

**A Consistent Direct-Iterative Inverse Design Method
for the Euler Equations**

by

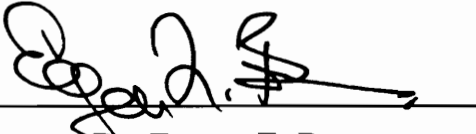
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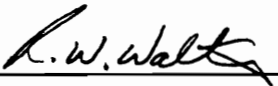
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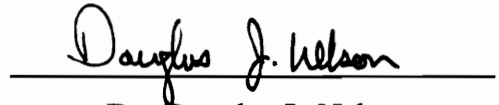
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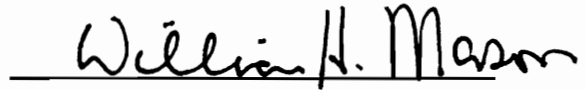
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(ABSTRACT)

A new, consistent direct-iterative method is proposed for the solution of the aerodynamic inverse design problem. Direct-iterative methods couple analysis and shape modification methods to iteratively determine the geometry required to support a target surface pressure. The proposed method includes a consistent shape modification method wherein the identical governing equations are used in both portions of the design procedure. The new shape modification method is simple, having been developed from a truncated, quasi-analytical Taylor's series expansion of the global governing equations. This method includes a unique solution algorithm and a design tangency boundary condition which directly relates the target pressure to shape modification. The new design method was evaluated with an upwind, cell-centered finite-volume formulation of the two-dimensional Euler equations. Controlled inverse design tests were conducted with a symmetric channel where the initial and target geometries were known. The geometric design variable was a channel-wall ramp angle, θ , which is nominally five degrees. Target geometries were defined with ramp angle perturbations of $\Delta\theta=2\%$, 10% , and 20% . The new design method was demonstrated to accurately predict the target geometries for subsonic, transonic, and supersonic test cases; $M=0.30$, 0.85 , and 2.00 . The supersonic test case efficiently solved the design tests and required very few iterations. A stable and convergent solution process was also demonstrated for the lower speed test cases using an under-relaxed geometry update procedure. The development and demonstration of the consistent direct-iterative method herein represent the important first steps required for a new research area for the advancement of aerodynamic inverse design methods.

Abstract

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Nomenclature

A	Metric Jacobian matrix
$\tilde{A}$	Approximate metric Jacobian matrix
$A \rightarrow I$	Normal matrix diagonals
b	Forcing function
BC	Boundary condition
BSR	Body Shape Rule
CFD	Computational Fluid Dynamics
CPU	Central Processing Unit
e	Specific internal energy
e_0	Total energy
F_q	Solution forcing function
F_N	Normal forcing function
GM	Garabedian and McFadden
h	Enthalpy
h_0	Stagnation enthalpy
H	Half-channel height
I	Identity matrix
INE	Incremental normal equation
J	Determinant of the coordinate variable transformation Jacobian matrix
JDIM	Maximum j spatial index
k	Total number of design variables
KDIM	Maximum k spatial index
LE	Leading edge

LHS	Left-hand-side
m	Total number of discrete equations
M	Mach number
MGM	Modified Garabedian and McFadden
n	Total number of physical coordinates
$\mathbf{n}$	Outward normal vector
N-S	Navier-Stokes
P	Pressure
q	Primitive variables
Q	Conserved variables
$\hat{\mathbf{R}}$	Residual vector
RHS	Right-hand-side
RMG	l_2 norm of geometry residual
RMP	l_2 norm of pressure residual
s	Dummy variable for x or y
SVD	Singular Value Decomposition
t	Time
T	Tangency boundary condition
TE	Trailing edge
u,v	Velocity components in the (x,y) directions
$\bar{u}$	Normal velocity
V	Total velocity
W	Local metric Jacobian matrix
x,y	Physical cartesian coordinates
$\bar{\mathbf{X}}$	Physical coordinate vector
z	Physical coordinate $\Delta\bar{\mathbf{X}}$ vector
∂	Partial derivative
$\ \bullet\ _2$	Euclidian l_2 norm

Greek Symbols

β	Design variable
θ	Ramp angle
γ	Ratio of specific heats
Δ	Finite difference
ε	Geometric constraint constant
ξ, η	Generalized computational coordinates
ρ	Density
ω	Relaxation factor

Indices, Subscripts, and Superscripts

a	Analysis
b	Back
B	Boundary
ce	Continuity and energy
cm	Continuity and momentum
d	Design
i,j	Matrix row and column indices
I	Interior
j,k	Spatial indices in the (ξ, η) directions
ls	Least-squared
me	Momentum and energy
me,ne	Geometric constraint variable indices
ref	Reference
t	Target
T	Transpose
∞	Infinity

1.0 Introduction

One of the primary goals of Computational Fluid Dynamics (CFD) research is to develop accurate and efficient methods for the analysis and design of aerospace vehicles [1]. The analysis and design of advanced aerospace vehicles require the accurate solution of the relevant fluid dynamic governing equations. The application of an accurate computational model to solve for the flow fields about these geometries requires an efficient numerical technique. Historically, CFD analysis methods have received greater attention and are presently considered more advanced than design methods. However, the full power and utility of CFD will not be realized until aerodynamic design methods are as advanced as analysis methods [2]. The purpose of this research was to contribute to the advancement of aerodynamic design methods.

Many aerodynamic design methods exist, and each possess unique strengths and specializations. In this introduction, only two broad categories of aerodynamic design are considered: optimization design and inverse design. Optimization design methods are considered more appropriate for the prediction of flow geometries which must effectively operate at both design and off-design conditions [3]. In contrast, inverse design methods are recommended for the prediction of geometries which operate almost exclusively at the design condition [4]. Optimization and inverse design methods are considered complimentary within the aerospace design community, as evidenced by the growing number of hybrid optimization/inverse design methods [5-8].

Direct-iterative methods are a popular sub-category of inverse design and are extensively used within the aerospace design community [9]. Improving these methods was the focus of this research. Direct-iterative methods determine the flow geometry required to support a set of boundary conditions (BC's) and a target surface pressure

profile. These design methods couple independent analysis and shape modification methods. The final design geometry is determined iteratively by successive design shape predictions. The analysis method determines the direct flow field solution about an initial shape estimate and the subsequent design shape predictions. The shape modification method uses the surface pressure residual, the difference between the computed and target pressures, to predict each intermediate geometry. The solution is converged when the surface pressure about the final shape estimate closely matches the target pressure.

Two important characteristics of direct-iterative methods are their accuracy and efficiency in obtaining a solution to the inverse design problem. The accuracy of the procedure is measured by the degree to which the surface pressure of the final shape estimate matches the target pressure. (However, the existence of the design solution must first be guaranteed by defining a physically possible target pressure [10,11]. The present research is focused on the design solution process and, therefore, assumes that the design solution exists.) The accuracy of direct-iterative methods is then solely dependent on the accuracy of the governing equations and numerical techniques used within the analysis portion of the method. Because any analysis method may be used, including the most physically and numerically accurate one available, the direct-iterative method can provide an accurate design solution. Research efforts for direct-iterative methods are, therefore, primarily focused on efficiency and shape modification techniques.

The efficiency of the direct-iterative method is measured by the computational cost required to achieve the design solution. Each design shape prediction and re-analysis defines a single analysis/design iteration. The computational cost of each analysis/design iteration is related to the governing equations and numerical techniques employed in both portions of the iteration. If the computational cost per iteration is comparable for different direct-iterative methods, then the most efficient method is the one that requires the least number of iterations. Minimizing the total number of analysis/design iterations may be achieved by using the most accurate shape modification method possible. The focus of the present research was then to explore the possibility of using the most accurate shape modification method possible within the direct-iterative design method.

Current direct-iterative design methods do not employ the most accurate shape

modification method possible. Instead, they use an inconsistent set of governing equations for analysis and shape modification. The relationships used within existing shape modification methods are typically based on potential flow theory, primarily because these are relatively easy to implement. However, the analysis methods commonly used for direct-iterative design solutions solve the Euler or Navier-Stokes (N-S) equations. The governing equations used in existing shape modification methods are, therefore, not as physically accurate as their counterparts used in the analysis method. This inconsistency is theorized to cause the poor convergence behavior exhibited in some of the existing direct-iterative design methods [9]. This disadvantage will only become more acute as analysis methods become more accurate relative to existing shape modification methods.

The objective of this research was to develop a *consistent* direct-iterative inverse design method which would include a more accurate approach to shape modification. Here, consistency refers to the use of the identical governing equations and numerical techniques for both the analysis and shape modification portions of the direct-iterative method. The implementation of this new design method then required the development of a new shape modification procedure which used the same governing equations and numerical techniques as the analysis method. This new shape modification method is, therefore, as physically and numerically accurate as the analysis method.

Two important aspects that influence the practicality of a consistent design method are the issues of computational cost and convergence behavior. In a consistent shape modification method, the solution of a global system of equations for each design shape prediction is a necessity; the governing equations are similarly global conservation laws. This is in contrast to existing methods which are applied only along the surface of the flow geometry. This implies that the consistent shape modification method will require more computational effort than the existing methods. However, as will be demonstrated in this research, the efficiency of the new method may still compare favorably with existing methods if the total number of analysis/design iterations is small. Furthermore, it will also be shown that the proposed new procedure of coupling consistent analysis/shape modification methods can yield a stable and convergent solution process.

An important characteristic of the proposed shape modification method is that the

procedure is Mach number independent, i.e. the method can be used for subsonic, transonic, and supersonic flows without any special modification. This inherent ability to be Mach number independent is due to the fact that the governing equations used for analysis, and, therefore, for shape modification, are applicable for all flow regimes. This is in sharp contrast to some existing shape modification methods which require a different shape modification procedure for each different flow regime.

To the best of the authors knowledge, this dissertation documents the first attempt to develop a consistent direct-iterative design method. This fact has an important implication; this dissertation introduces a new research area for inverse design. The primary objectives of this research were to develop a consistent direct-iterative method and to demonstrate it as an alternative solution method for the inverse design problem. The efforts herein should not be viewed as merely a simple extension of existing methods; the present objectives required an entirely new approach to shape modification. An upwind, cell-centered finite-volume formulation of the two-dimensional (2-D) Euler equations was selected for this investigation. These governing equations represent a significant advancement for shape modification methods. Development of the new shape modification method also presented numerous challenges and difficulties. Several new numerical techniques were required to complete the development. The contribution of this research is that it provides the foundation for future, comprehensive investigations of the efficiency characteristics of a consistent direct-iterative method relative to existing ones.

This dissertation continues with a review of current direct-iterative design methods available in the literature, and introduces the standard design equation. The previously developed standard design equation served as the model equation from which the consistent shape modification method was developed. A new solution algorithm, the incremental normal equation, and a modified version of the standard design equation are presented next for use within a direct-iterative design method. A new, design tangency BC which relates surface pressure changes to shape modification completes the development effort. Then, the geometry and test parameters used for the inverse design demonstrations are described. This is followed by the presentation and discussion of the results, and a summary concludes this dissertation.

2.0 Literature Review

Exhaustive reviews of the field of aerodynamic inverse design are available in the open literature and include those by Holst et al [3], Dulikravich [9], Slooff [12], and Volpe [13]. Direct-iterative methods represent only one of several inverse design procedures. The purpose of this research was to develop an advanced version of the direct-iterative method and, consequently, this literature review will focus only on this sub-category of inverse design. The following discussion will also introduce the standard design equation. This equation served as the model equation from which the new, consistent direct-iterative method was developed.

2.1 Direct-Iterative Design Methods

As mentioned previously, direct-iterative design methods couple independent analysis and shape modification methods to iteratively determine the design geometry. The key component of these design methods is the shape modification portion of the procedure. The shape modification method directly relates the surface pressure residual, the difference between the computed and target pressures, to the change in the geometry. Some existing shape modification methods transform the target pressure into target velocities or target coefficients of pressure. However, all of the existing shape modification methods are viewed as being driven by a specified target pressure. Also, modifications of the initial shape estimate may involve the entire flow surface or just the geometry within the design area. The design area is that portion of the initial shape estimate which is selected for modification.

In general, shape modification methods predict changes in the flow geometry

position, slope, or curvature. Within direct-iterative design methods, the next geometry estimate is obtained by integrating these changes along the geometry surface. This is the geometry update portion of the design procedure. Because this portion of the procedure is independent of both the analysis and shape modification methods, this process maintains control over each design shape prediction. The predicted geometry changes can be monitored, and systematically adjusted if necessary, to enforce geometric constraints on the next shape estimate before the re-analysis of the flow field. (Geometric constraints for inverse design will be discussed in Chapter 3.)

There are three types of shape modification methods used within the existing direct-iterative design methods: body shape rules, surface differential equations, and surface integrals. The following discussion will briefly review each of these methods.

2.1.1 Body Shape Rules

Body shape rules (BSR's) are simple expressions used for direct-iterative shape modification. These expressions are generally developed from the solution of potential flow theory about simple, model geometries. These shape modification methods are applied at discrete points within the design area on the geometry surface.

The first direct-iterative inverse design method was developed by Barger and Brooks in 1974 [14]. Their shape modification method uses a single BSR which relates the change in pressure on the flow surface to the corresponding change in the surface curvature. This BSR was developed from the momentum equation of fluid flow in streamline coordinates. To avoid a singularity at the stagnation point, the actual BSR recommended by Barger and Brooks relates the pressure residual to changes in the geometry curvature with a functional constant. This constant is a function of the free-stream Mach number, the surface curvature, and an empirical constant. The empirical constant is used to stabilize or accelerate the inverse design solution. This method was demonstrated in the design of subcritical and supercritical airfoils.

The first direct-iterative method developed for the design of both high and low speed geometries was reported by Davis in 1980 [15]. Davis's method performs shape

modification with separate BSR's which are uniquely defined for both supersonic and subsonic flows. The rationale for unique BSR's for these flow regimes was obtained from the classical wavy-wall solution of linearized, potential flow theory. This solution establishes that for supersonic flow, a change in the surface pressure is proportional to a change in the surface slope. Also, from this simple solution Davis determined that the BSR recommended by Barger and Brooks, which relates pressure changes to curvature changes, is most appropriate for subsonic design. Davis then suggested that BSR's based on these two principles may be used for an under-relaxed, iterative design procedure.

The direct-iterative method developed by Davis uses three BSR's, one for supersonic design and two for transonic design. For the design of geometries which operate with free-stream Mach numbers much larger than one, Davis uses the supersonic wavy-wall solution directly. This BSR is a function of the local, surface Mach number and was demonstrated in the design of a supersonic wing. The two BSR's which Davis recommends for transonic flow were developed from linearized potential flow theory applied to a thin airfoil. One of these BSR's is used when the local Mach number is less than unity, and relates pressure changes to curvature changes. The second transonic BSR is used when the Mach number is just over one, and relates pressure changes to slope changes. These two BSR's were demonstrated in the design of transonic airfoils.

Another direct-iterative method was developed by Campbell and Smith in 1987 for the design of transonic flow geometries [16]. This direct-iterative method is a hybrid subsonic/supersonic method, similar to the one developed by Davis. However, the shape modification method developed by Campbell and Smith contains only two BSR's. One BSR is for subsonic flows and relates pressure changes to curvature changes. This BSR was developed from the momentum equation in streamline coordinates and is similar to that recommended by Barger and Brooks. However, the functional constant in the Campbell and Smith subsonic BSR was altered, but similarly includes a relaxation parameter. This relaxation parameter is used to stabilize the design solution and to predict highly curved geometries more accurately.

The supersonic BSR recommended by Campbell and Smith was developed from thin airfoil theory and relates surface pressure changes to geometry slope changes. To

simplify the geometry update procedure, this supersonic BSR was differentiated to relate pressure changes to curvature changes. However, this differentiation process did not alter the pressure-to-slope relationship upon which this supersonic BSR is based. Both of these subsonic and supersonic BSR's were demonstrated by Campbell and Smith in the design of transonic airfoils and wings [16,17]. Their direct-iterative method was also reported in the design of nacelles and winglets by Lin et al [18], and Bell and Cedar [19].

The most recent direct-iterative method which uses BSR's was developed by Lee and Mason in 1991 [20]. Their direct-iterative method was specialized for the design of supersonic and hypersonic flow geometries. The fact that inviscid, high speed flows are unchanged by upstream disturbances is capitalized upon in this method. The design procedure is cast as a marching method in the streamwise direction. The design geometry is first determined on a single plane normal to the flow. The process then continues downstream to the next plane. Lee and Mason studied many BSR's for high speed flows and recommended one which is based on irrotational, conical flow theory. This BSR relates pressure changes to geometry slope changes. To stabilize or accelerate the design procedure, a root-finding technique was added to the shape modification method. This direct-iterative method was demonstrated in the design of hypersonic, minimum drag bodies at both supersonic and hypersonic free-stream Mach numbers.

2.1.2 Surface Differential Equations

Another popular sub-division of direct-iterative inverse design methods was originally developed by Garabedian and McFadden in 1982 [21]. The Garabedian and McFadden (GM) shape modification method is an auxiliary partial differential equation defined on the flow geometry surface. The differential equation is not related to any fluid dynamic governing equation, but rather, it relates the change in surface pressure to the change in surface position, slope, and curvature. Each of these relationships is expressed as a weighted partial derivative within the auxiliary equation. The weighting factors are user-supplied coefficients which are employed to stabilize or accelerate the design solution. A pseudo time-derivative is also used for the iterative solution of the differential

equation. Garabedian and McFadden solved this equation within a mapped computational plane with a finite-difference method. The displacement of the geometry normal to the original surface is obtained for each shape modification solution. This direct-iterative method was demonstrated in the design of supercritical wings.

Malone developed the Modified Garabedian and McFadden (MGM) shape modification method in 1985 [22,23]. The modified version of the differential equation is solved directly in the physical plane which obviates the complicated mapping function used in the GM method. This allows the MGM method to be applied to a wider variety of flow geometries. The solution of the MGM shape modification method predicts vertical displacements of the design geometry. These displacements are directly added to the current geometry estimate for the next design shape prediction. Malone originally demonstrated the MGM method in the design of supercritical airfoils and nacelles, while Birckelbaw used the method to design subsonic and transonic airfoils [24]. Malone later used the MGM method to design transonic airfoils [25], and Kinsey and Jolly used the method in the design of multi-element airfoils [26].

2.1.3 Surface Integrals

The third, and final, sub-division of direct-iterative inverse design methods was developed by Takanashi in 1984 [27,28]. This direct-iterative method was developed specifically for the design of transonic wings. The development of this shape modification method is based on the three-dimensional (3-D) full potential equations. The method is predicated upon the assumptions of small disturbance theory, symmetry of the wing at the wing-body juncture, and physical closure along the trailing edge of the wing. The final form of this shape modification method was presented as 2-D surface integrals defined over the wing. These integrals are functions of the surface pressure residual, and the wing is divided into discrete panel sections for the design procedure. The integrals are then solved over each panel implicitly for the 3-D shape modification solution. This method was demonstrated in the design of various transonic wings.

2.1.4 Summary

Existing direct-iterative methods represent a practical solution of the aerodynamic inverse design problem. Many of the example applications cited above provided an accurate inverse design solution. The more advanced of these applications used the Euler or N-S equations for the analysis portion of the design procedure. However, none of the existing shape modification methods represent a consistent application of these governing equations for both analysis and shape modification. Consistency requires that the shape modification method consider the entire flow field domain. Consistency also requires that both the analysis and shape modification methods use the identical governing equations and numerical techniques for the design procedure. Existing shape modification methods are only applied along the geometry surface, and each was developed from a model solution of the potential equations.

The remaining portion of this literature review will introduce the standard design equation. This equation has many uses within the aerodynamic design community. More importantly, the standard design equation served as the model equation from which the proposed consistent shape modification method was developed.

2.2 Quasi-Analytical Expansion

A truncated, quasi-analytical Taylor's series expansion of the global, fluid dynamic governing and boundary equations was developed by Taylor et al in 1991 [29]. This equation, the standard design equation, relates finite changes in the solution throughout the flow field to the corresponding finite changes in the flow geometry and grid. The relationship between the change in solution and the change in the geometry and grid is made via two Jacobian matrices. The equation is quasi-analytical in nature because the Jacobian matrices are derived from the discrete form of the global flow equations governing mass, momentum, and energy exchange. The standard design equation was originally proposed by Taylor for use within approximate analysis methods, sensitivity derivative evaluation, and inverse design methods.

Quasi-analytical approximate analysis methods include geometric and non-geometric solution predictions. Geometric solution predictions approximate a neighboring steady-state solution given a finite change in a geometric design variable. Geometric design variables include airfoil thickness or channel throat area. Successful demonstrations of these approximations have been reported for the 2-D Euler and N-S equations [29-31]. Non-geometric solution predictions approximate the finite change in the solution due to a perturbation of a non-geometric design variable. Examples of non-geometric design variables include inlet Mach number, angle of attack, and exit pressure ratio. Non-geometric solution predictions for neighboring steady-state solutions have been demonstrated for the 2-D Euler equations [32].

Quasi-analytical sensitivity derivative evaluation builds on the success of the approximate analysis methods by repeated use of the chain rule on the standard design equation. Sensitivity derivatives are used within optimization design methods to relate the change in the design variables to the required change in the geometry. Efficient evaluation of quasi-analytical sensitivity derivatives with respect to geometric and non-geometric design variables has been demonstrated for the 2-D Euler and N-S equations [30,31,33-35]. Quasi-analytical sensitivity derivative evaluation for the 3-D Euler equations has also recently been reported [36].

The standard design equation also represents a system of equations for consistent shape modification when used within a direct-iterative design method. Given a finite change in the solution, the corresponding finite change in the geometry and grid may be obtained with this equation. Because the standard design equation is quasi-analytical in nature, it provides a consistent relationship between these finite changes. The standard design equation has been successfully implemented for both approximate analysis and sensitivity derivative evaluation as noted above. However, to the best of the authors knowledge, this equation has not yet been used within any inverse design procedure. The purpose of the following chapters is to present the further development of this simple, yet powerful, technique for the remaining application of inverse design through consistent design shape prediction.

3.0 Consistent Shape Modification

This chapter presents the fundamental equations for the proposed, consistent direct-iterative inverse design method. The governing equations selected for this initial investigation will be presented first, followed by the standard design equation. The incremental normal equation will then be presented. This equation represents a method of solving the standard design equation for consistent shape modification.

3.1 Governing Equations

The governing equations used in the present research were the 2-D Euler equations. These equations represent a coupled set of non-linear, partial differential equations which govern the conservation of mass, momentum, and energy for inviscid fluid flow [37]. Within this research, the Euler equations were transformed from a cartesian, (x,y) , to a generalized, (ξ,η) , coordinate system [38]. The time dependent Euler equations are presented in Equation 3.1, where J is the determinant of the coordinate transformation Jacobian matrix.

$$\frac{1}{J} \frac{\partial Q}{\partial t} + \hat{R}(Q) = 0 \quad (3.1)$$

The steady-state form of the Euler equations is represented within Equation 3.1 by the residual vector, $\hat{R}$. At steady-state conditions the flow field is unchanged in time, and the residual is exactly zero. The residual vector is an explicit function of the conserved variable vector, Q . Elements of this vector include combinations of density, ρ , cartesian component velocities, u and v , and an energy term, e_0 . The total energy, e_0 ,

is a function of the specific internal energy, e , and cartesian component velocities; $e_0 = e + (u^2 + v^2)/2$. The conserved variable vector is presented in Equation 3.2.

$$Q = \{\rho, \rho u, \rho v, \rho e_0\}^T \quad (3.2)$$

Another common form of the solution variables is the primitive variable vector, $q = \{\rho, u, v, P\}^T$. This vector is an explicit function of the pressure, P . The calorically perfect, ideal gas law was used in this research to evaluate the pressure [39]. The ratio of specific heats was, therefore, constant. In this research, it was assumed to be equal to $\gamma = 1.4$. The pressure was then evaluated as $P = (\gamma - 1)\rho e$.

The Euler equations were cast in an integral, conservation law form within this research [40]. These equations were computationally described with a cell-centered, finite-volume formulation within a structured grid system [41]. The inviscid flux terms within the Euler equations were evaluated with Van Leer's upwind, flux-vector-splitting method [42]. The culmination of these numerical techniques defined the discrete form of the steady-state Euler equations. This form of the governing equations is represented by a vector of coupled non-linear, algebraic equations and is termed the discrete residual. This vector includes the discrete representation of all boundary equations and is a function of the discrete solution variable vector. The discrete residual and solution vectors are defined throughout the flow domain.

Generally, the functional dependence of the discrete residual vector is expressed only in terms of Q . This dependence was emphasized in Equation 3.1; $\hat{R} = \hat{R}(Q)$. However, the discrete residual is also a function of the discretized domain. The discretized domain is the grid, which on the boundaries includes the flow geometry. The dependence of $\hat{R}$ on the geometry/grid is generally not expressed because most analysis solutions use a stationary grid. The discrete values of the solution variables are the most important product of an analysis flow field solution.

However, the explicit dependence of the discrete residual on the geometry/grid becomes equally important when computational design methods are considered. For design purposes, the discrete residual must be explicitly defined as a function of both the solution vector and the geometry/grid vector, $\bar{X}$. The vector $\bar{X}$ represents the physical

coordinates of the discretized domain. One possible definition of $\bar{X}$ which was used in this research is presented in Equation 3.3.

$$\bar{X} = (x, y)^T \quad (3.3)$$

The required functional dependence of the residual vector for design purposes is then presented in Equation 3.4.

$$\hat{R} = \hat{R}(Q, \bar{X}) \quad (3.4)$$

The dual functional dependence of the residual vector on both Q and $\bar{X}$ is also an important factor within the development of the standard design equation. The standard design equation is the model equation from which the consistent direct-iterative method will be developed in the following sections of this dissertation.

3.2 Standard Design Equation

The standard design equation is developed from a Taylor's series expansion of the discrete residual vector between two steady-state solutions. The series expansion is taken with respect to both the Q and $\bar{X}$ variables and is presented in Equation 3.5.

$$\begin{aligned} \{\hat{R}_2(Q, \bar{X})\} &= \{\hat{R}_1(Q, \bar{X})\} \\ &+ \left[\frac{\partial \hat{R}_1(Q, \bar{X})}{\partial Q} \right] \{\Delta Q\} + \left[\frac{\partial \hat{R}_1(Q, \bar{X})}{\partial \bar{X}} \right] \{\Delta \bar{X}\} \\ &+ o \left[(\Delta Q)^2, \Delta Q \Delta \bar{X}, (\Delta \bar{X})^2 \right] \end{aligned} \quad (3.5)$$

The vectors ΔQ and $\Delta \bar{X}$ represent the finite differences between solutions and geometry/grids, respectively. These vectors are defined with respect to the end states of the series expansion and are presented in Equation 3.6.

$$\{\Delta Q\} = \{Q_2\} - \{Q_1\} \quad ; \quad \{\Delta \bar{X}\} = \{\bar{X}_2\} - \{\bar{X}_1\} \quad (3.6)$$

The two solutions and geometries/grids within the series expansion are assumed

to be closely related. The higher-order terms of the series expansion are, therefore, truncated. Also, both residual vectors within Equation 3.5 are exactly zero because the expansion is performed between steady-state solutions. The resulting truncated series expansion is presented in Equation 3.7 and is termed the standard design equation [29].

$$\left[\frac{\partial \hat{R}(Q, X)}{\partial Q} \right] \{ \Delta Q \} + \left[\frac{\partial \hat{R}(Q, X)}{\partial X} \right] \{ \Delta X \} = \{ 0 \} \quad (3.7)$$

(The subscripts have been dropped in Equation 3.7 for expedience, and will be shown subsequently only when necessary, for clarity.)

The standard design equation represents an equation for consistent shape modification. Given a finite change in the solution vector, ΔQ , the corresponding finite change in the geometry/grid vector, $\Delta \bar{X}$, may be determined. A consistent relationship between these vectors is guaranteed because the standard design equation was developed directly from the governing equations. The standard design equation also enables shape modification to be performed with the same numerical techniques originally selected for the analysis portion of the design procedure.

The vectors ΔQ and $\Delta \bar{X}$ are linearly related by two Jacobian matrices within the standard design equation. The first Jacobian matrix, $\partial \hat{R} / \partial Q$, is familiar, being used within analysis solution algorithms for implicit time integration. This matrix is commonly referred to as the flux Jacobian matrix. The second Jacobian matrix, $\partial \hat{R} / \partial \bar{X}$, is a relatively new matrix in the literature. The term metric Jacobian, while strictly a misnomer, will be used throughout this dissertation. Both Jacobian matrices are derived from the global, discrete residual vector and are therefore quasi-analytical expressions. Also, the proper linearization of all boundary equations is required within both the flux and the metric Jacobian matrices [34].

In summary, the standard design equation represents an equation for consistent shape modification which is straightforward in application. The functional dependence of the discrete residual on both the solution and the geometry/grid vectors is not a new concept. However, exploitation of this property in conjunction with the truncated Taylor

series expansion is a simple, yet powerful, tool which has not been fully explored. The following section of this chapter presents a method of solving the standard design equation for consistent shape modification.

3.3 Incremental Normal Equation

The standard design equation represents a global, linear system of equations for consistent shape modification. The solution of this equation to determine $\Delta\bar{X}$ requires the inversion of the metric Jacobian matrix. The present research is the first known use of the standard design equation for shape modification. Therefore, the present research is also the first known attempt to invert the metric Jacobian matrix. This section discusses two difficulties encountered in the solution of the standard design equation for shape modification. The incremental normal equation is then proposed as a simple solution method which circumvents these difficulties.

The first difficulty encountered in solving the standard design equation is the inversion of the metric Jacobian matrix. Both the flux and metric Jacobian matrices must be evaluated exactly to obtain the proper solution for $\Delta\bar{X}$. A direct or non-iterative inversion of the metric Jacobian matrix is also required which is costly in terms of computational storage and cost. The second difficulty encountered in obtaining the proper solution of $\Delta\bar{X}$ requires an examination of the size of the metric Jacobian matrix.

The Euler equations were transformed into a generalized coordinate system. In the present effort, (j,k) corresponds to the (ξ,η) directions, respectively. The constants JDIM and KDIM were defined as the maximum (j,k) dimensions within the discretized flow domain. The total number of domain governing and boundary equations within the discrete residual is represented by m. The total number of physical (x,y) coordinates within the domain is given by n. The coordinate positions of the geometry/gird are the unknowns for consistent shape modification. Both the number of equations and the coordinate unknowns are functions of JDIM and KDIM as defined in Equation 3.8.

$$m = (JDIM + 1)(KDIM + 1)(4) \quad ; \quad n = (JDIM)(KDIM)(2) \quad (3.8)$$

The values of m and n define the size of the flux and metric Jacobian matrices, and the ΔQ and $\Delta \bar{X}$ vectors within the standard design equation. There are approximately twice the number of equations, m , as there are unknowns, n , for the 2-D Euler equations. (In practice, boundary equations are not solved at the domain corners and, therefore, m should be reduced by sixteen. However, this will not be further noted.) The standard design equation is then re-written in Equation 3.9 to include the size of each Jacobian matrix and the vectors ΔQ and $\Delta \bar{X}$.

$$\left[\frac{\partial \hat{R}(Q, \bar{X})}{\partial Q} \right]_{m \times m} \{ \Delta Q \}_{m \times 1} + \left[\frac{\partial \hat{R}(Q, \bar{X})}{\partial \bar{X}} \right]_{m \times n} \{ \Delta \bar{X} \}_{n \times 1} = \{ 0 \}_{m \times 1} \quad (3.9)$$

It can be seen in Equation 3.9 that consistent inverse design with the Euler equations is an over-determined problem. The 2-D governing equation contains four partial differential equations but only two physical coordinate unknowns, (x, y) , for shape modification. Discrete governing and boundary equations are also defined at more positions within the flow domain than there are physical coordinate positions. These two factors produce a massively over-determined system of equations. An exact solution of the standard design equation for $\Delta \bar{X}$ is, therefore, not possible. (If the discrete vector $\bar{X}$ could be defined so that $n=m$ and still uniquely define the grid, then the over-determined nature of Equation 3.9 could be avoided. Re-definition of the discrete vector $\bar{X}$ was attempted early in this research effort without success.)

In summary, two difficulties exist in the solution of the standard design equation. These difficulties are the requirement of an exact metric Jacobian matrix inversion and the over-determined nature of the equation. Two simple techniques are now applied to this equation to overcome both difficulties and to obtain the best possible consistent shape modification solution. First, terms within the standard design equation are redefined as $A = \partial \hat{R} / \partial \bar{X}$, $z = \Delta \bar{X}$, and $b = (\partial \hat{R} / \partial Q) \Delta Q$, and the equation is restated in Equation 3.10.

$$A_{m \times n} z_{n \times 1} + b_{m \times 1} = 0 \quad (3.10)$$

3.3.1 Incremental Formulation

The first technique is to apply Newton's root finding method to the standard design equation. The standard design equation is a linear system of equations in ΔQ and $\Delta \bar{X}$. Newton's method is an iterative solution technique which is typically applied for the solution of non-linear systems of equations. However, this simple method was recently demonstrated to reduce the computational storage and cost required to solve a linear system of equations [43,44]. Newton's root finding method is applied to the standard design equation and is presented in Equation 3.11.

$$A_{m \times n} \Delta z_{n \times 1} = - (A_{m \times n} z_{n \times 1} + b_{m \times 1})_{m \times 1} \quad (3.11)$$

The standard design equation is now in iterative, or incremental form. An iterative solution of this equation may then be obtained starting with a reasonable estimate of $z = \Delta \bar{X}$. (In practice, this initial estimate is $z = \Delta \bar{X} = 0$.) As the solution converges, $\Delta z = \Delta(\Delta \bar{X})$ approaches zero, so that any approximations to the left-hand-side (LHS) metric Jacobian matrix will not affect the solution of the right-hand-side (RHS) equations. Approximations to the LHS metric Jacobian matrix may reduce the cost of its evaluation and also stabilize the inversion of this matrix. These approximations include inconsistent LHS/RHS numerical evaluation and the addition of a diagonal term. Also, any common iterative method for the solution of a linear system of equations can be used to solve Equation 3.11.

3.3.2 Normal Equation Technique

The over-determined nature of the standard design equation remains the second solution difficulty. The best possible solution for $\Delta \bar{X}$ within the standard design equation would be obtained if each equation in the over-determined system were satisfied in a least-squared sense. Singular value decomposition (SVD) is one popular least-squared solution method [45]. However, the SVD method is storage intensive and involves arbitrary tolerance filtering of the systems singular values. (The singular values of an

over-determined system of equations are analogous to the eigenvalues of a determined system of equations.) Both of these properties eliminate SVD from consideration for fluid dynamic problems which are typically large and include complex physical phenomena.

The normal equation method also obtains the least-squared solution of an over-determined linear system of equations [46]. Simple pre-multiplication of both sides of Equation 3.11 by the transpose of the metric Jacobian matrix, A^T , defines the normal equation technique. This second solution technique is also applied to the standard design equation and is presented in Equation 3.12.

$$\begin{bmatrix} A^T_{n \times m} & A_{m \times n} \end{bmatrix}_{n \times n} \Delta z_{n \times 1} = - A^T_{n \times m} (A_{m \times n} z_{n \times 1} + b_{m \times 1})_{m \times 1} \quad (3.12)$$

Equation 3.12 is the incremental normal equation (INE) and represents a solution method for the standard design equation [47]. The INE transforms the standard design equation into a determined system of equations for the solution of $\Delta z = \Delta(\bar{X})$. The shape modification solution for the vector $\bar{X}$ is obtained within the INE in a least-squared sense. The INE solution method is a key development within this research and is, therefore, an integral part of the proposed consistent direct-iterative design method.

The INE provides the best possible solution of the standard design equation because it obtains the least-squared solution for $\bar{X}$. The INE's also provide the flexibility to approximate the LHS matrix, which reduces the computational storage and cost required for this solution. Both metric Jacobian matrices within the normal matrix, $A^T A$, may be different in a numerical sense from either of the two which appear on the RHS of Equation 3.12. An efficient approximation of the normal matrix would use identical metric Jacobian matrices for its evaluation. This reduces the computational cost of its evaluation and also produces a symmetric normal matrix. The original normal matrix may then be replaced by an approximate one as presented in Equation 3.13.

$$\begin{bmatrix} A^T_{n \times m} & A_{m \times n} \end{bmatrix}_{n \times n} \rightarrow \begin{bmatrix} \tilde{A}^T_{n \times m} & \tilde{A}_{m \times n} \end{bmatrix}_{n \times n} \quad (3.13)$$

However, all Jacobian matrices on the RHS of equation 3.12 must be evaluated in a numerically consistent manner. The RHS portion of the INE represents the original

standard design equation cast in the normal equation format. This entire RHS must be numerically consistent to obtain a least-squared solution of the standard design equation. Therefore, the LHS and RHS sides of the INE's may be numerically distinct from each other. However, to obtain an efficient least-squared solution of the standard design equation each side of the INE must be independently numerically consistent.

Details of the metric Jacobian and normal matrices structure and elements are given in Appendix A. The following sections of this chapter present three other important issues for the solution of the INE's: the implementation of geometry/grid constraints, a recommended normal matrix approximation, and the INE error terms.

3.3.3 Geometry and Grid Constraints

Unlike analysis flow field solutions, physical constraints must be included within the solution of inverse design problems [10,11,48]. These constraints are both aerodynamic and geometric in nature. Generally, aerodynamic constraints for inverse design problems are concerned with the physical existence of the design solution. This investigation only considers inverse design solutions which are guaranteed to exist. Therefore, only geometric constraints are addressed within the present research.

Geometric constraints refer to the restriction of shape modification to certain conditions or limits. Examples of these constraints are the fixed length of a diffuser or the closed trailing edge of an airfoil. These types of constraints are applied locally at the flow geometry surface which occupies only a portion of the flow domain. In contrast, consistent shape modification is inherently global in nature which requires consideration of geometric constraints for the entire flow domain. Global constraints are applied along all free surface boundaries and within the interior of the grid. Examples of global geometric constraints are the outer boundary of an airfoil grid and the centerline of a combustion chamber. Global geometry/grid constraints are additional constraints which current design methods are not required to consider.

Therefore, both solid surface and global geometric constraints must be addressed within the proposed, direct-iterative design method. These combined geometry/grid

constraints are expressed in the present research as additional equations for each shape modification solution. In practice these equations take the form $\Delta \bar{X}_{j,k}=0$ because the initial geometry/grid naturally satisfies the constraints. An efficient solution of these equations would incorporate them within the INE's. However, the constraint equations cannot simply be added to the standard design equation. The constraints would not be exactly satisfied within the least-squared INE solution. The addition of the constraint equations to the standard design equation would also require dynamic storage definitions in the basic code structure. These storage requirements would be problem-dependent and would greatly increase the complexity of the code.

Geometry/grid constraints are satisfied within the present research with a two step process that maintains simple and general storage requirements [49]. The first step is to adjust the standard design equation to reflect each constraint equation. This is followed by replacing one equation within the standard design equation with each constraint equation. This method is applied to the standard design equation before it is cast into the INE. This method guarantees that each constraint is exactly satisfied and each constraint solution naturally appears within the shape modification solution vector $\Delta \bar{X}$. Implementation details for this method are given in Appendix B.

3.3.4 INE Singularity

Another important issue in the solution of the INE is the uniqueness of the least-squared solution for $\Delta \bar{X}$. A unique solution of the over-determined standard design equation requires a non-singular normal matrix. A non-singular normal matrix would be ensured if each equation within the standard design equation were linearly independent. Both geometric and aerodynamic singularity issues for the INE are discussed in this section. A simple method of guaranteeing a non-singular normal matrix is presented; this method is also an integral part of the proposed shape modification method.

The unknowns within the global shape modification solution are all of the physical (x,y) coordinate positions within the flow domain. To accurately resolve the flow field, viscous and highly non-linear inviscid problems require grid refinement. Within refined

grids, some coordinate positions may lie close together. If two grid points occupy the same point in space, then linear independence is violated for two equations within the standard design equation. Some degree of linear dependence can be tolerated within the standard design equation because it is massively over-determined. However, the normal matrix may be nearly singular if linear independence is violated often within the grid.

Another possible singularity problem exists for consistent shape modification within the INE which is aerodynamic in nature. For supersonic flow, the elements of the metric Jacobian matrix are linear combinations of cartesian flux terms [29]. This linear combination of flux terms is exactly zero for uniform supersonic flow. The normal matrix is a product of the metric Jacobian matrix. Therefore, zero elements within the metric Jacobian matrix would produce corresponding null elements within the normal matrix. The normal matrix may then be nearly singular if the uniform supersonic flow area is large. Other singularity problems may also exist for a solution of the INE. However, the examples cited are sufficient to warrant a systematic method of avoiding all singularity problems for consistent shape modification with the INE.

A number of options are available to guarantee a non-singular normal matrix and solve the standard design equation with the INE. One alternative is to test elements of the metric Jacobian matrix and identify equations which may produce a singularity. These equations may then be replaced with some benign identity statement, because they do not contribute to the system of equations. In practice, this is the evaluation of solution variable gradients by some tolerance level, and it is analogous to the SVD's method of filtering singular values. However, to avoid arbitrary filtering of data and to maintain a simple shape modification solution, another alternative is recommended.

One benefit of the incremental formulation is the flexibility to approximate the LHS normal matrix. This matrix may then be altered to avoid singularity problems. However, simply altering a singular normal matrix to initiate the INE solution is of no benefit. The original singularity problem would again be encountered as each shape modification solution converges. Therefore, a combination of two techniques is recommended. The first technique is to add the identity matrix to the LHS normal matrix as presented in Equation 3.14.

$$\left[\tilde{A}^T_{n \times m} \tilde{A}_{m \times n} \right]_{n \times n} = \left[A^T_{n \times m} A_{m \times n} \right]_{n \times n} + [I]_{n \times n} \quad (3.14)$$

This approximation ensures a non-singular normal matrix. The second technique is to restrict convergence of the INE solution to engineering accuracy. Combined, these techniques solve the standard design equation to an acceptable level of accuracy, and also provide a simple and robust solution of the INE.

3.3.5 Error Terms

It is important to understand the limitations of the shape modification method outlined above. Each solution of the standard design equation is strictly first-order accurate. The higher-order terms were truncated from the original Taylor's series expansion of the discrete residual vector. However, this truncation error represents the difference between the initial and final design shape estimates and their respective solutions. This error term is not equivalent to the standard, analysis solution truncation error. The truncation error of analysis solutions refers to the spatial accuracy of the flow field solution relative to the grid spacing.

There are three additional error terms within the INE solution of the standard design equation. The INE provides a least-squared solution of the standard design equation. Therefore, each equation within this system of equations is satisfied only in a least-squared sense. Additional errors are produced by requiring an incomplete convergence of the INE. Convergence restriction was required to avoid singularity problems. Also, the recommended method of implementing geometry/grid constraints within the INE removes equations from the original standard design equation. This technique violates the objective of consistency to some degree.

Another possible error term was avoided in this research by using van Leer's flux-vector-splitting method. The elements of the flux and metric Jacobian matrices are derived directly from van Leer's flux evaluation technique. Because this flux evaluation method is continuously differentiable, the Jacobian matrices are continuously defined for

all Mach numbers. This is in contrast to other popular flux evaluation methods which are not continuously differentiable. Use of these alternative methods would incur errors within the Jacobian matrices and, therefore, within the standard design equation.

No attempt is made in the present research to quantify the error terms within the solution of the INE mentioned above. The direct-iterative design method does not require an exact solution of the standard design equation. An iterative inverse design procedure only requires the best possible design shape prediction for each analysis/design iteration. The method of solving the standard design equation outlined above is currently considered the best possible method for consistent shape modification. The proposed method is also considered to be an advancement in shape modification methods relative to existing ones used within direct-iterative design procedures.

3.4 Summary

In summary, the INE was developed to solve the standard design equation for consistent shape modification. The INE, along with the associated constraint and singularity techniques, provides the best possible solution for each design shape prediction. The following chapter will present the details of implementing the INE within a direct-iterative inverse design procedure.

4.0 Design Shape Prediction

A solution method for the standard design equation was presented in the previous chapter. In the present chapter, the standard design equation is modified so that it may be employed within a direct-iterative inverse design procedure. A design tangency boundary condition for the Euler equations is also developed within this chapter. This new BC directly relates the target surface pressure to the shape modification process.

4.1 Modified Standard Design Equation

Direct-iterative methods iteratively determine the design geometry by successive modifications of the initial shape estimate. Each design shape prediction defines the current geometry. The current geometry and its respective analysis flow field solution, and the target surface pressure, P_t , constitute the available design information for each shape modification solution. The design information is sufficient to evaluate both the flux and metric Jacobian matrices within the standard design equation (Equation 3.7). These matrices are evaluated with the current geometry and solution. However, the design information is insufficient to evaluate the ΔQ vector which is also required within Equation 3.7. Therefore, the standard design equation may not be directly used within a direct-iterative inverse design procedure to predict $\Delta \bar{X}$.

The vector ΔQ represents the finite change in the solution variables between the current and design solutions. This vector is defined throughout the flow domain and is a function of the surface pressure residual. The pressure residual is the difference between the target and current surface pressures; $\Delta P_t = P_t - P$. However, ΔP_t represents only a small portion of ΔQ because the target pressure is only specified within the design

area. The design area is the portion of the initial flow geometry selected for shape modification. Therefore, it is not possible to evaluate the forcing function for the standard design equation, F_q . This vector is termed the solution forcing function and is presented in Equation 4.1.

$$\{F_q\}_{m \times 1} = \left[\frac{\partial \hat{R}(Q, \bar{X})}{\partial Q} \right]_{m \times m} \{\Delta Q\}_{m \times 1} \quad (4.1)$$

To obviate the requirement of a ΔQ vector within the standard design equation, it is necessary to approximate the entire solution forcing function. The change in solution between the current and design states is caused explicitly by a change in geometry and implicitly by ΔP_i . Insufficient information is available to estimate ΔQ due to both of these factors. However, it is possible to approximate Equation 4.1 by considering only the change in surface pressure. This approximation is developed by considering the residual to be an explicit function of Q and $\bar{X}$, and also a design variable vector, β . The expanded functional dependence of $\hat{R}$ for the approximation of F_q is presented in Equation 4.2.

$$\hat{R} = \hat{R}(Q, \bar{X}, \beta) \quad (4.2)$$

A Taylor's series expansion is performed on Equation 4.2 from one steady-state solution to the next. The series expansion is taken with respect to both the Q and β vectors, but not the geometry/grid vector $\bar{X}$. The steady-state residuals are zero, and the higher-order terms of the expansion are truncated. This truncated series expansion of Equation 4.2 is presented in Equation 4.3, where k is the length of the vector β .

$$\left[\frac{\partial \hat{R}(Q, \bar{X}, \beta)}{\partial Q} \right]_{m \times m} \{\Delta Q\}_{m \times 1} + \left[\frac{\partial \hat{R}(Q, \bar{X}, \beta)}{\partial \beta} \right]_{m \times k} \{\Delta \beta\}_{k \times 1} = \{0\}_{m \times 1} \quad (4.3)$$

(The truncated series expansion in Equation 4.3 was originally developed in Reference 32 for a special case of quasi-analytical approximate analysis.)

The second term in Equation 4.3 represents the change in the residual due to a perturbation of the design variables. This product, $(\partial \hat{R} / \partial \beta) \Delta \beta$, may be approximated by a finite difference expression as presented in Equation 4.4.

$$\left[\frac{\partial \hat{R}(Q, X, \beta)}{\partial \beta} \right]_{m \times k} \{ \Delta \beta \}_{k \times 1} \approx \{ \Delta \hat{R}(\beta) \}_{m \times 1} \quad (4.4)$$

This equation may be directly substituted into Equation 4.3 as presented in Equation 4.5.

$$\left[\frac{\partial \hat{R}(Q, X, \beta)}{\partial Q} \right]_{m \times m} \{ \Delta Q \}_{m \times 1} + \{ \Delta \hat{R}(\beta) \}_{m \times 1} = \{ 0 \}_{m \times 1} \quad (4.5)$$

The flux Jacobian matrices in Equation 4.5 and the standard design equation are identical. Both of these matrices are evaluated with steady-state solutions of the current geometry. The solution forcing function in the standard design equation may then be approximated as $F_q = (\partial \hat{R} / \partial Q) \Delta Q \approx -\Delta \hat{R}(\beta)$. This substitution process defines the modified standard design equation which is presented in Equation 4.6 [50].

$$- \{ \Delta \hat{R}(\beta) \}_{m \times 1} + \left[\frac{\partial \hat{R}(Q, X)}{\partial X} \right]_{m \times n} \{ \Delta X \}_{n \times 1} = \{ 0 \}_{m \times 1} \quad (4.6)$$

The forcing function for the modified standard design equation is the vector $\Delta \hat{R}(\beta)$ which represents the difference between two similar residual vectors where only the design variables have been perturbed. The design variable of interest for the inverse design problem is the surface pressure. A discrete pressure profile may then be directly substituted for β in Equation 4.6. Both discrete residuals within $\Delta \hat{R}(\beta)$ are evaluated with identical Q and $\bar{X}$ vectors as presented in Equation 4.7.

$$\{ \Delta \hat{R}(\beta) \}_{m \times 1} = \{ \hat{R}(Q, X, \beta + \Delta \beta) \}_{m \times 1} - \{ \hat{R}(Q, X, \beta) \}_{m \times 1} \quad (4.7)$$

The design variable perturbation, $\Delta \beta$, is analogous to the surface pressure residual, ΔP_i . The vector $\Delta \hat{R}(\beta)$ may also be subdivided into interior, $\Delta \hat{R}_i(\beta)$, and boundary, $\Delta \hat{R}_b(\beta)$, portions as presented in Equation 4.8.

$$\{ \Delta \hat{R}(\beta) \}_{m \times 1} = \{ \Delta \hat{R}_i(\beta), \Delta \hat{R}_b(\beta) \}_{m \times 1}^T \quad (4.8)$$

The formal evaluation of each residual vector in Equations 4.7 and 4.8 is not

necessary [32]. For inverse design purposes, the vector β is considered an externally specified surface pressure similar to a BC constant. While the interior residuals are functions of the surrounding solution vectors, they are not explicit functions of P_t . Therefore $\Delta\hat{R}_i(\beta)$ is exactly zero. The portion of $\Delta\hat{R}_B(\beta)$ which is outside of the design area is also not an explicit function of P_t , and is similarly zero. Only boundary equations which lie within the design area are functions of P_t . Therefore, $\Delta\hat{R}_B(\beta)$ is not equal to zero only in the design area. The evaluation of these boundary terms represents little computational effort and the vector $\Delta\hat{R}(\beta)$ is sparse.

Another benefit of the modified version of the standard design equation is the reduction of computational effort required to evaluate Jacobian matrices. The modified standard design equation does not require a flux Jacobian matrix. Therefore, significant savings in computational effort are incurred relative to the original equation. A reduction of approximately 80% in the cost of Jacobian evaluation is realized for a 2-D problem.

In summary, a simple modification of the standard design equation has been proposed. This new equation may be directly used for consistent shape modification within a direct-iterative design procedure. The modified standard design equation is only a function of the current geometry and solution, and the target pressure.

4.2 Design Tangency Boundary Condition

The modified standard design equation directly relates the target surface pressure to shape modification through the boundary equations. However, typical surface BC's are not functions of an externally specified target pressure. Therefore, before the modified standard design equation may be used for design purposes, a new surface BC, which is a function of P_t is required.

4.2.1 Inviscid Boundary Condition

The BC of interest for the Euler equations is the tangency condition which is applied on all solid surfaces. A common tangency BC vector which is applied for

analysis flow field solutions is presented in Equation 4.9. Within this analysis tangency BC, T_a , the symbol n denotes the outward normal vector as defined in Figure 1.

$$\{T_a\} = \left\{ \frac{\partial \rho}{\partial n} = 0, \frac{\partial h_0}{\partial n} = 0, \frac{\partial P}{\partial n} = 0, \bar{u} = 0 \right\}^T \quad (4.9)$$

Within T_a , the normal gradients of ρ , stagnation enthalpy, h_0 , and P are zero. The stagnation enthalpy is defined as $h_0 = \gamma/(\gamma-1)(P/\rho) + V^2/2$, where the total velocity, V , is defined with $V^2 = u^2 + v^2$. The outward normal velocity, $\bar{u}$, is also zero, and directly enforces the tangency condition. This velocity is an explicit function of the geometry surface direction cosines or metrics [29]. Other versions of T_a exist [51]; however, Equation 4.9 is a common version and the following development is easily repeated.

The objective is to modify T_a so that it is a function of P_t . This new design tangency BC, T_d , will then be used within the design area, and will be incorporated within the modified standard design equation. In order to maintain a consistent design method, it is necessary that the design tangency BC be equivalent to the analysis version, $T_d \rightarrow T_a$, at design convergence. Design convergence is achieved when the surface pressure residual is minimized, $\Delta P_t \rightarrow 0$. An equivalent form of T_a used in this research, which accounts for zero density and pressure gradients, is presented in Equation 4.10.

$$\{T_d\} = \left\{ \frac{\partial \rho}{\partial n} = 0, \frac{\partial V}{\partial n} = 0, \frac{\partial P}{\partial n} = 0, \bar{u} = 0 \right\}^T \quad (4.10)$$

The design tangency BC must only be a function of the current geometry and solution, and the surface pressure residual. This requirement was similarly imposed on the modified standard design equation. The specification of a pressure residual for the inverse problem produces a corresponding perturbation of surface density, $\Delta \rho$, total velocity, ΔV , and normal velocity, $\Delta \bar{u}$. All of these perturbations are functions of ΔP_t . The design tangency BC would then take the general form presented in Equation 4.11.

$$\{T_d\} = \left\{ \frac{\partial \rho}{\partial n} = +\Delta \rho, \frac{\partial V}{\partial n} = +\Delta V, \frac{\partial P}{\partial n} = +\Delta P_t, \bar{u} = +\Delta \bar{u} \right\}^T \quad (4.11)$$

The surface pressure must be identical to the target pressure at design convergence. The surface density and velocities will similarly become the corresponding correct values at design convergence. First-order BC's are typically applied at all free and solid surfaces within cell-centered finite-volume formulations. First-order BC's were also used within the present research. The discrete form of the surface pressure gradient, with $\Delta n=1$, is defined in Equation 4.12 using the boundary cell-face and first interior cell-center numbering as presented in Figure 1. Similar expressions are written for the density and total velocity gradients within T_d .

$$\frac{\partial P}{\partial n} = +\Delta P_t \rightarrow P_1 - P_2 = +(P_t - P_2) \quad (4.12)$$

The discrete form of the pressure gradient in Equation 4.12 is not a function of the geometry surface. This is similarly true for the discrete form of the density and total velocity gradients. Therefore, only the expression within T_s and T_d which contains $\bar{u}$ is an explicit function of $\bar{X}$. This property of the first-order boundary equations will later be shown to have an important impact on the proposed shape modification method.

4.2.2 One-Dimensional Euler Equations

The analysis tangency BC may be divided into an aerodynamic solution and a geometric condition. The geometric condition within T_s is the requirement of a tangent surface velocity. The first three expressions within T_s define the solution of an aerodynamic process for the analysis tangency BC. The aerodynamic solution is equivalent to the trivial solution of the one-dimensional (1-D) Euler equations between the first interior cell-center and the boundary cell-face. This aerodynamic solution provides a limit condition for the solution of $\Delta \rho$ and ΔV required in T_d .

An aerodynamic solution is also required for the design tangency BC. However, this new solution contains a known pressure perturbation, ΔP_t . Because the vector T_d must be equivalent to T_s at design convergence, it must also satisfy the 1-D Euler equations when $\Delta P_t \rightarrow 0$. The simple 1-D Euler equations are statements of conservation of mass, momentum, and energy for constant area [52]. These equations are presented in differential form in Equation 4.13.

$$\begin{aligned}\frac{d\rho}{\rho} + \frac{dV}{V} &= +0 \\ \frac{d\rho}{\rho} + 2\frac{dV}{V} &= -\frac{1}{\gamma M^2} \frac{dP}{P} \\ \frac{d\rho}{\rho} - (\gamma - 1)M^2 \frac{dV}{V} &= +\frac{dP}{P}\end{aligned}\tag{4.13}$$

It is proposed to approximate the finite changes in density and total velocity required in T_d with the differential change found with Equation 4.13; $\Delta\rho \approx d\rho$ and $\Delta V \approx dV$. However, the 1-D Euler equation with a specified dP , $dP = \Delta P_t$, represents a system of three equations and two unknowns. Therefore, just as the standard design equation for the solution of $\bar{\Delta X}$ is globally over-determined, the local solution for $\Delta\rho$ and ΔV at each boundary cell-face with Equation 4.13 is also over-determined.

There are at least four solution techniques which may be applied to Equation 4.13 to estimate $\Delta\rho$ and ΔV and satisfy the Euler 1-D equations at design convergence. Three solution techniques select combinations of two expressions in Equation 4.13 and solve for $\Delta\rho$ and ΔV . The three sets are denoted by me (momentum and energy), ce (continuity and energy), and cm (continuity and momentum). These notations define the two expressions which are satisfied within each solution set. However, these methods necessarily require that the expression which was not selected for each set be violated to some degree. The fourth solution technique is to satisfy each expression within Equation 4.13 in a least-squared sense. This method violates each 1-D Euler conservation law, but minimizes the errors, and is denoted by ls (least-squared).

4.2.3 Normal Forcing Function

A general format for the design tangency BC was presented in Equation 4.11. Given a surface pressure perturbation, a procedure was also outlined for determining the corresponding perturbations of density and total velocity required within T_d . The purpose of this section is to understand the relative importance of each expression within T_d as they relate to the proposed shape modification method. The following section of this chapter will present the final form of T_d used in this research.

The modified standard design equation, which now includes the design tangency BC, is solved with the incremental normal equation (INE). The INE (Equation 3.12) represents a global, iterative solution method. The forcing function for this equation is termed the normal forcing function, F_N . This vector is a product of the modified standard design equation pre-multiplied by the transpose of the global metric Jacobian matrix. The discrete form of this vector is presented in Appendix A. The global form of F_N is presented in Equation 4.14, where again $A = \partial \hat{R} / \partial \bar{X}$, $z = \Delta \bar{X}$, and $b = (\partial \hat{R} / \partial Q) \Delta Q$.

$$F_{N \ n \times 1} = A^T_{n \times m} (A_{m \times n} z_{n \times 1} + b_{m \times 1})_{m \times 1} \quad (4.14)$$

There are two products within F_N : $A^T A z$ and $A^T b$. The first product, $A^T A z$, appears within F_N only as a consequence of the iterative solution strategy. The second product within Equation 4.14, $A^T b$, is the solution forcing function, $b = F_q$, pre-multiplied by the transpose of the metric Jacobian matrix. The modified standard design equation requires the substitution $F_q = (\partial \hat{R} / \partial Q) \Delta Q \approx -\Delta \hat{R}(\beta)$ where $\Delta \beta = \Delta P_t$. Therefore, the target surface pressure is incorporated within the INE through the product $A^T b$. This product is only non-zero on the boundary within the design area. The profile P_t and the T_d BC are only specified within this region. Therefore, the product $A^T b$ within the design area is the focus for determining the relative importance of each expression within T_d .

For each coordinate position within the design area, the product $A^T b$ contains two portions, an interior and a boundary portion. The interior portion of $A^T b$ is a product of two local metric Jacobian matrices and two $\Delta \hat{R}(\beta)$ vectors from interior cell-centers. However, the interior portion of $\Delta \hat{R}(\beta)$ is zero. Therefore, the interior portion of $A^T b$

is also zero. The boundary portion of A^Tb is a product of two local metric Jacobian matrices and $\Delta\hat{R}(\beta)$ vectors which are derived from the T_d boundary equation. Within the design area, $\Delta\hat{R}_B(\beta) \neq 0$. However, the metric Jacobian matrix of T_d is sparse. Only the expression within T_d which contains $\bar{u}$ produces non-zero elements within the boundary metric Jacobian matrix. In the process of pre-multiplying the transpose of the sparse metric Jacobian matrix to the non-zero $\Delta\hat{R}_B(\beta)$ vector, all perturbations of density, total velocity, and pressure which appear within T_d are nullified.

Therefore, the perturbations of density, total velocity, and pressure which appear within the design tangency BC are irrelevant. However, the surface pressure residual, ΔP_t , is still required to determine the normal velocity perturbation within T_d . In retrospect, it was not surprising that $\bar{u} = \Delta\bar{u}$ was the only relevant term within T_d . This term is a function of $\bar{X}$ at the geometry surface, and the dependency is expressed through the local metric Jacobian matrix. In contrast, the aerodynamic portion of T_d is not a function of $\bar{X}$. The sparse boundary metric Jacobian matrix also reflects this property and nullifies the aerodynamic perturbations within T_d .

4.2.4 Influence Coefficients

The exclusive dependency of the design tangency BC on a normal velocity perturbation obviates the need for $\Delta\rho$, ΔV , and ΔP_t within T_d . This simplification does not relax the original requirements for T_d . However, this condition does provide flexibility in selecting an expression for $\Delta\bar{u}$. It is proposed to approximate $\Delta\bar{u}$ directly with the ΔV expression determined with the 1-D Euler equations. There is a unique expression for ΔV , and thus $\Delta\bar{u}$, for each of the four methods of solving Equation 4.13. Each of these expressions is substituted into T_d to obtain four different design tangency BC's. The final form of each design tangency BC is presented in Equation 4.15, where, for each unique $\Delta\bar{u}$ expression T_d becomes T_{mc} , T_{cc} , T_{cm} , or T_{ls} .

$$\{T_d\} = \left\{ \frac{\partial \rho}{\partial n} = 0, \frac{\partial P}{\partial n} = 0, \frac{\partial V}{\partial n} = 0, \bar{u} = +\Delta \bar{u} \right\}^T \quad (4.15)$$

Expressions for $\Delta \bar{u}$ may be expressed as influence coefficients, $(\Delta \bar{u}/V)/(\Delta P_t/P)$. These coefficients are functions of γ and Mach number, M , and are defined in Equations 4.16-4.19. The influence coefficients are also presented graphically in Figure 2.

$$\frac{\Delta \bar{u}_{me}/V}{\Delta P_t/P} = - \frac{1}{\gamma M^2} \left[\frac{\gamma M^2 + 1}{2 + (\gamma - 1)M^2} \right] \quad (4.16)$$

$$\frac{\Delta \bar{u}_{ce}/V}{\Delta P_t/P} = - \left[\frac{1}{1 + (\gamma - 1)M^2} \right] \quad (4.17)$$

$$\frac{\Delta \bar{u}_{cm}/V}{\Delta P_t/P} = - \frac{1}{\gamma M^2} \quad (4.18)$$

$$\frac{\Delta \bar{u}_{ls}/V}{\Delta P_t/P} = - \frac{1}{2\gamma M^2} \left[\frac{2\gamma(\gamma - 1)M^4 + (4\gamma - 1)M^2 + 3}{(\gamma - 1)^2 M^4 + 3(\gamma - 1)M^2 + 3} \right] \quad (4.19)$$

The influence coefficients are simple and they are a function of the local Mach number. Each coefficient is applicable for subsonic, transonic, and supersonic flows. The values of $\Delta \bar{u}$ required within T_d are easily evaluated with the design information. The total velocity, pressure, and Mach numbers in Equations 4.16-4.19 are evaluated at the first interior cell-center, and the Mach number is a function of the total velocity. Also, within Figure 2, $\Delta \bar{u}_{me} \approx \Delta \bar{u}_{ls}$ for all Mach numbers. Because the expression for $\Delta \bar{u}_{me}$ is much simpler than that for $\Delta \bar{u}_{ls}$, the T_{ls} tangency BC will not be further considered.

The sequence of the influence coefficients presented in Figure 2 will be shown to have an important impact on the results presented in the following chapters. For any Mach number less than one, the absolute value of the influence coefficients appear in ascending order as T_{ce} , T_{me} , and T_{cm} , respectively. In contrast, for any Mach number

greater than one, the absolute value of the influence coefficients appear in ascending order as T_{cm} , T_{mc} , and T_{cc} , respectively. At the sonic point, the absolute value of each influence coefficient in Equations 4.16-4.19 is equal to $1/\gamma$.

4.2.5 Transpiration Velocity

An expression for the vector $\Delta \hat{R}(\beta)$ is the only remaining definition required to implement the modified standard design equation. The original definition of $\Delta \hat{R}(\beta)$ was given in Equation 4.7. This vector is sparse because $\Delta \hat{R}_I(\beta)=0$, and P_t is only specified within the design area. The vector $\Delta \hat{R}_B(\beta)$ is also sparse because only the geometric condition within Equation 4.15 is a function of P_t . The definition of $\Delta \hat{R}_B(\beta)$ for the design tangency BC's developed in this research is presented in Equation 4.20.

$$\{\Delta \hat{R}_B(\beta)\} = \{0, 0, 0, -\Delta \bar{u}\}^T \quad (4.20)$$

The negative value of $\Delta \bar{u}$ is the only term which appears within $\Delta \hat{R}_B(\beta)$ and, therefore, $\Delta \hat{R}(\beta)$. This term represents the normal velocity into the flow field through the body surface, and is commonly termed a transpiration velocity. The proposed design method is, therefore, similar to existing transpiration velocity inverse design methods [53-56]. These are iterative methods which employ a two step shape modification procedure. The existing methods first use an altered form of the governing equations to determine $-\Delta \bar{u}$. Secondly, these methods use simple integration techniques to estimate the next design shape. However, none of the existing transpiration design methods represent a consistent application of the governing equations.

In contrast, the proposed consistent design method first approximates the transpiration velocity with a local application of the 1-D Euler equations. Secondly, this new method directly applies the global 2-D Euler equations to determine the next design shape estimate. The new method reverses the order in which the global governing equations are used. The present method also applies the governing equations directly for each design shape prediction. Therefore, both the existing and the proposed methods rely on transpiration velocities for the design procedure. However, differences in their

application of the governing equations represent an important distinction which further makes the proposed method unique within the design community.

4.3 Summary

In summary, the modified standard design equation was developed for use within an inverse design procedure. A design tangency BC which relates the target surface pressure to shape modification was also developed. These new procedures, together with the INE, represent the key developments within this research. These techniques will be demonstrated in the remaining chapters to provide an accurate inverse design solution.

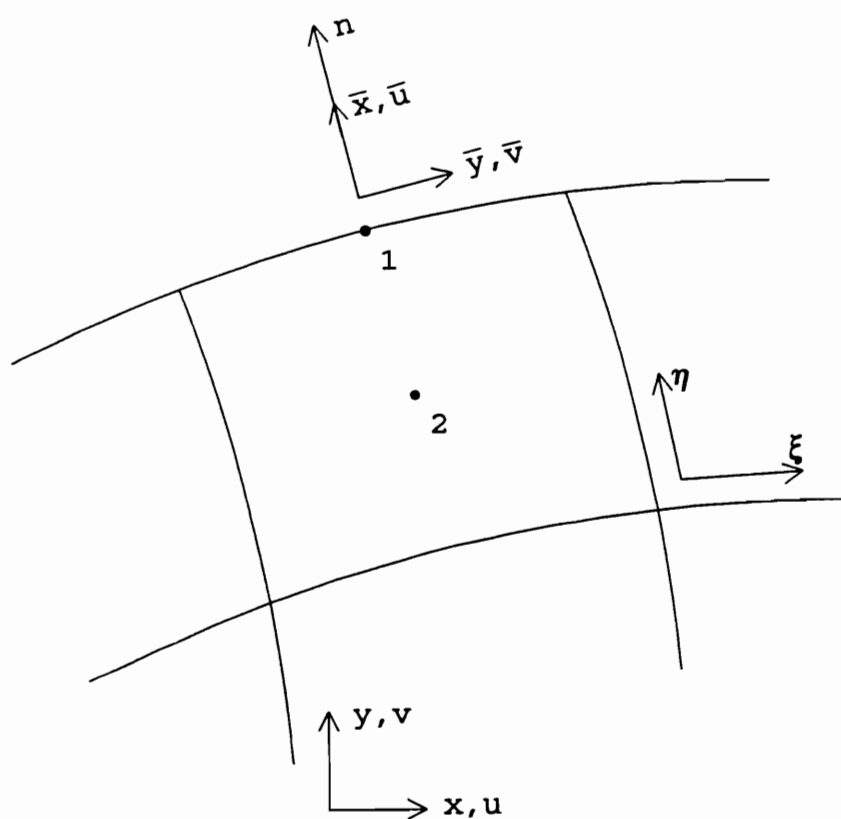


Figure 1 : Boundary Computational Cell

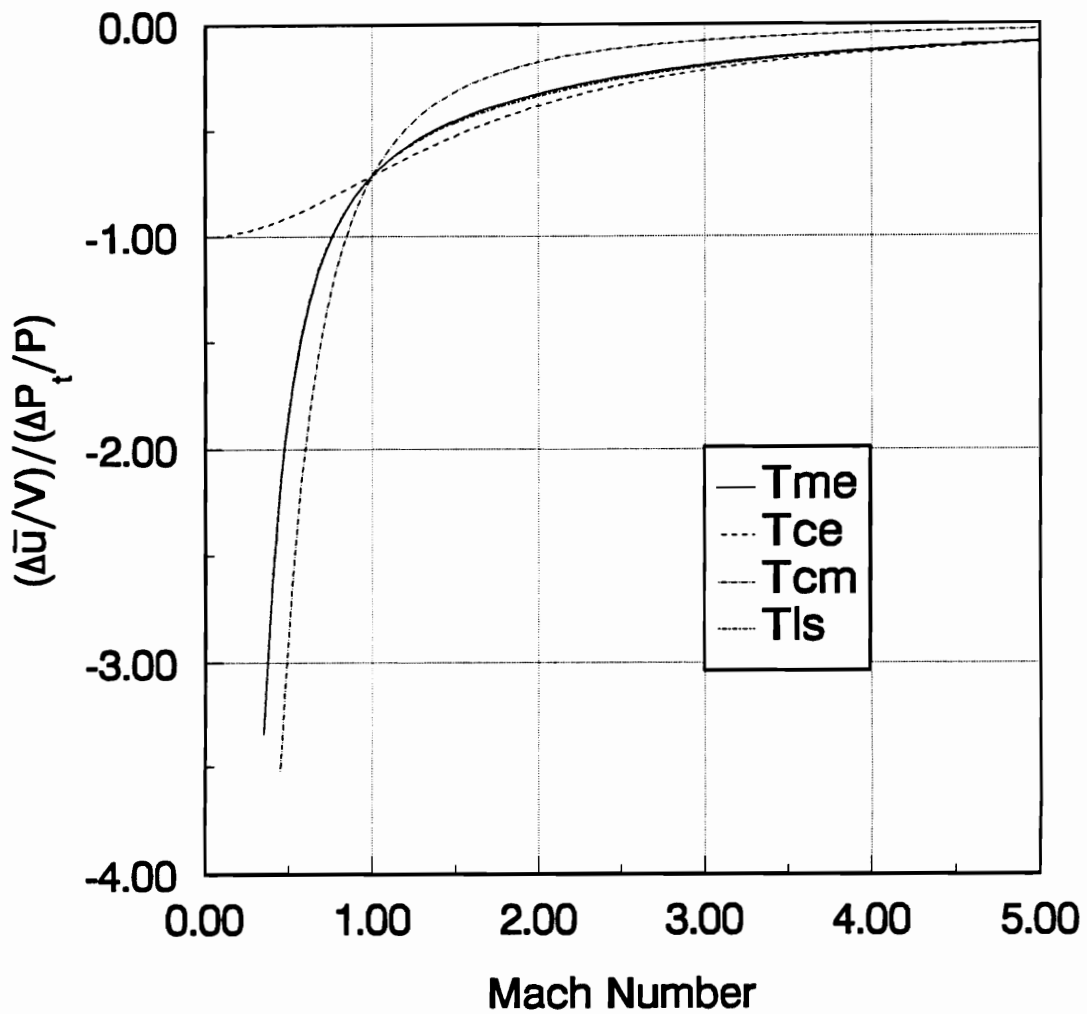


Figure 2 : Influence Coefficients

5.0 Design Algorithm

This chapter briefly reviews the direct-iterative inverse design algorithm. Those portions of the algorithm where the present research differs from the existing direct-iterative methods will also be emphasized.

The flow chart for the direct-iterative design method is presented in Figure 3. An initial shape estimate, a set of boundary conditions, and the target surface pressure are required before the application of the inverse design procedure. The analysis flow field solution about the initial geometry is then computed with this information. The iterative portion of the design method begins with a shape modification solution, which is then followed by a geometry update procedure. These two procedures define the current design shape prediction, and the analysis flow field solution about this geometry is then computed. The current surface pressure within the design area is then compared to the target pressure, and the design procedure is terminated if the pressure residual is sufficiently small. If the pressure residual does not satisfy the convergence criterion, then repeated analysis/design iterations are performed to obtain the design solution.

The direct-iterative inverse design method used in this research differs from the existing methods by utilizing the proposed consistent shape modification method. To summarize, this new method is comprised of the design tangency BC, the modified standard design equation, and the incremental normal equation. Implementation of the geometry/grid constraints and the normal matrix approximation are also an integral part of the proposed shape modification method. The following chapters of this dissertation demonstrate how these new numerical techniques accurately solve the aerodynamic inverse design problem.

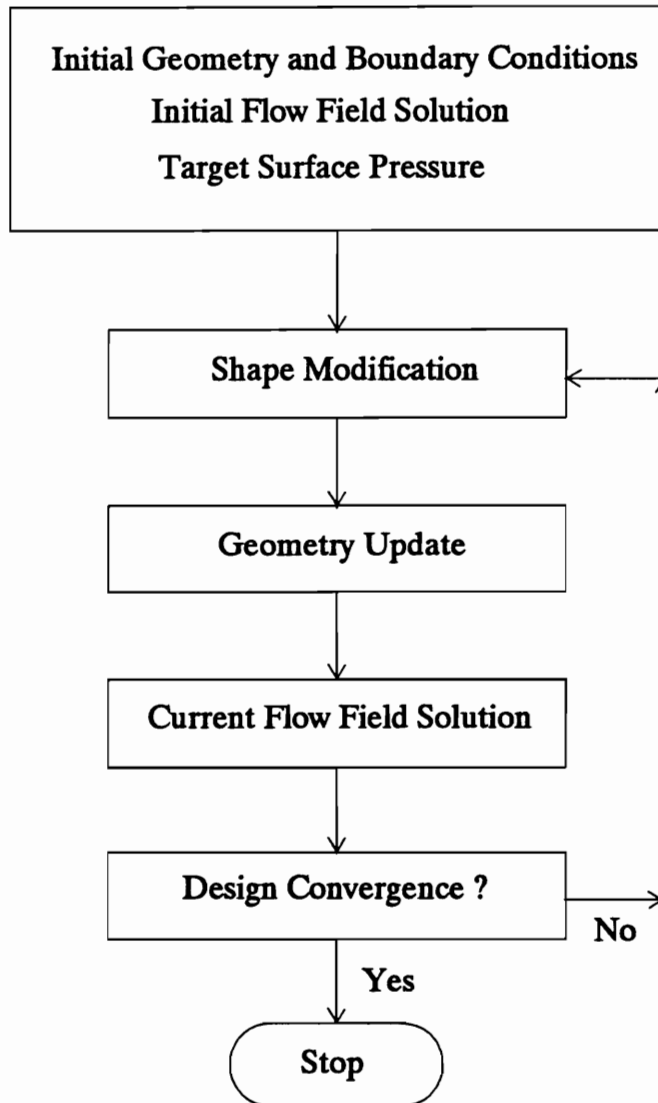


Figure 3 : Direct-Iterative Design Algorithm

6.0 Inverse Design Tests

The objective of this research was to develop and demonstrate a consistent direct-iterative inverse design method. The purpose of this chapter is to define the parameters for a series of test problems which are used to establish the ability of the new method to obtain an inverse design solution. Towards this end, a simple geometry was chosen, and the method was applied for three flow regimes; subsonic, transonic, and supersonic.

A series of controlled inverse design problems was selected for this research. Within controlled tests, the initial and target geometries and their respective analysis solutions are known prior to the application of the design procedure. These test conditions guarantee the existence of the design solution because the analysis solution of the target geometry is a physically possible end state of the design problem. The target surface pressure is obtained from the analysis solution of the target geometry. The objective of a controlled inverse design problem is then to reproduce the target geometry.

The geometry selected for this research was the symmetric channel presented in Figure 4. The reference length for the channel was the inlet half-height, H . The channel contained three equal length sections with a ramp in the middle section. The ramp angle, θ , was the geometric design variable. The initial channel was defined with $\theta=5.0^\circ$, and three target geometries were then defined with increased ramp angles of $\theta=5.1^\circ$, 5.5° , and 6.0° . These target ramp angles represent geometric design variable perturbations of $\Delta\theta=2\%$, 10% , and 20% . Only the lower portion of the symmetric channel was computationally modeled, and each half-channel grid contained 31 and 21 evenly distributed lines in the x and y directions, respectively. Three inlet Mach number test cases were then defined for this geometry; $M=0.30$, 0.85 , and 2.00 .

For each analysis flow field solution, the analysis tangency BC was applied along

the channel wall, and symmetry was enforced along the centerline. The inlet BC for the subsonic and transonic test cases fixed the stagnation enthalpy and entropy at free-stream values. Also, the vertical component of velocity was zero, and the pressure was extrapolated from the interior. The outlet BC for these test cases extrapolated the density and both components of velocity from the interior, and fixed the back pressure ratio at $P_t/P_\infty=1.0$. The inlet BC for the supersonic test case fixed the density, velocity, and pressure ratios at $\rho/\rho_\infty=1.0$, $u/u_\infty=1.0$, $v/u_\infty=0.0$, and $P/P_\infty=1.0$. The outlet BC for this test case extrapolated all primitive solution variables from the interior.

The design area for the channel geometry was the ramp section of the lower wall: $1.0 < x/H < 2.0$. The magnitude of the total velocity differed between the initial and target solutions along the entrance plate ($0.0 < x/H < 1.0$) and the exit plate ($2.0 < x/H < 3.0$) of the channel wall. These total velocity vectors were parallel for all geometries. However, the total velocity magnitude and direction differed along the ramp between the initial and target solutions. The difference in velocity direction was considered to produce a transpiration velocity in this area. This definition of the design area is consistent with the design tangency BC, in which only the change in body normal velocity is relevant.

The target surface pressure was, therefore, defined only along the ramp for the inverse design problem. This produced a discontinuous target pressure profile along the channel wall as presented in Figure 5. The resulting forcing function for the modified standard design equation was sparse, and the non-zero elements represented less than one percent of the total vector length. The BC's used for each shape modification solution were identical to those used for the analysis solutions except along the channel ramp. For each design shape prediction, the design tangency BC was applied within the design area.

Geometric constraints are also required for an inverse design problem. The global geometry/grid constraints used for the channel fixed the y positions of the inlet plane, the centerline, and the entrance plate. The x coordinates throughout the domain were also fixed. However, coordinate positions on the ramp, exit plate, and exit plane were free to move vertically. These constraints effectively fixed the channel inlet half-height and total length, while allowing the ramp and exit plate to change both shape and position.

All shape modification solutions were considered converged when the l_2 norm

of the $\Delta \bar{z} = \Delta(\Delta \bar{X})$ vector, normalized by the first iteration value, was reduced by two orders-of-magnitude. A banded-matrix, direct solver was used to compute the inverse of the normal matrix for these solutions [57]. A symmetric normal matrix was used within this research; however, the entire matrix between the outermost diagonal bands was used to compute its inverse. All analysis solutions of the initial and target geometries were considered converged when the normalized l_2 norm of the residual vector was reduced by twelve orders-of-magnitude. All re-analysis solutions began with the previous flow field solution and were considered converged when the normalized l_2 norm of the residual vector was reduced by six orders-of-magnitude.

A simple geometry update procedure was used within this research. The proposed shape modification method predicts the finite change in the current geometry/grid vector, $\Delta \bar{X}$. This vector was then simply added to the current vector, $\bar{X} = \bar{X} + \Delta \bar{X}$, to define the new geometry and grid. However, to ensure a smooth grid structure for each re-analysis, the interior portion of this new grid system was then re-generated. This process effectively used the shape modification solution only along the domain boundary which includes the channel geometry. While more advanced geometry update procedures are available [58], this simple method was sufficient for the objectives of this research.

The goal of these test problems was to accurately predict the target geometries with the new design method. Convergence of the design solution was monitored with the surface pressure residual within the design area and will be discussed in the next chapter. Success of the design solution was established by comparing the final design geometry to the known target geometry. The target geometry for these problems represents a measure of success not available within typical, un-controlled inverse design problems.

The comparisons between the design and target geometries will be presented in the next chapter as vertical displacements, Δy , along the channel wall with respect to the initial geometry. Monitoring the horizontal displacements, Δx , of the predicted channel wall was not required because the channel was held fixed in this coordinate direction. The Δy comparisons will be normalized by the reference length, Δy_{ref} , which is presented in Figure 6. This reference length is the difference between the initial and target geometry exit plate heights and, therefore, is uniquely defined for each target geometry.

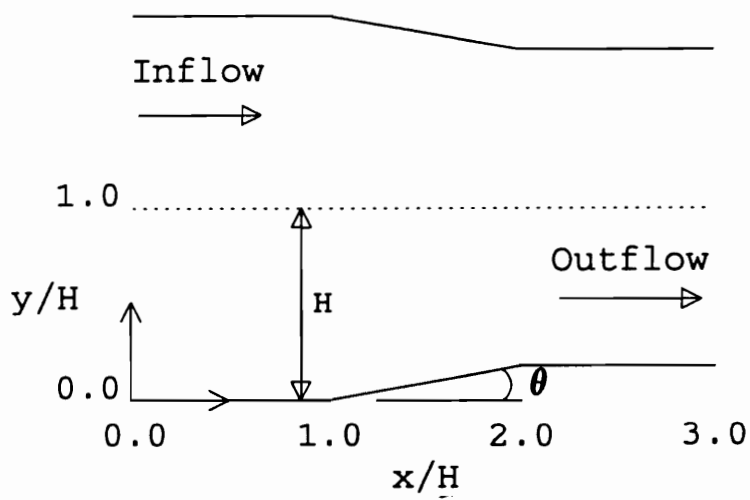


Figure 4 : Channel Geometry

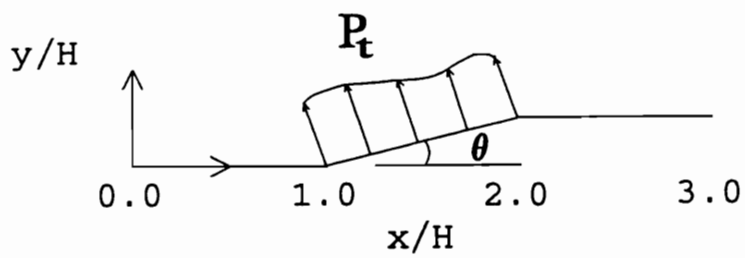


Figure 5: Channel Design Area

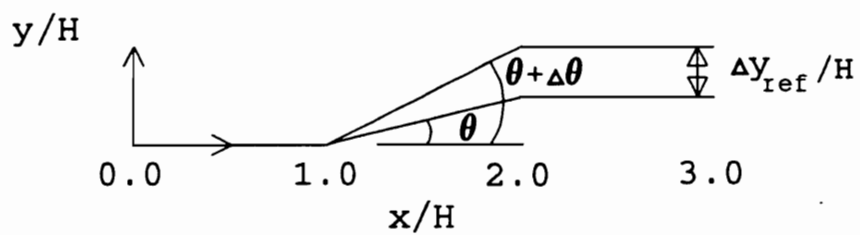


Figure 6: Channel Geometry Modification

7.0 Results and Discussion

The purpose of this chapter is to demonstrate the ability of the new, consistent direct-iterative method to solve the inverse design problem. The key component of this design procedure is the shape modification method. Therefore, it is important to demonstrate the ability of the consistent shape modification method to predict design shapes. The first, or initial, design shape prediction within the solution process will be presented for these purposes. This shape prediction is a function of the initial geometry and its direct solution and the target surface pressure. The results which follow will present the initial design shape prediction and then the final design geometry for the controlled inverse design test problems. Results for the supersonic test case will be discussed first, followed by those for the transonic and subsonic test cases.

7.1 Supersonic Test Case

7.1.1 Initial Design Shape Prediction

The consistent direct-iterative method requires the calculation of transpiration velocities which are then used in the modified standard design equation forcing function. These body-normal velocities are determined by the design tangency BC's and are explicit functions of the surface pressure residual. The pressure residual is the difference between the target and computed surface pressures, $\Delta P_t = P_t - P$. The initial surface pressure residuals for the supersonic test case are presented in Figure 7 for each target geometry. Each curve in Figure 7 is presented as a percentage of the initial surface pressure which is identical for each target geometry. The shape of these profiles is a

consequence of the flow field characteristics and relative change in channel geometries.

The flow field of the supersonic test case is characterized by a shock emanating from the ramp leading edge (LE) and an expansion fan that is located at the ramp trailing edge (TE). The initial and target geometry flow fields are identical along the entrance plate because the uniform supersonic flow is unchanged upstream of the shock. The surface pressure residuals are, therefore, zero in this area. The relative change in the ramp angle produces a difference in shock strength, and the pressure residuals are non-zero past the ramp LE. The pressure residuals are positive and relatively constant along the ramp for each target geometry. The average value of $\Delta P_t/P$ is 0.5%, 3.0%, and 6.0% in this area for the $\Delta\theta=2\%$, 10%, and 20% target geometries, respectively. Past the ramp TE, each flow field uniquely expands to the channel exit.

The initial transpiration velocities for the $\Delta\theta=10\%$ target geometry as determined by each design tangency BC are presented in Figure 8 for the supersonic test case. Each of these profiles is similar in shape to the surface pressure residual profiles. Along the ramp, the transpiration velocities are negative relative to the outward normal direction. These velocities are then imagined to flow into the flow field from the initial geometry. Along the ramp, the average value of the normal velocity for the T_{cm} , T_{mc} , and T_{ce} design BC's appear in ascending order respectively. This sequence is consistent with the sequence of absolute values for the influence coefficients presented in Figure 2, when the Mach number is greater than one. While Figure 8 only presents the transpiration velocities for one target geometry, the other target geometries had similar profiles.

The initial design shape predictions for the supersonic test case are presented in Figure 9 for all three target geometries and for all three design tangency BC's. Each profile in Figure 9 presents predicted shape changes, $\Delta y/\Delta y_{ref}$, along the channel wall from which the actual predicted geometry is easily inferred. In general, each predicted geometry contains a linear ramp, and the exit plate correctly extends from the ramp TE as a horizontal line. The continuous geometric shape change results are in contrast to the discontinuous P_t profile specified only along the ramp as presented in Figure 5.

Each predicted shape change profile in Figure 9 has a positive and relatively constant slope along the ramp. This characteristic is attributed to the positive and

relatively constant pressure residual in this area. Also, the exit plates shape and position were not guaranteed by the geometry/grid constraints, but correctly appeared within the solution. The predicted ramp TE height and the exit plate position for the T_{cm} , T_{mc} , and T_{ce} design BC's are on average 65 %, 110 %, and 130 % of the target value, respectively. This sequence of results is again consistent with the sequence of the influence coefficients for supersonic Mach numbers. Therefore, the average accuracy of each initial shape prediction is 65 %, 90 %, and 70 % for the T_{cm} , T_{mc} , and T_{ce} design BC's, respectively.

7.1.2 Inverse Design Solution

The initial design shape predictions represent the first shape estimate within the design procedure. Subsequent analysis/design iterations were performed with the consistent direct-iterative method to obtain the final design solution. Convergence of the design solution was measured by the l_2 norm of the surface pressure residual which is defined only along the ramp. The design solution was considered converged when this vector norm, $RMP = \|P_t - P\|_2$, was less than 1.0×10^{-3} . The following results will focus on the $\Delta\theta = 10\%$ target geometry. The results for all target geometries, which are similar to this one, will be summarized at the conclusion of this section.

The design solution convergence history for the $\Delta\theta = 10\%$ target geometry for the supersonic test case is presented in Figure 10. For each design tangency BC, RMP is presented as a function of analysis/design iterations. Each curve in Figure 10 monotonically decreases and asymptotically approaches the convergence limit. The total number of iterations required to achieve the design solution was 6, 7, and 9 for the T_{mc} , T_{ce} , and T_{cm} design BC's, respectively. Therefore, the design tangency BC which most accurately predicted a single design shape, T_{mc} , converged to the final shape in the fewest iterations. However, the total number of iterations required to achieve the design solution is very small for each design BC. This represents an important demonstration of the new method to solve the inverse design problem.

A comparison between the final surface pressure and the target pressure is required to establish the accuracy of the design solution. The final surface pressure is

determined with the analysis code on the last design shape estimate. Figure 11 presents the initial, target, and final design surface pressures for the $\Delta\theta=10\%$ target geometry. This comparison is for the geometry which was predicted with the T_{mc} design BC. The target pressure profile is exactly matched for this supersonic test problem. The solutions which used the T_{ce} and T_{cm} design BC's produced similar final design pressure profiles.

A controlled inverse design test may also monitor the geometric convergence of the design problem. This convergence criterion compares the target geometry to each successive predicted geometry within the design solution. This research used the difference in physical coordinate positions along the entire length of the channel wall to measure geometric convergence. This convergence criterion is termed the geometry residual, and the l_2 norm of this vector is defined as $RMG = \|\bar{x}_t - \bar{x}\|_2$. The geometry residual convergence history for the $\Delta\theta=10\%$ target geometry for the supersonic test case is presented in Figure 12. For each design tangency BC, RMG is presented as a function of analysis/design iterations.

Each geometry residual profile in Figure 12 monotonically converges and asymptotically approaches zero. These characteristics are similar to those exhibited by the pressure residual. While no geometric convergence limit was established for the design solution, Figure 12 provides an understanding of the new design methods behavior. Geometric convergence was achieved in two iterations for the T_{mc} BC. Both the T_{ce} and T_{cm} design BC's achieved the design solution in four iterations. This rapid geometric convergence rate is another important demonstration of the consistent direct-iterative method to solve the inverse design problem.

The convergence of the inverse design solution to the correct geometry may also be verified by comparing the predicted design shape to the target geometry. The final design shape estimate for the $\Delta\theta=10\%$ target geometry is compared to the known target geometry in Figure 13. The design profile in this figure was obtained with the T_{mc} design BC on the supersonic test problem. The profiles in this figure are coincident and, thus, the correct geometry was predicted. The T_{ce} and T_{cm} design BC's also predicted the target geometry with the same level of accuracy.

7.1.3 Summary

The design convergence history for the supersonic test case as measured by the pressure residual is summarized in Figure 14. The total number of analysis/design iterations required to achieve the design geometry is presented for each design tangency BC solution for each target geometry. The total number of iterations required to achieve the correct design geometry ranges from a minimum of three for the $\Delta\theta=2\%$ target geometry, to a maximum of twelve for the $\Delta\theta=20\%$ target geometry. While the number of iterations increases as $\Delta\theta$ becomes larger, the rate of increase is reduced for the larger geometric design variable perturbations. Therefore, all target geometries were correctly predicted in very few iterations using each design tangency BC for this test case.

The ability of the consistent direct-iterative method to solve the inverse design problem has been demonstrated for the supersonic test case. This was one of the primary objectives of this research. However, it is also important to understand the computational efficiency of the new method. The computational efficiency of the direct-iterative method is herein defined in terms of a design work unit, which is a ratio CPU times. The numerator of the design work unit is the total CPU time required to achieve the design solution. This term does not include the CPU time required to obtain the initial analysis solution. However, the initial analysis CPU time, which is required to initiate the solution process, is used as the denominator of the design work unit.

The computational efficiency of the consistent direct-iterative method as measured by the design work unit is summarized in Figure 15 for the supersonic test case. Each curve in Figure 15 represents a different design tangency BC and is presented as a function of $\Delta\theta$. The average value of the design work unit for the $\Delta\theta=2\%$ target geometry is just over unity. The average value of the design work unit is nearly two, and just under three for the $\Delta\theta=10\%$ and $\Delta\theta=20\%$ target geometries, respectively. The design work unit increases for larger values of $\Delta\theta$, but again the rate of increase is reduced for the largest geometric design variable perturbations. The typical design work unit cited in the literature is between one and three. Therefore, while no serious effort was made in this research to optimize the design work unit, the new method still exhibits

favorable computational efficiency for this test case.

Furthermore, while each design BC was demonstrated to solve the inverse design problem, one of the new BC's consistently outperformed the others. The T_{mc} design BC required the fewest iterations and the least computational cost to achieve the design solution for all target geometries. This may imply that conservation of momentum and energy is more important than continuity for the supersonic test case. However, the influence coefficient profile for the T_{mc} BC presented in Figure 2 is nearly coincident with the least-squared solution of the 1-D Euler equations. Therefore, both the T_{mc} and T_{ls} design BC's minimized the errors for each conservation law. This common characteristic, and the relative success of the T_{mc} design BC, then implies that each conservation law is equally important for high speed flows.

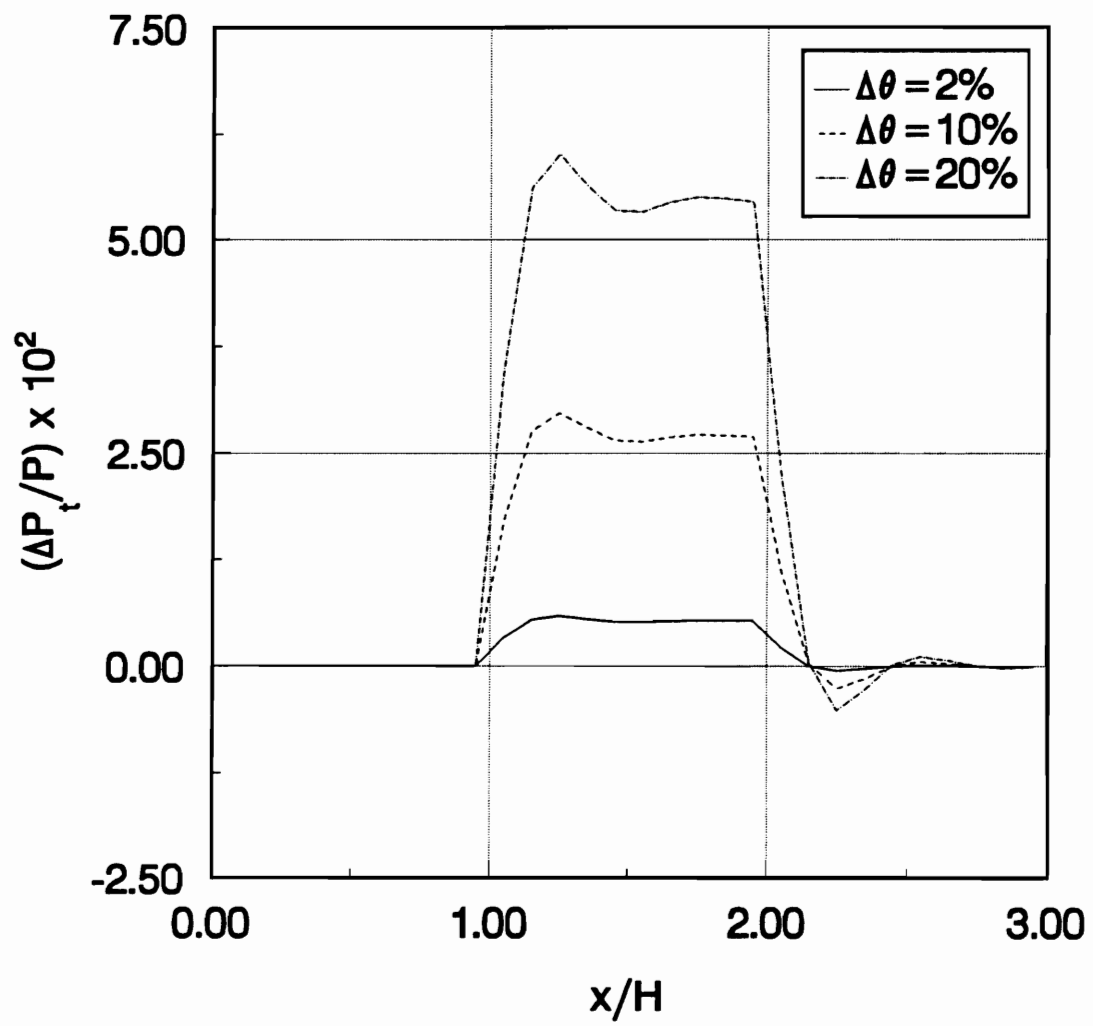


Figure 7 : Pressure Residual; $M=2.00$

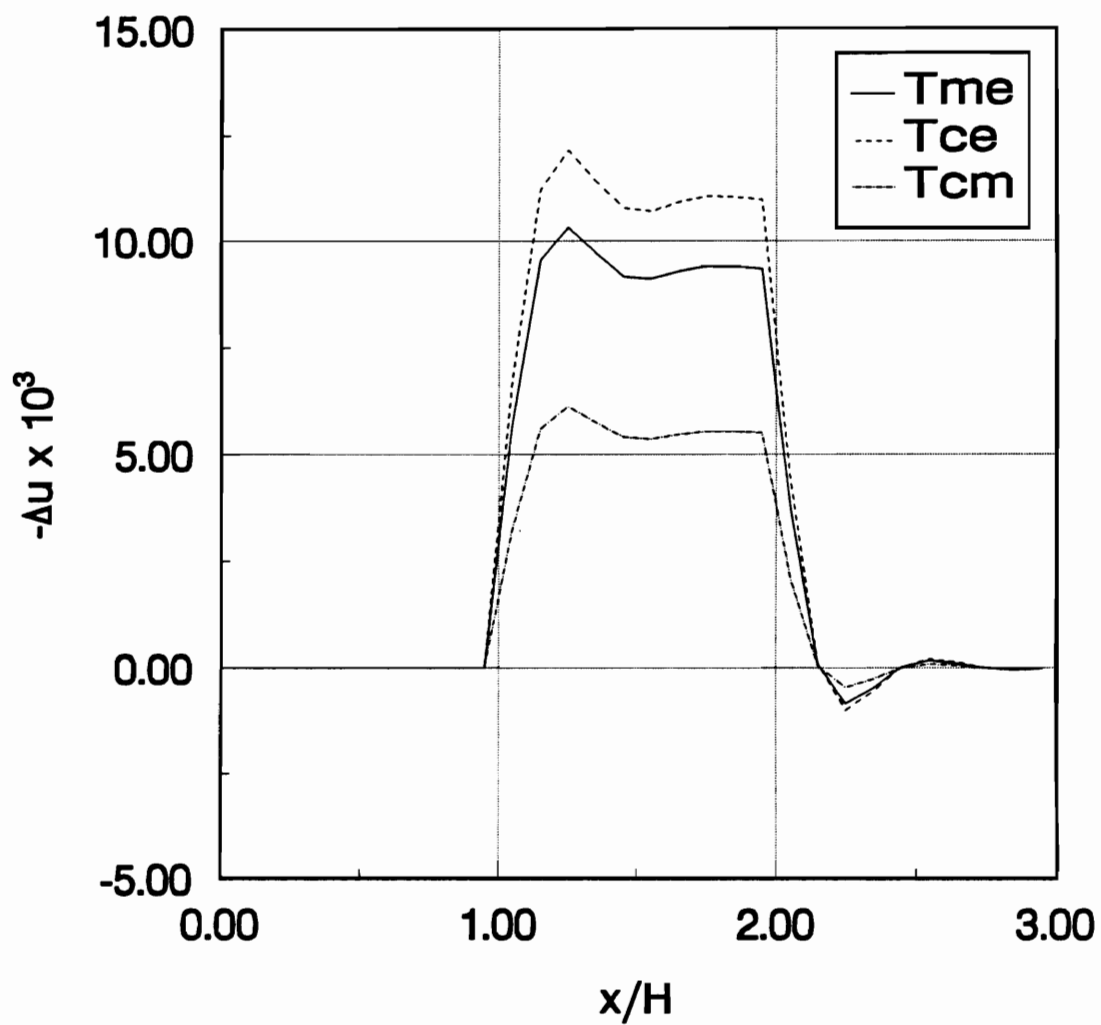


Figure 8 : Transpiration Velocity; $M=2.00$, $\Delta\theta=10\%$

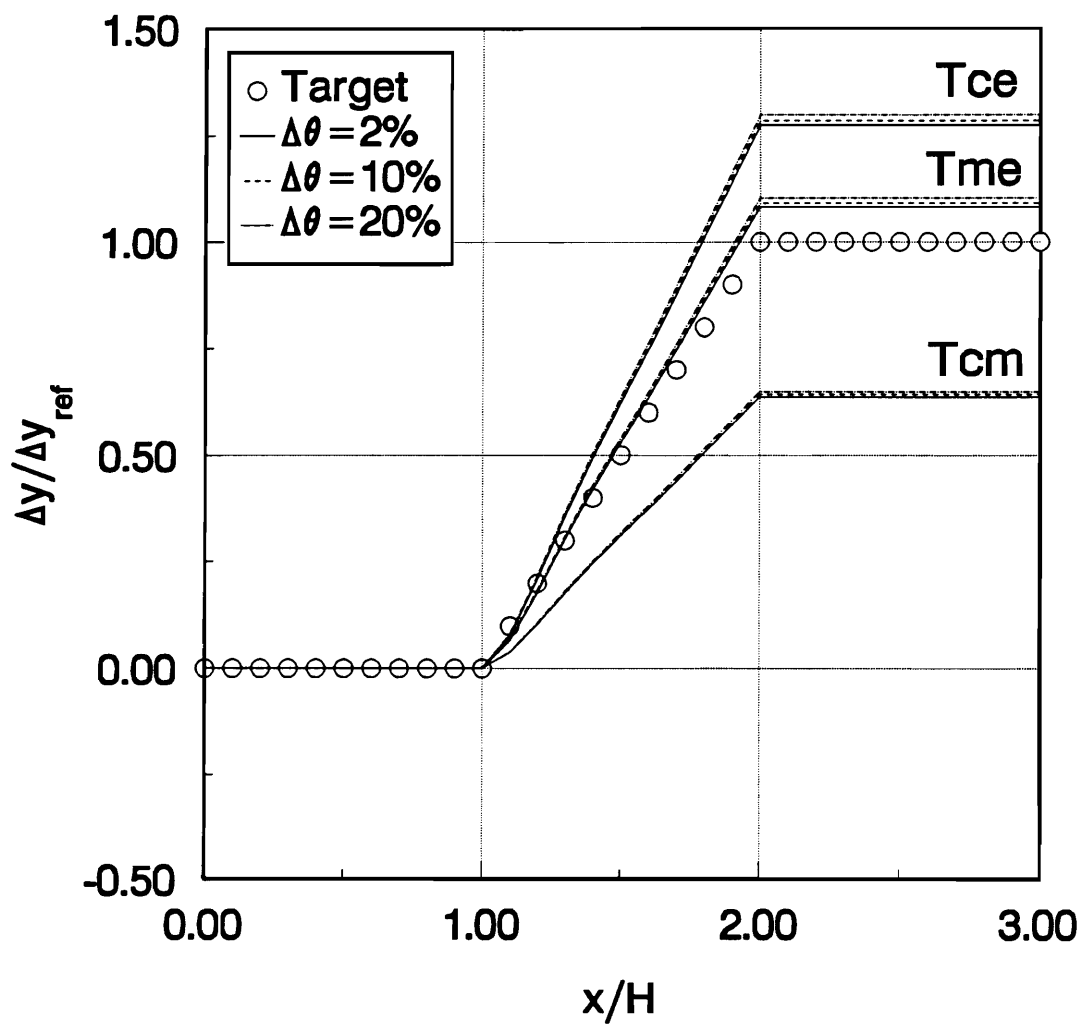


Figure 9 : Initial Design Shape Prediction; $M=2.00$

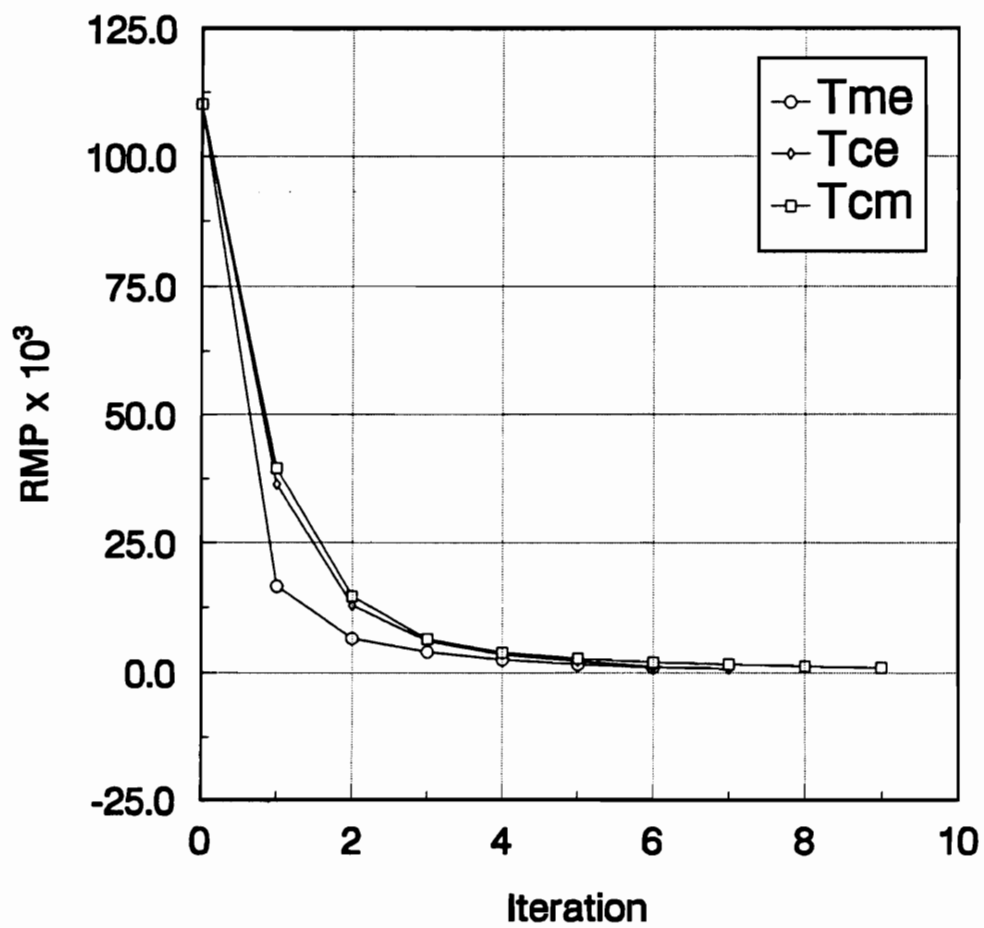


Figure 10 : Pressure Residual Convergence; $M=2.00$, $\Delta\theta=10\%$

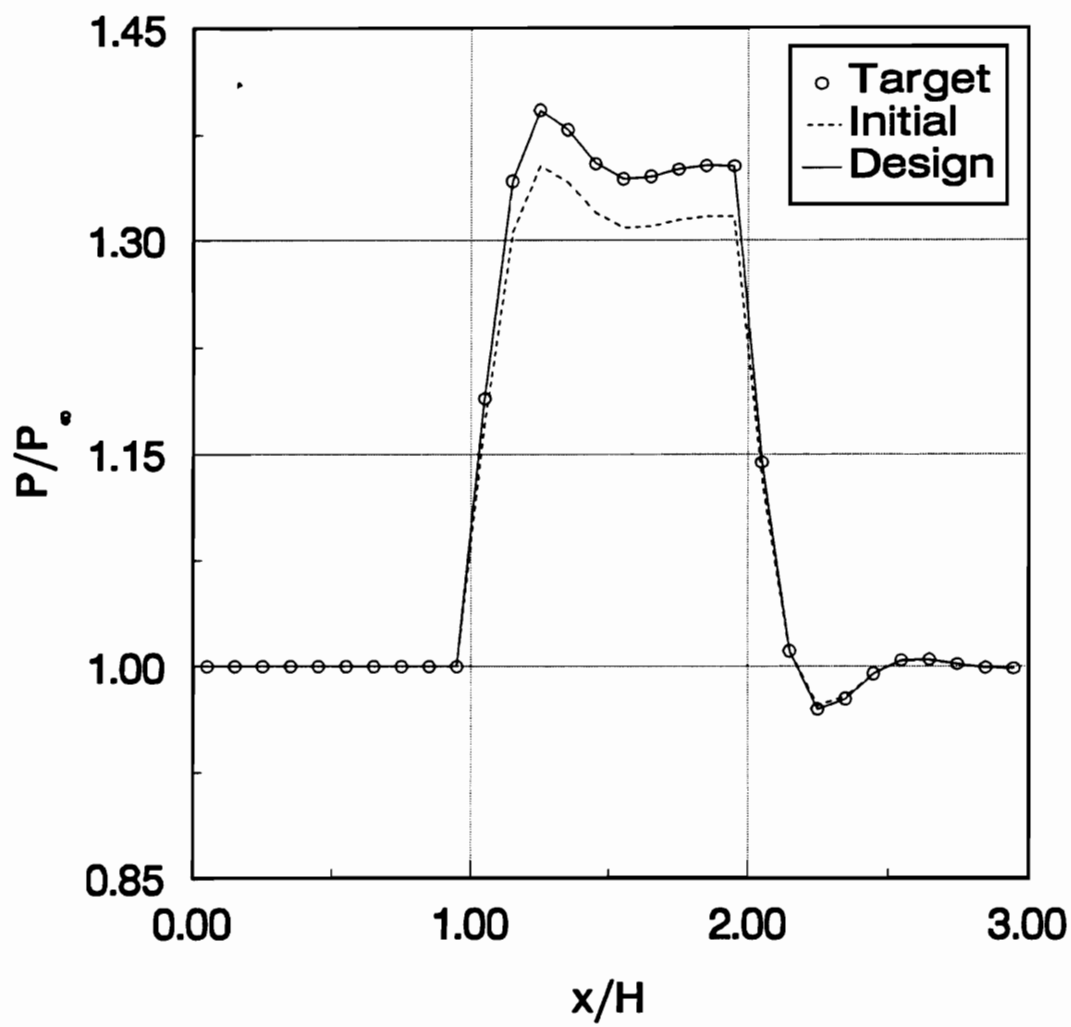


Figure 11 : Target Surface Pressure; $M=2.00$, $\Delta\theta=10\%$

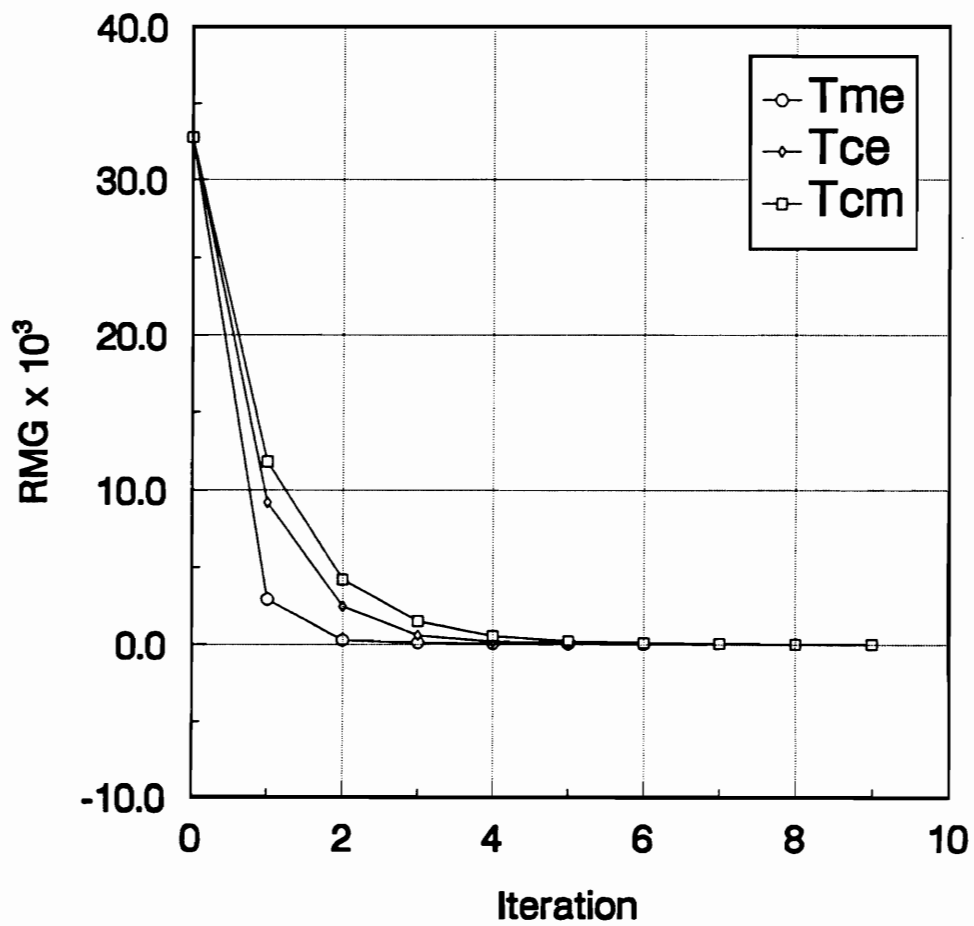


Figure 12 : Geometry Residual Convergence; $M=2.00$, $\Delta\theta=10\%$

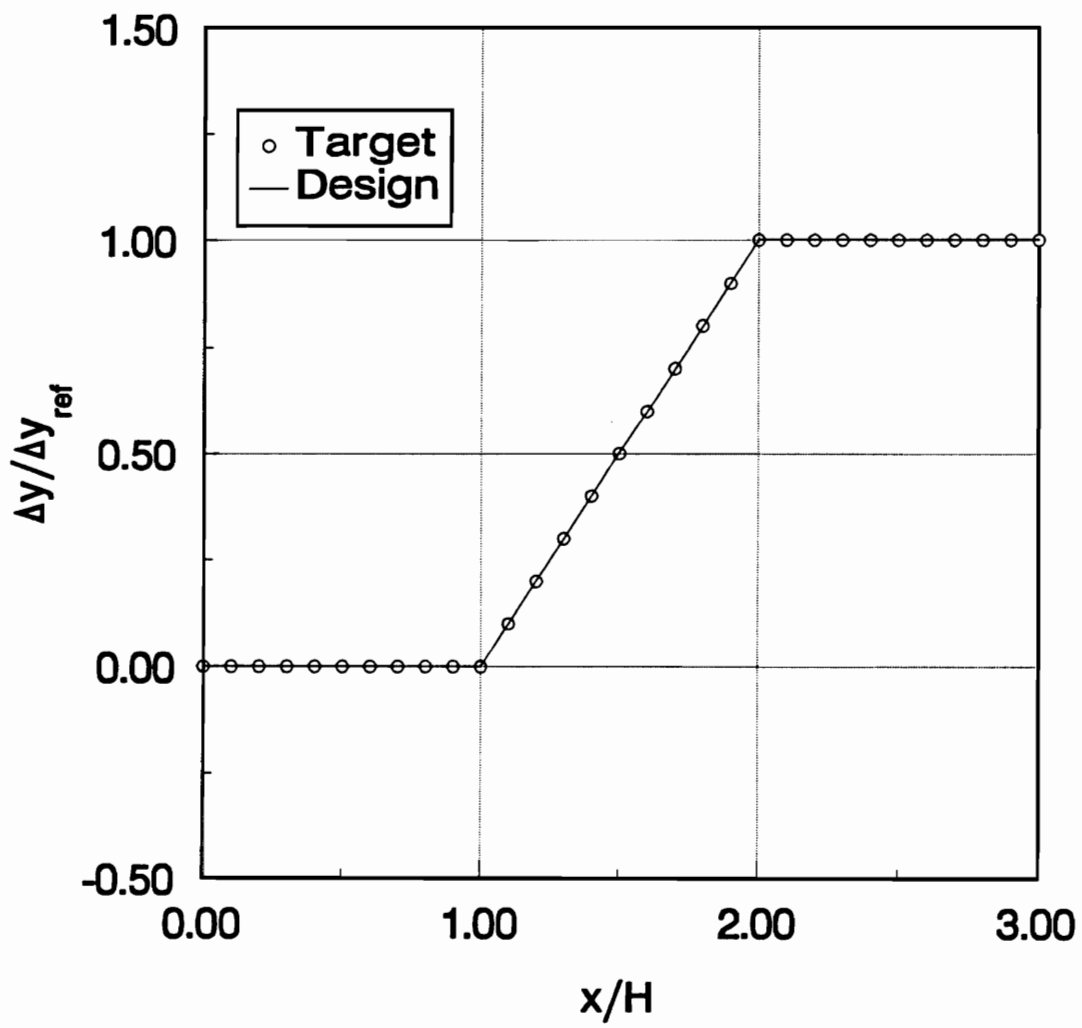


Figure 13 : Target Geometry; $M=2.00$, $\Delta\theta=10\%$

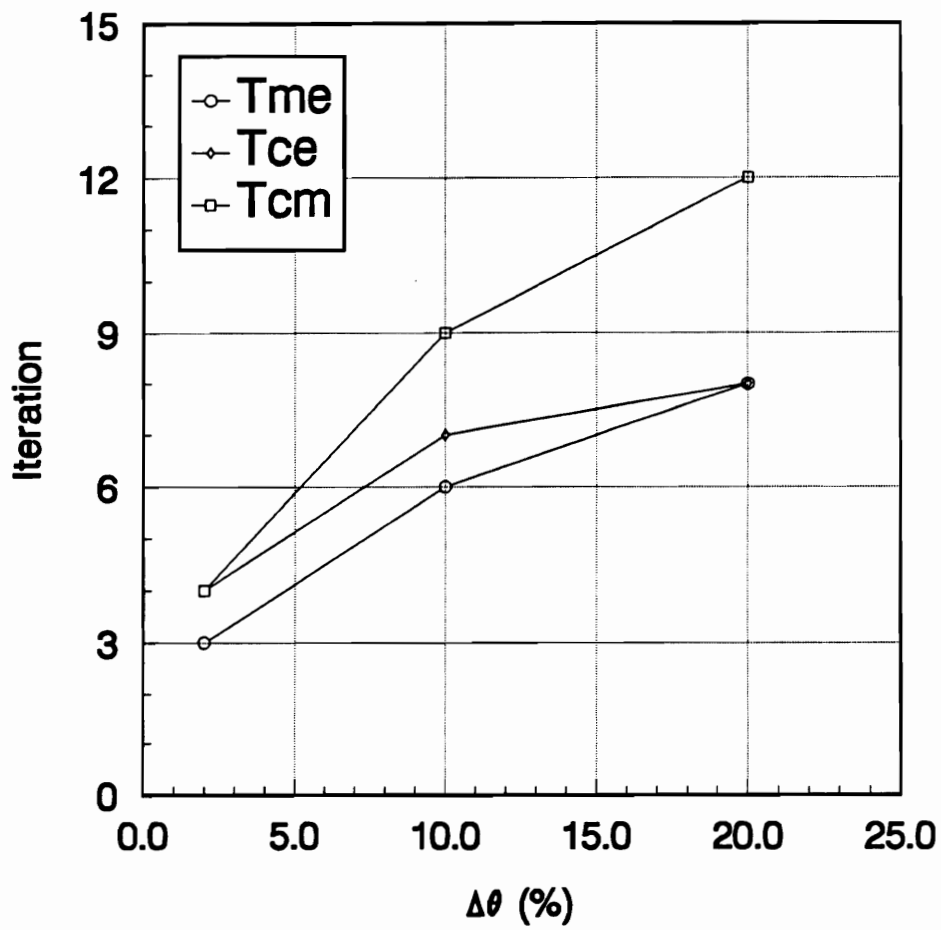


Figure 14 : Analysis/Design Iterations; M=2.00

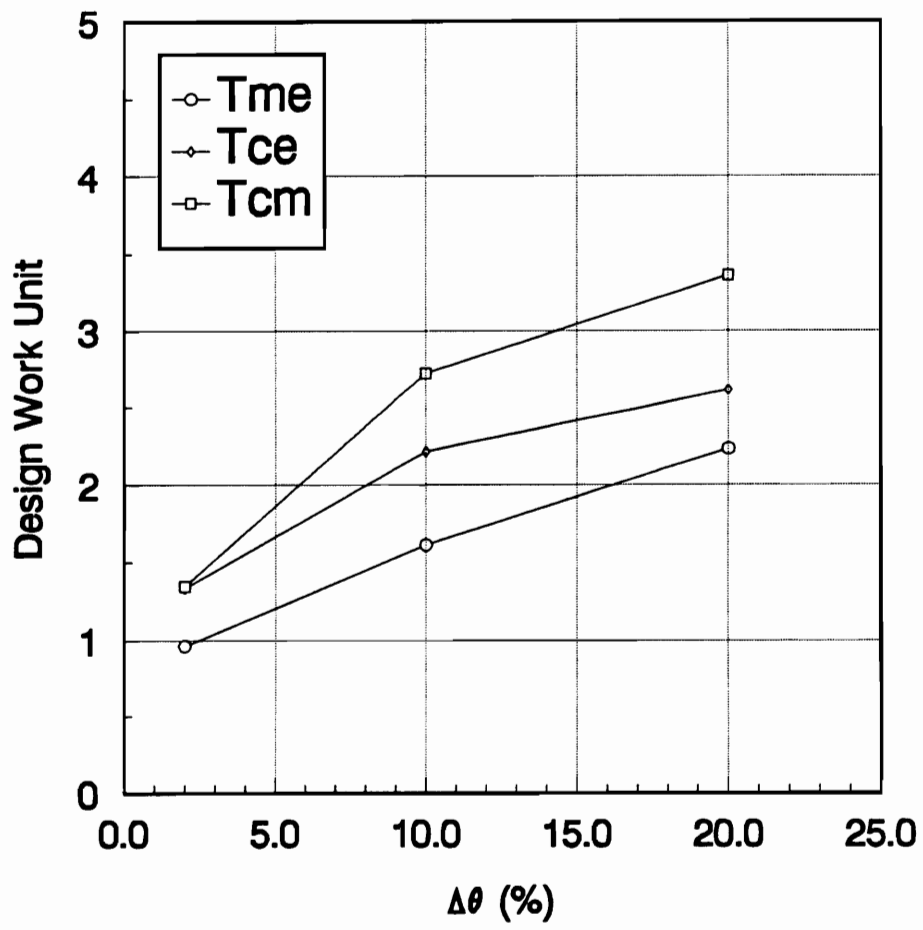


Figure 15 : Design Work Unit; M=2.00

7.2 Transonic Test Case

The following discussion will present the results for the transonic inlet Mach number test case; $M_\infty=0.85$. The surface pressure residuals and the transpiration velocities for this test case will be discussed first. These parameters will then be shown to have a strong influence on the initial design shape predictions. The consistent direct-iterative method will then be demonstrated to achieve the correct design geometry for the controlled inverse design problem in a transonic flow field.

7.2.1 Initial Design Shape Prediction

The dominant flow field characteristic of the transonic test case is the fixed exit, or back-pressure ratio of $P_b/P_\infty=1.0$. As the flow is accelerated through the channel, the surface pressure ratio is reduced from a maximum level which occurs on the inlet plane. The inlet-to-exit area ratio is progressively larger for each target geometry which requires greater acceleration of the flow through the channel. An average 30% reduction in the surface pressure ratio along the ramp is typical for both the initial and target geometries. However, the flow field for each target geometry must also achieve the exit pressure ratio of unity. Therefore, the inlet pressure ratio is larger for each target geometry relative to the initial inlet pressure ratio, and a positive pressure residual exists along most of the channel wall.

The surface pressure residual for the transonic test case is presented in Figure 16 for each target geometry. The pressure residual is positive and relatively constant along the entrance plate of the channel. Along the ramp, the pressure residual is rapidly reduced from its maximum level to near zero at the ramp TE. The maximum pressure residual is 2.5% relative to the initial surface pressure for the $\Delta\theta=20\%$ target geometry. Just upstream of the ramp TE, the pressure residual is negative but rapidly becomes zero along the exit plate. Also, while no shocks appear within this transonic flow field, the surface Mach number is greater than one just past the ramp TE.

The transpiration velocities from each design tangency BC for the transonic test

case are presented in Figure 17 for the $\Delta\theta=10\%$ target geometry. The general profile characteristics for the transpiration velocities are similar to those of the pressure residual profiles. The maximum transpiration velocity occurs near the ramp LE, and then is rapidly reduced to near zero at the ramp TE. Just upstream of the ramp TE, each transpiration velocity profile is negative. The transpiration velocity profiles coalesce near the ramp TE, where the surface Mach number is nearly one. This coalescence is consistent with Figure 2, where all influence coefficients are equivalent at the sonic point. Along the ramp, the transpiration velocities appear in an ascending order as T_{ce} , T_{mc} , and T_{cm} , respectively. This sequence is opposite to the sequence for the supersonic test case. However, the sequence in Figure 17 is consistent with the influence coefficient sequence for Mach numbers less than one. The transpiration velocity profiles for the $\Delta\theta=2\%$ and 20% target geometries are similar to those presented in Figure 17.

The initial design shape predictions for the transonic test case are presented in Figure 18 for each target geometry. The shape predictions from each design tangency BC solution contain a curved ramp profile. The predicted ramp is correctly shifted upward, but its curvature causes the prediction to overshoot the target ramp along most of its length. The exit plate correctly extends past the ramp TE as a horizontal line. The predicted ramp TE heights and the exit plate positions for the T_{ce} , T_{mc} , and T_{cm} design BC's are on average 75%, 100%, and 130% of the target position, respectively. This sequence is again consistent with the sequence of influence coefficients for Mach numbers less than one. Therefore, the average accuracy of each initial shape prediction is 75%, 100%, and 70% for the T_{ce} , T_{mc} , and T_{cm} design BC's respectively.

The initial shape modification profiles for this test case presented in Figure 18, $\Delta y/\Delta y_{ref}$, were strongly influenced by the initial pressure residual profiles presented in Figure 16. Just downstream of the ramp LE, the pressure residuals are positive and near their maximum values. The slope of the shape modification profiles are also positive and relatively large. As the pressure residual is rapidly reduced along the ramp, the slope of the $\Delta y/\Delta y_{ref}$ profile also approaches zero. Just upstream of the ramp TE, the pressure residuals are negative, and the shape modification profiles are sloped downward. This correspondence between the pressure residuals and the slope of the shape modification

profiles was also exhibited within the supersonic test case. For the supersonic test case, the pressure residuals and the slopes of the shape modification profiles are both positive and relatively constant along the ramp.

The transonic test case is essentially a high speed, subsonic test case because the Mach number along the ramp remains below the sonic value. As discussed in Chapter 2, for subsonic flows the proper relationship between the pressure residual and the change in geometry should be expressed as a change in curvature. However, after considering the initial shape predictions from both the supersonic test case and the present one, the consistent shape modification method proposed within this research relates pressure residuals to shape modification slopes. This was beneficial for the supersonic test case inverse design solutions where the pressure residual was rapidly and monotonically converged.

However, for the transonic test case, a stable and convergent inverse design solution was obtained only after a relaxation factor was applied to the geometry update procedure. Because most existing direct-iterative methods employ some form of a relaxation technique to stabilize the design solution, the application of one within this research is not unusual. The relaxation techniques employed in the existing methods are applied directly to the shape modification procedure. Within this research, the relaxation factor, ω , was applied to the global shape modification vector, $\Delta\bar{X}$. The relaxed version of the geometry update procedure was then defined as $\bar{X} = \bar{X} + \omega\Delta\bar{X}$. Therefore, the relaxation method used for the consistent design method is applied after the shape modification solution, which differs from the existing methods.

7.2.2 Inverse Design Solution

The consistent direct-iterative method with the relaxed geometry update procedure was used to obtain the correct design geometry for the transonic test case. The following results will focus on the $\Delta\theta=2\%$ target geometry design solution using the T_{mc} design tangency BC. This single inverse design problem is representative of each transonic design solution for the channel geometry. The pressure residual convergence history for

this test case is presented in Figure 19 using various relaxation factors. The initial pressure residual was reduced by 40% after the first iteration when $\omega=1.00$. The corresponding design shape predictions for this initial iteration were presented in Figure 18. However, the pressure residual increased for the second and subsequent iterations when $\omega=1.00$. A number of relaxation factors were then used to establish a convergent design solution. Through this study it was determined that any relaxation factor which was less than 0.50 was convergent. The optimum relaxation factor for this test case, the one which provided the largest convergence rate, was $\omega=0.25$.

Subsequent analysis/design iterations were then performed on the single transonic test problem using a relaxation factor of $\omega=0.25$. The l_2 norm of the surface pressure residual was reduced to 1.0×10^{-3} in 14 iterations, and the design work unit for this solution was just over five. Also, geometric convergence was established for this design solution in just 5 iterations. However, the initial surface pressure residual for the transonic test case was of the same order-of-magnitude as the convergence criterion. Therefore, a new convergence limit was established for this test case: $RMP < 1.0 \times 10^{-5}$. This convergence limit represents a two order-of-magnitude reduction from the initial level. To obtain the design solution with this convergence criterion required 110 iterations and a design work unit near 60. The correct surface pressure and geometry convergence are verified in Figures 20 and 21 for this more restrictive convergence limit. While the number of iterations required for this second design solution is not uncommon within the literature, the large design work unit is excessive.

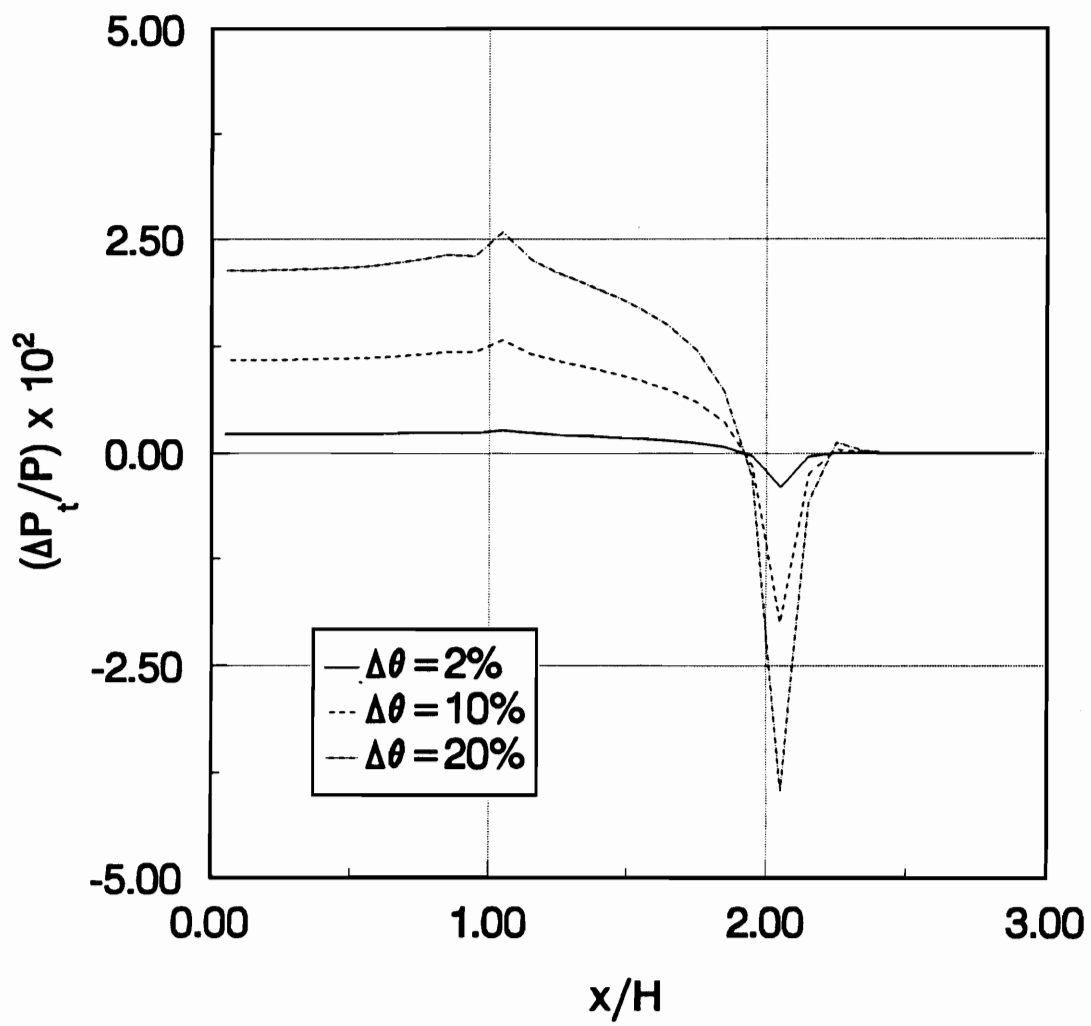


Figure 16 : Pressure Residual; $M=0.85$

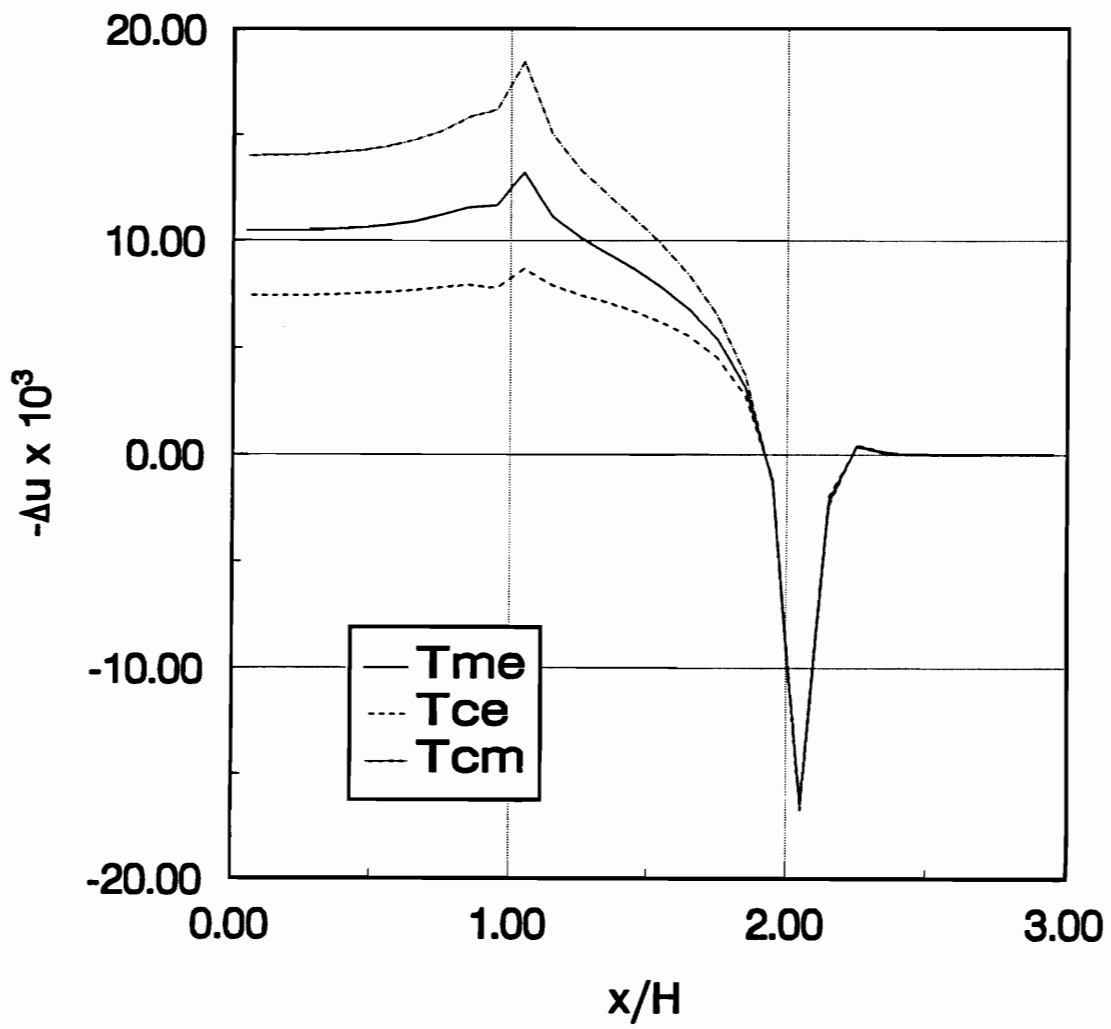


Figure 17 : Transpiration Velocity; $M=0.85$, $\Delta\theta=10\%$

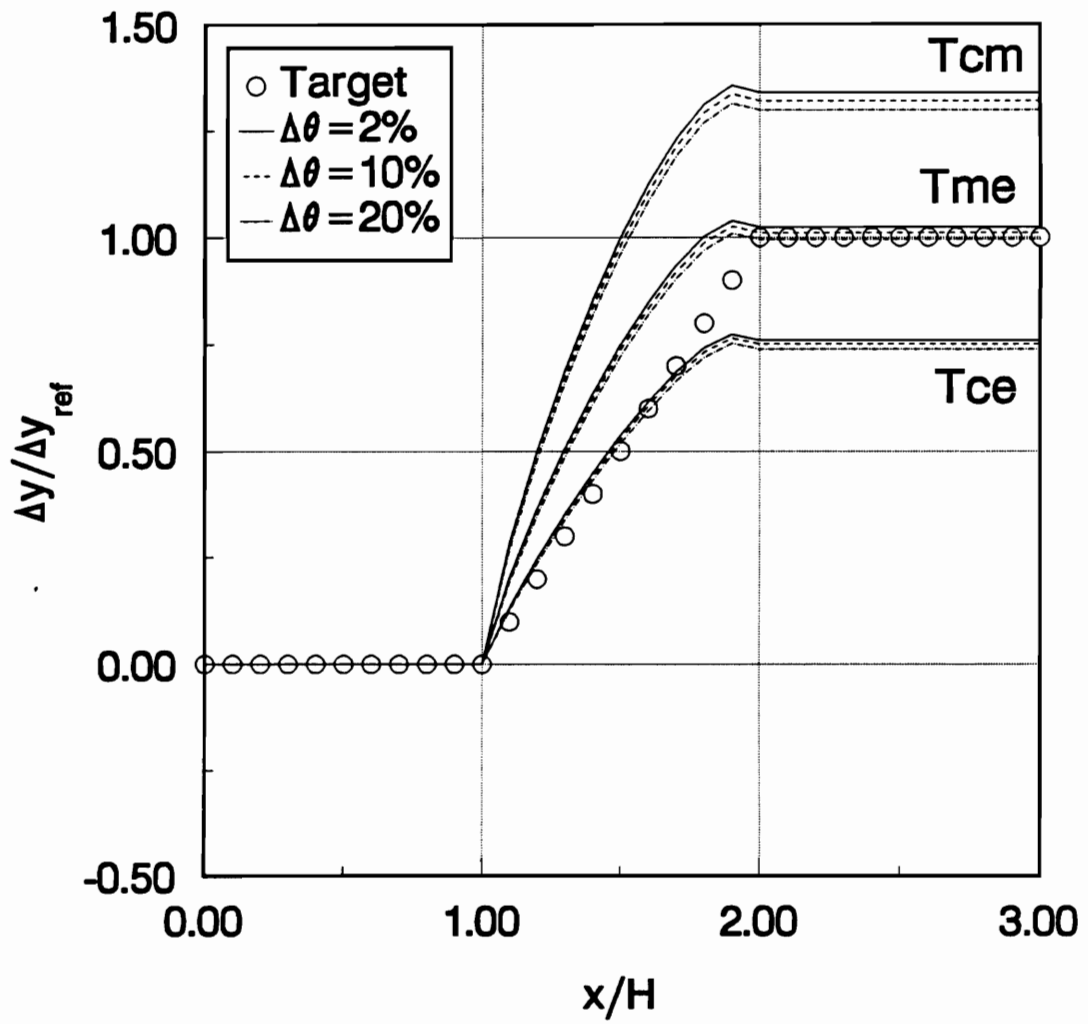


Figure 18 : Initial Design Shape Prediction; $M=0.85$

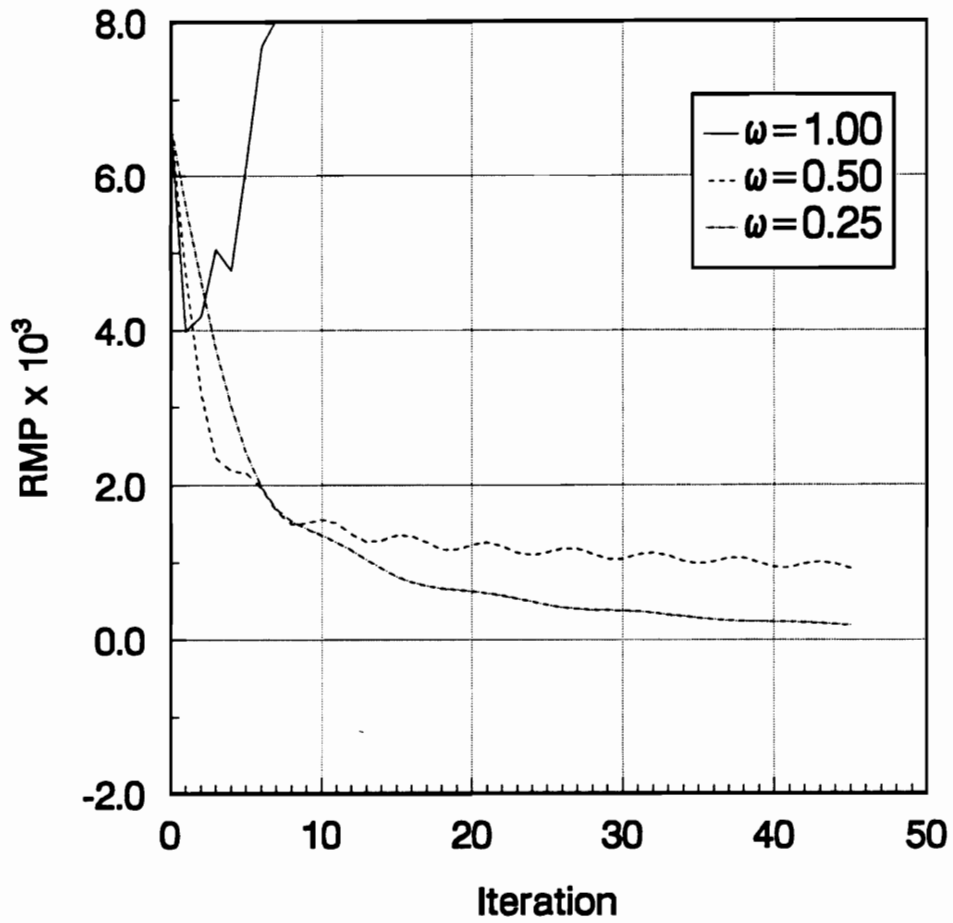


Figure 19 : Pressure Residual Convergence; $M=0.85$, $\Delta\theta=2\%$

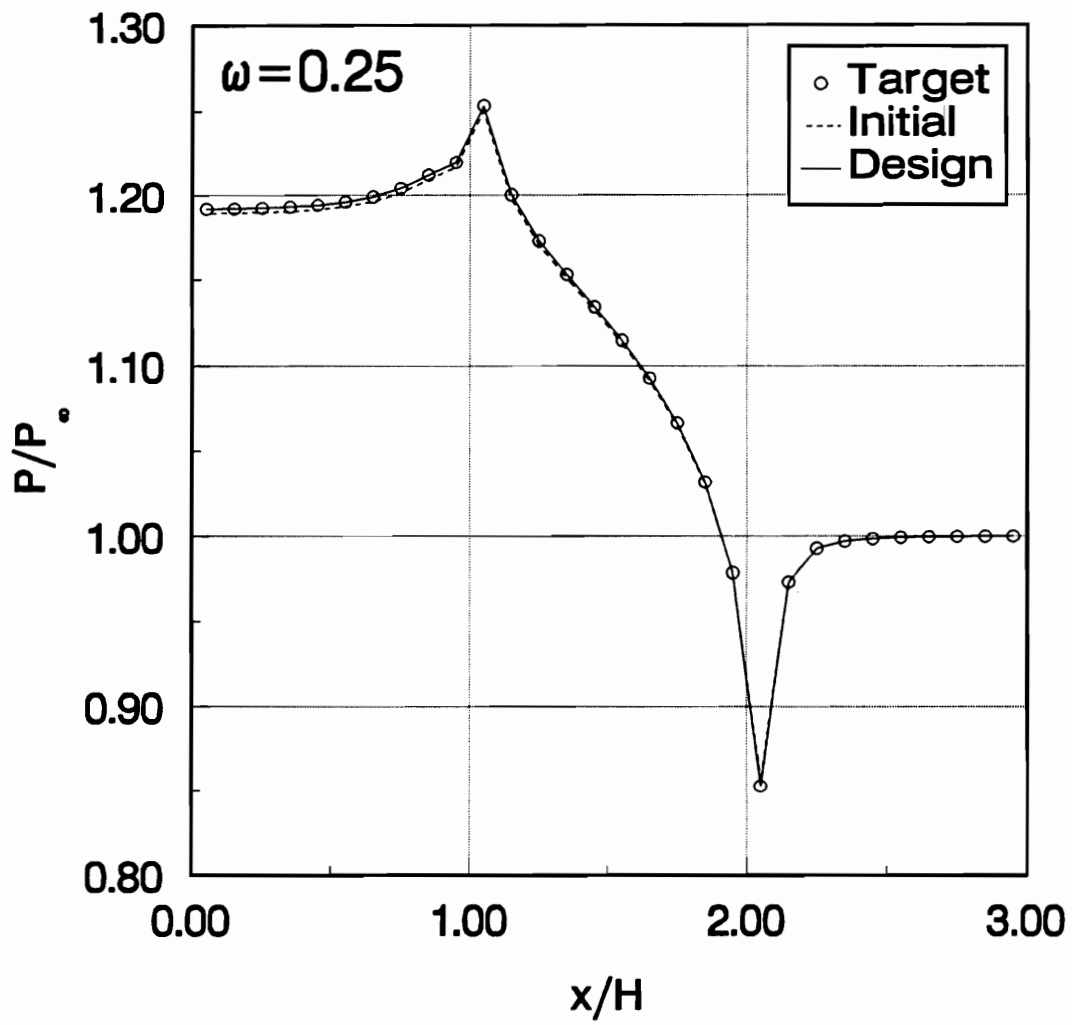


Figure 20 : Target Surface Pressure; $M=0.85$, $\Delta\theta=2\%$

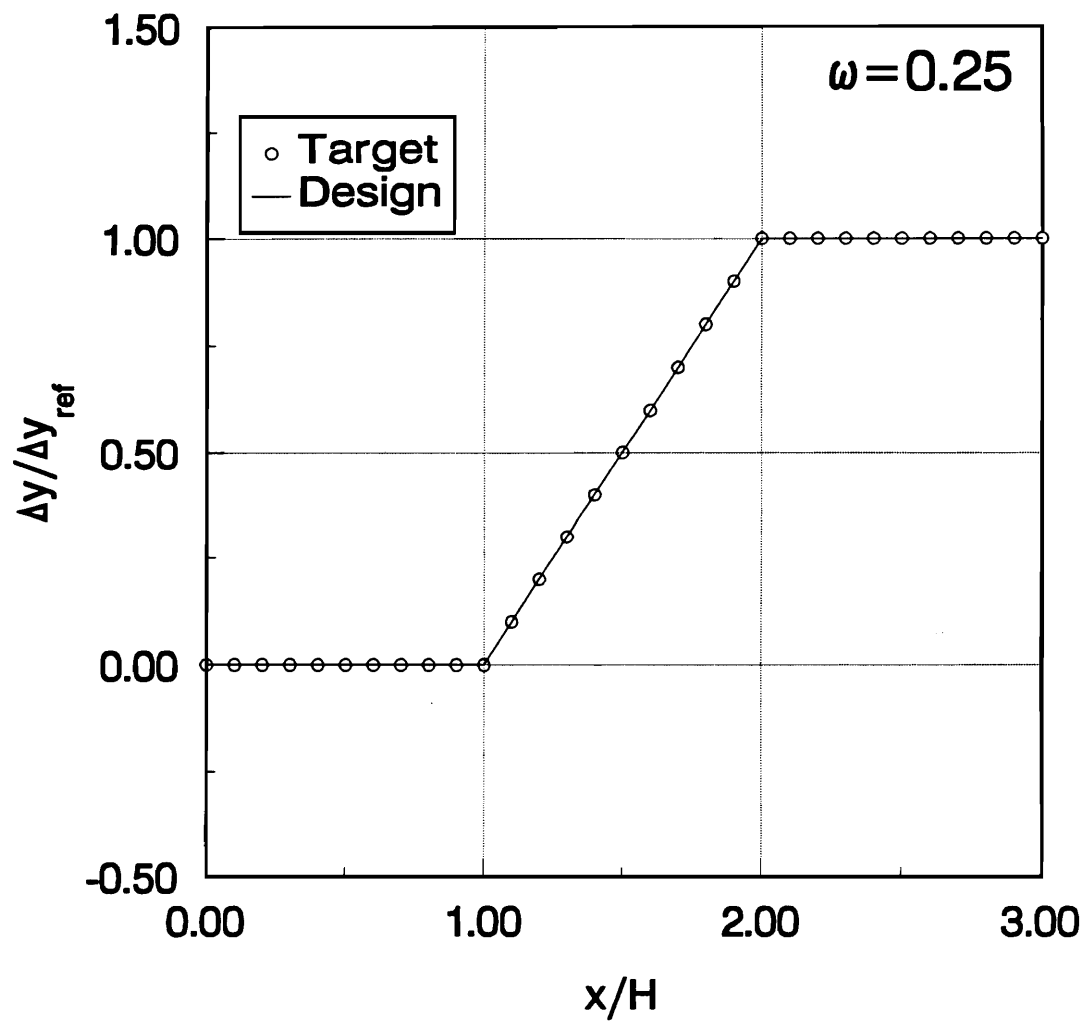


Figure 21 : Target Geometry; $M=0.85$, $\Delta\theta=2\%$

7.3 Subsonic Test Case

The following discussion will present the results for the subsonic inlet Mach number test case; $M_\infty=0.30$. The initial design shape predictions for the subsonic test case will be discussed first. The consistent direct-iterative method will then be demonstrated to achieve the correct design geometry for a single inverse design problem using a relaxation parameter within the geometry update procedure.

7.3.1 Initial Design Shape Prediction

The dominant flow field characteristic of the subsonic test case is again the fixed back-pressure ratio of $P_b/P_\infty=1.0$. The surface pressure ratio is reduced along the entrance plate and the ramp as the flow accelerates through the channel. An average 5% reduction along the ramp for this ratio is typical for all of the geometries. The fixed exit pressure ratio and the area ratio of the channel require that the inlet pressure ratios be larger for each target geometry relative to the initial geometry flow field. The surface pressure residuals are, therefore, positive along most of the channel wall.

The surface pressure residuals for the subsonic test case are presented in Figure 22 for each target geometry. The pressure residuals are positive and relatively constant along the entrance plate of the channel. Along the ramp, the pressure residuals are rapidly reduced from their maximum level to near zero at the ramp TE. In contrast to the transonic test case, the pressure residuals for the subsonic test case remain positive along the ramp. Just downstream of the ramp TE, the pressure residuals are negative, but rapidly become zero along the exit plate. This test case does not contain large values of the surface pressure residual. The maximum pressure residual is less than one-half of one percent relative to the initial surface pressure for all of the target geometries.

The transpiration velocities from each design tangency BC for the subsonic test case are presented in Figure 23 for the $\Delta\theta=10\%$ target geometry. The general profile characteristics for the transpiration velocities are similar to the pressure residual profiles. The maximum transpiration velocity occurs near the ramp LE. The profiles are then

rapidly reduced to near zero at the ramp TE. However, the transpiration velocities remain positive along the ramp and appear in ascending order as T_{ce} , T_{mc} , and T_{cm} , respectively. This sequence is again consistent with the influence coefficient sequence for Mach numbers less than one. The transpiration velocity profiles for the $\Delta\theta=2\%$ and 20% target geometries are similar to those presented in Figure 23.

The initial design shape predictions for the subsonic test case are presented in Figure 24 for each target geometry. In general, the predicted shape change from each design tangency BC is smooth and continuous. The predicted ramp is correctly shifted upward, but the ramp TE height is lower than the target value. Also, the exit plate correctly extends past the ramp TE as a horizontal line. The predicted ramp TE heights and the exit plate positions for the T_{ce} , T_{mc} , and T_{cm} design BC's are on average 10%, 40%, and 75% of the target position, respectively. This sequence, and the low T_{ce} shape prediction, are again consistent with the influence coefficient profiles in Figure 2.

Unlike the supersonic and transonic test cases, the initial design shape predictions for the subsonic test case are insensitive to variation of $\Delta\theta$ up to 20%. The shape modification profiles for each design BC in Figure 24 are nearly coincident for all target geometries. The streamwise variation of the influence coefficients is relatively large for this low speed test case, as implied in Figure 2. These parameters are constant for each target geometry, because they are evaluated with the initial geometry flow field solution. Also, the pressure residuals are very low for all of the target geometries. Therefore, large values of the influence coefficients, combined with low pressure residuals, did not produce noticeable variation in shape modification profiles between target geometries.

However, the initial shape modification profiles, $\Delta y/\Delta y_{ref}$, for the subsonic test case are strongly influenced by the shape of the initial pressure residual profiles. Just downstream of the ramp LE, the pressure residuals are positive and near their maximum values. Within this area, the slope of the shape modification profiles are also positive and relatively large. As the pressure residual is rapidly reduced along the ramp, the slope of the $\Delta y/\Delta y_{ref}$ profile also approaches zero. This correspondence between the pressure residuals and the slope of the shape modification profiles is similar to that exhibited by both the supersonic and transonic test cases.

7.3.2 Inverse Design Solution

The initial shape modification solutions in Figure 24 represent the first design geometry estimate for the subsonic inverse design solution. Because the pressure residual strongly influenced the slope of each shape modification profile, a relaxation factor was again required to obtain a stable design solution. The relaxation factor was applied to the geometry update portion of direct-iterative procedure in a similar fashion as the transonic test case. The consistent direct-iterative method with the relaxed geometry update procedure was then used to obtain the correct design geometry for the subsonic test case. The following results will focus on the $\Delta\theta=2\%$ target geometry design solution using the T_{cm} design tangency BC. This single inverse design problem is representative of each subsonic design solution for the channel geometry.

The pressure residual convergence history for the subsonic test case is presented in Figure 25 using various relaxation factors. The initial pressure residual was reduced after the first iteration when $\omega=1.00$. However, the pressure residual increased for the second and subsequent iterations for this solution. Several relaxation factors were then used to establish a convergent solution and the optimum factor for this test case was $\omega=0.25$. This relaxation factor is identical to that used for the transonic test case. This relaxation factor is not assumed to be universal for all subsonic and transonic test cases, but a consequence of the test conditions and geometry selected for this research.

Subsequent analysis/design iterations were then performed on the subsonic design problem using a relaxation factor of $\omega=0.25$. The initial l_2 norm of the surface pressure residual is less than 1.0×10^{-3} for this low-speed test case. Therefore, RMP was defined as 1.0×10^{-5} , and this convergence limit represents a two order-of-magnitude reduction from the initial level. This convergence criterion was also used for the transonic test case. To obtain the design solution with this convergence criterion required 75 iterations and a design work unit near 35. The correct surface pressure and geometry were obtained for this problem and these are verified in Figures 26 and 27. While the number of iterations required for this inverse design solution is again not uncommon in the literature, the large design work unit is excessive as in the transonic test case.

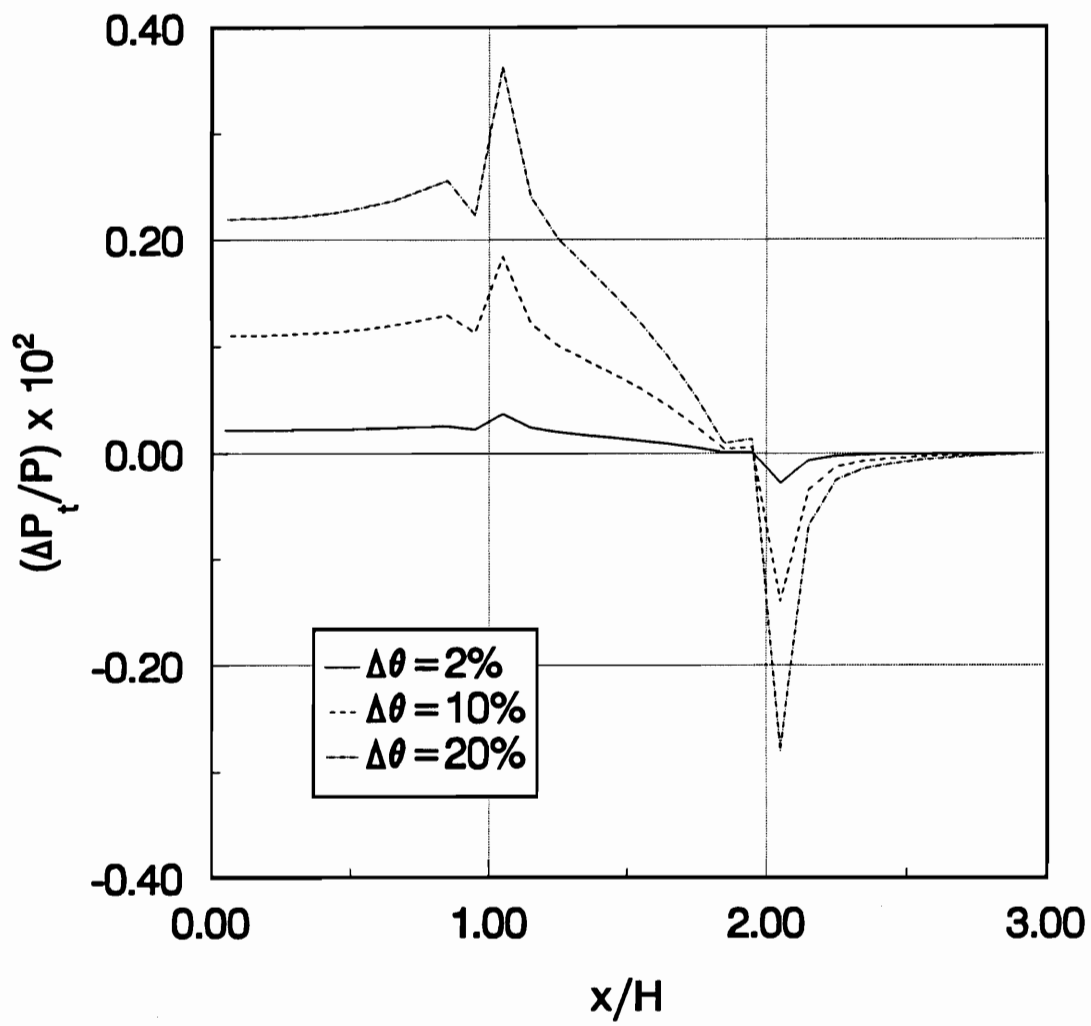


Figure 22 : Pressure Residual; $M=0.30$

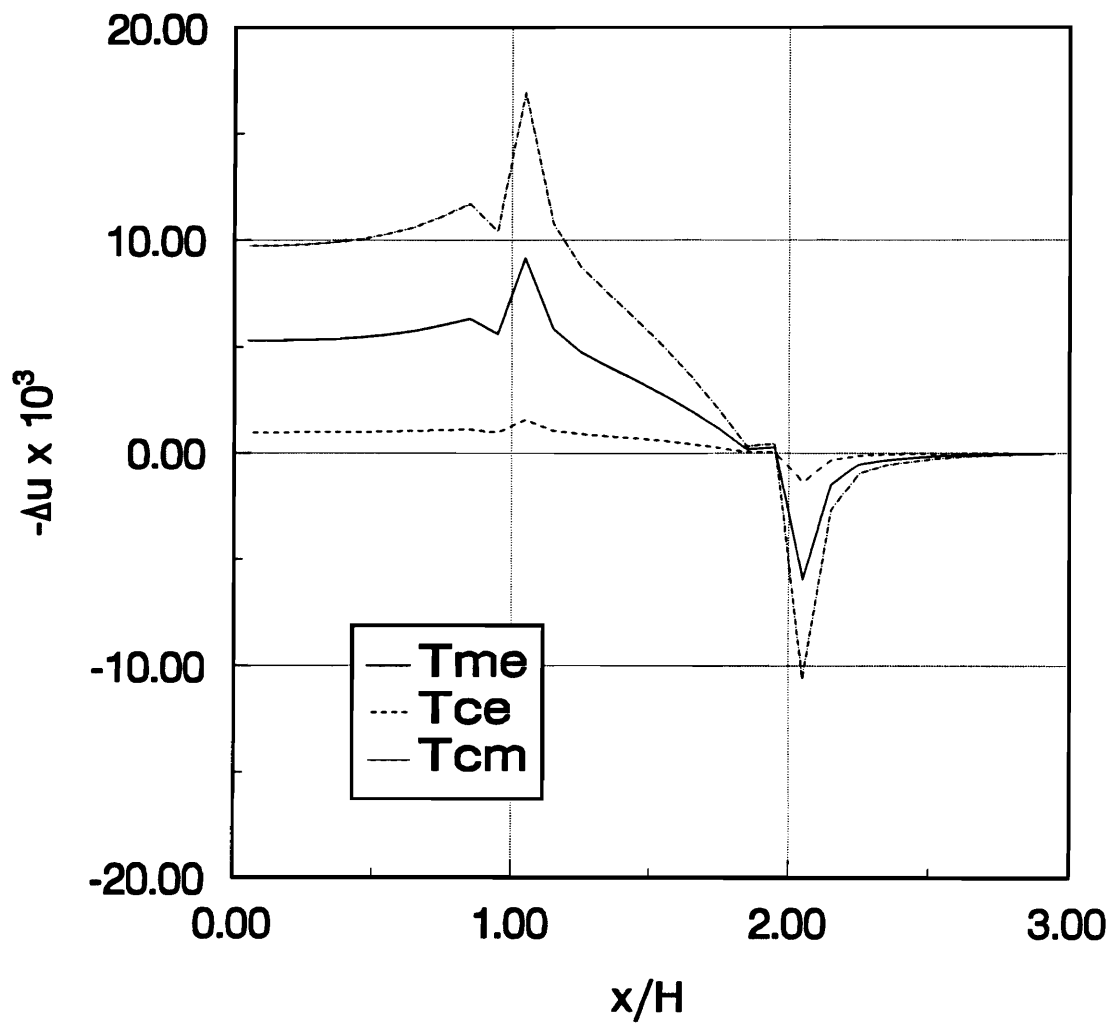


Figure 23 : Transpiration Velocity; $M=0.30$, $\Delta\theta=10\%$

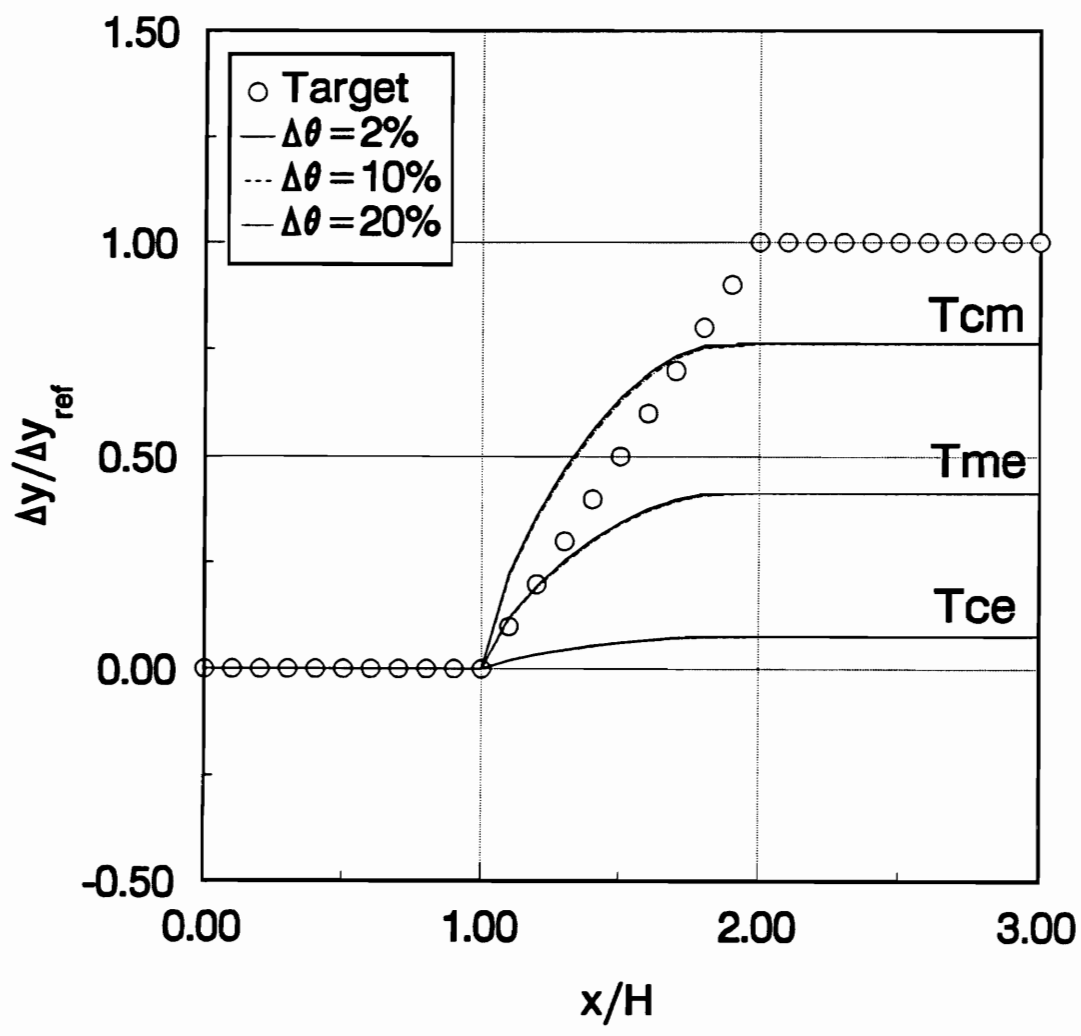


Figure 24 : Initial Design Shape Prediction; $M=0.30$

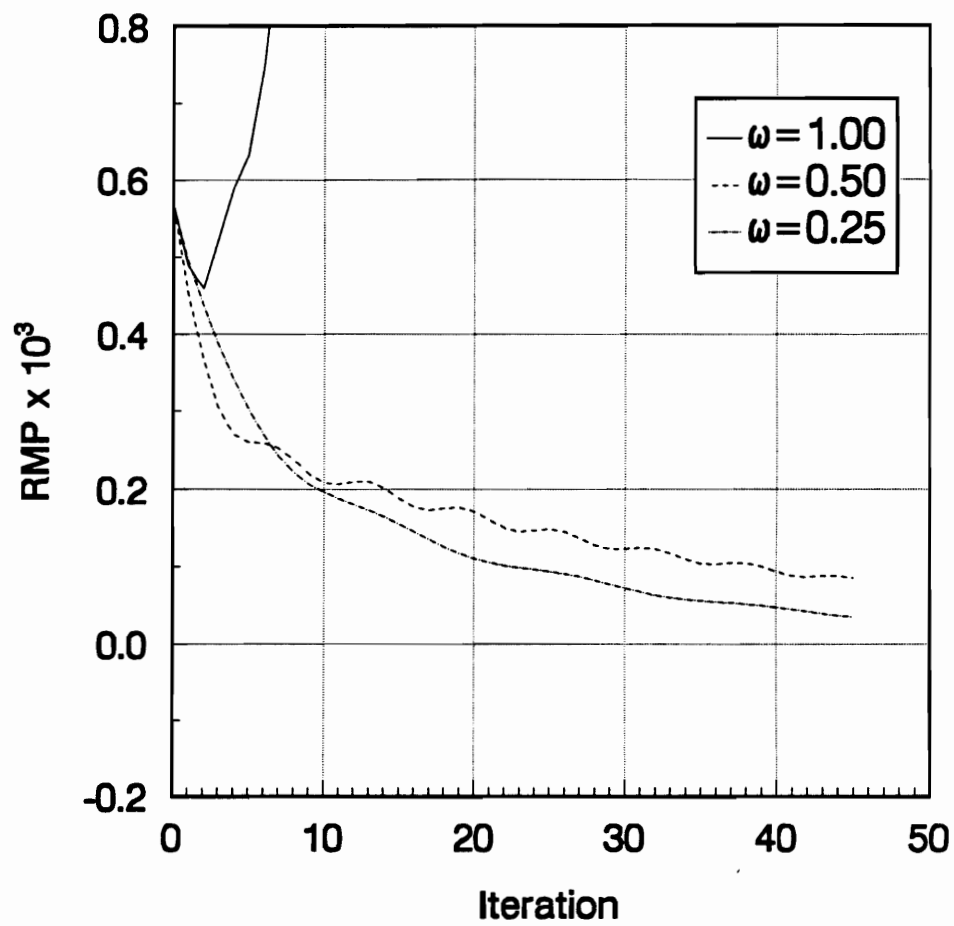


Figure 25 : Pressure Residual Convergence; $M=0.30$, $\Delta\theta=2\%$

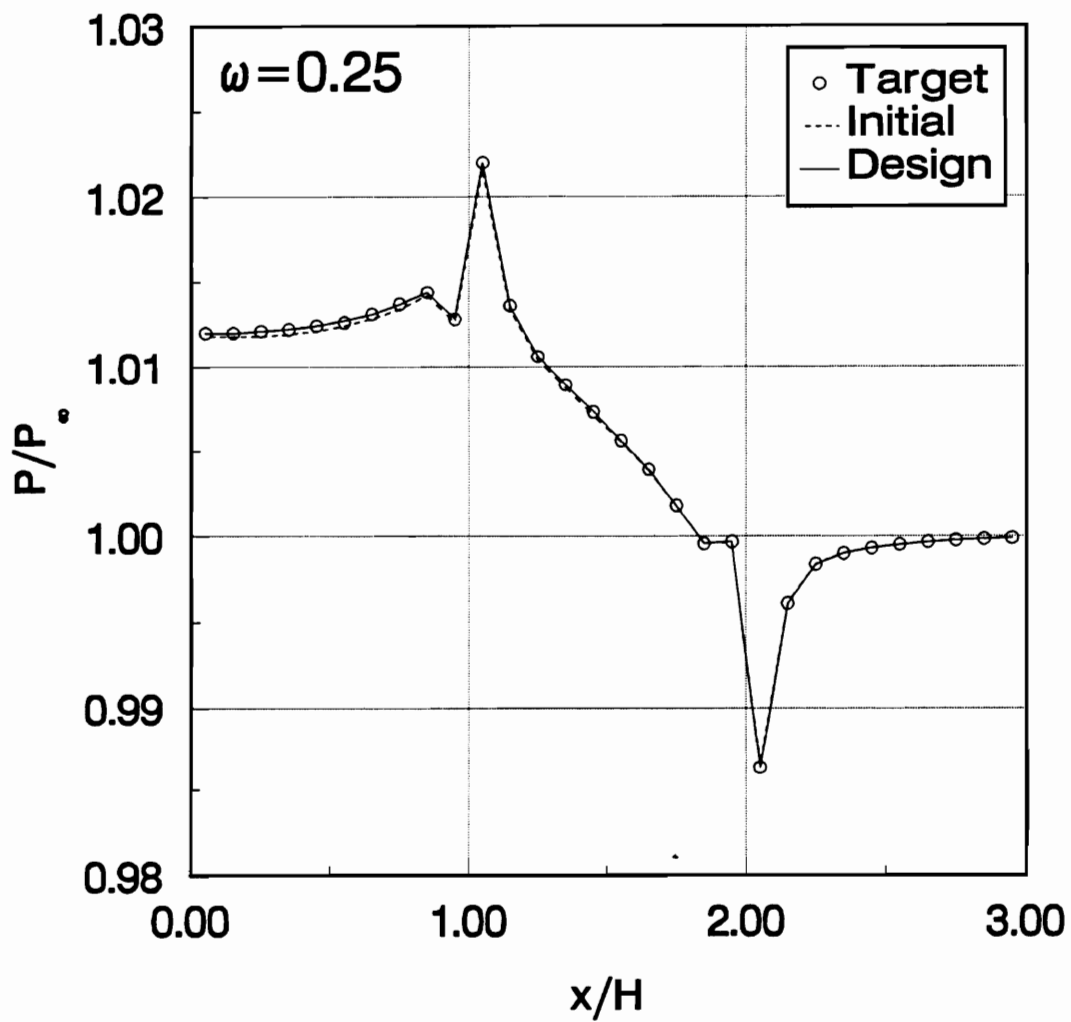


Figure 26 : Target Surface Pressure; $M=0.30$, $\Delta\theta=2\%$

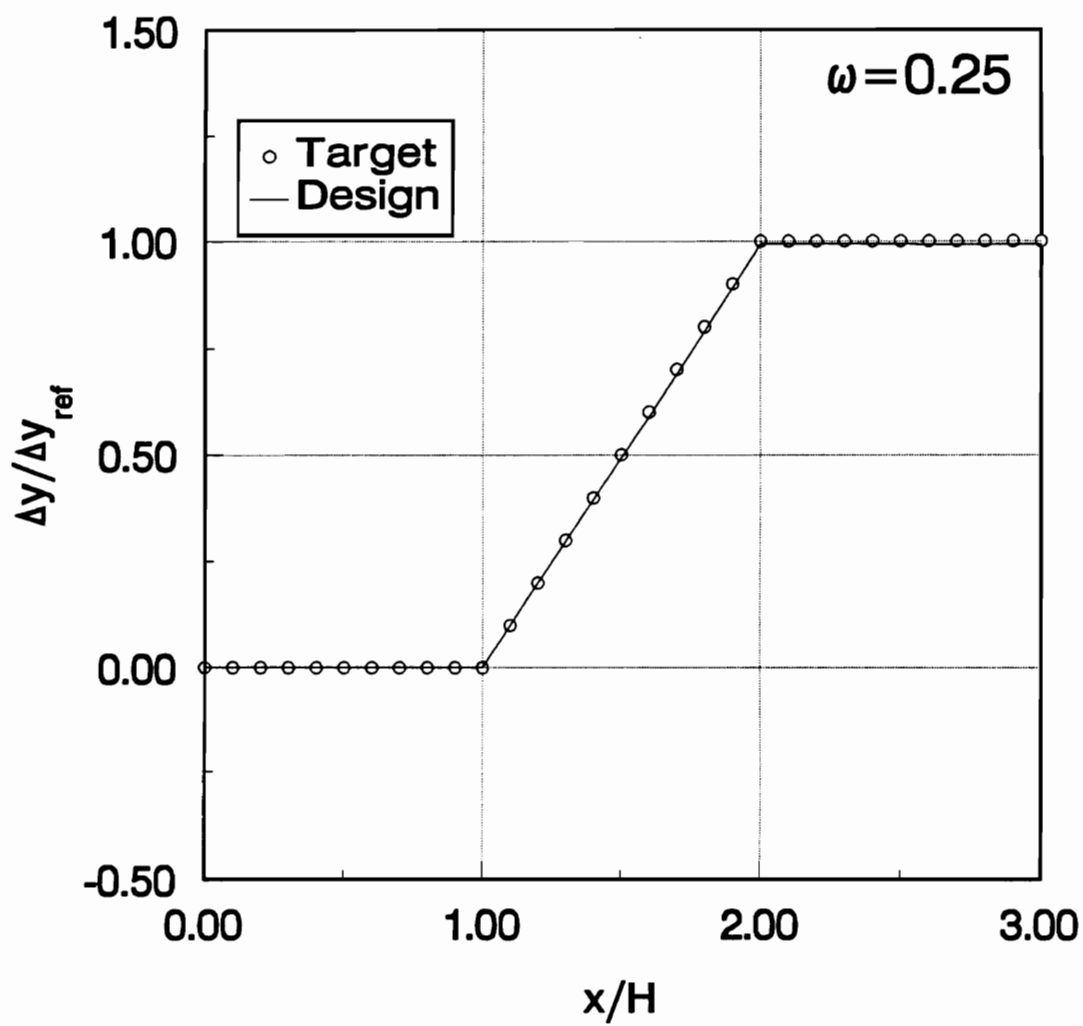


Figure 27 : Target Geometry; $M=0.30$, $\Delta\theta=2\%$

8.0 Summary and Recommendations

This chapter summarizes the development and demonstration of the consistent direct-iterative inverse design method. Recommendations for future testing and research efforts will then conclude this chapter.

8.1 Summary

A new, consistent direct-iterative method was proposed for the solution of the aerodynamic inverse design problem. The method includes a new, advanced shape modification method which is the key component of the direct-iterative design procedure. The design procedure is consistent in that it uses the identical governing equations and numerical techniques for both analysis and shape modification methods. The consistent application of the governing equations ensures that all relevant flow physics are incorporated within each design shape prediction. In contrast, the existing shape modification methods are based on potential flow theory, while the analysis portion of the design procedure typically solves the Euler or N-S equations. The proposed shape modification method is also Mach number independent and, therefore, may be applied within all flow regimes. This is in contrast to some existing methods which require different relationships for subsonic, transonic, and supersonic flows.

Consistent direct-iterative design represents a completely new approach to solving the inverse design problem. Several new numerical techniques were required to implement a consistent shape modification method for the 2-D Euler equations. The modified standard design equation relates a change in the surface pressure to a corresponding change in the geometry. The pressure residual, the difference between the

computed and target pressures, is incorporated into this equation through the boundary equations. However, the existing analysis tangency BC's are not explicit functions of an externally specified target pressure. Therefore, a new, design tangency BC which directly relates the target surface pressure to shape modification was also developed.

The standard design equation was modified to provide a consistent relationship between the surface pressure residual and each design shape prediction. However, this equation represents a global, over-determined system of equations which does not have an exact solution. To overcome this problem, the incremental normal equation (INE) was developed to obtain the least-squared solution of the modified standard design equation. The least-squared solution is the best possible approach for the over-determined system of equations. The INE also provides an iterative solution strategy which reduces the computational effort required to solve the modified standard design equation.

The consistent direct-iterative method was demonstrated to accurately solve inverse design problems on a simple geometry with subsonic, transonic, and supersonic flow conditions. A channel geometry which contained a ramp was selected for these tests. The channel-wall ramp angle, θ , was the geometric design variable and was nominally five degrees. Three target geometries were then defined with perturbed ramp angles; $\Delta\theta=2\%$, 10% , and 20% . The channel was selected to allow the identical geometry to be used for each of the subsonic, transonic, and supersonic test cases. The results of the method were then compared within each flow regime. The convergence of each inverse design solution was monitored by the reduction of the surface pressure residual. The success of the new method was established by comparing the predicted surface pressure and the channel geometry to the target pressures and geometry, respectively.

The proposed method accurately and efficiently predicted each target geometry for the supersonic test. The initial design shape prediction for this test case correctly related the surface pressure residual to the change in the slope of the channel ramp. The channel geometry was also correctly modified outside of the design area. The surface pressure residual was monotonically converged for each inverse design solution. The total number of analysis/design iterations required for pressure residual convergence increased for successively larger ramp angles. However, the total number of iterations required to

achieve each design solution was very low. The computational efficiency of the new method was measured by the ratio of design effort to the initial analysis effort. The computational efficiency of the new method was comparable to that of the existing direct-iterative methods for this high speed test case.

The proposed method also successfully solved the inverse design problem for the transonic and subsonic test cases. These two test cases had similar design solution characteristics. Significant geometry changes were predicted for each initial design shape prediction. These shape modifications shifted the channel ramp in the correct direction by the correct order-of-magnitude. The channel geometry was also correctly modified outside of the design area. A relaxation factor was then applied to the geometry update portion of the proposed design procedure. A relaxation method is a common feature of many existing direct-iterative methods. As used within the present method, this technique provided a stable design solution, and the surface pressure residual was monotonically converged. The total number of analysis/design iterations required to achieve each design solution was moderate. However, the consistent shape modification method requires additional computational effort relative to the existing methods. Therefore, even with a moderate number of iterations, an excessive amount of computational effort was required for each of the transonic and subsonic inverse design solutions.

The salient conclusion from these three test case results is that, in its present format, the proposed method is most appropriate for high speed design problems. The new shape modification method relates the surface pressure residual to the change in slope of the flow geometry. This relationship was demonstrated within each test case. The relationship also produced the superior efficiency characteristics of the supersonic test case relative to the lower speed problems. It is unclear which component of the proposed design method determines this behavior. The 1-D Euler equations were used to obtain the transpiration velocities, while the 2-D Euler equations were directly used for shape modification. These governing equations are applicable for both high and low speed flows. However, the combination of numerical techniques used within the proposed design method was shown to negate this general flow regime characteristic.

8.2 Recommendations

The consistent use of the governing equations within the direct-iterative method represents a new research area for the advancement of inverse design methods. This research represents only the first, important step required for an accurate and efficient consistent direct-iterative method. The fundamental equations for the new method were developed within this research, and the procedure was demonstrated as an alternative solution method for the inverse design problem. However, to fully realize the potential of the new design method, further testing and development efforts are recommended.

The consistent direct-iterative method accurately and efficiently solved the inverse design problem for the supersonic test case. Therefore, further testing of the new method in its present format is warranted for other high speed flow test cases. These tests should include various internal and external flow geometries which are commonly used for high speed flows. A direct comparison of the consistent method to the existing methods would also establish any efficiency improvements possible with the new method.

Additional development efforts for the proposed design method should include the investigation of alternative design tangency BC's. The design tangency BC's developed within this research represent only one possible solution. Alternative design BC's may also be developed and evaluated. To maintain a consistent design method, these new boundary equations must be identical to the analysis tangency BC at design convergence. Alternative design BC's may provide a more accurate relationship between the surface pressure residual and the geometry curvature change required for low speed flows.

Development efforts for the new design method should also include improving the computational efficiency of each INE solution. An improvement in this area would benefit both high and low speed applications of the proposed method. This research used a banded-matrix, direct inversion technique to solve the INE. The entire normal matrix, between the outermost diagonal bands, was used within the present solution method. A more efficient method would utilize a sparse, symmetric matrix inversion technique. This alternative solution technique would improve the computational efficiency of each shape modification solution and, therefore, the entire inverse design method.

Furthermore, an analytical investigation of the INE might provide several improvements in the consistent shape modification method. An analytical study of the INE might provide a better understanding of the approximations used within the modified standard design equation, and the flow physics included within the least-squared solution process. This would directly benefit the low speed application of the new design method. An analytical study of the INE, in conjunction with an alternative matrix inversion technique, might also provide an optimal convergence criterion for each shape modification solution. A better understanding of the INE singularity issue might also be obtained from an analytical investigation.

While many issues remain unsolved within this initial investigation, the potential of the proposed method has been demonstrated. Extensions of the method to three spatial dimensions and the inclusion of additional flow physics are also possible. The modified standard design equation and the INE are general in nature. There is no inherent limitation in these equations which would prohibit their application for 3-D flows or for viscous flows. Only the design tangency BC's presented in this research are limited to 2-D inviscid flows. However, even these may be directly extended to 3-D flows by including a third component within the total velocity. For an extension of the proposed method to viscous flows, new design BC's are required. The development of design viscous, or no-slip, BC's would follow the same procedure as that applied to the design tangency BC's. An aerodynamic process and limiting condition for a design version of the no-slip BC would be identified. The design no-slip BC would then be required to reduce to the original version at design convergence.

Appendix A. INE Structures

This appendix presents a structural and elemental description of the relevant matrices and system of equations within the incremental normal equation (INE). This includes the metric Jacobian and normal matrices, and the standard design equation. A brief review of the domain discretization employed within this research is also presented, as this technique directly determines the INE structures.

A.1 Domain Discretization

Within this research, the governing equations were transformed from a cartesian, (x,y) , to a generalized, (ξ,η) , coordinate system. The domain was then discretized into a structured computational grid system. The indices (j,k) within the grid system correspond to (ξ,η) coordinate lines, respectively. The constants JDIM and KDIM were defined as the maximum (j,k) dimensions of the spatial discretization. An illustration of the typical (j,k) th computational cell is presented in Figure A.1. The grid points surrounding each interior cell and boundary cell-face were labeled one through four, $\bar{X}_1 \rightarrow \bar{X}_4$. However, this labeling system is only for local designation purposes.

The discretized governing and boundary equations were then established as described in Chapters 3 and 4. The total number of discrete equations solved within the flow domain was $m = (JDIM + 1)(KDIM + 1)(4)$. This number includes four conservation laws and four boundary equations defined at each interior cell-center and boundary cell-face, respectively. The number m also includes sixteen equations for the domain corner positions. In practice, boundary equations were not defined at these locations. The total number of physical (x,y) coordinates within the flow domain was $n = (JDIM)(KDIM)(2)$.

A.2 Standard Design Equation

The consistent design procedure proposed within this research was developed from the standard design equation (Equation 3.7). The standard design equation is restated in Equation A.1 where again $A = \partial \hat{R} / \partial \bar{X}$, $z = \Delta \bar{X}$, and $b = F_q = (\partial \hat{R} / \partial Q) \Delta Q$.

$$A_{m \times n} z_{n \times 1} + b_{m \times 1} = 0 \quad (\text{A.1})$$

A single equation within the standard design equation is presented in Equation A.2. This equation is a function of four local metric Jacobian matrices, $W1 \rightarrow W4$, each of size 4×2 , and a local solution forcing function, F_q , which is a 4×1 vector. These matrices and the F_q vector are defined at computational cell-centers or boundary cell-faces within the flow domain. Equation A.2 also contains four local physical coordinate vectors, $\bar{X}_1 \rightarrow \bar{X}_4$, each of size 2×1 . The size of each term within Equation A.2 is a direct consequence of the four governing equations and two physical coordinates unknowns within the Euler equations. Details of the local metric Jacobian matrices for Van Leer's flux-vector-splitting method are given in Reference 29. Details of $F_q = (\partial \hat{R} / \partial Q) \Delta Q$ as used within this research were given in Chapter 4.

$$\begin{aligned} W1_{jk} \Delta \bar{X}_{1jk} + W2_{jk} \Delta \bar{X}_{2jk} + \\ W3_{jk} \Delta \bar{X}_{3jk} + W4_{jk} \Delta \bar{X}_{4jk} = -F_{qjk} \end{aligned} \quad (\text{A.2})$$

The standard design equation and the global metric Jacobian matrix are graphically presented in Figure A.2. The extra domain corner boundary equations are included within this figure to maintain a systematic metric Jacobian matrix structure. The metric Jacobian matrix is also sparse, containing four diagonals. The number of diagonals is associated with the four grid points which surround each computational cell. Within Figure A.2, the system of equations is shown so that KDIM controls the structure of the metric Jacobian matrix. For this example, the metric Jacobian matrix has JDIM column sections, each of length $(\text{KDIM})(2)$. There are also $\text{JDIM} + 1$ row sections in this matrix, each of length $(\text{KDIM} + 1)(4)$. Also, for simplicity $\text{JDIM} = 3$ within Figure A.2.

A.3 Incremental Normal Equation

The incremental normal equation (Equation 3.12) was developed within this research as an efficient, least-squared method of solution for the standard design equation. The INE is restated in Equation A.3 where $A^T A$ is the normal matrix.

$$\begin{bmatrix} A^T_{n \times m} & A_{m \times n} \end{bmatrix}_{n \times n} \Delta z_{n \times 1} = - A^T_{n \times m} (A_{m \times n} z_{n \times 1} + b_{m \times 1})_{m \times 1} \quad (\text{A.3})$$

A single equation within the INE is presented in Equation A.4. This equation is a function of nine coefficient matrices, A through I, each of which is a block 2X2 matrix. Each of these matrices is multiplied by a $\Delta z = \Delta(\Delta \bar{X})$ vector, each of size 2X1. Equation A.4 also contains a local normal forcing function, F_N , which is a 2X1 vector. Each matrix and vector within Equation A.4 is defined at grid points within the flow domain. The format of this equation produces a sparse, diagonal normal matrix structure.

$$\begin{aligned} & A_{jk} \Delta z_{j,k-1} + B_{jk} \Delta z_{jk} + C_{jk} \Delta z_{j,k+1} + \\ & D_{jk} \Delta z_{j-1,k-1} + E_{jk} \Delta z_{j-1,k} + F_{jk} \Delta z_{j-1,k+1} + \\ & G_{jk} \Delta z_{j+1,k-1} + H_{jk} \Delta z_{j+1,k} + I_{jk} \Delta z_{j+1,k+1} = - F_{N,jk} \end{aligned} \quad (\text{A.4})$$

The INE and the normal matrix are graphically presented in Figure A.3. Again, the example in this figure is for JDIM=3, and KDIM controls the normal matrix structure. Within this example, the normal matrix has JDIM row and column sections, each of length (KDIM)(2). The half bandwidth size of this normal matrix is also (KDIM)(2). The sparse, diagonal structure of the normal matrix is clearly evident within Figure A.3, and the molecule for this matrix is presented in Figure A.4. This molecule represents the correspondence of the normal matrix diagonals to the grid points surrounding a cluster of computational cells.

A benchmark of comparison for the normal matrix is the flux Jacobian matrix, $\partial \hat{R} / \partial Q$. The number of diagonals in the flux Jacobian matrix is a function of the solution variable extrapolation order used in each coordinate direction. In contrast, the structure of the normal matrix is fixed, being independent of the extrapolation order used for either

coordinate direction. Also, the bandwidth of the normal matrix is approximately one half of the bandwidth of the flux Jacobian matrix. Thus, the normal matrix is more simply defined and requires less computational storage relative to the flux Jacobian matrix.

A symmetric normal matrix would require even less computational storage requirements and was recommended in Chapter 3 for an efficient solution of the INE. The discrete representation of the coefficient matrices within a symmetric normal matrix are presented in Equation A.5. Each of these matrices is a linear combination of local metric Jacobian matrix multiplications. However, the matrices within Equation A.5 are defined at different locations. The coefficient matrices are defined at cell-corners, while the metric Jacobian matrices are defined at cell-centers. Therefore, a dual index set is used within Equation A.5; one for cell-corners and another for cell-centers. A graphical representation of Equations A.5 and of the dual set of indices is presented in Figure A.5.

$$\begin{aligned}
A_{jk} &= W2_{jk}^T W1_{jk} + W3_{j+1,k}^T W4_{j+1,k} \\
B_{jk} &= W1_{jk+1}^T W1_{jk+1} + W2_{jk}^T W2_{jk} \\
&\quad + W3_{j+1,k}^T W3_{j+1,k} + W4_{j+1,k+1}^T W4_{j+1,k+1} \\
C_{jk} &= W1_{jk+1}^T W2_{jk+1} + W4_{j+1,k+1}^T W3_{j+1,k+1} \\
D_{jk} &= W2_{jk}^T W4_{jk} \\
E_{jk} &= W1_{jk+1}^T W4_{jk+1} + W2_{jk}^T W3_{jk} \\
F_{jk} &= W1_{jk+1}^T W3_{jk+1} \\
G_{jk} &= W3_{j+1,k}^T W1_{j+1,k} \\
H_{jk} &= W4_{j+1,k+1}^T W1_{j+1,k+1} + W3_{j+1,k}^T W2_{j+1,k} \\
I_{jk} &= W4_{j+1,k+1}^T W2_{j+1,k+1}
\end{aligned} \tag{A.5}$$

The normal matrix is symmetric only if both global metric Jacobian matrices used for its evaluation are identical. Symmetry of the normal matrix was assumed in the derivation of Equation A.5. Symmetry of the main diagonal, B, and the lower diagonals symmetrical correspondence to the upper diagonals are emphasized in Equation A.6.

$$\begin{aligned}
B_{jk} &= B_{jk}^T \\
A_{jk} &= C_{jk-1}^T \\
F_{jk} &= G_{j-1,k+1}^T \\
E_{jk} &= H_{j-1,k}^T \\
D_{jk} &= I_{j-1,k-1}^T
\end{aligned} \tag{A.6}$$

The INE normal forcing function, F_N , is a product of the global metric Jacobian matrix and the standard design equation. The vector F_N is presented in Equation A.7.

$$F_{N \ n \times 1} = A_{n \times m}^T (A_{m \times n} z_{n \times 1} + b_{m \times 1})_{m \times 1} \tag{A.7}$$

A single equation within F_N is presented in Equation A.8. This equation is a function of four local metric Jacobian matrices and four local solution forcing function vectors. Again, each of these matrices and vectors is defined at cell-centers. In contrast, the local normal forcing function is defined at cell-corners.

$$\begin{aligned}
F_{N \ jk} &= W1_{jk+1}^T F_{q \ jk+1} + W2_{jk}^T F_{q \ jk} \\
&+ W3_{j+1,k}^T F_{q \ j+1,k} + W4_{j+1,k+1}^T F_{q \ j+1,k+1}
\end{aligned} \tag{A.8}$$

A graphical representation of Equation A.8 is presented in Figure A.6. The figure includes the dual set of cell-corner and cell-center indices used in the evaluation of F_N .

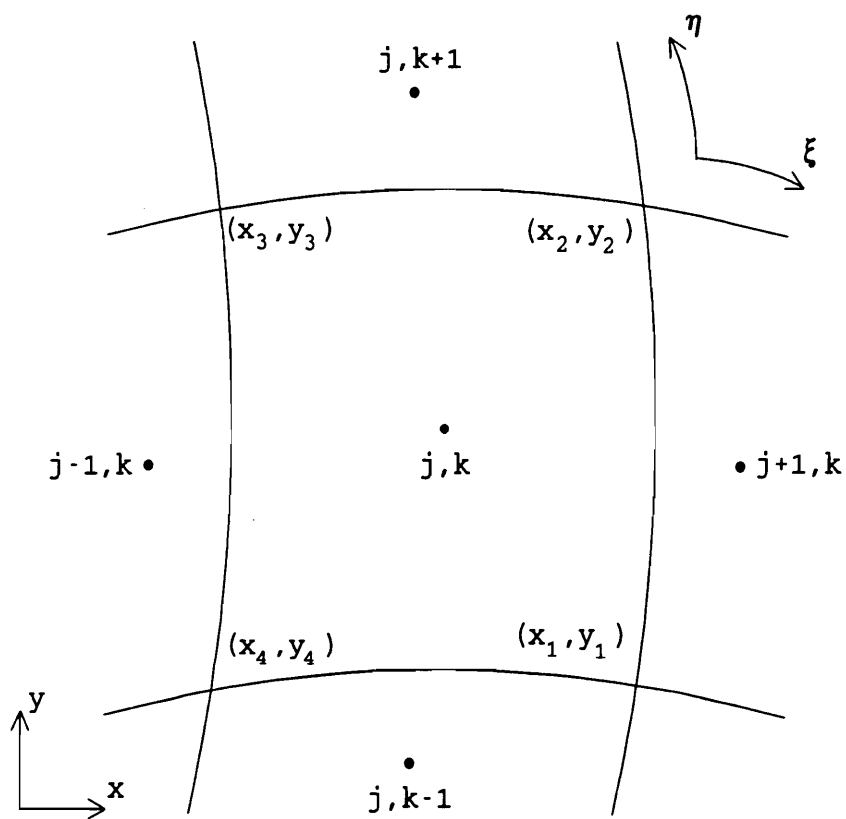


Figure A.1 : Typical (j,k) th Computational Cell

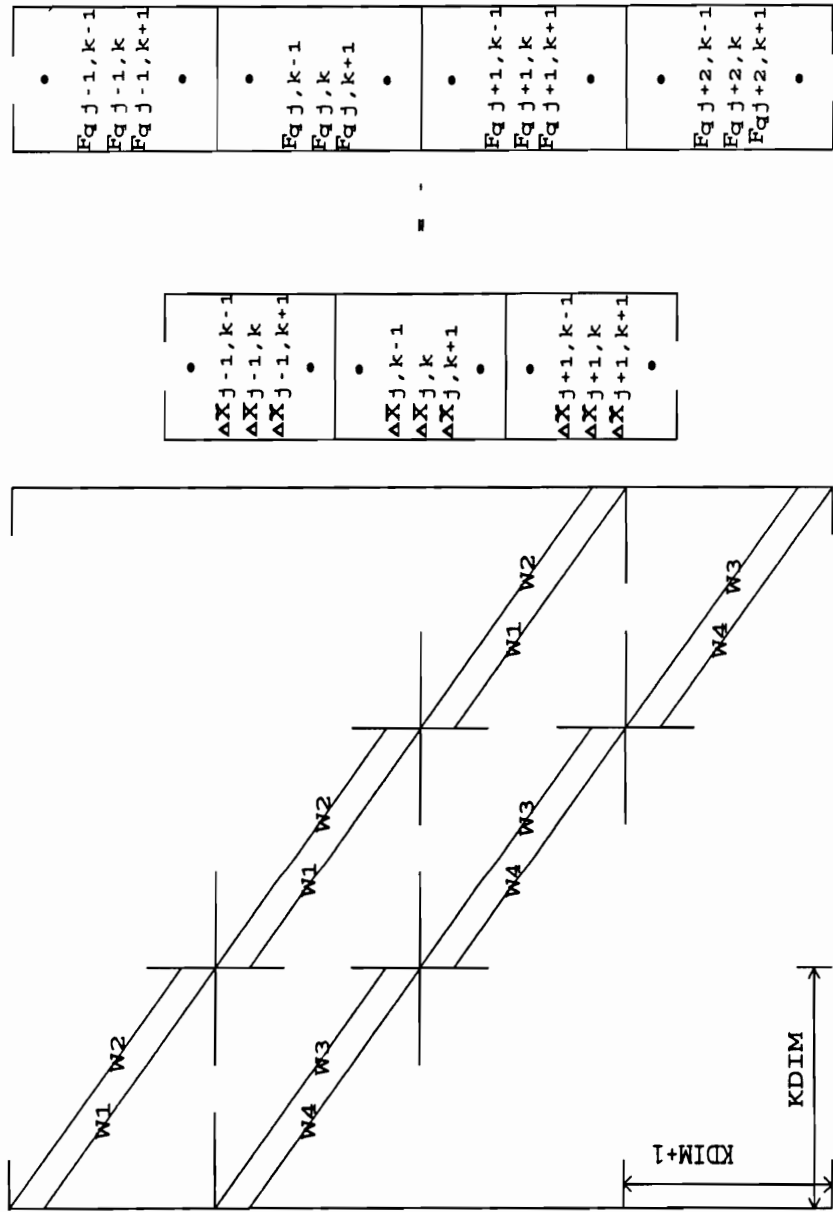


Figure A.2 : Standard Design Equation

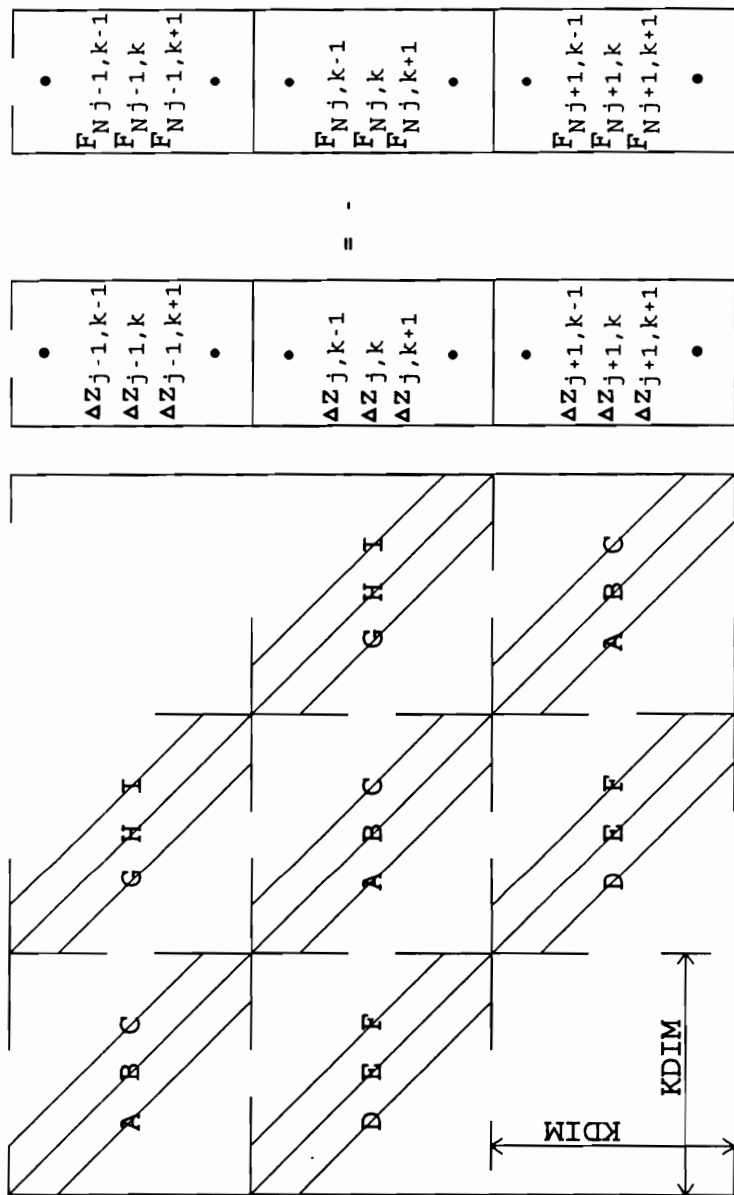


Figure A.3 : Incremental Normal Equation

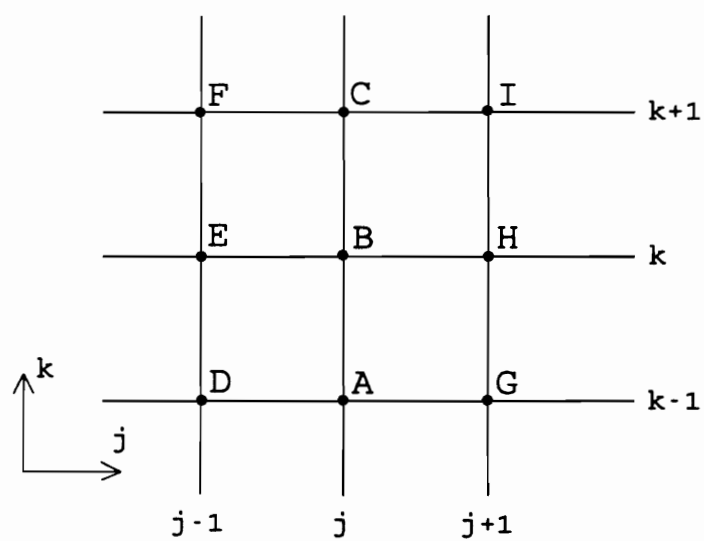


Figure A.4 : Normal Matrix Molecule

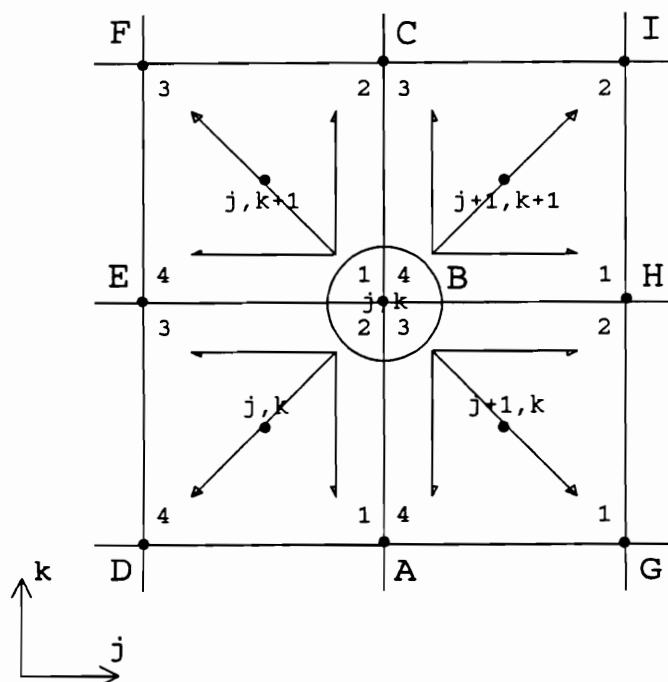


Figure A.5 : Normal Matrix Diagonals

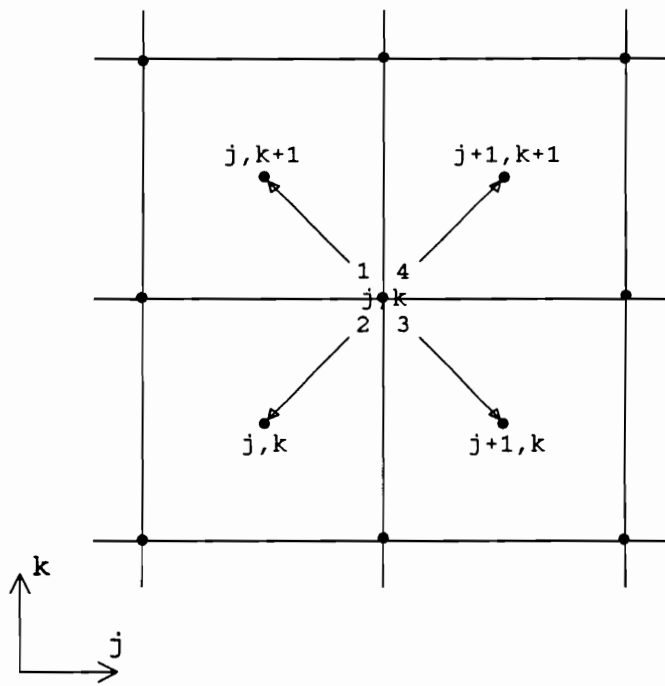


Figure A.6 : Normal Forcing Function

Appendix B. INE Constraints

This appendix presents the details for implementing geometry/grid constraints within the incremental normal equation (INE, Equation 3.12). The method used in this research included adjusting the standard design equation (Equation 3.7) for a single constraint. (The following procedure may be applied to either the original or the modified standard design equation.) This was followed by replacing one equation within the system with that constraint equation. The process was then repeated for the remaining constraint equations. In general, this is a simple method which maintains systematic computational storage requirements and guarantees that each constraint is exactly satisfied. Also, each constraint solution naturally appears within the shape modification solution vector $\Delta\bar{X}$.

In the present code, JDIM controls the storage sequence of the global metric Jacobian matrix. This is the opposite storage sequence of that represented in Figure A.2. However, this storage sequence results in a similar matrix structure. In practice, only the local metric Jacobian vectors, $W1 \rightarrow W4$, are stored within the code. This discussion, however, considers the entire metric Jacobian matrix as stored in a single matrix.

Each of the (JDIM)(KDIM) column pairs within the global metric Jacobian matrix is associated with a pair of x and y coordinate variables in that order. Each of the (JDIM+1)(KDIM+1) row sections of the standard design equation is associated with a set of four governing cell-center or boundary cell-face equations. The governing equations are ordered in a continuity, ξ -momentum, η -momentum, and energy equation sequence. The vector for the tangency boundary equation was given in Chapter 4.

The global metric Jacobian matrix and solution forcing function vector are represented by A_{mxn} and $F_{q\ mx1}$, respectively. Each element within this matrix and the F_q vector is denoted by A_{ij} and F_{qi} , respectively, where $i=1,m$ and $j=1,n$. The values of

m and n define the total number of discrete equations and coordinate unknowns within the flow domain. These constants were defined within Chapter 3 and Appendix A.

Within this research, each geometry/grid constraint was expressed as $\Delta s_{mc,nc} = \varepsilon$, where Δs is a generic variable for the change in either x or y. The value of ne determines the (j)th column of the global metric Jacobian matrix. Necessarily, this also determines the (j)th x or y coordinate variable. The sequence of operations presented in Equation B.1 was then used to adjust the standard design equation for a single geometry/grid constraint. This sequence of operations adjusts the F_q vector to reflect the constraint, and then zero's the appropriate column of the global metric Jacobian matrix.

$$\begin{aligned} F_{qi} &\rightarrow F_{qi} - A_{i,ne} * \varepsilon & ; & \quad i = 1, m \\ A_{i,ne} &\rightarrow 0 & ; & \quad i = 1, m \end{aligned} \tag{B.1}$$

The second step for implementing a single geometry/grid constraint requires replacing one equation within the standard design equation with that constraint equation. One method of implementing this step was suggested in Reference 47. However, this method only considered the first n, of the total m, equations within the standard design equation for replacement. Given the present storage sequence of the metric Jacobian matrix, constraint enforcement was concentrated in the lower half of the flow domain. Therefore, a governing or boundary equation within the lower section of the domain may be replaced by a constraint equation written for an upper domain position. This method was effective, but is considered inappropriate for a general design procedure.

Within this research, another method was used for replacing one equation within the metric Jacobian matrix for a single geometry/grid constraint. This new method identifies the important equations within the standard design equation which are directly a function of the coordinate variable within the constraint. There are sixteen governing equations surrounding each coordinate position within the interior of the flow domain. This new method only considers replacing one of these equations for a constraint written at an interior coordinate position. Similarly, there are a total of eight boundary equations surrounding each boundary coordinate position. The new method also only considers

replacing one of these equations for a constraint written at a boundary coordinate position. The remaining choice is which of the sixteen or eight equations to replace for each interior or boundary geometry/grid constraint, respectively.

Within this research, the continuity equation from the computational cell above and to the left of the constraint position was replaced for each constraint written for an interior x coordinate variable. The ξ -momentum equation from the identical position was replaced for each constraint written for an interior y coordinate variable. Similarly, the first equation within the boundary equation vector was replaced for each boundary x coordinate variable. Also, the second equation within the boundary equation vector was replaced for each boundary y coordinate variable. Boundary equation replacement was completed in a counter-clockwise manner around the flow domain, where the equations in front of the boundary coordinate position were replaced.

This selection process then identified the (me)th equation within the standard design equation for replacement. Necessarily, this determined the (i)th row of the metric Jacobian matrix for adjustment. The sequence of operations presented in Equation B.2 was then used to replace the (me)th equation with a single geometry/grid constraint. The sequence of operations within Equation B.2 erases one equation within the standard design equation, and then replaces it with the constraint equation. This guarantees that the constraint solution appears within the shape modification solution vector $\Delta\bar{X}$.

$$\begin{aligned} A_{me,j} &\rightarrow 0 \quad ; \quad j = 1, n \\ A_{me,ne} &\rightarrow 1 \quad \text{and} \quad F_{q\ me} \rightarrow \varepsilon \end{aligned} \tag{B.2}$$

There are many choices within the second step of the procedure outlined above. Several of these choices were investigated within this research, but only one produced a noticeable affect on the inverse design solution. The geometric condition within the design tangency BC, $\bar{u} = +\Delta\bar{u}$, is the only term which produces a non-zero element within the modified standard design equation forcing function. Therefore, this equation must not be replaced with a constraint equation.

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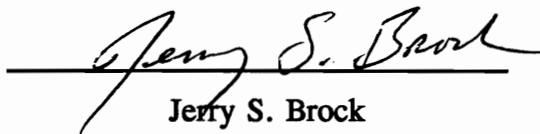
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