Is Quantum Gravity a Super-Quantum Theory?

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Abstract

We argue that quantum gravity should be a super-quantum theory, that is, a theory whose non-local correlations are stronger than those of canonical quantum theory. As a super-quantum theory, quantum gravity should display distinct experimentally observable super-correlations of entangled stringy states.

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1. Three Approaches to Quantum Gravity

Any quantum theory of gravity must necessarily be a theory which encompasses special relativity, quantum mechanics (QM), and Newtonian Gravity. Representing these three essential ingredients with the three fundamental constants c, $\hbar$, and G_N, we can classify the approaches to quantum gravity according to the ordering in which these constants are incorporated.

In the first approach, c and $\hbar$ are incorporated into relativistic quantum field theory (QFT), to which string theory adds G_N via the introduction of the length scale $\ell_s = \ell_P = \sqrt{G_N \hbar / c^3}$ [1]. String theory is a natural extension of QFT to an infinite number of fields. One of its massless excitations is the spin-2 graviton so features of general relativity (GR) emerge at long distances. However, despite recent advances in non-perturbative string theory, its basic principles are yet to be understood. The issues of background independence, the nature of the Big Bang singularity, the vacuum energy problem, as well as the origins of the Standard Model of particle physics and of dark matter remain unresolved. One thing is certain in this approach: gravity is emergent and not fundamental, and thus spacetime is as well. The nature of the underlying quantum degrees of freedom from which spacetime and gravity must emerge is still mysterious.

The second approach starts from GR, which unifies c and G_N, and attempts to quantize its dynamical degrees of freedom, *i.e.* spacetime itself. Perhaps the best example of this approach is given by loop quantum gravity [2]. In this approach the geometric nature of gravity is taken into account from the beginning. The resulting structure exploits the properties of non-local loop variables, yet is radically different from string theory, as string theory also features higher dimensionality of spacetime, supersymmetry, and supergravity. Loop gravity does not require these additional structures and essentially proposes a discrete fundamental structure for spacetime that is consistent with Lorentz symmetry and background independence. In this approach, matter and gravity do not go hand in hand as in string theory, and the origin of the matter sector is usually not considered to be one of its central questions.

The third avenue starts from a theory defined by $\hbar$ and G_N, a non-relativistic quantum theory of gravity [3]. One version of this unconventional, yet logically viable, option was advocated by us in Ref. [4], and begins with Matrix theory, a version of light-cone string field theory at infinite coupling, and also a natural regularization of the theory of membranes, viewed as solitonic objects in a strongly coupled string theory of a particular type [5]. Though Matrix theory does not look like a theory of gravity, being a supersymmetric Matrix QM with a non-Abelian gauge structure, it reproduces Newton's law of gravitation at long distances, and attains relativistic covariance in a limit defined by an infinite number of its quanta. To Matrix theory, we add a new ingredient: in order to address the question of background independence, we argue that the canonical geometric structure of QM, represented by complex projective spaces, must be made dynamical and generalized to the non-linear Grassmannians [6]. Essentially, our proposal is a generalized geometric Matrix

QM, which takes the central lessons of the first two approaches quite seriously. It uses Matrix theory, which appears in the context of non-perturbative string theory, and it emphasizes geometry and background independence, which are the hallmarks of loop quantum gravity.

In all three approaches, the attempts to incorporate the third constant into the formalisms suggest that fundamental departures from the conventional QM framework is inevitable for quantum gravity. What is the new framework of physics that quantum gravity demands? This new framework should not only deepen our understanding of gravity, but also shed light on the mysteries of QM, just as QM had shed light on the structure of classical mechanics. Independently of what this framework may be, we argue that it would be characterized by correlations which are stronger than those of QM in the context of Bell's inequalities [7].

2. Super-Quantum Correlations and Quantum Gravity

Here, we adopt the version of Bell's inequality as formulated in Ref. [8]. Let A and B represent the outcomes of measurements performed on some isolated physical system by detectors 1 and 2 which are placed at two causally disconnected spacetime locations. Assume that the only possible values of A and B are ± 1. Let $P(a, b) = \langle A(a)B(b) \rangle$ be the expectation value of the product $A(a)B(b)$ where a and b respectively denote the settings of detectors 1 and 2. Then, the upper bound, X, of the following combination of correlators, for arbitrary detector settings a, a', b, b', characterizes each underlying theory:

$$\left| P(a, b) + P(a, b') + P(a', b) - P(a', b') \right| \leq X . \tag{1}$$

This bound for classical hidden variable theories is $X_{\text{Bell}} = 2$, while that for QM is $X_{\text{QM}} = 2\sqrt{2}$. That is, QM correlations violate the classical Bell bound but are themselves bounded [9]. The maximum possible value of X is 4, and the requirement of relativistic causality does not preclude correlations which saturate this absolute bound [10].

What type of theory would predict such super-quantum correlations? Since the process of quantization increases the bound from $X_{\text{Bell}} = 2$ to $X_{\text{QM}} = 2\sqrt{2}$, we proposed in Ref. [11] the naive conjecture that another step of "quantization" would further increase the bound by a factor of $\sqrt{2}$ to 4.

What procedure would such a "double" quantization entail? Quantization demands that correlation functions of operators be calculated via the path integral

$$\langle\, \hat{A}(a)\hat{B}(b)\,\rangle \;=\; \int Dx \; A(a, x)\, B(b, x)\, \exp\left[\frac{i}{\hbar}S(x)\right] \;\equiv\; A(a) \star B(b) , \tag{2}$$

where x collectively denotes the classical dynamical variables of the system. In a similar fashion, we can envision performing another step of quantization by integrating over "paths" of quantum operators to define correlators of "super" quantum operators

$$\langle\!\langle\, \hat{\hat{A}}(a)\hat{B}(b)\,\rangle\!\rangle \;=\; \int D\hat{\phi} \; \hat{A}(a, \hat{\phi})\, \hat{B}(b, \hat{\phi})\, \exp\left[\frac{i}{\hbar}\tilde{S}(\hat{\phi})\right] , \tag{3}$$

where $\hat{\hat{\phi}}$ collectively denotes the dynamical quantum operators of the system. Here, $\langle\!\langle\hat{\hat{A}}(a)\hat{\hat{B}}(b)\rangle\!\rangle$ is an operator. To further reduce it to a number, we must calculate its expectation value in the usual way

$$\langle\!\langle\, \hat{\hat{A}}(a)\hat{\hat{B}}(b)\,\rangle\!\rangle \quad \rightarrow \quad \langle\!\langle\!\langle\, \hat{\hat{A}}(a)\hat{\hat{B}}(b)\,\rangle\!\rangle\!\rangle \;=\; \left\langle \int D\hat{\phi}\; \hat{A}(a,\hat{\phi})\, \hat{B}(b,\hat{\phi})\, \exp\left[\frac{i}{\hbar}\tilde{S}(\hat{\phi})\right]\right\rangle, \quad (4)$$

which would amount to replacing all the products of operators on the right-hand side with their first-quantized expectation values, or equivalently, replacing the operators with 'classical' variables except with their products defined via Eq. (2). Note that this is precisely the formalism of Witten's open string field theory (OSFT) [12], in which the action for the 'classical' open string field Φ is given formally as

$$S_W(\Phi) \;=\; \int \Phi \star Q_{\mathrm{BRST}}\Phi + \Phi \star \Phi \star \Phi \,, \tag{5}$$

where Q_{BRST} is the open string theory BRST cohomology operator ($Q^2_{\mathrm{BRST}} = 0$), and the star product is defined via a world-sheet path integral weighted with the Polyakov action and deformation parameter $\alpha' = \ell_s^2$. The fully quantum OSFT is then, in principle, defined by yet another path integral in the infinite dimensional space of the open string field Φ, i.e.

$$\int D\Phi \, \exp\left[\frac{i}{g_s}S_W(\Phi)\right] \,, \tag{6}$$

where g_s is the string coupling and all products are defined via the star-product. For reasons of unitarity, OSFT must contain closed strings, and therefore gravity. Thus, OSFT is a manifestly "doubly" quantized theory, and we argue that it, and the theory of quantum gravity it should become, would be characterized by super-quantum correlations when fully formulated.

Further insight can be obtained from toy models, such as those in Refs. [13] and [14]. Specifically, super-quantum correlations were found in Ref. [14]. There, the probabilities of individual outcomes were indeterminate while the expectation values of observables were determinable. This suggests that super-quantum correlations would result from a theory in which probability distributions themselves are probabilistically determined, again pointing to a "double" quantization.

"Double" quantization can also be qualitatively seen in loop quantum gravity, as spacetime itself is quantized before QFT is quantized on top of it. It is also likely present in the previously discussed geometric generalization of Matrix QM, as the additional quantum dynamics of the state space should represent a "second" quantization. Though super-correlations are not (currently) clearly present in these contexts, we expect that they should occur. If so, it would add further support to the notion that super-correlations are a generic aspect of quantum gravity.

3. Experimental Signatures?

In closing, let us offer some comments on possible experimental observations of such super-quantum violations of Bell's inequalities in quantum gravity.

The usual experimental setup for testing the violation of Bell's inequalities in QM involves entangled photons [15]. In OSFT, photons are the lowest lying massless states, but there is a whole Regge trajectory associated with them. The obvious experimental suggestion is to look for effects from entangled Reggeized photons. Such experiments are of course impossible at present, given their Planckian nature.

A more feasible place to look for super-quantum correlations could be in cosmological data. It is believed that quantum fluctuations seed the large scale structure of the Universe, i.e. galaxies and clusters of galaxies that we observe [16]. The simplest models use Gaussian quantum correlations, though non-Gaussian correlations are envisioned as well and are constrained by new data on the cosmic microwave background (CMB) from the Planck satellite [17]. While it is yet unclear how super-quantum correlations would affect the CMB data, we expect that they would leave "stringy" imprints on the large scale structure of the Universe and be observable at those scales.

Acknowledgements

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