A MULTIPLE STRESS, MULTIPLE COMPONENT STRESS SCREENING COST MODEL

by

Lori E. Seward

Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE
in
Industrial Engineering and Operations Research

APPROVED:

J. A. Nachlas, Chairman

M. R. Reynolds

H. D. Sherali

July, 1985

Blacksburg, Virginia

A MULTIPLE STRESS, MULTIPLE COMPONENT

STRESS SCREENING COST MODEL

by

Lori E. Seward

(ABSTRACT)

Environmental stress screening is used to enhance reliability by decreasing the number of failures experienced during customer use. It is suggested that added benefit can be gained by applying multiple stresses rather than a single stress, as is done presently. A further modification is to apply the stress at the assembly level, accelerating different types of components at the same time. Different component acceleration effects must then be considered.

The problem these modifications present is how to choose the appropriate stress levels and the time duration of the stress screen. A cost model is developed that trades off the cost of a field failure with the cost of applying a multiple stress, multiple component stress screen. The objective is to minimize this cost function in order to find an economical stress regimen.

The problem is solved using the software package GINO. The interesting result is that if a stress is used at all during the stress screen, the maximum amount of stress is the economic choice. Either the cost of stressing is low enough to justify the use of a stress, in which case the maximum amount of stress is used, or the cost is too high and the stress is not used at all.

ACKNOWLEDGEMENTS

I would like to thank my committee members, Dr. H. D. Sherali and Dr. M. R. Reynolds, for their excellent suggestions and advice. I thank Dr. S. Balachandran for the important information he supplied. Also, the IEOR faculty and staff deserve recognition for their understanding and support of their graduate students.

My committee chairman, Dr. J. A. Nachlas, deserves a special thanks for his constant and continued support throughout my undergraduate and graduate career. His expertise and never-ending optimism were essential to the completion of this thesis.

I would also like to thank . They each played a unique role in helping me start and finish this project. Finally, I need to thank my husband . May I be as patient and good-humored while he writes his thesis.

TABLE OF CONTENTS

Chapter		Page
I	Introduction	1
II	Literature Review	4
III	Model Development	11
IV	Model Solution	18
V	References	27
VI	Appendix	30

I. INTRODUCTION

Since environmental screening for reliability enhancement was first introduced by Watson & Wells in 1961, it has become an integral part of the production of most electrical components and systems. One of the original, and still most widely used, stresses is elevated temperature. Other environmental stresses, such as voltage, humidity, and thermal and power cycling, have also been used to create effective stress screens. Although the range of environmental stressing is broad, there has not been widespread use of multiple stresses to create a single stress screen.

There are many variables involved in multiple stress screening. Temperature ranges, voltage settings, ramp rates and dwell times (when talking of thermal or power cycling), and stress duration times are all variables of concern. Another variable of concern is the acceleration effect, or amount of time scale compression, experienced at each production level - component, assembly, and system. Furthermore, if stress screening is performed at the assembly or system level, consideration must be given to the different acceleration effects experienced by the individual components that make up the assembly or system.

By controlling the stress levels of the different stresses one can determine the acceleration effect experienced by each component. There is an optimum amount of time scale compression, determined by the stress regimen and duration of stress application, at which any further benefits derived from stress screening are not worth the effort needed to obtain them. Cost modeling is an appropriate way of determining this optimal stress screening regimen. The objective of the cost model is to trade off the costs associated with stress screening with the resulting failure costs when no stress screening is performed. The purpose of this paper is to present a cost model that will minimize the total expected cost associated with a multiple stress, multiple component stress screening plan.

Before pursuing the cost model, it is suggested that consistent definitions of terms and phrases used in reliability and stress screening are needed. There is discrepancy in the literature in the use of certain words dealing with quality, reliability, and stress screening. Classroom discussions, conversations with working engineers, and literature reviews are all used to provide a basis for definition selection.

Once this foundation of concepts is established, the formulation of the cost model can be completed. It is assumed that there are three quantifiable costs associated with stress screening. There is a cost for stress screening application, a cost for a failure during stress screening, and a cost for a failure once the unit is in customer use. A detailed description of the cost model is included in Chapter III of this thesis. The second chapter consists of a literature review of the books, papers, and articles researched for the formulation of both the definitions and the cost model. Chapter IV gives the solution methods used. An example problem is explained and the research findings are discussed.

A model has been suggested and a solution to a plausible example problem has been discussed. The results show that if a stress is used in the stress screen, the maximum amount of stress available is the economic choice. If the cost of stressing a particular characteristic, for example, voltage, is too high, then the decision is to use the nominal stress level. In other words, each stress variable will be at either its upper bound or lower bound. Further analysis shows that the solution of the minimization problem is not affected by the magnitude of the cost parameters. Rather, the ratio between the cost parameters is the important factor. The findings of this study conclude that it is indeed worthwhile to select a multiple stress regimen based on cost trade-offs.

II. LITERATURE REVIEW

This section reviews the literature relating to stress screening, cost models for stress screening, and reliability definitions. Articles dealing with reliability definitions will be discussed first followed by a review of the stress screening and stress testing literature and the articles on stress screening cost models.

While building a foundation of concise definitions dealing with quality and reliability, two papers on the meaning of these words were discovered.

Grant and Bell (May 1961) discuss the differences between quality and reliability. The main thrust of this article is that reliability is not a separate characteristic of a product but that it is one aspect of a high quality item. They rely heavily on dictionary quotations to justify their comments.

Paterson (October 1962) deals with the terms Quality Control, Quality Assurance, and Reliability. He asserts that reliability is an extension of Quality rather than a separate term. Paterson states "... I have never seen ... one such instance (product failure) which was not attributable to a shortcoming in design or a production irregularity."

These two articles contend that quality and reliability are not separate measures. Other sources, Vander Hamm (1969) and Rue (1976), do differentiate between measures of quality and reliability. The following paragraphs define the terms quality and reliability and discuss the differences between the words as well as their impact on the subject of environmental stress screening.

This author agrees that there are two definitions of the word quality. When referring to the general characteristics or attributes of a product, one may be describing the "qualities" of that product. Quality is also used to describe the goodness or badness of a product as compared to other "identical" items. In this case quality can be measured. It is the measure of conformance to physical design specifications. This latter definition of quality will be used throughout the remainder of this paper.

Quality and reliability describe two different types of measurements. Quality is the measure of conformance to physical design specifications. Reliability is the measure of performance in terms of operational design specifications. Although both quantities can be stated as probabilities, the probability of conformance is constant over time whereas the probability of performance changes over time.

Based on these definitions, there are two types of item failures. A quality failure is an item that cannot perform according to operational design specifications. A reliability failure is an item that stops performing according to the operational design specifications. This implies that a reliability failure has performed for some time, t, as opposed to a quality failure which never performs for any time t.

Differentiating between a quality failure and a reliability failure is important when one discusses stress screening. The object of stress screening is to "screen out" the early failures in a population in order to improve the reliability of a population. Environmental stressing is performed to increase the age of the items in a short amount of time and speed up the screening process.

It is argued that a quality failure cannot be accelerated whereas a reliability failure can be accelerated. Consider a population of defective items. Since these items do not conform to the physical design specifications, they will not perform according to the operational design specifications; and hence, one can conclude that the reliability of these defective items is zero over all time.

When the reliability of an item is zero, the chance of that item failing is the same at the nominal stress level as it is at an elevated stress level, namely, 100%. It is clear that creating a more severe environment will not accelerate a quality failure. The conclusion to this discussion of definitions is that when one talks of stress screening a population of items for reliability enhancement, one is referring only to the non-defective items in that population.

The literature on stresses other than elevated temperatures is scarce; however, much work has been done in the area of burn-in for stress screening. Vander Hamm (1969) discusses MIL-STD 781, known as AGREE. Vander Hamm criticizes most burn-in programs as assigning infant mortalities to the defective parts. He argues that any quality part should stand up under excessive stressing. Vander Hamm also states that past failure rates are most often too high because they reflect design faults and quality problems.

Rue (1976) follows the same reasoning as Vander Hamm in differentiating between quality defectives and reliability defectives. He considers the term "burn-in" to be a generic term, meaning to stress screen using any type of environmental stress. Rue's model of stress screening utilizes a screening strength. This screening strength measures the ability of a stress to screen out early life failures.

Reda, Brown, and Menze (1976) discuss multiple stress screening of electronic calculators. They conclude that high temperature is a good stress but that voltage, within the tolerable range, and vibration, also within the tolerable range, have basically no acceleration effect. This makes sense for electronic calculators. The tolerable ranges of most stresses would be quite small, thereby gaining little or no acceleration effect.

Peck and Trapp (1980) have put together an Accelerated Testing Handbook. This is a cookbook type treatment of accelerated testing. Peck and Trapp discuss multiple stresses, accelerated life testing, and stress screening; however, no stress functions for multiple stresses are included, nor is a cost model for stress screening provided.

Kuo and Kuo (November 1983) review the concerns and problems with burn-in for reliability enhancement. They discuss the advantages and disadvantages of screening a single component on an entire system. They also discuss the problem of ESD, or electrostatic discharge, caused by too much handling during burn-in. Kuo and Kuo give an extensive review of publications on burn-in and stress screening.

Another good review of burn-in is the book by Jensen and Petersen entitled Burn-In (1982). The authors discuss sequential burn-in, multilayer burn-in, accelerating burn-in tests, and Markovian burn-in. They also provide an excellent reference source for current literature on burn-in and multiple stress screening.

The basic form of the cost model described in this proposal is similar to many of the cost models found in the literature; however, the cost model developed here is based on multiple components and assumes a multiple stress environment. One general comment to be made about the cost models found in the literature is that none address multiple component, multiple stress screening.

Plesser and Field (August 1977) optimize burn-in costs for repairable systems. They assume a non-homogeneous Poisson process and that a repair action returns the system to "bad-as-old." Plesser and Field use many different time parameters in determining the optimum burn-in time. They assume a useful life, K, is independent of the burn-in time, t_b.

Kuo and Kuo (November 1983) give a good review of the basic concepts of burn-in. They discuss cost modeling and reference some general cost models. Their cost model is similar to the one described in this proposal; however, as mentioned previously, they do not consider multiple components or multiple stresses.

Chandrasekaran (1977) approaches the problem of optimizing a stress screening plan in two ways. He tries to maximize the mean residual life for failed parts that cannot be replaced, i.e., parts for a space ship, and he minimizes total cost for failed parts that are costly to replace. When minimizing total cost, Chandrasekaran deals with various types of replacement policies: replacement at failure, an age replacement policy, and different values for the age of replacement in the age replacement policy.

Stewart and Johnson (August 1972) use Bayesian analysis to determine optimal burn-in times and optimal replacement times. Stewart and Johnson address items which are prone to either early failures or wearout failures. They minimize the long-run average cost per unit time of service. They conclude that the true time-to-failure distribution cannot be known exactly; therefore, a posterior distribution of the parameters of the failure distribution is used to calculate the long-run average cost.

Canfield (June 1975) simplifies the solutions to burn-in problems and burn-in replacement problems. He assumes that all repairs return a unit to a like new condition. Canfield minimizes an average cost per time unit of maintaining a system, similar to Stewart and Johnson.

Weiss and Dishon (August 1971) assume a replacement policy where N devices are needed at the end of burn-in. If there are less than N devices, then items with zero burn-in time are chosen. This changes the life distribution. Weiss and Dishon are therefore obtaining the number of units to place on burn-in with respect to a given burn-in time, rather than determining the burn-in time.

Ekings (1976) describes the reliability program used for Xerox printers. He does not recommend 100% system burn-in or component burn-in. Ekings suggests a sample burn-in and then bases his reliability estimates on unscheduled maintenance per cumulative copies. This takes into account customer satisfaction. Ekings also contends that field data is most desirable in analyzing cause of failure and he does not talk of a distributed failure process; all failures can be attributed to a specific department, i.e., design, manufacturing, etcetera.

Kuo (October 1982) has described a cost model for optimal burn-in time where the decision variables are the duration of burn-in and the level of burn-in application (system level, unit level, or component level). He describes various costs and sets up two constrained problems. Kuo assumes a steady-state hazard rate and develops his two cost models based on the relationship between burn-in time duration, real time, and the time at which the steady-state hazard rate has been reached.

Washburn (November 1970) describes a total cost model, or what he calls a "utility function." He uses three basic cost factors: cost of burn-in, cost of failure during burn-in, and the sale price of the unit being placed on burn-in. He weights the sale price by an expression of system performance: $KP_E(t)$, where K is the minimum number of units to complete the mission and $P_E(t)$ is the system effectiveness. One can see that as system effectiveness is increased, the number of required units decreases and the total cost is minimized. Washburn uses a generalized gamma distribution because of its ability to take on the form of many different distributions. His suggestions for solution are graphing directly or a computer search.

Jensen and Petersen (1982) review the state-of-the-art in stress screening cost models and offer one of their own. The problem with Jensen and Petersen's model is that they assume the burn-in environment is invariable and they optimize with respect to the burn-in duration only. This is a good source of references, however, for stress screening cost models.

III. MODEL DEVELOPMENT

This section of the paper lays the foundation for the development of the cost model. The formulation of the cost model is described, and a characterization of the cost function is given. The following section discusses the solution method and gives an example problem.

Once the basis of definitions is established, the costs associated with environmental stress screening are defined. It is assumed that there are three costs associated with stress screening reliability enhancement. There is a cost of a field failure. This cost includes such things as part replacement, field engineer costs, and customer dissatisfaction. The second cost is a cost for stress application. This cost may entail a fixed cost plus any operating costs incurred during stressing. The third cost represents the cost of an in-house failure. This could include part replacement, delivery delays, or increased production.

These three cost factors are combined to represent the total expected cost of stress screening.

Expected Total Cost =

Cost of Field Failure x Probability of Field Failure

\+ Cost of Stress Application

\+ Cost of In-House Failure x Probability of In-House Failure.

This same equation written in mathematical terms is:

$$E[C_T] = C_F[F(T+\tau(S_1,S_2,\ldots,S_i,t)|\tau(S_1,S_2,\ldots,S_i,t)] + C_S(S_1,S_2,\ldots,S_i,t) + C_I[F(\tau(S_1,S_2,\ldots,S_i,t)]. \quad (1)$$

(The symbol, τ, will be used to represent the function $\tau(S_1,S_2,\ldots,S_i,t)$ throughout the rest of this paper.)

C_F and C_I are the costs of field failure and in-house failure. $F(T+\tau|\tau)$ is the conditional probability of failure in the field by time T, given in-house survival up to time τ. τ represents the equivalent power-on hours experienced during stress application. $F(\tau)$ is the probability of an in-house failure by time τ. The cost of stress application, C_S, is represented as a function of the multiple stresses, $S_1,S_2,\ldots,S_i$, and the duration of the stresses, t. C_S is assumed to be a nondecreasing function with respect to the decision variables.

It is desirable to express the total expected cost in terms of reliabilities rather than failure probabilities. $F(T+\tau|\tau)$ may be rewritten as the following:

$$
\begin{aligned}
&F(T + \tau|\tau)\\
&= \frac{F(T + \tau) - F(\tau)}{1 - F(\tau)}\\
&= \frac{[1 - R(T + \tau)] - [1 - R(\tau)]}{R(\tau)}\\
&= \frac{R(\tau) - R(T + \tau)}{R(\tau)}\\
&= 1 - \frac{R(T + \tau)}{R(\tau)}\,. \qquad (2)
\end{aligned}
$$

Stress screening is only applicable to components with decreasing hazard rates. All of the electronic components studied to date show a Weibull failure distribution, which is one of only a few failure distributions that display decreasing hazard rates. (For a discussion of failure distributions, reliability, and hazard rates, see Kapur and Lamberson, 1977.) For this reason, the Weibull distribution is chosen to describe the reliability of a component, where

$$R(\tau) = e^{-(\frac{\tau}{\Theta})^{\beta}}. \tag{3}$$

β is the shape parameter and Θ is the scale parameter of the reliability function. Since Θ and β are constant valued parameters, the reliability function can be simplified by the substitution of $\alpha = \Theta^{-\beta}$. The cost model then takes on the form:

$$\begin{aligned} E[C_T] &= C_F[1 - e^{-\alpha(T+\tau)^{\beta}+\alpha\tau^{\beta}}] \\ &+ C_S(S_1,S_2,\ldots,S_i,t) \\ &+ C_I[1 - e^{-\alpha\tau^{\beta}}]. \end{aligned} \tag{4}$$

A further refinement of the cost expression is to write the total cost per replaceable unit; however, each unit consists of j components. The reliability of the unit can be modeled using a series representation. The unit might not be physically modeled using a series representation; however, in terms of reliability measures the unit responds according to a series network. Therefore, the expected total cost per replaceable unit is now:

$$\begin{aligned} E[C_T] &= C_F[1 - e^{\sum_j -\alpha_j(T+\tau_j)^{\beta_j}+\alpha_j\tau_j^{\beta_j}}] \\ &+ C_S(S_1,S_2,\ldots,S_i,t) \\ &+ C_I[1 - e^{\sum_j -\alpha_j\tau_j^{\beta_j}}]. \end{aligned} \tag{5}$$

Each component may experience a unique number of power-on hours, resulting in j different values of τ. To calculate τ the following result is given by Nachlas, Gruber, and Binney (1985).

$$\tau_j = a_j(S_1,S_2,\ldots,S_i)t, \tag{6}$$

where a_j is the acceleration factor associated with component j and t is the duration of the stress screen. It has been suggested that

$$a_j\ (S_1,S_2,\ldots,S_i) = e^{\sum_i g_{ji}(S_i)}, \tag{7}$$

where $g_{ji}(S_i)$ is the stress function associated with stress i and component j. (For example, the Arrhenius equation is the stress function for elevated temperature.)

Substituting equations (6) and (7) into the cost model gives the total cost expression in its final form.

$$E[C_T] =$$

$$C_F[1 - e^{\sum_j -\alpha_j(te^{\sum_i g_{ji}(S_i)}+T)^{\beta_j} + \alpha_j(te^{\sum_i g_{ji}(S_i)})^{\beta_j}}]$$

$$+ C_S(S_1,S_2,\ldots,S_i,t)$$

$$+ C_I[1 - e^{\sum_j -\alpha_j(te^{\sum_i g_{ji}(S_i)})^{\beta_j}}]. \tag{8}$$

The entire cost expression is now in terms of the decision variables, $S_1,S_2,\ldots,S_i$ and t.

In order to show logical consistency of the cost model, it must be true that the optimum value of the decision variable t is equal to zero, at any value of the S_i, when $\beta_j = 1$ for all j. Restating this, there should be no stress screening performed when a negative exponential distribution is chosen for the failure distribution. (In the case of the negative exponential distribution, a constant hazard rate is present; therefore, stress screening does not enhance the reliability of a population.) The proof of this result follows.

Substituting the negative exponential distribution for the field and in-house failure probabilities gives:

$$F(\tau) = 1 - e^{-\alpha\tau}$$

and

$$F(T+\tau|\tau) = 1 - \frac{e^{-\alpha(\tau+T)}}{e^{-\alpha\tau}}$$

$$= 1 - e^{-\alpha T}. \qquad (9)$$

Writing the failure probabilities in terms of multiple components and substituting back into equation (1) provides the following cost expression.

$$E[C_T] = C_F[1 - e^{\sum_j -\alpha_j T}]$$
$$+ C_S(S_1, S_2, \ldots, S_i, t)$$
$$+ C_I[1 - e^{\sum_j -\alpha_j \tau_j}]. \qquad (10)$$

$C_F[1 - e^{\sum_j -\alpha_j T}]$ is independent of the decision variables and can be dropped from the cost expression.

$$E[C_T] = C_S(S_1, S_2, \ldots, S_i, t) + C_I[1 - e^{\sum_j -\alpha_j \tau_j}]. \qquad (11)$$

The expected total cost is now a non-decreasing function with respect to the decision variables. The optimum solution in this case would be to perform no stress screening, or choose $t = 0$. This result is a necessary condition in the verification of the cost model.

The stress cost function, $C_S(S_1,S_2,\ldots,S_i,t)$, may take on many forms. The form chosen here is a fixed cost per item plus a variable cost. The variable cost is a function of the amount of stress applied, $S - S_0$, S_0 being the nominal stress, and the stress duration time, t.

$$C_S(S_1,S_2,\ldots,S_i,t) = \sum_i p_i + q_i(S_i - S_{0i})t. \quad (12)$$

When characterizing the total expected cost function, the first question one asks is whether or not the function is convex. If the function is convex, any local minimum solution found is a global minimum, and therefore an optimal solution. Unfortunately this cost function is nonconvex.

In order to prove the convexity of a function, the following inequality must be true for all points, $\underline{X}_1$, $\underline{X}_2$, $\lambda \in [0,1]$, in the solution set, S.

$$f(\lambda\underline{X}_1 + (1-\lambda)\underline{X}_2) < \lambda f(\underline{X}_1) + (1-\lambda)f(\underline{X}_2). \quad (13)$$

With $f(\underline{X})$ being the expected total cost function and $\underline{X}$ taking the form

$$\underline{X} = (t,S_1,S_2,S_3), \quad (14)$$

two points, $\underline{X}_1$ and $\underline{X}_2$, have been found that violate this inequality. The following values were calculated using the function parameters given in Chapter IV.

$$\underline{X}_1 = (100,\ 25,\ 8,\ 350)$$

$$\underline{X}_2 = (200,\ 100,\ 8,\ 298)$$

$$\lambda = .8 \qquad 1-\lambda = .2$$

$$\lambda\underline{X}_1 + (1-\lambda)\underline{X}_2 = (120,\ 40,\ 8,\ 340)$$

$$f(\underline{X}_1) = .8526 \qquad f(\underline{X}_2) = 1.3180$$

$$f(\lambda\underline{X}_1 + (1-\lambda)\underline{X}_2) = .9539$$

$$\lambda f(\underline{X}_1) + (1-\lambda)f(\underline{X}_2) = .9457$$

$$.9539 > .9457.$$

Since $f(\lambda\underline{X}_1 + (1-\lambda)\underline{X}_2) \not\leq \lambda f(\underline{X}_1) + (1-\lambda)f(\underline{X}_2)$ for all $\underline{X}_1$, $\underline{X}_2 \varepsilon S$, $\lambda \varepsilon [0,1]$, then the cost function is concluded to be nonconvex.

Working with a nonconvex function does present some difficulties. Primarily, one can no longer say that any local minimum found is a global minimum, and therefore an optimal solution. However, the bounds on the decision variables are relatively close which makes the establishment of an optimal solution a bit easier than if there had been no bounds. The approach to take is to find a number of local minima within the bounds of the decision variables and then search over these points for the best solution. The optimization routine is described in Chapter IV.

The gradient vector is shown in the Appendix. The Appendix also includes a discussion of the nature of the partial derivatives with respect to each of the decision variables. These findings may be significant in certain instances.

IV. MODEL SOLUTION

There are no algorithms that are generally accepted over others for solving nonconvex problems; therefore, the selection of an optimization algorithm was fairly arbitrary and mainly based on ease of implementation. The generalized reduced gradient algorithm was chosen to minimize this cost function because it is a very general algorithm that has been known to solve a wide range of problems. The software package GINO, published by The Scientific Press, was purchased for use on the IBM personal computer. GINO utilizes a modified version of the GRG algorithm called GRG2, developed by Leon Lasdon and Allan Waren.

When finding the best solution to this problem a number of local minima emerge. Searching over these local minima produces the minimum solution and therefore an "optimal" solution. Since this function is nonconvex, one cannot claim to have found the true optimal solution. In individual cases, certain simplifications in the problem may be made in order to facilitate the comparison of local minima. These simplifications are discussed in the presentation of the following example problem.

Before discussing the specific example, it should be noted that this cost function does indeed represent the trade-off between a nonincreasing cost and nondecreasing costs. It must be realized that this cost model will behave as stated only when the components exhibit a decreasing hazard rate, as with the Weibull distribution. Referring to equation (15), the first expression of the model describes the cost due to field failures. As the function τ increases, this cost is nonincreasing. The second expression, cost of stress application, has been assumed to be a nondecreasing function. The third function is nondecreasing also since the probability expression is simply the cumulative distribution function. While this information might lead one to assume that the cost function is convex, it has been shown in Chapter III that the reverse is true.

To demonstrate the utility of the cost model, the following example problem was chosen. It is assumed that there are five components in a particular unit. The objective is to minimize the trade-off costs between failures in the field and in-house failures due to stress screening. Also, assume that there are three stresses that produce an acceleration effect on the five components. It is appropriate to note here that not every stress will produce an acceleration on every type of component. For example, an hermetically sealed unit, manufactured in a controlled environment, will not benefit from a humidity stress.

The problem is:

Minimize:

$$E[C_T] = C_F[1 - \exp(\sum_{j=1}^{5} - \alpha_j(te^{\sum_{i=1}^{3} d_{ji} + h_{ji}S_i} + T)^{\beta_j}$$

$$+ \alpha_j(te^{\sum_{i=1}^{3} d_{ji} + h_{ji}S_i})^{\beta_j})]$$

$$+ \sum_{i=1}^{3} (p_i + q_i(S_i - S_{oi})t)$$

$$+ C_I[1 - \exp(\sum_{j=1}^{5} - \alpha_j(te^{\sum_{i=1}^{3} d_{ji} + h_{ji}S_i})^{\beta_j})]$$

s.t. $0 < t < U_t$

$$S_{0i} < S_i < U_i \quad \text{for } i = 1, 2, 3. \tag{15}$$

The lower bound for the stress variable is the nominal stress level. It is assumed that subjecting an item to a stress at or below the nominal level gives no acceleration effect.

The values of the Weibull parameters, α_j, β_j, the stress function parameters, d_{ji}, h_{ji}, and the cost parameters, C_F, C_I, p_i, q_i, have been realistically constructed. The behavior of the types of components subjected to stress screening is fairly well-known (Nachlas, Gruber, Wiesel, 1984); therefore, the relative magnitude of the Weibull parameters can be obtained. No real-life cost data was obtainable for this study.

As stated previously, the form of the stress function,

$$g(s) = d + hs, \tag{16}$$

was suggested in a paper by Nachlas, Gruber, and Binney, 1985. Again, no real-life values for d and h were obtainable; however, it is felt that the numbers chosen for this example are sufficient to demonstrate the use of the cost model. The values of the model parameters are listed below. The effective stresses chosen for this example are humidity, power cycling, and temperature. Note that the stress function for temperature,

$$g(S) = \frac{E_A}{K}\left(\frac{1}{S_0} - \frac{1}{S}\right), \tag{17}$$

has been put into the form

$$g(S') = d + hS', \tag{18}$$

where $d = \frac{E_A}{KS_0}$, $h = \frac{-E_A}{K}$, $S' = \frac{1}{S}$,

and $K = 8.623 \times 10^{-5}$ (Boltzman's Constant.)

This was done so that the stress functions would all have the same form. Also, observe that when a stress is at its nominal level, $d_i = -h_i S_i$, or the resulting acceleration is $e^0 = 1$. This is a necessary characteristic of the stress function. There should be no increased acceleration effect at the nominal stress level.

Weibull parameters

$\alpha_1 = 2.25 \times 10^{-6}$	$\beta_1 = .40$
$\alpha_2 = 4 \times 10^{-6}$	$\beta_2 = .45$
$\alpha_3 = 6.25 \times 10^{-6}$	$\beta_3 = .50$
$\alpha_4 = 8.25 \times 10^{-6}$	$\beta_4 = .55$
$\alpha_5 = 9.4 \times 10^{-6}$	$\beta_5 = .60$

Stress Function parameters

$d_{11} = -.03875$	$h_{11} = 1.55 \times 10^{-3}$	
$d_{12} = -.304$	$h_{12} = .152$	
$d_{13} = 38.915$	$h_{13} = -11596.9$	$(E_A = 1)$
$d_{21} = -.0365$	$h_{21} = 1.46 \times 10^{-3}$	
$d_{22} = -.276$	$h_{22} = .138$	
$d_{23} = 38.915$	$h_{23} = -11596.9$	$(E_A = 1)$

$d_{31} = -.0345$ $h_{31} = 1.38 \times 10^{-3}$

$d_{22} = -.28$ $h_{32} = .14$

$d_{23} = 38.915$ $h_{33} = -11596.9$ $(E_A = 1)$

$d_{41} = -.038$ $h_{41} = 1.52 \times 10^{-3}$

$d_{42} = -.31$ $h_{42} = .155$

$d_{43} = 31.132$ $h_{43} = -9277.5$ $(E_A = .8)$

$d_{51} = -.037$ $h_{51} = 1.48 \times 10^{-3}$

$d_{52} = -.31$ $h_{52} = .155$

$d_{53} = 31.132$ $h_{53} = -9277.5$ $(E_A = .8)$

Cost parameters

C_F = \$100 per item field failure

C_I = \$1 per item in-house failure

$p_1 = .1$ (\$/per item) $q_1 = 9 \times 10^{-7}$ (\$/stress unit - time unit)

$p_2 = .07$ $q_2 = 8 \times 10^{-4}$

$p_3 = .05$ $q_3 = 2.25 \times 10^{-4}$

Nominal Stress levels

$S_{01} = 25\%$ $S_{02} = 2$ cycles/hour $S'_{03} = 33.557$

Bounds

$25 < S_1 < 75$ $2 < S_2 < 8$ $28 < S'_3 < 33.557$

$0 < t < 300$

Note: $S_3' = \frac{10,000}{S_3}$

$S_{03}' = 33.557$ converts to 298° Kelvin.

Scaling was done in order to have decision variable values between 1 and 100. Different scaling procedures were applied and were found to have no difference on the solution of the problem.

Using the GRG2 algorithm discussed earlier, numerous local minima were found. Searching over all local minimum solutions detected produced the solution of

$t^* = 22.44$ hours

$S_1^* = 75\%$ relative humidity

$S_2^* = 2$ power cycles/hour

$S_3^* = 350° \text{ K} \approx 77° \text{ C}$

The total expected cost is $.325 per item.

Notice that the stress levels are at their boundaries. S_1 and S_3 are at their upper bounds and S_2 is at its lower bound. Examining the cost function gives an intuitive explanation for this. The stress costs for S_1 and S_3 are lower than the stress cost for S_2. Also, the amount of acceleration gained from elevated temperature is far greater than that gained from power cycling or increased humidity.

$$g_{11}(S_1) = -.03875 + .00155\, S_1 \tag{19}$$

For $S_1 = 75$ $\quad\exp[g_{11}(S_1)] = 1.0806$

$$g_{12}(S_2) = -.304 + .152\, S_2 \tag{20}$$

For $S_2 = 8$ $\quad\exp[g_{12}(S_2)] = 2.4893$

$$g_{13}(S_3) = 38.915 - 11596.8 \left(\frac{1}{S_3}\right) \quad (21)$$

For $S_3 = 350$ $\exp[g_{13}(S_3)] = 324.1757$

The trade-off between cost of stress and amount of acceleration is the key factor in this cost model. The cost of power cycling was found to be greater than the cost saved from the power cycling acceleration. One can see that in most cases, elevated temperature will be the preferred stress, provided the cost of elevating the temperature is sufficiently low.

An analysis of the cost parameters and their effect on the solution was performed. It was found that the exact magnitude of the cost coefficients is not needed. Rather, the relationship between the cost parameters is necessary. Increasing or decreasing the costs does not change the solution as long as the ratio between the costs is maintained. Furthermore, the value of the fixed charges of stressing, p_i, does not affect the solution in any way. It is simply added to the objective function value. Both of these observations can be arrived at by examining the cost function. The fixed charges have no multipliers and therefore do not affect the values of t or S_i. The other cost parameters, C_F, C_I, q_i, are all multiplied by expressions containing both t and S_i. As long as the ratio between C_F, C_I, and q_i is held constant, the values of the decision variables will not be affected.

When solving this problem, in order to find as many local minima as possible, different starting solutions were used. Based on the findings of this research, it appears that the stress variables will take on either an upper or lower bound value. Either the cost of stressing is low enough to justify the use of a stress, in which case the maximum amount of stress is used, or the cost is too high and the stress is not used at all.

To reduce the number of minimum cost comparisons, it is suggested that starting solutions contain stress level values at their boundaries. This should help find the best solution much faster when a large number of stress variables are present. An examination of the stress cost coefficients should give an indication of which value the stress is likely to take.

At the outstart of this project undertaking, it was anticipated that direct implementation of this cost model and an analysis of those findings would add to the results of this thesis. Unfortunately that has not been the case. Had real-life data been available, further analysis would have taken place in verifying the suggested form of equation (12). The findings tend to show that the parameters chosen for the stress cost function are key factors in the solution of the problem; therefore, this is an area for further research.

Another area for further investigation is the form of equaton (7). At present, the best data available suggests the form

$$a = \exp[\sum_i d_i + h_i S_i]. \qquad (22)$$

(See Nachlas, Gruber and Binney, 1985.) Proof of any other type of relationship between stresses may lead to different results than those found during this study.

It is felt that although there are still areas open for investigation, the prime objective of this thesis has been met. A valid cost model for multiple stress, multiple component stress screening has been developed. The use of this model has been shown through an example problem. It is concluded that solving the model will lead one to selecting an economically superior stress screening regimen.

V. BIBLIOGRAPHY

1. J. Abadie, Integer and Nonlinear Programming, J. Abadie (Ed.), North Holland Publishing Company - Amsterdam, 1970.

2. Mokhtar S. Bazaraa and C. M. Shetty, Nonlinear Programming: Theory and Algorithms, John Wiley and Sons, New York, 1979.

3. Ronald Canfield, "Cost-Effective Burn-in and Replacement Times," IEEE Transactions on Reliability, Vol. R-24, No. 2, June 1975, pp. 154-156.

4. R. Chandrasekaran, "Optimal Policies for Burn-in Procedures," Opsearch, Vol. 14, No. 3, 1977, pp. 149-160.

5. J. Douglas Ekings, "Profit and Customer Satisfaction Equals the Specification for Commercial Reliability Programs," Proceedings 1976 Annual Reliability and Maintainability Symposium, 1976, pp. 63-71.

6. Eugene L. Grant and Lawrance F. Bell, "Some Comments on the Semantics of Quality and Reliability," Industrial Quality Control, Vol. XVII, No. 11, May 1961, pp. 14-17.

7. Gerald J. Hahn and Wayne Nelson, "A Comparison of Methods for Analyzing Censored Life Data to Estimate Relationships Between Stress and Product Life," IEEE Transactions on Reliability, Vol. R-23, No. 1, April 1974, pp. 2-10.

8. Finn Jensen and Niels Erik Petersen, Burn-In, John Wiley and Sons, New York, 1982.

9. K. C. Kapur and L. R. Lamberson, Reliability in Engineering Design, John Wiley and Sons, New York, 1977.

10. Thomas J. Kielpinski and Wayne Nelson, "Optimum Censored Accelerated Life Tests for Normal and Lognormal Life Distributions," IEEE Transactions on Reliability, Vol. R-24, No. 5, December 1975, pp. 310-320.

11. J. Kowalik and M. R. Osborne, Methods for Unconstrained Optimization Problems, American Elsevier, New York, 1968.

12. Way Kuo, "On Optimal Burn-in Modeling and its Application to an Electronic Product," Proceedings Third International Conference on Reliability and Maintainability, October 1982, pp. 517-524.

13. Way Kuo and Yue Kuo, "Facing the Headaches of Early Failures: A State-of-the-Art Review of Burn-in Decisions," Proceedings of the IEEE, Vol. 71, No. 11, November 1983, pp. 1257-1266.

14. Leon Lasdon and Allan Waren, GINO (General INteractive Optimizer), The Scientific Press, Palo Alto, CA, 1985.

15. Joel A. Nachlas, Blair A. Binney, and Stephen S. Gruber, "Aging Acceleration Under Multiple Stresses," Proceedings 1985 Annual Reliability and Maintainabililty Symposium, January 1985, pp. 438-440.

16. Joel A. Nachlas, Stephen S. Gruber, and Harry Z. Wiesel, "Sensitivity in Weibull System Reliability Models," Proceedings 1984 Annual Reliability and Maintainability Symposium, January 1984, pp. 428-433.

17. E. G. D. Paterson, "Quality Control vs. Quality Assurance vs. Reliability," Industrial Quality Control, Vol. 19, No. 4, October 1962, pp. 5-9.

18. D. S. Peck and O. D. Trapp, Accelerated Testing Handbook, Technology Associates, California, 1978.

19. K. T. Plesser and T. O. Field, "Cost-Optimized Burn-in Duration for Repairable Electronic Systems," IEEE Transactions on Reliability, Vol. 3-26, No. 3, August 1977, pp. 195-197.

20. M. R. Reda, S. G. Brown, and K. L. Menze, "High Temperature Burn-in and its Effect on Reliability," Proceedings 1976 Annual Reliability and Maintainability Symposium, 1976, pp. 72-75.

21. Herman D. Rue, "System Burn-in for Reliability Enhancement," Proceedings 1976 Annual Reliability and Maintainability Symposium, 1976, pp. 336-341.

22. Hiroshi Shiomi, "Application of Cumulative Degradation Model to Acceleration Life Test," IEEE Transactions on Reliability, Vol. R-17, No. 1, March 1968, pp. 27-33.

23. L. T. Stewart and J. D. Johnson, "Determining Optimum Burn-in and Replacement Times Using Bayesian Decision Theory," IEEE Transactions on Reliability, Vol. R-21, No. 3, August 1972, pp. 170-175.

24. R. L. Vander Hamm, "Environmental Testing - the Key to High Reliability," Proceedings 1969 Annual Reliability and Maintainability Symposium, 1969, pp. 27-33.

25. Leonard A. Washburn, "Determination of Optimum Burn-in Time: A Composite Criterion," IEEE Transactions on Reliability, Vol. R-19, No. 4, 1970, pp. 134-140.

26. G. S. Watson and W. T. Wells, "On the Possibility of Improving the Mean Useful Life of Items by Eliminating Those with Short Lives," Technometrics, Vol. 3, No. 2, May 1961, pp. 281-298.

27. George H. Weiss and Menachem Dishon, "Some Economic Problems Related to Burn-in Programs," IEEE Transactions on Reliability, Vol. R-20, No. 3, August 1971, pp. 190-195.

VI. APPENDIX

The gradient vector is shown here. For simplicity, only two decision variables are used: stress duration time, t, and one stress variable, s.

$$E[C_T] = C_F + C_I + C_s(s,t) - C_F R(T+\tau|\tau) - C_I R(\tau), \tag{a1}$$

where,

$$R(T+\tau|\tau) = \exp[-\alpha(te^{d+hs} + T)^{\beta} + \alpha(te^{d+hs})^{\beta}] \tag{a2}$$

$$R(\tau) = \exp[-\alpha(te^{d+hs})^{\beta}] \tag{a3}$$

$$C_s(s,t) = p + q(s-s_0)t \tag{a4}$$

$$\nabla E[C\ ]_T = \begin{bmatrix} q(s-s_0) \\ + C_F R(T+\tau|\tau)[\alpha\beta e^{d+hs}][(te^{d+hs}+T)^{\beta-1} - (te^{d+hs})^{\beta-1}] \\ + C_I R(\tau)[\alpha\beta e^{d+hs}][te^{d+hs})^{\beta-1}] \\ \\ qt \\ + C_F R(T+\tau|\tau)[\alpha\beta hte^{d+hs}][(te^{d+hs}+T)^{\beta-1} - (te^{d+hs})^{\beta-1}] \\ + C_I R(\tau)[\alpha\beta hte^{d+hs}][(te^{d+hs})^{\beta-1}] \end{bmatrix} \tag{a5}$$

It is desirable to show that the cost function is convex with respect to each of the decision variables. To do this, the second partial derivative must be shown to be greater than or equal to zero. Consider the second partial with respect to time.

$$\frac{d^2E[C_T]}{dt^2} = -C_F R(T+\tau|\tau)[\alpha\beta e^{d+hs}]^2 \, [(te^{d+hs}+T)^{\beta-1} - (te^{d+hs})^{\beta-1}]^2$$

$$+ C_F R(T+\tau|\tau)[\alpha\beta(1-\beta)(e^{d+hs})^2][(te^{d+hs})^{\beta-2} - (te^{d+hs}+T)^{\beta-2}]$$

$$- C_I R(\tau)[\alpha\beta e^{d+hs}(te^{d+hs})^{\beta-1}]^2$$

$$- C_I R(\tau)[\alpha\beta(1-\beta)(e^{d+hs})^2(te^{d+hs})^{\beta-2}]. \qquad \text{(a6)}$$

There are many variables that can affect the behavior of this derivative. The following relationships are known:

$C_F < C_I$,

$R(\tau) < R(T+\tau|\tau)$ as τ increases,

$0 < \beta < 1$, and

$(te^{d+hs}) < (te^{d+hs} + T)$.

Given these relationships, it is still not clear if this derivative is positive or not. Although no strong conclusions can be drawn from the gradient and Hessian matrices (the second derivative with respect to the stress variable proved as unpromising as this partial derivative), it may be possible for the cost function to be convex in each of the decision variables separately, for certain values of the function parameters. If this fact could be shown to be true, it might be beneficial to fix all decision variables except the one of particular interest. The problem could then be treated as a convex minimization problem. An investigation into this matter may prove fruitful; however, it is beyond the scope of this research.

The vita has been removed from the scanned document