APPENDICES

I. Dynamic Current Sharing for Paralleled Transformers

(i) Single Forward Diode (Fig. 3.2)

Let: $L_{p1} + L_{s1} = L + DL,\ R_{p1} + R_{s1} = R + DR,\ \ L_{p2} + L_{s2} = L - DL,\ R_{p2} + R_{s2} = R - DR$.

Assume: $i_{m1} = i_{m2} = 0$ and model the diodes have a body resistance of R_b

(a) $t<t_r$:

$$\begin{cases} \dfrac{V_{in}}{n} = (L+DL)\dfrac{di_1}{dt} + (R+DR)i_1 + R_D(i_1+i_2) - L_f\dfrac{di_f}{dt} - R_f i_f - R_b i_f \\ \dfrac{V_{in}}{n} = (L-DL)\dfrac{di_2}{dt} + (R-DR)i_2 + R_D(i_1+i_2) - L_f\dfrac{di_f}{dt} - R_f i_f - R_b i_f \end{cases} \qquad \text{(I-i)}$$

Therefore:

$$\begin{cases} \left[s(L+L_f+DL)+R+R_f+2R_D+DR\right]i_1 + (sL_f+R_f+2R_D)i_2 = \left[V_{in}/n + (R_f+R_D)I_o\right]/s \\ (sL_f+R_f+2R_D)i_1 + \left[s(L+L_f-DL)+R+R_f+2R_D-DR\right]i_2 = \left[V_{in}/n + (R_f+R_D)I_o\right]/s \end{cases}$$

Neglect Δ^2 terms, the s-domain solutions are:

$$i_{1,2}(s)=\frac{1}{L+2L_f}\frac{\left[(s+\frac{R}{L})\mp\frac{DL}{L}(s+\frac{DR}{DL})\right]\left[\frac{\frac{V_{in}}{n}+(R_f+R_D)I_o}{s}\right]}{\left[s+\frac{R+2R_f+4R_D}{L+2L_f}\right](s+\frac{R}{L})}+\frac{i_{1,2}(0)}{s+\frac{R}{L}}$$

$$\pm\frac{Ri_{1,2}(0)}{(L+2L_f)}\frac{\frac{DL}{L}-\frac{DR}{R}}{\left[s+\frac{R+2R_f+4R_D}{L+2L_f}\right](s+\frac{R}{L})}-\frac{I_o}{L+2L_f}\frac{(R_f+2R_D)(1\mp\frac{DL}{L})+(R\mp DR)\frac{L_f}{L}}{\left[s+\frac{R+2R_f+4R_D}{L+2L_f}\right](s+\frac{R}{L})}$$

Let $t_1=\frac{L+2L_f}{R+2R_f+4R_D}, t_2=\frac{L}{R}$. Since during this time interval: $t\ll t_1,t_2$,

$$\begin{cases} i_{1,2}=i_{1,2}(0)+\frac{I_o}{2}\left[1\mp\frac{DL}{L})\right]\frac{t}{t_r}; \\ i_f=I_o(1-\frac{t}{t_r}); \end{cases} \quad \text{(I-ii)}$$

where $t_r=I_o/\left(\frac{2}{L+L_f}\frac{V_{in}}{n}\right)$.

(b) DTs > t > t_r (i_f = 0):

$$\begin{cases} (L+DL)\frac{di_1}{dt}+(R+DR)i_1=(L-DL)\frac{di_2}{dt}+(R-DR)i_2 \\ i_1+i_2=0 \end{cases} \quad \text{(I-iii)}$$

$$i_{1,2}(s)=\frac{R\mp DR}{sL+R}\frac{I_o}{2s}+\frac{L}{sL+R}i_{1,2}(t_r)$$

$$\therefore \quad i_{1,2}(t) = (1 \mp \frac{DR}{R})\frac{I_o}{2}(1 - e^{-\frac{R}{L}(t-t_r)}) + i_{1,2}(t_r)e^{-\frac{R}{L}(t-t_r)} \ . \tag{I-iv}$$

(c) $DT_s + t_f > t > DT_s$ (secondary voltage changes polarity, similar to stage (a)):

$$\begin{cases} i_{1,2}(t) = i_{1,2}(DT_s) - \frac{I_o}{2}\left[1 \mp \frac{DL}{L})\right]\frac{t - DT_s}{t_f}; \\ i_f = I_o \frac{t}{t_f}; \end{cases} \tag{I-v}$$

where $t_f = I_o / \left[\frac{2}{L + L_f}\frac{V_r}{n}\right]$.

(d) $T_s > t > DT_s + t_f$ (similar to stage (b)):

$$i_{1,2}(t) = i_{1,2}(DT_s + t_f)e^{-\frac{R}{L}(t - DT_s - t_f)} \tag{I-vi}$$

Considering the current continuity among stages;

$$\begin{aligned} & i_{1,2}(t_r) = i_{1,2}(0) + \frac{I_o}{2}(1 \mp \frac{DL}{L}), \quad i_{1,2}(DT_s) = \frac{I_o}{2}(1 \mp \frac{DR}{R})(1 - e^{-\frac{R}{L}DT_s}) + i_{1,2}(t_r)e^{-\frac{R}{L}DT_s} \\ & i_{1,2}(DT_s + t_f) = i_{1,2}(DT_s) - \frac{I_o}{2}(1 \mp \frac{DL}{L}), \quad i_{1,2}(0) = i_{1,2}(DT_s + t_f)e^{-\frac{R}{L}D'T_s} \end{aligned} \tag{I-vii}$$

$$\therefore \quad i_{1,2}(0) = \mp \frac{I_o}{2}(\frac{DR}{R} - \frac{DL}{L})\frac{1 - e^{-\frac{R}{L}DT_s}}{e^{\frac{R}{L}D'T_s} - e^{-\frac{R}{L}DT_s}} \tag{I-viii}$$

Since t_r and t_f are very brief compared with switching period, therefore, the average current of each module can be approximated by:

$$\begin{aligned} I_{1,2} &= \frac{1}{T_s}\left[\int_{t_r}^{DT_s} i_{1,2}(t)dt + \int_{DT_s+t_f}^{T_s} i_{1,2}(t)dt\right] \\ &= \frac{I_o}{2}(1\mp\frac{DR}{R})D - \frac{I_o}{2}(1\mp\frac{DR}{R})\frac{L\cdot f_s}{R}(1-e^{-\frac{R}{L}DT_s}) \\ &\quad + \left[i_{1,2}(0)+\frac{I_o}{2}(1\mp\frac{DL}{L})\right]\frac{L\cdot f_s}{R}(1-e^{-\frac{R}{L}DT_s}) + i_{1,2}(0)e^{\frac{R}{L}D'T_s}\frac{L\cdot f_s}{R}(1-e^{-\frac{R}{L}D'T_s}) \end{aligned} \quad \text{(I-ix)}$$

$$I_{1,2} = \frac{I_o}{2}(1\mp\frac{DR}{R})D \quad \text{(I-x)}$$

$$\frac{I_1}{I_2} = \frac{R-DR}{R+DR} = \frac{R_2}{R_1} \quad \text{(I-xi)}$$

(ii) Separate Forward Diodes (Fig. 3.4)

(a) $t < t_r$:

$$\begin{cases} \frac{V_{in}}{n} = (L+DL)\frac{di_1}{dt} + (R+DR)i_1 + R_D i_1 - L_f\frac{di_f}{dt} - R_f i_f - R_D i_f; \\ \frac{V_{in}}{n} = (L-DL)\frac{di_2}{dt} + (R-DR)i_2 + R_D i_2 - L_f\frac{di_f}{dt} - R_f i_f - R_D i_f; \end{cases} \quad \text{(I-xii)}$$

$$i_{1,2} = \frac{1}{L+2L_f}\frac{V_{in}/n + (R_f + R_D)I_o}{s\left[s+\frac{R+2R_f+3R_D}{L+2L_f}\right](s+\frac{R+R_D}{L})}\cdot\left[(s+\frac{R+R_D}{L})\mp\frac{DL}{L}(s+\frac{DR}{DL})\right],$$

$$\begin{cases} i_{1,2} = \frac{I_o}{2}\left[1 \mp \frac{DL}{L})\right]\frac{t}{t_r}; \\ i_f = I_o\left(1 - \frac{t}{t_r}\right); \end{cases} \tag{I-xiii}$$

where $t_r = \frac{(L + 2L_f)I_o}{2\frac{V_{in}}{n}}$.

(b) $DT_s > t > t_r$:

$$\begin{cases} (L + DL)\frac{di_1}{dt} + (R + DR + R_D)i_1 = (L - DL)\frac{di_2}{dt} + (R - DR + R_D)i_2 \\ i_1 + i_2 = I_o \end{cases} \tag{I-xiv}$$

$$\therefore \qquad i_{1,2}(t) = (1 \mp \frac{DR}{R + R_D})\frac{I_o}{2}(1 - e^{-\frac{R+R_D}{L}(t-t_r)}) + i_{1,2}(t_r)e^{-\frac{R+R_D}{L}(t-t_r)} \tag{I-xv}$$

(c) $DT_s + t_f > t > DT_s$:

$$i_{1,2}(t) = i_{1,2}(DT_s) - \frac{I_o}{2}\left[1 \mp \frac{DL}{L})\right]\frac{t - DT_s}{t_f}, \tag{I-xvi}$$

where $t_f = I_o / \left[\frac{2}{L + L_f}\frac{V_r}{n}\right]$.

(d) $T_s > t > DT_s + t_f$ (similar to stage (II)):

$$i_{1,2}(t) = 0 \tag{I-xvii}$$

The average currents of the transformers are:

$$I_{1,2} = \frac{1}{T_s} \int_{t_r}^{DT_s} i_{1,2}(t)dt$$

$$= \frac{I_o}{2}(1 \mp \frac{DR}{R+R_D})\left[D - \frac{L \cdot f_s}{R}(1-e^{-\frac{R+R_D}{L}DT_s})\right] + \frac{I_o}{2}(1 \mp \frac{DL}{L})\frac{L \cdot f_s}{R+R_D}(1-e^{-\frac{R+R_D}{L}DT_s})$$

$$= \frac{I_o}{2}\left[(1 \mp \frac{DR}{R+R_D})D \pm \frac{L \cdot f_s}{R+R_D}(\frac{DR}{R+R_D} - \frac{DL}{L})(1-e^{-\frac{D(R+R_D)}{L \cdot f_s}})\right]. \qquad \text{(I-xviii)}$$

Not only that separate diodes reduces the resistance deviation, but also they eliminate the oscillation between modules during the off-time. Therefore under the condition $R + R_D \gg L \cdot f_s$, Eq. (I-xviii) is reduced to

$$I_{1,2} = \frac{I_o}{2}(1 \mp \frac{DR}{R+R_D})D\,. \qquad \text{(I-xix)}$$

II. Electro-Thermal Analysis of Current Sharing

Taking linear approximation, the *i*-*v* characteristic of a diode can be expressed as:

$$i = AT^2 e^{-qF_m/kT}(e^{qv/kT} - 1) \cong B(1 + KDT)e^{qv/kT_o} = C(DT)e^{qv/kT_o}\,. \tag{II-i}$$

With the same voltage drop, *v*, across two paralleled Schottkies, the current distribution is

$$I = I_1 + I_2 = (C_1 + C_2)e^{qv/kT_o}\,.$$

$$\therefore \quad \begin{cases} I_1 = \dfrac{C_1}{C_1 + C_2} I; \\ I_2 = \dfrac{C_1}{C_1 + C_2} I. \end{cases} \tag{II-ii}$$

Considering the dispersion of the device parameter between the two Schottkies, it can be assumed that

$$\begin{cases} C_1 = (B_o + \dfrac{DB}{2})(1 + KDT_1) = B_o(1 + \dfrac{d}{2})(1 + KDT_1); \\ C_2 = (B_o - \dfrac{DB}{2})(1 + KDT_2) = B_o(1 - \dfrac{d}{2})(1 + KDT_2). \end{cases} \tag{II-iii}$$

Thus, the current sharing unbalance is

$$D \equiv \frac{I_1 - I_2}{I} = \frac{C_1 - C_2}{C_1 + C_2} = \frac{d + K\left[(DT_1 - DT_2) + \dfrac{d}{2}(DT_1 + DT_2)\right]}{2 + K\left[(DT_1 + DT_2) + \dfrac{d}{2}(DT_1 - DT_2)\right]}\,. \tag{II-iv}$$

The electro-thermal relation ship can be derived by solving the thermal network, shown in Fig. 3.5:

$$R_b(P_1 - x) - R_b(P_2 + x) = R_c x \quad \Rightarrow \quad x = \frac{P_1 - P_2}{2 + \frac{R_c}{R_b}};$$
$$DT_1 = P_1 R_a + (P_1 - x)R_b = P_1(R_a + R_b) - \frac{R_b}{2 + \frac{R_c}{R_b}}(P_1 - P_2);$$

$$DT_2 = P_2(R_a + R_b) + \frac{R_b}{2 + \frac{R_c}{R_b}}(P_1 - P_2). \qquad \text{(II-v)}$$

The power loss of each individual diode is the product of its forward voltage drop and its conduction current:

$$\begin{cases} P_1 = vI_1 = \dfrac{C_1}{C_1 + C_2} vI; \\ P_2 = vI_2 = \dfrac{C_2}{C_1 + C_2} vI. \end{cases} \qquad \text{(II-vi)}$$

Combining Eqs. (II-v) and (II-vi), it can be derived that

$$\begin{cases} DT_1 + DT_2 = (R_a + R_b)(P_1 + P_2) = (R_a + R_b)vI \\ DT_1 - DT_2 = (R_a + R_b - \dfrac{2R_b}{2 + \frac{R_c}{R_b}})(P_1 - P_2) = (R_a + R_b - \dfrac{2R_b}{2 + \frac{R_c}{R_b}})vDI \, . \end{cases} \qquad \text{(II-vii)}$$

Combing Eqs. (II-iv) and (II-vii) gives the current sharing unbalance due to the electro-thermal interaction:

$$D = \frac{d + KvI\left[\frac{d}{2}(R_a + R_b) + (R_a + R_b - \frac{2R_b}{2 + \frac{R_c}{R_b}})D\right]}{2 + KvI\left[(R_a + R_b) + \frac{d}{2}(R_a + R_b - \frac{2R_b}{2 + \frac{R_c}{R_b}})D\right]} = \frac{(2 + a)\frac{d}{2} + bD}{(2 + a)\frac{d}{2} + b\frac{d}{2}D}, \qquad \text{(II-viii)}$$

where $\begin{cases} a = KvI(R_a + R_b) \\ b = KvI(R_a + R_b - \frac{2R_b}{2 + \frac{R_c}{R_b}}) \end{cases}$. Current unbalance, D, can be determined by solving the 2nd-order Eq. (II-viii)

$$D = \frac{-(2 + a - b) + \sqrt{(2 + a - b)^2 + (2 + a)bd^2}}{bd} \approx \frac{1}{bd}\frac{1}{2}\frac{(2 + a)bd^2}{(2 + a - b)} = \frac{(2 + a)}{(2 + a - b)}\frac{d}{2} \qquad \text{(II-ix)}$$

$$\therefore \qquad D = \left[\frac{1 + \frac{KvI(R_a + R_b)}{2}}{1 + \frac{KvIR_b}{2 + \frac{R_c}{R_b}}}\right]\frac{d}{2} \qquad \text{(II-x)}$$

By a uniform conduction approximation, the thermal coupling resistance can be calculated as

$$R_c = \frac{1}{K_{ht}}\frac{d}{w \cdot t}, \qquad \text{(II-xi)}$$

where K_{th} is the thermal conductivity of the heatsink material, w and t are the width and thickness of the heatsink respectively, and d is the distance between the two paralleled diodes. Therefore, Eq. (II-x) can be written as

$$D = \left[\frac{1 + \dfrac{KvI(R_a + R_b)}{2}}{1 + \dfrac{KvIR_b}{2 + \dfrac{d}{K_{th} wtR_b}}} \right] \frac{d}{2}, \qquad \text{(II-xii)}$$

i.e., the current distribution unbalance expressed in terms of the diode separation. Eqs. (II-x) and (II-xii) are plotted in Fig. 3.6 to graphically show the effects of the thermal coupling (or the diode separation) on the current unbalance.

III. Partial Element Equivalent Circuit Method

The loop inductance of the rectangular loop formed by four traces, shown in Fig. III.1(a), is by definition as

$$L_{loop} = \frac{1}{I}\oiint_{s} \vec{B} \cdot d\vec{s} , \tag{III-i}$$

where I is the conduction current, $\vec{B}$ is the magnetic flux intensity, and s is the loop area. Using the Maxwell equation, $\vec{B} = \nabla \times \vec{A}$, and the Stoke's theorem, $\oiint_{s}\left(\nabla \times \vec{X}\right) \cdot d\vec{s} = \oint_{c} \vec{X} \cdot d\vec{l}$, L_{loop} can be rewritten as

$$L_{loop} = \frac{1}{I}\oint_{c} \vec{A} \cdot d\vec{l} = \frac{1}{I}\left[\sum_{n=1}^{4} \int_{l_n} \vec{A} \cdot d\vec{l}\right]. \tag{III-ii}$$

where $\vec{A}$ is the magnetic potential, and can be decomposed into the contribution from each traces, i.e.,

$$\vec{A} = \sum_{m=1}^{4} \vec{A}_{l_m} , \tag{III-iii}$$

where $\vec{A}_{l_n}$ is the magnetic potential generated by the current flowing through l_n. Then, Eqs. ((III-ii) and (III-iii) give

$$L_{loop} = \frac{1}{I}\left[\sum_{n=1}^{4} \sum_{m=1}^{4} \int_{l_n} \vec{A}_{l_m} \cdot d\vec{l}\right]. \tag{III-iv}$$

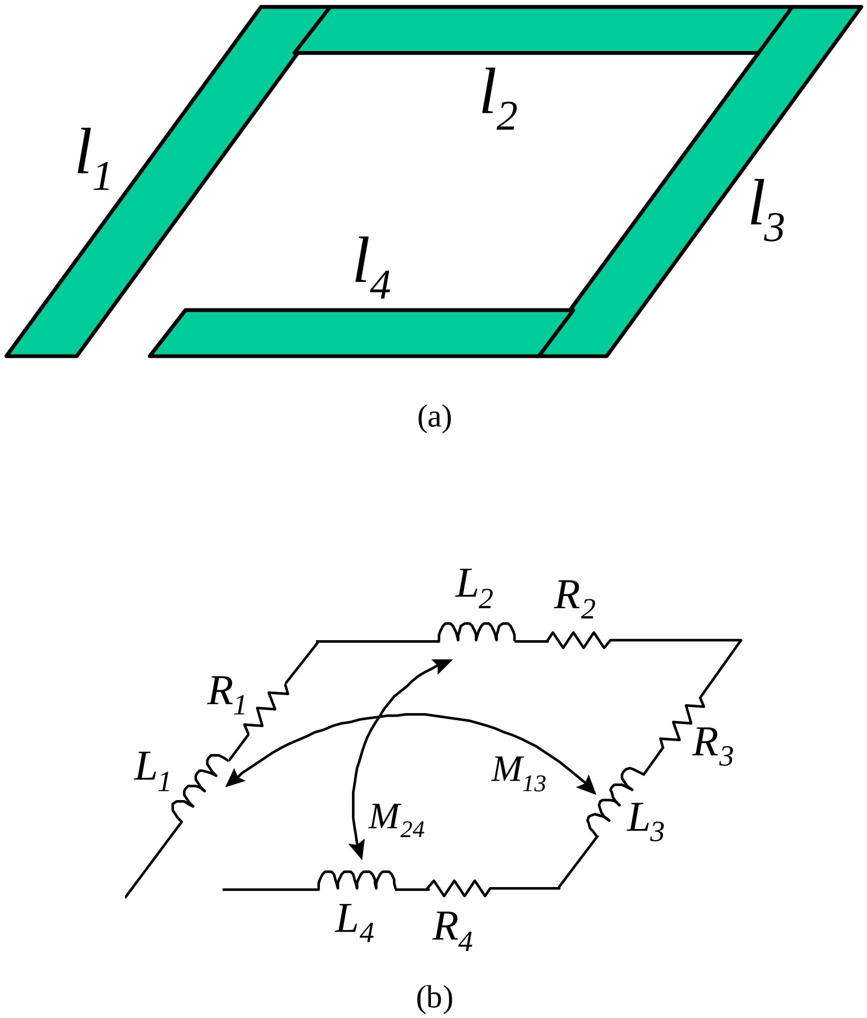

Figure III.1. A rectangular shaped loop described with PEEC method: (a) loop layout; (b) equivalent circuit representation.

Let $L_n = \frac{1}{I}\int_{l_n} \vec{A}_{l_n} \cdot d\vec{l}$, and $M_{mn} = \frac{1}{I}\int_{l_m} \vec{A}_{l_n} \cdot d\vec{l}$ ($m \neq n$), Eq. (III-iv) becomes

$$L_{loop} = \frac{1}{I}\left[\sum_{n=1}^{4} L_n + \sum_{n=1}^{4}\sum_{\substack{m=1 \\ m\neq n}}^{4} M_{mn}\right]. \tag{III-v}$$

Therefore, the loop inductance can be expressed as the combination of the trace partial self-inductance L_n, and partial mutual-inductance, M_{mn} ($M_{mn} = M_{nm}$). The representation of using equivalent circuit is shown in Fig. III.1(b). It should be noted that in this rectangular loop example, only the paralleled traces have mutual inductance between them, i.e., $M_{12} = M_{14} = M_{23} = 0$, because perpendicular traces have null induced fields.