Targeted Priority Mechanisms in Organ Transplantation

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Academic Abstract

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The persistent shortage of transplantable organs, compounded by high rates of organ under-utilization, necessitates innovative allocation mechanisms. This dissertation develops and analyzes targeted priority mechanisms, voluntary incentive-based programs designed to enhance access for disadvantaged patient groups and improve organ-recipient matching. Using a rigorous queueing-theoretic framework, I characterize patients' equilibrium participation strategies, identifying conditions under which no-, full-, and mixed-participation equilibria emerge. I further establish the necessary and sufficient conditions for their existence and uniqueness, highlighting how careful mechanism design can align individual incentives with socially optimal outcomes.

The study extends the analysis to class-separating allocations, demonstrating the feasibility of equilibria that improve social welfare while safeguarding non-participating patients' access to high-quality organs. A clinically detailed simulation of the U.S. kidney allocation system, focusing on elderly patients, illustrates the potential benefits: a targeted threshold of 84% KDPI yields approximately 220 additional annual transplants, reduces the waiting list by more than 450 patients, and prevents over 60 pre-transplant deaths annually, with minimal impact on graft survival rates. Overall, the findings provide both theoretical and practical guidance for the design of efficient, implementable allocation mechanisms.

General Audience Abstract

Targeted Priority Mechanisms in Organ Transplantation

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Targeted Priority Mechanisms in Organ Transplantation Ruochen Wang Every year, thousands of patients die while waiting for a life-saving organ transplant, even though many donated organs never get used. This research looks at how to make organ allocation fairer and more effective by introducing what are called targeted priority programs. These programs give certain groups of patients, such as older adults, earlier access to organs that are less likely to be used otherwise. In return, these patients give up priority for the highest-quality organs, making it easier to match every organ with the patient who can benefit most.

The study shows how patients might respond to such programs, and how the rules can be designed to encourage participation without disadvantaging those who are not eligible. A detailed computer model of the U.S. kidney transplant system suggests that, with the right design, targeted priority programs could prevent more than 60 deaths each year, reduce the waiting list by over 450 patients, and add about 220 extra transplants annually—all while keeping transplant success rates nearly the same.

Although no policy is perfect, this approach offers a promising way to make better use of donated organs and give more patients a second chance at life. By carefully choosing which patient groups to include and how to balance trade-offs, policymakers could create a system that is both more efficient and more equitable.

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Contents

Chapter 1

Introduction

1.1 Motivation

Transplantation stands as the most prevailing solution for organ failure. In 2023, around 40,000 individuals benefited from a deceased donor organ transplantation in the U.S., with a 60% increase over eight years prior OPTN (2024). In the U.S., deceased donor organs are allocated via national waiting lists, with more than 100,000 patients awaiting the life-saving procedure of transplantation Schladt and Israni (2023). Regrettably, on average, more than 70% of these patients do not receive a transplant Lewis et al. (2021), and those who do often endure prolonged waiting times Gill et al. (2005) due to a *critical imbalance* between organ supply and demand Lentine et al. (2023). Despite this urgent need, the underutilization of organs presents a significant challenge, with *a considerable portion of donated organs going unused*. In 2021 alone, more than 7,800 organs recovered for transplantation were not utilized Israni et al. (2023), accounting for almost 18% of all organs recovered for transplantation, with the highest rates for kidney (25%) and pancreas (26%).

Policymakers identify the underutilization as an "*opportunity to increase the number of transplants*" Israni et al. (2022). Although many unused organs are categorized as less-than-ideal, research demonstrates that these transplants still outperform continued waiting in terms of *enhanced survival outcomes*, including reduced mortality rates and improved life expectancy Gill et al. (2013), Lloveras et al. (2015), Pérez-Sáez et al. (2016). Consequently,

this has given rise to the term "better than dialysis" to characterize these organs, highlighting their comparative benefits over prolonged dialysis The Alliance (2024). Particularly, *older patients* and *those in centers with long median times to transplant* benefit most from accepting hard-to-place kidneys, suggesting the need for *targeted organ matching* strategies Massie et al. (2014), Pérez-Sáez et al. (2017). The relevance of such strategies is further heightened when considering patients' willingness to accept such organs. In 2021, almost two-thirds of U.S. patients over the age of 65 expressed willingness to accept hard-to-place kidneys, in contrast to just 16% of patients aged 18-34 years Lentine et al. (2023).

As a potential remedy for the underutilization of organs, the current system offers hard-to-place organs to patients who express their willingness to accept such organs during registration on the waiting list OPTN (2023). However, patients still have the right to reject such offers after a declaration of potential interest, and the current high nonuse rates suggest the need for more effective interventions Israni et al. (2023). Past literature has proposed various mechanisms to incentivize the utilization of available organs Cooper et al. (2019), Gordon et al. (2023), Reese et al. (2016), Stewart et al. (2017a), Tunç et al. (2022). Nonetheless, any implemented mechanisms, such as the Kidney Accelerated Placement (KAP) project OPTN (2019), leave space for improvements Noreen et al. (2022), and policymakers have called for *more sophisticated methodologies* to understand these processes Mohan and Schold (2022). A promising initiative was established in Europe in 1999, named The Eurotransplant Senior Program (ESP) Branger and Samuel (2016). ESP is a voluntary system, prioritizing kidneys from donors older than 65 for participating recipients within the same age group. Since its inception, ESP has seen not only an increase in elderly recipients (from 3.6% to 19.7%), transplants from elderly donors (from 10% to 14.2%) Cohen et al. (2005), De Fijter (2009), and hard-to-place organ transplants Bahde et al. (2014), but also a decrease in patient waiting times, while maintaining graft and patient survival rates almost unaffected Frei et al.

(2008). Our thesis explores mechanisms that *expand* upon the foundations laid by the ESP, offering an in-depth examination of various design alternatives.

We introduce *targeted priority mechanisms* designed to address the critical issue of organ nonuse and narrow the widening supply-demand gap, while simultaneously improving the efficiency of organ-recipient matching. Targeted priority mechanisms offer *strict priority* to a *specific group of candidates* over a *pre-defined set of organs* identified by the planner, provided these candidates *restrict their organ acceptance to this set*. Thus, participating candidates gain absolute priority for organs in the program but forfeit access to others. Participation is voluntary, and those who decline follow the baseline allocation. A distinctive aspect of these mechanisms is their ability to incentivize patients to align with the planner's optimal matching strategy, effectively rewarding them in a manner that further promotes this alignment. In contrast to conventional priority systems that often benefit the most advantaged or "premium" customers, our proposed mechanisms are specifically designed to help those who are disadvantaged under the current system. This shift from traditional incentive models underscores the importance of careful program design to protect the interests of patients outside the program's scope.

Aligned with the organ allocation principles established by the Final Rule (The Final Rule 1998), our proposed targeted priority mechanisms draw inspiration from Organ Procurement and Transplantation Network (OPTN) policies, aiming to improve the existing allocation system. These mechanisms are designed to address the underutilization of donated organs and to refine the matching between organs and waitlisted patients through voluntary means, thereby improving social welfare as a whole. Furthermore, with the U.S. Health Resources and Services Administration (HRSA) currently undertaking efforts to overhaul the OPTN system, there exists a timely opportunity to reevaluate and positively impact organ allocation practices (Adashi and Bayliss 2023, Doby et al. 2023, Pullen 2023).

1.2 Thesis Overview

The remainder of the thesis is organized as follows. In Chapter 2, we summarize the relevant literature and the differences between the existing literature and our study. In Chapter 3, we first analyze targeted priority mechanisms using a comprehensive, multiclass queueing model with reneging by incorporating the discrepancies between patients' waitlist mortality, post-transplant survival, and access to organs observed within the current systems. We characterize the strategic decisions of waitlisted candidates under targeted priority mechanisms and identify the participation decisions of eligible patients as well as the offer acceptance behavior of each patient class in equilibrium. We further establish the impact of the design parameters on the participation and offer acceptance equilibria adopted by patients, to establish optimally designed mechanisms. Moreover, it is shown that a well-designed targeted priority mechanism holds the potential to *distinctly separate* organ allocation between patient classes, without the need for compulsory organ acceptance or restricted organ access for any candidate. We establish that such mechanisms allow the planner to facilitate more efficient organ allocation by directing each organ to those who can benefit most, while still ensuring *voluntary participation*. Finally, we characterize the impact of these mechanisms on social welfare and show that they can improve social welfare *without adversely affecting* any patient group.

In Chapter 4, we expand on the queueing model and incentive mechanisms introduced in Chapter 3, extending patient classification from discrete to continuous types. In this generalized setting, we address the question of optimal patient selection for targeted program eligibility under continued eligibility restrictions. Additionally, we explore the optimal threshold for program organs when all patients are allowed to participate, forgoing access to non-program organs.

In Chapter 5, we assess the effectiveness and quantify the impact of the proposed mechanisms within a realistic setting through a case study. This study employs a validated, clinically detailed simulation model of the U.S. kidney transplantation system, with a focus on elderly patients (aged 65 and above at registration) for the targeted program, following an approach similar to the European Senior Program (ESP). Our numerical results reveal that even when the program offers priority access to kidneys traditionally seen as marginal, a substantial majority (80.5%) of elderly patients would opt to participate in equilibrium. These targeted priority mechanisms could enable up to 1,000 additional transplants per year and prevent up to 200 waitlist deaths annually. By encouraging substantial participation among elderly patients, these mechanisms help allocate higher-quality kidneys to younger patients who can derive greater long-term benefit from them. Consequently, post-transplant outcomes for nonelderly patients improve significantly, while their pre-transplant mortality remains stable compared to the existing system. Our findings highlight the unique potential of targeted priority programs to deliver dual benefits: offering the elderly priority access to harder-to-place organs while preserving quality organs for younger recipients. To build on these insights, we conduct further simulation experiments to address several questions: when eligibility for the targeted program expands beyond elderly patients (aged 65 and above), what is the optimal target threshold for program organs? Additionally, if eligible patients are granted the option to reverse their participation, how does this flexibility influence the equilibrium behaviors of different patient groups?

Finally, Chapter 6 provides a summary of this dissertation, and discuss limitations and possible research directions

Chapter 2

Literature Review

In this chapter, we review the relevant literature on organ allocation, incentive design, and targeted healthcare programs, highlighting key distinctions between existing work and our contributions.

Within the domain of organ transplantation, a substantial body of research has focused on optimizing allocation policies to improve system efficiency and fairness (Akan et al. 2012, Bertsimas et al. 2013, Su and Zenios 2006, Zenios et al. 2000). These studies often pursue objectives such as maximizing quality-adjusted life expectancy (QALE), graft survival, or match quality. However, they tend to advocate for comprehensive systemic redesigns and often overlook the behavioral responses of patients, particularly in terms of offer acceptance, under proposed changes. In contrast, our work focuses on incremental, incentive-compatible improvements within the existing allocation framework. We propose mechanisms that do not mandate compliance, but instead influence patient decisions under equilibrium through voluntary participation.

Several strands of literature explore the role of incentives in organ transplantation. For example, proposals for incentivized kidney exchange aim to improve both fairness and efficiency in living-donor programs (Kher and Jha 2020, Sönmez et al. 2020). Likewise, various initiatives encourage deceased organ donation by granting waiting list priority to individuals registered as donors (Dai et al. 2020, Kessler and Roth 2012, Stoler et al. 2017). Other strategies focus on increasing the utilization of hard-to-place kidneys by reducing acceptance

costs for transplant centers, adjusting performance metrics, or allocating these organs more efficiently (Reese et al. 2016, Stewart et al. 2017a). Notably, Tunç et al. (2022) propose offering re-transplantation priority to recipients of such organs. Departing from these designs, our targeted priority mechanism incentivizes disadvantaged patient subgroups by offering selective priority access to organs deemed most appropriate by the planner, without requiring system-wide mandates. This builds on precedents like the Eurotransplant Senior Program (ESP), which demonstrated the value of such targeted programs for elderly candidates and marginal organs (Bahde et al. 2014, Branger and Samuel 2016), and shows potential for broader application when designed strategically.

Our research is also situated within the literature on prioritization and strategic behavior in queueing systems, where arrival or service rates are influenced by agent decisions (Afeche 2013, Cui et al. 2019, Debo and Veeraraghavan 2014, Tunç et al. 2022, Wang et al. 2019, Yang and Debo 2019, Yang et al. 2017). Our contribution is twofold. First, we introduce a novel prioritization scheme that incentivizes voluntary participation by eligible patients, distinguishing it from mechanisms based on referrals, monetary contributions, or time-trading. Second, unlike premium services that offer access to better resources, our mechanism reallocates specific types of existing resources, potentially improving utilization without increasing supply or imposing mandates. Moreover, our setting differs from typical dynamic matching models (Abdulkadiroğlu and Grigoryan 2021, Agarwal 2015, Arnosti and Shi 2020, Ashlagi et al. 2023, Leshno 2022, Nguyen et al. 2021) due to the perishability and heterogeneity of organ offers, absence of mutual choice, and the ability of candidates to decline without strategic repositioning. These features complicate the allocation landscape and necessitate novel, context-aware mechanisms.

Beyond the transplantation setting, our work intersects with broader medical literature on targeted priority mechanisms for healthcare resource allocation. These mechanisms aim

to enhance both efficiency and equity by identifying patient subgroups with the greatest marginal benefit. Tools such as Programme Budgeting and Marginal Analysis (PBMA), Health Technology Assessment (HTA), and Multi-Criteria Decision Analysis (MCDA) have been developed to formalize these decisions (Barasa et al. 2021, Wisdom and et al. 2021). However, widespread adoption remains limited due to institutional inertia, fragmented implementation, and lack of transparency (Chalkidou and et al. 2016, Peacock and et al. 2006). Algorithmic frameworks have been proposed in high-stakes contexts like organ transplantation to improve allocation under fairness and utility constraints (Bertsimas et al. 2013), though long-term empirical evaluations remain scarce.

Complementing these allocation mechanisms are targeted treatment programs rooted in precision medicine, which aim to tailor clinical decisions to individual-level genetic or clinical characteristics. Notable examples include umbrella and basket trials, such as NCI-MATCH, which assign therapies based on tumor genomics (Li and et al. 2021, NCI 2015). Large-scale initiatives like the NIH's All of Us program seek to link multiomic and behavioral data to personalized interventions (NIH 2019). While the promise of precision medicine is substantial, its current impact remains limited, with only a small proportion of patients benefiting from approved genomic-based therapies (Marquart and et al. 2018). These limitations underscore the translational challenges of expanding precision approaches within real-world healthcare delivery systems.

Finally, sustaining the effectiveness of allocation and treatment programs requires deliberate strategies to boost patient engagement, particularly among underserved or low-compliance populations. Behavioral incentive programs, drawing on principles of behavioral economics, have demonstrated effectiveness in improving uptake of vaccinations, screenings, and chronic disease management (Giles and et al. 2023, Volpp and et al. 2008). In mental health and addiction treatment, contingency management approaches, such as voucher, based incentives—

have consistently improved treatment retention (Prendergast and et al. 2006). More recent innovations, including randomized-loss lotteries and peer-led interventions like Whole Health Action Management (WHAM), represent a growing ecosystem of incentive tools (SAMHSA 2019, Zhang and et al. 2018). These programs highlight the importance of integrating behavioral reinforcement with structural and clinical innovations to achieve population health goals.

Chapter 3

Targeted Priority Mechanisms for Exogenous Patient Groups

3.1 Introduction

Building upon the motivation introduced in the preceding section, this chapter presents a formal model of targeted priority mechanisms tailored for exogenous patient groups, subpopulations defined by externally assigned characteristics such as age, health risk, or center-specific access disparities. These groups are often systematically disadvantaged under the existing allocation system, particularly when it comes to hard-to-place organs. Our goal is to design a mechanism that operates within current allocation constraints while enhancing equity, utilization, and patient outcomes without requiring system-wide mandates or infrastructural changes.

The analytical foundation for this mechanism draws from several strands of literature. First, in the transplantation context, targeted policies such as the Eurotransplant Senior Program (ESP) have shown that priority matching based on age cohorts can lead to substantial gains in organ utilization and reduced waiting times, particularly for older recipients and marginal organs (Bahde et al. 2014, Branger and Samuel 2016, Frei et al. 2008). While ESP operates as a fixed matching rule, we generalize this idea to account for strategic patient behavior, introducing voluntary participation incentives that align individual decisions with planner-

defined organ-patient matches. The modeling approach also parallels work in queueing theory and market design, where patients or agents respond to changes in priority by optimizing over future arrival or service conditions (Afeche 2013, Debo and Veeraraghavan 2014, Tunç et al. 2022, Yang and Debo 2019). Our framework is distinctive in treating priority as a non-monetary and organ-specific reward rather than a universal speed-up, ensuring that participation is both self-selecting and targeted.

From a broader health policy perspective, this chapter contributes to the growing literature on targeted priority setting for health interventions and services, especially under conditions of scarcity and heterogeneity in resource quality. Formal prioritization tools such as Health Technology Assessment (HTA), Multi-Criteria Decision Analysis (MCDA), and Programme Budgeting and Marginal Analysis (PBMA) offer frameworks for evaluating trade-offs across populations (Barasa et al. 2021, Chalkidou and et al. 2016, Peacock and et al. 2006). However, they typically operate at a system-planning level and do not account for decentralized, real-time decision-making by patients. By embedding targeted prioritization into a dynamic, patient-interactive model, our approach operationalizes the principles of fair resource allocation while respecting autonomy and choice—features particularly relevant in the context of organ allocation, where offer acceptance is non-binding and decline behavior significantly affects overall system efficiency.

This chapter develops the formal game-theoretic model of our targeted priority mechanism, characterizing patient participation incentives, equilibrium acceptance strategies, and system-level outcomes. We derive comparative statics to examine how different design levers, such as the scope of the targeted organ set or the size of the prioritized group, affect both individual and social welfare. These results provide key insights for designing scalable, data-driven allocation programs that remain consistent with the OPTN's equity-driven goals and the ethical mandates of the Final Rule (The Final Rule 1998).

3.2 Model Formulation

We formulate an overloaded multiclass priority queueing model with reneging to study the organ transplant waiting list. Recognizing discrepancies between patients' access to organs within the current transplantation systems (Akan et al. 2012, Bertsimas et al. 2013), two types of candidates waiting for an organ transplant are considered: disadvantaged candidates (type D) who are disproportionately unlikely to receive organ offers, particularly, high-quality ones, and non-targeted candidates (type O), who are disproportionately likely to receive organ offers. The overall arrival rate of candidates is given by $\lambda := \lambda_D + \lambda_O$, where λ_D and λ_O denote that of type D and O, respectively. Candidates leave the waiting list by receiving a transplant or due to a non-transplant removal (e.g. death) while waiting, whichever happens first. Type D and O leave the waiting list by a non-transplant removal after respective exponential clocks with rates d_D and d_O.

Organs arrive at the waiting list with a normalized rate of $\mu = 1$, and can provide different post-transplant benefits, which is associated with quality score q, where $q \in [\underline{q}, \overline{q}]$ assumed to be drawn from a uniform distribution[1], which is in line with the clinical data (Stewart et al. 2017b). We model in our study a static randomized policy to allocate organs, and we introduce offer probabilities $p_i(q)$ to represent the likelihood of an organ with quality q being offered to a type i candidate. Then the overall arrival rate of organs to type i is given by $\mu_i = \int_{\underline{q}}^{\overline{q}} \frac{p_i(q)}{\overline{q}-\underline{q}}\, dq$, and to represent organ scarcity, we assume $\mu_i \leq \lambda_i$ for $i \in \{D, O\}$.

Organ offers do not always result in transplantation because transplant candidates can decline them. Patients are presumed to be risk-neutral, making decisions on organ offers jointly with their physicians. This decision-making follows a threshold-based approach, where ac-

[1]The assumption that the organ quality score q follows a uniform distribution is made for notational simplicity and practical relevance. All of the analytical results in this thesis hold for a general class of distributions $f(q)$.

ceptance is conditional on the organ satisfying predetermined quality benchmarks. The latter assumption is not only practically sound, reflecting that a patient willing to accept an organ of a certain quality would likely accept organs of higher quality, but also aligns with the existing literature (Alagoz et al. 2007, Howard 2002, Sandıkçı et al. 2013).

Two utility functions are of particular interest: the time candidates spend on the waiting list and their post-transplant life benefits, measured by QALE. Given a threshold q_i, the time spent on the waiting list by type i, $\Delta_i(q_i)$, is measured from arrival until departure from the list due to a transplant or a non-transplant removal. A quality-of-life score is used to account for the quality of life under different medical conditions, denoted as α and β for pre- and post-transplant periods, respectively. We assume that $\alpha < \beta$ since transplantation improves the quality of life. Letting $T_i(q)$ represent the average lifespan of a type i candidate after receiving an organ of quality q, the post-transplant life expectancy given q_i is $\mathbb{E}_i[T_i(q) \mid q \geq q_i]$. Since organ quality is correlated with post-transplant lifespan, we assume that $T_i(q)$ increases in q. In sum, expected QALE for type i candidates having threshold q_i can be written as $U_i(q_i) = \alpha\Delta_i(q_i) + \beta\pi_i(q_i) \cdot \mathbb{E}_i[T_i(q) \mid q \geq q_i]$, where $\pi_i(q_i)$ denotes the probability that a type i having threshold q_i receives an organ.

3.2.1 Targeted Priority Mechanisms

Targeted priority mechanisms offer *strict priority* to a *targeted group of candidates* (denoted as type D) over a pre-defined, *target set of organs* $q \in [\underline{q}, \overset{*}{q}]$ identified by the planner, provided these candidates *restrict their organ acceptance to this set*. Thus, under a targeted priority mechanism ($\overset{*}{q}$), participating candidates gain absolute priority for organs $q \in [\underline{q}, \overset{*}{q}]$ but forfeit access to organs $q \in (\overset{*}{q}, \bar{q}]$. Upon waitlist entry, type D candidates (also referred to as *eligible* candidates) can opt for the targeted priority mechanism, a decision assumed

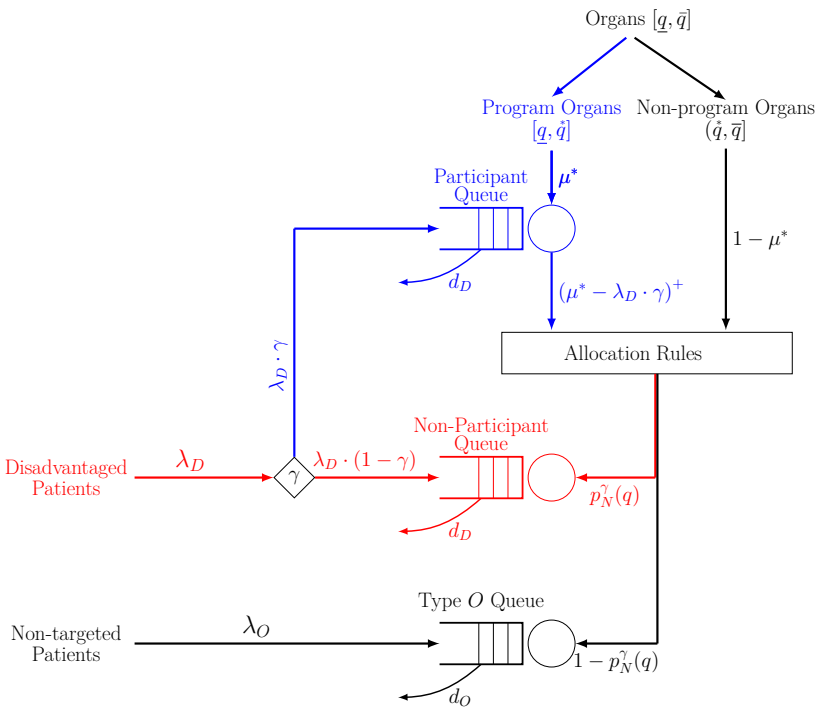


Figure 3.1: Schematic illustration of the targeted priority mechanisms.

to be irreversible, whereas type O (also referred to as *ineligible* candidates) is not allowed to participate in the program. Consequently, type D candidates are categorized into two subgroups: those who opt into the priority mechanism (P), and those who decline (N). Figure 3.1 provides a schematic illustration of the targeted priority mechanisms.

The target threshold $\overset{*}{q}$, set by the planner, influences both the participation rate of type D candidates and the offer acceptance thresholds set by each candidate type.

Let $\gamma(\overset{*}{q})$ denote the participating fraction of type D when the target set is given by $[\underline{q}, \overset{*}{q}]$. Participating candidates (P) have priority over non-participating (N) or ineligible (O) ones for organs of quality $q \leq \overset{*}{q}$. Those who do not join the program and those who are ineligible, continue receiving offers for organs of any quality q according to an adjusted offer process captured by the probabilities $p_N^{\gamma(\overset{*}{q})}(q)$ and $p_O^{\gamma(\overset{*}{q})}(q) := 1 - p_N^{\gamma(\overset{*}{q})}(q)$, respectively. We assume that the allocation probability for non-participating candidates, $p_N^{\gamma(\overset{*}{q})}(q)$, is decreasing in the

participation rate, $\gamma(\mathring{q}^*)$. This reflects that as the pool of non-participants diminishes, the allocation of organs to this group proportionally decreases.

We examine the targeted priority mechanisms under a fluid limit approximation. In particular, we analyze the scenario, where the arrival rates of candidates (λ_i) and organs (μ_i) are scaled up with a factor of $n \to \infty$, while the death rates (d_i) remain constant. This setting is a realistic approximation for the U.S. organ allocation systems, given that patients and organs are added to the waiting list at considerably higher rates than the waitlist mortality rates (Kwong et al. 2023, Lentine et al. 2023).

3.2.2 Discussion of Major Assumptions

Because of the complexity of organ allocation systems, we make several assumptions to keep our models analytically tractable while also preserving the fundamentals of the actual allocation systems. Most of these assumptions are derived from the literature on analytical approaches to organ transplantation (Akan et al. 2012, Bertsimas et al. 2013, Su and Zenios 2006, Zenios 1999, Zenios et al. 2000), to help provide key analytical insights on the impact of targeted priority mechanisms. We relax these assumptions in our case study in §**??** and offer a more realistic assessment of the proposed mechanisms using the most recent U.S. kidney allocation policy. In the following, we list and discuss the major assumptions of our analytical study.

Assumption 1. *Candidates of each type are homogeneous and they do not change their types while waiting.*

In the real allocation systems, individuals are inherently unique and their conditions or characteristics may change over time while still on the waiting list. In our theoretical model, we do not allow patients to move between types for analytical tractability.

Assumption 2. *Organs are allocated to each class according to a static randomized policy parameterized by* $p_i(q)$.

The organ allocation policies in the U.S. are complex and dynamic, and they depend on several donor and recipient characteristics (OPTN 2023). However, the static randomized policy $p_i(q)$ provides an approximation for the real allocation rules in a steady state and enables an analytically tractable model to study the *existing disparities* in patients' access to organs in the current allocation system. For example, in 2021, 27.8% of the high quality kidneys (KDPI $\leq$ 20%) were allocated to recipients between 18 to 34 years, whereas only 5.5% were allocated to older than 65 years (Lentine et al. 2023).

Assumption 3. *Organ offers are instantaneous.*

When a donor organ is procured for transplantation, potential recipients are categorized by priority levels and sequentially receive offers for the organ. The process of searching for a suitable candidate to accept the organ (called a *match run*) may lead to an increased cold ischemia time, which is detrimental to the organ quality (Cassuto et al. 2008, Peters-Sengers et al. 2019). Prolonged cold ischemia time can result in the organ going unused altogether. In our analytical models, we simplify this aspect of organ allocation through instantaneous offers, which might potentially lead to an underestimation of the nonuse rate of organs.

Assumption 4. *Waiting list positions are unobservable.*

Patients on the U.S. waiting lists can have approximate knowledge about their waiting list position using the data accessible to the public, which may potentially influence their offer acceptance decisions. Patients who are higher ranked on the waiting list may tend to be more selective in their acceptance behavior. In our theoretical model, we assume that the patients do not observe their waiting list positions. Note that, under this assumption, the specific order in which individuals are served within a type becomes inconsequential.

3.3 Strategic Decisions of Patients under Targeted Priority Mechanisms

Transplant candidates make strategic and rational decisions of opting into the targeted priority mechanism at their time of arrival, based on their eligibility to the program and their assessment of the expected benefits and harms of joining. An eligible candidate would opt into the program if she believes that she will be subjected to a higher expected utility by participating, when $\gamma^e(\overset{*}{q})$ fraction of other type D candidates opt in. Once candidates make a decision on opting in (if eligible) or not, they join their corresponding queue, and set their equilibrium offer acceptance thresholds $q_i^{e,\overset{*}{q}}$ to declare the type of organs that they are interested in. We consider symmetric participation equilibria and equilibrium acceptance thresholds among each candidate type, while considering potentially different equilibrium acceptance thresholds for participating and non-participating candidates (if they exist).

We are interested in characterizing the participation fraction $\gamma^e(\overset{*}{q})$ and the equilibrium acceptance thresholds $q_i^{e,\overset{*}{q}}$ under a targeted priority mechanism $(\overset{*}{q})$. We categorize the eligible patients' participation equilibria under three main categories, the first two of which are pure strategy Nash equilibria and the last one is mixed strategy equilibria:

(i) ***No-participation equilibrium:*** All disadvantaged patients refuse to participate in the program, i.e., $\gamma^e(\overset{*}{q}) = 0$, and maintain their access to all organs according to the existing allocation rules.

(ii) ***Full-participation equilibrium:*** All disadvantaged patients join the program, i.e., $\gamma^e(\overset{*}{q}) = 1$, and receive priority on organs of quality $q \leq \overset{*}{q}$ while foregoing their access to organs of quality $q > \overset{*}{q}$.

(iii) ***Mixed-participation equilibria:*** A nontrivial fraction of eligible candidates opt

into the program, i.e., $\gamma^e(\overset{*}{q}) \in (0,1)$, and have prioritized access to the organs $q \leq \overset{*}{q}$.

We characterize the equilibrium acceptance thresholds $q_i^{e,\overset{*}{q}}$ and participation fraction $\gamma^e(\overset{*}{q})$ under a targeted priority mechanism $(\overset{*}{q})$ in Theorem 3.1.

Theorem 3.1. *Under a targeted priority mechanism $(\overset{*}{q})$, candidates set their acceptance thresholds $q_i^{e,\overset{*}{q}}$ in equilibrium by solving the following set of equations:*

$$\beta T_O(q_O^{e,\overset{*}{q}}) = U_O(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha\Delta_O\Big(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta\pi_O(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q}))E\Big[T_O(q) \mid q \geqslant q_O^{e,\overset{*}{q}}\Big], \tag{3.1a}$$

$$\beta T_D(q_N^{e,\overset{*}{q}}) = U_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha\Delta_D\Big(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta\pi_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q}))E\Big[T_D(q) \mid q \geqslant q_N^{e,\overset{*}{q}}\Big], \tag{3.1b}$$

$$\beta T_D(q_P^{e,\overset{*}{q}}) = U_P(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha\Delta_D\Big(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta\pi_P(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q}))E\Big[T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_P^{e,\overset{*}{q}}\Big]. \tag{3.1c}$$

Furthermore, the equilibrium participation rate $\gamma^e(\overset{*}{q})$ can be characterized as follows:

(a) *A mixed-participation equilibrium solves* (3.1a)–(3.1c) *and the following equation:*

$$U_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) = U_P(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})). \tag{3.1d}$$

(b) *A no-participation equilibrium solves* (3.1a) *and* (3.1b) *with $\gamma^e(\overset{*}{q}) = 0$, and satisfies the following inequality:*

$$U_N(q_N^{e,\overset{*}{q}}, 0) > \lim_{\gamma \to 0^+} U_P(q_P^{e,\overset{*}{q}}, \gamma). \tag{3.1e}$$

(c) *A full-participation equilibrium solves* (3.1a) *and* (3.1c) *with $\gamma^e(\overset{*}{q}) = 1$, and satisfies the following inequality:*

$$U_P(q_P^{e,\overset{*}{q}}, 1) > \lim_{\gamma \to 1^-} U_N(q_N^{e,\overset{*}{q}}, \gamma). \tag{3.1f}$$

(d) *Under any participation equilibrium, the equilibrium acceptance thresholds* $q_i^{e,\overset{*}{q}}$ *are unique when they exist.*

The left-hand sides of equations (3.1a)–(3.1c) correspond to the transplant benefits, whereas the right-hand sides correspond to the utilities from waitlisting as a type O, N, or P, respectively, with the corresponding acceptance thresholds of $q_i^{e,\overset{*}{q}}$, $i \in \{O, N, P\}$, given that a $\gamma^e(\overset{*}{q})$ fraction of eligible candidates participate in the program. Therefore, Theorem 3.1 implies that the acceptance thresholds set by candidates in equilibrium correspond to the organ quality at which the benefits of accepting and rejecting an offered organ are equal. Furthermore, a mixed-participation equilibrium only exists when the eligible candidates are indifferent in their participation to the program, which is enforced by equation (3.1d), and all candidates set their acceptance thresholds by comparing transplant benefits to the utilities from waitlisting in their corresponding queue. Under a no-participation equilibrium, the benefits from participating should be inferior to those from not participating when no one else is participating, which is imposed by (3.1e). On the other hand, under a full-participation equilibrium, the benefits from participating should be superior to those from not participating when everyone else is participating, imposed by (3.1f). Further, Theorem 3.1 establishes that under any participation equilibrium, if there exist equilibrium acceptance thresholds $q_i^{e,\overset{*}{q}}$ solving the corresponding equations, then they exist uniquely.

The following corollary is a direct result of Theorem 3.1, and it implies that under a mixed-participation equilibrium, a type D candidate adopts the same acceptance thresholds in equilibrium and maintains the same utility from waitlisting whether they participate in the program or not.

Corollary 3.2. *Under a mixed-participation equilibrium* $\gamma^e(\overset{*}{q}) \in (0, 1)$*, the participating and non-participating type* D *candidates adopt the same acceptance thresholds in equilibrium, i.e.,* $q_P^{e,\overset{*}{q}} = q_N^{e,\overset{*}{q}}$.

Any $\overset{*}{q}$ with equilibrium participation rate $\gamma^e(\overset{*}{q}) = 0$ is of special interest, which we denote as *no-participation target thresholds*, and the characterization of such thresholds is important to make sure that the program can attract some participants, and the efforts of the planner will not be *futile*. Proposition 3.3 provides the necessary and sufficient conditions for a no-participation target threshold.

Proposition 3.3. *Under a targeted priority mechanism* $(\overset{*}{q})$*, no eligible candidates would participate in the program, i.e., the equilibrium participation rate satisfies* $\gamma^e(\overset{*}{q}) = 0$*, if and only if*

$$\beta T_D(\overset{*}{q}) < \frac{\alpha}{d_D} + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \lim_{\gamma \to 0^+} \int_{\overset{*}{q}}^{\bar{q}} \left(T_D(q) - \frac{\alpha}{d_D} \right) p_N^{\gamma}(q) dq. \tag{3.2}$$

The left- and right-hand sides of (3.2) correspond to the expected utility from participating and not participating in the program, respectively, when the participation rate converges to 0, i.e., $\gamma^e(\overset{*}{q}) \to 0^+$. Intuitively, the sufficiency of the condition (3.2) in Proposition 3.3 follows from that if, even, the maximum transplant benefit after participating in the program is less than the expected utility of not participating when no one else is participating, then no type D patients will participate in equilibrium. This is because participation would only provide prioritized access to organs of undesirably low quality so that not participating in the program, although may lead to not receiving a transplant, provides a higher utility in expectation. And the necessity of the condition (3.2) follows from that if a disadvantaged patient anticipates higher expected benefits by participating in the program while no other patient is participating, then $\overset{*}{q}$ can no longer be a no-participation target threshold since all individuals act in their best interest.

Furthermore, any $\overset{*}{q}$ with an equilibrium participation rate $\gamma^e(\overset{*}{q}) = 1$ is also of special interest, which we denote as *full-participation target thresholds*. By identifying such thresholds, the

planner can design *appealing* targeted priority mechanisms, where all eligible candidates are interested in participating, while concomitantly avoiding *needlessly increasing* the access of targeted candidates to the pool of high-quality organs, which may adversely make these candidates overly selective. Proposition 3.4 provides the necessary and sufficient conditions for the existence of a full-participation target threshold.

Proposition 3.4. *Under a targeted priority mechanism* $(\overset{*}{q})$*, all eligible candidates would participate in equilibrium, i.e., the equilibrium participation rate satisfies* $\gamma^e(\overset{*}{q}) = 1$*, if and only if*

$$\frac{\alpha}{d_D} + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\int_{q_P^e,\overset{*}{q}}^{\overset{*}{q}}\left(T_D(q)-\frac{\alpha}{d_D}\right)dq > \frac{\alpha}{d_D} + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\lim_{\gamma\to 1^-}\frac{\int_{\overset{*}{q}}^{\bar{q}}\left(T_D(q)-\frac{\alpha}{d_D}\right)p_N^{\gamma}(q)dq}{1-\gamma}. \tag{3.3}$$

Observe that the left- and right-hand sides of (3.3) in Proposition 3.4 represent the respected expected utilities of participating and not participating in the program when everyone else is joining, i.e., $\gamma^e(\overset{*}{q}) \to 1^-$. Intuitively, the sufficiency of (3.3) follows from that when the expected utility of not participating in the program is less than the benefits of participating even when all other patients choose to participate, then all type D would participate in the program in equilibrium. On the other hand, if a type D patient benefits more from not participating when all others are participating, then $\overset{*}{q}$ can no longer be a full-participation target threshold, implying the necessity of condition (3.3).

We are also interested in identifying any symmetric mixed-participation equilibria, where only a nontrivial fraction of eligible candidates participate in the program, i.e., $\gamma^e(\overset{*}{q}) \in (0,1)$, and we denote any $\overset{*}{q}$ leading to such an equilibrium as a *mixed-participation target threshold*. When designing targeted priority mechanisms, the planner needs to consider the harms and benefits of the program not only for those who are eligible for it but also for those who are not. Accordingly, the planner may potentially desire such equilibria over full-participation ones

because despite leading to a partial participation, a mixed-participation target threshold may yield the highest level of return from the concessions provided to the disadvantaged patients and may lead to the optimal overall social welfare. Proposition 3.5 provides the necessary and sufficient conditions for the existence of a mixed-participation equilibrium.

Proposition 3.5. *Under a targeted priority mechanism* $(\overset{*}{q})$*, a nontrivial fraction of eligible candidates would participate in equilibrium, i.e., there exists a mixed-participation equilibrium* $\gamma^e(\overset{*}{q}) \in (0,1)$*, if and only if the following two inequalities hold*

$$\frac{\alpha}{d_D} + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})} \lim_{\gamma\to 0^+} \int_{\overset{*}{q}}^{\bar{q}} \Big(T_D(q) - \frac{\alpha}{d_D}\Big) p_N^{\gamma}(q)dq \leq \beta T_D(\overset{*}{q}), \tag{3.4a}$$

$$\frac{\alpha}{d_D} + \frac{\beta}{\lambda_D\Big(\bar{q}-\underline{q}\Big)} \int_{q_P^{e,}\overset{*}{q}}^{\overset{*}{q}} \Big(T_D(q) - \frac{\alpha}{d_D}\Big) dq \leq \frac{\alpha}{d_D} + \frac{\beta}{\lambda_D\Big(\bar{q}-\underline{q}\Big)} \lim_{\gamma\to 1^-} \frac{\int_{\overset{*}{q}}^{\bar{q}} \Big(T_D(q) - \frac{\alpha}{d_D}\Big) p_N^{\gamma}(q)dq}{1-\gamma}. \tag{3.4b}$$

Observe that the conditions given in Proposition 3.5 are mutually exclusive and collectively exhaustive with those in Propositions 3.3 and 3.4. Therefore, the necessity of (3.4a) and (3.4b) follows from the sufficiency of the conditions provided in Propositions 3.3 and 3.4, implying that if any of them is violated, then one of the fixed equilibrium would be adopted. And the sufficiency of these conditions follows the necessity of the conditions in Propositions 3.3 and 3.4 along with Theorem 3.1 and Corollary 3.2, proving the existence of a mixed-participation equilibrium when no other type of equilibrium exists.

3.4 Optimal Design of Targeted Priority Mechanisms

When designing targeted mechanisms, the planner utilizes the target threshold $\overset{*}{q}$ to set the pool of organs on which the participating candidates would receive priority at the cost of limiting their interest to these organs. Accordingly, by varying $\overset{*}{q}$, the planner can make

the mechanism more (or less) attractive for those who are eligible to participate. On the two extremes, setting the target threshold too high (e.g., giving priority on all organs with $\overset{*}{q} = \overline{q}$) would make the program too attractive, incentivizing all disadvantaged patients to participate; whereas, setting it too low (e.g., asking participants to limit their interest to the lowest quality organs $\overset{*}{q} = \underline{q}$) would make the program too unappealing to participate at all. While implementing a completely unappealing mechanism would make the efforts of the planner futile, an overly attractive mechanism bears the risks of harming the social welfare through a potential mismatch between the high quality organs and disadvantaged patients and creating an overly selective participant class by assigning them too much priority. Therefore, characterizing the exact impact of the target threshold $\overset{*}{q}$ on the participation behavior adopted and the acceptance thresholds set in equilibrium is of critical importance for the effective and efficient design of targeted mechanisms.

Theorem 3.6 establishes the impact of the target threshold $\overset{*}{q}$ on the type of participation equilibria adopted by eligible candidates and shows that the feasible set of target thresholds can be separated into three disjoint intervals, each of which corresponds to a unique type of participation equilibrium.

Theorem 3.6. *Two thresholds $\overset{*}{q}_l$ and $\overset{*}{q}_u$, where $\underline{q} < \overset{*}{q}_l < \overset{*}{q}_u < \overline{q}$, determine how the target threshold $\overset{*}{q}$ regulates the type of the participation equilibrium adopted by eligible patients:*

(a) *Any $\overset{*}{q} \leq \overset{*}{q}_l$ is a no-participation target threshold, i.e., $\gamma^e(\overset{*}{q}) = 0$.*

(b) *Any $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$ is a mixed-participation target threshold, i.e., $\gamma^e(\overset{*}{q}) \in (0, 1)$.*

(c) *Any $\overset{*}{q} \geq \overset{*}{q}_u$ is a full-participation target threshold, i.e., $\gamma^e(\overset{*}{q}) = 1$.*

(d) *The two thresholds $\overset{*}{q}_l$ and $\overset{*}{q}_u$ are the respective unique solutions for the inequalities* (3.2) *and* (3.3) *to be satisfied as an equality.*

The results of Theorem 3.6 are intuitive in that the target threshold $\overset{*}{q}$ directly regulates the concessions given to the eligible candidates to attract them to the program. An important implication of Theorem 3.6 is that by selecting a target threshold $\overset{*}{q} > \overset{*}{q}_l$, the planner can guarantee some participation in the program. Further, the planner guarantees the full participation at $\overset{*}{q} = \overset{*}{q}_u$, and can therefore avoid *adversely generous* targeted mechanisms by keeping $\overset{*}{q} \leq \overset{*}{q}_u$. Finally, Theorem 3.6 provides the exact characterization of these two thresholds $\overset{*}{q}_l$ and $\overset{*}{q}_u$, which will later be used to establish the impact of several system parameters.

For the remainder of the structural analysis in the chapter, we make the following assumption:

Assumption 5. *Under a targeted priority mechanism* $(\overset{*}{q})$*, for any* $q \in (\overset{*}{q}, \overline{q}]$*,* $\frac{p_N^{\gamma}(q)}{1-\gamma}$ *is non-decreasing in* $\gamma \in (0,1)$*.*

Assumption 5 implies that under a targeted priority mechanism $(\overset{*}{q})$, the ratio of the non-participating disadvantaged patients' share of non-target organs, i.e., $p_N^{\gamma}(q)$ for $q \in (\overset{*}{q}, \overline{q}]$, to the rate of these patients, i.e., $(1-\gamma)$, does not decrease as more disadvantaged patients participate in the program. Although the non-participating disadvantaged patients' share of non-target organs would be expected to decline with increasing participation in the program, a decreasing ratio $\frac{p_N^{\gamma}(q)}{1-\gamma}$ in $\gamma \in (0,1)$ would imply that type N is penalized for their non-participating behavior over their share of organs that are not part of the program. Such penalties, however, do not align with the voluntary nature of the targeted mechanisms, and accordingly, they are prohibited in the rest of the structural analysis.

As shown in Theorem 3.6, any $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$ would result in a mixed-participation equilibrium. Proposition 3.7 shows that this mixed-participation equilibrium is unique for any given $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$, and establishes the impact of the target threshold $\overset{*}{q}$ on the equilibrium participation

$\gamma^e(\overset{*}{q}) \in (0,1)$.

Proposition 3.7. *For any targeted priority mechanism with $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$, the equilibrium participation rate of disadvantaged patients $\gamma^e(\overset{*}{q})$ is unique and increasing in the target threshold $\overset{*}{q}$.*

Together with Theorems 3.1 and 3.6, Proposition 3.7 illustrates that under any targeted priority mechanism $(\overset{*}{q})$, the equilibrium participation rate and acceptance thresholds of patients are uniquely characterized and the planner can directly influence the equilibrium participation rate of the program by controlling the target threshold.

3.4.1 Impact of $\overset{*}{q}$ on Equilibrium Behavior of Disadvantaged Patients

We are also interested in characterizing the impact of $\overset{*}{q}$ on the equilibrium acceptance thresholds of patients, which directly impact the utilization of available organs as well as the overall social welfare. Proposition 3.8 establishes the impact of $\overset{*}{q}$ on the organ acceptance behavior of disadvantaged patients in equilibrium.

Proposition 3.8. *For any targeted priority mechanism $(\overset{*}{q})$, the impact of the target threshold on the equilibrium acceptance behavior of disadvantaged patients is characterized as follows:*

(a) *$\overset{*}{q} \in [\underline{q}, \overset{*}{q}_l]$, then the selection of target threshold has no impact on the equilibrium acceptance thresholds of disadvantaged patients.*

(b) *$\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$, the impact of the target threshold $\overset{*}{q}$ on the equilibrium acceptance thresholds of disadvantaged patients $q_D^{e,\overset{*}{q}} := q_P^{e,\overset{*}{q}} = q_N^{e,\overset{*}{q}}$ is determined by the sign of the following expression*

$$(1-\gamma^e(\mathring{q}))\int_{\mathring{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))\cdot\frac{\partial}{\partial\gamma}\left(\frac{p_N^{\gamma}(q)}{1-\gamma}\right)\Big|_{\gamma=\gamma^e(\mathring{q})}dq-\frac{p_N^{\gamma^e(\mathring{q})}(\mathring{q})\int_{\mathring{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_N^{\gamma^e(\mathring{q})}(q)dq}{1-\gamma^e(\mathring{q})}. \tag{3.5}$$

The equilibrium acceptance threshold $q_D^{e,\mathring{q}}$ *decreases (increases) in the target threshold* $\mathring{q}$ *when the expression given in* (3.5) *is negative (positive).*

(c) $\mathring{q}\in[\mathring{q}_u,\bar{q}]$*, the equilibrium acceptance threshold of disadvantaged patients* $q_D^{e,\mathring{q}}:=q_P^{e,\mathring{q}}$ *increases in the target threshold* $\mathring{q}$*.*

As established in Theorem 3.6, when the target threshold $\mathring{q}$ is below the lower threshold $\mathring{q}_l$, no eligible candidate would participate in the program, and therefore, in this region, $\mathring{q}$ has no impact on the equilibrium acceptance behavior. When, on the other hand, the target threshold $\mathring{q}$ is above the upper threshold $\mathring{q}_u$, all eligible candidates would participate in the program, and increasing $\mathring{q}$ in this region would increase the pool of organs over which the participating patients are prioritized while their participation rate stays the same. Therefore, for any $\mathring{q}\geq\mathring{q}_u$, increasing the target threshold would make the disadvantaged patients more selective and result in an increase in their equilibrium acceptance threshold.

In the interval $(\mathring{q}_l,\mathring{q}_u)$, however, the impact of the target threshold on the equilibrium acceptance behavior of disadvantaged patients is less clear. In this region, increasing $\mathring{q}$ creates a trade-off: on one hand, it increases the pool of organs over which the participating patients are prioritized, on the other hand, it stiffens the competition among participating candidates by increasing the participation (see Proposition 3.7). Proposition 3.8 establishes that the impact is determined by the sign of the expression in (3.5), which can be best understood by understanding each term. As shown in the proof of Proposition 3.7, the first term represents the marginal increment in the expected utility of non-participating patients when more participate in the program. The second term corresponds to the decrement of the expected

utility of non-participants due to the marginal loss of the pool of organs. It also represents an approximation of the average expected utility that participants forgo. Therefore, the expression in (3.5) corresponds to the difference between the increment of the expected utility of disadvantaged patients when more of them participate and the average expected utility that participating ones forgo. As established in Theorem 3.1, under a mixed-participation equilibrium, all disadvantaged patients share the same expected utility whether they participate or not. Therefore, Proposition 3.8 implies that when the expression in (3.5) is positive (negative), the disadvantaged patients become more (less) selective in their offer acceptance behavior as the target threshold increases.

3.4.2 Impact of $\overset{*}{q}$ on Equilibrium Behavior of Non-targeted Patients

We also identify the influence of targeted priority mechanisms on the equilibrium offer acceptance behavior of non-targeted patients. This aspect is of particular importance since non-targeted patients are not included in the program. Therefore, any mechanism that inadvertently results in these patients experiencing a decline in their overall welfare or accepting lower quality organs could be perceived as inequitable and might pose significant challenges for practical implementation. Proposition 3.9 establishes the influence of the target threshold $\overset{*}{q}$ on the equilibrium acceptance threshold $q_O^{e,\overset{*}{q}}$ for non-targeted patients.

Proposition 3.9. *For any targeted priority mechanism with target threshold* $(\overset{*}{q})$*:*

(a) *When* $\overset{*}{q} \in [\underline{q}, \overset{*}{q}_l]$*, the selection of the target threshold* $\overset{*}{q}$ *has no impact on the equilibrium acceptance threshold of non-targeted patients* $q_O^{e,\overset{*}{q}}$*.*

(b) *When* $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$*,*

(b-i) *If* $\overset{*}{q}_l < \overset{*}{q} < q_O^{e,\overset{*}{q}}$*, then the equilibrium acceptance threshold of non-targeted patients* $q_O^{e,\overset{*}{q}}$ *is increasing in the target threshold* $\overset{*}{q}$*.*

(b-ii) *If* $q_O^{e,\overset{*}{q}} \leq \overset{*}{q} < \overset{*}{q}_u$*, and* $p_N^{\gamma}(q)$ *satisfies* $\frac{\partial^2 p_N^{\gamma}(q)}{\partial \gamma^2} \geq 0$ *for any* q*, then* $q_O^{e,\overset{*}{q}}$ *is unimodal in* $\overset{*}{q}$*, attaining its mode at* $q_O^{\max}$ *which is the unique solution of* $\frac{d\, q_O^{e,\overset{*}{q}}}{d\gamma} = 0$*.*

(c) *When* $\overset{*}{q} \in [\overset{*}{q}_u, \overline{q}]$*, the equilibrium acceptance threshold of non-targeted patients* $q_O^{e,\overset{*}{q}}$ *is non-increasing in the target threshold* $\overset{*}{q}$*.*

Thus, under a no-participation equilibrium ($\overset{*}{q} \leq \overset{*}{q}_l$), increasing the target threshold has no impact on the equilibrium offer acceptance behavior of non-targeted patients. Conversely, in a full-participation equilibrium ($\overset{*}{q} \geq \overset{*}{q}_u$), increasing the target threshold expands the pool of organs prioritized for disadvantaged patients, consequently reducing the non-targeted ones' access to organs and rendering them less selective.

The dynamics change under a mixed-participation equilibrium, where the target threshold's impact is non-monotonic. When the target threshold is lower than the non-targeted patients' equilibrium acceptance threshold ($\overset{*}{q} < q_O^{e,\overset{*}{q}}$), an increase in the target threshold prioritizes disadvantaged patients for organs less desirable to non-targeted patients, while also boosting disadvantaged participation (see Proposition 3.7). This reduces competition for the non-targeted group, making them more selective. However, when the target threshold falls within the range of $[q_O^{e,\overset{*}{q}}, \overset{*}{q}_u)$, it presents a trade-off for non-targeted patients. On one hand, a higher target threshold elevates disadvantaged patients' participation rate, improving non-targeted patients' access to non-program organs. On the other hand, it leads to participating patients having priority over a broader spectrum of organs, some of which are also desirable to non-targeted patients. Proposition 3.9 establishes that initially, as the target threshold approaches and surpasses $q_O^{e,\overset{*}{q}}$, the increased access to non-program organs predominates, leading to a rise in the non-targeted patients' equilibrium acceptance threshold. However, as

the target threshold continues to rise, the impact of prioritization becomes more significant, making non-targeted patients less selective in their offer acceptance. The condition on $p_N^{\gamma}(q)$ in Proposition 3.9(b-ii) indicates that the probability of allocating an organ of quality q decreases in a convex manner with the participation rate γ, which implies that an increase in disadvantaged patients' participation leads to a diminishing marginal decrease in the proportion of organs assigned to non-participants.

3.4.3 Numerical Illustrations of the Impact of $\overset{*}{q}$ on Equilibrium Behavior

We next illustrate our analytical findings over a numerical example, details of which are provided in Appendix A.3. Figure 3.2 displays both the equilibrium participation rate (left figure) and acceptance thresholds (right figure) of type O (dashed line) and type D (solid line) under various target thresholds of the program.

It shows that in region I (corresponding to $\overset{*}{q} \leq \overset{*}{q}_l$), no type D participates in the program, and thus the target threshold does not affect the equilibrium acceptance threshold of either types. However, as the target threshold increases beyond $\overset{*}{q}_l$ in region II, more disadvantaged

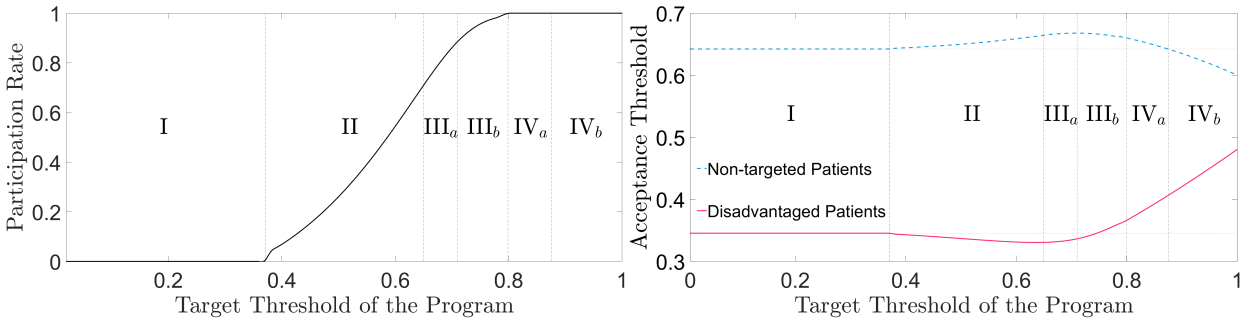


Figure 3.2: Acceptance Thresholds of Patients and Participation Rate into the Program vs. Target Threshold

patients participate in the program. In this region, participating type D gets prioritized access to organs that type O is less interested in, leaving type O with better access to relatively high-quality organs, thereby increasing type O's acceptance threshold. In region II, the stiffened competition in the program due to the flux of type D who are interested in enjoying the prioritized access dominates the increased pool of organs over which the participating patients are prioritized, and thus increasing target threshold decreases the acceptance threshold of type D.

When the target threshold further moves beyond region II to region III (III_a and III_b), its increase expands the already elevated participation rate while providing priority over relatively higher quality organs, thereby leading to an increase in the equilibrium acceptance threshold of type D. In region III_a, similar to region II, increasing the target threshold incentivizes participation and gives type O with better access to relatively high-quality organs, increasing their acceptance threshold. In region III_b, however, increasing the target threshold attracts a diminishing participation in the program, while also narrowing the pool of high quality organ offers to type O, making them less selective in their offer acceptance decisions.

When the target threshold increases beyond $\hat{q}_u^*$ entering region IV (IV_a and IV_b), all disadvantaged patients participate in the program. In region IV, increasing the target threshold prioritizes type D over organs also of high interest to type O, without increasing participation, thus making type D more selective, while making type O less selective in offer acceptance. Still, in regions II, III and IV_a, type O utilizes better quality organs in equilibrium compared to those before the implementation of the program. In IV_b, the utility of type O compared to the status quo. Still, in regions II, III, and IV_a, type O receives organs of better quality in equilibrium than those allocated before the program. The exception is region IV_b, where type O experiences a decline in organ quality relative to the existing baseline.

To summarize, regions II and III offer viable design options for the planner, while regions I and IV should generally be bypassed to avoid ineffective strategies or unwarranted prioritization. However, as explored in Section 3.5.2, certain exceptions exist where elevating the target threshold in region IV_a could advantageously serve disadvantaged patients without negatively impacting the utility of non-targeted ones.

3.5 Impact of the Targeted Priority Mechanisms

In this section, we conduct a comprehensive evaluation of how targeted priority mechanisms affect social welfare. It is evident that a no-participation equilibrium leaves social welfare unchanged, yet the effects of all-participation and mixed-participation equilibria on social welfare are not immediately apparent. The social planner's objective is to enhance overall utility through carefully designed incentive mechanisms. The effectiveness of these mechanisms is measured by a social welfare function, detailed as follows:

$$\begin{aligned} s(q_P^{e,\overset{*}{q}}, q_N^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}; \gamma^e(\overset{*}{q})) &= \sum_{i\in\{P,N,O\}} \frac{\lambda_i}{\lambda} \cdot U_i(q_i^{e,\overset{*}{q}}) \\ &= \frac{\gamma^e(\overset{*}{q}) \cdot \lambda_D}{\lambda} U_P(q_P^{e,\overset{*}{q}}) + \frac{(1-\gamma^e(\gamma^e())) \cdot \lambda_D}{\lambda} U_N(q_N^{e,\overset{*}{q}}) + \frac{\gamma^e(\overset{*}{q})\lambda_O}{\lambda} U_O(q_O^{e,\overset{*}{q}}). \end{aligned} \tag{3.6}$$

We first analyze the ability of targeted priority mechanisms to distinctly separate organ allocation among different patient classes (§3.5.1). This examination leads to the characterization of how these mechanisms influence social welfare, providing key insights into their optimal design (§3.5.2).

3.5.1 Class-Separating Allocations

Dynamic matching of scarce heterogeneous resources presents key challenges, such as determining how to prioritize "premium" customers to maximize social benefits without adversely affecting other groups, and encouraging adherence to socially optimal matching among self-interested customers without enforcing compulsory acceptance (Haviv and Oz 2018). A well-designed targeted priority mechanism holds the potential to distinctly separate organ allocation between patient types. Such mechanisms allow the planner to facilitate more efficient organ allocation by directing *high-quality organs* to *those who can benefit most*, while still ensuring *voluntary* participation.

In this context, we introduce the concept of *class-separating allocations*, which involves matching distinct patient groups to separate quality pools of organs, and we examine their influence on social welfare. Ideally, a class-separating allocation that benefits all patient types would be the most desirable outcome, as it would enhance social welfare without disadvantaging any group. Moreover, because such allocations are beneficial for all patient types, they have the potential to be naturally adopted in what we term *class-separating equilibrium*.

To comprehensively understand the conditions under which a separating allocation can improve overall welfare without adversely affecting either group, we initially examine the separating allocations within a broader context beyond our current setting. Subsequently, in the following section, we investigate whether a targeted priority mechanism can feasibly implement a separating equilibrium. Consider two classes of customers (patients in our setting): O and D, and a collection of goods (organs) assigned with specific types (quality), denoted as $q \in [\underline{q}, \overline{q}]$, which are allocated in status quo to the classes D and O with probabilities $p_D(q)$ and $p_O(q)$, respectively, where $p_D(q) + p_O(q) = 1$. For any good of type q, the respective

positive valuations (net benefit from transplantation) of class D and O are given by $\overline{T}_D(q)$ and $\overline{T}_O(q)$, both of which are increasing in q. We define a class-separating allocation with threshold q^T such that the goods of type $q \in [\underline{q}, q^T]$ are exclusively allocated to class D and those of $q \in \left[q^T, \bar{q}\right]$ are allocated to class O. Lemma 3.10 provides insights into the impact of the class-separating allocations on the utility of each class.

Lemma 3.10. *The class-separating allocations satisfy the following:*

(a) *There exists a unique threshold q_O^T such that the utility of class O remains unchanged, gets better or gets worse post the introduction of a class-separating allocation with $q^T = q_O^T$, $q^T < q_O^T$, or $q^T > q_O^T$, respectively.*

(b) *There exists a unique threshold q_D^T such that the utility of class D remains unchanged, gets better or gets worse post the introduction of a class-separating allocation with $q^T = q_D^T$, $q^T > q_D^T$, or $q^T < q_D^T$, respectively.*

Lemma 3.10 establishes the unique existence of thresholds q_O^T and q_D^T, the relative positioning of which dictates whether the class O and D are better or worse off under a class-separating allocation with q^T. It presents two distinct scenarios that may occur under a separating allocation. In the scenario where $q_O^T \leq q_D^T$, improving the overall utility without negatively affecting the utility of either class proves unachievable. On the other hand, when $q_O^T > q_D^T$, both classes are better off under any separating allocation with $q^T \in (q_D^T, q_O^T)$ compared to the status quo.

Theorem 3.11 establishes that if class O can utilize the same quality goods (organs) more effectively than class D, with the utility ratio between two classes being non-decreasing in quality, then there exists a class-separating allocation that increases overall utility by improving the utility of both classes compared to the status quo.

Theorem 3.11. *If* $\frac{\overline{T}_O(q)}{\overline{T}_D(q)}$ *is non-constant non-decreasing in* $q \in \left[\underline{q}, \bar{q}\right]$*, then we have* $q_O^T > q_D^T$ *and consequently any class-separating allocation with* $q^T \in (q_D^T, q_O^T)$ *increases the overall utility by improving the utility of both classes.*

In the context of organ transplantation, $\overline{T}_i(q) := T_i(q) - \frac{\alpha}{\beta d_i}$ reflects type i's net benefit of receiving a transplant of quality q as opposed to dying on the waitlist. Therefore, the condition that the utility ratio $\frac{\overline{T}_O(q)}{\overline{T}_D(q)}$ is non-decreasing in organ quality q is expected to hold, because the better the transplanted organ, the greater the difference between the expected net benefit from transplantation for non-targeted (e.g., younger) and disadvantaged (e.g., older) patients (Marconi et al. 2013, Ram et al. 2019).

Remark 3.12. In scenarios where customer classes have idiosyncratic preferences over different goods (e.g., non-monotone $\overline{T}_i(q)$), there may exist no class-separating allocation that improves the utility of both classes. Furthermore, in such scenarios, certain class-separating allocations could potentially lead to a decrease in utility for both customer classes.

3.5.2 Impact of the Targeted Priority Mechanisms on Social Welfare

In this section, we study the impact of targeted priority mechanisms on overall social welfare. To do this, we first define a few special equilibrium points. Let $q_D^{e_0}$ and $q_O^{e_0}$ denote the equilibrium offer acceptance thresholds of types D and O, respectively, in the status quo, i.e., $q_i^{e_0}$ satisfies the following:

$$\beta T_i(q_i^{e_0}) = \alpha \Delta_i(q_i^{e_0}) + \beta \pi_i(q_A^{e_0}) E[T_i(q) \mid q \geqslant q_i^{e_0}]. \tag{3.7}$$

Observe that $q_D^{e_0}$ and $q_O^{e_0}$ also correspond to the equilibrium acceptance thresholds when the program is introduced with a target threshold $\overset{*}{q} \leq \overset{*}{q}_l$ so that no patients participate in the program (see Theorem 3.6). Further, we define $q_O^{\bar{e}}$ denoting the equilibrium acceptance threshold of type O in the hypothetical scenario where they have absolute priority in the allocation of any organs, i.e., $q_O^{\bar{e}}$ satisfies the following:

$$\beta T_A(q_O^{\bar{e}}) = \alpha \frac{1 - \frac{\bar{q} - q_O^{\bar{e}}}{\lambda_A(\bar{q} - \underline{q})}}{d_A} + \frac{\beta}{\lambda_A(\bar{q} - \underline{q})} \int_{q_O^{\bar{e}}}^{\bar{q}} T_A(q) dq. \tag{3.8}$$

Theorem 3.13 establishes the necessary and sufficient conditions for the existence of targeted priority mechanisms that guarantee a class-separating equilibrium, shows that there always exists a targeted priority mechanism that improves social welfare, and provides the welfare-maximizing design for a set of systems.

Theorem 3.13. *The impact of the targeted priority mechanisms on social welfare is characterized as follows:*

(a) *For any system such that $\overset{*}{q}_u \leq q_O^{\bar{e}}$, the targeted priority mechanism with $\overset{*}{q} = q_O^{\bar{e}}$ leads to a class-separating equilibrium. Further, this mechanism maximizes social welfare if $\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ is non-decreasing in $q \in [q_O^{e_0}, \overline{q}]$, and the following inequality holds for all $\overset{*}{q}$:*

$$\overline{q} - q_O^{\bar{e}} \geq \int_{q_O^{e}, \overset{*}{q}}^{\overline{q}} \left(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\right) dq. \tag{3.9}$$

(b) *For any system such that $\overset{*}{q}_u > q_O^{\bar{e}}$, no targeted priority mechanism can guarantee a class-separating equilibrium. However, if $\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ for $q \in [q_O^{e_0}, \overline{q}]$, then there exists a targeted priority mechanism that improves social welfare compared to the status quo.*

Theorem 3.13 presents two scenarios for organ allocation systems, outlined by the relation-

ship between $\overset{*}{q}_u$, the minimum threshold required for full participation of type D patients, and $q_O^{\overline{e}}$, the equilibrium acceptance threshold for type O patients when given absolute priority over all organs. In any system where $\overset{*}{q}_u \leq q_O^{\overline{e}}$, achieving full participation from type D patients is feasible by allocating them organs of comparatively lower quality, while ensuring type O patients have exclusive access to higher quality organs; enabling the planner to establish a class-separating equilibrium. Further, in such cases, Theorem 3.13 establishes that setting $\overset{*}{q} = q_O^{\overline{e}}$ optimizes social welfare under some conditions. The optimality of $\overset{*}{q} = q_O^{\overline{e}}$ is intuitive because setting $\overset{*}{q} < q_O^{\overline{e}}$ risks underutilization of organs with quality $q \in (\overset{*}{q}, q_O^{\overline{e}})$, while $\overset{*}{q} > q_O^{\overline{e}}$ unnecessarily benefits type D, who would already fully participate at a lower threshold, leading to an inefficient allocation of high-quality organs. Conversely, in systems where $\overset{*}{q}_u > q_O^{\overline{e}}$, achieving full participation becomes challenging. Type D is unlikely to give up their access to higher quality organs unless they receive priority for organs that also significantly interest type O, and thus, the ability to establish a class-separating equilibrium is compromised. Despite this, Theorem 3.13 identifies a targeted priority mechanism that improves social welfare beyond the current baseline in this scenario.

Recall that $\overline{T}_i(q) := T_i(q) - \frac{\alpha}{\beta d_i}$ reflects type i's net benefit of receiving a transplant of quality q as opposed to the removal with no transplant. Therefore, the condition $\overline{T}_O(q)/\overline{T}_D(q) \geq 1$ for $q \in [q_O^{e_0}, \overline{q}]$ outlined in Theorem 3.13(a) and (b) implies that type O patients experience a greater net benefit than type D from transplants of any organ that they were willing to accept before the program's implementation (i.e., $q_O^{e_0}$). Additionally, Theorem 3.13(a) sets this ratio to be non-decreasing in q, implying that the better the organ, the greater the difference between the benefit for non-targeted and disadvantaged patients (similar to Theorem 3.11). Further, observe that the left-hand side of condition (3.9) given in Theorem 3.13(a) corresponds to $\int_{q_O^{\overline{e}}}^{\overline{q}} 1 \, dq$, which is the effective arrival rate of organs to type O in the hypothetical scenario where they have absolute priority in the allocation of all organs.

Therefore, condition (3.9) implies that the effective arrival rate of organs to type O under any targeted priority mechanism should be inferior to that of the absolute priority scenario.

A potential concern with the practical application of targeted priority mechanisms is the possibility of improving overall social welfare at the expense of type O's utility. This imbalance could pose significant challenges to implementation, as non-targeted patients have no autonomy over the program's initiation. Corollary 3.14 establishes the existence of easy-to-implement targeted priority mechanisms that improve overall welfare without sacrificing the utility of non-targeted patients.

Corollary 3.14. *There exists a targeted priority mechanism such that* $\overset{*}{q} = q_O^{e,\overset{*}{q}}$*, which satisfies the following:*

(a) *If* $q_D^{e,\overset{*}{q}} > q_D^{e_0}$*, then the mechanism improves the social welfare by increasing the utility of both patient groups.*

(b) *If* $q_D^{e,\overset{*}{q}} \leq q_D^{e_0}$*, then the mechanism increases the utilization of available organs. Further, if* $\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ *is non-decreasing in* $q \in [q_O^{e_0}, \overline{q}]$*, and the offer probabilities satisfy*

$$\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \left(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\right) dq \geq \int_{q_O^{e_0}}^{\bar{q}} p_O(q)\, dq, \tag{3.10}$$

then it increases the overall social welfare by providing non-targeted patients improved access to higher quality organs and increasing their utility.

To address concerns about compromising the utility of type O, the planner may consider granting type D priority only over organs that type O shows no interest in equilibrium. However, the range of organs that interest type O (defined by $q_O^{e,\overset{*}{q}}$) is directly affected by the target threshold. Corollary 3.14 first establishes the unique existence of a fixed point, $q_O^{e,\overset{*}{q}} = \overset{*}{q}$, providing a foundation for the desired design approach. By aligning $\overset{*}{q}$

with $q_O^{e,\overset{*}{q}}$, the planner can strategically balance type D's prioritized access to certain organs to encourage their participation, thereby channeling more higher-quality organs to type O, while maintaining type O's disinterest in the organs designated for the program.

Corollary 3.14 shows that if type D becomes more selective in equilibrium under the given targeted priority mechanism (given by $q_D^{e,\overset{*}{q}} > q_D^{e_0}$), then the mechanism improves the overall welfare by improving the utility of both patient groups. On the other hand, if they become less selective ($q_D^{e,\overset{*}{q}} < q_D^{e_0}$), then the program increases the utilization of organs. Further, under this scenario, if type O derives an increasingly higher net benefit from transplantation compared to type D (given by the non-decreasing $\overline{T}_O(q)/\overline{T}_D(q)$, similar to Theorems 3.11 and 3.13(a)); and the adjusted offer probabilities to type O (i.e., $1-p_N^{\gamma}(q)$) is designed such that the non-targeted patients are not penalized by being allocated fewer organs under the mechanism (given by the condition (3.10)), then the program increases the overall welfare by providing non-targeted patients improved access to higher quality organs and increasing their utility, leading to an easy-to-implement design in practice.

3.6 Conclusion

This chapter develops and analyzes a class of targeted priority mechanisms designed to improve the allocation efficiency and equity in organ transplantation systems, focusing on settings where disadvantaged patient groups are defined exogenously. Motivated by persistent disparities in organ access, particularly for older patients and those at centers with historically long wait times, we propose an incentive-compatible framework that grants strict priority to disadvantaged patients over a planner-defined set of organs, provided they voluntarily commit to accepting only organs from this set. Through this mechanism, we aim to increase utilization of hard-to-place organs, reduce wait times for under-served patients,

and improve overall system welfare without mandating participation or imposing harm on non-targeted groups.

We formally characterize the equilibrium participation decisions under these targeted mechanisms and derive analytical conditions under which voluntary alignment with the planner's matching preferences emerges. The results highlight the existence of distinct participation thresholds that segment patients based on their willingness to accept lower-quality organs in exchange for priority access. We show that under appropriate threshold settings, disadvantaged patients who are systematically underserved under the baseline allocation can achieve higher utility through program participation. Moreover, our findings indicate that such mechanisms can be designed to preserve the utility of non-targeted patients, making them appealing from a policy standpoint.

This chapter contributes to the broader literature on dynamic mechanism design in healthcare and strategic queueing systems (Afeche 2013, Cui et al. 2019, Debo and Veeraraghavan 2014, Tunç et al. 2022, Wang et al. 2019, Yang and Debo 2019, Yang et al. 2017), by offering an implementable and fairness-aware extension of classical prioritization schemes. Unlike time-trading or monetary-pricing schemes, the proposed targeted priority mechanism uses eligibility-based strict priority to incentivize behavioral alignment without resorting to coercion or economic exclusion. The mechanism also extends lessons from real-world programs such as the Eurotransplant Senior Program (ESP), offering a broader design framework that generalizes beyond age-based grouping and accommodates system-level constraints.

While this chapter provides foundational analytical insights, it operates under the assumption of exogenously specified disadvantaged groups. In practice, group definitions, such as age, comorbidity profiles, or geographic factors, are often subject to policy design. This motivates the generalization explored in the next chapter, where we endogenize group composition and explore optimal eligibility criteria alongside threshold design. Furthermore,

although our results offer tractable guidance, future empirical work and simulation-based validation will be essential for assessing robustness under real-world constraints, including multi-organ offers, stochastic organ arrivals, and patient heterogeneity beyond two types. The next chapter also builds on this framework to incorporate more flexible group definitions and further enrich the planner's design toolkit.

Chapter 4

Generalized Targeted Priority Mechanisms

4.1 Introduction

While targeted priority mechanisms offer promising avenues to reduce organ nonuse and improve equity, their performance depends critically on how disadvantaged patient groups are defined and how the priority thresholds are calibrated. In the previous chapter, we examined a setting where patient groups eligible for targeted programs were exogenously specified, such as by age or clinical risk, and we focused on optimizing targeted organ thresholds to ensure voluntary participation and improve system-wide welfare. However, in practical implementation, the choice of whom to prioritize is often flexible and contestable. This chapter builds upon that foundation by introducing a more general framework in which both the composition of the targeted group and the associated program thresholds are *jointly optimized*, allowing planners to fine-tune patients' eligibility criteria and organs' acceptance incentives for maximum efficiency and fairness.

This problem is highly relevant in healthcare resource allocation more broadly, where targeted programs frequently define eligibility via a fixed cutoff, e.g., age thresholds in colorectal cancer screening or income brackets for Medicaid eligibility. Such discretization, while administratively simple, may fail to account for the diversity within and across groups. In

transplantation specifically, programs like the Eurotransplant Senior Program (ESP) have fixed eligibility by donor and recipient age (65+), but recent discussions have called for more adaptive and data-informed criteria to maximize the marginal benefit of scarce organs (Bahde et al. 2014, Branger and Samuel 2016). Motivated by this, we explore both discrete and continuous definitions of patient groups. In the former, we examine how to optimally select an age threshold that delineates eligibility; in the latter, we allow the group to evolve as a function of patient characteristics, such as urgency or projected post-transplant benefit.

From an analytical standpoint, we build on the equilibrium framework introduced in the prior chapter by characterizing participation thresholds under endogenous group formation. We first study the conditions under which full- and no-participation thresholds exist, and how these thresholds vary with structural parameters like patient arrival and mortality rates. These insights are crucial, as the arrival rate of disadvantaged patients (λ_D) and their death rate (d_D) both shape the urgency and utility of participating in targeted programs. Related research in strategic queueing systems and healthcare operations underscores the importance of accounting for both *individual choice dynamics* and *group externalities* when designing priority mechanisms (Afeche 2013, Debo and Veeraraghavan 2014, Tunç et al. 2022, Yang and Debo 2019). Our results echo this: when disadvantaged patients become more numerous or face elevated mortality, the effectiveness of targeted incentives, and the equilibrium behavior they induce, can shift significantly.

The broader relevance of this work extends to the design of adaptive eligibility rules in healthcare interventions. Literature in behavioral health and incentive-driven engagement shows that static targeting rules often fail to maintain efficiency over time, especially when patient demographics evolve or respond to program structure (Giles and et al. 2023, Prendergast and et al. 2006, Volpp and et al. 2008). Our framework allows planners to anticipate these shifts by embedding group definition and participation behavior into a unified optimization

problem. Ultimately, this chapter provides theoretical foundations for adaptive program design, where *"who is targeted"* and *"how they are incentivized"* are treated as joint levers for improving outcomes in constrained healthcare systems.

4.2 Model Formulation

Building on the foundation laid in Chapter 3, this chapter presents a generalized model of targeted priority mechanisms in organ allocation systems. The prior chapter examined a setting in which patient groups eligible for targeted priority programs were exogenously specified and the policy design focused on optimizing organ quality thresholds to incentivize voluntary participation among disadvantaged patients without adversely affecting the outcomes of others. While this approach offered valuable insights into participation dynamics and utility trade-offs, it left open the question of how eligibility criteria and thresholds might be *jointly optimized* to maximize system-wide efficiency and equity.

In this chapter, we advance the modeling framework in two key directions. First, we endogenize the definition of the disadvantaged group by optimizing over possible age thresholds, maintaining intra-group homogeneity to preserve analytical tractability. Second, we further generalize the structure by allowing the group definition to vary continuously, thereby relaxing the assumption of within-group homogeneity. These generalizations enable the planner to identify eligibility criteria that not only promote program participation but also reflect the heterogeneity of the patient population and the quality distribution of available organs.

To ensure consistency with prior analysis and maintain interpretability, we retain the core structure of the model introduced in Chapter 3, with minor adjustments to notation and scope. As before, we formulate the system as a multiclass overloaded priority queue with reneging, approximating the transplant waiting list under conditions of persistent scarcity.

Recognizing persistent disparities in access to high-quality organs (Akan et al. 2012, Bertsimas et al. 2013), we continue to distinguish between two types of candidates: *disadvantaged* candidates (type D), who experience systematically lower access to favorable matches, and *non-targeted* candidates (type O), who benefit from preferential access under the current system. The aggregate arrival rate of patients is given by $\lambda = \lambda_D + \lambda_O$, where λ_D and λ_O denote the arrival rates of type D and O patients, respectively. Patients may exit the list either through transplantation or through non-transplant removal, such as death, with respective exponential clocks of rate d_D and d_O.

Organs arrive at a normalized rate of $\mu = 1$ and vary in post-transplant benefit, captured by a quality score $q \in [\underline{q}, \overline{q}]$, assumed to follow a uniform distribution for analytical convenience.[1] This modeling choice aligns with clinical data on the distribution of Kidney Donor Profile Index (KDPI) scores (Stewart et al. 2017b). We model allocation using a static randomized policy, whereby each organ of quality q is offered to type i with probability $p_i(q)$. Accordingly, the effective arrival rate of organs to type i candidates is given by $\mu_i = \int_{\underline{q}}^{\overline{q}} \frac{p_i(q)}{\overline{q}-\underline{q}}\, dq$. To represent realistic scarcity, we assume $\mu_i \leq \lambda_i$ for each $i \in \{D, O\}$.

Following prior literature (Alagoz et al. 2007, Howard 2002, Sandıkçı et al. 2013), we assume that patients make threshold-based acceptance decisions in consultation with their physicians, guided by the organ quality q. That is, a candidate accepts any offer exceeding their personal threshold and declines otherwise. This reflects clinical practice and is justified by empirical studies on patient decision-making under uncertainty. Each candidate's utility is derived from a combination of pre-transplant time and post-transplant benefit, adjusted by quality-of-life weights: α for time spent waiting and β for life after transplantation, with $\alpha < \beta$. Given an acceptance threshold q_i, the expected Quality-Adjusted Life Expectancy

[1] The uniform distribution assumption simplifies the exposition and does not restrict generality. All core results extend to more general quality distributions with density $f(q)$.

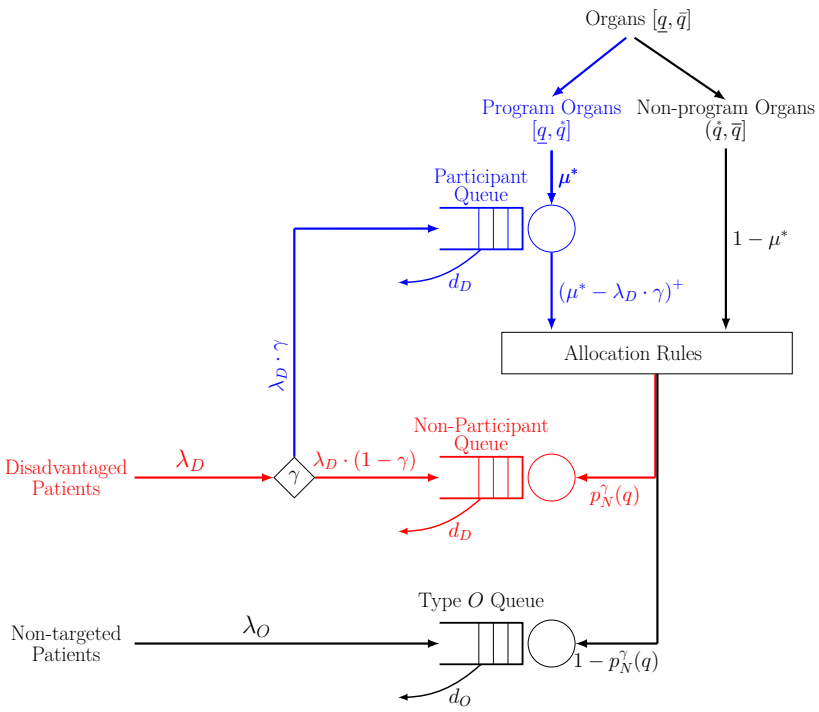


Figure 4.1: Schematic illustration of the generalized targeted priority mechanism.

(QALE) for a type i candidate is given by:

$$U_i(q_i) = \alpha \Delta_i(q_i) + \beta \pi_i(q_i) \cdot \mathbb{E}_i[T_i(q) \mid q \geq q_i],$$

where $\Delta_i(q_i)$ is the expected time on the waiting list and $\pi_i(q_i)$ is the probability of transplant under the threshold policy.

4.3 Generalized Targeted Priority Mechanisms

The core structure of the targeted priority mechanism remains consistent with Chapter 3: patients in the disadvantaged group (type D) may voluntarily participate in a program that offers them strict priority for a pre-defined set of organs, specifically those with quality $q \in [\underline{q}, \hat{q}]$, where $\hat{q}$ is set by the planner. In return, participating candidates restrict their

acceptance to this subset of organs and forgo access to higher-quality organs $q \in (\overset{*}{q}, \overline{q}]$. This mechanism is designed to improve utilization of hard-to-place organs while aligning patient incentives with system-level matching objectives.

Upon arrival, each type D candidate must decide whether to enroll in the targeted program. This choice is irreversible and results in one of two subgroups: P (participating) and N (non-participating). In contrast, type O candidates are not eligible to participate. Participants in P receive strict priority for all organs $q \leq \overset{*}{q}$ over candidates in both N and O, while non-participants and ineligible candidates are served under the residual allocation rule. These dynamics are illustrated in Figure 4.1.

Let $\gamma(\overset{*}{q})$ denote the endogenous fraction of type D candidates who opt into the program under threshold $\overset{*}{q}$. We assume that the allocation probability for non-participants, $p_N^{\gamma}(q)$, is decreasing in γ. This reflects a natural congestion externality: as more type D patients opt into the program, fewer remain outside it, and thus non-participants face increased competition for remaining organ offers. This structure ensures a meaningful trade-off in the participation decision and facilitates tractable equilibrium analysis.

To analyze system behavior at scale, we adopt a fluid limit approximation in which candidate and organ arrival rates are scaled proportionally by a factor $n \to \infty$, while death rates remain fixed. This approximation is consistent with real-world organ allocation systems, where arrival rates vastly exceed waitlist mortality rates (Kwong et al. 2023, Lentine et al. 2023). The fluid framework enables clean characterization of participation thresholds, allocation dynamics, and welfare effects under equilibrium behavior.

4.4 Discussion of Major Assumptions

To preserve analytical tractability while capturing essential features of organ allocation systems, we adopt a set of modeling assumptions that are standard in the theoretical literature on transplantation and dynamic matching under scarcity (Akan et al. 2012, Bertsimas et al. 2013, Su and Zenios 2006, Zenios 1999, Zenios et al. 2000). These assumptions are deliberately structured to reflect core patterns observed in real-world kidney allocation, while abstracting from institutional and clinical complexities that vary by region, organ type, and transplant center. In this chapter, we retain the primary assumptions introduced in Chapter 3, but extend their implications by endogenizing group definitions and jointly optimizing program parameters. In what follows, we restate and contextualize these assumptions in light of our generalized model, highlighting both their motivation and potential limitations.

Assumption 6. *Candidates within each group are homogeneous and maintain fixed characteristics while waiting.*

While real-world patients are heterogeneous across numerous clinical and demographic dimensions, and may experience changes in health status while awaiting transplantation, we model each patient group (e.g., by age or risk) as internally homogeneous. This abstraction is consistent with much of the literature on dynamic organ allocation models, where analytical clarity often requires grouping patients by static traits such as Estimated Post-Transplant Survival (EPTS) or age (Su and Zenios 2006, Zenios et al. 2000). In our extended model, this assumption allows us to explore the implications of jointly defining group boundaries and incentive thresholds without introducing further intra-group variance. While relaxing this assumption is feasible in simulation (see Chapter 5), it poses significant analytical complexity when modeling equilibrium participation behavior and threshold responses.

Assumption 7. *Organ allocation is governed by a static randomized policy, denoted by* $p_i(q)$.

Real-life organ allocation policies, such as those administered by the Organ Procurement and Transplantation Network (OPTN), use detailed deterministic algorithms that account for factors like wait time, donor-recipient matching, and geography (OPTN 2023). However, static probabilistic allocation rules are a widely used approximation in queueing-theoretic and game-theoretic transplant models, especially when studying long-run or steady-state behavior (Bertsimas et al. 2013). This abstraction enables us to isolate the effects of targeted prioritization by capturing disparities in access probabilities. For instance, in 2021, high-quality kidneys (KDPI $\leq 20\%$) were offered disproportionately to younger patients, with only 5.5% allocated to those aged 65 and older, compared to 27.8% for patients aged 18–34 (Lentine et al. 2023). Our model interprets $p_i(q)$ as a stylized representation of such structural disparities.

Assumption 8. *Organ offers are made instantaneously and decisions occur without delay.*

In reality, organ offers are sequenced across candidates via match runs, and delays may arise due to clinical evaluations, consent processes, and logistics. These delays are associated with extended cold ischemia time, which adversely affects organ viability and may lead to discard (Cassuto et al. 2008, Peters-Sengers et al. 2019). In our model, we abstract away from these operational frictions and assume that organ offers occur instantaneously and recipients respond immediately. While this may understate nonuse rates, it allows us to focus on equilibrium acceptance behavior and the role of incentives in shaping allocation outcomes. Future research could explore delayed acceptance dynamics and time-sensitive offer expiration using continuous-time models or simulations.

Assumption 9. *Patients do not observe their precise positions on the waiting list.*

Although patients can access some information about their waitlist status through platforms like the UNOS Patient Portal, the precise ranking and real-time availability of offers

are often opaque. We assume that patients form expectations based on average allocation probabilities and cannot condition their acceptance decisions on their exact queue position. This is consistent with modeling approaches in stochastic service systems and priority-based queues, where unobservable ranks simplify strategic decision-making (Afeche 2013, Yang and Debo 2019). The assumption also reflects a fairness principle: under such opacity, all patients within a group face identical decision environments, reinforcing the tractability and interpretability of our mechanism.

Assumption 10. *Under a targeted priority mechanism* $(\overset{*}{q})$*, for any* $q \in (\overset{*}{q}, \overline{q}]$*,* $\frac{\partial\left(\frac{p_N^{\gamma}(q)}{\lambda_D}\right)}{\partial \lambda_D}$ *is non-increasing in* λ_D*.*

This technical assumption ensures that the per-capita allocation probability for disadvantaged patients does not increase as the size of their group expands. It captures a congestion effect that is fundamental to voluntary program participation: if joining a targeted group increases competition among peers without raising access probability, then incentives must be carefully calibrated to ensure equilibrium participation. This condition aligns with existing work on capacity-constrained service systems and fairness-aware resource distribution (Tunç et al. 2022, Yang et al. 2017), where the addition of new agents reduces individual service rates unless compensated by increased priority or capacity. It also reflects a normative constraint often embedded in transplant policy: targeted programs should not penalize or crowd out existing participants without justification.

Taken together, these assumptions define a stylized yet flexible framework for analyzing targeted priority mechanisms under endogenous eligibility and dynamic participation. While they abstract from institutional nuances and biological heterogeneity, they allow us to derive transparent insights on how incentive design, group definition, and system parameters interact to influence patient behavior and social welfare. In Chapter 5, we relax several of these assumptions and evaluate the robustness of our results through a data-driven case study.

4.5 Strategic Decisions of Patients under Equilibrium

Before deriving our new analytical results, we recall the equilibrium characterization from Chapter 3. Upon arrival, each eligible type-D candidate chooses whether to join the targeted program, receiving priority for organs of quality $q \leq \overset{*}{q}$ in exchange for forgoing offers above $\overset{*}{q}$, by comparing expected QALE given a participation rate $\gamma^e(\overset{*}{q})$ to the QALE under the baseline allocation. Having decided, candidates enter either the priority queue (P) or the non-priority queue (N or O) and set a common acceptance threshold q_i^*, the lowest quality they will accept.

We consider symmetric Nash equilibria in which all members of each group share the same q_i^*. For a given cutoff $\overset{*}{q}$, there are three equilibrium regimes:

(i) **No-participation equilibrium** ($\gamma^e(\overset{*}{q}) = 0$): All disadvantaged patients decline the program and retain access to all organs under the baseline rules.

(ii) **Full-participation equilibrium** ($\gamma^e(\overset{*}{q}) = 1$): All disadvantaged patients join, gaining strict priority for $q \leq \overset{*}{q}$ but giving up access to $q > \overset{*}{q}$.

(iii) **Mixed-participation equilibrium** ($0 < \gamma^e(\overset{*}{q}) < 1$): A fraction $\gamma^e(\overset{*}{q})$ of disadvantaged patients enroll and receive priority for $q \leq \overset{*}{q}$, while the remainder, and all ineligible O patients, stay in the baseline queue.

The cutoff $\overset{*}{q}$ determines which organs (those with $q \leq \overset{*}{q}$) grant priority to enrolled candidates, but also restricts their acceptance set. By varying $\overset{*}{q}$, the planner can tune the program's attractiveness: at $\overset{*}{q} = \bar{q}$, priority extends to all organs, yielding $\gamma^e(\overset{*}{q}) = 1$; at $\overset{*}{q} = \underline{q}$, priority covers only the lowest-quality organs, yielding $\gamma^e(\overset{*}{q}) = 0$ (no participation); intermediate

values produce partial enrollment. Theorem 3.6 shows there are unique cutoffs

$$\underline{q} < \overset{*}{q}_l < \overset{*}{q}_u < \bar{q}$$

such that:

(a) If $\overset{*}{q} \leq \overset{*}{q}_l$, then $\gamma^e(\overset{*}{q}) = 0$ (no participation).

(b) If $\overset{*}{q}_l < \overset{*}{q} < \overset{*}{q}_u$, then $0 < \gamma^e(\overset{*}{q}) < 1$ (mixed participation).

(c) If $\overset{*}{q} \geq \overset{*}{q}_u$, then $\gamma^e(\overset{*}{q}) = 1$ (full participation).

These thresholds allow the planner to ensure at least some enrollment (by choosing $\overset{*}{q} > \overset{*}{q}_l$) or to prevent over-participation (by keeping $\overset{*}{q} \leq \overset{*}{q}_u$), guiding the design of effective and efficient targeted mechanisms. Therefore, it is helpful to identify the region in which those age cutoffs lie, in particular, to ensure that any chosen group contains patients with genuine incentives to participate. In Chapter 3, the existence of thresholds for full participation and no participation among disadvantaged patients was not thoroughly explored, as the disadvantaged patient group was exogenously specified. The following proposition addresses this gap by characterizing conditions under which the full-participation threshold does not exist, and demonstrating that a no-participation threshold always exists.

Proposition 4.1. *For any choice of disadvantaged patient group, $\overset{*}{q}_u$ does not exist if and only if:*

$$\int_{q_P^{e,}\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'))dq \leq \lim_{\gamma \to 1^-} \int_{\bar{q}^-}^{\bar{q}} (T_D(q) - T_D(q'))\left(-\frac{\partial p_N^{\gamma}(q)}{\partial \gamma}\right)dq.$$

And $\overset{*}{q}_l$ always exists for any choice of disadvantaged patients group.

As outlined in Proposition 4.1, the no-participation threshold $\overset{*}{q}_l$ always exists, regardless of the specification of the disadvantaged patient group. In contrast, the existence of the full-participation threshold $\overset{*}{q}_u$ is scenario-dependent. Importantly, the condition under which $\overset{*}{q}_u$ fails to exist is derived primarily for theoretical completeness and is unlikely to arise in realistic settings. For $\overset{*}{q}_u$ to be non-existent, it would require an implausible situation in which nearly all organs are made available with prioritized access to program participants, and yet, a non-negligible portion of disadvantaged patients still achieve higher expected utility by staying outside the program as the participation rate approaches one. This occurs because the allocation probability for non-participating disadvantaged patients diminishes sharply as participation approaches full saturation. Overall, Proposition 4.1 establishes that disadvantaged patients can always benefit from access to high-quality organs, ensuring the existence of $\overset{*}{q}_l$, while the non-existence of $\overset{*}{q}_u$ requires conditions that rarely hold in practice.

After discussing the existence of constraints on target thresholds, we also aim to explore the relationships between $\overset{*}{q}_l$, $\overset{*}{q}_u$ and various system parameters. By examining the connections between $\overset{*}{q}_l$ and parameters related to the selection of patient groups, a social planner can enhance system efficiency. A high $\overset{*}{q}_l$ can suggest a relatively cautious choice of the disadvantaged patient group, facilitating a more effective system design. Conversely, having a very low $\overset{*}{q}_l$ concerning the organ allocation system is not ideal. This situation implies that disadvantaged patients selected are extremely desperate to wait for organ transplantation in the queue and may hastily join the program even if the allocated organs are of relatively low quality. This rushed decision-making could negatively impact the well-being of disadvantaged patients and, consequently, overall patient welfare. The following proposition illustrates how the arrival rate λ_D and death rate d_D of disadvantaged patients affect the lower limit of the target threshold $\overset{*}{q}_l$.

Proposition 4.2. *The no-participation threshold $\overset{*}{q}_l$ satisfies the following:*

- $\overset{*}{q}_l$ *decreases in the arrival rate of disadvantaged patients,* λ_D.
- $\overset{*}{q}_l$ *increases in the death rate of the disadvantaged group,* d_D, *when* $\lambda_D < \frac{\int_{\overset{*}{q}_l}^{\bar{q}} p_N^0(q)dq}{\bar{q}-\underline{q}}$;
  $\overset{*}{q}_l$ *decreases in the death rate of the disadvantaged group,* d_D, *when* $\lambda_D > \frac{\int_{\overset{*}{q}_l}^{\bar{q}} p_N^0(q)dq}{\bar{q}-\underline{q}}$.

In Proposition 4.2, we establish that the lower limit of the target threshold, denoted as $\overset{*}{q}_l$, decreases as the arrival rate of disadvantaged patients λ_D increases. This conclusion is intuitive because, as the number of disadvantaged patients waiting for transplantation rises, the chances of an individual receiving an organ offer decrease. Consequently, each disadvantaged patient is likely to lower their acceptance threshold, leading to a simultaneous decrease in $\overset{*}{q}_l$. Proposition 4.2 also illustrates how $\overset{*}{q}_l$ changes in response to variations in the death rate of the disadvantaged group d_D. Initially, it may seem surprising that $\overset{*}{q}_l$ does not consistently decrease with the death rate, but this becomes more sensible when viewed from an individual perspective. The death rate of disadvantaged patients represents an intrinsic characteristic of the entire group. For an individual disadvantaged patient, the optimal strategy varies significantly depending on the arrival rate of the group. When the flow of disadvantaged patients is not substantial, those outside the program can still access organ offers relatively easily, and the death rate does not play a crucial role in determining their acceptance thresholds. However, as the arrival rate of disadvantaged patients increases, those outside the program begin to face more limited access to available organ transplantation. Consequently, the urgency of avoiding death compels disadvantaged patients to lower their acceptance threshold, resulting in a decrease in $\overset{*}{q}_l$.

We are also interested in showing the effects of λ_D and d_D on the upper limit of the target threshold for disadvantaged patients, denoted as $\overset{*}{q}_u$. Proposition 4.3 illustrates the connection between $\overset{*}{q}_u$ and λ_D.

Proposition 4.3. *The full-participation threshold* $\overset{*}{q}_u$ *satisfies the following:*

- $\overset{*}{q}_u$ *increases in the arrival rate of disadvantaged patients,* λ_D.
- $\overset{*}{q}_u$ *increases in the death rate of the disadvantaged group,* d_D, *when* $\lambda_D > \frac{\beta\left(\overset{*}{q}_u - q_P^{e,\overset{*}{q}}\right)}{\bar{q}-\underline{q}}$;
  $\overset{*}{q}_u$ *decreases in the death rate of the disadvantaged group,* d_D, *when* $\lambda_D < \frac{\beta\left(\overset{*}{q}_u - q_P^{e,\overset{*}{q}}\right)}{\bar{q}-\underline{q}}$.

In Proposition 4.3, we show that the full-participation threshold $\overset{*}{q}_u$ increases in the arrival rate of disadvantaged patients, λ_D. Intuitively, as λ_D grows, more disadvantaged patients compete for the same pool of organs, so the probability of receiving an organ falls for each individual. Although patients could respond by lower their personal acceptance thresholds, a high system-wide threshold is required to entice all patients to join the targeted priority program when competition is fiercer. Consequently, the system must set $\overset{*}{q}_u$ at a strictly higher quality level to secure full participation. We also characterize how $\overset{*}{q}_u$ responds to the disadvantaged death rate, d_D. When λ_D is moderate, an increase in d_D decreases $\overset{*}{q}_u$: higher mortality amplifies the survival benefit of joining the program, so a lower quality threshold suffices to secure full participation. However, when λ_D is large, the incremental gain in transplantation probability from participation becomes negligible; in this regime, raising the quality bar is again necessary to entice full participation, and thus $\overset{*}{q}_u$ increases in d_D.

4.6 Disadvantaged Patients under Mixed-Participation Equilibrium

In this section, we proceed to analyze how these thresholds vary with parameters characterizing the disadvantaged patient group, specifically, the arrival rate λ_D and the death rate d_D of patients eligible for the targeted program. The following result, stated in Proposition 4.4, shows that the equilibrium participation threshold for disadvantaged patients increases with the arrival rate of the disadvantaged group under a mild condition.

Proposition 4.4. *Assume* $\frac{\partial T_D(q)}{\partial \lambda_D} = c$ *for some constant* c*, for all* $q \in \left[\underline{q}, \bar{q}\right]$*, then* $q_D^{e,\overset{*}{q}}$ *is decreasing in* λ_D*.*

Proposition 4.4 can be interpreted as follows: under the assumption that the marginal change in post-transplant life expectancy with respect to the arrival rate of disadvantaged patients is constant across organ qualities, the equilibrium acceptance threshold for disadvantaged patients decreases with the group's arrival rate, provided that the system neither penalizes non-participation nor allocates transplant opportunities with disproportionately higher probability to individuals. This result holds under a mixed-participation equilibrium. It is also worth noting that as the arrival rate of disadvantaged patients increases, the average age within this group tends to decrease, making the group relatively less "disadvantaged". In the next proposition, we show that the equilibrium participation threshold $q_D^{e,\overset{*}{q}}$ increases in the disadvantaged death rate, d_D.

Proposition 4.5. $q_D^{e,\overset{*}{q}}$ *is increasing in* d_D*.*

Although Proposition 4.5 may initially appear counterintuitive, under the stated conditions, an increase in the disadvantaged group's death rate results in a smaller group size. This contraction effect outweighs the direct negative impact of higher mortality. As a result, patients in this group raise their equilibrium acceptance threshold in response to the increased death rate. In the following Proposition 4.6, we characterize how the equilibrium participation rate γ^* changes with respect to the death rate of disadvantaged patients.

Proposition 4.6. γ^* *is non-increasing in* d_D*.*

The intuition behind Proposition 4.6 parallels that of Proposition 4.5: as the death rate d_D increases, the disadvantaged group becomes smaller, which reduces the relative benefit of joining the targeted priority program. Consequently, disadvantaged patients become more reluctant to participate.

4.7 Simulation Study of Generalized Targeted Priority Mechanisms

In this section, we assess the performance of our generalized targeted priority mechanisms in a fully realistic setting by embedding them into a U.S. kidney-allocation simulator. Making use of the data on U.S. deceased-donor kidney transplants and the national waiting list, we can test our designs under conditions that relax the simplifying assumptions of our analytical model.

Over the past two decades, discrete-event simulation has been instrumental in shaping U.S. organ allocation policy. We build upon the flexible, high-fidelity framework of Sandıkçı et al. (2019), updating its parameters and priority rules to reflect the latest OPTN policy. Our simulated horizon spans January 2015 through January 2018, and all reported outcomes represent averages over 100 independent replications.

In our implementation, each kidney's quality score q in the analytical model is identified with the Kidney Donor Profile Index (KDPI), which ranges from 0–100% and inversely correlates with expected graft survival . A program threshold $\overset{*}{q}$ then partitions incoming kidneys: those with KDPI $\geq \overset{*}{q}$ are reserved for enrolled candidates, while kidneys with KDPI $< \overset{*}{q}$ follow the standard OPTN listing. Under the simulator's policy, each kidney is offered sequentially to up to 210 candidates before being labeled unused.

Eligible patients—defined here as those older than age threshold of programs, choose to enroll in the targeted-priority program with probability γ. Enrollees receive strict priority for all KDPI $\geq \overset{*}{q}$ offers and forfeit any KDPI $< \overset{*}{q}$ offers; non-enrollees continue under baseline OPTN rules. By varying γ and $\overset{*}{q}$, we recover each program's equilibrium participation rate, and hence its steady-state performance, in terms of transplant counts, nonuse rates,

wait-list mortality and size, and post-transplant graft survival, as well as spillovers to younger (ineligible) candidates.

4.7.1 Numerical Analysis of Equilibrium Program Participation

To understand how patients weigh their options under targeted priority mechanisms, we must select decision metrics that reflect their real-world trade-offs. Guided by our analytical framework, we first measure each patient's expected utility, combining both the time spent waiting and the post-transplant survival adjusted for quality of life. This comprehensive metric allows us to pinpoint the participation rate at which no eligible patient could improve their QALEs by switching strategies. Recognizing that some patients may instead focus on more concrete clinical outcomes, we then repeat our analysis using (i) normalized transplant rates, capturing the likelihood of receiving a transplant, and (ii) normalized pre-transplant mortality, capturing the risk of dying while waiting. By comparing equilibrium participation rates across these three perspectives, we capture the full spectrum of patient incentives and gain deeper insight into how program design drives uptake in practice.

When we raise the program's age threshold to 75 years old, we observe that across all three decision metrics, eligible patients always prefer participation. In other words, the equilibrium participation rate is 100%. This finding aligns with our theoretical predictions and suggests that pairing an age threshold of 75 years old with a KDPI cutoff of 84% makes the offer so attractive that no one opts out. If the goal is to temper participation while still improving social welfare, planners should consider lowering the age-eligibility bar whenever the organ-quality threshold is as lax as KDPI 84%.

For a visual comparison, the next set of figures shows how the equilibrium participation rate is determined under each metric: it is simply the point at which the participants' curve

intersects the non-participants' curve.

The expected utility is derived from the sum of quality scores of life multiplied by corresponding expected lengths of pre- and post-transplant life. In order to distinguish between the quality of pre- and post-transplant life, we utilize a ratio of $\alpha/\beta = 3/4$ aligning with the medical literature (Sayin et al. 2007, Tonelli et al. 2011, Wang et al. 2021). Figure 4.2 illustrates the identification of the equilibrium participation rate when the expected utility is the primary consideration for patients when the target threshold of the program is KDPI 84% and the age threshold is 65 years old. The equilibrium participation rate is identified at the intersection of these two curves, capturing the point at which patients are indifferent between enrolling and remaining on the standard waitlist.

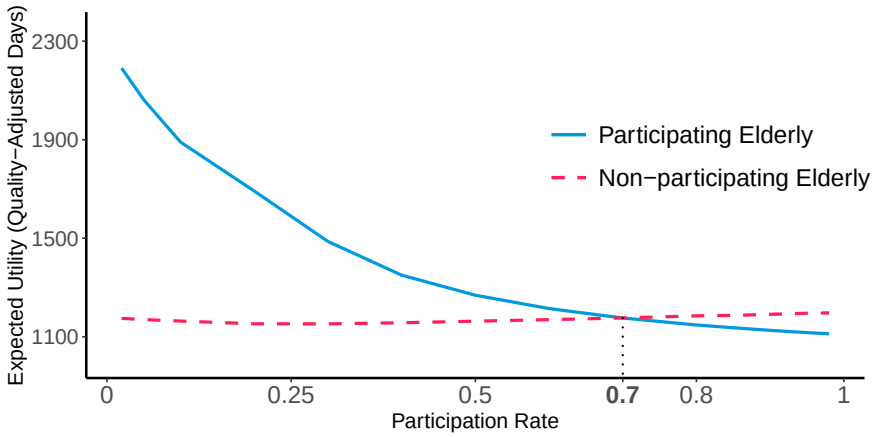


Figure 4.2: Expected utility of participating and non-participating elderly patients under the targeted priority program with a KDPI threshold of 84%, an age threshold of 65 years old.

When elderly patients prioritize the reduction of mortality, Figure 4.3 gives the answer. Figure 4.3 illustrates the identification of the equlibrium participation rate when mortality is the primary concern of patients.

When patients prioritize their probability of receiving transplantation when they are making participation decisions, Figure 4.4 illustrates that the participation rate can be even higher,

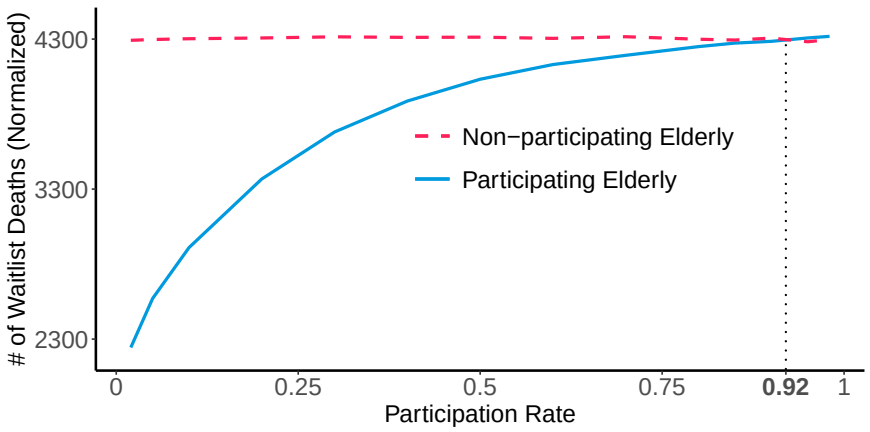


Figure 4.3: Waitlist deaths among participating and non-participating elderly under the targeted priority program with a KDPI threshold of 84%, an age threshold of 65 years old. The counts are normalized to the arrival rate of each patient group.

which highlights a specific benefit of the introduced program.

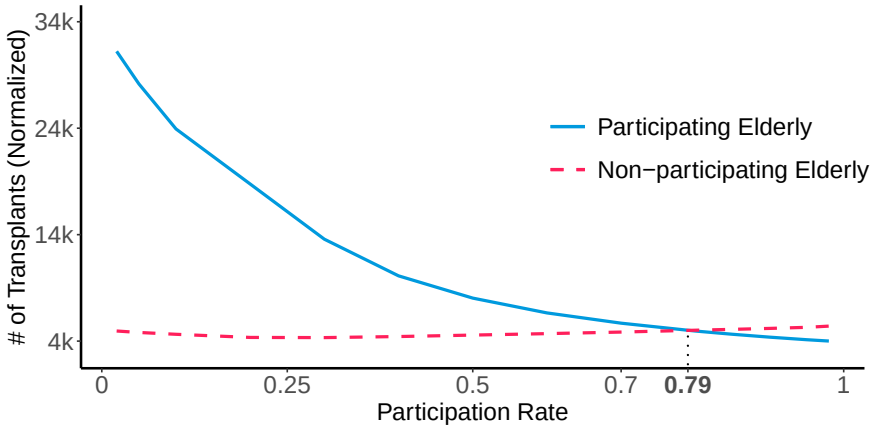


Figure 4.4: Transplant counts of participating and non-participating elderly under the targeted priority program with a KDPI threshold of 84%, an age threshold of 65 years old. The counts are normalized to the arrival rate of each patient group.

4.8 Conclusion

In this chapter, we have extended the theory of targeted priority mechanisms by endogenizing both the definition of the disadvantaged group and the design of the organ-quality threshold. Unlike the prior analysis in which eligibility was fixed exogenously, our generalized framework jointly improves the age threshold and the KDPI threshold, allowing planners to tailor programs to the underlying patient-and-organ distribution.

Analytically, we first established that for any choice of disadvantaged group there always exists a "no-participation'' cutoff $\underline{q}$, and under realistic conditions, a "full-participation'' cutoff $\bar{q}$, so that the planner's feasible KDPI thresholds fall into three regimes (no participation, mixed participation, full participation). We then showed how these bounds shift in response to key system parameters: as disadvantaged patients arrive more rapidly or face higher mortality, both the lower bound $\underline{q}$ and upper bound $\bar{q}$ move in predictable directions, reflecting stronger or weaker incentives to enroll. Finally, under mixed-participation equilibria we characterized how individual acceptance thresholds and overall enrollment respond to changes in patient-arrival and death rates. These results generalize our earlier equilibrium characterization (Chapter 3) to settings where "who" is targeted is itself a choice variable.

Numerically, embedding the generalized mechanism into a high-fidelity U.S. kidney-allocation simulator confirmed that jointly optimizing age cutoffs and KDPI thresholds can yield significant welfare gains. In particular, we found that programs with intermediate age thresholds and moderate KDPI thresholds produce the largest increases in transplant counts, and the greatest reductions in pre-transplant mortality. These gains extend to non-eligible patients, who benefit from spillover access to higher-quality kidneys when disadvantaged patients self-select into the program.

Overall, our generalized framework provides both a rich theoretical lens and actionable policy

guidance. By treating eligibility and organ thresholds as joint levers, planners can design adaptive, data-driven priority programs that dynamically balance efficiency and equity. We offer a blueprint for more nuanced, patient-centered targeting in transplantation, and, more broadly, in any setting where scarce health resources must be allocated under competing objectives.

Chapter 5

Evaluating Targeted Priority Mechanisms: A Comprehensive Simulation Study

5.1 Introduction

In this chapter, We evaluate the impact of our proposed targeted priority mechanisms within the U.S. kidney allocation system using a clinically detailed simulation model. The availability and comprehensiveness of the U.S. kidney transplant data provide an opportunity for in-depth analysis and comparison. Further, its diverse and dynamic patient population allows us to thoroughly evaluate the mechanism's adaptability to various real-life scenarios relaxing the assumptions of the theoretical models.

For over two decades, simulation models have been utilized to steer the development of organ allocation policies in the U.S. (Bertsimas et al. 2013, Harper et al. 2000, Shechter et al. 2005). Notably, the kidney-pancreas simulated allocation model (KPSAM) significantly influenced the two recent major policy updates of kidney allocation in 2014 (Israni et al. 2014) and 2021 (Israni et al. 2021). Building upon this, a previous work introduced a validated alternative for KPSAM (Sandıkçı et al. 2019), offering enhanced flexibility in assessing various policy

propositions and operating notably faster while retaining crucial clinical details. We evaluate and quantify the impact of the proposed targeted priority mechanisms by implementing them within the comprehensive simulation framework of Sandıkçı et al. (2019). To ensure the incorporation of the latest clinical details, we updated the simulation model according to the latest kidney allocation policies (OPTN 2023). We conduct simulations on the U.S. kidney allocation system spanning a three-year duration from January 2015 to January 2018 (using the patients and donors from the given framework, while simulating the most recent allocation policy), and report statistics derived from 100 independent replications of the simulation.

We implement the proposed targeted priority mechanisms by setting the quality of a kidney, denoted by q in the analytical model (§3.2), equivalent to the *KDPI score* between 0% and 100%, which combines donor characteristics into a singular metric summarizing the likelihood of graft failure post deceased-donor kidney transplant (Dahmen et al. 2019). The only difference, however, is that lower (higher) KDPI scores correlate to longer (shorter) estimated graft function and better (worse) quality of kidney. Correspondingly, the designated threshold of the priority program, denoted by $\hat{q}^*$ in the analytical model, defines the boundary for program-assigned kidneys. Any kidney with a KDPI higher than the program threshold (indicating inferior quality compared to a kidney with the program threshold KDPI) is prioritized for program participants. Under the simulation setup, organs are sequentially offered to 210 candidates before being classified as unused.

The participation strategy of eligible patients, defined by the participation rate γ, determines the likelihood of eligible patients opting into the program. In our simulation model, when patients arrive in the waiting list, they are offered the option of participating in the targeted priority program if their age is beyond the age threshold we set for eligible patients, and they are not eligible to join if they are younger. Following a similar approach to the ESP, the

effectiveness of which is validated in the European transplant system (Frei et al. 2008), we focus on *elderly patients* aged 65 and above at the point of registration for the program, and maintain the targeted patient group unchanged throughout the numerical study. We implement randomized participation strategies in the simulation such that any eligible patient arriving in the waiting list participate in the program with probability γ; they are prioritized if organ offers within the program arrive and never receive offers for organs outside of the program. And with probability $1-\gamma$, they choose not to participate and continue receiving offers for any organs based on the OPTN allocation policies.

5.2 Numerical Evaluation of the Equilibrium Program Participation

The selection of a metric that effectively captures patients' perceived utility in their decision-making processes is essential in identifying their equilibrium behavior within the organ allocation system. For this purpose, using a similar approach with our analytical models (§3.2), we use the expected combined pre- and post-transplant utility in quality adjusted life years. Compared to the conventional clinical metrics such as mortality or transplantation rates, the expected utility better comprehends the complexities of patients' decision process, combining the probability of and time until transplantation with post-transplant survival and quality of life. In the subsequent numerical analysis, this expected utility, derived from the sum of quality scores of life multiplied by corresponding expected lengths of pre- and post-transplant life, provides a basis for quantifying patients' decisions regarding the program participation. Distinguishing between the quality of pre- and post-transplant life, we utilize a ratio of $\alpha/\beta = 3/4$ aligning with the medical literature (Sayin et al. 2007, Tonelli et al. 2011, Wang et al. 2021).

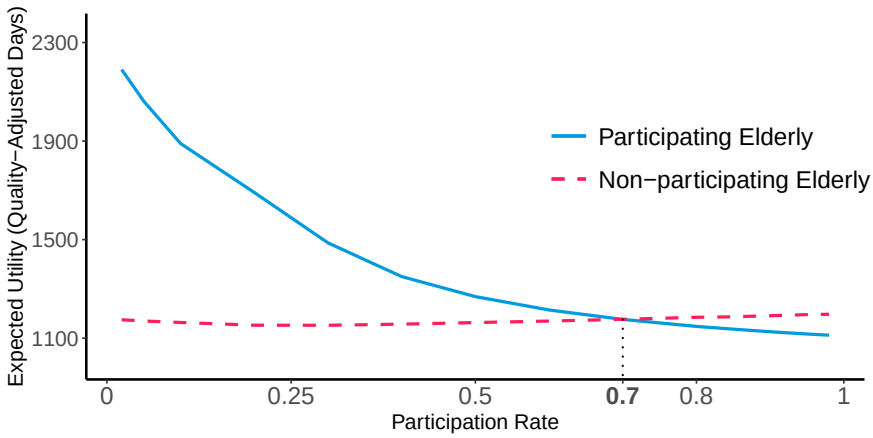


Figure 5.1: Expected utility of participating and non-participating elderly patients under the targeted priority program with a KDPI threshold of 84%.

Figure 5.1 illustrates the identification of the equilibrium participation rate when the expected utility is the primary consideration for patients and the program's target threshold is set at KDPI 84%. The equilibrium point is characterized by the participation rate equaling the expected utility of elderly patients participating in the program compared to those not participating, represented as the intersection point of plotted curves, given by $\gamma = 0.84$ (coincidentally). Figure 5.1 shows that when the participation rate of elderly patients is near 0, which may, for example, correspond to when the program is first introduced, the expected utility from participation strictly dominates the no-participation one. As the participation rate increases, the expected utility of participating patients decreases while remaining as the higher utility option until the participation rate reaches 0.84 in this case. Conversely, the expected utility of non-participating elderly patients remains similar and gradually rises with increasing participation, ultimately equaling that of participation when the participation rate hits 0.84. Both the observed trends and the high equilibrium participation rate are intuitive because elderly patients can hardly receive any high-quality organ offers while suffering from relatively low overall transplant access in the real allocation system (Chen et al. 2024, Poggio et al. 2021, Singh et al. 2016, Sørensen 2015), thereby making it ap-

pealing for them to participate in the program and have improved access to transplantation even for relatively high KDPI organs, i.e. KDPI $\geq 84\%$. This observation is also in line with the medical findings suggesting that elderly patients might benefit from transplanting organs with relatively high KDPI compared to remaining on dialysis (Chopra and Sureshkumar 2018, Jay et al. 2017, Massie et al. 2014), leading to the popular phrase of *better than dialysis* (The Alliance 2024).

Naturally, in the real allocation system, equilibrium is not expected to be reached instantly. Following the introduction of the program, there may be some elderly patients acting as explorers by participating in the program. Gradually, elderly patients, newly arrived and eligible for the program, along with their doctors can access statistics from both participants and non-participants in the program, enabling them to weigh the benefits of participation against non-participation, dynamically influencing the program's volume and pushing the system toward equilibrium through fluctuations in the long run. Similar patterns were observed in pilot programs developed to facilitate kidney transplants from HCV-infected donors, which have historically high nonuse rates, to uninfected recipients, followed by treatment to eradicate HCV (Durand et al. 2018, Gordon et al. 2023, Reese et al. 2018). Initially, the participation rate in the program was low, offering substantial benefits to early participants. However, as participation increased, the individual benefits saturated, leading to more stable participation rates.

Patients in the real allocation systems may potentially utilize different metrics other than the expected utility when making their participation decisions. For example, elderly patients may particularly prioritize the reduction of mortality. Figure 5.2 illustrates the identification of the equilibrium participation rate when mortality is the primary concern of patients; the point equaling the normalized mortality of participating elderly with those not participating. Similar to the case of expected utility, there is a significant discrepancy in mortality when

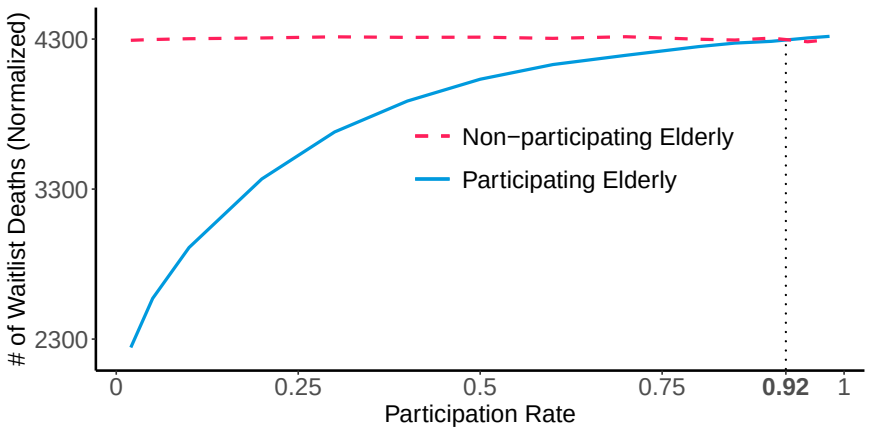


Figure 5.2: Waitlist deaths among participating and non-participating elderly under the targeted priority program with a KDPI threshold of 84%. The counts are normalized to the arrival rate of each patient group.

the participation rate is near 0. As the participation rate increases, the normalized mortality of participating patients grows with an increasingly slower pace while remaining the lower mortality option until the participation rate reaches 0.9. It is noteworthy that the equilibrium participation rate identified by normalized mortality is higher than that of utilizing expected utility, which highlights the predominant benefit of the program in reducing pre-transplant mortality.

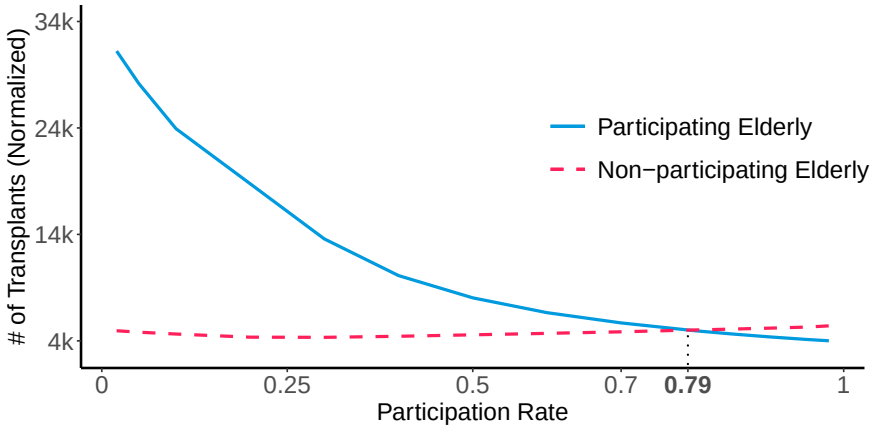


Figure 5.3: Transplant counts of participating and non-participating elderly under the targeted priority program with a KDPI threshold of 84%. The counts are normalized to the arrival rate of each patient group.

Patients may also prioritize their likelihood of receiving transplantation when making participation decisions. Figure 5.3 illustrates the equilibrium participation decisions when patients prioritize normalized transplantation as their primary objective. It demonstrates that the program consistently yields a higher normalized transplantation rate for participating elderly patients when the participation rate is below 0.94. Note that the equilibrium participation rate identified by normalized transplantation is even more aggressive than that of expected utility or normalized mortality, which highlights the improved transplant access as the most prominent benefit of targeted priority mechanisms for elderly patients.

5.3 Quantifying the Clinical Impact of Targeted Priority Mechanisms

The analytical results in §3.5 establish that carefully designed targeted priority mechanisms not only benefit eligible patients but also extend advantages to those ineligible for the program. We next quantify the impact of targeted priority mechanisms on various clinical outcomes such as transplant counts, pre-transplant deaths, post-transplant utility, and waitlist size under a realistic setting. Note that in the subsequent analysis, we limit our focus to the programs with target KDPI thresholds over 35% for two major reasons. First, as we later illustrate in §5.3 a targeted priority mechanism with too low KDPI thresholds would prioritize elderly patients when allocating high-quality organs that can be better utilized by nonelderly; thereby adversely impacting the matching quality between donor kidneys and patients. Second, the kidney allocation policy in the U.S. is customized based on KDPI categories (OPTN 2023) and elderly patients very rarely receive offers for kidneys with KDPI lower than 35% (Singh et al. 2016). Accordingly, the statistical models used in the simulation cannot offer reliable estimates for a match between elderly patients and kidneys with

KDPI lower than 35% due to the lack of existing data.

Equilibrium Participation to Targeted Priority Mechanisms

The participation rate of the elderly is expected to be directly influenced by the attractiveness of the program imposed by the target threshold. Figure 5.4 illustrates how the equilibrium participation rate changes as the target KDPI threshold varies, providing valuable insights into the program's effectiveness across varying thresholds. Consistent with our theoretical findings (§3.3), the equilibrium participation rate is non-increasing in the target threshold; all elderly patients participate in the program with a target threshold of KDPI 80% or lower, and thereafter the participation rate steadily decreases as the target threshold increases, eventually dropping to 0 when it reaches the maximum KDPI of 100%. These results highlight that a targeted priority mechanism in the U.S. kidney allocation system can be highly appealing to elderly patients. Even when the program offers priority over kidneys that are conventionally perceived as hard to place, such as those with KDPI over 85%, a considerable majority (80.5%) of the elderly patients would participate in the program in equilibrium.

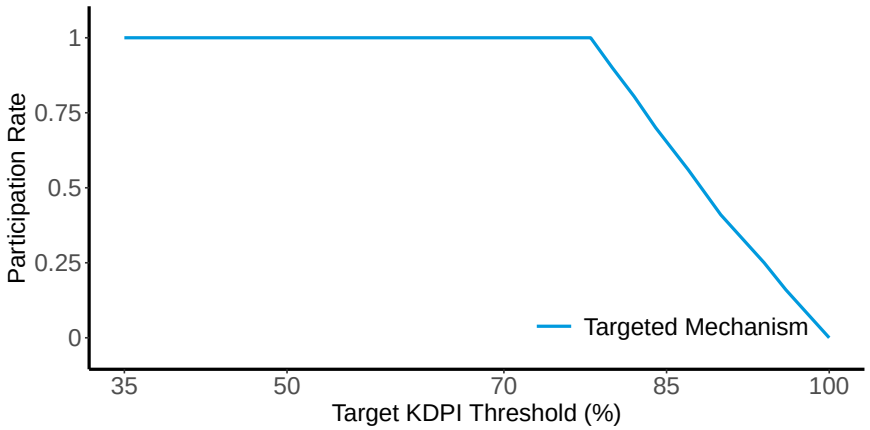


Figure 5.4: Equilibrium participation rates of elderly to targeted priority mechanisms with various target KDPI thresholds.

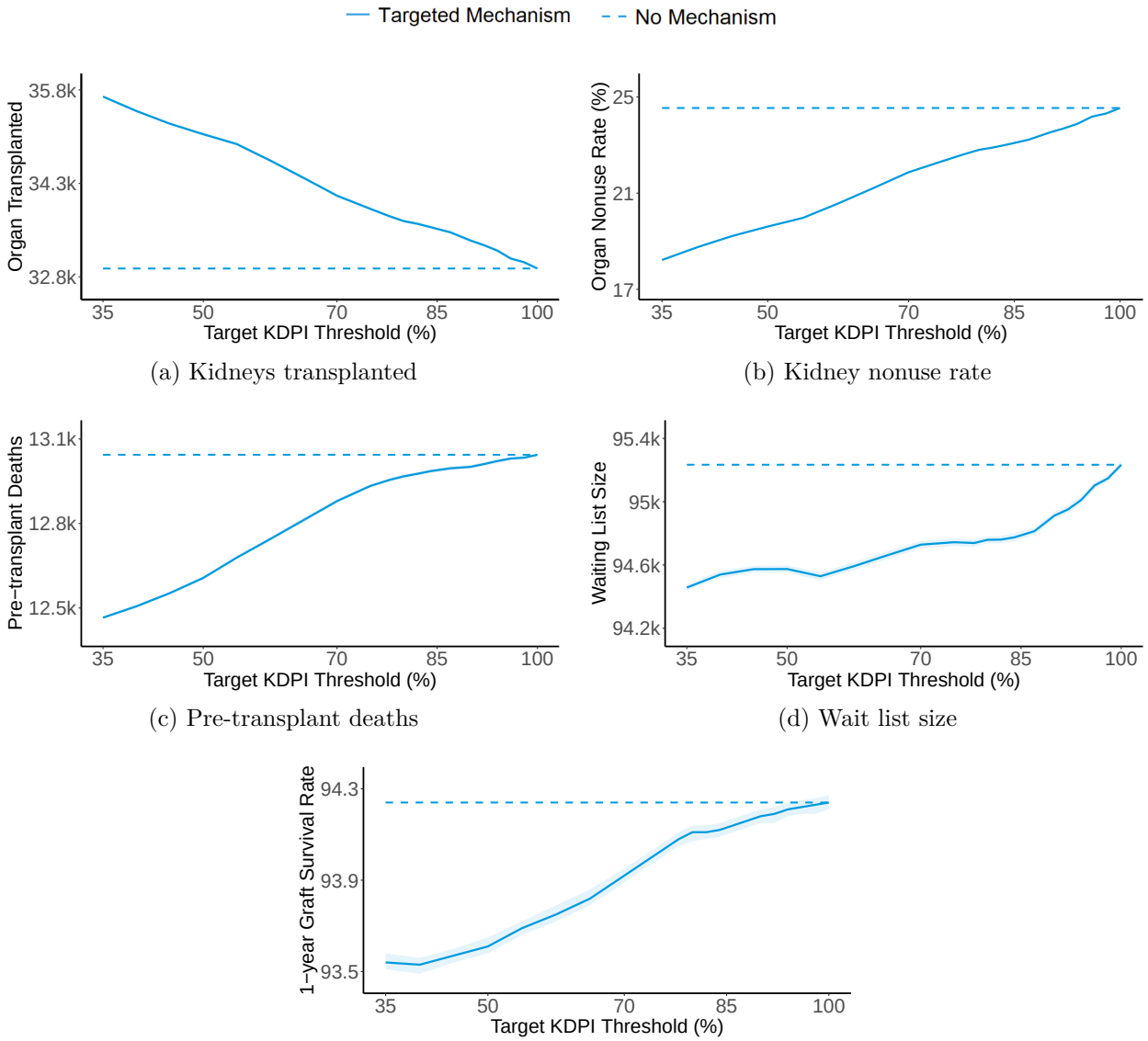
Impact of Targeted Priority Mechanisms on Key Transplant Outcomes

In this section, we explore the effectiveness of targeted priority programs in organ transplantation, focusing on their influence on key transplant outcomes. Figure 5.5 presents a comprehensive analysis of key metrics, including the total number of organs transplanted, patient mortality rates on the waitlist, the waiting list size at the end of the simulation period, and the average 1-year graft survival rate post-transplant. These outcomes are evaluated in relation to various KDPI thresholds set within the programs.

Figure 5.5a reveals that when targeted priority programs are in place, there is an observed annual increase of up to 920 in the number of organs transplanted, a striking contrast to the baseline system, as indicated by the dashed line in the figure. For instance, setting a target KDPI threshold at 84% results in additional 220 kidneys being transplanted annually. Complementing this, Figure 5.5b illustrates the trend in organ nonuse rates when targeted priority programs are implemented. As expected, the organ nonuse rate exhibits a clear decreasing pattern as more kidneys are accepted under the targeted priority program. Specifically, when the target KDPI threshold is set at 84%, the organ nonuse rate decreases to 23.04%, compared to 24.54% under the baseline. This decline in organ nonuse mirrors the corresponding rise in transplant volume, underscoring the efficiency gains introduced by targeted priority programs. Taken together, these trends highlight the pivotal role of targeted priority mechanisms in mitigating the alarming issue of organ underutilization, potentially enhancing the lives of hundreds of patients annually who would otherwise remain on dialysis.

Further analysis, as depicted in Figure 5.5c, reveals the life-saving impact of the targeted priority programs. By increasing the number of transplants, this innovative approach has the potential to avert up to 600 pre-transplant deaths among patients on the waiting list. For instance, with a KDPI target threshold of 84%, the program is estimated to save over

60 lives. Additionally, Figure 5.5c also reveals that by opting for lower target thresholds, social planners can enhance the program's life-saving capacity. Such programs significantly expand the pool of available organs, particularly benefiting elderly patients who often face higher risks of mortality while on the waiting list.



(a) Kidneys transplanted

(b) Kidney nonuse rate

(c) Pre-transplant deaths

(d) Wait list size

(e) Overall one-year graft survival rates

Figure 5.5: Impact of targeted priority mechanisms on key transplant outcomes. 99% confidence intervals of the point estimates are shaded around the lines.

The implementation of targeted priority programs also leads to a significant reduction in the size of organ transplant waiting lists, as illustrated in Figure 5.5d. This decrease is a direct result of the increased number of successful transplants facilitated by these programs. For instance, with a targeted KDPI threshold set at 84%, we observe a decrease of approximately 515 in the waiting list size. By effectively addressing the chronic imbalance between the supply and demand of deceased donor organs, targeted priority mechanisms play a crucial role in alleviating the backlog of patients who have been accumulating over the years on transplant waiting lists.

While there is an increasing emphasis in the medical field on reducing the high rates of organ nonuse, as discussed in §1, it is important to acknowledge that endorsing the transplantation of extremely low-quality organs or promoting inefficient organ-recipient matches solely for lowering nonuse rates might be ethically and medically questionable. However, Figure 5.5e presents a compelling case for the effectiveness of targeted priority programs in this regard. These programs demonstrate a significant capability to enhance the utilization of relatively lower quality organs, while maintaining comparable one-year graft survival rates. For instance, under a target KDPI threshold of 84%, the average one-year graft survival rate is observed to be over 94.13%, aligning closely with 94.24% observed in the baseline. This finding underscores the efficacy of targeted priority programs in increasing organ utilization without compromising the quality of transplant outcomes.

An interesting aspect of our findings is the non-monotonic behavior observed in transplant outcomes when the target KDPI threshold is set within the range of 35% to 40%. This particular pattern can be attributed to the limited data available on lower KDPI kidney offers and their subsequent transplants among elderly recipients, posing a challenge for the statistical models used in our simulation. More comprehensive data on lower KDPI transplants, especially in older recipients, may enhance the predictive accuracy and reliability of

simulation models in evaluating targeted priority programs in real-world transplant systems. However, as we illustrate in the next section, a mechanism with low KDPI thresholds adversely impacts the quality of donor-recipient matching, limiting the practical relevance of such programs.

Balancing Benefits and Fairness: The Effect of Targeted Priority Mechanisms on Ineligible Patients

The introduction of targeted priority mechanisms undeniably brings significant benefits, yet it is imperative to design these systems with an awareness of potential drawbacks. A critical aspect of this is the impact on nonelderly patients, who are ineligible to participate in these programs and thus have no influence over their operation. Implementing a program that results in a disadvantageous situation for nonelderly patients compared to the existing system could be perceived as unfair and potentially encounter substantial resistance in practical implementation. Therefore, it is essential for social planners to carefully consider how these mechanisms affect the access of nonelderly patients to organ offers and their subsequent health outcomes to ensure a fair and balanced allocation process.

While Figure 5.5c highlights a consistent decrease in overall pre-transplant deaths following the implementation of targeted priority programs, it is crucial to address an unintended consequence: the potential to increase mortality among waiting nonelderly. However, there is a silver lining. As shown in Figure 5.6, when the targeted priority mechanism is implemented with a KDPI threshold of 82% or higher, the pre-transplant mortality rate for nonelderly patients not only aligns with but, in some cases, is even slightly lower than the levels observed without the mechanism as marked by the dashed line.

As illustrated in §5.3, lowering the KDPI threshold effectively prioritizes elderly patients

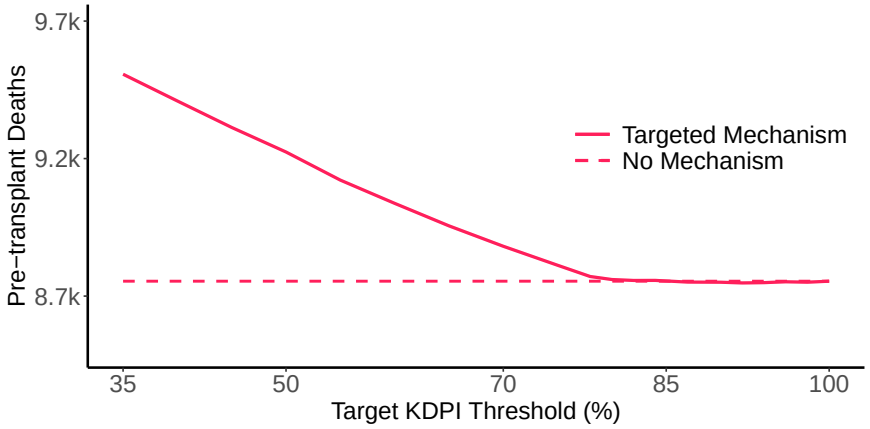


Figure 5.6: Waitlist deaths among nonelderly patients under targeted priority programs with various KDPI thresholds. 99% confidence interval of the point estimate is shaded around the line.

for a broader range of organ quality, thereby increasing their participation rates. Concomitantly, this shift redirects more high-quality, non-program organs towards nonelderly patients. Therefore, while a lower KDPI threshold might reduce overall access to transplants for nonelderly patients, those who do receive transplants are likely to experience improved post-transplant outcomes. This trade-off is illustrated in Figure 5.7 (as contrasted with Figure 5.6), where we observe a trend: as the program's KDPI threshold is lowered, the average post-transplant outcomes for nonelderly recipients improve with the one-year graft survival rate for these patients increasing up to 96.01%, a significant improvement from the 94.71% observed in the baseline.

The trade-off between pre-transplant mortality and post-transplant survival for nonelderly patients under varying KDPI thresholds presents a complex design choice. However, setting the threshold above 82% emerges as a win-win situation, where not only does pre-transplant mortality of nonelderly decrease, but also those receiving transplants enjoy enhanced post-transplant outcomes compared to the baseline. This dual benefit is clearly depicted in Figure 5.8, which shows the overall expected utility (combining pre- and post-transplant

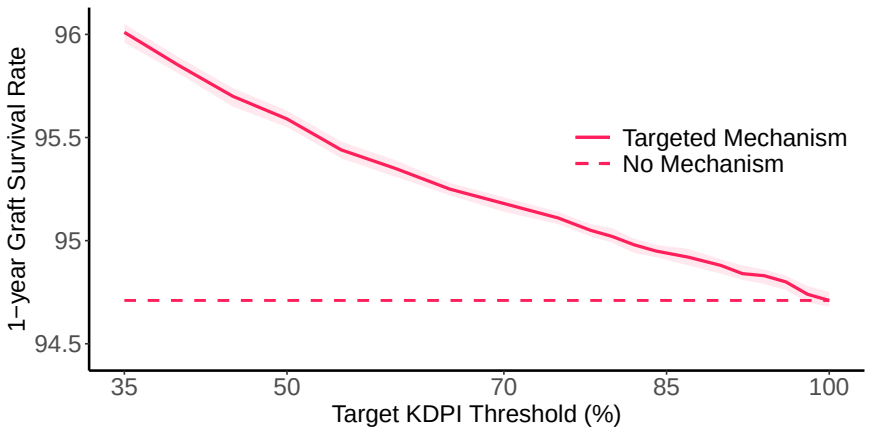


Figure 5.7: Average one-year graft survival rates among nonelderly transplant recipients under targeted priority programs with various KDPI thresholds. 99% confidence interval of the point estimate is shaded around the line.

quality-adjusted life years) of nonelderly patients against different KDPI thresholds. It demonstrates that the expected utility for nonelderly increases monotonically as the KDPI threshold rises up to 80%. At this point, most elderly patients choose to participate in the program, foregoing their access to higher quality organs, thereby benefiting the nonelderly. Beyond this threshold, the utility begins to decline as the participation drops and the program's advantages start to diminish. Significantly, any KDPI threshold above 76% assures better overall utility for nonelderly compared to the baseline. These insights resonate with the analytical findings in §3.4, underscoring that through well-designed targeted priority programs for elderly patients, policymakers can even enhance the well-being of nonelderly patients.

Targeted priority programs are uniquely positioned to improve outcomes for both elderly patients, who have the option to participate, and nonelderly patients, who are ineligible. By balancing the needs and benefits across these patient groups, these programs offer a promising, implementable solution to several critical challenges in organ transplantation, without compromising on fairness. This dual-benefit approach positions targeted priority programs

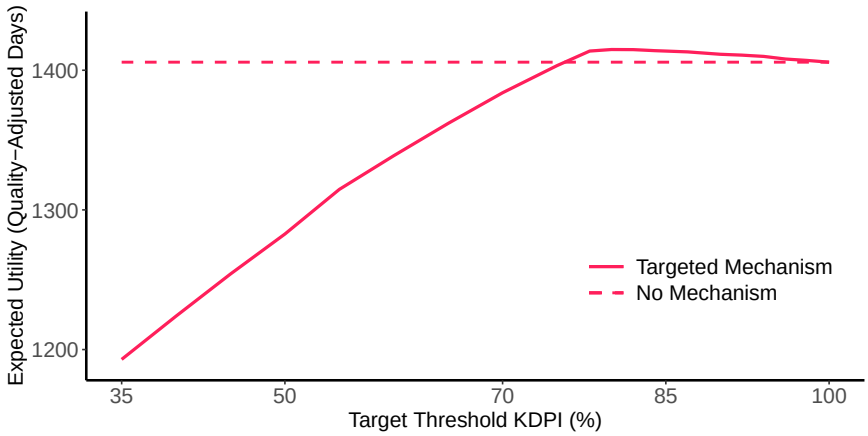


Figure 5.8: Nonelderly patients expected utility with different target thresholds under equilibrium. 99% confidence interval of the point estimate is shaded around the line.

as a pivotal tool in the ongoing effort to refine and optimize transplantation policies.

5.4 Sensitivity Analysis

Medical literature suggests that, compared with dialysis, kidney transplantation is associated with substantial reductions in mortality and cardiovascular risk, as well as clinically meaningful improvements in patients' quality of life (QoL). In §5.3, we identify the equilibrium participation rate of disadvantaged patients in the targeted priority program with a target KDPI of 84%, assuming that patients use QALE as the primary decision metric.

To further explore how equilibrium participation is influenced by the relative valuation of health states before and after transplantation, we conduct a sensitivity analysis by varying the ratio of pre-transplant to post-transplant quality-of-life scores, α/β, from 0.05 to 1. The resulting equilibrium participation rates are presented in Figure 5.9.

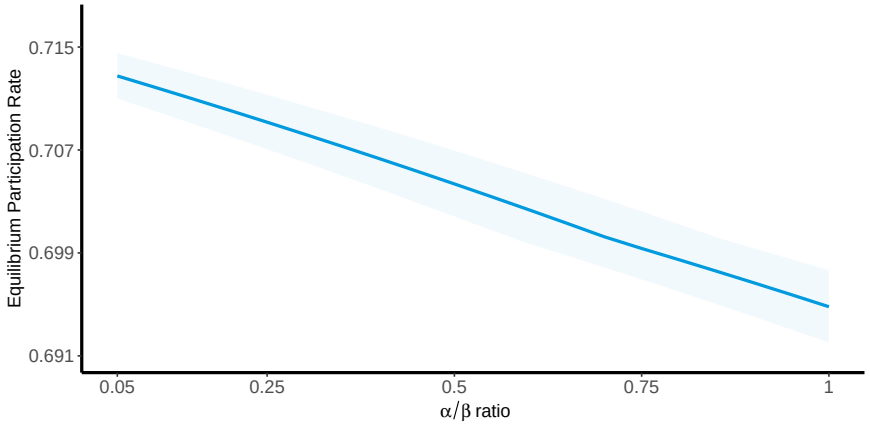


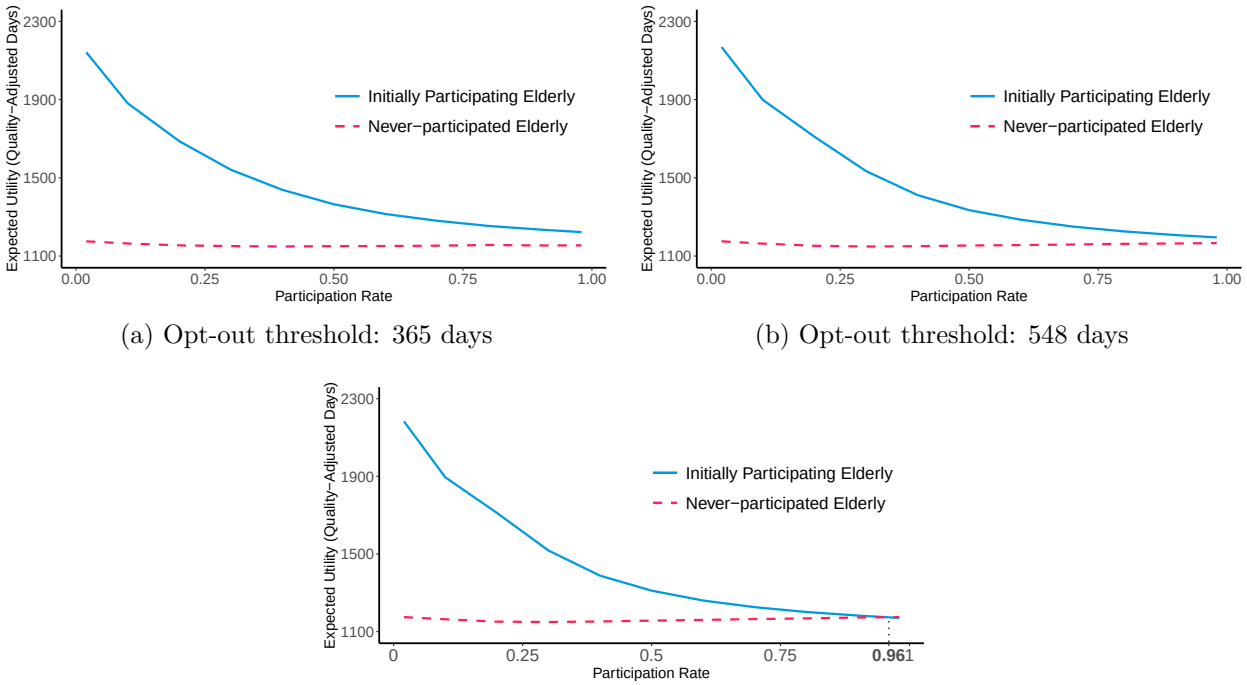
Figure 5.9: Sensitivity analysis of the ratio between pre-transplant and post-transplant quality-of-life scores.

5.5 Reneging Robustness

As we discussed in §5.3, selecting a metric that effectively captures patients' preferences is essential for identifying their equilibrium behavior within the organ allocation system. In that section, we employed three different metrics to evaluate the equilibrium participation rate of disadvantaged patients in the targeted priority program with a target KDPI threshold of 84%.

To further examine the robustness of the proposed mechanism, this subsection relaxes the participation constraint by allowing patients to opt out of the targeted program after waiting for a specified period—referred to as the "opt-out threshold"—and return to the general waitlist as non-participating disadvantaged patients, with their original waiting time preserved. This opt-out process mirrors the implementation in the Eurotransplant Senior Program (ESP).

We evaluate equilibrium participation rates under three opt-out thresholds: 1 year (365 days), 1.5 years (548 days), and 2 years (731 days). Figure 5.10a, Figure 5.10b, and Figure 5.10c illustrate the resulting equilibrium participation rates when expected utility is the



(a) Opt-out threshold: 365 days

(b) Opt-out threshold: 548 days

(c) Opt-out threshold: 731 days

Figure 5.10: Expected utility of participating and non-participating elderly patients under the targeted priority program with a KDPI threshold of 84%.

primary decision-making criterion and the program's target threshold is set at KDPI 84%, under opt-out thresholds of 365, 548, and 731 days, respectively.

We observe from Figure 5.10a that when participating disadvantaged patients are allowed to opt out of the program after waiting for 365 days, the expected utility of initially participating patients consistently exceeds that of non-participants across all nontrivial participation rates. This implies that, from an individual perspective, remaining outside the program is not a rational strategy when opting out is permitted after one year. In this setting, all patients would find it optimal to join the targeted program.

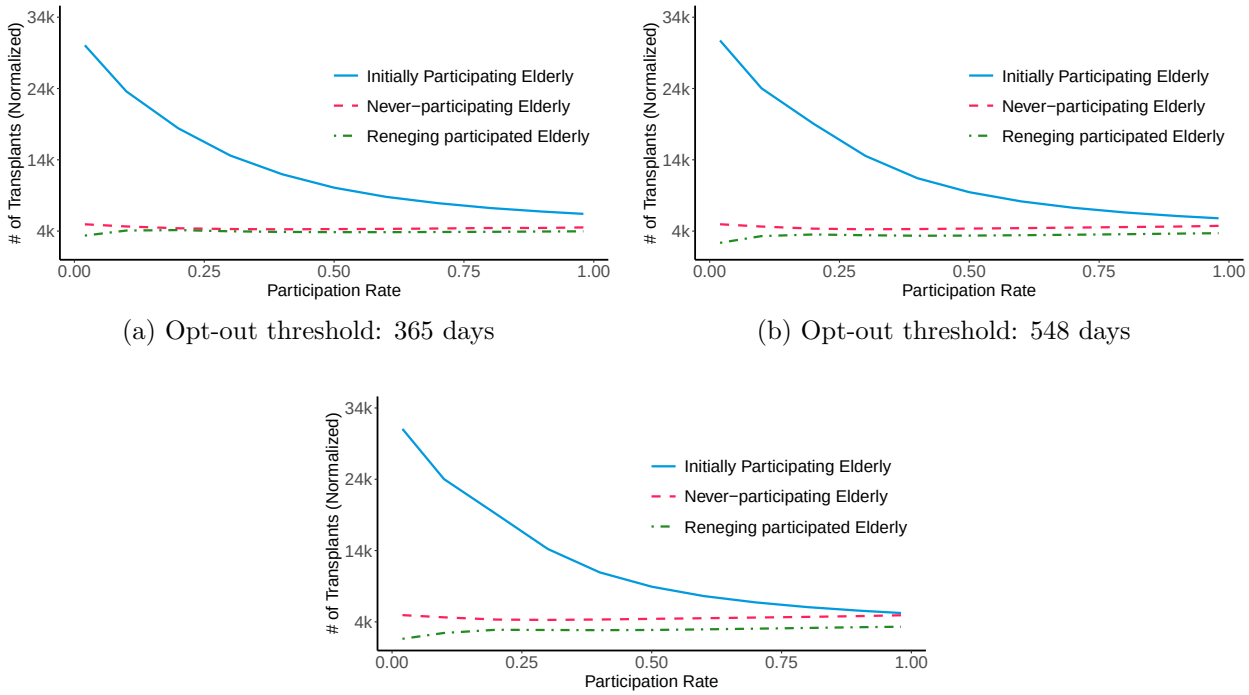
A similar outcome is observed when the opt-out threshold is extended to 548 days, as shown in Figure 5.10b. The expected utility for participating patients remains higher than that

of non-participants, reinforcing the result that full participation is the individually optimal behavior under this policy configuration.

When the opt-out threshold is extended to 731 days, the expected utility of initial participants continues to exceed that of non-participants across most participation rates. Nevertheless, as shown in Figure 5.10c, the equilibrium participation rate stabilizes at 0.96 rather than reaching full participation. This high—but not complete—participation level suggests that allowing patients to opt out while preserving their waiting time may constitute an overly generous policy, potentially weakening the mechanism's ability to differentiate between patient types. Notably, in contrast to the case without reneging shown in Figure 5.1, introducing the opt-out option for participating disadvantaged patients can lead to even higher equilibrium participation rates—including full participation under certain configurations. This result implies that targeted priority programs may become even more appealing when designed with appropriate opt-out provisions.

When analyzing subgroups within the participating disadvantaged population, it is noteworthy that patients who opt out of the program after initially participating do not experience a higher likelihood of receiving a transplant as a result of reneging. As shown in Figure 5.11a, even when the opt-out threshold is as short as 365 days, these patients have a lower normalized transplant rate compared to those who never participated in the program. This finding suggests that opting out does not confer a strategic advantage in terms of access to transplantation. This trend is consistently observed in Figure 5.11b and Figure 5.11c as well, reinforcing the conclusion that a rational patient—assuming self-interest maximization—would have no incentive to opt out of a well-structured targeted priority program with an appropriate KDPI threshold.

Patients in real-world allocation systems may rely on decision criteria other than expected utility when determining whether to participate in targeted programs. Notably, if patients



(a) Opt-out threshold: 365 days

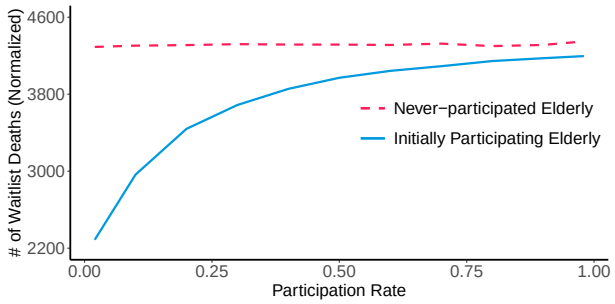
(b) Opt-out threshold: 548 days

(c) Opt-out threshold: 731 days

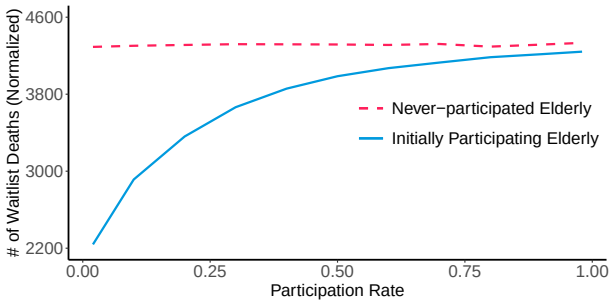
Figure 5.11: Transplant counts of initially participating, non-participating, and once-participated reneging elderly under the targeted priority program with a KDPI threshold of 84%. The counts are normalized to the arrival rate of each patient group.

place particular emphasis on reducing mortality risk, the advantage of participation remains robust across all three opt-out thresholds. As shown in Figures 5.12a–5.12c, initially participating elderly patients experience lower mortality compared to non-participants, further supporting the appeal of the targeted priority mechanism under alternative patient preferences.

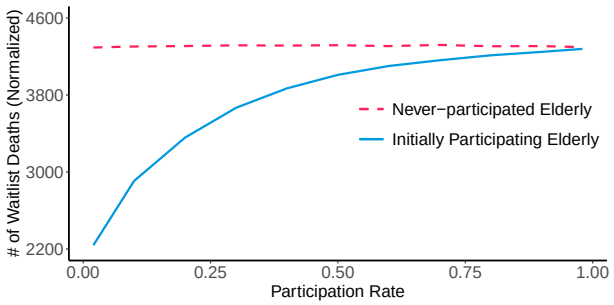
Patients may also prioritize their likelihood of receiving a transplant when making participation decisions. Similar to the findings above, across all three opt-out thresholds—shown in Figure 5.13a, Figure 5.13b, and Figure 5.13c—initially participating elderly patients consistently exhibit a higher probability of receiving a transplant compared to non-participants under any nontrivial participation rate. Notably, the longer the opt-out threshold, the



(a) Opt-out threshold: 365 days



(b) Opt-out threshold: 548 days

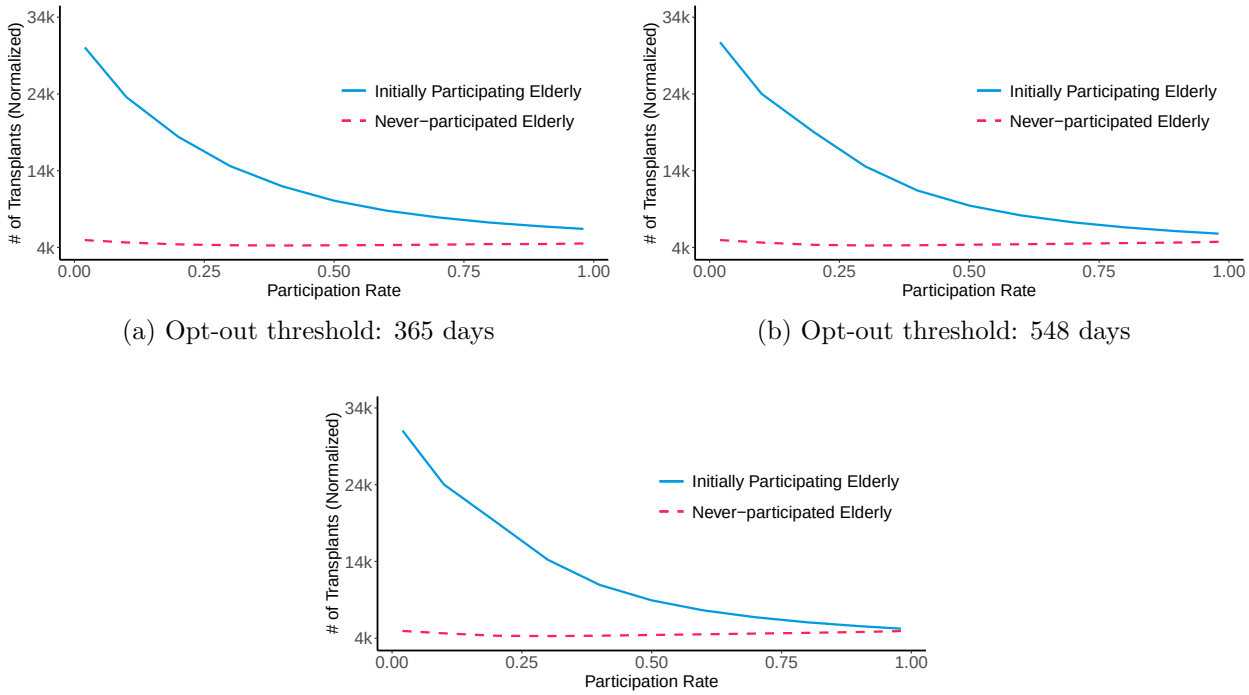


(c) Opt-out threshold: 731 days

Figure 5.12: Waitlist deaths among participating and non-participating elderly under the targeted priority program with a KDPI threshold of 84%. The counts are normalized to the arrival rate of each patient group.

smaller the gap between the two groups, highlighting that more lenient opt-out policies may diminish the differentiation in access between participants and non-participants.

To further examine the robustness of our proposed targeted priority mechanism when incorporating reneging behavior, we evaluate the equilibrium participation rates under an opt-out threshold of 548 days, assuming that patients prioritize expected utility. Figure 5.14 illustrates how the equilibrium participation rate changes as the target KDPI threshold varies. Consistent with our theoretical results, the equilibrium participation rate remains non-increasing in the target threshold, even when opting out is allowed. Specifically, all elderly patients participate in the program when the target KDPI threshold is set at 86% or lower. Beyond this point, the participation rate gradually declines as the threshold in-



(a) Opt-out threshold: 365 days

(b) Opt-out threshold: 548 days

(c) Opt-out threshold: 731 days

Figure 5.13: Transplant counts of participating and non-participating elderly under the targeted priority program with a KDPI threshold of 84%. The counts are normalized to the arrival rate of each patient group.

creases, ultimately dropping to zero when the maximum KDPI of 100% is reached. These findings suggest that introducing an opt-out option may enhance the appeal of targeted priority mechanisms within the U.S. kidney allocation system. Even when the program offers priority access to kidneys that are more difficult to place—such as those with KDPI above 85%—all elderly patients would still choose to participate in equilibrium under reasonable design choices.

Figure 5.15 presents a comprehensive analysis of key metrics, including the total number of organs transplanted, organ nonuse rates, patient mortality rates on the waitlist, the waiting list size at the end of the simulation period, and the average 1-year graft survival rate post-transplant. These outcomes are evaluated in relation to various KDPI thresholds set within

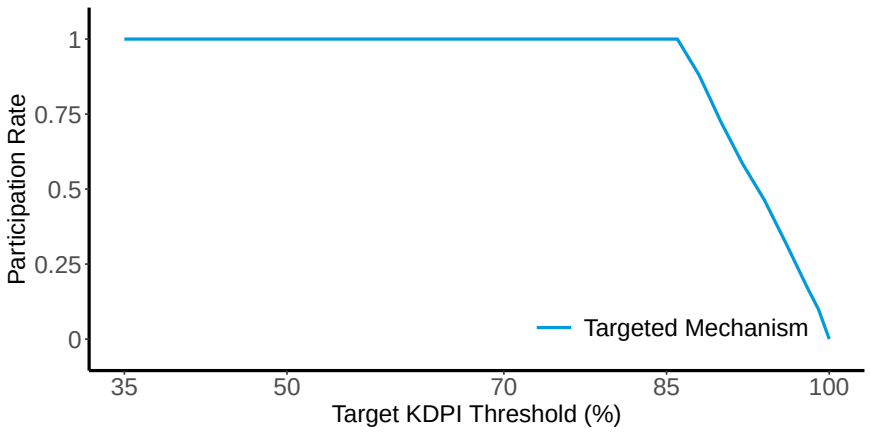
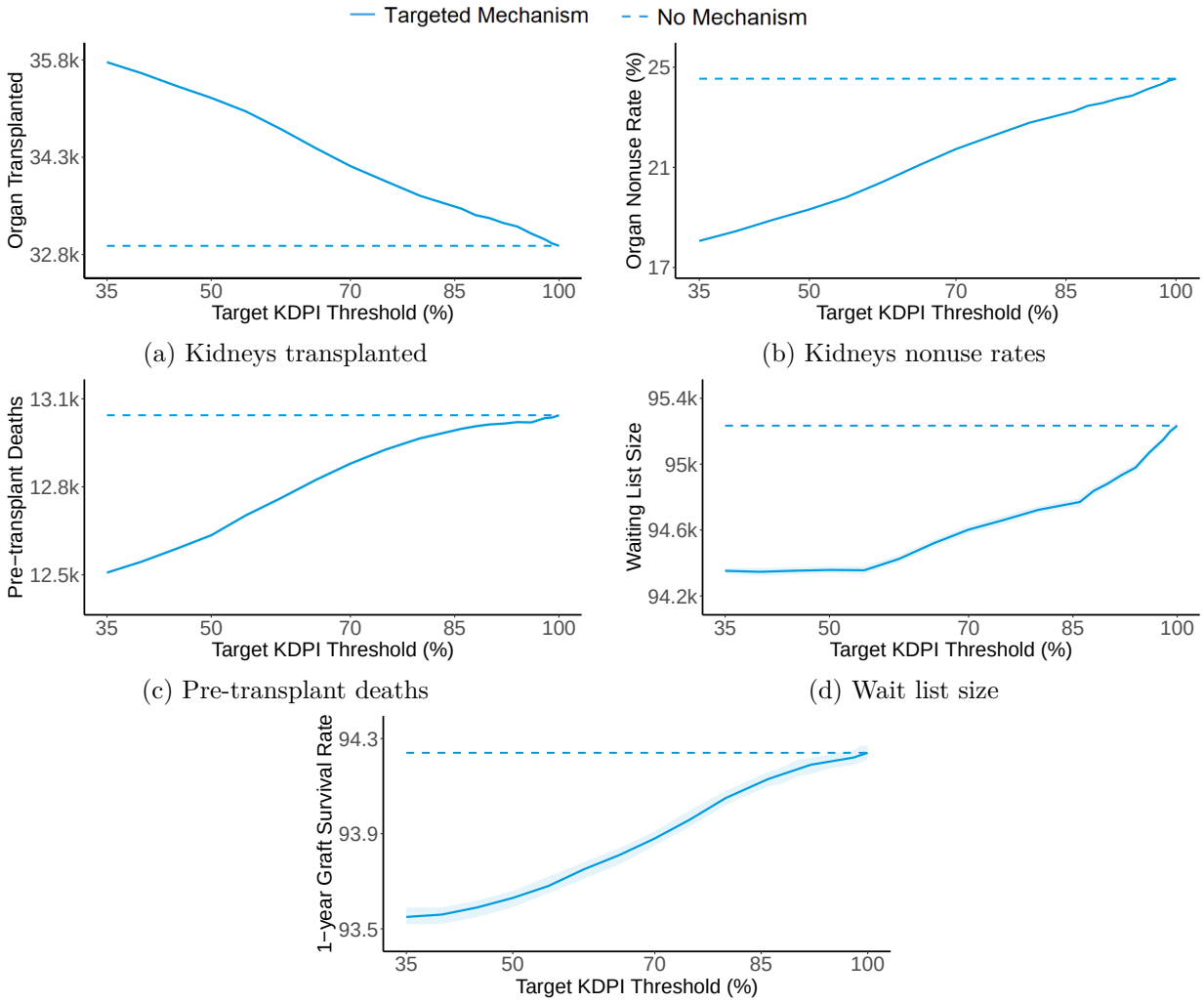


Figure 5.14: Equilibrium participation rates of elderly to targeted priority mechanisms with various target KDPI thresholds. Opt-out threshold is 548 days.

the programs.

Figure 5.15a reveals that when targeted priority programs are in place, there is an observed annual increase of up to 943 in the number of organs transplanted, a striking contrast to the baseline system, as indicated by the dashed line in the figure. For instance, setting a target KDPI threshold at 84% results in additional 207 kidneys being transplanted annually. Complementing this, Figure 5.15b illustrates the trend in organ nonuse rates when targeted priority programs are implemented. As expected, the organ nonuse rate exhibits a clear decreasing pattern as more kidneys are accepted under the targeted priority program. Specifically, when the target KDPI threshold is set at 84%, the organ nonuse rate decreases to 23.12%, compared to 24.54% under the baseline. As depicted in Figure 5.15c, reveals the life-saving impact of the targeted priority programs. By increasing the number of transplants, this innovative approach has the potential to avert up to 530 pre-transplant deaths among patients on the waiting list. For instance, with a KDPI target threshold of 84%, the program is estimated to save over 60 lives. The implementation of targeted priority programs also leads to a significant reduction in the size of organ transplant waiting lists, as illustrated in Figure 5.15d. This decrease is a direct result of the increased number of

successful transplants facilitated by these programs. For instance, with a targeted KDPI threshold set at 84%, we observe a decrease of approximately 470 in the waiting list size. While there is an increasing emphasis in the medical field on reducing the high rates of organ nonuse, as discussed in §1, it is important to acknowledge that endorsing the transplantation of extremely low-quality organs or promoting inefficient organ-recipient matches solely for lowering nonuse rates might be ethically and medically questionable. However, Figure 5.15e presents a compelling case for the effectiveness of targeted priority programs in this regard. These programs demonstrate a significant capability to enhance the utilization of relatively lower quality organs, while maintaining comparable one-year graft survival rates. For instance, under a target KDPI threshold of 84%, the average one-year graft survival rate is observed to be over 94.11%, aligning closely with 94.24% observed in the baseline. This finding underscores the efficacy of targeted priority programs in increasing organ utilization without compromising the quality of transplant outcomes.



(a) Kidneys transplanted

(b) Kidneys nonuse rates

(c) Pre-transplant deaths

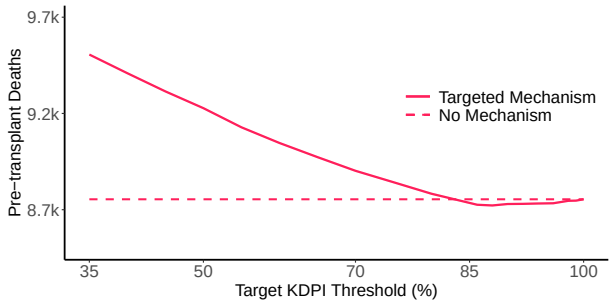
(d) Wait list size

(e) Overall one-year graft survival rates

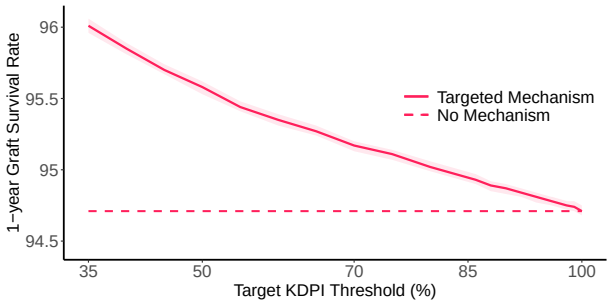
Figure 5.15: Impact of targeted priority mechanisms on key transplant outcomes. Opt-out threshold is 548 days. 99% confidence intervals of the point estimates are shaded around the lines.

As shown in Figure 5.16a, when the targeted priority mechanism is implemented with a KDPI threshold of 82% or higher, the pre-transplant mortality rate for nonelderly patients not only aligns with but is even slightly lower than the levels observed without the mechanism as marked by the dashed line.

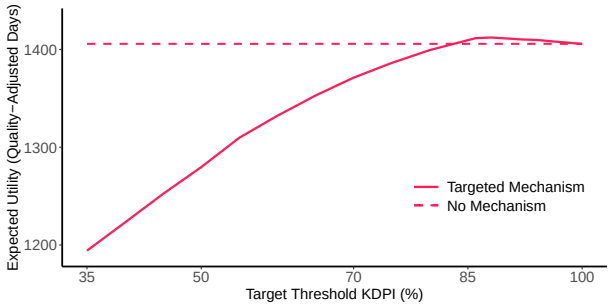
While a lower KDPI threshold might reduce overall access to transplants for nonelderly



(a) Pre-transplant deaths



(b) Average one-year graft survival rates



(c) Expected utility

Figure 5.16: Impact of targeted priority mechanisms on key transplant outcomes among nonelderly transplant recipients. Opt-out threshold is 548 days. 99% confidence intervals of the point estimates are shaded around the lines.

patients, those who do receive transplants are likely to experience improved post-transplant outcomes. This trade-off is illustrated in Figure 5.16b (as contrasted with Figure 5.16a), where we observe a trend: as the program's KDPI threshold is lowered, the average post-transplant outcomes for nonelderly recipients improve with the one-year graft survival rate for these patients increasing up to 96.01%, a significant improvement from the 94.71% observed in the baseline.

The trade-off between pre-transplant mortality and post-transplant survival for nonelderly patients under varying KDPI thresholds presents a complex design choice. However, setting the threshold above 84% emerges as a win-win situation, where not only does pre-transplant mortality of nonelderly decrease, but also those receiving transplants enjoy enhanced post-

transplant outcomes compared to the baseline. This dual benefit is clearly depicted in Figure 5.16c, which shows the overall expected utility (combining pre- and post-transplant quality-adjusted life years) of nonelderly patients against different KDPI thresholds. It demonstrates that the expected utility for nonelderly increases monotonically as the KDPI threshold rises up to 88%. At this point, most elderly patients choose to participate in the program, foregoing their access to higher quality organs, thereby benefiting the nonelderly. Beyond this threshold, the utility begins to decline as the participation drops and the program's advantages start to diminish. Significantly, any KDPI threshold above 84% assures better overall utility for nonelderly compared to the baseline. These insights resonate with the analytical findings in §3.4, underscoring that through well-designed targeted priority programs for elderly patients, policymakers can even enhance the well-being of nonelderly patients.

5.6 KDPI-EPTS Matching Extension

In addition to age-based eligibility, modern allocation systems often use patient-specific risk scores such as the Estimated Post-Transplant Survival (EPTS) index to determine who may be prioritized. In our extended KDPI–EPTS design, only candidates whose EPTS exceeds a pre-set cutoff are allowed to enroll; in return, those participants receive strict priority for kidneys whose KDPI falls above a specified threshold and forgo access to kidneys with better quality, whose KDPI is below the organ threshold.

We evaluate six program configurations by combining three KDPI cutoffs (75%, 80%, and 85%) with two EPTS eligibility rules (EPTS $\geq$ 75 and EPTS $\geq$ 80). Recognizing that patients may base their enrollment decisions on different outcomes, we simulate equilibrium behavior under three decision metrics: (1) expected quality-adjusted life years (combining

pre- and post-transplant utility), (2) pre-transplant mortality, and (3) transplant probability. This comparison reveals how the choice of evaluation criterion influences patient incentives and program uptake.

As an illustrative case, consider the KDPI=75, EPTS=75 program under the expected-utility metric. Figure 5.17 shows how the QALE for participating patients declines as more eligible candidates enroll, reflecting increased competition for the prioritized KDPI pool, yet remains above the baseline until about 66.3% enrollment. Beyond this point, the loss of access to higher-quality organs outweighs the benefit of priority, and further enrollment becomes counterproductive.

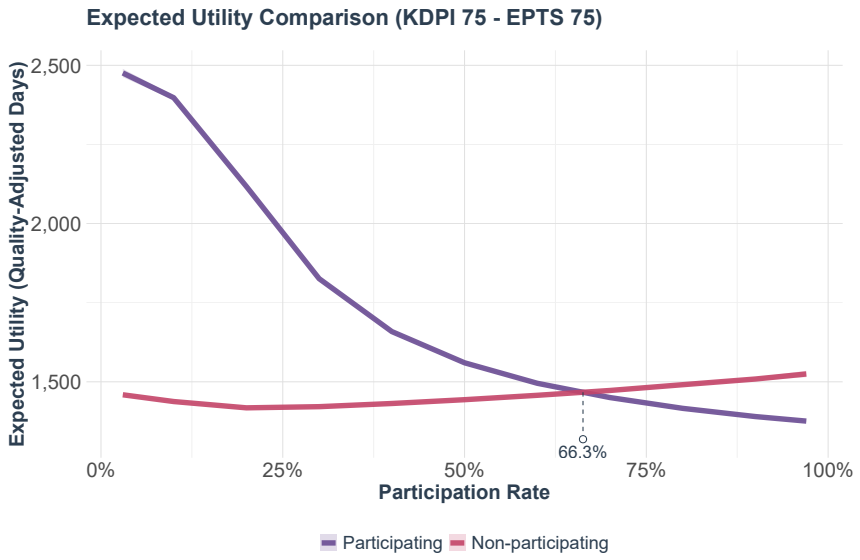


Figure 5.17: Expected utility of participating and non-participating eligible patients under the targeted priority program with a KDPI threshold of 75%, and a EPTS threshold of 75%.

When patients focus primarily on their probability of receiving a transplant, their enrollment incentives shift. Figure 5.18 shows the equilibrium participation rate under the normalized-transplantation metric for the KDPI 75 – EPTS 75 program. Consistent with our earlier findings, the cutoff at which participation maximizes transplant likelihood—approximately

72.6%, exceeds the 66.3% rate found under the expected-utility criterion. This higher equilibrium participation rate reflects participants' greater willingness to trade off organ quality when their priority is simply to secure a transplant.

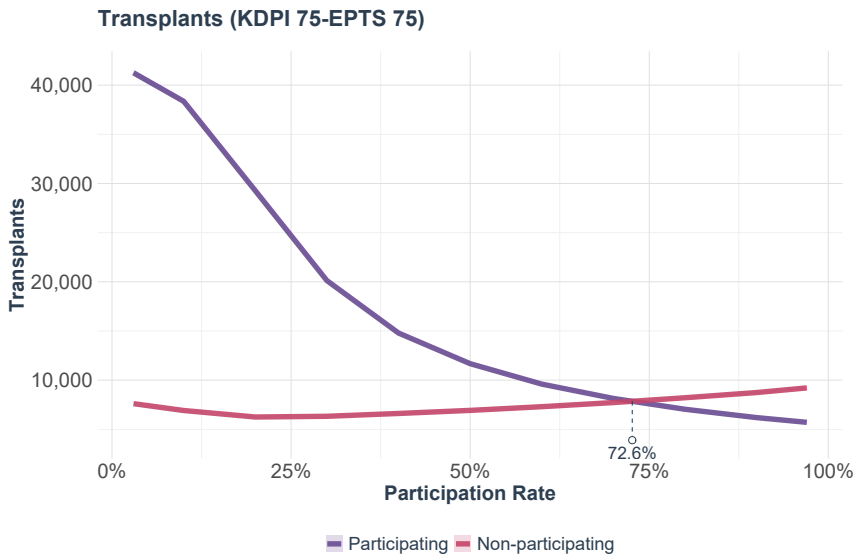


Figure 5.18: Transplant counts of participating and non-participating eligible patients under the targeted priority program with a KDPI threshold of 75%, and a EPTS threshold of 75%. The counts are normalized to the arrival rate of each patient group.

Patients may particularly prioritize the reduction of mortality. Figure 5.19 illustrates the identification of the equilibrium participation rate when mortality is the primary concern of patients; the point equaling the normalized mortality of participating patients with those not participating. Similar to the case of expected utility, there is a significant discrepancy in mortality when the participation rate is near 0. As the participation rate increases, the normalized mortality of participating patients grows with an increasingly slower pace while remaining the lower mortality option until the participation rate reaches 81.4%. It is noteworthy that the equilibrium participation rate identified by normalized mortality is higher than that of utilizing expected utility, which highlights the predominant benefit of the program in reducing pre-transplant mortality.

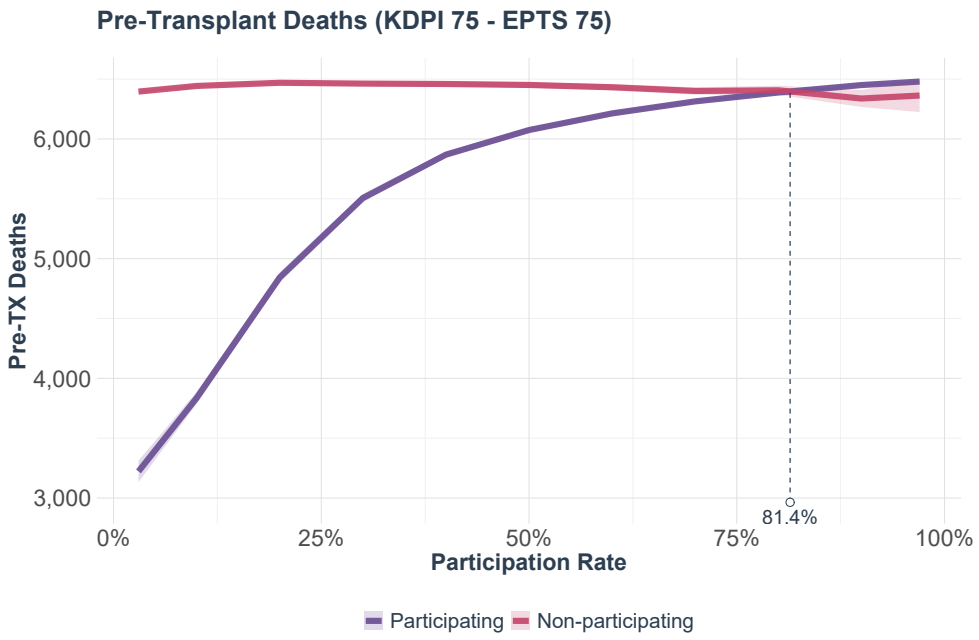


Figure 5.19: Waitlist deaths among participating and non-participating eligible patients under the targeted priority program with a KDPI threshold of 75%, and a EPTS threshold of 75%. The counts are normalized to the arrival rate of each patient group.

Similarly, we can identify the equilibrium participation rates of eligible patients into the targeted priority programs when different metrics are considered. To save space, we only show another case here, which is the program that the target threshold is set at KDPI 80, and the eligibility of patients for the program is set at EPTS 80. Figure 5.20 shows a similar pattern: participant QALEs fall as more candidates join the program, reflecting tougher competition for the priority pool. Enrollment remains beneficial up to about 60.4%, where the gain from priority exactly balances the opportunity cost of giving up access to better-quality kidneys.

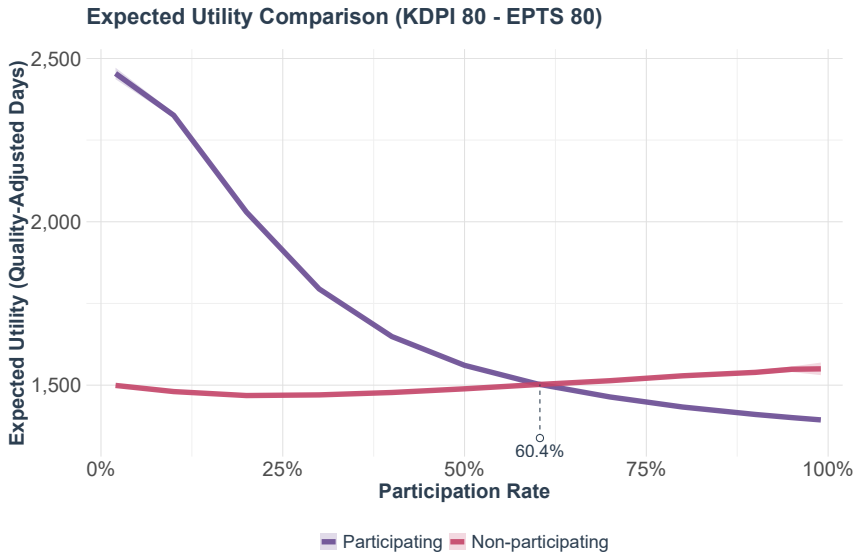


Figure 5.20: Expected utility of participating and non-participating eligible patients under the targeted priority program with a KDPI threshold of 80%, and a EPTS threshold of 80%.

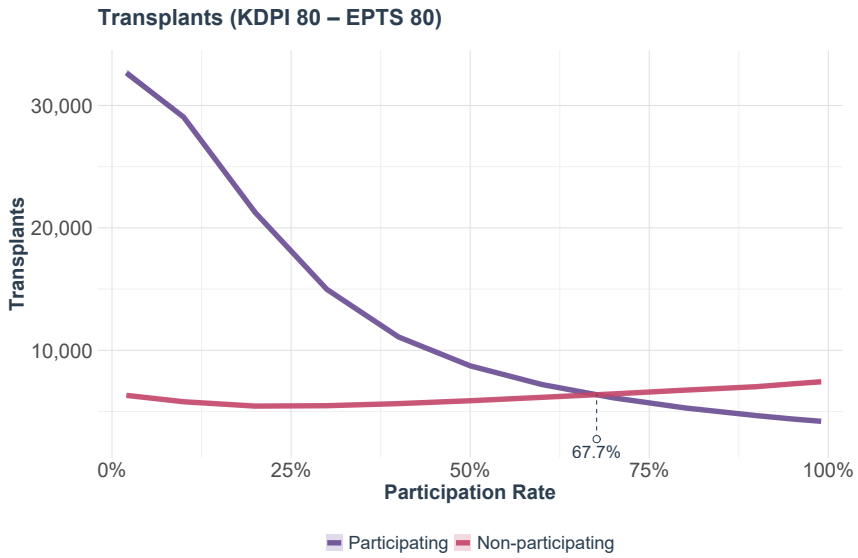


Figure 5.21: Transplant counts of participating and non-participating eligible patients under the targeted priority program with a KDPI threshold of 80%, and a EPTS threshold of 80%. The counts are normalized to the arrival rate of each patient group.

When patients base their decisions solely on transplant probability, their willingness to

enroll increases. Figure 5.21 plots the equilibrium participation rate under the normalized-transplantation metric for the KDPI 80-EPTS 80 program. Here, turnout peaks at about 67.7%, well above the 60.4% observed under the QALE-based metric, because participants are more inclined to sacrifice organ quality when their sole objective is securing a transplant.

Patients whose primary concern is minimizing mortality exhibit even stronger enrollment incentives. Figure 5.22 plots the equilibrium participation rate under the normalized-mortality metric for the KDPI 80–EPTS 80 program, defined by the intersection of participant and non-participant mortality curves. At low enrollment, joining the program sharply reduces mortality risk compared to remaining outside. As more patients enroll, the mortality advantage of participants shrinks—yet remains positive—until about 71.7% participation. Above this point, further enrollment offers no additional mortality benefit, and the curves cross. Notably, this 71.7% cutoff exceeds the QALE-based equilibrium rate, underscoring that patients who prioritize survival even more aggressively favor program enrollment.

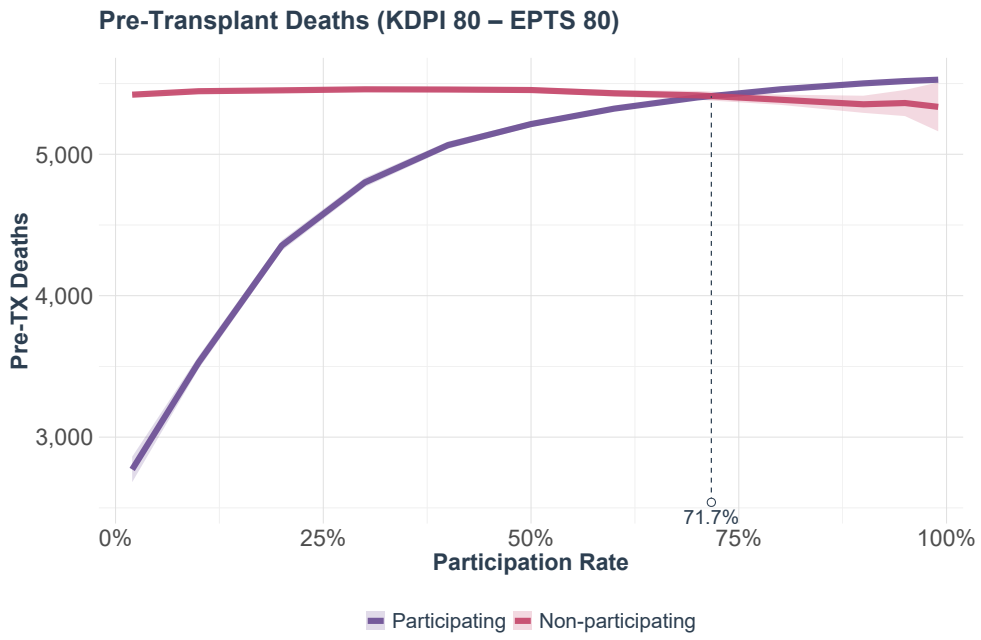


Figure 5.22: Waitlist deaths among participating and non-participating eligible patients under the targeted priority program with a KDPI threshold of 80%, and a EPTS threshold of 80%. The counts are normalized to the arrival rate of each patient group.

The choice of an outcome metric that truly reflects patients' trade-offs is critical for modeling their enrollment decisions in organ allocation. Building on our analytical framework, we adopt quality-adjusted life expectancy, which combine both pre- and post-transplant survival weighted by health-related quality-of-life. Unlike simpler clinical endpoints, such as raw mortality or transplant probability, QALEs integrate the chance of receiving a transplant, the waiting time until transplantation, post-transplant survival duration, and the quality of life experienced before and after surgery. The equilibrium outcomes are shown in table 5.23.

KDPI Group	EPTS Group	Anticipated Participation Rate(%)	Organ nonuse rate	Wait list size	Transplant	Pre-transplant mortality	Average Distance upon transplant(NM)	One-year graft survival rate(%)	Three-years graft survival rate(%)
NA	75	NA	24.54	95,234	32933.98	13,044	166	94.24	88.30
75	75	0.66	23.68	94,978	33312.44	13,004	173	94.16	88.18
NA	80	NA	24.54	95,234	32933.98	13,044	166	94.24	88.30
75	80	0.80	23.72	95,002	33294.47	13,008	174	94.15	88.17
80	80	0.60	23.82	95,013	33251.62	13,012	176	94.18	88.18
85	80	0.44	23.84	94,988	33240.33	13,018	180	94.22	88.27

Figure 5.23: Outcomes under targeted priority programs and benchmark cases

We first note that across both EPTS groups, the introduction of any KDPI cutoff reduces the organ nonuse rate by roughly 0.8–1.0 percentage points (from 24.54% at baseline to about 23.7–23.8%), translating into an additional 360–430 transplants annually (33,250–33,312 vs. 32,934). The largest increase, 378 extra transplants, occurs under the KDPI 75–EPTS 75 program, underscoring how a more generous kidney-quality guarantee combined with broader patient eligibility mobilizes the greatest incremental organ use.

This expansion in transplant volume also modestly lowers pre-transplant mortality: deaths fall from 13,044 in the no-program baseline to between 13,004 and 13,018 under the various designs, with the KDPI 75–EPTS 75 configuration again showing the largest absolute reduction (40 fewer deaths). Although a 0.3% decrease may appear small, it represents a clinically meaningful improvement when scaled to the national level.

At the same time, stricter KDPI thresholds demand wider organ sharing. Average distance to transplant increases by 7–8 NM under KDPI 75 (from 166 to 173–174 NM) and by up to 13–14 NM under KDPI 85 (179–180 NM). While these distances remain within current policy limits, they highlight a tangible logistical trade-off between higher utilization and transport burdens.

Crucially, one- and three-year graft survival rates remain essentially unchanged, around 94.15–94.24% at one year and 88.17–88.30% at three years, demonstrating that expanding marginal-kidney use does not compromise early outcomes.

Turning to EPTS eligibility, the EPTS 80 programs show slightly higher anticipated enrollment rates, as the more restrictive cutoff concentrates priority on those with the greatest urgency. However, this tighter eligibility yields somewhat smaller nonuse reductions (-0.82 percentage vs. -0.86 percentage) and transplant gains, indicating diminishing returns. Moreover, the higher enrollment under EPTS 80 contributes to even larger increases

in travel distance, up to 180 NM under KDPI 85, underscoring the cost of broader participation.

In sum, the KDPI 75–EPTS 75 configuration delivers the largest net benefit: it maximizes transplant volume and mortality reduction while maintaining graft survival and transport burdens within acceptable bounds. Restricting eligibility to EPTS 80 boosts enrollment but yields smaller utilization gains and slightly higher logistical costs, and raising the KDPI cutoff to 85 diminishes overall program impact. These insights will guide the optimal calibration of KDPI and EPTS parameters to balance organ use, patient benefit, and system sustainability.

5.7 Conclusion

In this chapter, we have implemented and evaluated our targeted priority mechanisms within a high-fidelity simulation of the U.S. kidney allocation system. By mapping our analytical parameters, KDPI thresholds and elderly eligibility, to the OPTN's allocation rules and using a validated KPSAM-style engine, we quantified the real-world impact of these programs on transplant volume, organ utilization, waitlist dynamics, and patient outcomes.

Our numerical experiments demonstrate that even modest KDPI cutoffs (e.g. 84%) yield substantial benefits: a 1.5–2.0 percentage reduction in organ nonuse translates to 200–900 additional transplants per year, 60–600 fewer pre-transplant deaths, and a 400–500 decrease in the active waitlist, all without compromising one- or three-year graft survival. Equilibrium participation analysis shows that targeted patients, with age-based or EPTS-based eligibility, are willing to enroll at rates exceeding 80% even for KDPI thresholds above 85%, underscoring the appeal of guaranteed access to marginal kidneys. We further establish that these gains extend to non-participating (nonelderly) patients: when KDPI thresholds exceed roughly 82%, nonelderly mortality falls and their post-transplant survival and overall QALEs

improve, thereby meeting key fairness considerations.

Sensitivity and reneging analyses confirm the robustness of our findings. Varying pre- and post-transplant quality-of-life weights has only minimal effect on equilibrium enrollment, and allowing participants to opt out (after 1–2 years) actually strengthens participation incentives without creating strategic withdrawal. Finally, our KDPI–EPTS extension shows that the KDPI 75–EPTS 75 program strikes a particularly effective balance between utilization, mortality reduction, and logistical costs, whereas more restrictive eligibility (EPTS ≥ 80) or higher KDPI cutoffs yield diminishing returns.

Taken together, these results validate that targeted priority mechanisms can be efficiently integrated into current U.S. policy to achieve hundreds of additional transplants, save dozens of lives annually, and improve equity across age and risk groups, without sacrificing graft outcomes or imposing undue burdens. Importantly, our KDPI–EPTS extension shows that the KDPI 75–EPTS 75 configuration strikes a notably effective balance between organ utilization, mortality reduction, and transport demands, while more restrictive eligibility (EPTS ≥ 80) or higher KDPI thresholds deliver diminishing returns. These findings provide clear, evidence-based guidance for policymakers seeking to refine allocation rules, ensuring that future reforms can be both impactful and equitable.

Chapter 6

Summary

The critical shortage of organs along with the alarming rates of organ underutilization pose a significant threat to thousands of patients awaiting life-saving transplants, highlighting the imperative for innovative and efficient incentive mechanisms. This thesis introduces a novel solution to this pressing issue through targeted priority mechanisms. These incentive-based voluntary mechanisms are crafted to improve the access of marginalized patients to transplantation and enhance the matching process between organ donors and recipients, aiming to improve overall social welfare and organ utilization. Our comprehensive analysis investigates the dynamics of these mechanisms, illustrating their potential through a rigorous queueing theoretic framework.

Our analysis of patients' equilibrium strategic decisions uncovers a complex interplay between the scarcity of organs and the dual consequences of participating in the program, namely, gaining prioritized access to organs of lower quality while giving up the opportunity to receive higher-quality ones. Through our analysis, we identify three potential equilibria: no-participation, full-participation, and mixed-participation; reflecting different patient responses to the introduction of targeted priority programs, and we establish the necessary and sufficient conditions for the existence and uniqueness of these equilibria. Our findings demonstrate that strategic program design can motivate disadvantaged patients towards participation, thereby nudging their behavior towards a socially optimal direction. Moreover, we characterize the relationship between the program's target threshold and the equilibrium

acceptance behavior of different patient groups, providing a roadmap for the optimal design.

We further analyze class-separating allocations and investigate the feasibility of achieving a separating equilibrium that benefits all patient types, thereby enhancing overall social welfare using targeted priority mechanisms. Addressing potential concerns that enhancing overall social welfare could inadvertently diminish the utility of patients ineligible to participate, our research focuses on designing mechanisms that are easy to implement. These mechanisms aim to strike a careful balance; they provide sufficient prioritized organ access to encourage eligible patients to join the program, while simultaneously directing higher-quality organs to those who can best utilize them. This ensures that non-targeted patients maintain their preference for non-program organs in equilibrium, effectively balancing the benefits across all patient groups without compromising the welfare of those unable to participate.

Our theoretical framework is complemented by practical insights derived from a case study using a clinically detailed simulation model for the U.S. kidney transplant system focusing on elderly patients aged 65 and above. Our findings reveal the program's capacity to significantly reduce kidney nonuse rates and pre-transplant mortality, while maintaining robust post-transplant graft survival rates. In particular, the targeted priority mechanism with a target KDPI threshold of 84% can facilitate 220 more transplants each year. As a result, such a mechanism can reduce the waiting list size by over 450 individuals during the simulation period and successfully prevent over 60 pre-transplant deaths annually. Furthermore, while the program encourages the utilization of relatively lower quality kidneys, the resulting post-transplant one-year graft survival rate remains stable at 94.12%, compared to the baseline of 94.24%. While we focus on targeting elderly patients for the program in our case study, our analysis reveals that with an appropriate design, these programs can also decrease pre-transplant mortality of nonelderly patients while also improving their one-year graft survival rate. Our findings imply that targeted priority mechanisms can successfully

facilitate an efficient matching of organs by directing each organ to those who can benefit most while maintaining voluntary participation.

While this study provides a promising outlook on the implementation of targeted priority mechanisms in organ transplantation, it also acknowledges several limitations. These include the assumption of homogeneity within each patient class and the omission of patients' awareness of their waiting list positions in the theoretical analysis. Moreover, we recognize that the careful selection of patient groups is crucial for maximizing the effectiveness of such programs in improving patient-organ matching and enhancing overall social welfare. To address these considerations, we extend our analysis to incorporate patient choice within these programs. Additionally, we explore a generalized version of the targeted priority mechanism in which eligibility is determined by the Estimated Post-Transplant Survival score rather than age. The potential applicability of targeted priority mechanisms beyond transplantation also opens promising directions for future research.

In conclusion, the targeted priority mechanisms proposed in this thesis not only help address the urgent issues of organ shortage and underutilization but also pave the way for more efficient and equitable organ allocation. By carefully designing and implementing such mechanisms, policymakers can significantly improve the transplantation practice, offering hope and improved outcomes to patients in dire need.

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Appendices

Appendix A

Appendix

A.1 Preliminaries

A.1.1 Preliminary Derivations

- Let $\mu_i(q_i)$ denote the effective arrival rate of organs to type-i candidates having an offer acceptance threshold q_i. We can express $\mu_i(q_i)$ by incorporating the offer probabilities for type i patients, focusing on the subset of organs that these patients are interested in, as follows:

$$\mu_i(q_i) = \frac{\overline{q} - q_i}{\overline{q} - \underline{q}} \cdot \int_{q_i}^{\overline{q}} p_i(q) \frac{1}{\overline{q} - q_i} \, dq = \frac{\int_{q_i}^{\overline{q}} p_i(q) \, dq}{\overline{q} - \underline{q}}. \tag{A.1}$$

We use the following asymptotic approximations for the expected pre-transplant life of type i and their probability of receiving a transplant:

$$\Delta_i(q_i) \approx \frac{1}{d_i} \cdot [1 - \pi_i(q_i)] \quad \text{and} \quad \pi_i(q_i) \approx \frac{\mu_i(q_i)}{\lambda_i}. \tag{A.2}$$

The formal proofs for the approximations given in (A.2) can be found in Zenios (1999). While the intuition of the approximation for $\pi_i(q_i)$ is clear, the intuition for $\Delta_i(q_i)$ can be better understood as follows. Let $\rho_i(n)$ denote the probability of having n type i candidates on the waiting list under steady-state. Then, we can write the removal rate of type-i candidates due to death as $\sum_{n=0}^{\infty} \rho_i(n) \cdot (n \cdot d_i) = d_i \cdot \sum_{n=0}^{\infty} n \cdot \rho_i(n)$, or

equivalently as $\lambda_i \cdot (1 - \pi_i(q_i))$ since $1 - \pi_i(q_i)$ is the probability leaving the waiting list without a transplant. Observing that $\sum_{n=0}^{\infty} n \cdot \rho_i(n)$ corresponds to the average queue length of type i, which equals $\lambda_i \Delta_i(q_i^e)$ using the Little's Law, the approximation $\Delta_i(q_i) \approx \frac{1}{d_i} \cdot [1 - \pi_i(q_i)]$ follows.

- We define q_i' as the organ quality that satisfies the equation $\beta T_i(q) = \frac{\alpha}{d_i}$ as q for type i, $i \in \{O, D\}$, for the proofs of analytical results in Section A.2. Note that β, α, d_i are constants for type i, and $T_i(q)$ is monotonically increasing in q, therefore for each type i, the solution q_i' is unique.

A.2 Proofs of Analytical Results

Theorem 3.1. *Under a targeted priority mechanism* $(\overset{*}{q})$*, candidates set their acceptance thresholds* $q_i^{e,\overset{*}{q}}$ *in equilibrium by solving the following set of equations:*

$$\beta T_O(q_O^{e,\overset{*}{q}}) = U_O(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha \Delta_O\Big(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta \pi_O(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) E\Big[T_O(q) \mid q \geqslant q_O^{e,\overset{*}{q}}\Big], \tag{3.1a}$$

$$\beta T_D(q_N^{e,\overset{*}{q}}) = U_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha \Delta_D\Big(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta \pi_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) E\Big[T_D(q) \mid q \geqslant q_N^{e,\overset{*}{q}}\Big], \tag{3.1b}$$

$$\beta T_D(q_P^{e,\overset{*}{q}}) = U_P(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha \Delta_D\Big(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta \pi_P(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) E\Big[T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_P^{e,\overset{*}{q}}\Big]. \tag{3.1c}$$

Furthermore, the equilibrium participation rate $\gamma^e(\overset{*}{q})$ *can be characterized as follows:*

(a) *A mixed-participation equilibrium solves* (3.1a)–(3.1c) *and the following equation:*

$$U_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) = U_P(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})). \tag{3.1d}$$

(b) *A no-participation equilibrium solves* (3.1a) *and* (3.1b) *with* $\gamma^e(\overset{*}{q}) = 0$*, and satisfies the*

following inequality:

$$U_N(q_N^{e,\overset{*}{q}}, 0) > \lim_{\gamma \to 0^+} U_P(q_P^{e,\overset{*}{q}}, \gamma). \tag{3.1e}$$

(c) *A full-participation equilibrium solves* (3.1a) *and* (3.1c) *with* $\gamma^e(\overset{*}{q}) = 1$*, and satisfies the following inequality:*

$$U_P(q_P^{e,\overset{*}{q}}, 1) > \lim_{\gamma \to 1^-} U_N(q_N^{e,\overset{*}{q}}, \gamma). \tag{3.1f}$$

(d) *Under any participation equilibrium, the equilibrium acceptance thresholds* $q_i^{e,\overset{*}{q}}$ *are unique when they exist.*

Proof. Proof of Theorem 3.1. Under a symmetric participation equilibrium $\gamma^e(\overset{*}{q})$, the symmetric equilibrium acceptance threshold $q_i^{e,\overset{*}{q}}$ corresponds to the quality of an organ that type i is indifferent between accepting and rejecting. Recall that when an organ is offered, accepting the organ provides the benefit of transplantation, whereas rejecting gives an opportunity to transplant a higher quality organ at the risk of dying without a transplant. Accordingly, under a participation equilibrium $\gamma^e(\overset{*}{q})$, a type O candidate sets their equilibrium acceptance threshold by solving

$$\beta T_O(q_O^{e,\overset{*}{q}}) = U_O(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha \Delta_O\Big(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta \pi_O\Big(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) E\Big[T_O(q) \mid q \geqslant q_O^{e,\overset{*}{q}}\Big],$$

a non-participating type D candidate, if exists, sets their threshold by solving

$$\beta T_D(q_N^{e,\overset{*}{q}}) = U_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha \Delta_N\Big(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta \pi_{D_N}\Big(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) E\Big[T_D(q) \mid q \geqslant q_N^{e,\overset{*}{q}}\Big],$$

and a participating type D candidate, if exists, sets their threshold by solving

$$\beta T_D(q_P^{e,\overset{*}{q}}) = U_P(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha\Delta_P\Big(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta\pi_{D_P}\Big(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) E\Big[T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_P^{e,\overset{*}{q}}\Big].$$

(a) A mixed-participation equilibrium exists when the eligible candidates are indifferent to participating in the program, which implies that a type D maintains the same utility whether they participate or not. Accordingly, we have

$$\begin{aligned} &\alpha\Delta_N\Big(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta\pi_{D_N}\Big(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) E[T_D(q) \mid q \geqslant q_N^e] \\ &= \alpha\Delta_P\Big(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta\pi_{D_P}\Big(q_P^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) E\Big[T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_P^{e,\overset{*}{q}}\Big], \end{aligned} \tag{A.3}$$

where the left- and right-hand sides of equation (A.3) correspond to a type D's expected utility from waitlisting by not participating (i.e., $U_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q}))$) and participating (i.e., $U_N(q_N^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q}))$) in the program, respectively.

(b) No disadvantaged patients participate in the program in equilibrium when the expected utility of not participating is greater than that of participating when no one else is participating, i.e.,

$$U_N(q_N^{e,\overset{*}{q}}, 0) > \lim_{\gamma \to 0^+} U_P(q_P^{e,\overset{*}{q}}, \gamma).$$

Under this equilibrium, no type D candidate participates in the program, i.e., $\gamma^e(\overset{*}{q}) = 0$, and accordingly, types O and N set their equilibrium offer acceptance thresholds by solving equations (3.1a) and (3.1b) at $\gamma^e(\overset{*}{q}) = 0$, respectively.

(c) All disadvantaged patients participate in the program in equilibrium when the expected utility of not participating is less than that of participating when everyone else is participating,

i.e.,

$$U_P(q_P^{e,\overset{*}{q}}, 1) > \lim_{\gamma \to 1^-} U_N(q_N^{e,\overset{*}{q}}, \gamma).$$

Under this equilibrium, all type D candidates participate in the program, i.e., $\gamma^e(\overset{*}{q}) = 1$, and accordingly, type O and type P set their acceptance thresholds by solving equations (3.1a) and (3.1c) at $\gamma^e(\overset{*}{q}) = 1$, respectively.

(d) Let $\mathcal{L}(q)$ and $\mathcal{R}(q)$ respectively denote the left- and right-hand sides of equation (3.1b). For notational simplicity, in the remainder of the proof, we use $\Delta_i\left(q_i^{e,\overset{*}{q}}\right)$ and $\pi_i\left(q_i^{e,\overset{*}{q}}\right)$ to denote $\Delta_i\left(q_i^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\right)$ and $\pi_i\left(q_i^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\right)$, respectively. Then we have,

$$\begin{aligned}
\mathcal{R}\left(q_N^{e,\overset{*}{q}}\right) &= \alpha\Delta_N\left(q_N^{e,\overset{*}{q}}\right) + \beta\pi_N\left(q_N^{e,\overset{*}{q}}\right)E\left(T_D(q) \mid q \geqslant q_N^{e,\overset{*}{q}}\right) \\
&= \alpha\frac{1}{d_D}\left(1 - \pi_N\left(q_N^{e,\overset{*}{q}}\right)\right) + \beta\pi_N\left(q_N^{e,\overset{*}{q}}\right)\int_{q_N^{e,\overset{*}{q}}}^{\bar{q}} T_D(q)\frac{q_N^{e,\overset{*}{q}}}{\int_{q_N^{e,\overset{*}{q}}}^{\bar{q}} p_N(q)dq}dq \\
&= \alpha\frac{1}{d_D}\left(1 - \frac{\int_{q_N^{e,\overset{*}{q}}}^{\bar{q}} p_N(q)dq}{\lambda_N(\bar{q} - \underline{q})}\right) + \beta\frac{1}{\lambda_N(\bar{q} - \underline{q})}\int_{q_N^{e,\overset{*}{q}}}^{\bar{q}} T_D(q)p_N(q)dq,
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\,\mathcal{R}\left(q_N^{e,\overset{*}{q}}\right)}{d\,q_N^{e,\overset{*}{q}}} &= \frac{\alpha}{d_D}\cdot\frac{p_N(q_N^{e,\overset{*}{q}})}{(\bar{q} - \underline{q})\cdot\lambda_N} - \frac{\beta}{\lambda_N(\bar{q} - \underline{q})}T_D(q_N^{e,\overset{*}{q}})p_N(q_N^{e,\overset{*}{q}}) \\
&= \frac{p_N(q_N^{e,\overset{*}{q}})}{\lambda_N(\bar{q} - \underline{q})}\left(\frac{\alpha}{d_D} - \beta T_D(q_N^{e,\overset{*}{q}})\right),
\end{aligned}$$

which implies that $\frac{d\,\mathcal{R}\left(q_N^{e,\overset{*}{q}}\right)}{d\,q_N^{e,\overset{*}{q}}} = 0$ when $q_N^{e,\overset{*}{q}} = q'_D$, where q'_D uniquely solves $\beta T_D(q) = \frac{\alpha}{d_D}$, which has a unique solution because $\beta T_D(q)$ is increasing for $q \in \left(\underline{q}, \bar{q}\right)$. Therefore, for $q \in (\underline{q}, q'_D)$, we have $\frac{d\,\mathcal{R}(q)}{d\,q} > 0$ and $\beta T_D(q) < \frac{\alpha}{d_N}$, on the other hand, for $q \in (q'_D, \bar{q})$, we have

$\frac{d\mathcal{R}(q)}{dq} < 0$ and $\beta T_D(q) > \frac{\alpha}{d_D}$.

Then

$$\begin{aligned}
\mathcal{R}(q'_D) &= \frac{\alpha}{d_D}(1-\pi_N(q'_D)) + \frac{1}{\lambda_N(\bar{q}-\underline{q})}\left(\int_{q'_D}^{\bar{q}} \beta T_D(q) p_N(q) dq\right) \\
&\geq \frac{\alpha}{d_D}(1-\pi_N(q'_D)) + \frac{1}{\lambda_N(\bar{q}-\underline{q})}\left(\int_{q'_D}^{\bar{q}} \beta T_D(q'_D) p_N(q) dq\right) \\
&= \frac{\alpha}{d_D}\left(1 - \frac{\int_{q'_D}^{\bar{q}} p_N(q) dq}{\lambda_N(\bar{q}-\underline{q})}\right) + \frac{1}{\lambda_N(\bar{q}-\underline{q})}\left(\int_{q'_D}^{\bar{q}} \beta T_D(q'_D) p_N(q) dq\right) \\
&= \frac{\alpha}{d_D} + \frac{1}{\lambda_N(\bar{q}-\underline{q})}\int_{q'_D}^{\bar{q}}\left(\beta T_D(q'_D) - \frac{\alpha}{d_D}\right) p_N(q) dq \\
&= \frac{\alpha}{d_D} = \beta T_N(q'_D) = \mathcal{L}(q'_D)
\end{aligned} \tag{A.4}$$

and

$$\begin{aligned}
\mathcal{R}(\bar{q}) &= \frac{\alpha}{d_D}(1-\pi_N(\bar{q})) + \beta \pi_N(\bar{q}) E[T_D(q) \mid q \geqslant \bar{q}] \\
&< \beta T_D(\bar{q})(1-\pi_N(\bar{q})) + \beta \pi_N(\bar{q}) T_D(\bar{q}) \\
&= \beta T_N(\bar{q}) = \mathcal{L}(\bar{q})
\end{aligned} \tag{A.5}$$

and

$$\begin{aligned}
\mathcal{R}(\underline{q}) &= \frac{\alpha}{d_D}(1-\pi_N(\underline{q})) + \pi_N(\underline{q}) \int_{\underline{q}}^{\bar{q}} \beta T_D(q) \frac{p_N(q)}{\int_{\underline{q}}^{\bar{q}} p_N(q) dq} dq \\
&> \beta T_D(\underline{q})(1-\pi_N(\bar{q})) + \pi_N(\underline{q}) \int_{\underline{q}}^{\bar{q}} \beta T_D(\underline{q}) \frac{p_N(q)}{\int_{\underline{q}}^{\bar{q}} p_N(q) dq} dq \\
&= \beta T_N(\underline{q}) = \mathcal{L}(\underline{q}).
\end{aligned} \tag{A.6}$$

Since $\mathcal{L}(q)$ is increasing for $q \in [\underline{q}, \bar{q}]$, and $\mathcal{R}(q)$ is increasing for $q \in [\underline{q}, q'_D)$ and decreasing for $q \in (q'_D, \bar{q}]$, using the intermediate value theorem, (A.4)–(A.6) conclude that if the

equilibrium threshold q_N^e exists, it is unique, in $(q'_D, \bar{q}]$.

Similarly, letting $\mathcal{L}(q)$ and $\mathcal{R}(q)$ denote the left- and right-hand sides of equation (3.1c), respectively, we have,

$$\begin{aligned}\mathcal{R}\Big(q_P^{e,\overset{*}{q}}\Big) &= \alpha\Delta_P\Big(q_P^{e,\overset{*}{q}}\Big) + \beta\pi_P\Big(q_P^{e,\overset{*}{q}}\Big)E\Big(T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_P^{e,\overset{*}{q}}\Big) \\ &= \frac{\alpha}{d_D}\left(1 - \frac{\overset{*}{q} - q_P^{e,\overset{*}{q}}}{\lambda_P(\bar{q} - \underline{q})}\right) + \frac{\beta}{\lambda_P(\bar{q} - \underline{q})}\int_{q_P^{e,\overset{*}{q}}}^{\overset{*}{q}} T_D(q)dq,\end{aligned}$$

and

$$\begin{aligned}\frac{d\,\mathcal{R}\Big(q_P^{e,\overset{*}{q}}\Big)}{d\,q_P^{e,\overset{*}{q}}} &= \frac{\alpha}{d_D}\cdot\frac{1}{(\bar{q} - \underline{q})\cdot\lambda_P} - \frac{\beta}{\lambda_P(\bar{q} - \underline{q})}T_D(q_P^{e,\overset{*}{q}}) \\ &= \frac{1}{\lambda_P(\bar{q} - \underline{q})}\left(\frac{\alpha}{d_D} - \beta T_D(q_P^{e,\overset{*}{q}})\right),\end{aligned}$$

which implies that $\frac{d\,\mathcal{R}\left(q_P^{e,\overset{*}{q}}\right)}{d\,q_P^{e,\overset{*}{q}}} = 0$ when $q_P^{e,\overset{*}{q}} = q'_D$, where q'_D uniquely solves $\beta T_D(q) = \frac{\alpha}{d_D}$, which has a unique solution as shown above. Therefore, similar to the previous case, for $q \in (\underline{q}, q'_D)$, we have $\frac{\partial\mathcal{R}(q)}{\partial q} > 0$ and $\beta T_D(q) < \frac{\alpha}{d_D}$; on the other hand, for $q \in (q'_D, \bar{q})$, we have $\frac{\partial\mathcal{R}(q)}{\partial q} < 0$ and $\beta T_D(q) > \frac{\alpha}{d_D}$.

Then, following similar steps, we have

$$\begin{aligned}\mathcal{R}(q'_D) &= \frac{\alpha}{d_D}\left(1 - \frac{\overset{*}{q} - q'_D}{\lambda_P(\bar{q} - \underline{q})}\right) + \frac{\beta}{\lambda_P(\bar{q} - \underline{q})}\int_{q'_D}^{\overset{*}{q}} T_D(q)dq \\ &\geqslant \frac{\alpha}{d_D} + \frac{1}{\lambda_P(\bar{q} - \underline{q})}\int_{q'_D}^{\overset{*}{q}}\left(\beta T_D(q'_D) - \frac{\alpha}{d_D}\right)dq \\ &= \beta T_P(q'_D) = \mathcal{L}(q'_D),\end{aligned} \tag{A.7}$$

and

$$\begin{aligned}\mathcal{R}(\overset{*}{q}) &= \frac{\alpha}{d_D}(1-\pi_P(\overset{*}{q})) + \beta\pi_P(\overset{*}{q})E[T_D(q) \mid \overset{*}{q} \geqslant q \geqslant \overset{*}{q}] \\ &< \beta T_D(\overset{*}{q})(1-\pi_P(\overset{*}{q})) + \beta\pi_P(\overset{*}{q})T_D(\overset{*}{q}) \\ &= \beta T_P(\overset{*}{q}) = \mathcal{L}(\overset{*}{q}), \end{aligned} \tag{A.8}$$

and finally,

$$\begin{aligned}\mathcal{R}(\underline{q}) &= \frac{\alpha}{d_D}\left(1 - \frac{\overset{*}{q}-\underline{q}}{\lambda_P(\bar{q}-\underline{q})}\right) + \frac{\beta}{\lambda_P(\bar{q}-\underline{q})}\int_{\underline{q}}^{\overset{*}{q}} T_D(q)dq \\ &> \beta T_D(\underline{q})\left(1 - \frac{\overset{*}{q}-\underline{q}}{\lambda_P(\bar{q}-\underline{q})}\right) + \frac{\beta}{\lambda_P(\bar{q}-\underline{q})}\int_{\underline{q}}^{\overset{*}{q}} T_D(\underline{q})dq \\ &= \beta T_P(\underline{q}) = \mathcal{L}\big(\underline{q}\big). \end{aligned} \tag{A.9}$$

Since $\mathcal{L}(q)$ is increasing for $q \in [\underline{q}, \bar{q}]$, and $\mathcal{R}(q)$ is increasing for $q \in [\underline{q}, q'_D)$ and decreasing for $q \in (q'_D, \overset{*}{q}]$, using the intermediate value theorem, (A.7)–(A.9) imply that if the equilibrium threshold $q_P^{e,\overset{*}{q}}$ exists, it is unique, in $(q'_D, \overset{*}{q}]$. Finally, the uniqueness of $q_O^{e,\overset{*}{q}}$ follows similarly, and its proof is omitted here for brevity.

□

Corollary 3.2. *Under a mixed-participation equilibrium $\gamma^e(\overset{*}{q}) \in (0,1)$, the participating and non-participating type D candidates adopt the same acceptance thresholds in equilibrium, i.e., $q_P^{e,\overset{*}{q}} = q_N^{e,\overset{*}{q}}$.*

Proof. Proof of Corollary 3.2. Theorem 3.1 establishes that a mixed-participation equilibrium satisfies equations (3.1b)–(3.1d). Using equation (3.1d) in equations (3.1b) and (3.1c), we have $\beta T_D(q_N^{e,\overset{*}{q}}) = \beta T_D(q_P^{e,\overset{*}{q}})$. And because $T_D(q)$ is increasing in q, we conclude that

$q_P^{e,\overset{*}{q}} = q_N^{e,\overset{*}{q}}$ under any mixed-participation equilibrium. □

Proposition 3.3. *Under a targeted priority mechanism* $(\overset{*}{q})$*, no eligible candidates would participate in the program, i.e., the equilibrium participation rate satisfies* $\gamma^e(\overset{*}{q}) = 0$*, if and only if*

$$\beta T_D(\overset{*}{q}) < \frac{\alpha}{d_D} + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \lim_{\gamma \to 0^+} \int_{\overset{*}{q}}^{\bar{q}} \left(T_D(q) - \frac{\alpha}{d_D} \right) p_N^{\gamma}(q) dq. \tag{3.2}$$

Proof. Proof of Proposition 3.3. First, observe that when $\gamma^e(\overset{*}{q}) = 0$, i.e., when no one is participating, if an individual type D participates in the program, the expected utility of her participation is $\beta T_D(\overset{*}{q})$. This is so because when there is no disadvantageous patient in the program, the first person participating will have fully prioritized access to all organs reserved for the program. Therefore, w.p. 1, the patient will receive an organ of quality $\overset{*}{q}$ with the expected utility $\beta T_D(\overset{*}{q})$. Note that $\overset{*}{q}$ should be greater than q'_D to make sure that $\beta T_D(\overset{*}{q}) > \frac{\alpha}{d_D}$. On the other hand, the expected utility of not participating in the program when $\gamma^e(\overset{*}{q}) = 0$, is given by:

$$\begin{aligned}
\beta T_D(q_N^{e,\overset{*}{q}}) &= \alpha \Delta_N \left(q_N^{e,\overset{*}{q}} \right) + \beta \pi_{D_N} \left(q_N^{e,\overset{*}{q}} \right) E \left(T_D(q) \mid q \geqslant q_N^{e,\overset{*}{q}} \right) \\
&= \frac{\alpha}{d_D} \left(1 - \frac{\int_{q_N^{e,\overset{*}{q}}}^{\bar{q}} p_D(q) dq}{\lambda_N(\bar{q} - \underline{q})} \right) + \frac{\beta}{\lambda_N(\bar{q} - \underline{q})} \int_{q_N^{e,\overset{*}{q}}}^{\bar{q}} T_D(q) p_D(q) dq \\
&= \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \int_{q_N^{e,\overset{*}{q}}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_D(q) dq.
\end{aligned} \tag{A.10}$$

Since patients act in their best interest, when $\gamma^e(\overset{*}{q}) = 0$, it follows that $q_N^{e,\overset{*}{q}} > \overset{*}{q}$, i.e.,

$$\beta T_D(q_N^{e,\overset{*}{q}}) > \beta T_D(\overset{*}{q}) \iff \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \int_{q_N^{e,\overset{*}{q}}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_D(q) dq > \beta T_D(\overset{*}{q}),$$

which implies that

$$\beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_D(q)dq > \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\int_{q_N^{e,\overset{*}{q}}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_D(q)dq,$$

and

$$\beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_D(q)dq > \beta T_D(\overset{*}{q}). \tag{A.11}$$

Because

$$\lim_{\gamma\to 0^+}\int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_N^{\gamma}(q)dq = \int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_D(q)dq,$$

together with (A.11), we have that when $\gamma^e(\overset{*}{q}) = 0$,

$$\beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\lim_{\gamma\to 0^+}\int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_N^{\gamma}(q)dq > \beta T_D(\overset{*}{q}). \tag{A.12}$$

On the other hand, when (A.12) holds, the expected utility of participation is $\beta T_D(q_P^{e,\overset{*}{q}})$ satisfying that

$$\beta T_D(q_P^{e,\overset{*}{q}}) \le \beta T_D(\overset{*}{q}) < \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\lim_{\gamma\to 0^+}\int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_N^{\gamma}(q)dq,$$

which further implies that

$$\beta T_D(q_P^{e,\overset{*}{q}}) < \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\lim_{\gamma\to 0^+}\int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_N^{\gamma}(q)dq = \beta T_D(q_N^{e,\overset{*}{q}}). \tag{A.13}$$

The inequality (A.13) implies that not participating in the program leads to a higher expected utility. Thus, no disadvantaged patients would participate in the program in equilibrium, i.e. $\gamma^e(\overset{*}{q}) = 0$. □

Proposition 3.4. *Under a targeted priority mechanism ($\overset{*}{q}$), all eligible candidates would participate in equilibrium, i.e., the equilibrium participation rate satisfies $\gamma^e(\overset{*}{q}) = 1$, if and only if*

$$\frac{\alpha}{d_D} + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \int_{q_P^{e,\overset{*}{q}}}^{\overset{*}{q}} \Big(T_D(q) - \frac{\alpha}{d_D} \Big) dq > \frac{\alpha}{d_D} + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \lim_{\gamma \to 1^-} \frac{\int_{\overset{*}{q}}^{\bar{q}} \Big(T_D(q) - \frac{\alpha}{d_D} \Big) p_N^{\gamma}(q) dq}{1 - \gamma}. \tag{3.3}$$

Proof. Proof of Proposition 3.4. The expected utility of an individual type D participating in the program, when all other type D candidates participate in the program, i.e. $\gamma^e(\overset{*}{q}) \to 1^-$, is given by:

$$\begin{aligned}
\beta T_D(q_P^{e,\overset{*}{q}}) &= \alpha \Delta_P \Big(q_P^{e,\overset{*}{q}} \Big) + \beta \pi_{D_P} \Big(q_P^{e,\overset{*}{q}} \Big) E \Big(T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_P^{e,\overset{*}{q}} \Big) \\
&= \beta T_D(q'_D) \left(1 - \frac{\overset{*}{q} - q_P^{e,\overset{*}{q}}}{\gamma \lambda_D(\bar{q} - \underline{q})} \right) + \frac{\beta}{\gamma^e(\overset{*}{q}) \lambda_D(\bar{q} - \underline{q})} \int_{q_P^{e,\overset{*}{q}}}^{\overset{*}{q}} T_D(q) dq \\
&= \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \int_{q_P^{e,\overset{*}{q}}}^{\overset{*}{q}} (T_D(q) - T_D(q'_D)) dq.
\end{aligned} \tag{A.14}$$

On the other hand, the expected utility of not participating in the program is given as follows:

$$\begin{aligned}
\beta T_D(q_N^{e,\overset{*}{q}}) &= \alpha \Delta_N \Big(q_N^{e,\overset{*}{q}} \Big) + \beta \pi_{D_N} \Big(q_N^{e,\overset{*}{q}} \Big) E \Big(T_D(q) \mid q \geqslant q_N^{e,\overset{*}{q}} \Big) \\
&= \beta T_D(q'_D) \left(1 - \frac{\int_{\overset{*}{q}}^{\bar{q}} p_N^{\gamma^e(\overset{*}{q})}(q) dq}{(1 - \gamma^e(\overset{*}{q})) \lambda_D(\bar{q} - \underline{q})} \right) + \frac{\beta}{(1 - \gamma^e(\overset{*}{q})) \lambda_D(\bar{q} - \underline{q})} \left(\int_{\overset{*}{q}}^{\bar{q}} T_D(q) p_N^{\gamma^e(\overset{*}{q})}(q) dq \right) \\
&= \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \lim_{\gamma \to 1^-} \frac{\int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma}(q) dq}{1 - \gamma}.
\end{aligned} \tag{A.15}$$

Therefore, if the following inequality holds, then participating in the program is associated with a higher utility than not participating, and all disadvantaged patients will participate

in the program in equilibrium.

$$\int_{q_P^{e,}}^{\overset{*}{q}} (T_D(q) - T_D(q'_D))dq > \lim_{\gamma \to 1^-} \frac{\int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma}(q) dq}{1-\gamma}.$$

The converse proof follows similarly, and is omitted here for brevity. □

Proposition 3.5. *Under a targeted priority mechanism $(\overset{*}{q})$, a nontrivial fraction of eligible candidates would participate in equilibrium, i.e., there exists a mixed-participation equilibrium $\gamma^e(\overset{*}{q}) \in (0,1)$, if and only if the following two inequalities hold*

$$\frac{\alpha}{d_D} + \frac{\beta}{\lambda_D(\bar{q} - \underline{q})} \lim_{\gamma \to 0^+} \int_{\overset{*}{q}}^{\bar{q}} \Big(T_D(q) - \frac{\alpha}{d_D} \Big) p_N^{\gamma}(q) dq \leq \beta T_D(\overset{*}{q}), \tag{3.4a}$$

$$\frac{\alpha}{d_D} + \frac{\beta}{\lambda_D\Big(\bar{q} - \underline{q}\Big)} \int_{q_P^{e,}}^{\overset{*}{q}} \Big(T_D(q) - \frac{\alpha}{d_D} \Big) dq \leq \frac{\alpha}{d_D} + \frac{\beta}{\lambda_D\Big(\bar{q} - \underline{q}\Big)} \lim_{\gamma \to 1^-} \frac{\int_{\overset{*}{q}}^{\bar{q}} \Big(T_D(q) - \frac{\alpha}{d_D} \Big) p_N^{\gamma}(q) dq}{1-\gamma}. \tag{3.4b}$$

Proof. Proof of Proposition 3.5. First, observe that (3.4a) and (3.4b) are the complements of (3.2) and (3.3), respectively. Therefore, if either of (3.4a) or (3.4b) does not hold, Propositions 3.3 and 3.4 establish that one of the two pure strategy equilibria –no participation or all participation– would be adopted, proving the necessity of the conditions (3.4a) and (3.4b). And the sufficiency of these conditions follow by the necessity of the conditions provided in Propositions 3.3 and 3.4 along with Theorem 3.1 and Corollary 3.2, proving the existence of a mixed-strategy equilibrium in the absence of other types of equilibrium. □

Theorem 3.6. *Two thresholds $\overset{*}{q}_l$ and $\overset{*}{q}_u$, where $\underline{q} < \overset{*}{q}_l < \overset{*}{q}_u < \bar{q}$, determine how the target threshold $\overset{*}{q}$ regulates the type of the participation equilibrium adopted by eligible patients:*

(a) *Any $\overset{*}{q} \leq \overset{*}{q}_l$ is a no-participation target threshold, i.e., $\gamma^e(\overset{*}{q}) = 0$.*

(b) *Any $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$ is a mixed-participation target threshold, i.e., $\gamma^e(\overset{*}{q}) \in (0,1)$.*

(c) *Any* $\overset{*}{q} \geq \overset{*}{q}_u$ *is a full-participation target threshold, i.e.,* $\gamma^e(\overset{*}{q}) = 1$.

(d) *The two thresholds* $\overset{*}{q}_l$ *and* $\overset{*}{q}_u$ *are the respective unique solutions for the inequalities* (3.2) *and* (3.3) *to be satisfied as an equality.*

Proof. Proof of Theorem 3.6.

(a) First, observe that the left-hand side of (3.2) is increasing in $\overset{*}{q}$ while the right-hand side is decreasing, therefore, there exists a unique solution (see Proposition 3.3) for the inequality (3.2) to be satisfied as an equality. Let $\overset{*}{q}_l$ denote this unique solution. Then the inequality (3.2) holds if and only if $\overset{*}{q} \leq \overset{*}{q}_l$, and thus, the proof follows by Proposition 3.3.

(c) Similarly, the right-hand side of equation (3.3) is a constant, while its left-hand side is increasing in $\overset{*}{q}$, and therefore, there exists a unique solution (see Proposition 3.4) for the inequality (3.3) to be satisfied as an equality, which we denote by $\overset{*}{q}_u$. Therefore, the inequality (3.3) holds if and only if $\overset{*}{q} \geq \overset{*}{q}_u$, and thus, the proof follows by Proposition 3.4.

(b) To prove (b), it suffices to prove that $\overset{*}{q}_l < \overset{*}{q}_u$, which is equivalent to showing that there exists no $\overset{*}{q}$ such that (3.1e) and (3.1f) simultaneously holds (see Theorem 3.1). We prove the result by contradiction. Assume that there exist $\overset{*}{q}$ satisfying both (3.1e) and (3.1f), i.e.,

$$U_P(q_P^{e,\overset{*}{q}}, 1) > \lim_{\gamma \to 1^-} U_N(q_N^{e,\overset{*}{q}}, \gamma), \tag{A.16a}$$

$$U_N(q_N^{e,\overset{*}{q}}, 0) > \lim_{\gamma \to 0^+} U_P(q_P^{e,\overset{*}{q}}, \gamma). \tag{A.16b}$$

Furthermore, we have

$$\lim_{\gamma \to 0^+} U_P(q_P^{e,\overset{*}{q}}, \gamma) \geq U_P(q_P^{e,\overset{*}{q}}, 1), \tag{A.17}$$

which follows by that under a fixed offer acceptance threshold, the utility of participating type P would decrease by increasing participation. Combining (A.16) and (A.17), we have

$$U_N(q_N^{e,\overset{*}{q}}, 0) > \lim_{\gamma \to 1^-} U_N(q_N^{e,\overset{*}{q}}, \gamma),$$

which gives a contradiction, because $U_N(q_N^{e,\overset{*}{q}}, 0) \leq \lim_{\gamma \to 1^-} U_N(q_N^{e,\overset{*}{q}}, \gamma)$ as under a fixed offer acceptance threshold, the utility of non-participating type N would increase by increasing participation. Thus, the proof follows.

(d) The proof follows by the definition of $\overset{*}{q}_l$ and $\overset{*}{q}_u$, combined with the proofs of parts (a)–(c).

□

Proposition 3.7. *For any targeted priority mechanism with $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$, the equilibrium participation rate of disadvantaged patients $\gamma^e(\overset{*}{q})$ is unique and increasing in the target threshold $\overset{*}{q}$.*

Proof. Proof of Proposition 3.7. We begin the proof by showing the uniqueness of $\gamma^e(\overset{*}{q})$. Theorem 3.1 and Proposition 3.5 establish that the equilibrium offer accepting thresholds, $q_i^{e,\overset{*}{q}}$, are unique under any mixed-participation equilibrium. For any targeted priority mechanism with $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$, the expected utility of a type D participating in the program is given by

$$\beta T_D(q'_D) + \frac{\beta}{\gamma^e(\overset{*}{q})\lambda_D(\bar{q} - \underline{q})} \int_{q_P^{e,\overset{*}{q}}}^{\overset{*}{q}} (T_D(q) - T_D(q'_D))dq. \tag{A.18}$$

Similarly, the expected utility of a disadvantaged patient not participating in the program

is given by

$$\beta T_D(q'_D) + \frac{\beta}{(1-\gamma^e(\overset{*}{q}))\lambda_D(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_N^{\gamma^e(\overset{*}{q})}(q)dq. \tag{A.19}$$

Therefore, given that $\frac{p_N^{\gamma}(q)}{1-\gamma}$ is non-decreasing in γ, the expected utility of a participating type D is decreasing in γ while that of a non-participating one is non-decreasing. Thus, the uniqueness of $\gamma^e(\overset{*}{q})$ follows.

Using Theorems 3.1 and 3.6, we conclude that for any targeted priority mechanism with $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$ the equilibrium offer acceptance threshold of type D, $q_D^{e,\overset{*}{q}}$, satisfies the following equations:

$$\begin{aligned}
F1 &:= \beta T_D(q_D^{e,\overset{*}{q}}) - \alpha\Delta_N\left(q_D^{e,\overset{*}{q}}\right) - \beta\pi_{D_N}\left(q_D^{e,\overset{*}{q}}\right)E(T_D(q) \mid q \geqslant \overset{*}{q}) = 0,\\
F2 &:= \beta T_D(q_D^{e,\overset{*}{q}}) - \alpha\Delta_P\left(q_D^{e,\overset{*}{q}}\right) - \beta\pi_{D_P}\left(q_D^{e,\overset{*}{q}}\right)E\left(T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_D^{e,\overset{*}{q}}\right) = 0,
\end{aligned}$$

which can be equivalently written as:

$$\begin{aligned}
F1 = \beta T_D(q_D^{e,\overset{*}{q}}) - \beta T_D(q'_D) - \frac{\beta}{(1-\gamma^e(\overset{*}{q}))\lambda_D(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}(T_D(q)-T_D(q'_D))p_N^{\gamma^e(\overset{*}{q})}(q)dq = 0,\\
F2 = \beta T_D(q_D^{e,\overset{*}{q}}) - \beta T_D(q'_D) - \frac{\beta}{\gamma^e(\overset{*}{q})\lambda_D(\bar{q}-\underline{q})}\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}(T_D(q)-T_D(q'_D))dq = 0.
\end{aligned}$$

Therefore, we have

$$\frac{\partial F1}{\partial q_D^{e,\overset{*}{q}}} = \beta \frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial q_D^{e,\overset{*}{q}}},$$
$$\frac{\partial F2}{\partial q_D^{e,\overset{*}{q}}} = \beta \frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial q_D^{e,\overset{*}{q}}} + \beta \frac{T_D(q_D^{e,\overset{*}{q}}) - T_D(q'_D)}{\gamma^e(\overset{*}{q})\lambda_D(\bar{q}-\underline{q})},$$
$$\frac{\partial F1}{\partial \gamma^e(\overset{*}{q})} = \frac{\beta}{\lambda_D(\bar{q}-\underline{q})(1-\gamma^e(\overset{*}{q}))^2}\left(\frac{\partial\Big(\int_{\overset{*}{q}}^{\bar{q}}(T_D(q'_D)-T_D(q))p_N^{\gamma^e(\overset{*}{q})}(q)dq\Big)}{\partial \gamma^e(\overset{*}{q})}(1-\gamma^e(\overset{*}{q})) + \int_{\overset{*}{q}}^{\bar{q}}(T_D(q'_D)-T_D(q))p_N^{\gamma^e}\right.$$
$$\frac{\partial F2}{\partial \gamma^e(\overset{*}{q})} = \frac{\beta}{\lambda_D\big(\bar{q}-\underline{q}\big)\big(\gamma^e(\overset{*}{q})\big)^2}\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}(T_D(q)-T_D(q'_D))dq,$$
$$\frac{\partial F1}{\partial \overset{*}{q}} = \frac{\beta}{\lambda_D\big(\bar{q}-\underline{q}\big)(1-\gamma^e(\overset{*}{q}))}(T_D(\overset{*}{q})-T_D(q'_D))p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q}),$$
$$\frac{\partial F2}{\partial \overset{*}{q}} = \frac{\beta}{\lambda_D\big(\bar{q}-\underline{q}\big)\gamma^e(\overset{*}{q})}(T_D(q'_D)-T_D(\overset{*}{q})).$$

Using the implicit function theorem, $\frac{\partial \gamma^e(\overset{*}{q})}{\partial \overset{*}{q}}$ is given by

$$\frac{\partial \gamma^*}{\partial \overset{*}{q}} = -\frac{\frac{\partial(F_1,F_2)}{\partial(\overset{*}{q},q_D^{e,\overset{*}{q}})}}{\frac{\partial(F_1,F_2)}{\partial(\gamma^e(\overset{*}{q}),q_D^{e,\overset{*}{q}})}} = -\frac{\frac{\partial F_1}{\partial \overset{*}{q}}\frac{\partial F_2}{\partial q_D^{e,\overset{*}{q}}} - \frac{\partial F_1}{\partial q_D^{e,\overset{*}{q}}}\frac{\partial F_2}{\partial \overset{*}{q}}}{\frac{\partial F_1}{\partial \gamma^e(\overset{*}{q})}\frac{\partial F_2}{\partial q_D^{e,\overset{*}{q}}} - \frac{\partial F_1}{\partial q_D^{e,\overset{*}{q}}}\frac{\partial F_2}{\partial \gamma^e(\overset{*}{q})}} =: -\frac{(num)}{(deno)}.$$

First, observe that the denominator is as follows:

$$(deno) = \frac{\beta^2}{\gamma^e(\overset{*}{q})(1-\gamma^e(\overset{*}{q}))\lambda_D^2(\bar{q}-\underline{q})}\left[\left(\frac{\partial \epsilon_1}{\partial \gamma^e(\overset{*}{q})} + \frac{\epsilon_1}{1-\gamma^e(\overset{*}{q})}\right)\frac{T_D(q_D^{e,\overset{*}{q}}) - T_D(q'_D)}{\bar{q}-\underline{q}}\right]$$
$$+ \frac{\beta^2}{\gamma^e(\overset{*}{q})(1-\gamma^e(\overset{*}{q}))\lambda_D(\bar{q}-\underline{q})}\cdot\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial(q_D^{e,\overset{*}{q}})}\left[\left(\frac{\partial \epsilon_1}{\partial \gamma^e(\overset{*}{q})} + \frac{\epsilon_1}{1-\gamma^e(\overset{*}{q})}\right)\gamma^e(\overset{*}{q}) - \frac{1-\gamma^e(\overset{*}{q})}{\gamma^e(\overset{*}{q})}\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}(T_D(q)-\right.$$

where

$$\epsilon_1 = \int_{\overset{*}{q}}^{\bar{q}}(T_D(q'_D)-T_D(q))p_N^{\gamma^e(\overset{*}{q})}(q)dq.$$

Therefore, $-(deno)$ is positive given that $\left(\frac{\partial \epsilon_1}{\partial \gamma^e(\overset{*}{q})} + \frac{\epsilon_1}{1-\gamma^e(\overset{*}{q})}\right)$ is negative, which holds under Assumption 5. Therefore the sign of $\frac{\partial \gamma^e(\overset{*}{q})}{\partial \overset{*}{q}}$ is the same as that of the numerator, which satisfies the following

$$\begin{aligned}(num) &= \frac{\partial F_1}{\partial \overset{*}{q}}\frac{\partial F_2}{\partial q_D^{e,\overset{*}{q}}} - \frac{\partial F_1}{\partial q_D^{e,\overset{*}{q}}}\frac{\partial F_2}{\partial \overset{*}{q}} \\ &= \frac{\beta^2(T_D(\overset{*}{q}) - T_D(q'_D))}{\lambda_D(\bar{q}-\underline{q})} \cdot \left[\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial (q_D^{e,\overset{*}{q}})}\left(\frac{1}{\gamma^e(\overset{*}{q})} + \frac{p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q})}{1-\gamma^e(\overset{*}{q})}\right) + \frac{T_D(q_D^{e,\overset{*}{q}}) - T_D(q'_D)}{\gamma^e(\overset{*}{q})(1-\gamma^e(\overset{*}{q}))\lambda_D\big(\bar{q}-\underline{q}\big)}\right] \\ &> 0,\end{aligned}$$

and thus, we conclude the proof that $\frac{\partial \gamma^e(\overset{*}{q})}{\partial \overset{*}{q}} > 0$. □

Proposition 3.8. *For any targeted priority mechanism $(\overset{*}{q})$, the impact of the target threshold on the equilibrium acceptance behavior of disadvantaged patients is characterized as follows:*

(a) *$\overset{*}{q} \in [\underline{q}, \overset{*}{q}_l]$, then the selection of target threshold has no impact on the equilibrium acceptance thresholds of disadvantaged patients.*

(b) *$\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$, the impact of the target threshold $\overset{*}{q}$ on the equilibrium acceptance thresholds of disadvantaged patients $q_D^{e,\overset{*}{q}} := q_P^{e,\overset{*}{q}} = q_N^{e,\overset{*}{q}}$ is determined by the sign of the following expression*

$$(1-\gamma^e(\overset{*}{q}))\int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) \cdot \frac{\partial}{\partial \gamma}\left(\frac{p_N^{\gamma}(q)}{1-\gamma}\right)\Big|_{\gamma=\gamma^e(\overset{*}{q})} dq - \frac{p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q})\int_{\overset{*}{q}}^{\bar{q}}(T_D(q) - T_D(q'_D))p_N^{\gamma^e(\overset{*}{q})}(q)dq}{1-\gamma^e(\overset{*}{q})}. \tag{3.5}$$

The equilibrium acceptance threshold $q_D^{e,\overset{*}{q}}$ decreases (increases) in the target threshold $\overset{*}{q}$ when the expression given in (3.5) is negative (positive).

(c) *$\overset{*}{q} \in [\overset{*}{q}_u, \overline{q}]$, the equilibrium acceptance threshold of disadvantaged patients $q_D^{e,\overset{*}{q}} := q_P^{e,\overset{*}{q}}$ increases in the target threshold $\overset{*}{q}$.*

Proof. Proof of Proposition 3.8.

(a)&(c) We first prove parts (a) and (c). Theorem 3.6 establishes that when $\overset{*}{q} \leq \overset{*}{q}_l$, no type D will participate in the program, and thus the selection of $\overset{*}{q}$ does not affect the equilibrium acceptance threshold. When, on the other hand, $\overset{*}{q} \geq \overset{*}{q}_u$, all type D participates in the program, and any increment of the target threshold of the program $\overset{*}{q}$ expands the pool of organs that the disadvantaged patients have priority over while keeping their participation rate the same. Therefore, for any given acceptance threshold, the right-hand side of (3.1f) would increase with increasing $\overset{*}{q}$, and thus, as per Theorem 3.1(c) the equilibrium acceptance threshold $q_D^{e,\overset{*}{q}}$ would also increase in $\overset{*}{q}$.

(b) To prove part (b), first, recall Theorem 3.6, which establishes that for any $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$, only a nontrivial fraction of type D participates in the program. Then, following similar steps to that of in the proof of Proposition 3.7, $\frac{\partial q_D^{e,\overset{*}{q}}}{\partial \overset{*}{q}}$ can be expressed as:

$$\frac{\partial q_D^{e,\overset{*}{q}}}{\partial \overset{*}{q}} = -\frac{\frac{\partial(F_1,F_2)}{\partial(\gamma^e(\overset{*}{q}),\overset{*}{q})}}{\frac{\partial(F_1,F_2)}{\partial(\gamma^e(\overset{*}{q}),q_D^{e,\overset{*}{q}})}} = -\frac{\frac{\partial F_1}{\partial \gamma^e(\overset{*}{q})}\frac{\partial F_2}{\partial \overset{*}{q}} - \frac{\partial F_1}{\partial \overset{*}{q}}\frac{\partial F_2}{\partial \gamma^e(\overset{*}{q})}}{\frac{\partial F_1}{\partial \gamma^e(\overset{*}{q})}\frac{\partial F_2}{\partial q_D^{e,\overset{*}{q}}} - \frac{\partial F_1}{\partial q_D^{e,\overset{*}{q}}}\frac{\partial F_2}{\partial \gamma^e(\overset{*}{q})}} =: -\frac{(num)}{(deno)} \tag{A.20}$$

And we have shown in the proof of Proposition 3.7 that $-(deno)$ is positive under Assumption 5. Hence the sign of $\frac{\partial q_D^{e,\overset{*}{q}}}{\partial \overset{*}{q}}$ is the same as that of the numerator, which is given by

$$\begin{aligned}(num) &= \frac{\partial F_1}{\partial \gamma^e(\overset{*}{q})}\frac{\partial F_2}{\partial \overset{*}{q}} - \frac{\partial F_1}{\partial \overset{*}{q}}\frac{\partial F_2}{\partial \gamma^e(\overset{*}{q})}, \\ &= \frac{\beta^2(T_D(\overset{*}{q}) - T_D(q'_D))}{\gamma^e(\overset{*}{q})(1-\gamma^e(\overset{*}{q}))\lambda_D^2(\bar{q}-\underline{q})^2}\left(-\frac{p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q})\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}(T_D(q)-T_D(q'_D))dq}{\gamma^e(\overset{*}{q})} - \left(\frac{\partial \epsilon_1}{\partial \gamma^e(\overset{*}{q})} + \frac{\epsilon_1}{1-\gamma^e(\overset{*}{q})}\right)\right),\end{aligned}$$

where

$$\epsilon_1 := \int_{\overset{*}{q}}^{\bar{q}} (T_D(q'_D) - T_D(q)) p_N^{\gamma^e(\overset{*}{q})}(q) dq.$$

Therefore, the expression controlling the sign of $\frac{\partial q_D^{e,\overset{*}{q}}}{\partial \overset{*}{q}}$ is given by

$$\left(-\frac{p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q}) \int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} (T_D(q) - T_D(q'_D)) dq}{\gamma^e(\overset{*}{q})} + \left(\frac{\partial \int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma^e(\overset{*}{q})}(q) dq}{\partial \gamma^e(\overset{*}{q})} + \frac{\int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma^e(\overset{*}{q})}(q) dq}{1 - \gamma^e(\overset{*}{q})} \right) \right). \tag{A.21}$$

Under a mixed equilibrium, the following holds using Theorem 3.1:

$$\frac{\int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma^e(\overset{*}{q})}(q) dq}{1 - \gamma^e(\overset{*}{q})} = \frac{\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} (T_D(q) - T_D(q'_D)) dq}{\gamma^e(\overset{*}{q})}.$$

Therefore, the expression in (A.21) can be equivalently written as:

$$\left(-\frac{p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q}) \int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma^e(\overset{*}{q})}(q) dq}{1 - \gamma^e(\overset{*}{q})} + \left(\frac{\partial \int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma^e(\overset{*}{q})}(q) dq}{\partial \gamma^e(\overset{*}{q})} + \frac{\int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma^e(\overset{*}{q})}(q) dq}{1 - \gamma^e(\overset{*}{q})} \right) \right). \tag{A.22}$$

Further simplifying the expression in (A.22), we conclude that the impact of the target threshold $\overset{*}{q}$ on the equilibrium acceptance thresholds of disadvantaged patients $q_D^{e,\overset{*}{q}} := q_P^{e,\overset{*}{q}} = q_N^{e,\overset{*}{q}}$ is determined by the following expression

$$(1 - \gamma^e(\overset{*}{q})) \int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) \cdot \frac{\partial}{\partial \gamma} \left(\frac{p_N^{\gamma}(q)}{1 - \gamma} \right) \Big|_{\gamma = \gamma^e(\overset{*}{q})} dq - \frac{p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q}) \int_{\overset{*}{q}}^{\bar{q}} (T_D(q) - T_D(q'_D)) p_N^{\gamma^e(\overset{*}{q})}(q) dq}{1 - \gamma^e(\overset{*}{q})}.$$

As a result, from (A.20), we conclude that the equilibrium acceptance threshold $q_D^{e,\overset{*}{q}}$ decreases (increases) in the target threshold $\overset{*}{q}$ when the expression given in (3.5) is negative (positive),

hence the proof follows.

□

Proposition 3.9. *For any targeted priority mechanism with target threshold* $(\overset{*}{q})$*:*

(a) *When* $\overset{*}{q} \in [\underline{q}, \overset{*}{q}_l]$*, the selection of the target threshold* $\overset{*}{q}$ *has no impact on the equilibrium acceptance threshold of non-targeted patients* $q_O^{e,\overset{*}{q}}$*.*

(b) *When* $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$*,*

(b-i) *If* $\overset{*}{q}_l < \overset{*}{q} < q_O^{e,\overset{*}{q}}$*, then the equilibrium acceptance threshold of non-targeted patients* $q_O^{e,\overset{*}{q}}$ *is increasing in the target threshold* $\overset{*}{q}$*.*

(b-ii) *If* $q_O^{e,\overset{*}{q}} \leq \overset{*}{q} < \overset{*}{q}_u$*, and* $p_N^{\gamma}(q)$ *satisfies* $\frac{\partial^2 p_N^{\gamma}(q)}{\partial \gamma^2} \geq 0$ *for any* q*, then* $q_O^{e,\overset{*}{q}}$ *is unimodal in* $\overset{*}{q}$*, attaining its mode at* $q_O^{\max}$ *which is the unique solution of* $\frac{d\, q_O^{e,\overset{*}{q}}}{d\, \gamma} = 0$*.*

(c) *When* $\overset{*}{q} \in [\overset{*}{q}_u, \overline{q}]$*, the equilibrium acceptance threshold of non-targeted patients* $q_O^{e,\overset{*}{q}}$ *is non-increasing in the target threshold* $\overset{*}{q}$*.*

Proof. Proof of Proposition 3.9. From Theorem 3.1, we have that the equilibrium offer acceptance threshold of type O, $q_O^{e,\overset{*}{q}}$, solves equation (3.1a), given by

$$\beta T_O(q_O^{e,\overset{*}{q}}) = U_O(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})) := \alpha \Delta_O\Big(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) + \beta \pi_O\Big(q_O^{e,\overset{*}{q}}, \gamma^e(\overset{*}{q})\Big) E\Big[T_O(q) \mid q \geqslant q_O^{e,\overset{*}{q}}\Big]. \tag{A.23}$$

(a) Theorem 3.6 establishes that any $\overset{*}{q} \leq \overset{*}{q}_l$ is a no-participation target threshold, and thus changing $\overset{*}{q}$ in this interval has no impact on the equilibrium offer acceptance thresholds of type D and O. Hence the proof follows.

(b) Theorem 3.6 establishes that any $\overset{*}{q}$ such that $\overset{*}{q}_l < \overset{*}{q} < \overset{*}{q}_u$ is a mixed participation threshold. If, in this case, $\overset{*}{q} \leq q_O^{e,\overset{*}{q}}$, then we can write equation (A.25) as follows:

$$F3 := \beta T_O(q_O^{e,\overset{*}{q}}) - \beta T_O(q'_O) - \frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}}(T_O(q)-T_O(q'_O))\Big(1-p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq = 0.$$

We can write its partial derivative in $\overset{*}{q}$ as follows

$$\begin{aligned}\frac{\partial F3}{\partial \overset{*}{q}} &= -\frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\frac{d\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}}(T_O(q)-T_O(q'_O))\Big(1-p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq}{d\,\overset{*}{q}} \\ &= \frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}}(T_O(q)-T_O(q'_O))\left(\frac{d\,p_N^{\gamma^e(\overset{*}{q})}(q)}{d\,\overset{*}{q}}\right)dq \\ &= \frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}}(T_O(q)-T_O(q'_O))\left(\frac{d\,p_N^{\gamma^e(\overset{*}{q})}(q)}{d\,\gamma^e(\overset{*}{q})}\frac{d\,\gamma^e(\overset{*}{q})}{d\,\overset{*}{q}}\right)dq.\end{aligned}$$

We further have

$$\begin{aligned}\frac{\partial F3}{\partial q_O^{e,\overset{*}{q}}} =& \beta\frac{d\,T_O(q_O^{e,\overset{*}{q}})}{d\,q_O^{e,\overset{*}{q}}} + \frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\left(\Big(T_O(q_O^{e,\overset{*}{q}})-T_O(q'_O)\Big)\Big(1-p_N^{\gamma^e(\overset{*}{q})}(q_O^{e,\overset{*}{q}})\Big) - \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}}(T_O(q)-T_O(q'_O))\left(\frac{d\,p_N^{\gamma^e(\overset{*}{q})}(q)}{d\,q_O^{e,\overset{*}{q}}}\right)dq\right) \\ =& \beta\frac{d\,T_O(q_O^{e,\overset{*}{q}})}{d\,q_O^{e,\overset{*}{q}}} + \frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\left(\Big(T_O(q_O^{e,\overset{*}{q}})-T_O(q'_O)\Big)\Big(1-p_N^{\gamma^e(\overset{*}{q})}(q_O^{e,\overset{*}{q}})\Big) - \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}}(T_O(q)-T_O(q'_O))\left(\frac{d\,p_N^{\gamma^e(\overset{*}{q})}(q)}{d\,\gamma^e(\overset{*}{q})}\frac{d\,\gamma^e(\overset{*}{q})}{d\,q_O^{e,\overset{*}{q}}}\right)dq\right).\end{aligned}$$

Proposition 3.7 establishes that under Assumption 5, we have $\frac{d\,\gamma^e(\overset{*}{q})}{d\,\overset{*}{q}} > 0$, and thus, $\frac{d\,\gamma^e(\overset{*}{q})}{d\,q_O^{e,\overset{*}{q}}} = 0$ and $\frac{d\,p_N^{\gamma^e(\overset{*}{q})}(q)}{d\,\gamma^e(\overset{*}{q})} < 0$ for any $\overset{*}{q} \in (\overset{*}{q}_l, \overset{*}{q}_u)$. Therefore, we have

$$\frac{\partial F3}{\partial \overset{*}{q}} < 0, \text{ and, } \frac{\partial F3}{\partial q_O^{e,\overset{*}{q}}} > 0,$$

which implies that

$$\frac{d\, q_O^{e,\overset{*}{q}}}{d\, \overset{*}{q}} = -\frac{\frac{\partial F3}{\partial \overset{*}{q}}}{\frac{\partial F3}{\partial q_O^{e,\overset{*}{q}}}} > 0.$$

If, on the other hand, we have $q_O^{e,\overset{*}{q}} \leq \overset{*}{q} < \overset{*}{q}_u$, then we can write equation (3.1a) as follows:

$$F4 := \beta T_O(q_O^{e,\overset{*}{q}}) - \beta T_O(q'_O) - \frac{\beta}{\lambda_O(\bar{q} - \underline{q})} \int_{\overset{*}{q}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq = 0.$$

We can write its partial derivative in γ as follows

$$\begin{aligned}\frac{\partial F4}{\partial \gamma} &= -\frac{\beta}{\lambda_O(\bar{q} - \underline{q})} \frac{d \int_{\overset{*}{q}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq}{d\, \gamma} \\ &= \frac{\beta}{\lambda_O(\bar{q} - \underline{q})} \Bigg(\big(T_O(\overset{*}{q}(\gamma)) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q}(\gamma))\Big) + \int_{\overset{*}{q}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\left(\frac{\partial\, p_N^{\gamma^e(\overset{*}{q})}(q)}{\partial\, \gamma}\right) dq \Bigg).\end{aligned} \tag{A.24}$$

Let $F4' := \big(T_O(\overset{*}{q}(\gamma)) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q}(\gamma))\Big)$ and $F4'' := \int_{\overset{*}{q}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\left(\frac{\partial\, p_N^{\gamma^e(\overset{*}{q})}(q)}{\partial\, \gamma}\right) dq$ respectively denote the two terms in the large parentheses in (A.24). We already showed in the previous parts of the proof that $\frac{\partial F4}{\partial \gamma} < 0$ for $\overset{*}{q} < q_O^{e,\overset{*}{q}}$, and $\frac{\partial F4}{\partial \gamma} > 0$ for $\overset{*}{q} > \overset{*}{q}_u$. Observe that $F4' > 0$ and is increasing in γ, because $T_O(\overset{*}{q}(\gamma))$ is increasing in γ and $p_N^{\gamma^e(\overset{*}{q})}(\overset{*}{q}(\gamma))$ is decreasing in γ. Moreover, $F4'' \leq 0$ and is increasing in γ when $\frac{\partial^2 p_N^{\gamma}(q)}{\partial \gamma^2} \geq 0$, because $\frac{\partial\, p_N^{\gamma^e(\overset{*}{q})}(q)}{\partial\, \gamma} < 0$ and $\frac{\partial \gamma}{\partial \overset{*}{q}} > 0$. Therefore, $\frac{\partial F4}{\partial \gamma}$ is increasing in $\overset{*}{q}$ for any $\overset{*}{q} \in [\overset{*}{q}_l, \overset{*}{q}_u]$. Thus, there exists a unique point $q_O^{max} \in [\overset{*}{q}_l, \overset{*}{q}_u]$ such that $\frac{\partial F4}{\partial \gamma} = 0$. Furthermore, we have

$$\frac{\partial F4}{\partial q_O^{e,\overset{*}{q}}} = \beta \frac{dT_O(q_O^{e,\overset{*}{q}})}{dq_O^{e,\overset{*}{q}}} > 0,$$

and

$$\frac{d\, q_O^{e,\overset{*}{q}}}{d\,\gamma} = -\frac{\frac{\partial F4}{\partial \gamma}}{\frac{\partial F4}{\partial q_O^{e,\overset{*}{q}}}}.$$

Therefore, we conclude that $q_O^{e,\overset{*}{q}}$ is increasing in $[\overset{*}{q}_l, q_O^{max}]$ and decreasing in $[q_O^{max}, \overset{*}{q}_u]$.

(c) First, recall from Theorem 3.6 that any $\overset{*}{q} \geq \overset{*}{q}_u$ is a full-participation target threshold. There are two possible cases when the system is under full participation equilibrium; namely, $\overset{*}{q} < q_O^{e,\overset{*}{q}}$ and $\overset{*}{q} \geq q_O^{e,\overset{*}{q}}$. When $\overset{*}{q} < q_O^{e,\overset{*}{q}}$, equation (3.1a) can be further written as

$$\beta T_O(q_O^{e,\overset{*}{q}}) - \beta T_O(q'_O) - \frac{\beta}{\lambda_O(\bar{q} - \underline{q})} \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\, dq = 0, \tag{A.25}$$

and therefore, changing $\overset{*}{q}$ does not affect $q_O^{e,\overset{*}{q}}$ when $q_O^{e,\overset{*}{q}} > \overset{*}{q} \geq \overset{*}{q}_u$. When $\overset{*}{q} \geq q_O^{e,\overset{*}{q}}$, on the other hand, equation (3.1a) can be written as

$$\beta T_O(q_O^{e,\overset{*}{q}}) - \beta T_O(q'_O) - \frac{\beta}{\lambda_O(\bar{q} - \underline{q})} \int_{\overset{*}{q}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\, dq = 0. \tag{A.26}$$

(A.26) implies that $T_O(q_O^{e,\overset{*}{q}}) = T_O(q'_O) + \frac{1}{\lambda_O(\bar{q}-\underline{q})} \int_{\overset{*}{q}}^{\bar{q}} (T_O(q) - T_O(q'_O))\, dq$, the right-hand side of which is decreasing in $\overset{*}{q}$ because $T_O(q)$ monotonically increases in q. Therefore, $q_O^{e,\overset{*}{q}}$ decreases in $\overset{*}{q}$ when $\overset{*}{q} \geq q_O^{e,\overset{*}{q}}$ and $\overset{*}{q} \geq \overset{*}{q}_u$. Combining these two cases, we conclude that $q_O^{e,\overset{*}{q}}$ is non-increasing in $\overset{*}{q}$ for $\overset{*}{q} \geq \overset{*}{q}_u$.

□

Lemma 3.10. *The class-separating allocations satisfy the following:*

(a) *There exists a unique threshold q_O^T such that the utility of class O remains unchanged, gets better or gets worse post the introduction of a class-separating allocation with $q^T = q_O^T$,*

$q^T < q_O^T$, or $q^T > q_O^T$, respectively.

(b) *There exists a unique threshold q_D^T such that the utility of class D remains unchanged, gets better or gets worse post the introduction of a class-separating allocation with $q^T = q_D^T$, $q^T > q_D^T$, or $q^T < q_D^T$, respectively.*

Proof. Proof of Lemma 3.10.

(a) Let q_O^T denote the solution q^T to the following equation:

$$\int_{\underline{q}}^{\bar{q}} p_O(q)\overline{T}_O(q)d\,q = \int_{q^T}^{\bar{q}} \overline{T}_O(q)d\,q. \tag{A.27}$$

Observe that any solution to (A.27) provides a threshold value that maintains the utility of the class O unchanged post the introduction of class-separating allocation with q^T. The existence and uniqueness of q_O^T simply follow by the monotonicity of $\overline{T}_O(q)$ in q.

For any $q^T < q_O^T$, we have

$$\int_{\underline{q}}^{\bar{q}} p_O(q)\overline{T}_O(q)d\,q = \int_{q_O^T}^{\bar{q}} \overline{T}_O(q)d\,q < \int_{q^T}^{\bar{q}} \overline{T}_O(q)d\,q,$$

which implies that class O is better off under any class-separating allocation with $q^T < q_O^T$. The remainder of the proof follows by symmetry and is omitted here for brevity.

(b) Similar to the previous part, let q_D^T denote the solution q^T to the following equation:

$$\int_{\underline{q}}^{\bar{q}} p_D(q)\overline{T}_D(q)d\,q = \int_{\underline{q}}^{q^T} \overline{T}_D(q)d\,q. \tag{A.28}$$

Observe that any solution to (A.28) provides a threshold value that maintains the utility of the class D unchanged post the introduction of class-separating allocation with q^T. The existence and uniqueness of q_D^T follow by the monotonicity of $\overline{T}_D(q)$ in q.

Furthermore, for any $q^T > q_D^T$, we have

$$\int_{\underline{q}}^{\bar{q}} p_D(q)\overline{T}_D(q)d\,q = \int_{\underline{q}}^{q_D^T} \overline{T}_D(q)d\,q < \int_{\underline{q}}^{q^T} \overline{T}_D(q)d\,q,$$

which implies that class D is better off under any class-separating allocation with $q^T > q_D^T$. The remainder of the proof follows by symmetry and is omitted here for brevity.

□

Theorem 3.11. *If $\frac{\overline{T}_O(q)}{\overline{T}_D(q)}$ is non-constant non-decreasing in $q \in \left[\underline{q}, \bar{q}\right]$, then we have $q_O^T > q_D^T$ and consequently any class-separating allocation with $q^T \in (q_D^T, q_O^T)$ increases the overall utility by improving the utility of both classes.*

Proof. Proof of Theorem 3.11. First, observe that Lemma 3.10 establishes that when $q_O^T \leq q_D^T$, the overall social welfare can not be improved without hurting the utility of either class. However, when $q_O^T > q_D^T$, any q^T such that $q_O^T > q^T > q_D^T$ improves the overall social welfare by increasing the utilities of both classes because the following holds;

$$\int_{\underline{q}}^{\bar{q}} p_O(q)\overline{T}_O(q)d\,q = \int_{q_O^T}^{\bar{q}} \overline{T}_O(q)d\,q < \int_{q^T}^{\bar{q}} \overline{T}_O(q)d\,q,$$
$$\int_{\underline{q}}^{\bar{q}} p_D(q)\overline{T}_D(q)d\,q = \int_{\underline{q}}^{q_D^T} \overline{T}_D(q)d\,q < \int_{\underline{q}}^{q^T} \overline{T}_D(q)d\,q.$$

Next, we prove that if $\frac{\overline{T}_O(q)}{\overline{T}_D(q)}$ is non-constant non-decreasing in $q \in \left[\underline{q}, \bar{q}\right]$, then we have $q_O^T > q_D^T$. To show the desired result, we prove that under a class-separating allocation with $q^T = q_D^T$, the utility of type O increases, which directly implies that $q^T = q_D^T < q_O^T$ using

Lemma 3.10. The following holds by the definition of q_D^T:

$$\int_{\underline{q}}^{\bar{q}} p_D(q)\overline{T}_D(q)dq = \int_{\underline{q}}^{q_D^T} \overline{T}_D(q)dq, \tag{A.29}$$

which, recalling that $p_O(q) + p_D(q) = 1$, can be equivalently written as

$$\int_{q^T}^{\bar{q}} (1 - p_O(q))\overline{T}_D(q)dq - \int_{\underline{q}}^{q_D^T} p_O(q)\overline{T}_D(q)dq = 0. \tag{A.30}$$

Defining $\alpha(q) := \frac{\overline{T}_O(q)}{\overline{T}_D(q)}$, we have

$$\begin{aligned}
\int_{q_D^T}^{\bar{q}} \overline{T}_O(q)dq - \int_{\underline{q}}^{\bar{q}} p_O(q)\overline{T}_O(q)dq &= \int_{q_D^T}^{\bar{q}} (1 - p_O(q))\overline{T}_O(q)dq - \int_{\underline{q}}^{q_D^T} p_O(q)\overline{T}_O(q)dq && \text{(A.31a)}\\
&= \int_{q_D^T}^{\bar{q}} \alpha(q)(1 - p_O(q))\overline{T}_D(q)dq - \int_{\underline{q}}^{q_D^T} \alpha(q)p_O(q)\overline{T}_D(q)dq && \text{(A.31b)}\\
&> \alpha\big(q_D^T\big) \int_{q^T}^{\bar{q}} (1 - p_O(q))\overline{T}_D(q)dq - \alpha\big(q_D^T\big) \int_{\underline{q}}^{q^T} p_O(q)\overline{T}_D(q)dq && \text{(A.31c)}\\
&= 0, && \text{(A.31d)}
\end{aligned}$$

where (A.31b) follows by the definition of $\alpha(q)$, (A.31c) follows because $\alpha(q)$ is positive, non-constant and non-decreasing, and finally, (A.31d) follows by (A.30). From (A.31), we have $\int_{q_D^T}^{\bar{q}} \overline{T}_O(q)dq > \int_{\underline{q}}^{\bar{q}} p_O(q)\overline{T}_O(q)dq$, which implies that the utility of type O increases under the class-separating allocation with $q^T = q_D^T$. Therefore, using Lemma 3.10 we conclude that $q^T = q_D^T < q_O^T$, hence the proof follows. □

Theorem 3.13. *The impact of the targeted priority mechanisms on social welfare is characterized as follows:*

(a) *For any system such that* $\overset{*}{q}_u \leq q_O^{\overline{e}}$*, the targeted priority mechanism with* $\overset{*}{q} = q_O^{\overline{e}}$ *leads to a class-separating equilibrium. Further, this mechanism maximizes social welfare if* $\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ *is non-decreasing in* $q \in [q_O^{e_0}, \overline{q}]$*, and the following inequality holds for all* $\overset{*}{q}$*:*

$$\overline{q} - q_O^{\overline{e}} \geq \int_{q_O^{e,\overset{*}{q}}}^{\overline{q}} \Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)\, dq. \tag{3.9}$$

(b) *For any system such that* $\overset{*}{q}_u > q_O^{\overline{e}}$*, no targeted priority mechanism can guarantee a class-separating equilibrium. However, if* $\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ *for* $q \in [q_O^{e_0}, \overline{q}]$*, then there exists a targeted priority mechanism that improves social welfare compared to the status quo.*

Proof. Proof of Theorem 3.13. Define q'_D and q'_O as the unique solutions to $\beta T_D(q'_D) = \frac{\alpha}{d_D}$ and $\beta T_O(q'_O) = \frac{\alpha}{d_O}$, respectively. Define $q_D^{e,q_O^{\overline{e}}}$ to be the acceptance threshold of disadvantaged patients when $\overset{*}{q}$ is set to be $q_O^{\overline{e}}$.

(a) We analyze the systems having $\overset{*}{q}_u \leq q_O^{\overline{e}}$ under two scenarios; $\overset{*}{q}_u = q_O^{\overline{e}}$ and $\overset{*}{q}_u < q_O^{\overline{e}}$.

- Under the first scenario, i.e. $\overset{*}{q}_u = q_O^{\overline{e}}$, setting $\overset{*}{q} = q_O^{\overline{e}}$ would result in all type D participating in the program ($\gamma^e(\overset{*}{q}) = 1$, see Theorem 3.6), while type O getting fully prioritized access to all organs that they are interested in, and therefore it would lead to a class-separating equilibrium.

  Any target threshold $\overset{*}{q} > q_O^{\overline{e}} = \overset{*}{q}_u$ would result in all type D participating in the program, i.e., $\gamma^e(\overset{*}{q}) = 1$ (see Theorem 3.6).

  Further, we have $q_O^{\overline{e}} > q_O^{e,\overset{*}{q}}$ for any $\overset{*}{q} > q_O^{\overline{e}}$ by Theorem 3.1 because

$$\beta T_O(q_O^{\overline{e}}) = \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\overline{q} - \underline{q})} \int_{q_O^{\overline{e}}}^{\overline{q}} \big(T_O(q) - T_O(q'_O)\big) dq,$$
$$\beta T_O(q_O^{e,\overset{*}{q}}) = \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\overline{q} - \underline{q})} \int_{q_O^{e,\overset{*}{q}}}^{\overline{q}} \big(T_O(q) - T_O(q'_O)\big) dq.$$

If for all $\overset{*}{q} > q_O^{\overline{e}}$, we have $q_D^{e,q_O^{\overline{e}}} > q_D^{e,\overset{*}{q}}$, then both patient groups are better off under the mechanism with $\overset{*}{q} = q_O^{\overline{e}}$, and therefore, $s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big) < s\Big(q_D^{e,q_O^{\overline{e}}}, q_O^{\overline{e}}\Big)$ by the definition of social welfare (see equation (3.6)). If, on the other hand, there exists $\overset{*}{q} > q_O^{\overline{e}}$ such that $q_D^{e,q_O^{\overline{e}}} \leq q_D^{e,\overset{*}{q}}$, then using Theorem 3.1, we have $q_O^{e,\overset{*}{q}}$ and $q_D^{e,\overset{*}{q}}$ satisfy:

$$\beta T_O(q_O^{e,\overset{*}{q}}) = \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)dq, \tag{A.32a}$$

$$\beta T_D(q_D^{e,\overset{*}{q}}) = \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} \big(T_D(q) - T_D(q'_D)\big)dq, \tag{A.32b}$$

and $q_O^{\overline{e}}$ and $q_D^{e,q_O^{\overline{e}}}$ satisfy:

$$\beta T_O(q_O^{\overline{e}}) = \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\int_{q_O^{\overline{e}}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)dq, \tag{A.33a}$$

$$\beta T_D(q_D^{e,q_O^{\overline{e}}}) = \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\int_{q_D^{e,q_O^{\overline{e}}}}^{q_O^{\overline{e}}} \big(T_D(q) - T_D(q'_D)\big)dq. \tag{A.33b}$$

Combining (A.32) and (A.33) with Theorem 3.1, we obtain

$$\begin{aligned}
&s\Big(q_D^{e,q_O^{\overline{e}}}, q_O^{\overline{e}}\Big) - s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big)\\
=&\Big(\lambda_O\beta T_O(q_O^{e,\overset{*}{q}}) + \lambda_D\beta T_D(q_D^{e,\overset{*}{q}})\Big) - \Big(\lambda_O\beta T_O(q_O^{e,\overset{*}{q}}) + \lambda_D\beta T_D(q_D^{e,\overset{*}{q}})\Big)\\
=&\frac{\beta}{\bar{q}-\underline{q}}\left(\int_{q_D^{e,q_O^{\overline{e}}}}^{q_D^{e,\overset{*}{q}}} \big(T_D(q) - T_D(q'_D)\big)dq + \int_{q_O^{\overline{e}}}^{\overset{*}{q}} \big(\big(T_O(q) - T_O(q'_O)\big) - \big(T_D(q) - T_D(q'_D)\big)\big)dq\right)\\
=&\frac{\beta}{\bar{q}-\underline{q}}\left(\int_{q_D^{e,q_O^{\overline{e}}}}^{q_D^{e,\overset{*}{q}}} \overline{T}_D(q)dq + \int_{q_O^{\overline{e}}}^{\overset{*}{q}} \big(\overline{T}_O(q) - \overline{T}_D(q)\big)dq\right)\\
>&0,
\end{aligned} \tag{A.34a}$$

where (A.34a) follows from that $q_D^{e,\overset{*}{q}} > q_D^{e,q_O^{\overline{e}}}$, $\overline{T}_D(q) > 0$ for $q \in [\underline{q}, \overline{q}]$, and $\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ for $q \in [q_O^{e_0}, \overline{q}]$. Therefore, for any targeted priority mechanism with $\overset{*}{q} > q_O^{\overline{e}}$, the overall

social welfare is less than that of $\overset{*}{q} = q_O^{\overline{e}}$.

On the other hand, any target threshold $\overset{*}{q} < q_O^{\overline{e}} = \overset{*}{q}_u$ would result in $q_O^{e,\overset{*}{q}} < q_O^{\overline{e}}$, because under this scenario not all type D participate in the program (i.e., $\gamma^e(\overset{*}{q}) < 1$), and therefore, type O has no longer full priority over organs $[q_O^{\overline{e}}, \overline{q}]$. If for this scenario, we have $q_D^{e,\overset{*}{q}} < q_D^{e,q_O^{\overline{e}}}$, then the mechanism with $\overset{*}{q} = q_O^{\overline{e}}$ increases the utility of both type D (given by $q_D^{e,\overset{*}{q}} < q_D^{e,q_O^{\overline{e}}}$ and Theorem 3.1) and type O (given by $q_O^{e,\overset{*}{q}} < q_O^{\overline{e}}$ and Theorem 3.1), it maximizes the social welfare. We next analyze the case when there exists a target threshold $\overset{*}{q} < q_O^{\overline{e}}$ such that $q_D^{e,\overset{*}{q}} > q_D^{e,q_O^{\overline{e}}}$. Under this case ($q_D^{e,\overset{*}{q}} > q_D^{e,q_O^{\overline{e}}}$), we have $\overset{*}{q} \leq q_O^{e,\overset{*}{q}} < q_O^{\overline{e}}$ or $q_O^{e,\overset{*}{q}} < \overset{*}{q} < q_O^{\overline{e}}$. First, if $\overset{*}{q} \leq q_O^{e,\overset{*}{q}} < q_O^{\overline{e}}$, then, from Theorem 3.1, we have $q_O^{e,\overset{*}{q}}$, $\overset{*}{q}$, and $q_D^{e,\overset{*}{q}}$ satisfy the following equations:

$$
\begin{aligned}
\beta T_O(q_O^{e,\overset{*}{q}}) &= \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\overline{q} - \underline{q})} \int_{q_O^{e,\overset{*}{q}}}^{\overline{q}} \big(T_O(q) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq, \\
\beta T_D(q_D^{e,\overset{*}{q}}) &= \beta T_D(q'_D) + \frac{\beta}{\gamma^e(\overset{*}{q})\lambda_D(\overline{q} - \underline{q})} \int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} \big(T_D(q) - T_D(q'_D)\big) dq, \\
\beta T_D(q_D^{e,\overset{*}{q}}) &= \beta T_D(q'_D) + \frac{\beta}{(1 - \gamma^e(\overset{*}{q}))\lambda_D(\overline{q} - \underline{q})} \int_{\overset{*}{q}}^{\overline{q}} \big(T_D(q) - T_D(q'_D)\big) p_N^{\gamma^e(\overset{*}{q})}(q) dq.
\end{aligned}
$$

In this case, the overall social welfare can be expressed as follows:

$$
\begin{aligned}
s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big) =& \lambda_O \beta T_O(q_O^{e,\overset{*}{q}}) + \lambda_D \beta T_D(q_D^{e,\overset{*}{q}}), \\
=& \beta\Big(\lambda_O T_O(q_O^{e,\overset{*}{q}}) + \gamma \lambda_D T_D(q_D^{e,\overset{*}{q}}) + (1 - \gamma)\lambda_D T_D(q_D^{e,\overset{*}{q}})\Big) \\
=& \beta\lambda_O T_O(q'_O) + \frac{\beta}{\overline{q} - \underline{q}} \int_{q_O^{e,\overset{*}{q}}}^{\overline{q}} \big(T_O(q) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq + \beta\lambda_D T_D(q'_D) \\
&+ \frac{\beta}{\overline{q} - \underline{q}} \int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} \big(T_D(q) - T_D(q'_D)\big) dq + \frac{\beta}{\overline{q} - \underline{q}} \int_{\overset{*}{q}}^{\overline{q}} \big(T_D(q) - T_D(q'_D)\big) p_N^{\gamma^e(\overset{*}{q})}(q) dq.
\end{aligned}
\tag{A.36}
$$

Similar to (A.36), we can write

$$s\Big(q_D^{e,q_O^{\bar{e}}},q_O^{\bar{e}}\Big) =\beta\lambda_O T_O(q'_O)+\frac{\beta}{\bar{q}-\underline{q}}\int_{q_O^{\bar{e}}}^{\bar{q}}\big(T_O(q)-T_O(q'_O)\big)dq+\beta\lambda_D T_D(q'_D)$$
$$+\frac{\beta}{\bar{q}-\underline{q}}\int_{q_D^{e,q_O^{\bar{e}}}}^{q_O^{\bar{e}}}\big(T_D(q)-T_D(q'_D)\big)dq. \tag{A.37}$$

Combining (A.36) and (A.37) with $\overset{*}{q}<q_O^{e,\overset{*}{q}}<q_O^{\bar{e}}$, $q_D^{e,\overset{*}{q}}>q_D^{e,q_O^{\bar{e}}}$, we obtain the following

$$s\Big(q_D^{e,q_O^{\bar{e}}},q_O^{\bar{e}}\Big)-s\Big(q_D^{e,\overset{*}{q}},q_O^{e,\overset{*}{q}}\Big) =\frac{\beta}{\bar{q}-\underline{q}}\left(\int_{q_D^{e,q_O^{\bar{e}}}}^{q_D^{e,\overset{*}{q}}}\overline{T}_D(q)dq+\int_{\overset{*}{q}}^{q_O^{e,\overset{*}{q}}}\overline{T}_D(q)\Big(1-p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq\right)$$
$$+\frac{\beta}{\bar{q}-\underline{q}}\int_{q_O^{e,\overset{*}{q}}}^{q_O^{\bar{e}}}\big(\overline{T}_D(q)-\overline{T}_O(q)\big)\Big(1-p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq$$
$$+\frac{\beta}{\bar{q}-\underline{q}}\int_{q_O^{\bar{e}}}^{\bar{q}}\big(\overline{T}_O(q)-\overline{T}_D(q)\big)p_N^{\gamma^e(\overset{*}{q})}(q)dq. \tag{A.38}$$

Since $\frac{\overline{T}_O(q)}{\overline{T}_D(q)}\geq 1$ is non-decreasing for $q\in\Big[q_O^{e,\overset{*}{q}},\overline{q}\Big]$, the following holds

$$\frac{\partial\Big(\frac{\overline{T}_O(q)}{\overline{T}_D(q)}\Big)}{\partial q}=\frac{\frac{\partial\overline{T}_O(q)}{\partial q}\overline{T}_D(q)-\frac{\partial\overline{T}_D(q)}{\partial q}\overline{T}_O(q)}{\overline{T}_D(q)^2}\geq 0, \tag{A.39}$$

which implies that

$$\frac{\partial\overline{T}_O(q)}{\partial q}\geq\frac{\partial\overline{T}_D(q)}{\partial q}. \tag{A.40}$$

Thus, $\overline{T}_O(q)-\overline{T}_D(q)\geq 0$ is non-decreasing for $q\in\Big[q_O^{e,\overset{*}{q}},\overline{q}\Big]$. Further, condition (3.10) implies that

Therefore, the following set of inequalities hold

$$\int_{q_O^{\overline{e}}}^{\overline{q}} \big(\overline{T}_O(q) - \overline{T}_D(q)\big) p_N^{\gamma^e(\overset{*}{q})}(q) dq + \int_{q_O^{e,\overset{*}{q}}}^{q_O^{\overline{e}}} \big(\overline{T}_D(q) - \overline{T}_O(q)\big) \Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq$$

$$\geq \big(\overline{T}_O(q_O^{\overline{e}}) - \overline{T}_D(q_O^{\overline{e}})\big) \int_{q_O^{\overline{e}}}^{\overline{q}} p_N^{\gamma^e(\overset{*}{q})}(q) dq + \int_{q_O^{e,\overset{*}{q}}}^{q_O^{\overline{e}}} \big(\overline{T}_D(q) - \overline{T}_O(q)\big) \Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq \tag{A.42a}$$

$$\geq \big(\overline{T}_O(q_O^{\overline{e}}) - \overline{T}_D(q_O^{\overline{e}})\big) \int_{q_O^{e,\overset{*}{q}}}^{q_O^{\overline{e}}} \Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq + \int_{q_O^{e,\overset{*}{q}}}^{q_O^{\overline{e}}} \big(\overline{T}_D(q) - \overline{T}_O(q)\big) \Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq \tag{A.42b}$$

$$\geq \int_{q_O^{e,\overset{*}{q}}}^{q_O^{\overline{e}}} \big(\overline{T}_O(q) - \overline{T}_D(q)\big) \Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq + \int_{q_O^{e,\overset{*}{q}}}^{q_O^{\overline{e}}} \big(\overline{T}_D(q) - \overline{T}_O(q)\big) \Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq \tag{A.42c}$$

$$= 0, \tag{A.42d}$$

where (A.42a) and (A.42c) follow from that $\overline{T}_O(q) - \overline{T}_D(q) \geq 0$ is non-decreasing for $q \in \Big[q_O^{e,\overset{*}{q}}, \overline{q}\Big]$, and (A.42b) follows from **??**. Observe that, the first line of the summation in (a) is always positive, and therefore (A.42) implies that $s\Big(q_D^{e,q_O^{\overline{e}}}, q_O^{\overline{e}}\Big) - s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big) > 0$ when $\overset{*}{q} \leq q_O^{e,\overset{*}{q}} < q_O^{\overline{e}}$. Following similar steps, we can also show that $s\Big(q_D^{e,q_O^{\overline{e}}}, q_O^{\overline{e}}\Big) - s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big) > 0$ when $q_O^{e,\overset{*}{q}} < \overset{*}{q} < q_O^{\overline{e}}$; we omitted those steps here for brevity.

In sum, the social welfare under the targeted priority mechanism with $\overset{*}{q} = q_O^{\overline{e}}$ is greater than that of any mechanism with $\overset{*}{q} > q_O^{\overline{e}}$, and those with $\overset{*}{q} < q_O^{\overline{e}}$. Therefore, the mechanism with $\overset{*}{q} = q_O^{\overline{e}}$ maximizes the social welfare.

- If, on the other hand, $\overset{*}{q}_u < q_O^{\overline{e}}$, then setting $\overset{*}{q} = q_O^{\overline{e}} > \overset{*}{q}_u$ would still result in all type D

participating in the program ($\gamma^e(\overset{*}{q}) = 1$), and leads to a class-separating equilibrium. Therefore, the proof follows by the proof of the first scenario, i.e. $\overset{*}{q}_u = q_O^{\overline{e}}$, since any target threshold $\overset{*}{q} > q_O^{\overline{e}}$ would still result in a donor-recipient mismatch, whereas any target threshold $\overset{*}{q} < q_O^{\overline{e}}$ would result in the nonuse of organs of quality $q \in \big(\overset{*}{q}, q_O^{\overline{e}}\big)$.

(b) For any system such that $\overset{*}{q}_u > q_O^{\overline{e}}$, any targeted priority mechanism with $\overset{*}{q} < \overset{*}{q}_u$ results in a nonempty set of non-participating type D ($\gamma^e(\overset{*}{q}) < 1$, see Theorem 3.6). On the other hand, by the definition of $q_O^{\overline{e}}$, any mechanism with $\overset{*}{q} \geq \overset{*}{q}_u > q_O^{\overline{e}}$ results in type O lowering their equilibrium offer acceptance threshold below $q_O^{\overline{e}} < \overset{*}{q}$. Therefore, in this case, no targeted priority mechanism can guarantee a class-separating equilibrium.

Next, we prove that if $\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ for $q \in [q_O^{e_0}, \overline{q}]$, which is equivalent to $T_O(q) - \frac{\alpha}{\beta d_O} \geq T_D(q) - \frac{\alpha}{\beta d_D}$, then setting $\overset{*}{q} = q_O^{e_0}$ improves the overall social welfare. Observe that for the targeted priority mechanism with $\overset{*}{q} = q_O^{e_0}$, if we have $q_D^{e_0} < q_D^{e,\overset{*}{q}}$, then $s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big) > s(q_D^{e_0}, q_O^{e_0})$ because $q_O^{e_0} < q_O^{e,\overset{*}{q}}$ (see Theorem 3.1), and thus we conclude that the mechanism improves the social welfare.

If, on the other hand, we have $q_D^{e_0} \geq q_D^{e,\overset{*}{q}}$, then $\overset{*}{q} = q_O^{e_0}$. Further, $q_O^{e,\overset{*}{q}}$, $\overset{*}{q}$, and $q_D^{e,\overset{*}{q}}$ satisfy the following equations

$$\beta T_O(q_O^{e,\overset{*}{q}}) = \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\bar{q} - \underline{q})} \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq, \tag{A.43a}$$

$$\beta T_D(q_D^{e,\overset{*}{q}}) = \beta T_D(q'_D) + \frac{\beta}{\gamma^e(\overset{*}{q})\lambda_D(\bar{q} - \underline{q})} \int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} \big(T_D(q) - T_D(q'_D)\big) dq, \tag{A.43b}$$

$$\beta T_D(q_D^{e,\overset{*}{q}}) = \beta T_D(q'_D) + \frac{\beta}{(1 - \gamma^e(\overset{*}{q}))\lambda_D(\bar{q} - \underline{q})} \int_{\overset{*}{q}}^{\bar{q}} \big(T_D(q) - T_D(q'_D)\big) p_N^{\gamma^e(\overset{*}{q})}(q) dq. \tag{A.43c}$$

On the other hand, $q_O^{e_0}$ and $q_D^{e_0}$ satisfy

$$\beta T_O(q_O^{e_0}) = \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\bar{q}-\underline{q})}\int_{q_O^{e_0}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)(1-p_D(q))dq, \tag{A.44a}$$

$$\beta T_D(q_D^{e_0}) = \beta T_D(q'_D) + \frac{\beta}{\lambda_D(\bar{q}-\underline{q})}\int_{q_D^{e_0}}^{\bar{q}} \big(T_D(q) - T_D(q'_D)\big)p_D(q)dq. \tag{A.44b}$$

Using (3.6), the change in overall social welfare after the introduction of the targeted priority program with a target threshold $\overset{*}{q} = q_O^{e_0}$ can be written as

$$\begin{aligned} s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big) - s(q_D^{e_0}, q_O^{e_0}) =& \lambda_O T_O(q_O^{e,\overset{*}{q}}) + \gamma\lambda_D T_D(q_D^{e,\overset{*}{q}}) + (1-\gamma)\lambda_D T_D(q_D^{e,\overset{*}{q}}) \\ & - \lambda_O T_O(q_O^{e_0}) - \lambda_D T_D(q_D^{e_0}). \end{aligned} \tag{A.45}$$

Using (A.43), we can write the following set of equations

$$\begin{aligned} & \lambda_O T_O(q_O^{e,\overset{*}{q}}) + \gamma\lambda_D T_D(q_D^{e,\overset{*}{q}}) + (1-\gamma)\lambda_D T_D(q_D^{e,\overset{*}{q}}) \\ =& \lambda_O T_O(q'_O) + \frac{1}{\bar{q}-\underline{q}}\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \overline{T}_O(q)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq + \lambda_D T_D(q'_D) + \frac{1}{\bar{q}-\underline{q}}\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} \overline{T}_D()dq \\ & + \frac{1}{\bar{q}-\underline{q}}\int_{\overset{*}{q}}^{\bar{q}} \overline{T}_D(q)p_N^{\gamma^e(\overset{*}{q})}(q)dq. \\ =& \lambda_O T_O(q'_O) + \frac{1}{\bar{q}-\underline{q}}\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \overline{T}_O(q)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq + \lambda_D T_D(q'_D) + \frac{1}{\bar{q}-\underline{q}}\int_{q_D^{e,\overset{*}{q}}}^{q_O^{e_0}} \overline{T}_D(q)dq \\ & + \frac{1}{\bar{q}-\underline{q}}\int_{q_O^{e_0}}^{\bar{q}} \overline{T}_D(q)p_N^{\gamma^e(\overset{*}{q})}(q)dq, \end{aligned} \tag{A.46}$$

where (A.46) follows because we have $\overset{*}{q} = q_O^{e_0}$.

Similarly, using (A.44) we can write

$$\begin{aligned}\lambda_O T_O(q_O^{e_0}) + \lambda_D T_D(q_D^{e_0}) =& \lambda_O T_O(q'_O) + \frac{1}{\bar{q}-\underline{q}} \int_{q_O^{e_0}}^{\bar{q}} \overline{T}_O(q)(1-p_D(q))dq \\ &+ \lambda_D T_D(q'_D) + \frac{1}{(\bar{q}-\underline{q})} \int_{q_D^{e_0}}^{\bar{q}} \overline{T}_D(q) p_D(q) dq. \end{aligned} \tag{A.47}$$

Combining (A.45)–(A.47), we have

$$\begin{aligned} s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big) - s(q_D^{e_0}, q_O^{e_0}) =& \frac{\beta}{\bar{q}-\underline{q}} \Big(\int_{q_O^{e_0}}^{\bar{q}} \big(\overline{T}_O(q) - \overline{T}_D(q)\big)\Big(p_D(q) - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq \\ &+ \int_{q_D^{e_0}}^{q_O^{e_0}} \overline{T}_D(q)(1-p_D(q))dq + \int_{q_D^{e,\overset{*}{q}}}^{q_D^{e_0}} \overline{T}_D(q) dq \Big) \\ =& \frac{\beta}{\bar{q}-\underline{q}} \Big(\int_{q_O^{e_0}}^{\bar{q}} \big(\overline{T}_O(q) - \overline{T}_D(q)\big)\Big(p_D(q) - p_N^{\gamma^e(\overset{*}{q})}(q)\Big) dq \\ &+ \int_{q_D^{e_0}}^{q_O^{e_0}} \overline{T}_D(q)(1-p_D(q))dq + \int_{q_D^{e,\overset{*}{q}}}^{q_D^{e_0}} \overline{T}_D(q) dq \Big) \\ >& 0, \end{aligned} \tag{A.48}$$

where (A.48) follows because $\overline{T}_O(q) - \overline{T}_D(q) > 0$ for $q \in [q_O^{e_0}, \overline{q}]$. Hence, we have $s\Big(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}\Big) > s(q_D^{e_0}, q_O^{e_0})$, and the proof follows.

□

Corollary 3.14. *There exists a targeted priority mechanism such that $\overset{*}{q} = q_O^{e,\overset{*}{q}}$, which satisfies the following:*

(a) *If $q_D^{e,\overset{*}{q}} > q_D^{e_0}$, then the mechanism improves the social welfare by increasing the utility of both patient groups.*

(b) *If $q_D^{e,\overset{*}{q}} \leq q_D^{e_0}$, then the mechanism increases the utilization of available organs. Further, if*

$\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ *is non-decreasing in* $q \in [q_O^{e_0}, \overline{q}]$*, and the offer probabilities satisfy*

$$\int_{q_O^{e,\overset{*}{q}}}^{\overline{q}} \left(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\right) dq \geq \int_{q_O^{e_0}}^{\overline{q}} p_O(q)\, dq, \tag{3.10}$$

then it increases the overall social welfare by providing non-targeted patients improved access to higher quality organs and increasing their utility.

Proof. Proof of Corollary 3.14. The existence of a targeted priority mechanism such that $\overset{*}{q} = q_O^{e,\overset{*}{q}}$ follows from Proposition 3.9, because we have $\overset{*}{q} > q_O^{e,\overset{*}{q}}$ for $\overset{*}{q} = \overline{q}$, while $q_O^{e,\overset{*}{q}} = q_O^{e_0} > \overset{*}{q}$ for $\overset{*}{q} = \underline{q} < \overset{*}{q}_l$.[1]

(a) For this case, using Theorem 3.1 and the definition of social welfare given in (3.6), we have the following inequalities

$$\begin{aligned} s(q_D^{e,\overset{*}{q}}, q_O^{e,\overset{*}{q}}) =& \lambda_D \beta T_D(q_D^{e,\overset{*}{q}}) + \lambda_O \beta T_O(q_O^{e,\overset{*}{q}}) && \text{(A.49a)} \\ >& \lambda_D \beta T_D(q_D^{e_0}) + \lambda_O \beta T_O(q_O^{e_0}) && \text{(A.49b)} \\ =& s(q_D^{e_0}, q_O^{e_0}), && \text{(A.49c)} \end{aligned}$$

where (A.49a) and (A.49c) follow by combining the results of Theorem 3.1 with (3.6), and (A.49b) follows because $q_D^{e,\overset{*}{q}} > q_D^{e_0}$ and $q_O^{e,\overset{*}{q}} = \overset{*}{q} > q_O^{e_0}$ (which follows by a similar set of steps to those provided in (A.32) and (A.33) in the proof of Theorem 3.13), and the overall arrival rates of types D and O are fixed. Hence, the proof follows.

(b) First, we show that $\gamma^e(\overset{*}{q}) > 0$ for $\overset{*}{q} = q_O^{e,\overset{*}{q}}$ under this case. Assume, on the contrary that $\gamma^e(\overset{*}{q}) = 0$, which implies that $\overset{*}{q} \leq \overset{*}{q}_l$ (see Theorem 3.6). If $q_D^{e_0} < \overset{*}{q} \leq \overset{*}{q}_l$, then any type D participating would enjoy an expected utility of $\beta T_D(\overset{*}{q}) > \beta T_D(q_D^{e_0})$, which contradicts with

[1] Under the assumption that $p_N^{\gamma}(q)$ satisfies $\frac{\partial^2 p_N^{\gamma}(q)}{\partial \gamma^2} \geq 0$ for any q, Proposition 3.9 implies the uniqueness of such a mechanism.

that $\gamma^e(\overset{*}{q}) = 0$ (see Proposition 3.3). And thus, we have $q_O^{e,\overset{*}{q}} = \overset{*}{q} \leq q_D^{e_0} < q_O^{e_0} = q_O^{e,\overset{*}{q}}$ (see Proposition 3.9(a)), which leads to a contradiction.

Therefore, we have that $\overset{*}{q} > \overset{*}{q}_l$ and $\gamma^e(\overset{*}{q}) > 0$, which together with the assumption that $p_N^{\gamma(\overset{*}{q})}(q)$ is decreasing in $\gamma(\overset{*}{q})$ imply that $p_N^{\gamma^e(\overset{*}{q})}(q) < p_N^{\gamma=0}(q) = p_D(q)$. Furthermore, following similar steps to those in the proof of Theorem 3.13, we can prove that $q_O^{e,\overset{*}{q}} > q_O^{e_0}$, using Theorem 3.1, since $p_N^{\gamma^e(\overset{*}{q})}(q) < p_D(q)$ and

$$\beta T_O(q_O^{e,\overset{*}{q}}) = \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\bar{q} - \underline{q})} \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq,$$
$$\beta T_O(q_O^{e_0}) = \beta T_O(q'_O) + \frac{\beta}{\lambda_O(\bar{q} - \underline{q})} \int_{q_O^{e_0}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)(1 - p_D(q))dq.$$

Observe that having $q_O^{e,\overset{*}{q}} > q_O^{e_0}$ implies that the utility of non-targeted patients group improves after the introduction of the program (see Theorem 3.1).

When $q_D^{e,\overset{*}{q}} \leq q_D^{e_0}$, we can write the flows of organs (see (A.1)) to non-targeted, participating disadvantaged, and non-participating disadvantaged patients, respectively, as follows:

$$\mu_O\Big(q_O^{e,\overset{*}{q}}\Big) = \frac{\int_{\overset{*}{q}}^{\bar{q}} \Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq}{\bar{q} - \underline{q}}, \quad \mu_P\Big(q_D^{e,\overset{*}{q}}\Big) = \frac{\overset{*}{q} - q_D^{e,\overset{*}{q}}}{\bar{q} - \underline{q}}, \quad \text{and} \quad \mu_N\Big(q_D^{e,\overset{*}{q}}\Big) = \frac{\int_{\overset{*}{q}}^{\bar{q}} p_N^{\gamma^e(\overset{*}{q})}(q)dq}{\bar{q} - \underline{q}}.$$

Similarly, the flow of organs to non-targeted and disadvantaged patients before the introduction of the program, respectively, are given as follows:

$$\mu_O(q_O^{e_0}) = \frac{\int_{q_O^{e_0}}^{\bar{q}} (1 - p_D(q))dq}{\bar{q} - \underline{q}}, \quad \text{and} \quad \mu_D(q_D^{e_0}) = \frac{\int_{q_D^{e_0}}^{\bar{q}} p_D(q)dq}{\bar{q} - \underline{q}}.$$

Therefore, the following holds:

$$\begin{aligned}\mu_O\left(q_O^{e,\overset{*}{q}}\right) &= \frac{\int_{\overset{*}{q}}^{\bar{q}}\left(1-p_N^{\gamma^e(\overset{*}{q})}(q)\right)dq}{\bar{q}-\underline{q}} \\ &> \frac{\int_{q_O^{e_0}}^{\bar{q}}(1-p_D(q))dq}{\bar{q}-\underline{q}} \qquad \text{(A.50a)} \\ &= \mu_O(q_O^{e_0}),\end{aligned}$$

where (A.50a) follows from the condition (3.10). Thus, the flow of organs allocated to non-targeted patients increases after the initiation of the program, and together with $q_O^{e,\overset{*}{q}} > q_O^{e_0}$, we have that non-targeted patients' utility improves along with their better access to higher quality organs.

Similar to the proof of Theorem 3.11, the overall social welfare before and after the introduction can be respectively written as follows:

$$s(q_O^{e_0}, q_D^{e_0}) = \lambda_O \beta T_O(q_O^{e_0}) + \lambda_D \beta T_D(q_D^{e_0}), \qquad \text{(A.51a)}$$

$$s\left(q_O^{e,\overset{*}{q}}, q_D^{e,\overset{*}{q}}\right) = \lambda_O \beta T_O(q_O^{e,\overset{*}{q}}) + \lambda_D \beta T_D(q_D^{e,\overset{*}{q}}). \qquad \text{(A.51b)}$$

Therefore, using (A.51) as well as (A.43) and (A.44) from the proof of Theorem 3.13, after some algebra, the program with $\overset{*}{q} = q_O^{e,\overset{*}{q}}$ satisfies the following:

$$
\begin{aligned}
s\Big(q_O^{e,\overset{*}{q}}, q_D^{e,\overset{*}{q}}\Big) =& \lambda_O \beta T_O(q_O^{e,\overset{*}{q}}) + \lambda_D \beta T_D(q_D^{e,\overset{*}{q}}) \\
=& \lambda_O \beta T_O(q'_O) + \frac{\beta}{\bar{q}-\underline{q}} \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \big(T_O(q) - T_O(q'_O)\big)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq + \lambda_D \beta T_D(q'_D) \\
&+ \frac{\beta}{\bar{q}-\underline{q}} \int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} \big(T_D(q) - T_D(q'_D)\big)dq + \frac{\beta}{\bar{q}-\underline{q}} \int_{\overset{*}{q}}^{\bar{q}} \big(T_D(q) - T_D(q'_D)\big) p_N^{\gamma^e(\overset{*}{q})}(q)dq \\
=& \lambda_O \beta T_O(q'_O) + \frac{\beta}{\bar{q}-\underline{q}} \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \overline{T}_O(q)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq + \lambda_D \beta T_D(q'_D) + \frac{\beta}{\bar{q}-\underline{q}} \int_{q_D^{e,\overset{*}{q}}}^{q_O^{e,\overset{*}{q}}} \overline{T}_D(q)dq \\
&+ \frac{\beta}{\bar{q}-\underline{q}} \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \overline{T}_D(q) p_N^{\gamma^e(\overset{*}{q})}(q)dq \\
>& \lambda_O \beta T_O(q'_O) + \lambda_D \beta T_D(q'_D) + \frac{\beta}{\bar{q}-\underline{q}} \int_{q_D^{e0}}^{q_O^{e0}} \overline{T}_D(q) p_D(q)dq \\
&+ \frac{\beta}{\bar{q}-\underline{q}} \left[\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \overline{T}_O(q)\Big(1 - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq + \int_{q_O^{e0}}^{q_O^{e,\overset{*}{q}}} \overline{T}_D(q)dq + \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \overline{T}_D(q) p_N^{\gamma^e(\overset{*}{q})}(q)dq \right]
\end{aligned} \tag{A.52a}
$$

$$
\begin{aligned}
=& \lambda_O \beta T_O(q'_O) + \lambda_D \beta T_D(q'_D) + \frac{\beta}{\bar{q}-\underline{q}} \int_{q_D^{e0}}^{q_O^{e0}} \overline{T}_D(q) p_D(q)dq + \frac{\beta}{\bar{q}-\underline{q}} \Bigg[\int_{q_O^{e0}}^{\bar{q}} \overline{T}_O(q)(1 - p_D(q))dq \\
&+ \int_{q_O^{e0}}^{\overline{q}} \overline{T}_D(q) p_D(q)dq + \int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \big(\overline{T}_O(q) - \overline{T}_D(q)\big)\Big(p_D(q) - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq \\
&+ \int_{q_O^{e0}}^{q_O^{e,\overset{*}{q}}} \big(\overline{T}_D(q) - \overline{T}_O(q)\big)(1 - p_D(q))dq \Bigg] \\
=& \lambda_O \beta T_O(q_O^{e0}) + \lambda_D \beta T_D(q_D^{e0}) + \frac{\beta}{\bar{q}-\underline{q}} \int_{q_D^{e0}}^{q_O^{e0}} \overline{T}_D(q) p_D(q)dq \\
&+ \frac{\beta}{\bar{q}-\underline{q}} \Bigg[\int_{q_O^{e,\overset{*}{q}}}^{\bar{q}} \big(\overline{T}_O(q) - \overline{T}_D(q)\big)\Big(p_D(q) - p_N^{\gamma^e(\overset{*}{q})}(q)\Big)dq \\
&+ \int_{q_O^{e0}}^{q_O^{e,\overset{*}{q}}} \big(\overline{T}_D(q) - \overline{T}_O(q)\big)(1 - p_D(q))dq \Bigg] \\
>& \lambda_O \beta T_O(q_O^{e0}) + \lambda_D \beta T_D(q_D^{e0}) + \frac{\beta}{\bar{q}-\underline{q}} \int_{q_D^{e0}}^{q_O^{e0}} \overline{T}_D(q) p_D(q)dq
\end{aligned} \tag{A.52b}
$$

$$
\begin{aligned}
>& \lambda_O \beta T_O(q_O^{e0}) + \lambda_D \beta T_D(q_D^{e0}) \\
=& s(q_O^{e0}, q_D^{e0})
\end{aligned} \tag{A.52c}
$$

where (A.52a) follows from that $q_D^{e,\overset{*}{q}} < q_D^{e_0}$ and $q_O^{e,\overset{*}{q}} > q_O^{e_0}$, and (A.52b) follows from condition (3.10) and that $\frac{\overline{T}_O(q)}{\overline{T}_D(q)} \geq 1$ is non-decreasing in $q \in [q_O^{e_0}, \overline{q}]$. Hence, the mechanism with $\overset{*}{q} = q_O^{e,\overset{*}{q}}$ increases the overall social welfare by providing non-targeted patients improved access to higher quality organs and increasing their utility.

□

A.3 Details of the Numerical Illustrations of the Impact of $\overset{*}{q}$ on Equilibrium Behavior

In §3.4.3, we present a numerical example to illustrate the analytical results concerning the impact of $\overset{*}{q}$ on patient equilibrium behavior. This section details the specifics of that numerical example. In particular, we set the organ allocation probability for non-participating type D individuals as $p_N^{\gamma}(q) = (0.5 - 0.04q)(1-\gamma)^{0.8}$ for any $q \in [0,1]$, adjusting based on the level of participation γ and the organ quality q. For both patient types D and O, the post-transplant life expectancy is modeled as a linear function of the organ quality, defined as $T_D(q) = T_O(q) = q$, the simplicity of which allows for straightforward comparisons. The parameters α and β represent the pre-transplant and post-transplant life factors, respectively, set with a ratio of $\frac{\alpha}{\beta} = \frac{3}{4}$. Moreover, the pre-transplant mortality rates are set to differ significantly between disadvantaged (type D) and non-targeted (type O) patients, with rates denoted as $d_D = \frac{20}{3}$ and $d_O = \frac{20}{9}$, respectively, to highlight the disparities in such health outcomes. With these selcetions, the equations $\beta T_D(q_D) = \frac{\alpha}{d_D}$ and $\beta T_O(q_O) = \frac{\alpha}{d_O}$ yield unique solutions for the quality $q_D = 0.2$ and $q_O = 0.6$, respectively. These solutions indicate the threshold levels of organ quality that equate the adjusted life expectancies to

the respective pre-transplant life for each patient type.

Proposition 4.1. *For any choice of disadvantaged patient group, $\overset{*}{q}_u$ does not exist if and only if:*

$$\int_{q_P^e, \overset{*}{q}}^{\bar{q}} \big(T_D(q) - T_D(q')\big) dq \leq \lim_{\gamma \to 1^-} \int_{\bar{q}^-}^{\bar{q}} \big(T_D(q) - T_D(q')\big) \left(-\frac{\partial p_N^{\gamma}(q)}{\partial \gamma}\right) dq.$$

And $\overset{*}{q}_l$ always exists for any choice of disadvantaged patients group.

Proof. For any choice of disadvantaged patients group, patients will not accept organs with quality lower than q'_D, since the expected utility of one disadvantaged patient receiving a transplant with quality q'_D is exactly equal to the expected utility of remaining on the waitlist and ultimately dying without transplantation. Therefore, it follows that $\overset{*}{q}_l > q'_D$. It is also true that $\overset{*}{q}_l < \bar{q}$. This is because, when the threshold for program organs is set to $\bar{q}$, and the participation rate $\gamma \to 0^+$, the very few patients who choose to join the targeted program receive absolute priority over all available organs. As a result, they are matched with high-quality organs and achieve significantly higher expected utility than those who remain outside the program. Hence, we conclude that $\overset{*}{q}_l \in (q'_D, \bar{q})$, which proves the existence of $\overset{*}{q}_l$. The condition $\overset{*}{q}_u > \overset{*}{q}_l$ is straightforward. The only scenario under which $\overset{*}{q}_u$ does not exist is when the threshold for program organs $\overset{*}{q} \to \bar{q}$, yet not all disadvantaged patients are incentivized to join the program. In such a case, the expected utility of non-participation remains at least as high as that of participation. The situation can be formally expressed as follows, after canceling common terms from the expected utility expressions for participation

and non-participation:

$$\begin{aligned}
&\int_{q_P^{e,*}}^{\bar{q}} \big(T_D(q) - T_D(q')\big) dq \\
&\leq \lim_{\gamma \to 1^-} \frac{\int_{\bar{q}^-}^{\bar{q}} (T_D(q) - T_D(q')) p_N^{\gamma}(q) dq}{1-\gamma} \\
&= \lim_{\gamma \to 1^-} \int_{\bar{q}^-}^{\bar{q}} \big(T_D(q) - T_D(q')\big) \left(-\frac{\partial p_N^{\gamma}(q)}{\partial \gamma}\right) dq
\end{aligned}$$

□

Proposition 4.2. *The no-participation threshold $\overset{*}{q}_l$ satisfies the following:*

- *$\overset{*}{q}_l$ decreases in the arrival rate of disadvantaged patients, λ_D.*
- *$\overset{*}{q}_l$ increases in the death rate of the disadvantaged group, d_D, when $\lambda_D < \frac{\int_{\overset{*}{q}_l}^{\bar{q}} p_N^0(q) dq}{\bar{q} - \underline{q}}$;*
  $\overset{*}{q}_l$ decreases in the death rate of the disadvantaged group, d_D, when $\lambda_D > \frac{\int_{\overset{*}{q}_l}^{\bar{q}} p_N^0(q) dq}{\bar{q} - \underline{q}}$.

Proof. Proof of Proposition 4.2. The lower bound of the target threshold $\overset{*}{q}_l$, and the arrival rate of disadvantaged patients λ_D satisfy the following equation:

$$F := T_D(\overset{*}{q}_l) - T_D(q') - \frac{\int_{\overset{*}{q}_l}^{\bar{q}} (T_D(q) - T_D(q')) p_N^0(q) dq}{\lambda_D\big(\bar{q} - \underline{q}\big)} = 0 \tag{A.53}$$

Therefore, we can get

$$\begin{aligned}
\frac{d\left(\overset{*}{q}_l\right)}{d\left(\lambda_D\right)} &= -\frac{F_{\lambda_D}}{F_{\overset{*}{q}_l}} \\
&= -\frac{\frac{\int_{\overset{*}{q}_l}^{\bar{q}} (T_D(q) - T_D(q')) p_N^0(q) dq}{\lambda_D^2\big(\bar{q} - \underline{q}\big)}}{\frac{d\, T_D(\overset{*}{q}_l)}{d\left(\overset{*}{q}_l\right)} + \frac{\big(T_D(\overset{*}{q}_l) - T_D(q')\big) p_N^0(\overset{*}{q}_l)}{\lambda_D\big(\bar{q} - \underline{q}\big)}}
\end{aligned}$$

By noticing that $\frac{dT_D(\overset{*}{q}_l)}{d(\overset{*}{q}_l)} > 0$ and $T_D(\overset{*}{q}_l) > T_D(q')$, we get that $\frac{d(\overset{*}{q}_l)}{d(\lambda_D)} < 0$, which means that $\overset{*}{q}_l$ decreases in λ_D.

Now we look into the impact of the death rate of disadvantaged patients d_D on $\overset{*}{q}_l$. Using the chain rule, we can get

$$\begin{aligned}\frac{d(\overset{*}{q}_l)}{d(d_D)} &= \frac{d(\overset{*}{q}_l)}{d(q')} \cdot \frac{d(q')}{d(d_D)} \\ &= \frac{d(q')}{d(d_D)} \cdot \left(-\frac{F_{q'}}{F_{\overset{*}{q}_l}}\right)\end{aligned}$$

q' satisfies that $\beta T_D(q') - \frac{\alpha}{d_D} = 0$, and therefore $\frac{d(q')}{d(d_D)} < 0$. From the previous proof, we know that $F_{\overset{*}{q}_l} > 0$. $-\frac{F_{q'}}{F_{\overset{*}{q}_l}}$ can be calculated as follows:

$$\begin{aligned}-\frac{F_{q'}}{F_{\overset{*}{q}_l}} &= \frac{\frac{dT_D(q')}{dq'} - \frac{\int_{\overset{*}{q}_l}^{\bar{q}} \frac{dT_D(q')}{dq'} \cdot p_N^0(q)dq}{\lambda_D(\bar{q}-\underline{q})}}{F_{\overset{*}{q}_l}} \\ &= \frac{1}{\lambda_{D(\bar{q}-\underline{q})}} \cdot \frac{\frac{dT_D(q')}{d(q')}\left(\lambda_D(\bar{q}-\underline{q}) - \int_{\overset{*}{q}_l}^{\bar{q}} p_N^0(q)dq\right)}{F_{\overset{*}{q}_l}}\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{d(\overset{*}{q}_l)}{d(d_D)} &= \frac{d(q')}{d(d_D)} \cdot \left(-\frac{F_{q'}}{F_{\overset{*}{q}_l}}\right) \\ &= -\frac{d(q')}{d(d_D)} \cdot \frac{1}{\lambda_D(\bar{q}-\underline{q})} \cdot \frac{\frac{dT_D(q')}{d(q')}\left(-\lambda_D(\bar{q}-\underline{q}) + \int_{\overset{*}{q}_l}^{\bar{q}} p_N^0(q)dq\right)}{F_{\overset{*}{q}_l}}\end{aligned}$$

Combining with the fact that $\frac{d(q')}{d(d_D)} < 0$, we know that $\overset{*}{q}_l$ increases in d_D when $\lambda_D < \frac{\int_{\overset{*}{q}_l}^{\bar{q}} p_N^0(q)dq}{\bar{q}-\underline{q}}$ and decreases in d_D when $\lambda_D > \frac{\int_{\overset{*}{q}_l}^{\bar{q}} p_N^0(q)dq}{\bar{q}-\underline{q}}$.

□

Proposition 4.3. *The full-participation threshold $\overset{*}{q}_u$ satisfies the following:*

- *$\overset{*}{q}_u$ increases in the arrival rate of disadvantaged patients, λ_D.*
- *$\overset{*}{q}_u$ increases in the death rate of the disadvantaged group, d_D, when $\lambda_D > \frac{\beta\left(\overset{*}{q}_u - q_P^{e,\overset{*}{q}}\right)}{\bar{q}-\underline{q}}$;*
  $\overset{*}{q}_u$ decreases in the death rate of the disadvantaged group, d_D, when $\lambda_D < \frac{\beta\left(\overset{*}{q}_u - q_P^{e,\overset{*}{q}}\right)}{\bar{q}-\underline{q}}$.

Proof. Proof of Proposition 4.3. The full-participation threshold $\overset{*}{q}_u$, and the arrival rate of disadvantaged patients λ_D satisfy the following equation:

$$F := T_D(q_P^{e,\overset{*}{q}}) - T_D(q') - \frac{\beta}{\lambda_D\big(\bar{q}-\underline{q}\big)}\int_{q_P^{e,\overset{*}{q}}}^{\overset{*}{q}_u}\big(T_D(q)-T_D(q')\big)dq = 0 \tag{A.54}$$

$\frac{d\overset{*}{q}_u}{d\lambda_D}$ can be expressed as:

$$\frac{d\overset{*}{q}_u}{d\lambda_D} = -\frac{F_{\lambda_D}}{F_{\overset{*}{q}_u}}$$

And we can write F_{λ_D} and $F_{\overset{*}{q}_u}$ as follows:

$$\begin{aligned} F_{\lambda_D} &= \frac{\beta\int_{q_P^{e,\overset{*}{q}}}^{\overset{*}{q}_u}(T_D(q)-T_D(q'))dq}{\lambda_D^2\big(\bar{q}-\underline{q}\big)} > 0, \\ F_{\overset{*}{q}_u} &= -\frac{\beta(T_D(\overset{*}{q}_u)-T_D(q'))(T_D(\overset{*}{q}_u)-T_D(q'))}{\lambda_D\big(\bar{q}-\underline{q}\big)} < 0. \end{aligned}$$

Therefore, $-\frac{F_{\lambda_D}}{F_{\overset{*}{q}_u}}$ is positive and $\overset{*}{q}_u$ increases in λ_D.

Next we look into the impact of the death rate of disadvantaged patients d_D on $\overset{*}{q}_u$. Using

the chain rule, we can get

$$\begin{aligned}\frac{d\overset{*}{q}_u}{dd_D} &= \frac{d\overset{*}{q}_u}{dq'} \cdot \frac{dq'}{dd_D} \\ &= \frac{dq'}{dd_D} \cdot \left(-\frac{F_{q'}}{F_{\overset{*}{q}_u}}\right)\end{aligned}$$

We know from the proof of Proposition 4.2 that $\frac{dq'}{dd_D}$ is negative. And we can calculate $F_{q'}$ as follows:

$$\begin{aligned}F_{q'} =& -\frac{d\,T_D(q')}{d\,(q')} + \frac{\beta}{\lambda_D\big(\bar{q}-\underline{q}\big)}\int_{q_P^{e,\overset{*}{q}}}^{\overset{*}{q}_u}\frac{d\,T_D(q')}{d\,(q')}dq \\ =& \frac{d\,T_D(q')}{d\,(q')}\left(-1+\frac{\beta\Big(\overset{*}{q}_u - q_P^{e,\overset{*}{q}}\Big)}{\lambda_D\big(\bar{q}-\underline{q}\big)}\right).\end{aligned}$$

Thus, when $q_P^{e,\overset{*}{q}} < \overset{*}{q}_u - \frac{\lambda_D\left(\bar{q}-\underline{q}\right)}{\beta}$, we have $F_{q'} > 0$; and conversely, when $q_P^{e,\overset{*}{q}} > \overset{*}{q}_u - \frac{\lambda_D\left(\bar{q}-\underline{q}\right)}{\beta}$, we have $F_{q'} < 0$. We already know from above that $F_{\overset{*}{q}_u} < 0$ and $\frac{dq'}{dd_D} < 0$. Therefore, when $q_P^{e,\overset{*}{q}} < \overset{*}{q}_u - \frac{\lambda_D\left(\bar{q}-\underline{q}\right)}{\beta}$, $\frac{d\overset{*}{q}_u}{dd_D} < 0$; and when $q_P^{e,\overset{*}{q}} > \overset{*}{q}_u - \frac{\lambda_D\left(\bar{q}-\underline{q}\right)}{\beta}$, $\frac{d\overset{*}{q}_u}{dd_D} > 0$.

□

Proposition 4.4. *Assume $\frac{\partial T_D(q)}{\partial \lambda_D} = c$ for some constant c, for all $q \in \big[\underline{q}, \bar{q}\big]$, then $q_D^{e,\overset{*}{q}}$ is decreasing in λ_D.*

Proof. The equilibrium point of disadvantaged patients, $q_D^{e,\overset{*}{q}}$, and the death rate of disadvantaged patients, λ_D, satisfy the following two equations:

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \alpha\Delta_N\Big(q_D^{e,\overset{*}{q}}\Big) - \beta\pi_{D_N}\Big(q_D^{e,\overset{*}{q}}\Big)E(T_D(q) \mid q \geqslant \overset{*}{q}) = 0 \quad (F1)$$

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \alpha\Delta_P\Big(q_D^{e,\overset{*}{q}}\Big) - \beta\pi_{D_P}\Big(q_D^{e,\overset{*}{q}}\Big)E\Big(T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_D^{e,\overset{*}{q}}\Big) = 0 \quad (F2)$$

And the equations can be written as follows:

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \beta T_D(q') - \frac{\beta}{(1-\gamma)\lambda_D(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)p_N(q)dq = 0 \quad (F1)$$

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \beta T_D(q') - \frac{\beta}{\gamma\lambda_D(\bar{q}-\underline{q})}\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}\big(T_D(q)-T_D(q')\big)dq = 0. \quad (F2)$$

We can firstly get

$$\frac{\partial F1}{\partial q_D^{e,\overset{*}{q}}} = \beta\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial q_D^{e,\overset{*}{q}}},$$
$$\frac{\partial F2}{\partial q_D^{e,\overset{*}{q}}} = \beta\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial q_D^{e,\overset{*}{q}}} + \frac{\beta\Big(T_D(q_D^{e,\overset{*}{q}}) - T_D(q')\Big)}{\gamma\lambda_D\big(\bar{q}-\underline{q}\big)}.$$

And similarly, we can get

$$\frac{\partial F1}{\partial\gamma} = -\frac{\beta}{\lambda_D(\bar{q}-\underline{q})(1-\gamma)^2}\bigg(\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)\frac{\partial p_N^{\gamma}(q)}{\partial\gamma}dq(1-\gamma) + \int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)p_N(q)dq\bigg),$$
$$\frac{\partial F2}{\partial\gamma} = \frac{\beta}{\lambda_D(\bar{q}-\underline{q})\gamma^2}\int_{q_D^{e,\overset{*}{q}}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)dq.$$

Furthermore, we can get

$$\begin{aligned}\frac{\partial F1}{\partial\lambda_D} &= \beta\bigg(\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial\lambda_D} - \frac{\partial T_D(q')}{\partial\lambda_D}\bigg)\\ &\quad - \frac{\beta}{(1-\gamma)\big(\bar{q}-\underline{q}\big)\lambda_D^2}\bigg[\lambda_D\int_{\overset{*}{q}}^{\bar{q}}\bigg(\Big(\frac{\partial T_D(q)}{\partial\lambda_D} - \frac{\partial T_D(q')}{\partial\lambda_D}\Big)p_N^{\gamma}(q) + \big(T_D(q)-T_D(q')\big)\frac{\partial p_N^{\gamma}(q)}{\partial\lambda_D}\bigg)dq\bigg]\\ &\quad + \frac{\beta}{(1-\gamma)\big(\bar{q}-\underline{q}\big)\lambda_D^2}\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)p_N^{\gamma}(q)dq,\\ \frac{\partial F2}{\partial\lambda_D} &= \beta\bigg(\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial\lambda_D} - \frac{\partial T_D(q')}{\partial\lambda_D}\bigg)\\ &\quad - \frac{\beta}{\gamma\big(\bar{q}-\underline{q}\big)\lambda_D^2}\bigg[\lambda_D\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}\Big(\frac{\partial T_D(q)}{\partial\lambda_D} - \frac{\partial T_D(q')}{\partial\lambda_D}\Big)dq - \int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}\big(T_D(q)-T_D(q')\big)dq\bigg].\end{aligned}$$

Then we can calculate the term

$$\frac{d\left(q_D^{e,\overset{*}{q}}\right)}{d\lambda_D} = \frac{\frac{\partial F1}{\partial \gamma}\frac{\partial F2}{\partial \lambda_D} - \frac{\partial F1}{\partial \lambda_D}\frac{\partial F2}{\partial \gamma}}{\frac{\partial F1}{\partial q_D^{e,\overset{*}{q}}}\frac{\partial F2}{\partial \gamma} - \frac{\partial F1}{\partial \gamma}\frac{\partial F2}{\partial q_D^{e,\overset{*}{q}}}}$$

We focus on the denominator first, $\frac{\partial F1}{\partial q_D^{e,\overset{*}{q}}}\frac{\partial F2}{\partial \gamma} - \frac{\partial F1}{\partial \gamma}\frac{\partial F2}{\partial q_D^{e,\overset{*}{q}}}$, and we notice that $\frac{\partial F1}{\partial q_D^{e,\overset{*}{q}}}$, $\frac{\partial F2}{\partial \gamma}$ and $\frac{\partial F2}{\partial q_D^{e,\overset{*}{q}}}$ are all positive. And we know

$$\begin{aligned}
-\frac{\partial F1}{\partial \gamma} &= \frac{\beta}{\lambda_D(\bar{q}-\underline{q})(1-\gamma)^2}\left(\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)\frac{\partial p_N^{\gamma}(q)}{\partial \gamma}dq(1-\gamma) + \int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)p_N^{\gamma}(q)dq\right) \\
&= \frac{\beta}{\lambda_D(\bar{q}-\underline{q})(1-\gamma)^2}\left(\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)\left(\frac{\partial p_N^{\gamma}(q)}{\partial \gamma}(1-\gamma)+p_N^{\gamma}(q)\right)dq\right)
\end{aligned}$$

When the participation is in a mixed equilibrium, we have that $\left(\frac{\partial p_N^{\gamma}(q)}{\partial \gamma}(1-\gamma)+p_N^{\gamma}(q)\right) \geq 0$ under the assumption that $\frac{p_N^{\gamma}(q)}{1-\gamma}$ is non-decreasing in γ. Therefore, we get the denominator is positive and the sign of $\frac{dq_D^{e,\overset{*}{q}}}{\partial \lambda_D}$ is decided by the numerator part. We now clearly write out the numerator part as follows:

$$\begin{aligned}
&\frac{\partial F1}{\partial \gamma}\frac{\partial F2}{\partial \lambda_D} - \frac{\partial F1}{\partial \lambda_D}\frac{\partial F2}{\partial \gamma} \\
=&\left[-\frac{\beta}{\lambda_D(\bar{q}-\underline{q})(1-\gamma)^2}\left(\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)\left(\frac{\partial p_N(q)}{\partial \gamma}(1-\gamma)+p_N(q)\right)dq\right)\right]\cdot\frac{\partial F2}{\partial \lambda_D} \\
&-\left[\frac{\beta}{\lambda_D(\bar{q}-\underline{q})\gamma^2}\int_{q_D^{e,\overset{*}{q}}}^{\bar{q}}\big(T_D(q)-T_D(q')\big)dq\right]\cdot\frac{\partial F1}{\partial \lambda_D}
\end{aligned}$$

Define $\frac{\partial T_D(q)}{\partial \lambda_D} - \frac{\partial T_D(q')}{\partial \lambda_D} := f(\lambda_D, q)$, then if $\frac{\partial T_D(q)}{\partial \lambda_D} = c$ for some constant c for all $q \in \big[\underline{q}, \bar{q}\big]$,

we have $f(\lambda_D, q) = 0$. Thus, we have the followings:

$$\frac{\partial F1}{\partial \lambda_D} = \frac{\beta}{(1-\gamma)(\bar{q}-\underline{q})\lambda_D^2} \int_{\overset{*}{q}}^{\bar{q}} \big(T_D(q) - T_D(q')\big)\left(p_N^{\gamma}(q) - \lambda_D \frac{\partial p_N^{\gamma}(q)}{\partial \lambda_D}\right) dq,$$
$$\frac{\partial F2}{\partial \lambda_D} = \frac{\beta}{\gamma(\bar{q}-\underline{q})\lambda_D^2} \left(\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}} \big(T_D(q) - T_D(q')\big) dq \right).$$

When $\frac{\partial\left(\frac{p_N^{\gamma}(q)}{\lambda_D}\right)}{\partial \lambda_D} < 0$, we have $p_N^{\gamma}(q) - \lambda_D \frac{\partial p_N^{\gamma}(q)}{\partial \lambda_D} > 0$, which further implies that $\frac{\partial F1}{\partial \lambda_D} > 0$. Recall that the sign of $\frac{dq_D^{e,\overset{*}{q}}}{d\lambda_D}$ is determined by the numerator part $\frac{\partial F1}{\partial \gamma}\frac{\partial F2}{\partial \lambda_D} - \frac{\partial F1}{\partial \lambda_D}\frac{\partial F2}{\partial \gamma}$, we can further get that $\frac{dq_D^{e,\overset{*}{q}}}{d\lambda_D} < 0$. Therefore, $q_D^{e,\overset{*}{q}}$ is decreasing in λ_D.

Observe that $\frac{\partial\left(\frac{p_N^{\gamma}(q)}{\lambda_D}\right)}{\partial \lambda_D} > 0$ implies that the allocation system becomes more likely to allocate organs to individual members of the disadvantaged group as the group size increases. In contrast, $\frac{\partial\left(\frac{p_N^{\gamma}(q)}{\lambda_D}\right)}{\partial \lambda_D} \leq 0$ suggests that as the disadvantaged group becomes larger, patients who are eligible for the program do not receive additional benefit for opting out of participation.

□

Proposition 4.5. *$q_D^{e,\overset{*}{q}}$ is increasing in d_D.*

Proof. Proof of Proposition 4.5. The equilibrium point of disadvantaged patients, $q_D^{e,\overset{*}{q}}$, and the death rate of disadvantaged patients, d_D, satisfy the following two equations:

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \alpha \Delta_N\left(q_D^{e,\overset{*}{q}}\right) - \beta \pi_{D_N}\left(q_D^{e,\overset{*}{q}}\right) E(T_D(q) \mid q \geqslant \overset{*}{q}) = 0 \quad (F1)$$
$$\beta T_D(q_D^{e,\overset{*}{q}}) - \alpha \Delta_P\left(q_D^{e,\overset{*}{q}}\right) - \beta \pi_{D_P}\left(q_D^{e,\overset{*}{q}}\right) E\left(T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_D^{e,\overset{*}{q}}\right) = 0 \quad (F2)$$

And the equations can be written as follows:

$$\beta T_D(q_D^{e,*q}) - \beta T_D(q') - \frac{\beta}{(1-\gamma^*)\lambda_D(\bar{q}-\underline{q})}\int_{*q}^{\bar{q}}\big(T_D(q)-T_D(q')\big)p_N(q)dq = 0 \quad (F1)$$

$$\beta T_D(q_D^{e,*q}) - \beta T_D(q') - \frac{\beta}{\gamma^*\lambda_D(\bar{q}-\underline{q})}\int_{q_D^{e,*q}}^{*q}\big(T_D(q)-T_D(q')\big)dq = 0 \quad (F2)$$

Assume that $T_D(q)$ does not change w.r.t. d_D,

$$\frac{d\lambda_D}{dd_D} = \frac{d\Big(\lambda\int_{d_D}^{\bar{d}} f(x)dx\Big)}{dd_D} = -\lambda f(d_D)$$
$$\frac{dp_N(q)}{dd_D} = \frac{dp_N(q)}{d\lambda_D}\frac{d\lambda_D}{dd_D} = -\lambda f(d_D)\frac{dp_N(q)}{d\lambda_D}$$

And

$$\frac{\partial(q_D^{e,*q})}{\partial(d_D)} = -\frac{\frac{\partial(F_1,F_2)}{\partial(\gamma^*,d_D)}}{\frac{\partial(F_1,F_2)}{\partial(\gamma^*,q_D^{e,*q})}}$$

$-\frac{\partial(F1,F2)}{\partial\left(\gamma^*,q_D^{e,*q}\right)} > 0$ is proved in the proof of Proposition 3.7, which means that we can focus on the numerator part of the above. We first calculate $\frac{\partial(F2)}{\partial(d_D)}$ as follows:

$$\begin{aligned}
\frac{\partial F1}{\partial d_D} &= \frac{\beta}{(1-\gamma^*)(\bar{q}-\underline{q})}\frac{\partial\left(\frac{\int_{*q}^{\bar{q}}(T_D(q')-T_D(q))p_N(q)dq}{\lambda_D}\right)}{\partial(d_D)} \\
&= \frac{\beta}{(1-\gamma^*)(\bar{q}-\underline{q})}\frac{\frac{\partial\int_{*q}^{\bar{q}}(T_D(q')-T_D(q))p_N(q)dq}{\partial d_D}\lambda_D - \int_{*q}^{\bar{q}}(T_D(q')-T_D(q))p_N(q)dq\frac{\partial\lambda_D}{\partial d_D}}{\lambda_D^2} \\
&= \frac{\beta}{(1-\gamma^*)(\bar{q}-\underline{q})}\frac{\int_{*q}^{\bar{q}}T_D(q')-T_D(q)\Big(-\frac{\partial p_N(q)}{\partial\lambda_D}\lambda\lambda_D f(d_D)\Big)dq - \int_{*q}^{\bar{q}}(T_D(q')-T_D(q))p_N(q)dq(-\lambda f(d_D))}{\lambda_D^2} \\
&= \frac{\beta\lambda f(d_D)}{(1-\gamma^*)\lambda_D^2(\bar{q}-\underline{q})}\int_{*q}^{\bar{q}}\big(T_D(q')-T_D(q)\big)\big(p_N(q)-\frac{\partial p_N(q)}{\partial\lambda_D}\lambda_D\big)dq.
\end{aligned}$$

And we obtain $\frac{\partial(F2)}{\partial(d_D)}$ as:

$$\begin{aligned}\frac{\partial(F2)}{\partial(d_D)} &= \frac{\beta}{\gamma^*(\bar{q}-\underline{q})}\frac{\partial\left(\frac{\int_{q_D^{e,*}}^{q^*}(T_D(q')-T_D(q))dq}{\lambda_D}\right)}{\partial(d_D)} \\ &= \frac{\beta}{\gamma^*(\bar{q}-\underline{q})}\frac{-\left(\int_{q_D^{e,*}}^{q^*}T_D(q')-T_D(q)dq\right)\frac{\partial\lambda_D}{\partial d_D}}{\lambda_D^2} \\ &= \frac{\beta\lambda f(d_D)}{\gamma^*\lambda_D^2(\bar{q}-\underline{q})}\left(\int_{q_D^{e,*}}^{q^*}T_D(q')-T_D(q)dq\right).\end{aligned}$$

Then we are able to calculate $\frac{\partial(F_1,F_2)}{\partial(\gamma^*,d_D)}$ as follows:

$$\begin{aligned}&\frac{\partial(F_1,F_2)}{\partial(\gamma^*,d_D)} \\ &= \frac{\partial F_1}{\partial\gamma^*}\frac{\partial F_2}{\partial d_D} - \frac{\partial F_1}{\partial d_D}\frac{\partial F_2}{\partial\gamma^*} \\ &= \left[\frac{\beta}{\lambda_D(\bar{q}-\underline{q})(1-\gamma^*)^2}\left(\frac{\partial\left(\int_{\underline{q}}^{\bar{q}}(T_D(q')-T_D(q))p_N(q)dq\right)}{\partial\gamma^*}(1-\gamma^*)+\int_{\underline{q}}^{\bar{q}}\big(T_D(q')-T_D(q)\big)p_N^{\gamma^*}(q)dq\right)\right] \\ &\cdot\left[\frac{\beta\lambda f(d_D)}{\gamma^*\lambda_D^2(\bar{q}-\underline{q})}\left(\int_{q_D^{e,*}}^{q^*}T_D(q')-T_D(q)dq\right)\right]- \\ &\left[\frac{\beta}{\lambda_D\big(\bar{q}-\underline{q}\big)(\gamma^*)^2}\int_{q_D^{e,*}}^{q^*}\big(T_D(q)-T_D(q')\big)dq\right] \\ &\cdot\left[\frac{\beta\lambda f(d_D)}{(1-\gamma^*)\lambda_D^2(\bar{q}-\underline{q})}\int_{\underline{q}}^{\bar{q}}\big(T_D(q')-T_D(q)\big)(p_N^{\gamma^*}(q)-\frac{\partial p_N^{\gamma^*}(q)}{\partial\lambda_D}\lambda_D)dq\right] \\ &= \frac{\beta^2\lambda f(d_D)}{(1-\gamma^*)\gamma^*\lambda_D^3(\bar{q}-\underline{q})^2}\left[\left(\frac{\partial\epsilon_1}{\partial\gamma^*}+\frac{\epsilon_1}{1-\gamma^*}\right)(-\epsilon_2)-\left(\frac{\epsilon_2}{\gamma^*}\right)\int_{\underline{q}}^{\bar{q}}(T_D(q')-T_D(q))\left(p_N^{\gamma^*}(q)-\frac{\partial p_N^{\gamma^*}(q)}{\partial\lambda_D}\lambda_D\right)dq\right] \\ &= \frac{-\epsilon_2\lambda\beta^2 f(d_D)}{(1-\gamma^*)\gamma^*\lambda_D^3(\bar{q}-\underline{q})^2}\left[\left(\frac{\partial\epsilon_1}{\partial\gamma^*}+\frac{\epsilon_1}{1-\gamma^*}\right)+\left(\frac{1}{\gamma^*}\right)\int_{\underline{q}}^{\bar{q}}(T_D(q')-T_D(q))\left(p_N^{\gamma^*}(q)-\frac{\partial p_N^{\gamma^*}(q)}{\partial\lambda_D}\lambda_D\right)dq\right],\end{aligned}$$

where $\epsilon_1 := \int_{\underline{q}}^{\bar{q}}(T_D(q')-T_D(q))p_N^{\gamma^*}(q)dq$, and $\epsilon_2 := \int_{q_D^{e,*}}^{q^*}(T_D(q)-T_D(q'))dq$. Under the

assumption that $\left(\frac{\partial \epsilon_1}{\partial \gamma^*} + \frac{\epsilon_1}{1-\gamma^*}\right) < 0$, which is part of the sufficient conditions that ensure the existence of the randomized equilibrium, we look into the term $p_N^{\gamma^*}(q) - \frac{\partial p_N^{\gamma^*}(q)}{\partial \lambda_D}\lambda_D$.

If $\frac{\partial\left(\frac{p_N^{\gamma}(q)}{\lambda_D}\right)}{\partial \lambda_D} \leq 0$, then we have

$$p_N^{\gamma^*}(q) - \frac{\partial p_N^{\gamma^*}(q)}{\partial \lambda_D}\lambda_D > 0.$$

Therefore, from above, we can get $\frac{\partial(F_1,F_2)}{\partial(\gamma^*,d_D)} > 0$, i.e., $\frac{\partial(q_D^{e,*}{}^{*}_{q})}{\partial(d_D)} > 0$. Now we remove the assumption that $T_D(q)$ does not change w.r.t. d_D, but instead assume that $\frac{\partial T_D(q)}{\partial d_D} = F(d_D)$, we again look into $\frac{\partial(F1)}{\partial(d_D)}, \frac{\partial(F2)}{\partial(d_D)}$:

$$\begin{aligned}
\frac{\partial F1}{\partial d_D} &= 0 + \frac{\beta}{(1-\gamma^*)(\bar{q}-\underline{q})} \frac{\partial\left(\frac{\int_{\underline{q}}^{\bar{q}}(T_D(q')-T_D(q))p_N^{\gamma^*}(q)dq}{\lambda_D}\right)}{\partial(d_D)} \\
&= \frac{\beta}{(1-\gamma^*)(\bar{q}-\underline{q})} \frac{\frac{\partial \int_{\underline{q}}^{\bar{q}}(T_D(q')-T_D(q))p_N^{\gamma^*}(q)dq}{\partial d_D}\lambda_D - \int_{\underline{q}}^{\bar{q}}(T_D(q')-T_D(q))p_N^{\gamma^*}(q)dq\frac{\partial \lambda_D}{\partial d_D}}{\lambda_D^2} \\
&= \frac{\beta\lambda f(d_D)}{(1-\gamma^*)\lambda_D^2(\bar{q}-\underline{q})}\int_{\underline{q}}^{\bar{q}}\big(T_D(q')-T_D(q)\big)(p_N^{\gamma^*}(q) - \frac{\partial p_N^{\gamma^*}(q)}{\partial \lambda_D}\lambda_D)dq,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial(F2)}{\partial(d_D)} &= \frac{\beta}{\gamma^*(\bar{q}-\underline{q})} \frac{\partial\left(\frac{\int_{q_D^{e,*}{}^{*}_{q}}^{\bar{q}}(T_D(q')-T_D(q))dq}{\lambda_D}\right)}{\partial(d_D)} \\
&= \frac{\beta}{\gamma^*(\bar{q}-\underline{q})} \frac{-\left(\int_{q_D^{e,*}{}^{*}_{q}}^{\bar{q}} T_D(q')-T_D(q)dq\right)\frac{\partial \lambda_D}{\partial d_D}}{\lambda_D^2} \\
&= \frac{\beta\lambda f(d_D)}{\gamma^*\lambda_D^2(\bar{q}-\underline{q})}\left(\int_{q_D^{e,*}{}^{*}_{q}}^{\bar{q}} T_D(q')-T_D(q)dq\right)
\end{aligned}$$

We can see that they are of the same forms as the ungeneralized $T_D(q)$ ones, therefore the derivations still hold for the generalized $T_D(q)$ version. □

Proposition 4.6. *γ^* is non-increasing in d_D.*

Proof. Proof of Proposition 4.6. The participation rate of disadvantaged patients opt into the program, γ, and the death rate of disadvantaged patients, d_D, also satisfy the following two equations:

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \alpha\Delta_N\Big(q_D^{e,\overset{*}{q}}\Big) - \beta\pi_{D_N}\Big(q_D^{e,\overset{*}{q}}\Big)E(T_D(q) \mid q \geqslant \overset{*}{q}) = 0 \quad (F1)$$

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \alpha\Delta_P\Big(q_D^{e,\overset{*}{q}}\Big) - \beta\pi_{D_P}\Big(q_D^{e,\overset{*}{q}}\Big)E\Big(T_D(q) \mid \overset{*}{q} \geqslant q \geqslant q_D^{e,\overset{*}{q}}\Big) = 0 \quad (F2)$$

And the equations can be written as follows:

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \beta T_D(q') - \frac{\beta}{(1-\gamma^*)\lambda_D(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q) - T_D(q')\big)p_N^{\gamma^*}(q)dq = 0 \quad (F1)$$

$$\beta T_D(q_D^{e,\overset{*}{q}}) - \beta T_D(q') - \frac{\beta}{\gamma^*\lambda_D(\bar{q}-\underline{q})}\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}\big(T_D(q) - T_D(q')\big)dq = 0 \quad (F2)$$

As we do in the proof of Proposition 4.5, we look into $\frac{\partial(\gamma^*)}{\partial(d_D)}$:

$$\frac{\partial(\gamma^*)}{\partial(d_D)} = -\frac{\frac{\partial(F_1,F_2)}{\partial(d_D,q_D^{e,\overset{*}{q}})}}{\frac{\partial(F_1,F_2)}{\partial(\gamma^*,q_D^{e,\overset{*}{q}})}}$$

Similar as above, we focus on the numerator, $\frac{\partial(F_1,F_2)}{\partial(d_D,q_D^{e,\overset{*}{q}})}$:

$$
\begin{aligned}
&\frac{\partial(F_1,F_2)}{\partial(d_D,q_D^{e,\overset{*}{q}})}\\
=&\frac{\partial F_2}{\partial q_D^{e,\overset{*}{q}}}\frac{\partial F_1}{\partial d_D}-\frac{\partial F_1}{\partial q_D^{e,\overset{*}{q}}}\frac{\partial F_2}{\partial d_D}\\
=&\left[\beta\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial q_D^{e,\overset{*}{q}}}+\beta\frac{T_D(q_D^{e,\overset{*}{q}})-T_D(q')}{\gamma\lambda_D(\bar{q}-\underline{q})}\right]\left[\frac{\beta\lambda f(d_D)}{(1-\gamma)\lambda_D^2(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q')-T_D(q)\big)(p_N(q)-\frac{\partial p_N(q)}{\partial\lambda_D}\lambda_D)dq\right]\\
&-\left[\beta\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial q_D^{e,\overset{*}{q}}}\right]\left[\frac{\beta\lambda f(d_D)}{\gamma\lambda_D^2(\bar{q}-\underline{q})}\left(\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}T_D(q')-T_D(q)dq\right)\right]\\
=&\frac{\beta^2\lambda f(d_D)}{\lambda_D^2\big(\bar{q}-\underline{q}\big)}\left(\frac{\partial T_D(q_D^{e,\overset{*}{q}})}{\partial q_D^{e,\overset{*}{q}}}\left(-\frac{\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}T_D(q')-T_D(q)dq}{\gamma}+\frac{\int_{\overset{*}{q}}^{\bar{q}}(T_D(q')-T_D(q))(p_N(q)-\frac{\partial p_N(q)}{\partial\lambda_D}\lambda_D)dq}{1-\gamma}\right)\right)\\
&+\frac{\beta^2\lambda f(d_D)}{\lambda_D^2\big(\bar{q}-\underline{q}\big)}\left(\frac{T_D(q_D^{e,\overset{*}{q}})-T_D(q')}{(1-\gamma)\gamma\lambda_D(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q')-T_D(q)\big)(p_N(q)-\frac{\partial p_N(q)}{\partial\lambda_D}\lambda_D)dq\right). \quad \text{(A.55)}
\end{aligned}
$$

We know that under a mixed-participation equilibrium, the following holds:

$$
\frac{\int_{q_D^{e,\overset{*}{q}}}^{\overset{*}{q}}T_D(q')-T_D(q)dq}{\gamma}=\frac{\int_{\overset{*}{q}}^{\bar{q}}(T_D(q')-T_D(q))(p_N(q)-\frac{\partial p_N(q)}{\partial\lambda_D}\lambda_D)dq}{1-\gamma}.
$$

Thus, the whole term (A.55) can be simplified to

$$
\frac{\beta^2\lambda f(d_D)}{\lambda_D^2\big(\bar{q}-\underline{q}\big)}\left(\frac{T_D(q_D^{e,\overset{*}{q}})-T_D(q')}{(1-\gamma)\gamma\lambda_D(\bar{q}-\underline{q})}\int_{\overset{*}{q}}^{\bar{q}}\big(T_D(q')-T_D(q)\big)(p_N(q)-\frac{\partial p_N(q)}{\partial\lambda_D}\lambda_D)dq\right). \quad \text{(A.56)}
$$

We know that $T_D(q_D^{e,\overset{*}{q}})>T_D(q')$ and $T_D(q')<T_D(q)$ for $q\in[\overset{*}{q},\bar{q}]$. And under **??**, $p_N(q)-\frac{\partial p_N(q)}{\partial\lambda_D}\lambda_D\geq 0$. Therefore, (A.56) ≤ 0, which further implies that $\frac{\partial(F_1,F_2)}{\partial(d_D,q_D^{e,\overset{*}{q}})}\leq 0$, and $\frac{\partial(\gamma^*)}{\partial(d_D)}\leq 0$. □