

**SELECTION OF PURE ERROR GENERATORS FOR SIMULATION
EXPERIMENTS**

by

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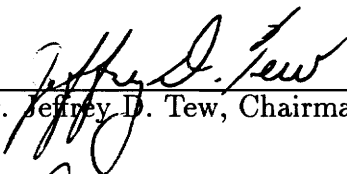
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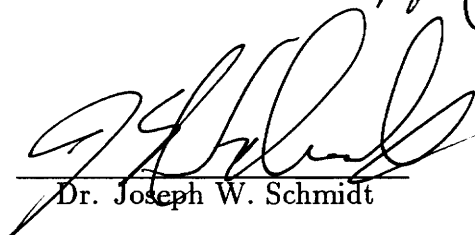
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(ABSTRACT)

To reduce the variance of metamodel coefficients, simulation analysts often advocate the use of correlation induction strategies. Under certain conditions, these strategies have been shown to reduce the variance of metamodel coefficients without producing significant bias in the coefficient estimates. Although these procedures are very useful for estimating metamodels, the application of many statistical analysis techniques is inappropriate unless the analyst is assured that a pure error component is present in the response. Crenshaw and Tew have demonstrated the absence of pure error in experiments in which all random number streams are used to induce correlations. Mihram argues that a pure error component can be maintained by selecting the seeds for at least one random component randomly and non-repetitively for all design points and replications.

In this thesis, random components for which seeds are randomly and non-repetitively selected are referred to as pure error generators. This thesis examines the selection of pure error generators in the context of univariate response, replicated simulation experiments. To assess the impact of pure error generator selection, we give the results of an extensive series of Monte-Carlo experiments in which the Schruben-Margolin strategy is applied for each possible pure error generator in each of two simulation models. To determine causes for the differences in pure error generator performance,

four pure error generator selection methods are outlined, tested, and compared to the results of the Monte-Carlo experiments. The results strongly suggest the importance of careful pure error generator selection and indicate that the primary difference in their performance is related to their ability to maintain the prescribed correlation structures of the correlation induction strategy.

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Chapter 1

1. Introduction

A simulation model is a computer model designed to mimic elements of the causal behavior of a system. There are many types of simulation models, but we restrict our attention to discrete event, stochastic simulation models in this thesis. Discrete event, stochastic simulation models have one or more *random components* and a definable system state that is completely described by a set of state variables whose values change at discrete points in time (see Law and Kelton 1991, pp. 7-13).

Simulation models are often used to analyze system behavior or system alternatives. To facilitate this analysis, classical statistical methods provide a powerful set of techniques provided the simulation response is independent and normally distributed with constant variance. That is, there must be at least some *pure error* in the response. In particular, metamodels are often used for the analysis of simulation output. Metamodels are mathematical, black-box representations of the causal and time dependent behavior of simulation models and usually have the form of least squares regression models. Thus, under the given assumptions the general linear models techniques of classical statistics are applicable.

To reduce the sample variance of metamodel coefficients, simulation analysts often advocate the use of correlation induction strategies. Under certain conditions, these strategies have been shown to reduce the sample variance of metamodel coefficients without producing significant bias in the coefficient estimates. However, Crenshaw and Tew (1992) have demonstrated that, for correlation induction strategies in which all *random number streams* are used to induce positive correlations, no pure error is present in the response. Crenshaw and Tew also show that the apparent

randomness in the response is due to *lack-of-fit* variation and *random block effects*. The absence of pure error in the response causes the variance to be non-homogeneous which results in a violation of the stated assumptions. Thus, classical statistical methods are not appropriate for the analysis of such experiments.

Mihram (1974) was the first to recognize the absence of pure error in responses from simulation experiments in which all random number streams are used to induce positive correlations. Mihram argues that a pure error component can be maintained by selecting the *seeds* for at least one *random component* randomly and non-repetitively for all design points and replications of a simulation experiment.

In this thesis, random components for which seeds are randomly and non-repetitively selected are referred to as pure error generators. This thesis focuses on the selection of pure error generators for univariate response, replicated simulation experiments. In particular, we examine the importance of carefully selecting pure error generators and illustrate four methods for selecting pure error generators as vehicles for studying pure error in simulation responses. In Chapter 2, we present and discuss the basic concepts and notation. In Chapter 3, we identify the problem of selecting pure error generators and describe four possible pure error generator selection methods. In Chapter 4, we present two examples and empirically examine performance of the four selection methods. In Chapter 5, we draw conclusions concerning the selection of pure error generators and give some suggestions for future research.

Chapter 2

2. Background and Literature Review

The purpose of this chapter is to identify the problem of selecting a pure error generator (PEG) and to present the basic concepts and notation. In this thesis, we approach the problem of selecting a PEG as design of simulation experiments problem, so the first three sections of this chapter are devoted to the review of some experimental design and simulation concepts. Section 2.4 is devoted to a review of the literature on pure error generation in simulation and the identification of the PEG selection problem.

2.1. Regression and Experimental Design

Most of the content of this section is commonly found in textbooks on linear statistical models, experimental design, regression, and response surface methodology. The descriptions and notation given here are principally drawn from Myers (1978, Chapters 3, 6, 7, and 8), Myers and Milton (1990, Chapter 3) and Myers (1991, Chapters 3, 5, and 6).

2.1.1. Linear Regression

Consider a regression model of the form

$$\mathbf{y} = \mathbf{\xi}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \tag{1}$$

where $\mathbf{y}$ is a $(n \times 1)$ vector of responses, $\mathbf{\xi}$ is a $(n \times p)$ data matrix, $\boldsymbol{\beta}$ is a $(p \times 1)$ vector of regression model coefficients, and $\boldsymbol{\epsilon}$ is a $(n \times 1)$ vector of error terms with mean $\mathbf{0}_{n \times 1}$ and variance $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}_{n \times n}$. Each row of $\mathbf{\xi}$ consists of a 1 in the first position, the

levels of k factors, and possibly some terms that are products of the k factors such that $p = 1 + k + l$, where l is the number of product terms included in the model.

The principal goal of regression is to estimate the unknown model coefficients, β , and make statistical inferences concerning the significance of the model coefficients and the ability of the model to predict responses at arbitrary locations in ξ -space. In this thesis, we restrict our attention to ordinary least squares (OLS) estimators (see Myers 1990, pp. 83-95). The OLS estimator of model coefficients is given by

$$\hat{\beta} = (\xi' \xi)^{-1} \xi' y. \quad (2)$$

When statistical inferences are needed on estimates obtained using OLS estimators, the assumption that ϵ is normally distributed is required.

2.1.2. Experimental Design

When the levels of the k factors are controllable, it is possible to design an experiment by preselecting those levels of the k factors that are of interest to the analyst. We will consider one particular class of experimental designs called 2^k factorial designs. A 2^k experimental design consists of k factors, each at two levels, arranged in all possible factor-level combinations. Often, to simplify analysis of designed experiments, the levels of the factors are centered and scaled such that

$$x_i = 2 \left(\frac{\xi_i - \bar{\xi}}{d_i} \right), \quad (3)$$

where d_i is the spacing between the low and high levels of ξ_i . To illustrate, consider a 2^2 factorial experiment replicated r times for the purpose of estimating the regression model given by

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \epsilon. \quad (4)$$

The corresponding design matrix can be written as

$$\mathbf{X} = \begin{bmatrix} \mathbf{1}_r & \mathbf{1}_r & \mathbf{1}_r & \mathbf{1}_r \\ \mathbf{1}_r & -\mathbf{1}_r & -\mathbf{1}_r & \mathbf{1}_r \\ \mathbf{1}_r & \mathbf{1}_r & -\mathbf{1}_r & -\mathbf{1}_r \\ \mathbf{1}_r & -\mathbf{1}_r & \mathbf{1}_r & -\mathbf{1}_r \end{bmatrix}, \quad (5)$$

where $\mathbf{1}_r$ is an $r \times 1$ column vector of ones. In this case, the design matrix, $\mathbf{X}$, is said to be orthogonal since $\mathbf{X}'\mathbf{X} = n\mathbf{I}$. The orthogonality property of 2^k designs leads to many desirable variance properties including the satisfaction of several optimality criteria (see Box, Hunter, and Hunter 1978, p. 504; Khuri and Cornell 1987, pp. 375-380; and Box and Draper 1987, Chapter 14).

2.1.3. Efficiency and Optimality Criteria

The purpose of designing experiments is to obtain the best estimated model possible in the fewest number of experimental runs. The quality of parameter estimates is usually determined by the variance of the estimate: the lower the variance of the estimate, the better the estimate. One way to measure the quality of an experimental design is precision of the design which is written as

$$\frac{n\text{var}[y(\mathbf{x}_0)]}{\sigma^2} = \frac{n\text{var}[\mathbf{x}_0'\hat{\boldsymbol{\beta}}]}{\sigma^2}, \quad (6)$$

where

$$\text{var}[y(\mathbf{x}_0)] = \sigma^2 \mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0 \quad (7)$$

is the variance of prediction at some point, $\mathbf{x}_0$, in x -space (see Myers 1976, pp. 146-147). Thus, the ratio of the precisions of a pair of competing designs is a measure of the relative efficiency of the designs. The covariance matrix of model coefficient estimators is given by

$$\text{cov}[\hat{\beta}] = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\Sigma\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} \quad (8)$$

which reduces to

$$\text{cov}[\hat{\beta}] = \frac{\sigma^2}{n}\mathbf{I} \quad (9)$$

when $\mathbf{X}$ is orthogonal and $\Sigma = \sigma^2\mathbf{I}$. Thus, we can rewrite equation (6) as

$$\frac{n\text{var}[y(\mathbf{x}_0)]}{\sigma^2} = \mathbf{x}_0'\mathbf{x}_0 \quad (10)$$

which leads to several optimality properties for two-level orthogonal designs such as 2^k factorials. In particular, two-level orthogonal designs are D-optimal, E-optimal, and G-optimal among designs with constant r and σ (see Khuri and Cornell 1987, pp. 378-379).

A D-optimal design is one for which the determinant of $\text{cov}[\hat{\beta}]$ is minimized. This implies that for D-optimal designs, the volume of the confidence ellipsoid given by

$$(\hat{\beta} - \beta)\mathbf{V}^{-1}(\hat{\beta} - \beta) \leq \frac{(n-1)p}{(n-p)n}F_{1-\alpha,p,n-p} \quad (11)$$

is minimized, where

$$\mathbf{V} = \widehat{\text{cov}[\hat{\beta}]} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{S}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1} \quad (12)$$

is the *sample* coefficient covariance matrix and $\mathbf{S}$ is the *sample* covariance matrix of $\mathbf{y}$.

An E-optimal design is one for which the sum of the eigenvalues of $\text{cov}[\hat{\beta}]$ is minimized. If $\Sigma = \sigma^2\mathbf{I}$ then the sum of the eigenvalues is simply the sum of the sample variances of the model coefficients. Assuming that all competing designs are

scaled such that the maximum distance of any design point from the center of the design is 1, all alternative designs have $\mathbf{X}'\mathbf{X} \geq \mathbf{I}_{n \times n}$. Since the variance of these estimates cannot be less than $\frac{\sigma^2}{n}$, two level orthogonal designs are E-optimal (see Khuri and Cornell p. 79 and p. 378).

A G-optimal design is one for which maximum coefficient variance is minimized. For 2^k designs, the variances of the model coefficient estimates are all equal to $\frac{\sigma^2}{n}$, so two level orthogonal designs are G-optimal.

2.1.4. Pure Error and Lack Of Fit

The experimental error, ϵ , represents the inability of a model to determine $\mathbf{y}$ exactly. Experimental error can be thought of as a composite of all of the extraneous factors in an experiment that are uncontrolled or simply not understood. The assumption that $\epsilon \sim N_n(\mathbf{0}_{n \times 1}, \sigma^2 \mathbf{I}_{n \times n})$ depends on these extraneous sources of variation being uncorrelated with the factors that are controlled in the experiment. If this is not the case, then at least some of the experimental error will be location dependent and is called lack-of-fit (LOF) (see Myers 1990, pp. 117-120).

To illustrate, consider a postulated model given by

$$\mathbf{y} = \mathbf{X}_1 \boldsymbol{\beta}_1 + \epsilon^* \quad (13)$$

and the theoretically correct, or *true* model given by

$$\mathbf{y} = \mathbf{X}_1 \boldsymbol{\beta}_1 + \mathbf{X}_2 \boldsymbol{\beta}_2 + \epsilon, \quad (14)$$

where $\mathbf{X}_1$ is $(n \times p)$ and corresponds to the p parameters of the postulated model, $\boldsymbol{\beta}_1$ is $(p \times 1)$ and contains the parameters of the postulated model, $\mathbf{X}_2$ is $(n \times (m - p))$ and corresponds to the additional parameters of the true model, m is the number of

distinct design points, β_2 is $((m - p) \times 1)$ and contains the additional parameters of the true model, and $\epsilon^* = \mathbf{X}_2\beta_2 + \epsilon$.

If the true model is fit to the data, then the total variability in the response can be written as

$$\mathbf{y}'\mathbf{y} = \mathbf{y}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} + SS_{PE}, \quad (15)$$

where SS_{PE} represents the variability in the response explained by pure error. If we fit the postulated model, then the total variability in the response can be written as

$$\mathbf{y}'\mathbf{y} = \mathbf{y}'\mathbf{X}_1(\mathbf{X}'_1\mathbf{X}_1)^{-1}\mathbf{X}'_1\mathbf{y} + \mathbf{y}'\mathbf{X}_2(\mathbf{X}'_2\mathbf{X}_2)^{-1}\mathbf{X}'_2\mathbf{y} + SS_{PE}. \quad (16)$$

If the additional factors in $\mathbf{X}_2$ are functions, such as products, of the factors in $\mathbf{X}_1$, then $\mathbf{y}'\mathbf{X}_2(\mathbf{X}'_2\mathbf{X}_2)^{-1}\mathbf{X}'_2\mathbf{y}$ is LOF variation (see Myers 1990, p. 119 and Khuri and Cornell 1987, pp. 38-43). In practice it is common for small amounts of LOF variation to be included in the estimate of error variance. Obviously, the assumption that the error variance is homogeneous is in jeopardy if LOF is included in its estimate since LOF depends on the location in x -space.

2.1.5. Blocking

Often, experimenters attempt to hold extraneous factors constant. This reduces the total variation in the response by eliminating variation due to the extraneous factors. However, some extraneous factors cannot be held constant. In this case, the experimenter may attempt to limit the number of levels that such a factor can take on in an experiment. This is known as blocking. Blocking factors can take on levels and have effects similar to non-blocking factors. Thus, they can be included in regression models as separate terms or their effects can be combined with LOF

(see Myers 1990, pp. 272-275). In either case, the variation due to blocking factors is removed from experimental error variation which results in a reduction of σ^2 (see Anderson and McLean 1974, Chapter 5).

2.2. Simulation Models

Most of the content of this section is commonly found in textbooks on simulation modeling and analysis such as Law and Kelton (1991) and Bratley, Fox, and Schrage (1987). The notation and descriptions given here are principally drawn from Law and Kelton (1991).

A simulation model can be defined as a set of mathematical and logical relationships and assumptions that is designed to mimic the behavior of a real world system over time. Several types of simulation models are commonly used, but we restrict our attention to discrete event, stochastic simulation models. Discrete event, stochastic simulation models contain one or more random components and have a definable state that is completely described by a set of state variables whose values change at discrete of finite points in time. Similar definitions of discrete event, stochastic simulation models can be found in Law and Kelton (1991, pp. 7-13), Bratley, Fox, and Schrage (1987, pp. 11-13), and Pritsker (1986, p. 52).

The random components of a system are modeled via *random variate generators*. Random variate generators are mathematical functions that use elements of *random number streams* as inputs to produce sequences of numbers that appear to follow a probability distribution specified by the nature of the random component. Random number streams are sequences of real numbers that are constructed to appear to be IID and uniformly distributed on the interval $[0,1]$. We will assume that each random component has a separate, dedicated random number stream. Because of the

functional relationship between random numbers and random variates, the behavior of a simulation model is completely determined by the random numbers used in the simulation. That is, the same random numbers will always produce the same responses provided the random numbers are used in the same order.

2.3. Designed Simulation Experiments

The control that simulation analysts have over random numbers used in a simulation experiment has led to the development of a set of specialized experimental design techniques known as variance reduction techniques (VRT). Typically, they are employed in order to reduce the variance of $\hat{\beta}$. In this section, we consider the *common random numbers* technique, the *antithetic variates* technique and a strategy that combines the two VRTs called the Schruben-Margolin correlation-induction strategy. In depth discussions of variance reduction techniques can be found in Wilson (1983 and 1984), Law and Kelton (1991, Chapter 11), Bratley, Fox, and Schrage (1987, Chapter 2), and Kleijnen (1975a). Nelson (1987) presents applications of variance reduction techniques in the context of two examples. Kleijnen (1975b) examines issues related to optimal computer time allocation for VRTs.

Although it is possible to design experiments for *batch means* experiments, we restrict our attention to replicated simulation experiments. Each replication of a simulation experiment is produced by selecting a set of *seeds* for the corresponding random number streams used in the experiment and then executing the simulation. A seed indicates the first element of a random number stream that is used in a simulation run. Subsequent random numbers are selected sequentially from the random number stream. Thus, randomly selecting a seed from a practically infinite population of possible seeds is equivalent to randomly selecting a starting point for sampling from the stream. Concerning the seeds and random number streams used in all of the

simulation experiments given in this thesis, we make the following assumptions:

1. The elements of all random number streams are IID $U[0,1]$.
2. The effect of any randomly selected seed on any response has mean zero for simulation runs of sufficient length.

Of course, random number streams are really *pseudorandom* since they are generated from mathematical functions. However, from a statistical standpoint these random number generators appear to be IID $U[0,1]$ (see Law and Kelton 1991, Chapter 7 and Bratley, Fox, Schrage 1987, Chapter 6). The second assumption simply means that it is impossible to select a *bad* seed when the length of a simulation run is of sufficient length such that a large number of random numbers are drawn from the corresponding stream.

In order to emphasize the dependency of the response on the random numbers used in the simulation experiment, we rewrite the regression model and denote the metamodel as

$$\mathbf{y}(\mathbf{R}) = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}(\mathbf{R}) \quad (17)$$

where $\mathbf{R}$ is an $(n \times g)$ matrix of random number streams and represents the set of random numbers used to generate the response. For an n design point simulation experiment, $\mathbf{R}$ is given by

$$\mathbf{R} = \mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_n \quad (18)$$

where $\mathbf{R}_i = (\mathbf{r}_{i1}, \mathbf{r}_{i2}, \dots, \mathbf{r}_{ig})$ ($i = 1, 2, \dots, n$) and $\mathbf{r}_{ij} = (r_{ij1}, r_{ij2}, \dots)'$ ($i = 1, 2, \dots, n$ and $j = 1, 2, \dots, g$), is the potentially infinite, random number stream used at the

i th design point.

We write the theoretical response at the i th design point as

$$y_i(\mathbf{R}_i) = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \varepsilon_i(\mathbf{R}_i); \text{ for } i = 1, 2, \dots, n. \quad (19)$$

2.3.1. Common Random Numbers

The common random numbers (CRN) technique is easy to implement and is often used for the purpose of reducing the variance of metamodel coefficient estimates. CRN is based on the idea of blocking on random numbers.

If r replications of an experiment are performed for m model configurations such that a randomly and independently selected set of random number seeds are used for each replication, and within each replication the same random numbers are used for all design points, then we have blocked the experimental material into r blocks of size m . Thus, a portion of the variability that would have otherwise been treated as random error can be attributed to a blocking factor (see Kleijnen 1985). This reduces the variance of parameter estimates and increases the power of statistical inferences.

In CRN, the same random numbers are used for each design point resulting in a positive homogeneous correlation induced among responses. In the simple case where $m = 2$ and $k = 1$, it is easy to show that this induced correlation results in a variance reduction. Suppose we want to estimate the model

$$y = \beta_0 + \beta_1 x. \quad (20)$$

The least squares estimates of the model coefficients are given by

$$\hat{\beta}_0 = \frac{\bar{y}_1 + \bar{y}_2}{2} \quad (21)$$

and

$$\hat{\beta}_1 = \frac{\bar{y}_1 - \bar{y}_2}{x_1 - x_2}, \quad (22)$$

where $\bar{y}_i$ ($i = 1, 2$) is the mean of the response at the x_i level of x .

The variances of these estimates are given by

$$\text{var}[\hat{\beta}_0] = \frac{1}{4n}(\text{var}[y_1] + \text{var}[y_2] - 2\text{cov}[y_1, y_2]) \quad (23)$$

and

$$\text{var}[\hat{\beta}_1] = \frac{1}{n(x_1 - x_2)^2}(\text{var}[y_1] + \text{var}[y_2] - 2\text{cov}[y_1, y_2]), \quad (24)$$

respectively.

Thus, when a positive correlation is induced between y_1 and y_2 a variance reduction occurs. See Law and Kelton (1991, pp. 620-628) and Wright and Ramsey (1979) for applications of the common random numbers technique.

2.3.2. Antithetic Variates

The antithetic variates (AV) technique is based on the idea of creating a set of observations by averaging two negatively correlated sets of observations in order to reduce the variance on the estimate of the population mean. The new set of observations can then be used to reduce the variance of metamodel coefficient estimates.

AV is implemented by making one set of n runs with a set of seeds, $\mathbf{R}$, and a second set of n runs with a set of seeds, $\bar{\mathbf{R}} = \mathbf{1} - \mathbf{R}$. Assuming the runs are made in

a paired and synchronized fashion and that all random components have monotonic distributions, each pair of antithetic responses will be negatively correlated. Synchronization implies that each random number that results from the same seed or antithetic seed will be used to generate random variates for the same random component and in the same order. Thus, under the given assumptions, the average of each pair of observations will be closer to population mean than either of the individual observations.

To illustrate, let y_{1i} and y_{2i} ($i=1,2,\dots,n$) be the responses for the i th antithetic pair of runs, and let $\bar{y}_i$ be the average of these responses. An unbiased estimate of the population mean is given by

$$\hat{\mu} = \frac{\sum_{j=1}^n \bar{y}_i}{n}, \quad (25)$$

where n is the number of antithetic paired runs. The variance of $\hat{\mu}$ is given by

$$\text{var}[\hat{\mu}] = \frac{\text{var}[y_1] + \text{var}[y_2] + 2\text{cov}[y_1, y_2]}{4n}. \quad (26)$$

Thus, if a negative correlation has successfully been induced between the pair of observations, $\text{cov}[y_1, y_2]$ will be negative and a variance reduction in our estimate of the population mean will result. See Cheng (1982) for a detailed discussion of AV.

2.3.3. Schruben-Margolin Strategy

Schruben and Margolin (1978) devised a correlation-induction strategy that effectively combines CRN and AV for the purpose of reducing the variance of first-order metamodel coefficient estimators. They show that, under certain restrictions and for a large class of experimental designs, their strategy is optimal is the estimation of β to the use of independent random number streams and to the use of CRN. Additional

comments on these experimental strategies and their comparison to the Schruben-Margolin correlation-induction strategy can be found in Tew (1986, 1989) and Tew and Wilson (1992a).

The Schruben-Margolin strategy is applicable when $\mathbf{X}$ is orthogonally blockable into two blocks of size n_1 and n_2 such that $n_1 + n_2 = n$. If we let $\mathbf{X} = [\mathbf{1}_n \ \mathbf{T}]$ then $\mathbf{X}$ is orthogonally blockable into two blocks if $\mathbf{X}$ is orthogonal and there exists an $n \times 2$ matrix, $\mathbf{W}$, of zeros and ones such that $\mathbf{T}'\mathbf{W} = \mathbf{0}_{(k-1) \times 2}$ and $\mathbf{1}_n' \mathbf{W} = [n_1 \ n_2]$.

Schruben and Margolin suggest the following assignment rule for designed simulation experiments that are orthogonally blockable into two blocks:

If $\mathbf{X}$ is orthogonally blockable into two blocks of sizes n_1 and n_2 , preferably chosen to be as nearly equal in size as possible, then for all n_1 observations in the first block, use a set of randomly selected seeds $\mathbf{R}$, and for all n_2 observations in the second block, use $\overline{\mathbf{R}}$.

At each design point in the experiment, Schruben and Margolin decompose the error term ε_i into a pure error term ε_i° and random block effect term b_i such that $\varepsilon_i = \varepsilon_i^\circ + b_i$. They propose the following first order model for experiments that employ their strategy:

$$\mathbf{y}(\mathbf{R}) = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{B}(\mathbf{R}) + \boldsymbol{\varepsilon}^\circ(\mathbf{R}), \quad (27)$$

where all terms are defined as before except $\mathbf{B}$ which is defined as

$$\mathbf{B} = \begin{bmatrix} b(\mathbf{R}) \\ b(\overline{\mathbf{R}}) \end{bmatrix}_{2 \times 2}. \quad (28)$$

Here, $b(\mathbf{R})$ and $b(\overline{\mathbf{R}})$ are random block effects for the blocks using $\mathbf{R}$ and $\overline{\mathbf{R}}$ respectively.

In addition to the usual linear models assumptions, Schruben and Margolin also make the following eight assumptions at each design point:

1. $E[\varepsilon_i^\circ(\mathbf{R}_i)] = E[b_i(\mathbf{R}_i)] = 0$;
2. $\text{var}[y_i(\mathbf{R}_i)] = \sigma^2$;
3. $\text{cov}[b_i(\mathbf{R}_i), b_j(\mathbf{R}_j)] = \rho_+ \sigma^2$ for $i \neq j$, where $0 \leq \rho_+ \leq 1$;
4. $\text{var}[\varepsilon_i^\circ(\mathbf{R}_i)] = \sigma^2(1 - \rho_+)$;
5. $\text{cov}[b_i(\mathbf{R}_i), b_j(\overline{\mathbf{R}}_i)] = -\rho_- \sigma^2$, where $-1 \leq -\rho_- \leq 0$;
6. $\text{cov}[b_i(\mathbf{R}_i), \varepsilon_j^\circ(\mathbf{R}_i)] = \text{cov}[b_i(\mathbf{R}_i), \varepsilon_j^\circ(\overline{\mathbf{R}}_i)] = 0$;
7. $\text{cov}[\varepsilon_i^\circ(\mathbf{R}_i), \varepsilon_j^\circ(\mathbf{R}_i)] = \text{cov}[\varepsilon_i^\circ(\mathbf{R}_i), \varepsilon_j^\circ(\overline{\mathbf{R}}_i)] = 0$ for $i \neq j$;
8. $\Sigma = \|\text{cov}[y_i, y_j]\|$ is positive definite.

These assumptions imply the following basic requirements for successful implementation of the Schruben-Margolin strategy: (1) the response variance is homogeneous at all design points, (2) there is a homogeneous positive correlation, ρ_+ between all pairs of responses that are generated from common random numbers, (3) there is a homogeneous negative correlation, ρ_- , between all pairs of responses that are generated from antithetic random numbers.

The assumptions can also be summarized as:

$$\Sigma_y^{sm} = \sigma^2 \left[\begin{array}{cccc|cccc} 1 & \rho_+ & \cdots & \rho_+ & \rho_- & \rho_- & \cdots & \rho_- \\ \rho_+ & 1 & \cdots & \rho_+ & \rho_- & \rho_- & \cdots & \rho_- \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_+ & \rho_+ & \cdots & 1 & \rho_- & \rho_- & \cdots & \rho_- \\ \hline \rho_- & \rho_- & \cdots & \rho_- & 1 & \rho_+ & \cdots & \rho_+ \\ \rho_- & \rho_- & \cdots & \rho_- & \rho_+ & 1 & \cdots & \rho_+ \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_- & \rho_- & \cdots & \rho_- & \rho_+ & \rho_+ & \cdots & 1 \end{array} \right], \quad (29)$$

where Σ_y^{sm} is the necessary covariance structure of the responses.

When the assumptions are satisfied, Schruben and Margolin prove the following:

1. The Schruben-Margolin strategy is D-optimal among (1) independent replications, (2) CRN, and (3) the Schruben-Margolin strategy provided

$$[1 + (n - 1)\rho_+ - (2/n)n_1n_2(\rho_+ + \rho_-)](1 - \rho_+)^{(k+1)} < 1.$$

2. The Schruben-Margolin strategy is superior to the use of CRN with respect to the $\text{var}[\hat{\beta}_0]$ and equivalent with respect to the variance of all other first-order model coefficients.
3. The Schruben-Margolin strategy is superior to the use of independent replications with respect to the variance of all first-order model coefficients.
4. The variances of the model coefficients are minimized when $n_1 = n_2$.

Additional discussion on the use of CRN and antithetic variates in combination is given in Kleijnen (1975b, p. 1177). Issues related to the satisfaction of the assumptions required by the Schruben-Margolin strategy are addressed in Tew (1986) and Tew and Wilson (1992b).

2.4. Pure Error in Simulation Experiments

Classical statistics provides a powerful set of techniques for the analysis of simulation experiments. However, the appropriateness of these techniques depends on the presence of pure error in the simulation response. A simulation model is a computer representation of a real system. Thus, the system that is used for experimentation is completely known and controlled by the experimenter. In this context, the meaning of pure error seems somewhat unclear.

To illustrate, consider a production process where orders for finished products arrive to a work-station and are subsequently processed on one of several identical machines. Suppose we are interested in studying the relationship given by

$$y = \beta_0 + \beta_1 x + \varepsilon, \tag{30}$$

where y is the average number of orders in queue for a day and x is the number of machines at the work-station.

An efficient way to conduct a two design point experiment would be to observe the operation of the system at one level of x and keep track of the sample responses as well as the time between arrivals, which operators are present, and other factors that might influence the performance of the machines and operators. We could then block on the interarrival times, operators, and other factors by forcing these factors to be close to the same for the other level of x . Although, some of the variation in the response that would have otherwise been attributed to error could be assigned to blocking factors, there would still be unexplained sources of variation in our response that we could treat as pure error. These sources of variation might be caused by measurement errors or our inability to reproduce the same operating conditions at both levels of x .

Now consider a situation in which we want to estimate the same regression model using a simulation model. In simulation, measurement error does not exist, and we have the absolute ability to reproduce operating conditions because we are using a computer model and because we have control over the random numbers used in an experiment. Thus, it is possible to block out all pure error variation by using the same random numbers for each design point (see Crenshaw and Tew 1992).

Mihram (1974) was the first to identify the lack of pure error in simulation responses that employ common random numbers for all random components. Mihram represents a simulation response as a function of a set of “environmental conditions”, $\mathbf{x}$, and a set of seeds, S , used to drive the simulation run that produces $y(\mathbf{x}; S)$. When independent replications are used to generate the responses, Mihram suggests the model given by

$$y(\mathbf{x}) = \mu(\mathbf{x}) + \varepsilon(\mathbf{x}, S), \quad (31)$$

where $\mu(\mathbf{x})$ is the mean response for a given $\mathbf{x}$. Mihram asserts that the use of CRN is equivalent to the idea of blocking in classical experimental design and suggests a model for simulation experiments that employ CRN given by

$$y(\mathbf{x}) = \mu(\mathbf{x}) + b(\mathbf{x}, s^*) + \varepsilon(\mathbf{x}, S - s^*), \quad (32)$$

where s^* represents the set of seeds that are not randomly and nonrepetitively selected, $S - s^*$ represents the set of seeds that are randomly and nonrepetitively selected and $b(\mathbf{x}, s^*)$ is a random block effect due to the repetitive use of s^* . Thus, when $S - s^*$ is empty, there is no pure error in the response. Mihram goes on to note that classical statistical analysis techniques are only valid in simulation experiments when a pure error component exists. This implies that the seeds for at least one

random component must be selected randomly and nonrepetitively in any simulation experiment that employs CRN.

Crenshaw and Tew (1992) provide empirical evidence to support Mihram. They examine the Nozari, Arnold, and Pegden (1987) statistical analysis procedure which was developed to analyze experiments that employ the Schruben-Margolin strategy. The procedure requires the assumption that a pure error component is present in the response. Crenshaw and Tew demonstrate that, in the absence of pure error, the Nozari, Arnold, and Pegden procedure is invalid. Crenshaw and Tew also assert that the correct model for experiments that employ the Schruben-Margolin strategy is given by

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{B}(\mathbf{R}), \quad (33)$$

instead of the one presented in Schruben and Margolin (1978) which is given by

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{B}(\mathbf{R}) + \boldsymbol{\varepsilon}^\circ, \quad (34)$$

where $\boldsymbol{\varepsilon}^\circ$ is assumed to be $N_n(\mathbf{0}_{n \times 1}, \sigma_{\varepsilon^\circ}^2 \mathbf{I}_{n \times n})$. Crenshaw and Tew show that, in experiments that employ the Schruben-Margolin strategy without a PEG, the resulting experimental error is comprised entirely of LOF. To demonstrate this, they conduct a series of experiments using a pair simulation models of known functional form. The first of these models has functional form given by

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon_1 + \varepsilon_2, \quad (35)$$

where β_0 , β_1 , and β_2 are fixed and ε_1 and ε_2 are generated by sampling from $N(0, \sigma_1^2)$ and $N(0, \sigma_2^2)$ random variate generators respectively. Using the Schruben-Margolin strategy, they proceed to conduct a 2^2 factorial experiment and to fit a regression

model of the form given by

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon. \quad (36)$$

Crenshaw and Tew assert that the true model for the experiment is given by

$$y = \beta_0 + b + \beta_1 x_1 + \beta_2 x_2 + \varepsilon_1 + \varepsilon_2, \quad (37)$$

where b is a random block effect and is assumed to be $N(0, \sigma_b^2)$. Their results indicate that the variances of β_1 and β_2 for the experiment are not significantly different from zero, and thus, no pure error component is present.

Next, they conduct a similar experiment using a simulation model of the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + (x_1^2 x_2^2)(\varepsilon_1 + \varepsilon_2), \quad (38)$$

where $(x_1^2 x_2^2)(\varepsilon_1 + \varepsilon_2)$ is location dependent and therefore nonhomogeneous. Again, they fit the model given by equation (36). Their results indicate that the variances of β_1 and β_2 are quite different from zero in this case. Thus, they conclude that this is a result of the nonhomogeneous variance in the simulation model or equivalently the LOF between the simulation model and the postulated model.

Thus, as suggested by Mihram (1974) and supported by Crenshaw and Tew (1992), the seeds for at least one random component in a given simulation model should be selected randomly and nonrepetitively across all design points in order to insure valid statistical analysis of simulation responses.

2.5. Correlated Replicates Strategies

Recently, several new simulation experimental designs for fitting first-order and second-order metamodels have been developed. Tew (1991) proposes three designs for fitting first-order metamodels that employ CRN and AV to correlate replicates. Under certain conditions specific to each design, Tew proves that all three are superior to the Schruben-Margolin strategy with respect to the variance of metamodel coefficients. All three designs employ PEGs which are denoted in the following discussion by $\mathbf{R}_{2ij}$ ($i = 1, 2, \dots, m$; $j = 1, 2, \dots, 2r$; where m is the number of design points and $2r$ is the number of replications).

The first of the three strategies is a variation of the Schruben-Margolin strategy that employs common random numbers within block/replicate combinations and antithetic variates between pairs of replicates within blocks. The layout of this design, which Tew calls Correlated Replicates I, is given in Table 1.

In addition to assumptions (1) and (8) from the Schruben-Margolin strategy, Correlated Replicates I requires the following assumptions:

1. $\text{var}[y_{ij}(\mathbf{R}_{ij})] = \sigma^2$;
2. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kj}(\mathbf{R}_{kj})] = \sigma^2 \rho_+$ where $0 \leq \rho_+ \leq 1$, $i \neq k$ and $1 \leq i, k \leq q$ or $q+1 \leq i, k \leq m$;
3. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kl}(\mathbf{R}_{kl})] = -\sigma^2 \rho'_-$ where $-1 \leq -\rho'_- \leq 0$, $1 \leq i, k \leq q$ or $q+1 \leq i, k \leq m$, $j = 1, 3, \dots, 2r-1$, and $l = j+1$;
4. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kl}(\mathbf{R}_{kl})] = 0$ otherwise.

Table 1					
Correlated Replicates I					
Design Point	1	2	Replicate		
1	$y_{11}(R_{111}, R_{211})$	$y_{12}(\bar{R}_{111}, R_{212})$	$\dots$	$2r-1$	$2r$
2	$y_{21}(R_{111}, R_{221})$	$y_{22}(\bar{R}_{111}, R_{222})$	$\dots$	$y_{1,2r-1}(R_{11,2r-1}, R_{21,2r-1})$	$y_{1,2r-1}(\bar{R}_{11,2r-1}, R_{21,2r})$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$y_{2,2r-1}(R_{11,2r-1}, R_{22,2r-1})$	$y_{1,2r-1}(\bar{R}_{11,2r-1}, R_{21,2r})$
$q = \frac{m}{2}$	$y_{q1}(R_{111}, R_{2q1})$	$y_{q2}(\bar{R}_{111}, R_{2q2})$	$\dots$	$y_{q,2r-1}(R_{11,2r-1}, R_{2q,2r-1})$	$y_{1,2r-1}(\bar{R}_{11,2r-1}, R_{21,2r})$
$q+1$	$y_{q+1,1}(R_{1,q+1,1}, R_{2,q+1,1})$	$y_{q+1,2}(\bar{R}_{1,q+1,1}, R_{2,q+1,2})$	$\dots$	$y_{q+1,2r-1}(R_{1,q+1,2r-1}, R_{2,q+1,2r-1})$	$y_{q+1,2r-1}(\bar{R}_{1,q+1,2r-1}, R_{2,q+1,2r})$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
m	$y_{m,1}(R_{1,q+1,1}, R_{2,m,1})$	$y_{m,2}(\bar{R}_{1,q+1,1}, R_{2,m,2})$	$\dots$	$y_{m,2r-1}(R_{1,q+1,2r-1}, R_{2,m,2r-1})$	$y_{m,2r}(\bar{R}_{1,q+1,2r-1}, R_{2,m,2r})$

These assumptions imply the covariance structure given by

$$\Sigma_I = \text{cov}[\bar{\mathbf{y}}] = \left(\frac{\sigma^2}{2r}\right) (1 - \rho_-) \mathbf{I}_m + \left(\frac{\sigma^2}{2r}\right) (\rho_+ + \rho'_-) \begin{bmatrix} \mathbf{E}_q & \mathbf{0}_q \\ \mathbf{0}_q & \mathbf{E}_q \end{bmatrix}, \quad (39)$$

where $\bar{\mathbf{y}} = \sum_{i=1}^{2r} \frac{\mathbf{y}}{2r}$, $\mathbf{E}_q$ is a $q \times q$ matrix of ones, and $\mathbf{0}_q$ is a $q \times q$ matrix of zeros. From equation (39) Tew obtains the metamodel coefficient covariance matrix given by

$$\text{cov}[\hat{\boldsymbol{\beta}}]_I = \left(\frac{\sigma^2}{2rm}\right) \left[\frac{\frac{m}{2}(\rho_+ - \rho'_-) + (1 - \rho_+)}{\mathbf{0}_k} \middle| \frac{\mathbf{0}'_k}{(1 - \rho_+) \mathbf{I}_k} \right], \quad (40)$$

where $\mathbf{0}_k$ is a $k \times 1$ column vector of zeros.

In the second of the three strategies, common random numbers is employed across design points within replicates and antithetic variates is employed between pairs of replicates. The layout of this design, which Tew calls Correlated Replicates II, is given in Table 2.

In addition to assumptions (1) and (8) from the Schruben-Margolin strategy, Correlated Replicates II requires the following assumptions:

1. $\text{var}[y_{ij}(\mathbf{R}_{ij})] = \sigma^2$;
2. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kj}(\mathbf{R}_{kj})] = \sigma^2 \rho_+$ where $0 \leq \rho_+ \leq 1$, $i \neq k$, $1 \leq i, k \leq m$;
3. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kl}(\mathbf{R}_{kl})] = -\sigma^2 \rho'_-$ where $-1 \leq -\rho_- \leq 0$, $1 \leq i, k \leq m$, $j = 1, 3, \dots, 2r - 1$, and $l = j + 1$;
4. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kl}(\mathbf{R}_{kl})] = 0$ otherwise.

These assumptions imply the covariance structure given by

$$\Sigma_{II} = \text{cov}[\bar{\mathbf{y}}] = \left(\frac{\sigma^2}{2r}\right) (1 - \rho_+) \mathbf{I}_m + \left(\frac{\sigma^2}{2r}\right) (\rho_+ - \rho_-) \mathbf{E}_m. \quad (41)$$

Table 2		Correlated Replicates II					
Design Point		Replicate					
		1	2	3	4	$2r - 1$	$2r$
1	$y_{11}(R_{111}, R_{211})$	$y_{12}(\bar{R}_{111}, R_{212})$	$y_{13}(R_{113}, R_{213})$	$y_{14}(\bar{R}_{113}, R_{214})$	$\dots$	$y_{1,2r-1}(R_{11,2r-1}, R_{21,2r-1})$	$y_{1,2r}(\bar{R}_{11,2r-1}, R_{21,2r})$
2	$y_{21}(R_{111}, R_{221})$	$y_{22}(\bar{R}_{111}, R_{222})$	$y_{23}(R_{113}, R_{223})$	$y_{24}(\bar{R}_{113}, R_{224})$	$\dots$	$y_{2,2r-1}(R_{11,2r-1}, R_{22,2r-1})$	$y_{2,2r}(\bar{R}_{11,2r-1}, R_{22,2r})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$y_{m1}(R_{111}, R_{2m1})$	$y_{m2}(\bar{R}_{111}, R_{2m2})$	$y_{m3}(R_{113}, R_{2m3})$	$y_{m4}(\bar{R}_{113}, R_{2m4})$	$\dots$	$y_{m,2r-1}(R_{11,2r-1}, R_{2m,2r-1})$	$y_{m,2r}(\bar{R}_{11,2r-1}, R_{2m,2r})$

From equation (41) Tew obtains the metamodel coefficient covariance matrix given by

$$\text{cov}[\hat{\beta}]_{II} = \left(\frac{\sigma^2}{2rm} \right) \left[\frac{m(\rho_+ - \rho_-) + (1 - \rho_+)}{\mathbf{0}_k} \middle| \frac{\mathbf{0}'_k}{(1 - \rho_+)\mathbf{I}_k} \right]. \quad (42)$$

In the third of the three strategies, common random numbers is employed across design points and pairs of replications and antithetic variates is employed between pairs of replicates. The layout of this design, which Tew calls Correlated Replicates III, is given in Table 3.

In addition to assumptions (1) and (8) from the Schruben-Margolin strategy, Correlated Replicates III requires the following assumptions:

1. $\text{var}[y_{ij}(\mathbf{R}_{ij})] = \sigma^2$;
2. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kj}(\mathbf{R}_{kj})] = \sigma^2 \rho_+$ where $0 \leq \rho_+ \leq 1$, $i \neq k$ and $1 \leq i, k \leq m$;
3. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kl}(\mathbf{R}_{kl})] = \sigma^2 \rho'_+$ for $|l - j| = 2, 4, \dots, 2r - 2$;
4. $\text{cov}[y_{ij}(\mathbf{R}_{ij}), y_{kl}(\mathbf{R}_{kl})] = -\sigma^2 \rho'_-$ where $-1 \leq -\rho_- \leq 0$, $|l - j| = 1, 3, \dots, 2r - 1$.

These assumptions imply the covariance structure given by

$$\Sigma_{III} = \text{cov}[\bar{y}] = \left(\frac{\sigma^2}{2r} \right) (1 - \rho'_+)\mathbf{I}_m + \frac{\sigma^2}{2r}(\rho'_+ - \rho'_-)\mathbf{E}_m, \quad (43)$$

From equation (38) Tew obtains the metamodel coefficient covariance matrix given by

$$\text{cov}[\hat{\beta}]_{III} = \left(\frac{\sigma^2}{2rm} \right) \left[\frac{\frac{m}{2}(\rho'_+ - \rho'_-) + (1 - \rho'_+)}{\mathbf{0}_k} \middle| \frac{\mathbf{0}'_k}{(1 - \rho'_+)\mathbf{I}_k} \right], \quad (44)$$

Table 3						
Correlated Replicates III						
Design Point	1	2	3	Replicate 4	...	2r - 1
1	$y_{11}(R_{111}, R_{211})$	$y_{12}(\bar{R}_{111}, R_{212})$	$y_{13}(R_{111}, R_{213})$	$y_{14}(\bar{R}_{111}, R_{214})$	...	$y_{1,2r-1}(R_{111}, R_{21,2r-1})$
2	$y_{21}(R_{111}, R_{221})$	$y_{22}(\bar{R}_{111}, R_{222})$	$y_{23}(R_{111}, R_{223})$	$y_{24}(\bar{R}_{111}, R_{224})$	...	$y_{2,2r-1}(R_{111}, R_{22,2r-1})$
:	:	:	:	:	:	:
m	$y_{m1}(R_{111}, R_{2m1})$	$y_{m2}(\bar{R}_{111}, R_{2m2})$	$y_{m3}(R_{111}, R_{2m3})$	$y_{m4}(\bar{R}_{111}, R_{2m4})$	...	$y_{m,2r-1}(R_{111}, R_{2m,2r-1})$
						$y_{m,2r}(\bar{R}_{111}, R_{2m,2r})$

Note that all three designs depend on PEGs in order to avoid repeating realizations of the response. For example, if a PEG is not used in Correlated Replicates III only two realizations of the response result and no pure error is present in the response.

Tew (1992) presents two correlated replicates designs for second-order metamod-els. Tew shows that his designs are superior to common random number with respect to the variance of metamodel coefficients and demonstrates that the metamodel parameter estimates obtained using his designs are unbiased. Again, both designs rely on PEGs to insure the presence of pure error in the response.

Chapter 3

3. Methodology

The need for pure error in simulation responses was identified by Mihram (1974) and the consequences of failing to include pure error in simulation responses have been studied by Crenshaw and Tew (1992). Mihram suggests that, to obtain a pure error component in simulation responses, the seeds for at least one random component should be selected randomly and non-repetitively for all simulation runs. Thus, the need for using a PEG in simulation experiments has been established in the literature.

In this chapter, the problem of selecting *efficient* PEGs is identified in the context of univariate response, replicated simulation experiments. We assume that simulation experiments have one of two goals: (1) obtain the *best* estimates of model parameters with the minimum experimental effort and (2) obtain the *best* predictive models with the minimum experimental effort. Referring to equations (7) and (8), there are two factor which affect these goals: (1) the experimental design, $\mathbf{X}$ and (2) the pure error variance, σ^2 . Thus, for a given experimental design the most efficient PEG is the one that yields the smallest pure error variance. However, since the effectiveness of correlation induction strategies depends in part on degree to which specified correlation structures are maintained, the effect of PEG selection on this correlation structure must be considered.

In Section 3.1, we give three measure of PEG efficiency that include the effect of PEG selection on correlation structure of the responses. In Section 3.2, we propose and discuss four methods for selecting efficient PEGs.

3.1. Measures of Pure Error Generator Efficiency

One particular measure of interest in simulation experiments is the sample variance of metamodel coefficient estimators. Low sample variances of metamodel coefficient estimators lead to more powerful inferences concerning the coefficients and to models with better predictive capability.

Recalling that $\mathbf{V}$ is the sample metamodel coefficient covariance matrix given by equation (12), we use three measures to compare PEG selections: (1) determinant of $\mathbf{V}$, (2) trace of $\mathbf{V}$, and (3) maximum diagonal element of $\mathbf{V}$. The reader should note the similarity of these three measures to D-optimality, E-optimality, and G-optimality, respectively. Of these three measures, we place emphasis on the determinant of $\mathbf{V}$ since it is the only measure that accounts for the non-diagonal elements of $\mathbf{V}$. Because some simulation experimental designs, such as the Schruben-Margolin and correlated replicates strategies, require a specific correlation structure, departures from these correlation structures may manifest themselves in inflated metamodel coefficient variance or covariance estimates. Thus, the three aggregate coefficient variance measures given here may provide a means of detecting and measuring these departures.

3.2. Methods for Selecting Efficient Pure Error Generators

One way to select an efficient PEG is to conduct a set of pilot experiments and estimate the desired metamodel and metamodel coefficient covariance matrix, $\mathbf{V}$. The efficiency of each PEG could then be measured using one of the efficiency measures given in Section 3.1, and an efficient PEG could be selected.

For the purposes of studying PEGs and their efficiencies, we illustrate four PEG selections methods. All four methods assume that the necessary correlation structure can be maintained when any random component is employed as a PEG. Ignoring the

necessary correlation structure, these four methods attempt to isolate the random component that contributes the least to the variability of the response. If the correlation structure is maintained, then the random component that contributes little to the variability of the response should lead to a small pure error variance when it is employed as a PEG. This should, in turn lead to small metamodel coefficient variance estimates. The principal difference in the four methods is the approach taken to determine the contribution of the random component to the variability of the response.

Before proceeding to describe the four PEG selection methods, the following assumptions are made concerning the characteristics of the simulation models and the experiments in which they will be used:

1. The run length of any simulation experiment where a selected PEG is used must be sufficiently long such that at least a small sample of random variates is generated from the PEG.
2. Only one random component is required to generate pure error in the response.
3. The only basis for the selection of a particular PEG is the observed efficiency of that PEG.
4. All random components contribute positively to the variance of the response.
5. All random components have continuous distributions except for branching components.

3.2.1. Variance Method 1

In Variance Method 1 (V1), we attempt to isolate the contribution of each random component to the variability of the response by making each of g random components constant, one a time, estimating a corresponding set of g response variances, and comparing these variance estimates. Thus, this method requires the experimenter to conduct a series of g experiments, where g is the number of random components in the given simulation model. The purpose of the i th experiment ($i = 1, 2, \dots, g$) is to estimate σ_{-i}^2 , where σ_{-i}^2 is the variance of the response when random component i is made constant.

For non-branching components, estimates of σ_{-i}^2 can be obtained by altering the simulation model such that the i th random component takes its mean. Using the mean is equivalent to making the variance of the random component equal to zero. Thus, using the mean allows the effect of the variability contributed by the component to be determined by its absence.

For branching components, all possible branches must be considered to estimate σ_{-i}^2 . For each branch, an estimate of $\sigma_{-i,j}^2$ can be obtained, where $\sigma_{-i,j}^2$ is the variance of the response when the i th random component is set to the j th branch. Note that if a branching component has h branches then

$$\text{var}[y] = E[\text{var}[y|B]] + \text{var}[E[y|B]], \quad (45)$$

where, in this case, B is the random variable that maps the h branches into the real numbers and $B = \{1, 2, \dots, h\}$ is the corresponding sample space (see Ross 1989, p. 93). Thus, an estimate of the variance of σ^2 is given by

$$\hat{\sigma}_{-i}^2 = \sum_{j=1}^h p_{ij} \hat{\sigma}_{-i,j}^2 + \sum_{j=1}^h \frac{(\bar{y}_{ij} - \bar{\bar{y}}_i)^2}{h-1} \quad (46)$$

where p_{ij} is the branch probability of the j th branch of random component i , $\bar{y}_{ij}$ is the mean of the response when the j th branch of random component i is taken, and $\bar{\bar{y}}_i = \sum_{j=1}^h \frac{\bar{y}_{ij}}{h}$.

The selection procedure for V1 can be summarized as follows:

1. Obtain estimates of $\sigma_{-i,j}^2$ and $\bar{y}_{ij}$ for each branching component.
2. Obtain variance estimates, σ_{-i}^2 , directly for non-branching components and with equation (46) for branching components.
3. Select as the PEG the random component that, when held constant, yields the highest variance estimate.

Because we are assuming that all random components contribute positively to the variance of the response, selecting the random component that yields the highest variance estimate is equivalent to selecting the random component that yields a variance estimate that is closest to the variance of the response when nothing is held constant.

Note that there is no need for a ranking and selection procedure here since we are assuming that we only need one random component to generate pure error and that any random component can serve as a PEG. Selecting the random component that yields the highest variance estimate is adequate since it is at least as good as any other. That is, it would be impossible to statistically show that any other random component is better.

To illustrate V1, consider the activity network depicted in Figure 1. Random

component definitions for the example activity network are given in Table 4. Entities arrive to node 1 with interarrival times determined by samples from the distribution of random component 1 and are subsequently routed to node 2 or node 3 with probability p_{11} and p_{12} respectively. Entities that are routed to node 3 receive service of length determined by a sample from the distribution of random component 4 and depart the system. Entities that are routed to node 2 receive service of length determined by a sample from the distribution of random component 3 and are subsequently routed to node 4 or node 5 with probability p_{21} and p_{22} respectively. Entities routed to node 4 receive service of length determined by a sample from the distribution of random component 6 and depart the system. Entities routed to node 5 receive service of length determined by a sample from the distribution of random component 7 and depart the system.

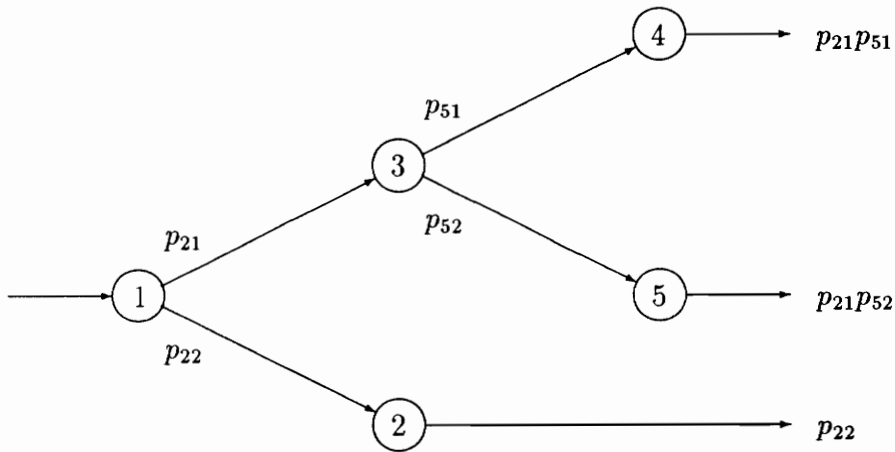


Figure 1: Example Activity Network

Table 4	
Random Component Definitions for the Example Activity Network	
Random Component	Definition
1	interarrival time
2	branching from node 1
3	service time at node 2
4	service time at node 3
5	branching from node 2
6	service time at node 4
7	service time at node 5

In order to obtain the 7 variance estimates required for V1, step one of the stated procedure is applied to random components 2 and 5. These are branching components. For random component 2 the branch is set to node 2, and a number of replications are made, resulting in estimates of $\sigma_{-2,1}^2$ and $\bar{y}_{21}$. The branch is then set to node 3 and a number of replications are made, resulting in estimates of $\sigma_{-2,2}^2$ and $\bar{y}_{22}$. Using equation (46), the variance estimate for random component 2 is given by

$$\hat{\sigma}_{-2}^2 = p_{21}\hat{\sigma}_{-2,1}^2 + p_{22}\hat{\sigma}_{-2,2}^2 + (\bar{y}_{21} - \bar{\bar{y}}_2)^2 + (\bar{y}_{22} - \bar{\bar{y}}_2)^2$$

Repeating this procedure for both branches at random component 5, an estimate of σ_{-5}^2 is obtained.

Next, step two of the stated procedure is applied to the remaining random components. For random component 1 the component is set to the mean interarrival time, and a number of replications are made, resulting in an estimate of σ_{-1}^2 . The variance estimates for the remaining random components are obtained in a similar fashion. The PEG selection is made by selecting the random component that yielded the highest variance estimate.

3.2.2. Variance Method 2

In Variance Method 2 (V2), we attempt to isolate the contribution of each random component to the variability of the response by making all random components in the model constant except the one whose variance contribution is of interest. This method requires the experimenter to conduct a series of g experiments, where g is the number of random components in the given simulation model. The purpose of the i th experiment ($i=1,2,\dots,g$) is to estimate σ_{-i}^2 , where σ_{-i}^2 is the variance of the response when all components except the i th are made constant.

When no branching components are present in the simulation model, estimates of σ_{-i}^2 can be obtained by altering the simulation model such that all random component except the i th are set to their means. When branching components are present in the given simulation model, all possible path through the network must be considered in order to estimate σ_{-i}^2 . For the j th path we can estimate $\sigma_{-i,j}^2$, the variance of the response when all random components except the i th are made constant and the j th path is taken. Using equation (45) with $B = \{1, 2, \dots, f\}$, an estimate of σ_{-i}^2 is given by

$$\hat{\sigma}_{-i}^2 = \sum_{j=1}^f p_j \hat{\sigma}_{-i,j}^2 + \sum_{j=1}^f \frac{(\bar{y}_{ij} - \bar{\bar{y}}_i)^2}{f-1} \quad (47)$$

where p_j is the path probability corresponding to the j th path, f is the number of paths, $\bar{y}_{ij}$ is the mean of response when random component i is random and the j th path is taken, and $\bar{\bar{y}}_i = \sum_{j=1}^f \frac{\bar{y}_{ij}}{f}$.

The selection procedure for V2 can be summarized as follows:

1. Obtain estimates of $\sigma_{-i,j}^2$ and $\bar{y}_{ij}$, for each random component/path combination.

2. Use equation (47) to obtain variance estimates, $\hat{\sigma}_{-i}^2$, for all random components.
3. Select as the PEG the random component that yields the lowest variance estimate.

To illustrate, consider again the activity network example given in Section 3.2.1. In order to obtain the 7 variance estimates required for V2, step one of the stated procedure is applied to each random component. Note that there are three paths: (1) node 1 - node 2 - node 4, (2) node 1 - node 2 - node 5, and (3) node 1 - node 3. The corresponding path probabilities are $p_{21}p_{51}$, $p_{21}p_{52}$, and p_{22} respectively. For RC_1 the path is set to combination 1 and a number of replications are made, resulting in estimates of $\sigma_{-i,1}^2$ and $\bar{y}_{11}$. The path is then set to combination 2 and a number of replications are made, resulting in estimates of $\sigma_{-i,2}^2$ and $\bar{y}_{12}$. The path is then set to combination 3 and a number of replications are made, resulting in estimates of $\sigma_{-i,3}^2$ and $\bar{y}_{13}$. Applying equation (47), the variance estimate for random component 1 is given by

$$\hat{\sigma}_{-i}^2 = p_{21}p_{51}\hat{\sigma}_{-i,1}^2 + p_{21}p_{52}\hat{\sigma}_{-i,2}^2 + p_{22}\hat{\sigma}_{-i,3}^2 + \left[\frac{(\bar{y}_{11} - \bar{\bar{y}}_1)^2 + (\bar{y}_{12} - \bar{\bar{y}}_1)^2 + (\bar{y}_{13} - \bar{\bar{y}}_1)^2}{2} \right].$$

This procedure is repeated for each component, and the PEG selection is made by selecting the random component that yields the lowest variance estimate.

3.2.3. Simple Correlation Method

In the simple correlation method (SC), we attempt to determine which random component is least correlated with the response. The simple correlation between y and $\bar{z}_i$ is given by

$$\rho_{y\bar{z}_i} = \frac{\sigma_{y\bar{z}_i}}{\sqrt{\sigma^2\sigma_{\bar{z}_i}^2}}, \quad (48)$$

where $\bar{z}_i$ is the mean, over an entire simulation run, of the random variates generated from random component i ; y is the response from the simulation run; $\sigma_{y\bar{z}_i}$ is the covariance of y and $\bar{z}_i$; σ^2 is the variance of y ; and $\sigma_{\bar{z}_i}^2$ is the variance of $\bar{z}_i$. Thus, simple correlation is simply a scaled covariance and measures the variability explained by a linear relationship between y and $\bar{z}_i$ (see Anderson 1984, pp. 103-125). If the linear relationship between y and $\bar{z}_i$ is strong, then simple correlation provides a good measure of the variability in the response due to $\bar{z}_i$.

The simple correlation method requires the experimenter to make r independent replications and to estimate $\rho_{y\bar{z}_i}$ ($i = 1, 2, \dots, g$). The maximum likelihood estimator of $\rho_{y\bar{z}_i}$ is given by

$$\hat{\rho}_{y\bar{z}_i} = \frac{\sum_{j=1}^r (y_j - \bar{y})(\bar{z}_j - \bar{\bar{z}})}{\sqrt{\sum_{j=1}^r (y_j - \bar{y})^2} \sqrt{\sum_{j=1}^r (\bar{z}_j - \bar{\bar{z}})^2}} \quad (49)$$

(see Anderson 1984, p. 103). The PEG selection is made by selecting the random component with the smallest sample simple correlation.

For the example activity network given in Section 3.2.1, the set of 7 simple correlation estimates are obtained by calculating the mean random variate generated from each of the 7 random components for each of r independent runs. The mean random variate generated for each run can be calculated within the simulation model just as other statistics are calculated, or a record of the random variates generated during the run can be output and averaged. Equation (49) is then applied, and the random component that has the smallest sample simple correlation with the response is selected as the PEG.

3.2.4. Partial Correlation Method

In the partial correlation method (PC), we attempt to determine which random

component is least correlated with the response. However in this case, we attempt to adjust our estimate of the correlation for extraneous sources of variation by using partial correlation. The partial correlation between y and $\bar{z}_i$ is given by

$$\rho_{y\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g} = \frac{\sigma_{y\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}}{\sqrt{\sigma_{y|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}^2 \sigma_{\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}^2}}, \quad (50)$$

where $\bar{z}_i$ and y are defined as before; $\sigma_{y\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}$ is the covariance of y and $\bar{z}_i$ given all other variables are fixed; $\sigma_{y|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}^2$ is the variance of y given all other variables save $\bar{z}_i$ are fixed; $\sigma_{\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}^2$ is the variance of $\bar{z}_i$ given all other variables, save y , are fixed. Partial correlation is similar to simple correlation except that partial correlation is the additional variability explained by adding $\bar{z}_i$ to a linear model that includes $\bar{z}_1, \bar{z}_2, \dots, \bar{z}_{i-1}, \bar{z}_{i+1}, \dots, \bar{z}_g$ (see Anderson 1984, pp. 125-134). Again, if the linear relationship between y and $\bar{z}_i$ is strong, then partial correlation provides a good measure of the variability in the response due to $\bar{z}_i$. To simplify the notation, we write the partial correlation between y and $\bar{z}_i$ as $\rho_{p_{y\bar{z}_i}}$.

The partial correlation method requires the experimenter to make r independent replications and to estimate $\rho_{p_{y\bar{z}_i}}$ ($i = 1, 2, \dots, g$). An unbiased, maximum likelihood estimator of $\rho_{p_{y\bar{z}_i}}$ is given by

$$\hat{\rho}_{p_{y\bar{z}_i}} = \frac{\hat{\sigma}_{y\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}}{\sqrt{\hat{\sigma}_{y|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}^2 \hat{\sigma}_{\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}^2}}, \quad (51)$$

where $\hat{\sigma}_{y\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}$ is the (1,2) or (2,1) element of $\mathbf{S}_{11.2}$, $\hat{\sigma}_{y|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}^2$ is the (1,1) element of $\mathbf{S}_{11.2}$, $\hat{\sigma}_{\bar{z}_i|\bar{z}_1,\bar{z}_2,\dots,\bar{z}_{i-1},\bar{z}_{i+1},\dots,\bar{z}_g}^2$ is the (2,2) element of $\mathbf{S}_{11.2}$, and the conditional covariance matrix, $\mathbf{S}_{11.2}$, is given by

$$\mathbf{S}_{11.2} = \mathbf{S}_{11} - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{21} \quad (52)$$

(see Anderson 1984, pp. 125-127). In this case, $\mathbf{S}_{11}$ is the sample covariance sub-matrix for the two variables for which the partial correlation estimate is needed, $\mathbf{S}_{22}$ is the sample covariance sub-matrix for the remaining variables, and $\mathbf{S}_{12}$ and $\mathbf{S}_{21}$ contain the estimates of the covariance between the two variables for which the partial correlation estimate is needed and the remaining variables. The PEG selection is made by selecting the random component that results in the smallest sample partial correlation.

For the activity network example given in Section 3.2.1, the set of 7 partial correlation estimates are obtained by calculating the mean random variate generated from each of the 7 random components for each of r runs. The mean random variate generated for each run can be calculated within the simulation model just as other statistics are calculated, or a record of the random variates generated during the run can be output and averaged. Equation (51) is then applied, and the random component that has the smallest estimated partial correlation with the response is selected as the PEG.

Chapter 4

4. Examples and Results

In this chapter, we present the results of the four PEG selection procedures outlined in Chapter 3 in the context of univariate simulation experiments in which the Schruben-Margolin strategy is employed for the purpose of reducing the sample variance of first-order metamodel coefficients.

In order to evaluate the relative performance of each PEG selection method, we compare the PEG selections suggested by the four methods to the results of Monte Carlo experiments. Each Monte Carlo experiment consists of making 50 simulation runs at each design point of a Schruben-Margolin experiment for each possible PEG and observing the efficiency of each PEG in terms of the PEG efficiency measures in Section 3.1. PEG selections were made at each design point in order to isolate the effect of design point location on the performance of the PEG selection methods.

The results of the PEG selection procedures and corresponding Monte Carlo analyses are given for each of two simulation models. The first model is a simple serial queuing system. The second model is a comparatively complex model of a hospital facility. Both models were implemented on an IBM 3080 computer using the SLAM II simulation language (see Pritsker 1986). The computer code for these two models is presented in Appendix I and Appendix II, respectively.

4.1. Serial Queuing Example

Consider the queuing network depicted in Figure 2. Random component definitions are given in Table 5. Customers arrive to a set of three serial, infinite capacity queues according to a Poisson process with mean arrival rate 0.01 arrivals per time

unit. Service times at the three queues all have truncated normal distributions with means 100, 100, and 25 time units, respectively, and standard deviations 1, 2, and 8 time units, respectively.



Figure 2: Serial Queuing Example

Table 5	
Random Component Definitions for the Serial Queuing Example	
Random Component	Definition
1	interarrival time
2	service time at node 1
3	service time at node 2
4	service time at node 3

The purpose of the experiment was to estimate a set of first-order metamodels of the form

$$y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \varepsilon, \tag{53}$$

where y is a response, x_1 is the mean time between arrivals, and x_2 is the mean service time of the first server. The following five responses were considered in order to study the effects of response type on the performance of the PEG selection procedures:

1. time in system (y_1),
2. time between departures (y_2),
3. percent utilization of server 1 (y_3),
4. percent utilization of server 2 (y_4),
5. percent utilization of server 3 (y_5).

In order to estimate the model given by equation (53) for each of the five responses, a 2^2 factorial design with two orthogonal blocks was used so that the Schruben-Margolin strategy could be employed. The design points are given in Table 6.

Table 6		
Uncoded Factor Levels for the Serial Queuing Example		
Design Point	ξ_1	ξ_2
1	110	110
2	90	90
3	110	90
4	90	110

The centered and scaled design matrix is given by

$$\mathbf{X} = \begin{bmatrix} 1_r & 1_r & 1_r \\ 1_r & -1_r & -1_r \\ 1_r & 1_r & -1_r \\ 1_r & -1_r & 1_r \end{bmatrix}. \quad (54)$$

4.1.1. PEG Selections

All simulation runs made to conduct the PEG selection procedures were initialized to the empty and idle state and were of length determined by the time required to process 500 customers.

V1 was conducted by making 100 simulation runs for each of the four required variance estimates $(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \dots, \hat{\sigma}_4^2)$. The process was repeated at each of the four design points, and PEG selections were made at each design point based on these estimates. Thus, the experiment resulted in 16 sets of 100 observations. In addition, the time in system (y_1), time between departures (y_2), percent utilization of server 1 (y_3), percent utilization of server 2 (y_4), and percent utilization of server 3 (y_5) were observed simultaneously for each run and a separate variance estimate was calculated for each response. All 80 resulting variance estimates are given in Appendix VII with 95% confidence intervals. Individual observation are not given in this thesis due to the size of the data sets. Instead, seed values are given in Appendix III so that the results can be reproduced.

V2 was conducted by making 100 replications for each of the four required variance estimates $(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \dots, \hat{\sigma}_4^2)$. The process was repeated at each of the four design points, and PEG selections were made at each design point based on these estimates. Thus, the experiment resulted in 16 sets of 100 observations. In addition, all five responses ($y_1, y_2, \dots, y_5$) were observed simultaneously for each run and a separate variance estimate was calculated for each response. All 80 resulting variance estimates are given in Appendix VII with 95% confidence intervals. A record of all seeds used for this procedure is given in Appendix III.

SC was conducted by making a set of 400 simulation runs and obtaining the four

required simple correlation estimates ($\hat{\rho}_{yx_1}, \hat{\rho}_{yx_2}, \dots, \hat{\rho}_{yx_4}$) from the same set. This process was repeated at each of the four design points, and PEG selections at each design point were based on these simple correlation estimates. In addition, all five responses ($y_1, y_2, \dots, y_5$) were observed simultaneously for each run and a separate simple correlation estimate was calculated for each response. All 80 resulting simple correlation estimates are given in Appendix VII with 95% confidence intervals. A record of all seeds used for this procedure is given in Appendix III.

PC was conducted by making a set of 400 simulation runs and obtaining the four required partial correlation estimates ($\hat{\rho}_{p_{yx_1}}, \hat{\rho}_{p_{yx_2}}, \dots, \hat{\rho}_{p_{yx_4}}$) from the same set. This process was repeated at each of the four design points, and PEG selections at each design point were based on these partial correlation estimates. In addition, all five responses ($y_1, y_2, \dots, y_5$) were observed simultaneously for each run and a separate partial correlation estimate was calculated for each response. All 80 resulting partial correlation estimates are given in Appendix VII with 95% confidence intervals. A record of all seeds used for this procedure is given in Appendix III.

Table 7 - Table 10 summarize the results of the PEG selections procedures for each design point/response combination. The number corresponding to each design point/response combination is random component number, corresponding to Table 5, of the PEG selected by the given method. Numbers separated by slashes, such as 2/3/4, indicate that given method was unable to make a single selection. This occurred exclusively in V2 because some near zero (to 5 significant digits) variance estimates were obtained.

Table 7					
PEG Selections for the Serial Queuing Example Variance Method 1					
	Response				
Design Point	y_1	y_2	y_3	y_4	y_5
1	2	4	4	4	3
2	3	3	3	3	3
3	3	2	3	4	2
4	3	3	3	3	2

Table 8					
PEG Selections for the Serial Queuing Example Variance Method 2					
	Response				
Design Point	y_1	y_2	y_3	y_4	y_5
1	3	3	3/4	4	3
2	3	3	2/3/4	4	3
3	2	2	3	2/4	2/3
4	4	3	3/4	2/3/4	2

Table 9					
PEG Selections for the Serial Queuing Example Simple Correlation Method					
	Response				
Design Point	y_1	y_2	y_3	y_4	y_5
1	3	2	2	2	3
2	2	4	4	4	3
3	4	2	2	2	2
4	4	2	4	4	3

Table 10					
PEG Selections for the Serial Queuing Example Partial Correlation Method					
	Response				
Design Point	y_1	y_2	y_3	y_4	y_5
1	4	3	3	3	3
2	4	4	4	4	3
3	2	2	4	4	3
4	4	2	3	2	2

4.1.2. Monte Carlo Analysis

In order to check the efficiency of the PEG selections made by the four PEG selection methods, the Schruben-Margolin strategy was employed to estimate the metamodel given by equation (53) for each of the five responses. For each possible PEG, 50 runs were made at each of the four design points given in Table 6. Each run was initialized to the empty and idle state and was of length determined by the time to process 500 customers. The seeds were set up in the manner prescribed in the description of the Schruben-Margolin strategy except that, for the stream corresponding to the i th PEG, the seed values were selected randomly for each run. Thus, the $g-1$ random number streams corresponding to the $g-1$ random components that were not PEGs were initialized using the seed matrix $\mathbf{R}$ at design points 1 and 2 and were initialized using $\bar{\mathbf{R}}$ at design points 3 and 4. The remaining random number stream corresponding to the PEG was randomly initialized for each run. A complete record of the runs made and corresponding seeds used in the Monte Carlo analysis of the serial queuing example is given in Appendix IV.

Table 11 summarizes the observed efficiency of each PEG in terms of determinant of the sample metamodel coefficient covariance matrix, $\mathbf{V}$. The number corresponding to each PEG/response combination is the determinant of $\mathbf{V}$ for that PEG and

response. Boxed elements indicate the lowest determinant of $\mathbf{V}$ for each response. A complete summary, including sample response correlation matrices, sample meta-model coefficient covariance matrices, and estimated metamodels is given in Appendix IX.

Table 11					
Observed Efficiency of PEGs for the Serial Queuing Example det[$\mathbf{V}$]					
	Responses				
PEG	y_1	y_2	y_3	y_4	y_5
1	31.9194E 14	0.7200	1.4979	0.1984	5.2171E-05
2	1.6505E 14	0.3667	0.0187	0.0825	2.2166E-05
3	1.9975E 14	0.3542	0.0063	0.0974	2.1046E-05
4	2.0552E 14	0.5106	0.0657	0.1445	29.0705E-05

Table 12 summarizes the observed efficiency of each PEG in terms of trace of the sample metamodel coefficient covariance matrix, $\mathbf{V}$. The number corresponding to each PEG/response combination is the trace of $\mathbf{V}$ for that PEG and response. Boxed elements indicate the lowest trace of $\mathbf{V}$ for each response.

Table 12					
Observed Efficiency of PEGs for the Serial Queuing Example trace[$\mathbf{V}$]					
	Responses				
PEG	y_1	y_2	y_3	y_4	y_5
1	524501.0000	5.8922	4.1713	3.6074	0.2273
2	237639.2500	5.4393	2.7867	3.1774	0.2012
3	258326.5620	3.6830	2.1836	2.2018	0.1379
4	253767.3750	5.4204	3.4492	3.2570	0.2573

Table 13 summarizes the observed efficiency of each PEG in terms of maximum diagonal element of the sample metamodel coefficient covariance matrix, $\mathbf{V}$. The

number corresponding to each PEG/response combination is the maximum diagonal element of $\mathbf{V}$ for that PEG and response. Boxed elements indicate the lowest trace of $\mathbf{V}$ for each response.

Table 13					
Observed Efficiency of PEGs for the Serial Queuing Example max diagonal of $\mathbf{V}$					
PEG	Responses				
	y_1	y_2	y_3	y_4	y_5
1	213511.1870	2.1215	1.6122	1.2985	0.0823
2	101420.0602	2.5916	1.2197	1.5277	0.0971
3	123087.8750	1.7354	1.1684	1.0182	0.0632
4	115116.0620	3.1504	1.2848	1.8807	0.1329

4.2. Hospital Example

In this section, the results of two of the four PEG selection methods are given for the hospital facility depicted in Figure 3. Although a thorough discussion of the results of the PEG selections for the serial queuing example are reserved for Chapter 5, we note here that SC and PC both performed as well as the V1 and V2. Because V1 and V2 are cumbersome to implement for models with branching components, only SC and PC were conducted for this example. Random component definitions are given in Table 14. Note that this is the same system studied by Schruben and Margolin (1978); Hussey, Houck, and Myers (1987); and Tew and Wilson (1992).

Consider a hospital facility where patients arrive according to a Poisson process with mean arrival rate of 3.3 patients per day. Seventy-five percent of the arriving patients require intensive care for a period of time that is lognormally distributed with mean $\mu = 3.4$ days and standard deviation $\sigma = 3.5$ days. The remaining twenty-five percent of arriving patients require coronary care and for period of time that is

lognormally distributed with mean $\mu = 3.8$ days and standard deviation $\sigma = 1.6$ days. Twenty-seven percent of intensive care patients will depart the facility after their care is complete. The remaining seventy-three percent will need intermediate care for a period of time that is lognormally distributed with mean $\mu = 15$ and standard deviation $\sigma = 7$ days. Twenty percent of patients leaving CCU will depart the facility after their care is complete. The remaining eighty percent require intermediate care for a period of time that is lognormally distributed with mean $\mu = 17$ days and $\sigma = 3$ days.

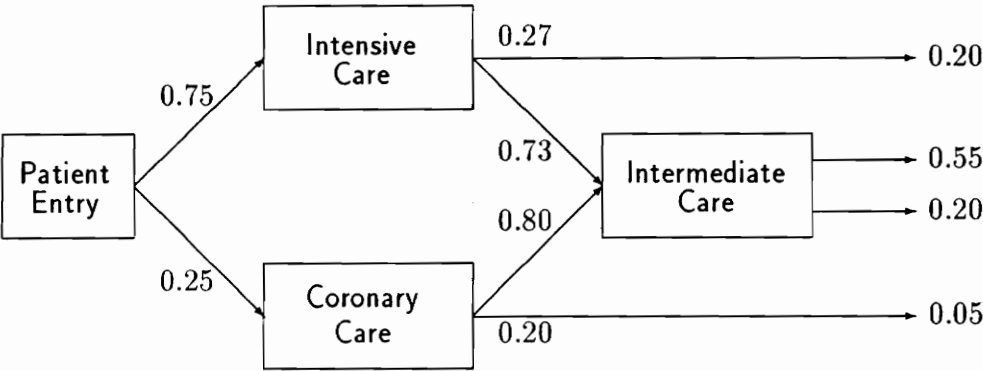


Figure 3: Hospital Example

Table 14	
Random Component Definitions for the Hospital Example	
Random Component	Definition
1	interarrival time
2	branching from patient entry
3	service time at ICU
4	service time at CCU
5	branching from ICU
6	branching from CCU
7	service time at intermediate care for ICU customers
8	service time at intermediate care for CCU customers

The hospital facility has three care units with limited numbers of beds in each unit. Patients that require care from a unit that is full will be turned away.

The purpose of the experiment was to estimate a set of first-order metamodels of the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon, \quad (55)$$

where y is a response and x_i ($i = 1, 2, 3$) is the number of beds in the intensive, coronary, and intermediate care units, respectively. The following five responses were considered in order to study the effects of response type on the performance of the PEG selection procedures:

1. time in system for non-balking customers (y_1),
2. balking rate in barks per month (y_2),
3. percent utilization per bed in the ICU (y_3),
4. percent utilization per bed in the CCU (y_4),

5. percent utilization per bed in the intermediate care unit (y_5).

The response of primary interest is the balking rate since it was the response used by Schruben and Margolin (1978) and Tew and Wilson (1992). In order to estimate the models a 2^3 factorial design with two orthogonal blocks was used so that the Schruben-Margolin strategy could be employed. The design points are given in Table 15.

Table 15			
Uncoded Factor Levels for the Hospital Example			
Design Point	ξ_1	ξ_2	ξ_3
1	13	4	15
2	15	6	15
3	15	4	17
4	13	6	17
5	15	4	15
6	13	6	15
7	13	4	17
8	15	6	17

The centered and scaled design matrix is given by

$$\mathbf{X} = \begin{bmatrix} \mathbf{1}_r & -\mathbf{1}_r & -\mathbf{1}_r & -\mathbf{1}_r \\ \mathbf{1}_r & \mathbf{1}_r & \mathbf{1}_r & -\mathbf{1}_r \\ \mathbf{1}_r & \mathbf{1}_r & -\mathbf{1}_r & \mathbf{1}_r \\ \mathbf{1}_r & -\mathbf{1}_r & \mathbf{1}_r & \mathbf{1}_r \\ \hline \mathbf{1}_r & \mathbf{1}_r & -\mathbf{1}_r & -\mathbf{1}_r \\ \mathbf{1}_r & -\mathbf{1}_r & \mathbf{1}_r & -\mathbf{1}_r \\ \mathbf{1}_r & -\mathbf{1}_r & -\mathbf{1}_r & \mathbf{1}_r \\ \mathbf{1}_r & \mathbf{1}_r & \mathbf{1}_r & \mathbf{1}_r \end{bmatrix}. \quad (56)$$

4.2.1. PEG Selections

All simulation runs made to conduct the PEG selection procedures were of 1200 simulated days in length and were *warmed up* for 300 time units prior to the collection

of statistics.

SC was conducted by making a set of 800 simulation runs and obtaining the eight required simple correlation estimates $(\hat{\rho}_{yx_1}, \hat{\rho}_{yx_2}, \dots, \hat{\rho}_{yx_8})$ from the same set. This process was repeated at each of the eight design points, and PEG selections at each design point were based on these simple correlation estimates. In addition, all five responses $(y_1, y_2, \dots, y_5)$ were observed simultaneously for each run and a separate simple correlation estimate was calculated for each response. All 320 resulting simple correlation estimates are given in Appendix VIII with 95% confidence intervals. A complete record of the seeds used for this procedure is given in Appendix V.

PC was conducted by making a set of 800 simulation runs and obtaining the eight required partial correlation estimates $(\hat{\rho}_{p_{yx_1}}, \hat{\rho}_{p_{yx_2}}, \dots, \hat{\rho}_{p_{yx_8}})$ from the same set. This process was repeated at each of the eight design points, and PEG selections at each design point were based on these partial correlation estimates. In addition, all five responses $(y_1, y_2, \dots, y_5)$ were observed simultaneously for each run and a separate partial correlation estimate was calculated for each response. All 320 resulting partial correlation estimates are given in Appendix VIII with 95% confidence intervals. A complete record of the seeds used for this procedure is given in Appendix V.

Table 16 - Table 17 summarize the results of the PEG selection procedures for each design point/response combination. The number corresponding to each design point/response combination is random component number, corresponding to Table 11, of the PEG selected by the given method.

Table 16					
PEG Selections for the Hospital Example Simple Correlation Method					
	Response				
Design Point	y_1	y_2	y_3	y_4	y_5
1	8	8	8	6	6
2	4	8	4	3	6
3	8	8	7	8	2
4	8	8	8	8	8
5	8	5	4	3	6
6	2	4	4	6	4
7	2	8	7	6	2
8	8	4	7	7	3

Table 17					
PEG Selections for the Hospital Example Partial Correlation Method					
	Response				
Design Point	y_1	y_2	y_3	y_4	y_5
1	2	8	8	7	4
2	8	4	7	3	4
3	2	2	8	8	3
4	2	4	4	7	4
5	2	3	8	8	3
6	2	4	4	8	2
7	2	4	8	5	2
8	4	3	8	6	2

4.2.2. Monte Carlo Analysis

In order to check the efficiency of the PEG selections made by the four PEG selection methods, the Schruben-Margolin strategy was employed to estimate the metamodel given by equation (55) for each of the five responses. For each possible PEG, 50 runs were made at each of the eight design points given in Table 15. Each run was *warmed up* for 300 simulated days and was of 1200 days in length. The seeds were set up in the manner prescribed in the description of the Schruben-Margolin strategy

except that, for the stream corresponding to the i th PEG, the seed values were selected randomly for each run. Thus, the $g - 1$ random number streams corresponding to the $g - 1$ random components that were not PEGs were initialized using the seed matrix $\mathbf{R}$ at design points 1 and 2 and were initialized using $\overline{\mathbf{R}}$ at design points 3 and 4. The remaining random number stream corresponding to the PEG was randomly initialized for each run. A complete record of the runs made and corresponding seeds used in the Monte Carlo analysis of the serial queuing example is given in Appendix VI.

Table 18 summarizes the observed efficiency of each PEG in terms of determinant of the sample metamodel coefficient covariance matrix, $\mathbf{V}$. The number corresponding to each PEG/response combination is the determinant of $\mathbf{V}$ for that PEG and response. Boxed elements indicate the lowest determinant of $\mathbf{V}$ for each response. A complete summary, including sample response correlation matrices, sample metamodel coefficient covariance matrices, and estimated metamodels is given in Appendix X.

Table 18					
Observed Efficiency of PEGs for the Hospital Example					
det[$\mathbf{V}$]					
Responses					
PEG	y_1	y_2	y_3	y_4	y_5
1	2.9325E -12	2298.4438E -6	1.1771E -4	1.7024E -4	108.0954E -10
2	8.9643E -12	13.6728E -6	0.2272E -4	15.6141E -4	9.9990E -10
3	5.0135E -12	0.4292E -6	1.8545E -4	0.0000E -4	10.8761E -10
4	0.3087E -12	0.1062E -6	0.0000E -4	0.2586E -4	0.2546E -10
5	25.5351E -12	28.4660E -6	0.0000E -4	0.0000E -4	12.6426E -10
6	1.2376E -12	0.6288E -6	0.0000E -4	0.0000E -4	0.0717E -10
7	0.7183E -12	0.4379E -6	0.0000E -4	0.0000E -4	8.0889E -10
8	0.1650E -12	0.0906E -6	0.0000E -4	0.0000E -4	0.0592E -10

Table 19 summarizes the observed efficiency of each PEG in terms of trace of the sample metamodel coefficient covariance matrix, $\mathbf{V}$. The number corresponding to each PEG/response combination is the trace of $\mathbf{V}$ for that PEG and response. Boxed elements indicate the lowest trace of $\mathbf{V}$ for each response.

Table 19					
Observed Efficiency of PEGs for the Hospital Example					
trace[$\mathbf{V}$]					
PEG	Responses				
	y_1	y_2	y_3	y_4	y_5
1	0.0083	0.9174	0.5871	1.2990	0.0818
2	0.0108	0.5584	0.4562	0.8507	0.0643
3	0.0092	0.4335	0.5761	0.6824	0.0791
4	0.0095	0.3204	0.3672	0.5560	0.0769
5	0.0110	0.4172	0.4550	0.5712	0.0729
6	0.0137	0.4356	0.4146	0.5670	0.0670
7	0.0088	0.3233	0.5067	0.5378	0.0781
8	0.0097	0.4242	0.4909	0.5245	0.0695

Table 20 summarizes the observed efficiency of each PEG in terms of maximum diagonal element of the sample metamodel coefficient covariance matrix, $\mathbf{V}$. The number corresponding to each PEG/response combination is the maximum diagonal element of $\mathbf{V}$ for that PEG and response. Boxed elements indicate the lowest trace of $\mathbf{V}$ for each response.

Table 20					
Observed Efficiency of PEGs for the Hospital Example max diagonal of $\mathbf{V}$					
	Responses				
PEG	y_1	y_2	y_3	y_4	y_5
1	0.0058	0.3096	0.3644	1.1251	0.0645
2	0.0072	0.4576	0.3275	0.3333	0.0550
3	0.0062	0.4016	0.3012	0.6566	0.0700
4	0.0084	0.2974	0.3614	0.4248	0.0710
5	0.0059	0.2712	0.4504	0.5439	0.0639
6	0.0122	0.3966	0.4087	0.5421	0.0651
7	0.0072	0.2854	0.5029	0.5100	0.0698
8	0.0089	0.4052	0.4865	0.4874	0.0667

Chapter 5

5. Discussion and Conclusions

In this chapter, we discuss the data given in Chapter 4 and draw some conclusions. In Section 5.1, we discuss empirical evidence concerning the potential benefits of careful PEG selection. In Section 5.2, we review the performance of the four proposed PEG selection methods. In Section 5.3, we draw final conclusions and discuss areas of possible future research.

5.1. Observed Pure Error Generator Efficiency

One of the most important conclusion that can be drawn from this thesis is that PEG selection does affect the sample variance metamodel coefficients. Depending on the model, the response, and the measure used, the results summarized in Tables 11 - 13 and Tables 18 - 20 indicate that the effect of PEG selection on the sample variance of metamodel coefficients can be quite significant.

Consider the determinant of $\mathbf{V}$ for both the serial queuing example (Table 11) and the hospital example (Table 18). In all 10 cases, the ratio of the lowest determinant to highest determinant is not greater than $\frac{1}{2}$ and is extremely close to 0 in several cases, indicating that the highest determinant is invariably much larger than the lowest for the given examples. With the exception of y_5 in the hospital example, the same observation holds true, although less dramatically, for the trace of $\mathbf{V}$ and the maximum diagonal of $\mathbf{V}$.

Thus, careful PEG selection has great potential benefits with respect to the sample variance of metamodel coefficients for experiments in which a PEG is used.

5.2. Performance of Proposed Pure Error Generator Selection Methods

To assess the performance of the four PEG selection methods, we use the ratio of determinant of $\mathbf{V}$ for each PEG to the determinant of $\mathbf{V}$ for the PEG with the highest determinant. We refer to these ratios as relative performance ratios to simplify the discussion. Relative performance ratios for the hospital and serial queuing examples are given in Tables 21 and 22, respectively.

5.2.1. Serial Queuing Example

Comparing the contents of Tables 7 - 10 with Table 21, it is clear that all four PEG selection methods have successfully avoided selection of the least efficient PEG for all five responses. In addition, the PEG selections made by the four methods are of approximately the same performance on average. The average performance ratios across design points and responses for V1, V2, SC, and PC are 0.231, 0.257, 0.259, and 0.261 respectively.

In most cases, differences in the PEGs selected by each method and for each design point can be attributed to random sampling and the unknown distributional properties of the determinant of $\mathbf{V}$. For example, the determinants of $\mathbf{V}$ for y_2 and PEGs 2 and 3 are 0.3667 and 0.3542, respectively. The difference in these determinants is not very large and may, in fact, be statistically insignificant. Thus, the selection of PEG 2, in this case, does not represent a failure in the selection method.

In some cases, differences in the PEGs selected by each method and for each design point are less easily attributed to random sampling. For example, the determinant of $\mathbf{V}$ for y_2 and PEG 4 is 0.5106 which is substantially higher than the determinant of $\mathbf{V}$ for y_2 and PEGs 2 and 3. Note that for y_2 , PEG 4 was selected by V1, SC, and PC for one out of four design points each. This and other fail-

ures in the PEG selection methods might be attributable to failures in the assumed Schruben-Margolin correlation structure given by equation (29). In fact, from a subjective standpoint, failures in the Schruben-Margolin correlation structure occurred for about half of the PEG selections. This can be checked by reviewing the sample response correlation matrices given in Appendix IX and comparing them to equation (29). Since all selections were made assuming the Schruben-Margolin correlation structure, the potential for failures in the PEG selection methods existed for about half of the selections. In particular, the selection of PEG 4 for responses y_2 , y_3 , and y_4 may have occurred because of the failures in the Schruben-Margolin correlation structure exhibited in Tables IX.33, IX.35, and IX.37, respectively.

Table 21					
Relative Performance Ratios for the Serial Queuing Example					
	Responses				
PEG	y_1	y_2	y_3	y_4	y_5
1	1.000	1.000	1.000	1.000	0.179
2	0.052	0.509	0.012	0.416	0.076
3	0.063	0.492	0.004	0.491	0.072
4	0.064	0.709	0.044	0.728	1.000

5.2.2. Hospital Example

As noted in Section 4.2, V1 and V2 were not conducted for this example because they are cumbersome to implement for models with branching components and because they failed to out-perform SC and PC in the serial queuing example. Comparing the contents of Tables 18 - 20 with Table 22, it is clear that both SC and PC have successfully avoided selection of the least efficient PEG for all responses and

design points. Again, many of the differences in the PEGs selected by each method and for each design point can be attributed to random sampling.

Note that, aside from y_5 for which failures in required correlation structure occurred for all PEGs, four failures in the required correlation structure occurred: (1) y_1 - PEG 5, (2) y_2 - PEG 1, (3) y_4 - PEG 1, and (4) y_4 - PEG 2. In cases 1, 2, and 4, the indicated response/PEG combination corresponds to the highest determinant of $\mathbf{V}$. In case 3, the indicated response/PEG combination corresponds to the second highest determinant of $\mathbf{V}$ for that response. Thus, a large portion of the sample variance of metamodel coefficients can be attributed to failures in the required correlation structure.

Table 22					
Relative Performance Ratios for the Hospital Example					
	Responses				
PEG	y_1	y_2	y_3	y_4	y_5
1	0.115	1.000	0.635	0.109	1.000
2	0.351	0.006	0.123	1.000	0.093
3	0.196	0.000	1.000	0.000	0.101
4	0.012	0.000	0.000	1.000	0.002
5	1.000	0.012	0.000	0.017	0.117
6	0.048	0.000	0.000	0.000	0.001
7	0.028	0.000	0.000	0.000	0.075
8	0.006	0.000	0.000	0.000	0.001

5.3. Conclusions

While the results given in this thesis cannot be generalized to all simulation models, the empirical evidence suggests three important conclusions: (1) careful PEG

selection has great potential benefits with respect to the variance of metamodel coefficient estimates in replicated univariate simulation experiments that employ a PEG, (2) PEG selection methods that consider only the contribution of the PEG to the variance of the response perform well assuming the required correlation structure is maintained, and (3) failures in required correlations structures due to the use of certain PEGs have potentially significant effects on the sample variance of metamodel coefficients.

These three conclusions suggest several areas for future research. In particular, the development of selection methods that consider the effect of potential PEGs on required correlation structures should be considered.

The development of other methods to measure variability contributions by random components should also be considered. One possible method, involves the use of polynomial or non-linear regression models to measure the variability explained by random components. This is similar to the idea presented in the description of the simple and partial correlation methods except that the assumption that there is a strong linear relationship between y and x_i is no longer needed.

Other areas of possible research include the application and development of PEG selection methods for multivariate and batch means simulation experiments.

Appendix I

This appendix contains the combined SLAM II and FORTRAN computer code for the serial queuing model described in section 4.1. Model specific variable definitions and logic comments are given within the code.

```
PROGRAM MAIN
C*****
C                               Serial Queuing Model
C*****
COMMON/SCOM1/ATRIB(100),DD(100),DDL(100),DTNOW,II,MFA,
*MSTOP,NCLNR,NCRDR,NPRNT,NNRUN,NNSET,NTAPE,SS(100),
*SSL(100),TNEXT,TNOW,XX(100)
COMMON QSET(10000)
DIMENSION NSET(10000)
EQUIVALENCE (NSET(1),QSET(1))
NNSET=10000
NCRDR=5
NPRNT=6
NTAPE=7
CALL SLAM
STOP
END

SUBROUTINE EVENT(I)
INCLUDE (SLMSCOM1)
REAL TSYS,STSYS,AVGTSYS,SATRIB(4),X(4),Y1,Y2,TBD,TLD,STBD,AVGTBD
C*****
C  AVGTSYS: Average time in system
C  AVGTBD: Average time between departures
C  II: Number of customers that have departed
C  SATRIB(i): Sum of ATRIB(i) for all customers that have departed
C  STBD: Sum of the time between delays for all customers that have
C         departed
C  STSYS: Sum of the time in system for all customers that have
C         departed
C  TBD: Time between departures
C  TLD: Time of last departure
C  TSYS: Time in system for departing customer
C  X(i): Average value of ATRIB(i)
C  Y1: Average time in system
C  Y2: Average time between departures
C  AA AVG(i): Average utilization of servers at queue i
C*****
GO TO (1,2),I
C*****
C Update customer count, calculate and sum the time between departures
C and time in system for departing customer, and sum the random variates
C for departing customers.
```

```

C*****
1  II=II+1
   TSYS=TNOW-ATRIB(5)
   STSYS=STSYS+TSYS
   IF(II.GT.1) THEN
       TBD=TNOW-TLD
       TLD=TNOW
       STBD=STBD+TBD
   ELSE
       TLD=TNOW
   ENDIF
   DO 10 J=1,4
       SATRIB(J)=SATRIB(J)+ATRIB(J)
10  CONTINUE
   RETURN
C*****
C Average random variates and responses.
C*****
2  DO 20 J=2,4
    X(J)=SATRIB(J)/II
20  CONTINUE
    X(1)=SATRIB(1)/(II-1)
    Y1=STSYS/II
    Y2=STBD/(II-1)
C*****
C Write observation to a data file.
C*****
    WRITE(88,30) Y1,Y2,AAAVG(1),AAAVG(2),AAAVG(3),X(1),X(2),X(3),X(4)
    II=0
    TSYS=0
    STSYS=0
    AVGTSYS=0
    DO 40 J=1,4
        SATRIB(J)=0
        X(J)=0
40  CONTINUE
    TBD=0
    TLD=0
    STBD=0
    AVGTDB=0
30  FORMAT(2F8.3,3F8.4,4F8.3)
    RETURN
    END

GEN,ZEIMER,SERIAL QUEUES,11/04/1991,100,NO,NO,,NO,NO;
LIMITS,3,5,500;
NETWORK;
;*****
ATTRIB(1): Time between the arrivals
ATTRIB(2): Service time at first server
ATTRIB(3): Service time at second server
ATTRIB(4): Service time at third server

```

```

;*****
; Create entities.
;*****
CR CREATE,,,5;
  ASSIGN,TRIB(1)=XX(2);
  ASSIGN,XX(2)=EXPON(100.0,1);
  GOON,2;
  ACT,,,ASN1;
  ACT,XX(2),,CR;
;*****
; Assign random variates to arriving entities.
;*****
ASN1 ASSIGN,TRIB(2)=RNORM(100.0,1.0,2);
  ASSIGN,TRIB(3)=RNORM(100.0,2.0,3);
  ASSIGN,TRIB(4)=RNORM(25.0,8.0,4);
;*****
; Entities receive service.
;*****
Q1 QUEUE(1);
  ACT(1)/1,TRIB(2);
Q2 QUEUE(2);
  ACT(1)/2,TRIB(3);
Q3 QUEUE(3);
  ACT(1)/3,TRIB(4);
E1 EVENT,1;
G1 GOON,1;
;*****
; End simulation.
;*****
  ACT,,II.GE.500,TRM;
  ACT,,II.LT.500;
  TERM;
  TRM EVENT,2;
  TERM,1;
  END;
;*****
; Set seeds for first run.
;*****
SEEDS,470079(1),467951(2),908929(3),376975(4);
FIN;

```

Appendix II

This appendix contains the combined SLAM II and FORTRAN computer code for the hospital model described in section 4.2. Model specific variable definitions and logic comments are given within the code.

```
PROGRAM MAIN
C*****
C                                     Hospital Model
C*****
COMMON/SCOM1/ATRIB(100),DD(100),DDL(100),DTNOW,II,MFA,
*MSTOP,NCLNR,NCRDR,NPRNT,NNRUN,NNSET,NTAPE,SS(100),
*SSL(100),TNEXT,TNOW,XX(100)
COMMON QSET(10000)
DIMENSION NSET(10000)
EQUIVALENCE (NSET(1),QSET(1))
NNSET=10000
NCRDR=5
NPRNT=6
NTAPE=7
CALL SLAM
STOP
END

SUBROUTINE EVENT(I)
INCLUDE (SLMSCOM1)
REAL TSYS,STSYS,AVGTSYS,SATRIB(8),X(8),Y1,Y2,NS(3)
INTEGER NBALKS,N(8)
C*****
C  AVGTSYS: Average time in system for customers that do not balk
C  II: Number of customers that do not balk
C  N(i): Number of observations corresponding to SATRIB(i)
C  NBALKS: Current number of balks
C  NS(i): Number of servers at queue i
C  SATRIB(i): Sum of ATRIB(i) for all customers that have departed
C  STSYS: Sum of the time in system for all customers that have
C          departed
C  TSYS: Time in system for departing customer
C  X(i): Average value of ATRIB(i)
C  Y1: Average time in system
C  Y2: Average balks per month
C  AA AVG(i): Average utilization of servers at queue i
C*****
GO TO (1,2,3),I
C*****
C Count the number of customers using each random component and sum
C the values of corresponding random variates. Also sum the time in
C system for departing customers.
C*****
```

```

1  IF(ATRIB(11).EQ.1) THEN
    DO 10 J=1,3
      N(J)=N(J)+1
      SATRIB(J)=SATRIB(J)+ATRIB(J)
10  CONTINUE
    N(5)=N(5)+1
    SATRIB(5)=SATRIB(5)+ATRIB(5)
    ELSE IF(ATRIB(11).EQ.2) THEN
      DO 20 J=1,3
        N(J)=N(J)+1
        SATRIB(J)=SATRIB(J)+ATRIB(J)
20  CONTINUE
    N(5)=N(5)+1
    SATRIB(5)=SATRIB(5)+ATRIB(5)
    N(7)=N(7)+1
    SATRIB(7)=SATRIB(7)+ATRIB(7)
    ELSE IF(ATRIB(11).EQ.3) THEN
      DO 30 J=1,2
        N(J)=N(J)+1
        SATRIB(J)=SATRIB(J)+ATRIB(J)
30  CONTINUE
    N(4)=N(4)+1
    SATRIB(4)=SATRIB(4)+ATRIB(4)
    N(6)=N(6)+1
    SATRIB(6)=SATRIB(6)+ATRIB(6)
    N(8)=N(8)+1
    SATRIB(8)=SATRIB(8)+ATRIB(8)
    ELSE IF(ATRIB(11).EQ.4) THEN
      DO 40 J=1,2
        N(J)=N(J)+1
        SATRIB(J)=SATRIB(J)+ATRIB(J)
40  CONTINUE
    N(4)=N(4)+1
    SATRIB(4)=SATRIB(4)+ATRIB(4)
    N(6)=N(6)+1
    SATRIB(6)=SATRIB(6)+ATRIB(6)
    ENDIF
    II=II+1
    TSYS=TNOW-ATRIB(10)
    STSYS=STSYS+TSYS
    RETURN

C*****
C Average random variates and responses.
C*****
2  DO 60 J=1,8
    X(J)=SATRIB(J)/N(J)
60  CONTINUE
    Y1=STSYS/II
    Y2=NBALKS/40.0
C*****
C Set number of servers at each queue (different for each design point).

```

```

C*****
      NS(1)=15.0
      NS(2)=6.0
      NS(3)=17.0
C*****
C Write observation to a data file.
C*****
      WRITE(90,1000) Y1,Y2,AAAVG(1)/NS(1),AAAVG(2)/NS(2),AAAVG(3)/NS(3)
      WRITE(91,1010) X(1),X(2),X(3),X(4),X(5),X(6),X(7),X(8)
      TSYS=0
      STSYS=0
      AVGTSYS=0
      NBALKS=0
      DO 70 J=1,8
          II=0
          N(J)=0
          SATRIB(J)=0
          X(J)=0
      70 CONTINUE
      1000 FORMAT(5F10.4)
      1010 FORMAT(8F9.4)
      RETURN
C*****
C Count the number of balks.
C*****
      3  NBALKS=NBALKS+1
      RETURN
      END

GEN,ZEIMER,HOSPITAL EX,2/24/1992,100,NO,NO,,NO,NO;
LIMITS,3,11,100;
NETWORK;
;*****
  ATRIB(1): Time between the arrivals
  ATRIB(2): U[0,1] used for branching from arrival
  ATRIB(3): Service time at ICU
  ATRIB(4): Service time at CCU
  ATRIB(5): U[0,1] used for branching from ICU
  ATRIB(6): U[0,1] used for branching from CCU
  ATRIB(7): Service time at intermediate care for ICU customers
  ATRIB(8): Service time at intermediate care for CCU customers
  ATRIB(9): Service time at intermediate care
  ATRIB(10): System entry time
  ATRIB(11): Path through network (1=ICU-TERM, 2=ICU-INTRC-TERM,
           3=CCU-TERM, 4=CCU-INTRC-TERM)
;*****
; Create patients.
;*****
CR  CREATE,,10;
    ASSIGN,ATRIB(1)=XX(2);
    ASSIGN,XX(2)=EXPON(0.303,1);
    GOON,2;

```

```

    ACT,,ASN1;
    ACT,XX(2),,CR;
;*****
; Assign random variates to arriving entities.
;*****
ASN1 ASSIGN,TRIB(2)=UNFRM(0.0,1.0,2);
    ASSIGN,TRIB(3)=RLOGN(3.4,3.5,3);
    ASSIGN,TRIB(4)=RLOGN(3.8,1.6,4);
    ASSIGN,TRIB(5)=UNFRM(0.0,1.0,5);
    ASSIGN,TRIB(6)=UNFRM(0.0,1.0,6);
    ASSIGN,TRIB(7)=RLOGN(15.0,7.0,7);
    ASSIGN,TRIB(8)=RLOGN(17.0,3.0,8);
;*****
; Arriving customers go to ICU or CCU.
;*****
GN1 GOON,1;
    ACT,0.0,TRIB(2).LE.0.75,ICU;
    ACT,0.0,,CCU;
;*****
; ICU customers receive service if beds are available.
;*****
ICU  QUEUE(1),0,0,BALK(FAIL);
    ACT(15)/1,TRIB(3);
;*****
; Customers completing service at ICU go to intermediate care or leave.
;*****
    GOON,1;
    ACT,,TRIB(5).LE.0.27,ASN2;
    ACT;
    ASSIGN,TRIB(9)=TRIB(7);
    ASSIGN,TRIB(11)=2;
    ACT,,INTRC;
ASN2 ASSIGN,TRIB(11)=1;
    ACT,,EV1;
;*****
; CCU customers receive service if beds are available.
;*****
CCU  QUEUE(2),0,0,BALK(FAIL);
    ACT(6)/2,TRIB(4);
;*****
; Customers completing service at CCU go to intermediate care or leave.
;*****
    GOON,1;
    ACT,,TRIB(6).LE.0.20,ASN3;
    ACT;
    ASSIGN,TRIB(9)=TRIB(8);
    ASSIGN,TRIB(11)=3;
    ACT,,INTRC;
ASN3 ASSIGN,TRIB(11)=4;
    ACT,,EV1;
;*****
; Intermediate care customers receive service if beds are available.

```

```

;*****
INTRC QUEUE(3),0,0,BALK(FAIL);
      ACT(17)/3,TRIB(9);
;*****
; Update statistics as customers depart.
;*****
EV1 GOON,1;
    ACT,,TNOW.LE.300,TERM;
    ACT;
    EVENT,1;
TERM TERMINATE;
;*****
; Count balking customers.
;*****
FAIL GOON,1;
    ACT,,TNOW.LE.300,TERM;
    ACT;
    EVENT,3;
    TERMINATE;
;*****
; End Simulation.
;*****
    CREATE,,1500,,1,1;
    EVENT,2;
    TERMINATE;
    ENDNETWORK;
;*****
; Set seeds for first run.
;*****
SEEDS,834722(1),863222(2),102423(3),039421(4),712103(5),881726(6),
      363627(7),635728(8);
FINISH;

```

Appendix III

This appendix contains the seeds used for PEG selection methods in the serial queuing example (section 4.1). The sets of seeds given in Tables III.1 - III.16 were randomly selected and used to initialize each set of 100 runs. For variance method 1 and 2, each set of 100 runs corresponds to one of the four required variance estimates denoted in the far right column. For the simple and partial correlation methods, all 400 runs were used to simultaneously calculate the required correlation estimates.

Table III.1					
PEG Selection Seeds for the Serial Queuing Example Design Point 1, Variance Method 1					
	Random Number Stream				
Runs	1	2	3	4	$\hat{\theta}$
1-100		395547	708949	278965	$\hat{\sigma}_{-1}^2$
101-200	846295		913809	127455	$\hat{\sigma}_{-2}^2$
201-300	651235	253857		101935	$\hat{\sigma}_{-3}^2$
301-400	723455	424997	323729		$\hat{\sigma}_{-4}^2$

Table III.2					
PEG Selection Seeds for the Serial Queuing Example Design Point 1, Variance Method 2					
	Random Number Stream				
Runs	1	2	3	4	$\hat{\theta}$
1-100	944727				$\hat{\sigma}_{-1}^2$
101-200		707725			$\hat{\sigma}_{-2}^2$
201-300			915143		$\hat{\sigma}_{-3}^2$
301-400				292839	$\hat{\sigma}_{-4}^2$

Table III.3				
PEG Selection Seeds for the Serial Queuing Example Design Point 1, Simple Correlation Method				
	Random Number Stream			
Runs	1	2	3	4
1-100	673397	478301	614237	466377
101-200	635407	618211	746907	112837
201-300	561237	657121	982337	786537
301-400	343097	837621	917377	827557

Table III.4				
PEG Selection Seeds for the Serial Queuing Example Design Point 1, Partial Correlation Method				
	Random Number Stream			
Runs	1	2	3	4
1-100	377691	887439	756125	102293
101-200	675401	657319	103855	745623
201-300	405781	250479	923485	563403
301-400	783241	747229	273775	643623

Table III.5					
PEG Selection Seeds for the Serial Queuing Example Design Point 2, Variance Method 1					
	Random Number Stream				
Runs	1	2	3	4	$\hat{\theta}$
1-100		683945	559033	783659	$\hat{\sigma}_{-1}^2$
101-200	848757		103743	678029	$\hat{\sigma}_{-2}^2$
201-300	343307	562335		448369	$\hat{\sigma}_{-3}^2$
301-400	146667	786015	224043		$\hat{\sigma}_{-4}^2$

Table III.6					
PEG Selection Seeds for the Serial Queuing Example Design Point 2, Variance Method 2					
	Random Number Stream				
Runs	1	2	3	4	$\hat{\theta}$
1-100	927513				$\hat{\sigma}_{-1}^2$
101-200		923473			$\hat{\sigma}_{-2}^2$
201-300			983027		$\hat{\sigma}_{-3}^2$
301-400				189321	$\hat{\sigma}_{-4}^2$

Table III.7				
PEG Selection Seeds for the Serial Queuing Example Design Point 2, Simple Correlation Method				
	Random Number Stream			
Runs	1	2	3	4
1-100	673397	478301	614237	466377
101-200	776287	216031	762767	500087
201-300	535447	728371	730097	900127
301-400	100787	761241	201027	634607

Table III.8				
PEG Selection Seeds for the Serial Queuing Example Design Point 2, Partial Correlation Method				
	Random Number Stream			
Runs	1	2	3	4
1-100	673397	478301	614237	466377
101-200	434387	959571	855757	674607
201-300	554027	771801	448997	875317
301-400	027387	567191	777717	148487

Table III.9					
PEG Selection Seeds for the Serial Queuing Example Design Point 3, Variance Method 1					
	Random Number Stream				
Runs	1	2	3	4	$\hat{\theta}$
1-100		923287	790103	378565	$\hat{\sigma}_{-1}^2$
101-200	770617		497119	284963	$\hat{\sigma}_{-2}^2$
201-300	820827	405015		237549	$\hat{\sigma}_{-3}^2$
301-400	994625	900387	177939		$\hat{\sigma}_{-4}^2$

Table III.10					
PEG Selection Seeds for the Serial Queuing Example Design Point 3, Variance Method 2					
	Random Number Stream				
Runs	1	2	3	4	$\hat{\theta}$
1-100	609117				$\hat{\sigma}_{-1}^2$
101-200		107439			$\hat{\sigma}_{-2}^2$
201-300			693435		$\hat{\sigma}_{-3}^2$
301-400				757187	$\hat{\sigma}_{-4}^2$

Table III.11				
PEG Selection Seeds for the Serial Queuing Example Design Point 3, Simple Correlation Method				
	Random Number Stream			
Runs	1	2	3	4
1-100	973407	941995	525299	920539
101-200	461427	658021	519255	433441
201-300	857547	121615	880959	120637
301-400	215953	488677	798951	465813

Table III.12				
PEG Selection Seeds for the Serial Queuing Example Design Point 3, Partial Correlation Method				
	Random Number Stream			
Runs	1	2	3	4
1-100	989923	633341	940881	889439
101-200	577083	259593	110119	142593
201-300	812059	529663	879335	382391
301-400	450025	524893	283647	817157

Table III.13					
PEG Selection Seeds for the Serial Queuing Example Design Point 4, Variance Method 1					
	Random Number Stream				
Runs	1	2	3	4	$\hat{\theta}$
1-100		629405	918475	990365	$\hat{\sigma}_{-1}^2$
101-200	193785		921115	845521	$\hat{\sigma}_{-2}^2$
201-300	976621	396297		153141	$\hat{\sigma}_{-3}^2$
301-400	457601	996645	943651		$\hat{\sigma}_{-4}^2$

Table III.14					
PEG Selection Seeds for the Serial Queuing Example Design Point 4, Variance Method 2					
	Random Number Stream				
Runs	1	2	3	4	$\hat{\theta}$
1-100	261963				$\hat{\sigma}_{-1}^2$
101-200		686743			$\hat{\sigma}_{-2}^2$
201-300			939095		$\hat{\sigma}_{-3}^2$
301-400				752527	$\hat{\sigma}_{-4}^2$

Table III.15				
PEG Selection Seeds for the Serial Queuing Example Design Point 4, Simple Correlation Method				
	Random Number Stream			
Runs	1	2	3	4
1-100	583555	934135	626575	374697
101-200	552563	933729	883849	844431
201-300	448767	691417	551025	863897
301-400	335463	249701	144077	707007

Table III.16				
PEG Selection Seeds for the Serial Queuing Example Design Point 4, Partial Correlation Method				
	Random Number Stream			
Runs	1	2	3	4
1-100	554877	724037	402561	231165
101-200	436653	836171	142615	274819
201-300	357549	534283	240559	368249
301-400	470079	467951	908929	376975

Appendix IV

This appendix contains the seeds used for the Monte Carlo analysis in the serial queuing example (section 4.1.2). Each set of 50 runs corresponds to one of the four design points used in the experiment. Note that '*' indicates that a prefix number from 1-50 corresponding to the replication number should be added. Also note that '-' indicates an antithetic seed.

Table IV				
Monte Carlo Analysis Seeds for the Serial Queuing Example				
PEG 1				
	Random Number Stream			
Runs	1	2	3	4
1-50	591247	*088965	*858123	*387025
51-100	715393	*088965	*858123	*387025
101-150	741711	-*088965	-*858123	-*387025
151-200	912969	-*088965	-*858123	-*387025
PEG 2				
	Random Number Stream			
Runs	1	2	3	4
1-50	*687005	383117	*401347	*732481
51-100	*687005	523209	*401347	*732481
101-150	-*687005	403237	-*401347	-*732481
151-200	-*687005	697353	-*401347	-*732481
PEG 3				
	Random Number Stream			
Runs	1	2	3	4
1-50	*368665	*905533	109507	*580823
51-100	*368665	*905533	918265	*580823
101-150	-*368665	-*905533	637767	-*580823
151-200	-*368665	-*905533	251375	-*580823
PEG 4				
	Random Number Stream			
Runs	1	2	3	4
1-50	*580475	*531843	*236269	364643
51-100	-*580475	-*531843	-*236269	746163
101-150	-*580475	-*531843	-*236269	702973
151-200	*580475	*531843	*236269	299791

Appendix V

This appendix contains the seeds used for PEG selection methods in the hospital example (section 4.2.1). The sets of seeds given in Tables V.1 - V.16 were randomly selected and used to initialize each set of 100 runs.

Table V.1								
PEG Selection Seeds for the Hospital Example Design Point 1, Simple Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	948721	394179	438445	017635	973373	827783	627359	252757
101-200	321422	768087	678277	486043	437675	576423	535153	326285
201-300	925117	500287	014207	884667	377671	585699	885699	707619
301-400	410781	840513	813523	171105	554171	537131	091403	088557
401-500	804673	302789	271390	353107	134011	498555	510293	638801
501-600	541211	363997	851853	001167	480349	566979	084179	118595
601-700	148017	188055	197701	608805	142153	851687	708837	516385
701-800	395707	051611	651089	343847	109631	607071	984007	693035

Table V.2								
PEG Selection Seeds for the Hospital Example Design Point 1, Partial Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	855537	577907	881693	633647	968595	515317	643241	882015
101-200	123685	137769	221795	226435	658587	751047	282877	551643
201-300	552671	083941	186341	274043	159987	090055	357375	466079
301-400	613859	897625	306903	162777	259849	226825	718267	007823
401-500	939059	149251	076165	670241	829693	451273	040275	747389
501-600	613253	804659	829073	507987	418577	232983	318465	918361
601-700	991439	946785	865717	856487	270409	662037	905107	298369
701-800	454687	348457	215684	177840	047008	975201	635621	892495

Table V.3								
PEG Selection Seeds for the Hospital Example Design Point 2, Simple Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	124407	001421	655415	158903	760851	250857	015757	547597
101-200	439009	691935	870095	771229	504485	420645	087623	249971
201-300	284325	330741	425693	690037	411181	387701	662679	078943
301-400	130125	045015	698577	455281	013803	066583	328809	261845
401-500	590495	208999	923505	848383	284025	938297	212507	356055
501-600	081533	801939	942669	121951	732351	213233	472507	230519
601-700	924963	124551	678819	639863	937173	352820	911441	615179
701-800	485361	637189	027375	132723	809349	011983	702803	308639

Table V.4								
PEG Selection Seeds for the Hospital Example Design Point 2, Partial Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	280757	739619	422791	538711	723535	126355	067863	119243
101-200	307411	573035	279869	054093	668481	421203	156027	177023
201-300	520711	656785	686793	318951	625923	738733	495449	654575
301-400	300665	345553	625633	553005	165551	510369	472825	930297
401-500	053469	582289	482907	740093	872263	878467	719081	883455
501-600	038383	930159	582339	015873	743005	989103	403531	144481
601-700	166567	718015	753047	044195	568083	468475	266625	999143
701-800	215297	218675	981319	133093	451205	849985	554993	041881

Table V.5								
PEG Selection Seeds for the Hospital Example Design Point 3, Simple Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	187793	551419	657211	574297	669057	275061	172803	060013
101-200	036217	675533	574161	834737	863697	974945	533391	207997
201-300	264757	665081	945347	277623	602981	974919	439403	457033
301-400	425749	395283	433955	115457	083697	242877	353893	660503
401-500	342153	815485	107983	200887	707959	303255	428939	183319
501-600	965401	380797	423661	184141	552863	308651	224957	874331
601-700	529031	595325	703503	887269	205073	011049	820393	054351
701-800	410149	026295	785151	808325	186257	027387	836691	430281

Table V.6								
PEG Selection Seeds for the Hospital Example Design Point 3, Partial Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	588845	081907	023009	550711	315813	537193	517347	780853
101-200	550931	662069	743987	886399	970975	395193	027655	501283
201-300	428373	801499	672051	761281	872107	167915	207267	809995
301-400	144991	587819	799211	079519	569431	496761	111285	476693
401-500	147687	711585	028485	323391	726673	269463	046881	215782
501-600	269477	170021	221945	919167	878489	205697	903917	815633
601-700	754355	052513	710885	807091	102449	342853	862595	712199
701-800	262641	084329	602597	915855	792861	366765	457917	754739

Table V.7								
PEG Selection Seeds for the Hospital Example Design Point 4, Simple Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	605669	182215	161235	701195	744485	806189	189409	844331
101-200	434673	904657	724403	536385	863445	906639	327105	127035
201-300	115379	527009	679125	848211	391305	977415	836593	116309
301-400	931493	853375	000259	466473	398071	712801	518905	816121
401-500	659723	854041	336493	704075	578649	120681	801887	435065
501-600	226921	518711	791881	800881	607993	357519	569411	101043
601-700	925579	191227	012619	073793	236723	449067	682985	051271
701-800	482857	665205	684284	721152	400351	179431	716907	780579

Table V.8								
PEG Selection Seeds for the Hospital Example Design Point 4, Partial Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	815581	516999	239145	870621	916119	096033	264151	223983
101-200	797869	626629	457237	658771	685985	239939	452555	045011
201-300	964845	096037	722361	827971	109949	538145	916545	640115
301-400	692299	100731	569493	688879	345781	834891	316719	203063
401-500	392669	589311	590497	927543	304931	578559	838869	594529
501-600	574539	878071	253847	759035	436355	437595	181733	491815
601-700	900403	425177	399047	560251	368299	671531	524019	051157
701-800	637541	719827	110677	730121	955641	947857	694381	439305

Table V.9								
PEG Selection Seeds for the Hospital Example Design Point 5, Simple Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	938773	591419	409985	202793	703429	526143	270999	744279
101-200	924701	134645	736491	920743	885773	939667	046495	049679
201-300	152733	062309	091741	012853	370071	065451	349897	002219
301-400	597853	903315	510175	003031	824841	054369	380321	967955
401-500	309833	167471	068039	407985	589849	762247	197157	075397
501-600	130893	092143	856641	979987	721247	386709	989571	525651
601-700	593723	023699	034565	558589	413717	741411	248131	695041
701-800	095665	952375	884077	367965	880487	324459	023125	043753

Table V.10								
PEG Selection Seeds for the Hospital Example Design Point 5, Partial Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	999023	287605	641701	169643	154855	160471	552361	945843
101-200	408613	929953	688711	124835	440917	855027	648347	814657
201-300	344243	776419	894809	455183	006305	205575	273215	542785
301-400	091761	483289	877655	674667	340909	411017	095371	925065
401-500	550817	281515	767501	531231	882667	552973	144663	589399
501-600	702763	796455	448031	904281	011931	603515	977879	516505
601-700	267847	239581	979039	773867	901333	907179	590759	039243
701-800	672625	988029	652687	186059	690313	629303	120239	191191

Table V.11								
PEG Selection Seeds for the Hospital Example Design Point 6, Simple Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	974229	718413	324619	166073	797051	428261	939291	835851
101-200	275483	329825	671267	008181	622095	113311	717321	424679
201-300	207063	604305	412469	331503	496027	403649	175781	420963
301-400	400763	858571	935537	825149	913095	545749	532663	067797
401-500	419579	968371	748391	449063	969885	751725	264011	101575
501-600	260177	331935	044031	309377	473917	575401	810269	711799
601-700	397013	999143	095593	927101	328723	372177	152323	697461
701-800	183637	041463	751625	645553	665629	913601	374055	683803

Table V.12								
PEG Selection Seeds for the Hospital Example Design Point 6, Partial Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	470461	880191	192957	962727	841609	736853	439149	091373
101-200	283007	493477	029233	485527	543413	680017	123381	293377
201-300	614679	266331	323553	628381	074025	314307	115873	518715
301-400	707839	794635	565141	329405	586203	494611	316547	348649
401-500	759723	437205	066533	771901	779595	103675	546067	363875
501-600	003333	063977	590751	851487	343631	129777	347285	952485
601-700	102769	818771	384573	280011	422187	471077	669489	375701
701-800	830551	151645	571959	609525	138611	029015	124911	781659

Table V.13								
PEG Selection Seeds for the Hospital Example Design Point 7, Simple Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	426573	675307	175987	036171	254015	113423	921013	738321
101-200	596179	878883	703843	989697	149619	599087	457571	696791
201-300	913111	656489	570333	195105	993753	322789	104431	041557
301-400	689707	897451	790043	475921	545279	940157	340189	166587
401-500	844399	787319	080389	266481	826355	994195	938951	844933
501-600	353563	089969	822487	072353	764289	074935	596747	117853
601-700	104653	484469	845031	751475	790055	468367	288537	894527
701-800	473887	854859	406465	319397	905649	281293	162939	611291

Table V.14								
PEG Selection Seeds for the Hospital Example Design Point 7, Partial Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	068905	584489	844039	901271	963345	222697	008699	481681
101-200	519535	649259	005577	043667	582467	218019	965415	034721
201-300	566577	840045	247863	068367	301173	361149	626407	465931
301-400	405675	453085	545907	605639	833329	430615	454645	675511
401-500	513147	907373	858387	879027	349661	186325	492419	464265
501-600	653315	699263	842871	735909	427213	720939	111051	993189
601-700	240867	134139	935389	224805	840223	236123	760341	134687
701-800	001075	440305	009321	981165	751737	202365	362333	287823

Table V.15								
PEG Selection Seeds for the Hospital Example Design Point 8, Simple Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	262241	498129	393839	772301	276259	479927	216067	703201
101-200	973625	831985	973119	830213	848531	996833	783993	146715
201-300	866991	828069	975275	435177	335661	612893	235345	297905
301-400	436571	907619	814365	271387	660575	265783	584576	838399
401-500	096943	423863	908673	460323	198831	343311	879587	594433
501-600	439863	125893	579441	051045	366149	715947	648853	866513
601-700	698561	690615	026809	180387	554869	234215	767525	973685
701-800	202851	671533	589465	098665	421299	264075	326999	604117

Table V.16								
PEG Selection Seeds for the Hospital Example Design Point 8, Partial Correlation Method								
Random Number Stream								
Runs	1	2	3	4	5	6	7	8
1-100	073577	793827	592887	125955	698443	451473	715965	735451
101-200	775685	720927	254509	439295	919137	555951	255089	821969
201-300	963083	396697	790297	063993	953363	998043	453975	800023
301-400	828297	771653	540455	392885	882653	751341	960421	376997
401-500	502833	816813	203781	277291	754129	297697	360219	242667
501-600	972423	686847	992791	622811	694521	195757	615283	950013
601-700	266401	944627	229333	915123	996563	918117	336761	502123
701-800	852213	645093	908569	547691	643661	224255	999185	396961

Appendix VI

This appendix contains the seeds used for the Monte Carlo analysis in the hospital example (section 4.2.2). Each set of 50 runs corresponds to one of the four design points used in the experiment. Note that '*' indicates that a prefix number from 1-50, corresponding to the replication number, should be added. Also note that '-' indicates an antithetic seed.

Table VI.1								
Monte Carlo Analysis Seeds for the Hospital Example , PEG 1-4								
PEG 1								
Random Number Stream								
DP	1	2	3	4	5	6	7	8
1	355061	*862561	*858123	*523253	*750449	*565851	*605277	*671777
2	356929	*862561	*858123	*523253	*750449	*565851	*605277	*671777
3	003569	*862561	*858123	*523253	*750449	*565851	*605277	*671777
4	219707	*862561	*858123	*523253	*750449	*565851	*605277	*671777
5	218417	-*862561	-*858123	-*523253	-*750449	-*565851	-*605277	-*671777
6	040053	-*862561	-*858123	-*523253	-*750449	-*565851	-*605277	-*671777
7	665333	-*862561	-*858123	-*523253	-*750449	-*565851	-*605277	-*671777
8	754623	-*862561	-*858123	-*523253	-*750449	-*565851	-*605277	-*671777
PEG 2								
Random Number Stream								
DP	1	2	3	4	5	6	7	8
1	*316753	208421	*223841	*530271	*463805	*783153	*598501	*867023
2	*316753	048527	*223841	*530271	*463805	*783153	*598501	*867023
3	*316753	938375	*223841	*530271	*463805	*783153	*598501	*867023
4	*316753	374187	*223841	*530271	*463805	*783153	*598501	*867023
5	-*316753	348379	-*223841	-*530271	-*463805	-*783153	-*598501	-*867023
6	-*316753	417007	-*223841	-*530271	-*463805	-*783153	-*598501	-*867023
7	-*316753	088973	-*223841	-*530271	-*463805	-*783153	-*598501	-*867023
8	-*316753	432517	-*223841	-*530271	-*463805	-*783153	-*598501	-*867023
PEG 3								
Random Number Stream								
DP	1	2	3	4	5	6	7	8
1	*355061	*956787	815443	*124541	*900839	*412933	*599309	*273263
2	*355061	*956787	812485	*124541	*900839	*412933	*599309	*273263
3	*355061	*956787	167237	*124541	*900839	*412933	*599309	*273263
4	*355061	*956787	891927	*124541	*900839	*412933	*599309	*273263
5	-*355061	-*956787	238615	-*124541	-*900839	-*412933	-*599309	-*273263
6	-*355061	-*956787	416017	-*124541	-*900839	-*412933	-*599309	-*273263
7	-*355061	-*956787	178005	-*124541	-*900839	-*412933	-*599309	-*273263
8	-*355061	-*956787	424205	-*124541	-*900839	-*412933	-*599309	-*273263
PEG 4								
Random Number Stream								
DP	1	2	3	4	5	6	7	8
1	*572683	*196883	*951479	104025	*491241	*317181	*842577	*089757
2	*572683	*196883	*951479	649473	*491241	*317181	*842577	*089757
3	*572683	*196883	*951479	742193	*491241	*317181	*842577	*089757
4	*572683	*196883	*951479	024799	*491241	*317181	*842577	*089757
5	-*572683	-*196883	-*951479	430415	-*491241	-*317181	-*842577	-*089757
6	-*572683	-*196883	-*951479	281745	-*491241	-*317181	-*842577	-*089757
7	-*572683	-*196883	-*951479	408527	-*491241	-*317181	-*842577	-*089757
8	-*572683	-*196883	-*951479	546699	-*491241	-*317181	-*842577	-*089757

Table VI.2								
Monte Carlo Analysis Seeds for the Hospital Example , PEG 5-8								
PEG 5								
Random Number Stream								
DP	1	2	3	4	5	6	7	8
1	*248237	*392033	*911009	*110577	019585	*807235	*460731	*878909
2	*248237	*392033	*911009	*110577	451413	*807235	*460731	*878909
3	*248237	*392033	*911009	*110577	247293	*807235	*460731	*878909
4	*248237	*392033	*911009	*110577	113619	*807235	*460731	*878909
5	-*248237	-*392033	-*911009	-*110577	018049	-*807235	-*460731	-*878909
6	-*248237	-*392033	-*911009	-*110577	842259	-*807235	-*460731	-*878909
7	-*248237	-*392033	-*911009	-*110577	174777	-*807235	-*460731	-*878909
8	-*248237	-*392033	-*911009	-*110577	554845	-*807235	-*460731	-*878909
PEG 6								
Random Number Stream								
DP	1	2	3	4	5	6	7	8
1	*751393	*717943	*286811	*498107	*308833	695231	*035621	*780405
2	*751393	*717943	*286811	*498107	*308833	468313	*035621	*780405
3	*751393	*717943	*286811	*498107	*308833	657795	*035621	*780405
4	*751393	*717943	*286811	*498107	*308833	454055	*035621	*780405
5	-*751393	-*717943	-*286811	-*498107	-*308833	501861	-*035621	-*780405
6	-*751393	-*717943	-*286811	-*498107	-*308833	517451	-*035621	-*780405
7	-*751393	-*717943	-*286811	-*498107	-*308833	595411	-*035621	-*780405
8	-*751393	-*717943	-*286811	-*498107	-*308833	259769	-*035621	-*780405
PEG 7								
Random Number Stream								
DP	1	2	3	4	5	6	7	8
1	*313033	*415327	*063169	*588167	*600903	*744201	232817	*844469
2	*313033	*415327	*063169	*588167	*600903	*744201	383617	*844469
3	*313033	*415327	*063169	*588167	*600903	*744201	897417	*844469
4	*313033	*415327	*063169	*588167	*600903	*744201	331539	*844469
5	-*313033	-*415327	-*063169	-*588167	-*600903	-*744201	676439	-*844469
6	-*313033	-*415327	-*063169	-*588167	-*600903	-*744201	489929	-*844469
7	-*313033	-*415327	-*063169	-*588167	-*600903	-*744201	563735	-*844469
8	-*313033	-*415327	-*063169	-*588167	-*600903	-*744201	212625	-*844469
PEG 8								
Random Number Stream								
DP	1	2	3	4	5	6	7	8
1	*894185	*841503	*996989	*867905	*057973	*668143	*742247	772719
2	*894185	*841503	*996989	*867905	*057973	*668143	*742247	780331
3	*894185	*841503	*996989	*867905	*057973	*668143	*742247	810061
4	*894185	*841503	*996989	*867905	*057973	*668143	*742247	743957
5	-*894185	-*841503	-*996989	-*867905	-*057973	-*668143	-*742247	021743
6	-*894185	-*841503	-*996989	-*867905	-*057973	-*668143	-*742247	969187
7	-*894185	-*841503	-*996989	-*867905	-*057973	-*668143	-*742247	627751
8	-*894185	-*841503	-*996989	-*867905	-*057973	-*668143	-*742247	971041

Appendix VII

This appendix contains the estimates of all parameter estimates used for the PEG selections in the serial queuing example (section 4.1.1). Confidence intervals are included with the estimates for completeness. Confidence intervals on all variance estimates are given by

$$\frac{(n-1)\hat{\sigma}^2}{\chi_{1-\alpha/2, n-1}^2} \leq \sigma^2 \leq \frac{(n-1)\hat{\sigma}^2}{\chi_{\alpha/2, n-1}^2} \quad (57)$$

where n is the number of observations used to calculate $\hat{\sigma}^2$, α is the specified type I error rate, and $\chi_{p, n-1}^2$ is the p , $n-1$ point of the χ^2 distribution (see Neter, Wasserman, and Kutner 1990, 16).

Confidence intervals on all simple correlation estimates are given by

$$\tanh \left\{ \frac{1}{2} \ln \left[\frac{1 + \hat{\rho}_{y\bar{z}_i}}{1 - \hat{\rho}_{y\bar{z}_i}} \right] - \frac{z_\alpha}{\sqrt{n-3}} \right\} \leq \rho \leq \tanh \left\{ \frac{1}{2} \ln \left[\frac{1 + \hat{\rho}_{y\bar{z}_i}}{1 - \hat{\rho}_{y\bar{z}_i}} \right] + \frac{z_\alpha}{\sqrt{n-3}} \right\} \quad (58)$$

where n is the number of observations used to calculate $\hat{\rho}_{y\bar{z}_i}$ and z_α is the α point of the standard normal distribution (see Anderson 1989, 123-124).

Confidence intervals on all partial correlation estimates are given by

$$\tanh \left\{ \frac{1}{2} \ln \left[\frac{1 + \hat{\rho}_{p_y\bar{z}_i}}{1 - \hat{\rho}_{p_y\bar{z}_i}} \right] - \frac{z_\alpha}{\sqrt{n-6}} \right\} \leq \rho \leq \tanh \left\{ \frac{1}{2} \ln \left[\frac{1 + \hat{\rho}_{p_y\bar{z}_i}}{1 - \hat{\rho}_{p_y\bar{z}_i}} \right] + \frac{z_\alpha}{\sqrt{n-6}} \right\} \quad (59)$$

where n is the number of observations used to calculate $\hat{\rho}_{p_y\bar{z}_i}$ and z_α is the α point of the standard normal distribution (see Anderson 1989; 123-124, 133).

Table VII.1			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 1, Response 1			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	35.43118	45.97112	62.03584
2	425544.06200	552132.75000	745077.62500
3	318939.43700	413816.06200	558425.50000
4	328266.43700	425917.43700	574755.93700
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	385893.43700	500687.18700	675654.06200
2	31.96176	41.46959	55.96127
3	0.00611	0.00793	0.01070
4	0.12213	0.15846	0.21383
	316593.43700	410772.06200	554317.93700
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.86076	-0.83301	-0.80033
2	-0.11538	-0.01753	0.08066
3	-0.10667	-0.00871	0.08942
4	-0.08686	0.01129	0.10922
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.84092	-0.80949	-0.77262
2	-0.15132	-0.05370	0.04496
3	-0.09036	0.00814	0.10648
4	-0.09585	0.00260	0.10100

Table VII.2			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 1, Response 2			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00098	0.00127	0.00172
2	6.65682	8.63706	11.65531
3	6.30375	8.17896	11.03713
4	9.01198	11.69282	15.77892
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	6.37953	8.27729	11.16981
2	0.00066	0.00085	0.00115
3	0.00002	0.00003	0.00004
4	0.00034	0.00045	0.00060
	6.52801	8.46992	11.42977
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.80921	0.84057	0.86715
2	-0.09599	0.00208	0.10011
3	-0.08061	0.01758	0.11543
4	-0.04774	0.05055	0.14787
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	0.81988	0.84973	0.87498
2	-0.13988	-0.04204	0.05662
3	-0.06707	0.03156	0.12958
4	-0.20072	-0.10436	-0.00600

Table VII.3			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 1, Response 3			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00062	0.00080	0.00108
2	4.55901	5.91521	7.98230
3	4.36465	5.66302	7.64198
4	6.06540	7.86971	10.61980
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	4.39633	5.70412	7.69745
2	0.00066	0.00085	0.00115
3	0.00000	0.00000	0.00000
4	0.00000	0.00000	0.00000
	4.45181	5.77611	7.79459
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.87047	-0.84450	-0.81384
2	-0.08443	0.01374	0.11164
3	-0.11536	-0.01750	0.08069
4	-0.14680	-0.04946	0.04883
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.87667	-0.85174	-0.82225
2	-0.03409	0.06455	0.16194
3	-0.13255	-0.03458	0.06406
4	0.01160	0.10990	0.20609

Table VII.4			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 1, Response 4			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00539	0.00699	0.00944
2	3.76417	4.88392	6.59063
3	3.56766	4.62895	6.24656
4	4.98951	6.47377	8.73605
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	3.60297	4.67476	6.30837
2	0.00047	0.00061	0.00082
3	0.00502	0.00651	0.00879
4	0.00000	0.00000	0.00000
	3.66751	4.75851	6.42138
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.86879	-0.84251	-0.81149
2	-0.09953	-0.00149	0.09658
3	-0.08178	0.01641	0.11428
4	-0.14805	-0.05073	0.04756
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.87625	-0.85124	-0.82166
2	-0.06164	0.03701	0.13494
3	-0.07393	0.02467	0.12279
4	0.01241	0.11070	0.20687

Table VII.5			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 1, Response 5			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.07648	0.09923	0.13391
2	0.32720	0.42453	0.57288
3	0.39017	0.50623	0.68314
4	0.31288	0.40596	0.54782
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.22475	0.29161	0.39352
2	0.00003	0.00003	0.00005
3	0.00001	0.00002	0.00003
4	0.10010	0.12988	0.17527
	0.31111	0.40366	0.54472
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.78635	-0.74580	-0.69885
2	-0.07764	0.02057	0.11838
3	-0.11244	-0.01455	0.08362
4	0.35545	0.43823	0.51419
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.87524	-0.85004	-0.82024
2	-0.06137	0.03728	0.13521
3	-0.13376	-0.03581	0.06284
4	0.68774	0.73632	0.77834

Table VII.6			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 2, Response 1			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	127.75285	165.75613	223.68024
2	854151.50000	1108240.00000	1495518.00000
3	923581.62500	1198324.00000	1617082.00000
4	776070.37500	1006931.87000	1358807.00000
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	1093464.00000	1418743.00000	1914528.00000
2	145.81464	189.19084	255.30429
3	0.00709	0.00920	0.01242
4	0.10427	0.13529	0.18256
	833795.68700	1081829.00000	1459878.00000
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.88171	-0.85782	-0.82954
2	-0.11039	-0.01247	0.08569
3	-0.12697	-0.02928	0.06897
4	-0.07957	0.01863	0.11647
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.89484	-0.87334	-0.84779
2	-0.02891	0.06971	0.16699
3	-0.05722	0.04144	0.13929
4	-0.11200	-0.01373	0.08481

Table VII.7			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 2, Response 2			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00202	0.00262	0.00354
2	0.15045	0.19520	0.26342
3	0.29569	0.38365	0.51772
4	0.22715	0.29472	0.39771
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.17749	0.23029	0.31077
2	0.00157	0.00204	0.00275
3	0.00002	0.00003	0.00004
4	0.00039	0.00051	0.00068
	0.14567	0.18901	0.25506
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.22608	0.31710	0.40263
2	0.04101	0.13851	0.23339
3	-0.02012	0.07809	0.17480
4	-0.08600	0.01216	0.11008
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	0.16220	0.25653	0.34621
2	0.10534	0.20167	0.29425
3	0.00103	0.09944	0.19595
4	-0.02958	0.06905	0.16634

Table VII.8			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 2, Response 3			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00000	0.00000	0.00000
2	0.11984	0.15549	0.20982
3	0.22893	0.29702	0.40082
4	0.17962	0.23305	0.31449
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.14155	0.18366	0.24785
2	0.00000	0.00000	0.00000
3	0.00000	0.00000	0.00000
4	0.00000	0.00000	0.00000
	0.11543	0.14977	0.20211
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.40418	-0.31876	-0.22783
2	-0.15855	-0.06145	0.03683
3	-0.17604	-0.07936	0.01884
4	-0.10889	-0.01095	0.08720
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.34757	-0.25797	-0.16370
2	-0.20398	-0.10772	-0.00940
3	-0.19951	-0.10311	-0.00473
4	-0.16556	-0.06825	0.03038

Table VII.9			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 2, Response 4			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00720	0.00934	0.01261
2	0.09823	0.12745	0.17199
3	0.18960	0.24601	0.33197
4	0.15806	0.20508	0.27675
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.11645	0.15109	0.20389
2	0.00105	0.00136	0.00183
3	0.00573	0.00743	0.01002
4	0.00000	0.00000	0.00000
	0.10023	0.13004	0.17548
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.40513	-0.31978	-0.22890
2	-0.22950	-0.13447	-0.03690
3	0.00202	0.10005	0.19618
4	-0.10189	-0.00388	0.09421
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.34702	-0.25739	-0.16310
2	-0.29290	-0.20025	-0.10387
3	-0.00239	0.09606	0.19266
4	-0.16486	-0.06753	0.03110

Table VII.10			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 2, Response 5			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.06681	0.08668	0.11697
2	0.09360	0.12144	0.16388
3	0.09547	0.12387	0.16716
4	0.00920	0.01193	0.01610
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00709	0.00920	0.01241
2	0.00007	0.00010	0.00013
3	0.00001	0.00001	0.00002
4	0.08624	0.11190	0.15100
	0.08635	0.11203	0.15118
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.19190	-0.09565	0.00243
2	-0.12150	-0.02373	0.07450
3	-0.08116	0.01703	0.11489
4	0.93538	0.94661	0.95594
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.35110	-0.26172	-0.16761
2	-0.29469	-0.20213	-0.10581
3	-0.19319	-0.09660	0.00184
4	0.94448	0.95420	0.96225

Table VII.11			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 3, Response 1			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.10193	0.13225	0.17846
2	21616.58980	28046.98050	37848.10550
3	47325.95310	61404.23440	82862.12500
4	41363.26170	53667.79690	72422.18750
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	60962.26560	79097.00000	106737.75000
2	0.00116	0.00150	0.00203
3	0.00713	0.00926	0.01249
4	0.07014	0.09100	0.12280
	35933.28910	46622.54300	62914.96480
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.73183	-0.68277	-0.62667
2	-0.00757	0.09055	0.18694
3	-0.13850	-0.04100	0.05728
4	-0.10903	-0.01110	0.08705
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.71860	-0.66738	-0.60896
2	-0.09958	-0.00117	0.09727
3	-0.10887	-0.01056	0.08796
4	-0.18040	-0.08346	0.01509

Table VII.12			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 3, Response 2			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00053	0.00069	0.00093
2	14.47833	18.78526	25.34984
3	13.43356	17.42970	23.52057
4	14.31640	18.57516	25.06631
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	16.29581	21.14340	28.53204
2	0.00000	0.00001	0.00001
3	0.00003	0.00004	0.00005
4	0.00045	0.00058	0.00079
	15.72909	20.40810	27.53978
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.97297	0.97774	0.98168
2	-0.08929	0.00884	0.10680
3	-0.06120	0.03708	0.13464
4	-0.06478	0.03349	0.13111
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	0.97161	0.97664	0.98079
2	-0.10303	-0.00465	0.09382
3	-0.09210	0.00638	0.10474
4	-0.12810	-0.03006	0.06857

Table VII.13			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 3, Response 3			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00131	0.00170	0.00230
2	8.31613	10.78998	14.56057
3	8.36618	10.85491	14.64820
4	8.18541	10.62036	14.33169
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	10.67367	13.84882	18.68834
2	0.00093	0.00120	0.00163
3	0.00002	0.00002	0.00003
4	0.00006	0.00008	0.00011
	8.99498	11.67076	15.74916
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.99610	-0.99525	-0.99422
2	-0.09388	0.00421	0.10222
3	-0.13324	-0.03565	0.06262
4	-0.12898	-0.03132	0.06694
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.99607	-0.99521	-0.99417
2	-0.02465	0.07395	0.17113
3	-0.06102	0.03763	0.13555
4	-0.11218	-0.01391	0.08463

Table VII.14			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 3, Response 4			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00622	0.00807	0.01089
2	9.02508	11.70981	15.80185
3	8.97724	11.64775	15.71810
4	9.42378	12.22712	16.49992
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	10.69301	13.87391	18.72220
2	0.00000	0.00000	0.00000
3	0.00587	0.00761	0.01028
4	0.00000	0.00000	0.00000
	9.89026	12.83237	17.31668
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.98141	-0.97741	-0.97257
2	-0.10169	-0.00367	0.09442
3	-0.11262	-0.01473	0.08344
4	-0.13119	-0.03357	0.06470
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.98054	-0.97634	-0.97125
2	-0.08742	0.01110	0.10940
3	-0.00033	0.09810	0.19464
4	-0.08958	0.00892	0.10725

Table VII.15			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 3, Response 5			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.06948	0.09015	0.12165
2	0.66589	0.86397	1.16589
3	0.62515	0.81112	1.09456
4	0.58546	0.75962	1.02507
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00668	0.00867	0.01170
2	0.00000	0.00000	0.00000
3	0.00000	0.00000	0.00000
4	0.05768	0.07484	0.10099
	0.66035	0.85679	1.15620
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.92643	-0.91115	-0.89287
2	-0.10082	-0.00279	0.09529
3	-0.17982	-0.08323	0.01495
4	0.23641	0.32689	0.41175
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.98009	-0.97580	-0.97059
2	-0.08402	0.01452	0.11278
3	-0.10003	-0.00162	0.09682
4	0.81444	0.84512	0.87109

Table VII.16			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 4, Response 1			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	592.85449	769.21411	1038.01880
2	699171.93700	907158.06200	1224167.00000
3	795666.87500	1032357.69000	1393119.00000
4	704247.00000	913742.75000	1233053.00000
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	845459.93700	1096963.00000	1480300.00000
2	0.70914	0.92009	1.24162
3	612.46436	794.65723	1072.35327
4	0.08129	0.10548	0.14233
	737559.50000	956964.93700	1291380.00000
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.87732	-0.85261	-0.82339
2	-0.00730	0.09082	0.18721
3	-0.04544	0.05285	0.15013
4	-0.08318	0.01500	0.11289
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.89710	-0.87603	-0.85098
2	-0.18317	-0.08630	0.01223
3	-0.06300	0.03565	0.13360
4	-0.08522	0.01332	0.11160

Table VII.17			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 4, Response 2			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00644	0.00836	0.01128
2	0.45588	0.59149	0.79819
3	0.71254	0.92451	1.24758
4	0.32020	0.41545	0.56063
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.47955	0.62221	0.83964
2	0.00000	0.00000	0.00000
3	0.00747	0.00970	0.01309
4	0.00034	0.00044	0.00059
	0.76746	0.99577	1.34374
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.29727	0.38413	0.46468
2	-0.14533	-0.04796	0.05033
3	0.02026	0.11808	0.21366
4	-0.08612	0.01204	0.10996
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	0.26879	0.35775	0.44066
2	-0.09397	0.00450	0.10288
3	0.02553	0.12364	0.21939
4	-0.10486	-0.00650	0.09198

Table VII.18			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 4, Response 3			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00061	0.00080	0.00107
2	4.13546	5.36566	7.24071
3	6.33514	8.21969	11.09209
4	3.24369	4.20861	5.67932
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	5.15627	6.69014	9.02803
2	0.00053	0.00068	0.00092
3	0.00000	0.00000	0.00000
4	0.00000	0.00000	0.00000
	4.54920	5.90247	7.96511
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.87471	-0.84952	-0.81975
2	-0.00537	0.09273	0.18906
3	-0.05992	0.03836	0.13590
4	-0.10165	-0.00363	0.09446
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.89186	-0.86979	-0.84358
2	-0.15007	-0.05242	0.04624
3	-0.12480	-0.02670	0.07191
4	-0.12693	-0.02887	0.06975

Table VII.19			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 4, Response 4			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.00000	0.00000	0.00000
2	0.43963	0.57040	0.76973
3	0.66157	0.85837	1.15832
4	0.28963	0.37579	0.50712
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.45257	0.58720	0.79240
2	0.00000	0.00000	0.00000
3	0.00000	0.00000	0.00000
4	0.00000	0.00000	0.00000
	0.70811	0.91876	1.23982
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.47040	-0.39035	-0.30393
2	-0.05028	0.04801	0.14538
3	-0.10745	-0.00950	0.08864
4	-0.10325	-0.00525	0.09285
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.44163	-0.35880	-0.26991
2	-0.10183	-0.00344	0.09502
3	-0.10469	-0.00633	0.09215
4	-0.09082	0.00767	0.10601

Table VII.20			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Serial Queuing Example Design Point 4, Response 5			
Variance Method 1			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.08906	0.11556	0.15594
2	0.12761	0.16557	0.22343
3	0.11783	0.15288	0.20631
4	0.01906	0.02472	0.03336
Variance Method 2			
PEG	Lower 95 % CL	$\hat{\sigma}^2$	Upper 95 % CL
1	0.02819	0.03657	0.04935
2	0.00000	0.00000	0.00000
3	0.00048	0.00062	0.00084
4	0.08116	0.10531	0.14211
	0.16590	0.21525	0.29047
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.25295	-0.15884	-0.06175
2	-0.07560	0.02262	0.12041
3	-0.10339	-0.00539	0.09271
4	0.81592	0.84627	0.87197
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.44005	-0.35709	-0.26809
2	-0.10406	-0.00569	0.09279
3	-0.21917	-0.12341	-0.02529
4	0.86051	0.88406	0.90384

Appendix VIII

This appendix contains the estimates of all parameters used for the PEG selections in the hospital example (section 4.2.1). Confidence intervals are included with the estimates for completeness.

Confidence intervals on all simple correlation estimates are given by

$$\tanh \left\{ \frac{1}{2} \ln \left[\frac{1 + \hat{\rho}_{y\bar{z}_i}}{1 - \hat{\rho}_{y\bar{z}_i}} \right] - \frac{z_\alpha}{\sqrt{n-3}} \right\} \leq \rho \leq \tanh \left\{ \frac{1}{2} \ln \left[\frac{1 + \hat{\rho}_{y\bar{z}_i}}{1 - \hat{\rho}_{y\bar{z}_i}} \right] + \frac{z_\alpha}{\sqrt{n-3}} \right\} \quad (60)$$

where n is the number of observations used to calculate $\hat{\rho}_{y\bar{z}_i}$ and z_α is the α point of the standard normal distribution (see Anderson 1989, 123-124).

Confidence intervals on all partial correlation estimates are given by

$$\tanh \left\{ \frac{1}{2} \ln \left[\frac{1 + \hat{\rho}_{p_{y\bar{z}_i}}}{1 - \hat{\rho}_{p_{y\bar{z}_i}}} \right] - \frac{z_\alpha}{\sqrt{n-6}} \right\} \leq \rho \leq \tanh \left\{ \frac{1}{2} \ln \left[\frac{1 + \hat{\rho}_{p_{y\bar{z}_i}}}{1 - \hat{\rho}_{p_{y\bar{z}_i}}} \right] + \frac{z_\alpha}{\sqrt{n-6}} \right\} \quad (61)$$

where n is the number of observations used to calculate $\hat{\rho}_{p_{y\bar{z}_i}}$ and z_α is the α point of the standard normal distribution (see Anderson 1989; 123-124, 133).

Table VIII.1			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 1, Response 1			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.16307	0.22979	0.29442
2	0.01412	0.08335	0.15179
3	0.48205	0.53354	0.58136
4	0.09076	0.15908	0.22590
5	0.33542	0.39554	0.45245
6	0.08233	0.15079	0.21783
7	0.30191	0.36362	0.42229
8	-0.01645	0.05293	0.12180
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	0.12740	0.19529	0.26136
2	0.11997	0.18802	0.25431
3	0.62401	0.66475	0.70189
4	0.13258	0.20035	0.26626
5	0.63469	0.67451	0.71075
6	0.42015	0.47585	0.52798
7	0.66285	0.70016	0.73400
8	0.16457	0.23154	0.29638

Table VIII.2			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 1, Response 2			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.62354	-0.57926	-0.53128
2	-0.10747	-0.03844	0.03096
3	-0.05234	0.01704	0.08625
4	-0.04914	0.02024	0.08943
5	-0.03371	0.03569	0.10475
6	-0.05715	0.01221	0.08146
7	0.16231	0.22905	0.29370
8	-0.07049	-0.00118	0.06814
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.67612	-0.63646	-0.59312
2	-0.05763	0.01204	0.08159
3	0.04034	0.10965	0.17791
4	-0.07973	-0.01017	0.05949
5	0.19134	0.25753	0.32139
6	0.12811	0.19598	0.26203
7	0.24988	0.31404	0.37545
8	0.03091	0.10032	0.16876

Table VIII.3			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 1, Response 3			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.33638	-0.27344	-0.20807
2	-0.18755	-0.11979	-0.05090
3	0.43398	0.48860	0.53964
4	-0.10642	-0.03738	0.03202
5	-0.10004	-0.03094	0.03846
6	-0.12581	-0.05699	0.01237
7	-0.08122	-0.01197	0.05739
8	-0.06091	0.00844	0.07771
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.46837	-0.41219	-0.35269
2	-0.19562	-0.12774	-0.05864
3	0.48461	0.53614	0.58396
4	-0.14566	-0.07682	-0.00724
5	-0.14492	-0.07607	-0.00648
6	-0.11865	-0.04944	0.02025
7	-0.09642	-0.02698	0.04272
8	-0.05046	0.01923	0.08873

Table VIII.4			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 1, Response 4			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.38171	-0.32088	-0.25729
2	0.10166	0.16978	0.23631
3	-0.03199	0.03741	0.10645
4	0.18982	0.25577	0.31942
5	-0.12701	-0.05821	0.01115
6	-0.08283	-0.01359	0.05578
7	-0.05467	0.01470	0.08393
8	-0.13630	-0.06762	0.00170
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.33819	-0.27505	-0.20944
2	0.17144	0.23822	0.30282
3	-0.05903	0.01064	0.08020
4	0.21462	0.28006	0.34299
5	-0.14956	-0.08078	-0.01122
6	-0.03896	0.03074	0.10015
7	-0.07076	-0.00114	0.06849
8	-0.12686	-0.05775	0.01192

Table VIII.5			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 1, Response 5			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.34258	-0.27991	-0.21476
2	-0.06775	0.00157	0.07088
3	-0.14988	-0.08141	-0.01216
4	-0.11168	-0.04270	0.02669
5	-0.08462	-0.01540	0.05397
6	-0.06924	0.00008	0.06939
7	0.23164	0.29620	0.35816
8	-0.02634	0.04305	0.11203
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.33276	-0.26938	-0.20358
2	-0.10225	-0.03286	0.03685
3	-0.21775	-0.15041	-0.08164
4	-0.08615	-0.01663	0.05305
5	0.09232	0.16091	0.22798
6	0.05346	0.12262	0.19061
7	0.20266	0.26849	0.33191
8	0.01690	0.08642	0.15511

Table VIII.6			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 2, Response 1			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.19053	0.25646	0.32008
2	0.00753	0.07681	0.14535
3	0.47435	0.52636	0.57471
4	-0.00096	0.06836	0.13703
5	0.29699	0.35892	0.41784
6	0.18158	0.24778	0.31174
7	0.28602	0.34843	0.40789
8	0.02746	0.09659	0.16480
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{py\bar{z}_i}$	Upper 95 % CL
1	0.15876	0.22588	0.29093
2	0.14931	0.21668	0.28205
3	0.59957	0.64238	0.68152
4	0.15139	0.21871	0.28401
5	0.62829	0.66866	0.70544
6	0.46025	0.51342	0.56292
7	0.62050	0.66154	0.69897
8	0.09920	0.16766	0.23454

Table VIII.7			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 2, Response 2			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.65644	-0.61511	-0.57010
2	-0.05231	0.01707	0.08628
3	-0.01676	0.05262	0.12149
4	-0.04763	0.02176	0.09094
5	-0.10535	-0.03630	0.03310
6	-0.02222	0.04717	0.11611
7	0.13476	0.20219	0.26775
8	-0.07655	-0.00727	0.06208
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.67605	-0.63638	-0.59303
2	0.02307	0.09254	0.16112
3	0.04823	0.11746	0.18556
4	-0.07223	-0.00262	0.06701
5	0.16312	0.23013	0.29502
6	0.11998	0.18803	0.25432
7	0.25088	0.31500	0.37637
8	-0.03372	0.03599	0.10535

Table VIII.8			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 2, Response 3			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.41317	-0.35399	-0.29184
2	-0.15349	-0.08508	-0.01586
3	0.43083	0.48564	0.53688
4	-0.05563	0.01374	0.08298
5	-0.16592	-0.09773	-0.02861
6	-0.09612	-0.02698	0.04241
7	-0.03683	0.03257	0.10166
8	-0.02767	0.04173	0.11073
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.50368	-0.44983	-0.39250
2	-0.22929	-0.16226	-0.09370
3	0.48232	0.53401	0.58199
4	-0.09515	-0.02570	0.04400
5	-0.19887	-0.13106	-0.06200
6	-0.11220	-0.04291	0.02679
7	-0.09215	-0.02267	0.04703
8	-0.10471	-0.03535	0.03436

Table VIII.9			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 2, Response 4			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.41702	-0.35805	-0.29608
2	0.24510	0.30916	0.37053
3	-0.07322	-0.00393	0.06540
4	0.20329	0.26882	0.33195
5	-0.18078	-0.11288	-0.04391
6	-0.13755	-0.06889	0.00043
7	-0.08797	-0.01877	0.05061
8	-0.12449	-0.05566	0.01371
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.36346	-0.30147	-0.23682
2	0.28743	0.35005	0.40969
3	-0.05971	0.00995	0.07952
4	0.21809	0.28341	0.34620
5	-0.04439	0.02531	0.09476
6	-0.10344	-0.03407	0.03564
7	-0.07690	-0.00732	0.06233
8	-0.08948	-0.01998	0.04971

Table VIII.10			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 2, Response 5			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.38596	-0.32535	-0.26194
2	-0.08929	-0.02010	0.04928
3	-0.08489	-0.01567	0.05370
4	-0.03060	0.03880	0.10782
5	-0.11463	-0.04568	0.02371
6	-0.05829	0.01107	0.08032
7	0.23322	0.29772	0.35961
8	-0.04465	0.02474	0.09389
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.34506	-0.28222	-0.21686
2	0.01890	0.08840	0.15705
3	-0.11889	-0.04968	0.02001
4	-0.06457	0.00507	0.07466
5	0.11000	0.17826	0.24484
6	0.04334	0.11262	0.18082
7	0.28411	0.34687	0.40667
8	-0.00122	0.06841	0.13738

Table VIII.11			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 3, Response 1			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.24811	0.31206	0.37330
2	0.05326	0.12212	0.18983
3	0.41498	0.47075	0.52300
4	0.04060	0.10961	0.17758
5	0.39157	0.44871	0.50240
6	0.04266	0.11165	0.17958
7	0.29616	0.35812	0.41708
8	-0.01147	0.05789	0.12670
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	0.10975	0.17801	0.24460
2	0.04656	0.11581	0.18395
3	0.46301	0.51600	0.56531
4	0.13068	0.19850	0.26447
5	0.45533	0.50882	0.55865
6	0.30004	0.36210	0.42110
7	0.51480	0.56420	0.60987
8	0.07360	0.14249	0.21003

Table VIII.12			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 3, Response 2			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.66434	-0.62375	-0.57949
2	-0.11441	-0.04546	0.02393
3	-0.05748	0.01188	0.08113
4	-0.10900	-0.03999	0.02941
5	-0.09959	-0.03049	0.03891
6	-0.08156	-0.01231	0.05705
7	0.16766	0.23425	0.29871
8	-0.07074	-0.00143	0.06789
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.59296	-0.54587	-0.49506
2	-0.05273	0.01695	0.08647
3	-0.03634	0.03337	0.10275
4	-0.01888	0.05081	0.12001
5	0.15622	0.22341	0.28854
6	0.00172	0.07133	0.14025
7	0.26691	0.33039	0.39102
8	-0.00062	0.06900	0.13796

Table VIII.13			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 3, Response 3			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.41672	-0.35774	-0.29576
2	-0.23122	-0.16454	-0.09632
3	0.43760	0.49199	0.54279
4	-0.15703	-0.08868	-0.01948
5	-0.17149	-0.10340	-0.03433
6	-0.18769	-0.11994	-0.05105
7	-0.03266	0.03674	0.10579
8	-0.12031	-0.05142	0.01796
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.40167	-0.34160	-0.27860
2	-0.20169	-0.13395	-0.06493
3	0.45505	0.50856	0.55841
4	-0.11937	-0.05017	0.01952
5	-0.12182	-0.05265	0.01703
6	-0.13152	-0.06247	0.00718
7	-0.02531	0.04439	0.11366
8	-0.06311	0.00654	0.07613

Table VIII.14			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 3, Response 4			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.30313	-0.23883	-0.17237
2	0.19338	0.25922	0.32274
3	-0.10363	-0.03456	0.03484
4	0.12934	0.19689	0.26262
5	-0.11421	-0.04525	0.02414
6	-0.11148	-0.04249	0.02691
7	-0.11533	-0.04639	0.02300
8	-0.05398	0.01539	0.08461
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.33144	-0.26800	-0.20215
2	0.19257	0.25872	0.32253
3	-0.04530	0.02440	0.09386
4	0.16345	0.23045	0.29533
5	-0.10612	-0.03677	0.03294
6	-0.08106	-0.01150	0.05817
7	-0.05130	0.01839	0.08790
8	-0.07575	-0.00616	0.06349

Table VIII.15			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 3, Response 5			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.33494	-0.27194	-0.20652
2	-0.07440	-0.00511	0.06423
3	-0.11535	-0.04641	0.02298
4	-0.13743	-0.06877	0.00055
5	-0.11912	-0.05022	0.01916
6	-0.05166	0.01772	0.08693
7	0.23792	0.30225	0.36394
8	0.00684	0.07612	0.14467
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.36860	-0.30685	-0.24241
2	-0.03688	0.03283	0.10222
3	-0.07350	-0.00390	0.06574
4	-0.10320	-0.03382	0.03589
5	0.06490	0.13392	0.20166
6	0.03383	0.10321	0.17160
7	0.17390	0.24061	0.30512
8	-0.07439	-0.00479	0.06485

Table VIII.16			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 4, Response 1			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.21140	0.27666	0.33947
2	0.01367	0.08291	0.15136
3	0.40510	0.46146	0.51432
4	0.03456	0.10363	0.17171
5	0.33904	0.39898	0.45569
6	0.15059	0.21763	0.28268
7	0.30153	0.36325	0.42194
8	-0.03125	0.03815	0.10718
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{P_{y\bar{z}_i}}$	Upper 95 % CL
1	0.19829	0.26426	0.32785
2	0.13134	0.19914	0.26509
3	0.57598	0.62071	0.66173
4	0.13937	0.20698	0.27267
5	0.63673	0.67637	0.71244
6	0.41033	0.46662	0.51937
7	0.64436	0.68333	0.71876
8	0.14803	0.21543	0.28084

Table VIII.17			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 4, Response 2			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.65856	-0.61743	-0.57262
2	-0.06382	0.00552	0.07481
3	-0.01755	0.05183	0.12071
4	-0.03593	0.03347	0.10255
5	-0.10478	-0.03572	0.03368
6	-0.05323	0.01615	0.08537
7	0.17104	0.23754	0.30188
8	-0.06705	0.00228	0.07158
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.69731	-0.65972	-0.61851
2	-0.01544	0.05424	0.12339
3	0.06338	0.13242	0.20019
4	-0.03590	0.03381	0.10319
5	0.17144	0.23822	0.30282
6	0.06471	0.13373	0.20147
7	0.25221	0.31628	0.37759
8	-0.02835	0.04135	0.11065

Table VIII.18			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 4, Response 3			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.41592	-0.35689	-0.29487
2	-0.16566	-0.09746	-0.02834
3	0.47489	0.52686	0.57517
4	-0.03204	0.03736	0.10640
5	-0.17407	-0.10603	-0.03699
6	-0.14835	-0.07986	-0.01060
7	-0.05904	0.01032	0.07958
8	-0.06834	0.00098	0.07029
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.49361	-0.43908	-0.38111
2	-0.28616	-0.22094	-0.15368
3	0.51868	0.56780	0.61318
4	-0.05865	0.01102	0.08058
5	-0.13820	-0.06925	0.00037
6	-0.12027	-0.05108	0.01861
7	-0.10335	-0.03397	0.03574
8	-0.13706	-0.06809	0.00154

Table VIII.19			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 4, Response 4			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.39519	-0.33505	-0.27205
2	0.18797	0.25398	0.31770
3	-0.08427	-0.01504	0.05433
4	0.15491	0.22184	0.28675
5	-0.18514	-0.11733	-0.04841
6	-0.17063	-0.10253	-0.03345
7	-0.03725	0.03215	0.10124
8	-0.06728	0.00205	0.07135
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{P_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.36926	-0.30755	-0.24313
2	0.31439	0.37579	0.43405
3	-0.05339	0.01629	0.08581
4	0.20853	0.27417	0.33735
5	-0.15408	-0.08538	-0.01585
6	-0.08266	-0.01311	0.05656
7	-0.07717	-0.00759	0.06206
8	-0.09234	-0.02287	0.04683

Table VIII.20			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 4, Response 5			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.40330	-0.34359	-0.28097
2	-0.02941	0.03999	0.10900
3	-0.13973	-0.07110	-0.00179
4	-0.09873	-0.02962	0.03978
5	-0.10685	-0.03781	0.03159
6	-0.00869	0.06066	0.12943
7	0.23510	0.29953	0.36134
8	-0.03423	0.03517	0.10423
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.38905	-0.32832	-0.26475
2	-0.10665	-0.03731	0.03240
3	-0.10516	-0.03580	0.03391
4	-0.09986	-0.03045	0.03925
5	0.08431	0.15303	0.22030
6	0.01060	0.08016	0.14895
7	0.29196	0.35438	0.41379
8	-0.02458	0.04512	0.11438

Table VIII.21			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 5, Response 1			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.21132	0.27658	0.33939
2	0.02322	0.09239	0.16068
3	0.41625	0.47195	0.52412
4	0.02627	0.09541	0.16364
5	0.37002	0.42835	0.48331
6	0.11892	0.18670	0.25274
7	0.27651	0.33932	0.39924
8	-0.01783	0.05155	0.12043
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	0.19832	0.26429	0.32788
2	0.05995	0.12903	0.19688
3	0.61313	0.65480	0.69284
4	0.12927	0.19712	0.26313
5	0.64414	0.68313	0.71858
6	0.39171	0.44908	0.50297
7	0.66018	0.69773	0.73180
8	0.10786	0.17616	0.24280

Table VIII.22			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 5, Response 2			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.59756	-0.55107	-0.50089
2	-0.09945	-0.03034	0.03906
3	-0.01388	0.05549	0.12433
4	-0.02228	0.04711	0.11605
5	-0.06312	0.00622	0.07550
6	-0.05749	0.01187	0.08112
7	0.10805	0.17605	0.24241
8	-0.03347	0.03593	0.10498
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.64183	-0.59897	-0.55238
2	-0.00921	0.06045	0.12953
3	-0.01243	0.05724	0.12636
4	-0.01025	0.05941	0.12850
5	0.18759	0.25390	0.31790
6	0.08438	0.15310	0.22037
7	0.29178	0.35421	0.41363
8	-0.00136	0.06827	0.13724

Table VIII.23			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 5, Response 3			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.43267	-0.37459	-0.31341
2	-0.25130	-0.18521	-0.11740
3	0.42731	0.48234	0.53381
4	-0.06685	0.00248	0.07178
5	-0.17847	-0.11052	-0.04152
6	-0.09873	-0.02962	0.03978
7	-0.09856	-0.02945	0.03995
8	-0.04618	0.02321	0.09238
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.45602	-0.39907	-0.33886
2	-0.18372	-0.11558	-0.04633
3	0.48790	0.53921	0.58680
4	-0.04662	0.02308	0.09255
5	-0.15657	-0.08791	-0.01840
6	-0.05207	0.01762	0.08713
7	-0.09388	-0.02442	0.04528
8	-0.06875	0.00088	0.07050

Table VIII.24			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 5, Response 4			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.34956	-0.28720	-0.22231
2	0.15475	0.22169	0.28660
3	-0.06523	0.00410	0.07339
4	0.16484	0.23151	0.29607
5	-0.17889	-0.11095	-0.04196
6	-0.06436	0.00498	0.07427
7	-0.03844	0.03096	0.10006
8	-0.08860	-0.01940	0.04998
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.28846	-0.22332	-0.15613
2	0.18150	0.24799	0.31222
3	-0.07797	-0.00839	0.06127
4	0.21540	0.28081	0.34371
5	-0.13212	-0.06308	0.00657
6	-0.11590	-0.04666	0.02304
7	-0.04178	0.02792	0.09735
8	-0.06758	0.00205	0.07166

Table VIII.25			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 5, Response 5			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.36237	-0.30061	-0.23622
2	-0.10993	-0.04093	0.02847
3	-0.01747	0.05191	0.12079
4	-0.14397	-0.07541	-0.00613
5	-0.05260	0.01678	0.08600
6	-0.06855	0.00077	0.07008
7	0.14460	0.21179	0.27704
8	-0.07025	-0.00094	0.06838
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.30575	-0.24126	-0.17457
2	-0.04421	0.02549	0.09494
3	-0.09232	-0.02285	0.04685
4	-0.03145	0.03825	0.10758
5	-0.02372	0.04598	0.11523
6	0.02010	0.08960	0.15823
7	0.21848	0.28378	0.34655
8	-0.00611	0.06354	0.13257

Table VIII.26			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 6, Response 1			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.16643	0.23306	0.29757
2	0.01990	0.08909	0.15743
3	0.38583	0.44329	0.49732
4	0.03567	0.10473	0.17279
5	0.34643	0.40600	0.46230
6	0.16554	0.23219	0.29673
7	0.36692	0.42542	0.48056
8	0.02065	0.08984	0.15817
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{py\bar{z}_i}$	Upper 95 % CL
1	0.15026	0.21760	0.28293
2	0.06336	0.13240	0.20018
3	0.59723	0.64023	0.67956
4	0.08446	0.15318	0.22045
5	0.61987	0.66097	0.69845
6	0.43707	0.49173	0.54277
7	0.65567	0.69363	0.72809
8	0.19537	0.26144	0.32514

Table VIII.27			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 6, Response 2			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.65013	-0.60822	-0.56262
2	-0.10331	-0.03424	0.03516
3	-0.04495	0.02444	0.09360
4	-0.11410	-0.04514	0.02425
5	-0.04671	0.02268	0.09185
6	-0.01371	0.05566	0.12449
7	0.19491	0.26070	0.32416
8	-0.06707	0.00226	0.07156
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.67106	-0.63091	-0.58708
2	-0.09227	-0.02280	0.04690
3	0.05627	0.12540	0.19333
4	-0.05533	0.01435	0.08389
5	0.17335	0.24007	0.30460
6	0.06961	0.13856	0.20619
7	0.26807	0.33150	0.39207
8	0.06916	0.13812	0.20576

Table VIII.28			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 6, Response 3			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.40877	-0.34935	-0.28698
2	-0.19457	-0.12697	-0.05817
3	0.45001	0.50362	0.55361
4	-0.08446	-0.01523	0.05414
5	-0.21440	-0.14727	-0.07876
6	-0.09047	-0.02129	0.04810
7	-0.07012	-0.00081	0.06851
8	-0.13824	-0.06959	-0.00028
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.46333	-0.40683	-0.34704
2	-0.20666	-0.13904	-0.07010
3	0.49627	0.54699	0.59399
4	-0.05522	0.01446	0.08400
5	-0.13112	-0.06207	0.00758
6	-0.10902	-0.03970	0.03000
7	-0.08490	-0.01537	0.05431
8	-0.06054	0.00912	0.07869

Table VIII.29			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 6, Response 4			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.38870	-0.32823	-0.26494
2	0.13066	0.19818	0.26387
3	-0.12116	-0.05228	0.01710
4	0.16062	0.22740	0.29211
5	-0.12720	-0.05840	0.01096
6	-0.05951	0.00985	0.07911
7	-0.13354	-0.06482	0.00452
8	-0.10052	-0.03142	0.03798
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.35458	-0.29217	-0.22717
2	0.15751	0.22467	0.28976
3	0.01963	0.08913	0.15777
4	0.25380	0.31781	0.37904
5	-0.06739	0.00224	0.07185
6	-0.06387	0.00578	0.07537
7	-0.02727	0.04243	0.11172
8	-0.01682	0.05286	0.12203

Table VIII.30			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 6, Response 5			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.36542	-0.30380	-0.23953
2	-0.07514	-0.00586	0.06348
3	-0.13470	-0.06600	0.00333
4	-0.09554	-0.02640	0.04299
5	0.01827	0.08748	0.15585
6	-0.02543	0.04396	0.11293
7	0.20132	0.26691	0.33012
8	-0.00226	0.06707	0.13575
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.38523	-0.32431	-0.26057
2	-0.10865	-0.03933	0.03037
3	-0.15419	-0.08549	-0.01596
4	-0.14070	-0.07178	-0.00217
5	0.05566	0.12480	0.19275
6	0.07144	0.14036	0.20795
7	0.25864	0.32245	0.38346
8	0.03683	0.10618	0.17451

Table VIII.31			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 7, Response 1			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.21399	0.27916	0.34186
2	0.04769	0.11662	0.18444
3	0.44379	0.49779	0.54819
4	0.13348	0.20094	0.26654
5	0.36972	0.42807	0.48305
6	0.10748	0.17549	0.24186
7	0.24550	0.30955	0.37091
8	0.01488	0.08411	0.15254
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{py\bar{z}_i}$	Upper 95 % CL
1	0.18353	0.24996	0.31411
2	0.13338	0.20113	0.26701
3	0.61742	0.65873	0.69641
4	0.11234	0.18055	0.24706
5	0.64310	0.68218	0.71771
6	0.39894	0.45590	0.50935
7	0.66308	0.70037	0.73419
8	0.19056	0.25677	0.32066

Table VIII.32			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 7, Response 2			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.65341	-0.61180	-0.56651
2	-0.13491	-0.06621	0.00312
3	-0.04438	0.02501	0.09416
4	-0.05896	0.01040	0.07966
5	-0.04507	0.02432	0.09348
6	-0.11202	-0.04304	0.02635
7	0.12941	0.19696	0.26269
8	-0.02408	0.04531	0.11427
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.70459	-0.66772	-0.62726
2	-0.02059	0.04910	0.11832
3	-0.04828	0.02141	0.09090
4	-0.10196	-0.03257	0.03714
5	0.19451	0.26060	0.32434
6	0.10203	0.17044	0.23725
7	0.24564	0.30996	0.37156
8	0.04383	0.11311	0.18130

Table VIII.33			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 7, Response 3			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.44443	-0.38704	-0.32648
2	-0.24858	-0.18241	-0.11454
3	0.43826	0.49261	0.54337
4	-0.01843	0.05095	0.11984
5	-0.20115	-0.13370	-0.06499
6	-0.07424	-0.00495	0.06439
7	-0.06998	-0.00067	0.06865
8	-0.06086	0.00849	0.07776
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.47923	-0.42375	-0.36489
2	-0.21469	-0.14727	-0.07845
3	0.46089	0.51402	0.56348
4	-0.12518	-0.05605	0.01362
5	-0.15785	-0.08921	-0.01971
6	-0.10732	-0.03798	0.03173
7	-0.13524	-0.06624	0.00340
8	-0.04339	0.02631	0.09576

Table VIII.34			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 7, Response 4			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.36863	-0.30716	-0.24302
2	0.28231	0.34488	0.40452
3	-0.10419	-0.03513	0.03427
4	0.10896	0.17694	0.24327
5	-0.14514	-0.07660	-0.00732
6	-0.14563	-0.07709	-0.00782
7	-0.06634	0.00299	0.07229
8	-0.09999	-0.03089	0.03851
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.38877	-0.32803	-0.26445
2	0.27150	0.33479	0.39520
3	-0.04488	0.02482	0.09428
4	0.24553	0.30985	0.37146
5	-0.16883	-0.10039	-0.03099
6	-0.06622	0.00342	0.07302
7	-0.08283	-0.01329	0.05638
8	-0.14818	-0.07938	-0.00981

Table VIII.35			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 7, Response 5			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.35967	-0.29778	-0.23328
2	-0.07107	-0.00176	0.06756
3	-0.07095	-0.00164	0.06768
4	-0.09549	-0.02635	0.04304
5	-0.05528	0.01409	0.08332
6	-0.11826	-0.04935	0.02003
7	0.13550	0.20291	0.26845
8	-0.03660	0.03280	0.10188
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.37590	-0.31451	-0.25037
2	-0.06945	0.00017	0.06979
3	-0.17021	-0.10180	-0.03241
4	-0.06116	0.00850	0.07807
5	0.10553	0.17387	0.24058
6	0.00351	0.07311	0.14201
7	0.19591	0.26196	0.32564
8	0.01520	0.08473	0.15345

Table VIII.36			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 8, Response 1			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	0.18503	0.25113	0.31496
2	0.01260	0.08184	0.15030
3	0.43185	0.48660	0.53778
4	0.10699	0.17501	0.24140
5	0.28310	0.34563	0.40524
6	0.12340	0.19108	0.25699
7	0.26481	0.32810	0.38858
8	0.09175	0.16005	0.22685
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	0.11814	0.18623	0.25258
2	0.10951	0.17778	0.24438
3	0.62018	0.66125	0.69870
4	0.11877	0.18685	0.25318
5	0.62729	0.66775	0.70461
6	0.45505	0.50856	0.55841
7	0.62890	0.66922	0.70595
8	0.17104	0.23783	0.30244

Table VIII.37			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 8, Response 2			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.60216	-0.55605	-0.50625
2	-0.05415	0.01522	0.08445
3	0.00096	0.07027	0.13891
4	-0.05897	0.01039	0.07965
5	-0.10143	-0.03234	0.03706
6	-0.01826	0.05112	0.12001
7	0.15490	0.22183	0.28674
8	0.04699	0.11593	0.18377
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.69503	-0.65721	-0.61576
2	0.00372	0.07332	0.14221
3	0.01716	0.08668	0.15536
4	-0.04791	0.02178	0.09126
5	0.09922	0.16768	0.23456
6	0.11485	0.18301	0.24945
7	0.27356	0.33677	0.39708
8	0.03560	0.10496	0.17331

Table VIII.38			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 8, Response 3			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.40219	-0.34242	-0.27974
2	-0.17795	-0.10999	-0.04099
3	0.46116	0.51404	0.56329
4	-0.07000	-0.00069	0.06863
5	-0.14034	-0.07172	-0.00242
6	-0.14504	-0.07649	-0.00721
7	-0.07625	-0.00697	0.06238
8	0.00270	0.07200	0.14061
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.49651	-0.44217	-0.38438
2	-0.23261	-0.16567	-0.09717
3	0.49443	0.54528	0.59241
4	-0.05764	0.01203	0.08158
5	-0.14754	-0.07873	-0.00916
6	-0.09499	-0.02554	0.04416
7	-0.11579	-0.04654	0.02316
8	-0.04106	0.02864	0.09807

Table VIII.39			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 8, Response 4			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.35621	-0.29416	-0.22952
2	0.21286	0.27807	0.34082
3	-0.00220	0.06713	0.13581
4	0.15347	0.22044	0.28539
5	-0.17225	-0.10418	-0.03512
6	-0.07518	-0.00590	0.06344
7	-0.02437	0.04502	0.11398
8	-0.11514	-0.04619	0.02320
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.33448	-0.27117	-0.20543
2	0.32206	0.38309	0.44095
3	-0.13240	-0.06336	0.00629
4	0.27314	0.33636	0.39669
5	-0.18151	-0.11332	-0.04405
6	-0.14007	-0.07114	-0.00153
7	-0.11662	-0.04738	0.02231
8	-0.08670	-0.01718	0.05250

Table VIII.40			
Estimates and Confidence Intervals Used for the PEG Selection Methods in the Hospital Example Design Point 8, Response 5			
Simple Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{y\bar{z}_i}$	Upper 95 % CL
1	-0.32851	-0.26523	-0.19958
2	-0.08175	-0.01251	0.05685
3	-0.04543	0.02396	0.09312
4	-0.07239	-0.00309	0.06624
5	-0.07564	-0.00636	0.06298
6	-0.03459	0.03481	0.10387
7	0.22107	0.28600	0.34841
8	0.02156	0.09074	0.15905
Partial Correlation Method			
PEG	Lower 95 % CL	$\hat{\rho}_{p_{y\bar{z}_i}}$	Upper 95 % CL
1	-0.34317	-0.28025	-0.21482
2	-0.07326	-0.00366	0.06598
3	-0.17580	-0.10750	-0.03816
4	-0.03094	0.03876	0.10809
5	0.08340	0.15214	0.21944
6	0.03236	0.10175	0.17016
7	0.22065	0.28588	0.34856
8	0.05561	0.12475	0.19270

Appendix IX

This appendix contains the sample response correlation and metamodel coefficient covariance matrices for the serial queuing example (section 4.1.2). All estimated metamodels are given in Table IX.41.

Table IX.1
Sample Response Correlation Matrix
Serial Queuing Example
PEG 1, Response 1

1.0000	0.0440	0.0296	-0.0588
0.0440	1.0000	0.4827	-0.2952
0.0296	0.4827	1.0000	-0.1263
-0.0588	-0.2952	-0.1263	1.0000

Table IX.2
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 1, Response 1

154514.4370	-108394.5000	30258.0781
-108394.5620	213511.1870	-27412.7031
30258.0937	-27412.6562	156475.3750

Table IX.3
Sample Response Correlation Matrix
Serial Queuing Example
PEG 1, Response 2

1.0000	0.3328	0.1324	0.2761
0.3328	1.0000	-0.0310	0.2036
0.1324	-0.0310	1.0000	-0.0024
0.2761	0.2036	-0.0024	1.0000

Table IX.4
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 1, Response 2

2.1215	1.8001	-0.8335
1.8001	1.7465	-0.8874
-0.8335	-0.8874	2.0242

Table IX.5
Sample Response Correlation Matrix
Serial Queuing Example
PEG 1, Response 3

1.0000	0.2584	0.0105	0.0135
0.2584	1.0000	-0.0388	0.1779
0.0105	-0.0388	1.0000	-0.0467
0.0135	0.1779	-0.0467	1.0000

Table IX.6
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 1, Response 3

1.6122	0.7592	-0.6409
0.7592	1.2043	-0.2525
-0.6409	-0.2525	1.3548

Table IX.7
Sample Response Correlation Matrix
Serial Queuing Example
PEG 1, Response 4

1.0000	0.3132	0.1079	0.1676
0.3132	1.0000	-0.0402	0.1425
0.1079	-0.0402	1.0000	-0.0103
0.1676	0.1425	-0.0103	1.0000

Table IX.8
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 1, Response 4

1.2985	1.0808	-0.5910
1.0808	1.0658	-0.5878
-0.5910	-0.5878	1.2431

Table IX.9
Sample Response Correlation Matrix
Serial Queuing Example
PEG 1, Response 5

1.0000	0.5155	−0.1671	−0.8036
0.5155	1.0000	−0.1342	−0.3619
−0.1671	−0.1342	1.0000	0.2359
−0.8036	−0.3619	0.2359	1.0000

Table IX.10
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 1, Response 5

0.0823	0.0677	−0.0371
0.0677	0.0667	−0.0367
−0.0371	−0.0367	0.0784

Table IX.11
Sample Response Correlation Matrix
Serial Queuing Example
PEG 2, Response 1

1.0000	0.8917	−0.5732	−0.6690
0.8917	1.0000	−0.5234	−0.6493
−0.5732	−0.5234	1.0000	0.6419
−0.6690	−0.6493	0.6419	1.0000

Table IX.12
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 2, Response 1

77361.0625	−24319.2969	5875.4258
−24319.3594	58858.1719	−56961.7031
5875.2812	−56961.7187	101420.0620

Table IX.13
Sample Response Correlation Matrix
Serial Queuing Example
PEG 2, Response 2

1.0000	0.5067	−0.0569	−0.0300
0.5067	1.0000	−0.4458	−0.0937
−0.0569	−0.4458	1.0000	0.4117
−0.0300	−0.0937	0.4117	1.0000

Table IX.14
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 2, Response 2

1.6892	1.2997	−1.2517
1.2997	1.1586	−1.1794
−1.2517	−1.1794	2.5916

Table IX.15
Sample Response Correlation Matrix
Serial Queuing Example
PEG 2, Response 3

1.0000	0.9903	−0.4688	−0.1447
0.9903	1.0000	−0.4559	−0.1471
−0.4688	−0.4559	1.0000	0.3813
−0.1447	−0.1471	0.3813	1.0000

Table IX.16
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 2, Response 3

1.2197	0.3911	−0.3855
0.3911	0.8017	−0.7715
−0.3855	−0.7715	0.7653

Table IX.17
Sample Response Correlation Matrix
Serial Queuing Example
PEG 2, Response 4

1.0000	0.4913	−0.0756	−0.0654
0.4913	1.0000	−0.4692	−0.1350
−0.0756	−0.4692	1.0000	0.4180
−0.0654	−0.1350	0.4180	1.0000

Table IX.18
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 2, Response 4

0.9754	0.7419	−0.7582
0.7419	0.6743	−0.7132
−0.7582	−0.7132	1.5277

Table IX.19
Sample Response Correlation Matrix
Serial Queuing Example
PEG 2, Response 5

1.0000	0.6508	−0.0357	−0.8241
0.6508	1.0000	−0.4165	−0.5107
−0.0357	−0.4165	1.0000	0.1716
−0.8241	−0.5107	0.1716	1.0000

Table IX.20
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 2, Response 5

0.0631	0.0462	−0.0484
0.0462	0.0410	−0.0444
−0.0484	−0.0444	0.0971

Table IX.21
Sample Response Correlation Matrix
Serial Queuing Example
PEG 3, Response 1

1.0000	0.9083	−0.5144	−0.5647
0.9083	1.0000	−0.4377	−0.4732
−0.5144	−0.4377	1.0000	0.6825
−0.5647	−0.4732	0.6825	1.0000

Table IX.22
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 3, Response 1

123087.8750	−29058.7812	3056.9180
−29058.8281	42856.7344	−41951.2344
3056.8906	−41951.1875	92382.0000

Table IX.23
Sample Response Correlation Matrix
Serial Queuing Example
PEG 3, Response 2

1.0000	0.6444	−0.2476	0.1243
0.6444	1.0000	−0.4454	0.0677
−0.2476	−0.4454	1.0000	0.3571
0.1243	0.0677	0.3571	1.0000

Table IX.24
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 3, Response 2

1.2019	0.7667	−0.4229
0.7667	0.7456	−0.6050
−0.4229	−0.6050	1.7354

Table IX.25
Sample Response Correlation Matrix
Serial Queuing Example
PEG 3, Response 3

1.0000	0.9952	−0.4474	0.0313
0.9952	1.0000	−0.4204	0.0367
−0.4474	−0.4204	1.0000	0.3273
0.0313	0.0367	0.3273	1.0000

Table IX.26
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 3, Response 3

1.1684	0.2011	−0.1348
0.2011	0.5119	−0.5001
−0.1348	−0.5001	0.5033

Table IX.27
Sample Response Correlation Matrix
Serial Queuing Example
PEG 3, Response 4

1.0000	0.6250	−0.2767	0.0526
0.6250	1.0000	−0.4613	0.0284
−0.2767	−0.4613	1.0000	0.3155
0.0526	0.0284	0.3155	1.0000

Table IX.28
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 3, Response 4

0.7034	0.4343	−0.2864
0.4343	0.4802	−0.4066
−0.2864	−0.4066	1.0182

Table IX.29
Sample Response Correlation Matrix
Serial Queuing Example
PEG 3, Response 5

1.0000	0.7579	−0.4754	−0.7790
0.7579	1.0000	−0.5672	−0.5092
−0.4754	−0.5672	1.0000	0.4826
−0.7790	−0.5092	0.4826	1.0000

Table IX.30
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 3, Response 5

0.0455	0.0281	−0.0179
0.0281	0.0293	−0.0255
−0.0179	−0.0255	0.0632

Table IX.31
Sample Response Correlation Matrix
Serial Queuing Example
PEG 4, Response 1

1.0000	0.9202	−0.4804	−0.7879
0.9202	1.0000	−0.4278	−0.7187
−0.4804	−0.4278	1.0000	0.6466
−0.7879	−0.7187	0.6466	1.0000

Table IX.32
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 4, Response 1

90740.0625	−7349.5703	−3087.0859
−7349.5312	47911.2812	−56113.9062
−3087.1719	−56113.9219	115116.0620

Table IX.33
Sample Response Correlation Matrix
Serial Queuing Example
PEG 4, Response 2

$$\left[\begin{array}{cc|cc} 1.0000 & 0.6820 & -0.2475 & 0.1311 \\ 0.6820 & 1.0000 & -0.5901 & 0.1014 \\ \hline -0.2475 & -0.5901 & 1.0000 & -0.0434 \\ 0.1311 & 0.1014 & -0.0434 & 1.0000 \end{array} \right]$$

Table IX.34
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 4, Response 2

$$\left[\begin{array}{ccc} 1.2668 & 0.9604 & -0.8009 \\ 0.9604 & 1.0031 & -1.1766 \\ -0.8009 & -1.1766 & 3.1504 \end{array} \right]$$

Table IX.35
Sample Response Correlation Matrix
Serial Queuing Example
PEG 4, Response 3

$$\left[\begin{array}{cc|cc} 1.0000 & 0.9828 & -0.6851 & 0.1136 \\ 0.9828 & 1.0000 & -0.6387 & 0.1080 \\ \hline -0.6851 & -0.6387 & 1.0000 & -0.1892 \\ 0.1136 & 0.1080 & -0.1892 & 1.0000 \end{array} \right]$$

Table IX.36
Sample Metamodel Coefficient Covariance Matrix, (V)
Serial Queuing Example
PEG 4, Response 3

$$\left[\begin{array}{ccc} 1.0278 & 0.2046 & -0.1448 \\ 0.2046 & 1.2848 & -1.1798 \\ -0.1448 & -1.1798 & 1.1366 \end{array} \right]$$

Table IX.37
Sample Response Correlation Matrix
Serial Queuing Example
PEG 4, Response 4

1.0000	0.6627	−0.2519	0.1547
0.6627	1.0000	−0.6220	0.0908
−0.2519	−0.6220	1.0000	−0.1040
0.1547	0.0908	−0.1040	1.0000

Table IX.38
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 4, Response 4

0.7541	0.5378	−0.5544
0.5378	0.6221	−0.7953
−0.5544	−0.7953	1.8807

Table IX.39
Sample Response Correlation Matrix
Serial Queuing Example
PEG 4, Response 5

1.0000	0.3695	−0.3904	0.0571
0.3695	1.0000	−0.5855	0.1258
−0.3904	−0.5855	1.0000	−0.0639
0.0571	0.1258	−0.0639	1.0000

Table IX.40
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Serial Queuing Example
PEG 4, Response 5

0.0540	0.0287	−0.0235
0.0287	0.0704	−0.0497
−0.0235	−0.0497	0.1329

Table IX.41				
Estimated Metamodels for the Serial Queuing Example				
Response	PEG	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
1	1	2567.96	-1540.28	787.97
1	2	2641.44	-1567.97	730.42
1	3	2518.92	-1491.74	716.63
1	4	2717.29	-1590.82	817.29
2	1	108.79	3.27	3.05
2	2	109.06	3.55	2.67
2	3	109.37	3.81	2.70
2	4	108.89	3.32	3.27
3	1	93.74	-4.25	4.66
3	2	93.81	-4.72	4.69
3	3	93.43	-4.85	4.77
3	4	93.83	-4.32	4.31
4	1	92.00	-2.79	-2.71
4	2	91.79	-3.02	-2.43
4	3	91.52	-3.23	-2.42
4	4	91.93	-2.82	-2.88
5	1	22.99	-0.70	-0.67
5	2	22.94	-0.76	-0.61
5	3	22.87	-0.81	-0.60
5	4	22.97	-0.67	-0.72

Appendix X

This appendix contains the sample response correlation and metamodel coefficient covariance matrices for the hospital example (section 4.2.2). The estimated metamodels corresponding to the sample metamodel coefficient covariance matrices are given in Table X.81.

Table X.1
Sample Response Correlation Matrix
Hospital Example
PEG 1, Response 1

1.0000	0.6373	0.7694	0.7182	-0.3159	-0.2650	-0.33850	-0.3895
0.6373	1.0000	0.7565	0.7528	-0.2278	-0.2928	-0.21270	-0.2512
0.7694	0.7565	1.0000	0.7415	-0.3400	-0.2894	-0.26230	-0.3211
0.7182	0.7528	0.7415	1.0000	-0.1975	-0.3230	-0.23850	-0.3391
-0.3159	-0.2278	-0.3400	-0.1975	1.0000	0.6511	0.74160	0.6256
-0.2650	-0.2928	-0.2894	-0.3230	0.6511	1.0000	0.64650	0.6500
-0.3385	-0.2127	-0.2623	-0.2385	0.7416	0.6465	1.00000	0.7366
-0.3895	-0.2512	-0.3211	-0.3391	0.6256	0.6500	0.73660	1.0000

Table X.2
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 1, Response 1

0.0058	0.0001	0.0000	-0.0001
0.0001	0.0008	0.0002	0.0002
0.0000	0.0002	0.0008	0.0000
-0.0001	0.0002	0.0000	0.0008

Table X.3
Sample Response Correlation Matrix
Hospital Example
PEG 1, Response 2

1.0000	-0.1138	0.5606	0.2404	0.0211	0.1122	-0.03340	-0.0213
-0.1138	1.0000	-0.0487	0.1954	0.2527	-0.2363	-0.12540	-0.0979
0.5606	-0.0487	1.0000	0.3295	0.1026	-0.1068	-0.21530	0.1626
0.2404	0.1954	0.3295	1.0000	-0.1171	-0.0246	-0.07170	0.0852
0.0211	0.2527	0.1026	-0.1171	1.0000	0.2673	-0.03160	0.0293
0.1122	-0.2363	-0.1068	-0.0246	0.2673	1.0000	0.21520	0.1596
-0.0334	-0.1254	-0.2153	-0.0717	-0.0316	0.2152	1.00000	0.1920
-0.0213	-0.0979	0.1626	0.0852	0.0293	0.1596	0.19200	1.0000

Table X.4
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 1, Response 2

0.3096	-0.0030	-0.0027	0.0012
-0.0030	0.2143	0.0017	-0.0273
-0.0027	0.0017	0.1731	0.0536
0.0012	-0.0273	0.0536	0.2205

Table X.5
Sample Response Correlation Matrix
Hospital Example
PEG 1, Response 3

1.0000	0.6052	0.7418	0.6540	−0.1817	−0.0791	−0.20830	−0.2378
0.6052	1.0000	0.6318	0.6970	−0.1023	−0.1710	−0.30410	−0.2102
0.7418	0.6318	1.0000	0.7223	−0.2737	−0.2817	−0.44950	−0.2888
0.6540	0.6970	0.7223	1.0000	−0.1620	−0.1040	−0.17120	−0.1535
−0.1817	−0.1023	−0.2737	−0.1620	1.0000	0.5760	0.55450	0.4535
−0.0791	−0.1710	−0.2817	−0.1040	0.5760	1.0000	0.75160	0.6162
−0.2083	−0.3041	−0.4495	−0.1712	0.5545	0.7516	1.00000	0.6643
−0.2378	−0.2102	−0.2888	−0.1535	0.4535	0.6162	0.66430	1.0000

Table X.6
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 1, Response 3

0.3644	−0.0329	0.0293	−0.0227
−0.0329	0.0838	0.0096	−0.0112
0.0293	0.0096	0.0632	0.0123
−0.0227	−0.0112	0.0123	0.0758

Table X.7
Sample Response Correlation Matrix
Hospital Example
PEG 1, Response 4

1.0000	0.6913	0.7901	0.7220	0.4540	0.5015	0.53900	0.4087
0.6913	1.0000	0.6966	0.7919	0.5303	0.5540	0.46280	0.4861
0.7901	0.6966	1.0000	0.7691	0.5229	0.5934	0.50020	0.4879
0.7220	0.7919	0.7691	1.0000	0.5550	0.5915	0.49360	0.4590
0.4540	0.5303	0.5229	0.5550	1.0000	0.7569	0.71690	0.7348
0.5015	0.5540	0.5934	0.5915	0.7569	1.0000	0.76680	0.7632
0.5390	0.4628	0.5002	0.4936	0.7169	0.7668	1.00000	0.6567
0.4087	0.4861	0.4879	0.4590	0.7348	0.7632	0.65670	1.0000

Table X.8
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 1, Response 4

1.1251	0.0135	0.0695	0.0026
0.0135	0.0595	0.0178	0.0013
0.0695	0.0178	0.0761	−0.0034
0.0026	0.0013	−0.0034	0.0383

Table X.9
Sample Response Correlation Matrix
Hospital Example
PEG 1, Response 5

1.0000	0.7221	0.6566	0.6000	0.5565	0.5472	0.57070	0.5354
0.7221	1.0000	0.5563	0.6536	0.6495	0.6225	0.55410	0.6765
0.6566	0.5563	1.0000	0.5652	0.6596	0.4211	0.55040	0.5098
0.6000	0.6536	0.5652	1.0000	0.5766	0.5411	0.44170	0.5930
0.5565	0.6495	0.6596	0.5766	1.0000	0.6596	0.55590	0.6313
0.5472	0.6225	0.4211	0.5411	0.6596	1.0000	0.38930	0.6257
0.5707	0.5541	0.5504	0.4417	0.5559	0.3893	1.00000	0.6148
0.5354	0.6765	0.5098	0.5930	0.6313	0.6257	0.61480	1.0000

Table X.10
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 1, Response 5

0.0645	0.0009	0.0026	0.0004
0.0009	0.0041	−0.0002	0.0005
0.0026	−0.0002	0.0074	−0.0009
0.0004	0.0005	−0.0009	0.0057

Table X.11
Sample Response Correlation Matrix
Hospital Example
PEG 2, Response 1

1.0000	0.7154	0.7407	0.5431	-0.0887	-0.1231	-0.15860	-0.3002
0.7154	1.0000	0.6924	0.5732	-0.2109	-0.1595	-0.14920	-0.3783
0.7407	0.6924	1.0000	0.5857	-0.2883	-0.3114	-0.27450	-0.4813
0.5431	0.5732	0.5857	1.0000	-0.1458	-0.2028	-0.14820	-0.2262
-0.0887	-0.2109	-0.2883	-0.1458	1.0000	0.6182	0.77800	0.6955
-0.1231	-0.1595	-0.3114	-0.2028	0.6182	1.0000	0.69740	0.6003
-0.1586	-0.1492	-0.2745	-0.1482	0.7780	0.6974	1.00000	0.6934
-0.3002	-0.3783	-0.4813	-0.2262	0.6955	0.6003	0.69340	1.0000

Table X.12
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 2, Response 1

0.0072	-0.0006	-0.0008	-0.0005
-0.0006	0.0009	-0.0002	0.0001
-0.0008	-0.0002	0.0014	0.0001
-0.0005	0.0001	0.0001	0.0013

Table X.13
Sample Response Correlation Matrix
Hospital Example
PEG 2, Response 2

1.0000	0.8996	0.8698	0.8735	-0.4450	-0.5290	-0.47510	-0.4774
0.8996	1.0000	0.8904	0.8926	-0.4596	-0.5256	-0.44030	-0.4538
0.8698	0.8904	1.0000	0.8629	-0.4693	-0.5573	-0.48950	-0.5099
0.8735	0.8926	0.8629	1.0000	-0.5235	-0.5842	-0.54740	-0.5113
-0.4450	-0.4596	-0.4693	-0.5235	1.0000	0.8804	0.91130	0.8738
-0.5290	-0.5256	-0.5573	-0.5842	0.8804	1.0000	0.87420	0.8756
-0.4751	-0.4403	-0.4895	-0.5474	0.9113	0.8742	1.00000	0.8567
-0.4774	-0.4538	-0.5099	-0.5113	0.8738	0.8756	0.85670	1.0000

Table X.14
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 2, Response 2

0.4576	0.0258	0.0070	-0.0199
0.0258	0.0287	-0.0023	0.0045
0.0070	-0.0023	0.0440	0.0052
-0.0199	0.0045	0.0052	0.0280

Table X.15
Sample Response Correlation Matrix
Hospital Example
PEG 2, Response 3

1.0000	0.7241	0.7634	0.7661	-0.3527	-0.3160	-0.38110	-0.3750
0.7241	1.0000	0.8690	0.7614	-0.3597	-0.2951	-0.45870	-0.4334
0.7634	0.8690	1.0000	0.7851	-0.4270	-0.3209	-0.44570	-0.4828
0.7661	0.7614	0.7851	1.0000	-0.3875	-0.4226	-0.56100	-0.4663
-0.3527	-0.3597	-0.4270	-0.3875	1.0000	0.7294	0.67980	0.7818
-0.3160	-0.2951	-0.3209	-0.4226	0.7294	1.0000	0.75960	0.7329
-0.3811	-0.4587	-0.4457	-0.5610	0.6798	0.7596	1.00000	0.6993
-0.3750	-0.4334	-0.4828	-0.4663	0.7818	0.7329	0.69930	1.0000

Table X.16
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 2, Response 3

0.3275	0.0044	-0.0101	-0.0206
0.0044	0.0500	0.0108	-0.0016
-0.0101	0.0108	0.0427	-0.0006
-0.0206	-0.0016	-0.0006	0.0361

Table X.17
Sample Response Correlation Matrix
Hospital Example
PEG 2, Response 4

1.0000	0.2070	0.1778	0.3801	-0.0809	-0.1456	-0.04270	-0.1457
0.2070	1.0000	-0.0222	0.2931	-0.1386	-0.1156	-0.17590	-0.1069
0.1778	-0.0222	1.0000	0.2067	-0.0653	0.0297	0.09320	0.0714
0.3801	0.2931	0.2067	1.0000	0.0016	0.1145	-0.20800	-0.0529
-0.0809	-0.1386	-0.0653	0.0016	1.0000	0.3730	0.23330	0.1553
-0.1456	-0.1156	0.0297	0.1145	0.3730	1.0000	0.28870	0.2691
-0.0427	-0.1759	0.0932	-0.2080	0.2333	0.2887	1.00000	0.3633
-0.1457	-0.1069	0.0714	-0.0529	0.1553	0.2691	0.36330	1.0000

Table X.18
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 2, Response 4

0.3333	-0.0103	0.0227	0.0069
-0.0103	0.1432	0.0033	0.0287
0.0227	0.0033	0.1914	-0.0262
0.0069	0.0287	-0.0262	0.1828

Table X.19
Sample Response Correlation Matrix
Hospital Example
PEG 2, Response 5

1.0000	0.7222	0.6857	0.8437	0.5294	0.5049	0.43100	0.5686
0.7222	1.0000	0.8901	0.8504	0.6108	0.5737	0.34470	0.5983
0.6857	0.8901	1.0000	0.8074	0.5800	0.3680	0.32720	0.5951
0.8437	0.8504	0.8074	1.0000	0.5233	0.4688	0.36390	0.5169
0.5294	0.6108	0.5800	0.5233	1.0000	0.7122	0.64170	0.8353
0.5049	0.5737	0.3680	0.4688	0.7122	1.0000	0.73290	0.7633
0.4310	0.3447	0.3272	0.3639	0.6417	0.7329	1.00000	0.7033
0.5686	0.5983	0.5951	0.5169	0.8353	0.7633	0.70330	1.0000

Table X.20
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 2, Response 5

0.0550	0.0040	0.0023	0.0000
0.0040	0.0049	0.0001	0.0004
0.0023	0.0001	0.0022	−0.0007
0.0000	0.0004	−0.0007	0.0022

Table X.21
Sample Response Correlation Matrix
Hospital Example
PEG 3, Response 1

1.0000	0.6646	0.7354	0.6569	-0.1982	-0.1506	-0.24020	-0.1447
0.6646	1.0000	0.7057	0.6219	-0.2679	-0.1487	0.00370	-0.0320
0.7354	0.7057	1.0000	0.6385	-0.2834	-0.2293	-0.19640	-0.1576
0.6569	0.6219	0.6385	1.0000	-0.1965	-0.1196	-0.25600	-0.0986
-0.1982	-0.2679	-0.2834	-0.1965	1.0000	0.4784	0.51290	0.5146
-0.1506	-0.1487	-0.2293	-0.1196	0.4784	1.0000	0.63780	0.5266
-0.2402	0.0037	-0.1964	-0.2560	0.5129	0.6378	1.00000	0.5938
-0.1447	-0.0320	-0.1576	-0.0986	0.5146	0.5266	0.59380	1.0000

Table X.22
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 3, Response 1

0.0062	-0.0003	0.0002	0.0001
-0.0003	0.0010	-0.0003	0.0002
0.0002	-0.0003	0.0011	0.0002
0.0001	0.0002	0.0002	0.0009

Table X.23
Sample Response Correlation Matrix
Hospital Example
PEG 3, Response 2

1.0000	0.9651	0.9660	0.9637	-0.6787	-0.6397	-0.67690	-0.6527
0.9651	1.0000	0.9658	0.9674	-0.6899	-0.6575	-0.68070	-0.6570
0.9660	0.9658	1.0000	0.9611	-0.6415	-0.6114	-0.63940	-0.6194
0.9637	0.9674	0.9611	1.0000	-0.6546	-0.6241	-0.66940	-0.6282
-0.6787	-0.6899	-0.6415	-0.6546	1.0000	0.9665	0.96080	0.9671
-0.6397	-0.6575	-0.6114	-0.6241	0.9665	1.0000	0.95100	0.9692
-0.6769	-0.6807	-0.6394	-0.6694	0.9608	0.9510	1.00000	0.9663
-0.6527	-0.6570	-0.6194	-0.6282	0.9671	0.9692	0.96630	1.0000

Table X.24
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 3, Response 2

0.4016	-0.0062	0.0057	0.0052
-0.0062	0.0088	-0.0003	0.0019
0.0057	-0.0003	0.0133	-0.0005
0.0052	0.0019	-0.0005	0.0098

Table X.25
Sample Response Correlation Matrix
Hospital Example
PEG 3, Response 3

1.0000	0.3969	0.5657	0.5809	-0.1702	-0.2473	-0.36710	-0.1801
0.3969	1.0000	0.6997	0.6249	-0.3279	-0.3552	-0.28510	-0.2388
0.5657	0.6997	1.0000	0.5816	-0.2888	-0.3311	-0.35600	-0.2613
0.5809	0.6249	0.5816	1.0000	-0.4286	-0.4322	-0.45500	-0.2381
-0.1702	-0.3279	-0.2888	-0.4286	1.0000	0.4800	0.58780	0.5319
-0.2473	-0.3552	-0.3311	-0.4322	0.4800	1.0000	0.66180	0.5561
-0.3671	-0.2851	-0.3560	-0.4550	0.5878	0.6618	1.00000	0.5341
-0.1801	-0.2388	-0.2613	-0.2381	0.5319	0.5561	0.53410	1.0000

Table X.26
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 3, Response 3

0.3012	0.0124	0.0165	0.0130
0.0124	0.0947	-0.0051	-0.0022
0.0165	-0.0051	0.1094	0.0333
0.0130	-0.0022	0.0333	0.0708

Table X.27
Sample Response Correlation Matrix
Hospital Example
PEG 3, Response 4

1.0000	0.9476	1.0000	0.9476	-0.1421	-0.3105	-0.14210	-0.3105
0.9476	1.0000	0.9476	1.0000	-0.1038	-0.3054	-0.10380	-0.3054
1.0000	0.9476	1.0000	0.9476	-0.1421	-0.3105	-0.14210	-0.3105
0.9476	1.0000	0.9476	1.0000	-0.1038	-0.3054	-0.10380	-0.3054
-0.1421	-0.1038	-0.1421	-0.1038	1.0000	0.8647	1.00000	0.8647
-0.3105	-0.3054	-0.3105	-0.3054	0.8647	1.0000	0.86470	1.0000
-0.1421	-0.1038	-0.1421	-0.1038	1.0000	0.8647	1.00000	0.8647
-0.3105	-0.3054	-0.3105	-0.3054	0.8647	1.0000	0.86470	1.0000

Table X.28
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 3, Response 4

0.6566	0.0000	0.0014	0.0000
0.0000	0.0000	0.0000	0.0000
0.0014	0.0000	0.0258	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.29
Sample Response Correlation Matrix
Hospital Example
PEG 3, Response 5

1.0000	0.7115	0.7889	0.9102	0.5511	0.5911	0.41500	0.5815
0.7115	1.0000	0.8031	0.8031	0.5117	0.5039	0.48100	0.4624
0.7889	0.8031	1.0000	0.7123	0.5205	0.5267	0.36900	0.6058
0.9102	0.8031	0.7123	1.0000	0.3682	0.4799	0.36900	0.4796
0.5511	0.5117	0.5205	0.3682	1.0000	0.7918	0.68400	0.7369
0.5911	0.5039	0.5267	0.4799	0.7918	1.0000	0.86100	0.8303
0.4150	0.4810	0.3690	0.3690	0.6840	0.8610	1.00000	0.6669
0.5815	0.4624	0.6058	0.4796	0.7369	0.8303	0.66690	1.0000

Table X.30
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 3, Response 5

0.0700	-0.0002	0.0022	-0.0050
-0.0002	0.0046	-0.0005	-0.0001
0.0022	-0.0005	0.0022	0.0007
-0.0050	-0.0001	0.0007	0.0023

Table X.31
Sample Response Correlation Matrix
Hospital Example
PEG 4, Response 1

1.0000	0.8874	0.8890	0.8860	-0.2528	-0.1718	-0.22640	-0.2322
0.8874	1.0000	0.9002	0.9278	-0.2468	-0.2645	-0.27200	-0.2917
0.8890	0.9002	1.0000	0.9097	-0.2075	-0.2303	-0.21360	-0.2573
0.8860	0.9278	0.9097	1.0000	-0.2117	-0.2175	-0.21520	-0.2282
-0.2528	-0.2468	-0.2075	-0.2117	1.0000	0.8674	0.92530	0.8986
-0.1718	-0.2645	-0.2303	-0.2175	0.8674	1.0000	0.87810	0.8551
-0.2264	-0.2720	-0.2136	-0.2152	0.9253	0.8781	1.00000	0.8847
-0.2322	-0.2917	-0.2573	-0.2282	0.8986	0.8551	0.88470	1.0000

Table X.32
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 4, Response 1

0.0084	-0.0002	-0.0004	0.0000
-0.0002	0.0004	0.0000	0.0001
-0.0004	0.0000	0.0003	-0.0001
0.0000	0.0001	-0.0001	0.0003

Table X.33
Sample Response Correlation Matrix
Hospital Example
PEG 4, Response 2

1.0000	0.9787	0.9753	0.9820	-0.7554	-0.7327	-0.75750	-0.7351
0.9787	1.0000	0.9739	0.9821	-0.7573	-0.7381	-0.76450	-0.7341
0.9753	0.9739	1.0000	0.9795	-0.7740	-0.7637	-0.78010	-0.7517
0.9820	0.9821	0.9795	1.0000	-0.7535	-0.7438	-0.75970	-0.7286
-0.7554	-0.7573	-0.7740	-0.7535	1.0000	0.9794	0.98290	0.9861
-0.7327	-0.7381	-0.7637	-0.7438	0.9794	1.0000	0.98020	0.9724
-0.7575	-0.7645	-0.7801	-0.7597	0.9829	0.9802	1.00000	0.9761
-0.7351	-0.7341	-0.7517	-0.7286	0.9861	0.9724	0.97610	1.0000

Table X.34
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 4, Response 2

0.2974	-0.0056	0.0043	-0.0029
-0.0056	0.0085	0.0010	0.0026
0.0043	0.0010	0.0064	-0.0014
-0.0029	0.0026	-0.0014	0.0081

Table X.35
Sample Response Correlation Matrix
Hospital Example
PEG 4, Response 3

1.0000	0.9876	0.9876	1.0000	−0.4372	−0.4008	−0.40080	−0.4372
0.9876	1.0000	1.0000	0.9876	−0.4548	−0.4199	−0.41990	−0.4548
0.9876	1.0000	1.0000	0.9876	−0.4548	−0.4199	−0.41990	−0.4548
1.0000	0.9876	0.9876	1.0000	−0.4372	−0.4008	−0.40080	−0.4372
−0.4372	−0.4548	−0.4548	−0.4372	1.0000	0.9779	0.97790	1.0000
−0.4008	−0.4199	−0.4199	−0.4008	0.9779	1.0000	1.00000	0.9779
−0.4008	−0.4199	−0.4199	−0.4008	0.9779	1.0000	1.00000	0.9779
−0.4372	−0.4548	−0.4548	−0.4372	1.0000	0.9779	0.97790	1.0000

Table X.36
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 4, Response 3

0.3614	−0.0079	0.0000	0.0000
−0.0079	0.0058	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.37
Sample Response Correlation Matrix
Hospital Example
PEG 4, Response 4

1.0000	0.8186	0.8344	0.8152	-0.2836	-0.3718	-0.35860	-0.3728
0.8186	1.0000	0.8683	0.8702	-0.4425	-0.5820	-0.50250	-0.6052
0.8344	0.8683	1.0000	0.8332	-0.3563	-0.5192	-0.48540	-0.4959
0.8152	0.8702	0.8332	1.0000	-0.3618	-0.4638	-0.38950	-0.4692
-0.2836	-0.4425	-0.3563	-0.3618	1.0000	0.7756	0.78440	0.7469
-0.3718	-0.5820	-0.5192	-0.4638	0.7756	1.0000	0.84150	0.8423
-0.3586	-0.5025	-0.4854	-0.3895	0.7844	0.8415	1.00000	0.8459
-0.3728	-0.6052	-0.4959	-0.4692	0.7469	0.8423	0.84590	1.0000

Table X.38
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 4, Response 4

0.4248	-0.0415	-0.0158	-0.0060
-0.0415	0.0497	-0.0130	-0.0018
-0.0158	-0.0130	0.0380	0.0101
-0.0060	-0.0018	0.0101	0.0435

Table X.39
Sample Response Correlation Matrix
Hospital Example
PEG 4, Response 5

1.0000	0.6987	0.8509	0.8971	0.3963	0.4928	0.38490	0.4218
0.6987	1.0000	0.8633	0.8973	0.4539	0.4736	0.41530	0.5296
0.8509	0.8633	1.0000	0.8625	0.4363	0.4866	0.32630	0.4139
0.8971	0.8973	0.8625	1.0000	0.4186	0.5172	0.31730	0.3954
0.3963	0.4539	0.4363	0.4186	1.0000	0.8373	0.85240	0.8118
0.4928	0.4736	0.4866	0.5172	0.8373	1.0000	0.85350	0.8605
0.3849	0.4153	0.3263	0.3173	0.8524	0.8535	1.00000	0.8413
0.4218	0.5296	0.4139	0.3954	0.8118	0.8605	0.84130	1.0000

Table X.40
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 4, Response 5

0.0710	0.0000	0.0007	0.0016
0.0000	0.0031	0.0005	0.0003
0.0007	0.0005	0.0028	0.0002
0.0016	0.0003	0.0002	0.0001

Table X.41
Sample Response Correlation Matrix
Hospital Example
PEG 5, Response 1

1.0000	0.4466	0.3430	0.3721	-0.0749	0.0549	0.14070	0.3153
0.4466	1.0000	0.1726	0.3217	0.0452	-0.0697	0.03720	0.2499
0.3430	0.1726	1.0000	0.1981	-0.2158	-0.1822	-0.08930	-0.0879
0.3721	0.3217	0.1981	1.0000	-0.1117	-0.0740	0.15650	0.0455
-0.0749	0.0452	-0.2158	-0.1117	1.0000	0.4994	0.29990	0.2911
0.0549	-0.0697	-0.1822	-0.0740	0.4994	1.0000	0.64670	0.3515
0.1407	0.0372	-0.0893	0.1565	0.2999	0.6467	1.00000	0.4324
0.3153	0.2499	-0.0879	0.0455	0.2911	0.3515	0.43240	1.0000

Table X.42
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 5, Response 1

0.0059	-0.0007	0.0001	-0.0001
-0.0007	0.0021	-0.0001	-0.0004
0.0001	-0.0001	0.0013	0.0000
-0.0001	-0.0004	0.0000	0.0018

Table X.43
Sample Response Correlation Matrix
Hospital Example
PEG 5, Response 2

1.0000	0.8563	0.7978	0.8371	−0.6344	−0.5613	−0.70360	−0.6536
0.8563	1.0000	0.7720	0.8398	−0.6079	−0.5515	−0.69350	−0.6307
0.7978	0.7720	1.0000	0.7805	−0.5517	−0.4789	−0.64360	−0.6415
0.8371	0.8398	0.7805	1.0000	−0.5850	−0.5154	−0.62930	−0.6770
−0.6344	−0.6079	−0.5517	−0.5850	1.0000	0.8567	0.80150	0.7735
−0.5613	−0.5515	−0.4789	−0.5154	0.8567	1.0000	0.87630	0.7869
−0.7036	−0.6935	−0.6436	−0.6293	0.8015	0.8763	1.00000	0.8095
−0.6536	−0.6307	−0.6415	−0.6770	0.7735	0.7869	0.80950	1.0000

Table X.44
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 5, Response 2

0.2712	−0.0131	0.0080	−0.0242
−0.0131	0.0553	0.0006	−0.0038
0.0080	0.0006	0.0414	−0.0008
−0.0242	−0.0038	−0.0008	0.0493

Table X.45
Sample Response Correlation Matrix
Hospital Example
PEG 5, Response 3

1.0000	0.9874	0.9874	1.0000	−0.3869	−0.3781	−0.37810	−0.3869
0.9874	1.0000	1.0000	0.9874	−0.3975	−0.3851	−0.38510	−0.3975
0.9874	1.0000	1.0000	0.9874	−0.3975	−0.3851	−0.38510	−0.3975
1.0000	0.9874	0.9874	1.0000	−0.3869	−0.3781	−0.37810	−0.3869
−0.3869	−0.3975	−0.3975	−0.3869	1.0000	0.9846	0.98460	1.0000
−0.3781	−0.3851	−0.3851	−0.3781	0.9846	1.0000	1.00000	0.9846
−0.3781	−0.3851	−0.3851	−0.3781	0.9846	1.0000	1.00000	0.9846
−0.3869	−0.3975	−0.3975	−0.3869	1.0000	0.9846	0.98460	1.0000

Table X.46
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 5, Response 3

0.4504	−0.0064	0.0000	0.0000
−0.0064	0.0046	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.47
Sample Response Correlation Matrix
Hospital Example
PEG 5, Response 4

1.0000	0.9198	1.0000	0.9198	-0.1234	-0.1500	-0.12340	-0.1500
0.9198	1.0000	0.9198	1.0000	-0.1164	-0.1645	-0.11640	-0.1645
1.0000	0.9198	1.0000	0.9198	-0.1234	-0.1500	-0.12340	-0.1500
0.9198	1.0000	0.9198	1.0000	-0.1164	-0.1645	-0.11640	-0.1645
-0.1234	-0.1164	-0.1234	-0.1164	1.0000	0.8963	1.00000	0.8963
-0.1500	-0.1645	-0.1500	-0.1645	0.8963	1.0000	0.89630	1.0000
-0.1234	-0.1164	-0.1234	-0.1164	1.0000	0.8963	1.00000	0.8963
-0.1500	-0.1645	-0.1500	-0.1645	0.8963	1.0000	0.89630	1.0000

Table X.48
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 5, Response 4

0.5439	0.0000	0.0047	0.0000
0.0000	0.0000	0.0000	0.0000
0.0047	0.0000	0.0273	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.49
Sample Response Correlation Matrix
Hospital Example
PEG 5, Response 5

1.0000	0.7545	0.7787	0.7464	0.4199	0.5331	0.50430	0.5335
0.7545	1.0000	0.7058	0.8238	0.4901	0.3644	0.38540	0.5488
0.7787	0.7058	1.0000	0.8155	0.4184	0.4564	0.43180	0.4427
0.7464	0.8238	0.8155	1.0000	0.4583	0.4833	0.33790	0.5312
0.4199	0.4901	0.4184	0.4583	1.0000	0.7161	0.75490	0.7814
0.5331	0.3644	0.4564	0.4833	0.7161	1.0000	0.76020	0.8372
0.5043	0.3854	0.4318	0.3379	0.7549	0.7602	1.00000	0.7257
0.5335	0.5488	0.4427	0.5312	0.7814	0.8372	0.72570	1.0000

Table X.50
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 5, Response 5

0.0639	0.0027	−0.0001	0.0026
0.0027	0.0030	0.0010	0.0001
−0.0001	0.0010	0.0040	−0.0001
0.0026	0.0001	−0.0001	0.0020

Table X.51
Sample Response Correlation Matrix
Hospital Example
PEG 6, Response 1

1.0000	0.8562	0.8341	0.8669	-0.0214	-0.0201	-0.01050	-0.0105
0.8562	1.0000	0.8338	0.8196	0.0183	0.0801	0.04510	0.0451
0.8341	0.8338	1.0000	0.7973	-0.0941	-0.0455	-0.05090	-0.0509
0.8669	0.8196	0.7973	1.0000	-0.1422	-0.1230	-0.14280	-0.1428
-0.0214	0.0183	-0.0941	-0.1422	1.0000	0.8695	0.87210	0.8721
-0.0201	0.0801	-0.0455	-0.1230	0.8695	1.0000	0.86600	0.8660
-0.0105	0.0451	-0.0509	-0.1428	0.8721	0.8660	1.00000	1.0000
-0.0105	0.0451	-0.0509	-0.1428	0.8721	0.8660	1.00000	1.0000

Table X.52
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 6, Response 1

0.0122	-0.0003	0.0000	-0.0004
-0.0003	0.0004	-0.0001	0.0000
0.0000	-0.0001	0.0004	-0.0001
-0.0004	0.0000	-0.0001	0.0006

Table X.54
Sample Response Correlation Matrix
Hospital Example
PEG 6, Response 2

1.0000	0.9514	0.9487	0.9377	-0.5879	-0.5677	-0.58420	-0.5842
0.9514	1.0000	0.9425	0.9279	-0.5953	-0.5612	-0.60080	-0.6008
0.9487	0.9425	1.0000	0.9347	-0.6318	-0.6064	-0.62950	-0.6295
0.9377	0.9279	0.9347	1.0000	-0.6695	-0.6553	-0.68310	-0.6831
-0.5879	-0.5953	-0.6318	-0.6695	1.0000	0.9533	0.96190	0.9619
-0.5677	-0.5612	-0.6064	-0.6553	0.9533	1.0000	0.96200	0.9620
-0.5842	-0.6008	-0.6295	-0.6831	0.9619	0.9620	1.00000	1.0000
-0.5842	-0.6008	-0.6295	-0.6831	0.9619	0.9620	1.00000	1.0000

Table X.55
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 6, Response 2

0.3966	0.0021	-0.0043	-0.0236
0.0021	0.0103	-0.0034	-0.0014
-0.0043	-0.0034	0.0117	-0.0022
-0.0236	-0.0014	-0.0022	0.0169

Table X.55
Sample Response Correlation Matrix
Hospital Example
PEG 6, Response 3

1.0000	0.9843	0.9843	1.0000	-0.5044	-0.5071	-0.50440	-0.5044
0.9843	1.0000	1.0000	0.9843	-0.5247	-0.5338	-0.52470	-0.5247
0.9843	1.0000	1.0000	0.9843	-0.5247	-0.5338	-0.52470	-0.5247
1.0000	0.9843	0.9843	1.0000	-0.5044	-0.5071	-0.50440	-0.5044
-0.5044	-0.5247	-0.5247	-0.5044	1.0000	0.9879	1.00000	1.0000
-0.5071	-0.5338	-0.5338	-0.5071	0.9879	1.0000	0.98790	0.9879
-0.5044	-0.5247	-0.5247	-0.5044	1.0000	0.9879	1.00000	1.0000
-0.5044	-0.5247	-0.5247	-0.5044	1.0000	0.9879	1.00000	1.0000

Table X.56
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 6, Response 3

0.4087	-0.0007	-0.0021	0.0021
-0.0007	0.0046	-0.0010	0.0010
-0.0021	-0.0010	0.0007	-0.0007
0.0021	0.0010	-0.0007	0.0007

Table X.57
Sample Response Correlation Matrix
Hospital Example
PEG 6, Response 4

1.0000	0.9378	1.0000	0.9378	-0.2188	-0.3243	-0.32430	-0.3243
0.9378	1.0000	0.9378	1.0000	-0.3028	-0.4078	-0.40780	-0.4078
1.0000	0.9378	1.0000	0.9378	-0.2188	-0.3243	-0.32430	-0.3243
0.9378	1.0000	0.9378	1.0000	-0.3028	-0.4078	-0.40780	-0.4078
-0.2188	-0.3028	-0.2188	-0.3028	1.0000	0.9235	0.92350	0.9235
-0.3243	-0.4078	-0.3243	-0.4078	0.9235	1.0000	1.00000	1.0000
-0.3243	-0.4078	-0.3243	-0.4078	0.9235	1.0000	1.00000	1.0000
-0.3243	-0.4078	-0.3243	-0.4078	0.9235	1.0000	1.00000	1.0000

Table X.58
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 6, Response 4

0.5421	0.0034	-0.0154	-0.0034
0.0034	0.0040	-0.0035	-0.0040
-0.0154	-0.0035	0.0169	0.0035
-0.0034	-0.0040	0.0035	0.0040

Table X.59
Sample Response Correlation Matrix
Hospital Example
PEG 6, Response 5

1.0000	0.8409	0.9039	0.8957	0.4001	0.4928	0.56940	0.5694
0.8409	1.0000	0.9753	0.9575	0.4492	0.6078	0.63220	0.6322
0.9039	0.9753	1.0000	0.8729	0.2727	0.4733	0.49260	0.4926
0.8957	0.9575	0.8729	1.0000	0.4436	0.4902	0.52160	0.5216
0.4001	0.4492	0.2727	0.4436	1.0000	0.8855	0.86580	0.8658
0.4928	0.6078	0.4733	0.4902	0.8855	1.0000	0.93260	0.9326
0.5694	0.6322	0.4926	0.5216	0.8658	0.9326	1.00000	1.0000
0.5694	0.6322	0.4926	0.5216	0.8658	0.9326	1.00000	1.0000

Table X.60
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 6, Response 5

0.0651	0.0004	0.0035	-0.0002
0.0004	0.0006	0.0003	0.0001
0.0035	0.0003	0.0010	0.0002
-0.0002	0.0001	0.0002	0.0003

Table X.61
Sample Response Correlation Matrix
Hospital Example
PEG 7, Response 1

1.0000	0.8122	0.7693	0.8156	−0.3936	−0.4572	−0.39180	−0.3893
0.8122	1.0000	0.8358	0.8218	−0.2942	−0.3190	−0.23300	−0.2396
0.7693	0.8358	1.0000	0.8092	−0.1435	−0.2243	−0.21990	−0.2357
0.8156	0.8218	0.8092	1.0000	−0.1531	−0.2202	−0.21470	−0.1559
−0.3936	−0.2942	−0.1435	−0.1531	1.0000	0.7860	0.82370	0.7835
−0.4572	−0.3190	−0.2243	−0.2202	0.7860	1.0000	0.83740	0.7945
−0.3918	−0.2330	−0.2199	−0.2147	0.8237	0.8374	1.00000	0.7798
−0.3893	−0.2396	−0.2357	−0.1559	0.7835	0.7945	0.77980	1.0000

Table X.62
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 7, Response 1

0.0072	0.0000	0.0003	0.0006
0.0000	0.0006	−0.0001	0.0001
0.0003	−0.0001	0.0007	0.0001
0.0006	0.0001	0.0001	0.0003

Table X.63
Sample Response Correlation Matrix
Hospital Example
PEG 7, Response 2

1.0000	0.9527	0.9459	0.9321	−0.6882	−0.6980	−0.69160	−0.6837
0.9527	1.0000	0.9520	0.9163	−0.6355	−0.6408	−0.63540	−0.6262
0.9459	0.9520	1.0000	0.9308	−0.5821	−0.5991	−0.60230	−0.5832
0.9321	0.9163	0.9308	1.0000	−0.6664	−0.6722	−0.66680	−0.6566
−0.6882	−0.6355	−0.5821	−0.6664	1.0000	0.9328	0.96650	0.9485
−0.6980	−0.6408	−0.5991	−0.6722	0.9328	1.0000	0.95280	0.9444
−0.6916	−0.6354	−0.6023	−0.6668	0.9665	0.9528	1.00000	0.9604
−0.6837	−0.6262	−0.5832	−0.6566	0.9485	0.9444	0.96040	1.0000

Table X.64
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 7, Response 2

0.2854	0.0194	−0.0105	0.0093
0.0194	0.0141	−0.0042	0.0006
−0.0105	−0.0042	0.0131	0.0015
0.0093	0.0006	0.0015	0.0108

Table X.65
Sample Response Correlation Matrix
Hospital Example
PEG 7, Response 3

1.0000	0.9874	0.9874	1.0000	-0.1302	-0.1158	-0.11580	-0.1302
0.9874	1.0000	1.0000	0.9874	-0.1586	-0.1377	-0.13770	-0.1586
0.9874	1.0000	1.0000	0.9874	-0.1586	-0.1377	-0.13770	-0.1586
1.0000	0.9874	0.9874	1.0000	-0.1302	-0.1158	-0.11580	-0.1302
-0.1302	-0.1586	-0.1586	-0.1302	1.0000	0.9824	0.98240	1.0000
-0.1158	-0.1377	-0.1377	-0.1158	0.9824	1.0000	1.00000	0.9824
-0.1158	-0.1377	-0.1377	-0.1158	0.9824	1.0000	1.00000	0.9824
-0.1302	-0.1586	-0.1586	-0.1302	1.0000	0.9824	0.98240	1.0000

Table X.66
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 7, Response 3

0.5029	-0.0132	0.0000	0.0000
-0.0132	0.0038	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.67
Sample Response Correlation Matrix
Hospital Example
PEG 7, Response 4

1.0000	0.9419	1.0000	0.9419	-0.3332	-0.3425	-0.33320	-0.3425
0.9419	1.0000	0.9419	1.0000	-0.3456	-0.3692	-0.34560	-0.3692
1.0000	0.9419	1.0000	0.9419	-0.3332	-0.3425	-0.33320	-0.3425
0.9419	1.0000	0.9419	1.0000	-0.3456	-0.3692	-0.34560	-0.3692
-0.3332	-0.3456	-0.3332	-0.3456	1.0000	0.9043	1.00000	0.9043
-0.3425	-0.3692	-0.3425	-0.3692	0.9043	1.0000	0.90430	1.0000
-0.3332	-0.3456	-0.3332	-0.3456	1.0000	0.9043	1.00000	0.9043
-0.3425	-0.3692	-0.3425	-0.3692	0.9043	1.0000	0.90430	1.0000

Table X.68
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 7, Response 4

0.5100	0.0000	-0.0012	0.0000
0.0000	0.0000	0.0000	0.0000
-0.0012	0.0000	0.0278	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.69
Sample Response Correlation Matrix
Hospital Example
PEG 7, Response 5

1.0000	0.7967	0.6125	0.7220	0.4791	0.3929	0.56630	0.5414
0.7967	1.0000	0.6992	0.8498	0.5431	0.5624	0.66020	0.7256
0.6125	0.6992	1.0000	0.7396	0.4312	0.3657	0.25520	0.3867
0.7220	0.8498	0.7396	1.0000	0.5530	0.4795	0.48530	0.6061
0.4791	0.5431	0.4312	0.5530	1.0000	0.8403	0.75450	0.8010
0.3929	0.5624	0.3657	0.4795	0.8403	1.0000	0.71430	0.8336
0.5663	0.6602	0.2552	0.4853	0.7545	0.7143	1.00000	0.6421
0.5414	0.7256	0.3867	0.6061	0.8010	0.8336	0.64210	1.0000

Table X.70
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 7, Response 5

0.0698	0.0002	0.0040	−0.0030
0.0002	0.0027	0.0008	0.0006
0.0040	0.0008	0.0042	0.0002
−0.0030	0.0006	0.0002	0.0014

Table X.71
Sample Response Correlation Matrix
Hospital Example
PEG 8, Response 1

1.0000	0.8582	0.9043	0.9112	-0.1477	-0.0183	-0.15080	-0.1287
0.8582	1.0000	0.8950	0.8605	-0.0591	0.0589	-0.02110	-0.0253
0.9043	0.8950	1.0000	0.9366	-0.0503	0.0692	-0.04240	0.0116
0.9112	0.8605	0.9366	1.0000	-0.1063	-0.0081	-0.14230	-0.1139
-0.1477	-0.0591	-0.0503	-0.1063	1.0000	0.8151	0.85860	0.8669
-0.0183	0.0589	0.0692	-0.0081	0.8151	1.0000	0.89530	0.9176
-0.1508	-0.0211	-0.0424	-0.1423	0.8586	0.8953	1.00000	0.9196
-0.1287	-0.0253	0.0116	-0.1139	0.8669	0.9176	0.91960	1.0000

Table X.72
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 8, Response 1

0.0089	0.0001	-0.0001	0.0000
0.0001	0.0003	0.0000	0.0000
-0.0001	0.0000	0.0002	0.0000
0.0000	0.0000	0.0000	0.0003

Table X.73
Sample Response Correlation Matrix
Hospital Example
PEG 8, Response 2

1.0000	0.9886	0.9886	1.0000	−0.3008	−0.2930	−0.29300	−0.3008
0.9886	1.0000	1.0000	0.9886	−0.3073	−0.3012	−0.30120	−0.3073
0.9886	1.0000	1.0000	0.9886	−0.3073	−0.3012	−0.30120	−0.3073
1.0000	0.9886	0.9886	1.0000	−0.3008	−0.2930	−0.29300	−0.3008
−0.3008	−0.3073	−0.3073	−0.3008	1.0000	0.9883	0.98830	1.0000
−0.2930	−0.3012	−0.3012	−0.2930	0.9883	1.0000	1.00000	0.9883
−0.2930	−0.3012	−0.3012	−0.2930	0.9883	1.0000	1.00000	0.9883
−0.3008	−0.3073	−0.3073	−0.3008	1.0000	0.9883	0.98830	1.0000

Table X.74
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 8, Response 2

0.4865	−0.0026	0.0000	0.0000
−0.0026	0.0044	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.75
Sample Response Correlation Matrix
Hospital Example
PEG 8, Response 3

1.0000	0.9886	0.9886	1.0000	−0.3008	−0.2930	−0.29300	−0.3008
0.9886	1.0000	1.0000	0.9886	−0.3073	−0.3012	−0.30120	−0.3073
0.9886	1.0000	1.0000	0.9886	−0.3073	−0.3012	−0.30120	−0.3073
1.0000	0.9886	0.9886	1.0000	−0.3008	−0.2930	−0.29300	−0.3008
−0.3008	−0.3073	−0.3073	−0.3008	1.0000	0.9883	0.98830	1.0000
−0.2930	−0.3012	−0.3012	−0.2930	0.9883	1.0000	1.00000	0.9883
−0.2930	−0.3012	−0.3012	−0.2930	0.9883	1.0000	1.00000	0.9883
−0.3008	−0.3073	−0.3073	−0.3008	1.0000	0.9883	0.98830	1.0000

Table X.76
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 8, Response 3

0.4865	−0.0026	0.0000	0.0000
−0.0026	0.0044	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.77
Sample Response Correlation Matrix
Hospital Example
PEG 8, Response 4

1.0000	0.9087	1.0000	0.9087	−0.2509	−0.2631	−0.25090	−0.2631
0.9087	1.0000	0.9087	1.0000	−0.2884	−0.3160	−0.28840	−0.3160
1.0000	0.9087	1.0000	0.9087	−0.2509	−0.2631	−0.25090	−0.2631
0.9087	1.0000	0.9087	1.0000	−0.2884	−0.3160	−0.28840	−0.3160
−0.2509	−0.2884	−0.2509	−0.2884	1.0000	0.8645	1.00000	0.8645
−0.2631	−0.3160	−0.2631	−0.3160	0.8645	1.0000	0.86450	1.0000
−0.2509	−0.2884	−0.2509	−0.2884	1.0000	0.8645	1.00000	0.8645
−0.2631	−0.3160	−0.2631	−0.3160	0.8645	1.0000	0.86450	1.0000

Table X.78
Sample Metamodel Coefficient Covariance Matrix, (V)
Hospital Example
PEG 8, Response 4

0.4874	0.0000	0.0077	0.0000
0.0000	0.0000	0.0000	0.0000
0.0077	0.0000	0.0371	0.0000
0.0000	0.0000	0.0000	0.0000

Table X.79
Sample Response Correlation Matrix
Hospital Example
PEG 8, Response 5

1.0000	0.9025	0.9494	0.8385	0.4972	0.4969	0.31410	0.3715
0.9025	1.0000	0.9377	0.9172	0.5117	0.5734	0.47850	0.4728
0.9494	0.9377	1.0000	0.8492	0.4913	0.4774	0.33130	0.3628
0.8385	0.9172	0.8492	1.0000	0.4881	0.4131	0.24870	0.2967
0.4972	0.5117	0.4913	0.4881	1.0000	0.8900	0.82520	0.9139
0.4969	0.5734	0.4774	0.4131	0.8900	1.0000	0.86700	0.9467
0.3141	0.4785	0.3313	0.2487	0.8252	0.8670	1.00000	0.9168
0.3715	0.4728	0.3628	0.2967	0.9139	0.9467	0.91680	1.0000

Table X.80
Sample Metamodel Coefficient Covariance Matrix, (**V**)
Hospital Example
PEG 8, Response 5

0.0667	0.0032	-0.0001	-0.0052
0.0032	0.0006	-0.0003	0.0000
-0.0001	-0.0003	0.0015	0.0001
-0.0052	0.0000	0.0001	0.0008

Table X.81					
Estimated Metamodels for the Hospital Example					
Response	PEG	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
1	1	12.06	-0.05	-0.05	0.23
1	2	12.08	-0.05	-0.04	0.24
1	3	12.11	-0.05	-0.05	0.23
1	4	12.10	-0.05	-0.05	0.23
1	5	12.09	-0.04	-0.05	0.24
1	6	12.08	-0.02	-0.02	0.21
1	7	12.10	-0.04	-0.05	0.23
1	8	12.05	-0.04	-0.05	0.23
2	1	45.50	-0.25	-0.32	-1.79
2	2	45.63	-0.29	-0.38	-1.80
2	3	45.71	-0.29	-0.39	-1.80
2	4	45.64	-0.29	-0.39	-1.81
2	5	45.76	-0.28	-0.41	-1.81
2	6	45.40	-0.12	-0.21	-1.98
2	7	45.69	-0.26	-0.40	-1.81
2	8	45.61	-0.29	-0.38	-1.80
3	1	58.76	-3.35	0.03	0.01
3	2	58.73	-3.40	0.01	-0.05
3	3	58.75	-3.34	-0.07	-0.02
3	4	58.56	-3.38	0.00	0.00
3	5	58.64	-3.39	0.00	0.00
3	6	57.73	-2.54	0.84	-0.84
3	7	58.70	-3.40	0.00	0.00
3	8	58.69	-3.39	0.00	0.00
4	1	54.98	0.06	-5.95	-0.03
4	2	55.03	-0.03	-6.00	0.09
4	3	54.73	0.00	-5.94	0.00
4	4	55.18	-0.04	-5.96	0.00
4	5	55.15	0.00	-6.00	0.00
4	6	53.53	1.48	-4.49	-1.48
4	7	55.07	0.00	-5.97	0.00
4	8	55.09	0.00	-5.94	0.00
5	1	95.38	0.05	0.13	-0.21
5	2	95.39	0.03	0.15	-0.19
5	3	95.40	0.03	0.12	-0.20
5	4	95.39	0.05	0.13	-0.21
5	5	95.39	0.05	0.13	-0.21
5	6	95.47	-0.01	0.09	-0.15
5	7	95.39	0.05	0.12	-0.21
5	8	95.35	0.04	0.12	-0.21

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Vita

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A handwritten signature in black ink, reading "Michael A. Zeimer". The signature is written in a cursive style with a large, stylized 'M' and 'Z'.

four pure error generator selection methods are outlined, tested, and compared to the results of the Monte-Carlo experiments. The results strongly suggest the importance of careful pure error generator selection and indicate that the primary difference in their performance is related to their ability to maintain the prescribed correlation structures of the correlation induction strategy.