

Chapter 2: Formulation

2.1 Assumptions

Before building the computer model to analyze the structure, some assumptions were necessary. First, the breakwater is modeled as if in air with a net buoyant force acting upwards on the structure through its center of mass. Therefore, the fluid inertia and damping effects that would be added to the structure if a more realistic analysis were conducted are neglected. The reason for this is that it is very difficult, and beyond the scope of this thesis, to analyze the nonlinear fluid-structure-interaction problem of the cylinder performing large motions due to waves. Thus, the wave forces are represented as regular sinusoidal waves, as will be explained in detail in Chapter 4. A third assumption made involves the damping in the system. Linear damping is used and it is assumed that this is an adequate model of the fluid damping. Finally, the mooring lines are assumed to be massless and are modeled as taut, linearly elastic springs and compressionless springs. The mooring lines are assumed to be rigidly attached to the seafloor.

2.2 Equilibrium Configuration of Breakwater

The basic configuration of the breakwater is arranged to take advantage of symmetry. The profile, plan, and end views of the structure in equilibrium are shown in Figures 2.1-2.3, respectively. A rendered three-dimensional drawing of the breakwater is also provided in Figure 2.4. Figure 2.1 shows that the origin of the global X-Y-Z axes is located on the ocean floor directly below the cylinder's center of mass. The X axis is parallel to the axis of the cylinder and the Y axis is upward. The x-y-z axes are fixed in the cylinder with their origin at the center of mass, the x axis along the cylinder's axis, and the y axis upward in equilibrium. The cylinder's axis is parallel to the ocean floor and is parallel to the shoreline or to the structure the breakwater would be designed to protect. The length of the cylinder is L and the radius is R . The equilibrium position of the center of mass of the cylinder is defined by the global coordinates X_{c_eq} , Y_{c_eq} , and Z_{c_eq} . Alternatively, this location can be described as $(0, Y_{c_eq}, 0)$.

Four mooring lines connected to the ends of the cylinder are symmetrically attached to the ocean floor. As shown in the drawings of the breakwater, at equilibrium the lines are connected to the cylinder ends in the horizontal plane of the structure's center of mass. Each mooring line is then connected to the ocean floor a distance, a , away from the cylinder in the X direction, and a distance, b , away from the cylinder in the Z direction. The dimensions used for this breakwater are the same as those used in Mays et al. (1999): $L = 30$ ft, $R = 5$ ft, $a = 10$ ft, $b = 10$ ft, and $l_i = 20$ ft, the natural (unstrained) length of each mooring line ($i = 1,2,3,4$).

2.3 Model of the Mooring Lines

As mentioned in Section 2.1, the mooring lines are modeled first as taut, linearly elastic springs ("regular" springs). The mass of the mooring lines is neglected, which was proved to be a valid assumption by Mays (1997) with regard to equilibrium. He showed that regardless of whether the lines were modeled as massless springs or as multiple springs with lumped masses, the static equilibrium position of the cylinder was not significantly changed. It is important to note that this is the mooring line model that will be used to determine the equilibrium position of the breakwater and the structure's modes of vibration. Numerous forcing analyses will be conducted using this model. A second model using compressionless springs to represent the slackening effects of true mooring lines will also be considered under forcing and will be explained in detail in Chapter 5.

2.4 Equations of Motion

The derivation of the EOM's of this structure is detailed in Mays (1997). A summary of the applicable steps taken in this lengthy derivation is provided as follows.

2.4.1 General Breakwater Configuration

Before the EOM's are shown, many terms need to be described. The six degrees of freedom of the structure are defined as follows: surge is represented by X , heave by Y , sway by Z , pitch by ψ , yaw by θ , and roll by ϕ . The motion of the cylinder is traced by the location of X_c , Y_c , Z_c , ψ ,

θ , and ϕ at any point in time, with respect to the origin of the X-Y-Z axes. The subscript c denotes the location of the cylinder's center of mass.

Figure 2.5 shows the breakwater in an arbitrary state. The local axes of the cylinder are the x-y-z axes. The point where the spring and the cylinder connect is called Point A_i (a_{i1}, a_{i2}, a_{i3}), where subscript i denotes the specific mooring line. In the X-Y-Z global axes, Point A_i is located at ($X_c + a_{i1}, Y_c + a_{i2}, Z_c + a_{i3}$) in equilibrium, and is determined by adding the two vectors \mathbf{r}_c and \mathbf{A} during motion. Mays (1997) developed the transformation matrix that allowed Point A_i to be defined as a function of $X_c, Y_c, Z_c, \psi, \theta$, and ϕ . The dimensions for point A_i are: $a_{11} = 15$ ft, $a_{12} = 0$, $a_{13} = 5$ ft, $a_{21} = 15$ ft, $a_{22} = 0$, $a_{23} = -5$ ft, $a_{31} = -15$ ft, $a_{32} = 0$, $a_{33} = 5$ ft, $a_{41} = -15$ ft, $a_{42} = 0$, and $a_{43} = -5$ ft.

2.4.2 Lagrange's Equations

Mays (1997) used Lagrange's Equations to derive the EOM's of the breakwater. First, the kinetic and potential energies, T and V respectively, of the mooring lines and the cylinder were developed. However, in the present case, the mooring line energy only consists of potential energy since the lines are massless. The Lagrangian was then found for the mooring line and for the cylinder using the following equations:

$$L_i = T_i - V_i \quad (2.1)$$

$$L_c = T_c - V_c \quad (2.2)$$

where L_i is the Lagrangian for the mooring lines and L_c is the Lagrangian for the cylinder. The total Lagrangian of the system is then

$$L = L_i + L_c. \quad (2.3)$$

Lagrange's Equations of Motion neglecting damping and forcing are then derived from

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{Q}_j} \right) - \frac{\partial L}{\partial Q_j} = 0 \quad j = 1, 2, 3, \dots, N \quad (2.4)$$

or

$$\frac{d}{dt} \left(\frac{\partial L_i}{\partial \dot{Q}_j} \right) - \frac{\partial L_i}{\partial Q_j} + \frac{d}{dt} \left(\frac{\partial L_c}{\partial \dot{Q}_j} \right) - \frac{\partial L_c}{\partial Q_j} = 0 \quad j = 1,2,3,\dots,N \quad (2.5)$$

where Q_j represents each of the N independent variables in the system. Therefore, in the present case, six Lagrange's EOM's would result from the above procedure, one for each of the six degrees of freedom. The equations presented here are corrected versions of the equations of Mays (1997) along with the simplification that the principal moments of inertia I_{yy} and I_{zz} are equal.

The final six EOM's are:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{X}_c} \right) - \frac{\partial L}{\partial X_c} = 0 \quad (2.6)$$

or,

$$\begin{aligned} m_c \ddot{X}_c + \sum_{i=1}^4 \{ K_i [X_c + a_{i1} \cos \theta \cos \psi + a_{i2} (\sin \phi \sin \theta \cos \psi - \sin \psi \cos \phi) + a_{i3} (\sin \theta \cos \phi \cos \psi + \\ \sin \phi \sin \psi) - X_i] - K_i \{ [X_c + a_{i1} \cos \theta \cos \psi + a_{i2} (\sin \phi \sin \theta \cos \psi - \sin \psi \cos \phi) + \\ a_{i3} (\sin \theta \cos \phi \cos \psi + \sin \phi \sin \psi) - X_i]^2 + [Y_c + a_{i1} \sin \psi \cos \theta + a_{i2} (\sin \psi \sin \phi \sin \theta + \\ \cos \phi \cos \psi) + a_{i3} (\sin \theta \cos \phi \sin \psi - \sin \phi \cos \psi) - Y_i]^2 + [Z_c - a_{i1} \sin \theta + a_{i2} \sin \phi \cos \theta + \\ a_{i3} \cos \theta \cos \phi - Z_i]^2 \}^{-1/2} [X_c + a_{i1} \cos \theta \cos \psi + a_{i2} (\sin \phi \sin \theta \cos \psi - \sin \psi \cos \phi) + \\ a_{i3} (\sin \theta \cos \phi \cos \psi + \sin \phi \sin \psi) - X_i] \} = 0; \end{aligned} \quad (2.7)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{Y}_c} \right) - \frac{\partial L}{\partial Y_c} = 0 \quad (2.8)$$

or,

$$\begin{aligned} m_c \ddot{Y}_c - w_c + \sum_{i=1}^4 \{ K_i [Y_c + a_{i1} \sin \psi \cos \theta + a_{i2} (\sin \psi \sin \phi \sin \theta + \cos \phi \cos \psi) + a_{i3} (\sin \theta \cos \phi \sin \psi \\ - \sin \phi \cos \psi) - Y_i] - K_i \{ [X_c + a_{i1} \cos \theta \cos \psi + a_{i2} (\sin \phi \sin \theta \cos \psi - \sin \psi \cos \phi) + \\ a_{i3} (\sin \theta \cos \phi \cos \psi + \sin \phi \sin \psi) - X_i]^2 + [Y_c + a_{i1} \sin \psi \cos \theta + a_{i2} (\sin \psi \sin \phi \sin \theta + \\ \cos \phi \cos \psi) + a_{i3} (\sin \theta \cos \phi \sin \psi - \sin \phi \cos \psi) - Y_i]^2 + [Z_c - a_{i1} \sin \theta + a_{i2} \sin \phi \cos \theta + \\ a_{i3} \cos \theta \cos \phi - Z_i]^2 \}^{-1/2} [Y_c + a_{i1} \sin \psi \cos \theta + a_{i2} (\sin \psi \sin \phi \sin \theta + \cos \phi \cos \psi) \end{aligned} \quad (2.9)$$

$$+ a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - Y_i\} = 0;$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{Z}_c} \right) - \frac{\partial L}{\partial Z_c} = 0 \quad (2.10)$$

or,

$$\begin{aligned} m_c \ddot{Z}_c + \sum_{i=1}^4 \{ & K_i [Z_c - a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - Z_i] - K_{i1} \{ [X_c + a_{i1}\cos\theta\cos\psi + \\ & a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - X_i]^2 + [Y_c + \\ & a_{i1}\sin\psi\cos\theta + a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - Y_i]^2 + \\ & [Z_c - a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - Z_i]^2 \}^{-1/2} [Z_c - a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - \\ & Z_i] \} = 0; \end{aligned} \quad (2.11)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\Psi}} \right) - \frac{\partial L}{\partial \Psi} = 0 \quad (2.12)$$

or,

$$\begin{aligned} (I_{xx}\sin^2\theta + I_{yy}\cos^2\theta) \ddot{\Psi} - (I_{xx}\sin\theta) \ddot{\phi} - I_{xx}(\dot{\phi} \dot{\theta} \cos\theta - 2\dot{\psi} \dot{\theta} \sin\theta\cos\theta) - I_{yy}(2\dot{\psi} \dot{\theta} \cos\theta\sin\theta) \\ + \sum_{i=1}^4 \{ K_i [X_c + a_{i1}\cos\theta\cos\psi + a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \\ \sin\phi\sin\psi) - X_i] [-a_{i1}\cos\theta\sin\psi - a_{i2}(\sin\phi\sin\theta\sin\psi + \cos\psi\cos\phi) - a_{i3}(\sin\theta\cos\phi\sin\psi - \\ \sin\phi\cos\psi)] + K_i [Y_c + a_{i1}(\sin\psi\cos\theta) + a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \\ \sin\phi\cos\psi) - Y_i] [a_{i1}\cos\theta\cos\psi + a_{i2}(\cos\psi\sin\phi\sin\theta - \cos\phi\sin\psi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \\ \sin\phi\sin\psi)] - K_{i1} \{ [X_c + a_{i1}\cos\theta\cos\psi + a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi \\ + \sin\phi\sin\psi) - X_i]^2 + [Y_c + a_{i1}\sin\psi\cos\theta + a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \\ \sin\phi\cos\psi) - Y_i]^2 + [Z_c - a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - Z_i]^2 \}^{-1/2} \{ [X_c + a_{i1}\cos\theta\cos\psi \\ + a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - X_i] [-a_{i1}\cos\theta\sin\psi - \\ a_{i2}(\sin\phi\sin\theta\sin\psi + \cos\psi\cos\phi) - a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi)] + [Y_c + a_{i1}\sin\psi\cos\theta + \end{aligned} \quad (2.13)$$

$$a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - Y_i][a_{i1}\cos\theta\cos\psi + a_{i2}(\cos\psi\sin\phi\sin\theta - \cos\phi\sin\psi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi)]\} = 0;$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \quad (2.14)$$

or,

$$I_{yy}\ddot{\theta} + I_{xx}(\dot{\phi}\dot{\psi}\cos\theta - \dot{\psi}^2\cos\theta\sin\theta) + I_{yy}\dot{\psi}^2\cos\theta\sin\theta + \sum_{i=1}^4 \{K_i[X_c + a_{i1}\cos\theta\cos\psi + a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - X_i][-a_{i1}\cos\psi\sin\theta + a_{i2}\sin\phi\cos\theta\cos\psi + a_{i3}\cos\theta\cos\phi\cos\psi] + K_i[Y_c + a_{i1}\sin\psi\cos\theta + a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - Y_i][-a_{i1}\sin\psi\sin\theta + a_{i2}\sin\psi\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi\sin\psi] + K_i[Z_c - a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - Z_i](-a_{i1}\cos\theta - a_{i2}\sin\phi\sin\theta - a_{i3}\sin\theta\cos\phi) - K_{i1}\{[X_c + a_{i1}\cos\theta\cos\psi + a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - X_i]^2 + [Y_c + a_{i1}\sin\psi\cos\theta + a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - Y_i]^2 + [Z_c - a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - Z_i]^2\}^{-1/2}\{[X_c + a_{i1}\cos\theta\cos\psi + a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - X_i][-a_{i1}\cos\psi\sin\theta + a_{i2}\sin\phi\cos\theta\cos\psi + a_{i3}\cos\theta\cos\phi\cos\psi] + [Y_c + a_{i1}\sin\psi\cos\theta + a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - Y_i][-a_{i1}\sin\psi\sin\theta + a_{i2}\sin\psi\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi\sin\psi] + [Z_c - a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - Z_i](-a_{i1}\cos\theta - a_{i2}\sin\phi\sin\theta - a_{i3}\sin\theta\cos\phi)\}\} = 0;$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = 0 \quad (2.16)$$

or,

$$-(I_{xx}\sin\theta)\ddot{\psi} + I_{xx}\ddot{\phi} - I_{xx}\dot{\psi}\dot{\theta}\cos\theta + \sum_{i=1}^4 \{K_i[X_c + a_{i1}\cos\theta\cos\psi + a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - X_i](a_{i2}(\cos\phi\sin\theta\cos\psi + \sin\psi\sin\phi) - a_{i3}(\sin\theta\sin\phi\cos\psi - \sin\psi\cos\phi)) + K_i[Y_c + a_{i1}(\sin\psi\cos\theta) + a_{i2}(\sin\psi\sin\phi\sin\theta +$$

$$\begin{aligned}
& \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - Y_i][a_{i2}(\sin\psi\cos\phi\sin\theta - \sin\phi\cos\psi) - \\
& a_{i3}(\sin\theta\sin\phi\sin\psi + \cos\phi\cos\psi)] + K_i[Z_c - a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - \\
& Z_i](a_{i2}\cos\phi\cos\theta - a_{i3}\cos\theta\sin\phi) - K_i l_i \{ [X_c + a_{i1}\cos\theta\cos\psi + a_{i2}(\sin\phi\sin\theta\cos\psi - \\
& \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - X_i]^2 + [Y_c + a_{i1}\sin\psi\cos\theta + \\
& a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - Y_i]^2 + [Z_c - \\
& a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - Z_i]^2 \}^{-1/2} \{ [X_c + a_{i1}\cos\theta\cos\psi + \\
& a_{i2}(\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + a_{i3}(\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - \\
& X_i][a_{i2}(\cos\phi\sin\theta\cos\psi + \sin\psi\sin\phi) - a_{i3}(\sin\theta\sin\phi\cos\psi - \sin\psi\cos\phi)] + [Y_c + \\
& a_{i1}\sin\psi\cos\theta + a_{i2}(\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + a_{i3}(\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) \\
& - Y_i][a_{i2}(\sin\psi\cos\phi\sin\theta - \sin\phi\cos\psi) - a_{i3}(\sin\theta\sin\phi\sin\psi + \cos\phi\cos\psi)] + [Z_c - \\
& a_{i1}\sin\theta + a_{i2}\sin\phi\cos\theta + a_{i3}\cos\theta\cos\phi - Z_i](a_{i2}\cos\phi\cos\theta - a_{i3}\cos\theta\sin\phi) \} \} = 0,
\end{aligned} \tag{2.17}$$

where

$$I_{xx} = \frac{1}{2} m_c R^2 \tag{2.18}$$

$$I_{yy} = \frac{1}{4} m_c R^2 + \frac{1}{12} m_c L^2 \tag{2.19}$$

$$I_{zz} = I_{yy} \tag{2.20}$$

are the principal moments of inertia for a solid cylinder and

X_i = X coordinate of the base of spring i; $X_1 = 25$ ft, $X_2 = 25$ ft, $X_3 = -25$ ft, $X_4 = -25$ ft

Y_i = Y coordinate of the base of spring i; $Y_1 = 0$, $Y_2 = 0$, $Y_3 = 0$, $Y_4 = 0$

Z_i = Z coordinate of the base of spring i; $Z_1 = 15$ ft, $Z_2 = -15$ ft, $Z_3 = 15$ ft, $Z_4 = -15$ ft

$K_i = \frac{EA}{l_i}$; stiffness of mooring line i

E = Modulus of elasticity of the mooring line material

A = Cross sectional area of the mooring line

m_c = mass of the cylinder

w_c = net buoyancy of the cylinder

2.5 Nondimensionalization

In order to simplify the problem and make it easy to consider different dimensions and properties of the structure, the equations were nondimensionalized. Lowercase letters are used to represent some of the nondimensional quantities and tildes are used in others. The cylinder's radius, R , and mass, m_c , will be the reference length and reference mass, respectively. Gravitational acceleration is defined as g and nondimensional time is denoted as τ . The nondimensional quantities used are:

$$x_c = \frac{X_c}{R}, \quad y_c = \frac{Y_c}{R}, \quad z_c = \frac{Z_c}{R} \quad (2.21, 2.22, 2.23)$$

$$\psi = \Psi, \quad \theta = \Theta, \quad \phi = \Phi \quad (2.24, 2.25, 2.26)$$

$$i_{xx} = \frac{I_{xx}}{m_c R^2}, \quad i_{yy} = \frac{I_{yy}}{m_c R^2}, \quad i_{zz} = \frac{I_{zz}}{m_c R^2} \quad (2.27, 2.28, 2.29)$$

$$k_i = \frac{K_i R}{m_c g}, \quad \tilde{l}_i = \frac{l_i}{R}, \quad \tilde{a}_{ij} = \frac{a_{ij}}{R} \quad (2.30, 2.31, 2.32)$$

$$\tilde{a} = \frac{a}{R}, \quad \tilde{b} = \frac{b}{R}, \quad \tilde{L} = \frac{L}{R} \quad (2.33, 2.34, 2.35)$$

$$\tilde{w} = \frac{w_c}{m_c g}, \quad f = \frac{F}{m_c g}, \quad \tau = t \sqrt{\frac{g}{R}} \quad (2.36, 2.37, 2.38)$$

$$\tilde{\omega} = \omega \sqrt{\frac{R}{g}}, \quad c = C \frac{R}{(m_c g)} \sqrt{\frac{g}{R}} \quad (2.39, 2.40)$$

where F = force applied to the cylinder

t = time

ω = frequency of vibration

C = damping coefficient

Therefore, the nondimensional quantities that describe the shape of the breakwater are

$$\tilde{a} = \tilde{b} = 2, r = 1, \tilde{L} = 6, \text{ and } \tilde{l}_1 = 4.$$

2.5.1 Nondimensional Equations of Motion

Before nondimensionalizing the EOM's, the acceleration terms are uncoupled so that each equation involves the acceleration term for one degree of freedom. This form of the EOM's is required for the FORTRAN program used for analysis, which will be described in Chapter 3. For Equations 2.7, 2.9, 2.11, and 2.15, this is simply a matter of isolating the acceleration terms on one side of the equation, setting them equal to the rest of the terms in the equations.

However, the acceleration terms $\ddot{\psi}$ and $\ddot{\phi}$ in Equations 2.13 and 2.17 are coupled; therefore, these equations are put in matrix form in order to isolate the two terms. This matrix is shown in Equation 2.41, where f_4 and f_6 (for the 4th and 6th EOM's) represent all of the non-acceleration associated terms in Equations 2.13 and 2.17, respectively:

$$[\mathbf{A}] \begin{bmatrix} \ddot{\psi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} f_4 \\ f_6 \end{bmatrix} \quad (2.41a)$$

where

$$[\mathbf{A}] = \begin{bmatrix} I_{xx} \sin^2 \theta + I_{yy} \cos^2 \theta & -I_{xx} \sin \theta \\ -I_{xx} \sin \theta & I_{xx} \end{bmatrix} \quad (2.41b)$$

Therefore, to solve for $\ddot{\psi}$ and $\ddot{\phi}$, the inverse of \mathbf{A} is found and is shown in Equation 2.43:

$$[\mathbf{A}]^{-1} = [\mathbf{B}] \quad (2.42a)$$

where

$$[\mathbf{B}] = \begin{bmatrix} \frac{1}{I_{yy} \cos^2 \theta} & \frac{\sin \theta}{I_{yy} \cos^2 \theta} \\ \frac{\sin \theta}{I_{yy} \cos^2 \theta} & \frac{I_{xx} \sin^2 \theta + I_{yy} \cos^2 \theta}{I_{xx} I_{yy} \cos^2 \theta} \end{bmatrix} \quad (2.42b)$$

Notice that both \mathbf{A} and \mathbf{B} are symmetric. Finally, $\ddot{\psi}$ and $\ddot{\phi}$ become

$$\begin{bmatrix} \ddot{\psi} \\ \ddot{\phi} \end{bmatrix} = [\mathbf{B}] \begin{bmatrix} f_4 \\ f_6 \end{bmatrix} \quad (2.43)$$

or,

$$\ddot{\psi} = B_{11}f_4 + B_{12}f_6 \quad (2.44)$$

$$\ddot{\phi} = B_{12}f_4 + B_{22}f_6 \quad (2.45)$$

The nondimensional forms of the six EOM's solved for the acceleration terms are:

$$\begin{aligned} \ddot{x}_c = & \sum_{i=1}^4 \{-k_i[x_c + \tilde{a}_{i1} \cos\theta \cos\psi + \tilde{a}_{i2} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) + \tilde{a}_{i3} (\sin\theta \cos\phi \cos\psi + \\ & \sin\phi \sin\psi) - x_i] + k_i \tilde{l}_i \{ [x_c + \tilde{a}_{i1} \cos\theta \cos\psi + \tilde{a}_{i2} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) + \\ & \tilde{a}_{i3} (\sin\theta \cos\phi \cos\psi + \sin\phi \sin\psi) - x_i]^2 + [y_c + \tilde{a}_{i1} \sin\psi \cos\theta + \tilde{a}_{i2} (\sin\psi \sin\phi \sin\theta + \\ & \cos\phi \cos\psi) + \tilde{a}_{i3} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi) - y_i]^2 + [z_c - \tilde{a}_{i1} \sin\theta + \tilde{a}_{i2} \sin\phi \cos\theta + \\ & \tilde{a}_{i3} \cos\theta \cos\phi - z_i]^2\}^{-1/2} [x_c + \tilde{a}_{i1} \cos\theta \cos\psi + \tilde{a}_{i2} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) + \\ & \tilde{a}_{i3} (\sin\theta \cos\phi \cos\psi + \sin\phi \sin\psi) - x_i]\}; \end{aligned} \quad (2.46)$$

$$\begin{aligned} \ddot{y}_c = & \tilde{w} - \sum_{i=1}^4 \{k_i[y_c + \tilde{a}_{i1} \sin\psi \cos\theta + \tilde{a}_{i2} (\sin\psi \sin\phi \sin\theta + \cos\phi \cos\psi) + \tilde{a}_{i3} (\sin\theta \cos\phi \sin\psi \\ & - \sin\phi \cos\psi) - y_i] + k_i \tilde{l}_i \{ [x_c + \tilde{a}_{i1} \cos\theta \cos\psi + \tilde{a}_{i2} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) + \\ & \tilde{a}_{i3} (\sin\theta \cos\phi \cos\psi + \sin\phi \sin\psi) - x_i]^2 + [y_c + \tilde{a}_{i1} \sin\psi \cos\theta + \tilde{a}_{i2} (\sin\psi \sin\phi \sin\theta + \\ & \cos\phi \cos\psi) + \tilde{a}_{i3} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi) - y_i]^2 + [z_c - \tilde{a}_{i1} \sin\theta + \tilde{a}_{i2} \sin\phi \cos\theta + \\ & \tilde{a}_{i3} \cos\theta \cos\phi - z_i]^2\}^{-1/2} [y_c + \tilde{a}_{i1} \sin\psi \cos\theta + \tilde{a}_{i2} (\sin\psi \sin\phi \sin\theta + \cos\phi \cos\psi) \\ & + \tilde{a}_{i3} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi) - y_i]\}; \end{aligned} \quad (2.47)$$

$$\begin{aligned} \ddot{z}_c = & \sum_{i=1}^4 \{-k_i[z_c - \tilde{a}_{i1} \sin\theta + \tilde{a}_{i2} \sin\phi \cos\theta + \tilde{a}_{i3} \cos\theta \cos\phi - z_i] + k_i \tilde{l}_i \{ [x_c + \tilde{a}_{i1} \cos\theta \cos\psi + \\ & \tilde{a}_{i2} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) + \tilde{a}_{i3} (\sin\theta \cos\phi \cos\psi + \sin\phi \sin\psi) - x_i]^2 + [y_c + \\ & \tilde{a}_{i1} \sin\psi \cos\theta + \tilde{a}_{i2} (\sin\psi \sin\phi \sin\theta + \cos\phi \cos\psi) + \tilde{a}_{i3} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi) - y_i]^2 + \end{aligned} \quad (2.48)$$

$$[z_c - \tilde{a}_{11} \sin\theta + \tilde{a}_{12} \sin\phi \cos\theta + \tilde{a}_{13} \cos\theta \cos\phi - z_i]^2 \}^{-1/2} [z_c - \tilde{a}_{11} \sin\theta + \tilde{a}_{12} \sin\phi \cos\theta + \tilde{a}_{13} \cos\theta \cos\phi - z_i] \};$$

$$\ddot{\psi} = \tilde{B}_{11} \tilde{f}_4 + \tilde{B}_{12} \tilde{f}_6 \quad (2.49)$$

$$\begin{aligned} \ddot{\theta} = & \frac{1}{i_{yy}} \{ -i_{xx} (\dot{\phi} \dot{\psi} \cos\theta - \dot{\psi}^2 \cos\theta \sin\theta) - i_{yy} \dot{\psi}^2 \cos\theta \sin\theta - \sum_{i=1}^4 \{ k_i [x_c + \tilde{a}_{11} \cos\theta \cos\psi + \\ & \tilde{a}_{12} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) + \tilde{a}_{13} (\sin\theta \cos\phi \cos\psi + \sin\phi \sin\psi) - x_i] [-\tilde{a}_{11} \cos\psi \sin\theta + \\ & \tilde{a}_{12} \sin\phi \cos\theta \cos\psi + \tilde{a}_{13} \cos\theta \cos\phi \cos\psi] - k_i [y_c + \tilde{a}_{11} \sin\psi \cos\theta + \tilde{a}_{12} (\sin\psi \sin\phi \sin\theta \\ & + \cos\phi \cos\psi) + \tilde{a}_{13} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi) - y_i] [-\tilde{a}_{11} \sin\psi \sin\theta + \tilde{a}_{12} \sin\psi \sin\phi \cos\theta \\ & + \tilde{a}_{13} \cos\theta \cos\phi \sin\psi] - k_i [z_c - \tilde{a}_{11} \sin\theta + \tilde{a}_{12} \sin\phi \cos\theta + \tilde{a}_{13} \cos\theta \cos\phi - z_i] (-\tilde{a}_{11} \cos\theta - \\ & \tilde{a}_{12} \sin\phi \sin\theta - \tilde{a}_{13} \sin\theta \cos\phi) + k_i \tilde{l}_1 \{ [x_c + \tilde{a}_{11} \cos\theta \cos\psi + \tilde{a}_{12} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) \\ & + \tilde{a}_{13} (\sin\theta \cos\phi \cos\psi + \sin\phi \sin\psi) - x_i]^2 + [y_c + \tilde{a}_{11} \sin\psi \cos\theta + \tilde{a}_{12} (\sin\psi \sin\phi \sin\theta + \\ & \cos\phi \cos\psi) + \tilde{a}_{13} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi) - y_i]^2 + [z_c - \tilde{a}_{11} \sin\theta + \tilde{a}_{12} \sin\phi \cos\theta + \\ & \tilde{a}_{13} \cos\theta \cos\phi - z_i]^2 \}^{-1/2} \{ [x_c + \tilde{a}_{11} \cos\theta \cos\psi + \tilde{a}_{12} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) + \\ & \tilde{a}_{13} (\sin\theta \cos\phi \cos\psi + \sin\phi \sin\psi) - x_i] [-\tilde{a}_{11} \cos\psi \sin\theta + \tilde{a}_{12} \sin\phi \cos\theta \cos\psi + \\ & \tilde{a}_{13} \cos\theta \cos\phi \cos\psi] + [y_c + \tilde{a}_{11} \sin\psi \cos\theta + \tilde{a}_{12} (\sin\psi \sin\phi \sin\theta + \cos\phi \cos\psi) + \\ & \tilde{a}_{13} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi) - y_i] [-\tilde{a}_{11} \sin\psi \sin\theta + \tilde{a}_{12} \sin\psi \sin\phi \cos\theta + \\ & \tilde{a}_{13} \cos\theta \cos\phi \sin\psi] + [z_c - \tilde{a}_{11} \sin\theta + \tilde{a}_{12} \sin\phi \cos\theta + \tilde{a}_{13} \cos\theta \cos\phi - z_i] (-\tilde{a}_{11} \cos\theta - \\ & \tilde{a}_{12} \sin\phi \sin\theta - \tilde{a}_{13} \sin\theta \cos\phi) \} \} = 0; \end{aligned} \quad (2.50)$$

$$\ddot{\phi} = \tilde{B}_{12} \tilde{f}_4 + \tilde{B}_{22} \tilde{f}_6 \quad (2.51)$$

where

$$\begin{aligned} \tilde{f}_4 = & i_{xx} (\dot{\phi} \dot{\theta} \cos\theta - 2 \dot{\psi} \dot{\theta} \sin\theta \cos\theta) + i_{yy} (2 \dot{\psi} \dot{\theta} \cos\theta \sin\theta) - \sum_{i=1}^4 \{ k_i [x_c + \tilde{a}_{11} \cos\theta \cos\psi + \\ & \tilde{a}_{12} (\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi) + \tilde{a}_{13} (\sin\theta \cos\phi \cos\psi + \sin\phi \sin\psi) - x_i] [-\tilde{a}_{11} \cos\theta \sin\psi - \\ & \tilde{a}_{12} (\sin\phi \sin\theta \sin\psi + \cos\psi \cos\phi) - \tilde{a}_{13} (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi)] - k_i [y_c + \tilde{a}_{11} (\sin\psi \cos\theta) + \end{aligned}$$

$$\begin{aligned}
& \tilde{a}_{i_2} (\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - y_i][\tilde{a}_{i_1} \cos\theta\cos\psi + \\
& \tilde{a}_{i_2} (\cos\psi\sin\phi\sin\theta - \cos\phi\sin\psi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi)] + k_i \tilde{l}_i \{ [x_c + \\
& \tilde{a}_{i_1} \cos\theta\cos\psi + \tilde{a}_{i_2} (\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - x_i]^2 + \\
& [y_c + \tilde{a}_{i_1} \sin\psi\cos\theta + \tilde{a}_{i_2} (\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - \\
& y_i]^2 + [z_c - \tilde{a}_{i_1} \sin\theta + \tilde{a}_{i_2} \sin\phi\cos\theta + \tilde{a}_{i_3} \cos\theta\cos\phi - z_i]^2 \}^{-1/2} \{ [x_c + \tilde{a}_{i_1} \cos\theta\cos\psi + \\
& \tilde{a}_{i_2} (\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - x_i][-\tilde{a}_{i_1} \cos\theta\sin\psi - \\
& \tilde{a}_{i_2} (\sin\phi\sin\theta\sin\psi + \cos\psi\cos\phi) - \tilde{a}_{i_3} (\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi)] + [y_c + \tilde{a}_{i_1} \sin\psi\cos\theta + \\
& \tilde{a}_{i_2} (\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - y_i][\tilde{a}_{i_1} \cos\theta\cos\psi + \\
& \tilde{a}_{i_2} (\cos\psi\sin\phi\sin\theta - \cos\phi\sin\psi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi)] \} \}
\end{aligned} \tag{2.52}$$

and

$$\begin{aligned}
\tilde{f}_6 = & \dot{i}_{xx} \dot{\psi} \dot{\theta} \cos\theta + \sum_{i=1}^4 \{ k_i [x_c + \tilde{a}_{i_1} \cos\theta\cos\psi + \tilde{a}_{i_2} (\sin\phi\sin\theta\cos\psi - \\
& \sin\psi\cos\phi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - x_i] (\tilde{a}_{i_2} (\cos\phi\sin\theta\cos\psi + \sin\psi\sin\phi) \\
& - \tilde{a}_{i_3} (\sin\theta\sin\phi\cos\psi - \sin\psi\cos\phi)] + k_i [y_c + \tilde{a}_{i_1} (\sin\psi\cos\theta) + \tilde{a}_{i_2} (\sin\psi\sin\phi\sin\theta + \\
& \cos\phi\cos\psi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - y_i][\tilde{a}_{i_2} (\sin\psi\cos\phi\sin\theta - \sin\phi\cos\psi) - \\
& \tilde{a}_{i_3} (\sin\theta\sin\phi\sin\psi + \cos\phi\cos\psi)] + k_i [z_c - \tilde{a}_{i_1} \sin\theta + \tilde{a}_{i_2} \sin\phi\cos\theta + \tilde{a}_{i_3} \cos\theta\cos\phi - \\
& z_i] (\tilde{a}_{i_2} \cos\phi\cos\theta - \tilde{a}_{i_3} \cos\theta\sin\phi) - k_i \tilde{l}_i \{ [x_c + \tilde{a}_{i_1} \cos\theta\cos\psi + \tilde{a}_{i_2} (\sin\phi\sin\theta\cos\psi - \\
& \sin\psi\cos\phi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - x_i]^2 + [y_c + \tilde{a}_{i_1} \sin\psi\cos\theta + \\
& \tilde{a}_{i_2} (\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) - y_i]^2 + [z_c - \\
& \tilde{a}_{i_1} \sin\theta + \tilde{a}_{i_2} \sin\phi\cos\theta + \tilde{a}_{i_3} \cos\theta\cos\phi - z_i]^2 \}^{-1/2} \{ [x_c + \tilde{a}_{i_1} \cos\theta\cos\psi + \\
& \tilde{a}_{i_2} (\sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi) - \\
& x_i][\tilde{a}_{i_2} (\cos\phi\sin\theta\cos\psi + \sin\psi\sin\phi) - \tilde{a}_{i_3} (\sin\theta\sin\phi\cos\psi - \sin\psi\cos\phi)] + [y_c + \\
& \tilde{a}_{i_1} \sin\psi\cos\theta + \tilde{a}_{i_2} (\sin\psi\sin\phi\sin\theta + \cos\phi\cos\psi) + \tilde{a}_{i_3} (\sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi) \\
& - y_i][\tilde{a}_{i_2} (\sin\psi\cos\phi\sin\theta - \sin\phi\cos\psi) - \tilde{a}_{i_3} (\sin\theta\sin\phi\sin\psi + \cos\phi\cos\psi)] + [z_c - \\
& \tilde{a}_{i_1} \sin\theta + \tilde{a}_{i_2} \sin\phi\cos\theta + \tilde{a}_{i_3} \cos\theta\cos\phi - z_i] (\tilde{a}_{i_2} \cos\phi\cos\theta - \tilde{a}_{i_3} \cos\theta\sin\phi) \} \}
\end{aligned} \tag{2.53}$$

2.6 Calculation of Equilibrium Height

The net buoyant force acting upward on the cylinder while in its static equilibrium state causes the four mooring lines to be in tension. The equilibrium height of the center of mass of the cylinder from the ocean floor, y_{c_eq} , is the only unknown coordinate of the structure and must be determined. From Figure 2.1, it can be seen that the nondimensional stretched length of each of the mooring lines at equilibrium is

$$\tilde{l}_0 = \sqrt{\tilde{a}^2 + \tilde{b}^2 + y_{c_eq}^2} \quad (2.54)$$

and the force in each spring at equilibrium is

$$\tilde{N}_i = k_i(\tilde{l}_0 - \tilde{l}_i) \quad (2.55)$$

The vertical equilibrium of the breakwater can now be established as

$$\tilde{w} = \sum_{i=1}^4 \frac{k_i(\tilde{l}_0 - \tilde{l}_i)y_{c_eq}}{\tilde{l}_0} \quad (2.56)$$

The relationship between equilibrium height, y_{c_eq} , and spring stiffness, k , where $k = k_i$ is the same for each mooring line, is shown in Figure 2.6. This graph shows that the equilibrium height rapidly drops as k increases for small spring stiffnesses. For a spring stiffness of 2.79, the equilibrium height is 3.0. After this point, the equilibrium height starts to gradually decrease until it plateaus at around 2.83. The standard stiffness of the mooring line used for this thesis is $k_i = 50$, for $i = 1, 2, 3$, and 4. This stiffness yields an equilibrium height of $y_{c_eq} = 2.838401$ and an equilibrium stretched length of $\tilde{l}_0 = 4.007059$ for each mooring line. The strain that results in the springs at this state is approximately 0.18 %. In comparison, the strains at 50% of the breaking strength for some typical mooring line materials are 0.5 % for steel, 1-1.5% for Kevlar, 2-4 % for polyester, 5-17.5 % for dacron, 15 % for polyethylene, 25 % for polypropylene, and 37.5 % for nylon (Skop 1988) and (D'Souza 1993).

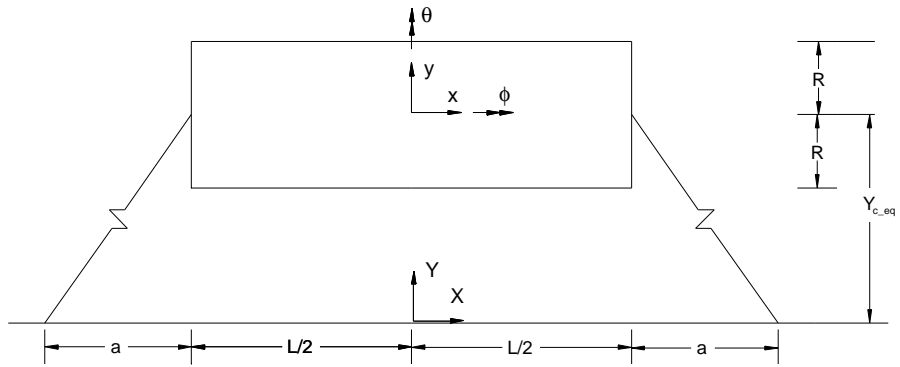


Figure 2.1. Profile View of Breakwater

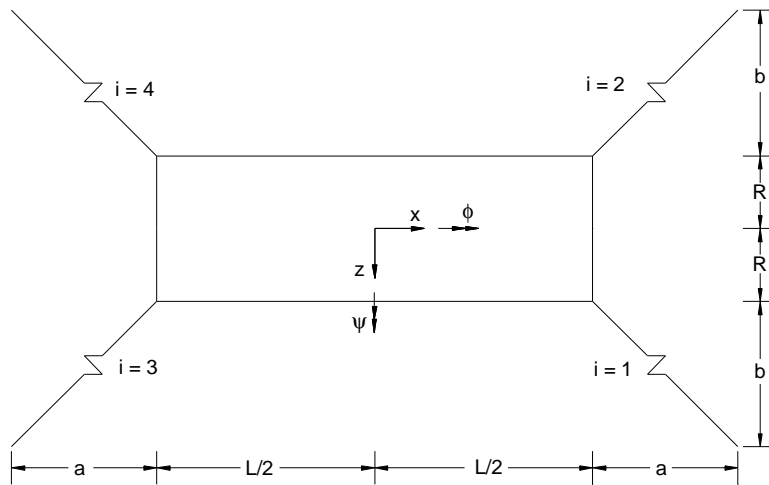


Figure 2.2. Plan View of Breakwater

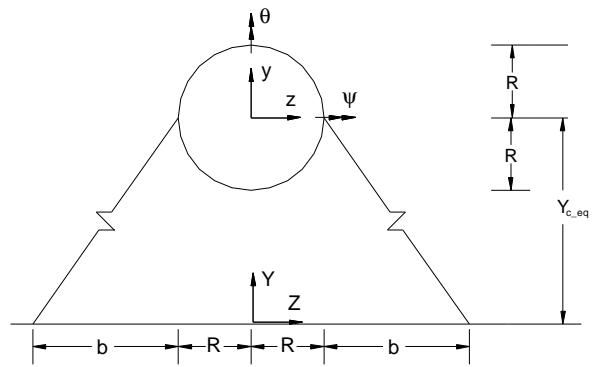


Figure 2.3. End View of Breakwater

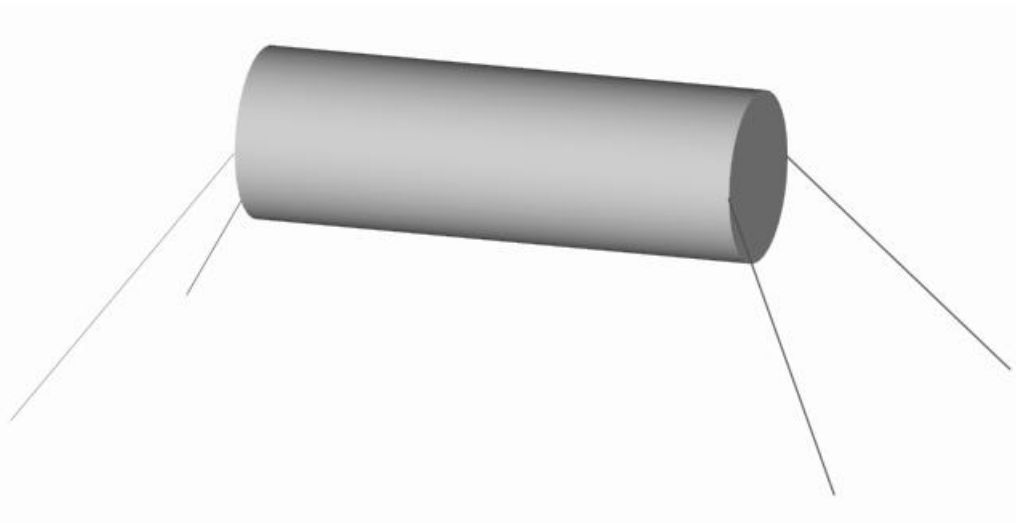


Figure 2.4. Isometric View of Breakwater

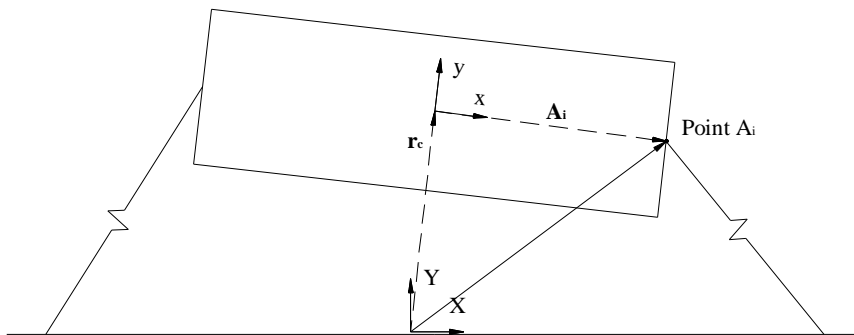


Figure 2.5. Breakwater in an Arbitrary State

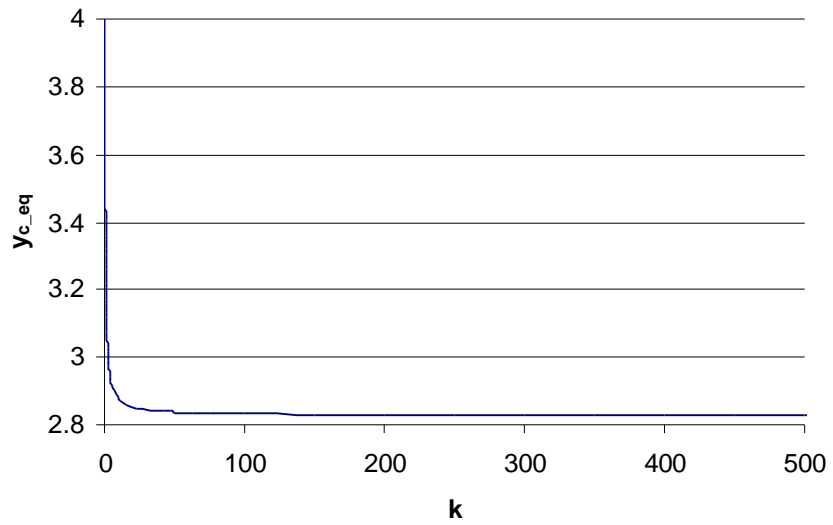


Figure 2.6. y_{c_eq} vs. k