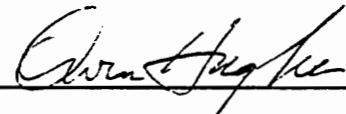


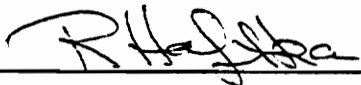
Elastic and Inelastic Analysis of Panel Collapse by Stiffener Buckling

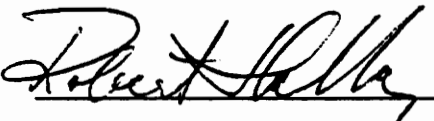
by
Ming Ma

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in
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Elastic and inelastic analysis of panel collapse by stiffener buckling

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Aerospace and Ocean Engineering

ABSTRACT

A method is developed for analyzing the flexural-torsional and lateral-torsional buckling ("tripping") behavior of flanged stiffeners subjected to axial force, end moment, lateral pressure and any combination of these. The effects of cross-sectional distortion, postbuckling behavior of the plate (incorporated by considering the plate effective width), initial imperfections and plasticity are included.

The method uses the Rayleigh-Ritz approach. Based on an assumed strain distribution, a displacement field is obtained for the tripping model, and the total potential energy functional is then derived. The strain distribution assumptions coincide with van der Neut's assumption. However, unlike the somewhat obscure differential equation approach given by van der Neut, this study provides a simple, clear, energy approach. Also the resulting method is applicable in the inelastic range, which is not possible with van der Neut's approach.

Both the rigid web case and the flexible web case are studied. The effect of plate rotational restraint in the elastic range is accounted for. The method requires only four degrees of freedom and therefore the solution process is rapid. In order to verify the method in the elastic range, a number of sample stiffened panels are analyzed using the ABAQUS finite element program; the results are in quite good agreement. An inelastic tripping model is then developed based on the established elastic model, using deformation theory. Results obtained using the inelastic tripping method are shown to be in good agreement with experimental results, and to be more accurate than other methods.

In loving memory of my mother Lan-feng Hou

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Notation

A_k	Zone cross-section area of stiffener (Fig. 5.5-1)
A_f	Cross-section area of stiffener flange
A_w	Cross-section area of stiffener web
A_p	Cross-section area of plate
A_s	$A_f + A_w$, total cross-section area of stiffener
A	$A_f + A_w + A_p$, total cross-section area of per stiffener pitch
A_{uv}, A_{uw}	Cross-sectional parameters due to axial shortening
$\left. \begin{array}{l} \alpha_{\theta\theta}^a, \alpha_{ww}^a, \alpha_{\theta w}^a \\ \alpha_{\theta\theta}^m, \alpha_{ww}^m, \alpha_{\theta w}^m \\ \alpha_{\theta\theta}^q, \alpha_{ww}^q, \alpha_{\theta w}^q \end{array} \right\}$	Parameters in geometric stiffness
$\left. \begin{array}{l} \alpha_{v_T v_T}^T, \alpha_{ww}^T \\ \alpha_{\varphi_T \varphi_T}^T, \alpha_{\varphi_B \varphi_B}^T \\ \alpha_{w \varphi_B}^T, \alpha_{v_T \varphi_T}^T \\ \alpha_{v_T \varphi_B}^T, \alpha_{\varphi_T \varphi_B}^T \end{array} \right\}$	Tangent cross-section parameters in geometric stiffness
b, b_e, b_σ	Uniform stiffener spacing, plate effective width and plate effective breadth
β	Plate slenderness ratio
$\left. \begin{array}{l} b_f, h, b \\ t_f, t_w, t_p \end{array} \right\}$	defined in the Figure N-1.

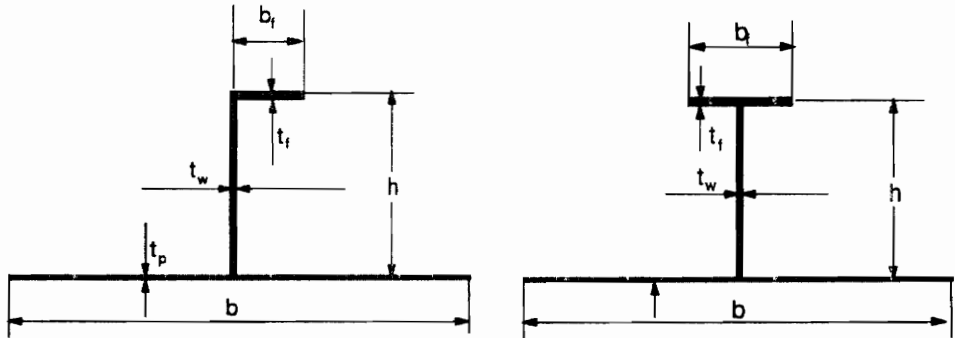


Fig. N-1, Geometric parameters of stiffened panels

δ	Vector of degrees of freedom ($\delta = \{\bar{\varphi}_B \quad \bar{v}_T \quad \bar{\varphi}_T \quad \bar{w}\}^T$)
δ_0	Vector of initial deformation ($\delta_0 = \{\bar{\varphi}_{0B} \quad \bar{v}_{0T} \quad \bar{\varphi}_{0T} \quad \bar{w}_0\}^T$)
D_p	Flexural rigidity of plate
D_w	Flexural rigidity of web
e	Distance between the neutral axis and the load position
E	Young's modulus
E_t	Tangent modulus
E_{ts}	Structural tangent modulus
ϵ^c	Plate mid-surface strains. $\epsilon^c = [\epsilon_x^c \quad \epsilon_x^c \quad \gamma_{xy}^c]^T$
ϵ^l	Linear portion of the strains
ϵ^{nl}	Nonlinear portion of the strain
$\epsilon_f^l, \epsilon_w^l, \epsilon_p^l$	Linear strains of flange, web and plating with respect to x -axis
$\epsilon_f, \epsilon_w, \epsilon_p$	Strains of flange, web and plating with respect to x -axis
$\epsilon_r, \bar{\epsilon}_r$	Residual strains and generalized residual strain
$\epsilon_a, \epsilon_q, \epsilon_m$	Strains due to axial compression, uniform lateral pressure and pure bending
ϵ_d	Strain due to initial deformation
ϵ_e	Strain due to eccentricity of applied axial load
G	Elastic shear modulus
G_t	Tangent shear modulus
g_{ij}	Element of geometric stiffness matrix K_G ($i, j=1..4$)
f_1, f_2, f_3	Shape function defined in equation (2.2-3).
I	Effective vertical moment of inertia of stiffener and associated effective width of plating
I_p	Polar moment of inertia of stiffener about toe
I_{zy}	Product of inertia of panel respect to yz -plane
I_z	Moment of inertia of panel respect to z -axis

I_f	Moment of inertia of stiffener flange about web plane (z-axis)
J_s	St. Venant torsion constant for the stiffener
J_w	St. Venant torsion constant for the web
J_f	St. Venant torsion constant for the flange
$I^T, I_z^T, I_{zy}^T, J_f^T$	Tangent cross-section properties
κ^c	Plate mid-surface curvatures. $\kappa^c = [\kappa_x^c \quad \kappa_x^c \quad \kappa_{xy}^c]^T$
k	Zone index.
k_{ij}	Element of linear stiffness matrix K_L ($i, j = 1..4$)
K_G	Geometric stiffness matrix
K_G^a, K_G^m, K_G^q	Coefficient of geometric stiffness matrix due to axial compression, end moment and uniform lateral pressure respectively
$K_{GW}^a, K_{GF}^a, K_{GP}^a$	Coefficient of geometric stiffness matrix of web, flange and plating due to axial compression
$K_{GW}^m, K_{GF}^m, K_{GP}^m$	Coefficient of geometric stiffness matrix of web, flange and plating due to end moment
$K_{GW}^q, K_{GF}^q, K_{GP}^q$	Coefficient of geometric stiffness matrix of web, flange and plating due to uniform lateral pressure
$K_{GW}^{\bar{v}_T}, K_{GF}^{\bar{v}_T}, K_{GP}^{\bar{v}_T}$	Geometric stiffness matrix of web, flange and plating due to initial sideways bending
$K_{GW}^{\bar{w}}, K_{GF}^{\bar{w}}, K_{GP}^{\bar{w}}$	Geometric stiffness matrix of web, flange and plating due to initial vertical bending
K_G^L	Geometric stiffness matrix due to eccentricity of axial compression
K_L^0	Linear stiffness matrix
K_L	Generalized linear stiffness matrix
K_{LW}	Linear stiffness matrix of web
K_{LFP}	Linear stiffness matrix of flange and plating
K_L^{upper}	Upper bound linear stiffness matrix

K_G^{upper}	Upper bound geometric stiffness matrix
$K_G^{\bar{v}_T}$	Geometric stiffness matrix due to initial sideways bending
$K_G^{\bar{w}}$	Geometric stiffness matrix due to initial vertical bending
$\bar{K}_G$	Geometric stiffness matrix including plate rotational restraint
$\bar{K}_L$	Linear stiffness matrix including plate rotational restraint
K_L^p	Linear stiffness matrix due to plate-out-of plane deformation
K_{2L}	Linear stiffness matrix of two stiffeners including plate-out-of-plane deformation
K_{2G}	Geometric stiffness matrix of two stiffeners including plate-out-of-plane deformation
K_{LS}	Linear stiffness matrix of two stiffeners with plate deforming S-shape
K_{GS}	Geometric stiffness matrix of two stiffeners with plate deforming S-shape
K_{LU}	Linear stiffness matrix of two stiffeners with plate deforming U-shape
K_{GU}	Geometric stiffness matrix of two stiffeners with plate deforming U-shape
K_T	Tangent stiffness matrix
K_{ϕ_B}	Plate rotational spring
k_{spring}	Plate rotational spring constant
l	Length of stiffener between transverse supports
λ	$\frac{m\pi}{l}$
m	Mode number = number of half waves lengthwise
M	Vertical bending moment
M_{cr}	Elastic tripping moment
M_{cr}^a	Tripping moment of the angle stiffener
M_{cr}^t	Tripping moment of the tee stiffener
N_f, N_w, N_p	Number of flange, web and plate zones, respectively
Π_T	Total potential energy

Π_a	Total potential energy due to axial deformation
$\Pi_{plate-out-of-plane}$	Total potential energy due to plate-out-of-plane deformation
Π	Total potential energy due to bending-torsional deformation
p	Uniform lateral pressure
q	Equivalent line load
q_{cr}^a	Tripping uniform lateral pressure of the angle stiffener
q_{cr}^t	Tripping uniform lateral pressure of the tee stiffener
Ω	Potential energy of the applied load
ρ	Curvature
ρ_{v_T}	Curvature due to sideways bending
ρ_w	Curvature due to vertical bending
ρ_e	Curvature due to eccentricity of applied axial load
σ	Applied axial stress
σ_{avg}	Average plate stress
$\sigma_a, \sigma_m, \sigma_q$	Stress due to axial compression, uniform lateral pressure and pure bending
σ_d	Stress due to initial deformation
$\sigma_f, \sigma_w, \sigma_p$	Axial stress of the flange, web and plate
σ_{cr}	Elastic tripping stress
σ_{cr}^a	Tripping stress of angle stiffeners
$\sigma_{cr}^a{}_{iso}$	Tripping stress of isolated pinned angle stiffeners
σ_{cr}^t	Tripping stress of tee stiffeners
σ_m	Mean stress
σ_{pbe}	Elastic plate buckling stress
σ_{spl}	Structural proportional limit
σ_r	Residual stress
$\bar{\sigma}_r$	Generalized residual stress

σ_U	Euler buckling stress
σ_{ult}	Ultimate stress
σ_Y	Yield stress
σ_{Ys}	Yield stress of stiffener
σ_{Yp}	Yield stress of plating
σ_{Ym}	Mean yield stress
S_f	$\int_{A_s} y dA$ of stiffener
$S_{\theta\theta}, S_{ww}, S_{\theta w}$	Parameters in stiffness
θ	defined in Fig. 3.1-1
U_T	Total strain energy during deformation
U_T^f, U_T^w, U_T^p	Total strain energy of flange, web and plating
U	Strain energy due to bending-torsional deformation
$u_{web-out-of-plane}$	Web strain energy due to out-of-plane deformation
V	Work done due to bending-torsional deformation
y_o, z_o	Defined in Fig. 2.1-4 and 2.1-5
$\left. \begin{matrix} x, y, z \\ v_T, \phi_T, \phi_B \end{matrix} \right\}$	Defined in Fig. 2.2-1
$\left. \begin{matrix} v_{IT}, \phi_{IT}, \phi_{IB} \\ v_{rT}, \phi_{rT}, \phi_{rB} \end{matrix} \right\}$	Defined in Fig. 3.3-9
$\left. \begin{matrix} u_f, v_f, w_f \\ u_w, v_w, w_w \\ u_p, v_p, w_p \end{matrix} \right\}$	Displacements of flange, web and plating
w_{pout}	Plate-out-of-plane displacement

1.0 Introduction

A stiffened panel is the most common structural member in marine structures, and especially in ships. The stiffeners must carry local bending moments arising from cargo loads and water pressure (with either the stiffener flange or the plating in compression) and the panel cross section must carry axial compression arising from the overall bending of the hull girder. The way in which a stiffened panel will buckle when subjected to these loads falls into three categories:

1. Plate buckling. Plates may buckle between stiffeners, as shown in Fig. 1.1-1, and the stiffeners adjacent to them carry most of the additional load beyond first buckling. Local buckling of the plating between stiffeners is not very sensitive to stiffener shape. Moreover, local plate buckling usually does not result in the immediate collapse of a panel. The prediction of plate buckling is well established for many load cases and has been used successfully for design.

2. Panel overall buckling or Euler buckling. The stiffeners buckle along with the plating, as shown in Fig. 1.1-2. This happens mostly when the plating is under compression, but overall buckling may also occur when the stiffener is under compression. However, stiffeners are more susceptible to either flexural-torsional and lateral-torsional buckling; in ship structures the latter is often referred to as “tripping”. If there is no lateral load, then the direction of collapse —towards the plate or towards the stiffener— is decided mainly by the initial imperfection of the panel. Panel overall buckling is mainly a column type of collapse, and the Perry-Robertson formula (page 396 in [H3]) can accurately predict the failure.

3. Stiffener tripping or lateral-torsional buckling. As shown in Fig. 1.1-3, a stiffener may buckle by twisting about its line of attachment to the plating. This is commonly referred to as “tripping” and it occurs when the stiffener is under compression. The plate may rotate somewhat to accommodate the stiffener rotation, and the direction of rotation often alternates between adjacent stiffeners. Tripping is distinguished from plate

buckling by the fact that in the latter case the stiffener flange remains nearly unrotated and straight (Fig. 1.1-1).

1.1 Motivation

Tripping is regarded as panel collapse because once tripping occurs the plating is left with no stiffening and therefore collapse follows immediately. Also, tripping is often a quite sudden phenomenon and hence it is an undesirable mode of collapse. In many design codes tripping is ignored or is considered as a secondary mode of failure. This is mainly because for typical panel proportions and no imperfections, linear elastic buckling theory predicts a higher stiffener tripping stress than plate buckling stress. This is borne out by linear elastic finite element analysis and folded plate analysis of perfect panels. However, practical experience and realistic experiments [S3] have shown that stiffener tripping is a common failure in stiffened panel structures. Also, tripping and plate buckling often interact. The buckling sequence is usually plate buckling first, followed by stiffener tripping. This is because plate buckling stress is generally lower than stiffener torsional buckling stress. However, the buckling sequence may occur in either order, depending on the stiffener and plating proportions. It is very important to consider this mode of buckling and to provide an adequate margin of safety.

Local plate buckling and panel overall buckling have been well studied and can be predicted quite well. However, there is very little information in the literature on the tripping of stiffened panels. The few formulations that exist are mainly adaptations of Timoshenko's lateral-torsional thin-walled beam theory [T2], the main modification to that theory being an enforced axis of rotation, instead of the natural rotational axis of the beam cross section (Bleich [B4], Gjelsvik [G2]).

In the earliest work on the subject (Windenburg [W1], Bleich [B4]), one of the basic assumptions that has been made in deriving explicit expressions for stiffener torsional buckling stress is that the stiffener is attached to the plating in a simply supported manner,

and therefore the plate does not provide any rotational restraint. At first look this may appear to be a conservative criterion, especially when the stiffener is considered in isolation. However, plate flexural buckling stress is generally lower than stiffener torsional buckling stress. Thus, when the plate starts to buckle it will twist the stiffener to accommodate the plate's rotation. This will destabilize the stiffener, so that its torsional buckling stress will be reduced.

In actual practice, plate post-buckling and plasticity complicate the response considerably. After plate initial buckling, further loading of the panel will result in plate post-buckling behavior, and the elastic large-deflection response is quickly modified by plasticity. Plasticity usually begins first and grows most rapidly in the region of the junction between stiffener and plate, and the stiffener will then act as an isolated column. This isolated action was shown in Smith's test [S2], in which the stiffener buckled in its prime mode (one half wave) while the plate buckled in several half waves.

Conversely, in some cases the plating provides rotational restraint to the stiffener, even after plate buckling, depending on the relative proportions and the degree of plasticity, and some treatments have considered this [A1, F2].

Research on stiffened panels has been carried out in the aircraft industry, but it is not possible to apply the results directly to steel structures because of the different material properties and structural configurations, and because most of the work has been on purely elastic buckling. Unlike aircraft structures, steel structures such as ship panels tend to buckle in the plastic range.

1.2 Literature Review

There are three main types of stiffeners in ship panels:

- flat-bar or bulb flat stiffeners (Fig. 3.0-2(A))
- tee stiffeners
- angle stiffeners

(1) flat-bar stiffeners

Panels with flat bar and bulb section stiffeners have been given much attention in naval and civil engineering. It is common practice to use these types of stiffeners in merchant vessels. Tests have been performed for ship-type structures at Cambridge University [R2] and Det norske Veritas [D2]. An extensive series of tests relating to the civil engineering field (box girder bridge) was carried out at Manchester University by Horne [H2]. Murray's [M4] experiments showed that the collapse of an axially compressed stiffened panel can be very abrupt if the panel deflects away from the stiffener. The stiffeners trip at or near the maximum compressive load, after which the load carrying capacity of the panel falls rapidly. Michelutti [M2, M3] tested 14 full-scale nominally identical panels, with bulb-flat stiffeners. An interactive failure curve of axial load versus transverse bending moment was obtained. It was shown that when the transverse loading was arranged so that the tip of the stiffener was in compression, the criterion of failure which gave reliable results was that of first yield at that point. Murray [M4] suggested a rigid-plastic model from which a set of P - δ curves was obtained. A sensitivity study showed a 'threshold' of load at about $0.6P_Y$, where P_Y is the squash load ($=\sigma_Y \times \text{area of cross-section of panel}$). It was believed that even the most carefully constructed test specimen of the given dimensions would fail at about $0.6P_Y$. The experimentally observed collapse loads of two panels were $0.63P_Y$. Bhat and Wierzbicki [B3] modified the Shanley rigid-plastic model and analyzed post-ultimate-load behavior of a compressed panel. For this part of the response, their results agreed better with the experimental results than Murray's [M4] solution. Other formulas [A1, F1] based on the energy method have been used to provide data for design. To study the nonlinear aspects of the problem comprehensively, Caridis and Frieze [C2] used the finite difference method. The results were in better agreement with tests than current ship structural design formulas. A conclusion was drawn that it is necessary to allow for the effect of plate rotational restraint, particularly for slender flat-bar stiffeners. Ronalds [R3] proposed a design approach supported by a non-linear finite element analysis.

(2) Tee stiffeners

Panels with T-stiffeners are used extensively in warships. Only a few tests have been performed on this type. Some experimental results were obtained from full-scale tests of multi-bay panels (grillages) by Smith [S2]. Two of the grillages, 1a and 1b, failed by tripping. The grillages were nominally identical except 1b was subjected to lateral pressure. A photograph taken at an early stage in the collapse of Grillage 1a shows that plate buckling occurred before panel failure. In the fully developed collapse the plate buckled into three half waves while the stiffener buckled into one half wave. Small-scale one bay panels were also tested by Smith et al. [S5]. Photographs show that the stiffeners buckled in less than one half wave. Faulkner et al. [F3] decided to take the mean of the lower-bound and upper-bound solutions to be the explicit elastic tripping solution. Inelastic effects were approximated using the Ostenfeld-Bleich quadratic parabola. Adamchak [A1] used the Rayleigh-Ritz method to analyze the isolated beam and considered plating as a rotational spring. A classical one-dimensional plastic reduction method was used to get ultimate strength after elastic tripping stress was found. The results were in good agreement with Smith's experimental results [S3]. Danielson et al. [D1] integrated the plating and stiffener together to give an elastic tripping stress. However, the plate received more attention than the stiffener, and the critical stress obtained is the stiffened plate buckling stress rather than the stiffener tripping stress. Drymakis [D3] compared the results obtained from several elastic torsional buckling design formulae for tee-bar stiffeners including some of those mentioned above. He found that Bleich's [B4] formula predicted generally higher stress than any of the other methods. Overall an appreciable amount of scatter was noted when buckling stress was plotted against several geometrical parameters.

(3) Angle stiffeners

Panels with angle stiffeners are popular because they have a substantial flange and yet also have a flat profile on one side, which has fabrication advantages. However, because of the unsymmetrical form of the cross section, in-plane compression in the presence of initial deformation will always be associated with lateral-torsional deformation of the stiffeners, tending to increase outer-fiber stress and hence reduce the loads at which yield and elasto-plastic collapse occurs. Bleich [B4] discusses the earliest results from tests carried out in the field of torsional buckling of panel stiffeners. Experimental work on torsional buckling was first conducted on aluminum panels used in aircraft structures by Wagner and Pretscher. In the 1930s and 1940s, further work was carried out on panels with Z-section and bulb stiffeners. Later tests were conducted using magnesium and steel alloys also for aircraft-type structures. Argyris [A2] attacked the problem in the early fifties. He was aware of the deformation of the cross-section of the stiffeners and accounted for this flexibility by a spring against twist between the plate and the stiffener with rigid cross-section. With this model, however, the destabilizing effect of the free flange is underestimated. van der Neut [V1] proposed a simple method, suitable for evaluation with a pocket calculator, which yields a good approximation of buckling stress and mode. However, both Argyris and van der Neut did not consider plastic behavior or plate post-buckling phenomena. Ostapenko [O1] used the Rayleigh-Ritz method to analyze the tripping strength of asymmetrical stiffeners subjected to a combination of axial and lateral loads. Although first yielding was used as the criterion of ultimate strength, the method is essentially an elastic model. As Smith [S3] pointed out, evaluation of initial elastic buckling behavior may provide a pointer to the likely failure mode, but gives no direct indication of inelastic collapse strength.

In addition to these approximate formulations which are intended for designers, there are several analysis methods that are intended to give a general solution and are based on computers. Folded plate analysis was first introduced by Wittrick and Williams [W2, W3]. The basic idea is to determine the stiffness matrix that relates the amplitudes of

the edge forces and moments to the amplitudes of the corresponding edge displacements and rotations for a single flat component. Once this stiffness matrix is known, the equations of equilibrium of the line junction result in a series of homogeneous simultaneous equations relating the displacements of, and the rotations about, all the line junctions to be constructed. The determinant of the coefficients in these equations must be zero at instability, which gives the buckling criterion as a determinantal equation. With four equations of equilibrium at each line junction, the calculation is quite fast.

The computer code VIPASA and its successor PASCO were implemented based on the method, and are mainly used in the aircraft industry since they can only handle elastic material and initial buckling. Stoll [S6] extended the VIPASA code and made it available for post-buckling analysis. For ship structures, Smith [S2] was the first to use folded plate analysis to solve stiffened plate problems.

The finite strip method has many similarities with folded plate analysis. The main difference is that the finite strip method uses the energy method to get equilibrium algebraic equations, while folded plate analysis uses differential equations. The finite difference method and finite element method are able to model and study the local behavior of a stiffened panel as the grid-mesh is reduced. One of the finite difference techniques, dynamic relaxation, has been used to solve a plate type of problem. The governing equations were originally derived by Von Karman and later modified by Marguerre to allow for initial geometrical imperfections. Panels stiffened by flat bars were analyzed by Caridis and Frieze [C2] using the dynamic relaxation method. Since the finite difference method directly uses governing equations and is able to analyze local deformation, it can give better results than the finite strip method and the folded plate analysis. However, as stiffener configurations get more complicated, such as tee-stiffeners and angle stiffeners, the finite difference method requires more boundary conditions and may cause difficulties in solution convergence. Finally, the finite element method has been around for years and is capable of doing virtually any structural configuration. Notwithstanding the many unanswered questions regarding geometric and material

nonlinearities, the finite element method is probably the most reliable and accurate numerical method.

1.3 Present Study

Failure of steel panels under axial compression and lateral load involves a variety of problems ranging from material properties and manufacturing imperfections to initial instability and post-buckling phenomena. Despite the importance of tripping collapse, present understanding is far from complete.

The objective of this research is to derive an improved model of tripping or lateral-torsional buckling, and to develop a practical method for predicting the tripping collapse of longitudinally stiffened panels, mainly under axial compression but also able to handle any combination of axial compression, pure bending, and uniform lateral pressure.

Unlike most existing tripping research, this study presents numerous finite element method (ABAQUS) results to verify the proposed elastic model. The study begins with an elastic stiffener tripping model. Based on the assumptions of the proposed strain distribution, a displacement field is obtained and the total potential energy functional is derived. For the model without plate rotational restraint, both the rigid web case and the flexural web case are studied. Then the study explores the effect of plate rotational restraint, and a previously unresolved question regarding plate mode shape is resolved. Finally, the material nonlinearity is incorporated in an inelastic buckling model. Because the buckling model involves only four degrees of freedom, the solution is quite fast, making the method suitable for design.

Comparisons with experimental results show that the model is more accurate than previous design methods, and is definitely suitable for estimating the ultimate strength of a panel for design purposes. As with any design method, it is necessarily a compromise between accuracy and practicality. The agreement with experimental results shows that it has achieved a good balance between the two.

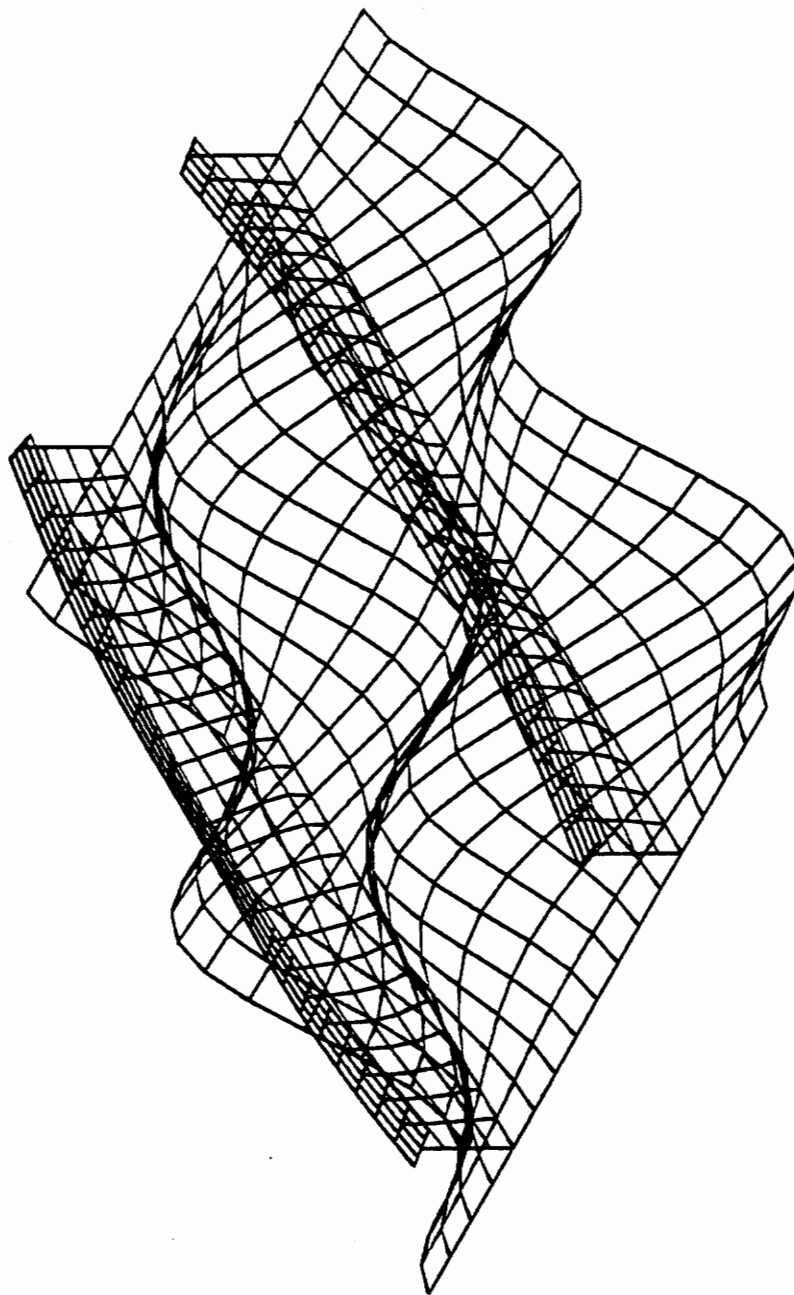


Fig. 1.1-1 Panel plate buckling mode

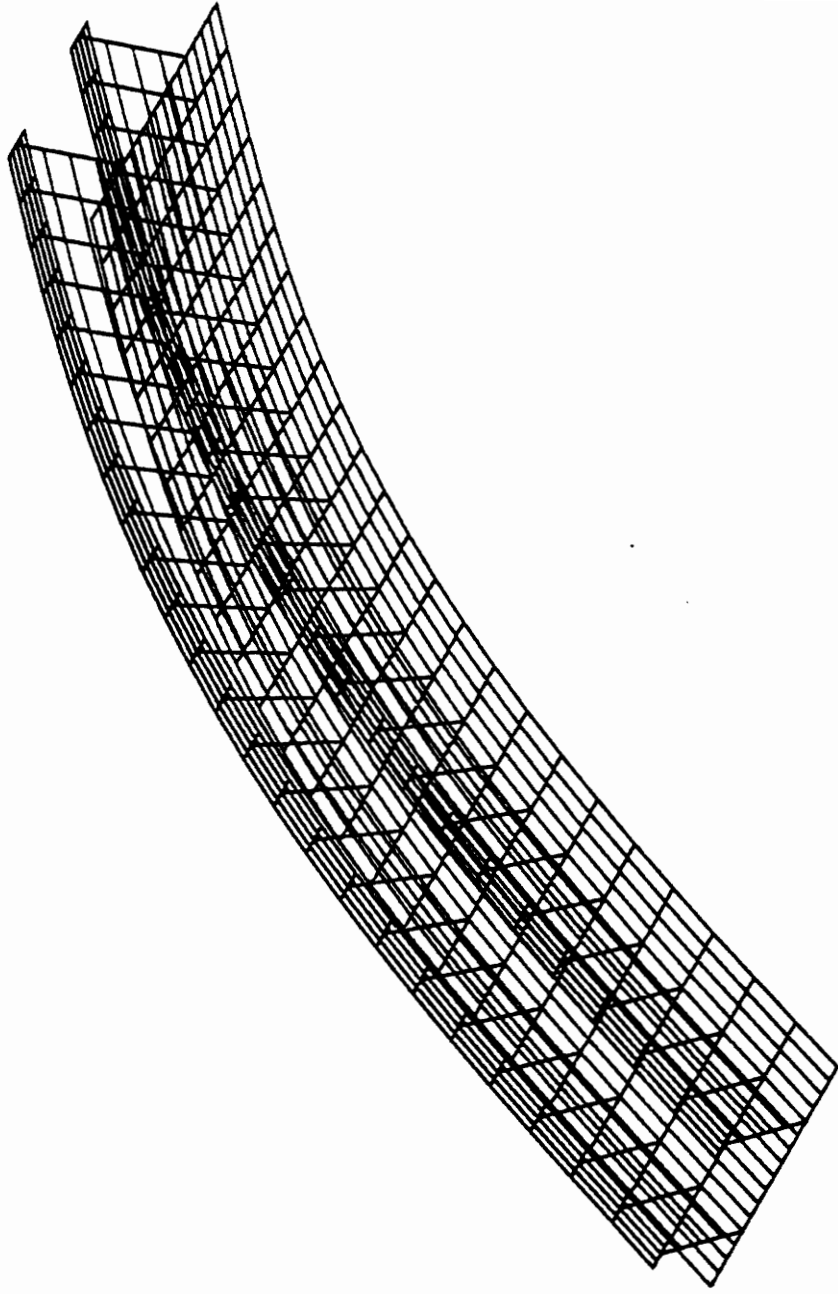


Fig. 1.1-2 Panel Euler buckling mode

ABAQUS

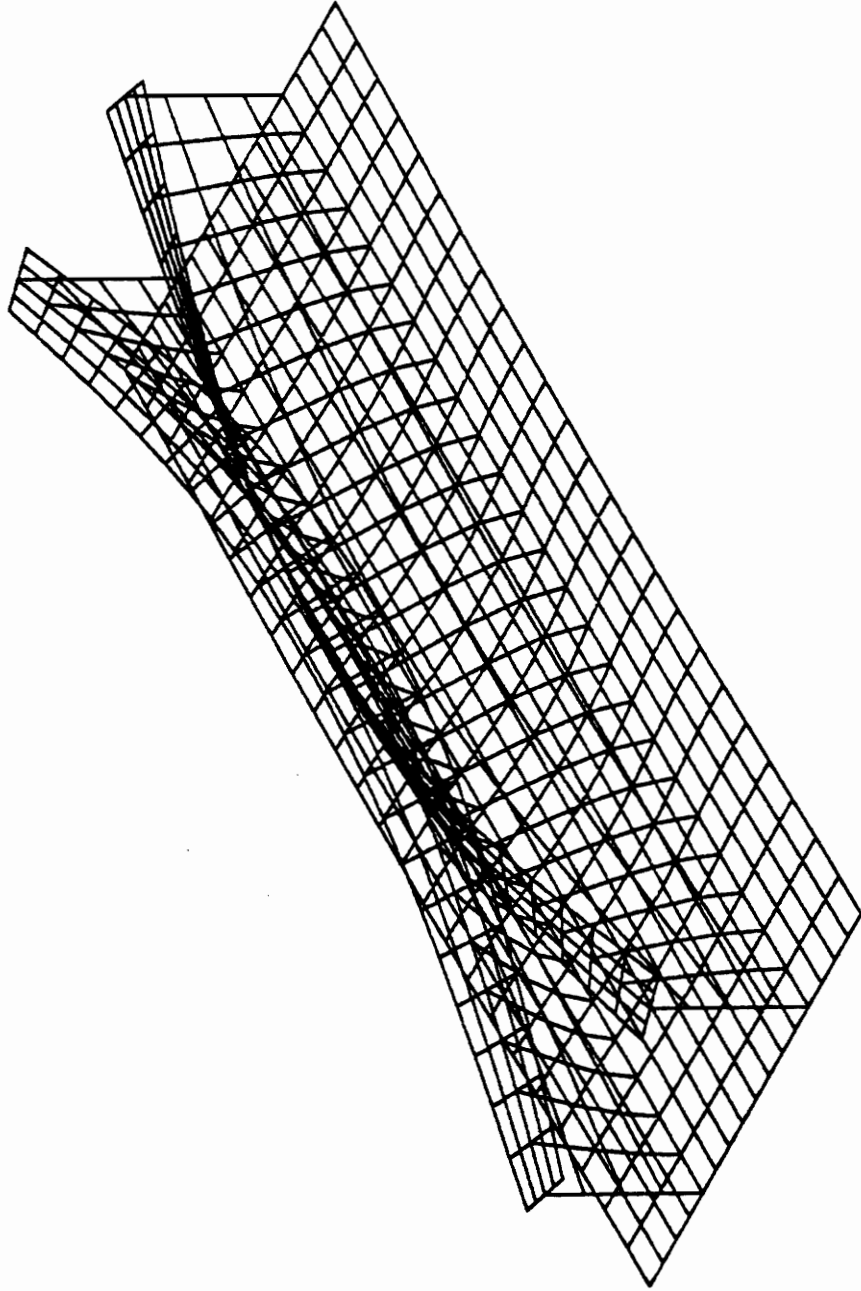


Fig. 1.1-3 Panel tripping mode

2.0 Model for tripping analysis

A panel stiffener and its associated plating can be considered as a special beam-column, where the stiffener flange is free to deflect vertically and sideways but the plating is restricted from deflecting sideways. The energy method will be used throughout this study. Therefore, an appropriate set of displacement fields must first be established. Of the previous studies of tripping, the great majority [A1, B4, D1, F1, F2] dealt with symmetric stiffeners and the plate inplane deformation was ignored, since Euler buckling and tripping are uncoupled for a panel with symmetric stiffeners. However, tripping of asymmetric stiffeners is always accompanied by overall flexural Euler buckling, as indicated by van der Neut [V1]. The plate inplane deformation can no longer be ignored, even if the plate provides no rotational restraint. Therefore, unlike symmetric stiffened panels, the plate must be included in the analytical model.

2.1 Strain distribution assumptions

For ordinary beam-columns (no plating attached to the stiffener), it is well known that vertical bending, sideways bending and torsion are closely coupled for an unsymmetric cross-section beam, and only sideways bending and torsion are closely coupled for a monosymmetric cross-section beam (tee-beam or unsymmetric I-beam). Therefore it is logical to extend these essential characteristics to beams that are attached to plating; i.e., for an asymmetric stiffener (angle and bulb-flat) with plating, vertical bending, sideways bending and torsion are expected to be closely coupled, while for a symmetric stiffener (tee) with plating, only sideways bending and torsion are expected to be closely coupled. The strain distribution of flexural (vertical) bending has been well-established. However, for an unsymmetric beam that is attached to plating, the stress or strain distribution of sideways bending of an asymmetric stiffener has not been clear. Rothwell [R1] and Ostapenko [O1] assumed that bending strain due to sideways bending

of the flange goes to zero at the junction of the web and the plate (see Fig. 2.1-1). The present study has found that, under this assumption, the critical tripping stress for unsymmetric stiffeners is almost independent of overall Euler buckling stress (Fig.2.1-3) whereas in reality, and also as shown by van der Neut [V1], they are closely coupled. In Fig. 2.1-3, when the stiffener is pinned, which is modeled as simply supported at the base of the web in ABAQUS (Fig. 3.1-2), it implies that Euler buckling would never occur and the upper bound of tripping stress is obtained. When the plate width is taken as a finite value (203.2 mm in the example), then the coupling between Euler buckling and tripping causes the tripping stress to be considerably smaller than the upper bound tripping stress. However, this is not the case if the strain assumption of Rothwell [R1] and Ostapenko [O1] is used.

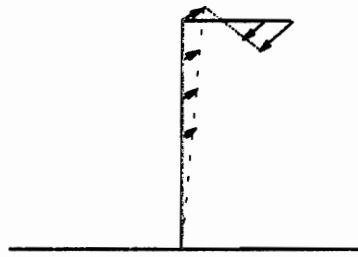


Fig. 2.1-1, Rothwell and Ostapenko's strain distribution for flange bending

In contrast, the sideways bending strain distribution assumption of the present study is shown in Fig. 2.1-2. With the present assumption the critical tripping stress changes as the width of plating changes and, as shown in Fig. 2.1-3, there is much better agreement with the ABAQUS fine mesh finite element analysis. Details of the ABAQUS modeling are given in Section 3.1.1(d).

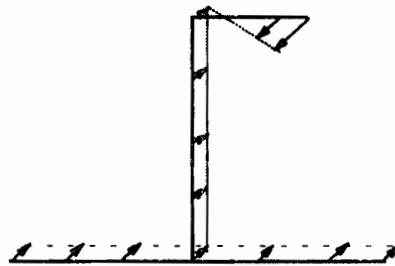
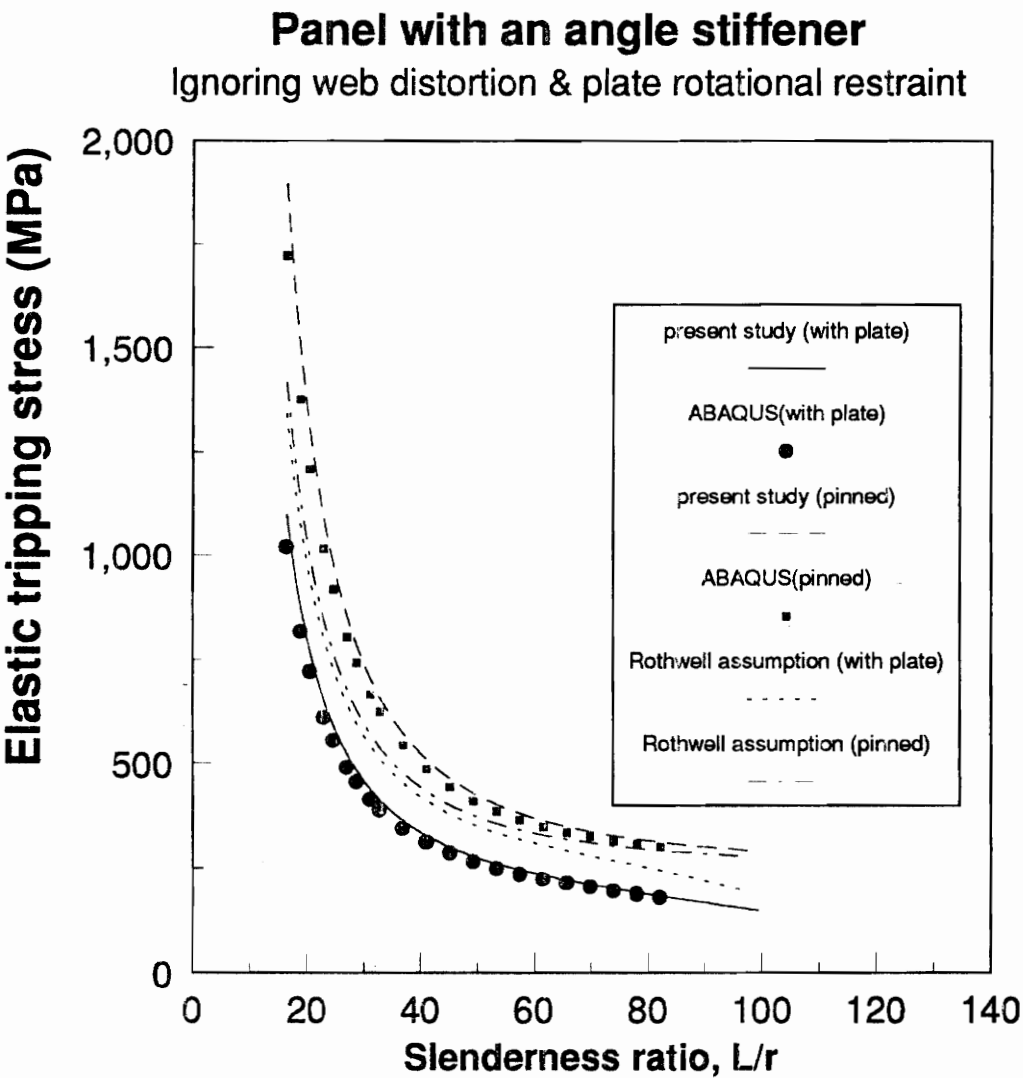


Fig. 2.1-2, Proposed strain distribution for flange bending

Comparison of strain distribution assumptions



Note: For the case where the stiffener is pinned, the slenderness ratio is assumed to be the same as the case where the plate width is a finite value (203.2 mm)

Fig. 2.1-3

The strain distributions used herein for axial compression, vertical bending and flange sideways bending are shown in Fig. 2.1-4 and Fig. 2.1-5.

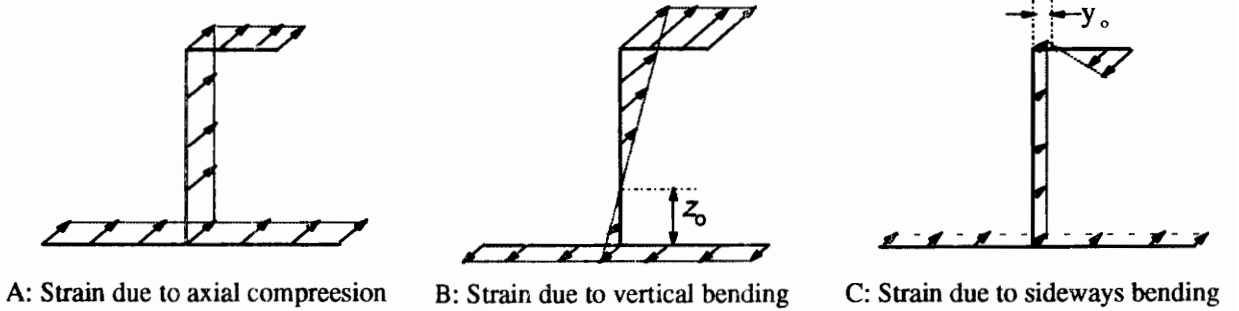


Fig. 2.1-4. Strain Distribution of Angle

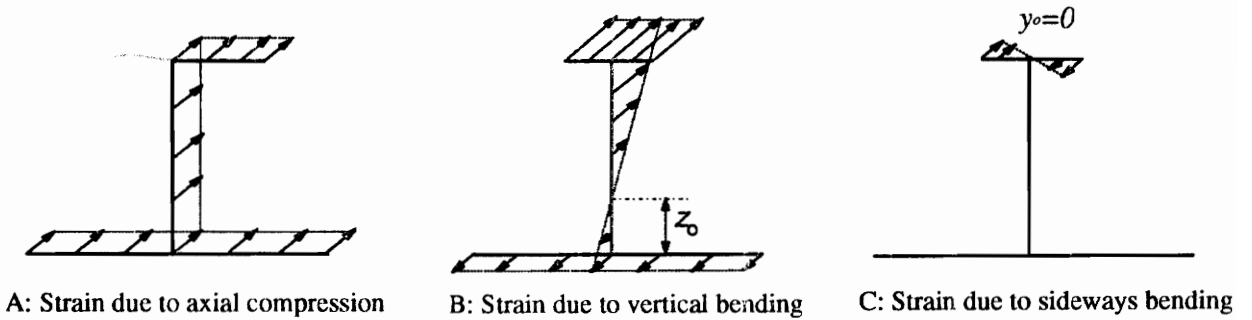


Fig. 2.1-5. Strain Distribution of Tee

2.2 Displacement field

The deformation of the panel cross section can be described approximately by a displacement field having five degrees of freedom ($u, v_r, w, \phi_r, \phi_b$). The variable u is the uniform axial displacement along the x -direction, and the cross-section deformations v_r, w, ϕ_r and ϕ_b are shown in Fig. 2.2-1, where w is the flexural displacement of the element with respect to the neutral axis z , v_r is the lateral displacement of the flange with respect to the y axis, ϕ_b is the rotation at the web baseline, and ϕ_r is the rotation at the tip of the web. It is also assumed that the flange cross section remains straight (undistorted) during deformation. A displacement function is needed which gives the values of $\tilde{u}, \tilde{v}$ and $\tilde{w}$ anywhere in the panel in terms of the five variables u, v_r, w, ϕ_b and ϕ_r . Since the stability

analysis requires nonlinearity to be taken into account, the strategy of the study to obtain the full strain distributions (linear and nonlinear) in terms of the five degrees of freedom is as follows: first the linear part of the strain distribution shown in Fig. 2.1-4 and Fig. 2.1-5 is represented by using basic beam theory, then the displacement field is obtained, and finally the full strain distribution is obtained by substituting the displacement field into nonlinear plate theory [M1].

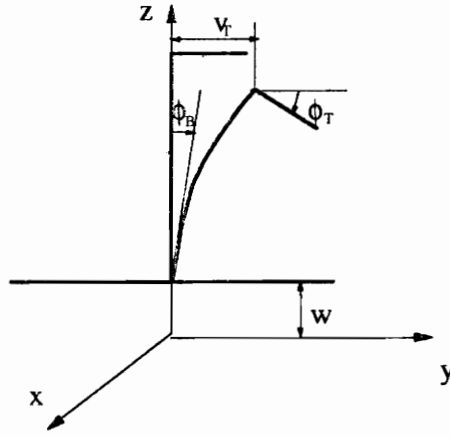


Fig. 2.2-1 Cross-section deformation of flexible web

Based on the strain distribution assumption of Fig. 2.1-4 and Fig. 2.1-5, the linear part of the strain distribution in the x -direction for the flange, web and plate is as follows:

$$\begin{aligned}\epsilon_f^l &= (z_o - h)w_{,xx} + (y_o - y)v_{T,xx} + u_{,x} \\ \epsilon_w^l &= (z_o - z)w_{,xx} + y_o v_{T,xx} + u_{,x} \\ \epsilon_p^l &= z_o w_{,xx} + y_o v_{T,xx} + u_{,x}\end{aligned}\tag{2.2-1}$$

where, for example, $u_{,x}$ denotes the partial derivative of u with respect to x . h is the height of the stiffener, which is shown in Fig. N-1. z_o is the distance between the neutral axis of vertical bending and the plating (Fig. 2.1-4 and Fig. 2.1-5), and is given by

$$z_o = \frac{A_f + \frac{1}{2}A_w}{A}h\tag{2.2-2}$$

where A_f , A_w and A are the cross-section area of flange, web and total cross-section area of flange, web and plate, respectively. y_o is the distance between the neutral axis of lateral bending and the web center (Fig. 2.1-4 and Fig. 2.1-5), and is given by

$$y_o = \begin{cases} \frac{1}{2} b_f^2 t_f / A & \text{for angles} \\ 0 & \text{for tees} \end{cases} \quad (2.2-3)$$

The displacements of the flange, web and plate in the x -direction are then obtained from $u = \int \epsilon' dx$, and the displacements in the y and z directions (v , w) are obtained from kinematic conditions. They are,

for the stiffener web:

$$\begin{aligned} u_w &= (z_o - z) w_{,x} + y_o v_{T,x} + u \\ w_w &= w(x) \\ v_w &= v(x, z) \end{aligned} \quad (2.2-4)$$

for the plate:

$$\begin{aligned} u_p &= z_o w_{,x} + y_o v_{T,x} + u \\ w_p &= w(x) \\ v_p &= 0 \end{aligned} \quad (2.2-5)$$

and for the stiffener flange:

$$\begin{aligned} u_f &= (z_o - h) w_{,x} + (y_o - y) v_{T,x} + u \\ w_f &= w(x) - y \phi_T(x) \\ v_f &= v_T(x) \end{aligned} \quad (2.2-6)$$

2.3 Strain-displacement relationship

The principal complexity in dealing with large deformations is the nonlinearity of the equations. Another major problem is that in the general theory the local stresses are difficult to identify because the deformations cause changes in the areas with respect to which the stresses are measured. Thus it is of paramount importance to specify in the general case whether one is operating in terms of the original undeformed coordinates or the deformed set of coordinates that moves with the deforming body. However, for practical buckling problems such precision is not required. What we do require is sufficiently large nonlinearity to allow projections of planar forces to have a significant

component out of their initial plane of action. The strains derived here are based on the displacement fields in Eq. (2.2-4) through Eq. (2.2-6). If the linear effect is the only consideration, Eq. (2.2-1) will be obtained. However, for an adequate analysis of tripping, the nonlinear effect has to be included. Geometrically nonlinear plate theory [M1] is assumed to govern the behavior of each component. The inplane strain components, $\{\epsilon\} = [\epsilon_x, \epsilon_y, \gamma_{xy}]^T$, are restricted to be small compared to unity, but rotation amplitudes are permitted to be moderately large, including inplane rotations. The expression for the distribution of inplane strains in a flat plate is given by

$$\{\epsilon(x, y, z)\} = \{\epsilon^c(x, y) + z\{\kappa^c(x, y)\}\} \quad (2.3-1)$$

where the mid-surface strains, $\{\epsilon^c\} = [\epsilon_x^c \quad \epsilon_y^c \quad \gamma_{xy}^c]^T$, and the mid-surface curvatures, $\{\kappa^c\} = [\kappa_x^c \quad \kappa_y^c \quad \kappa_{xy}^c]^T$, are given by

$$\{\epsilon^c\} = \begin{Bmatrix} u_{,x} + 0.5(v_{,x}^2 + w_{,x}^2) \\ v_{,y} + 0.5(u_{,y}^2 + w_{,y}^2) \\ u_{,y} + v_{,x} + w_{,x}w_{,y} \end{Bmatrix} \quad \text{and} \quad \{\kappa^c\} = - \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix}$$

Thus

$$\{\epsilon\} = \begin{Bmatrix} u_{,x} - zw_{,xx} + 0.5(v_{,x}^2 + w_{,x}^2) \\ v_{,y} - zw_{,yy} + 0.5(u_{,y}^2 + w_{,y}^2) \\ u_{,y} + v_{,x} - 2zw_{,xy} + w_{,x}w_{,y} \end{Bmatrix} = \epsilon^l + \epsilon^{nl} \quad (2.3-2)$$

where ϵ^l is the linear portion of the strain and ϵ^{nl} is the nonlinear portion of the strain. The linear portion of the strain in the x -direction is the same as Eq. (2.2-1). Details of the full strain distribution of each component are given in Appendix B.

2.4 Total Potential Energy

The total potential energy of a stiffened panel subjected to edge loading is the sum of the strain energy U_T and the potential energy of the applied load, Ω :

$$\Pi_T = U_T + \Omega$$

The strain energy for a three-dimensional isotropic medium referred to arbitrary orthogonal coordinates is

$$U_T = \frac{1}{2} \int_V (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz}) dv$$

In accordance with the basic approximations of thin-plate theory, γ_{xz} , γ_{yz} and σ_z may be omitted:

$$\begin{aligned} U_T &= \frac{1}{2} \int_V (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy}) dv \\ &= U_T^f + U_T^w + U_T^p \end{aligned}$$

The full details of the derivation are given in Appendix B. The strain energy expression obtained by using the displacement assumptions (Eqs. (2.2-4)~(2.2-6)) can written as

$$\begin{aligned} U_T &= \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_0^l J_f \varphi_{T,x}^2 dx + u_{web-out-of-plane} \\ &\quad + \frac{1}{2} E \int_0^l A u_{,x}^2 dx + \frac{1}{2} E \int_0^l 2 A_{uv} u_{,x} v_{T,xx} dx + \frac{1}{2} E \int_0^l 2 A_{uw} u_{,x} w_{,xx} dx \\ &\quad + \frac{1}{2} \int_{A_f} \sigma \int_0^l (v_{T,x}^2 + [(w - y \varphi_T)_{,x}]^2) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA \\ &\quad + \Pi_{plate-out-of-plane} \end{aligned}$$

where I , I_z , I_{zy} , J_f , A , A_{uv} and A_{uw} are cross-sectional properties, $u_{web-out-of-plane}$ is the web out-of-plane strain energy, and $\Pi_{plate-out-of-plane}$ is the plate out-of-plane total potential energy. They are defined in Eq. (B-6). Since A_{uv} and A_{uw} are zero, the potential energies due to axial deformation and flexural-torsional deformation are uncoupled. Thus

$$\Pi_T = \Pi + \Pi_a \quad (2.4-1)$$

where

$$\begin{aligned} \Pi &= \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_0^l J_f \varphi_{T,x}^2 dx + u_{web-out-of-plane} \\ &\quad + \frac{1}{2} \int_{A_f} \sigma \int_0^l (v_{T,x}^2 + [(w - y \varphi_T)_{,x}]^2) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA \\ &\quad + \Pi_{plate-out-of-plane} \end{aligned}$$

is the potential energy due to bending-torsional deformation and

$$\Pi_a = \frac{1}{2} E \int_0^l A u_{,x}^2 dx - P \int_0^l u_{,x} dx \quad (2.4-2)$$

is the potential energy due to axial deformation. Equation (2.4-2) yields

$$\sigma_x = \frac{P}{A}$$

For convenience, let compressive stress be positive. Then the strain energy U and the work done V due to the bending-torsional deformation may be written as

$$\Pi = U - V \quad (2.4-3)$$

where

$$\begin{aligned} U &= \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_0^l J_f \phi_{T,x}^2 dx \\ &\quad + u_{\text{web-out-of-plane}} + \Pi_{\text{plate-out-of-plane}} \\ V &= \frac{1}{2} \int_{A_f} \sigma \int_0^l \left(v_{T,x}^2 + [(w - y \phi_T)_{,x}]^2 \right) dx dA + \frac{1}{2} \int_{A_r} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA \end{aligned}$$

The total potential energy so formed approximates the actual total potential energy for the kind of deformation restrictions embodied in Kirchhoff's assumptions. And, since undeformed geometry is used for stress and external loads, the functional is limited to small deformations.

The total potential energy depends on the nature of the displacement functions, i.e., $u(x)$, $v_T(x)$, $w(x)$, $\phi_T(x)$ and $\phi_B(x)$. Different types of functions result in different values for Π_T . Out of all possibilities, one particular function provides a minimum value for Π_T and that is the one which represents the configuration for stable equilibrium. By applying the principle of stationary total potential energy, the governing differential equations of tripping may be obtained, and the displacement function which makes Π_T stationary is also the function which satisfies the differential equations. However, the exact solution to the characteristic equations is often difficult, if not impossible, to obtain. It is necessary to

resort to approximate methods, which are often capable of being carried to any desired degree of accuracy provided suitable computing facilities are available. Since approximate methods based on total potential energy rather than the differential equations will be used in the study, the governing differential equations are not given here.

2.5 Displacement assumptions for buckling analysis

As shown in Section 2.4, the interaction between axial shortening and bending is completely negligible, and they can be regarded as uncoupled. Therefore, the displacements of the panel can be separated into displacements due to bending and displacements due to axial shortening. Since buckling involves only flexural deflection (w in Fig. 2.2-1) and sideways deflection (v_T in Fig. 2.2-1), the axial strain ϵ_o can be ignored in the buckling analysis. Therefore the linear portion of strain due to bending becomes

$$\begin{aligned}\epsilon_f^I &= (z_o - h)w_{,xx} + (y_o - y)v_{T,xx} \\ \epsilon_w^I &= (z_o - z)w_{,xx} + y_o v_{T,xx} \\ \epsilon_p^I &= z_o w_{,xx} + y_o v_{T,xx}\end{aligned}\tag{2.5-1}$$

and then the displacements corresponding to the above strain assumptions are,

for the stiffener web:

$$\begin{aligned}u_w &= (z_o - z)w_{,x} + y_o v_{T,x} \\ w_w &= w(x) \\ v_w &= v(x, z)\end{aligned}\tag{2.5-2}$$

for the plate:

$$\begin{aligned}u_p &= z_o w_{,x} + y_o v_{T,x} \\ w_p &= w(x) \\ v_p &= 0\end{aligned}\tag{2.5-3}$$

and for the stiffener flange:

$$\begin{aligned}
u_f &= (z_o - h)w_{,x} + (y_o - y)v_{T,x} \\
w_f &= w(x) - y\phi_T(x) \\
v_f &= v_T(x)
\end{aligned}
\tag{2.5-4}$$

Again, for tee stiffeners, $y_o=0$. Note that the strain assumption, Eq. (2.5-1), is identical to van der Neut's (see Appendix A), although the derivation is different. Equation (2.4-3), giving the total potential energy for the (uncoupled) flexural-torsional deformation, could also have been derived from the (uncoupled) flexural-torsional displacements of Eqs. (2.5-2)~(2.5-4). In fact, it is common practice to ignore the axial effect in buckling analysis.

2.6 Model without plate rotational restraint

One of the most difficult aspects of tripping is the rotational restraint of the plating. The usual practice is to assume that the rotational spring constant of the plate interacts linearly with the axial inplane stress in the stiffeners [A1, F1]. This approach implies (or assumes) that the rotational restraint disappears when the axial stress reaches the plate buckling stress. Since the plate axial or lengthwise buckling stress is generally lower than the stiffener tripping stress, the plate buckling modes usually come first. Because of the kinematic continuity condition between the stiffener and the plate, it is almost impossible to isolate the stiffener tripping mode, even with the most complicated finite element analysis. In order to isolate the tripping mode in a theoretical analysis, it is necessary to incorporate additional boundary conditions. Therefore, in order to obtain a good understanding of tripping, it is best to begin with a simplified tripping model, which ignores the rotational restraint between the stiffener and the plate. A model of this type is presented in Section 3.1 and 3.2, and it is used to illustrate some basic properties of asymmetric stiffener tripping and to provide the lower bound of tripping failure (ignoring rotational restraint). This model incorporates six assumptions, of which the first two correspond to the exclusion of plate rotational restraint.

- (1) The stiffener and the plate are connected by pin hinges; i.e., at the base line of the web, the displacements u , v , w of the web and the plate are equal, but the rotation angle can be different.
- (2) The plate out-of-plane deformation is ignored.
- (3) The interaction of axial deformation and bending deformation is ignored.
- (4) The flange cross-section remains as a straight line during buckling.
- (5) The panel strain distributions are obtained from Eqs. (2.5-2) ~ (2.5-4).
- (6) Shear deformation is ignored.

2.7 Model with plate rotational restraint

Since tripping is essentially rotation of the stiffener, the plate rotational restraint plays an important role in tripping failure, at least in the elastic range. The inclusion of the plate rotational restraint is done by incorporating the plate out-of-plane strain energy. Of the above assumptions, only 3, 4, 5 and 6 apply to the model with plate rotational restraint. Details will be presented in Section 3.3.

2.8 Method for inelastic tripping

In Chapter 5 the model will be further generalized to include inelastic effects, using deformation theory rather than flow theory. It will be shown that by using an incremental piecewise linear approach, a basic strategy of inelastic tripping analysis can be developed that is similar to that of elastic tripping analysis. Both analyses use the same total potential energy expression. The main difference between them lies in the evaluation of the cross-sectional properties. In the elastic range, the stress distribution in the cross-section is linear, and therefore the cross-sectional properties can be obtained in closed form. In the inelastic range the nonlinear distribution and growth of the stress make it necessary to use numerical integration to evaluate the cross-sectional properties. For this purpose, the

cross-section is discretized into small zones and the properties at the zone centroid are considered to represent those of the zone. For computational simplicity, the cross-sectional properties are determined by a transformed area concept instead of using instantaneous tangent modulus. Details will be presented in Chapter 5.

2.9 Rayleigh-Ritz model vs. segmented beam (finite element) model

Among approximate methods, the Rayleigh-Ritz method and the finite element method are commonly used. The total potential energy expression (Eq. (2.4-1)) can be used for both the Rayleigh-Ritz method and segmented beam model, depending on the selection of the lengthwise shape function and the degrees of freedom. In the Rayleigh-Ritz method, since the approximate solution is assumed for the whole domain, only four degrees of freedom are required and the solution is rapid. This method ignores the effect of axial shortening and interaction. In the segmented beam model, the axial shortening effect can be incorporated by transforming the local displacements into global displacements. With the aid of an updated Lagrangian formulation, it can handle relatively large deformation, but it requires many more degrees of freedom than the Rayleigh-Ritz method. The effect of axial shortening and interaction only becomes important with large deformations, as occur in the post buckling regime. Of its very nature, stiffener tripping occurs before the deformations have become large. Therefore in order to simply predict the occurrence of tripping (as opposed to tracing its aftermath), the Rayleigh-Ritz method is sufficient. Since the model developed in the present study is meant for practical design, the Rayleigh-Ritz method was used as the basis.

3.0 Bifurcation analysis of tripping

After obtaining the total potential energy, there are two classes of techniques to obtain the critical load, differential equation methods and approximate methods. The governing differential equations can be obtained by using the principle of total potential energy, and the critical load can then be obtained from the eigenvalue analysis of the general solution of the differential equations. However, this technique often fails when the characteristic equations of the differential equations are too sophisticated to be solved. In such a situation the energy approach, which has the inherent ability to converge to the exact solution, can be used to obtain an approximate solution. In the following, the Rayleigh-Ritz method is used to obtain an approximate solution of the critical load. In this method an assumed displacement function satisfying the essential boundary conditions is used in the expression for the total potential energy function. By using such an assumed displacement function, a continuous structural system is converted to a system having a finite number of degrees of freedom. The principle of stationary potential energy then leads directly to the equilibrium configuration, and only ordinary differential calculus has to be used in the process.

The total potential energy in the buckled state is given by

$$\Pi = U - V \quad (3.0-1)$$

where

$$U = \frac{1}{2} E \int_0^l I_{w,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{f,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{f,xx} dx + \frac{1}{2} G \int_0^l J_f \phi_{T,x}^2 dx$$

$$+ u_{web-out-of-plane} + \Pi_{plate-out-of-plane}$$

$$V = \frac{1}{2} \int_{A_f} \sigma \int_0^l (v_{f,x}^2 + (w - y \phi_T)_{,x}^2) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA$$

in which

$$I = \int_{flange} (z_o - h)^2 dA + \int_{web} (z_o - z)^2 dA + \int_{plate} z_o^2 dA \quad (3.0-2)$$

is the moment of inertial of the panel with respect to the y-axis.

$$I_z = \int_{flange} (y - y_o)^2 t_f dy + \int_{web} y_o^2 t_w dz + \int_{plate} y_o^2 t_p dy \quad (3.0-3)$$

is the moment of inertial of the panel with respect to the z-axis.

$$I_{zy} = \int_{web} (z_o - z) y_o dy + \int_{flange} (z_o - h)(y_o - y) dy + \int_{plate} z_o y_o dy \quad (3.0-4)$$

is the product of inertia of the panel with respect to the yz-plane.

For the model without plate rotational restraint, the total potential energy of the plate bending $\Pi_{plate-out-of-plane}$ is assumed to be zero. For the model with plate rotational restraint, $\Pi_{plate-out-of-plane}$ will be taken into account. The total energy expression is valid for both the rigid web case and the flexible web case. The difference lies in the expression of the web out-of-plane strain energy, $u_{web-out-of-plane}$. The rigid web without plate rotational restraint case will be considered first, in Section 3.1.

The most significant nonlinear influence in the elastic behavior of structures is the influence of axial forces on the flexural stiffness of members. Tensile axial forces can be considered as increasing the flexural stiffness, while compressive force decreases the flexural stiffness. For the cases where the external load is end moment or uniform lateral pressure, the work done during buckling is calculated in terms of the axial stress caused by the end moment or lateral pressure.

3.0.1 Sectional properties of stiffeners in a stiffened Panel

a) Flexural properties

By performing the necessary operations, the flexural sectional properties of stiffeners in a stiffened panel given by Eq. (3.0-2) through (3.0-4) are:

for asymmetric stiffeners, i.e. angle stiffeners and bulb-flat stiffeners

$$\begin{cases} I_{zy} = z_o y_o t_p b + y_o \left(z_o h - \frac{h^2}{2} \right) t_w + (z_o - h) \left(y_o b_f - \frac{b_f^2}{2} \right) t_f \\ I_z = A_p y_o^2 + A_w y_o^2 + A_f \left(y_o^2 - b_f y_o + \frac{b_f^2}{3} \right) \\ I = (z_o - h)^2 t_f b_f + \left(z_o^2 h - z_o h^2 + \frac{h^3}{3} \right) t_w + z_o^2 t_p b \end{cases} \quad (3.0-5)$$

and for symmetric stiffeners, i.e. tee stiffeners

$$\begin{cases} I_{zy} = 0 \\ I_z = \frac{b_f^3 t_f}{12} = I_f \\ I = (z_o - h)^2 t_f b_f + \left(z_o^2 h - z_o h^2 + \frac{h^3}{3} \right) t_w + z_o^2 t_p b \end{cases} \quad (3.0-6)$$

b) Torsional properties

Torsional rigidity, which depends on the profile of beam cross sections, is an important factor in lateral-torsional buckling. A beam cross section may be solid, or an assembly of thin plates, where “thin” means that the plate thickness is small compared to other cross-sectional dimensions. The torsional rigidity of a solid cross section is quite different from that of a thin plate, as will be shown in the following.

(1) Torsional rigidity of a rectangular cross section

The exact torsional rigidity of a rectangular section is given by Timoshenko [T3] as

$$J = \frac{1}{3} a^3 b \left(1 - \frac{192}{\pi^5} \frac{a}{b} \sum_{n=1,3,5}^{\infty} \frac{1}{n^5} \tanh \frac{n\pi b}{2a} \right) \quad (3.0-7)$$

where a and b are illustrated in Fig. 3.0-1, and $b > a$.

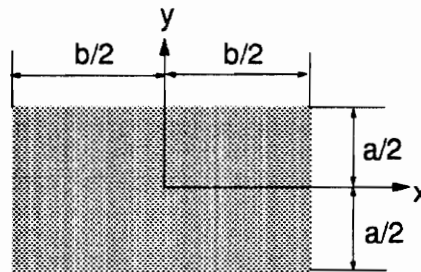


Fig. 3.0-1 Rectangular cross section

The series on the right side of Eq. (3.0-7) converges very rapidly. For example, for $a=b$ the exact solution is $2.25(a/2)^4$, and the result using the first three terms is $2.24953(a/2)^4$. For a narrow rectangular section as in the web of a stiffener, Eq. (3.0-7) also reduces quickly to the limit value of $a^3b/3$. For example if $h/t_w = b/a = 10$, Eq. (3.0-7) gives $0.312a^3b$.

(2) Torsional rigidity of the flange

Torsional rigidity of a thin flange

The torsional rigidity of a thin flange (tee and angle stiffeners) is given by

$$J_f = \frac{t_f^3 b_f}{3} \quad (3.0-8)$$

Torsional rigidity of a thick flange (bulb-flat stiffeners)

For bulb-flat stiffeners (Fig. 3.0-2(A)), an equivalent cross section (Fig. 3.0-2(B)) is calculated such that the sum of the errors in the moment of inertia and position of centroid is smallest.

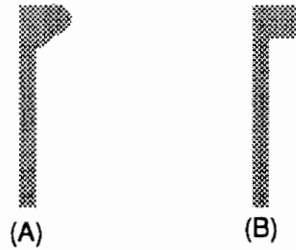


Fig. 3.0-2 Profile of bulb-flat stiffener (A) and the equivalent section (B)

Since the ratio b_f/t_f of the flange cross section is generally between 1 and 2, the torsional rigidity of the flange can no longer be expressed as $J_f = t_f^3 b_f / 3$, as in Eq. (3.0-8), because this seriously over-estimates the torsional rigidity. For example, for $b_f/t_f = 1.5$, the exact solution is $0.196t_f^3 b_f$, and Eq. (3.0-8) gives $0.333t_f^3 b_f$. Therefore the torsional rigidity of a thick flange (bulb-flat stiffeners) is given by

$$J_f = \frac{1}{3} t_f^3 b_f \left(1 - \frac{192}{\pi^5} \frac{t_f}{b_f} \sum_{n=1,3,5}^{\infty} \frac{1}{n^5} \tanh \frac{n\pi b_f}{2t_f} \right) \quad (3.0-9)$$

where $b_f > t_f$

(2) Torsional rigidity of the web

For the rigid web case, the out of plane bending energy of the web can be expressed as

$$u_{web-out-of-plane} = \frac{1}{2} G \int_0^l J_w \theta_{,x}^2 dx$$

(For derivation, see Appendix B)

Since the web is a thin plate, the torsional rigidity of the web is given by

$$J_w = \frac{t_w^3 h}{3} \quad (3.0-10)$$

For the flexible web case, the torsional rigidity of the web is embodied in the web out-of-plane strain energy $u_{web-out-of-plane}$ and cannot be written as an explicit expression. The implementation of the flexible web case will be discussed in Section 3.2.

3.1 Rigid Web Case

In beam-column buckling analysis, good results can be obtained even though, for simplicity, the cross-section of the beam-column is considered as undeformed. Therefore, it is worthwhile to first examine the rigid web case as have other researchers [A1, B4]. Lateral-torsional or “tripping” instability is characterized by a twisting of the stiffener about its line of attachment to the plating. This deformation pattern involves both sideways and vertical deflections (v , w) and rotation, as shown in Fig. 2.2-1. If the cross sectional profile of the web at any transverse cross section is not allowed to curve, then the web is regarded as rigid and the rotation and sideways deflection are coupled (Fig. 3.1-1). For small displacement

$$\begin{aligned} v_T &= h\theta \\ v_w &= z\theta \\ \phi_T &= \phi_B = \theta \end{aligned} \quad (3.1-1)$$

From practical experience, it is known that the rigid web assumption is valid when $h/t_w < 20$. Therefore it is worthwhile to develop a theory based on this assumption.

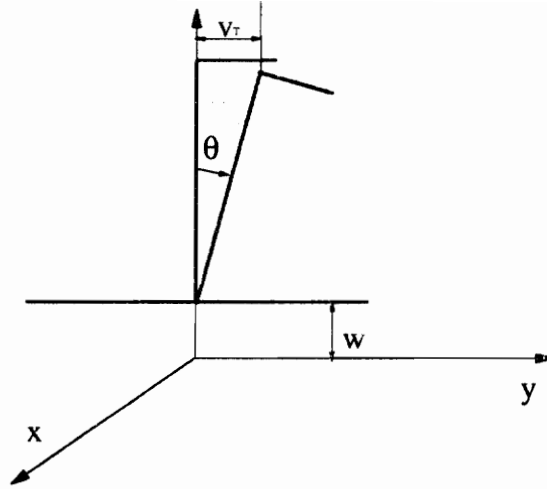


Fig. 3.1-1 Cross-section deformation of rigid web

For the rigid web case, the out of plane bending energy of the web is then simplified as

$$U_{web-out-of-plane} = \frac{1}{2} G \int_0^l J_w \theta_{,x}^2 dx \quad (3.1-2)$$

(For derivation, see Appendix B)

The total potential energy can be written as

$$\Pi = U - V \quad (3.1-3)$$

where the strain energy stored during buckling is

$$U = \frac{1}{2} E \int_0^l I_w w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z h^2 \theta_{,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} h w_{,xx} \theta_{,xx} dx + \frac{1}{2} G \int_0^l J_s \theta_{,x}^2 dx$$

and the work done during the buckling is

$$V = \frac{1}{2} \int_{A_f} \sigma \int_0^l (h^2 \theta_{,x}^2 + w_{,x}^2 - 2 y \theta_{,x} w_{,x} + y^2 \theta_{,x}^2) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + z^2 \theta_{,x}^2) dx dA$$

The flexural sectional properties I , I_z and I_{zy} are given by Eq. (3.0-5) for angle and bulb-flat stiffeners, and by Eq. (3.0-6) for tee stiffeners. The torsional rigidity of the stiffener is

$$J_s = J_w + J_f \quad (3.1-4)$$

where J_w is given by Eq. (3.0-10). J_f is given by Eq. (3.0-8) for angle and tee stiffeners, and by Eq. (3.0-9) for bulb-flat stiffeners.

Since a stiffened panel in a typical ship is usually a multi-bay structure, it is logical to select a mode shape incorporating simply supports at both ends of a bay in the panel. One of the simplest choices is to assume

$$\begin{aligned} w &= C_1 \sin \frac{m\pi x}{l} = C_1 \sin \lambda x \\ \theta &= C_2 \sin \frac{m\pi x}{l} = C_2 \sin \lambda x \end{aligned} \quad (3.1-5)$$

Substituting the mode shapes into equation (3.1-3) and performing the necessary operations, the total potential energy of a stiffened panel can be obtained. The strain energy stored during buckling (U) will be the same regardless of the load conditions, and is given in Eq. (3.1-6):

$$U = \frac{1}{4} \lambda^2 l \left[C_2^2 (GJ_s + EI_z h^2 \lambda^2) + C_1^2 EI \lambda^2 + 2 C_1 C_2 EI_y h \lambda^2 \right] \quad (3.1-6)$$

However the work done during buckling (V) will differ, depending on the load conditions. In the following, the critical load for tripping under different load conditions will be discussed.

3.1.1 Tripping under axial compression

Axial loading, which arises from the overall bending of the hull girder, is the most common (and usually dominant) tripping load for a stiffened panel, and will receive extra attention in this study. When the applied load is uniform axial compressive force, the stress is

$$\sigma_f = \sigma_w = \sigma_p = \sigma$$

Then the work done during buckling is

$$\begin{aligned} V &= \frac{1}{2} \int_{A_f} \sigma \int_0^l (h^2 \theta_{,x}^2 + w_{,x}^2 - 2y \theta_{,x} w_{,x} + y^2 \theta_{,x}^2) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + z^2 \theta_{,x}^2) dx dA \\ &= \frac{1}{2} \left(\alpha_{\theta\theta}^a \int_0^l \theta_{,x}^2 dx + 2\alpha_{\theta w}^a \int_0^l \theta_{,x} w_{,x} dx + \alpha_{ww}^a \int_0^l w_{,x}^2 dx \right) \end{aligned}$$

where

$$\alpha_{\theta\theta}^a = \int_{A_f} \sigma_f (h^2 + y^2) dA + \int_{A_w} \sigma_w z^2 dA = \sigma I_p$$

$$\alpha_{ww}^a = \int_{A_f} \sigma_f dA + \int_{A_w} \sigma_w dA + \int_{A_p} \sigma_p dA = \sigma A$$

$$\alpha_{\theta w}^a = \int_{A_f} \sigma_f (-y) dA = \sigma S_f$$

in which for asymmetric stiffeners, i.e. angle stiffeners and bulb-flat stiffeners

$$\begin{cases} I_p = \frac{h^3 t_w}{3} + \frac{t_f b_f^3}{3} + h^2 t_f b_f \\ S_f = -\frac{t_f b_f^2}{2} \\ A = A_f + A_w + A_p \end{cases} \quad (3.1-7)$$

and for symmetric stiffeners, i.e. tee stiffeners

$$\begin{cases} I_p = \frac{h^3 t_w}{3} + \frac{t_f b_f^3}{12} + h^2 t_f b_f \\ S_f = 0 \\ A = A_f + A_w + A_p \end{cases} \quad (3.1-8)$$

The total potential energy of the panel is

$$\begin{aligned} \Pi &= U - V \\ &= \frac{1}{4} \lambda^2 [C_2^2 (GJ_s + EI_z h^2 \lambda^2 - \sigma I_p) + C_1^2 (EI \lambda^2 - \sigma A) + 2C_1 C_2 (EI_{xy} h \lambda^2 - \sigma S_f)] \end{aligned}$$

For the system to be in equilibrium, the first variation of total potential energy must vanish, i.e., $\delta \Pi = 0$. Hence the derivative of Π with respect to $(C_1, C_2)^T$ must be zero.

That is

$$\begin{bmatrix} EI \lambda^2 - \sigma A & EI_{xy} h \lambda^2 - \sigma S_f \\ EI_{xy} h \lambda^2 - \sigma S_f & GJ_s + EI_z h^2 \lambda^2 - \sigma I_p \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The equilibrium state is stable when $\delta^2 \Pi > 0$. At the limit of stability, the second variation of total potential energy is zero. Hence the condition for critical state may be written as

$$\det \begin{bmatrix} EI \lambda^2 - \sigma A & EI_{xy} h \lambda^2 - \sigma S_f \\ EI_{xy} h \lambda^2 - \sigma S_f & GJ_s + EI_z h^2 \lambda^2 - \sigma I_p \end{bmatrix} = 0 \quad (3.1-9)$$

a) Asymmetric stiffeners (angle stiffeners and bulb-flat stiffeners)

For a panel with asymmetric stiffeners, since I_{zy} and S_f are non-zero, Eq. (3.1-9) clearly shows that Euler buckling and tripping are closely coupled. The critical condition occurs when

$$I_p A (\sigma_T - \sigma) (\sigma_U - \sigma) - (EI_{zy} h \lambda^2 - \sigma S_f)^2 = 0$$

where

$$\sigma_T = \frac{GJ_s + EI_z h^2 \lambda^2}{I_p}$$

$$\sigma_U = \frac{EI \lambda^2}{A}$$

The smaller root is the critical tripping stress due to axial compression,

$$\sigma_{cr}^a = \frac{\sigma_U + \sigma_T - \frac{2S_f EI_{zy} h}{I_p A} \lambda^2 - \sqrt{\left(\sigma_U + \sigma_T - \frac{2S_f EI_{zy} h}{I_p A} \lambda^2 \right)^2 - 4 \left(1 - \frac{S_f^2}{I_p A} \right) \left(\sigma_U \sigma_T - \frac{(EI_{zy} h)^2}{I_p A} \lambda^4 \right)}}{2 \left(1 - \frac{S_f^2}{I_p A} \right)} \quad (3.1-10)$$

where the flexural properties I , I_z and I_{zy} are given by Eq. (3.0-5), and A , S_f and I_p are given by Eq. (3.1-7). The stiffener torsional rigidity J_s is given by Eq. (3.1-4).

b) Isolated pinned asymmetric stiffener

Some studies, for example Smith and Kirkwood [S4], evaluated the tripping stress for an isolated stiffener pinned at the base of the web as a reference value. This assumption implies that Euler buckling would never happen, and therefore the critical tripping stress obtained in this case can be regarded as the upper bound value without considering plate rotational restraint. Although it has little practical use, we will give it here for the sake of comparison. Since $w = 0$ for this case, then the total potential energy (3.1-3) becomes

$$\Pi = \frac{1}{4} \lambda^2 I C_2^2 (GJ_s + EI_z h^2 \lambda^2 - \sigma I_p)$$

Thus for an isolated pinned asymmetric stiffener, the critical tripping stress is

$$\sigma_{cr_iso}^a = \frac{GJ_s + EI_z h^2 \lambda^2}{I_p} \quad (3.1-11)$$

where I_p is given in Eq. (3.1-7) and

$$I_z = A_w y_o^2 + A_f (y_o^2 - b_f y_o + \frac{b_f^2}{3}) \quad (3.1-12)$$

The stiffener torsional rigidity J , is given by Eq. (3.1-4).

c) Isolated symmetric stiffeners (tee stiffeners)

For a panel with tee stiffeners, a similar method can be used to obtain the tripping stress. Here again the applied load is uniform axial force. The stability equation becomes

$$\det \begin{bmatrix} EI\lambda^2 - \sigma A & 0 \\ 0 & GJ_s + EI_f h^2 \lambda^2 - \sigma I_p \end{bmatrix} = 0 \quad (3.1-13)$$

It can be seen from Eq. (3.1-13) that tripping and Euler buckling are uncoupled. Therefore the critical tripping stress is the same with or without plating, and is given as

$$\sigma'_{cr} = \frac{GJ_s + EI_f h^2 \lambda^2}{I_p} \quad (3.1-14)$$

where I_f is given by Eq. (3.0-6); I_p is given by Eq. (3.1-8); and J , is given by Eq. (3.1-4). Eq. (3.1-14) is given in many books. Because of the uncoupling of Euler buckling and tripping, Eq. (3.1-14) also applies to pinned tee stiffeners.

d) Comparison with ABAQUS

In order to verify the above theory, ABAQUS was employed. Because of the lengthwise symmetry of the structure and its response, it was only necessary to model one-half of the length of the panel. Flange and web are modeled as 4 x 24 S4R5 elements (4-node shell element with 5 degrees of freedom per node, recommended for thin shell), and the plate is modeled as 8 x 24 S4R5 elements if applicable. A full ABAQUS model for a panel with an angle stiffener is illustrated in Fig. 3.1-14.

For the case of axial load, the dimensions of the tee stiffener are set equal to those from NCRE [S3] test model 1a. The angle stiffener is assumed to have the same size of web and flange. In the NCRE test the plate buckled before tripping occurred. Therefore an

effective width of one third of the plate width of the NCRE 1a panel is used for the ABAQUS model. These same dimensions were used in the theoretical calculations.

The dimensions of the panel are as follows: web 138.2 mm $\times$ 7.1 mm; flange 76.2 mm $\times$ 14.2 mm; and plate 203.2 mm $\times$ 7.9 mm. Young's modulus is 2.066E5 N/mm², and Poisson's ratio is 0.3. The ratio of t_w/h is 0.05. It will be shown in Section 3.2 (see Fig. 3.2-5) that the validity of the rigid web assumption deteriorates for values of t_w/h below this value. Typical ship stiffeners are in the range from 0.03 to 0.06. Symbolic illustration of the ABAQUS models, showing only one row of elements, is given in Fig. 3.1-2 through Fig. 3.1-8. For the full ABAQUS model, refer to Fig. 3.1-14.

1. Angle stiffener

1). Isolated angle-stiffener pinned at the base of the web

The base of the web is simply supported ($v=w=0$). This boundary condition is intended to prevent Euler buckling and to give the upper bound of the critical tripping stress without considering plate rotational restraint. Figure 3.1-2 illustrates the boundary and load conditions. Note that only one element in the lengthwise direction is drawn here.

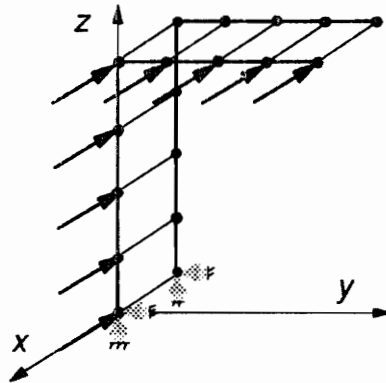


Figure 3.1-2, ABAQUS model of isolated angle stiffener

2). Angle stiffener with plating

The stiffener size is as above; the plate size is 203.2 mm $\times$ 7.9 mm. At the base of the stiffener w is unrestricted, and v is constrained to be zero. The nodes have only a pin connection at the junction of web and plating, which implies that there is no rotational

restraint on the stiffener. This is done by using the 'PIN' option of '*MPC (multi-point constraint)' in ABAQUS.

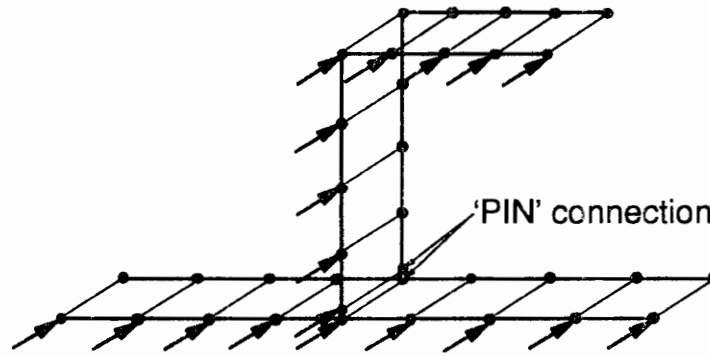


Figure 3.1-3, ABAQUS model of angle stiffener with plating

3). Isolated angle-stiffener with v constrained to be zero and w unrestricted

The stiffener size is as above. This case can be considered as having a plate width equal to zero. This boundary condition is intended to give the lower bound of the critical tripping stress, and to validate the present study.

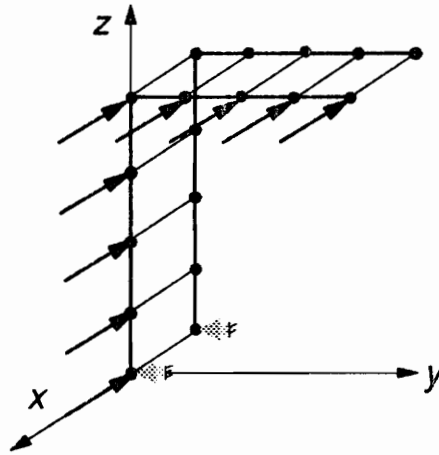


Figure 3.1-4, ABAQUS model of angle stiffener without plating

ABAQUS was executed for a series of models with different lengths. The results and the comparison with the present study are shown in Fig. 3.1-9. The buckling mode shape obtained from ABAQUS shows that tripping and Euler buckling of angle stiffeners are indeed coupled (Fig. 3.1-15).

2. Tee stiffener

For the tee stiffener, it can be seen from Eq. (3.1-13) that Euler buckling is uncoupled from tripping. In order to verify the theory, two models were analyzed with ABAQUS. The purpose of the two models is to show that there is virtually no difference between the pinned model, in which the critical stress is the upper bound, and the model with plating, in which the critical stress is not the upper bound, and also to validate the critical stress predicted by Eq. (3.1-14). For the first model, the base of the web is pinned, as shown in Fig. 3.1-5, which isolates the stiffener and gives the upper bound case, in which Euler buckling cannot occur.

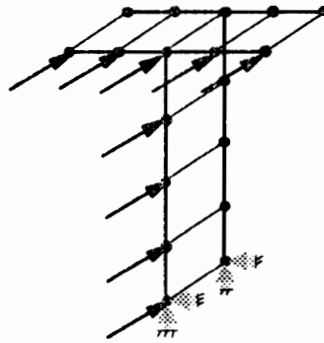


Figure 3.1-5, ABAQUS model of isolated tee stiffener

For the second model, the plate size is 203.2 mm x 7.9 mm. The nodes have only a pin connection at the junction of web and plating, which implies that there is no rotational restraint on the stiffener. The stiffener sizes are the same as the angle stiffener in both models.

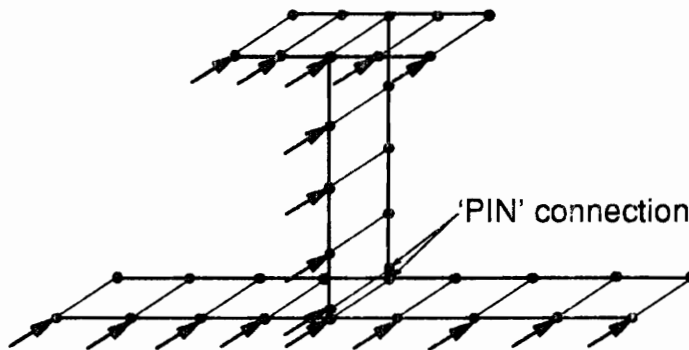


Figure 3.1-6, ABAQUS model of tee stiffener with plating

ABAQUS was run for a series of models with different lengths. As shown in Fig. 3.1-10, the results indicated that the elastic tripping stress is almost independent of the panel plating width, whereas the critical stress with angle stiffeners depends on the plating width (Fig. 3.1-9). The buckling mode shape obtained from ABAQUS shows that tripping and Euler buckling are indeed uncoupled (Fig. 3.1-16). The comparison with the present study is shown in Fig. 3.1-11. The discrepancy of ABAQUS and the present study is partly due to ignoring web distortion. The flexible web case will be studied in Section 3.2.

3. Bulb-flat stiffener

The dimensions of the stiffener are as follows: web 151 mm $\times$ 8 mm; flange 21.9 mm $\times$ 18 mm. Young's modulus is 2.07E5 N/mm², and Poisson's ratio is 0.3.

1). Isolated bulb-flat stiffener pinned at the base of the web

The base of the web is simply supported ($v=w=0$). The boundary condition was intended to give the upper bound of the critical tripping stress, and to validate the present study. The web is modeled as shell elements (thin-shell element S4R5) and the flange is modeled as beam elements (2 node space beam element B31). The shell nodes at the tip of the web are connected to the beam nodes of the flange by the rigid beam connections. This is done by using the 'BEAM' option of '*MPC' in ABAQUS. Figure 3.1-7 illustrates the boundary and load conditions. Note that only one element in the lengthwise direction is drawn here.

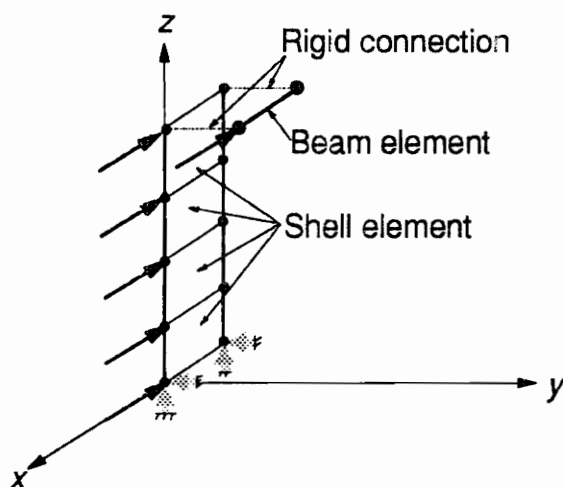


Figure 3.1-7, ABAQUS model of isolated bulb-flat stiffener

2). Isolated bulb-flat stiffener with v constrained to be zero and w unrestricted

The stiffener size is as above. This is the lower bound case, in which the plate width can be considered as zero.

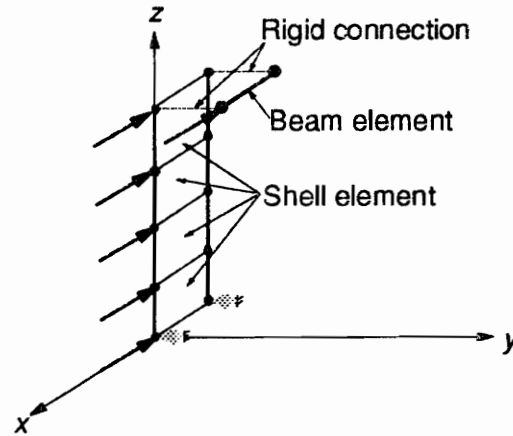


Figure 3.1-8, ABAQUS model of bulb-flat stiffener without plating

ABAQUS was run for a series of models with different lengths. The results and the comparison with the present study are shown in Fig. 3.1-8.

e) Comments

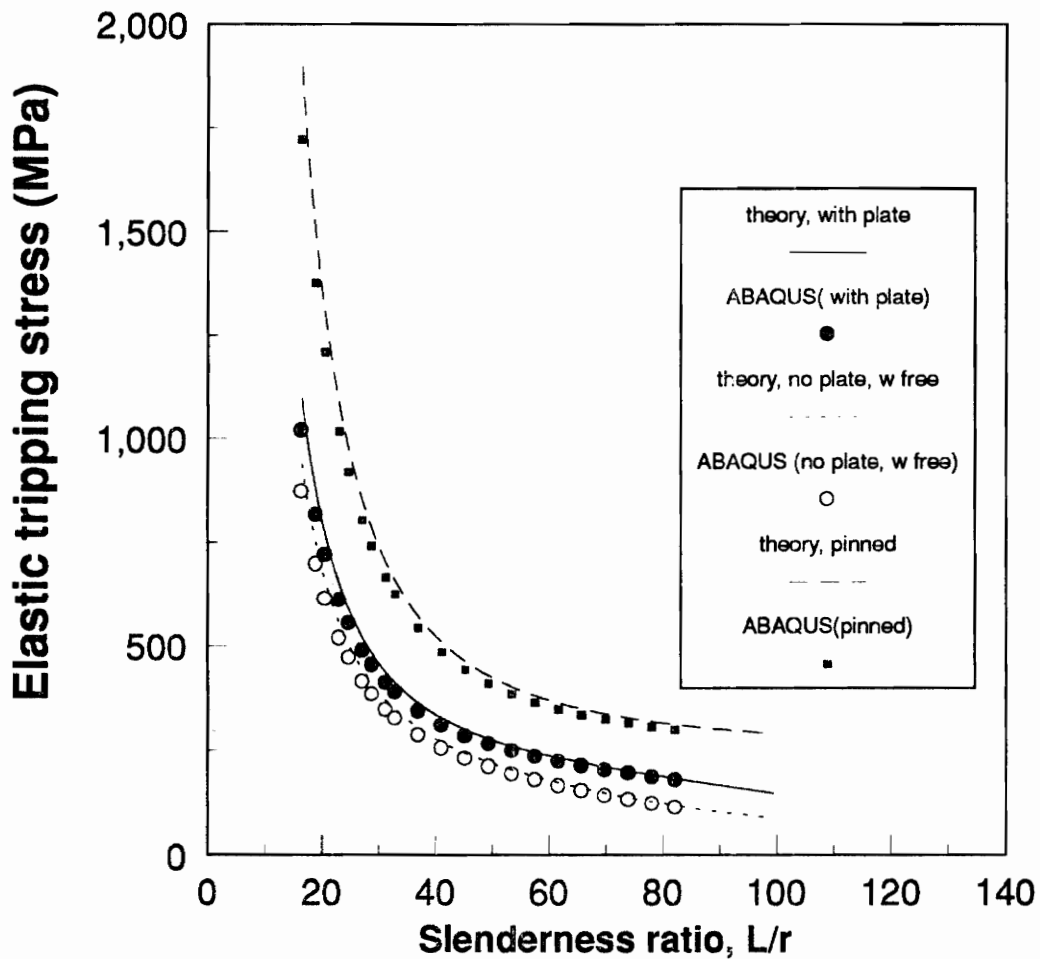
Fig.3.1-13 gives several curves of buckling stress vs. slenderness ratio. It is worthwhile to comment on some general properties of angle stiffeners at this stage, although the web distortion and plate rotational restraint will play a major role in tripping, as we will discuss later.

- For an isolated pinned stiffener, the tripping stress of the angles is always higher than that of tees. This is because the warping constant of angles is larger than that of tees: $I_z h^2$ of equation (3.1-11) vs. $I_z h^2$ of equation (3.1-14)
- For panels with moderate plate width, the tripping stress of angles is higher than tees when the panel slenderness ratio is small. This is also due to the considerably larger warping constant. However, when the panel slenderness ratio increases, the tripping stress of angles becomes lower than that of tees. This is because of the coupling of tripping and Euler buckling.

Comparison with ABAQUS for rigid web case -angle stiffener with and without plating under axial loading

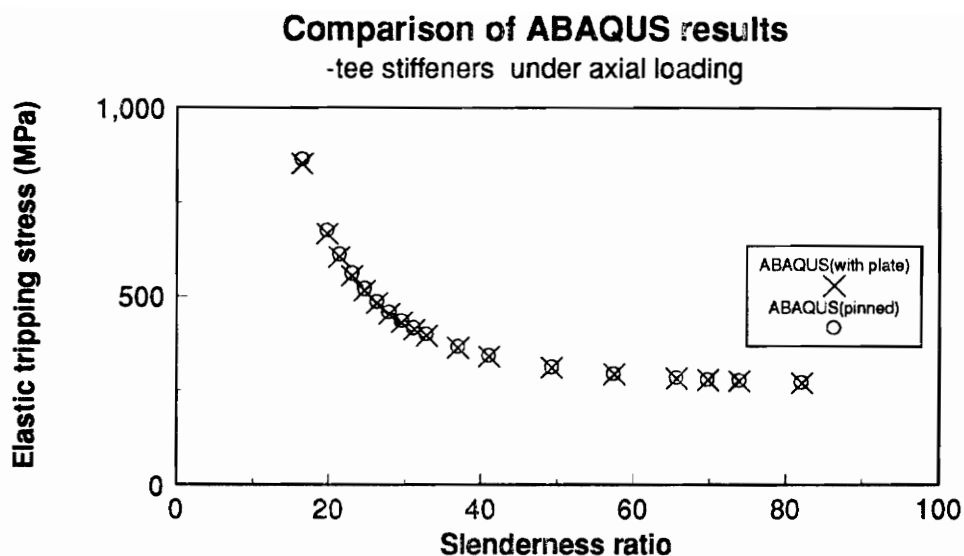
Panel with an angle stiffener

Ignoring web distortion and plate rotational restraint



Note: For the case where the stiffener is pinned and the case where there is no plate, the slenderness ratio is assumed to be the same as the case where the plate width is a finite value (203.2 mm)

Fig. 3.1-9



Note: For the case where the stiffener is pinned, the slenderness ratio is assumed to be the same as the case where the plate width is a finite value (203.2 mm)

Fig. 3.1-10

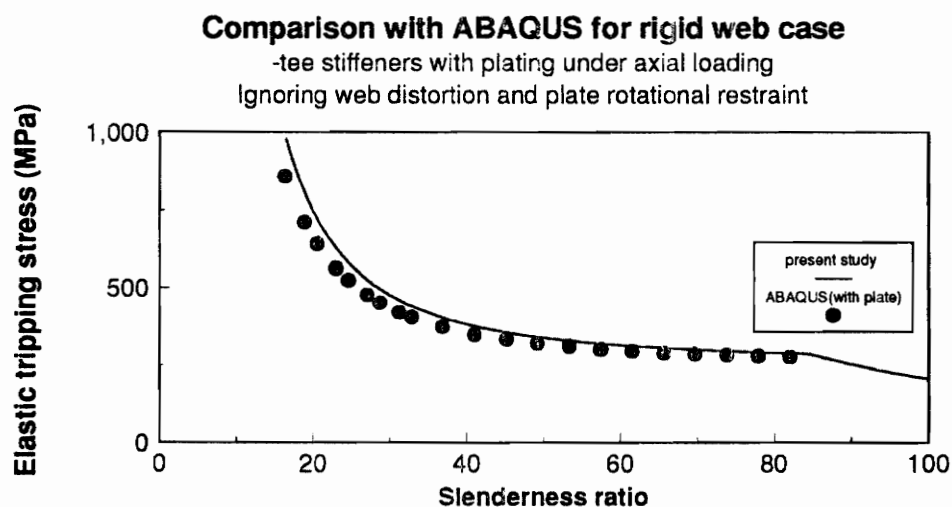


Fig. 3.1-11

Comparison with ABAQUS for rigid web case
-bulb-flat stiffener under axial loading

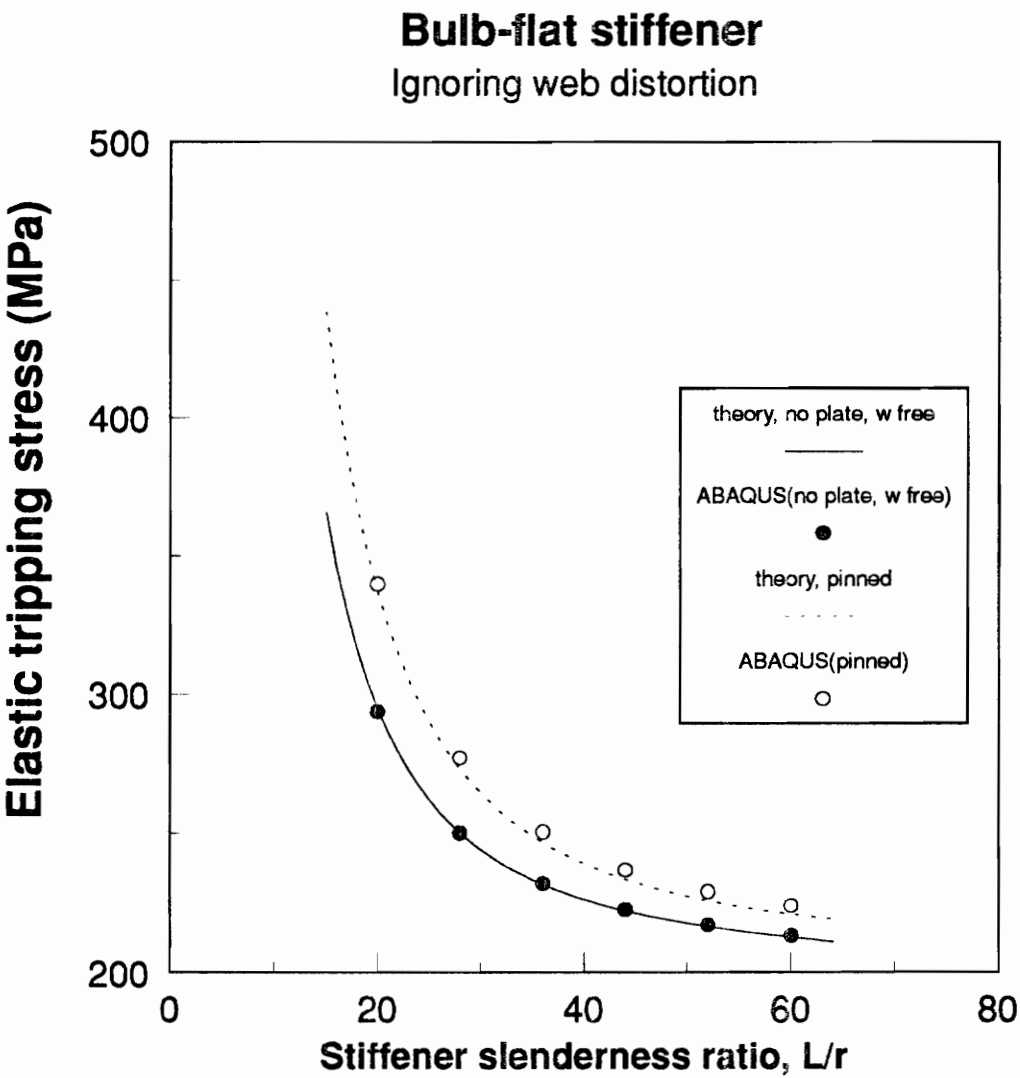
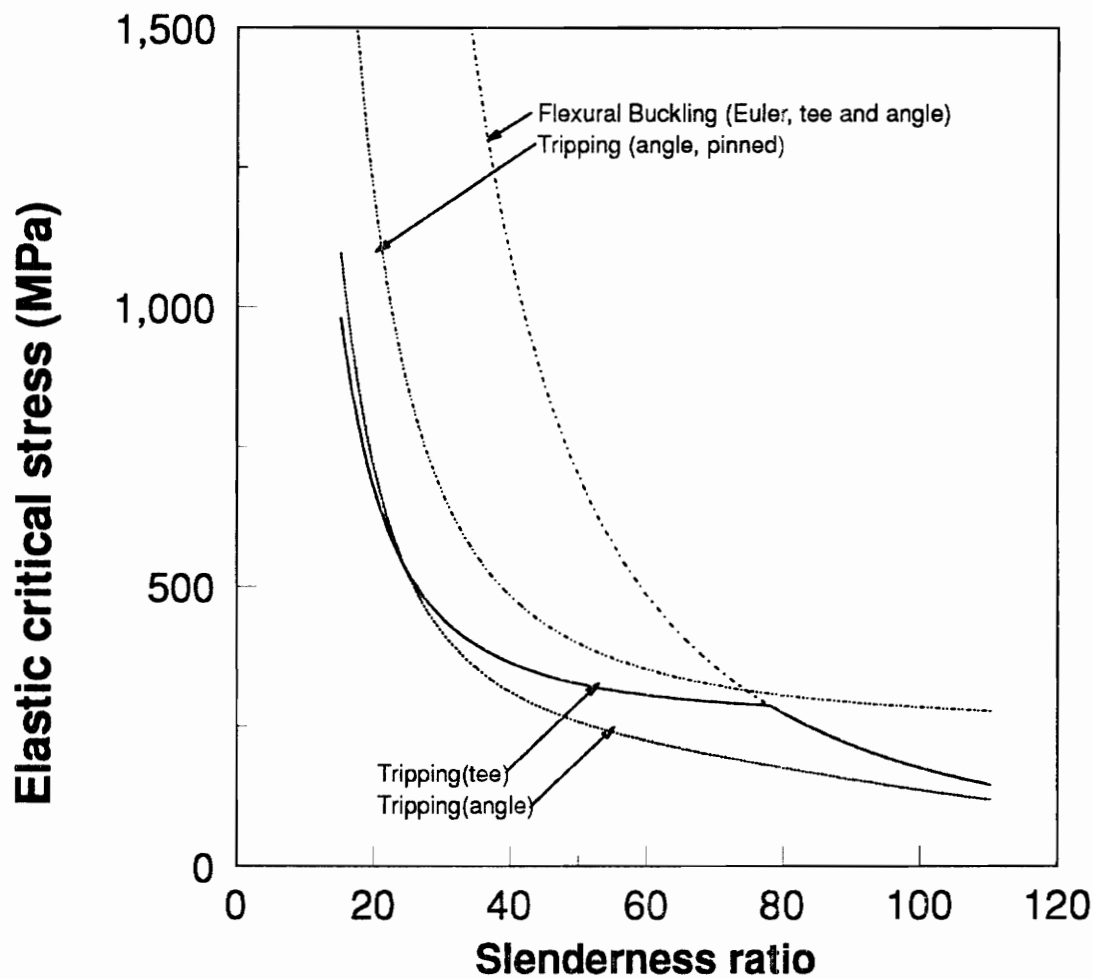


Fig. 3.1-12

Comparison of critical stress under axial loading

Ignoring web distortion and plate rotational restraint



Note: For the case where the stiffener is pinned, the slenderness ratio is assumed to be the same as the case where the plate width is a finite value (203.2 mm)

Fig. 3.1-13

ABAQUS

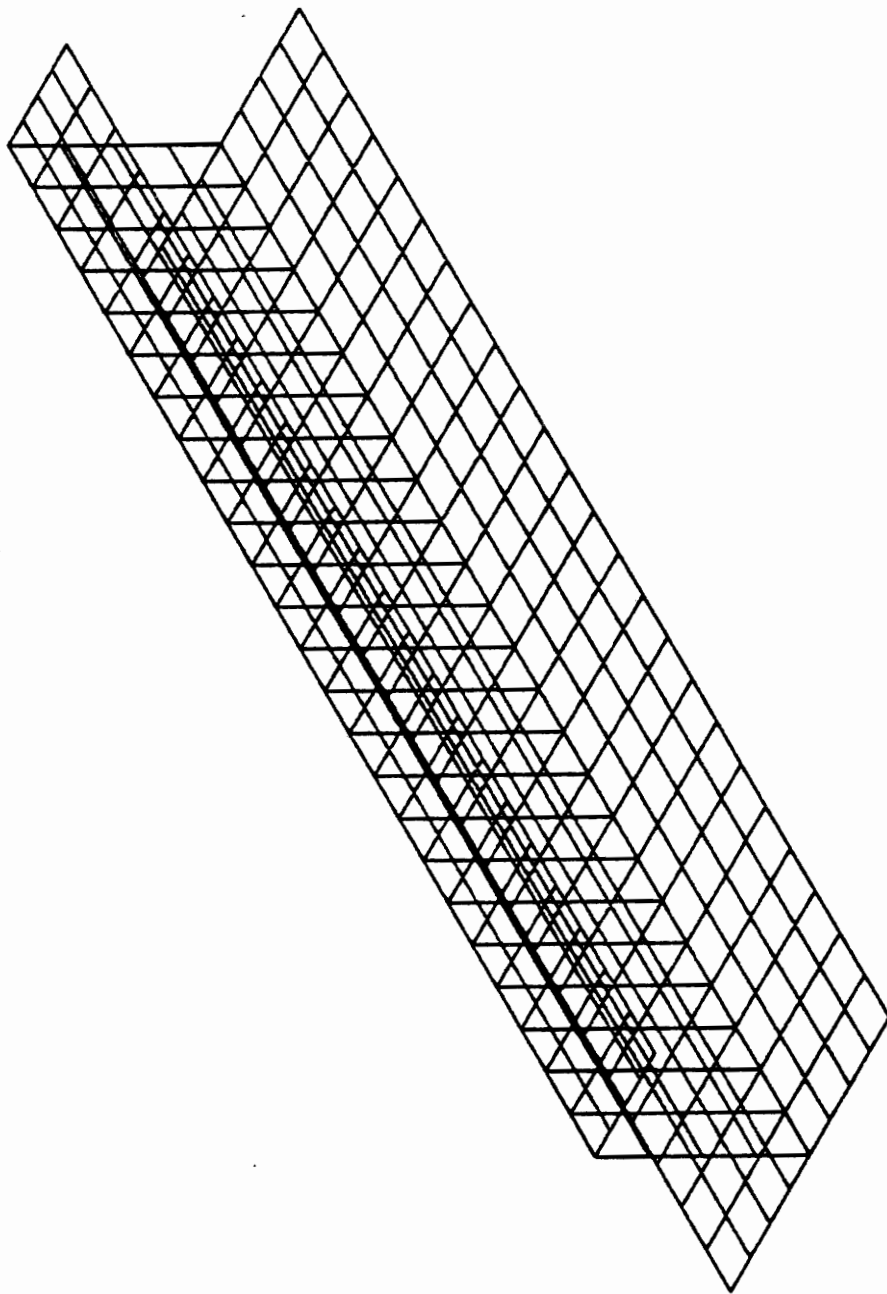


Fig. 3.1-14 Full ABAQUS model of a panel with an angle stiffener

ABAQUS

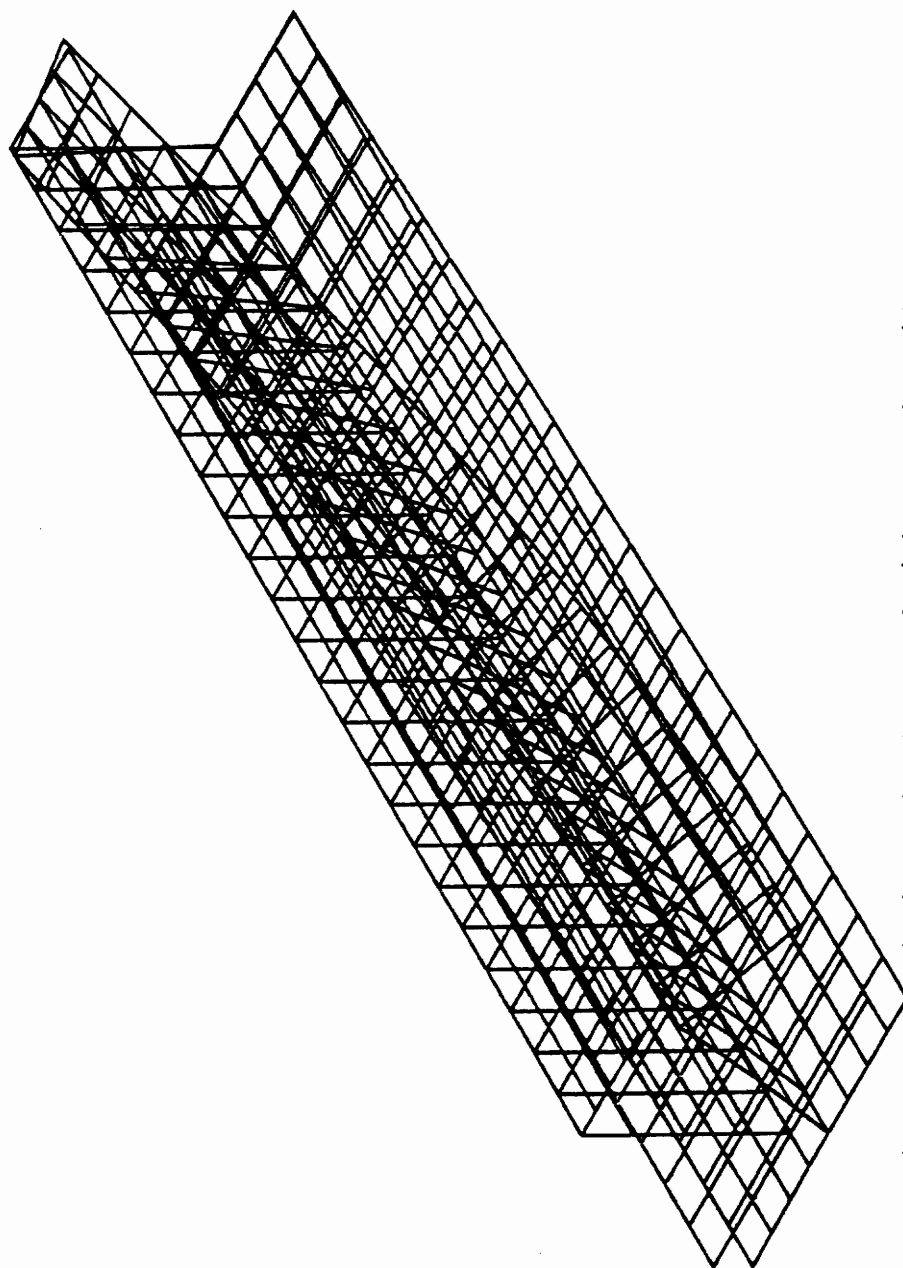


Fig. 3.1-15 Tripping mode of a panel with an angle stiffener

ABAQUS

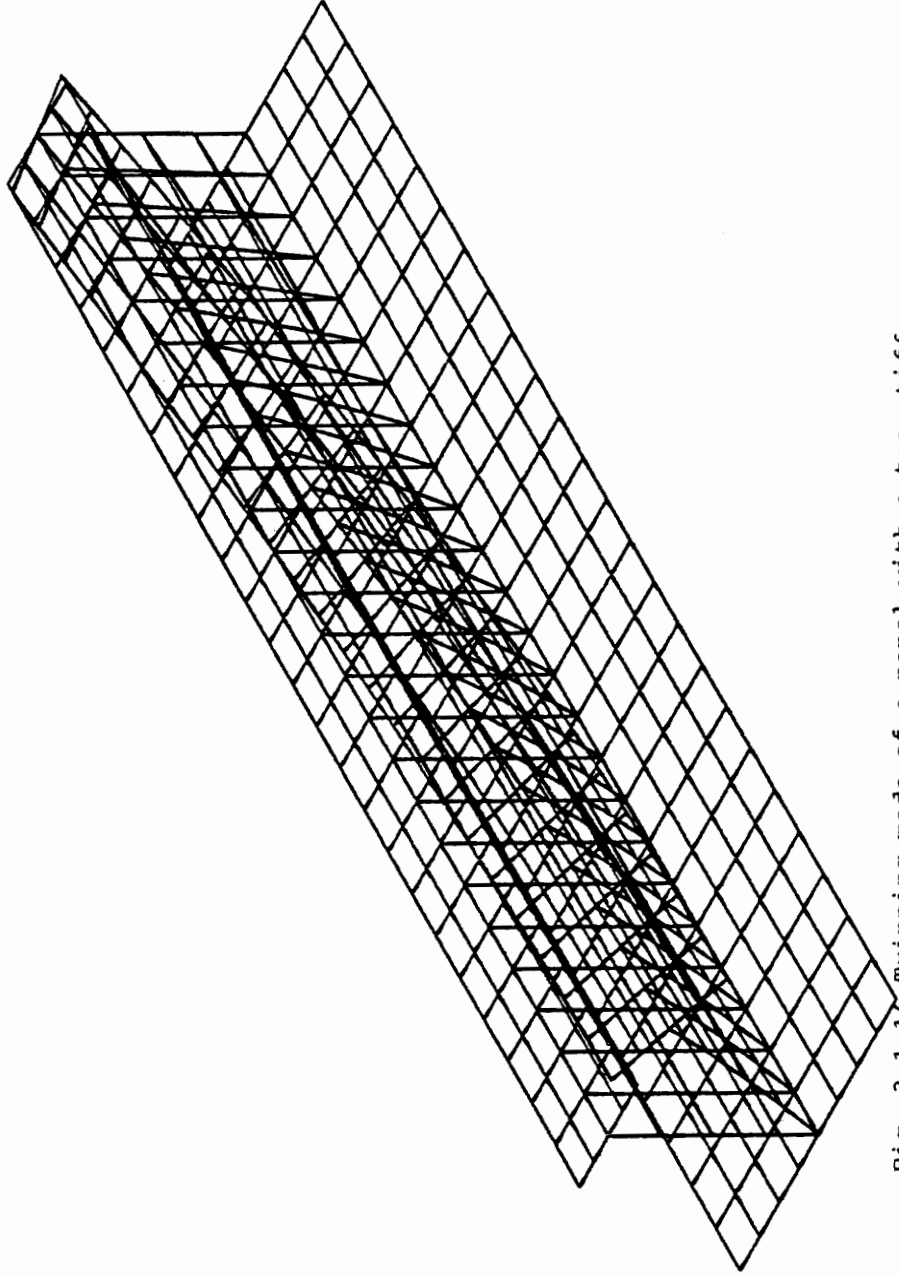
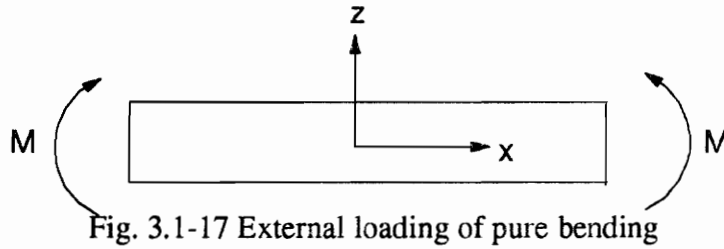


Fig. 3.1-16 Tripping mode of a panel with a tee stiffener

3.1.2 Tripping under end moment

A transversely loaded member that is bent about its major axis may buckle sideways if its compression flange does not have sufficient stiffness in the lateral direction. That is, sideways buckling occurs in a beam when the compression flange, which is in effect a column on an elastic foundation, becomes unstable. At the critical load there is a tendency for the compression flange to bend sideways, and for the remainder of the cross section, which is stable, to restrain it from doing so.



If the plate-stiffener combination is loaded by a constant moment M (positive as indicated in Fig. 3.1-17), the stress distribution will no longer be constant over the cross section but rather will vary linearly from plate to flange, as shown in Fig. 3.1-18.

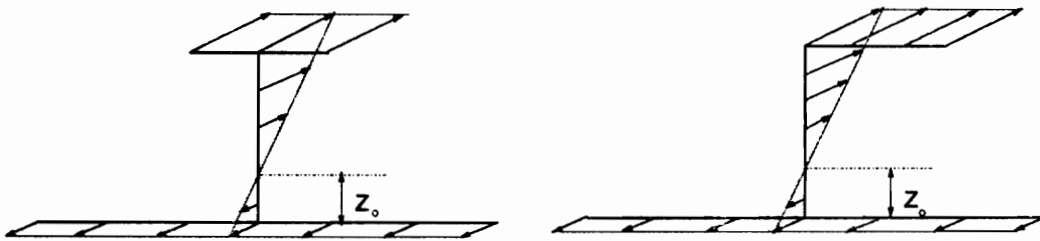


Figure 3.1-18 Stress distribution under external bending moment

The bending moment is

$$M = \int_A \sigma z dA$$

and the values of the bending stress σ corresponding to the bending moment are given by

$$\begin{cases} \sigma_f = -\frac{M}{I}(z_o - h) \\ \sigma_w = -\frac{M}{I}(z_o - z) \\ \sigma_p = -\frac{M}{I}z_o \end{cases}$$

where I is the vertical moment of inertia of the stiffener and its associated effective breadth of plating. If the same mode shape, Eq. (3.1-5), is assumed, then the expression of internal work will be the same as (3.1-6). The external work is

$$\begin{aligned} V &= \frac{1}{2} \int_{A_f} -\frac{M}{I}(z_o - h) \int_0^l (h^2 \theta_{,x}^2 + w_{,x}^2 - 2y \theta_{,x} w_{,x} + y^2 \theta_{,x}^2) dx dA \\ &\quad + \frac{1}{2} \int_{A_p} -\frac{M}{I} z_o \int_0^l w_{,x}^2 dx dA \\ &\quad + \frac{1}{2} \int_{A_w} -\frac{M}{I} (z_o - z) \int_0^l (w_{,x}^2 + z^2 \theta_{,x}^2) dx dA \\ &= \frac{1}{2} \left(\alpha_{\theta\theta}^m \int_0^l \theta_{,x}^2 dx + 2\alpha_{\theta w}^m \int_0^l \theta_{,x} w_{,x} dx + \alpha_{ww}^m \int_0^l w_{,x}^2 dx \right) \end{aligned}$$

where

$$\alpha_{\theta\theta}^m = \int_{A_f} \sigma_f (h^2 + y^2) dA + \int_{A_w} \sigma_w z^2 dA = M \frac{S_{\theta\theta}}{I}$$

$$\alpha_{ww}^m = \int_{A_f} \sigma_f dA + \int_{A_w} \sigma_w dA + \int_{A_p} \sigma_p dA = M \frac{S_{ww}}{I}$$

$$\alpha_{\theta w}^m = \int_{A_f} \sigma_f (-y) dA = M \frac{S_{w\theta}}{I}$$

The total potential energy is

$$\Pi = U - V$$

$$= \frac{1}{4} \lambda^2 l \left[C_2^2 (GJ_s + EI_z h^2 \lambda^2 - \frac{M}{I} S_{\theta\theta}) + C_1^2 (EI \lambda^2 - \frac{M}{I} S_{ww}) + 2C_1 C_2 (EI_{zy} h \lambda^2 - \frac{M}{I} S_{w\theta}) \right]$$

where for asymmetric stiffeners (angle and bulb-flat stiffeners)

$$\begin{cases} S_{\theta\theta} = -(z_o - h)t_f \left(h^2 b_f + \frac{b_f^3}{3} \right) - t_w \left(z_o \frac{h^3}{3} - \frac{h^4}{4} \right) \\ S_{ww} = -(z_o - h)t_f b_f - z_o t_p b - t_w \left(z_o h - \frac{h^2}{2} \right) \\ S_{w\theta} = (z_o - h)t_f \frac{b_f^2}{2} \end{cases} \quad (3.1-15)$$

and for tees

$$\begin{cases} S_{\theta\theta} = -(z_o - h)t_f \left(h^2 b_f + \frac{b_f^3}{12} \right) - t_w \left(z_o \frac{h^3}{3} - \frac{h^4}{4} \right) \\ S_{ww} = -(z_o - h)t_f b_f - z_o t_p b - t_w \left(z_o h - \frac{h^2}{2} \right) \\ S_{w\theta} = 0 \end{cases} \quad (3.1-16)$$

a) Asymmetric stiffeners

The critical condition occurs when

$$\frac{S_{\theta\theta} S_{ww}}{I^2} (M_\theta - M)(M_w - M) - \left(EI_{zy} h \lambda^2 - M \frac{S_{w\theta}}{I} \right)^2 = 0$$

where I and I_{zy} are given in Eq. (3.0-5); $S_{\theta\theta}$, $S_{\theta w}$ and S_{ww} are given in Eq. (3.1-15) and

$$M_\theta = \frac{I}{S_{\theta\theta}} (GJ_s + EI_z h^2 \lambda^2)$$

$$M_w = \frac{I}{S_{ww}} EI \lambda^2$$

in which I_z is given in Eq. (3.0-5), and J_s is given by Eq. (3.1-4).

The smaller root is the critical moment

$$M_{cr}^a = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (3.1-17)$$

where

$$\begin{cases} a = 1 - \frac{S_{w\theta}^2}{S_{\theta\theta}S_{ww}} \\ b = M_{\theta} + M_w - \frac{2IS_{w\theta}EI_{zy}h}{S_{\theta\theta}S_{ww}}\lambda^2 \\ c = M_wM_{\theta} - \frac{I^2(EI_{zy}h)^2}{S_{\theta\theta}S_{ww}}\lambda^4 \end{cases} \quad (3.1-18)$$

b) Tee stiffeners

For a panel with tee stiffeners

$$M'_{cr} = M_{\theta} = \frac{I}{S_{\theta\theta}}(GJ_s + EI_f h^2 \lambda^2) \quad (3.1-19)$$

where $S_{\theta\theta}$ is defined in Eq. (3.1-16); I and I_f are given in Eq. (3.0-6); and J_s is given by Eq. (3.1-4). A similar expression is also given by Adamchak [A1].

c) Comparison with ABAQUS

To verify the above theory, ABAQUS was again employed. One-half length of the panel was modeled. The flange and web are modeled as 4 x 24 S4R5 elements; the plate is modeled as 8 x 24 S4R5 elements. The dimensions of the panel are as follows: web 152.4 mm x 7.1 mm; flange 76.2 mm x 14.2 mm; plate 426.7 mm x 10.2 mm; Young's modulus is 2.066E5 N/mm²; and Poisson's ratio is 0.3. With those dimensions, the neutral axis is at height 38.1 mm. Symbolic illustrations of the ABAQUS models, showing only one row of elements, are given in Fig. 3.1-19 and Fig. 3.1-20.

1. Panel with angle-stiffener

The panel is simply supported at both ends, and the lateral deflection of the plate is prevented ($v=0$ at the base of the web). The nodes have only a pin connection at the junction of the web and the plating. Nodal forces are imposed such that $\Sigma F_x = 0$.

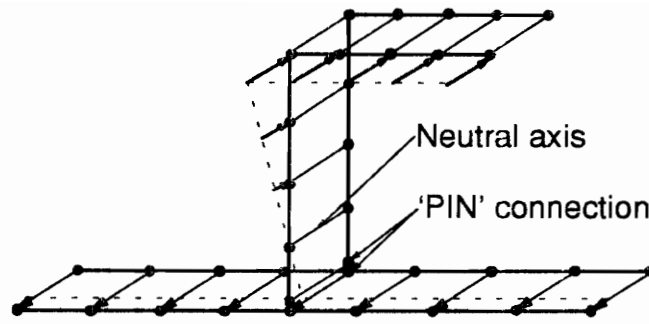


Figure 3.1-19 ABAQUS model of panel with angle stiffener under pure bending

ABAQUS was run for a series of models with different lengths. The results and the comparison with the present study are shown in Fig. 3.1-21(A).

2. Panel with tee-stiffener

Boundary conditions and loading are the same as for angles. The result and the comparison with the present study are shown in Fig.3.1-21(B).

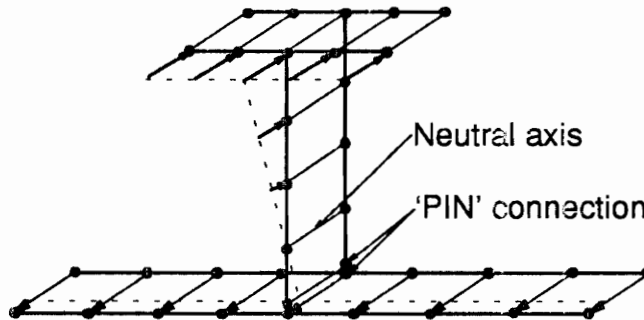
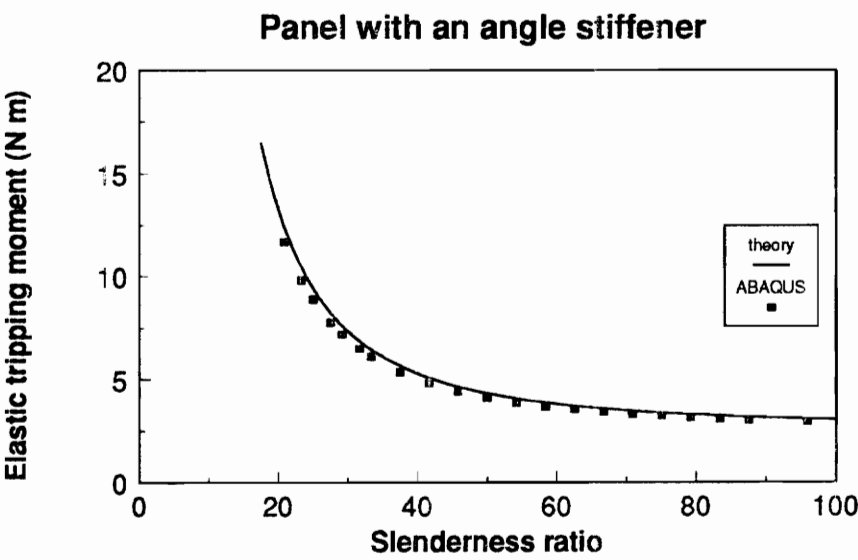
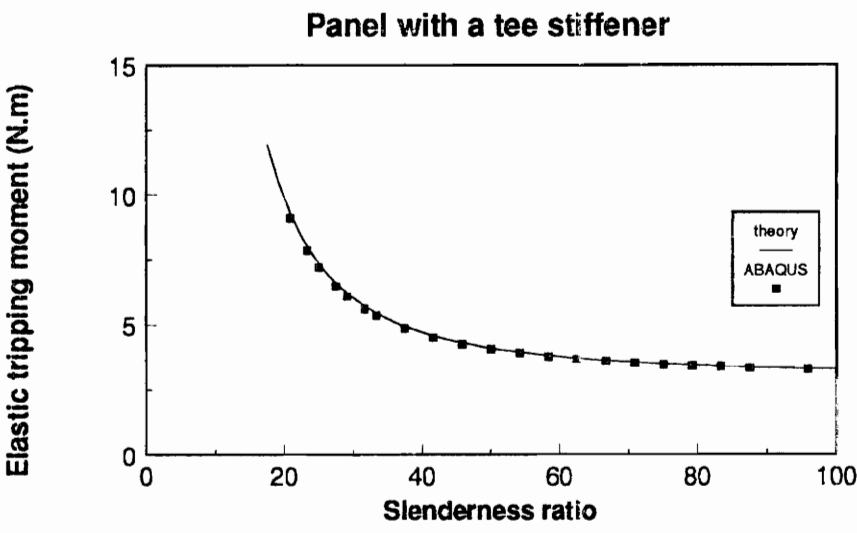


Figure 3.1-20 ABAQUS model of panel with tee stiffener under pure bending

Comparison with ABAQUS for rigid web case -panel under pure bending



(A)



(B)

Fig. 3.1-21

3.1.3 Tripping under uniform lateral pressure

Many stiffened plate components of a ship routinely experience lateral loading such that tripping collapse due to such loading (or at least in conjunction with inplane loading) is a definite possibility. Since tripping happens only when the stiffener flange is under compression, the study will consider the case when the stiffener panel is under downward loading.

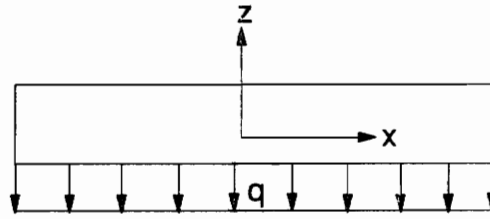


Fig. 3.1-22 Uniform lateral load

For a single stiffener and its associated width of plating, simply supported at its ends, the axial stress components due to a uniform lateral loading q (positive as shown in Fig.3.1-22) are as follows

$$\sigma_f = -\frac{q}{I}(z_o - h)\frac{(lx - x^2)}{2}$$

$$\sigma_w = -\frac{q}{I}(z_o - z)\frac{(lx - x^2)}{2}$$

$$\sigma_p = -\frac{q}{I}z_o\frac{(lx - x^2)}{2}$$

$$q = p \times b$$

where p is the uniform lateral pressure of the plating, and q is the equivalent line load. I is the vertical moment of inertia of the stiffener and its associated effective breadth of plating and b is the plate width. Compressive stress is assumed to be positive. Again, if the same mode shape, Eq. (3.1-5), is assumed, then the expression of internal work will be the same as (3.1-6). The external work becomes

$$\begin{aligned}
V &= \frac{1}{2} \int_{A_f} -\frac{q}{I} (z_o - h) \frac{(lx - x^2)}{2} \int_0^l (h^2 \theta_{,x}^2 + w_{,x}^2 - 2y \theta_{,x} w_{,x} + y^2 \theta_{,x}^2) dx dA \\
&+ \frac{1}{2} \int_{A_p} -\frac{q}{I} z_o \frac{(lx - x^2)}{2} \int_0^l w_{,x}^2 dx dA \\
&+ \frac{1}{2} \int_{A_w} -\frac{q}{I} (z_o - z) \frac{(lx - x^2)}{2} \int_0^l (w_{,x}^2 + z^2 \theta_{,x}^2) dx dA \\
&= \frac{1}{2} \left(\alpha_{\theta\theta}^q \int_0^l \frac{(lx - x^2)}{2} \theta_{,x}^2 dx + 2\alpha_{\theta w}^q \int_0^l \frac{(lx - x^2)}{2} \theta_{,x} w_{,x} dx + \alpha_{ww}^q \int_0^l \frac{(lx - x^2)}{2} w_{,x}^2 dx \right)
\end{aligned}$$

where

$$\alpha_{\theta\theta}^q = \int_{A_f} \sigma_f (h^2 + y^2) dA + \int_{A_w} \sigma_w z^2 dA = q \frac{S_{\theta\theta}}{I}$$

$$\alpha_{ww}^q = \int_{A_f} \sigma_f dA + \int_{A_w} \sigma_w dA + \int_{A_p} \sigma_p dA = q \frac{S_{ww}}{I}$$

$$\alpha_{\theta w}^q = \int_{A_f} \sigma_f (-y) dA = q \frac{S_{w\theta}}{I}$$

Thus the total potential energy is

$$\Pi = U - V$$

$$\begin{aligned}
&= C_2^2 \frac{l}{4} \lambda^2 \left(GJ_s + EI_z h^2 \lambda^2 - \frac{q}{I} S_{\theta\theta} \frac{(l\lambda)^2 - 3}{12\lambda^2} \right) + C_1^2 \frac{l}{4} \lambda^2 \left(EI\lambda^2 - \frac{q}{I} S_{ww} \frac{(l\lambda)^2 - 3}{12\lambda^2} \right) \\
&+ 2C_1 C_2 \frac{l}{4} \lambda^2 \left(EI_{zy} h \lambda^2 - \frac{q}{I} S_{w\theta} \frac{(l\lambda)^2 - 3}{12\lambda^2} \right)
\end{aligned}$$

The equilibrium equations are obtained by using the principle of total potential energy:

$$\begin{bmatrix} EI\lambda^2 - q \frac{S_{ww}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} & EI_{zy} h \lambda^2 - q \frac{S_{w\theta}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} \\ EI_{zy} h \lambda^2 - q \frac{S_{w\theta}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} & GJ_s + EI_z h^2 \lambda^2 - q \frac{S_{\theta\theta}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

a) Asymmetric stiffeners

The critical condition occurs when

$$\frac{S_{\theta\theta}S_{ww}((l\lambda)^2-3)^2}{144I^2}(q_w-q)(q_\theta-q)-\left(EI_{zy}h\lambda^2-q\frac{S_{w\theta}}{I}\frac{(l\lambda)^2-3}{12\lambda^2}\right)^2=0$$

where

$$q_\theta = \frac{I}{S_{\theta\theta}} \frac{12\lambda^2}{(l\lambda)^2-3} (GJ_s + EI_z h^2 \lambda^2)$$

$$q_w = \frac{I}{S_{ww}} \frac{12\lambda^2}{(l\lambda)^2-3} EI \lambda^2$$

$S_{\theta\theta}, S_{ww}, S_{w\theta}$ are defined in Eq. (3.1-15); I, I_z , and I_{zy} are given by Eq. (3.0-5); and J_s is given by Eq. (3.1-4). The smaller root is the critical lateral load of flexural-torsional buckling

$$q_{cr}^a = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (3.1-20)$$

where

$$\begin{cases} a = 1 - \frac{S_{w\theta}^2}{S_{\theta\theta}S_{ww}} \\ b = q_\theta + q_w - 2 \frac{IS_{w\theta}EI_{zy}h\lambda^2}{S_{\theta\theta}S_{ww}} \frac{12\lambda^2}{(l\lambda)^2-3} \\ c = q_w q_\theta - \frac{I^2(EI_{zy}h)^2\lambda^4}{S_{\theta\theta}S_{ww}} \left(\frac{12\lambda^2}{(l\lambda)^2-3} \right)^2 \end{cases}$$

b) Tee stiffeners

For panels with tee-stiffeners

$$q_{cr}' = q_\theta = \frac{I}{S_{\theta\theta}} \frac{12\lambda^2}{(l\lambda)^2-3} (GJ_s + EI_f h^2 \lambda^2) \quad (3.1-21)$$

where $S_{\theta\theta}$ is defined in Eq. (3.1-16); I and I_f are given by Eq. (3.0-6); and J_s is given by Eq. (3.1-4).

c) Comparison with ABAQUS

To verify the above theory, ABAQUS was again employed. The geometric dimensions and the modeling of the panel are the same as for the pure bending case. Symbolic illustration of the ABAQUS models, showing only one row of elements, are given in Fig. 3.1-23 and Fig. 3.1-24. A uniform lateral pressure is applied to plating downwards.

1. Panel with an angle-stiffener

The panel is simply supported at both ends, and the sideways deflection of the plate was prevented ($v=0$ at the base of the web). The nodes have only a pin connection at the junction of the web and the plating.

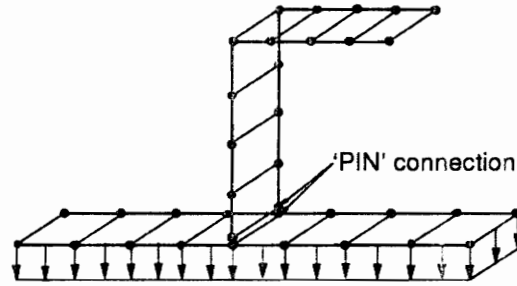


Fig.3.1-23 ABAQUS model of panel with angle stiffener under uniform lateral pressure

ABAQUS was run for a series of models with different lengths. The results and the comparison with the present study are shown in Fig. 3.1-25(A).

2. Panel with a tee-stiffener

Boundary conditions and loading are the same as for angles, and the result and the comparison with the present study are shown in Fig.3.1-25(B).

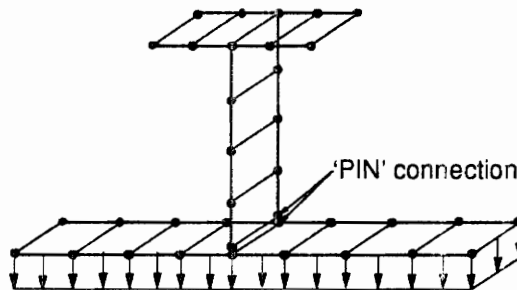
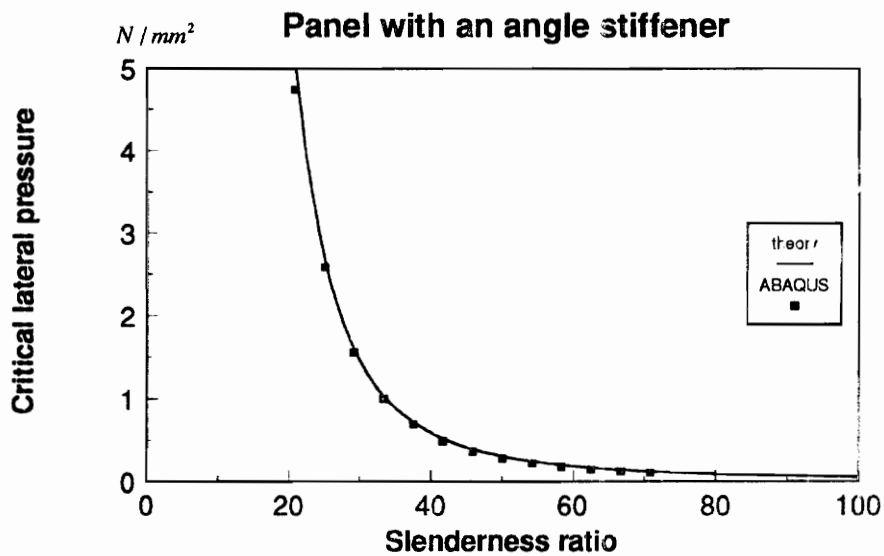
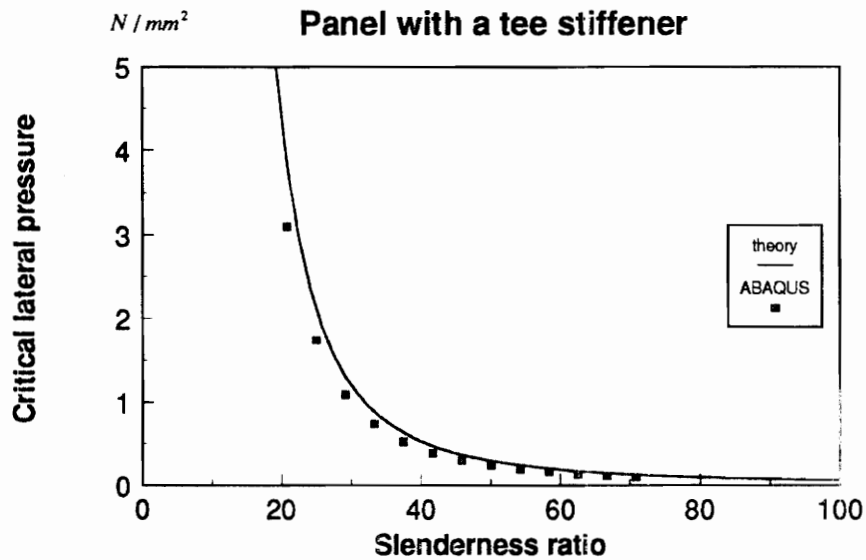


Fig.3.1-24 ABAQUS model of panel with tee stiffener under uniform lateral pressure

Comparison with ABAQUS for rigid web case -panel under uniform lateral loading



(A)



(B)

Fig. 3.1-25

3.1.4 Tripping under combined loading

In many practical situations axial loading, end moments and lateral pressure will occur simultaneously which naturally leads to the subject of tripping under combined loading. For a single stiffener and its associated width plating (simply supported at its ends) the axial stress components due to combined loading are as follows:

$$\sigma_f = \sigma - \frac{q}{I}(z_o - h) \frac{(lx - x^2)}{2} - \frac{M}{I}(z_o - h)$$

$$\sigma_w = \sigma - \frac{q}{I}(z_o - z) \frac{(lx - x^2)}{2} - \frac{M}{I}(z_o - z)$$

$$\sigma_p = \sigma - \frac{q}{I}z_o \frac{(lx - x^2)}{2} - \frac{M}{I}z_o$$

where I is the vertical moment of inertia of the stiffener and its associated effective breadth of plating, and compressive stress is positive. If the same mode shape, Eq. (3.1-5), is assumed, then the expression of internal work will be the same as (3.1-6). The external work leads to

$$\begin{aligned} V &= \frac{1}{2} \int_{A_f} \left(\sigma - \frac{q}{I}(z_o - h) \frac{(lx - x^2)}{2} - \frac{M}{I}(z_o - h) \right) \int_0^l (h^2 \theta_{,x}^2 + w_{,x}^2 - 2y \theta_{,x} w_{,x} + y^2 \theta_{,x}^2) dx dA \\ &+ \frac{1}{2} \int_{A_p} \left(\sigma - \frac{q}{I}(z_o - z) \frac{(lx - x^2)}{2} - \frac{M}{I}(z_o - z) \right) \int_0^l w_{,x}^2 dx dA \\ &+ \frac{1}{2} \int_{A_w} \left(\sigma - \frac{q}{I}z_o \frac{(lx - x^2)}{2} - \frac{M}{I}z_o \right) \int_0^l (w_{,x}^2 + z^2 \theta_{,x}^2) dx dA \\ &= \frac{1}{2} \left(\alpha_{\theta\theta}^a \int_0^l \theta_{,x}^2 dx + 2\alpha_{\theta w}^a \int_0^l \theta_{,x} w_{,x} dx + \alpha_{ww}^a \int_0^l w_{,x}^2 dx \right) \\ &+ \frac{1}{2} \left(\alpha_{\theta\theta}^q \int_0^l \frac{(lx - x^2)}{2} \theta_{,x}^2 dx + 2\alpha_{\theta w}^q \int_0^l \frac{(lx - x^2)}{2} \theta_{,x} w_{,x} dx + \alpha_{ww}^q \int_0^l \frac{(lx - x^2)}{2} w_{,x}^2 dx \right) \\ &+ \frac{1}{2} \left(\alpha_{\theta\theta}^m \int_0^l \theta_{,x}^2 dx + 2\alpha_{\theta w}^m \int_0^l \theta_{,x} w_{,x} dx + \alpha_{ww}^m \int_0^l w_{,x}^2 dx \right) \end{aligned}$$

where $\alpha_{\theta\theta}^a, \alpha_{\theta w}^a, \alpha_{ww}^a, \alpha_{\theta\theta}^q, \alpha_{\theta w}^q, \alpha_{ww}^q, \alpha_{\theta\theta}^m, \alpha_{\theta w}^m, \alpha_{ww}^m$ are defined in previous sections.

The total potential energy is

$$\Pi = U - V$$

$$\begin{aligned} &= \frac{l}{4} \lambda^2 C_2^2 \left(GJ_s + EI_z h^2 \lambda^2 - \sigma I_p - q \frac{S_{\theta\theta}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta\theta}}{I} \right) \\ &+ \frac{l}{4} \lambda^2 C_1^2 \left(EI\lambda^2 - \sigma A - q \frac{S_{ww}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{ww}}{I} \right) \\ &+ \frac{l}{4} \lambda^2 2C_1 C_2 \left(EI_{zy} h \lambda^2 - \sigma S_f - q \frac{S_{\theta w}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta w}}{I} \right) \end{aligned}$$

and the equilibrium equations are

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

where

$$\begin{aligned} c_{11} &= EI\lambda^2 - \sigma A - q \frac{S_{ww}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{ww}}{I} \\ c_{22} &= GJ_s + EI_z h^2 \lambda^2 - \sigma I_p - q \frac{S_{\theta\theta}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta\theta}}{I} \\ c_{12} &= EI_{zy} h \lambda^2 - \sigma S_f - q \frac{S_{\theta w}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta w}}{I} \end{aligned}$$

a) Asymmetric stiffeners

The critical condition occurs when

$$\begin{aligned} &\left[EI\lambda^2 - \sigma A - q \frac{S_{ww}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{ww}}{I} \right] \left[GJ_s + EI_z h^2 \lambda^2 - \sigma I_p - q \frac{S_{\theta\theta}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta\theta}}{I} \right] \\ &- \left[EI_{zy} h \lambda^2 - \sigma S_f - q \frac{S_{\theta w}}{I} \frac{(l\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta w}}{I} \right]^2 = 0 \end{aligned}$$

where $S_{\theta\theta}$, $S_{\theta w}$ and S_{ww} are given in Eq. (3.1-15); I , I_{zy} and I_z are given by Eq. (3.0-5); J_s is defined in Eq. (3.1-4); and A , S_f and I_p are given by Eq. (3.1-7). The smaller root is the critical value of the live load.

Case 1: Uniform lateral pressure and pure bending moment are dead loads. Axial compressive force is the live load; then the critical axial stress is

$$\sigma_{cr}^a = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (3.1-22)$$

where

$$\begin{cases} a = AI_p - S_y^2 \\ b = -I_p \left[EI\lambda^2 - q \frac{S_{wy}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{wy}}{I} \right] - A \left[GJ_s + EI_z h^2 \lambda^2 - q \frac{S_{\theta\theta}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta\theta}}{I} \right] \\ \quad + 2S_f \left[EI_{zy} h \lambda^2 - q \frac{S_{\theta w}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta w}}{I} \right] \\ c = \left[EI\lambda^2 - q \frac{S_{wy}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{wy}}{I} \right] \left[GJ_s + EI_z h^2 \lambda^2 - q \frac{S_{\theta\theta}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta\theta}}{I} \right] \\ \quad - \left[EI_{zy} h \lambda^2 - q \frac{S_{\theta w}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} - M \frac{S_{\theta w}}{I} \right]^2 \end{cases}$$

Case 2: Axial compressive force and pure bending moment are dead loads. Uniform lateral pressure force is the live load; then the critical uniform lateral pressure is

$$q_{cr}^a = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (3.1-23)$$

where

$$\begin{cases} a = \left(\frac{(\lambda)^2 - 3}{12\lambda^2} \right)^2 \left(\frac{S_{wy} S_{\theta\theta}}{I^2} - \left(\frac{S_{\theta w}}{I} \right)^2 \right) \\ b = -\frac{S_{\theta\theta}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \left[EI\lambda^2 - \sigma A - M \frac{S_{wy}}{I} \right] - \frac{S_{wy}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \left[GJ_s + EI_z h^2 \lambda^2 - \sigma I_p - M \frac{S_{\theta\theta}}{I} \right] \\ \quad + 2 \frac{S_{\theta w}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \left[EI_{zy} h \lambda^2 - \sigma S_f - M \frac{S_{\theta w}}{I} \right] \\ c = \left[EI\lambda^2 - \sigma A - M \frac{S_{wy}}{I} \right] \left[GJ_s + EI_z h^2 \lambda^2 - \sigma I_p - M \frac{S_{\theta\theta}}{I} \right] - \left[EI_{zy} h \lambda^2 - \sigma S_f - M \frac{S_{\theta w}}{I} \right]^2 \end{cases}$$

Case 3: Uniform lateral pressure and axial compressive force are the dead loads. Pure bending moment is the live load; then the critical end moment is

$$M_{cr}^a = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (3.1-24)$$

$$\begin{cases} a = \frac{S_{ww}S_{\theta\theta}}{I^2} - \left(\frac{S_{\theta w}}{I}\right)^2 \\ b = -\frac{S_{\theta\theta}}{I} \left[EI\lambda^2 - \sigma A - q \frac{S_{ww}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \right] - \frac{S_{ww}}{I} \left[GJ_s + EI_z h^2 \lambda^2 - \sigma I_p - q \frac{S_{\theta\theta}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \right] \\ \quad + 2 \frac{S_{\theta w}}{I} \left[EI_{zy} h \lambda^2 - \sigma S_f - q \frac{S_{\theta w}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \right] \\ c = \left[EI\lambda^2 - \sigma A - q \frac{S_{ww}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \right] \left[GJ_s + EI_z h^2 \lambda^2 - \sigma I_p - q \frac{S_{\theta\theta}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \right] - \left[EI_{zy} h \lambda^2 - \sigma S_f - q \frac{S_{\theta w}}{I} \frac{(\lambda)^2 - 3}{12\lambda^2} \right]^2 \end{cases}$$

b) Tee stiffener

For a tee-stiffener

$$\sigma'_{cr} = \frac{GJ + EI_f h^2 \lambda^2}{I_p} - \frac{S_{\theta\theta}}{I_p I} \left(q \frac{(\lambda)^2 - 3}{12\lambda^2} + M \right) \quad (3.1-25)$$

$$M'_{cr} = \frac{I}{S_{\theta\theta}} (GJ_s + EI_f h^2 \lambda^2) - \left(\sigma I_p \frac{I}{S_{\theta\theta}} + q \frac{(\lambda)^2 - 3}{12\lambda^2} \right) \quad (3.1-26)$$

$$q'_{cr} = \frac{I}{S_{\theta\theta}} \frac{12\lambda^2}{(\lambda)^2 - 3} (GJ_s + EI_f h^2 \lambda^2) - \frac{12\lambda^2}{(\lambda)^2 - 3} \left(\sigma I_p \frac{I}{S_{\theta\theta}} + M \right) \quad (3.1-27)$$

where $S_{\theta\theta}$ is defined in Eq. (3.1-16); I , and I_f are given by Eq. (3.0-6); J_s is defined in Eq. (3.1-4); and A and I_p are given by Eq. (3.1-8).

c) Comparison with ABAQUS

To verify the above theory, ABAQUS was again employed. The geometric dimensions and the modeling of the panel are the same as for the pure bending case. The panel is subjected to a static uniform lateral pressure (dead load) and an axial compression (live load) since this load combination is most common in ship structures. Symbolic illustrations of the ABAQUS models, showing only one row of elements, are given in Fig. 3.1-26 and Fig. 3.1-27.

1. Panel with angle-stiffener

The panel was simply supported at both ends, and the lateral deflection of the plate was prevented ($v=0$ at the base of the web). The nodes have only a pin connection at the junction of the web and the plating. Uniform lateral pressure is applied to plating downwards acting as a dead load, uniform axial nodal forces are the live load.

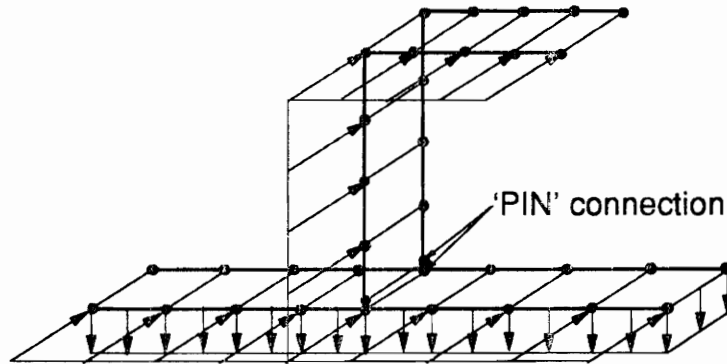


Fig. 3.1-26 ABAQUS model of panel with angle stiffener under combined loading

ABAQUS was run for a series of models with different lengths. The results and the comparison with the present study are shown in Fig. 3.1-28(A).

2. Panel with tee-stiffener

Boundary conditions and loading are the same as for angle stiffeners. The results and the comparison are shown in Fig.3.1-28(B). The discrepancy of ABAQUS and the present study is partly due to ignoring web distortion. This also occurs in the case of axial compression (Fig. 3.1-11). The flexible web case will be studied in Section 3.2.

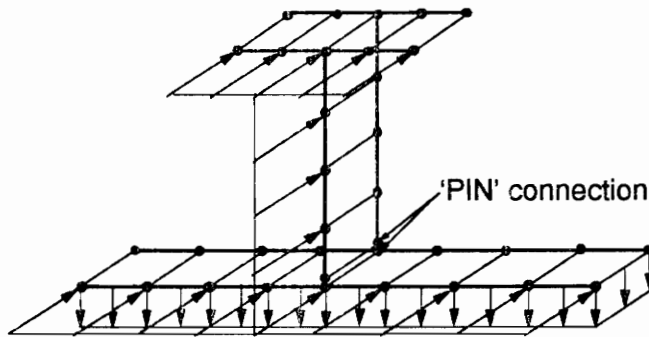


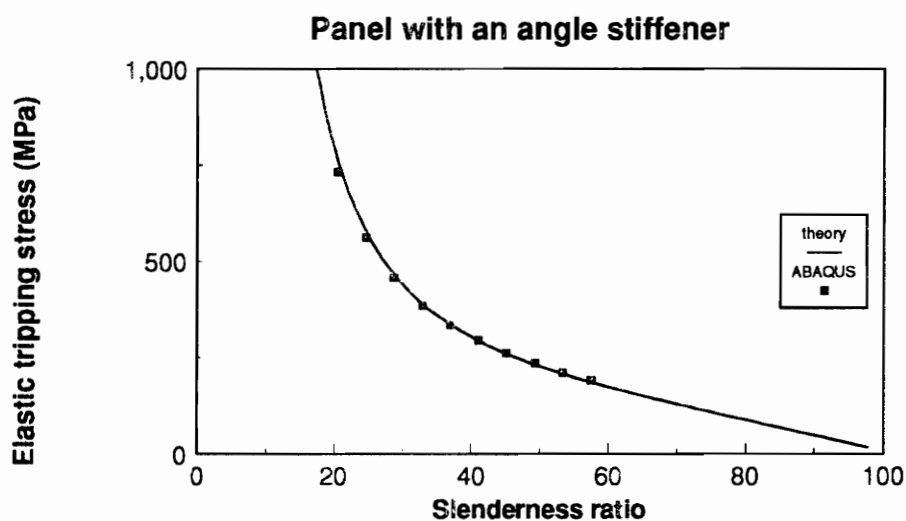
Fig. 3.1-27 ABAQUS model of panel with tee stiffener under combined loading

Comparison with ABAQUS for rigid web case

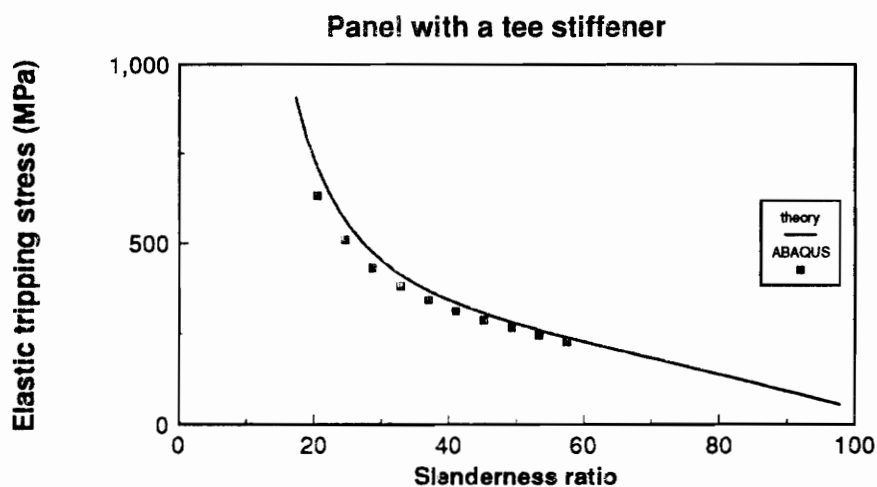
-panel under combined loading

dead load-- uniform lateral pressure (0.1033 MPa)

live load--axial compression



(A)



(B)

Fig. 3.1-28

3.2 Flexible Web Case

In the previous developments the web of the stiffener was assumed to be rigid in cross section. In certain circumstances, for example, when the ratio of t_w/h is small, or the plate rotational restraint is included, this assumption can lead to buckling predictions which are considerably higher than the true values. It is clear that a solution which takes into account web distortion is preferable.

3.2.1 Displacement field and total potential energy

The flexural displacement v_w of the web, of which the maximum value is v_T in Fig. 2.2-1, is obtained by assuming that the web buckles out-of-plane as a cubic curve, so that

$$v_w = \phi_B(x)f_1(z) + v_T(x)f_2(z) + \phi_T(x)f_3(z) \quad (3.2-1)$$

in which f_1 , f_2 and f_3 are cubic polynomials. They can be obtained by the compatibility identities

$$\begin{cases} \phi_T = -\left(\frac{\partial v_w}{\partial z}\right)_{z=h} \\ \phi_B = \left(\frac{\partial v_w}{\partial z}\right)_{z=0} \\ v_T = (v_w)_{z=h} \end{cases} \quad (3.2-2)$$

By using Eq.(3.2-1) and Eq. (3.2-2), the polynomials can be written as

$$\begin{cases} f_1 = h\left(\frac{z}{h} - 2\left(\frac{z}{h}\right)^2 + \left(\frac{z}{h}\right)^3\right) \\ f_2 = 3\left(\frac{z}{h}\right)^2 - 2\left(\frac{z}{h}\right)^3 \\ f_3 = -h\left(\left(\frac{z}{h}\right)^2 - \left(\frac{z}{h}\right)^3\right) \end{cases} \quad (3.2-3)$$

The displacements of a panel for the flexible web case will be the same as for the rigid web case except for the sideways displacement v_w , which is given by Eq. (3.2-1), and therefore will not be written here.

The out of plane bending energy of the flexible web can be written as

$$U_{web-out-of-plane} = \frac{1}{2} \frac{Et_w^3}{12(1-\nu^2)} \iint (v_{w,xx}^2 + v_{w,zz}^2 + 2\nu v_{w,xx} v_{w,zz} + 2(1-\nu) v_{w,xz}^2) dx dz$$

The total potential energy is

$$\Pi = U - V \quad (3.2-4)$$

where

$$\begin{aligned} U &= \frac{1}{2} E \int_0^l I_{w,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_l J_f \phi_{T,x}^2 dx \\ &\quad + \frac{1}{2} \frac{Et_w^3}{12(1-\nu^2)} \iint (v_{w,xx}^2 + v_{w,zz}^2 + 2\nu v_{w,xx} v_{w,zz} + 2(1-\nu) v_{w,xz}^2) dx dz \\ V &= \frac{1}{2} \int_{A_f} \sigma \int_0^l (v_{T,x}^2 + (w - y \phi_T)_{,x}^2) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA \end{aligned}$$

It is assumed that displacements and twists vary sinusoidally lengthwise along the member.

Then the buckling deformations can be written as

$$\begin{Bmatrix} \phi_B \\ v_T \\ \phi_T \\ w \end{Bmatrix} = \begin{Bmatrix} \bar{\phi}_B \\ \bar{v}_T \\ \bar{\phi}_T \\ \bar{w} \end{Bmatrix} \sin \frac{m\pi x}{l} = \{\delta\} \sin \lambda x \quad (3.2-5)$$

where $\bar{w}, \bar{\phi}_B, \bar{v}_T, \bar{\phi}_T$ are the maximum amplitudes of buckling displacements, and m is a positive integer representing the number of harmonics into which the beam buckles. Substituting the mode shapes into equation (3.2-4) and performing the necessary operations, the linear stiffness matrix and the geometric matrix may be obtained.

3.2.2 Linear stiffness matrix

The linear stiffness matrix can be obtained from the strain energy stored during buckling,

$$\begin{aligned}
U &= \frac{1}{2} E \int_0^l I_{w,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_l J_f \phi_{T,x}^2 dx \\
&+ \frac{1}{2} \frac{Et_w^3}{12(1-\nu^2)} \iint (v_{w,xx}^2 + v_{w,zz}^2 + 2\nu v_{w,xx} v_{w,zz} + 2(1-\nu)v_{w,xz}^2) dx dz \\
&= \frac{1}{2} \{\delta\}^T [K_L^0] \{\delta\}
\end{aligned}$$

where $[K_L^0]$ is given by

$$[K_L^0] = [K_{LFP}] + [K_{LW}] \quad (3.2-6)$$

in which

$$[K_{LW}] = \frac{l}{2} D_w \begin{bmatrix} \frac{h^3}{105} \lambda^4 + \frac{4}{h} + \frac{4h}{15} \lambda^2 & \frac{13h^2}{420} \lambda^4 - \frac{6}{h^2} - \frac{1}{5} \lambda^2 & -\frac{h^3}{140} \lambda^4 + \frac{2}{h} - \frac{h}{15} \lambda^2 & 0 \\ \frac{13h}{35} \lambda^4 + \frac{12}{h^3} + \frac{12}{5h} \lambda^2 & -\frac{11h^2}{210} \lambda^4 - \frac{6}{h^2} - \frac{1+5\nu}{5} \lambda^2 & 0 & 0 \\ \frac{h^3}{105} \lambda^4 + \frac{4}{h} + \frac{4h}{15} \lambda^2 & 0 & 0 & 0 \\ \text{sym} & & & \end{bmatrix}$$

$$[K_{LFP}] = \frac{l}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ & EI_z \lambda^4 & 0 & EI_{zy} \lambda^4 \\ & & GJ_f \lambda^2 & 0 \\ \text{sym} & & & EI \lambda^4 \end{bmatrix}$$

$$\lambda = \frac{m\pi}{l}$$

$$D_w = \frac{Et_w^3}{12(1-\nu^2)}$$

I , I_z and I_{zy} are given by Eq. (3.0-5) for angle and bulb-flat stiffeners and by Eq. (3.0-6) for tee stiffeners. J_f is given by Eq. (3.0-8) for thin flange stiffeners and by Eq. (3.0-9) for bulb-flat stiffeners.

3.2.3 Geometric stiffness matrix

The geometric matrix can be obtained from the work done during buckling

$$\begin{aligned}
 V &= \frac{1}{2} \int_{A_f} \int_0^l (v_{T,x}^2 + (w - y\phi_T)_{,x}^2) dx dA + \frac{1}{2} \int_{A_p} \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA \\
 &= \frac{1}{2} \{\delta\}^T [K_G] \{\delta\}
 \end{aligned} \tag{3.2-7}$$

Since σ is a function of x depending on the load condition, the geometric stiffness matrix $[K_G]$ will be different according to the load conditions and is given in the following.

1) Geometric stiffness due to axial compressive load

The stress distribution is

$$\sigma_f = \sigma_w = \sigma_p = \sigma$$

Thus

$$[K_G] = \sigma [K_G^a]$$

where $[K_G^a]$ is given by

$$[K_G^a] = [K_{GW}^a] + [K_{GF}^a] + [K_{GP}^a] \tag{3.2-8}$$

in which

$$[K_{GW}^a] = \frac{1}{2} I \lambda^2 t_w \begin{bmatrix} \frac{h^3}{105} & \frac{13h^2}{420} & -\frac{h^3}{140} & 0 \\ & \frac{13h}{35} & -\frac{11h^2}{210} & 0 \\ & & \frac{h^3}{105} & 0 \\ \text{sym} & & & h \end{bmatrix}$$

$$[K_{GP}^a] = \frac{1}{2} I \lambda^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ & & 0 & 0 \\ \text{sym} & & & t_p b_p \end{bmatrix}$$

where for angle stiffener and bulb-flats

$$[K_{GF}^a] = \frac{1}{2} I \lambda^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & t_f b_f & 0 & 0 \\ & & \frac{t_f b_f^3}{3} & -\frac{t_f b_f^2}{2} \\ sym & & & t_f b_f \end{bmatrix}$$

for tee-stiffener

$$[K_{GF}^a] = \frac{1}{2} I \lambda^2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & t_f b_f & 0 & 0 \\ & & \frac{t_f b_f^3}{12} & 0 \\ sym & & & t_f b_f \end{bmatrix}$$

2) Geometric stiffness matrix due to pure bending

The stress distribution is

$$\sigma_f = -\frac{M}{I}(z_o - h)$$

$$\sigma_w = -\frac{M}{I}(z_o - z)$$

$$\sigma_p = -\frac{M}{I}z_o$$

Then

$$[K_G] = M[K_G^m]$$

where $[K_G^m]$ is given by

$$[K_G^m] = [K_{GW}^m] + [K_{GF}^m] + [K_{GP}^m] \quad (3.2-9)$$

in which

$$[K_{GW}^m] = \frac{1}{2} l \lambda^2 \frac{t_w}{I} \begin{pmatrix} \begin{bmatrix} \frac{h^3}{105} & \frac{13h^2}{420} & -\frac{h^3}{140} & 0 \\ \frac{13h^2}{35} & -\frac{11h^2}{210} & \frac{h^3}{105} & 0 \\ \text{sym} & & & h \end{bmatrix} + \begin{bmatrix} \frac{h^4}{280} & \frac{h^3}{60} & -\frac{h^4}{280} & 0 \\ \frac{2h^2}{7} & -\frac{h^3}{28} & \frac{h^4}{168} & 0 \\ \text{sym} & & & \frac{h^2}{2} \end{bmatrix} \end{pmatrix}$$

$$[K_{GP}^m] = \left(-\frac{z_0}{I} \right) [K_{GP}^p]$$

$$[K_{GF}^m] = \left(-\frac{z_0 - h}{I} \right) [K_{GF}^p]$$

3) Geometric stiffness matrix due to uniform lateral load

The stress distribution is

$$\sigma_f = -\frac{q}{I} (z_0 - h) \frac{(lx - x^2)}{2}$$

$$\sigma_w = -\frac{q}{I} (z_0 - z) \frac{(lx - x^2)}{2}$$

$$\sigma_p = -\frac{q}{I} z_0 \frac{(lx - x^2)}{2}$$

Then

$$[K_G] = q [K_G^q]$$

where $[K_G^q]$ is given by

$$[K_G^q] = [K_{GW}^q] + [K_{GF}^q] + [K_{GP}^q] \quad (3.2-10)$$

in which

$$[K_{GW}^q] = \frac{(l\lambda)^2 - 3}{12\lambda^2} [K_{GW}^m]$$

$$[K_{GP}^q] = \frac{(l\lambda)^2 - 3}{12\lambda^2} [K_{GP}^m]$$

$$[K_{GF}^q] = \frac{(l\lambda)^2 - 3}{12\lambda^2} [K_{GF}^m]$$

4) Geometric stiffness matrix due to axial load eccentricity

In lab tests, in order to promote tripping, the applied axial compressive load is sometimes not aligned with the neutral axis of the cross-section. The load eccentricity induces end moments proportional to axial load, and therefore an additional geometric stiffness due to the load eccentricity should be taken into account. The moment due to the eccentricity is

$$M = Pe = \sigma A e$$

Then the geometric stiffness matrix is

$$[K_G^L] = M[K_G^m] = \sigma A e [K_G^m]$$

where e is the distance between the neutral axis and the load position and A is the area of cross-section.

The total potential energy can then be written as

$$\begin{aligned} \Pi &= U - V \\ &= \frac{1}{2} \{\delta\}^T ([K_L] - [K_G]) \{\delta\} \end{aligned}$$

where

$$[K_L] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ & k_{22} & k_{23} & k_{24} \\ & & k_{33} & k_{34} \\ sym & & & k_{44} \end{bmatrix} \quad (3.2-11)$$

$$[K_G] = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ & g_{22} & g_{23} & g_{24} \\ & & g_{33} & g_{34} \\ sym & & & g_{44} \end{bmatrix} \quad (3.2-12)$$

The stiffness elements k_{ij} and g_{ij} ($i, j=1..4$) will be defined in equations (3.2-13)~(3.2-18) according to different loading conditions.

3.2.4 Single loading case

For the case of single loading,

$$[K_L] = [K_L^0]$$

and $[K_G]$ is chosen as either $[K_G^a] + Ae[K_G^m]$, or $[K_G^m]$ or $[K_G^q]$ corresponding to axial force, end moments, and lateral uniform load, respectively. $[K_G^a]$, $[K_G^m]$ and $[K_G^q]$ are defined in Eq. (3.2-8), Eq. (3.2-9) and Eq. (3.2-10), respectively. $[K_L^0]$ is defined in Eq. (3.2-6).

3.2.5 Combined loading case

For the case of combined loading

A) If axial compressive force is the live load, then

$$[K_L] = [K_L^0] - q[K_G^q] - M[K_G^m] \quad (3.2-13)$$

$$[K_G] = \sigma([K_G^a] + Ae[K_G^m]) \quad (3.2-14)$$

B) If end moments are the live load, then

$$[K_L] = [K_L^0] - \sigma([K_G^a] + Ae[K_G^m]) - q[K_G^q] \quad (3.2-15)$$

$$[K_G] = M[K_G^m] \quad (3.2-16)$$

C) If lateral uniform load is the live load, then

$$[K_L] = [K_L^0] - \sigma([K_G^a] + Ae[K_G^m]) - M[K_G^q] \quad (3.2-17)$$

$$[K_G] = q[K_G^q] \quad (3.2-18)$$

3.2.6 Buckling solution

The usual stability condition

$$[[K_L] - [K_G]] = 0 \quad (3.2-19)$$

must be satisfied. This eigenproblem is of order of 4×4 , and so an analytical solution would be quite tedious. Therefore it was solved by a numerical eigenvalue routine, which gives a very rapid solution. The critical value of σ , q , or M that results in a zero determinant of $[K_L] - [K_G]$ is the eigenvalue, while the corresponding eigenvector $\{\delta\}$

describes the buckled shape at the mid-point of the relevant harmonic n . The accuracy of the model was verified by comparisons with ABAQUS. The same four types of load, axial compression, end moment, uniform lateral pressure, and combined loading which were studied in the rigid web case are analyzed again using the flexible web formulation. Stiffener and plate dimensions and details about the ABAQUS modeling of the panel were given in Section 3.1. Comparisons of ABAQUS with rigid web formulation and the flexible web formulation are shown in Figures 3.2-1 through 3.2-5. As can be seen, the agreement for the latter case is quite good. The next section examines the influence of t_w/h on the accuracy of the rigid web formulation.

3.2.7 Limitation of rigid web theory

In order to demonstrate the importance of including web distortion, a series of models with different web thickness was also analyzed. The results and the comparison between the rigid web case and the flexible web case are shown in Fig. 3.2-6. The panel was subjected to axial compression. The dimensions of the panel are as follows:

$$\left\{ \begin{array}{l} l = 1524mm \\ t_f = 14.224mm \\ b_f = 76.2mm \\ h = 138.176mm \\ b = 203.2mm \\ t_p = 7.874mm \end{array} \right.$$

For typical ship stiffeners the ratio t_w/h is in the range from 0.03 to 0.06. In all of the rigid web examples (Section 3.1) and in the flexible web examples given thus far, the value of t_w/h is 0.05. This is a relatively large value, which explains the good agreement shown in Figures 3.2-1 through 3.2-5. As shown in Fig. 3.2-6, the agreement deteriorates progressively as t_w/h decreases. It will be shown in the next section that web distortion becomes even more important when the plate rotational restraint is taken into account.

**Comparison of ABAQUS with flexible web and rigid web theory
-panel under axial loading**

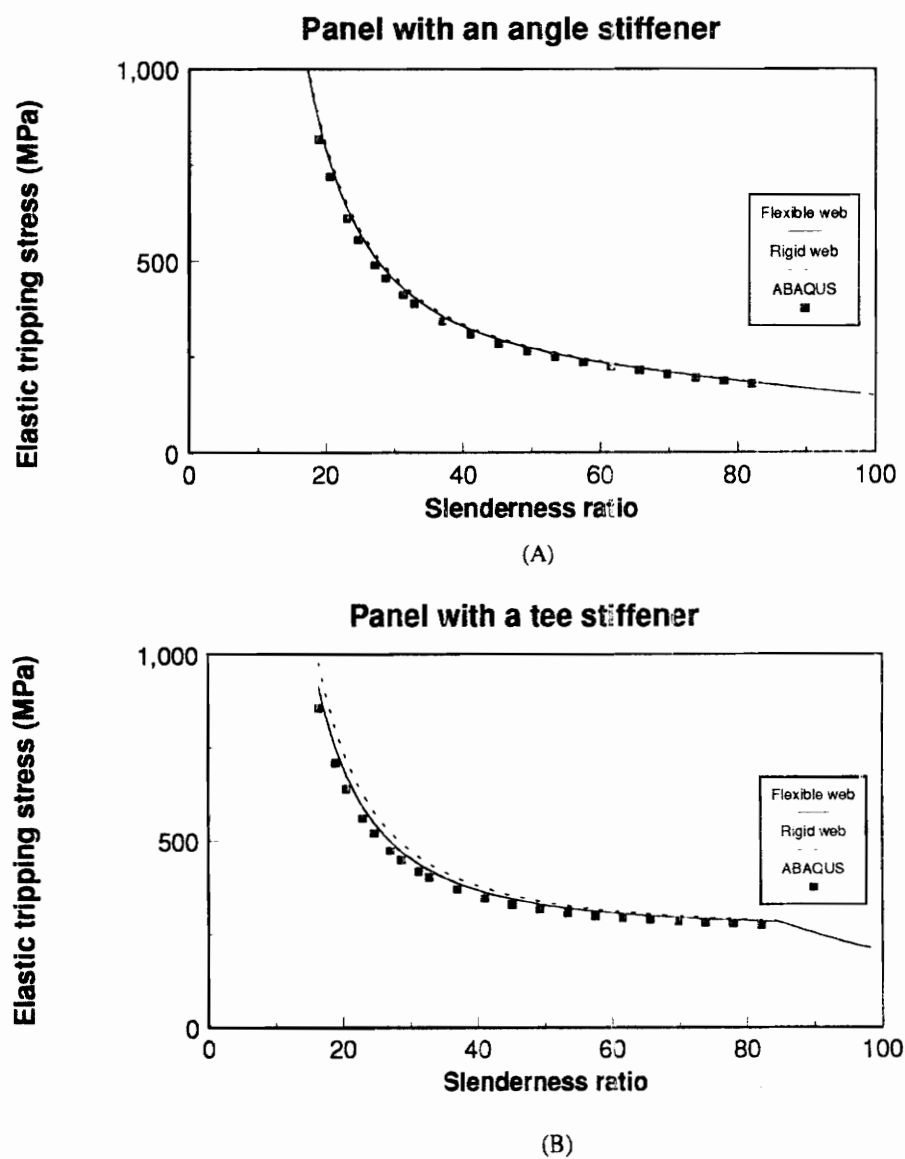
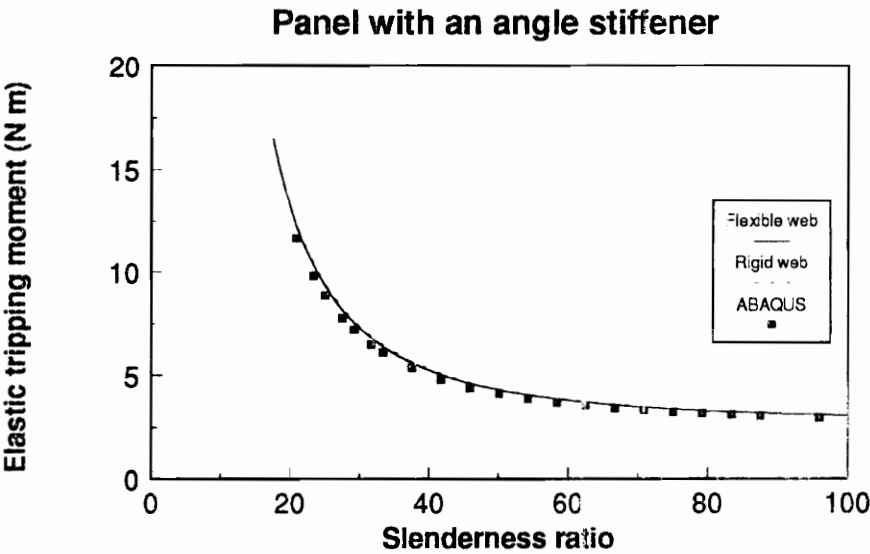


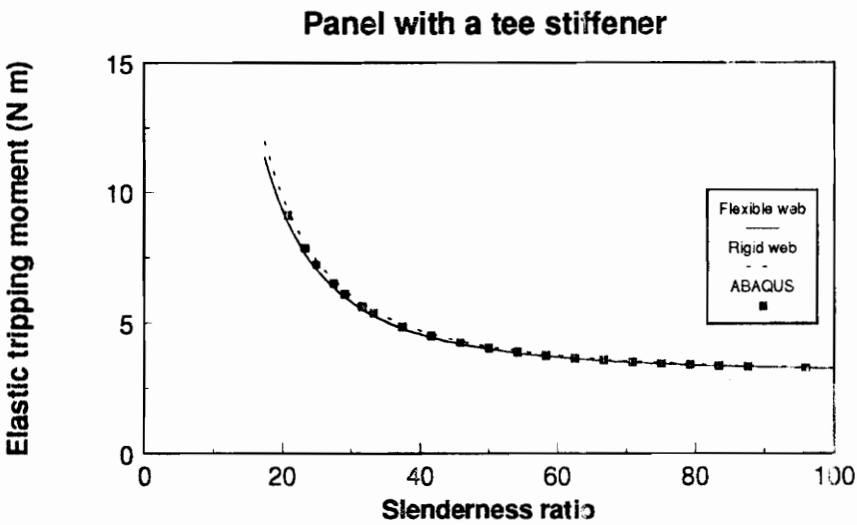
Fig.3.2-1

Comparison of ABAQUS with flexible web and rigid web theory

-panel under pure bending



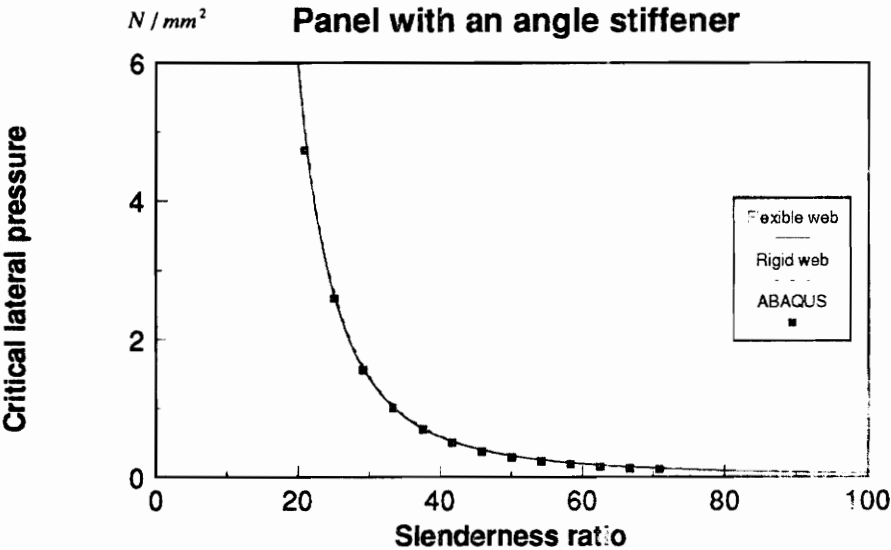
(A)



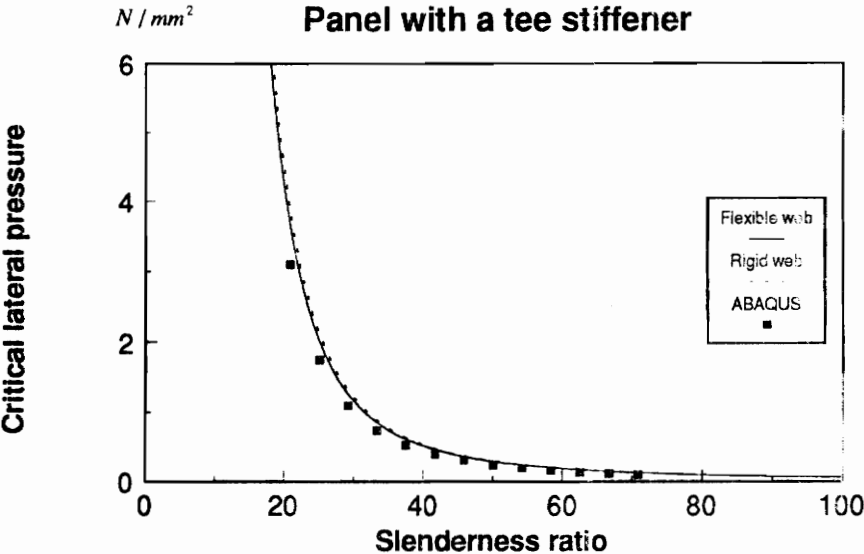
(B)

Fig. 3.2-2

**Comparison of ABAQUS with flexible web and rigid web theory
-panel under uniform lateral loading**



(A)



(B)

Fig. 3.2-3

Comparison of ABAQUS with flexible web and rigid web theory
-panel under combined loading
dead load-- uniform lateral pressure (0.1033 MPa)
live load--axial compression

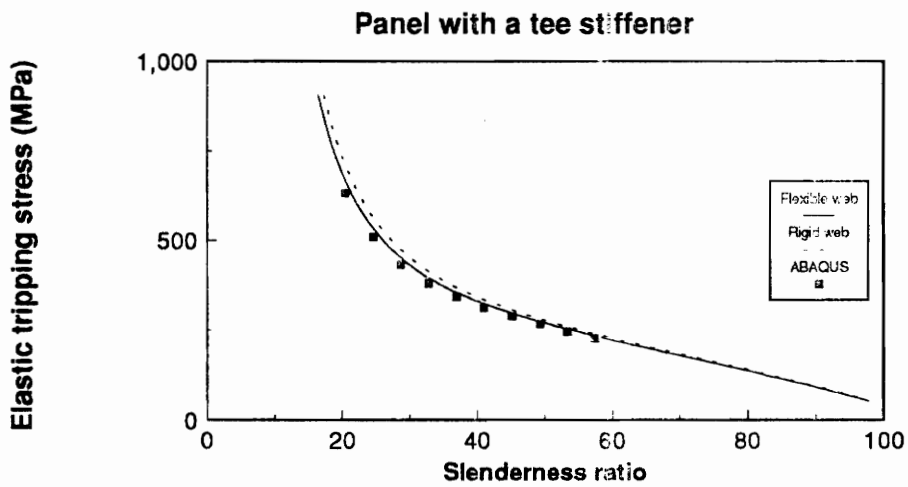
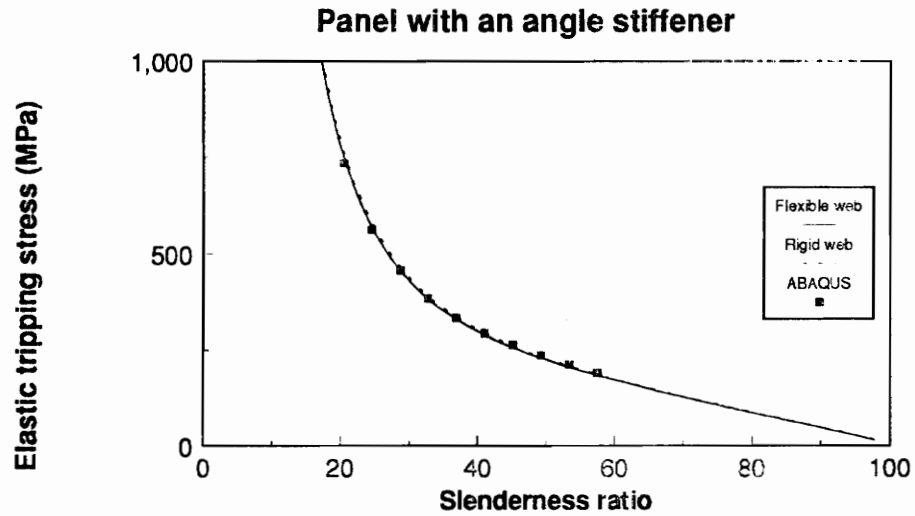


Fig. 3.2-4

Comparison of ABAQUS with flexible web and rigid web theory
-bulb-flat stiffener under axial loading

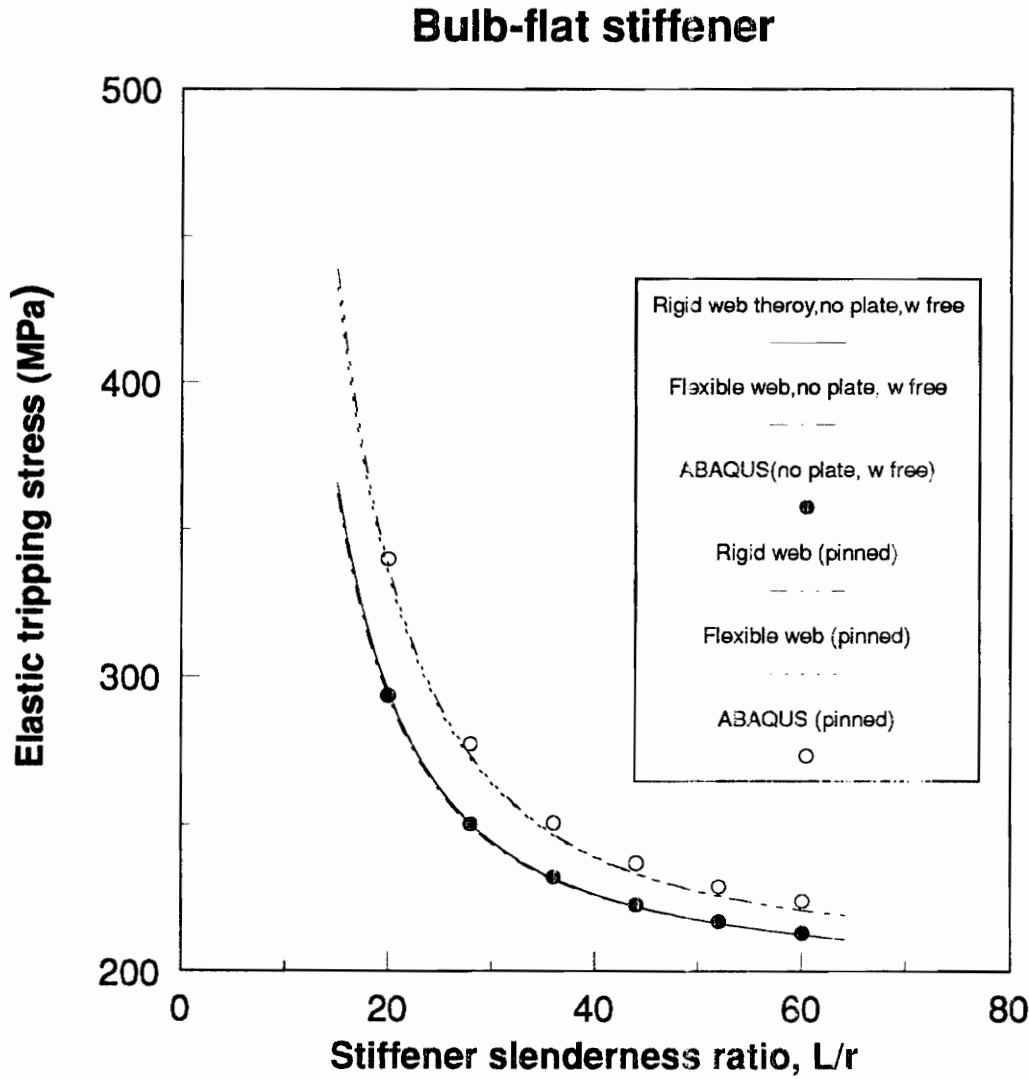


Fig. 3.2-5

Effect of web thickness

comparison of ABAQUS with flexible web and rigid web theory

-panel under axial loading

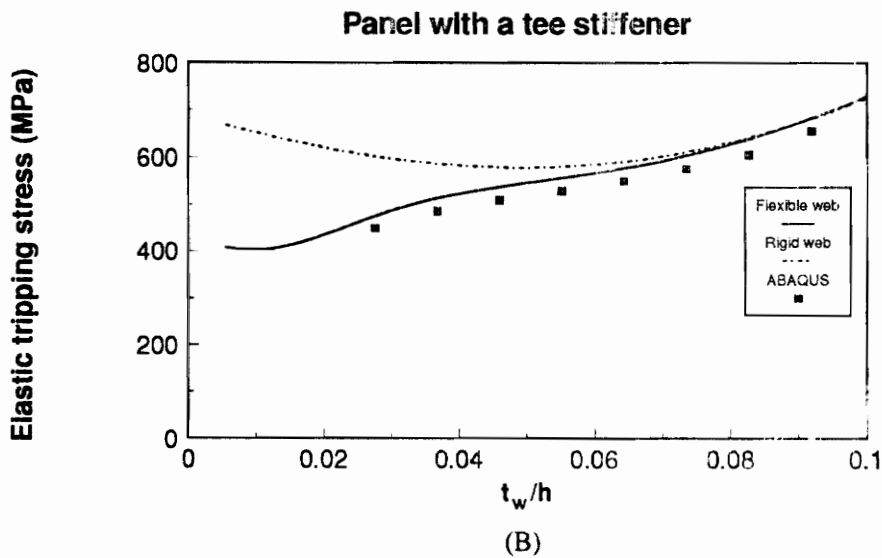
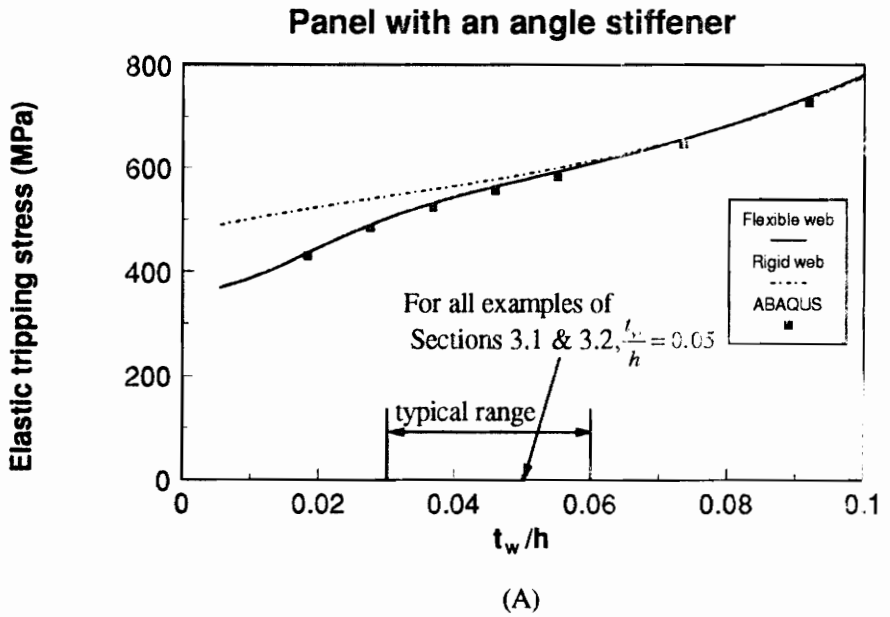


Fig. 3.2-6

3.3 Effect of Plate Rotational Restraint

Stiffener tripping is defined as buckling of the stiffener by twisting about its line of attachment to the plating. Therefore the plate rotational restraint has a dominant effect upon the elastic tripping stress. In this section the plate rotational restraint is included in the analysis. The study begins by showing the upper-bound rotational restraint of plating under different load conditions, which gives the general tripping characteristics of a panel with flanged stiffeners. Then the study investigates the case of a plate with a single stiffener. It is found that, although rigid web formulation gives acceptable results for the case without plate rotational restraint, it seriously over-estimates the critical stress when considering plate rotational restraint. Finally, the study explores the plate deformed shape of panel with multiple stiffeners, and shows that the critical plate deformed mode is an S-shape for a plate with asymmetric stiffeners (angles and bulb-flats) and a U-shape for a plate with tee stiffeners.

3.3.1 Upper-bound rotational restraint

The upper-bound case, where the plate provides total rotational restraint, is obtained by imposing $\phi_b=0$ on the web lateral displacement function (3.2-1). Thus

$$v_w = v_T(x)f_2(z) + \phi_T(x)f_3(z) \quad (3.3-1)$$

in which f_2 and f_3 are defined in Eq. (3.2-3). The total potential energy and the buckling equations can be established in a similar way as in Section 3.2, and the eigenvalue equation becomes

$$|K_L^{upper} - K_G^{upper}| = 0 \quad (3.3-2)$$

where

$$[K_L^{upper}] = \begin{bmatrix} k_{22} & k_{23} & k_{24} \\ & k_{33} & k_{34} \\ sym & & k_{44} \end{bmatrix}$$

$$[K_G^{upper}] = \begin{bmatrix} g_{22} & g_{23} & g_{24} \\ & g_{33} & g_{34} \\ sym & & g_{44} \end{bmatrix}$$

The stiffness elements k_{ij} and g_{ij} ($i,j=2,3,4$) are defined in Eq. (3.2-11) and Eq. (3.2-12), respectively. Figures 3.3-1 through 3.3-4 show the results of elastic critical tripping load and the associated mode shapes (i.e., the number of half waves) for axial compression, end moment and uniform lateral pressure, where for angle stiffeners and tee stiffeners, dimensions of the panel are as follows:

$$\left\{ \begin{array}{l} t_w = 7.1 \text{ mm} \\ t_f = 14.2 \text{ mm} \\ b_f = 76.2 \text{ mm} \\ h = 138.2 \text{ mm} \\ b = 203.2 \text{ mm} \\ t_p = 7.9 \text{ mm} \end{array} \right. \quad (3.3-3)$$

and for bulb-stiffeners, dimensions of the panel are

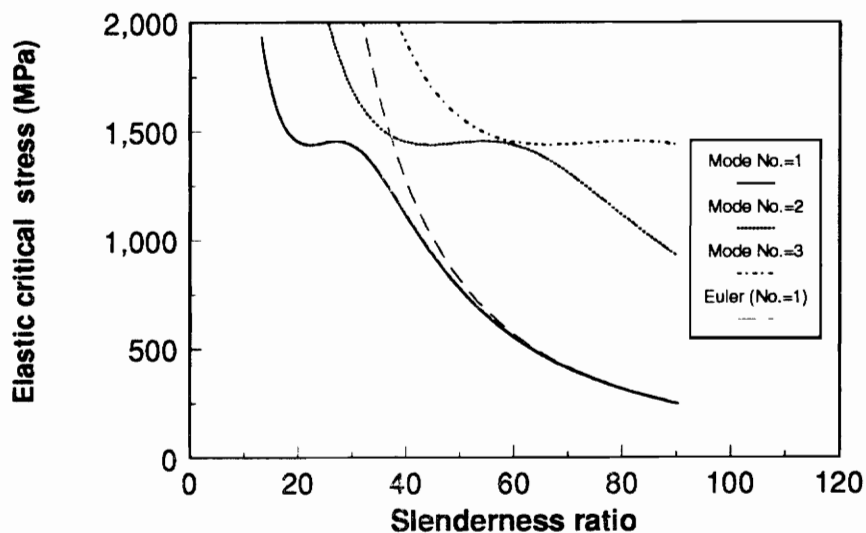
$$\left\{ \begin{array}{l} t_w = 8 \text{ mm} \\ t_f = 17.9 \text{ mm} \\ b_f = 21.9 \text{ mm} \\ h = 156 \text{ mm} \\ b = 300 \text{ mm} \\ t_p = 10 \text{ mm} \end{array} \right. \quad (3.3-4)$$

The upper-bound analysis provides the trends of stiffener failure mode. For example, for a panel under axial compressive load, angle stiffeners generally fail at mode one (one half wave, Fig. 3.3-1(A)), while tee stiffeners might fail at mode two (two half waves, Fig. 3.3-1(B)). The tee-stiffener tripping is just column buckling of the flange on an elastic foundation supplied by the web. An even higher mode (more than two half waves) may occur if the stiffener is more slender. For bulb-flat stiffeners, stiffener tripping is usually more critical than Euler buckling , and the failure mode is often several half waves (Fig. 3.3-4(A)).

Upperbound critical stress - stiffened panel under axial loading

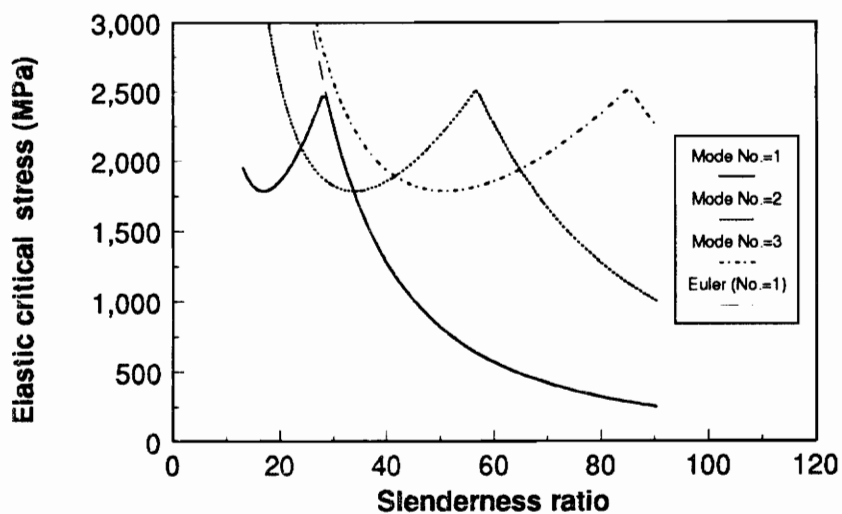
(Slenderness ratio is based on plate width=203.2 mm)

Panel with an angle stiffener



(A)

Panel with a tee stiffener



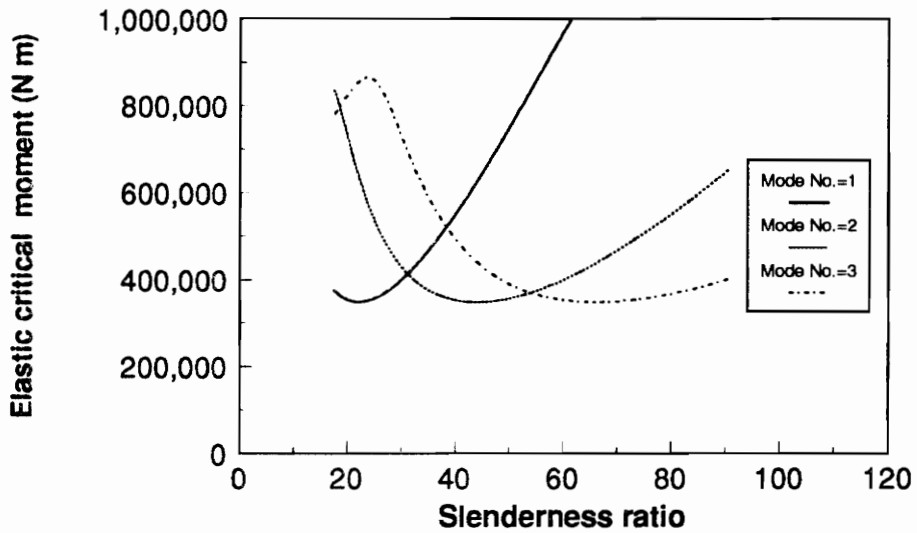
(B)

Fig. 3.3-1

Upperbound critical end moment - stiffened panel under pure bending

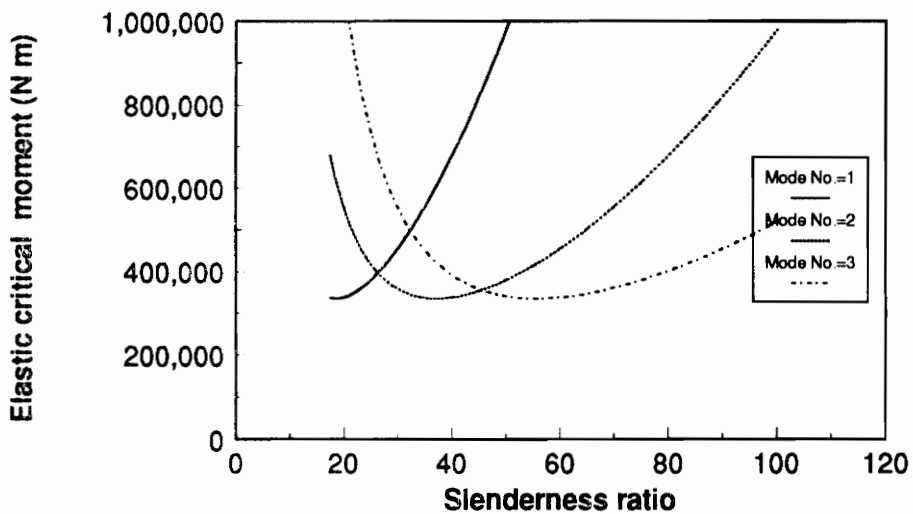
(slenderness ratio is based on plate width=203.2 mm)

Panel with an angle stiffener



(A)

Panel with a tee stiffener



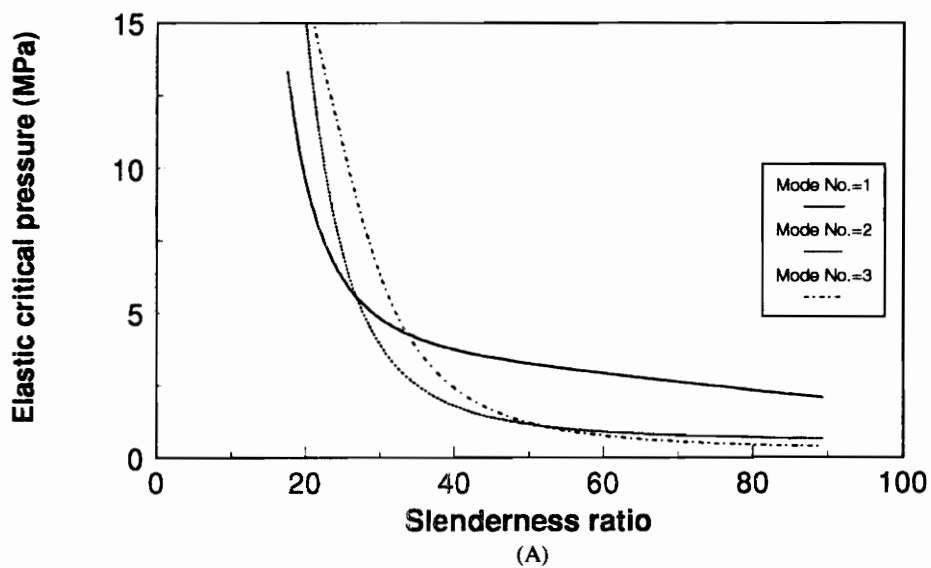
(B)

Fig. 3.3-2

Upperbound critical pressure -stiffened panel under uniform lateral pressure

(Slenderness ratio is based on plate width=203.2 mm)

Panel with an angle stiffener



Panel with a tee stiffener

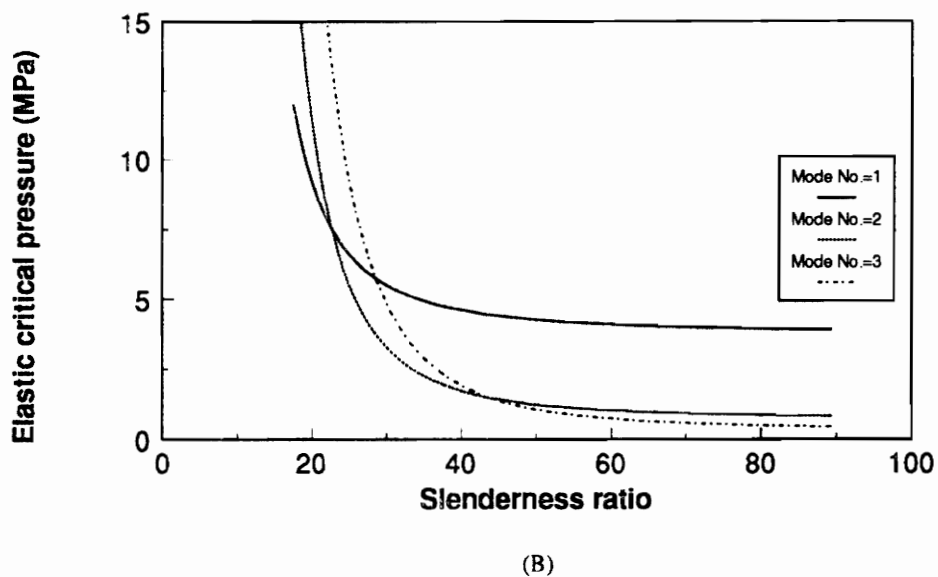


Fig. 3.3-3

Upperbound critical load- panel with bulb-flat stiffeners

(slenderness ratio is based on plate width=300 mm)

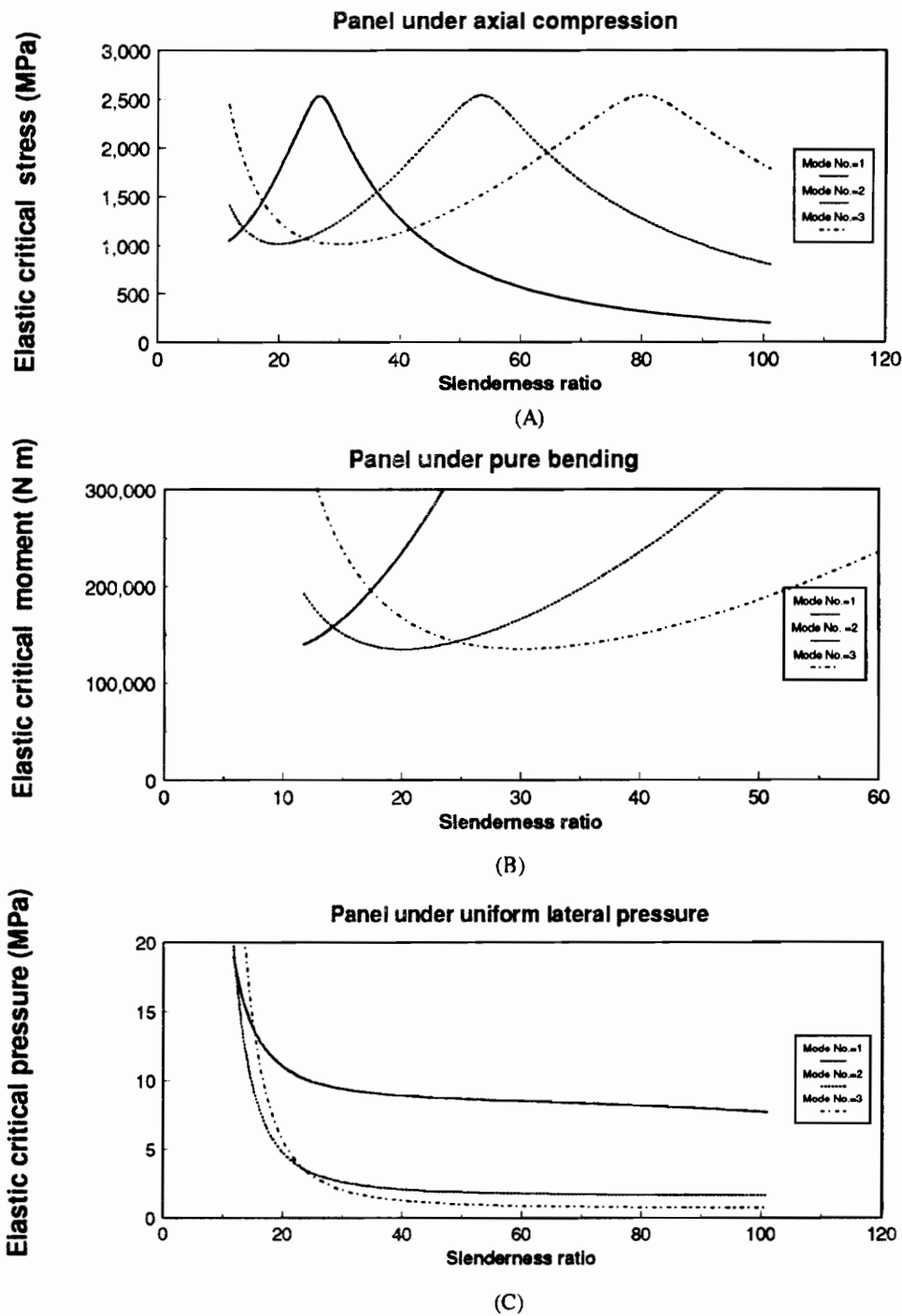


Fig. 3.3-4

3.3.2 Rotational restraint for a typical (isolated) stiffener

In reality, the plate cannot totally prevent the rotation of the stiffener at the base of the web, and it will largely accommodate the stiffener's rotation. Therefore, the total potential energy of the plate should be taken into account. For a rigid web panel with plate rotational restraint, the total potential energy can be written as

$$\begin{aligned}\Pi = & \frac{1}{2}E \int_0^l I w_{,xx}^2 dx + \frac{1}{2}E \int_0^l I_z h^2 \theta_{,xx}^2 dx + \frac{1}{2}E \int_0^l 2I_z h w_{,xx} \theta_{,xx} dx + \frac{1}{2}G \int_0^l J_s \theta_{,x}^2 dx \\ & - \frac{1}{2} \int_{A_f} \sigma \int_0^l (h^2 \theta_{,x}^2 + w_{,x}^2 - 2y \theta_{,x} w_{,x} + y^2 \theta_{,x}^2) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + z^2 \theta_{,x}^2) dx dA \\ & + \Pi_{\text{plate-out-of-plane}}\end{aligned}$$

For the flexible web case, the total potential energy can be written as

$$\Pi = \frac{1}{2} \begin{Bmatrix} \varphi_B & v_T & \varphi_T & w \end{Bmatrix} [K_L - K_G] \begin{Bmatrix} \varphi_B & v_T & \varphi_T & w \end{Bmatrix}^T + \Pi_{\text{plate-out-of-plane}}$$

where K_L , K_G are the linear stiffness matrix and geometric stiffness matrix defined in Section 3.2, and

$$\begin{aligned}\Pi_{\text{plate out of plane}} = & \frac{D_p}{2} \iint (w_{\text{pout},xx}^2 + w_{\text{pout},yy}^2 + 2\nu w_{\text{pout},xx} w_{\text{pout},yy} + 2(1-\nu) w_{\text{pout},xy}^2) dx dy \\ & - \frac{1}{2} \iint t_p (\sigma_p w_{\text{pout},x}^2) dx dy\end{aligned}\tag{3.3-5}$$

where

$$D_p = \frac{Et_p^3}{12(1-\nu^2)}$$

and w_{pout} denotes the plate out-of-plane displacement function. $\Pi_{\text{plate-out-of-plane}}$ can be calculated easily if a proper plate displacement function is chosen. There are two possible mode shapes between stiffeners, the U-shaped mode and the S-shaped mode, as shown in Fig. 3.3-5.

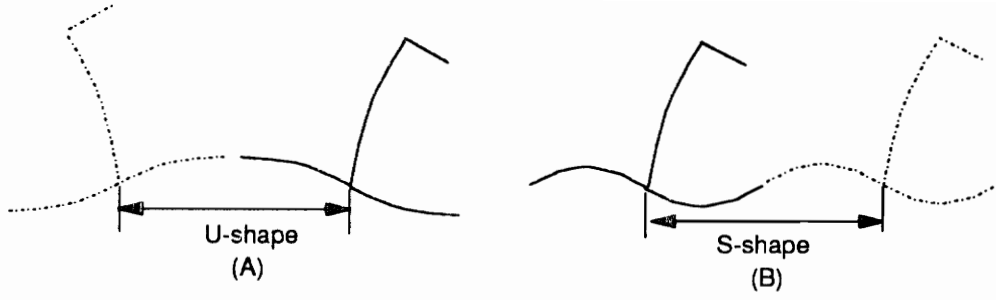


Fig. 3.3-5 plate possible mode shapes

In the expression of $\Pi_{plate-out-of-plane}$, the terms involving $w_{pout,xx}^2, w_{pout,xx}w_{pout,yy}, w_{pout,xy}^2$ are very small compared to the term $w_{pout,yy}^2$, and therefore can be ignored. The destabilizing term, $\iint t_p (\sigma_p w_{pout,x}^2) dx dy$, which in a finite element analysis would be the geometric stiffness of the plate, is also negligible when the plate deforms as S-shape or when the plate spacing b is small as occurs in heavily stiffened panels (van der Neut [V2]). This same term, $\iint t_p (\sigma_p w_{pout,x}^2) dx dy$, does become significant for the case of wide plating with U-shape mode, but in this case $\Pi_{plate-out-of-plane}$ is dominant in Π , and the critical stress obtained will be the stiffened plate buckling stress rather than the stiffener tripping stress. Since the plate buckling stress is not of interest in this study, then in order to obtain the stiffener tripping stress, $\iint t_p (\sigma_p w_{pout,x}^2) dx dy$ will be ignored, as was done by Adamchak [A1]. Therefore, Eq. (3.3-5) may be simplified as

$$\Pi_{plate out of plane} = \frac{D_p}{2} \iint w_{pout,yy}^2 dx dy$$

For the U-shaped mode, the plate displacement can be assumed either as a sine function

$$w_{pout} = \varphi_B \frac{b}{\pi} \sin\left(\frac{\pi y}{b}\right), \quad \left(-\frac{b}{2} < y < \frac{b}{2}\right)$$

or as a polynomial

$$w_{pout} = \begin{cases} \varphi_B b \left(\frac{y}{b} + \left(\frac{y}{b}\right)^2 \right), & \left(-\frac{b}{2} < y < 0\right) \\ \varphi_B b \left(\frac{y}{b} - \left(\frac{y}{b}\right)^2 \right), & \left(0 < y < \frac{b}{2}\right) \end{cases} \quad (3.3-6)$$

For the S-shaped mode, the plate out-of plane displacement can again be assumed either as a sine function

$$w_{pout} = \varphi_B \frac{b}{2\pi} \sin\left(\frac{2\pi y}{b}\right), \quad \left(-\frac{b}{2} < y < \frac{b}{2}\right)$$

or as a polynomial

$$w_{pout} = \begin{cases} \varphi_B b \left(\frac{y}{b} + 3\left(\frac{y}{b}\right)^2 + 2\left(\frac{y}{b}\right)^3 \right), & \left(-\frac{b}{2} < y < 0\right) \\ \varphi_B b \left(\frac{y}{b} - 3\left(\frac{y}{b}\right)^2 + 2\left(\frac{y}{b}\right)^3 \right), & \left(0 < y < \frac{b}{2}\right) \end{cases} \quad (3.3-7)$$

Both sine and polynomial shape functions satisfy essential (or principal) boundary conditions, which in general are prescribed displacements and rotation angles. However, if the sine shape function is chosen, the moment at the junction of plate and stiffener will be zero, which is not correct physically (refer to Fig. 3.3-6). When applying the Rayleigh-Ritz method, the only requirement for the shape function is to satisfy essential boundary conditions. However, better results will be obtained if the shape function does not violate the natural boundary conditions, which are prescribed values of higher derivatives of the field quantity. For this reason, a polynomial shape function was chosen for this study. Numerical solutions also showed that polynomial shape function gives better results (see Fig. 3.3-7 and Fig. 3.3-8).

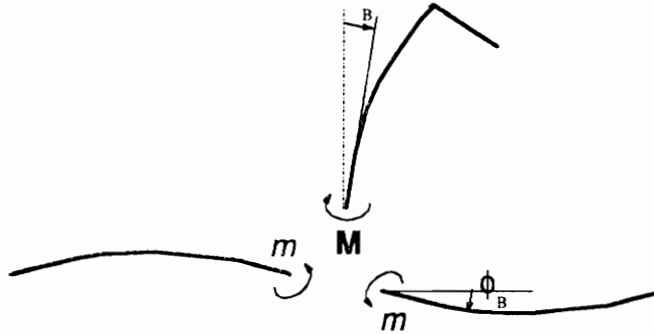


Fig. 3.3-6 joint moment

If the polynomial shape function is chosen, then

$$\Pi_{plate-out-of-plane} = \frac{1}{2} D_p \frac{4}{b_p} \int_0^l \varphi_B^2 dx \quad \text{for U-shaped mode}$$

$$\Pi_{plate-out-of-plane} = \frac{1}{2} D_p \frac{12}{b_p} \int_0^l \varphi_B^2 dx \quad \text{for S-shaped mode}$$

If $\varphi_B = \bar{\varphi}_B \sin \lambda x$ then

$$\Pi_{plate-out-of-plane} = \begin{cases} \frac{3l}{b_p} D_p \bar{\varphi}_B^2 & \text{for } S\text{-shape} \\ \frac{l}{b_p} D_p \bar{\varphi}_B^2 & \text{for } U\text{-shape} \end{cases}$$

The critical tripping load can be obtained by a similar method as Section 3.2, and the eigenvalue equation becomes

$$|\bar{K}_L - \bar{K}_G| = 0 \quad (3.3-8)$$

where

$$\begin{aligned} [\bar{K}_L] &= \begin{bmatrix} k_{11} + k_{spring} & k_{12} & k_{13} & k_{14} \\ & k_{22} & k_{23} & k_{24} \\ & & k_{33} & k_{34} \\ sym & & & k_{44} \end{bmatrix} \\ [\bar{K}_G] &= \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ & g_{22} & g_{23} & g_{24} \\ & & g_{33} & g_{34} \\ sym & & & g_{44} \end{bmatrix} \\ k_{spring} &= \begin{cases} \frac{6l}{b_p} D_p & \text{for } S\text{-shape} \\ \frac{2l}{b_p} D_p & \text{for } U\text{-shape} \end{cases} \end{aligned} \quad (3.3-9)$$

Figures 3.3-7 and 3.3-8 show the comparison of the above model for both flexible and rigid web cases, and the results of ABAQUS. It can be seen that the rigid web model overestimates the critical stress, and this is another reason that the flexible web theory should be considered (the first reason is the effect of t_w/h , see Section 3.2.7).

**Panel with a single angle stiffener
under axial loading**

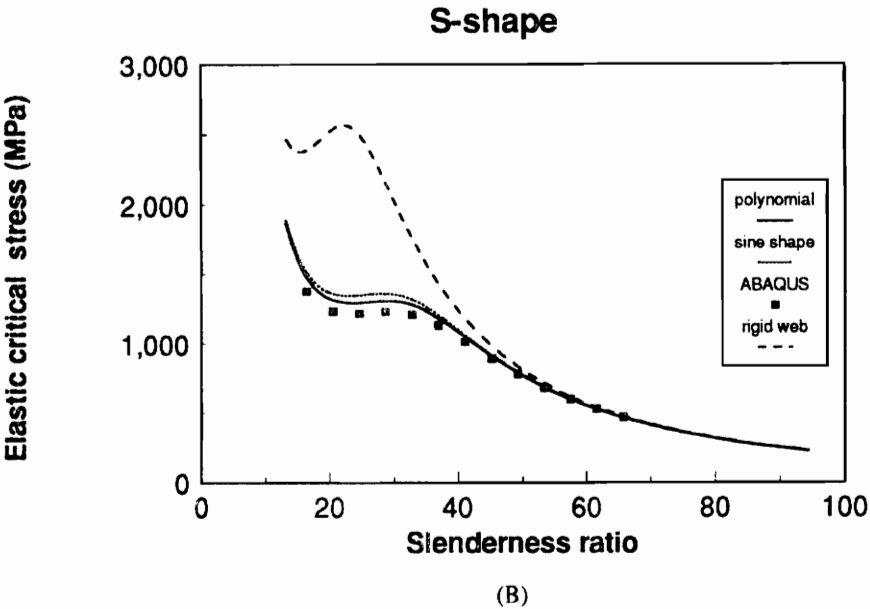
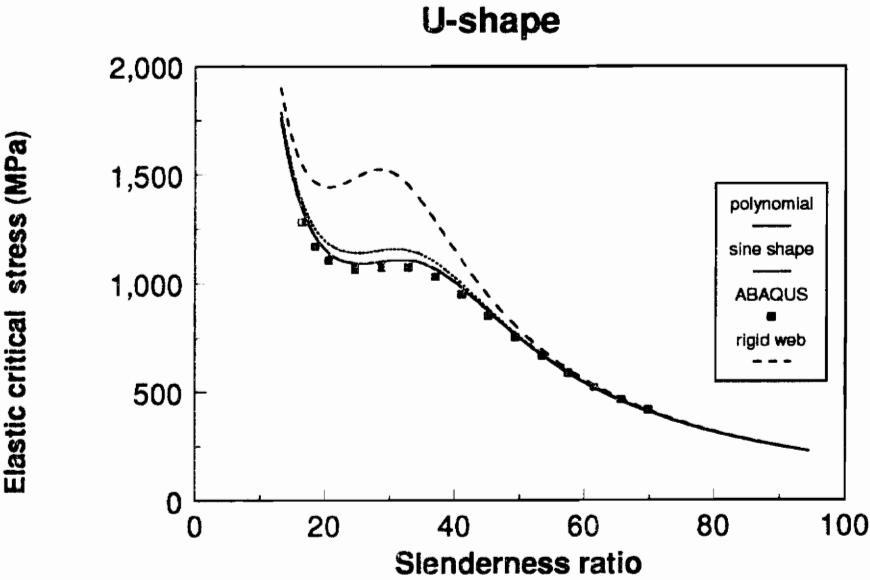
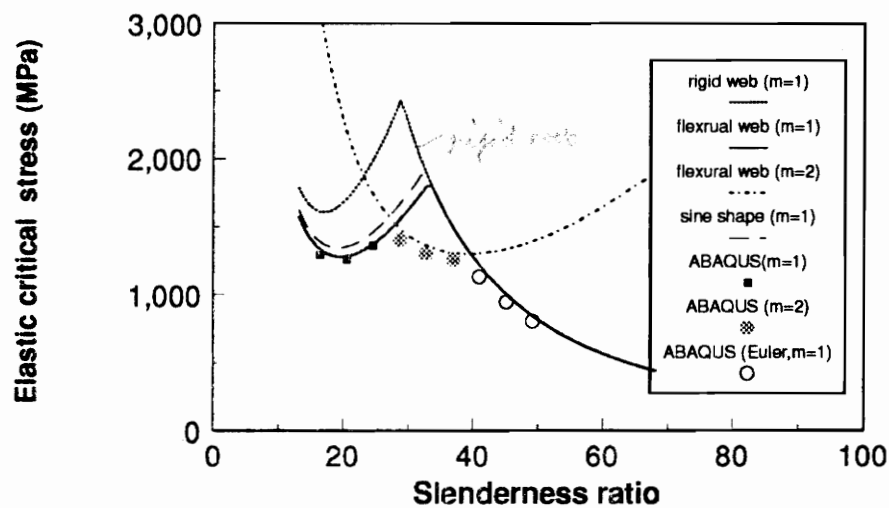


Fig. 3.3-7

Panel with a single tee stiffener
under axial loading

U-shape



S-shape

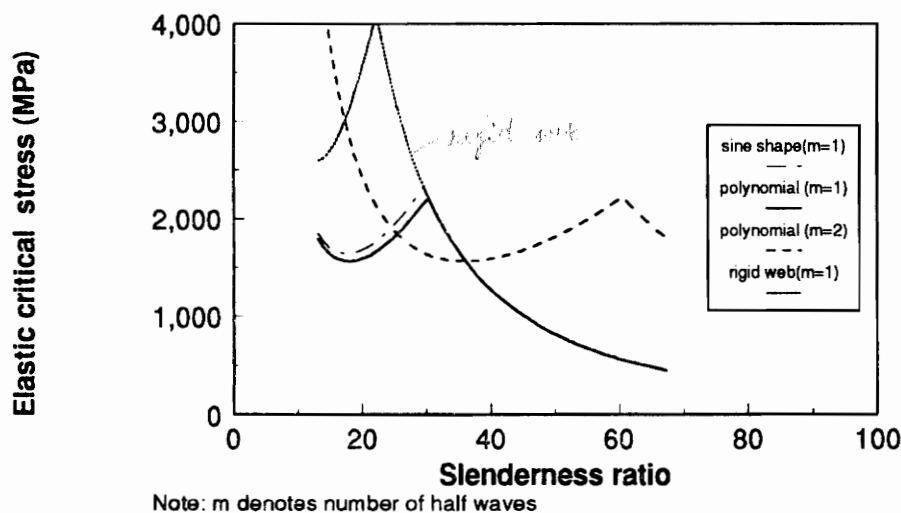


Fig. 3.3-8

3.3.3 Rotational restraint for multiple stiffeners

For a panel with a single stiffener, the plate deflection always is U-shaped since it takes less strain energy. However, for multiple stiffeners, the mode shape depends on the type of stiffener. For tees it has long been recognized that the U-shape is always the critical mode. The U-shape and the S-shape are shown in Figs. 3.3-9 and 3.3-10, in which subscript l denotes left, and r denotes right.

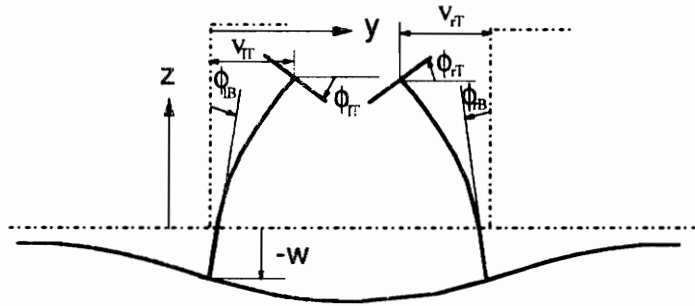


Fig. 3.3-9 U-shape for tees

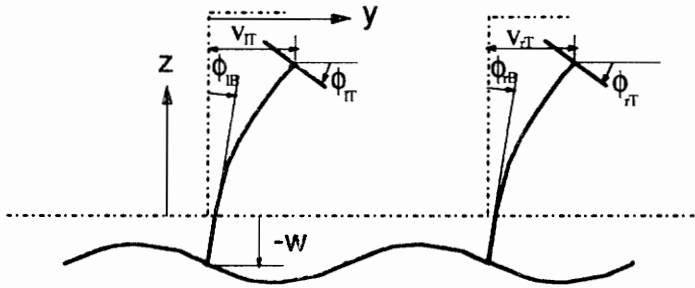


Fig. 3.3-10 S-shape for tees

For angles, the situation has until now been unresolved. van der Neut [V1] assumed that the deflection $w_{pout}(y)$ is exactly repeated for each stiffener; i.e., he proposed the S-shaped mode (Fig. 3.3-11). For this mode, the bending energy in the plate is larger than for the U-shaped mode (Fig. 3.3-12), and therefore the plate rotational restraint is greater. However, in the case of U-shape, the plate deflection $w_{pout}(y)$ alternates and the stiffeners do not have identical deflected shapes. van der Neut theorized that the alternating shape of the plating would cause a greater restraint on w , and less rotational restraint on the

stiffener compared to the S-shaped mode. Beforehand, one cannot decide which one of the two modes is critical. Based on a numerical example performed by W.H. Wittrick (using the VIPASA program), van der Neut proposed that the S-shape is the lowest mode for a panel with angle stiffeners.

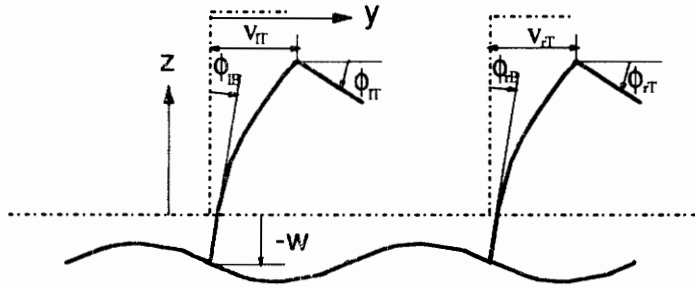


Fig. 3.3-11 S-shape for angles

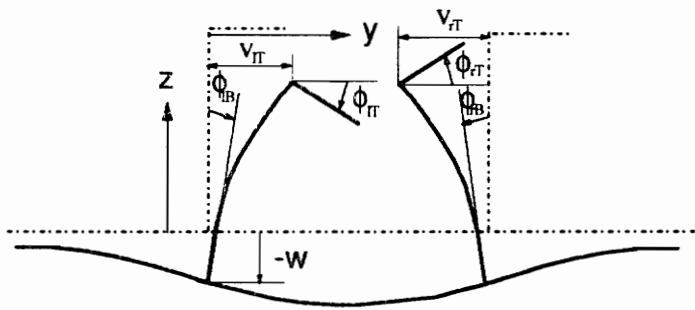


Fig. 3.3-12 U-shape for angles

Unlike previous studies, the present study does not assume or predetermine the mode shape. It makes only one basic assumption: that the vertical deflection (w in Fig. 3.3-11) at the plate-stiffener junction is the same for each stiffener. The critical mode shape will be decided by the total potential energy method. The plate displacement function is assumed as follows:

$$w_{pout} = \begin{cases} b \left(\phi_{IB} \frac{y}{b} + (2\phi_{IB} + \phi_{rB}) \left(\frac{y}{b} \right)^2 + (\phi_{IB} + \phi_{rB}) \left(\frac{y}{b} \right)^3 \right), & (-b < y < 0) \\ b \left(\phi_{IB} \frac{y}{b} - (2\phi_{IB} + \phi_{rB}) \left(\frac{y}{b} \right)^2 + (\phi_{IB} + \phi_{rB}) \left(\frac{y}{b} \right)^3 \right), & (0 < y < b) \end{cases} \quad (3.3-10)$$

The polynomial is a general shape function, and it can be reduced to either the U-shape or the S-shape. For example, for the U-shape case, when $\phi_{IB} = -\phi_{rB} = \phi_B$, shape function

(3.3-10) reduces to (3.3-6), and for the S-shape case, when $\varphi_{IB} = \varphi_{rB} = \varphi_B$, shape function (3.3-10) reduces to (3.3-7). In the study that follows, $\varphi_{IB}, \varphi_{rB}$ are treated as independent variables. Whether the plate displacement w_{pout} deforms as U-shape or S-shape, or neither of them, is completely decided by minimum strain energy. The shape function (3.3-10) can also be used for a panel with tee-stiffeners.

Solution when mode shape is a free variable

The total potential energy can be written as

$$\begin{aligned} \Pi = & \frac{1}{2} \{ \varphi_{IB} \quad v_{IT} \quad \varphi_{IT} \quad w_I \} [K_L - K_G] \{ \varphi_{IB} \quad v_{IT} \quad \varphi_{IT} \quad w_I \}^T \\ & + \frac{1}{2} \{ \varphi_{rB} \quad v_{rT} \quad \varphi_{rT} \quad w_r \} [K_L - K_G] \{ \varphi_{rB} \quad v_{rT} \quad \varphi_{rT} \quad w_r \}^T \\ & + \Pi_{plate-out-of-plane} \end{aligned}$$

where K_L , K_G are linear stiffness matrix and geometric stiffness matrix defined in the previous section, and

$$\Pi_{plate-out-of-plane} = \frac{1}{2} \{ \varphi_{IB} \quad \varphi_{rB} \} [K_L^p] \{ \varphi_{IB} \quad \varphi_{rB} \}^T \quad (3.3-11)$$

where

$$[K_L^p] = \begin{bmatrix} k_{11}^p & k_{15}^p \\ k_{15}^p & k_{55}^p \end{bmatrix}$$

Assuming that $w_I = w_r = w$, then the total potential energy of the panel with two stiffeners is

$$\Pi = \frac{1}{2} \{ \delta \}^T [K_{2L} - K_{2G}] \{ \delta \}$$

where

$$\{ \delta \} = \{ \varphi_{IB} \quad v_{IT} \quad \varphi_{IT} \quad w \quad \varphi_{rB} \quad v_{rT} \quad \varphi_{rT} \}^T$$

$$[K_{2L}] = \begin{bmatrix} K_L & \\ & K_L \end{bmatrix} + [K_L^P] = \begin{bmatrix} k_{11} + k_{11}^P & k_{12} & k_{13} & k_{14} & k_{15}^P & & & \\ & k_{22} & k_{23} & k_{24} & & & & \\ & & k_{33} & k_{34} & & & & \\ & & & 2k_{44} & k_{14} & k_{24} & k_{34} & \\ & & sym & & k_{11} + k_{55}^P & k_{12} & k_{13} & \\ & & & & & k_{22} & k_{23} & \\ & & & & & & k_{33} & \end{bmatrix}$$

$$[K_{2G}] = \begin{bmatrix} K_G & \\ & K_G \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & & & & \\ & g_{22} & g_{23} & g_{24} & & & & \\ & & g_{33} & g_{34} & & & & \\ & & & 2g_{44} & g_{14} & g_{24} & g_{34} & \\ & & sym & & g_{11} & g_{12} & g_{13} & \\ & & & & & g_{22} & g_{23} & \\ & & & & & & g_{33} & \end{bmatrix}$$

From the principle of total potential energy,

$$[K_{2L} - K_{2G}]\{\delta\} = \{0\} \quad (3.3-12)$$

A nontrivial solution of Eq. (3.3-12) for $\{\delta\}$ requires that

$$|K_{2L} - K_{2G}| = 0 \quad (3.3-13)$$

which defines a critical buckling load and a corresponding buckled shape obtained from the eigenvector $\{\delta\}$.

Solution for specified mode shape

(a) S-shape

To obtain complete curves for both the U-shape and S-shape cases, it is required that some solutions be obtained in which the shape is predetermined. The S-shape is imposed by setting

$$\varphi_{lB} = \varphi_{rB} = \varphi_B$$

$$w_l = w_r = w$$

The total potential energy becomes

$$\Pi = \frac{1}{2} \{ \varphi_B \quad v_{\pi} \quad \varphi_{\pi} \quad w \quad v_{rT} \quad \varphi_{rT} \} [K_{LS} - K_{GS}] \{ \varphi_B \quad v_{\pi} \quad \varphi_{\pi} \quad w \quad v_{rT} \quad \varphi_{rT} \}^T$$

where subscript *S* denotes S-shape,

$$[K_{LS}] = \begin{bmatrix} 2k_{11} + k_{11}^p + k_{55}^p + 2k_{15}^p & k_{12} & k_{13} & 2k_{14} & k_{12} & k_{13} \\ & k_{22} & k_{23} & k_{24} & & \\ & & k_{33} & k_{34} & & \\ & & & 2k_{44} & k_{24} & k_{34} \\ & sym & & & k_{22} & k_{23} \\ & & & & & k_{33} \end{bmatrix}$$

$$[K_{GS}] = \begin{bmatrix} 2g_{11} + g_{11}^p + g_{55}^p + 2g_{15}^p & g_{12} & g_{13} & 2g_{14} & g_{12} & g_{13} \\ & g_{22} & g_{23} & g_{24} & & \\ & & g_{33} & g_{34} & & \\ & & & 2g_{44} & g_{24} & g_{34} \\ & sym & & & g_{22} & g_{23} \\ & & & & & g_{33} \end{bmatrix}$$

and the solution is obtained by

$$|K_S - G_S| = 0 \quad (3.3-14)$$

(b) U-shape

The U-shape is imposed by setting

$$\begin{aligned} \varphi_{lB} &= -\varphi_{rB} \\ w_l &= w_r = w \end{aligned}$$

The total potential energy becomes

$$\Pi = \frac{1}{2} \{ \varphi_B \quad v_{\pi} \quad \varphi_{\pi} \quad w \quad v_{rT} \quad \varphi_{rT} \} [K_{LU} - K_{GU}] \{ \varphi_B \quad v_{\pi} \quad \varphi_{\pi} \quad w \quad v_{rT} \quad \varphi_{rT} \}^T$$

where subscript *U* denotes U-shape,

$$[K_{LU}] = \begin{bmatrix} 2k_{11} + k_{11}^p + k_{55}^p - 2k_{15}^p & k_{12} & k_{13} & 0 & -k_{12} & -k_{13} \\ & k_{22} & k_{23} & k_{24} & & \\ & & k_{33} & k_{34} & & \\ & & & 2k_{44} & k_{24} & k_{34} \\ & sym & & & k_{22} & k_{23} \\ & & & & & k_{33} \end{bmatrix}$$

$$[K_{GU}] = \begin{bmatrix} 2g_{11} + g_{11}^p + g_{55}^p - 2g_{15}^p & g_{12} & g_{13} & 0 & -g_{12} & -g_{13} \\ & g_{22} & g_{23} & g_{24} & & \\ & & g_{33} & g_{34} & & \\ & & & 2g_{44} & g_{24} & g_{34} \\ & sym & & & g_{22} & g_{23} \\ & & & & & g_{33} \end{bmatrix}$$

and the eigenvalue equation becomes

$$|K_{LU} - K_{GU}| = 0 \quad (3.3-15)$$

Results and discussion

Angle stiffeners

A panel with two stiffeners is analyzed. Dimensions of one stiffener and its associated plating are given by Eq. (3.3-3). The study showed that the S-shaped mode gives lower critical stress than U-shaped mode. Eq. (3.3-13), Eq. (3.3-14) and Eq. (3.3-8) where S-shape is chosen, give the same results if round-off error is ignored. Therefore, to solve a panel with multiple stiffeners, one could use either Eq. (3.3-13) where plate mode shape is not predetermined, or Eq. (3.3-8) where plate mode shape is predetermined (S-shape for angles). Results of Eq. (3.3-13), where the mode shape is a free variable, Eq. (3.3-15), where the plate mode shape is imposed as U-shape, and ABAQUS results are shown in Fig. 3.3-21. It is clear that for multiple angle stiffeners the critical stress of U-shape is higher than that of S-shape. For the situation of thick plates, the critical stress of S-shape and U-shape tends to be the same since the rotational restraint of the plate is so dominant that the deformed shape of the plate has little influence on the tripping stress.

Tee stiffeners

For tee stiffeners, the U-shaped mode always gives lower critical stress (Fig. 3.3-22). Eq. (3.3-13), Eq. (3.3-15) and Eq. (3.3-8) where U-shape is chosen, give the same results if round-off error is ignored. Thus, either Eq. (3.3-13) where plate mode shape is not predetermined, or Eq. (3.3-8) where a proper plate mode shape is predetermined (U-

shape for tees), can be used to decide the critical load. Results of Eq. (3.3-13), where the mode shape is a free variable, Eq. (3.3-14), where the plate mode shape is imposed as S-shape, and ABAQUS are shown in Fig. 3.3-22. It is clear that for multiple tee stiffeners the critical stress of S-shape is higher than that of U-shape.

Bulb-flat stiffeners

For bulb-flat stiffeners, the critical stress of S-shape is lower than that of U-shape. In this respect, a panel with bulb-flat stiffeners is similar to angle stiffeners. However, bulb-flat stiffeners tend to buckle into more than one half wave. A panel with two stiffeners is analyzed. Dimensions of one stiffener and its associated plating are given by Eq. (3.3-4). Results of Eq. (3.3-13), where the mode shape is a free variable, and ABAQUS results are shown in Fig. 3.3-23.

3.3.4 Numerical study using ABAQUS

To verify the above theory, ABAQUS was employed. Because the plate rotational restraints provide an elastic foundation for the stiffeners, the critical mode might not be one half wave over the stiffener length. This means the response of the structure may not be symmetric, and therefore it was necessary to model the full length of the panel. Young's modulus is $2.066E5 \text{ N/mm}^2$, Poisson's ratio is 0.3.

1) Panel with a single stiffener

A panel with one stiffener is modeled. The flange and web are modeled as $4 \times 24 \text{ S4R5}$ elements, and the plate is modeled as $8 \times 24 \text{ S4R5}$ elements. The dimensions of the panel are given by Eq. (3.3-3).

For a panel with an angle stiffener, symbolic illustrations of the ABAQUS models showing only one row of elements are given in Fig. 3.3-13 & 3.3-14. For full models, please refer

to Fig. 3.1-14. For a panel with a tee stiffener, ABAQUS models are similar to those models with an angle stiffener (please refer to Fig. 3.1-6).

Boundary conditions for the U-shaped mode:

Boundary condition 1. The panel is simply supported at both ends.

Boundary condition 2. At each end of the plate, the axial displacement (x-direction) of each node is equal. This is done by using the *EQUATION option of ABAQUS.

Boundary condition 3. For the two rows of nodes along the left and right edges of the plate, the rotation angle (x-direction) of the nodes is set to be zero.

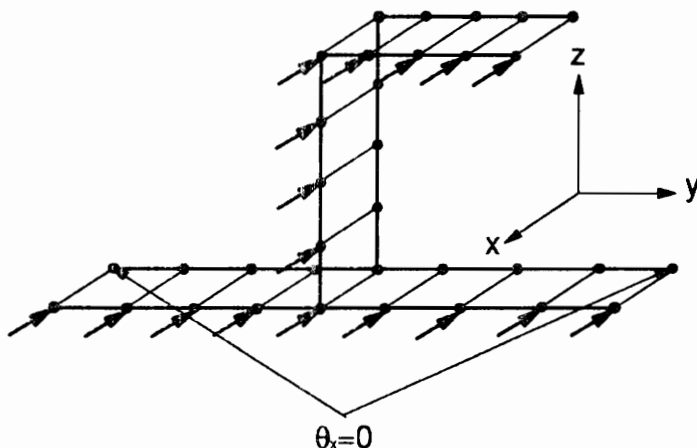
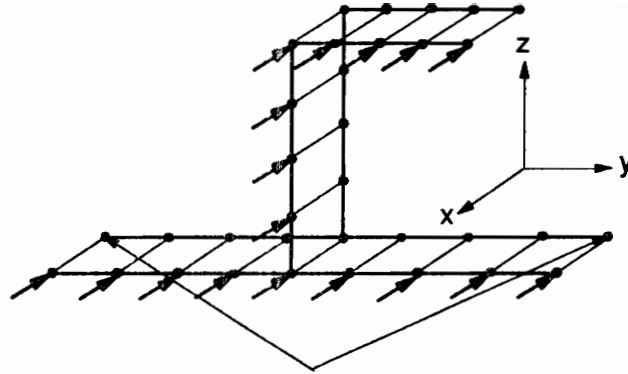


Fig. 3.3-13 U-shape ABAQUS model of panel with a single angle stiffener

Boundary conditions for the S-shaped mode:

Boundary condition 1. and Boundary condition 2. are the same as for U-shape.

Boundary condition 3. For the two rows of nodes along the left and right edges of the plate, the flexural displacement (z-direction) of the corresponding pairs of nodes is equal, and the rotation angle (x-direction) of the corresponding pairs of nodes is equal.



For corresponding pairs of nodes $\theta_{ix} = \theta_{rx}$, $w_i = w_r$

Fig. 3.3-14 S-shape ABAQUS model of panel with a single angle stiffener

ABAQUS was run for a series of models with different lengths. The results and the comparison with the present study are shown in Fig. 3.3-19.

Angle stiffener

For the angle stiffener, the critical mode is always one half wave lengthwise. Figure 3.3-19(A) shows that as the panel slenderness ratio increases the critical stress curve approaches the Euler buckling curve because of the relatively small flange sideways deflection v_w (Fig. 2.2-1). When the panel slenderness ratio is small, the critical stress curve is quite separate from the Euler curve. The sideways deflection v_w dominates over the vertical deflection w (Fig. 2.2-1) and hence the tripping mode is dominant. For panels of intermediate slenderness ratios there is a gradual transition from one mode to the other at an almost constant value of critical stress. Argyris [A2] and van der Neut [V1] found a similar transition behavior.

Tee stiffener

For the tee stiffener, the Euler buckling mode and tripping mode are uncoupled (Fig. 3.3-19(B)). When the panel slenderness ratio is large the critical stress curve matches the Euler buckling curve and the critical mode is one half wave. When the panel slenderness ratio is small the tripping mode becomes the critical mode. Also, in the tripping range, the

critical mode changes from one half wave to two half waves as the panel slenderness ratio increases. The result is that there are two “*knuckles*” for the tee stiffener. ABAQUS shows good agreement with the method developed in this study.

Bulb-flat stiffener

For the bulb-flat stiffener, the flange is modeled as 1 x 24 B31 space beam elements, the web is modeled as 4 x 24 S4R5 elements, and the plate is modeled as 8 x 24 S4R5 elements. Dimensions of the panel are given in Eq. (3.3-4)

A symbolic illustration of the ABAQUS model, showing only one row of elements, is given in Fig. 3.3-15. The plate boundary condition is taken as S-shape as described above. The study shows that the critical tripping mode changes from one half wave to more waves as the panel length increases.

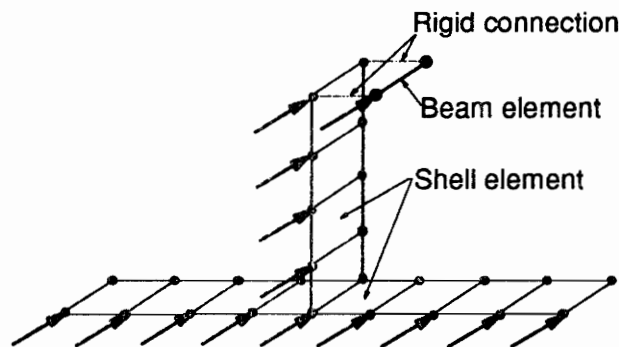


Fig. 3.3-15 ABAQUS model of panel with a single bulb-flat stiffener

ABAQUS was run for a series of models with different lengths. Fig. 3.3-20 shows that ABAQUS has good agreement with the method developed in this study.

2) Panel with multiple stiffeners

a) Angle stiffeners

A panel with two angle stiffeners is modeled. Each flange and web are modeled as 4 x 24 S4R5 elements, and the plate is modeled as 16 x 24 S4R5 elements. Dimensions of one

stiffener and its associated plating are given by Eq. (3.3-3). A symbolic illustration of the ABAQUS model, showing only one row of elements, is given in Fig. 3.3-16. For a full ABAQUS model, please refer to Fig. 3.3-25.

The ABAQUS model involved the following boundary conditions:

Boundary condition 1. The panel is simply supported at both ends.

Boundary condition 2 At each end of the plate, the axial displacement (x-direction) of each node is equal. This is done by using the *EQUATION option of ABAQUS.

Boundary condition 3. For the two rows of nodes along the bottom of the web, the flexural displacement (z-direction) of the corresponding pairs of nodes is equal.

Boundary condition 4. For the two rows of nodes along the left and right edges of the plate, the flexural displacement (z-direction) of the corresponding pairs of nodes is equal.

Boundary condition 5. For the two rows of nodes along the left and right edges of the plate, the rotational angle (θ_x) of the corresponding pairs of nodes is equal.

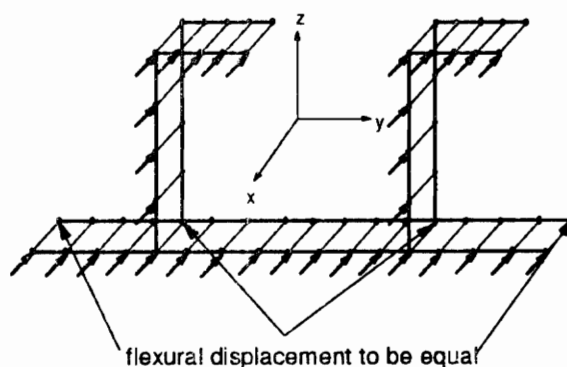


Fig. 3.3-16 ABAQUS model of panel with two angle stiffeners

These boundary conditions are equally valid for both the U-shape and the S-shape cases. ABAQUS was run for a series of models with different lengths. The comparison with the S-shape predictions (Eq.3.3-14), the U-shape predictions (Eq. 3.3-15), the ABAQUS results with one angle stiffener where the plating is forced to be S-shape, and the ABAQUS results with two angle stiffeners are shown in Fig. 3.3-21.

Buckling analysis using ABAQUS gives many possible modes including plate buckling modes. From the study of Section 3.1, we know that for a panel with angle stiffeners, tripping is always coupled with flexural buckling. This condition will be checked against each buckling mode in order to identify the stiffener tripping mode. For the panel with the above geometric dimensions, it can be seen that the plate does deform as S-shape (Fig. 3.3-26).

In order to further verify that the S-shaped mode is the correct mode, another boundary condition which favors the U-shaped mode was introduced.

Boundary condition 5. The x -direction rotation of the corresponding base node of the stiffeners is opposite to each other.

With this new boundary condition, the plate's buckled shape was the U-shape, and the lowest critical stress was 964.2MPa, which is lower than the previous S-shape solution. The mode shape is shown in Fig. 3.3-27. However, although it involves rotation of the stiffeners, this mode shape is *not* tripping, but rather plate buckling for the following reasons:

- (1) From the work in Section 3.1, we know that tripping of angle stiffeners is always accompanied by flexural buckling. But in this case, the stiffener only rotates; the vertical displacement is completely negligible and so there is no flexure.
- (2) This failure mode is in fact simply plate buckling, and the stiffeners are forced to rotate by the plating. This is confirmed by the fact that a "plating only" model buckles (Fig. 3.3-28) at a slightly lower critical stress (936.6MPa). The slightly larger value of 964.2MPa is due to the rotational restraint exerted by the stiffeners on the plating.
- (3) This is further confirmed by the fact that for a single stiffener, the critical mode is the U-shape and the critical stress is 1067.5MPa. The U-shape single stiffener model is less stiff than the two-stiffener model, which gives the S-shaped mode, and it is the lowest

artificial tripping stress. But the stress (964.2MPa) is even smaller than the U-shape single stiffener tripping stress.

b) Tee stiffeners

As introduced in Chapter 1, whether a panel fails as Euler buckling, plate buckling or tripping is strongly dependent on the panel dimensions. In the previous example, the stiffener tripping mode occurs later than the plate buckling mode. However, this is not always the case. To illustrate how tripping can be the first buckling mode, a panel with two tee stiffeners is modeled. Each flange and web are modeled as 4 x 24 S4R5 elements, and the plate is modeled as 16 x 24 S4R5 elements. The panel dimensions are as follows:

$$\left\{ \begin{array}{l} t_f = 7.62mm \\ b_f = 50.8mm \\ h = 152.4mm \\ t_w = 5.08mm \\ b = 203.2mm \\ t_p = 10.16mm \end{array} \right.$$

A symbolic illustration of the ABAQUS model, showing only one row of elements, is given in Fig. 3.3-17. The boundary conditions are the same as those for the panel with angle stiffeners, and again these boundary conditions are equally valid for both the U-shape and the S-shape cases.

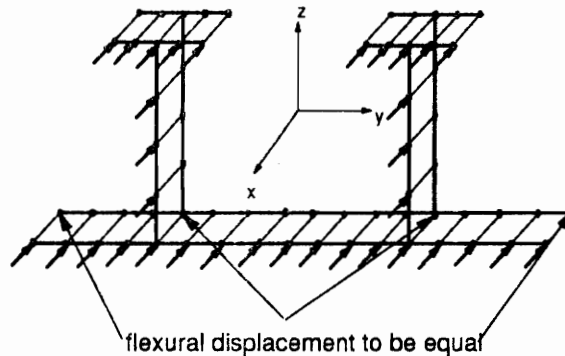


Fig. 3.3-17 ABAQUS model of panel with two tee stiffeners

ABAQUS was run for a series of models with different lengths, and the results show that the stiffener tripping mode is the first buckling mode. The comparison with the S-shape predictions (Eq.3.3-14), the U-shape predictions (Eq. 3.3-15), the ABAQUS results with one angle stiffener where the plating is forced to be S-shape, and the ABAQUS results with two angle stiffeners are shown in Fig. 3.3-22. It can be seen that the plate does deform as U-shape (Fig. 1.1-3).

c) Bulb-flat stiffeners

A panel with two bulb-flat stiffeners is modeled. Each web is modeled as 4×24 shell elements (S4R5); each flange is modeled as 1×24 beam elements (B31), and the plate is modeled as 16×24 shell elements (S4R5). Dimensions of one stiffener and its associated plating are given in Eq. (3.3-4).

A symbolic illustration of the ABAQUS model is shown in Fig. 3.3-18. The boundary conditions are the same as those for the panel with angle stiffeners, and again these boundary conditions are equally valid for both the U-shape and the S-shape cases.

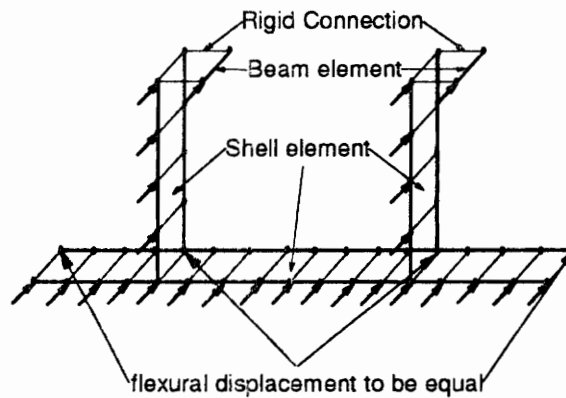


Fig. 3.3-18 ABAQUS model of panel with two bulb-flat stiffeners

ABAQUS was run for a series of models with different lengths, and the results show that the stiffener tripping mode is the first buckling mode. The comparison with the S-shape predictions (Eq.3.3-14), the ABAQUS results with one angle stiffener where the plating is

forced to be S-shape, and the ABAQUS results with two angle stiffeners are shown in Fig. 3.3-23.

d) Comments

Figure 3.3-24 shows the predictions of Euler stress, tripping stress of angle stiffeners, tripping stress of tee stiffeners, and tripping stress of bulb-flat stiffeners. It can be seen that the tripping stress of angles is higher than tees when the panel slenderness ratio is small, and is lower than tees when the panel slenderness ratio is large. For an equivalent sectional area of the flange, the tripping stress of bulb-flat stiffener is lower than that of both angle stiffeners and tee stiffeners, and the critical tripping mode is usually more than one half wave. Dimensions of the panel with angle stiffeners and tee stiffeners are

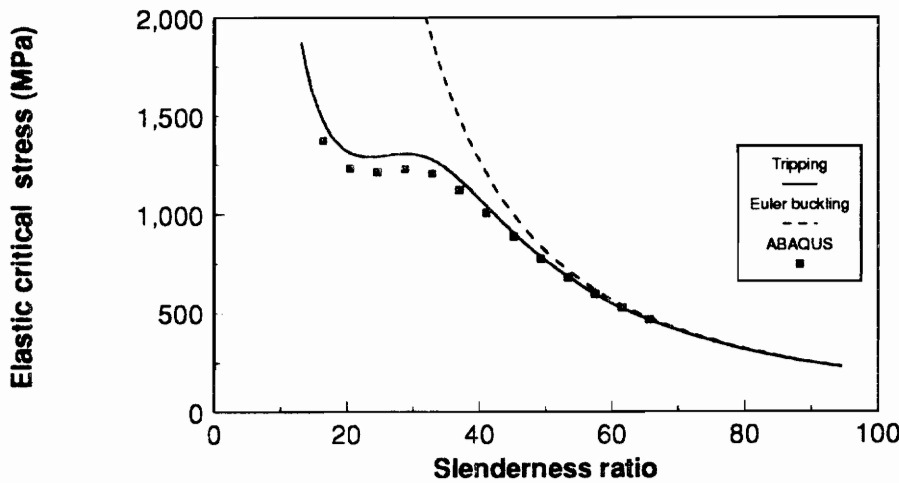
$$\left\{ \begin{array}{l} t_f = 14.2mm \\ b_f = 76.2mm \\ h = 138.2mm \\ t_w = 7.1mm \\ b = 203.2mm \\ t_p = 7.9mm \end{array} \right.$$

and dimensions of the panel with bulb-flat stiffeners are

$$\left\{ \begin{array}{l} t_f = 25.4mm \\ b_f = 42.7mm \\ h = 138.2mm \\ t_w = 7.1mm \\ b = 203.2mm \\ t_p = 7.9mm \end{array} \right.$$

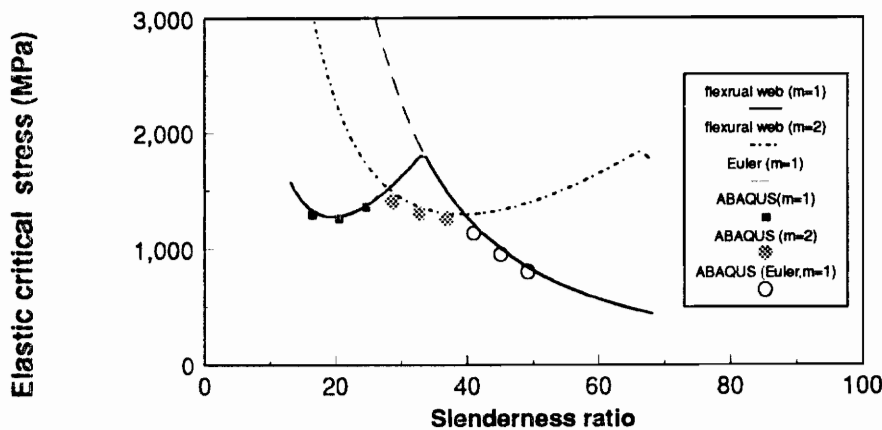
Axial Loading

Panel with a single angle stiffener S-shape



(A)

Panel with a single tee stiffener U-shape

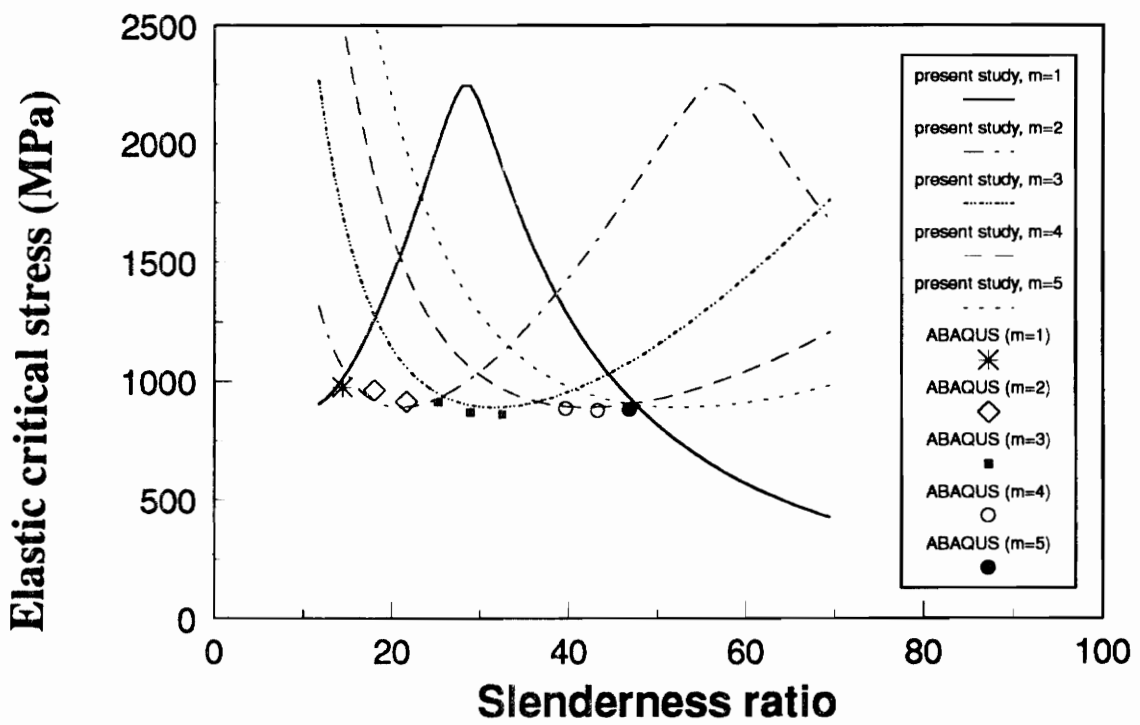


Note: m denotes number of half waves

(B)

Fig. 3.3-19

Panel with a bulb-flat stiffener under axial loading



Note:
plate deforms as S-shape
m denotes number of half waves

Fig. 3.3-20

Panel with angle stiffeners under axial loading

-Comparison of Euler stress, tripping stress of S-shape plate, and tripping stress of U-shape plate of a 2-stiffeners model with 1-angle-stiffener ABAQUS model results and 2-angle-stiffener ABAQUS model results

Panel with angle stiffeners

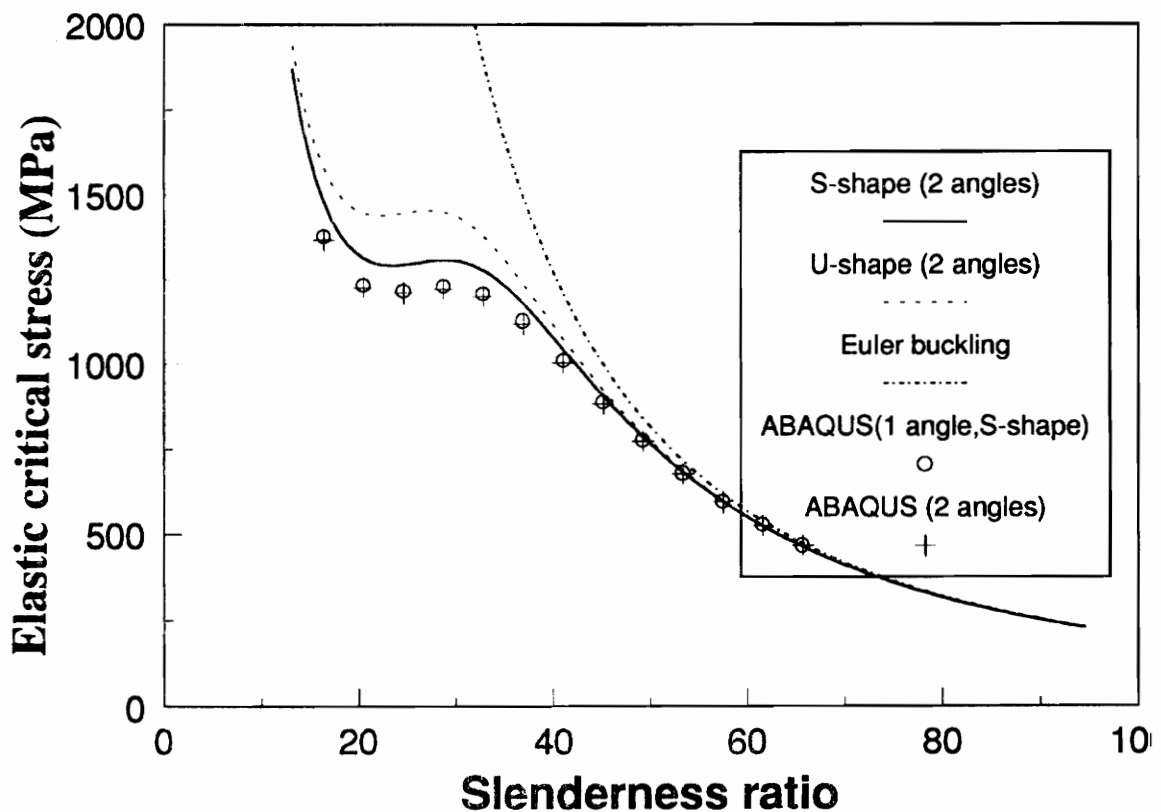
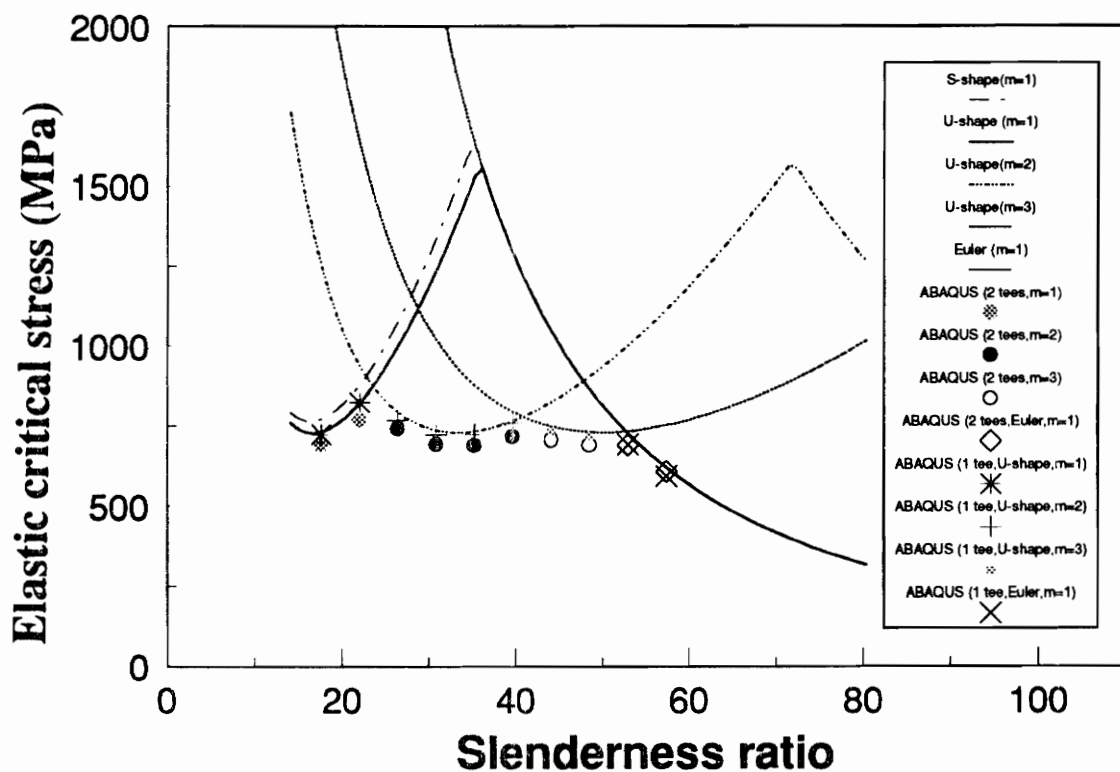


Fig. 3.3-21

Panel with tee stiffeners under axial loading

-Comparison of Euler stress, tripping stress of S-shape plate, and tripping stress of U-shape plate of a 2-stiffener model with 1-tee-stiffener ABAQUS model results and 2-tee-stiffener ABAQUS model results

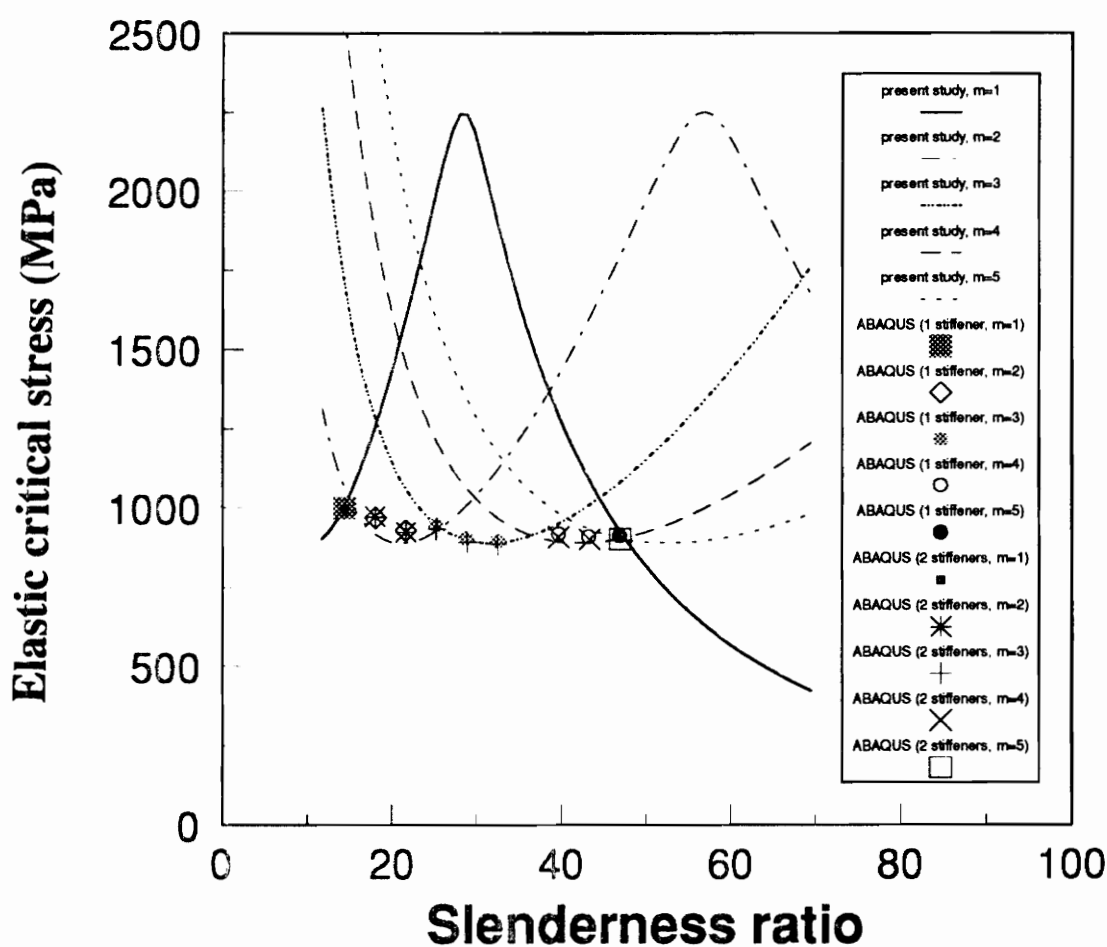
Panel with tee stiffeners



Note: m denotes number of half waves

Fig. 3.3-22

Panel with bulb-flat stiffeners under axial loading

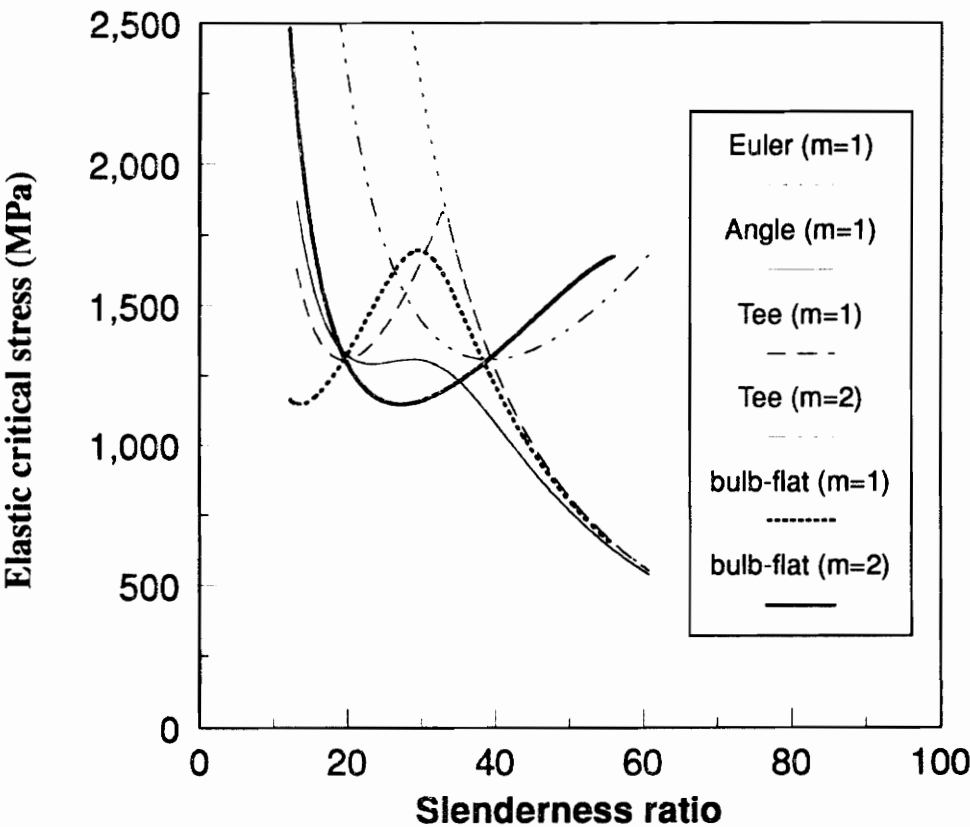


Note:
plate deforms as S-shape
 m denotes number of half waves

Fig. 3.3-23

Comparison of critical stress under axial loading

-Angle stiffener, tee stiffener and bulb-flat stiffener with plate rotational restraint



Note:
m denotes number of half waves

Fig. 3.3-24

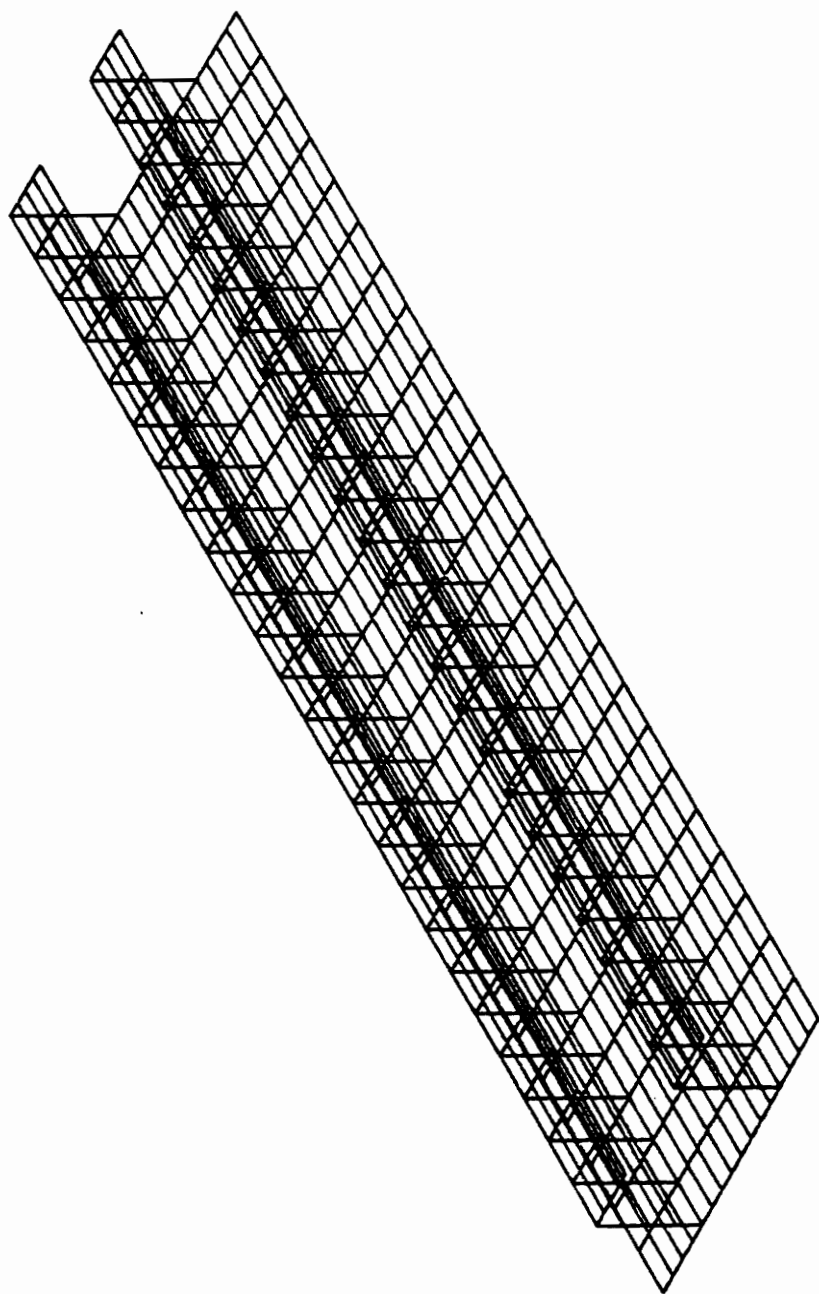


Fig. 3.3-25 Full model of a panel with two angle stiffeners

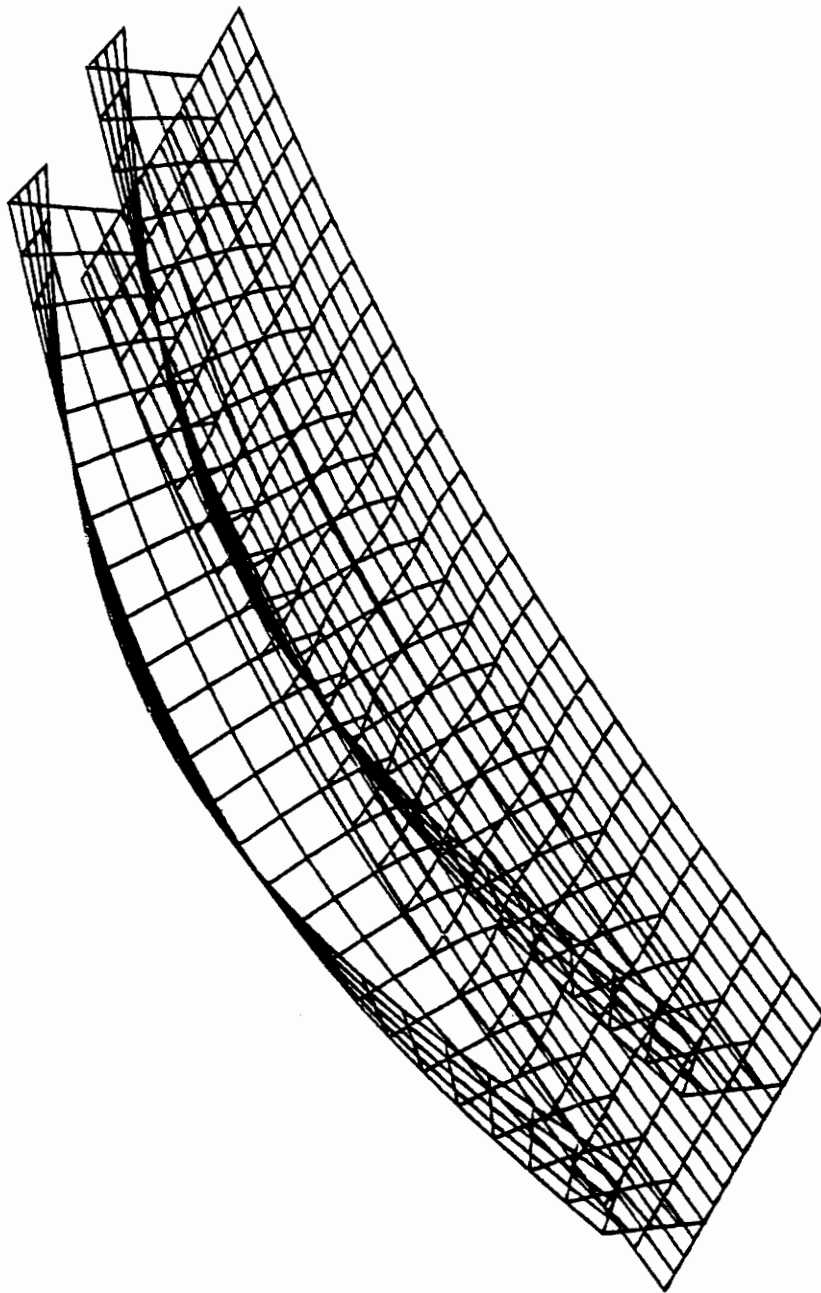


Fig. 3.3-26 S-shape mode of a panel with two angle stiffeners

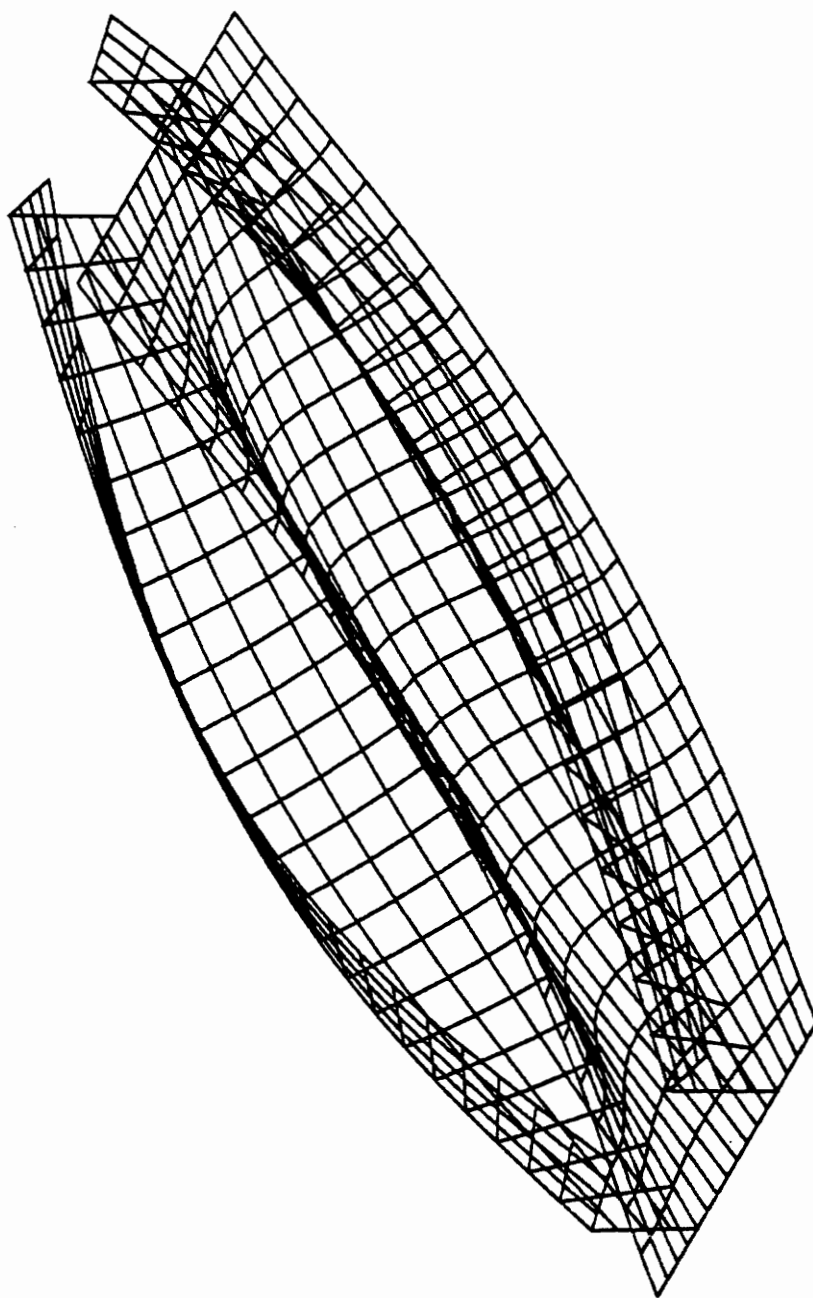


Fig. 3.3-27 U-shape mode of a panel with two angle stiffeners

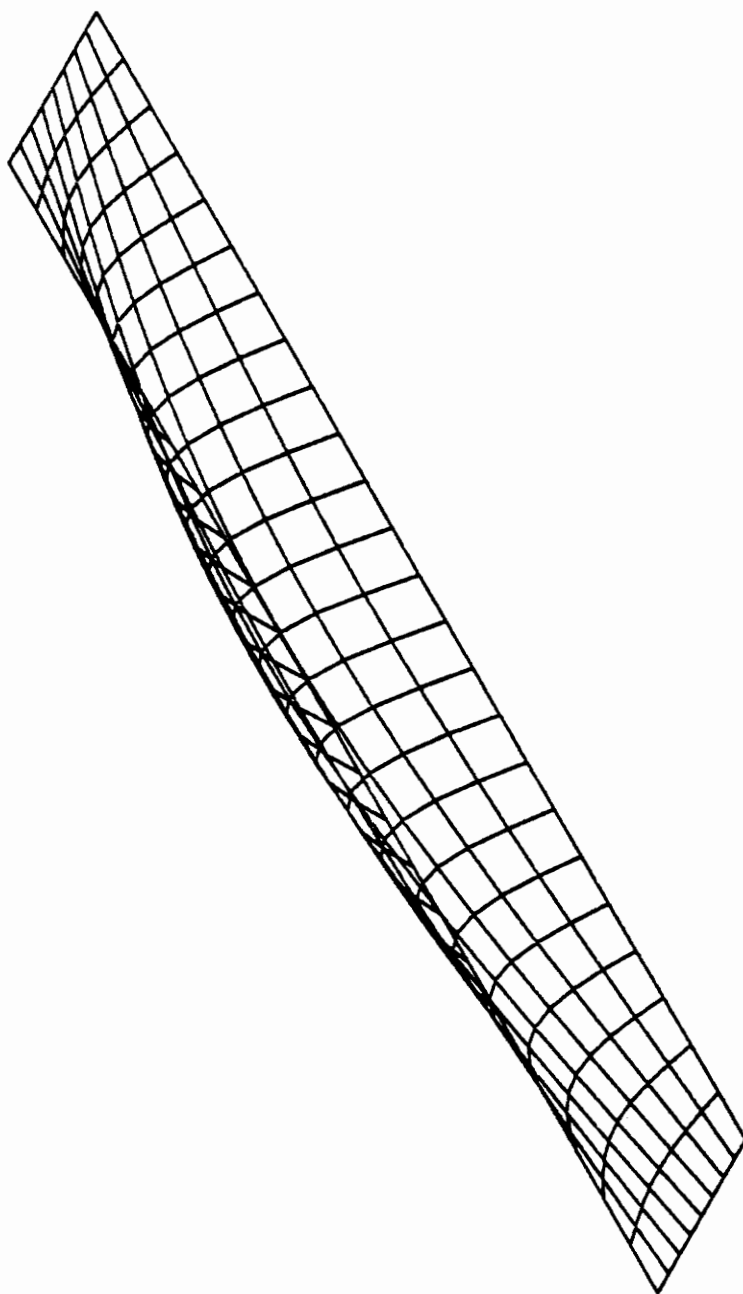


Fig. 3.3-28 Plate buckling mode

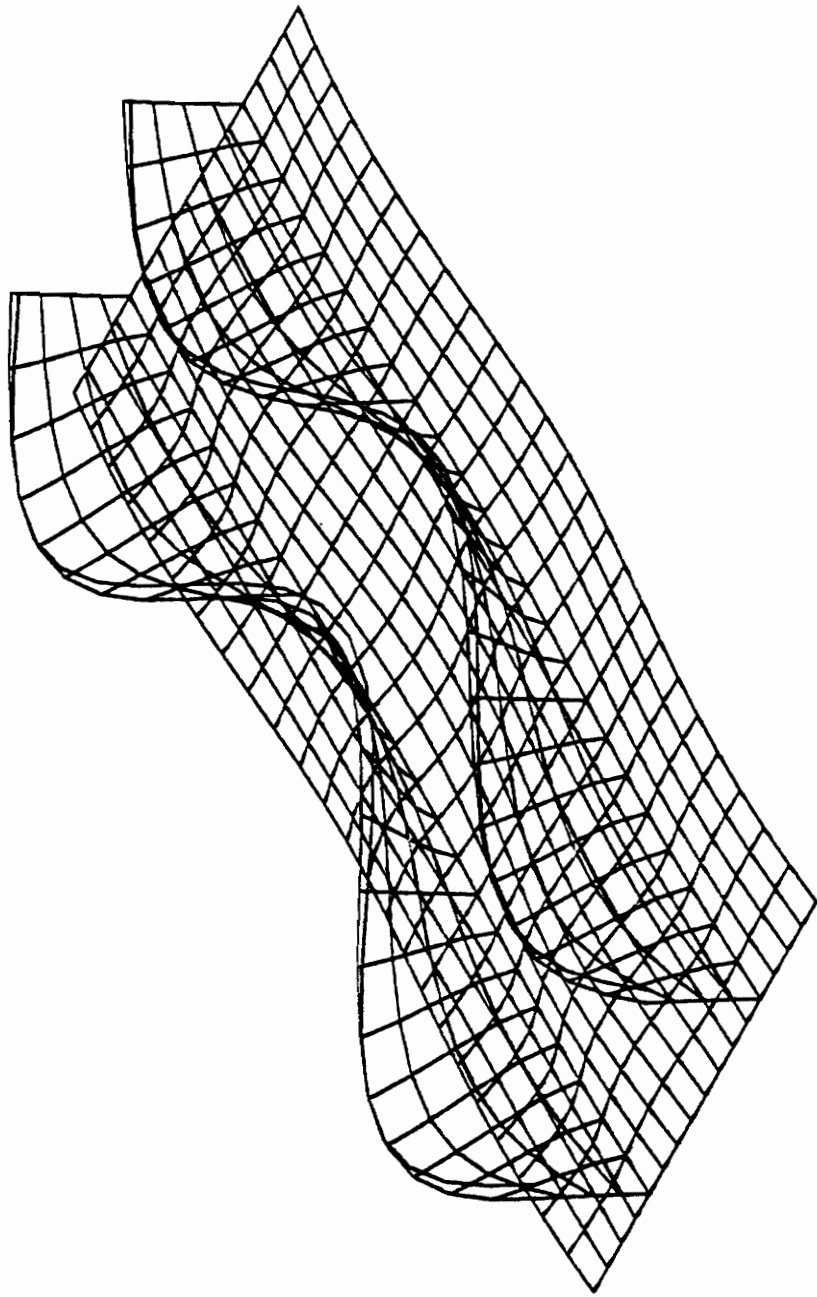


Fig. 3.3-29 S-shape mode of a panel with two bulb-flat stiffeners

4.0 Allowance for Imperfection; Second-Order Theory

In previous sections the load that corresponds to a state of bifurcation equilibrium of a perfect beam-column has been determined. This analysis used *first-order* (linear) buckling theory [M5], which assumes that the applied stresses have no influence upon the deflections until the load reaches the critical load, and the buckling stress is obtained as the solution of an eigenvalue problem. However, in practice panel stiffeners are seldom perfectly straight, nor is the plating. The lateral and vertical deflections of an imperfect beam-column will occur as soon as the load is applied. The theory that takes into account the effect of deflections on the equilibrium conditions is called *second-order* theory.

4.1 Second-order elastic analysis

Two types of imperfection are considered to be most important, the flexural deflection (w in Fig. 2.2-1) and flange sideways deflection (v_T in Fig. 2.2-1). The rotation angles ϕ_B and ϕ_T are considered linearly proportional to v_T ; i. e.

$$\phi_{0B} = \frac{v_{0T}}{h}$$

$$\phi_{0T} = \phi_{0B}$$

The pattern of initial imperfections is assumed to correspond to the displacement shape. Initial imperfections of the plate are indirectly considered by the use of the effective width concept, which will be discussed in Chapter 5:

$$\begin{Bmatrix} \phi_{0B} \\ v_{0T} \\ \phi_{0T} \\ w_0 \end{Bmatrix} = \begin{Bmatrix} \bar{\phi}_{0B} \\ \bar{v}_{0T} \\ \bar{\phi}_{0T} \\ \bar{w}_0 \end{Bmatrix} \sin \frac{m\pi x}{a} = \{\delta_0\} \sin \lambda x \quad (4.1-1)$$

where $\bar{w}_0, \bar{\phi}_{0B}, \bar{v}_{0T}, \bar{\phi}_{0T}$ are the maximum amplitudes of initial imperfection. Assume that displacements due to deformation are

$$\begin{Bmatrix} \Phi_B \\ v_T \\ \Phi_T \\ w \end{Bmatrix} = \begin{Bmatrix} \bar{\Phi}_B \\ \bar{v}_T \\ \bar{\Phi}_T \\ \bar{w} \end{Bmatrix} \sin \frac{m\pi x}{a} = \{\delta\} \sin \lambda x \quad (4.1-2)$$

where $\bar{w}, \bar{\Phi}_B, \bar{v}_T, \bar{\Phi}_T$ are the maximum amplitudes of deformation. Then the total displacements are $\{\delta + \delta_0\}$.

The total potential energy is

$$\Pi = U - V \quad (4.1-3)$$

where

$$\begin{aligned} U &= \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_0^l J_f \Phi_{T,x}^2 dx \\ &\quad + \frac{1}{2} \frac{E t_w^3}{12(1-\nu^2)} \iint (\nu^2 w_{,xx} + v^2 w_{,zz} + 2\nu v w_{,xx} v_{w,zz} + 2(1-\nu) v^2 w_{,xz}) dx dz \\ &= \frac{1}{2} \{\delta\}^T [K_L] \{\delta\} \\ V &= \frac{1}{2} \int_{A_f} \sigma \int_0^l (v_{T,x}^2 + (w - y \Phi_T)_{,x}^2) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA \\ &= \frac{1}{2} \{\delta + \delta_0\}^T [K_G] \{\delta + \delta_0\} \end{aligned}$$

The equilibrium becomes,

$$[K_L] \{\delta\} + [\bar{K}_G] \{\delta\} = -[\bar{K}_G] \{\delta_0\} \quad (4.1-4)$$

where matrix K_L is given by Eq. (3.2-11). $\bar{K}_G$ is the generalized geometric stiffness. The matrix $\bar{K}_G$ should be evaluated using the stress resultants in the final equilibrium position, which are directly dependent on the displacements $\{\delta\}$. This technique was used by Rajasekaran [R4], but in his case the beam-column was discretized into many segments lengthwise and he used the updated Lagrangian formulation. Since the model in this study has the equivalent of only one element, the updated Lagrangian formulation cannot be used, and therefore the method is restricted to small initial imperfections and deflections.

The curvatures due to the deformation are

$$\begin{aligned}\rho_{v_T} &= \frac{\partial^2 v_T}{\partial x^2} = -\bar{v}_T \lambda^2 \sin \lambda x \\ \rho_w &= \frac{\partial^2 w}{\partial x^2} = -\bar{w} \lambda^2 \sin \lambda x\end{aligned}\tag{4.1-5}$$

The stress distribution may be expressed in terms of the strain

$$\sigma = E\varepsilon$$

where the strains in the flange, web and plate are

$$\begin{aligned}\varepsilon_f &= (z_o - h)\rho_w + (y_o - y)\rho_{v_T} \\ \varepsilon_w &= (z_o - z)\rho_w + y_o\rho_{v_T} \\ \varepsilon_p &= z_o\rho_w + y_o\rho_{v_T}\end{aligned}$$

Letting compressive stress be positive, the stress of each component is

$$\begin{aligned}\sigma_f &= \sigma + E(z_o - h)\bar{w}\lambda^2 \sin \lambda x + E(y_o - y)\bar{v}_T\lambda^2 \sin \lambda x \\ \sigma_w &= \sigma + E(z_o - z)\bar{w}\lambda^2 \sin \lambda x + Ey_o\bar{v}_T\lambda^2 \sin \lambda x \\ \sigma_p &= \sigma + Ez_o\bar{w}\lambda^2 \sin \lambda x + Ey_o\bar{v}_T\lambda^2 \sin \lambda x\end{aligned}\tag{4.1-6}$$

Then the geometric stiffness matrix consists of the ordinary geometric stiffness and two additional matrices which depend on structural deformations:

$$[\bar{K}_G] = [K_G] + [K_G^{\bar{w}}] + [K_G^{\bar{v}_T}]\tag{4.1-7}$$

where matrix $[K_G]$ is the ordinary geometric stiffness defined in Eq. (3.2-11). $[K_G^{\bar{w}}]$ is given by

$$[K_G^{\bar{w}}] = [K_{GW}^{\bar{w}}] + [K_{GF}^{\bar{w}}] + [K_{GP}^{\bar{w}}]\tag{4.1-8}$$

in which

$$[K_{GW}^w] = -\frac{2}{3}\lambda^3 E\bar{w}t_w \begin{pmatrix} \begin{bmatrix} \frac{h^3}{105} & \frac{13h^2}{420} & -\frac{h^3}{140} & 0 \\ 0 & \frac{13h}{35} & -\frac{11h^2}{210} & 0 \\ 0 & 0 & \frac{h^3}{105} & 0 \\ \text{sym} & & & h \end{bmatrix} + \begin{bmatrix} \frac{h^4}{280} & \frac{h^3}{60} & -\frac{h^4}{280} & 0 \\ 0 & \frac{2h^2}{7} & -\frac{h^3}{28} & 0 \\ 0 & \frac{h^4}{168} & 0 & 0 \\ \text{sym} & & & \frac{h^2}{2} \end{bmatrix} \end{pmatrix}$$

$$[K_{GP}^w] = \frac{4\lambda}{3l} E\bar{w}z_o [K_{GP}^p]$$

$$[K_{GF}^w] = \frac{4\lambda}{3l} E\bar{w}(z_o - h) [K_{GF}^p]$$

and $[K_G^{\bar{v}_T}]$ is given by

$$[K_G^{\bar{v}_T}] = [K_{GW}^{\bar{v}_T}] + [K_{GF}^{\bar{v}_T}] + [K_{GP}^{\bar{v}_T}]$$

in which

$$[K_{GW}^{\bar{v}_T}] = \frac{4\lambda}{3l} E\bar{v}_T y_o [K_{GW}^p]$$

$$[K_{GP}^{\bar{v}_T}] = \frac{4\lambda}{3l} E\bar{v}_T y_o [K_{GP}^p]$$

for angles and bulb-flats

$$[K_{GF}^{\bar{v}_T}] = \frac{2}{3}\lambda^3 E\bar{v}_T \begin{pmatrix} y_o \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & t_f b_f & 0 & 0 \\ 0 & 0 & \frac{t_f b_f^3}{3} & -\frac{t_f b_f^2}{2} \\ \text{sym} & & & t_f b_f \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{t_f b_f^2}{2} & 0 & 0 \\ 0 & 0 & \frac{t_f b_f^4}{4} & -\frac{t_f b_f^3}{3} \\ \text{sym} & & & \frac{t_f b_f^2}{2} \end{bmatrix} \end{pmatrix}$$

for tees

$$[K_{GF}^{\bar{v}_T}] = \frac{2}{3} l \lambda^3 E\bar{v}_T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{t_f b_f^3}{24} & 0 \\ \text{sym} & & & 0 \end{bmatrix}$$

4.2 Solution Techniques

In a second-order analysis the equilibrium equations are formulated with respect to the deformed geometry of the structure, which is not known in advance and is constantly changing with the applied loads. Therefore it is necessary to employ an iterative technique to obtain solutions. In a numerical implementation one of the most popular solution techniques is the incremental load approach. In this approach, the applied load is divided into increments and applied successively to the structure. The deformed configuration of the structure at the end of each cycle is used as the basis for the equilibrium equation for the next cycle. Within each particular cycle of calculation the structure is assumed to behave linearly. In effect, the nonlinear response of the structure as a result of geometric change is approximated by a series of linear analyses. The geometry of the structure used in the analysis for a specific cycle is the deformed geometry of the structure corresponding to the previous cycle of calculation. Because of the linearization process, equilibrium may be violated and the external force may not always balance the internal force. The unbalanced force must be reapplied to the structure and the process repeated until equilibrium is satisfied. A corresponding diagram of axial compressive load versus the maximum deflection is shown in Fig. 4.2-1, where the dimensions of the panel are given in Section 3.1.

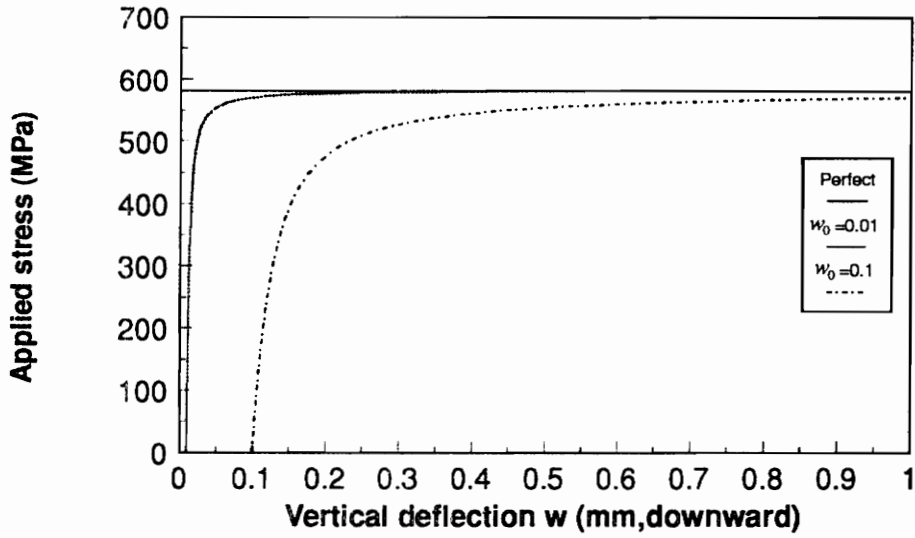
Since Eq. (4.1-4) is only of order four, one can use either the Newton-Raphson method or the Gauss-Seidel iterative method to obtain equilibrium at each load increment. An algorithm using the Gauss-Seidel iterative method is illustrated in the following.

1. Initially, a small-displacement solution of Eq. (4.1-4) is obtained for the first load increment, with K_G^w and K_G^v set to be zero. The stresses are computed from the known displacements and geometric stiffness. The process is repeated until the configuration of the model maintains equilibrium with the applied load.

2. For any r th load step, a new load increment is applied and the geometric stiffness (K_G) is calculated from the applied stress.
3. For any i th iterative step, the applied stress remains unchanged and therefore K_G is unchanged. However, $K_G^{\bar{w}}$ and $K_G^{\bar{v}_r}$ are updated as the displacement changes.
4. Compute the determinant of the tangent stiffness ($K_T = K_L + \bar{K}_G$). If the determinant is less than zero, the method of bisection is used to converge on the critical load. If the determinant is positive then proceed to the next step.
5. Solve Eq. (4.1-4). The iteration is continued (steps 3 to 5) until the difference of displacement for the i th iteration and that for the $(i-1)$ th iteration is less than the allowable tolerance value. When the displacements converge, return to step 2 to begin the next increment.

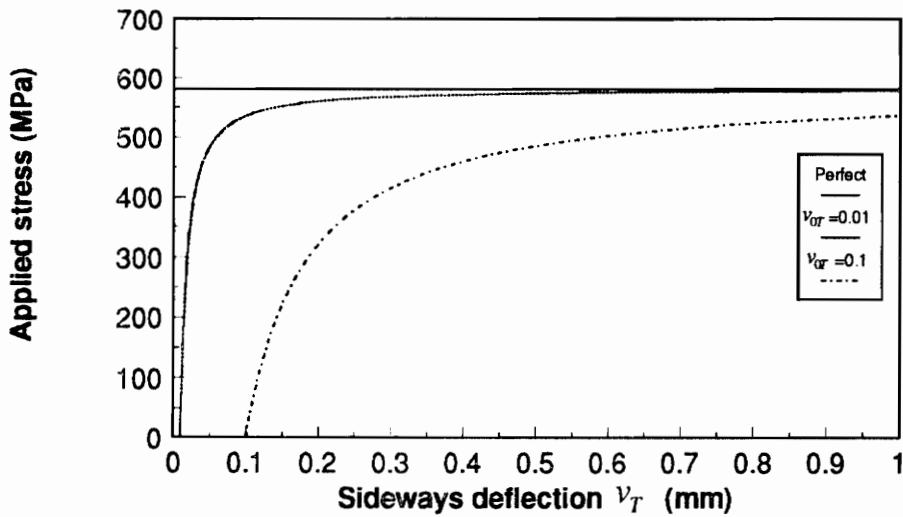
Force-displacement relation for perfect and imperfect panel

Panel with an angle stiffener



(A)

Panel with an angle stiffener



(B)

Fig. 4.2-1

5.0 Inelastic buckling

As indicated in Chapter 2, the tripping model in this study is regarded as a special beam-column, where the stiffener flange is free to deflect vertically and sideways but the plating is restricted from deflecting sideways. The tripping model has some features in common with ordinary beam-columns, and the solution techniques and some theories for ordinary beam-columns can be used for the tripping model. Long slender beam-columns buckle in the elastic range, whereas buckling of short to intermediate length beam-columns occurs at an applied load that may be relatively close to the yield load because the effective stiffness is reduced by yielding within the member. Yielding of the member is affected by its in-plane loading and by any residual stresses that are induced during its manufacture. Although restricted in application, research work on inelastic buckling of beam-columns has been reasonably plentiful, and is usually based on computer methods of analysis. However, very few attempts have been made towards an inelastic tripping analysis. One reason is due to the lack of a good elastic tripping model; the other is because of the difficulty of estimating the plate effective width due to plate buckling.

The inelastic buckling formulation in this study uses the Rayleigh-Ritz method, and it requires only four degrees of freedom. Because the cross-sectional and lengthwise stress distribution are not constant due to the presence of geometric eccentricity, residual stress, and/or uniform lateral pressure, the cross-section is subdivided into small zones, and ten Gauss points are chosen lengthwise. The cross-section properties are first evaluated at each Gauss point cross-section, then the linear stiffness matrix and the geometric matrix are obtained by integrating the cross-section properties lengthwise. The material nonlinearity, residual stress and plate postbuckling strength are taken into account during the calculation of the cross-section properties, and the effect of plate rotational restraint is also incorporated. The geometric stiffness matrix and eccentricity vector, combined with

the linear stiffness matrix, constitutes the system of equilibrium equations. Finally, an incremental and iterative scheme is used to solve these equations.

5.1 Deformation theory

Most engineering materials initially respond elastically. Elastic behavior means that the deformation is fully recoverable so that, when the load is removed, the specimen returns to its original shape. If the load exceeds some elastic limit the deformation is no longer fully recoverable. In a uniaxial stress state, the elastic limit of ductile materials corresponds to the yield stress σ_y on a stress-strain curve. In combined stress states, the elastic limit is defined by a yield surface. Increasing a load beyond the initial yield surface and into the work-hardening range results in both elastic and plastic deformations. With the development of plastic deformation, a new yield surface, called *subsequent yield surface* or *loading surface*, evolves. If the stress point moves inside the subsequent loading surface, the material behaves elastically, and no plastic deformation takes place. Since loading surfaces change according to the way the plastic deformation was produced, the stress-strain behavior of work-hardening material is *loading path dependent* or *loading history dependent*. There are two types of theories dealing with work-hardening materials. *Deformation (total) theory* ignores the loading history dependency, and assumes that the final state of stress is uniquely determined by the final state of strain. Because of its relative simplicity, deformation theory has been used extensively in engineering practice for solving elastic-plastic problems. In contrast to deformation theory, loading history dependency is assumed in the *flow (or incremental) theory*. In this theory, the plastic strain increments are related to the stress increment and to the final state of stress, plastic strain. It is a well established fact that deformation theory is incapable of tracing the totality of the load-deformation history, especially during a reversal of loading.

The general validity of *deformation theory* is limited to monotonically increasing loading in which: (1) the stress components are increased nearly proportionally in a loading process, known as *proportional loading*; and (2) no unloading occurs. Because no strain reversal occurs, the relationship between increment in stress and increment in strain may be determined from the tangent modulus. Since this study deals only with a beam-column (stiffener and effective width of plating) the stress in the member is essentially uniaxial, and the effective strain for the determination of the tangent modulus is considered to be the longitudinal strain. Hence

$$\Delta\sigma = E_t\Delta\epsilon$$

By virtue of the above assumptions, deformation theory is essentially a piecewise elastic theory and this is used in the further development of the plastic buckling model.

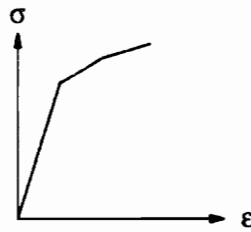


Fig. 5.1-1 Stress-strain diagram

Assumptions: The following assumptions are made for the inelastic model, in addition to the assumptions made in Section 2.1 for the elastic buckling model.

1. The load is static and is incremented gradually, in successively smaller steps.
2. There is no local buckling (of the stiffener cross section).
3. Strain reversal does not occur.

5.2 Stress-strain relationship

Trilinear model

The stress-strain curve assumed for the structural steel is shown in Fig. 5.2-1. It is a trilinear idealization. In this model the tangent modulus is assumed to be constant, and the stress-strain relation is approximated by three straight lines.

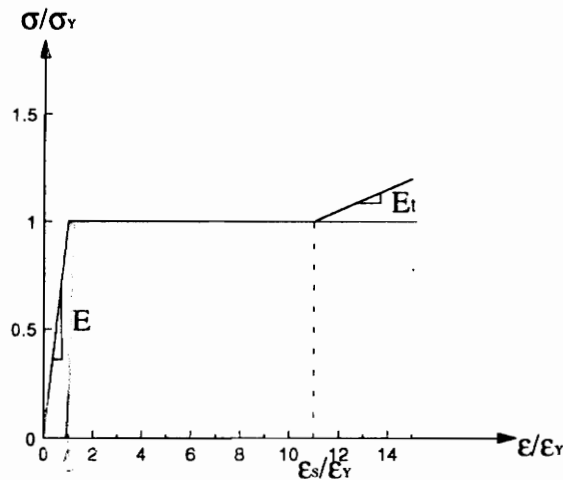


Fig. 5.2-1 Idealized trilinear strain-stress model

The existence of an upper yield stress is ignored.

As an example, some nominal values for E , E_t , σ_y and ϵ_s are given for steel by Trahair

[T1]:

$$E = 200,000 \text{ N/mm}^2$$

$$E_t = E / 33$$

$$\sigma_y = 250 \text{ N/mm}^2$$

$$\epsilon_y = 1250 \times 10^{-6}$$

$$\epsilon_s = 11\epsilon_y \quad \checkmark$$

5.3 Residual stress

Metal members are usually heated at some stage during the fabrication process. As they cool down, some parts of the cross-section lose heat more rapidly than others. This nonuniform cooling process leaves the finished member with a pattern of self-equilibrating stress locked into it. The locked-in stress is termed residual stress and it can have a considerable effect on the strength of panel stiffeners made from hot-rolled or welded steel sections. Systematic studies in the 1950s and 1960s have established that residual stress has in fact a major effect on the load capacity of short steel columns while the effect of imperfection is secondary in comparison [B1]. The effect of residual stress on the critical

load of a straight axially loaded column is easy to understand in principle. As the applied thrust increases, those parts of the section which have the highest compressive residual stress will yield soonest, leaving only a proportion of the cross-section in an elastic state. It is this elastic part of the section which determines the flexural rigidity and hence the critical load. The critical load is, therefore, less than it would have been in the absence of residual stress.

Residual stress in hot-rolled beam-columns

Longitudinal residual stresses are induced in hot-rolled steel beam-columns during cooling after rolling, and as a result of any mechanical straightening process. During cooling, the tips of the flanges cool more rapidly than the regions where the web joins the flanges, since they have a greater surface area in relation to their volume. When early cooling regions have cooled sufficiently the steel becomes stiff; at the same time, the steel at the web/flange junctions remains hot and plastic. Subsequent shrinkage of the high temperature late-cooling regions during final cooling are resisted by the regions already cooled, and induce residual compression in them. Equilibrating residual tensions are induced in the late-cooling regions. Idealized and measured distributions of residual stress in hot-rolled beam-columns are often approximately parabolic, being compressive at the early-cooling regions, and tensile at the late-cooling regions. The effect depends mainly on geometry, and is most pronounced in wide flange sections, in which the residual compressive stress in the flange tips can be as high as 80 MPa for mild steel. The effect is diminished for steels having a high yield strength.

Residual stress in welded stiffened panels

Residual stresses are induced in welded stiffened panels by uneven heating and cooling. During welding the metal is in a perfectly plastic state. At any time during and after the instant of welding, some plastic fibers in the cross section will be cooling and other elastic fibers will be heating. The action of the loads on the elastic fibers due to changes in

temperature is not counteracted by the plastic fibers until the plastic fibers have cooled sufficiently to become elastic. Consequently the metal near the welds acquires a very high longitudinal tensile stress (approaching the yield stress) which is balanced by a relatively low compressive residual stress σ_r elsewhere in the section. The magnitudes and distributions of residual stress due to welding again vary with the cross-section geometry and the process. An idealized distribution is given as

$$\frac{\sigma_r}{\sigma_Y} = \frac{2\eta}{\frac{b}{t} - 2\eta} \tag{5.3-1}$$

where ηt denotes the width of the tension yield zone and b the width between longitudinal welds in the section (Fig. 5.3-1). The effect of residual stress is somewhat diminished for higher yield steels due to a narrower tension zone.

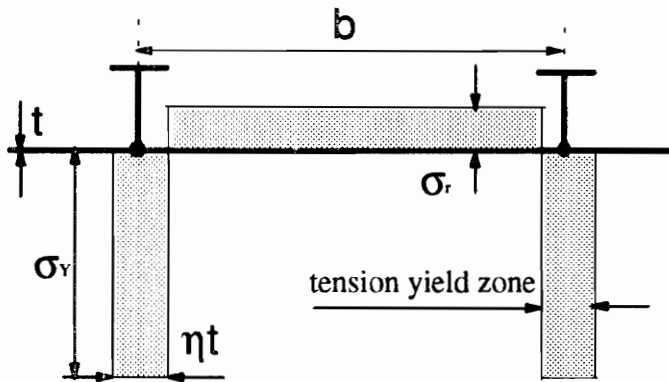


Fig. 5.3-1 Idealized residual stresses in welded plate

5.4 Effective width and effective breadth

A plate ~~with~~ reinforced with stiffeners can be represented as a structure consisting of a grid of beams. In this manner, it can be examined easily with calculation methods based on the beam theory, without resorting to complex numerical methods. This is especially useful during the design phase. However, the simplification makes it necessary to assume that the hypotheses on which “simple beam theory” is based are valid. In calculating the beam cross-sectional properties the plate is not considered fully effective because of the

plate non-uniform stress distributions. Two concepts are introduced to take account the effect: the effective width due to the plate buckling and the effective breadth due to the shear lag.

Effective width due to buckling

The plating effective width b_e plays an important role in calculation of the tripping stress. Since slender panels are normally designed such that plate buckling precedes overall buckling, when the latter occurs the plate will not be fully effective over the width b . As a result, the compressive stress within the plate is no longer uniform. Nevertheless, the plate is still supporting the load and so, although it has buckled, it has not yet collapsed. However, because of the deflected shape of the buckled portion of the plate, the axial stiffness is substantially reduced in this region and hence the overall stiffness of the plate is reduced. As the load is further increased the plating effective width decreases, the load shirking of the center region becomes more pronounced, and the maximum stress at the stiffener location increases ever more rapidly, until finally this stress reaches the yield stress (squashing) or the stiffener buckling stress (inelastic buckling), whereupon collapse occurs.

The effective width due to buckling has long been a vexing question, mainly because in most cases it was being discussed and applied in the difficult context of the ultimate strength of panels which were not slender and which therefore did not buckle elastically. Several formulas have been proposed for computing the effective width as a function of the width-thickness ratio and the average or edge stress. The following effective width formula [G1] was selected for use in this study:

$$\frac{b_e}{b} = \sqrt{\frac{\sigma_{pbe}}{\sigma}} \left(1 - 0.22 \sqrt{\frac{\sigma_{pbe}}{\sigma}} \right) \quad (5.4-1)$$

where σ_{pbe} is the classical elastic plate buckling stress and σ is the applied stress. Eq. (5.4-1) is based on experimental results and incorporates the effects of residual stress and initial

imperfections. The maximum effective width is the actual width ($b_e=b$) and the minimum width is obtained when the edge stress (σ_e) reaches the yield stress of the plate. The plate effective width is assumed to be constant lengthwise. The implementation of Eq. (5.4-1) is

$$b_e = \begin{cases} b \sqrt{\frac{\sigma_{pbe}}{\sigma}} (1 - 0.22 \sqrt{\frac{\sigma_{pbe}}{\sigma}}) & \text{if } \sigma > 0.1936 \sigma_{pbe} \text{ \& } \sqrt{\frac{\sigma_{pbe}}{\sigma}} (1 - 0.22 \sqrt{\frac{\sigma_{pbe}}{\sigma}}) < 1 \\ b & \text{otherwise} \end{cases}$$

Peak stress

It is also important to note here that the critical tripping stress is a *peak stress* at the stiffener location and that when the effective width of the plating b_e is less than b this stress will be greater than the corresponding average stress in the plating, σ_{avg} , and also greater than the overall mean stress of the total plate-beam cross section, σ_m . These definitions of stress are illustrated in Figure 5.4-1.

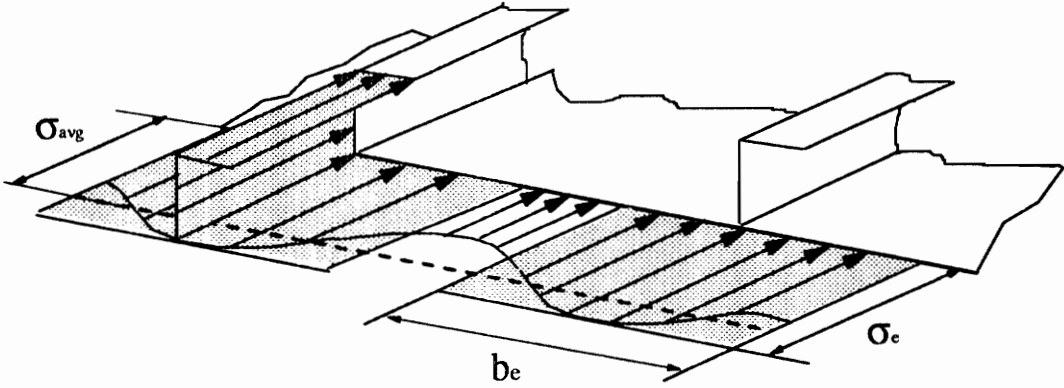


Fig.5.4-1 Peak stress of stiffened panel

Average plate stress

$$\sigma_{avg} = \frac{b_e}{b} \sigma_e$$

Overall mean stress

$$\sigma_m = \frac{A_s + b_e t_p}{A_s + b t_p} \sigma_e \quad (5.4-2)$$

Effective breadth due to shear lag

In any flange-and web type of beam under lateral loads, these loads are absorbed by the web of the beam and then transmitted to the flange by means of shear in the plane of the flange. This effect gives rise to a non-uniform distribution of the consequent direct stress across the flange, as shown in Fig. 5.4-2. The bending stress remote from a web “lag behind” the stress near the web.

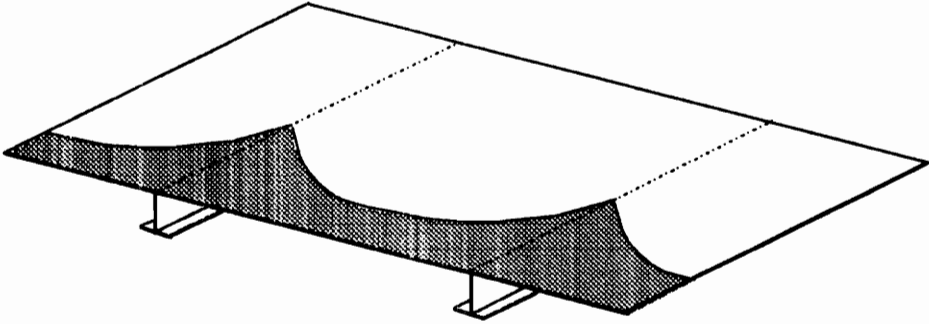


Fig. 5.4-2 Shear lag in the plating

In a simplified beam solution this effect is taken into account by introducing an artificial effective breadth b_e on which the distribution of stresses is assumed to be uniform across the breadth and equal to the maximum value. Boote and Mascia [B6] performed numerical analysis to a set of models of stiffened plates using the finite-element method. The results were then compared with those supplied by a simple calculation performed by using a beam model. The following expression is proposed for effective breadth,

$$\frac{b_e}{b} = 1 - \exp\left((0.005t - 0.649) \cdot (L/b)^{(-0.027t + 1.067)}\right) \quad (5.4-3)$$

5.5 Total potential energy

Due to the presence of residual stresses and/or lateral load, some fibers in the cross section yield before buckling, and the effective area of the cross section will change as a consequence. To calculate the cross-sectional properties the cross section is discretized into small zones, as illustrated in Fig. 5.5-1, and the properties at the zone centroid are

considered to represent those of the zone. Therefore, the cross-sectional properties of the complete section cannot be expressed analytically but rather must be calculated by numerical integration.

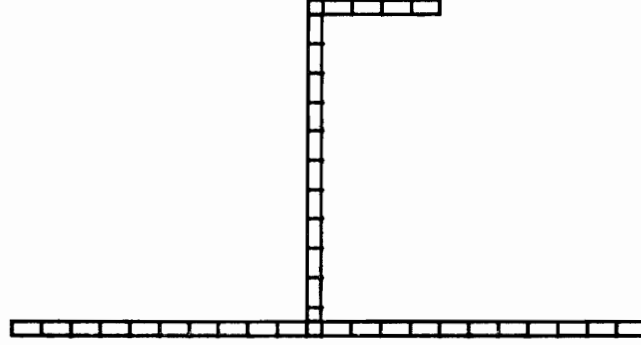


Fig. 5.5-1 Idealized discretization of the cross section of the beam-column

The total potential energy is

$$\Pi = U - V$$

where U is the strain energy stored during buckling, and V is the work done during buckling:

$$\begin{aligned} U &= \frac{1}{2} E \int_0^l I^T(x) w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z^T(x) v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2I_{zy}^T(x) w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_l J_f^T(x) \phi_{T,x}^2 dx \\ &\quad + \frac{1}{2} \iint (D_{11}(x) v_{w,xx}^2 + D_{22}(x) v_{w,zz}^2 + 2D_{12}(x) v_{w,xx} v_{w,zz} + D_{33}(x) v_{w,xz}^2) dx dz \\ &\quad + \frac{1}{2} \int_0^l K_{\phi_B} \phi_B^2 dx \\ &= \frac{1}{2} \{\delta\}^T [K_L] \{\delta\} \end{aligned}$$

where K_{ϕ_B} is the plate rotational spring defined in Eq. (3.3-16), $I^T(x)$, $I_z^T(x)$, $I_{zy}^T(x)$, and $J_f^T(x)$ are the tangent modulus section properties due to panel in-plane deformation at length x . They are constant only if there is no eccentricity and no lateral pressure. To evaluate the in-plane tangent modulus section properties, the plate thickness in each region can be modified by the ratio E_f/E and the transformed section can be treated as though it possesses a constant modulus. Figure 5.5-2 shows a typical transformed section.

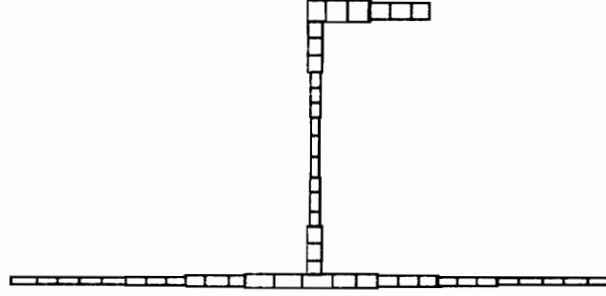


Fig. 5.5-2 Transformed panel cross-section

The tangent section properties at location x along the length may be evaluated as

$$I^T = \int_{\text{flange}} (z_o - h)^2 \frac{E_t}{E} dA + \int_{\text{web}} (z_o - z)^2 \frac{E_t}{E} dA + \int_{\text{plate}} z_o^2 \frac{E_t}{E} dA$$

$$= \sum_{k=1}^{N_f} (z_o - h)^2 \frac{E_t}{E} A_k + \sum_{k=1}^{N_w} (z_o - z_k)^2 \frac{E_t}{E} A_k + \sum_{k=1}^{N_p} z_o^2 \frac{E_t}{E} A_k$$

$$I_z^T = \int_{\text{flange}} (y - y_o)^2 \frac{E_t}{E} t_f dy + \int_{\text{web}} y_o^2 \frac{E_t}{E} t_w dz + \int_{\text{plate}} y_o^2 \frac{E_t}{E} t_p dy$$

$$= \sum_{k=1}^{N_f} (y_k - y_o)^2 \frac{E_t}{E} A_k + \sum_{k=1}^{N_w} y_o^2 \frac{E_t}{E} A_k + \sum_{k=1}^{N_p} y_o^2 \frac{E_t}{E} A_k$$

$$I_{zy}^T = \int_{\text{flange}} (z_o - h)(y_o - y) \frac{E_t}{E} dA + \int_{\text{web}} (z_o - z)y_o \frac{E_t}{E} dA + \int_{\text{plate}} z_o y_o \frac{E_t}{E} dA$$

$$= \sum_{k=1}^{N_f} (z_o - h)(y_o - y_k) \frac{E_t}{E} A_k + \sum_{k=1}^{N_w} (z_o - z_k)y_o \frac{E_t}{E} A_k + \sum_{k=1}^{N_p} z_o y_o \frac{E_t}{E} A_k$$

$$J_f^T = J_f$$

To evaluate the out-of-plane plate bending properties, the method used by Bradford [B5] is adopted here. For the elastic range, the out-of plane bending constants D_{11} , D_{22} , D_{12} and D_{33} are given by

$$[D] = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} = \frac{t^3}{12} \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 4G \end{bmatrix}$$

For the inelastic range, the material properties used here are based largely on results obtained by Handelman and Prager [H1] with G_t , the tangent shear modulus, being modified according to Lay [L1]. As a result of the work of Handelman and Prager,

$$E_x = E_t$$

$$E_y = \frac{4EE_t}{3E_t + E}$$

$$\nu_x = \frac{(2\nu - 1)E_t + E}{2E}$$

$$\nu_y = \frac{4\nu_x E}{3E_t + E}$$

and, according to Lay,

$$G_t = \frac{4EE_t}{4E_t(1 + \nu) + E}$$

in which E_x and E_y are the moduli in the x and y direction; ν_x and ν_y are Poisson's ratio in the x and y directions; E_t is the tangent modulus; E is the elastic modulus; and ν is Poisson's ratio in the elastic range. Therefore, the out-of-plane bending constants are

$$[D] = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} = \frac{t^3}{12} \begin{bmatrix} \frac{E_x}{1 - \nu_x \nu_y} & \nu_y \frac{E_x}{1 - \nu_x \nu_y} & 0 \\ \nu_x \frac{E_y}{1 - \nu_x \nu_y} & \frac{E_y}{1 - \nu_x \nu_y} & 0 \\ 0 & 0 & 4G_t \end{bmatrix}$$

Note that $D_{12} = D_{21}$

The work done during buckling is

$$\begin{aligned} V &= \frac{1}{2} \int_{A_f} \int_0^l (\nu_{T,x}^2 + (w - y\varphi_T)_{,x}^2) dx dA + \frac{1}{2} \int_{A_p} \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \int_0^l (w_{,x}^2 + \nu_{w,x}^2) dx dA \\ &= \frac{1}{2} \left(\int_0^l \alpha_{\nu_T \nu_T}^T(x) \nu_{T,x}^2 dx + \int_0^l \alpha_{ww}^T(x) w_{,x}^2 dx + \int_0^l \alpha_{\varphi_T \varphi_T}^T(x) \varphi_{T,x}^2 dx + \int_0^l \alpha_{\varphi_B \varphi_B}^T(x) \varphi_{B,x}^2 dx \right. \\ &\quad \left. + 2 \int_0^l \alpha_{w\varphi_T}^T(x) w_{,x} \varphi_{T,x} dx + 2 \int_0^l \alpha_{\nu_T \varphi_T}^T(x) \nu_{T,x} \varphi_{T,x} dx + 2 \int_0^l \alpha_{\nu_T \varphi_B}^T(x) \nu_{T,x} \varphi_{B,x} dx + 2 \int_0^l \alpha_{\varphi_T \varphi_B}^T(x) \varphi_{T,x} \varphi_{B,x} dx \right) \\ &= \frac{1}{2} \{\delta\}^T [\bar{K}_G] \{\delta\} \end{aligned}$$

where for a cross-section at location x along the length,

$$\alpha_{v_T v_T}^T = \int_{A_f} \sigma_f dA + \int_{A_w} \sigma_w f_2^2 dA = \sum_{k=1}^{N_f} \sigma_f^k A_k + \sum_{k=1}^{N_w} \sigma_w^k f_2^2 A_k$$

$$\alpha_{ww}^T = \int_{A_f} \sigma_f dA + \int_{A_w} \sigma_w dA + \int_{A_p} \sigma_p dA = \sum_{k=1}^{N_f} \sigma_f^k A_k + \sum_{k=1}^{N_w} \sigma_w^k A_k + \sum_{k=1}^{N_p} \sigma_p^k A_k$$

$$\alpha_{\varphi_T \varphi_T}^T = \int_{A_f} \sigma_f y^2 dA + \int_{A_w} \sigma_w f_3^2 dA = \sum_{k=1}^{N_f} \sigma_f^k y_k^2 A_k + \sum_{k=1}^{N_w} \sigma_w^k f_3^2 A_k$$

$$\alpha_{\varphi_B \varphi_B}^T = \int_{A_w} \sigma_w f_1^2 dA = \sum_{k=1}^{N_w} \sigma_w^k f_1^2 A_k$$

$$\alpha_{w \varphi_T}^T = \int_{A_f} \sigma_f (-y) dA = \sum_{k=1}^{N_f} \sigma_f^k (-y_k) A_k$$

$$\alpha_{v_T \varphi_T}^T = \int_{A_w} \sigma_w f_2 f_3 dA = \sum_{k=1}^{N_w} \sigma_w^k f_2 f_3 A_k$$

$$\alpha_{v_T \varphi_B}^T = \int_{A_w} \sigma_w f_1 f_2 dA = \sum_{k=1}^{N_w} \sigma_w^k f_1 f_2 A_k$$

$$\alpha_{\varphi_T \varphi_B}^T = \int_{A_w} \sigma_w f_1 f_3 dA = \sum_{k=1}^{N_w} \sigma_w^k f_1 f_3 A_k$$

in which N_f , N_w and N_p are the total number of flange, web and plate zones, respectively, k is the zone index, and f_1 , f_2 , and f_3 are the shape functions defined in Eq. (3.2-3).

5.6 Load application

It has been observed previously [S3, S4] that as compressive load increases in single-span stiffened panels, a loss of average stiffness occurs in the plating as a result of local buckling and/or yielding (both of which are influenced by residual stress), causing a shift of the neutral axis towards the stiffener flange and a consequent eccentricity in the applied load, which usually leads to compression failure of the plating (Fig. 5.6-1). In this respect tests on single-span panels with fixed alignment of axial load are unrepresentative of the behavior of continuous multi-bay panels.

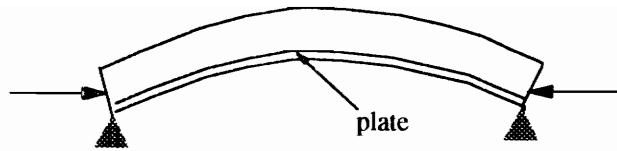


Fig. 5.6-1 Failure mode in single-span panel under fixed axial load aligned with neutral axis

In order to trigger tripping on single-span panels in lab tests, the axial load is usually applied on the stiffener side of the neutral axis (Fig. 5.6-2). The load eccentricity imposes an additional bending moment on the panel stiffener, and the bending moment changes as the neutral axis shifts.

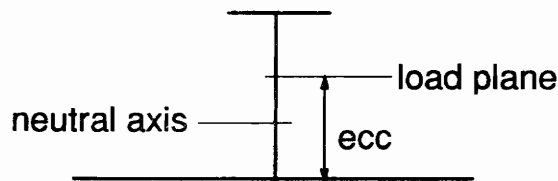


Fig. 5.6-2 Eccentricity of load plane

In multi-bay grillages with stiff transverse frames and reinforced end-bays, the effect of load eccentricity on interframe collapse is likely to be small. Since the external load is applied to the reinforced end bays, the panel in the middle bay, in which collapse usually occurs, can be considered as being subjected to a uniform strain loading rather than a fixed point load. When the plate in the middle bay buckles, the neutral axis shifts towards the stiffener flange as a result of a loss of average stiffness in the plating. However, because the loading in the middle bay is considered as uniform strain loading, the stiffener and its associated effective plating will take more loads than the rest of the area as shown in Fig. 5.4-1. Consequently the effective loading plane is also shifted towards the stiffener flange. Thus tests on orthogonally stiffened grillages should in this respect give a satisfactory representation of actual ship bottom and deck structures.

5.7 Method of solution

Due to the complex behavior characteristics in the inelastic range, an incremental and iterative algorithm is developed to determine the various coefficients of the stability equations, and to solve them for the buckling load. The inelastic buckling method is best illustrated in the following algorithms. Strain is the basic incremented variable and the strain and stress of each zone are required to be stored and updated after each computational cycle. The algorithm can deal with any combination of axial compression, pure bending, and uniform lateral pressure. However, because the evaluation of strain and stress depends entirely on the loading condition, separate algorithms are given depending on which of the loads is the live load. Geometric eccentricity and residual stress are also incorporated into the algorithms. Because the distribution and the magnitude of residual stress are often not available, an empirical formula, Eq. (5-4), is used to represent the effect of plate residual stress. Therefore the actual cross section discretization used in the algorithms is shown in Fig. 5.7-1 rather than in Fig. 5.5-1. If the actual residual stress is known, then the algorithms can account for it by discretizing the plating as in Fig. 5.5-1.

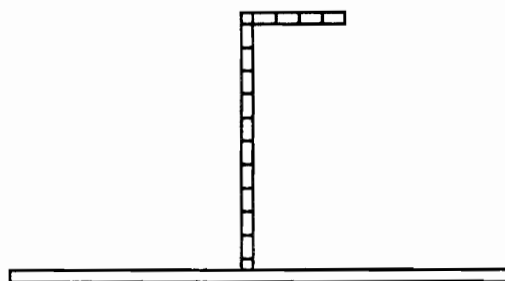


Fig. 5.7-1 Actual discretization of the cross section of the beam-column

Algorithm A. Inelastic buckling under axial compressive load

When the live load is a uniform axial compressive force, then the algorithm is as follows:

Step 1. Determine the critical buckling mode by elastic tripping analysis, and calculate the plate elastic buckling stress σ_{pbe} :

$$\sigma_{pbe} = 3.62E \left(\frac{t_p}{b} \right)^2$$

The increment of the strain is the outer-loop computational cycle. For the first strain increment ($i=1$), assume $E_i = E$ for all zones. Therefore, y_0 and z_0 can be easily obtained by Eqs. (A-2) and (A-3). Then apply the first incremental uniform axial strain $\{\Delta\epsilon_a\}_1$.

Step 2. The total strain of a zone $\{\epsilon_i\}_i$ at current strain increment i , is equal to

$$\begin{aligned}\{\epsilon_a\}_i &= \{\epsilon_a\}_{i-1} + \{\Delta\epsilon_a\}_i \\ \{\epsilon_d\}_i &= \{\epsilon_d\}_{i-1} + \{\Delta\epsilon_d\}_i \\ \{\epsilon_e\}_i &= \{\epsilon_e\}_{i-1} + \{\Delta\epsilon_e\}_i \\ \{\epsilon_t\}_i &= \{\epsilon_a\}_i + \{\epsilon_d\}_i + \{\epsilon_e\}_i + \epsilon_m + \epsilon_q + \epsilon_r\end{aligned}$$

where $\{\epsilon_a\}_{i-1}$ is the applied uniform strain (live load) in the previous cycle and is stored for every cycle, and $\{\epsilon_d\}_{i-1}$ is the strain due to structural deformation in the previous cycle and is stored for every cycle. For a single span panel $\{\epsilon_e\}_{i-1}$ is the strain due to axial load eccentricity in the previous cycle and is stored for every cycle. For multi-span panels $\{\epsilon_e\}_{i-1}$ is set to zero. $\{\Delta\epsilon_a\}_i$ is the current incremental uniform axial strain, which is selected at the start of the solution, and $\{\Delta\epsilon_d\}_i$ is the incremental strain due to the deformation, which is computed by the following method:

$$\{\Delta\epsilon_d\}_i = \{\epsilon_d\}_i - \{\epsilon_d\}_{i-1}$$

where the flange, web and plate components of $\{\epsilon_d\}_i$ are

$$\begin{aligned}\{\epsilon_f\}_i &= (\{z_o\}_i - h)\{\rho_w\}_i + (\{y_o\}_i - y)\{\rho_{v_T}\}_i \\ \{\epsilon_w\}_i &= (\{z_o\}_i - z)\{\rho_w\}_i + \{y_o\}_i\{\rho_{v_T}\}_i \\ \{\epsilon_p\}_i &= \{z_o\}_i\{\rho_w\}_i + \{y_o\}_i\{\rho_{v_T}\}_i\end{aligned}$$

in which

$$\begin{aligned}\{\rho_{v_T}\}_i &= \left\{ \frac{\partial^2 v_T}{\partial x^2} \right\}_i = -\{\bar{v}_T\}_i \lambda^2 \sin \lambda x \\ \{\rho_w\}_i &= \left\{ \frac{\partial^2 w}{\partial x^2} \right\}_i = -\{\bar{w}\}_i \lambda^2 \sin \lambda x\end{aligned}$$

The incremental strain due to the axial load eccentricity of a single span panel, $\{\Delta\epsilon_e\}_i$, is computed by

$$\{\Delta\epsilon_e\}_i = \{\epsilon_e\}_i - \{\epsilon_e\}_{i-1}$$

where the flange, web and plate components of $\{\epsilon_e\}_i$ are

$$\{\epsilon_f\}_i = (\{z_o\}_i - h)\{\rho_e\}_i$$

$$\{\epsilon_w\}_i = (\{z_o\}_i - z)\{\rho_e\}_i$$

$$\{\epsilon_p\}_i = \{z_o\}_i\{\rho_e\}_i$$

in which

$$\{\rho_e\}_i = \frac{\{\epsilon_a\}_i \{A_{eff}\}_i \{ecc\}_i}{\{I\}_i}$$

$$\{e\}_i = ecc - \{z_o\}_i$$

ϵ_r is the residual strain. ϵ_m is the strain due to end moments (dead load) and ϵ_q is the strain due to uniform lateral pressure (dead load). The summation of ϵ_m , ϵ_q and ϵ_r can be regarded as a generalized residual strain $\bar{\epsilon}_r$. Assuming that elastic unloading will take place in the zone if the generalized residual stress and bending stress are released, the stress due to the generalized residual strain can then be written as

$$\bar{\sigma}_r(x, y, z) = E\bar{\epsilon}_r$$

where

$$\bar{\epsilon}_r = \epsilon_m + \epsilon_q + \epsilon_r$$

The applied stresses in the current cycle are then obtained from

$$\{\sigma_a\}_i = \{\sigma_a\}_{i-1} + E_t\{\Delta\epsilon_a\}_i$$

$$\{\sigma_d\}_i = \{\sigma_d\}_{i-1} + E_t\{\Delta\epsilon_d\}_i$$

$$\{\sigma\}_i = \{\sigma_a\}_i + \{\sigma_d\}_i + \bar{\sigma}_r$$

where $\{\sigma_a\}_{i-1}$ is the applied uniform stress (live load) in the previous cycle and is stored for every cycle; $\{\sigma_d\}_{i-1}$ is the stress due to structural deformation in the previous cycle and is stored for every cycle, $\{\sigma\}_i$ are the total stresses in the current cycle, which include the

residual stress and the stress due to the dead load. The effective width of the plate can then be evaluated by using Eq. (5.4-2). Thus, the cross-section properties will be based on the plate effective width $\{b_e\}_i$ rather than the actual width b . It is noted that strains ϵ_m , ϵ_q and $\{\epsilon_d\}_i$ are functions of the location of the neutral axis z_o and/or the moment of inertia (I), which depends on the plate effective width. Thus, an iterative algorithm is needed.

The values of ϵ_m and/or ϵ_q can be obtained from

$$\epsilon = \frac{\sigma}{E}$$

where σ is the stress of each component. For end moment M the component stresses are

$$\begin{cases} \sigma_f = -\frac{M}{I}(z_o - h) \\ \sigma_w = -\frac{M}{I}(z_o - z) \\ \sigma_p = -\frac{M}{I}z_o \end{cases}$$

and for uniform lateral pressure q they are

$$\begin{cases} \sigma_f = -\frac{q}{I}(z_o - h)\frac{(lx - x^2)}{2} \\ \sigma_w = -\frac{q}{I}(z_o - z)\frac{(lx - x^2)}{2} \\ \sigma_p = -\frac{q}{I}z_o\frac{(lx - x^2)}{2} \end{cases}$$

Note that the stresses and the strains should be calculated at ten Gauss points along the length (x -direction), and this also applies to the following steps.

Step 3. In this inner-loop, denoted by j , $\{\Delta\epsilon_a\}_i$, $\{\epsilon_a\}_{j-1}$, $\{\epsilon_d\}_{j-1}$, $\{\sigma_a\}_{j-1}$ and $\{\sigma_d\}_{j-1}$ are assumed to remain unchanged.

Step 3.1. Let $\{\bar{b}_e\}_1 = \{b_e\}_i$ and $\{\bar{\epsilon}_t\}_1 = \{\epsilon_t\}_i$ for the first cycle ($j=1$)

Step 3.2. For the plate effective width $\{\bar{b}_e\}_{j-1}$ and the total strain distribution

$\{\bar{\epsilon}_t\}_{j-1}$, obtain $\{\bar{z}_o\}_{j-1}$ and $\{I\}_{j-1}$ by computing the following two steps.

Step 3.3. Calculate the effective thickness of a zone. If the value of $\{\epsilon_i\}_j$ is less than ϵ_y , the full thickness t is used for the zone. In the strain-hardening regions where ϵ_i is greater than ϵ_s , the reduction in stiffness is simulated by a reduced thickness $t \frac{E_t}{E}$ based on the tangent modulus ratio. In the yield regions where ϵ_i is between ϵ_y and ϵ_s , tangent modulus theory says that E_t is zero. However, experimental evidence indicates that zero values are too low for the tangent moduli for the yield regions, and Trahair [T1] proposed that the tangent moduli for the yielded regions should conservatively be taken as equal to the strain-hardening values.

Step 3.4. To determine the updated neutral axes $\{y_o\}_i$ and $\{z_o\}_i$, the strain assumptions in Figures 2.1-4 and 2.1-5 are again used. However, the thickness of the components (flange, web and plate) is no longer a constant. The net axial force due to vertical and sideways bending F_{x1} and F_{x2} in the cross-section corresponding to $\{y_o\}_i$ and $\{z_o\}_i$ may then be found by numerical integration over the cross-sectional area A as

$$F_{x1}(z_o) = \int_A \sigma dA$$

$$F_{x2}(y_o) = \int_A \sigma dA$$

respectively, where the effective thickness of each zone is decided in step 3.3. Since the net force should be zero ($F_{x1}=0$ and $F_{x2}=0$), the equations are solved to give the value $\{y_o\}_i$ and $\{z_o\}_i$ by the bisection method. The positions $\{y_o\}_i$ and $\{z_o\}_i$ are shown in Fig. 5.7-2. and Fig. 5.7-3, respectively.

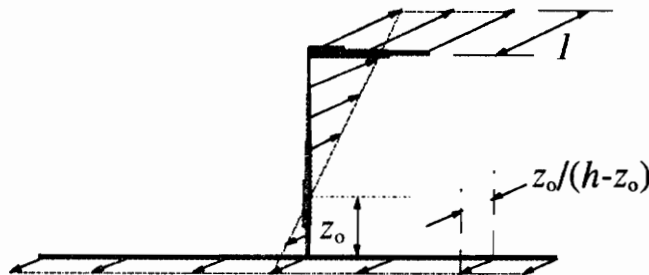


Figure 5.7-2 Neutral axis of vertical bending of transformed panel cross-section

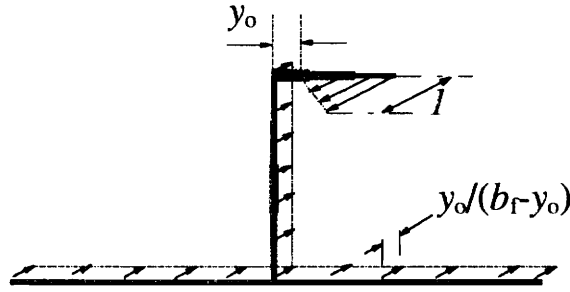


Figure 5.7-3 Neutral axis of sideways bending of transformed panel cross-section

Step 3.5. Obtain the new strain $\{\epsilon_m\}_j$, $\{\epsilon_q\}_j$ and $\{\epsilon_d\}_j$. Compute the new generalized stress and the new total strain $\{\bar{\epsilon}_t\}_j$. Then compute the new plate effective width $\{\bar{b}_e\}_j$. Repeat steps 3.2 to 3.5 until $\{\bar{b}_e\}_j$ converges.

Step 3.6. Let $\{b_e\}_i = \{\bar{b}_e\}_j$ and $\{\epsilon_t\}_i = \{\bar{\epsilon}_t\}_j$ etc. for the last cycle ($j=n$)

Step 3.7. The plate effective width b_e is taken as an average width at the 10 Gauss point sections.

Step 4. Compute the cross-sectional properties $I^T(x)$, $I_z^T(x)$, $I_{zy}^T(x)$, and $J^T(x)$, etc., the geometric stiffness properties $\alpha_{v_T v_T}^T(x)$, $\alpha_{w w}^T(x)$, $\alpha_{\phi_T \phi_T}^T(x)$, etc., and the plate rotational spring stiffness K_{ϕ_b} . This is done by numerical integration over the cross-section and then lengthwise to calculate the linear stiffness matrix $[K_L]$ and geometric stiffness matrix $[\bar{K}_G]$. Note that since the properties $I^T(x)$, $I_z^T(x)$, $I_{zy}^T(x)$, $J^T(x)$, $\alpha_{v_T v_T}^T(x)$, $\alpha_{\phi_T \phi_T}^T(x)$ and $\alpha_{w w}^T(x)$ etc. are functions of x , it is necessary to do numerical integration along the lengthwise direction.

Step 5. The equation $[K_L]\{\delta\} + [\bar{K}_G]\{\delta\} = -[\bar{K}_G]\{\delta_0\}$ is solved and the determinant $[[K_L] + [\bar{K}_G]]$ is tested at each value of $\{\epsilon_a\}_i$. If the determinant changes sign, the method of bisection is used to converge on the critical uniform strain ϵ_a , and then the average critical stress is given by

$$\sigma_{cr} = \frac{\sum_{k=1}^{N_f} \{\sigma_p\}_f^k A_f^k + \sum_{k=1}^{N_w} \{\sigma_p\}_w^k A_w^k + \sum_{k=1}^{N_p} \{\sigma_p\}_p^k A_p^k}{A}$$

where the applied uniform stress $\{\sigma_p\}$ is the stress stored in each zone at the last cycle. The ultimate applied load is then given by

$$\sigma_{ult} = \frac{A_s + b_e t_p}{A_s + b t_p} \sigma_{cr}$$

Repeat step 3 through step 5 until the solution converges at this cycle.

Step 6. Increment $\{\Delta\epsilon_p\}_i$ for the next cycle, and begin again at step 2. Since the matrices $[K_L]$ and $[\bar{K}_G]$ are only of order four, the solution process is rapid.

Algorithm B. Inelastic buckling under pure bending

When the live load is a pair of end moments, then the inelastic buckling algorithm is as follows:

Step 1. For the first outer-loop computational cycle ($i=1$), a small incremental curvature $\{\Delta\rho\}_i$ is applied to the panel, and in this case the total strain ϵ_i of each zone can be assumed to be in the elastic range. Therefore, y_0 and z_0 can be obtained by Eqs. (A-2) and (A-3).

Step 2. The constant curvature $\{\rho\}_i$ of the current cycle is given by

$$\begin{aligned} \{\rho\}_i &= \{\rho\}_{i-1} + \{\Delta\rho\}_i \\ \{\epsilon_m\}_i &= (\{z_o\}_i - z) \{\rho\}_i \\ \{\Delta\epsilon_m\}_i &= \{\epsilon_m\}_i - \{\epsilon_m\}_{i-1} \\ \{\Delta\epsilon_d\}_i &= \{\epsilon_d\}_i - \{\epsilon_d\}_{i-1} \end{aligned}$$

where $\{\epsilon_m\}_{i-1}$ is the strain due to the end moments in the previous cycle and is stored for every cycle; $\{\epsilon_d\}_{i-1}$ is the strain due to structural deformation in the previous cycle and is stored for every cycle. The total strain $\{\epsilon_i\}_i$ is calculated from the current end moment strain $\{\epsilon_m\}_i$, the uniform axial strain ϵ_a (dead load), the uniform lateral pressure strain ϵ_q (dead load) and the residual strain ϵ_r by

$$\{\epsilon_t\}_i = \{\epsilon_m\}_i + \{\epsilon_d\}_i + \epsilon_a + \epsilon_q + \epsilon_r$$

Assuming that elastic unloading will take place in the zone if the generalized residual stress is released, the stress due to the generalized residual strain can then be written as

$$\bar{\sigma}_r(x, y, z) = E\bar{\epsilon}_r$$

where

$$\bar{\epsilon}_r = \epsilon_a + \epsilon_q + \epsilon_r$$

The applied stresses in the current cycle are obtained from

$$\{\sigma_m\}_i = \{\sigma_m\}_{i-1} + E_t\{\Delta\epsilon_m\}_i$$

$$\{\sigma_d\}_i = \{\sigma_d\}_{i-1} + E_t\{\Delta\epsilon_d\}_i$$

$$\{\sigma\}_i = \{\sigma_m\}_i + \{\sigma_d\}_i + \sigma_r$$

where $\{\sigma_m\}_{i-1}$ is the stress due to the end moments in the previous cycle and is stored for every cycle, and $\{\sigma_d\}_{i-1}$ is the stress due to structural deformation in the previous cycle and is stored for every cycle. Again, ten Gauss points along the length are used as discussed in algorithm A.

Step 3. Do step 4 of algorithm A.

Step 4. The equation $[K_L]\{\delta\} + [\bar{K}_G]\{\delta\} = -[\bar{K}_G]\{\delta_0\}$ is solved and the determinant $[[K_L] + [\bar{K}_G]]$ is tested at each value of $\{\rho\}_i$. If the determinant changes sign, the method of bisection is used to converge on the critical curvature ρ , then the critical moment is obtained from

$$M_{cr} = \int_A z \sigma_m dA = \sum_{k=1}^{N_f} z_k \{\sigma_m\}_f^k A_f^k + \sum_{k=1}^{N_w} z_k \{\sigma_m\}_w^k A_w^k + \sum_{k=1}^{N_p} z_k \{\sigma_m\}_p^k A_p^k$$

where the stress $\{\sigma_m\}$ due to end moments is the stress stored in each zone at the last cycle.

Step 5. Increment $\{\Delta\rho\}_i$ for the next cycle, and begin again at step 2.

Algorithm C. Inelastic buckling under lateral uniform pressure

When the panel is primarily subjected to lateral uniform pressure (live load), the inelastic buckling algorithm is basically the same as algorithm B. However, instead of assuming the curvature ρ to be constant as in the pure bending case above, the curvature varies lengthwise. Since $M(x) = \frac{lx - x^2}{2}q$, it is logical to assume the curvature $\rho \propto \frac{lx - x^2}{2}$.

Thus let

$$\rho(x) = \rho_0 \frac{lx - x^2}{2} \frac{1}{l^2}$$

where ρ_0 is a constant and the factor $\frac{1}{l^2}$ has the purpose of non-dimensionalizing $\frac{lx - x^2}{2}$.

Ten Gauss points are used along the length. For a given ρ_0 , the curvature $\rho(x)$ will be calculated at each Gauss point.

Step 1. Determine the effective breadth using Eq. (5.4-3).

Step 2. For a constant parameter $\{\Delta\rho_0\}_i$ of the current cycle, do step 1 through step 4 of algorithm B at each Gauss point. Note that the generalized residual strain is now $\bar{\epsilon}_r = \epsilon_a + \epsilon_m + \epsilon_r$

Step 3. The equation $[K_L]\{\delta\} + [\bar{K}_G]\{\delta\} = -[\bar{K}_G]\{\delta_0\}$ is solved and the determinant $[[K_L] + [\bar{K}_G]]$ is tested at each value of $\{\rho_0\}_i$. If the determinant changes sign, the method of bisection is used to converge on the critical curvature ρ_0 . The critical moment at each Gauss point is then determined from

$$\bar{M}(x) = \int_A z \sigma_q(x) dA = \sum_{k=1}^{N_f} z_k \{\sigma_q\}_f^k A_f^k + \sum_{k=1}^{N_w} z_k \{\sigma_q\}_w^k A_w^k + \sum_{k=1}^{N_p} z_k \{\sigma_q\}_p^k A_p^k$$

Since the load on each stiffener is a uniform lateral load q , the equivalent bending moment is

$$M(x) = q \frac{lx - x^2}{2}$$

Therefore, one way to approximately make $M(x)$ to be equal to $\bar{M}(x)$ is to let

$$\int_0^l M(x) dx = \int_0^l \overline{M}(x) dx$$

which yields

$$q_{cr} = \frac{12}{l^3} \int_0^l \overline{M}(x) dx$$

where $\int_0^l \overline{M}(x) dx$ is obtained by numerical integration.

Step 4. Increment $\{\Delta p_o\}_i$ for the next cycle, and begin again at step 2.

5.8 Comparison with a non-linear incremental finite element method (ABAQUS)

The method developed in this chapter is essentially an inelastic buckling analysis. It is not intended to trace the load-displacement history. However, to show that the method agrees fully with non-linear incremental finite element methods, an example is solved by both the present study and by ABAQUS.

NCRE [S3] test model 1a is used for the comparison. For the ABAQUS model, the flange and web are modeled as 4 x 24 S4R5 elements, and the plate is modeled as 8 x 24 S4R5 elements. The dimensions of the panel are as follows: panel length 1219.2mm, web 138.2 mm x 7.1 mm; flange 76.2 mm x 14.2 mm; and plate 609.6 mm x 7.9 mm. The material is assumed to be elasto-plastic. Young's modulus is 2.066E5 N/mm², Poisson's ratio is 0.3 and yield stress is 243.4 N/mm². Mises yield criterion is used and no flow law is applied. The geometric initial imperfection is set such that the plate has three half sine waves and the stiffener has one half sine wave, and the magnitude is assumed to be 1.5mm. Residual stress is ignored. The panel is simply supported at the ends. Axial loads are applied at the two ends of the panel, and the RIKS algorithm is used to obtain nonlinear equilibrium solution. For the present inelastic model, since the geometric initial

imperfection of the plate is inherently incorporated into Eq. (5.4-1), only the imperfection of the stiffener needs to be considered. The load-displacement history by both the present method and by ABAQUS is shown in Fig. 5.8-1. The collapse load is reached very soon after the flange undergoes yield.

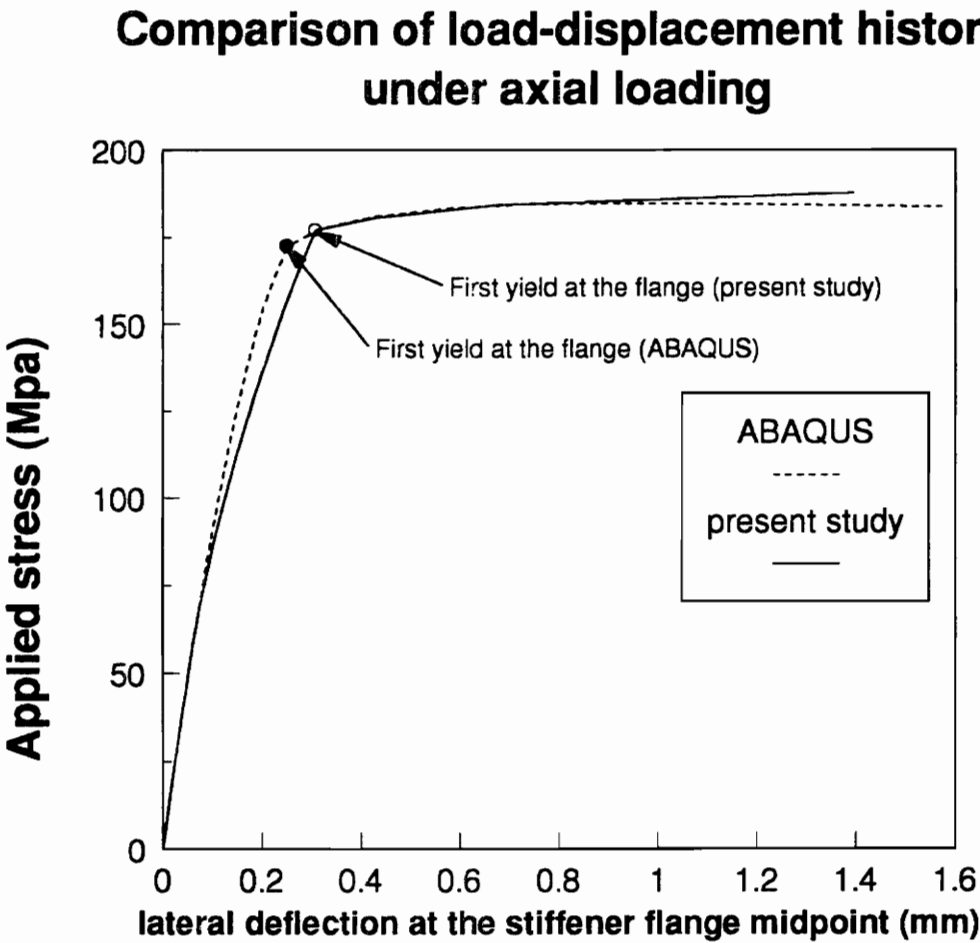


Fig. 5.8-1

6.0 Brief Discussion of a Plasticity Reduction Method

6.1 Structural tangent modulus method for a column

The tangent modulus method (Euler-Engesser theory) is widely used in inelastic column buckling. In ship structures, because of the existence of residual stress, the structural tangent modulus (average value over the entire cross-section) theory is used instead of the ordinary tangent modulus theory. For materials that have a definite yield plateau, one of the most useful representations of the structural tangent modulus is the Ostenfeld-Bleich parabola:

$$E_s = \frac{\sigma_{av}(\sigma_Y - \sigma_{av})}{\sigma_{spl}(\sigma_Y - \sigma_{spl})} E \quad \text{for } (\sigma_{spl} < \sigma_{av} < \sigma_Y)$$

$$E_s = E \quad \text{for } (\sigma_{spl} > \sigma_{av}) \quad (6.1-1)$$

$$E_s = 0 \quad \text{for } (\sigma_{av} > \sigma_Y)$$

where σ_Y is the yield stress, σ_{av} is the average stress of the cross-section, and σ_{spl} is the structural proportional limit. The structural proportional limit may be expressed as a fraction P_r of the yield stress:

$$\sigma_{spl} = P_r \sigma_Y$$

For a rolled, "universal column" (wide flange) section, a typical value of P_r is 0.5.

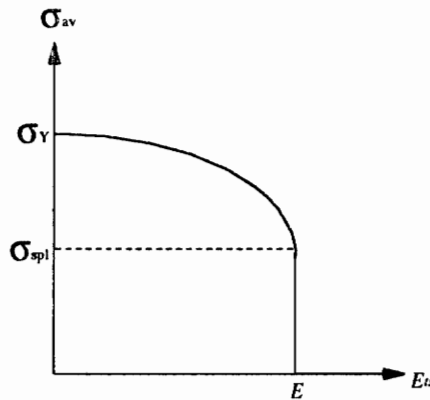


Fig. 6.1-1 Ostenfeld-Bleich curve

The inelastic buckling stress is given by

$$\sigma_{cr} = \frac{E_s}{E} \sigma_{cre} \quad (6.1-2)$$

where σ_{cre} is the elastic critical stress of column buckling.

To use the Ostenfeld-Bleich curve, one has to know the average stress, σ_{av} . Assuming $\sigma_{av} = \sigma_{cre}$ and substituting Eq. (6.1-1) into Eq.(6.1-2), the inelastic buckling stress can be written as

$$\sigma_{cr} = \begin{cases} \sigma_{cre} & \sigma_{cre} \leq P_r \sigma_Y \\ \sigma_Y \left(1 - P_r (1 - P_r) \frac{\sigma_Y}{\sigma_{cre}} \right) & \sigma_{cre} > P_r \sigma_Y \end{cases} \quad (6.1-3)$$

For a column the ultimate applied stress is the same as the inelastic buckling stress, i.e.,

$$\sigma_{ult} = \sigma_{cr} \quad (6.1-4)$$

6.2 Structural tangent modulus method for a panel stiffener

The tangent modulus and the structural tangent modulus methods are meant for inelastic column buckling. Tripping of panel stiffeners is a much more complex phenomenon than column buckling, and therefore neither the Euler-Engesser nor the structural tangent modulus method is strictly applicable. As indicated by Smith [S3]:

“As in the case of inelastic flexural buckling, use of tangent moduli in conjunction with elastic buckling analysis cannot provide a reliable estimate of collapse load unless ‘structural’ tangent moduli are established referring to the appropriate stiffener geometry and allowing correctly for initial deformation and residual stresses; such tangent moduli can only be obtained from rigorous analysis of inelastic tripping behaviour, or empirically from tripping test data.”

“Unless based directly on reliable test data or rigorous analysis such factors are no more than guesswork.”

However, because of the lack of other practical methods, the structural tangent modulus method has been used in some cases [F2, A1]. Therefore it is briefly summarized here, and its predictions will be compared with experimental results in Chapter 8.

For an ordinary beam-column σ_{av} is simply taken as the elastic Euler stress since the cross-section of a beam-column is fully effective and the stress is considered as uniformly distributed. For a stiffened panel, the plate flange of a panel stiffener is usually not fully effective because plate buckling results in a non-uniform stress distribution (Fig. 5.4-1). However, the rationale of effective width of buckled plating is such that the stress distribution of the stiffener and its associated effective plating can be regarded as uniform and equal to the peak stress σ_e (see Fig. 5.4-1). Since the Ostenfeld-Bleich curve is obtained from column tests, and not from panel tests, it is important to use the average stress in the stiffener, which is σ_e . It would be incorrect to use the panel overall average stress, which is given as σ_m in Eq. (5.4-2). Thus the inelastic buckling stress is given by

$$\sigma_{cr} = \begin{cases} (\sigma_e)_{cr} & (\sigma_e)_{cr} \leq P_r \sigma_Y \\ \sigma_Y \left(1 - P_r (1 - P_r) \frac{\sigma_Y}{(\sigma_e)_{cr}} \right) & (\sigma_e)_{cr} > P_r \sigma_Y \end{cases} \quad (6.2-1)$$

where $(\sigma_e)_{cr}$ is the critical elastic peak stress of the stiffened panel.

To obtain the ultimate applied stress of the panel, the overall average stress of the full panel cross section must be considered. The ultimate applied stress is then given by

$$\sigma_{ult} = \frac{A_s + b_e t_p}{A_s + b t_p} \sigma_{cr} \quad (6.2-2)$$

where b_e is given by

$$\frac{b_e}{b} = \frac{2}{\beta} - \frac{1}{\beta^2}$$

and β is the plate slenderness ratio

$$\beta = \frac{b}{t} \sqrt{\frac{\sigma_Y}{E}}$$

The above scheme is suggested by Faulkner [F2] and Adamchak [A1].

7.0 Comparisons with Experimental Data

Experimental data on tripping of panel stiffeners are very limited. The Naval Construction Research Establishment (NCRE) grillage tests conducted by Smith [S3] provide data on two full-scale steel grillages whose ultimate failure was ascertained to have been caused primarily by longitudinal stiffener tripping. Some further tests done at the same establishment (but renamed as the Admiralty Marine Technology Establishment, AMTE) by Smith et al. [S4, S5] provide data on four small scale panels under longitudinal compression. Also, in a series of panel tests directed by Faulkner [F3], two of the panels were reported as failing by tripping.

7.1 Laboratory tests

(a) NCRE Tests

The test grillages 1a and 1b were each fabricated as a five bay grillage; i.e., five interframe longitudinal stiffened panels. The pertinent scantlings (mean values) of the two grillages are given in Table 7.1. A summary of initial deformation is given in Table 7.2. The weld-induced residual strains were not measured for grillages 1a and 1b.

Table 7.1 NCRE Panel Data

Grillage	1a (Tee)	1b (Tee)
panel length l	48 in.	48 in.
plate width b	24 in.	24 in.
plate thickness t_p	0.315 in.	0.310 in.
web height h	6.05 in.	6.0 in. *
web thickness t_w	0.284 in.	0.28 in. *
flange thickness t_f	0.56 in. *	0.56 in. *
flange width b_f	3.11 in.	3.0 in. *
σ_s (mean yield stress of stiffener)	37408 psi	36960 psi
σ_p (mean yield stress of plate)	36736 psi	37184 psi

* Nominal values

The two grillages were nominally identical, the only difference being the presence of a 15 psi lateral loading on grillage 1b.

Table 7.2 NCRE Panel Initial Deformations

Central Interframe Deflection of Longitudinal Stiffeners (as a fraction of frame spacing)									
Gillage #	Vertical Deflection (Upward)			Vertical Deflection (Downward)			Horizontal Deflection of Stiffener Flange		
	Max	Mean	Cov	Max	Mean	Cov	Max	Mean	Cov
1a	0.0015	0.0007	0.77	0.0015	0.0013	0.12	-	-	-
1b	0.0015	0.0011	0.33	-	-	-	0.0010	0.0004	0.76

(b) AMTE Tests

Among the four pairs of small-scale panels, panels 1D, 2D and 3D are reinforced by bulb-flat stiffeners, tee stiffeners and angle stiffeners, respectively. Tee and angle stiffeners were fabricated by welding the flanges to the webs. Yield strengths of plating and stiffeners were evaluated by tensile tests on coupons cut from the plate and bar material used for panel construction. Compressive yield strengths were assumed to be 5% higher than the measured tensile values. Panel scantlings, load eccentricity and assumed yield stress are given in Table 7.3, where the bulb-flat stiffener dimensions are obtained from the equivalent cross section. Weld-induced residual stresses and initial distortions are given in Table 7.4. Young's modulus is assumed to be 2.07E5 N/mm².

Table 7.3 AMTE Panel Data

	1D (Bulb-flat)	2D (Tee)	3D (Angle)
panel length l	238 mm	238 mm	238 mm
plate width b	97 mm	97 mm	97 mm
plate thickness t_p	1.7 mm	1.7 mm	1.7 mm
web height h	23.23 mm	25.95 mm	25.95 mm
web thickness t_w	1.2 mm	1.2 mm	1.2 mm
flange thickness t_f	3.711 mm	2.1 mm	2.1 mm
flange width b_f	5 mm	13.5 mm	13.5 mm
load eccentricity ecc	5.98mm	7.71mm	6.91mm
Assumed Compressive σ_{yp}	235 N/mm ²	235 N/mm ²	235 N/mm ²
Assumed Compressive σ_{yw}	250 N/mm ²	250 N/mm ²	250 N/mm ²
Assumed Compressive σ_{yf}	303 N/mm ²	244 N/mm ²	244 N/mm ²

Table 7.4 Deformations and Residual Stresses in AMTE Panels

Panel #	Amplitude of Initial Deformation (mm)						Residual Stress (N/mm ²)			
	Plating*		Stiffener Bending +		Stiffener Tripping*		Plating		Stiffener Outer-fibre	
	Mean	Cov	Mean	Cov	Mean	Cov	Mean	Cov	Mean	Cov
1D	0.10	0.36	0.12	0.18	0.12	0.58	-41.8	0.43	12.5	0.72
2D	0.07	0.34	0.13	0.67	0.17	0.46	-41.6	0.23	4.2	6.5
3D	0.14	0.19	0.30	0.29	0.45	0.46	-52.2	0.28	61.6	0.72

* Deformation is maximum value measured over short span $L/3$. (direction ignored)

+ Deformation measured over full span L . (positive towards stiffener outstand).

(c) Faulkner's Tests

These models were of single bay construction, simply supported on the loaded edges and free to deflect out-of plane on the unloaded edges. Among 13 models which were stiffened by tee stiffeners, two failed in tripping.

Table 7.5 Data for Faulkner's Test Panels

	Model #P3 (Angle)	Model #P7 (Angle)
panel length l	638 mm.	1275 mm.
plate width b	221 mm.	221 mm.
plate thickness t_p	2.54 mm.	2.54 mm.
web height h	54.1 mm.	54.1 mm.
web thickness t_w	4.90 mm.	4.90 mm.
flange thickness t_f	6.10 mm.	6.10 mm.
flange width b_f	12.7 mm.	12.7 mm.
load eccentricity ecc	17.02mm	17.02mm
E (Young's Modulus)	190000(N/mm ²)	190000(N/mm ²)
σ_{ys} (Yield Stress)	247(N/mm ²)	245(N/mm ²)
σ_{yp} (Yield Stress)	256(N/mm ²)	270(N/mm ²)

7.2 Comparison

Figures 7.2-1 and 7.2-2 compares the six experimental results with the values predicted by the procedure developed in Chapter 5, and with the predictions of the other two methods. For the panels with multiple bays (NCRE 1a and NCRE 1b), the effective line of the

applied load follows any shift of the neutral axis, and therefore, there is no extra end moment to apply to the panel. For the panels with a single bay, the effective line of the applied load remains unchanged, and consequently the extra end moments due to the load eccentricity would change as the neutral axis of the cross-section shifts. In welded panels of ship structures, the residual stress of the stiffeners is much less important than that of the plating and is ignored in the calculations. The effect of plate residual stress is incorporated in Eq. (5-4). The geometric eccentricity is taken as the maximum measured value.

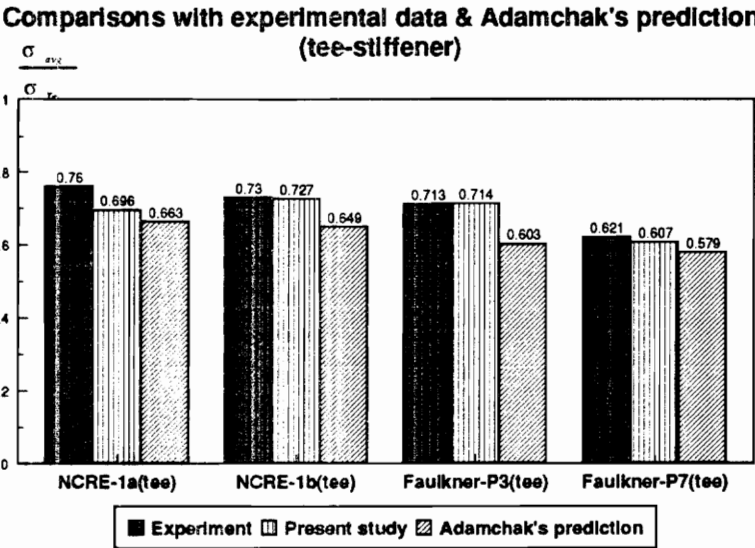


Fig. 7.2-1

Comparisons with experimental data & AMTE prediction

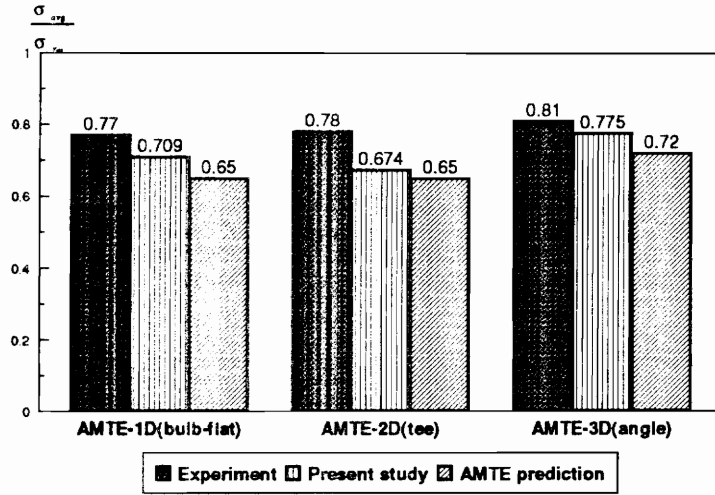


Fig. 7.2-2

In figures 7.2-1 and 7.2-2, the mean yield stress σ_{ym} is given by

$$\sigma_{ym} = \frac{\sigma_{yp}bt + \sigma_{ys}(b_f t_f + h t_w)}{bt + b_f t_f + h t_w}$$

Of the values shown for Adamchak's method, the values for NCRE-1a and NCRE-1b are taken from [A1]. The values for Faulkner-P3 and Faulkner-P7 are calculated by the flexible web formulations provided in [A1], where the plate rotational restraint is assumed to be zero since the plate buckling stress is much less than the yield stress, which would assure that the plate buckles before inelastic tripping. The load eccentricity effect which would decrease the prediction values is also ignored. Panels AMTE-1D and AMTE-3D are not relevant since Adamchak's method only deals with tee stiffeners. For panel AMTE-2D, the plate rotational restraint is taken as zero and the load eccentricity is also ignored.

8.0 Summary, Conclusions and Recommendations

8.1 Summary and Conclusions

A method is developed for analyzing the tripping behavior of flanged stiffeners subjected to axial force, end moment, lateral pressure and combined loads. The effects of cross-sectional distortion, postbuckling behavior of the plate (incorporated by considering the plate effective width), initial imperfections and plasticity are included.

The method uses the Rayleigh-Ritz approach. Based on an assumed strain distribution, a displacement field is obtained for the tripping model, and the total potential energy functional is then derived. The strain distribution assumptions coincide with van der Neut's [V1] assumption. However, unlike the somewhat obscure differential equation approach given by van der Neut, this study provides a simple, clear, energy approach. Also the resulting method is applicable in the inelastic range, which is not possible with van der Neut's approach.

Both the rigid web case and the flexible web case are studied. The effect of plate rotational restraint in the elastic range is also carefully studied. The method uses only four degrees of freedom and therefore the solution process is rapid. In order to verify the method in the elastic range, a number of sample stiffened panels are analyzed using ABAQUS; the results are in quite good agreement. An inelastic tripping model is then developed based on the established elastic model, using deformation theory. Results obtained using the inelastic tripping method are shown to be in good agreement with experimental results and more accurate than the predictions of other methods.

The following observations and conclusions can be drawn from the study:

- The rigid web assumption is good only if (a) the plate is quite thin and buckles well before tripping, such that it does not provide any rotational restraint, and (b) the ratio of web thickness to height is greater than 0.05.

- Euler buckling and tripping of a panel with asymmetric stiffeners (angles and bulb-flats) are closely coupled.
- Euler buckling and tripping of a panel with symmetric stiffeners (tees) are not coupled.
- For stiffener tripping the plate deformed shape is an S-shape for asymmetric stiffeners (angles and bulb-flats) and a U-shape for tee stiffeners (see Figs. 3.3-26 and 1.1-3).
- The tripping stress of angles is higher than tees when the panel slenderness ratio is small, and is lower than tees when the panel slenderness ratio is large.
- For an equivalent sectional area of the flange, the tripping stress of a bulb-flat stiffener is lower than that of both angle stiffeners and tee stiffeners, and the critical tripping mode is usually more than one half wave.

8.2 Recommendations for future work

In spite of the encouraging results obtained in this study, research in this field has by no means reached finality. Recommendations for further work on the basis of the completed study can be put into two categories: (1) further experimental work to provide a more complete assessment of the method; (2) further development of the method to consider more effects and to extend it to more general geometry and loading conditions.

1. Experimental work

- Since there are almost no test results available on the tripping of angle and bulb-flat stiffeners, such tests are sorely needed for obtaining a more complete validation of the method.
- In particular, tests are needed on multi-span stiffened panels with and without uniform lateral pressure.

2. Further development of the method

- Effect of shear stress
- Plate rotational restraint after plate buckling
- Destabilizing effect of plating
- Comparison with nonlinear incremental ABAQUS results, especially in plastic range.

- Different buckling modes in plate, web and flange
- Tripping of flat-bar stiffeners. The method developed in this study should be extended so as to handle tripping of these stiffeners.

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Appendix A. Derivation of stress distribution

A.1 Derivation of tripping stress distribution (van der Neut [V1]):

The normal stress originates from the vertical deflection w and lateral deflection v , and the normal stress is assumed to be linear in y and z . Therefore the longitudinal displacement u is also linear in y and z . Assume the longitudinal displacement at the root of the web ($y=z=0$) is (refer to Fig. 2.2-1)

$$u = u_1$$

where subscript ₁ refers to the plate. Then the longitudinal displacement of the web has

$$u = u_1 - w'z$$

where superscript ' denotes the derivative with respect to x . The longitudinal displacement of the flange is

$$u = u_1 - w'h - v'y$$

The condition that at buckling the normal force remains constant, therefore that the integral of the stress increments over the total cross section vanishes, yields

$$u_1' = \left[(1-q)hw + \frac{S}{A}v \right]''$$

where

$$A = A_f + A_w + A_l$$

is the total area per stiffener pitch. A_f , A_w and A_l are the cross section area of the flange, web and plate, respectively.

$$q = \frac{(A_l + \frac{1}{2}A_w)}{A}$$

$$S = \int_{A_f} y dA$$

w is the deflection due to bending.

Hence the normal stresses of flange, web and plate are

$$\begin{cases} \sigma_f = E \left[-qhw + \left(\frac{S}{A} - y \right) v \right]'' \\ \sigma_w = E \left[\{ (1-q)h - z \} w + \frac{S}{A} v \right]'' \\ \sigma_1 = E \left[(1-q)hw + \frac{S}{A} v \right]'' \end{cases}$$

A.2 Tripping stress distribution of this study

The assumption of linear tripping strain distribution due to bending of the present study is given by Equation (2.5-1):

$$\epsilon_f^I = (z_o - h)w_{,xx} + (y_o - y)v_{T,xx}$$

$$\epsilon_w^I = (z_o - z)w_{,xx} + y_o v_{T,xx}$$

$$\epsilon_p^I = z_o w_{,xx} + y_o v_{T,xx}$$

Then the stress distribution due to bending is

$$\begin{cases} \sigma_f = E \left[(z_o - h)w_{,xx} + (y_o - y)v_{T,xx} \right] \\ \sigma_w = E \left[(z_o - z)w_{,xx} + y_o v_{T,xx} \right] \\ \sigma_p = E \left[z_o w_{,xx} + y_o v_{T,xx} \right] \end{cases} \quad (\text{A-1})$$

where y_o and z_o are determined by equilibrium. By Fig 2.1-4 (B)

$$\sum F_{x1} = 0$$

This leads to

$$z_o = \frac{A_f + \frac{1}{2}A_w}{A} h = (1-q)h \quad (\text{A-2})$$

By Fig 2.1-4(C)

$$\sum F_{x2} = 0$$

This leads to

$$y_o = \frac{\frac{1}{2} b_f^2 t_f}{A} = \frac{S}{A} \quad (\text{A-3})$$

Substitute y_o and z_o into Equation (A-1); this yields

$$\begin{cases} \sigma_f = E \left[-q h w_{,xx} + \left(\frac{S}{A} - y \right) v_{T,xx} \right] \\ \sigma_w = E \left[((1-q)h - z) w_{,xx} + \frac{S}{A} v_{T,xx} \right] \\ \sigma_p = E \left[(1-q) h w_{,xx} + \frac{S}{A} v_{T,xx} \right] \end{cases}$$

which is identical to the equations given by van der Neut.

Appendix B. Derivation of total potential energy

The total potential energy of a stiffened panel subjected to edge loading is the sum of the strain energy U_T and the potential energy of the applied load, Ω :

$$\Pi_T = U_T + \Omega$$

The strain energy for a three-dimensional isotropic medium referred to arbitrary orthogonal coordinates may be written

$$U_T = \frac{1}{2} \int_v (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \sigma_z \epsilon_z + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz}) dv$$

After omitting γ_{xz} , γ_{yz} and σ_z in accordance with the basic approximations of thin-plate theory, the strain energy becomes

$$\begin{aligned} U_T &= \frac{1}{2} \int_v (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy} \gamma_{xy}) dv \\ &= U^f + U^w + U^p \end{aligned}$$

It is assumed that the strains and curvatures are everywhere much less than unity. The finite-strain expressions for the in-plane strain components of the mid-surface are given by [M1]

$$\begin{aligned} \epsilon_x^c &= u_{,x} + 0.5(u_{,x}^2 + v_{,x}^2 + w_{,x}^2) \\ \epsilon_y^c &= v_{,y} + 0.5(u_{,y}^2 + v_{,y}^2 + w_{,y}^2) \\ \gamma_{xy}^c &= u_{,y} + v_{,x} + u_{,x}u_{,y} + v_{,x}v_{,y} + w_{,x}w_{,y} \end{aligned} \quad (B-1)$$

In the plate theory of von Karman, only displacement gradients $w_{,x}$ and $w_{,y}$ are expected to achieve significantly large amplitudes, so of the nonlinear terms in Eq. (B-1), only $w_{,x}^2$, $w_{,y}^2$ and $w_{,x}w_{,y}$ are retained. However, in the present application, gradients of u and v , in addition to gradients of w , may become significantly large due to in-plane rotation, especially for the flange component. The terms $u_{,x}$ and $v_{,y}$ are of higher orders than the other terms, therefore the second order terms involving $u_{,x}$ and $v_{,y}$ are ignored. Hence

$$\begin{aligned}
\varepsilon_x^c &= u_{,x} + 0.5(v_{,x}^2 + w_{,x}^2) \\
\varepsilon_y^c &= v_{,y} + 0.5(u_{,y}^2 + w_{,y}^2) \\
\gamma_{xy}^c &= u_{,y} + v_{,x} + w_{,x}w_{,y}
\end{aligned} \tag{B-2}$$

The derivation can be found in many books (for example, [S6]). The strains in an arbitrary location of a plate component can be written as

$$\{\varepsilon\} = \begin{Bmatrix} u_{,x} - zw_{,xx} + 0.5(v_{,x}^2 + w_{,x}^2) \\ v_{,y} - zw_{,yy} + 0.5(u_{,y}^2 + w_{,y}^2) \\ u_{,y} + v_{,x} - 2zw_{,xy} + w_{,x}w_{,y} \end{Bmatrix} \tag{B-3}$$

For the flange:

$$\begin{cases} \varepsilon_{fx} = u_{f,x} + 0.5v_{f,x}^2 + 0.5w_{f,x}^2 - zw_{f,xx} \\ \varepsilon_{fy} = v_{f,y} + 0.5u_{f,y}^2 + 0.5w_{f,y}^2 - zw_{f,yy} \\ \gamma_{fxy} = u_{f,y} + v_{f,x} + w_{f,x}w_{f,y} - 2zw_{f,xy} \end{cases}$$

By Hooke's law

$$\begin{cases} \sigma_{fx} = \frac{E}{1-\nu^2} (\varepsilon_{fx} + \nu\varepsilon_{fy}) \\ \sigma_{fy} = \frac{E}{1-\nu^2} (\varepsilon_{fy} + \nu\varepsilon_{fx}) \\ \tau_{fxy} = \frac{E}{2(1+\nu)} \gamma_{fxy} \end{cases}$$

Since the flange acts as a beam, σ_y is assumed to be equal to zero, thus

$$\varepsilon_{fy} = -\nu\varepsilon_{fx}$$

and by eliminating ε_y , it is possible to write the following stress-strain relationship:

$$\begin{aligned}
\sigma_{fx} &= E\varepsilon_{fx} \\
\tau_{fxy} &= G\gamma_{fxy}
\end{aligned}$$

where for an isotropic material $G=E/[2(1+\nu)]$ is the shear modulus.

Substituting the above into U^f , noting that $\int zdz = 0$, and ignoring 4th order terms,

$$\begin{aligned}
U^f &= \frac{1}{2} \int_v (E \epsilon_x^2 + G \gamma_{xy}^2) dv \\
&= \frac{1}{2} \int_v E [u_{f,x} - z w_{f,xx} + 0.5 v_{f,x}^2 + 0.5 w_{f,x}^2]^2 dv \\
&\quad + \frac{1}{2} \int_v G [u_{f,y} + v_{f,x} - 2z w_{f,xy} + w_{f,x} w_{f,y}]^2 dv \\
&= \frac{1}{2} \int_v E (u_{f,x}^2 + z^2 w_{f,xx}^2) dv + \frac{1}{2} \int_v G ((u_{f,y} + v_{f,x})^2 + 4z^2 w_{f,xy}^2) dv \\
&\quad + \frac{1}{2} \int_v (\sigma_x (v_{f,x}^2 + w_{f,x}^2) + 2\tau_{xy} w_{f,x} w_{f,y}) dv
\end{aligned} \tag{B-4}$$

where

$$\begin{aligned}
\sigma_x &= E u_{f,x} \\
\tau_{xy} &= G (u_{f,y} + v_{f,x})
\end{aligned}$$

By the flange displacement assumption (Eq. 2.2-6)

$$\begin{cases}
u_{f,y} = -v_{T,x} = -v_{f,x} \\
u_{f,x} = (z_o - h) w_{,xx} + (y_o - y) v_{T,xx} + u_{,x} \\
w_{f,x} = w_{,x} - y \phi_{T,x} \\
w_{f,xx} = w_{,xx} - y \phi_{T,xx} \\
w_{f,xy} = -\phi_{T,x}
\end{cases}$$

Also note that $\int z^2 dz = t^3 / 12$, and $\tau_{xy} \approx 0$. Then Eq. (B-4) becomes

$$\begin{aligned}
U^f &= \frac{1}{2} E \int_{flange} [(z_o - h)^2 w_{,xx}^2 + (y_o - y)^2 v_{T,xx}^2 + 2(z_o - h)(y_o - y) w_{,xx} v_{T,xx}] dv \\
&\quad + \frac{1}{2} E \int_{flange} [u_{,x}^2 + 2(z_o - h) w_{,xx} u_{,x} + 2(y_o - y) v_{T,xx} u_{,x}] dv \\
&\quad + \frac{1}{2} E \int_l \frac{t_f^3}{12} (w_{,xx} - y \phi_{T,xx})^2 dx dy + \frac{1}{2} \int_l G \frac{t_f^3 b_f}{3} (\phi_{T,x})^2 dx \\
&\quad + \frac{1}{2} \int_l \sigma_x t_f (v_{T,x}^2 + (w_{,x} - y \phi_{T,x})^2) dx
\end{aligned}$$

Since the term

$$\frac{1}{2} E \int_I \frac{t_f^3}{12} (w_{,xx} - y \phi_{T,xx})^2 dx dy$$

is quite small compared to other terms, it can be ignored. Thus

$$\begin{aligned} U^f = & \frac{1}{2} E \int_{flange} [(z_o - h)^2 w_{,xx}^2 + (y_o - y)^2 v_{T,xx}^2 + 2(z_o - h)(y_o - y) w_{,xx} v_{T,xx}] dv \\ & + \frac{1}{2} G \int_I J_f \phi_{T,x}^2 dx + \frac{1}{2} E \int_{flange} [u_{,x}^2 + 2(z_o - h) w_{,xx} u_{,x} + 2(y_o - y) v_{T,xx} u_{,x}] dv \\ & + \frac{1}{2} \int_I \sigma_x t_f (v_{T,x}^2 + (w_{,x} - y \phi_{T,x})^2) dx \quad \checkmark \end{aligned}$$

where

$$J_f = \frac{t_f^3 b_f}{3}$$

For the web the strain is

$$\begin{cases} \epsilon_{wx} = u_{w,x} + 0.5 v_{w,x}^2 + 0.5 w_{w,x}^2 - y v_{w,xx} \\ \epsilon_{wz} = w_{w,z} + 0.5 u_{w,z}^2 + 0.5 v_{w,z}^2 - y v_{w,zz} \\ \gamma_{wxz} = u_{w,z} + w_{w,x} + v_{w,x} v_{w,z} - 2 y v_{w,xz} \end{cases}$$

and the stress may be obtained by

$$\begin{cases} \sigma_{wx} = \frac{E}{1-\nu^2} (\epsilon_{wx} + \nu \epsilon_{wz}) \\ \sigma_{wz} = \frac{E}{1-\nu^2} (\epsilon_{wz} + \nu \epsilon_{wx}) \\ \tau_{wxz} = \frac{E}{2(1+\nu)} \gamma_{wxz} \end{cases}$$

By the web displacement assumption (Eq. 2.2-4)

$$\begin{cases} u_{w,z} = -w_{,x} = -w_{w,x} \\ u_{w,x} = (z_o - z) w_{,xx} + y_o v_{T,xx} + u_{,x} \\ w_{w,z} = 0 \end{cases} \quad \text{web displacement assumption}$$

$$\begin{aligned} U^w = & \frac{1}{2} \int_v (\sigma_{wx} \epsilon_{wx} + \sigma_{wz} \epsilon_{wz} + \tau_{wxz} \gamma_{wxz}) dv \\ = & u_{web-inplane} + u_{web-out-of-plane} + u_{wg} \end{aligned}$$

where

$$\begin{aligned}
 u_{web-inplane} &= \frac{1}{2} \frac{Et_w}{1-\nu^2} \iint \left(u_{w,x}^2 + w_{w,z}^2 + 2\nu u_{w,x} w_{w,z} + \frac{1-\nu}{2} (u_{w,z} + w_{w,x})^2 \right) dx dz \\
 &= \frac{1}{2} \frac{Et_w}{1-\nu^2} \iint u_{w,x}^2 dx dz \\
 &\approx \frac{1}{2} Et_w \iint u_{w,x}^2 dx dz
 \end{aligned} \tag{B-5}$$

Note that Eq. (B-5) can also be derived in a same manner as for the flange; i.e., by assuming σ_y to be zero, the strain energy due to in-plane deformation becomes

$$u_{web-inplane} = \frac{1}{2} Et_w \iint [(z_o - z)w_{,xx} + y_o v_{T,xx} + u_{,x}]^2 dx dz$$

and the strain energy due to out-of-plane deformation is

$$\begin{aligned}
 u_{web-out-of-plane} &= \frac{1}{2} \frac{Et_w^3}{12(1-\nu^2)} \iint (v_{w,xx}^2 + v_{w,zz}^2 + 2\nu v_{w,xx} v_{w,zz} + 2(1-\nu)v_{w,xz}^2) dx dz \\
 &= \frac{1}{2} \iint \left(D_x v_{w,xx}^2 + D_z v_{w,zz}^2 + 2D_{xz} v_{w,xx} v_{w,zz} + G 4 \frac{t_w^3}{12} v_{w,xz}^2 \right) dx dz
 \end{aligned}$$

where

$$D_x = \frac{Et_w^3}{12(1-\nu^2)}$$

$$D_z = \frac{Et_w^3}{12(1-\nu^2)}$$

$$D_{xz} = \nu \frac{Et_w^3}{12(1-\nu^2)}$$

$$G = \frac{E}{2(1+\nu)}$$

and

$$u_{wg} = \frac{1}{2} \iint t_w (\sigma_{wx} v_{w,x}^2 + \sigma_{wz} v_{w,z}^2 + 2\tau_{wxz} v_{w,x} v_{w,z}) dx dz$$

For the plate, the strain is

$$\begin{cases} \epsilon_{px} = u_{p,x} + 0.5v_{p,x}^2 + 0.5w_{p,x}^2 - zw_{p,xx} \\ \epsilon_{py} = v_{p,y} + 0.5u_{p,y}^2 + 0.5w_{p,y}^2 - zw_{p,yy} \\ \gamma_{pxy} = u_{p,y} + v_{p,x} + w_{p,x}w_{p,y} - 2zw_{p,xy} \end{cases}$$

and the stress is

$$\begin{cases} \sigma_{px} = \frac{E}{1-\nu^2}(\epsilon_{px} + \nu\epsilon_{py}) = \frac{E}{1-\nu^2}\epsilon_{px} \\ \sigma_{py} = \frac{E}{1-\nu^2}(\epsilon_{py} + \nu\epsilon_{px}) = \frac{E}{1-\nu^2}\nu\epsilon_{px} \\ \tau_{fxy} = \frac{E}{2(1+\nu)}\gamma_{fxy} = 0 \end{cases}$$

By the plating displacement assumption (Eq. 2.2-5)

$$\begin{cases} u_{p,y} = 0 \\ v_{p,x} = 0 \\ u_{p,x} = z_o w_{,xx} + y_o v_{T,xx} + u_{,x} \\ v_{p,y} = 0 \end{cases}$$

The strain energy of the plating is

$$U^P = u_{plate-inplane} + \Pi_{plate-out-of-plane}$$

where the strain energy of the plating due to in-plane deformation is

$$\begin{aligned} u_{plate-inplane} &= \frac{1}{2} \int_V (\sigma_{px}\epsilon_{px} + \sigma_{py}\epsilon_{py} + \tau_{pxy}\gamma_{pxy}) dv \\ &= \frac{1}{2} \frac{E}{1-\nu^2} \int_{A_p} [z_o w_{,xx} + y_o v_{T,xx} + u_{,x}]^2 dxdA + \frac{1}{2} \int_{A_p} \sigma_x w_{,x}^2 dxdA \\ &\approx \frac{1}{2} E \int_{A_p} [z_o w_{,xx} + y_o v_{T,xx} + u_{,x}]^2 dxdA + \frac{1}{2} \int_{A_p} \sigma_x w_{,x}^2 dxdA \end{aligned}$$

The total strain energy can be obtained by summation of each component strain energy:

$$\begin{aligned}
U_T &= U^f + U^w + U^p \\
&= \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_l J_f \phi_{T,x}^2 dx + u_{web-out-of-plane} \\
&\quad + \frac{1}{2} E \int_0^l A u_{,x}^2 dx + \frac{1}{2} E \int_0^l 2 A_{uv} u_{,x} v_{T,xx} dx + \frac{1}{2} E \int_0^l 2 A_{uw} u_{,x} w_{,xx} dx + \Pi_{plate-out-of-plane} \\
&\quad + \frac{1}{2} \int_{A_f} \sigma \int_0^l \left(v_{T,x}^2 + [(w - y \phi_T)_{,x}]^2 \right) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA
\end{aligned}$$

where

$$u_{web-out-of-plane} = \frac{1}{2} \iint \left(D_x v_{w,xx}^2 + D_z v_{w,zz}^2 + 2 D_{xz} v_{w,xx} v_{w,zz} + G 4 \frac{t_w^3}{12} v_{w,x}^2 \right) dx dz$$

$$\left\{ \begin{aligned}
I &= \int_{A_w} (z_o - z)^2 dA + \int_{A_p} z_o^2 dA + \int_{A_f} (z_o - h)^2 dA \\
I_z &= \int_{A_w} y_o^2 dA + \int_{A_p} y_o^2 dA + \int_{A_f} (y - y_o)^2 dA \\
I_{zy} &= \int_{A_p} z_o y_o dA + \int_{A_w} (z_o - z) y_o dA + \int_{A_f} (z_o - h)(y_o - y) dA \\
A &= \int_{A_f} dA + \int_{A_w} dA + \int_{A_p} dA \\
A_{uv} &= \int_{A_f} (y_o - y) dA + \int_{A_w} y_o dA + \int_{A_p} y_o dA = y_o A - \frac{1}{2} b_f^2 t_f = 0 \\
A_{uw} &= \int_{A_f} (z_o - h) dA + \int_{A_w} (z_o - z) dA + \int_{A_p} z_o dA = z_o A - h b_f t_f - \frac{1}{2} h^2 t_w = 0
\end{aligned} \right. \quad (B-6)$$

$$\begin{aligned}
\Pi_{plate-out-of-plane} &= \frac{1}{2} \frac{E t_p^3}{12(1-\nu^2)} \iint (w_{pout,xx}^2 + w_{pout,yy}^2 + 2\nu w_{pout,xx} w_{pout,yy} + 2(1-\nu) w_{pout,xy}^2) dx dy \\
&\quad - \frac{1}{2} \iint t_p (\sigma_p w_{pout,x}^2) dx dy
\end{aligned}$$

The form of the expression for the potential energy of the edge loads depends on the nature of the loads. For uniformly distributed edge load p , the potential energy may be written

$$\begin{aligned}
\Omega &= p(u(l) - u(0)) \\
&= p \int_A u_{,x} dx
\end{aligned}$$

Since A_{uw} and A_{uw} are zero, the strain energy is

$$\begin{aligned}
 U_T &= U^f + U^w + U^p \\
 &= \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_l J_f \phi_{T,x}^2 dx + u_{web-out-of-plane} \\
 &\quad + \frac{1}{2} \int_{A_f} \sigma \int_0^l \left(v_{T,x}^2 + [(w - y \phi_T)_{,x}]^2 \right) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA \\
 &\quad + \Pi_{plate-out-of-plane} + \frac{1}{2} E \int_0^l A u_{,x}^2 dx
 \end{aligned}$$

and the total potential energy may be written as

$$\Pi_T = \Pi + \Pi_a$$

where

$$\begin{aligned}
 \Pi &= \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_l J_f \phi_{T,x}^2 dx + u_{web-out-of-plane} \\
 &\quad + \frac{1}{2} \int_{A_f} \sigma \int_0^l \left(v_{T,x}^2 + [(w - y \phi_T)_{,x}]^2 \right) dx dA + \frac{1}{2} \int_{A_p} \sigma \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \sigma \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA \\
 &\quad + \Pi_{plate-out-of-plane}
 \end{aligned}$$

and

$$\Pi_a = \frac{1}{2} E \int_0^l A u_{,x}^2 dx - P \int_0^l u_{,x} dx \quad \checkmark$$

Since axial displacement is uncoupled with bending displacements,

$$\sigma_x = \frac{P}{A}$$

For convenience, let the compressive stress be positive. Then the total potential energy due to bending may be written as

$$\Pi = U - V$$

where

$$\begin{aligned}
 U &= \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2 I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_l J_f \phi_{T,x}^2 dx \\
 &\quad + u_{web-out-of-plane} + \Pi_{plate-out-of-plane}
 \end{aligned}$$

$$V = \frac{1}{2} \int_{A_f} \int_0^l \left(v_{T,x}^2 + [(w - y\phi_T)_{,x}]^2 \right) dx dA + \frac{1}{2} \int_{A_p} \int_0^l w_{,x}^2 dx dA + \frac{1}{2} \int_{A_w} \int_0^l (w_{,x}^2 + v_{w,x}^2) dx dA$$

Rigid web case

If the cross sectional profile of the web at any transverse cross section is not allowed to curve, then the web is regarded as rigid and the rotation and sideways deflection are coupled (Fig. 3.1-1). For small displacements

$$v_w = z\theta$$

$$\phi_T = \phi_{Tc} = \theta$$

Then the web strain energy due to out-of-plane bending is

$$u_{web-out-of-plane} = \frac{1}{2} \iint \left(D_x v_{w,xx}^2 + D_z v_{w,zz}^2 + 2D_{xz} v_{w,xx} v_{w,zz} + G4 \frac{t_w^3}{12} v_{w,xz}^2 \right) dx dz$$

$$= \frac{1}{2} \iint \left(D_x z^2 \theta_{,xx}^2 + G4 \frac{t_w^3}{12} \theta_{,x}^2 \right) dx dz$$

The first term is very small compared to the second term; therefore

$$u_{web-out-of-plane} \approx \frac{1}{2} \int_l G4 \frac{t_w^3}{12} \theta_{,x}^2 dx = \frac{1}{2} GJ_w \int_l \theta_{,x}^2 dx$$

Thus for the rigid web case, the total potential energy is

$$\Pi = \frac{1}{2} EI \int_0^l w_{,xx}^2 dx + \frac{1}{2} EI_z h^2 \int_0^l \theta_{,xx}^2 dx + \frac{1}{2} 2EI_{zy} h \int_0^l w_{,xx} \theta_{,xx} dx + \frac{1}{2} G(J_f + J_w) \int_l \theta_{,x}^2 dx$$

$$- \frac{1}{2} (\alpha_{ww} \int_0^l w_{,x}^2 dx + \alpha_{\theta\theta} \int_0^l \theta_{,x}^2 dx + 2\alpha_{\theta w} \int_0^l \theta_{,x} w_{,x} dx)$$

where

$$\alpha_{ww} = \int_{A_f} \sigma_f dA + \int_{A_w} \sigma_w dA + \int_{A_p} \sigma_p dA$$

$$\alpha_{\theta\theta} = \int_{A_f} \sigma_f (y^2 + h^2) dA + \int_{A_w} \sigma_w z^2 dA$$

$$\alpha_{\theta w} = \int_{A_w} \sigma_w (-y) dA$$

Flexible web case

For the flexible web case, since

$$v_w = \varphi_B(x)f_1(z) + v_T(x)f_2(z) + \varphi_T(x)f_3(z)$$

then the web strain energy due to out-of-plane bending is a function of φ_B , φ_T and v_T , and is given by

$$u_{web-out-of-plane} = \frac{1}{2} \iint (D_{11} v_{w,xx}^2 + D_{22} v_{w,zz}^2 + 2D_{12} v_{w,xx} v_{w,zz} + D_{33} v_{w,xz}^2) dx dz$$

Thus for the flexible web case, the total potential energy is

$$\begin{aligned} \Pi = & \frac{1}{2} E \int_0^l I w_{,xx}^2 dx + \frac{1}{2} E \int_0^l I_z v_{T,xx}^2 dx + \frac{1}{2} E \int_0^l 2I_{zy} w_{,xx} v_{T,xx} dx + \frac{1}{2} G \int_0^l J_f \varphi_{T,x}^2 dx + u_{web-out-of-plane} \\ & - \frac{1}{2} (\alpha_{v_T v_T} \int_0^l v_{T,x}^2 dx + \alpha_{ww} \int_0^l w_{,x}^2 dx + \alpha_{\varphi_T \varphi_T} \int_0^l \varphi_{T,x}^2 dx + \alpha_{\varphi_B \varphi_B} \int_0^l \varphi_{B,x}^2 dx \\ & + 2\alpha_{w\varphi_T} \int_0^l w_{,x} \varphi_{T,x} dx + 2\alpha_{v_T \varphi_T} \int_0^l v_{T,x} \varphi_{T,x} dx + 2\alpha_{v_T \varphi_B} \int_0^l v_{T,x} \varphi_{B,x} dx + 2\alpha_{\varphi_T \varphi_B} \int_0^l \varphi_{T,x} \varphi_{B,x} dx) \end{aligned}$$

where

$$\alpha_{v_T v_T} = \int_{A_f} \sigma_f dA + \int_{A_w} \sigma_w f_2^2 dA$$

$$\alpha_{ww} = \int_{A_f} \sigma_f dA + \int_{A_w} \sigma_w dA + \int_{A_p} \sigma_p dA$$

$$\alpha_{\varphi_T \varphi_T} = \int_{A_f} \sigma_f y^2 dA + \int_{A_w} \sigma_w f_3^2 dA$$

$$\alpha_{\varphi_B \varphi_B} = \int_{A_w} \sigma_w f_1^2 dA$$

$$\alpha_{w\varphi_T} = \int_{A_f} \sigma_f (-y) dA$$

$$\alpha_{v_T \varphi_T} = \int_{A_w} \sigma_w f_2 f_3 dA$$

$$\alpha_{v_T \varphi_B} = \int_{A_w} \sigma_w f_1 f_2 dA$$

$$\alpha_{\varphi_T \varphi_B} = \int_{A_w} \sigma_w f_1 f_3 dA$$

For the elastic model, the above integral can be calculated by hand, or by mathematical software such as Maple or Mathematica. For the inelastic model, numerical integration should be used to obtain the cross sectional properties.

Vita

Ming Ma was born in Wuhan, P. R. China on May 8, 1963. He received the degree Bachelor of Engineering in Ship engineering in 1984, and the degree Master of Engineering in Ship Engineering in 1987 from Wuhan University of Water Transportation Engineering, Wuhan, P. R. China. In 1989, he went to the University of Southern Mississippi to study Mathematics and received the degree of Master of Science in Mathematics in 1991. He began his doctoral study in 1991 at Virginia Polytechnic Institute and State University.