

ANALYSIS AND CONTROL OF LANDSLIDES

by

Liang-Song Liu

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APPROVED

APPROVED:

Dr. Louis A. Pardue
Director of Graduate Studies

Prof. Russell R. Brinker
Head of The Department of
Civil Engineering

Prof. John W. Whittemore
Dean of Engineering and
Architecture

Prof. James H. Schaub
Major Professor

May 17, 1957

Blacksburg, Virginia

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b	= slope inclination
c	= cohesion per unit area
d	= D/H = depth factor
F	= factor of safety for cohesion
F_s	= factor of safety
H, h	= height or depth
h_{cr}	= critical height of an unsupported vertical cut, tensile stresses neglected
h_{cr}'	= critical height of an unsupported vertical cut, tensile stresses considered
H_w	= depth of water
H_w'	= elevation of ground water level above the toe
L	= length
M_o	= overturning moment
M_r	= resisting moment
M_1 and M_2	= parameters
N_s	= stability number in general
N_o	= stability number for simple slopes in purely cohesive soil
N_{cf}	= stability number for simple slopes when
$N_g, N_q, N_h, N_v,$ and N_m	= load ratios
q	= surcharge (load per unit area)
q_u	= unconfined compressive strength
R, r', r	= radius or moment arm
$R_c = \sqrt{c} H$	= critical radius
s	= shearing strength
W	= weight
$X_o = x_o H$ $Y_o = y_o H$	} coordinates of critical center

Θ = angle

ϕ = angle of internal friction

β = angle

γ = unit weight of soil

γ_e, γ_d = effective unit weight

γ' = submerged unit weight of soil

γ_{sat} = saturated unit weight of soil

γ_w = unit weight of water

$\lambda_{c\phi} = \frac{\gamma H \tan \phi}{c}$ = parameter

$M_q, M_t, M_w, M_d,$ and M_e = reduction factors

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ANALYSIS AND CONTROL OF LANDSLIDES

I. SYNOPSIS

Landslides are the natural phenomena of earth movement involving not only hillsides and mountainsides but also cuts and fills of roadways, channels and other engineering structures. The purpose of the thesis is to study classifications of landslides, the causes, the methods of analysis and the methods of control of the slide type of earth movement. In addition, there is a mathematical example to illustrate the various methods of analysis.

Many systems of classification and cause of landslides have been developed. Those of Baltzer, Heim, Ladd and Terzaghi are discussed in this paper. Actually, no slide will fit into any one class of these systems, or be due to any one cause, for its character is a compound one.

There have been various methods suggested for the analysis of landslides; all of them are based on simplifying assumptions and use trial and error methods or semi-mathematical methods to reach a satisfactory result. In this paper, the basic methods of analysis, as well as the most recent technique, have been explained briefly. Since most of the methods used require many complex steps to prove, the proofs are referred to by a footnote which indicates the source of the proof for those who are interested in further research.

II. DEFINITION OF LANDSLIDES

Landslides have been defined by Terzaghi ⁽¹⁾ as follows: "The term landslides refers to a rapid displacement of a mass of rock, residual soil, or settlement adjoining a slope in which the center of gravity of the moving mass advances in a downward and outward direction."

Ladd ⁽²⁾ has defined landslides as, "Landslides are superficial earth movements having essentially a horizontal component, usually involving slopes varying from very gentle to very steep, but rarely, involving vertical cliffs at one extreme, and level but unsupported ground at the other."

A landslide implies that the soil mass has lost its static stability of equilibrium and is seeking a more stable condition by means of movement. The loss of static stability may be caused by an increase of the load on the mass or a decrease of the strength of the soil. The change from equilibrium to a movement of the soil mass may require only a slight change in the balance of forces. The cause of these changes could include the addition of water to the soil mass or other disturbing factors, such as underground settlement, explosions, or earthquakes.

III. CAUSES OF LANDSLIDES

The causes of landslides can be divided into external and internal ones. External causes are those which produce an increase of shearing stress at unaltered shearing resistance of the material adjoining the slope. They include a steepening or heightening of the slope by erosion or man-made excavation. They also include the deposition of material along the upper edge of slopes, and earthquake shocks. If an external cause leads to a landslide, we can conclude that it increased the shearing stresses along the potential surface of sliding to the point of failure. The slide at Bekkelaget, Sweden, 1953 (3), was an example of a slide of this type. On the morning of the 7th of October, 1953, a slide occurred at Bekkelaget, Oslo. The railway and the road slid out for a distance of 100 meters together with the adjacent lower natural slope. The slide occurred in a normally consolidated quick clay, and the stability analysis has demonstrated that the shear strength was mobilized simultaneously along the whole sliding surface in spite of its length and compound form. The sliding surface passes through gravel, a dried crust and a very soft extra quick clay. The primary cause of the slide was the weight of the embankment for the road and railway. After the slide, pore-water pressure was measured on the site. In the slipplane the pore-water pressure corresponded to the total overburden pressure. Measurements taken outside the slide indicated a distribution of pore-water pressure

corresponding to a ground-water level just below the ground surface. Thus, it is concluded that the slide was not caused by an excess pressure in the pore-water.

Internal causes of slides are those which lead to a slide without any change in surface conditions and without the assistance of an earthquake shock. If a slope fails in spite of the absence of an external cause, we must assume that the shearing resistance of the material has decreased. The most common causes of such a decrease are an increase of pore-water pressure and the progressive decrease of the cohesion of the material within the slope. The slide at Jackfield, England, 1951 ⁽⁴⁾ was an example of an internally caused slide. The slide occurred during the winter of 1951-1952 on the right bank of the stream at Jackfield. The appearance of the slide suggested that it was a relatively shallow movement and that it was probably caused by the variation in shear stress and effective pressure associated with seasonal changes in water table.

Intermediate between the landslides due to external and internal causes are those due to rapid drawdown of the water level, to subsurface erosion, to underground settlement, and spontaneous liquefaction.

IV. CLASSIFICATION OF LANDSLIDES

There have been proposed many different systems of classification for landslides. Most classifications however, are based on the cause and effect of movement rather than the mechanics of the movement. One major primary classification treats landslides in consolidated materials and those in unconsolidated materials separately. Another classification divides the movement into those with a slip surface and those without a slip surface, the former being known as slides and the latter as flows. (Fig. 1) Some of the more well-known classifications of landslides are as follows:

1. Baltzer ⁽⁵⁾ , in a treatise entitled "Ueber Bergsturze in den Alpen", 1875, classified Swiss landslides as

- A. Rock Falls
- B. Earth Slips
- C. Mud Streams
- D. Mixed Falls

2. Heim ⁽⁶⁾ , 1882, suggested the following comprehensive classification:

Movement involving detritus

- A. Soil Slips
- B. Earth Slides or falls of greater magnitude than A.

Movement involving solid rock

- C. Rock Slips
- D. Rock Falls

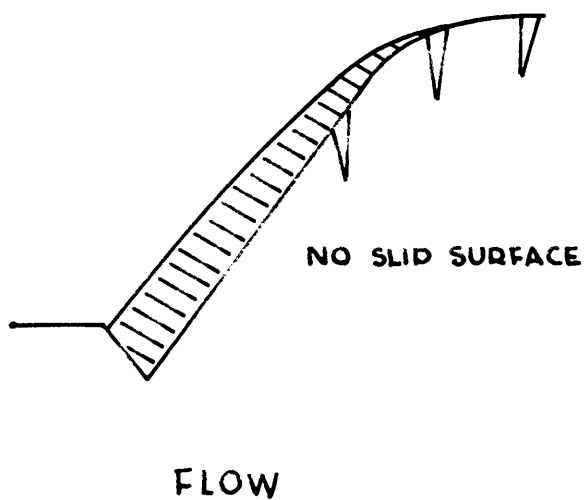
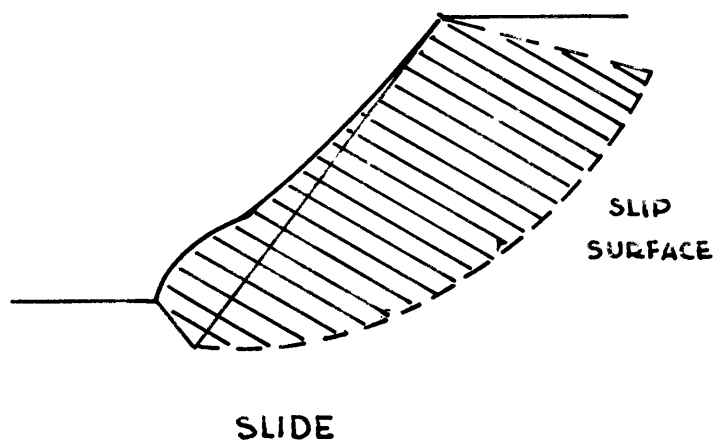


Fig.1. Differentiation between slide & flow

E. Compound Slides, with respect to character of movement and of materials

F. Unclassified and special cases

3. Dr. Karl Terzaghi's Classification, 1925 (7)

Dry movements - earth movements involving completely effective static friction

A. Creeps - continuous mass movements

B. Landslides - spontaneous mass movements

C. Settling Flows - flows resulting from a rapid change in porosity

Plastic movement - mass movement partly or entirely without static friction

D. Disruption Flows - flows resulting from disruption through swelling and dislocation

E. Flows - resulting from hydrostatic overload in the contained water called forth by overload

F. Squeezing out - reduction of resistance through a tremendous overload in hydrostatic pressure

4. Ladd's Classification (2)

A. Flows - This type of slide includes those which actually flow as semi-liquids, where clayey materials or volcanic ash or decomposition products become water saturated and there is freedom for movement

B. Slope Readjustments - Slides belonging to this class are geographically distributed over extensive areas of the

world and, in frequency, they are far more common than all other types. They have been important moulders of topographic form, moving successively down slopes to streams where their material is gradually removed by erosion.

C. Undermined Strata - This class excludes cases belonging clearly to subsidence and due to undermining by erosion where it leads solely to rock falls and talus accumulation.

D. Structural Slides - This type of slide includes, primarily, all of those whose movement is upon definite, pre-existing structural planes.

E. Clay ejection from ancient clay - filled caves opened by cut (rare).

For the convenience of analysis, the term slide will be defined as all landslides which involve unconsolidated material in which the movement is along a slip-surface, and the term flow will be defined as those movements which do not have a slip-surface. However, "creep" which was defined by Terzaghi (1), as a continuous downhill movement which proceeds at an average rate of less than one foot per decade, could be treated as a special case of a flow.

V. METHODS OF ANALYSIS

1. Analysis of the stability of unsupported vertical banks

(1) Plane surface of failure (Fig. 2)

$$W = \frac{1}{2} h \frac{h}{\tan \theta} \cdot \gamma = \frac{h^2 \gamma}{2 \tan \theta}$$

At failure $W \sin \theta = \frac{c \cdot h}{\sin \theta} + W \cos \theta \tan \phi$

$$\frac{h^2 \gamma}{2 \tan \theta} \sin \theta = \frac{c \cdot h}{\sin \theta} + \frac{h^2 \gamma}{2 \tan \theta} \cos \theta \tan \phi$$

$$\frac{h^2 \gamma}{2} \left(\frac{\sin \theta}{\tan \theta} - \frac{\cos \theta \tan \phi}{\tan \theta} \right) = \frac{c \cdot h}{\sin \theta}$$

$$\frac{h^2 \gamma}{2} (\cos \theta \sin \theta - \cos^2 \theta \tan \phi) = c \cdot h$$

$$h = \frac{2c}{\gamma} \left(\frac{1}{\cos \theta \sin \theta - \cos^2 \theta \tan \phi} \right)$$

for min. h $\cot 2\theta = -\tan \phi = \cot (90^\circ + \phi)$

$$\therefore \theta = 45^\circ + \frac{\phi}{2}$$

$$\sin \theta \cos \theta - \cos^2 \theta \tan \phi = \frac{1}{2 \tan \theta}$$

$$\therefore h = \frac{2c}{\gamma} \cdot 2 \tan \theta = \frac{4c}{\gamma} \tan \left(45^\circ + \frac{\phi}{2} \right)$$

for C = $\frac{\gamma u}{2 \tan (45^\circ + \frac{\phi}{2})}$ $\therefore h_{cr} = \frac{2 \gamma u}{\gamma} \quad \dots (1)$

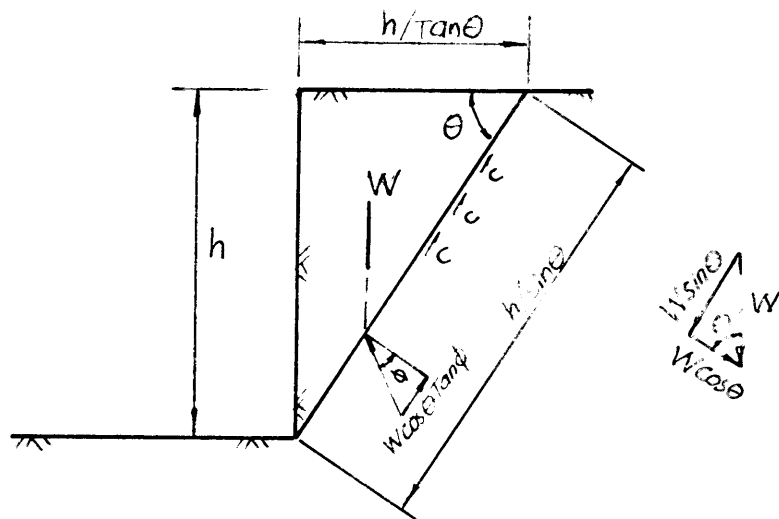


Fig.2. Critical height of vertical bank

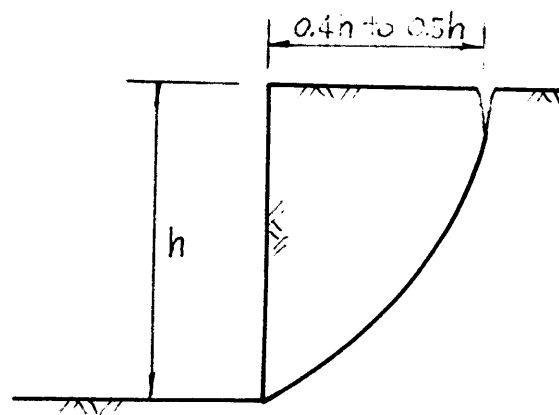


Fig.3.
Tensile cracks effect critical height of vertical bank

(2) Curve Surface of Failure - derived by Fellenius

$$h_{cr} = \frac{3.865}{\gamma} \quad (2)$$

(3) Tensile Crack and Their Effect (Fig. 3)

It has been observed during the study of many slope failures that vertical cracks appear near the top of the slope prior to failure. The cracks are noticeable from the top of the slope extending back to the upper end of the failure surface that later develops. It seems likely that these cracks are the result of tensile forces on the soil resulting from the force of gravity acting on the mass of the slope. Soil, having little if any tensile strength, readily develops failure planes or cracks under these forces.

The formation of tension cracks near the potential sliding surface destroys any shear strength of the soil in this region and thereby decreases the resisting force along the slide surface. In addition, the tension cracks may be the source of other forces that tend to create failure. Surface water entrapped in the cracks can exert an appreciable driving force on the soil mass. Expansion and contraction of cohesive soils within the cracked zone and the action of frost within the cracks can further weaken the soil and hasten a failure.

Terzaghi estimates the maximum depth to which tensile cracks may reach down from the surface to be one-half of the unsupported height

of the cut. Therefore the critical height of a bank unweakened by tensile cracks, will be reduced by such cracks to a value h'_{cr} , which can be determined in accordance with the conservative assumption of Terzaghi (8), from

$$h'_{cr} = h_{cr} - \frac{1}{2} h'_{cr}$$

$$h'_{cr} = \frac{2}{3} h_{cr}$$

$$\text{from (2)} \quad h'_{cr} = \frac{2}{3} \frac{3.865}{\gamma} = \frac{2.585}{\gamma} = \frac{1.2926}{\gamma} \quad (3)$$

2. Analysis of the stability of sloping face bank

(1) Plane surface of failure

As Fig. 4, if the (internal friction angle) was increased to without any failure, the height would be increased by semi-empirical (9)

$$t = \frac{4c(2\theta+0.2)\cos\theta}{\gamma\cos^2\phi}$$

$$\text{Since } h-d = b+t, \quad d = h\cos\theta\tan\phi, \quad b = \frac{4c}{\gamma} \left(\frac{1+\sin\phi}{\cos\phi} \right)$$

$$\therefore h(1-\cot\theta\tan\phi) = \frac{4c}{\gamma} \left(\frac{1+\sin\phi}{\cos\phi} \right) + \frac{4c}{\gamma} \cdot \frac{(2\theta+0.2)\cos\theta}{\cos^2\phi}$$

$$h = \frac{4c}{\gamma\cos\phi(1-\cot\theta\tan\phi)} \left[1+\sin\phi + \frac{\cos\theta(2\theta+0.2)}{\cos\phi} \right] \quad \dots (4)$$

(2) Circular arc surface of failure

Actually, the usual surface of failure is not a plane surface but a curved one. Various assumptions of the shape of this curve have been made for the purpose of analysis. The circular arc and the logarithmic spiral are the most common assumptions. The former is the most

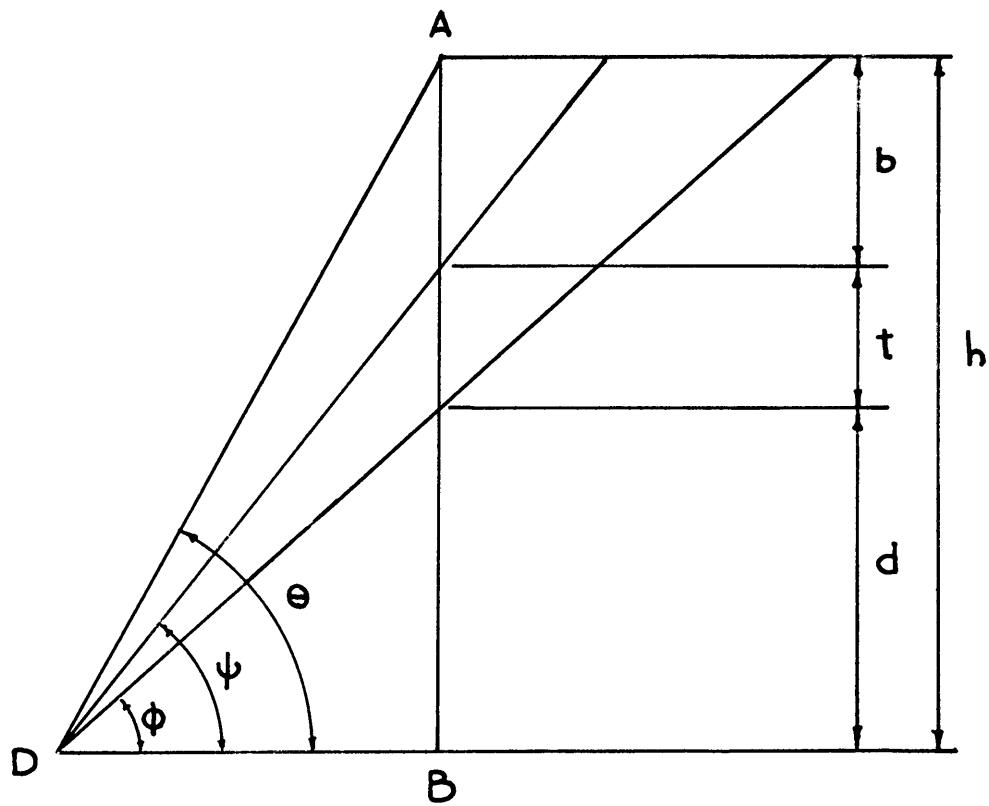


Fig. 4. Critical height of slope face ⁽⁹⁾
(After Hennes)

popular one, but though the latter has some theoretical advantages, it takes much more time to compute than the former. Taylor has shown that the results obtained by the logarithmic spiral differ only slightly from those obtained by use of the circular arc.

The analysis of an earth slope by the circular arc surface of failure was first proposed by K. E. Petterson in 1916 during a study of a quay failure in Gothenburg, Sweden (10). Since then many proposals based on this assumption have been developed, but all of them have utilized trial and error methods to locate the most critical center graphically and semi-graphically rather than a purely mathematical solution. The usual methods for the analysis of landslides on the assumption of a circular arc of failure are shown in the following outline.

i. Methods of slices and area moments

A. Using trial and computation, locate the most unfavorable surface and its center by means of the following steps.

B. Divide the cross-section above the slide surface into vertical slices. A width of slice of about $H/4$ to $H/2$ is generally satisfactory.

C. Compute the overturning moment of each slice due to its weight.

D. Compute the resisting moment due to the cohesion and friction forces along the sliding surface.

In illustration see Fig. 5

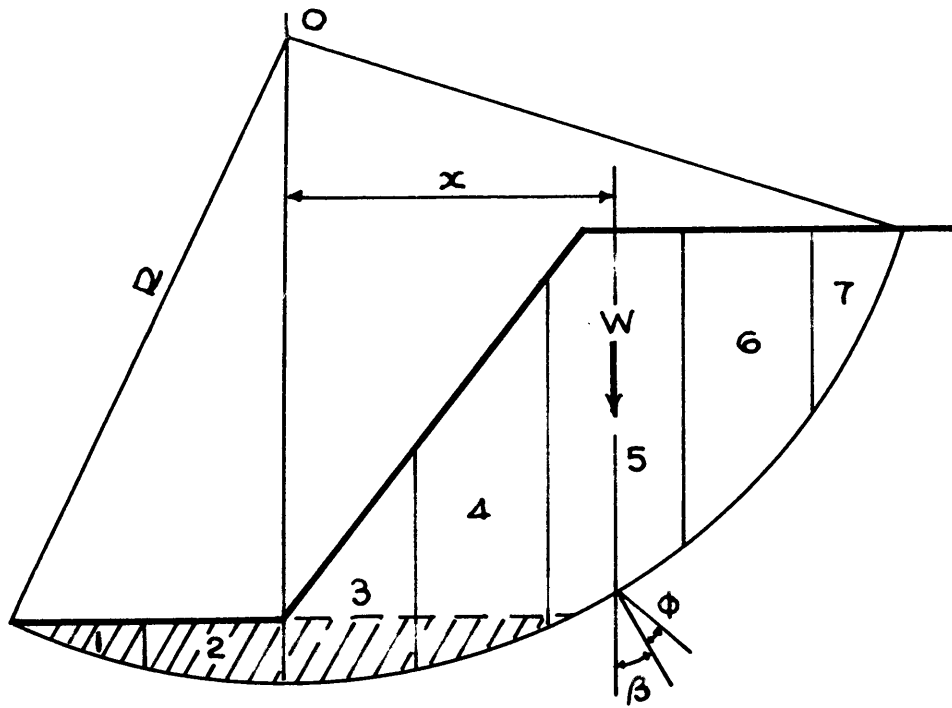


Fig 5. Method of slices

By the method of slices

$$M_o = \sum_3^7 Wx - \sum_1^2 Wx$$

$$M_r = R \sum_1^7 cL - R \sum_1^7 W \cos \beta \tan \phi$$

$$F_s = \frac{M_r}{M_o} \quad - \text{factor safety against sliding}$$

By the method of area moments

The area below ground surface (shaded portion) is omitted since the moment of this area about point O equals zero.

$$M_o = \sum_3^7 Wx$$

$$M_r = R \sum_1^7 cL - R W \cos \beta \tan \phi$$

$$F_s = \frac{M_r}{M_o}$$

The experience of numerous investigators has lead to the development of many useful hints to assist in the initial choice of a trial center. These guides are of great help in reducing the work of a trial and error solution. Failure surfaces have been classified for this and other purposes as deep seated and toe slides. The former consist of those failure surfaces that extend below and beyond the toe of the slope while the latter have failure surfaces passing through

the toe.

Fellenius and other Swedish investigators arrived at the conclusion that the most critical deep seated or base failure sliding surface in a homogenous clay with $\phi = 0$ had a center of rotation lying on the bisector of the slope. It was observed also that the most likely critical center was one which had an angle of approximately 153 degrees between the radii to the ends of the slide surface.

In the case of a toe circle failure the following table and sketch (Fig. 6) serve to indicate a method of selecting a starting point that will be at or near the critical center of rotation.

i°	B_1°	B_2°
60	29	40
45	28	37
33.8	26	35
26.6	25	35
18.4	25	35
11.3	25	37

In most cases it has been found that slope angles of greater than 53 degrees are most likely to fail by a toe slide while those with slope angles of less than 53 degrees are prone to fail by the deep seated type of failure arc. In all cases, the initial selection of the critical center of rotation selected by these methods should be verified by trial centers at points surrounding the initial choice.

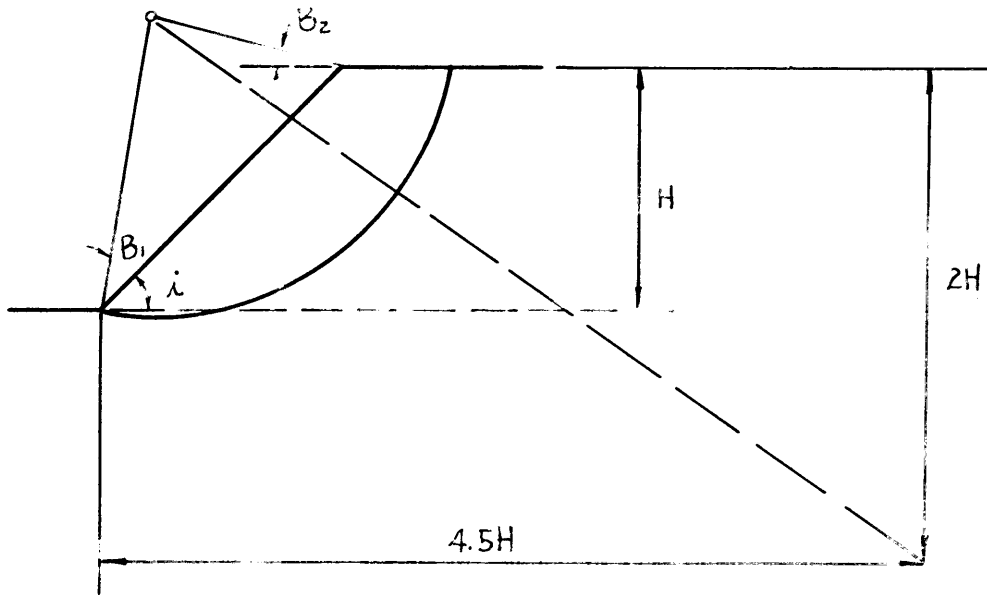


Fig. 6 Locating of critical center, $\Phi=0$

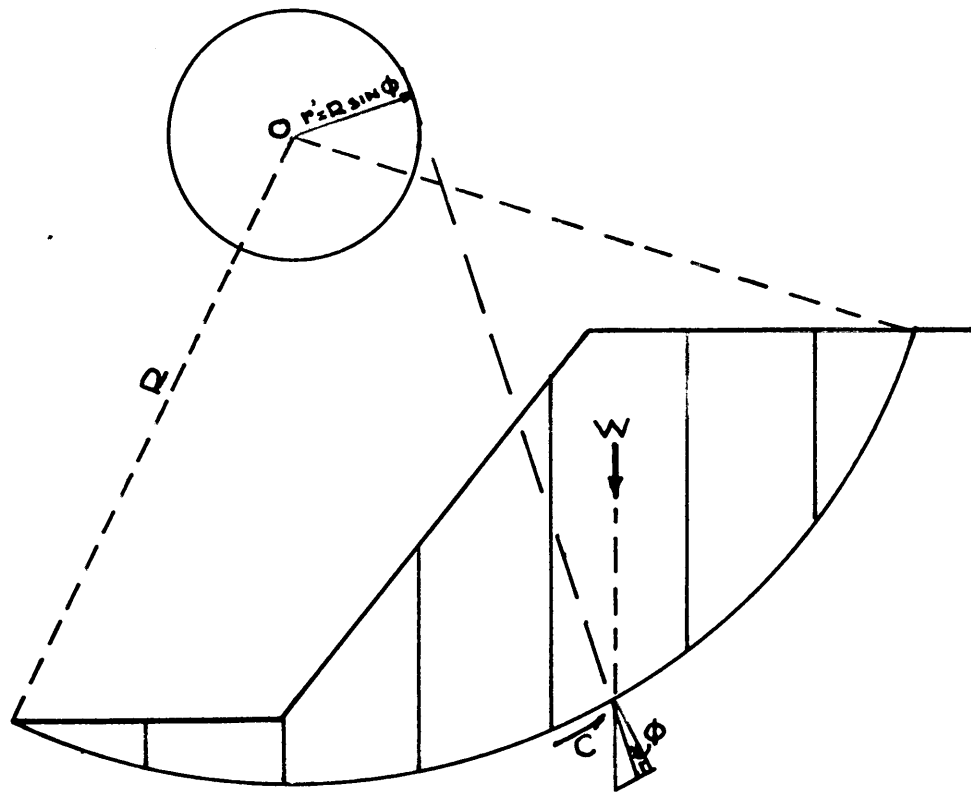


Fig. 7 Method of ϕ -circle

ii. Method of the ϕ -circle

As shown by Fig. 7, after a center of rotation has been selected, the mass above the rupture plane is divided into vertical slices with the forces on opposite sides of each slice assumed to be equal and opposite. A statically determinate problem results. Semi-graphical methods have been developed using a circle with a center at the assumed center of rotation and with a radius $r' = R \sin \phi$. The resultants of all slices, formed by the weight W and by the friction resistance along the corresponding segment of the failure surface will then be tangent to the ϕ - circle of radius r' . With the help of the ϕ - circle, the force polygon for the analysis of the safety against sliding can be drawn.

Taylor, (11) using the assumptions of a homogeneous soil, a constant angle of slope, a level top surface, the soil shearing strength as expressed by Coulomb's equation, and constant cohesion along the entire rupture surface, developed the semi-mathematical solution in two cases which is expressed in the following.

A. Failure circle passing through toe slope (Fig. 8)

$$\frac{C}{F \gamma H} = \frac{\frac{1}{2} \csc^2 \alpha (y \csc^2 \beta - \cot \beta) + \cot \alpha - \cot \beta}{Z \cot \alpha + \cot \beta + Z} \quad - - (5)$$

F = Factor of safety for cohesion

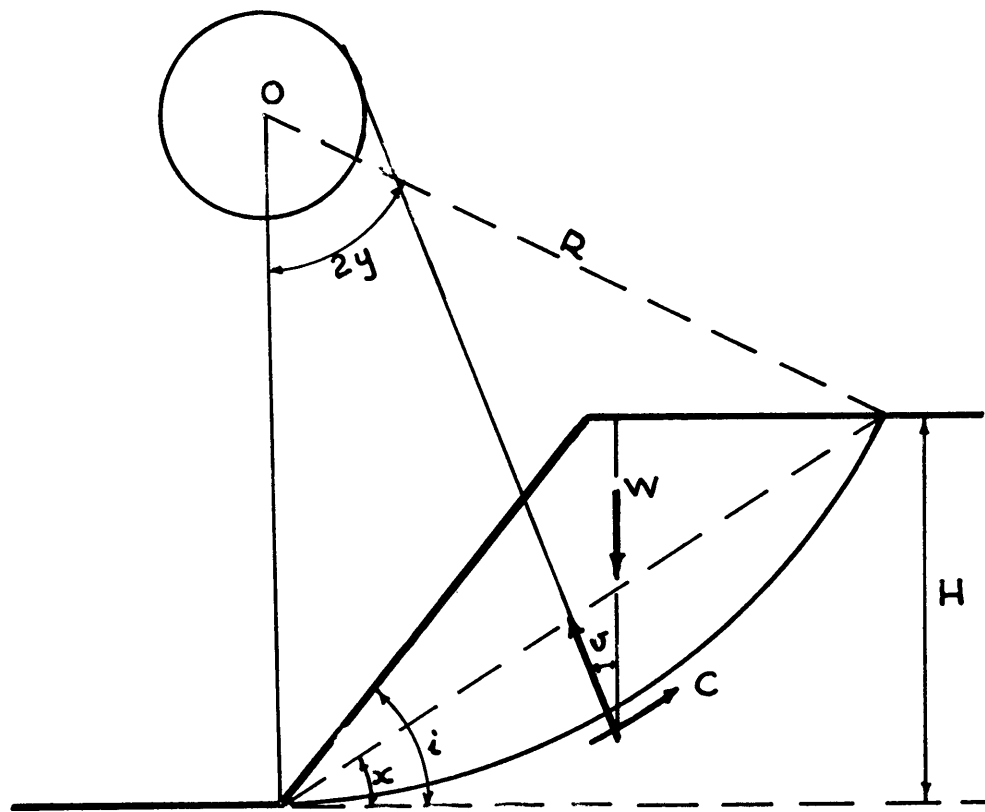


Fig 8 Failure circle passing through toe of slope⁽¹¹⁾
(After Taylor)

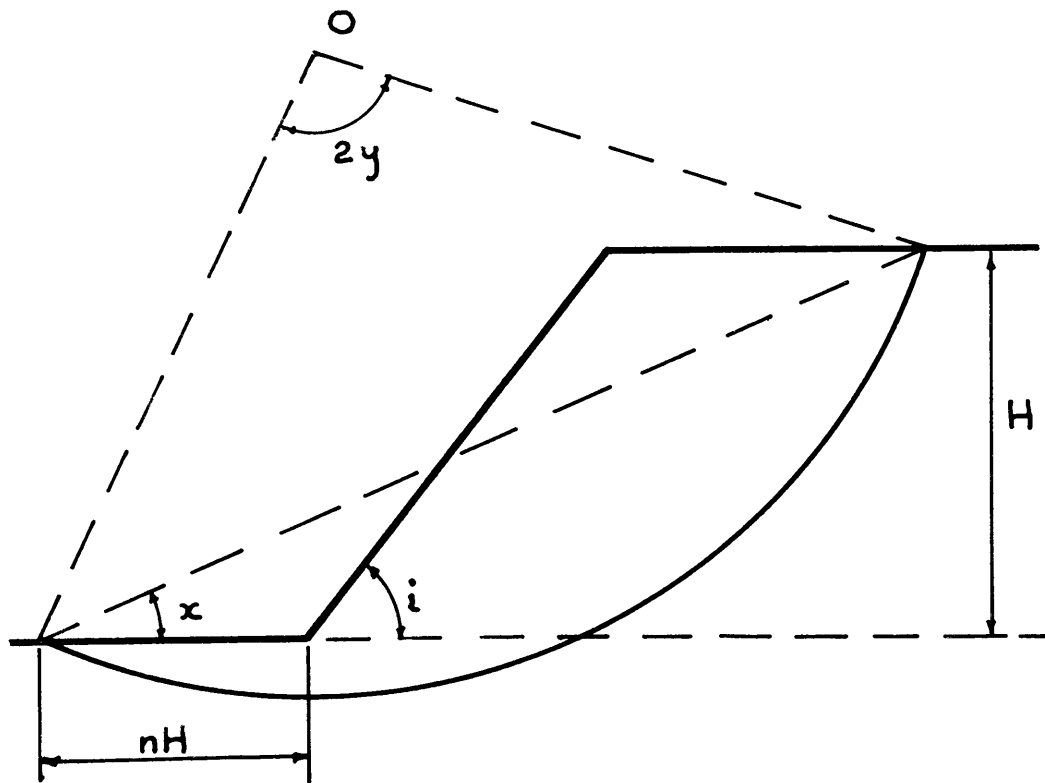


Fig 9. Failure of circle passing below toe of slope (11)
(After Taylor)

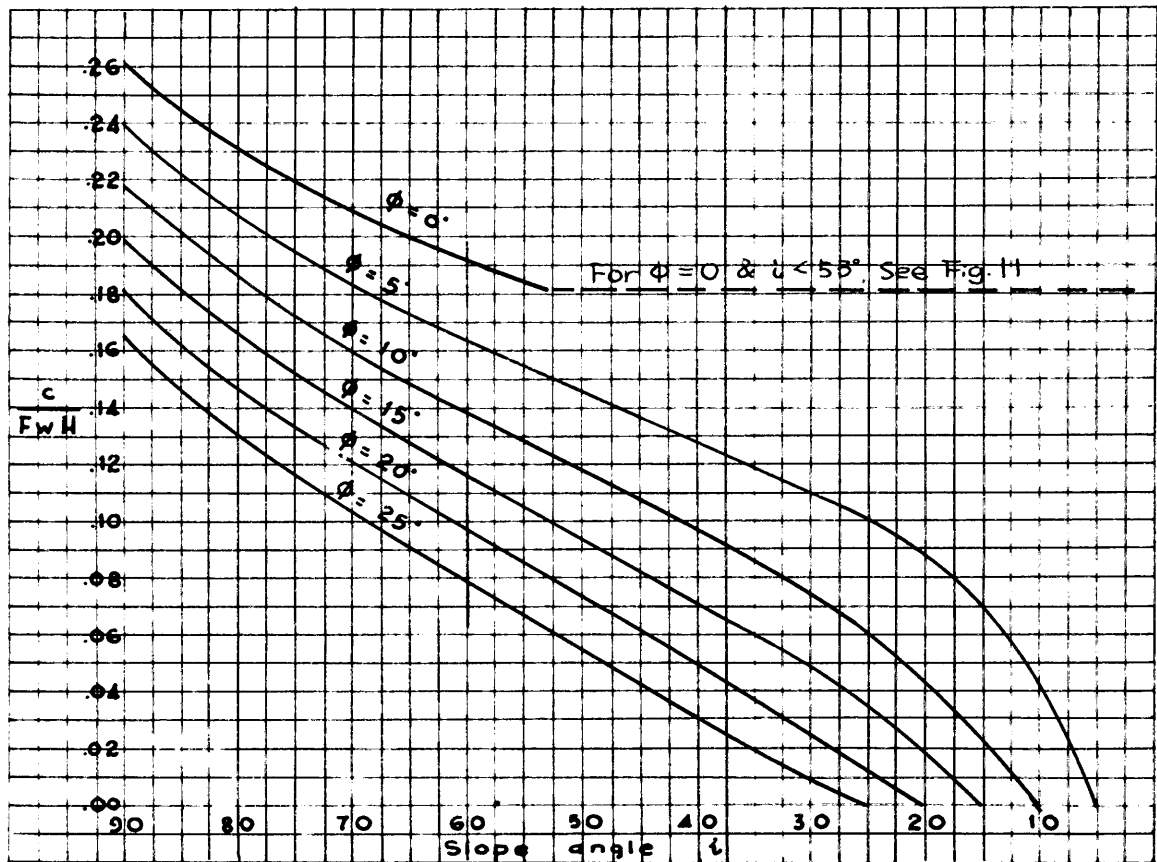


Fig 10 Stability number (I) (11)
(After Taylor)

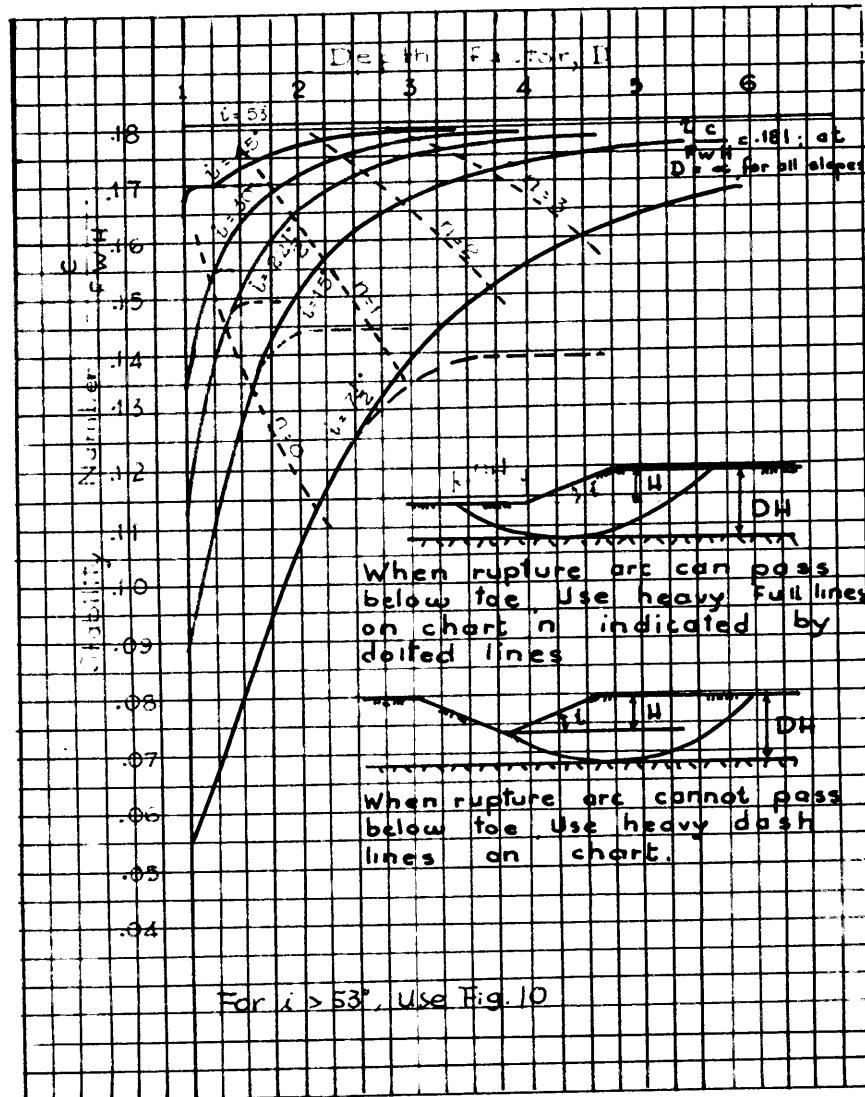


Fig 11 Stability number (II) (11)
(After Taylor)

B. Failure circle passing below toe of slope (Fig. 9)

$$\frac{C}{F \gamma H} = \frac{\frac{1}{2} \csc^2 \alpha (\gamma \csc^2 \beta - \cot \beta) + \cot \alpha - \cot i - 2n}{2 \cot \alpha + \cot \beta + 2} \quad - - (6)$$

By Fig. 10 or Fig. 11, if the internal friction angle and slope angle are known, the stability number $\frac{C}{F \gamma H}$ could be found for toe circle and base circle respectively.

(3) Logarithmic Spiral Surface of Failure

$$\text{The curve of failure } r = r_1 e^{\theta \tan \phi} \quad - - (7)$$

i. Taylor's semi-mathematical solution (11)

a. Curve of failure passing through the toe of slope
(Fig. 12)

$$\frac{c}{F \gamma H} = \frac{\tan \phi}{3 g^2 (m^2 - 1)} \left[\frac{2 g^3 \{ (m^3 \sin j - \sin g) - 3 \tan \phi (m^3 \cos j + \cos g) \}}{9 \tan^2 \phi + 1} \right. \\ \left. + g^3 \sin^3 g (\cot^2 j - \cot^2 g) + 3 m g \cos j (\cot i - \cot j) - \cot^2 i + \cot^2 j \right] \quad - (8)$$

$$m = e^{z \tan \phi} \quad - - - - - (9)$$

$$g = \frac{r_1}{H} = \frac{1}{\sin t \sqrt{1 + m^2 - 2 m \cos z}} \quad - - - - - (10)$$

$$j = t + \sin^{-1} \left[\frac{\sin z}{\sqrt{1 + m^2 - 2 m \cos z}} \right] \quad - - - - (11)$$

$$q = \pi - z - j \quad - - - - - (12)$$

c = Cohesion

F = Factor of safety for cohesion

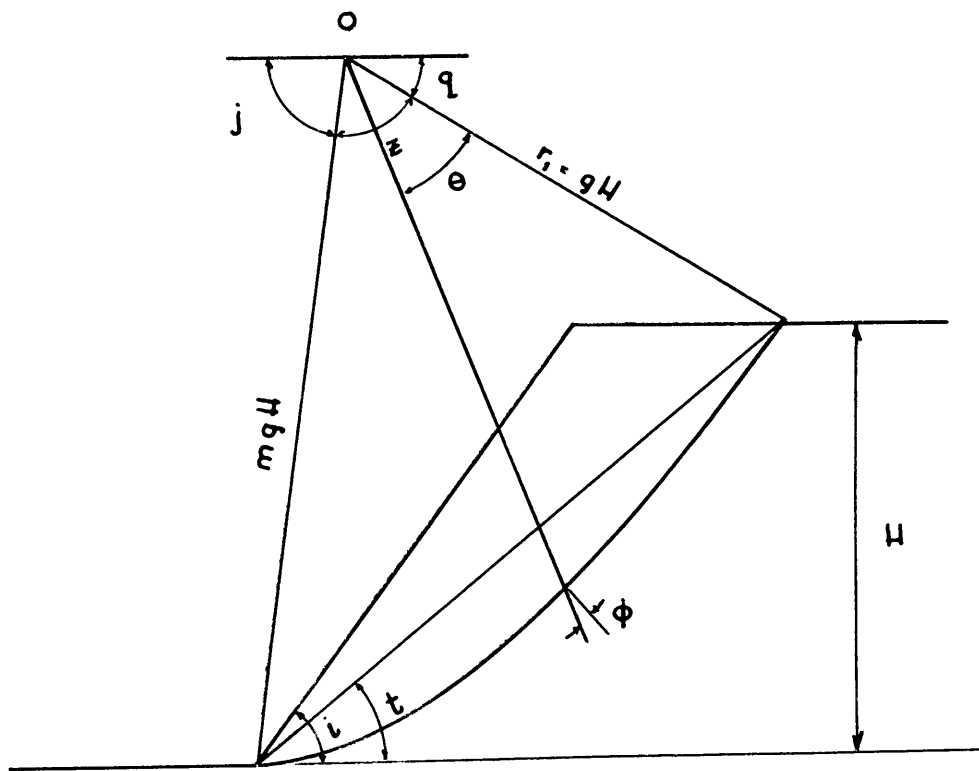


Fig. 12 Logarithmic spiral - toe failure⁽¹¹⁾
(After Taylor)

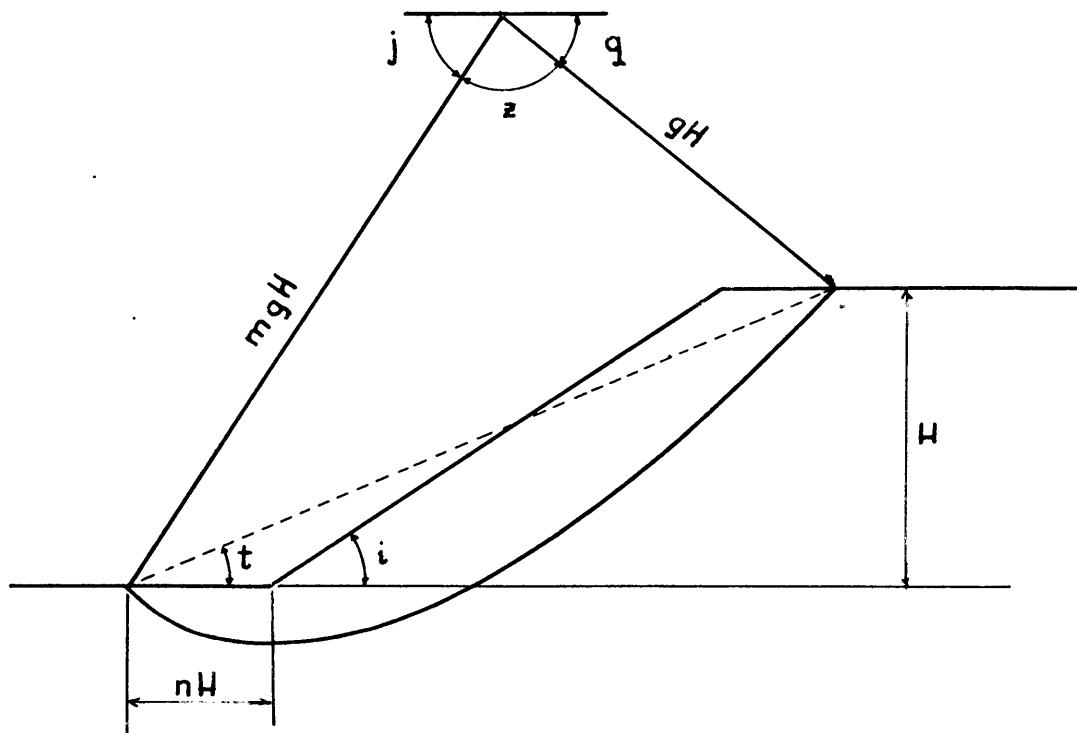


Fig.13 Logarithmic spiral - base failure⁽¹¹⁾
(After Taylor)

For any chosen values of z and t the dependent variables m , g , j , and q may be obtained from above expressions, then the critical value for $\frac{c}{F \gamma H}$ can be found. Taylor has investigated various slopes by trial and error methods for both the circular arc and logarithmic spiral surfaces of failure. He found that for a chosen slope angle the z and t in the logarithmic spiral surface of failure are approximately equal to the $2y$ and x respectively in the circular arc assumption.

b. Curve of failure passing below the toe of slope

(Fig. 13)

$$\frac{c}{F \gamma H} = \frac{\tan \phi}{3g^2(m^2-1)} \left[\frac{2g^3 \{ (m^2 \sin j - \sin q) - 3 \tan \phi (m^2 \cos j + \cos q) \}}{9 \tan^2 \phi + 1} \right. \\ \left. + g^3 \sin^3 q (\cot^2 j + \cot^2 q) + 3mg \cos^2 j (mg - \csc j) - \frac{1}{4} \cot^2 i + \cot^2 j \right] - (13)$$

ii. Frohlich's method of analysis (12)

The curve of failure $r = r_e e^{\theta \tan \phi}$ (Fig. 14)

$$F_s = \frac{M \text{ (resisting)}}{M \text{ (driving)}}$$

M (resisting) - The moment of all resisting forces acting upon the sliding mass long the sliding surface, turning about an axis, whose location is compatible with the start of the sliding movement. These forces are the sum of frictional, cohesive, and normal stresses.

M (driving) - The moment of all the other forces acting upon the sliding mass above the sliding surface, turning about the same axis. These forces are earth weight, hydrostatic uplift, surcharges, seepage pressure, earthquake actions, etc.

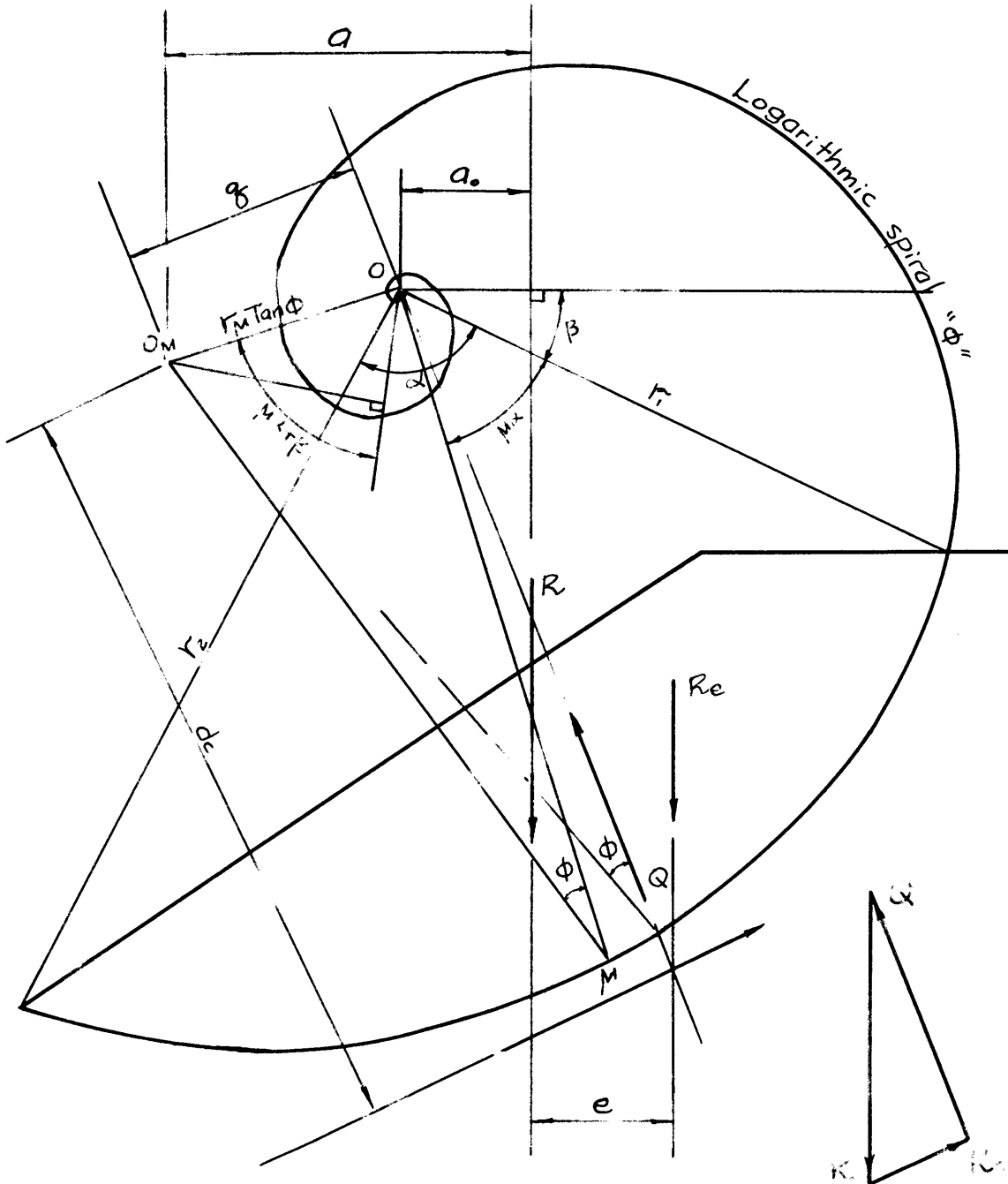


Fig. 14.
Frölich's analysis on logarithmic spiral surface of failure (I)⁽¹²⁾

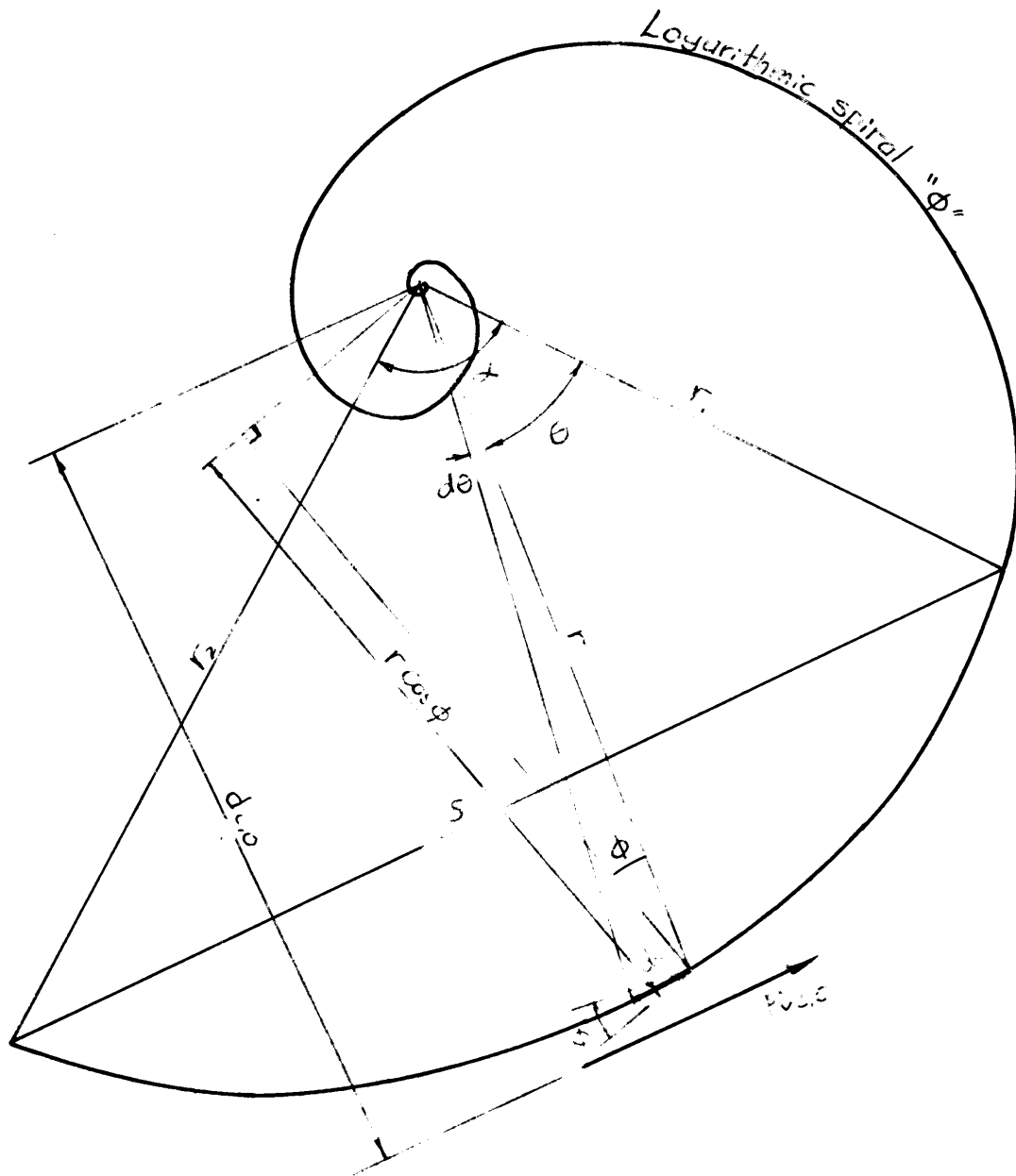


Fig. 15.
Fröhlich's analysis on logarithmic spiral surface of failure (II)⁽¹²⁾

a. Graphical Method (Fig. 14)

R - Resultant of all driving forces acting upon the sliding mass above arc AB.

R_c - Resultant of the cohesional stresses $c \times ds$ acting upon the sliding mass along arc AB at the instant when sliding begins, this available resistance may be determined by means of a polygon of forces.

ϕ - Angle of internal friction

Q - Resultant of all oblique stresses along the sliding surface AB at the first moment of sliding, and passing through the point O of the logarithmic spiral.

$$R_e = R$$

$$Re(a + e) = Q \cdot q + R_c \cdot d_c$$

$$F.S. = \frac{Q \cdot q + R_c \cdot d_c}{R \cdot a} = \frac{R_e(a + e)}{R \cdot a} = 1 + \frac{e}{a} \quad \text{-- (14)}$$

(1) By trial, locate the most critical spiral curve.

(2) The center O_μ of rotation may be chosen at the arc O_1, O_2 of the evolute of the given logarithmic spiral (choose point μ at the arc AB, for instance at the axis of symmetry of the angle α between OA and OB, trace the radius vector $\mu-O$ and erect the normal to this radius at O, where the tangent T at μ , it is the average location of the center of rotation of the sliding mass.)

b. Analytic Method (Fig. 15)

c = Cohesion along the arc AB

$R_{c.o}$ = Resultant of cohesional stresses

By assumption, $R_{c,o}$ is parallel to the chord $AB = s$

$$\therefore R_{c,o} = sc$$

ds = element of arc

$$ds \cos \phi = r d\theta$$

$$\therefore ds = \frac{r d\theta}{\cos \phi}$$

$$r = r_1 e^{\theta \tan \phi}$$

the moment produced by $R_{c,o}$ is $R_{c,o} \cdot d_{c,o}$

the moment produced by $c \cdot ds$ is $\int_{\theta=0}^{\theta=\alpha} c ds r \cos \phi$

$$R_{c,o} \cdot d_{c,o} = \int_{\theta=0}^{\theta=\alpha} c ds \cdot r \cos \phi = \int_0^\alpha c r_1^2 e^{2\theta \tan \phi} d\theta$$

$$\begin{aligned} s \cdot c \cdot d_{c,o} &= c r_1^2 \left[\frac{1}{2 \tan \phi} e^{2\theta \tan \phi} \right]_0^\alpha \\ &= c r_1^2 \left[\frac{1}{2 \tan \phi} e^{2\alpha \tan \phi} - \frac{1}{2 \tan \phi} \right] \\ &= c \left[\frac{r_2^2}{2 \tan \phi} - \frac{r_1^2}{2 \tan \phi} \right] \end{aligned}$$

$$\therefore d_{c,o} = \frac{r_2^2 - r_1^2}{2s \tan \phi}$$

$$\text{Since } R(a_0 + e) = R_{c,0} \cdot d_{c,0}$$

$$\begin{aligned} e &= \frac{R_{c,0} \cdot d_{c,0}}{R} - a_0 \\ &= \frac{s \cdot c}{R} \left[\frac{r_2^2 - r_1^2}{2S \tan \phi} \right] - a_0 \\ &= \frac{c(r_2^2 - r_1^2)}{2R \tan \phi} - a_0 \end{aligned} \quad \text{--- (15)}$$

Substitute (15) into (14)

$$\text{F. S.} = 1 + \frac{\frac{c(r_2^2 - r_1^2)}{2R \tan \phi} - a_0}{a}$$

$$\text{Since } a = a_0 + r_\mu \tan \phi \sin(\mu\alpha + \beta)$$

$$r_\mu = r_1 e^{\mu\alpha \tan \phi}$$

$$\therefore \text{F.S.} = 1 + \frac{\frac{c}{R} \cdot \frac{r_2^2 - r_1^2}{2 \tan \phi} - a_0}{a_0 + r_\mu e^{\mu\alpha \tan \phi} \tan \phi \sin(\mu\alpha + \beta)} \quad \text{--- (16)}$$

In practical cases, the values c & ϕ are known from laboratory investigations. The angles α, β , the distances r_1, r_2, a , and the force R are represented in Fig. 16. The value of μ which determines the location of the center of rotation varies between the extreme limits of zero and unity. For practical computations, the average value $\mu = 0.5$ gives fair results.

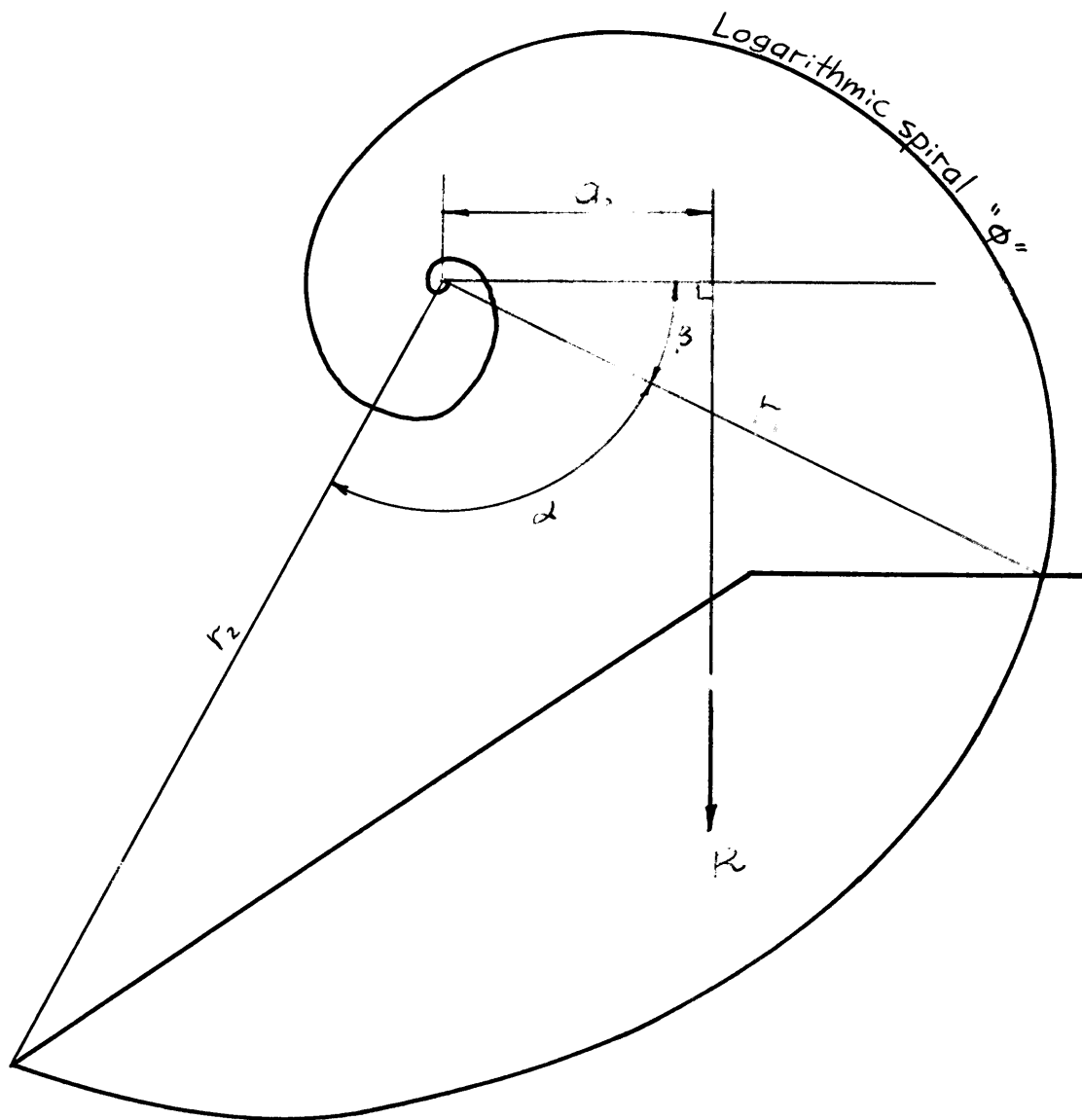


Fig. 16.
Fröhlich's analysis on logarithmic spiral surface of failure (III)⁽¹²⁾

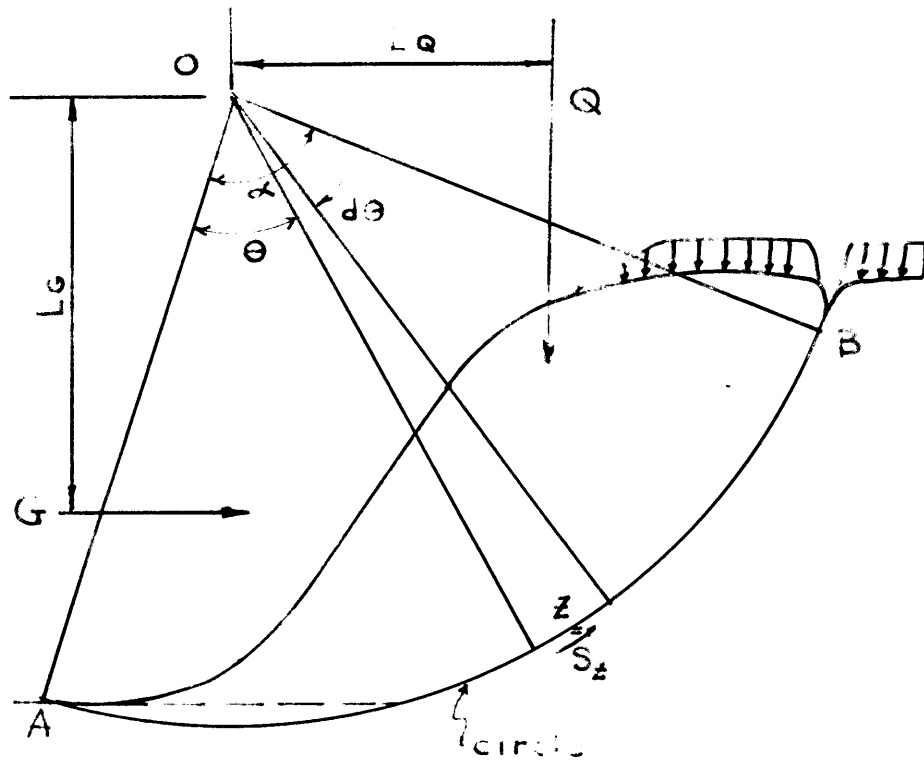


Fig. 17.
Principal elements used for defining $F_s^{(13)}$
(After Junbu)

(4) Janbu's Method (13)

i. $\phi = 0$ (Fig. 17)

M_r = available resisting moment about O

$$= \int_0^\alpha S_z R^2 d\theta$$

M_o = total overturning moment about O

$$= Q L_Q - G L_G$$

S_z = the shear strength of the soil at point Z

Q = resultant of all vertical forces between points A. & B, including the weight of the soil

G = resultant of all horizontal force, including hydrostatic pressure in tension cracks

L_Q & L_G are moment arms about O of forces Q and G, respectively

$$F_r = \frac{M_r}{M_o}$$

$$F_s = F_r, \text{ min.}$$

F.S. = F_s = The ratio between the shear strength available and the average shear stress necessary for equilibrium along the critical surface of sliding

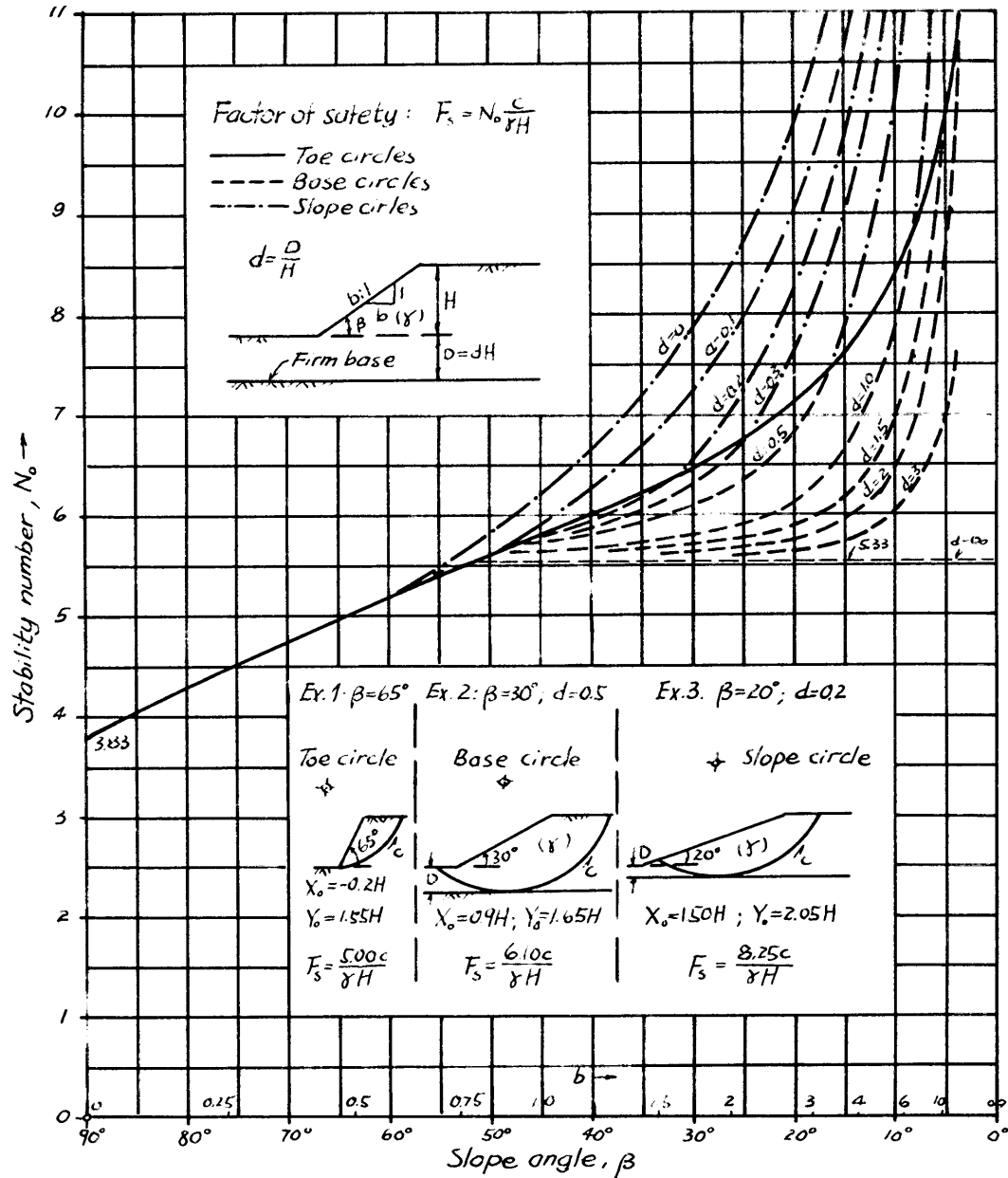


Fig. 18. Stability number for simple slopes when $\phi = 0$ (13) (After Janbu)

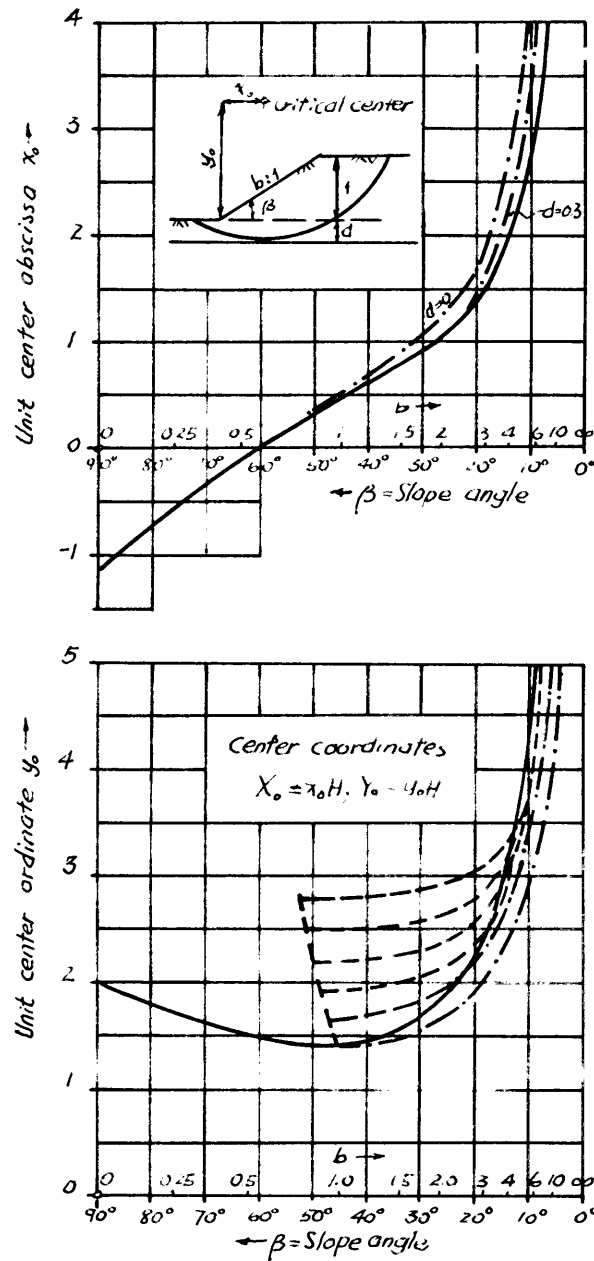


Fig. 19. Critical center coordinates for simple slopes when $\phi = 0$ (13) (After Janbu)

(A) Analysis of Simple Slopes

$$F_s = \frac{N \cdot c}{\gamma H} \quad - - - - - (17)$$

$$X_o = x_o \cdot H \quad Y_o = y_o \cdot H \quad - - - (18)$$

Procedures of locating the critical center

- 1) Using Fig. 18 and by knowing the slope angle β and the depth factor $d = D/H$, find the stability number, then compute F_s .
- 2) Using Fig. 19, determine the unit coordinates x_o and y_o , and compute the X_o and Y_o .

a. Effect of Surcharge (Fig. 20)

$$F_s = M_q N_o \frac{c}{\gamma H + q} \quad - - - (19)$$

b. Effect of Partial Submergence (Fig. 21)

$$F_s = M_w N_o \frac{c}{\gamma H - \gamma_w H_w} \quad - - - (20)$$

c. Effect of Tension Cracks (Fig. 22)

$$F_s = M_t N_o \frac{c}{\gamma H} \quad - - - (21)$$

d. Combination of Surcharge, Submergence and Tension Cracks (Fig. 23)

$$F_s = \frac{N_s c}{\gamma H + q - \gamma_w H_w} \quad - - - (22)$$

$$N_s = M_q M_w M_t N_o = M_d N_o \quad - - - (23)$$

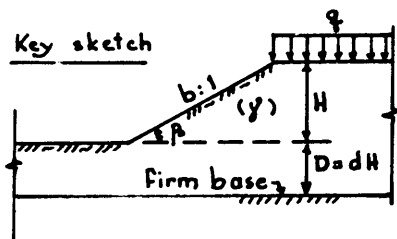
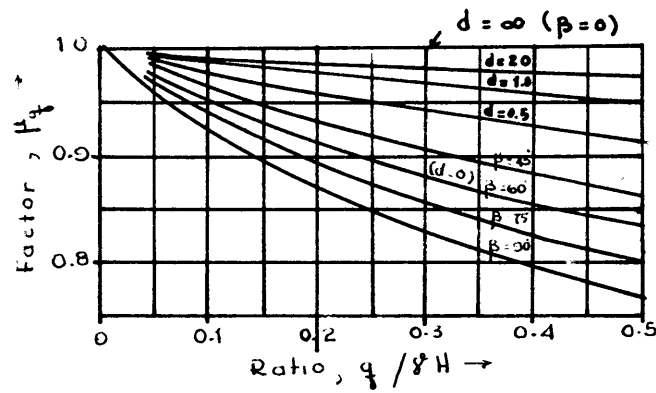
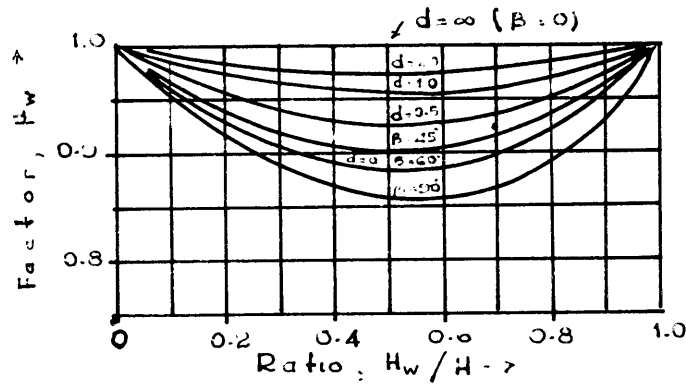


Fig. 20. Reduction factor μ_q (13)



Key Sketch

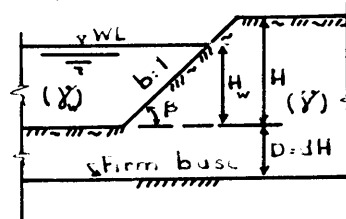
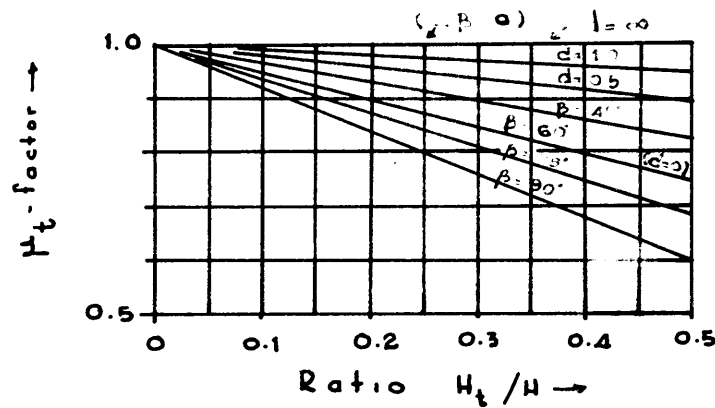
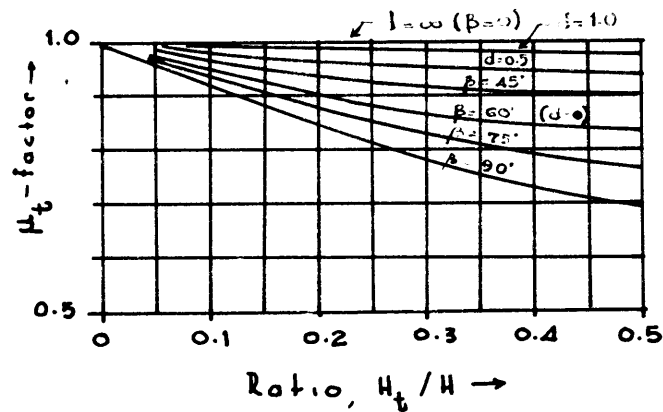


Fig. 21. Reduction factor μ_w (13)
(After Janbu)



Full hydrostatic pressure is acting in the tension cracks



The hydrostatic pressure is equal to zero

Key sketch

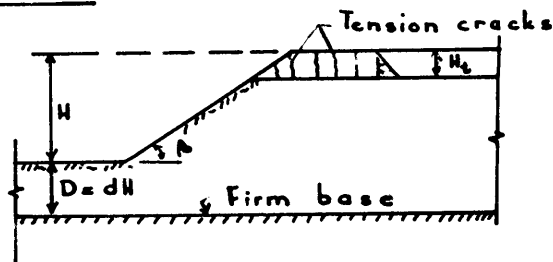


Fig. 22. Reduction factor M_t (13)
 (After Junbu)

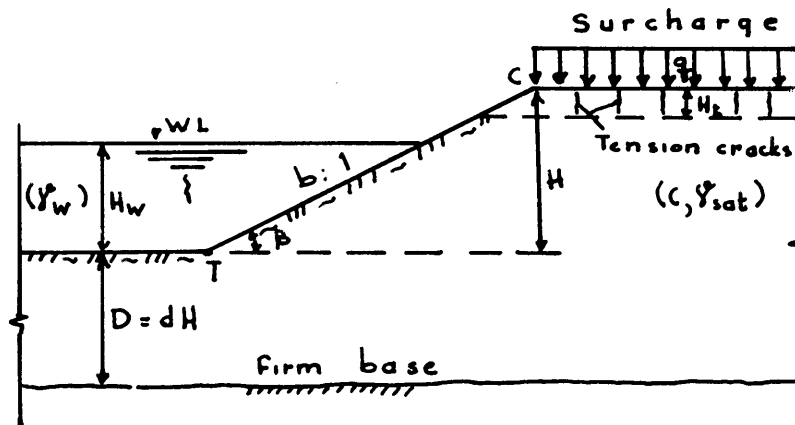


Fig. 23.

Combination of surcharge, submergence & tension cracks⁽¹³⁾
(After Janbu)

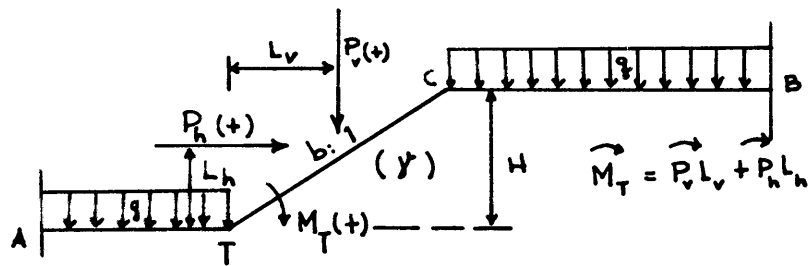


Fig. 24. Simplified load system⁽¹³⁾
(After Janbu)

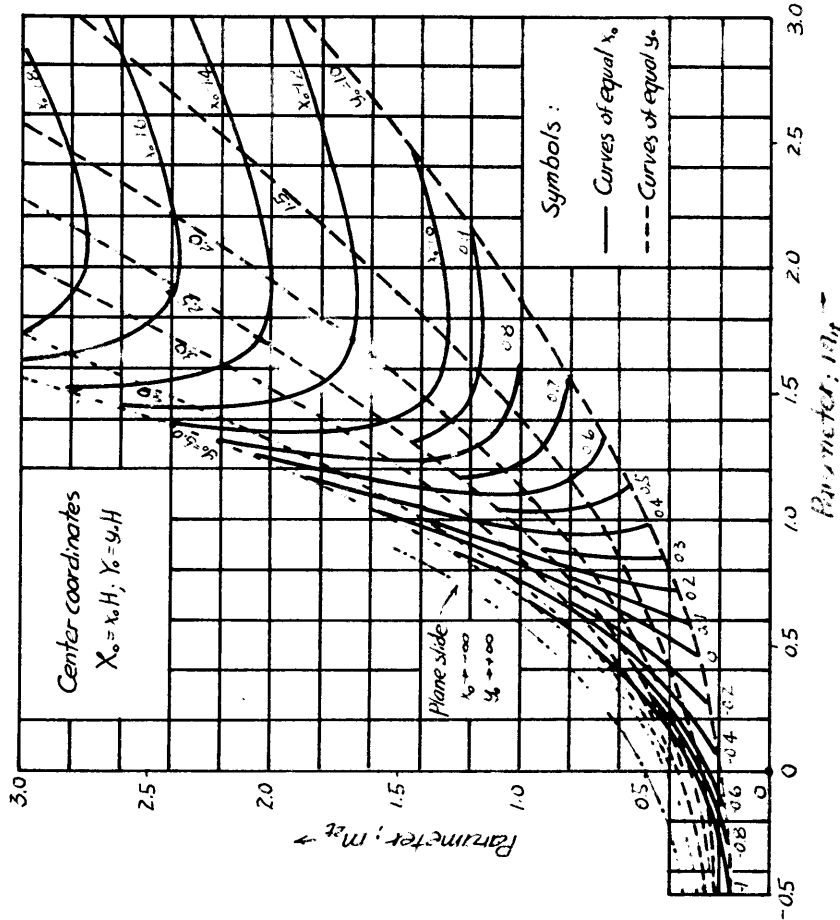


Fig. 16. Critical center coordinates for toe circles, loaded slopes, $\phi = 0$. (13) (After Janbu)

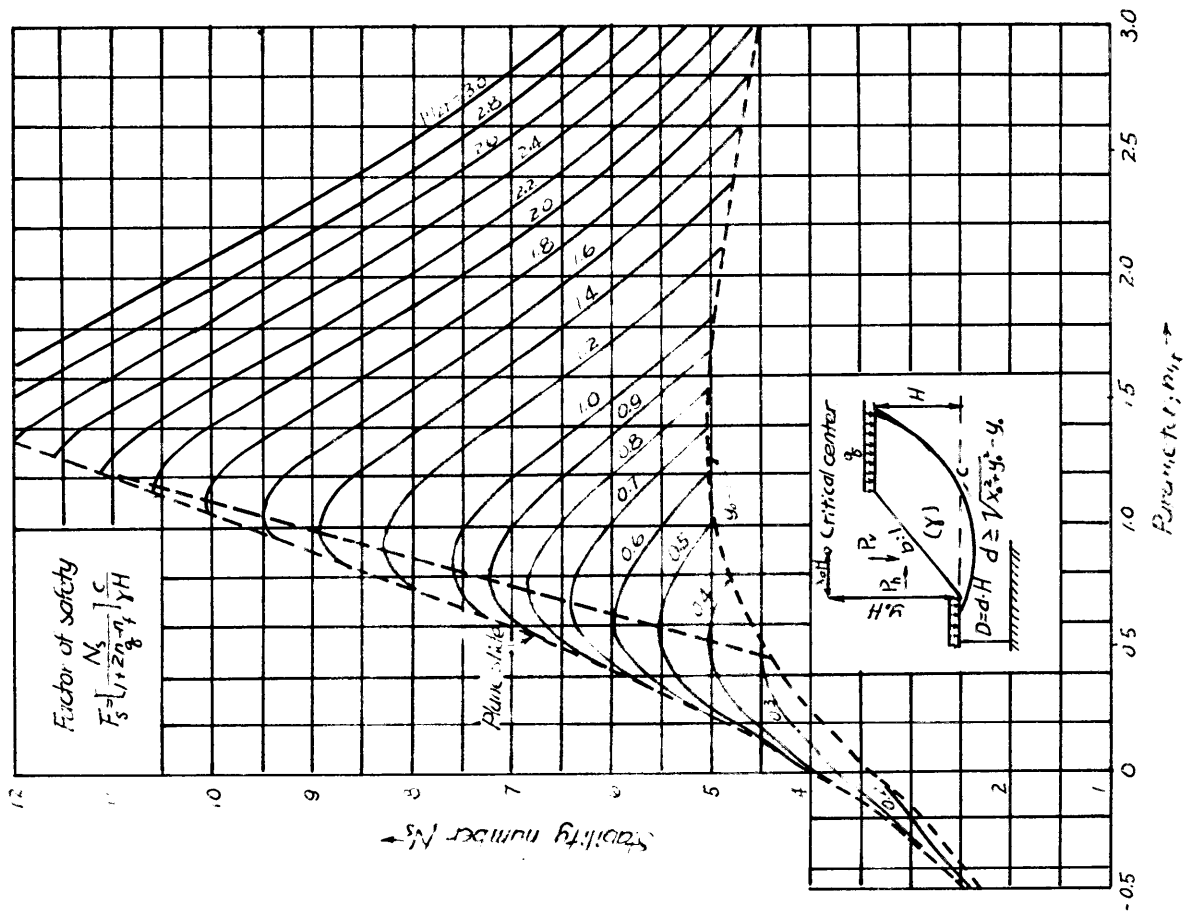


Fig. 15. Stability number for toe circles, loaded slopes, $\phi = 0$. (13) (After Janbu)

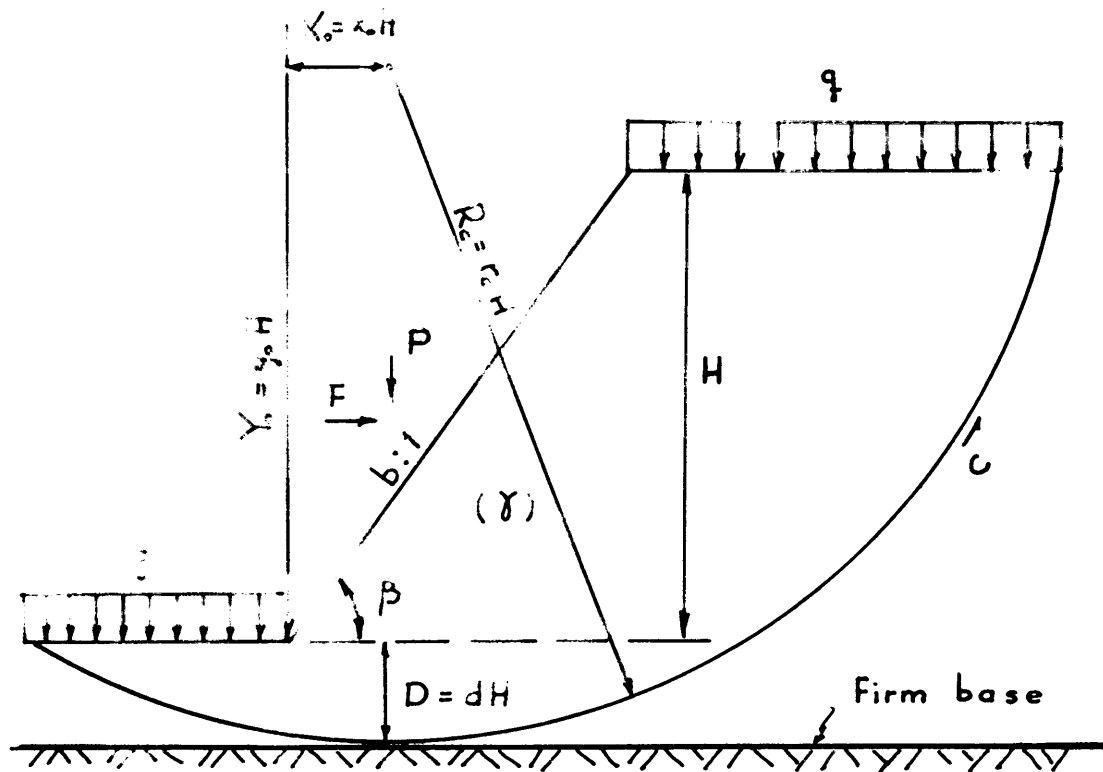


Fig. 27
Illustration for critical center (13)
(After Junbu)

(B) Analysis of Loaded Slopes

a. Toe circle analysis

$$F.S. = \left[\frac{N_s}{1 + 2n_q - n_h} \right] \frac{c}{\gamma H} \quad \text{--- (24)}$$

$$M_{1t} = \frac{(1 + 2n_q)b - n_v}{1 + 2n_q - n_h}$$

$$M_{2t} = \frac{(\frac{1}{3} + n_q)(1 + b^2) - n_M}{1 + 2n_q - n_h}$$

$$n_q = \frac{q}{\gamma H}, \quad n_g = \frac{g}{\gamma H}, \quad n_v = \frac{2P_v}{\gamma H^2}, \quad n_h = \frac{2P_h}{\gamma H^2}, \quad n_M = \frac{2M_T}{\gamma H^3}$$

M_T = moment of forces P_v & P_h about the toe

Procedures for locating the critical center

- 1) Compute M_{1t} & M_{2t}
- 2) Use Fig. 25 to find N_s , then compute F.S.
- 3) Use Fig. 26 to find x_o & y_o .
- 4) Locate the critical center by

$$X_o = x_o H$$

$$Y_o = y_o H$$

b. Base circle analysis (Fig. 27)

$$M_{1b} = \frac{1 - 2n_q - n_h}{1 - n_q - n_g}$$

$$M_{2b} = \frac{\frac{1}{3}(1 + b^2) + n_q - n_M}{1 + n_q - n_g} + \frac{bn_q(n_v - bn_g) - \frac{1}{4}(b - n_v)^2}{(1 + n_q - n_g)^2}$$

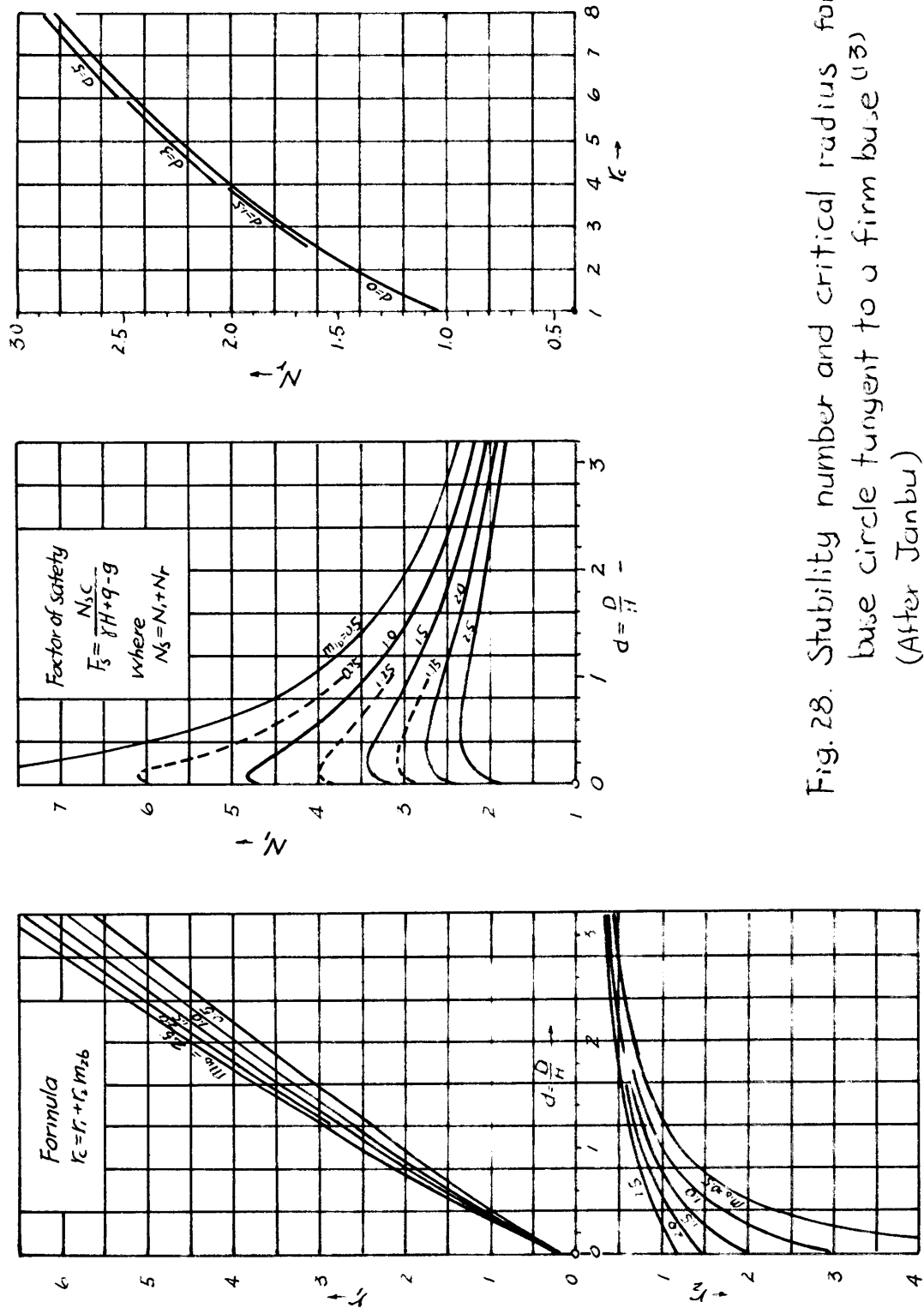


Fig. 28. Stability number and critical radius for base circle tangent to a firm base (13)
(After Janbu)

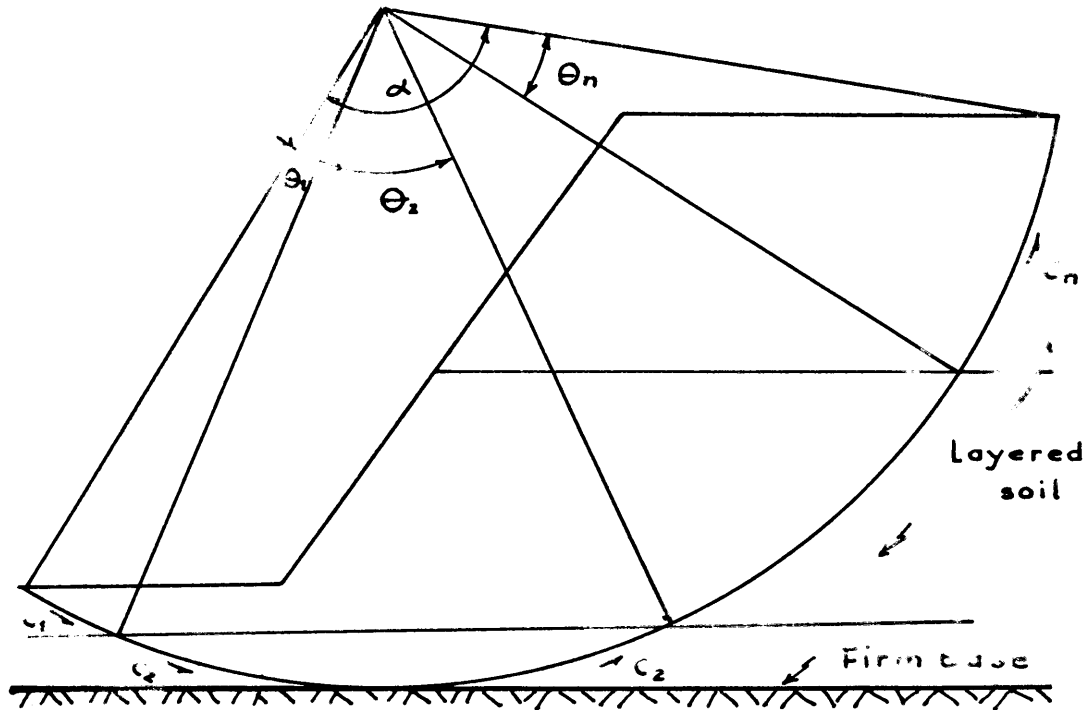


Fig. 29

Average shear strength for layered soil when $q = 0$ ⁽¹³⁾
(After Janbu)

Procedures for locating the critical center

1) Compute M_{1b} & M_{2b}

2) Use Fig. 28 to find N_g , then compute F.S. by

$$F.S. = \left[\frac{N_s}{1 + n_q - n_g} \right] \frac{c}{\gamma H} \quad \text{--- (25)}$$

3) The coordinates of the critical center are

$$X_o = \left[\frac{(1 + 2n_q)b - n_v}{2(1 + n_q - n_g)} \right] H \quad \text{--- (26)}$$

$$Y_o = (r_c - d) H \quad \text{--- (27)}$$

$$R_c = r_c H \quad \text{--- (28)}$$

$$r_c = r_1 + r_2 M_{2b} \quad \text{--- (29)}$$

c. Analysis of layered soil (Fig. 29)

Average shear strength

$$c = \frac{\sum_{i=1}^n \theta_i c_i}{\alpha} \quad \text{--- (30)}$$

The other steps are the same as in the previous articles.

ii. $\phi > 0$

a. Simple slopes for zero neutral stress

$$F.S. = \frac{N_c c}{\gamma H} \quad \text{--- (31)}$$

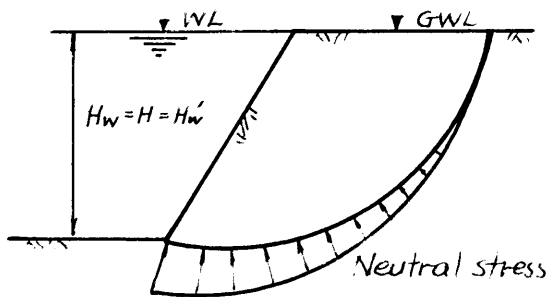


Fig. 31
Complete submergence⁽¹³⁾

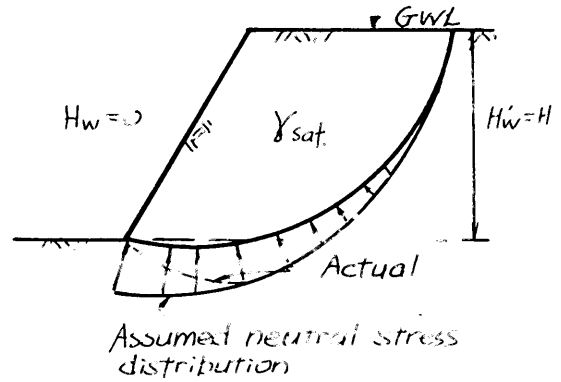


Fig. 32
Complete sudden drawdown⁽¹³⁾

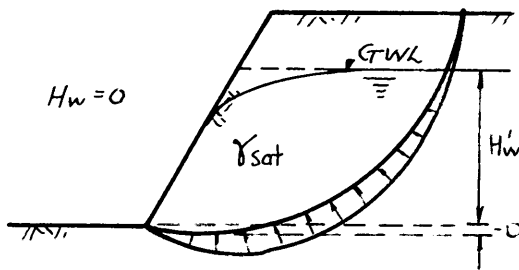


Fig. 33
Steady seepage⁽¹³⁾

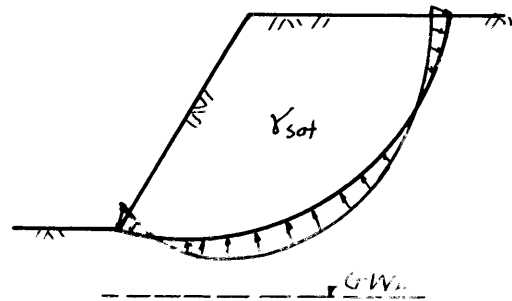


Fig. 34
Zero boundary neutral force⁽¹⁵⁾

(After Janbu)

When the slope angle and $\lambda_{c\phi}$ are known, Figure 30 may be used. The F.S. and the coordinates of the critical center can be found by

$$\lambda_{c\phi} = \frac{\gamma H \tan \phi}{c} \quad \text{--- (32)}$$

$$X_o = x_o H$$

$$Y_o = y_o H$$

b. Modified formula for simple neutral stress condition

$$F.S. = \frac{N_{cf} C}{\gamma_d H} \quad \text{--- (33)}$$

$$\lambda_{c\phi} = \frac{\gamma_e H \tan \phi}{c} \quad \text{--- (34)}$$

1) Complete submergence (Fig. 31)

$$\gamma_d = \gamma_e = \gamma' = \gamma - \gamma_w$$

2) Sudden drawdown (Fig. 32)

$$\gamma_d = \gamma, \quad \gamma_e = \gamma'$$

3) Steady seepage (Fig. 33)

$$\gamma_d = \gamma, \quad \gamma_e = \gamma - \frac{H'_w + D}{H + D} \gamma_w$$

4) Zero boundary neutral force (Fig. 34)

$$\gamma_d = \gamma_e = \gamma$$

5) Partially submerged (Fig. 20)

$$F_s = \frac{M_w N_{cf} c}{\gamma_{sat} H - \gamma_w H_w}$$

$$\lambda_{c\phi} = \frac{(\gamma_{sat} H - \gamma_w H_w) \tan \phi}{c \cdot \mu'_w}$$

c. Combination of surcharge, tension cracks, submergence and steady seepage

$$F_s = \frac{N_{cf} c}{P_d} \quad \text{----- (35)}$$

$$\lambda_{c\phi} = \frac{P_e \tan \phi}{c}$$

$$P_d = \frac{\gamma_{sat} H + q - \gamma_w H_w}{M_d}$$

$$M_d = M_w \mu_q M_x$$

$$P_e = \frac{\gamma_{sat} H + q - \gamma_w H_w'}{M_e}$$

$$M_e = M_q \mu'_w$$

1. Using Fig. 30, β and $\lambda_{c\phi}$ are known, and N_{cf} may be found
2. Compute P_d with μ'_w interpolated from Fig. 21
3. Determine the F_s .

VI. NUMERICAL EXAMPLES

A. COMPUTATIONS:

Example 1. Given soil condition as Figure 45.

$$\phi_1 = 0, C_1 = 250 \text{ lbs/sq.ft.} \quad \gamma_1 = 105 \text{ lbs/cu.ft.}$$

$$\phi_2 = 0, C_2 = 600 \text{ lbs/sq.ft.} \quad \gamma_2 = 108 \text{ lbs/cu.ft.}$$

$$D = 60 \text{ ft}$$

Design a safe slope for 20 foot (H 20 ft.)

cut, use $F_s = 1.4$.

(1) By Janbu's Method

a. Base circle of failure

$$\text{Assume } \gamma = \gamma_2 = 108 \text{ lbs./cu. ft.}$$

$$C = C_2 = 600 \text{ lbs./sq. ft.}$$

$$\text{Compute the depth factor } d = D/H = 60/20 = 3$$

Assume various slope value b, by Figures 18 and 19

find N_o , x_o and y_o corresponding to each slope value.

$$\text{Then } F_s = \frac{N_o C}{\gamma H}$$

$$X_o = x_o H$$

$$Y_o = y_o H$$

<u>b</u>	<u>d</u>	<u>C</u>	<u>γ</u>	<u>H</u>	<u>N_o</u>	<u>F_s</u>	<u>x_o</u>	<u>y_o</u>	<u>X_o</u>	<u>Y_o</u>
1	3	600	108	20'	5.54	1.54	0.5	2.8	10'	56'
1.5	3	600	108	20'	5.60	1.55	unnecessary	unnecessary		
2	3	600	108	20'	5.60	1.55	unnecessary	unnecessary		

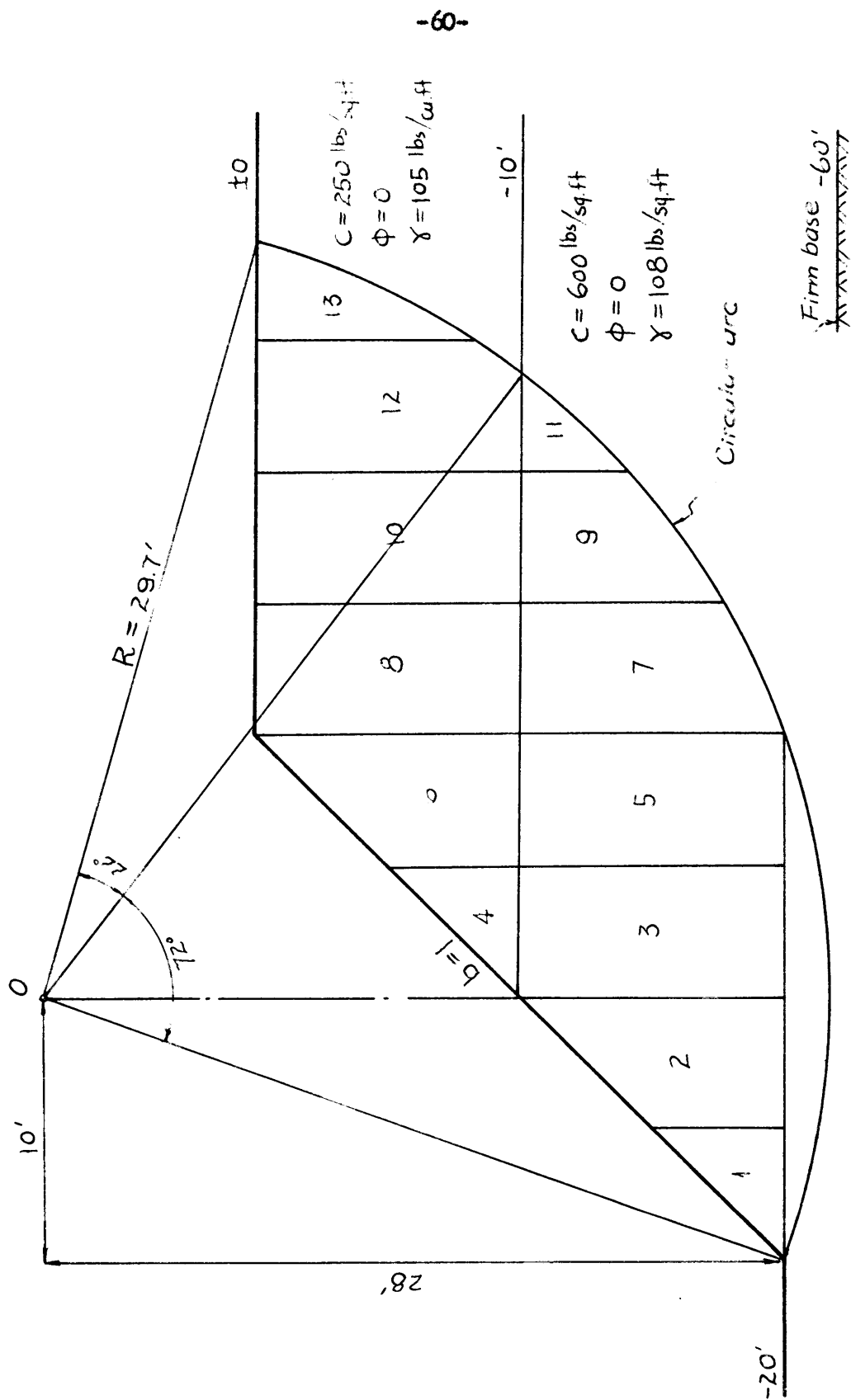


Fig. 45 Illustration for Example 1

b. Toe circle failure

Assume $\gamma = \gamma_z = 108 \text{ lbs.cu. ft.}$

Compute $C = \frac{\sum \theta_i c_i}{\alpha}$ when critical center is located.

Other procedures are same as the computation on a., base circle of failure by Janbu's Method, as indicated in VI, 1a.

<u>b</u>	<u>d</u>	<u>c</u>	<u>Y</u>	<u>H</u>	<u>N.</u>	<u>F_s</u>	<u>α.</u>	<u>y.</u>	<u>X.</u>	<u>Y.</u>
1	3	521	108	20	5.8	<u>1.40</u>	0.5	1.4	10'	28'

As shown by the previous computations on page , for $b = c \cot \beta = 1$, the most possible failure is toe circle. Therefore, the following analysis is based on the case of toe circle of failure.

(2) By Method of Slices

As shown in Figure 45, use the critical center and the possible toe circle of failure found by Janbu's Method to find the F_s by the method of slices.

<u>No.</u>	<u>Area(sq. ft.)</u>	<u>γ_1 or γ_2 ($\frac{\text{lb}}{\text{ft}^3}$)</u>	<u>W (lb.)</u>	<u>r (ft.)</u>	<u>M (ft-lb)</u>
1	12.5	108	1,350	6.7	9,040
2	37.5	108	4,050	2.2	8,910
	Total				<u>17,950</u>
3	50.0	108	5,400	2.0	10,800
4	12.5	105	1,310	3.3	4,325
5	50.0	108	5,400	2.5	40,500
6	37.5	105	3,940	7.8	30,700
7	44.4	108	4,800	12.4	59,600
8	50.0	105	5,250	12.5	65,600
9	29.6	108	3,200	17.1	54,700
10	50.0	105	5,250	17.5	91,700
11	7.8	108	840	21.2	17,800
12	48.9	105	5,140	22.4	115,000
13	15.6	105	1,640	26.2	43,000
	Total				<u>533,725</u>

$$M_o = 533,725 - 17,950 = 515,775 \text{ ft-lb}$$

$$\begin{aligned} M &= R \times (c_1 \times L_1 + c_2 \times L_2) \\ &= 29.7 (250 \times 11.4 + 600 \times 37.4) \\ &= 750,000 \text{ ft-lb} \end{aligned}$$

$$F_s = \frac{M_r}{M_o} = \frac{750,000}{516,000} = 1.45$$

Example 2. Given soil conditions as shown in Figure 46.

$$\phi = 10^\circ, \quad c = 250 \text{ lbs/sq.ft.}, \quad \gamma = 105 \text{ lbs/cu.ft.}$$

Design a maximum height for an economical and reasonably safe slope.

(1) By Janbu's Method

According to Janbu's investigations, the critical slip circle will intersect the toe if the dimensionless parameter $\lambda_{c\phi} > 0$. Base circle analysis for $\phi > 0$ has not been considered herein. For this example

$$\lambda_{c\phi} = \frac{\gamma H \tan \phi}{c} = \frac{105 \times \tan 10^\circ \times H}{250} = 0.074 H$$

$\lambda_{c\phi}$ is always greater than zero, therefore, the analysis is based on the failure by a toe circle only. Assume a height H , then compute $\lambda_{c\phi}$, by Figure 30. For various slope values b , find N_{cf} and calculate F_s .

$$a. \quad H = 18', \quad \lambda_{c\phi} = 1.33, \quad F_s = .132 N_{cf}$$

<u>b</u>	<u>$\lambda_{c\phi}$</u>	<u>N_{cf}</u>	<u>X.</u>	<u>Y.</u>	<u>F_s</u>
0.8	1.33	7.7	unnecessary		1.02
1.0		8.2	unnecessary		1.07
1.5		9.5	unnecessary		1.25
2.0		10.3	unnecessary		1.36

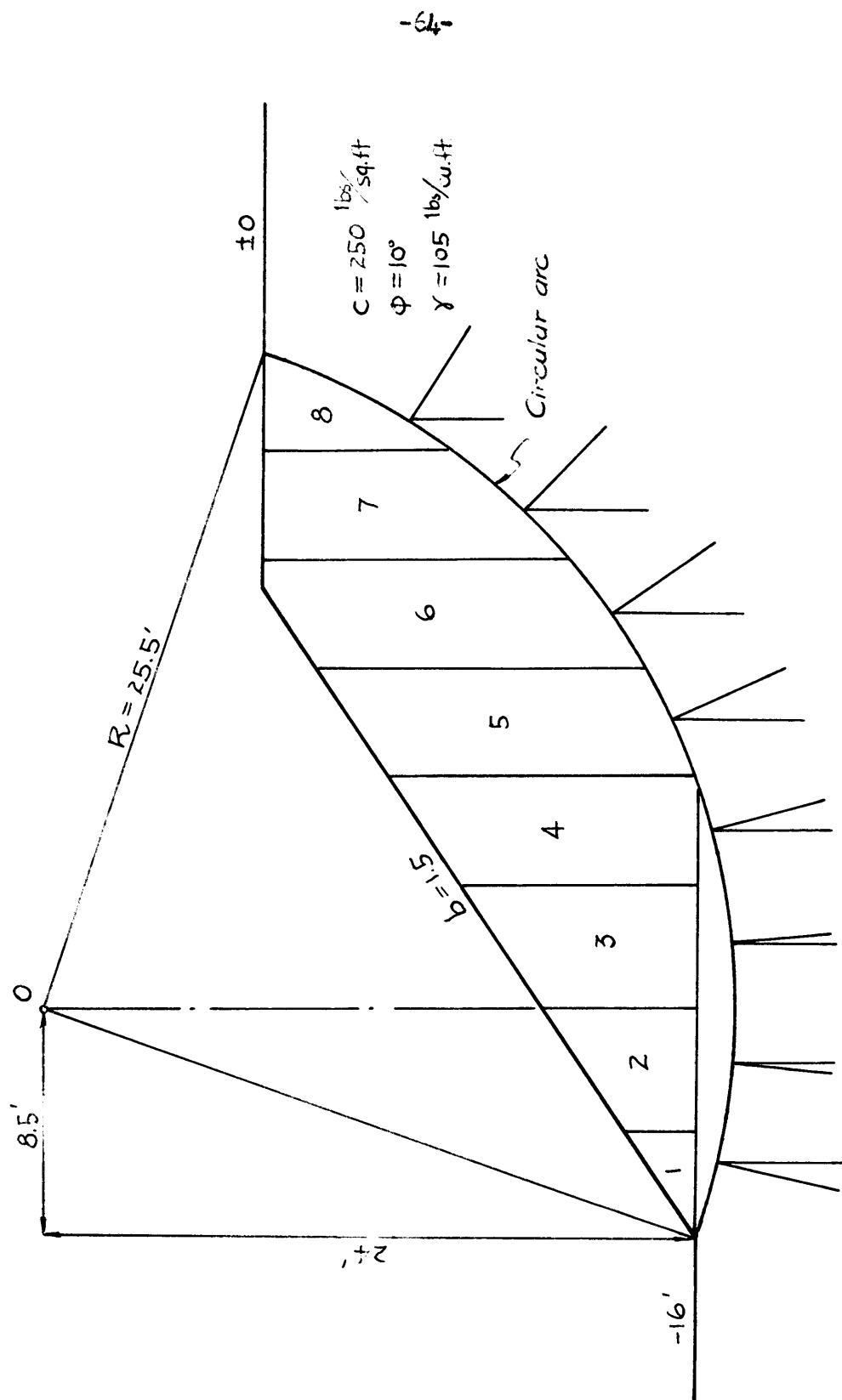


Fig. 46. Illustration for Example 2

b. $H = 16'$, $\lambda_{c\phi} = 1.13$, $F_s = .143N_{cf}$

<u>b</u>	<u>$\lambda_{c\phi}$</u>	<u>N_{cf}</u>	<u>X_o</u>	<u>Y_o</u>	<u>F_s</u>
0.8	1.13	7.4	unnecessary		1.09
1.0		7.9	unnecessary		1.17
1.5		9.2	<u>8.5'</u>	<u>24.0'</u>	<u>1.36</u>
2.0		10.0	unnecessary		1.48

The reasonable answer for this example is:

$$H = 16'$$

$$b = \cot \beta = 1.5$$

$$F_s = 1.36$$

The following analyses are based upon the above figures.

(2) By Method of Slices

As shown in Figure 46, the overturning moment is computed as listed in following table:

<u>No.</u>	<u>Area</u>	<u>Y</u>	<u>W</u>	<u>r</u>	<u>M</u>	<u>β</u>	<u>$\cos \beta$</u>	<u>$\cos \beta \cdot W$</u>	<u>$\tan \phi$</u>
1	5.34	105	560	-5.83	-3,250	13.0	.9744	546	.1763
2	18.74	1,960	-1.98	-3,880	4.6	.9968	1,954		
3	32.25	3,330	2.41	8,140	5.5	.9954	3,365		
4	40.00	4,200	6.57	27,500	15.0	.9659	4,050		
5	47.10	4,940	10.55	50,400	24.6	.9092	4,500		
6	48.30	5,010	14.50	73,200	34.9	.8210	4,110		
7	36.80	3,760	18.30	68,800	46.0	.6947	2,615		
8	13.05	1,370	21.71	29,800	58.2	.5270	722		

$$M_o = 257,840$$

$$21,660$$

	<u>M</u>	<u>cos</u>
From previous page,	$M_o = 257,840$	$21,860$
$R \cdot \sum \frac{1}{r} W \cos \beta \tan 10^\circ = 25.5 \times 21,860 \times .1763 = 103,000$		
$R \cdot \sum \frac{1}{r} c h = 25.5 \times 250 \times 40.5 = 258,000$		
M_r		$= 361,000$

$$F_s = \frac{M_r}{M_o} = \frac{361,000}{257,840} = 1.40$$

(3) By Taylor's Method (circular arc)

Since $\phi = 10^\circ$

$$\theta = \text{slope angle} = \cot^{-1} 1.5 = 33^\circ 41'$$

From Figure 10, found

$$\frac{c}{F \gamma H} = 0.083$$

Using $c = 250 \text{ lb/sq.ft.}$ $\gamma = 105 \text{ lb/cu.ft.}$ and $H = 16$

in the above expression,

F = Factor of Safety for Cohesion

$$= \frac{250}{105 \times 16 \times 0.083} = 1.79$$

Assume $F_s = 1.40$

$$\phi_o = \frac{10}{1.4} = 7.15^\circ$$

From Figure 10, found

$$\frac{c}{F \gamma H} = 0.106$$

$$\frac{250}{1.4 \times 105 \times H} = 0.106, \quad H = 16.05' \quad (\text{checked})$$

Therefore $F_s = 1.40$

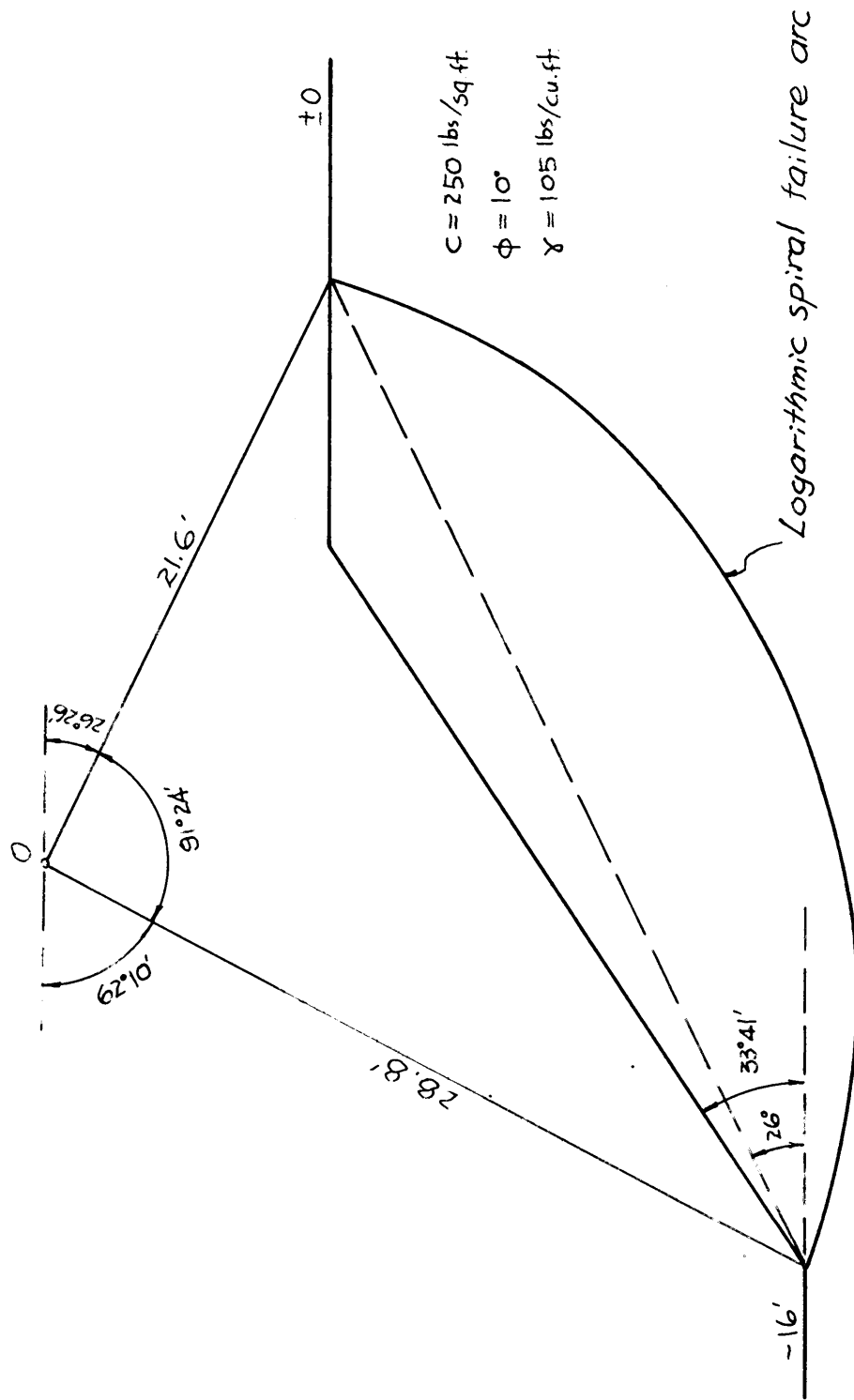


Fig 47 Illustration for Example 2

(4) By Taylor's Method (logarithmic spiral)

As shown in Figure 47, $\phi = 10^\circ$, $i = 33^\circ 41'$

Assume $Z = 91^\circ 24' = 1.594 \text{ rad.}$ and $t = 26^\circ$

By equations 8, 9, 10, 11 and 12.

$$m = e^{Z \tan \phi} = e^{1.594 \times 1.1763} = e^{0.281} = 1.334$$

$$\begin{aligned} g &= \frac{1}{\sin t \sqrt{1+m^2-2m \cos Z}} \\ &= \frac{1}{\sin 26^\circ \sqrt{1+(1.334)^2-2 \times 1.334 \cos 91^\circ 24'}} \\ &= \frac{1}{0.4348 \sqrt{1+1.780-2 \times 1.334 \times (-0.0244)}} = 1.35 \end{aligned}$$

$$j = t + \sin^{-1} \left[\frac{\sin Z}{\sqrt{1+m^2-2m \cos Z}} \right] = 26^\circ + 36^\circ 10' = 62^\circ 10'$$

$$q = 180^\circ - Z - j = 26^\circ 26'$$

$$\frac{\tan 10^\circ}{3g^2(m^2-1)} = \frac{.1763}{3 \times 1.82 \times 0.78} = 0.0414 \quad (A)$$

$$\frac{2 \times g^3 \{ (m^3 \sin j - \sin q) - 3 \tan \phi (m^3 \cos j + \cos q) \}}{9 \tan^2 \phi + 1} = 2.165 \quad (B)$$

$$\begin{aligned} g^3 \sin^3 q (\cot^2 j - \cos^2 q) + 3mg \cos j (\cot^2 i - \cot^2 j) - \cot^2 i + \cot^2 j \\ \approx -0.365 \end{aligned} \quad (C)$$

$$\begin{aligned} \frac{C}{F \&H} &= (A) [(B) + (C)] = 0.0414 [2.165 - 0.365] \\ &= 0.075 \end{aligned}$$

$$\therefore F = \frac{250}{105 \times 16 \times 0.075} = \underline{1.99}$$

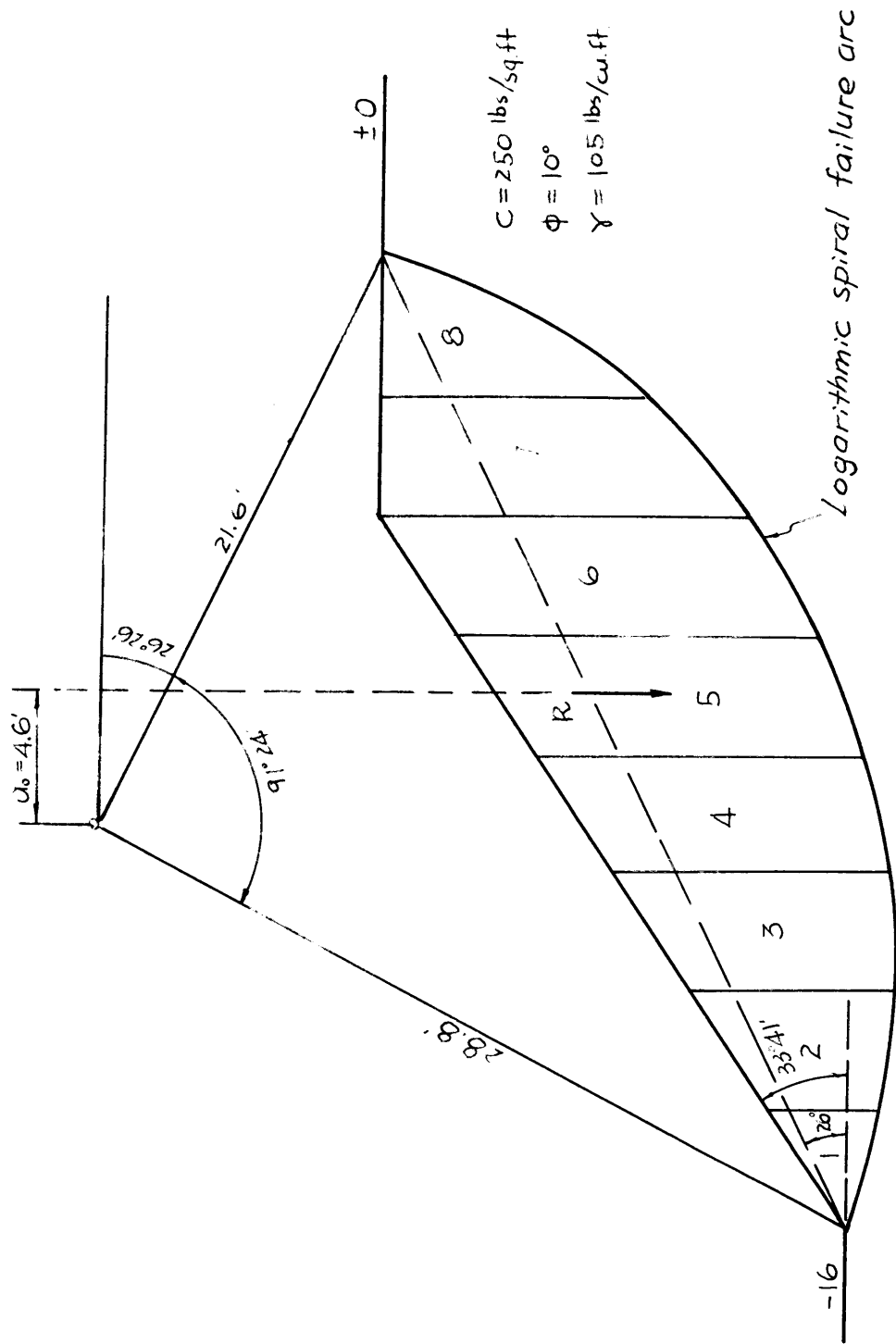


Fig. 48 Illustration for Example 2

(5) By Frohlich's Method

As shown in Figure 48, $\phi = 10^\circ$, $i = 33^\circ 41'$, $H = 16'$, $c = 250$

Assume $\alpha = 91^\circ 24' = 1.594$ radians and $t = 26$

From the previous calculation $m = 1.334$

$$g = 1.35$$

Then $r_1 = gH = 1.35 \times 16 = 21.6$

$$r_2 = mgH = 1.334 \times 1.35 \times 16 = 28.8$$

After drawing the logarithmic spiral curve of failure,
compute the total driving force above the curve

$$R = 27,117 \text{ lbs}$$

$$a_0 = 4.60$$

$$\beta = 26^\circ 26'$$

By equation 16

$$\begin{aligned} F_s &= 1 + \frac{\frac{c}{R} \cdot \frac{r_2^2 - r_1^2}{2 \tan \phi} - a_0}{a_0 + r_1 e^{\mu \alpha \tan \phi} \tan \phi \sin(\mu \alpha + \beta)} \\ &= 1 + \frac{\frac{250}{27,117} \cdot \frac{(28.8)^2 - (21.6)^2}{2 \times 1.763} - 4.60}{4.60 + 21.6 e^{0.5 \times 1.594 \times 1.763} \tan 10^\circ \sin(0.5 \times 91^\circ 24' + 26^\circ 26')} \\ &= 1 + \frac{9.45 - 4.60}{4.60 + 4.16} \\ &\approx 1 + 0.55 \\ &= \underline{1.55} \end{aligned}$$

Comparison of the results of analysis

<u>Methods</u>	<u>Factors of safety</u>			
	<u>F_s</u>		<u>c/F_sH</u>	<u>F</u>
	<u>Ex. 1</u>	<u>Ex. 2</u>	<u>Ex. 2</u>	<u>Ex. 2</u>
<u>Circular arc</u>				
Janbu's	1.40	1.36	---	---
Slices	1.45	1.40	---	---
Taylor's	---	1.40	0.083	1.79
<u>Logarithmic spiral</u>				
Taylor's	---	---	0.075	1.99
Frohlich's	---	1.55	---	---

On Example 1, Janbu's and Slices method give very similar results. The variation in results is $\pm 1.75\%$. No solution could be found by Taylor's methods using the circular arc or logarithmic spiral surface of failure or by Frohlich's logarithmic spiral method, as these methods do not consider a layered soil condition.

On Example 2, the variations in the results for the F_s is from -4.73% to +8.58%, for C/F_sH is $\pm 5.06\%$, and for F it is $\pm 5.29\%$. Since most of the calculation work has been done by slide rule and the figures for graphical computation are drawn to a limited scale, the variation of the results does not appear unreasonable.

B. DISCUSSION OF RESULTS:

Various computational methods of analysis for landslides have been discussed in previous paragraphs. They indicate the progress that has been made in the analysis of landslides, such as the change from the assumption of a plane surface of failure to a curved surface of failure and from trial and error methods to semi-mathematical methods of computation. Actually, however, when these methods of analysis are applied to a practical problem with the usual complicated soil conditions, a satisfactory analysis is not always possible.

The assumption of a circular arc surface of failure is for convenience in practical analysis. Janbu's method, the most recent technique to use this assumption, utilizes a mathematical solution to locate the critical center of failure circle. It does not give, however, a solution for the layered soil condition where the angles of internal friction of the strata are greater than zero. Most methods of analysis using the circular arc surface of failure, such as Taylor's and Janbu's case of $\phi > 0$, are based upon the assumption that the soil conditions are homogeneous and isotropic. No practical problem is identical with this assumption. For this reason, the method of slices is used extensively for the analysis of the more complex problem.

The logarithmic spiral surface of failure has its theoretical advantages, but the difficulty with both the graphical and the semi-mathematical methods of analysis is the time required to locate the

critical surface of failure and to compute the related forces acting upon the sliding soil mass. Also, it has not permitted an analysis for the case of layered soil conditions. For these reasons, it is rarely used to analyze the practical problem.

For those cases where a mathematical or graphical analysis of slope stability is reasonable, it appears that the best tool presently available is Janbu's method. Computations by this technique are adequate to locate a critical center of rotation which may then be quickly verified by the method of slices. This approach reduces the time and labor otherwise necessary to define the critical center. If Janbu's method is not applicable the moment area or slices methods may be utilized for the stability analysis.

VII. METHODS OF CONTROL

The complexity of the analysis of slides in either natural or man made slopes is so great that there is no one method of control or correction that is satisfactory for all cases. Baker⁽¹⁵⁾ indicates seventeen different techniques suitable as corrective measures for use in stabilizing or controlling sliding areas. (See Table 1) The methods to be discussed in this section will be limited to the more common and more economical procedures.

TABLE 1. FACTORS IN THE DESIGN OF CORRECTIVE MEASURES

<u>Corrective Measure</u>	<u>Best Application</u>	<u>Principles Involved</u>
1. Relocation	All types	Structure moved to location where mass movement is not present.
2. Excavate, drain and backfill	Shallow soil or deep soil in combination with lightweight backfill.	The sliding mass is removed and the area stabilized by the removal of water and by improving foundation conditions. Permanent solution.
3. Drainage: Surface Subsurface Jacked-in-place pipe Tunnels	Subsurface is good method for shallow soil. Tunnel and jacking good for deep soil when water source is deep.	Removal of water results in: (1) A lower density of the soil; (2) a higher soil cohesion; (3) an elimination of lubrication of slip plane; (4) a removal of excess hydrostatic pressure; (5) a removal of seepage forces. Permanent solutions.
4. Removal of Material: Entirely Partial at toe Partial at top	Shallow soil. Clear road for traffic. Deep soil.	Entire slide mass is removed. Permanent solution. Condition relieved temporarily. Expedient solution. A large part of the main source of shearing force is removed. Permanent solution.
5. Buttress at toe	Good foundation at toe in shallow or deep soil. Buttress should extend below the slip plane.	Large mass blocks the mass movement and any further movement involves displacement of the buttress. Permanent solution.

Table 1. (continued)

<u>Corrective Measure</u>	<u>Best Application</u>	<u>Principles Involved</u>
6. Bridging	Steep hillside locations with a narrow failure parallel to road in deep soil.	The troublesome area is bridged and future movement passes under the structure. Permanent solution.
7. Cribbing - timber, concrete or metal	Shallow soil with good foundations.	A retaining mass, with or without additional lateral restraint, placed in path of the mass movement. Further movement involves displacement of the retaining mass. Permanent solution.
8. Retaining wall of stone or concrete	Shallow soil with good foundations.	A retaining mass with lateral restraint, placed in path of the mass movement. Further movement involves displacement of the retaining wall. Permanent solution.
9. Piling: Floating Fixed-no provision for preventing extrusion. Fixed-provision for preventing extrusion.	Shallow soil to hold temporarily. Shallow soil with numerous large rocks. Shallow soil.	Piling offers a retaining influence. For soil masses the piling may tend to "pin" the moving and stable materials, and bring about a more lengthy slip-surface with a corresponding increase in shearing resistance. Floating piles are expedients. All others are permanent solutions.
10. Sealing joint planes and open seams.	Deep source of water.	The same benefits are sought as for a drainage solution. Permanent solution.

Table 1. (Continued)

Corrective Measure	Best Application	Principles Involved
11. Cementation of loose material	Permeable material only, unless "column" principle is to be used.	An attempt to improve the shear characteristics of the soil. For "columns" the weight of the structure is carried to firm foundations. Some lateral restraint is offered. Permanent solution.
12. Chemical treatment --- flocculation	Permeable material.	Same principles as 11, except chemicals are used. Permanent solution.
13. Tie-rod-ding slopes	Shallow soil with large rock fragments.	The retaining device insufficient to withstand movement is anchored to bedrock. Permanent solution.
14. Blasting	Shallow soil underlain by good rock.	An attempt is made to disrupt the sliding surface as well as to develop a drainage system. An expedient solution.
15. Reshaping slide material	Face of slide has open fissures. Bench- es cause ponding of water.	See 3. Method generally used in combination with other actions.
16. Slope treatment	Shallow soil in combination with other methods or any erosion problem.	See 3. Method generally used in combination with other actions.
17. Lightweight Fill	In cases where loading is critical.	Reduction of the weight causing movement. Rarely a solution in itself.

A. Drainage

One of the most common causes of landslides or embankment failures is excess moisture which acts to reduce the shear strength of the mass and contributes, on occasion, a hydrostatic force to the driving forces that are acting on the mass. It would appear, therefore, that a large number of unstable slopes may be stabilized or controlled by properly designed drainage systems. Experience has indicated that drainage facilities to control the inflow of surface water to the slide area and internal drainage within the sliding mass are frequently successful stabilizing measures. It should be realized, however, that though drainage is valuable, it is not always successful nor can the design of the drainage system be standardized. Each design must be developed on the basis of the particular problem.

The following discussion illustrates several methods of drainage that have proven to be successful for slide control.

1. Control of surface water

Rost (16) reports a slide in Monterey County, California that developed shortly after the construction of a highway cut. In this case, surface water appeared to be softening the upper surface of the cut slope to the extent that sliding was occurring. The corrective action consisted of placing a blanket of pervious material over the cut and installing an interceptor drain along the toe. The details are indicated in Fig. 35. The pervious material provided a fast drainage route out of the unstable area for surface waters, while

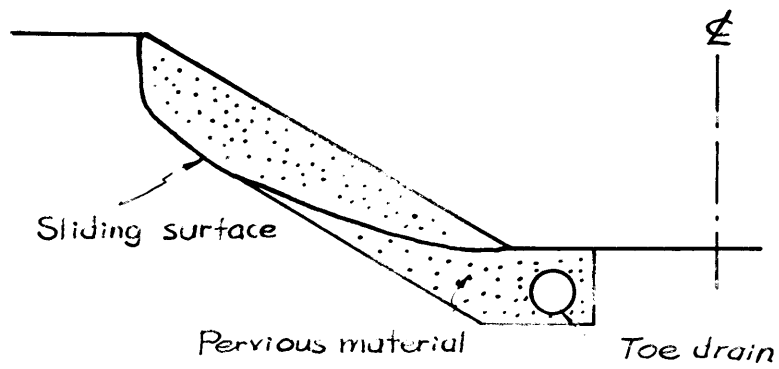


Fig. 35. Typical drain installation for surface water

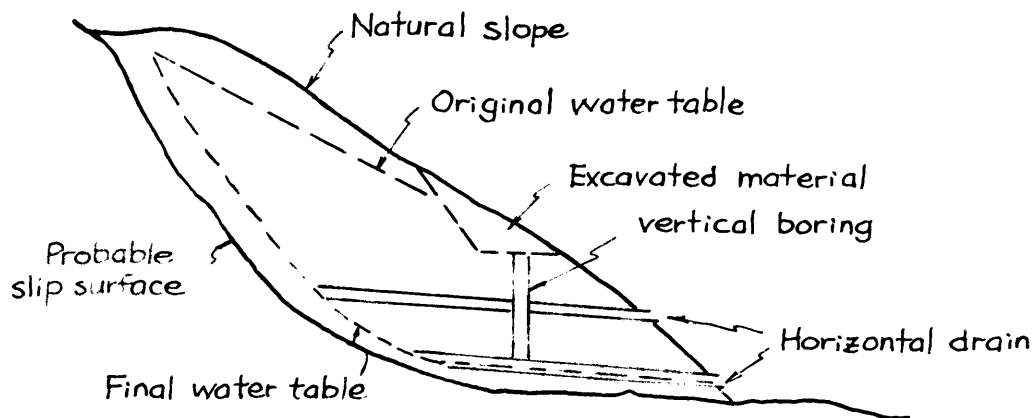


Fig. 36. Typical horizontal drain installation

the toe drain provided a means of removal of the waters collected by the blanket.

In similar situations many agencies have found the use of an interceptor ditch at the head of the slope to be successful in preventing the inflow of surface water.

2. Horizontal Drainage

As used by most agencies, horizontal drainage consists of a series of horizontal perforated drain pipes placed into the slide mass. The drains serve to collect and remove impounded waters that exert hydrostatic forces on the sliding mass and also serve to reduce the shear strength of the material within the slide mass. (Fig. 36)

Arnco Drainage and Metal Products, Inc. (17) have reported the use of this scheme to successfully stabilize a cut slope in the Yellowstone National Park. The sliding mass in this case extended approximately 350 feet up the slope from the toe of a 25 feet deep cut and was about 200 feet in extent along the toe. Exploratory borings indicated the presence of free water under abnormally high pressure beneath the slide. Four 6 inch diameter holes were driven into the slide area on a slope of 2 to 15 degrees above the horizontal. A two inch perforated steel pipe was inserted into each hole. Measured drainage from one of these drains totaled 21,600 gallons per day which decreased to a steady flow of about 17,000 gallons per day in the first two weeks. All drains carried appreciable quantities of water from the slide area. No further movement has been reported from this area.

Other references to similar experiences in the use of horizontal drains may be found in the publications of the Armco Co. (18) and those of various engineers of the California Highway Department. (19, 20)

3. Tunnels

In those cases, where exceedingly large volumes of water must be removed from the slide mass or where very extensive drainage facilities must be utilized tunnels have been used. During the construction of Lookout Point Dam, on the Willamette River in Oregon, the U.S. Army Engineers utilized an extensive system of drainage tunnels to stabilize a slope that was threatening to destroy a re-located highway and railroad. The tunnels were driven by normal mining methods at a slight slope above the horizontal and formed a collecting system for a series of horizontal drains driven into the unstable area from within the tunnel. (21)

4. Typical Drainage Installations

Several typical drainage installations that have been successfully used for slope stabilization are shown in Fig. 37, 38 and 39.

B. Piling

Another method to prevent the landslide of a slope is to force the surface of failure into the deeper underground stratum by the use of piles. It is suggested that long enough piles be used to penetrate past the previous critical position of sliding surface. Then the most dangerous position of sliding surface will be moved down to near the

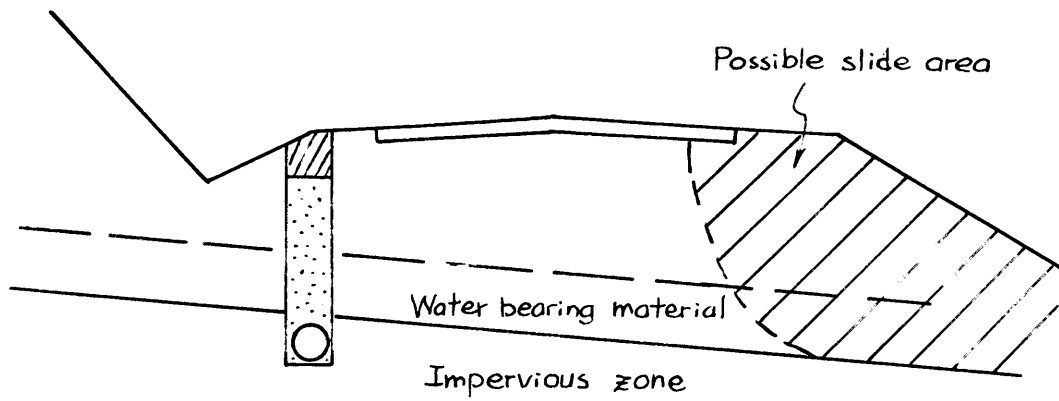


Fig. 37. Typical drainage installation (I)

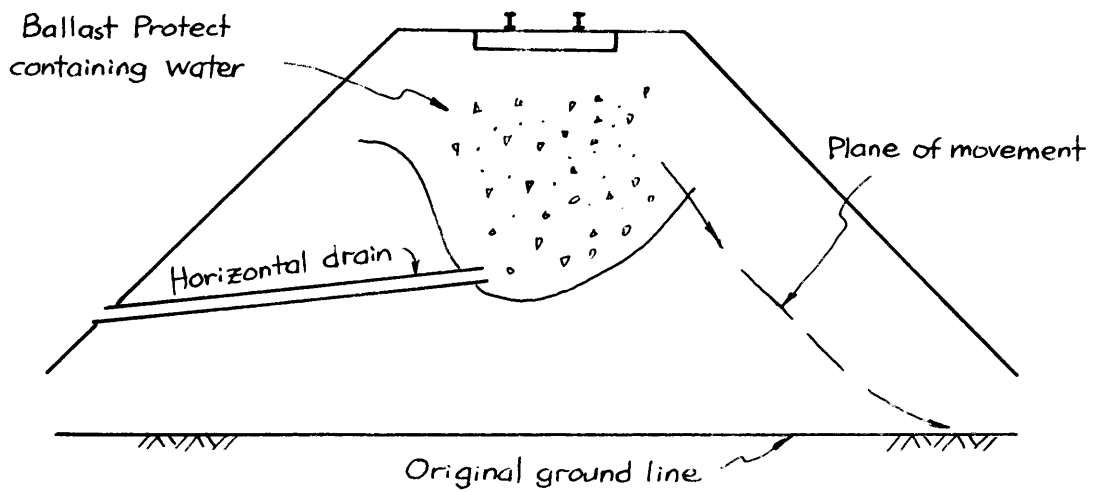


Fig 38. Typical drainage installation (II)

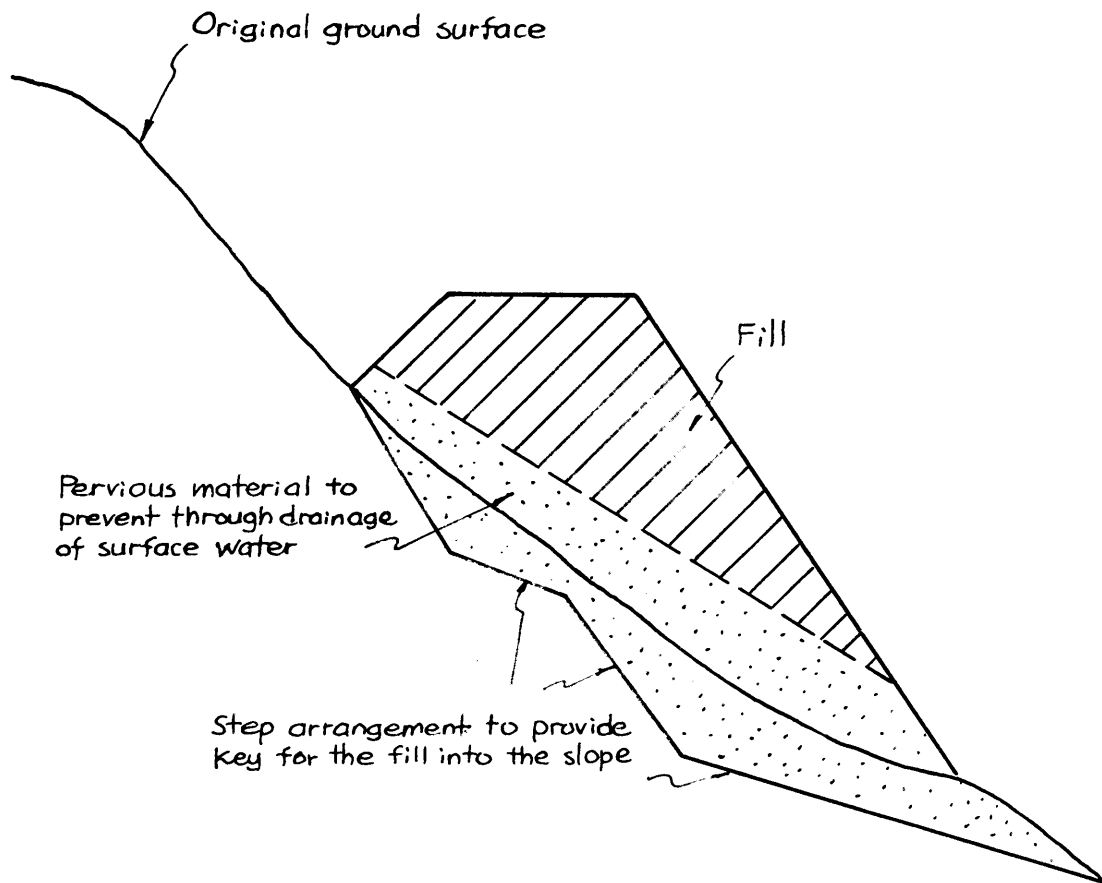


Fig. 39. Typical drainage installation (II)

foot of the piles, and the piles will provide the shearing resistance to raise the factor of safety. Hennes (9) has derived the spacing of piles to prevent slides as follows: (Fig. 40 and 41)

$$\sum t_c - \sum p_c \tan \phi - c L_c - \frac{A_{fp}}{D} = \sum t_f - \sum p_f \tan \phi - c L_f$$

$$D = \frac{A_{fp}}{\sum t_c - \sum t_f + (\sum p_c - \sum p_f) \tan \phi + c (L_f - L_c)} \quad \text{--- (36)}$$

L - length of sliding arc

t - tangential force along sliding surface caused by soil mass

p - vertical force perpendicular to sliding surface caused by soil mass

ϕ - angle of internal friction

c - cohesion of soil

A - cross section area of one pile

D - distance between centers of piles in the direction parallel to the top of the bank

f_p - allowable shear stress of pile

The subscript c distinguishes forces along the original critical sliding surface from those along a sliding surface below the foot of the piles, the latter being marked by a subscript f.

It is also possible that the above criterion will give a spacing so great that the soil mass will shear off on each side of the pile and slide down between the piles. Therefore, another criterion has been established based on the assumption that the total cross section area of piles must be so large as to not exceed the

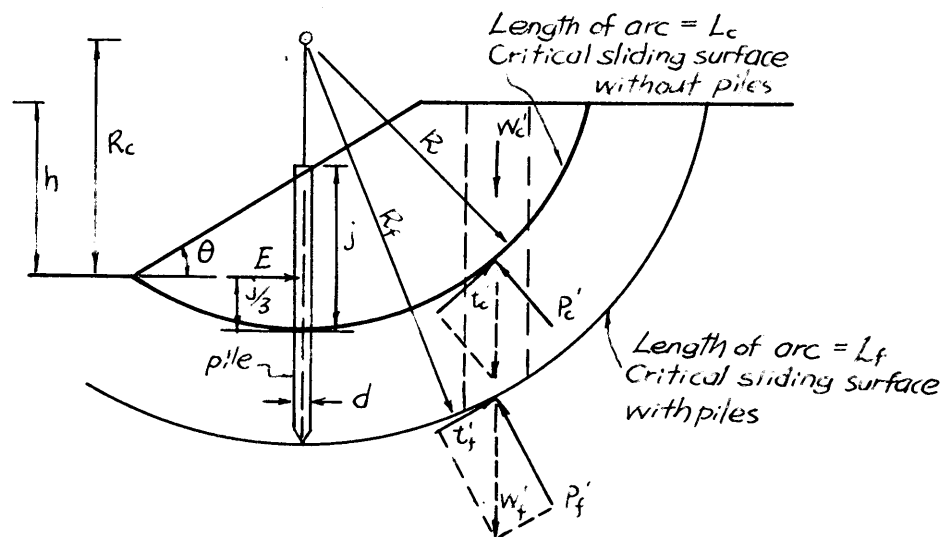


Fig 40. Section view of piling

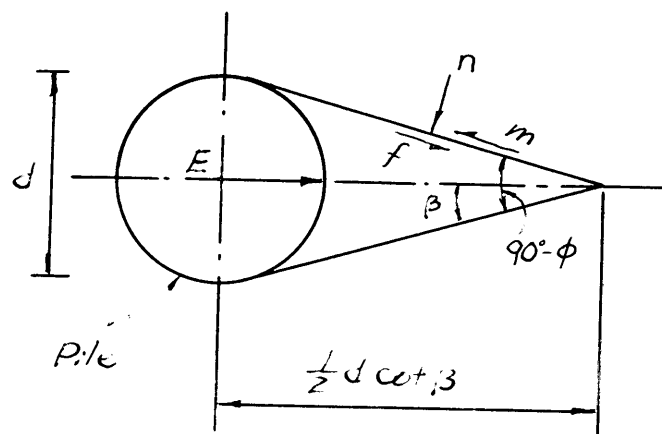


Fig. 41. Plane view of piling (9)
(After Hennes)

lateral bearing value of the soil. As Fig. 40 and 41, the summation of moments about the center of the arc of radius of R_c equals zero:

$$E(R - \frac{1}{3})d = D(\bar{\Sigma} t_c - \bar{\Sigma} p_c \tan \phi - c h_c) R$$

$$E = \frac{D(\bar{\Sigma} t_c - \bar{\Sigma} p_c \tan \phi - c h_c) R}{d(R - \frac{1}{3})}$$

If F is the desired factor of safety against sliding, then

$$E = \frac{F D(\bar{\Sigma} t_c - \bar{\Sigma} p_c \tan \phi - c h_c) R}{d(R - \frac{1}{3})} \quad \dots (37)$$

As Fig.

$$n = \frac{1}{2} E \sin \beta$$

$$m = \frac{1}{2} E \cos \beta$$

$$f = n \tan \phi + c j \left(\frac{d}{2 \cos \beta} \right)$$

For equilibrium $\bar{\Sigma} F_x = 0$, $m = f$

$$E \cos \beta = E \sin \beta \tan \phi + \frac{c j d}{\cos \beta}$$

$$E (\cos \beta - \sin \beta \tan \phi) = \frac{c j d}{\cos \beta}$$

$$E = \frac{c j d}{(\cos \beta - \sin \beta \tan \phi) \cos \beta} = 2 c j d$$

Substitute E into ()

$$2 c j d = \frac{F D(\bar{\Sigma} t_c - \bar{\Sigma} p_c \tan \phi - c h_c) R}{d(R - \frac{1}{3})}$$

$$\frac{D}{d^2} = \frac{2 c (3R - j) j}{3 F (\bar{\Sigma} t_c - \bar{\Sigma} p_c \tan \phi - c h_c) R} \quad \dots (38)$$

C. Retaining Walls

As indicated in Table 1 , there are several types of retaining devices suitable for the control of sliding masses. In all of these methods the principle involved is similar - an artificial blocking device is placed in the path of the slide between the area to be protected and the toe of the slide.

This system of control is of limited use. To be successful, the blocking device must be extended sufficiently deep to penetrate the stable material below the sliding mass and must be anchored firmly to the stable material. If these criteria are not satisfied, the slide may conceivably move the retaining structure down the slope with the moving mass. Structurally, the device must be able to withstand the thrust of the moving material. The above considerations imply that this type of control is expensive and applicable only in restricted cases. A failure of the installation will result in complete loss of all of the investment in the control structure.

On the advantage side of the retaining device solution, the restricted area necessary for the installation must be considered. The limited space required will permit the minimum right-of-way damage to the area. In addition, this type of construction may be utilized to exert a control over only that part of a sliding area that may be critical.

The design of a retaining structure is a typical retaining wall design problem and offers no exceptional problems. It should be

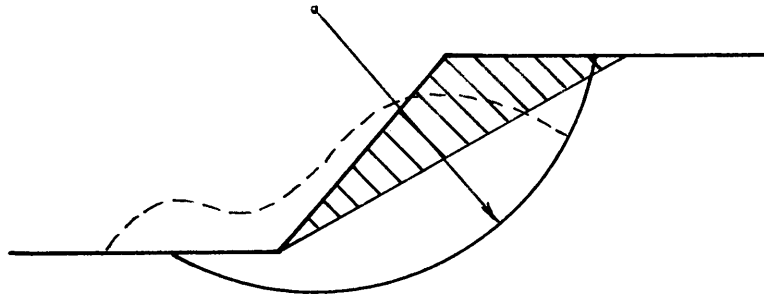


Fig.42. Stabilizing a slide by removal of material

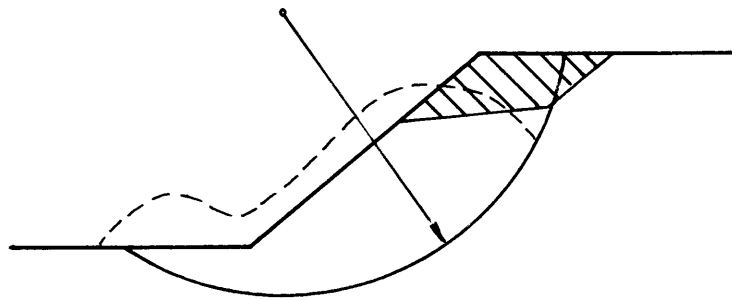


Fig. 43 Stabilizing a slide by removal of material

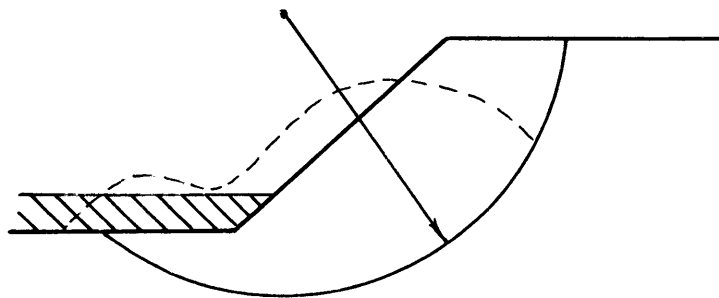


Fig. 44. Stabilizing a slide by addition of material

realized at all times, however, that there is a primary necessity of providing adequate drainage through the structure for impounded waters within the controlled mass.

D. Removal or Addition of Material

Any corrective measures which decreases the overturning or sliding moment or increases the resisting moment will be of assistance in stabilizing a slide area. The most usual applications of this method are illustrated by Figures 42, 43 and 44.

Terzaghi (22) discusses the use of toe or counter weight fill to eliminate the tendency of a slide type failure through an embankment constructed over a soft foundation. The principle of the counterweight fill is simple. The moment of the added load of the counterweight fill acts in opposition to the driving moment of the potential slide. Design is accomplished through two considerations; (1) the counterweight fill must be stable within itself and (2) the load exerted by the fill must be sufficient to balance the excess driving force of the slide mass.

The value of reducing the slope angle of an unstable slope and the removal of load from the slide mass is obvious from an inspection of Figures 42 and 43. Any modification of these procedures that accomplishes a reduction in the driving force on the unstable area is advantageous.

VIII. CONCLUSIONS

Since the earliest recorded history of man, the stability of natural and artificial slopes has been of major concern to builders. Early attempts to avoid slope failure are lost in antiquity but it may reasonably be assumed that a great deal of engineering effort was expended on these problems.

The development of modern concepts of slope analysis was given its major impetus by the Swedish investigators just after the turn of the Twentieth Century. Since that time numerous techniques of mathematical and graphical analysis have been advanced and are in use. A discussion of the most representative and useful of these techniques has been presented. Examples of typical design problems using the selected methods has been presented to illustrate both the procedure of analysis and the difficulties associated with each.

A comparison of the results of the illustrative examples of the methods of analysis indicates a remarkable uniformity among them. With considerations of time and mathematical complexity, it appears that the method of Janbu is ideally suited for locating the most critical center of rotation for a particular case. Once the critical center has been established a detailed analysis for the problem in question may be accomplished by the method of slices or one of the other similar procedures. For the case of a stratum with , Janbu's method is of little assistance unless simplifying assumptions may be made. If this is not possible, the only reasonable approach

to the problem is one of trial and error to determine the minimum factor of safety using the method of slices.

The section of this report devoted to the control of slides serves to illustrate the more typical situation of a movement, that is not subject to a theoretical analysis. The great majority of slides that occur in natural ground pass through complex soil stratifications that do not permit the simplifying assumptions needed for the various methods of mathematical and graphical analysis. In these cases, the procedures for the control and stabilization of the moving material depend exclusively on the characteristics of the particular location. Each slide is a separate problem whose solution is not necessarily similar to any other.

The control section of this report has presented a summary of the various methods for the field control of slides that may be used. The choice of the applicable method is made only after extensive study of the nature of the particular slide. The more common control procedures have been illustrated by reference to the case histories of actual slides that have been successfully controlled.

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