

TORSIONAL STRESS CONCENTRATION FACTOR
for
HOLLOW STEPPED SHAFTS

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146642

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INTRODUCTION

With current trends in the field of engine design, notably in the case of gas turbines, moving relentlessly toward lighter and more compact units, the use of hollow shafts becomes more and more prevalent. This progress is, however, severely hampered at present by the lack of adequate design data for this type of shaft.

Historically the study of stress concentration in shafts due to keyways, splines, fillets and other such irregularities, has been limited largely to the case of solid shafts. The stress concentration factor for solid, stepped shafts has been evaluated by several methods prior to this writing and the results made available in the form of an easily applied formula¹. Later works on the subject credit this formula with a good degree of accuracy.

In spite of the exhaustive study made of solid shafting, there has been no attempt known to the author to evaluate the stress concentration existing in the fillets of shafts of hollow cross-section. This problem thus became the subject of this investigation.

Several methods of approaching the problem were considered; use of the electrical analogy of torsion, photoelasticity in three dimensions, and the use of SR-4 resis-

1. Sonntag's Formula, Appendix I

tance gages.

The electrical analogy having been used successfully by L. S. Jacobson² was considered but was rejected in view of the comprehensive technique required. In addition, the necessary machining of appropriate sections was prohibitive to the investigation.

Investigation by the methods of three dimensional photoelasticity was carefully considered with a view towards utilizing the newly developed "Fostorite". Equipment was lacking to adequately pursue this approach, and it was found that sufficient photoelastic material would place the expense considerably beyond reason.

The decision was thus made to employ SR-4 gages in the experimental work. Together with existing auxilliary equipment, they were thought to combine the necessary factors of flexibility, accuracy and economy.

Since the stress distribution in an elastic body was known to be largely independant of its actual dimensions, but dependent rather on the shape or configuration, the results were to be interpreted in terms of nondimensional ratios and thus be applicable in general.

2. Bibliography, Appendix II

OBJECT AND SCOPE

It was the purpose of this Thesis to determine the torsional stress concentration factor for hollow stepped shafts of various wall thicknesses. A relation was sought between this factor and the relative wall thickness.

Connecting fillets of two sizes were studied. The ratios of fillet radius to smaller shaft diameter being 0.50 and 0.375.

The investigation of hollow shafts was limited to cases where the ratio of bore to smaller shaft diameter is between 0.625 and 0.25. The investigation was extended to include the case of solid cross section for the larger fillet section.

THE INVESTIGATION

The problem defined, it was then decided to use electric resistance gages to measure the strains corresponding to the pertinent stresses. Since the tests were restricted to the region of nearly linear elasticity, the stress concentration factor was accurately represented by the ratio of strains.

It can be easily shown that, for a shaft of rotational symmetry, the maximum shearing stress on a particular section is numerically equal to the tensile or compressive stress that occurs in the free surface at an angle of 45 degrees to the generatrix. With this knowledge, it was possible to measure the maximum strain on a given transverse section. The section of maximum stress, however, could be computed only with great difficulty.

To determine the maximum shear stress present in the specimen, and indirectly the critical section, gages were placed so as to determine the maximum strains occurring at several sections on the fillet and on the smaller shaft. Rough preliminary tests indicated that the greatest strain occurred in the fillet where the angle¹ was approximately three degrees.

1. Refer to Figure 5, Page 14 .

Accordingly, gages were then concentrated in this general area with an angular spacing of one degree. A gage was also placed at the midpoint of the smaller diameter shaft to serve as a reference point of zero stress concentration. The strain readings thus obtained were then used to plot strain-position curves for each of the tests from which the maximum strain was obtained.

A series of tests were run for hollow stepped shafts of various wall thicknesses and also for a solid shaft. The latter test was made to serve as a basis of comparison with previous work on solid shafts. The same specimen was used for all tests, the wall thickness controlled by successive drilling operations.

The general method of investigation having been determined, certain preliminary work presented itself. This included control tests performed on the specimen material as well as the development of a satisfactory experimental technique.

THE SPECIMEN

The specimen used in the subsequent investigation was machined from dural in the V.P.I. Machine Shop. In the photograph, Figure 1, it is shown installed in the testing machine. Gages are omitted on one end for clarity.

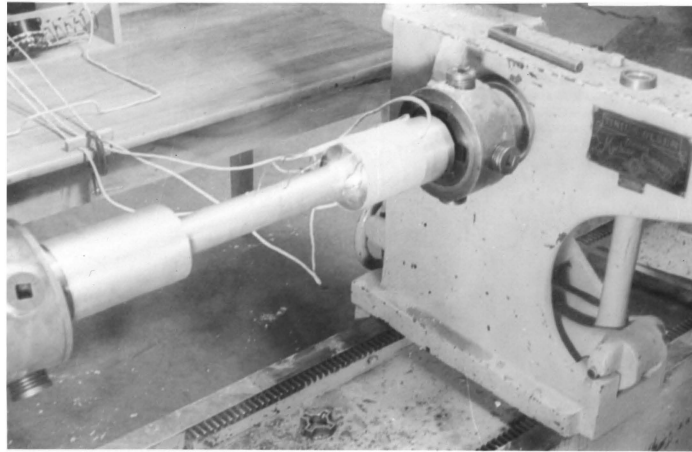
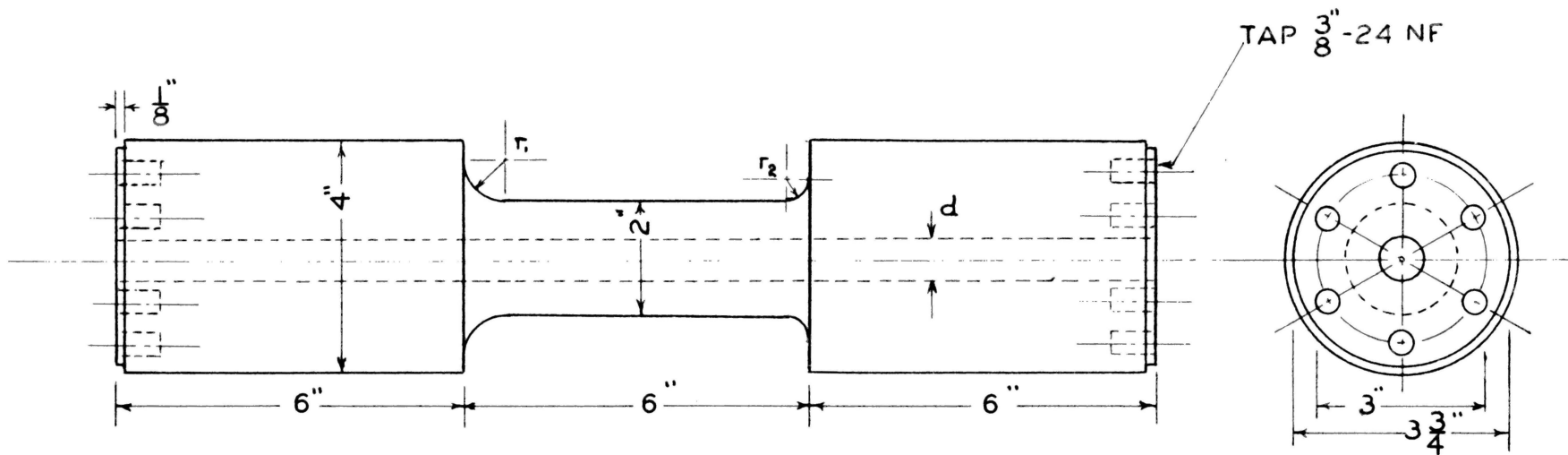


Figure 1, Test Specimen

A detail of the specimen may be found on Page 8 , Figure 2. The two fillet radii r_1 and r_2 were one inch and three quarters inch respectively.

Duralumin alloy 24 ST was selected as the most suitable material for the specimen, since it had two marked advantages over other easily obtainable material. The comparatively low modulus of elasticity permitted larger, and hence more accurately recorded strain values within the limits of the tests. Furthermore, since relatively large dimensions were required for accuracy, the good machinability and lightness of this metal also recommended its choice.

In order to estimate the degree of uniformity of the specimen, a hardness traverse was performed on the end



DURAL TORSION SPECIMEN
 Figure 2

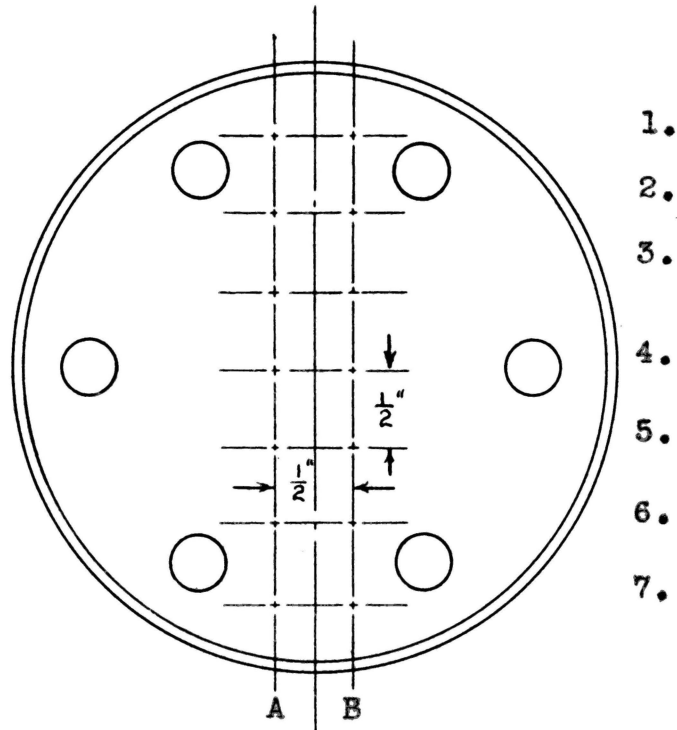
of the finished specimen. Standard Brinell tests were made using the 500 kg load and 30 second application. The location of test points, as well as the data obtained, may be found on page 10 , figure 3.

The average B.H.N. was found to be 83.3 with a maximum variation of 5.4% from the average.

Inasmuch as the actual measurements were of strain, rather than stress, it was important that the loading be confined to that range in which the variation in slope of the stress-strain curve from the highest to the lowest stressed portion is not significant. It was also important to avoid undue yielding which would injure the gages. Since the same gages were preserved throughout the investigation, the cumulative effect of even apparently slight yielding presented a definite danger.

Accordingly data was obtained, figure 4, page 11, and a load-indicated strain curve plotted. As a result of this test, it was thought advisable to hold indicated strain values to approximately one thousand microinches per inch.

HARDNESS TEST DATA, Figure 3



Test Stations

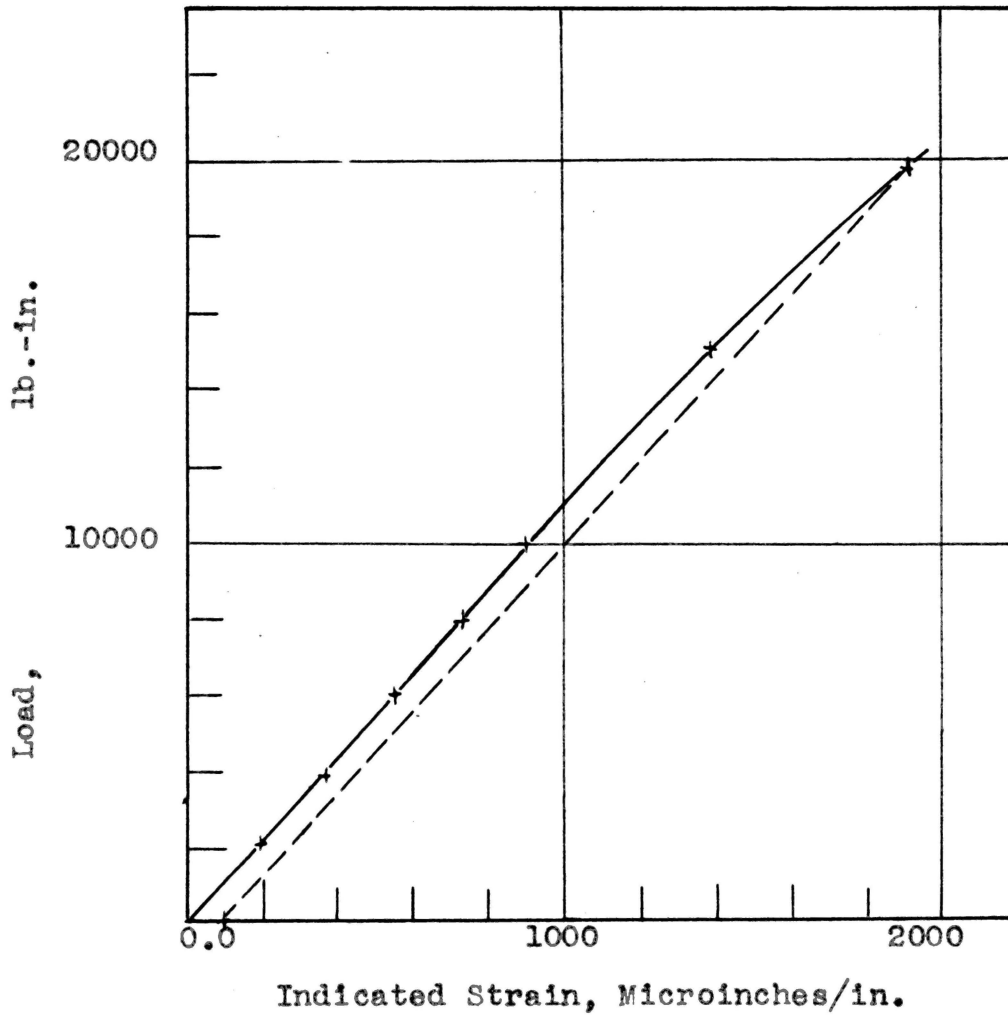
Data

Station	1	2	3	4	5	6	7
Diameter of A	2.74	2.72	2.73	2.82	2.83	2.77	2.71
Impression B	2.70	2.62	2.69	2.78	2.73	2.83	2.68
Average Diam.	2.72	2.67	2.71	2.80	2.78	2.80	2.70
Station Average	84.4	87.7	85.1	79.6	80.8	79.6	85.7

STRESS - STRAIN TEST, Figure 4

Data

Torque	0	2000	3910	5910	7880	10010	15080	19830	0
Strain Read.	979	1174	1346	1530	1707	1879	2364	2891	1062
Indicated Strain	0	195	367	551	728	900	1385	1912	83



GAGE PLACEMENT

One of the greatest difficulties encountered in the investigation, was that of accurately positioning the gages at the desired points on the specimen. It was essential to the very nature of the investigation that both inclination and axial position be carefully preserved.

Several methods were proposed, and tried unsuccessfully, to achieve this aim. Of the several jigs and templates employed, most suffered from inadequate referencing.

To remedy this defect the following method was developed and did enjoy some measure of success.

In this method, the gages were all placed with reference to the surface of the larger shaft. With reference to Figure 5 on page 14, the axial position of each gage was determined as the distance L along the surface from the gage point to the reference. The values of L were then computed for the various angular positions.

These values were then plotted, Figures 8 and 9, pages 18 & 19, and a line passed through each point parallel to a reference line. This resulted in a series of lines each at a distance from the reference corresponding to one axial gage position. Pairs of lines inclined at 45 degrees and each intersecting on one of the parallels were then added.

A template was then constructed of light gage steel as shown in figure 7, page 16. This was secured on the previously constructed chart so that its arms terminated on the reference line.

With the aid of a lowpower magnifier the gages were then placed on the chart, and under the template, with the center of each gage directly over one set of cross lines. While held in this position with tape, the gages were then tack-cemented to the template.

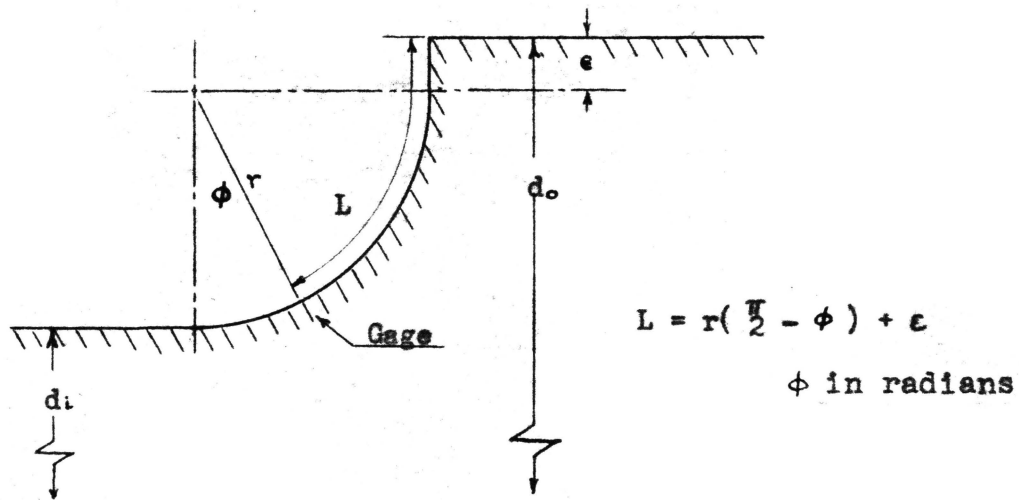
It may assist the reader at this time to refer to figure 6 on page 15 , covering the basic operations.

When the cement was thoroughly dry, the template was removed from the chart. A thin metal band was next wrapped around the large end of the specimen so that it extended beyond the edge an eighth inch.

The template was then formed so as to roughly conform to the fillet outline. The arms were curved to fit the fillet and the main portion curved to conform to the smaller shaft. The template was now installed on the specimen, the arms pressed snugly against the retaining band and into the fillet, the main portion wrapped around the smaller shaft and spring secured.

The preceding operations resulted in placing all gages at their desired axial position and with the

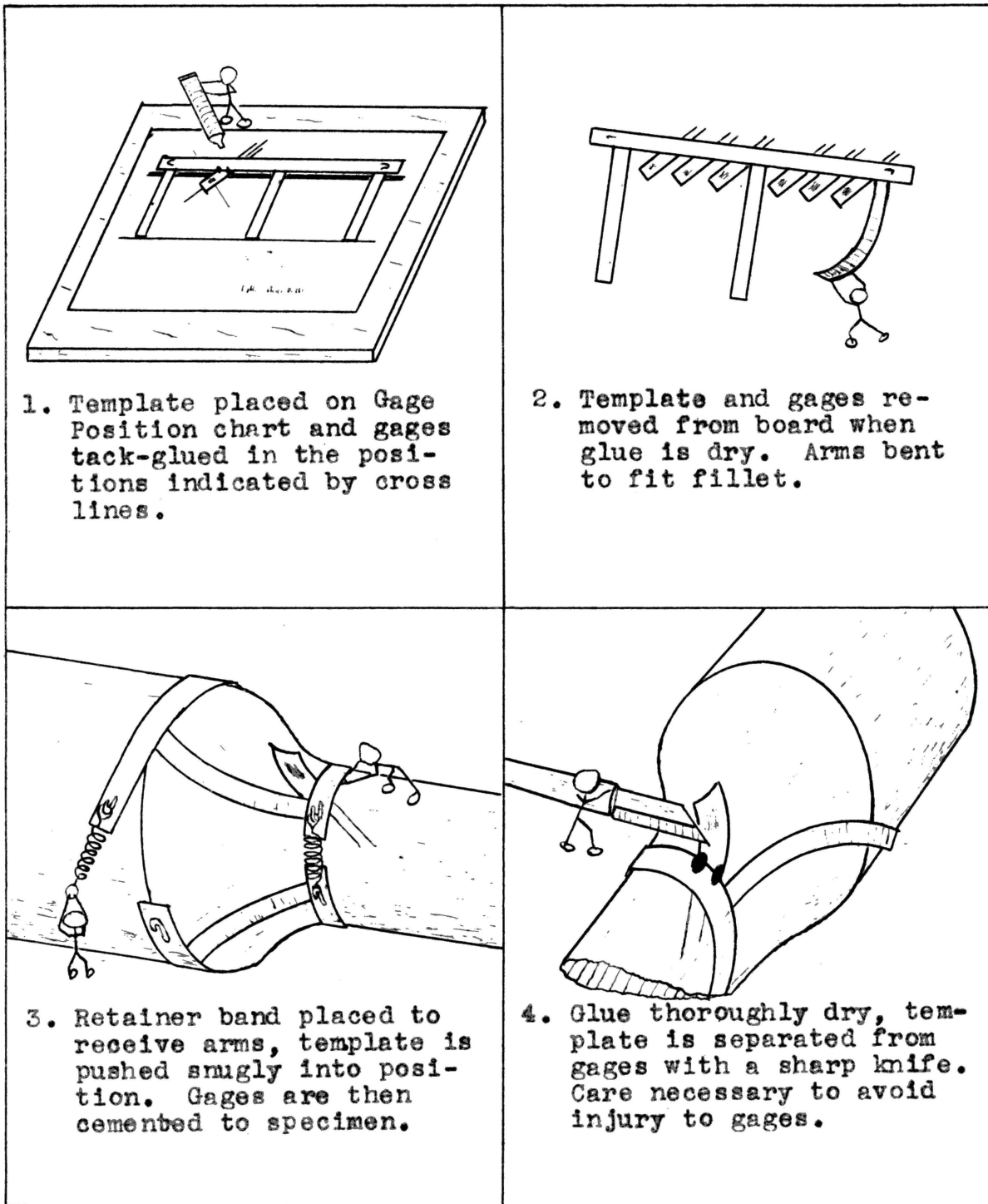
GAGE LOCATION Figure 5



TABULATED CALCULATIONS

$r^I = 1.000''$ $e^I = -0.031''$
 $r^{II} = 0.750''$ $e^{II} = +0.219''$

ϕ	$\frac{\pi}{2} - \phi$	$r^I \left(\frac{\pi}{2} - \phi \right)$	$r^{II} \left(\frac{\pi}{2} - \phi \right)$	L^I	L^{II}
+				1.557	1.410
0	1.57080	1.57080	1.17810	1.540	1.397
1	1.55335	1.55335	1.16501	1.522	1.384
2	1.53589	1.53589	1.15192	1.505	1.371
3	1.51844	1.51844	1.13883	1.487	1.358
4	1.50099	1.50099	1.12574	1.470	1.345
5	1.48353	1.48353	1.11265	1.453	1.332
6	1.46608	1.46608	1.09956	1.435	1.319
7	1.44863	1.44863	1.08647	1.418	1.306
8	1.43117	1.43117	1.07338	1.400	1.292
9	1.41372	1.41372	1.06029	1.383	1.279
10	1.39627	1.39627	1.04720	1.365	1.266



METHOD OF PLACING GAGES

Figure 6

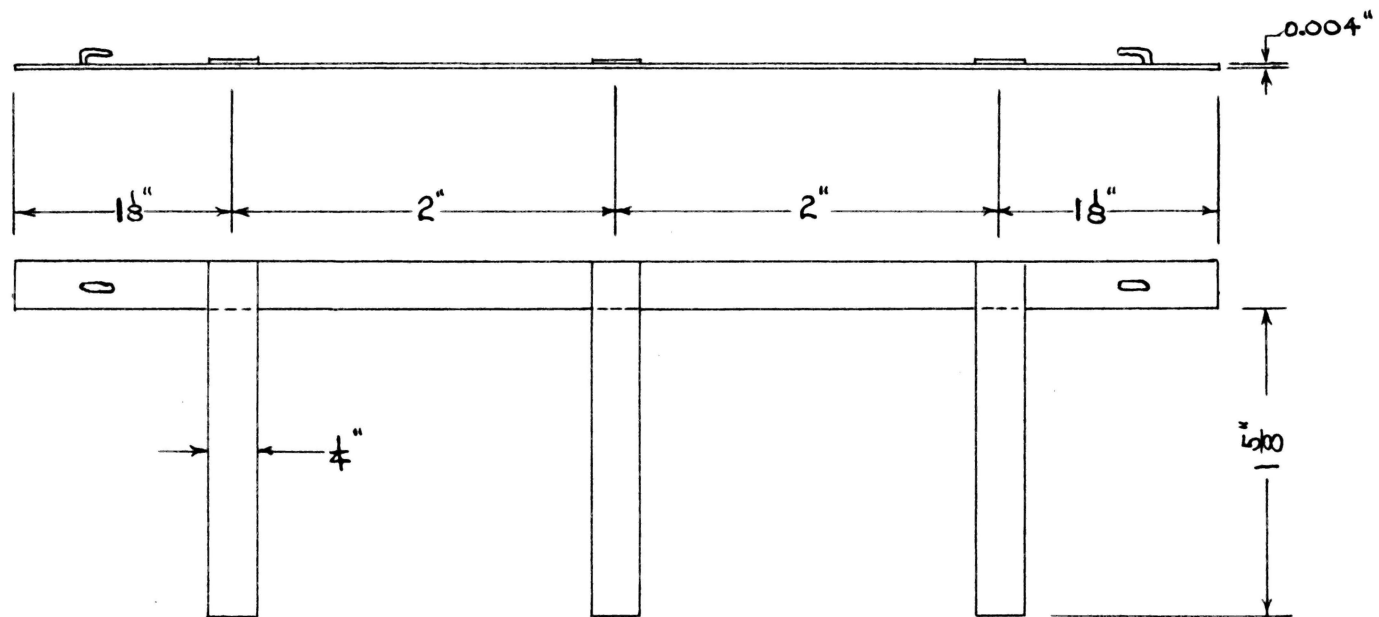


Figure 7, GAGE TEMPLATE

desired inclination of 45 degrees to a generatrix.

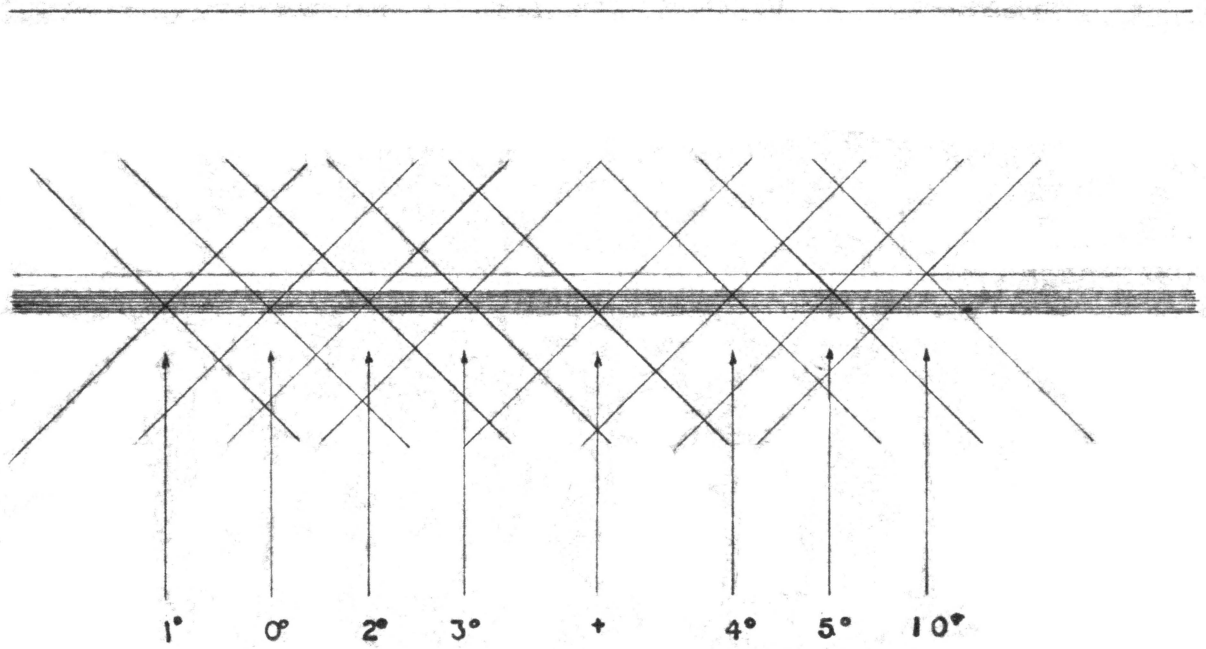
It remained only to float glue under each gage and press firmly in place for a few seconds to complete the installation. When the glue band had completely dried the template was removed by cutting the glue tackings. A razor knife such as is used in model construction was found convenient for this.

Gages were then applied in the manner described to the larger fillet. These gages were placed at the positions 0, 1, 2, 3, 4, 5, 10 degrees of ϕ . This series of gages, however, were destroyed in the first drilling operation. Both cutting oil and centrifugal force served to render them useless for the later tests.

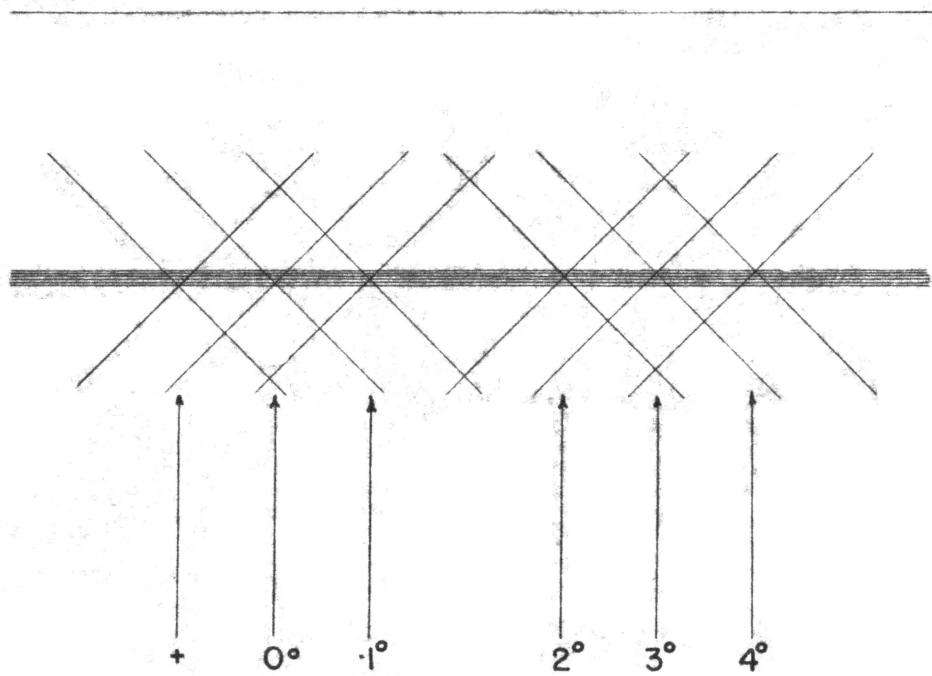
For the remainder of the investigation gages placed at 0, 1, 2, 3, 5 and 10 degrees on the larger fillet and at +, 0, 1, 2, 3 and 4 degrees on the smaller were used. The designation + refers to a position beyond the fillet a distance equal to the arc length of one degree.

APPARATUS

The experimental work was performed on a Tineus-Olsen 60,000 in.-lb. torsion testing machine, located in the Materials Testing Laboratory at the Virginia Polytechnic Institute. Calibration of this machine just prior to the testing period revealed no impairment of its operational



GAGE POSITION - 1" FILLET
Figure 8



GAGE POSITION - $\frac{3}{4}$ " FILLET
Figure 9

characteristics.

All gages used were Baldwin Southworth SR 4, type A-8 of 1/8" gage length. A standard balancing unit was employed whose smallest calibration of 10 microinches per inch permitted visual interpolation to units.

A twenty circuit rotary selector switch facilitated the reading of the several gages. Equipped with silver contacts, variation of contact resistance was nil.

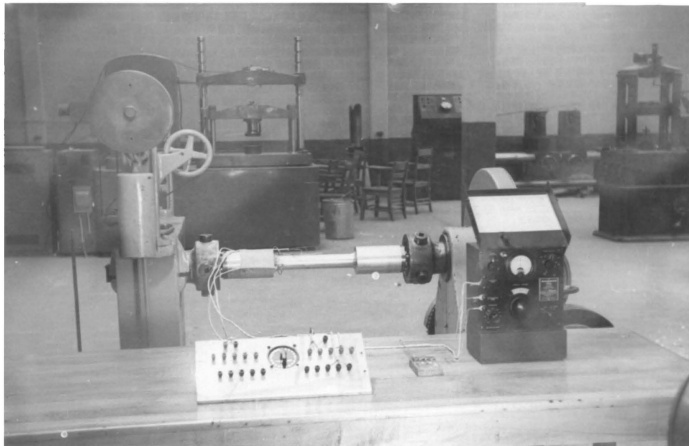
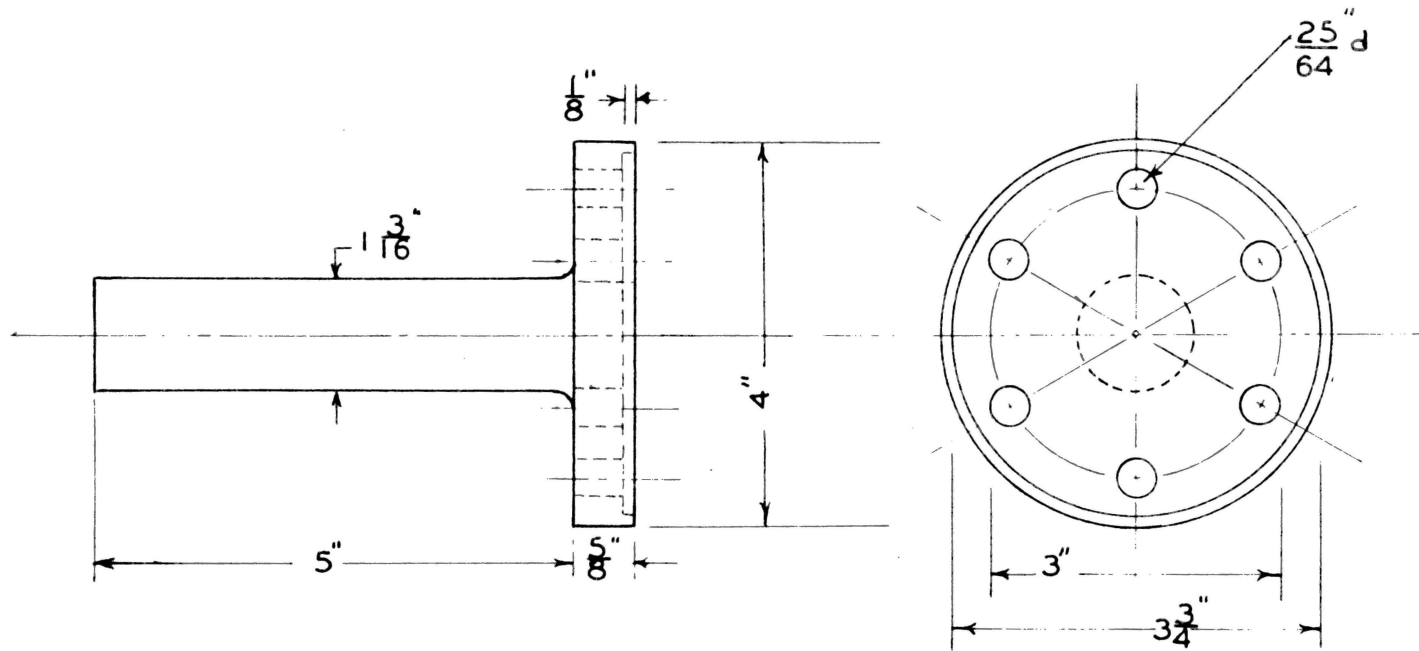


Figure 10, General View of Apparatus

A pair of steel reducing flanges were employed to enable the testing machine to grip the large ends of the specimen. Longitudinal alignment was insured by a matching recess and tenion joint between flange and specimen. Figure 11, on page 21, shows these flanges in detail.



LOADING FLANGE

Figure 11

TEST PROCEDURE

After placing the gages on the larger fillet, the solid specimen was placed in the testing machine. Since jaw type grips were used, the specimen was centered by measurements from the grip periphery.

A small base load was first applied and strain measurements taken at this load. It had been found more convenient to index later strains to this base load rather than zero load since the latter required complete removal from the machine. Further, it was thought that strain differences obtained in this manner would not contain the error due to clamping in the grips, an error sensibly constant throughout the run for any one gage.

The load was then increased in steps, and the gage readings recorded at each loading.

For the remainder of the tests, however, a slightly different procedure was followed. The specimen having been bored to the desired inside diameter, it was placed in the testing machine as before.

A major load was then applied corresponding to roughly 1000 microinches per inch indicated strain, and readings taken. The specimen was then released to a base load of 2000 lb.-in. and the gages were read. The major load was then reapplied, and readings taken.

Base load strains were again measured.

When this cycle was completed, the specimen was removed from the grips and then replaced. The complete cycle was performed a total of four times, a complete removal from the machine grips effected between each. This was done in order to minimize the effect of any misalignment introduced when installing the specimen.

After all four testing cycles had been completed, the specimen was removed from the testing machine and prepared for the next drilling.

It was found necessary to unsolder and remove all wiring from the gages for this operation. The gage lead wires were taped securely to the specimen and the whole center portion covered with craft paper. This was also taped at the edges to prevent the entrance of oil.

The stated procedure was then repeated after each counter boring operation.

During each of the various tests it was found necessary to ground the case of the strain indicator. A severe electrostatic condition accompanied by erratic behavior of the indicator sometimes occurred when this was not done.

Data and Calculations: Solid Shaft

Gage Position Neutral		Fillet Radius = 1.00"					
Torque		0	1	2	4	5	10
10210	1556	1693	1886	2127	1522	2058	1230
14030	1785	1942	2144	2393	1762	2300	1464
15930	1902	2066	2273	2529	1886	2426	1585
17880	2015	2182	2393	2658	2000	2547	1702
Δe_a	229	249	258	266	240	242	234
k		1.086	1.124	1.160	1.047	1.057	1.021
Δe_b	117	124	129	136	124	126	121
k		1.060	1.103	1.162	1.060	1.077	1.033
Δe_c	113	116	120	129	114	121	117
k		1.026	1.062	1.152	1.009	1.071	1.034
Average k		1.057	1.096	1.158	1.039	1.068	1.029

Data: Inside Diameter = 0.50 in.

Gage Position		1" Fillet Radius						Neutral	3/4" Fillet Radius					
Run	Torque	0	1	2	3	5	10		0	1	2	3	4	
1.	17000	1269	1580	1753	1513	2147	1457	1238	1292	2083	1533	2092	1562	1308
	2000	231	479	598	348	1000	403	223	172	981	398	907	389	142
	17000	1248	1561	1741	1503	2133	1433	1227	1283	2072	1528	2081	1552	1297
	2000	217	467	590	338	997	393	213	163	973	392	900	381	132
2.	17000	1223	1567	1709	1460	2129	1437	1201	1272	2073	1533	2056	1522	1263
	2000	177	439	553	303	982	380	183	152	948	392	875	379	98
	17000	1224	1557	1710	1467	2128	1433	1201	1267	2077	1533	2048	1528	1262
	2000	170	429	548	297	973	369	175	141	943	382	866	344	89
3.	17000	1251	1561	1740	1514	2162	1468	1240	1302	2078	1541	2099	1568	1317
	2000	197	457	567	326	1017	423	213	182	962	410	911	378	137
	17000	1232	1542	1723	1493	2158	1453	1222	1279	2064	1532	2079	1550	1297
	2000	169	427	547	303	998	397	185	150	959	387	883	358	111
4.	17000	1243	1527	1736	1501	2119	1417	1203	1247	2067	1511	2047	1541	1273
	2000	178	416	558	309	968	366	173	122	937	367	860	357	98
	17000	1233	1528	1726	1488	2118	1413	1198	1242	2061	1508	2042	1537	1269
	2000	168	409	551	301	967	361	167	117	931	362	853	352	92

Data: Inside Diameter = 0.75 in.

Gage Position		1" Fillet Radius						Neutral	3/4" Fillet Radius					
Run	Torque	0	1	2	3	5	10		0	1	2	3	4	
1.	17000	1462	2007	1869	1660	1377	1640	1292	1647	1357	1881	1268	1469	1719
	2000	363	948	672	431	172	533	122	572	183	697	38	259	516
	17000	1467	2009	1872	1666	1382	1643	1293	1647	1357	1888	1270	1472	1723
	2000	362	947	673	432	171	533	221	572	188	698	36	257	516
2.	17000	1480	2013	1900	1703	1386	1637	1313	1676	1329	1851	1317	1525	1777
	2000	383	961	707	474	163	517	237	567	161	654	66	301	568
	17000	1477	2009	1893	1698	1383	1632	1310	1666	1327	1848	1313	1523	1772
	2000	379	953	702	471	159	512	231	557	160	652	61	299	564
3.	17000	1450	1993	1852	1651	1393	1657	1287	1723	1363	1887	1274	1472	1726
	2000	357	926	662	431	177	538	217	592	192	689	33	258	525
	17000	1447	1983	1854	1653	1387	1643	1287	1709	1357	1880	1272	1471	1727
	2000	352	923	662	430	172	532	213	583	187	686	27	258	524
4.	17000	1435	1968	1847	1644	1323	1577	1263	1690	1287	1838	1270	1460	1693
	2000	339	907	657	422	137	493	200	570	149	666	30	243	500
	17000	1434	1963	1848	1643	1321	1575	1262	1683	1292	1839	1266	1453	1692
	2000	342	910	660	423	136	493	202	567	152	667	27	247	502

Data: Inside Diameter = 1.00 in.

Gage Position		1" Fillet Radius						Neutral		3/4" Fillet Radius				
Run	Torque	0	1	2	3	5	10			0	1	2	3	4
1.	15000	1473	1813	1910	1553	1277	1679	1313	1683	1417	1822	1183	1408	1697
	2000	493	763	847	480	191	697	343	658	340	737	87	337	629
	15000	1478	1818	1917	1560	1273	1683	1312	1688	1417	1823	1183	1409	1698
	2000	489	762	842	477	193	697	345	657	343	742	87	334	626
2.	15000	1552	1872	1983	1609	1267	1683	1333	1724	1467	1864	1193	1413	1706
	2000	547	797	897	518	167	678	357	662	374	760	97	349	647
	15000	1543	1867	1979	1605	1269	1679	1330	1716	1468	1867	1197	1418	1713
	2000	547	795	897	518	170	680	359	663	373	761	99	351	647
3.	15000	1540	1853	1978	1606	1254	1669	1326	1698	1459	1847	1188	1420	1717
	2000	542	787	897	523	173	678	360	660	366	753	98	351	657
	15000	1534	1851	1973	1607	1253	1663	1326	1693	1443	1837	1191	1427	1722
	2000	540	788	896	520	172	678	359	659	367	753	98	351	654
4.	15000	1540	1792	1963	1643	1297	1658	1331	1711	1400	1813	1197	1433	1738
	2000	540	743	877	547	211	672	358	673	318	734	105	357	678
	15000	1541	1796	1962	1641	1297	1654	1330	1711	1398	1812	1197	1432	1739
	2000	1539	739	874	543	213	669	360	674	320	736	107	357	677

Data: Inside Diameter = 1.25 in.

Gage Position		1" Fillet Radius						Neutral	3/4" Fillet Radius					
		0	1	2	3	5	10		0	1	2	3	4	
1.	12000	1210	1527	1587	1252	1093	1424	1133	1639	1450	1513	1943	1190	1602
	2000	363	630	673	336	167	587	300	777	559	603	1020	283	700
	12000	1207	1523	1587	1256	1092	1427	1130	1642	1457	1507	1943	1188	1603
	2000	361	630	675	339	167	586	298	779	560	600	1021	284	699
2.	12000	1207	1509	1589	1259	1087	1416	1133	1632	1449	1507	1944	1192	1600
	2000	358	613	678	343	166	580	303	765	563	597	1024	287	701
	12000	1203	1503	1589	1258	1083	1412	1127	1631	1444	1498	1943	1187	1599
	2000	353	607	673	339	163	577	303	757	558	591	1023	288	693
3.	12000	1202	1500	1587	1257	1073	1400	1123	1627	1437	1493	1933	1182	1593
	2000	352	607	670	336	153	568	290	758	552	587	1010	269	688
	12000	1202	1501	1587	1254	1072	1398	1120	1631	1435	1490	1032	1180	1593
	2000	347	603	667	330	152	567	289	757	549	592	1007	267	687
4.	12000	1206	1463	1602	1277	1039	1357	1113	1623	1417	1462	1923	1173	1592
	2000	356	582	682	348	130	540	289	763	535	561	1011	277	697
	12000	1206	1466	1600	1277	1040	1359	1112	1623	1418	1462	1924	1172	1592
	2000	359	587	683	350	133	543	288	767	539	564	1011	274	700

Calculations: Inside Diameter = 0.50 in.

Gage Position		1" Fillet Radius						Neutral	3/4" Fillet Radius					
Run	Item	0	1	2	3	5	10		0	1	2	3	4	
1.	Δe_a	1038	1101	1155	1165	1147	1054	1015	1120	1102	1135	1185	1173	1166
	Δe_b	1017	1082	1143	1155	1133	1030	1004	1111	1091	1130	1174	1163	1155
	Δe_c	1031	1094	1151	1165	1136	1040	1014	1120	1099	1136	1181	1171	1165
	$\Sigma \Delta e$	3086	3277	3449	3485	3416	3124	3033	3351	3292	3401	3540	3507	3486
	k	1.017	1.080	1.137	1.149	1.126	1.030		1.105	1.085	1.121	1.167	1.156	1.149
2.	Δe_a	1046	1128	1156	1157	1147	1057	1018	1120	1125	1141	1181	1143	1165
	Δe_b	1047	1118	1157	1164	1146	1053	1018	1115	1129	1141	1173	1149	1164
	Δe_c	1054	1128	1162	1170	1155	1064	1026	1126	1134	1151	1182	1184	1173
	$\Sigma \Delta e$	3147	3374	3475	3491	3448	3174	3062	3361	3388	3433	3536	3476	3502
	k	1.028	1.102	1.135	1.140	1.126	1.037		1.098	1.106	1.121	1.155	1.135	1.144
3.	Δe_a	1054	1104	1173	1188	1145	1045	1027	1120	1116	1131	1188	1190	1180
	Δe_b	1035	1085	1156	1167	1141	1030	1009	1097	1102	1122	1168	1172	1160
	Δe_c	1063	1115	1176	1190	1160	1056	1037	1129	1125	1145	1196	1192	1186
	$\Sigma \Delta e$	3152	3304	3505	3545	3446	3131	3073	3346	3343	3398	3552	3554	3526
	k	1.026	1.075	1.141	1.154	1.121	1.019		1.089	1.088	1.106	1.156	1.157	1.147
4.	Δe_a	1065	1111	1178	1192	1151	1051	1030	1125	1130	1144	1187	1184	1175
	Δe_b	1055	1112	1168	1179	1150	1047	1025	1120	1124	1141	1182	1180	1171
	Δe_c	1065	1119	1175	1187	1151	1052	1031	1125	1130	1146	1189	1185	1177
	$\Sigma \Delta e$	3185	3342	3521	3558	3452	3150	3086	3370	3384	3431	3558	3549	3523
	k	1.032	1.083	1.141	1.153	1.119	1.021		1.092	1.097	1.112	1.153	1.150	1.142
Average k		1.023	1.085	1.139	1.149	1.123	1.030		1.096	1.094	1.115	1.158	1.150	1.146

Calculations: Inside Diameter = 0.75 in.

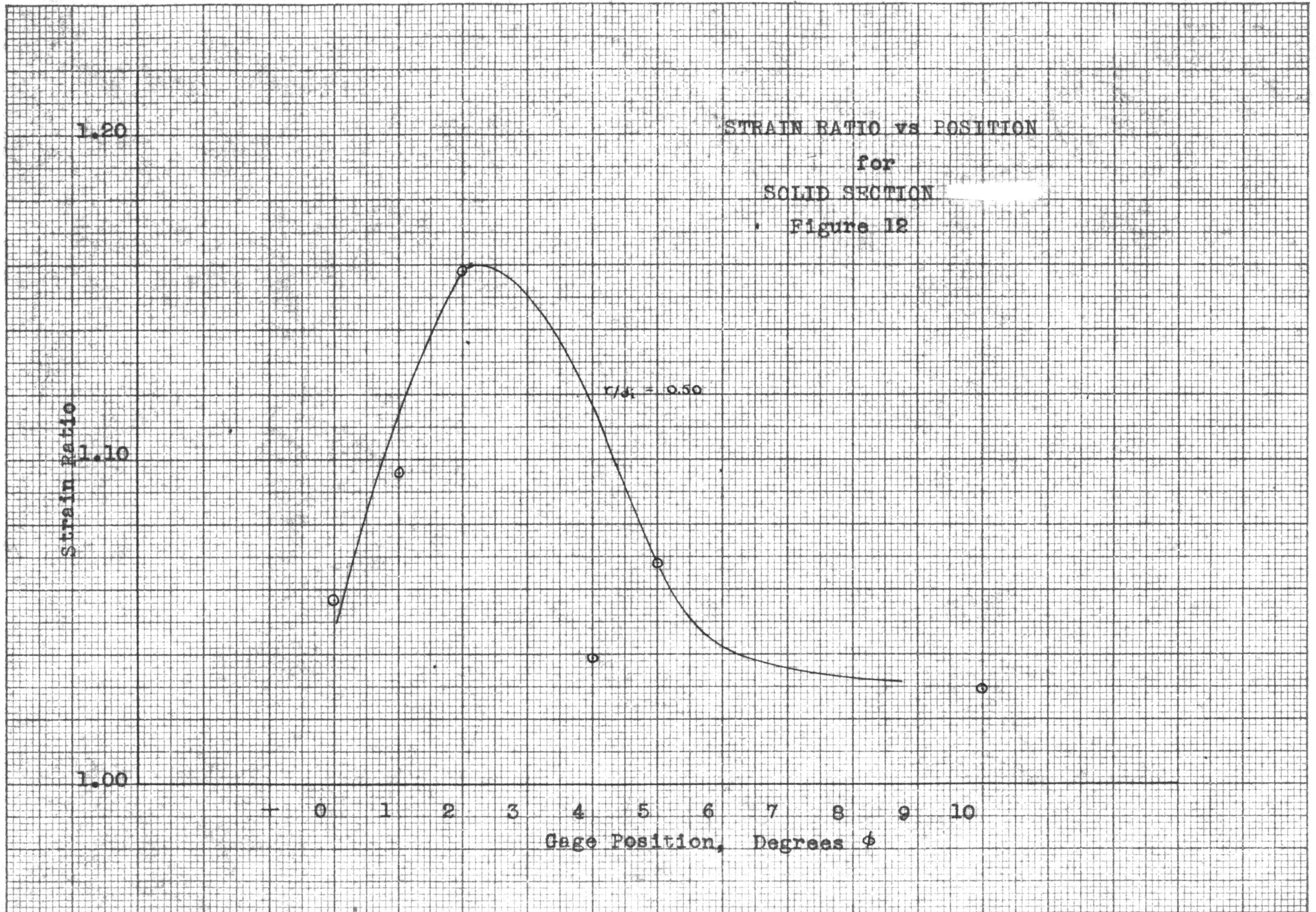
Gage Position		1" Fillet Radius						Neutral	3/4" Fillet Radius					
Run	Item	0	1	2	3	5	10		0	1	2	3	4	
1.	Δe_a	1099	1164	1197	1229	1205	1107	1070	1075	1174	1184	1230	1210	1203
	Δe_b	1104	1170	1200	1235	1210	1110	1071	1075	1174	1191	1232	1213	1207
	Δe_c	1105	1174	1199	1234	1210	1110	1072	1075	1169	1190	1234	1215	1207
	$\Sigma \Delta e$	3308	3508	3596	3698	3625	3327	3213	3225	3517	3565	3696	3638	3617
	k	1.031	1.093	1.120	1.152	1.129	1.037		1.005	1.093	1.110	1.151	1.132	1.126
2.	Δe_a	1097	1168	1193	1229	1223	1120	1076	1109	1168	1197	1251	1224	1209
	Δe_b	1094	1163	1186	1224	1220	1115	1073	1099	1166	1194	1247	1222	1204
	Δe_c	1098	1164	1191	1227	1224	1120	1079	1109	1167	1196	1252	1224	1208
	$\Sigma \Delta e$	3289	3495	3570	3680	3667	3355	3228	3317	3501	3587	3750	3670	3621
	k	1.019	1.082	1.107	1.149	1.137	1.039		1.027	1.087	1.112	1.162	1.137	1.123
3.	Δe_a	1093	1176	1190	1220	1216	1119	1070	1131	1171	1198	1241	1214	1201
	Δe_b	1090	1165	1192	1222	1210	1105	1070	1117	1165	1191	1239	1213	1202
	Δe_c	1095	1166	1192	1223	1215	1111	1074	1126	1170	1194	1245	1213	1203
	$\Sigma \Delta e$	3278	3507	3574	3665	3641	3335	3214	3374	3506	3583	3725	3640	3606
	k	1.019	1.092	1.110	1.142	1.133	1.037		1.050	1.092	1.115	1.159	1.132	1.122
4.	Δe_a	1096	1149	1190	1222	1186	1084	1063	1120	1138	1172	1240	1217	1193
	Δe_b	1095	1150	1191	1221	1184	1082	1062	1113	1143	1173	1236	1210	1192
	Δe_c	1092	1150	1188	1220	1185	1082	1060	1116	1140	1172	1237	1206	1190
	$\Sigma \Delta e$	3283	3449	3569	3663	3555	3248	3185	3349	3421	3517	3715	3633	3575
	k	1.031	1.083	1.121	1.150	1.117	1.020		1.052	1.073	1.103	1.167	1.140	1.123
Average k		1.025	1.088	1.114	1.148	1.129	1.033		1.036	1.086	1.110	1.160	1.135	1.123

Calculations: Inside Diameter = 1.00"

Gage Position		1" Fillet Radius						Neutral	3/4" Fillet Radius					
Run	Item	0	1	2	3	5	10		0	1	2	3	4	
1.	$\Delta\epsilon_a$	980	1050	1063	1073	1086	982	970	1025	1077	1085	1096	1071	1068
	$\Delta\epsilon_b$	985	1055	1070	1080	1082	986	969	1030	1077	1086	1096	1072	1069
	$\Delta\epsilon_c$	989	1056	1075	1083	1080	986	967	1031	1074	1081	1096	1075	1072
	$\Sigma\Delta\epsilon$	2954	3161	3208	3236	3248	2954	2906	3086	3228	3252	3288	3218	3209
	k	1.017	1.089	1.104	1.113	1.118	1.017		1.062	1.112	1.119	1.132	1.107	1.104
2.	$\Delta\epsilon_a$	1005	1075	1086	1091	1100	1005	976	1062	1093	1104	1096	1064	1059
	$\Delta\epsilon_b$	996	1070	1082	1087	1102	1001	973	1054	1094	1107	1100	1069	1066
	$\Delta\epsilon_c$	996	1072	1082	1087	1099	999	971	1053	1095	1106	1098	1067	1066
	$\Sigma\Delta\epsilon$	2997	3217	3250	3265	3301	3005	2920	3169	3282	3317	3294	3200	3191
	k	1.027	1.101	1.113	1.117	1.130	1.028		1.085	1.123	1.137	1.128	1.096	1.092
3.	$\Delta\epsilon_a$	998	1066	1081	1083	1081	991	966	1038	1093	1094	1090	1069	1060
	$\Delta\epsilon_b$	992	1064	1076	1084	1080	985	966	1033	1077	1084	1093	1076	1065
	$\Delta\epsilon_c$	994	1063	1077	1087	1081	985	967	1034	1076	1084	1093	1076	1068
	$\Sigma\Delta\epsilon$	2984	3193	3234	3254	3242	2961	2899	3105	3246	3262	3276	3221	3193
	k	1.030	1.101	1.117	1.123	1.118	1.021		1.071	1.120	1.126	1.130	1.111	1.100
4.	$\Delta\epsilon_a$	1000	1049	1086	1096	1086	986	973	1038	1082	1079	1092	1076	1060
	$\Delta\epsilon_b$	1001	1053	1085	1094	1086	982	972	1038	1080	1078	1092	1075	1061
	$\Delta\epsilon_c$	1002	1057	1088	1098	1084	985	970	1037	1078	1076	1090	1075	1062
	$\Sigma\Delta\epsilon$	3003	3159	3259	3288	3256	2953	2915	3113	3240	3233	3274	3226	3183
	k	1.030	1.083	1.117	1.128	1.116	1.012		1.067	1.111	1.108	1.123	1.107	1.092
Average k		1.026	1.094	1.113	1.120	1.121	1.020		1.071	1.117	1.123	1.128	1.105	1.097

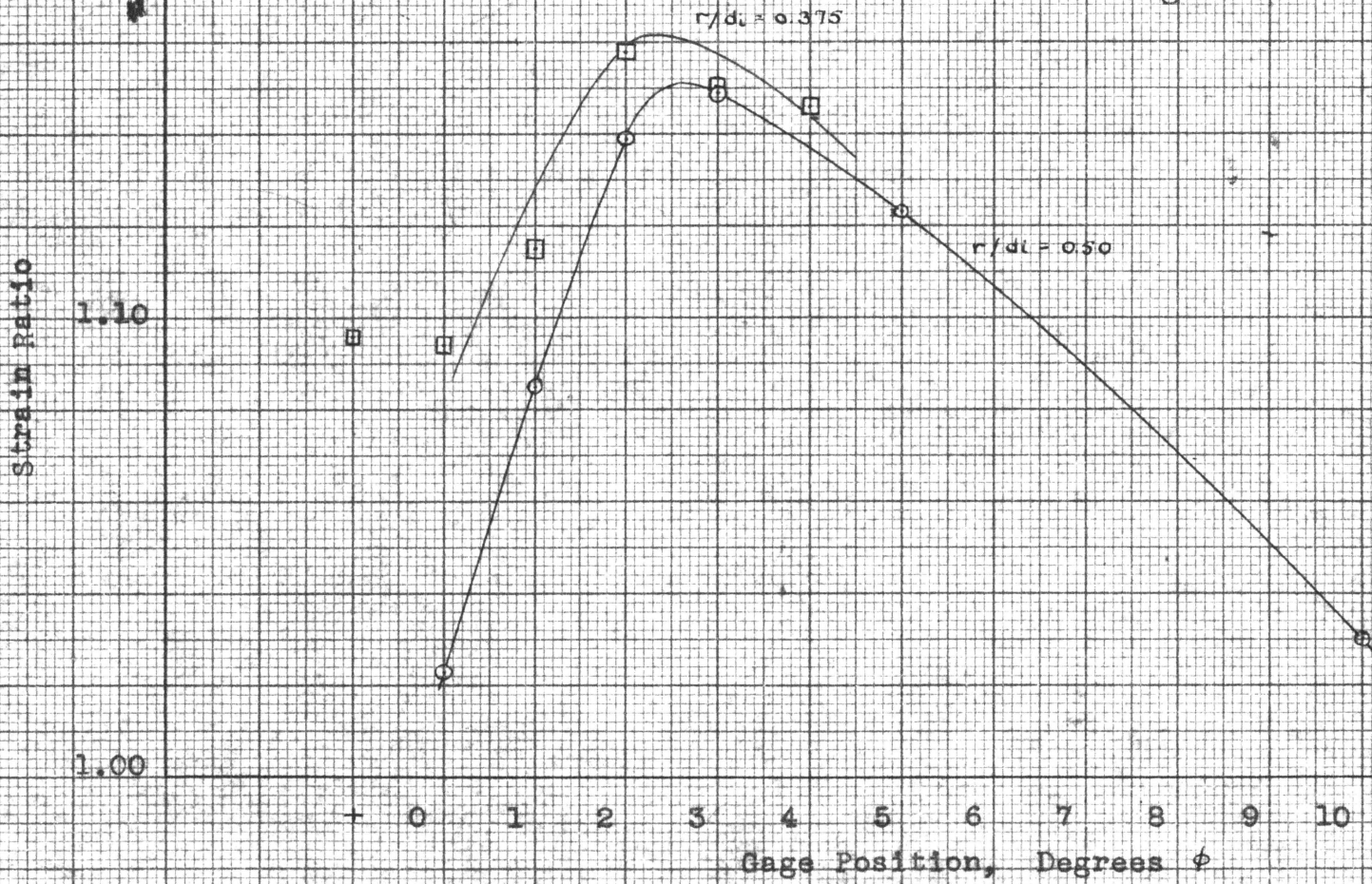
Calculations: Inside Diameter = 1.25"

Gage Position		1" Fillet Radius						Neutral	3/4" Fillet Radius					
Run	Item	0	1	2	3	5	10		0	1	2	3	4	
1.	Δ_{ea}	847	897	914	916	926	837	833	862	891	910	923	907	902
	Δ_{eb}	844	893	914	920	925	840	830	865	898	904	922	905	903
	Δ_{ec}	846	893	912	917	925	841	832	863	897	907	922	904	904
	$\Sigma \Delta \epsilon$	2537	2683	2740	2753	2776	2518	2495	2590	2686	2721	2768	2716	2709
	κ	1.027	1.078	1.100	1.103	1.113	1.010		1.038	1.077	1.090	1.110	1.087	1.087
2.	Δ_{ea}	849	896	911	916	921	836	830	867	886	910	920	905	899
	Δ_{eb}	845	890	911	915	917	832	824	866	881	901	919	900	898
	Δ_{ec}	850	896	916	919	920	835	824	874	886	907	920	899	906
	$\Sigma \Delta \epsilon$	2544	2682	2738	2750	2758	2503	2498	2607	2653	2718	2759	2704	2703
	κ	1.027	1.082	1.103	1.109	1.113	1.010		1.053	1.070	1.097	1.113	1.092	1.092
3.	Δ_{ea}	850	893	917	921	920	832	833	869	885	906	923	913	905
	Δ_{eb}	850	894	917	918	919	830	830	873	883	903	922	911	905
	Δ_{ec}	855	898	920	924	920	831	831	874	886	898	925	913	906
	$\Sigma \Delta \epsilon$	2555	2683	2754	2763	2759	2493	2494	2616	2654	2707	2770	2737	2716
	κ	1.023	1.076	1.102	1.107	1.105	1.000		1.047	1.063	1.085	1.111	1.097	1.088
4.	Δ_{ea}	850	881	920	929	909	817	824	860	882	901	912	896	895
	Δ_{eb}	850	884	918	929	911	819	823	860	883	901	913	895	895
	Δ_{ec}	847	879	917	927	908	816	824	856	879	898	913	898	892
	$\Sigma \Delta \epsilon$	2547	2644	2755	2785	2728	2452	2471	2576	2644	2700	2738	2689	2682
	κ	1.032	1.071	1.114	1.127	1.104	.995		1.043	1.070	1.093	1.108	1.088	1.086
Average κ		1.025	1.077	1.105	1.113	1.109	1.004	1.000	1.045	1.070	1.091	1.111	1.091	1.088



STRAIN RATIO vs POSITION
for
INSIDE DIAMETER = 0.50"

Figure 13



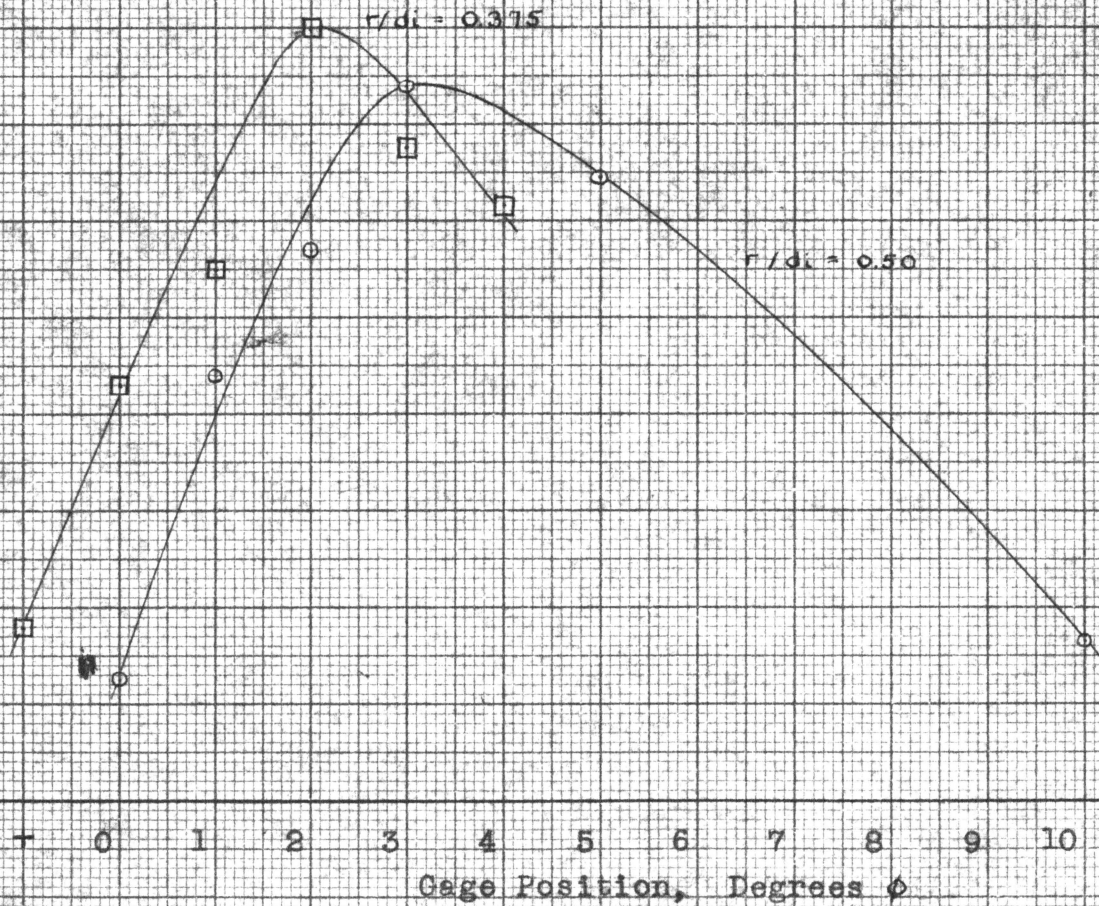
STRAIN RATIO vs. POSITION
for
INSIDE DIAMETER = 0.75"
Figure 14

STRAIN RATIO

1.20

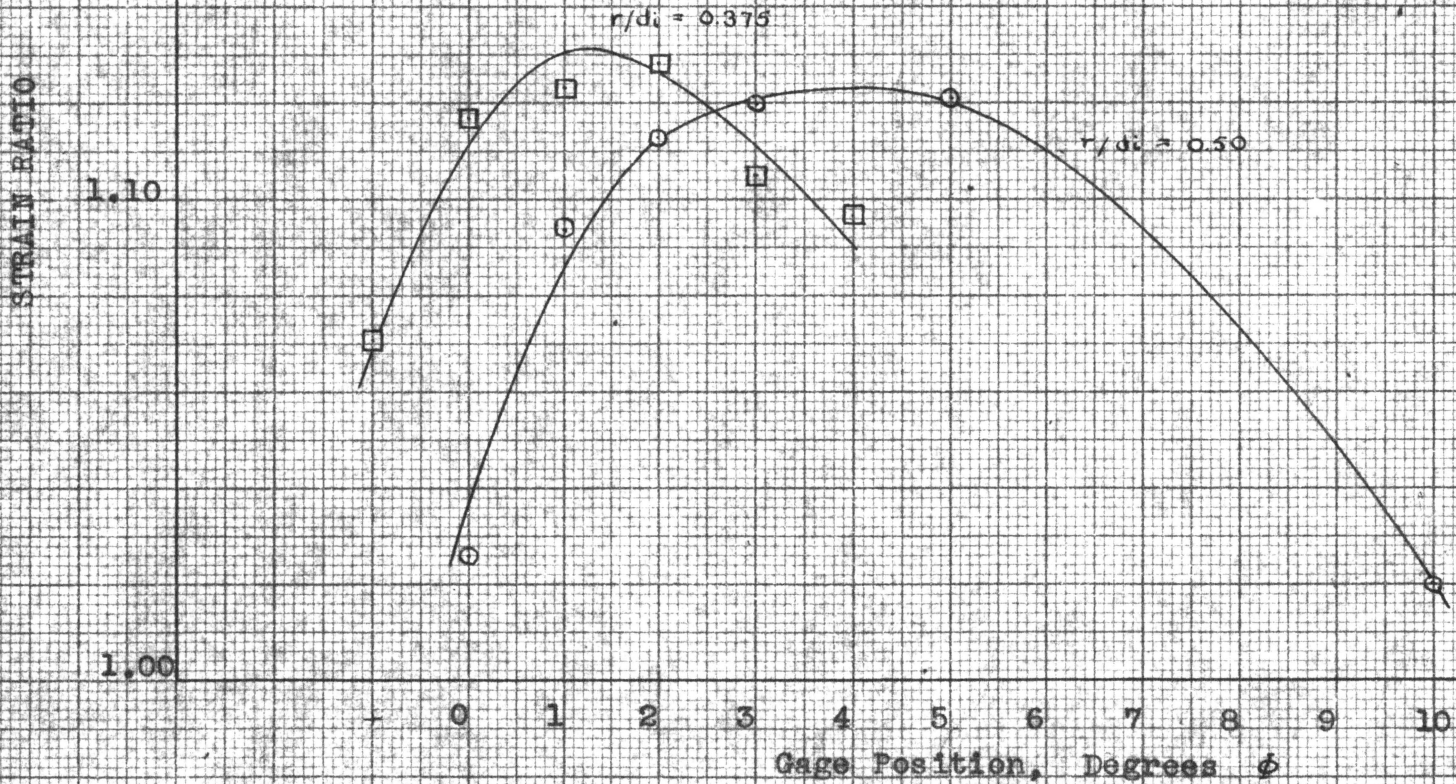
1.10

1.00



STRAIN RATIO vs POSITION
for
INSIDE DIAMETER = 1.00"

Figure 15



STRAIN RATIO vs POSITION
for
INSIDE DIAMETER = 1.25"
Figure 16

STRAIN RATIO

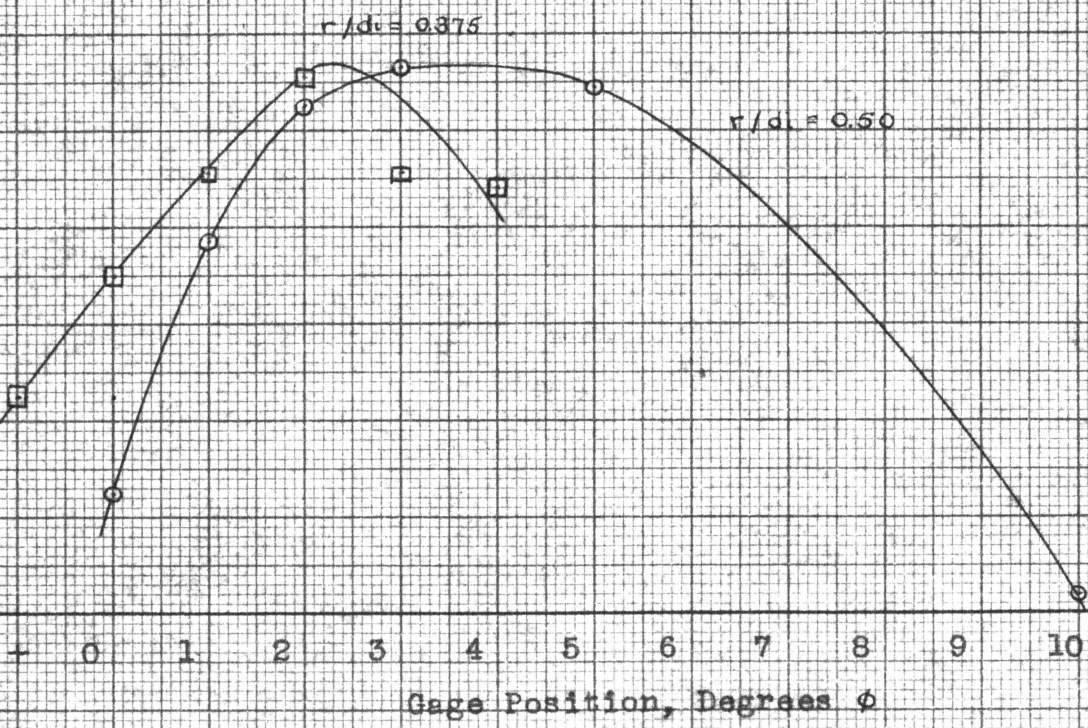
1.20

1.10

1.00

$r/d_i = 0.375$

$r/d_i = 0.50$

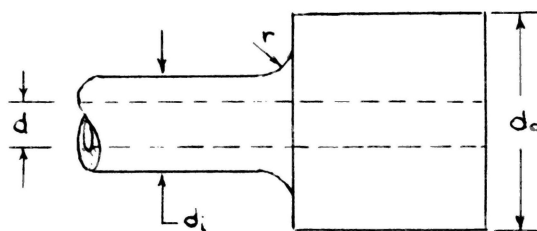


Gage Position, Degrees ϕ

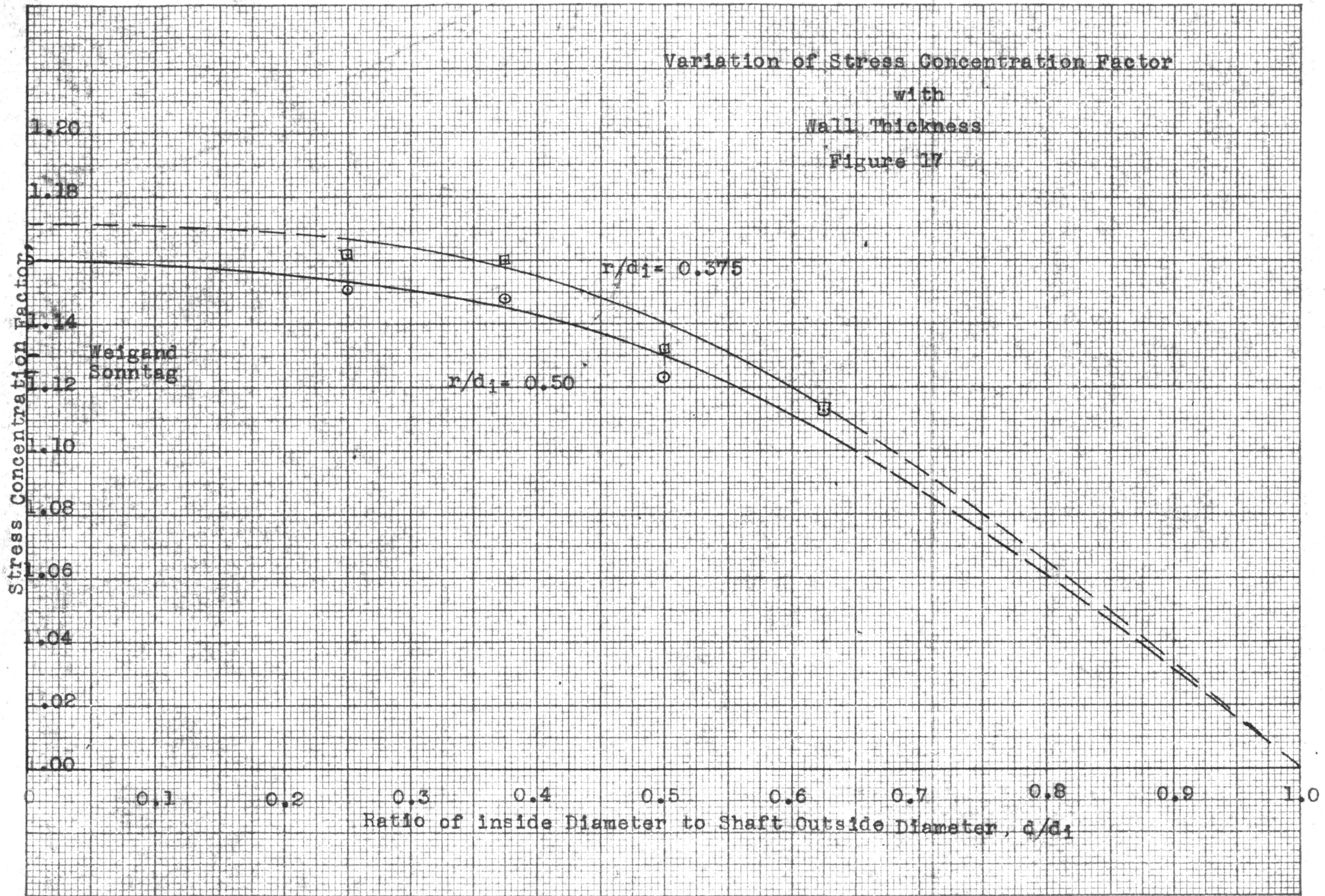
RESULTS

The values of stress concentration factor determined in this investigation are tabulated below. The values described as measured are those actually computed from the experimental data. In addition, values taken from the faired curve, Figure 17, page 39, are also presented.

r/d_i	r/d_i	k (measured)	k (faired)
0.50	0.00	1.160	1.160
	0.25	1.151	1.154
	0.375	1.148	1.145
	0.50	1.123	1.130
	0.625	1.113	1.106
0.375	0.25	1.162	1.167
	0.375	1.160	1.158
	0.50	1.132	1.140
	0.625	1.114	1.115



Key to Symbols



DISCUSSION

ACCURACY

Previous work on the problem of torsional stress concentration being limited to the study of stepped shafts of solid cross-section, the results of the test of the solid shaft with $r/d_i = 0.5$ form the only basis of comparison with the results of other investigators.

The value of the stress concentration factor determined for this case was compared with the value predicted by Sonntag's¹ Formula and also with the extrapolated results of A. Weigand². This extrapolation was necessary since $r/d_i = 0.5$ was beyond the scope of that investigation. Other papers read on this subject dealt largely with values of r/d_i of the order of 0.1 and did not lend themselves to extrapolation to the values required.

It should be noted that the fillet size studied was at the limit of validity for the Sonntag Formula. This formula is a fair representation of the accumulative experimental data, prior to Mr. Weigand's Investigation.

Source	Stress Concentration Factor	r/G ave.
Sonntag Formula	1.125	-
A. Weigand (extr.)	1.13	7.7
Author's Value	1.16	8.0

1. Appendix I
2. Appendix II

It is seen that the results of this investigation are somewhat higher than either previously determined value although not to a significant degree. However, both the method of Mr. Weigand and of the author suffer from a common source of error, that of finite gage length. The ratio of fillet radius to gage length (G) may be taken as a rough index, to the extent of this error.

In view of the fact that the stress varies as a pointwise function of position along the fillet surfact, rising to a maximum at one section, it is inevitable that error be present in any attempt to measure this maximum with a gage of finite length. The severity of this restriction may be judged from Figure 18, page 44.

It has been observed that the strain-position curve rises to a maximum over a comparatively short interval. The one-eighth inch gage used in this investigation clearly blankets this interval, extending over 7.15 degrees. Considerable "averaging" is thus to be expected, with the measured strains being somewhat smaller than the true value at any point.

Unfortunately, the extent of this error is not known to the author. Further investigation is thus indicated with a view toward increasing the r/G ratio. Since smaller gages appear impractical at the present, increased specimen size appears the more attractive approach.

Error in the results of this investigation due to incorrect gage position were thought to be of a minor degree, since the relative positions were maintained by the use of a template. While the position of the maximum stress might well vary as much as one degree, the numerical value of this maximum is thought to be truly represented in this respect.

The gages proper were controlled to a variation of three per cent according to the manufacturer's specifications. The resultant accuracy of the several gages used was expected to be slightly greater.

RESULTS

The values of the stress concentration factor determined in this investigation were seen to be reasonably compatible with the earlier works on solid shafts.

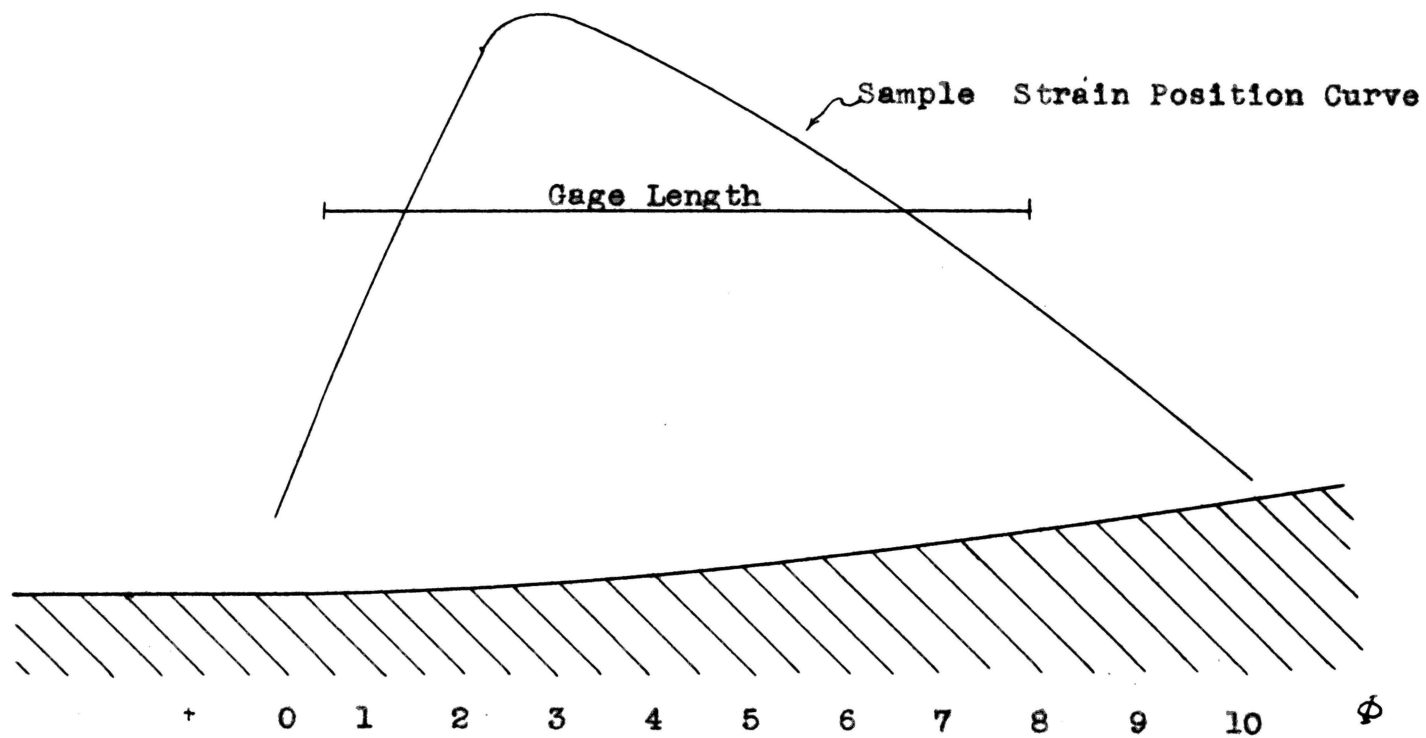
One of the more significant features is the establishment of a decreasing relationship between the stress concentration factor and the d/d_f ratio. This relation is presented graphically, figure 17, page 39.

The character of this relationship remains largely independent of the averaging error. That this trend was observed for both fillets tested served to substantiate its truth.

From the general trend of the data obtained, it may be expected that the stress concentration factor approaches

unity as the d/d_1 ratio approaches unity. For this hypothetical limit of zero wall thickness the stress concentration factor becomes Mathematically indeterminate, zero divided by zero, since no stress can be transmitted. The manner of approach to this limit must therefore serve to establish the limiting value of the stress concentration factor.

The choice of unity appears a reasonable one on the strength of the data available, however, further investigation is indicated to satisfactorily establish this limit. Definite knowledge regarding this limiting value would undoubtedly prove to be a great step toward a more complete understanding of the problem.



RELATIVE SIZE OF GAGE LENGTH, Figure 18

CONCLUSIONS

1. The method of placing SR-4 gages by means of the intermediate template described was seen to be capable of good accuracy.
2. The stress concentration factor decreases with an increasing d/d_1 ratio, all other conditions constant.
3. Further investigation of the problem is indicated with greater r/G ratios, to definitely establish the values of stress concentration factor determined.

SUMMARY

In this investigation the stress concentration factor was evaluated for a hollow stepped shaft with variable wall thickness and two fillet sizes. Electric strain gages of the SR-4 type were employed to measure strains in the fillet section.

Values of the stress concentration factor were determined with the ratio of inside to outside diameter ranging from 0.25 to 0.625. A decreasing relation was established between stress concentration and the internal diameter.

A satisfactory method of positioning gages accurately on the complex fillet surface was developed. This method featured an intermediate steel template with positive referencing.

NOMENCLATURE

r	Fillet Radius
d _o	Large shaft diameter
d _i	Small shaft diameter
d	Diameter of longitudinal hole
φ	Angular position of gage
L	Arc Length to gage position
k	Stress concentration factor
G	Gage Length
e	Indicated unit strain

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation and gratitude to Prof. D. H. Pletta for his generous contribution of time and enthusiastic advice throughout the entire investigation.

APPENDIX I

STRESS CONCENTRATION FACTOR BY SONNTAG FORMULA

The stress concentration factor for the larger fillet, and solid cross section is calculated on the basis of Sonntag's Formula.

$$k = B [1.5 + 3.0A] \left[\frac{1+4A}{1+6A} \right] + [1 - B - 2AB] \left[1 + \frac{1}{12A} \right]$$

$$\text{for } B < \frac{1}{1+2A}$$

$$\text{where } A = r/d_i \quad B = d_i/d_o$$

$$\therefore A = \frac{1.0}{2.0} = 0.50$$

$$B = \frac{2.0}{4.0} = 0.50$$

$$\frac{1}{1+2A} = \frac{1}{1+2(0.5)} = 0.50$$

$$\begin{aligned} \therefore k &= 0.5 [1.5 + (3.0 \times 0.5)] \left[\frac{1 + (4.0 \times 0.5)}{1 + (6.0 \times 0.5)} \right] + [1 - 0.5 - (2 \times 0.5 \times 0.5)] \left[1 + \frac{1}{12(0.5)} \right] \\ &= 0.5 [3.0] [0.75] + 0 \end{aligned}$$

$$k = 1.125$$

APPENDIX II

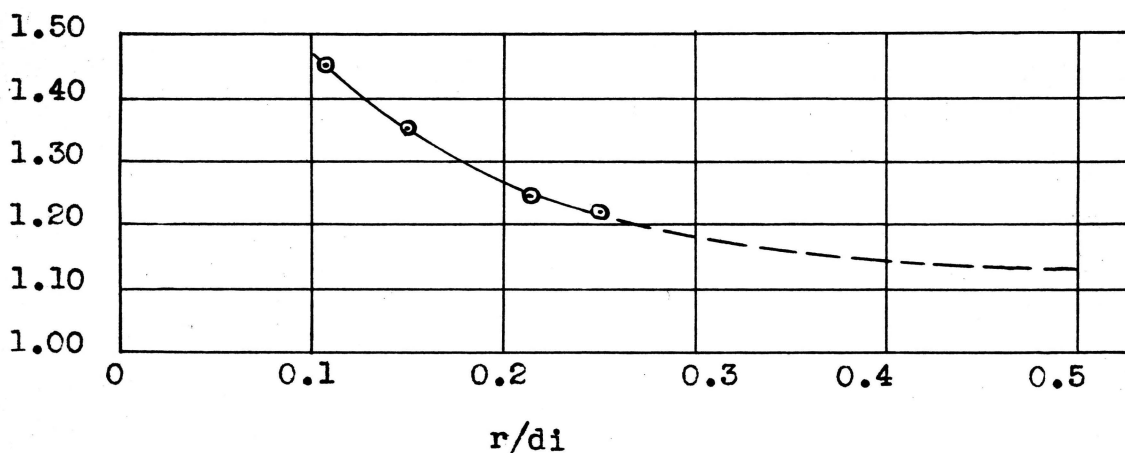
EXTRAPOLATION FROM A PRIOR INVESTIGATION

The results of a comparatively recent investigation of solid shafts by A. Weigand were extrapolated to include the case of $d_i/d_o = 0.5$ and $r/d_i = 0.5$

The following faired values were taken from N.A.C.A. Technical Memorandum 1179.

$d_i/d_o = 0.5$

r/d_i	0.250	0.214	0.150	0.107
k	1.22	1.25	1.36	1.45



The approximate value of k thus determined was 1.13

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