

transducer such as was used in the present study. The correction given by Corcos (1963) is the method most commonly used to correct experimental wall pressure spectra.

2.4. Pressure Transducer

The Endevco model 8507-C2 pressure transducer (figure 10) was used for the present measurements. It is a high sensitivity piezoresistive, circular-deflection-type, pressure transducer that has a nominally flat frequency response from 0-70 kHz with a rated sensitivity of 130 mV/psi and a full scale output of 300 mV. The transducer contains an active four-arm strain gage bridge that operates with a nominal 10.0 V excitation voltage and is diffused into a sculpted silicon diaphragm.

2.5. Measurements in a Two-Dimensional Boundary Layer and a Wing-Body Junction Flow

The calibration, data acquisition and reduction, and measurement uncertainty of the measurements in the two-dimensional boundary layers is identical to that of the measurements in the wing-body junction flow. Therefore, the description of these procedures for both sets of measurements are grouped together in this section.

2.5.1. Calibration

The nominally flat frequency response ($0 \text{ kHz} < f < 70 \text{ kHz}$) of the Endevco model 8507-C2 pressure transducer is degraded by the response of the pinhole mask that is necessary in order to resolve small-scale pressure fluctuations. Whether or not the degree of degradation is significant depends on flow conditions such as pressure and temperature (equations 35 - 38). The power spectrum of pressure fluctuations beneath the 2-D, $Re_\theta = 23400$ flow without any calibration applied is shown in figure 11. This power spectrum is typical of the power spectra measured in the two-dimensional and wing-body junction flows. The resonant peak due to the pinhole is near 28 kHz. Figure 12 shows the expected amplitude and phase response of the pinhole mask. The amplitude response shown in figure 12 is that of a Helmholtz resonator (equation 35) that is attenuated at high frequencies using the values given by Corcos (1963), assuming that $U_C = 14u_\tau$ (Ha, 1993), in order to account for the finite pinhole size (§2.3.2). The

phase response shown in figure 12 is that of a Helmholtz resonator (equation 36). The maximum deviation of the expected amplitude response, within the frequency range of interest here ($0 \text{ Hz} < f < 20 \text{ kHz}$), is 17% (0.7 dB). The maximum phase shift, within the frequency range of interest here, is 0.8° . Since these values are within acceptable uncertainties of p spectral values, only a static (0 Hz) calibration was performed.

All of the power spectral levels measured in the two-dimensional and wing-body junction flows follow a well defined power law decay at the highest frequencies. Therefore, upward deviations of the high frequency spectral levels from the power law decay are due only to the dynamic response of the pinhole mask. The frequency at which the spectral level starts to deviate from the high frequency spectral power law-type decay is the upper limit of valid spectral levels. Spectral levels calculated at frequencies higher than this upper limit were discarded in the present study.

The static calibration was done using a beaker-tubing arrangement (figure 13). The basic principle is to apply both positive and negative pressures to the beaker using the input tube. The water inside the beaker holds the applied pressure field in the beaker. One of the output tubes is connected to a Datametrix Barocel pressure transducer type 590D-100T-3Q8-H5X-4D (range = 100 torr) which has a known sensitivity and measures the applied pressure. The other output tube is connected to the pressure transducer back-pressure vent tube (figure 13). The static sensitivity of the transducer was 0.0237 mV/Pa during the measurements in the higher Re_θ flows ($Re_\theta = 23400(2\text{-D}), 23200(3\text{-D})$). During the measurements in the lower Re_θ flows ($Re_\theta = 7300(2\text{-D}), 5940(3\text{-D})$), the static sensitivity of the transducer-amplifier system was 0.0243 mV/Pa.

2.5.2. Data Acquisition and Reduction

The pressure transducer was mounted within a housing unit designed for these experiments (figure 14). Access to the flow field was provided through a 0.5 mm diameter pinhole (figure 15) which was used to decrease spatial averaging (§2.3). The housing unit was mounted flush with the surface of the test section and rigidly supported from the laboratory floor beneath the wind tunnel. The diameter of the housing unit was 1.55 cm while the hole in the test

surface was 1.65 cm in diameter. The resulting gap between the housing unit and the test surface mechanically isolated the housing unit from the wind tunnel which prevented tunnel vibration from contaminating the surface pressure measurements. The gap was covered with 0.03 mm thick cellophane tape in order to provide continuity of the surface while maintaining mechanical isolation of the housing unit. The tape did not significantly contribute to surface roughness because the thickness of the tape is smaller than the viscous sublayer ($\leq 2.3 \nu / u_\tau$).

The transducer signal was amplified by a Measurements Group model 2310 strain gage conditioning amplifier and stored to 12-bit precision by an IBM-type PC using a RC Electronics ISC-16 A/D converter. The surface pressure fluctuations were sampled at 67 kHz. At each measurement station, 512 records of 32768 contiguous samples per record were acquired. The total sampling period at each measurement station was at least 16 minutes in order to insure the stationary of the measured signal^{¶¶}. During post-processing, each of the contiguous records was divided into 2 sub-records and the time-delay subtraction was carried out. The time-delayed power spectra were calculated using a C program called TIM-DLY.C (appendix A) which uses the FFT algorithm given by Press *et al.* (1994). Additional bin averaging (Bendat and Piersol, 1986) was performed to produce the final spectrum using a C program called BIN-AV.C (appendix B). The final spectral values were calculated using at least 1024 averages. The Poisson Equation Term Ratio which is discussed in chapter 4 was computed using a C program called P-TERM.C (appendix C) and the data of Ölçmen and Simpson (1996: $Re_\theta = 5940, 7300$) and the data of Ölçmen *et al.* (1998: $Re_\theta = 23200, 23400$). The normalization of the p spectra was done using Microsoft Excel v5.0. Also, the values of p' were calculated by integrating each p spectrum using Microsoft Excel v5.0. Numerical integrations were calculated using the composite trapezoidal rule.

2.5.3. Measurement Uncertainty

The experimental uncertainty for the spectral power density of surface pressure fluctuations is primarily due to the statistical convergence uncertainty and the uncertainty

^{¶¶} A time interval was allowed to elapse in between the acquisition of each contiguous record of data in order to lengthen the total sample period. These time intervals were not necessarily equal to one another.

introduced by assuming that the transfer function of the pressure transducer-pinhole combination is equal to 1 within the frequency range of interest in the present study. Following the analysis of Bendat and Piersol (1986), the normalized statistical convergence uncertainty is

$$\varepsilon_{\text{convergence}} = \frac{1}{\sqrt{\text{Number of ensembles averaged}}} \quad (39)$$

The p spectra presented here were calculated by averaging at least 1024 ensembles. Therefore, the upper limit of $\varepsilon_{\text{convergence}} = \pm 3\%$. The maximum deviation of the expected amplitude response of the pressure transducer-pinhole combination (figure 12) is a conservative estimate of the uncertainty due to the assumption of a transfer function equal to 1 within the frequency range of interest. This value is $\varepsilon_{\text{transfer}} = \pm 17\%$. Following the analysis of Kline and McClintock (1953), the combined uncertainty due to the statistical convergence uncertainty and the uncertainty due to the transfer function is

$$\varepsilon_{\text{spectral value}} = \sqrt{\varepsilon_{\text{convergence}}^2 + \varepsilon_{\text{transfer}}^2} \quad (40)$$

The uncertainty of the p spectral values is estimated to be $\pm 17.3\%$ using equation 40. Values of p' were calculated as the square root of $\overline{p^2}$. Each value of $\overline{p^2}$ was calculated by numerically integrating the spectral power density of p using the composite trapezoidal rule. Therefore, the uncertainty of p' and $\overline{p^2}$ is due to the uncertainty of the individual p spectral values that were integrated. Following the analysis of Kline and McClintock (1953), the uncertainty of $\overline{p^2}$ is

$$\varepsilon_{\overline{p^2}} = \varepsilon_{\text{spectral value}} \frac{\sqrt{\sum_{i=1}^N (\Phi_i(\omega_i))^2 (\Delta\omega_i)^2}}{\sum_{i=1}^N (\Phi_i(\omega_i)) (\Delta\omega_i)} \quad (41)$$

where $\Phi_i(\omega_i)$ and $\Delta\omega_i$ are each individual spectral value and corresponding bin width, respectively. The uncertainty of $\overline{p^2}$ is estimated to be $\pm 14.3\%$ using equation 40. Following the analysis of Kline and McClintock (1953), the uncertainty of p' is

$$\varepsilon_{p'} = \sqrt{\frac{1}{2} \varepsilon_{\overline{p^2}}^2} \quad (42)$$

The uncertainty of p' is estimated to be $\pm 10.1\%$ using equation 42.

2.6. Measurements in Flow Around a 6:1 Prolate Spheroid

2.6.1. Calibration

The static calibration was performed using the procedure described in §2.5.1. The sensitivity of the pressure transducer was 0.0233 mV/Pa. A coarse dynamic calibration was done using a GenRad model 1986 Omnical Sound Level Calibrator. The calibration was done using a sound pressure level (SPL) of 114 dB at 125 Hz, 250 Hz, 500 Hz, 1 kHz, 2 kHz, and 4 kHz. A Hewlett-Packard model HP 3478A true-RMS multimeter was used to measure the output AC voltage of each pressure transducer. The results of the coarse dynamic calibration are shown in figure 16. Note that the coarse dynamic calibration was performed on the pressure transducer alone, without the pinhole mask. The purpose of the coarse dynamic calibration on the pressure transducer alone was to verify that the frequency response of transducer alone is constant.

The pinhole mask used to reduce spatial averaging (Corcos, 1963) significantly affected the dynamic response of the pressure transducer system. For the conditions of the prolate spheroid flow, the theoretical estimate of the resonant frequency (equation 38) is 11.28-11.54 kHz and the theoretical estimate of the damping factor (equation 37) is 0.0115-0.0122 depending on ϕ location. Figure 17 shows a representative power spectrum ($\alpha = 10^\circ$, $x/L = 0.772$, $\phi = 150^\circ$) of the pressure transducer signal with a time-delay subtraction ($\Delta t = 29$ ms) and the static calibration applied. Figure 18 shows this power spectrum corrected assuming the theoretical transducer frequency response. Clearly the theoretical correction is insufficient. The source of the high frequency (≈ 33 kHz) spike in the power spectrum (figure 18) is unknown. Such a spike was observed in all of the p spectra in the prolate spheroid flow. The spike must be due to some measurement error since there is no natural phenomenon present to cause it. Therefore, for each spectrum, the minimum high frequency spectral estimate (before the spike) was located. All spectral estimates at higher frequencies than where this minimum occurred were discarded.

The frequency response of the transducer-pinhole mask system that was used for the p measurements on a prolate spheroid was modeled after the theoretical frequency response of a Helmholtz resonator (equation 35). The velocity data at $\alpha = 10^\circ$, $x/L = 0.772$, $\phi = 150^\circ$ show that $\overline{v^2}/u_\tau^2$ (figure 19) is similar to that in a two-dimensional boundary layer. Additionally,

Chesnakas and Simpson (1997) show that the mean velocity profile is collateral near the wall which suggests that the structure of the boundary layer is nearly two-dimensional near the wall. McGrath and Simpson (1987) showed that the pressure spectrum beneath 2-D boundary layers collapse at high frequencies when normalized using viscous scales. Their normalized p spectra can be approximated by the four curves shown in figure 20. It is assumed here that, near the resonant frequency of the transducer-pinhole mask, the pressure spectrum at $\alpha = 10^\circ$, $x/L = 0.772$, $\phi = 150^\circ$ follows the curves in figure 20 when normalized using viscous scales based on the behavior of $\overline{v^2}/u_\tau^2$ and the mean velocity at this location. It is also assumed here that the frequency response of the pinhole mask is constant below 1 kHz and equal to the static sensitivity. These two assumptions were used along with the mathematical form of the Helmholtz resonator to construct a transfer function for the transducer-pinhole combination. Therefore, the resulting transfer function will produce a p spectrum (at $\alpha = 10^\circ$, $x/L = 0.772$, $\phi = 150^\circ$) that is similar to that of McGrath and Simpson (1987) near the resonant frequency of the transducer-pinhole mask.

The initial approximation of the transfer function for the transducer-pinhole system at $\alpha = 10^\circ$, $x/L = 0.772$, $\phi = 150^\circ$ is illustrated in figure 21. The transfer function is defined such that the measured signal multiplied by the static sensitivity divided by the transfer function yields the true pressure fluctuation. The approximate transfer function is unity from 0-1 kHz. The approximate transfer function is given by the uncorrected pressure spectrum, normalized using viscous scales, divided by the curves that fit the data of McGrath and Simpson (1987) for frequencies higher than 6 kHz. The low frequency variation is blended to the high frequency variation in the range 1-6 kHz. This procedure yields a table of values that approximate the transfer function. The initial approximation (figure 21) was used only to determine the likely variation of the transfer function near resonance being that it is not adequately described by the theoretical response of a Helmholtz resonator.

The center frequency of the resonant peak of the uncorrected pressure spectra varies with azimuthal position. Table 1 shows this variation at $\alpha = 10^\circ$. A table of values for the transfer function would require adjustment at each measurement location due to the variation of resonance frequency with azimuthal position. Such an adjustment would have a large number of degrees of freedom which would degrade the validity of such a procedure. Therefore, it is desirable to

describe the frequency response with a function that has limited degrees of freedom. The approximate transfer function shown in figure 21 can be described by

$$\left(\frac{p_{\text{MEASURED}}}{p_{\text{TRUE}}} \right)^2 = \frac{2 \left(\frac{f}{f_n} \right) + 1}{\left[1 - \left(\frac{f}{f_n} \right)^2 \right]^2 + 4 \zeta^2 \left(\frac{f}{f_n} \right)^2} \quad (43)$$

where f_n and ζ are determined by fitting the curve (figure 22). Equation 22 was modeled after the theoretical curve for a Helmholtz resonator and can be adjusted to fit the data at other azimuthal locations with only 2 degrees of freedom.

The data at each measurement station were taken at different times. Also, the transducer was removed from the model and moved to another physical location in order to switch from measuring p at $x/L = 0.600$ to measuring p at $x/L = 0.722$. The values of $f_n (= \omega_n / 2\pi)$ and ζ for a theoretical Helmholtz resonator do not adequately model the observed spectral behavior. Therefore, the values of f_n and ζ were determined by inspection for each spectrum. The variation of f_n and ζ with azimuthal position is not large. The peak in the uncorrected spectrum is centered about the actual resonant frequency. The resonance of the transducer-pinhole system is the most likely cause for this sharp peak. The damping factor (ζ) was determined such that the pressure spectrum varies smoothly at frequencies near resonance. It was assumed that any “waviness” in the pressure spectrum at frequencies near resonance is most likely due to the resonance of the transducer-pinhole system.

While it is not standard to vary the transfer function from measurement to measurement, the variation of the transfer function is not large and it was done judiciously in order to decrease the likelihood of discarding valid data. Tables 1 and 2 show the values of f_n and ζ used for each pressure spectrum. Figure 23 shows representative transfer functions that bound the transfer functions that were used for the p data at $x/L = 0.600$. The maximum difference between the functions shown in figure 23 is 1.2 dB at $f = 8500$ Hz. Figure 24 shows representative transfer functions that bound the transfer functions that were used for the p data at $x/L = 0.772$. The maximum difference between the functions shown in figure 24 is 1.2 dB at $f = 20$ kHz. The measurement uncertainty due to this calibration procedure is the subject of §2.6.3.

2.6.2. Data Acquisition and Reduction

The pressure transducer was mounted to the optical access window (figure 8). Access to the flow field was provided through a 0.5 mm diameter pinhole (figure 25) which was used to decrease spatial averaging (§2.3). The Corcos correction (Corcos, 1963) was applied to the power spectra to account for the spatial resolution issues discussed in §2.3.2. Figure 26 shows the correction that was applied to the power spectra at $\alpha = 20^\circ$, $x/L = 0.772$ which extend through the full range of d^+ used here ($38 < d^+ < 92$). Gravante *et al.* (1998) report that the maximum allowable sensing diameter to avoid spectral attenuation at high frequencies is in the range $12 < d^+ < 18$. In order to have $d^+ \leq 18$ here would require a pinhole of 0.09 mm which, in turn, would lower the Helmholtz resonant frequency of the pinhole to 1860 Hz. Such a low resonant frequency would cause significant attenuation of the high frequency spectral values, thus offsetting any benefit of better spatial resolution. Since the Corcos correction has been found to recover the true pressure spectrum for the flow conditions here (Schewe, 1983; Lueptow, 1995), it was used here rather than a smaller d^+ pinhole.

The surface pressure fluctuations were sampled at 71 kHz and stored to 12-bit precision by an IBM-type PC using a RC Electronics ISC-16 A/D converter. At each measurement location, 25 records of 16384 contiguous samples per record were acquired. The total sampling period at each measurement station was at least 5 minutes in order to insure the stationarity of the measured signal^{§§}. During post-processing, each contiguous record was divided into 8 sub-records and the time-delay subtraction was carried out. The time-delayed power spectra were calculated using a C program called TIM-DLY.C (appendix A) which uses the FFT algorithm given by Press *et al.* (1994). Additional bin averaging (Bendat and Piersol, 1986) was performed to produce the final spectrum using a C program called BIN-AV.C (appendix B). The final spectral values were calculated using at least 500 averages. The application of the pressure transducer frequency response function and the normalization of the p spectra was done using Microsoft Excel v5.0. Also, the values of p' were calculated by integrating each p spectrum using

^{§§} A time interval was allowed to elapse in between the acquisition of each contiguous record of data in order to lengthen the total sample period. These time intervals were not necessarily equal to one another.

Microsoft Excel v5.0. Numerical integrations were calculated using the composite trapezoidal rule.

The surface pressure-velocity covariance measurements acquired by the author that are presented in chapter 5 are part of the data set of Chesnakas and Simpson (1997), but have not been published. Since these measurements are presented in this dissertation for the first time, some details of the associated data acquisition and reduction are included here. Velocity measurements for the surface pressure-velocity covariance were made radially and 1 mm windward of the pressure pinhole using a three-orthogonal-velocity-component, fiber-optic LDV probe. The LDV probe was mounted inside the model with all laser beams passing through a window (figure 8). The probe can be remotely traversed ± 2.5 cm in both the axial direction and normal to the major axis of the model. Positioning in the circumferential direction (ϕ) was achieved by rotating the model about its major axis on the sting. The Doppler frequency of each of the LDV signals was analyzed using 3 Macrodyne model FDP3100 frequency domain signal processors operating in coincidence mode. The flow was seeded using polystyrene latex spheres 0.7 μm in diameter. The particle velocity and the pressure at the window pressure tap were sampled simultaneously and stored with 16-bit precision. The sample rate varied from 40 samples/sec near the surface to 250 samples/sec in the outer part of the measurement region. Boundary layer velocity profiles were measured at 10-14 circumferential locations in the range $90^\circ \leq \phi \leq 180^\circ$. Each profile consisted of 17-19 radial locations from 0.007 cm above the model surface out to 2.5-3.0 cm. At each of these locations 16384 coincident 3-D velocity-surface pressure realizations were acquired.

The surface pressure data acquired simultaneously with the 3-D velocity data were used to compute the surface pressure-velocity covariance. They were not used to compute pressure spectra. Separate surface pressure measurements were carried out at a higher sampling rate (71 kHz) and used to compute the pressure spectra. Pressure fluctuations due to acoustics and vibration do not affect the surface pressure-velocity covariance since only p contributions due to turbulence correlate with the turbulent velocity fluctuations.

The outer layer/free-stream velocity measurements presented in chapter 5 are part of the data set of Goody *et al.* (1998), but some have not been published. Since these measurements are presented in this dissertation for the first time, some detail of the associated data acquisition and reduction are included here. Measurements of the mean velocity, Reynolds stresses, and triple products were made by Goody *et al.* (1998) in the outer boundary layer and free-stream using a miniature 4-sensor hot-wire probe that consists of two orthogonal X-wire arrays (Auspex Corp. AVOP-4-100). The *quad-wire* probe has 5 μm tungsten sensors 0.8 mm in length and a measurement volume of 0.5 mm³ (figure 27). The quad-wire sensors were operated separately, each using a Dantec 56C17/56C01 anemometer unit. Anemometer outputs were read by an IBM-type PC through an Analogic 12-bit HSDAS-12 A/D converter buffered by four $\times 10$ buck-and-gain amplifiers. A traversing gear mounted in the wind tunnel allowed the horizontal and vertical position of the probe to be controlled from the computer. A probe holder positioned the tip of the probe well upstream of the traverse gear. The probe was held parallel to the free-stream direction for all velocity measurements. A detailed description of the quad-wire measurement system in addition to related calibration and reduction procedures is given by Devenport *et al.* (1997), and Wittmer *et al.* (1998). Profiles of the complete flow velocity vector were measured at 5° ϕ increments (circumferential) from $\phi = 130^\circ$, 140° to $\phi = 180^\circ$. Each profile consisted of 11-13 radial locations from 0.60-0.75 cm to 8-12 cm above the model surface.

2.6.3. Measurement Uncertainty

The experimental uncertainty for the spectral power density of surface pressure fluctuations is primarily due to the statistical convergence uncertainty and the uncertainty of the transfer function of the pressure transducer-pinhole combination. Following the analysis of Bendat and Piersol (1986), the normalized statistical convergence uncertainty is given by equation 39. The p spectra presented here were calculated by averaging at least 500 ensembles. Therefore, the upper limit of $\epsilon_{\text{convergence}} = \pm 4.5\%$. The uncertainty of the transfer function of the pressure transducer-pinhole combination has two parts. The first part is the uncertainty of the p spectrum of McGrath and Simpson (1987) which was used to determine the shape of the transfer function near resonance. The uncertainty given by McGrath and Simpson (1987) is $\epsilon_{\text{McGrath and Simpson (1987)}} = \pm 1.5 \text{ dB } (\pm 41\%)$. The second part is conservatively estimated as the maximum difference among the transfer functions used at a particular axial (x/L) location which is

$\epsilon_{\text{max difference}} = \pm 1.2 \text{ dB}$ (32%). Following the analysis of Kline and McClintock (1953), the combined uncertainty due to the statistical convergence uncertainty and the uncertainty due to the transfer function is

$$\epsilon_{\text{spectral value}} = \sqrt{\epsilon_{\text{convergence}}^2 + \epsilon_{\text{McGrath and Simpson (1987)}}^2 + \epsilon_{\text{max difference}}^2} \quad (44)$$

The uncertainty of the p spectral values is estimated to be $\epsilon_{\text{spectral value}} = \pm 52.2\%$ ($\pm 1.8 \text{ dB}$) using equation 44.

Values of p' were calculated as the square root of $\overline{p^2}$. Each value of $\overline{p^2}$ was calculated by numerically integrating the spectral power density of p using the composite trapezoidal rule and adding an Analytical Integral Contribution (AIC). The AIC is described in §5.4.1 where it is noted that the AIC gives a lower bound on the true value of $\overline{p^2}$. The uncertainty of $\overline{p^2}$ due to the AIC was determined, using a “jitter” analysis (Moffat, 1982), to be $\epsilon_{\text{AIC}} = +8\%$. Since the uncertainty of $\overline{p^2}$ due to the AIC is only positive, the lower bound of the uncertainty of $\overline{p^2}$ is only due to $\epsilon_{\text{spectral value}}$. The lower bound of the uncertainty of $\overline{p^2}$ is estimated to be -15.1% using equation 40. The upper bound of the uncertainty of $\overline{p^2}$ is due to $\epsilon_{\text{spectral value}}$ and ϵ_{AIC} . Following the analysis of Kline and McClintock (1953), the upper bound of the uncertainty of $\overline{p^2}$ is

$$\epsilon_{\overline{p^2}}(\text{upper bound}) = \sqrt{\epsilon_{\text{AIC}}^2 + \epsilon_{\text{spectral value}}^2 \frac{\sum_{i=1}^N ((\Phi_i(\omega_i)) \Delta\omega_i)^2}{\left(\sum_{i=1}^N (\Phi_i(\omega_i)) \Delta\omega_i \right)^2}} \quad (45)$$

where $\Phi_i(\omega_i)$ and $\Delta\omega_i$ are each individual spectral value and corresponding bin width, respectively. The upper bound of the uncertainty of $\overline{p^2}$ is estimated to be $+17\%$. Therefore, the uncertainty of $\overline{p^2}$ is estimated to be $+17\% / -15\%$. The uncertainty of p' is estimated to be $+12\% / -10.7\%$ using equation 42.

The p measurements made simultaneously with velocity were not adjusted for the response of the pressure transducer at high frequencies. In consideration of the rapid decay of the p spectrum with frequency and the loss of correlation between surface and interior fluctuations as

frequency increases, the uncertainty that this introduces in the surface pressure-velocity covariance is very small. Using the definition of R_{pu} , R_{pv} , and R_{pw} given in the List of Symbols and neglecting the uncertainty of the surface pressure-velocity covariances, the uncertainty of R_{pu} , R_{pv} , and R_{pw} is due to the uncertainty of p' , u' , v' , and w' . The uncertainty of p' is estimated above as $\varepsilon_{p'} = +12\% / -10.7\%$. The uncertainty of u' , v' , and w' is given by Chesnakas and Simpson (1997) as $\varepsilon_{u'} = \varepsilon_{v'} = \varepsilon_{w'} = \pm 1.5\%$. Therefore, following the analysis of Kline and McClintock (1953), the uncertainty in the correlation coefficients is equal and is calculated using

$$\begin{aligned}\varepsilon_{R_{pu}, R_{pv}, R_{pw}} &= \sqrt{\varepsilon_{p'}^2 + \varepsilon_{u', v', w'}^2} \\ &= +12.1\% / -10.8\%\end{aligned}\tag{46}$$