Chapter 5

RESULTS AND DISCUSSION

The results obtained using the base Fuzzy GoDEA model and its variations are presented in this chapter. First, the Fuzzy GoDEA Base Model and its variations with the solution outcomes are summarized. Second, the efficiency and effectiveness goals are explained with respect to the results. Third, a discussion of the results obtained for the feasible models is presented. Fourth, the achievement of the efficiency and effectiveness goals is compared across the variations of the Fuzzy GoDEA Model. Finally, the performance of the packaging line is evaluated.

5.1 APPLICATION DATA

The derivation of the data used for the application was presented in Chapter 4. The complete data set is reported in Appendix A and descriptive statistics on the data are provided in Appendix B. Girod 's (1996) results for the packaging line application were obtained utilizing input/output data for 48 production days. Girod (1996) reported that a reorganization of the packaging line management took place approximately around day 22. To gain insights into the possible impact of this reorganization the data set of 48 observations is divided into two halves of 24 observations each. Thus, the Fuzzy GoDEA model variations are applied to three data sets, namely production days 1-24, production days 25-48 and production days 1-48. Table 5.1 displays the summary of the average input and output data values for the three data sets.

Table 5.1 Average Input/Output Data for the Packaging Line

Output/Inputs	*Prod. Days 1-24*	*Prod. Days 25-48*	*Prod. Days 1-48*
PCF	**129061.93**	**169458.62**	**149260.27**
DLR	**1310.42**	**1663.79**	**1487.10**
RWK	**7240.25**	**11017.00**	**9128.63**
RML	**5880.87**	**6720.67**	**6300.77**

5.2 FUZZY GODEA MODELS SUMMARY

Table 5.2 presents a summary of the model variations utilized in this research to capture different decision-making scenarios. The outcomes of the models are included in the summary for comparison across models.

Table 5.2 Summary of the Fuzzy GoDEA Model Variations

No.	*Variation*	*Model*	*Outcome*
1.	Base	Max Σ μ's St Fuzzy DEA Constraints Fuzzy GT Constraints $0 \leq \mu \leq 1$, $\lambda \geq 0$ $\Sigma\, \lambda = 1$	Feasible - Optimal
2.	Variation 1	Max Σ (Weighted DEA μ's + Weighted GT μ's) St Fuzzy DEA Constraints Fuzzy GT Constraints $0 \leq \mu \leq 1$, $\lambda \geq 0$ $\Sigma\, \lambda = 1$	Feasible - Optimal
3.	Variation 2	***Stage 1*** Max Σ GT μ's St Fuzzy GT constraints $0 \leq \mu \leq 1$, $\lambda \geq 0$ $\Sigma\, \lambda = 1$	Feasible - Optimal

		Stage 2 Max Σ DEA μ's St Fuzzy DEA constraints Fuzzy GT constraints *(with GT μ values fixed form Stage 1)* $0 \le \mu \le 1,\ \lambda \ge 0$ $\Sigma\ \lambda = 1$	*Infeasible*
4.	Variation 3	***Stage 1*** Max Σ DEA μ's St Fuzzy DEA constraints $0 \le \mu \le 1,\ \lambda \ge 0$ $\Sigma\ \lambda = 1$ ***Stage 2*** Max Σ GT μ's St Fuzzy DEA constraints *(with DEA μ values fixed from Stage 1)* Fuzzy GT constraints $0 \le \mu \le 1,\ \lambda \ge 0$ $\Sigma\ \lambda = 1$	Feasible - Optimal *Infeasible*
5.	Variation 4	Max Σ DEA μ's St Fuzzy DEA constraints Crisp GT Constraints *(with GTs = Sum of Individual Bounds)* $0 \le \mu \le 1,\ \lambda \ge 0$ $\Sigma\ \lambda = 1$	Feasible - Optimal
6	Variation 5	Max Σ GT μ's St Crisp DEA constraints *(with RHS = Observed DMU Values)* Fuzzy GT Constraints $0 \le \mu \le 1,\ \lambda \ge 0$ $\Sigma\ \lambda = 1$	Feasible - Optimal
7.	Variation 6	***Stage 1*** Max Σ DEA μ's St Fuzzy DEA constraints $0 \le \mu \le 1,\ \lambda \ge 0$ $\Sigma\ \lambda = 1$	Feasible - Optimal

		Stage 2 Min Σ GT Deviations *(i.e., Min NPCF+PDLR+PRWK+PRML)* St Fuzzy DEA constraints *(with DEA μ values fixed from Stage 1)* Crisp GT constraints *(with GTs = Sum of Individual Bounds)* $0 \leq \mu \leq 1, \lambda \geq 0$ $\Sigma \lambda = 1$	Feasible - Optimal
8.	Variation 7	***Stage 1*** Min Σ GT Deviations *(i.e., Min NPCF+PDLR+PRWK+PRML)* St Crisp GT constraints *(with GTs = Sum of Individual Bounds)* $0 \leq \mu \leq 1, \lambda \geq 0$ $\Sigma \lambda = 1$ ***Stage 2*** Max Σ DEA μ's St Fuzzy DEA constraints Crisp GT constraints *(with deviations fixed from Stage 1)* $0 \leq \mu \leq 1, \lambda \geq 0$ $\Sigma \lambda = 1$	Feasible - Optimal *Infeasible*
9.	Variation 8	***Stage 1*** Min Σ DEA Deviations *{i.e., Min (d^- +d^+)}* St Crisp DEA constraints *(with RHS = Observed DMU Values)* $0 \leq \mu \leq 1, \lambda \geq 0$ $\Sigma \lambda = 1$ ***Stage 2*** Max Σ GT μ's St Crisp DEA constraints *(with deviations substituted from Stage 1)* Fuzzy GT constraints $0 \leq \mu \leq 1, \lambda \geq 0$	Feasible - Optimal Feasible - Optimal

		$\Sigma\, \lambda = 1$	
10.	Variation 9	***Stage 1*** Max Σ GT μ's St Fuzzy GT constraints $0 \leq \mu \leq 1,\ \lambda \geq 0$ $\Sigma\, \lambda = 1$ ***Stage 2*** Min Σ DEA Deviations *{i.e., Min ($d^- + d^+$)}* St Crisp DEA constraints Fuzzy GT constraints *(with GT μ* values *fixed from Stage 1)* $0 \leq \mu \leq 1,\ \lambda \geq 0$ $\Sigma\, \lambda = 1$	Feasible - Optimal Feasible - Optimal

The summary in Table 5.2 provides a snapshot of the behavior of the models for the packaging line application. Detailed results for the feasible model variations for the three data sets are reported in Appendices C, D, and E. The discussion of results for the model variations categorized by the efficiency and effectiveness goals is presented next.

5.3 EFFICIENCY GOAL

The efficiency goal for this research was measured using DEA type constraints for the individual DMU representations *i.e.,* each input (DLR, RWK, RML) and output (PCF) for each DMU was measured relative to all the DMUs in the data set. The Fuzzy GoDEA model is not designed to provide an efficiency score for each DMU. The efficiency score can readily be obtained by conventional DEA analysis. However, the achievement of the efficiency goal through the DEA type constraints in the Fuzzy GoDEA formulation provides additional useful insights. The results obtained when the efficiency constraints are fuzzy reveal two dimensions of interest. Each constraint is constructed by comparing the composite input/output unit with the observed input/output for each DMU. In conventional BCC evaluation this composite unit is a convex combination of efficient units. If the DMU under consideration is efficient then the composite unit is basically the DMU itself and thus the associated activity level is equal to one. If the DMU under consideration is inefficient then the activity levels associated with the efficient DMUs in the composite unit sum up to one. In other words the activity levels denote the percentage contribution of each efficient unit in the definition of the efficient reference unit on the efficient frontier for the inefficient DMU.

The membership functions associated with the fuzzy efficiency (DEA type) constraints represent the degree of satisfaction of these constraints. The activity levels (λ) associated with the inputs and output are free to take on non-negative values (with the restriction that the sum of the activity levels equals one) that would make the composite unit less than or equal to the observed values for the assessed DMU for the inputs and greater than or equal to the observed values for the assessed DMU for the output. When the membership function for such a constraint equals one or is very close to one it implies that the DEA input or output inequality is satisfied crisply. Consequently, it follows that all the membership functions associated with the efficiency constraints should attain the value one to achieve a crisp DEA type evaluation. If any of these membership functions achieves a value less than one then it signifies a relaxation of the DEA structure for that particular DMU. This is the fuzzy dimension associated with the efficiency goal in the Fuzzy GoDEA formulation. In the present application the DEA structure was maintained crisply for all variations of the Fuzzy GoDEA model.

The activity levels (λ) require additional analysis following the evaluation of the membership functions. In the absence of an efficiency score the activity levels for each DMU reveal whether it is efficient or inefficient. For a DMU to be 100% efficient the activity level associated with it in the composite unit must attain the value one. This implies that such a DMU is its own "reference set" as it is 100% efficient relative to all the members of the data set. An inefficient DMU is one with a reference set (or peers) that consists of *other* DMUs. In conventional DEA this reference set would contain only efficient DMUs. The Fuzzy GoDEA formulation provides a departure from conventional DEA in this regard. The reference set for an inefficient DMU is allowed to have *inefficient* DMUs in addition to the efficient DMUs. In every variation the reference sets for inefficient DMUs typically includes only units evaluated as efficient in *that* variation. The BCC efficiency scores are used in this research to characterize the behavior of the BCC-inefficient DMUs as peers in the Fuzzy GoDEA model variations. Typically, the BCC-inefficient peers for an inefficient DMU will display relatively small amounts of inefficiency *i.e.*, will have high BCC efficiency scores. However, the presence of a large number of BCC-inefficient DMUs in the reference sets cannot be ruled out. A high frequency of BCC-inefficient peer units can be attributed to a large variation in the data (*e.g.*, the minimum and maximum values of an input/output are very spread out). In the present application, data set 1-48 shows a much higher variation in the data ranges than data sets 1-24 and 25-48. This results in a high variation of the BCC efficiency scores for data set 1-48 as compared to data sets 1-24 and 25-48. Consistent with this observation, data sets 1-24 and 25-48 display a low frequency of BCC-inefficient peer units while data set 1-48 displays a much higher frequency. It is not the purpose of the Fuzzy GoDEA model to corroborate the results of conventional DEA analysis. The fuzzy formulation applied in this research aims to relax the DEA evaluation and to allow relative efficiency comparison with not only 100% BCC-efficient DMUs but also less than 100% BCC-efficient units. Alternately, the concept of a *crisp efficient frontier* that envelops the data is modified to allow a *thick* frontier.

Moreover, the inclusion of DMUs less than 100% BCC-efficient makes a case that it is more realistic for an inefficient unit to attain the input/output levels of *near BCC-efficient* units before trying to achieve the input/output levels of efficient units. The quantification of a *near BCC-efficient unit* is open to subjectivity. The results obtained from the application of this research to the three data sets suggest that the decision-maker would have to make a subjective decision regarding the threshold for near BCC-efficient and BCC-inefficient units. This philosophy is in line with the fuzzy concepts proposed in this research where the decision-maker seeks for a compromise that provides a *satisfycing* level of all goals rather than an optimal achievement level of all goals. The peers for the inefficient DMUs for the three data sets are displayed in Tables 5.3, 5.4 and 5.5.

When the efficiency constraints are crisp they either have slack variables or deviation variables to reflect the inefficiency depending on the Fuzzy GoDEA variation characteristics. For the one stage approach with fuzzy effectiveness constraints and crisp efficiency constraints the inefficiency is reflected via the slack variables. An efficient DMU has all slack variables associated with the inputs and output equal to zero. An inefficient DMU will have at least one non-zero slack variable. Consequently, it follows that an inefficient DMU can become efficient by reducing its slack variables to zero.

The goal programming approach minimizes the positive and negative deviations of the composite unit from the observed values for each DMU. The deviation variables replace the slack/surplus variables and the contraction/expansion factors in conventional DEA. A DMU for which all deviations are zero and the activity level associated with itself equals one is considered efficient *i.e.,* the efficient DMU acts as its own reference unit. It is possible for a DMU to have an activity level of one as its peer that corresponds to another DMU *i.e.,* the activity level is associated with an efficient DMU in the data set. This implies that such a DMU is located on a facet of the efficient frontier but has some inefficiency associated with one or more of its input/output variables. This inefficiency is represented by the non-zero deviation variables for this DMU. Consequently, it follows that such a DMU can become efficient by reducing its deviation variables to zero.

The deviation variables represent the amounts of inefficiency in the variations that employ a 2-stage goal programming approach with crisp efficiency constraints (Variations 8 and 9). Only one of the negative or positive deviation for each constraint can assume a non-zero value. It is intuitive that for an inefficient DMU the associated slack (Variation 5) and deviation variables (Variations 8 and 9) representing a DMU's inefficiency will be identical when that DMU has the same activity levels in both the cases.

Table 5.3 **Fuzzy Peer Table for Production Days 1-24**

Prdn Day	*Base*	*Variation 4*	*Variation 5*	*Variation 6*	*Variation 8*	*Variation 9*	*DEA (BCC Input Reducing)*	
							Peers	*Eff. Score*
1	-	-	-	-	-	3,6	-	**1**
2	-	1,12,14, 16,21	6,7	-	-	3,6,7	1,3,6	**0.882**
3	-	-	-	-	-	-	-	**1**
4	1,11,12, 21, 22	1,11,12, 21,22	3,6,7	1,2,9,22	1,2,9,10, 22	3,6,7	1,3,6,7	**0.783**
5	-	-	3,6,7	-	-	3,6,7	1,3,6,7	**0.744**
6	-	-	-	-	-	-	-	**1**
7	-	-	-	-	-	-	-	**1**
8	-	-	1,3,19	-	-	3,6	1,19	**0.755**
9	-	-	1,6,19	-	-	3,6	1,19	**0.951**
10	-	-	3,6,7	-	-	3,6,7	1,3,6,7	**0.797**
11	-	-	3,6,7	-	-	3,6,7	3,6,19, 21	**0.833**
12	-	-	1,3,7	-	-	3,7	1,7,24	**0.733**
13	1,11,19, 21, 22	1,9,19, 21,22	3,6,7	1,2,11, 19,22	1,2,11, 19,22	3,6,7	1,3,6,7	**0.806**
14	-	-	3,7	-	-	3,7	1,3,24	**0.811**
15	-	-	3,7	-	-	3,7	1,24	**0.798**
16	-	-	1,3,7,24	-	-	3,7	1,24	**0.843**
17	-	-	3,7	-	-	3,7	1,7,24	**0.759**
18	1,7,15,16, 20	1,2,12, 15,24	3,7	1,2,14, 16,22	1,2,7, 12,14	3,7	1,7,24	**0.804**
19	-	-	-	-	-	3	-	**1**
20	-	-	7,21	-	-	7	3,7,21	**0.714**
21	-	-	-	-	-	7	-	**1**
22	-	-	7,21	-	-	7	3,21	**0.807**
23	-	-	6,7,21	-	-	3,6,7	6,7,21	**0.982**
24	-	-	-	-	-	7	-	**1**

Table 5.4 Fuzzy Peer Table for Production Days 25-48

Prdn Day	*Base*	*Variation 4*	*Variation 5*	*Variation 6*	*Variation 8*	*Variation 9*	*DEA (BCC Input Reducing)*	
							Peers	*Eff Score*
25	-	-	47,48	-	-	27,48	29,47,48	**0.996**
26	-	-	-	-	-	27,48	-	**1**
27	-	-	-	-	-	-	-	**1**
28	-	-	37,42,48	-	-	27,37	27,47,48	**0.654**
29	-	-	-	-	-	37,48	-	**1**
30	-	-	37,42	-	-	37	27,37	**0.978**
31	29,34,35, 37,48	29,35,37, 46,48	37,48	25,30,35, 37,46	25,30,35, 37,46	27,37	27,37	**0.973**
32	-	-	47,48	-	-	27,48	27,47	**0.901**
33	25,28,30, 35,44	25,27,28, 46,47	37,48	25,26,28, 30,40	25,26,28, 30,40	27,48	27,29,47, 48	**0.892**
34	-	-	37,48	-	-	27,37,48	27,29,37	**0.975**
35	-	-	27,37,48	-	-	27,37	27,37	**0.976**
36	-	-	37,48	-	-	27,37	27,47	**0.945**
37	-	-	-	-	-	-	-	**1**
38	-	-	37,42	-	-	37	27,47	**0.767**
39	25,37,40, 42,46	25,34,40, 42,46	37,42,48	25,37,40, 42,46	25,35,37, 40,46	37,48	37,40,48	**0.945**
40	-	-	-	-	-	37,48	-	**1**
41	-	-	37,42	-	-	37	37,42	**0.937**
42	-	-	-	-	-	37	-	**1**
43	29,33,35, 42,44	26,30,41, 42,44	37,42	28,30,35, 42,44	41,42,44, 45,46	37	27,37	**0.750**
44	-	-	37,42	-	-	37	27,29,37	**0.621**
45	28,38,42, 44,47	28,30,42, 44,48	37,42	25,28,41, 42,44	25,28,41, 42,44	37	27,29,37	**0.691**
46	-	-	37,48	-	-	27,37	27,29,47	**0.719**
47	-	-	-	-	-	48	-	**1**
48	-	-	-	-	-	-	-	**1**

Table 5.5 Fuzzy Peer Table for Production Days 1-48

Prdn Day	*Base*	*Variation 4*	*Variation 5*	*Variation 6*	*Variation 8*	*Variation 9*	*DEA (BCC Input Reducing)*	
							Peers	*Eff. Score*
1	-	-	-	-	-	48	-	**1**
2	2,21,22, 30, 44	1,12,21	48	-	-	48	1,27,37,4 8	**0.728**
3	-	-	19,48	-	-	48	19,48	**0.937**
4	1,11,21, 22, 46	1,11,21, 22,46	37,48	1,2,5,11, 22	1,2,5,11, 22	48	1,19,48	**0.668**
5	6,8,11, 22,46	1,8,11, 21,46	1,48	1,8,11, 22,46	1,8,11,22, 46	48	1,19,48	**0.657**
6	-	-	-	-	-	48	-	**1**
7	1,22,27, 40, 48	1,21,37, 40,46	37,48	1,21,40, 42,46	1,19,21, 40,41	37,48	1,37,48	**0.837**
8	-	-	1,19,48	-	-	48	1,19	**0.755**
9	-	-	1,6,48	-	-	49	1,6,19	**0.951**
10	6,8,11, 22,44	1,11,21, 22,44	48	1,2,11, 40,46	1,2,11,40, 46	48	1,19,48	**0.668**
11	-	-	6,48	-	-	48	1,6,19,48	**0.776**
12	1,16,21, 22, 46	1,16,22, 42,44	1,48	1,2,14,22, 44	1,2,14,22, 44	48	1,27,48	**0.635**
13	1,11,19, 21, 22	3,8,11, 12,20	48	1,2,11,19, 22	1,2,11,19, 22	48	1,19,48	**0.720**
14	-	-	37,48	-	-	37,49	1,37	**0.695**
15	-	-	48	-	-	48	1,37	**0.708**
16	1,14,35, 46	1,14,35, 46	42,48	1,15,35, 41,46	1,15,35, 41,46	42	1,37	**0.747**
17	1,6,20, 37,44	1,16,22, 42,46	48	1,2,28,30, 44	1,2,28,30, 44	48	1,27,37,4 8	**0.643**
18	14,22,42, 44	1,21,22, 38,44	48	1,22,28, 41,46	1,22,28, 41,46	48	1,27,48	**0.686**
19	-	-	-	-	-	48	-	**1**
20	1,14,22, 42, 46	8,14,22, 42,46	37,48	1,15,22, 43,44	1,15,22, 43,44	37,48	1,27,48	**0.585**
21	-	-	40,42,48	-	-	37	40,42,48	**0.986**
22	-	-	37,48	-	-	37	42,48	**0.706**
23	6,21,39, 41, 44	1,6,21, 40,44	6,48	6,9,21,40, 46	6,21,40, 46,48	48	1,6,48	**0.853**
24	1,7,16, 35,41	1,16,25, 35,44	37,48	1,31,35, 41,46	2,19,35, 42	37,48	1,37	**0.880**

Prdn Day	*Base*	*Variation 4*	*Variation 5*	*Variation 6*	*Variation 8*	*Variation 9*	*DEA (BCC Input Reducing)*	
							Peers	*Eff Score*
25	1,21,27, 29,37	1,21,34, 37,40	1,48	1,2,37,40	1,2,37,40	48	1,19,37,4 8	**0.887**
26	1,16,22, 46	1,16,22, 46	1,48	1,2,14,22, 46	1,2,14,22, 46	48	1,19,48	**0.694**
27	-	-	-	-	-	29,37,48	-	**1**
28	-	-	48	-	-	48	1,19,48	**0.631**
29	-	-	1,37,48	-	-	37,48	1,37,48	**0.958**
30	-	-	37	-	-	37	1,37	**0.975**
31	1,35,37, 46,48	1,16,35, 37,47	37,48	1,35,37, 41,46	1,35,37, 41,46	37,48	1,37	**0.952**
32	1,8,15,47	1,8,15,47	1,48	1,8,15,28, 47	1,8,15,28, 47	48	1,27,48	**0.745**
33	1,14,22, 27,46	1,3,15,22, 42	37,48	1,21,30, 41	1,21,30, 41	37,48	1,27,48	**0.832**
34	-	-	37,48	-	-	37,48	1,37	**0.954**
35	-	-	1,37,48	-	-	37,48	1,37	**0.950**
36	-	-	37,48	-	-	37,48	1,27,37	**0.910**
37	-	-	-	-	-	-	-	**1**
38	14,22,30, 46	14,28,41, 47	37	15,22,36, 41,45	15,22,36, 41,45	37	1,27,37	**0.734**
39	-	1,21,37, 40,46	37,42,48	2,21,37, 40,46	2,21,37, 40,46	37,48	37,40,48	**0.945**
40	-	-	-	-	-	37,48	37,40,48	**1**
41	-	-	37,42	-	-	37	37,42	**0.937**
42	-	-	-	-	-	37	-	**1**
43	21,34,41, 42,44	21,34,41, 42,44	37	21,34,41, 42,44	1,35,42, 44,46	37,48	1,37	**0.743**
44	-	-	37	-	-	37,48	1,37	**0.591**
45	1,16,22, 42,44	1,16,22, 42,44	37,48	2,15,22, 30,44	2,15,22, 30,44	37,48	1,37	**0.669**
46	-	-	48	-	-	48	1,37	**0.637**
47	-	-	1,19,48	-	-	48	1,19,48	**0.914**
48	-	-	-	11,19,48	1,6,19,21, 48	-	-	**1**

5.4 EFFECTIVENESS GOAL

The effectiveness goal in the Fuzzy GoDEA formulation is measured through the achievement of global targets for the three inputs (DLR, RWK, and RML) and one output (PCF). The efficient contribution of each DMU for each of DLR, RWK, RML, and PCF is aggregated and compared with the respective global target. The decision-maker's goal is to exceed or equal the output target with the aggregate efficient output unit and to be less than or equal the input target with the aggregate efficient input unit.

When the effectiveness constraints are fuzzy the decision-maker allows for a specified tolerance violation of the global targets. Accordingly, the global targets are so chosen that they *cannot* be met crisply simultaneously. In other words an *ideal* benchmark is chosen for each global target. In the application presented in this research the global targets for the fuzzy effectiveness goals are computed by aggregating the efficient BCC projections for each input (BCC input reducing model) and output (BCC output increasing model). It is intuitive that these aggregate efficient projections can be achieved simultaneously only in the event that *all* DMUs are evaluated as 100% efficient. This, of course, is impossible in relative efficiency measurement unless the values of all observed inputs and outputs are identical across DMUs. The fuzzy effectiveness goals then provide the decision-maker with a measure of the degree of satisfaction related to the achievement of each global target. The membership functions associated with the fuzzy effectiveness constraints reflect this degree of satisfaction. The closer the aggregate contribution to the global target the higher will be the value of the membership function.

In case of crisp effectiveness constraints the goal programming formulation associates positive and negative deviations with respect to achievement of the global target. However, in this case the global targets have to be redefined due to the unattainable nature of the efficient BCC projections. As in the fuzzy scenario, the decision-maker's goal in the crisp case is to exceed or equal the output target with the aggregate efficient output unit and to stay within or equal the input target with the aggregate efficient input unit. The efficient BCC projections as global targets provide an infeasible region for satisfaction of these crisp constraints. However, the sum of the individual bounds on the inputs and outputs for each DMU provide the decision-maker with one reasonable method of specifying the global targets in the crisp case. The decision-maker's objective would be to ensure at least the satisfaction of these global targets as the sum of the individual bounds can be considered as the *risk-free* global values of inputs and output. These risk-free values take on the same meaning in the global sense as for the individual inputs and output as explained in Chapter 4. Accordingly, the decision-maker would aim to achieve at least these risk free values in the crisp sense. The results for the achievement of the effectiveness goals for the three data sets across the model variations are reported in Appendices C, D, and E.

5.5 RESULTS FOR THE FUZZY GODEA MODEL VARIATIONS

The results for the variations of the Fuzzy GoDEA model are presented in this section.

5.5.1 Fuzzy GoDEA Base Model

The results for the Fuzzy GODEA Base Model (Model 3.3, Chapter 3) for data sets 1-24, 25-48, and 1-48 are presented in Appendices C.1, D.1, and E.1. An optimal solution is realized for the problem where the achievement of the fuzzy efficiency constraints and fuzzy effectiveness constraints is maximized. All the membership functions associated with the fuzzy efficiency constraints for all three data sets are equal to one. This indicates that each of the efficiency constraints is satisfied at the target and, therefore, the DEA structure holds crisply for each data set.

For the data set 1-24 DMUs 4, 13 and 18 are inefficient. DMUs 1, 7, 19 and 21 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. Also, DMUs 11, 16, 20 and 22 operate at 80% or more efficiency in the BCC evaluation and also appear in the reference sets for the inefficient DMUs. However, DMU 12 is 73.3% efficient in the BCC evaluation and appears in the reference sets for inefficient DMU 4 while DMU 15 is 79.8% efficient in the BCC evaluation and appears in the reference set for inefficient DMU 18.

For the data set 25-48 DMUs 31, 33, 39, 43 and 45 are inefficient. DMUs 29, 37, 40, 42, 47 and 48 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. Also, DMUs 25, 30, 33, 34 and 35 operate at 89% or more efficiency in BCC evaluation and also appear in the reference sets for the inefficient DMUs. However, DMUs 28, 38, 44 and 46 are 65.4%, 76.7%, 62.1%, and 71.9% efficient respectively in the BCC evaluation and also appear in the reference sets for inefficient DMUs 33, 39, 43 and 45. DMU 28 is a peer for DMUs 33 and 45, DMU 38 is a peer for DMU 45, DMU 44 is a peer for DMUs 33, 43 and 45, and DMU 46 is a peer for DMU 39.

21 DMUs are inefficient for the data set 1-48. DMUs 4, 13 and 18 from data set 1-24 and DMUs 31, 33, 43 and 45 from data set 25-48 continue being inefficient. Notably, DMU 39 (with a BCC efficiency score of 94.5%) from data set 25-48 is now evaluated as efficient from its previous inefficient status. DMUs 1, 6, 19, 27, 37, 40, 42 and 48 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. The average BCC score for data set 1-48 (82.2%) is the least of the three data sets and a higher frequency of BCC-inefficient peers is visible. DMUs 7, 21, 29, 30 34, 35 and 41 operate at 83% or more efficiency in BCC evaluation and also appear in the reference sets for the inefficient DMUs. However, a large number of DMUs with a BCC score in the neighborhood of 60% appear as peers for the inefficient DMUs. The decrease in the average BCC score for data set 1-48 suggests a high variation in the data. In addition, it can be seen from the Table 5.5 that the BCC scores for the second half (25-48) of the observations are much closer to one while the first half (1-24) are much lower. Further, 13 inefficient units belong to observations (1-24) while only 8 inefficient units belong to observations 25-48. Consequently, the inefficient DMUs from observations 1-24 have more BCC-inefficient units as peers than the inefficient DMUs from observations 25-48. The fuzzy formulation adopts a higher relaxation in the choice of peers for the inefficient DMUs for data set 1-48 given its overall low efficiency performance as displayed by the BCC scores.

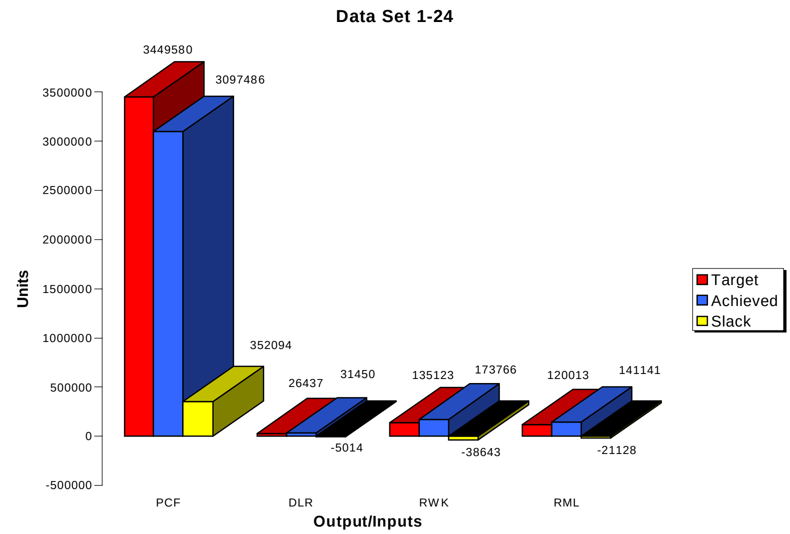
A low degree of achievement is observed from the solutions obtained for the membership functions associated with the fuzzy effectiveness constraints. All the membership functions for the achievement of the global targets are identical for the base model and Variation 8. This implies a low degree of satisfaction for the decision-maker regarding the achievement of the global targets for the inputs and output. The results for the membership functions achieved for the effectiveness constraints are shown in Appendices C.1.b, D.1.b, and E.1.b.

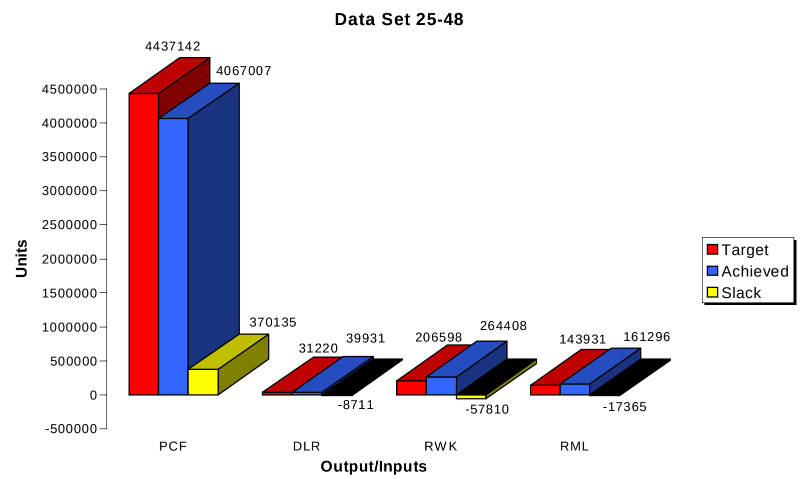
For the data set 1-24, the membership functions associated with PCF, DLR, RWK and RML are 0.55, 0.40, 0.81 and 0.83 respectively. The achievement of the global target for PCF is short by 352094 packages while the achievement of the global targets for DLR, RWK and RML is exceeded by 5014 dollars, 38643 rework pieces and 21128 NNSS pieces respectively.

For the data set 25-48, the membership functions associated with PCF, DLR, RWK and RML are 0.33, 0.26, 0.80 and 0.87 respectively. The achievement of the global target for PCF is short by 370135 packages while the achievement of the global targets for DLR, RWK and RML is exceeded by 8711 dollars, 57810 rework pieces and 17365 NNSS pieces respectively.

For the data set 1-48, the membership functions associated with PCF, DLR, RWK and RML are 0.35, 0.27, 0.71 and 0.80 respectively. The achievement of the global target for PCF is short by 1147014 packages while the achievement of the global targets for DLR, RWK and RML is exceeded by 17,249 dollars, 164,646 rework pieces and 56,874 NNSS pieces respectively.

The graphical representations for the achievement of the global targets for the Fuzzy GoDEA Base Model are shown in Figure 5.1.





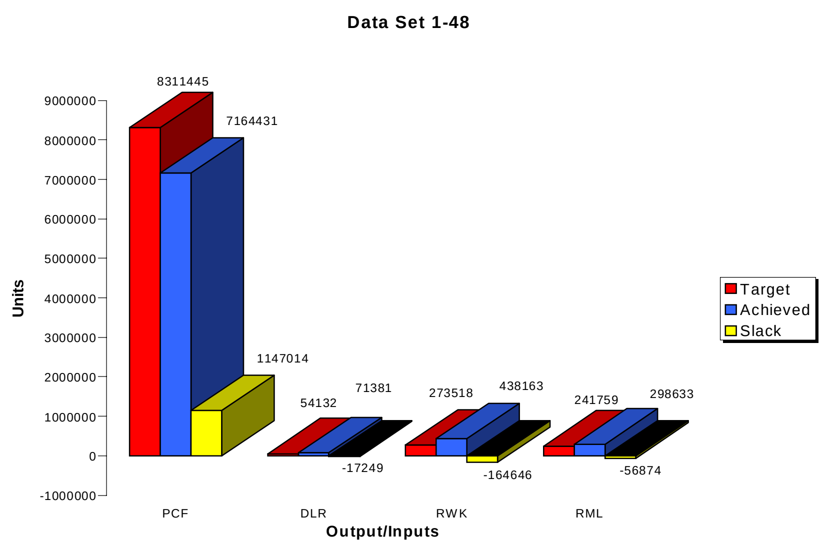


Figure 5.1 Output/Inputs Global Target Achievement: Fuzzy GoDEA Base Model

5.5.2 Fuzzy GoDEA: *Variation 1*

The objective function in this variation maximizes the weighted sum of the membership functions associated with the fuzzy efficiency and fuzzy effectiveness constraints. Arbitrary weights were assigned to the membership functions to reflect higher/lower importance for the efficiency and effectiveness goals. The solution obtained in this variation was identical to that obtained for the base model. In the absence of sufficient knowledge regarding the decision-maker's preferences and a rigorous weighting scheme that accurately reflects such preferences no definite conclusions were possible from the results for this variation. Possible weighting procedures their impacts for this variation and are recommended for future research in Chapter 7. In the present research a goal programming approach utilizing a combination of fuzzy and crisp goals was used to reflect the decision-maker's priorities. This approach was found suitable for implementation, as it does not require the decision-maker to quantify the priorities but only choose the relative order of importance. The following variations capture the different scenarios of relative importance of the efficiency goals and the combinations of fuzzy and crisp constraints.

5.5.3 Fuzzy GoDEA: *Variation 2*

In this variation both the efficiency and effectiveness goals are fuzzy. The Stage 1 problem maximizes the achievement of the fuzzy global targets. The solutions for the membership functions associated with the achievement of the global targets are passed as constraints to the Stage 2 problem where the fuzzy efficiency constraints are maximized. The Stage 1 problem for all three data sets provided optimal solutions for the membership functions associated with the fuzzy global targets. However, the Stage 2 problem was found to be infeasible in all three cases. The optimal solution for the achievement of the fuzzy global targets created an infeasible region for the maximization of the fuzzy efficiency constraints.

5.5.4 Fuzzy GoDEA: *Variation 3*

In this variation both the efficiency and effectiveness goals are fuzzy but the solution order is reversed from Variation 2. The Stage 1 problem maximizes the achievement of the efficiency goal. The solutions for the membership functions associated with the efficiency goal are passed as constraints to the Stage 2 problem where the fuzzy effectiveness constraints are maximized. Again, the Stage 1 problem for all three data sets provided optimal solutions for the membership functions associated with the fuzzy efficiency constraints. However, the Stage 2 problem was found to be infeasible in all three cases. The optimal solution to the achievement of the fuzzy efficiency constraints created an infeasible region for the maximization of the fuzzy effectiveness constraints.

5.5.5 Fuzzy GoDEA: *Variation 4*

The results for Variation 4 for data sets 1-24, 25-48, and 1-48 are presented in Appendices C.2, D.2, and E.2. An optimal solution is realized for the problem where the achievement of the fuzzy efficiency constraints is maximized. The crisp effectiveness constraints act as additional constraints for the problem. All the membership functions associated with the fuzzy efficiency constraints are equal to one. This indicates that each of the efficiency constraints is satisfied at the target and, therefore, the DEA structure holds crisply for each data set.

For the data set 1-24, DMUs 2, 4, 13 and 18 are inefficient. DMUs 1, 19, 21, 24 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. Also, DMUs 2, 9, 11, 14, 16 and 22 operate at 80% or more efficiency in the BCC evaluation and also appear in the reference sets for the inefficient DMUs. However, DMU 12 is 73.3% efficient in the BCC evaluation and appears in the reference sets for inefficient DMUs 2, 4, and 18 while DMU 15 is 79.8% efficient in the BCC evaluation and appears in the reference set for inefficient DMU 18.

For the data set 25-48, DMUs 31, 33, 39, 43 and 45 are inefficient. DMUs 26, 27, 29, 37, 40, 42, 47 and 48 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. Also, DMUs 25, 30, 34, 35 and 41 operate at 93% or more efficiency in BCC evaluation and also appear in the reference sets for the inefficient DMUs. However, DMUs 28, 44 and 46 are 65.4%, 62.1%, and 71.9% efficient respectively in the BCC evaluation and also appear in the reference sets for inefficient DMUs 31, 33, 39, 43 and 45. DMU 28 is a peer for DMUs 33 and 45, DMU 44 is a peer for DMUs 43 and 45, and DMU 46 is a peer for DMUs 31, 33 and 39.

For the data set 1-48, the are similar to those obtained in the Base Model. All the 21 DMUs evaluated inefficient in the Base Model are inefficient in this variation. However, DMU 39 was efficient in the base model but is inefficient in this variation and continues to remain inefficient for all subsequent variations. DMUs 1, 6, 37, 40 and 42 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. DMUs 3, 21, 25, 34, 35, 41 and 47 operate at 88% or more efficiency in BCC evaluation and also appear in the reference sets for the inefficient DMUs. As in the Base Model a large number of DMUs with a BCC score in the neighborhood of 60% appear as peers for the inefficient DMUs.

The crisp effectiveness constraints are achieved as shown in Appendices C.2.b, D.2.b, and E.2.b for the three data sets. The output slack shows the amount of excess output production that would be achieved with respect to the global output target. The input surplus variables show the amount of input reduction that would be achieved with respect to the global input targets. Therefore, the slack and surplus variables associated with the global target constraints denote the improvements possible with respect to the effectiveness goal when the fuzzy efficiency goal is maximized. However, the global targets in this case represent the risk-free bounds for the inputs and output. Therefore these results cannot be compared directly with the results obtained when the global target constraints are fuzzy. Instead the absolute achievement of the global target for each input and output should be compared with the absolute achievement of the global targets for the fuzzy effectiveness constraints. A comparison of achievement of global targets is provided in Section 5.7, Table 5.6. Figure 5.2 graphically depicts the global targets, the achieved values and the augmentation/reduction of output/input possible for the three data sets.

For data set 1-24, the excess number of PCF that can be produced is 4,25,197 packages, the reduction possible in DLR, RWK, and RML is 3,298 dollars, 159,708 rework pieces and 100,102 NNSS pieces respectively.

For data set 25-48, the excess number of PCF that can be produced is 184,651 packages, the reduction possible in DLR, RWK, and RML is 3,082 dollars, 233,946 rework pieces and 115,842 NNSS pieces respectively.

For data set 1-48, the excess number of PCF that can be produced is 609,847 packages, the reduction possible in DLR, RWK, and RML is 6,380 dollars, 393,654 rework pieces and 219,752 NNSS pieces respectively.

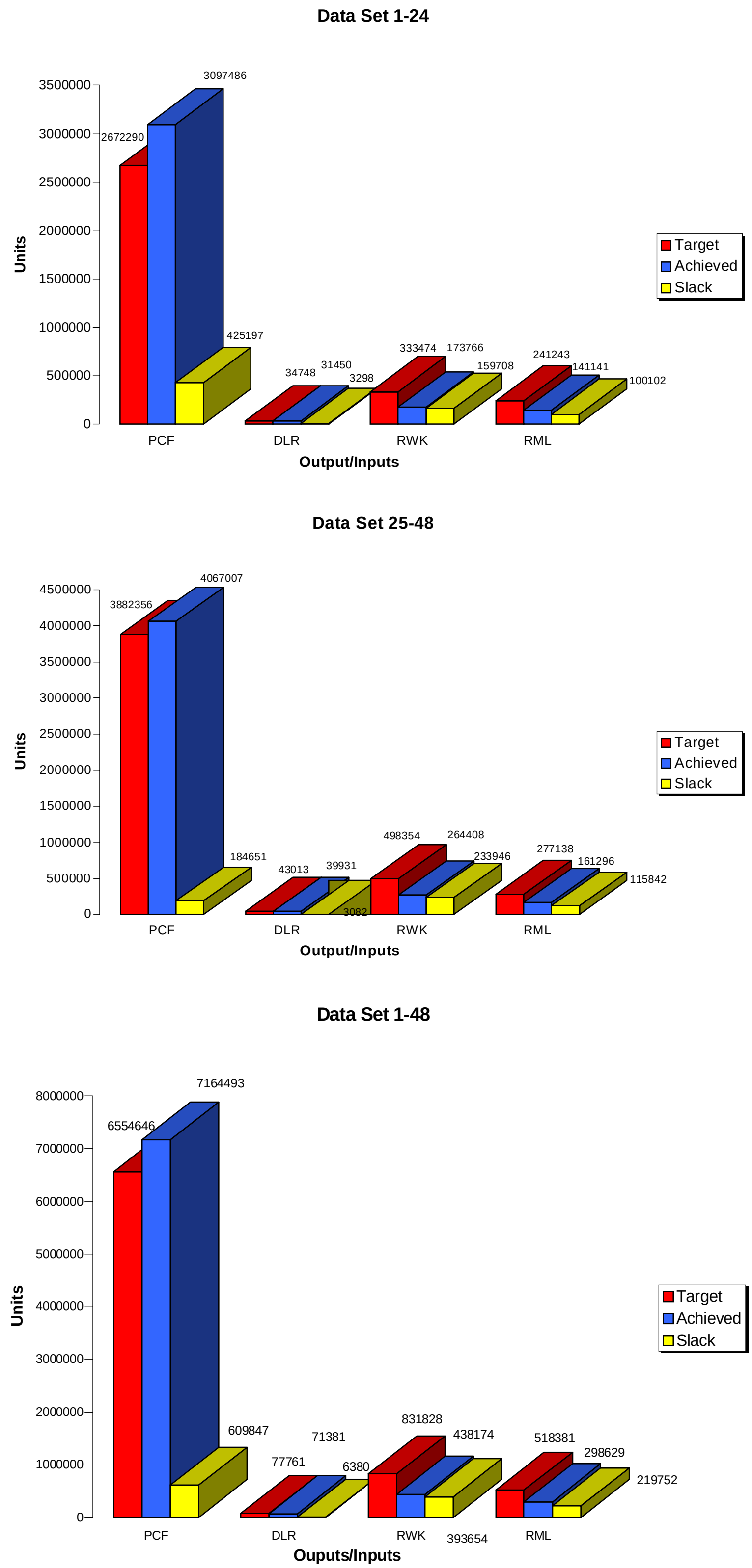


Figure 5.2 Output/Inputs Global Target Achievement: Fuzzy GoDEA Variation 4

5.5.6 Fuzzy GoDEA: *Variation 5*

The results for Variation 5 for data sets 1-24, 25-48, and 1-48 are presented in Appendices C.3, D.3, and E.3 respectively. An optimal solution is achieved for the problem where the achievement of the fuzzy effectiveness constraints is maximized. The crisp efficiency constraints act as additional constraints for the problem. The realizations of the membership functions associated with the fuzzy effectiveness constraints are shown in Appendices C.3.b, D.3.b, and E.3.b. These membership functions indicate the degree of satisfaction for the decision-maker with respect to the achievement of each effectiveness constraint or global target. Figure 5.3 graphically depicts the achievement of the global targets for PCF, DLR, RWK and RML for this variation.

For the data set 1-24, the membership functions associated with inputs DLR and RWK are equal to one. This implies that the global targets for these inputs are achieved at the specified target level. For output PCF and input RML the membership functions are 0.80 and 0.86 respectively. The achievement of the global target for PCF is short by 161,676 packages while the achievement of the global target for RML is exceeded by 17,215 NNSS pieces.

For the data set 25-48, only the membership function associated with input RWK is equal to one. This implies that the global target for RWK is achieved at the specified target level. For output PCF and inputs DLR and RML the membership functions are 0.95, 0.86, and 0.87 respectively. The achievement of the global target for PCF is short by 25,631 packages. The achievement of the global target for DLR is exceeded by 1,696 dollars while the global target for RML is exceeded by 17,365 NNSS pieces.

For the data set 1-48, the membership functions associated with inputs DLR and RWK are equal to one. This implies that the global targets for these inputs are achieved at the specified target level. For output PCF and input RML the membership functions are 0.66 and 0.91 respectively. The achievement of the global target for PCF is short by

599,771 packages while the achievement of the global target for RML is exceeded by -24,121 NNSS pieces.

For the crisp efficiency constraints the values of slack variables are shown in Appendices C.3.a, D.3.a, and E.3.a for the three data sets. The slack variables denote the amount of inefficiency for each input and output. For an efficient DMU all slack variables are equal to zero, as it has no inefficiency.

For the data set 1-24, the crisp DEA constraints reveal DMUs 1, 3, 6, 7, 19, 21, and 24 as efficient and their associated slack variables are equal to zero. DMUs 5, 13, 14, 17 and 18 are inefficient and have identical activity levels in this variation and Variation 9, and consequently, display the same amount of inefficiency in both cases. Also, DMUs 10 and 12 have very similar activity levels in both variations and thus, display very similar amounts of inefficiency in both cases.

For the data set 25-48, the crisp DEA constraints reveal DMUs 26, 27, 29, 37, 40, 42, 47 and 48 as efficient and their associated slack variables are equal to zero. No inefficient DMUs have identical activity levels in this variation compared with Variation 9. Only DMU 38 has similar activity levels in the two variations and shows similar amounts of inefficiency.

For the data set 1-48, the crisp DEA constraints reveal DMUs 1, 6, 19, 27, 37, 40, 42 and 48 as efficient and the rest as inefficient. Inefficient DMUs 31 and 33 show identical amounts of inefficiency while DMU 45 shows similar inefficiency with Variation 9. In all three cases the efficient DMUs are confirmed by the BCC evaluation. However, the reference sets for the inefficient DMUs differ with respect to some peers from the BCC evaluation. This is expected due to the absence of slack and surplus variables in Variation 5, which may affect the choice of the peer units (See Section 3.1, page 76).

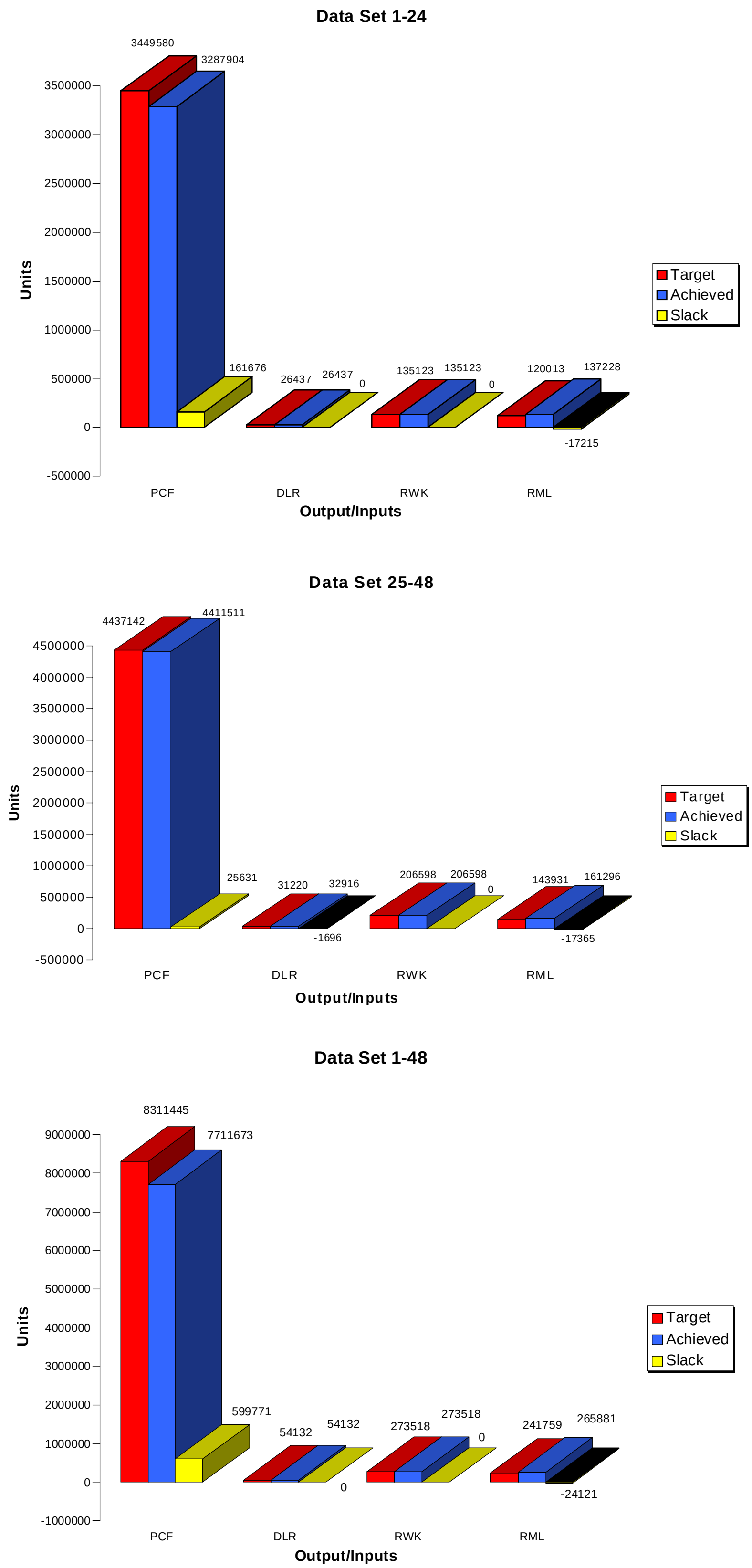


Figure 5.3. Output/Inputs Global Target Achievement: Fuzzy GoDEA Variation 5

5.5.7 Fuzzy GoDEA: *Variation 6*

The results for Variation 6 for data sets 1-24, 25-48, and 1-48 are presented in Appendices C.4, D.4, and E.4. Stage 1 of this variation maximizes the achievement of the fuzzy efficiency constraints. All the membership functions associated with the fuzzy efficiency constraints are equal to one and indicate that the DEA structure holds crisply for each data set.

The optimal membership function values associated with the fuzzy efficiency constraints are introduced as constraints for the Stage 2 problem. The Stage 2 problem is then a typical goal programming problem. The objective of the Stage 2 problem is to minimize the deviations from the achievement of the crisp effectiveness constraints. The decision-maker is only concerned with minimizing the negative deviation from the output target and positive deviation from the input targets, and therefore, only these deviation variables are minimized in the objective function.

For the data set 1-24, DMUs 4, 13, and 18 are inefficient. DMUs 1 and 19 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. Also, DMUs 2, 9, 11, 14, 16 and 22 operate at 80% or more efficiency in the BCC evaluation and also appear in the reference sets for the inefficient DMUs. No DMU that is evaluated as inefficient in the BCC evaluation shows up as a peer for any of the inefficient units for this variation.

For the data set 25-48, DMUs 31, 33, 39, 43 and 45 are inefficient. DMUs 26, 37, 40 and 42 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. Also, DMUs 25, 30, 35 and 41 operate at 93% or more efficiency in the BCC evaluation and also appear in the reference sets for the efficient DMUs. However, DMUs 28, 44 and 46 are 65.4%, 62.1%, and 71.9% efficient respectively in the BCC evaluation and also appear in the reference sets for inefficient DMUs 31, 33, 39, 43 and 45. DMU 28 is a peer for DMUs 33, 43,and 45, DMU 44 is a peer for DMUs 43 and 45 and DMU 46 is a peer for DMUs 31 and 39.

For the data set 1-48, 22 DMUs are inefficient. DMU 2 attains efficient compared to its inefficient status in the Base Model and Variation 4 whereas DMU 48 is deemed inefficient (and is BCC efficient). Interestingly, DMU 48 also appears as a peer for itself with a 95% activity level. This indicates that DMU 48's inefficiency is negligible. DMUs 4, 13 and 18 from data set 1-24 and DMUs 31, 33, 43 and 45 from data set 25-48 continue being inefficient. DMUs 1, 6, 19, 37, 40, and 42 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. DMUs 9, 21, 30, 34, 35, 36, 41 and 47 operate at 91% or more efficiency in BCC evaluation and also appear in the reference sets for the inefficient DMUs. As in the Base Model a large number of DMUs with a BCC score in the neighborhood of 60% appear as peers for the inefficient DMUs.

The crisp effectiveness constraints are achieved as shown in Appendices C.4.b, D.4.b, and E.4.b for the three data sets. These results are identical to those obtained from Variation 4 for data sets 1-24 and 25-48. For data set 1-48 the results are identical for Variation 4 and Variation 6 for DLR, RWK and RML but differ for PCF. The positive output deviation (PPCF) shows the amount of excess output production that would be achieved with respect to the global output target. The negative input deviation variables (NDLR, NRWK, and NRML) show the amount of input reduction that would be achieved with respect to the global input targets. Therefore, the deviation variables associated with the global target constraints denote the possible improvement in meeting the effectiveness goal when the fuzzy efficiency goal is maximized. Figure 5.4 graphically depicts the global targets, the achieved values and the augmentation/reduction of output/input possible for the three data sets.

For data set 1-24, the excess number of PCF that can be produced is 4,25,197, the reduction possible in DLR, RWK and RML is 3,298 dollars, 159,708 rework pieces, and 100,102 NNSS pieces respectively.

For data set 25-48, the excess number of PCF that can be produced is 184,651, the reduction possible in DLR, RWK and RML is 3,082 dollars, 233,946 rework pieces, and 115,842 NNSS pieces respectively.

For data set 1-48, the excess number of PCF that can be produced is 607,467, the reduction possible in DLR, RWK and RML is 6,380 dollars, 393,654 rework pieces, and 219,752 NNSS pieces respectively.

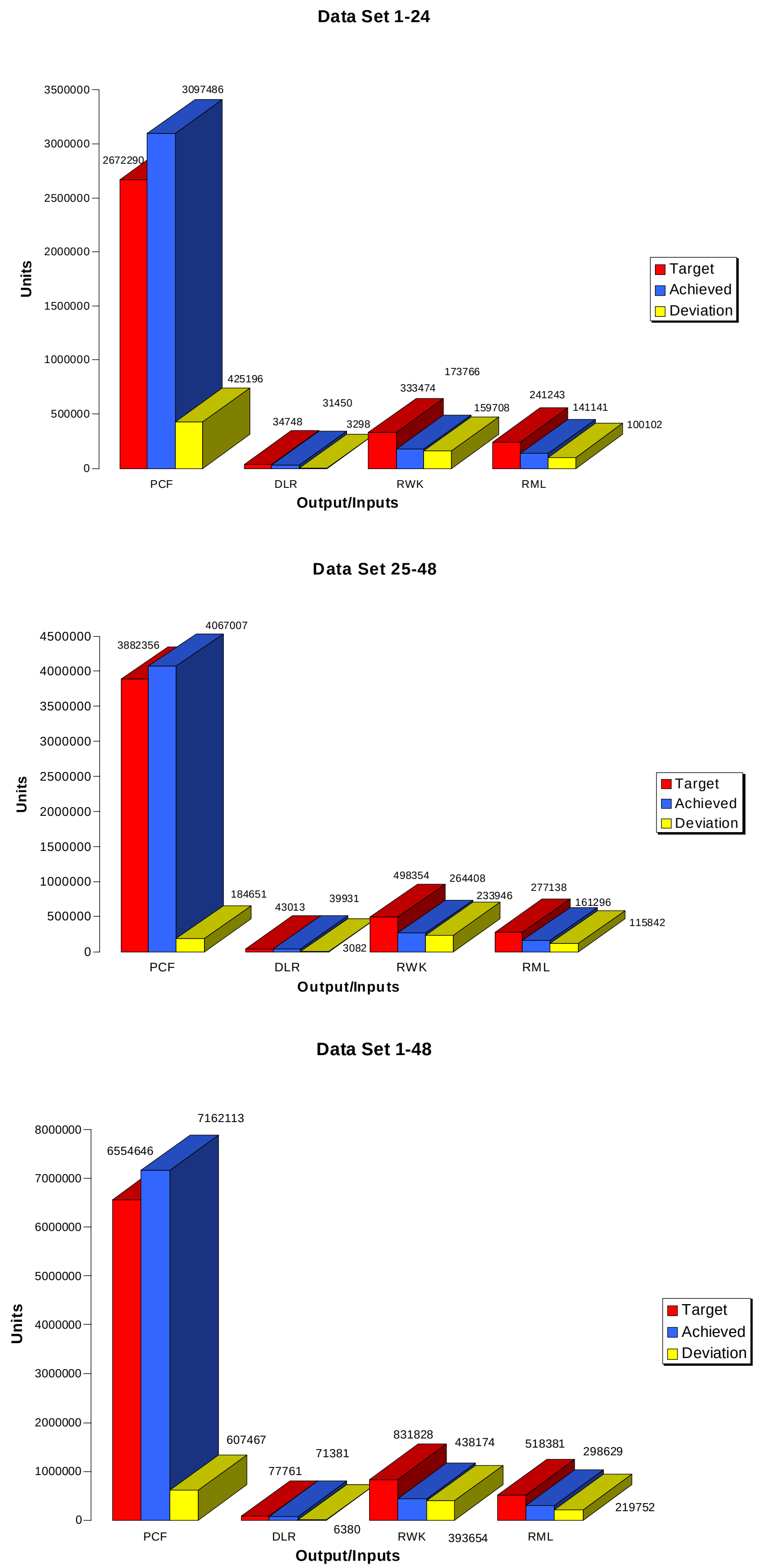


Figure 5.4 Output/Inputs Global Target Achievement: Fuzzy GoDEA Variation 6

5.5.8 Fuzzy GoDEA: *Variation 7*

In this variation the order of achieving the efficiency and effectiveness goals is reversed form Variation 6. Stage 1 of this variation minimizes the positive and negative deviations from the global targets. As explained in Variation 6 the decision-maker is only concerned with minimizing the negative deviation from the output target and positive deviation from the input targets, and therefore, only these deviation variables are minimized in the objective function. An optimal solution was obtained for all three data sets in Stage 1 with all the deviation variables equal to zero. In Stage 2 the achievement of the fuzzy efficiency goal is maximized. Stage 2 was found to be infeasible in all cases. The optimal solution for the deviation variables when introduced as constraints to the Stage 2 problem created an infeasible region. The effectiveness constraints from Stage 1 were found to limit the solution space in the Stage 2 problem. Consequently, a solution is possible only with violation of the bounds on the membership functions. In other words, a solution for the Stage 2 problem can be achieved if the membership functions are allowed to be free. However, this violates the definition of the membership function and renders the efficiency constraints meaningless.

5.5.9 Fuzzy GoDEA: *Variation 8*

The results for Variation 8 for data sets 1-24, 25-48, and 1-48 are presented in Appendices C.5, D.5, and E.5. Stage 1 of this variation minimizes the positive and negative deviations associated with achievement of the crisp efficiency constraints. All the deviation variables equal zero in the solution for Stage 1. This implies that each composite input/output unit for each assessed DMU is equal to the observed input/output for that DMU. The solutions to the deviation variables from Stage 1 are then supplied as constraints for the Stage 2 problem.

The achievement of the fuzzy effectiveness constraints is maximized in the Stage 2 problem with the solution to the deviations from the crisp efficiency targets as additional constraints. The realizations of the membership functions associated with the fuzzy effectiveness constraints are shown in Appendices C.5.b, D.5.b, and E.5.b. These membership functions indicate the degree of satisfaction for the decision-maker with respect to the achievement of each effectiveness constraint or global target

For the data set 1-24, all the membership functions are less than 1. For PCF the membership function is 0.55. The achievement of the global target for PCF is short by 352,097 packages. For DLR the membership function is 0.40 and the achievement of the global target is exceeded by 5,012 dollars. For the RWK the membership function is 0.81 and the achievement of the global target is exceeded by 38678 rework pieces. For the RML the membership function is 0.83 and the achievement of the global target is exceeded by 21094 NNSS pieces.

For the data set 25-48, also all the membership functions are less than 1. For PCF the membership function is 0.33. The achievement of the global target for PCF is short by 370,137 packages. For DLR the membership function is 0.26 and the achievement of the global target is exceeded by 8,711 dollars. For the RWK the membership function is 0.80 and the achievement of the global target is exceeded by 57,808 rework pieces. For RML the membership function is 0.87 and the achievement of the global target is exceeded by 17,365 NNSS pieces.

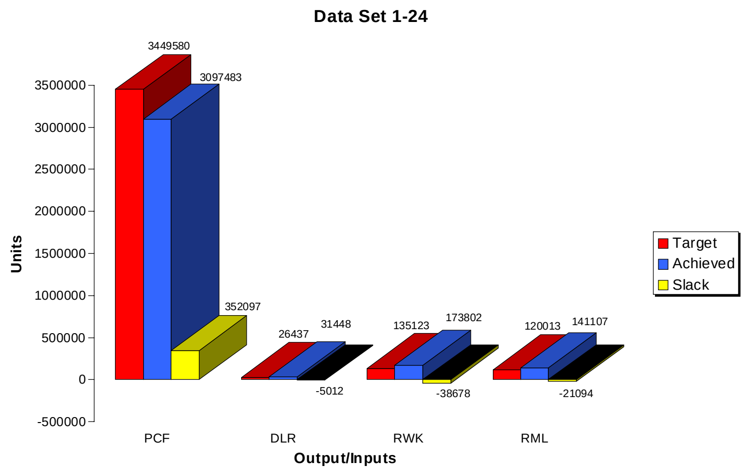
For the data set 1-48, all membership functions are less than 1. For PCF the membership function is 0.35. The achievement of the global target for PCF is short by 1,149,298 packages. For DLR the membership function is 0.27 and the achievement of the global target is exceeded by 17,249 dollars. For the RWK the membership function is 0.71 and the achievement of the global target is exceeded by 164,646 rework pieces. For RML the membership function is 0.87 and the achievement of the global target is exceeded by 56,874 NNSS pieces.

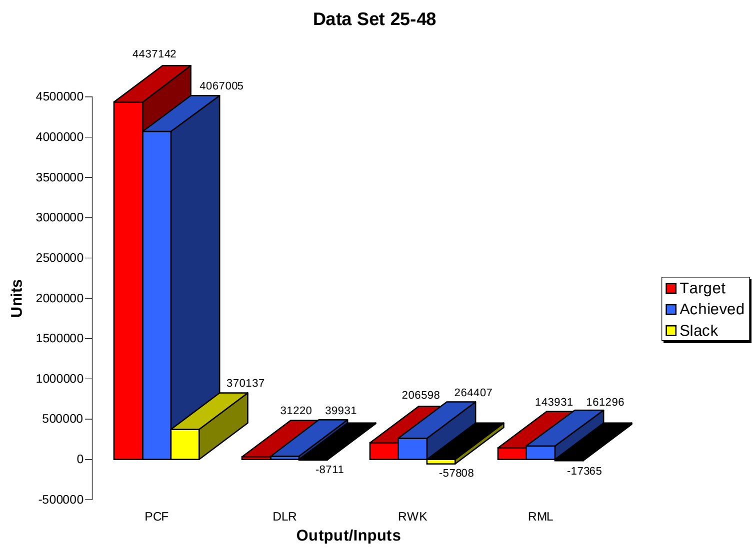
For all three data sets the achievement of the global targets i.e., the membership functions for PCF, DLR, RWK and RML are identical for the Base Model and Variation 8. Figure 5.5 graphically displays the results for the achievement of the global targets.

For the data set 1-24, the crisp DEA constraints reveal DMUs 4, 13 and 18 as in efficient and the rest as efficient. DMUs 1, 7 and 19 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. Also, DMUs 2, 9, 11, 14 and 22 operate at 80% or more efficiency in the BCC evaluation and also appear in the reference sets for the inefficient DMUs. However, DMU 10 is 75.5% efficient in the BCC evaluation and appears in the reference sets for inefficient DMU 4 while DMU 12 is 73.3% efficient in the BCC evaluation and appears in the reference sets for inefficient DMU 18.

For the data set 25-48, the crisp DEA constraints reveal DMUs 31, 33, 39, 43 and 45 as inefficient and the rest as efficient. DMUs 26, 37, 40 and 42 are efficient in the BCC evaluation and appear in the reference sets for the inefficient DMUs. Also, DMUs 25, 30, 35 and 41 operate at 93% or more efficiency in the BCC evaluation and also appear in the reference sets for the efficient DMUs. However, DMUs 28, 44, 45 and 46 are 65.4%, 62.1%, 69.1% and 71.9% efficient respectively in the BCC evaluation and also appear in the reference sets for inefficient DMUs 31, 33, 39, 43 and 45. DMU 28 is a peer for DMUs 33 and 45, DMU 44 is a peer for DMUs 43 and 45, DMU 45 is a peer for DMU 43 and DMU 46 is a peer for DMUs 31, 39, 43 and 45.

For the data set 1-48, the results for the efficiency constraints are identical to those obtained from Variation 6. The same sets of DMUs are deemed efficient and inefficient in both variations. Furthermore, both variations choose the same peer units for the inefficient DMUs except for DMUs 7, 24, 25, 43, and 48. DMU 48 again has itself as a peer with an activity level of 91%, which signifies negligible inefficiency.





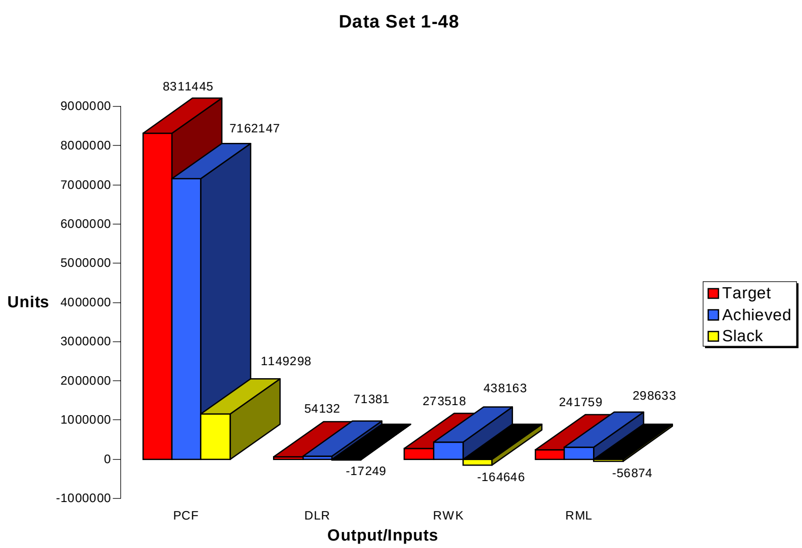


Figure 5.5 Output/Inputs Global Target Achievement: Fuzzy GoDEA Variation 8

5.5.10 Fuzzy GoDEA: *Variation 9*

The results for Variation 9 for data sets 1-24, 25-48, and 1-48 are presented in Appendices C.6, D.6, and E.6. Stage 1 of this variation maximizes the achievement of the fuzzy effectiveness constraints. The realizations of the membership functions associated with the fuzzy effectiveness constraints are shown in Appendices C.6.b, D.6.b, and E.6.b. These membership functions indicate the degree of satisfaction for the decision-maker with respect to the achievement of each effectiveness constraint or global target.

The optimal membership function values associated with the fuzzy effectiveness constraints are introduced as constraints for the Stage 2 problem. The Stage 2 problem is then a typical goal programming problem. The objective of the Stage 2 problem is to minimize the positive and negative deviations associated with the achievement of the crisp efficiency constraints. The solutions for the deviation variables obtained in Stage 2 are displayed in Appendices C.6.a, D.6.a, and E.6.a.

For data set 1-24, all the deviation variables for DMUs 3, 6 and 7 are equal to zero. Also, the activity level corresponding to the observed input/output values for each of DMUs 3, 6 and 7 is equal to one. Therefore, DMUs 3, 6 and 7 are efficient. DMU 19 has the activity level associated with DMU 3's input/output values equal to one but shows positive deviation for PCF (30,206), positive deviation for DLR (295), negative deviation for RWK (2,506) and positive deviation for RML (1,268). This implies that the efficiency targets were exceeded by amounts equal to the positive deviations and unattained by shortfalls equal to the negative deviations. Physically, the deviation amounts are interpreted as follows. For the output (PCF) a non-zero *positive* deviation for a DMU implies that an increase in output production equal to the deviation amount would render an efficient level of output production. For the inputs (DLR, RWK, and RML) a non-zero *negative* deviation for a DMU implies that a decrease in input consumption equal to the deviation amount would render an efficient level of input usage.

However, DMU 19 does lie on the same facet of the efficient frontier as DMU 3, which appears as its peer. The deviation amounts (see Appendix C.6.a) for PCF (+30,208) and RWK (-2,506) convey the required increase in PCF production and decrease in RWK quantity that would make DMU 19 as efficient as DMU 3. Similarly, DMUs 20, 21 and 22 lie on the same facet of the efficient frontier as DMU 7. DMU 20 shows positive deviation for PCF (11,758), negative deviation for DLR (333), negative deviation for RWK (5,104) and negative deviation for RML (2,171). DMU 21 shows negative deviation for PCF (45,625), negative deviation for DLR (287), negative deviation for RWK (2,242) and negative deviation for RML (4,767). DMU 22 shows negative deviation for PCF (12,106), negative deviation for DLR (258), negative deviation for RWK (6,464) and negative deviation for RML (5,846).

For data set 25-48, all the deviation variables for DMUs 27, 37 and 48 equal zero. Also, the activity level corresponding to the observed input/output values for each DMU is equal to one. Therefore, DMUs 27, 37 and 48 are efficient. DMU 30 has the activity level associated with DMU 37's input/output values equal to one but shows positive deviation for PCF (3,279), negative deviation for DLR (42), negative deviation for RWK (3,233) and negative deviation for RML (26). Therefore, an increase in PCF production and decrease in DLR, RWK, and RML usage equal to the deviation amounts would render DMU 30 efficient. However, DMU 30 does lie on the same facet of the efficient frontier as DMU 37, which appears as its peer. Similarly, DMUs 38, 41, 42, 43, 44 and 45 lie on the same facet of the efficient frontier as DMU 37 while DMU 47 lies on the same facet of the efficient frontier as DMU 48.

For data set 1-48, 32 DMUs have one of DMUs 37, 42 or 48 as the only peer unit. DMU 48 appears 23 times while DMU 37 appears 6 times and DMU 42 appears only once as a peer for these 32 DMUs. DMUs 37 and 48 are BCC efficient and also appear as peers for 16 inefficient DMUs while DMU 29 (with a BCC efficiency score of 96%) appears as a peer only for DMU 27. This clearly indicates DMUs 37 and 48 to be the most influential observations in this variation. DMU 37 is the only observation with all deviation variables equal to zero and has an activity level equal to one. This makes DMU

37 the only efficient DMU in this data set. All of the 32 DMUs with only one peer have non-zero deviations associated with all of PCF, DLR, RWK and RML. DMU 48 is an exception as it has only a positive PCF deviation (2,380) which is one of the smallest positive output deviations observed for this data set. This implies a very small amount of output inefficiency for DMU 48 and is consistent with the negligible inefficiency displayed by DMU 48 in Variations 6 and 8. The Fuzzy GoDEA formulation in general, and this variation in particular, thus demonstrates an acute discrimination of relative levels of (in)efficiency between observations and does not rule out observations that are near-efficient as peers. This is submitted as a major contribution in the present research effort.

When a DMU exhibits *negative* deviation for the output (PCF) or *positive* deviation for an input (DLR, RWK, and RML) it implies better performance than the composite unit for that output/input. For example, DMUs 13, 21, 22 and 24 for data set 1-24, DMU 36, 41 and 42 for data set 25-48, and DMUs 21, 41 and 42 for data set 1-48 display negative PCF deviations. All these DMUs have a high BCC efficiency score (81% - 100%). This confirms that they have efficient levels of output production. Also, DMUs 9, 11, and 23 for data set 1-24 and DMUs 1, 6, 8, 9, and 19 for data set 1-48 display positive input deviations for some or all inputs. All these DMUs also have a high BCC efficiency score (83%-100%) except for DMU 8 (75.5%). DMU 8's high DLR usage can be attributed to its slightly lower BCC efficiency score but has below average usage for RWK and RML thus enabling efficient consumption levels of these inputs.

For the data set 1-24, the membership functions associated with DLR and RWK are equal to one. This implies that the global targets for DLR and RWK are achieved at the specified target level. The membership function is 0.86 for PCF and 0.85 for RML. The target achievement for PCF is short by 107,266 while the target achievement for RML is exceeded by 17,700 NNSS pieces.

For the data set 25-48, the membership functions associated with PCF and RWK are equal to one. This implies that the global targets for PCF and RWK are achieved at the specified target level. The membership function is 0.94 for both DLR and RML. The global target for DLR is exceeded by 762 dollars while the global target for RML is exceeded by 8,595 NNSS pieces.

For the data set 1-48, the membership function associated with DLR is equal to one. This implies that the global target for DLR is achieved at the specified target level. The membership functions for RWK, RML and PCF are 0.99, 0.89, and 0.83 respectively. The global targets for RWK and RML are exceeded by 3,852 rework pieces and 30,262 NNSS pieces respectively while the global target for PCF is short by 297953 packages. Figure 5.6 graphically displays the results for the achievement of the global targets.

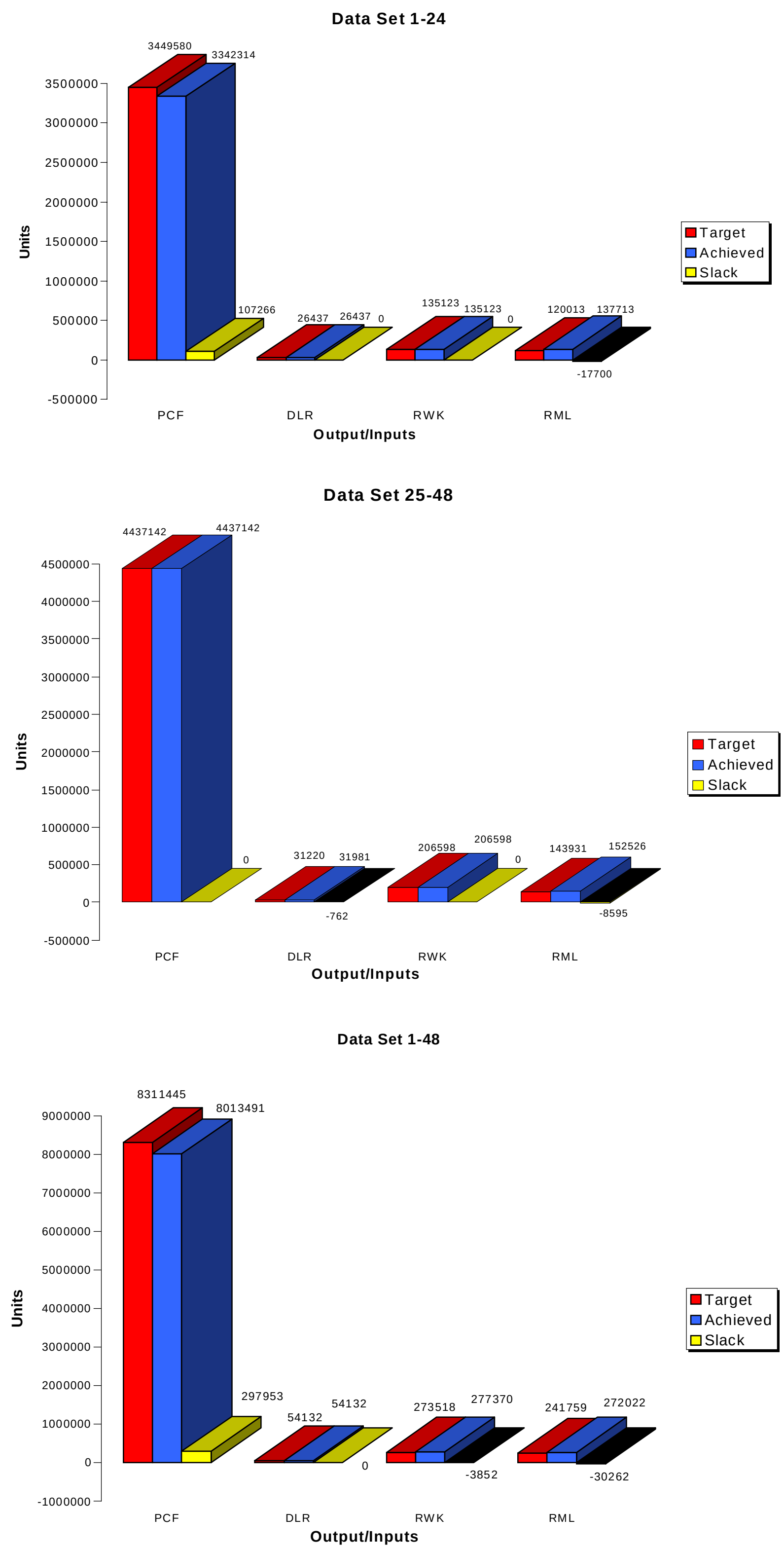


Figure 5.6 Output/Inputs Global Target Achievement: Fuzzy GoDEA Variation 9

5.6 EFFICIENCY ACHIEVEMENT ACROSS FUZZY GODEA VARIATIONS

The evaluation of efficient DMUs for the packaging line data differs across the Fuzzy GoDEA model variations. Figure 5.7 shows the number (percentage) of inefficient DMUs by variations for the three data sets. The fuzzy peer tables (Tables 5.3, 5.4 and 5.5) show the efficient and inefficient units across the variations along with the BCC input reducing evaluation. The choice of a subset of efficient and inefficient units evaluated by the BCC model is confirmed across the variations for all three data sets. This provides one form of validation for the fuzzy efficiency measurement approach used in this research.

For data set 1-24, DMUs 3, 6 and 7 are efficient across the variations as well as in the BCC evaluation. DMUs 1, 19, 21 and 24 are efficient in the BCC evaluation but are found inefficient in Variation 9. In Variation 9, DMU 19 is found to lie on the same facet of the efficient frontier as DMU 3 while DMUs 21 and 24 are found to lie on the same facet of the efficient frontier as DMU 7. This indicates a higher discerning power of Variation 9 in evaluating efficient DMUs. This, however, limits BCC efficient DMUs 3, 6 and 7 to be the peers in Variation 9. DMUs 4, 13 and 18 are found to be inefficient across the variations as well as in the BCC evaluation. The choice of peers for the inefficient DMUs differs across the variations depending on (i) the solution stage assigned to the efficiency constraints and (ii) the fuzzy or crisp nature. For example, the peers for DMU 4 chosen by Variations 5 and 9 (Stage 1) that have crisp efficiency constraints are a subset of the peers chosen by the BCC model. However, Variations 4 and 6 (Stage 1) with fuzzy efficiency constraints and Variation 8 (Stage 1) with crisp efficiency constraints and positive and negative deviation variables display similarity in their choice of peers but differ significantly from the BCC evaluation.

For data set 25-48, DMUs 27, 37 and 48 are efficient across the variations as well as in the BCC evaluation. DMUs 26, 29, 40, 42 and 47 are efficient in the BCC evaluation but are found inefficient in Variation 9. Also, in Variation 9, DMU 42 lies on the same facet of the efficient frontier as DMU 37 and DMU 47 lies on the same facet as DMU 48. DMUs 31, 33, 39, 43 and 45 are found to be inefficient across the variations as well as in the BCC evaluation. Again, the choice of peers for the inefficient DMUs differs across the variations. However, Variations 6 and 8 display an identical choice of peers for DMUs 31, 33, 39 and 45 but different peers for DMU 21.

For data set 1-48, only DMU 37 is efficient across the variations as well as in the BCC evaluation. DMUs 1, 6, 19, 27, 40 and 42 are efficient in the BCC evaluation and in all the Fuzzy GoDEA variations except Variation 9 while DMU 48 is efficient in the BCC evaluation and in all variations except Variations 6 and 8. As for the other two data sets, the choice of peers for the inefficient DMUs differs across the variations. However, Variations 6 and 8 display an identical choice of peers for DMUs 4, 5, 10, 12, 13, 16, 17, 18, 20, 25, 26, 31, 32, 33, 38, 39 and 45 but different peers for the remainder of the DMUs.

The Fuzzy GoDEA Base Model and Variations 4, 6 and 8 have fuzzy efficiency (DEA representation) constraints and identify both BCC efficient and BCC-inefficient units as peers for the inefficient units evaluated by each model variation. Variations 5 and 9 have crisp efficiency constraints and identify only BCC-efficient units as peers for the inefficient units identified by them. These observations are consistent with the objective of this research to allow relaxation of the DEA structure through fuzzy constraints.

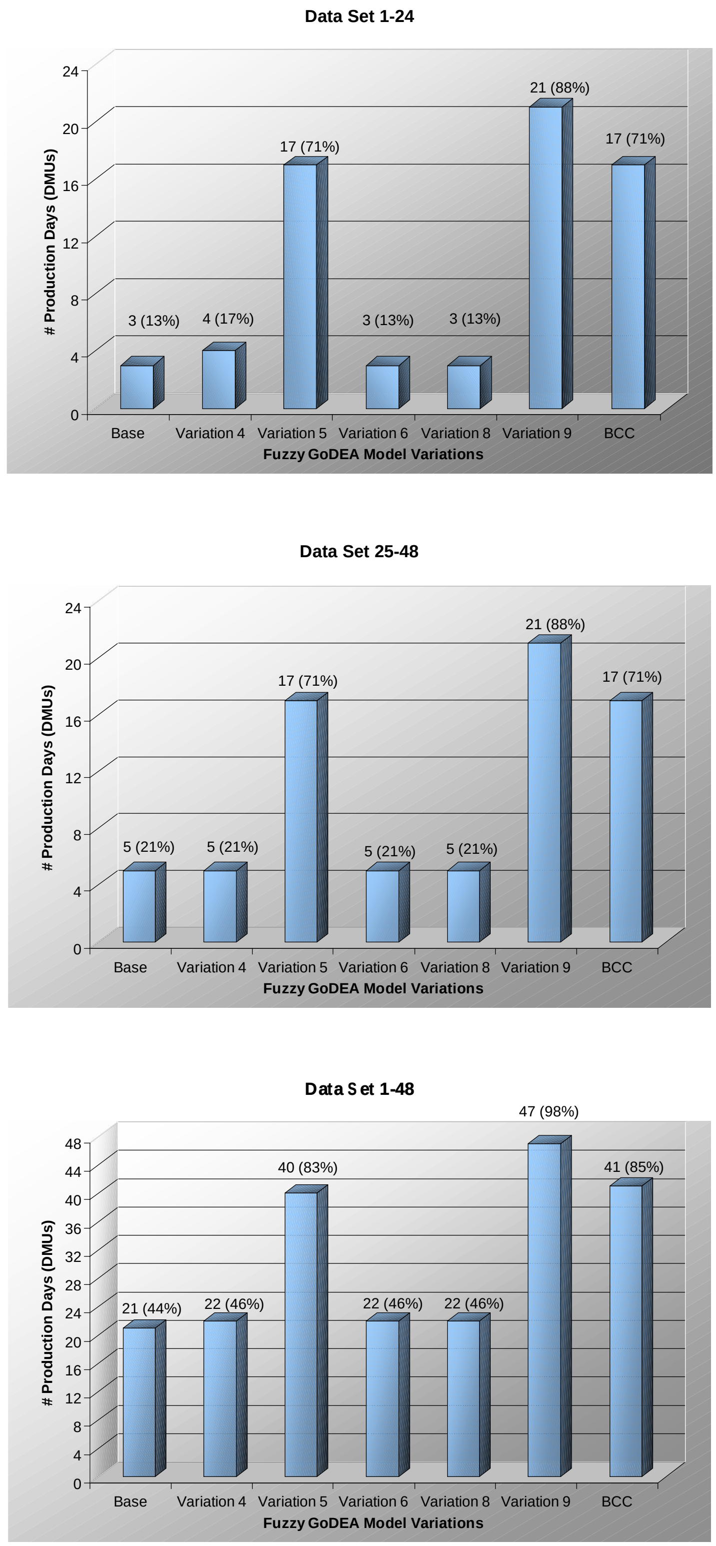


Figure 5.7 Number (Percentage) of Inefficient Units by Variation

5.7 EFFECTIVENESS ACHIEVEMENT ACROSS FUZZY GODEA VARIATIONS

The achievement of the effectiveness goal or global targets for the inputs and output for the packaging line data differs across the variations of the Fuzzy GoDEA model. Detailed results for the effectiveness goals were presented in Sections 5.5.1 - 5.5.10 and are reported in Appendices C, D and E. Table 5.6 shows the global target achievements for the three data sets across the Fuzzy GoDEA model variations.

Table 5.6 Output/Input Global Target Achievement for Fuzzy GoDEA Models

Data Set	***Output/ Inputs***	***Base Model***	***Variation 4***	***Variation 5***	***Variation 6***	***Variation 8***	***Variation 9***
1-24	PCF	3097486	3097486	3287904	3097486	3097483	3342314
	DLR	31450	31450	26437	31450	31448	26437
	RWK	173766	173766	135123	173766	173802	135123
	RML	141141	141141	137228	141141	141107	137713
25-48	PCF	4067007	4067007	4411511	4067007	4067005	4437142
	DLR	39931	39931	32916	39931	39931	31981
	RWK	264408	264408	206598	264408	264407	206598
	RML	161296	161296	161296	161296	161296	152526
1-48	PCF	7164431	7164493	7711673	7162113	7162147	8013491
	DLR	71381	71381	54132	71381	71381	54132
	RWK	438163	438174	273518	438174	438163	277370
	RML	298633	298629	265881	298629	298633	272022

Variations 4 and 6 have crisp effectiveness constraints with the global targets representing the decision-maker's risk-free scenario. Variation 6 has a second level priority attached to the effectiveness constraints. For data sets 1-24 and 25-48, the results for the achievement of the global targets are identical for these variations. For data set 1-48, the results for the inputs are identical. However, Variation 4 achieves a better target achievement for PCF (2,380 packages more) than Variation 6.

The Base Model and Variations 5, 8 and 9 have fuzzy effectiveness constraints with the global targets representing the decision-maker's ideal benchmarks. The Base Model and Variation 5 have equal priority for the efficiency and effectiveness constraints. Variation 8 has a second level priority while Variation 9 has a first level priority attached to the effectiveness constraints. For all three data sets the Base Model and Variation 8 have identical global target achievement results and show the lowest achievement for all inputs DLR, RWK, and RML and output PCF. For data set 1-24, achievement for DLR and RWK is maximized at the global targets and achievement for RML is approximately the same for Variations 5 and 9. However, the achievement for PCF is lower in Variation 5 than in Variation 9. For data set 25-48, achievement for RWK is maximized at the global target for both Variations 5 and 9 but achievement for PCF, DLR, and RML is greater in Variation 9 than in Variation 5. For data set 1-48, achievement for DLR is maximized at the global target for both Variations 5 and 9. The achievement for RWK is almost equal for the two Variations. However, the achievement for RML is greater in Variation 5 than in Variation 9 but the achievement for PCF is greater in Variation 9 than in Variation 5.

5.8 ACTIVITY LEVELS (λ) ACROSS FUZZY GoDEA VARIATIONS

The activity levels associated with the DMUs in the Fuzzy GoDEA variations denote the efficient input and output activity. By definition of DEA (see Chapter 2), the activity levels for an assessed DMU are the same for the inputs and the outputs. The activity levels can be considered to represent the efficient contribution of the peers for an inefficient DMU. For an efficient DMU, the activity level is associated with itself is equal to 1. This means that such a DMU is its own peer. The activity levels in this research are restricted so that for each DMU the sum of its activity levels is equal to one. This is the BCC convexity constraint that is maintained in the Fuzzy GoDEA formulation and enables modeling of variable returns to scale (see Chapter 2).

Mathematically, the activity levels act as input/output coefficients for construction of the composite unit that acts as a reference point on the efficient frontier. An efficient DMUs lies on the frontier and therefore, is the reference unit. The composite unit provides a point on the frontier as a benchmark for an inefficient DMU. In this case a convex virtual or composite unit is constructed with the activity levels denoting the percentage contribution of selected peers. While conventional DEA allows only efficient units to participate as peers for inefficient units, the Fuzzy GoDEA model allows units that may be evaluated efficient and inefficient by conventional DEA evaluation. This feature of the Fuzzy GoDEA methodology is submitted as a major research contribution.

In the results for the Fuzzy GoDEA variations presented in this chapter BCC-inefficient peers have been identified. The activity levels for all the three data sets are reported in Appendices C, D, and E. The decision-maker can analyze these activity levels to observe the change in peer contribution for the different variations. The activity levels also help to distinguish influential observations (*e.g.*, DMU 22 in data set 1-24, DMU 25 in data set 25-48, and DMU 21 in data set 1-48) by their frequency as a peer unit and by their numerical strength. Further, undesirable DMU behavior as a peer (e.g., very low BCC-efficiency score) can also be monitored by analyzing the activity levels for such DMUs.

5.9 EVALUATION OF THE PACKAGING LINE PERFORMANCE

The Fuzzy GoDEA model and its variations are employed in this research to assess the performance of a newspaper preprint insertion packaging line in a fuzzy environment or a combination of fuzzy and crisp environment. The decision-maker seeks to maximize two goals, namely efficiency and effectiveness. The decision-maker's different scenarios are captured by both fuzzy goals or by a combination of one crisp and the other fuzzy. The decision-maker can consider both goals equally important. Alternately, the relative importance of achievement of the goals assigned by the decision-maker is reflected through priority levels where the more important goal is optimized first and the other goal is optimized next constrained by the solution obtained for the first goal. The above outlined alternatives result in nine variations of the Fuzzy GoDEA Base model.

The Fuzzy GoDEA Base model and its nine variations were applied to data derived from Girod (1996). The results were obtained for three data sets, namely, production days 1-24, production days 25-28, and production days 1-48. It is submitted that the Fuzzy GoDEA methodology developed in this research provides the decision-maker with a tool to measure and analyze the simultaneous achievement of the efficiency and effectiveness goals. The efficiency goal is measured at the local level (production day) and the effectiveness goal is measured at the global level (global input/output targets) in two dimensions: (i) fuzzy or crisp and (ii) with or without priority. The achievement of the efficiency and effectiveness goals under different scenarios was presented in preceding sections of this chapter.

At the local level, on a production day basis, the decision-maker can use the fuzzy peer tables (Tables 5.3, 5.4 and 5.5) to identify inefficient units for different scenarios and make resource reallocation decisions, explore possible efficiency enhancement interventions and further investigate root causes of inefficiency. For example, DMU 4 is inefficient across all variations for data set 1-24 and data set 1-48. The above average

RML consumption (6,141 NNSS pieces) and a significantly low PCF production (122,720 packages) explain this inefficiency. Similarly, for data set 25-48, DMU 45 is inefficient across all variations. The above average consumption of DLR (2,306 dollars), RWK (13,215 rework pieces), and RML (8,564 NNSS pieces) for a marginally above average PCF production (175,644 packages) explain the inherent inefficiency. For data set 1-48, DMU 20 is inefficient across all variations and has an above average consumption of DLR (1,771 dollars), RWK (12,159 rework pieces), and RML (8,923 NNSS pieces) for a marginally above average PCF production (153,300 packages). The decision-maker can investigate the root causes for such inefficient observations by analyzing the operational records for the packaging line. Girod (1996) tabulated the symptoms and root causes of inefficiency for severely inefficient DMUs. A similar approach can be adopted to analyze the characteristics of inefficient DMUs. For example, the records showed that for DMU 4 the preprint insertion machine operated at high speeds (greater than 20,000 cycles/hour) causing a high number of NNSS faults (3,000 to 5,000 multiple feeds). Moreover, DMU 4 experienced a high volume of preprint shortage (14,000 NNSS pieces) due to a deficient preprint tracking system. Table 5.7 provides a tabular example (Girod (1996)) that shows symptoms and root causes information for DMUs 4, 17 and 20. Further, the fuzzy peer tables allow the decision-maker to visualize the transition of a DMU from efficient to inefficient or vice versa depending on the characteristics of the adopted Fuzzy GoDEA variation. Tables 5.3, 5.4, and 5.5 show the transition behavior of DMU 2 for data set 1-24, DMU 25 for data set 25-48, DMU 39 for data set 1-48.

Table 5.7 Symptoms and Root Causes for Inefficient DMUs (from Girod (1996))

Production Day (DMU)	Symptoms	Root Cause
4	1. High raw material consumption driven by excessive amount of NNSS faults (3,000 to 5,000 multiple feeds). 2. Significant preprint shortage (14,000 pieces).	1. Preprint Insertion Machine operating at high speed (greater than 20,000 cycles/hour). 2. Preprint Tracking System deficiency.
17	1. High direct labor utilization (47% downtime). 2. Significant preprint shortage (22,000 pieces).	1. Bundle Stacker mechanical failure causing repetitive package jams (at least 30 minutes). 2. Stopped packaging line to set up Hopper Loader and adjust one Printer Hopper (at least 30 minutes). 3. Wait for NNSS or Preprints (at least 45 minutes). 4. Preprint Tracking System deficiency.
20	1. High raw material consumption driven by large amount of NNSS faults (13,000 unopened).	1. Unknown. Assumed related to NNSS quality problem most probably attributable to NNSS printing process.

Thus, the evaluation of inefficient and efficient DMUs by the Fuzzy GoDEA methodology along with the conventional BCC evaluation affords the decision-maker the ability to compare inefficient DMUs with efficient or near-efficient DMUs and implement a best practices approach to improve efficiency.

At the global level the achievement of the global input and output targets was detailed in Sections 5.3 and 5.7 and also categorized by variations in Sections 5.5.1-5.5.10. The achievement of the global targets appears greater for data sets 1-24 and 25-48 individually as compared with data set 1-48. This suggest that the line reorganization around production day 22 does seem to have impacted the efficiency and effectiveness performance overall. In the absence of more detailed information further analysis and conclusions on this possible impact is beyond the scope of this research.

The Fuzzy GoDEA model and its variations allow the objective of achieving *satisfying* versus *optimizing* aspiration levels as demonstrated in this research. In order to extract further information from the fuzzy formulations the decision-maker requires detailed information regarding the operations for each production day to find the causes for inefficiency at the local level and to failure to meet effectiveness targets at the global level.