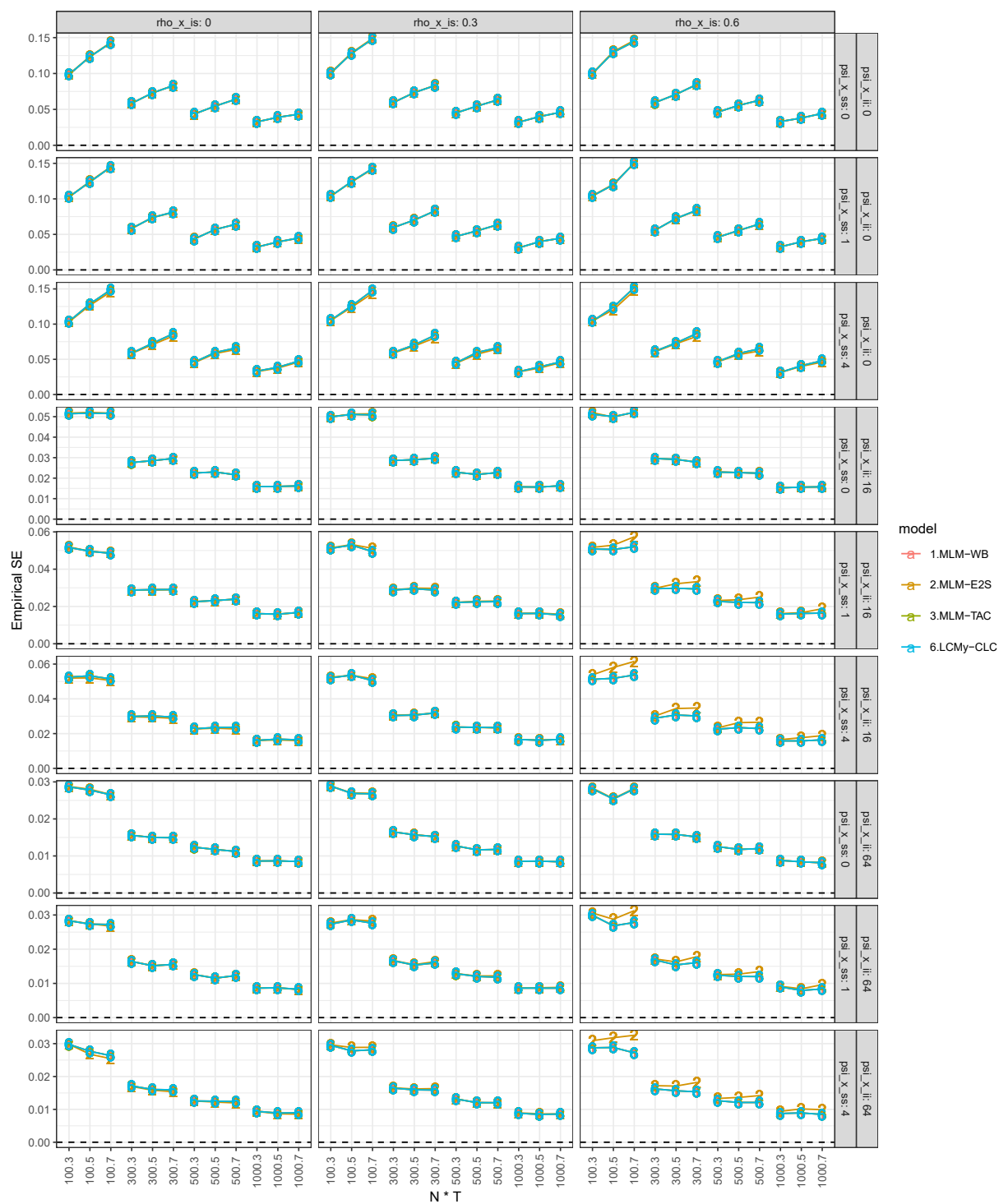
Empirical SE for the Between-Person Effect β_{II} in Simulation 5

Figure E.43

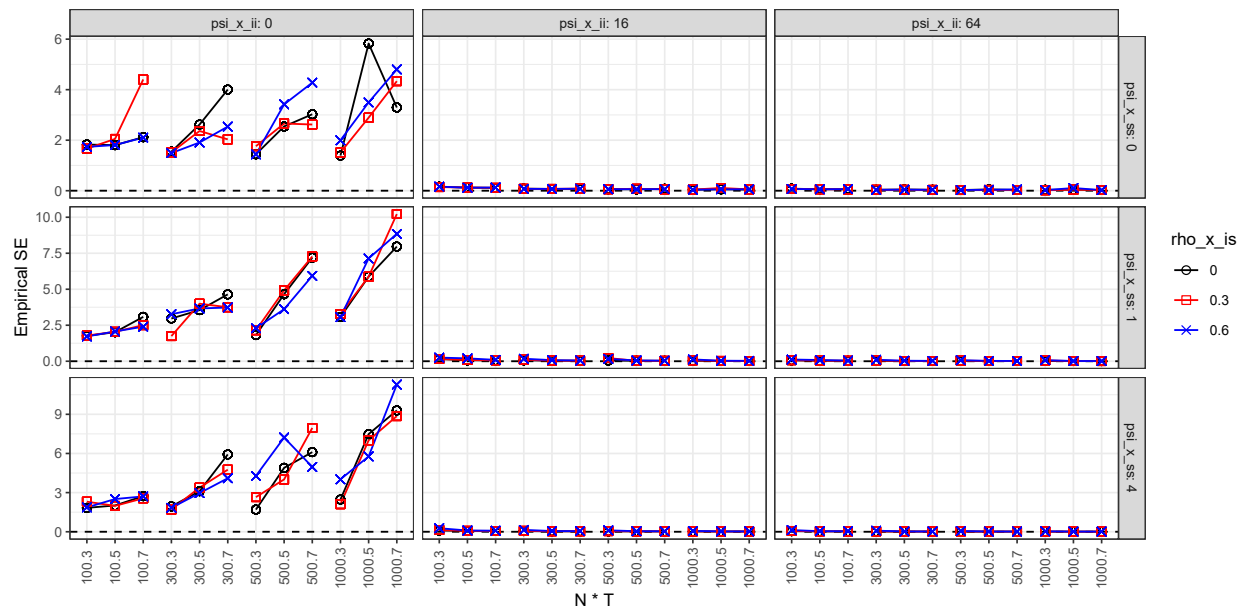
Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)

Figure E.44

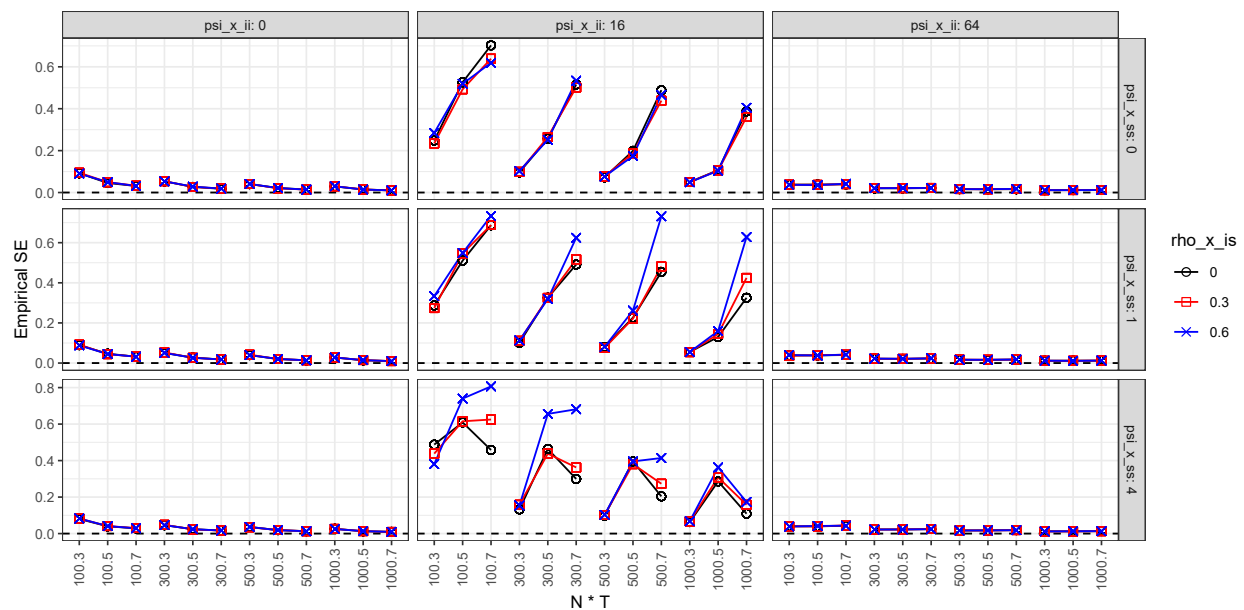
Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)

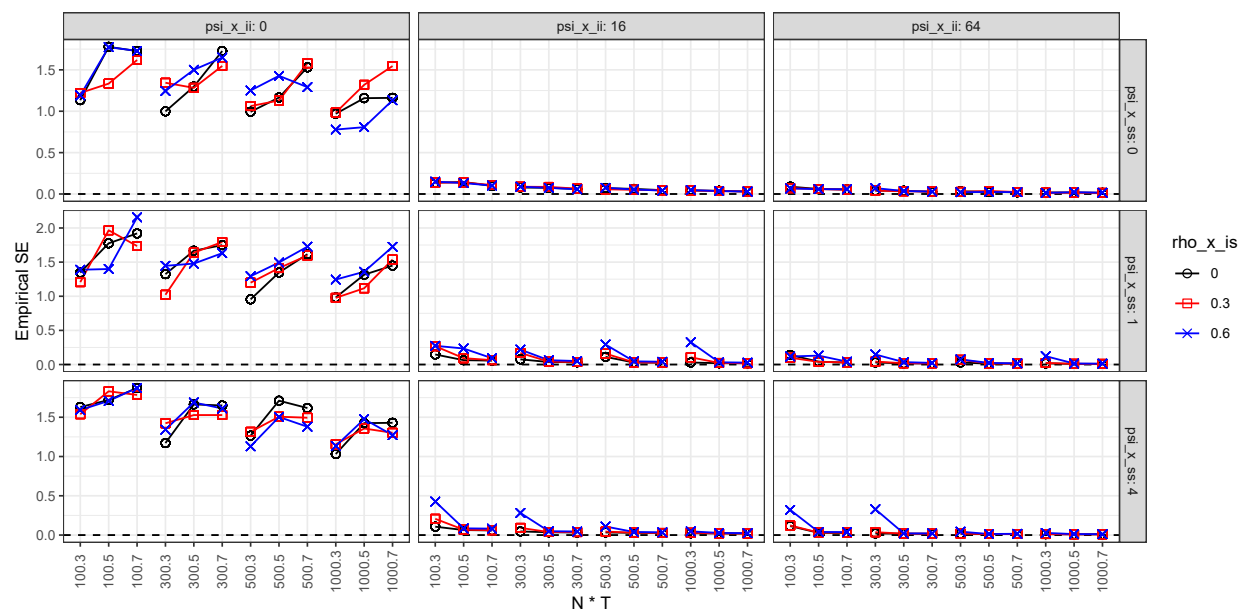
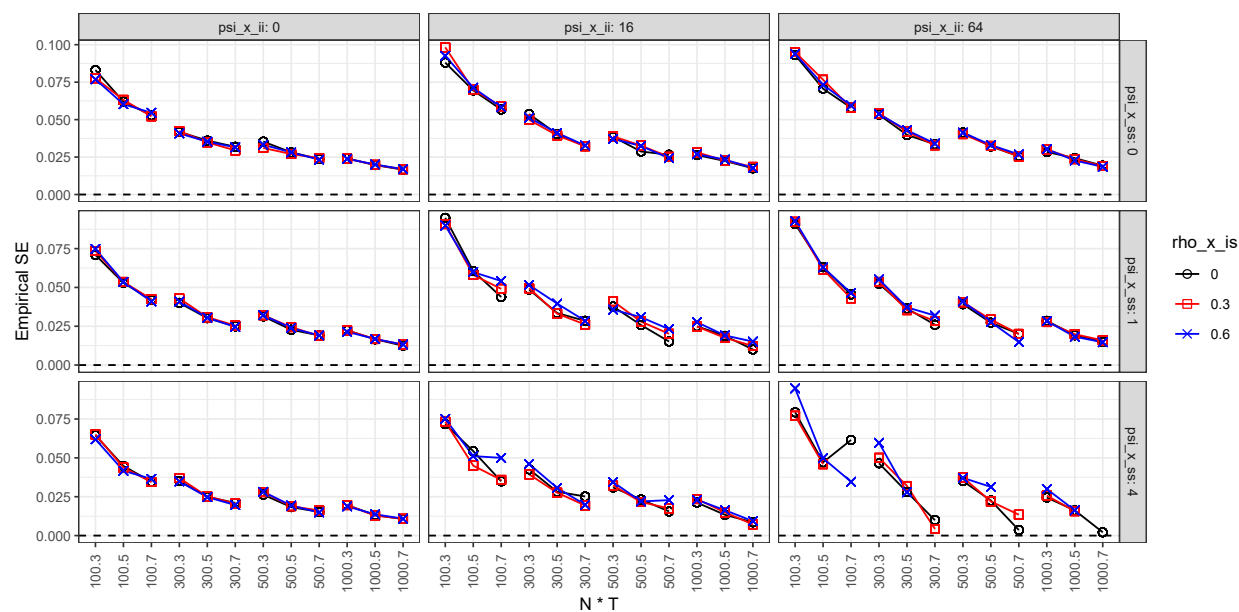
Figure E.45*Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (LCM_{xy}-RP)***Figure E.46***Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (LCM_x-RIy)*

Figure E.47

Empirical SE for the Between-Person Effect β_{II} in Simulation 5 (LCM_X-CLC)

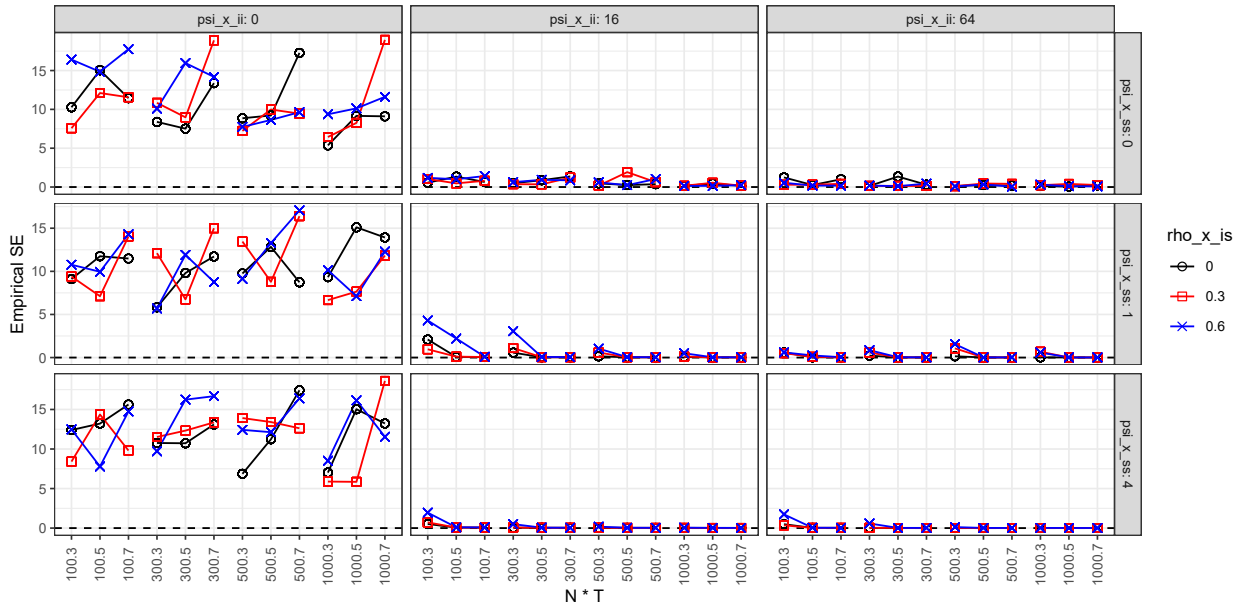


Figure E.48

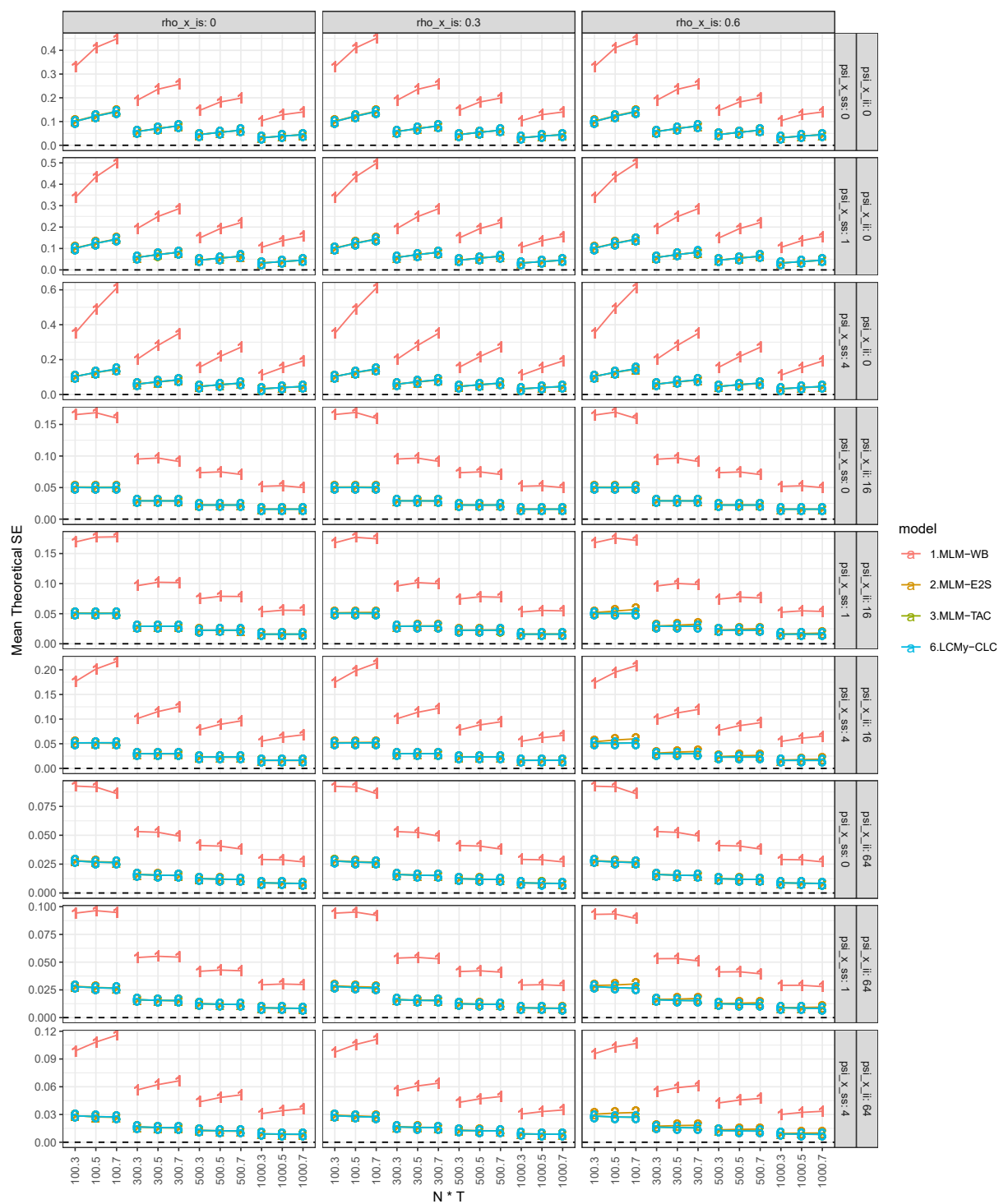
Theoretical SE for the Between-Person Effect β_{II} in Simulation 5

Figure E.49

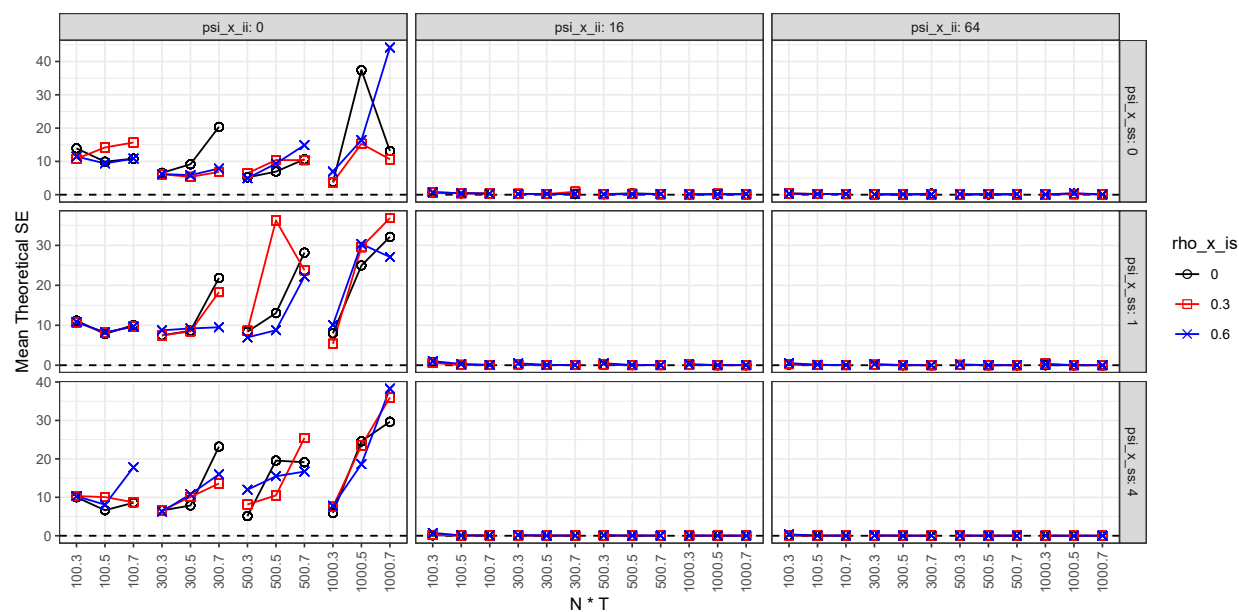
Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)

Figure E.50

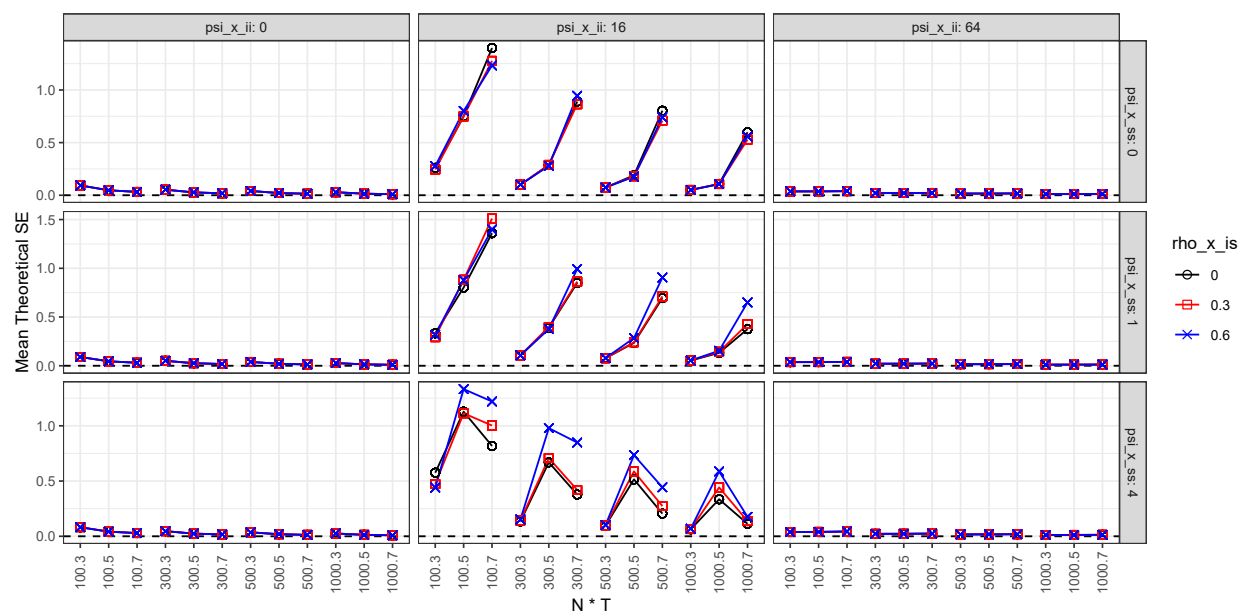
Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)

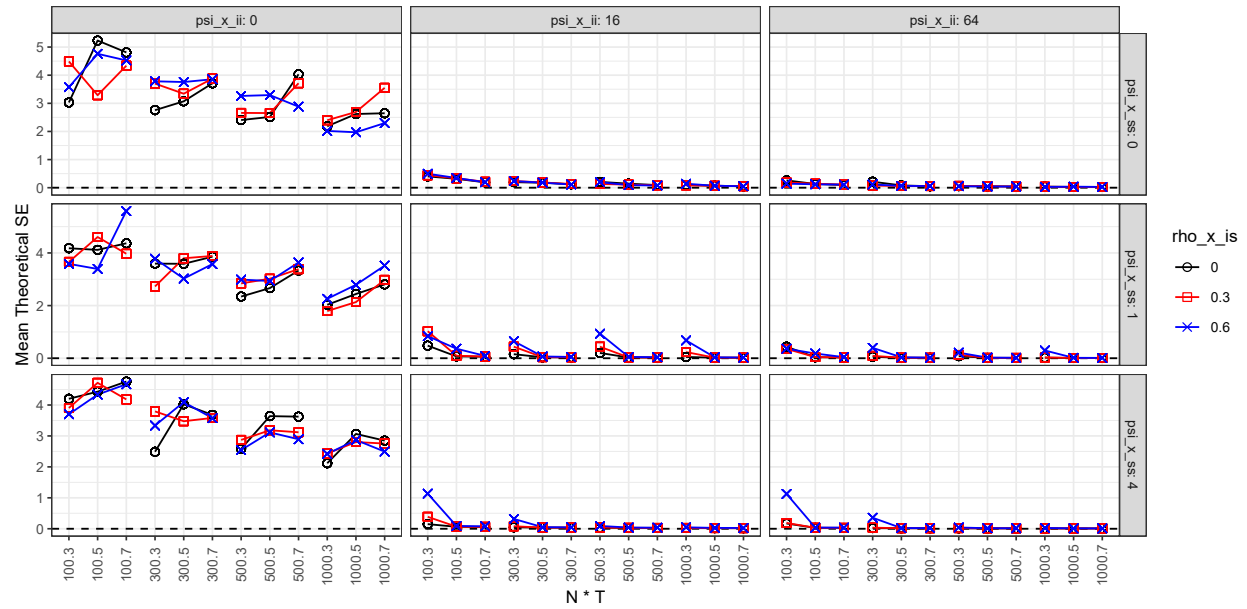
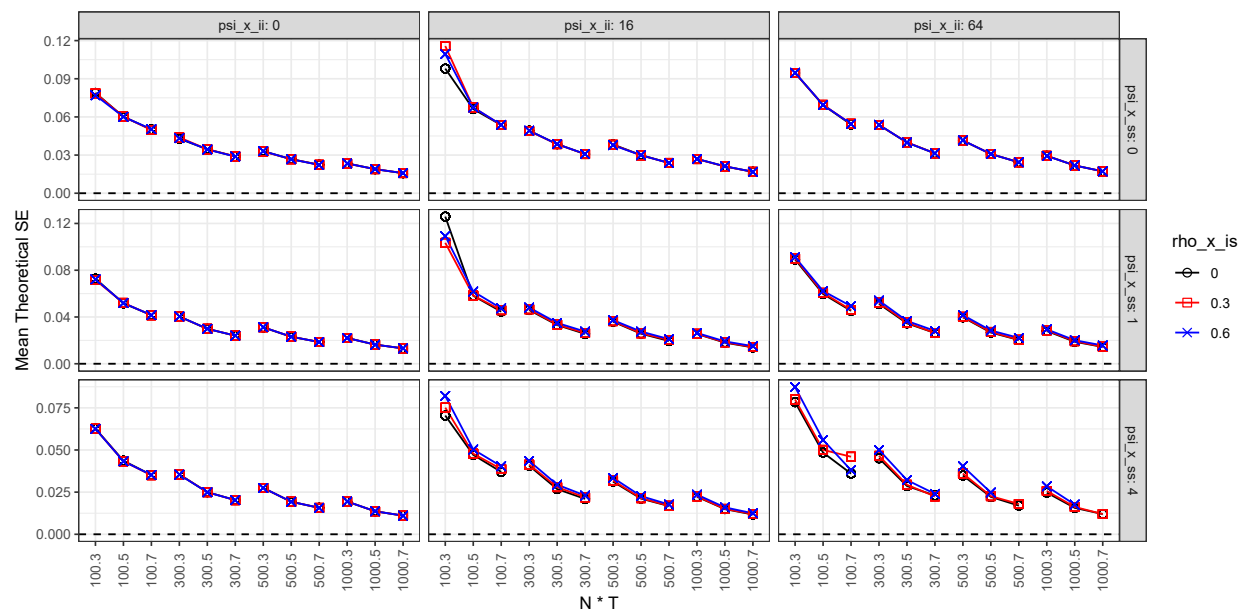
Figure E.51*Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)***Figure E.52***Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-Rly)*

Figure E.53

Theoretical SE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)

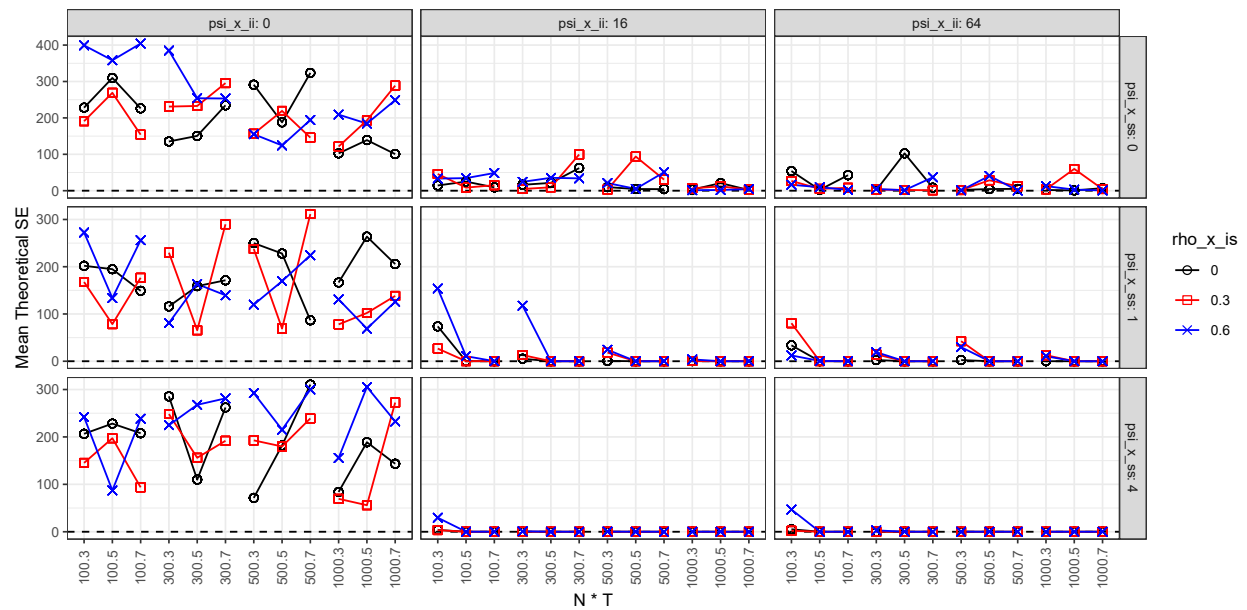


Figure E.54

SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (MLM-WB)

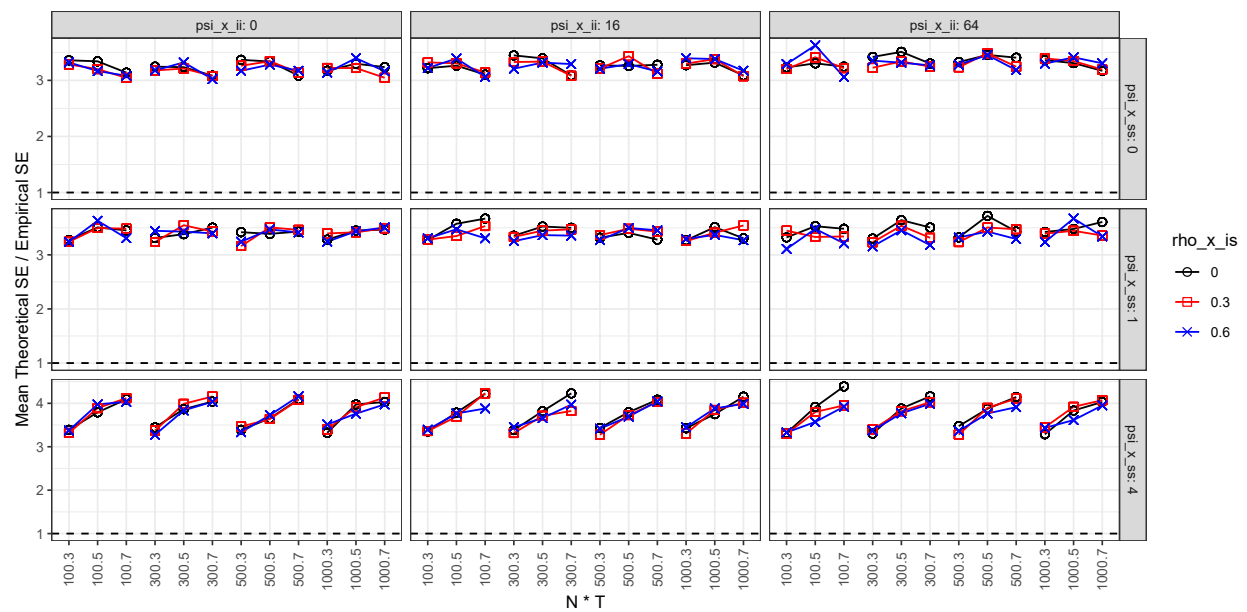


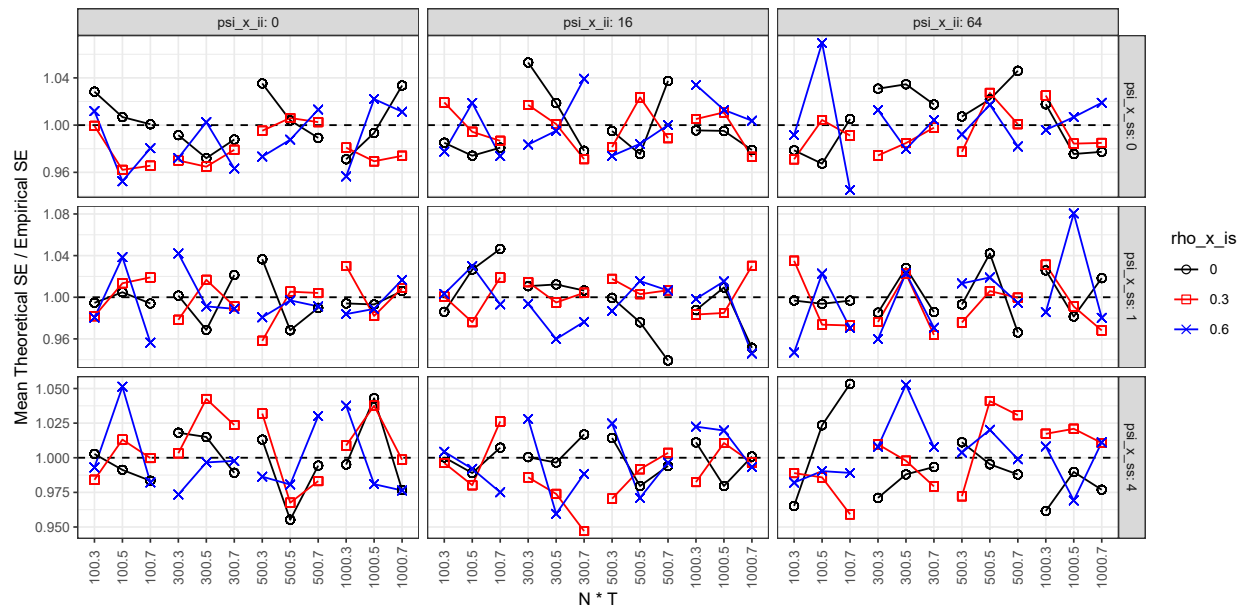
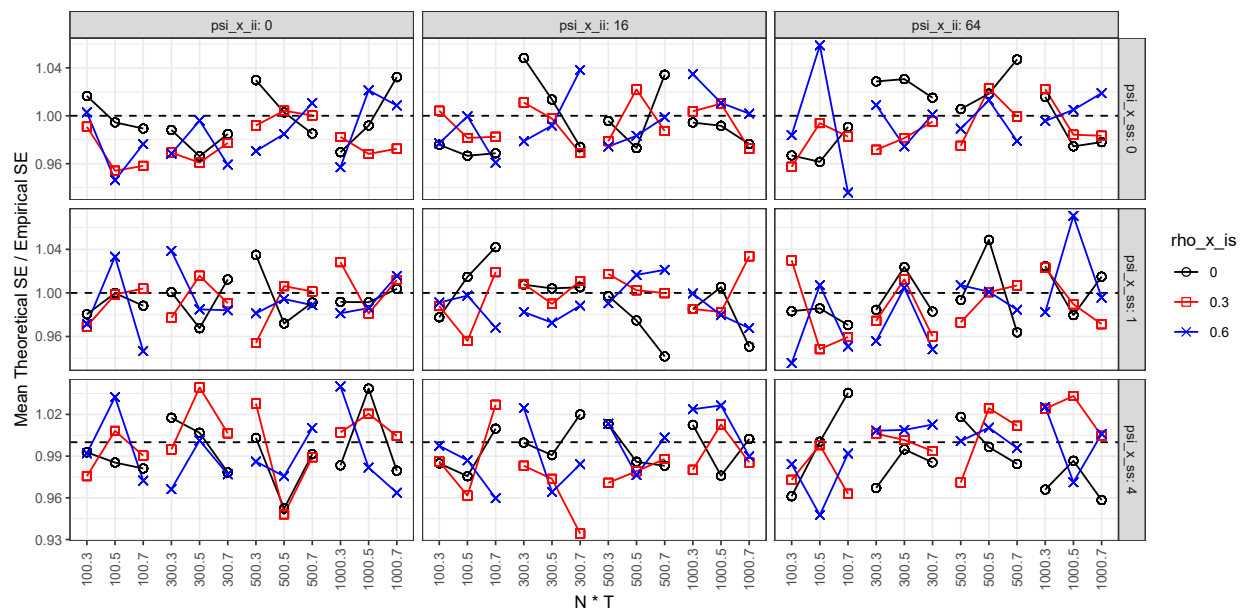
Figure E.55*SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (MLM-E2S)***Figure E.56***SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (MLM-TAC)*

Figure E.57

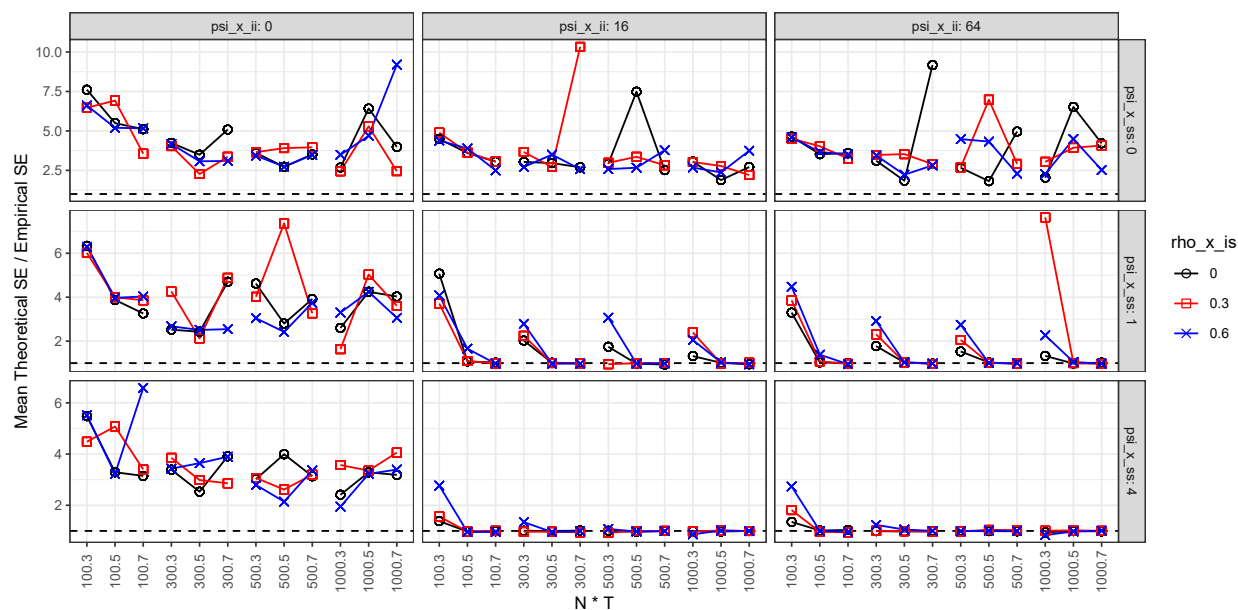
SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)

Figure E.58

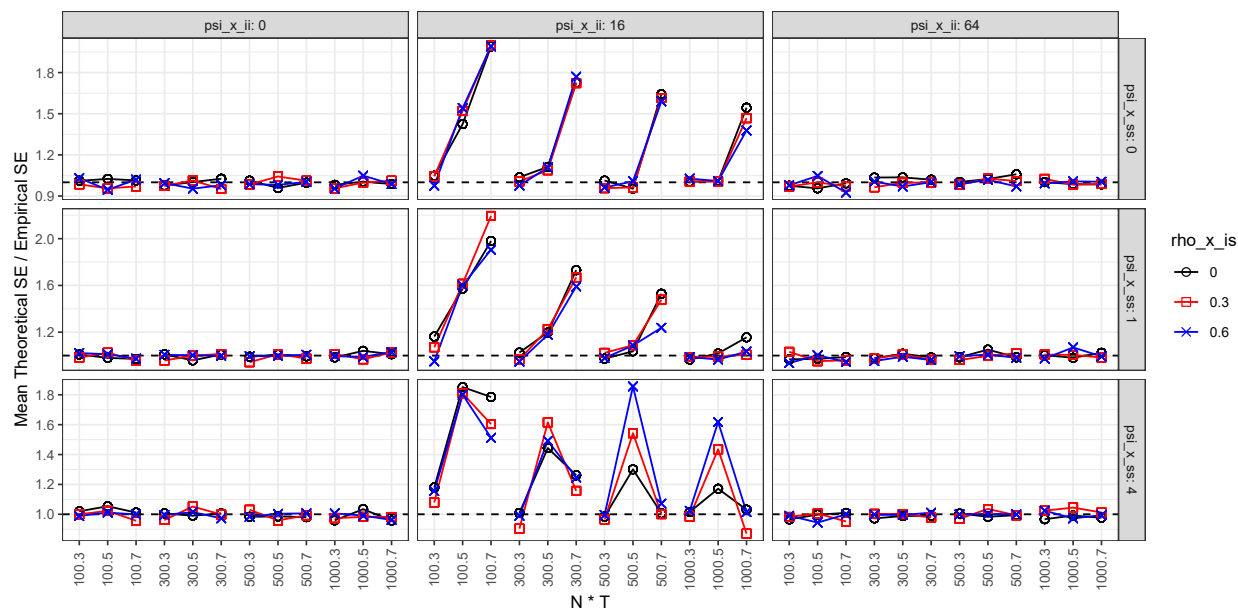
SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMy-Rlx)

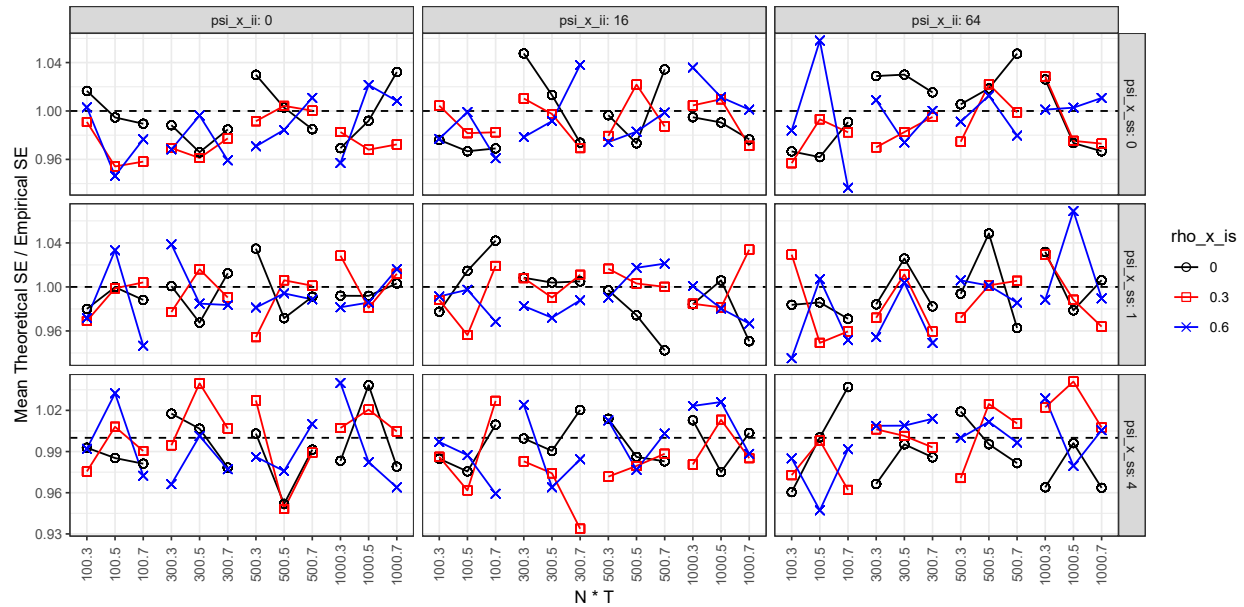
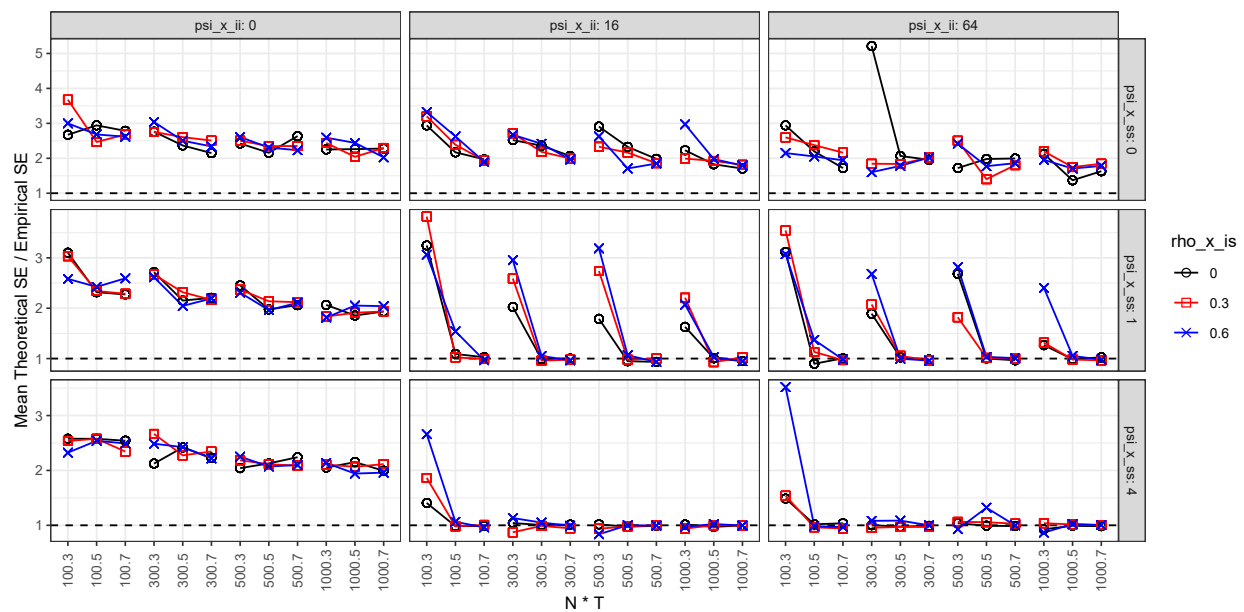
Figure E.59*SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMy-CLC)***Figure E.60***SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)*

Figure E.61

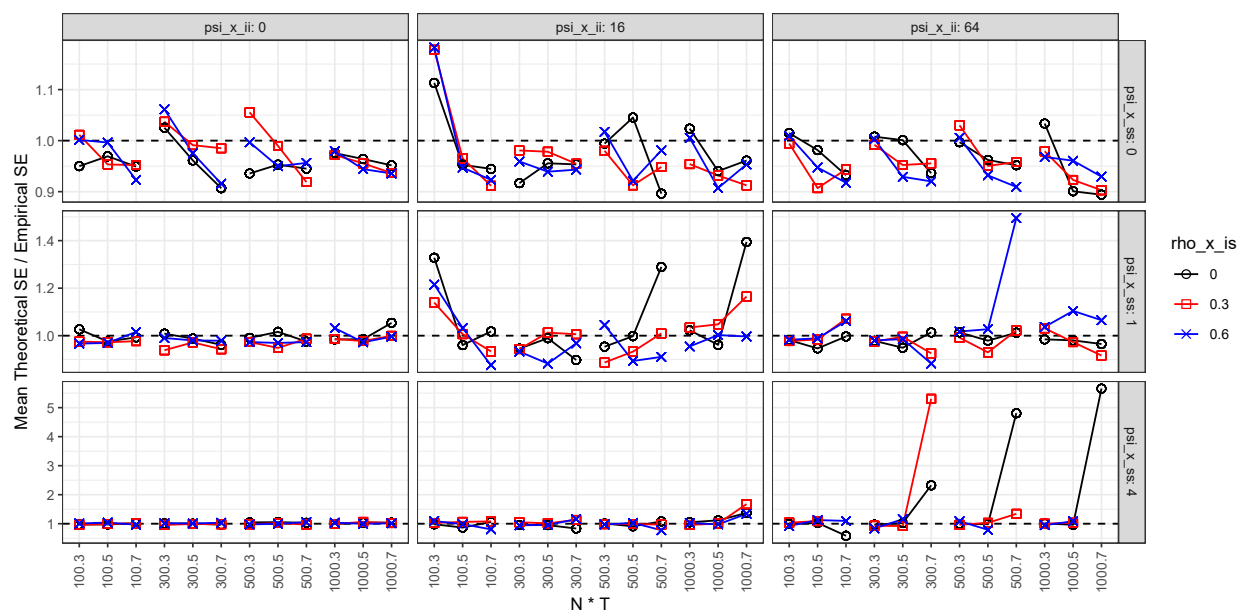
SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMx-Rly)

Figure E.62

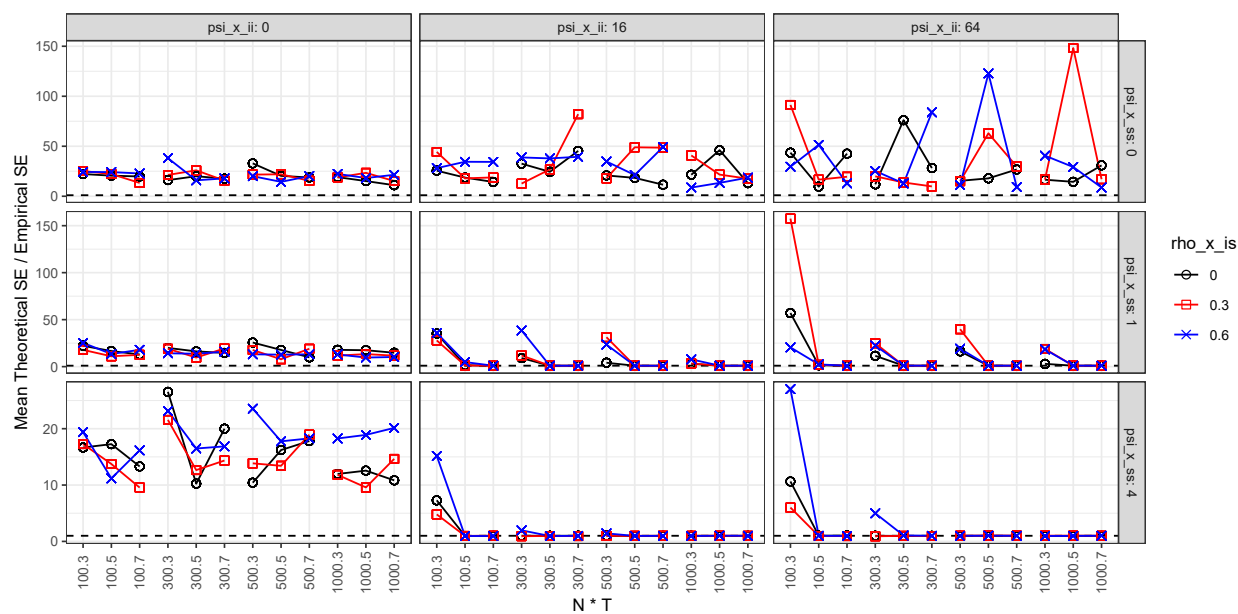
SE Ratio for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)

Figure E.63

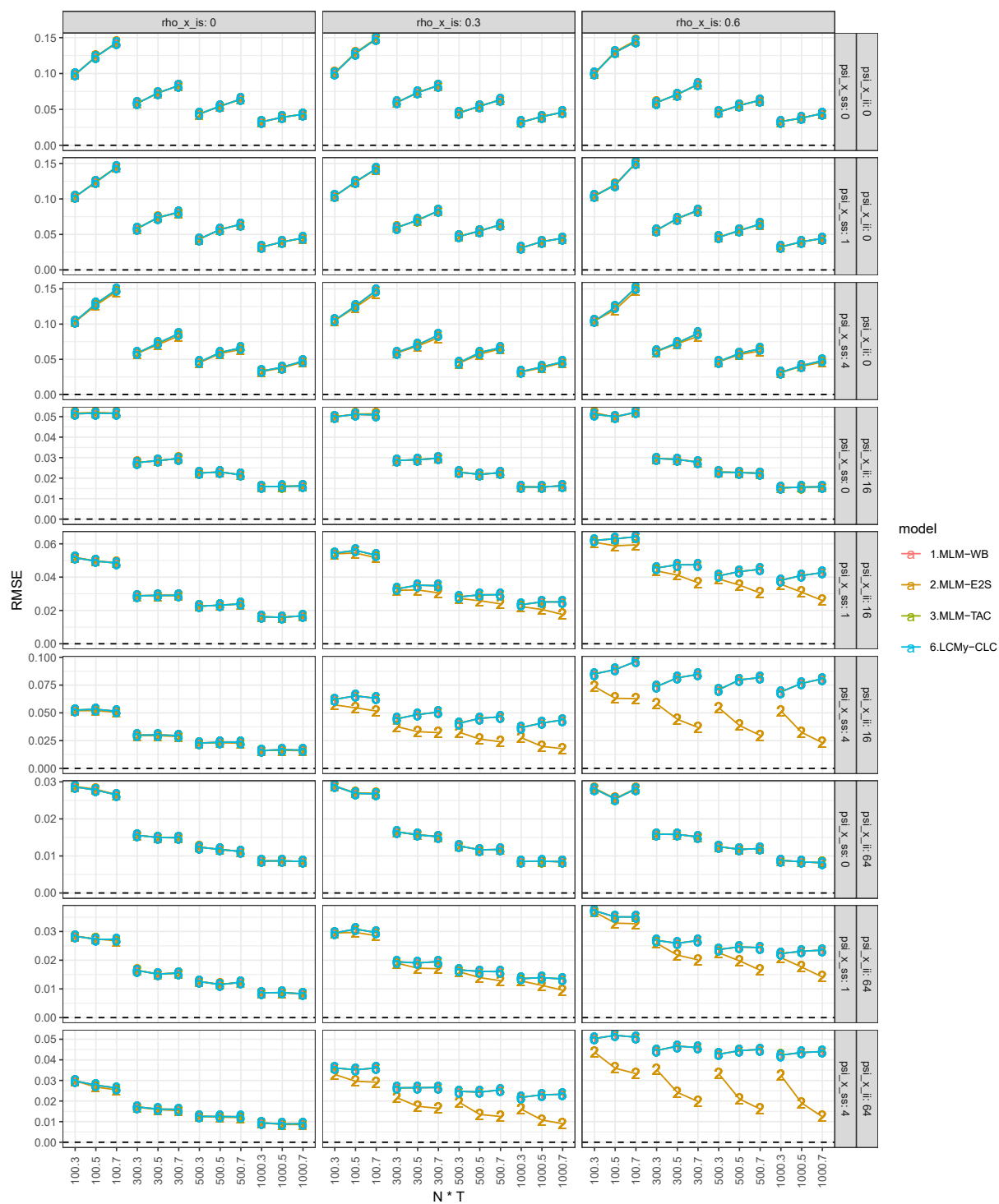
RMSE for the Between-Person Effect β_{II} in Simulation 5

Figure E.64

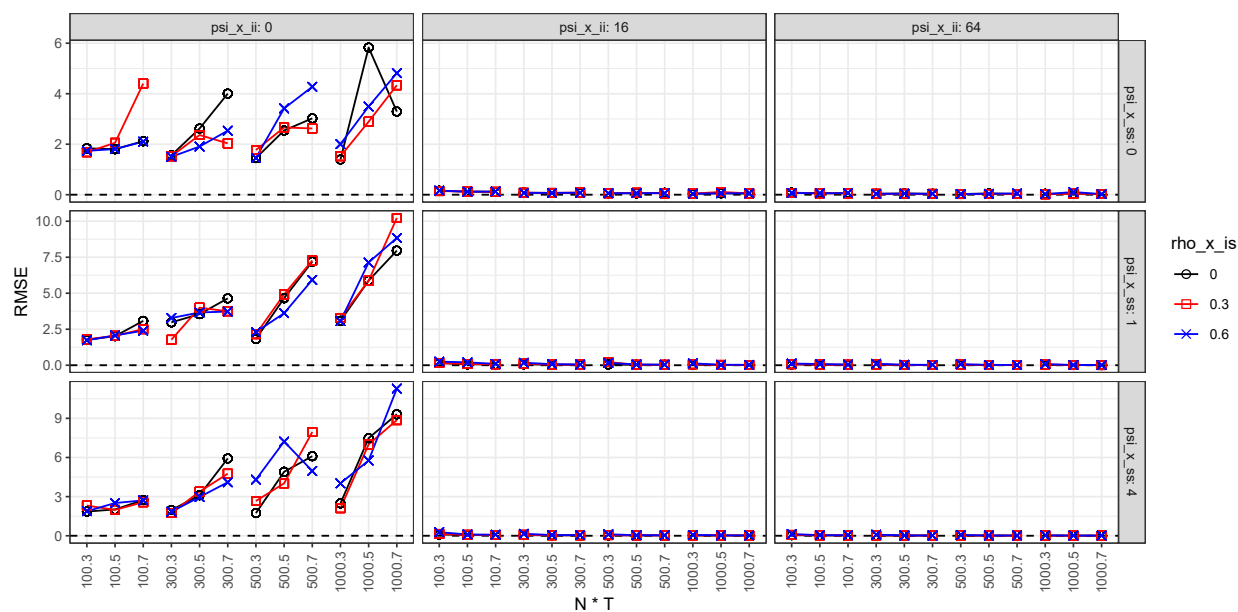
RMSE for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)

Figure E.65

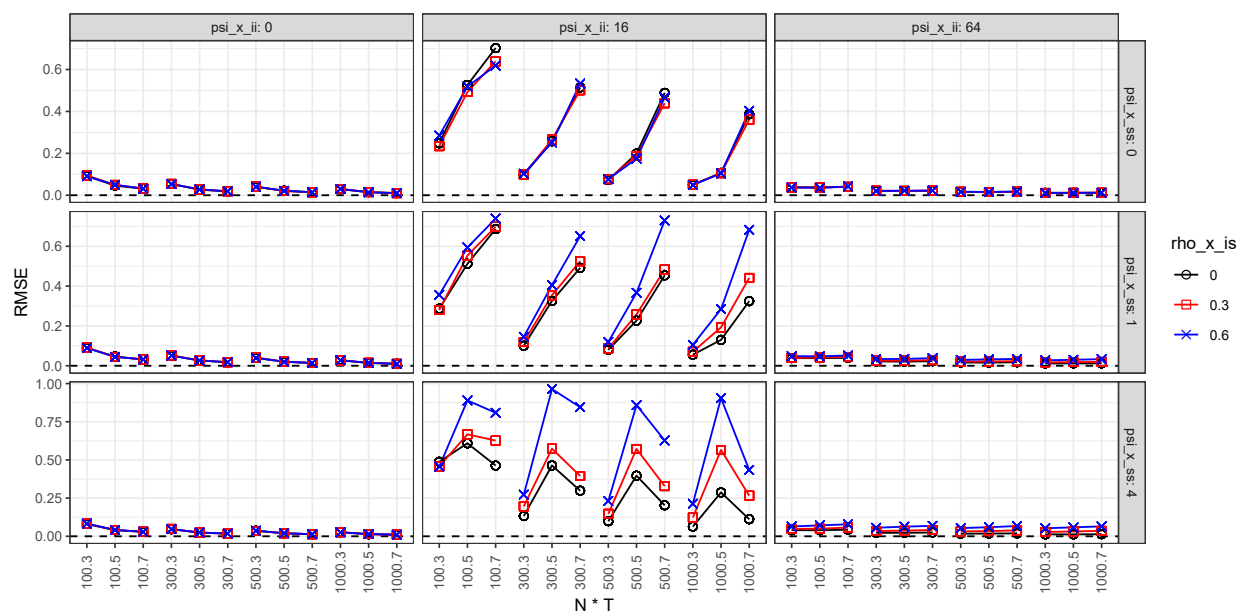
RMSE for the Between-Person Effect β_{II} in Simulation 5 (LCMy-RIx)

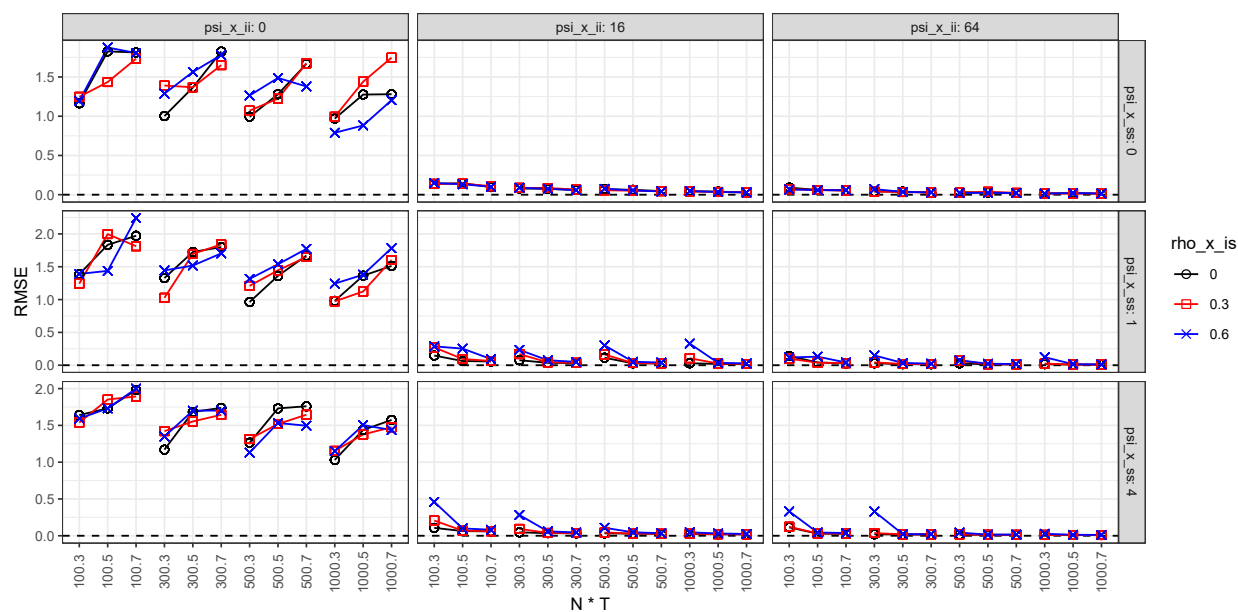
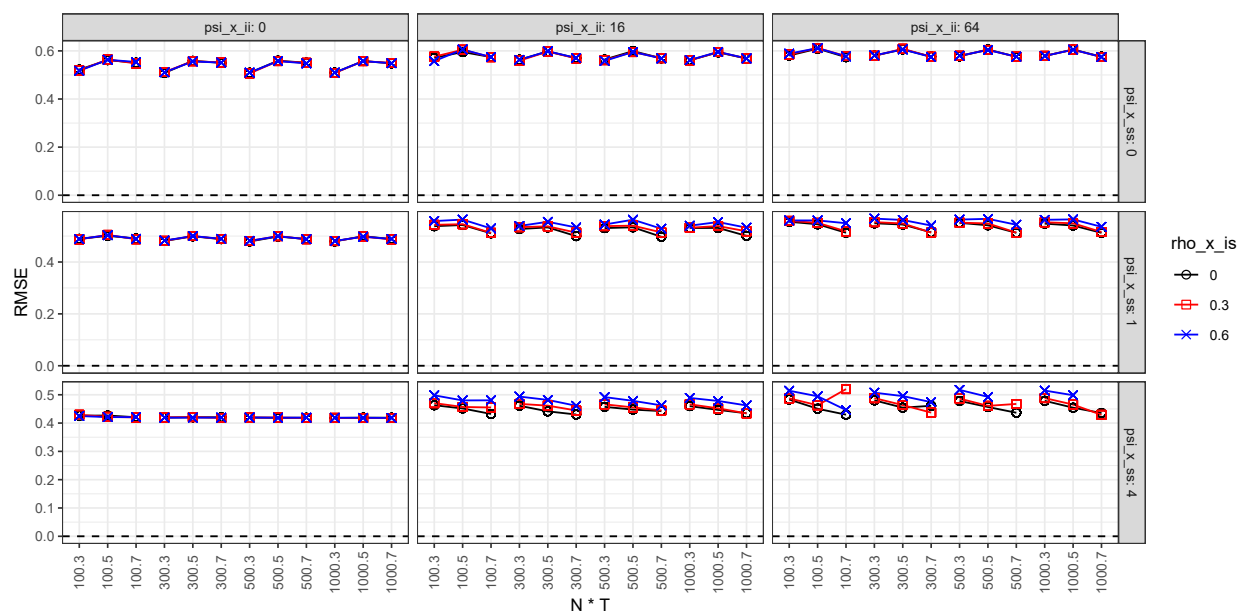
Figure E.66*RMSE for the Between-Person Effect β_{II} in Simulation 5 (LCM_{xy}-RP)***Figure E.67***RMSE for the Between-Person Effect β_{II} in Simulation 5 (LCM_x-Rly)*

Figure E.68

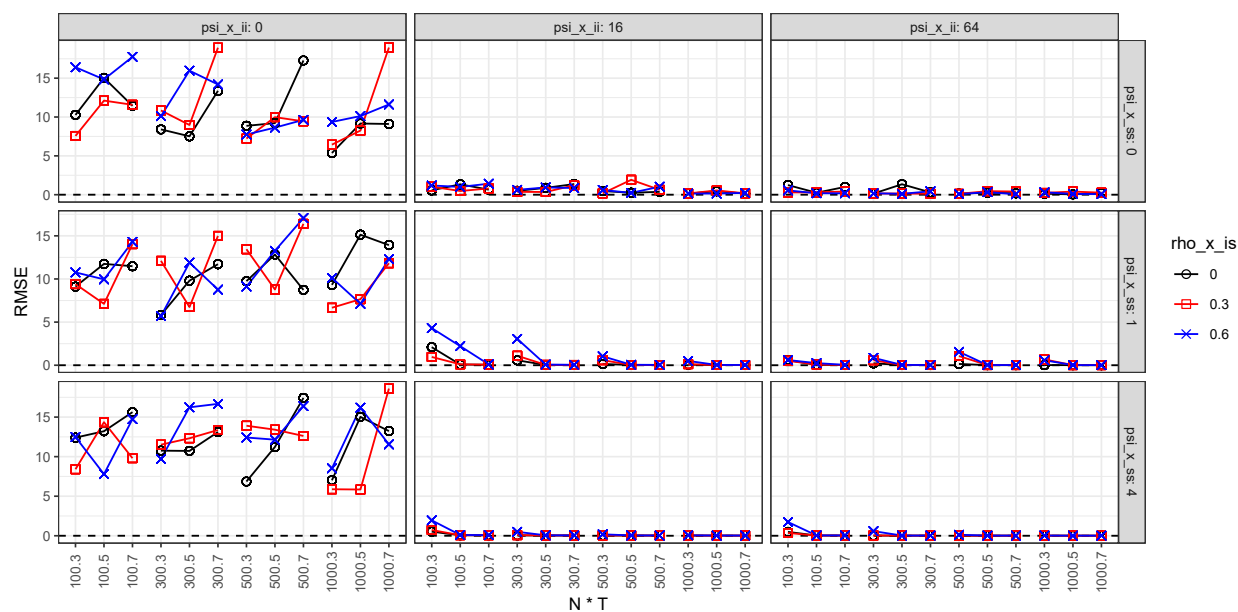
RMSE for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)

Figure E.69

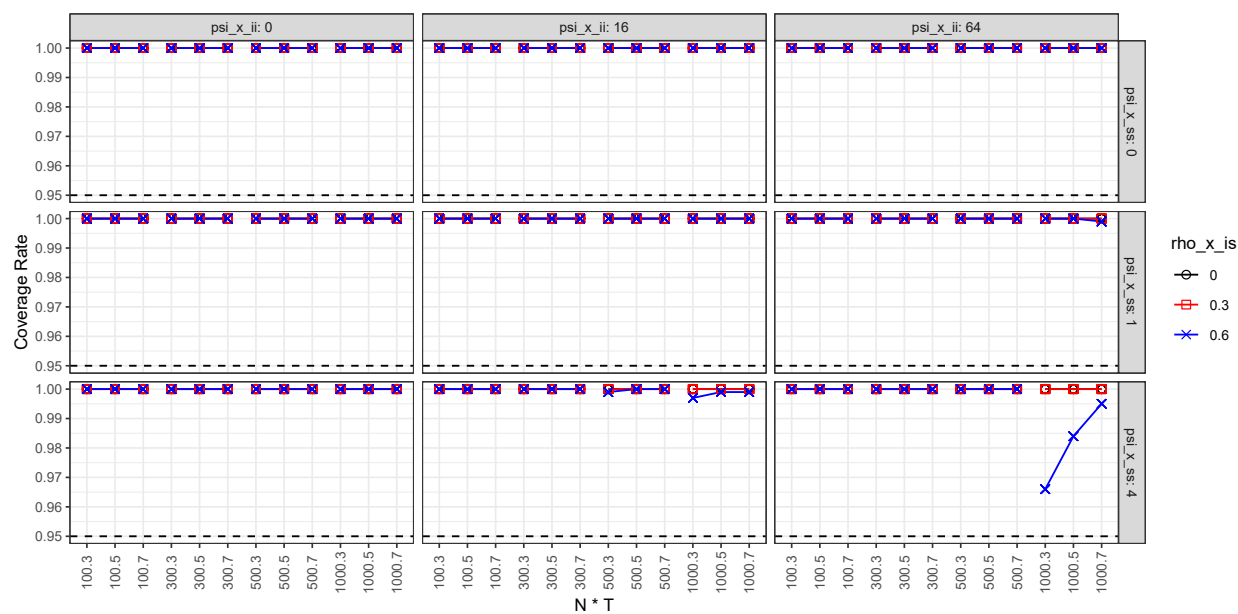
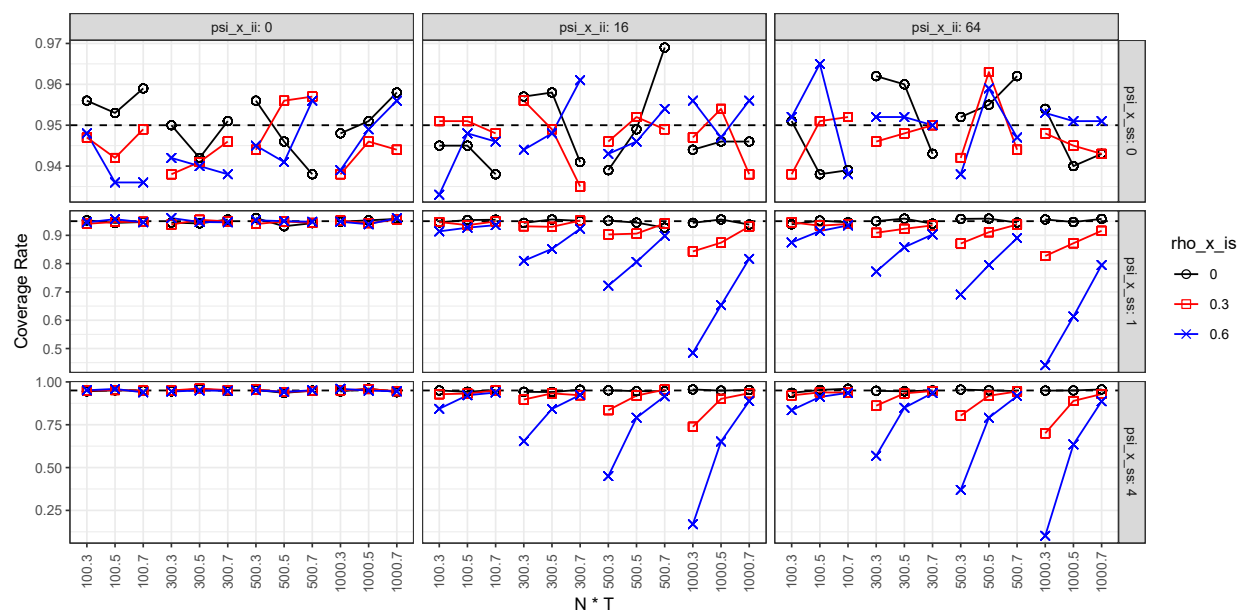
Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (MLM-WB)

Figure E.70

Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (MLM-E2S)

**Figure E.71**

Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (MLM-TAC)

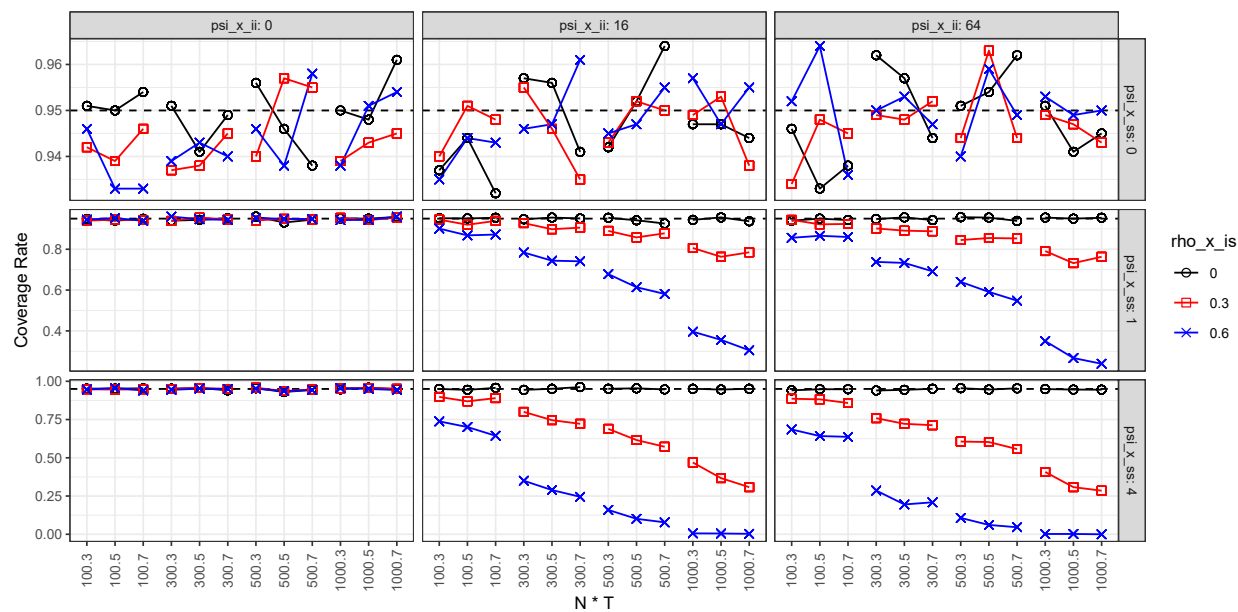


Figure E.72

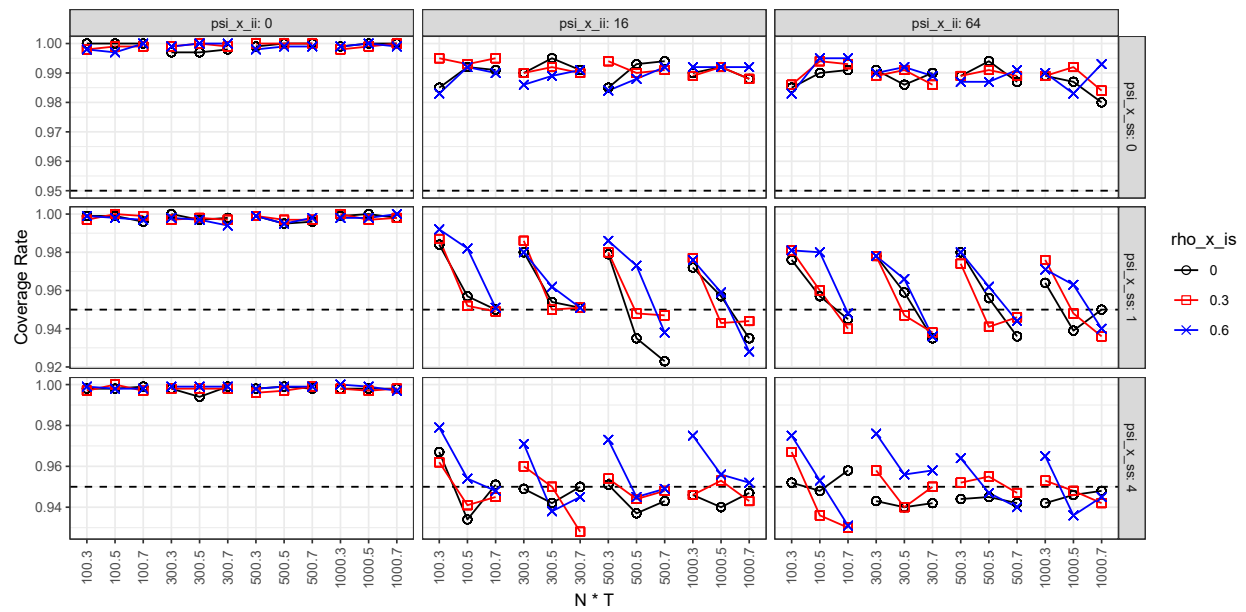
Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)

Figure E.73

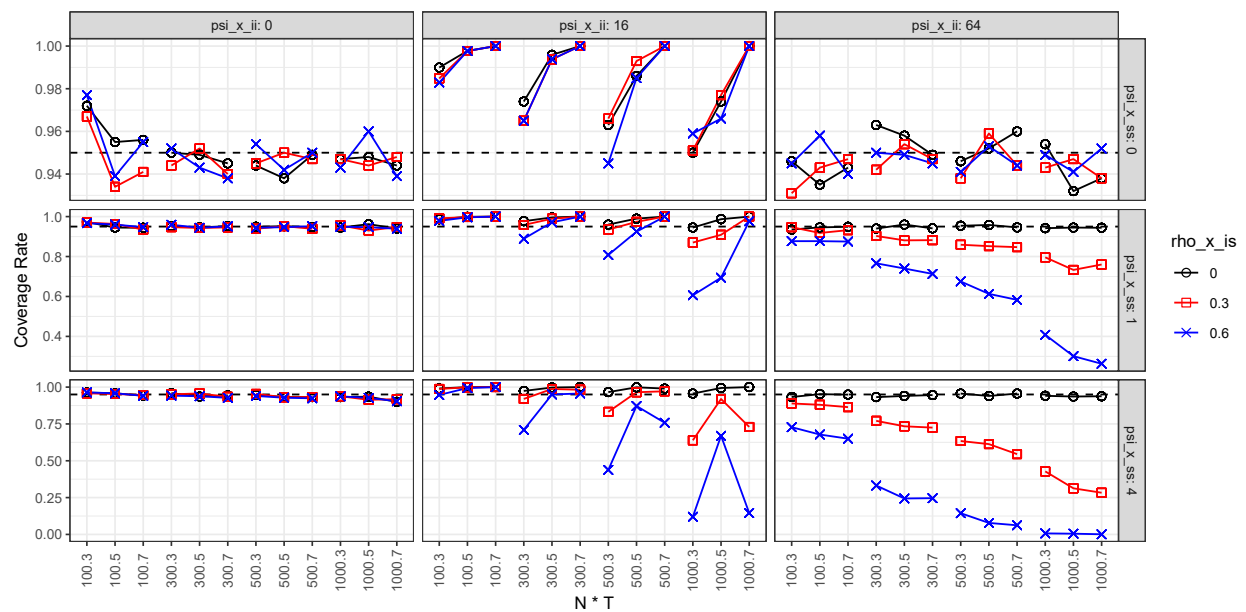
Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMy-Rlx)

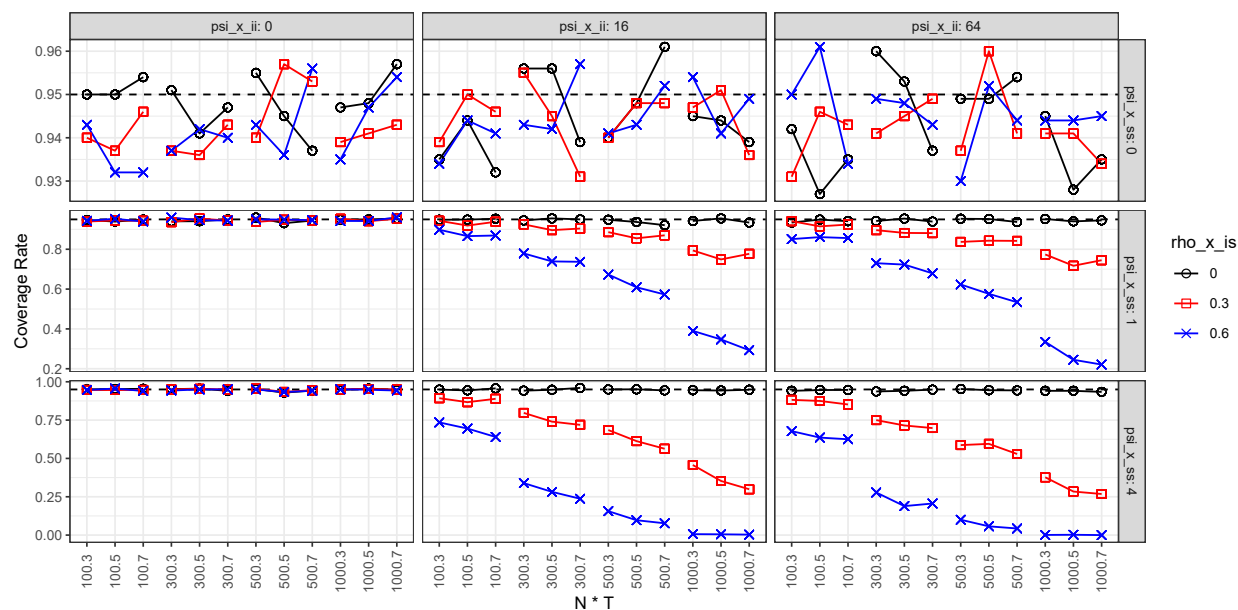
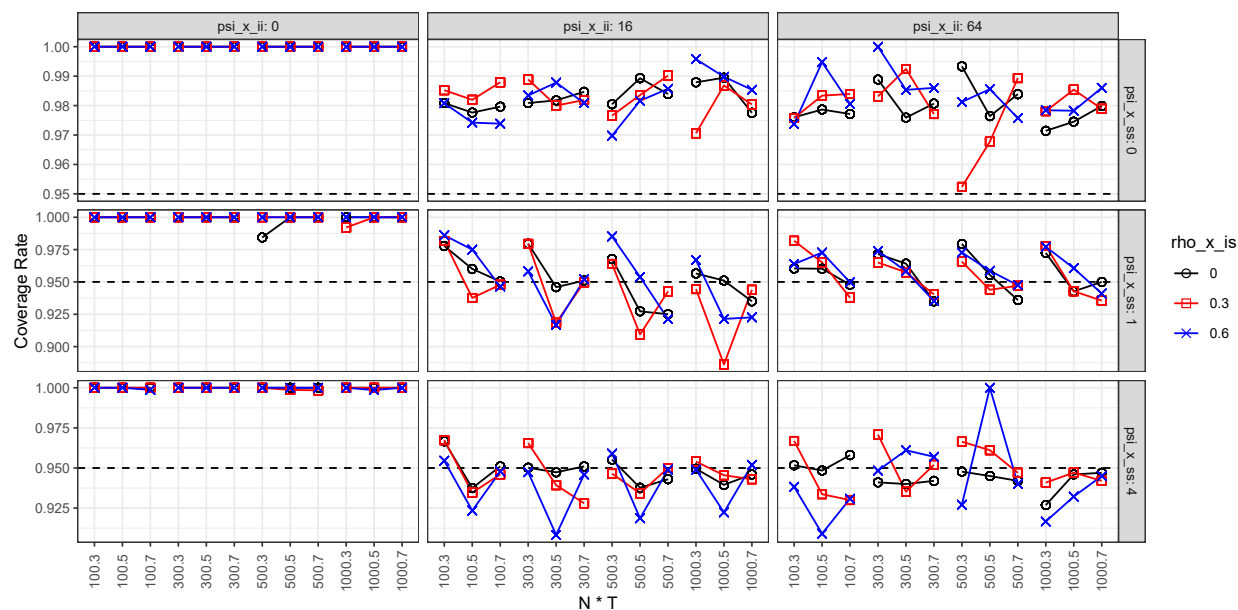
Figure E.74*Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMy-CLC)***Figure E.75***Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)*

Figure E.76

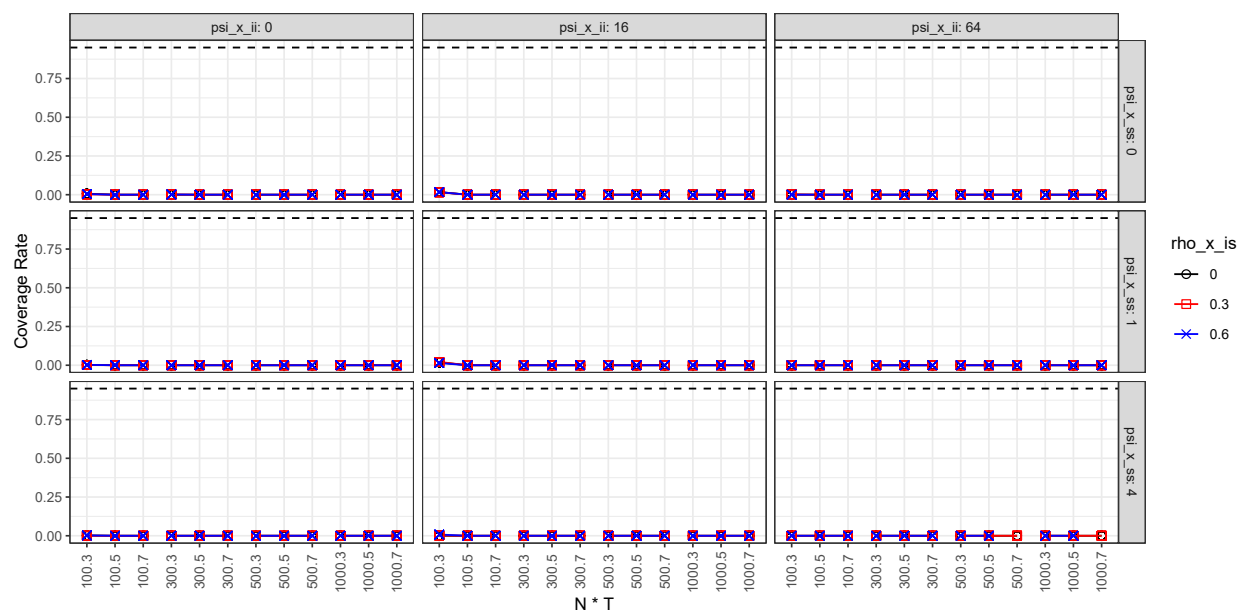
Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMx-Rly)

Figure E.77

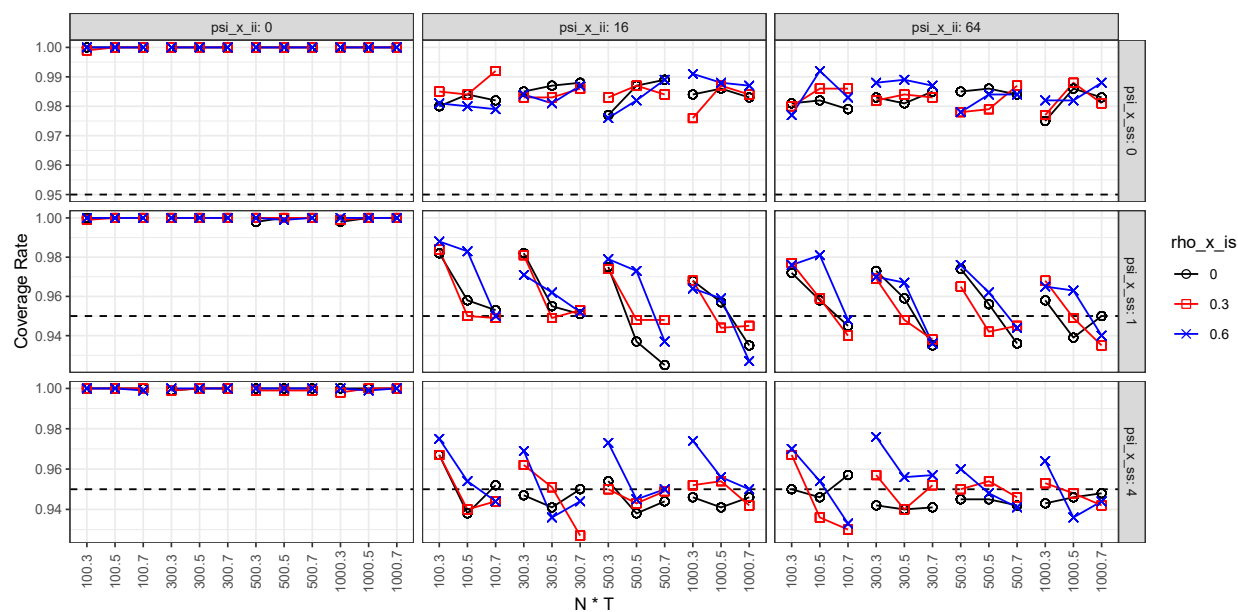
Coverage Rate for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)

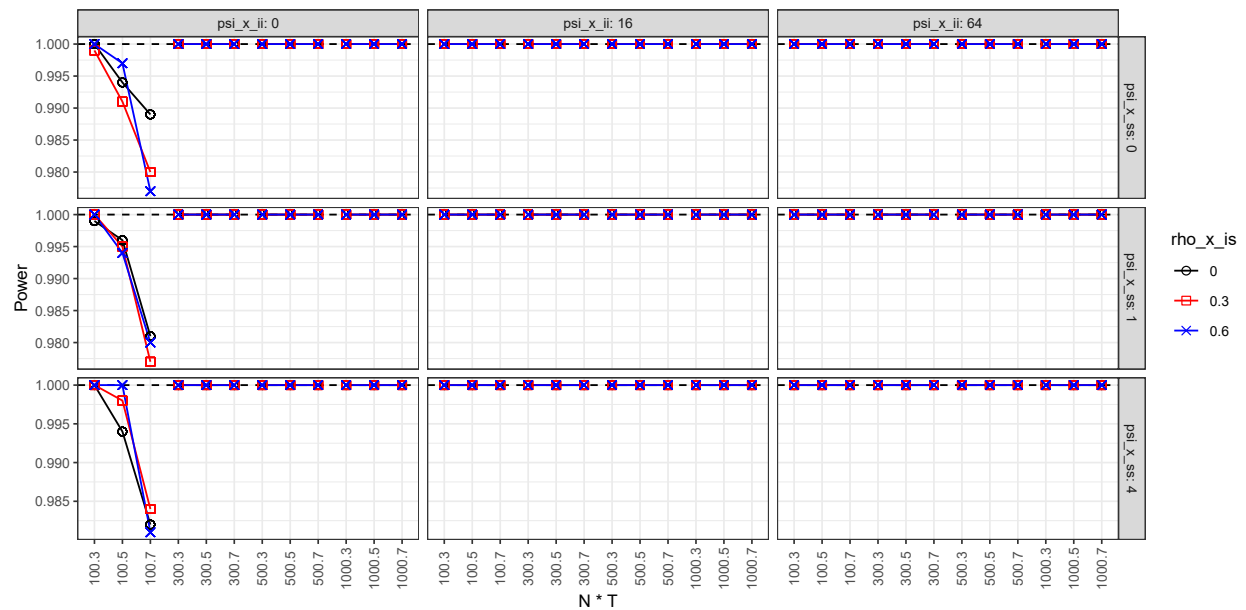
Figure E.78*Power for the Between-Person Effect β_{II} in Simulation 5 (MLM-WB)***Figure E.79***Power for the Between-Person Effect β_{II} in Simulation 5 (MLM-E2S)*

Figure E.80

Power for the Between-Person Effect β_{II} in Simulation 5 (MLM-TAC)

Figure E.81

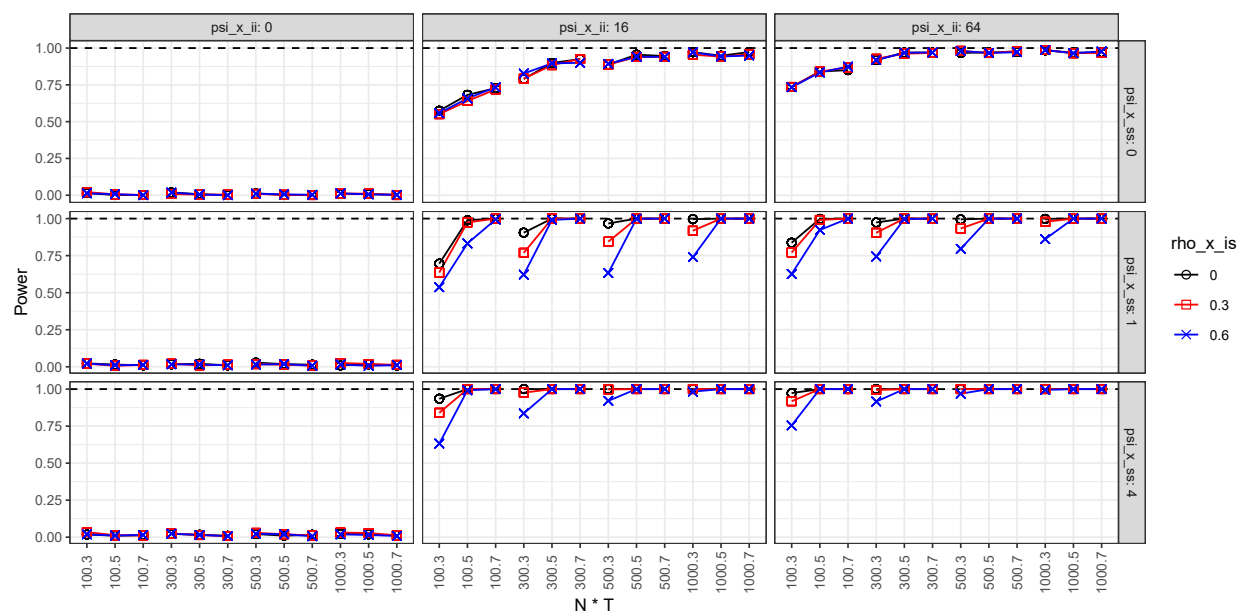
Power for the Between-Person Effect β_{II} in Simulation 5 (MLM-MV)

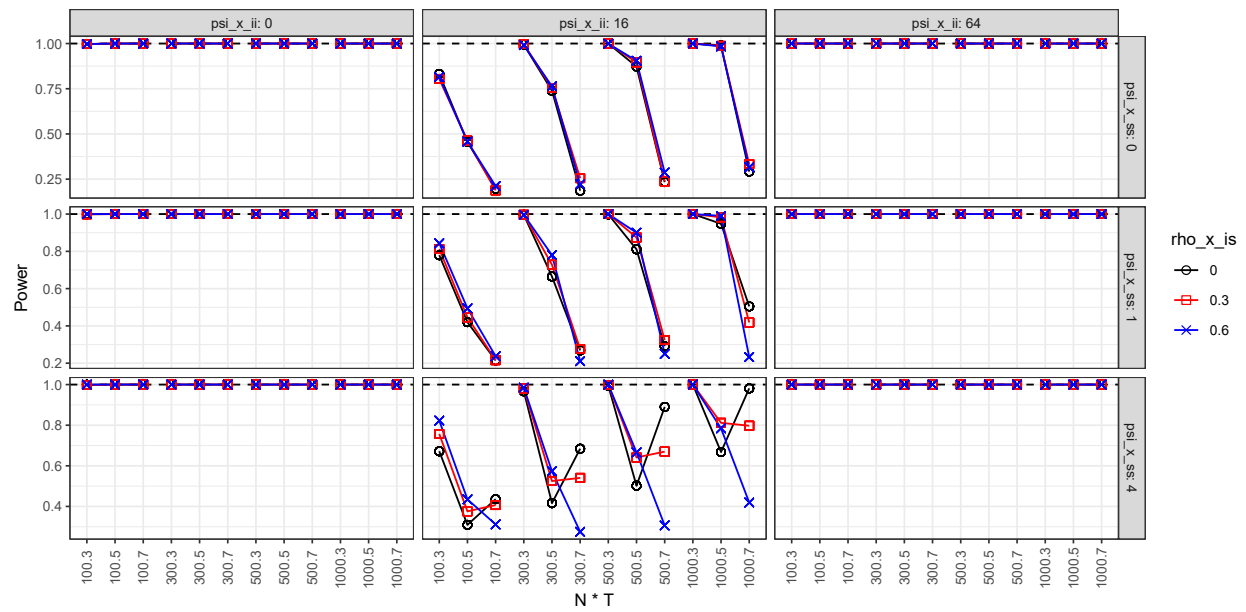
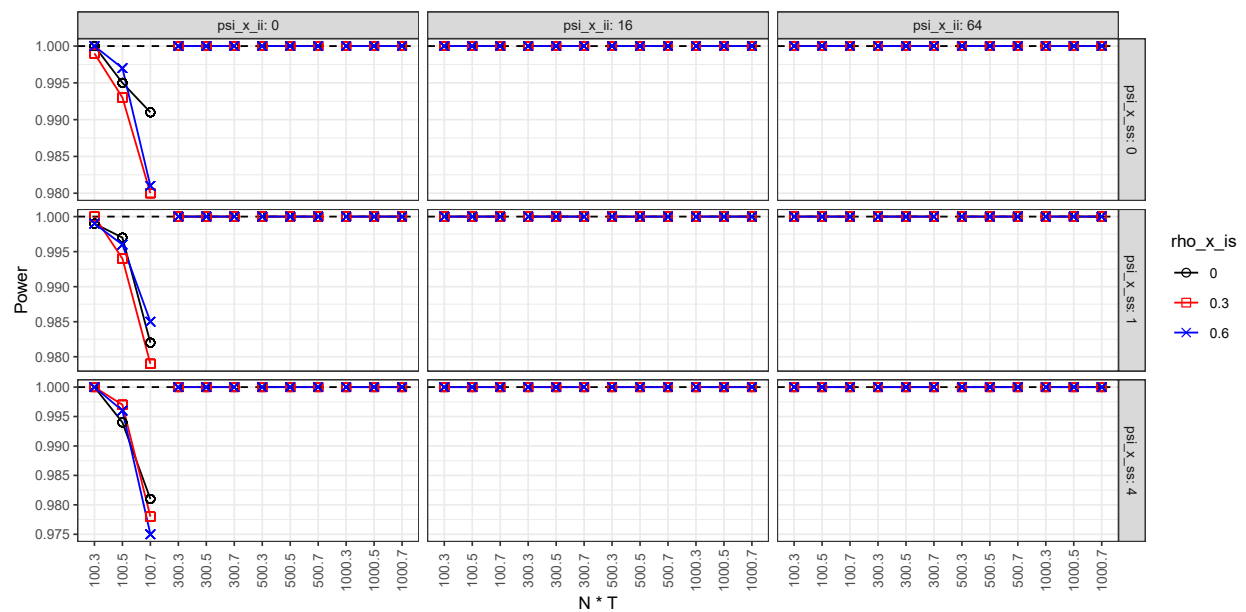
Figure E.82*Power for the Between-Person Effect β_{II} in Simulation 5 (LCMy-Rlx)***Figure E.83***Power for the Between-Person Effect β_{II} in Simulation 5 (LCMy-CLC)*

Figure E.84

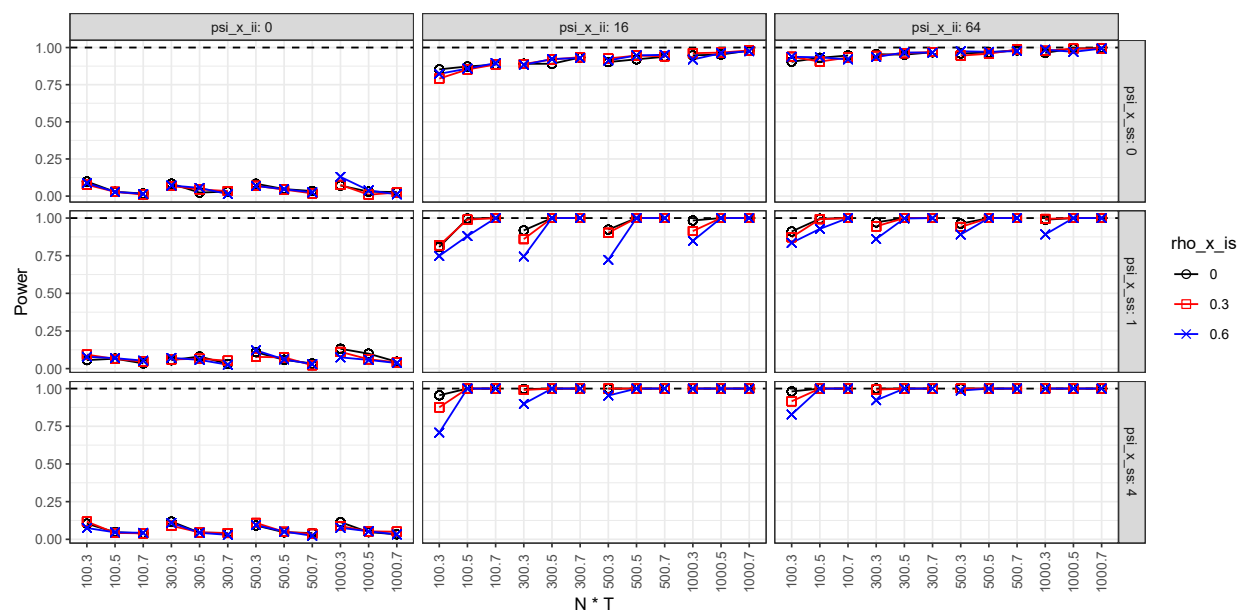
Power for the Between-Person Effect β_{II} in Simulation 5 (LCMxy-RP)

Figure E.85

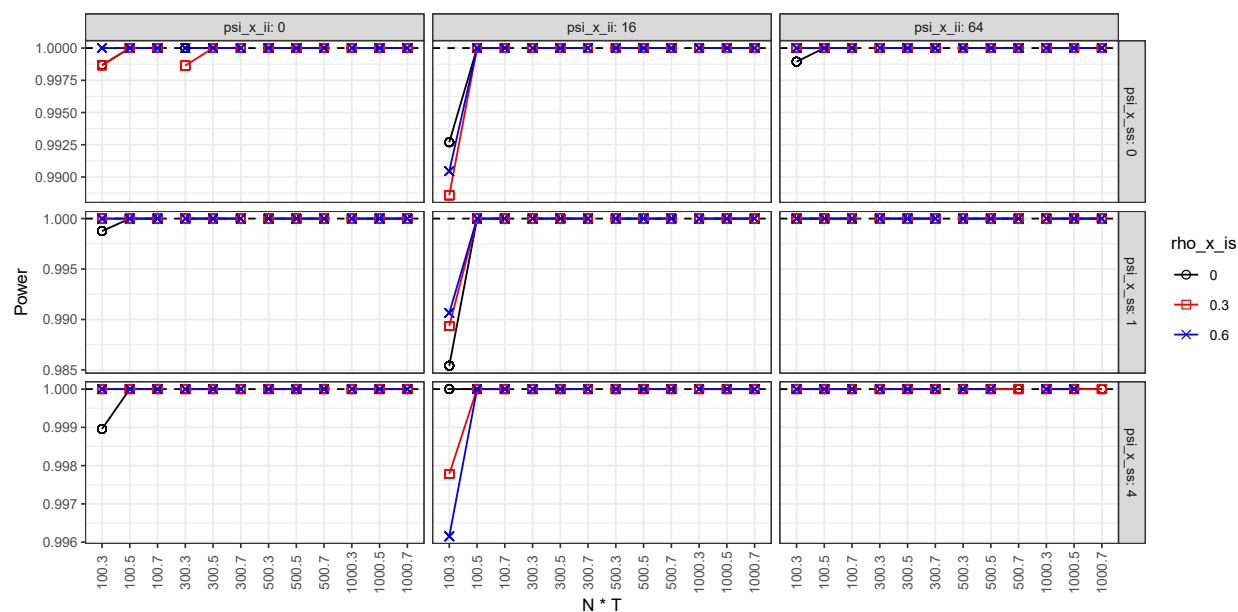
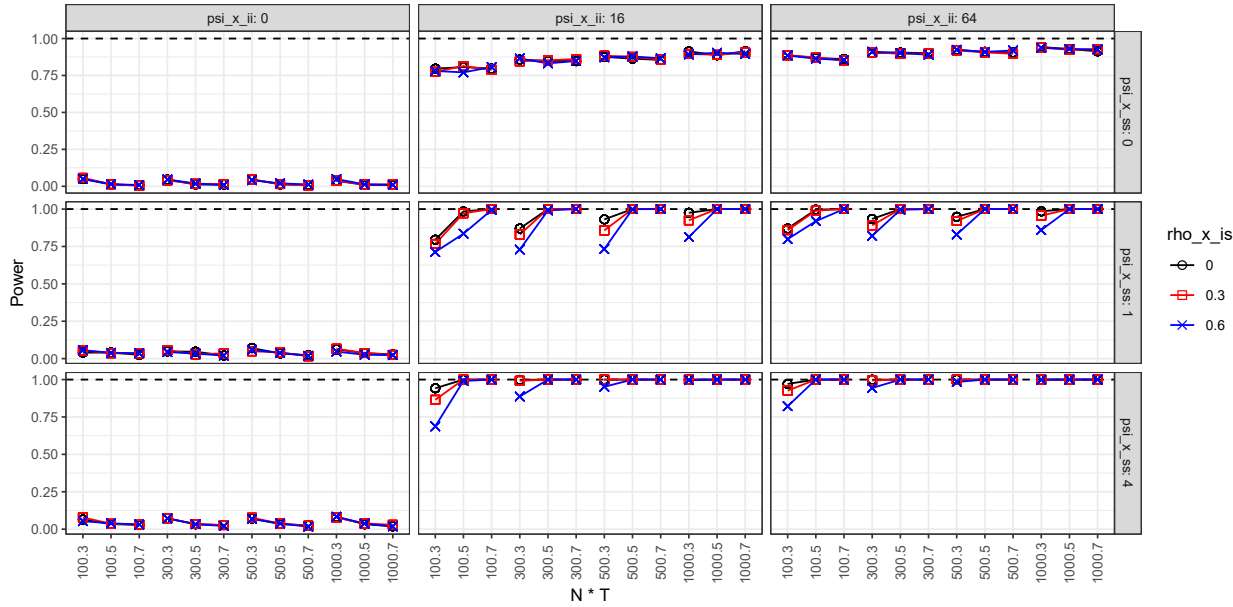
Power for the Between-Person Effect β_{II} in Simulation 5 (LCMx-Rly)

Figure E.86

Power for the Between-Person Effect β_{II} in Simulation 5 (LCMx-CLC)



E.4 Between-Person Effect of Intercept on Slope

Figure E.87

Bias for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S)

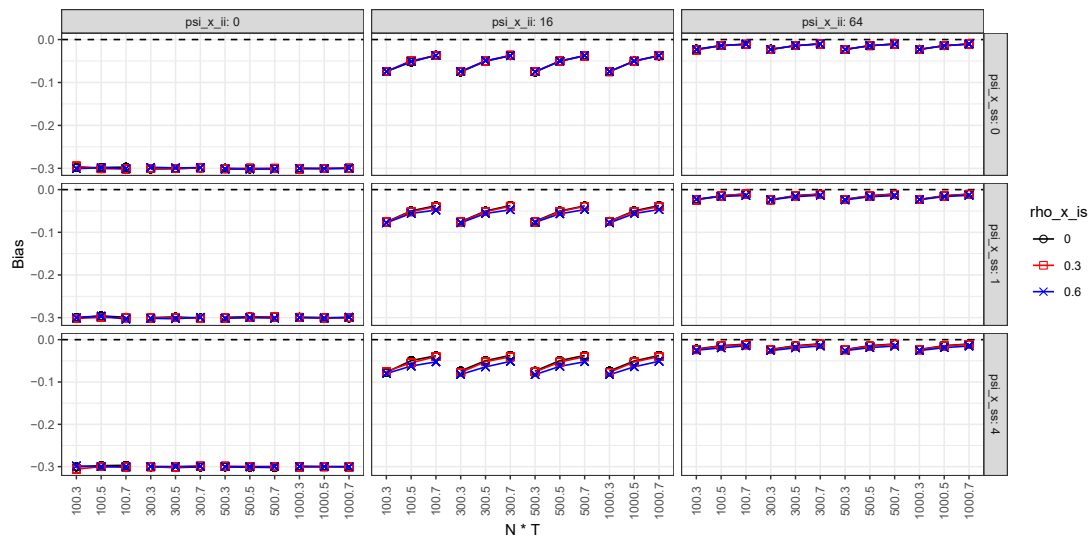


Figure E.88

Bias for the Between-Person Effect β_{SI} in Simulation 5 (MLM-TAC)

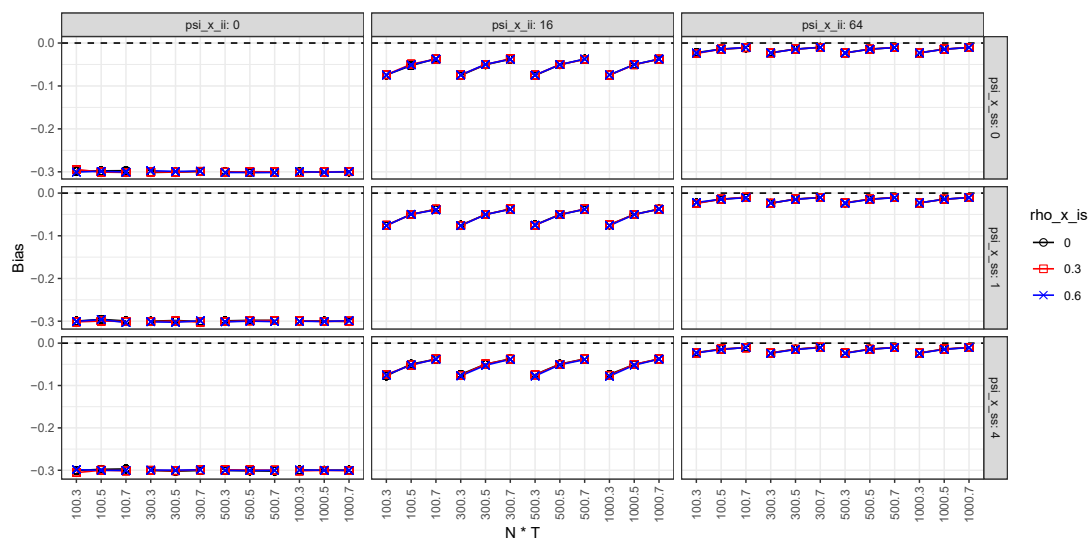


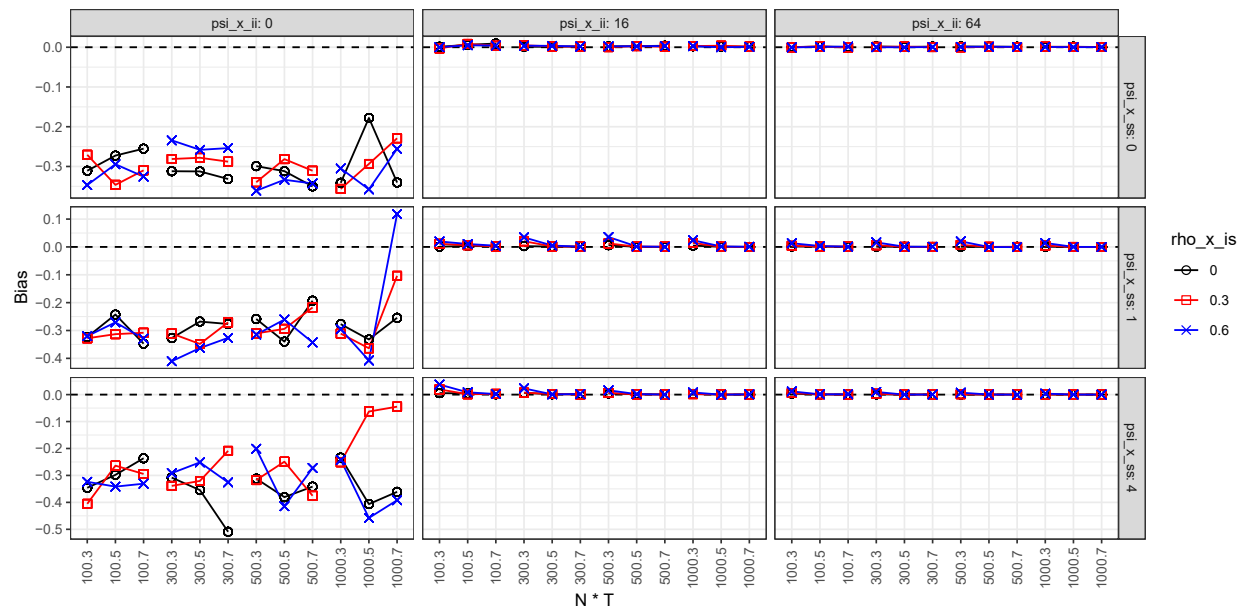
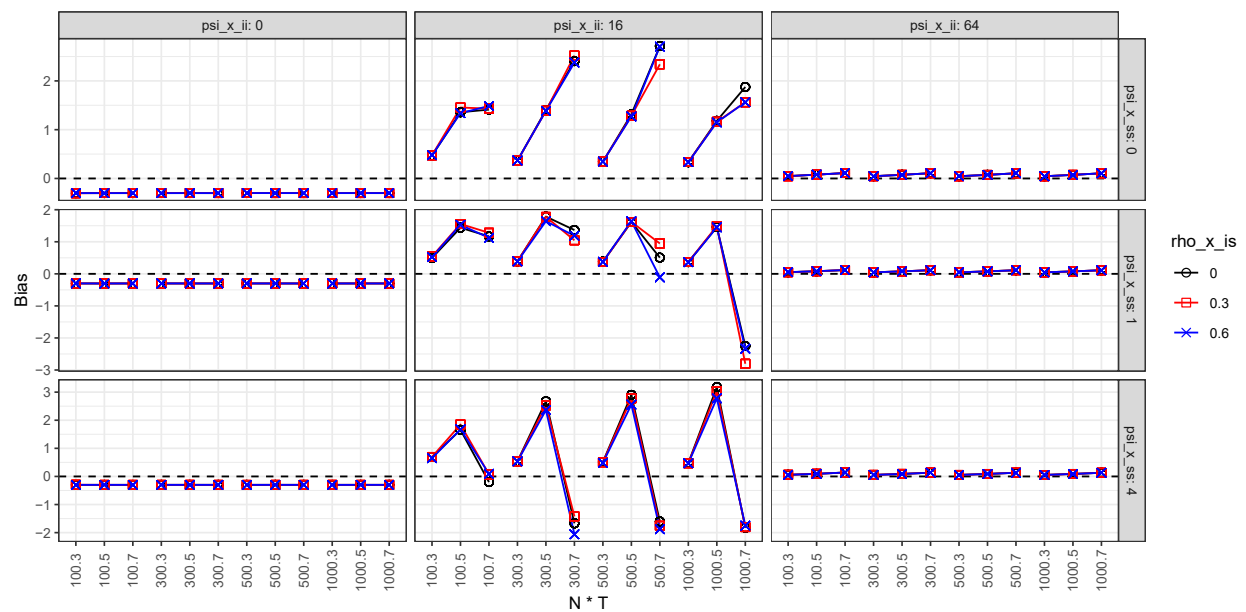
Figure E.89*Bias for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV)***Figure E.90***Bias for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx)*

Figure E.91

Bias for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-CLC)

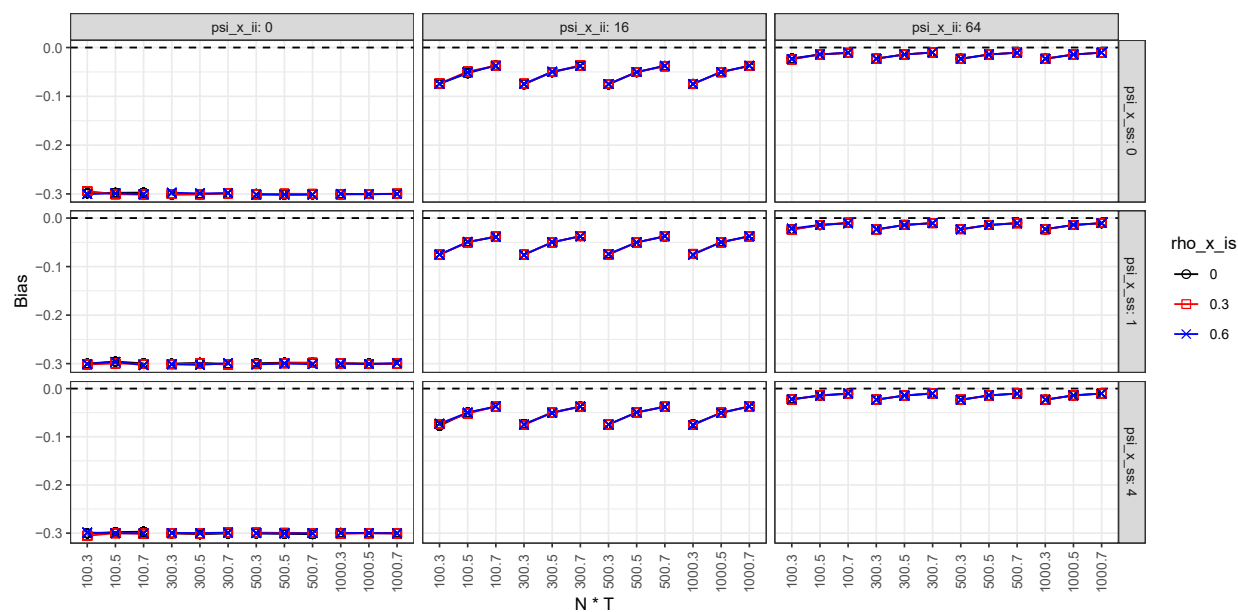


Figure E.92

Bias for the Between-Person Effect β_{SI} in Simulation 5 (LCMxy-RP)

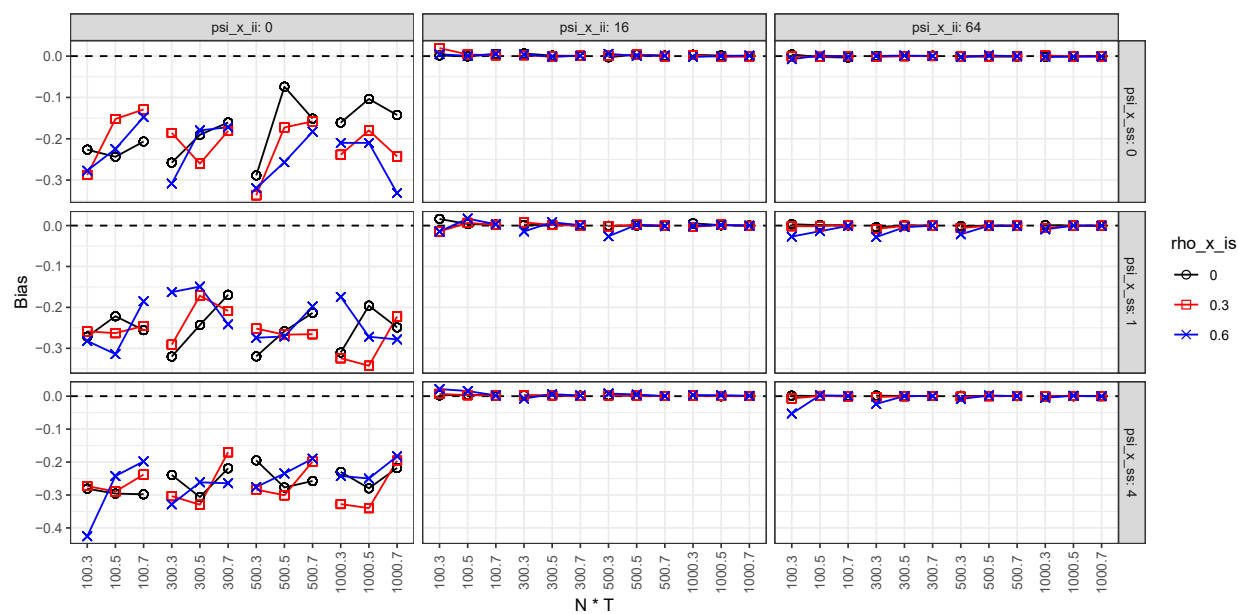


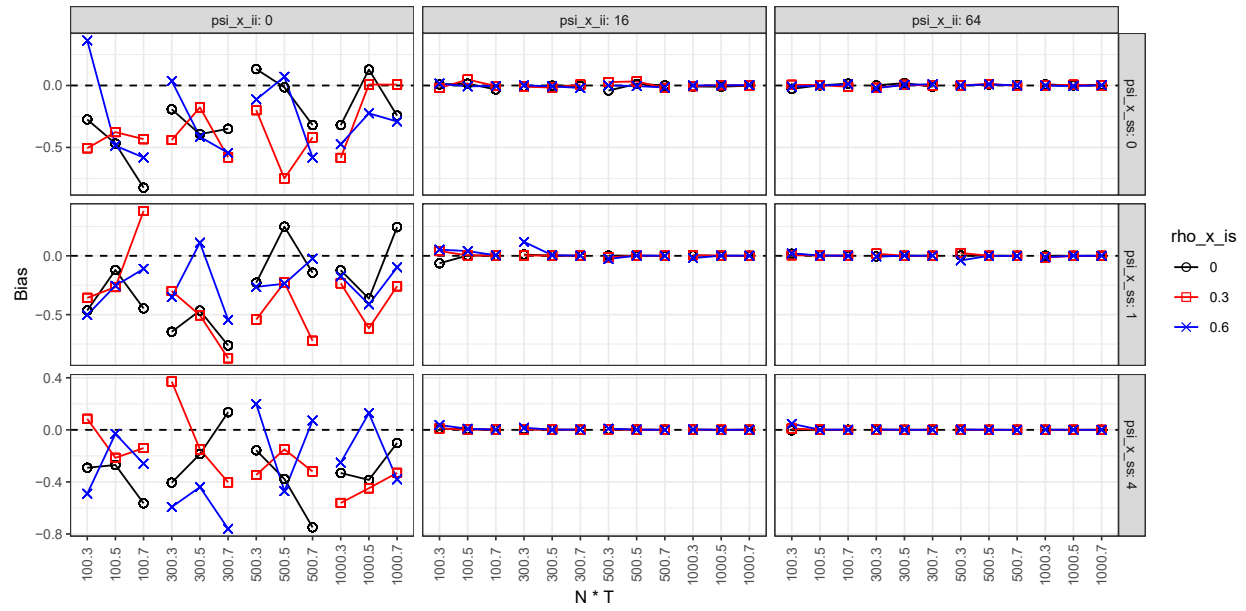
Figure E.93*Bias for the Between-Person Effect β_{SI} in Simulation 5 (LCMx-CLC)*

Figure E.94

Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM-TAC, LCMMy-CLC)

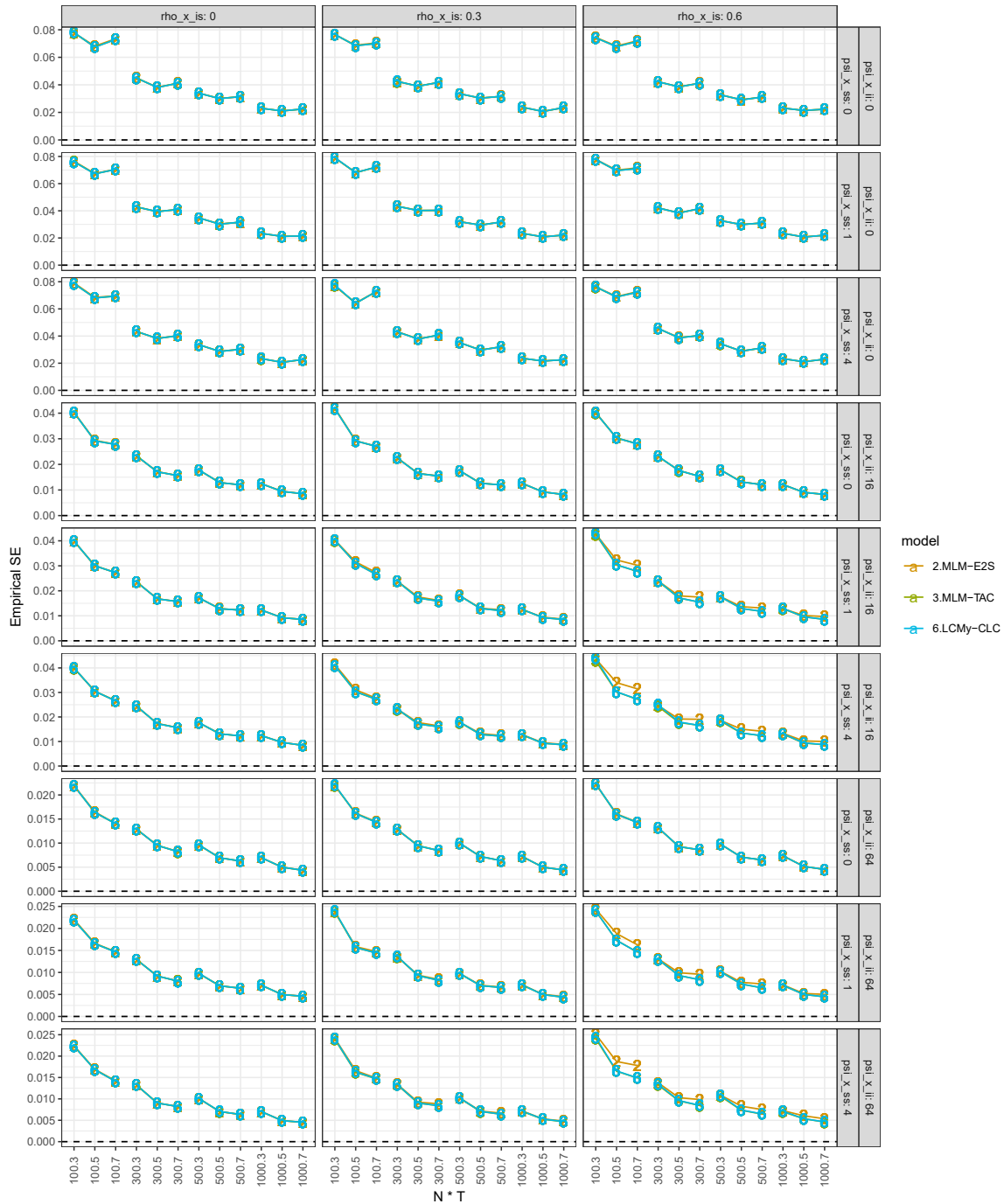


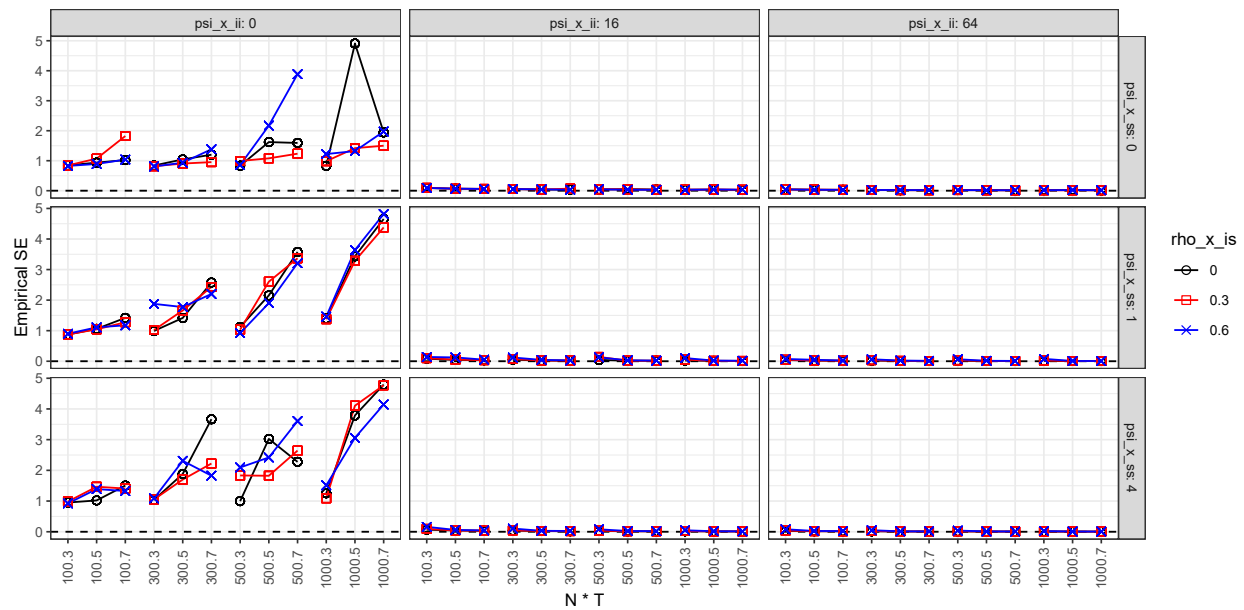
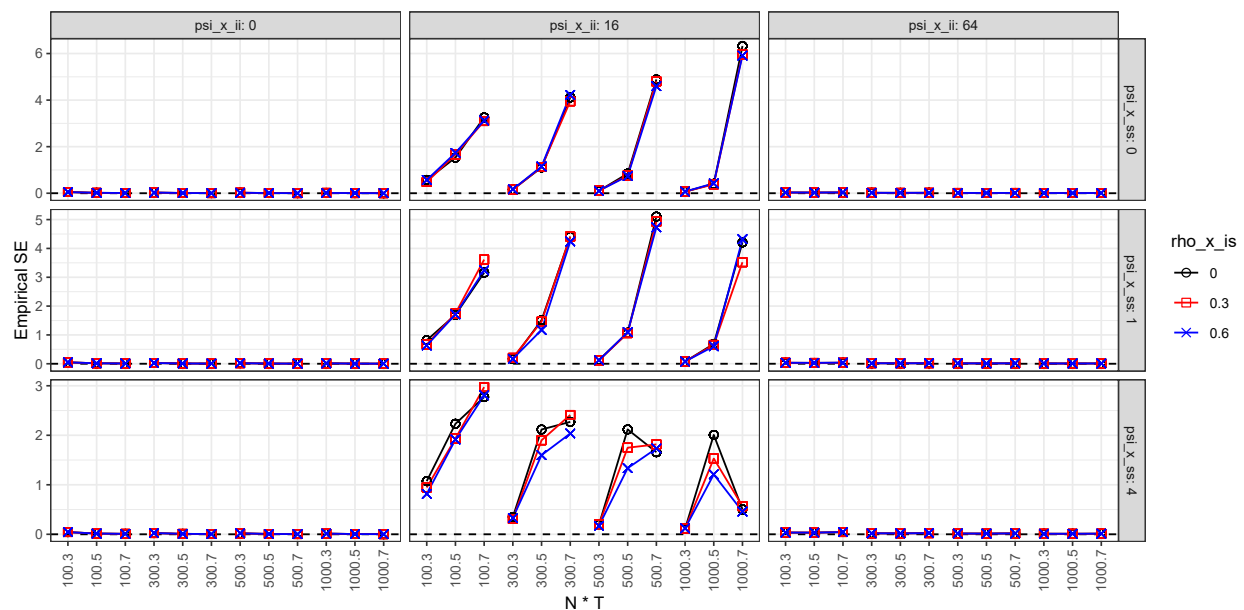
Figure E.95*Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV)***Figure E.96***Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx)*

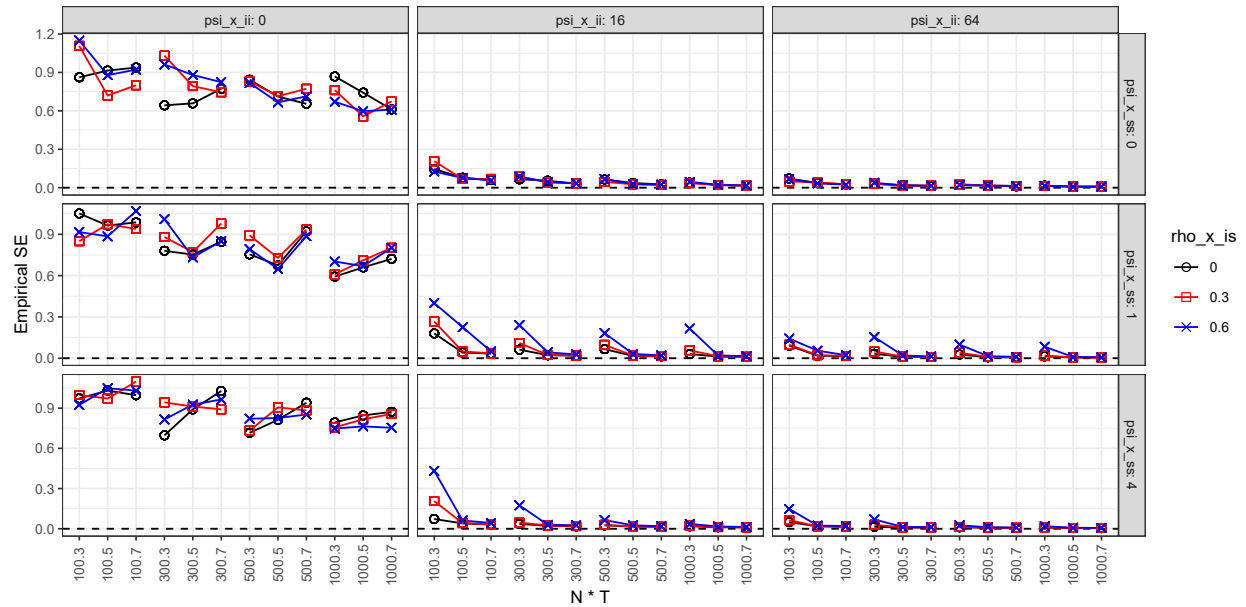
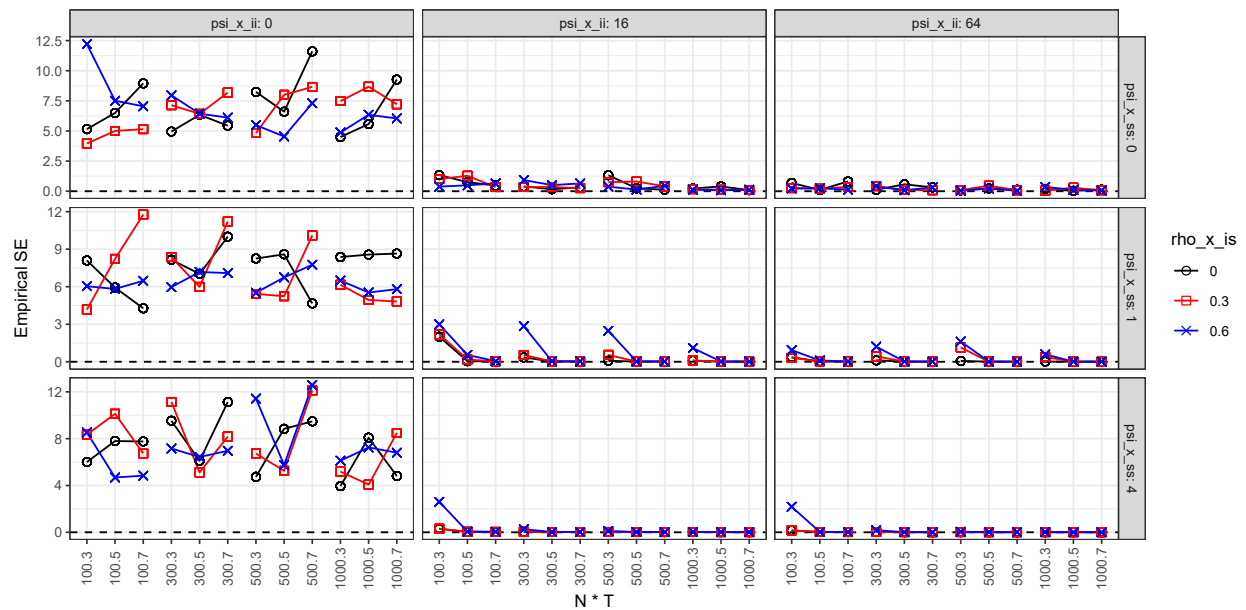
Figure E.97*Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMxy-RP)***Figure E.98***Empirical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMx-CLC)*

Figure E.99

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM-TAC, LCMMy-CLC)

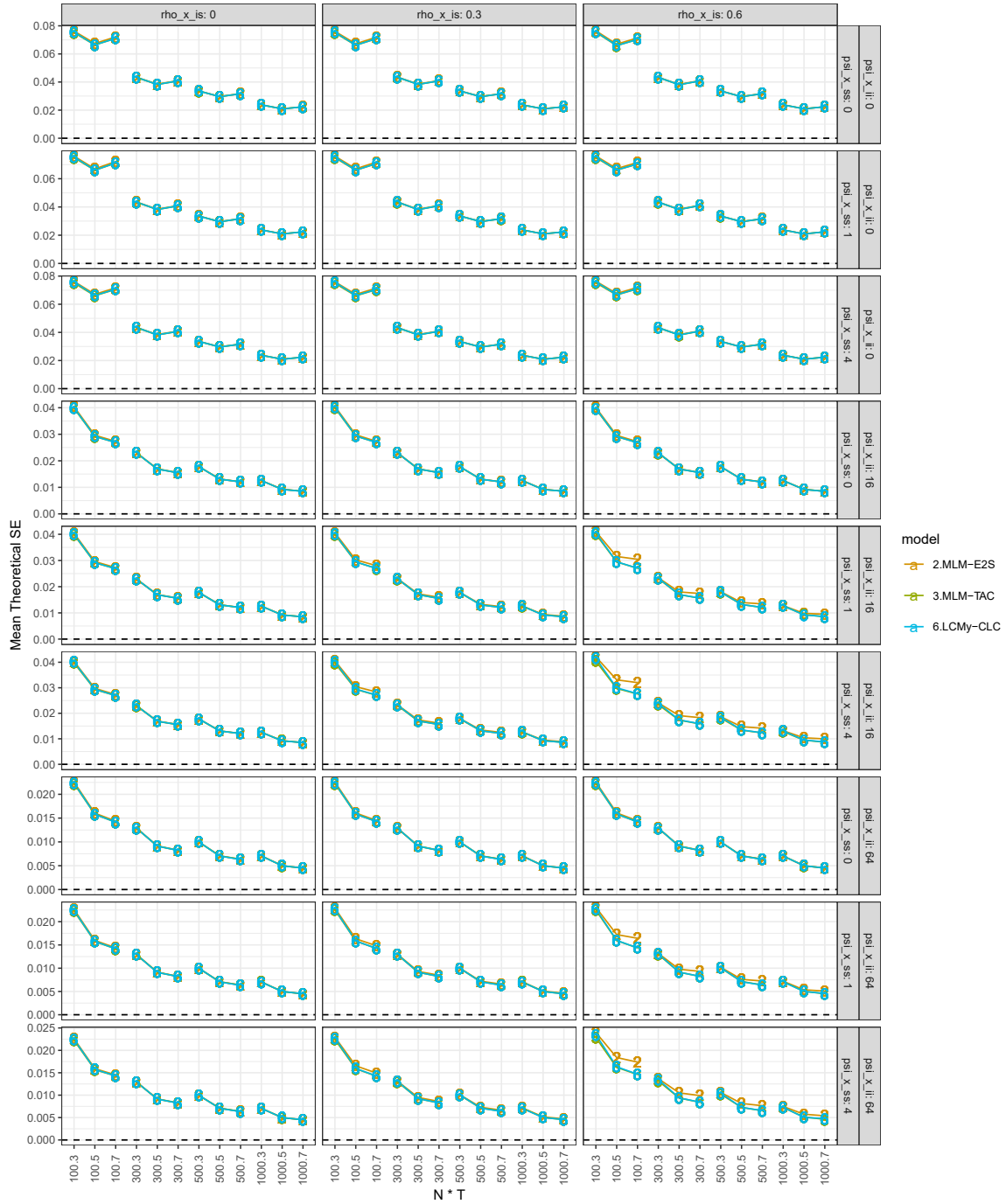


Figure E.100

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV)

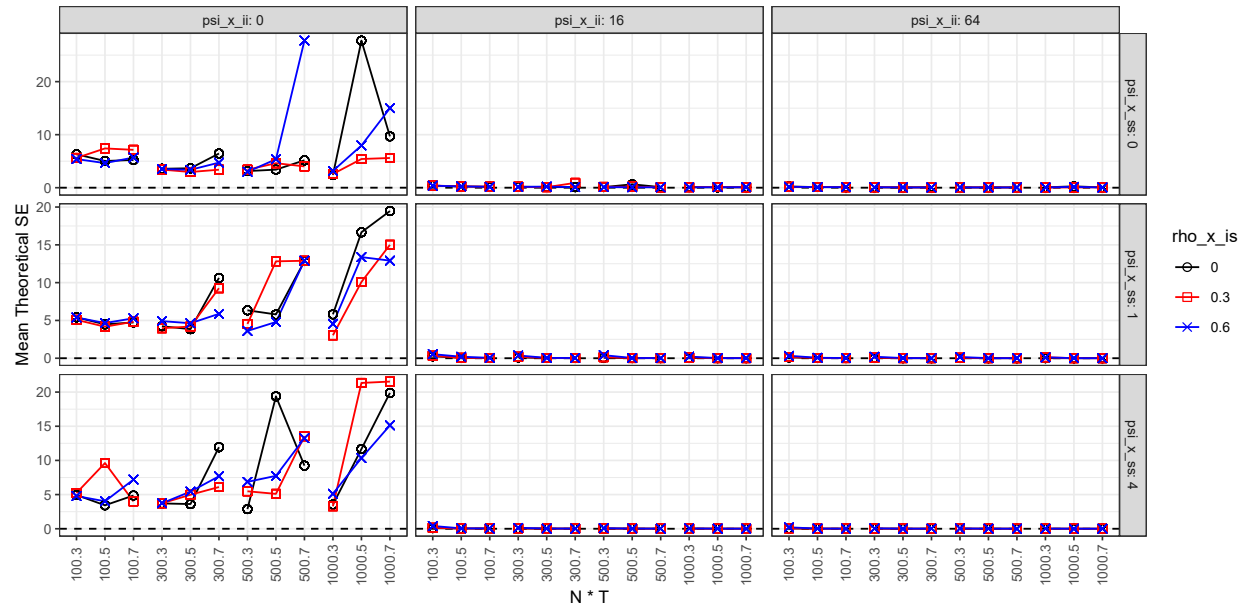


Figure E.101

Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx)

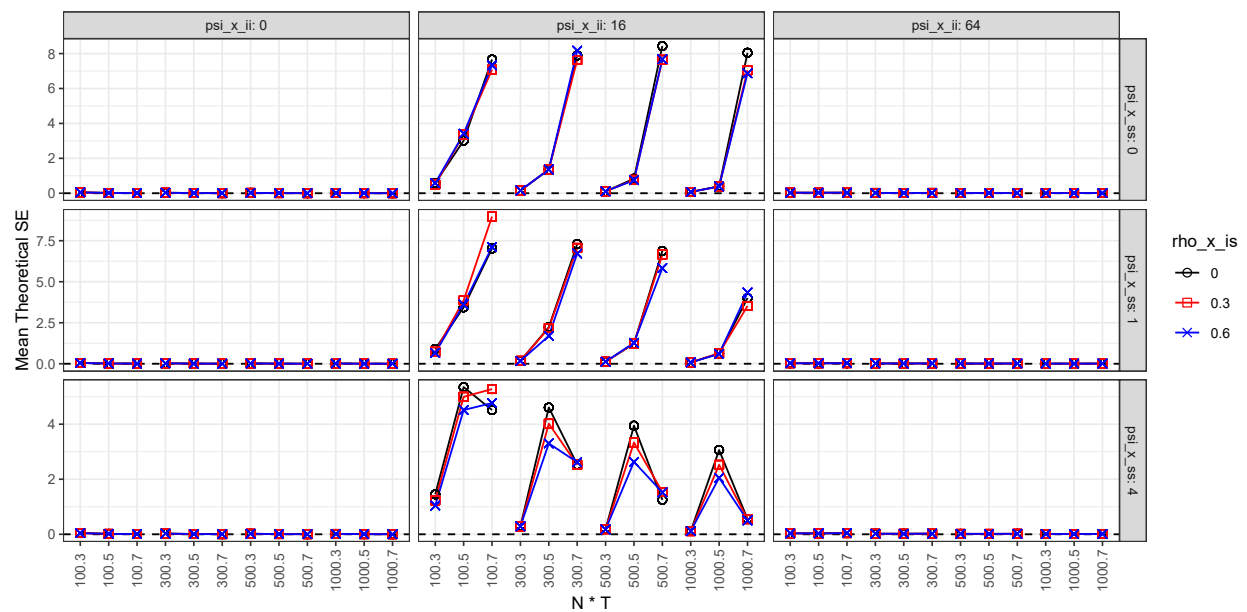


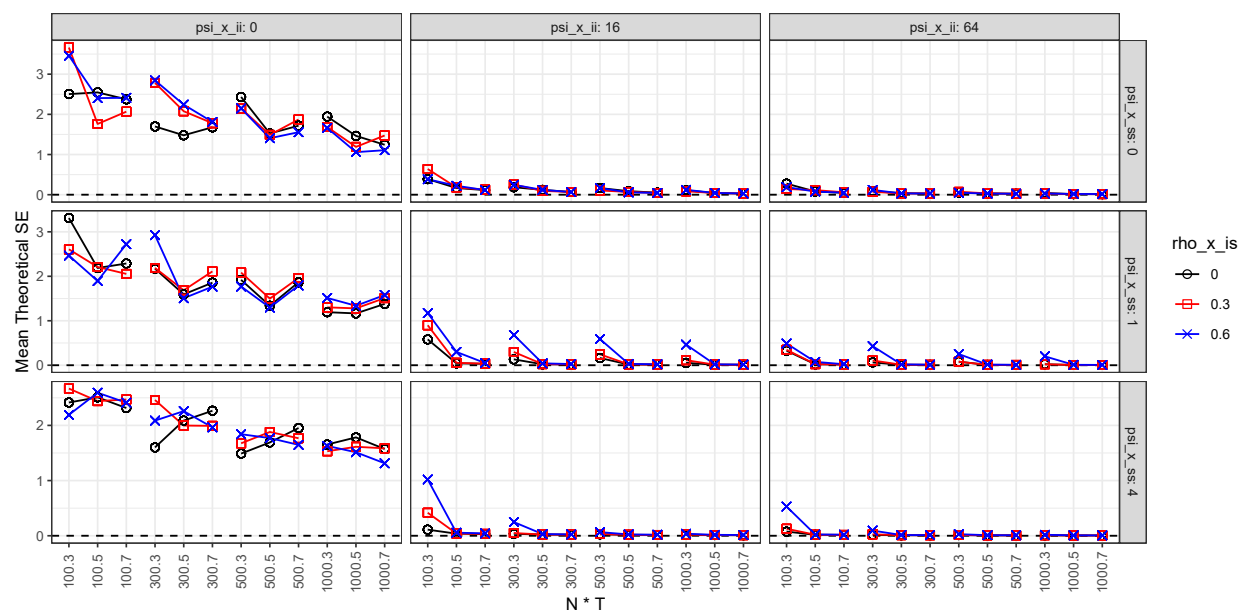
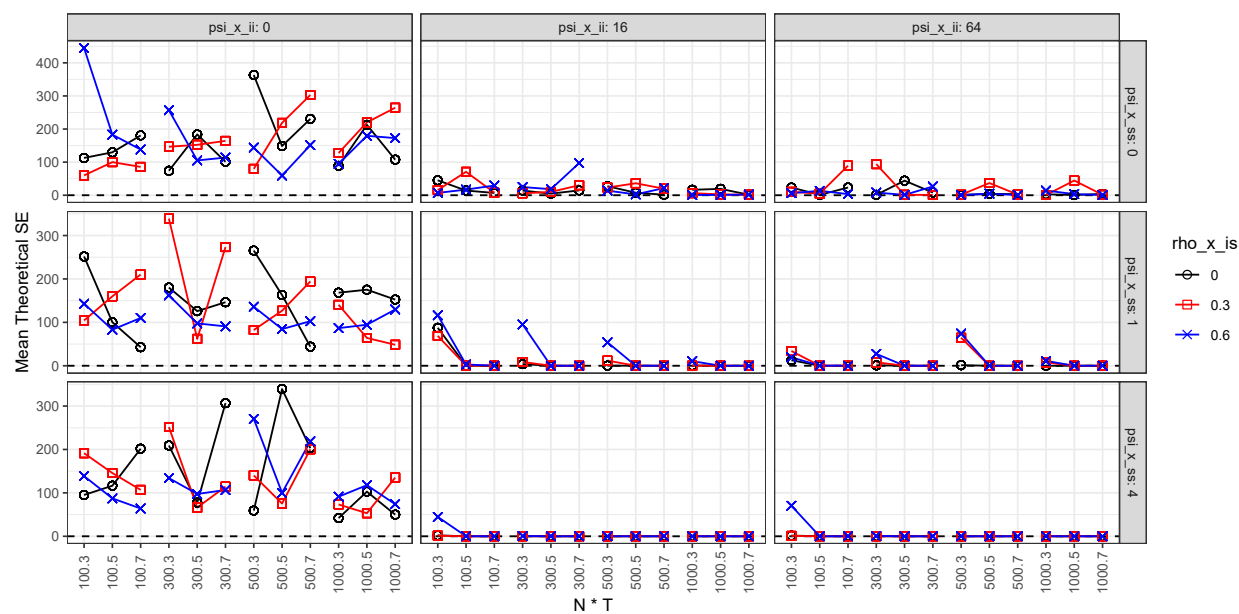
Figure E.102*Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCM_{xy}-RP)***Figure E.103***Theoretical SE for the Between-Person Effect β_{SI} in Simulation 5 (LCM_x-CLC)*

Figure E.104

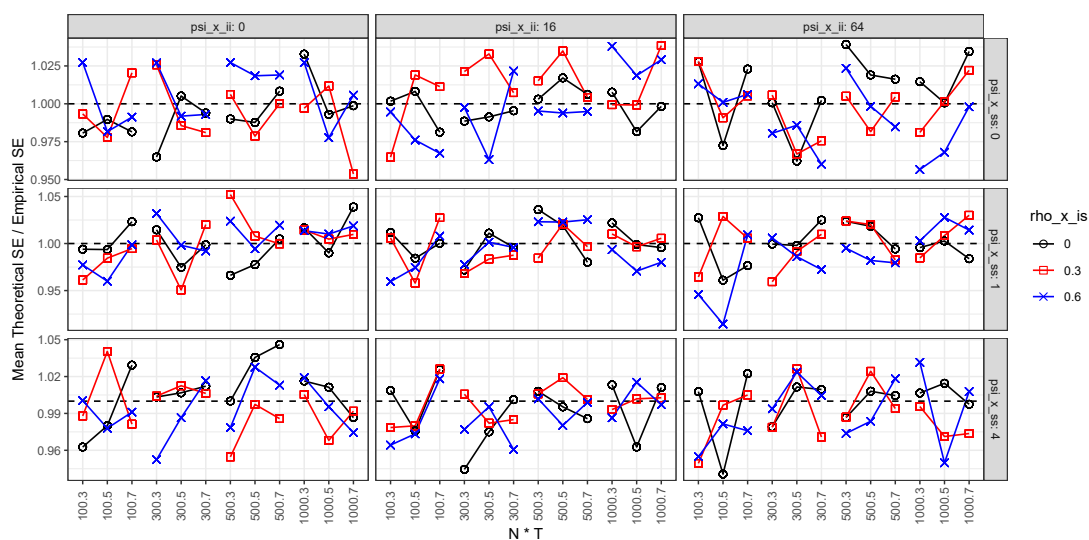
SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S)

Figure E.105

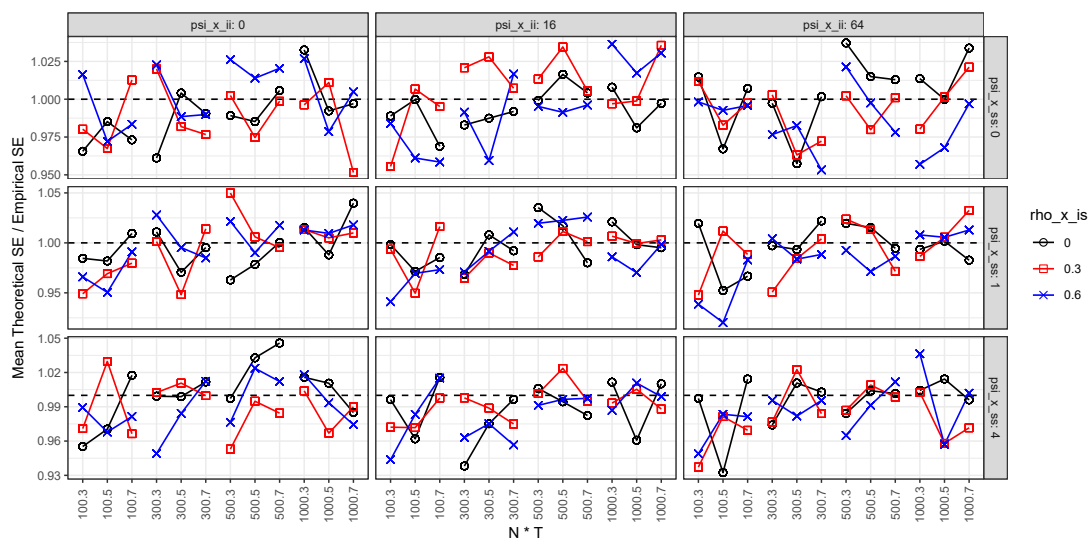
SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (MLM-TAC)

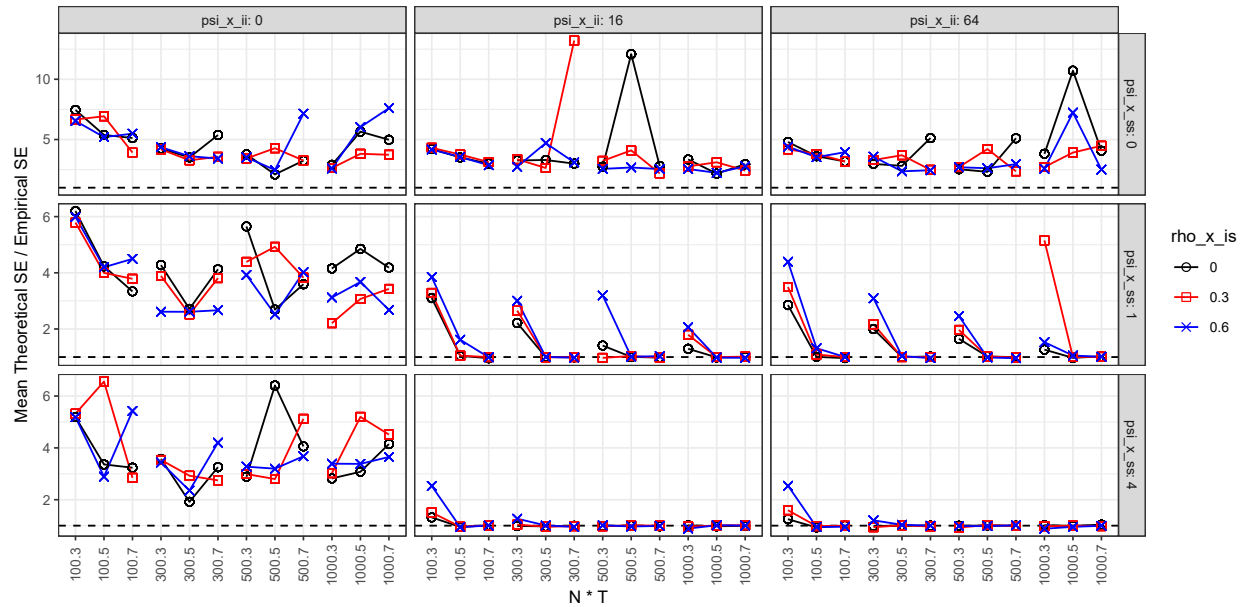
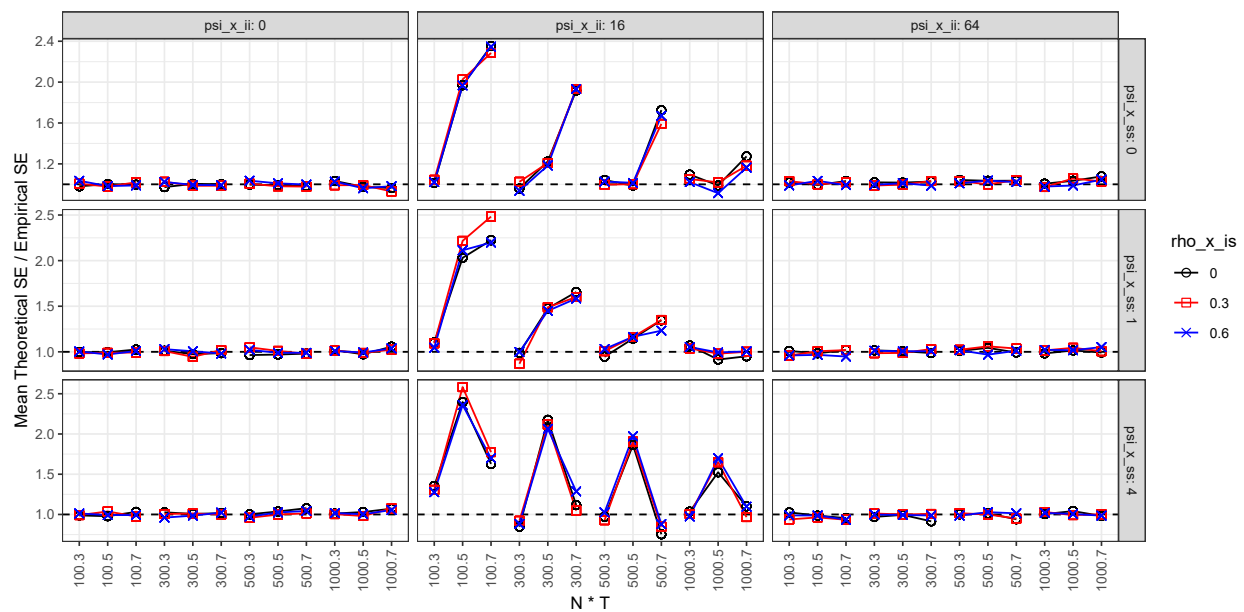
Figure E.106*SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV)***Figure E.107***SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-Rlx)*

Figure E.108

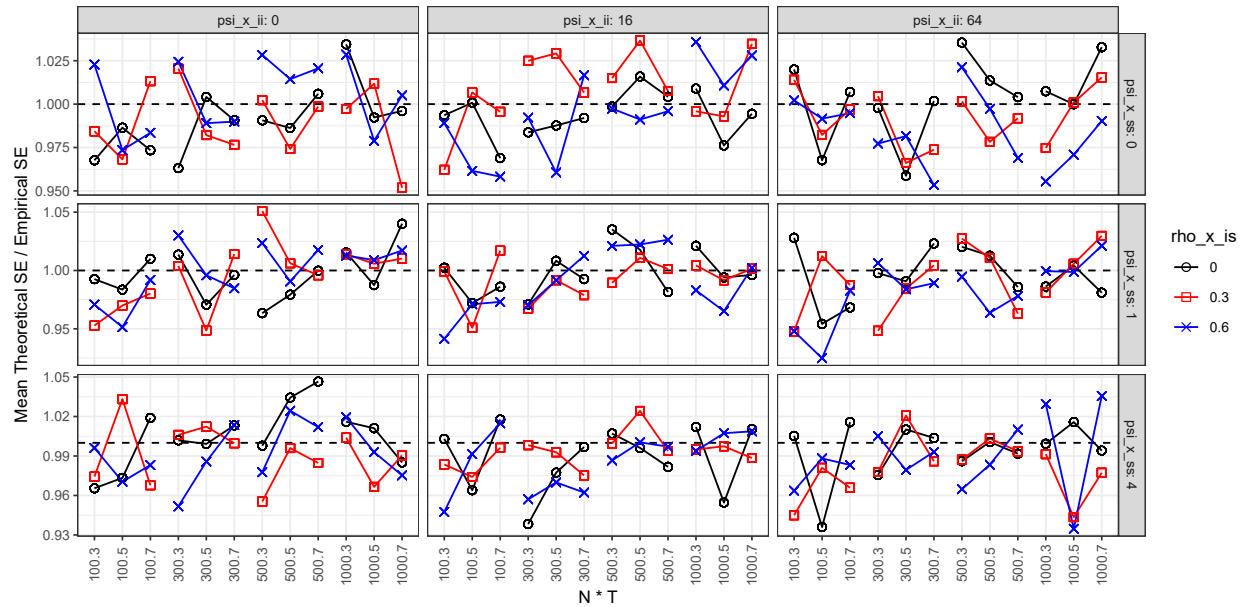
SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-CLC)

Figure E.109

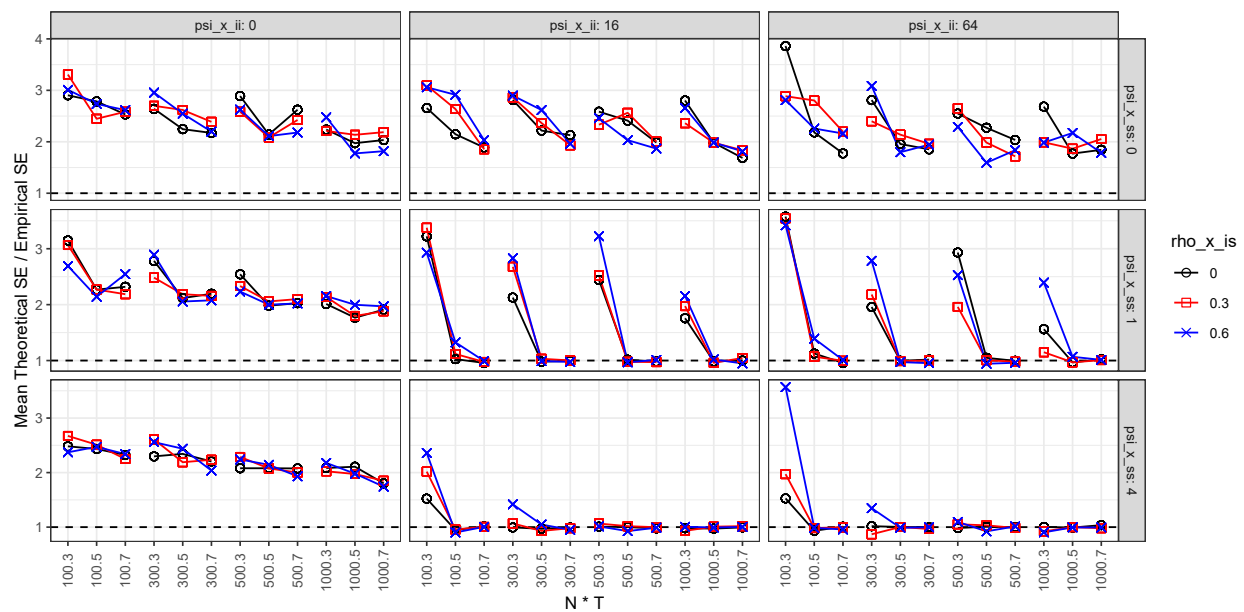
SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (LCMxy-RP)

Figure E.110

SE Ratio for the Between-Person Effect β_{SI} in Simulation 5 (LCM_X-CLC)

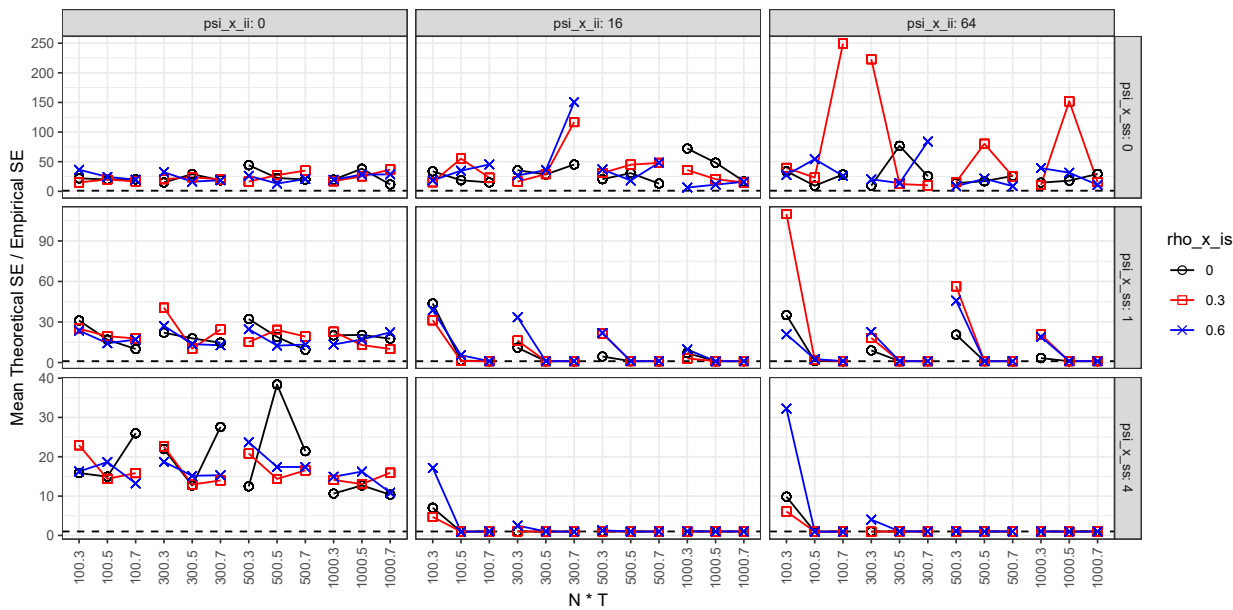


Figure E.111

RMSE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM-TAC, LCMY-CLC)

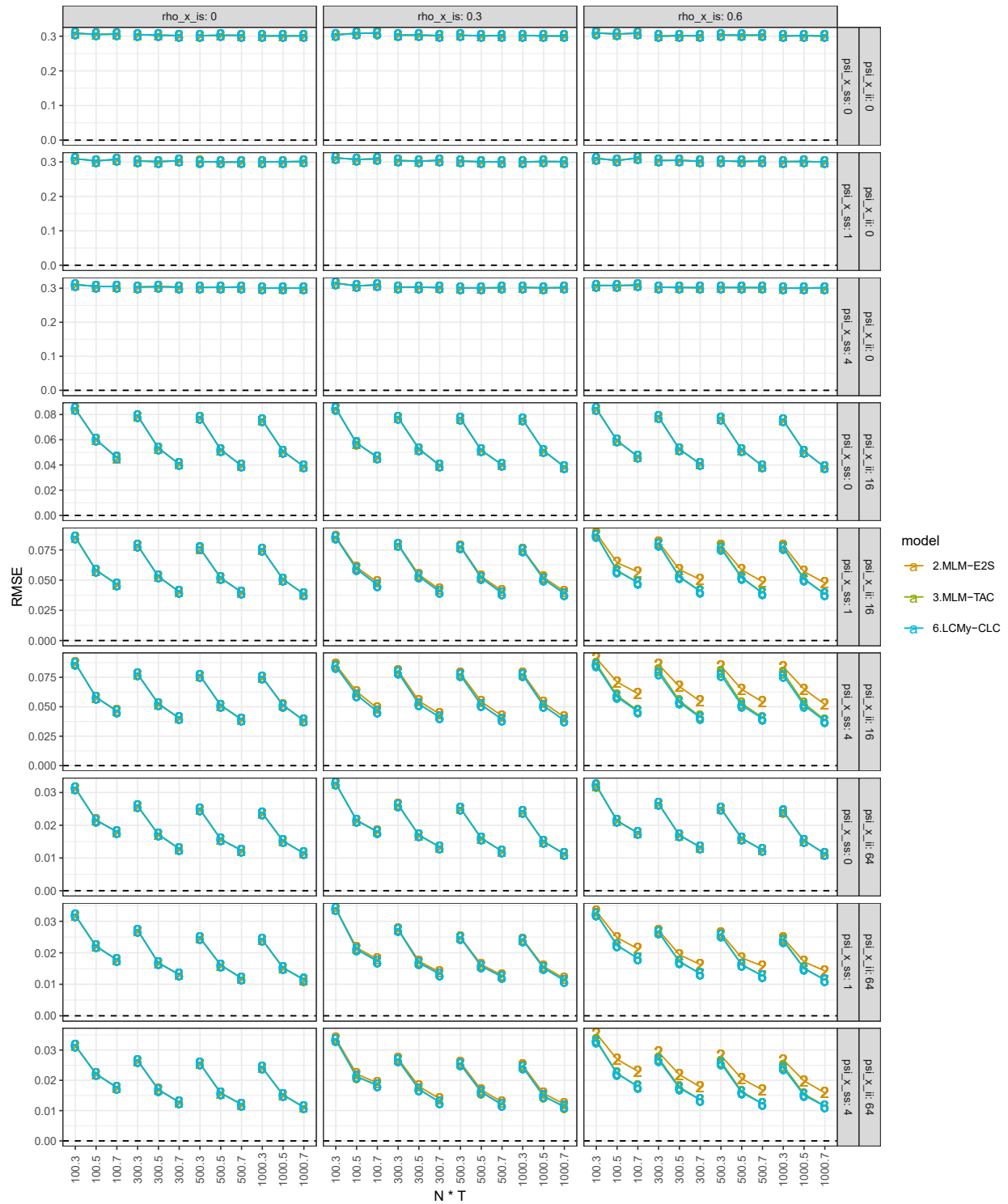


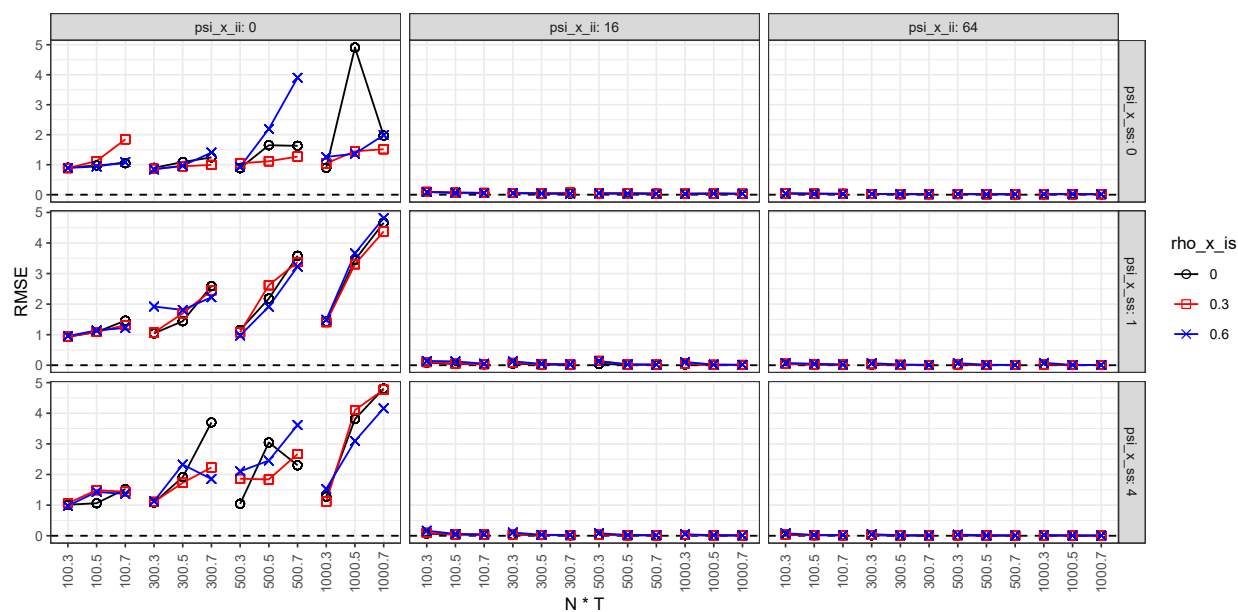
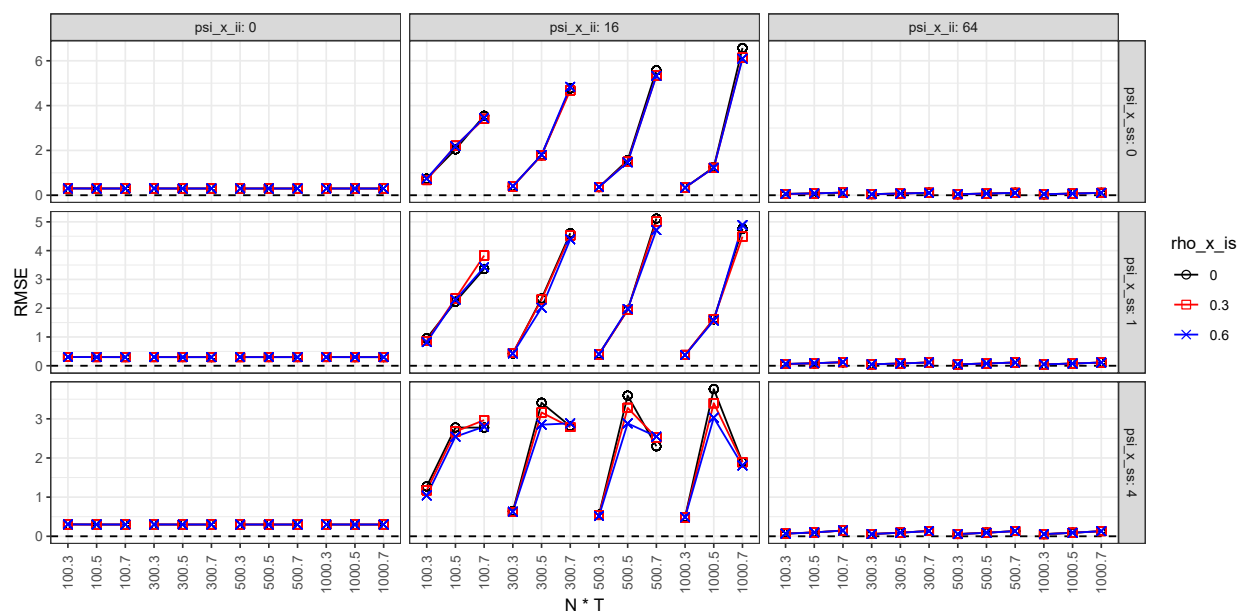
Figure E.112*RMSE for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV)***Figure E.113***RMSE for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-Rlx)*

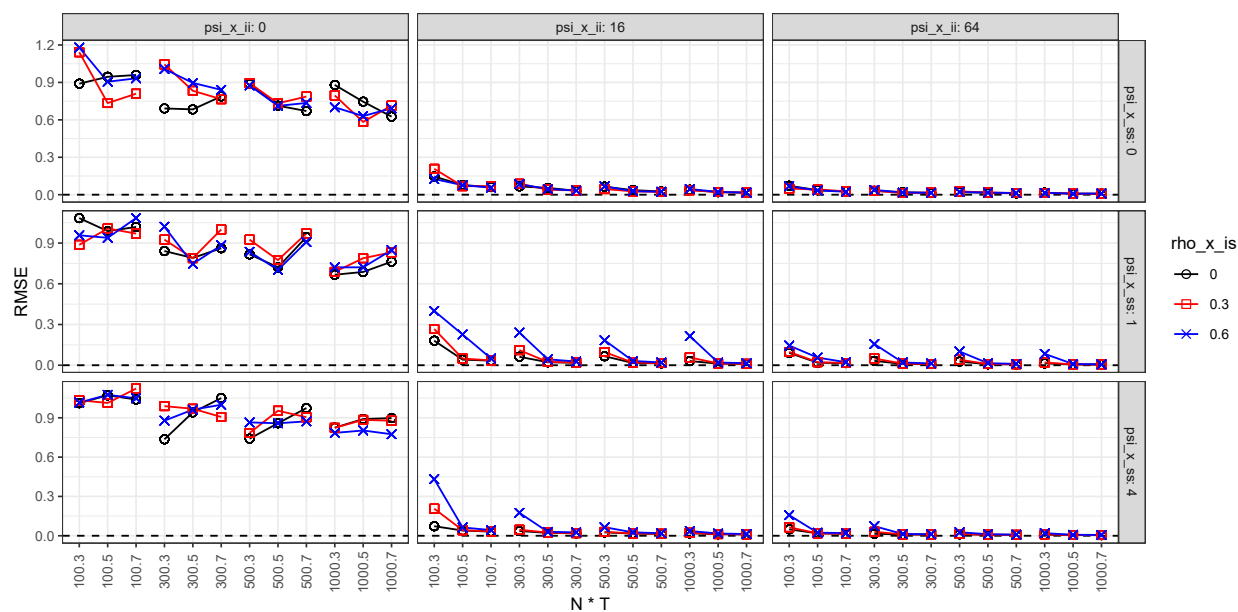
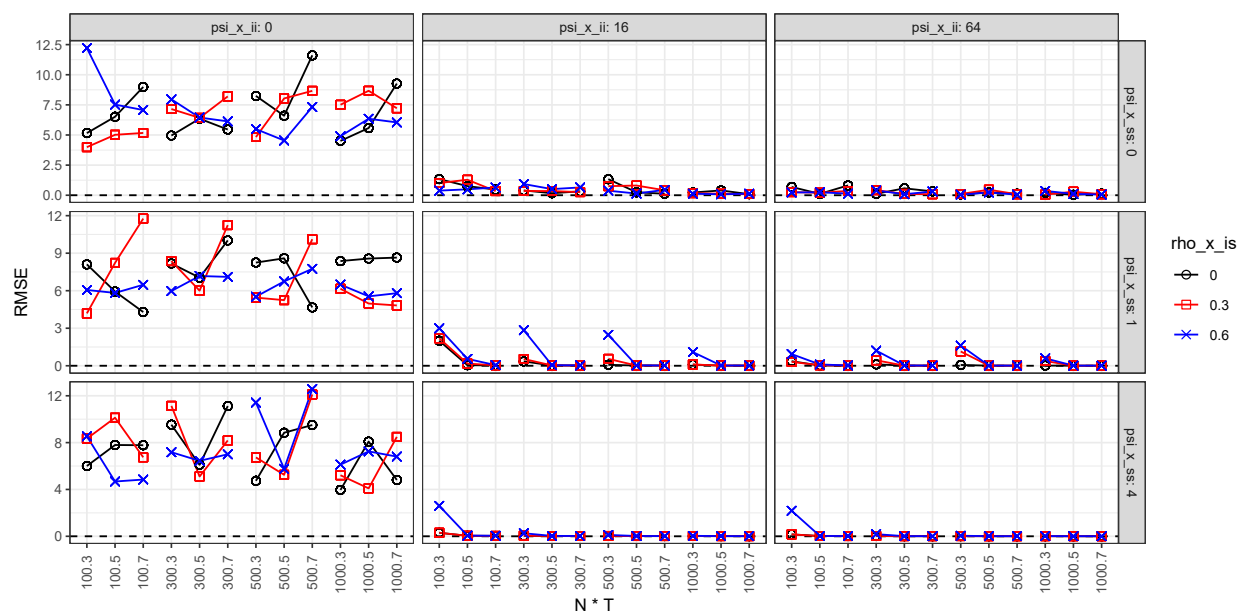
Figure E.114*RMSE for the Between-Person Effect β_{SI} in Simulation 5 (LCM_{xy}-RP)***Figure E.115***RMSE for the Between-Person Effect β_{SI} in Simulation 5 (LCM_x-CLC)*

Figure E.116

Coverage Rate for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM-TAC, LCMMy-CLC)

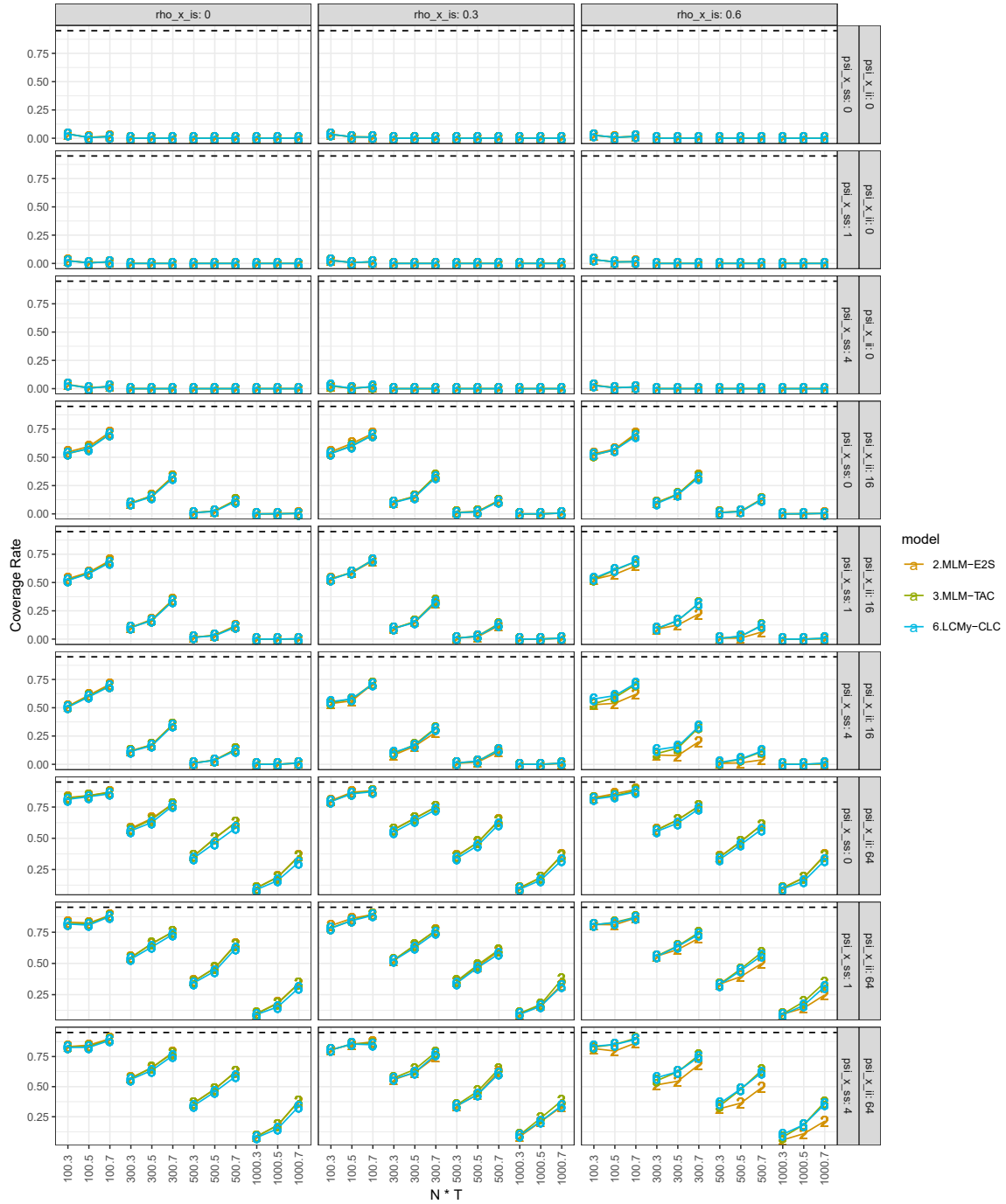


Figure E.117

Coverage Rate for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV, LCMxy-RP, LCMx-CLC)

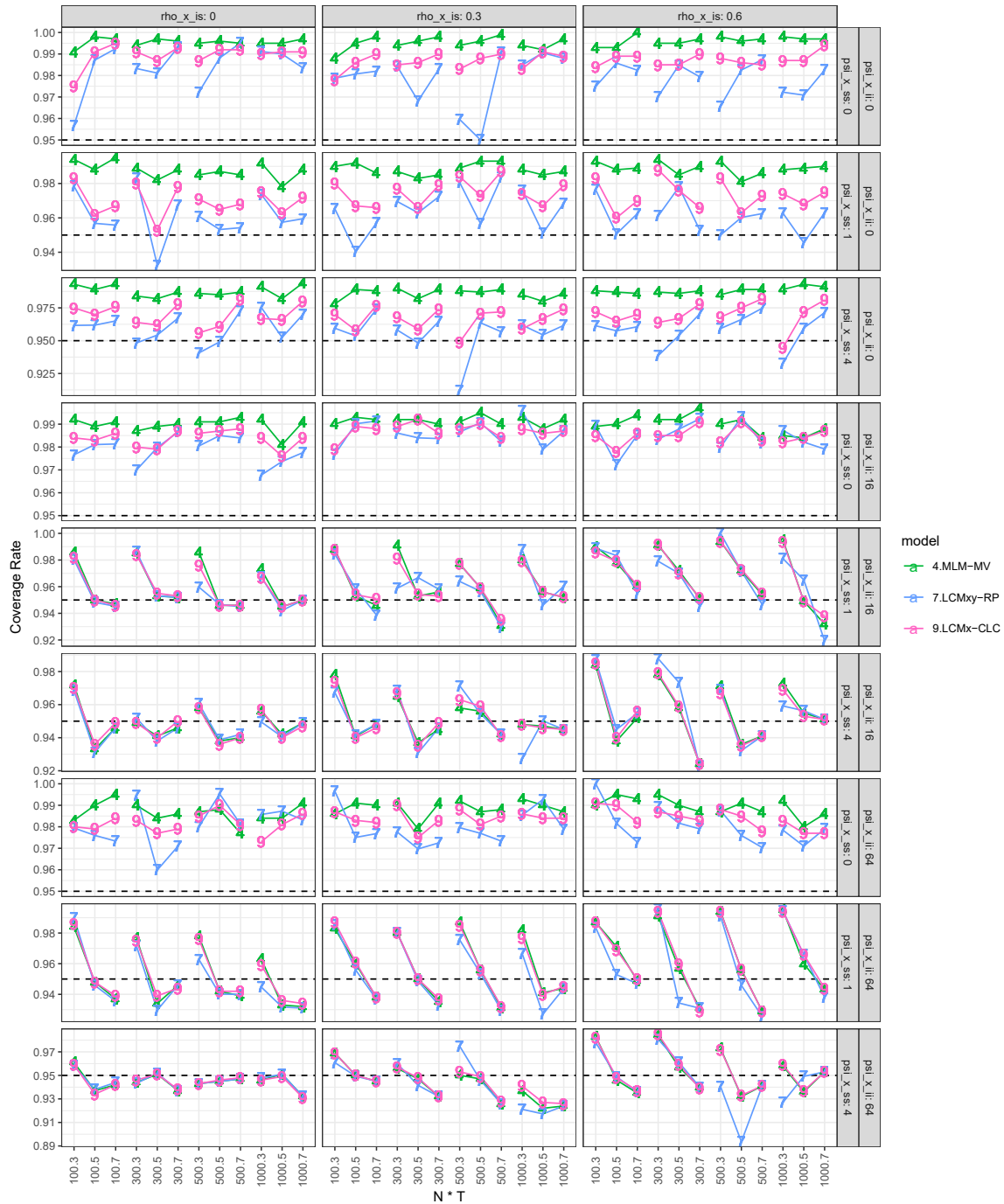


Figure E.118
Coverage Rate for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-RIx)

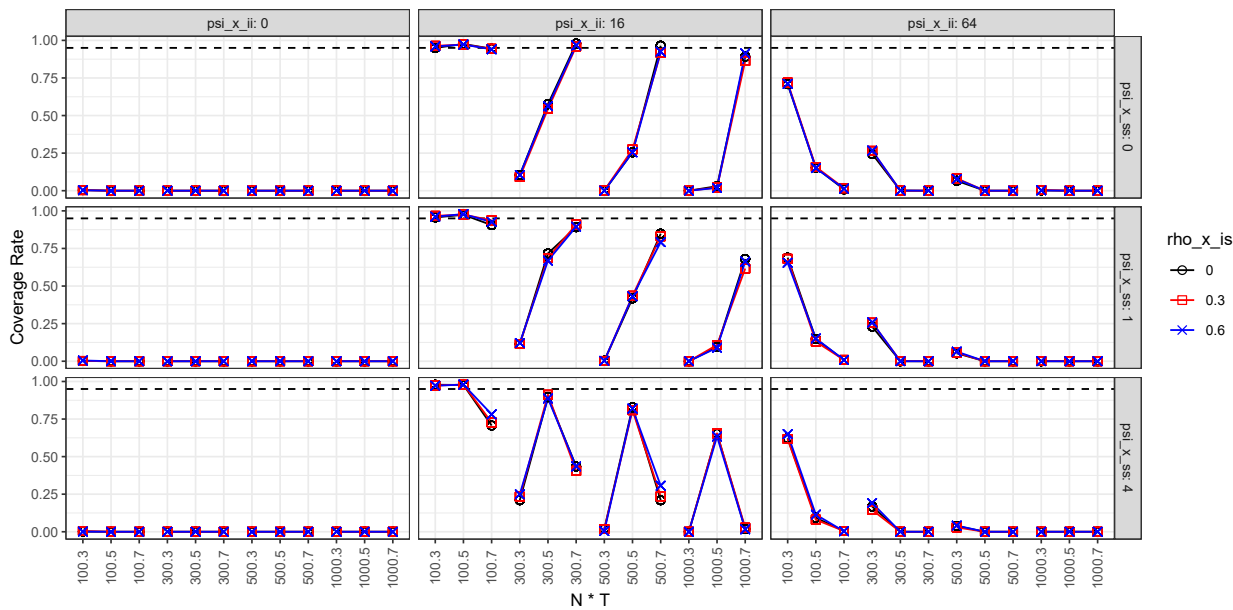


Figure E.119

Power for the Between-Person Effect β_{SI} in Simulation 5 (MLM-E2S, MLM-TAC, LCMy-CLC)

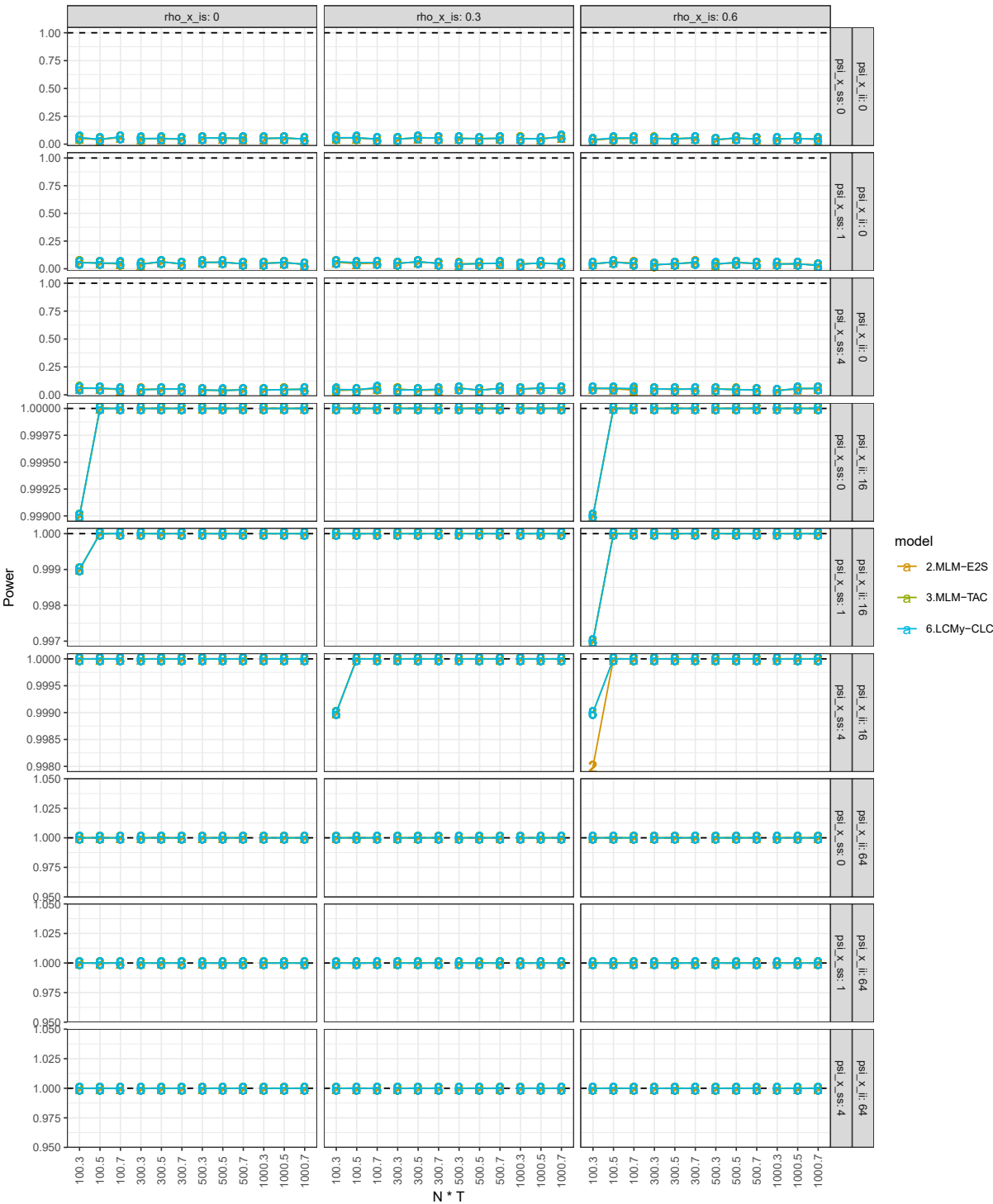
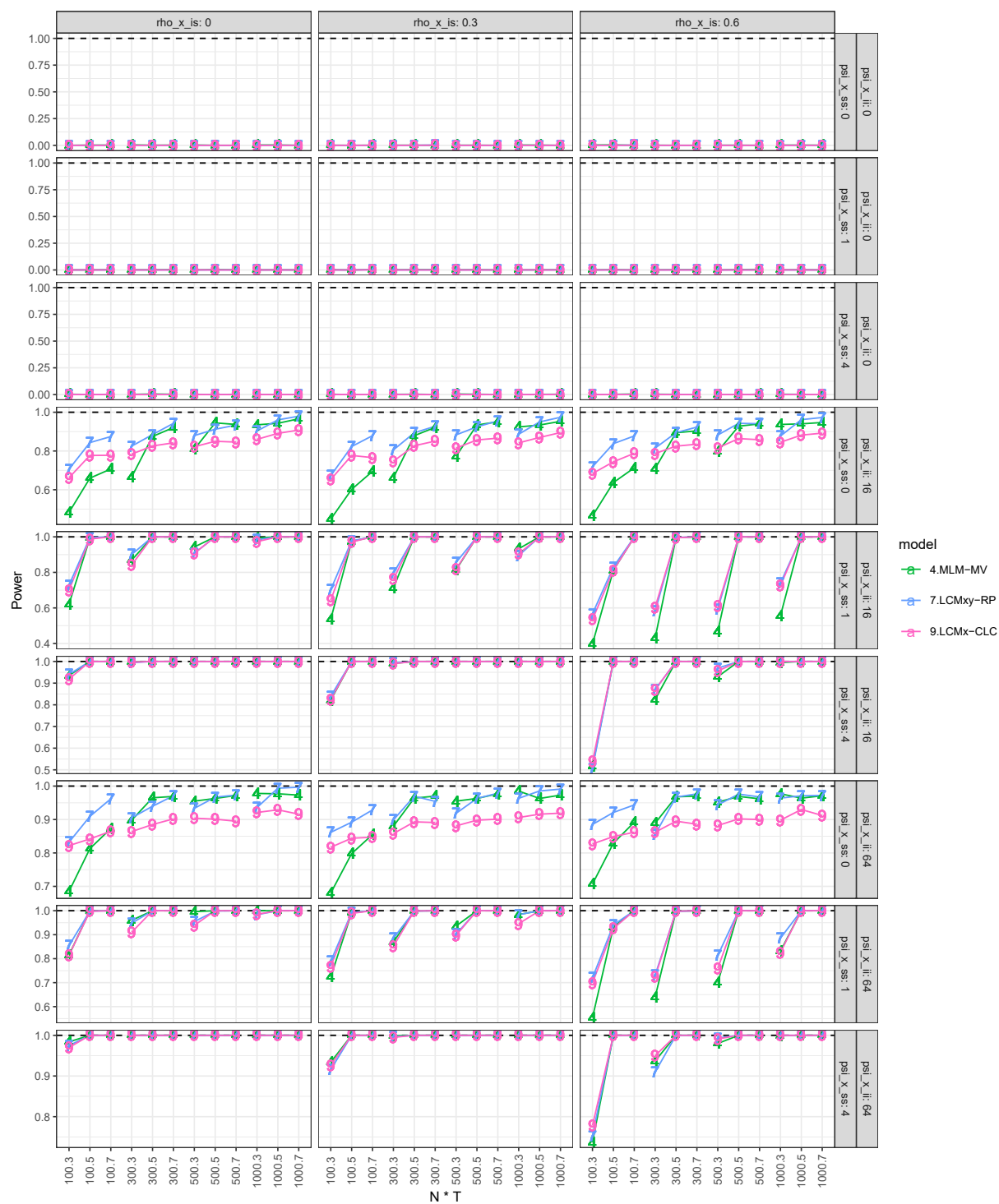
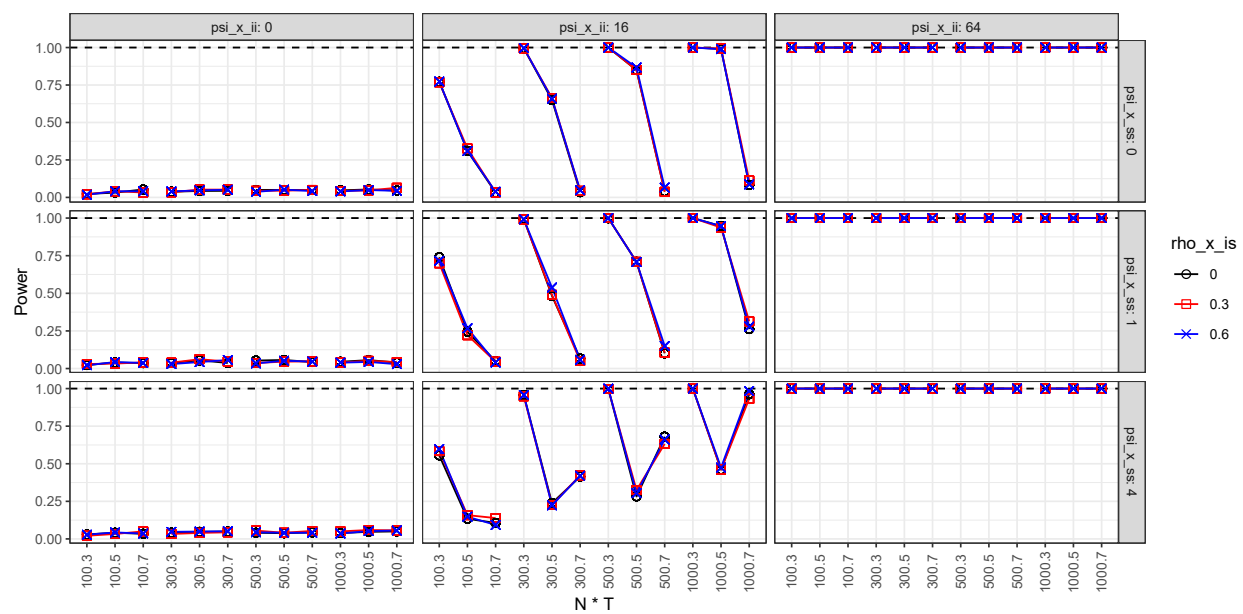


Figure E.120

Power for the Between-Person Effect β_{SI} in Simulation 5 (MLM-MV, LCM_{xy}-RP, LCM_x-CLC)



Power for the Between-Person Effect β_{SI} in Simulation 5 (LCMy-Rlx)



E.5 Between-Person Effect of Slope on Intercept

Figure E.122

Bias for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)

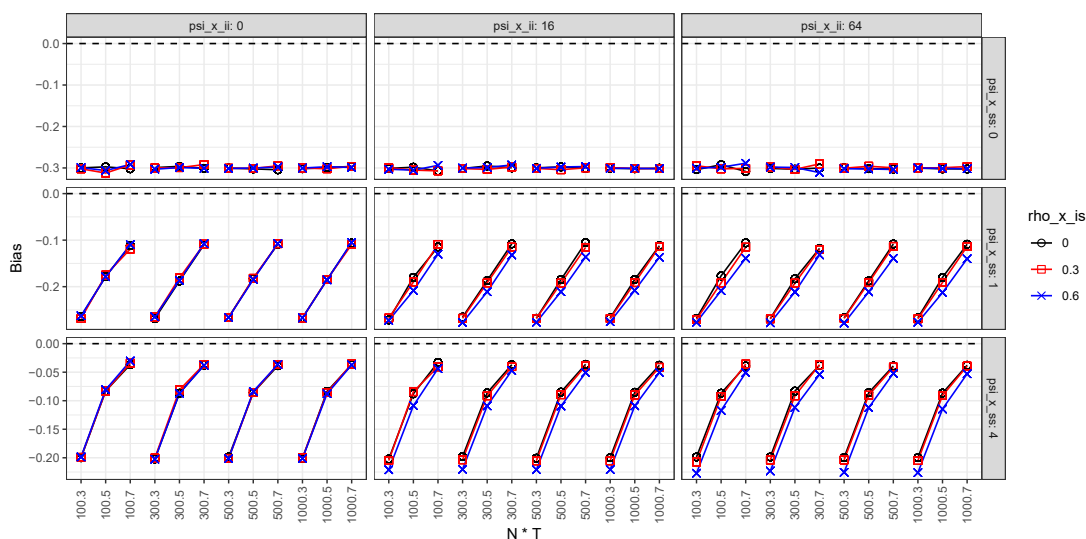


Figure E.123

Bias for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)

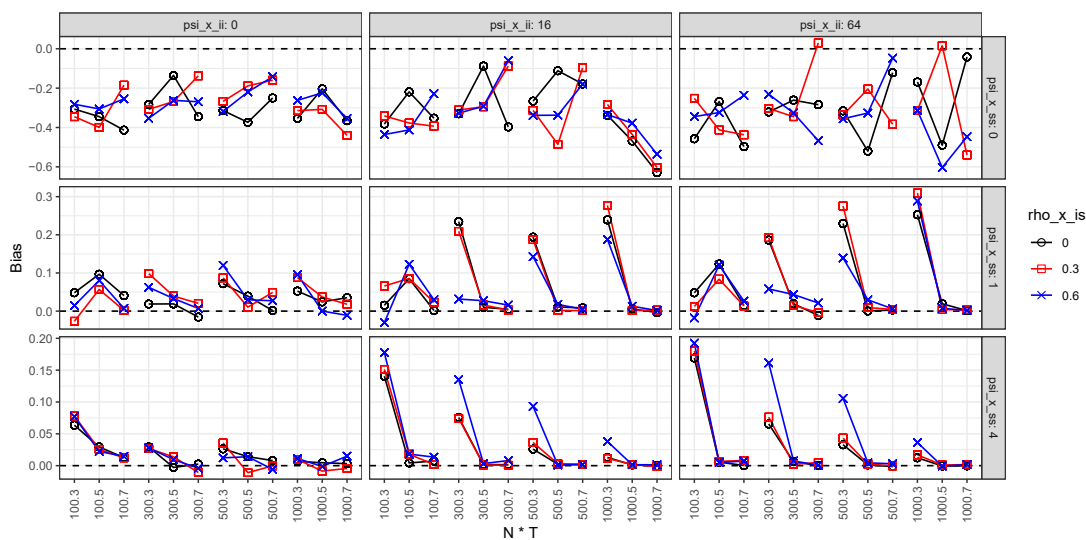


Figure E.124

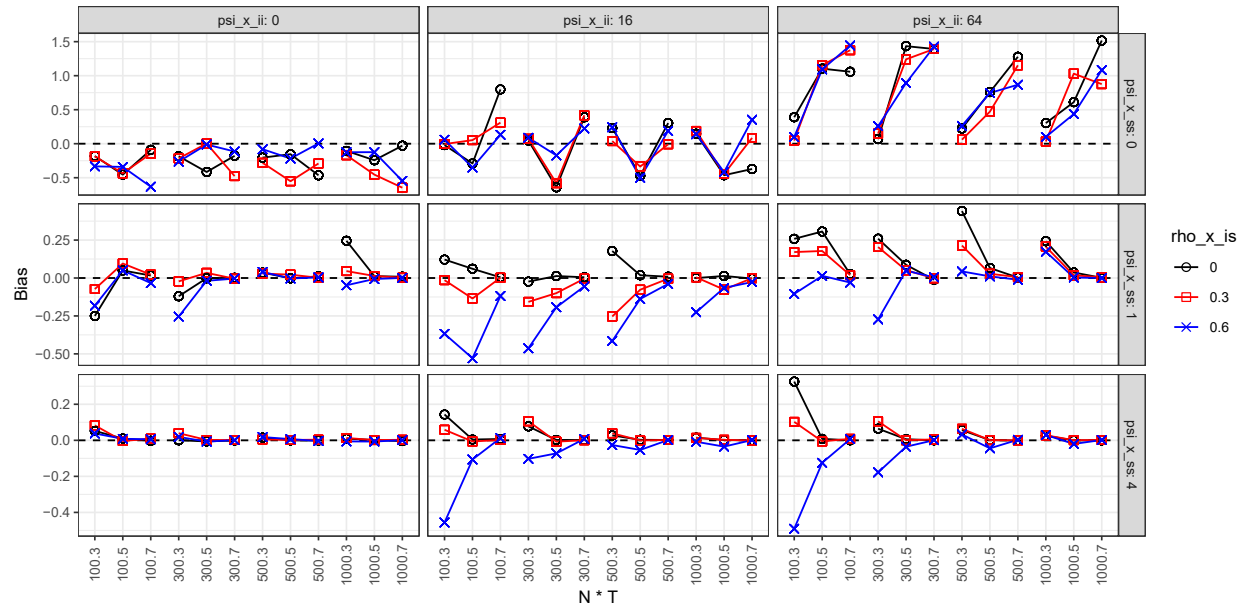
Bias for the Between-Person Effect β_{IS} in Simulation 5 (LCMxy-RP)

Figure E.125

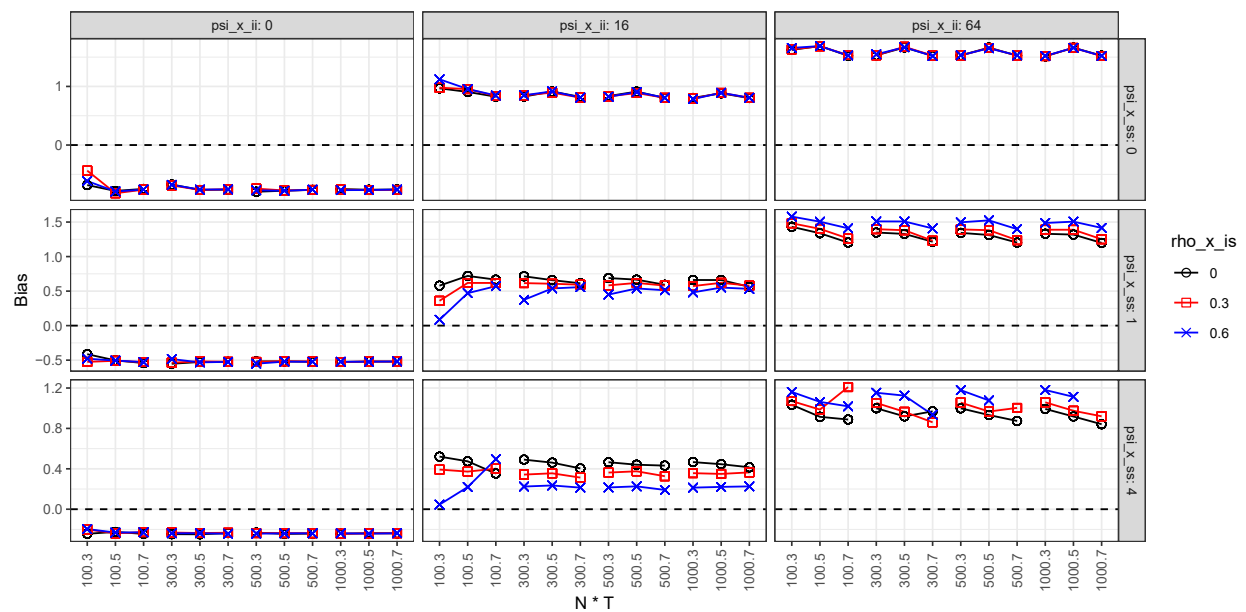
Bias for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-Rly)

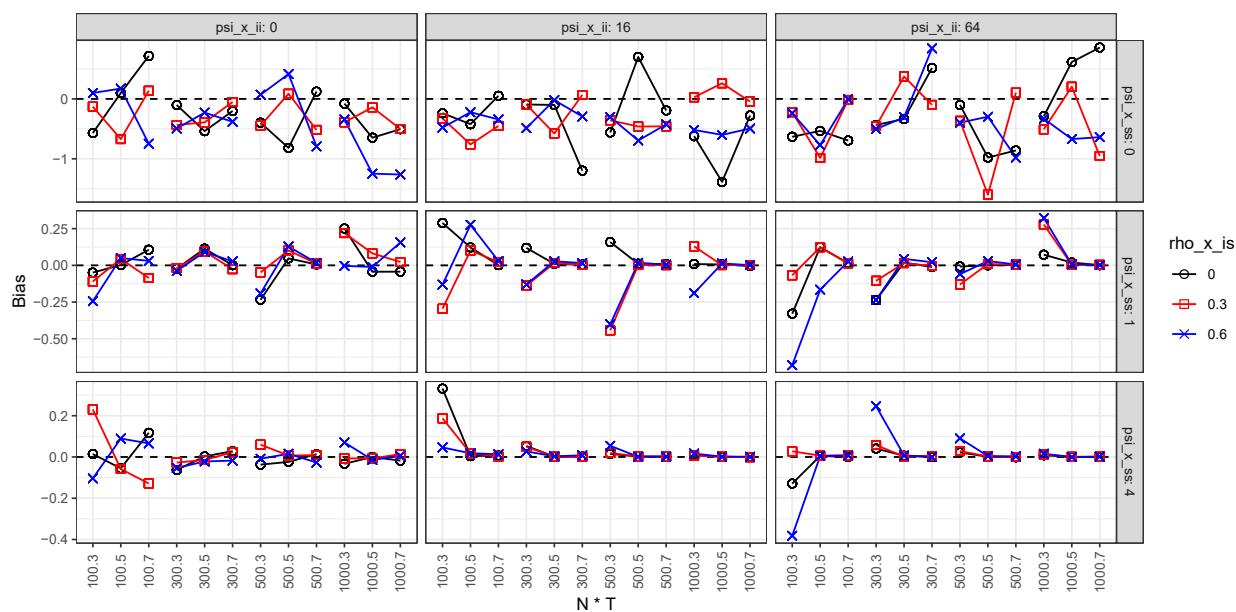
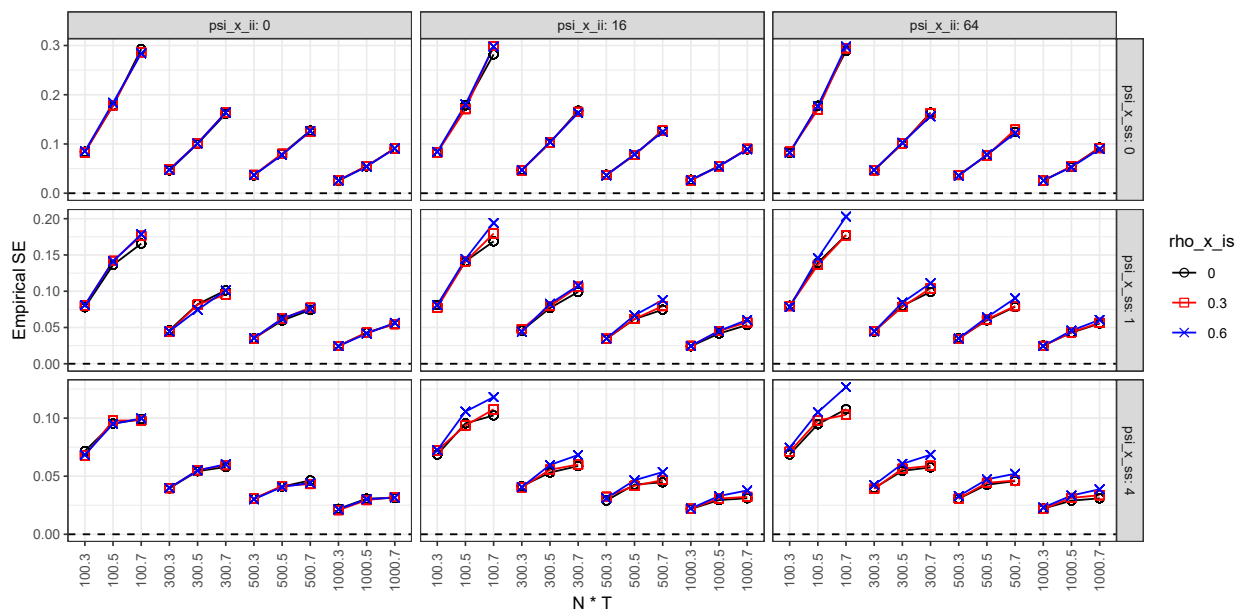
Figure E.126*Bias for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-CLC)***Figure E.127***Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)*

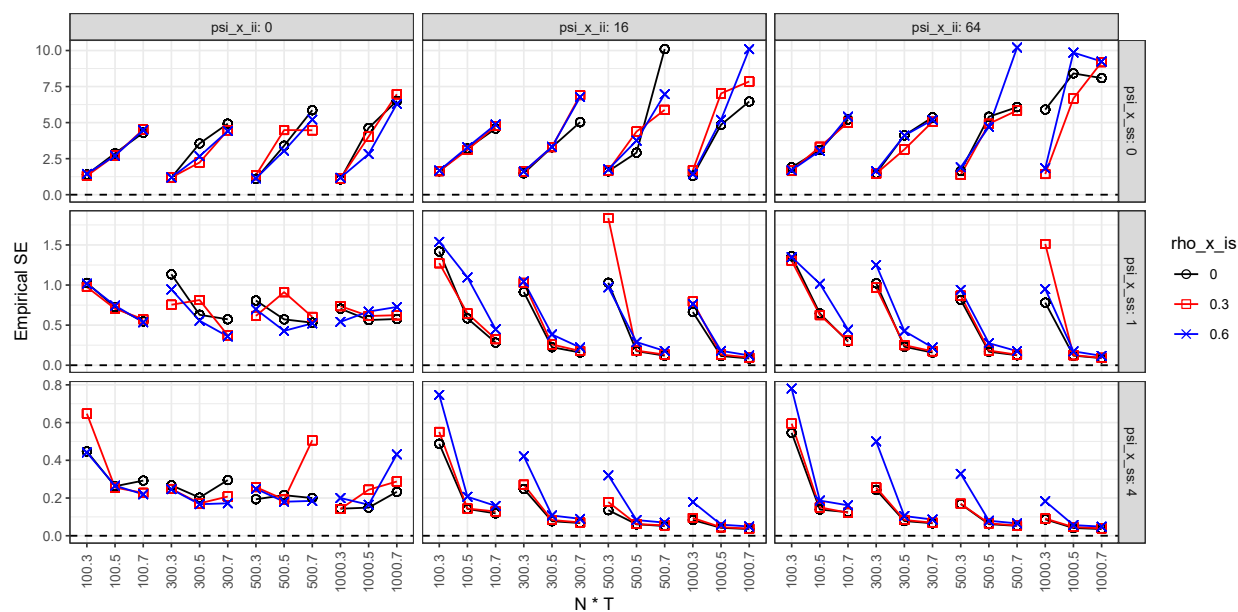
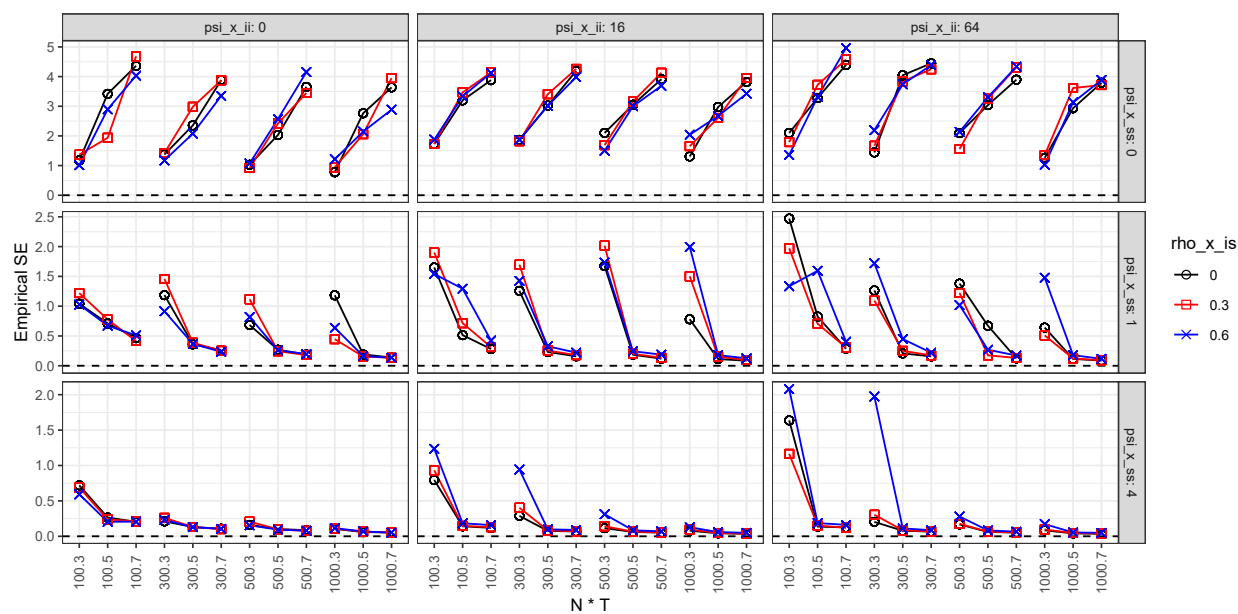
Figure E.128*Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)***Figure E.129***Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMxy-RP)*

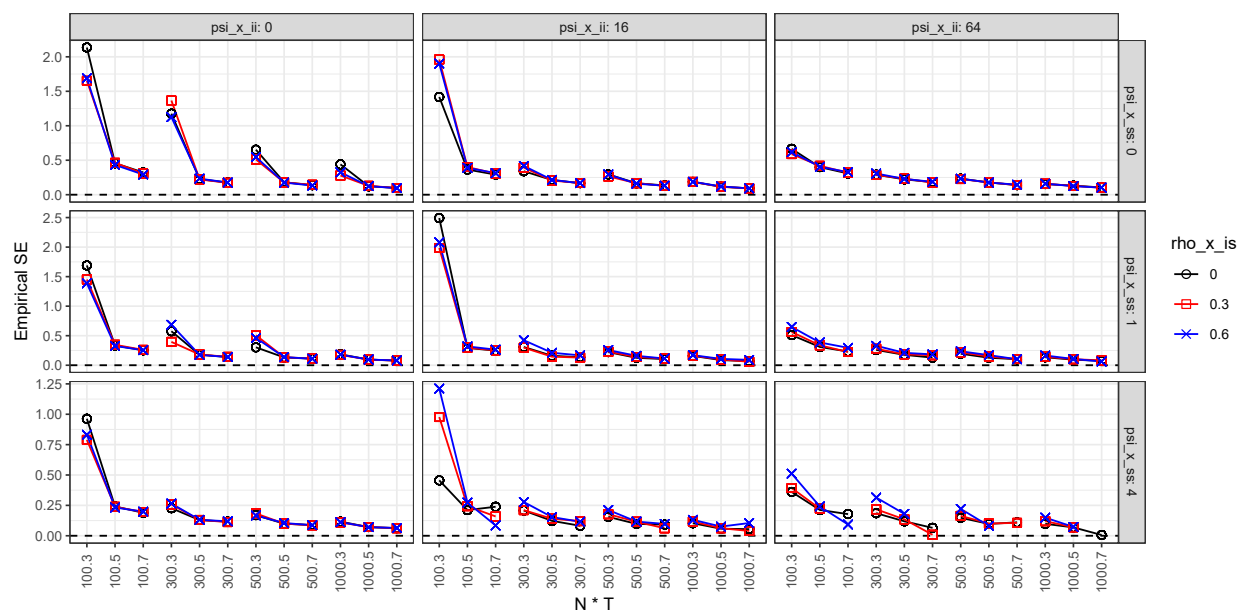
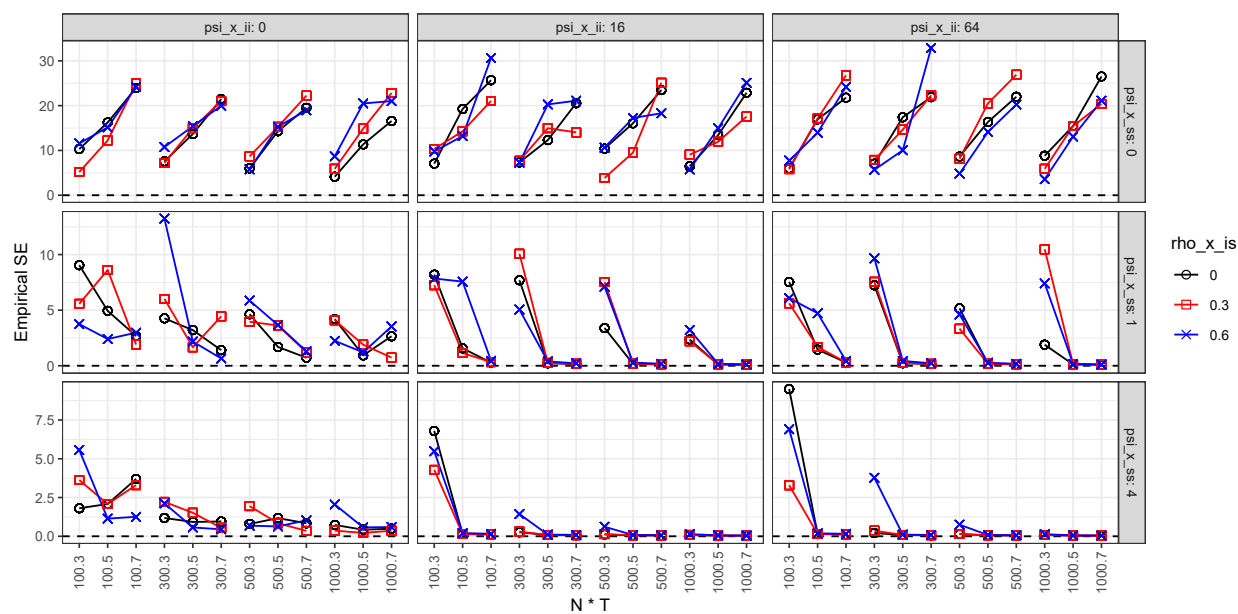
Figure E.130*Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-Rly)***Figure E.131***Empirical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-CLC)*

Figure E.132

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)

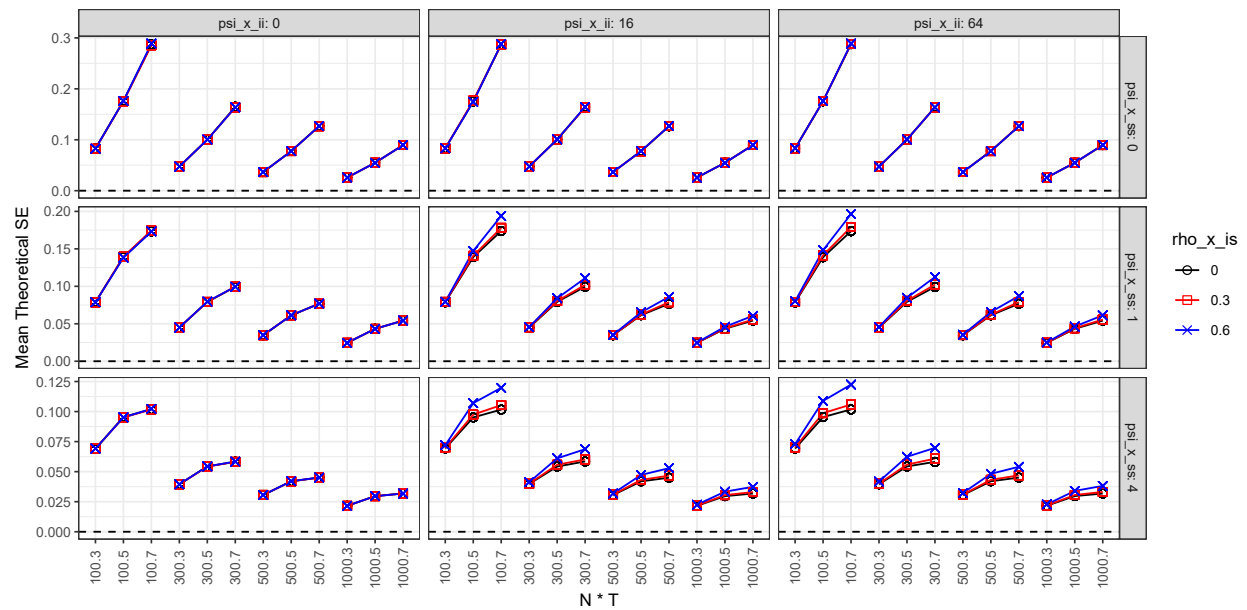


Figure E.133

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)

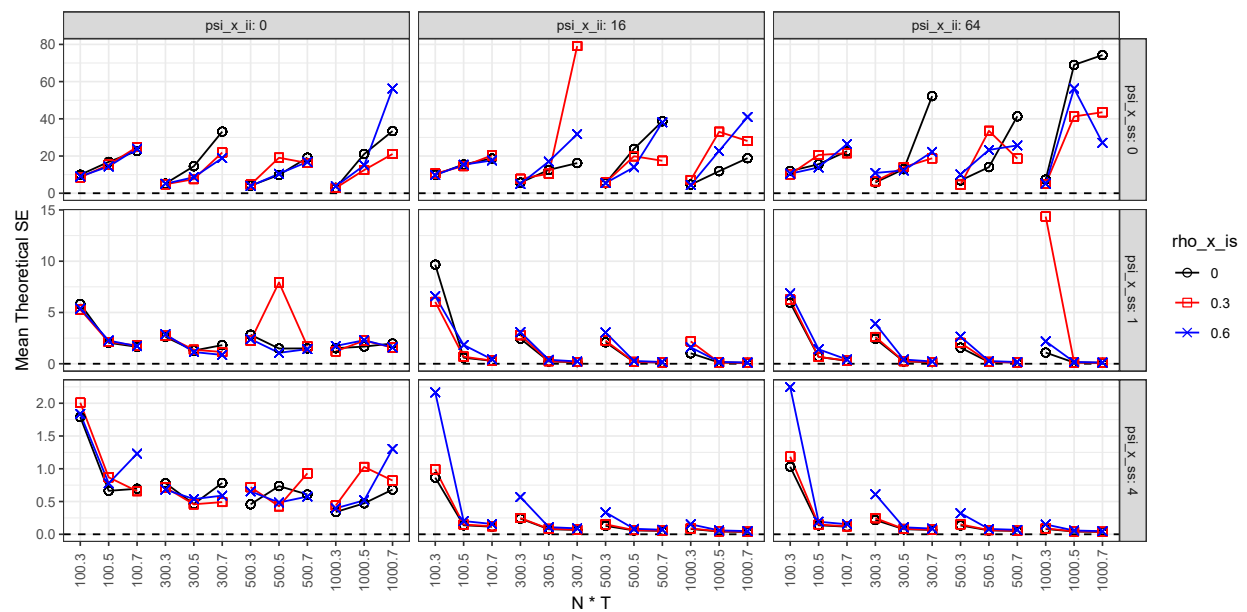
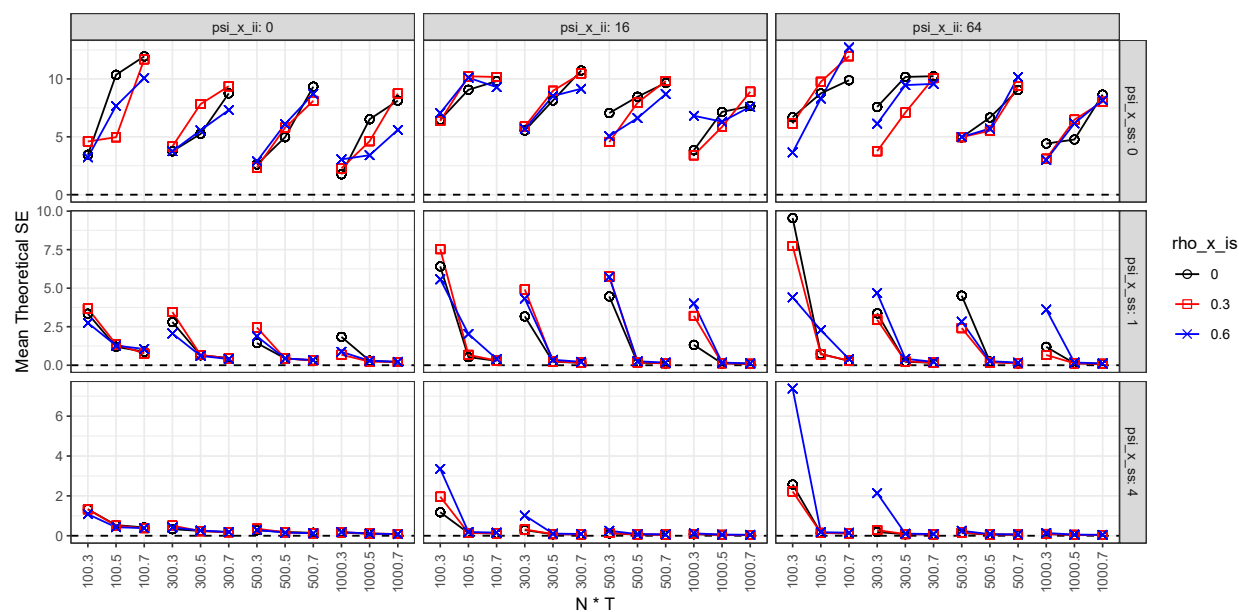


Figure E.134

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCM_{xy}-RP)

**Figure E.135**

Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCM_x-Rly)

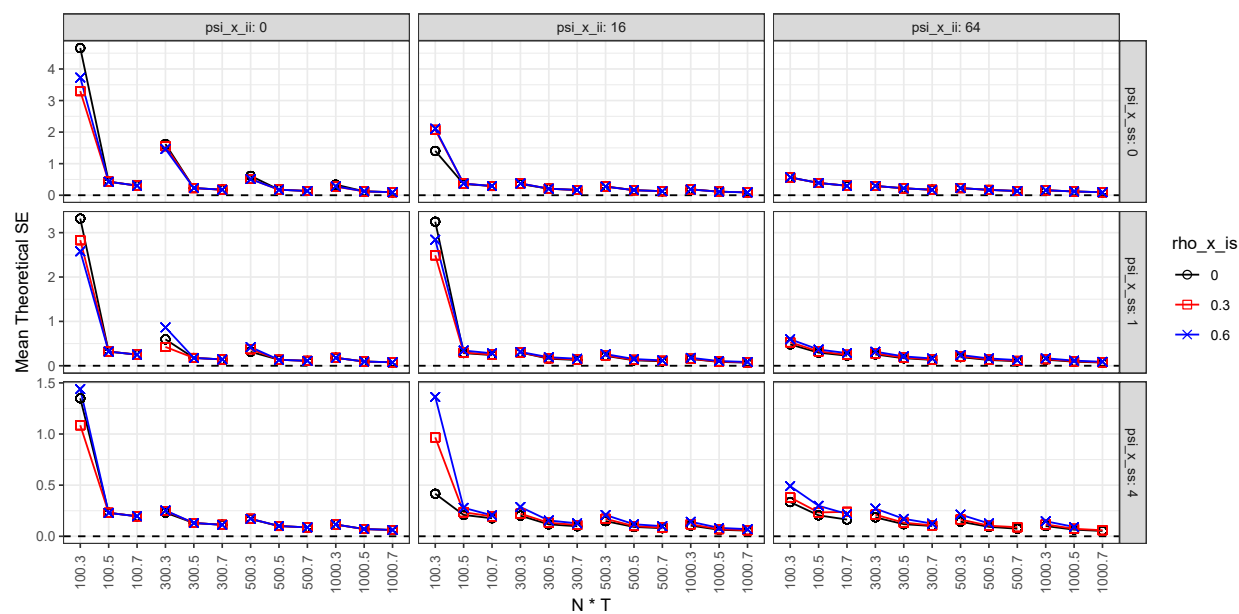


Figure E.136

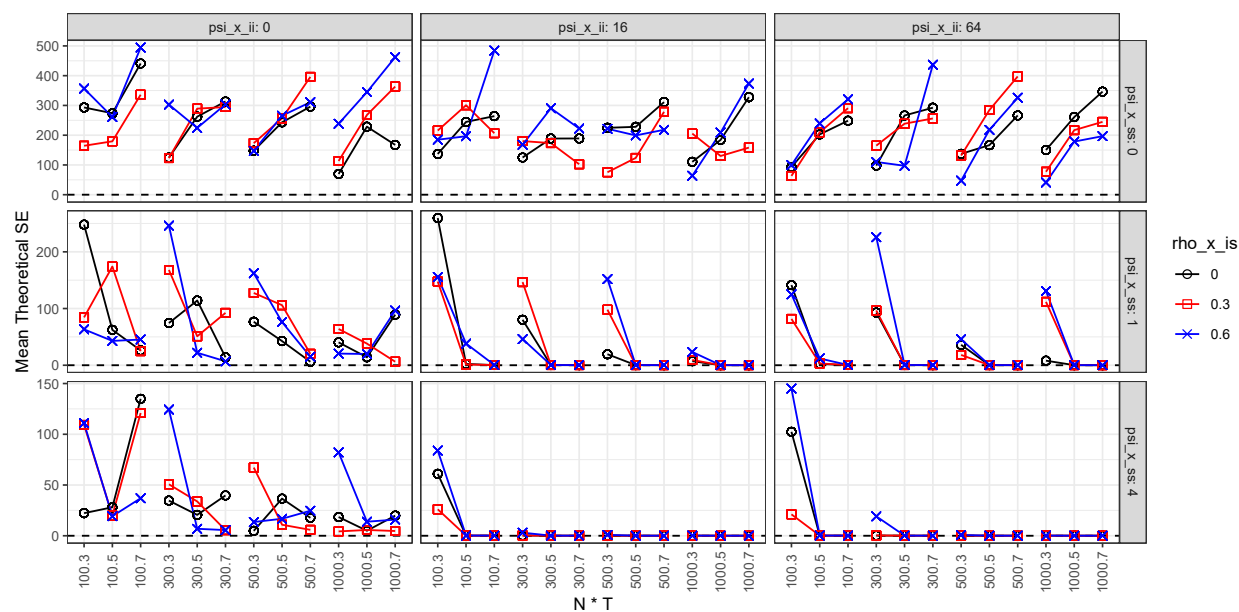
Theoretical SE for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-CLC)

Figure E.137

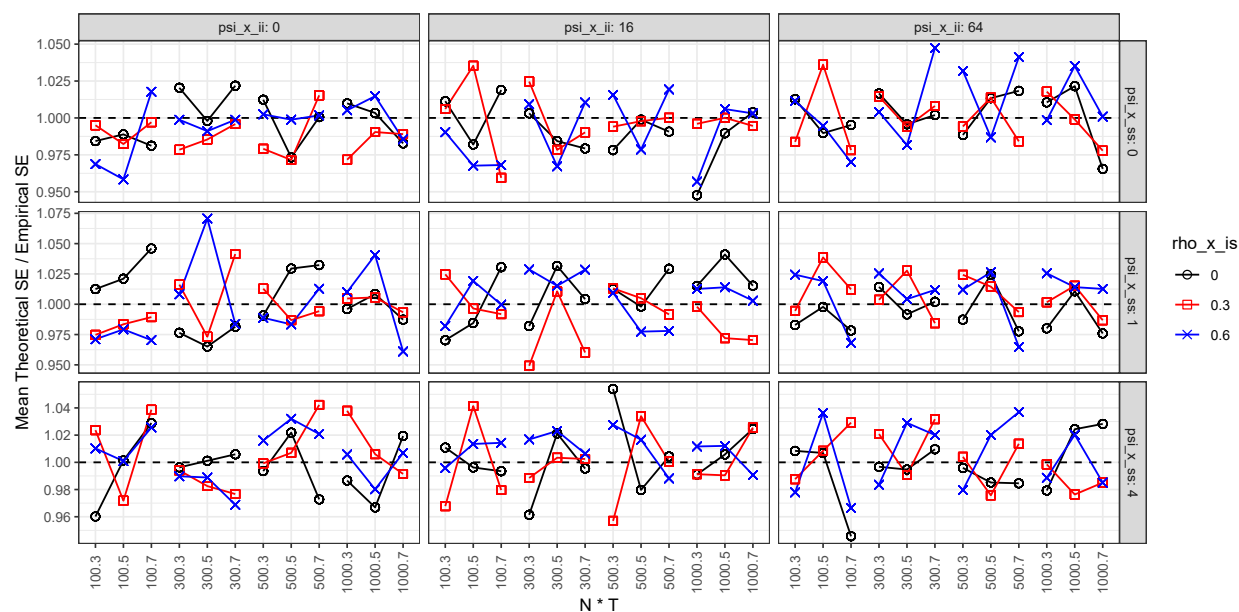
SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)

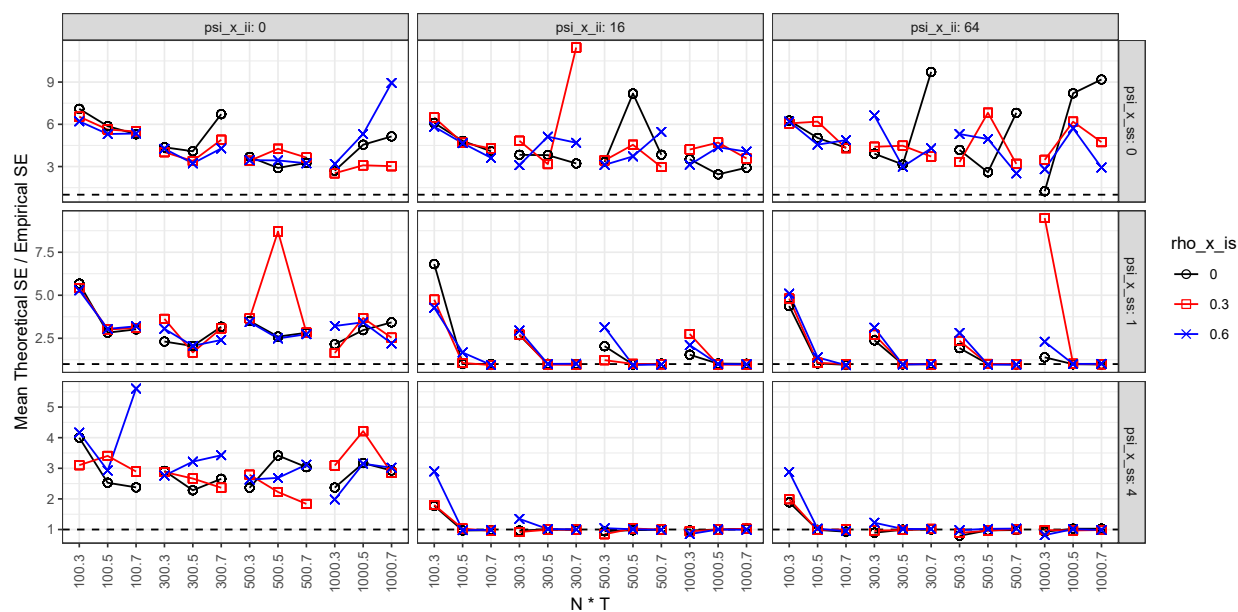
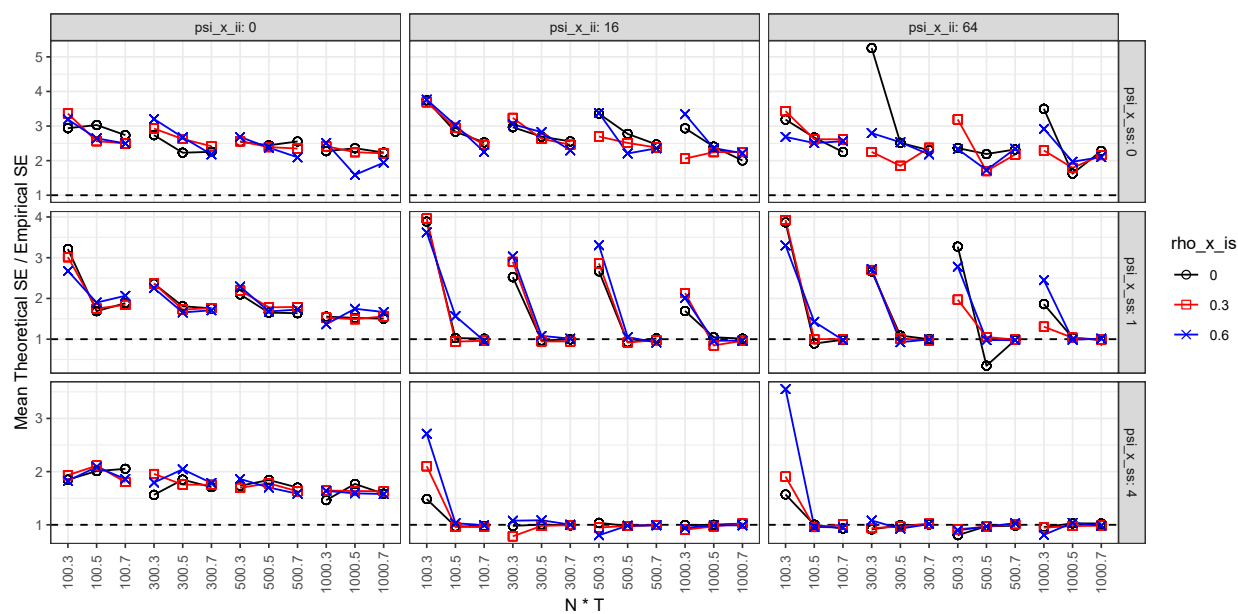
Figure E.138*SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)***Figure E.139***SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (LCM_{xy}-RP)*

Figure E.140

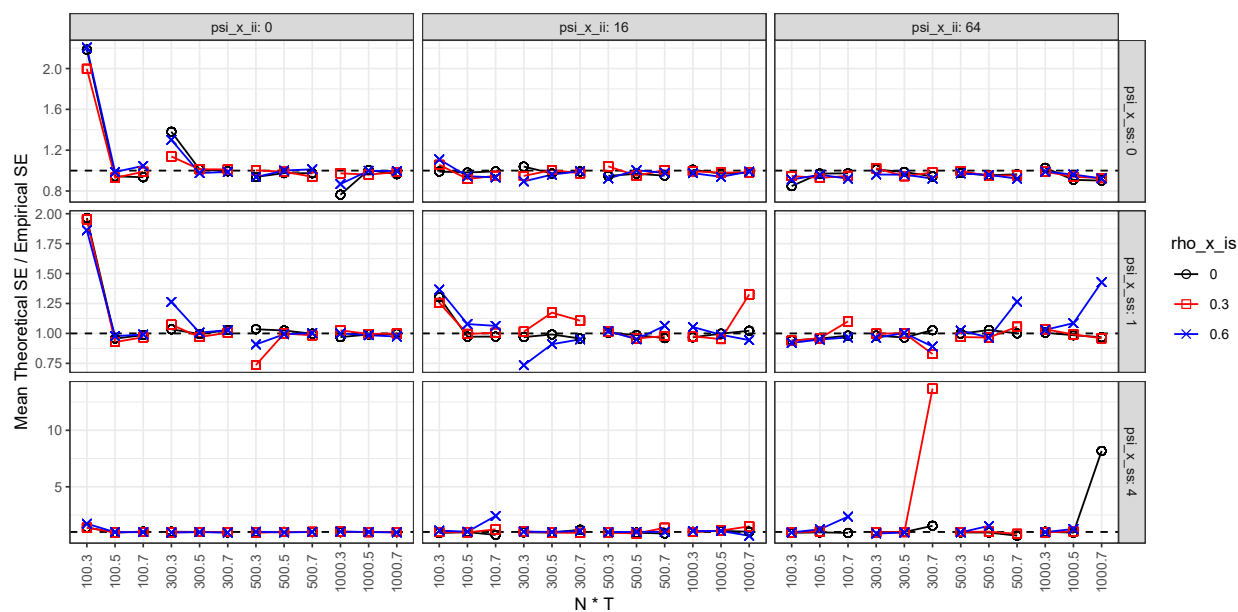
SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (LCM_x-RI_y)

Figure E.141

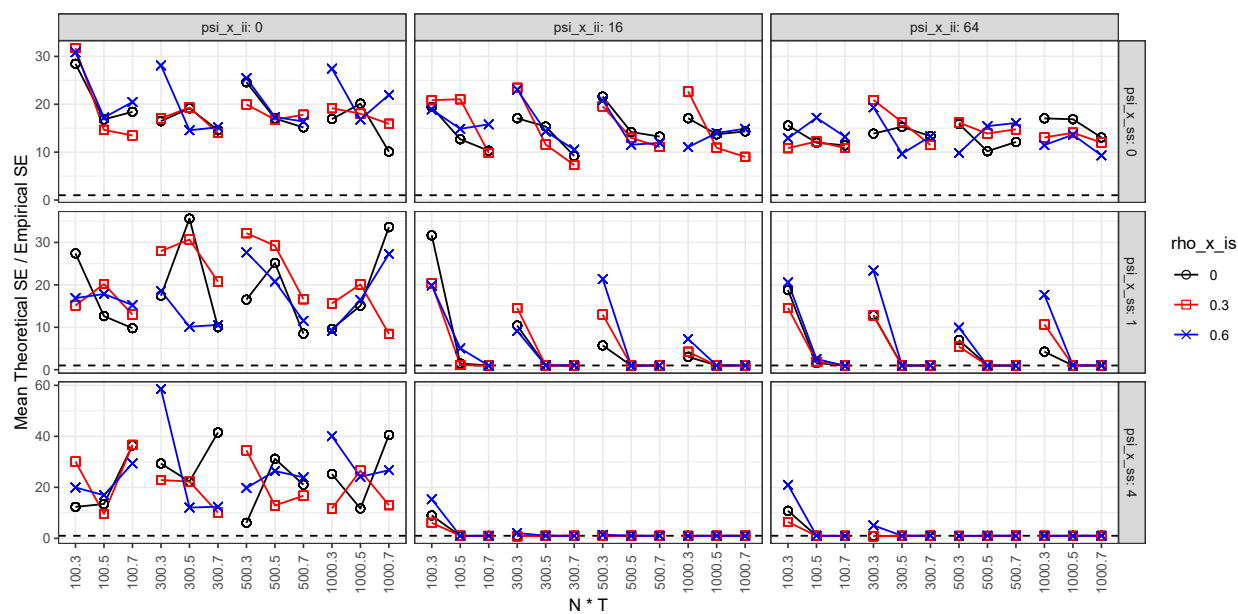
SE Ratio for the Between-Person Effect β_{IS} in Simulation 5 (LCM_x-CLC)

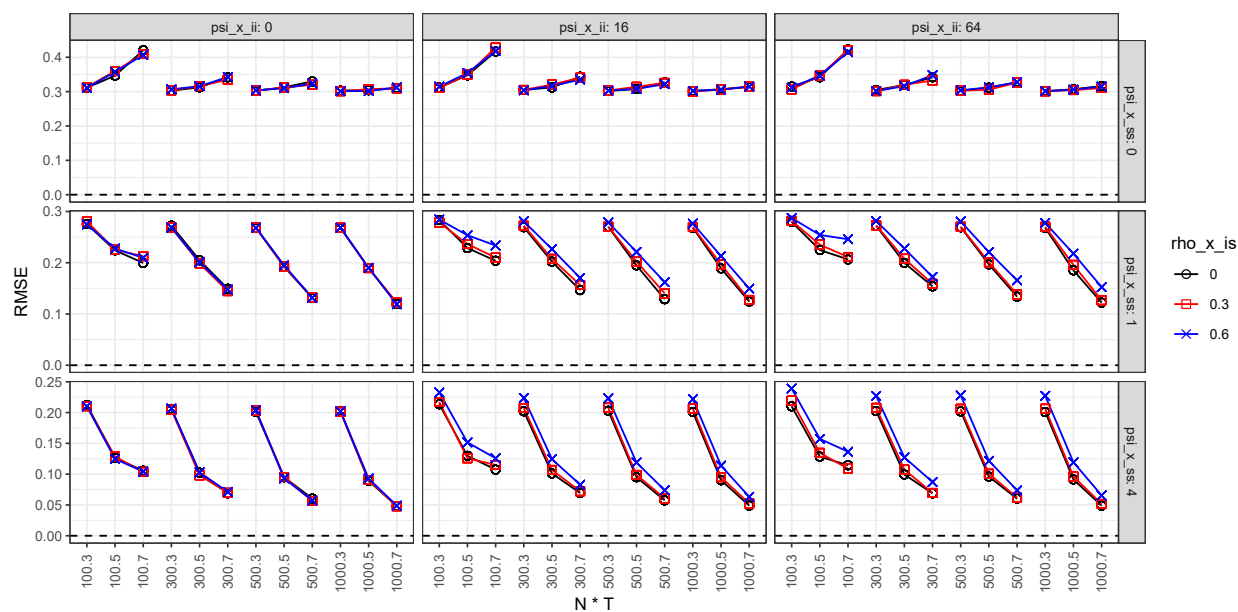
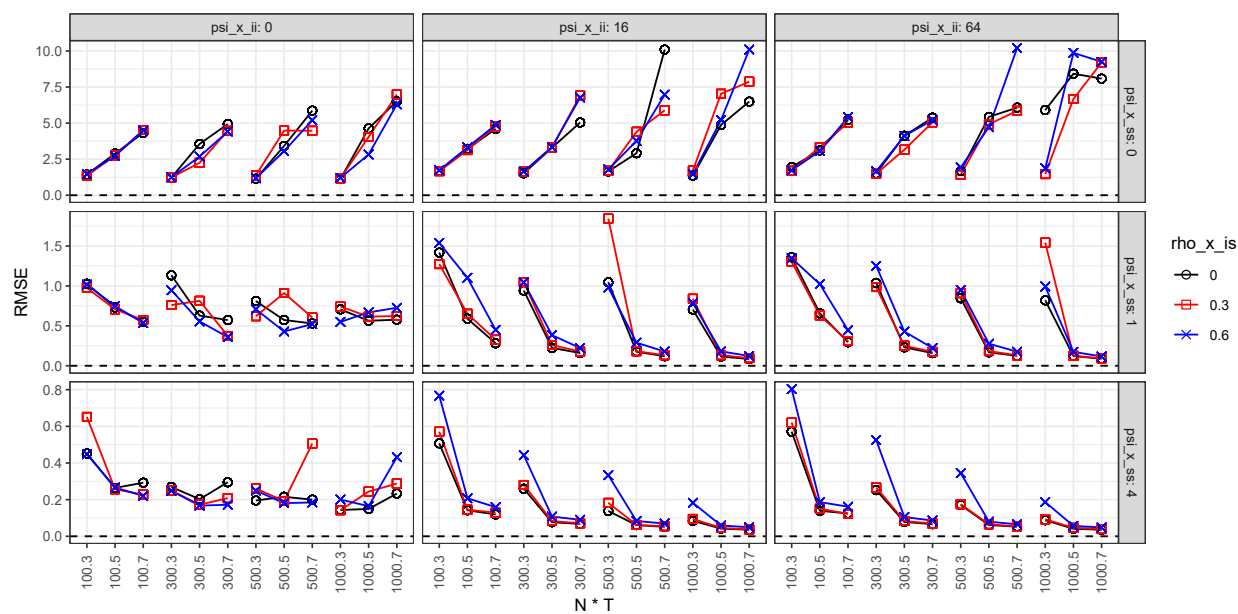
Figure E.142*RMSE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)***Figure E.143***RMSE for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV)*

Figure E.144

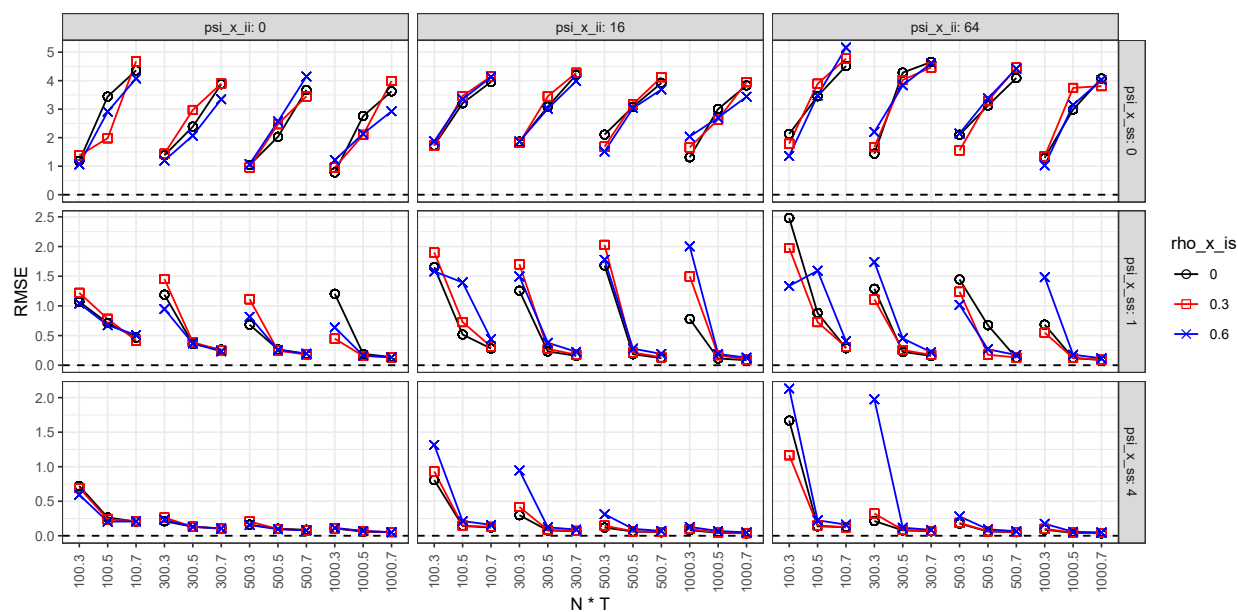
RMSE for the Between-Person Effect β_{IS} in Simulation 5 (LCM_{xy}-RP)

Figure E.145

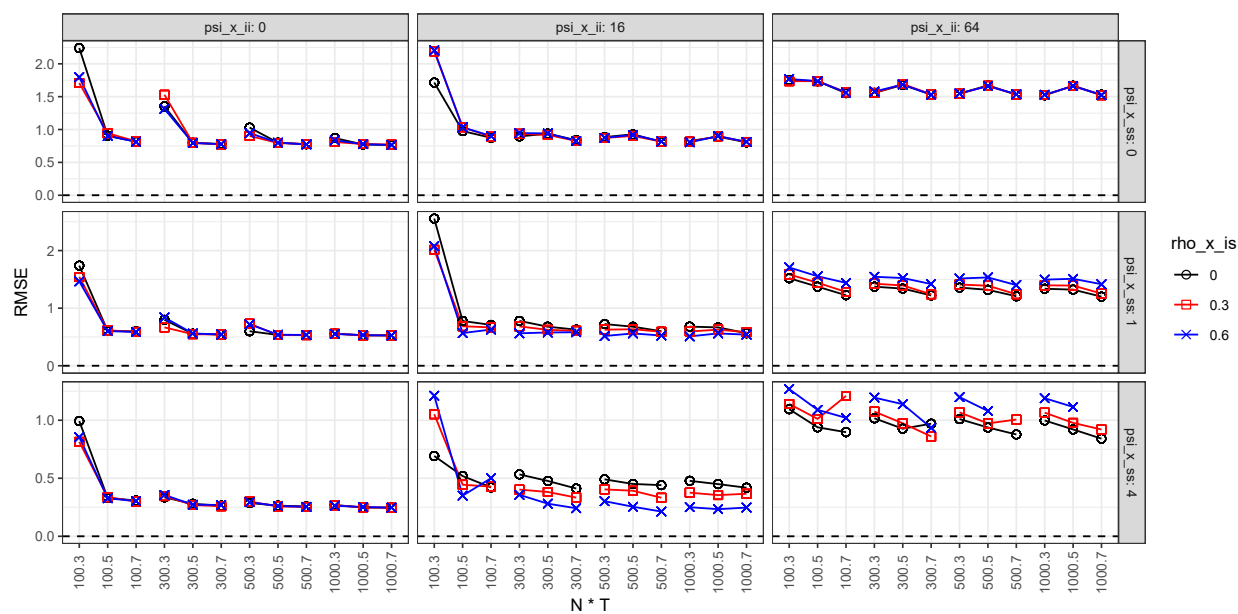
RMSE for the Between-Person Effect β_{IS} in Simulation 5 (LCM_x-Rly)

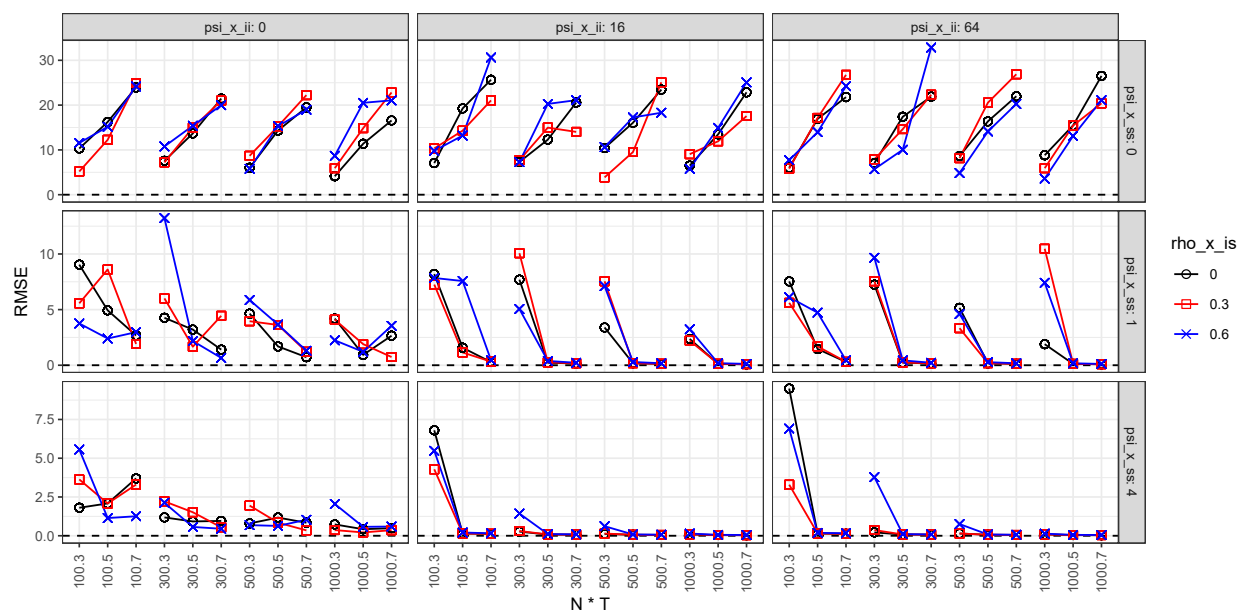
Figure E.146*RMSE for the Between-Person Effect β_{IS} in Simulation 5 (LCM_X-CLC)*

Figure E.147

Coverage Rate for the Between-Person Effect β_{IS} in Simulation 5 (MLM-MV, LCM_{xy}-RP, LCM_x-CLC)

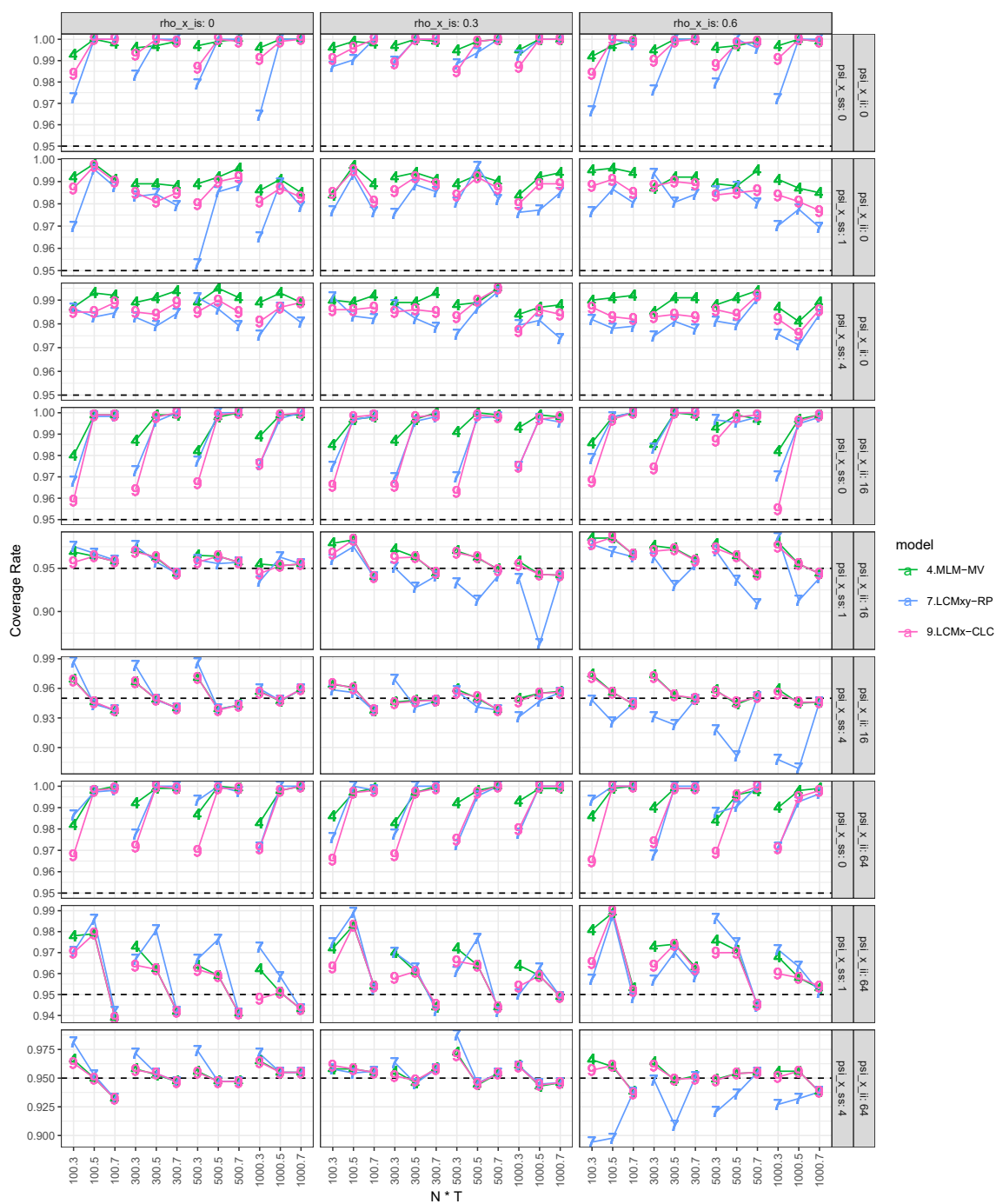
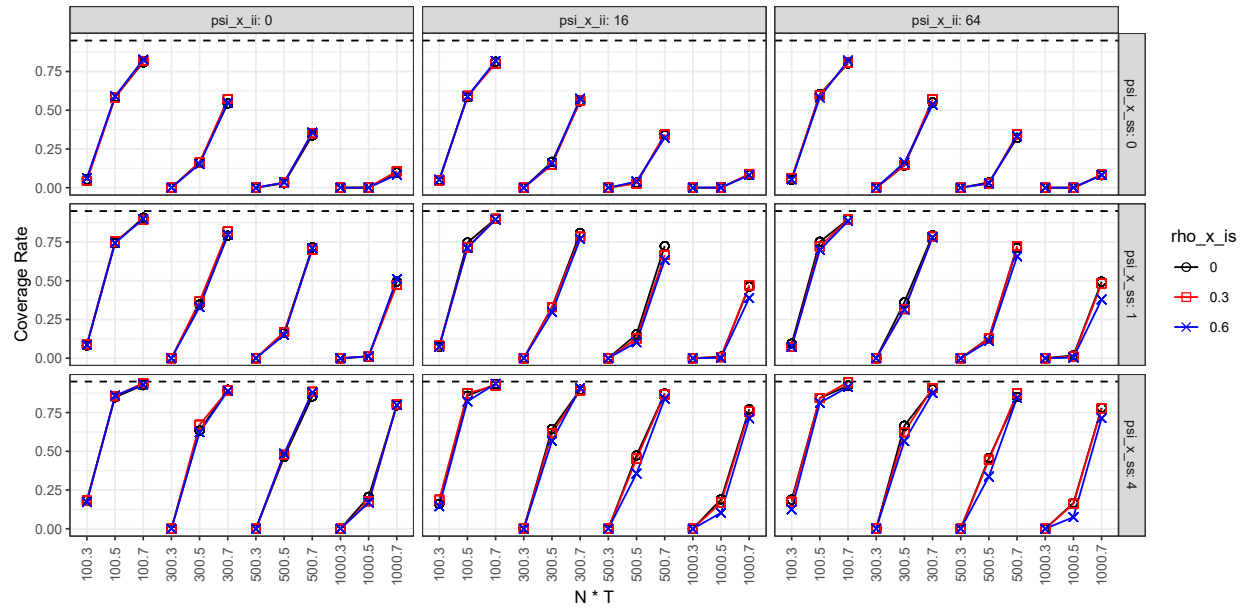


Figure E.148

Coverage Rate for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S)

**Figure E.149**

Coverage Rate for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-Rly)

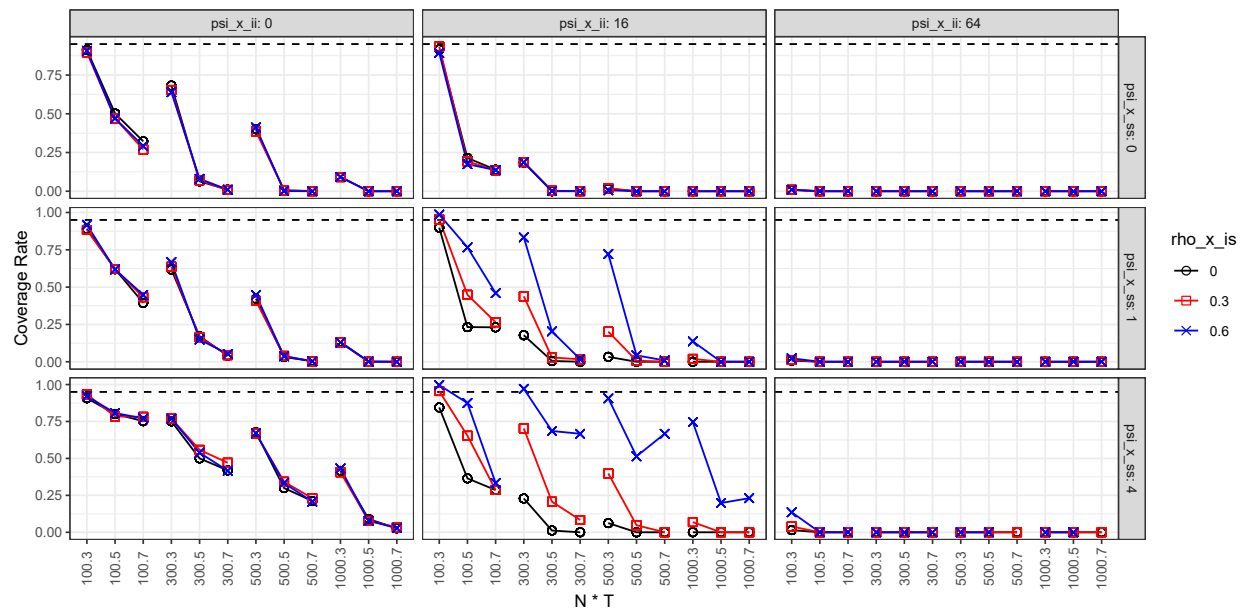


Figure E.150

Power for the Between-Person Effect β_{IS} in Simulation 5 (MLM-E2S, MLM-MV, LCMxy-RP, LCMx-CLC)

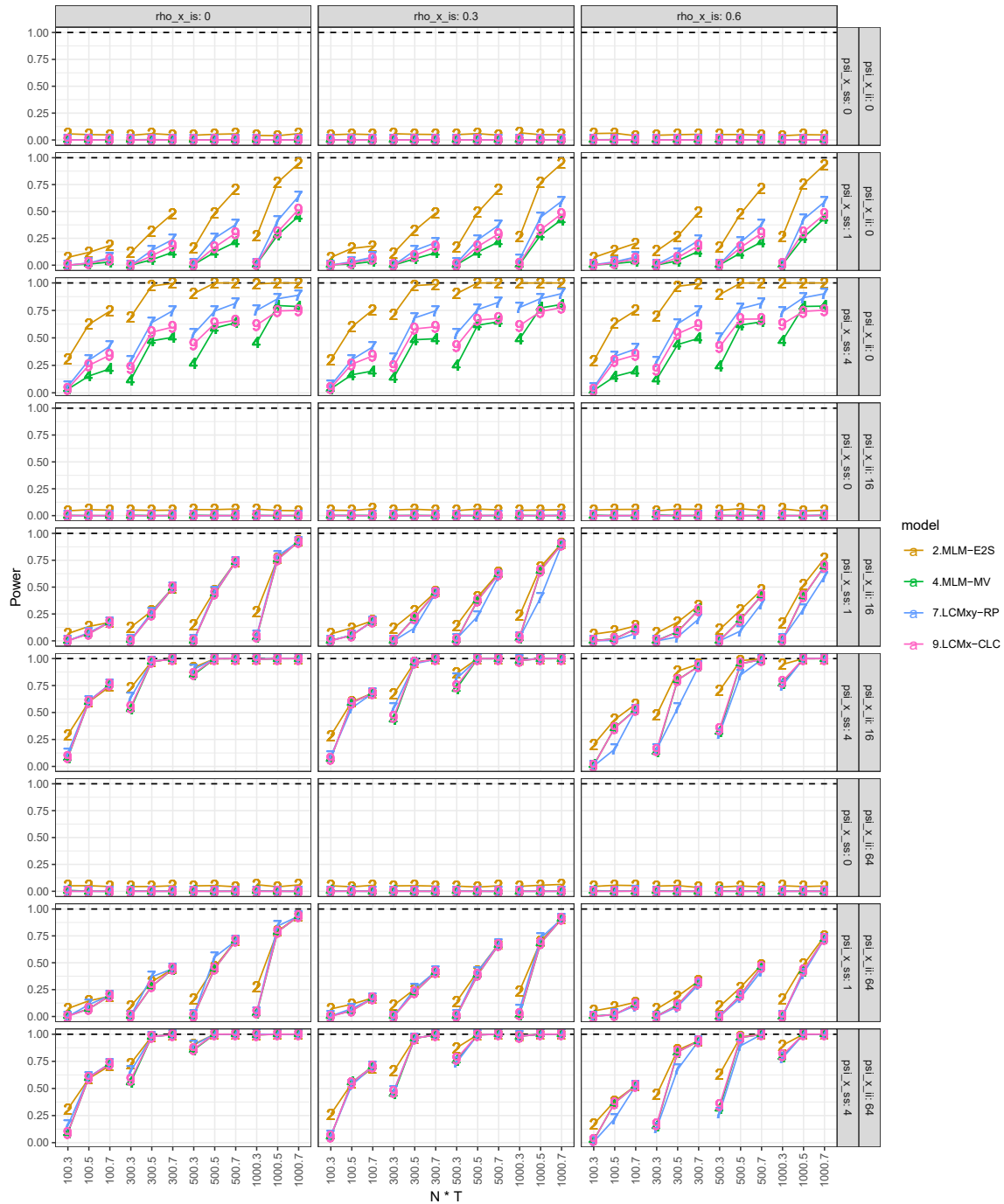
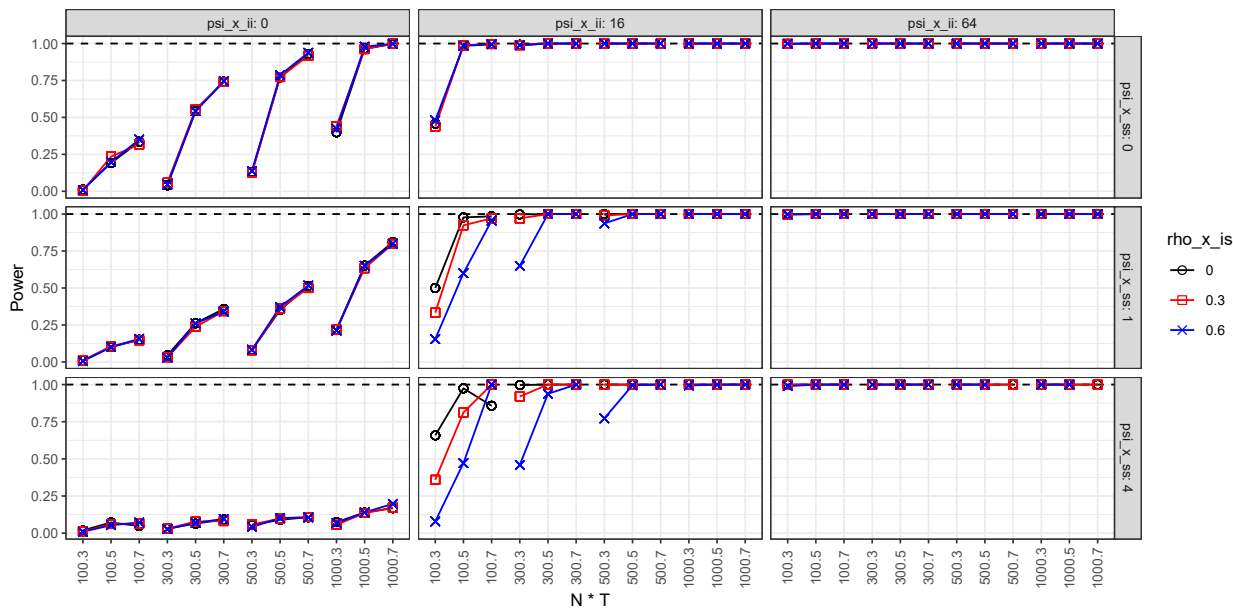


Figure E.151

Power for the Between-Person Effect β_{IS} in Simulation 5 (LCMx-Rly)



E.6 Between-Person Effect of Slope on Slope

Figure E.152

Bias for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)

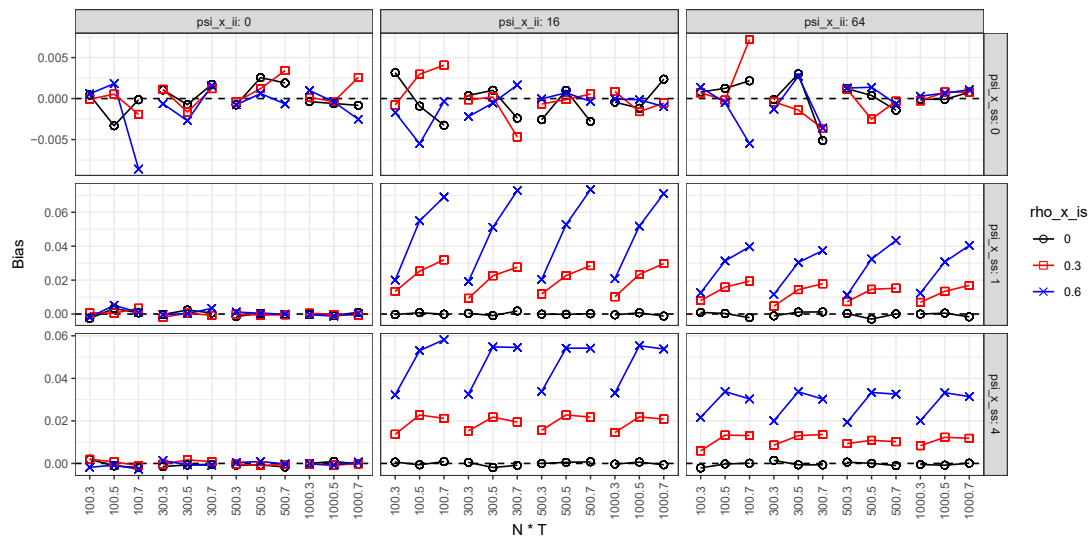


Figure E.153

Bias for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)

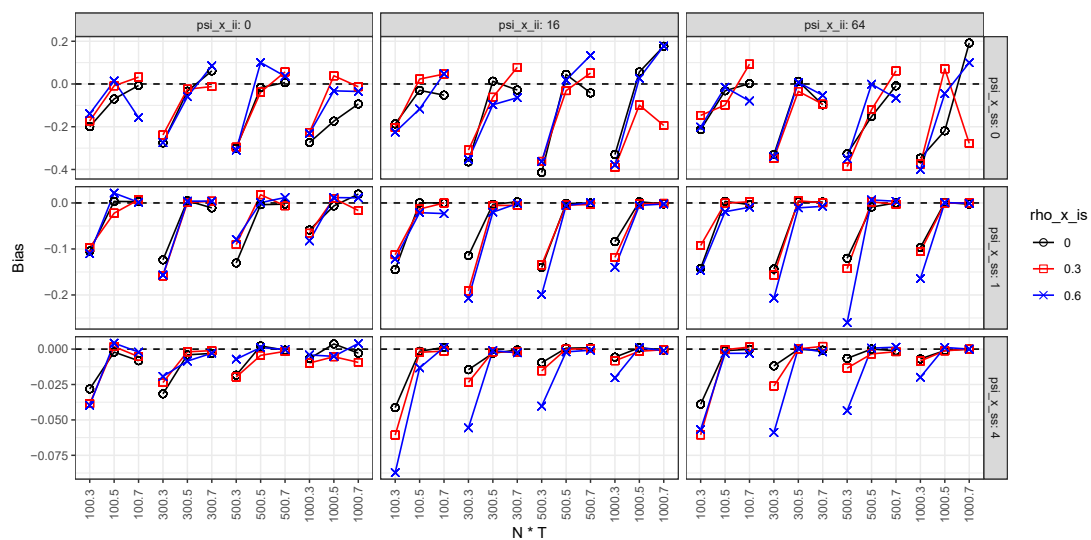


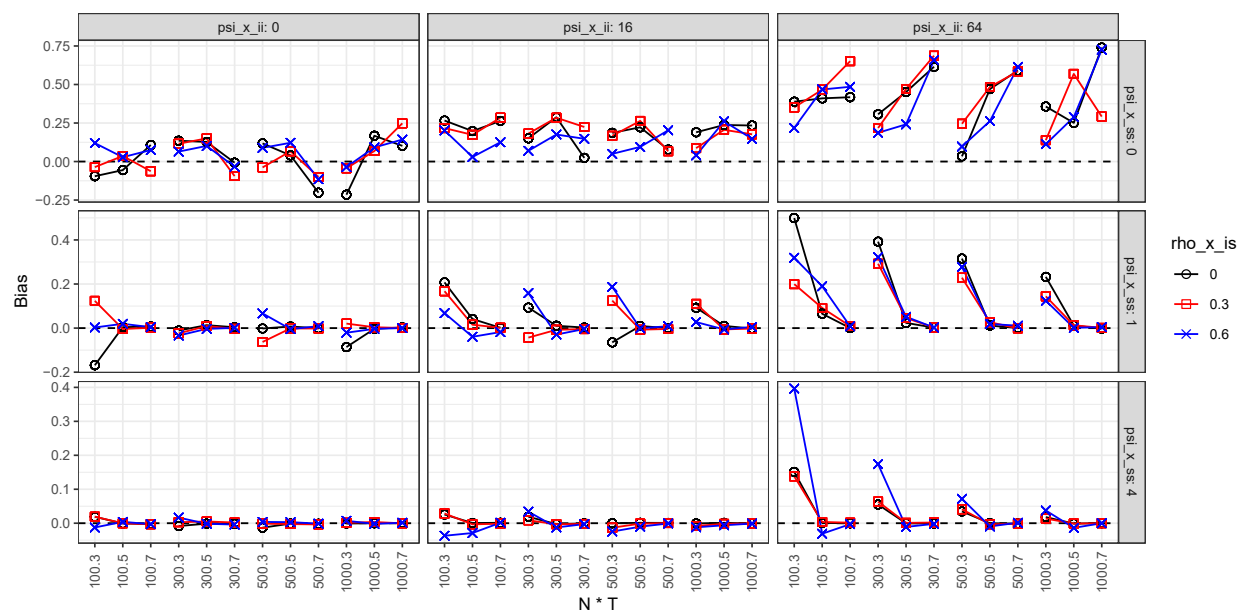
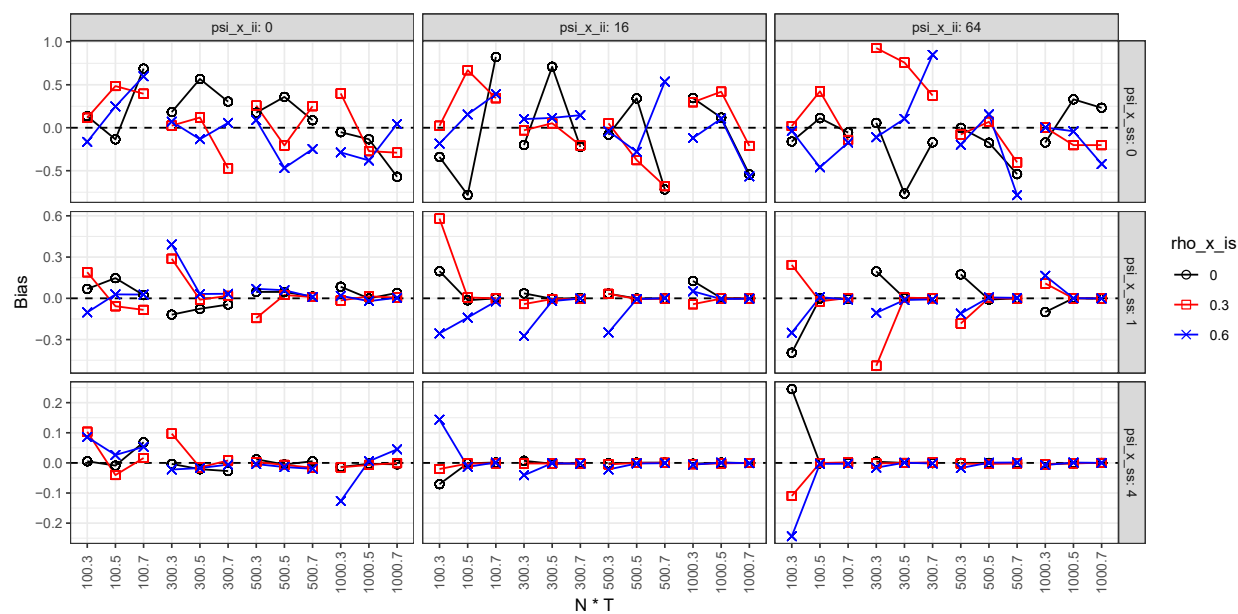
Figure E.154*Bias for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)***Figure E.155***Bias for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)*

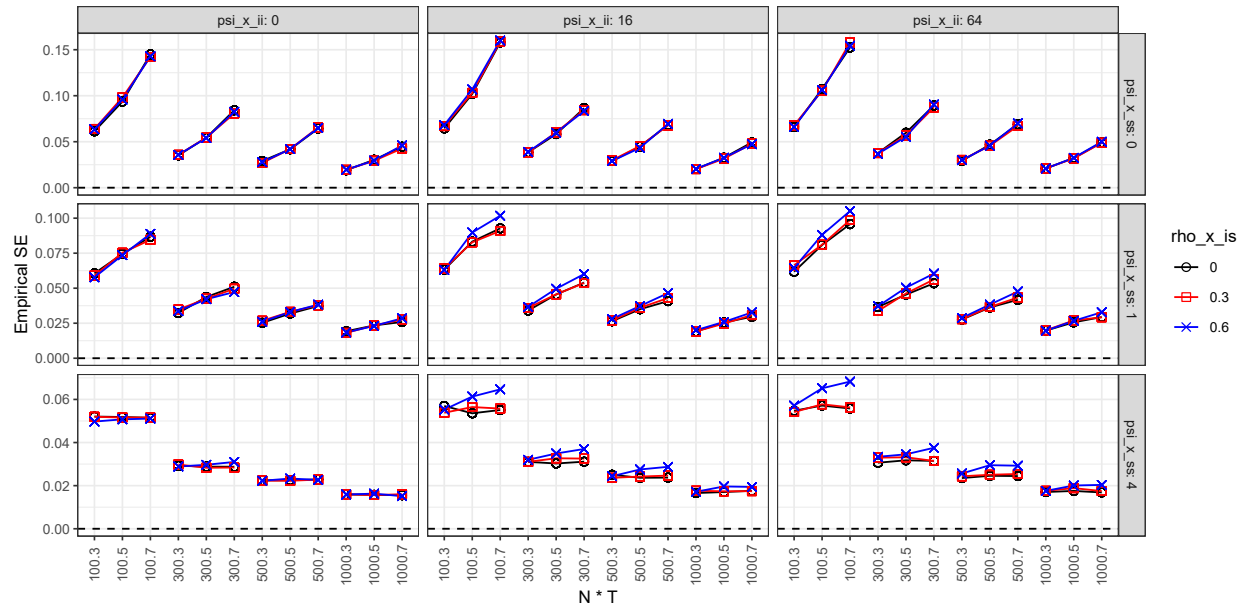
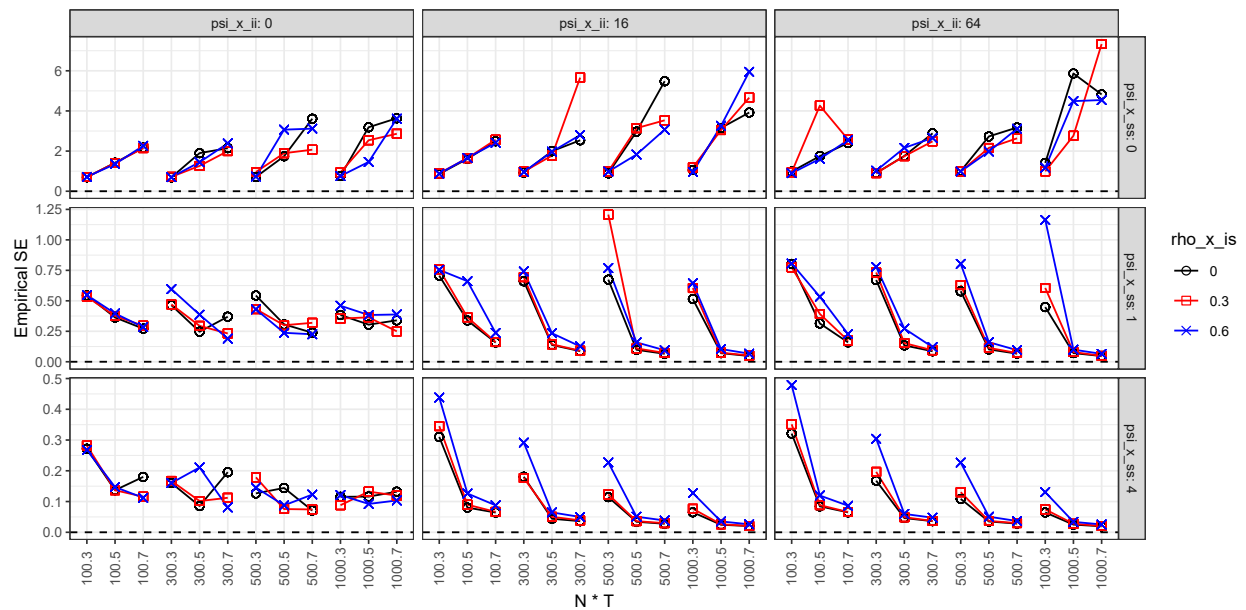
Figure E.156*Empirical SE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)***Figure E.157***Empirical SE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)*

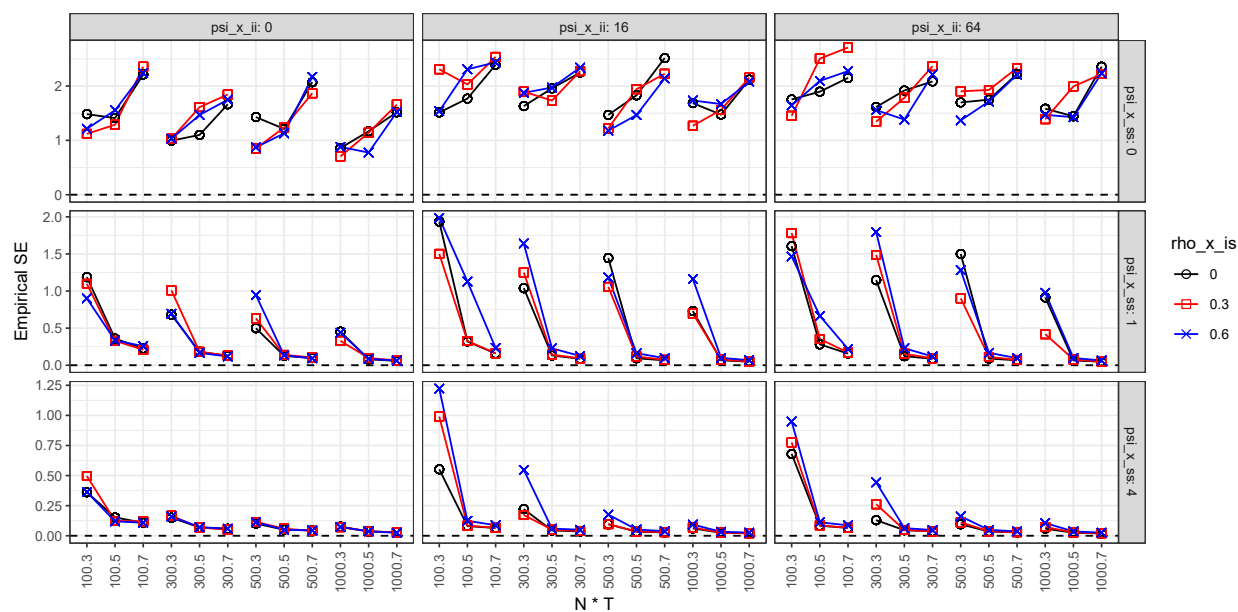
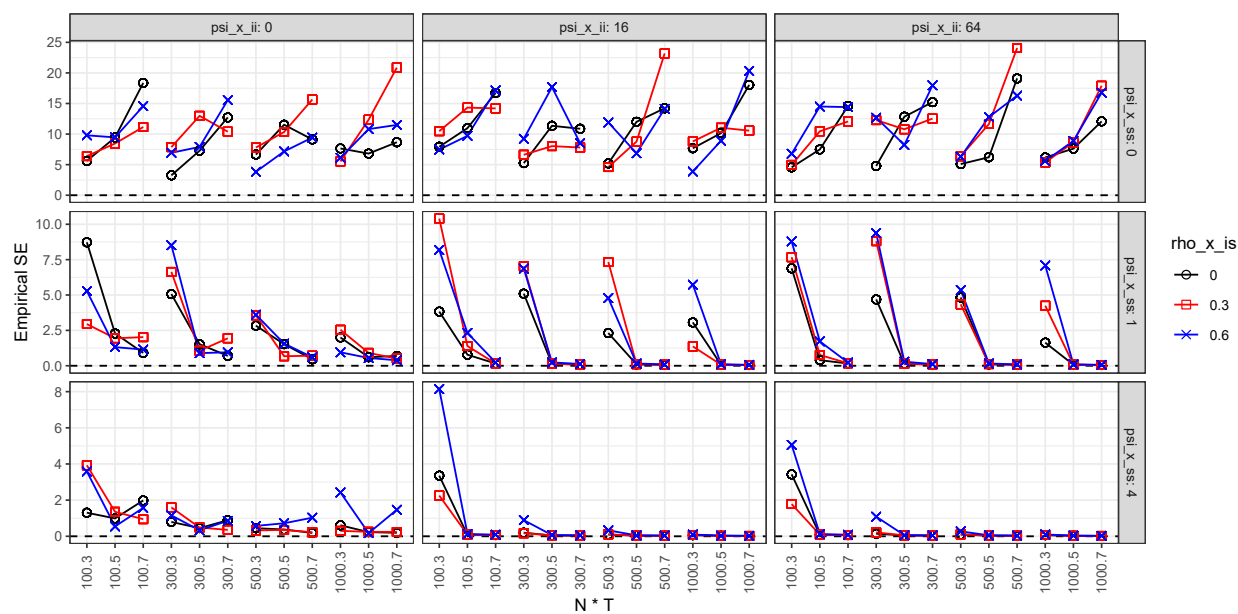
Figure E.158*Empirical SE for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)***Figure E.159***Empirical SE for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)*

Figure E.160

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)

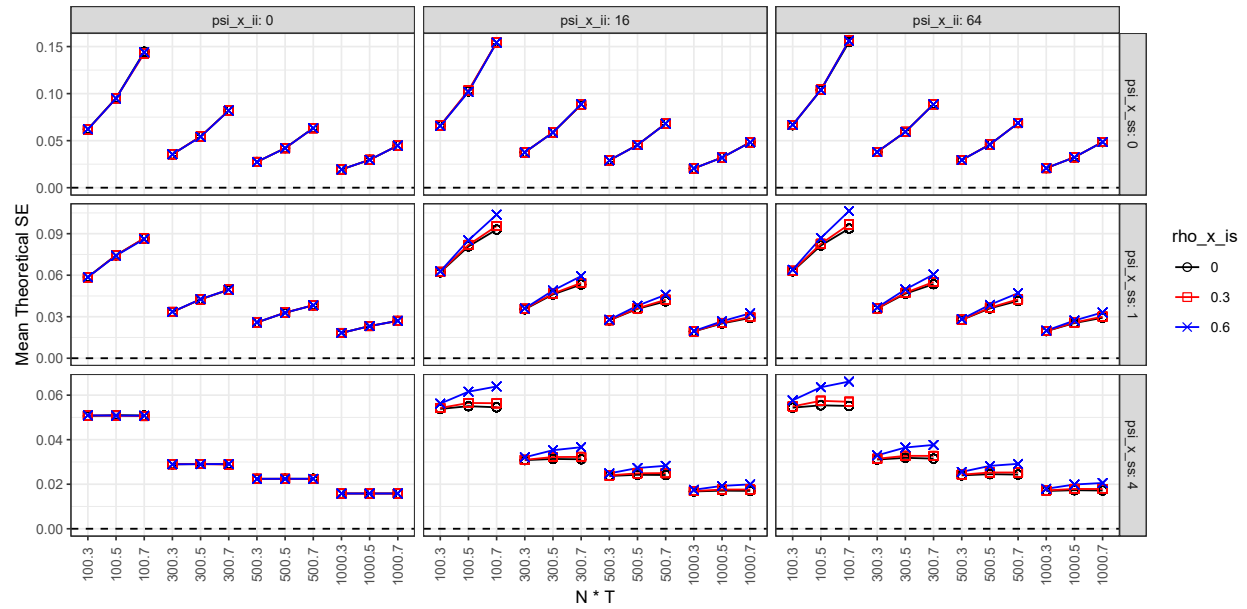


Figure E.161

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)

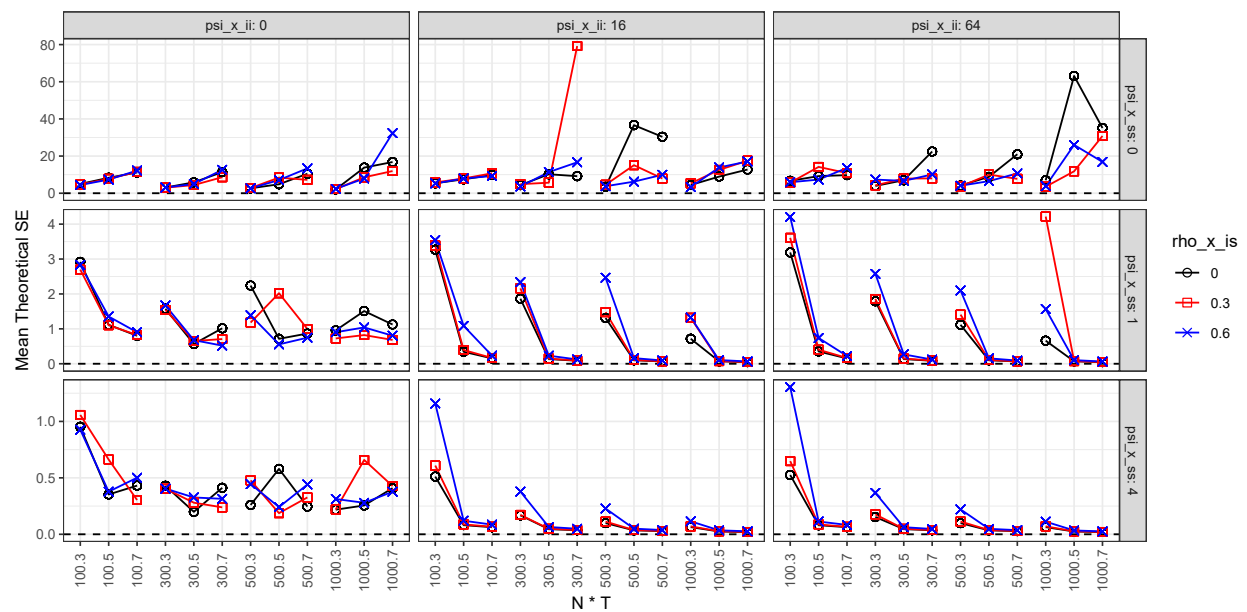
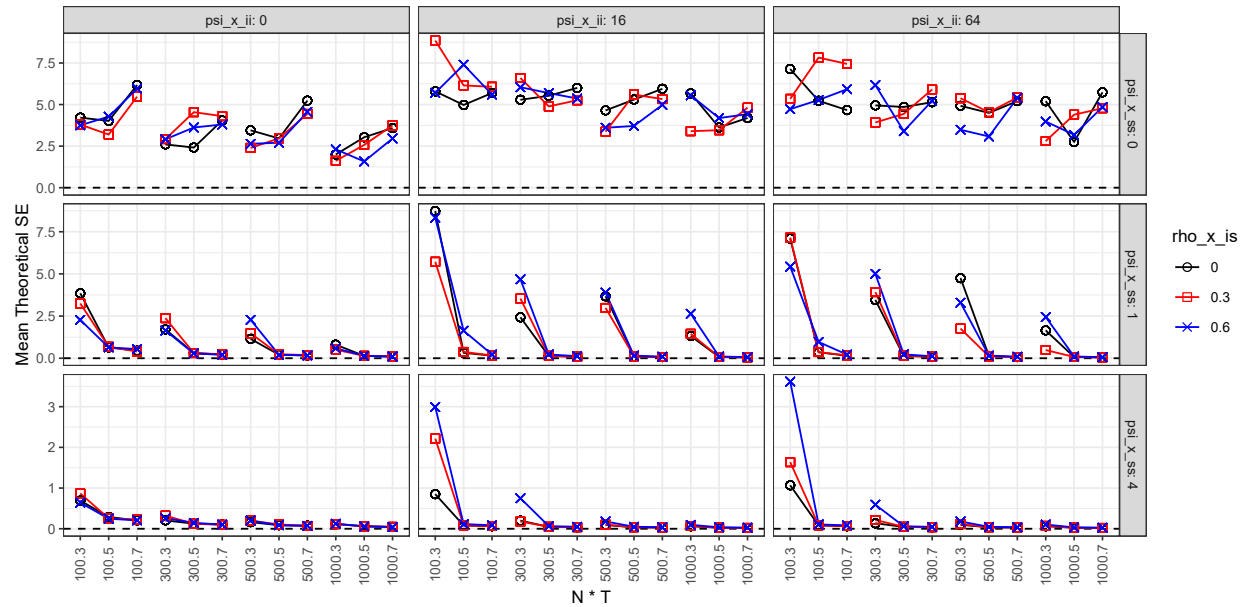


Figure E.162

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 5 (LCM_{xy}-RP)

**Figure E.163**

Theoretical SE for the Between-Person Effect β_{SS} in Simulation 5 (LCM_x-CLC)

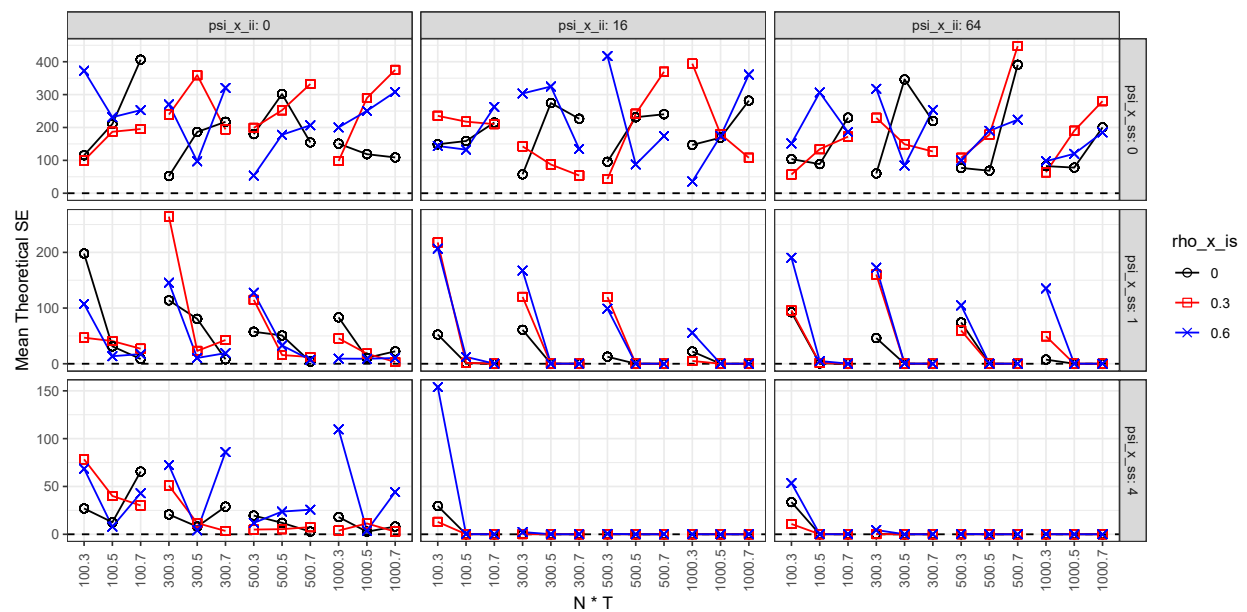


Figure E.164

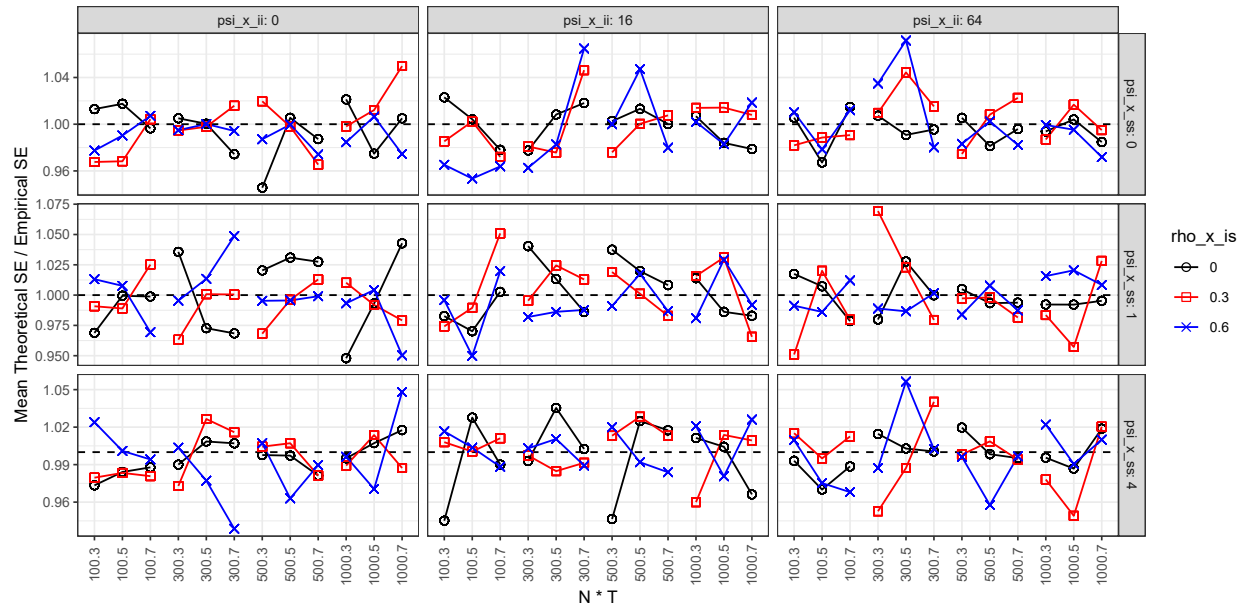
SE Ratio for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)

Figure E.165

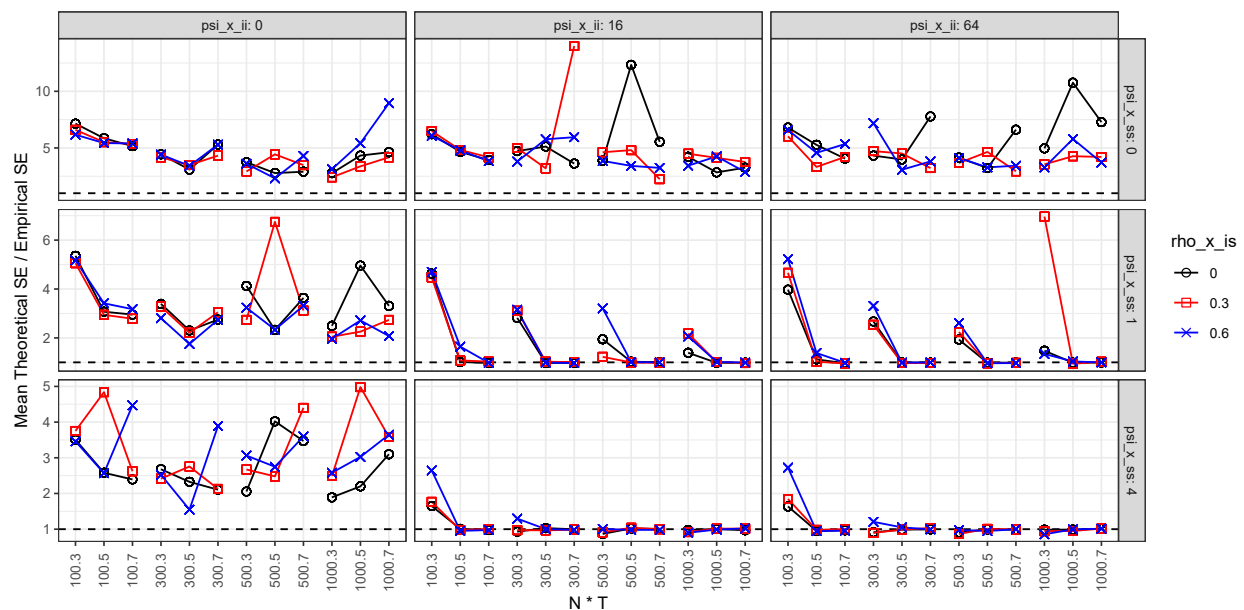
SE Ratio for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)

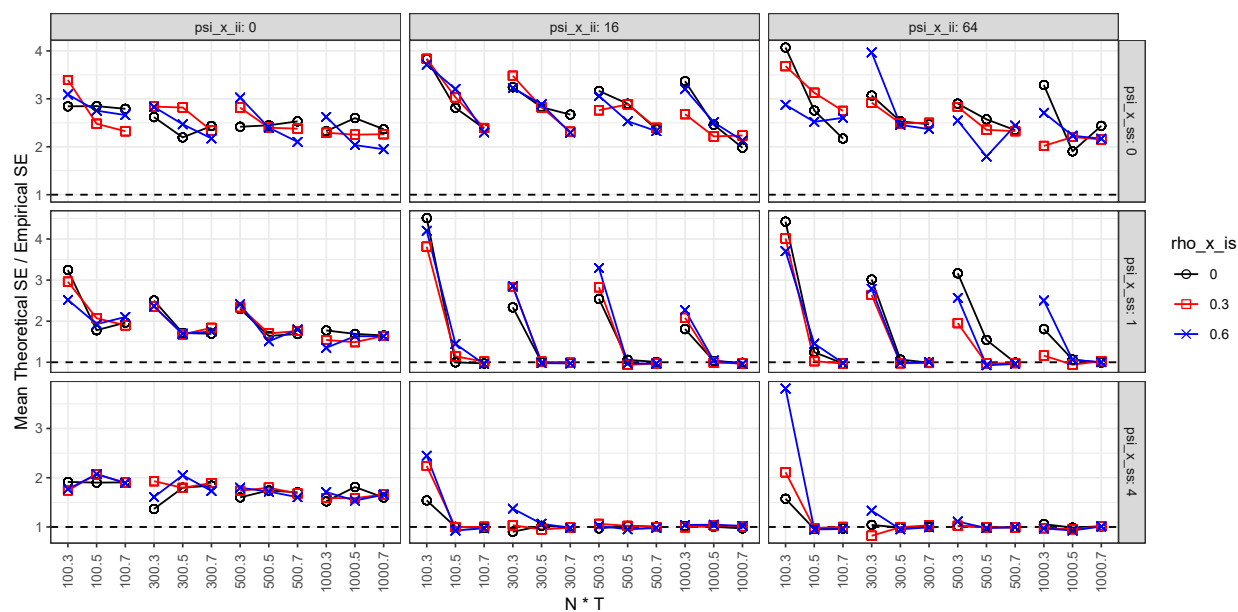
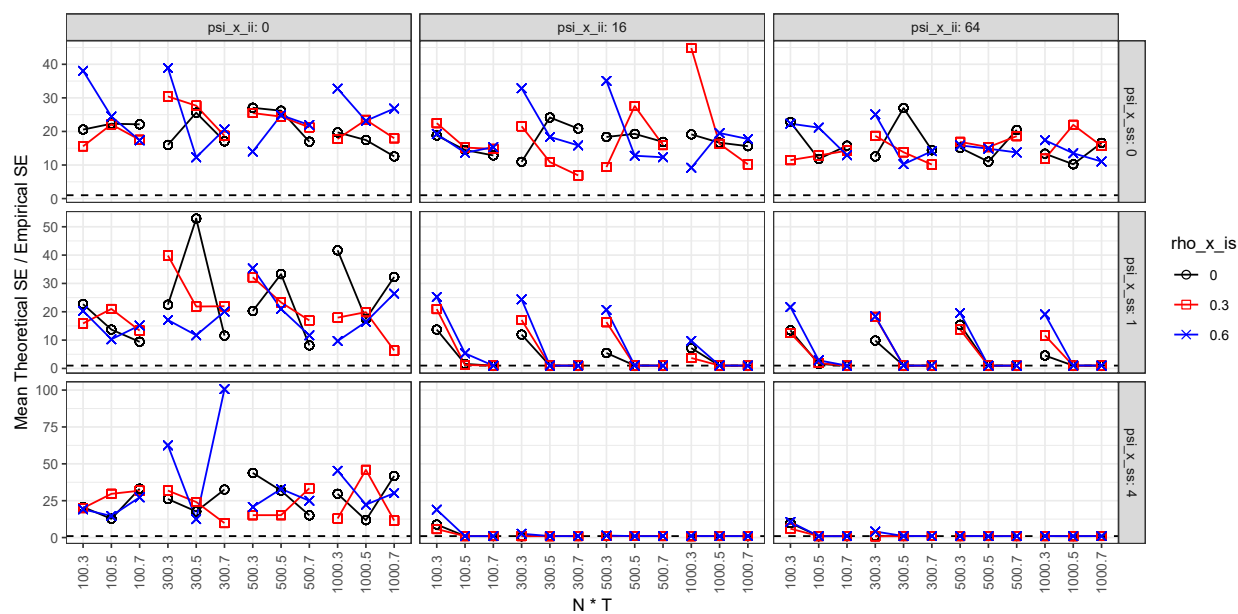
Figure E.166*SE Ratio for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)***Figure E.167***SE Ratio for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)*

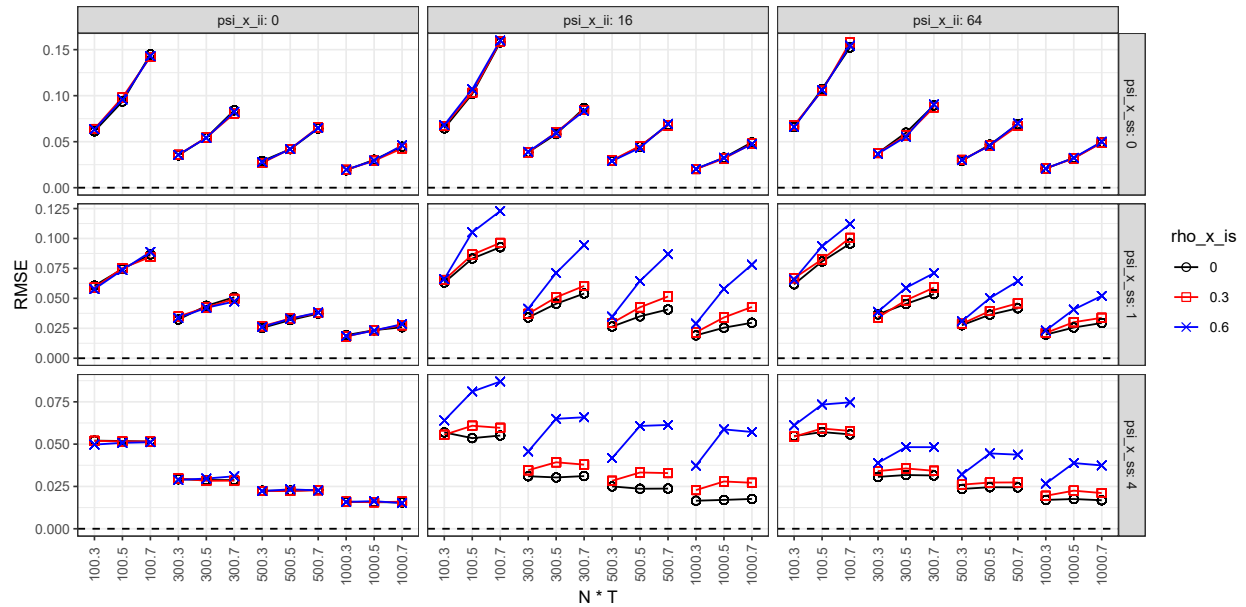
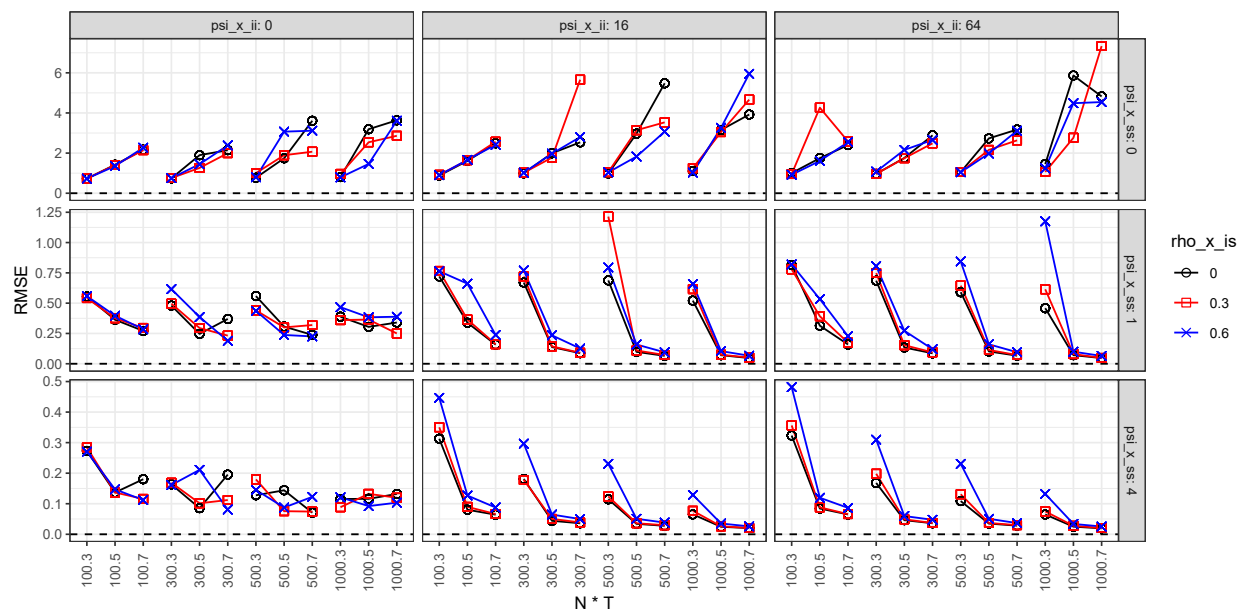
Figure E.168*RMSE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-E2S)***Figure E.169***RMSE for the Between-Person Effect β_{SS} in Simulation 5 (MLM-MV)*

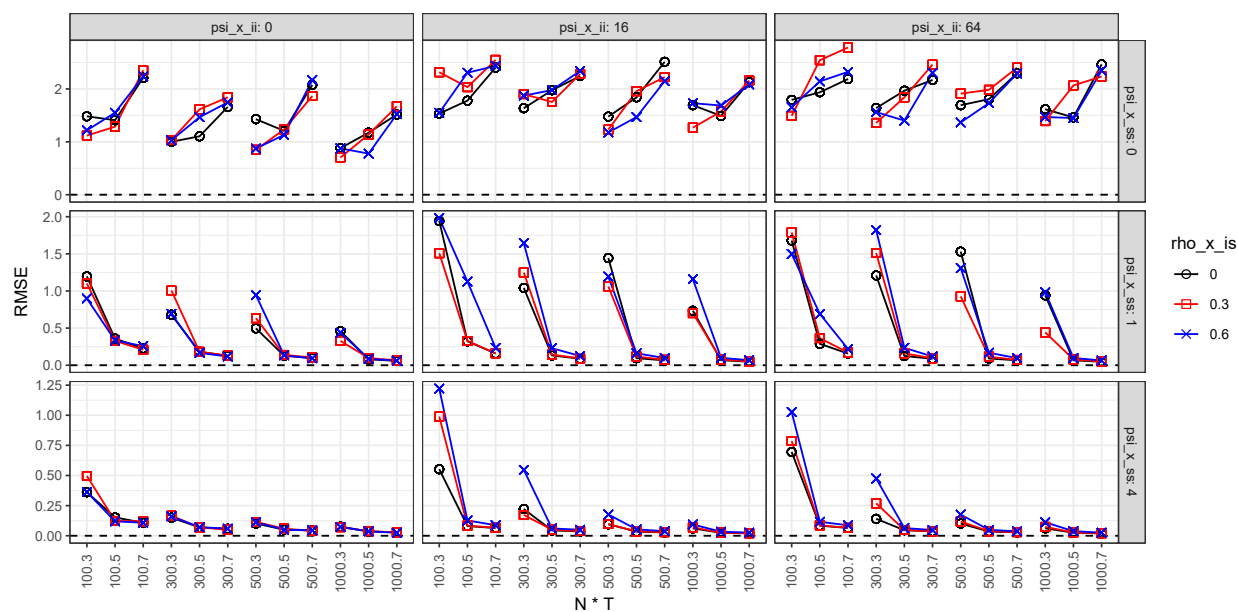
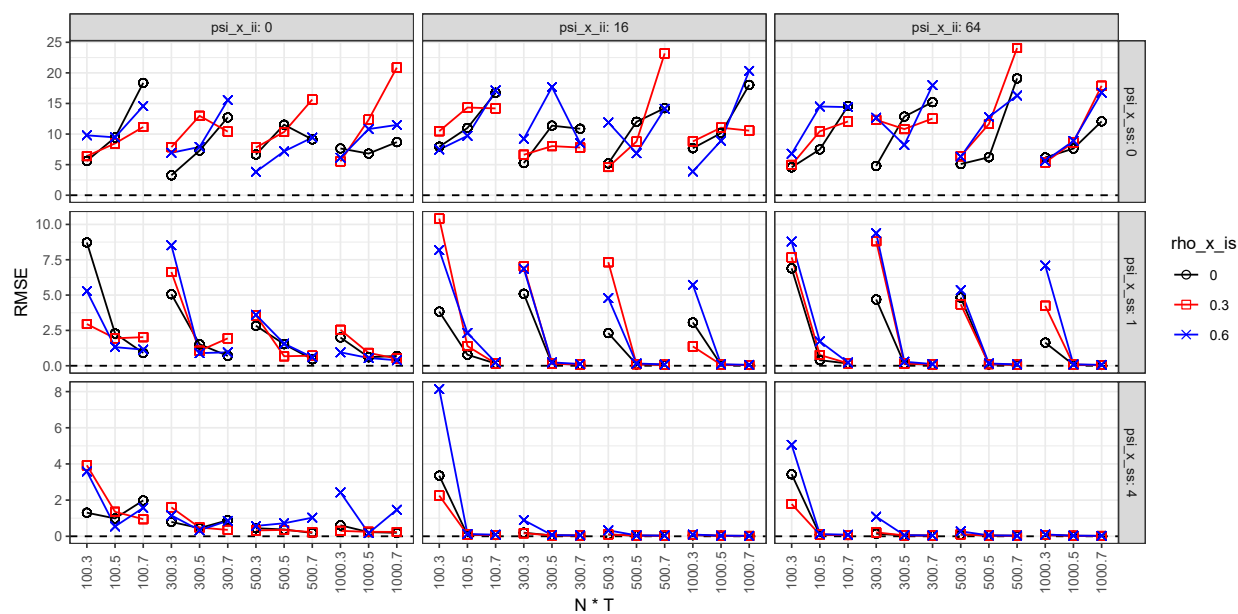
Figure E.170*RMSE for the Between-Person Effect β_{SS} in Simulation 5 (LCMxy-RP)***Figure E.171***RMSE for the Between-Person Effect β_{SS} in Simulation 5 (LCMx-CLC)*

Figure E.172

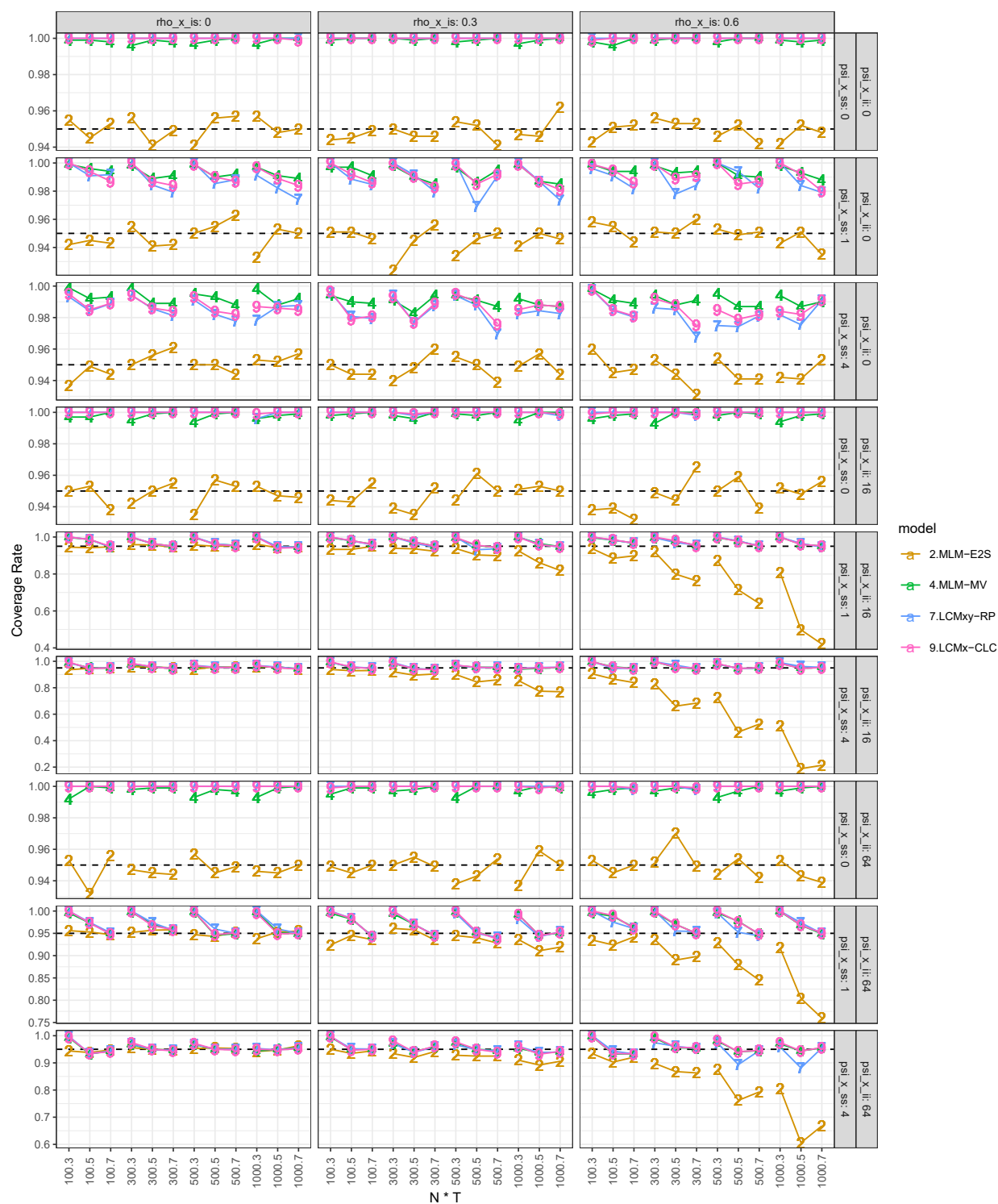
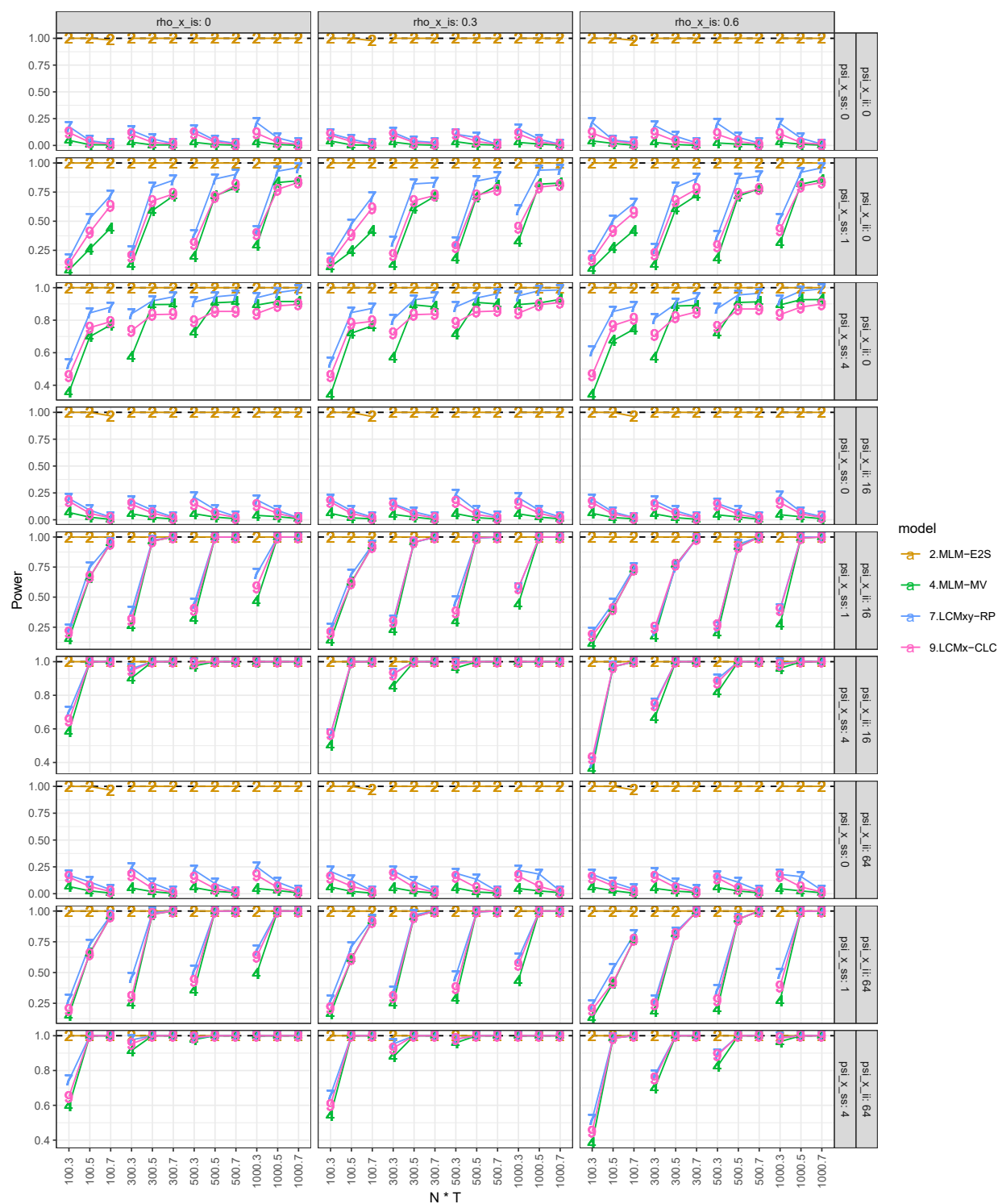
Coverage Rate for the Between-Person Effect β_{SS} in Simulation 5

Figure E.173

Power for the Between-Person Effect β_{SS} in Simulation 5

Appendix F

R Script for the Simulations

F.1 simulation.R

```
## Chapter 5: Simulation Studies
## simulation.R
## Kazuki Hori

## Simulation Number
simnum <- 1 # one of 1, 2, 3, 4, and 5
set.seed(simnum)

## Packages
library("foreach")
library("doParallel")
library("iterators")
library("mvtnorm")
library("lmerTest")
library("MplusAutomation")
library("tidyverse")

## Import user functions
source("checkers.R")

# Frequently used objects
vn <- c("term", "label", "est", "se", "stat", "pvalue") # variables in outputs
mod_mp <- c("MLM_MV", "LCMy_RIx", "LCMy_CLC",
           "LCMxy_RP", "LCMx_RIy", "LCMx_CLC") # models for Mplus

## -----
```

```

## Experimental Factors
N <- c(100, 300, 500, 1000)
T <- c(3, 5, 7)
gam <- switch(simnum, c(-0.6, 0, 0.6), c(-0.6, 0, 0.6),
              c(-0.6, 0, 0.6), c(-0.6, 0, 0.6), 0.6)
bii <- switch(simnum, c(0, 0.6), c(0, 0.6), c(0, 0.6), c(0, 0.6), 0.6)
bsi <- switch (simnum, 0, c(0, 0.3), 0, c(0, 0.3), 0.3)
bis <- switch (simnum, 0, 0, c(0, 0.3), c(0, 0.3), 0.3)
bss <- switch (simnum, 0, 0, 0, c(0, 0.6), 0.6)
psi_xii <- switch(simnum, 16, 16, 16, 16, c(0, 16, 64))
psi_xss <- switch(simnum, 0, 0, 1, 1, c(0, 1, 4))
rho_xis <- switch(simnum, 0, 0, 0.3, 0.3, c(0, 0.3, 0.6))

## Full Factorial Design
cond_ttbl <-
  crossing(N = N, T = T, gam = gam,
           bii = bii, bsi = bsi, bis = bis, bss = bss,
           psi_xii = psi_xii, psi_xss = psi_xss, rho_xis = rho_xis) %>%
  mutate(psi_xis = rho_xis * sqrt(psi_xii * psi_xss),
         cid = 1:n()) %>%
  as.data.frame()

## Export the condition table
write.csv(cond_ttbl, sprintf("sim%i_conditions.csv", simnum),
          row.names = FALSE, quote = FALSE)

## Other Parameters
tau_ix <- 50
tau_sx <- switch(simnum, 0, 0, 5, 5, 5)
tau_x <- c(tau_ix, tau_sx)
sgm_x2 <- 16
dlt_iy <- 100
dlt_sy <- switch(simnum, 0, 10, 0, 10, 10)
dlt <- c(dlt_iy, dlt_sy)
tht_yii <- 4
tht_yss <- switch(simnum, 0, 1, 0, 1, 1)
tht_yis <- 0.2 * sqrt(tht_yii * tht_yss)
tht <- matrix(c(tht_yii, tht_yis, tht_yis, tht_yss), 2, 2)
omg_y2 <- 4

```

```

## true parameter values
true_values <-
  cond_tbl %>%
  rename(gamma = gam,
         beta_ii = bii,
         beta_si = bsi,
         beta_is = bis,
         beta_ss = bss) %>%
  select(cid, gamma:psi_xis) %>%
  gather(label, true_value, -cid) %>%
  as.data.frame()

## -----

## Prepare for parallel computing
nc <- detectCores() # number of CPU cores (threads)
R <- 1000 # number of replications
cl <- makeCluster(nc)
registerDoParallel(cl)

## Monte Carlo Simulation
monitoring_file <- sprintf("sim%i_monitor.txt", simnum)
t0 <- proc.time()
for (i in 1:nrow(cond_tbl)) {

  cond <- cond_tbl[i, ] # the i-th condition

  ## Information of the current condition
  cat(as.character(Sys.time()),
      " Simulation ", simnum,
      " Current Condition: ", cond$cid,
      " (Out of " , nrow(cond_tbl), " Conditions; ",
      sprintf("%.01f%%", 100 * (cond$cid - 1) / nrow(cond_tbl)), " done; ",
      round((proc.time() - t0)[3] / 60, 1), " mins)\n", sep = "",
      file = monitoring_file, append = TRUE)

  ## Model Specification
  ## MLM-MV
  mod_MLM_MV <-

```



```

str_c("%WITHIN%",
      "Sx | X ON ctime;",
      "Sy | Y ON ctime;",
      "X Y (sxx syy);",
      "X WITH Y (sxy);",
      "%BETWEEN%",
      "[X Sx Y Sy];",
      "X Sx Y Sy (xixi xsxs sisi sysy);",
      "X WITH Sx Y Sy (xixs xiyi xiys);",
      "Sx WITH Y Sy (xsyi xsys);",
      "Y WITH Sy (yiys);",
      sep = "\n")
mod_const_MLM_MV <-
  str_c("NEW (gamma beta_ii beta_is beta_si beta_ss);",
        "gamma = sxy / sxx;",
        "beta_ii = (xsxs*xiyi - xixs*xsyi) / (xixi*xsxs - xixs**2);",
        "beta_is = (xixi*xsyi - xixs*xiyi) / (xixi*xsxs - xixs**2);",
        "beta_si = (xsxs*xiys - xixs*xsys) / (xixi*xsxs - xixs**2);",
        "beta_ss = (xixi*xsys - xixs*xiys) / (xixi*xsxs - xixs**2);",
        sep = "\n")

## LCMY-CLC
mod_LCMY_CLC <-
  str_c(str_c("Iy BY Y1-Y", cond$T, "@1;"),
        str_c("Sy BY ",
              str_c("Y", 1:cond$T,
                    "@", 1:cond$T - mean(1:cond$T),
                    collapse = " "),
              ";"),
        str_c("[Iy Sy Y1-Y", cond$T, "@0;"),
        "Iy Sy (yiyi ysys);",
        str_c("Y1-Y", cond$T, " (syy);"),
        str_c("Y1-Y", cond$T, " PON X1-X", cond$T, " (gam);"),
        str_c("Iy WITH X1-X", cond$T, " (iyx1-iyx", cond$T, ");"),
        str_c("Sy WITH X1-X", cond$T, " (syx1-syx", cond$T, ");"),
        str_c("X1-X", cond$T, " (",
              str_c("p", 1:cond$T, 1:cond$T, collapse = " "), ")"),
        str_c(map_chr(1:(cond$T-2),
                      ~ str_c("X", .x, " WITH X", .x+1, "-X", cond$T,

```

```

      " (p", .x, .x+1, "-p", .x, cond$T, ");");),
    collapse = "\n"),
  str_c("X", cond$T-1, " WITH X", cond$T, " (p", cond$T-1, cond$T, ");"),
  sep = "\n")
mod_const_LCMY_CLC <-
  str_c("NEW (gamma miyx msyx mcov mvar delta0 delta1 beta_ii beta_si);",
    "gamma = gam;",
    str_c("miyx = (", str_c("iyx", 1:cond$T, collapse = "+"), ")/", cond$T, ";"),
    str_c("msyx = (", str_c("syx", 1:cond$T, collapse = "+"), ")/", cond$T, ";"),
    str_c("mcov = 2 * (",
      exec(str_c,
        map(1:(cond$T-1),
          ~ str_c("p", .x, (.x+1):cond$T, collapse = "+")),
        collapse = "+\n"),
      ") / ", cond$T^2, ";"),
    str_c("mvar = (", str_c("p", 1:cond$T, 1:cond$T, collapse = "+"), ")/", cond$T^2, ";"),
    "delta0 = miyx / (mcov + mvar);",
    "delta1 = msyx / (mcov + mvar);",
    "beta_ii = delta0 + gamma;",
    "beta_si = delta1;",
    sep = "\n")

## LCMY-RIx
mod_LCMY_RIx <-
  str_c(str_c("Iy BY Y1-Y", cond$T, "@1;"),
    str_c("Sy BY ",
      str_c("Y", 1:cond$T,
        "@", 1:cond$T - mean(1:cond$T),
        collapse = " "),
      ";"),
    str_c("[Iy Sy Y1-Y", cond$T, "@0;"),
    "Iy Sy (iyiy sysy);",
    str_c("[Y1-Y", cond$T, "@0;"),
    str_c("Y1-Y", cond$T, " (syy);"),
    "Iy WITH Sy (iysy);",
    str_c("Ix BY X1-X", cond$T, "@1;"),
    "[Ix];", "Ix;",
    str_c("ex", 1:cond$T, " BY X", 1:cond$T, "@1;", collapse = "\n"),
    str_c("Y1-Y", cond$T, " PON ex1-ex", cond$T, " (gam);"),

```

```

str_c("ex1-ex", cond$T, "@0;"),
str_c("ex1-ex", cond$T, " (sxx);"),
str_c("Ix WITH ex1-ex", cond$T, "@0;"),
str_c(map_chr(seq(1, cond$T-2),
  ~ str_c("ex", .x, " WITH ex", .x+1, "-ex", cond$T, "@0;")),
  collapse = "\n"),
str_c("ex", cond$T-1, " WITH ex", cond$T, "@0;"),
str_c("X1-X", cond$T, "@0;"),
str_c("X1-X", cond$T, "@0;"),
"Iy Sy ON Ix (bii bsi);",
sep = "\n")
mod_const_LCMxy_RIx <-
str_c("NEW (gamma beta_ii beta_si);",
  "gamma = gam;",
  "beta_ii = bii;",
  "beta_si = bsi;",
  sep = "\n")

## LCMxy-RP
mod_LCMxy_RP <-
str_c(str_c("Ix BY X1-X", cond$T, "@1;"),
  str_c("Sx BY ",
    str_c("X", 1:cond$T, "@",
      1:cond$T - mean(1:cond$T), collapse = " "), ";"),
  "[Ix Sx];", "Ix Sx;", "Ix WITH Sx;",
  str_c("ex", 1:cond$T, " BY X", 1:cond$T, "@1;", collapse = "\n"),
  str_c("Y1-Y", cond$T, " PON ex1-ex", cond$T, " (gam);"),
  str_c("ex1-ex", cond$T, "@0;"),
  str_c("ex1-ex", cond$T, " (sxx);"),
  str_c("Ix WITH ex1-ex", cond$T, "@0;"),
  str_c("Sx WITH ex1-ex", cond$T, "@0;"),
  str_c(map_chr(seq(1, cond$T-2),
    ~ str_c("ex", .x, " WITH ex", .x+1,
      "-ex", cond$T, "@0;")),
    collapse = "\n"),
  str_c("ex", cond$T-1, " WITH ex", cond$T, "@0;"),
  str_c("Iy BY Y1-Y", cond$T, "@1;"),
  str_c("Sy BY ",
    str_c("Y", 1:cond$T, "@",

```

```

1:cond$T - mean(1:cond$T), collapse = " "), ";"),
"Iy Sy";", "Iy Sy;", "Iy WITH Sy;",
str_c("X1-X", cond$T, "@0;"),
str_c("X1-X", cond$T, "@0;"),
str_c("Y1-Y", cond$T, "@0;"),
str_c("Y1-Y", cond$T, " (sy);"),
"Iy ON Ix Sx (bii bis);",
"Sy ON Ix Sx (bsi bss);",
sep = "\n")
mod_const_LCMxy_RP <-
str_c("NEW (gamma beta_ii beta_is beta_si beta_ss);",
"gamma = gam;",
"beta_ii = bii;",
"beta_is = bis;",
"beta_si = bsi;",
"beta_ss = bss;",
sep = "\n")

## LCMx-RIy
mod_LCMx_RIy <-
str_c(str_c("Ix BY X1-X", cond$T, "@1;"),
str_c("Sx BY ",
str_c("X", 1:cond$T, "@",
1:cond$T - mean(1:cond$T), collapse = " "), ";"),
"Ix Sx";", "Ix Sx;", "Ix WITH Sx;",
str_c("ex", 1:cond$T, " BY X", 1:cond$T, "@1;", collapse = "\n"),
str_c("Y1-Y", cond$T, " PON ex1-ex", cond$T, " (gam);"),
str_c("ex1-ex", cond$T, "@0;"),
str_c("ex1-ex", cond$T, " (sxx);"),
str_c("Ix WITH ex1-ex", cond$T, "@0;"),
str_c("Sx WITH ex1-ex", cond$T, "@0;"),
str_c(map_chr(seq(1, cond$T-2),
~ str_c("ex", .x, " WITH ex", .x+1,
"-ex", cond$T, "@0;")),
collapse = "\n"),
str_c("ex", cond$T-1, " WITH ex", cond$T, "@0;"),
str_c("Iy BY Y1-Y", cond$T, "@1;"),
"Iy];", "Iy;",
str_c("X1-X", cond$T, "@0;"),

```

```

      str_c("X1-X", cond$T, "@0;"),
      str_c("[Y1-Y", cond$T, "@0];"),
      str_c("Y1-Y", cond$T, " (sy);"),
      "Iy ON Ix Sx (bii bis);",
      sep = "\n")
mod_const_LCMx_RIy <-
  str_c("NEW (gamma beta_ii beta_is);",
        "gamma = gam;",
        "beta_ii = bii;",
        "beta_is = bis;",
        sep = "\n")

## LCMx-CLC
mod_LCMx_CLC <-
  str_c(str_c("Ix BY X1-X", cond$T, "@1;"),
        str_c("Sx BY ",
              str_c("X", 1:cond$T, "@",
                    1:cond$T - mean(1:cond$T), collapse = " "), ";"),
        "[Ix Sx];", "Ix Sx (xii xss);", "Ix WITH Sx (xis);",
        str_c("ex", 1:cond$T, " BY X", 1:cond$T, "@1;", collapse = "\n"),
        str_c("Y1-Y", cond$T, " PON ex1-ex", cond$T, " (gam);"),
        str_c("ex1-ex", cond$T, "@0;"),
        str_c("ex1-ex", cond$T, " (sxx);"),
        str_c("Ix WITH ex1-ex", cond$T, "@0;"),
        str_c("Sx WITH ex1-ex", cond$T, "@0;"),
        str_c("Ix WITH Y1-Y", cond$T, " (pi1-pi", cond$T, ");"),
        str_c("Sx WITH Y1-Y", cond$T, " (ps1-ps", cond$T, ");"),
        str_c(map_chr(seq(1, cond$T-2),
                      ~ str_c("ex", .x, " WITH ex", .x+1,
                              "-ex", cond$T, "@0;")),
              collapse = "\n"),
        str_c("ex", cond$T-1, " WITH ex", cond$T, "@0;"),
        str_c("X1-X", cond$T, "@0;"),
        str_c("X1-X", cond$T, "@0;"),
        str_c("[Y1-Y", cond$T, "];"),
        str_c("Y1-Y", cond$T, ";"),
        str_c("Y1-Y", cond$T, " (",
              str_c("p", 1:cond$T, 1:cond$T, collapse = " "), ")"),
        str_c(map_chr(1:(cond$T-2),

```

```

      ~ str_c("Y", .x, " WITH Y", .x+1, "-Y", cond$T,
              " (p", .x, .x+1, "-p", .x, cond$T, ");"),
      collapse = "\n"),
  str_c("Y", cond$T-1, " WITH Y", cond$T, " (p", cond$T-1, cond$T, ");"),
  sep = "\n")
mod_const_LCMx_CLC <-
  str_c("NEW (gamma mpi mps mdpi mdps beta_ii beta_is beta_si beta_ss);",
        "gamma = gam;",

        str_c("mpi = (", str_c("pi", 1:cond$T, collapse = "+"),
              ")/", cond$T, ";"),
        str_c("mps = (", str_c("ps", 1:cond$T, collapse = "+"),
              ")/", cond$T, ";"),
        str_c("mdpi = 2*(",
              exec(str_c,
                  map(seq(1, cond$T-1),
                      ~ str_c("pi", seq(.x+1, cond$T),
                              "-pi", .x, ")/",
                              seq(.x+1, cond$T) - .x,
                              collapse = "+")),
                  collapse = "+\n"),
              ") / ", cond$T*(cond$T-1), ";"),
        str_c("mdps = 2*(",
              exec(str_c,
                  map(seq(1, cond$T-1),
                      ~ str_c("ps", seq(.x+1, cond$T),
                              "-ps", .x, ")/",
                              seq(.x+1, cond$T) - .x,
                              collapse = "+")),
                  collapse = "+\n"),
              ") / ", cond$T*(cond$T-1), ";"),
        "beta_ii = (xss*mpi - xis*mps) / (xii*xss - xis**2);",
        "beta_is = (xii*mps - xis*mpi) / (xii*xss - xis**2);",
        "beta_si = (xss*mdpi - xis*mdps) / (xii*xss - xis**2);",
        "beta_ss = (xii*mdps - xis*mdpi) / (xii*xss - xis**2);",
        sep = "\n")

## Replications
suppressMessages({

```

```

res <-
  foreach(r = 1:R, .combine = c,
    .errorhandling = "pass",
    .export = c("i", ls(pattern = "mod_.*")),
    .packages = c("lme4", "lmerTest",
      "MplusAutomation",
      "tidyverse", "mvtnorm")) %dopar% {

  ## Data Generation (DGP = LCM-RPO)
  psi_x <- with(cond, matrix(c(psi_xii, psi_xis, psi_xis, psi_xss), 2, 2))
  ISx <- rmvnorm(cond$N, tau_x, psi_x)
  Lambda <- cbind(1, 1:cond$T - mean(1:cond$T))
  eps_x <- with(cond, matrix(rnorm(N * T, 0, sqrt(sgm_x2)), N, T))
  X <- tcrossprod(ISx, Lambda) + eps_x
  colnames(X) <- str_c("X", 1:cond$T)
  B <- with(cond, matrix(c(bii, bsi, bis, bss), 2, 2))
  zeta_y <- rmvnorm(cond$N, sigma = tht)
  Delta <- matrix(dlt, cond$N, 2, byrow = TRUE)
  ISy <- Delta + ISx %*% t(B) + zeta_y
  nu_y <- with(cond, matrix(rnorm(N * T, 0, sqrt(omg_y2)), N, T))
  eps_y <- cond$gam * eps_x + nu_y
  Y <- tcrossprod(ISy, Lambda) + eps_y
  colnames(Y) <- str_c("Y", 1:cond$T)
  d_w <- data.frame(X, Y, id = 1:cond$N)
  d_l <-
    d_w %>%
    gather(variable, value, -id) %>%
    mutate(time = parse_number(variable),
      variable = str_sub(variable, 1, 1)) %>%
    spread(variable, value) %>%
    group_by(id) %>%
    mutate(Xm = mean(X), Xc = X - Xm, ctime = time - mean(time)) %>%
    as.data.frame()

  ## Estimation
  ## Models for lmer
  ## MLM-WB
  res_MLM_WB <- lmer(Y ~ Xc + Xm + (1 | id), data = d_l, REML = FALSE)

```

```

## MLM-E2S
# By Individual OLS Regressions
res_MLM_E2S <-
  d_l %>%
  group_by(id) %>%
  nest() %>%
  mutate(lmres = map(data, ~ lm(X ~ ctime, data = .x)),
         xint = map_dbl(lmres, ~ coef(.x)[1]),
         xslp = map_dbl(lmres, ~ coef(.x)[2]),
         xeps = map(lmres, resid)) %>%
  select(-lmres) %>%
  unnest(c("data", "xeps")) %>%
  as.data.frame() %>%
  lmer(Y ~ xeps + xint + xslp + ctime + xint:ctime + xslp:ctime +
       (1 + ctime | id), data = .)

## MLM-TAC
res_MLM_TAC <- lmer(Y ~ Xc + Xm*ctime + (1 + ctime | id),
                    data = d_l, REML = FALSE)

## Models for Mplus
res_mp <- vector("list", length(mod_mp)) %>% set_names(mod_mp)
for (m in mod_mp) {
  ## TITLE, VARIABLE, ANALYSIS, and OUTPUT statements
  title <- str_c(m, ": Simulation", simnum,
                "Condition", cond$cid,
                "Replication", r, "\n", sep = " ")
  output <- "CINTERVAL;\n"

  ## MODEL
  mod <- eval(parse(text = str_c("mod_", m)))

  ## MODEL CONSTRAINT
  mod_const <- eval(parse(text = str_c("mod_const_", m)))

  ## ANALYSIS and USEVARIABLES statements and DATA in R
  if(m == "MLM_MV") {
    d <- d_l
    usevar <- c("id", "X", "Y", "ctime")
  }
}

```



```

variable <- str_c("CLUSTER = id;",
                 "WITHIN = ctime;\n",
                 sep = "\n")
analysis <- str_c("TYPE = TWOLEVEL RANDOM;",
                 "ESTIMATOR = ML;\n", sep = "\n")
data_file <- str_interp("sim${simnum}_c${03i}{i}_r${04i}{r}_l.dat")
## Mplus Object
mp_obj <- mplusObject(TITLE = title,
                     rdata = d,
                     VARIABLE = variable,
                     usevariables = usevar,
                     ANALYSIS = analysis,
                     MODEL = mod,
                     MODELCONSTRAINT = mod_const,
                     OUTPUT = output)
} else {
  d <- d_w
  usevar <- names(d_w)[names(d_w) != "id"]
  variable <- NULL
  analysis <- str_c("TYPE = GENERAL;",
                   "ESTIMATOR = ML;",
                   "ITERATIONS = 10000;",
                   sep = "\n")
  data_file <- str_interp("sim${simnum}_c${03i}{i}_r${04i}{r}_w.dat")
  mp_obj <- mplusObject(TITLE = title,
                      rdata = d,
                      usevariables = usevar,
                      ANALYSIS = analysis,
                      MODEL = mod,
                      MODELCONSTRAINT = mod_const,
                      OUTPUT = output)
}
## Estimateion
inp_file <- str_interp("sim${simnum}_c${03i}{i}_r${04i}{r}_m.inp")
res_mp[[m]] <- mplusModeler(object = mp_obj,
                          dataout = data_file,
                          modelout = inp_file,
                          run = 1,
                          writeData= "always",

```

```

                                hashfilename = FALSE)

} # m

## Extract Estimates
## Models for lmer
## MLM-WB
est_MLM_WB <-
  summary(res_MLM_WB) %>%
  pluck("coefficients") %>%
  as.data.frame() %>%
  rownames_to_column("term") %>%
  mutate(model = "MLM_WB",
         cid = cond$cid,
         r = r,
         sim = simnum,
         label = c("tau_xi", "gamma", "beta_ii")) %>%
  rename(est = Estimate, se = `Std. Error`,
         stat = `t value`, pvalue = `Pr(>|t|)`)
est_MLM_WB <-
  confint(res_MLM_WB, method = "Wald") %>%
  as.data.frame() %>%
  rownames_to_column() %>%
  drop_na() %>%
  set_names(c("term", "ci_l", "ci_u")) %>%
  inner_join(est_MLM_WB, by = "term") %>%
  left_join(true_values, by = c("cid", "label")) %>%
  mutate(ci_covered = ci_l < true_value & true_value < ci_u)

## MLM-E2S
est_MLM_E2S <-
  summary(res_MLM_E2S) %>%
  pluck("coefficients") %>%
  as.data.frame() %>%
  rownames_to_column("term") %>%
  mutate(label = case_when(
    term == "(Intercept)" ~ "delta_iy",
    term == "xeps" ~ "gamma",
    term == "xint" ~ "beta_ii",
    term == "xslp" ~ "beta_is",

```

```

    term == "xint:ctime" ~ "beta_si",
    term == "xslp:ctime" ~ "beta_ss",
    term == "ctime" ~ "delta_sy")) %>%
  rename(est = Estimate, se = `Std. Error`,
          stat = `t value`, pvalue = `Pr(>|t|)`)
est_MLM_E2S <-
  confint(res_MLM_E2S, method = "Wald") %>%
  as.data.frame() %>%
  rownames_to_column() %>%
  drop_na() %>%
  set_names(c("term", "ci_l", "ci_u")) %>%
  inner_join(est_MLM_E2S, by = "term") %>%
  mutate(model = "MLM_E2S", cid = cond$cid, r = r, sim = simnum) %>%
  left_join(true_values, by = c("cid", "label")) %>%
  mutate(ci_covered = ci_l < true_value & true_value < ci_u)

## MLM-TAC
est_MLM_TAC <-
  summary(res_MLM_TAC) %>%
  pluck("coefficients") %>%
  as.data.frame() %>%
  rownames_to_column("term") %>%
  mutate(label = c("delta_ii", "gamma", "beta_ii", "delta_sy", "beta_si")) %>%
  rename(est = Estimate, se = `Std. Error`,
          stat = `t value`, pvalue = `Pr(>|t|)`)
est_MLM_TAC <-
  confint(res_MLM_TAC, method = "Wald") %>%
  as.data.frame() %>%
  rownames_to_column() %>%
  drop_na() %>%
  set_names(c("term", "ci_l", "ci_u")) %>%
  inner_join(est_MLM_TAC, by = "term") %>%
  mutate(model = "MLM_TAC", cid = cond$cid, r = r, sim = simnum) %>%
  left_join(true_values, by = c("cid", "label")) %>%
  mutate(ci_covered = ci_l < true_value & true_value < ci_u)

## Models for Mplus
est_mp <- vector("list", length(mod_mp)) %>% set_names(mod_mp)
for (m in mod_mp) {

```

```

par_est <- res_mp[[m]]$results$parameters$unstandardized
par_ci <- res_mp[[m]]$results$parameters$ci.unstandardized
if (ncol(par_est) > 3 && "se" %in% names(par_est)) {
  est_mp[[m]] <-
    par_est %>%
    data.frame() %>%
    select(paramHeader, param, est, se, est_se, pval) %>%
    set_names(vn) %>%
    mutate(across(c(est, se, stat, pvalue), as.numeric))
  est_mp[[m]] <-
    par_ci %>%
    data.frame() %>%
    transmute(term = paramHeader, label = param,
              ci_l = low2.5, ci_u = up2.5) %>%
    mutate(across(c(ci_l, ci_u), as.numeric)) %>%
    inner_join(est_mp[[m]], by = c("term", "label")) %>%
    mutate(label = tolower(label), model = m,
           df = Inf, cid = cond$cid, r = r, sim = simnum) %>%
    left_join(true_values, by = c("label", "cid")) %>%
    mutate(ci_covered = ci_l < true_value & ci_u > true_value)
} else {
  est_mp[[m]] <-
    par_est %>%
    data.frame() %>%
    mutate(se = NA, stat = NA, pvalue = NA, df = Inf,
           cid = cond$cid, r = r, sim = simnum,
           ci_l = NA, ci_u = NA, ci_covered = NA,
           model = m,
           param = tolower(param),
           term = paramHeader, label = param,
           across(c(est, se, stat, pvalue), as.numeric)) %>%
    select(-paramHeader, -param) %>%
    left_join(true_values, by = c("cid", "label"))
}
} # m

## Combine all estimates
est_all <-
  bind_rows(est_mp) %>%

```

```

bind_rows(est_MLM_WB, est_MLM_E2S, est_MLM_TAC) %>%
mutate(across(c(est, se, stat, pvalue, ci_l, ci_u), as.numeric))

## Frequency of Convergence
conv_info <-
  list(MLM_WB = res_MLM_WB,
        MLM_E2S = res_MLM_E2S,
        MLM_TAC = res_MLM_TAC) %>%
  map(slot, "optinfo") %>%
  map(pluck, "conv", "opt") %>%
  map(as.data.frame) %>%
  map_df(set_names, "converged", .id = "model") %>%
  mutate(converged = converged == 0,
         cid = cond$cid, r = r, sim = simnum)
conv_info <-
  res_mp %>%
  map_df(conv_checker, .id = "model") %>%
  gather(model, converged) %>%
  mutate(cid = cond$cid, r = r, sim = simnum) %>%
  rbind(conv_info) %>%
  as.data.frame()

## Check the positive-definiteness of the model-implied covariance matrices
pd_covmat <-
  list(MLM_WB = res_MLM_WB,
        MLM_E2S = res_MLM_E2S,
        MLM_TAC = res_MLM_TAC) %>%
  map(isSingular) %>%
  map(isFALSE) %>%
  map(as.data.frame) %>%
  map_df(set_names, "is_pd", .id = "model") %>%
  mutate(cid = cond$cid, r = r, sim = simnum)
pd_covmat <-
  res_mp %>%
  map_df(pd_checker, .id = "model") %>%
  gather(model, is_pd) %>%
  mutate(cid = cond$cid, r = r, sim = simnum) %>%
  rbind(pd_covmat) %>%
  as.data.frame()

```

```

## Goodness-of-fit indices
gof_list <-
  res_mp[names(res_mp) != "MLM_MV"] %>%
  map(pluck, "results", "summaries") %>%
  map_df(data.frame, .id = "model") %>%
  mutate(cid = cond$cid, r = r, sim = simnum) %>%
  as.data.frame()

## Remove data files, input files, and output files
dir(pattern = sprintf("sim%i_c%03i_r%04i", simnum, cond$cid, r),
  full.names = TRUE) %>% file.remove()

## Return results
out <- list(est = est_all,
            conv = conv_info,
            pd = pd_covmat,
            gof = gof_list)
return(list(out))
} # foreach
}) # suppressMessages

## Export results
outname <- sprintf("./output/simulation%i/sim%i_c%03i.rds",
  simnum, simnum, cond$cid)
saveRDS(res, outname)

gc(); Sys.sleep(3); gc(); Sys.sleep(3); gc() # free RAM
} # i

## Last update
cat(as.character(Sys.time()), " Simulation ", simnum,
  " Finished! (",
  round((proc.time() - t0)[3] / 60, 1), " mins)\n", sep = "",
  file = monitoring_file, append = TRUE)

stopCluster(cl) # Stop parallel computation

```

F.2 checkers.R

```
## Chapter 5: Simulation Studies
## checkers.R
## Kazuki Hori

## Functions used in the simulations
library("tidyverse") # install tidyverse if not installed

## Convergence checker for Mplus
conv_checker <- function(x) {
  if (length(x) == 0) {
    return(TRUE)
  } else {
    map_lgl(x$results$errors,
            ~ str_detect(string = str_c(.x, collapse = " "),
                          pattern = "NO CONVERGENCE",
                          negate = TRUE)) %>% all()
  }
}

## non-negative definiteness checker
pd_checker <- function(x) {
  if (length(x$results$warnings) == 0) {
    return(TRUE)
  } else {
    map_lgl(x$results$warnings,
            ~ str_detect(string = str_c(.x, collapse = " "),
                          pattern = "NOT POSITIVE DEFINITE")) %>%
    any() %>% isFALSE()
  }
}
```