A MODEL OF THE ALUMINUM INDUSTRY
IN THE UNITED STATES

by

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I. INTRODUCTION

The situation presently faced by the aluminum industry in the United States can be summarized in the following three statements.

(1) The industry consumes over 5% of the nation's production of electricity [6]. Most (78%) of the power is purchased and, due to the present energy crisis, some utility companies are planning not to renew the existing supply contracts when they expire in the next two decades [18].

(2) The U.S. has 32% of the world's production capacity [10] and consumes 40% of the world's reported consumption [41]. To achieve this consumption, the U.S. imports about 90% of its bauxite, 35% of its alumina and 10% of its aluminum [42]. The bauxite supplying countries have recently organized themselves forming the International Bauxite Association and have applied cartel like pricing procedures in the bauxite that they export, doubling its price.

(3) In the past five years, the production capacity of the industry did not increase according to the historical rate due to the 1974-75 recession and the supply of aluminum from the Government Stockpile. Thus, the industry was left with inefficient capacity to face a forecasted 5% per year aluminum demand increase rate [1] and the constant competition from materials like steel, copper, paper and plastics, which aluminum had previously substituted. Moreover, the capital costs in new facilities have increased tremendously in the past five years [16, 5].

As a result of this scenario the aluminum industry has been the subject of several studies in recent years. Some of them deal with specific issues, for example, one on energy requirements prepared by the Battelle Columbus Laboratories [2]. Others deal with the entire industry, for example, one prepared by Charles River and Associates [16] which includes an econometric model.

Nevertheless to the present day, there has been no model developed to represent the aluminum industry as a complete time-dynamic system, in order to determine optimal trends under predetermined scenarios. The objective of the present research is to present such a model, discuss its applicability and flexibility and demonstrate its capability by examining some basic cases.

The report is organized as follows. Chapter II introduces the reader to the aluminum industry in the United States, describes the aluminum process and presents the most crucial scenarios faced by the industry. Chapter III is a literature review of the modeling of mineral industries. Chapter IV presents the structure of the model developed in this research and a small prototype example. Chapter V shows how the aluminum industry is represented by the model. Chapters VI and VII show the implementation of the model and the data structure. Chapter VIII presents the results obtained from the cases studied. Finally Chapter IX presents the conclusions extracted from the sample runs and suggests future research. A complete printout of the programs and data are given in an Appendix.

II. THE ALUMINUM INDUSTRY

A. The Aluminum Process

Aluminum is a silvery white metal, light, nontoxic, easy to machine and cast. It is non-magnetic, corrosion resistant, non-sparking and a very good conductor of electricity. It stands second among metals in the scale of malleability and sixth in ductility. In its commercial utilization it is usually alloyed with small amounts of copper, silicon or other elements to achieve properties that provide a wide variety of applications.

The extraction of aluminum from primary sources starts with the mining of ores containing alumina. The ore presently mined is bauxite which contains between forty and sixty percent alumina [1]. The alumina is present either in the monohydrate form (diaspore, boehmite) or in the trihydrate form (gibbsite). In addition it contains silicon, iron and titanium oxides.

There are other, lower graded, alumina sources presently unexploited which could potentially supply the market in the future. These are: bauxite clays, alunite, anorthosite, kaolin clays and laterite [1].

Bauxite is obtained principally (90% world wide) by open pit mining. In some cases blasting is required. In other locations the over burden is easily stripped; shovels and draglines are used to load the ore. The ore is transported by trucks, rail cars or belt conveyors to the mills where it is crushed, washed and dried to different degrees of moisture. Bauxite consumed by the chemical, refractory and abrasive industry is calcined before shipping.

The refining of bauxite to alumina is done using the Bayer process. This process was patented in 1888 by Karl Bayer and has been improved and slightly modified throughout the years. Nevertheless the basic process -- that is, the caustic leach of bauxite at elevated temperatures and pressures, separation of the sodium aluminate and selective precipitation of the alumina as alumina trihydrate (Al2O3, 3H2O) -- remains unchanged.

The silica content of the bauxite and the water of hydration have led to modifications in the original process. The American Bayer is suited for low silica, trihydrate ores. The modified Bayer process can treat bauxite with high monohydrate contents, operating at higher temperatures and pressures. For higher silica bauxite, the combination process adds a further step of sintering and leaching to recover the alumina precipitated with the silica [1, 2, 3].

The process, whose flow sheet is shown in Figure 2.1, starts with the preparation of the ore by grinding bauxite in spent liquor solution. It is then mixed with reconstituted caustic solution and heated. The mixture is allowed to digest for about an hour while the alumina contents of the bauxite dissolves as sodium aluminate. Silica is rejected from the solution and precipitates as a complex sodium aluminum silicate. Some alumina is precipitated with the silica. The precipi-

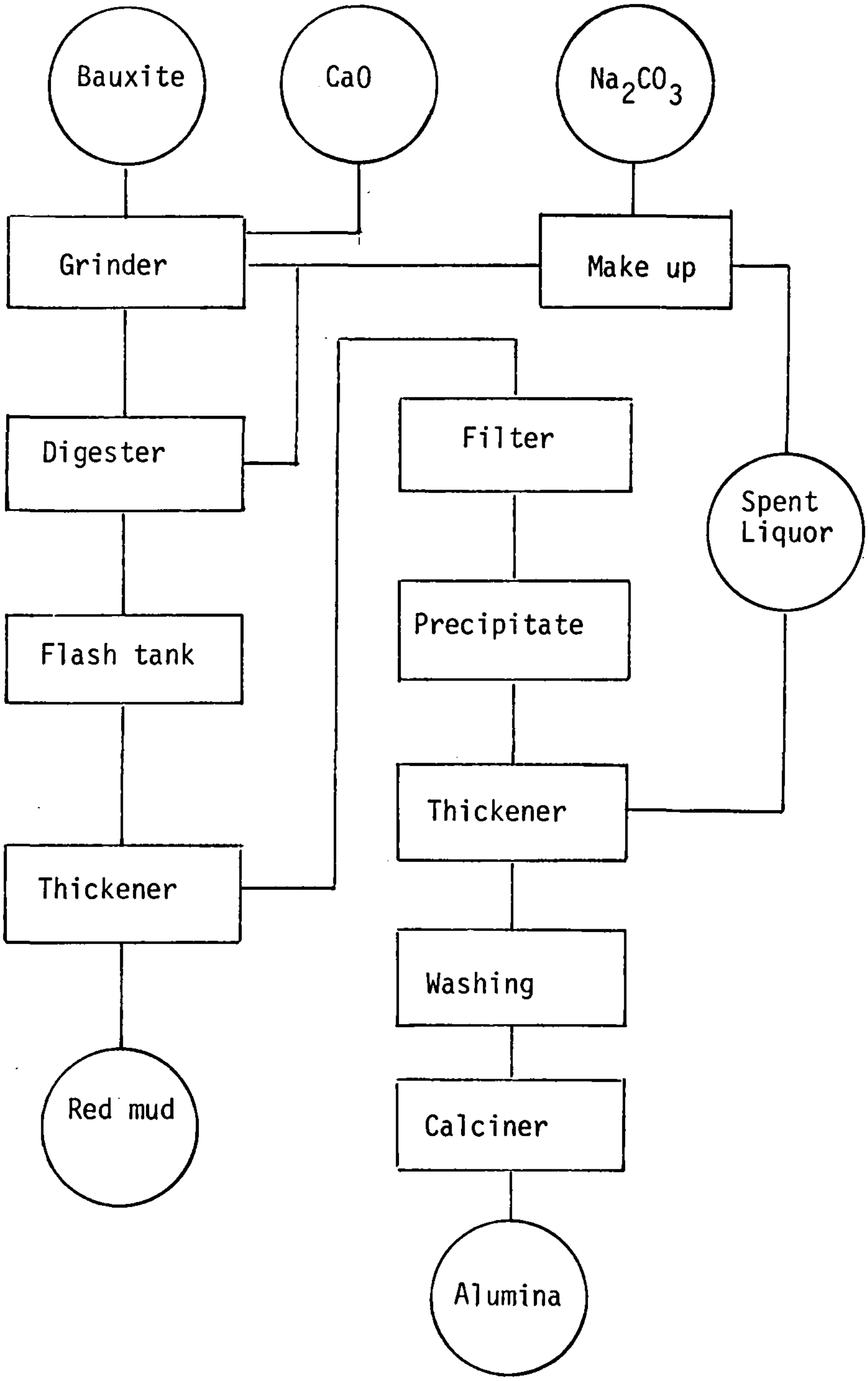


Figure 2.1 Bayer process for producing alumina

tate, called red mud, is separated by settling and filtration and is further washed to recover caustic liquor. In the combination process the red mud is further treated to recover some of the alumina precipitated.

The remaining solution is cooled and seeded with alumina trihydrate crystals to crystallize. The crystals are separated and calcined to remove water of hydration. The spent caustic solution, containing about 50% of the originally dissolved alumina, is recirculated.

Environmental problems are present with the disposal of the red mud.

Processes to recover alumina from other ores include the hydrochloric acid-ion exchange process for the kaolin clay [3, 4], the lime-sinter process for anorthosite [3, 4], and the H plus method which uses sulfuric and hydrochloric acid, to process several types of clay [1]. The concept is uniformly common, i.e. isolate the alumina from the other materials and then extract it. Presently, all existing plants are pilot plants.

Alumina is reduced to aluminum in an electrolytic process discovered by Charles Hall in the U.S. and Paul Herroult in France in 1886. In the process aluminum is deposited as a liquid on the cathode while oxygen is liberated at the anode. A flow sheet of the Hall-Herroult process is shown in Figure 2.2.

The process takes place in cells, which are large steel boxes, lined with refractory material and carbon. The cathode made of

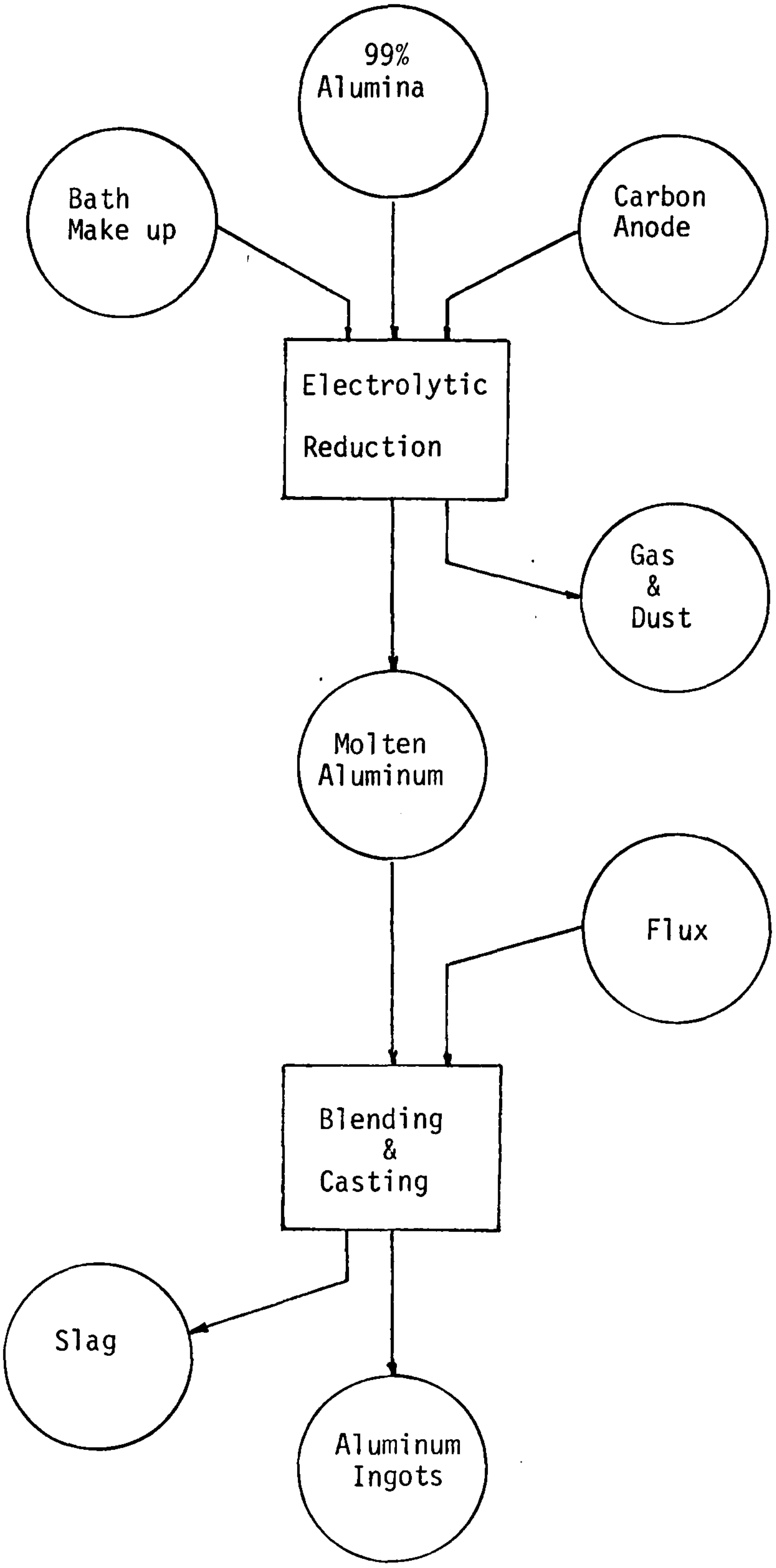


Figure 2.2 Hall-Herroult process for producing Aluminum

graphite is the bottom liner, and the molten aluminum deposits as a layer over it. The anode is made of carbon and it is consumed in the process by reacting with the liberated oxygen. Anodes and cathodes are connected to busses that distribute the current. Cells, arranged in series, form the potlines.

Two types of anodes are used by the industry: Soderberg (or continuous) and prebaked. In the first case the anode is formed above the cell, where the heat of the cell and the heat produced by the electric current, bake a mixture of petroleum pitch and petroleum coke that is continuously supplied. Soderberg anodes may be either vertical or horizontal. In the second case, prebaked anodes are prepared in separate facilities, sometimes outside the plant. The preparation of prebaked anodes consumes less energy and these anodes permit a better control of the effluent gases (fluoride gases) of the smelter.

Alumina is continuously supplied to the electrolyte. The electrolyte is a pre-eutectic formed by cryolite (80-85%), calcium fluoride (5-7%), aluminum fluoride (5-7%) and alumina (2-8%). Magnesium or lithium fluoride may also be present in small amounts to improve conductivity. Working temperature is about 950°C.

The cells use direct current ranging from 100,000 to 200,000 amperes [1, 5] and normal voltage drop per cell ranges between 4 and 5 volts. Power consumption ranges from 6 to 9 KWH per lb of aluminum, the lower figures obtained with computerized control of prebaked electrodes in large cells.

There are many factors which affect the voltage drop and hence the power consumption. Some of the more prominent are: Concentration of alumina of less than 2% which causes voltage drops on the order of 20 volts or more. The back reaction at the anode, caused by the presence of aluminum in the electrolyte due to turbulence, which reduces Faraday efficiencies (aluminum is oxidized back to alumina); if the distance between electrodes increases, the reaction decreases but the voltage drop increases. Furthermore there is a voltage drop at each connector of the electric circuit.

Research is currently under progress in areas like the so called refractory cathodes (i.e. titanium diboride) to reduce voltage drop, and in better control of the process. In addition, a process that electrolytically reduces aluminum chloride is presently being tested in small experimental plants. Power consumption for this process has been reported as 4.5 to 5 KWH per lb of aluminum [2], in contrast to the 6 to 9 KWH per lb required in the original process.

The molten aluminum deposited on the cathode is extracted by vacuum techniques and cast into ingots or billets or transported in molten form.

Before the metal reaches the end use market it may be wrought, cast or turned to flake and powder for explosives or flares. Aluminum is wrought by hot or cold rolling into sheet, plate or foil, by further rolling into rod, bar, wire or cable, by drawing or welding into tubes and by extrusion into pipe, tube, rod or other shapes. Aluminum castings are principally made from sand, die and permanent molds.

The fabricated product then reaches the industrial consumers.

Another form of supply to the aluminum cycle is scrap. Scrap is continuously generated during the fabrication processes and the industrial consumption of the metal. These cuttings, borings and turnings are, for the most part, recycled within the mills. This scrap is known as runaround scrap and never enters the scrap market. Those users which do not own melting facilities generally sell their scrap, either to dealers or to primary or secondary producers. This scrap is known as new scrap.

Scrap is also generated by the obsolescence of end use products. These may be collected at junk yards by scrap dealers and classified according to purity. This is known as old scrap.

Aluminum scrap is the source of secondary aluminum. This aluminum is processed by secondary smelters, primary producers and also fabricators. Its energy requirements are substantially lower than those of primary aluminum (8×10^6 BTU per ton of secondary aluminum compared to 240×10^6 BTU per ton of primary aluminum) [2].

Scrap which requires refining is segregated before processing. It is usually crushed or shredded. In some instances it is dried. It then passes through a magnetic separator and goes into a reverberatory furnace. Here the scrap is molten, blended with other scrap to reach alloy specifications, degassed and refined (removal of magnesium). After these steps the metal is poured into ingots, or sometimes it is shipped in molten form to the users. Secondary aluminum is used principally for castings.

B. The Aluminum Industry in the U.S.

The aluminum industry in the United States originated with the Aluminum Company of America (Alcoa) simultaneously (1893) with the discovery of the Hall process. Alcoa was the sole producer until the beginning of the Second World War. At this time the government built the refineries and smelters for war needs and contracted their operations to Kaiser Aluminum and Chemical Co. and Reynolds Metals Co.

These companies, Alcoa first, Kaiser and Reynolds following, took advantage of their position in the market and purchased bauxite deposits (domestic and foreign) along with hydroelectric power sources, completing a backward integration [6].

By mid-century Alcoa and its Canadian partner, Aluminum Limited (Alted), which owned Alcan, accounted for over 70% of world production capacity. They were severed in 1950 when Judge Knox required the stockholders who owned shares from both these companies to choose between one or the other. From that moment, Alcoa and Alted became two separate companies. The Korean War enabled three more firms to enter the market: Anaconda, Ormet and Harvey (later acquired by Martin Marietta).

Year	
1955	Anaconda Aluminum Co., a wholly owned subsidiary of Anaconda Co.
1958	Ormet; owned 50% by Revere Copper & Brass, 50% by Olin Mathieson, but two thirds of output controlled by Olin, one third by Revere. Olin's interests later acquired by Conalco
1958	Harvey Aluminum Inc., controlling interest obtained by Martin Marietta in 1969; later became a 100% owned subsidiary of Martin Marietta
1963	Conalco (Consolidated); originally owned 100% by Alusuisse. Phelps Dodge later acquired 40% interest
1966	Howmet; owned 40% by Pechiney (it owned 50% of Intalco). Pechiney now owns 100% of Howmet
1966	Amax; owned 50% of Intalco. Later became Alumax, now owned 50% by Amax, 45% by Mitsui and 5% by Nippon Steel
1969	National-Southwire; output divided between National Steel, 50%, and Southwire, 50%. National Steel owns 20.2% of Southwire
1971	Noranda Aluminum Incorporated

Figure 2.3

Entry of New Firms into Primary Aluminum Production

In the 1960's the two largest firms outside North America, Pechiney Ugine Kuhlman and Suisse Aluminum (Alusuisse), entered the American market with totally or partially owned smelters. Figure 2.3 shows how these entrances took place.

Presently there are 12 firms which own smelting facilities in the United States in 31 different plants, with a total production capacity per year of 5,175,000 short tons. Figure 2.4 and Table 2.1 show their location, individual capacities and ownership. Of these firms, seven are vertically integrated from mine to smelter (Alcoa, Kaiser, Reynolds, Martin Marietta, Noranda, Anaconda, and Revere). The smelters are located in three distinct areas: The Pacific Northwest, the east along the Mississippi and Ohio rivers and the northeast, at or near Massena, New York.

Three main criteria determine the location of a smelter: proximity to end use market, availability of low cost power and access to a river system for the transportation of alumina [5]. These factors are present in a greater or lesser degree in each of these regions. The Pacific Northwest and the Northeast have access to (what once was) inexpensive hydroelectric power, the southeast obtains inexpensive electric power from TVA, and the Ohio and Mississippi rivers, on the one hand, and the St. Lawrence river, on the other hand, allow the access of ships transporting alumina to the eastern and northeastern plants.

Five companies own refining plants in the U.S. All of them use the Bayer process, modified according to the origin of the bauxite processed.

Table 2.1

Smelters in the U.S.

Company and Plant	Location	Capacity at year end 1976 (short tons)	Ownership
Aluminum Co. of America (Alcoa)			Self, 100%
Alcoa	Tenn.	270000	
Palestine	Texas	15000	
Badin	N.C.	120000	
Evansville	Ind.	280000	
Massena	N.Y.	205000	
Pt.Comfort	Texas	180000	
Rockdale	Texas	295000	
Vancouver	Wash.	115000	
Wenatchee	Wash.	195000	
	Total	1675000	
Anaconda Aluminum Co.			Self, 100%
Columbia Falls	Mont.	180000	
Sebree	Ktky.	120000	
	Total	300000	
Consolidated Aluminum Inc.			Swiss Aluminum Ltd. 50%, Phelps Dodge Corp. 50%
New Johnsonville	Tenn.	145500	
Lake Charles	La.	36500	
	Total	182000	
Eastalco Aluminum Co.			Pechiney, Ugine Kuhlman 70%, Howmet Corp. 30%
Frederick	Md.	176000	
	Total	176000	
Intalco Aluminum Corp.			Pechiney, Ugine Kuhlman 35%, Amax Inc.25% Howmet Corp. 15%, Mitsui & Co. 25%
Fernale	Wash.	261000	
	Total	261000	

Table 2.1 (cont'd)

Company and Plant	Location		Capacity at year end 1976 (short tons)	Ownership
Kaiser Aluminum & Chemical Corp.				Self, 100%
Chalmette	La.		260000	
Mead	Wash.		220000	
Ravenswood	W.Va.		163000	
Tacoma	Wash.		81000	
		Total	724000	
Martin Marietta Aluminum Inc.				Martin Marietta 87.2%, Private interest 12.8%
The Dalles	Ore.		90000	
Goldendale	Wash.		110000	
		Total	200000	
National Southwire Aluminum Co.				National Steel Corp. 50% Southwire Corp. 50%
Hawesvill	Ktky.		180000	
		Total	180000	
Noranda Aluminum Inc.				Noranda Mines Ltd. 100%
New Madrid	Mo.		140000	
		Total	140000	
Ormet Corp.				Swiss Aluminum Ltd. 40%, Revere Copper & Brass Inc. 34%, Phelps Dodge Corp. 26%
Hannibal	Ohio		250000	
		Total	250000	
Revere Copper & Brass Inc.				Self, 100%
Scottsboro	Ala		112000	
		Total	112000	

Table 2.1 (cont'd)

Company and Plant	Location	Capacity at yearend 1976 (short tons)	Ownership
Reynolds Metal Co.			Self, 100%
Arkadelphia	Ark.	68000	
Jones Mills	Ark.	125000	
Listerhill	Ala.	202000	
Longview	Wash.	210000	
Massena	N.Y.	126000	
Corpus Christi	Texas	114000	
Troutdale	Ore.	130000	
	Total	975000	

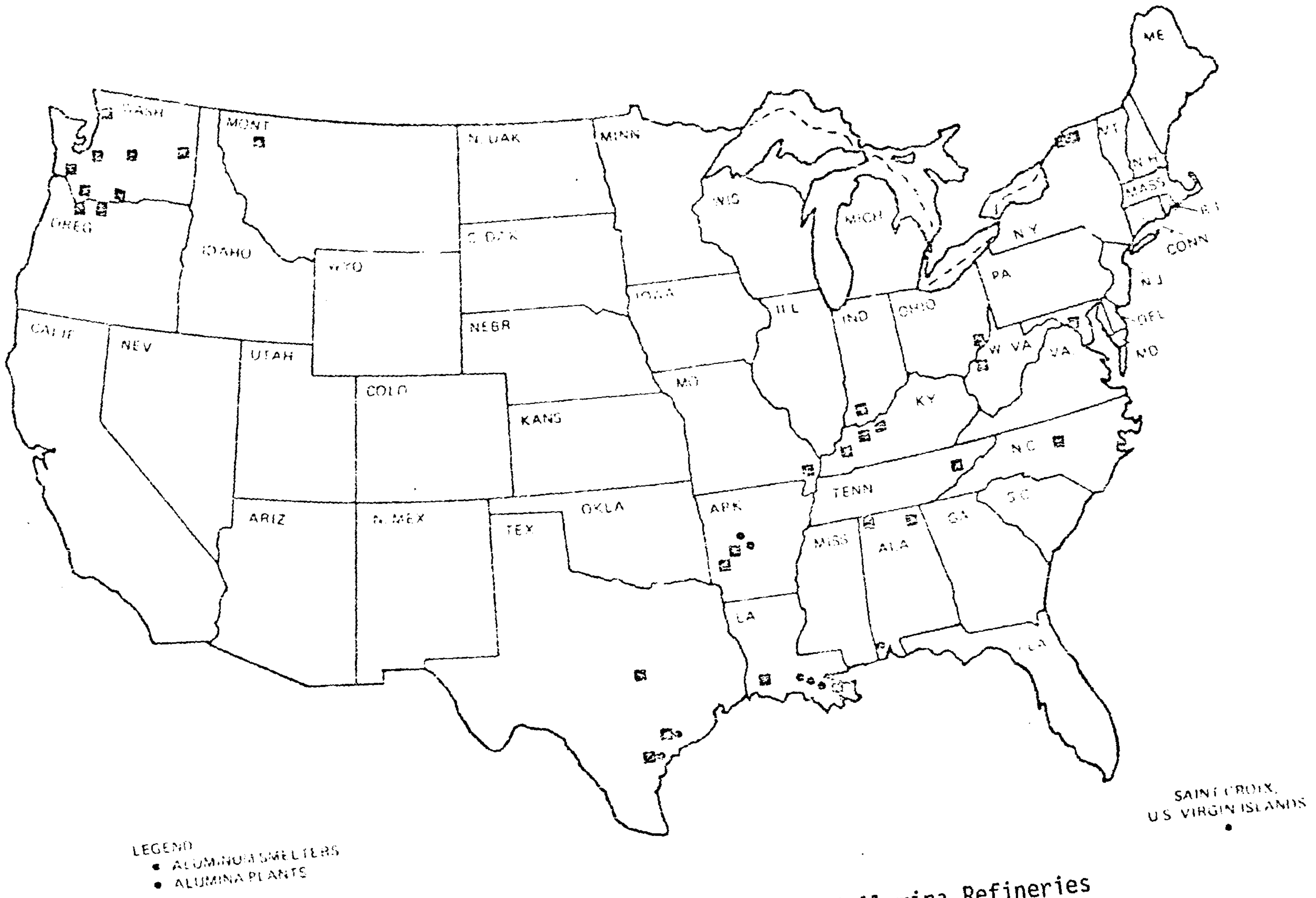


Figure 2.4: Distribution of Aluminum Smelters and Alumina Refineries

These plants are located in the Gulf coast and neighboring states in order to receive the bauxite coming from the Caribbean and South American countries and the gas from Louisiana and Texas. One plant is located in the Virgin Islands. The plants in Arkansas process domestic ore. Figure 2.4 and Table 2.2 show their location and individual capacity.

Existing capacity is 7,700,000 tons of alumina per year, approximately equivalent to 4,000,000 tons of aluminum. Thus, when working at the full 5,175,000 tons of smelting capacity, at least 23% of the alumina must be imported.

Bauxite in the U.S. is relatively scarce. There are mines in Arkansas (3.0 million tons per year capacity but only 2 million tons production), Alabama and Georgia (272,00 tons combined). The Arkansas mines are open pit while the others are underground. Arkansas bauxite, which is the only one processed to alumina, has a very high silica content and has to be refined using the combination Bayer process. In some instances it must be blended with imported, low silica bauxite, to reach proper processing grade.

Many American companies own mines in bauxite producing countries. In other cases there are commercial agreements, or long range contracts with the governments or the private companies that own the mines; there is very little, if any, bauxite in the open market. As a matter of interest Table 2.3 lists the imports from these countries in 1973 and 1974. In these years American production was 1897 and 1979 thousand

Table 2.2

Refineries in the U. S.

Company	Location	Alumina Capacity 1000 tons/
Alcoa	Bauxite, Ark.	375
Alcoa	Mobile, Ala.	1,025
Alcoa	Pt. Comfort, Tex.	1,350
Martin Marietta	St. Croix, V. I.	370
Kaiser	Baton Rouge, La.	1,025
Kaiser	Gramercy, La.	800
Ormet	Burnside, La.	600
Reynolds	Hurricane Creek, Ark.	840
Reynolds	Corpus Christi, Tex.	1,385
		7,700

long tons, respectively.

American bauxite reserves are 40 million long tons, enough to supply the market at the 1973 level of demand for only two years. World reserves as of 1975, were 24 billion long tons, with approximately 25% in Australia, 25% in Guinea and 25% in the Western Hemisphere (Brazil, Jamaica, Surinam, and Guyana). Reserves are constantly increasing due to new discoveries [1, 9]. Of these countries, Australia will not sell its bauxite but will sell alumina instead.

American firms own alumina refining plants in most bauxite producing countries. Alcoa, Kaiser, Reynolds, Revere and Noranda own facilities in Jamaica, Brazil, Surinam, Australia and Guinea [8]. In addition, the U.S. imports alumina from Guyana, which recently nationalized all mines and refineries in its territory. During the years 1972-1975, alumina imports averaged 3370 thousand long tons per year. Australia's contribution was 56%, Jamaica 25% and Surinam 12%.

Primary aluminum imports, of which 80% are in the form of ingots [10, 11, 12], come principally from Canada, since Alcan owns fabricating plants in the U.S. Until 1975, approximately 80% of the imports came from this origin. In 1976 a considerable fraction of imports came from Ghana.

American companies own or have interests in smelting facilities in Canada, Mexico, Brazil, United Kingdom, Surinam, Australia, Ghana, Venezuela, Germany, Norway and India. This capacity was 1455 thousand tons in 1975 [15]. Moreover, Alcan owns smelting facilities through-

Figure 2.5: Location of Fabricating Plants

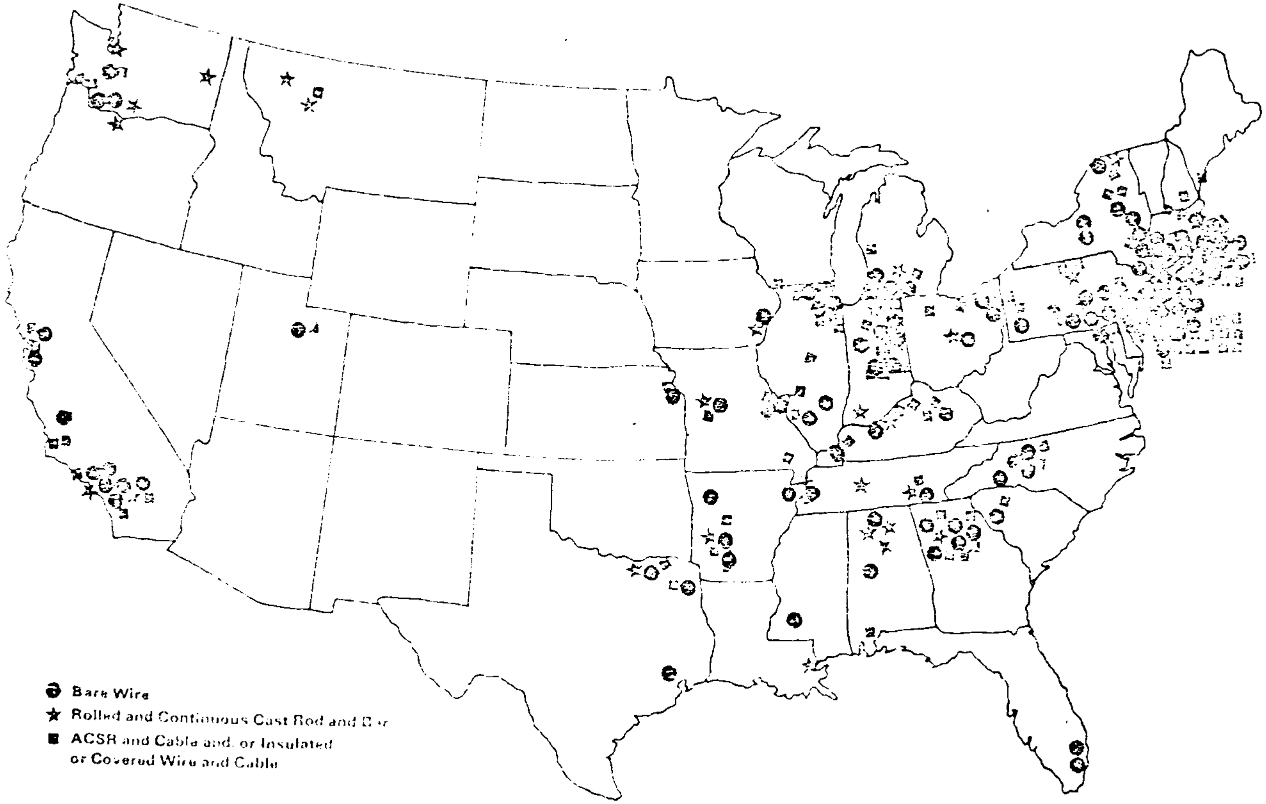


Figure 2.5 (cont'd): Location of Fabricating Plants

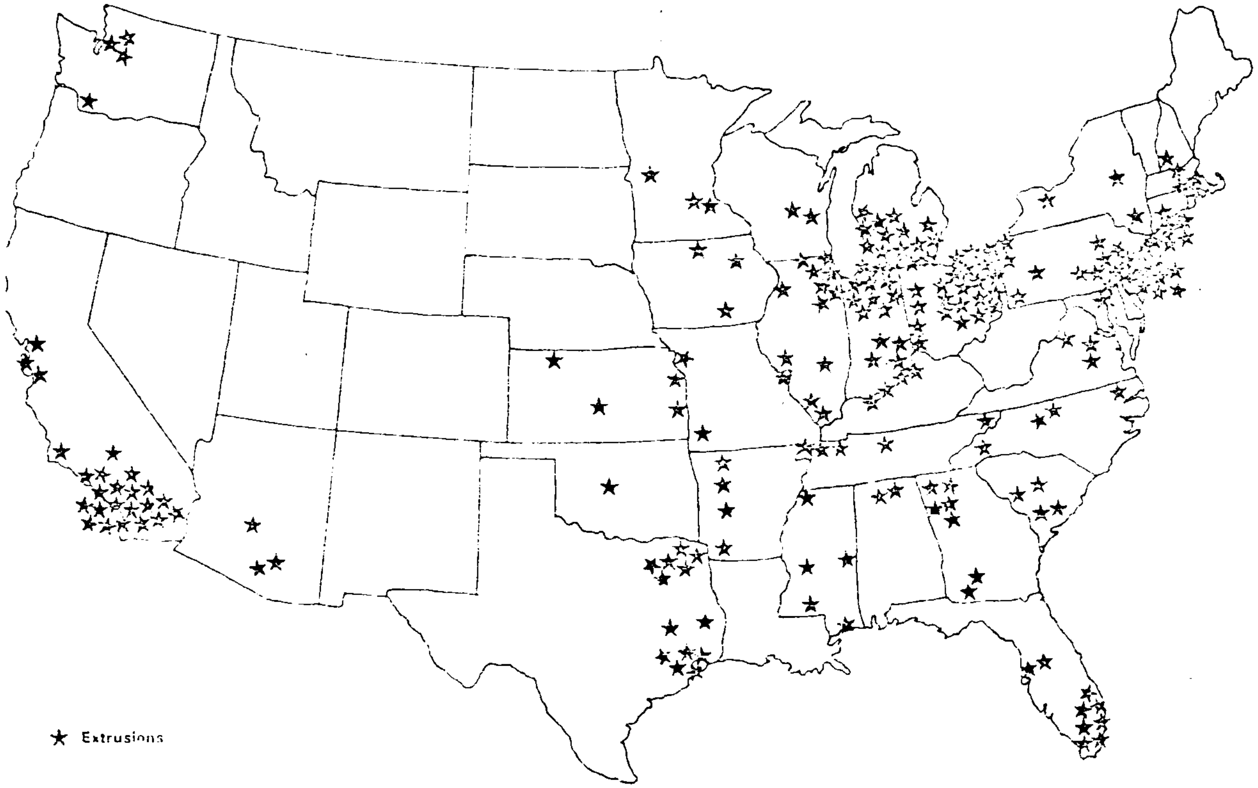


Figure 2.5 (cont'd): Location of Fabricating Plants

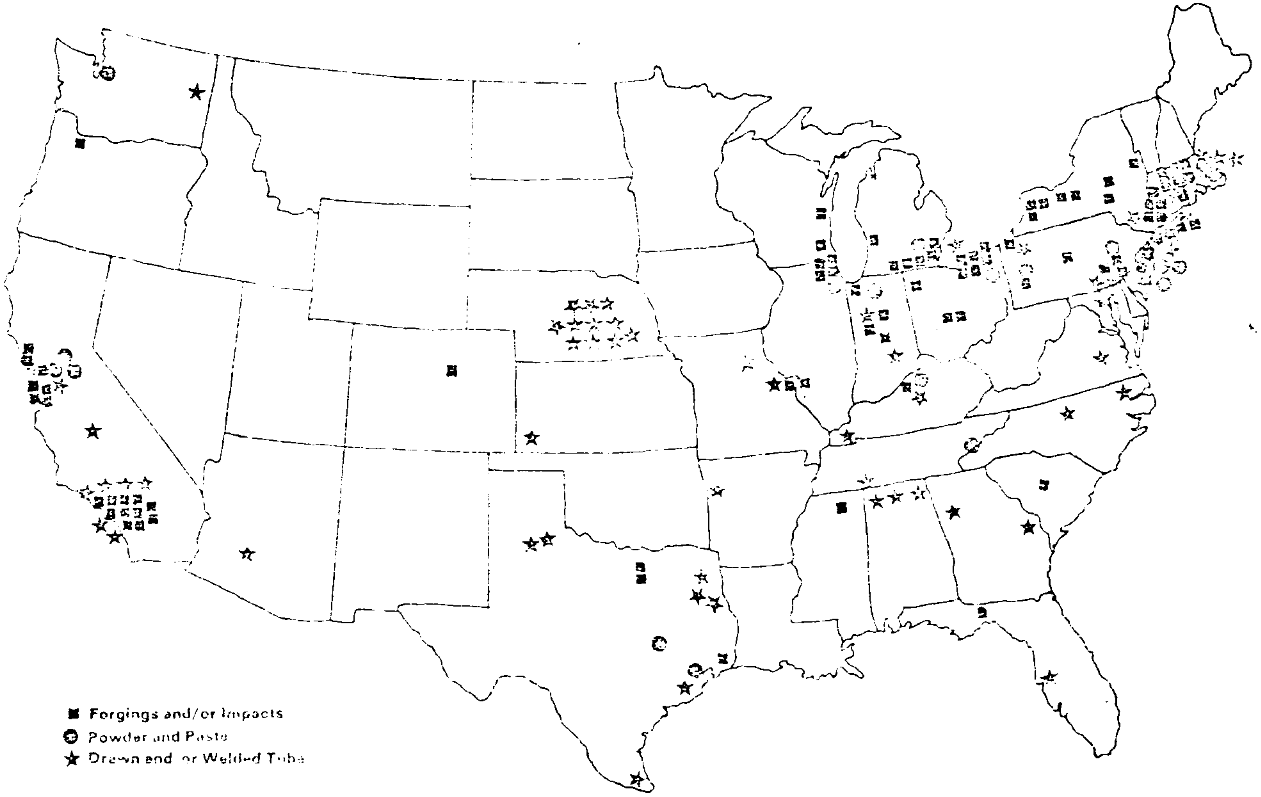


Figure 2.5 (cont'd): Location of Fabricating Plants

out the whole world for a capacity share of more than 2.1 million tons [15].

Fabrication plants are evenly distributed across the country as shown in Figure 2.5. These plants are owned by primary producers and independent fabricators. The latter own extrusion and wire mills principally; primary producers own most of the other mills [6].

Historically, the principal aluminum markets have been: building and construction, transportation, consumer durables, electrical, machinery and equipment and containers and packaging. These markets further transform the ingot or milled product into canopies, awnings, sidings, insulated wire, cans, refrigerators, airplane bodies, cooking utensils, etc. Figure 2.6 shows the distribution of milled products and Figure 2.7 shows the breakdown of aluminum demand by end use market in 1975. The U.S. also exports aluminum, as ingot or mill products and alumina.

Aluminum scrap, the other source of aluminum metal, is used by the industry to obtain secondary aluminum. The secondary aluminum industry originated in the United States after World War I, but it was not until after World War II, with the inclusion of the smelter (instead of remelters), that it became an important element of the aluminum market.

Of the scrap generated during the fabrication and industrial consumption processes, 90% is runaround scrap, and the remaining 10% enters the market as new scrap. The recovery of new scrap has accounted for an average of 16% of the total shipments of the industry

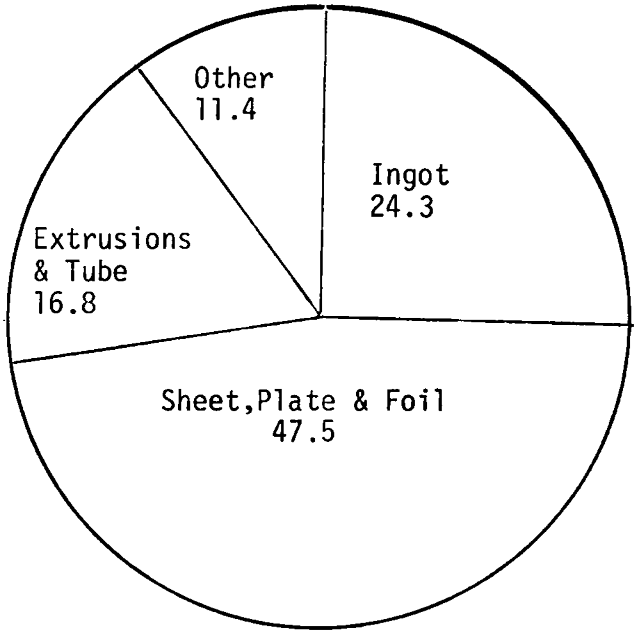


Figure 2.6

Distribution of milled products

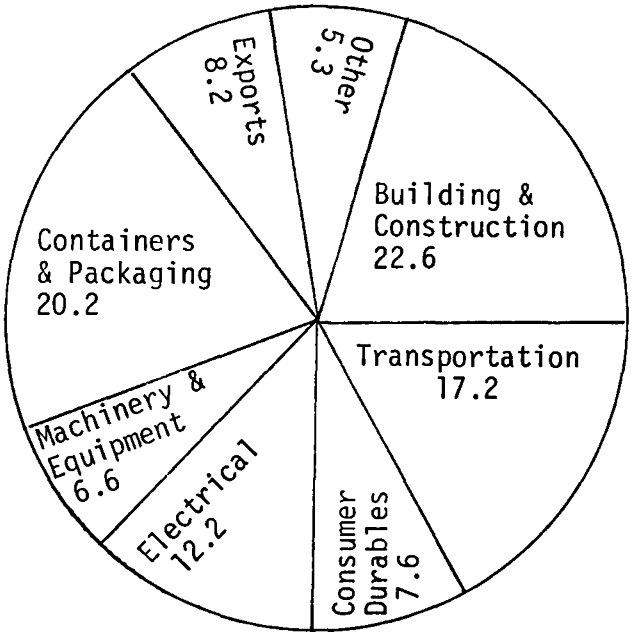


Figure 2.7

Breakdown of demand

in the last twenty years [11]. Old scrap recovery, i.e. the recovery out of discarded end products, accounted for an average of 6% of the total shipments, largely beverage cans and foil.

Secondary smelters consume more than 60% of the recovered scrap. This scrap is of low quality and needs refining. Purer, and hence more expensive scrap, is consumed by primary producers and fabricators. The cost of scrap accounts for at least 65% of the cost of secondary aluminum [13].

A study made by the Bureau of Mines shows that, on a broad basis, the obsolescence of aluminum end use products consumed on a certain year follows a Poisson distribution with a mean of 13 years. Of the obsolete aluminum only 15% is actually recovered as old scrap. Higher aluminum prices and better recovery techniques may increase this fraction.

Secondary smelting facilities are plentiful in the U.S. They are distributed near scrap and end use markets. A typical concentration occurs in the Great Lakes area as shown in Figure 2.8. Their number is continually changing. According to the Aluminum Association, in 1974 there were 92 plants and in 1975, 88 plants [11].

Secondary aluminum is produced as ingot or billet, and 80% of it goes to the casting industry. A very small fraction is actually milled; this comes mostly from the recycling of aluminum cans and foil which are rolled to produce sheet to produce more cans. The end use markets are the same as those mentioned for the primary metal.

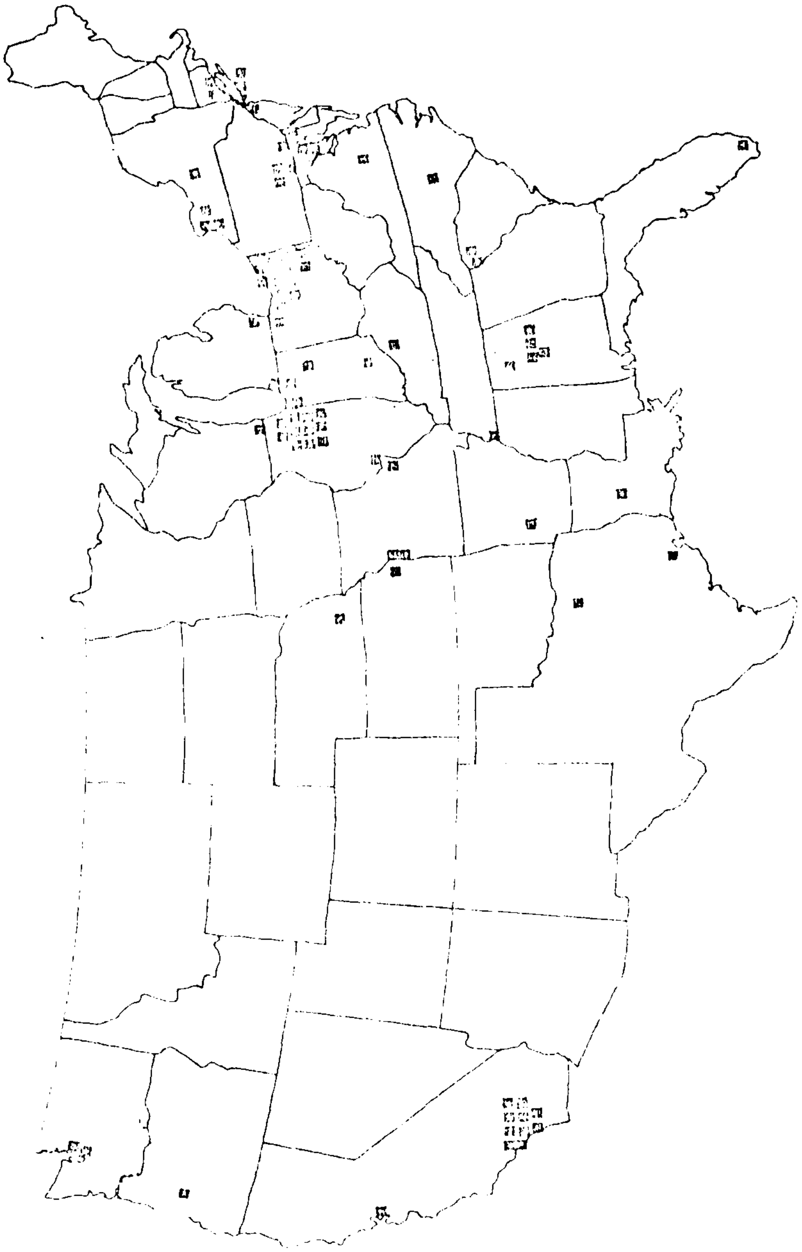


Figure 2.8: Location of Secondary Smelters in 1975

Aluminum uses cover a wide scope. As bauxite, its end uses account for 5% of the total use on an element basis. It is consumed by the chemical abrasives and refractory industries. As alumina, its consumption accounts for 7% of the total use of aluminum on an element basis. The alumina consumers are the same as the bauxite consumers. In both cases their uses are specific and therefore they have almost no competitors. The remaining 88% is consumed as metal. The aluminum metal consumers are: the building and construction sector (awnings, canopies, frames), the transportation sector (engine parts, transmission parts, bodies), the electrical sector (steel reinforced cable, towers), the packaging and containers sector (cans, caps, closures), the consumer durables sector (refrigerators, air conditioners, furniture) and the industrial sector (special machinery). Most of these uses are not unique and aluminum has to compete with copper, steel, glass, wood and plastics.

Aluminum consumption grew at a rate of 8.1% per annum from 1965 to 1975. This rate was obtained principally by substituting for other materials. For example, in 1955 an average automobile used 30 pounds of aluminum, today it uses 85 pounds. The major factors of this growth were the intrinsic characteristics of aluminum (i.e. resistance to corrosion, light weight and high strength to weight ratio), its ease of fabrication, its low cost and its stable price. The Bureau of Mines predicts a growth demand of 5.1% per annum until 1985, decreasing then to 4.5% per annum until the year 2000 [1].

An important characteristic of the Aluminum industry in the U.S. is that it consumes over 5% of the national production of electricity [6]. The most energy demanding stage is the smelting of alumina, which in 1971 consumed almost 61 trillion KWH. In fact all stages in the aluminum cycle are energy intensive, as shown in Table 2.4 [10].

Power requirements for smelters were satisfied in 1975, 51% by hydroelectric sources and 49% by thermal sources (including nuclear). Furthermore 78% was purchased power and 22% was generated by captive power plants [15].

An additional feature of the industry is the fact that aluminum is one of the four controlled materials in the Department of Commerce Defense Materials System. In other words, defense orders have priority in the industry. In past years a Government Stockpile of bauxite and aluminum was maintained but under the aluminum metal disposal program (November 1965 -- December 1975), metal stocks have been reduced to zero.

The government and industry have been investigating the possibility of extracting alumina from domestic ores. A private company has a pilot plant in Colorado using alunite. Research carried out by the Bureau of Mines in a pilot plant in Nevada, with the collaboration of certain metal producers, have found anorthosite and kaolin clay as the two most promising ores. Laterite bauxites have been used in the Bayer Process.

Anorthosite has an ore grade of 27% alumina. Existing reserves consist of 599,490 million tons, with 153,8000 million tons in the state of Wyoming. Kaolin clay reserves total 3,238 million tons, with 1,025 million tons in the state of Georgia. Kaolin clay has an average ore grade of 33% alumina. Both ores account for enough reserves to supply the U.S. at the 1973 level of demand, for more than 100 years [16].

The United States, in 1974, accounted for 31.5% of the world's smelter capacity [10]; it produced 33.7% and consumed 36.8% of the world's primary aluminum [12]. As far as alumina is concerned, the U.S. produced 24% of the total world production and its refining capacity was 21.7% of the world's refining capacity. Moreover, American bauxite production was 2.5% of the world's bauxite production.

An important statistic is the share of non-communist world capacity of the three largest American companies, Alcoa, Kaiser and Reynolds. Their share of bauxite capacity is 50.63% and their share of alumina capacity is 46.76%. Their share of the non-communist world smelting capacity is 36.16% and if we include Alcan, the corresponding share is 53.27%.

C. Scenario Faced by the Industry

The aluminum industry is faced with decisions on investments, technologies, inventory strategies and demand satisfaction, but there are three areas where the policy issues are more delicate than usual. These are: bauxite supply, energy requirements and capital costs.

Table 2.3
Bauxite Imports
(thousand long tons)

	1973	1974
Australia	359	--
Dominican Republic	1101	1283
Guinea	164	1256
Guyana	483	606
Haiti	696	586
Jamaica	7273	7766
Surinam	2651	2811

Table 2.4

Energy Consumption in the Manufacturing of Aluminum
(figures in 10^9 BTU)

Bauxite	300
Alumina	130,336
Hot metal production	711,312
Holding, casting and melting	61,958
Fabrication	125,490
Total	1,029,396

1. Bauxite Supply

The United States imports 90% of its bauxite requirements.

In March 1974 seven bauxite producing countries, Australia, Guinea, Guyana, Jamaica, Sierra Leone, Surinam and Yugoslavia, formed the International Bauxite Association (IBA). As of November 1975 four new members had joined: the Dominican Republic, Haiti, Ghana and Indonesia. Together, these countries account for more than 80% of the world's production.

Two of the stated objectives of the IBA as outlined in Article III of the Final Act of the International Conference of Bauxite Producing Countries, are: "to promote the orderly and rational development of the bauxite industry and to secure for member countries fair and reasonable returns from the exploitation ... of bauxite ... bearing in mind the recognized interests of consumers." The present objective of the IBA is to establish a "uniform" price policy for bauxite among its members [16].

Aside from the fact of IBA ownership of most of the world's reserves, the formation of the IBA was precipitated by two principal factors: the success of the Organization of Petroleum Exporting Countries (OPEC), and the direct and indirect effects of OPEC's policy in the bauxite producing countries. That is, the increased petroleum prices resulted in a tremendous burden in the balance of payments of the bauxite producing countries, since all of them import most of their petroleum requirements. Moreover, since most of the IBA members are developing countries, OPEC's policy resulted in an increased cost of their imports of manufactured products.

Jamaica emerged as the leader of the IBA, and it was the first country to raise its taxes from a pre-1973 average of $1.80 per long ton to about $15 per long ton and further impose an obligatory minimum tonnage on the companies mining in its territory. Other IBA countries quickly followed Jamaica's example, applying different formulas for their taxes, most of them a function of the price of aluminum. Even in countries like Guinea or Guyana, where the government has partial or total interests in the mining, c.i.f. prices of bauxite increased from a pre-IBA average of $12 per ton to a present value of $30 per ton. The only exception is Australia which refused to raise its taxes by the same high margin, although it prefers to export alumina rather than bauxite [16].

One of the goals pursued by the IBA countries is downstream integration. They may do this by encouraging the installation by private companies of refineries and smelters, and, in some cases, they may try to gain equity interests in existing or new facilities.

The principal impact of IBA policy on the U.S. is therefore a higher price and lower availability of bauxite. There are, however, countries with high bauxite reserves which do not belong to the IBA, e.g. Brazil. Furthermore, there are IBA countries like Australia which due to their economical and cultural links with the U.S. may not follow IBA's general taxation policy.

Several alternative bauxite supply scenarios may, thus, be formulated. For example:

a) Brazil and other non-IBA countries with high bauxite reserves, may choose to join the IBA and follow its "uniform" price policy.

b) IBA and non-IBA countries may choose to export alumina rather than bauxite, resulting in a reduced supply of bauxite.

c) The IBA may fail, due to either economical and political pressures or due to misunderstanding between its members from within. Bauxite prices may decline as they enter a competitive market.

d) The IBA may have partial success. In other words, only a fraction of the member countries, say the Caribbean and South American, may follow the uniform policy. This may be due to the existence of fully developed American owned mines within their territories and the reluctance of the American firms to abandon their facilities. Therefore, although some bauxite prices may be lower there could still well be imports from the Caribbean and South American countries.

Moreover, the upper limit of bauxite price is that at which it may be marginally replaced by domestic ores. In the previous sections the current status of research was discussed, and it is to be expected that further research will lower operating costs still further. Therefore, the ceiling price of bauxite should be considered in all possible IBA scenarios.

2. Energy Requirements

It is worthwhile to recall some statistics before presenting the scenario. The aluminum industry consumes an average of 239 million BTU per ton of aluminum, of which 175 million are consumed in the electrolytic reduction, equivalent to 15,000 KWH per ton [2].

All of the Pacific Northwest smelting capacity (1,600,000 tons) is supplied by the Bonneville Power Administration (BPA) with inexpensive hydroelectric power. Recently BPA notified eight of the ten aluminum smelters that their contracts, expiring between the years 1986 and 1993, will not be renewed.

The industry faces three different choices: Cease production, build captive power plants or purchase electricity from a power park installed by a utility company.

In the first case, the industry would actually have to replace 31% of the present capacity or choose to import the metal or some combination of both. If it chooses to rebuild the capacity, it may do it either by installing new facilities or expanding present ones, in either case the investment will be very large.

In the second case, building captive power plants requires the approval by the regulatory agencies. Current lead times are upwards of six years. Furthermore, any power plant would have to be thermal since there are no hydroelectric sources available.

Finally, the third case is considered. The Pacific Northwest Utilities Commission Conference (PNUCC) has suggested that the aluminum plants give up their present low cost power contracts and place that power in the PNUCC pool. The plan would require Congressional amendment of the Bonneville Power Act and the plants would have their power supply assured for the next 35 years, at a higher price than it is presently paying [18].

In addition, the industry has to consider the higher costs of fuel and electricity and the way they are going to supply the increased refining and smelting capacity that has to be installed if they plan to meet the demands with domestic production. Present research in the Hall cell has reduced its electricity consumption. Moreover, a reduction process which electrolyzes aluminum chloride instead of alumina, developed by Alcoa has been reported to consume 30% less power than the present processes.

3. Capital Costs

In the 1971-1975 time period, the American industry has been increasing its smelting capacity at an average of 1.86% per year compared to an average rate for 1960-1972 of 6% per year. The consequence of this slowdown is that the industry is presently working at nearly full capacity and, if demand is to be met with domestic production, massive investments will be necessary.

New smelting capacity may cost, on the average, $1,800 per ton per year of aluminum capacity [19], assuming that the electric power is purchased. For captive power plants, the cost may be around $600 per KW of installed capacity [20]. Furthermore, expansion of existing smelters may cost $1,100 per ton of aluminum capacity [20].

III. LITERATURE REVIEW

Most of the mineral models that have been developed are of the econometric type. Econometric models can be defined as models where "... a causal linkage between distinct economic variables and the dependent variable is found through statistical analysis. As generally accepted in use, econometric models are based on structural relations between supply, demand and inventories, using price as the market clearing mechanism" [22].

A recent publication which summarizes the state of the art in minerals modeling [21] dedicates half of its pages to discuss econometric models. In addition, it presents general considerations regarding minerals modeling as a whole. This fact is discussed in some detail in the papers presented by Wassily Leontief and Leonard Fischman.

Leontief [24] focuses attention on data. He 'fears' models that are insensitive to data and argues that models, which are theoretically excellent, may not work because it is impossible to obtain the data required. In addition, he contends that another common source of error is the practice of averaging data. Regarding the dynamic aspect of modeling, Leontief considers time as another variable in a static matrix that may include several time periods.

His position on data is not shared by everyone. In the same publication Meadows mentions that tendencies of society dominate long run paths, and tendencies are better considered by relationships.

Leonard Fischman [23] presents a model for long range modeling which, in his words, is linear, cross sectional with respect to time and sequential. In his approach, the time horizon under study is divided into periods. For each period (about a decade apart) a series of linear relationships represent the scenario under study. These blocks are almost independent of one another although the equations are very similar. The exogenous variables and the parameters are forecasted separately and applied to the corresponding block of equations. These blocks are fully interlocked in the sense that the dependent variable of one equation becomes an independent variable of the following equation. The forecasted parameters simulate the progress in technology and the institutional changes. In this representation all units are the actual physical units, in contrast with the constant dollars of econometric models. The independence of these blocks of equations allows the model to be solved for a particular time without the necessity of solving it for any other time (as in time dynamic models). Fischman concedes that the model has its drawbacks in representing economies of scale and in comprehensiveness. For more details, the reader is referred to [23].

In the same publication, Saxton and Ayres present a different model [21]. The authors emphasize the importance of technology in any forecasting model and discuss the difficulties of a realistic representation of this fact. In addition they mention that a source of minerals many times forgotten is recycled scrap. Their model shows a mineral industry as a network, where flows include aggregated values, environmental constraints and predetermined inter-industry relationships.

The most complete model of the aluminum industry existing today is the one prepared by Charles River and Associates [16]. The report includes a very good description of the aluminum industry at all stages, and, in particular, of the U.S. dependency on foreign bauxite.

The CRA model uses an econometric approach, making a very detailed study of existing relations in the industry, although it does not include new technology. It covers the whole world, although for the mining formulation, it relates all data to Jamaican mining operations and Jamaica bauxite (Jamaica is the principal bauxite supplier to the U.S.). Endogenous variables include demand, list price and transaction price of aluminum and old and new scrap recovery. Forecasts of the exogenous variables are an input to the model. The output of the model consists of the forecast of aluminum demand and list prices, under different scenarios, up to the year 1990. Results given by this model may be considered reasonable if future relationships between the different factors of the industry do not change significantly from the historical trend.

In addition to the Charles River Model, Synergy Inc. presented a joint Aluminum-Copper model to the Bureau of Mines [28]. This is another econometric model whose objective is to describe the interactions between the industries.

Another model of the metals industry reviewed was one developed by the Stanford Research Institute [22]. They present a linear model of the zinc industry as a sample of the metals industry. The model is static. Although, by forecasting the exogenous variables and the parameter values, the model can be used for any point in time. The model divides the world into thirty regions, which are considered as producing and consuming regions. The different processes in the zinc industry are analyzed in the flow from ore or scrap to metal, and transportation of intermediate products is allowed between the regions. The objective function formulated is cost minimization.

Other models reviewed, not related to the metals industry but to energy were: Project Independence (PIES) [25], Brookhaven [26], and Lorendas [27].

The Brookhaven model is formulated as a transportation model between energy producing and energy consuming sectors, with efficiency coefficients in the supply and demand constraints. It includes constraining equations representing environmental factors and technical features of the energy system. The PIES model makes a detailed description of the different processes and uses the information provided by the dual variables to reach equilibrium.

Both of the above models are static, that is, they look at the world at one point in time. On the other hand, the Lorendas model has a time dynamic formulation. This means that it studies a certain time span, divided into time periods. The variables used can be classified either as dynamic (or investment) variables or static (or operational) variables. The dynamic variables may appear in more than one time period, therefore affecting decisions at different points in time.

IV. THE MODEL

A. Model Used -- General Characteristics

The basic structure of the model used in this research corresponds to a time dynamic, linear model as described in references [27] and [29].

The main characteristic of this type of model is the way in which it achieves dynamism. It divides the time horizon under study into time periods and the variables are defined for each time period. Variables defined for any time period may appear in subsequent time periods, interlocking them.

The variables used by the model fall into two categories: investment and operational variables. The investment variables represent the building of production facilities while the operational variables reflect the level of usage of the facilities. Dividing the variables into these two classes helps in various aspects.

a) Improved comprehension in the representation of the building efforts and computation of the investment costs.

b) Better representation of technology improvements by treating separately the building of the facilities and their operation.

c) An adequate fitting into the time dynamic formulation, since investments made in a certain time period increase production capacity in further time periods.

The relationship between investment and operational variables, that is the constraining equations or inequalities, may be classified into seven different categories.

a) Process Description: These equations represent the chemical or physical transformation of intermediate products in the processing plants. They include the concepts of material balance and process yield.

b) Supply and Distribution Balance: These equations represent the channels by which intermediate processes are linked in the model. They are transportation balances linking internal and external supply sources with internal demand sinks.

c) Capacity Constraints: These equations represent the limitation to production activities by the capacity of the facilities used for that production process. Since the facilities' capacity include the capacity built by the model, these equations are clear representatives of the dynamism of the model, where investments in one time period increase the operating capacity in subsequent time periods.

d) Demand Stipulations: These equations are the driving force of the model. They link the end use market demand for different products with the processes or external sources which supply these products. The representation of the optimal way of meeting these inelastic demands is the objective of the model.

e) Externally Imposed Restrictions: These are simple to formulate and very important in the solution of the model. Political, historical, geographical and financial considerations, that are beyond the system's domain, influence the system's behavior. For example: federal regulations may restrict the location of new facilities, forecasted strikes may reduce productivity, etc. These are the constraints that the analyst manipulates to simulate the different scenarios.

f) Inventory Balance Rows: These equations represent the inventory balance between consecutive time periods.

g) Technological Improvement Balance Rows: These equations represent improvements in technology of existing facilities. The capacity technologically improved from, say, type A to type B, is deleted from the existing type A capacity and added to the existing type B capacity. It shall be assumed that these improvements take place between successive time periods.

Furthermore, to achieve a better representation of the flows of intermediate and end products, the geographic area under consideration (i.e. the United States) is divided into distinct regions. Several factors affect this division, among them: cost structure, availability of materials and political restrictions.

Following the presentation made in [27], the overall mathematical structure of the model is as follows:

Let X_j and Y_j represent the set of vectors of Operational and Investment variables respectively in time period j

Let A_j, D_j, C_j and B_j be matrices associated with these categories of variables in time period j

Then, the following set of equations represent the basic structure of the model:

a) $A_j^1 X_j \geqslant L_j^1$

b) $A_j^2 X_j - B_j Y_j \leqslant L_j^2$

c) $D_j [X_j + X_{j+1}] = 0$

d) $C_j (Y_j + Y_{j+1}) = 0$

Set (a) represents the process descriptions, the supply and distribution balance, the demand stipulations and the externally imposed restrictions. Set (b) represents the capacity constraint rows. Set (c) represents the inventory balance rows. Finally, set (d) represents the technology improvement rows. For all practical purposes this set of equations can be represented in a condensed form as follows:

$$A_jX_j + B_jY_j + D_j(X_j + X_{j+1}) + C_j(Y_j + Y_{j+1}) \geq L_j$$

With this symbolism the overall dynamic structure of the model is shown in Figure 4.1, where the summation symbols placed in front of the equations indicate the representation of all producing and consuming regions.

The structure presented in Figure 4.1 shows the interlocking between the equations of the different time periods. This interlocking occurs as a direct consequence of the carry over effect of the capacity building activities (Y_j). Furthermore, each equation shows the interlocking between variables of different time periods (in this case,

$$\begin{aligned}
&\sum [A_1X_1 + D_1(X_1 + X_2) && + B_1^0Y_1 && + C_1(Y_1 + Y_2) \geq L_1] \\
&\sum [\quad A_2X_2 + D_2(X_2 + X_3) && + B_1^1Y_1 + B_2^0Y_2 && + C_2(Y_2 + Y_3) \geq L_2] \\
&\sum [\qquad A_3X_3 + D_3(X_3 + X_4) && + B_1^2Y_1 + B_2^1Y_2 + B_3^0Y_3 && + C_3(Y_3 + Y_4) \geq L_3]
\end{aligned}$$

Figure 4.1

Basic structure of the formulation

successive time periods). For a more detailed illustration of this concept, the reader is referred to [27].

The availability of new facilities is not instantaneous, but follows a certain pattern, characteristic of the facility, which includes building time and time to reach full operation. This results in a gradual addition of capacity. For a detailed explanation of how this feature is represented by the model, the reader is referred to Appendix I.

The objective function is formulated in a cost minimization context, although there is enough flexibility to accept other objectives, such as profit maximization or energy consumption minimization. It may be argued that a cost minimization, since it represents a perfect market status [30] and the aluminum industry is a practical oligopoly, may not represent real trends of the industry. Nevertheless, cost minimization is a formulation that will show basic trends in the industry.

Furthermore, for the computation of the objective function, the time value of money is considered. All expenditures are present valued to the beginning of the model using, as discount rate, the historical pretax rate of return of the industry [6]. Due to its formulation, the model guarantees, for all investments, a rate of return at least equal to the discount rate. For the calculation of the rate of return, the price considered is the value of the dual variable of the balance row which represents the product resulting from the investment (see Appendix I). Moreover, discounting the operating expenditures and the investment expenditures requires different methods represented by different discount factors. The computation of these factors is shown in Appendix I.

Costs represented in the model are the out of pocket costs: this strategy represents the real environment of the decision making process. That is, depreciation is not considered as a cost and existing facilities are considered sunk costs. Furthermore, since the model does not represent the actual financing of the investments, no interest charges are added to the costs and it is assumed that the required capital is generated by the industry.

Another feature of the cost structure is economies of scale. Due to the linear structure of the model, costs have to be proportional to the level of the variable that they affect and some assumptions are necessary.

Operating costs are calculated as those of a plant whose capacity is what economic studies consider minimum economic capacity to be (i.e. 400,000 tons per year for alumina refineries). For larger capacities above this minimum, the bias caused by fixed costs is almost negligible. Similarly, when dealing with new facilities, investment costs are calculated for the minimum economic capacity of a new plant. The computation of investment costs corresponds to the minimum economic capacity of a new plant. The computation of investment costs for expansion of existing facilities do not vary significantly with capacity.

The results of the model may suggest capacities greater or lower than the minimum economic capacity. In the first case the computation of the operating costs of these facilities may be considered correct, since it may be assumed that all plants built will have, at least, that minimum capacity. In the second case, and especially if capacities are significantly lower than the minimum economic capacity, the results may be questionable. This is not likely to happen if the time periods are long and if the rate of growth in demand and the present level of demand are large. Nevertheless, the analyst has enough information to see trends, and investments can be forced to required levels.

B. Building of the Model -- Brief Example

To illustrate the building of the model, a prototype example with a simplified aluminum industry is presented. The basic characteristics of this industry are:

a) Two sources of bauxite: Bauxite is supplied by existing domestic mines with reserves R^B and by imports -- the imports are considered unlimited. Existing domestic mine capacity is M^0 and these mines may be expanded up to 25% of their existing capacity.

b) Two sources of alumina: Alumina is supplied by domestic refineries with an existing capacity of R^0 and by imports -- the imports are considered unlimited. New refineries may be built.

c) Two sources of aluminum: Aluminum is supplied by domestic smelters. These smelters belong to two different technological categories, type 1 and type 2, with existing capacity S^{01} and S^{02} respectively. Type 2 smelters have lower operating cost than type 1 and existing type 1 smelters may be changed to become type 2 smelters (but not vice versa).

d) Inventory: An inventory of bauxite is carried with an initial stock of Y^0.

Figure 4.2 shows this simplified industry. In this prototype example only two time periods are considered. It is assumed that there is no lag for the availability of new facilities and that improvements in technology become effective in the time period following their introduction.

Let

M_j, R_j, S_j^i = capacity built for mines, refineries and type i smelters respectively in time period j

B_j^D, L_j^D, A_j^{Di} = production of domestic bauxite, alumina and aluminum (from smelter type i) in time period j

E_j^i = existing smelting capacity of smelter type i in time period j. The changes in this variable are due to technological improvements. For time period 1, $E_1^i = S^{0i}$.

I_j^{ik} = smelting capacity improved from type i technology to type k technology in time period j

B_j^I, L_j^I = imports of bauxite and alumina respectively in time period j

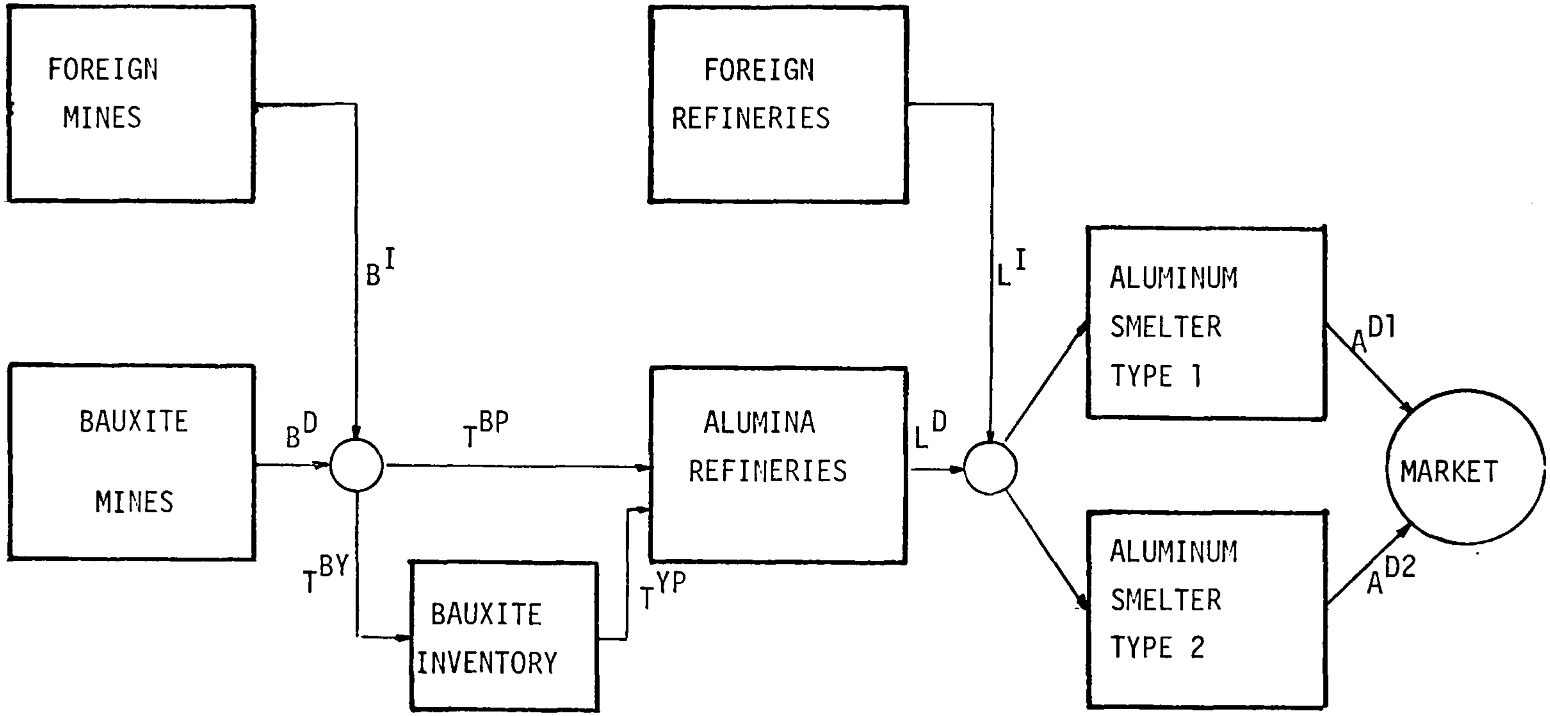


Figure 4.2
Simplified Aluminum Industry

Y_j = bauxite inventory at the end of time period j

D_j = aluminum demand in time period j

T_j^{BP}, T_j^{BY} = transportation of bauxite to process and to inventory respectively in time period j

T_j^{YP} = transportation of bauxite from inventory to process in time period j

θ_j = duration of time period j

Let it be further assumed that all processes are 100% efficient, that all products are expressed in the same units (tons of aluminum equivalent), and that there is only one region. Then, the model is illustrated by the following set of equations

(1) Capacity constraint for domestic production in time period 1

$$\begin{aligned}
-M_1 + B_1^D &\le M^0 \\
-R_1 + L_1^D &\le R^0 \\
-S_1^1 - E_1^1 + A_1^{D1} &\le 0 \\
-S_1^2 - E_1^2 + A_1^{D2} &\le 0
\end{aligned}$$

(2) Capacity constraint for domestic production in time period 2

$$\begin{aligned}
-M_1 - M_2 + B_2^D &\le M^0 \\
-R_1 - R_2 + L_2^D &\le R^0 \\
-S_1^1 - S_2^1 - E_2^1 + A_2^{D1} &\le 0 \\
-S_1^2 - S_2^2 - E_2^2 + A_2^{D2} &\le 0
\end{aligned}$$

It can be noticed how capacity built in time period 1 interacts in time period 2.

(3) Technological improvement balance

$$E_1^1 - I_1^{12} - E_2^1 = 0$$

$$E_1^2 + I_1^{12} - E_2^2 = 0$$

Since only two time periods are considered, there are no improvements allowed in time period 2.

(4) Production and transportation balance for any time period

$$\begin{aligned} -B_j^I - B_j^D + T_j^{BD} + T_j^{BY} &\leq 0 \\ -T_j^{BP} - T_j^{YP} + L_j^D &\leq 0 \\ -L_j^D - L_j^1 + A_j^{D1} + A_j^{D2} &\leq 0 \end{aligned}$$

They are simple material balances that carry the ore through the different processes until it reaches its final form.

(5) Demand satisfaction balance for any time period

$$A_j^{D1} + A_j^{D2} \geq D_j$$

(6) Inventory balance

$$Y^0 + \theta_1 T_1^{BY} - \theta_2 T_1^{YP} - Y_1 = 0$$

$$Y_1 + \theta_2 T_2^{BY} - \theta_2 T_2^{YP} - Y_2 = 0$$

The transportation vectors must be multiplied by the length of the time period to represent the actual storage or depletion of the inventory.

(7) Reserves depletion balance (through all time periods)

$$\theta_1 B_1^D + \theta_2 B_2^D \leq R^B$$

(8) Mine expansion constraint (through all time periods)

$$M_1 + M_2 < 0.25 \, M^0$$

This prototype example presents the basic structure of the full scale model, showing how the time dynamic aspect of the formulation works in the capacity constraints and how the time factor pervades in all equations.

V. REPRESENTATION OF THE ALUMINUM INDUSTRY BY THE MODEL

A. General Considerations and Assumptions

1) Bauxite and other alumina bearing ores

For the past ten years no major discoveries of bauxite reserves have been made in the U.S. and it is a general assumption [16, 8] that no new discoveries will be made. Therefore it is assumed that domestic bauxite reserves are known and fixed. Furthermore, it is assumed that the ore grade and the silica concentration in the existing reserves will maintain their present values regardless of depletion.

Only two alternative alumina bearing ores are considered, kaolin clay and anorthosite. In both cases, due to their size, the reserves are assumed infinite. Furthermore, the location of new mines is restricted, in the case of kaolin clay, to Georgia, and in the case of anorthosite, to Wyoming. New York has twice the reserves of Wyoming, but state regulations make the building of mines in Wyoming more probable than in New York.

No new mines are allowed in the Arkansas bauxite reserves. Nevertheless expansion of existing mines is allowed. This expansion is limited to 15% of the present capacity. Expansions are allowed on new kaolin clay and anorthosite mines, but these expansions can only take place once the original mine is in full operation.

Imported bauxite is assumed to come entirely from the following countries: Jamaica, Surinam, Haiti, Dominican Republic, Guyana, Guinea, Australia, and Brazil. Haiti and the Dominican Republic are considered as one region called Caribbean. The quality of the bauxite (i.e. alumina and silica content and proportion of monohydrate alumina) is considered an attribute of each producing region. Furthermore it is assumed that this quality, per producing region, is maintained throughout the duration of the model. Imported bauxite is assumed to be low silica.

Each bauxite exporting country is represented by a segmented supply curve consisting of two segments. The first segment represents the forecasted capacity of the mines that supply the U.S. including developing capacity such as that in Brazil. The cost of bauxite belonging to the first segment is assumed to be the current transaction price. If desired, a price forecast can be readily implemented to simulate pricing policies. The second segment corresponds to the whole bauxite mining capacity owned by American firms excluding the ones included in the first segment. The cost of this segment is arbitrarily assumed to be 30% higher than the cost of the first segment. In addition, the representation of bauxite imports also simulates commercial ties.

An inventory is carried for bauxite imported from Jamaica, Surinam and Guinea. It includes Government Stockpile, consumers' stocks and producers' stocks. The initial stock is that of December 1976. No carrying cost is charged on the inventory except that implied by the time value of money. Bauxite inventory can supply any of the bauxite demand sectors.

The bauxite demand sectors are the refineries and the chemical, abrasives and refractories industries. The model groups these last three industries into one general sector called 'chemical'. The 'chemical' sector, following historical trends, is supplied with bauxite coming from Surinam and Guyana. American bauxite from Alabama and Georgia also supplies the 'chemical' sector; nevertheless it is considered constant [8] and it is not represented by the model.

It is assumed that bauxite is not exported.

2) Alumina

Alumina is obtained by the processing of alumina bearing ores in the refineries.

Domestic bauxite refineries are considered to be located in one sole region, namely the Southeast of the U.S. These refineries are divided into two types, those which process high silica (domestic) bauxite and those which process low silica (imported) bauxite. The first group includes the two Arkansas refineries while the second group accounts for the remaining refineries.

High silica refineries have historically consumed low silica bauxite from Surinam and Guyana to meet process grade specifications. Actually, only one of the two Arkansas refineries consumes low silica bauxite. Nevertheless it is assumed that the ratio of low silica to high silica bauxite consumed by these refineries remains constant.

Low silica refineries may process any type of low silica bauxite, although the efficiency and cost of the operation depends on the origin of the bauxite.

It is assumed that raw materials such as limestone and sodium carbonate and utilities, such as gas, will always be available. It is also assumed that the operation of the refineries includes the correct disposal of the red and brown mud to meet pollution regulations. This latter assumption is reflected in the operating cost.

Presently the model does not allow increments of existing refinery capacity. Nevertheless the installation of new bauxite refineries, of either type, is allowed.

Domestic refineries designed to process kaolin clay and anorthosite may be installed near the mines following present location trends. It is further assumed that they will operate with present technology. The kaolin clay refinery uses the hydrochloric acid-ion exchange process while the anorthosite refinery uses the lime-soda sinter process. No capacity increments are allowed in these refineries.

Alumina is assumed to be imported from Jamaica, Surinam, Guyana, Guinea and Australia. As in bauxite, each alumina exporting country is represented by a segmented supply curve. The first segment represents the forecasted capacity of the present suppliers, including any new refinery built to supply the U.S. The cost of alumina belonging to the first segment, for each country, is the current transaction price of alumina. The second segment (for each country) represents the capacity of all alumina refineries owned by American companies (excluding the capacity included in the first segment). The cost of alumina from this segment is arbitrarily assumed to be 20% higher than the first segment. In addition alumina supply per country follows commercial ties.

An inventory of alumina is carried by the model. It is supplied exclusively by the bauxite refineries. Inventory can supply any of the aluminum smelting regions. The demand of the 'chemical' sector, which consists of the same individual consumers as the bauxite 'chemical' sector, is satisfied exclusively by alumina from bauxite refineries. Similarly, the exports are satisfied exclusively by alumina from bauxite refineries.

Imported alumina and alumina refined from non-bauxite ores can supply the aluminum smelters, but not the other sectors.

3) Aluminum

Aluminum smelters are assumed to be located in three distinct regions, namely: Pacific Northwest, Northeast and Southeast. These regions account for 32%, 29%, and 39% of the present smelting capacity, respectively.

Six types of smelters are considered, five of these types use the Hall-Herroult process and the remaining type uses the Alcoa process. Those using the Hall-Herroult process are characterized by their electrode technology and are classified in: Horizontal Soderberg anodes, vertical Soderberg anodes, prebaked anodes, computerized control with prebaked anodes and titanium diboride cathodes. The power consumption of these smelters is assumed to be 9, 8, 7, 6 and 5 KWH per pound of aluminum respectively. The power consumption of smelters using the Alcoa process is assumed to be 4.5 KWH per pound of aluminum [5].

The building of new smelters is allowed if their nominal power consumption is less than 8 KWH/lb. The building of incremental smelting capacity is allowed only in smelters using the Hall process. The increments can be of any type, although they cannot exceed 50% of the original smelting capacity. Increments of new smelters can take place only after the smelter is in full operating conditions. Presently there is only one small experimental plant using the Alcoa chloride process and increments are not allowed in this type of smelter.

Any of the existing smelting capacity using the Hall process can be technologically improved in the sense of less power consumption. Following a logical observation, only improvements to the 'computerized control with prebaked anodes' type and to the 'titanium diboride cathode' type are considered. These improvements are arbitrarily considered to take place in the transition from one time period to the next and some technological improvements may include an increase in capacity.

The smelting process, which is the reduction of alumina to aluminum, is considered 100% efficient, and its output is molten aluminum metal. Two interrelated operations are considered in the smelting process: the production of aluminum and the consumption of electricity to achieve this production.

The consumption of electricity is restricted to the electricity available from the utility companies (purchased electricity) plus that generated by captive power plants. The available electric power from the utility companies is represented by extrapolating existing contracts (with TVA & BPA for example). Captive power plants include existing ones and those built by the model. All captive power plants are assumed to be thermal (presently 82% of captive power capacity is thermal) [15].

A domestic aluminum inventory is maintained, which follows some predesigned inventory policy and can supply any of the demand sectors. As in the case of bauxite and alumina, there are no carrying costs, except that implied by the time value of money.

Aluminum can be imported too. The model considers only one foreign supply region, not particularly identified. All imports are assumed at a common price and they are limited by the capacity of smelters owned by American companies (including Alcan) in foreign countries. Imports represent a useful tool to control the marginal cost of primary aluminum and to be able to simulate alternative foreign pricing policies. Aluminum is assumed to be imported entirely in ingot form.

In the case of secondary aluminum the model deals separately with the metal obtained from old aluminum scrap and the metal obtained from new aluminum scrap.

An analysis made on new scrap recovery showed that in the last 15 years the average fraction of new scrap recovered with respect to total shipments by the industry was 16% with a sample standard deviation of 1.8%. Following an optimistic policy it is assumed that a maximum of 18% of the total shipments may be recovered as new scrap.

For old scrap, an analysis prepared by the Bureau of Mine (BOM), a copy of which was given personally to the author, was followed. In this analysis the BOM presented the rate of obsolscence of aluminum products consumed from 1930 until 1976. This function is not the same for all years, but for the last five years it showed the same pattern (approximately a Poisson distribution with mean 13 years). Historically, only 15% of the obsolete aluminum is recovered as old scrap. Hence the model follows this rate of obsolescence given by the BOM and assumes that 15% of the obsolete aluminum is available at the present price of old scrap, a further 7% is available at a higher price and a further 5% is available at an even higher price. The setting of these prices, except for the first segment, is arbitrary.

The cost of old and new scrap is part of the operating cost to obtain secondary aluminum. Following an observation that the cost of scrap accounts for at least 65% of the cost of secondary aluminum [13], a single operating cost (excluding the cost of scrap) was assumed for secondary aluminum which is approximately that of secondary smelters. No imports or exports of scrap are considered.

Due to the existence of a large number of secondary smelters and the fact that many facilities used to process different types of scrap may produce secondary aluminum, it is assumed that secondary production capacity will always exist and no specific computation of secondary smelters is made. Furthermore, no specific region is considered for the production of secondary aluminum; rather, these facilities are assumed evenly distributed across the U.S.

4) Fabrication and Marketing

The traditional fabrication operations yield the following products:

a) Ingot

b) Sheet, plate and foil, including heat treatable and non heat treatable sheet and plate

c) Extrusions and tubes, which include extruded rod and bar, extruded shapes, extruded pipe and tube, drawn tube, welded tube and rolled and continuous rod and bar

d) Wire and cable, which include aluminum cable steel reinforced (ACSR), bare cable, bare wire and insulated or covered wire and cable

e) Various, which include forgings, powders, impacts and flakes for destructive uses.

Following historical trends published in reference [11], primary aluminum can be fabricated into any of the above mentioned forms while secondary aluminum is either fabricated into ingot or for destructive uses (as in the various classification).

The end use markets considered are: Building and Construction, Transportation, Consumer Durables, Electrical, Machinery and Equipment and Containers and Packaging. In addition a group 'Others' is included to account for miscellaneous demands. The group 'Exports' is included to cover the foreign trade.

Table 5.1
Fraction by Which Each Fabricated Form Satisfies each end use Market (as percentage)
Sample Mean and Sample Standard Deviation

	Ingot		Sheet, Plate and Foil		Extrusions and Tubes		Wire and Cable		Various	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
Building and Construction	6.87	1.75	50.10	2.71	41.80	2.93	1.2	-	.03	-
	7.30	1.01	52.17	1.85	39.39	2.14	1.2	-	.04	-
Transport.	46.76	2.12	32.69	1.12	16.72	1.70	.42	-	3.41	.50
	46.71	2.52	32.08	0.69	17.63	1.86	.53	-	3.05	.40
Consumer Durables	31.07	1.36	47.73	1.73	19.40	1.16	1.6	-	0.3	-
	31.31	1.43	46.84	2.11	20.20	0.92	1.25	-	0.4	-
Electrical	15.09	2.53	15.44	1.51	15.84	1.73	53.32	1.68	0.5	-
	15.27	1.66	15.55	1.75	14.86	.56	53.93	1.65	0.5	-
Machinery and Equipment	37.08	2.24	30.20	2.62	27.97	1.79	2.58	.22	1.6	-
	35.73	2.06	32.07	1.70	27.58	2.11	2.24	.21	1.6	-
Containers and Packaging	1.04	-	98.80	0.35	-	-	.16	-	-	-
	1.01	-	98.83	0.38	-	-	.16	-	-	-

NOTE: The upper figure corresponds to 11 years estimate and the lower figure to last 5 years estimate.

In studying the shipments of the different fabricated forms to the end use markets it was found that there was a consistent pattern in the proportion in which each fabricated form satisfied each end use market. Table 5.1 shows this pattern. Therefore, it was assumed that each end use sector was consistently supplied by the same combination of fabricated forms.

Due to the number of mills and foundries existing in the U.S., their distribution and the fact that many of them do not work exclusively with aluminum products, the fabrication capacity was not modeled but it was assumed that fabricating capacity always exists. Furthermore, again due to their distribution, no specific region was considered for the mills but their location was assumed to be evenly distributed.

Finally, exports are assumed to be either in ingot or mill products form.

B. Representation

The representation of the U.S. aluminum industry by the model follows the flow diagram of Figure 5.1. This diagram represents the whole aluminum cycle.

The time horizon considered by the model is eighteen years long, divided in nine time periods of two years each. A time period length of two years is appropriate for the aluminum industry since it is short enough to show response to irregular variations in prices or demands and long enough to make the value of the investment variables reasonable. Furthermore, nine time periods permit the development of a manageable model.

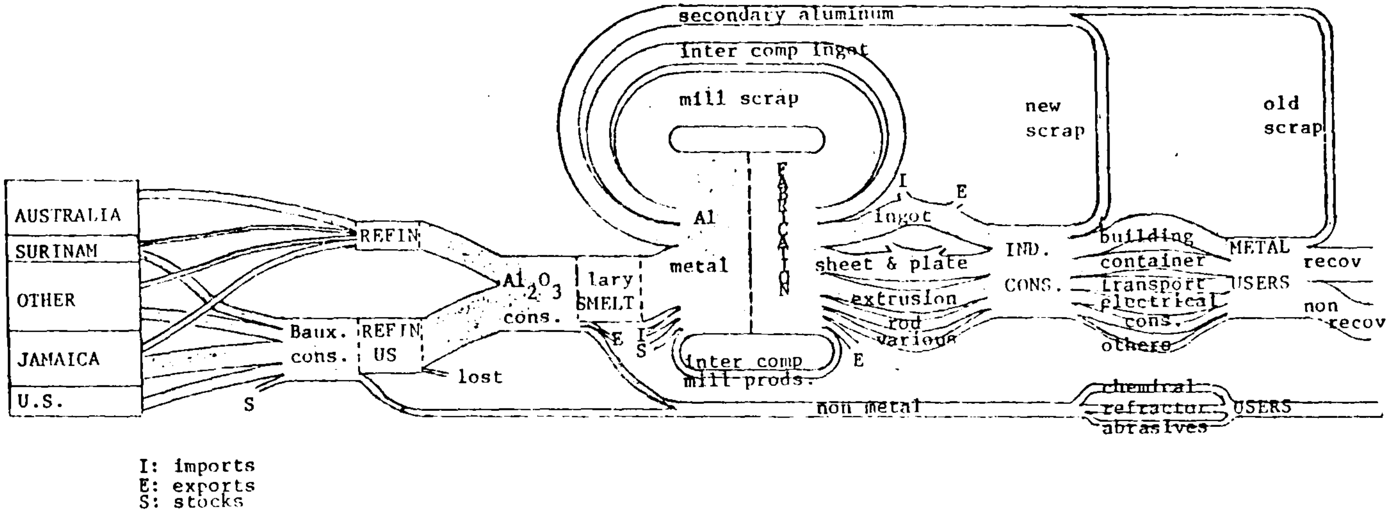


Figure 5.1
The Aluminum Cycle

In this section the investment and operating variables will be presented and the representation of the most important relationships will be described. The whole set of equations that comprise the model are presented in AppendixIII with the vectors and equations represented by their mnemonic structure. The mnemonic structure of the model is presented in Appendix II.

The investment variables considered by the model are the following:

1) New mines of anorthosite and kaolin clay, per region
2) Mine expansion of existing bauxite mines and newly built kaolin clay and anorthosite mines -- per region
3) New alumina refineries, per type and per region
4) New aluminum smelters, per type and per region
5) Expansion of existing and new aluminum smelters, per type and per region
6) Technological improvement of existing smelters, per type and per region
7) Captive power plants, per region

The operating variables considered are the following:

a) Process
    1) Mine production, per ore type and per region
    2) Alumina production, per refinery type, ore processed and region
    3) Primary aluminum production, per smelter type and per region
    4) Secondary aluminum production, per type of scrap processed
    5) Power generation, per type (captive or purchased) and per region

b) Transportation

   1) Transportation of imported bauxite, per segment of bauxite supply curve, country of origin, bauxite type and consuming sector or region
   2) Transportation of imported alumina, per segment of alumina supply curve, country of origin and consuming sector or region
   3) Transportation of domestic alumina, per origin and consuming sector or region
   4) Transportation of imported primary aluminum, per fabricating process and consuming sector
   5) Transportation of domestic primary and secondary aluminum, per fabricating process and consuming sector
   6) Transportation from inventory of bauxite, alumina and aluminum. It follows the same pattern as domestic transports except that the origin is inventory.

c) Inventory

   1) Inventory at the beginning of the time period -- per product

d) Distribution

   1) Distribution of bauxite, alumina and aluminum, per end use market and in the case of aluminum, per fabricated form

Furthermore these variables are represented by different vectors for different time periods. The units of these vectors are:

1) Investment: tons of output product per year (Megawatt in the case of power plants)
2) Operating:
    a) Process: ton of output product per year (MWhr/year in the case of power generation)
    b) Transportation: ton of product per year
    c) Inventory: ton of product
    d) Distribution: ton of product per year

Investment variables are characterized by their investment and building profiles, described in Chapter IV. In addition, new facilities and existing facilities maintain their production capacity without the need of specific upkeep functions, and the maintenance of the facilities is included in the operating cost.

The expansion of new facilities is allowed only after these facilities reach full operating capacity. This is represented by identifying each facility with its vintage v, and verifying the value of the capacity availability factor f_{vt} (see Appendix I for its definition) for the time period t considered. If this factor is lower than 1, it means that the facility has not reached full capacity, and the generation of the expansion vector is not allowed.

A concept that is widely used in the model is the representation of segmented supply curves. They are used in the supply of imported bauxite and alumina and in the supply of old scrap. Figure 5.2 shows a segmented supply curve. Referring to Figure 5.2 and recalling that the model is formulated as cost minimization,

Let

x_i = product purchased in segment i
p_i = price of segment i, where $p_{i+1} > p_i$
q_i = upper limit to segment i

The model always chooses the least expensive segment first and any variable x_i will not be basic unless x_{i-1} is at its upper bound. The bounds for the variables x_i are:

Lower bound:	0	for all i
Upper bound:	q_i	if i = 1
	$q_i - q_{i-1}$	otherwise

To simulate the commercial ties in the importation of bauxite, alumina and aluminum, the transportation of each of these products from their producing regions is lower bounded. The lower bounds correspond to the lowest import of each product in the period 1970 to 1974. These lower bounds are active for four time periods, after which they are removed.

The representation of policies or external constraints such as inventory policy or the limitation to purchase electric power is obtained by bounding the corresponding variables. Inventory lower

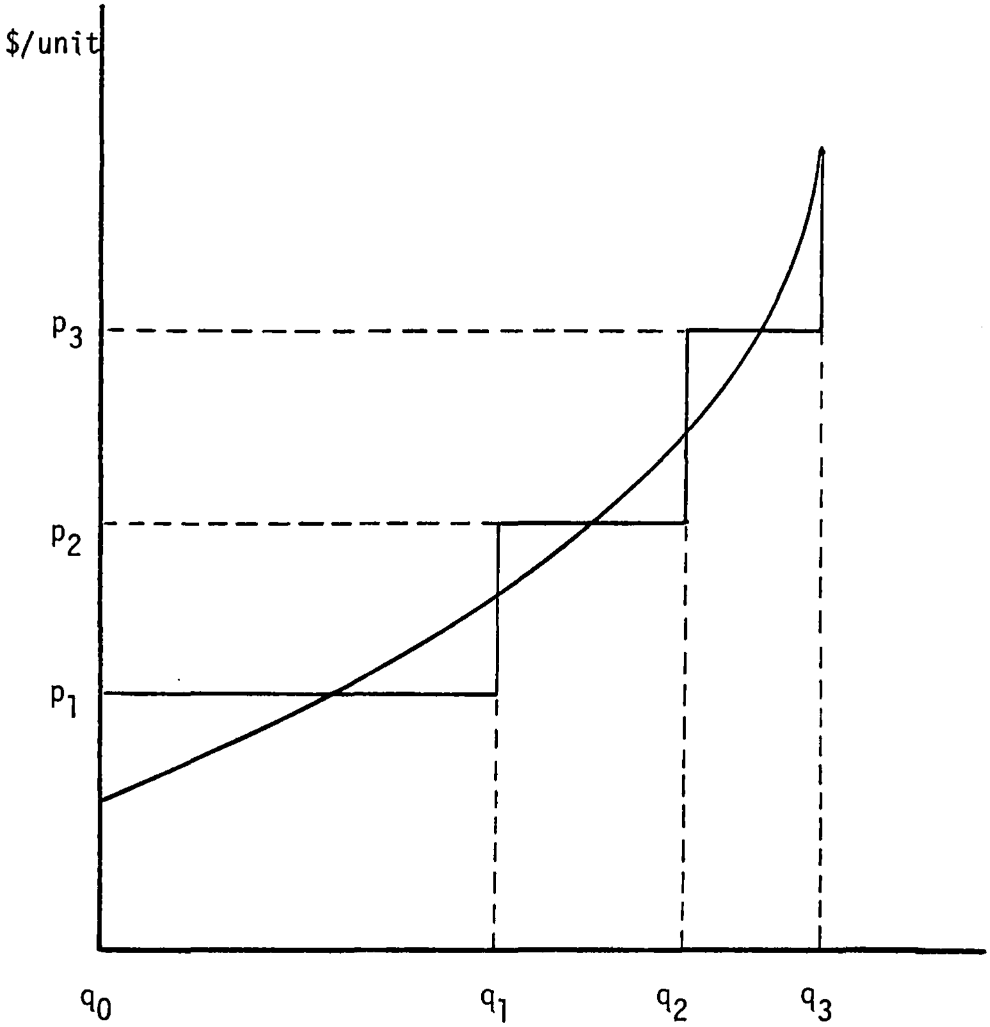


Figure 5.2
Segmented Supply Curve

bounds force the inventories of each product to follow historical trends. The upper bound to the purchase of electric power in the Pacific Northwest region follows the expiration of the power supply contracts between the Bonneville Power Administration and the aluminum companies.

The transportation of aluminum, either primary or secondary, either domestic or imported or from inventory follows a particular pattern. These vectors satisfy the end use market demands, though the demand is not as aluminum metal as it leaves the smelters but as some fabricated form of aluminum. Therefore these vectors represent the supply of aluminum from some particular source, through some particular fabricating process, to some particular market. The fabrication cost is added to the transportation cost.

It was mentioned above that the aluminum demand is per end use market and per fabricated form. This figure was obtained by multiplying the forecasted end-use demand per market by the historical fraction with which each fabricated form satisfied the particular market's demand.

The end use market demand for each fabricated form is represented by lower bounding the distribution vectors with the calculated demands. Since the model is formulated as a cost minimization these vectors will be at their lower bounds in the optimal solution. This enables the analysis of their reduced cost, which represent the marginal cost of each fabricated form to each end use-market.

The technological improvement of smelters, from a high power consumption rate to a lower one, is represented by a series of balance equations as shown in the prototype example of Section IV.B. If it is assumed that x_t^j is the original smelter capacity of type j in time period t and that y_t^{jk} is the capacity technologically improved from type j to type k, the following equation shows how this balance takes place

$$\sum_{i \in I} \Phi_{ij} \; y_t^{ij} + x_t^j - \sum_{k \in K} y_t^{jk} = x_{t+1}^j$$

where I = set of smelter types with higher power consumption than j

K = set of smelter types with lower power consumption than j

Φ_{ij} = increase in capacity due to the technological improvement i-j

The cost of the improvement vector y_t^{jk}, although a typical investment cost, is discounted as an operating cost.

The consumption of low silica bauxite by low and high silica refineries is represented by means of two constraining equations taking advantage of the fact that both low silica and high silica refineries are located in the same region.

Let

X_n^L = shipment of low silica bauxite, from country n to the refining region

y^L = production of alumina from the low silica refineries

y^H = production of alumina from the high silica refineries

N = set of all bauxite exporting countries

H = set of bauxite exporting countries that supply high silica refineries [H is a subset of N]

Then the two equations are:

$$(1)\ -\sum_{n \varepsilon H} \Phi_n X_n^L \quad + \quad \eta y^H \quad \leq 0$$

$$(2)\ -\sum_{n \varepsilon N} \Phi_n x_n \quad + \quad \eta y^H + y^L \leq 0$$

where Φ_n = yield of alumina per unit of bauxite of origin n

η = fraction of alumina from high silica refineries produced from low silica bauxite

The production of secondary aluminum from old scrap is the most elaborate part of the model. It is assumed that as much secondary aluminum from old scrap is produced as old scrap is recovered, since there is no explicit inventory of old scrap.

For this calculation, two functions were extracted from the Bureau of Mines report mentioned in the previous section:

(a) $\alpha(t)$ = the availability of obsolete aluminum originated in metal consumed prior to the beginning of the model at time period t

(b) $f(t-v)$= the rate at which products shipped in time period v are obsolete in time period t

If it is assumed that:

Φ_i = fraction of the obsolete aluminum that can be recovered as old scrap at price p_i

x_t^i = old scrap recovered at price p_i in time period t

y_t = new scrap recovered in time period t

s_t = domestic shipments of aluminum to the end use markets in time period t

If it is recalled that, of the metal shipped to the end use markets only that part that is not transformed into new scrap by the production process $(s_t - y_t)$ actually reaches the consumers and hence, is the fraction of the shipments that may generate old scrap, then the following equations are obtained for time period 1

$$x_1^i \leq \Phi_i \, [\alpha(1) + f_{(1-1)} \, (s_1 - y_1)] \tag{1}$$

for time period 2

$$x_2^i \leq \Phi_i \, [\alpha(2) + \sum_{j=1}^{2} f_{(2-j)} \, (s_j - y_j)] + \Phi_i[\alpha(1) + f_{(1-1)} \, (s_1 - y_1)] - x_1^i \tag{2}$$

since, whatever old scrap available in time period 1, which was not recovered in time period 1, is still available in time period 2. If it is assumed that all available old scrap at times prior to the beginning of the model has been consumed, then, the following equation shows the relationship for old scrap at any time period t.

$$x_t^i \leq \Phi_i[\alpha(t) + \sum_{v=1}^{t} f_{(t-v)}(s_v - y_v)] + \sum_{z=1}^{t-1} \left[\Phi_i[\alpha(z) + \sum_{v=1}^{z} f_{(z-v)}(s_v - y_v)] - x_z^i\right] \tag{3}$$

which is the generalization of equation (2). And the equation used by the model is a further simplification of (3)

$$\sum_{j=1}^{t} x_j^i \leq \Phi_i \sum_{j=1}^{t} \left[\alpha(j) + \sum_{v=1}^{j} f_{(j-v)}[S_j - y_j] \right] \tag{4}$$

The model has guaranteed feasibility (i.e. there will always be a way of satisfying the demands) for two reasons. First, aluminum imports, although upper bounded, can be considered unlimited for all practical purposes. Second, the reserves of kaolin clay and anorthosite are assumed to be infinite.

To represent the financial capabilities of the industry, the investments are limited to half a billion 1977 dollars per year for each of the stages of the aluminum cycle (i.e. mining, refining and smelting).

C. Results Provided by the Model

The present model provides information ranging from the optimal allocation of sources for the demanded products to the marginal cost of each of these products for any time period.

With these results, a picture of the optimal evolution of the industry can be traced, within the limitations of the formulation. If the most important features of the industry are represented, as well as the most important external factors, then the solution of the model simulates the optimal reaction of the industry in the time horizon considered.

The coefficients entered in the matrix can be easily modified to simulate changes in scenarios. This feature allows a very simple way of analyzing the sensitivity of the model.

Furthermore, the results obtained are one way of validating the model. That is, if the results obtained for the first time period differ considerably from the present performance of the industry, it indicates that there may be an error in the data or in the formulation.

Results in this type of model should be considered as trends and not as definitive answers since several assumptions had to be made to arrive to this prototype of the aluminum industry.

The present version of the model supplies the analyst with the following results, for the stages of mining, refining, smelting and marketing:

1) Mining:

a) Building of new mines per time period and per mine type (or region)

b) Building of incremental mine capacity per mine vintage

c) Total new mine capacity per time period and per mine type (or region)

d) Available mine capacity per time period and per mine type (or region)

e) Mine production per time period and per mine type (or region)

f) Imports of bauxite per time period and per country

g) Shipments of imported bauxite per country of origin and destiny, per time period

h) Bauxite inventory status per time period and per bauxite origin

i) Investments in mines per time period, in 1977 dollars

2) <u>Refining</u>:

a) Building of new refineries per refinery type and per time period

b) Available refining capacity per refinery type and per time period

c) Refinery production per refinery type and per time period

d) Alumina imports per country of origin and per time period

e) Shipments of domestic alumina per producing region and consuming region or sector, per time period

f) Shipments of imported alumina per producing region and consuming region or sector, per time period

g) Alumina inventory status per time period

h) Investments in refineries per time period, in 1977 dollars

i) Marginal cost of domestic alumina per time period, in 1977 dollars per ton

3) <u>Smelting</u>:

a) Original smelter capacity per time period taking account of the technological improvements, per smelter type and per region

b) Building of new smelters, per time period, smelter type and region

c) Expansion of new and existing smelters, per time period, smelter type and region

d) Available smelting capacity per time period, smelter type and region

e) Smelter production, per time period, smelter type and region

f) Secondary aluminum production, per type of scrap and time period

g) Primary aluminum inventory status, per time period

h) Building of captive power plants,.per region and time period

i) Investments in smelters and power plants, per time period

j) Marginal cost of primary and secondary aluminum

4) <u>Marketing</u>:

a) Shipments of aluminum, per time period, per origin, to each end market through each fabrication process

b) Marginal cost of each fabricated form that reaches the end use market, per time period

These results are the ones explicitly displayed by the model although, if desired, the value of the dual variable on imports of bauxite or alumina, or the reduced cost of purchased electricity are easily obtained from the general solution of the model.

VI. IMPLEMENTATION OF THE MODEL

A. Computer Language Used -- General Characteristics

The aluminum model was developed on an IBM 370/158 computer using the MPS-III mathematical programming system.

In using the MPS-III system, there are four hierarchical levels of control that must be considered, namely, Job Control, MPS Procedure Control, DATAFORM and input data.

Job Control, implemented through the system JCL, will not be considered in this report.

The MPS Procedure Control consists of a series of procedures, each performing a clearly identified function, i.e. the READY statement initializes the LP matrix for VARIFORM to determine the optimal solution which is then printed by the SOLUTION procedure. The variety of procedures is large and the user specifies the sequence of these procedures in the Procedure Control Language. Moreover, at this level, values of parameters may be set (i.e. the discount rate).

DATAFORM is a high-level programming language which is used to interface with all components of the MPS-III system. It performs the classical logical, character and arithmetic operations but, in addition, it is built in such a way that it provides special tools for the building and storing of the LP matrix and the generation of reports. Furthermore, modification of existing LP matrices is easily done with an appropriate DATAFORM program without having to regenerate the whole matrix.

The input data that the DATAFORM programs manipulate to create the matrices and generate the reports, are presented in the form of one or two dimensional tables. These tables contain all of the models input data.

In addition, a special procedure in the PCL, PRINTRAN [31], provides a way of validating the model in the sense that it represents what it is intended to represent. By means of this procedure, the whole series of equations that comprise the model are printed in their mnemonic structure with the original matrix coefficients. The correspondence of the proposed equations and the systems printout was verified for the aluminum model. The printout cannot be presented in this report due to its size, but it can be readily generated following the corresponding programming procedure.

The optimal solution provides valuable information for both rows and variables. For the rows it provides the row activity, the slack activity, the row's upper and lower bound and its dual activity. For the variables it provides the activity level, its input cost, its upper and lower bounds and its reduced cost.

The LP matrix of the model consists of 1485 rows and 4351 variables. There are 17700 non-zero elements. The compilation and execution of the matrix generation program takes 14 minutes of CPU time and the determination of an optimal solution from zero level takes 40 minutes of CPU time.

For further information on the MPS-III and the MPS-III-Dataform language, the reader is referred to [32] and [33].

B. Matrix Generation

The generation of the linear programming matrix relies on two basic concepts: first, the mnemonic structure chosen for rows and columns and second, the fact that the model is data driven.

The mnemonic structure is a sine qua non condition to be able to manipulate a large model. The vectors and the equations are identified by an eight character name which itself is a concatenation of different basic structures which identify activity, resource, region and time. For a detailed presentation of the mnemonic structure of the model, the reader is referred to Appendix II.

The fact that the model is data driven means that the structure of the data tables is, perhaps, the main foundation of the building of the matrix. Each table, identified by a mnemonic structure pertaining to tables, carries data on processing costs, processing efficiencies, etc. Furthermore, the head and the stub of these tables are the basic elements of the mnemonic structure of the vectors and the equations. Therefore, when generating the intersection of vectors and equations, the elements of the corresponding tables are extracted by masking the name of the equation or the vector to extract the basic mnemonic element which is the head or the stub in that table.

The matrix elements are generated either by row or by column. The aluminum model was built by column, generating all the row intersections for each column. Following, an example of this operation is presented:

Let it be assumed that the intersections of the vector that represents the production of alumina from Jamaican bauxite in the southeastern U.S. in a certain time period t, are generated. Let it be further assumed that the intersection with the objective function (operating cost) is desired.

The vector is represented mnemonically by: PRLBJE.t

The operating cost table is

T: CO2UR.. = A, J, N, C, Y, G, L, B
E = 90, 98, 90, 90, 90, 90, 90, 90

The origin of the bauxite, J, and the processing region, E, define the element in the table that corresponds to the desired value.

Dataform language includes mathematical operations. This allows the computation of discount factors and available capacity factors by means of subroutines. In addition, the implementation of the model using the dataform language allows an overall flexible structure. Changes in the formulation, changes in the matrix coefficients, addition and deletion or vectors and equations are very easily done.

VII. DATA

A. Required Data

Data required for the development and execution of the model can be divided into five categories:

1) Decision data
2) Constraining data
3) Process definition data
4) Demand data
5) Miscellaneous data

Decision data consist of investment cost in: new facilities, expansion of existing facilities and technological improvements, processing costs, transportation costs, importation costs and distribution costs.

Constraining data include the existing capacities of the different facilities, the ore reserves in mines, the restrictions to imports, the availability of capital, the availability of purchased electric power, the inventory policy and any other external restriction.

Process definition data include process efficiencies and yields, and process requirements (i.e. electric power).

Demand data are the forecasted demands of each end use sector for bauxite, alumina and aluminum.

Miscellaneous data include precalculated figures, like the fraction of each fabricated form required by each aluminum end use market, the rate of obsolescence of aluminum, the availability of obsolete aluminum originating in consumption prior to the beginning of the model and the historical fraction of new scrap availability. They also include building and investment profiles and tables of permissible links (i.e. which ores may be processed by each refinery).

In addition, there is a data table, not related with the aluminum industry, that contains information essential to the development of the model. This table defines the number of time periods and the length of each time period.

B. Data Manipulation and Maintenance

All data, except for a few cases, are organized and stored in table format. The table format is essential for the generation of the matrix. The exceptions are: the discount rate, the historical fraction of shipments of low silica bauxite from Guyana and Surinam to the high silica refineries and the increase in capacity of smelters due to improvements of their technology, which are part of the matrix generation program.

A mnemonic structure, different from the one used to identify equations and vectors, is used to identify the tables. The rows and columns of these tables are identified by the basic elements of the mnemonic structure of vectors and equations. The mnemonic structure of the tables is presented in Appendix IV.

Representation in tables makes locating data simple. Furthermore, all elements in the tables are expressed in the units customarily used by the industry and all necessary conversion factors are included or calculated by the matrix generation program.

All data which are functions of time, such as demand, are represented by the average value in the corresponding time period.

Maintenance of the data is simple thanks to the tabular representation. The addition of new data is very simple and the model structure readily absorbs new information.

The data tables are presented in Appendix V.

C. Data Gathering and Sources

To analyze the computation of the required data, the outline presented in section VII-A will be followed

a) Decision data: They are the most difficult data to obtain since the private companies do not, in general, disclose them and there are very few publications that report or estimate them directly or indirectly.

Cost data was obtained from federal publications from the Bureau of Mines [1, 4, 8, 10 and 17] and the Council on Wage and Price Stability [6]. Cost data was also obtained from reports concerning directly or indirectly with the aluminum industry [2, 3, 5, 7, 13, 16, 19 and 40] and from several journals [34, 35, 36 and 37].

These data may be considered the most uncertain in all the model since they are based mostly on individual estimates. In many cases different estimates for the same cost were obtained and the decision on which cost to adopt was based on the reliability of the source and the comparison with other known data. Furthermore, only data posterior to 1973, that is, after the oil embargo by the OPEC, were considered.

Since the application of indices for updating costs did not always work, it was considered that most of the related costs kept their relative values from 1974 to 1977, therefore with the estimates of some costs for 1977 and their relationships with the other costs, all costs were updated.

b) Constraining data: These data are easier to obtain and in most cases, estimates from different sources, coincide. Data on existing capacities and reserves were obtained from [1, 7, 9, 11, 12, 15 and 38]. Future capacities of the bauxite, alumina and aluminum exporting countries are forecasted for the next ten years in [15, 39, 40]. For later years the forecast consists of time series analysis, application of regression analysis and general criteria.

Restrictions in the availability of electric power were obtained from [18 and 20].

All other constraining data necessary for the correct development of the model, were personal estimates of the author.

c) Process definition data: These comprise mostly technical data and the main source of reference were technical reports.

The computation of the yield of alumina by bauxite and other ores was done considering the alumina content of the ore, the silica content (in the case of bauxite) and the process. These data appear in [1, 3, 4, 7, and 16]. The yield of aluminum by alumina was considered to be the same for all types of smelters and the data source is [2].

The electric power requirements, which is a function of the smelter type, was obtained from [5 and 14].

d) Demand data: This is the driving force of the model and therefore, very important. There are not many sources of these data and alternative forecasts show significant differences. The demand forecasted by Charles River and associates [16] was adopted as a variable rate of growth demand, while the forecast of the Bureau of Mines [1] was adopted as the steady growth demand.

e) Miscellaneous: Since there is no common denominator in the type of data grouped by this section, they will be listed, together with the computation method and the source.

The fraction of fabricated product per end use market was discussed in section V-B, the source being [11].

The rate of obsolescence of aluminum and the available obsolete aluminum resulting from consumption prior to the beginning of the model were calculated from a personal report given to the author by the Bureau of Mines [17]. This report consisted of a table with all shipments of aluminum to the end use consumers from 1930 to 1976 and the rate at which these shipments became obsolete in later years.

The building profiles and investment profiles were estimates of the author based on data from the journals [34, 35, 36, 37].

VIII. RESULTS

A. Reference Case

The reference case is based on the forecast of aluminum demand prepared by Charles River and Associates [16]. The breakdown of demand by end use sector was obtained following historical trends. Furthermore, the historical breakdown was subjectively perturbed following market forecasts as suggested by the Bureau of Mines [1]. Table 8.1 shows the demands used in the reference case.

The main characteristics of the reference case are:

(1) The pricing policy of imported bauxite assumes the success of the IBA cartel. Prices for the first segment of the imported bauxite supply curves are shown in Table 8.2. It is assumed that Brazil does not join the IBA.

(2) Only one segment is represented for the supply curve of imported bauxite and alumina. The upper bounds on imports of bauxite and alumina, for the first segment of the supply curve, are shown in Tables 8.3 and 8.4.

(3) Imported alumina prices maintain their 1976 level. Table 8.5 shows the prices of the first segment of the imported alumina supply curve.

(4) Bauxite inventory policy allows the gradual depletion of the December 1976 stocks of 22.8 million tons.

(5) The BPA power supply contracts for smelters in the Pacific Northwest expire and cannot be renegotiated.

(6) Primary aluminum imports are upper bounded. Table 8.6 shows these bounds

(7) Primary aluminum imports price is $.70 per pound, constant throughout the time horizon.

(8) Primary aluminum inventory policy forces a lower bound on primary aluminum stocks that increases at a rate of 50,000 tons per year, starting from the stocks existing on December 1976.

(9) Investments are limited to 500 million 1977 dollars per year for every processing stage (mines, refineries, smelters)

(10) The discount rate is 18%

For the analysis of the solution of the reference case, the three main stages of the aluminum cycle are treated individually.

a) Bauxite and other alumina bearing ores

There is no development of non bauxite ores and the existing bauxite mines are not expanded.

There is no domestic bauxite production during the first two time periods. From the third time period until the end of the model, domestic bauxite production is at the 1976 level.

Imported bauxite from the IBA countries remain at their lower bound for the first three time periods. From the third time period to the last the level of consumption of bauxite increases at a rate of 2.7% per year, but Jamaica's relative share decreases while Guinea and Surinam increase theirs'. Imports from Brazil, a non IBA country, increase to their upper bound. Figure 8.1 shows the development of bauxite imports.

Bauxite inventory is depleted following the inventory policy. There are no shipments to inventory.

b) Alumina

There is practically no expansion of the existing refining capacity and domestic alumina production is below capacity for the first five periods as shown in Figure 8.2.

Alumina imports reach their upper bound in the third time period and remain on this level until the end of the time horizon. Figure 8.3 shows the breakdown of alumina imports.

Some alumina is shipped into inventory during the periods 89-90 and 91-92, to be used in the last time period, 93-94.

The Northwest smelting region is supplied mostly by Australian alumina and marginally by domestic alumina. The Northeast smelting region is supplied, entirely, by imported alumina. The Southeast smelting region is supplied, principally, by domestic alumina and marginally by imported alumina.

c) Aluminum

Primary aluminum smelting capacity increases at a rate of 3.2% per year (compared to the rate of growth of demand of 4.6% per year), to reach a capacity of 9.05 million tons per year in 1994. Smelters with Soderberg anodes are not improved technologically and they become idle smelting capacity in the Pacific Northwest in latter years. All prebaked anode smelters are technologically improved and, in 1994, smelters with titanium diboride cathodes represent 65% of the active smelting capacity while smelters with computerized control on prebaked anodes represent 18% of the same capacity. Smelters using the Alcoa process are built after 1990 and they represent 500,000 tons per year of smelting capacity in 1994. The development of primary aluminum smelting capacity is shown in Figure 8.4.

The breakdown of aluminum supply is shown in Figure 8.5. The increased importance of secondary aluminum and of imported primary aluminum is notable. In 1994 their share of aluminum supply is 25% and 23% respectively compared to 20% and 13% in 1976.

Moreover, Figure 8.6 shows the marginal cost of primary and secondary aluminum. The high demand and the lack of smelting facilities in 1977-1980, forced the importation of primary aluminum, as shown by a marginal cost of 70¢. In the third time period, with appropriate facilities, the cost drops significantly and starts a steady increase, following the overall demand rate until importing starts again (above the lower bound) in 1987-88. It is noteworthy that, in the last time period, when imports reach their upper bound, the marginal cost of both primary and secondary aluminum increase significantly.

The influence of the expiration of the power supply contracts in the Pacific Northwest is noticed in the decrease of the relative share of domestic primary aluminum production by this region. In 1977 the share is 33% while in 1994 it drops to 20%. Furthermore, the model builds captive power plants in the Pacific Northwest for a total of 1347 MW installed capacity.

The trend followed by the model to satisfy the end use market aluminum demand in the different fabricated forms is to allocate secondary aluminum to satisfy the ingot and 'various' demand while primary aluminum satisfies the demand of the remaining milled forms. The model shows the marginal cost of the distribution of each fabricated form to each end use market.

As far as investments are concerned, there are no investments at the mining and refining stages, but at the smelting stage investments are at or near the upper bound in the first five time periods and are null only in the last time period.

Summarizing the reference case, the following conclusion can be extracted from the results:

(1) Alumina imports are preferred over bauxite imports, at the current prices

(2) Secondary aluminum becomes more attractive as the marginal cost of primary aluminum increases

(3) Smelters with Soderberg anodes are not attractive for technological improvement

(4) Smelters with titanium diboride cathodes become the most attractive smelter type. Even though it is assumed that, with a longer time horizon, the Alcoa process would be attractive too.

(5) Investment requirements are large

(6) Non bauxite ores are not attractive

Table 8.1

Aluminum Demand
Reference Case Demand
(MM ton/yr)

	B	T	D	E	M	C	O	F	TOTAL
1977-78	1.73	1.30	.65	.94	.58	1.65	.36	.60	7.81
1979-80	2.05	1.56	.74	1.15	.49	1.80	.41	.57	8.77
1981-82	2.06	1.51	.80	1.03	.64	1.51	.40	.55	8.50
1983-84	2.13	1.64	.74	1.06	.66	1.56	.41	.66	8.86
1985-86	2.18	2.00	.64	1.09	.64	2.09	.45	.82	9.91
1987-88	2.84	2.20	.84	1.26	.74	2.10	.53	.84	11.35
1989-90	3.02	2.44	.93	1.39	.81	2.44	.58	1.04	12.65
1991-92	2.97	2.97	1.16	1.55	1.03	2.58	.65	1.16	14.07
1993-94	3.71	3.27	1.34	1.78	1.19	2.82	.74	1.19	16.04

where

B = Building and Construction

T = Transportation

D = Consumer Durables

E = Electrical

M = Machinery and Equipment

C = Containers and Packaging

O = Others

F = Foreign Demand

Table 8.2

Bauxite Price

(First segment of supply curve)

Reference case

($/ton)

	Jamaica	Surinam	Caribbean	Guyana	Guinea	Australia	Brazil
77-78	28.5	30.2	30.8	34.3	26.5	38	--
79-80	28.5	30.2	30.8	34.3	26.5	38	20
81-82	28.5	30.2	30.8	34.3	26.5	38	22
83-84	32	32	32	35	29	40	25
85-86	32	32	32	32	32	40	30
87-88	32	32	32	32	32	40	30
89-90	32	32	32	32	32	40	30
91-92	32	32	32	32	32	40	30
93-94	32	32	32	32	32	40	30

Table 8.3

Bauxite imports

(upper limits to the first segment of supply curve)

Reference case

(1000 tons/year)

	Jamaica	Surinam	Guyana	Guinea	Australia	Caribbean	Brazil	TOTAL
77-78	8700	3200	700	3200	400	2100	0	18300
79-80	8700	3300	750	3300	400	2100	600	19150
81-82	8700	3500	820	3400	500	2100	1000	20020
83-84	9000	3600	820	3500	500	2100	1200	20720
85-86	9000	3600	820	3800	500	2100	1500	21320
87-88	9000	3600	820	3800	500	2100	1500	21320
89-90	9000	3800	820	3800	500	2100	2000	22020
91-92	9000	3800	820	3800	500	2100	2000	22020
93-94	9000	4000	800	4000	500	2100	2000	22420

Table 8.4

Alumina imports

(upper limit to first segment of supply curve)

Reference case

(1000 tons/year)

	Jamaica	Surinam	Guyana	Guinea	Australia	Total
77-78	1500	1490	100	200	3200	6490
79-80	2000	1490	100	400	3500	7490
81-82	2000	1490	100	600	3500	7690
83-84	2200	1490	100	1000	3800	8590
85-86	2500	1500	100	1000	3800	8900
87-88	2500	1500	100	1400	4100	9600
89-90	2500	1500	100	1400	4500	10000
91-92	2500	1600	100	1800	5000	11000
93-94	2500	1600	100	1800	5000	11000

Table.8.5
Alumina price
(first segment of supply curve)

Reference case
($/ton)

	Australia	Jamaica	Surinam	Guinea	Guyana
77-78	141	133	132	135	132
79-80	141	133	132	135	132
81-82	141	133	132	135	132
83-84	141	133	132	135	132
85-86	141	133	132	135	132
87-88	141	133	132	135	132
89-90	141	133	132	135	132
91-92	141	133	132	135	132
93-94	141	133	132	135	132

Table 8.6
Aluminum imports
Upper bounds
Reference case
('000 tons per year)

1977-78	2200
1979-80	2360
1981-82	2580
1983-84	2760
1985-86	2950
1987-88	3140
1989-90	3330
1991-92	3520
1993-94	3700

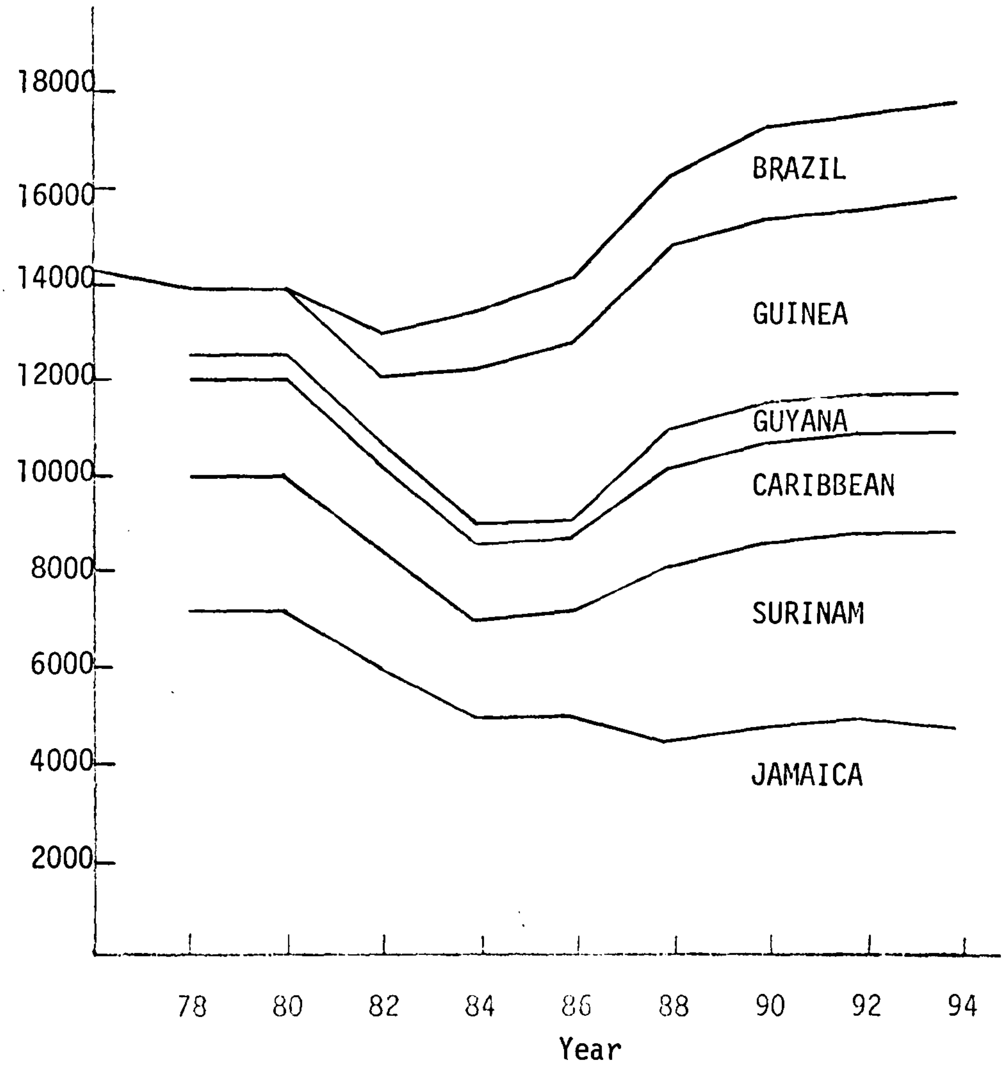


Figure 8.1
Bauxite Imports
Reference Case ('000 ton/yr)

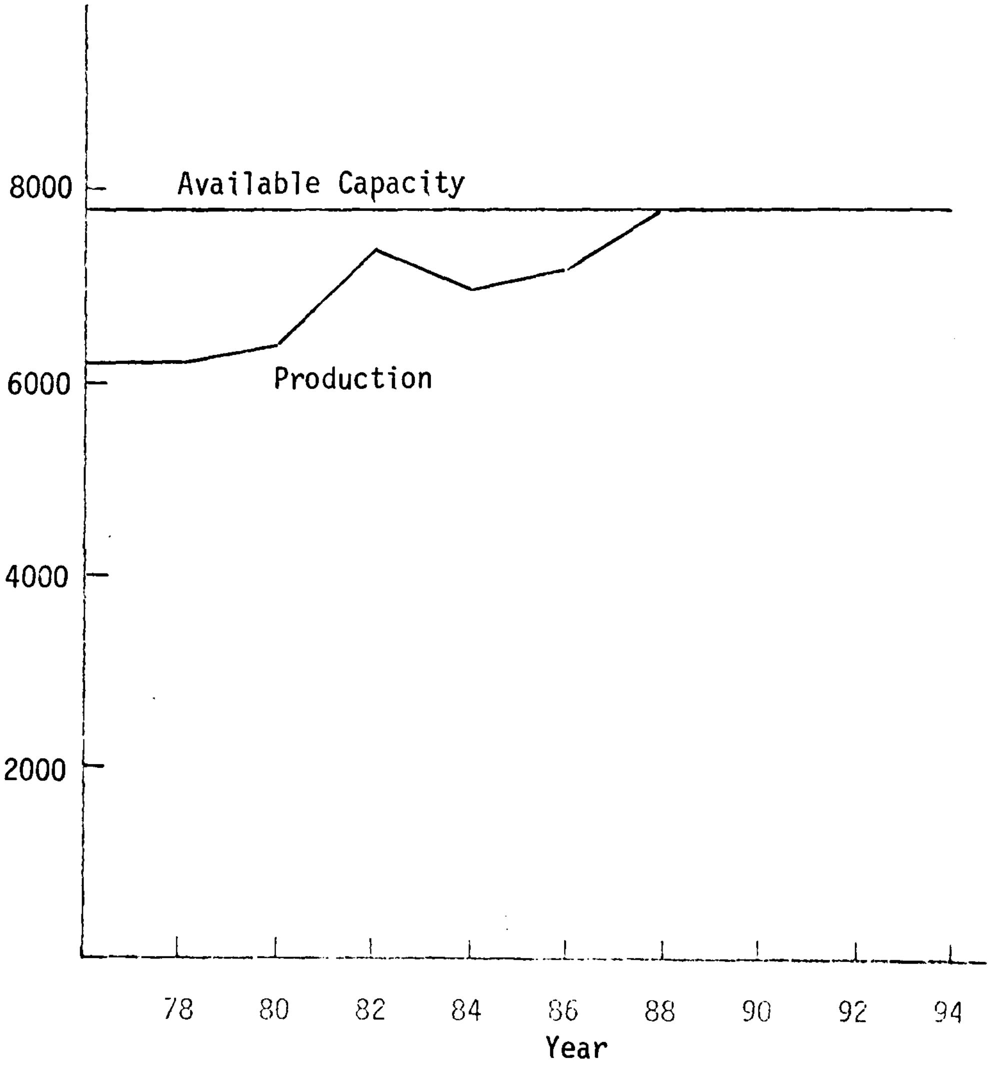


Figure 8.2
Domestic Alumina Production
Reference Case ('000 ton/yr)

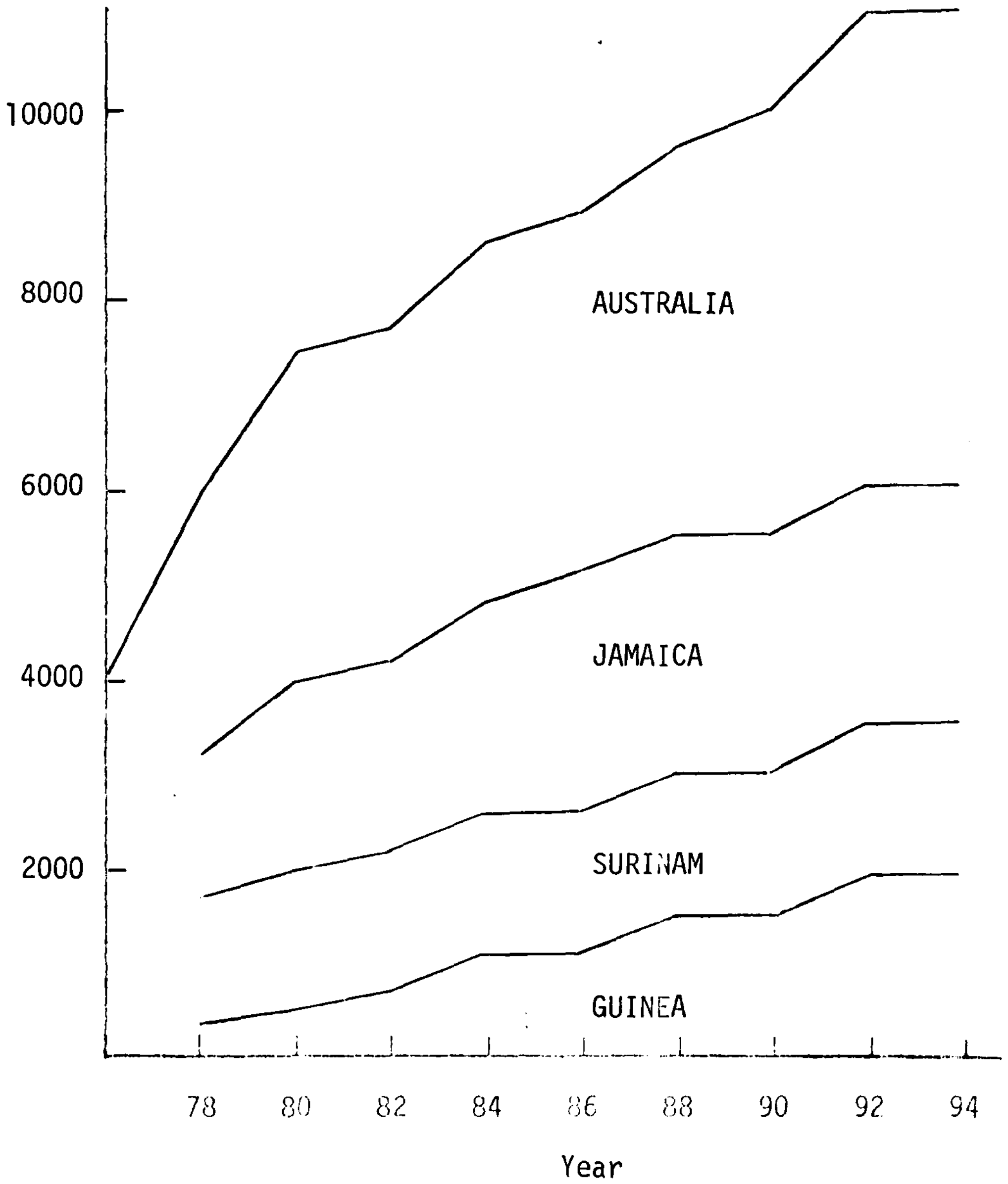


Figure 8.3
Alumina Imports
Reference Case ('000 ton/yr)

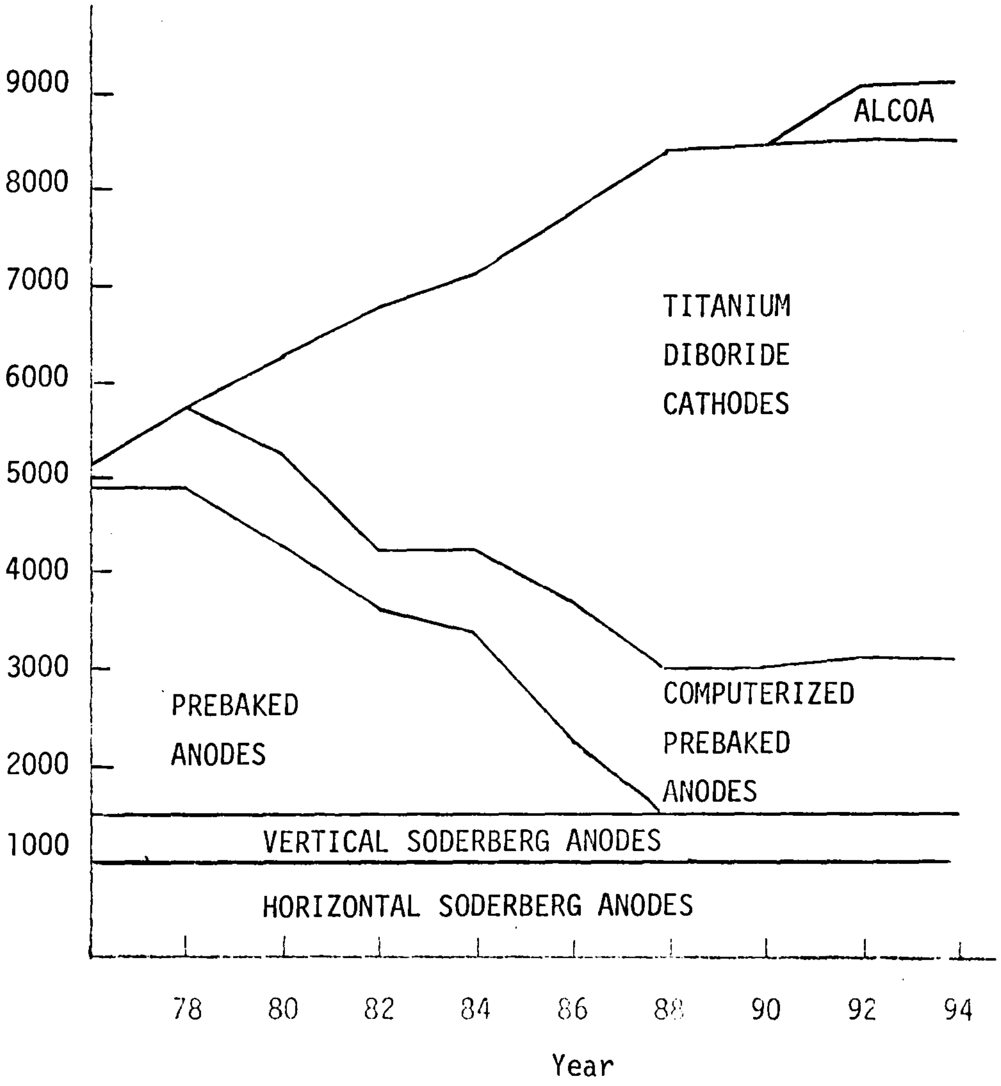


Figure 8.4
Smelter Capacity Development
Reference Case ('000 ton/yr)

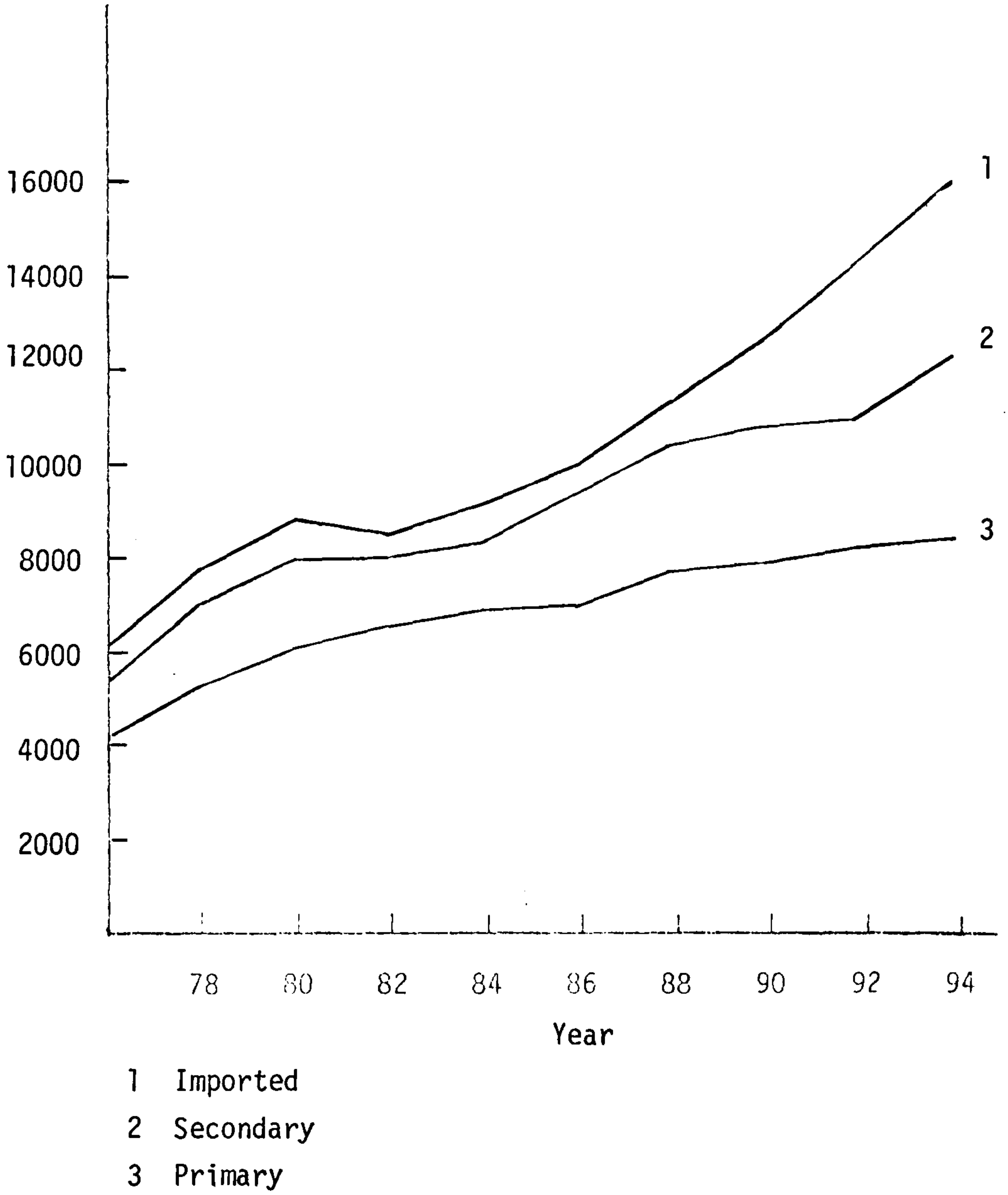


Figure 8.5
Aluminum Supply
Reference Case ('000 ton/yr)

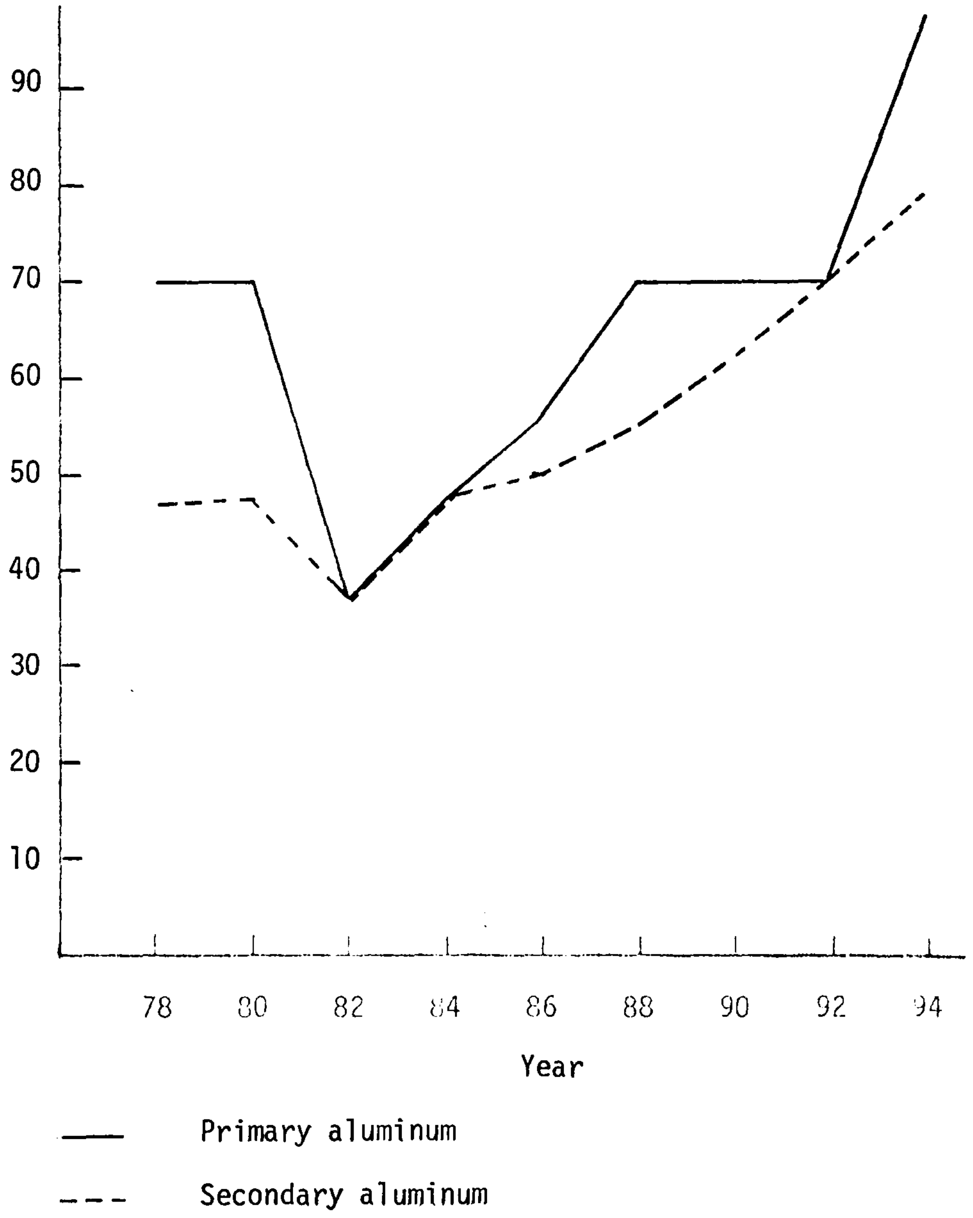


Figure 8.6

Aluminum Marginal Cost

Reference Case (¢/lb)

B. Case Studies

In all cases studied, contrasting with the reference case, the supply curve for bauxite and alumina consists of two segments. The price of the first segment was shown in Tables 8.4 and 8.5. The price of the second segment is shown in Table 8.7 for bauxite and in Table 8.8 for alumina. The upper bound on shipments corresponding to the second segment is shown in Table 8.9 for bauxite and Table 8.10 for alumina.

The cases studied can be divided into two main groups, depending on the aluminum demand forecast that they utilize. The first group uses the Charles River & Associates forecast [16], which is the one used by the reference case and represents an irregular rate of growth of aluminum demand. The second group uses the Bureau of Mines forecast [1], which represents a uniform rate of growth of aluminum demand.

1) Charles River demand

Four cases were run using this forecast, and their variations with respect to the reference case are the following:

Case 1: The price of imported primary aluminum is 60¢ per pound

Case 2: Idem case 1, plus the upper bound to primary aluminum imports is assumed to grow at a rate of 5% per year, starting with the capacity of American owned smelters in foreign countries in 1976.

Case 3: Idem case 2, but the growth rate is assumed to be 6% per year.

Case 4: Idem case 3 but the price of imported primary alumina is increased; the price of the second segment of the original supply curve becomes the price of the new first segment and the new price of the second segment is shown in Table 8.11.

From the analysis of the outcome of these four cases, the following observations are made:

Alumina imports are preferred rather than importing bauxite and processing it to alumina domestically, even at high imported alumina prices. Figures 8.7 and 8.8 show the imports of bauxite and alumina respectively for cases (3) and (4). As reference, the 1976 level of imports is included in both figures. Moreover, the processing of domestic bauxite is attractive in both cases.

With the CRA forecast, which predicts a demand of 16 million tons of aluminum for 1994, the aluminum imports of the reference case and of case (2) cannot satisfy the requirements of imported aluminum in the last time period. Moreover, a change in the price of imported aluminum from 70¢ per pound to 60¢ per pound, does not change the amount imported significantly. Figure 8.9 shows the primary aluminum imports of the reference case (70¢ per pound) and case (1) (60¢ per pound), both having the same imported alumina price. It is interesting to analyze Figure 8.9 together with Figure 8.10, which shows the evolution of the marginal cost of primary and secondary aluminum for the same cases. The sharp increase in demand of the first two time periods cannot be covered by the existing smelting capacity and the difference has to be imported, thus the marginal cost is the import price. When domestic smelters are able to satisfy the demand requirements, the marginal cost drops significantly to 38¢ per pound. As demand increases, the marginal cost of primary aluminum increases too, until aluminum imports become attractive once more (in the sixth time period for both cases). From that point onwards, aluminum imports grow at a rate of 26% per year until the end of the time horizon. Since in both these cases, imports cannot satisfy the demand requirements of the last time period, the marginal costs of both primary and secondary aluminum show a sharp increase in 1993-94.

With a rate of growth of American foreign smelters capacity of 6% per year the demand requirements of imported primary aluminum in the last time period are satisfied. The average imports in this period are 5.4 million tons per year, which represent 34% of the total aluminum supply.

The evolution of the marginal cost of primary aluminum in the reference case and in case (4) show the same pattern, and their values are very similar. This fact shows that moderate variations in the price of alumina have no major effects in the cost of aluminum.

The building of smelting capacity in all cases studied shows the same pattern as in the reference case. Nevertheless it is interesting to see the reaction of the Pacific Northwest smelting region to the price and the limitation of primary aluminum imports, since its purchased electric power availability is reduced considerably towards the end of the time horizon. Figure 8.11 shows the production and the available capacity of smelters in the Pacific Northwest for cases (1) and (2). Complementing Figure 8.11, Figure 8.12 shows the building of captive power plants in the Pacific Northwest for the same cases. It is recalled that, in case (1), imports of primary aluminum are constrained to 3.7 million tons in the last time period, and this constraint is tight, and in case (3), imports are practically unconstrained and the industry imports 5.4 million tons of primary aluminum in the last time period.

The building of captive power plants when aluminum imports at 60¢ per lb are practically unlimited shows that it is attractive to build captive power plants in the Pacific Northwest to make up for the shortage of available power resulting from the expiration of the Bonneville Power Administration contracts.

Bureau of Mines Demand

The Bureau of Mines forecasts a rate of growth of aluminum demand of 5.1% per year until 1985, decreasing then to 4.5% per year until the year 2000. This forecast results in an aluminum demand of 14.65 million tons in 1994, compared to the 16 million tons forecasted by CRA.

All cases studied with this demand forecast include the assumption of a rate of growth of American owned smelting capacity in foreign countries of 6%, which means practically unlimited primary aluminum importing capacity. In addition, the cases studied in this section have the following differences with the reference case.

Case 5: Primary aluminum importing price 60¢ per pound

Case 6: Idem Case 5 plus a high price for alumina imports

Case 7: Idem Case 6 but primary aluminum importing price returns to 70¢ per pound

Case 8: Idem Case 7 plus a diminution of bauxite prices in the third time period simulating the failure of the IBA cartel. The bauxite prices of the two segments of the supply curve are shown in Tables 8.12 and 8.13.

Case 9: Idem Case 7 plus a diminution of refining costs of kaolin clay and anorthosite, from $145 per ton of alumina and $180 per ton of alumina to $100 per ton of alumina and $140 per ton of alumina

Analyzing the outcome of these cases, the following observations are made:

Bauxite imports are not attractive at the IBA prices, even when alumina prices are high. Nevertheless, if the IBA fails in maintaining a uniform pricing policy (case 8), bauxite imports become attractive, as shown in Figure 8.13.

The drop in bauxite imports results in a decrease of domestically produced alumina, as shown in Figure 8.14, except when the bauxite cartel pricing policy fails. When this happens, domestic alumina production reaches full capacity in the second time period. From that point onwards the model prefers to import alumina rather than to build new refining facilities.

It is interesting to notice in case (8) the breakdown of bauxite imports as shown in Figure 8.15. Jamaican imports share decreases from 51% in 1977 to 28% in 1994. Meanwhile, the share of Guinea and Brazil, increase from 10% and 0% in 1977 to 22% and 11% in 1994.

The building of smelting capacity does not vary significantly with the change of the price of imported primary aluminum (within the range covered by this analysis) as shown in Figure 8.16. Even though, it is interesting to notice the proportion of domestic production of primary aluminum with respect to total aluminum supply. In 1977 this proportion was 75% for all cases studied; in 1994, for case (7), (imports of primary aluminum at 70¢/lb) the proportion decreased to 55% and for case (6) (imports of primary aluminum at 60¢/lb), the proportion decreased even lower, to 47%. Figure 8.17 shows the imports of primary aluminum for case (6) and case (7).

The production of primary aluminum in the Pacific Northwest is very sensitive to changes in the importing price of primary aluminum. Figure 8.18 shows the production of primary aluminum from the Pacific Northwest for case (6) and case (7). In case (6) (importing price 60¢/lb), the model builds captive power plants for a total of 115 MW capacity and in case (7) (importing price 70¢/lb), the total capacity of the captive power plants built by the model is 1250 MW. In both cases the production of aluminum in 1994 comes solely from smelters whose power consumption is lower than 5.2 KWH per pound of aluminum.

In addition, Figure 8.19 shows the available smelter capacity per region. This figure shows the preponderance of the Northeast and the Southeast as smelting regions. From a similar level of capacity in 1977, by 1994 the Northeast and the Southeast have (each) 50% more available capacity than the Pacific Northwest.

To show the effect of IBA's bauxite pricing policy in the price of aluminum, Figure 8.20 shows the marginal cost of primary aluminum under IBA policy (case 7), and without IBA policy (case 8).

As far as secondary aluminum production is concerned, it seems to be independent of the price of primary aluminum imports within the range covered by this analysis. Figure 8.21 shows the evolution of the marginal cost and Figure 8.22 shows the production of secondary aluminum for case (7). Case (6) is not represented since it overlaps with case (7).

The reduction of the refining cost of kaolin clay and anorthosite (case (9)), did not make the opening of mines and the building of refineries attractive. Since a reduction in the operating cost of kaolin clay and anorthosite refineries of 31% and 22% respectively had no effect in the sense of making these ores attractive, the possibility of obtaining alumina from domestic non bauxite ores is no longer considered in the present analysis.

Table 8.7

Bauxite Imports

Price of the second segment of the supply curve

($/ton)

	Jamaica	Surinam	Caribbean	Guyana	Guinea	Australia	Brazil
77-78	32	32	32	36	32	42	--
79-80	32	32	32	36	32	42	30
81-82	32	32	32	36	32	42	30
83-84	36	36	36	40	36	45	32
85-86	36	36	36	36	36	45	36
87-88	36	36	36	36	36	45	36
89-90	36	36	36	36	36	45	36
91-92	36	36	36	36	36	45	36
93-94	36	36	36	36	36	45	36

Table 8.8
Alumina Imports
Price of the second segment of supply curves
($/ton)

	Australia	Jamaica	Surinam	Guinea	Guyana
77-78	151	143	142	145	142
79-80	151	143	142	145	142
81-82	151	143	142	145	142
83-84	151	143	142	145	142
85-86	151	143	142	145	142
87-88	151	143	142	145	142
89-90	151	143	142	145	142
91-92	151	143	142	145	142
93-94	151	143	142	145	142

Table 8.9
Bauxite Imports
(Upper limit to second segment of supply curve)
('000 tons/yr)

	Jamaica	Surinam	Caribbean	Guyana	Guinea	Australia	Brazil
77-78	6000	2800	--	--	4800	5000	--
79-80	6000	2800	--	--	4800	5000	400
81-82	6000	2800	--	--	6600	5000	1000
83-84	6000	2800	--	--	6600	5000	1000
85-86	6000	2800	--	--	7500	5000	1500
87-88	6000	2800	--	--	7500	5000	1500
89-90	6000	2800	--	--	7500	5000	1500
91-92	6000	2800	--	--	7500	5000	1500
93-94	6000	3000	--	--	8000	6000	2000

Table 8.10
Alumina Imports
(Upper limit to second segment of supply curve)
('000 ton/yr)

	Australia	Jamaica	Surinam	Guinea	Guyana
77-78	3000	1500	--	--	200
79-80	3000	1500	--	--	200
81-82	3000	1500	--	--	200
83-84	3000	1600	--	--	200
85-86	3000	1600	--	--	200
87-88	3500	1600	--	--	200
89-90	3500	1600	--	--	200
91-92	4000	1800	--	--	200
93-94	4000	1800	--	--	200

Table 8.11
Alumina Imports
Price of the second segment of the supply curve
High price
($/ton)

	Australia	Jamaica	Surinam	Guinea	Guyana
77-78	171	163	162	165	162
79-80	171	163	162	165	162
81-82	171	163	162	165	162
83-84	171	163	162	165	162
85-86	171	163	162	165	162
87-88	171	163	162	165	162
89-90	171	163	162	165	162
91-92	171	163	162	165	162
93-94	171	163	162	165	162

Table 8.12
Bauxite Price
First segment supply curve
Failure of IBA cartel policy
($/ton of bauxite)

	Jamaica	Surinam	Caribbean	Guyana	Guinea	Australia	Brazil
1977-78	28.5	30.2	30.8	34.3	26.5	38	--
1979-80	28.5	30.2	30.8	34.3	26.5	38	20
1981-82	18	16	18	18	22	30	19
1983-84	18	16	18	18	22	30	19
1985-86	18	16	18	18	22	30	19
1987-88	18	16	18	18	22	30	19
1989-90	18	16	18	18	22	30	19
1991-92	18	16	18	18	22	30	19
1993-94	18	16	18	18	22	30	19

Table 8.13
Bauxite Prices
Failure of IBA cartel policy
Second segment supply curve
($/ton of bauxite)

	Jamaica	Surinam	Caribbean	Guyana	Guinea	Australia	Brazil
1977-78	32	32	32	36	32	42	--
1979-80	32	32	32	36	32	42	30
1981-82	24	24	24	25	29	40	26
1983-84	24	24	24	25	29	40	26
1985-86	24	24	24	25	29	40	26
1987-88	24	24	24	25	29	40	26
1989-90	24	24	24	25	29	40	26
1991-92	24	24	24	25	29	40	26
1993-94	24	24	24	25	29	40	26

Figure 8.7

Bauxite Imports

Case Studies 3 and 4 ('000 ton/yr)

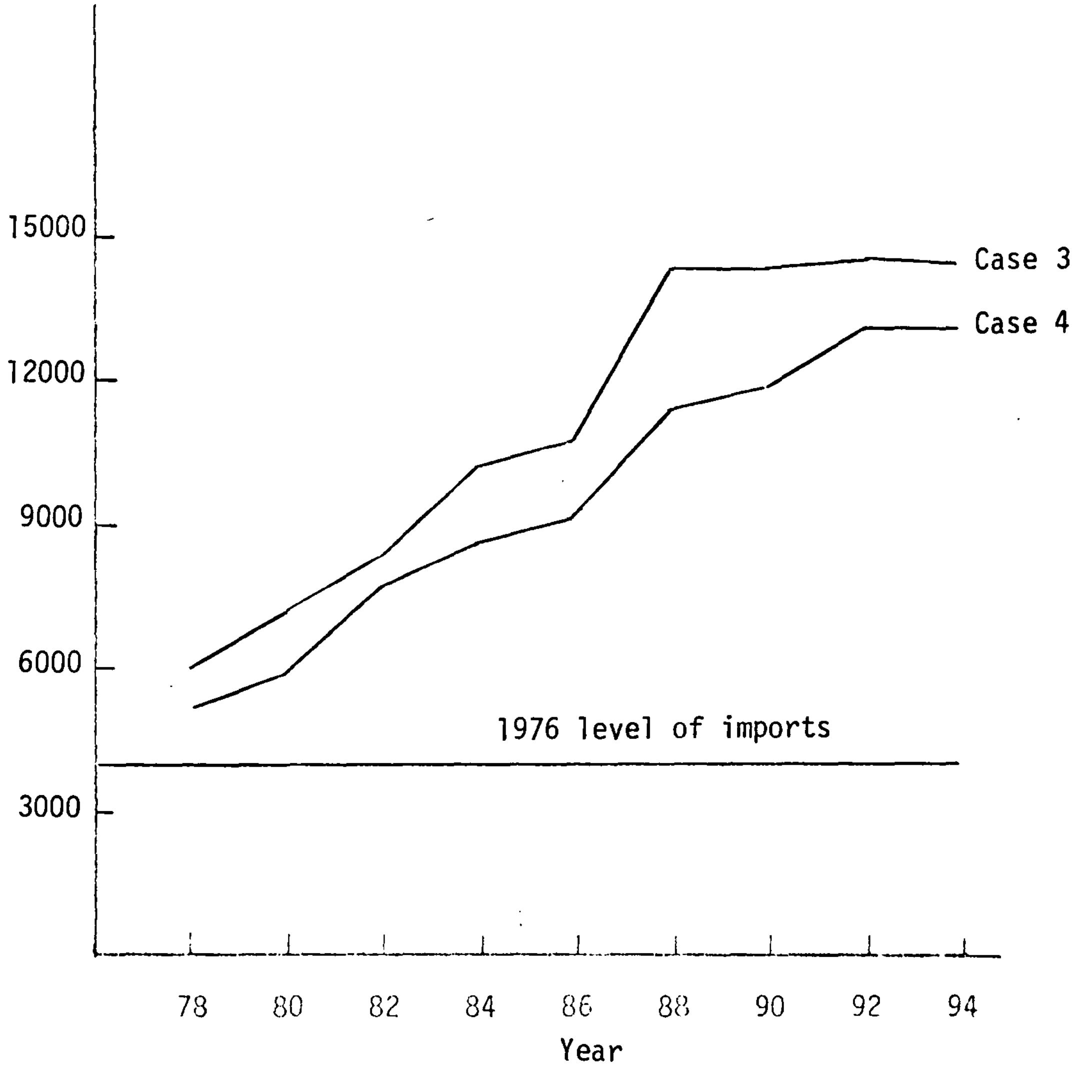
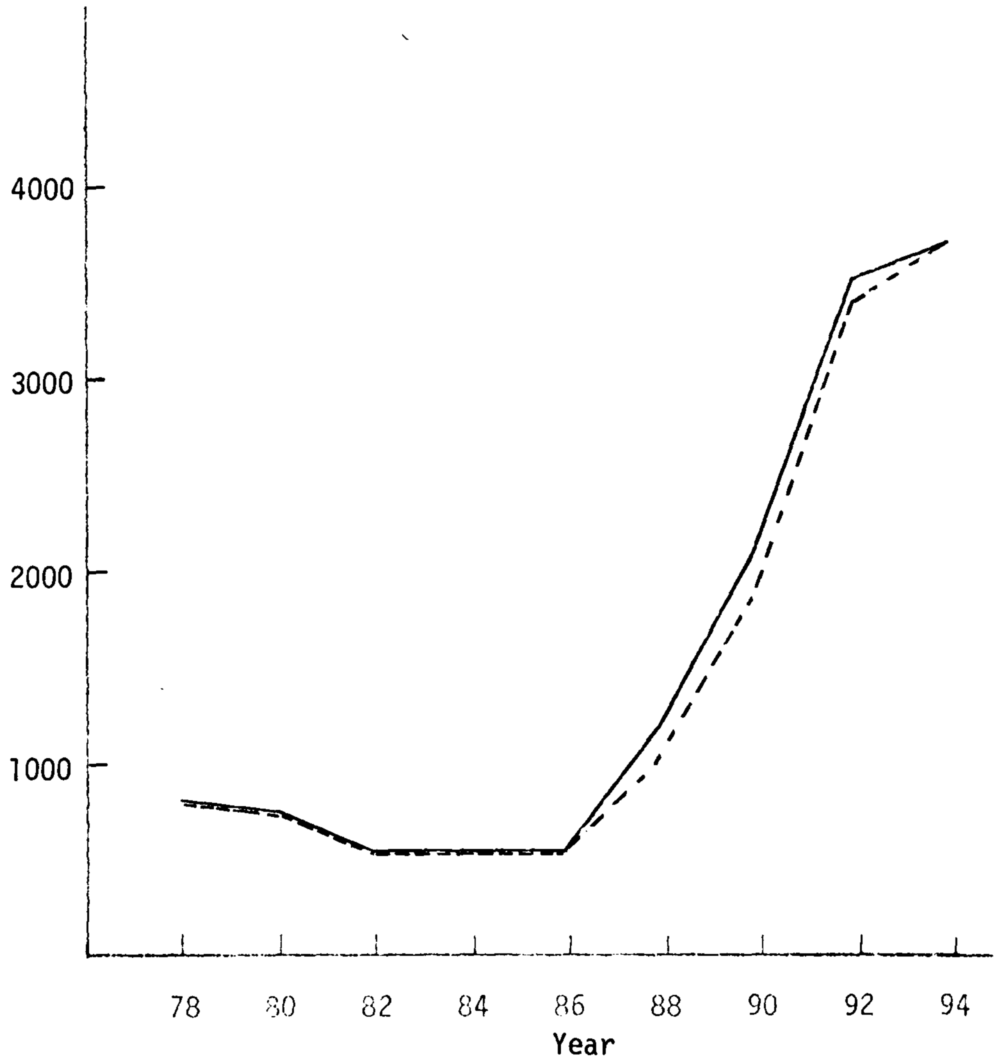


Figure 8.8

Alumina Imports

Case Studies 3 and 4 ('000 ton/yr)



—— Reference Case

--- Case Study 1

Figure 8.9

Aluminum Imports

Reference Case and Case Study 1 ('000 ton/yr)

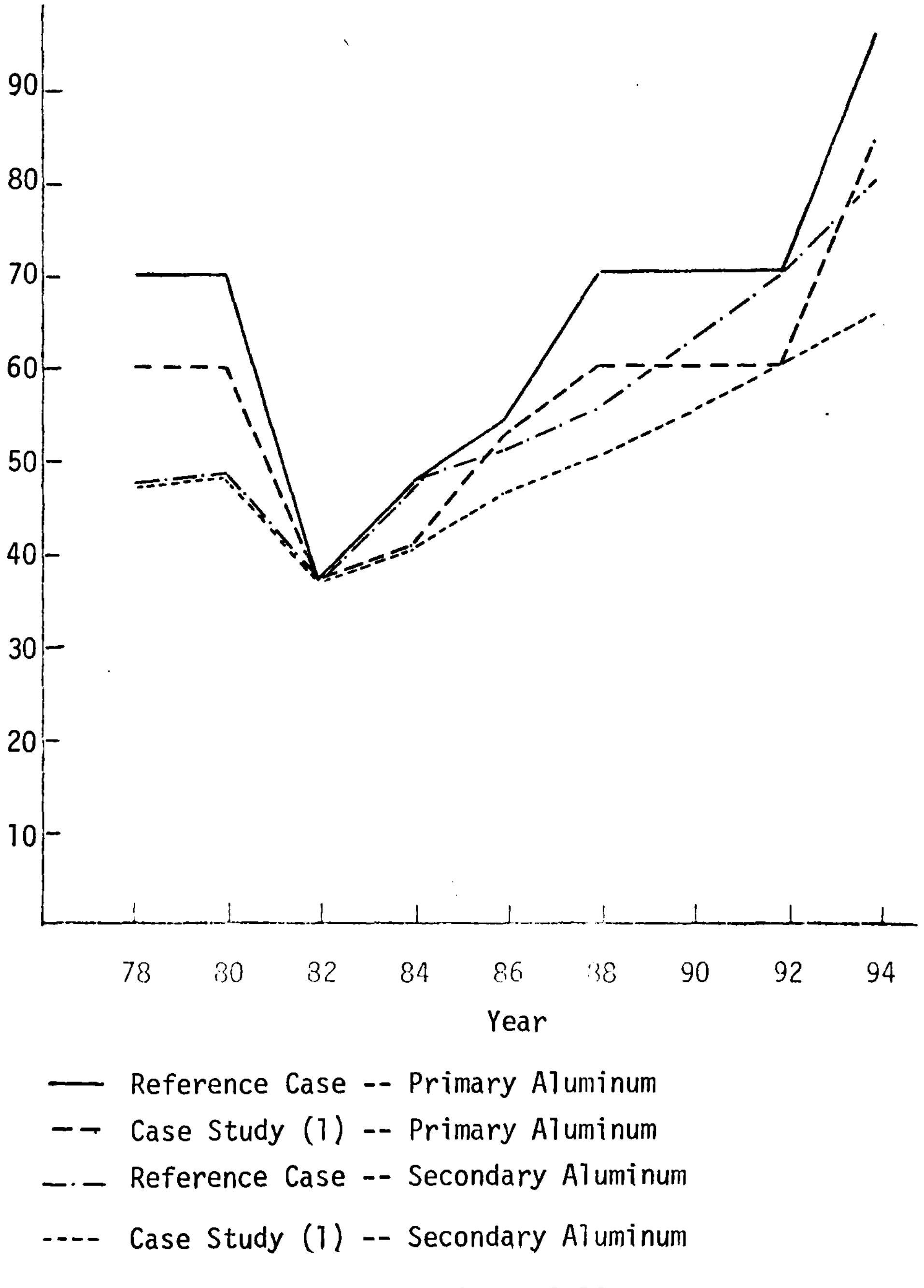


Figure 8.10

Aluminum Marginal Cost
Reference Case and Case Study 1 (¢/lb)

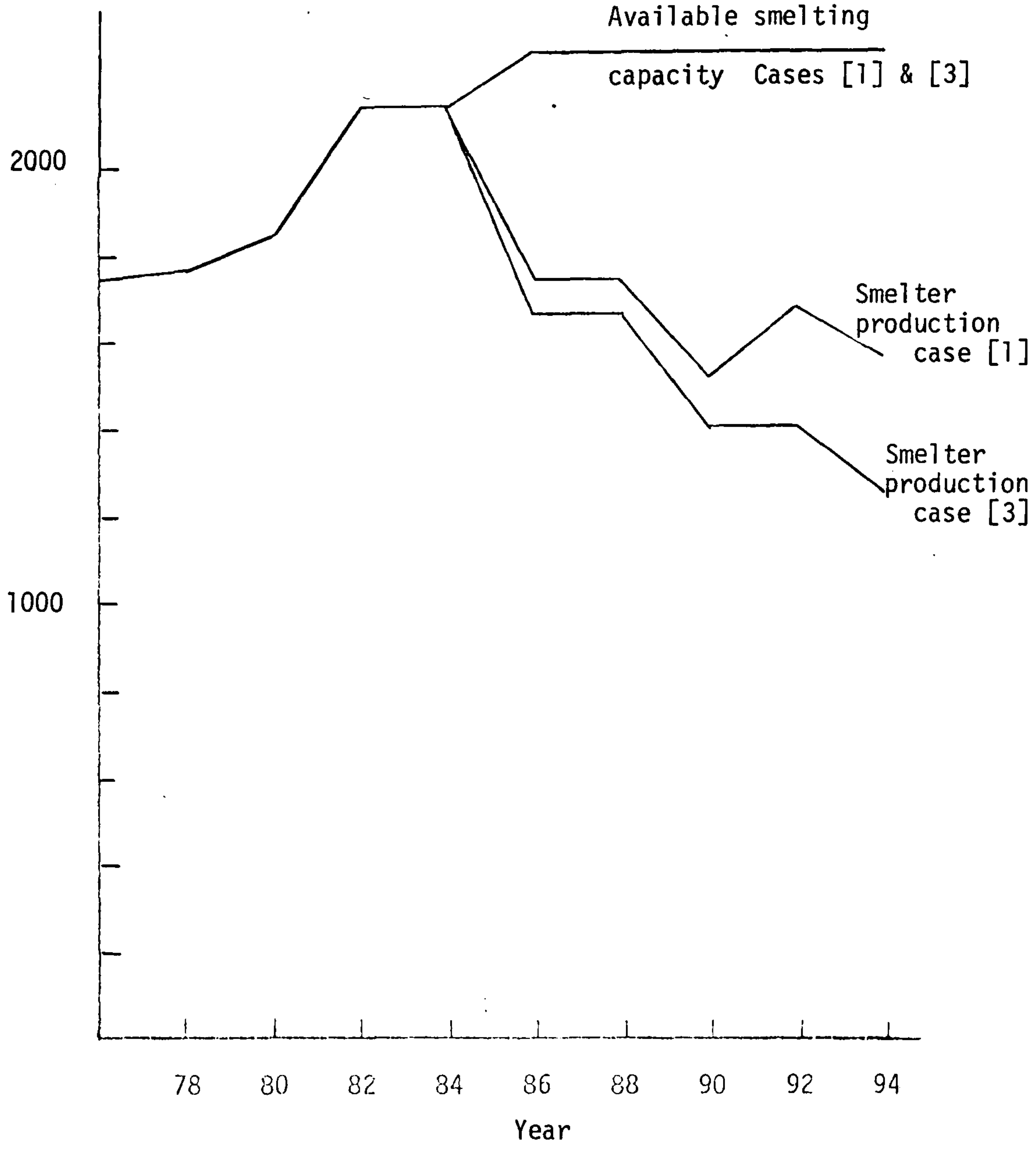


Figure 8.11

Aluminum Production and Available Capacity in the Pacific Northwest

Case Studies 1 and 3 ('000 ton/yr)

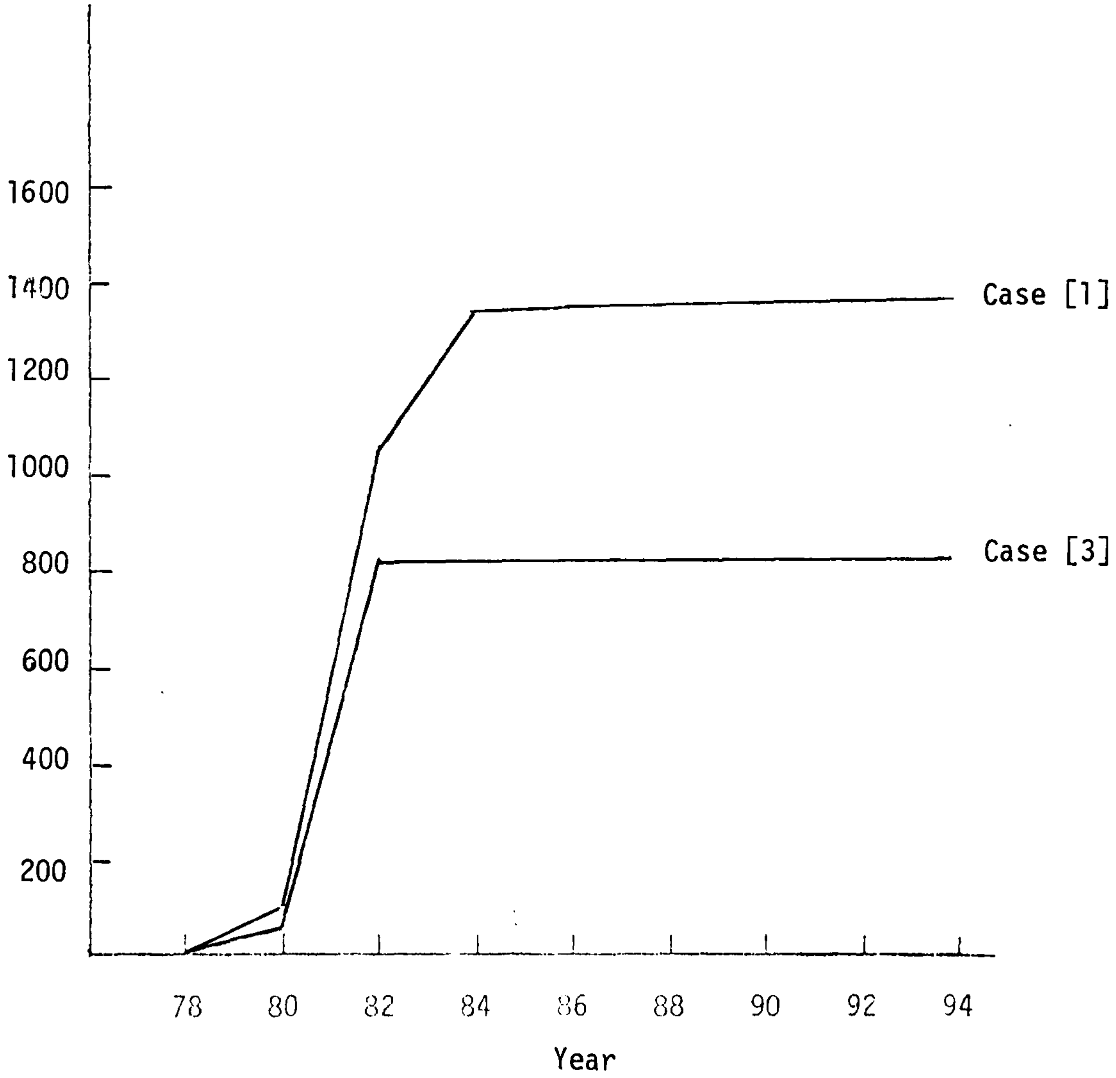


Figure 8.12

Building of Captive Power Plants in the Pacific Northwest

Case Studies 1 and 3 (MW)

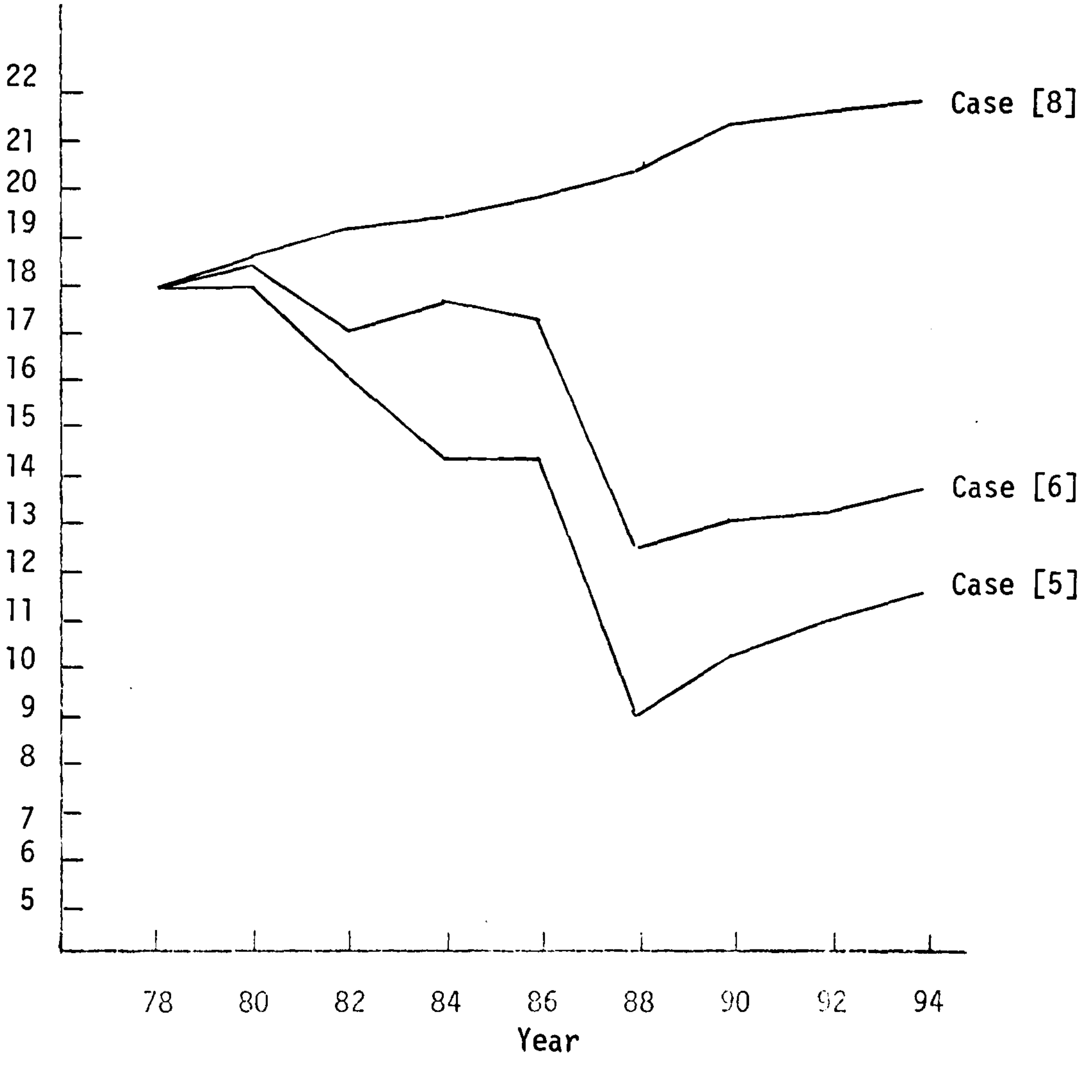


Figure 8.13

Bauxite Imports

Case Studies 5, 6 and 8 ('000,000 ton/yr)

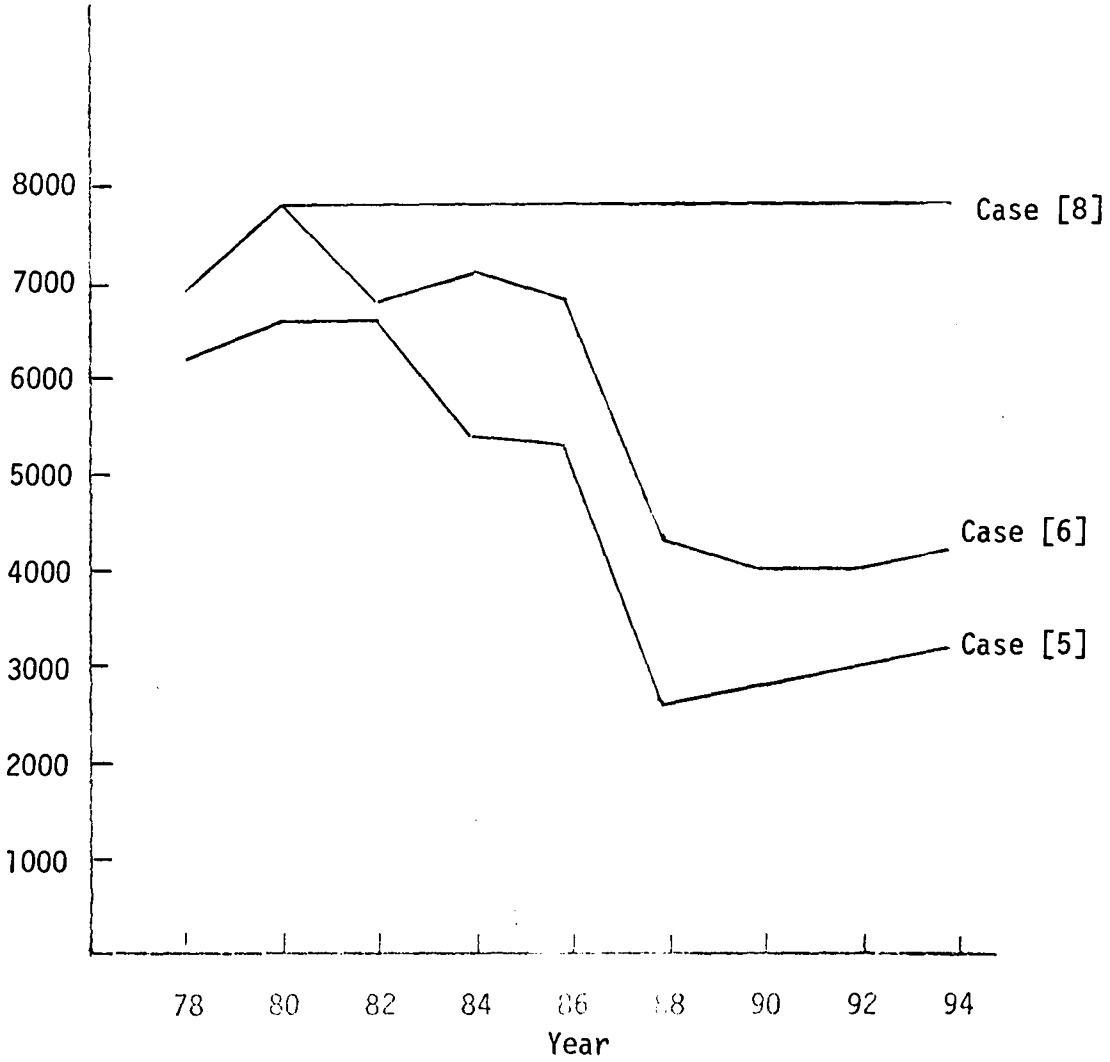


Figure 8.14

Domestic Alumina Production

Case Studies 5, 6 and 8 ('000 ton/yr)

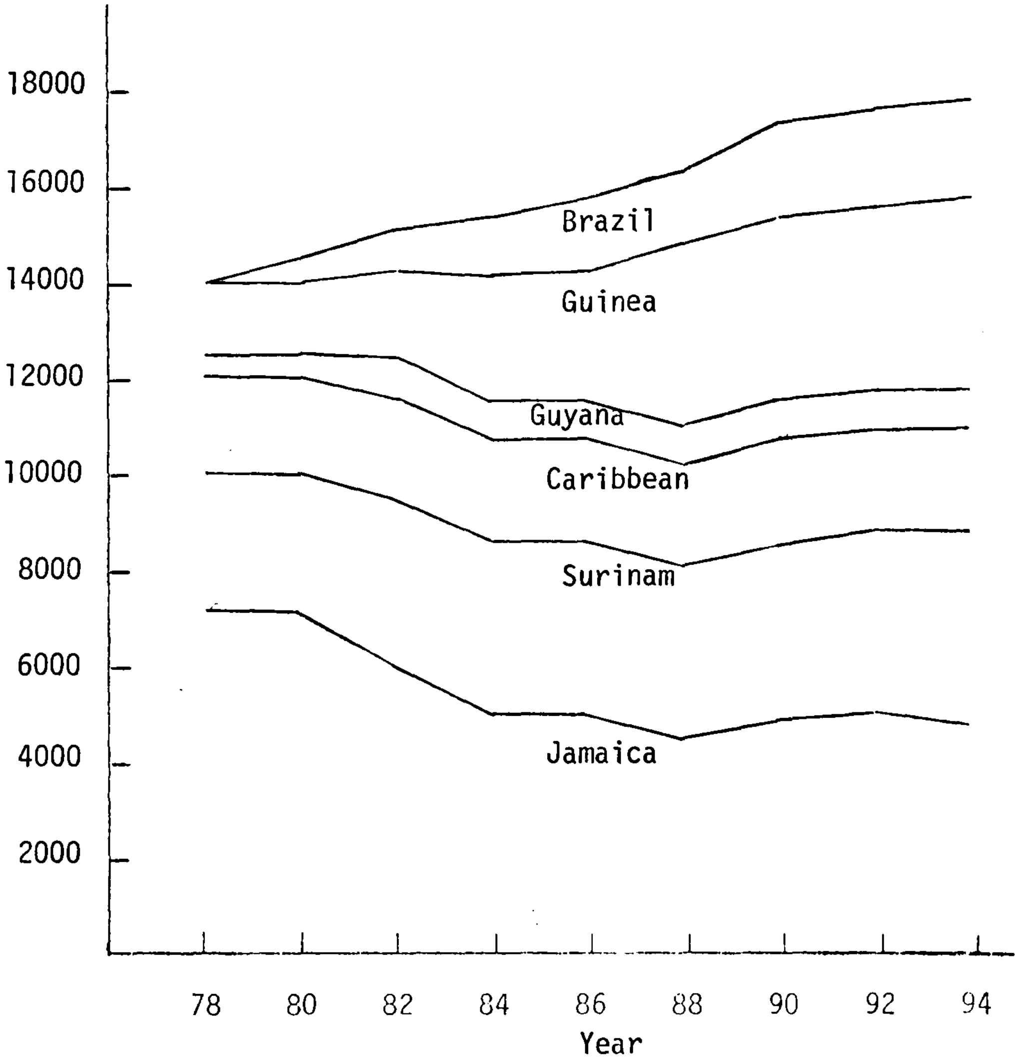


Figure 8.15

Bauxite Imports Breakdown

Case Study 8 ('000 ton/yr)

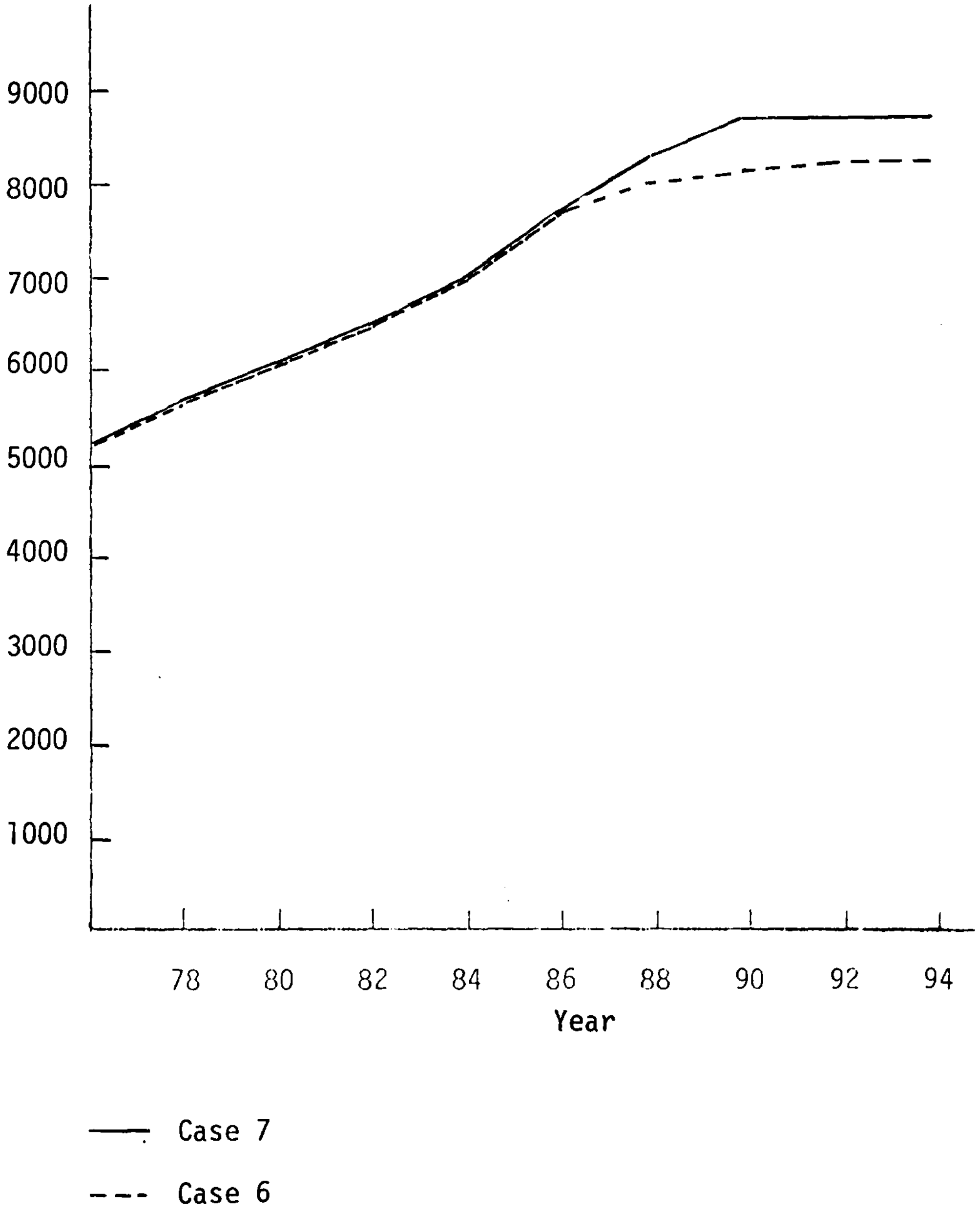
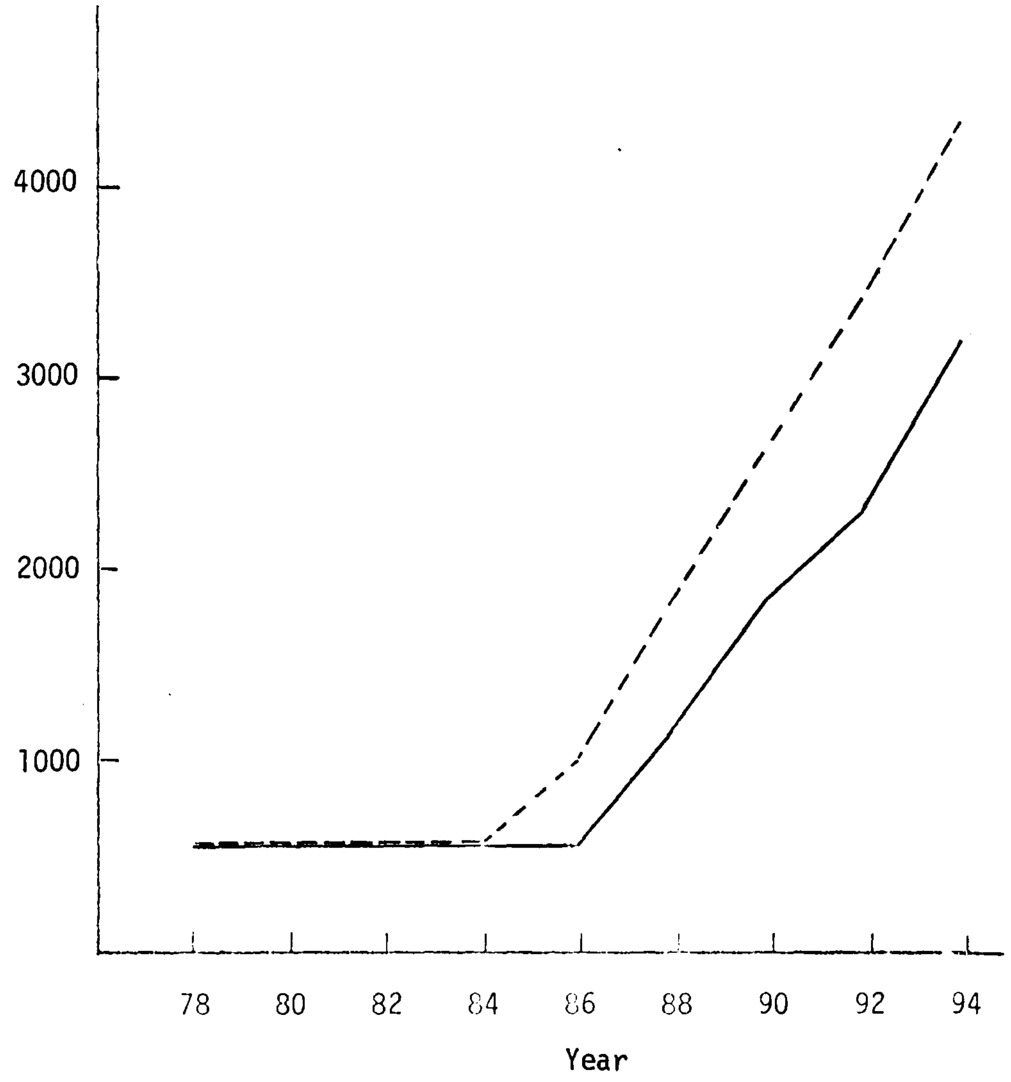


Figure 8.16

Smelting Capacity Building

Case Studies 6 and 7 ('000 ton/yr)



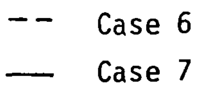


Figure 8.17

Aluminum Imports

Case Studies 6 and 7 ('000 ton/yr)

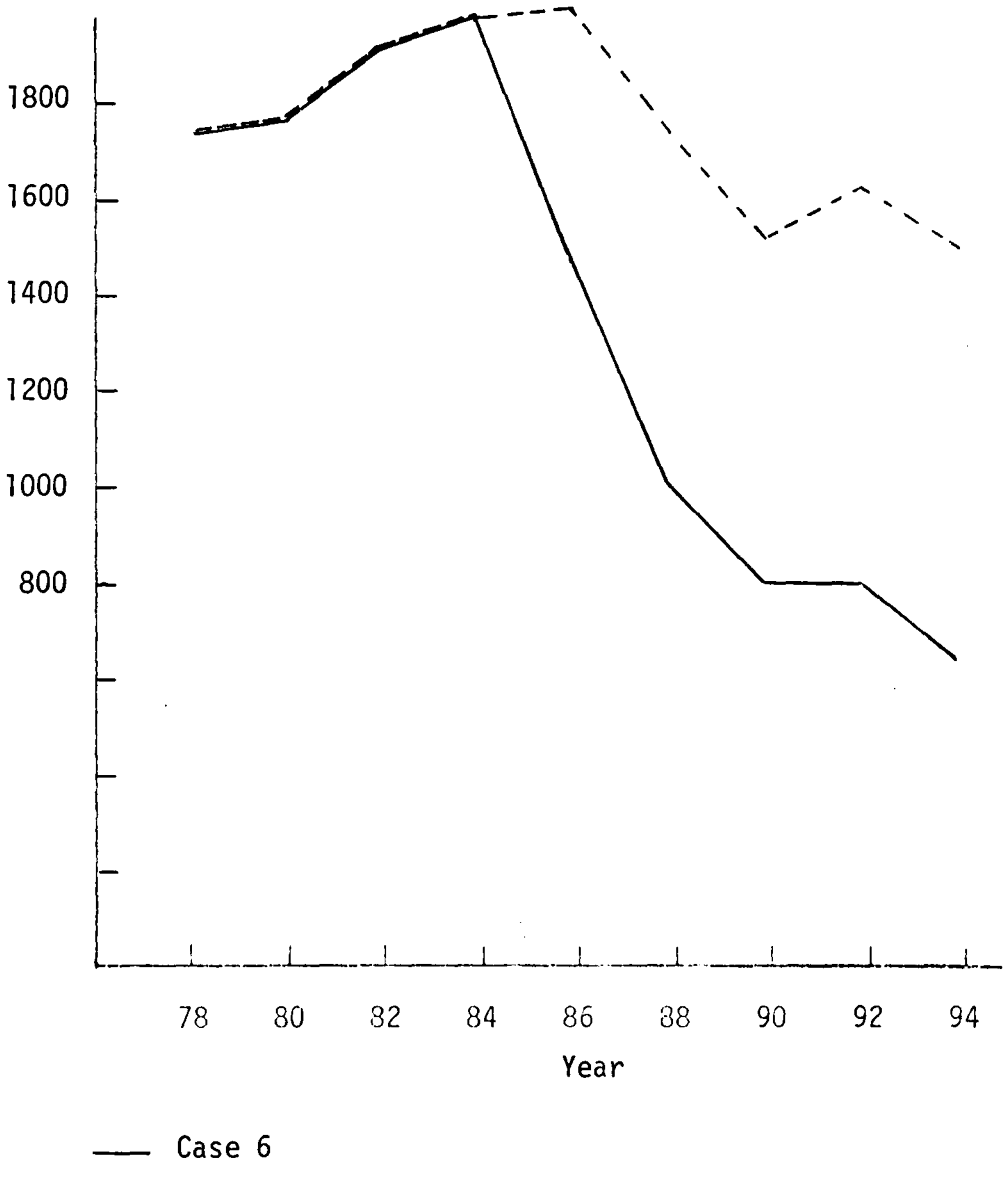


Figure 8.18

Aluminum Production -- Pacific Northwest

Case Studies 6 and 7 ('000 ton/yr)

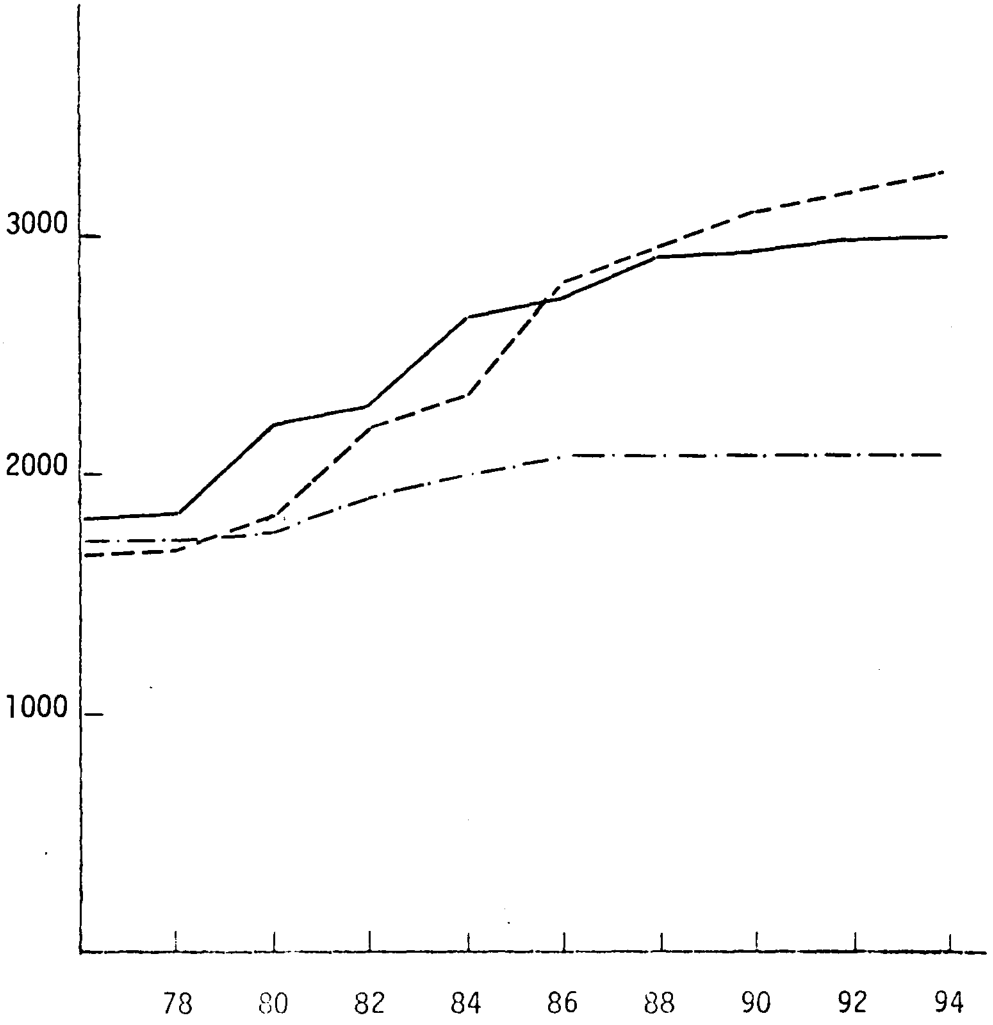


Figure 8.19

Available Smelter Capacity

Case Study 6 ('000 ton/yr)

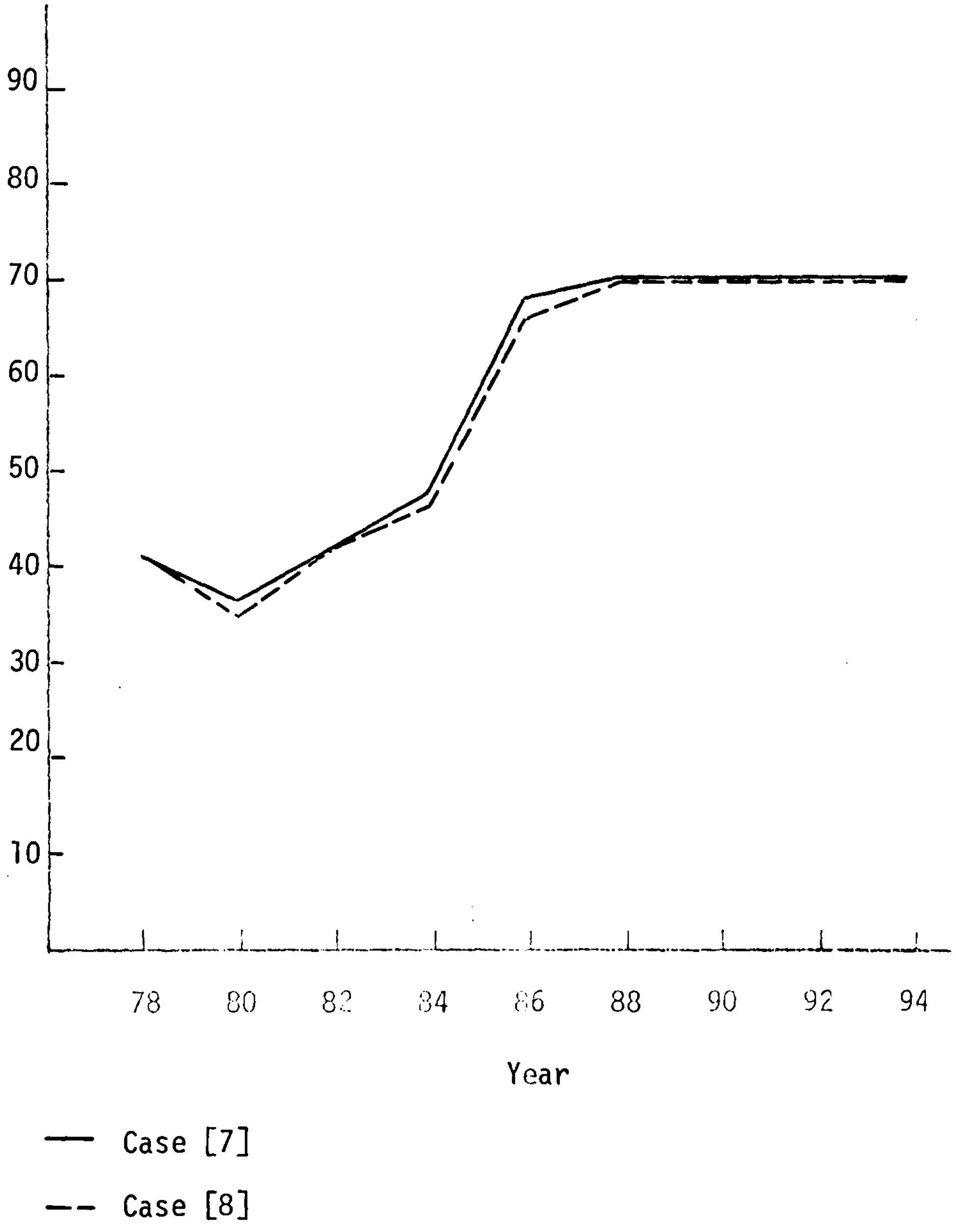


Figure 8.20

Primary Aluminum Marginal Cost

Case Studies 7 and 8 (¢/lb)

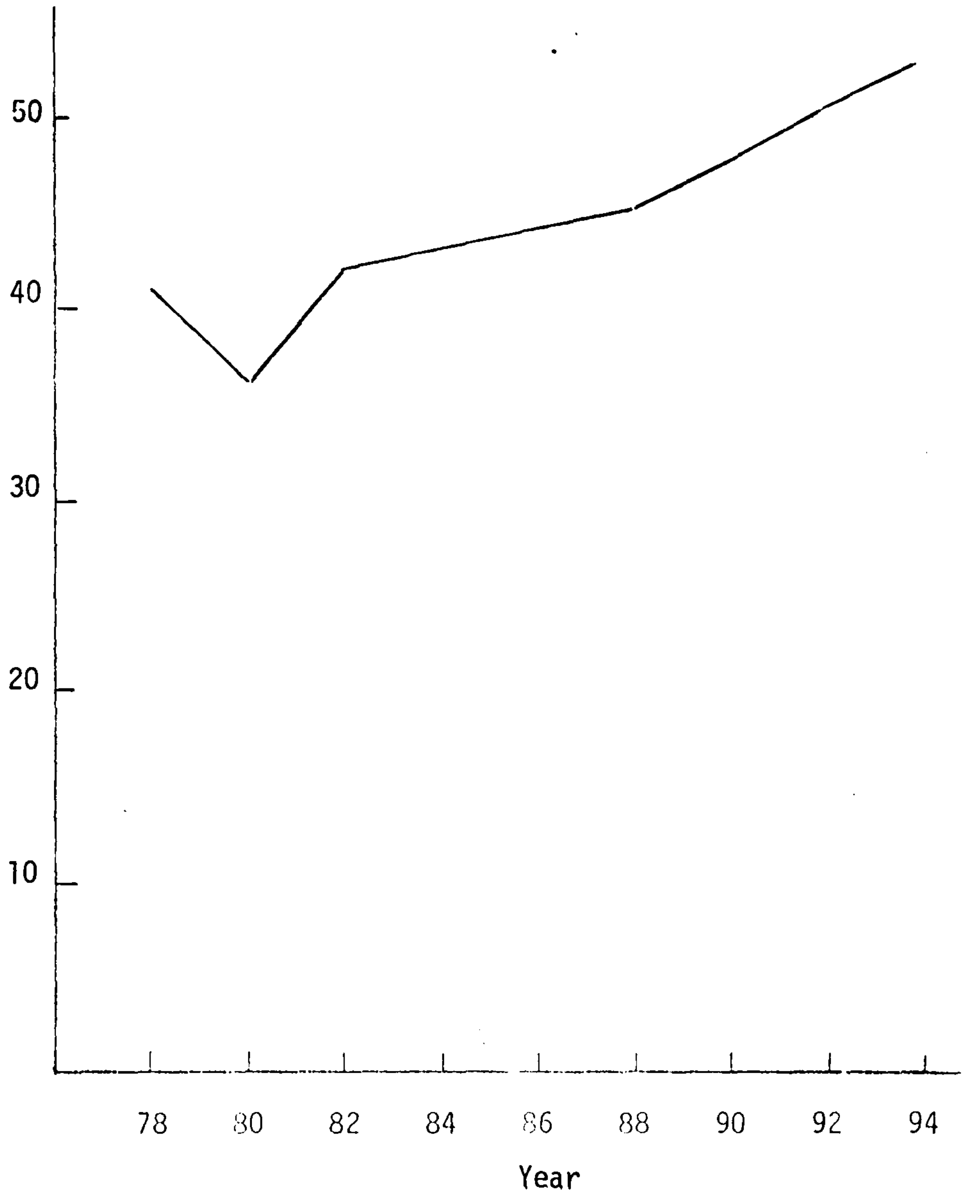


Figure 8.21

Secondary Aluminum -- Marginal Cost

Case Study 7 (¢/lb)

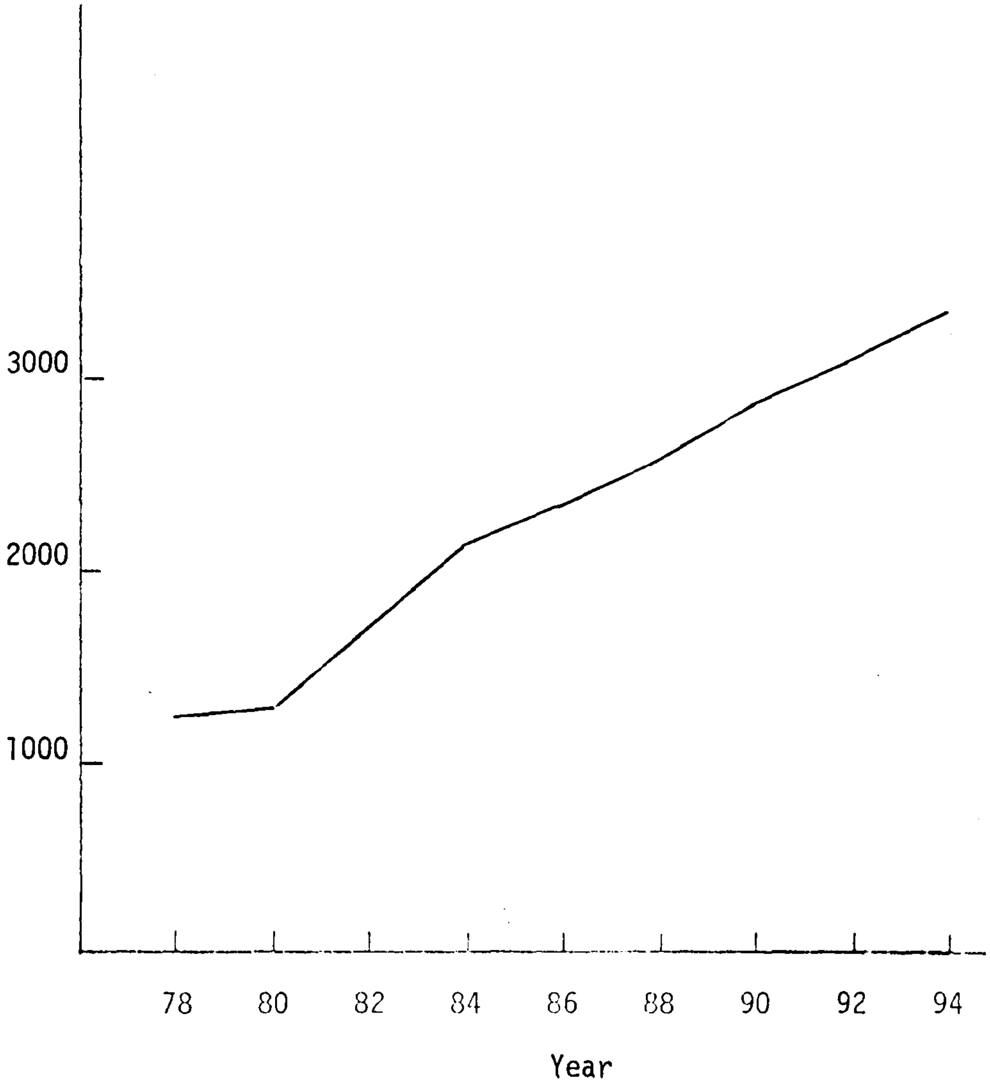


Figure 8.22

Secondary Aluminum Production
Case Study 7 ('000 ton/yr)

IX. CONCLUSIONS

The linear model developed in this paper appears to be a valid representation of the aluminum industry in the United States. The results obtained for the first time period (1977-1978) are very close to the present statistics of the industry. Nevertheless it is recalled that the model, due to its characteristics, shows optimal paths to solve predetermined long range scenarios within the assumptions intrinsic to the modeling activity. Therefore it is the opinion of the author that, since the most delicate and important aspects of the industry are represented in detail, the results obtained can be considered as the trends that the industry must follow to satisfy its objectives under the corresponding scenarios.

The model is data driven. That is, rows and columns of the matrix are the result of existing data. Therefore, if new sectors are to be added, the rows and columns that represent them are readily generated, provided the existence of the data, and the alterations that have to be made to the formulation are minor (if any). Data is stored in separate tables, making the maintenance and updating of the model quick and simple.

From the computation point of view, the model is very efficient. The present version of the model, which consists of 1485 rows and 4351 variables, takes 40 minutes of CPU time to arrive to an optimal solution with no advanced basis. To solve different cases of the same problem, the average execution time to arrive to an optimal solution (from an advanced basis) is 9 minutes of CPU time.

The cases studied were purposely chosen to demonstrate the capacity and the flexibility of the formulation. The conclusions arrived at after the analysis of the ten cases can, by no means, be considered an exhaustive analysis of the aluminum industry. Nevertheless, it is the opinion of the author, that they fulfill the purpose of showing the reaction of the model to the different scenarios.

The result of this study can be summarized as follows:

a) Mining and Refining

1) Non bauxite ores are not attractive under the scenarios studied
2) Variations in the price of bauxite and alumina do not affect, significantly, the cost of primary aluminum
3) The share of imported bauxite of Jamaica decreases while the share of Guinea and Brazil (or other non-IBA countries) increases
4) Bauxite refineries do not expand their capacity
5) Alumina imports increase in attractiveness
6) The domestic bauxite mines maintain their present production

b) Smelting and Marketing

7) The rate of growth of domestic smelting capacity is lower than the rate of growth of aluminum demand
8) Smelters with Soderberg anodes are not attractive to be technologically improved and they may become idle smelting

capacity as the cost of electric power increases

9) New smelters (and expansions) require a power consumption of 6 KWH per pound of aluminum or less to be attractive
10) Smelters using the Alcoa process are marginally attractive
11) Smelters with prebaked anodes are bound to be technologically improved
12) If the contracts with the Bonneville Power Administration cannot be renegotiated, the Pacific Northwest may become a marginal smelting region
13) If the average power consumption of the Pacific Northwest smelters is at most 5 KWH per pound of aluminum, the building of captive power plants is attractive even with a price of imported primary aluminum of 60¢ per pound
14) The importance of secondary aluminum and of imported primary aluminum in the total supply of aluminum increases
15) Investments in foreign countries may become more attractive
16) Techniques to recover aluminum from old scrap should be improved
17) The industry needs considerable capital for investments

Further Research

The development of the model had to be terminated at some stage. Nevertheless, there are many areas where the present model can be improved and augmented. Following, the most important are listed:

1) Include other non bauxite ores that can be refined in the existing refineries (i.e. laterite)
2) Allow low silica bauxite to be refined at high silica bauxite refineries
3) Allow expansion of the existing refineries
4) Model the energy requirements of the refineries (bauxite and non bauxite)
5) Divide refineries according to their pollution characteristics and include an investment vector to simulate the inclusion of pollution control devices
6) Breakdown many of the operating costs that are presently constant into operating cost per time period to simulate technological advances
7) Improve the scrap formulation. Presently all scrap is assumed to be smelted to obtain secondary aluminum. Actually some fraction is only remelted
8) Keep a new scrap inventory -- Allow imports of scrap
9) Include a specific formulation for can recycling
10) Improve the fabrication formulation, especially the fabrication cost. Perhaps it is better to make a further breakdown of the fabricated forms.
11) In the report section, add the following reports:
    a) Marginal cost of the financial rows (value of money)
    b) Compute the average power consumption per smelting region in the production of aluminum

12) Expand the financial formulation

13) If good demand curves are obtained, formulate the equilibrium supply-demand

14) Expand the model into a world model

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APPENDICES

APPENDIX I

DISCOUNT AND AVAILABILITY FACTORS -- RATE OF RETURN ON INVESTMENTS

I-A The Availability factor

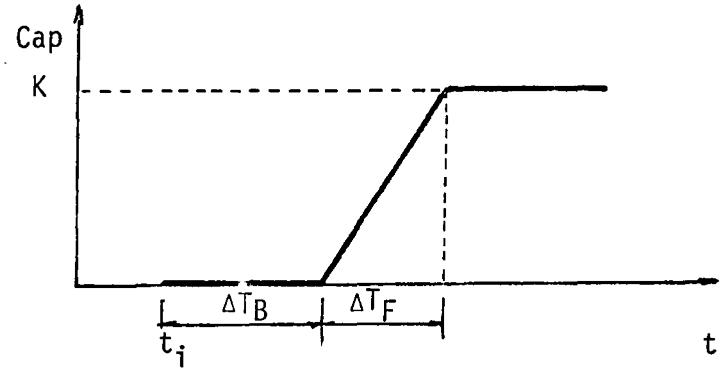
I-B The discount rate as rate of return on investments

I-C Computation of investment and operating costs

I-A The Availability Factor

Facilities built by the model do not appear instantaneously. It is assumed that from the beginning of the time period a time ΔT_B elapses until the facility may start to operate and then another time ΔT_F must pass before it reaches full production capacity. It is further assumed that the rate of growth of available capacity in the time ΔT_F follows a linear pattern.

Let it be assumed that the capacity availability function for a facility of capacity K is $y_K(t)$. Then the following graph represents the rate of growth of capacity for the new facility



where t_i = vintage of the facility

$$\text{and}\quad y_K(t) = \begin{cases} 0 & , \ t \le t_i + \Delta T_B \\ \dfrac{K}{\Delta T_F}\, t & , \ t_i + \Delta T_B \le t \le t_i + \Delta T_B + \Delta T_F \\ K & , \ t \ge t_i + \Delta T_B + \Delta T_F \end{cases}$$

therefore, the average available capacity of a facility of vintage i in time period j, starting at t_j and of duration θ_j, $(\overline{C}_{ij})$, is represented by the following expression

$$\overline{C}_{ij} = \int_{t_j}^{t_j+\theta_j} \frac{y_K(t)}{\theta_j}$$

if $y_K(t)$ is replaced by some combination of the three segments of the curve

$$\overline{C}_{ij} = \int_{\theta_j} \frac{[0dt + K\frac{t}{\Delta T_F}dt + Kdt]}{\theta_j} = K\int_{\theta_j} \frac{[0dt + \frac{t}{\Delta T_F}dt + dt]}{\theta_j}$$

$$\overline{C}_{ij} = K\,f_{ij}$$

where $f_{ij} = \int_{\theta_j} \frac{0dt + \frac{t}{\Delta T_F}dt + dt}{\theta_j}$ is the capacity availability factor

Therefore the average available capacity for any time period, is proportional to the total capacity and the capacity availability factor is solely a function of the vintage i of the facility and the time period j considered.

NOTE: any capacity availability function $y_K(t)$ that can be expressed as $K.z(t)$ fulfills the requirements for the generation of a capacity availability factor.

I-B The discount rate as rate of return on investments

An example will be presented to show that, on a simplified basis, the rate of return on investments made by a time dynamic model is at least equal to the discount rate at which costs are present valued when the price considered is the dual variable of the product balance row.

Let there be a system that has to satisfy final demands of a certain product. This demand may be satisfied by purchasing the product at a certain price P or manufacturing it at a certain cost C. Let it be further assumed that no manufacturing facilities exist presently but a facility may be built at a cost C^I per capacity unit and it can start to produce instantaneously.

Let two time periods be considered, allowing investments only in the first time period. Total investments are limited to I$ and the discount rate considered is i.

The objective is to minimize the cost to satisfy the demands

Let y_j = investment in time period j

x_j = operation in time period j

z_j = product purchased in time period j

$C_j^I = C^I(P/F,\ i,\ t_j)$ = discounted investment cost for time period j (including salvage value)

$C_j = C(P/F,\ i,\ t_j)$ = discounted operating cost for time period j

$P_j = P(P/F,\ i,\ t_j)$ = discounted price for time period j

d_j = demand in time period j

one further assumption is that $d_1 > I/C^I$, which forces that at all time periods there will be some product that is purchased. Therefore the formulation of this model is

$$\begin{array}{llll}
\min & C_1^I y_1 + C_1 x_1 + P_1 z_1 + C_2 x_2 + P_2 z_2 & & \text{associated dual variable} \\
\text{s.t.} & & & \\
 & -y_1 + x_1 & \le 0 & w_1 \\
 & -y_1 + x_2 & \le 0 & w_2 \\
 & x_1 + z_1 & \ge d_1 & w_3 \\
 & x_2 + z_2 & \ge d_2 & w_4 \\
 & C^I y_1 & \le I & w_5
\end{array}$$

For the first analysis let it be assumed that all variables are basic. That is, the investment is made but some product must still be purchased. Therefore, the thesis is:

$$(w_3 - C_1)x_1 + (w_4 - C_2)x_2 - C_1^I y_1 \ge 0$$

which is the justification of an investment by the present worth method

Proof:

$$z_1(w_3 - P_1) = 0 \quad \text{by complementary slackness}$$

$$\therefore \quad w_3 = P_1 \quad \text{since } z_1 > 0$$

Similarly

$$z_2\,(w_4 - P_2) = 0$$

$$\therefore \quad w_4 = P_2 \quad \text{since } z_2 > 0$$

$\therefore$ The dual variable of the balance row is equal to the market price of the product.

$$(w_4 - w_2 - C_2)x_2 = 0$$

$$w_4 - w_2 = C_2 \qquad \text{since } x_2 > 0$$

$$w_2 = w_4 - C_2 = P_2 - C_2$$

Similarly $\qquad w_1 = P_1 - C_1$

Again by complementary slackness

$$-w_5C_1^I + w_2 + w_1 - C_1^I = 0$$

$$w_5 = \frac{w_2}{C_1^I} + \frac{w_1}{C_1^I} - 1$$

replacing

$$w_5 = \frac{(P_2 - C_2)}{C_1^I} + \frac{(P_1 - C_1)}{C_1^I} - 1$$

Since all variables are basic

$$y_1 = x_1 = I/C^I = x_2$$

if numerator and denominator are multiplied by the discount factor $(P/F, i, t_1)$

$$y_1 = x_1 = x_2 = \frac{I(P/F, i, t_1)}{C_1^I} = \frac{I_1}{C_1^I}$$

substituting $$w_5 = \frac{(P_2 - C_2)x_2}{I_1} + \frac{(P_1 - C_1)x_1}{I_1} - 1$$

$$I_1 w_5 = (P_2 - C_2)x_2 + (P_1 - C_1)x_1 - I_1$$

$$I_1 w_5 = (P_2 - C_2)x_2 + (P_1 - C_1)x_1 - C_1^I y_1$$

since $I_1 > 0$ and $w_5 \geq 0$

$$(P_2 - C_2)x_2 + (P_1 - C_1)x_1 - C_1^I y_1 \geq 0$$

or $$(w_4 - c_2)x_2 + (w_3 - C_1)x_1 - C_1^I y_1 \geq 0$$

Let it be assumed now that the investment is not made, that is $y_1 = 0$, and the demands have to be satisfied by purchased products. This means that the investment is not justified for any capacity Y

or $$(w_3 - C_1)\, Y + (w_2 - C_2)\, Y - C_1^I Y < 0$$

$$(w_4 - P_2)z_2 = 0$$

$$w_4 = P_2 \quad \text{since } z_2 > 0$$

$$(w_3 - P_1)z_1 = 0$$

$$w_3 = P_1 \quad \text{since } z_1 > 0$$

$$w_4 - w_2 \leq C_2 \quad \text{by dual feasibility}$$

$$w_2 \geq P_2 - C_2$$

similarly

$$w_1 \geq P_1 - C_1$$

$$\text{if } y_1 = 0 \qquad -w_5 = 0$$

$$\text{and } w_2 + w_1 - C_1^I \leq 0$$

substituting the previous inequalities

$$(P_2 - C_2) + (A - C_1) - C_1^I \leq 0$$

and for any number Y

$$Y(P_2 - C_2) + Y(P_1 - C_1) - YC_1^I \leq 0$$

$$\text{or} \quad Y(w_4 - C_2) + Y(w_3 - C_1) - YC_1^I \leq 0$$

which means that the investment is not justified for any capacity Y.

The proof could be extended for all types of time dynamic models, but this is not within the scope of the present research.

I-C Computation of investment and operating costs

The model considers a time value of money for the industry. This is reflected in the discount rate which represents the historical pretax rate of return of the aluminum industry.

The objective function of the model computes the summation of the present value of all expenditures using the above mentioned discount rate. For this purpose, the computation of the present value of investment and operating costs are treated separately since they follow different discounting patterns.

Investment Costs Discount

Each facility considered by the model has an associated annual investment profile $X = x_1, x_2, \ldots x_f$ where $\sum_{i=1}^{F} x_i = 1$.

Furthermore, each facility is linearly depreciated and a depreciation life of 20 years is assumed for all types of new facilities.

If the discount rate considered is i and if discounting is assumed continuous, given a cost C and a time period j starting at t_j, the present value of the investment per unit capacity is computed as follows:

$$\text{Present value} = C \sum_{t_j+0.5}^{t_j+F-0.5} X_{t-t_j+.5}\, e^{-it} - C[1 - \frac{T - t_j}{20}]e^{-iT}$$

$$= C \sum_{t_j+0.5}^{t_j+F-0.5} X_{t-t_j+.5}\, e^{-it} - [1 - \frac{T - t_j}{20}]e^{-iT}$$

$$= Cf_j^i$$

where T = duration of the model

and

$$f_j^i = \left[\sum_{t_j+0.5} x_{t-t_j+.5}\, e^{-it} - [1 - \frac{T - t_j}{20}]e^{-iT} \right]$$

is the investment discount factor.

Operating Costs Discount

Operating costs are assumed uniform throughout any time period. Therefore, assuming continuous discounting and an average operating cost of C \$/year, the present value of the operating expenditures in time period j, starting at tj with duration θj is as follows.

$$\text{Present value} = \int_{t_j}^{t_j+\theta_j} C\, e^{-it} dt = \frac{-C}{i}\left[e^{-i[t_j+\theta_j]} - e^{-it_j} \right]$$

$$= C\, f_j^i$$

where i = discount rate

and

$$f_j^i = \frac{1}{i}\left[e^{-it_j} - e^{-i[t_j+\theta_j]} \right]$$ is the operating discount factor.

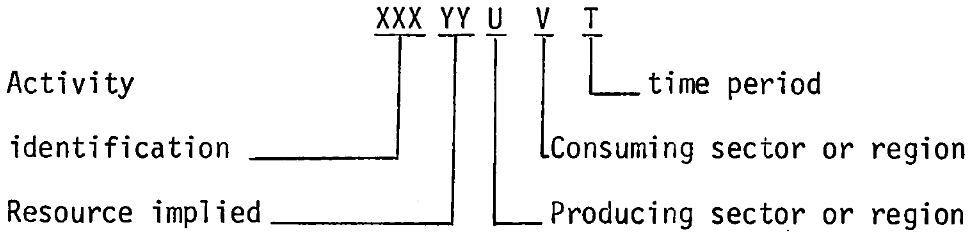
APPENDIX II

Detailed Mnemonic Structure

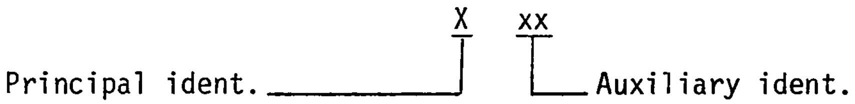
Each vector and each equation of the model is identified by an eight character name. The general structure for equation names, and vector names is very similar but they will be treated separately for better comprehension.

1 Activities

General representation



1.1 Activity Identification



1.1.1 Principal Identifiers

P :	process or production	(operational variable)
T :	transportation	(operational variable)
D :	distribution	(operational variable)
Y :	inventory	
M :	New or incremental Mining facilities	(investment variable)
R :	New Refining facilities	(investment variable)

S : New or existing smelting fac. (investment variable)

I : Technology improvement vector (investment variable)

X : Expansion vector (investment variable)

E : Electric power plant (investment variable)

1.1.2 Auxiliary Identifiers

a) Investments

		Units
M.j	New mining facilities -- vintage j	ton ore/yr
Mij	Incremental mining facilities -- segment i, vintage j	ton ore/yr
Rj.	New refinery -- type j	ton Alumina/yr
Snv	Smelter type n, vintage v	ton Al/yr
Xnv	Expansion on smelter type n, vintage v	ton Al/yr
Inm	Improvement of smelter type n to type m	ton Al/yr
E..	Power plant (thermal)	MW

b) Inventory

Y..	Stocks at beginning of time period	ton

c) Processing

PM.	Mine production	ton ore/yr
PRi	Production out of refinery type i	ton Alumina/yr
PSi	Production out of smelter type i	ton Al/yr
PC.	Production out of new scrap	ton Al/yr
PCi	Production out of old scrap (segment i)	ton Al/yr
PE.	Production of electricity from captive power plants	MWhr/yr
PEP	Production of electricity (purchased)	MWhr/yr

Transportation	Units
TY. : From inventory	ton/yr
TP. : Domestic product	ton/yr
TIi : Imported product -- segment i	ton/yr

In the Aluminum transportation, since we account for fabrication

TYj : From inventory, through fabricating process j	ton/yr
TPj : From domestic producers, through fabricating process j	ton/yr
TIj : Imported, through fabricating process j	ton/yr

Distribution to end demand

Dnm : Distribution to end sector n through fabricating process m (if no fabrication process m=.)	ton/yr

1.2 Resources

a) Mining

BL : Bauxite Low Silica

BH : Bauxite High Silica

N. : Anorthosite

C. : Kaolin Clay

b) Refinery

L. : Alumina

Bn : Bauxite from region n

c) Smelter

A. : Aluminum

NS : New Scrap

OS : Old Scrap

1.3 Regions

a) Mining

Domestic	A: Arkansas	(Bauxite)
	W: Wyoming	(Anorthosite)
	G: Georgia	(Kaolin Clay)
Foreign	J: Jamaica	(Bauxite)
	N: Surinam	(Bauxite)
	C: Caribbean	(Bauxite)
	G: Guinea	(Bauxite)
	Y: Guyana	(Bauxite)
	L: Australia	(Bauxite)
	B: Brazil	(Bauxite)

b) Refinery

Domestic	E: South East U.S.
	W: Wyoming
	G: Georgia
Imported	L: Australia
	J: Jamaica
	N: Surinam
	G: Guinea
	Y: Guyana

c) Smelters

Domestic 1: North West U.S.

2: North East U.S.

3: South East U.S.

Imported F: Foreign

d) Demand sectors

Y: inventory

Q: chemical

F: foreign

B: building and construction

T: transportation

E: electrical

C: containers and packaging

D: consumer durables

M: machinery and equipment

O: others

1.4 Miscellaneous

a) Fabrication Process

I: Ingot

P: Plate, Sheet and Foil

T: Extruded rod and bar, shapes, pipe and tube, drawn and welded tube, rolled continuous cast rod and bar

C: Wire and cable

V: Various

b) Refinery Types

L: Bauxite Low Silica

H: Bauxite High Silica

N: Anorthosite

C: Kaolin Clay

c) Smelter Types

1: Horizontal Soderberg anodes

2: Vertical Soderberg anodes

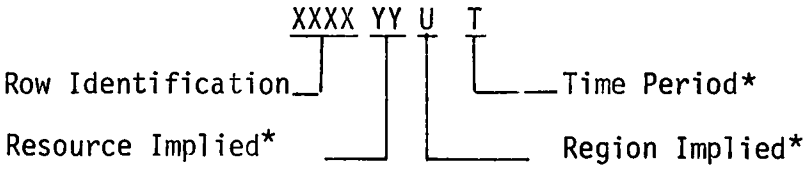
3: Prebaked anodes

4: Computerized control with Prebaked anodes

5: Computerized control with Titanium diboride cathodes

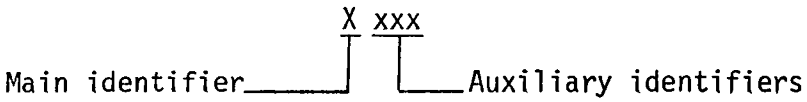
6: Alcoa reduction process

2 Rows General Representation



* The same as in the activity case

2.1 Row identification



2.1.1 Main identifiers

K: Capacity

P: Process balance

B: Other balances

F: Financial row

2.1.2 Auxiliary Identifiers

a) K Capacity

Kxxx Production limitation due to available capacity, where xxx is any of the investment identifiers defined previously

KTIi Upper limit to imports, segment i [in the case of aluminum i=@]

KS@n Limitation to expansion of smelter vintage n, during all time periods

Note: vintages, either of mines or smelters, do not appear in xxx

KMnv Expansion limitation on mines vintage v, segment m

b) B Balances

BDmn: Demand balance of end use sector m through fabrication process n, if no fabrication process is applicable, n=.

BE..: Energy requirement balance

BPSi: Scrap Supply balance, segment i (if applicable)

BR..: Reserves supply balance

BTI@: Lower limit on imports

BTP.: }
BTP@: } Domestic transportation balance

BVn@: Improvement balance on existing smelters, type n

BY..: Inventory balance

P Process

PRN.: Process balance non bauxite ore n refinery

PRBm: Process balance bauxite refinery, bauxite origin m

PS@.: Process balance, smelter

Miscellaneous

Activities

Investment activities are expressed in terms of output product capacity and the resource element of the mnemonic identifier identifies this product.

Transportation activities are expressed in terms of the transported product which is identified by the resource element of the mnemonic identifier.

Production vectors are expressed in terms of the output product but the resource element of the mnemonic structure indicates the input to the process.

Rows

Capacity rows are expressed in terms of the output product and the resource element of the mnemonic identifier identifies this product.

Process balance rows are expressed in terms of the output of the process but the resource element of the mnemonic structure identifies the input to the process.

APPENDIX III

Equations of the Model

The equations that comprise the model are presented following the path of aluminum from ore to end market. The variables and the equations are identified by the same mnemonic structure used by the model, therefore the reader is strongly recommended to read Appendix II before proceeding with this Appendix.

Each equation is presented first by its mnemonic identification, then a brief explanation of its meaning and finally the equation development. Some elements that are common to many equations, are the following:

t and h: represent time period

v: represents vintage, i.e. time period in which investment was made

r: represents producing region or sector

u: represents consuming region or sector

f^n_{vt}: available capacity factor for facility n, of vintage v, in time period t

θ_t: length of time period t in years

T: last time period

@: all elements (i.e. all time periods, all resources)

1 KM..n.rt Production limitation by capacity Non bauxite ores

$$-\sum_{v=1}^{t} f_{vt}^{n} M.vn.r.v - \sum_{v=1}^{t} \sum_{h=v'}^{t} \sum_{j=1}^{2} f_{vt}^{nj} M_{jvn.r.h} + PM.n.r.t \leq 0$$

where n = ore type

j = segment of the mine increment (if applicable)

$v' = v : f_{vv'}^{n} = 1$, that is, the mine has to be in full operating conditions before an expansion can take place

2 KM..BHrt Production limitation by capacity Domestic bauxite (high silica)

$$-\sum_{h=1}^{t} \sum_{j=1}^{2} f_{ht}^{BH} M_{j}0\ BHr.h + PM.BHr.t \leq M_{0}^{BHr}$$

where j = segment of mine increment (if applicable)

M_{0}^{BHr} = existing mining capacity in region r

3 BR..BHr@ Limitation on the depletion of domestic bauxite reserves

$$\sum_{h=1}^{T} \theta_{h} PM.BHr.h \leq R_{0}^{BHr}$$

where R_{0}^{BHr} = existing reserves of domestic bauxite in region r

4 KTIjBLrt Limitation to bauxite imports (low silica)

$$\sum_{u} TIjBLrut \leq U_{rjt}^{BL}$$

where j = segment of supply curve

U^{BL}_{rjt} = upper limit to the imports of bauxite (low silica) from country r, supply curve segment j, during time period t

5 BTI@BLrt Lower limits to bauxite imports (low silica)

$$\sum_{u} \sum_{j} \text{TIjBLrut} \geq L^{BL}_{rt}$$

where j = segment of supply curve

L^{BL}_{rt} = lower limit to the imports of bauxite (low silica) from country r during time period t

6 BY..BLrt Inventory balance

$$\text{Y..BLrYt} + \theta_t[\sum_{j} \text{TIjBLrYt} - \sum_{u} \text{TY.BLrut}] - \text{Y..BLrY t+1} = 0$$

where j = segment of supply curve

NOTE: when t = T, t+1 = @

7 BDQ.BLUt Demand satisfaction of the chemical sector of the United States by low silica bauxite

$$\sum_{m} \sum_{j} \text{TIjBLmQt} + \sum_{m} \text{TY.BLmQt} - \text{DQ.BL.Ut} \geq 0$$

where m = country of origin of bauxite (selected countries)

j = segment in supply curve

8 PRn.n.rt Production of alumina balance Non bauxite ore

$$- \Phi_n \text{ PM.n.r.t} + \text{PRnn.r.t} \le 0$$

where n = ore type

Φ_n = yield of alumina per unit weight of ore, includes process efficiency

9 PRH.BHrt Production of alumina balance, from domestic bauxite (high silica bauxite in high silica refineries)

$$- \Phi_m \text{ PM.BHm.t} + \gamma_m \text{PRH.Bmr.t} \le 0$$

where m = region where domestic bauxite is mined

Φ_m = yield of alumina per unit weight of domestic bauxite from region m

γ_m = fraction of the alumina produced by high silica bauxite refineries that comes from domestic region m (there is only one domestic region m)

10 PRH.BLrt Production of alumina balance, from imported bauxite (low silica bauxite in high silica refineries)

$$- \sum_m \sum_j \Phi_m \text{ TIjBLmrt} - \sum_m \Phi_m \text{TY.BLmrt} + (1-\gamma_s)\text{PRHBsr.t} \le 0$$

where m = country of origin of the bauxite (selected countries); s = domestic bauxite region

Φ_m = yield of alumina per unit weight of imported bauxite from country m

γ_s = idem (9)

11 PRLmBLrt Production of alumina balance, from imported bauxite (low silica bauxite in low silica refineries)

$$- \Phi_m \left[\sum_j TIjBLmrt + TY.BLmrt \right] + \eta_m \xi_m \; PRHBsr.t + PRLBmr.t \le 0$$

where j = supply segment

m = country of origin of bauxite (all countries)

s = domestic bauxite region

Φ_m = yield of alumina per unit weight of imported bauxite from country m

η_m = 1 , if country m supplies high silica bauxite refineries

0 , otherwise

ξ_m = historical fraction of the high silica refineries output that comes from country m such that

$\xi_m = (1-\gamma_s)$, where s is the domestic bauxite region

12 KRn.L.rt Production limitation by capacity of non bauxite refineries

$$- \sum_{v=1}^{t} f^n_{vt} \; Rn.L.r.v \; + PRnn.r.t \le 0$$

where n = type of ore treated

13 KRn.L.rt Production limitation by capacity of bauxite refineries

$$- \sum_{v=1}^{t} f^n_{vt} \; Rn.L.r.v + \sum_m PRnBmr.t \le R^{nr}_0$$

where n = type of refinery (high or low silica)

m = origin of the bauxite

R_0^{nr}= existing capacity of refinery type n in region r

14 KTIjL.rt Limitation to alumina imports

$$\sum_u \text{TIjL.rut} \leq U^L_{rjt}$$

where j = segment of supply curve

U^L_{rjt} = upper limit to the imports of alumina from country r, supply curve segment j, during time period t

15 BTI@L.rt Lower limit to alumina imports

$$\sum_u \sum_j \text{TIjL.rut} \geq L^L_{rt}$$

where j = segment in supply curve

L^L_{rt}= lower limit in alumina imports from country r during time period t

16 BY..L.Yt Alumina inventory balance

$$\text{Y..L..Yt} + \theta_t \left[\text{TP.L.rYt} - \sum_u \text{TY.L.Yut} \right] - \text{Y..L..Y(t+1)} = 0$$

NOTE: When t = T, t+1 = @

Only one alumina refinery region supplies to inventory

17 BDu.L.Ut Demand satisfaction balance (of sector u)

$$- TP.L.rut + Du.L..Ut \leq 0$$

NOTE: Only one alumina refinery region supplies the demand sectors

18 BTP.L.rt Transportation balance from non bauxite refineries

$$- PRnn.r.t + \sum_{u} TP.L.rut \leq 0$$

where n = non bauxite ore processed

19 BTP.L.rt Transportation balance from bauxite refineries

$$-\sum_{n} \sum_{m} PRnBmr.t + \sum_{u} TP.L.\ rut \leq 0$$

where n = type of refinery

m = origin of the bauxite processed

20 KSn.A.r.t. Production limitation by capacity of smelters

$$-SnOA.r.t - \sum_{h=1}^{t} f^{x}_{ht} XnOA.r.h - \sum_{v=1}^{t} f^{s}_{vt}\ SnvA.r.v -$$

$$- \sum_{v=1} \sum_{h=v'} f^{x}_{ht}\ XnvA.r.h + PSnL.r.t \leq 0$$

where n = smelter type

$v' = v: f^{s}_{vv'} = 1$

21 KS@vA.r.@ Limitation to the expansion of smelters

$$-\ \alpha \sum_{n} \mathrm{SnvA.r.v} + \sum_{h \geq v} \sum_{n} \mathrm{XnvA.r.h} \leq 0$$

where n = smelter type

α = fraction of smelter capacity that can be expanded

NOTE: In the case of vintage 0, the first term of the equation becomes a right hand side with the existing capacity multiplied by α

22 BVn@A.rt Smelter improvement balance

$$\mathrm{Sn0A.r.t} + \sum_{m<n} \eta_{mn}\ \mathrm{ImnA.r.t} - \sum_{p>n} \mathrm{InpA.r.t} - \mathrm{Sn0A.r.(t+1)} = 0$$

where n, m, p = smelter type

$\eta_{m,n}$ = increase in capacity when improved from type m to type n

NOTE: the equation holds up to t = T - 1

23 PS@.L.rt Smelter production balance

$$-\ \phi_L \left[\sum_{m} \sum_{j} \mathrm{TIjL.mrt} + \sum_{p} \mathrm{TP.L.prt} + \mathrm{TY.L.Yrt} \right] + \sum_{n} \mathrm{PSnL.r.t} \leq 0$$

where m = alumina exporting country

j = segment of supply curve

p = domestic refinery region

n = smelter type

Φ_L = aluminum production per ton of alumina, including efficiency (considered the same for all smelter types)

24 BE..A.rt Power requirement balance

$$- \text{PE.A.r.t} - \text{PEPA.r.t} + \sum_{n} \lambda_n \text{PSnL.r.t} \leq 0$$

where n = smelter type

λ_n = power requirements per unit production of aluminum from type n smelters

NOTE: the vector PEPA.r.t is upper bounded

25 KE..A.rt Production limitation by capacity of electric power from captive power plants

$$- \rho \sum_{v=1}^{t} f^E_{vt} \, \text{E..A.r.v} + K \, \text{PE.A.r.t} \leq \rho E^r_0$$

where ρ = factor of use

K = unit conversion term (MW into KWH/yr)

E^r_0 = existing captive power capacity in region r

26 BTP@A.rt Transportation balance of domestic primary aluminum

$$- \sum_{n} \text{PSnL.r.t} + \sum_{u} \sum_{m} \text{TPmA.rut} + \text{TP.A.rYt} \le 0$$

where n = smelter type

m = fabrication process

27 KTI@A.rt Limitation to aluminum imports

$$\sum_{m} \sum_{u} \text{TImA.rut} \le U^{A}_{rt}$$

where m = fabrication process

U^{A}_{rt} = upper limit to aluminum imports from country r in time period t

28 BTI@A.rt Lower limit to aluminum imports

$$\sum_{m} \sum_{u} \text{TImA.rut} \ge L^{A}_{rt}$$

where m = fabrication process

L^{A}_{rt} = lower limit to aluminum imports from country r in time period t

29 BY..A..t Inventory balance of primary aluminum

$$\text{Y..A..Yt} + \theta_{t}\left[\sum_{r} \text{TP.A.rYt} - \sum_{u} \sum_{m} \text{TYmA.Yut}\right] - \text{Y..A..Y(t+1)} = 0$$

where m = fabrication process

NOTE: when t=T, t+1 = @

30 BPS.NSrt Production balance of secondary aluminum from new scrap

$$\text{PC.NSr.t} - \Phi_{NS} \sum_{n} \sum_{m} \text{DnmA.r.t} \leq 0$$

where n = end use market

m = fabrication process

Φ_{NS} = fraction of shipments recycled as new scrap

NOTE: in this case r = U or the United States -- similarly in following equations 31, 32, 33

31 BPSjOSrt Production balance of secondary aluminum from old scrap

$$\sum_{h=1} \left[\text{PCjOSr.h} - \Phi_j \left[\alpha(h) + \sum_{v=1} (\text{DmnA.rv} - \text{PC.NSr.v}) f(h-v) \right] \right] \leq 0$$

where j = old scrap supply segment

Φj = fraction of the total available old scrap that is recycled in segment j

$\alpha(h)$ = available old scrap in time period h originated in shipments prior to the beginning of the model

f(h-v) = fraction of the shipments made in time period v available as old scrap in time period h

32 BTP@@Srt Transportation balance of secondary aluminum

$$-\left[\text{PC.NSr.t} + \sum_{j} \text{PCjOSr.t} \right] + \sum_{m} \sum_{u} \text{TPm@Srmt} \leq 0$$

where j = segment in old scrap supply curve

m = fabricating process

NOTE: Not all fabricating processes are allowed for secondary aluminum

33 BDmnA.rt Demand satisfaction balance of aluminum

$$DmnA..rt - \sum_{P} TPmA.pnt - TYmA.Ynt - TImA.fnt -$$

$$\delta_m TPm@Srnt \leq 0$$

where m = fabricating process

n = end use market

p = smelting region

f = foreign primary aluminum origin

$$\delta m = \begin{cases} 1 \text{, when } m = I \text{ or } V \\ 0 \text{, otherwise} \end{cases}$$

34 FI.M@@.t Limitation to total investments in mines

$$\sum_{r} \sum_{m} C_{nn} M.tnnr.t + \sum_{r} \sum_{nn} \sum_{v} C^{I}_{nn} MIvnnr.t \leq I_{Mt}$$

where nn = type of mine

C_{nn}&$C_{nn}{}^{I}$ = investment cost per unit capacity for new mines and increments, mine type nn

I_{Mt} = total allowable investment expenditures in time period t for mines

35 FI.R@@.t Limitation to total investment in refineries

$$\sum_{r} \sum_{n} C_{nr} Rn.L.r.t \leq I_{Rt}$$

where n = refinery type

C_n = investment cost per unit capacity for new refineries

I_{Rt} = total allowable investment expenditures in time period t for refineries

36 FI.S@@.t Limitation to total investment in smelters et.al

$$\sum_r \sum_n C_n^S \text{SntA.r.t} + \sum_r \sum_n \sum_v C_n^X \text{XnvA.r.t} +$$

$$\sum_v \sum_m \sum_n C_{nm}^I \text{InmA.r.t.} + \sum_r C^E \text{ E..A.r.t.} \leq \text{Ist}$$

where m, n = smelter type

C_n^S, C_n^X, C_{nm}^I, C^E = investment cost per unit capacity of new smelters, expansion to smelters, improvement of smelters and power plants

I_{St} = total allowable investment expenditures in time period t for smelters et.al

APPENDIX IV

Data Tables

The name of the data tables consists of seven characters. Whenever they have similar meaning, their mnemonic structure is similar.

Following the data categories defined in section VII-1, the name of the data tables is presented,

1 Decision Data

1.1 Investment Costs (CI.U)

CI.UM..	($/ton ore/year)	new mines -- per ore type and region
CI.UMI.	($/ton ore/year)	mine expansion -- per ore type and region
CI.UR..	($/ton alumina/year)	new refineries -- per refinery type and region
CI.US..	($/ton aluminum/yr)	new smelters -- per smelter type and region
CI.USI.	($ /ton aluminum/year)	smelter expansion -- per smelter type and region
CI.UIMP	($/ton aluminum/yr)	improvement of smelters -- per improvement and region
CI.UE..	($/KW)	new power plants -- per region

1.2 Operating Costs (CO.U)

CO.UM..	($/ton ore)	mine production -- per ore type,region)

CO.UR..	($/ton alumina)	refinery production -- per refinery type and region
CO.US..	($/ton aluminum)	smelter production -- per smelter type and region
CO.UE..	($/KW)	electricity production -- per region
CO..SN	($/ton aluminum)	secondary aluminum from new scrap production
CO..SO.	($/ton aluminum)	secondary aluminum from old scrap production -- per segment of old scrap supply curve
CO.UDD.	($/ton product)	distribution cost, per end use product and end use market

1.3 Transportation Costs (CT.U and CTtU)

CTtUB..	($/ton bauxite)	imported bauxite (cost and freight) -- per country of origin and segment in time period t
CTtULIs	($/ton alumina)	imported alumina (cost and freight) -- per country of origin and region of use -- for segment s in time period t
CT.UI..	($/ton aluminum)	imported aluminum -- includes fabrication
CT.UD..	($/ton aluminum)	domestic aluminum -- per fabricating process and smelting region
CTYUD..	($/ton aluminum)	domestic aluminum from inventory -- per fabrication process
CT..S@.	($/ton aluminum)	secondary aluminum -- per fabrication process

2 Constraining Data

2.1 Reserves (REVU)

REVU...	(MM ton of ore)	domestic bauxite reserves -- per region

2.2 Existing Capacity (Rx.U)

RM.U...	(MM ton ore/year)	mine capacity -- per ore and region
RR.U...	(MM ton alumina/yr)	refineries capacity -- per type and region
RS.U...	(MM ton aluminum/yr)	smelters capacity -- per type and region
RE.U...	(MW)	captive power plants -- per region

2.3 Restrictions to Imports (xxxtIbB)

MB@tIUB	(M ton dry ore/year)	upper bound to bauxite imports -- per country of origin and segment in time period t
MB@tILB	(M ton dry ore/year)	lower bound to bauxite imports -- per country of origin in time period t
RL.tIUB	(M ton alumina/year)	upper bound to alumina imports -- per country of origin and segment -- in time period t
RL.tILB	(M ton alumina/year)	lower bound to alumina imports -- per country of origin in time period t
SA.tIUB	(M ton aluminum/year)	upper bound to aluminum imports in time period t
SA.tILB	(M ton aluminum/year)	lower bound to aluminum imports in time period t

2.4 Limitations to expansion (xI.UB..)

MI.UO..	(fraction)	Limitation to mine expansion as fraction of existing mine capacity -- per mine type
SI.UB..	(fraction)	Limitation to smelter expansion as fraction of existing smelter capacity -- per region

2.5 Availability of services

BD.UPE.	(10^2 MW)	purchased power -- per region and time period
RFI@...	(MMM 1977$)	investment capital -- per investment and time period

2.6 Inventory Policy

Y@@LBD.	(M ton)	lower bounds to inventories -- per product and time period

3 Process Definition Data

3.1 Process Yields (YL)

YL..BI.	(fraction)	yield of alumina per unit weight of foreign bauxite -- per country of origin
YL..@P.	(fraction)	yield of alumina per unit weight of domestic ore -- per ore type
RH..BL.	(fraction)	fraction of low silica bauxite processed in high silica refineries
YLP....	(fraction)	yield of aluminum per unit weight of alumina

3.2 Process Requirements (RQ)

RQELS..	(KWH)	electric energy required to produce 1 lb of aluminum -- per smelter type

4 Demand Data

BDtUDEM (MM ton/year)	demand per product and end use market during time period t

5 Miscellaneous

BPU.xxx (fraction)	investment profile, in fraction per year, where xxx stands for the investment as in (1-1) of this appendix
BTU.xxx (years)	building profile for investment xxx, where B is the building time and F the additional time until it reaches full capacity
S..U... (fraction)	fraction of each fabricated form that satisfies unit demand of each end use market
FIV.... (fraction)	average fraction of consumed aluminum that becomes obsolete per time period following its consumption
ALFA... (M ton/year)	average available obsolete aluminum from shipments prior to the beginning of the model -- per time period
OSUB... (fraction)	fraction of obsolete aluminum consumed as old scrap -- per segment
SA.2F.. (fraction)	old scrap and new scrap generated as a fraction of total shipments
TIME... (years)	establishes the number of time periods, the years per time period and the year in which each time period starts

All other tables are 0-1 truth tables, that help in the building of the matrix.

APPENDIX V

Computer code

A Job Control Language

```
//B0769ALU JOB 409F1,KELLY,CMSNOTE=ALUAR
/*LONGKEY KELLY
/*PRIORITY IDLE
/*JOBPARM LINES=20
// EXEC PGM=IEFBR14
//MAPSFILE DD DSN=A409F1.MAPSFILE,DISP=(NEW,CATLG),
// UNIT=SYSDA,VOL=SER=USR305,SPACE=(TRK,(76,25))
//ALUMLOAD DD DSN=A409F1.ALUMLOAD,DISP=(NEW,CATLG),
// UNIT=SYSDA,VOL=SER=USR305,SPACE=(TRK,(500))
//  EXEC MSSMPS,REGION=350K,TIME.CPC=(0,10),TIME.EXEC=(100),
// SYSACT=3000,ACTFILE=3000,LODLIB2=FIXS
//CPC.SYSIN DD *
```

```
//EXEC.STEPLIB   DD   DSN=AFU661.UTIL.LOADMODS,DISP=SHR
//*EXEC.SYSUDUMP DD SYSOUT=A
//EXEC.MAPSFILE DD DSN=A409F1.MAPSFILE,
//       DISP=(OLD,KEEP)
//EXEC.ALUMACT DD UNIT=SYSDA,SPACE=(TRK,(500))
//EXEC.ALUMLOAD DD DSN=A409F1.ALUMLOAD,
//       DISP=(OLD,KEEP)
//EXEC.TRANFILE  DD  UNIT=SYSDA,SPACE=(TRK,(20,10)),
//      DSN=&TRANFILE,DISP=(NEW,DELETE),
//      DCB=(RECFM=VB,LRECL=204,BLKSIZE=3088)
//SORTLIB    DD   DSN=SM01.SORTLIB,DISP=SHR
//SORTWK01   DD   UNIT=SYSDA,SPACE=(TRK,(20,10))
//SORTWK02   DD   UNIT=SYSDA,SPACE=(TRK,(20,10))
//SORTWK03   DD   UNIT=SYSDA,SPACE=(TRK,(20,10))
//SORTMSG    DD   SYSOUT=A
//EXEC.SYSIN DD *
```

B Procedure Control Language

```
         PROGRAM('ND')
         INITIALZ
         TITLE('ALUMINUM  VOMO')
ACTFILE  DC('ALUMACT')
ACTPROB  DC('ALUM.B')
CASE     DC('ALUM.B8')
BASIS    DC('ALUM.B8')
BOUNDS   DC('BOUND1')
DFRATE   DC(0.18)
TIMEPER  DC(3.0)
DEBUG    DC('ND')
*                 *INITIALIZE CR CELLS
         MOVE(XACTFILE,ACTFILE)
         MOVE(XACTPROB,ACTPROB)
         MOVE(XOBJ,'OBJ')
         MOVE(XRHS,'RHS1')
         XFREQ1=300
         MVADR (XDOFREQ1,SAVIT)
         XFREQLGO=300
*        LOAD ('TO',ACTFILE,'FROM','ALUMLOAD')
*                 *COMPILATION OF MATRIX GENERATION
*                 *AND REPORT PROGRAMS
         MOVE(XDATA,'ALUMGEN')
         CPDF('NAME','ALUMGEN','XREF')
         MOVE(XDATA,'ALUMREP')
         CPDF('NAME','ALUMREP')
         MOVE(XDATA,'ALUMTAB')
         EXDF('NAME','ALUMGEN')
         UNLOAD ('TO','ALUMLOAD','FROM',ACTFILE)
         LISTFILE ('DTFM',ACTFILE)
*        MOVE (XDATA,'MODPROG')
*        CPDF ('FILE','MODS','NAME','MODS','XREF')
```

```
*           EXDF ('NAME','MODS')
            READY('BOUND',BOUNDS)
*           TRANCOL('STANDARD','TRANFILE')
*           CLOSEF('TRANFILE')
*           PRINTRAN
*           READY('BOUND',BOUNDS)
*           RESET ('NAME',BASIS)
            VARIFORM
            PRESERVE ('NAME',CASE)
            SOLUTION
            RECORD('ANSWERS',CASE)
            FREECORE
            MOVE(XDATA,'REPTAB')
            EXDF('NAME','ALUMBER')
            EXIT
SAVIT       PRESERVE ('NAME',CASE)
            CONTINUE
            PEND
```

C Matrix generation program

```
NAME          ALUMGEN
         EJECT
         READTAB Z:TABLES,SYSIN,U:XDATA
*                           *NAME VARIABLE DEFINITIONS
         NAME    N:CASE=H:CASE
         NAME    N:DEBUG=H:DEBUG
         NAME    N:BOUNDS=H:BOUNDS
         NAME    N:OBJ=U:XOBJ
         NAME    N:RHS=U:XRHS
         NAME    N:FILE=H:ACTFILE
         NAME    N:PROB=H:ACTPROB
         CALC    V:DFRATE=I:DFRATE
         CALC    V:TIMEPER=I:TIMEPER
         EJECT
*                           *TALLY ON TIME PERIODS
         DIMEN   V:TMX=T:TIME...(HEAD)
         IF      N:DEBUG<NM>YES,BUG1
         CALC    V:TMX=V:TIMEPER
BUG1     CALC    V:T=1
         NAME    N:TMX=T:TIME...(0,V:TMX)
TIME     NAME    N:T=T:TIME...(0,V:T)
         DFO     RSLT,DFRATE,1
*                           *DFO IS THE OPERATIONAL DISCOUNT
*                           *FACTOR SUBROUTINE
*                           ********************************
*                           *TALLY ON CONTINENTS
         DIMEN   V:CTMX=Z:CONT...(STUB)
         CALC    V:CONT=1
CONT     NAME    N:CONT=Z:CONT...(V:CONT,0)
         EJECT
*                           *MINING FORMULATION
*                           *MINING INVESTMENTS
```

```
         NAME      N:S=M..R&N:CONT&..
*                                *LOOP ON AALLOWABLE REGION/ORE COMBINATIONS
         LOOP      T:N:S(=1,=2)<EQ>1,MR1
         NAME      N:R= =1
         NAME      N:OR= =2
         IF        N:R<NM>A,MR2
*                                *IN REGION A ONLY MINE
*                                *INCREMENTS ARE ALLOWED
         NAME      N:S1=MI.&N:CONT&0..
         LOOP      T:N:S1(0,=10),MR3
         NAME      N:INC= =10
         NAME      N:COL=M&N:INC&0&N:OR&N:R&.&N:T
         NAME      N:INV=MI.
         DFI       RESULT,DERATE,T,OR,INV,CONT,TMX
         NAME      N:S2=CI.&N:CONT&M&N:INC&.
         CALC      V:S1=V:RESULT*T:N:S2(N:R,N:OR)
         COL       N:COL,
                   N:OBJ<N>=V:S1
*                                *FINANCIAL CONSTRAINT
         COL       N:COL,
                   FI.M@@.&N:T<L>=T:N:S2(N:R,N:OR)
*                                *CAPACITY AVAILABILITY FACTORS
         LOOP      T:TIME...(0,=11)<HIEQ>N:T,MR4
         NAME      N:J= =11
         AVL       FIJ,T,J,OR,INV,CONT
         COL       N:COL,
                   KM..&N:OR&N:R&N:J<L>=-V:FIJ
MR4      CONTINUE
*                                *TOTAL POSSIBLE INCREMENT
         COL       N:COL,
                   KM&N:INC&0&N:OR&N:R&@<L>= 1
MR3      CONTINUE
```

```
         GOTO     MR5
*                            *NON BAUXITE MINE INVESTMENTS
MR2      NAME     N:COL=M.&N:T&N:OR&N:R&.&N:T
         NAME     N:INV= M..
         DFI      RESULT,DFRATE,T,OR,INV,CONT,TMX
         NAME     N:S1=CI.&N:CONT&N:INV
         COL      N:COL,
                  N:OBJ<M>= V:RESULT*T:N:S1(N:R,N:OR)
*                            *FINANCIAL CONSTRAINT
         COL      N:COL,
                  FI.M@@.&N:T<L>= T:N:S1(N:R,N:OR)
*                            *AVAILABLE MINING CAPACITY
         LOOP     T:TIME...(0,=12)<HIEQ>N:T,MR6
         NAME     N:J= =12
         AVL      FIJ,T,J,OR,INV,CONT
         COL      N:COL,
                  KM..&N:OR&N:R&N:J<L>= -V:FIJ
MR6      CONTINUE
*                            *MINES INCREMENTS LIMIITATIONS
         NAME     N:S2=MI.&N:CONT&@..
         AVL      FIJ,T,TMX,OR,INV,CONT
         IF       V:FIJ<LT>1,MR70
         LOOP     T:N:S2(0,=15),MR7
         NAME     N:INC= =15
         COL      N:COL,
                  KM&N:INC&N:T&N:OR&N:R&@<L>= -T:N:S2(N:OR,N:INC)
MR7      CONTINUE
MR70     CONTINUE
*                            *MINE INCREMENTS BUILDING
*                            *RESTRICTED TO MINE IN FULL OPERATION
         LOOP     T:TIME...(0,=16)<LOEQ>N:T,MR8
         NAME     N:VT= =16
```

```
         NAME     N:S2= MI.&N:CONT&O..
         LOOP     T:N:S2(0,=17),MR9
         NAME     N:INC= =17
         NAME     N:COL=M&N:INC&N:VT&N:OR&N:R&.&N:T
         NAME     N:INV=M..
         AVL      FIJ,VT,T,CR,INV,CONT
         IF       V:FIJ<LT>1,MR5
         NAME     N:INV=MI.
         DFI      RESULT,DFRATE,T,OR,INV,CONT,TMX
         NAME     N:S3=CI.&N:CONT&M&N:INC&.
         CALC     V:S1=V:RESULT*T:N:S3(N:R,N:OR)
         COL      N:COL,
                  N:OBJ<N>= V:S1
*                              *FINANCIAL CONSTRAINT
         COL      N:COL,
                  FI.M@@.&N:T<L>=T:N:S3(N:R,N:OR)
*                              *AVAILABLE MINING CAPACITY
         LOOP     T:TIME...(0,=18)<HIEQ>N:T,MR10
         NAME     N:J= =18
         AVL      FIJ,T,J,OR,INV,CONT
         COL      N:COL,
                  KM..&N:OR&N:R&N:J<L>= -V:FIJ
MR10     CONTINUE
*                              *TOTAL AVAILABLE INCREMENT
         COL      N:COL,
                  KM&N:INC&N:VT&N:OR&N:R&@<L>=1
MR9      CONTINUE
MR8      CONTINUE
         EJECT
*                              *MINING OPERATIONS
MR5      NAME     N:COL=PM.&N:OR&N:R&.&N:T
         NAME     N:S1=CO.&N:CONT&M..
```

```
          COL      N:COL,
                   N:OBJ<N>=V:RSLT*T:N:S1(N:R,N:CR)
*                                *CAPACITY CONSTRAINT
          COL      N:COL,
                   KM..&N:OR&N:R&N:T<L>=1
*                                *ALUMINA PRODUCTION BALANCE
*                                *YIELD PER ORE
          NAME     N:S2=M..&N:CONT&RR.
          LOOP     T:N:S2(N:K,=19)<EQ>1,MR11
          NAME     N:S1= =19
          NAME     N:S3=M..&N:CONT&OT.
          LOOP     T:N:S3(N:OR,=20)<EQ>1,MR12
          NAME     N:S4= =20
          COL      N:COL,
                   PR&N:S4&.&N:OR&N:S1&N:T<L>=-T:YL..@P.(N:OR,1)
MR12      CONTINUE
MR11      CONTINUE
*                                *RESERVES DEPLETION LIMITS
          IF       N:OR<NM>BH,MR1
          COL      N:COL,
                   BR..&N:OR&N:F&@<L>=T:TIME...(YTP,N:T)
MR1       CONTINUE
          EJECT
*                                *IMPORTED BAUXITE
          LOOP     T:MB@.I..(=1,=2)<EQ>1,MI1
          NAME     N:R= =1
          NAME     N:U= =2
*                                *TYPE OF BUXITE IMPORTED
          LOOP     T:MB@.T..(N:R,=3)<EQ>1,MI2
          NAME     N:OR= =3
          NAME     N:S=MB@&N:T&IUB
*                                *SEGMENT OF IMPORTATION
```

```
         LOOP     T:N:S(N:R,=4),MI3
         NAME     N:SG= =4
         IF       N:SG<HIEQ>2,MI3
*********************DEBUG*********************************
         NAME     N:COL=TI&N:SG&B&N:OR&N:R&N:U&N:T
         NAME     N:S1=CT&N:T&N:CONT&B..
         COL      N:COL,
                  N:OBJ<N>=V:RSLT*T:N:S1(N:R,N:SG)
*                              *UPPER & LOWER BOUNDS COSTRAINTS
         COL      N:COL,
                  KTI&N:SG&B&N:OR&N:R&N:T<L>=1,
                  BTI@B&N:OR&N:R&N:T<L>= -1
         IF       N:U<NM>Q<AND>N:U<NM>Y,MI4
         IF       N:U<MT>0,MI5
*                              *INVENTORY OF BAUXITE
         COL      N:COL,
                  BY..B&N:OR&N:R&N:T<E>=-T:TIME...(YTP,N:T)
         GOTO     MI3
*                              *SATISFYING DEMAND OF SECTOR Q
MI5      COL      N:COL,
                  BD&N:U&.B&N:OR&N:CONT&N:T<L>=-1
         GOTO     MI3
*                              *SUPPLYING REFINERIES
MI4      COL      N:COL,
                  PR&N:OR&N:R&B&N:OR&N:U&N:T<L>=-T:YL..BI.(N:R,1)
         IF       T:MB&.I..(N:R,Q)<NE>1,MI3
*                              *LOW SILICA TO HIGH SILICA REF.
         COL      N:COL,
                  PRH.B&N:OR&N:U&N:T<L>=-T:YL..BI.(N:R,1)
MI3      CONTINUE
         EJECT
*                              *INVENTORY COLUMNS
```

```
          IF        N:U<NM>Y,MI2
          NAME      N:COL=Y..B&N:OR&N:R&N:U&N:T
*                                 *INVENTORY HAS NO MORE COST
*                                 *THAN THE TIME VALUE OF MONEY
*                                 *INVENTORY BALANCE
          COL       N:COL,
                    BY..B&N:OR&N:R&N:T<E>=-1
          IF        V:T<LE>1,MI6
          CALC      V:T1=V:T-1
          NAME      N:T1=T:TIME...(0,V:T1)
          COL       N:COL,
                    BY..B&N:OR&N:R&N:T1<E>=1
MI6       CONTINUE
*                                 *TRANSPORTS OF IMPORTED
*                                 *BAUXITE FROM INVENTORY
          LOOP      T:MB@.I..(N:R,=6)<EQ>1,MI7
          NAME      N:U1= =6
*                                 *INVETORY CAN SUPPLY ALL BAUXITE
*                                 *DEMAND SECTORS EXCEPT INVENTORY
          IF        N:U1<MT>N:U,MI7
          NAME      N:COL=T&N:U&.B&N:OR&N:R&N:U1&N:T
*                                 *TRANSPORTS FROM INVENTORY HAVE
*                                 *MARGINAL COST
          COL       N:COL,
                    N:OBJ<N>=0.001*V:RSLT
          COL       N:COL,
                    BY..B&N:OR&N:R&N:T<E>=T:TIME...(YTP,N:T)
          IF        N:U1<MT>O,MI8
          COL       N:COL,
                    PR&N:OR&N:R&B&N:OR&N:U1&N:T<L>=-T:YL..BI.(N:R,1)
          IF        N:R<NM>N,MI7
          COL       N:COL,
```

```
                 PRH.B&N:OR&N:UL&N:T<L>= -T:YL..BI.(N:R,1)
        GOTO     MI7
MI8     COL      N:COL,
                 BDQ.B&N:OR&N:CONT&N:T<L>= -1
MI7     CONTINUE
MI2     CONTINUE
MI1     CONTINUE
        EJECT
*                               *REFINING-INVESTMENTS
        NAME     N:S=R..&N:CONT&RP.
*                               *LOOP ON ADMISSIBLE REGION/REF. TYPE
*                               *COMBINATIONS
        LOOP     T:N:S(=1,=2)<EQ>1,RF1
        NAME     N:R= =1
        NAME     N:TYP= =2
        NAME     N:COL=R&N:TYP&.L.&N:R&.&N:T
        NAME     N:INV=R..
        DFI      RESULT,DFRATE,T,TYP,INV,CONT,TMX
        NAME     N:S1=CI.&N:CONT&N:INV
        COL      N:COL,
                 N:OBJ<N>=V:RESULT*T:N:S1(N:R,N:TYP)
*                               *FINANCIAL CONSTRAINT
        COL      N:COL,
                 FI.R@@.&N:T<L>=T:N:S1(N:R,N:TYP)
*                               *AVAILABLE CAPACITY FACTORS
        LOOP     T:TIME...(0,=10)<HIEQ>N:T,RF2
        NAME     N:J= =10
        AVL      FIJ,T,J,TYP,INV,CONT
        COL      N:COL,
                 KR&N:TYP&.L.&N:R&N:J<L>= -V:FIJ
RF2     CONTINUE
        IF       N:TYP<NT>L<OR>N:TYP<NT>H,RF3
```

```
          NAME      N:COL=PR&N:TYP&N:TYP&.&N:R&.&N:T
*                                 *REFINING NON BAUXITE ORES
          NAME      N:S1=CO1&N:CONT&N:INV
          COL       N:COL,
                    N:OBJ<N>=V:RSLT*T:N:S1(N:R,N:TYP)
*                                 *CAPACITY PRODUCTION & TRANSPORTATION BALANCE
          COL       N:COL,
                    KR&N:TYP&.L.&N:R&N:T<L>=1,
                    PR&N:TYP&.&N:TYP&.&N:R&N:T<L>=1,
                    BTP.L.&N:R&N:T<L>= -1
          GOTO      RF1
          EJECT
*                                 *REFINING BAUXITE ORES
RF3       LOOP      T:M32.T..(=4,N:TYP)<EQ>1,RF4
          NAME      N:K= =4
          NAME      N:COL=PR&N:TYP&B&N:K&N:R&.&N:T
          NAME      N:S1=CO2&N:CONT&N:INV
          COL       N:COL,
                    N:OBJ<N>=V:RSLT*T:N:S1(N:R,N:K)
*                                 *PRODUCTION,TRANSPORTATION & CAPACITY BALANCE
          COL       N:COL,
                    KR&N:TYP&.L.&N:R&N:T<L>=1,
                    BTP.L.&N:R&N:T<L>= -1
          IF        N:TYP<NT>H,RF5
          COL       N:COL,
                    PR&N:TYP&N:K&B&N:TYP&N:R&N:T<L>=1
          GOTO      RF4
*                                 *WE ASSUME THAT SURINAM PROVIDES 75%
*                                 *OF THE LO BAUXITE CONSUMED BY HI REFINERIES
RF5       COL       N:COL,
                    PRLNBL&N:P&N:T<L>=T:RH..BL.(1,1)*0.75,
                    PRLYBL&N:R&N:T<L>=T:RH..BL.(1,1)*0.25,
```

```
                 PRH.BL&N:R&N:T<L>=T:RH..BL.(1,1),
                 PRH.BH&N:R&N:T<L>=1-T:RH..BL.(1,1)
RF4       CONTINUE
RF1       CONTINUE
          EJECT
*                                  *TRANSPORTATION OF ALUMINA
          NAME     N:S=R..&M:CONT&RT.
*                                  *LOOP THROUGH THE ALLOWABLE
*                                  *TRANSPORTATION LINKS
          LOOP     T:N:S(=1,=2)<EQ>1,TL1
          NAME     N:R= =1
          NAME     N:U= =2
          NAME     N:COL=TP.L.&N:R&N:U&N:T
          NAME     N:S1=CT.&N:CONT&LD.
          COL      N:COL,
                   N:OBJ<N>=V:RSLT*T:N:S1(N:R,N:U)
*                                  *TRANSPORTATION BALANCE
          COL      N:COL,
                   BTP.L.&N:R&N:T<L>=1
          IF       N:U<MT>Q<OR>N:U<MT>F,TL2
          IF       N:U<MT>Y,TL3
*                                  *ALUMINA TO SMELTERS
          COL      N:COL,
                   PS3.L.&N:U&N:T<L>=-T:YLP....(1,1)
          GOTO     TL1
*                                  *S\TISFYING END USE DEMANDS
TL2       COL      N:COL,
                   BD&N:U&.L.&N:CONT&N:T<L>=-1
          GOTO     TL1
*                                  *TRANSPORTATION TO INVENTORY
TL3       COL      N:COL,
                   BY..L..&N:T<E>=-T:TIME...(YTP,N:T)
```

```
TL1        CONTINUE
           EJECT
*                                    *ALUMINA INVENTORY
           NAME       N:COL=Y..L..Y&N:T
           COL        N:COL,
                      BY..L..&N:T<E>=-1
           IF         V:T<LE>1,LY1
           CALC       V:T1=V:T-1
           NAME       N:T1=T:TIME...(0,V:T1)
           COL        N:COL,
                      BY..L..&N:T1<E>=1
LY1        CONTINUE
           EJECT
*                                    *IMPORTED ALUMINA
           NAME       N:S=RL.&N:T&IUB
           LOOP       T:N:S(=1,=2),IL1
           NAME       N:R= =1
           NAME       N:SG= =2
           IF         N:SG<HIEQ>2,IL1
*******************************DEBUG********************************
           NAME       N:S1=CT&N:T&N:CONT&LI&N:SG
*                                    *LOOP THROUGH THE SMENTING REGIONS
           LOOP       T:N:S1(N:R,=C),IL2
           NAME       N:U= =3
           NAME       N:COL=TI&N:SG&L.&N:R&N:U&N:T
           COL        N:COL,
                      N:OBJ<N>=V:RSLT*T:N:S1(N:R,N:U)
*                                    *UPPER AND LOWER BOUNDS &
*                                    *ALUMINUM PRODUCTION
           COL        N:COL,
                      KTI&N:SG&L.&N:R&N:T<L>=1,
                      BTI@L.&N:R&N:T<L>=-1,
```

```
                 PS@.L.&N:U&N:T<L>=-T:YLP....(1,1)
IL2     CONTINUE
IL1     CONTINUE
*                                *TRANSPORTATION FROM INVENTORY
        NAME     N:S=S..&N:CCNT&RT.
        LOOP     T:N:S(=1,0),TYL1
        NAME     N:U= =1
        NAME     N:COL=TY.L.Y&N:U&N:T
        NAME     N:S1=CT.&N:CONT&LD.
        COL      N:COL,
                 N:OBJ<N>=V:RSLT*(T:N:S1(Y,N:U)+0.001)
        COL      N:COL,
                 BY..L..&N:T<E>=T:TIME...(YTP,N:T),
                 PS@.L.&N:U&N:T<L>=-T:YLP....(1,1)
TYL1    CONTINUE
        EJECT
*                                *SMELTING INVESTMENT
        NAME     N:S=S..&N:CCNT&RT.
*                                *LOOP ON ALLOWABLE REGION/TYPE LINKS
        LOOP     T:N:S(=1,0),SM20
        NAME     N:R= =1
        LOOP     T:N:S(N:R,=2)<EQ>1,SM1
        NAME     N:TYP= =2
        NAME     N:INV=S..
        NAME     N:COL=S&N:TYP&0A.&N:R&.&N:T
*                                *CAPACITY INTERSECTION OF
*                                *EXISTING SMELTERS
        COL      N:COL,
                 KS&N:TYP&.A.&N:R&N:T<L>= -1
        IF       N:TYP<MT>6,SM2
*                                *THE SMELTER MUST BE PART OF
*                                *AN IMPROVEMENT EQUATION
```

```
SM5       IF        V:T<EQ>V:TMX,SM6
*                             *NO IMPROVEMENTS IN LAST
*                             *TIME PERIOD
          COL       N:COL,
                    BV&N:TYP&@A.&N:R&N:T<E>=1
*                             *IMPROVEMENTS APPEAR AFTER
*                             *FIRST TIME PERIOD
SM6       IF        V:T<LE>1,SM4
          CALC      V:T1=V:T-1
          COL       N:COL,
                    BV&N:TYP&@A.&N:R&N:T1<E>=-1
SM4       CONTINUE
SM2       CONTINUE
          EJECT
*                             *NEW SMELTERS INVESTMENTS
*                             *NO SODERBERG ANNCDES ALLOWED
          IF        N:TYP<LOEQ>2,SM30
          NAME      N:COL=S&N:TYP&N:T&A.&N:R&.&N:T
          DFI       RESULT,DFRATE,T,TYP,INV,CONT,TMX
          NAME      N:S1=CI.&N:CONT&N:INV
          COL       N:COL,
                    N:OBJ<N>=V:RESULT*T:N:S1(N:R,N:TYP)
*                             *FINANCIAL CONSTRAINT
          COL       N:COL,
                    FI.S@@.&N:T<L>=T:N:S1(N:R,N:TYP)
*                             *AVAILABLE CAPACITY FACTORS
          LOOP      T:TIME...(0,=10)<HIEQ>N:T,SM8
          NAME      N:J= =10
          AVL       FIJ,T,J,TYP,INV,CONT
          COL       N:COL,
                    KS&N:TYP&.A.&N:R&N:J<L>=-V:FIJ
SM8       CONTINUE
```

```
*                             *NEW SMELTER CAPACITY SET A
*                             *LIMIT TO EXPANSIONS
        IF      N:TYP<MT>6,SM30
        AVL     FIJ,T,TMX,TYP,INV,CONT
        IF      V:FIJ<LT>1,SM30
        NAME    N:S1=SI.&N:CONT&B..
        COL     N:COL,
                KS@&N:T&A.&N:R&@<L>=-T:N:S1(N:R,I)
SM30    CONTINUE
        EJECT
*                             *EXPANSION IS NOT ALLOWED IN
*                             *TYPE 6 SMELTERS
        IF      N:TYP<MT>6,SM9
*                             *NOR IN TYPES 1 OR 2
        IF      N:TYP<LOEQ>2,SM9
*                             *EXPANSION OF CAPACITY
        NAME    N:VT=0
        DIMEN   V:VTX=T:TIME...(0,N:T)
        CALC    V:VTX=V:VTX+1
        CALC    V:VT=1
SMTLY   NAME    N:COL=X&N:TYP&N:VT&A.&N:R&.&N:T
        NAME    N:INV=SI.
        DFI     RESULT,DFRATE,T,TYP,INV,CONT,TMX
        NAME    N:S2=CI.&N:CONT&N:INV
        COL     N:COL,
                N:OBJ<N>=V:RESULT*T:N:S2(N:R,N:TYP),
                KS@&N:VT&A.&N:R&@<L>=1
*                             *FINANCIAL CONSTRAINT
        COL     N:COL,
                FI.S@@.&N:T<L>= T:N:S2(N:R,N:TYP)
*                             *AVAILABLE CAPACITY FACTORS
        LOOP    T:TIME...(0,=10)<HIEQ>N:T,SM10
```

```
         NAME    N:J= =10
         AVL     FIJ,T,J,TYP,INV,CONT
         COL     N:COL,
                 KS&N:TYP&.A.&N:R&N:J<L>=-V:FIJ
SM10     CONTINUE
         NAME    N:INV=S..
         NAME    N:I=T:TIME...(0,V:VT)
         AVL     FIJ,I,T,TYP,INV,CONT
         IF      V:FIJ<LT>1,SM9
         NAME    N:VT=N:I
         TALLY   V:VT,V:VTX,SMTLY
SM9      CONTINUE
         EJECT
*                          *NO IMPROVEMENT PERMITTED
*                          *ON TYPE 6
         IF      N:T<MT>N:TMX,SM11
         IF      N:TYP<HIEQ>6,SM11
         LOOP    T:IMPROVE(=5,=6)<EQ>1;SM12
         NAME    N:FROM= =5
         NAME    N:TO= =6
         NAME    N:COL=I&N:FROM&N:TO&A.&N:R&.&N:T
*                          *ASSUME THE IMPROVEMENT IS
*                          *DONE DURING THE TIME PERIOD
         NAME    N:HEAD=N:FROM&N:TO
         NAME    N:S1=CI.&N:CONT&IMP
         COL     N:COL,
                 N:OBJ<N>=V:RSLT*T:N:S1(N:R,N:HEAD)/
                 T:TIME...(YTP,N:T)
*                          *FINANCIAL CONSTRAINT
         COL     N:COL,
                 FI.S@@.&N:T<L>=T:N:S1(N:R,N:HEAD)
         IF      N:TYP<MT>N:FROM,SM13
```

```
          IF        N:TYP<MT>N:TO,SM14
          GOTO      SM12
SM13      COL       N:COL,
                    BV&N:TYP&@A.&N:R&N:T<E>=-1
          GOTO      SM12
SM14      CALC      V:CAP=1
          IF        N:TO<NM>5,SM15
          CALC      V:CAP=1.3
SM15      COL       N:COL,
                    BV&N:TYP&@A.&N:R&N:T<E>=V:CAP
SM12      CONTINUE
SM11      CONTINUE
          EJECT
*                                 *SMELTER PRODUCTION
          NAME      N:COL=PS&N:TYP&L.&N:R&.&N:T
          NAME      N:S1=CO.&N:CONT&S..
          COL       N:COL,
                    N:OBJ<N>=V:RSLT*T:N:S1(N:R,N:TYP)
*                                 *CAPACITY,PRODUCTION,TRANSPORTATION,
*                                 *ELECTRICITY BALANCES
          COL       N:COL,
                    KS&N:TYP&.A.&N:R&N:T<L>=1,
                    PS@.L.&N:R&N:T<L>=1,
                    BTP@A.&N:R&N:T<L>=-1,
                    BE..A.&N:R&N:T<L>=T:RQELS..(Y,N:TYP)*2
SM1       CONTINUE
          EJECT
*                                 *BUILDING OF CAPTIVE
*                                 *POWER PLANTS
          NAME      N:COL=E..A.&N:R&.&N:T
          NAME      N:INV=E..
          DFI       RESULT,DFRATE,T,R,INV,CONT,TMX
```

```
         NAME     N:S1=CI.&N:CONT&N:INV
         COL      N:COL,
                  N:OBJ<N>=V:RESULT*T:N:S1(N:R,E)*1000
*                              *FINANCIAL CONSTRAINT
         COL      N:COL,
                  FI.S@@.&N:T<L>= T:N:S1(N:R,E)*1000
*                              *AVAILABLE CAPACITY FACTORS
         LOOP     T:TIME...(0,=10)<HIEQ>N:T,SM23
         NAME     N:J= =10
         AVL      FIJ,T,J, ,INV,CONT
         COL      N:COL,
                  K&N:INV&A.&N:R&N:J<L>=-V:FIJ*0.85
SM23     CONTINUE
         NAME     N:COL=PE.A.&N:R&.&N:T
         NAME     N:S1=CO&N:T&N:CONT&N:INV
         COL      N:COL,
                  N:OBJ<N>=V:RSLT*T:N:S1(N:R,C)
*                              *CAPACITY & PRODUCTION BALANCE
         COL      N:COL,
                  K&N:INV&A.&N:R&N:T<L>=1/8760,
                  B&N:INV&A.&N:R&N:T<L>=-1
*                              *PURCHASED POWER
         NAME     N:COL=PEPA.&N:R&.&N:T
         COL      N:COL,
                  N:OBJ<N>=V:RSLT*T:N:S1(N:R,P),
                  BC..A.&N:R&N:T<L>=-1
         EJECT
*                              *TRANSPORTATION OF PRIMARY ALUMINUM
         NAME     N:S1=S..&N:CONT&...
*                              *LINK ALL PROCESSES AND END MARKETS
*                              *WITH THE CORRESPONDENT FACTORS
         LOOP     T:N:S1(=10,=11),SM24
```

```
         NAME     N:PR= =11
         NAME     N:MK= =10
         NAME     N:COL=TP&N:PR&A.&N:R&N:MK&N:T
         NAME     N:S2=CT.&N:CONT&D..
         COL      N:COL,
                  N:OBJ<N>=V:RSLT*T:N:S2(N:R,N:PR)
*                              *TRANSPORTATION BALANCE &
*                              *END DEMAND SATISFACTION
         COL      N:COL,
                  BTP@A.&N:R&N:T<L>= 1,
                  BD&N:MK&N:PR&A.&N:CONT&N:T<L>=-1
SM24     CONTINUE
*                              *TRANSPORTATION TO INVENTORY
         NAME     N:COL=TP.A.&N:R&Y&N:T
         COL      N:COL,
                  BY..A..&N:T<E>= -T:TIME...(YTP,N:T),
                  BTP@A.&N:R&N:T<L>=1
SM20     CONTINUE
         EJECT
*                              *INVENTORY
         NAME     N:COL=Y..A..Y&N:T
         COL      N:COL,
                  BY..A..&N:T<E>=-1
         IF       V:T<LE>1,SM80
         CALC     V:T1=V:T-1
         NAME     N:T1=T:TIME...(0,V:T1)
         COL      N:COL,
                  BY..A..&N:T1<E>=1
SM80     CONTINUE
         EJECT
*                              *IMPORTED PRIMARY ALUMINUM
*                              *ONLY ONE SUPPLYING REGION
```

```
          NAME     N:S=S..&N:CONT&...
*                                *LOOP THROUGH ALLOWABLE
*                                *PROCESS/END MARKET  LINKS
          LOOP     T:N:S(=1,=2),IPA1
          NAME     N:MK= =1
          NAME     N:PR= =2
          NAME     N:COL=TI&N:PR&A.F&N:MK&N:T
          NAME     N:S1=CT.&N:CONT&I..
          COL      N:COL,
                   N:OBJ<N>=V:RSLT*T:N:S1(F,N:PR)
*                                *UPPER & LOWER BOUNDS
*                                *AND SUPPLY BALANCE
          COL      N:COL,
                   KTI@A.F&N:T<L>=1,
                   BTI@A.F&N:T<L>=-1,
                   BD&N:MK&N:PR&A.&N:CONT&N:T<L>=-1
*                                *TRANSPORTATION FROM INVENTORY
          NAME     N:COL=TY&N:PR&A.Y&N:MK&N:T
          NAME     N:S2=CTY&N:CONT&D..
          COL      N:COL,
                   N:OBJ<N>=V:RSLT*(T:N:S2(N:CONT,N:PR)+0.001)
          COL      N:COL,
                   BY..A..&N:T<F>=T:TIME...(YTP,N:T),
                   BD&N:MK&N:PR&A.&N:CUNT&N:T<L>=-1
IPA1      CONTINUE
          EJECT
*                                *SCRAP VECTORS
          LOOP     T:SA.2F..(=1,0),SCR1
          NAME     N:S= =1
          IF       N:S<MT>0,SCR2
*                                *NEW SCRAP VECTOR
          NAME     N:COL=PC.&N:S&S&N:CONT&.&N:T
```

```
          NAME     N:S1=CC..S&N:S&.
          COL      N:COL,
                   N:OBJ<N>=T:N:S1(N:CONT,1)*V:RSLT
*                                *SCRAP BALANCE & MARKET SUPPLY
          COL      N:COL,
                   BPS.&N:S&S&N:CONT&N:T<L>=1,
                   BTP@@S&N:CONT&N:T<L>=-1
*                            *OLD SCRAP BALANCE
          LOOP     T:OSUB...(0,=10),SCR5
          NAME      N:SG= =10
          LOOP     T:TIME...(0,$10),SCR6
          CALC     V:J= $10
          IF       V:J<LT>V:T,SCR6
          NAME     N:J=T:TIME...(0,V:J)
          CALC     V:S=0
          LOOP     T:TIME...(0,$11),SCR7
          CALC     V:I= $11
          IF       V:I<GT>V:J,SCR7
          IF       V:I<LT>V:T,SCR7
          CALC     V:TI=V:I-V:T+1
          CALC     V:S=V:S+T:OSUB...(N:CONT,N:SG)*
                    T:FIV....(FR,V:TI)
SCR7      CONTINUE
          COL      N:COL,
                     BPS&N:SG&OS&N:CONT&N:J<L>=V:S
SCR6      CONTINUE
SCR5      CONTINUE
          GOTO     SCR1
SCR2      NAME     N:S1=CO..S&N:S&.
          LOOP     T:N:S1(0,=2),SCR3
          NAME     N:SG= =2
          NAME     N:COL=PC&N:SG&N:S&S&N:CONT&.&N:T
```

```
         COL      N:COL,
                  N:OBJ<N>=V:RSLT*T:N:S1(N:CCNT,N:SG)
*                           *OLD SCRAP BALANCE
         LOOP     T:TIME...(0,=10)<HIEQ>N:T,SCR4
         NAME     N:J= =10
         COL      N:COL,
                      BPS&N:SG&US&N:CONT&N:J<L>=1
SCR4     CONTINUE
*                        *SCRAP TRANSPORTATION BALANCE
         COL      N:COL,
                  BTP@@S&N:CONT&N:T<L>=-1
SCR3     CONTINUE
SCR1     CONTINUE
         EJECT
*                             *SCRAP TRANSPORT TO END MARKETS
*                             *ONLY THOSE LINKS ALLOWED
         LOOP     T:S..2AF.(=1,1)<EQ>1,SCT1
         NAME     N:PR= =1
         NAME     N:S=S..&N:CCNT&...
         LOOP     T:N:S(=2,0),SCT2
         NAME     N:MK= =2
         NAME     N:COL=TP&N:PR&@S&N:CONT&N:MK&N:T
         COL      N:COL,
                  N:OBJ<N>=T:CT..S@.(1,N:PR)*V:RSLT
*                             *BALANCE & SUPPLY CONSTRAINTS
         COL      N:COL,
                  BTP@@S&N:CONT&N:T<L>=1,
                  BD&N:MK&N:PR&A.&N:CONT&N:T<L>=-1
SCT2     CONTINUE
SCT1     CONTINUE
         EJECT
*                             *DEMAND OF THE DIFFERENT PRODUCTS
```

```
        NAME    N:TBL=D@..&N:CONT&..
*                               *LOOP ON ALLOWABLE LINKS
        LOOP    T:N:TBL(=1,=2)<EQ>1,DMD1
        NAME    N:OR= =1
        NAME    N:MK= =2
        NAME    N:COL=D&N:MK&.&N:OR&.&N:CONT&N:T
        IF      N:OR<MT>A.,DMD2
        COL     N:COL,
                BD&N:MK&.&N:OR&N:CONT&N:T<L>=1
        GOTO    DMD1
DMD2    NAME    N:S=S..&N:CONT&...
        LOOP    T:N:S(0,=3),DMD3
        NAME    N:PR= =3
        NAME    N:COL=D&N:MK&N:PR&N:OR&.&N:CONT&N:T
        COL     N:COL,
                BD&N:MK&N:PR&N:OR&N:CONT&N:T<L>=1
        IF      N:MK<MT>F,DMD3
        COL     N:COL,
                BPS.NS&N:CONT&N:T<L>=-T:SA.2F..(N,1)
DMD3    CONTINUE
*                               *OLD SCRAP BALANCE
        LOOP    T:OSUB...(0,=10),DMD6
        NAME    N:SG= =10
        LOOP    T:TIME...(0,$10),DMD4
        CALC    V:J= $10
        IF      V:J<LT>V:T,DMD4
        NAME    N:J=T:TIME...(0,V:J)
        CALC    V:S=0
        LOOP    T:TIME...(0,$11),DMD5
        CALC    V:I= $11
        IF      V:I<GT>V:J,DMD5
        IF      V:I<LT>V:T,DMD5
```

```
         CALC      V:TI=V:I-V:T+1
         CALC      V:S=V:S+T:CSUB...(N:CONT,N:SG)*
                   T:FIV....(FR,V:TI)
DMD5     CONTINUE
         COL       N:COL,
                   BPS&N:SG&GS&N:CONT&N:J<L>=-V:S
DMD4     CONTINUE
DMD6     CONTINUE
DMD1     CONTINUE
         EJECT
*********************************************************************************
*                            *TIME DEPENDENT RHS
*                            *ORE FORMULATION
         NAME      N:S=M..R&N:CONT&..
         LOOP      T:N:S(=1,=2)<EQ>1,RHSOR1
         NAME      N:R= =1
         NAME      N:OR= =2
         NAME      N:S1=RM.&N:CONT&...
         RHS       N:RHS,
                   KM..&N:OR&N:R&N:T<L>=T:N:S1(N:R,N:OP)*1000000
RHSOR1   CONTINUE
*                            *BAUXITE INVENTORY
         LOOP      T:MB@.I..(=1,Y)<EQ>1,RHSYB1
         NAME      N:R= =1
         RHS       N:RHS,
                   BY..BL&N:R&N:T<E>=0
RHSYB1   CONTINUE
*                            *IMPORTED BAUXITE UPPER BOUNDS
         NAME      N:S=MB@&N:T&IUB
         LOOP      T:N:S(=1,=2),RHSIBU1
         NAME      N:R= =1
         NAME      N:SG= =2
```

```
         RHS     N:RHS,
                 KTI&N:SG&BL&N:R&N:T<L>=T:N:S(N:R,N:SG)*1000
RHSIBU1  CONTINUE
*                          *IMPORTED BAUXITE LOWER BOUNDS
         NAME    N:S=MB@&N:T&ILB
         LOOP    T:N:S(=1,0),RHSIBL1
         NAME    N:R= =1
         RHS     N:RHS,
                 BTI@BL&N:R&N:T<L>=-T:N:S(N:R,@)*1000
RHSIBL1  CONTINUE
         EJECT
*                          *REFINERY CAPACITY & PRODUCTION
         NAME    N:S=R..&N:CONT&RP.
         LOOP    T:N:S(=1,=2)<EQ>1,RHSRK1
         NAME    N:R= =1
         NAME    N:TYP= =2
         NAME    N:S1=RR.&N:CONT&...
         RHS     N:RHS,
                 KR&N:TYP&.&L.&N:R&N:T<L>= T:N:S1(N:R,N:TYP)*1000000
         NAME    N:S2=M..&N:CONT&OT.
         LOOP    T:N:S2(=3,N:TYP)<EQ>1,RHSRK2
         NAME    N:OR= =3
         RHS     N:RHS,
                 PR&N:TYP&.&N:OK&N:R&N:T<L>=0
RHSRK2   CONTINUE
*                          *TRANSPORTATION OF DOMESTIC ALUMINA
         RHS     N:RHS,
                 BTP.L.&N:R&N:T<L>=0
RHSRK1   CONTINUE
*                          *IMPORTS OF ALUMINA
         NAME    N:S1=RL.&N:T&IUB
         NAME    N:S2=RL.&N:T&ILB
```

```
         LOOP      T:N:S1(=1,0),RHSLI1
         NAME      N:R= =1
         RHS       N:RHS,
                   BTI@L.&N:R&N:T<L>=-T:N:S2(N:R,@)*1000
         LOOP      T:N:S1(0,=2),RHSLI2
         NAME      N:SG= =2
         RHS       N:RHS,
                   KTI&N:SG@L.&N:R&N:T<L>=T:N:S1(N:R,N:SG)*1000
RHSLI2   CONTINUE
RHSLI1   CONTINUE
*                              *INVENTORY BALANCE
         RHS       N:RHS,
                   BY..L..&N:T<E>=0
         EJECT
*                              *SMELTERS CAPACITY & PRODUCTION
         NAME      N:S=S..&N:CONT&RT.
         LOOP      T:N:S(=1,0),RHSSK10
         NAME      N:R= =1
         LOOP      T:N:S(N:R,=2)<EQ>1,RHSSK1
         NAME      N:TYP= =2
         RHS       N:RHS,
                   KS&N:TYP&.A.&N:R&N:T<L>=0,
                   PS@.L.&N:R&N:T<L>=0
*                              *IMPROVEMENT BALANCE
         IF        N:TYP<MT>6,RHSSK2
         IF        V:T<EQ>V:TMX,RHSSK2
         RHS       N:RHS,
                   BV&N:TYP&@A.&N:R&N:T<E>=0
RHSSK2   CONTINUE
RHSSK1   CONTINUE
*                              *ELECTRIC POWER CAPACITY CONSTRAINTS
         NAME      N:S1=RE.&N:CONT&...
```

```
         RHS      N:RHS,
                  KE..A.&N:R&N:T<L>=T:N:S1(N:R,E),
                  BE..A.&N:R&N:T<L>=0
*                                   *PRIMARY ALUMINUM TRANSPORTATION BALANCE
         RHS      N:RHS,
                  BTP@A.&N:R&N:T<L>=0
RHSSK10  CONTINUE
         EJECT
*                                   *NEW  SCRAP PRODUCTION BALANCE
         RHS      N:RHS,
                  BPS.NS&N:CONT&N:T<L>=0
*                                   *IMPORTED ALUMINUM BLUNDS
         NAME     N:S=SA.&N:T&IUB
         RHS      N:RHS,
                  KTI@A.F&N:T<L>=T:N:S(F,1)*1000
         NAME     N:S=SA.&N:T&ILB
         RHS      N:RHS,
                  BTI@A.F&N:T<L>=-T:N:S(F,1)*1000
*                                   *INVENTORY BALANCE
         RHS      N:RHS,
                  BY..A..&N:T<E>=0
*                                   *SCRAP TRANSPORTATION BALANCE
         RHS      N:RHS,
                  BTP@@S&N:CONT&N:T<L>=0
         EJECT
*                                   *DEMANDS
         NAME     N:S=D@..&N:CONT&..
         LOOP     T:N:S(=1,=2)<EQ>1,RHSDM1
         NAME     N:OF=  =1
         NAME     N:MK=  =2
         IF       N:OR<MT>A.,RHSDM2
         RHS      N:RHS,
```

```
                 BD&N:MK&.&N:OR&N:CONT&N:T<L>=0
         GOTO    RHSDM1
RHSDM2   NAME    N:S1=S..&N:CONT&...
         LOOP    T:N:S1(0,=3),RHSDM3
         NAME    N:PR= =3
         RHS     N:RHS,
                 BD&N:MK&N:PR&N:OR&N:CONT&N:T<L>=0
RHSDM3   CONTINUE
RHSDM1   CONTINUE
*                             *OLD SCRAP
         LOOP    T:OSUB...(0,=10),ZZ1
         NAME    N:SG= =10
         CALC    V:S=0
         LOOP    T:TIME...(0,$11),ZZ3
         CALC    V:I= $11
         IF      V:I<GT>V:T,ZZ3
         CALC    V:S=V:S+T:CSUB...(N:CONT,N:SG)*
                 T:ALFA...(N:CONT,V:I)
ZZ3      CONTINUE
         RHS     N:RHS,
                 BPS&N:SG&OS&N:CONT&N:T<L>=V:S*1000
ZZ1      CONTINUE
*                             *FINANCIAL CONSTRAINTS
         LOOP    T:RFI@...(=10,0),FI1
         NAME    N:INV= =10
         RHS     N:RHS,
                 FI.&N:INV&@@.&N:T<L>= T:RFI@...(N:INV,N:T)*
                 1000000000
FI1      CONTINUE
         EJECT
*                             *BOUNDS
************************************************************************
```

```
*                                *BOUNDS ON INVENTORY
          LOOP     T:Y@@....(0,=2),BD2
          NAME     N:OR= =2
          CALC     V:INF=INF
          IF       N:T<NM>1,BD1
          CALC     V:INF=T:Y@@LBD.(N:T,N:OR)*1000
BD1       BOUND    N:BOUNDS,
                   Y..&N:OR&Y&N:T=<T:Y@@LBD.(N:T,N:OR)*1000,
                   V:INF>
BD2       CONTINUE
*                                *BOUNDS ON DEMAND
          NAME     N:S1=BD&N:T&N:CONT&DEM
          NAME     N:S2=D@..&N:CONT&..
          LOOP     T:N:S2(=1,=2)<EQ>1,BD4
          NAME     N:OR= =1
          NAME     N:MK= =2
          IF       N:OR<MT>A.,BD10
          BOUND    N:BOUNDS,
                   D&N:MK&.&N:OR&.&N:CONT&N:T= <T:N:S1(N:OR,N:MK)*
                   1000000,INF>
          GOTO     BD4
BD10      NAME     N:S3=S..&N:CONT&...
          LOOP     T:N:S3(0,=3),BD40
          NAME     N:PR= =3
          BOUND    N:BOUNDS,
                   D&N:MK&N:PR&N:OR&.&N:CONT&N:T=<T:N:S1(N:OR,N:MK)*
                   1000000*0.01*T:N:S3(N:MK,N:PR),INF>
BD40      CONTINUE
BD4       CONTINUE
*                                *BOUND ON EXISTING SMELTERS
          NAME     N:S1=S..&N:CONT&RT.
          LOOP     T:N:S1(=1,=2)<EQ>1,BD5
```

```
        NAME    N:R= =1
        NAME    N:TYP= =2
        NAME    N:S2=RS.&N:CONT&...
        NAME    N:TM=1
        IF      N:TYP<NM>6,BD11
        NAME    N:TM=N:T
BD11    BOUND   N:BOUNDS,
                S&N:TYPR0A.&N:RS.&N:TM=<T:N:S2(N:R,N:TYP)*1000000,
                T:N:S2(N:R,N:TYP)*1000000>
BD5     CONTINUE
        NAME    N:S1=BD.&N:CONT&PE.
*                               *PURCHASED POWER BOUNDS
        LOOP    T:N:S1(=1,0),BD6
        NAME    N:R= =1
        BOUND   N:BOUNDS,
                PEPA.&N:R&.&N:T=<T:N:S1(N:R,N:T)*1000000>
BD6     CONTINUE
        EJECT
*****************************************************************************
********************** END TO THE TALLIES **************************************
*****************************************************************************
        TALLY   V:CONT,V:CTMX,CONT
        TALLY   V:T,V:TMX,TIME
*****************************************************************************
        EJECT
*
*                               *TIME INDEPENDENT ELEMENTS
        CALC    V:CONT=1
CONT1   NAME    N:CONT=Z:CCNT...(V:CUNT,0)
*                               *INVENTORY BALANCE VECTOR
        NAME    N:T=T:TIME...(0,V:TMX)
        LOOP    T:Y@@....(0,=1),XX1
```

```
           NAME     N:OR= =1
           NAME     N:COL=Y..&N:OR&Y@
           COL      N:COL,
                    BY..&N:OR&N:T<E>=1
           BOUND    N:BOUNDS,
                    Y..&N:OR&Y@=<T:Y@@LBD.(@,N:OR)*1000,INF>
XX1        CONTINUE
*                                *RHS ELEMENTS
*                                *EXISTING RESERVES
           NAME     N:S=REV&N:CONT&...
           RHS      N:RHS,
                    BR..BHA@<L>=T:N:S(A,BH)*1000000
*                                *MINE INCREMENTS
           NAME     N:S=M..R&N:CONT&..
           NAME     N:S1=MI.&N:CONT&0..
           LOOP     T:N:S(=1,=2)<EQ>1,YY1
           NAME     N:R= =1
           NAME     N:OR= =2
           LOOP     T:N:S1(0,=3),YY2
           NAME     N:INC= =3
           IF       N:OR<NM>BH,YY3
           RHS      N:RHS,
                    KM&N:INC&0&N:OR&N:R&@<L>=T:N:S1(N:OR,N:INC)*
                    T:RM.U...(N:R,N:OR)*1000000
           GOTO     YY2
YY3        CONTINUE
           LOOP     T:TIME...(0,=4),YY5
           NAME     N:T= =4
           RHS      N:RHS,
                    KM&N:INC&N:T&N:OR&N:R&@<L>=0
YY5        CONTINUE
YY2        CONTINUE
```

```
YY1       CONTINUE
*                             *SMELTER  INCREMENTS
          NAME     N:S=SI.&N:CONT&B..
          LOOP     T:N:S(=1,0),YY6
          NAME     N:R= =1
          CALC     V:CAP=0
          CALC     V:CAP=V:CAP+T:RS.U...(N:R,$10)
          RHS      N:RHS,KS@0A.&N:R&@<L>=
                   T:N:S(N:R,I)*V:CAP*1000000
          LOOP     T:TIME...(0,-2),YY7
          NAME     N:VT= =2
          RHS      N:RHS,
                   KS@&N:VT&A.&N:R&@<L>=0
YY7       CONTINUE
YY6       CONTINUE
          TALLY    V:CONT,V:CTMX,CONT1
          EJECT
*                             *COMMITTED SMELTER EXPANSIONS
          BOUND    N:BOUNDS,
                   X40A.1.1=<10000,INF>,
                   X40A.1.2=<93000,INF>,
                   X40A.2.1=<35000,INF>,
                   X40A.2.2=<89000,INF>,
                   X40A.3.2=<20500,INF>
          EJECT
*                             *DEBUGGING STATEMENTS
          IF       N:DEBUG<NM>YES,ENF
          GOTO     ENF
          LOOP     T:TIME...(0,=1)<LOEQ>N:TMX,ZZZZ
          NAME     N:T= **** ***&=1
          MATTAB   T:DEBUG=ALLI,ALLC(N:T)
          SPACE    NP
```

```
          DISPLAY TABLE=T:DEBUG
ZZZZ      CONTINUE
ENF       ENFILE  N:FILE(N:PROB)=MATRIX
          EXIT
          EJECT
*                              *GENERAL SUBROUTINES
*INVESTMENT DISCOUNT FACTORS
DFI       CONTINUE
          NAME    N:DFI.R=N:#4
          CALC    V:DFI.T=T:TIME...(STP,N:#3)+0.5
          NAME    N:DFI.BP=BP&N:#6&.&N:#5
          CALC    V:DFI.DF=V:#2
          CALC    V:TMAX=T:TIME...(STP,N:#7)+T:TIME...(YTP,N:#7)
          CALC    V:SV.BP=(1-(V:TMAX-V:DFI.T)/20.)*
                  FV:DEXP(-V:DFI.DF*V:TMAX)
          CALC    V:#1=0
          LOOP    T:N:DFI.BP(N:DFI.R,$101),DFI.LP
          CALC    V:#1=V:#1+T:N:DFI.BP(N:DFI.R,$101)*
                  FV:DEXP(-V:DFI.DF*V:DFI.T)
          CALC    V:DFI.T=V:DFI.T+1.0
DFI.LP    CONTINUE
          CALC    V:#1=V:#1-V:SV.BP
          NEXT
*==================================================================
*OPERATIONAL DISCOUNT FACTOR
DFO       CONTINUE
          CALC    V:DF.X1=0
          DIMEN   V:DF.T=T:TIME...(0,N:#3),DF.BADD
          IF      V:DF.T<EQ>1,DF.OX1
          CALC    V:DF.X=1
          CALC    V:DF.T=V:DF.T-1
DF.LPO    CALC    V:DF.X1=V:DF.X1+T:TIME...(YTP,V:DF.X)
```

```
         TALLY    V:DF.X,V:DF.T,DF.LPO
         CALC     V:DF.X1=-V:DF.X1*V:#2
DF.OX1   CALC     V:DF.X2=V:DF.X1-(T:TIME...(YTP,N:#3)*V:#2)
         CALC     V:#1=-(FV:DEXP(V:DF.X2)-FV:DEXP(V:DF.X1))/V:#2
         NEXT
DF.BADO  NOTE     INVALID TIME PERIOD PASSED TO DFO SUBROUTINE
         DISPLAY NAME=N:#3
         EXIT
         EJECT
*+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
*AVAILABLE CAPACITY SUBROUTINE
AVL      CONTINUE
         NAME     N:AV.LD=DT&N:#6&.&N:#5
         CALC     V:AV.TS=T:TIME...(STP,N:#3)
         CALC     V:AV.TE=V:AV.TS+T:TIME...(YTP,N:#3)
         CALC     V:AV.TB=T:TIME...(STP,N:#2)+T:N:AV.LD(N:#4,B)
         CALC     V:AV.TF=V:AV.TB+T:N:AV.LD(N:#4,F)
         CALC     V:#1=0
         IF       V:AV.TB<GE>V:AV.TE,AV.ZERO
         IF       V:AV.TF<LE>V:AV.TS,AV.ONE
         CALC     V:AV.T1=0
         IF       V:AV.TB<GE>V:AV.TS,AV.1
         CALC     V:AV.T1=V:AV.TS-V:AV.TB
AV.1     CALC     V:AV.T2=V:AV.TE-V:AV.TB
         IF       V:AV.TF<GE>V:AV.TE,AV.2
         CALC     V:AV.T2=V:AV.TF-V:AV.TB
AV.2     CALC     V:#1=(1.0/(2.0*(V:AV.TF-V:AV.TB)))*
                  ((V:AV.T2*V:AV.T2)-(V:AV.T1*V:AV.T1))
         IF       V:AV.TF<GE>V:AV.TE,AV.DONE
         CALC     V:#1=V:#1+(V:AV.TE-V:AV.TF)
AV.DONE  CALC     V:#1=V:#1/(V:AV.TE-V:AV.TS)
         NEXT
```

```
AV.ONE    CALC     V:#1=1
AV.ZERO   NEXT
ENDATA
```

D Data tables

```
NAME          ALUMTAB
         TABLE   T:TIME...= 1,2,3,4,5,6,7,8,9
                 YTP= 2,2,2,2,2,2,2,2,2
                 STP= 0,2,4,6,8,10,12,14,16
         TABLE   Z:CONT...
                 U
         TABLE   T:Y@@....= BLJ,BLN,BLG,L..,A..
                 U= 9922,5936,0,0,2000
         TABLE   T:Y@@LBD.= BLJ,BLN,BLG,L..,A..
                 1= 14300,8500,0,0,2200
                 2= 12700,8500,0,0,2300
                 3= 10000,6200,0,0,2400
                 4=  8000,4500,0,0,2500
                 5=  6000,3000,0,0,2600
                 6=  4000,2000,0,0,2700
                 7=  2700,1600,0,0,2800
                 8=  2700,1600,0,0,2900
                 9=  2700,1600,0,0,3050
                 @=  2700,1600,0,0,3150
         TABLE   T:M..PU..= BH,N.,C.
                 A= 1,0,0
                 N= 0,1,0
                 G= 0,0,1
         TABLE   T:MI.UD..= 1
                 BH= .15
                 N.= .15
                 C.= .15
         TABLE   T:MB@.I..= Q,E,Y
                 J= 0,1,1
                 N= 1,1,1
                 C= 0,1,0
                 Y= 1,1,0
```

```
                G= 0,1,1
                L= 0,1,0
                B= 0,1,0
TABLE           T:MB@.T..= H,L
                A= 1,0
                J= 0,1
                N= 0,1
                C= 0,1
                Y= 0,1
                G= 0,1
                L= 0,1
                B= 0,1
TABLE           T:MB@1IUB= 1,2
                J= 8700,6000
                N= 3200,2800
                C= 2100,0
                Y= 700,0
                G= 3200,4800
                L= 400,5000
                B= 0,0
TABLE           T:MB@2IUB= 1,2
                J= 8700,6000
                N= 3300,2800
                C= 2100,0
                Y= 750,0
                G= 3300,4800
                L= 400,5000
                B= 600,400
TABLE           T:MB@3IUB= 1,2
                J= 8700,6000
                N= 3500,2800
                C= 2100,0
```

```
          Y= 820,0
          G= 3400,6600
          L= 500,5000
          B= 1000,1000
TABLE     T:MB@4IUB= 1,2
          J= 9000,6000
          N= 3600,2800
          C= 2100,0
          Y= 820,0
          G= 3500,6600
          L= 500,5000
          B= 1200,1000
TABLE     T:MB@5IUB= 1,2
          J= 9000,6000
          N= 3600,2800
          C= 2100,0
          Y= 820,0
          G= 3800,7500
          L= 500,5000
          B= 1500,1500
TABLE     T:MB@6IUB= 1,2
          J= 9000,6000
          N= 3600,2800
          C= 2100,0
          Y= 820,0
          G= 3800,7500
          L= 500,5000
          B= 1500,1500
TABLE     T:MB@7IUB= 1,2
          J= 9000,5000
          N= 3800,2800
          C= 2100,0
```

```
         Y= 820,0
         G= 3800,7500
         L= 500,5000
         B= 2000,1500
TABLE    T:MB@8IUB= 1,2
         J= 9000,6000
         N= 3800,2800
         C= 2100,0
         Y= 820,0
         G= 3800,7500
         L= 500,5000
         B= 2000,1500
TABLE    T:MB@9IUB= 1,2
         J= 9000,6000
         N= 4000,3000
         C= 2100,0
         Y= 800,0
         G= 4000,8000
         L= 500,6000
         B= 2000,2000
TABLE    T:MB@1ILB= @
         J= 7200
         N= 2800
         C= 2000
         Y= 540
         G= 1400
         L= 0
         B= 0
TABLE    T:MB@2ILB= @
         J= 7200
         N= 2800
         C= 2000
```

```
          Y= 500
          G= 1400
          L= 0
          B= 0
TABLE     T:MB@3ILB= @
          J= 6000
          N= 2400
          C= 1800
          Y= 400
          G= 1400
          L= 0
          B= 0
TABLE     T:MB@4ILB= @
          J= 5000
          N= 2000
          C= 1600
          Y= 300
          G= 1400
          L= 0
          B= 0
TABLE     T:MB@5ILB= @
          J= 5000
          N= 2000
          C= 1600
          Y= 300
          G= 1400
          L= 0
          B= 0
TABLE     T:MB@6ILB= @
          J= 0
          N= 0
          C= 0
```

```
          Y=  0
          G=  0
          L=  0
          B=  0
TABLE     T:MB@7ILB=  @
          J=  0
          N=  0
          C=  0
          Y=  0
          G=  0
          L=  0
          B=  0
TABLE     T:MB@8ILB=  @
          J=  0
          N=  0
          C=  0
          Y=  0
          G=  0
          L=  0
          B=  0
TABLE     T:MB@9ILB=  @
          J=  0
          N=  0
          C=  0
          Y=  0
          G=  0
          L=  0
          B=  0
TABLE     T:YL..BI.=  1
          J=  .45
          N=  .44
          C=  .47
```

```
          Y= .44
          G= .50
          L= .45
          B= .46
TABLE     T:YL..@P.= 1
          BH= .36
          N.= .20
          C.= .20
TABLE     T:M..URR.= E,W,G
          A= 1,0,0
          W= 0,1,0
          G= 0,0,1
TABLE     T:M..UOT.= H,L,N,C
          BH= 1,0,0,0
          BL= 1,1,0,0
          N.= 0,0.1,0
          C.= 0,0,0,1
TABLE     T:R..URP.= H,L,N,C
          E= 1,1,0,0
          W= 0,0,1,0
          G= 0,0,0,1
TABLE     T:R..URT.= 1,2,3,Q,F,Y
          E= 1,1,1,1,1,1
          W= 1,1,1,0,0,0
          G= 1,1,1,0,0,0
TABLE     T:RL.1IUB= 1,2
          L= 3200,3000
          J= 1500,1500
          N= 1490,0
          G= 200,0
          Y= 100,200
TABLE     T:RL.2IUB= 1,2
```

```
               L= 3500,3000
               J= 2000,1500
               N= 1490,0
               G= 400,0
               Y= 100,200
TABLE          T:RL.3IUB= 1,2
               L= 3500,3000
               J= 2000,1500
               N= 1490,0
               G= 600,0
               Y= 100,200
TABLE          T:RL.4IUB= 1,2
               L= 3800,3000
               J= 2200,1600
               N= 1490,0
               G= 1000,0
               Y= 100,200
TABLE          T:RL.5IUB= 1,2
               L= 3800,3000
               J= 2500,1600
               N= 1500,0
               G= 1000,0
               Y= 100,200
TABLE          T:RL.6IUB= 1,2
               L= 4100,3500
               J= 2500,1600
               N= 1500,0
               G= 1400,0
               Y= 100,200
TABLE          T:RL.7IUB= 1,2
               L= 4500,3500
               J= 2500,1600
```

```
          N= 1500,0
          G= 1400,0
          Y= 100,200
TABLE     T:RL.8IUB= 1,2
          L= 5000,4000
          J= 2500,1800
          N= 1600,0
          G= 1800,0
          Y= 100,200
TABLE     T:RL.9IUB= 1,2
          L= 5000,4000
          J= 2500,1800
          N= 1600,0
          G= 1800,0
          Y= 100,200
TABLE     T:RL.1ILB= a
          L= 1500
          J= 750
          N= 400
          G= 0
          Y= 0
TABLE     T:RL.2ILB= a
          L= 1500
          J= 750
          N= 400
          G= 100
          Y= 0
TABLE     T:RL.3ILB= a
          L= 1500
          J= 700
          N= 400
          G= 100
```

```
            Y= 0
TABLE       T:RL.4ILB= @
            L= 1500
            J= 700
            N= 350
            G= 100
            Y= 0
TABLE       T:RL.5ILB= @
            L= 0
            J= 0
            N= 0
            G= 0
            Y= 0
TABLE       T:RL.6ILB= @
            L= 0
            J= 0
            N= 0
            G= 0
            Y= 0
TABLE       T:RL.7ILB= @
            L= 0
            J= 0
            N= 0
            G= 0
            Y= 0
TABLE       T:RL.8ILB= @
            L= 0
            J= 0
            N= 0
            G= 0
            Y= 0
TABLE       T:RL.9ILB= @
```

```
                 L= 0
                 J= 0
                 N= 0
                 G= 0
                 Y= 0
TABLE            T:RH..BL.= 1
                 1= .39
TABLE            T:S..URT.= 1,2,3,4,5,6
                 1= 1,1,1,1,1,1
                 2= 1,1,1,1,1,1
                 3= 1,1,1,1,1,1
TABLE            T:SI.UB..= I
                 1= .50
                 2= .50
                 3= .50
TABLE            T:YLP....= 1
                 1= 0.52
TABLE            T:IMPROVE= 1,2,3,4,5
                 1= 0,0,0,1,1
                 2= 0,0,0,1,1
                 3= 0,0,0,1,1
                 4= 0,0,0,0,1
                 5= 0,0,0,0,0
TABLE            T:ROELS..= 1,2,3,4,5,6
                 Y= 9,8,7,6,5,4.5
TABLE            T:S..U...= I,P,T,C,V
                 B= 7.30,52.17,39.39,1.40,.04
                 T= 46.76,32.08,17.63,0.48,3.05
                 D= 31.07,46.84,20.20,1.59,0.30
                 E= 15.27,15.44,14.86,53.93,0.50
                 M= 35.73,32.07,27.97,2.58,1.65
                 C= 1.04,98.80,0.0,0.16,0.0
```

```
          O= 46.57,13.28,13.35,26.41,0.59
          F= 44,41.80,9.01,3.76,1.43
TABLE     T:SA.2F..= 1
          N= .18
          O= .06
TABLE     T:OSUB...= 1,2,3
          U= .15,.06,.04
TABLE     T:FIV....=0,1,2,3,4,5,6,7,8,9
          FR= .015,.01,.01,.015,.05,.09,.115,.095,.035,.01
TABLE     T:S..2AF.= 1
          I= 1
          P= 0
          T= 0
          C= 0
          V= 1
TABLE     T:SA.1IUB= 1
          F= 2200
TABLE     T:SA.2IUB= 1
          F= 2360
TABLE     T:SA.3IUB= 1
          F= 2580
TABLE     T:SA.4IUB= 1
          F= 2760
TABLE     T:SA.5IUB= 1
          F= 2950
TABLE     T:SA.6IUB= 1
          F= 3140
TABLE     T:SA.7IUB= 1
          F= 3330
TABLE     T:SA.8IUB= 1
          F= 3520
TABLE     T:SA.9IUB= 1
```

```
         F= 3700
TABLE    T:SA.1ILB= 1
         F= 560
TABLE    T:SA.2ILB= 1
         F= 560
TABLE    T:SA.3ILB= 1
         F= 560
TABLE    T:SA.4ILB= 1
         F= 560
TABLE    T:SA.5ILB= 1
         F= 560
TABLE    T:SA.6ILB= 1
         F= 560
TABLE    T:SA.7ILB= 1
         F= 560
TABLE    T:SA.8ILB= 1
         F= 560
TABLE    T:SA.9ILB= 1
         F= 560
TABLE    T:D@..U..= Q,F,B,T,D,E,M,C,O
         BL= 1,0,0,0,0,0,0,0,0
         L.= 1,1,0,0,0,0,0,0,0
         A.= 0,1,1,1,1,1,1,1,1
TABLE    T:BPU.M..= 1,2,3,4
         BH= 0,0,0,0
         N.= .4,.3,.2,.1
         C.= .4,.3,.2,.1
TABLE    T:BPU.MI.= 1,2
         BH= .8,.2
         N.= .8,.2
         C.= .8,.2
TABLE    T:BPU.R..= 1,2,3,4
```

```
            H= .65,.25,.05,.05
            L= .65,.25,.05,.05
            N= .70,.20,.08,.02
            C= .70,.15,.10,.05
TABLE       T:BPU.S..= 1,2,3
            1= .7,.2,.1
            2= .7,.2,.1
            3= .7,.2,.1
            4= .7,.2,.1
            5= .7,.2,.1
            6= .7,.2,.1
TABLE       T:BPU.SI.= 1,2
            1= .8,.2
            2= .8,.2
            3= .8,.2
            4= .8,.2
            5= .8,.2
TABLE       T:BPU.E..= 1,2,3
            1= .80,.15,.05
            2= .80,.15,.05
            3= .80,.15,.05
TABLE       T:BTU.M..= B,F
            BH= 3,1
            N.= 3,1
            C.= 3,1
TABLE       T:BTU.MI.= B,F
            BH= 1.5,.5
            N.= 1.5,.5
            C.= 1.5,.5
TABLE       T:BTU.R..= B,F
            H= 2,.8
            L= 2,.8
```

```
              N= 2,1
              C= 2,1
        TABLE T:BTU.S..= B,F
              1= 2,1
              2= 2,1
              3= 2,1
              4= 2,1
              5= 2,1
              6= 2,1
        TABLE T:BTU.SI.= B,F
              1= 1.5,.5
              2= 1.5,.5
              3= 1.5,.5
              4= 1.5,.5
              5= 1.5,.5
        TABLE T:BTU.E..= B,F
              1= 5.4,.6
              2= 5.4,.6
              3= 5.4,.6
        TABLE T:CI.UM..= BH,N.,C.
              A= 0,0,0
              W= 0,90,0
              G= 0,0,90
        TABLE T:CI.UM1.=BH,N.,C.
              A= 27,0,0
              W= 0,27,0
              G= 0,0,27
*                 *THIS TABLE IS NOT USED FOR THE TIME BEING
        TABLE T:CI.UM2.= BH,N.,C.
              A= 42,0,0
              W= 0,42,0
              G= 0,0,42
```

```
TABLE     T:CO.UM..= BH,N.,C.
          A= 14,0,0
          W= 0,7.5,0
          G= 0,0,2.5
TABLE     T:CT1UB..= 1,2
          J=28.5,32
          N=30.2,32
          C=30.8,32
          Y=34.3,36
          G= 26.5,32
          L= 38,42
          B= 0,0
TABLE     T:CT2UB..=1,2
          J=28.5,32
          N= 30.2,32
          C=30.8,32
          Y=34.3,36
          G=26.5,32
          L= 38,42
          B= 20,30
TABLE     T:CT3UB..= 1,2
          J= 28.5,32
          N= 30.2,32
          C= 30.8,32
          Y= 34.3,36
          G= 26.5,32
          L= 38.0,42
          B= 22,30
TABLE     T:CT4UB..= 1,2
          J= 32,36
          N=32,36
          C=32,36
```

```
        Y=35,40
        G=29,36
        L=40,45
        B=25,32
TABLE   T:CT5UB..= 1,2
        J= 32,36
        N= 32,36
        C= 32,36
        Y= 32,36
        G= 32,36
        L= 40,45
        B= 30,36
TABLE   T:CT6UB..= 1,2
        J= 32,36
        N= 32,36
        C= 32,36
        Y= 32,36
        G= 32,36
        L= 40,45
        B= 30,36
TABLE   T:CT7UB..= 1,2
        J= 32,36
        N= 32,0
        C= 32,36
        Y= 32,36
        G= 32,36
        L= 40,45
        B= 30,36
TABLE   T:CT8UB..= 1,2
        J= 32,36
        N= 32,36
        C= 32,36
```

```
        Y= 32,36
        G= 32,36
        L= 40,45
        B= 30,36
TABLE   T:CT9UR..= 1,2
        J= 32,36
        N= 32,36
        C= 32,36
        Y= 32,36
        G= 32,36
        L= 40,45
        B= 30,36
TABLE   T:CI.UR..= H,L,N,C
        E= 450,550,0,0
        W= 0,0,550,0
        G= 0,0,0,770
TABLE   T:CO1UR..= N,C
        W= 180,0
        G= 0,145
TABLE   T:CO2UR..= A,J,N,C,Y,G,L,B
        E= 90,98,90,90,90,90,90,90
TABLE   T:CT1ULI1= 3,2,1
        L= 142,142,138
        J= 130,130,140
        N= 128,128,140
        G= 132,132,142
        Y= 128,128,140
TABLE   T:CT2ULI1= 3,2,1
        L= 142,142,138
        J= 130,130,140
        N= 128,128,140
        G= 132,132,142
```

```
         Y= 128,128,140
TABLE    T:CT3ULI1= 3,2,1
         L= 142,142,138
         J= 130,130,140
         N= 128,128,140
         G= 132,132,142
         Y= 128,128,140
TABLE    T:CT4ULI1= 3,2,1
         L= 142,142,138
         J= 130,130,140
         N= 128,128,140
         G= 132,132,142
         Y= 128,128,140
TABLE    T:CT5ULI1= 3,2,1
         L= 142,142,138
         J= 130,130,140
         N= 128,128,140
         G= 132,132,142
         Y= 128,128,140
TABLE    T:CT6ULI1= 3,2,1
         L= 142,142,138
         J= 130,130,140
         N= 128,128,140
         G= 132,132,142
         Y= 128,128,140
TABLE    T:CT7ULI1= 3,2,1
         L= 142,142,138
         J= 130,130,140
         N= 128,128,140
         G= 132,132,142
         Y= 128,128,140
TABLE    T:CT8ULI1= 3,2,1
```

```
         L= 142,142,138
         J= 130,130,140
         N= 128,128,140
         G= 132,132,142
         Y= 128,128,140
TABLE    T:CT9ULI1= 3,2,1
         L= 142,142,138
         J= 130,130,140
         N= 128,128,140
         G= 132,132,142
         Y= 128,128,140
TABLE    T:CT1ULI2= 3,2,1
         L= 152,152,148
         J= 140,140,150
         N= 138,138,150
         G= 142,142,152
         Y= 138,138,150
TABLE    T:CT2ULI2= 3,2,1
         L= 152,152,148
         J= 140,140,150
         N= 138,138,150
         G= 142,142,152
         Y= 138,138,150
TABLE    T:CT3ULI2= 3,2,1
         L= 152,152,148
         J= 140,140,150
         N= 138,138,150
         G= 142,142,152
         Y= 138,138,150
TABLE    T:CT4ULI2= 3,2,1
         L= 152,152,148
         J= 140,140,150
```

```
          N=  138,138,150
          G=  142,142,152
          Y=  138,138,150
TABLE     T:CT5ULI2=  3,2,1
          L=  152,152,148
          J=  140,140,150
          N=  138,138,150
          G=  142,142,152
          Y=  138,138,150
TABLE     T:CT6ULI2=  3,2,1
          L=  152,152,148
          J=  140,140,150
          N=  138,138,150
          G=  142,142,152
          Y=  138,138,150
TABLE     T:CT7ULI2=  3,2,1
          L=  152,152,148
          J=  140,140,150
          N=  138,138,150
          G=  142,142,152
          Y=  138,138,150
TABLE     T:CT8ULI2=  3,2,1
          L=  152,152,148
          J=  140,140,150
          N=  138,138,150
          G=  142,142,152
          Y=  138,138,150
TABLE     T:CT9ULI2=  3,2,1
          L=  152,152,148
          J=  140,140,150
          N=  138,138,150
          G=  142,142,152
```

```
        Y= 138,138,150
TABLE   T:CT.ULD.= 1,2,3,Q,F,Y
        E= 12,7,4,0,0,0
        W= 8,6,12,0,0,0
        G= 12,7,4,0,0,0
        Y= 12,7,10,0,0,0
TABLE   T:CI.US..= 1,2,3,4,5,6
        1= 1750,1750,1750,1750,1800,1750
        2= 1750,1750,1750,1750,1800,1750
        3= 1750,1750,1750,1750,1800,1750
TABLE   T:CI.USI.= 1,2,3,4,5
        1= 0,0,1100,1000,1250
        2= 0,0,1100,1000,1250
        3= 0,0,1100,1000,1250
TABLE   T:CO.US..= 1,2,3,4,5,6
        1= 320,320,320,288,280,283
        2= 310,310,310,288,280,283
        3= 310,310,310,288,290,283
TABLE   T:CI.UIMP= 14,15,24,25,34,35,45
        1= 1200,1500,1200,1500,500,700,150
        2= 1200,1500,1200,1500,500,700,150
        3= 1200,1500,1200,1500,500,700,150
TABLE   T:CI.UE..= E
        1= 600
        2= 600
        3= 600
TABLE   T:CO1UE..= P,C
        1= 0.01,0.012
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:CO2UE..= P,C
        1= 0.01,0.012
```

```
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:C03UE..= P,C
        1= 0.01,0.012
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:C04UE..= P,C
        1= 0.01,0.012
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:C05UE..= P,C
        1= 0.01,0.012
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:C06UE..= P,C
        1= 0.01,0.012
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:C07UE..= P,C
        1= 0.01,0.012
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:C08UE..= P,C
        1= 0.01,0.012
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:C09UE..= P,C
        1= 0.01,0.012
        2= 0.012,0.012
        3= 0.014,0.012
TABLE   T:CT.UI..= I,P,T,C,V
        F= 1460,1640,1620,1880,1580
```

```
TABLE     T:CT.UD..= I,P,T,C,V
          1= 85,265,245,505,205
          2= 80,260,240,500,200
          3= 80,260,240,500,200
TABLE     T:CTYUD..= I,P,T,C,V
          U= 86,266,246,506,206
TABLE     T:CO..SN.= 1
          U= 700
TABLE     T:CL..SU.= 1,2,3
          U= 800,900,1050
TABLE     T:CT..SO.= I,P,T,C,V
          1= 78,0,0,C,200
TABLE     T:CC.UDD.= G,F,B,T,D,E,M,C,O
          BL= 20,0,0,0,0,0,0,0,0
          L.= 20,30,0,0,0,0,0,0,0
          A.= 0, 30,20,20,20,20,20,20,20
TABLE     T:RM.U...= BH,N.,C.
          A= 3.12,0,0
          W= 0,0,0
          G= 0,0,0
TABLE     T:RR.U...= H,L,N,C
          E= 1.215,6.585,0,0
          W= 0,0,0,0
          G= 0,0,0,0
TABLE     T:RS.U...= 1,2,3,4,5,6
          1= 0.291,0.27,1.176,0,0,0
          2= 0.126,0,1.5195,0,0,0
          3= 0.644,0.18,0.688,0.270,0,0.015
TABLE     T:ALFA...= 1,2,3,4,5,6,7,8,9,10
          U= 2792,3117,3453,3801,3582,2966,1956,996,587,517
TABLE     T:RE.U...= E
          1= 0
```

```
          2= 270
          3= 1920
TABLE     T:REVU...= BH
          A= 44
TABLE     T:BD1UDEM= G,F,B,T,D,E,M,C,O
          BL= 1.30,0,0,0,0,0,0,0,0
          L.= 1.03,1.00,0,0,0,0,0,0,0
          A.= 0,0.60,1.73,1.30,0.65,0.94,0.53,1.65,0.35
TABLE     T:BD2UDEM= G,F,B,T,D,E,M,C,O
          BL= 1.42,0,0,0,0,0,0,0,0
          L.= 1.11,1.00,0,0,0,0,0,0,0
          A.= 0, 0.57,2.05,1.56,0.74,1.15,0.49,1.30,0.41
TABLE     T:BD3UDEM= G,F,B,T,D,E,M,C,O
          BL= 1.55,0,0,0,0,0,0,0,0
          L.= 1.20,1.20,0,0,0,0,0,0,0
          A.= 0,0.55,2.06,1.51,0.80,1.03,0.64,1.51,0.40
TABLE     T:BD4UDEM= G,F,B,T,D,E,M,C,O
          BL= 1.72,0,0,0,0,0,0,0,0
          L.= 1.30,1.10,0,0,0,0,0,0,0
          A.= 0,0.66,2.13,1.64,0.74,1.06,0.66,1.56,0.41
TABLE     T:BD5UDEM= G,F,B,T,D,E,M,C,O
          BL= 1.85,0,0,0,0,0,0,0,0
          L.= 1.41,1.15,0,0,0,0,0,0,0
          A.= 0,0.32,2.13,2.00,0.64,1.09,0.64,2.09,0.45
TABLE     T:BD6UDEM= G,F,B,T,D,E,M,C,O
          BL= 1.98,0,0,0,0,0,0,0,0
          L.= 1.52,1.08,0,0,0,0,0,0,0
          A.= 0,0.84,2.84,2.20,0.84,1.26,0.74,2.10,0.53
TABLE     T:BD7UDEM= G,F,B,T,D,E,M,C,O
          BL= 2.15,0,0,0,0,0,0,0,0
          L.= 1.65,1.06,0,0,0,0,0,0,0
          A.= 0,1.04,3.02,2.44,0.93,1.39,0.81,2.44,0.53
```

```
TABLE     T:BD8UDEM= G,F,B,T,D,E,M,C,O
          BL= 2.33,0,0,0,0,0,0,0,0
          L.= 1.78,1.20,0,0,0,0,0,0,0
          A.= 0,1.16,2.97,2.97,1.16,1.55,1.03,2.58,0.65
TABLE     T:BD9UDEM= G,F,B,T,D,E,M,C,O
          BL= 2.56,0,0,0,0,0,0,0,0
          L.= 1.93,1.20,0,0,0,0,0,0,0
          A.= 0,1.19,3.71,3.27,1.34,1.78,1.19,2.32,0.74
TABLE     T:SD.UPE.= 1,2,3,4,5,6,7,8,9
          1= 27,27,27,27,16,10.4,7.2,7.2,5.7
          2= 22,23.3,24.7,26.2,27.8,29.4,31.2,33,35
          3= 14.25,15.1,16,17,18,19,20.2,21.4,22.7
TABLE     T:RFI@...= 1,2,3,4,5,6,7,8,9
          M= 1,1,1,1,1,1,1,1,1
          R= 1,1,1,1,1,1,1,1,1
          S= 1,1,1,1,1,1,1,1,1
ENDATA
```

E Report generation program

```
NAME          ALUMREP
         EJECT
*                             *SUMMARY OF REPORTS
         NAME    N:CASE=H:CASE
         NAME    N:OBJ=U:XOBJ
         READTAB Z:RPTBL,SYSIN,U:XDATA
         FORM    T:TOTAL=T/1,Z:TIME(STUB)
         NAME    T:TOTAL(1,0)=TOTAL
         CALC    V:DFRATE=I:DFRATE
NEWPAGE  CONTINUE
         PRINT   FPAGE,N:CASE
         NEXT
         EJECT
*                MINE OPENING REPORT
         NEWPAGE
         FORM    Z:TXT=Z:INIT<OR>Z:TIME,Z:TIME
         FILL    Z:TXT=Z:TIME
         FILL    Z:TXT=Z:INIT
         FORM    T:BODY=Z:BDR,Z:TIME(STUB)
         FORM    T:BODY1=Z:BDR,Z:TXT(STUB)
         FORM    T:BODY2=T:BODY,T:BODY
         NAME    N:MASK=M*******
         SOLTAB  T:SOL=N:CASE,ALLC(N:MASK),X
         DIMEN   V:SMX=T:SOL(STUB)
         CALC    V:S=1
MNRP1    NAME    N:R=MASK(T:SOL(V:S,0),0000 0*00)
         NAME    N:T=MASK(T:SOL(V:S,0),0000 000*)
         NAME    N:V=MASK(T:SOL(V:S,0),00*0 0000)
         NAME    N:INC=MASK(T:SOL(V:S,0),0*00 0000)
         CALC    T:BODY2(N:R,N:T)=T:BODY2(N:R,N:T)+T:SOL(V:S,1)/
                 1000000
         IF      N:INC<NM>.,MNRP2
```

```
         CALC     T:BODY(N:R,N:T)=T:BODY(N:R,N:T)+T:SOL(V:S,1)/
                  1000000
         GOTO     MNRP3
MNRP2    CALC     T:BODY1(N:R,N:V)=T:BODY1(N:R,N:V)+T:SOL(V:S,1)/
                  1000000
MNRP3    TALLY    V:S,V:SMX,MNRP1
         PRINT    FMNRP
         SPACE    DS
         HEADING  1,'NEW MINES          (MM TON ORE/YR)'
         SPACE    SS
         TABULATEFTAB,STUB=Z:BDR(A1-A3),
                  HEAD=Z:TIME(A1),
                  BODY=T:BODY
         SPACE    DS
         HEADING  1,'INCREMENTS PER MINE VINTAGE    (MM TON ORE/YR)'
         SPACE    SS
         TABULATEFTAB,STUB=Z:BDR(A1-A3),
                  HEAD=Z:TXT(A1),
                  BODY=T:BODY1
         SPACE    DS
         HEADING  1,'TOTAL NEW MINE CAPACITY        (MM TON ORE/YR)'
         SPACE    SS
         TABULATEFTAB,STUB=Z:BDR(A1-A3),
                  HEAD=Z:TIME(A1),
                  BODY=T:BODY2
         EJECT
*                                 *DOMESTIC MINE PRODUCTION REPORT
         NEWPAGE
         PRINT    FMNPR
         SPACE    DS
         FORM     T:BODY=Z:ORE(STUB),Z:TIME(STUB)
         NAME     N:MASK=PM****.*
```

```
         SOLTAB   T:SLL=N:CASE,ALLC(N:MASK),X
         DIMEN    V:SMX=T:SOL(STUB)
         CALC     V:S=1
MNPR1    NAME     N:OR=MASK(T:SUL(V:S,0),000**000)
         NAME     N:T=MASK(T:SOL(V:S,0),0000 000*)
         CALC     T:BODY(N:OR,N:T)=T:BODY(N:OR,N:T)+T:SOL(V:S,1)/1000000
         TALLY    V:S,V:SMX,MNPR1
         HEADING 1,'PER TYPE OF ORE         (MM TON ORE/YR)'
         SPACE    SS
         TABULATEFTAB,STUB=Z:ORE(A1-A3),
                  HEAD=Z:TIME(A1),
                  BODY=T:BODY
         SPACE    DS
         SPACE    DS
         EJECT
*                            *AVAILABLE MINE CAPACITY
         PRINT    FAVMNK
         SPACE    DS
         FORM     T:BODY=Z:ORE,Z:TIME(STUB)
         NAME     N:MASK1=KM..****
         NAME     N:MASK2=PK.***.*
         SOLTAB   T:SUL1=N:CASE,ALLR(N:MASK1),L
         SOLTAB   T:SOL2=N:CASE,ALLC(N:MASK2),X
         DIMEN    V:SMX=T:SOL1(STUB)
         CALC     V:S=1
AVMNK1   NAME     N:T1=MASK(T:SOL1(V:S,0),0000 000*)
         NAME     N:T2=MASK(T:SOL2(V:S,0),0000 000*)
         NAME     N:TYP1=MASK(T:SOL1(V:S,0),0000**00)
         NAME     N:TYP2=MASK(T:SOL2(V:S,0),000**000)
         CALC     T:BODY(N:TYP1,N:T1)=T:BODY(N:TYP1,N:T1)+T:SOL1(V:S,1)/
                  1000000
         CALC     T:BODY(N:TYP2,N:T2)=T:BODY(N:TYP2,N:T2)+T:SOL2(V:S,1)/
```

```
                 1000000
         TALLY   V:S,V:SMX,AVMNK1
         HEADING 1,'(MM TON/YR)'
         SPACE   SS
         TABULATEFTAB,STUB=Z:ORE(A1-A3),
                 HEAD=Z:TIME(A1),
                 BODY=T:BODY
         EJECT
*                            *IMPORTED BAUXITE
         NEWPAGE
         FORM    T:BODY=Z:BIR<OR>T:TOTAL,Z:TIME(STUB)
         NAME    N:MASK=TI*8****
         SOLTAB  T:SOL=N:CASE,ALLC(N:MASK),X
         DIMEN   V:SMX=T:SOL(STUB)
         CALC    V:S=1
IMPBX1   NAME    N:R=MASK(T:SOL(V:S,0),C0000*00)
         NAME    N:T=MASK(T:SOL(V:S,0),0000 000*)
         CALC    T:BODY(N:R,N:T)=T:BODY(N:R,N:T)+T:SOL(V:S,1)/1000000
         CALC    T:BODY(TOTAL,N:T)=T:BODY(TOTAL,N:T)+T:SOL(V:S,1)/
                 1000000
         TALLY   V:S,V:SMX,IMPBX1
         PRINT   FBXIMP
         SPACE   DS
         HEADING 1,'PER COUNTRY OF ORIGIN     (MM DRY TON/YR)'
         SPACE   SS
         TABULATEFTAB,STUB=Z:BIR(A1-A3),
                 HEAD=Z:TIME(A1),
                 BODY=T:BODY
         SPACE   SS
         PRINT   FTOT,TOTAL,T:BODY(TOTAL,$12)
         EJECT
*                            *BAUXITE INVENTORY
```

```
        NEWPAGE
        PRINT   FBXINV
        SPACE   DS
        FORM    T:BODY=Z:BXINV<OR>T:TOTAL,Z:TIME(STUB)
        NAME    N:MASK=Y..B**Y*
        SOLTAB  T:SOL=N:CASE,ALLC(N:MASK),X
        DIMEN   V:SMX=T:SOL(STUB)
        CALC    V:S=1
BXINV1  NAME    N:R=MASK(T:SOL(V:S,0),0000 0*00)
        NAME    N:T=MASK(T:SOL(V:S,0),0000 000*)
        IF      N:T<MT>0,BXINV2
        CALC    T:BODY(N:R,N:T)=T:BODY(N:R,N:T)+T:SOL(V:S,1)/1000000
        CALC    T:BODY(TOTAL,N:T)=T:BODY(TOTAL,N:T)+T:SOL(V:S,1)/
                1000000
BXINV2  TALLY   V:S,V:SMX,BXINV1
        HEADING 1,'PER ORIGIN OF ORE      (MM DRY TON/YR)'
        HEADING 2,'(AT THE BEGINNING OF THE TIME PERIOD)'
        SPACE   SS
        TABULATEFTAB,STUB=Z:BXINV(A1-A3),
                HEAD=Z:TIME(A1),
                BODY=T:BODY
        SPACE   SS
        PRINT   FTOT,TOTAL,T:BODY(TOTAL,$12)
        EJECT
*                                  *ALUMINA REFINERS
        NEWPAGE
        FORM    T:BODY=Z:REF<OR>T:TOTAL,Z:TIME(STUB)
        NAME    N:MASK=R*.L.*.*
        SOLTAB  T:SOL=N:CASE,ALLC(N:MASK),X
        DIMEN   V:SMX=T:SOL(STUB)
        CALC    V:S=1
PFBG1   NAME    N:TYP=MASK(T:SOL(V:S,0),0*00 0000)
```

```
          NAME     N:T=MASK(T:SOL(V:S,0),0000 000*)
          CALC     T:BODY(N:TYP,N:T)=T:BODY(N:TYP,N:T)+T:SOL(V:S,1)/
                   1000
          CALC     T:BODY(TOTAL,N:T)=T:BODY(TOTAL,N:T)+T:SOL(V:S,1)/
                   1000
          TALLY    V:S,V:SMX,RFBGL
          PRINT    FRFBG
          SPACE    DS
          HEADING  1,'PER REFINERY TYPE      (M TONS/YR)'
          SPACE    SS
          TABULATE              FTAB,STUB=Z:REF(A1-A3),
                   HEAD=Z:TIME(A1),
                   BODY=T:BODY
          SPACE    SS
          PRINT    FTOT,TOTAL,T:BODY(TOTAL,$12)
          SPACE    DS
          SPACE    DS
          EJECT
*                               *AVAILABLE REFINERY CAPACITY
          PRINT    FAVRF
          SPACE    DS
          FORM     T:BODY=Z:REF,Z:TIME(STUB)
          NAME     N:MASK1=KR*.L.**
          NAME     N:MASK2=PF****.*
          SOLTAB   T:SOL1=N:CASE,ALLR(N:MASK1),L
          SOLTAB   T:SOL2=N:CASE,ALLC(N:MASK2),X
          DIMEN    V:SMX=T:SOL2(STUB)
          DIMEN    V:SMM=T:SOL1(STUB)
          CALC     V:S=1
AVRF1     IF       V:S<GT>V:SMM,AVRF2
          NAME     N:T1=MASK(T:SOL1(V:S,0),0000000*)
          NAME     N:TYP1=MASK(T:SOL1(V:S,0),00*00000)
```

```
         CALC     T:BODY(N:TYP1,N:T1)=T:BODY(N:TYP1,N:T1)+T:SOL1(V:S,1)/
                  1000
AVRF2    NAME     N:T2=MASK(T:SOL2(V:S,0),0000000*)
         NAME     N:TYP2=MASK(T:SOL2(V:S,0),00*00000)
         CALC     T:BODY(N:TYP2,N:T2)=T:BODY(N:TYP2,N:T2)+T:SOL2(V:S,1)/
                  1000
         TALLY    V:S,V:SMX,AVRF1
         HEADING 1,'(M TON/YR)'
         SPACE    SS
         TABULATEFTAB,STUB=Z:REF(A1-A3),
                  HEAD=Z:TIME(A1),
                  BODY=T:BODY
         EJECT
*                               *ALUMINA REFINERIES PRODUCTION
         NEWPAGE
         FORM     T:BODY=T:BODY<OR>T:TOTAL,T:BODY
         NAME     N:MASK=PR****.*
         SOLTAB   T:SOL=N:CASE,ALLC(N:MASK),X
         DIMEN    V:SMX=T:SOL(STUB)
         CALC     V:S=1
RFPD1    NAME     N:TYP=MASK(T:SOL(V:S,0),00*0 0000)
         NAME     N:T=MASK(T:SOL(V:S,0),0000 000*)
         CALC     T:BODY(N:TYP,N:T)=T:BODY(N:TYP,N:T)+T:SOL(V:S,1)/
                  1000
         CALC     T:BODY(TOTAL,N:T)=T:BODY(TOTAL,N:T)+T:SOL(V:S,1)/
                  1000
         TALLY    V:S,V:SMX,RFPD1
         PRINT    FRFPR
         SPACE    DS
         HEADING 1,'PER REFINERY TYPE         (M TON/YR)'
         SPACE    SS
         TABULATEFTAB,STUB=Z:REF(A1-A3),
```

```
                  HEAD=Z:TIME(A1),
                  BODY=T:BODY
         SPACE    SS
         PRINT    FTOT,TOTAL,T:BODY(TOTAL,$12)
         EJECT
*                           *IMPORTED ALUMINA
         NEWPAGE
         FORM     T:BODY=Z:LIR<CR>T:TOTAL,Z:TIME(STUB)
         NAME     N:MASK=TI*L.***
         SOLTAB   T:SOL=N:CASE,ALLC(N:MASK),X
         DIMEN    V:SMX=T:SOL(STUB)
         CALC     V:S=1
LIMP1    NAME     N:R=MASK(T:SOL(V:S,0),0000 0*00)
         NAME     N:T=MASK(T:SOL(V:S,0),0000000*)
         CALC     T:BODY(N:R,N:T)=T:BODY(N:R,N:T)+T:SOL(V:S,1)/
                  1000
         CALC     T:BODY(TOTAL,N:T)=T:BODY(TOTAL,N:T)+T:SOL(V:S,1)/
                  1000
         TALLY    V:S,V:SMX,LIMP1
         PRINT    FLIMP
         SPACE    DS
         HEADING  1,'PER COUNTRY OF ORIGIN         (M TON/YR)'
         SPACE    SS
         TABULATEFTAB,STUB=Z:LIR(A1-A3),
                  HEAD=Z:TIME(A1),
                  BODY=T:BODY
         SPACE    SS
         PRINT    FTOT,TOTAL,T:BODY(TOTAL,$12)
         EJECT
*                           *ALUMINA INVENTORY
         NEWPAGE
         FORM     T:BODY=A/1,Z:TIME(STUB)
```

```
          NAME     T:BODY(1,0)=STOCKS
          LOOP     Z:TIME(=1,0),LINV1
          NAME     N:T= =1
          NAME     N:COL=Y..L..Y&N:T
          CALC     T:BODY(1,N:T)= X:(N:CASE,N:COL)/1000
LINV1     CONTINUE
          PRINT    FLINV
          SPACE    DS
          HEADING 1,'AT THE BEGINNING OF THE TIME PERIOD     (M TON)'
          SPACE    SS
          TABULATEFTAB,HEAD=Z:TIME(A1),
                   BODY=T:BODY
          EJECT
*                            *SMELTER REPORTS
          FORM     T:ACUL=Z:ADR<OR>T:TOTAL,Z:SMTYP(STUB)
          LOOP     Z:TIME(=1,0),SM1
          NAME     N:T= =1
          NEWPAGE
          PRINT    FSMBG
          SPACE    SS
          PRINT    FTITLE1,Z:TIME(N:T,$2)
          SPACE    DS
          FORM     T:BODY=Z:ADR<OR>T:TOTAL,Z:SMTYP(STUB)
          FORM     T:BODO=T:BODY,T:BODY
          NAME     N:MASK=S*UA.*.&N:T
          HEADING 1,'ORIGINAL SMELTER CAPACITY WITH IMPROVEMETS'
          HEADING 2,'PER TIME PERIOD                 (M TON/YR)'
          COMPUT   BODY,MASK,CASE,SMTYP,ADR
          CALC     T:BODO($4,$5)=T:BODO($4,$5)+T:BODY($4,$5)
          NAME     N:MASK=S*&N:T&A.*.&N:T
          HEADING 1,'BUILDING OF NEW SMELTERS'
          HEADING 2,'                            (M TON/YR)'
```

```
          COMPUT  BODY,MASK,CASE,SMTYP,ADR
          CALC    T:ACUL($4,$5)=T:ACUL($4,$5)+T:BODY($4,$5)
          NAME    N:MASK=X**A.*.&N:T
          HEADING 1,'EXPANSION OF EXISTING & NEW SMELTERS'
          HEADING 2,'IN THE YEAR OF THE INVESTMENT   (M TON/YR)'
          COMPUT  BODY,MASK,CASE,SMTYP,ADR
          CALC    T:ACUL($4,$5)=T:ACUL($4,$5)+T:BODY($4,$5)
          CALC    T:BODO($4,$5)=T:BODO($4,$5)+T:ACUL($4,$5)
          NEWPAGE
          HEADING 1,'OVERALL SMELTER CAPACITY'
          HEADING 2,'                  (M TON/YR)'
          SPACE   SS
          TABULATEFTAB,HEAD=Z:SMTYP(A1/A2/A3),
                  STUB= Z:ADR(A1-A3),
                  BODY=T:BODO
          SPACE   SS
          PRINT   FTOT,TOTAL,T:BODO(TOTAL,$7)
          EJECT
*                             *AVAILABLE SMELTER CAPACITY
*                             *PER TIME PERIOD
          SPACE   DS
          FORM    T:BODY=Z:ADR,Z:SMTYP(STUB)
          NAME    N:T= =1
          CALC    T:BODY($4,$5)=0
          NAME    N:MASK1=KS*.A.*&N:T
          NAME    N:MASK2=PS*L.*.&N:T
          SOLTAB  T:SOL1=N:CASE,ALLR(N:MASK1),L
          SOLTAB  T:SOL2=N:CASE,ALLC(N:MASK2),X
          DIMEN   V:SMX=T:SOL2(STUB)
          CALC    V:S=1
AVSMK2    NAME    N:R1=MASK(T:SOL1(V:S,0),0000 00*0)
          NAME    N:R2=MASK(T:SOL2(V:S,0),0000 0*00)
```

```
            NAME     N:TYP1=MASK(T:SOL1(V:S,0),00*0 0000)
            NAME     N:TYP2=MASK(T:SOL2(V:S,0),00*0 0000)
            CALC     T:BODY(N:R1,N:TYP1)=T:BODY(N:R1,N:TYP1)+T:SOL1(V:S,1)/
                     1000
            CALC     T:BODY(N:R2,N:TYP2)=T:BODY(N:R2,N:TYP2)+T:SOL2(V:S,1)/
                     1000
            TALLY    V:S,V:SMX,AVSMK2
            HEADING 1,'AVAILABLE SMELTER CAPACITY'
            HEADING 2,'(M TON/YR)'
            SPACE    SS
            TABULATEFTAB,STUB=Z:ADR(A1-A3),
                     HEAD=Z:SMTYP(A1/A2/A3),
                     BODY=T:BODY
            SPACE    DS
            EJECT
*                                 *SMELTER PRODUCTION
            SPACE    DS
            FORM     T:BODY=T:BODY<OR>T:TOTAL,T:BODY
            NAME     N:MASK=PS*L.**&N:T
            HEADING 1,'SMELTEF PRODUCTION'
            HEADING 2,'             (M TON/YR)'
            SOLTAB   T:SOL=N:CASE,ALLC(N:MASK),X
            DIMEN    V:SMX=T:SOL(STUB)
            CALC     V:S=1
SM5         NAME     N:R=MASK(T:SOL(V:S,0),0000 0*00)
            NAME     N:TYP=MASK(T:SOL(V:S,0),00*0 0000)
            CALC     T:BODY(N:R,N:TYP)=T:BODY(N:R,N:TYP)+T:SOL(V:S,1)/
                     1000
            CALC     T:BODY(TOTAL,N:TYP)=T:BODY(TOTAL,N:TYP)+T:SOL(V:S,1)/
                     1000
            TALLY    V:S,V:SMX,SM5
            SPACE    SS
```

```
         TABULATEFTAB,HEAD=Z:SMTYP(A1/A2/A3),
                  STUB=Z:ADR(A1-A3),
                  BODY=T:BODY
         SPACE    SS
         PRINT    FTOT,TOTAL,T:BODY(TOTAL,$5)
SM1      CONTINUE
         EJECT
*                            *REFINED ALUMINUM INVENTORY
         NEWPAGE
         FORM     T:BODY=A/1,Z:TIME(STUB)
         NAME     T:BODY(1,0)=STOCKS
         LOOP     Z:TIME(=1,0),AINV
         NAME     N:T= =1
         NAME     N:MASK=Y..A..Y&N:T
         CALC     T:BODY(1,N:T)=X:(N:CASE,N:MASK)/1000
AINV     CONTINUE
         PRINT    FAINV
         SPACE    DS
         HEADING 1,'AT THE BEGINNING OF THE TIME PERIOD  (M TONS)'
         SPACE    SS
         TABULATEFTAB,HEAD=Z:TIME(A1),
                  BODY=T:BODY
         EJECT
*                            *BUILDING OF CAPTIVE POWER PLANTS
         FORM     T:BODY=Z:ADR<OR>T:TOTAL,Z:TIME(STUB)
         NAME     N:MASK=E..A.*.*
         SOLTAB   T:SOL=N:CASE,ALLC(N:MASK),X
         DIMEN    V:SMX=T:SOL(STUB)
         CALC     V:S=1
PWR1     NAME     N:T=MASK(T:SOL(V:S,0),0000 000*)
         NAME     N:R=MASK(T:SOL(V:S,0),0000 0*00)
         CALC     T:BODY(N:R,N:T)=T:BODY(N:R,N:T)+T:SOL(V:S,1)
```

```
         CALC    T:BODY(TOTAL,N:T)=T:BODY(TOTAL,N:T)+T:SOL(V:S,1)
         TALLY   V:S,V:SMX,PWR1
         PRINT   FPWR
         SPACE   SS
         HEADING 1,'IN (MW)'
         SPACE   SS
         TABULATEFTAB,STUB=Z:ADR(A1-A3),
                 HEAD=Z:TIME(A1),
                 BODY=T:BODY
         SPACE   SS
         PRINT   FTOT,TOTAL,T:BODY(TOTAL,$4)
         EJECT
*                            *SECONDRY ALUMINUM PRODUCTION
         NEWPAGE
         FORM    T:BODY=Z:SCRAP<OR>T:TOTAL,Z:TIME(STUB)
         NAME    N:MASK=PC**SU.*
         SOLTAB  T:SOL=N:CASE,ALLC(N:MASK),X
         DIMEN   V:SMX=T:SOL(STUB)
         CALC    V:S=1
SCR1     NAME    N:TYP=MASK(T:SOL(V:S,0),000* 0000)
         NAME    N:T=MASK(T:SOL(V:S,0),0000 000*)
         CALC    T:BODY(N:TYP,N:T)=T:BODY(N:TYP,N:T)+T:SOL(V:S,1)/
                 1000
         CALC    T:BODY(TOTAL,N:T)=T:BODY(TOTAL,N:T)+T:SOL(V:S,1)/
                 1000
         TALLY   V:S,V:SMX,SCR1
         PRINT   FSCRAP
         SPACE   DS
         HEADING 1,'PRODUCTION OUT OF SCRAP    (M TONS/YR)'
         SPACE   SS
         TABULATEFTAB,HEAD=Z:TIME(A1),
                 STUB=Z:SCRAP(A1-A3),
```

```
                 BODY=T:BODY
        SPACE    SS
        PRINT    FTOT,TOTAL,T:BODY(TOTAL,$7)
        EJECT
*                              *TRANSPORTATION REPORTS
*                              *TRANSP IS THE SUBROUTINE COMMON
*                              *TO THESE REPORTS
*                              *TRANSPORTATION OF IMPORTED BAUXITE
        LOOP     Z:TIME(=1,0),TRBX1
        NAME     N:T= =1
        NEWPAGE
        PRINT    FTRBX
        SPACE    SS
        PRINT    FTITLE1,Z:TIME(N:T,$5)
        SPACE    DS
        FORM     T:BODY=Z:BIR<OR>T:TOTAL,Z:BXCONS(STUB)
        NAME     N:MASK=TI*B***&N:T
        HEADING 1,'(M TON/YR)'
        TRANSP   MASK,CASE,BODY,BIR,BXCONS
TRBX1   CONTINUE
        EJECT
*                              *TRANSPORTATION OF DOMESTIC ALUMINA
        NEWPAGE
        PRINT    FLDTR
        SPACE    SS
        FORM     T:BODY=Z:LDR<OR>T:TOTAL,Z:ADR(STUB)<OR>Z:LSECT(STUB)
        FORM     Z:HEAD=Z:ADR<OR>Z:LSECT,Z:ADR
        FILL     Z:HEAD=Z:ADR
        FILL     Z:HEAD=Z:LSECT
        LOOP     Z:TIME(=1,0),LDTR1
        NAME     N:T= =1
        PRINT    FTITLE1,Z:TIME(N:T,$4)
```

```
          SPACE    DS
          CALC     T:BODY($5,$6)=0
          NAME     N:MASK=TP.L.**&N:T
          HEADING  1,'(M TON/YR)'
          TRANSP   MASK,CASE,BODY,LDR,HEAD
          IF       LINE(X)+10<LE>55,LDTR1
          NEWPAGE
LDTR1     CONTINUE
          EJECT
*                            *IMPORTED ALUMINA TRANSPORTATION
          NEWPAGE
          PRINT    FLITR
          SPACE    SS
          FORM     T:BODY=Z:LIR<OR>T:TOTAL,Z:ADR(STUB)
          LOOP     Z:TIME(=1,0),LITR1
          NAME     N:T= =1
          CALC     T:BODY($5,$6)=0
          PRINT    FTITLE1,Z:TIME(N:T,$4)
          SPACE    SS
          NAME     N:MASK=TI*L.**&N:T
          HEADING  1,'(M TON/YR)'
          TRANSP   MASK,CASE,BODY,LIR,ADR
          IF       LINE(X)+15<LE>55,LITR1
          NEWPAGE
LITR1     CONTINUE
          EJECT
*                            *TRANSPORTATION OF PRIMARY ALUMINUM
*                            *PRALUM IS A SUBROUTINE USED IN THIS REPORT
*                            *DOMESTIC ALUMINUM TRANSPORTATION
          FORM     T:BODY=Z:MK<OR>T:TOTAL,Z:FB(STUB)<OR>Z:TOTAL(STUB)
          FORM     Z:HEAD=Z:FB<OR>Z:TOTAL,Z:FB
          FILL     Z:HEAD=Z:FB
```

```
         FILL     Z:HEAD=Z:TOTAL
         LOOP     Z:TIME(=1,0),ATR1
         NAME     N:T=  =1
         NEWPAGE
         PRINT    FATR
         SPACE    SS
         PRINT    FTITLE1,Z:TIME(N:T,$4)
         SPACE    DS
         LOOP     Z:ADR(=2,0),ATR2
         NAME     N:R=  =2
         CALC     T:BODY($7,$8)=0
         NAME     N:MASK=TP*A.&N:R&*&N:T
         HEADING  1,Z:ADR(N:R,$9)
         HEADING  2,'(M TON/YR)'
         PRALUM   MASK,CASE,BODY,MK,HEAD
         IF       LINE(X)+20<LE>55,ATR2
         NEWPAGE
ATR2     CONTINUE
         EJECT
*                           *IMPORTED ALUMINUM TRANSPORTATION
         NEWPAGE
         PRINT    FATR
         PRINT    FTITLE1,Z:TIME(N:T,$10)
         SPACE    DS
         CALC     T:BODY($5,$6)=0
         NAME     N:MASK=TI*A.F*&N:T
         HEADING  1,'IMPORTED ALUMINUM'
         HEADING  2,'(M TON/YR)'
         PRALUM   MASK,CASE,BCDY,MK,HEAD
         EJECT
*                           *SECONDARY ALUMINUM TRANSPORTATION
         SPACE    DS
```

```
         CALC     T:BODY($5,$6)=0
         NAME     N:MASK=TP*&SU*&N:T
         HEADING  1,'SECONDARY ALUMINUM'
         HEADING  2,'(M TON/YR)'
         PRALUM   MASK,CASE,BODY,MK,HEAD
         NEWPAGE
*                              *TRANSPORTATION FROM INVENTORY
         SPACE    SS
         CALC     T:BODY($5,$6)=0
         NAME     N:MASK=TY*A.Y*&N:T
         HEADING  1,'ALUMINUM FROM INVENTORY'
         HEADING  2,'(M TON/YR)'
         PRALUM   MASK,CASE,BODY,MK,HEAD
ATR1     CONTINUE
         EJECT
*                              *DUAL VARIABLE REPORTS
         NEWPAGE
         FORM     T:BODD=Z:MK,Z:FB(STUB)
         PRINT    FMRG
         SPACE    SS
         LOOP     Z:TIME(=1,0),DVA1
         NAME     N:T=  =1
         DFO      RSLT,DFRATE,T
         NAME     N:MASK=D*+A..URN:T
         SOLTAB   T:SOL=N:CASE,ALLC(N:MASK),D
         DIMEN    V:SMX=T:SOL(STUB)
         CALC     V:S=1
         CALC     T:BODD($4,$5)=0
DVA3     NAME     N:MK=MASK(T:SOL(V:S,0),0*00 0000)
         NAME     N:FB=MASK(T:SOL(V:S,0),00*0 0000)
         CALC     T:BODD(N:MK,N:FB)=T:SOL(V:S,1)/(2000*V:RSLT)
         TALLY    V:S,V:SMX,DVA3
```

```
        HEADING 1,Z:TIME(N:T,$7)
        HEADING 2,'(1977$/LB)'
        SPACE   SS
        TABULATEFTAB,STUB=Z:MK(A1-A3),
                HEAD=Z:FD(A1/A2/A3),
                BODY=T:BODC
        SPACE   DS
        IF      LINE(X)+20<LE>55,DVA1
        NEWPAGE
DVA1    CONTINUE
        EJECT
        NEWPAGE
        PRINT   FMRG
        SPACE   SS
        LOOP    Z:PRICE(=2,0),DVA5
        NAME    N:MAT= =2
        NAME    N:TITLE=MASK(N:MAT,0*00 0000)&DR
        FORM    T:BODY=Z:N:TITLE,Z:TIME(STUB)
        NAME    N:MASK=BTP&N:MAT&**
        SOLTAB  T:SOL=N:CASE,ALLR(N:MASK),P
        DIMEN   V:SMX=T:SOL(STUB)
        CALC    V:S=1
DVA6    NAME    N:R=MASK(T:SOL(V:S,0),0000 00*0)
        NAME    N:T=MASK(T:SOL(V:S,0),0000 000*)
        DFO     RSLT,DFRATE,T
        CALC    T:BODY(N:R,N:T)=T:SOL(V:S,1)/(2000*V:RSLT)
        TALLY   V:S,V:SMX,DVA6
        HEADING 1,Z:PRICE(N:MAT,$5)
        HEADING 2,'(1977 $/TON)'
        SPACE   SS
        TABULATEFTAB,STUB=Z:N:TITLE(A1-A3),
                HEAD=Z:TIME(A1),
```

```
                 BODY=T:BODY
         SPACE   DS
DVA5     CONTINUE
         EJECT
*                                *FINANCIAL INVESTMENT REPORT
         NEWPAGE
         FORM    T:BODY=Z:INVEST<OR>T:TOTAL,Z:TIME(STUB)
         NAME    N:MASK=FI.*@@.*
         SOLTAB  T:SOL=N:CASE,ALLR(N:MASK),RA
         DIMEN   V:SMX=T:SOL(STUB)
         CALC    V:S=1
FIV1     NAME    N:T=MASK(T:SOL(V:S,0),0000 000*)
         NAME    N:INV=MASK(T:SOL(V:S,0),000* 0000)
         CALC    T:BODY(N:INV,N:T)=T:SOL(V:S,1)/1000000000
         CALC    T:BODY(TOTAL,N:T)=T:BODY(TOTAL,N:T)+T:BODY(N:INV,N:T)
         TALLY   V:S,V:SMX,FIV1
         PRINT   FINV
         SPACE   SS
         HEADING 1,'(MMM 1977 $)'
         SPACE   SS
         TABULATE FTAB,STUB=Z:INVEST(A1-A3),
                 HEAD=Z:TIME(A1),
                 BODY=T:BODY
         SPACE   SS
         PRINT   FTOT,TOTAL,T:BODY(TOTAL,$9)
         SPACE   NP
         EXIT
         EJECT
*                                *COMPUT SUBROUTINE
COMPUT   CONTINUE
         CALC    T:#1($1,$2)=0
         SOLTAB  T:SOL=N:#3,ALLC(N:#2),X
```

```
         DIMEN    V:SMX=T:SOL(STUB)
         CALC     V:S=1
CMP1     NAME     N:R=MASK(T:SOL(V:S,0),0000 0*00)
         NAME     N:TYP=MASK(T:SOL(V:S,0),0*00 0000)
         CALC     T:#1(N:R,N:TYP)=T:#1(N:R,N:TYP)+T:SOL(V:S,1)/1000
         CALC     T:#1(TOTAL,N:TYP)=T:#1(TOTAL,N:TYP)+T:SOL(V:S,1)/1000
         TALLY    V:S,V:SMX,CMP1
         SPACE    SS
         TABULATEFTAB,HEAD=Z:#4(A1/A2/A3),
                  STUB=Z:#5(A1-A3),
                  BODY=T:#1
         SPACE    SS
         PRINT    FTOT,TOTAL,T:#1(TOTAL,$7)
         SPACE    DS
         NEXT
         EJECT
*                                 *TRANSP SUBROUTINE
TRANSP   CONTINUE
         SOLTAB   T:SOL=N:#2,ALLC(N:#1),X
         DIMEN    V:SMX=T:SOL(STUB)
         CALC     V:S=1
TRP      NAME     N:R=MASK(T:SOL(V:S,0),0000 0*00)
         NAME     N:U=MASK(T:SOL(V:S,0),0000 00*0)
         CALC     T:#3(N:R,N:U)=T:#3(N:R,N:U)+T:SOL(V:S,1)/1000
         CALC     T:#3(TOTAL,N:U)=T:#3(TOTAL,N:U)+T:SOL(V:S,1)/1000
         TALLY    V:S,V:SMX,TRP
         SPACE    SS
         TABULATEFTAB,STUB=Z:#4(A1-A3),
                  HEAD=Z:#5(A1/A2/A3),
                  BODY=T:#3
         SPACE    SS
         PRINT    FTOT,TOTAL,T:#3(TOTAL,$6)
```

```
         SPACE    DS
         NEXT
         EJECT
*                              *PRALUM SUBROUTINE
PRALUM   CONTINUE
         SOLTAB   T:SOL=N:#2,ALLC(N:#1),X
         DIMEN    V:SMX=T:SOL(STUB)
         CALC     V:S=1
ATR      NAME     N:FB=MASK(T:SOL(V:S,0),00*0 0000)
         NAME     N:MK=MASK(T:SOL(V:S,0),0000 00*0)
         CALC     T:#3(N:MK,N:FB)=T:#3(N:MK,N:FB)+T:SOL(V:S,1)/1000
         CALC     T:#3(TOTAL,N:FB)=T:#3(TOTAL,N:FB)+T:SOL(V:S,1)/1000
         CALC     T:#3(N:MK,TOTAL)=T:#3(N:MK,TOTAL)+T:SOL(V:S,1)/1000
         CALC     T:#3(TOTAL,TOTAL)=T:#3(TOTAL,TOTAL)+T:SOL(V:S,1)/1000
         TALLY    V:S,V:SMX,ATR
         SPACE    SS
         TABULATEFTAB,STUB=Z:#4(A1-A3),
                  BODY=T:#3,
                  HEAD=Z:#5(A1/A2/A3)
         SPACE    SS
         PRINT    FTOT,TOTAL,T:#3(TOTAL,$4)
         SPACE    DS
         NEXT
         EJECT
DFC      CONTINUE
         CALC     V:DF.X1=0
         DIMEN    V:DF.T=T:TIME...(0,N:#3),DF.BADD
         IF       V:DF.T<EQ>1,DF.GX1
         CALC     V:DF.X=1
         CALC     V:DF.T=V:DF.T-1
DF.LPO   CALC     V:DF.X1=V:DF.X1+T:TIME...(YTP,V:DF.X)
         TALLY    V:DF.X,V:DF.T,DF.LPO
```

```
        CALC    V:DF.X1=-V:DF.X1*V:#2
DF.OX1  CALC    V:DF.X2=V:DF.X1-(T:TIME...(YTP,N:#3)*V:#2)
        CALC    V:#1=-(FV:DEXP(V:DF.X2)-FV:DEXP(V:DF.X1))/V:#2
        NEXT
DF.BADD NOTE    INVALID TIME PERIOD PASSED TO DFO SUBROUTINE
        DISPLAY NAME=N:#3
        EXIT
        EJECT
***************************************************************
*                                *FORMAT STATEMENTS
*                                ******************
FPAGE   FORMAT  (1H1,T43,'CASE NAME IS',3X,A8)
FTAB    FORMAT  (1H ,A24,T26,12Z10.3)
FTOT    FORMAT  (1H ,A8,T26,12Z10.3)
FTITLE1 FOPMAT    (1HO,3A8)
FMNRP   FORMAT  (1HO,T43,'MINE BUILDING REPORT')
FMNPR   FORMAT  (1HO,T43,'MINE PRODUCTION REPORT')
FBXIMP  FORMAT  (1HO,T43,'BAUXITE IMPORTS')
FBXINV  FORMAT  (1HO,T43,'BAUXITE INVENTORY STATUS')
FRFBG   FORMAT  (1HO,T43,'REFINERY BUILDING REPORT')
FRFPR   FORMAT  (1HO,T43,'REFINERY PRODUCTION REPORT')
FLIMP   FORMAT  (1HO,T43,'ALUMINA IMPORTS')
FLINV   FORMAT  (1HO,T43,'ALUMINA INVENTORY STATUS')
FSMBG   FORMAT  (1HO,T43,'SMELTER BUILDING REPORT')
FAINV   FORMAT  (1HO,T43,'ALUMINUM INVENTORY STATUS')
FSCRAP  FORMAT  (1HO,T43,'SECONDARY ALUMINUM PRODUCTION')
FTRBX   FORMAT  (1HO,T43,'TRANSPORTATION OF IMPORTED BAUXITE')
FAVMNK  FORMAT  (1HO,T43,'AVAILABLE MINE CAPACITY')
FLDTR   FORMAT  (1HO,T43,'DOMESTIC ALUMINA TRANSPORTATION')
FLITR   FORMAT  (1HO,T43,'IMPORTED ALUMINA TRANSPORTATION')
FAVRF   FORMAT  (1HO,T43,'AVAILABLE REFINERY CAPACITY')
FAVSM   FORMAT  (1HO,T43,'AVAILABLE SMELTER CAPACITY')
```

```
FATR      FORMAT  (1HO,T38,'FLOW OF PRIMARY ALUMINUM'/1H ,T38,
                  'FROM THE PRODUCING REGIONS TO THE CONSUMING MARKETS'/
                  1H ,T38,'THROUGH DIFFERENT FABRICATION PROCESSES')
FPWR      FORMAT  (1HO,T43,'BUILDING OF CAPTIVE POWER PLANTS')
FMRG      FORMAT  (1HO,T43,'MARGINAL COSTS')
FINV      FORMAT  (1HO,T43,'INVESTMENTS')
ENDATA
```

F Report tables

```
NAME         REPTAB
        TABLE   T:TIME...= 1,2,3,4,5,6,7,8,9
                YTP= 2,2,2,2,2,2,2,2,2
                STP= 0,2,4,6,8,10,12,14,16
        TEXT    Z:BDR= A(3)
                A= 'ARKANSAS'
                W= 'WYOMING'
                G= 'GEORGIA'
        TEXT    Z:BIR=A(3)
                J= 'JAMAICA'
                N= 'SURINAM'
                C= 'CARIBBEAN'
                Y= 'GUYANA'
                G= 'GUINEA'
                L= 'AUSTRALIA'
                B= 'BRAZIL'
        TEXT    Z:ORE= A(3)
                BH= 'BAUXITE   HI SILICA'
                BL= 'BAUXITE   LO SILICA'
                N.= 'ANORTHOSITE'
                C.= 'KAOLIN CLAY'
        TEXT    Z:TIME= A(1)
                1= '77-78'
                2= '79-80'
                3= '81-82'
                4= '83-84'
                5= '85-86'
                6= '87-88'
                7= '89-90'
                8= '91-92'
                9= '93-94'
        TEXT    Z:INIT= A(1)
```

```
         O= '1976'
TEXT     Z:BXINV= A(3)
         J= 'JAMAICA'
         N= 'SURINAM'
         G= 'GUINEA'
TEXT     Z:LDR= A(3)
         E= 'SOUTHEAST  U.S.'
         W= 'WYOMING'
         G= 'GEORGIA'
TEXT     Z:LIR= A(3)
         L= 'AUSTRALIA'
         J= 'JAMAICA'
         N= 'SURINAM'
         G= 'GUINEA'
         Y= 'GUYANA'
TEXT     Z:REF= A(3)
         H= 'HI SILICA BAUXITE'
         L= 'LO SILICA BAUXITE'
         N= 'ANORTHOSITE'
         C= 'KAOLIN CLAY'
TEXT     Z:ADR= A(3)
         1= 'N.WEST  U.S.'
         2= 'N.EAST  U.S.'
         3= 'S.EAST  U.S.'
TEXT     Z:AIR= A(3)
         F= 'FOREIGN ORIGIN'
TEXT     Z:SCRAP= A(3)
         N= 'NEW SCRAP'
         O= 'OLD SCRAP'
TEXT     Z:SMTYP= A(3)
         1= 'HORIZ.  SODBG.'
         2= 'VERT.   SODBG.'
```

```
          3= 'PREBKD. ANODES'
          4= 'COMPUT. PREBKD.'
          5= 'COMPUT. TI.B2'
          6= 'ALCOA   PROCESS'
TEXT      Z:FB= A(3)
          I= 'INGOT'
          P= 'SHEET    PLATE &   FOIL'
          T= 'EXTRUSN.   &      TUBES'
          C= 'WIRE       &        CABLE'
          V= 'VARIOUS'
          .= '  NO      MILLING'
TEXT      Z:MK= A(3)
          B= 'BUILDING & CONSTRUCTION'
          T= 'TRANSPORTATION'
          D= 'CONSUMER DURABLES'
          E= 'ELECTRICAL'
          M= 'MACHINERY & EQUIPMENT'
          C= 'PACKAGING & CONTAINERS'
          O= 'OTHERS'
          F= 'EXPORTS'
          Y= 'INVENTORY'
TEXT      Z:BXCONS= A(3)
          E= 'REFINERY'
          Y= 'STOCKS'
          Q= 'CHEMICAL'
TEXT      Z:LSECT= A(3)
          Q= 'CHEMICAL'
          Y= 'STOCKS'
          F= 'EXPORTS'
TEXT      Z:TOTAL= A(3)
          TOTAL= 'TOTAL'
TEXT      Z:PRICE= A(3)
```

```
                    .L.= 'DMTC ALUMINA'
                    @A.= 'DMTC PRIMARY ALUMINUM'
                    @@S= 'DMTC SECONDARY ALUMINUM'
          TEXT      Z:INVEST= A(3)
                    M='MINES'
                    R='REFINERIES'
                    S='SMELTER & POWER PLANTS'
          TEXT      Z:@DR= A(3)
                    U= 'UNITED STATES'
ENDATA
```

G Sample of matrix modification program

```
//EXEC.MODS DD *
NAME            MODPROG
*
*                   * THIS PROGRAM CHANGES BAUXITE PRICES
*                   * AFTER THE THIRD TIME PERIOD SIMULATING
*                   * A FAILURE OF THE IBA.
*
           REMOTE   U:XACTFILE,U:XACTPROB
           READTAB  Z:Z,MODS,MODTAB
           NAME     N:OBJ=U:XOBJ
           CALC     V:DFRATE=I:DFRATE
           LOOP     T:TIME...(0,=1)<HIEQ>3,BB1
           NAME     N:T= =1
           DFO      RSLT,DFRATE,T
           LOOP     T:TBL1(=2,=3),BB2
           NAME     N:R= =2
           NAME     N:SG= =3
           LOOP     T:TBL2(N:R,=4)<EQ>1,BB3
           NAME     N:U= =4
           NAME     N:COL=TI&N:SG&BL&N:R&N:U&N:T
           RCOL     N:COL,
                    N:OBJ<N>=V:RSLT*T:TBL1(N:R,N:SG)
BB3        CONTINUE
BB2        CONTINUE
BB1        CONTINUE
           ENDREM
           EXIT
DFO        CONTINUE
           IF       V:#2<NE>0.0,DFO.10
           CALC     V:#1=1.0
           GOTO     DFO.11
DFO.10     CALC     V:DF.X1=0
```

```
          DIMEN    V:DF.T=T:TIME...(0,N:#3),DF.BADO
          IF       V:DF.T<EQ>1,DF.OX1
          CALC     V:DF.X=1
          CALC     V:DF.T=V:DF.T-1
DF.LPO    CALC     V:DF.X1=V:DF.X1+T:TIME...(YTP,V:DF.X)
          TALLY    V:DF.X,V:DF.T,DF.LPO
          CALC     V:DF.X1=-V:DF.X1*V:#2
DF.OX1    CALC     V:DF.X2=V:DF.X1-(T:TIME...(YTP,N:#3)*V:#2)
          CALC     V:#1=-(FV:DEXP(V:DF.X2)-FV:DEXP(V:DF.X1))/V:#2
DFO.11    NEXT
DF.BADO   NOTE     INVALID TIME PERIOD PASSED TO DFC SUBROUTINE
          DISPLAY  NAME=N:#3
          EXIT
ENDATA
NAME           MODTAB
          TABLE    T:TIME...= 1,2,3,4,5,6,7,8,9
                   YTP= 2,2,2,2,2,2,2,2,2
                   STP= 0,2,4,6,8,10,12,14,16
          TABLE    T:TBL1= 1,2
                   J= 18,24
                   N= 16,24
                   C= 18,24
                   Y= 18,25
                   G= 22,29
                   L= 30,40
                   B= 19,26
          TABLE    T:TBL2= E,Y,Q
                   J= 1,1,0
                   N= 1,1,1
                   C= 1,0,0
                   Y= 1,0,1
                   G= 1,1,0
```

```
                L= 1,0,0
                B= 1,0,0
ENDATA
```

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A MODEL OF THE ALUMINUM INDUSTRY IN THE UNITED STATES

by

Miguel A. Kelly

(ABSTRACT)

The energy crisis, the high capital requirements and the high price of the raw materials are critically affecting the short and long run decisions of the aluminum industry in the United States. To study the effect of these and other factors in the optimal development of the industry, a time dynamic linear model was developed. The model represents the industry in the United States interacting with the raw materials supplying countries.

To demonstrate the capability of the model, nine case studies were run representing nine probable scenarios. The results, which should be approached from both a qualitative and a quantitative point of view, are discussed and some limited conclusions are derived.