

Classical Element Feedback Control for Spacecraft Orbital Maneuvers

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(ABSTRACT)

The recent addition of autonomous formation flying spacecraft to the world's satellite fleet provides new motivation to study feedback control techniques. In this thesis, we develop nonlinear orbit control laws for use in spacecraft orbital maneuvers, and spacecraft formation flying. We apply these new control laws to a number of sample maneuvers, including formation establishment and formation keeping maneuvers for NASA-Goddard's Leonardo-BRDF formation, and coupled orbit, and attitude maneuvers for HokieSat, a spacecraft designed, and built by students at Virginia Tech to fly in the Ionospheric Observation Nanosatellite Formation (ION-F). To provide target orbit states for feedback control, we develop and apply an algorithm to calculate a formation master orbit representing the geometric center of the formation. We also define a new technique for choosing orbital element feedback gains which appropriately scales the gains for orbit maintenance, and provides an excellent starting point for gain optimization. The orbital element feedback control law, augmented by mean motion control, and applied with appropriate gains, forces asymptotic convergence to a spacecraft target orbit, for a large variety of spacecraft maneuvers.

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Chapter 1

Introduction

In the past twenty years, spacecraft formation flight has been the topic of a great deal of research. Clusters of satellites flying in close proximity can decrease mission costs and increase performance, reliability, adaptability and survivability of space missions. Distribution of satellite systems over multiple spacecraft can also result in weight and launch cost reductions, and increased mission versatility.

Spacecraft flying in close proximity is not a new concept. Gemini VI accomplished the first space rendezvous on December 15, 1965, and performed station-keeping with Gemini VII over three orbits at distances of 0.3 meter to 90 meters. Gemini VIII followed with the first docking of two spacecraft in orbit on March 16, 1966. The first automated rendezvous was performed by Soviet spacecraft Cosmos 186 and Cosmos 188 on October 27, 1967, when the two unmanned spacecraft performed a preprogrammed closure and docking maneuver.¹ Since the 1960s, numerous manned and unmanned spacecraft have performed operations in close proximity, mostly for the purpose of docking for re-supply or crew transfer, and observation, as in the case of Shuttle flyarounds of the International Space Station. Autonomous formation flight became a reality for spacecraft on May 17, 2001, when NASA-Goddard's Earth Observing-1 spacecraft performed its first autonomous maneuver to maintain a one minute in-track separation with Landsat7. These early and recent space encounters have paved the way for spacecraft formation flight for use in a number of varied scientific, military, and satellite service operations.

1.1 Motivation

The recent addition of autonomous, formation flying spacecraft to the world's satellite fleet provides new motivation to study feedback control techniques. Traditionally, spacecraft orbits have been controlled by engineers on the ground, who command the spacecraft to perform open-loop thrust profiles designed to force optimal maneuver trajectories. Linear feedback control is used primarily to counter unexpected perturbations, and maintain the optimal trajectory.

The primary goal of this research is to develop nonlinear feedback control laws to au-

tonomously maneuver and maintain spacecraft formations, without the use of computationally expensive optimization techniques. Intuitively, the use of nonlinear control as opposed to linear control should result in significant fuel savings. This savings should be particularly evident when feedback control is used to perform large maneuvers, where the nonlinear dynamics effects become pronounced.

The funding for this research comes from NASA's Goddard Space Flight Center (GSFC), and the Air Force, through the ION-F and Leonardo projects. A brief description of these missions follows.

1.1.1 ION-F

The Ionospheric Observation Nanosatellite Formation (ION-F) consists of three spacecraft, one each designed and built by students at Virginia Tech, Utah State, and the University of Washington. The mission has three primary objectives: to conduct a multi-point study on properties of the ionosphere;² to test and demonstrate the concept of formation flying for use in space missions;³ and to provide students with the valuable opportunity to design, build, and operate a real spacecraft. ION-F is sponsored by the Air Force and NASA and will launch on the Space Shuttle in 2003.

HokieSat, the Virginia Tech nanosatellite, presents an interesting control problem. A pulsed plasma thruster (PPT) system has been developed specifically for the ION-F mission by the University of Washington and Primex Aerospace Corporation.^{4,5} Figure 1.1 shows the layout of these thrusters in the HokieSat spacecraft. Thrusters T_2 and T_3 in the figure are the primary source of translation control, whereas thrusters T_1 and T_4 provide yaw steering to augment the magnetic torque coil system. The thrusters produce an impulse-bit of $56 \mu\text{N}\cdot\text{s}$ and fire at a rate of 1 Hz, giving a total translational impulse-bit of $112 \mu\text{N}\cdot\text{s}$. As there are no thrusters pointed in the zenith-nadir direction, and the spacecraft must remain nadir-pointing for inter-satellite communications, radial thrusting is prohibited.

The attitude determination and control system of HokieSat is described in Ref. 6. The attitude is primarily controlled by a system of magnetic torque coils, with thrusters T_1 and T_4 providing a level of redundancy for yaw steering. For the preliminary formation control laws described in this paper we assume that the spacecraft can be slewed in order to provide thrust in any allowable direction. Specifically, we do not allow thrust in the radial direction, since the spacecraft does not have thrusters mounted on the nadir or zenith faces and must remain nadir-pointing.

All of the ION-F spacecraft use a combined GPS-based navigation and crosslink communications system designed and manufactured by the Applied Physics Laboratory.⁷ This system allows each spacecraft to determine its orbit and communicate this and other information to the other spacecraft allowing for autonomous operations.

Additional information about the HokieSat spacecraft may be found in Ref. 8.

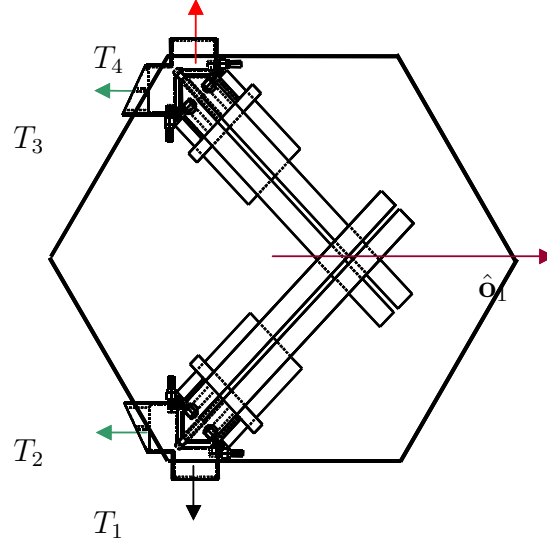


Figure 1.1: HokieSat pulsed plasma thruster (PPT) layout viewed from zenith direction

1.1.2 Leonardo-BRDF

Leonardo-BRDF is a mission concept currently under consideration by scientists at NASA's Goddard Space Flight Center (GSFC). The mission concept involves the use of formation flying satellites to study atmospheric phenomena, specifically the instantaneous net radiation flux, or albedo, at the Earth's upper atmosphere. The Bi-directional Reflectance Distribution Function (BRDF) characterizes the angle and intensity of sunlight reflected off of a surface, and is dependent on the position of the sun, atmospheric conditions, and the type of surface. Scientists are interested in BRDF because of its significant impact on climate phenomena. Previous BRDF studies have resulted in large errors, as a result of angular radiance variations. The Leonardo formation seeks to minimize these errors by measuring the atmospheric radiation flux from multiple spacecraft, positioned in a formation chosen to evenly distribute azimuth view angles, and optimize zenith view angles for BRDF measurement.

Additional information on the Leonardo-BRDF mission, and BRDF may be found in Ref. 9.

1.2 Problem Definition

The goal of this research is to develop and implement nonlinear orbit control laws and formation flying strategies for use in spacecraft formation flying missions. These control laws should be applicable to real satellites, and therefore must work in the presence of constraints such as thrust magnitude and direction limitations. They should also seek to minimize the fuel consumption, both of the individual formation flyers, and the formation as a whole. The formation flying strategies should cover issues including the definition of reference, or target trajectories, and the definition of appropriate nonlinear feedback control gains.

1.3 Approach

We approach formation flying control in two steps: target orbit definition; and target orbit rendezvous. We assume that desired relative formation geometries are known, and most likely chosen to optimize the performance of mission operations other than formation control. In the case of the Leonardo mission, the relative formation geometries have been designed by GSFC scientists to optimize the BRDF measurement accuracy.

To maintain and maneuver a formation of spacecraft, we first develop a method for defining the absolute state of the formation, and the relative states of the spacecraft within the formation. In Chapter 3 we define a formation master orbit, positioned in the geometric center of the desired formation. From this master orbit, we define differential orbital element sets, used to calculate the desired spacecraft states at any given time. During simulation, we integrate the individual spacecraft states, as well as the master orbit state, and calculate the target orbit states at every time step.

To force convergence of the formation flying spacecraft to the desired orbit states, we use nonlinear feedback control of a variety of state-space variables. In Chapter 4, we use Lyapunov's direct method to develop 3 nonlinear orbit control laws which use cartesian position and velocity error feedback, classical orbital element error feedback, and equinoctial orbital element error feedback. We treat each maneuver as a rendezvous between a maneuvering spacecraft, and a passive, target orbit, or virtual spacecraft.

1.4 Overview

In Chapter 2, we present a literature review, including previous works on both individual spacecraft dynamics and control, and spacecraft formation dynamics and control. In Chapter 3 we present a brief review of spacecraft dynamics, and introduce some formation flying concepts. In Chapter 4, we develop various nonlinear control laws, and discuss some issues associated with the application of these control laws. In Chapter 5, we describe the formation flying simulation software, and present the results of control law application in some sample maneuvers, including some simple maneuvers for the Ionospheric Observation Nanosatellite Formation, and formation establishment and formation keeping maneuvers for the Leonardo formation.

Chapter 2

Literature Review

In this chapter, we present a review of spacecraft formation dynamics and control literature. We introduce some applications of spacecraft formation flight and some of the terminology that appears in the literature. We then discuss articles on formation dynamics and associated perturbations, some basic formation geometries, and control strategies for maintaining and maneuvering spacecraft formations.

2.1 Applications

One major function in which a cluster of small, formation flying satellites provides benefits over the traditional single, large satellite is deep space and Earth remote sensing. Image resolution is inversely proportional to instrument aperture.¹⁰ Formation flying clusters take advantage of this fact by placing measurement devices at separation distances not practical for individual larger satellites.

Some examples of proposed formation flying missions in various stages of development are ALPHA, Aqua, Aura, Auroral Lites, Auroral Multiscale Midex (AMM), CloudSat, ClusterII, GRACE, ION-F, Leonardo-BRDF, Magnetospheric Multiscale (MMS), MUSIC, Power Sail, ST3, Parasol, PICASSO, SMART-2, Starlight, TechSat21, Terrestrial Planet Finder (TPF), and TOPSAT. Missions for these spacecraft include extra-solar planet studies (TPF), formation flying optical interferometry tests (Starlight), magnetosphere and solar wind studies (AMM and Cluster II), and a variety of other science studies and technology demonstrations.

2.2 Terminology

The simplest example of formation flight in space is the rendezvous problem, in which an interceptor spacecraft attempts to rendezvous with a passive target spacecraft. Most formation flying control papers attack the problem of controlling the relative orbit of two or more spacecraft, one

of which does not perform primary formation control and follows some reference trajectory, and the rest of which perform orbit control to maintain the relative formation. The nomenclature for spacecraft flying in formation is almost as varied as the application. The controlled spacecraft appears in the literature as the follower, the slave, the deputy, the interceptor, the chase satellite, the free-flying satellite, the collector, the daughter, and so on. The uncontrolled spacecraft appears as the leader, the lead, the master, the chief, the target, the reference, the head, or virtual head, the control, the combiner, the mother, and so on. Many of these names are convenient for specific formations based on their missions. For example, the formation described in Ref. 11 uses interferometry to combine data at a central spacecraft (the combiner) after reflected light from the target has been collected by the rest of the satellites in the formation (the collectors). The combiner is the leader in this formation, and the collectors are the followers, maintaining the required formation with respect to the combiner. In this paper, we refer mostly to the actively maneuvering spacecraft as the controlled spacecraft, the desired or target orbit of the controlled spacecraft as the reference orbit, and the satellite, or virtual satellite from which the reference orbits of all of the spacecraft in a formation are defined as the master spacecraft, or master orbit.

2.3 Formation Dynamics and Perturbations

The motion of spacecraft formations can be divided into absolute and relative motion. The absolute (or inertial) motion of the individual spacecraft is described in numerous texts.^{12–14} The relative motion of bodies orbiting in close proximity is included briefly in these texts, mostly in the form of the Clohessy-Wiltshire, or Hill’s equations. A more detailed description of the dynamics of formation flying spacecraft is available in journal articles and conference papers dating back to the 1980s.

Karlgard and Lutze¹⁵ present an approximate solution to the second-order relative motion equations for spacecraft in near-circular, Keplerian orbits. This nonlinear solution provides obvious benefits over the linear approximation, including improved accuracy (especially in the transverse direction), and re-coupling of in-plane and out-of-plane motion.

Researchers at Texas A&M University have provided numerous conference and journal articles characterizing the relative motion of formation flying satellites.^{16–20} The authors present methods for accommodating nonlinearity and eccentricity perturbation effects in order to more accurately predict the long-term relative motion of satellite formations, and thus minimize the fuel necessary to maintain and control them.

Sedwick, Miller, and Kong²¹ present a detailed discussion of the environmental perturbative forces encountered by formation flying spacecraft, including differential and absolute non-spherical earth effects, and atmospheric drag. The authors assume differential drag effects are negligible, compared to differential J_2 effects. This assumption is appropriate because the spacecraft in the formation have identical or nearly identical ballistic coefficients. The authors discuss the impact of ΔV on mission life, both in terms of the bulk motion of the formation,

and the differential, or relative motion of the individual spacecraft, and conclude that the J_2 acceleration is the predominant perturbation for TechSat21, followed distantly by differential accelerations resulting from solar pressure, atmospheric drag, and magnetic field interaction.

2.4 Formation Geometries

Folta and Quinn²² describe three formation types for use in the EO-1 Landsat7 formation: the close formation, the ideal formation, and the dynamic formation. The close formation consists of spacecraft flying with a separation of less than one kilometer in the velocity direction, and is intended for missions making simultaneous, multi-point observations. This formation type has also been called the in-plane²³ formation, and while it is often modeled with a transverse-direction separation only, inspection shows that there is also a small separation in the radial direction. For this reason, we refer to this formation as the anomaly shift formation, and model it as a shift in true anomaly, as shown in Figure 2.1.

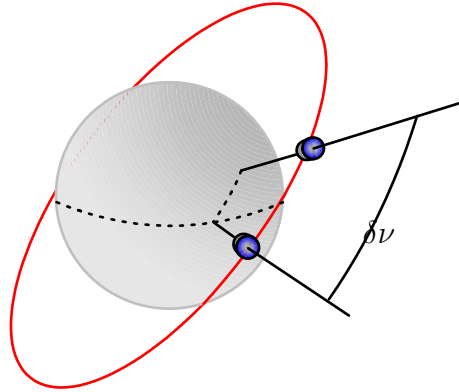


Figure 2.1: The anomaly shift formation

The ideal formation involves separation distances on the order of 1 to 100 kilometers, and is intended for missions making time-lapse observations of the same point. The ideal formation separation is mostly in the transverse direction, but also in the orbit-normal direction to compensate for the rotation of the earth, and to ensure that each formation flyer follows the same ground track. The ideal formation is often called the same ground track, or in-track formation, and is usually modeled as a shift in true anomaly and right ascension, as shown in Figure 2.2.

The dynamic formation is similar to the ideal formation, except that the maneuvering spacecraft is allowed to drift with respect to the non-maneuvering reference satellite as a result of differential atmospheric drag effects. In the case of EO-1 and Landsat7, differential drag effects are the largest source of error, as the two spacecraft are distinctly different in shape, size, and mass, and therefore have different ballistic coefficients.²⁴ The formation is maintained by allowing EO-1 to drift from a carefully selected relative orbit until it either reaches the limit of its control box, or Landsat7 performs a maneuver, at which point EO-1 performs a maneuver to return itself to the predetermined relative orbit.

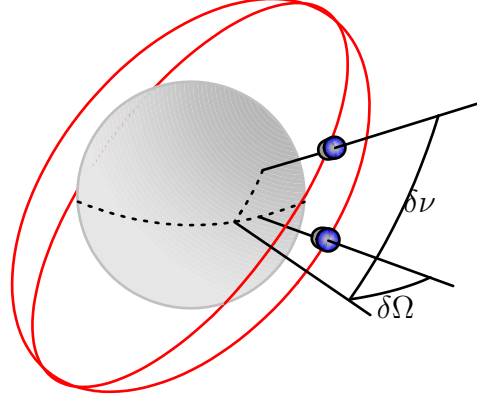


Figure 2.2: The same ground track formation

The rotating formation type involves a cluster of spacecraft that travel in elliptical paths about a central, reference orbit. Hughes and Hall^{25,26} develop an analytical method for creating rotating formations of n spacecraft that maintain a constant elliptical shape as viewed from Earth, and develop mission performance measures to evaluate the formations. Hughes and Mailhe⁹ continue to develop rotating formation design techniques, and evaluate the use of rotating formations for the Leonardo-BRDF formation.

Sabol, Burns, and McLaughlin²³ describe four basic satellite formation designs, and approximate the ΔV required to compensate for “realistic” differential perturbations. The authors’ primary goal is to quantify the formation control maneuvers required to compensate for perturbations, for each of the described formation designs. They model polar orbits, near 800 km altitude, with inter-satellite spacings of approximately 1 km. Perturbations modeled include gravity field of degree and order 21, luni-solar third body point mass effects, atmospheric drag, and solar radiation pressure. The authors conclude that the in-track and in-plane formations are stable or relatively stable, and require little to no formation-keeping maneuvers, whereas the circular and projected circular formations are highly unstable due to oblateness effects.

Schaub and Alfrend^{27,28} develop an analytic method for defining J_2 invariant relative orbits for spacecraft formations. The authors assume spacecraft with equal ballistic coefficients (and therefore negligible differential drag effects), and use mean orbit elements and Brouwer’s artificial satellite theory without drag to find J_2 invariant orbits. The method described matches the drift rates of two neighboring orbits up to the first order by constraining the values of δa , δe , and δi , and provides the most significant fuel savings for non-circular, non-polar orbits.

Vadali and Vaddi²⁹ discuss the formation establishment problem, the objective of which is to maximize the inclination difference between two formation flying spacecraft while satisfying relative mean orbit elements and rate constraints. The authors find that satisfaction of rate constraints conflicts with the objective of maximizing the inclination difference.

Chao, *et al*³⁰ assume a cluster consisting of a “centersat” in some reference orbit and with higher area-to-mass ratio, and “subsats” that follow the reference orbit. The centersat makes up the separation caused by area-mass ratio differential by applying drag compensation to target

a reference orbit that matches the average decay rate of subsats.

2.5 Control Strategies

Control of satellite formations should be considered in terms of two operational modes: formation keeping, and formation maneuvering. The difference between the two modes is, at first, subtle. It is not difficult to think of formation keeping as a subcategory of formation maneuvering. However, when we consider the application of orbit control to establish large formations, or to reconfigure them (formation maneuvering), as compared to the mitigation of well known perturbations to maintain formation geometries (formation keeping), the distinction becomes more clear.

2.5.1 Formation Keeping

Formation keeping controllers have been developed by Vassar and Sherwood,³¹ for two satellites in a circular orbit, Leonard, Hollister, and Bergmann,³² for two satellites with drag panels to generate differential drag, and Middour,³³ for along track formation keeping of low eccentricity formation flyers.

A more recent formation keeping control, the Folta-Quinn (FQ) Formation Flying Algorithm²² uses Lambert's two point boundary value problem and the 'C*' guidance and navigation matrix to maintain a formation of two spacecraft. Carpenter, Folta and Quinn^{22,34} use the FQ algorithm, in conjunction with a linear-quadratic-Gaussian (LQG) control³⁵ to develop a decentralized control for autonomous formation flying for use in the New Millenium Program and the Earth Observing-1 (EO-1) mission. The formation model includes 2 spacecraft, one with real-time GPS and autonomous control (EO-1), and one with orbit determination and maneuver generation performed on the ground (Landsat-7). The authors conclude that autonomous, decentralized control is not only feasible, but beneficial over traditional orbit control procedures. The preliminary results²⁴ of the EO-1 experiment show that the FQ formation flying algorithm has succeeded in providing robust, closed-loop, autonomous control of NASA's first formation flying spacecraft.

2.5.2 Formation Maneuvering

Schaub and Alfriend^{28,36} use mean orbital element differences to control the relative orbits of formation flying spacecraft with identical ballistic coefficients. The authors' use of mean orbital element differences stresses long-term behavior over short-term deviation, by forcing the control to compensate for secular and long period drifts, while ignoring short period oscillations. A modified form of Gauss' variational equations of motion are used to develop a sequential, impulsive algorithm to correct orbit element errors. The control law establishes the desired

formation in a nearly fuel-optimal sense by requesting impulses when $\nu = 0$, and $\nu = \pi/2$, only. This impulse timing allows correction of each element, with minimal impact on the other elements. The authors also develop a cartesian, Lyapunov-control law, and apply it using relative position and velocity tracking vectors calculated from mean orbital elements. A comparison of the two nonlinear feedback control laws shows that the cartesian form is effective for short time-frame, high thrust maneuvers, while the orbital element form performs best in multiple-orbit or low-thrust maneuvers.

Ilgen³⁷ develops Lyapunov-optimal feedback control laws using Guass’s form of Lagrange’s planetary equations (LPE) in classical and equinoctial orbital-element forms. The author presents equations of motion for the equinoctial elements in terms of thrust components in the equinoctial coordinate axes. While the author does not discuss formation-flying control directly, the paper provides an excellent starting point for the development of an elemental, Lyapunov-optimal, formation flying feedback control law.

In Ref. 38 the authors develop non-impulsive control for transfers between elliptic orbits, using feedback of the angular momentum vector, $\mathbf{L}$, and the Laplace (or eccentricity) vector, $\mathbf{A}$. The authors suggest that the use of angular momentum and Laplace vectors in nonlinear control law derivation is superior to elemental feedback because $\mathbf{L}$ and $\mathbf{A}$ are: 1) globally well defined, while the elements have singularities; 2) R^3 valued, and therefore have a “nice (Lie)-algebraic structure”, while the elements have no useful algebraic structure. The authors do not discuss gain selection, nor do they comment on controlling the angular position within an orbit. Future work suggestions include investigation of J_2 effects, and optimization through gain selection. While the authors do not address formation-flying control directly, it may be possible to expand their approach to include an in-orbit angular position parameter, and thus control the full six degree-of-freedom, formation control problem.

In a series of papers, Kechichian^{39–41} presents minimum-time solutions for optimal, low-thrust rendezvous of spacecraft in a variety of orbits using equinoctial orbit elements. In the third of these papers, the author incorporates first order oblateness effects in a minimum-time, constant acceleration transfer to geosynchronous orbit. Again, these papers do not directly discuss spacecraft formation flying control, but may provide valuable insight into the development of time-optimal formation flying control solutions.

2.6 Summary

In this chapter, we have presented a review of formation flying literature, including a summary of some formation flying terminology used by various authors, and a review of previous works discussing formation dynamics and perturbations, formation geometries, and formation control strategies. In Chapter 3, we present a more thorough review of spacecraft dynamics in cartesian and orbital element terms, and discuss some formation flying concepts including the definition of a formation master orbit.

Chapter 3

Formation Dynamics

The dynamics of space objects is most often described by two-body motion in a Keplerian gravitational field, with corrections to compensate for forces from perturbations such as non-spheroidal central body effects, atmospheric drag, solar and lunar gravitational effects, and solar radiation pressure. In this chapter we present a brief description of Keplerian orbits, and the effect of some perturbations on formation dynamics. We conclude the chapter with a discussion of some formation flying concepts, including formation models and formation master orbit definitions.

3.1 Keplerian Motion

The standard Earth-centered inertial frame is designated as $\mathcal{F}_i$ and has unit vectors $\hat{\mathbf{i}}_1$, $\hat{\mathbf{i}}_2$ and $\hat{\mathbf{i}}_3$ in the inertial x , y and z directions, respectively. The perifocal frame, $\mathcal{F}_p$ has unit vectors $\hat{\mathbf{p}}_1$, $\hat{\mathbf{p}}_2$ and $\hat{\mathbf{p}}_3$ with $\hat{\mathbf{p}}_1$ in the periapsis direction, $\hat{\mathbf{p}}_3$ in the orbit normal direction, and $\hat{\mathbf{p}}_2 = \hat{\mathbf{p}}_3 \times \hat{\mathbf{p}}_1$. The orbit frame, $\mathcal{F}_o$, has unit vectors $\hat{\mathbf{o}}_1$, $\hat{\mathbf{o}}_2$ and $\hat{\mathbf{o}}_3$ with $\hat{\mathbf{o}}_3$ in the nadir direction, $\hat{\mathbf{o}}_2$ in the direction of the negative orbit normal, and $\hat{\mathbf{o}}_1 = \hat{\mathbf{o}}_2 \times \hat{\mathbf{o}}_3$, which is the velocity direction for a circular orbit. The spherical frame, $\mathcal{F}_s$, is a permutation of the orbital frame, and has unit vectors $\hat{\mathbf{s}}_1$ in the radial direction, $\hat{\mathbf{s}}_2$ in the co-velocity direction and $\hat{\mathbf{s}}_3$ in the orbit normal direction.

3.1.1 Cartesian Coordinates

In cartesian form, the equations of motion for a point-mass satellite take the form

$$\ddot{\mathbf{r}} = -\frac{\mu}{\|\mathbf{r}\|^3}\mathbf{r} + \vec{\mathbf{a}}_p \quad (3.1)$$

where $\mathbf{r}$ is the position vector of the satellite measured from the center of mass of the primary body, μ is the gravitational parameter of that body, and $\vec{\mathbf{a}}_p$ includes perturbation accelerations caused by non-spheroidal gravitational effects of the central body, atmospheric drag, third body

effects and so on. If $\vec{\mathbf{a}}_p = \vec{\mathbf{0}}$ then the equations of motion reduce to the ideal Keplerian situation where the central body is a perfect sphere and all other disturbances are zero.

3.1.2 Classical Orbital Elements

We can also express the motion of a point-mass satellite in terms of orbital elements. We define the classical orbital elements, using Bate, Mueller, and White's notion,¹² as the semi-major axis, a , the eccentricity, e , the inclination, i , the right ascension of the ascending node, Ω , the argument of periapse, ω , and the true anomaly, ν . The equations of motion of a controlled spacecraft in terms of the classical orbital element set are given by Gauss's form of Lagrange's planetary equations, as follows:¹⁴

$$\frac{da}{dt} = \frac{2a^2}{h} \left(e \sin \nu \mathbf{u}_r + \frac{p}{r} \mathbf{u}_\theta \right) \quad (3.2)$$

$$\frac{de}{dt} = \frac{1}{h} \{ p \sin \nu \mathbf{u}_r + [(p+r) \cos \nu + re] \mathbf{u}_\theta \} \quad (3.3)$$

$$\frac{di}{dt} = \frac{r \cos \theta}{h} \mathbf{u}_h \quad (3.4)$$

$$\frac{d\Omega}{dt} = \frac{r \sin \theta}{h \sin i} \mathbf{u}_h \quad (3.5)$$

$$\frac{d\omega}{dt} = \frac{1}{he} [-p \cos \nu \mathbf{u}_r + (p+r) \sin \nu \mathbf{u}_\theta] - \frac{r \sin \theta \cos i}{h \sin i} \mathbf{u}_h \quad (3.6)$$

$$\frac{dM}{dt} = n + \frac{b}{ahe} [(p \cos \nu - 2re) \mathbf{u}_r - (p+r) \sin \nu \mathbf{u}_\theta] \quad (3.7)$$

Where $\mathbf{u}_r$, $\mathbf{u}_\theta$, and $\mathbf{u}_h$ are the radial, transverse, and orbit normal control acceleration components, M is the mean anomaly, $\theta = \omega + \nu$, is the argument of latitude, and b , h , and p are the semi-minor axis, the angular momentum, and the semi-latus rectum, respectively. The mean motion, n , is calculated from the gravitational parameter, μ , and the semi-major axis, a , as:

$$n = \sqrt{\frac{\mu}{a^3}} \quad (3.8)$$

3.1.3 Equinoctial Orbital Elements

Gauss's form of Lagrange's planetary equations contains singularities for circular or equatorial orbits. For orbits with near zero inclination or eccentricity, we can use the nonsingular, equinoctial elements. We define the equinoctial orbital element set, using Battin's notation,¹⁴ as the semi-major axis, a , unnamed elements P_1 , P_2 , Q_1 , and Q_2 , and the mean longitude, l . Some authors refer to these elements as a , h , k , p , q , and λ .^{37,39} The non-singular equinoctial elements are defined in terms of the classical orbital elements as:

$$a = a \quad (3.9)$$

$$P_1 = e \sin \varpi \quad (3.10)$$

$$P_2 = e \cos \varpi \quad (3.11)$$

$$Q_1 = \tan \frac{i}{2} \sin \Omega \quad (3.12)$$

$$Q_2 = \tan \frac{i}{2} \cos \Omega \quad (3.13)$$

$$l = \varpi + M \quad (3.14)$$

where ϖ , the longitude of pericenter, is defined as

$$\varpi = \Omega + \omega \quad (3.15)$$

In addition to the mean longitude, l , we also need the true longitude, L , defined as

$$L = \varpi + \nu \quad (3.16)$$

The reverse transformation is given by:

$$a = a \quad (3.17)$$

$$e = (P_1^2 + P_2^2)^{1/2} \quad (3.18)$$

$$i = 2 \tan^{-1} (Q_1^2 + Q_2^2)^{1/2} \quad (3.19)$$

$$\Omega = \tan^{-1} \left(\frac{Q_1}{Q_2} \right) \quad (3.20)$$

$$\omega = \tan^{-1} \left(\frac{P_1}{P_2} \right) - \tan^{-1} \left(\frac{Q_1}{Q_2} \right) \quad (3.21)$$

$$M = l - \tan^{-1} \left(\frac{P_1}{P_2} \right) \quad (3.22)$$

The equations of motion of a controlled spacecraft in terms of the nonsingular equinoctial orbital elements are:

$$\frac{da}{dt} = \frac{2a^2}{h} \left[(P_2 \sin L - P_1 \cos L) \mathbf{u}_r + \frac{p}{r} \mathbf{u}_\theta \right] \quad (3.23)$$

$$\frac{dP_1}{dt} = \frac{r}{h} \left\{ -\frac{p}{r} \cos L \mathbf{u}_r + \left[P_1 + \left(1 + \frac{p}{r} \right) \sin L \right] \mathbf{u}_\theta - P_2 (Q_1 \cos L - Q_2 \sin L) \mathbf{u}_h \right\} \quad (3.24)$$

$$\frac{dP_2}{dt} = \frac{r}{h} \left\{ -\frac{p}{r} \sin L \mathbf{u}_r + \left[P_2 + \left(1 + \frac{p}{r} \right) \cos L \right] \mathbf{u}_\theta - P_1 (Q_1 \cos L - Q_2 \sin L) \mathbf{u}_h \right\} \quad (3.25)$$

$$\frac{dQ_1}{dt} = \frac{r}{2h} (1 + Q_1^2 + Q_2^2) \sin L \mathbf{u}_h \quad (3.26)$$

$$\frac{dQ_2}{dt} = \frac{r}{2h} (1 + Q_1^2 + Q_2^2) \cos L \mathbf{u}_h \quad (3.27)$$

$$\begin{aligned} \frac{dl}{dt} = & n - \frac{r}{h} \left\{ \left[\frac{a}{a+b} \left(\frac{p}{r} \right) (P_1 \sin L + P_2 \cos L) + \frac{2b}{a} \right] \mathbf{u}_r \right. \\ & \left. + \frac{a}{a+b} \left(1 + \frac{p}{r} \right) (P_1 \cos L - P_2 \sin L) \mathbf{u}_\theta + (Q_1 \cos L - Q_2 \sin L) \mathbf{u}_h \right\} \end{aligned} \quad (3.28)$$

3.2 Perturbation Effects

Whereas orbital perturbations have a significant impact on the absolute motion of individual spacecraft, the effect of perturbations on the relative motion of formation flying spacecraft can be quite limited. For formations in Earth orbit, the two most significant perturbations are Earth oblateness effects, and atmospheric drag effects.

For formations of identical spacecraft (equal size and mass), the differential drag effect is negligible, and the most significant perturbation to the formation geometry is the differential J_2 effect. Earth oblateness causes regression of the node, or rotation of the orbit about the inertial $\hat{\mathbf{i}}_3$ vector, as well as secular rotation of the orbit about the orbit normal vector. The secular drifts of Ω , ω , M due to J_2 perturbations are:

$$\frac{\overline{d\Omega}}{dt} = -\frac{3}{2}J_2 \left(\frac{r_{eq}}{p}\right)^2 n \cos i \quad (3.29)$$

$$\frac{\overline{d\omega}}{dt} = \frac{3}{4}J_2 \left(\frac{r_{eq}}{p}\right)^2 n(5 \cos^2 i - 1) \quad (3.30)$$

$$\frac{\overline{dM}}{dt} = \frac{3}{4}J_2 \left(\frac{r_{eq}}{p}\right)^2 \eta n(3 \cos^2 i - 1) \quad (3.31)$$

where $\eta = \sqrt{1 - e^2}$. Notice that the secular drift due to J_2 effects varies with inclination. Formations of spacecraft with no inclination variation will drift as a group, experiencing no relative deformation. Formations with inclination variations, such as the most recent Leonardo formation, encounter differential secular drift, which must be mitigated to maintain the formation geometry.

A more detailed description of formation perturbation effects is available in Reference 21.

3.3 Formation Flying Concepts

In this thesis, we study the control of various formations of multiple spacecraft, in which some or all of the spacecraft are free to maneuver and establish formations. We calculate the desired orbits of the maneuvering spacecraft from a central orbit, which we refer to as the master orbit. We refer to the desired orbits as reference, target, or virtual orbits.

We now introduce some basic formation geometries and models, and a method for calculating the formation master orbit and desired spacecraft orbits.

3.3.1 Formation Models

The ION-F formation flying mission includes two simple formation types: the anomaly shift formation, and the same ground track formation, both illustrated in Fig. 3.1. The anomaly shift

formation consists of two or more spacecraft in identical orbits separated by some change in true anomaly, $\delta\nu$.

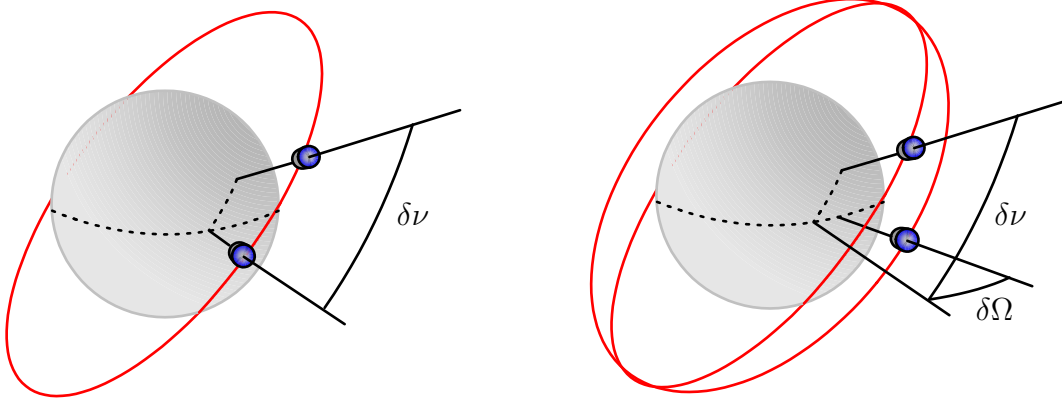


Figure 3.1: Anomaly shift and same ground track formations

The same ground track formation consists of two or more spacecraft orbits separated by shifts in true anomaly, $\delta\nu$, and right ascension, $\delta\Omega$. The value of $\delta\Omega$ orients the orbits so that all of the spacecraft in the formation share the same ground track:

$$\delta\Omega = \omega_e \delta t \quad (3.32)$$

where ω_e and δt are the angular velocity of the Earth and the time between satellite ground passes, respectively.

The proposed Leonardo formation is much more complex than the formations shown in Figure 3.1. The most current version of the Leonardo formation includes spacecraft in orbits with inclinations varying from about 0.5 to 5.0 degrees, with right ascension and argument of latitude variations resulting in satellite separation distances on the order of hundreds of kilometers. The Leonardo formation is described in more detail in Section 3.3.2.

Figure 3.2 shows three models for controlling and simulating formation dynamics. The three models increase in complexity from a simple target-interceptor problem to cooperative, multi-craft maneuvering problem. The target-interceptor problem simply involves maneuvering one spacecraft to rendezvous with another.

The intermediate model is a leader-follower (or master-slave) type formation where the reference orbit of the active spacecraft (the follower) is determined by a formation planner from the leader's orbit. In the case of an anomaly shift formation, the active spacecraft targets a reference orbit defined by a $\delta\nu$ shift of the leader's orbit. This model is only required if differential perturbations, such as Earth oblateness or orbit control at the leader spacecraft, are applied, causing the reference orbit to drift with respect to leader's. If no perturbations are applied in orbit propagation, this model is identical to the target-interceptor problem, as the reference orbit follows a path identical to the leader's.

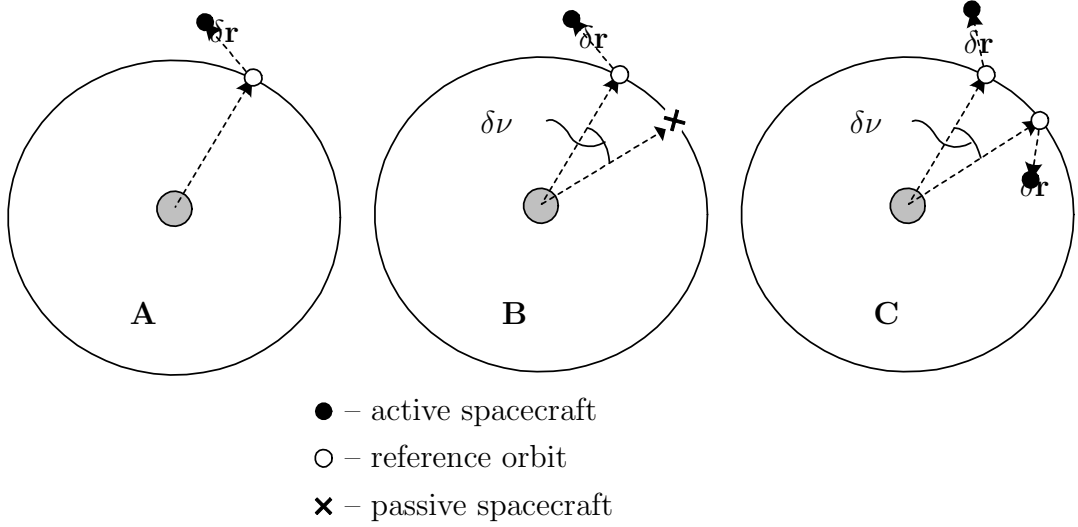


Figure 3.2: Formation simulation models: A) target-interceptor model; B) leader-follower model; C) cooperative model

The cooperative, multi-craft maneuvering problem involves the complicated task of determining reference orbits for two active spacecraft from the inertial orbits of the two spacecraft, and a predetermined relative formation type. In Ref. 29, the authors show that the use of two actively controlled spacecraft as opposed to a leader-follower type formation improves the efficiency of formation control by minimizing both the time, and the total number of thrust impulses required to establish the desired formation.

The desired relative orbit can be chosen in many ways, possibly to optimize the mission performance in some sense,²⁵ or to minimize the effect of gravitational perturbations on the formation geometry²⁸ in an effort to minimize formation keeping propellant consumption. For this study, we assume that the formation geometry is already defined, and seek to find ways to model, maneuver, and maintain that formation geometry.

3.3.2 The Master Orbit

To implement the cooperative formation control problem we must develop a method for calculating the desired orbits of all of the spacecraft in the formation as a function of the current location of each spacecraft, and the required formation geometry. We would like to define a master orbit, close to the center of the cluster, for use as a reference point in the calculation of the individual satellite target orbits. Starting with the position and velocity vectors of each of the formation flyers, and the required relative geometry, we can calculate the individual reference orbits in three steps: 1) initialize the formation; 2) calculate the orbit which remains closest to the center of the formation throughout the formation's motion (the master orbit); 3) define orbital element offsets used to calculate the desired orbits from the master orbit.

Initializing the Formation

Formation definitions often come in relative terms, such as differential orbital elements, and leave the absolute positioning general. We study the control of two formations for the Leonardo mission, both of which can be fully described in terms of some absolute orbital elements, and some relative, or differential, orbital elements. The first Leonardo formation, given in Table 3.1, consists of six spacecraft in circular orbits with matching altitudes and inclinations, and varying node vectors and arguments of latitude. Note that the semi-major axis, eccentricity, and inclination are given in absolute terms, while the remaining elements are given in relative terms.

Table 3.1: First Leonardo formation.

s/c	a [km]	e	i [°]	$\Delta\Omega$ [°]	$\Delta\theta$ [°]
1	6778	0	5	0.0	5.0
2	6778	0	5	2.0	3.5
3	6778	0	5	-4.0	2.0
4	6778	0	5	4.0	-2.0
5	6778	0	5	-2.0	-3.5
6	6778	0	5	0.0	-5.0

The second Leonardo formation, defined by Table 3.2, consists of six spacecraft in 400 km, circular orbits, with varying inclinations, node vectors, and arguments of latitude. Note that in this formation, the inclination varies between spacecraft, but is still given in absolute terms.

Table 3.2: Second Leonardo formation.

s/c	a [km]	e	i [°]	$\Delta\Omega$ [°]	$\Delta\theta$ [°]
1	6778	0	0.4783	16.86	-9.99
2	6778	0	0.9771	-20.88	25.52
3	6778	0	4.4108	166.10	-160.40
4	6778	0	4.5866	17.42	-13.58
5	6778	0	0.6153	17.52	-14.66
6	6778	0	2.4481	-157.80	160.13

These formations are defined in terms of both relative and absolute parameters. For both formations, the value of Ω and θ are not important, as long as their relative values match those given in the tables. To initialize these formations, we choose reference values of Ω and θ at random, and construct the individual orbits from the defined offset values.

Defining the Master Orbit

Once the formation has been initialized, it is convenient to define a master orbit which remains in the center of the formation as the spacecraft complete their orbits. For simple formations such as the anomaly shift formation, where the argument of latitude, θ , is the only parameter that differs between spacecraft, this master orbit is easily found. Since five of the orbital parameters are shared by all spacecraft in the formation, the sixth, the argument of latitude of the master, θ_m , can be found by averaging:

$$\theta_m = \frac{1}{n} \sum \theta_j \quad (3.33)$$

For more complex formations such as the one described in Table 3.2, the definition of a central master orbit requires a more robust, less intuitive method of calculation. We derive this method by considering some conserved properties of the formation: the total angular momentum vector, and the formation equivalents of the node and eccentricity vectors.

For a single spacecraft, the classical orbital elements are calculated from the position and velocity vectors by manipulation of these three vectors. We present this method as a baseline for the calculation of a formation master orbit.

Algorithm rv2oe

Following Bate, Mueller, and White,¹² the orbital elements of a spacecraft are found from the position and velocity vectors, $\mathbf{r}$ and $\mathbf{v}$, using the three fundamental vectors, $\mathbf{h}$, $\mathbf{n}$, and $\mathbf{e}$, as follows:

1. Calculate the angular momentum vector, $\mathbf{h}$,

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} \quad (3.34)$$

2. Calculate the node vector, $\mathbf{n}$,

$$\mathbf{n} \equiv \mathbf{K} \times \mathbf{h} \quad (3.35)$$

3. Calculate the eccentricity vector, $\mathbf{e}$,

$$\mathbf{e} = \frac{1}{\mu} \left[\left(v^2 - \frac{\mu}{r} \right) \mathbf{r} - (\mathbf{r} \cdot \mathbf{v}) \mathbf{v} \right] \quad (3.36)$$

4. Calculate the inclination, i , the angle between the vectors $\mathbf{K}$, and $\mathbf{h}$,

$$\cos i = \frac{h_K}{h} \quad (3.37)$$

(i is less than 180°)

5. Calculate the right ascension of the ascending node (RAAN), Ω , the angle between $\mathbf{I}$ and $\mathbf{n}$,

$$\cos \Omega = \frac{n_I}{n} \quad (3.38)$$

(if $n_J > 0$, Ω is less than 180°)

6. Calculate the argument of periapsis, ω , the angle between $\mathbf{n}$ and $\mathbf{e}$ (assuming e is non-zero),

$$\cos \omega = \frac{\mathbf{n} \cdot \mathbf{e}}{ne} \quad (3.39)$$

(if $e_K > 0$, ω is less than 180°)

7. Calculate the true anomaly, ν , the angle between $\mathbf{e}$ and $\mathbf{r}$ (again, assuming e is non-zero),

$$\cos \nu = \frac{\mathbf{e} \cdot \mathbf{r}}{er} \quad (3.40)$$

(if $\mathbf{r} \cdot \mathbf{v} > 0$, ν is less than 180°)

8. Calculate the argument of latitude, θ , the angle between $\mathbf{n}$ and $\mathbf{r}$,

$$\cos \theta = \frac{\mathbf{n} \cdot \mathbf{r}}{nr} \quad (3.41)$$

(if $r_K > 0$, θ is less than 180°)

The orbital elements of the master orbit can be found from the position and velocity vectors, $\mathbf{r}_j$ and $\mathbf{v}_j$, of individual formation flyers in a similar way, using the three fundamental vectors of the cluster: the total angular momentum vector, $\mathbf{h}_T$, the total node vector, $\mathbf{n}_T$, and the total eccentricity vector, $\mathbf{e}_T$.

Algorithm OrbitMaster

Following the same basic steps as for the single satellite orbit element calculation, we calculate the formation master orbit elements as follows:

1. Calculate the total angular momentum vector, $\mathbf{h}_T$,

$$\mathbf{h}_T = \sum \mathbf{r}_j \times \mathbf{v}_j = \sum \mathbf{h}_j \quad (3.42)$$

2. Calculate the total node vector, $\mathbf{n}_T$,

$$\mathbf{n}_T = \sum \mathbf{K} \times \mathbf{h}_j = \sum \mathbf{n}_j \quad (3.43)$$

3. Calculate the total eccentricity vector, $\mathbf{e}_T$,

$$\mathbf{e}_T = \sum \frac{1}{\mu} \left[\left(v_j^2 - \frac{\mu}{r_j} \right) \mathbf{r}_j - (\mathbf{r}_j \cdot \mathbf{v}_j) \mathbf{v}_j \right] = \sum \mathbf{e}_j \quad (3.44)$$

4. Calculate the inclination of the master orbit, i_m ,

$$\cos i_m = \frac{h_{TK}}{h_T} \quad (3.45)$$

(i_m is less than 180°)

5. Calculate the RAAN of the master orbit, Ω_m ,

$$\cos \Omega_m = \frac{n_{TI}}{n_T} \quad (3.46)$$

(if $n_{Tj} > 0$, Ω_m is less than 180°)

6. Calculate the argument of periapsis of the master, ω_m (assuming e_T is non-zero),

$$\cos \omega_m = \frac{\mathbf{n}_T \cdot \mathbf{e}_{ave}}{n_T e_{ave}} \quad (3.47)$$

(if $e_{ave_K} > 0$, ω_m is less than 180°)

Where $\mathbf{e}_{ave}$ is the average eccentricity vector.

7. Calculate the true anomaly of the master, ν_m (again, assuming e_{ave} is non-zero),

$$\cos \nu_m = \frac{\mathbf{e}_{ave} \cdot \mathbf{r}_T}{e_{ave} r_T} \quad (3.48)$$

(if $\mathbf{r}_T \cdot \mathbf{v}_T > 0$, ν is less than 180°)

where $\mathbf{r}_T$ is the component of the average radius vector, $\mathbf{r}_{ave}$, perpendicular to the total angular momentum vector, $\mathbf{h}_T$:

$$\mathbf{r}_{ave} = \frac{1}{n} \sum_{j=1}^n \mathbf{r}_j \quad (3.49)$$

$$\mathbf{r}_T = \mathbf{r}_{ave} - (\mathbf{r}_{ave} \cdot \hat{\mathbf{h}}_T) \hat{\mathbf{h}}_T \quad (3.50)$$

where $\hat{\mathbf{h}}_T$ designates a unit vector in the $\mathbf{h}_T$ direction.

8. Calculate the argument of latitude of the master, θ_m , the angle between $\mathbf{n}_T$ and $\mathbf{r}_T$,

$$\cos \theta_m = \frac{\mathbf{n}_T \cdot \mathbf{r}_T}{n_T r_T} \quad (3.51)$$

(if $r_{TK} > 0$, θ_T is less than 180°)

Calculating the Desired Orbits

Given these definitions for the master orbit elements, using Equations (3.42–3.51), we can calculate the desired states of the formation flyers at any time by propagating the master orbit to the desired time, and adding orbital element offsets slightly modified from those in Tables 3.1–3.2. These new offsets, $\Delta\Omega$, and, $\Delta\theta$, are calculated by subtracting the initialized spacecraft orbital element values from the master orbital elements:

$$\Delta\Omega_j = \Omega_j - \Omega_m \quad (3.52)$$

$$\Delta\theta_j = \theta_j - \theta_m \quad (3.53)$$

We calculate the desired orbit of the formation flyers from these new offsets as in Tables 3.1–3.2, except this time we use the master orbit for the reference values of Ω and θ .

Example: Master Orbit Calculation

To calculate the master orbit of a cluster of spacecraft in the second Leonardo formation, start by choosing reference values of $\Omega = 0$, and $\theta = 0$. For these reference values, the orbital elements of the Leonardo spacecraft are given in Table 3.3, along with the corresponding master orbit elements.

Table 3.3: Second Leonardo formation.

s/c	a [km]	e	i [°]	Ω [°]	θ [°]
1	6778	0	0.4783	16.86	-9.99
2	6778	0	0.9771	-20.88	25.52
3	6778	0	4.4108	166.10	-160.40
4	6778	0	4.5866	17.42	-13.58
5	6778	0	0.6153	17.52	-14.66
6	6778	0	2.4481	-157.80	160.13
master	6778	0	0.2497	98.24	266.14

As shown in the table, the orbit that best defines the center of the formation has an inclination of about 0.25° , and right ascension and argument of latitude of about 98° and 266° , respectively. Notice that these values are nowhere near the average values, and that the master inclination is very nearly zero. This result is not surprising, as the orbital planes of the individual spacecraft are fairly evenly dispersed around the equator by varying values of Ω .

With the master orbit identified, we can calculate the orbital element offsets in Equations (3.52–3.53) by subtracting the initialized spacecraft orbital element values from the master orbital elements. The new offsets, given in Table 3.4, can then be used to calculate the desired state of any of the spacecraft in the formation at any time by propagating the master orbit, and offsetting the Ω_i , and θ_i values.

Table 3.4: Sample offset values for second Leonardo formation.

s/c	a [km]	e	i [°]	$\Delta\Omega$ [°]	$\Delta\theta$ [°]
1	6778	0	0.4783	-81.3764	83.8718
2	6778	0	0.9771	-119.1164	119.3748
3	6778	0	4.4108	67.8636	-66.5352
4	6778	0	4.5866	-80.8164	80.2848
5	6778	0	0.6153	-80.7164	79.2046
6	6778	0	2.4481	-256.0364	-106.0052

Notice that these offsets do not match those given in Table 3.2, and apply only to the formation generated using the reference values $\Omega = 0$, and $\theta = 0$.

Figure 3.3 shows a plot of the second Leonardo formation, plotted in the orbital frame of the formation master (δr_1 is in the velocity direction, δr_2 is in the negative orbit normal direction, and δr_3 is in the nadir direction). The ‘*’ in the plot is the formation master (which is stationary in this frame), and the ‘o’s are the positions of the formation flyers at a given time in the orbit.

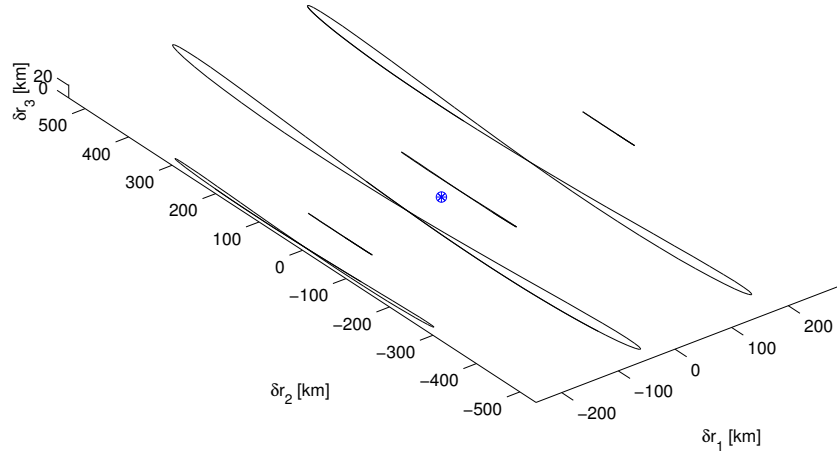


Figure 3.3: The second Leonardo formation, in the frame of the formation master orbit

The definition of a central formation master orbit is essential to formation flying operations. We calculate the target spacecraft orbits as a function of the formation master orbital elements, and predefined formation geometries. We also use the master orbit calculation algorithm to re-center the formation after periods of differential drift due to perturbation effects. The master orbit defined in this section is one of many possible choices for the formation master. In Section 5.3.2, we will see that other formation master orbit definitions can result in improved fuel

efficiency for the formation.

3.4 Summary

We have introduced spacecraft orbital dynamics and perturbations, and defined a formation master orbit which represents the geometric center of a formation. Using the methods described in this chapter, we can now define target orbits for formation flying spacecraft. In the following chapter, we develop nonlinear control laws, which use both cartesian and orbital element feedback to force maneuvering spacecraft to acquire predefined target orbits, like the ones defined in this chapter.

Chapter 4

Orbit Control

To control the orbit of formation flying spacecraft, we develop 3 nonlinear feedback control laws based on feedback of: a) inertial cartesian position and velocity errors; b) classical orbital element errors; c) equinoctial orbital element errors.

After defining the various control laws, we discuss control application issues including thrust magnitude and direction constraints, and feedback gain selection.

4.1 Lyapunov Stability

Whereas linear control techniques can be highly effective in the control of nonlinear systems, nonlinear control strategies can be beneficial in a variety of ways. For example, nonlinear controllers may: a) increase the region of state space in which we can effectively control the system, b) improve control robustness to parametric uncertainty, c) obtain truer optimality results, d) enable control of systems which are not linearly controllable, and e) allow us to preserve and exploit physical insight.

Lyapunov Control

The direct method, or second method, of Lyapunov is a valuable tool in the analysis of both linear and non-linear systems. The stability of the system

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) \tag{4.1}$$

where

$$\mathbf{z} = \mathbf{x} - \mathbf{x}^* \tag{4.2}$$

is analyzed using Lyapunov's stability approach by constructing an energy-like scalar Lyapunov function of the state, $V(\mathbf{z})$. For such a system, the equilibrium point, $\mathbf{x}^*$, is globally asymptotically stable if there is a $V(\mathbf{z})$, continuous in the 1st derivative, that satisfies the following

conditions:^{42–44}

$$\begin{aligned}
V(\mathbf{0}) &= 0 \\
V(\mathbf{z}) &> 0 \quad \forall \mathbf{z} \neq \mathbf{0} \\
\dot{V}(\mathbf{z}) &< 0 \quad \forall \mathbf{z} \neq \mathbf{0} \\
\lim_{\|\mathbf{z}\| \rightarrow \infty} V &= \infty
\end{aligned} \tag{4.3}$$

While there is no foolproof method for generating Lyapunov functions, quadratic and logarithmic forms are often effective starting points for a trial-and-error approach.

4.2 Cartesian Lyapunov Control

To implement a cartesian Lyapunov control, we choose a quadratic Lyapunov function, $V(\mathbf{x})$, that satisfies the Lyapunov criteria for guaranteed asymptotic stability, where $\mathbf{x}$ is the inertial, cartesian, state-error vector.

Taking the time derivative of V , and writing $\dot{\mathbf{x}}$ as a vector function, $\mathbf{f}$, of $\mathbf{x}$, and the control, $\mathbf{u}$, we get

$$\dot{V} = \frac{\partial V}{\partial \mathbf{x}} \dot{\mathbf{x}} = \frac{\partial V}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{4.4}$$

To drive the position and velocity error of a formation flying satellite to zero, we define the state vector $\mathbf{x}$ as

$$\mathbf{x} = \begin{bmatrix} \delta \mathbf{r} \\ \delta \mathbf{v} \end{bmatrix} \tag{4.5}$$

where $\delta(\cdot)$ describes the relative value of parameters of the controlled satellite with respect to the reference, or target satellite (in this case, the relative position and velocity):

$$\delta(\cdot) = (\cdot)_{\text{controlled}} - (\cdot)_{\text{reference}} \tag{4.6}$$

Note here that the *reference* refers to the desired state of the controlled spacecraft, which is calculated from the master orbit (Section 3.3.2).

We choose a positive definite Lyapunov function, V , as shown in Eq. (4.7), so that $\dot{V}$ can only be negative for decreasing state vector magnitudes:

$$V(\delta \mathbf{r}, \delta \mathbf{v}) = \frac{1}{2} k_1 \delta \mathbf{r}^T \delta \mathbf{r} + \frac{1}{2} k_2 \delta \mathbf{v}^T \delta \mathbf{v} \tag{4.7}$$

where k_1 and k_2 are some positive gains. The time derivative of this Lyapunov function is

$$\dot{V}(\delta \mathbf{r}, \delta \mathbf{v}) = [k_1 \delta \mathbf{r} + k_2 \delta \mathbf{v}]^T \delta \mathbf{f} \tag{4.8}$$

We now have an equation for the Lyapunov function in terms of the relative position and velocity, and the vector function $\delta \mathbf{f}$:

$$\delta \mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \delta \mathbf{f}_r \\ \delta \mathbf{f}_v \end{bmatrix} \tag{4.9}$$

The time rate of change of the relative position, $\delta \mathbf{r}$, is simply the relative velocity:

$$\delta \mathbf{f}_r = \delta \dot{\mathbf{r}} = \delta \mathbf{v} \quad (4.10)$$

Similarly, the time rate of change of the relative velocity, $\delta \mathbf{v}$, is the relative acceleration, plus the control acceleration. The control term shows up only in the $\delta \mathbf{f}_v$ equation:

$$\delta \mathbf{f}_v = \delta \ddot{\mathbf{r}} + \mathbf{u} = \ddot{\mathbf{r}} - \ddot{\mathbf{r}}_{\text{ref}} + \mathbf{u} = -\mu \left(\frac{\mathbf{r}}{r^3} - \frac{\mathbf{r}_{\text{ref}}}{r_{\text{ref}}^3} \right) + \mathbf{u} \quad (4.11)$$

Re-writing Eq. (4.8), we get

$$\dot{V}(\delta \mathbf{r}, \delta \mathbf{v}) = k_1 \delta \mathbf{r}^T \delta \mathbf{v} + k_2 \delta \mathbf{v}^T \delta \mathbf{f}_v \quad (4.12)$$

To control the system we must choose a control that cancels the position error from Eq. (4.10), cancels the relative gravitational force from Eq. (4.11), and satisfies the original criteria for guaranteed asymptotic stability given by Eq. (4.3):

A cartesian Lyapunov control acceleration that satisfies these requirements is:

$$\mathbf{u}_{\text{cart}} = \mu \left(\frac{\mathbf{r}}{r^3} - \frac{\mathbf{r}_{\text{ref}}}{r_{\text{ref}}^3} \right) - \frac{k_1}{k_2} \delta \mathbf{r} - k_3 \delta \mathbf{v} \quad (4.13)$$

Notice that substituting the Lyapunov control, $\mathbf{u}_{\text{cart}}$, into $\delta \mathbf{f}_v$ in Eq. (4.12) gives the negative semi-definite Lyapunov derivative:

$$\dot{V}(\delta \mathbf{r}, \delta \mathbf{v}) = -k \delta \mathbf{v}^T \delta \mathbf{v} \leq 0 \quad (4.14)$$

LaSalle's Invariance Principle shows that the largest invariant set where $\dot{V}$ is zero includes only the origin. Therefore the system is globally, asymptotically stable under the control given in Eq. (4.13).

4.3 Orbital Element Lyapunov Control

Schaub and Alfriend^{28,36} have shown that Lyapunov control based on orbital element feedback performs well in low-thrust, multi-orbit maneuvers. Unfortunately, the equations of motion required in the development of their Lyapunov control contain singularities for circular, and equatorial orbits. To resolve this issue, we develop a Lyapunov feedback control law using the nonsingular, equinoctial variables.

Ilgen³⁷ develops Lyapunov optimal feedback control techniques for low thrust orbit transfer between orbits of different semi-major axis, eccentricity, and inclination using both classical, and equinoctial element sets. While the author does not discuss formation-flying control directly, the paper provides an excellent starting point for the development of classical and equinoctial, Lyapunov-optimal, formation flying feedback control law.

4.3.1 Classical Orbital Element Control

In a straightforward expansion of Ilgen's work, we develop a technique to control the full, six element relative motion of formation flying spacecraft in both classical and equinoctial orbital element forms. For systems of the form

$$\dot{\mathbf{x}} = [\mathbf{g}(\mathbf{x})]\mathbf{u} \quad (4.15)$$

we can write the derivative as

$$\frac{d}{dt} [V(\mathbf{x} - \mathbf{x}^*)] = \dot{V}(\mathbf{x} - \mathbf{x}^*) = \frac{\partial V}{\partial \mathbf{x}} \dot{\mathbf{x}} = \frac{\partial V}{\partial \mathbf{x}} [\mathbf{g}(\mathbf{x})]\mathbf{u} = V_{\mathbf{x}}[\mathbf{g}(\mathbf{x})]\mathbf{u} \quad (4.16)$$

Setting the control, $\mathbf{u}$, to be

$$\mathbf{u} = -\mathbf{K}[\mathbf{g}(\mathbf{x})]^T V_{\mathbf{x}}^T \quad (4.17)$$

where $\mathbf{K}$ is a positive definite gain matrix, the derivative becomes

$$\dot{V}(\mathbf{x}) = -V_{\mathbf{x}}[\mathbf{g}(\mathbf{x})][\mathbf{K}][\mathbf{g}(\mathbf{x})]^T V_{\mathbf{x}}^T = -\mathbf{K} \left| [\mathbf{g}(\mathbf{x})]^T V_{\mathbf{x}}^T \right|^2 \quad (4.18)$$

This derivative is negative semi-definite, and therefore, by Lyapunov's direct method, the system described in Eq. (4.15) is globally, asymptotically stable when the control given in Eq. (4.17) is applied, provided we choose an appropriate positive-definite Lyapunov function, and that we can show, using LaSalle's Invariance Principal, that the maximum invariant set for which $\dot{V} = 0$ includes only the origin.

Gauss's form of Lagrange's Planetary Equations (LPE) takes the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) = [\mathbf{g}(\mathbf{x})]\mathbf{u} + \mathbf{h}(\mathbf{x}) \quad (4.19)$$

where $\mathbf{x}$ is the $[6 \times 1]$ classical element set, $[a, e, i, \Omega, \omega, M]$, $\mathbf{u}$ is a $[3 \times 1]$ control acceleration vector in Hill's frame, and $\mathbf{f}(\mathbf{x}, \mathbf{u})$ is

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \frac{2a^2 e S \nu}{h} & \frac{2a^2 p}{hr} & 0 \\ \frac{p S \nu}{h} & \frac{(p+r) C \nu + r e}{h} & 0 \\ 0 & 0 & \frac{r C(\omega + \nu)}{h} \\ 0 & 0 & \frac{r S(\omega + \nu)}{h} \\ -\frac{p C \nu}{h e} & \frac{(p+r) S \nu}{h e} & -\frac{r S(\omega + \nu) C i}{h S i} \\ \frac{b(p C \nu - 2 r e)}{a h e} & -\frac{b(p+r) S \nu}{a h e} & 0 \end{bmatrix} \begin{bmatrix} u_r \\ u_\theta \\ u_h \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ n \end{bmatrix} \quad (4.20)$$

where the sine and cosine functions are abbreviated as $S(\cdot)$ and $C(\cdot)$.

We can modify these equations of motion to match the form of Eq. (4.15) by replacing the mean anomaly, M , with the mean anomaly at epoch, $M_0 = M - nt$, so that the state vector is $[a, e, i, \Omega, \omega, M_0]$. In this way, we remove the mean motion term from the equations of motion. We can choose the epoch time as the time of periapsis passage of the target orbit, so that the time since epoch, t , is given by

$$t = \frac{M_{\text{ref}}}{n_{\text{ref}}} \quad (4.21)$$

and the desired mean anomaly at epoch, $M_{0_{\text{ref}}}$ is always zero.

We complete the control derivation by defining a quadratic, positive-definite Lyapunov function:

$$V = \frac{1}{2} [K_1 \delta a^2 + K_2 \delta e^2 + K_3 \delta i^2 + K_4 \delta \Omega^2 + K_5 \delta \omega^2 + K_6 \delta M_0^2] \quad (4.22)$$

Note that in Eq. (4.22), we assume a diagonal gain matrix, $\mathbf{K}$, and incorporate it into the Lyapunov function. Taking the derivative of the Lyapunov function with respect to the state, $\mathbf{x}$, we get

$$V_x = \begin{bmatrix} K_1 \delta a & K_2 \delta e & K_3 \delta i & K_4 \delta \Omega & K_5 \delta \omega & K_6 \delta M_0 \end{bmatrix} \quad (4.23)$$

Substituting into Eq. (4.17) yields the following classical orbital element Lyapunov control law:

$$\mathbf{u}_{\text{cl}} = -[\mathbf{g}(\mathbf{x})]^T V_x^T = - \begin{bmatrix} \frac{2a^2 e S \nu}{h} & \frac{2a^2 p}{hr} & 0 \\ \frac{p S \nu}{h} & \frac{(p+r) C \nu + r e}{h} & 0 \\ 0 & 0 & \frac{r C(\omega+\nu)}{h} \\ 0 & 0 & \frac{r S(\omega+\nu)}{h} \\ -\frac{p C \nu}{h} & \frac{(p+r) S \nu}{h} & -\frac{r S(\omega+\nu) C i}{h} \\ \frac{b(p C \nu - 2 r e)}{a h e} & -\frac{b(p+r) S \nu}{a h e} & 0 \end{bmatrix}^T \begin{bmatrix} K_1 \delta a \\ K_2 \delta e \\ K_3 \delta i \\ K_4 \delta \Omega \\ K_5 \delta \omega \\ K_6 \delta M_0 \end{bmatrix} \quad (4.24)$$

The time derivative of the Lyapunov function under this choice of control is negative semi-definite. To prove asymptotic stability, we apply LaSalle's invariance theorem. The time derivative of the Lyapunov function is always zero when $\mathbf{x} = \mathbf{0}$, and could be zero when the trigonometric functions of $\theta = \omega + \nu$ are zero, which occurs when $\theta = k\pi/2$, where k is an integer. The set where $\theta = k\pi/2$ is not an invariant set, because ν is time-varying and therefore trajectories that start in the set do not remain in the set. Therefore, the largest invariant set where $\dot{V} = 0$ is the set $\mathbf{x} = \mathbf{0}$. So by LaSalle's invariance principle, the system is asymptotically stable under the choice of control given in Eq. (4.24). Furthermore, the system is globally asymptotically stable because V goes to infinity as the state error goes to infinity.

4.3.2 Equinoctial Orbital Element Control

The classical orbital element formation control law works well for low-thrust, multi-orbit maneuvering of spacecraft in non-circular, non-equatorial orbits. To find such a control for spacecraft in nearly circular or equatorial orbits, we first introduce the equinoctial element set, whose equations of motion contain no singularities.

The non-singular equinoctial elements are defined in terms of the classical orbital elements in Section 3, and repeated here for convenience:

$$a = a \quad (4.25)$$

$$P_1 = e \sin \varpi \quad (4.26)$$

$$P_2 = e \cos \varpi \quad (4.27)$$

$$Q_1 = \tan \frac{i}{2} \sin \Omega \quad (4.28)$$

$$Q_2 = \tan \frac{i}{2} \cos \Omega \quad (4.29)$$

$$l = \varpi + M \quad (4.30)$$

where ϖ , the longitude of pericenter, is defined as

$$\varpi = \Omega + \omega \quad (4.31)$$

In addition to the mean longitude, l , we also need the true longitude, L , defined as

$$L = \varpi + \nu \quad (4.32)$$

We can once again manipulate the mean longitude, l , to get the mean longitude at epoch, l_0 , by replacing M with $M_0 + nt$:

$$l_0 = \varpi + M_0 = \varpi + M - nt = l - nt \quad (4.33)$$

Here we define epoch as the time where the reference mean longitude is zero, and thus the time since epoch is

$$t = \frac{l_{\text{ref}}}{n_{\text{ref}}} \quad (4.34)$$

Thus we remove the mean motion from the equations of motion of the longitude term. From Battin,¹⁴ the equinoctial variational equations in a Gauss-like form are:

$$\dot{\mathbf{x}} = \begin{bmatrix} \frac{2a^2(P_2SL - P_1 \cos L)}{h} & \frac{2a^2p}{hr} & 0 \\ -\frac{pCL}{h} & \frac{r[P_1 + (1 + \frac{p}{r})SL]}{h} & -\frac{P_2(Q_1CL - Q_2SL)r}{h} \\ \frac{p \sin L}{h} & \frac{r[P_2 + (1 + \frac{p}{r})CL]}{h} & \frac{P_1(Q_1CL - Q_2SL)r}{h} \\ 0 & 0 & \frac{r(1 + Q_1^2 + Q_2^2)SL}{2h} \\ 0 & 0 & \frac{r(1 + Q_1^2 + Q_2^2)CL}{2h} \\ -\frac{pa[(P_1SL + P_2CL) + \frac{2b}{a}]}{h(a+b)} & -\frac{ra(1 + \frac{p}{r})(P_1CL - P_2SL)}{h(a+b)} & -\frac{r(Q_1CL - Q_2SL)}{h} \end{bmatrix} \begin{bmatrix} \mathbf{u}_r \\ \mathbf{u}_\theta \\ \mathbf{u}_h \end{bmatrix} \quad (4.35)$$

where the state $\mathbf{x}$, is the $[6 \times 1]$ equinoctial element set, $[a, P_1, P_2, Q_1, Q_2, l_0]$, and where

$$b = a\sqrt{1 - P_1^2 - P_2^2} \quad (4.36)$$

$$h = nab \quad (4.37)$$

$$\frac{p}{r} = 1 + P_1 \sin L + P_2 \cos L \quad (4.38)$$

$$\frac{r}{h} = \frac{h}{\mu(1 + P_1 \sin L + P_2 \cos L)} \quad (4.39)$$

We choose a candidate Lyapunov function as:

$$V = \frac{1}{2} [K_1 \delta a^2 + K_2 \delta P_1^2 + K_3 \delta P_2^2 + K_4 \delta Q_1^2 + K_5 \delta Q_2^2 + K_6 \delta l_0^2] \quad (4.40)$$

whose partial derivative with respect to the state, $\mathbf{x}$, is given by

$$V_x = \begin{bmatrix} K_1 \delta a & K_2 \delta P_1 & K_3 \delta P_2 & K_4 \delta Q_1 & K_5 \delta Q_2 & K_6 \delta l_0 \end{bmatrix} \quad (4.41)$$

Substituting into Eq. (4.17) yields the following equinoctial orbital element Lyapunov control law:

$$\mathbf{u}_{eq} = - \begin{bmatrix} \frac{2a^2(P_2SL - P_1CL)}{h} & \frac{2a^2p}{hr} & 0 \\ -\frac{pCL}{h} & \frac{r[P_1 + (1 + \frac{p}{r})SL]}{h} & -\frac{P_2(Q_1CL - Q_2SL)r}{h} \\ \frac{pSL}{h} & \frac{r[P_2 + (1 + \frac{p}{r})CL]}{h} & \frac{P_1(Q_1CL - Q_2SL)r}{h} \\ 0 & 0 & \frac{r(1 + Q_1^2 + Q_2^2)SL}{2h} \\ 0 & 0 & \frac{r(1 + Q_1^2 + Q_2^2)CL}{2h} \\ -\frac{pa[(P_1SL + P_2CL) + \frac{2b}{a}]}{h(a+b)} & -\frac{ra(1 + \frac{p}{r})(P_1CL - P_2SL)}{h(a+b)} & -\frac{r(Q_1CL - Q_2SL)}{h} \end{bmatrix}^T \begin{bmatrix} K_1 \delta a \\ K_2 \delta P_1 \\ K_3 \delta P_2 \\ K_4 \delta Q_1 \\ K_5 \delta Q_2 \\ K_6 \delta l_0 \end{bmatrix} \quad (4.42)$$

By Lyapunov's direct method and LaSalle's invariance principle, these nonlinear control laws guarantee global asymptotic convergence of the orbital elements to the target values. While our decision to replace the mean anomaly and mean longitude (M and l) with the corresponding terms at epoch (M_0 and l_0) allows us to express the system dynamics in a convenient form, it also eliminates a useful natural component of the nonlinear dynamics: the differential mean motion, δn . Next, we develop an anomaly control law that forces mean motion error, in order to passively correct anomaly angle errors.

4.3.3 Mean Motion Control

Observe that an effective method for correcting mean anomaly error is to force a semi-major axis error, changing the mean motion such that the in-plane angle is corrected. Consider an uncontrolled spacecraft in 2-body motion, where the orbital element dynamics simplify to

$$\frac{dM}{dt} = n \quad (4.43)$$

and thus the relative dynamics are

$$\frac{d}{dt}(\delta M) = \frac{dM}{dt} - \frac{dM^*}{dt} = n - n^* = \delta n \quad (4.44)$$

where

$$\delta n = n - n^* = \sqrt{\frac{\mu}{a^3}} - \sqrt{\frac{\mu}{a^{*3}}} \quad (4.45)$$

To drive the mean anomaly error, δM , to zero, we want

$$\delta \dot{M} = -K \delta M \quad (4.46)$$

where K is some positive gain. Combining Eqs. (4.45) and (4.46), and using canonical units,

$$-K\delta M = \sqrt{\frac{1}{a^3}} - \sqrt{\frac{1}{a^{*3}}} \quad (4.47)$$

Solving for a , we define a new target semi-major axis, a^{**} , which forces the mean anomaly error to zero:

$$a^{**} = \left(-K\delta M + \frac{1}{a^{*3/2}} \right)^{-2/3} \quad (4.48)$$

Notice that as δM goes to zero, the new target semi-major axis goes to the original target value, a^* .

We will see in Section 5.3.2 that this mean motion control should only be applied to control a spacecraft which has already established the correct orbital plane, using the feedback control given in Eq. (4.24).

4.4 Control Application Issues

To apply the control laws developed in this chapter, we must first address some control application issues. Real spacecraft have numerous constraints limiting the application of thrust. It is not always possible for thrusters to deliver the exact magnitude and direction of thrust requested by the control law. In this section, we discuss the impact of constrained thrust magnitude and direction on control law design, as well as a method for choosing appropriate gains for the nonlinear control.

4.4.1 Constrained Thrust Magnitude

Since most spacecraft have fixed magnitude thrusters, we must develop some way of discretizing the control acceleration requested by the control laws. For example, if the orbital element feedback control law requests acceleration of magnitude A in the $\hat{a}$ direction, we must determine whether or not to thrust, and if so, in what direction.

This decision is quite simple for the elemental control laws, as we have full scaling freedom in our choice of gains. We can simply calculate a desired thrust direction and magnitude from the control law, and thrust if and only if the desired thrust magnitude is greater than the available thrust magnitude. Mathematically, we would thrust in the $\hat{a}$ direction if and only if

$$A > \frac{\overline{T}}{m} \quad (4.49)$$

where $\overline{T}$ is the available thrust magnitude, and m is the spacecraft mass.

The decision is slightly more difficult for the cartesian control law, as we do not have full scaling freedom.

Cartesian Control Scaling

The cartesian control given in Eq. (4.13) has three components, easily distinguishable as a nonlinear dynamics component, and proportional and derivative components. The non-linear dynamics component, $\mathbf{u}_{\text{dyn}}$, given by

$$\mathbf{u}_{\text{dyn}} = \mu \left(\frac{\mathbf{r}}{r^3} - \frac{\mathbf{r}_{\text{ref}}}{r_{\text{ref}}^3} \right) \quad (4.50)$$

negates the relative natural dynamics, allowing the remaining components to behave as a simple proportional-derivative (PD) control. Note that in the control definition, only the PD components have associated gains. This property of the control law means that we can only guarantee asymptotic stability with this controller if the entire $\mathbf{u}_{\text{dyn}}$ component is applied. This fact is critical when we apply this control law in fixed thrust magnitude problems. When scaling the control acceleration given by Eq. (4.13), we only have the freedom to scale the proportional and derivative components by choosing appropriate gains. For this reason we redefine the cartesian Lyapunov control as:

$$\mathbf{u}_{\text{cart}} = \mathbf{u}_{\text{dyn}} + S \hat{\mathbf{u}}_{\text{PD}} = \mu \left(\frac{\mathbf{r}}{r^3} - \frac{\mathbf{r}_{\text{ref}}}{r_{\text{ref}}^3} \right) + S \left(\frac{-k_P \delta \mathbf{r} - k_D \delta \mathbf{v}}{\| -k_P \delta \mathbf{r} - k_D \delta \mathbf{v} \|} \right) \quad (4.51)$$

where PD gains, k_P and k_D , and scaling factor S are chosen by the control designer. We introduce the scaling factor as a third gain so that the control can be scaled as needed without changing the ratio of the PD gains, or scaling the $\mathbf{u}_{\text{dyn}}$ component. Figure 4.1 illustrates one possible scaling method for a sample formation maneuver in which the magnitude of $\mathbf{u}_{\text{dyn}}$ is less than the available thrust acceleration, T/m . The dotted circle in the figure represents the available thrust acceleration. Sketch A in the figure shows an improperly scaled control vector, $\mathbf{u}_{\text{cart}}$, in which the entire control is scaled. Sketch B shows an appropriately scaled thrust vector in which the scaling factor, S , is computed using the law of cosines:

$$\left(\frac{T}{m} \right)^2 = S^2 + u_{\text{cart}}^2 - 2S u_{\text{cart}} \cos \theta \quad (4.52)$$

where

$$\theta = \pi - \cos^{-1} (\hat{\mathbf{u}}_{\text{dyn}} \cdot \hat{\mathbf{u}}_{\text{PD}}) \quad (4.53)$$

and thus

$$S = \frac{1}{2} \left[2u_{\text{cart}} \cos \theta \pm \sqrt{(u_{\text{cart}})^2 - 4 \left(u_{\text{dyn}}^2 - \left(\frac{T}{m} \right)^2 \right)} \right] \quad (4.54)$$

When applied to formation maneuvers in which $\mathbf{u}_{\text{dyn}}$ is negligible, the control law given by Eq. (4.51) behaves identically to a PD control. As the size of formation maneuvers increase, the $\mathbf{u}_{\text{dyn}}$ component of the control becomes critical to the success of the maneuver. Problems arise in the application of this control when the $\mathbf{u}_{\text{dyn}}$ component of the requested control becomes

larger than the available thruster acceleration. If the thrusters are incapable of negating the differential acceleration because they cannot provide sufficient thrust, or the maneuver is too large, the cartesian Lyapunov control will not successfully complete the maneuver, and another control law must be employed.

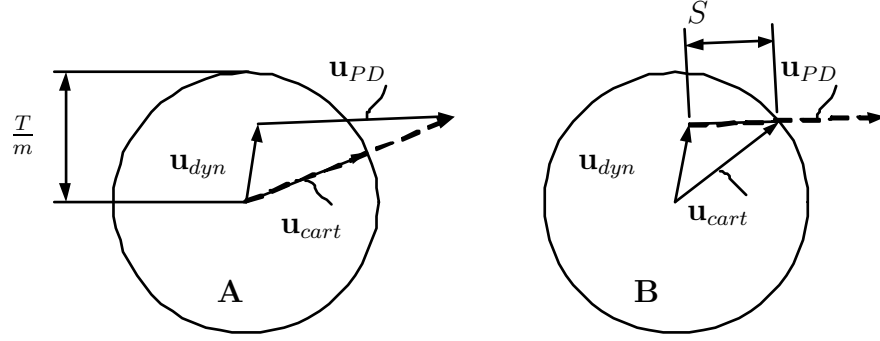


Figure 4.1: Cartesian Lyapunov control scaling techniques: A) incorrectly scaled control vector; B) properly scaled control vector

4.4.2 Constrained Thrust Direction

In the case of the HokieSat mission, we must not only scale the thrust magnitude, but constrain the thrust direction. HokieSat's orbit control consists of four pulsed plasma thrusters (PPTs) aligned in the local horizontal frame, with no thrust available in the radial direction.

This constraint prevents the use of the cartesian control law, which in its current form cannot be easily modified to compensate for a lack of radial thrust. The classical and equinoctial Lyapunov control laws, however can both be modified to exclude radial thrusting by setting the terms in the first column of the $\mathbf{g}$ matrix in the control definition to zero, thus requesting only transverse and orbit normal thrust.

4.4.3 Gain Selection

Intuitively, nonlinear control of formation flying spacecraft should provide improvements over more traditional, linear feedback control laws such as LQR and H_∞ . One major obstacle in the use of nonlinear controls such as the Lyapunov-based controls developed in this paper is the lack of a well defined method for choosing gains. Whereas numerous tools exist for gain selection for linear feedback control methodologies, nonlinear gain selection techniques are often limited to trial-and-error type approaches. In the full state feedback problem, the orbital element feedback control laws require us to select a $[6 \times 6]$ gain matrix. Even if we choose to use only the diagonal terms, we are unlikely to find 6 gains that perform well in our control law using a trial-and-error approach.

Limited State Feedback

To help us develop a more effective gain selection process for the full-state feedback problem, we explore the use of our nonlinear control laws in more limited control problems. The orbital element Lyapunov control law can be used to control specific elements only, by setting the gains corresponding to the uncontrolled elements to zero. For example, the equinoctial Lyapunov control law can be used to perform a circle-to-circle orbit raising maneuver using feedback of the semi-major axis, a , and equinoctial eccentricity terms, P_1 and P_2 as given by Equation (4.55).

$$\mathbf{u} = - \begin{bmatrix} \frac{2a^2(P_2SL - P_1CL)}{h} & \frac{2a^2p}{hr} & 0 \\ -\frac{pCL}{h} & \frac{r[P_1 + (1 + \frac{p}{r})SL]}{h} & -\frac{P_2(Q_1CL - Q_2SL)r}{h} \\ \frac{pSL}{h} & \frac{r[P_2 + (1 + \frac{p}{r})CL]}{h} & \frac{P_1(Q_1CL - Q_2SL)r}{h} \end{bmatrix}^T \begin{bmatrix} K_1 \frac{\delta a}{R_e^2} \\ K_2 \delta P_1 \\ K_2 \delta P_2 \end{bmatrix} \quad (4.55)$$

The selection of gains for this problem is quite simple, as there are only 2 gains to choose (we use identical feedback gains for P_1 and P_2 because both represent the eccentricity, and therefore should be weighted identically). For a sample orbit raising maneuver from 7,000 km to 10,000 km, and using 1 N thrusters, fired continuously, trial-and-error tuning yields gains $K_1 = 0.7$ and $K_2 = 1.174$. Figure 4.2 shows a three-dimensional plot of this maneuver.

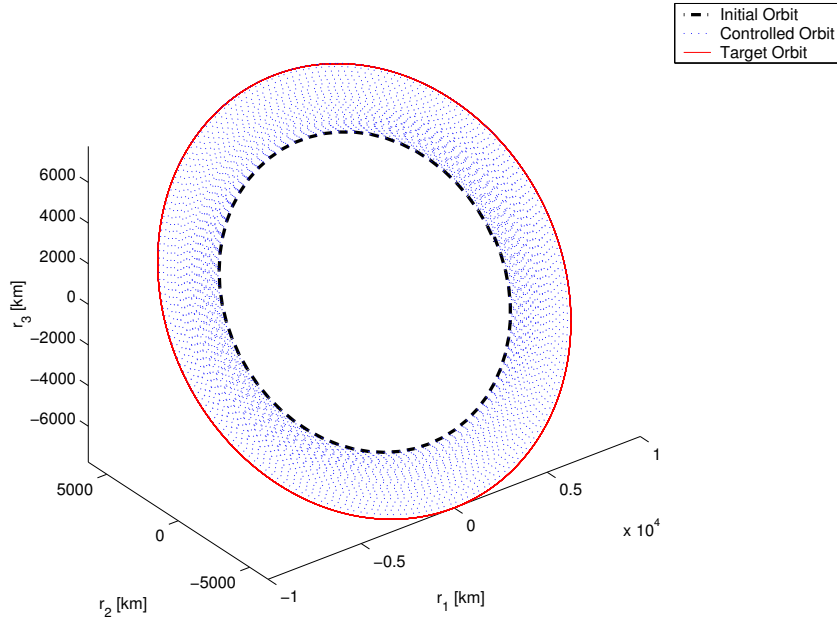


Figure 4.2: Circle-to-circle orbit raising maneuver

We can expand this problem to include inclination feedback by applying a variation of the equinoctial element control law, given by Equation (4.56).

$$\mathbf{u} = - \begin{bmatrix} \frac{2a^2(P_2SL - P_1CL)}{h} & \frac{2a^2p}{hr} & 0 \\ -\frac{pCL}{h} & \frac{r[P_1 + (1 + \frac{p}{r})SL]}{h} & -\frac{P_2(Q_1CL - Q_2SL)r}{h} \\ \frac{pSL}{h} & \frac{r[P_2 + (1 + \frac{p}{r})CL]}{h} & \frac{P_1(Q_1CL - Q_2SL)r}{h} \\ 0 & 0 & \frac{r(1 + Q_1^2 + Q_2^2)SL}{2h} \\ 0 & 0 & \frac{r(1 + Q_1^2 + Q_2^2)CL}{2h} \end{bmatrix}^T \begin{bmatrix} K_1 \frac{\delta a}{R_e^2} \\ K_2 \delta P_1 \\ K_2 \delta P_2 \\ K_3 \frac{(\sqrt{Q_1^2 + Q_2^2} - \tan \frac{i^*}{2})Q_1}{\sqrt{Q_1^2 + Q_2^2}} \\ K_3 \frac{(\sqrt{Q_1^2 + Q_2^2} - \tan \frac{i^*}{2})Q_2}{\sqrt{Q_1^2 + Q_2^2}} \end{bmatrix} \quad (4.56)$$

Notice that instead of feeding back the equinoctial inclination-like elements directly, we take advantage of the identity

$$\sqrt{Q_1^2 + Q_2^2} = \tan \frac{i^*}{2} \quad (4.57)$$

to control only the inclination error, and not the right ascension error associated with these elements (see Equations (4.28–4.29)).

Figure 4.3 shows an example of this type of maneuver, in which the same circle-to-circle orbit raising maneuver is performed, this time with an inclination shift from 28° to 60° . Plot A of the figure shows the maneuver performed with gains $K_1 = 1$, $K_2 = 1$, and $K_3 = 50$. Plot B shows the maneuver performed with gains $K_1 = 1$, $K_2 = 1$, and $K_3 = 5$. The different values of K_3 have a predictable effect on the maneuver; because we have weighted the inclination more heavily in plot A, the trend in the maneuver is to correct the out-of-plane error, and then complete the orbit raising. Notice that in plot B, where the semi-major axis and inclination are more equally weighted, the control forces orbit raising to occur first, and then completes the inclination maneuver. The maneuver illustrated in plot A requires about 13% more fuel than maneuver B. This fuel savings is easily explained by the fact that the ΔV required for simple plane change maneuvers is directly proportional to the initial speed. Maneuver B saves fuel by performing the majority of the plane change maneuver after it has already increased the semi-major axis (and thus decreased the speed).

Full State Feedback

To choose appropriate gains for the full orbital element feedback problem, we strive to understand the response of the individual elements to thrust impulses. The effect of radial, transverse, and orbit-normal thrust impulses on the osculating orbital elements varies throughout the orbit as a function of the true anomaly, as given by Gauss's form of Lagrange's planetary equations. For example, the change in semi-major axis resulting from impulsive thrust acceleration, $(T/m)\Delta t$, in the radial and transverse directions, respectively is

$$\Delta a_r = \frac{2a^2 e \sin \nu}{h} \frac{T}{m} \Delta t \quad (4.58)$$

$$\Delta a_\theta = \frac{2a^2 (1 + e \cos \nu)}{h} \frac{T}{m} \Delta t \quad (4.59)$$

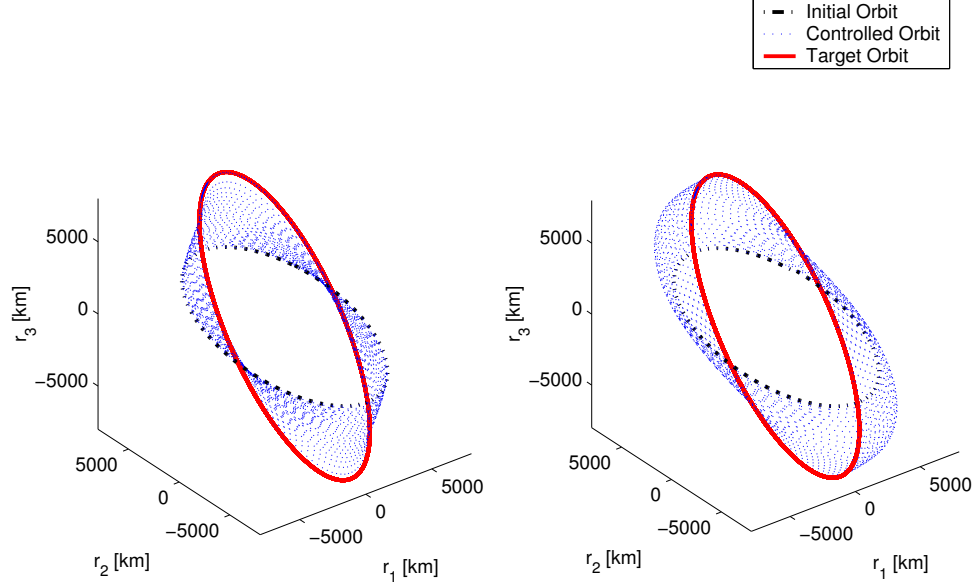


Figure 4.3: Combination orbit raising and inclination shift maneuver. A) $K_1 = 1$, $K_2 = 1$, and $K_3 = 50$ B) $K_1 = 1$, $K_2 = 1$, and $K_3 = 5$

where T , m , and Δt are the thrust force, spacecraft mass, and thrust duration. The maximum change in the semi-major axis results from an impulse in the transverse direction at periape (where ν is 0):

$$\Delta a_{max} = \frac{2a^2(1+e)}{h} \frac{T}{m} \Delta t \quad (4.60)$$

We can use this knowledge as a guide for selecting the gain, K_a , associated with the semi-major axis error feedback. From Eq. (4.24), the transverse Lyapunov control acceleration resulting from classical semi-major axis error, δa , is

$$\mathbf{u}_{a_\theta}(\delta a) = - \left(\frac{2a^2(1+e \cos \nu)}{h} \right) K_a \delta a \quad (4.61)$$

We would like to choose K_a such that the maximum value of $\mathbf{u}_{a_\theta}(\Delta a)$ is equal to the available thrust acceleration magnitude. In other words, we would like to choose the gain such that the control on-off logic asks for transverse thrust, only when the semi-major axis error is at least Δa_{max} . Furthermore, we would like for the control to ask for thrust in the presence of this error only once in the orbit, at the point where the control required to correct the error is minimized. Mathematically, we would like to choose the minimum value of K_a such that the magnitude of the control given in Eq. (4.61) is equal to the available control acceleration, T/m :

$$|\mathbf{u}_{a_\theta}(\Delta a_{max})| = \left| - \left(\frac{2a^2(1+e \cos \nu)}{h} \right) K_a \frac{2a^2(1+e)}{h} \frac{T}{m} \Delta t \right| = T/m \quad (4.62)$$

This minimum value of K_a occurs at periapse, and is given by the equation

$$K_a = \frac{h^2}{4a^4(1+e)^2} \frac{1}{\Delta t} \quad (4.63)$$

With this choice, our thruster on-off logic will request thrust when the semi-major axis error is greater than Δa_{max} , and will do nothing when the error is less than Δa_{max} . Choosing all of the gains in this way allows us to properly weight the elemental errors, and also prevents chatter in the system, as the control law will not attempt to control elemental errors more precisely than is possible with the available impulse.

With the exception of the K_ω , and K_M , the remaining control gains are easy to calculate using the same method used to find K_a , and are given by the following equations:

$$K_e = \frac{h^2}{4p^2} \frac{1}{\Delta t} \quad (4.64)$$

$$K_i = \left[\frac{h + eh \cos(\omega + \arcsin(e \sin \omega))}{p(-1 + e^2 \sin^2 \omega)} \right]^2 \frac{1}{\Delta t} \quad (4.65)$$

$$K_\Omega = \left[\frac{h \sin i(-1 + e \sin(\omega + \arcsin(e \cos \omega)))}{p(1 - e^2 \cos^2 \omega)} \right]^2 \frac{1}{\Delta t} \quad (4.66)$$

The maximum value of $\Delta\omega$ occurs when transverse thrust is applied near $\nu = \pi/2$. Therefore, the appropriate selection of K_ω comes from the transverse ω control equation:

$$u_{\omega_\theta}(\delta\omega) = -\frac{p \sin \nu}{he} \left(1 + \frac{1}{1 + e \cos \nu} \right) K_\omega \delta\omega \quad (4.67)$$

$$K_\omega(\nu) = \min \left[\frac{e^2 h^2 \csc^2 \nu}{p^2 \left(1 + \frac{1}{1 + e \cos \nu} \right)^2} \frac{1}{\Delta t} \right] \quad (4.68)$$

Equation (4.68) is minimized on ν when the function

$$f(\nu) = \frac{\csc^2 \nu}{\left(1 + \frac{1}{1 + e \cos \nu} \right)^2} \quad (4.69)$$

is minimized, or the first derivative of $f(\nu)$ with respect to ν is zero, and its second derivative is positive.

$$\frac{df}{d\nu} = -\frac{2(1 + e \cos \nu) \csc \nu (e + e(3 + e \cos \nu) \cot^2 \nu + 2 \cot \nu \csc \nu)}{(2 + e \cos \nu)^3} = 0 \quad (4.70)$$

To find the roots of the first derivative, we expand the numerator of $f(\nu)$ in a first order Taylor series about $e = 0$, and solve for the approximate root:

$$g(\nu, e) = 2(1 + e \cos \nu) \csc \nu (e + e(3 + e \cos \nu) \cot^2 \nu + 2 \cot \nu \csc \nu) \quad (4.71)$$

$$g(\nu, 0) = 4 \cot \nu \csc^2 \nu \quad (4.72)$$

Equation (4.72) is zero when $\nu = \pi/2$. The first order approximation of the root is thus

$$\nu \simeq \pi/2 + \nu_1 e \quad (4.73)$$

where ν_1 is the root of the following equation

$$\left. \frac{dg(\pi/2 + \nu_1 e, e)}{de} \right|_{e=0} = 4\nu_1 - 2 = 0 \quad (4.74)$$

The approximate root of $df/d\nu$ is

$$\nu \simeq \frac{\pi}{2} + \frac{e}{2} \quad (4.75)$$

Substituting into Eq. (4.68), we get the approximate minimum value of K_ω :

$$\min(K_\omega) \simeq \frac{e^2 h^2 \sec^2 \frac{e}{2}}{p^2 \left(1 + \frac{1}{1-e \sin \frac{e}{2}}\right)^2} \frac{1}{\Delta t} \quad (4.76)$$

Since the root is only valid to $\mathcal{O}(e)$, we expand K_ω in a Taylor series expansion about $e = 0$, and arrive at the final selection of the gain K_ω :

$$K_\omega = \frac{e^2 h^2}{4p^2} \left(1 - \frac{e^2}{4}\right) \frac{1}{\Delta t} \quad (4.77)$$

The final gain, K_M , found using the same Taylor series approximations, also occurs at $\nu \simeq \frac{\pi}{2} + \frac{e}{2}$, and is given by the following equation:

$$K_M = \frac{a^2 e^2 h^2}{4b^2 p^2} \left(1 - \frac{e^2}{4}\right) \Delta t \quad (4.78)$$

We have defined a positive definite, diagonal gain matrix for use in classical orbital element Lyapunov control that: 1) guarantees asymptotic convergence to an error envelope, 2) appropriately weights the elemental errors, and 3) eliminates the problem of chattering by properly defining the size of the error envelope. These gains should be calculated as a function of the target orbit at the beginning of a maneuver, and held constant throughout the maneuver.

4.4.4 Circular Orbits

While the equinoctial control law avoids the singularities associated with circular and equatorial orbits, the lack of physical meaning makes the equinoctial elements difficult to deal with. In the case of circular orbits, the argument of periaapse and the mean anomaly are undefined, and singularities appear in the classical element equations of motion of those variables. We can modify the classical orbital element Lyapunov control to deal with these issues by: 1) setting the gains K_ω and K_M to zero so that the errors associated with those gains do not contribute to the total requested thrust; 2) using the argument of latitude error, $\delta\theta$, in place of the δM in the mean motion control. These modifications are acceptable as long as the eccentricity error remains small, so that $\delta\theta$ is a good approximation of the in-plane-angle error.

4.5 Summary

We have developed a number of nonlinear feedback control laws to maneuver spacecraft orbits. Because the classical element control law is functional in the presence of thrust magnitude and direction constraints, and because we have defined a method for choosing gains for this control, it is the most likely candidate for application in real space missions. The use of mean motion control in conjunction with classical element feedback results in periods of drifting, in which in-plane angular errors are corrected without thrust application. Fuel consumption is a major concern for formation flying spacecraft (and any spacecraft). In the next chapter, we present results of spacecraft maneuver simulations in which classical element feedback control is applied.

Chapter 5

Results

To test the application of spacecraft control laws, we have developed a Matlab modeling environment. Using this modeling environment we test the control laws defined in the previous section in a number of generic simulations, HokieSat formation maneuvering simulations, a Leonardo formation establishment simulation, and some sample Leonardo formation keeping simulations.

5.1 Simulation

The Matlab modeling environment simulates the motion of two or more formation flying spacecraft in the presence of a Keplerian gravitational field using the f and g expressions,¹² also known as the Lagrangian Coefficients.¹⁴ We simulate thrust by applying sequential impulsive ΔV s at every time step for which the requested thrust magnitude is greater than the available thrust magnitude. This method of thrust discretization is ideal for use with pulsed plasma thrusters, which apply thrust in short, almost impulsive bursts. Error increases with time step size for this method, but is well within tolerance for this study.

The Matlab modeling environment flows as follows:

1. Initialize spacecraft orbits (Section 3.3.2)
2. Calculate the master orbit and define relative desired orbit parameters (Section 3.3.2)
3. Calculate the feedback control gains (Section 4.4.3)
4. Calculate desired spacecraft orbits from master orbit (Section 3.3.2)
5. Calculate desired thrust direction and magnitude (Section 4.3)
6. Apply appropriately scaled, impulsive ΔV in the requested thrust direction, $\hat{\alpha}$ (Section 4.4.1):

$$\mathbf{v}_{t+1} = \mathbf{v}_t + \hat{\alpha} \frac{T}{m} \Delta t \quad (5.1)$$

For example, for a 150 kg spacecraft, with 1 N thrusters, and a step size of 10 seconds, we apply impulsive ΔV in the $\hat{a}$ direction with magnitude:

$$\Delta V = \frac{T}{m} \Delta t = \frac{1\text{N}}{150\text{kg}} 10\text{s} = 0.06667\text{m/s} \quad (5.2)$$

7. Step spacecraft states (including formation master orbit) one time step forward using f and g method.
8. Repeat steps 4-7 while $t < t_{max}$.

5.2 Generic Orbit Maneuvers

In addition to controlling the full orbital motion of a spacecraft, the orbital element feedback control law can control individual elements of the orbit, as shown in Section 4.4.3. In any maneuver performed using the orbital element feedback control law, the control gains should be scheduled so that the spacecraft first acquires the desired orbit, and then establishes its angular position within that orbit. For the first three simulations in this section, we demonstrate the use of the orbital element control law to perform orbit transfer maneuvers. For these simulations, we allow feedback of some or all of the first five elements, which describe the size, shape, and orientation of the orbit in inertial space, but no feedback of the in-plane angular error. In each of these sample maneuvers, we use gains calculated as a function of the target orbit, as described in Section 4.4.3.

5.2.1 Orbit Raising Maneuver

To perform a simple circle-to-circle orbit raising maneuver, we allow feedback of only the semi-major axis and eccentricity errors. The initial conditions for this maneuver are: $a_0 = 7,000$ km; $e_0 = 0$. The target elements are: $a^* = 8000$ km; $e^* = 0$. Figure 5.1 shows a three dimensional plot of this maneuver, performed by a 150 kg spacecraft with a 1 Newton thruster, pointed instantaneously in whatever direction the control law commands.

Using 1 N scaled thrust, the classical element control law completes the circle-to-circle orbit raising maneuver in 0.98 days, with a total ΔV of 501.3 m/s.

5.2.2 LEO to GEO Maneuver

To perform a maneuver from a low earth orbit (LEO) parking orbit, to a geostationary orbit (GEO), we feedback semi-major axis, eccentricity, and inclination errors. The initial parking orbit is chosen to have semi-major axis, $a_0 = 6,700$ km, eccentricity, $e_0 = 0$, and inclination, $i_0 = 28.4^\circ$. The target orbit elements for this simulation are: $a^* = 42,100$ km, $e^* = 0$, $i^* = 0$.

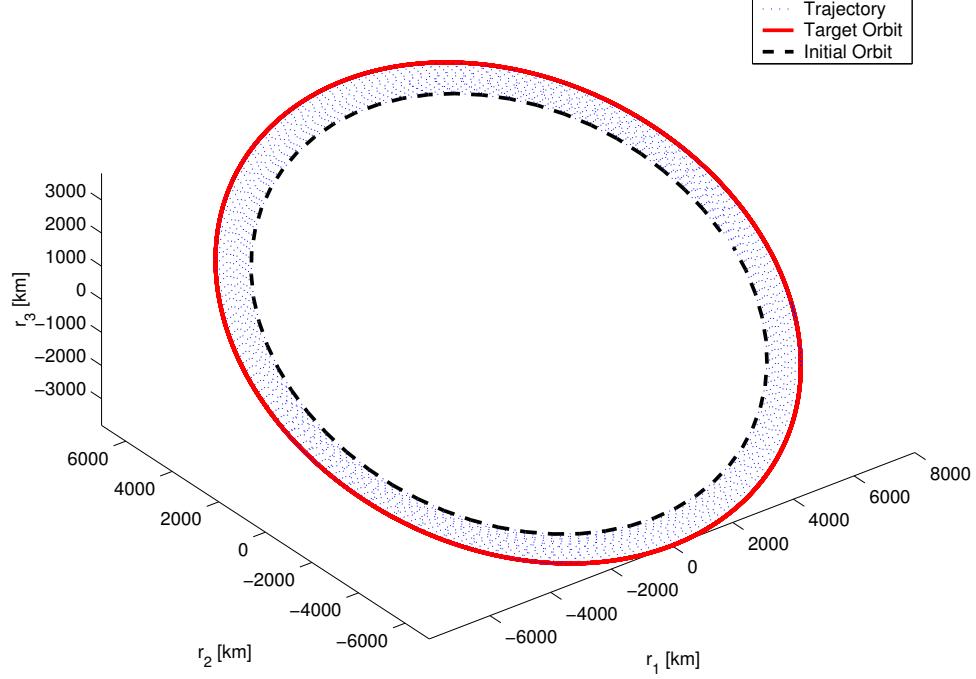


Figure 5.1: Circle-to-circle orbit raising maneuver from 7000–8000 km, using 1 N, scaled, classical orbital element feedback control

Using 1 N scaled thrust, the classical element control law completes the LEO to GEO maneuver in 11.18 days, with a total ΔV of 6.84 km/s.

5.2.3 LEO to Molniya Maneuver

To perform a maneuver from a LEO parking orbit to a Molniya orbit, we allow feedback of δa , δe , δi , $\delta \Omega$, and $\delta \omega$. The Molniya target orbit elements are: $a^* = 26,000$ km; $e^* = 0.75$, $i^* = 63.4^\circ$, $\Omega^* = 0^\circ$, and $\omega^* = 270^\circ$.

Using 5 N scaled thrust, the classical element control law completes the LEO to Molniya orbit transfer in 6.56 days, with a total ΔV of 18.13 km/s. This simulation is an interesting display of the applicability of the orbital element feedback control law to a large variety of orbital maneuvers. Further inspection of the periapee altitude during the maneuver reveals frequent values far less than the radius of Earth. By targeting the Molniya orbit with this control law, and selection of gains, we have forced the spacecraft to strike the Earth.

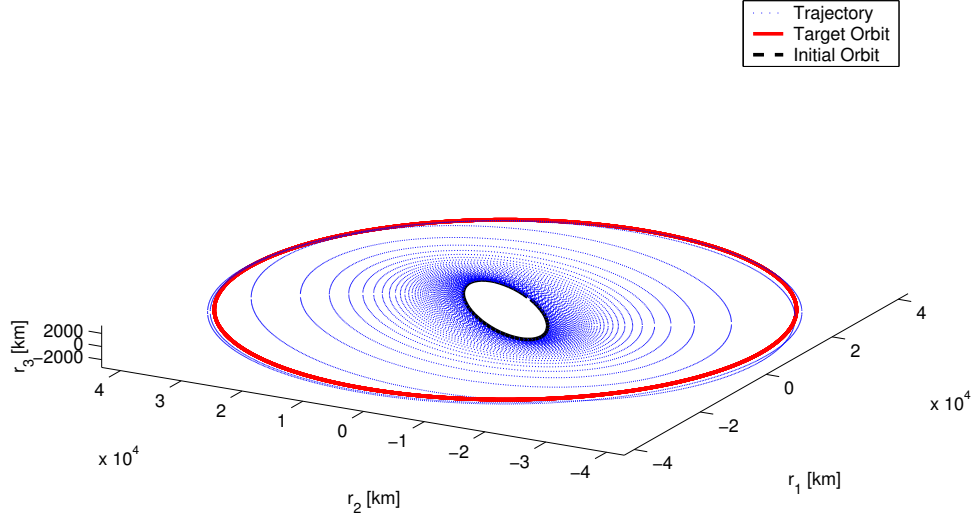


Figure 5.2: Maneuver from inclined, low earth orbit, to geostationary orbit, using 1 N, scaled, classical orbital element feedback control

5.2.4 Phasing Maneuver

Once the maneuvering spacecraft has acquired the desired orbit, we use mean motion control to complete the maneuver, correcting the spacecraft's angular position within the orbit. Figure 5.4 shows a phasing maneuver, with initial argument of latitude error, $\delta\theta = 1 \times 10^{-4}$ radians. This initial error corresponds to a velocity-direction error of about 700 meters, with the satellite ahead of the target point in its orbit, and is one of the formation maneuvers HokieSat will perform with its ION-F counterparts. Figure 5.4 shows the results of this phasing maneuver, performed by a 20 kg spacecraft with 112 μN maximum thrust. In this simulation, we assume the thrusters can be pointed instantaneously in the direction commanded by the control law.

The four subplots of Figure 5.4 show, a) the position error components; b) the argument of latitude error; c) the applied thrust; d) the maneuvering semi-major axis, the target semi-major axis, and the mean motion control target semi-major axis. Notice that the controller initially applies thrust in the velocity direction, to raise the semi-major axis to the initial value of a^{**} . As the value of a increases, the mean motion, n , decreases, and the target point begins to catch up with the spacecraft. As the $\delta\theta$ error is corrected, the value of a^{**} begins to decrease to a^* . Notice in subplot c), that as $\delta\theta$ decreases, the controller occasionally requests thrust, mostly in the negative velocity direction, to target the decreasing value of a^{**} . At the end of the simulation, the spacecraft converges to within 25 meters of the target orbit, and no more thrust is applied. Using 112 μN scaled thrust, the classical element control law completes the 1×10^{-4} radian phasing maneuver in 0.47 days, with a total ΔV of 0.072 m/s.

Figure 5.5 shows the results of an the same phasing maneuver, this time performed with thrust limited to the transverse and orbit-normal directions. Notice in subplot c) that no radial

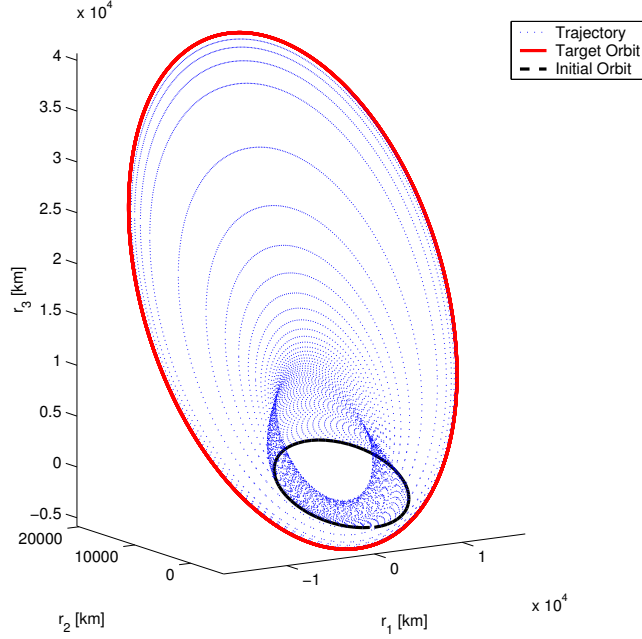


Figure 5.3: Maneuver from low earth parking orbit, to Molniya orbit, using 1 N, scaled, classical orbital element feedback control

thrust applied in this case. The classical element control completes this constrained thrust direction phasing maneuver in 1.34 days, with a total ΔV of 0.067 m/s. For this maneuver, the lack of radial thrust has extended the maneuver duration, but has decreased the required ΔV . This result is not surprising. Transverse thrust is more effective than radial thrust for semi-major axis control. By not allowing radial thrust we can increase control efficiency for this maneuver.

5.3 Leonardo Maneuvers

Two maneuvers of interest to the Leonardo formation are the formation establishment, and formation keeping maneuvers. In the formation establishment maneuver, the spacecraft are initially positioned in a parking orbit, and must use orbit control laws to initialize the formation. In the formation keeping maneuver, the spacecraft are initially in a perturbed arrangement, and must re-establish the formation geometry using orbit control.

5.3.1 Leonardo Formation Establishment

The Leonardo formation establishment problem involves the maneuvering of 6 spacecraft from some post-launch parking orbit into the desired formation geometry. We assume the spacecraft

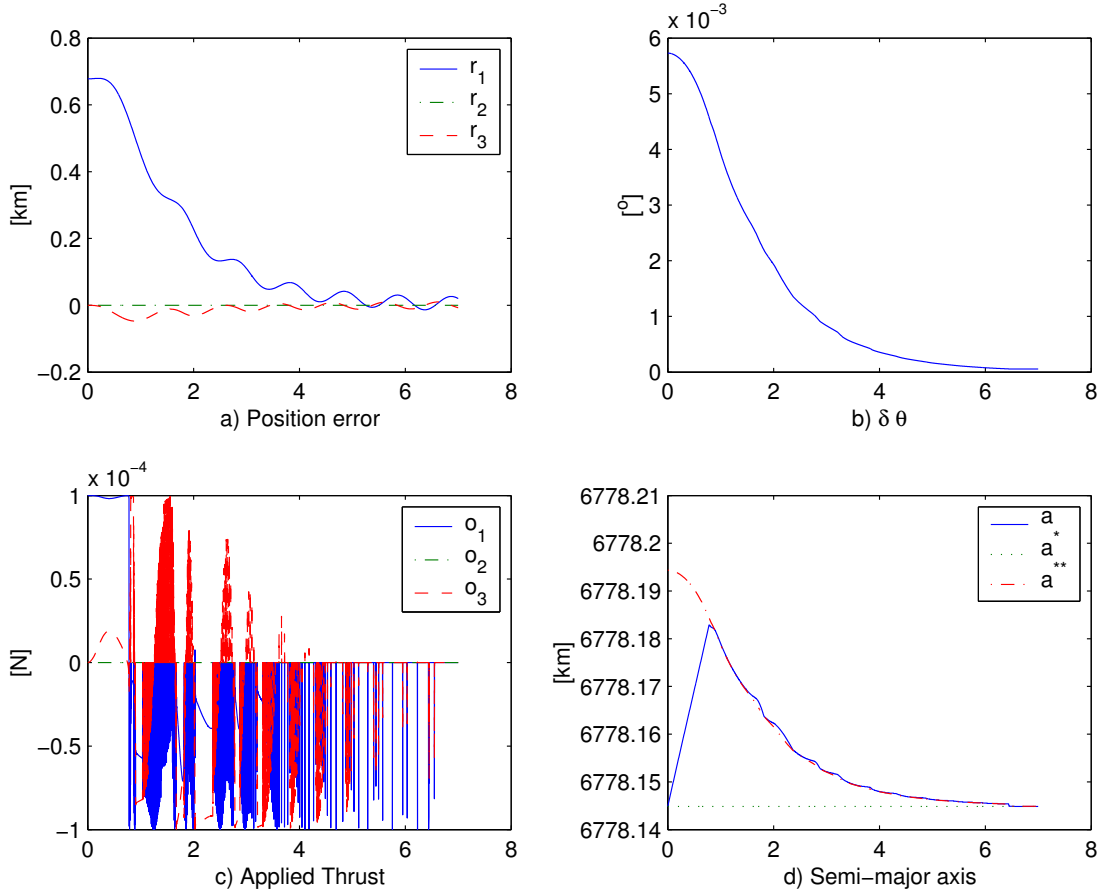


Figure 5.4: Phasing maneuver, using $112\mu\text{N}$, scaled, classical orbital element control

are launched as a stack, and therefore share the same initial conditions. One choice for the initial parking orbit is the formation master orbit defined in Section 3.3.2. This master orbit is the geometric center of the formation and therefore a logical choice for the initial parking orbit.

Cartesian Lyapunov Control

Figure 5.6 shows a three dimensional plot of the application of unbounded cartesian Lyapunov control to the formation establishment problem. The control forces rapid convergence of the spacecraft to their desired orbits in just under 2 orbits.

Figure 5.7 shows a typical thrust magnitude requested by one of the spacecraft during the maneuver, plotted on a semi-log scale. Notice that the control asks for hundreds of Newtons of thrust initially, and more than one Newton for a large portion of the maneuver. This example is a good illustration of the failure of the cartesian control in low thrust situations. If we constrain

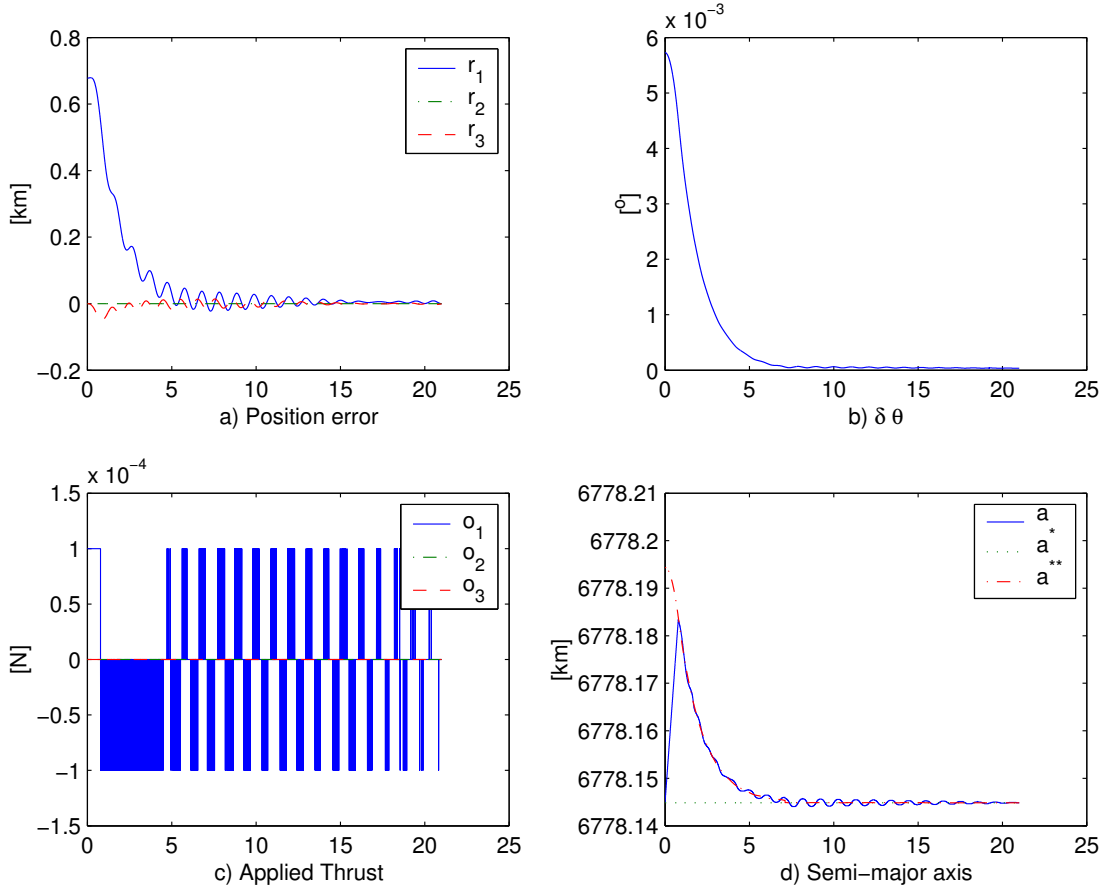


Figure 5.5: Phasing maneuver, using $112\mu\text{N}$, scaled, classical orbital element control

the thrust magnitude in the simulation, the cartesian control fails to complete the maneuver, as the relative dynamics term in the control law (see Eq. (4.51)) cannot be overcome.

Figure 5.8 shows the formation establishment simulation with thrust scaled to 30 Newtons. The six sub-plots show the orbital frame components of the position error, in kilometers. Notice that the spacecraft starting closest to their respective target orbits (spacecraft 2, 4, and 5, in subplots b, d, and e) successfully complete the maneuver, while the remaining spacecraft fail to reach their desired orbits. For this formation, the 30 Newton thrust level is insufficient to complete the formation establishment maneuver, using the cartesian Lyapunov control law.

Elemental Lyapunov Control

Figure 5.9 shows a three dimensional plot of the application of scaled classical orbital element Lyapunov control to the formation establishment problem, for one of the spacecraft in the

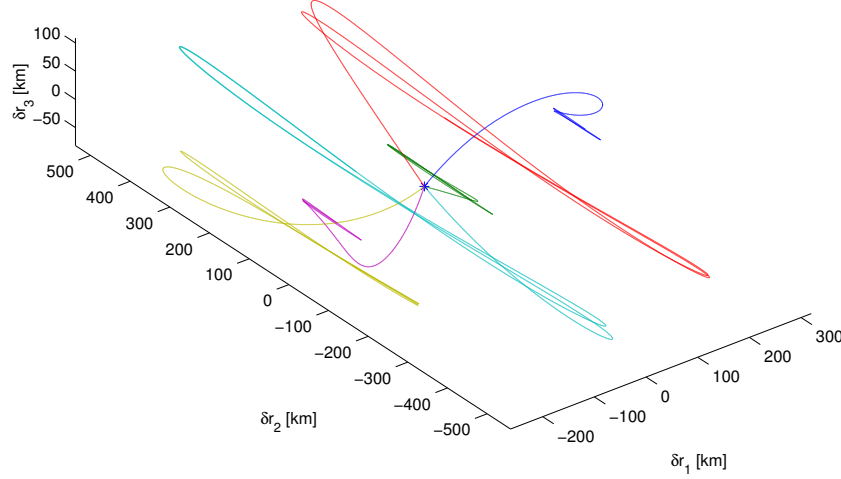


Figure 5.6: Formation establishment using unbounded cartesian Lyapunov control

Leonardo formation. In the simulation, a 150-kg spacecraft is maneuvered from the formation master orbit to the desired orbit defined in Table 3.4 as spacecraft 1, with thrust magnitude constrained to 1 Newton. For this simulation we use a mean motion control gain of $K_n = 1 \times 10^{-4}$.

For this simulation the target orbit is nearly circular. For this reason, the gains K_ω and K_M have been set to zero so that the errors associated with the argument of periapee and mean anomaly terms, $\delta\omega$ and δM (which are undefined for circular orbits), do not contribute to the requested thrust. For the same reason, the argument of latitude error, $\delta\theta$ is used in the mean motion control.

Notice in Figure 5.9 that the out-of-plane error is corrected after just over three orbits, but the radial and in-track errors remain for the duration of the simulation. This is not a surprising result. The choice of K_n has created a small semi-major axis error (negative δr_3 represents error in the positive radial direction). Close inspection of the δr_1 data reveals that this small semi-major axis error is gradually correcting the in-track error. If the simulation is allowed to run long enough, the in-track error will eventually decrease to the error envelope, and the semi-major axis error, shown by radial error, will be corrected. This approach provides significant fuel savings, as the control law inherently allows drifting arcs in the maneuvering spacecraft's trajectory.

We can complete the formation establishment maneuver much more quickly by using gain scheduling with the classical elemental control law. Figure 5.10 shows the same maneuver, this time with $K_n = 1 \times 10^{-4}$ for the first 3.1 orbits, and $K_n = 0.1$ for the remainder of the

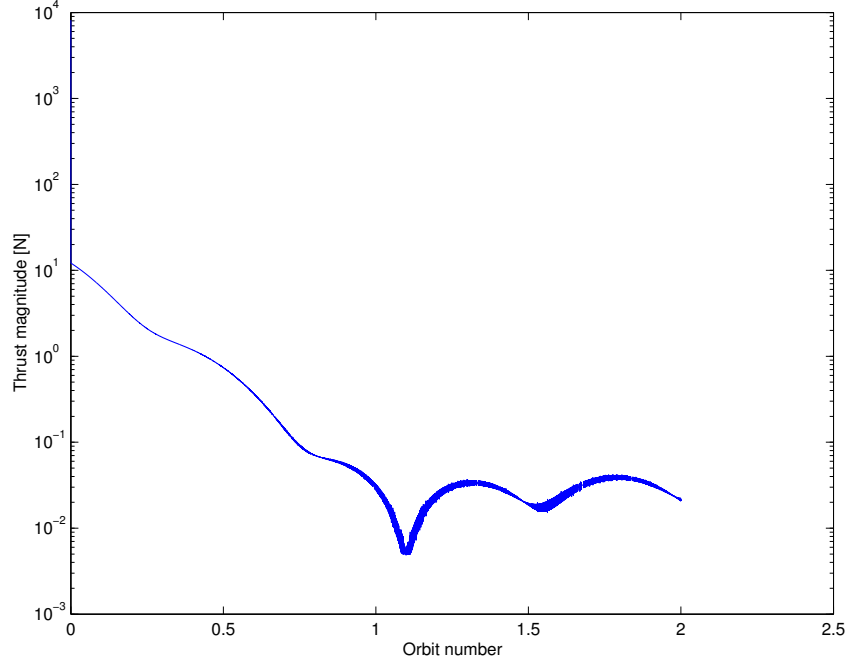


Figure 5.7: Semi-log plot of unbounded cartesian-controller-requested applied thrust magnitude for the formation establishment maneuver

simulation, as long as δM is greater than 5×10^{-6} radians (an angular error corresponding to an in-track position error of about 30 meters). We also schedule the eccentricity gain, K_e , during the phasing stage of the maneuver, by setting it to 1/5 the value calculated using the method described in Section 4.4.3. This gain reduction is necessary during the phasing maneuver to prevent chattering.

Notice that after 3.1 orbits, the increase in K_n immediately causes an increase in δa . This semi-major axis error decreases the velocity of the maneuvering spacecraft until the target point catches up, at which point the radial error is corrected to within the error envelope. This effect is better illustrated in Figure 5.11, which shows the evolution of the elemental errors over the course of the maneuver, as well as the components of the total thrust caused by each error. Notice that the majority of the out-of-plane error is corrected in the first three orbits, with the majority of the total requested thrust coming from the right ascension and inclination errors, $\delta\Omega$ and δi . Also notice that after the out-of-plane error is corrected, the majority of the requested thrust comes from the semi-major axis error, $\delta a = a - a^{**}$. Recall that a^{**} is the target semi-major axis defined by the mean motion control, in Eq. (4.48).

The left-hand plots in Figure 5.11 show the components of thrust requested by the control as a result of the respective element errors. Notice that in this simulation, the magnitude of the requested thrust is often quite large: much greater than 1 Newton. Whereas thrust saturation was a major concern in the cartesian control law application, it is of no concern in the application of the elemental control law, which allows total freedom in gain selection, and therefore allows

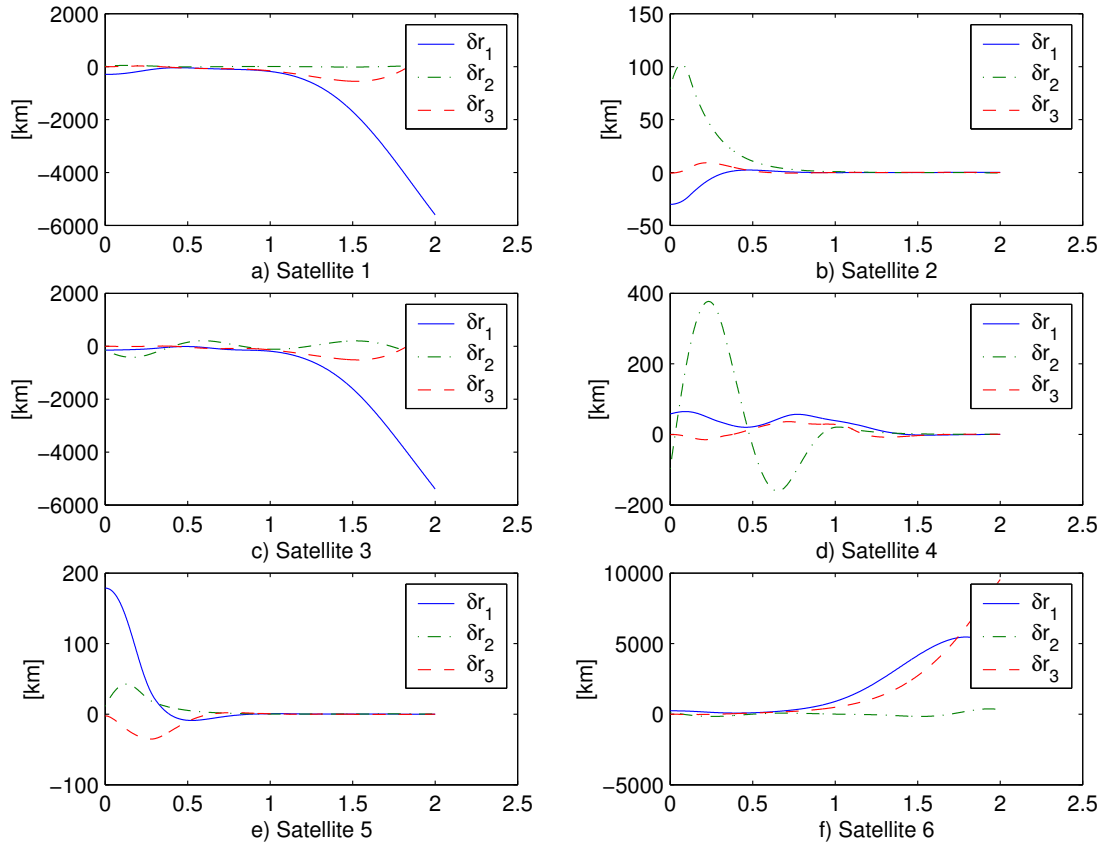


Figure 5.8: Formation establishment maneuver using 30-N constrained magnitude cartesian Lyapunov control law

the requested control to be scaled as needed.

Figure 5.12 shows the values of the semi-major axis, a , the target orbit semi-major axis, a^* , and the semi-major axis requested by the mean motion control, a^{**} , for the duration of the maneuver. Recall that the elemental control law feeds back $\delta a = a - a^{**}$. In the initial stage of the maneuver, the semi-major axis remains within a few kilometers of the target value. Once the phasing part of the maneuver begins, the mean motion control forces δa up to almost 20 kilometers. As the in-track error is corrected, the value of a^{**} gradually decreases to the original target value of a^* . Notice that after about 5.5 orbits, the value of a starts to make steps towards the target value. These steps correspond to impulses correcting the semi-major axis. The level areas in between the step changes correspond to coasting arcs, where no thrust is being applied as the in-track error is slowly corrected by a constant level of semi-major axis error.

Table 5.1 lists the number of thrust impulses required by each spacecraft to establish the formation, using 1-N, scaled, classical element control. Notice that spacecraft 3, 4, and 6 require a significantly higher number of thrust impulses (and thus more fuel) to reach their respective orbits within the formation. This is an important observation, as it means that during this maneuver the ballistic coefficients of the spacecraft will change non-uniformly throughout the

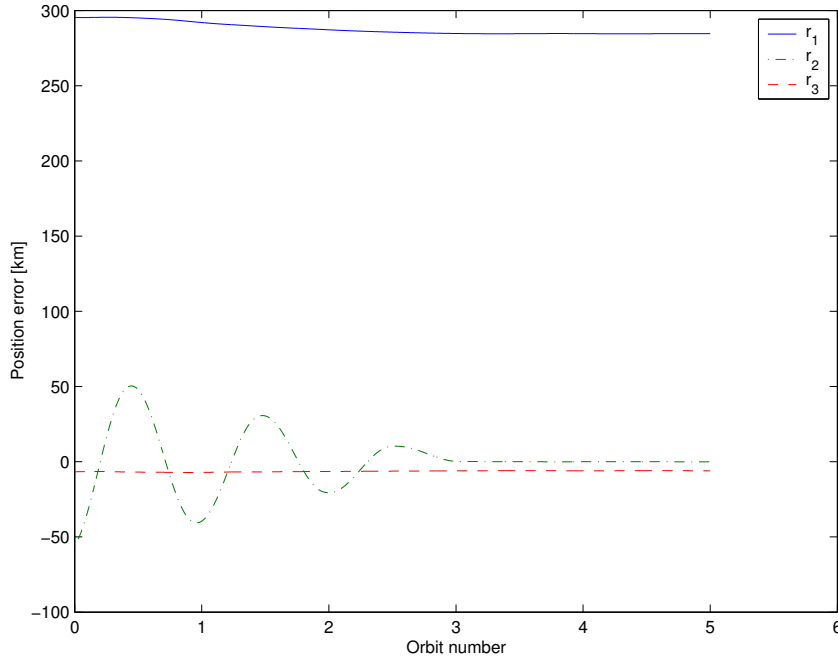


Figure 5.9: Formation establishment using 1-N scaled classical orbital element Lyapunov control

formation. This non-uniform ballistic coefficient distribution results in differential atmospheric drag on the formation, and should be avoided.

Table 5.1: Thrust impulses required to establish formation, using 1-N, scaled, classical orbital element feedback Lyapunov control

s/c	Impulses	% of Max Impulses
1	5626	40.52
2	4232	30.48
3	13881	99.99
4	13883	100.00
5	4216	30.37
6	11971	86.23

While the formation establishment maneuver is an interesting problem, low-thrust feedback control might not be the best choice for this stage of the mission. For a formation of this size, a more practical approach might be to use a launch vehicle upper stage to place each spacecraft in the stack, one at a time, on or near its desired orbit, as is commonly done with spacecraft constellations. Of course, the use of an upper stage loses its appeal as the difference between the orbital planes of the formation flyers increases. Ultimately, we want to choose a method that minimizes the total propellant mass at launch, but also maintains mass symmetry amongst the formation flyers, reducing the effect of differential drag.

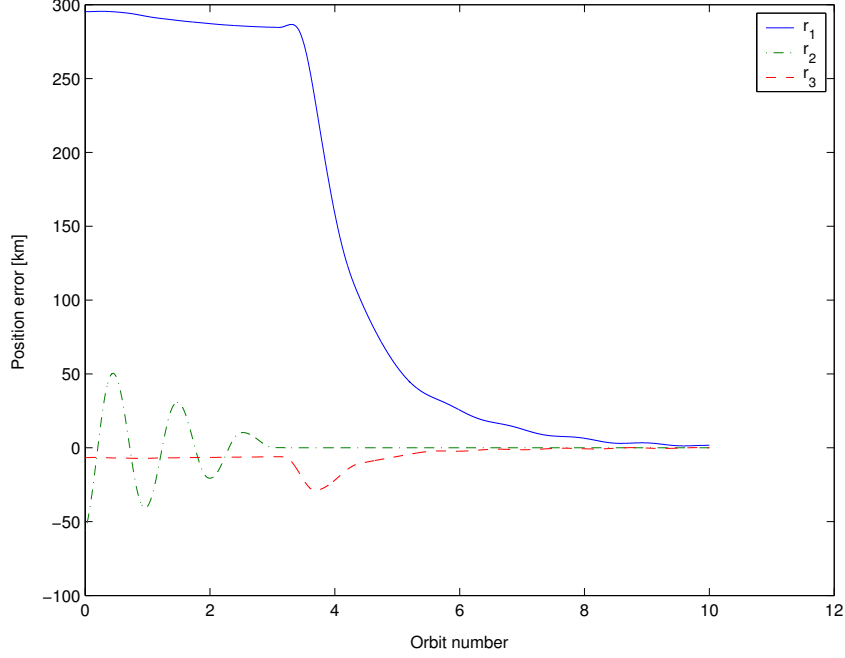


Figure 5.10: Formation establishment using 1-N scaled orbital element Lyapunov control law with gain scheduling

5.3.2 Leonardo Formation Keeping

The formation keeping problem involves the maintenance of the formation geometry in the presence of differential orbital perturbations such as non-spherical central body effects, atmospheric drag, and solar radiation pressure. We assume that the Leonardo spacecraft are, for the most part, identical in size and mass, and therefore that the differential effects of atmospheric drag and solar radiation pressure are negligible. This assumption may prove unwise when we consider the effects of formation geometry on the propellant mass required by each spacecraft.

The secular drifts of Ω , ω , M due to J_2 perturbations are presented in Section 3.2, and repeated here for convenience:

$$\frac{\overline{d\Omega}}{dt} = -\frac{3}{2}J_2 \left(\frac{r_{eq}}{p}\right)^2 n \cos i \quad (5.3)$$

$$\frac{\overline{d\omega}}{dt} = \frac{3}{4}J_2 \left(\frac{r_{eq}}{p}\right)^2 n(5 \cos^2 i - 1) \quad (5.4)$$

$$\frac{\overline{dM}}{dt} = \frac{3}{4}J_2 \left(\frac{r_{eq}}{p}\right)^2 \eta n(3 \cos^2 i - 1) \quad (5.5)$$

Where $\eta = \sqrt{1 - e^2}$. As shown in Eqs. (5.3–5.5), the magnitude of secular drift varies with inclination. Because the Leonardo spacecraft have different inclinations, the formation will

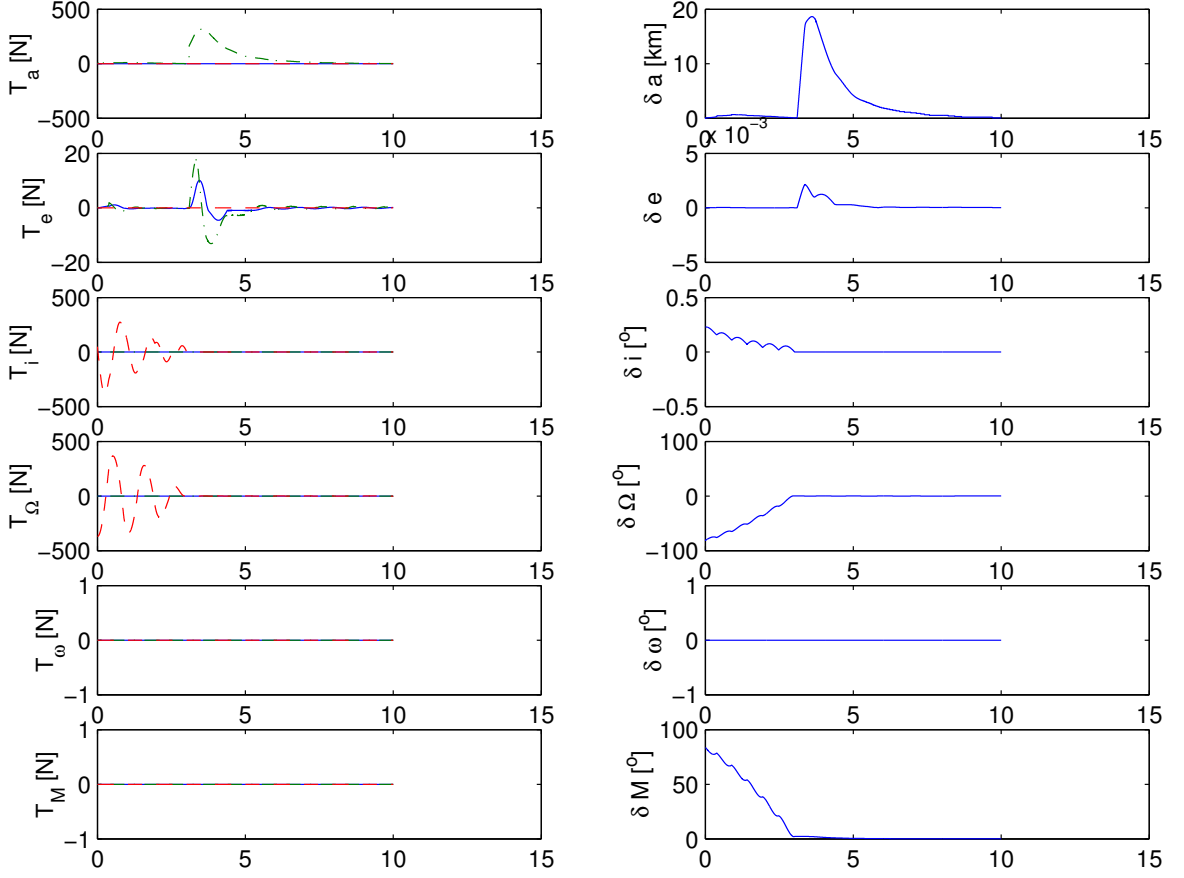


Figure 5.11: Elemental errors and corresponding thrust components during scaled elemental establishment maneuver with scaling

experience differential secular drift due to Earth oblateness effects.

While it may be ideal, from a control engineer's perspective, to correct orbital errors resultant from perturbation effects as they appear (indeed, a simple way to do this is to simply allow the spacecraft to thrust whenever the control law requests), other spacecraft operations, such as performance of science missions, may require a much less aggressive approach. Discussions with NASA-Goddard engineers suggest that Leonardo formation keeping operations might occur every two weeks. For this reason, we simulate a maneuver from initial conditions defined by two weeks of secular J_2 drift. These initial conditions are calculated by applying drift to the appropriate spacecraft orbital elements, and recalculating the formation master orbit and desired orbital states from the perturbed formation states (Section 3.3.2). The initial elemental errors for this maneuver are given in Table 5.2.

Notice that the elemental errors are not well distributed amongst the spacecraft. This imbalance is a result of our master orbit calculation. Because we calculate the desired target

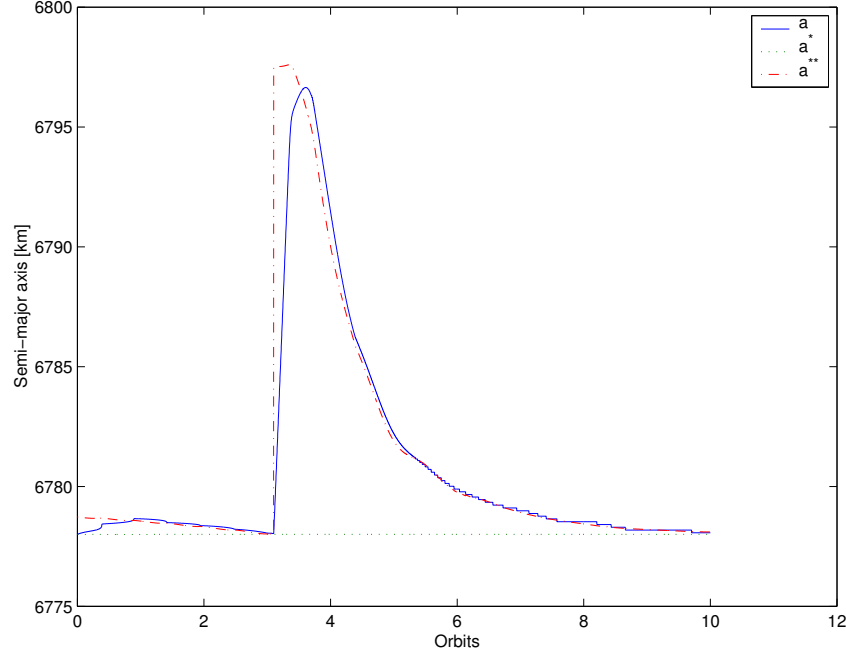


Figure 5.12: Evolution of semi-major axis error for mean motion control in the formation establishment maneuver

orbits as a function of the master orbit, we prefer to keep the formation master close to the center of the desired orbit locations. When we calculate the master orbit from the perturbed spacecraft orbits to find a central point, we calculate a master orbit with inclination, $i \approx 0.2499^\circ$. When the master orbit is calculated from the desired spacecraft orbits, we calculate a master orbit with inclination, $i \approx 0.2497^\circ$. The two inclinations vary by about $1.35 \times 10^{-4}^\circ$. Recall that the desired orbit inclinations are absolute, and therefore are not calculated as a function of the master inclination. If we calculate desired orbits from a master orbit defined with the wrong inclination, the resulting errors are shifted by a negative $\delta\Omega$ and a positive $\delta\theta$, as apparent in the data in Table 5.2. We correct this error by shifting the master orbit's Ω and θ such that the average values of $\delta\Omega$ and $\delta\theta$ are zero. This correction results in new initial elemental errors,

Table 5.2: Elemental errors after two weeks of secular J_2 drift

s/c	$\delta\Omega[^\circ]$	$\delta\theta[^\circ]$
1	-0.4981	1.3881
2	-0.4857	1.2884
3	-0.1681	-1.2479
4	-0.1410	-1.4643
5	-0.4956	1.3675
6	-0.3992	0.5967

given in Table 5.3.

Table 5.3: Elemental errors after two weeks of secular J_2 drift, corrected by averaging

s/c	$\delta\Omega[^\circ]$	$\delta\theta[^\circ]$
1	-0.1335	1.0667
2	-0.1211	0.9670
3	0.1965	-1.5693
4	0.2236	-1.7857
5	-0.1310	1.0461
6	-0.0346	0.2753

Figure 5.13 illustrates the simulation of a two-week- J_2 -error, formation keeping maneuver, using classical orbital element Lyapunov control, scaled to 0.01 N. The figure shows the orbital-frame components of position error for all six spacecraft in the formation. As before, we use gain scheduling, with an emphasis on the out-of-plane error in the first stage of the maneuver, and the in-plane angular error in the second stage. We define the end of the first stage, and the beginning of the second stage as the point where the control law no longer requests thrust for 30 consecutive time steps. At this time, the elemental errors are within the error envelope, and mean motion control commences. Notice that gain scheduling is applied on different schedules for each of the spacecraft. In fact, individual gains should be calculated for each spacecraft using the methods described in Section 4.4.3.

Figure 5.14 shows the 3D position of the Leonardo satellites during a formation keeping maneuver, plotted in the orbital frame of the master orbit. Notice that the initial conditions for the maneuver have replaced the formation master in the center of the formation, so that the bulk drift of the formation is ignored, and only the differential secular drift is corrected. Also notice that during the two weeks of secular drift, the highly inclined orbits have been overtaken by their nearest neighbors, orbits with lower inclinations, and therefore higher secular drift rates.

Table 5.4 shows the initial values of $\delta\Omega$ and $\delta\theta$, and the thrust impulses required to re-establish the formation after two weeks of J_2 drift. The results show a significant imbalance in fuel consumption, with spacecraft 3 and 4 performing the majority of the thrusting. This result is not surprising, as the thrust count for each spacecraft seems to correlate well with the initial elemental errors. While there may be a choice of initial conditions which spreads the thrust more evenly throughout the formation, it is unlikely that any other choice will result in fewer total thrust impulses for the formation as a whole. For this formation, it appears that large differences in fuel usage are unavoidable.

Comparing spacecraft 1 with spacecraft 2, we notice that while spacecraft 1 starts with larger initial elemental errors, it requires fewer thrust impulses. Inspection of the gains used in this simulation reveals a possible explanation of this result. Recall that the K_Ω gain is a

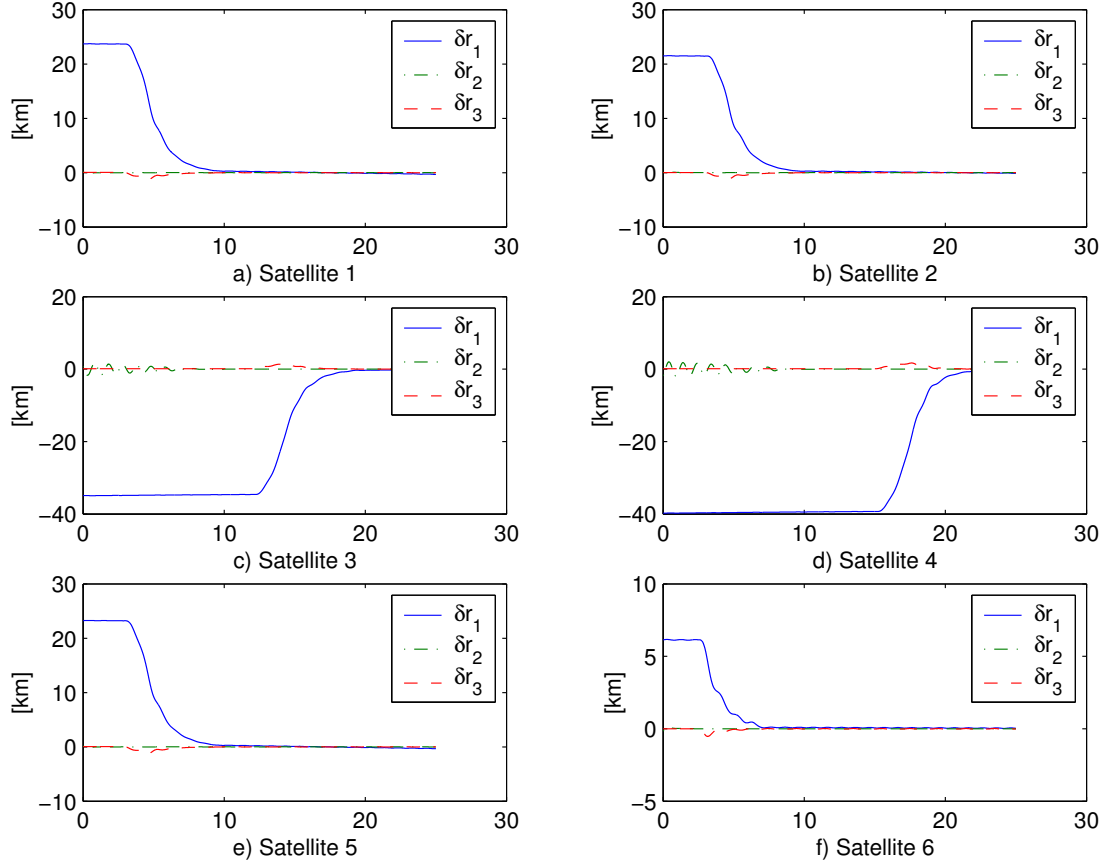


Figure 5.13: Formation keeping maneuver using classical element control with gain scheduling, and initial conditions defined by two weeks of J_2 perturbations

function of inclination. For spacecraft in circular orbits, the gain is:

$$K_{\Omega} = \left[\frac{h \sin i}{p} \right]^2 \quad (5.6)$$

This dependence of K_{Ω} on the inclination makes a noticeable difference in thrust results for this formation of non-coplanar orbits. As the inclination decreases, the value of K_{Ω} decreases. This is an ideal arrangement; we would like K_{Ω} to be zero when the inclination is zero, as the right ascension of the ascending node, Ω , is undefined for equatorial orbits.

Table 5.5 shows the gains for each of the Leonardo spacecraft, as used in this simulation. Notice that the only gain which varies between spacecraft is the K_{Ω} gain.

The spacecraft in orbits with lower inclinations have noticeably lower K_{Ω} values. This variation is actually the result of the original equations of motion, and or gain selection methodology. The time rate of change of Ω as a result of orbit normal thrust is

$$\frac{d\Omega}{dt} = \frac{r \sin \theta}{h \sin i} u_{\theta} \quad (5.7)$$

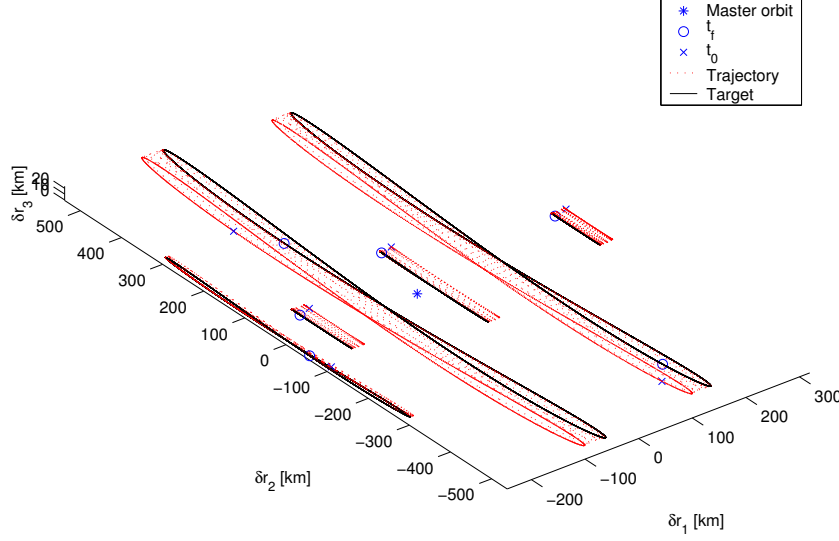


Figure 5.14: 3D plot of formation keeping maneuver, plotted in the orbital frame of the formation master

Notice that decreasing inclination results in increasing $\dot{\Omega}$. Therefore, we should not be surprised that spacecraft with lower inclinations require less thrust to correct the same Ω errors. This observation suggests that a shift of the master Ω towards the highly inclined orbits may reduce the total thrust use.

Table 5.6 shows the results of a formation keeping maneuver with $\delta\Omega$ values shifted by 0.1° toward the spacecraft with higher inclinations. The results of this shift are quite startling. While there is little improvement in the thrust distribution, the total thrust count has decreased by almost 10 percent. Furthermore, the thrust count has gone down for every spacecraft in the formation, including those whose initial orbital element errors increased as a result of the Ω shift. The cause of this result is quite simple; by shifting the right ascension angle of the master

Table 5.4: Initial conditions, and thrust impulses required to re-establish formation after two weeks of J_2 drift, using 0.01-N, scaled, classical orbital element feedback Lyapunov control

s/c	$i[^\circ]$	$\delta\Omega[^\circ]$	$\delta\theta[^\circ]$	Impulses	% of Max
1	0.4783	-0.1335	1.0667	4744	40.5158
2	0.9771	-0.1211	0.9670	4791	40.9172
3	4.4108	0.1965	-1.5693	10248	87.5224
4	4.5866	0.2236	-1.7857	11709	100.0000
5	0.6153	-0.1310	1.0461	5070	43.3000
6	2.4481	-0.0346	0.2753	2428	20.7362

Table 5.5: Gains for Leonardo formation, calculated using method suggested in Section 4.4.3

s/c	K_a	K_e	K_i	K_Ω	K_ω	K_ν
1	16.806	18.980	75.921	0.005	0	0
2	16.806	18.980	75.921	0.022	0	0
3	16.806	18.980	75.921	0.449	0	0
4	16.806	18.980	75.921	0.485	0	0
5	16.806	18.980	75.921	0.009	0	0
6	16.806	18.980	75.921	0.139	0	0

point (and thus the target points, which are defined from the master), we have actually brought the target orbit closer to the spacecraft in question.

Table 5.6: Data from formation keeping maneuver performed with $\delta\Omega$ values shifted by 0.1°

s/c	$i[^\circ]$	$\delta\Omega[^\circ]$	$\delta\theta[^\circ]$	Impulses	% of Max
1	0.4783	-0.2335	1.0667	4728	48.2892
2	0.9771	-0.2211	0.9670	4973	50.7915
3	4.4108	0.0965	-1.5693	8271	84.4755
4	4.5866	0.1236	-1.7857	9791	100.0000
5	0.6153	-0.2310	1.0461	4969	50.7507
6	2.4481	-0.1346	0.2753	2912	29.7416

This effect is illustrated in Figure 5.15. Notice that an increase of the right ascension error magnitude results in a decrease in the position error magnitude. The target point (‘ $\circ$ ’) in the orbit rotated the most about the inertial z-axis (up), is much closer to the spacecraft (‘ $*$ ’) than is the target point in the orbit rotated less about the z-axis. Because the element θ is measured from the node, and because the orbit is inclined so little, increasing the angle between the target node and the actual node, while holding $\delta\theta$ constant, actually brings the target point closer to the spacecraft. As a result, less thrust is required to perform the maneuver.

This effect is also illustrated by the results of previous simulations. In Figure 5.11, notice that during the first phase of the maneuver, where we have selected gains to correct the out-of-plane error, the value of $\delta\theta$ decreases dramatically. This correction of the in-plane angular error is not a result of mean motion control, but a result of the movement of the node, the point from which θ is measured. As the $\delta\Omega$ error is corrected, the $\delta\theta$ error decreases, because the points from which θ and θ^* are measured get closer together. This observation is a strong argument in favor of our decision to use gain scheduling for this control. For this formation (and others with low inclinations), it is unwise to allow feedback of $\delta\theta$ initially, as the points from which θ and θ^* are measured can vary greatly until the spacecraft has acquired the appropriate, target orbital plane.

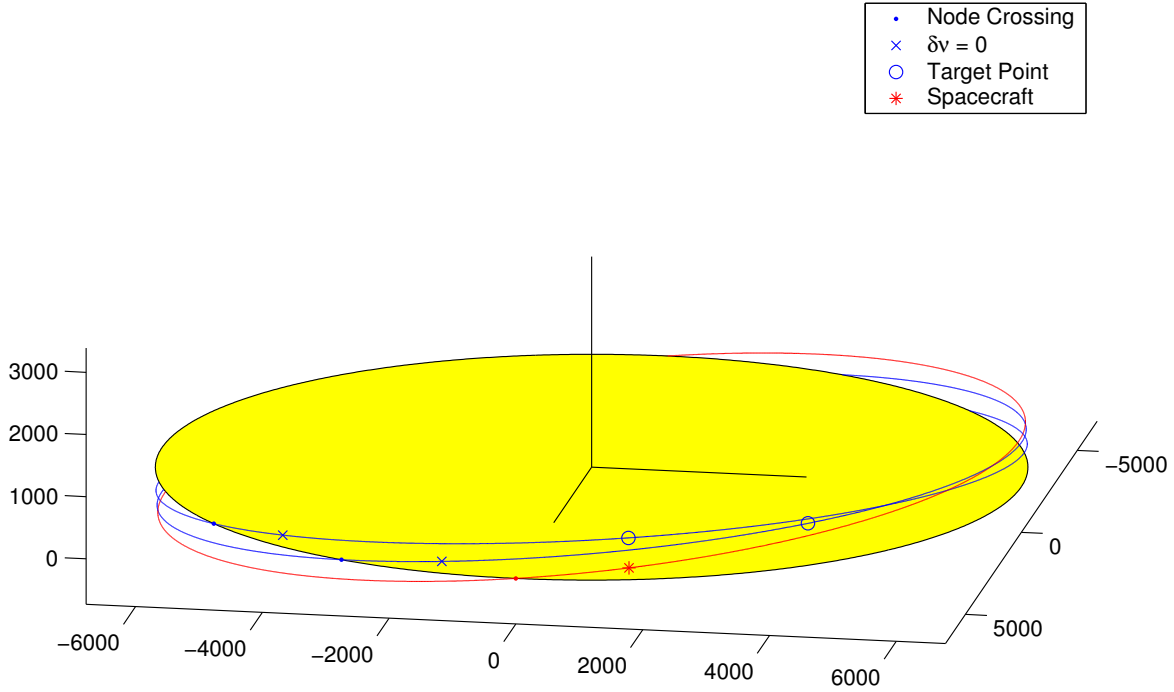


Figure 5.15: The effect of a right ascension shift of the target orbit on the distance between the spacecraft and target orbits. If $\delta\theta$ is held constant, the target orbit with a greater $\delta\Omega$ ends up closer to the spacecraft.

5.3.3 Propellant Usage

The thrust impulses tabulated in the Leonardo formation maneuvers are based on the number of impulsive ΔV s applied during each simulation. These impulsive ΔV s are calculated based on a spacecraft mass of 150 kg, thruster total impulses, I , of 1.0 or 0.01 N-s, and integrator step size, Δt of 10 seconds.

$$\Delta V = \frac{T}{m} \Delta t \quad (5.8)$$

At each time step for which thrust is applied, a ΔV of either 0.0667 m/s, or 0.000667 m/s is added in the direction of the requested thrust. The actual mass of propellant used in each maneuver is dependent on the specific impulse of the thrusters used to perform the maneuver.

Hughes and Mailhe⁹ list the expected ΔV budget for the Leonardo formation as on the order of 400 m/s per spacecraft. Table 5.7 shows the ΔV used in these simulations for formation establishment, and formation keeping. Notice that ΔV required to establish the formation is well above the 400 m/s limit. The formation keeping ΔV requirement of about 0.25 m/s/day, allows for about 4 years of formation keeping maneuvers, assuming the entire 400 m/s ΔV budget is spent on formation keeping. Note that these ΔV values could be reduced by refining of gains, and, or target orbit definitions.

An interesting question we can now pose is: How does the length of drift time between

Table 5.7: Total ΔV used for Leonardo formation establishment and formation keeping

s/c	ΔV Establish [m/s]	ΔV Keep [m/s/day]
1	375.0667	0.2251
2	282.1333	0.2368
3	925.4000	0.3939
4	925.5333	0.4662
5	281.0667	0.2366
6	798.0667	0.1387

formation keeping maneuvers effect the ΔV required per day to maintain the formation?. If we allow the formation to drift for more, or less than two weeks, how will the fuel consumption change? Figure 5.16 shows the maximum, minimum, and average ΔV required, as a function of drift time, to perform the formation keeping maneuver. The plot shows generally decreasing fuel consumption per day as drift duration increases. This trend of increasing fuel efficiency continues for higher drift durations, until well after the time at which the formation is no longer truly a formation, and could not possibly perform the necessary tasks.

The fact that ΔV per day of drift decreases with increasing drift durations is probably a result of the nature of the mean motion control. As shown in Figure 5.17, a major part of the initial error is in the $\delta\theta$ term. As we allow the formation to drift for more days, the initial elemental errors increase linearly. The fuel required to correct the linearly increasing errors seems to increase at a lower rate, probably because of the coasting arcs inherent in the mean motion control.

5.4 HokieSat Attitude Constrained Phasing Maneuver ¹

To test the orbit control law in the presence of thrust interruption, we simulate integrated orbit and attitude control in a 1×10^{-4} radian phasing maneuver. The simulation is written in C++ and performed on a Linux machine. The simulation emulates the software environment that will run on HokieSat's onboard computer, and involves four processes interacting in the same way that the flight processes will interact. The four processes are Orbit Determination, Orbit Control, Attitude Determination, and Attitude Control.

We enforce orbit and attitude coupling by allowing the attitude control process to interrupt orbit control, either when the thruster pointing error is greater than 5° , or the spacecraft's angular rate is greater than 0.0001 rad/sec. As HokieSat's orbit control thrusters point in only one direction, the attitude control system must slew the spacecraft about the nadir vector to appropriately point the thrusters. In the simulation, we apply scaled attitude control based on

¹This work was performed as a class project for AOE 5984: Nonlinear Control, and includes contributions from Matthew M. Berry.

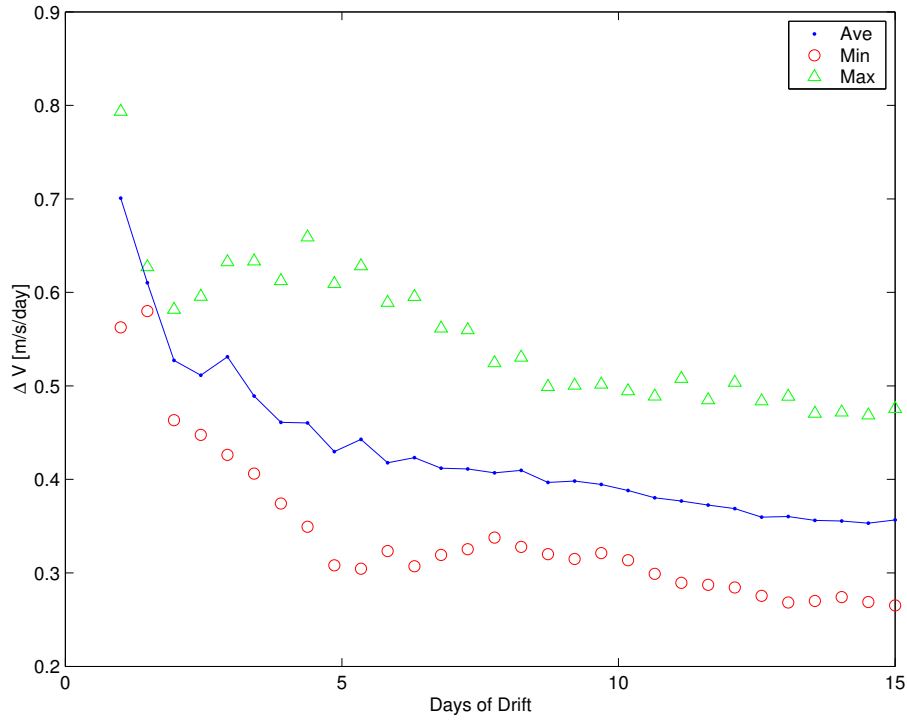


Figure 5.16: Formation keeping ΔV as a function of drift duration

feedback of modified Rodrigues parameters, as developed by Tsiotras.⁴⁵ Magnetic interference prohibits simultaneous orbit and attitude control. For this reason, we do not apply torque during thruster operations.

Figures 5.18–5.22 show the results of a 14 orbit simulation, with a maximum torque value of 5×10^{-5} N-m. This value corresponds to an approximate best case torque for HokieSat.

Figure 5.18 shows the position error in the orbital frame. After 12 orbits, the position error decreases to within a 20 meter error envelope. This error envelope is acceptable, as HokieSat's onboard orbit determination will be accurate to only approximately 20 m, so any error less than 20 m can be considered within the noise.

Figure 5.19 shows the applied thrust direction. When no thrust is applied, the graph shows zeros. Solid bars on the graph represent areas where thrust is frequently turned on and off. Notice that the applied thrust switches directions several times throughout the maneuver, with thrust mostly applied in the $\pm \hat{o}_1$ direction. Also notice that some thrust is applied in the out of plane direction, a result of thruster pointing error. Thrust is no longer applied after approximately 12.5 orbits, when the orbital error decreases to within a tolerable error envelope. The constant thrust applied in the first orbit increases the semi-major axis. The thrust is in the negative velocity direction in orbits 1 through 4, correcting the semi-major axis as the mean anomaly error decreases. In orbits 5 through 12, the thrust direction changes twice per orbit,

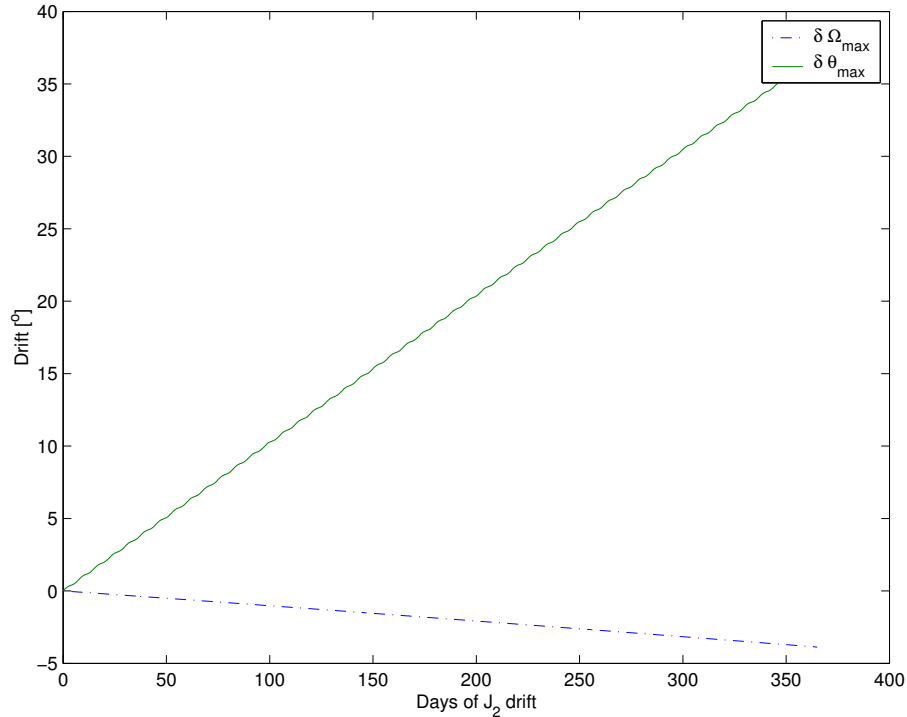


Figure 5.17: Formation keeping initial error as a function of drift duration

attempting to correct the eccentricity error induced by the phasing maneuver thrusts.

Figures 5.20–5.22 show the applied torque, the angular error, and the angular velocity. At each thrust direction change, there is a corresponding spike in these graphs, which represent a 180° slew maneuver. The angular velocity shown in Figure 5.22 has spikes representing yaw maneuvers in both clockwise and counter-clockwise directions. The direction of these maneuvers is nearly arbitrary because it is a 180° turn. An interesting source of attitude and orbit control coupling would be introduced here if momentum wheels were being used for attitude control. In that case, switching the direction of the yaw maneuver would be desirable to minimize momentum wheel speeds.

Figures 5.23–5.26 show the applied thrust direction, applied torque, angular error, and angular velocity around the first of the 180° slew maneuvers. In the zone from 0.8 to 0.9 orbits, the slew maneuver is being performed, and thus no thrust is applied. We also see three angular corrections in the plots, between 0.9 and 1.05 orbits. These attitude errors are not caused by drift, but by step changes in the requested thrust direction, possibly including brief coasting periods where the control requests no thrust.

Figures 5.27–5.29 show the position difference, thrust direction, and angular error results for a simulation using a maximum torque magnitude of 5×10^{-6} N-m. This low torque value represents a worse case scenario for the HokieSat attitude control system. Figure 5.27 shows

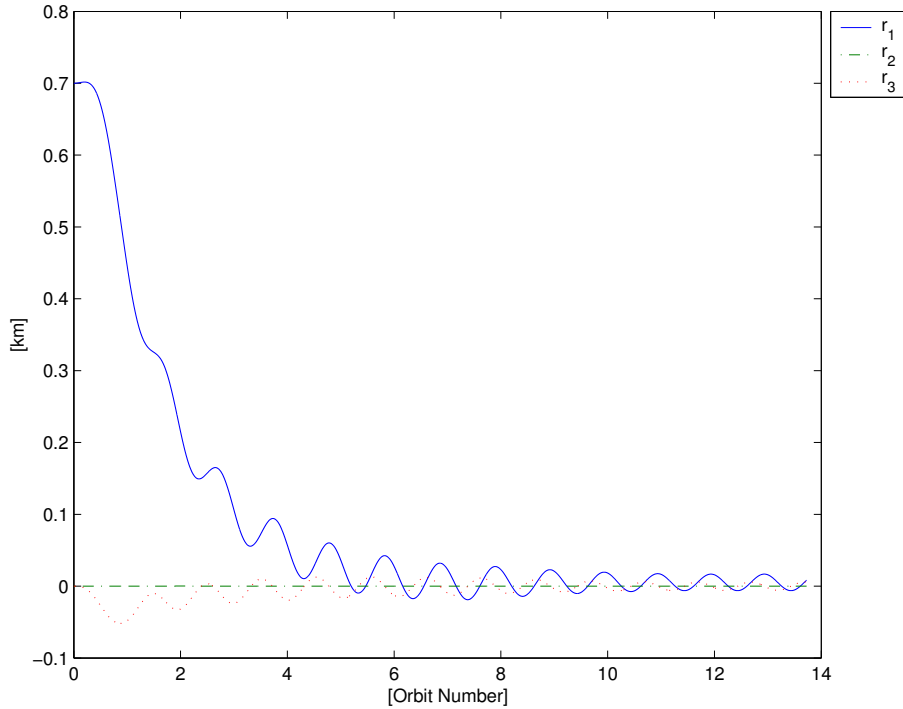


Figure 5.18: Position error in orbital frame of the target, during HokieSat phasing maneuver with maximum torque value ($g_{max} = 5 \times 10^{-5}$ N-m)

that the position error reaches a minimum after about eight orbits, with a final error amplitude of approximately 60 m. While this error is larger than the higher torque case, 60 m is still considered an acceptable value.

Figure 5.28 shows the thrust direction during the maneuver. No thrust is applied after nine orbits. Notice that the thrust changes direction four times, yet the angular error, shown in Figure 5.29, shows six peaks where the angle is off by 180° . The third peak begins just as the second peak ends, and the fifth peak begins at the end of the fourth peak. This indicates that the requested thrust direction changes before the attitude control system can align the thrusters. Despite this problem, the orbit control is still able to force convergence of the orbit error to within the tolerable error envelope. In fact, the lower torque case performs the maneuver with 13% fewer thrust impulses. Because the mean motion control corrects the orbit even when no thrust is applied, the low torque case can perform well for this maneuver. This will not be true for orbit maneuvers which change orbital elements other than the mean anomaly.

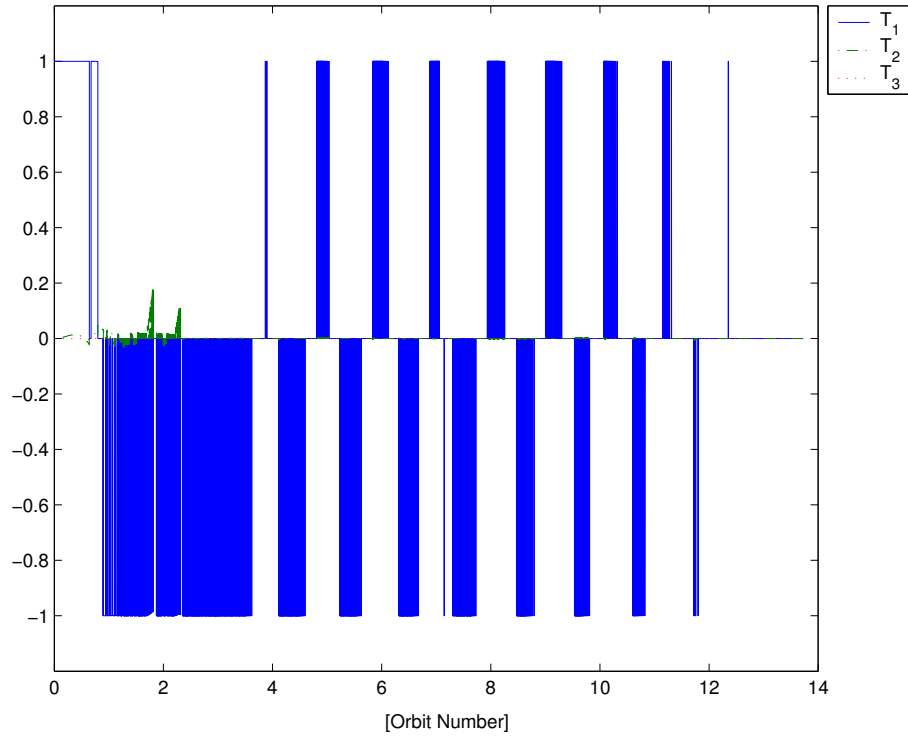


Figure 5.19: Applied thrust direction in spacecraft orbital frame, during HokieSat phasing maneuver with high maximum torque value ($g_{max} = 5 \times 10^{-5} \text{N-m}$)

5.5 Summary

We have demonstrated the use of classical orbital element feedback control for spacecraft maneuvers, including some generic orbit transfer maneuvers, and formation establishment and keeping maneuvers for the Leonardo-BRDF and ION-F formations. In the next chapter, we draw conclusions from these results, and make some recommendations for further study.

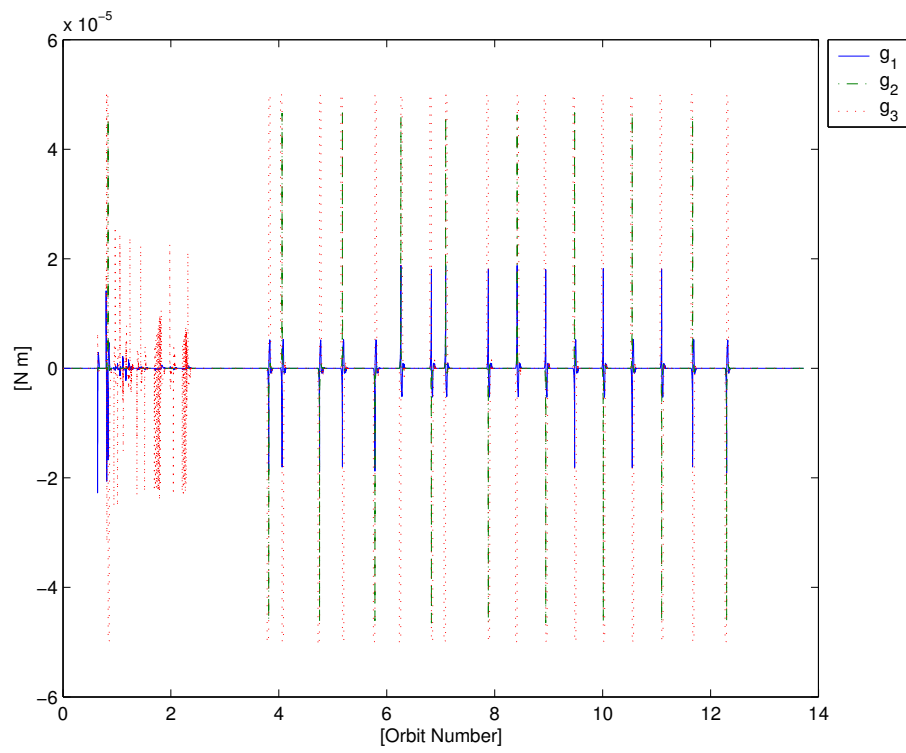


Figure 5.20: Applied torque in spacecraft orbital frame, during HokieSat phasing maneuver with high maximum torque value ($g_{max} = 5 \times 10^{-5}$ N-m)

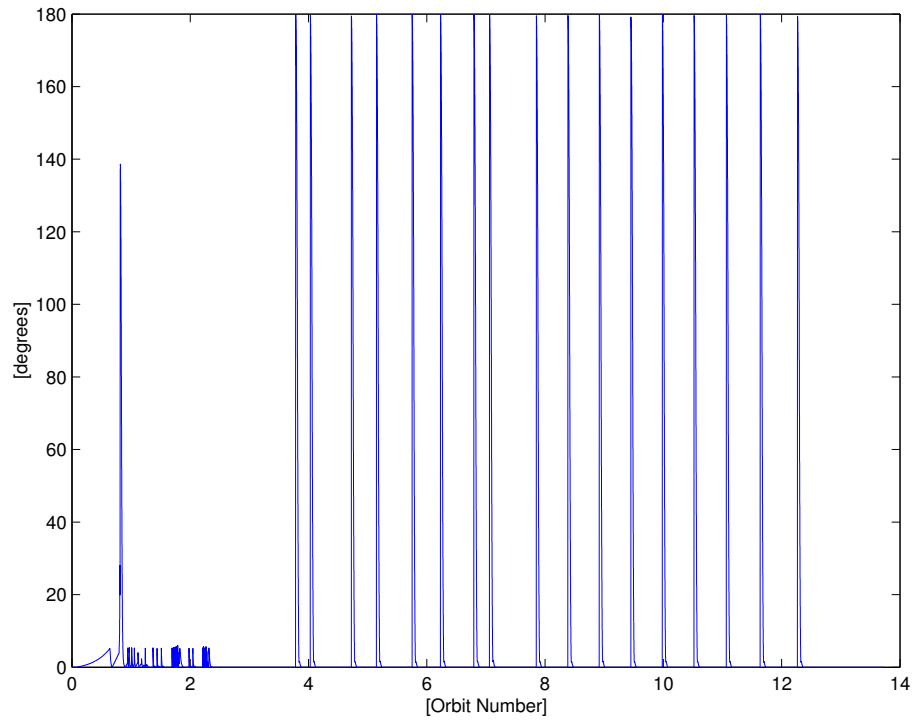


Figure 5.21: Angular attitude error during HokieSat phasing maneuver with high maximum torque value ($g_{max} = 5 \times 10^{-5}$ N-m)

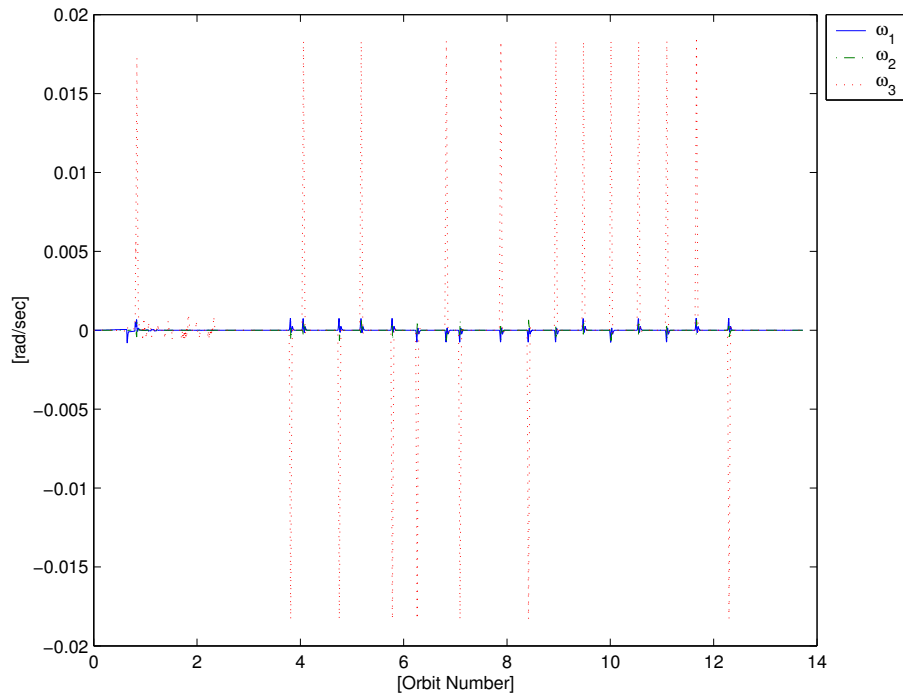


Figure 5.22: Angular velocity error during HokieSat phasing maneuver with high maximum torque value ($g_{max} = 5 \times 10^{-5}$ N-m)

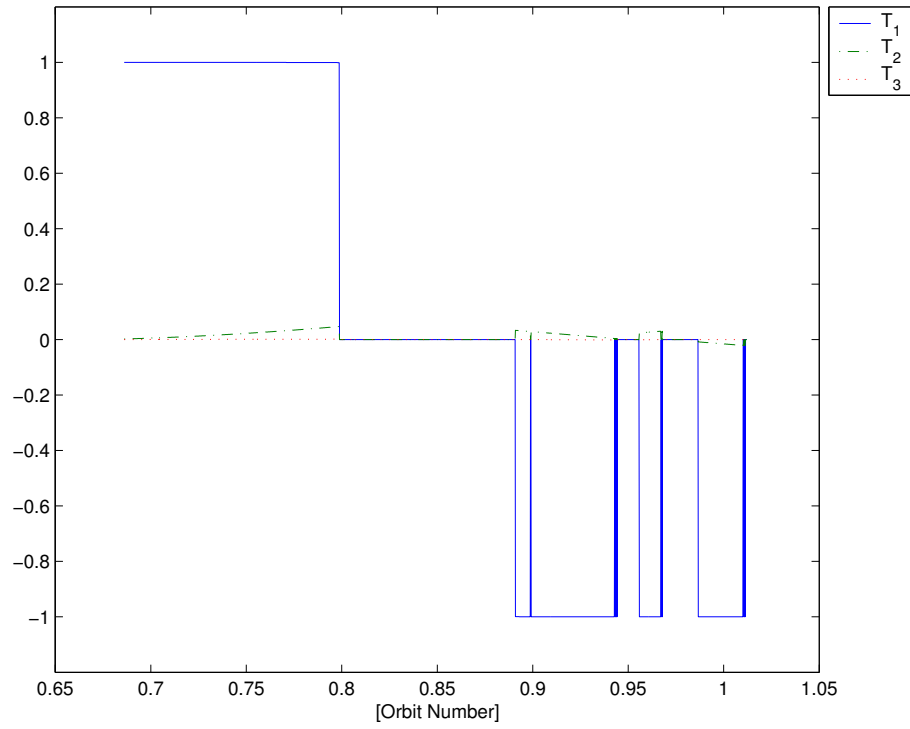


Figure 5.23: Zoom-in of applied thrust direction in spacecraft orbital frame, during HokieSat phasing maneuver with high maximum torque value ($g_{max} = 5 \times 10^{-5}$ N-m)

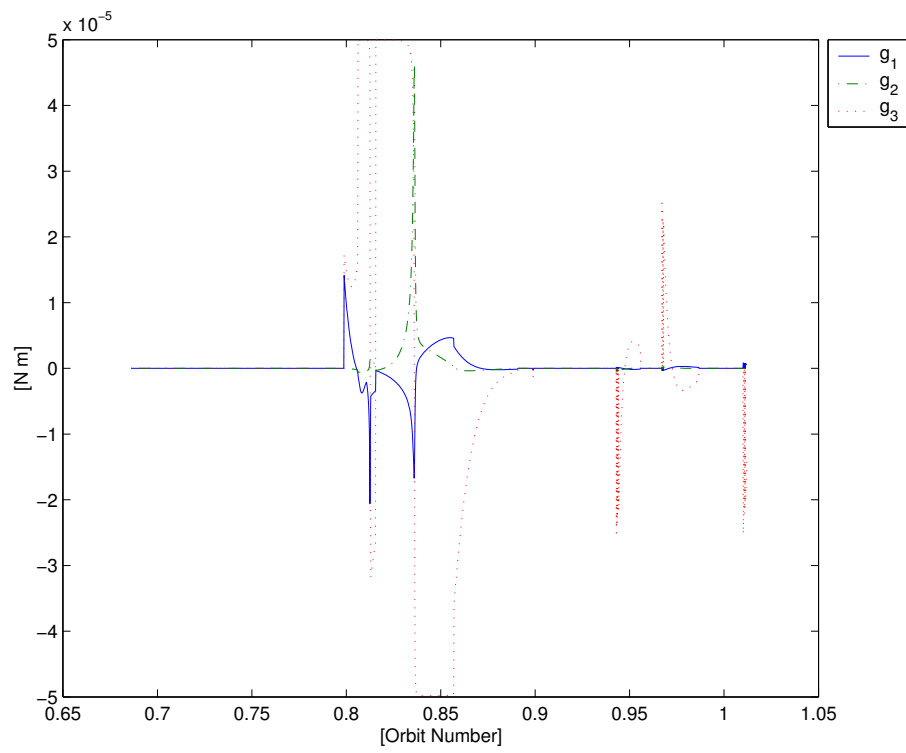


Figure 5.24: Zoom-in of applied torque, in spacecraft orbital frame, during HokieSat phasing maneuver with high maximum torque value ($g_{max} = 5 \times 10^{-5} \text{N-m}$)

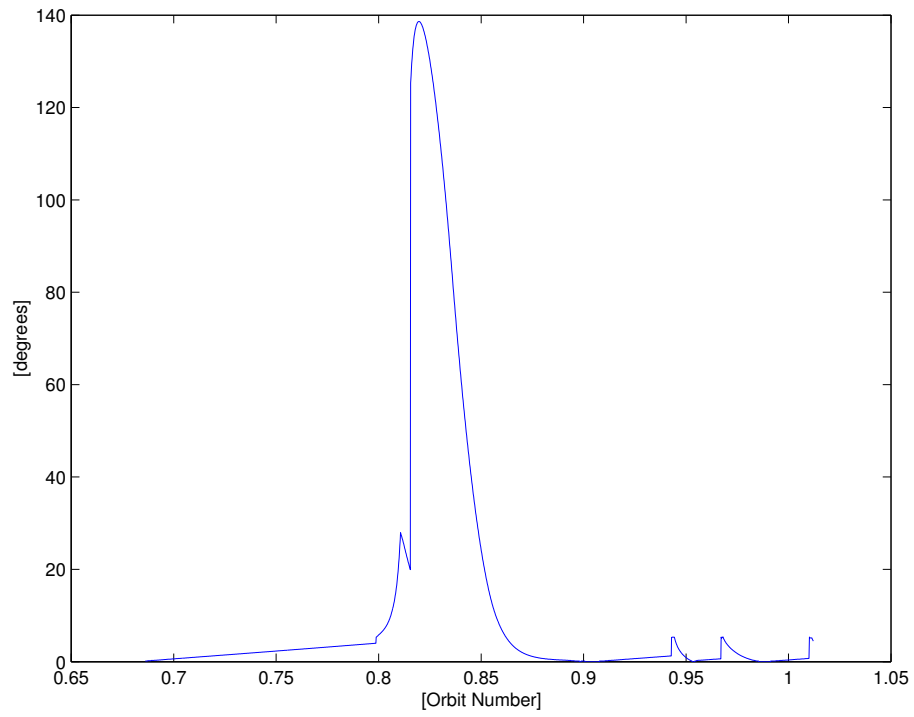


Figure 5.25: Zoom-in of angular attitude error during HokieSat phasing maneuver with high maximum torque value ($g_{max} = 5 \times 10^{-5}$ N-m)

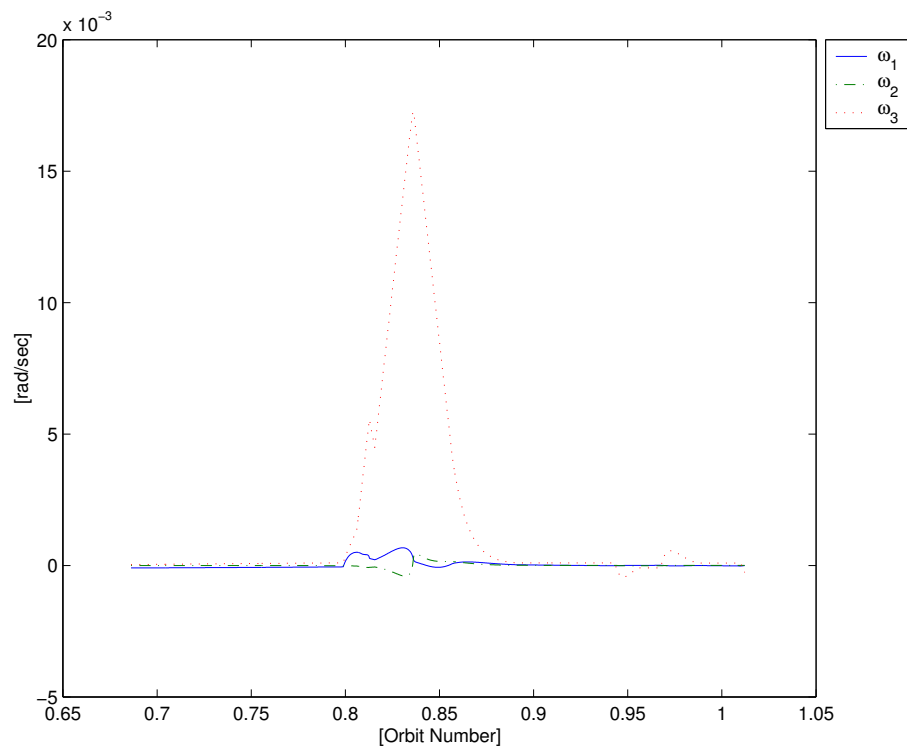


Figure 5.26: Zoom-in of angular velocity error during HokieSat phasing maneuver with high maximum torque value ($g_{max} = 5 \times 10^{-5} \text{N-m}$)

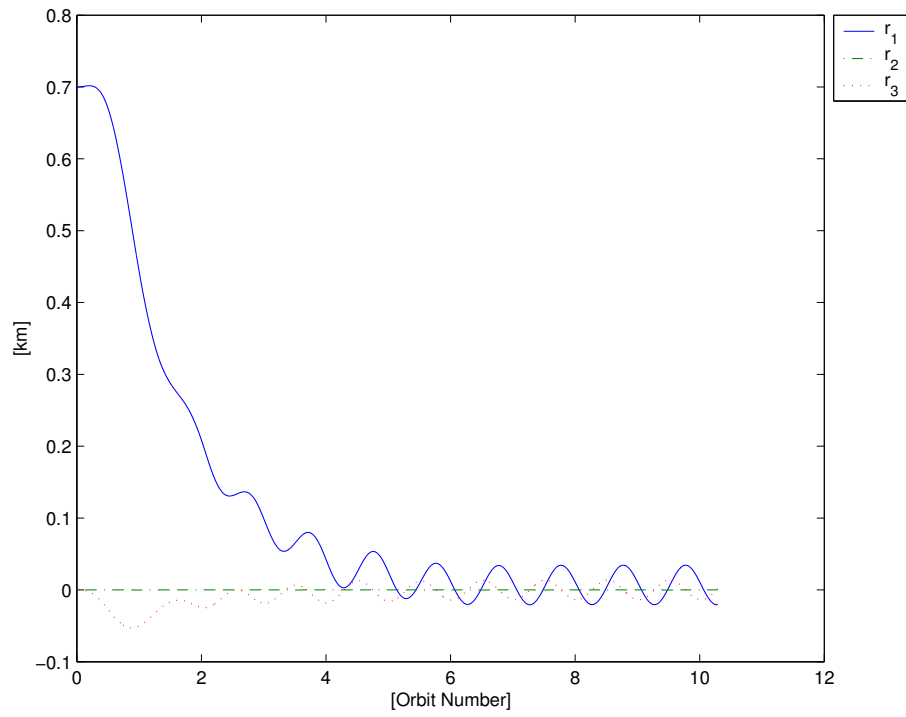


Figure 5.27: Position error in orbital frame of the target, during HokieSat phasing maneuver with low maximum torque value ($g_{max} = 5 \times 10^{-6}$ N-m)

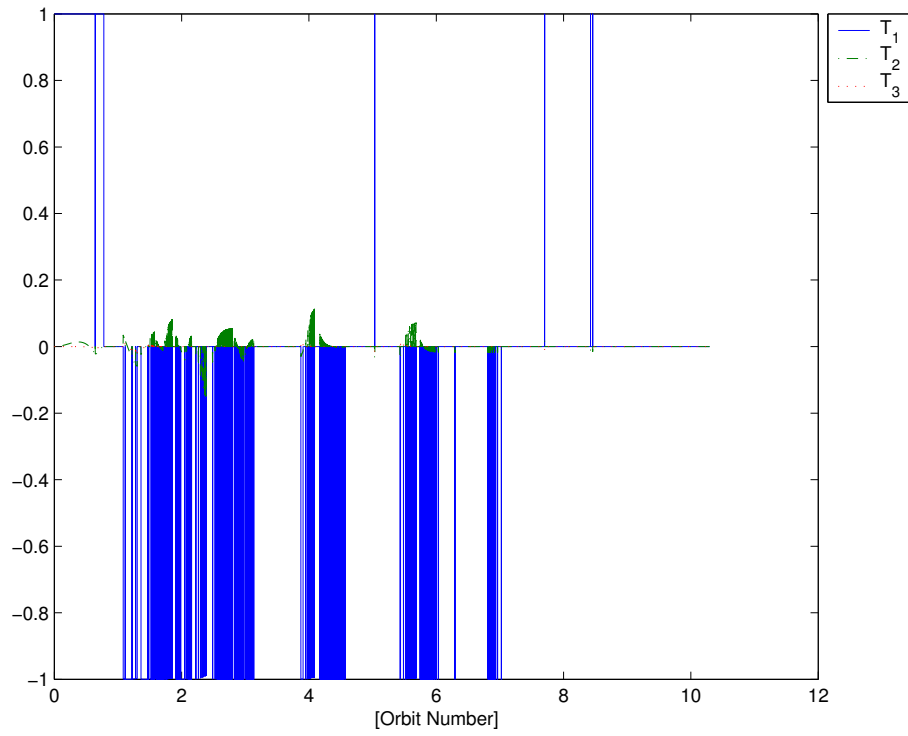


Figure 5.28: Applied thrust direction in spacecraft orbital frame, during HokieSat phasing maneuver with low maximum torque value ($g_{max} = 5 \times 10^{-6}$ N-m)

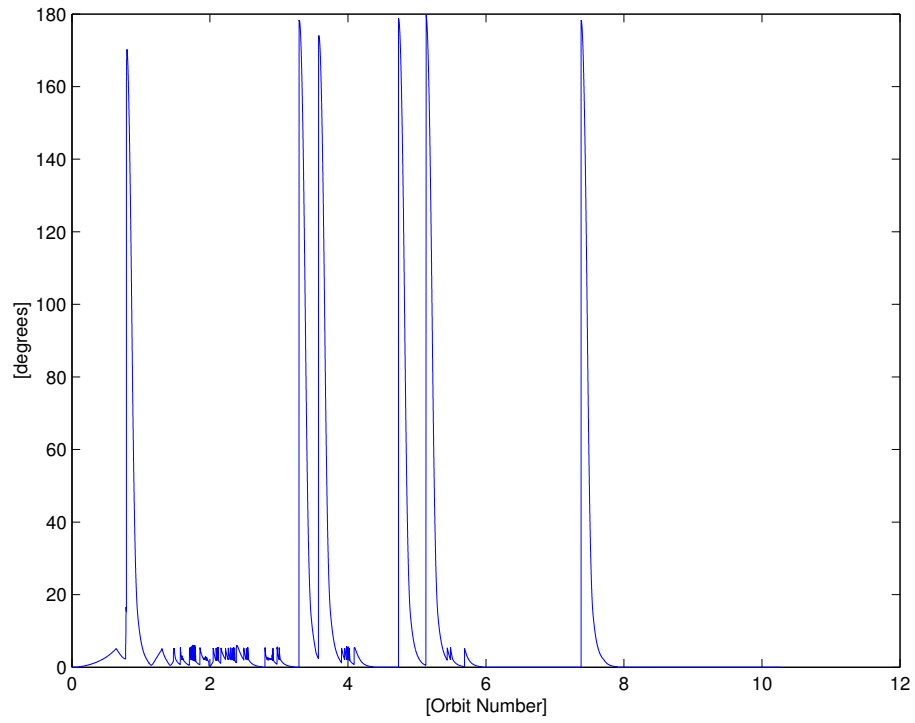


Figure 5.29: Angular attitude error during HokieSat phasing maneuver with low maximum torque value ($g_{max} = 5 \times 10^{-6}$ N-m)

Chapter 6

Conclusions and Recommendations for Future Study

We develop nonlinear orbit control laws for use in spacecraft orbital maneuvers, and spacecraft formation flying. This nonlinear orbital element feedback control law, augmented by mean motion control, successfully and efficiently maintains and modifies the full Keplerian motion of a large variety of spacecraft orbits. Unlike some previous element feedback controls, this law controls both the orbit size, shape, and orientation, and the spacecraft's angular position within the orbit. The control also takes advantage of physical insight to minimize fuel consumption, by utilizing differential mean motion to correct in-plane angle errors.

One major limitation to nonlinear feedback control is the absence of any works describing gain selection techniques. We define a new technique for choosing orbital element feedback gains which appropriately scales the gains for orbit maintenance, and provides an excellent starting point for gain optimization. While these gains may not provide optimal results for every maneuver, they do provide efficient, chatter-free thrust performance.

To provide target orbit states for feedback control, we develop and apply an algorithm to calculate a formation master orbit representing the geometric center of the formation. This master orbit serves as an appropriate starting point for a formation establishment maneuver, and also a point from which we can calculate the desired spacecraft orbital elements. Simulation results show that for the Leonardo formation, the definition of the master orbit at the geometric center of the formation does not provide fuel optimal results. Future studies should seek to find other analytical methods for calculating a fuel optimal formation master orbit. These studies should consider the possibility that spacecraft may not occupy the same position in the formation throughout the mission, but may trade places in the formation to minimize and evenly distribute the fuel consumption of the formation.

Simulations of nonlinear, orbital element feedback control in Leonardo formation establishment and formation keeping maneuvers show positive results, and provide some interesting new insight. One important observation is the problem of non-uniform fuel consumption throughout

the formation during both formation establishment and formation keeping maneuvers. This is significant because it virtually guarantees that at some point in the mission, the spacecraft in the formation will have different masses, and therefore different ballistic coefficients. The inevitable result is that differential drag effects must be considered in the relative motion of this formation.

Another interesting conclusion drawn from the simulation results is that the total thrust required to complete a maneuver is not necessarily proportional to the initial elemental errors. Because of the interaction between the elements, increasing some elemental errors might actually result in decreased thrust use. This observation suggests that the geometric center of the formation is not the best choice for a master reference orbit, and that the master formation orbit definition should be refined.

Simulation of integrated orbit and attitude control for the HokieSat mission reveals significant coupling between the two systems. If appropriate thruster pointing for orbit control requires attitude control maneuvers, as in the HokieSat mission, the performance of the orbital element control law may be compromised. For the simple phasing maneuver, the coupling of the two systems via attitude interruption does not negatively effect the performance of the orbit control law. This result is completely due to the nature of the mean motion control, which requires minimal thrust-direction switching to successfully perform its duties. The mean motion control corrects the majority of the angular error during coasting periods, where no thrust is applied. For more complex, non-coasting maneuvers, the orbital element control is more sensitive to thrust interruption, and more innovative methods of integrated orbit and attitude control are required.

The most important work remaining in the development of these control laws is the modification to include non-spherical earth effects. Schaub and Alfried^{28,36} have suggested the use of mean orbital elements, as defined by Brouwer.⁴⁶ Feedback of mean orbital elements, as opposed to osculating elements, allows the control to target orbit averaged elements, correcting errors due to secular and long-period variations, while ignoring short-period oscillations. While the introduction of J_2 effects changes the formation dynamics dramatically, the control laws developed here require little modification to be effective in a non-spheroidal gravitational field.

Introduction of natural J_2 effects also gives us a new source of control. By varying the inclination of an orbit, we can change the rate of secular drift of the ascending node. We can control the $\delta\Omega$ error by inducing inclination error, in an approach similar to the mean motion control. Recall that the uncontrolled, secular time rate of change of Ω due to J_2 effects is:

$$\dot{\Omega} = -\frac{3}{2}J_2 \left(\frac{r_{eq}}{p}\right)^2 n \cos i \quad (6.1)$$

Assuming $\delta a = 0$, and thus $\delta n = 0$, the relative dynamics are given by the equation:

$$\delta\dot{\Omega} = -\frac{3}{2}J_2 \left(\frac{r_{eq}}{p}\right)^2 n(\cos i - \cos i^*) \quad (6.2)$$

To control the $\delta\Omega$ error, we would like $\dot{\delta\Omega}$ to be:

$$\dot{\delta\Omega} = -K_{J_2}\delta\Omega \quad (6.3)$$

where K_{J_2} is some positive gain. Combining Eqs. (6.2) and (6.3), we can force the desired $\delta\Omega$ rate by targeting a new inclination, i^{**} :

$$i^{**} = \cos^{-1} \left[\frac{2}{3J_2n} \left(\frac{p}{r_{eq}} \right)^2 K_{J_2}\delta\Omega + \cos i^* \right] \quad (6.4)$$

This precession control could result in significant fuel savings. Instead of using thrust impulses, the control allows the spacecraft to coast into the appropriate orbital plane by forcing a small inclination error (and thus a differential secular drift), and correcting that error as the right ascension error is driven to zero. Notice that, as in the mean motion control, the new target inclination, i^{**} converges to the original target inclination, i^* , as the right ascension error goes to zero.

Other remaining work includes refining of the formation master orbit definition. While we are confident that the desired orbits should be calculated from some central master point, it is not clear that the geometric center of the formation, as we define in Section 3.3.2, is the best choice. Future studies should optimize the definition of the master reference point by minimizing the total fuel consumption required to achieve the orbits defined from this point. Note that future iterations of this master point may or may not be natural orbit2, as presented here.

Attitude and orbit dynamics and control coupling is an area requiring significant future work. As spacecraft pointing requirements and formation relative position requirements become more strict, the tasks of attitude and orbit determination and control subsystems become more and more coupled. We can no longer consider the dynamics and control of spacecraft in terms of two subsystems. Rather, we must explore the concept of unified attitude and orbit dynamics and control.

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Vita

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