

Design and steady-state analysis of the switched reluctance motor drive

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(ABSTRACT)

In the last two decades there has been a revival of interest in variable reluctance drives mainly due to the advent of high power semiconductor devices and improvements in the understanding of the principles of electromagnetic energy conversion. In particular, the switched reluctance motor (SRM) has received attention mainly due to its simple construction and robustness when compared to other variable-speed drives. Considerable research has been done on topics ranging from the design to the control of the motor. Due to the high nonlinearity of the machine, even the prediction of the steady-state performance of the drive has been difficult. In the attempt to overcome the nonlinearity problem, researchers have resorted to computer solutions. The Finite Element Analysis (FEA) method has been used to predict the steady state performance of the motor. While this has improved the accuracy of performance prediction, it is very time intensive. This is unacceptable in an industrial situation where the PC is the main design tool. The need for analytical methods therefore exists. An analytical method for the steady-state performance prediction of the SRM drive based on an improved method for estimating the maximum and minimum inductances is described. This method is extended to include determination of the motor inductances at any rotor position and subsequently, the prediction of the steady-state average torque. The work also proposes an analytical method for determining the core losses of the SRM. Considerable effort is also dedicated to the

design and analysis of new and old SRM converter topologies with particular attention to the criteria governing the choice of converter topology, the prediction of key waveforms, the criteria for selecting power semiconductor devices and the determination of device ratings. A novel method for the direct steady-state analysis of the SRM drive without going through the transient solution is proposed. The effect of motor geometry on converter ratings is also investigated for the common SRM pole combinations. Novel methods for the measurement and instrumentation of SRM are also described. Theoretical predictions are verified by experimental results using a 6/4 pole prototype switched reluctance motor.

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1.0 INTRODUCTION

1.1 *The switched reluctance motor drive*

The switched reluctance motor drive belongs to the family of modern variable speed drives which have received considerable attention in the last two decades [1,2,3,]. Although the first motor operating on the variable reluctance principle was built as early as 1842 [4], it was not until the early 1960's that the on-going research efforts were started. The current research efforts were motivated by the invention of high power semiconductor devices and improved understanding of the principles of electromagnetic energy conversion [5]. When compared to the modern variable speed induction motor drive, the SRM drive is simpler in construction and has at least equivalent performance in terms of torque per unit volume, efficiency and volt-ampere requirements [6,7]. Its torque to inertia ratio surpasses that of the induction motor mainly because of the absence of rotor windings [8]. The complete absence of windings, brushes or permanent magnets in the rotor is one of the most outstand-

ing features of the SRM. This feature makes the rotor mechanically and thermally very robust.

The switched reluctance motor is doubly salient with even but unequal number of poles in the stator and rotor. Each stator pole carries a concentrated winding similar to the field coils of a d.c. motor. Except for the single phase SRM, windings on diametrically opposite poles are connected in series and constitute one phase of the motor. By exciting the phases in the proper sequence, the motor can be made to operate as a four quadrant drive using a unipolar converter. A position sensor is normally mounted on the shaft, the output of which is used to determine the appropriate switching instants for each phase. The single phase SRM has the same number of poles on the stator and rotor and carries a single circular winding which embraces all the poles [9].

The torque output of the SRM is proportional to the square of the applied current and therefore only unidirectional currents are required. This makes the converter relatively simple when compared to other variable speed drives. Several converter topologies have been proposed in the literature [66,69,70,71]. The distinguishing feature of these topologies lies in the method of regeneration used. The basic converter circuit has only one switch per phase which is connected in series with the phase winding. Therefore, there is no possibility of 'shoot-through' problems common in multi-phase bridge inverters for variable-speed a.c. drives. For the majority of SRM converter topologies, individual phases operate in total independence from other phases. This permits the drive to continue to operate at reduced power without

the problems of negative sequence fields and high short circuit currents even if one phase becomes inoperative.

As a double salient motor, the SRM falls in the same category as the variable reluctance stepper motor. Its operation is based on the tendency of a magnetic circuit to align itself in the direction of minimum reluctance. Unlike the stepper motor however, the SRM is designed for efficient power conversion at variable speed rather than as a torque motor [12]. Its pole geometry and the strategy for control is therefore different. Usually, the number of poles is small, the most common stator/rotor pole combinations being 4/2, 6/4, 8/6 and 12/10 [11]. Further, the conduction angle is not held constant but is modulated as a function of both torque and speed.

The operation of the motor is as follows. When a stator phase is excited, the rotor moves so as to align itself with the magnetic axis of the excited phase. This position corresponds to the minimum reluctance and therefore gives the maximum phase inductance. The minimum reluctance position is referred to as the fully aligned position. The position corresponding to maximum reluctance is called the fully unaligned position and occurs when a pair of rotor poles approaching alignment is half a rotor pole pitch away from the axis of the excited stator poles. Just prior to full alignment, the next stator phase is excited so that the rotor continues to move towards alignment with the currently excited phase and so on. Depending on the converter topology and the mode of operation, the previously excited phase may be turned off prior to or after the next phase is excited. This sequence is repeated so that the motor runs at a smooth speed. The direction of rotation is opposite to the switching

sequence. Reversal of direction is effected by reversing the switching sequence. Torque production is dependent on the change of co-energy as the rotor moves from the fully unaligned to the fully aligned position. For given motor dimensions, the change of co-energy is dependent on the magnitude and waveform of the phase current.

The basic function of the SRM drive controller is to provide the switching signals to the appropriate phase switch at the right time and to modulate the conduction (dwell) angle as a function of speed and/or torque. To perform these functions, the controller requires the rotor position and phase current as inputs. A shaft-mounted rotor position sensor is normally used. Recently, there has been a study on sensorless methods for estimating rotor position [13,14,72]. The controller can be implemented using analog, microprocessor or personal computer- based techniques [15,16,17,18].

1.2 Historical background and previous research

The history of the switched reluctance motor drive dates back to more than 150 years. The first machine operating on the principle of electromagnetic energy conversion through synchronous commutation of the machine reluctance was reported in 1835, shortly after the discovery of the electromagnet in 1830 [4]. The machine was later patented in the United States in 1838 under the name 'electromagnetic engine'. It consisted of electromagnets which formed the stator, and a rotor made of solid iron masses constituting the rotor poles. To operate the machine, the electromagnets

were magnetized and demagnetized by applying unipolar currents. When the electromagnets were energized, the rotor poles moved and aligned themselves with the excited electromagnets. The rotor continued to rotate under its own inertia when the electromagnets were de-energized. In some cases, flywheels were used to smoothen the speed. Cam driven switches served both as position sensors and power controllers.

Several researchers in the United Kingdom and the United States built similar motors in the period between 1840 and 1859 [19,20]. The largest motor in this period was built in the United States in 1851. The machine, intended for traction purposes, had an output greater than 8 h.p. and cost the U.S. government \$30,000 and the inventor \$6000 [21]. The exorbitant cost of the drive dictated against any further development. But cost was not the only problem [22]. First, the wooden frame that supported the electromagnets was not strong enough to withstand the pulsating forces of the drive. To reduce these forces, the air-gap was made larger resulting in the loss of output. Secondly, there was no provision for energy recovery. The energy stored in the magnetic field during the working stroke was wasted in the form of arcs at the commutating switches. This resulted in severe contact wear. Thirdly, the rotor was made of solid iron and therefore the eddy current losses were high. Fourthly, the machines had a low energy conversion efficiency because they were operated in the linear mode. Finally, the switching duty cycle was constant offering no possibility of dwell angle modulation. These problems cumulatively led to the collapse of the efforts to develop the electromagnetic engine especially after the development of the ring-wound armature (d.c.) machine in 1860.

The currently on-going research efforts on the SRM drive started in the 1960's after the advent of the high-power semiconductor devices like thyristors and , later, power transistors. This was complimented by extensive research on reluctance motors and the variable reluctance stepper motor [23,24,25,26]. Between 1961 and 1965, a 3-phase SRM drive was developed using a cycloconverter for power conditioning [27]. Development of the drive was abandoned because the cost for providing the necessary commutating circuits for the thyristors was considered to be unacceptable. In 1963, researchers at the University of Leeds in England introduced the segmented rotor reluctance motor [28]. The machine had a distributed winding on the stator and a segmented rotor with no windings. Although this was a singly salient machine, it demonstrated the possibility of electromagnetic energy conversion without any winding on the rotor.

The first modern switched reluctance motor was patented in 1973 [29]. The motor was a 4/2 pole SRM with the poles designed so as to produce a uniform rate of change of flux linkages during a working stroke. The motor was designed to produce its optimum torque when saturated so that energy conversion could be maximized. This was the first time that the idea of exploiting magnetic non-linearity to increase the specific output of a reluctance machine was exploited although the problem of poor energy conversion efficiency in singly excited linear magnetic systems had been described as early as 1851 [30]. At about the same time, an axial air-gap reluctance motor for battery powered traction application was developed in the U.S.A. [31]. At that time, the cost of the motor and mechanical problems hindered further development.

Up to this time, a thorough and comprehensive treatment of the theory and design of this drive had been lacking. In 1977, an attempt to provide the missing link using a 3-phase SRM as an illustration was described [32]. The analysis provided insight into methods for predicting current waveforms, power output, switching angles and optimum pole and air-gap dimensions. However, the analysis failed to recognize the importance of advancing the phase current into a phase so that by the time alignment begins, the current shall have attained the commanded value. This is a necessary condition for maximum energy conversion efficiency. The analysis also neglected the effect of the negative torque produced during regeneration if the conduction angle is too large such that current flows through the phase winding when the rate of change of inductance with rotor position is negative.

Another factor which contributed to the development of the SRM drive in the last two decades is the increased understanding of factors which affect energy conversion efficiency. Based on some unpublished works of C. W. Carter, linear permeance factors for doubly salient structures were derived [33]. Using these factors, a method for determining the limiting torque of a doubly salient structure like the SRM was proposed [34]. The authors showed the dependence of the motor torque on the effective rotor volume, the excitation current and the shape of the current waveform. A more accurate method which takes the non-linearity of the motor into consideration was later proposed in 1977 [35]. The method predicts motor torque and current waveform from measured or predicted magnetization data. To predict the magnetization data, the same authors proposed an analytical method for estimating the maximum and minimum inductances of the motor from the motor dimensions, excitation current and winding data [36]. The method assumes that in the unaligned (minimum inductance) position, the flux lines consist of straight line segments and

concentric circular arcs. The permeability of iron is assumed to be infinite in this position so that all the applied m.m.f. drops across the air-gap. In the aligned (maximum inductance) position, the flux lines are assumed to link the stator and rotor perfectly without any leakage. Errors obtained with this method were attributed to the failure to accurately predict the end-leakage flux.

Although most of the research effort has concentrated on the motor, some work has also been done on the converter. In 1979, researchers at the Nottingham University in England described the principle features of thyristor SRM converters [37]. Using linear analysis, they proposed a method for predicting phase current waveforms and r.m.s. current. A follow-up paper on SRM converter circuits and component ratings was published by the same authors in 1980 [10].

In recognition of the inevitable interdependence between the motor and converter, there has been recent efforts to understand the SRM converter beyond the linear model. In 1985, a method for estimating the volt-ampere requirements of the SRM converter was proposed [38]. The method assumes a flat-topped current and approximates the nonlinear magnetization characteristic of the motor by a piecewise-linear curve. Using this approach, it was established that the maximum possible energy conversion efficiency when operating in the linear mode is 50 %. The same fact had been established earlier for a general singly excited magnetic storage system [29]. Converter circuit topologies have also been explored; the main difference between them being in the method of regeneration used. Accordingly, there are single-switch-per phase, two-switch-per phase and quasi-two-switch-per phase topologies. These are conveniently referred to here as SSPP, TSPP and QTSPP topologies. The SSPP converters have only one switch per phase. Energy stored in

the winding during the working stroke is either recovered using a bifilar winding or dissipated in a resistor. The TSPP topologies on the other hand have two switches per phase. When both switches are off, -1 p.u. supply voltage is applied to the phase winding so that energy recovery is accelerated. There is therefore almost perfect phase independence. The QTSP topologies have one commutating switch in addition to the single phase switches. When this switch and a phase switch are both turned off, -1 p.u. supply voltage is applied to the phase winding as is the case in TSPP topologies, thus accelerating energy recovery. However, the commutating switch is turned on again when the next phase is excited thus slowing down the energy recovery process in much the same way as in a SSPP converter. In 1985, a transistorized QTSP converter which was called the 'C-Dump' SRM converter was reported [11]. The name of the converter derives from the fact that the energy trapped in the phase winding during a working stroke is first dumped into a capacitor after turn-off. Later, the energy is either fed back to the source by a resonant process performed by an auxiliary circuit or is dissipated in a resistor. The resonant type was found to be energy efficient at the expense of increased circuit complexity. It is also limited to those applications where the source can accept the waveform of the resonant current. Various other forms of QTSP converters have also been proposed [3,37,40]. However, a thorough nonlinear analysis and experimental verification of the capabilities and limitations of the various topologies is currently lacking.

In 1980, a broad treatment of the SRM drive as a variable speed drive was described [1]. Guidelines to the selection of the number of phases, pole arcs, pole combinations, pole pitch ratio and control strategy were described in addition to a broad treatment of the principle of operation and the steady state characteristics and waveforms. Although the author used the linear model in most illustrations, the au-

thor recognized the importance of operating in saturation in order to increase the energy conversion efficiency. This work has become a valuable reference to many researchers. Since then there has been subsequent efforts to establish a comprehensive design procedure for the SRM. The first in 1984, was an extension of a method developed earlier for variable reluctance stepper motors [12,41]. The procedure was applied to the design of a two-teeth-per pole SRM. Later in 1985, a step-by-step procedure for the design of switched reluctance motors for non-servo applications was proposed [42]. The procedure is very similar to that used for the design of conventional rotating machines. It is therefore easily adaptable by experienced design engineers. A Computer-Aided-Design (CAD) algorithm based on this procedure was recently described [43,44].

Among the most outstanding developments in this decade was the use of Finite Element Analysis (FEA) to determine the magnetic field distribution of the SRM and its application in predicting the performance of the motor. The first work was described in 1985 [45]. Using a 2-dimensional FEA method, the inductances, flux linkages and motor torque were predicted assuming rectangular blocks of current. This method was later used to determine and compare the performance of the single- and two-teeth-per pole switched reluctance motors [46,47]. Although the 2-dimensional FEA method neglects end effects, it provides a systematic tool for dealing with the non-linearity of the SRM. For example, using the method, it was shown for the first time, that up to 30 % more torque can be produced if the phase current is applied well in advance of the onset of alignment [47].

A comparison between the 2-dimensional and 3-dimensional FEA methods was described recently [48]. The authors observed, that for a finite element mesh with the

same degree of fineness, the values of inductance and flux density obtained in the aligned position are almost the same for the two methods. The corresponding values in the unaligned position were found to differ considerably mainly because the 2-dimensional method neglects end effects which are more severe in this position. However, these differences were observed to have minor effect on the predicted torque.

Considerable effort has also been devoted to the comparison between the variable speed SRM drive and its main competitor - the variable speed induction motor drive [6,7,8]. The simplicity and robustness of the SRM is its strongest feature. It has been shown that the SRM is superior to the induction motor in torque/inertia ratio and, for some designs, power density. The converter is also simpler considering the fact that no reversal of current is required during the cycle as is the case in the inverter of an induction motor drive. Apart from having low inertia, the absence of a winding in the rotor has the advantage of eliminating the problems associated with imperfections in the casting and fatigue faults in the cage of an induction motor. The SRM is therefore capable of operating at higher speeds than the induction motor [7]. One researcher compared the SRM drive to a variable speed d.c. motor [6]. The superiority of the SRM drive in this case lies mainly in the difference in the costs of the motors. The cost of a d.c. motor is high and the maintenance cost is considerable whereas the cost of a SRM is likely to decrease dramatically once the design and manufacturing technology matures.

A single phase version of the SRM was recently described [9]. The motor has equal number of poles on the stator and rotor. The stator carries a simple circular winding which embraces all the poles. The stator poles are simultaneously magnetized and

demagnetized. When the winding is energized, the rotor poles move to align themselves with the stator poles; when de-energized, the rotor continues to rotate under its own inertia. A parking magnet is provided so that the motor can be started from rest in any direction of rotation. The motor was operated at a maximum duty cycle of 25 % giving a maximum measured efficiency of 28 % at 500 r.p.m.

Though most of the research on the SRM drive has concentrated on variable-speed applications, there has also been some focus on servo applications [49]. Two prototypes, designed to produce rectangular and trapezoidal torque profiles from trapezoidal and rectangular current waveforms respectively, were built and tested. These were unsuccessful because it was not possible to obtain the desired current waveforms at high speeds. The design was later modified so that the applied current consisted of half sinusoids. The torque versus rotor position profile was therefore sinusoidal giving an average-to-peak torque ripple of less than 5 %.

The control of the drive has also been of interest to researchers. The controllers developed in the early stages of the development of the drive were mainly analog [15]. Later, microprocessor-based controllers were proposed [16,17,50,51]. Recently, a highly flexible personal computer-based controller was proposed [18]. Though not cost effective at low power levels, the PC-based controller offers extensive flexibility which is useful in determining the best control strategy for the drive. Sensorless controllers have also been proposed recently [13,14,72]. These controllers operate without a shaft-mounted position sensor; the rotor position being estimated from terminal parameters and/or the inductance profile. The effectiveness of these controllers is yet to be fully verified.

1.3 Contributions

The rapidly growing interest in the switched reluctance drive has created the need for accurate analytical methods for the design and steady-state analysis of the drive. Since the converter and motor operate as an integral unit, there is also the need to examine alternative SRM converter topologies and develop design and analysis procedures for the same. This research work contributes to these objectives through the following focuses:

1. Development of an improved analytical method for determining the motor inductance at all rotor positions and the subsequent prediction of the steady-state average torque.
2. A comparative analysis between the analytical and FEA methods and between these methods and experimental measurements.
3. Development of a novel method for estimating the core losses in the SRM.
4. Development of a PC-based method for the direct prediction of the steady-state waveforms of the SRM drive, inclusive of the input filter and switching-aid networks.
5. Design and steady-state analysis of two new SRM converter configurations and one low cost topology for low performance drives.

6. Investigation of the effect of motor geometry on converter ratings and performance.
7. Development and comparison of measurement and instrumentation techniques for switched reluctance motors.

1.4 Organization

This dissertation is presented in ten chapters, each of which is an introduction and a conclusion. This arrangement is intended to provide the reader with a summary of the materials covered in each chapter. The material is supported by experimental verification based on a 6/4 pole laboratory prototype SRM, details of which are given in Appendix A.

Chapter 2 describes an analytical method for the prediction of the inductance profile and the steady-state average torque of the motor. Using this method, the motor inductance at any rotor position can be predicted from machine dimensions, the magnetization characteristic of the core material and electric loading. The effects of both local and bulk saturation are taken into consideration at all rotor positions. The use of a 2-dimensional finite element analysis method in the development of the analytical method is described. Based on this method, the maximum average power and torque capability of the SRM for given excitation current and speed is predicted.

In Chapter 3 an analytical method for estimating the core losses of a switched reluctance motor is described. A procedure for determining the flux linkages waveforms in the different parts of the core is described. Using Fourier analysis, the sinusoidal

components of the flux density functions are determined and used in conjunction with the loss characteristics of the core material to estimate core losses.

Chapter 4 describes a computer-based method for predicting the key waveforms of the drive. The method is unique in that the steady-state solution of the system equations is obtained directly without going through the transient solution.

Chapter 5 describes the design of a novel, energy-efficient, 4-switch, 3-phase converter configuration which has most of the main features of the asymmetrical bridge (two-switch-per-phase) converter. A comparison between this and other converter topologies is presented.

Chapter 6 deals with the design and steady-state analysis of a minimum switch converter configuration suitable for low performance applications. The converter is unique in that the stored energy is partially dumped in a resistor instead of regenerating into the d.c. link as is the case with most other converter configurations. Closed form expressions for determining device ratings are given. Simulation and experimental results for the key waveforms of the converter and drive are also given.

Chapter 7 describes the design and steady-state analysis of a novel converter configuration hereafter referred to as the "Inter-phase Recycling" converter. The converter differs from other topologies in that the stored energy in one phase is directly recycled to the other phase windings rather than regenerating to the d.c. link as in most other configurations.

Chapter 8 deals with the measurement and instrumentation of the SRM. Various methods for measuring inductance are studied and compared. The hardware implementation of the various measurement techniques is also described. Experimental results are presented and compared to analytical and FEA results.

Chapter 9 is a study on the effect of the motor geometry on the converter ratings. The effect of the main design parameters of the motor on the r.m.s. current ratings of the

switching devices is investigated and used to demonstrate the need for an integral approach to the design of the SRM drive.

Chapter 10 provides a comprehensive summary of the major conclusions arising from the dissertation work. The chapter ends with some suggestions on possible areas of future research work.

2.0 ANALYTICAL PREDICTION OF SRM INDUCTANCE PROFILE AND STEADY-STATE AVERAGE TORQUE

The electromagnetic torque of a switched reluctance motor (SRM) is dependent on the coenergy change and hence on the rate of change of inductance with rotor position. The motor inductance is a function of both excitation current and rotor position. The non-linearity of the motor increases the difficulty in developing an analytical method for predicting the flux linkages, coenergy change and inductances at all rotor positions. This chapter describes an improved analytical method for determining the SRM terminal inductance, flux linkages and coenergy change per phase at any position of the rotor from the geometrical dimensions of the motor and core material characteristics. The effects of both local and bulk saturation are included at all excitation levels and rotor positions. It is shown that the phase inductance, even in the fully unaligned rotor position, is a function of the excitation current and not constant as it is generally considered. Simulation and measured results for the prototype

SRM are given and shown to be in good agreement. Compared to an earlier method for predicting maximum and minimum inductances, the proposed method represents a significant improvement in the prediction of the minimum inductance. Further, the method applies to inductance prediction at all rotor positions.

2.1 Introduction

An analytical method for estimating the minimum and maximum inductances of a switched reluctance motor was proposed in [36]. The minimum inductance occurs at the fully unaligned position which is defined as that position of the rotor where the axis of a pair of rotor poles is exactly half a rotor pole pitch away from the axis of a pair of excited stator poles. The maximum inductance occurs at the fully aligned position which is defined as the position of the rotor when the axis of a pair of rotor poles exactly coincides with the axis of a pair of excited stator poles. Rotor positions intermediate between these extreme positions are hereafter referred to as the overlapping and non-overlapping rotor positions to signify the existence or nonexistence of overlap between stator and rotor poles, respectively. The variation of the phase inductance with rotor position as the a rotor pole moves between the extreme positions is referred to as the inductance profile.

The method described in [36] provides a useful guide to the prediction of motor inductances and the maximum steady-state average torque. However, it has the following limitations:

- i. The method is not applicable to the prediction of inductance at rotor positions other than the fully aligned and unaligned. In practice, each phase is excited at a duty cycle less than unity; prediction of the average steady-state torque therefore requires knowledge of inductances at intermediate rotor positions.
- ii. The method neglects the the m.m.f. drop in the iron core at all non-overlapping rotor positions. As the degree of saturation increases, the m.m.f. drop in the iron core becomes very significant, particularly as the leading tip of a rotor pole approaches the tip of an excited stator pole.
- iii. The stator interpolar space is assumed to be completely filled by windings. In practice, due to the shape of the winding, some space between the winding and the stator yoke remains unfilled.

A Finite Element Analysis (FEA) method for the switched reluctance motor was proposed recently [45,46,47]. Though FEA methods are generally more accurate than analytical methods, they require far more set-up and execution times on a computer. This is impractical in a typical industrial situation. Moreover, it is advantageous for a machine designer to know of the quantitative relationships between parameters, however inaccurate they may be. Therefore, the need for an improved analytical method capable of predicting the motor inductance at every rotor position and level of excitation exists.

The proposed improved method considers the SRM magnetic circuit to be nonlinear at all rotor positions and levels of excitation. Based on the experience from FEA, the flux distribution traversed by a rotor pole as it moves from the fully unaligned to the fully aligned positions (one stroke), is described by two regions:

1. Region I: between the fully unaligned position and the point at which the leading tip of a rotor pole coincides with the trailing tip of an excited stator pole (tip-to-tip position).
2. Region II: between the tip-to-tip position and the fully aligned position.

Compared to the method described in [36], the outstanding features of the proposed improved method are:

1. The method applies to all rotor positions.
2. Saturation effects are included at all rotor positions. Consequently, inductance at all rotor positions inclusive of the fully unaligned position is a function of excitation current.
3. Includes the effect of pole tip saturation.
4. Includes leakage flux at the fully aligned position.

2.2 General approach

Figure 1 shows a 6/4 pole SRM with a pair of rotor poles approaching alignment with the excited stator poles of phase A.

As the rotor moves from the minimum inductance position to full alignment, the flux linkages between the stator and the rotor increase. In SRM design, the stator interpolar space has to be wide enough to ensure minimum flux leakage between adjacent stator poles. A ratio, $\frac{t_s}{\lambda_s} \simeq 0.4$ (where t_s is the stator pole width and λ_s is the stator pole pitch) has been found to be optimal for doubly salient motors [55]. The pole geometry shown in Figure 1 on page 22 satisfies these conditions. Thus, in the fully unaligned position, there is no overlap between the rotor and the excited stator poles. Overlap between stator and rotor begins when the axis of the rotor poles approaching alignment is exactly half a stator pole pitch away from the axis of the excited poles. As the stator and rotor approach each other, the m.m.f. drop in the iron core becomes significant and can not be neglected. Thus, even in the absence of overlap, the magnetic circuit is nonlinear, the degree of nonlinearity depending on the level of excitation. The problem is further complicated by the fact that the degree of saturation is not uniform over all iron parts. Two types of saturation effects are considered:

- (i) bulk saturation and
- (ii) local saturation

Bulk saturation affects the whole magnetic circuit and occurs whenever the excitation level is raised beyond the value corresponding to the linear region of the B-H curve of the core material. This type of saturation occurs in all types of electrical machines.

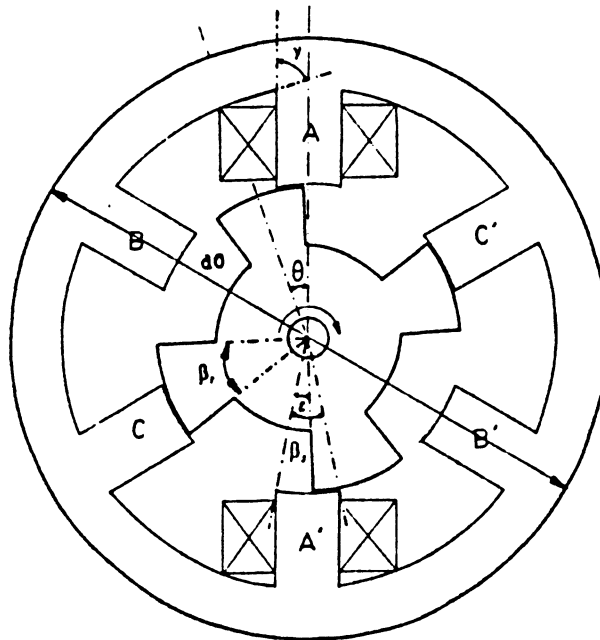


Figure 1. 6/4 pole SRM approaching alignment

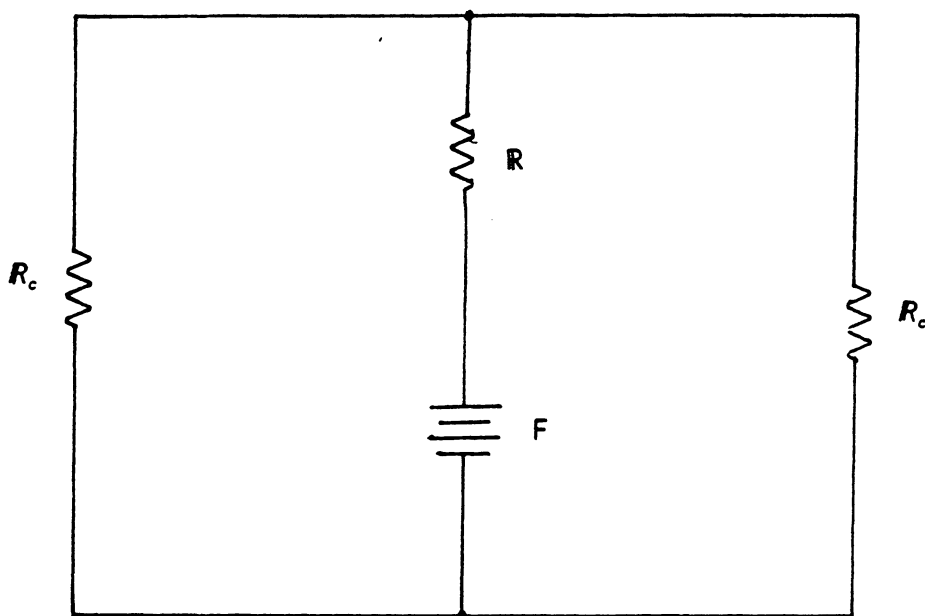
Local saturation, on the other hand, is more specific to doubly salient machines. It occurs in the pole tips at the threshold of overlap or during partial (small) overlap. Under these conditions, intense fields develop in the pole tip regions resulting in a distortion of the effective airgap. The severity of this effect is dependent on the level of excitation and the shape of the pole tips. The complexity of the problem is compounded by the fact that both bulk and local saturation exist simultaneously at all practical levels of excitation. It is therefore difficult to separate the two. Further, the exact area of the pole tip which is subjected to local saturation is not known since the exact field distribution in this region cannot be analytically determined in advance. The range (of rotor positions) within which local saturation is significant is also a function of the excitation level and pole tip shape. An approximate technique for incorporating this effect is presented in the next section.

The intense fields at the beginning of the overlap between stator and rotor poles results in a sharp increase in inductance. As the overlap length increases, most of the flux flows through the overlapped region thus reducing the effects of local saturation while increasing bulk saturation. This results in a monotonic decrease in the rate of change of inductance with rotor position as full alignment is approached. The rate of change of inductance with rotor position is also a function of the level of excitation; lower values correspond to higher levels of excitation.

2.2.1 Procedure for determining inductance

Figure 2 shows a lumped parameter model of the magnetic circuit of the SRM.

The m.m.f. applied to a phase winding at any position of the rotor is given by



R is the sum of the stator poles, rotor poles, airgap and rotor core reluctances, R_c is the reluctance of one half of the stator back-iron and F is the applied m.m.f. per phase

Figure 2. Lumped parameter model of the SRM

$$F = F_g + F_i \quad (2.1)$$

where

F_g = m.m.f. drop in the airgap

F_i = m.m.f. drop in the iron

F = total m.m.f. applied

In order to determine F_g and F_i , the m.m.f. equation is re-written in terms of the magnetic field strength and flux path length as

$$F = \sum H_g l_g + \sum H_i l_i \quad (2.2)$$

where

H_g = magnetic field strength in the airgap

H_i = magnetic field strength in the iron parts

l_g = length of flux line in air

l_i = length of flux line in iron

Determination of the magnetic field strength in the iron parts of the motor requires an accurate fit of the B-H curve of the core material. Several B-H curve fitting methods have been proposed in the literature [43,52,53,54]. For the core material used in this study, the spline fit was found to be most appropriate with a maximum error of 4 % [43].

2.3 Equations for the calculation of inductance

The geometry of the SRM is such that different parts of the core experience different flux densities. On this basis, five different sections of the magnetic circuit can be distinguished: the stator poles, rotor poles, stator back-iron, the rotor body and the airgap. Further, except for the fully aligned position, the magnetic circuit is not symmetrical about the axis of the excited stator poles. The result is that the magnetic circuit is not the same for all flux paths. Therefore, the procedure for calculating inductance has to be repeated on a path-to-path basis. The procedure is described below. Assuming that the flux linkages Φ_k linked with path 'k' (at a fixed value of excitation current and rotor position) is known, the flux density in part 'j' of the motor can be found using

$$B_{jk} = \frac{\Phi_k}{N_k a_j} \quad (2.3)$$

where

B_{jk} = flux density in part 'j' due to flux path 'k'

a_j = cross-sectional area of core section 'j'

N_k = number of turns per phase linked with flux path 'k'

It is assumed that

- i. flux is uniformly distributed over the cross-section of the core, that is, the flux density is constant over a cross-section provided that the excitation is kept constant.

- ii. hysteresis is neglected
- iii. a rectangular block of current with amplitude I_p is applied

Both Φ_k and B_{jk} are unknown at this stage. Equation (2.3) is therefore solved iteratively. An initial value of Φ_k is assumed and substituted in equation (2.3) for the airgap section of the flux path (linear) so that

$$B_{ag} = \mu_0 H_g = \frac{\Phi_k}{N_k a_{ag}} \quad (2.4)$$

where

B_{ag} = airgap flux density

a_{ag} = airgap cross-sectional area

μ_0 = permeability of free space

Using (2.4), the flux densities in the iron parts of magnetic circuit can be calculated in terms of B_{ag} as

$$B_{jk} = B_{ag} \left[\frac{a_{ag}}{a_j} \right] \quad (2.5)$$

Once the flux densities are known, the corresponding values of H are obtained from the B-H curve of the core material. Values of the path length and cross-sectional area are determined from the geometric dimensions of the motor. The values of H_{jk} and l_{jk} so obtained are substituted into the right-hand-side of equation (2.2) and compared with the left-hand-side. If the difference is larger than a user-defined tolerance, the flux linkage estimate is adjusted and the procedure repeated until the m.m.f. equation is satisfied. The corresponding value of Φ_k is then used to calculate the path inductance using

$$L_k = \frac{\Phi_{kf}}{I_p} \quad (2.6)$$

where

L_k = inductance due to flux path 'k'

Φ_{kf} = flux linkage satisfying m.m.f. equation for path 'k'

I_p = peak value of the on-time phase current

Since half-machine symmetry is assumed in determining the flux linkages, the total flux linkages equals twice the value calculated from equation (2.3). This procedure is repeated for all the paths defined at a given rotor position. A user-friendly computer program for implementing this algorithm has been developed [43]. A flow chart of the procedure is shown in Figure 3.

The phase inductance is calculated from the arithmetic sum of the individual path inductances, that is,

$$L_{ph} = \sum_k L_k \quad (2.7)$$

Detailed equations for the two regions I and II and their respective diagrams are given in Appendix B.

2.3.1 Local saturation effects

Figure 4 shows the assumed distribution in the tip sections of a rotor and a stator pole just prior to the beginning of overlap.

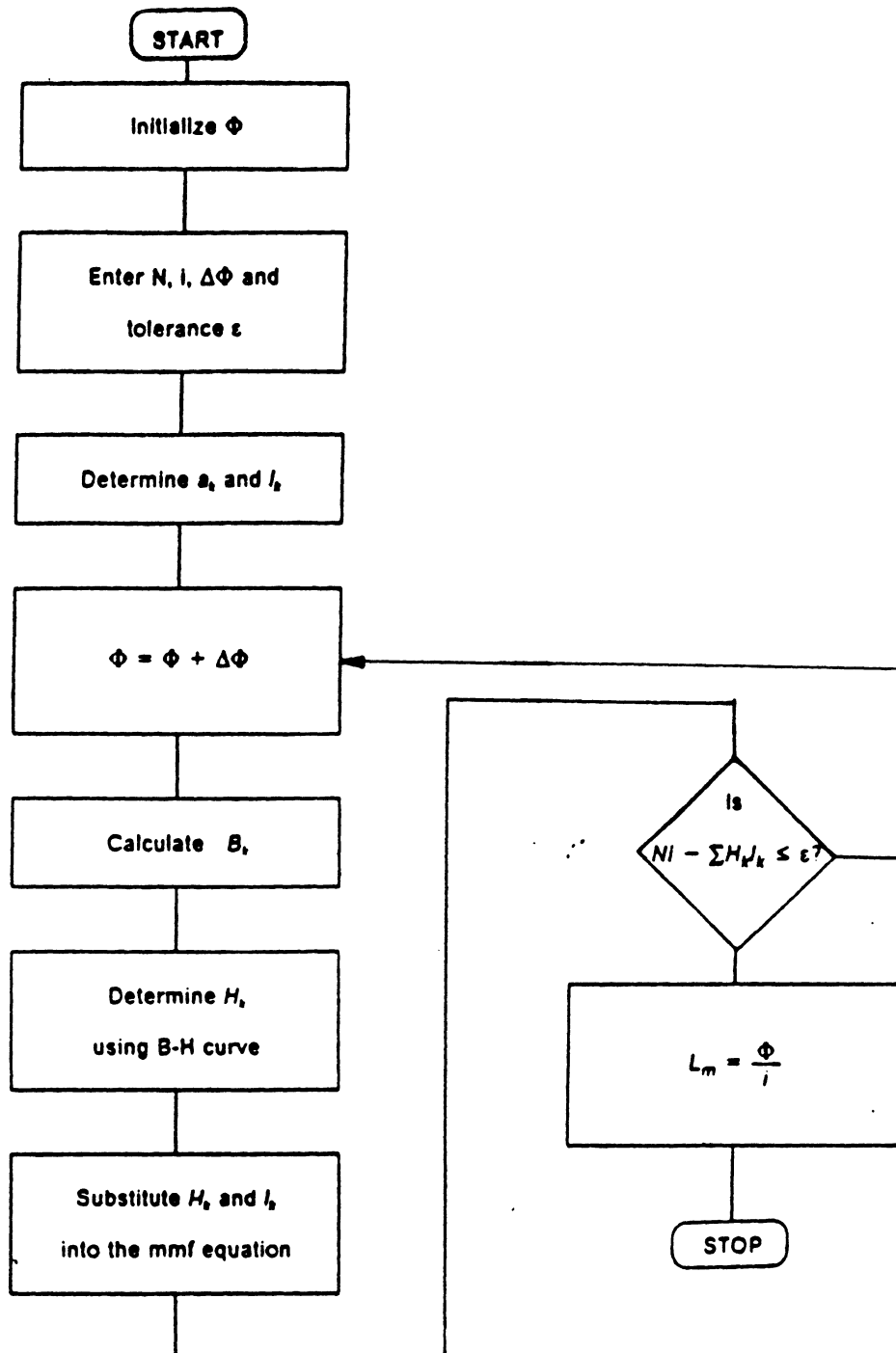


Figure 3. Flow chart of the inductance prediction procedure

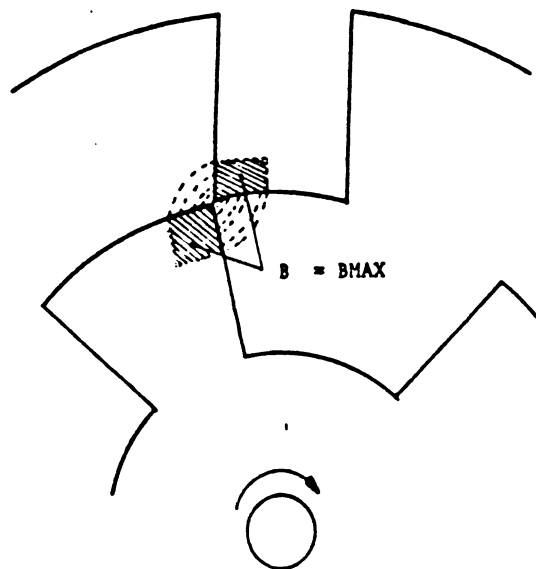


Figure 4. Assumed flux distribution in the tip sections

The tip sections experience higher magnetic fields than the other parts of the iron core. It is assumed that the flux density in these regions equals the maximum value for the iron core material. In this model, the area subjected to local saturation is approximated by the (shaded) parallelograms on the stator and rotor poles shown in Figure 4 on page 30. The two areas are assumed to be equal for a fixed rotor position. Local saturation is assumed to be present at all non-overlapping rotor positions and at small overlap lengths for overlap angles in the range $\varepsilon \leq 0.03\lambda_r$ where λ_r is the rotor pole pitch and ε is the overlap angle [1].

2.4 Inductance prediction by Finite Element Analysis

A two-dimensional PC-based finite element analysis package was used to verify the accuracy of the analytical method. The method is based on Maxwell's fourth equation in steady state given by

$$\nabla \times \bar{H} = \bar{J} \quad (2.8)$$

where $\bar{H}$ is the magnetic field strength in the field region under consideration and $\bar{J}$ is the current density within the field. A 'bar' above a symbol signifies a vector quantity. Equation (2.8) can be expressed in terms of the flux density as

$$\nabla \times \left(\frac{\bar{B}}{\mu} \right) = \bar{J} \quad (2.9)$$

where μ is the permeability of the core material.

In the two dimensional case, the assumption is that the flux density vector has components only in the x and y directions. Thus it is convenient to define a vector magnetic potential $\bar{A}$ such that

$$\bar{B} = \nabla \times \bar{A} \quad (2.10)$$

By this definition, $\bar{A}$ has components only in the z direction. This means that the potential A_z can be treated as a scalar thus simplifying determination of the magnetic field strength. Substituting equation (2.10) into equation (2.9) for the two dimensional case, the vector equation is reduced into a scalar partial differential equation given by

$$\frac{\partial}{\partial x} \left[\frac{1}{\mu} \frac{\partial A_z}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{1}{\mu} \frac{\partial A_z}{\partial y} \right] = -J_z \quad (2.11)$$

The solution of equation (2.11) gives the value of the magnetic vector potential within the field which is then applied to equations (2.10) and (2.8) to obtain $\bar{B}$ and $\bar{H}$ respectively.

Once $\bar{A}$, $\bar{B}$ and $\bar{H}$ are known, inductance is calculated using either of one of two methods namely,

1. The Flux Linkage Method or
2. The Energy Method

Accordingly, the relevant equations are, for the flux linkage method

$$L = \frac{Z}{i^2} \int \bar{A} \bar{J} d\bar{S} \quad (2.12)$$

and, for the energy method,

$$L = \frac{2}{i^2} \int \int_0^B \bar{H} d\bar{B} d\bar{U} \quad (2.13)$$

The flux linkage method is based on a linear concept and is therefore of limited use in magnetic materials which operate at high flux densities as is the case in the switched reluctance motor. The energy method has therefore been used.

In order to solve equation (2.11), the magnetic device is discretized into triangular or rectangular segments within which approximating functions are used to describe the parameter of interest. In this case, triangular segments have been used. Solution for the magnetic vector potential within a segment is performed using the weighted residual method. In the weighted residual method, an approximating function is evaluated in each iteration and compared to the differential equation. The difference between the two values is the residual. Various methods for choosing the weighting function for the residual are described in the literature [58]. The solution therefore entails minimization of this residual.

2.5 Prediction of steady-state average torque

The torque produced by a switched reluctance motor is proportional to the rate of change of coenergy as the rotor moves from one rotor position to another. This is shown in equation (2.8) where W is coenergy, T is torque and θ is rotor position.

$$T(\theta, i) = \frac{\delta}{\delta \theta} W(\theta, i) \quad (2.14)$$

Coenergy is a function of both rotor position and excitation current and therefore, the torque produced has to be expressed in terms of the same parameters. Equation (2.8) can also be expressed in terms of the flux linkages since

$$W(\theta, i) = \int_0^i \Phi(\theta, i) di \quad (2.15)$$

assuming that the rotor position is held constant. Substituting (2.9) into (2.8), the equation for torque can be expressed in terms of flux linkages as

$$T(\theta, i) = \frac{\delta}{\delta \theta} \left[\int_0^i \Phi(\theta, i) di \right] \quad (2.16)$$

The values of flux linkages at different values of current and rotor position are obtained in the process of calculating inductance as explained in the foregoing sections. Thus, given two rotor positions, the coenergy change is obtained by numerical integration of the corresponding flux linkages versus current characteristics and taking the difference between the two results. The resulting torque is then obtained using

equation (2.14). Figure 5 shows typical flux linkages characteristics for the prototype motor for the fully aligned and fully unaligned positions and the coenergy change for a current of 20 A. The simple trapezoidal rule with an integration step of 0.1 A has been used in this study.

The maximum steady-state average torque, $\bar{T}_{\max}$, is then given by:

$$\bar{T}_{\max} = W_{\max} \times \frac{\text{Number of strokes per revolution}}{2\pi} \quad (2.17)$$

where $W_{\max}$ is the coenergy change per phase between the fully unaligned and aligned rotor positions. A stroke is defined as one movement of the rotor between these extreme positions. For P_s stator poles and P_r rotor poles, the number of strokes per revolution is given by

$$S = \frac{P_s}{2} P_r \quad (2.18)$$

Thus, the general equation for the steady-state average torque becomes:

$$\bar{T} = W \times S = \frac{W P_s P_r}{4\pi} \quad (2.19)$$

The average electromagnetic power at angular speed ω_m is then

$$\bar{P} = \omega_m \bar{T} = \frac{W P_s P_r n_m}{120} \quad (2.20)$$

where

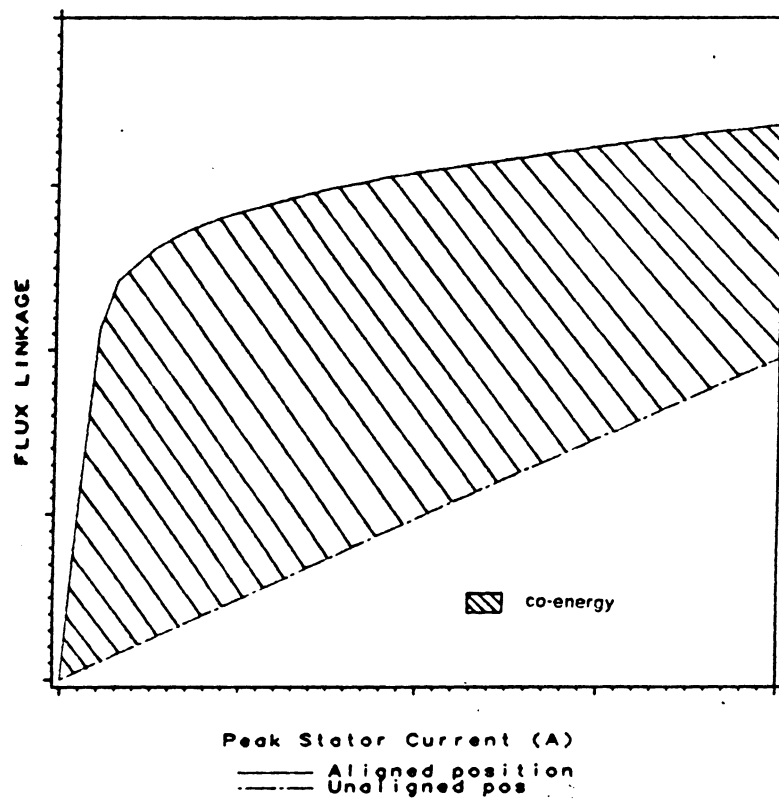


Figure 5. Typical flux linkages characteristics

n_m = motor speed in r.p.m.

Equations (2.19) and (2.20) represent the electromagnetic torque and power respectively, which the motor can produce at a given excitation current and conduction angle assuming a rectangular block of current is switched into the winding throughout the on-time for each phase. To obtain the output power, the friction and windage losses have to be subtracted. Addition of the copper, core and stray losses to $\bar{P}$ gives the input power.

2.6 Results

2.6.1 Comparison of predicted and measured inductance values

The inductance per phase of the prototype 6/4 pole switched reluctance motor was predicted and measured at 5 ° intervals between the fully aligned (0 °) and the fully unaligned (45 °) rotor positions and for excitation currents from 0 to 20 A. The instantaneous voltage and current values were recorded at intervals of 100 μ s using a Data Acquisition System (DAS). Details about the inductance measurement technique are given in chapter 8. For the fully aligned positions, predictions by three different methods were performed:

1. The analytical method described in [36]

2. The proposed analytical method

3. Finite Element Analysis

Due to the excessively long computer time required to implement a single finite element analysis solution, predictions by this method were performed for only a few values of current. Extensive work on the prediction of SRM inductance by the FEA method is available in the literature [45,46,47].

2.6.1.1 Inductance at the extreme rotor positions

Figures 6 and 7 show the simulated and measured inductances at the fully aligned and fully unaligned positions, respectively.

Table 1 summarizes these results for excitation currents of 1 A, 10 A and 20 A strategically selected to represent light, medium and very deep saturation. It is observed that at the fully aligned position, there is excellent agreement between the simulated and measured results for the whole range of the phase current. Using the measured inductance as a reference, the prediction error by the proposed method is 7.4 % at 20 A (very deep saturation) and less than 1 % at 1 A (low excitation). The prediction error by the method described in [1] is 24.8 % at 20 A and 3.3 % at 1 A. Compared to finite element analysis, the prediction error by the proposed method is 8.7 % at the deep saturation of 20 A.

Considerable variation of the inductance at the fully unaligned position is observed in both the predicted and measured results. The unaligned inductance increases by

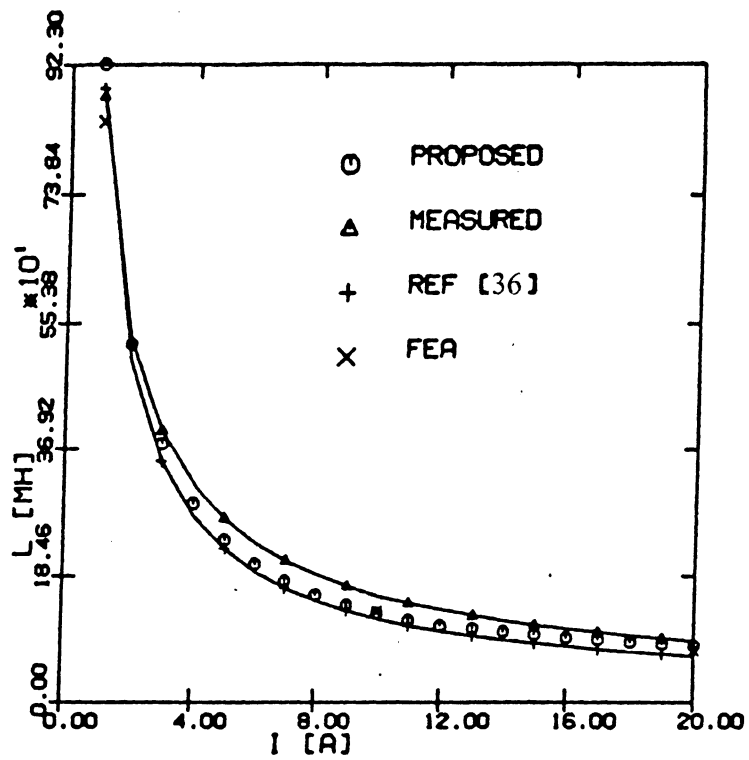


Figure 6. Predicted and measured maximum inductances

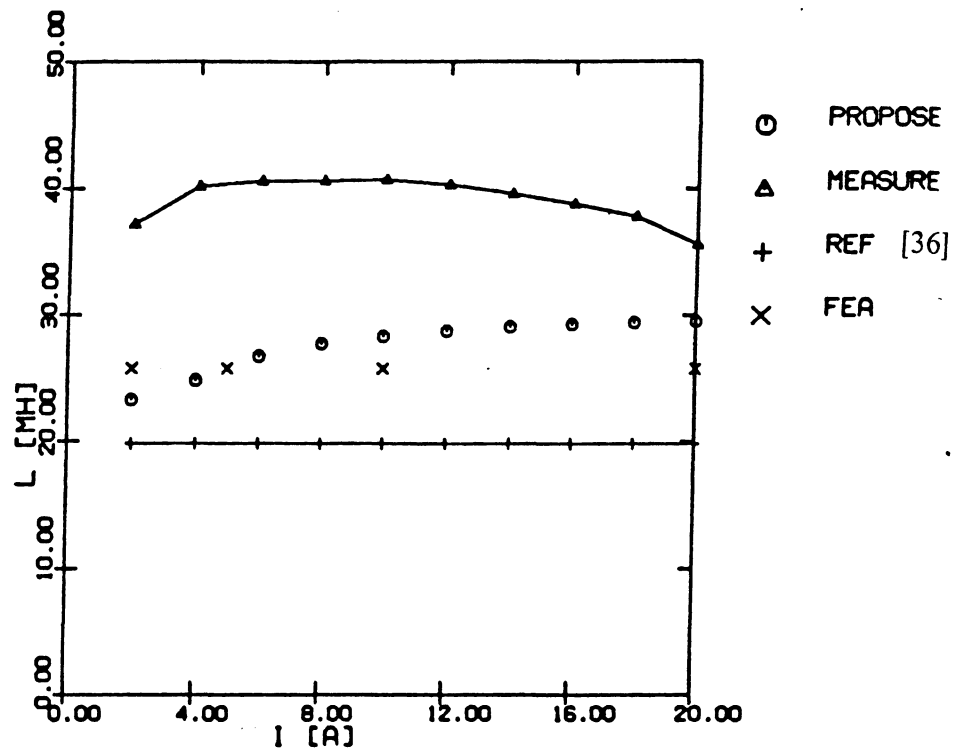


Figure 7. Predicted and measured minimum Inductances

Table 1. Maximum inductance by various methods

Method	I_p [A]	L_s [mH]	L_v [mH]	T_{avg} [Nm]
Proposed	1	923.0	23.2	0.88
"	10	130.0	28.3	18.77
"	20	82.4	29.5	38.6
Ref. [36]	1	844.8	19.9	0.93
"	10	123.2	19.9	18.2
"	20	67.3	19.9	37.0
FEA	1	843.0	25.8	0.80
"	10	133.0	25.8	18.5
"	20	75.5	25.8	37.8
Measured	1	879.0	35.5	0.90
"	10	155.0	40.7	20.3
"	20	89.0	35.6	41.6

over 9 % for a current change from 2 to 10 A which is the designed operating range of the prototype motor. The net increase is lower at higher currents due to saturation of the core and particularly the pole tips.

The new method introduces significant improvement in the prediction of the minimum inductance when compared to the method proposed in [36]. Reasonable agreement between finite element analysis and the proposed analytical method is also observed. The maximum prediction error is 14.5 % at 20 A excitation. However, when compared to the measured results, the error lies between 17.1 % at 20 A and 38.3 % at 4 A. The corresponding errors by the earlier method [36] are 43.9 % and 50.9 % at 20 A and 4 A respectively.

The analytical method was also tested on a 8/6 pole switched reluctance motor designed for the same output power as the 6/4 pole SRM. Figures 8 and 9 compare the inductances for the 6/4 and 8/6 pole motors at the fully aligned and unaligned positions respectively.

The 8/6 pole motor has a higher inductance in the fully unaligned position because the interpolar airgap sections in this machine are shorter. The path reluctances are therefore lower than in the 6/4 pole machine resulting in higher flux linkages. At very deep saturation, this difference in inductance values is lower due to tip saturation which is higher for the 8/6 pole SRM. The two machines have been designed with the same core length-to-bore diameter ratio so that the end leakage effects are approximately the same for the two motors.

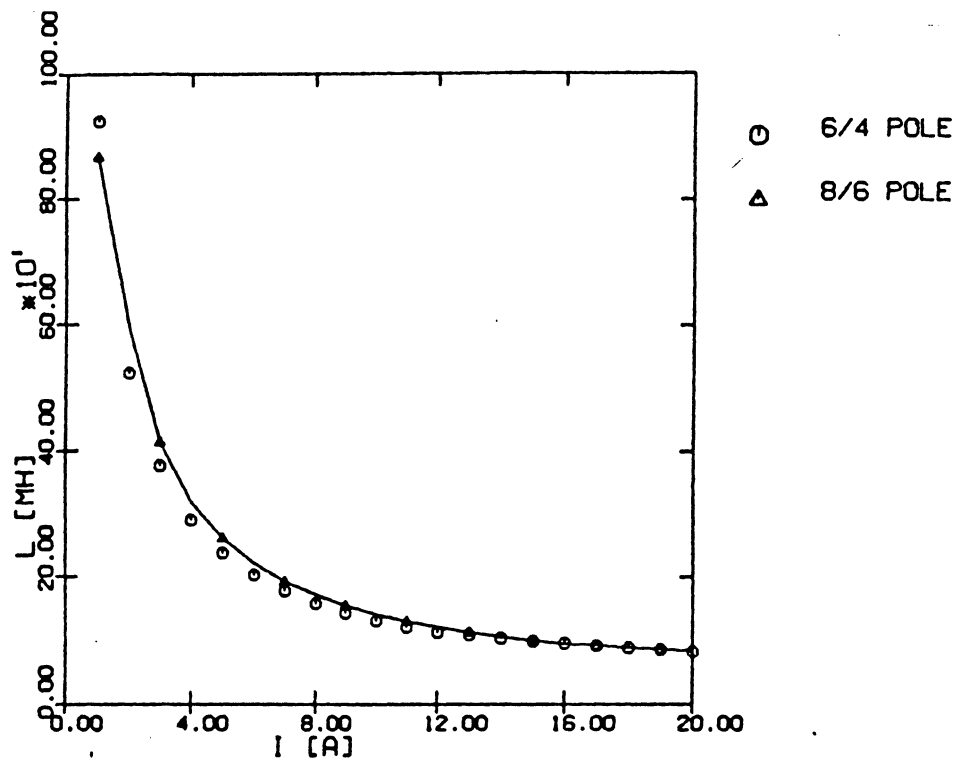


Figure 8. Comparison of maximum Inductances for the 6/4 and 8/6 SRM's

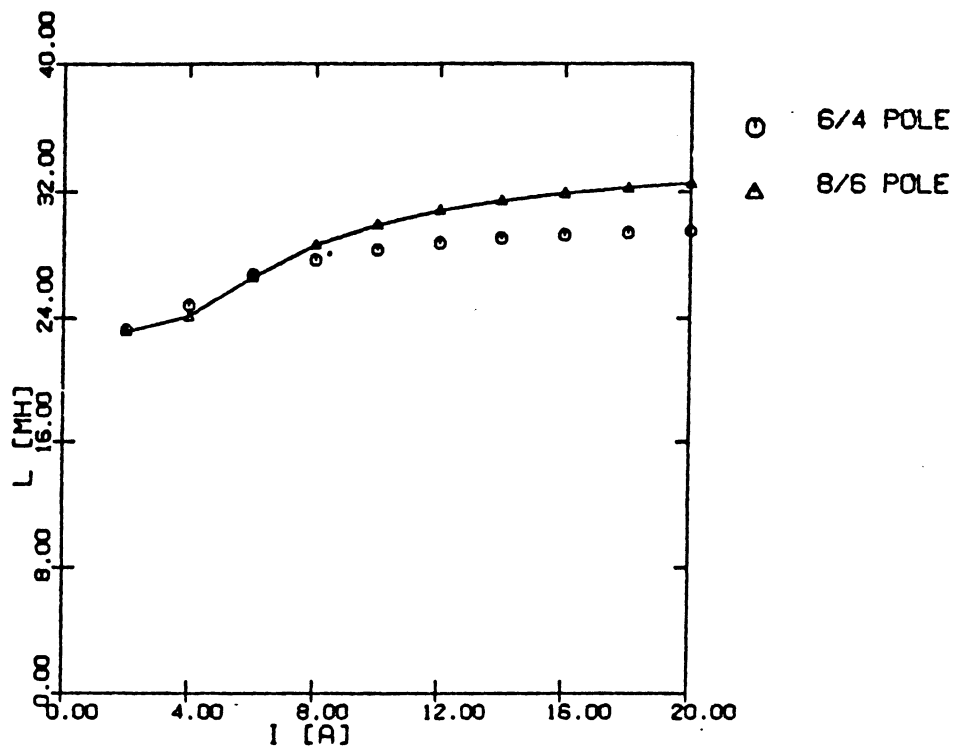


Figure 9. Comparison of minimum inductances for the 6/4 and 8/6 SRM's

Since the two-dimensional methods show good agreement, the error is largely attributed to the end effects which have been neglected in these models. End leakage effects are more severe in the fully unaligned position because, in this position, the reluctance of the end paths becomes comparable to that of the airgap paths. A three dimensional finite element analysis study reported recently, estimates the end-leakage inductances to be 7.5 % and 35 % of the total inductance in the fully aligned and fully unaligned positions, respectively [48]. Other possible sources of error include the distortion of the magnetic properties of the core material due to cutting stresses and non-uniformity of the air-gap.

2.6.1.2 Inductance at intermediate rotor positions

Figure 10 shows a comparison between the measured and predicted inductances at 0, 5, 15, 25 and 45 degree rotor positions at current intervals of 2 A.

At large overlap angles (0 and 5 degree positions), the error in the predicted inductance (based on the measured value) is about 5 % at 2 A and increases to 12 % at 20 A (deep saturation). However, the error increases as the degree of overlap between stator and rotor poles decreases. At 25 ° the error at 20 A is 32.7 % and increases to a maximum of 38.3 % at 4 A when the rotor is in the fully unaligned position. The fact that the error is a function of the degree of overlap suggests that end effects are significantly responsible for the error. The other sources of error stated in section 2.5.1.1 also apply in this case.

Figure 11 shows the variation of inductance with rotor position at 0, 10, 20, 30, 35, 40, and 45 degrees and excitation currents of 1 A, 5A, 10 A and 20 A.

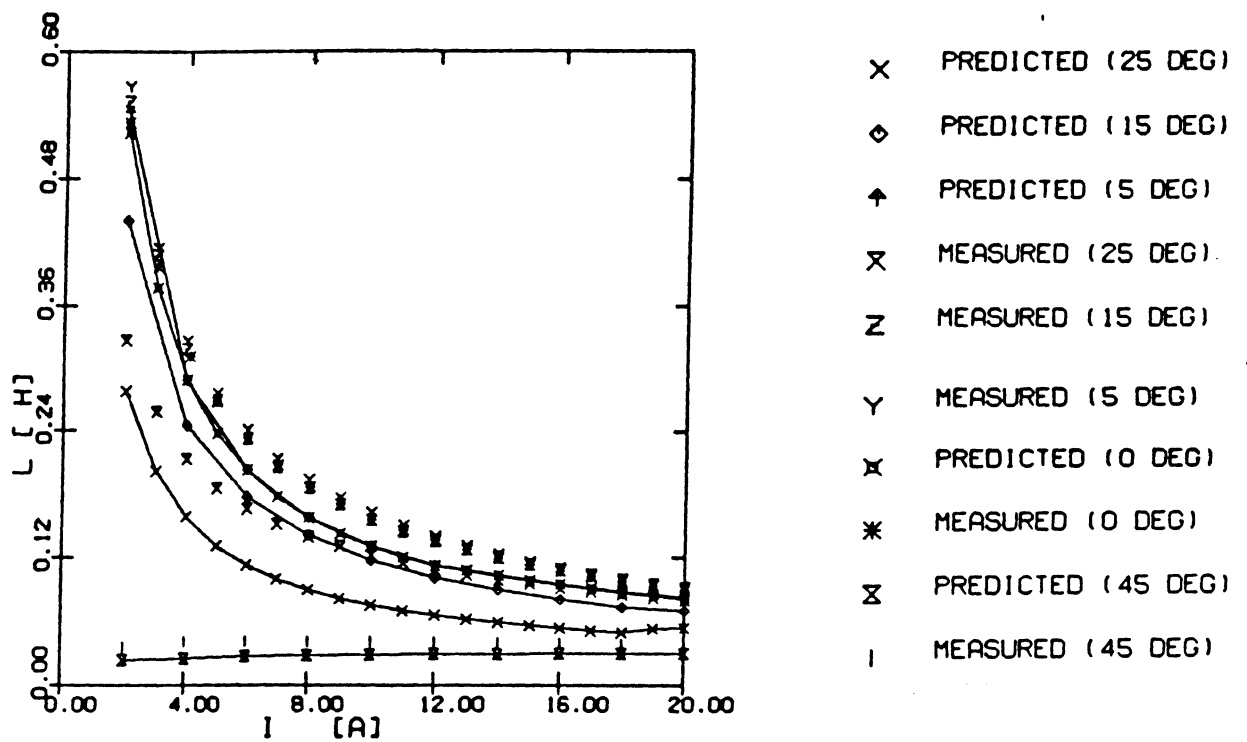


Figure 10. Inductance versus excitation current and rotor position

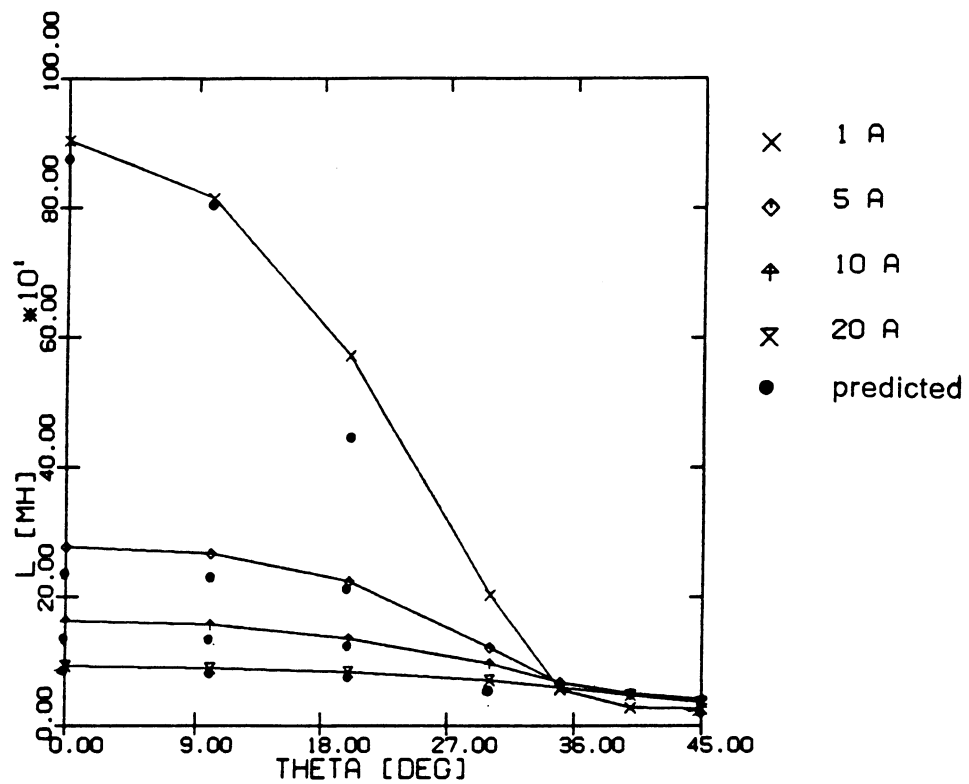


Figure 11. Inductance-rotor position (measured and predicted)

It is observed that the rate of change of inductance with rotor position in the non-overlapping positions is comparable to that at large overlap angles. This implies that considerable torque can be produced even in the unaligned positions by operating the motor with a large advance angle.

2.6.2 Maximum steady-state average torque

Figure 12 compares the maximum average steady-state torque predicted from the flux linkages characteristics obtained by the proposed analytical method, the method described in [36], finite element analysis and measurements for the prototype SRM. Numerical integration of the flux linkages characteristics to obtain coenergy is performed using the trapezoidal method. A step length of 0.1 A (corresponding to 10 points per decade) was found to be optimal. It is observed that the results by the finite element method are in good agreement with the analytical results. The proposed method gives a prediction error of 2.2 % at 20 A. The torque computed from measured flux linkages is 7.2 % and 9.1 % higher than that predicted by the proposed method and finite element analysis, respectively. The same error for the earlier method is 10.9 %. The prediction errors discussed above do not reflect the large differences in the values of inductance at the fully unaligned position predicted by different methods. This is because, the coenergy at the fully unaligned position is small compared to that at the fully aligned position. Torque is proportional to the difference between the two values and therefore less sensitive to variations in the lower value. The sensitivity of the predicted torque to errors in the predicted value of the minimum inductance becomes more significant at small values of the conduction period since the magnitude of the associated coenergy change is low.

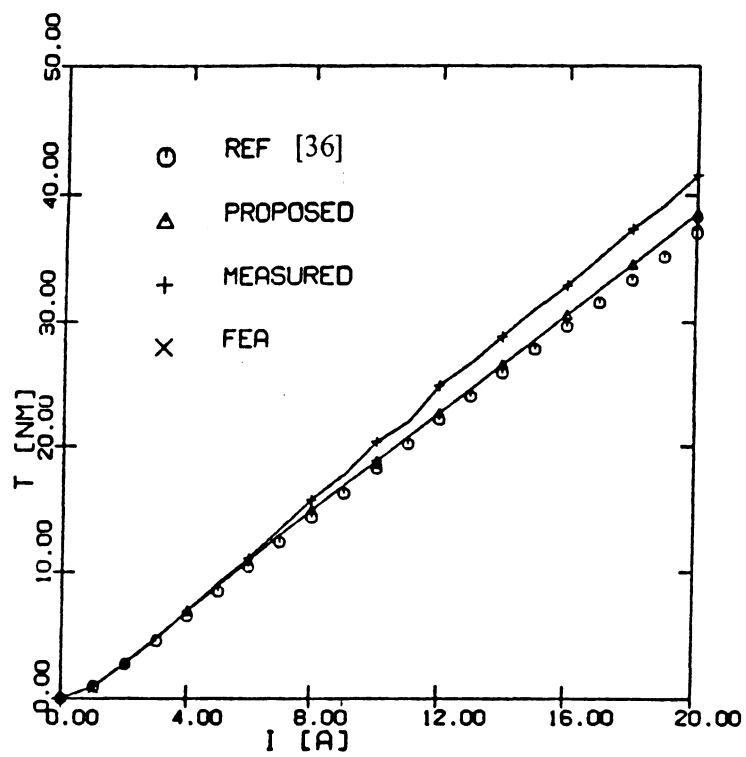


Figure 12. Comparison of steady-state average torque by various methods

2.7 Conclusion

An improved analytical method for determining the inductance profile of a switched reluctance motor has been proposed. The method takes into full consideration the nonlinearity of the magnetic circuit incorporating the effects of both bulk and local (tip) saturation. It is shown, that at all practical levels of excitation, both bulk and local (tip) saturation effects have to be taken into full consideration. Contrary to linear theory, it is also shown that inductance is not constant in the unaligned positions. The slope of the inductance profile in this region has been observed to be comparable to that at large overlap angles which provides extra flexibility in optimizing the control of the SRM drive. A comparison of the proposed analytical method with an earlier method shows that the new method results in significant improvement in the accuracy of the predicted inductance especially in the unaligned position. In the fully aligned position, good agreement between the predicted and finite element analysis results is observed for both analytical methods. When compared to measured results, a maximum error of 7.4 % is observed in the fully aligned position and 38.3 % in the fully unaligned position. The corresponding errors by an earlier method are 24.8 % and 50.9 %. This error is largely attributed to end effects which have been neglected in the two-dimensional model and inaccurate B-H characterization of the core material. Prediction of steady-state average torque using the analytical method has been shown to be of the same order of accuracy as the two dimensional finite element analysis method. The prediction error by both methods is less than 10 % in deep saturation, when compared to torque computed using measured flux linkages. The predicted maximum steady- state average torque shows little sensitivity to errors in the predicted minimum inductance due to the large difference in the magnitudes of

the coenergies at the two extreme positions. The sensitivity is more significant when the commutation of the phase currents is effected at well in advance of the full alignment rotor position.

3.0 ESTIMATION OF SWITCHED RELUCTANCE MOTOR LOSSES

This chapter describes a novel method of predicting SRM losses under the operating conditions inclusive of saturation. First, a procedure for determining the waveforms of the flux linkages in different parts of the motor is derived. Fourier analysis is then applied to decompose the flux linkages waveforms into the constituent sinusoidal components which are then used in conjunction with the loss characteristics of the core material to estimate the core losses for a given excitation current and speed. The dependence of these losses on the magnitude and waveform of the excitation current, the switching frequency and the conduction period per phase is investigated for a peaky and a flat-topped current waveform. A procedure for estimating the copper losses is also described. Current measurements are performed using a Data Acquisition System and compared to measurements by analog d.c. meters. The results are summarized in tabular and graphical forms.

3.1 Introduction

The losses in a switched reluctance motor (SRM) mainly comprise of iron and copper losses. Estimation of core losses is complicated by the fact that the frequency of flux reversals is different for different parts of the core. The complexity increases as the number of phases increases. An approximate but simple procedure for estimating these losses has been developed and is described in this chapter. First, a general procedure for determining the flux waveforms for any pole combination is presented. Fourier analysis is then used to extract the a.c. components of flux. The core losses corresponding to each component are then determined. It is shown that a number of flux harmonics are significantly present. Data acquisition and processing techniques are used for the measurement and evaluation of the phase currents. These methods are described and compared to d.c. meter measurements. Experimental verification based on the prototype SRM is given.

3.2 Determination of flux waveforms

In the SRM, windings on diametrically opposite stator poles are connected in series and therefore simultaneously excited. Figure 13 shows the basic circuit of the SRM drive.

The voltage equation for one phase is given by

$$V = R i + \frac{d\Phi}{dt} \quad (3.1)$$

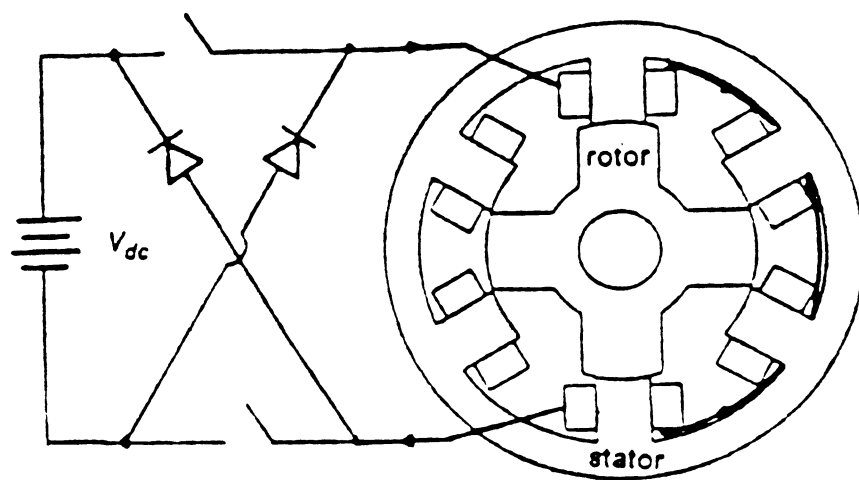


Figure 13. Basic circuit of the SRM drive

where V is the d.c. supply voltage, R is the stator winding resistance per phase and Φ is the flux linkages.

Since the resistive drop is much smaller than the supply voltage above 0.1 p.u. speed, the rate of change of flux linkages with respect to time can be considered to be approximately linear. The flux can therefore be approximated by triangular waveforms, positive slope corresponding to the time when the phase winding is excited and negative slope corresponding to the regeneration period during which a negative voltage equal to the supply voltage is applied across the phase winding. Since only the stator winding is excited, the flux waveforms in the different parts of the core are determined by the switching sequence in the stator. The normal switching sequence for an m -phase motor is to excite the phases successively, that is, 1,2,3,..., $m-1$, m ; 1,2,3..etc. In this case, each commutation step constitutes one working stroke and each cycle constitutes a switching period, T_s . Since a phase winding is excited whenever a rotor pole tends to alignment, the number of cycles (periods) for a stator phase per revolution equals the number of rotor poles. The switching period per phase is therefore given by

$$T_s = \frac{2\pi}{\omega_m P_r} = \frac{\lambda_r}{\omega_m} \quad (3.2)$$

where ω_m is angular speed of the rotor, P_r is number of rotor poles and λ_r is rotor pole pitch.

The stroke period T_w depends on the number of of phases. Each phase is excited once per cycle so that

$$T_w = \frac{T_s}{m} \quad (3.3)$$

Thus for any pole combination, the flux frequency in the stator poles is determined from T_s as

$$f_{ph} = \frac{1}{T_s} = \frac{\omega_m P_r}{2\pi} \quad (3.4)$$

The d.c. supply switching frequency is then m-times the phase switching frequency and is therefore given by

$$f_s = \frac{m \omega_r P_r}{2\pi} \quad (3.5)$$

These two frequencies provide the basis for constructing the flux waveforms in all parts of the core.

It has been observed earlier that the period and form of the flux variations in the core differs from part to part [1]. Since the exciting currents are usually unipolar, the flux variations in the stator poles are unidirectional. This is however not the case in the other parts of the core. The flux waveforms in these parts are examined below.

3.2.1 Flux waveforms in the stator core

During commutation, only some sections of the stator back iron experience flux reversal. Thus, except for the 4/2 pole combination, the stator core flux in the SRM consists of multiple frequencies, the number of frequencies depending on the number

of phases. Neglecting leakage effects, the flux waveforms in the stator core for an m -phase SRM can be constructed from the stator pole flux waveform. Table 2 shows the flux polarities in the $2m$ stator core segments of a general m -phase SRM for one switching period.

This sequence is repeated in every period. A segment is defined as the part of the stator core between two adjacent stator poles. A transition from plus to minus represents reversal of flux whereas a plus (or a minus) represents a unipolar triangular flux pulse. An illustration for a 6/4 pole SRM is given in Figure 14.

It is observed that the flux, in the segments bounded by the poles corresponding to phases 1 and 2, is always unipolar irrespective of the number of phases. The flux waveform in these segments is obtained by a direct summation of the stator pole flux waveforms. Note that the stator pole flux waveforms of adjacent phases (poles) are phase-shifted by one stroke period T_w . An illustration for a 6/4 pole SRM is given in Figure 15 where Φ_{s1} , Φ_{s2} and Φ_{s3} represent the flux linkages of stator phases 1, 2 and 3 respectively, and Φ_{b1} represents the flux linkages in the stator core segments with non-reversing flux (type 1 segments).

The resulting waveform has a large d.c. component with a low amplitude a.c. ripple of frequency f_s . The amplitude of the ripple depends on the conduction angle and the level of excitation.

The other segments in the stator core experience one reversal in each switching period. However, these reversals do not occur at the same time. Thus each reversal constitutes a different frequency. There will therefore be an additional $m-2$ flux frequencies in the back iron. Again, the corresponding flux waveform is constructed by direct summation of the stator phase waveforms with the appropriate polarities and

Table 2. Flux polarities in the stator core

	Stroke number						
	1	2	3		m-1	m	m+1
Segment Time	T_w	$2T_w$	$3T_w$		$(m-1) T_w$	T_s	$(m + 1) T_w$
	Flux polarities						
1	+	+	+		+	+	+
2	+	+	+		+	-	+
3	+	+	+		-	-	+
m-1	+	+	-		-	-	+
m	+	-	-		-	-	+
m+1	-	-	-		-	-	-

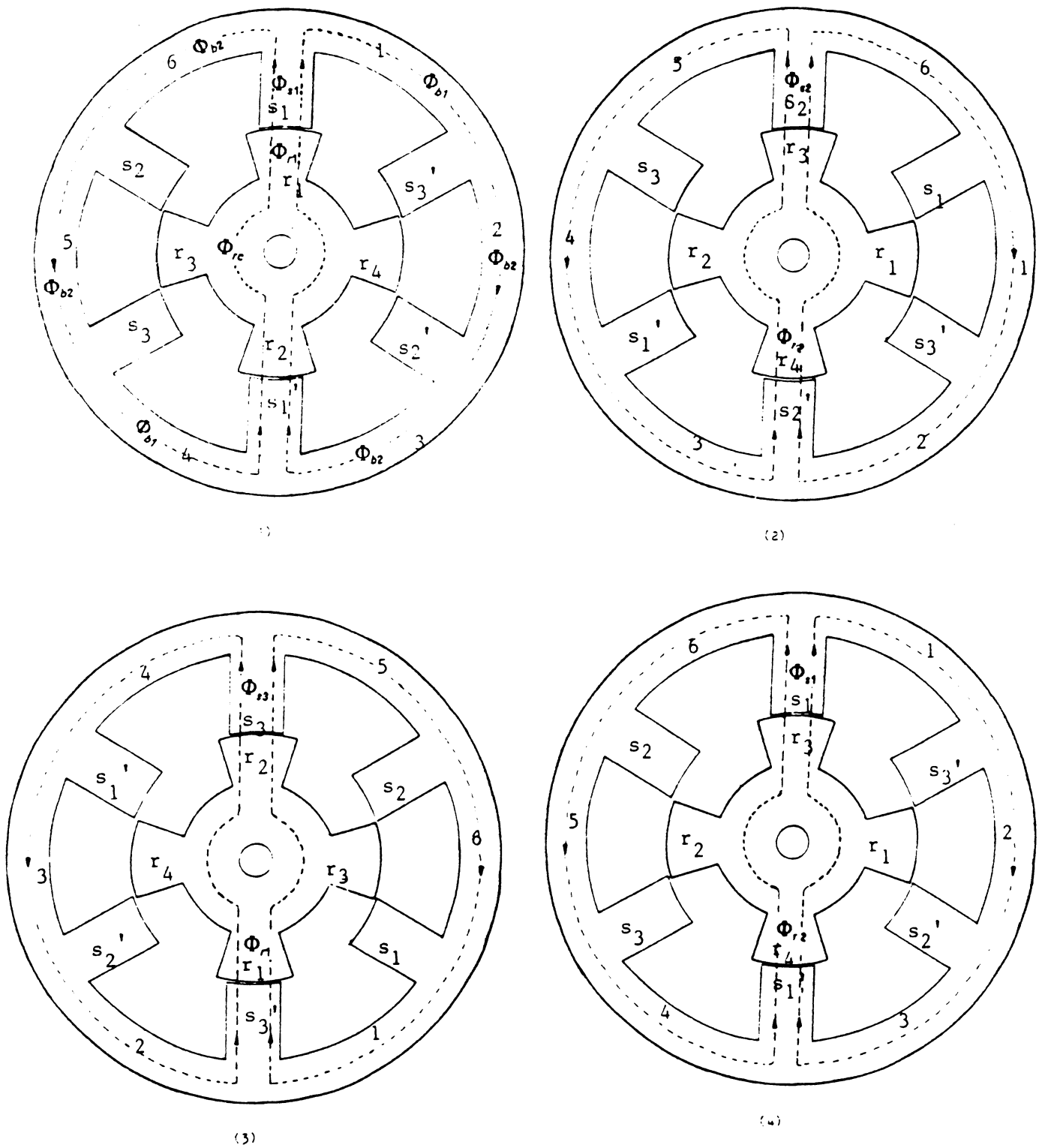


Figure 14. Flux in a 3-phase SRM during one switching period

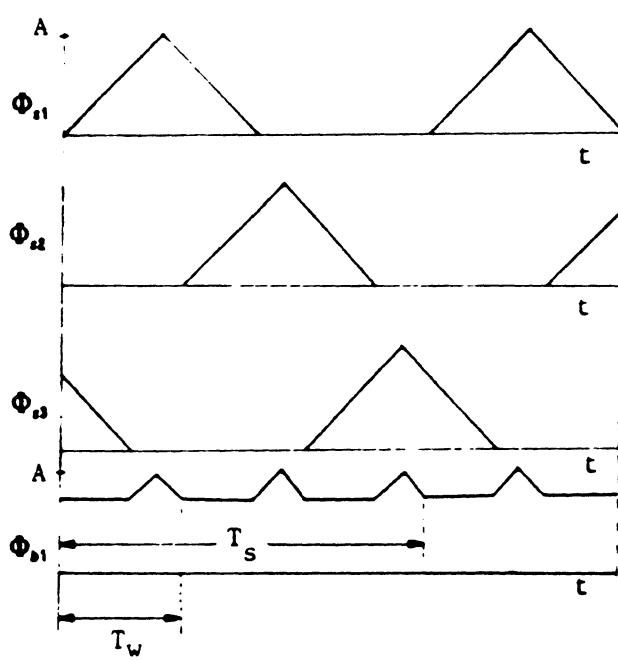


Figure 15. Unipolar stator core flux waveform

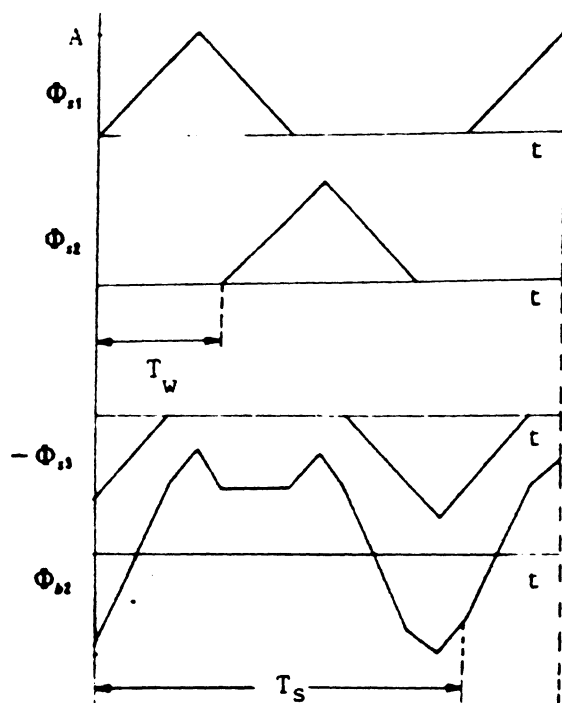


Figure 16. Bipolar stator core flux waveform

phase. Figure 16 gives an illustration for a 6/4 pole SRM where Φ_{b2} represents the flux linkages in the stator core segments with bipolar flux (type 2 segments). Since these flux waveforms are bidirectional, they are responsible for most of the iron losses in the stator. The number of different flux frequencies occurring in the stator core of a m-phase SRM can be obtained using [1]

$$c_f = \text{INTEGER} \left[\frac{m+2}{2} \right] \quad (3.6)$$

where c_f is the number of frequencies. So, a 4/2 pole SRM will have 1 frequency; a 6/4 pole 2 frequencies, an 8/6 pole 3 frequencies etc. The frequencies of these flux waveforms are given by

$$f_{sc} = k \times f_{ph} \quad 1 \leq k \leq m \quad (3.7)$$

3.2.2 Flux waveforms in the rotor

1) The rotor poles

For an m-phase SRM with P_r rotor poles, each pair of rotor poles sets into alignment with a stator pole at a periodic interval given by

$$T_r = T_w \left(\frac{P_r}{2} \right) \quad (3.8)$$

Further, the flux polarity in the pole reverses every half a revolution of the rotor. Thus, over a complete revolution, the flux waveform describes a bipolar envelope with a frequency

$$f_{r1} = \frac{f_{ph}}{P_r} \quad (3.9)$$

In addition, from equation (3.8), there will be a carrier frequency

$$f_{r2} = \frac{1}{T_r} = \frac{2}{T_w P_r} \quad (3.10)$$

The rotor pole flux waveform can therefore be synthesized from the stator pole waveforms by observing the phase relationship stated in equation (3.8). Figure 17 shows an example for a 6/4 pole SRM where Φ_{r1} represents the flux linkages in one pair of rotor poles.

2) The rotor core

Flux is established in the rotor core every time a stator phase is excited. Further, all the flux through the rotor poles also flows through the rotor core. The flux waveform for this part is therefore constructed by direct addition of the rotor pole flux waveforms. The flux in the rotor core, similar to the flux in the rotor poles, is bipolar and has the same fundamental frequency. Figure 18 shows an illustration for a 6/4 pole SRM.

In summary, the fundamental flux frequencies in an m-phase SRM are

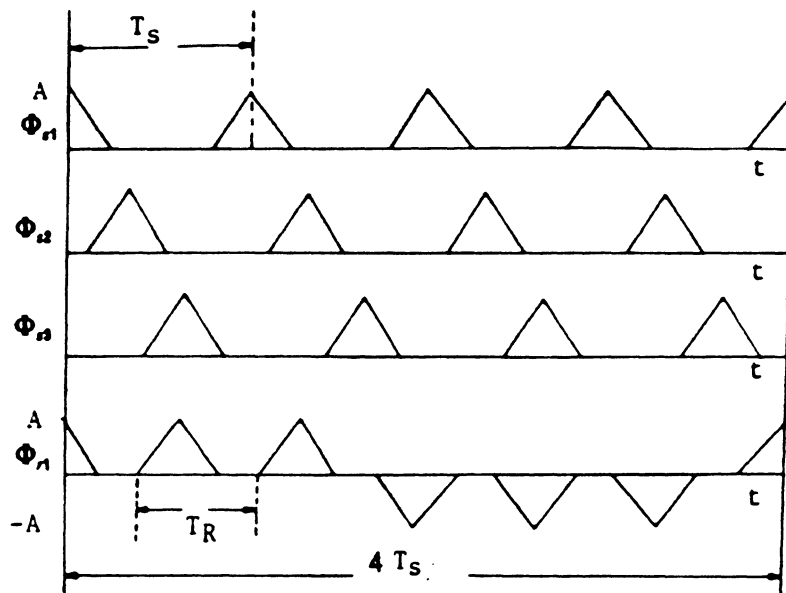


Figure 17. Flux linkages in the rotor poles

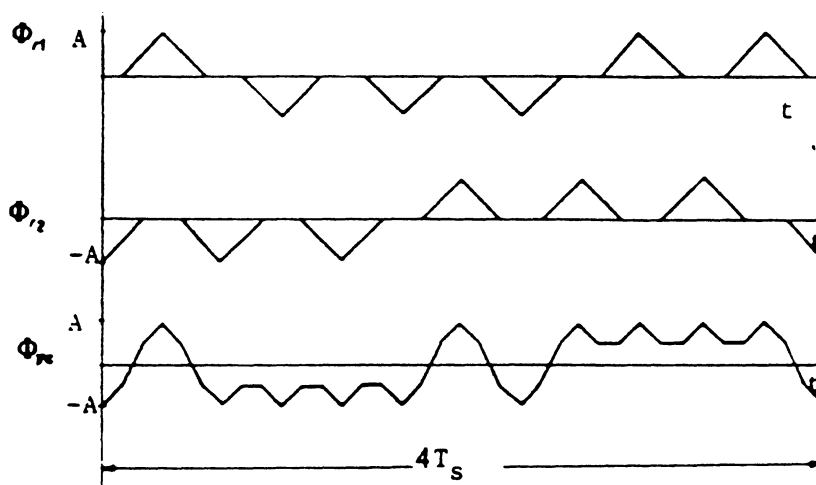


Figure 18. Flux linkages in the rotor core

Core part Frequency

Stator poles: $f_{sp} = f_{ph}$

Rotor poles: $f_{rp} = f_{ph}/P_r$

Rotor core: $f_{rc} = f_{ph}/P_r$

Stator core: $f_{sc} = k \times f_{ph}; \quad 1 \leq k \leq m$

3.3 Harmonic Analysis

In the previous section, it was shown that the stator pole flux is unipolar and that in general, the flux waveforms in the core parts are not sinusoidal and therefore contain multiple harmonic frequencies. It is therefore necessary to determine the loss-causing sinusoidal components of various frequencies in each of the waveforms presented in section 2. This has been accomplished using Fourier Series analysis as described in the following subsection.

3.3.1 Stator waveforms

Since the stator core flux waveforms are constituted by direct addition of the stator pole waveforms, it is sufficient to perform Fourier Series expansion on the stator pole flux waveforms only. The harmonic spectrum of the core fluxes can be obtained by point- by-point addition of the pole waveforms for each harmonic. The analysis is first

performed for the general case where the rates of rise and fall of fluxes are unequal as would be the case in a typical drive. regeneration. The origin of the waveform is set to be at the peak point of the flux waveform as shown in Figure 3.5 such that an even function is achieved when $t_r = t_f$.

The general Fourier expansion for period T is given by

$$\Phi_{s1}(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t] \quad (3.11)$$

where

$$a_0 = \frac{2}{T} \left[\int_{-t_r}^0 \Phi_{11}(t) dt + \int_0^{t_f} \Phi_{12}(t) dt \right] \quad (3.12)$$

$$a_n = \frac{2}{T} \left[\int_{-t_r}^0 \Phi_{11}(t) \cos n\omega t dt + \int_0^{t_f} \Phi_{12}(t) \cos n\omega t dt \right] \quad (3.13)$$

$$b_n = \frac{2}{T} \left[\int_{-t_r}^0 \Phi_{11}(t) \sin n\omega t dt + \int_0^{t_f} \Phi_{12}(t) \sin n\omega t dt \right] \quad (3.14)$$

where

$$\omega = \frac{2\pi}{T} \quad (3.15)$$

$$\Phi_{11}(t) = A \left[1 + \frac{t}{t_r} \right] \quad (3.16)$$

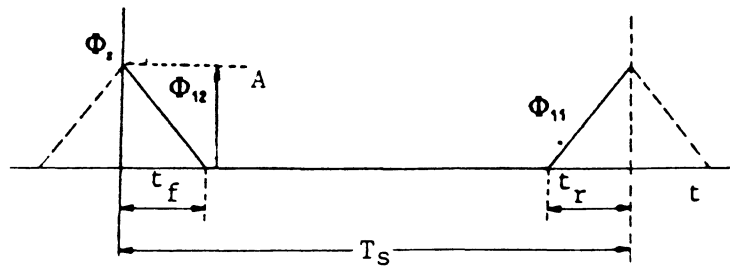


Figure 19. Reference flux linkages waveform in the stator poles

$$\Phi_{12}(t) = A \left[1 - \frac{t}{t_f} \right] \quad (3.17)$$

t_r is the time taken for the flux to rise to the maximum value, t_f is the time taken for the flux to fall to zero, A is the peak value of the flux linkage function and n is the order of a harmonic.

After the integration and arrangement of terms, the Fourier coefficients become

$$\begin{aligned} a_n = & C_1 [\sin n\omega t_r + \sin n\omega t_f] \\ & + C_2 [t_f + t_r - t_f \cos n\omega t_r - t_r \cos n\omega t_f] \\ & - C_2 [n\omega t_r t_f (\sin n\omega t_r - \sin n\omega t_f)] \end{aligned} \quad (3.18)$$

and

$$\begin{aligned} b_n = & C_1 [\cos n\omega t_r - \cos n\omega t_f] \\ & + C_2 [t_f \sin n\omega t_r - n\omega t_f t_r (\cos n\omega t_r + \cos n\omega t_f)] \\ & - C_2 t_r \sin n\omega t_f \end{aligned} \quad (3.19)$$

and

$$a_0 = \frac{A}{T} (t_r + t_f) \quad (3.20)$$

where

$$C_1 = \frac{2 A}{n \omega T} \quad (3.21)$$

and

$$C_2 = \frac{2 A}{(n \omega)^2 T t_r t_f} \quad (3.22)$$

For the case when the recovered energy is fed back to the dc supply, $t_r = t_f = t_c$ and the stator pole flux linkage function becomes an even function. In this case, $b_n = 0$ and a_n simplifies to

$$\begin{aligned} a_n = & 2 C_1 \sin n \omega t_c \\ & + 2 C_2 t_c [1 - \cos n \omega t_c] \end{aligned} \quad (3.23)$$

and

$$a_0 = \frac{2 A t_c}{T} \quad (3.24)$$

The Fourier expansion for other phases can be derived from $\Phi_{s1}(t)$ by phase shifting as given below

$$\begin{aligned} \Phi_{s2}(t) = & \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n \omega(t + T/m) \\ & + \sum_{n=1}^{\infty} b_n \sin n \omega(t + T/m) \end{aligned} \quad (3.25)$$

$$\begin{aligned}\Phi_{s3}(t) = & \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega(t + 2T/m) \\ & + \sum_{n=1}^{\infty} b_n \sin n\omega(t + 2T/m)\end{aligned}\quad (3.26)$$

$$\begin{aligned}\Phi_{sm}(t) = & \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega(t + \frac{m-1}{m}T) \\ & + \sum_{n=1}^{\infty} b_n \sin n\omega(t + \frac{m-1}{m}T)\end{aligned}\quad (3.27)$$

As stated earlier on, the stator core flux spectrum can be constructed from a direct point-by-point summation of the stator pole waveforms for each harmonic. The functions can also be expressed in terms of the rotor position by substituting

$$t = \frac{\theta}{\omega_m} \quad (3.28)$$

and setting $T = T_s$ as defined in equation (3.2).

3.3.2 The rotor

The rotor pole flux waveform is an odd function and therefore contains only sine terms. The corresponding Fourier expansion is of the form

$$\Phi_r(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t \quad (3.29)$$

where

$$b_n = \frac{4}{T} \int_0^{T/2} \Phi_r(t) \sin n\omega t \quad (3.30)$$

$\Phi_r(t)$ is described by $2m$ functions given by

$$\Phi_{2k-1}(t) = -\frac{A}{t_c} (t - t_{k-1}) \quad (3.31)$$

and

$$\Phi_{2k}(t) = -A + \frac{A}{t_c} (t - t_k); \quad k = 1, 2, 3 \dots 2m \quad (3.32)$$

t_k are the points of discontinuity shown on Figure 20 and T is the rotor flux period given by $1/f_r$.

The equations for determining the Fourier coefficients in the rotor are given in Appendix C. Similar to the stator core flux spectrum, the rotor core flux spectrum is

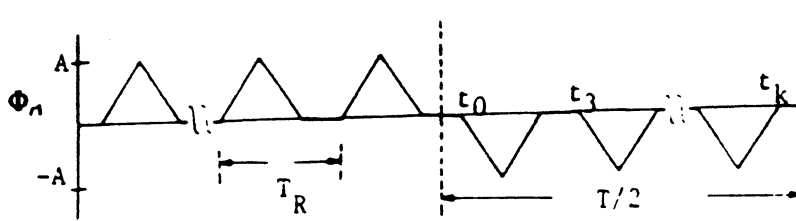


Figure 20. Reference flux linkages waveform in the rotor poles

obtained by a point-by-point addition of the rotor pole waveforms for each harmonic. In all subsequent derivations, it is assumed that $t_r = t_f = t_c$.

3.4 Calculation of losses

3.4.1 Iron losses

Manufacturers of lamination core steel provide data showing the variation of core loss (in watts per pound or kilogram) as a function of flux density and frequency. The base frequency for the SRM is determined from the desired operating speed. Using Fourier analysis the harmonic frequencies and their relative amplitudes are determined. To determine the contribution of each harmonic to the core loss, the peak amplitude is used as a weighting factor for the flux density in the respective part of the motor. Thus, for a harmonic of order n , the weighted peak flux density is given by

$$\hat{B}_n = \frac{\hat{A}_n}{A} \hat{B} \quad (3.33)$$

where A is the peak value of the flux linkages function used in the Fourier analysis (equation (3.17)), $\hat{A}_n$ is the peak value of the n -th order harmonic from Fourier analysis, $\hat{B}$ is the flux density in the core for a given excitation current and $\hat{B}_n$ is the peak flux density for a harmonic of order n . The flux density is obtained by the analytical method described in section 2.3.

By choosing $A = 1$, the expression is further simplified. Once the flux density and frequency are known, the corresponding loss factor (watts/pound) is looked up from the loss characteristic and multiplied by the appropriate weight of iron.

In most cases the loss characteristics given by the manufacturer are logarithmic. The variation of core loss as a function of frequency can be fitted using linear logarithmic polynomials of the form

$$p = W_i \left[\log^{-1} \{ a \log(f) - b \} \right] \quad (3.34)$$

where a and b are constants representing the slope and y intercept, respectively, W_i is the core weight and p is the loss factor (watts/lb). The iron weight for each part is calculated from motor dimensions. It is to be noted that whereas the whole of the stator and rotor core iron is continuously active, only a pair each of the stator and rotor poles is active at any instant.

Once the loss factors and the iron weights are known, the iron losses for a given operating point are calculated from

$$P_i = \sum p_k \times W_{ik} \quad (3.35)$$

where P_i is the total core losses (watts), p_k is the loss factor in part 'k' of the core (watts/lb) and W_{ik} is the iron weight of part 'k' contributing to the losses.

3.4.2 Copper losses

Copper losses constitute a considerable part of SRM losses particularly at high excitation levels. Accurate estimation of SRM copper losses at all speeds is complicated by the fact that the current waveform in the SRM is not sinusoidal. The waveform is dependent on operating conditions, particularly the excitation current, speed and the switching strategy. For the case of flat-topped currents, the current waveform can be approximated to a rectangular block. With this assumption it has been shown that the r.m.s. current per phase, assuming unity duty cycle and zero rise time is given by [42]

$$I = \frac{I_p}{\sqrt{(m)}} \quad (3.36)$$

where I_p is the peak value of the on-time stator current per phase.

The copper losses are generally given by

$$P_{cu} = m I^2 R_{ph} \quad (3.37)$$

where R_{ph} is the resistance per stator phase and I is the r.m.s. value of the phase current.

For a precise prediction of the copper losses, the actual stator current waveform has to be known. A method for the prediction of the SRM current waveform has been developed and is described in chapter 4. The method requires the phase inductance

as an input. This can be predicted using Finite Element Analysis [45] or analytical methods proposed in chapter 2.

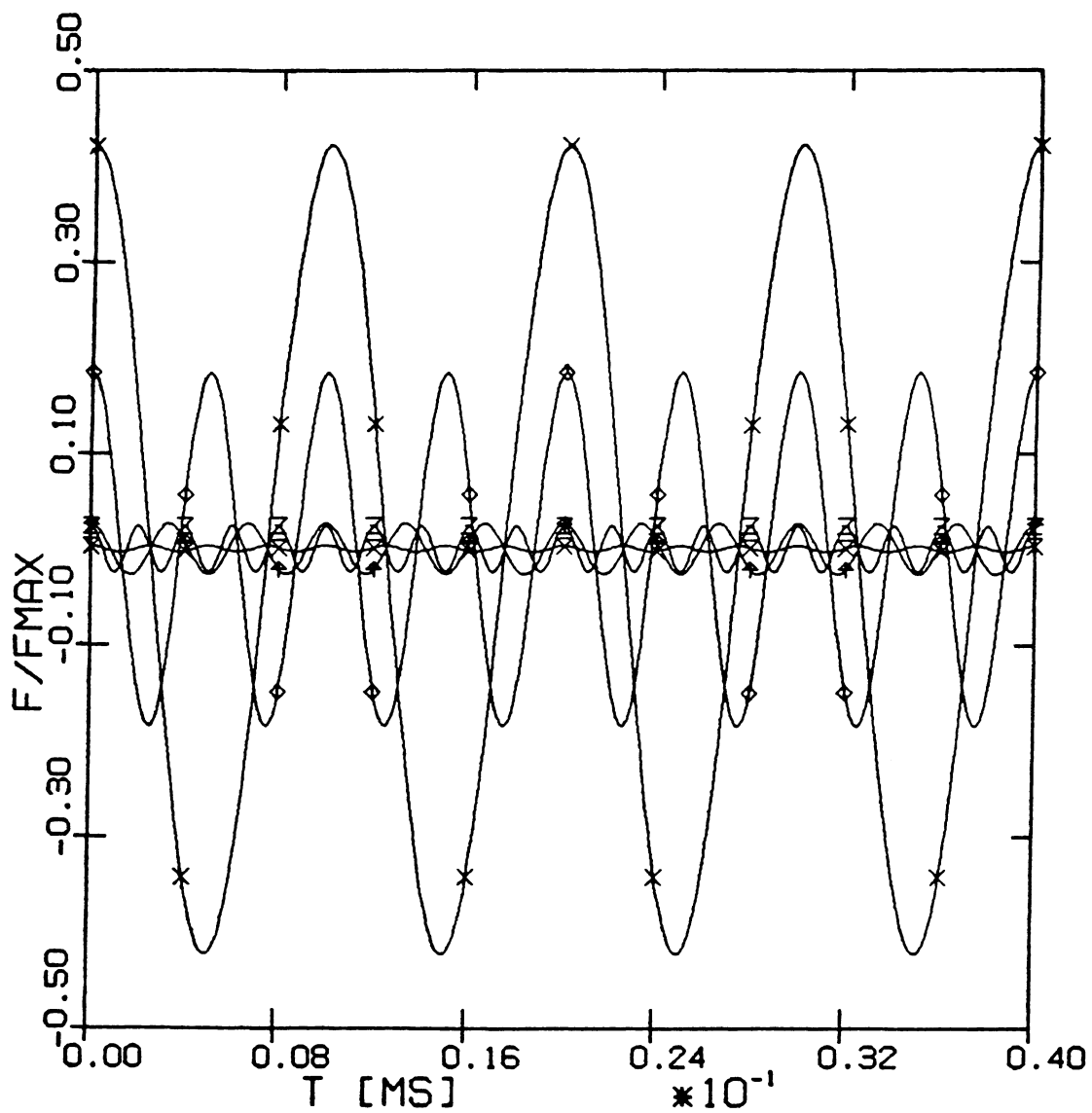
3.5 Simulation results

3.5.1 Harmonic spectra of flux waveforms

Figures 21 to 25 show the harmonic spectra in a 6/4 pole SRM for a switching period of 10 ms, and a conduction period of 2.7 ms. This corresponds to a speed of 1500 r.p.m. and a conduction angle of 24.3° .

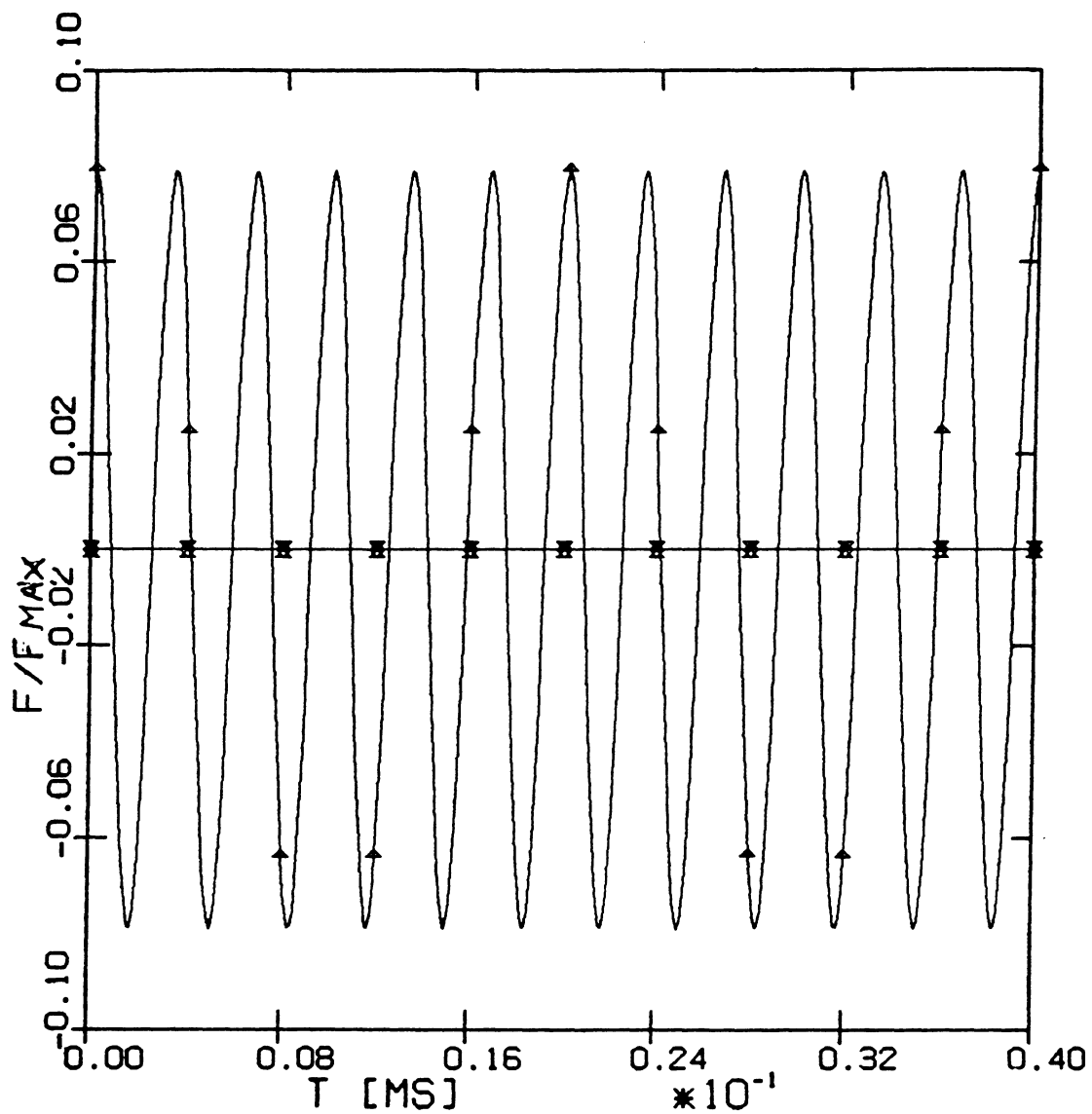
The peak value of the flux linkage function is normalized to $A = 1$. The first five harmonics are shown in each case. It is observed that:

- i. Harmonics of all orders are significantly present in the rotor and stator.
- ii. The fundamental flux component in the stator poles has a peak amplitude less than half of the overall peak. This was observed to be the case at all frequencies.
- iii. The fundamental component of flux in the core segments with non-reversing flux is less than 10 % of the overall peak. The value increases when the conduction angle is decreased.



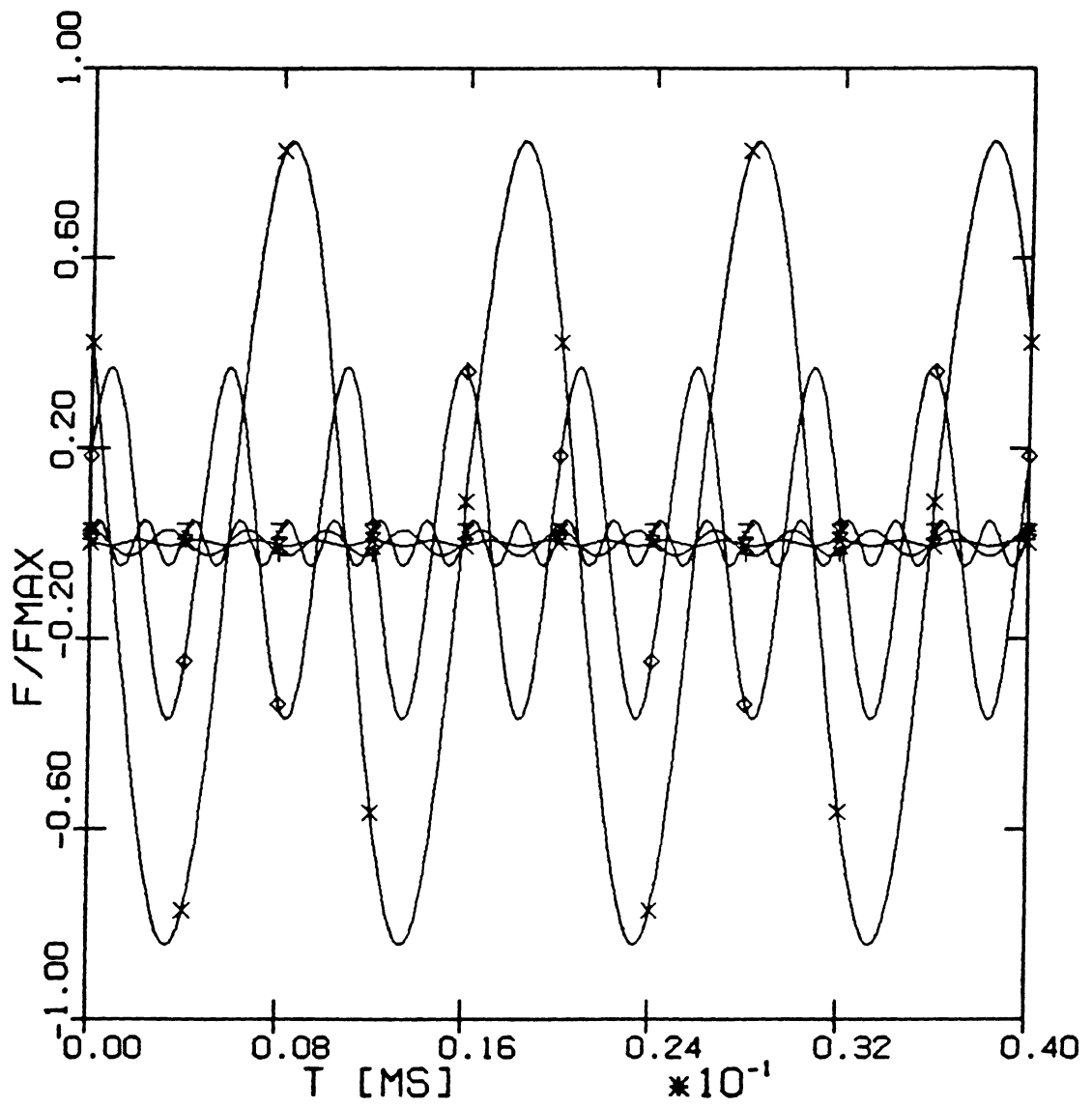
x	N = 1	X	N = 4
◇	N = 2	Z	N = 5
+	N = 3		

Figure 21. Normalized harmonic spectrum of the stator pole flux



x	N = 1	x	N = 4
◇	N = 2	z	N = 5
↑	N = 3		

Figure 22. Normalized harmonic spectrum of the unipolar stator core flux



x	N = 1	x	N = 4
◇	N = 2	z	N = 5
+	N = 3		

Figure 23. Normalized harmonic spectrum of the bipolar stator core flux

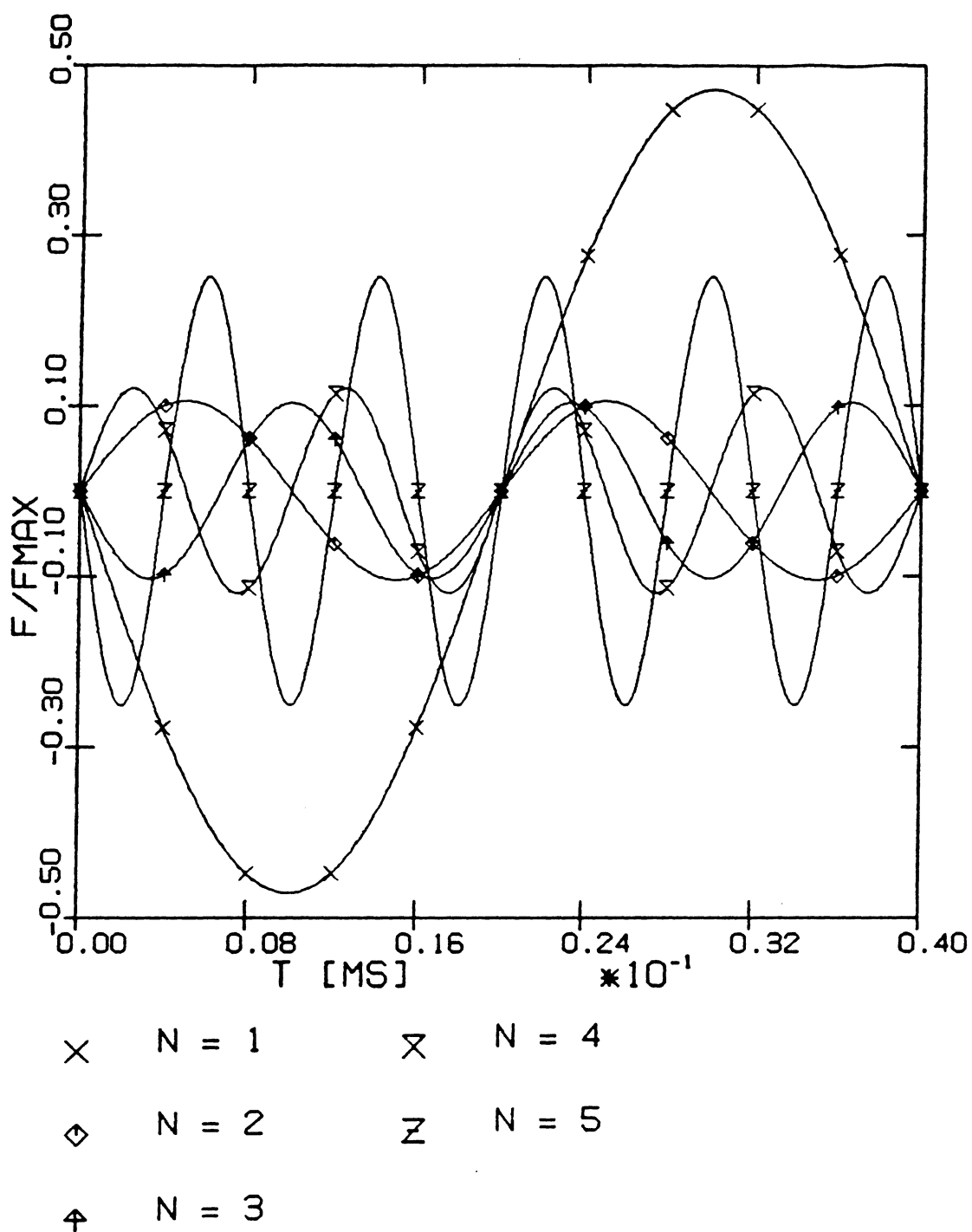


Figure 24. Normalized harmonic spectrum of the rotor pole flux

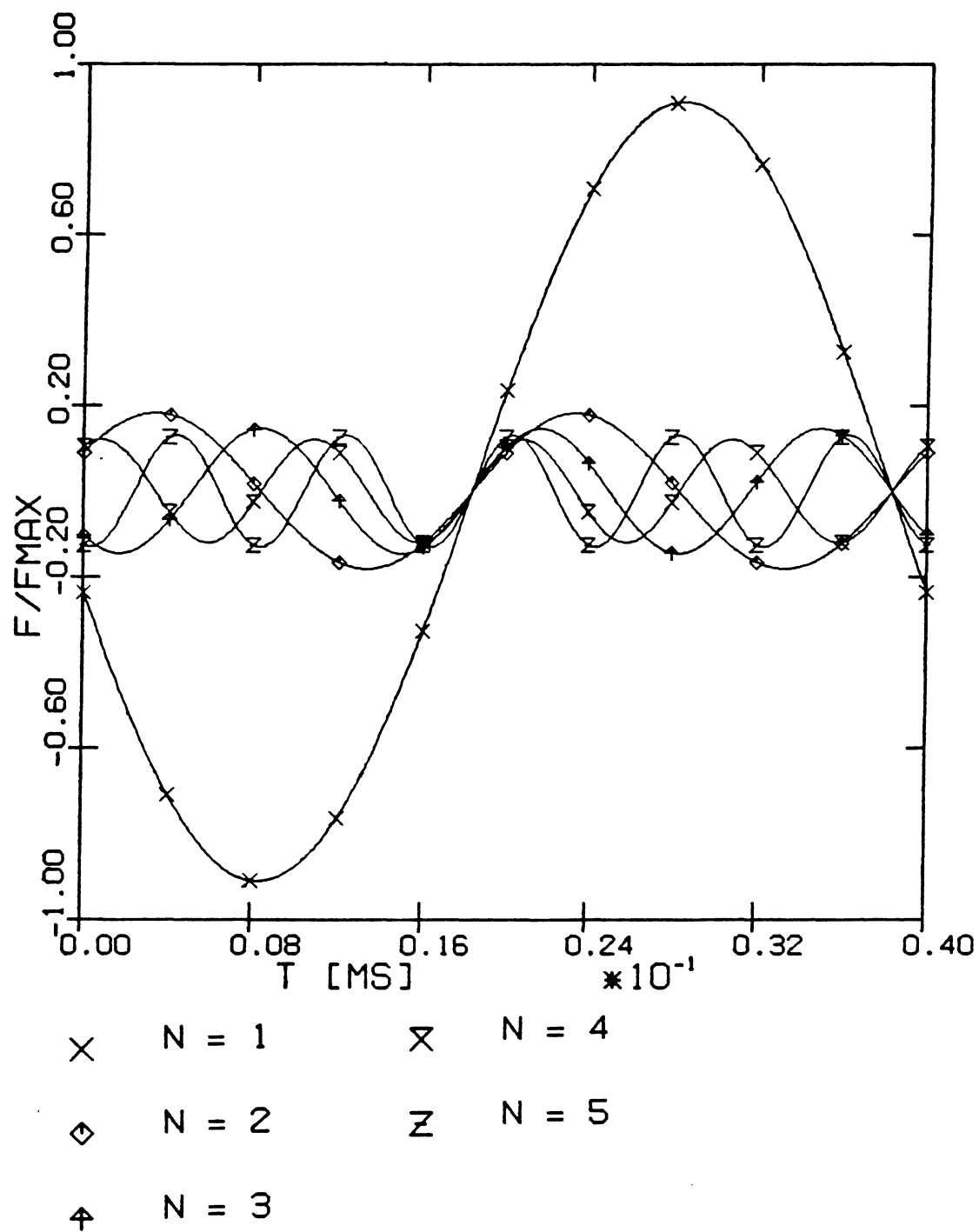


Figure 25. Normalized harmonic spectrum of the rotor core flux

- iv. In the rotor core and poles, all harmonics are very significant. In this particular case, the peak amplitude of the 12-th harmonic for example, is 14.2 % of the peak amplitude of the fundamental. The corresponding figures for other parts of the core are 13.75 % for the rotor poles, 0.58 % for the stator poles and .29 % for the stator core.

Considering the weighted harmonic content, the resulting error when all harmonics higher than order 12 are neglected is less than 1 % in all parts of the core. The weighted harmonic content is calculated from

$$g = \sqrt{\left[\sum_{n=1}^k \left[\frac{F_n}{F_1 n} \right]^2 \right]} \quad (3.38)$$

where F_n is the effective value of the n-th harmonic given by

$$F_n = \sqrt{\left[\sum_{t=0}^T f_n(t)^2 \frac{n \Delta t}{T} \right]}; \quad n = 1, 2, \dots, k \quad (3.39)$$

T is the period of the fundamental frequency in the respective core part. The overall effective value (including all harmonics) in a given core part is calculated as

$$F = \sqrt{\left[\sum_{n=1}^k [F_n]^2 \right]} \quad (3.40)$$

3.5.2 Effect of excitation level

The flux linkages in the SRM are proportional to the level of excitation. Increase in stator currents results in increased flux linkages leading to core saturation. The losses in the core will therefore be a function of excitation level. Variation of excitation level occurs every time the load is changed or during a commanded change in current such as would occur during chopping. This variation also occurs as the rotor moves from the unaligned to the fully aligned position. Variation in the excitation level is simulated by varying the peak amplitude A of the flux linkage function. Figure 26 shows the change in the r.m.s. value of the a.c. component of flux linkage when A is varied from 0.1 to 1.0.

The conduction angle and switching period are kept constant at 2.7 ms and 10 ms respectively. It is observed that the effective flux linkages in all parts of the core fall linearly as A is decreased. The rotor core and the stator core segments with reversing flux show higher sensitivity to these changes.

3.5.3 Effect of conduction angle and switching period

Conduction angle modulation is one of the control strategies used in multi-quadrant SRM drives. Figure 27 shows the variation of the r.m.s. value of the flux linkages in the different parts of the core when the conduction angle is varied from 9° to 45° with the peak amplitude $A = 1$ and a switching period of 10 ms.

The effective flux linkages fall as the conduction angle is decreased. Again, the rotor core is observed to be more sensitive to these variations. By inference, the rotor losses increase as the conduction angle increases. The stator core segments with

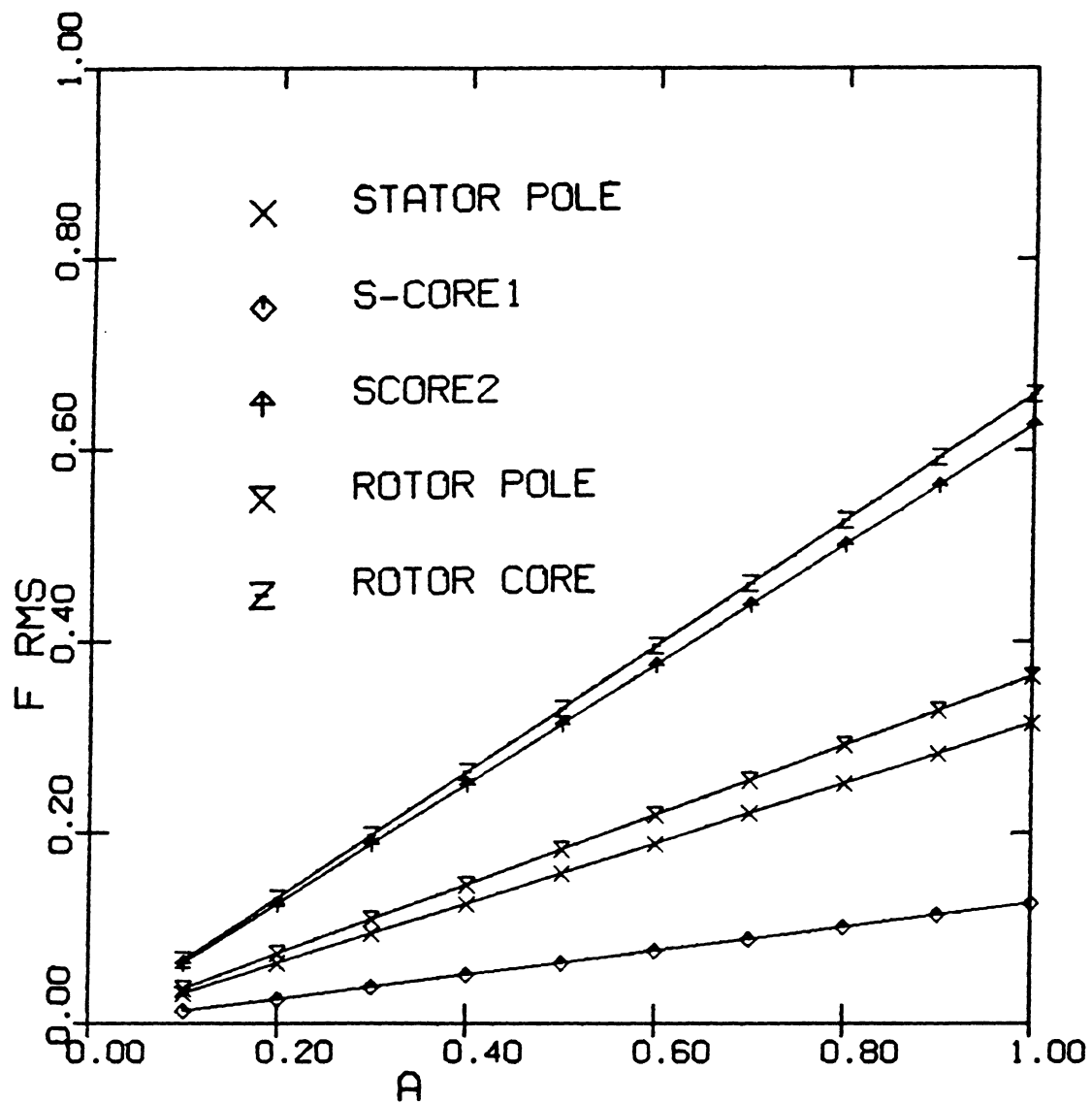


Figure 26. Effective value of flux linkages versus excitation level

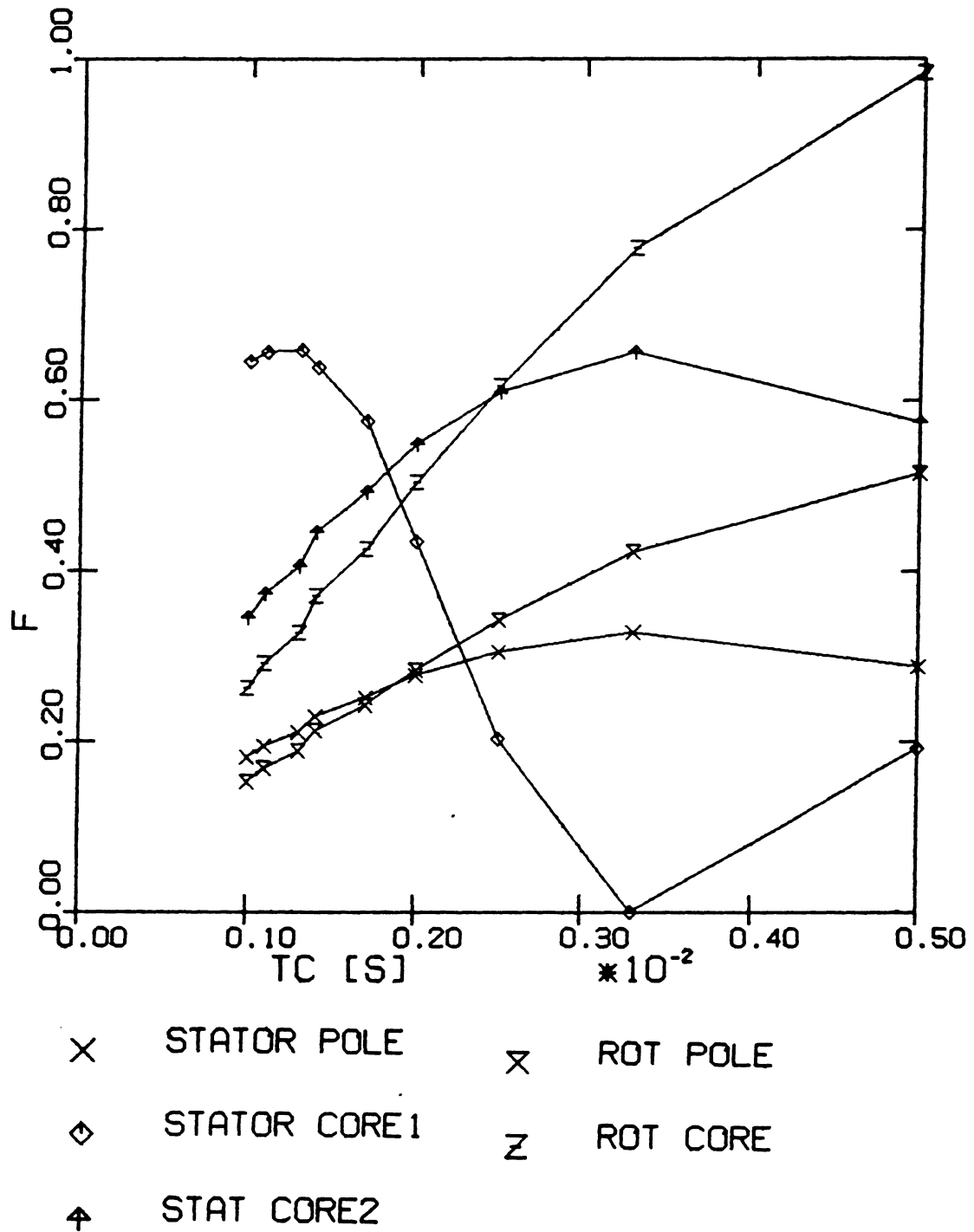


Figure 27. Flux linkages versus conduction period

non-reversing flux show an interesting characteristic. For values of conduction period smaller than T_w , the effective value of flux falls in inverse proportion to the conduction angle, becoming zero at the threshold of overlap. Beyond this point it increases again while the effective values in the other core parts fall. This implies that the loss component due to this frequency is high for small values of conduction angle and also when the conduction overlap between phases is large. In the 8/6 pole SRM, conduction overlap is inevitable and therefore this loss component is significant.

Figure 28 shows the effect of varying the switching period while maintaining a constant conduction angle.

The simulation is done for a speed range of 300-2500 r.p.m. It can be observed that, except for the stator segments with non-reversing flux, the effective flux decreases as the speed decreases. In the non-reversing flux stator segments, the flux linkage is observed to be more dependent on the duty cycle; at low values of duty cycle (ratio between conduction angle and switching period), the effective value is high. Due to its high frequency, this flux component has significant contribution to the core losses at low speeds.

3.5.4 Effect of Chopping

When operating in the constant torque region, it is necessary to maintain a constant current. To achieve this, the converter is operated in the chopping. This is normally implemented by chopping the current between two extreme levels (hysteresis control) or by using a PWM scheme. Chopping introduces a high frequency flux component which results in increased core loss. When the hysteresis method is used, the chop-

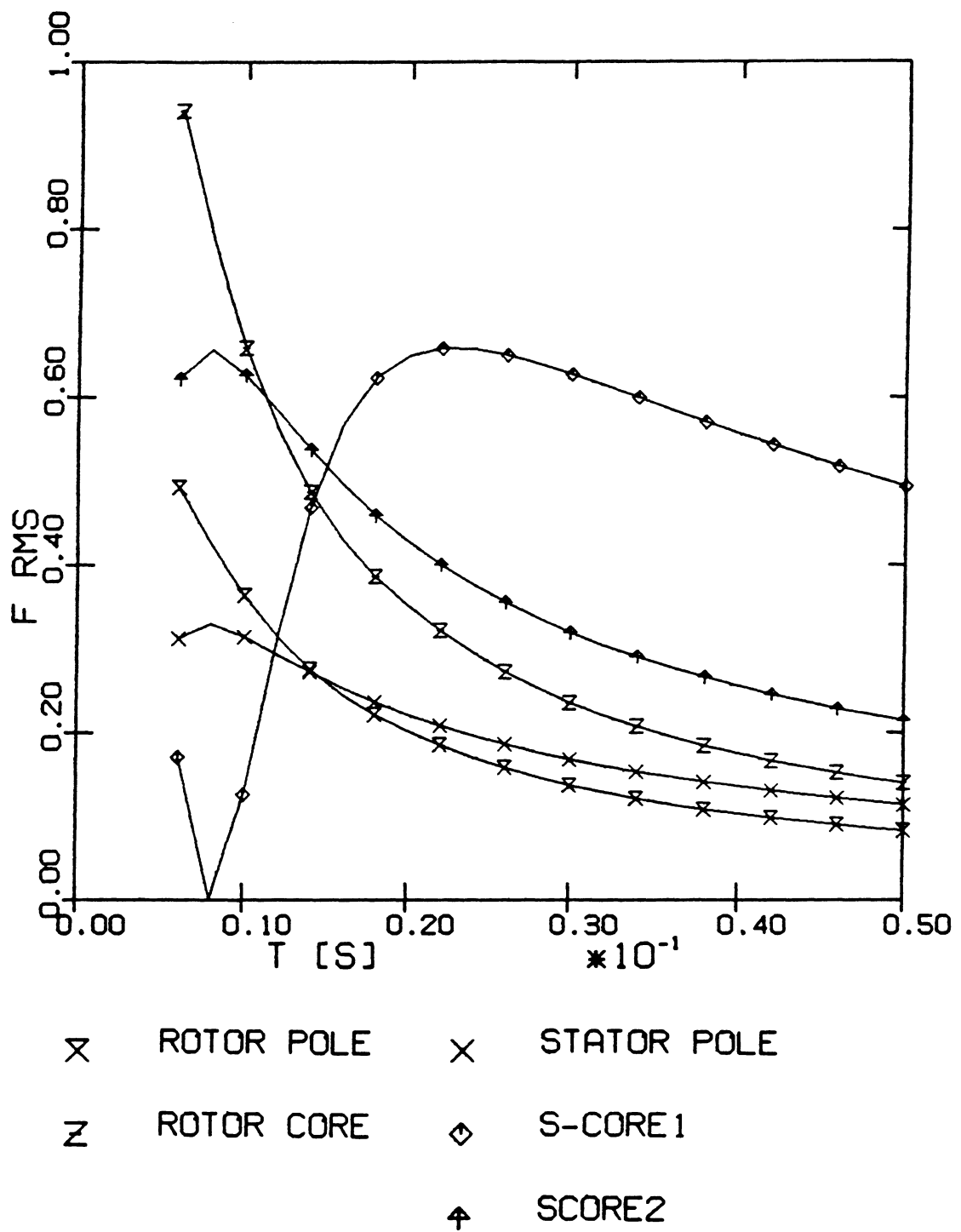


Figure 28. Flux linkages versus switching period

ping frequency depends on the magnitude of the specified current excursions, the magnitude of the average current and the rotor position. Taking the 6/4 pole SRM whose details are given in Appendix A and assuming a commanded average current of 10 A, a 20 % chopping window would result in current excursions between 12 A and 8 A. At low speed, this would result in 2.6 % excursions in flux linkages and a frequency of about 160 Hz. The core losses in the motor will therefore vary accordingly. Copper losses will also vary due to changes in the r.m.s. value of current as well as increased resistance due to skin effect, particularly if the chopping window is too small or, in the case of a PWM scheme, if the modulation frequency is too high.

3.6 *Measurements and Experimental results*

The 6/4 pole laboratory prototype SRM was used for experimental verification. The motor was coupled to a d.c. dynamometer for measuring the output power. As mentioned earlier, accurate measurements of motor currents can not be obtained with ordinary a.c. or d.c. meters because the current waveforms are neither sinusoidal nor pure d.c. and they vary with operating conditions namely, speed, saturation level and conduction angle. To alleviate the problem, a Data Acquisition System was used. In order to ensure uniformity in the accuracy of the data, 1024 data points per period were acquired. All measurements were made at a fixed voltage of 180 V d.c. The r.m.s. current was calculated from the acquired data using

$$I = \sqrt{\left[\frac{A_I}{T_s} \right]} \quad (3.43)$$

The average current is given by

$$\bar{i} = \frac{A_A}{T_s} \quad (3.44)$$

where A_A is the area enclosed between the current waveform and the time axis, A_I is the area enclosed between the square of the current waveform and the time axis and T_s is the switching period. Areas A_I and A_A were calculated using Simpson's rule. Simpson's rule was chosen because the error by this method is proportional to the fourth power of the time step used and the third derivative of the function at the end points. Since the current function is smooth at both ends and the time step is small (1/1024 of a period), the error is low. The input power to the motor is calculated from the average current and the d.c. supply voltage. The difference between this value and the output (shaft) power of the motor gives the total losses in the motor. By subtracting the copper losses, the friction and windage losses and the stray-load losses from this value, the core losses are obtained. The copper losses are calculated using equation (3.39) and the r.m.s. current calculated above. A friction and windage losses versus speed characteristic was obtained by driving the SRM at no load using a permanent magnet d.c. motor of a known torque constant and stator resistance. The stray load losses were estimated to be 6 % of the total motor losses [62].

Two types of current waveforms were investigated; a peaky and a flat-topped current waveform [38]. The peaky waveform is obtained by operating with a large advance angle. In this case, the advance angles were 11.95° and 0° and the conduction angles were 25.27° and 18.05° for the peaky and flat-topped currents, respectively. The conduction angle for the flat-topped current has to be small otherwise the current be-

comes peaky just before turn-off due to the low inductance slope. Figures 29 and 30 show oscillograms of the current and gating signal in one phase for the peaky and flat-topped waveforms, respectively.

Table 3 compares average current readings obtained using a d.c. meter to those obtained by the Data Acquisition System.

For the flat-topped current, the difference between the two readings is less than 10 % for all the readings whereas for the peaky current, up to 15 % error is observed. Figures 31 and 32 shows the variation of core and copper losses with excitation for the two types of current waveforms. Data samples for the same are shown in Tables 4 and 5.

The phase resistance is 3.4Ω . It is shown that operating with a peaky current waveform results in relatively higher copper losses when compared to the flat-topped current. The error obtained when the r.m.s. current is estimated using equation (3.38) and the appropriate duty cycle is observed to be larger for the peaky current. This accounts for the observed low copper losses compared to the predictions. In order to obtain accurate predictions, the actual current waveform has to be used. This has been predicted using the method described in chapter 4 and used to construct the flux linkages waveform for the peaky current waveform. The resulting waveform is shown in Figure 33 for an operating point of 1095 r.p.m, a peak current of 5.1 A and a conduction angle of 25.3° . Close agreement between the assumed and the actual waveforms is observed (approximately linear rise and fall of flux linkages). The inductance was determined using the analytical method described in chapter 2.

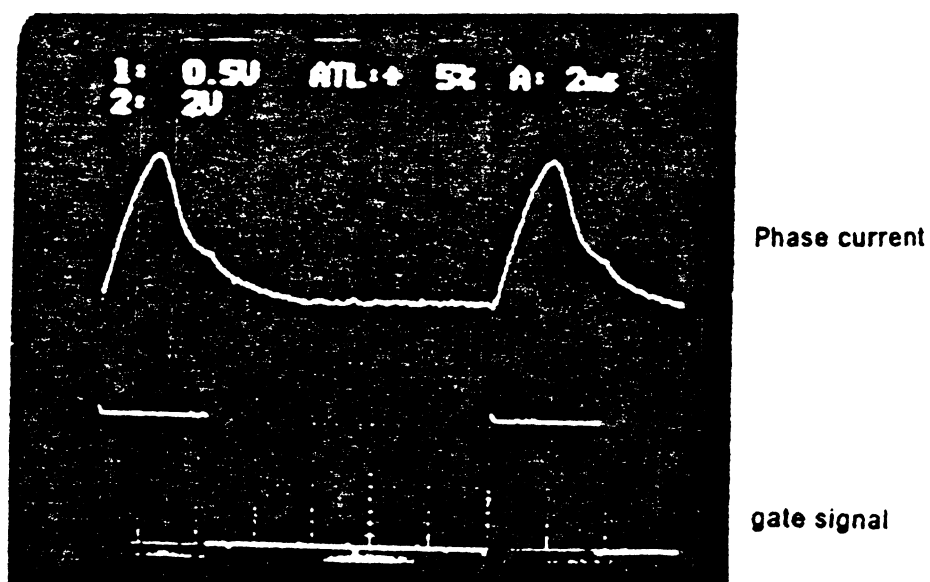


Figure 29. Peaky current waveform (Current scale: 2 A/div.)

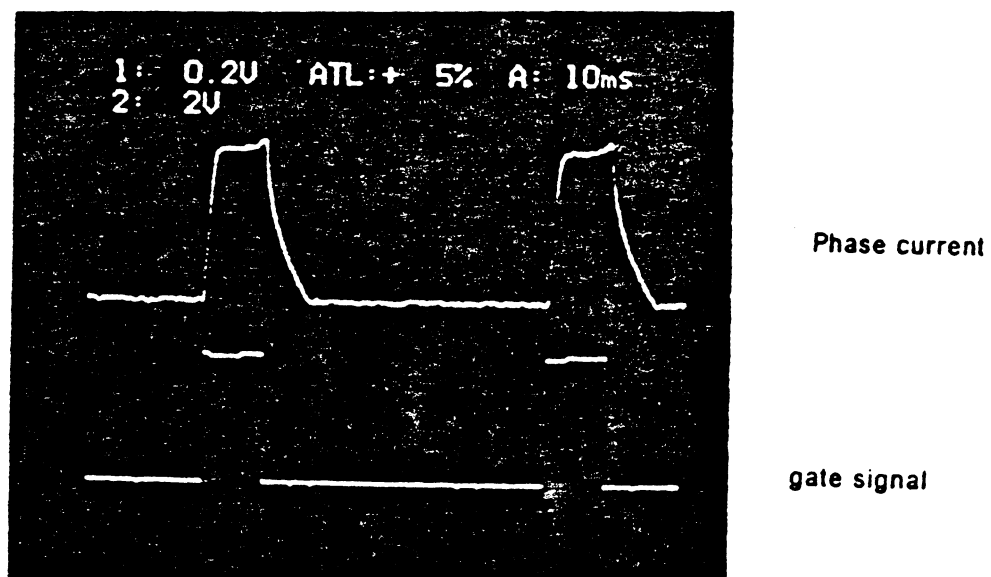


Figure 30. Flat-topped current waveform (Current scale: 0.8 A/div.)

Table 3. Comparison of current measurements by DAS and analog d.c. meter

Flat-topped current			Peaky current		
Peak current [A]	Average current [A]		Peak current [A]	Average current [A]	
	D.C Meter	Data Acqsn.		D.C Meter	Data Acqsn.
1.07	0.25	0.230	2.67	0.42	0.441
1.54	0.35	0.328	3.28	0.55	0.546
1.83	0.42	0.397	4.04	0.60	0.694
2.13	0.50	0.464	4.94	0.90	0.888
2.54	0.57	0.542	5.66	1.05	1.051

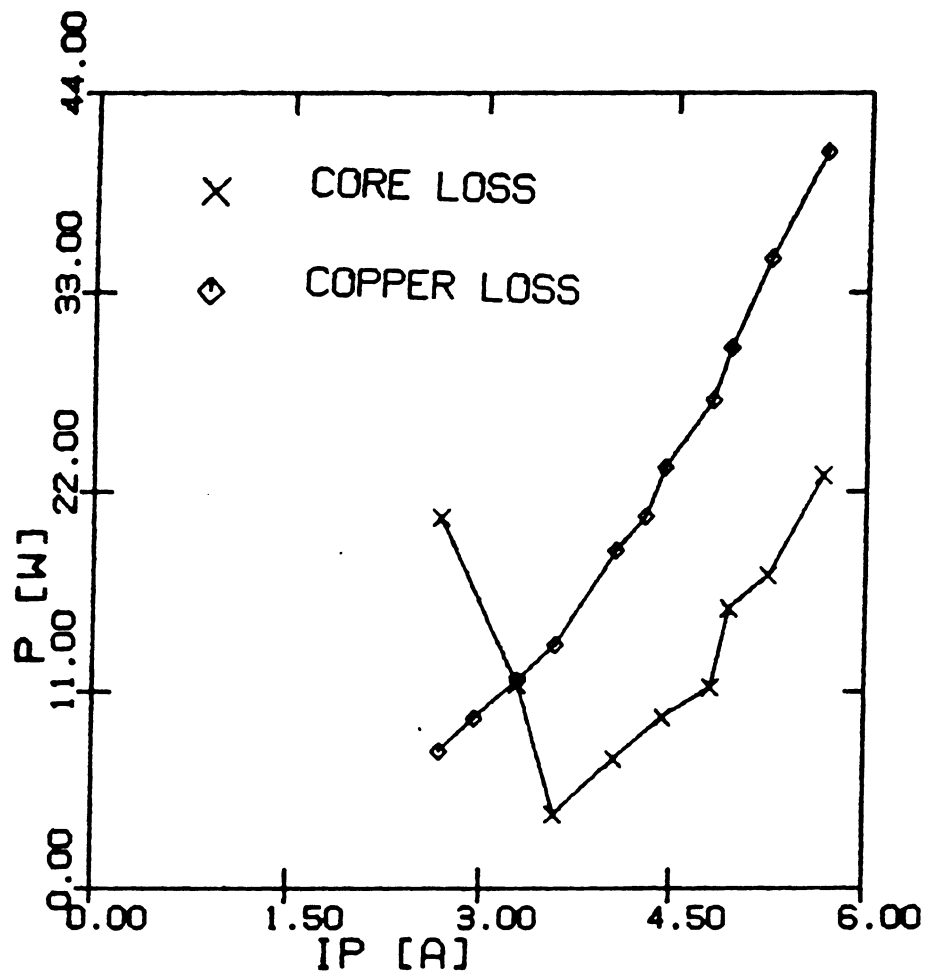


Figure 31. Measured losses for peaky current waveform (988-2324 r.p.m.)

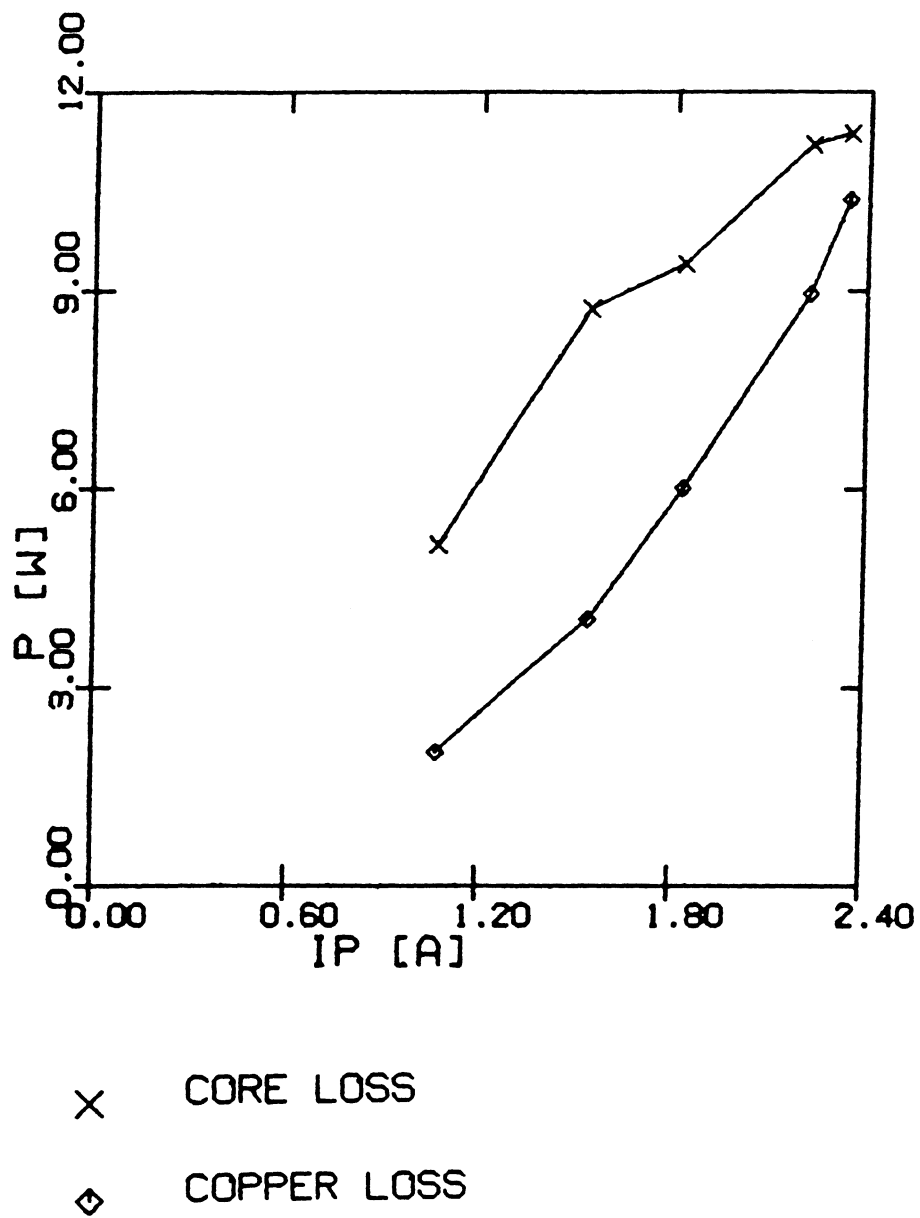


Figure 32. Measured losses for flat-topped current waveform (486-955 r.p.m.)

Table 4. R.m.s. current and losses for flat-topped current waveform

Peak current [A]	R.M.S. current by DAS [A]	R.M.S. current by [38] [A]	Iron losses [W]	Copper losses [W]	Speed [rpm]	$\frac{P_{cu}}{P_{in}}$ [%]
1.07	0.446	0.618	5.6	2.0	955	3.4
1.54	0.629	0.889	9.6	4.0	729	4.8
1.83	0.768	1.057	10.4	6.0	615	5.9
2.13	0.888	1.230	10.5	8.0	550	6.9
2.54	1.051	1.466	10.70	11.3	486	8.0

DAS stands for Data Acquisition System.

P_{in} and P_{cu} are the input power and copper losses respectively.

Table 5. R.m.s. current and losses for peaky current waveform

Peak current [A]	R.M.S. current by DAS [A]	R.M.S. current by [38] [A]	Iron losses [W]	Copper losses [W]	Speed [rpm]	$\frac{P_{cu}}{P_{in}}$ [%]
2.67	0.870	1.30	20.6	7.7	2324	4.5
3.28	1.067	1.59	11.3	11.6	1806	5.2
4.04	1.356	1.96	7.3	18.8	1458	7.1
4.94	1.712	2.40	15.5	29.9	1150	9.0
5.66	2.000	2.66	22.9	40.8	1004	10.6

DAS stands for Data Acquisition System.

P_{in} and P_{cu} are the input power and copper losses respectively.

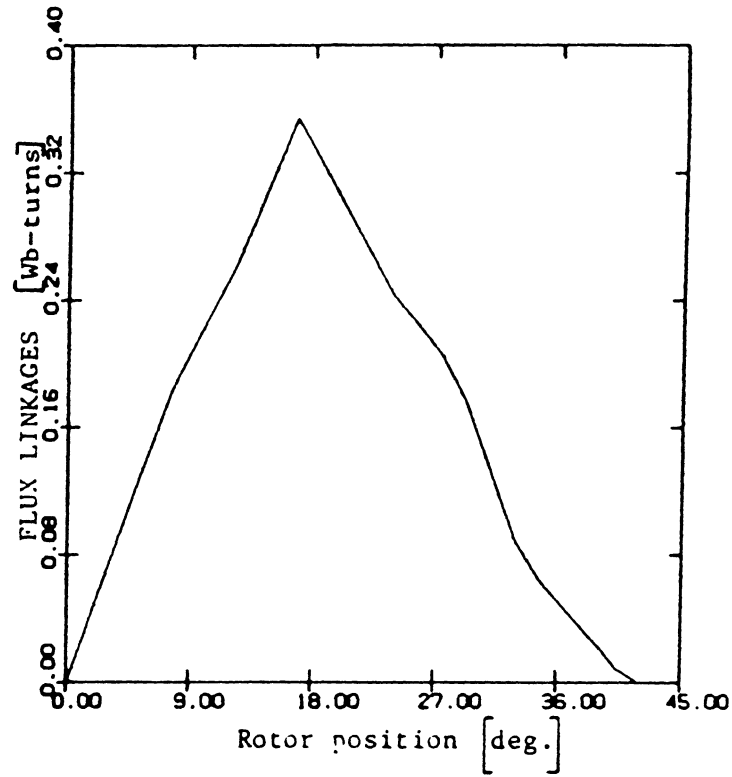


Figure 33. Flux linkages waveform reconstructed from measured current

3.6.1 Effect of saturation

Figures 34 and 35 show a comparison between the measured core losses, core losses predicted on the basis of the voltage waveform and those predicted using the actual current waveform.

It is observed that at high phase currents, the prediction based on the voltage waveform gives a larger error and tends to deviate from the expected trend as the degree of saturation increases, predicting decreasing core losses as the current increases. The error arises from the fact that as the degree of saturation increases, the maximum flux linkages do not occur at the turn-off point as the voltage-based approach would suggest. This effect is particularly significant in the case of the peaky current waveform where the current at the turn off point may be less than half of the peak on-time current achieved. The tendency then is for the point of maximum flux linkages to occur prior to turn-off. Consequently, the flux linkages waveform based on the current waveform has a short rise time and a long fall time resulting in a Fourier spectrum different from that of the voltage-based flux linkages waveform.

The difference between the measured and the predicted value can be partly attributed to the change in the magnetic properties of the core material due to the stresses induced during cutting of laminations and also due to imperfections in stacking the laminations. The combined effect of these errors has been observed to result in increasing the core losses by as much as 25 % [62]. Other possible sources of error include errors in the predicted values of inductance, imperfections in the motor core structure, particularly the air-gap and a probable error in the estimated value of stray load losses.

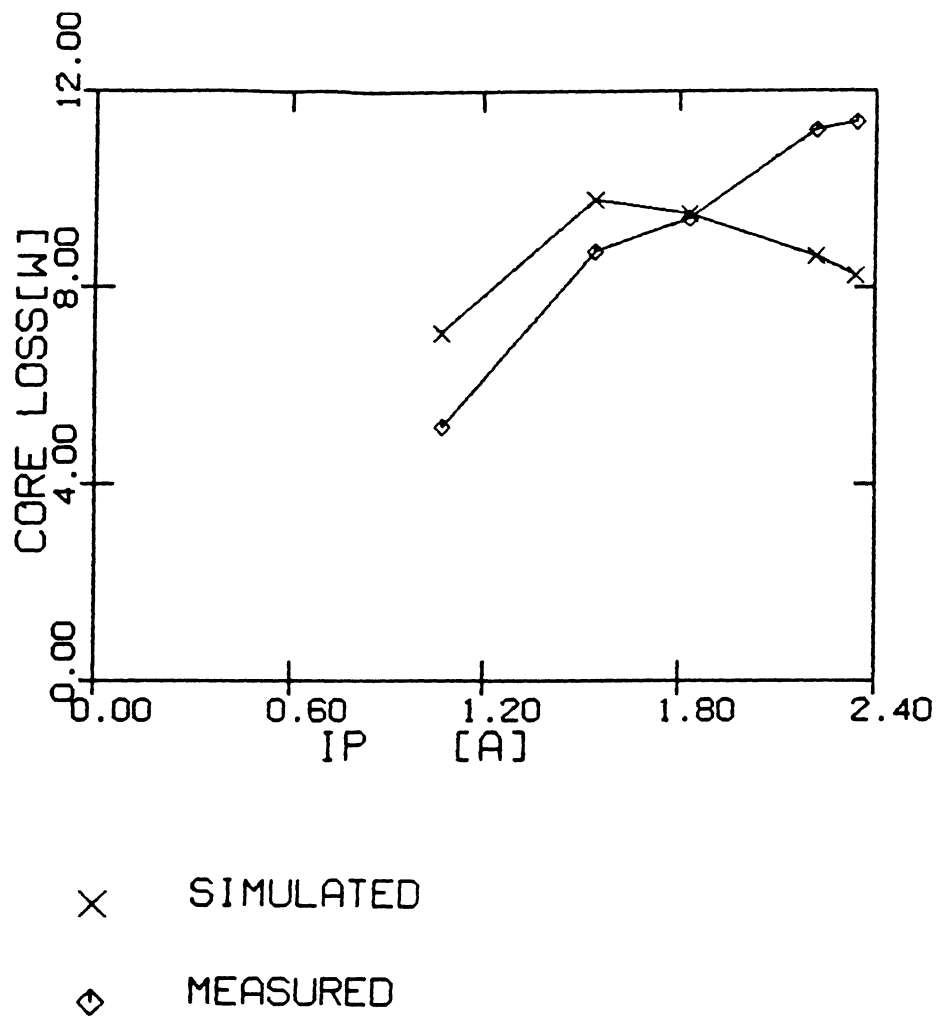
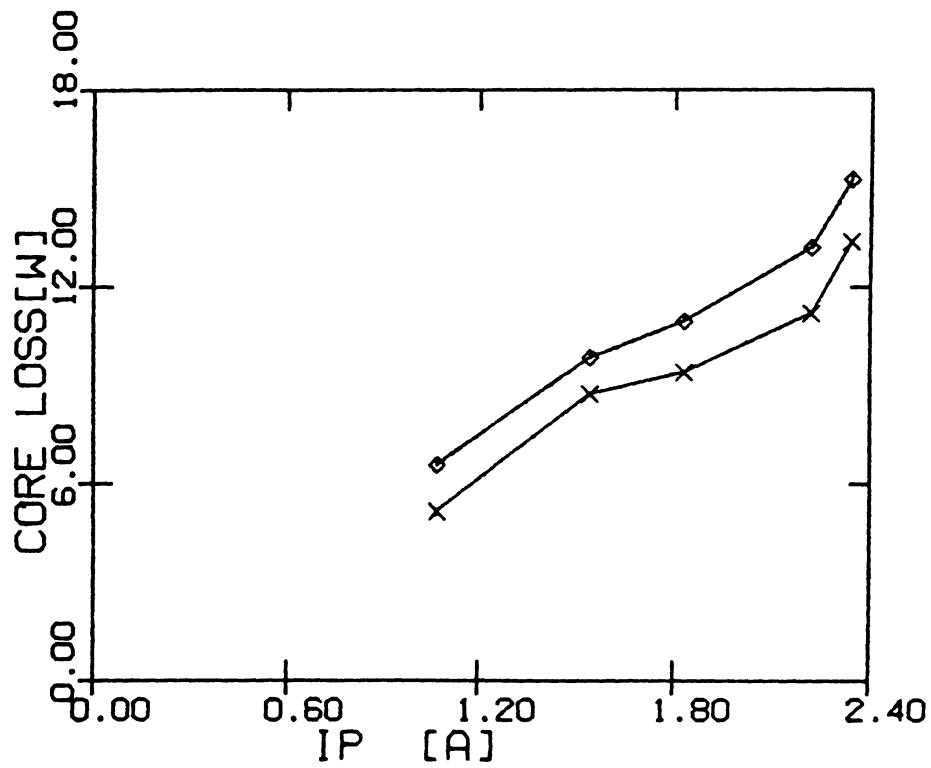


Figure 34. Measured and predicted core losses based on voltage waveform



X SIMULATED

◇ MEASURED

Figure 35. Measured and predicted core losses based on current waveform

3.7 Conclusion

A method for determining the losses in a switched reluctance motor has been described. Fourier analysis has been used to determine the harmonic spectra of the fluxes in different parts of the core. It is shown that harmonics of all orders exist and have a very significant effect on the core losses particularly in the rotor core. The maximum core losses occur in the rotor. It is also shown that, unlike conventional machines, core losses are not constant but vary widely with stator current, conduction angle and speed. Core losses increase considerably as the level of excitation and speed increase. Conduction angle modulation produces very significant changes in the flux spectrum in the motor. The high frequency component in the stator core is very sensitive to this change particularly during overlap and at low duty cycle. At low excitation, core losses are higher than copper losses; however, as the current increases, copper losses dominate. The problems associated with measurement of motor currents have been discussed and a solution is proposed. It is observed that the values of average current measured using a d.c. current analog meter differ by as much as 15 % from those obtained using a (digital) Data Acquisition System (DAS). Since the DAS is based on the actual current waveform, it is proposed as the most reliable method for measuring the SRM currents.

4.0 STEADY-STATE ANALYSIS OF THE VARIABLE SPEED SWITCHED RELUCTANCE MOTOR DRIVE

The principle of operation of the SRM drive demands that the motor and converter be treated as one unit. Little has been done to develop a complete analysis of this motor-converter combination. This chapter presents an approach to the steady-state analysis of the drive including the effects of stator winding resistance, input filter parameters and snubber circuits which are often neglected. The analysis yields current, voltage, torque and back-e.m.f. waveforms providing guidelines to the optimal design of the drive. The method is unique in that the steady-state solution of the system equations is obtained directly without going through the transient solution. Experimental verification is provided for a 6/4 pole prototype SRM drive and shown to be in good agreement with the simulated results. The approach can be applied to any other motor-converter combination with minimal modifications.

4.1 Introduction

So far, a complete steady state analysis of the drive which includes all the parts of a typical SRM drive has not been reported. Most of the previous work assumes a linear model of the motor and neglect:

- i. the resistance of the stator winding,
- ii. effects of snubber circuits on device stresses and
- iii. effect of the input filter.

Omission of some or all of these factors in the analysis of the drive results in varying degrees of inaccuracy depending on the type of drive and operating point. For example, the assumption of negligible winding resistance is justified for high voltage drives but results in a large error in the case of battery-powered drives.

The general voltage equation for a phase winding is given by:

$$V_{dc} = R_e i + L(\theta, i) \frac{di}{dt} + i \frac{dL(\theta, i)}{dt} \pm M(\theta, i) \frac{di'}{dt} \quad (4.1)$$

where

V_{dc} = dc source voltage

R_e = sum of the stator winding and switch resistances

$L(\theta, i)$ = inductance per phase

i = instantaneous phase current in the excited phase

i' = phase current in the previously excited phase

θ = rotor position

$M(\theta, i)$ = mutual inductance between adjacent phases.

The four terms in the equation are here conveniently labelled as the resistive, reactive, back e.m.f. and mutual coupling components. At low speeds, the magnitude of the resistive component is comparable to that of the other three components and can therefore not be neglected in the analysis. At high speeds, the reactive and back e.m.f. components dominate. Both components are functions of the phase inductance and act so as to oppose the flow of current thereby slowing down the rise of current in the winding. Thus by performing steady-state analysis, the designer can predict the maximum possible on-time current and accordingly adjust the design so as to conform to specifications. Conversely, if the peak on-time current is specified, the designer can predict whether or not the specified current can be achieved at the specified supply voltage and rated speed. Knowing the instantaneous phase current and inductance, the instantaneous torque per phase can be determined and used to predict the steady-state average torque.

The presence of capacitive snubber circuits significantly alters the switching stresses on the power semiconductor devices. When a phase switch is turned on, the current rise is delayed until the snubber capacitor discharges (into the switch). At turn-off, the rate of rise of switch voltage is limited by the snubber circuit. Through steady-state analysis, the snubber circuit can be adjusted to produce a desired switching effect.

For an off-line SRM drive, the waveform of the source current determines the magnitude and spectrum of the harmonic distortion transmitted to the utility system by the drive. The proposed analysis provides the source current and filter capacitor voltage

waveforms. It is shown that the source current consists of peaky, discontinuous pulses whose amplitude depend on the applied voltage, the switching strategy and the load. Further studies on ways to minimize the resulting harmonic distortion are suggested.

A complete steady-state analysis of SRM drive must therefore include all of the above considerations. This chapter describes a systematic approach for the steady-state analysis of the variable speed switched reluctance drive. Starting with the voltage equations of the drive, the state equations for each mode of operation are derived. Solution of the equations is effected using the initial steady-state values and boundary matching techniques. The solution requires the inductance profile as an input. Methods for the prediction of this profile are given in chapter 2. An illustration for the prototype 6/4 pole motor fed from a MOSFET converter is given.

4.2 General analysis procedure

The basic circuit shown given in Figure 13 on page 54 simply represents the major parts of an SRM drive. In a practical drive, additional circuit elements are required. There are a minimum of two additional requirements on the circuit:

- i. Snubber circuits across the switches to reduce the switching stresses particularly at turn-off.
- ii. An input filter across the supply.

The input filter, snubber circuits and the winding inductance together form a fourth or higher order system of equations depending on the converter configuration used. The interaction between the different states varies according to the mode of operation. The SRM drive can therefore be represented by the equivalent circuit shown in Figure 36 where $L(\theta, i)$, $M(\theta, i')$, i , i' and R_e are as described earlier. E is the forcing function, the value of which depends on the mode of operation.

The induced voltage function is described by

$$e = i \frac{dL(\theta, i)}{dt} \pm M(\theta, i') \frac{di'}{dt} \quad (4.2)$$

Since the output torque of the drive is dependent on the rotor position, it is convenient to express the voltage equation in the θ domain by applying the transformations of the form:

$$\frac{dL(\theta, i)}{dt} = \frac{dL(\theta, i)}{d\theta} \omega \quad (4.3)$$

$$\frac{di}{dt} = \frac{di}{d\theta} \omega \quad (4.4)$$

where ω is the angular speed of the motor.

These transformations apply only to the steady state condition. For example, equation (4.1) becomes:

$$V_{dc} = R_e i + L(\theta, i) \frac{di}{d\theta} \omega + i \frac{dL(\theta, i)}{d\theta} \omega \pm M(\theta, i') \frac{di'}{d\theta} \omega \quad (4.5)$$

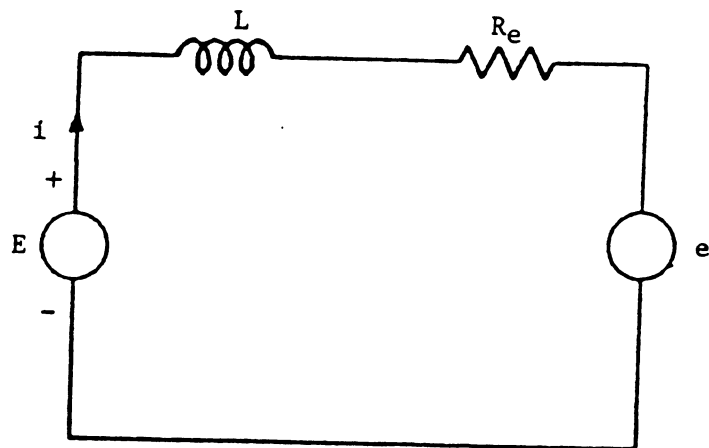


Figure 36. Approximate equivalent circuit of the SRM drive

4.2.1 Solution of the system equations

Irrespective of the order of the system, the general form of the system equations is:

$$\dot{x} = Ax + Bv \quad (4.6)$$

Equation (4.6) has a solution of the form

$$x = X(0)e^{A(t-t_0)} + \int_{t_0}^t e^{A(t-\tau)} B(\tau) v(\tau) d\tau \quad (4.7)$$

Due to the large number of state variables and the existence of many modes of operation, computer solution of the equations is a necessity. Two of the most commonly used numerical methods for a computer solution are the predictor-corrector method and the Runge- Kutta method. In both the methods, accuracy is dependent on the integration step used. The predictor-corrector method is simple and self-starting. However, most of the formulas used are truncated series. Therefore, the truncation error has to be continually checked and appropriate adjustment on the time step implemented. This takes considerable computing time. On the other hand, the Runge-Kutta method is not self-starting and therefore requires an initial solution. If the initial steady-state vector is known, direct evaluation of the steady-state performance of the drive can be implemented without having to go through the transient solution. This saves computer time. The evaluation of the steady-state performance directly without going through the initial transients has been successfully applied to induction machines controlled by series connected semiconductor switches [56,57]. However, the technique proposed in these references is only applicable to the case

where the excitation consists of alternate periods of a normal sine wave and periods of open circuit operation. In the case of the switched reluctance motor, the excitation is not sinusoidal. Further, the waveforms of the states cannot be explicitly defined because many intermediate variables are dependent on operating conditions. Hence, numerical integration is used to solve the system of equations.

A fourth order Runge-Kutta integration method has been employed in this case. Direct evaluation of the steady-state performance can still be achieved because the initial steady state vector for a switched reluctance motor is explicitly known. At the end of each cycle, the phase and source currents decrease to zero before the phase is excited again. The snubber and input filter capacitors charge up to 1 p.u. d.c. link voltage in single-switch-per-phase converter configurations. In the two-switch-per-phase configurations, the snubber capacitors charge up to 0.5 p.u. d.c. link voltage. An illustration is given for a 6/4 pole switched reluctance motor fed by a novel, four-switch converter configuration.

4.3 System study

4.3.1 Description of the circuit

Figure 37 shows the motor-converter configuration analyzed.

The configuration has the following unique features:

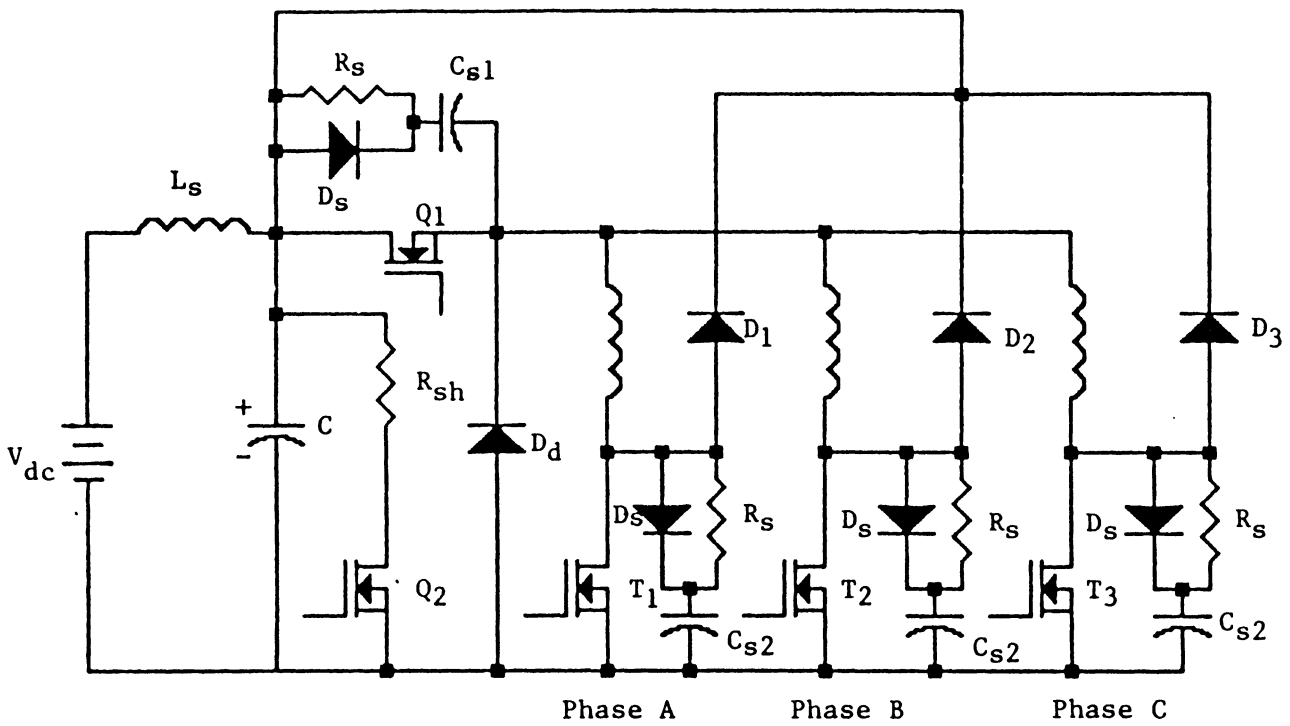


Figure 37. The 4-switch, 3-phase SRM converter configuration

- i. it requires only one switch per phase and one commutation switch
- ii. it is energy efficient since energy stored in the winding is fed back to the d.c. link
- iii. it has only four switches and diodes (3-phase) but has most of the advantages of the two-switch-per-phase configuration.

Since all the phases are identical, a description is given for only one phase.

Considering phase A, Q_1 and T_1 are turned on simultaneously. Since the output torque and power are proportional to $dL/d\theta$, turn-on is effected prior to position θ_1 , that is, before a rotor pole starts aligning with the excited stator pole. A simplified representation of the switching sequence is shown in Figure 38.

During the chopping period, the main switch, Q_1 , is maintained ON while the phase switch, T_1 , is switched on and off. The switching frequency of the phase switch in this condition depends on the width of the hysteresis band ($I_{\max} - I_{\min}$), shown in Figure 38. Chopping can also be implemented using the main switch only or both the main and phase switches.

In motoring operation, the switches are turned off in advance of the falling inductance region. Ideally, the current is expected to fall to zero before $dL/d\theta$ becomes negative. If current flows in the phase winding when $dL/d\theta$ is negative, negative torque is produced which in effect reduces the steady-state average torque of the motor. At speeds above base speed, the motor back e. m. f. delays the rise of current and also prolongs the time during which current falls to zero with the consequence that at large conduction angles, current flows in the phase winding when the rate of change

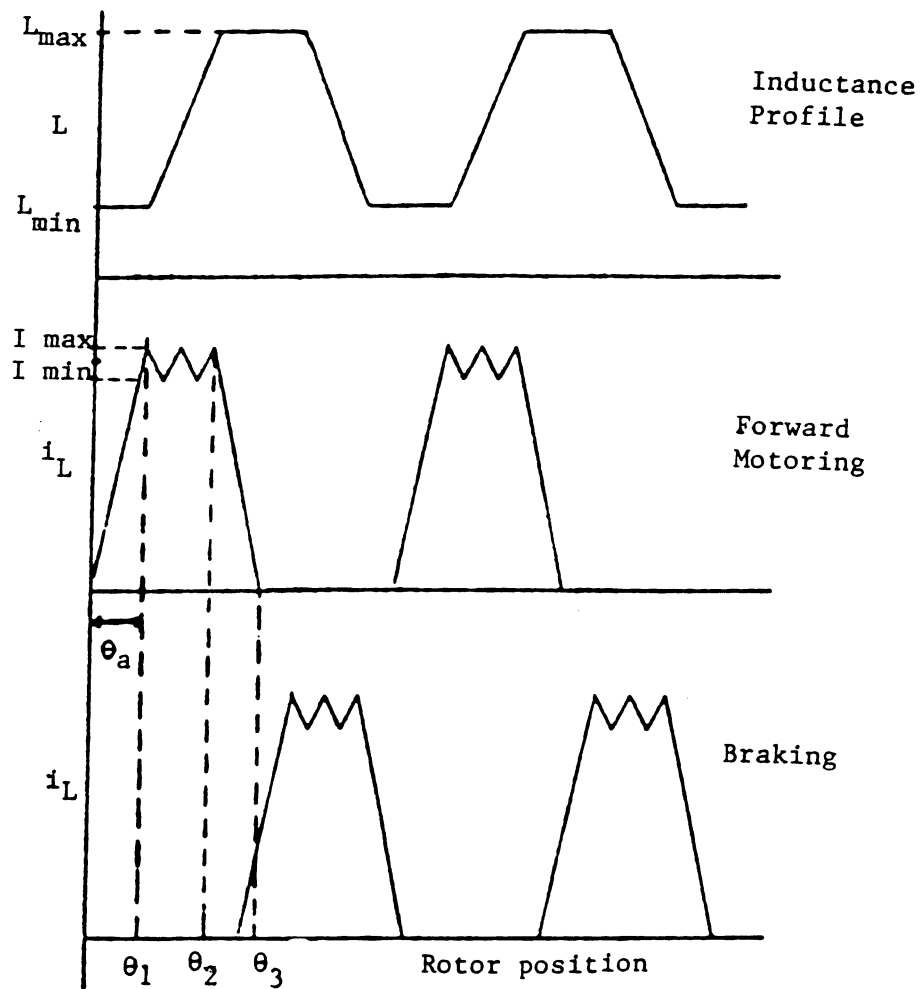


Figure 38. Simplified switching waveforms for one phase

of inductance with rotor position is negative. The switching strategy must therefore be designed to avoid this undesirable operating condition.

When the switches are turned off, the snubber capacitors take up the current and limit the rate of rise of voltage across the switches. Once the phase switch snubber capacitor charges up to slightly above the d.c. link voltage, the energy recovery diode D1 and shunt diode D_d snap on dumping part of the stored energy into the filter capacitor C. The diodes have to be of the fast recovery type. This mode continues until the stored energy is depleted or the next phase is turned on.

If the voltage across the input filter capacitor exceeds a pre-set value, the clamping switch Q_2 is turned on so that the extra energy is dissipated in the resistor in series with the switch. This checks the overcharging of the capacitor which may occur during regeneration.

4.3.2 Steady-state equations

In steady state, all voltages and currents are periodic and the speed is constant. It is assumed that the rotor inertia is large enough to smooth out any speed variation which may arise due to time-varying components contained in the electromagnetic torque. Since all the phases are identical, only one phase is required for the analysis. Figure 39 shows the circuit diagram of the drive for one phase of the motor.

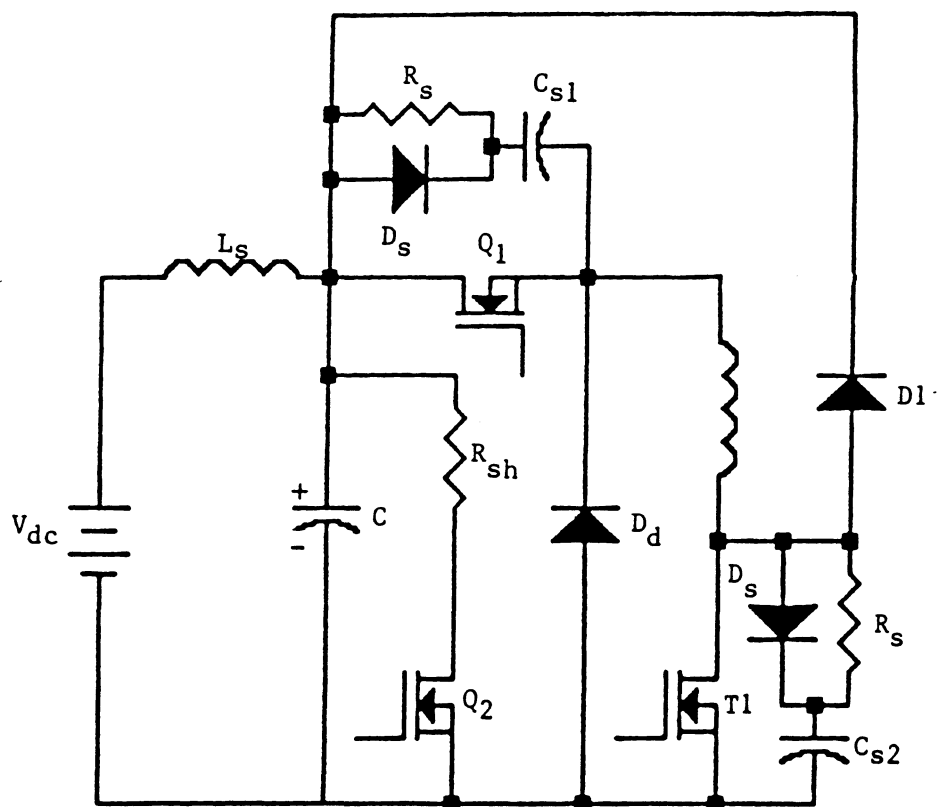


Figure 39. Single phase circuit of the 4-switch, 3-phase SRM drive

At constant speed, the circuit reduces to a fifth order piecewise-linear system with seven different topological modes. These modes are described by seven sets of linear differential equations of the form shown in equation (4.6) where

$$X = \begin{bmatrix} i_s \\ i_L \\ v_c \\ v_{cs1} \\ v_{cs2} \end{bmatrix} \quad (4.8)$$

and

$$V = \begin{bmatrix} V_{dc} \\ V_D \end{bmatrix} \quad (4.9)$$

i_s and i_L are the source and phase currents respectively, v_c is the input filter capacitor voltage, v_{cs1} and v_{cs2} are the main and phase switch snubber capacitor voltages and V_{dc} and V_d are the dc source and diode drop voltages, respectively. It is assumed that the design of the motor is such that mutual coupling between phases is negligible.

The initial steady state vector in this case is given by

$$X(0) = \begin{bmatrix} 0 \\ 0 \\ V_{dc} \\ \frac{V_{dc}}{2} \\ \frac{V_{dc}}{2} \end{bmatrix} \quad (4.10)$$

Matrices A and B are uniquely defined for each topological mode. The elements of these matrices are functions of circuit elements and the speed of the motor. To facilitate the understanding of the description, the switching period is divided into three regions as follows:

Region I: Q_1 and T_1 ON (Phase and main switches on)

Region II: Chopping region (Main switch on, phase switch chopping)

Region III: Q_1 and T_1 OFF (Phase and main switches off)

The different modes and their respective state equations are described in the following section.

Region I.

When the main and phase switches are turned on, the snubber capacitors discharge. At the same time, energy is transferred from the source to the motor. Thus the phase

current starts from zero and rises at a rate mainly determined by the applied d.c voltage and the value of the minimum inductance. This operational mode is characterized by the following equations:

$$\begin{bmatrix} \dot{i}_s \\ \dot{i}_L \\ \dot{v}_c \\ \dot{v}_{cs1} \\ \dot{v}_{cs2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-1}{\omega L_s} & 0 & 0 \\ 0 & -\frac{(R_e + k\omega)}{\omega L} & \frac{1}{\omega L} & 0 & 0 \\ \frac{1}{C\omega} & \frac{-1}{\omega C} & 0 & 0 & 0 \\ 0 & \frac{R_D}{\omega R' C_s} & 0 & \frac{-1}{\omega R' C_s} & 0 \\ 0 & \frac{R_D}{\omega R' C_s} & 0 & 0 & \frac{-1}{\omega R' C_s} \end{bmatrix} \begin{bmatrix} i_s \\ i_L \\ v_c \\ v_{cs1} \\ v_{cs2} \end{bmatrix} + \begin{bmatrix} \frac{1}{\omega L_s} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ V_D \end{bmatrix} \quad (4.11)$$

where

$$R_e = R_L + 2R_D \quad (4.12)$$

$$R' = R_s + R_D \quad (4.13)$$

and

$$k = \frac{dL}{d\theta} \quad (4.14)$$

R_D , R_L , and R_s represent MOSFET ON resistance, phase winding and snubber circuit resistances respectively.

This mode ends when the motor current attains the pre-set maximum value $I_{\max}$ or when Q_1 and T_1 are switched off.

Region II:

In the chopping region, the main switch is maintained ON while the phase switch is switched on and off within a preset window such that the current oscillates around a desired value of peak current. Three different modes exist in this region:

Mode 2. Phase snubber capacitor charging

$$\begin{bmatrix} \dot{i}_s \\ \dot{i}_L \\ \dot{v}_c \\ \dot{v}_{cs1} \\ \dot{v}_{cs2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-1}{\omega L_s} & 0 & 0 \\ 0 & -\frac{(R+k\omega)}{\omega L} & \frac{1}{\omega L} & 0 & \frac{-1}{\omega L} \\ \frac{1}{C\omega} & \frac{-1}{\omega C} & 0 & 0 & 0 \\ 0 & \frac{R_D}{\omega R' C_s} & 0 & \frac{-1}{\omega R' C_s} & 0 \\ 0 & \frac{1}{\omega C_s} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_s \\ i_L \\ v_c \\ v_{cs1} \\ v_{cs2} \end{bmatrix} + \begin{bmatrix} \frac{1}{\omega L_s} & 0 \\ 0 & \frac{-1}{\omega L} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ V_D \end{bmatrix}$$

(4.15)

where

$$R = R_D + R_L \quad (4.16)$$

Mode 3. Freewheeling

When the phase snubber capacitor is charged-up to $V_c + V_D$, diode D_1 becomes forward biased. The motor current thus freewheels through path $Q_1LR_LD_1Q_1$. The mode equations are:

$$\begin{bmatrix} \dot{i}_s \\ \dot{i}_L \\ \dot{v}_c \\ \dot{v}_{cs1} \\ \dot{v}_{cs2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-1}{\omega L_s} & 0 & 0 \\ 0 & -\frac{(R + k\omega)}{\omega L} & 0 & 0 & 0 \\ \frac{1}{C\omega} & 0 & 0 & 0 & 0 \\ 0 & \frac{R_D}{\omega R' C_s} & 0 & \frac{-1}{\omega R' C_s} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_s \\ i_L \\ v_c \\ v_{cs1} \\ v_{cs2} \end{bmatrix} + \begin{bmatrix} \frac{1}{\omega L_s} & 0 \\ 0 & \frac{-1}{\omega L} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ V_D \end{bmatrix} \quad (4.17)$$

This mode ends when the phase switch is turned on again, that is, when $i_L = i_{L \min}$ or when Q_1 is also switched off.

Mode 4. Chopping, Q_1 and T_1 ON

This mode begins when $i_L = i_{L \min}$. At this point the phase switch is switched on again. The phase current rises as in mode 1 until $i_L = i_{L \max}$. The difference between this mode and mode 1 is that in this case, the voltage across the main switch snubber capacitor is zero at the beginning of the mode. The state equations below reflect this difference.

$$\begin{bmatrix} \dot{i}_s \\ \dot{i}_L \\ \dot{v}_c \\ \dot{v}_{cs1} \\ \dot{v}_{cs2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-1}{\omega L_s} & 0 & 0 \\ 0 & -\frac{(R_e + k\omega)}{\omega L} & \frac{1}{\omega L} & 0 & 0 \\ \frac{1}{\omega C} & \frac{-1}{\omega C} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{R_D}{\omega R' C_s} & 0 & 0 & \frac{-1}{\omega R' C_s} \end{bmatrix} \begin{bmatrix} i_s \\ i_L \\ v_c \\ v_{cs1} \\ v_{cs2} \end{bmatrix} + \begin{bmatrix} \frac{1}{\omega L_s} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ V_D \end{bmatrix}$$

(4.18)

Region III

This region begins when the conduction time for a given phase expires. This instant is normally selected in such a way that there is enough time for the phase current to decrease to zero before the rate of change of inductance with rotor position becomes negative. This operational mode is characterized by three topological modes.

Mode 5. Q_1, T_1 OFF, $i_L > 0$, snubber capacitors charging

$$\begin{bmatrix} \dot{i}_s \\ \dot{i}_L \\ \dot{v}_c \\ \dot{v}_{cs1} \\ \dot{v}_{cs2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-1}{\omega L_s} & 0 & 0 \\ 0 & -\frac{(R_L + k\omega)}{\omega L} & \frac{1}{\omega L} & \frac{-1}{\omega L} & \frac{-1}{\omega L} \\ \frac{1}{\omega C} & \frac{-1}{\omega C} & 0 & 0 & 0 \\ 0 & \frac{-1}{\omega C_s} & 0 & 0 & 0 \\ 0 & \frac{-1}{\omega C_s} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_s \\ i_L \\ v_c \\ v_{cs1} \\ v_{cs2} \end{bmatrix} + \begin{bmatrix} \frac{1}{\omega L_s} & 0 \\ 0 & \frac{-2}{\omega L} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ V_D \end{bmatrix} \quad (4.19)$$

Mode 6. Q_1, T_1 OFF, $i_L > 0$, energy fed back to the source

When the snubber capacitors charge up to $V_c + V_D$, diodes D_1 and D_d become forward biased and the energy stored in the motor winding is fed back to the source. In effect, this energy is dumped into the input filter capacitor C causing V_c to rise. In heavy load operation, the energy stored in the winding will be high. If left uncontrolled, the voltage across the capacitor may rise well above the forward blocking capability of the switches resulting in improper operation of the converter and possible damage to the switches. In this case Q_2 will clamp the voltage to a preset value damping the extra energy in the shunt resistor R_{sh} . The mode equations are summarized below:

$$\begin{bmatrix} \dot{i}_s \\ \dot{i}_L \\ \dot{v}_c \\ \dot{v}_{cs1} \\ \dot{v}_{cs2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-1}{\omega L_s} & 0 & 0 \\ 0 & -\frac{(R_L + k\omega)}{\omega L} & \frac{-1}{\omega L} & 0 & 0 \\ \frac{1}{\omega C} & \frac{-1}{\omega C} & \frac{-\lambda}{\omega C R_B} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_s \\ i_L \\ v_c \\ v_{cs1} \\ v_{cs2} \end{bmatrix} + \begin{bmatrix} \frac{1}{\omega L_s} & 0 \\ 0 & \frac{-2}{\omega L} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ V_D \end{bmatrix} \quad (4.20)$$

where

$$R_B = R_{sh} + R_D \quad (4.21)$$

and

$$\lambda = \begin{bmatrix} 1 \rightarrow V_c > V_{c \max} \\ 0 \rightarrow V_c \leq V_{c \max} \end{bmatrix} \quad (4.22)$$

R_{sh} is the shunt resistance across the input filter capacitor and $V_{c \max}$ is the maximum allowable voltage across the input filter capacitor. The clamp is activated whenever this voltage level is exceeded.

Mode 7. Q_1, T_1 OFF, $i_L = 0$, input filter capacitor charging

Under light load operation, the input filter capacitor may not charge up to the desired level during regeneration (mode 6). In that case, the extra charge is supplied by the dc source once the phase current decreases to zero. The equations for this mode are:

$$\begin{bmatrix} i_s \\ i_L \\ \dot{v}_c \\ \dot{v}_{cs1} \\ \dot{v}_{cs2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-1}{\omega L_s} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\omega C} & 0 & \frac{-1}{\omega C R_B} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_s \\ i_L \\ v_c \\ v_{cs1} \\ v_{cs2} \end{bmatrix} + \begin{bmatrix} \frac{1}{\omega L_s} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ V_D \end{bmatrix}$$

(4.23)

The advance angle θ_a is calculated under the assumption that the phase current attains the desired peak value when the leading rotor pole tip coincides with the trailing tip of the excited stator pole for a given direction of rotation. On the linear inductance profile, this corresponds to position θ_1 in Figure 4.3 The advance angle under these assumptions is given by

$$\theta_a = \frac{2I\omega L_{\min}}{2V_{dc} - R_e I} \quad (4.24)$$

where

$$I = \frac{i_{\max} + i_{\min}}{2} \quad (4.25)$$

4.4 Results

Figure 39 shows the flow chart of the computer algorithm used for the steady-state analysis of the prototype SRM drive shown in Figure 38 on page 114.

A fourth order Runge-Kutta integration routine is used. The inductance profile was predicted using a two-dimensional finite element analysis method. The simulation results are based on a d.c. link voltage of 180 V, a phase resistance of 3.4 ohms and an input filter capacitor of 1000 μF . The source inductance for the experimental set-up was measured to be 52.5 μH . The snubber circuit resistance and capacitance values were 220 ohms and 0.047 μF , respectively. Figures 41, 42 and 43 summarize the simulation results. Figures 41 and 42 show the inductance profile, phase current, back-e.m.f. and torque for an average on-time current of 5.1 A per phase and speeds of 1800 and 500 r.p.m. respectively.

In both cases, the conduction angle is $\theta_a + 24^\circ$. Using equation (4.15), θ_a is 8.1° at 1800 r.p.m. and 2.25° at 500 r.p.m. for the predicted minimum inductance of 25.8 mH. It is observed that at 1800 r.p.m., considerable phase current flows in the negative slope region of the inductance profile resulting in negative torque production. A reduction of the conduction angle is therefore necessary. It is also observed that at this speed, the chopping mode is never accessed due to the opposing effect of the back-e.m.f. which is proportional to the operating speed and the motor inductance. At the lower speed of 500 r.p.m., the maximum on-time current is initially exceeded resulting in chopping. The chopping window was set to be 2 A peak-to-peak corresponding to 20 % excursions from the average on-time current.

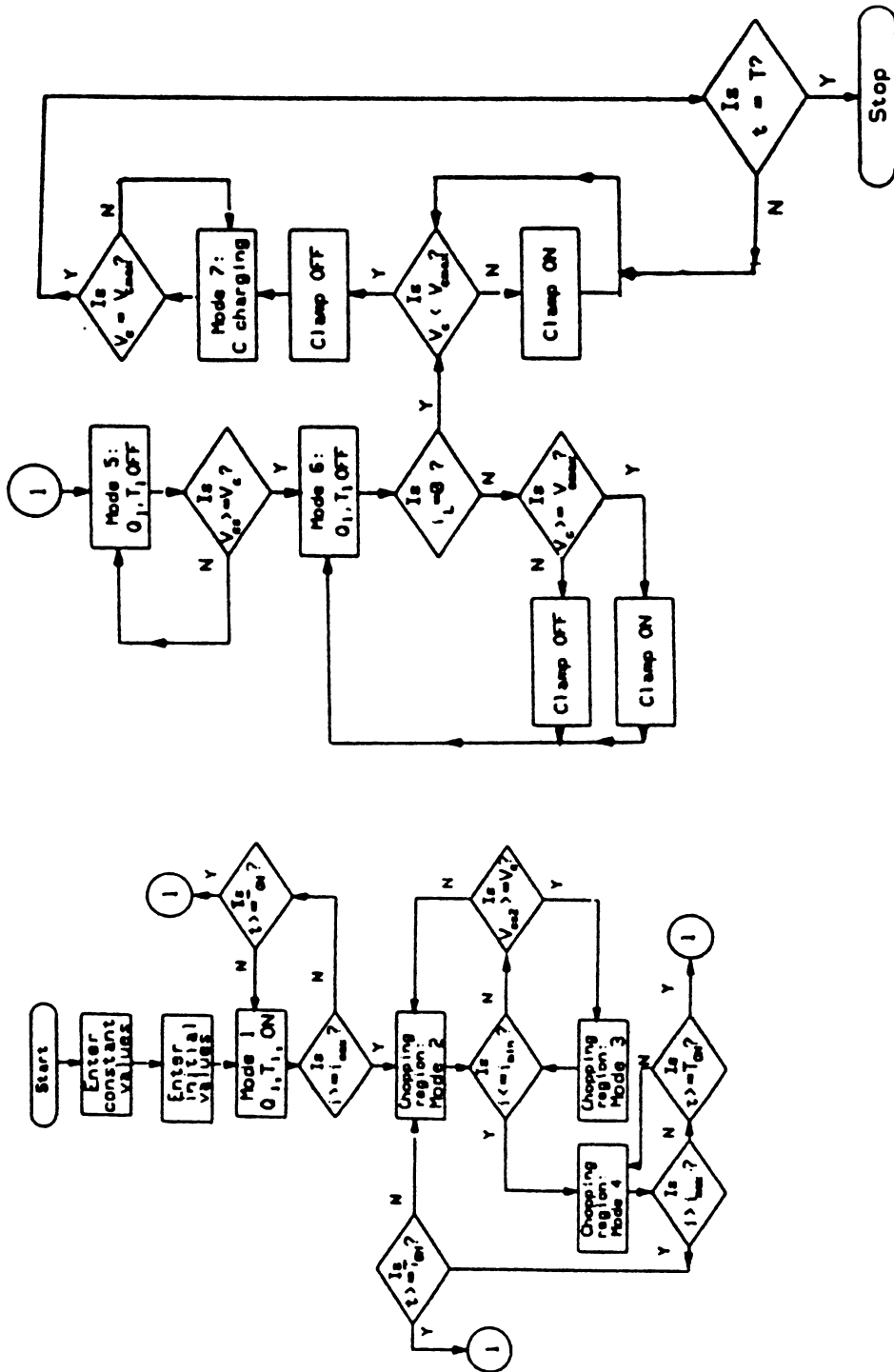


Figure 40. Flow chart of the computer solution routine

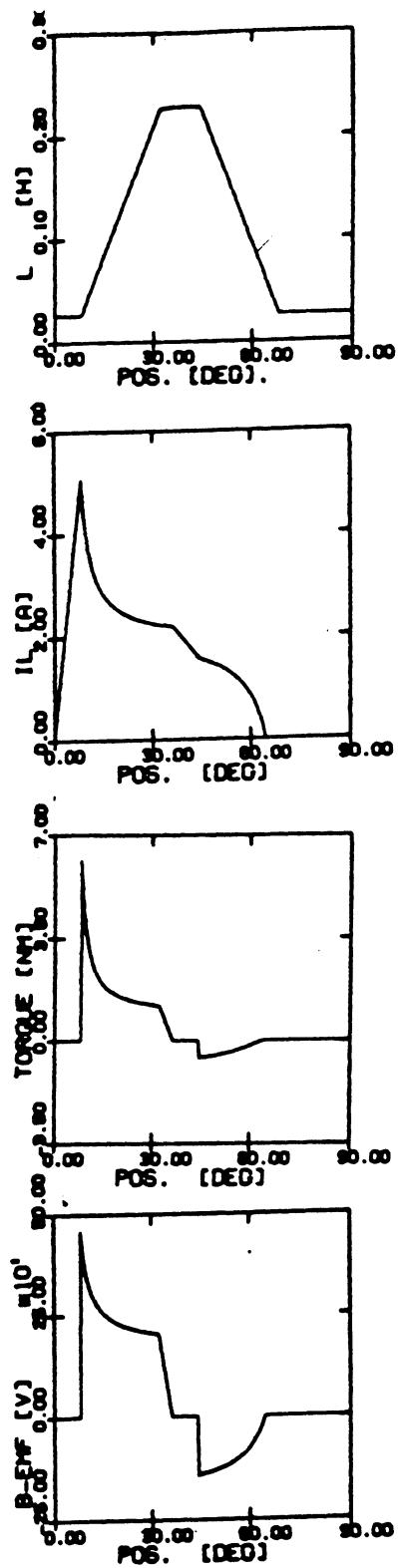


Figure 41. Simulated waveforms at 1800 r.p.m., 5.1 A and 180 V

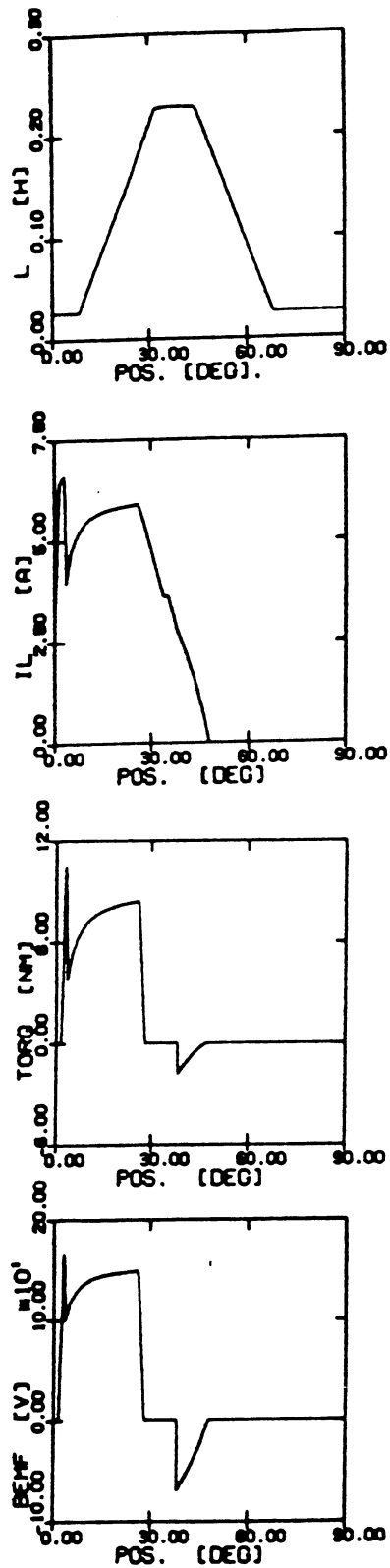


Figure 42. Simulated waveforms at 500 r.p.m., 5.1 A and 180 V

Figure 43 shows the phase current, source current and filter capacitor voltage at 1800 r.p.m. and the same conditions as in Figure 41 on page 128 and Figure 42 on page 129.

It is observed that the source current is discontinuous and its amplitude is a function of the operating conditions. The pulsed nature of the source current is bound to generate harmonics in the a.c. system (for off-line converters). Harmonics in the a.c. system result in various adverse effects including electromagnetic interference, increased transmission losses, disturbances to other appliances on the same grid system and noise. A large filter inductor is therefore required so as to maintain a continuous input current. Alternatively, the current waveform on the a.c. side can be improved by introducing a high frequency resonant link between the a.c. source and the rectifier.

Figure 45 compares simulated and measured results at 180 V, 5.1 A peak current, 12 ° advance angle, 25° conduction angle and a speed of 1101 r.p.m. A six-step approximation of the inductance profile was used. This profile is shown in Figure 44 and compared to the three-step profile used in Figure 41 on page 128 and Figure 42 on page 129.

The current waveform was acquired by means of a Data Acquisition System (DAS) at a time-step of 100 μ s. The peak on-time current predicted by the analytical method is 4.5 % higher than the measured value. However, there is a 4.8 ° error in the prediction of the rotor position corresponding to the peak current. The error is attributed to the inaccuracy of the inductance profile which assumes a constant on-time current throughout the conduction period. At lower currents, the inductance is larger than predicted resulting in a lower rate of rise of current. This observation applies to all

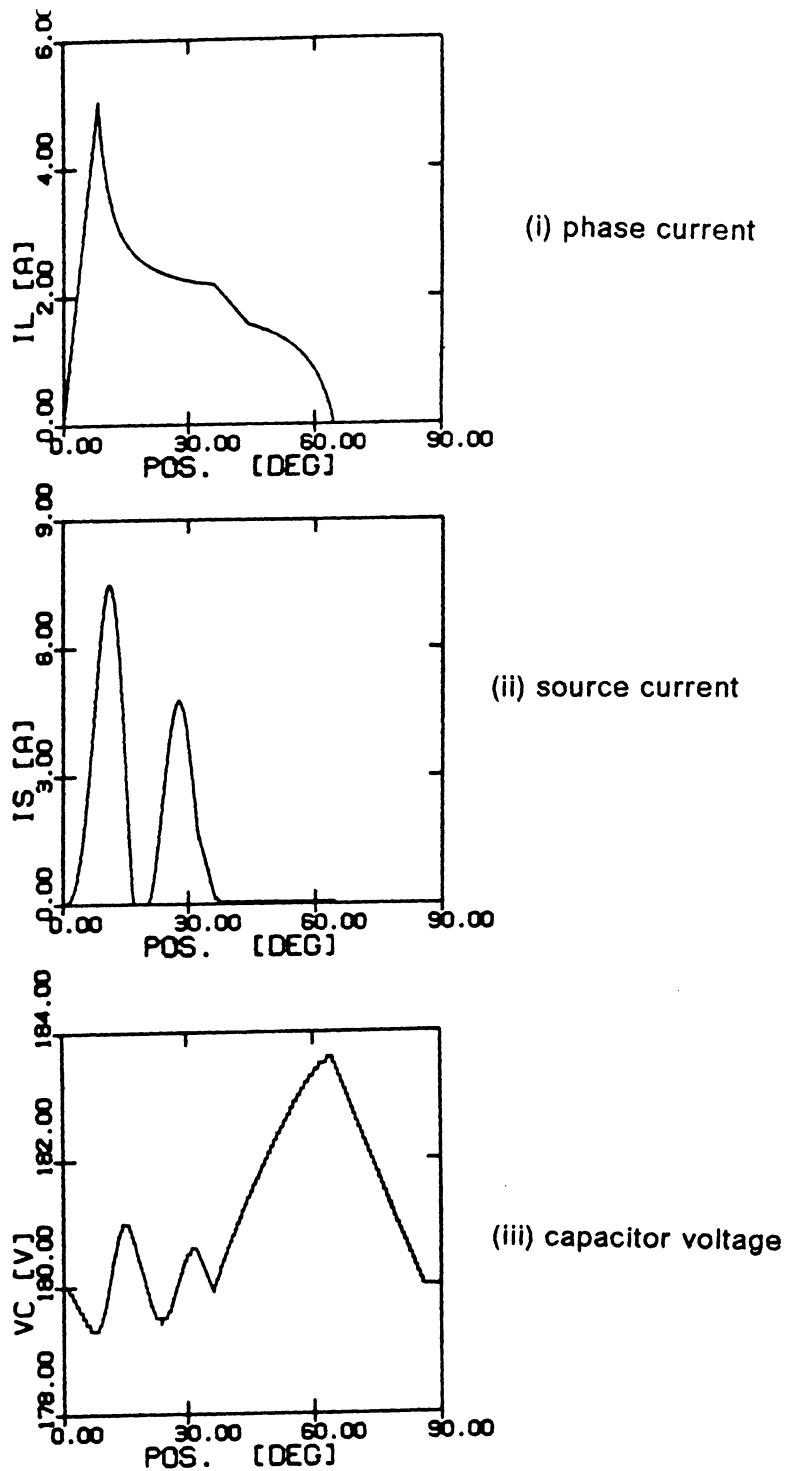


Figure 43. Input filter waveforms at 1800 r.p.m., 5.1 A and 180 V

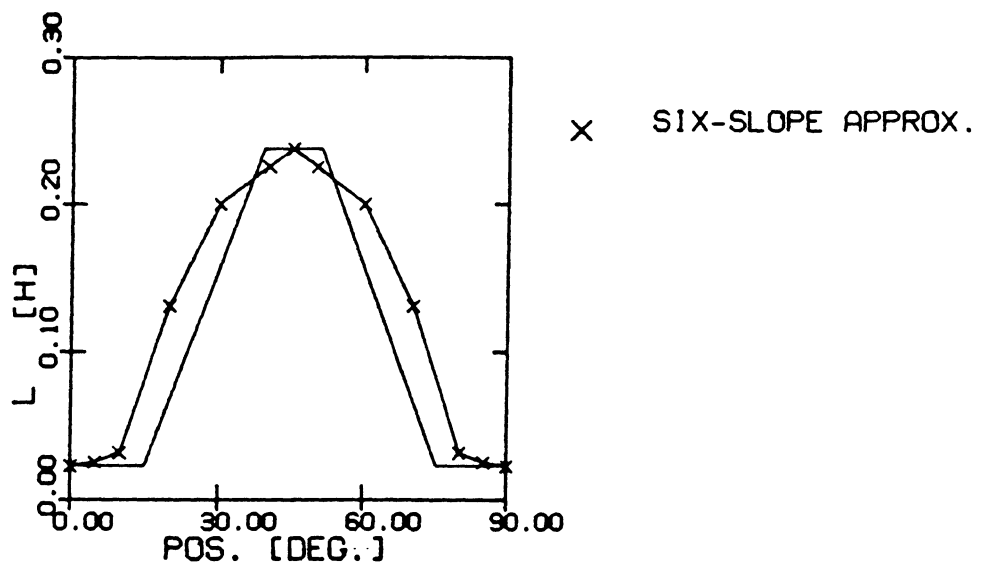


Figure 44. Comparison of inductance profile approximations at 5.1 A

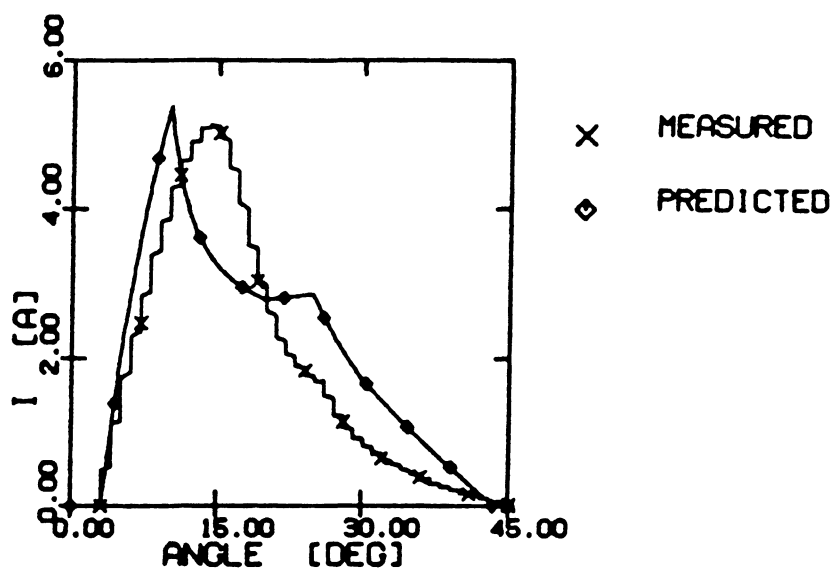


Figure 45. Measured and predicted currents at 1101 r.p.m. and 5.1 A.

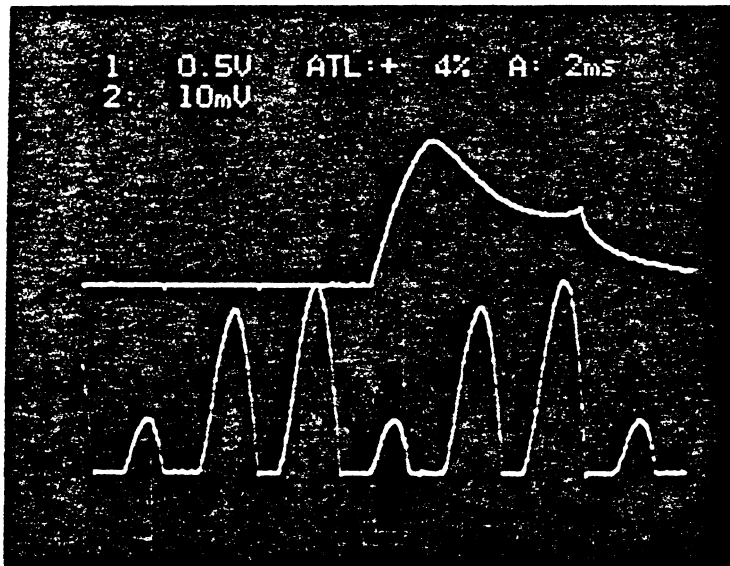
other rotor positions. It is also observed that the predicted time taken for the current to decay to zero after turn-off is within 1 % of the measured value.

Figure 46 shows the measured source current for the same operating conditions as in Figure 45 on page 133.

As predicted, the source current is pulsed. The number and magnitude of the pulses depends on the applied voltage and the source and winding inductances. The difference in the number of pulses per switching period between the measured and predicted waveforms is attributed to the inaccuracy of the inductance profile as explained earlier.

4.5 Conclusion

A novel approach to the steady-state analysis of the switched reluctance motor drive is proposed. The method has the advantage that the steady-state solution of system equations is obtained directly without going through the transient solution. It allows for the inclusion of all the parameters of a practical drive circuit including the input filter and switching-aid networks. An illustration is provided for a 6/4 pole prototype SRM drive fed from a novel, four-switch converter. Experimental verification based on the prototype drive shows that despite the assumptions taken, the predicted waveforms closely match the measurements. Comparison between predicted and measured values shows that the accuracy of the predicted values is very dependent on the accuracy of the inductance profile. It is shown that using a piecewise-linear approximation of the inductance profile, the predicted peak on-time current is within 5 % of the measured value. The predicted time taken for the winding current to decay



Top trace: Winding current per phase (2 A/division)

Bottom trace: Source current (2 A/division)

Figure 46. Measured source and winding currents at 1101 r.p.m. and 5.1 A.

to zero after turn-off is within 1 % of the measured value. Both predictions and measurements show that the source current is pulsed. For off-line converters, pulsed source current translates to a.c. side harmonics which are undesirable. Further studies on techniques for minimizing this effect are proposed. Overall, the proposed method provides a reliable basis for a better understanding of the operation of the practical SRM drive and a guideline towards optimal design of the converter, controller and motor.

5.0 DESIGN OF A SINGLE-SWITCH-PER-PHASE CONVERTER FOR SWITCHED RELUCTANCE MOTOR DRIVES

This chapter discusses the design of the novel 4-switch, 3-phase converter described in chapter 4. Although the main focus is on this particular converter, most of the principles described are applicable to other SRM converter topologies [66]. The converter is intended for use in low power variable speed SRM drives with peak ratings up to 5 kW.

5.1 *Introduction*

In chapter 4, a procedure for the steady state analysis of the variable speed switched reluctance motor drive was described. The drive circuit consisted of 6/4 pole

switched reluctance motor driven from a 4-switch, 3-phase converter which has most of the key features of the two-switch per phase converter. It was shown that the current and voltage waveforms are dictated by the operating conditions of the drive and the design parameters of the motor. The current and voltage stresses on the power semiconductor devices are therefore functions of the operating conditions. Optimal sizing of the converter is essential because the converter cost constitutes the major portion of the cost of the entire drive system. In this chapter, closed form expressions for the determination of the device ratings are given. It is assumed that the converter is supplied from rectified a.c. (off-line converter). The design therefore includes the sizing of the input filter components.

When a phase winding is excited, part of the input electrical energy is converted into mechanical energy while the rest is stored in the magnetic field. The amount of energy converted into mechanical energy at a constant supply voltage depends on the magnitude and duration of the excitation current and the switching strategy. When the winding is de-energized, the energy stored in the magnetic field is depleted either by feeding it back to the source or by dissipative means. The method by which the field energy is processed when a phase winding is de-energized is a main distinguishing factor between different converter topologies. An overview of five different topologies is given and discussed with particular reference to the switching device ratings, component count, method of regeneration and range of operation. Using the converter described in chapter 4 as an illustration, normalized curves for the r.m.s. current rating of the power devices are given. It is assumed that a PWM technique is applied for chopping. On this basis, it is shown that the r.m.s. current ratings are dependent on the chopping strategy used. For a given strategy, these ratings are

also sensitive to the size of the hysteresis window and the switching strategy. The chapter also includes a study on the input filter requirements of the drive. Simulation and experimental results are given showing that the capacitor current ripple is excessively high if no filter inductor is provided. It is shown that a very large filter inductor is necessary in order to obtain a continuous input current. Operation with conduction overlap between phases with the inevitable consequence of a reduced torque capability is suggested as a possible technique of minimizing the source current ripple. Experimental verification is given based on the 6/4 pole prototype SRM.

5.2 Topologies

A single-switch-per phase converter is most appropriate for inexpensive applications due to its relatively low component count and simplicity of the drive circuits when compared to the two-switch-per phase configuration. Various types of single switch per phase converter topologies have been proposed in the literature [10,11]. These include the bifilar winding converter, the C-Dump converter, the split supply converter, the dump resistor converter and the Williams converter [69]. The advantages and the disadvantages of each topology are summarized in Table 6.

In the bifilar winding, the split supply and the 4-switch, 3-phase converters, -1 p.u. supply voltage is applied to the phase winding immediately after turn-off. The circuits are therefore more energy efficient and regenerate faster when compared to the dump resistor converter where the recovered energy is dissipated in a resistor. On the other hand, the dump resistor configuration is the most compact and the cheapest due to the low number, of components involved. This configuration is discussed fur-

Table 6. Comparison of SRM converter configurations

Converter Topology	Switch voltage	Positive attributes	Negative attributes
Bifilar winding Converter	$> 2 V_s$	- fast regeneration - low component count - energy efficient	- many connections - high turn-off spike - extra copper
C-Dump Converter	$> V_s$	- energy efficient	- high component count - 2 storage capacitors - limited dwell angle - 4 extra switches
Dump Resistor Converter	$> V_s$	- simple - low component count - easy to implement	- low efficiency - slow regeneration - limited dwell angle
Dual supply Converter	$> V_s$	- low component count - fast regeneration	- phases dependent - even phases only - 2 supplies required
Regenerative Converter	$> V_s$	- energy efficient - fast regeneration - low component count - easy to implement	- 1 extra switch - limited dwell angle

ther in Chapter 6. In the C-dump converter, the recovered energy is temporarily stored in a dump capacitor and later returned to the source by a resonant process. This requires extra switches and thus the component count is high. The 4-switch, 3-phase converter topology shown in Figure 47 has one switch per phase and one extra switch for commutation, but has most of the advantages of the two-switch-per phase converter shown in Figure 48.

This is achieved by the use of one additional switch (hereafter called the main switch) in series with all the phases. When the main switch and a phase switch are turned on, +1 p.u. d.c supply voltage is applied to the motor winding; when both switches are off, the energy stored in the winding inductance is transferred to the input filter capacitor C. In this mode, the voltage across the phase winding reverses to -1 p.u. The application of -1 p.u. voltage accelerates the rate of fall of current in a phase winding making it possible to operate at higher speeds. In constant torque operation, the current is maintained constant by chopping using only the phase switch or only the main switch or both the main and phase switches. The ratings of the switching devices and freewheeling diodes depend on the chopping method employed. A limitation on this topology is encountered for large values of conduction angle when overlap between phases occurs. In such circumstances, the main switch is not available for energy recovery. The complete circuit diagram of the converter including the snubber circuits is given in Figure 37 on page 112. For the convenience of the reader, the basic circuit excluding the snubber circuits is given in Figure 49.

A voltage clamp is provided across the input capacitor such that the current is diverted to a 'clamp resistor' in case the capacitor voltage exceeds some preset maximum. This clamp may be eliminated if the switching devices, input filter capacitor

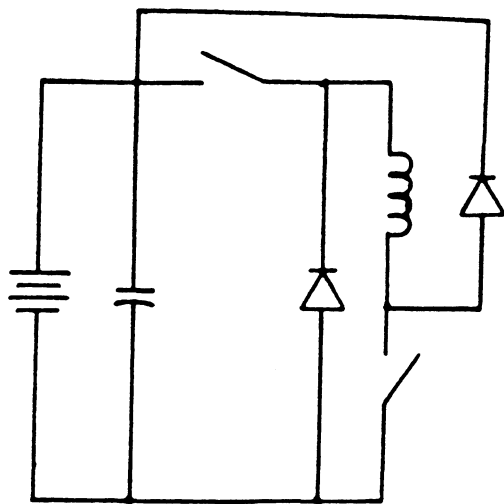


Figure 47. Single phase circuit of the 4-switch, 3-phase converter

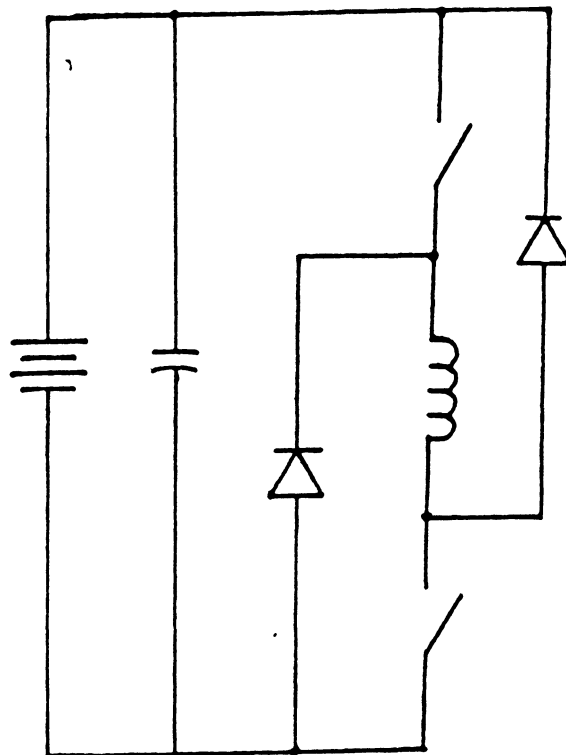


Figure 48. Single phase circuit of the two-switch-per-phase converter

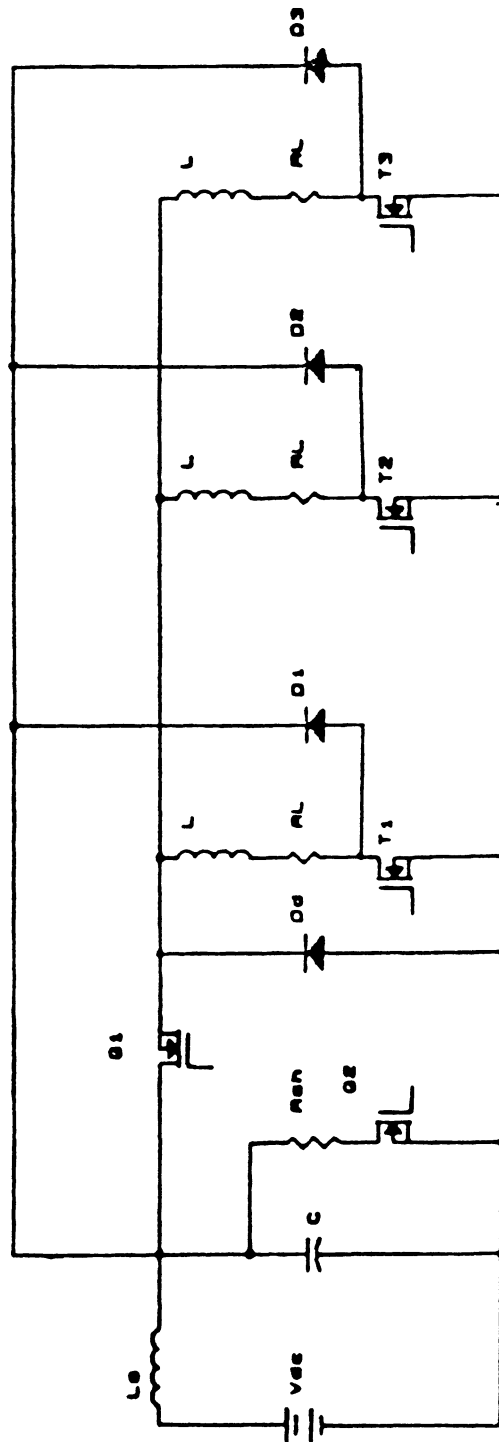


Figure 49. Basic circuit of the 4-switch, 3-phase converter

and the freewheeling diodes are adequately rated to withstand the maximum capacitor voltage at rated load.

5.3 *Design conciderations*

5.3.1 Criteria for the selection of type of switching devices

The factors considered in choosing the type of switching devices for a variable speed drive converter are the power level, supply voltage, peak current, switching speed and cost. Others include simplicity of driving circuits, paralleling possibility, robustness, ratio of peak to continuous current rating and availability. Although low power SCR and GTO devices exist in the market, their driving and (force) commutation requirements and the relatively low switching speed render them unattractive for low power, high performance applications. Thus, in the power range under consideration, the MOSFET, power transistor and the Insulated Gate Thyristor (IGT) are the suitable candidates. The relative attributes of these devices are summarized in Table 7.

For SRM drive applications, the MOSFET is the most attractive due to its high pulsed/continuous current rating; typically in the order of 4:1 for a 500 V , 10 A device. Measurements show that the ratio between peak and r.m.s. current ratings in a typical SRM varies between 2:1 to 3:1 over a wide range of operating conditions [65].

Table 7. Comparison of power semiconductor devices for SRM drives.

Attribute	Type of Device		
	MOSFET	Power Transistor	IGT
Max. frequency	25 MHz	200 kHz	50 kHz
Forward blocking voltage [V]	≤ 1000	≤ 1400	≤ 1200
dv/dt range(kV/us)	50-200	3-10	50-100
di/dt range(kA/us)	10-30	0.5-2	2-5
Max operating temperature (° C)	200	150	200
Conduction losses	high	very low	low
Peak/rms rating	high	very low	moderate
Paralleling	simple	complex	simple
Switching requirements	low pulsed gate current	moderate continuous current	low pulsed gate current

5.3.2 Current rating of power devices

Two types of current ratings have to be observed for SRM converter switches: the pulsed (peak) current rating and the continuous current rating. The pulsed current rating has to be at least equal to the maximum on-time current per phase of the drive. The maximum value of the on-time current occurs in the constant torque region whereby the current is maintained at the value corresponding to maximum torque by chopping. For small current excursions during chopping, the current waveform can be approximated by periodic rectangular blocks of current. For a known peak power output P_d , the peak current per phase can be calculated using [42]

$$I_p = \frac{P_d}{m \eta k_d V_{dc}} \quad (5.1)$$

where I_p is the rated on-time current, V_{dc} is the rated d.c. supply voltage, η is the predicted efficiency of the drive system, k_d is the duty cycle and m is the number of phases. The duty cycle is defined as the ratio of conduction time to switching period per phase. The continuous current rating has to be equal to or higher than the r.m.s. current per phase at the maximum torque. Assuming rectangular blocks of current, the r.m.s. current per phase is given by

$$I_{ph} = \sqrt{\left[\frac{1}{\lambda_r} \int_0^{\theta_c} (I_p)^2 d\theta \right]} \quad (5.2)$$

where θ_c is the dwell (conduction) angle per phase (in radians), λ_r is the switching period (in radians) per phase and θ is the rotor position.

For P_r rotor poles,

$$\lambda_r = \frac{2 \pi}{P_r} \quad (5.3)$$

For one phase switching period, each phase switch is excited once. Thus, the maximum dwell angle for each phase (assuming non-overlapping phase switch conduction) is given by

$$\theta_m = \frac{\lambda_r}{m} = \frac{2 \lambda_r}{P_s} \quad (5.4)$$

Thus, the r.m.s. current rating of the phase switches neglecting the effect of chopping is given by

$$I_{ph} = \frac{I_p}{\sqrt{(m)}} \quad (5.5)$$

The main switch conducts during every stroke. Its r.m.s. current rating under the same assumptions is therefore equal to the rated on-time current per phase of the drive.

5.3.2.1 Effect of chopping

During chopping, the current in a phase is maintained constant either by using Pulse Width Modulation (PWM) or by the hysteresis method whereby the current is switched between two levels. As stated earlier, there are three possible chopping strategies for this converter; (i) using only the phase switch (strategy I), (ii) using only the main switch (strategy II) and (iii) using both the main and phase switches (strategy III). The current stresses on the switching devices and freewheeling diodes depend on the

chopping strategy. Figure 50 shows simplified SRM current waveforms corresponding to chopping strategy I.

It is assumed that a PWM technique is employed so that the switching frequency is constant. Other assumptions are

1. The current ripple during chopping has a constant amplitude
2. The rise and fall of currents are linear

However, the simplified waveforms provide a basis for comparing the normalized ratings of the devices for the three chopping strategies. The normalizing current is the peak current corresponding to the flat-topped waveform at rated voltage, speed and torque. The voltage equation when both the main and phase switches are conducting is given by

$$V_{dc} = R i + i \omega_m \frac{dL}{d\theta} + L \omega_m \frac{di}{d\theta} \quad (5.6)$$

where V_{dc} is the input voltage, R is the sum of the winding and switch resistances, i is the winding current, ω_m is the angular speed of the motor, L is the phase inductance and θ is the rotor position. Assuming a linear inductance profile, $dL/d\theta$ can be represented by a constant $k(i)$ which equals zero in the fully unaligned and aligned positions. In the intermediate positions, its value depends on the instantaneous current and rotor position. Thus, the peak current of the flat-topped current under the stated conditions is given by

$$\hat{i} = \frac{V_{dc}}{R + k(\hat{i})\omega_{mr}} \quad (5.7)$$

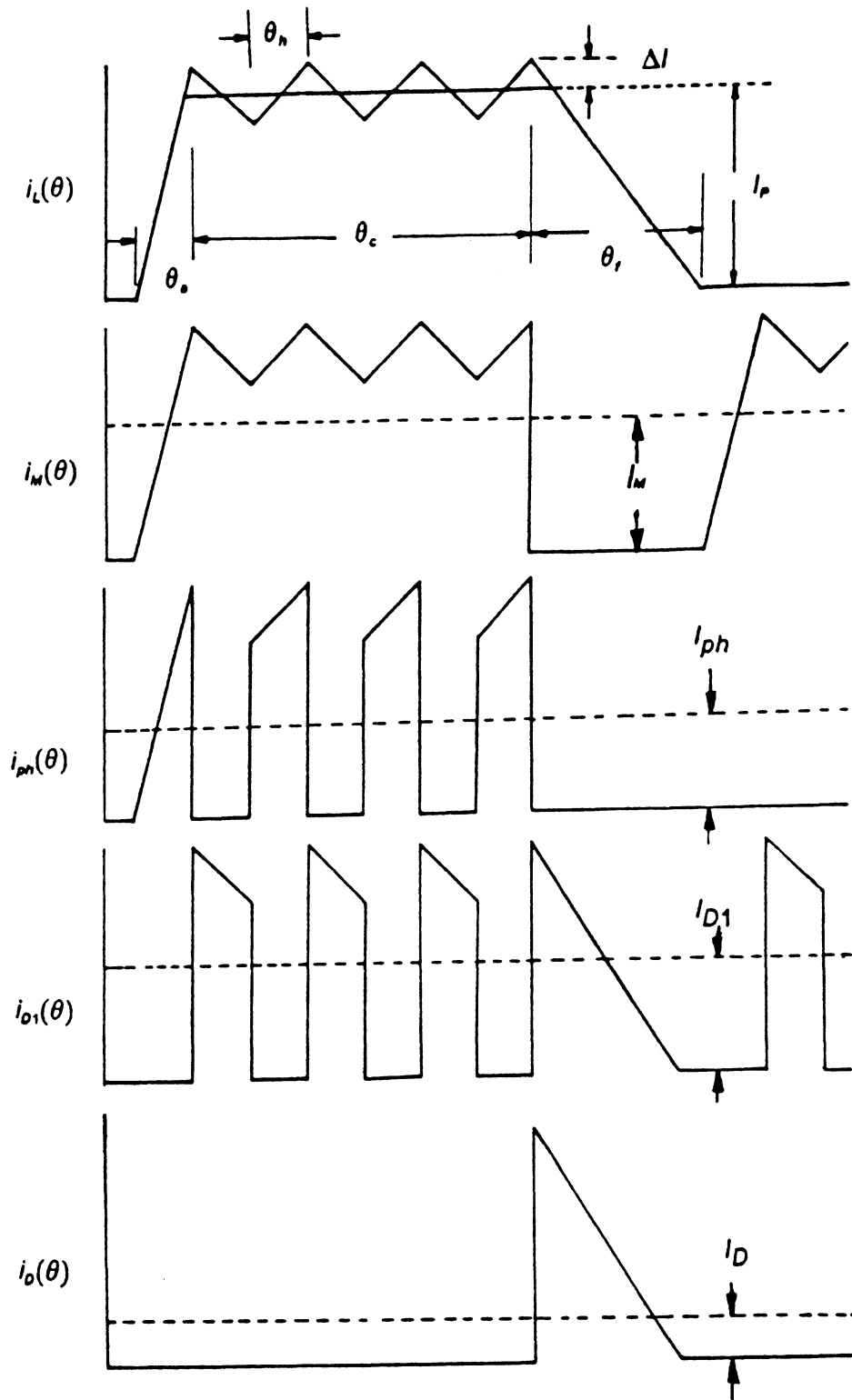


Figure 50. Current waveforms by strategy I chopping

where V_{dc} and ω_{mr} are the nominal d.c. supply voltage and angular speed respectively, and $k(\hat{I})$ is the inductance slope corresponding to the peak current $\hat{I}$. Chopping is initiated whenever the current exceeds $\hat{I}$. Applying equation (5.2) to the waveforms shown in Figure 50 on page 150, the r.m.s. current per phase is determined as

$$I_{ph} = \sqrt{\frac{1}{\lambda_r} \left[\int_0^{\theta_a} \left[\frac{(I + \Delta I)}{\theta_a} \theta \right]^2 d\theta + c_n \int_0^{\theta_h} \left[I - \Delta I \left(1 - 2 \frac{\theta}{\theta_h} \right) \right]^2 d\theta \right]} \quad (5.8)$$

where I is the average on-time current per phase, ΔI is the mean-peak amplitude of current ripple, θ_h is one half of the chopping period in radians, θ_a is the advance angle in radians and c_n is the number of chopping periods per stroke per phase. Generally, the average on-time current assuming rectangular blocks of current equals I_p . Taking λ_r as the normalizing factor for angles, integration of equation (5.8) gives

$$\bar{I}_{ph} = \sqrt{\left[\frac{[\bar{I} + \Delta \bar{I}]^2}{3} \bar{\theta}_a + \bar{\theta}_c \left[\frac{3\bar{I}^2 + 7\Delta \bar{I}^2}{6} \right] \right]} \quad (5.9)$$

where the 'bar' denotes normalized values and $\bar{\theta}_c$ is the period in radians during which chopping occurs. In a similar way, the r.m.s. current rating of the freewheeling diodes D_1, D_2 and D_3 is determined by

$$\bar{I}_{D1} = \sqrt{\left[\frac{[\bar{I} + \Delta \bar{I}]^2}{3} \bar{\theta}_f + \bar{\theta}_c \left[\frac{\bar{I}^2 + 4\bar{I} \Delta \bar{I} + 3\Delta \bar{I}^2}{2} \right] \right]} \quad (5.10)$$

where $\bar{\theta}_f$ is the normalized conduction period of a freewheeling diode per stroke. The rating for the shunt diode D_d is likewise determined as

$$\bar{I}_D = \sqrt{\left[\frac{[\bar{I} + \Delta \bar{I}]^2}{3} \bar{\theta}_f \right]} \quad (5.11)$$

The device current ratings for the other chopping strategies are easily derived using the same waveforms applied to the appropriate devices. These are summarized in Table 8.

In a practical drive, due to the nonlinear characteristics of the motor, the current ripple amplitude varies with the rotor position, decreasing gradually as the rotor moves from the fully unaligned to the fully aligned position. Further, the rise and fall times of currents are exponential. Thus, the assumptions invoked in the foregoing derivations result in a marginal over-rating of the power semiconductor devices.

5.3.3 Voltage rating

The voltage rating of the switching devices is dependent on the available d.c. supply voltage. Irrespective of the chopping strategy used, the minimum off-state, forward blocking voltage rating equals $V_{dc} + \Delta V_L$ where ΔV_L is a voltage margin to allow for a voltage spike at turn-off due to lead inductances. This margin also includes provision for the maximum capacitor voltage. During the on-time of the switches, the freewheeling diodes are reverse-biased by the d.c. link voltage. Therefore, the minimum peak reverse voltage rating of the diodes is the same as the forward blocking voltage of the MOSFETS. The diodes must have fast recovery characteristics in both the forward and reverse directions. Fast forward recovery is necessary to ensure fast transfer of current from a phase switch to the diode at turn-off whereas fast reverse recovery ensures that the diode blocks fast enough to avoid a short circuit of the supply at the beginning of the next stroke. The effect is particularly significant at high speeds and during chopping when the time available for recovery is short.

Table 8. R.m.s. current ratings for various chopping strategies

Device	Strategy I	Strategy II	Strategy III
Phase switch	I_{ph}	$\frac{I_M}{\sqrt{m}}$	I_{ph}
Main switch	I_M	$I_{ph}\sqrt{m}$	$I_{ph}\sqrt{m}$
Diodes D1,D2,D3	I_{D1}	$\frac{I_D}{\sqrt{m}}$	I_{D1}
Diode D_d	I_D	$I_{D1}\sqrt{m}$	$I_{D1}\sqrt{m}$

5.3.4 Input filter design

The switched reluctance motor draws a pulsating current from the d.c source. The frequency of the unipolar pulsations in the d.c. link equals m-times the phase switching frequency, where m is the number of phases. If the d.c source is obtained from rectified a.c., the current pulsations introduce harmonics into the a.c. system. On the d.c. side, the pulsating current results in fluctuations in the input voltage. To limit these effects, an L-C filter is introduced between the rectifier output and the input to the converter as shown in Figure 49 on page 144. The filter also takes care of the voltage ripple in the rectifier output.

5.3.4.1 Determination of filter components

Figure 51 shows the equivalent circuit of the input filter where L_f and C_f are the inductance and capacitance respectively, and R_d and R_e are the inductor resistance and equivalent series resistance of the capacitor respectively.

For an average input voltage V_1 , the current at the input to the converter has the waveform shown in Figure 52 (b) for the winding current waveform shown in Figure 52 (a).

Assuming that the inductor L_f is sufficiently large so that the input current i_1 is continuous and has negligible ripple, the capacitor current will be given by

$$i_c(t) = I_1 - i_2(t) \quad (5.12)$$

where I_1 is average value of the source current (filter inductor current) and $i_2(t)$ is the instantaneous current at the input to the converter (d.c. link current). Figure 52 (c)

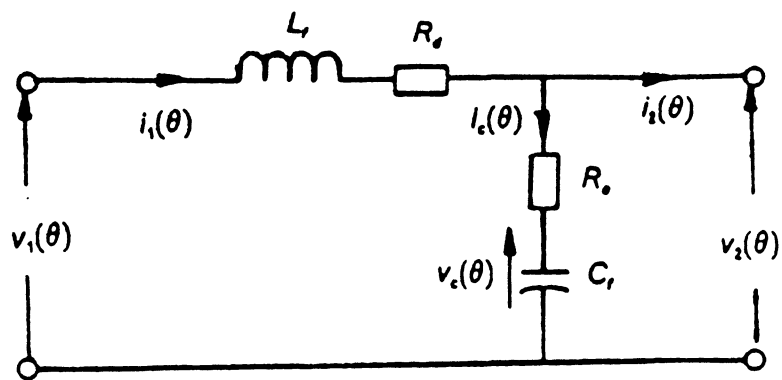


Figure 51. Equivalent circuit of the input filter

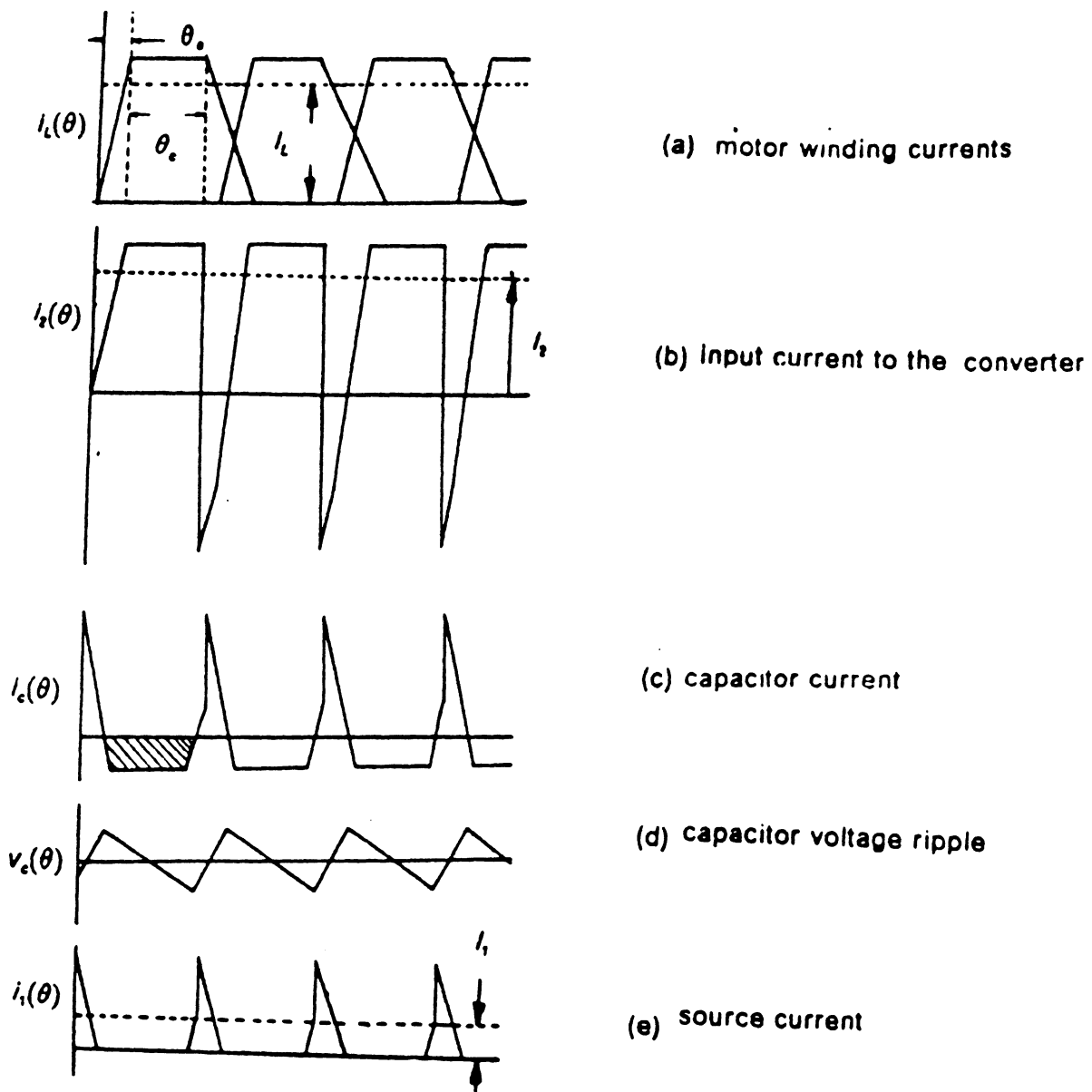


Figure 52. Current waveforms at the input to the converter

shows a typical waveform of the capacitor current. For a specified peak-to-peak input voltage ripple ΔV_{pp} , the charge balance equation for the capacitor is given by

$$\Delta Q_c = C_f \Delta V_{pp} \quad (5.13)$$

where ΔQ_c is the charge stored in or delivered by the capacitor as it charges or discharges by a voltage equal to ΔV_{pp} . This charge equals the area under the $i_c(t)$ curve for the respective interval. The minimum value of C_f is calculated using equation (5.13) for the largest value of ΔQ_c . The worst case occurs during constant torque operation when the current is maintained at a constant maximum value by chopping. Assuming that the current excursions during chopping are absorbed by the capacitor, the waveform of $i_2(t)$ during this mode of operation is approximated by a flat-topped waveform as shown in Figure 52 (b). In steady state, the change in the capacitor charge is the same for both the charging and discharging intervals. Thus, from Figure 52 (c), the charge delivered by the capacitor during the discharge period (shaded area) is given by

$$\Delta Q_{cc} = \left[\frac{1}{2} (\theta_a - \theta_1) + \theta_c \right] \frac{(I_p - I_1)}{\omega_m} \quad (5.14)$$

where

$$\theta_1 = \frac{I_1}{I_p} \theta_a \quad (5.15)$$

Combining equations (5.13), (5.14) and (5.15), the filter capacitance at rated speed ω_{mr} is given by

$$C_f = \frac{I_p - I_1}{\omega_{mr} \Delta V_{pp}} \left[\frac{I_p - I_1}{2 I_p} \theta_a + \theta_c \right] \quad (5.16)$$

Having determined the capacitor value, the inductance required to satisfy a specified input current ripple Δi_{pp} is given by

$$L_f = \frac{\Delta V_{pp}}{\Delta i_{pp}} \left[\frac{\theta_c + \theta_a - \theta_1}{\omega_{mr}} \right] \quad (5.17)$$

It is normally desirable to limit the size of the capacitor so as to improve the reliability of the drive. The value of the filter inductance is therefore large. The inductor resistance and the ESR of the capacitor provide some damping effect in the circuit while at the same time dissipating energy resulting in reduced drive efficiency.

5.3.5 Thermal considerations

The total converter losses are given by

$$P_L = P_{sc} + P_{ss} + P_{dc} + P_{ds} + P_{rf} \quad (5.18)$$

where P_{sc} and P_{ss} are the MOSFET conduction and switching losses respectively, P_{dc} and P_{ds} are the diode conduction and switching losses respectively, and P_{rf} is the power loss in the rectifier. Equations for the calculation of these losses for each device are available in the literature. The conduction losses in the MOSFET switches constitute the highest proportion of losses in the converter. Due to the fast switching times of the MOSFET, switching losses are much lower than conduction losses.

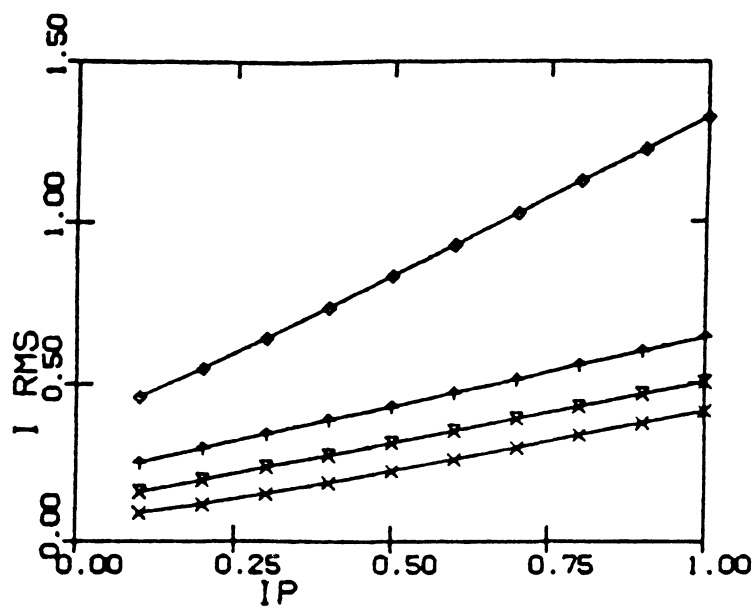
However, in constant torque operation, these losses are significant due to the higher switching (chopping) frequency. The switching losses are also dependent on the chopping strategy. Diode losses also consist of conduction and switching losses, the latter depending on the speed of recovery of the diodes.

5.4 *Simulation results*

5.4.1 Comparison of chopping strategies

Figures 53 to 55 show the normalized curves of r.m.s. current rating as a function of average on-time current per phase for the three chopping strategies.

It is observed that whereas strategy I results in the highest rating for the main switch while the ratings of the other devices are comparable to those obtained by strategies II and III. The r.m.s. current rating of the MOSFETS is the most critical factor in selecting the chopping strategy since the conduction losses and the device cost are directly proportional to it. On this basis, strategy III is superior to the other two and using this strategy, the r.m.s. current rating of the phase switches is the lowest. For this strategy, the sensitivity of the current rating to the magnitude of the current excursion ΔI and the advance angle are described in the following section.



- X PHASE SWITCH
- ◇ MAIN SWITCH
- ▲ DIODES D1.D2.D3
- X DIODE D0

Figure 53. Device current ratings by strategy I

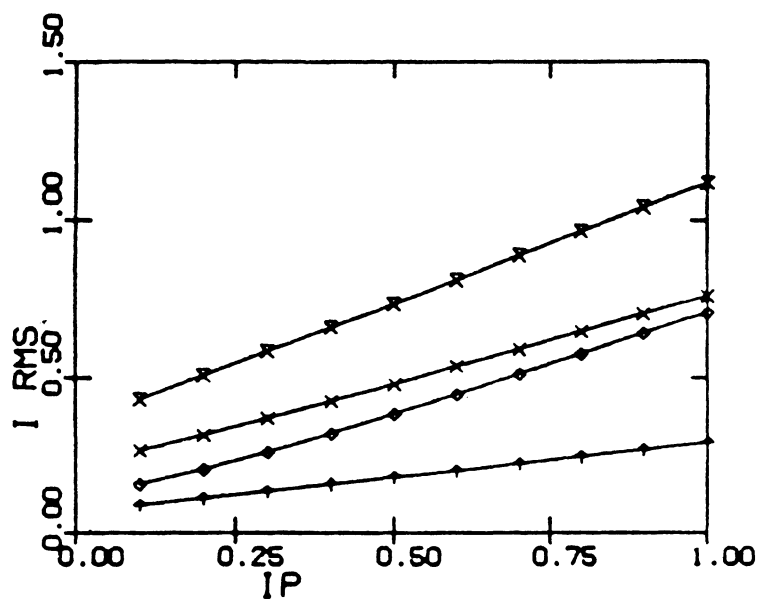


Figure 54. Device current ratings by strategy II

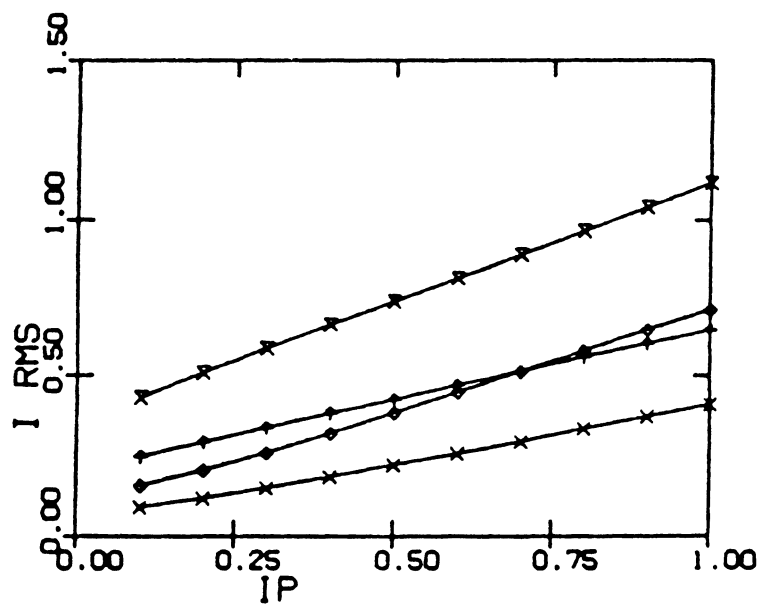


Figure 55. Device current ratings by strategy III

5.4.2 Sensitivity to size of hysteresis window

Figures 56 to 59 show the effect of the current excursion ΔI on the r.m.s. current rating of the various devices for normalized current excursions between 5 and 20 percent with chopping strategy III.

The corresponding current rating when the effect of chopping is neglected are also shown on the same axes. It is observed that when chopping is neglected, the phase switches can be over-rated by as much as 36 % while the freewheeling diodes D_1 , D_2 , and D_3 are under-rated by as much as 53 % at $\Delta I = 5 \%$.

5.4.3 Sensitivity to advance angle

Figures 60 and 61 show the sensitivity of the phase and main switch current ratings to changes in the advance angle.

The advance angle is the angle by which turn on of a phase is advanced prior to the onset of alignment between an approaching rotor pole and the excited stator pole. It is assumed that the chopping duration is constant. It is observed that the advance angle has the effect of increasing the current ratings of the transistors. For example, the rating of the phase switches at $I_p = 1.00$ is increased by 19 % when the advance angle is 0.1 p.u. corresponding to 9° for a 3-phase SRM drive. Since the chopping duration is assumed constant, the advance angle has no effect on the ratings of the freewheeling diodes. This is true at small values of advance angle; however, as the advance angle increases, the rate of rise of current increases so that the desired

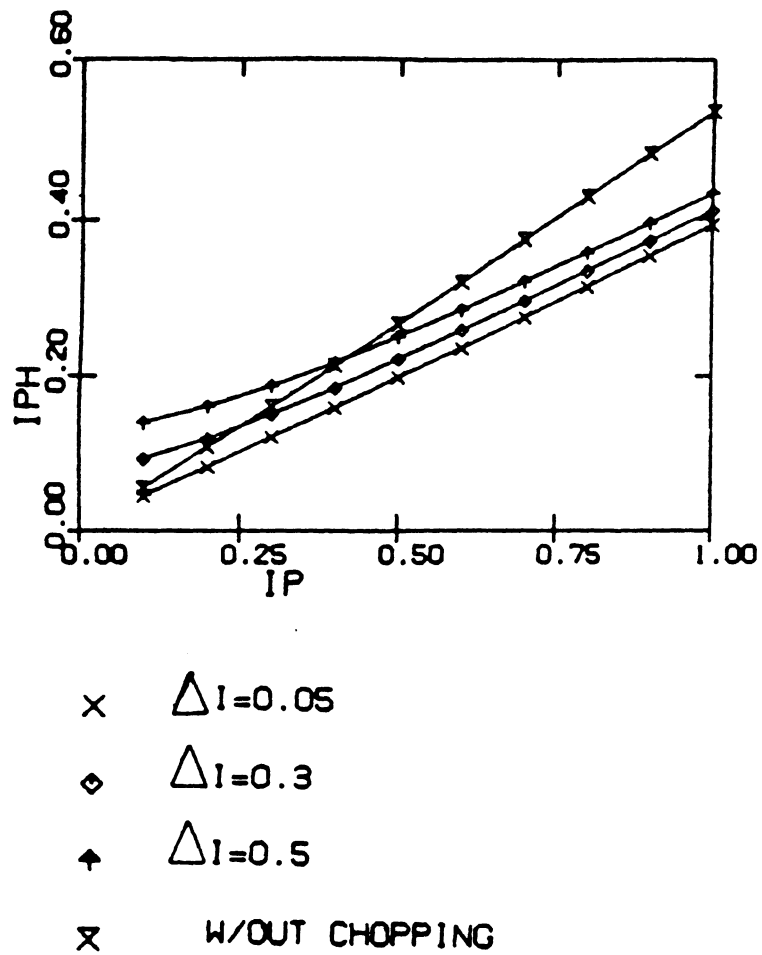


Figure 56. Sensitivity of phase switch current rating to hysteresis window size

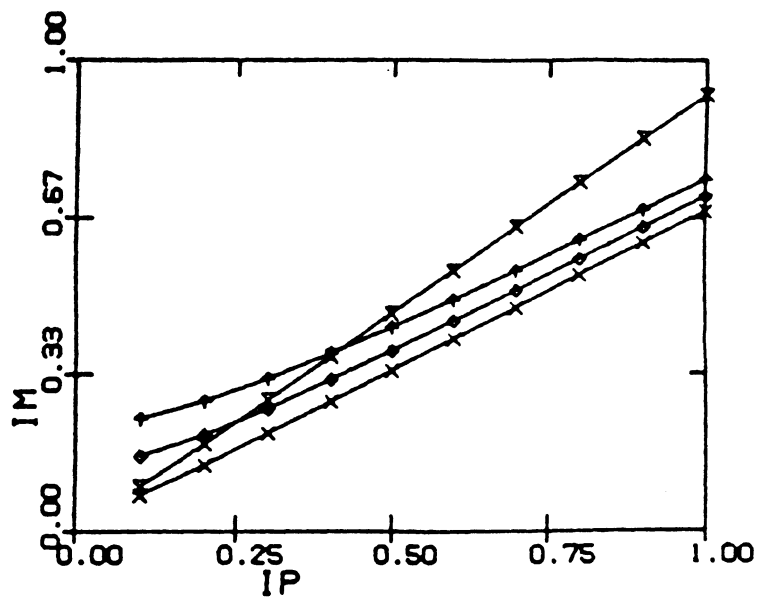


Figure 57. Sensitivity of main switch current rating to hysteresis window size

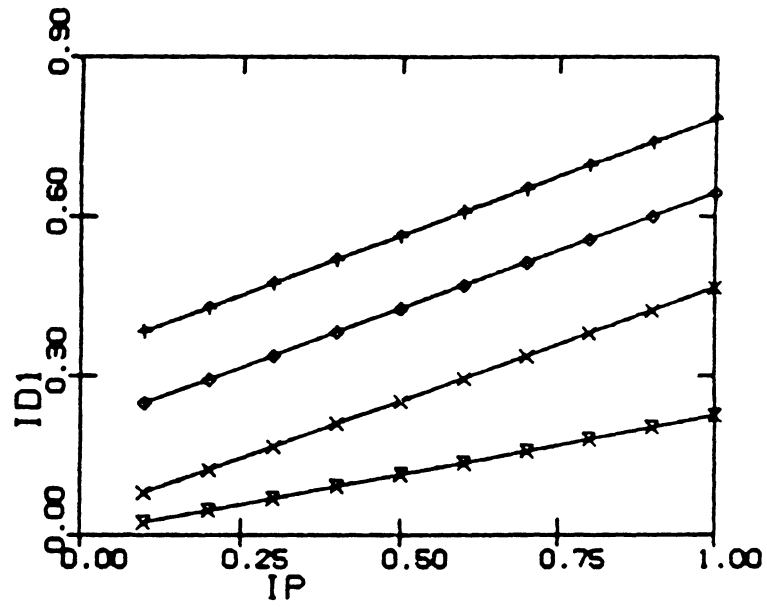


Figure 58. Sensitivity of diodes D1, D2, D3 current rating to hysteresis window size

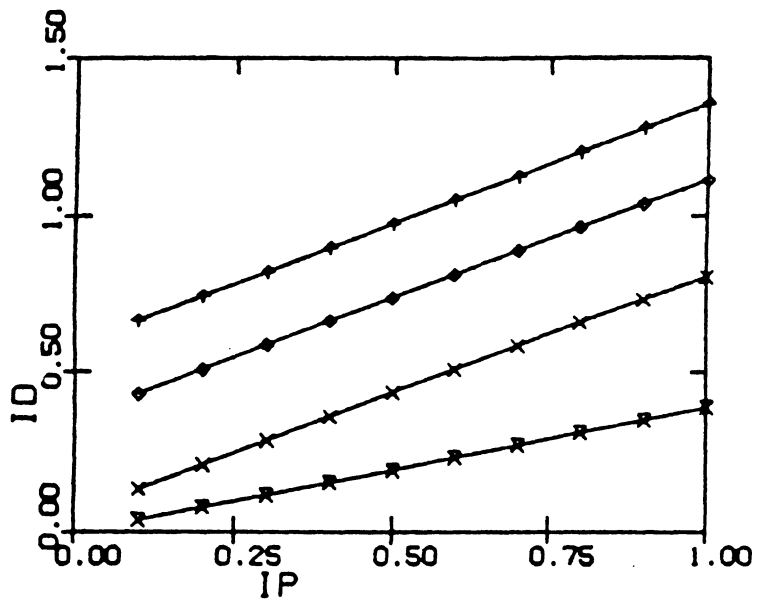


Figure 59. Sensitivity of diode DD current rating to hysteresis window size

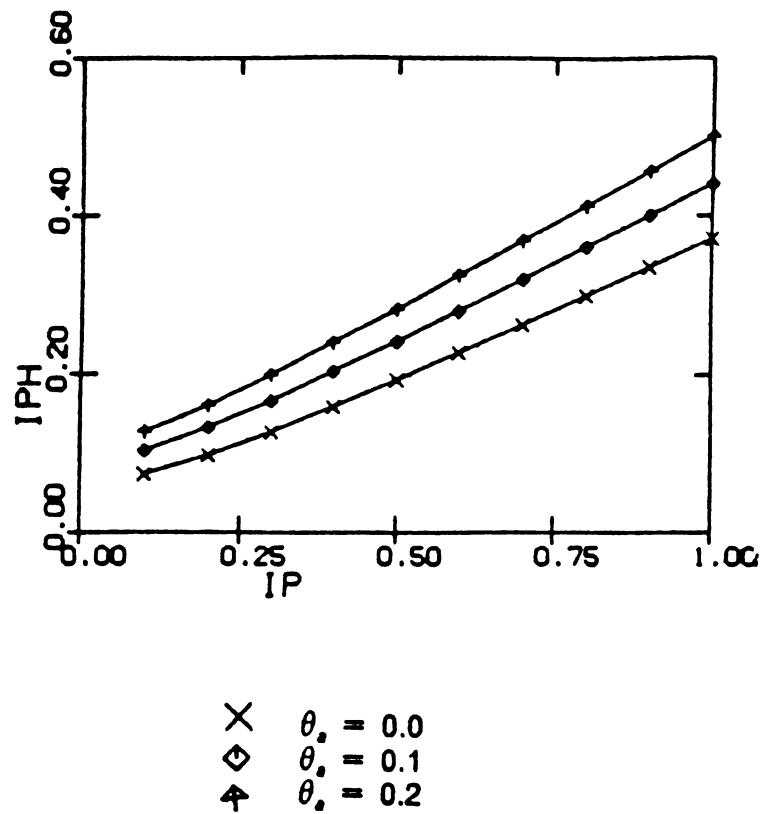


Figure 60. Sensitivity of phase switch current rating to advance angle

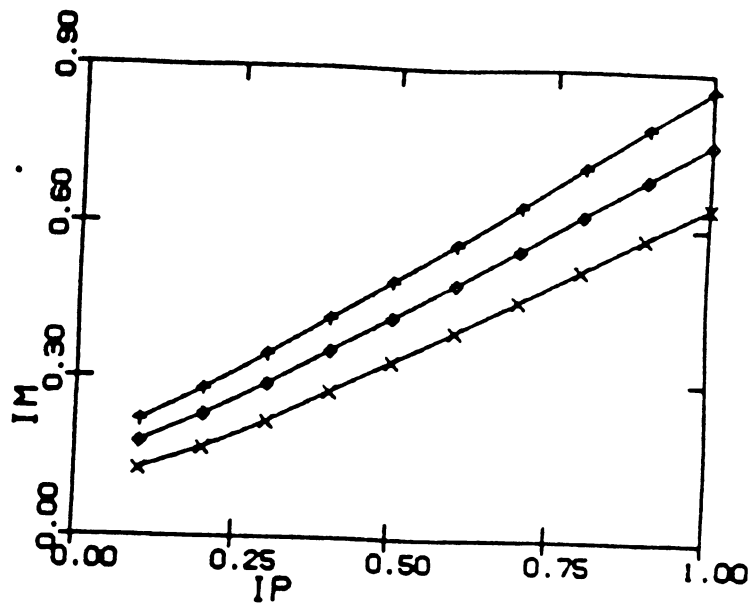


Figure 61. Sensitivity of main switch current rating to advance angle

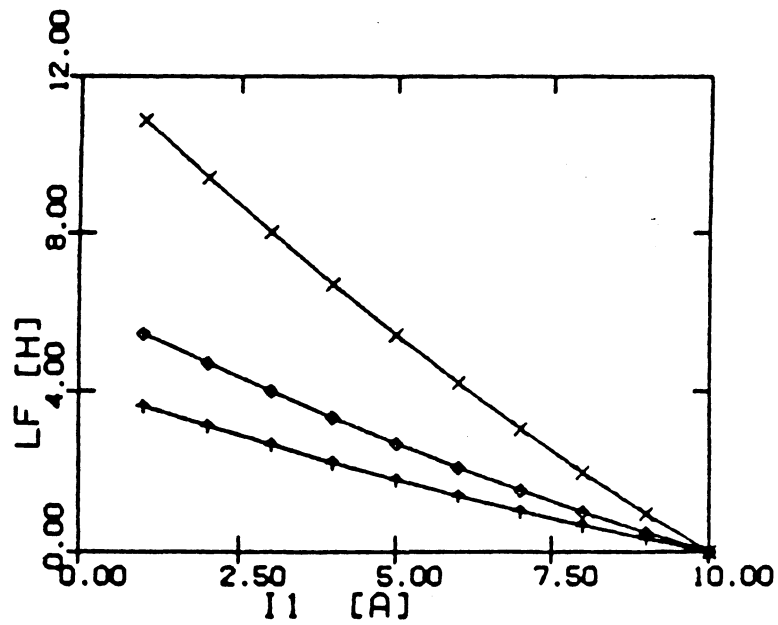
current level is attained faster. In this case, chopping is also advanced resulting in increased current stress on the freewheeling diodes.

5.4.4 Filter components

Figure 62 shows the variation of the input filter inductor with the average input current for 1000, 2000 and 3000 μF capacitors obtained by solving equations (5.16) and (5.17). The calculation is based on a peak current of 10 A and an input current ripple of 1 A. θ_a and θ_c are 5 and 24 degrees respectively. The curves show that the inductance required to maintain a continuous input current can be reduced by operating with a large average input current. This can be achieved by operating with a large conduction overlap between the phases. The inductance value is also dependent on the value of capacitance especially for low magnitudes of average input current.

5.5 Experimental Results

A 6/4 pole laboratory prototype SRM fed from the 4-switch, 3phase converter and controlled by a personal computer-based controller [64] has been built and tested. The drive is designed for a maximum d.c. supply voltage of 300 V and a continuous d.c. current of 10 A. Each phase switch consists two 500 V, 8 A MOSFETs connected in parallel. The freewheeling and shunt diodes are rated at 600 V, 12 A. The effect of the input filter is demonstrated for flat-topped non-overlapping motor currents. In Figure 63, the filter capacitor voltage and current, the source current and the phase



- X CF=1000 UF
- ◇ CF=2000 UF
- ↑ CF=3000 UF

Figure 62. Filter inductance as a function of average source current

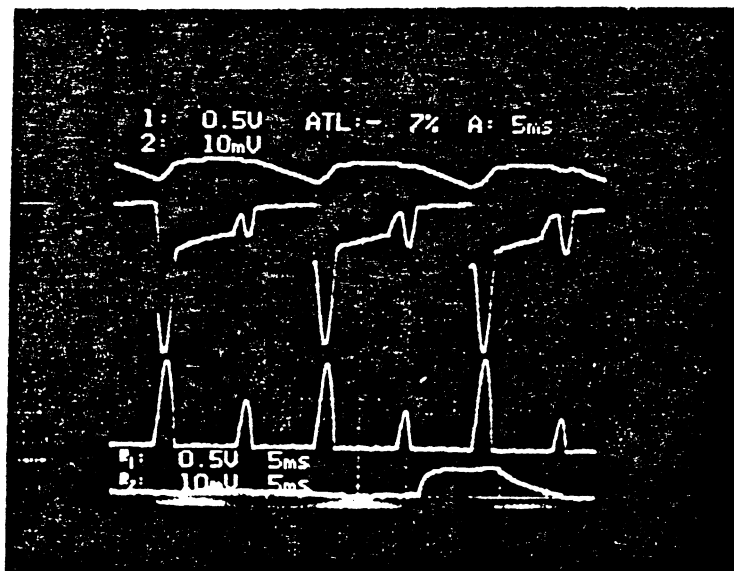
winding current are shown for a filter capacitance of $1200\ \mu\text{F}$ and no (external) filter inductor. The source inductance is $52.5\ \mu\text{H}$.

It is observed that the capacitor supplies most of the energy requirements of the drive since the input current is discontinuous. This results in a large capacitor current ripple which imposes a severe limitation to the lifetime of the capacitor. In order to transfer a higher proportion of the load to the source, a inductor of $18.8\ \text{mH}$ was inserted between the rectifier and the capacitor. Figure 64 shows the corresponding waveforms at the same scales as in Figure 63.

The input current is still discontinuous but the current ripple amplitude decreases by 59 % from $3.7\ \text{A}$ to $1.5\ \text{A}$. This implies that in order to ensure continuous input current in all modes of operation, a large input filter inductor is necessary. The same problem will be encountered in other converter topologies. A possible solution is to operate with large conduction overlap between the phases. However, such a switching strategy results in a large negative torque component in each stroke with the consequence that the torque capability of the drive is reduced.

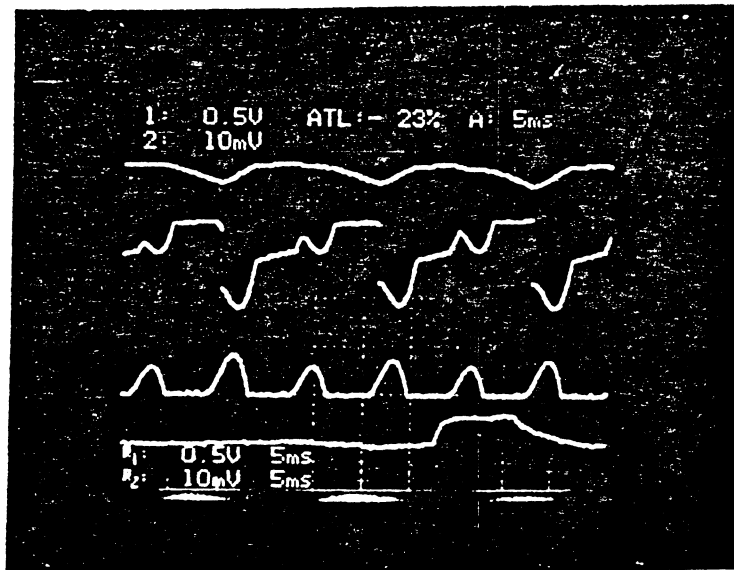
5.6 Conclusion

The design and performance of a unipolar single switch per phase SRM converter has been described. Design characteristics for determining the device current ratings are given. Based on a simplified linear analysis, it is shown that the current ratings of the power devices are strongly dependent on the chopping strategy. Criteria for the selection of the switching devices have been discussed. It is concluded that at the present level of power semiconductor device technology, the MOSFET has advantage



Traces (top to bottom): (i) capacitor voltage ripple (0.5 V/div.) (ii) capacitor current (2 A/div.) (iii) source current (2 A/div.) (iv) phase winding current (2 A/div.)

Figure 63. Measured input filter waveforms: $L_f = 52.5 \mu\text{H}$



Traces (top to bottom): (i)capacitor voltage ripple (0.5 V/div.) (ii)capacitor current (2 A/div.) (iii)source current (2 A/div.) (iv)phase winding current (2 A/div.)

Figure 64. Measured input filter waveforms: $L_f = 18.8$ mH

over other devices for low power drives, mainly due to the simplicity of its drive circuit, ease of paralleling and high peak/r.m.s. current rating. The importance of an input filter for the SRM converter has been demonstrated and a procedure for its design is developed. It is shown that using a normal filter size, the source current is discontinuous, the bulk of the energy being provided by the capacitor. This implies that the SRM drive requires a large filter inductor so as to improve the reliability of the drive and also to minimize the a.c. side harmonics. Operation with a large conduction overlap between the phases is suggested as a solution with the inevitable consequence of a reduced torque capability of the drive.

6.0 MEASUREMENT AND INSTRUMENTATION OF A SWITCHED RELUCTANCE MOTOR

The inductance profile of a switched reluctance motor (SRM) is an important parameter in the verification of the design and performance of the drive. Due to the nonlinear operation and double salient structure of the motor, this profile is a function of both the rotor position and excitation current. This makes the measurement of this parameter difficult. This chapter describes and compares various techniques for the measurement of SRM inductance for any rotor position and level of excitation. Results for an experimental prototype are compared to the analytical and finite element analysis predictions. It is shown that the proposed flux linkages method is the most appropriate for the SRM and can be applied to the direct measurement of the coenergy change for the steady-state operation of the motor.

6.1 Introduction

Growing interest in switched reluctance motor drives has raised many analysis, design and measurement issues. One of the most important aspects of study is the measurement of motor parameters for use in the analysis and design of the converter, prediction of the performance of the motor drive etc. This chapter is concerned with the measurement of these parameters.

Torque production in the switched reluctance motor is a function of the change in coenergy as the rotor moves from one position to another with excitation in a stator winding. The change in coenergy is directly proportional to the rate of change of flux linkages as a function of the rotor position and excitation current. Furthermore, for a fixed supply voltage, the instantaneous value of the winding current is dependent on the instantaneous phase inductance. Hence the need for the measurement of the inductance profile of the motor is important for determining the drive performance and design modification [75].

Both the inductance and electromagnetic torque can be determined from the flux linkages versus current characteristics. Hence, for the purpose of design and performance verification, it is sufficient to measure only these characteristics. This chapter describes a simple method for the measurement of SRM flux linkages, hereafter referred to as the Flux Linkages (FL) method. A comparison between this method and the High Frequency Superposition (HFS) method for the measurement of inductance is given. In the case of the flux linkages method, two alternatives are considered: (i) application of a step voltage and simultaneous measurement of rising

current, voltage and/or flux linkages and (ii) measurement of decaying current and voltage when a steady-state current in the motor winding is interrupted. The results are compared to the predictions by analytical and finite element methods and are shown to be in good agreement.

6.2 *Methods of measurement*

The dependence of the phase inductance of the SRM on both the excitation current and rotor position requires that the measurement of inductance be performed while the phase winding is excited with the appropriate d.c. current. This can be achieved either by direct measurement of the winding impedance or by measuring the flux linkages [74]. A flux linkages-based method for measuring the inductance in large electrical machines has been proposed in the literature [61]. The method employs a flux meter for measuring the change in flux linkages when current is switched into a phase winding (or when the winding is de-energized). Inductance is then calculated by dividing the change in flux linkages by the corresponding change in current. The problem with this method is that to obtain the inductance profile (for various currents and rotor positions), the test circuit set-up has to be repeated for each value of current and rotor position. This is time consuming and subject to a non-uniform error. Secondly the method is not suitable for small machines of the SRM type since the low values of flux linkages elude the accuracy of a flux meter. An alternative flux linkages method suitable for both small and large machines is proposed. Several implementation alternatives are considered and compared.

6.2.1 The High Frequency Superposition (HFS) method

Figure 65 shows the circuit diagram of the HFS method for measuring the phase inductance. Capacitor C_2 is used to block the d.c. current from flowing into the a.c. circuit.

In order to avoid interaction between the a.c. and d.c. circuits, it is necessary that the d.c. source be ripple-free. One phase of the motor winding is connected between points X and Y as shown. The operation of the circuit is as follows. First, the rotor is mechanically fixed at the desired position. Switch S_1 is then closed and the d.c. voltage is gradually increased until the desired d.c. current is established in the winding. Next, switch S_2 is closed and a high frequency a.c. voltage is applied to the already excited test winding. Several measurements of the a.c. voltage and current in the winding are recorded. The procedure is repeated for other values of excitation current and rotor position.

Assuming that the winding resistance per phase is known, the inductance is then calculated by

$$L(\theta, I) = \frac{1}{2\pi f} \sqrt{\left[\left\{ \frac{V_w}{i_w} \right\}^2 - R^2 \right]} \quad (6.1)$$

where $L(\theta, I)$ is the inductance per phase at rotor position θ and d.c. excitation current I , f is the frequency of the injected a.c. current, V_w and i_w are the a.c. voltage and current in the winding and R is the winding resistance per phase.

The method works well when applied to linear magnetic circuits. However, problems arise when applied to the SRM because:

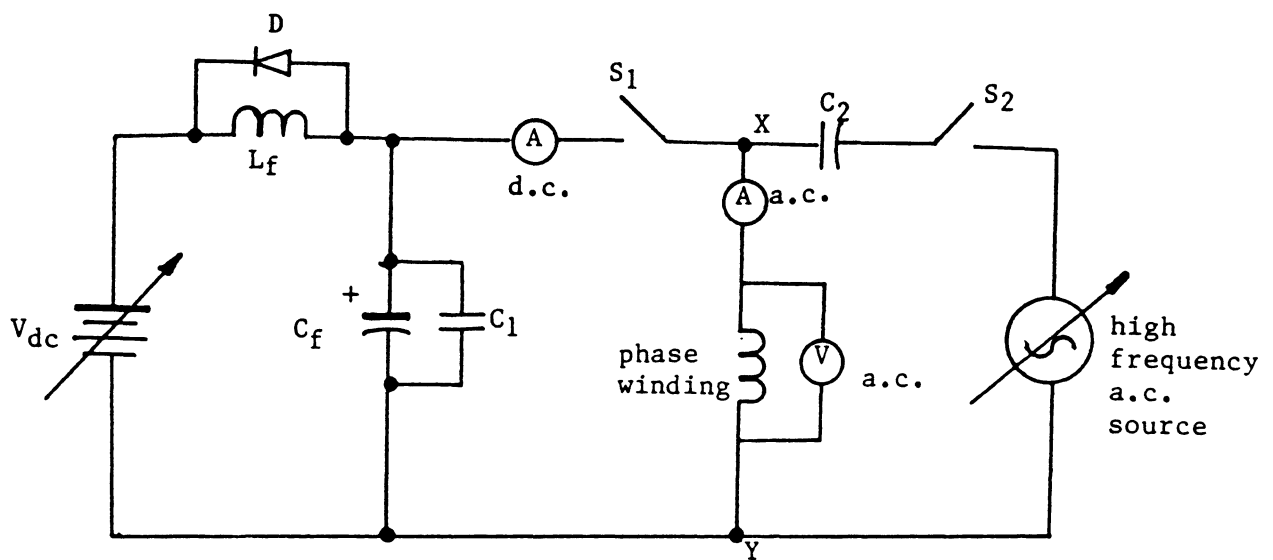


Figure 65. Implementation circuit for the HFS method

- i. The SRM is a nonlinear electromagnetic circuit; the waveform of the injected a.c. current is therefore not sinusoidal resulting in inaccuracies in the measurement unless high accuracy, true r.m.s. instruments are used.
- ii. A separate set-up is required for each rotor position and excitation current.

6.2.2 The Flux Linkages (FL) method

The flux linkages method is based on the voltage equation of the SRM. The voltage equation for one phase is given by

$$v(t) = R i(t) + \frac{d\Psi(t)}{dt} \quad (6.2)$$

where $v(t)$ is the instantaneous phase voltage, $i(t)$ is the instantaneous phase current and $\Psi(t)$ is the instantaneous flux linkages. From (6.2), the instantaneous flux linkages are obtained as

$$\Psi(t) - \Psi(0) = \int_0^t \{ v(t) - R i(t) \} dt \quad (6.3)$$

where $\Psi(0)$ is the flux linkages at $t = 0$. Thus, if the winding resistance per phase is known, the flux linkages at any instant can be obtained from terminal measurements of the winding voltage and current. The integration to obtain flux linkages can be

performed using either software or hardware. From the flux linkages, inductance is calculated as

$$L(t) = \frac{\Psi(t) - \Psi(0)}{i(t)} \quad (6.4)$$

Compared to the HFS method, this method is more efficient since only one test set-up per rotor position is required. In the HFS method, additional set-ups for each value of excitation current are required.

6.3 *Measurement Techniques*

Measurement of flux linkages can be implemented in two ways:

- i. By applying a constant voltage to the winding and measuring the rising current.
- ii. By first establishing a steady d.c. current in the winding and then measuring the decaying current when the circuit is de-energized.

The merits and demerits of each of these two measuring techniques are discussed in the following sections.

6.3.1 Measurement of rising current at constant voltage

In this case, the initial flux linkages $\Psi(0) = 0$ as both $v(t)$ and $i(t)$ are zero at time $t = 0$. The measurement procedure is as follows:

- i. The rotor is fixed at the desired position mechanically.
- ii. With the other phases open-circuited, a step voltage is applied to the test phase winding and the instantaneous winding voltage and current are recorded simultaneously. In case of hardware integration, the flux linkages proportional to a voltage at the output of the integrator and the instantaneous current are recorded. The recording is terminated when the current attains its steady-state value.
- iii. Flux linkages and inductance are calculated using equations (6.3) and (6.4).

In practice, the voltage across the winding does not remain constant throughout the measurement period due to regulation of the supply. A large electrolytic capacitor across the supply is therefore necessary. A large capacitor is also necessary so as to minimize the effect of oscillations due to the LC circuit formed by the capacitor and the motor and/or the source inductances. Such oscillations distort the accuracy of the measurements.

6.3.2 Measurement of decaying current

The main advantage of this procedure is that the initial transients are left to die out before measurements begin. After the rotor is fixed to the desired position, a step voltage is applied to the test winding as before except that no measurements are made at this time. Once the current attains its steady-state value, the winding is de-energized and measurements are initiated. A reverse diode of the fast recovery type has to be connected across the test winding to provide a path for the stored energy once the winding is disconnected from the source. The voltage across the winding during the measurement period is therefore equal to the forward drop of the diode.

6.4 Instrumentation for the flux linkages method

Two alternatives for measuring flux linkages are considered:

1. Direct measurement through hardware integration.
2. Indirect measurement through acquisition of current and voltage.

6.4.1 Direct measurement of flux linkages

Figure 66 shows the circuit for the direct measurement of flux linkages.

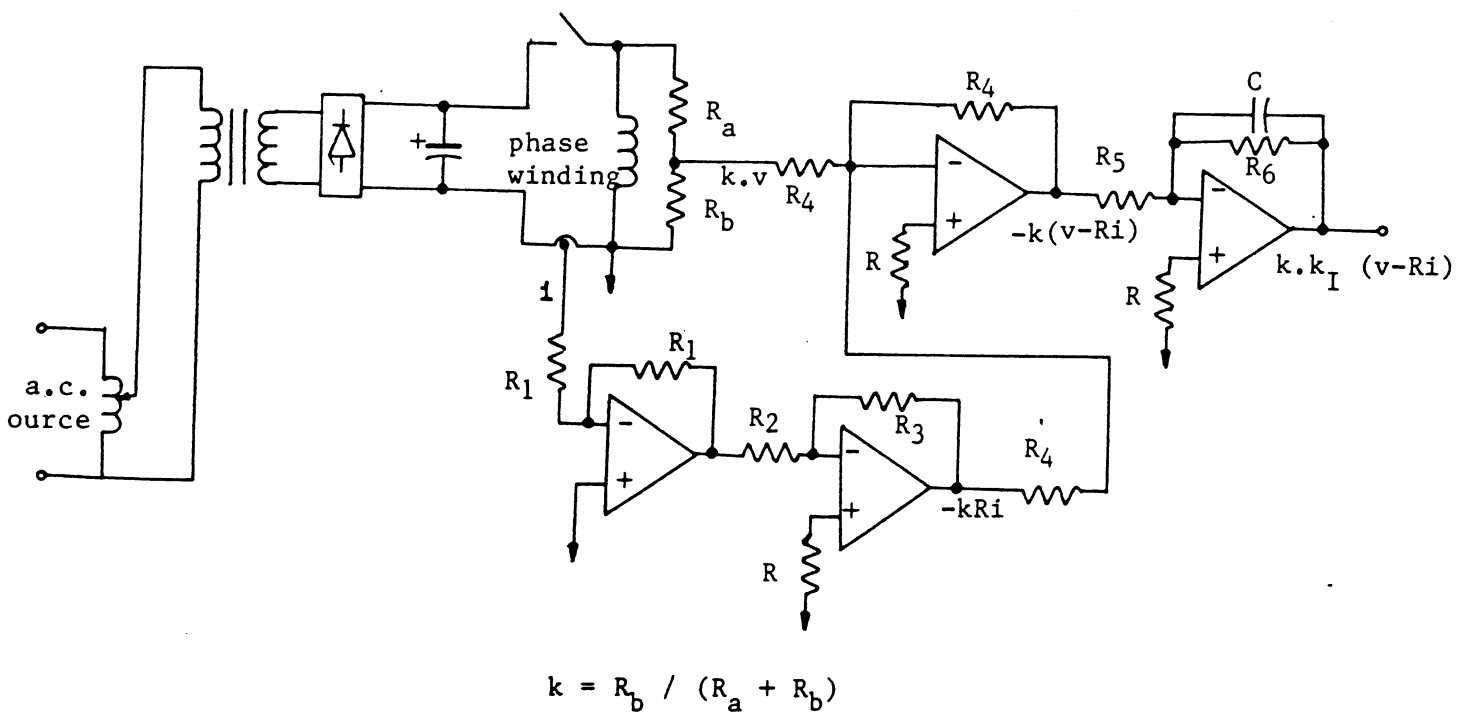


Figure 66. Implementation circuit Direct Flux Linkages method

Using a potential divider, the voltage across the winding is sensed. A voltage proportional to the resistive voltage drop in the winding is subtracted from the sensed voltage and the resulting voltage is fed to an integrator to obtain a flux linkages-proportional voltage. The integration time constant has to be selected such that the integration period is greater than the rise time of the current.

If the winding voltage is scaled down by a factor 'k', the voltage equation becomes

$$k v(t) = k \left[R i(t) + \frac{d\Psi(t)}{dt} \right] \quad (6.5)$$

The output of the integrator includes a factor due to the integrator gain. Thus, with reference to equation (6.3), the output of the integrator is given by

$$v_I(t) = k k_I \int_0^t [v(t) - R i(t)] dt \quad (6.6)$$

where k_I is the integrator gain. The flux linkages are then calculated from

$$\Psi(t) - \Psi(0) = \frac{v_I(t)}{k k_I} \quad (6.7)$$

The flux linkages versus current characteristic can be observed on an oscilloscope operating in the x-y mode or recorded on an x-y recorder. For good accuracy, the characteristic can be recorded using a Data Acquisition System (DAS) with a high sampling rate.

6.4.2 Indirect measurement

The electronic processing circuits in Figure 66 can be eliminated by direct acquisition of the winding voltage and current. Numerical integration is then applied on the acquired data to obtain flux linkages. The accuracy of a numerical integration routine is very sensitive to the step-length. Therefore, for good results, the time-step for data acquisition has to be much smaller than the time constant of the test winding. A ratio of 10:1 has been found to be satisfactory.

The indirect method is more reliable than the direct measurement method since the errors are limited to the accuracy of the data acquisition equipment and the integration routine, the errors of which can be quantified. The direct method is prone to additional errors such as the temperature sensitivity of the amplifiers and the gain-setting resistors and integrator imperfections.

6.4.3 Measurement of Coenergy

The same method used for measuring the flux linkages characteristic at a fixed rotor position can be used for measuring the coenergy change per phase when the motor is in operation. The coenergy change equals the area enclosed by the flux linkages versus current trajectory over one switching period. Using this value, the steady-state average torque and power output, T and P , respectively, can be calculated using

$$T = \frac{W N_S N_R}{4 \pi} \quad (6.8)$$

and

$$P = T \omega \quad (6.9)$$

where W is the coenergy change per stroke, N_S and N_R are the number of stator and rotor poles, respectively and ω is the angular speed. A stroke is defined as the movement of a pair of rotor poles from the fully unaligned to the fully aligned positions.

6.5 *Experimental Results*

The methods described in the foregoing sections were implemented on a 6/4 pole laboratory prototype SRM designed for a peak output power of 2.7 kW at a nominal speed of 1800 r.p.m. and a peak on-time current of 10 A. Details on the prototype motor are given in Appendix A. The winding resistance per phase for the prototype SRM is 3.4 Ω . Table 9 compares measured maximum inductance at 1 and 10 A by the High Frequency Superposition method, the Flux Linkages, rising current method and the Flux Linkages, decaying current method. A Data Acquisition System (DAS) was used to acquire data at a time step of 100 μ s for the rising current technique and 50 μ s for the decaying current technique. A full comparison for the range of 0-10 amperes is given in Figure 67.

HFS, FLRS, FLRH, FLD and FEA stand for high frequency superposition, flux linkages by rising current and software integration, flux linkages by rising current and hardware integration, flux linkages by decaying current and finite element analysis methods, respectively.

Table 9. Comparison of maximum inductance values by various methods

i [A]	IIFS [mH]	FLRS [mH]	FLRII [mH]	FLD [mH]	FEA [mH]
1	506.5	895	646	561	843
10	115.3	162	149.7	130	133

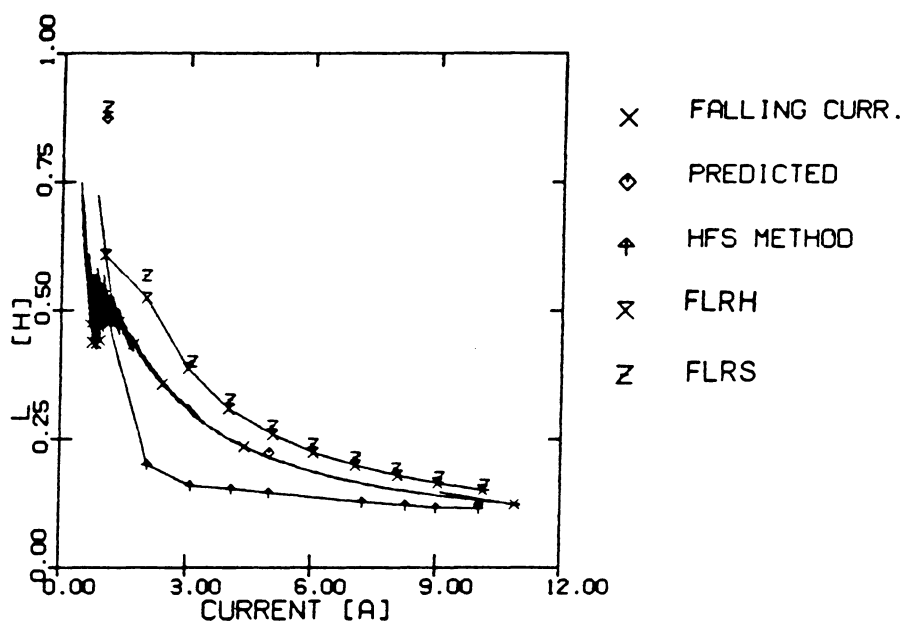


Figure 67. Comparison of measured maximum inductances by various methods

The finite element analysis values were obtained using a PC-based two-dimensional finite element analysis package. It is observed that the high frequency superposition method gives the largest error both at low and high saturation. Taking the FEA values as a reference, the error is 39.9 % at the low current of 1 A and 13.3 % at the nominal current of 10 A. The error is mainly attributed to the distortion of the winding current and voltage waveforms due to saturation of the iron resulting in inaccurate measurements. Due to this large error, this method was not pursued any further.

The results by the flux linkages method are in close agreement with the FEA results. However, the degree of accuracy depends on the measurement technique and the excitation current range. Figures 68 to 71 show voltage, current, flux linkages and inductance characteristics obtained using the Flux Linkages by rising current and numerical integration technique.

The instantaneous winding voltage and current were acquired using a standard Data Acquisition System (DAS) with a time step of $100\ \mu\text{s}$. Numerical integration to obtain the flux linkages was implemented using the trapezoidal rule.

Figure 72 shows the flux linkages versus current characteristics directly measured through hardware integration.

The corresponding inductance characteristics are shown in Figure 73 and compared with the results in Figure 71 for the 0° , 20° and 45° rotor positions representing the fully aligned, partially aligned and fully unaligned rotor positions, respectively.

In Figure 73, the Direct Measurement technique refers to the direct measurement of flux linkages through hardware integration of the winding voltage function. The Indirect Measurement technique refers to the determination of flux linkages through numerical integration of the winding voltage function. It is observed that the Direct

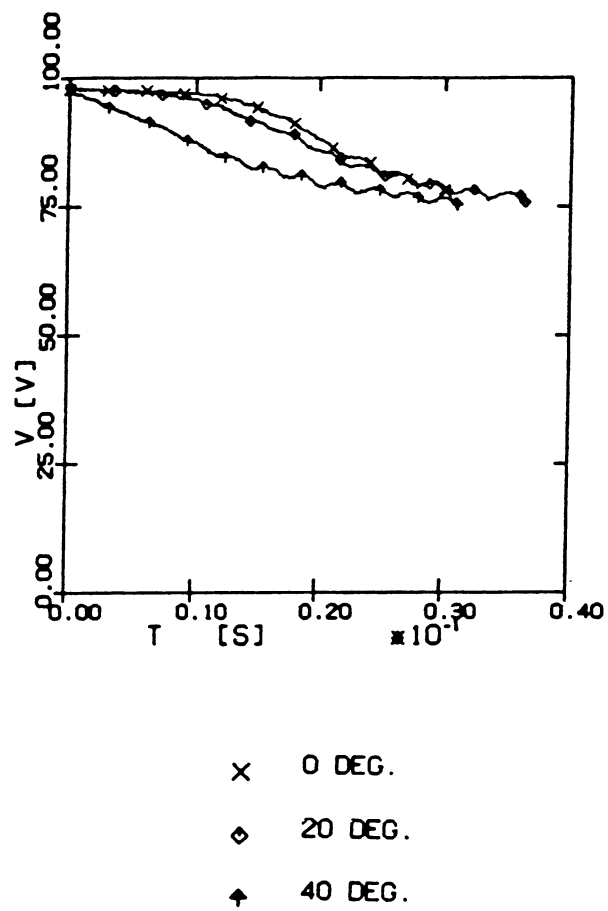


Figure 68. Measured voltage-time characteristic by the Indirect Flux Linkages method

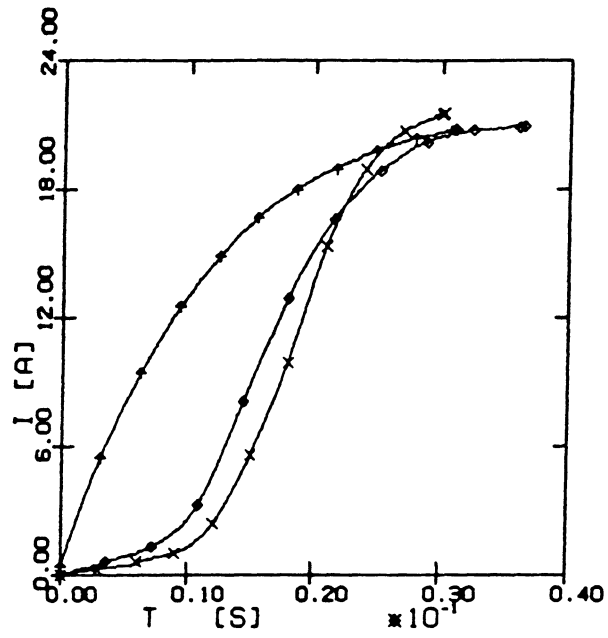


Figure 69. Measured current-time characteristic by the Indirect Flux Linkages method

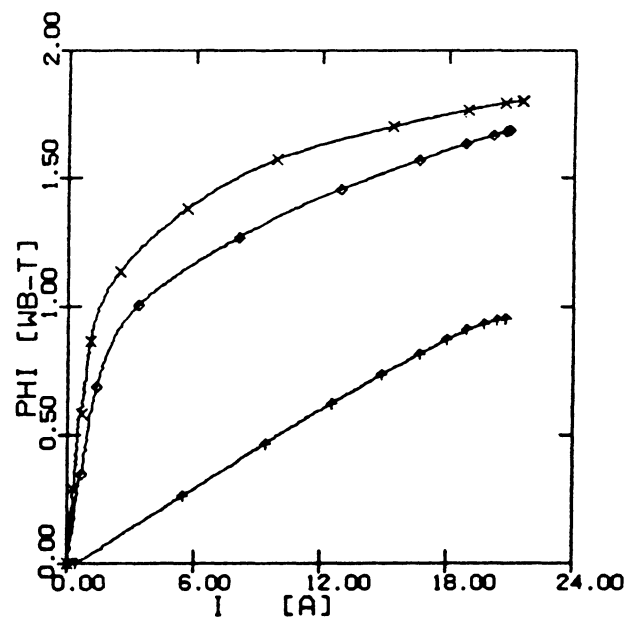


Figure 70. Derived flux linkages-current characteristic by the Indirect Flux Linkages method

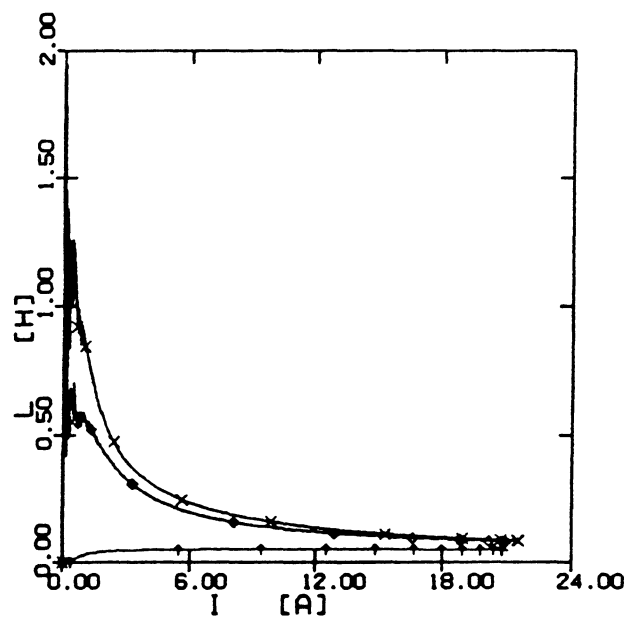


Figure 71. Derived Inductance-current characteristic by the Indirect Flux Linkages method

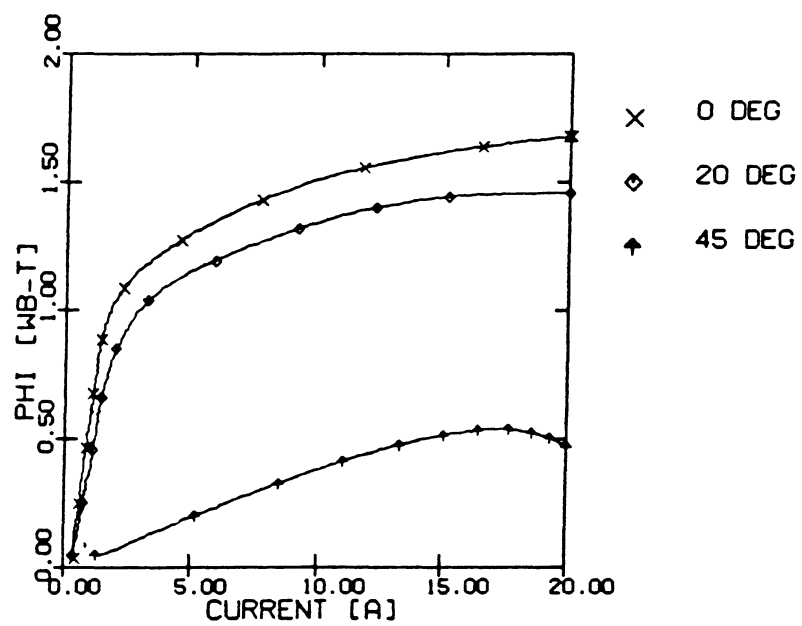


Figure 72. Measured flux linkages by the Direct Method at various rotor positions

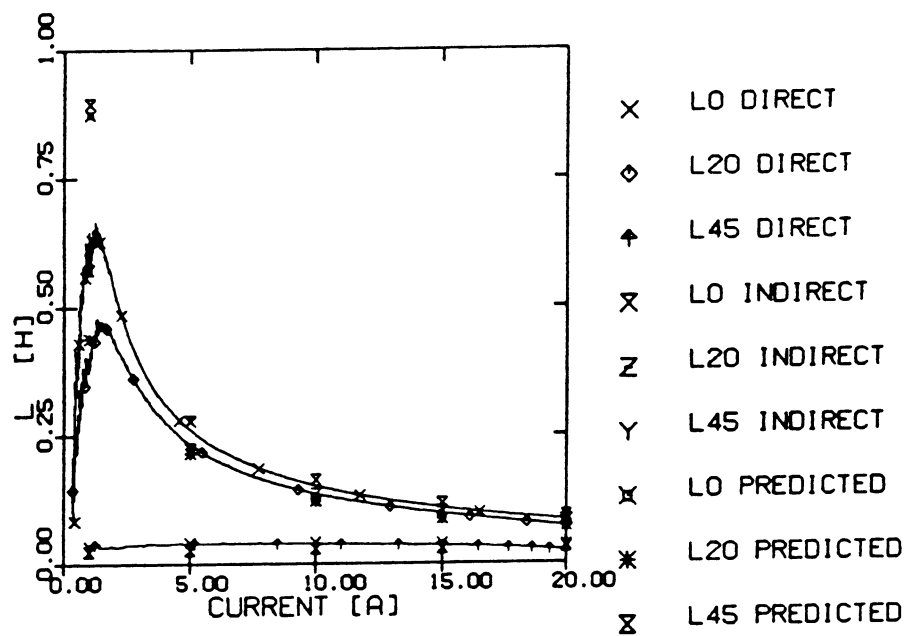


Figure 73. Comparison of Inductances by the Direct, Indirect and analytical methods

Measurement technique gives a large error at low currents but offers higher accuracy at higher currents. For example, in the fully aligned position, the errors by the direct approach based on the FEA predictions are 23.3 % and 12.5 % at 1 A and 10 A, respectively. The same errors for the Indirect method are 6.1 % and 21.8 %. The error by the Direct Measurement technique is partly attributed to imperfections of the operational amplifiers used in the hardware circuit and the effects of initial switching transients on the integration process. The results indicate that the direct measurement technique is more appropriate for measurements at higher excitation currents, typically above 10 % of the nominal current of the motor. At lower currents, the indirect method gives higher accuracy.

Figure 74 to 77 shows the results by measuring the falling current and voltage across the winding when the source is disconnected after steady-state conditions are achieved. The measured currents and voltages are shown in Figures 74 and 75 and the calculated flux linkages and inductances are shown in Figures 76 and 77 for the 0 °, 20 ° and 45 ° rotor positions as before.

A MOSFET switch operating at a low frequency was used for disconnecting the d.c. source from the winding so as to improve the accuracy of the measurements. Due to contact resistance, a mechanical switch was found to introduce an intolerable time delay between the beginning of the voltage fall and the onset of the decaying current. When compared to the predicted values of inductance, this technique is the most accurate at high currents. A comparison with the rising current technique is given in Figure 78 for the fully aligned position.

At high currents, typically above 50 % of the nominal current, the decaying current technique offers superior accuracy. For example, at the nominal current, the error by the falling current technique is 2.25 % as compared to the FEA results.

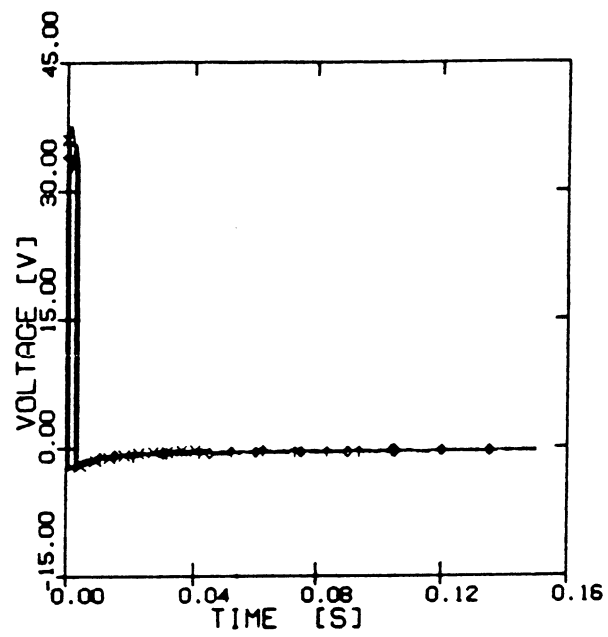


Figure 74. Measured voltage-time characteristic by the falling current method

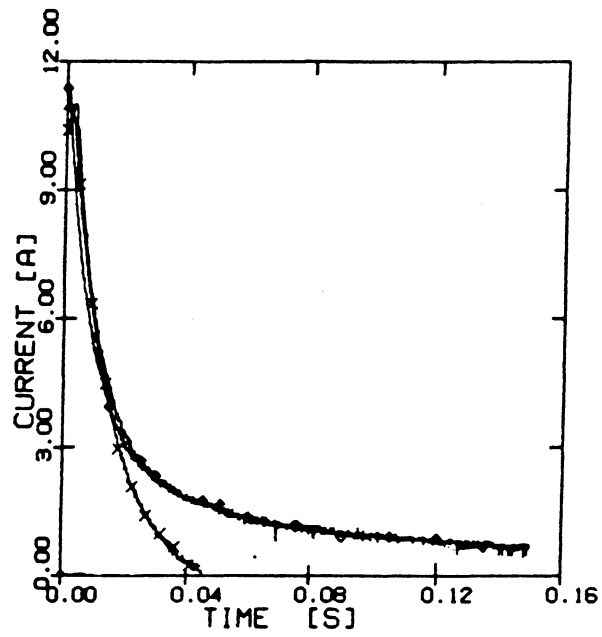


Figure 75. Measured current-time characteristic by the falling current method

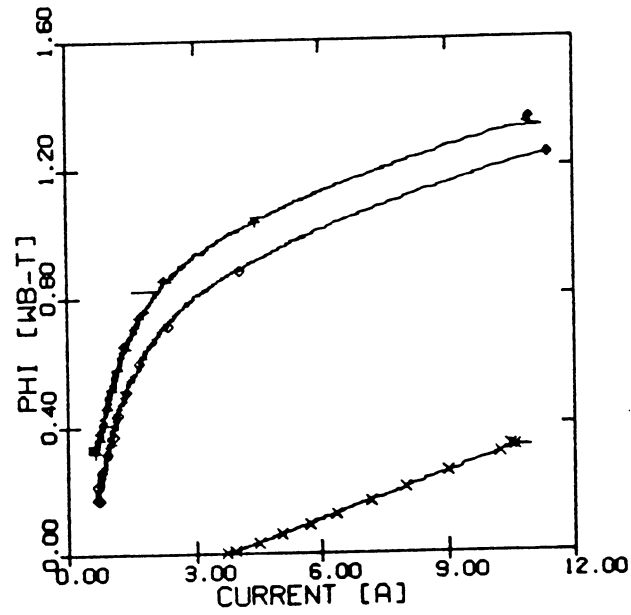
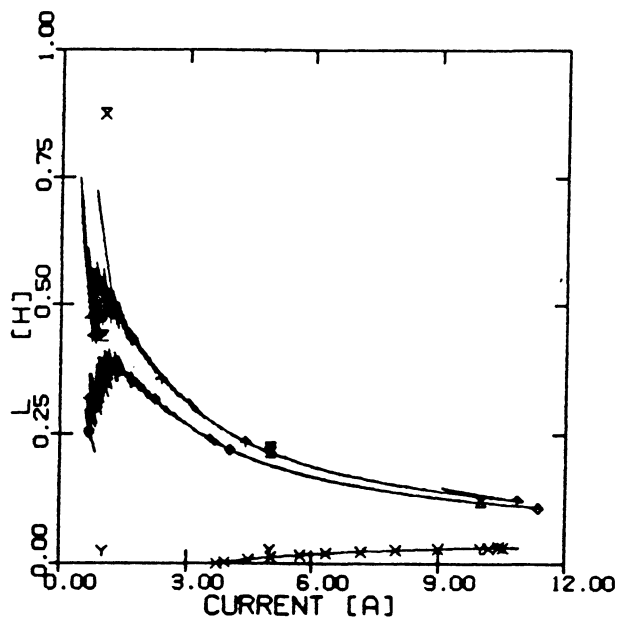


Figure 76. Derived flux linkages-current characteristic by the falling current method



X 45 DEG: MEASURED

◇ 20 DEG: MEASURED

↑ 0 DEG: MEASURED

x 0 DEG: PREDICTED

Z 20 DEG: PREDICTED

Y 45 DEG: PREDICTED

Figure 77. Derived Inductance-current characteristic by the falling current method

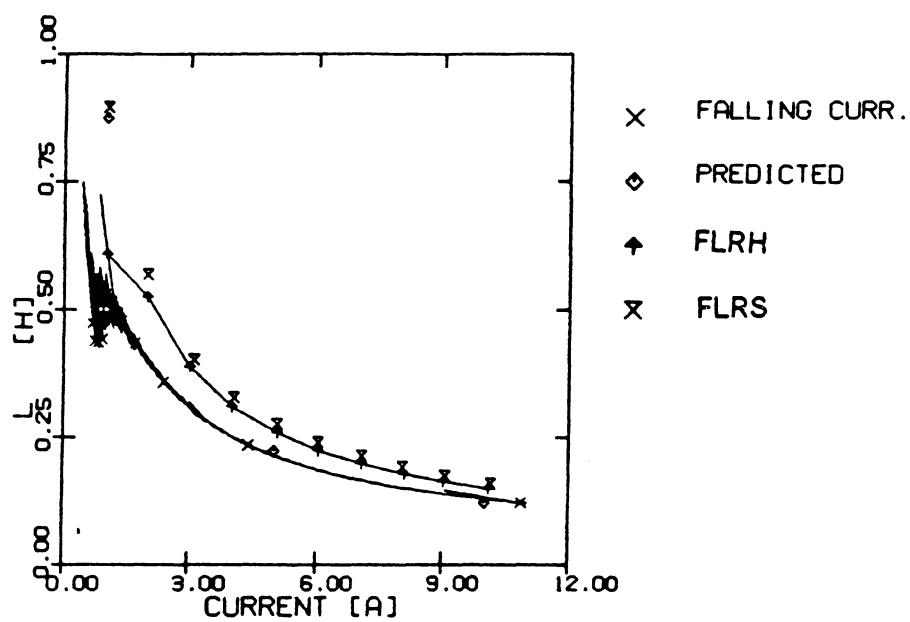


Figure 78. Comparison of predicted and measured inductances by various flux linkages methods

As stated earlier, the hardware implemented for the Direct Measurement technique can be used for the direct measurement of coenergy change when the motor is in normal operation. Figure 79 shows an oscillograph of flux linkages versus current for one phase of the SRM drive under normal operation for a conduction angle of 25 degrees, and an advance angle of 12 degrees.

The advance angle is the angle by which the switch-on of the winding current is advanced from the rotor position corresponding to the beginning of overlap between a rotor pole and an excited stator pole. The operating point corresponds to a d.c. supply voltage of 150 V, peak current of 4.5 A and speed of 600 r.p.m. The enclosed area represents the coenergy change per phase as a pair of rotor poles moves through one rotor pole pitch under the influence of the magnetic field set up by the current in one stator phase. In this particular case, the scaling factors are 32.14 and 10 for the flux linkages and current, respectively, resulting in a coenergy change per phase of 1.74 Joules. According to equations (6.7) and (6.8), this gives an average steady-state torque of 3.32 Nm for the prototype motor and an average steady-state airgap power of 209 watts. The main advantage of this technique is that it provides a fast, on-line visual method for examining the performance of the drive (e.g. on an oscilloscope) as a function of changes in the control strategies.

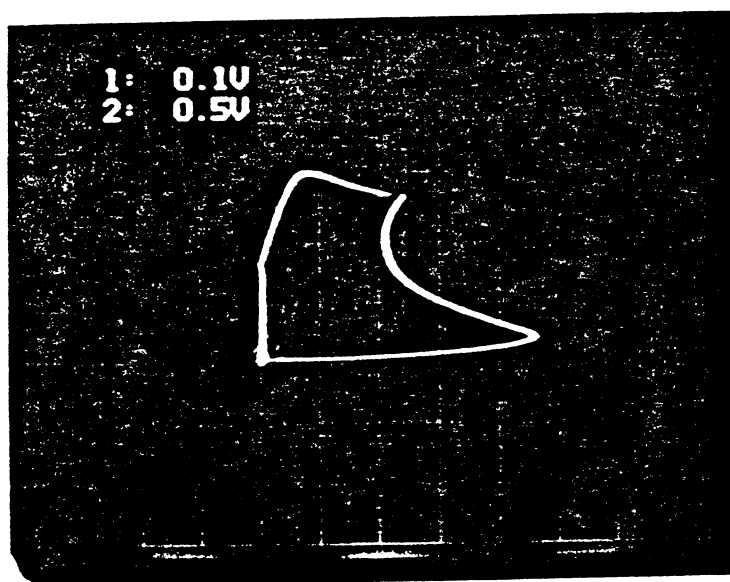


Figure 79. Measured flux linkages-current trajectory: x-axis: 2 A/div., y-axis: .23 Wb-t/div.

6.6 CONCLUSIONS

A number of measurement and instrumentation techniques for evaluating the parameters and performance of a switched reluctance motor have been presented and compared. Experimental results based on a 6/4 pole SRM are compared to the FEA predictions. The High Frequency Superposition method for measuring the inductance is shown to be inappropriate for the switched reluctance motor due to the nonlinearity of the motor which results in a distortion of the waveform of the injected high frequency a.c. signal. An alternative method based on the measurement of the instantaneous flux linkages is proposed. Three alternative approaches to the implementation of the proposed method are described and compared. These are the direct measurement of flux linkages by hardware integration of the winding voltage function when a step voltage is applied, the indirect determination of flux linkages through measurement of the rising voltage and current followed by off-line numerical integration of the winding voltage function and the measurement of decaying current when the winding is de-energized followed by numerical integration. It is shown that the decaying current approach gives the highest accuracy particularly at high currents. The rising current method gives the best accuracy at low currents. The direct measurement method has been observed to be in good agreement with predictions for currents in excess of 10 % of the nominal peak current of the motor. This technique has the advantage that it can be used for the direct measurement of coenergy change per phase when the motor is in steady-state operation thus providing a fast and on-line visual method for studying the performance of the SRM drive, particularly the effect of various control parameters on the performance.

7.0 ANALYSIS AND DESIGN OF A LOW COST CONVERTER FOR SWITCHED RELUCTANCE MOTOR DRIVES

The emergence of new applications for switched reluctance motor (hereafter referred to as SRM) drives has created a need for low cost, compact converter topologies. This chapter describes the analysis and design of a low cost, one-switch per phase converter topology suitable for low performance applications. The converter has the advantage of minimum possible number of semiconductor devices in the power circuit without the attendant need for a bifilar winding. Consequently, its drive requirements are minimal and the windings are simple. This is achieved through the elimination of the additional semiconductor switch(es) normally required for regeneration. Energy stored in the phase windings during conduction time is partially dissipated in a resistor while the rest is converted to mechanical power. Although the basic circuit topology has been known for sometime, no effort has so far been reported on its analysis and design. This chapter gives a complete steady-state analy-

sis of the drive including the input filter parameters. Closed form expressions for determining the current and voltage stresses on the semiconductor devices are derived and they are cast in normalized form for use in selecting the ratings of the power devices. Experimental verification of the key results is provided from a 6/4 pole SRM drive system.

7.1 Introduction

Recently there has been growing interest in the switched reluctance motor drive for such applications as automotive, appliances, pumps, fans etc. Cost and space constraints in these applications have mandated the need for low cost, compact converters and controllers. Various converter configurations have been proposed in the literature [10,37,63]. The distinguishing factor in all the converters has been in the management of stored energy and its impact on the efficiency of the drive. The minimum switch configuration is obtained by dissipating a part of the stored energy in a resistor instead of regenerating to the d.c. link. Figure 80 shows the basic configuration for one phase.

Since all the switches share a common source (emitter) connection, the drive requirements are also minimum. This configuration, hereafter referred to as the minimum switch configuration, is therefore very appropriate for the applications cited above. The circuit has been known for some time but the analysis, performance in the SRM drive and design of the circuit are not available. The present chapter addresses these aspects of the low cost converter for switched reluctance motor drives.

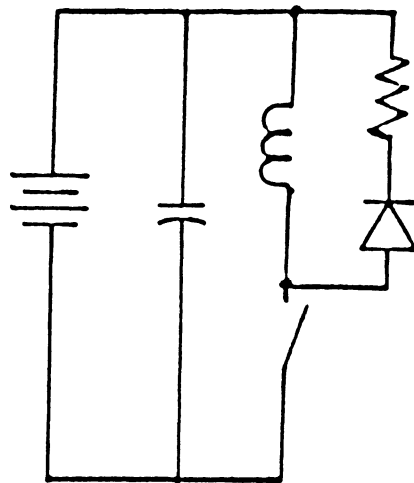


Figure 80. Single phase circuit of the minimum switch SRM converter

Two topologies of the minimum switch configuration can be implemented:

- i. With a common dump resistor and R-C snubber for all the switches.
- ii. Without a dump resistor but with separate snubbers for each switch.

The voltage rating of the semiconductor switches depends on the topology used and the value of the dump resistor which also determines the current rating of the free-wheeling diodes. The merits and demerits of each topology are discussed further in the following sections. It is shown that the elimination of the dump resistor results in an excessive current stress in the freewheeling diodes and a low drive efficiency. The performance equations are derived for the dump resistor configuration. Normalized characteristics for the selection of the power semiconductor device ratings are also provided. The analytical results are supported by experimental measurements based on the 6/4 pole prototype SRM.

7.2 Minimum switch converter topologies

7.2.1 The Dump-resistor topology

The complete circuit of the dump resistor topology is shown in Figure 81 for a 3-phase SRM drive.

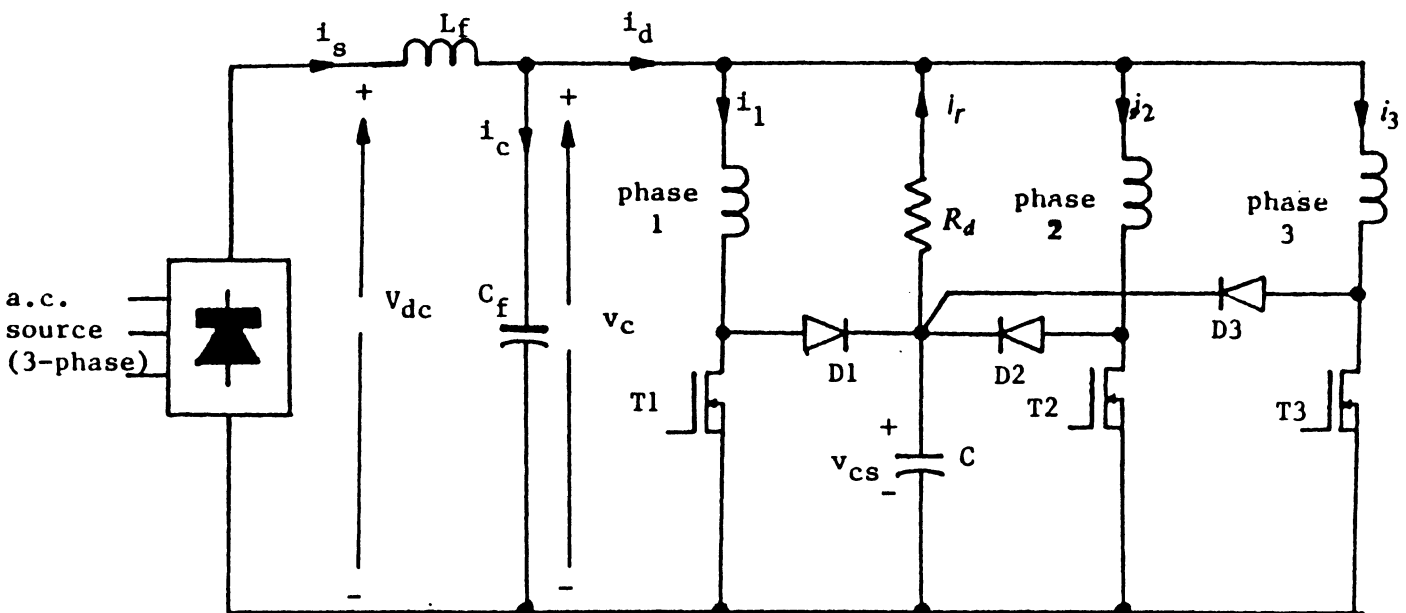


Figure 81. Three phase minimum switch converter with dump resistor

Only one dump-resistor and snubber circuit are necessary. Since all the switches share a common point, only one power supply is necessary for all the gate drive circuits. The operation of the circuit is as follows. When a phase switch is turned on, the d.c. link voltage is applied to the winding. Energy is therefore transferred from the source to the SRM. Part of this energy is converted to mechanical energy whereas some is stored in the magnetic field. At turn-off, the stored energy freewheels through the winding, the freewheeling diode and the dump resistor so that part of the energy is dissipated in the winding and dump resistances while the rest is converted to mechanical energy. During this period, the voltage across the switch equals the sum of the d.c. link voltage and the voltage drop across the resistor. The current through the freewheeling diode and the power dissipated in the dump resistor are functions of the resistance in the freewheeling path and the value of the snubber capacitor. Thus the sizing of these components is critical to the voltage rating of the phase switches, the current and voltage ratings of the freewheeling diodes and the power rating of the dump resistor. From the point of view of the motor, the value of these components has a direct effect on torque production and efficiency since they determine the time taken for the winding current to fall to zero after turn-off.

7.2.2 Topology without dump-resistor

The dump resistor in Figure 81 can be eliminated provided that the diodes are appropriately rated. Separate snubbers for each phase have to be provided and more time has to be allowed for the winding current to fall to zero. The resulting circuit is shown in Figure 82

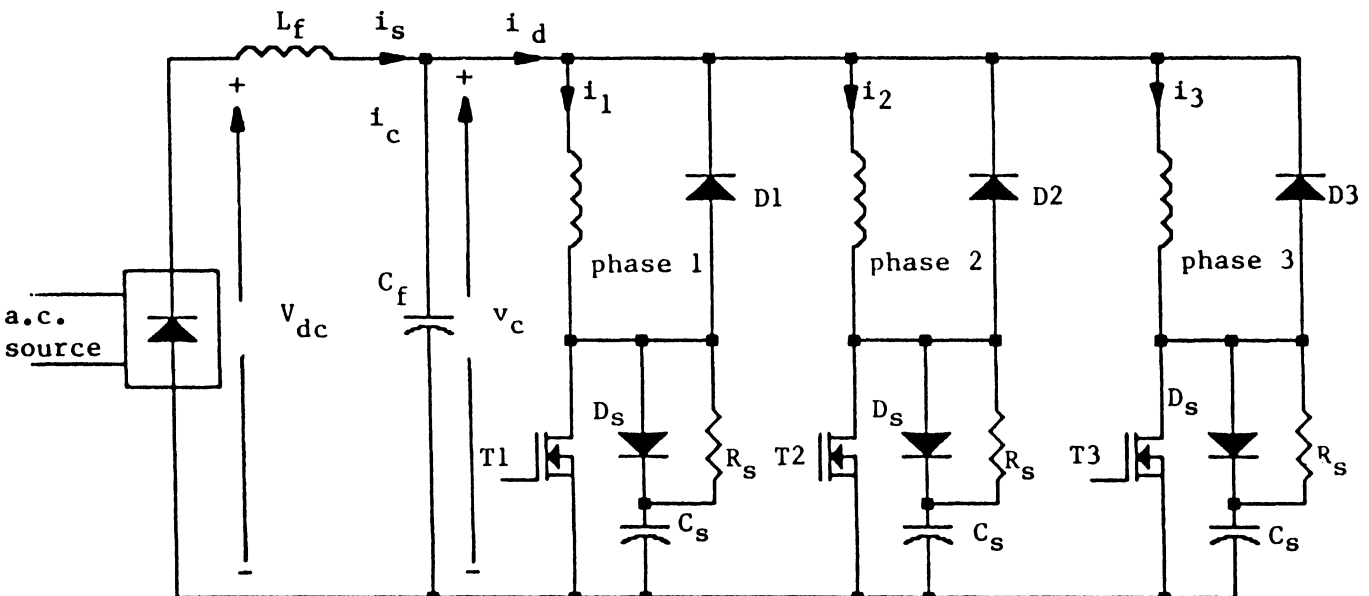


Figure 82. Three phase minimum switch converter without dump resistor

This topology is inappropriate because of the excessively long time required for the current to fall to zero even in the absence of mutual coupling effects. A prolonged fall time results in continuous conduction of the freewheeling diodes and consequently higher current ratings of these diodes. Thus despite the advantage of low voltage rating of the power switches, this topology is inappropriate where the semiconductor devices are constrained by size and cost.

7.3 Analysis

The voltage equation per phase during the on-time is given by

$$V_{dc} = (R_w + R_s)i + \omega_m \left[L \frac{di}{d\theta} + i \frac{dL}{d\theta} - M \frac{di'}{d\theta} \right] \quad (7.1)$$

During the off-time, the equation becomes

$$0 = (R_w + R_d)i + \omega_m \left[L \frac{di}{d\theta} + i \frac{dL}{d\theta} - M \frac{di'}{d\theta} \right] + V_d \quad (7.2)$$

where V_{dc} is the d.c.-link voltage, R_w is the winding resistance per phase, R_s and R_d are the MOSFET on-resistance and dump resistance respectively, L is the inductance per phase, M is the mutual inductance between phases, ω_m is the angular speed of the drive, i is the instantaneous winding current per phase, i' is the current in an adjacent phase winding, θ is the mechanical position of the rotor and V_d is the forward voltage drop of a freewheeling diode. The input filter equations are given by

$$v_c = V_{dc} - \omega_m L_f \frac{di_s}{d\theta} \quad (7.3)$$

$$i_s = i + \omega_m C_f \frac{dv_c}{d\theta} \quad (7.4)$$

where i_s and v_c are the source current and filter capacitor voltage, respectively and L_f and C_f are the input filter inductance (including source inductance) and capacitance, respectively. The mutual inductance may be neglected in most of the switched reluctance motors. Analysis of the drive involves the solution of (7.1) to (7.4) in steady-state. The procedure for obtaining a direct steady-state solution without going through the transient solution is described in chapter 4.

7.4 Design equations

Figure 83 shows the worst-case current waveforms in the switches and freewheeling diodes for one phase assuming a linear inductance profile.

The current and voltage stresses on the individual devices and their maximum ratings are derived in the following sections.

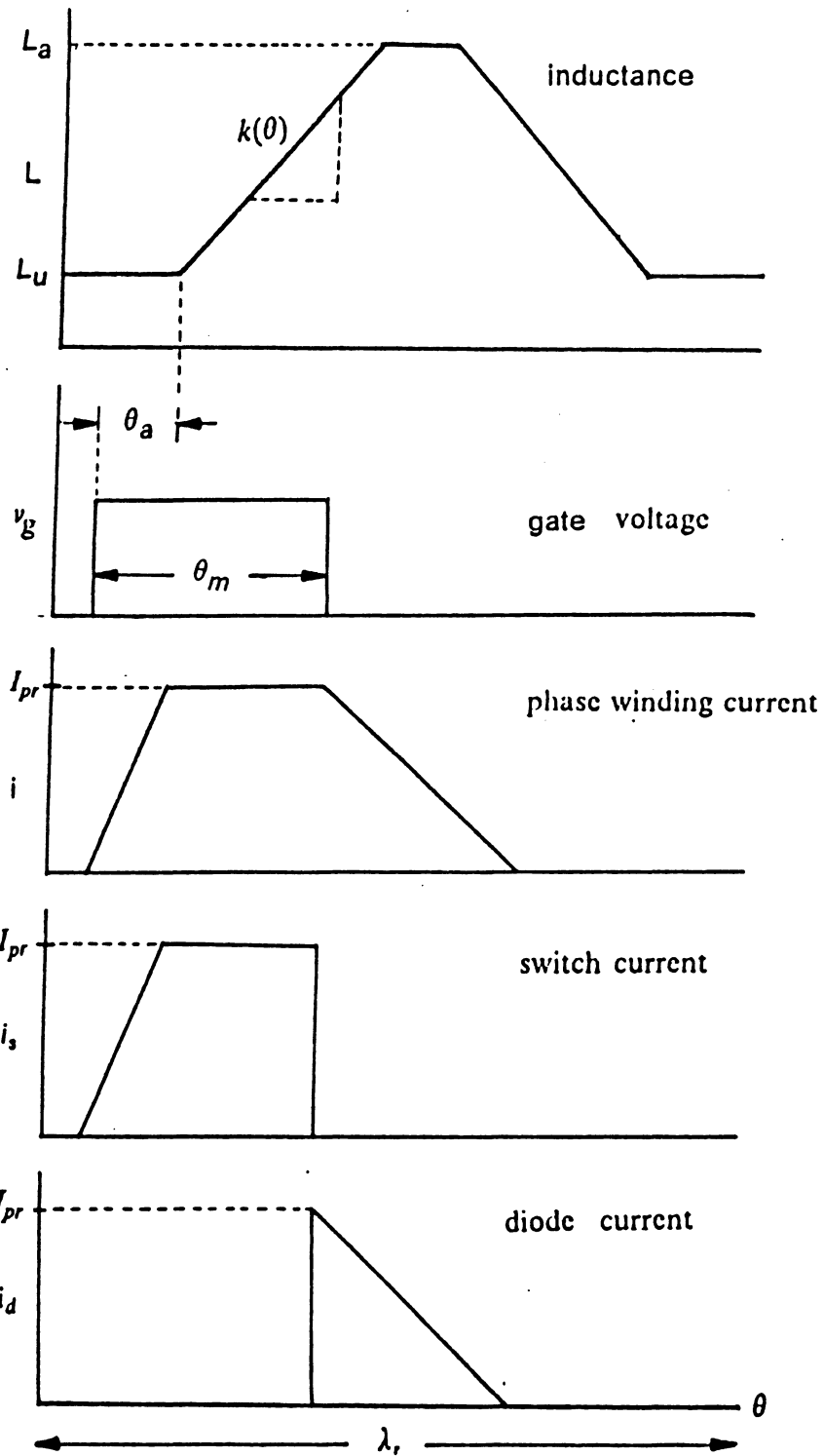


Figure 83. Worst-case waveforms of the minimum switch converter

7.4.1 Switch stresses and ratings

7.4.1.1 Continuous ratings

The worst-case switch current waveform is a trapezoidal block of amplitude I_{pr} , where I_{pr} is the rated on-time current per phase for the drive. The r.m.s. current per switch for a conduction angle θ_c is therefore given by

$$I_s = \sqrt{\frac{1}{\lambda_r} \left[\int_0^{\theta_a} \left[\frac{I_{pr}}{\theta_a} \theta \right]^2 d\theta + \int_{\theta_a}^{\theta_c} I_{pr}^2 d\theta \right]} \quad (7.5)$$

where θ_a is the advance angle defined as the angle by which turn-on is advanced prior to the beginning of overlap between a pair of rotor poles and a pair of excited stator poles. λ_r is the rotor pole pitch given by

$$\lambda_r = \frac{2\pi}{P_r} \quad (7.6)$$

where P_r is the number of rotor poles. The maximum conduction period per phase θ_m is given by

$$\theta_m = \frac{\lambda_r}{m} \quad (7.7)$$

where m is the number of phases given by

$$m = \frac{P_s}{2} \quad (7.8)$$

P_s is the number of stator poles. The duty cycle k_d is defined as

$$k_d = \frac{\theta_c}{\theta_m} \quad (7.9)$$

Substituting (7.6), (7.7), (7.8) and (7.9) into (7.5), the equation for the r.m.s. current of the switch is obtained as

$$I_s = I_{pr} \left[\sqrt{\frac{2 k_d}{P_s} - \frac{4 k_d \theta_a}{3 P_s \theta_c}} \right] \quad (7.10)$$

The maximum r.m.s. current per switch corresponds to unity duty cycle. Therefore, the r.m.s. current rating of the phase switches without chopping is given by

$$I_{sr} = I_{pr} \sqrt{\frac{2}{P_s} \left[1 - \frac{2}{3} \left\{ \frac{\theta_a}{\theta_c} \right\} \right]} \quad (7.11)$$

If chopping is employed, the switch can be significantly derated, depending on the chopping strategy used [66]. The maximum voltage stress on a phase switch occurs immediately after turn-off when operating at the rated on-time current. The magnitude of this voltage equals the sum of the d.c. link voltage, the forward voltage drop of the freewheeling diode and the voltage drop across the dump resistor and written as

$$V_{sr} = V_{dc} + V_d + I_{pr} R_d \quad (7.12)$$

7.4.1.2 Peak current rating

The peak on-time current is normally maintained at the desired value by means of a chopping strategy employing PWM or hysteresis techniques. In low performance and

low cost drives, the extra circuitry required for these current control schemes is not economical. Without chopping, the peak on-time current is dependent on the motor impedance and the applied voltage. Usually, turn-on is effected prior to the beginning of the overlap period during which time the inductance has its minimum value and the rate of change of the inductance with rotor position is negligible. Under these conditions and assuming negligible mutual inductance, the solution for equation (7.1) is given by

$$i(\theta) = \frac{V_{dc}}{R_e} \left[1 - \exp\left\{ -\frac{\theta}{\omega_m \tau_r} \right\} \right] \quad (7.13)$$

where

$$R_e = R_w + R_s \quad (7.14)$$

and

$$\tau_r = \frac{L_u}{R_e} \quad (7.15)$$

L_u is the minimum inductance per phase. The peak current occurs at the onset of overlap. Thus, the peak on-time steady-state current without chopping is given by

$$I_{prn} = \left[\frac{V_{dcn}}{R_{en}} \right] \left[1 - \exp\left\{ -\frac{\theta_a}{\omega_m \tau_r} \right\} \right] \quad (7.16)$$

where

$$I_{prn} = \frac{I_{pr}}{I_{rated}} , p.u.$$

$$V_{dcn} = \frac{V_{dc}}{V_{rated}} , p.u.$$

$$R_{en} = \frac{R_e}{Z_{rated}} , p.u.$$

ω_m is angular speed of the drive. Equation (7.16) can be expressed in a normalized form as

$$\frac{\tau_a}{\tau_r} = \ln \left[1 - \frac{I_{prn} R_{en}}{V_{dcn}} \right] \quad (7.17)$$

where τ_a is the advance time in seconds. The above equation can be used to determine the appropriate value of θ_a so as to limit the peak on-time current to a desired value.

7.4.2 Diode ratings

When a phase switch is on, the corresponding freewheeling diode is reverse-biased by a voltage equal to the sum of the d.c. link voltage and the voltage drop across the dump resistor. Hence the peak reverse blocking voltage rating of the freewheeling diodes must be greater than this value. Knowing the rated peak on-time current and the dump resistor value, the peak reverse voltage rating of the freewheeling diodes can be determined using equation (7.12). The peak (pulsed) current rating of the di-

odes has to be equal to the rated peak on-time current of the drive. However, the r.m.s. (continuous) current rating is not so obvious. A diode conducts only when the corresponding phase switch is off. Assuming a piecewise-linear inductance profile, the solution of equation (7.2) gives

$$i_d(\theta) = I_{pr} \exp \left\{ -\frac{\theta}{\omega_m \tau_f} \right\} \quad (7.18)$$

where

$$\tau_f = \frac{L(\theta)}{[R_w + R_d + k(\theta)]} \quad (7.19)$$

$k(\theta)$ is the rate of change of the inductance as a function of the rotor position and $L(\theta)$ is the inductance per phase at the turn-off point. The values of $k(\theta)$, $L(\theta)$ and τ_f depend on the rotor position at turn-off. There are three possible conditions:

- i. Turn-off during full overlap between the stator and rotor poles.
- ii. Turn-off prior to the onset of full overlap.
- iii. Turn off after the full overlap period (negative slope).

Condition (iii) is undesirable since it results in the production of negative torque. Assuming unequal stator and rotor pole arcs, condition (i) is the most preferable since it ensures maximum conduction angle without necessarily producing significant negative torque. When a pair of rotor poles fully overlaps a pair of excited stator

poles, the inductance for the corresponding phase is approximately constant and has its maximum value. Thus the time constant for the falling current is given by

$$\tau_f = \frac{L_a}{R_w + R_d} \quad (7.20)$$

where L_a is the inductance at the fully aligned position.

Assuming that the current falls linearly to 2 % of its maximum value within 4 time constants (2 % criterion), the r.m.s. value of the diode current is given by

$$I_d = \sqrt{\left[\frac{P_r}{2\pi} \int_0^{4\tau_f\omega_m} I_p^2 \left[\frac{4\omega_m\tau_f - \theta}{4\omega_m\tau_f} \right]^2 d\theta \right]} \quad (7.21)$$

After evaluating the integral and substituting the expression for τ_f given in (7.20), the expression for the r.m.s. current rating of the of the freewheeling diodes is

$$I_{dr} = I_{pr} \sqrt{\left[\frac{2L_a\omega_m P_r}{3\pi(R_w + R_d)} \right]} \quad (7.22)$$

7.4.3 Dump resistor ratings

7.4.3.1 Power Rating

During the freewheeling period a part of the stored energy is converted to mechanical energy whereas the rest is dissipated in the dump resistor and winding resistance, the ratio between the two depending on the dump resistor value, the snubber capacitance and the operating point. The power rating of the resistor is calculated for

the worst case where all the stored energy (at rated current and speed) is dissipated in the dump resistor. Since power is dissipated in the dump resistor every time a phase switch is turned off, the total power dissipated equals m -times the power dissipated by one phase where m is the number of phases as defined in equation (7.8). Further, the current through the resistor equals the freewheeling diode current. Thus the power rating of the dump resistor, P_{dr} , is given by

$$P_{dr} = m I_{dr}^2 R_d \quad (7.23)$$

7.4.3.2 Resistor value

The value of the power resistor is critical to the voltage rating of the phase switches, the current and voltage ratings of the freewheeling diodes and the power rating of the dump resistor itself. It also determines the fall time of the winding current which is critical to the performance of the drive. The relationships between the dump resistor value and these critical parameters are given in equations (7.12), (7.20), (7.22) and (7.23) for the diode and switch voltage ratings, the current fall time, the the diode r.m.s. current rating and the dump resistor power rating, respectively. Figure 84 shows the normalized curves of the critical parameters as a function of the dump resistor value for a typical SRM drive.

The normalizing factors are V_{dc} and I_{pr} for the voltage and current ratings, respectively. The switching period per phase and the r.m.s. input power were used as normalizing factors for the current fall time and the dump resistor power rating, respectively. It is observed that while large values of the dump resistor result in low values for the diode current rating and the current fall time, they result in increasingly

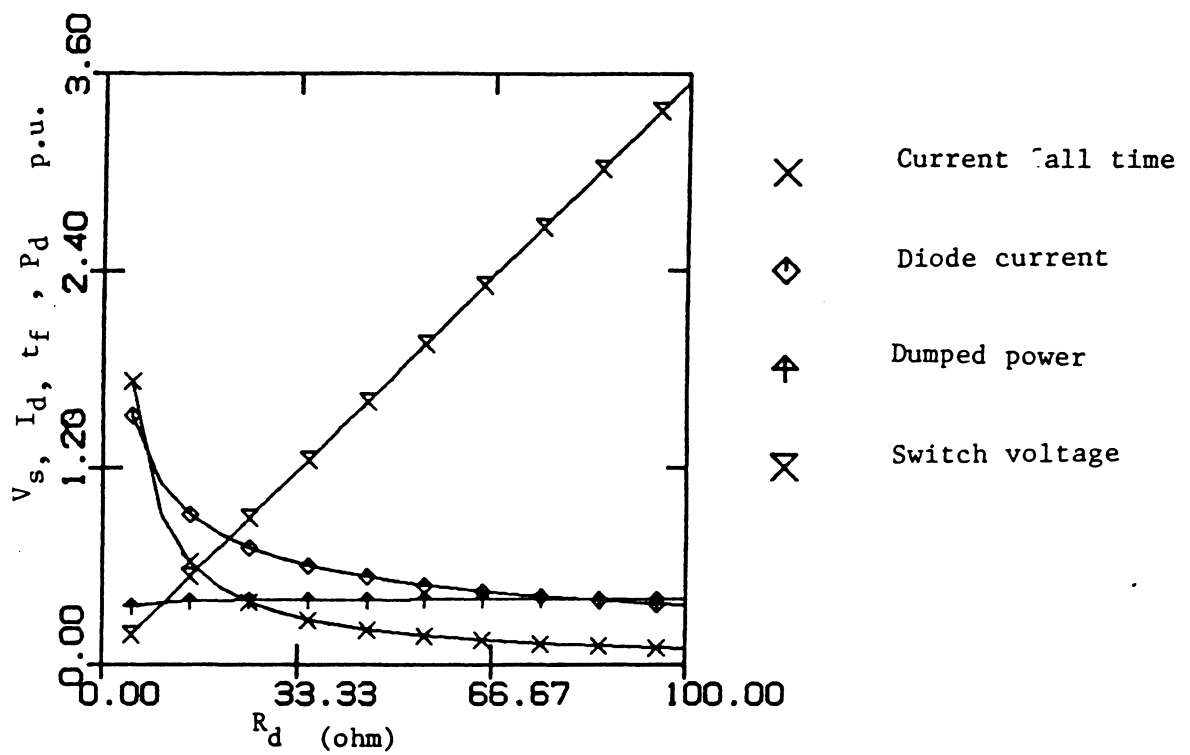


Figure 84. Converter ratings as a function of the dump resistor value

large values of the diode and switch voltage ratings. The power rating shows very little sensitivity to the resistor value. Thus the voltage rating of the switches and diodes and the current fall time are the most critical factors in the determination of the resistor value. The switch voltage can be significantly reduced by using a snubber capacitor connected as shown in Figure 81.

7.4.4 Snubber design

The capacitive snubber shown in Figure 81 limits the rate of voltage rise and the maximum voltage across the phase switch. The sizing of the capacitor depends on the maximum allowable voltage across the switch, the value of the dump resistor and the peak on-time current per phase. During on-time, the capacitor voltage equals the d.c. link voltage. When a phase switch is turned off, the current is diverted into the snubber capacitor. When the capacitor voltage rises above the d.c. link voltage, part of the winding current is diverted to the dump resistor, the magnitude of which depends on the dump resistor value. The switch voltage equals the capacitor voltage plus the diode forward voltage drop. Diodes D1, D2 and D3 have to be of the fast recovery type so as to ensure immediate transfer of current from the switch to the capacitor.

Assuming that turn-off is effected during the full overlap period, the effective circuit immediately after turn-off is a simple RLC circuit consisting of the winding resistance and inductance and the snubber capacitor. The corresponding voltage equations are given by:

$$(R_w + L_a \frac{d}{dt}) i + v_{cs} = V_{dc} \quad (7.24)$$

$$v_{cs} - V_{dc} = R_d i_r \quad (7.25)$$

$$C_s \frac{dv_{cs}}{dt} = i - i_r \quad (7.26)$$

where i_r is the dump resistor current, C_s is the snubber capacitance and v_{cs} is the snubber capacitor voltage. Immediately after turn-off, the winding current equals the peak on-time current and the capacitor voltage equals the d.c. link voltage. Applying these initial conditions to equations (7.24) to (7.26) and solving, the expression for the capacitor voltage is obtained as

$$\begin{aligned} v_{cs} = & \frac{I_p}{C_s(x_1 - x_2)} [e^{-x_2 t} - e^{-x_1 t}] \\ & + \frac{V_{dc}}{C_s L_a} \left[\frac{1}{x_1 x_2} + \frac{1}{x_1^2 - x_1 x_2} e^{-x_1 t} + \frac{1}{x_2^2 - x_1 x_2} e^{-x_2 t} \right] \\ & + \frac{V_{dc}}{C_s R_d} \left[\frac{\tau_a}{x_1 x_2} + \frac{\tau_a - x_1}{x_1^2 - x_1 x_2} e^{-x_1 t} + \frac{\tau_a - x_2}{x_2^2 - x_1 x_2} e^{-x_2 t} \right] \\ & - V_{dc} \left[\frac{\tau_a - x_1}{x_1^2 - x_1 x_2} e^{-x_1 t} + \frac{\tau_a - x_2}{x_2^2 - x_1 x_2} e^{-x_2 t} \right] \end{aligned} \quad (7.27)$$

where

$$\tau_a = \frac{R_w}{L_a} \quad (7.28)$$

$$x_1 = \sigma + \beta \quad (7.29)$$

$$x_2 = \sigma - \beta \quad (7.30)$$

$$\sigma = \frac{L_a + C_s R_w R_d}{2C_s R_d L_a} \quad (7.31)$$

and

$$\beta = \sqrt{\left[\frac{L_a + C_s R_w R_d}{2C_s R_d L_a} \right]^2 - \frac{R_d + R_w}{C_s R_d L_a}} \quad (7.32)$$

It is obvious from equation (7.27) that the peak capacitor voltage occurs sometime after turn-off. The time interval between the turn-off point and the instant at which the peak capacitor voltage occurs is determined by the first term in equation (7.27) and is given by

$$t_p = \frac{1}{2\beta} \ln \left[\frac{(\sigma + \beta)^2}{\sigma^2 - \beta^2} \right] \quad (7.33)$$

7.5 Results

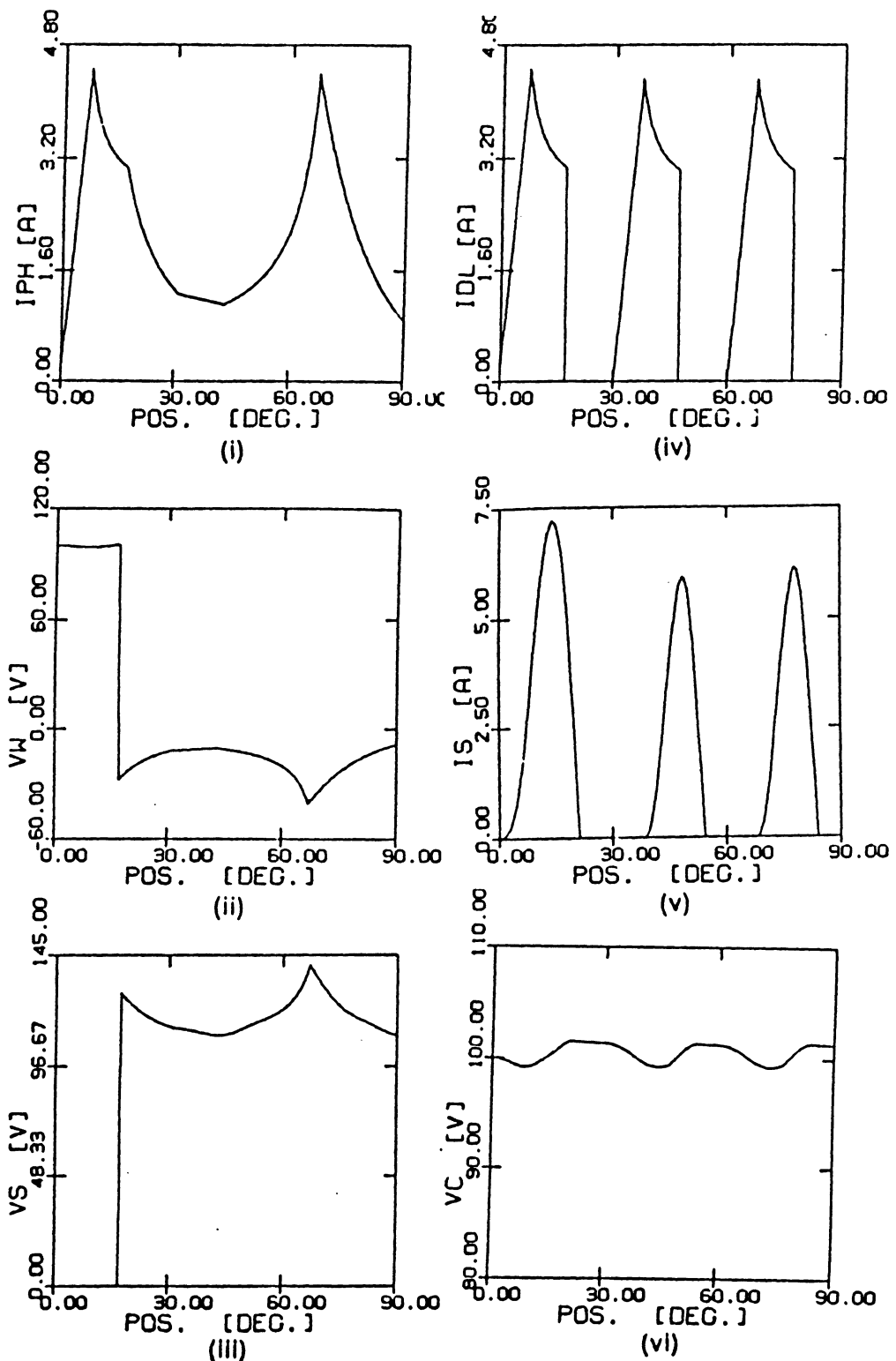
The analysis and design procedure described in the preceding sections was applied to two prototype 6/4 pole switched reluctance motors rated at 300 V, 10.65 A, 3475 r.p.m. (Prototype I) and 300 V, 10 A, 1800 r.p.m. (Prototype II), respectively. The Prototype I motor has maximum and minimum inductances of 53 mH and 3 mH, re-

spectively at the rated current. The winding resistance per phase is 0.615 ohm. Prototype II has maximum and minimum inductances of 133 mH and 40.7 mH, respectively and a phase resistance of 3.4 ohms. MOSFET switches with a drain-to-source resistance of 0.85 ohm were used in the converter circuit. Figures 85 to 88 are based on Prototype I whereas Figure 89 is based on Prototype II. Figure 85 shows the simulated waveforms for the winding current and voltage per phase, switch voltage, d.c. link current, source current (at the rectifier output) and input filter capacitor voltage at 100 V d.c. and 2746 r.p.m. The corresponding experimental waveforms measured using a Data Acquisition System are shown in Figure 86.

In both cases, a dump resistor value of 10 ohms was used. It is observed that both the simulated and measured results show that the diodes conduct continuously because the dump resistor value is too low. However, the voltage stress on the switches and diodes is relatively low. The maximum voltage stress on the switch is only 125 % of the d.c. link voltage. Figure 87 shows the winding current and voltage per phase and the switch voltage when the dump resistor value is increased to 100 ohms while the other operating conditions remain the same. The current fall time is proportionately lower. However, the switch voltage rating is more than three times the d.c. link voltage. The voltage spike across the switch is reduced by 50 % when a 16 μ F, snubber capacitor is used. The corresponding waveforms are shown in Figure 88.

Figure 89 shows typical curves for the variation of the peak capacitor voltage as a function of the capacitor value and the dump resistance. A comparison of the predicted and measured values are summarized in Table 10 for $R_d = 100$ ohms and capacitor values of 1, 4, 10, 13 and 20 μ F.

The measurements were conducted at a d.c. bus voltage of 100 V, a peak on-time current of 1 A and a conduction angle of 25 degrees. The advance angle is 12 de-



(i) phase winding current (phase 1) (iv) d.c. link current
(ii) phase winding voltage (phase 1) (v) source current
(iii) phase switch voltage (phase 1) (vi) filter capacitor voltage

Figure 85. Simulated waveforms at 100 V d.c. and $R_d = 10$ ohms

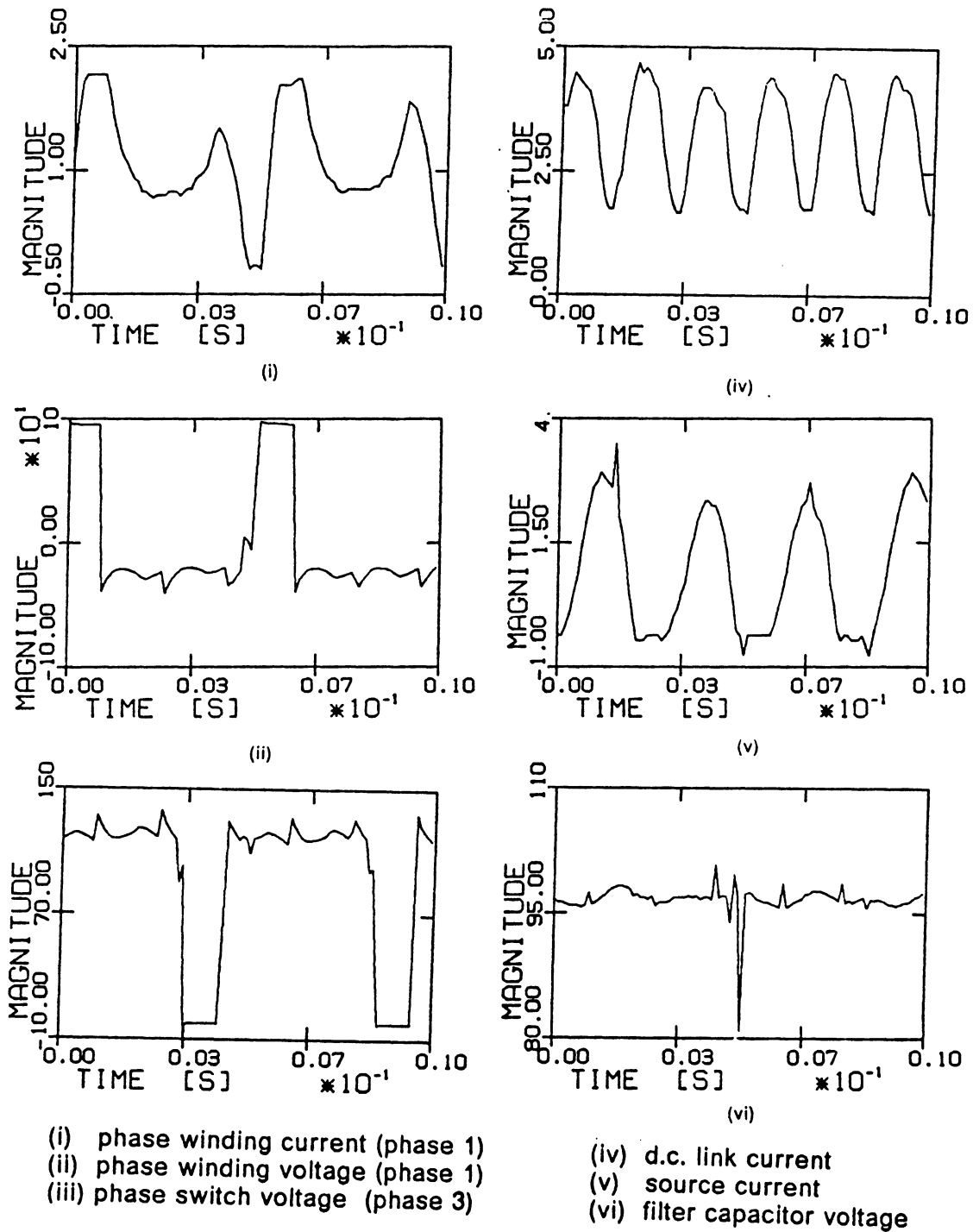


Figure 86. Measured waveforms at 100 V d.c. and $R_d = 10$ ohms

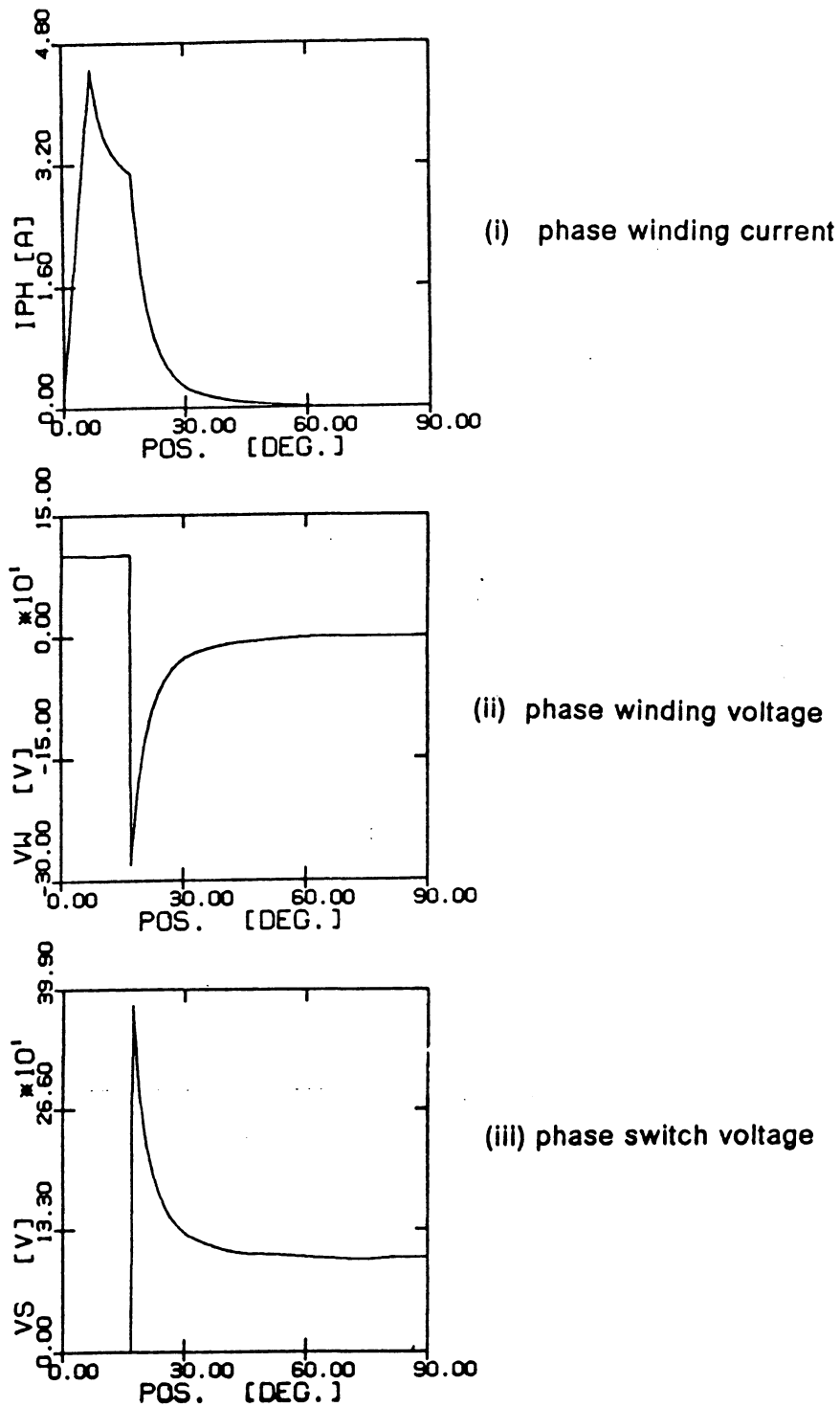


Figure 87. Simulated waveforms for $R_d = 100$ ohms without snubber capacitor

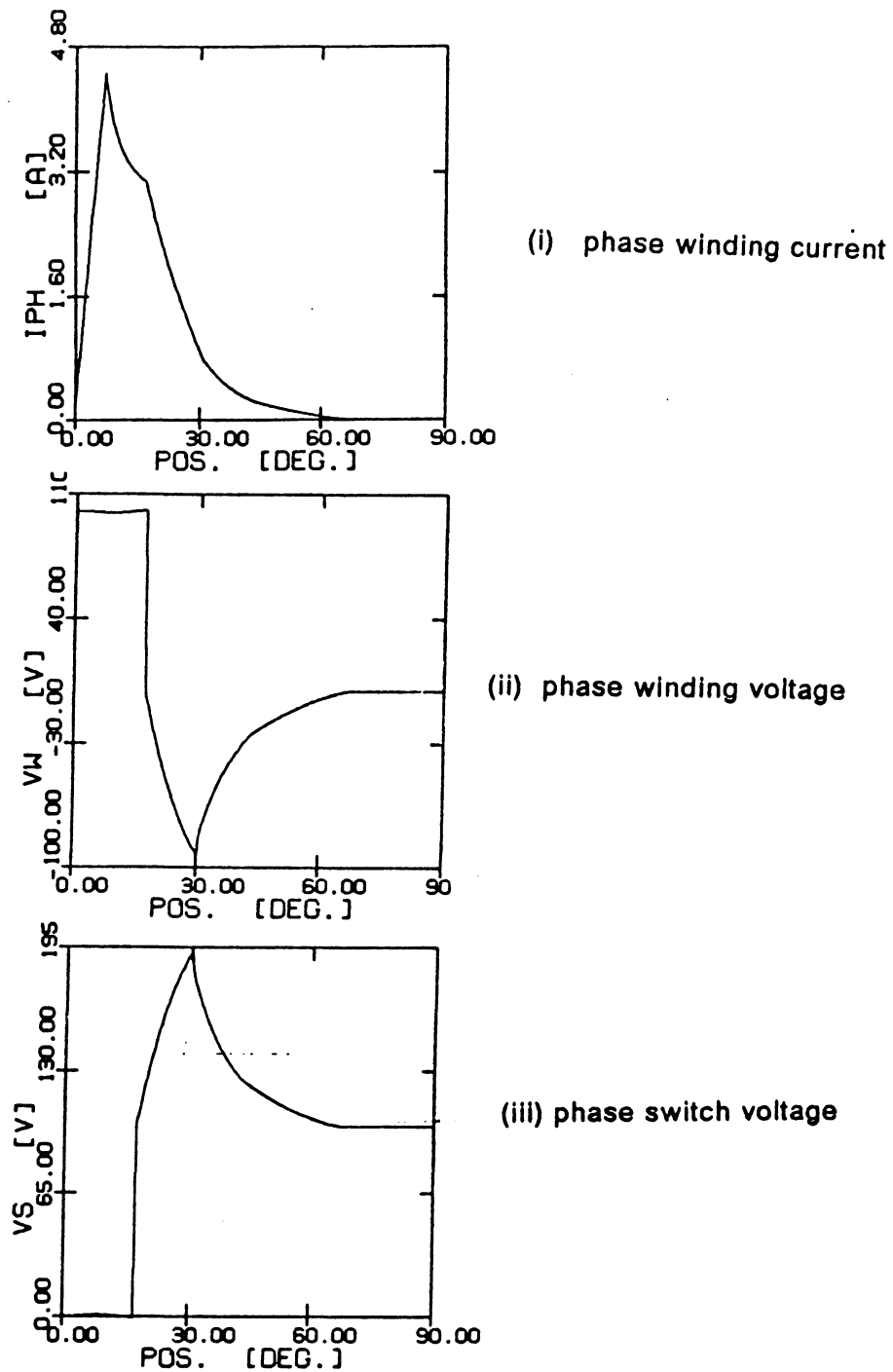
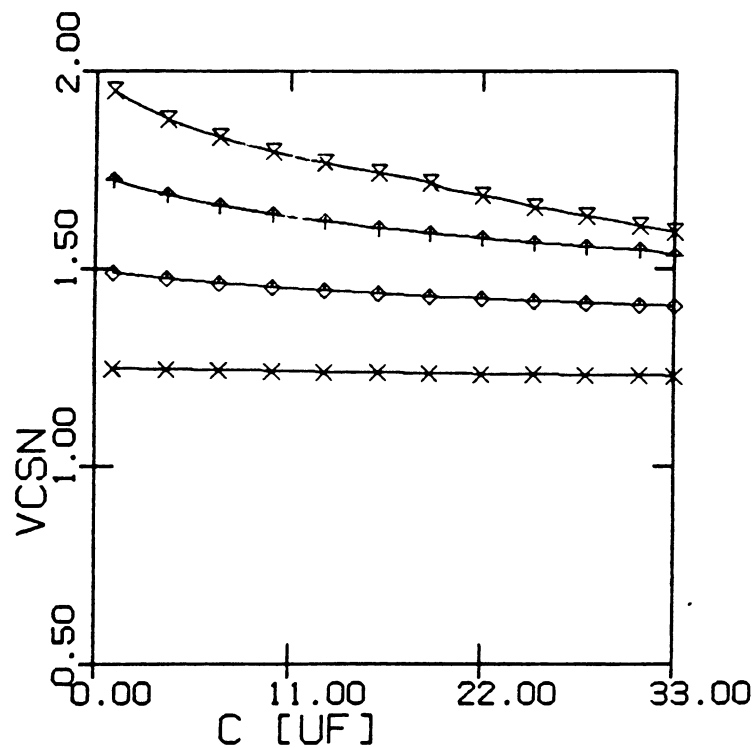


Figure 88. Simulated waveforms for $R_d = 100$ ohms with a $16 \mu\text{F}$ snubber capacitor



- × RD = 25 OHM
- ◇ RD = 50 OHM
- ⤴ RD = 75 OHM
- ⌵ RD = 100 OHM

Figure 89. Measured and predicted peak capacitor voltage as a function of capacitor and resistor values

Table 10. Peak capacitor voltage versus capacitor value; $R_d = 100$ ohms

C [UF]	0	1	4	10	13	20
VCN (measured)	> 4.0	2.15	2.02	1.85	1.80	1.75
VCN (predicted)	2.0	1.95	1.88	1.80	1.77	1.40

degrees and the cut-off point is 17 degrees prior to full alignment. It is observed that the use of a large snubber capacitor allows for the reduction of the dump resistor value without a severe penalty on the voltage stress across the switch. The difference between the measured and predicted voltages is due to the ringing effect caused by lead inductances and the effect of the motional e.m.f. which has been neglected in the derivation of the equations. If the cut-off point is well in advance of full overlap, the rate of change of inductance is considerable and therefore the motional e.m.f. component becomes significant.

7.6 Conclusion

The analysis and design of a minimum switch converter suitable for low performance switched reluctance motor drives has been described with particular attention to the ratings of the power semiconductor devices. The converter is inexpensive and compact. This is achieved by dumping a part of the stored energy in a resistor and thereby eliminating the extra circuitry required for regenerating into the d.c link. Both the analysis and measurements show that the voltage ratings of the switches and diodes and the r.m.s. current rating of the diodes are critically dependent on the value of the dump resistor. The performance of the drive is also dependent on this value due to the dependence of the current fall time on the resistor value. A very low value of the dump resistor results in not only a high current rating of the freewheeling diodes but also the production of significant negative torque. On the other hand, a large value of the resistor results in high voltage stresses on the switches and diodes. Therefore both the device voltage ratings and the current fall time are critical pa-

rameters in determining the value of the dump resistor. The use of a single capacitive snubber significantly reduces the voltage stress on the switches. The exact value to be used depends on the forward blocking voltage of the phase switches and the value of the dump resistor. It is believed that this converter topology is ideal for hp drive systems where cost may be a major criterion in the use of variable speed drives. The switch ratings can be minimized by coordinating the design of the switched reluctance motor in terms of the proper choice of inductance and its profile.

8.0 ANALYSIS AND DESIGN OF A NEW CONVERTER TOPOLOGY FOR SWITCHED RELUCTANCE MOTOR DRIVES

The emergence of new applications for switched reluctance motor drives (SRMD's) has created the need for energy efficiency maximization. This chapter proposes a new, energy efficient SRMD converter configuration. Unlike all currently known configurations, the proposed converter eliminates the need for a regenerative brake. Energy stored in a previously excited winding is recycled into the winding of the phase which is to be excited next. Since the rate of change of inductance with rotor position is positive in both phases during this period, the recycled energy is directly converted into useful mechanical energy (positive torque during forward motoring). Closed form expressions for determining device stresses and ratings at any operating point are given. Steady-state analysis to obtain key waveforms of the SRMD fed from the proposed converter is described and simulation results are verified and compared with experimental results.

8.1 Introduction

In most applications, the SRMD is usually part of a larger system. There is therefore the need to maximize energy efficiency in the sub-systems so as to improve the overall system efficiency. The efficiency of the drive is very dependent on the converter configuration. Converter configurations are mainly distinguished by the way the stored energy is managed which impacts the drive efficiency and cost. In most of the currently existing converter configurations, the stored energy is regenerated into the d.c. link. This requires extra circuitry and does not ensure the full utilization of the recoverable energy. In some cases, part of the stored energy is dissipated as heat resulting in poor energy utilization. In battery powered applications, regeneration of the stored energy into the source is complicated and costly due to the extra circuitry required for the control of the charging current.

The proposed converter ensures maximum energy utilization by recycling the energy stored in a previously excited phase to the next phase. Figure 90 shows the circuit diagram of the converter for three phases.

Each phase consists of one main switch, one freewheeling switch and two diodes; similar to the standard two-switch-per-phase (TSPP) topology. Taking phase A for example, T_1 , D_1 , D_{ac} and T_{ab} constitute a phase unit. Unlike the TSPP converter, the freewheeling diodes in the proposed converter need not be of the fast recovery type resulting in a considerable saving in device costs. This chapter deals with the analysis and design of the converter inclusive of the input filter. Closed form expressions for determining the power semiconductor device ratings are given. Steady-state

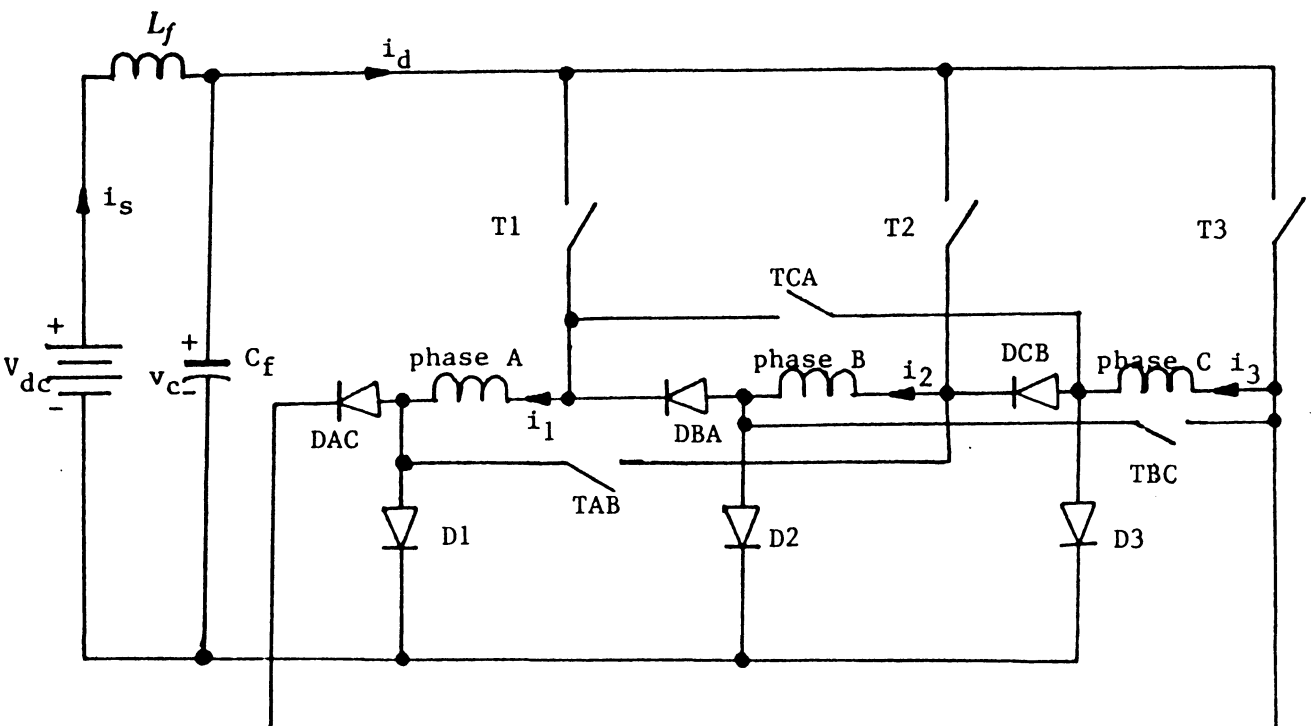


Figure 90. The Interphase Recycling Converter (three phase)

waveforms of the key parameters of the drive are also given and supported by experimental measurements.

8.2 Principle of operation

When a phase switch, say T1 is turned on, the d.c. link voltage is applied to the winding of phase A through switch T1 and diode D1. Energy is therefore transferred from the source to the motor. In this period, diodes D_{ba} and D_{ac} are reverse biased. The freewheeling switch T_{ab} is maintained in the off-state. When T1 is turned off, T_{ab} is simultaneously turned on and takes over the current. T_{ab} can also be turned on prior to the turn-off of T1. With T1 and D1 in the off-state, the current freewheels through the windings of phase A and B via switch T_{ab} and diode D_{ba} . The energy stored in the winding of phase A is thus partly transferred to phase B and converted into mechanical energy. The rest of the energy is dissipated in the winding and switch resistances. In steady-state operation, the inductance slope for phases A and B are positive in this period provided that turn-off of T1 is executed sufficiently in advance of the full alignment position. The current therefore produces positive torque in both phases. If T_{ab} is a device with a body diode (e.g. MOSFET), a blocking diode has to be connected in series with it so as to avoid a short-circuit of the d.c. supply when T2 is turned on.

8.3 Analysis

In one switching period, each phase encounters two main modes of operation. Considering phase A for example, the two modes are:

- i) **Mode 1: - T_1 and T_{ca} on.** Energy transferred from the source to the motor (phase A) while the energy stored in the previously excited phase (phase C) is recycled (to phase A).
- ii) **Mode 2: - T_1 off, T_{ab} on.** Energy stored (in phase A) is recycled (to phase B).

The analytical equations describing the two modes of operation are derived for one phase (phase A). The state variables of interest are i_1 , i_2 , i_3 representing the currents in phases A, B and C, respectively, and i_s and v_c representing the source current and input filter capacitor voltage, respectively.

8.3.1 Mode equations

The mode equations are developed on the assumption that steady-state conditions already exist. The simulation begins when switch T_1 is turned on.

8.3.1.1 Mode 1: T1 and

T_{ca} ON When T1 is turned on, the d.c. link voltage is applied to the winding of phase A. In the meantime, the stored energy in phase C circulates through phases C and A via switch T_{ca} and diode D_{ac} . Furthermore, energy stored in phase B during the previous stroke continues to recycle into phase C via switch T_{bc} . The state equations during this period are:

$$\frac{di_s}{dt} = \frac{V_{dc} - v_c}{L_f} \quad (8.1)$$

$$\frac{dv_c}{dt} = \frac{i_s - i_1}{C_f} \quad (8.2)$$

$$\frac{di_1}{dt} = \left[\frac{v_c - i_1(R_w + R_s + k_1) - V_d}{L_1} \right] \quad (8.3)$$

$$\frac{di_2}{dt} = - \left[\frac{(k_2 + k_3 2 R_w + R_s) i_2 + V_d}{L_2 + L_3} \right] \quad (8.4)$$

$$\frac{di_3}{dt} = - \left[\frac{i_3 \{k_1 + k_3 + 2 R_w + R_s\} + 2 V_d}{L_1 + L_3} \right] \quad (8.5)$$

where V_{dc} is the d.c link voltage, L_f and C_f are the input filter inductance and capacitance, respectively, L_1 , L_2 and L_3 are the phase inductances for phases A, B and C, respectively, R_w and R_s are the winding and switch on-time resistances, respectively, V_d is the forward voltage drop across a diode and k_1 , k_2 and k_3 are the rates of change of inductance as a function of time for phases A, B and C, respectively. This mode ends when switch T1 is turned off.

8.3.1.2 Mode 2: T1 off,

T_{ab} on The energy recycling period, begins when switch T1 is turned off. To facilitate the transfer of the stored energy from phase A to phase B, switch T_{ab} must be turned on prior to, or at the instant switch T1 is turned off. The length of the energy recycling period depends on the winding inductance profile and resistance per phase. The state equations during this period assuming that the recycling period is completed before the next phase switch (T2) is turned on are given by

$$\frac{di_s}{dt} = \frac{V_{dc} - v_c}{L_f} \quad (8.6)$$

$$\frac{dv_c}{dt} = -\frac{i_s}{C_f} \quad (8.7)$$

$$\frac{di_1}{dt} = -\left[\frac{i_1 \{k_1 + k_2 + 2R_w + R_s\} + 2V_d}{L_1 + L_2} \right] \quad (8.8)$$

$$\frac{di_2}{dt} = -\frac{di_1}{dt} \quad (8.9)$$

$$\frac{di_3}{dt} = -\left[\frac{i_3 (2R_w + R_s + k_2 + k_3) + V_d}{L_2 + L_3} \right] \quad (8.10)$$

This mode ends when the stored energy is depleted. Switch T_{ab} has to be turned off before T2 is turned on. The current diverts into the loop which includes the windings of phases A, B and C and diodes D_{ac} , D_{cb} and D_{ba} . The high impedance of this path is the main cause of the circulating current observed in both the simulated and measured results. A similar set of equations can be derived for each of phases B and

C. The above equations can be expressed in terms of the rotor position by substituting:

$$dt = \frac{d\theta}{\omega_m} \quad (8.11)$$

The steady state waveforms of the state variables and the derived parameters is obtained by solving the state equations as described in chapter 4.

8.4 Design equations

Figure 91 shows the worst-case current waveforms in the power semiconductor devices for one phase.

It is assumed that the desired peak on-time current of the drive is determined based on the desired output power and the rated d.c. voltage. The following sections describe the determination of the r.m.s. current ratings for the various devices shown in Figure 90. For convenience, switches T1, T2 and T3 and diodes D1, D2 and D3 are hereafter collectively referred to as the main switches and main diodes, respectively, whereas switches T_{ab} , T_{bc} and T_{ca} and diodes D_{ac} , D_{ba} and D_{cb} are referred to as the recycling switches and diodes, respectively.

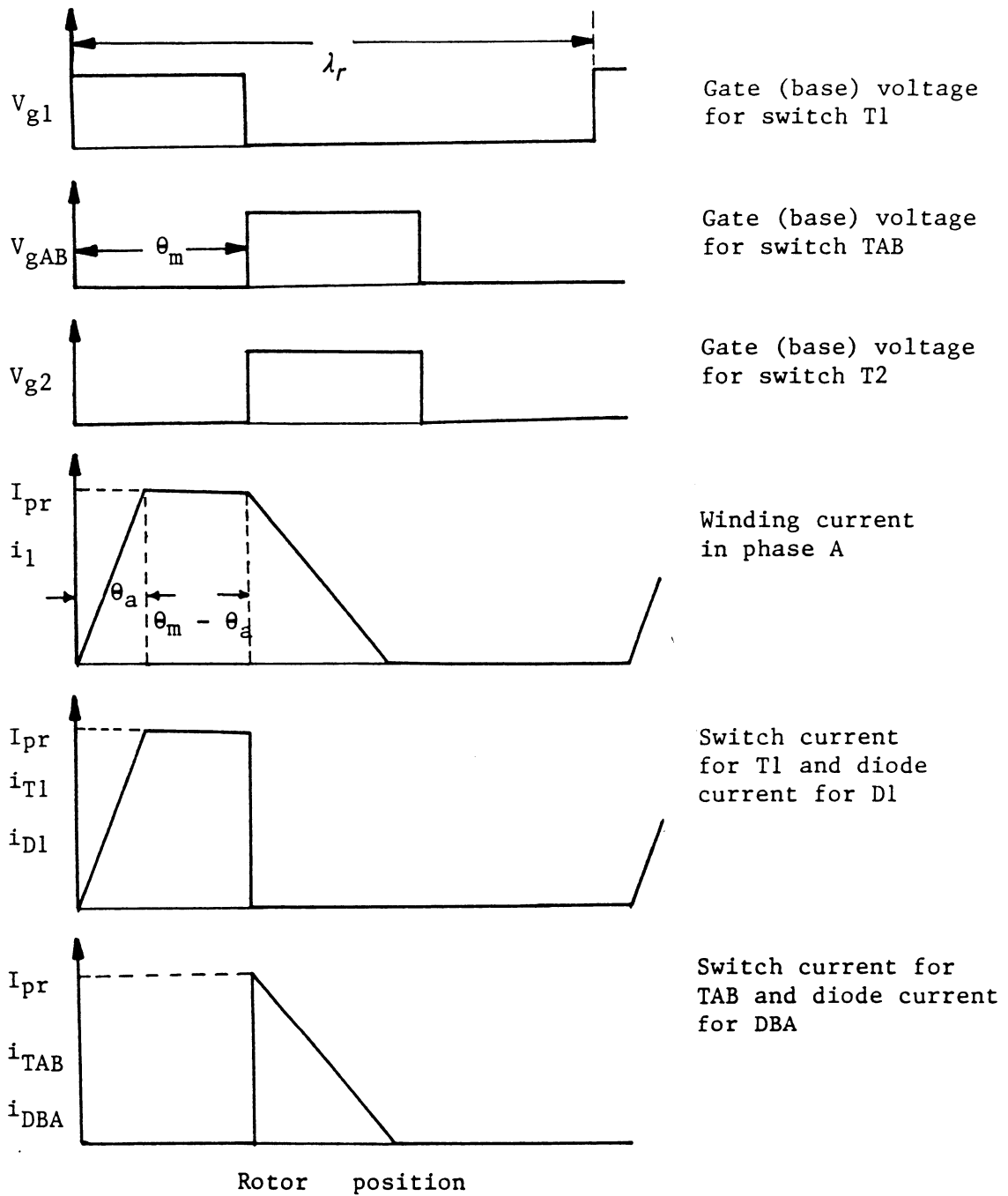


Figure 91. Simplified worst-case waveforms

8.4.1 The main switches and diodes

The main switches and diodes conduct during the period when energy is being transferred from the source to load. Their peak current rating is therefore equal to the rated on-time phase current of the drive. The r.m.s. current rating of these devices depends on the number of stator and rotor poles. Referring to Figure 91, the expression for the r.m.s. current of the phase switch is given by:

$$I_{sr} = \sqrt{\frac{P_r}{2\pi} \left[\int_0^{\theta_a} \left\{ \frac{I_{pr}}{\theta_a} \theta \right\}^2 d\theta + \int_{\theta_a}^{\theta_m} I_{pr}^2 d\theta \right]} \quad (8.12)$$

where I_{pr} is the rated on-time phase current, P_r is the number of rotor poles, θ_m is the maximum on-time period in radians without conduction overlap between the phases and I_{sr} is the rated r.m.s. current of the main switches and diodes. The maximum conduction period θ_m is given by

$$\theta_m = \frac{4\pi}{P_s P_r} \quad (8.13)$$

After evaluating the integral and substituting the expression for θ_m given in equation (8.13), the expression for the r.m.s. current becomes

$$I_{sr} = I_{pr} \left[\sqrt{\left[\frac{2}{P_s} - \frac{P_r \theta_a}{3\pi} \right]} \right] \quad (8.14)$$

In the off state, the main switch blocks a maximum voltage equal to the d.c. link voltage plus the winding voltage. Provided that the winding current is non-zero, the winding voltage has a maximum value equal to the d.c. link voltage plus a margin due to ringing. Therefore, the maximum voltage stress across a main switch equals

$2 V_{dc}$ plus a margin for the turn-off spike and ringing. The freewheeling switches must also have a forward blocking capability greater than twice the d.c. link voltage and a reverse blocking capability greater than V_{dc} . There is no specific requirement on the blocking capabilities of the main diodes since their cathodes are always maintained at zero potential. For the same reason, these diodes need not be of the fast recovery type.

8.4.2 The recycling switches and diodes

The voltage equation during the energy recycling period for one phase (phase A recycling into phase B) is given by

$$0 = L_e \frac{di_1}{d\theta} + i_1 \{ \omega_m k_e + R_e \} + V_d \quad (8.15)$$

where

$$L_e = L_1(\theta) + L_2(\theta) \quad (8.16)$$

$$R_e = 2 R_w + R_s \quad (8.17)$$

$$k_e = k_1(\theta) + k_2(\theta) \quad (8.18)$$

By applying Laplace transformation to equation (8.15), the expression for the instantaneous current per phase in the recycling devices is given by

$$i_r(\theta) = I_0 \exp - \left\{ \frac{\theta}{\omega_m \tau_e} \right\} \quad (8.19)$$

where I_0 is the winding current immediately after turn-off and τ_e is given by

$$\tau_e = \frac{L_e}{R_e + \omega_m k_e} \quad (8.20)$$

The current immediately after the turn off of the main switch equals the winding current immediately prior to turn off. The worst-case condition corresponds to turn off at the rated peak on-time phase current. Thus the expression for the instantaneous current per phase in the recycling devices is given by

$$i_r(\theta) = I_{pr} \exp - \left\{ \frac{\theta}{\omega_m \tau_e} \right\} \quad (8.21)$$

The time constant τ_e is critical to the performance of the drive. If τ_e is large, current will continue to flow during the negative inductance slope region resulting into the production of negative torque. The denominator of the expression for τ_e includes the rate of change of inductance with rotor position. Thus, the magnitude of the time constant depends on the turn-off point. Three cases are considered:

- i. turn off and recycling during partial overlap, positive slope
- ii. turn off and recycling during full overlap
- iii. turn off and recycling during partial overlap, negative slope

The expressions for τ_e corresponding to the three cases based on the simplified inductance profile are given by:

Case (i)

$$\tau_e = \frac{2 L_u + k \theta}{R_e + k} \quad (8.22)$$

Case (ii)

$$\tau_e = \frac{L_a + L_u}{R_e} \quad (8.23)$$

Case (iii)

$$\tau_e = \frac{L_a + L_u}{R_e + k_e} \quad (8.24)$$

where L_u is the per phase inductance value at the fully unaligned position, and k is the rate of change of inductance with rotor position. It is observed that case (iii) gives the minimum value of τ_e . However, operation in this mode is undesirable due to the resulting negative torque. Case (ii) is therefore the practical expression for predicting the current fall time.

Once the value of τ_e is known, the r.m.s. current rating of the recycling devices is obtained by integrating the square of the expression for current given in equation (8.21). Thus, assuming that the current falls to zero within four time constants, the rated r.m.s. current of the recycling devices is given by:

$$I_{rr} = \sqrt{\left[\frac{P_r}{2\pi} \int_0^{4\omega_m \tau_e} \{I_{pr} \exp - (\frac{\theta}{\omega_m \tau_e})\}^2 d\theta \right]} \quad (8.25)$$

After evaluating the integral, the expression for the rated r.m.s. current of the recycling devices becomes

$$I_{rr} = I_{pr} \sqrt{\frac{\tau_e \omega_m P_r}{4 \pi}} \quad (8.26)$$

By substituting the expression for τ_e given in equation (8.23), equation (8.26) becomes:

$$I_{rr} = I_{pr} \sqrt{\left[\frac{\omega_m (L_a + L_u)}{R_e} \frac{P_r}{4 \pi} \right]} \quad (8.27)$$

The forward blocking voltage of the recycling switches has to be equal to that of the main switches as stated earlier. The reverse blocking voltage has to be greater than twice the rated d.c. link voltage. If this is not the case, a short circuit of the supply will occur when the main switch of the next phase is turned on. Therefore, if switches with a reverse diode such as MOSFET's are used, a fast recovery diode with a peak reverse voltage (PRV) rating greater than $2V_{dc}$ has to be connected in series with the recycling switch of each phase. The recycling diodes must also have a PRV rating greater than $2V_{dc}$.

8.5 Results

Figure 92 shows simulated waveforms for the phase current and voltage based on a 6/4 pole prototype SRM for a conduction period of 24° , an advance angle of 12° , a d.c. bus voltage of 300 V and a peak on-time current of 10 A.

The simulated conditions correspond to the rated operating conditions of the motor under the assumption that the on-time current is constant. When phase A is de-energized (T1 turned off), current begins to rise in phase B as the current in phase

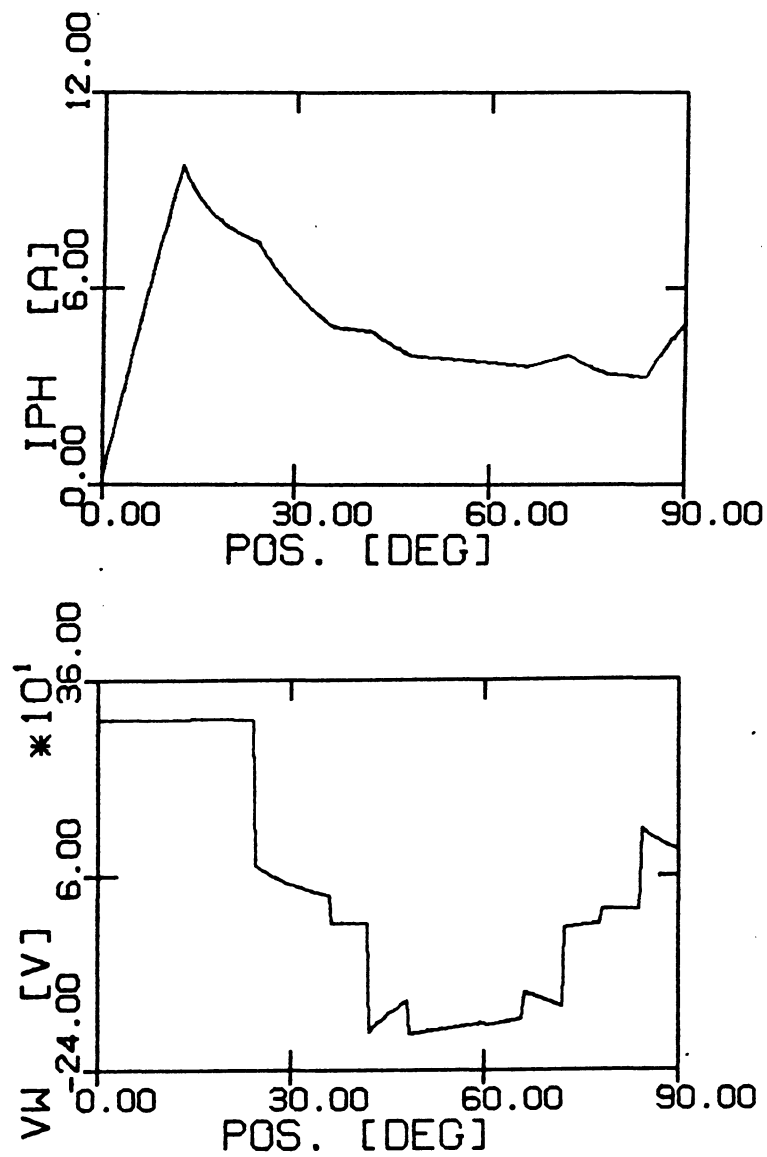


Figure 92. Simulated waveforms at the rated conditions

A falls. For the simulated switching conditions, there is a delay between the turn-off of T1 and the turn-on of T2. Thus, prior to or as soon as T1 is turned off, T_{ab} is turned on so that the energy stored in A freewheels into phase B. When T3 is turned off, the stored energy is recycled to phase A causing the current in this phase to rise as observed. The maximum amplitude of this current depends on the amount of stored energy and the inductances of phases A and C during the recycling period. It is also observed that there is a continuous flow of current in the phase windings at all times. The magnitude of this current has been observed to increase in direct proportion to the d.c. bus voltage and the phase current prior to commutation. From the simulated waveforms, the minimum value of the circulating current at the rated conditions is 33 % of the peak value. The negative torque resulting from this current is a limitation to the range of application of the proposed configuration. The effect of the circulating current is considerably reduced for this particular prototype motor when a resistor is introduced in the recycling path of each phase. As shown in equations (8.22) to (8.24), additional resistance in the recycling path accelerates the rate of decay of the phase current. Figures 93 and 94 show the predicted and measured waveforms, respectively, for a conduction angle of 30 degrees, an advance angle of 14 degrees and a d.c. bus voltage of 50 V when a 33 ohms resistor is introduced in the freewheeling path.

VS1 and VS2 represent the voltage across the main and recycling switches for phase 1, respectively, whereas I_{PH}, V_{W1}, I_D and I_S represent the phase current, phase winding voltage, d.c. link current and source current, respectively. The circulating current in this case is less than 10 % of the peak throughout the off-time. In this case, the recycling switch of a previously excited phase and the main switch of the succeeding phase are turned on and off simultaneously. It is observed that the circulat-

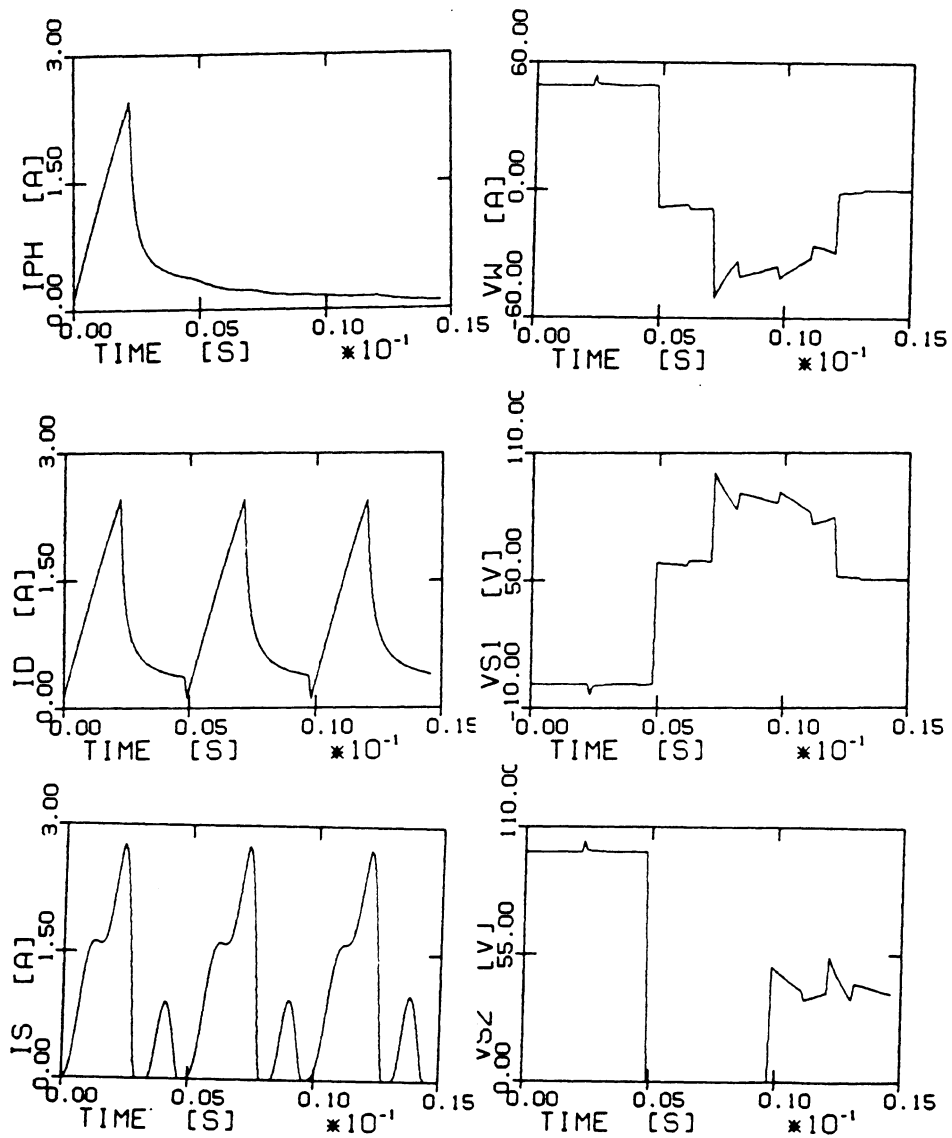


Figure 93. Simulated waveforms with a 33 ohm resistor in the recycling path

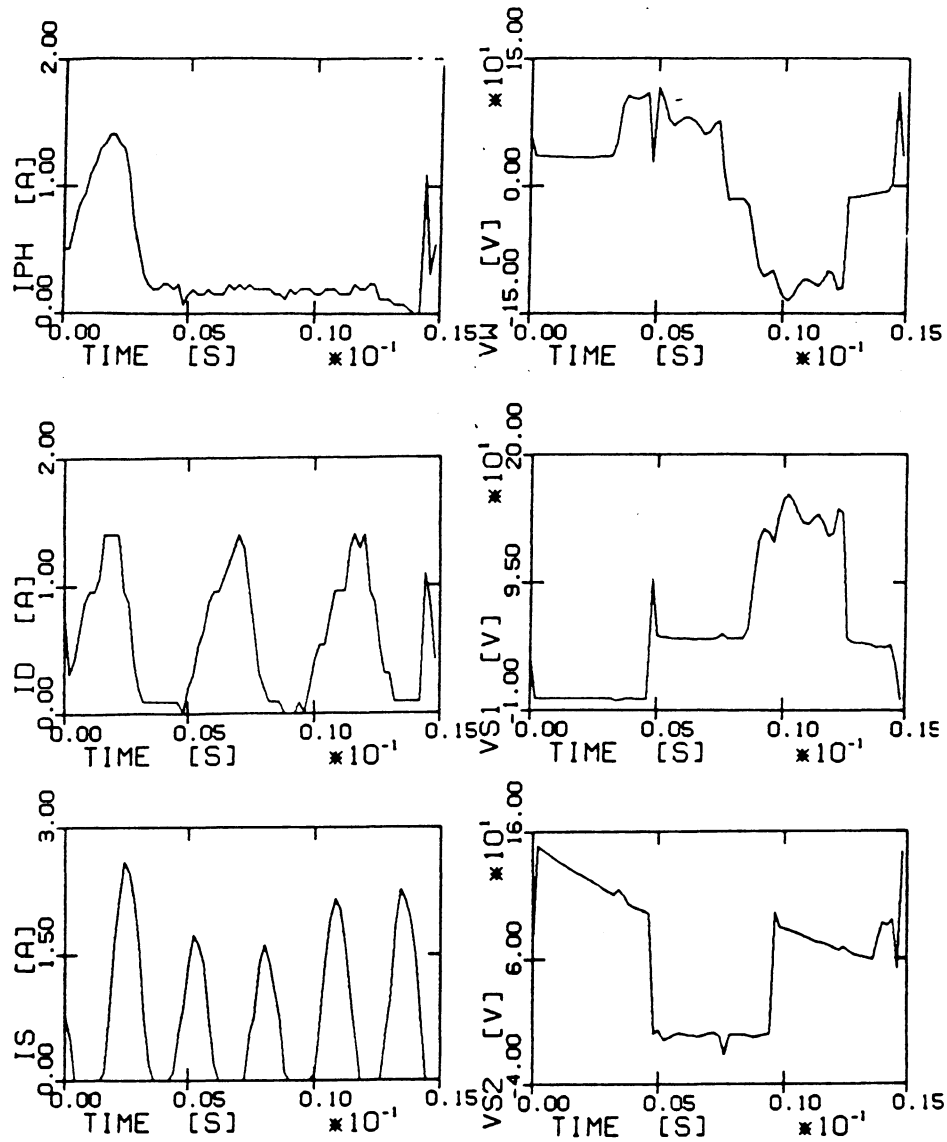


Figure 94. Measured waveforms with a 33 ohm resistor in the recycling path

ing current is considerably lower resulting in significantly higher positive torque. A motor speed of 1013 r.p.m. was obtained at a d.c. link supply voltage of 50 V.

The voltage across a main switch rises to about three times the d.c. link voltage when the recycling switch of the corresponding switch is opened. The same phenomenon is observed for the recycling switch. The sudden voltage rise is due to the high di/dt caused by the interruption of the circulating current path. The magnitude of this voltage is proportional to the magnitude of the circulating current. The effect can therefore be minimized by coordinating the motor and converter design so that the aligned inductance is minimized.

Compared to the two-switch-per-phase (TSPP) converter, the proposed converter has the advantage that only half of the switches carry the rated current. The r.m.s. current rating of the other switches is lower and is approximately equal to that of the free-wheeling diodes of the TSPP converter. The voltage rating is however twice that of the TSPP converter. A comparison of the main features of the two converters is summarized in Table 11.

8.6 Conclusion

A new, energy efficient SRMD converter configuration has been proposed. The configuration has the advantage that the energy stored in a winding is directly recycled to the succeeding phase instead of regenerating to the d.c. link or dumping it in a resistor. Closed form expressions to determine device stresses for the proposed

Table 11. Comparison of the Interphase Recycling and Two-switch-per-phase (TSPP) converters

Item	TSPP Converter	Interphase Converter
# of switches	P_s	P_s
# of power diodes	P_s	P_s
# of leads (minimum)	P_s	P_s
# of snubber circuits (minimum)	P_s	P_s
Switch voltage rating	$V_{dc} + \Delta V$	$2V_{dc} + \Delta V$
Phase switch r.m.s. current (top switch)	$I_{pr} \sqrt{\left[\frac{2}{P_s} \right]}$	$I_{pr} \sqrt{\left[\frac{2}{P_s} \right]}$
Phase switch r.m.s. current (bottom switch)	$I_{pr} \sqrt{\left[\frac{2}{P_s} \right]}$	$I_{pr} \sqrt{\left[\frac{P_r \omega_m (L_a + L_u)}{4\pi R_e} \right]}$
Possible # of phases	Any	Any
Phase independence	Full	Partial

converter configuration have been derived. A complete steady-state analysis of the drive fed from this converter is given and supported by experimental verification. The concept of interphase energy recycling has a great potential in applications where compactness and energy efficiency are major considerations. It is noted that the design of the switched reluctance motor has to be coordinated with the converter to minimize the aligned inductance. This will allow the fast decay of current in the phase winding when the succeeding phase is energized.

9.0 SENSITIVITY OF CONVERTER RATINGS TO SRM POLE GEOMETRY

In this chapter, the effects of the SRM pole geometry on the converter ratings is studied. It is shown that the converter ratings and the pole geometry are linked through the motor inductances. The study therefore begins by examining the effect of the pole geometry on the motor inductances and applies this information to determine the effect on the ratings of the converter power semiconductor devices. Using simplified inductance and current waveforms, the effects of the pole arc ratio and the pole arc-to-pole pitch ratio on the r.m.s. current ratings of the switches and diodes is investigated from the constant power and constant torque points of view. The 6/4 pole prototype SRM is used for illustration. Normalized curves showing the change in the converter ratings as the pole geometry is varied are given.

9.1 Introduction

The performance of a switched reluctance motor is critically dependent on the non-linear properties of the magnetic material used for the core and the geometry of the motor. Due to this non-linearity, both the current and voltage waveforms are non-sinusoidal, the shape and magnitudes being dependent on the saturation level of the motor. Thus, from the viewpoint of the converter, the ratings of the power semiconductor devices also depend on the nonlinear characteristics of the motor and its geometry. This chapter examines the sensitivity of the converter ratings to the motor parameters. In particular, the effects of rotor-to-stator pole arc ratio and the pole arc-to-pole pitch ratio on the peak and r.m.s current ratings of the power semiconductor devices are examined.

Apart from power output considerations, the design of a SRM drive requires that:

- i. the inductance profile be such as to allow the rated on-time current to be switched into the winding and maintained at a constant value throughout the conduction period
- ii. the rate of change of inductance with rotor position at the rated current be large enough to produce the desired output torque.

In a recent study, it was shown that the the output of the motor is very dependent on the pole arc-to-pole pitch ratio [67]. Any effect on the motor has consequences on the converter. Optimal design of the converter is particularly important because the

converter constitutes a very significant proportion of the overall cost of the drive. The effect of the motor geometry on the converter is conveyed through the inductance. When a phase winding is excited from a constant d.c. source, the magnitude and waveform of the current is dependent on the winding inductance. As will be shown in the following sections, the inductance is a function of various motor parameters. At constant power, the D^2L product is fixed and so is the electric loading. The peak on-time current is also fixed if the d.c. supply is specified. Under these conditions, the minimum stator pole height is also fixed. However, the magnetic loading is largely dependent on the effective air-gap which, in turn depends on the pole geometry. It has been shown that for maximum torque production, the pole arc to pole pitch ratio should be between 0.35 and 0.45 [67]. In this study, these findings are extended to include the sensitivity of the converter ratings to both the pole arc ratio and the pole-arc-to-pole-pitch ratio. First, the equations for the current ratings of the converter devices are reviewed showing their dependence on the motor inductances. The variation of the motor inductances with the pole arc ratio is then determined for the range between 0.6 and 2.0 in steps of 0.05. Similarly, the variation of inductance with the pole arc-to-pole-pitch ratio is determined for the range of 0.1 to 0.6 in steps of 0.02. The analytical method described in chapter 2 is used for the prediction of motor inductance. The inductance values are then substituted into the current equations. The results are summarized by normalized curves.

9.2 Converter equations

The expression for the r.m.s. current rating of a phase switch for the case where each phase switch serves only one phase winding is derived in chapter 7 and is given by

$$I_{sr} = I_{pr} \sqrt{\frac{2}{P_s} \left[1 - \left(\frac{\theta_a}{\theta_c} \right) \right]} \quad (9.1)$$

The time taken for the current to rise to the maximum value depends on several factors. It is a function of the inductance of the motor, the applied voltage and the speed of operation. Assuming that the current rises linearly to the peak value, the rise time t_r is given by

$$t_r = \frac{L(\theta) I_{pr}}{V_{dc} - [R_w + k(\theta) \omega_{mr}] I_{pr}} \quad (9.2)$$

where $L(\theta)$ and $k(\theta)$ are the inductance and rate of change of inductance as a function of rotor position at the point of maximum current, respectively, R_w is sum of the winding and switch resistances and V_{dc} is the rated d.c. bus voltage.

The rate of change of inductance based on the simplified inductance profile is given by

$$k(\theta) = \frac{L_a - L_u}{\beta_s} \quad (9.3)$$

where L_u and β_s are the minimum inductance per phase and the stator pole arc respectively. In this study, it is assumed that the turn-on point is advanced such that the peak on-time current is attained during the periods where the rate of change of

inductance is the minimum. The worst case occurs when switch-on is effected during the full overlap period as would be during forward braking. In that case, $L(\theta)$ equals the maximum inductance and $k(\theta) = 0$. The rise time is then given by

$$t_r = \frac{L_a I_{pr}}{V_{dc} - R_e I_{pr}} \quad (9.4)$$

The corresponding rating for the freewheeling diode depends on the method of energy recovery. In this study, the two-switch per phase (asymmetrical bridge) converter is assumed. The stored energy is therefore directly regenerated to the d.c. link. Assuming a linear fall of the current, the diode current rating is given by:

$$I_{dr} = I_{pr} \sqrt{\left[\frac{P_r \omega_{mr}}{2\pi} \int_0^{t_f} \left[1 - \frac{t}{t_f} \right]^2 dt \right]} \quad (9.5)$$

where t_f is time taken for the current to fall to zero after turn-off.

The current fall time is also a function of the winding inductance. Since the stored energy is regenerated into the d.c. link, -1 p.u. d.c. link voltage is applied to the winding during this period. Assuming that turn-off is effected during the maximum inductance region, the fall time when rated on-time current is switched off is given by:

$$t_f = \frac{I_{pr} L_a}{V_{dc}} \quad (9.6)$$

The r.m.s. current rating of the freewheeling diodes is therefore given by:

$$I_{dr} = I_{pr} \sqrt{\left[\frac{\omega_{mr} L_a I_{pr} P_r}{6\pi V_{dc}} \right]} \quad (9.7)$$

The preceding equations show that the relationship between the converter ratings and the motor geometry is largely based on the winding inductance, the number of poles and the pole arcs. The winding inductance is a function of the motor dimensions. This relationship is further examined in the following section.

9.3 Inductance and motor dimensions

The dependence of inductance on the motor geometry arises mainly due to the double salient structure of the motor. In the fully aligned position, the m.m.f. equation for a peak on-time current I_{pr} is given by

$$L_a I_{pr} = B A_g T_{ph} \quad (9.8)$$

where A_g is the cross-sectional area of the airgap given by:

$$A_g = D L \frac{\beta_s}{2} \quad (9.9)$$

B is the flux density in the airgap when the poles are fully aligned. D , L and T_{ph} are the bore diameter, length of the stator poles and the number of turns per phase, respectively. When equation (9.9) is substituted into equation (9.8), the m.m.f. equation becomes:

$$L_a I_{pr} = \frac{B D L T_{ph} \beta_s}{2} \quad (9.10)$$

Equation (9.10) provides an approximate method for predicting the maximum allowable inductance in the fully aligned position for a specified on-time current. In that case, the flux density is assumed to be equal to the allowable maximum value for the core material. However, errors arise when the pole arcs are varied over a wide range. Variation of the motor pole arc has a significant effect on the flux density. Since the flux density function is non-linear, the magnetization (B-H) characteristic of the core material has to be used in determining the sensitivity of the motor inductance to the pole geometry. The results of the study are described in the following sections.

9.4 *Simulation results*

9.4.1 Sensitivity to pole arc ratio

The dimensions and ratings of the prototype SRM were used to investigate the effect of the pole geometry on the converter ratings. The motor has a pole arc-to-pole pitch ratio of 0.4 in both the stator and rotor and a rotor-to-stator pole arc ratio of 1.5. Figures 95 to 97 show the sensitivity of the aligned inductance, switch r.m.s. current and diode r.m.s. current ratings, respectively, when the rotor-to-stator pole arc ratio is varied between 0.6 and 2.0 for three values of the rotor pole arc corresponding to pole arc-to-pole-pitch ratios of 0.2667, 0.333 and 0.4. The corresponding results when the stator pole arc is kept constant while the rotor pole arc is varied are given in Figures 98 to 100.

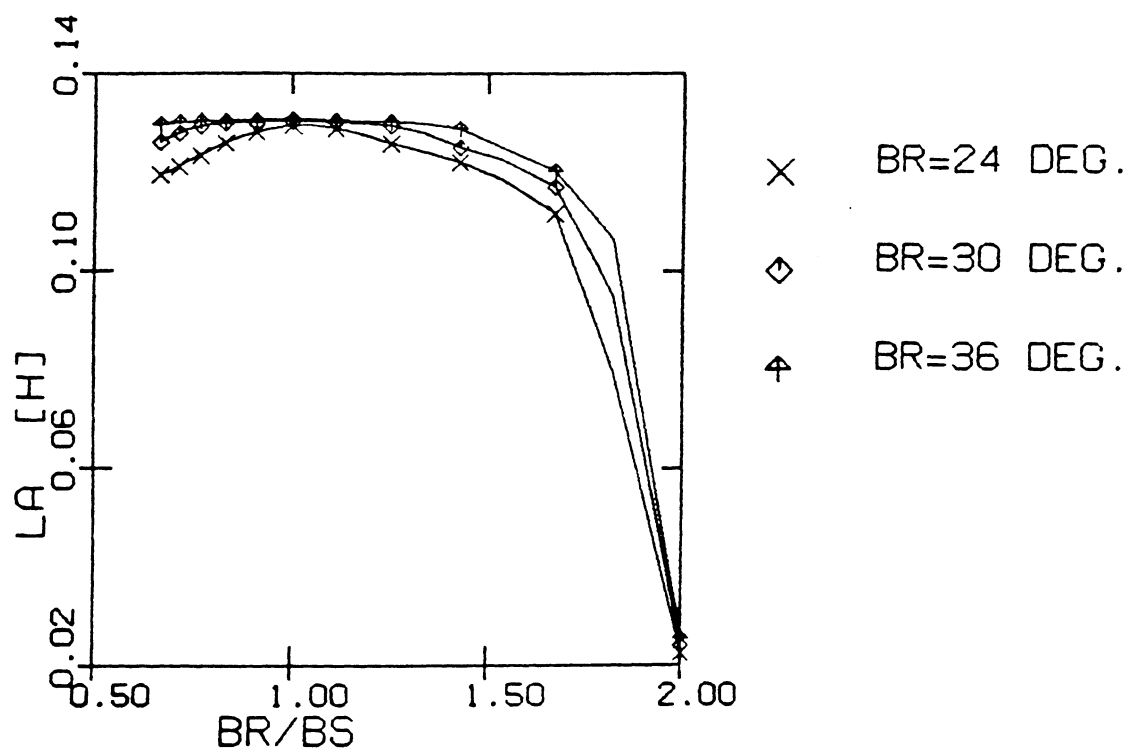


Figure 95. Sensitivity of aligned Inductance to pole arc ratio: β_r constant

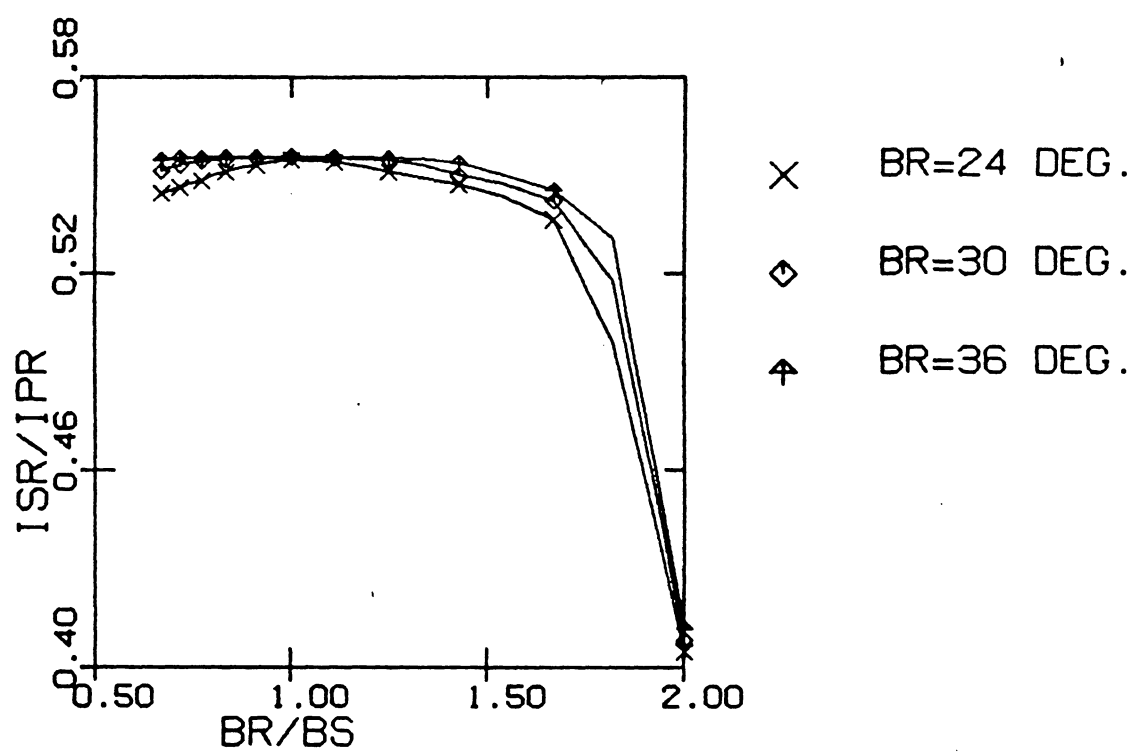


Figure 96. Sensitivity of switch r.m.s. current rating to pole arc ratio: β_r constant

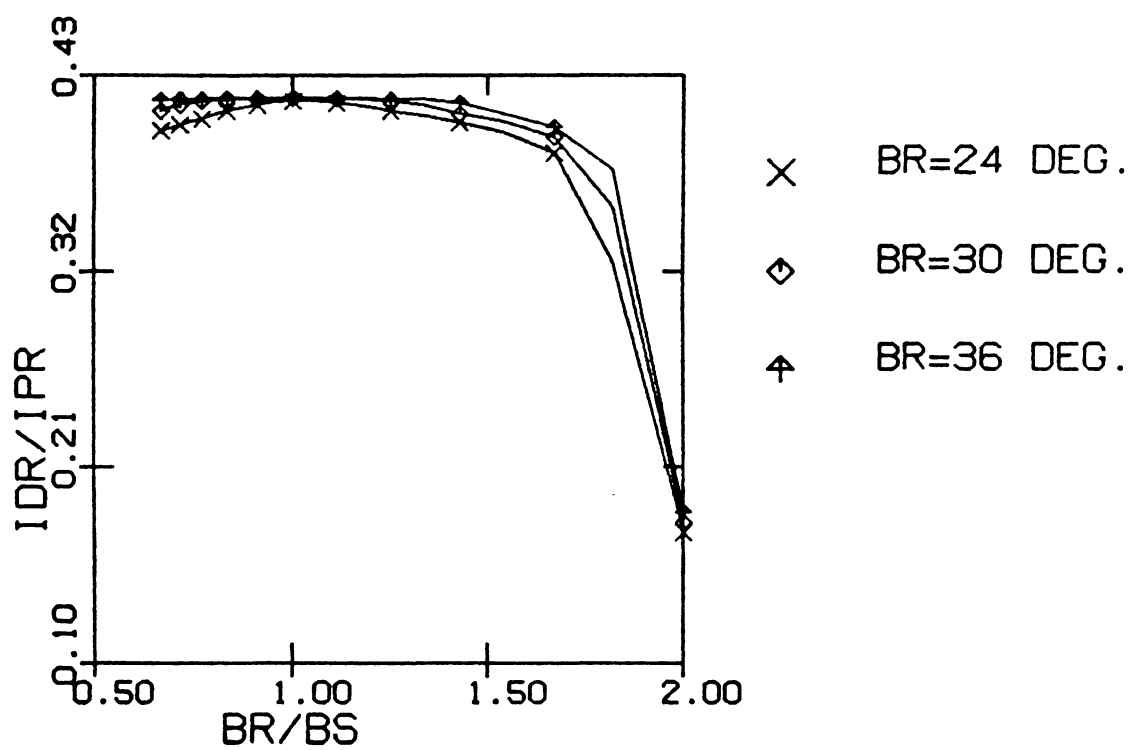


Figure 97. Sensitivity of diode r.m.s. current rating to pole arc ratio: β_r constant

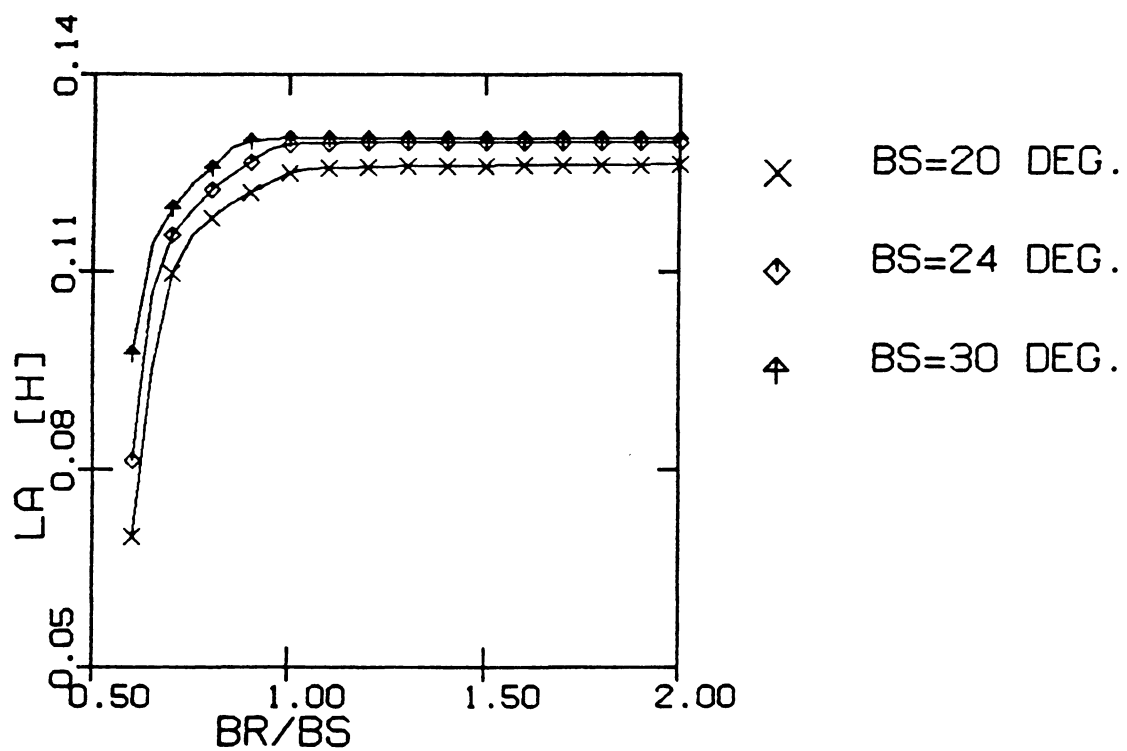


Figure 98. Sensitivity of aligned inductance to pole arc ratio: β_s constant

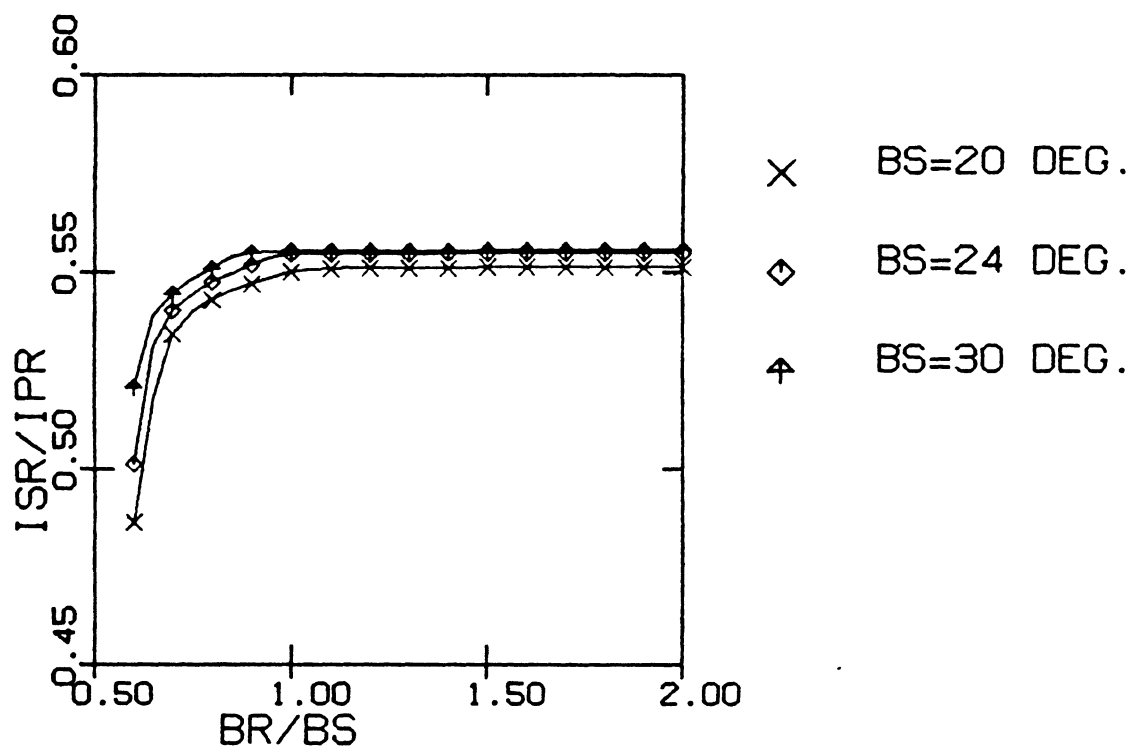


Figure 99. Sensitivity of switch r.m.s. current rating to pole arc ratio: β_s constant

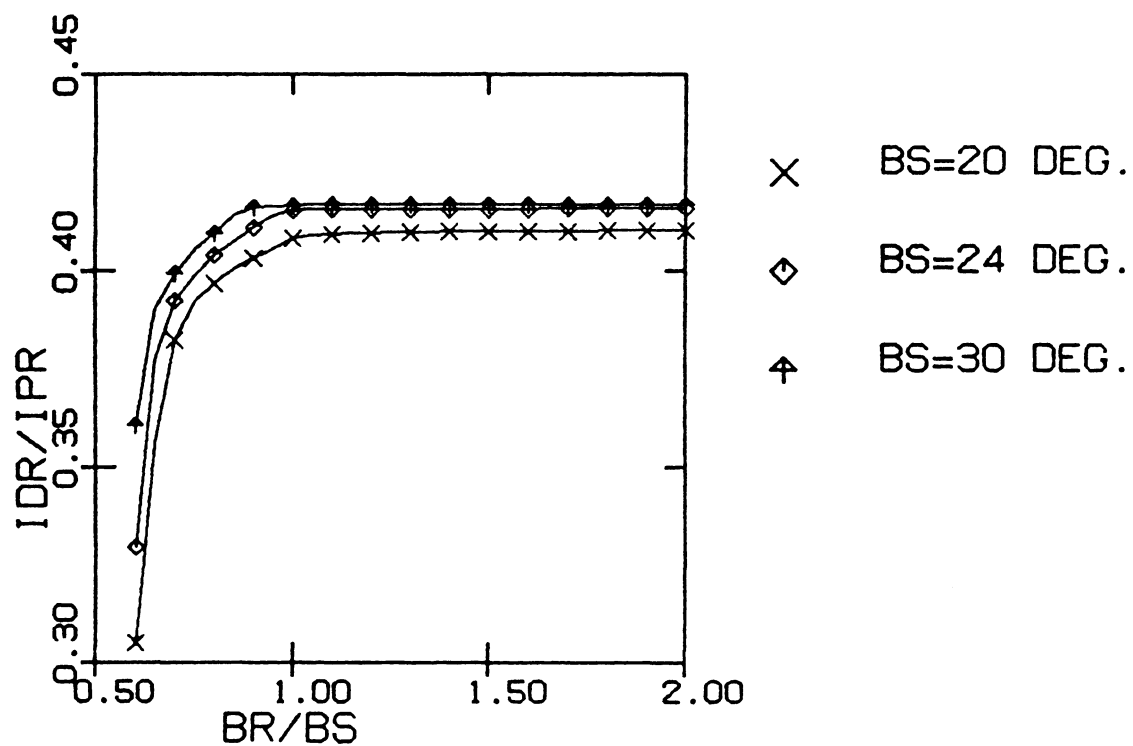


Figure 100. Sensitivity of diode r.m.s. current rating to pole arc ratio: β_s constant

I_{SR} , I_{DR} , and I_{PR} represent the rated switch r.m.s. current, diode r.m.s. current and switch on-time current, respectively, while L_A represents the inductance at the fully aligned position. B_R and B_S are the rotor and stator pole arcs, respectively. In both cases, it is observed that increasing the pole arc ratio beyond unity does not produce any significant effect on the current ratings of the power semiconductor devices. Comparing Figures 95 to 97 to Figures 98 to 100, one observes that the device ratings are more sensitive to variations in the stator pole arc. The observed fall in the r.m.s. current ratings of the devices at very low values of the stator pole arc are attributed to the relatively lower rise and fall times of current due to the reduced inductance. The relatively lower inductance is caused by the increased flux density in the stator poles which results in lower permeability of the iron core.

9.4.2 Sensitivity to the pole arc-to-pole pitch ratio

Figures 101 to 103 show the aligned inductance, switch r.m.s. current and diode r.m.s. current as a function of the stator pole arc-to-pole-pitch ratio. The ratio is varied between 0.1 and 0.6 in steps of 0.02 for three values of the rotor pole arc corresponding to rotor pole arc-to-pole pitch ratios of 0.267, 0.333 and 0.40.

T_S and T_R represent the stator and rotor pole pitches, respectively. It is observed that below a stator pole arc-to-pole pitch ratio of 0.4, the aligned inductance decreases dramatically irrespective of the value of the rotor pole arc. Further, provided that the rotor pole arc-to-pole pitch ratio is equal to or greater than 0.4, the device current ratings decrease irrespective of the stator pole arc-to-pole pitch ratio.

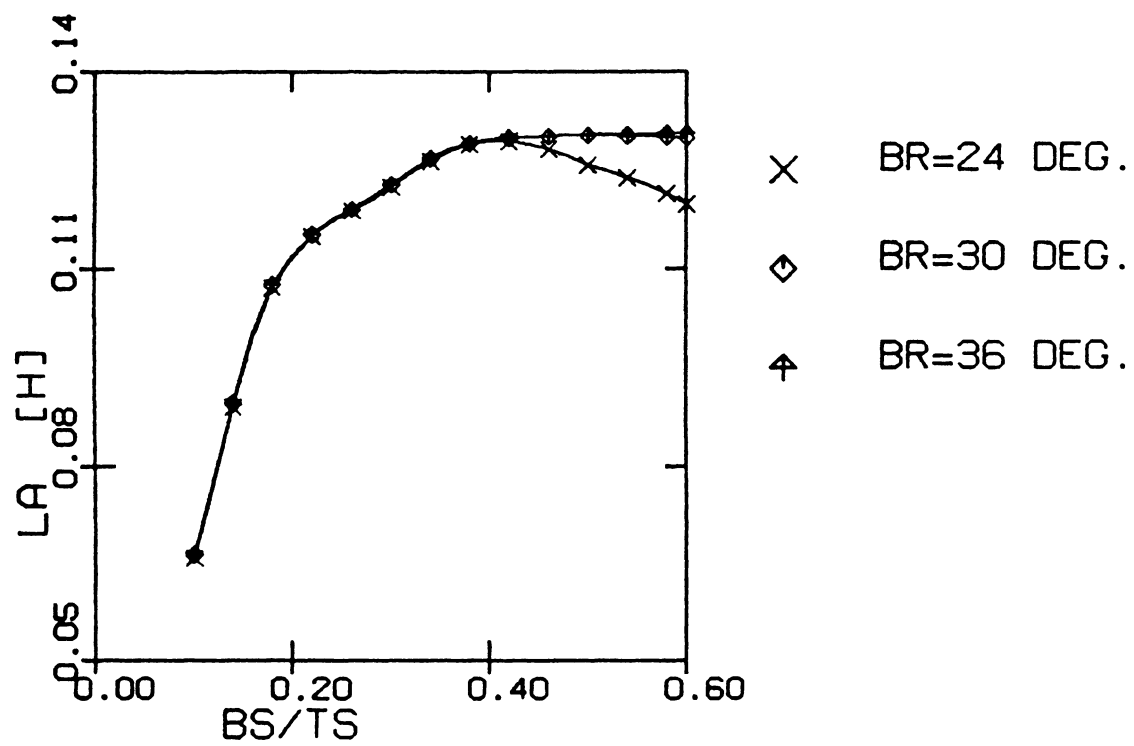


Figure 101. Sensitivity of aligned Inductance to the pole arc-to-pole pitch ratio

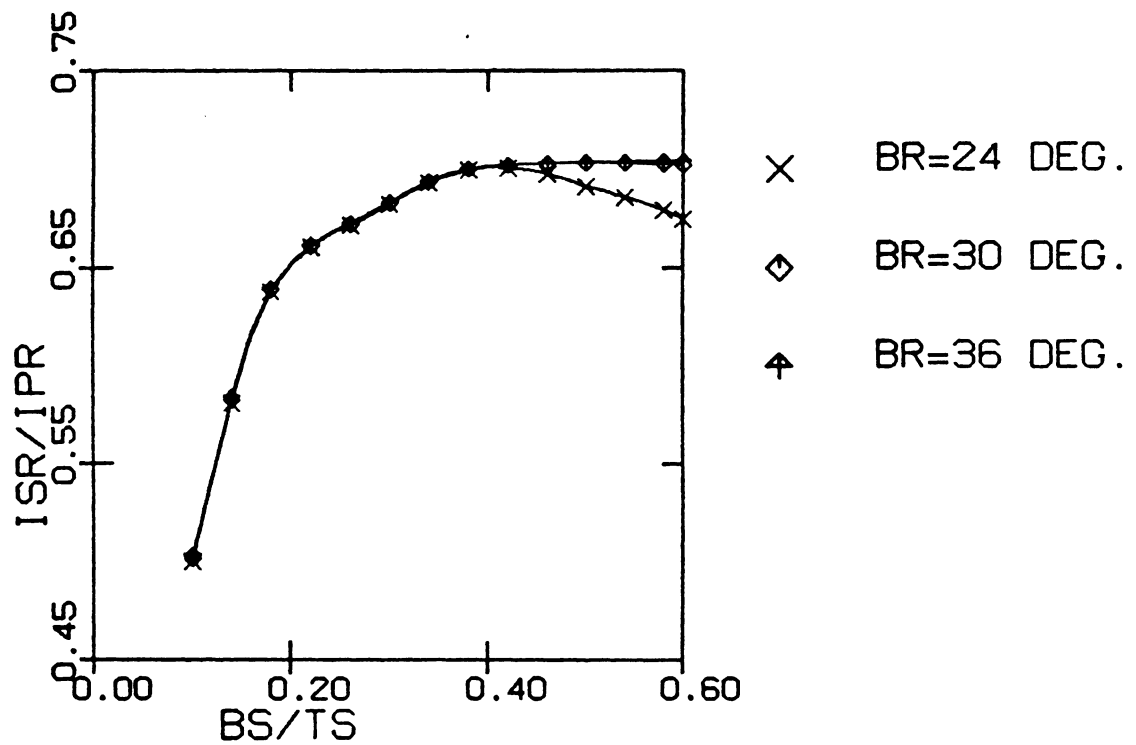


Figure 102. Sensitivity of switch r.m.s. current rating to the pole arc-to-pole pitch ratio

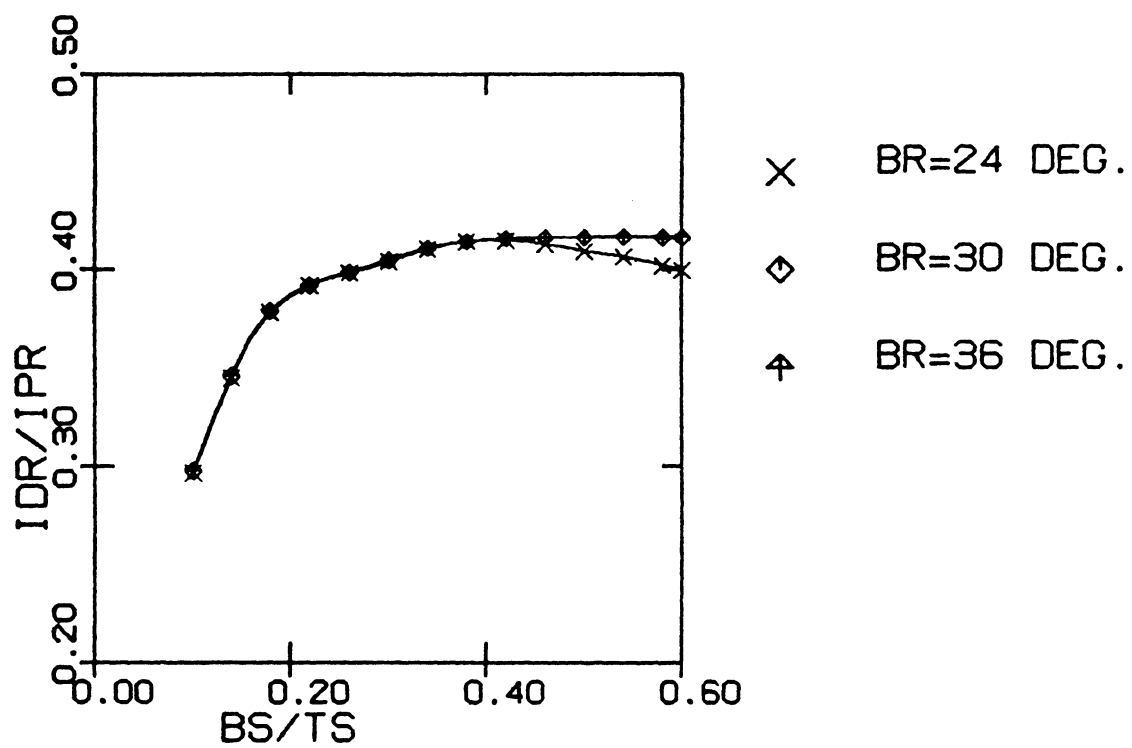


Figure 103. Sensitivity of diode r.m.s. current rating to the pole arc-to-pole pitch ratio

9.5 Conclusion

A theoretical study of the sensitivity of the converter ratings to the pole geometry of the SRM has been described. It is shown that the linkage between the motor geometry and the converter is through the motor inductance and the stator pole arc. From a constant power point of view, the converter ratings have been observed to be insensitive to the pole arc ratio when the rotor-to-stator pole arc ratio is between 1.0 and 1.25. When the stator pole arc-to-pole pitch ratio is taken as the variable, it is observed that if this ratio is greater than 0.35, the converter r.m.s. current ratings are insensitive to changes in the pole arc provided that the rotor pole arc-to-pole pitch ratio is equal to or greater than 0.33. At constant torque, the r.m.s. current rating of the converter switches is observed to increase dramatically as the stator pole arc increases. Thus, the results suggest that values of rotor-to-stator pole arc ratio much greater than unity result in unnecessary increase in converter ratings. In view of the fact that the torque output of the motor is also the maximum around unity pole arc ratio, it is recommended that in spite of other requirements (e.g. self-starting from any position), the rotor-to-stator pole arc ratio be between 1.0 and 1.25. The recommendation of a stator pole arc-to-pole pitch ratio between 0.35 and 0.5 suggested in relation to optimum motor output [67] has also been found to be equally applicable to the converter provided that the rotor pole arc-to-pole pitch ratio is between 0.33 and 0.4.

10.0 CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

10.1 *Conclusions*

The work described in this dissertation constitutes the following original contributions to research efforts on switched reluctance motor drives:

1. An improved method for the analytical prediction of SRM inductance and steady-state average torque has been developed and verified experimentally. The method is applicable at all rotor positions and levels of excitation.
2. Novel techniques for the measurement and instrumentation of SRM parameters have been developed. Results compare well with finite element analysis and measured results.

3. A novel method for the prediction of SRM core losses is proposed. It is shown that the core losses represent between 2 and 13 % of the input power depending on the level of saturation, switching frequency and current waveform.
4. A procedure for the complete steady-state analysis of the SRM drive has been developed and verified experimentally. Using this method, the necessity for an input filter is demonstrated.
5. Steady-state analysis and design of three novel converters are described and experimentally verified.
6. The sensitivity of converter ratings to pole geometry is studied. Optimum values of rotor-to-stator pole arc ratio are shown to be between 1.0 and 1.25 and optimum values of stator pole arc-to-pole pitch ratio are between 0.35 and 0.5.

10.2 Suggestions for further work

In the course of the research work, several concerns which require further studies have arisen. The two most outstanding ones are:

1. Further studies into techniques for minimizing a.c. side harmonic distortion due to the pulsating current at the converter input. The use of a high frequency resonant link is suggested.

2. Development of an integrated Computer Aided Design (CAD) package which includes motor design, inductance and steady-state torque prediction and design optimization.

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Appendix A. Main dimensions and parameters of the 6/4 pole SRM

Number of stator poles	6	
Number of rotor poles	4	
Stator pole arc	24	degrees
Rotor pole arc	36	degrees
Stator outer diameter	190	mm
Bore diameter	122.0	mm
Core length	60.37	mm
Shaft diameter	28.56	mm
Rotor pole height	27.25	mm
Stator back-iron thickness	10.15	mm
Airgap length	0.25	mm
Number of turns per phase	536	
Supply voltage	300	V
Rated peak current	10	A

Core Material	E140	Steel
Rated speed	1800	r.p.m.
Wire gauge	AWG 19	

Appendix B. Equations for the prediction of phase inductance

The equations derived in this section apply to all rotor positions and levels of excitation under the assumptions stated below.

B.1 Assumptions

- i. the air-gap sections of the flux lines consist of concentric arcs and straight line segments
- ii. the flux lines enter and leave the iron surface normally
- iii. the flux lines in the stator and rotor poles run parallel to the pole axes

- iv. the flux lines in the stator back-iron and rotor body are concentric
- v. the shaft is purely nonmagnetic
- vi. the pole arc to pitch ratio is 0.4
- vii. the zero degrees rotor position is the fully aligned position
- viii. the pole windings are cylindrical; consequently, the stator interpolar space is only partially filled with windings.

The assumed flux paths are shown in Figures 104 and 105 for regions I and II respectively.

The identification and disposition of the assumed flux paths was mainly based on experience gained through finite element analysis (FEA). The distinction between the paths is based on the length and shape of the air-gap sections. Each flux path consists of iron sections and two unequal airgap sections. Thus, the m.m.f. equation per path is given by

$$F_j = F_i + F_{g1} + F_{g2} \quad (B1)$$

where F_j is the total m.m.f. of the path, F_i is the m.m.f. drop in the iron core for that path, F_{g1} is the m.m.f. drop in the air-gap section between the upper stator pole and the rotor and F_{g2} is the m.m.f. drop in the air-gap section between the lower stator pole and the rotor. Equation (B1) can be written in terms of the flux linkages and dimensions of the motor as

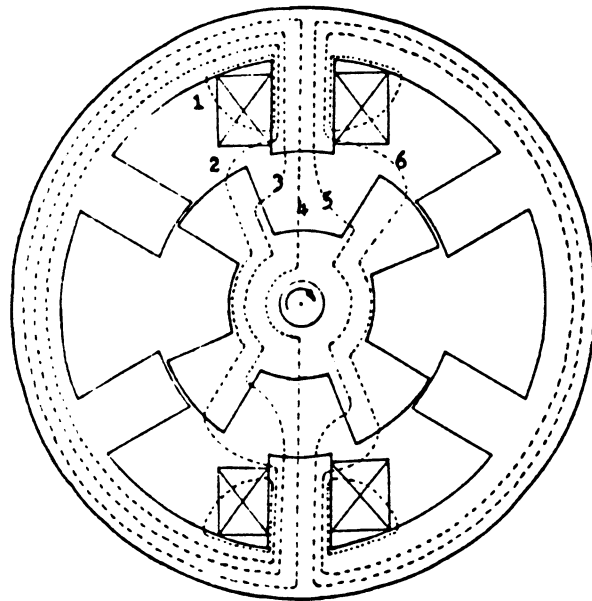


Figure 104. Assumed flux paths for the non-overlapping region (region I)

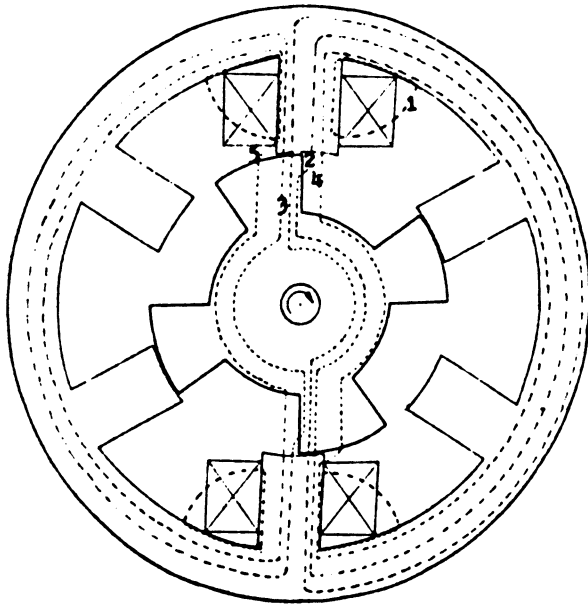


Figure 105. Assumed flux paths for the overlapping region (regionII)

$$F_j = \sum \frac{\Phi_{jk}}{N_j \mu a_{jk}} l_{jk} \quad (B2)$$

where Φ_{jk} is the flux linkages due to path 'j' in section 'k', a_{jk} is the cross-sectional area in section 'k' normal to the flux lines of path 'j', N_j is the number of turns per phase linked with path 'j', l_{jk} is the length of path 'j' in section 'k' of the motor and μ is the permeability. In the air-gap sections, $\mu = \mu_0$ whereas in the iron sections, it is a nonlinear function of the level of excitation.

Since the magnetic circuit of the motor is always symmetrical about the axis perpendicular to the axis of the excited stator poles, it is sufficient to derive the equations with reference to one half of the motor only. Subscripts 1,2,3...refer to the different flux paths as shown in Figures 104 and 105. The following parameters are common to all subsequent equations unless otherwise stated.

$$\alpha = \theta - \frac{\beta_r}{2} \quad (B3)$$

$$\varepsilon = \frac{\beta_s}{2} - \alpha \quad (B4)$$

$$l_0 = \left(\frac{d}{2} + g \right) \varepsilon \quad (B5)$$

$$t = \frac{2 g_i}{\pi - (2 \theta - \beta_r)} \quad (B6)$$

$$t' = \left(\frac{d}{2} + g \right) \left[\tan \alpha - \sin \frac{\beta_s}{2} \right] \quad (B7)$$

$$\eta = \phi - \theta - \frac{\beta_r}{2} \quad (B8)$$

$$b = \frac{2 g_i}{\pi - 2 \eta} \quad (B9)$$

$$e = \left(\frac{d}{2} + g \right) \left[\tan \eta - \sin \frac{\beta_s}{2} \right] \quad (B10)$$

$$t_s = \left(\frac{d}{2} + g \right) \beta_s \quad (B11)$$

$$h = \frac{d}{2} \sin \alpha - \left(\frac{d}{2} + g \right) \sin \frac{\beta_s}{2} \quad (B12)$$

$$x = \frac{d}{2} \sin \left(\theta - \frac{\beta_r}{2} \right) - \left(\frac{d}{2} + g \right) \sin \frac{\beta_s}{2} + \frac{h_s}{8} \quad (B13)$$

$$y = g + \frac{d}{2} (1 - \cos \alpha) \quad (B14)$$

$$h_2 = \frac{d}{2} \sin \eta - \left(\frac{d}{2} + g \right) \sin \frac{\beta_s}{2} \quad (B15)$$

$$y_2 = g + \frac{d}{2} (1 - \cos \eta) \quad (B16)$$

$$x_2 = h_2 + \frac{h_s}{8} \quad (B17)$$

B.2 Region I

$$(0.5 \lambda_r \geq \theta \geq 0.5 (\beta_s + \beta_r))$$

With reference to Figure 104, the following equations are derived for the non-overlapping rotor positions. Six distinct flux paths are assumed.

B.2.1 M.m.f. per path

Path 1

$$F_1 = \left[\frac{(\frac{1}{2} \gamma + \frac{\sin 2 \gamma}{4})(\frac{3}{4} h_s)^2 - w x \cos \gamma}{w h_w} \right] N i \quad (B18)$$

Path 2

$$F_2 = \left[w h_w + x y - \frac{\pi}{4} x^2 \right] \frac{N i}{w h_w} \quad (B19)$$

Path 3

$$F_3 = F_4 = F_5 = N i \quad (20)$$

Path 6

$$F_6 = \left[w h_w + x_2 y_2 - \frac{\pi}{4} x_2^2 \right] \frac{N i}{w h_w} \quad (B21)$$

B.2.2 Mean path length and cross-sectional area

Stator pole:

$$l_{s1} = \frac{5}{8} h_s \quad (B22)$$

$$l_{s2} = l_{s6} = \frac{7}{8} h_s \quad (B23)$$

$$l_{s3} = l_{s4} = l_{s5} = h_s \quad (B24)$$

$$a_s = L \left(\frac{d}{2} + g \right) \beta_s \quad (B25)$$

Rotor pole:

$$a_r = \left(\frac{d}{2} - g_i + g \right) \beta_r L \quad (B26)$$

$$l_{r2} = l_{r6} = (g_i - g) \quad (B27)$$

$$l_{r3} = g_i - g - \left(\frac{t - t'}{2} \right) \quad (B28)$$

$$l_{r5} = g_i - g - \left(\frac{b - e}{2} \right) \quad (B29)$$

Rotor core:

$$l_{rc} = \frac{\pi}{2} \left(\frac{dsh}{2} + \frac{d}{2} + g - g_i \right) \quad (B30)$$

$$a_{rc} = \frac{d}{2} L \left(\frac{d}{2} + g - g_i - \frac{dsh}{2} \right) \quad (B31)$$

Stator core:

Path 1

$$l_{sc1} = \frac{5}{8} \gamma h_s \quad (B32)$$

Paths 2-6

$$l_{sc} = \frac{\pi}{2} (d_0 - c) \quad (B33)$$

$$a_{sc} = c L \quad (B34)$$

Air-gap sections

$$l_{g1} = \frac{5}{8} \gamma h_s \quad (B35)$$

$$a_{g1} = \frac{1}{4} h_s L \quad (B36)$$

$$l_{g2} = (g + \frac{h_s}{4} + \frac{d}{2} (1 - \cos \alpha)) \frac{\pi}{2} \quad (B37)$$

$$a_{g2} = a_{g1} \quad (B38)$$

$$l_{g3} = (\frac{\pi}{2} - \alpha) \left[\frac{t + t'}{2} \right] \quad (B39)$$

$$a_{g3} = (t - t') L \quad (B40)$$

$$l_{g4} = g_i \quad (B41)$$

$$a_{g4} = \left[(\frac{d}{2} + g) \beta_s - (t - t') \right] L \quad (B42)$$

$$l_{g5} = (\frac{\pi}{2} - \eta) \left[\frac{b + e}{2} \right] \quad (B43)$$

$$a_{g5} = [b - e] L \quad (B44)$$

$$l_{g6} = \frac{\pi}{2} \left[h_2 + \frac{h_s}{8} \right] \quad (B45)$$

$$a_{g6} = \frac{h_s}{8} L \quad (B46)$$

Pole tips

The effect of tip saturation is included by assuming a saturated rectangular region at the stator pole tips approaching alignment. With reference to -- Figure id 'f4' unknown --, this area is given by

$$a_t = (t - t') \frac{h_s}{8} \quad (B47)$$

This area is assumed equal for all the four pole tips. The mean length of the flux paths in these regions is approximated by a circular arc inscribed in the saturated region and is given by

$$l_t = \frac{\pi}{8} h_s \quad (B48)$$

The flux density in these regions is assumed to be equal to the maximum permissible flux density for the core material.

B.3 Region II

$$(0.5(\beta_s + \beta_r) \geq \theta \geq 0)$$

The analytical model assumes five different flux paths as shown in Figure 105. The existence of paths 2, and 5 is mainly due to the tip saturation effect which is assumed

to be significant for overlap angles $\varepsilon \leq 0.03\phi$ where ϕ is the rotor pole pitch in radians. These flux paths are neglected at larger overlap angles.

The equations for the calculation of mean path lengths and cross-sectional area in the iron parts are the same as those given for Region I. Further, all the equations for path 1 remain the unchanged. The m.m.f. linked with path 1 is the same as given by equation B18 above. With the exception of path 5, all the other paths are linked with all the ampere-turns per phase. The equation for the m.m.f. linked with path 5 equals that for path 2 in the unaligned positions and is given in equation (B19). The equations for the airgap sections are given below.

B.3.1 Mean path length and cross-sectional area

The equations for the mean path lengths and cross-sectional areas of the air-gap sections for one half of the motor are as follows:

$$l_{g2} = \left(\frac{\pi}{2} - \alpha \right) t + g \quad (B49)$$

$$a_{g2} = (t - g) L \quad (B50)$$

$$l_{g3} = g \quad (B51)$$

$$a_{g3} = \frac{d}{2} \varepsilon L \quad (B52)$$

$$l_{g4} = g_i \quad (B53)$$

$$a_{g4} = (t_s - l_0 - t) L \quad (B54)$$

$$l_{g5} = \frac{\pi}{2} \left(y + \frac{h_s}{8} \right) \quad (B55)$$

$$a_{g5} = L \frac{h_s}{4} \quad (B56)$$

Pole tips

The area under tip saturation is assumed to be a rectangle whose sides are formed by the overlap length and the span of path 5 on the stator pole side. The path length in the saturated area equals the length of an arc with a radius equal to the length of the overlapped section of the stator pole width. Thus, the mean path length l_t and cross-sectional area a_t under tip saturation are given by:

$$a_t = l_0 \frac{h_s}{8} \quad (B57)$$

$$l_t = \frac{\pi}{2} l_0 \quad (B58)$$

Appendix C. Equations for the Fourier coefficients of the rotor flux waveforms

These equations are developed for the general case where t_r is not equal to t_r . For the 6/4 SRM, the rotor pole flux linkage function is described by six linear functions of the form shown in equations (3.31) and (3.32). t_0 to t_5 are the points of discontinuity shown in Figure 20 on page 73. The corresponding Fourier coefficients are:

$$\begin{aligned}
 b_{n1} = & \frac{-4 A}{n^2 \omega^2 T t_r} [\sin n \omega t_1 - \sin n \omega t_0] \\
 & \frac{+ 4 A}{n^2 \omega^2 T t_r} [(n \omega t_1 \cos n \omega t_1 - n \omega t_0 \cos n \omega t_0)] \\
 & - \frac{4 A t_0}{n \omega T t_r} [\cos n \omega t_1 - \cos n \omega t_0]
 \end{aligned} \tag{C1}$$

$$b_{n2} = \frac{4 A}{n \omega T} [\cos n \omega t_2 - \cos n \omega t_1]$$

$$\begin{aligned}
& + \frac{4 A}{n^2 \omega^2 T t_f} [\sin n \omega t_2 - \sin n \omega t_1] \\
& - \frac{4 A}{n^2 \omega^2 T t_f} [(n \omega t_2 \cos n \omega t_2 - n \omega t_1 \cos n \omega t_1)] \\
& + \frac{4 A t_1}{n \omega T t_f} [\cos n \omega t_2 - \cos n \omega t_1]
\end{aligned} \tag{C2}$$

$$\begin{aligned}
b_{n3} &= \frac{-4 A}{n^2 \omega^2 T t_f} [\sin n \omega t_4 - \sin n \omega t_3] \\
& + \frac{4 A}{n^2 \omega^2 T t_f} [(n \omega t_4 \cos n \omega t_4 - n \omega t_3 \cos n \omega t_3)] \\
& - \frac{4 A t_3}{n \omega T t_f} [\cos n \omega t_4 - \cos n \omega t_3]
\end{aligned} \tag{C3}$$

$$\begin{aligned}
b_{n4} &= \frac{4 A}{n \omega T} [\cos n \omega t_5 - \cos n \omega t_4] \\
& + \frac{4 A}{n^2 \omega^2 T t_f} [\sin n \omega t_5 - \sin n \omega t_4] \\
& - \frac{4 A}{n^2 \omega^2 T t_f} [(n \omega t_5 \cos n \omega t_5 - n \omega t_4 \cos n \omega t_4)] \\
& + \frac{4 A t_4}{n \omega T t_f} [\cos n \omega t_5 - \cos n \omega t_4]
\end{aligned} \tag{C4}$$

$$b_{n5} = \frac{-4 A}{n^2 \omega^2 T t_f} [\sin n \omega t_7 - \sin n \omega t_6]$$

$$\begin{aligned}
& \frac{+4A}{n^2\omega^2 T t_r} \left[(n\omega t_7 \cos n\omega t_7 - n\omega t_6 \cos n\omega t_6) \right] \\
& - \frac{4A t_6}{n\omega T t_r} \left[\cos n\omega t_7 - \cos n\omega t_6 \right]
\end{aligned} \tag{C5}$$

$$\begin{aligned}
b_{n6} = & \frac{4A}{n\omega T} \left[\cos n\omega t_8 - \cos n\omega t_7 \right] \\
& + \frac{4A}{n^2\omega^2 T t_f} \left[\sin n\omega t_8 - \sin n\omega t_7 \right] \\
& - \frac{4A}{n^2\omega^2 T t_f} \left[(n\omega t_8 \cos n\omega t_8 - n\omega t_7 \cos n\omega t_7) \right] \\
& + \frac{4A t_7}{n\omega T t_f} \left[\cos n\omega t_8 - \cos n\omega t_7 \right]
\end{aligned} \tag{C6}$$

$$b_n = b_{n1} + b_{n2} + b_{n3} + b_{n4} + b_{n5} + b_{n6} \tag{C7}$$

The flux linkage functions for the two pairs of rotor poles are are obtained as

$$\Phi_{r1} = \sum_{n=1}^{\infty} b_n \sin n\omega t \tag{C8}$$

and

$$\Phi_{r2} = \sum_{n=1}^{\infty} b_n \sin n\omega(t + T_w) \tag{C9}$$

The rotor core flux linkages function Φ_{rc} is obtained by a point-by-point summation of the two functions given in equations (C8) and (C9) above.

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