NONLINEAR EARTHQUAKE ANALYSIS OF WALL PIER BRIDGES

by

Camille Amine Issa

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APPROVED:

Dr. R.M.Barker, Chairman

Dr. S.M.Holzer

Dr. M.P.Kamat

Dr. R.H.Plaut

Dr. A.E.Somers,Jr.

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Camille Amine Issa

Committe Chairman: Dr. R.M. Barker

Civil Engineering Department

(ABSTRACT)

Accurately predicting the response of complex bridge structures to strong earthquake ground motion requires the use of sophisticated nonlinear dynamic analysis computer programs not generally available to the bridge design engineer. The analytical tools that have been developed are generally applicable to bridges whose substructures can be idealized as beam-columns. Bridges with wall piers do not belong to this category.

The major objective of this study is to develop an analysis tool capable of simulating the effects of earthquakes on monolithic concrete wall pier bridges. Thus, after surveying the literature, a mathematical model is developed for the geometrically nonlinear earthquake

analysis of wall pier bridges. Mixed plate elements are used to model the wall pier. The plate element has eight nodes and the degrees of freedom per node are three displacements and three moments. Beam elements are used to model the bridge deck. The beam element accounts for shear deformation and it has two nodes with three displacements and three rotations as degrees of freedom per node. A transitional element is used to join the beam elements to the plate elements. The equation of dynamic equilibrium is solved using the Newmark method with modified Newton-Raphson type iteration at each time step.

The mixed plate element is used to model two plate structures and the results are compared with analytical and other finite element solutions. A two span wall pier bridge is modeled using the structural elements developed in this study. The digitized time history for the N-S component of the El Centro Earthquake of May 18, 1940, is used to seismically excite the bridge model.

ACKNOWLEDGEMENTS

The author is indebted to Dr. R. M. Barker for his support and guidance. The author is also very much obliged to Dr. S. M. Holzer, Dr. M. P. Kamat, Dr. R. H. Plaut, and Dr. A. E. Somers, Jr. for reviewing this dissertation and serving on his committee. Special thanks to Dr. J. N. Reddy for his assistant in developing the mixed plate element. Also, special thanks is in order to Dr. M. P. Singh for providing earthquake time history data files.

Finally, the author extends his appreciation to all his friends at VPI. Last, but not least, the author wishes to dedicate this Dissertation to his beloved parents, wife, son, sister and brothers: in short, the whole ISSA family.

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Chapter I

INTRODUCTION

Since the beginning of recorded history, many cities of the ancient world were described as being totally destroyed by the massive destructive power of earthquakes. The San Fernando, California, earthquake in 1971 is one of the most recent destructive earthquakes that occurred in the United States and diverted national attention to the earthquake problem and what might face the nation in the future. In addition to the structural damage in buildings, it caused substantial damage to the bridge system in the area.

It is estimated that during this century alone, earthquakes in the entire United States have caused about $5 billion in property damage and life losses of about 1400 people [1]. Of all the natural hazards, earthquakes constitute the greatest threat to life. A summary of death tolls of the greatest earthquake disasters is found in Table 1.1.

Although there is a risk that lives might be lost as a direct result of bridge failures during an earthquake, an even larger concern is that collapsed bridges might cause the earthquake region to be cut off from outside help during the disaster, thereby greatly hampering the rescue efforts

TABLE 1.1

Great Earthquake Disasters [2].

Date	Region	No. of Deaths
1556, Jan 23	China:Shansi	830,000
1693, Jan 9	Italy:Sicily	60,000
1730, Dec 30	Japan:Hokaido	137,000
1737, Mar 12	India:Calcutta	300,000
1755, Nov 1	Portugal:Lisbon	60,000
1759, Oct 30	Syria	30,000
1783, Feb 5	Italy:Calabria	30,000
1797, Feb 4	Ecuador:Peru	40,000
1812, Mar 26	Venezuala:Caracas	20,000
1857, Dec 16	Italy	19,000
1868, Aug 13	Ecuador:Peru	40,000
1891, Oct 28	Japan:Mino-Owari	7,500
1896, Jan 15	Japan:Riku-Ugo	27,000
1905, Apr 4	India:Kangra	12,000
1906, Apr 16	San Francisco	478
1906, Aug 17	Chile:Valparaiso	20,000
1907, Oct 21	Central Asia	12,000
1908, Dec 28	Italy:Messina	83,000
1915, Jan 10	Italy:Avezzano	29,000
1920, Dec 16	China:Shansi	100,000
1923, Sep 1	Japan:Tokyo	99,000
1925, Mar 16	China:Yunnan	9,000
1927, May 22	China:Non-Shan	200,000
1934, Jan 15	India:Bihar	11,000
1935, May 30	India:Quetta	30,000
1939, Jan 25	Chile	28,000
1939, Dec 26	Turkey:Erzincan	30,000
1948, Jan 28	Japan:Fuki	5,500
1949, Aug 5	Ecuador:Ambato	6,000
1960, Feb 29	Morocco:Agadir	12,000
1960, May 2	Chile:Concepcion	6,000
1964, Mar 25	Southern Alaska	66
1970, May 31	Peru:Chimbote	50,000
1971, Feb 9	San Fernando	62
1972, Apr 10	Iran:Fars	6,000
1972, Dec 23	Nicaragua	5,000
1974, May 31	Peru	50,000
1976, Feb 4	Guatemala	22,000
1976, Jul 27	China:Hopeh	100,000
1980, Oct 10	Algeria:Al-Asnam	25,000

and increasing the death toll. Because bridges are essential for crossing both manmade and natural obstacles, it is crucial that they continue to function safely, following an earthquake, when the protection of lives and property depends on the efficient movement of emergency traffic. Bridges exposed to earthquakes must remain accessible and mantain their structural integrity.

To aid in the above objective, and because seismologists estimate that more than 35 of the nation's 50 states have the potential for ground motions of magnitude sufficient to cause serious bridge damage, critical areas related to the seismic design of bridges have attracted intense interest and are the subject of federal and state-sponsored research studies. Research, especially during the past few years, has contributed to improved techniques for consideration of earthquake effects in designing new bridges and retrofitting old ones. Primary among these activities is the Federal Highway Administration's Earthquake Engineering Research Program, which has recently capped off more than a decade of concentrated effort with the production of comprehensive guidelines for the seismic design of highway bridges.

In addition to all those humanitarian reasons mentioned previously, bridges represent the highest cost per mile in the nation's highway system. Thus it is crucial to protect this national investment from the massive destructive power of earthquakes.

The general concern of this research is to determine the effects of earthquakes on the dynamic response of bridges. The effects of earthquakes on wall pier bridges will be specifically addressed. Presently, all other types of bridges have been extensively researched (see CHAPTER II), however, little could be found concerning wall pier bridges (Figure 1.1). In the recently published Guide Specifications for Seismic Design of Highway Bridges, there were twenty-three bridges redesigned to meet the new seismic design specifications. The types of bridges redesigned and cost increases associated with their redesign are summarized in Tables 1.2 and 1.3. Out of those twenty-three bridges, seven were wall pier bridges with a construction cost increase ranging from 4.5 to 44.8 percent. The highest cost increase is for monolithic concrete wall pier bridges. Yet, there is not a convenient analysis tool available to perform an earthquake analysis of this type of bridge. Therefore, the major objective in this study is to develop an analysis tool capable of simulating the effects of earthquakes on monolithic concrete wall pier bridges.

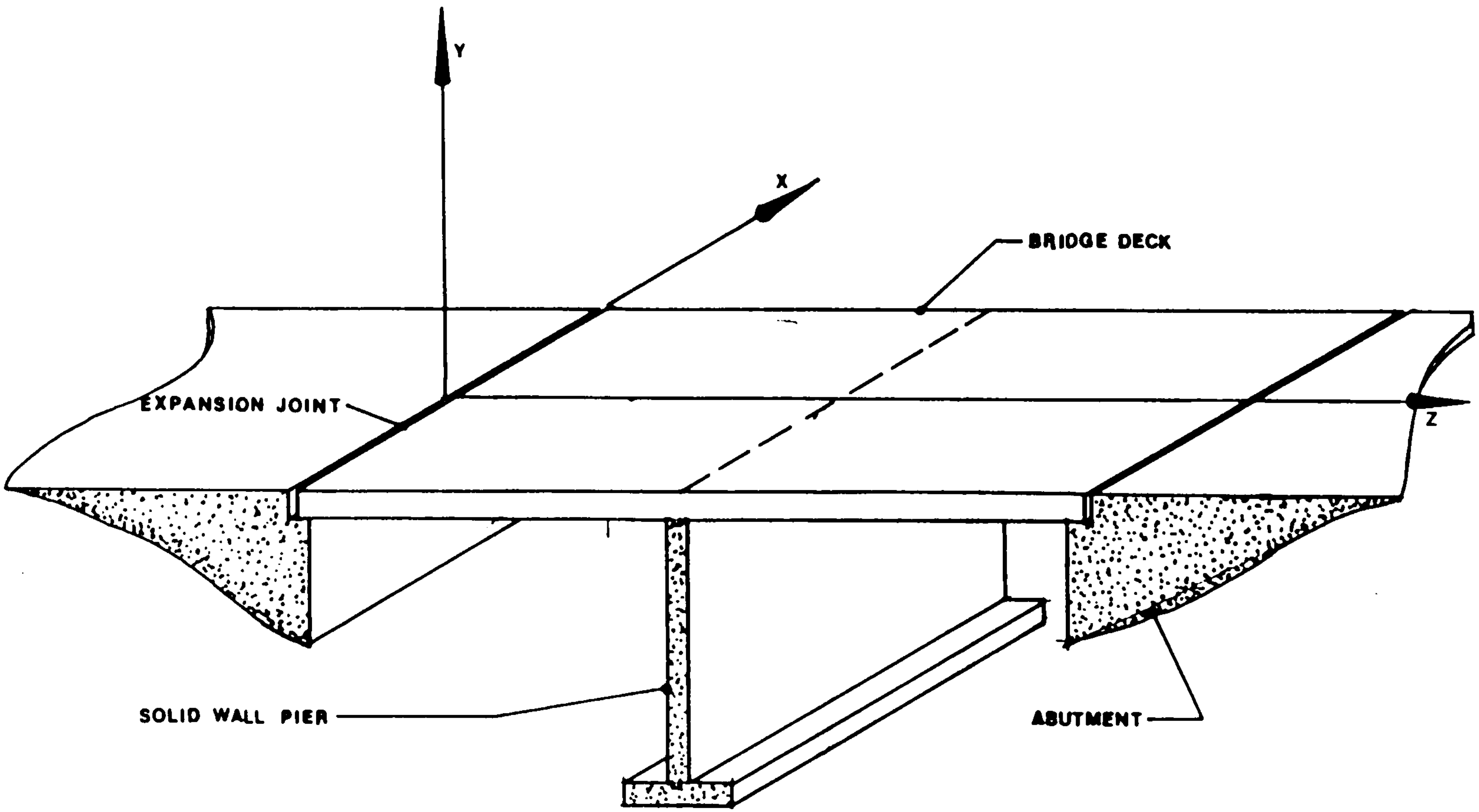


Figure 1.1: Typical Wall Pier Bridge

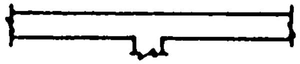
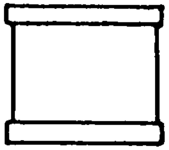
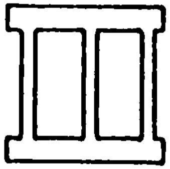
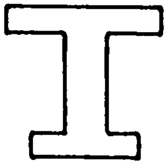
TABLE 1.2

Types of Bridges Redesigned [108].

SUBSTRUCTURE (BENTS AND PIERS)	SUPERSTRUCTURE			
	STEEL	CONCRETE	STEEL	CONCRETE
PIER	1	2	2	2
MULTI COLUMN BENT	2	1	3	6
SINGLE COLUMN BENT	-	-	-	2
OTHER	-	2	-	-

TABLE 1.3

Cost Increase by Bridge Types [108].

SUBSTRUCTURE (BENTS AND PIERS)		SUPERSTRUCTURE			
					
		STEEL	CONCRETE	STEEL	CONCRETE
PIER		5.1	5.1	4.5	44.8
MULTI COLUMN BENT		0.5	1.9	0.3	4.7
SINGLE COLUMN BENT		-	-	-	3.6
OTHER		-	0.0	-	-

Chapter II

BRIDGES AND EARTHQUAKES

2.1 HISTORICAL BACKGROUND

The problem of bridge vibration has been studied since the 1847 Chester Rail Bridge collapse in England [3]. Since that time, the study of vibration of bridges due to moving vehicles was a major concern of bridge engineers. Significant progress was achieved in understanding this problem during the early 1950s with the availability of computers and suitable measuring equipment. A thorough review of that progress by the end of 1959 has been given by Wright and Green [4]. Most recently, Huang [5] has reviewed the whole subject of "Vibration of Bridges".

Not only moving vehicles but also wind and seismic excitations can cause bridge vibration and damage. The wind effect is more predominant on suspension bridges than on any other type and can cause aerodynamic instability of the bridge. This problem has been studied since the collapse of the Tacoma Narrows Bridge [6]. Most recently, Abdel-Ghaffar [7] has reviewed the whole subject of effect of wind on suspension bridges and presented a mathematical model for suspension bridges which proved to be very reliable when compared to the experimental data obtained from instrumentation on the Golden Gate Bridge's vibration.

Very little damage to bridges is known to have resulted directly from seismically induced vibrational effects prior to the San Fernando, California, Earthquake of February 9, 1971. During this earthquake approximately $6.5 million in bridge damage occurred and most of it was due to vibrational effects and inadequate connections between structural elements [8]. Before the San Fernando Earthquake, most bridge damage on a worldwide basis was caused by permanent ground displacements that resulted in:

1. Tilting, settlement, and overturning of the substructure.
2. Displacement of the supports and anchor bolt breakage.
3. Settlement of the approach fills and wing wall damage.

The San Fernando Earthquake (6.6 magnitude on the Richter Scale) was an important turning point in seismic design of highway bridges. This event increased public awareness of the potential for earthquake-induced damage to the transportation system and resulted in recognition of the need to design highway bridges that are more resistant to the damaging effect of seismic forces induced by ground motion. This earthquake was the first real "test case" of existing U.S. seismic design provisions for structures in, and close to, an earthquake region.

2.2 LITERATURE REVIEW

2.2.1 Documented Earthquake Damages to Bridges

Damages sustained by highway bridges as a result of earthquakes have been recorded in many countries, and a considerable number of investigations have been made. Iwasaki, Penzien, and Clough [9] surveyed the literature of seismic effects that had been reported by 1972. This included references to:

1. The characteristics of strong ground motions at bridge sites.
2. Bridge-bearing capacity.
3. The effects of soil stability and hydraulic pressures on piers or abutments.
4. Dynamic properties and dynamic analyses of bridges.
5. Field measurements.
6. Laboratory experiments.
7. Specifications for earthquake-resistant bridge designs.

In general, two types of bridge damage occurred during these earthquakes [9]. For those bridges supported on low abutments and piers, damage usually results from foundation failure. For those on high abutments and/or columns, however, damage results from severe superstructure motions. These motions may result in:

1. Excessive inelastic deformation in columns.
2. Impact at the expansion joints between the superstructure.
3. Disastrous separation of the superstructure from the piers or columns.

Since 1965, high long-span bridges have become more popular in bridge design. It has been reported [10-15] that damage to such structures was very severe during the San Fernando Earthquake. Again, extensive documentation of bridge damage and failures in the 1964 Alaska, 1971 San Fernando, 1976 Guatemala, and 1978 Japan earthquakes by Cooper [16-19] has pointed out critical weaknesses in bridge supports and substructures during strong seismic activity. Typical bridge damage reported includes collapsed spans, loss of pier bearing support, abutment and column or pier movement, joint movement, and column and foundation failure. The documentation of earthquakes can be developed into rational design procedures that might ensure the structural integrity of the bridge system during periods of ground motion. To emphasize the importance of documentation, it is helpful to summarize the modes of bridge failure during the following earthquakes:

The 1964 Alaska Earthquake[19,20]:

On March 25, 1964, South-Central Alaska suffered an earthquake of unusually large magnitude: between 8.3 and 8.6 on the Richter Scale. Due to this earthquake, the primary damage to the Alaskan bridges is attributed to span shortening, horizontal pier movement, and relative vertical movement between piers and abutments. Abutment fills settled, and in some cases bridge abutments moved together toward the center of a stream causing expansion joint collapse and severe superstructure shortening. Ten bridges totally collapsed, 26 experienced partial collapse and were beyond repair, and 60 others experienced some form of repairable damage. Soil liquifaction, evident wherever severe damage occurred, played a major role in foundation displacement and bridge damage. Liquifaction is thought to have occurred during seismic loading, thus accounting for most of the pier movement at river crossings. Damage to many bridge piers was caused when piers and pier caps displaced longitudinally to the point where spans fell off bearing supports. Those bridges founded on piles that were driven through sands and silts experienced severe damage, those bridges having foundations supported directly on bedrock suffered little damage, and those bridges founded on piles in gravelly sands and gravel performed relatively well.

<u>The 1971 San Fernando Earthquake</u>[<u>10-16,19</u>]:

On February 9, 1971, the San Fernando valley suffered an earthquake of magnitude 6.6 on the Richter Scale. During this earthquake, several major bridges collapsed. These spectacular collapses highlighted the weaknesses in the design detail of columns and connections. A unique feature of the earthquake was the significant amount of vertical acceleration, averaging approximately two-thirds that of the horizontal. Seven bridges either totally collapsed or were so badly damaged that they had to be replaced. Sixty additional structures experienced moderate to extensive, but repairable, damage. The enormity of the vertical component of force caused some reinforced concrete bridge columns to burst in the middle as though they had been failed in a huge testing machine. This kind of failure makes a strong case for providing more spiral ties that can retain the crushed concrete and provide continued support to the structure. The integrity of footings was severely jeopardized by the kind of forces produced by this earthquake. In several cases, the connection between the base of a column and the cap of a pile footing failed if the connection was not adequately designed. The grinding back and forth and the pounding completely reduced the pile cap to nothing more than rock, dust, and rubble. Main column reinforcement, typically number 18 bars, pulled out completely because of inadequate connection details. Because of this kind of failure, additional top steel has since been added by bridge designers to the footing and tied into the column. Also, anchorage details have since been improved.

<u>The 1976 Guatemala Earthquake</u>[17,19]:

On February 4, 1976, the city of Guatemala and its surroundings suffered an earthquake of magnitude 7.5 on the Richter Scale. This earthquake was of particular interest because it vividly demonstrated the importance of using hinge restraining devices to tie the superstructure together. The superstructure of one major bridge collapsed and that of another rotated, in plan, and fell off its bearings. A third structure that had restrainer bars designed into the superstructure suffered no appreciable damage. Eleven other bridges suffered varying degrees of repairable damage. Typical damage included lateral displacement of the superstructure, cracked abutments caused by deck impact, and tipped or fallen rocker bearings.

The most severely damaged bridge was a five-span, simply supported steel plate girder structure [17,19]. The earthquake caused the three center spans of the superstructure to fall off the bearing supports. The abutments and column bents were undamaged with the exception of extremely minor hairline cracks around the base of one of the columns. The bridge collapsed principally because of the bearing details used and the lack of longitudinal and transverse superstructure restraint at the supports. Thus the fixed bearings could be seen in place atop of free standing piers. If appropriate hinge restrainers had been in place (thus tying the structure together), the bridge probably would not have collapsed.

The bridge that performed well was constructed in 1972 [17,19]. The bridge is of interest because it incorporates seismic hinge and abutment restrainers, and although minor damage occurred, the bridge remained in service following the earthquake. The performance of this eight-span concrete bridge indicated that incorporating new seismic design techniques in new construction can provide adequate resistance against major earthquakes. The generally good performance of the bridge is attributed to the use of seismic restraint mechanisms.

<u>The 1978 Japan Earthquake</u>[18,19]:

On June 2, 1978, the island of Honsha in Japan suffered an earthquake of magnitude 7.5 on the Richter Scale, thus making it the most powerful earthquake that hit Japan since 1964. Approximately two dozen bridges were damaged significantly during this earthquake. Massive reinforced concrete piers were badly cracked and damaged. They responded essentially as nonductile, rigid bodies to the earthquake motion. In spite of the major damage to these elements, the designs proved satisfactory because the bridges did not collapse but remained operational for emergency use. Although this kind of construction proved successful, it is too expensive for use in the United States where the philosophy is to use more ductile, energy-absorbing designs. Pier tilting and abutment movement were evident at several sites and probably caused extensive damage to bearing devices . This kind of movement is exceptionally difficult to control and indicates the need to perform indepth geotechnical investigations, particularly in areas with high water tables. The most spectacular damage occurred to a 574m (1885ft) 15-span structure constructed in 1956. It suffered major damage to the superstructure, bearings, and abutment. One drop-in plate girder span fell off its bearings and dropped to the ground. The girders at the adjacent pier displaced longitudinally 533mm (21 in), allowing the unrestrained suspended span to drop off a 457mm (18 in) hinge seat. Extensive bearing and abutment damage emphasized the extensive relative motion that occurred between the superstructure and substructure.

2.2.2 Research Activities in Japan

Prior to the San Fernando Earthquake, most of the research activities on seismic effects on bridges had been conducted in Japan [9]. These research activities can be classified into four groups:

1. Field Testing of Bridges: Extensive investigations have been carried out to determine the dynamic properties of bridge structures such as natural periods, mode shapes, and damping characteristics. Numerous dynamic tests have been conducted on actual bridges, usually with excitations small in comparison with those caused by major earthquakes [21]. During the period of 1958 to 1969, twenty six-highway bridges in Japan weere tested dynamically. From the results of these tests the following conclusions may be deduced:

   a) The fundamental natural periods of horizontal motions are about 0.07 to 1.0 seconds for foundations, and about 0.1 to 1.3 seconds for overall structures. The fundamental periods get noticeably longer as pier heights increase.

   b) Damping ratios of bridge structures for horizontal vibrations are approximately 0.02 per period for ordinary bridge structures including foundations, piers, and superstructures and are about 0.013 for highrise piers alone and for bridges with highrise piers.

c) The results obtained from field tests are for small amplitude vibration. Based on the results of some bridges, it appears that the dynamic properties vary significantly with even small changes in amplitude. Therefore, dynamic properties for large amplitude oscillation can differ appreciably from those described above. The natural periods tend to get longer and the damping ratios become higher as the amplitude of oscillation increases.

2. Instrumentation: Observation of the dynamic response of bridges during strong motion earthquakes was initiated in Japan after the development of accelerographs in 1953. As of March, 1969, a total of 106 accelerographs had been placed on highway bridges and on the ground near these bridges [22]. By March, 1970, this number increased to 119 [23]. From the recorded results in References [24-29], the following observations can be made:

   a) In most cases, maximum accelerations at pier crests are greater than those measured on the ground surface.

b) Ratios of pier to ground accelerations tend to decrease with intensity level of ground acceleration.

c) Accelerations at crests of abutments are usually less than the ground accelerations.

d) Ratios of response acceleration to ground acceleration are in the approximate range 2-4 and are nearly independent of substructure height,i.e., no distinct relation appears to exist between substructure height and their response.

e) Intensities of ground accelerations have greater significance on bridge response than does substructure height.

f) When subjected to moderate ground accelerations, the ratios of response acceleration to ground acceleration for highrise bridges are usually in the range 1-4.

g) Highrise bridges are expected to respond as linear structures for moderate ground motions.

3. Dynamic Analyses: The heavy damages sustained by bridges during the 1964 Niigata Earthquake emphasized the importance of dynamic effects during earthquakes. Thus, in assessing earthquake resistance of major highway bridges (References [30] to [38]), dynamic

analyses were conducted even though designs were normally carried out using the conventional seismic coefficient method or a modified seismic method. Two methods of analysis were commonly used, namely, the response spectrum method, in which maximum values of structural response are determined using appropriate response spectrum curves, and the time history method, which generates complete histories of response using selected accelerograms as the input excitation. In applying either of these methods in the design process, it is important to:

a) Establish accurate mathematical models for the overall structure, including substructures, foundations, and surrounding soils.

b) Select appropriate seismic excitation for the bridge site.

c) Properly interpret the results of analysis in terms of prototype behavior.

4. Experimental Models: To study the dynamic behavior of bridges, investigators have conducted experimental studies using laboratory models [21,35,39-42] and subjected them to dynamic excitation using shaking tables and exciters (Japan is a leader in the world of building scale models of structures and using shaking tables to simulate earthquake motion).

2.2.3 Research Activities in the United States

Following the San Fernando Earthquake, the Federal Highway Administration (FHWA) program of earthquake engineering research (initiated in 1969) went into high gear to develop improved design standards for new bridges and procedures for upgrading existing bridges. A major research project sponsored by FWHA entitled "An Investigation of the Effectiveness of Existing Bridge Design Methodology in Providing Adequate Structural Resistance to Seismic Disturbances" was initiated at the Earthquake Engineering Research Center, University of California, Berkeley. The main purpose of the Berkeley research was to develop improved design standards for new bridges and it consisted of the following six phases:

1. A thorough review of the world's literature on seismic effects on highway bridge structures, including damage to bridges during the San Fernando earthquake [9].
2. An analytical investigation of the dynamic response of long multiple-span highway overcrossings of the type which suffered heavy damage during the 1971 San Fernando earthquake [43,44].
3. An analytical investigation of the dynamic response of short, single, and multiple span highway

overcrossings of the type which suffered heavy damage during the 1971 San Fernando Earthquake [45,46].

4. Detailed model experiments on a shaking table to provide dynamic response data similar to prototype behavior which was used to verify the validity of theoretical response predictions [47-49].
5. Correlation of dynamic response data obtained from shaking table experiments and the theoretical response. Modification of analytical procedures as found necessary to achieve correlation [50,51].
6. Preparation of recommendations for changes in seismic design specifications and methdology as necessary to provide adequate protection of reinforced concrete highway bridges against severe damage in future earthquakes [52,53].

Besides the work done at Berkeley, numerous other studies at a smaller scale, but of equal importance, were being conducted all around the nation to improve the seismic design of bridges. Abdel-Ghaffar and his colleagues [54-57] studied the earthquake response of suspension bridges. Heins and Lin [58] were concerned with the seismic design of curved concrete box girder bridges. Douglas and his colleagues [59,60] experimentally applied seismic design loads on a highway bridge. Ghobarah and Tso [61] were concerned with the seismic analysis of skewed highway bridges. In References [62-65] the major concern was the computer simulation of the seismic response of bridges.

While the major concern of the above investigators was to improve the seismic design of new highway bridges, others [66-71] were busy developing procedures for retrofitting the existing bridges. The retrofit measures, which are means to increase the seismic resistance of existing bridges, that were investigated are [66-69]:

1. Restricting longitudinal, vertical, and lateral relative displacements of the superstructure at expansion joints, bearing seats, and so on by means of cables, tie bars, shear keys, extra anchor bolts, and metal stoppers (Figures 2.1 and 2.2).
2. Restricting rigid body motion of the superstructure by connecting it (e.g., with high-strength steel cables as in Figure 2.1) to a supporting or an adjacent foundation or pier cap, by enlarging bearing areas, or by placing stoppers at edges of bearing areas.
3. Reducing induced vibrations by installing energy absorbing devices such as elastomeric bearing pads at bearing seats or adapting a Japanese shock absorber (Figure 2.3) type of damper that allows slow

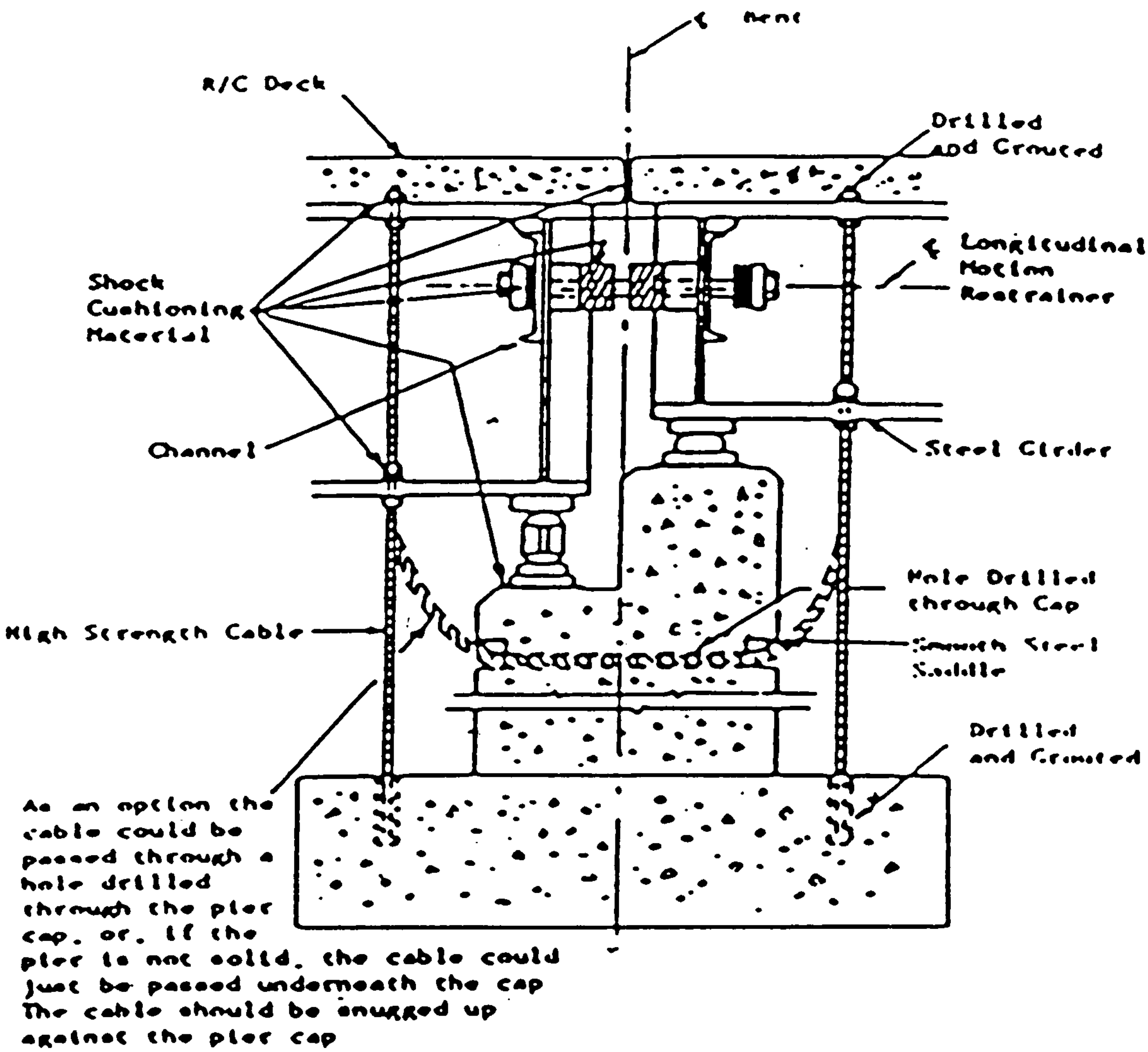


Figure 2.1: Relative longitudinal motion restrainer and high-strength cable for preventing uplift of superstructure[67].

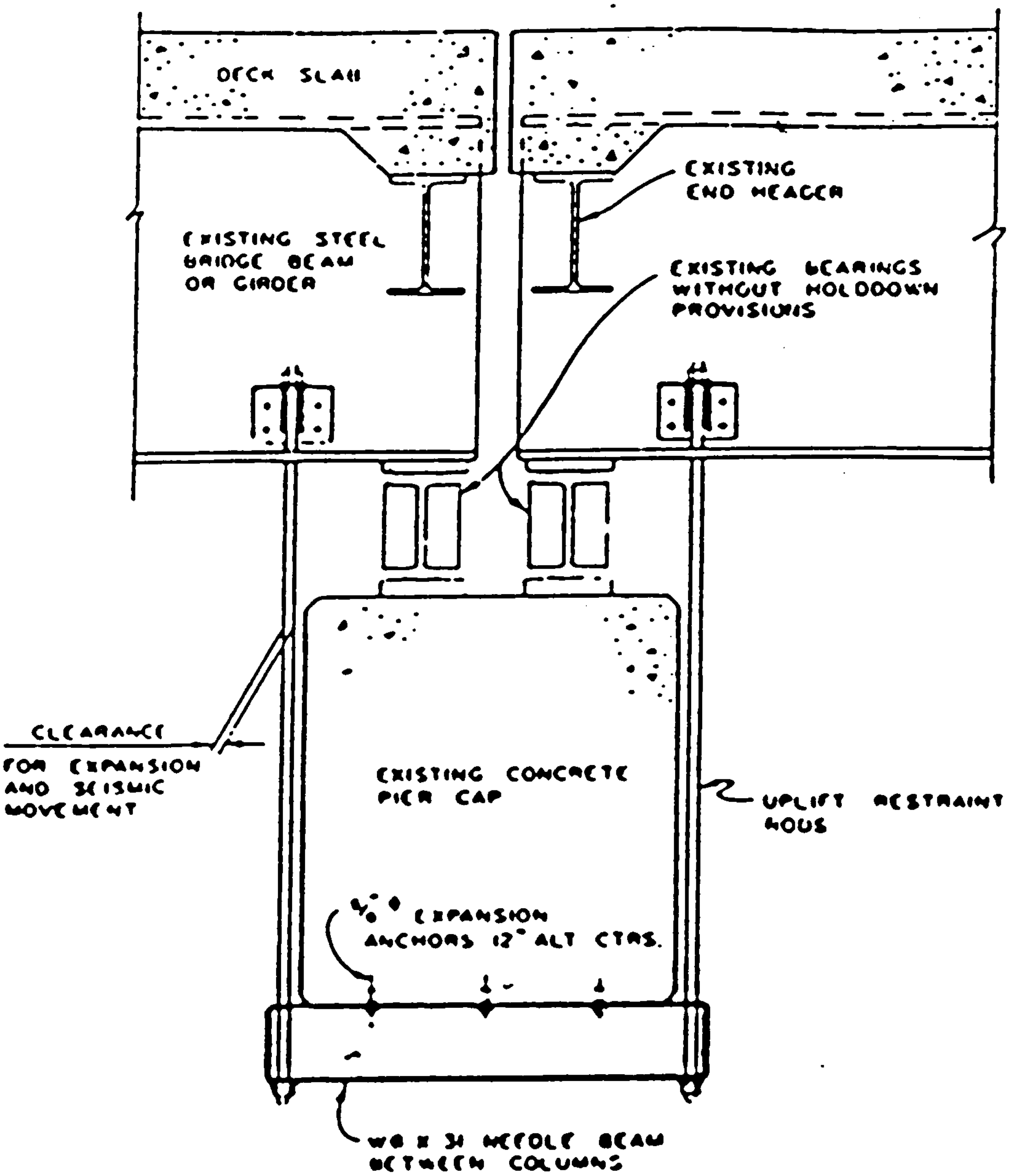


Figure 2.2: Steel girder vertical restrainer[67].

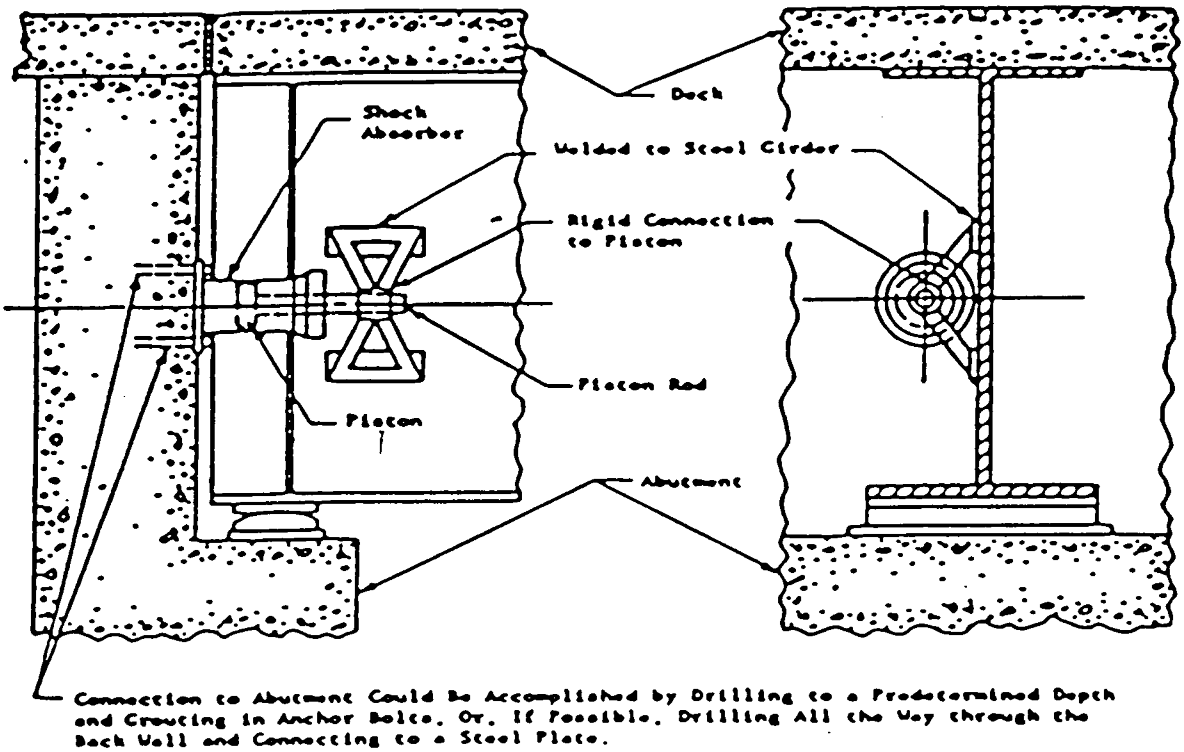


Figure 2.3: Possible adaptation of Japanese shock absorber type of damper[67].

movement, such as displacement due to creep, shrinkage, and temperature change, with negligible resistance but that develops a large resistance in the case of a rapid displacement, i.e., high velocity, such as that caused by an earthquake.

4. Strengthening supporting structures. As a specific example, the strength of an existing column (Figure 2.4) can be increased by adding longitudinal and spiral reinforcement to the exterior of the column and then bonding the added reinforcement with a new layer of high-strength concrete by using pressure grouting procedures or gunite.

With very few exceptions, existing highway bridges in the nation have not been designed to resist motions and forces which may be generated by the occurrence of earthquakes in the surrounding areas. As a result, many bridges may be expected to fail in some major way during their remaining life if subjected to strong motion seismic loads. Such bridge failures are clearly undesirable and raise the question as to which bridges should be retrofitted and to what degree so as to improve their survivability with respect to probable earthquake excitations. To help in that decision making, an outline (Figure 2.5) and flow chart (Figure 2.6) were developed in References [67] to [71] to

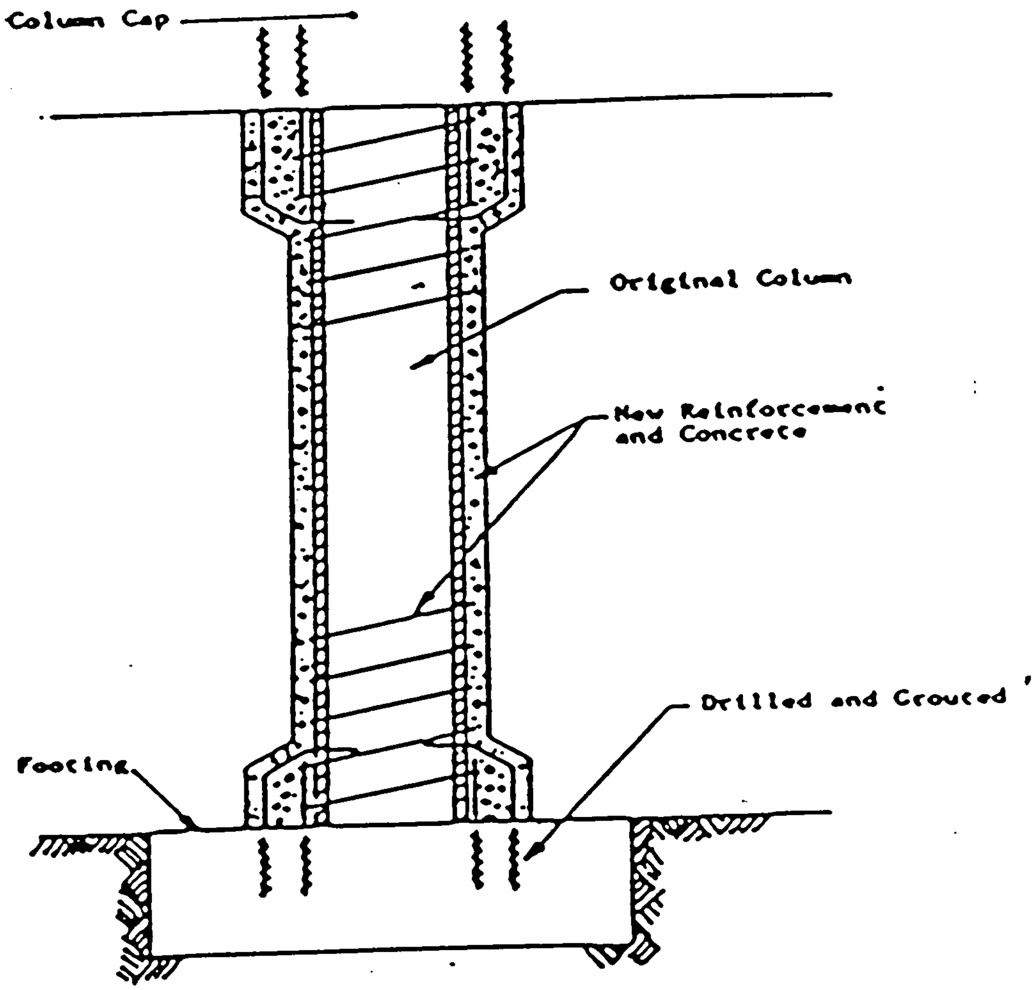


Figure 2.4: Srengthening of column[67].

1. Initial Screening

   a. Identify primary routes
   b. Identify structures required to remain in service
   c. Evaluate alternate available routes
   d. Investigate likelihood of damage due to seismic exposure
   e. Evaluate structural vulnerability based on:
      (1) age
      (2) condition
      (3) code under which designed
      (4) structure type
      (5) details
   f. Make preliminary selection

2. Develop Retrofit Analysis/Design Criteria

   a. Seismic forces
   b. Displacement
   c. Load combinations
   d. Allowable element capacities

3. Perform Structural Investigation of Selected Bridges

   a. Simplified analysis
   b. Computer analysis
      (1) Linear
      (2) Nonlinear
   c. Evaluate analytical results

4. Identify Retrofit Measures

   a. Superstructure
   b. Substructure
   c. Soil condition

5. Design Retrofit Details

6. Evaluate Effect of Retrofit Details on Overall Seismic Response of Bridges

7. Make Final Selection of Bridges for Retrofit Based on Economic Considerations

Figure 2.5: Outline of Retrofit Process[69].

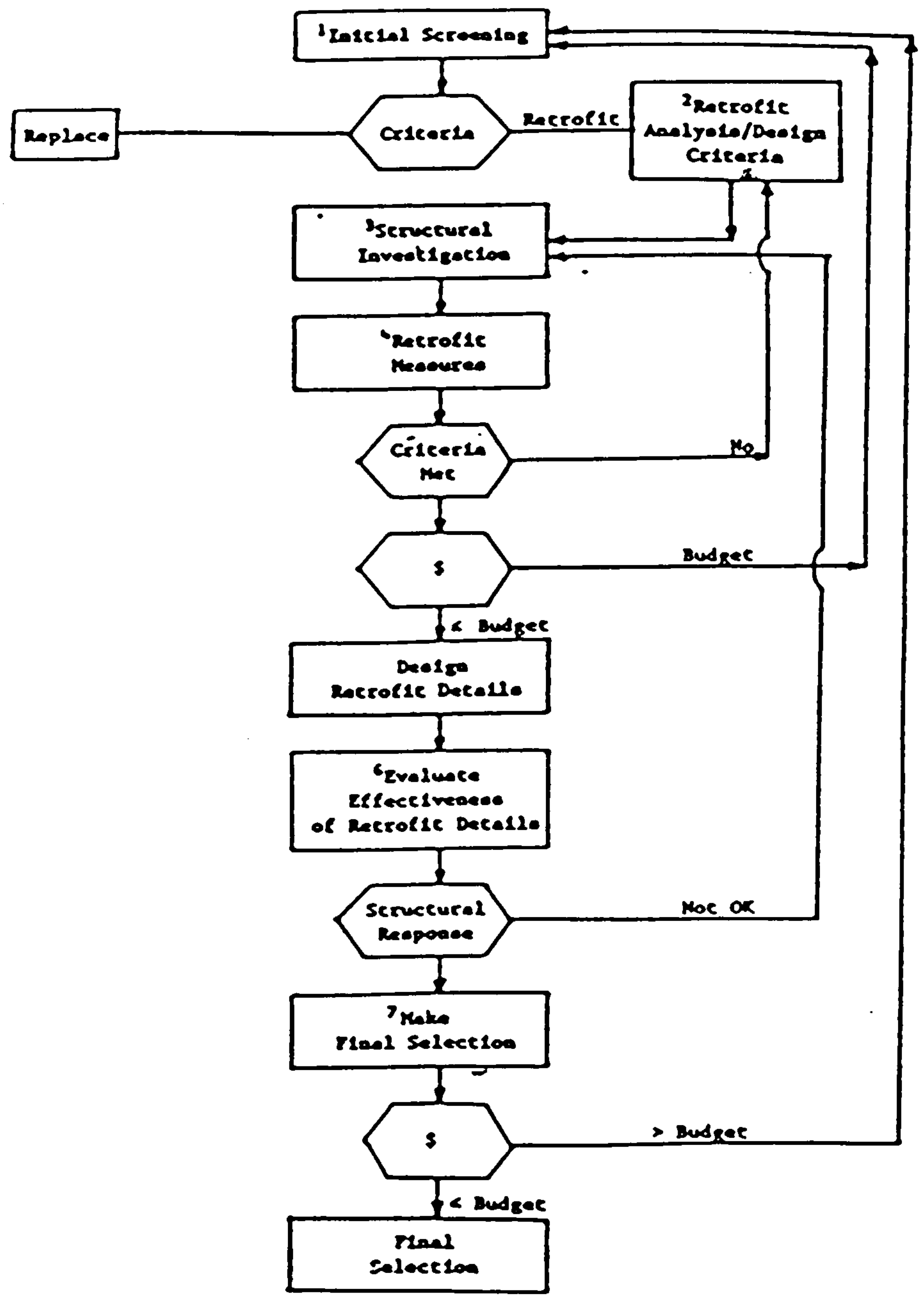


Figure 2.6: Flow Chart of Retrofit Process[69]. (Numbers refer to steps in Figure 2.5)

assist in the process of decision making whether to upgrade the existing bridge to increase its seismic resistance capabilities. The method is based mainly on the importance of the bridge and budget restraints. The importance of the bridge will depend on the following factors [68]:

1. Administration/transportation system effects.
2. Social/survival effects.
3. Security/defense effects.
4. Economical/personal effects.

2.2.4 Research Activities in New Zealand

Despite the fact that most of the discussion following the San Fernando Earthquake event concentrated on research done in this country, in addition to the work done in Japan [72-91], New Zealand engineers [92-106] were equally active in researching the topic of improving the seismic design of bridges. While the United States is clearly the leader in the field of using computers to simulate the response of bridges to earthquakes, it remains behind New Zealand in the field of experimental studies and directly designing according to what is developed in the laboratories. Earthquake engineers in New Zealand like to use mechanical devices in their designs to help dissipate the energy induced into the structure by earthquakes. They develop these devices in the laboratories and test them on large scale models. After succeeding in this process, these designs will often be directly implemented in actual structures. These research activities are of interset to American engineers; thus a team of 16 researchers and praticing design professionals visited New Zealand to assess design practices and implementation of research developments. As a result of that trip, a report was written [107] from which the following conclusions were drawn:

1. There is a strong interaction between New Zealand researchers and design practitioners. Consequently, research findings are often promptly implemented or adopted as guidelines or design provisions, and research laboratories are mobilized to solve needed practical problems.
2. There is a strong emphasis on experimental research, particularly full-scale structural components models.
3. The New Zealand engineers appear to be more willing to use new ideas in the seismic design of bridges such as energy absorbing bearings, rocking bents, uplift in footings, friction slabs, and other innovative devices or approaches. This is the result of the close liaison that exists between the research community and the design engineers.

4. New Zealand researchers have developed an advanced technology using base isolators and energy absorbers as an integral part of the design of earthquake resistant structures. The concept has been adopted and installed in several bridges in New Zealand, including the innovatively designed South Rangitikei Rail Bridge (Figure 2.7), which incorporates hysteretic dampers at the base of the twin-legged piers [106].
5. The capacity design procedure has been developed in New Zealand and is now used extensively. This procedure involves selecting the areas where damage will occur in the structure and confining damage to these areas by providing sufficient capacity in the remainder of the structure. The areas where damage is to occur are suitably designed and detailed for energy dissipation under severe deformations. All other structural elements are then provided with sufficient strength so that the chosen means of energy dissipation can be maintained.

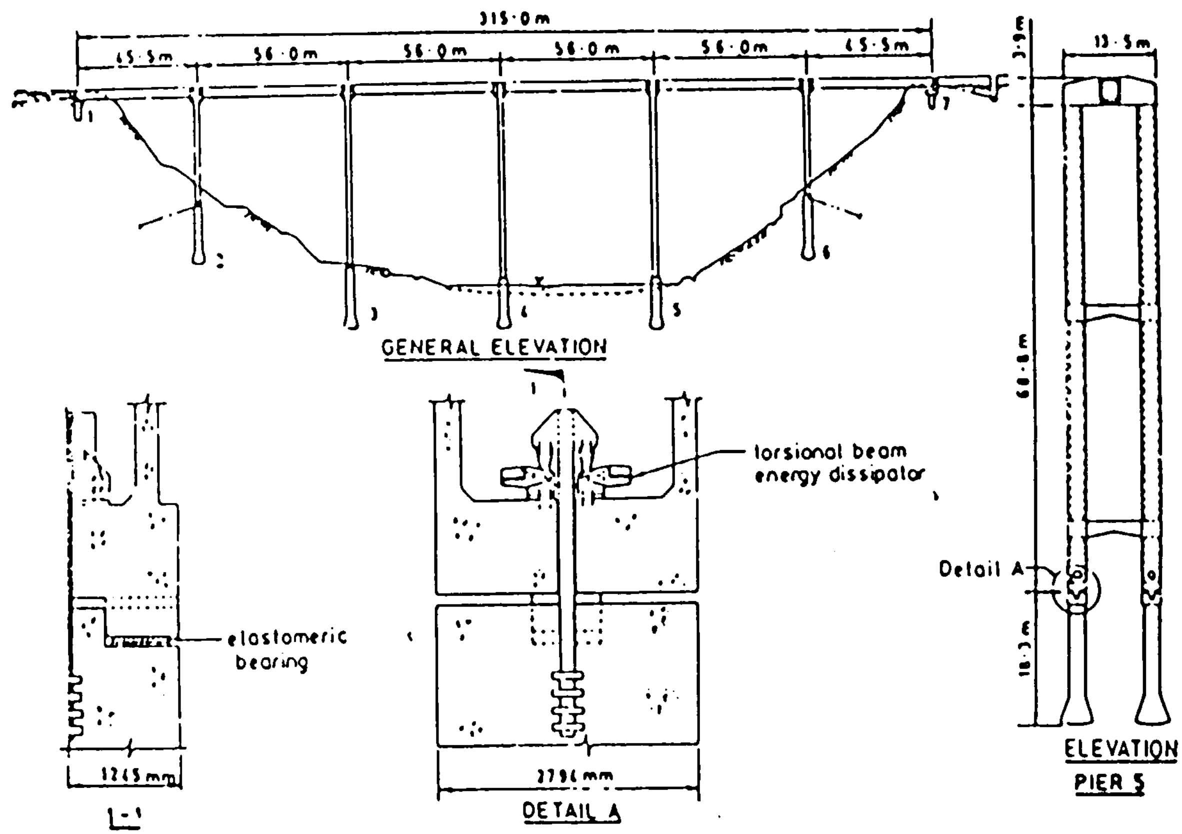


Figure 2.7: South Rangitikei, New Zealand Rail Bridge Construction[106].

2.2.5 Design Philosophies

In the United States, the development of seismic design criteria has been predicated on certain basic concepts which include the following [108,109]:

1. Hazard to life should be minimized.
2. Bridges may suffer damage but have low probability of collapse due to earthquake motions.
3. Function of essential bridges should be maintained.
4. Design ground motions have low probability of being exceeded during normal lifetime of bridge.
5. Provisions should be applicable to all of the United States.
6. Ingenuity of design should not be restricted.

The above concepts constitute the basis on which the seismic design of bridges procedures were developed. Certain principles that were followed are [108,109]:

1. Small to moderate earthquakes should be resisted within the elastic range of the structural components without significant damage.
2. Realistic seismic ground motion intensities and forces are used in the design procedures.
3. Exposure to shaking from large earthquakes should not cause collapse of all or part of the bridge. Where possible, damage that does occur should be readily detectable and accessible for inspection and repair.

There are two different philosophies that are currently being used to satisfy the above principles: the United States and the New Zealand Seismic Design Philosophies. The New Zealand Seismic Design Philosophy currently favored is one of limiting, or at least reducing, the horizontal force which can develop between the foundation and the structure above during earthquake shaking, and to achieve this, two main approaches have been used [107]:

1. The first method ensures that the structure can deform in a ductile manner beyond its elastic limit, thereby limiting the seismic loading that it will be required to react to. A plastic mechanism must be able to form and, due to the nature of the bridge structures, the plastic hinges would usually form in the piers or foundations. These must therefore be capable of considerable plastic deformation. Design precautions are normally taken to cause hinging to occur in the piers rather than in the foundations where possible, thus reducing the likelihood of damage occurring in less accessible parts.
2. The second method is based on increasing energy dissipation during earthquake motions by introducing specially developed devices between the piers and the superstructure. Such devices effectively increase the

damping in the structure, thereby reducing its elastic response to earthquake shaking. In such case the principles used in the above method have been applied to the design of the piers to give improved performance in the event of excessive earthquake motions. The ductile demand on such piers would not normally be very great, however.

On the other hand, the United States Design Philosophy is based on designing the bridge in such a way that it will resist the energy induced into the structure due to earthquakes. This concept is evident in the design approach of bridge abutments, which are utilized to remove energy from the structure even though the earthquake induced forces at the abutment may cause heavy damage. Whereas, in New Zealand, the abutments are designed to avoid the dissipation of energy into the soils. Also, in New Zealand, the emphasis seems to be on using energy absorbing bearings, rocking bents, uplift in footings, friction slabs, and other innovative devices or approaches which reflect their design philosophy of letting the structure respond to the earthquake motion instead of resisting it, which is contrary to the United States approach of literally resisting the earthquake ground motion. A point of agreement is that both philosophies advocate the principle of attempting to achieve ductile action in column members; the only difference is that in New Zealand the approach is to compute the ductile capacity, whereas in the United States the approach is to presume that the standard reinforcement used will provide adequate ductile action.

2.2.6 Design Codes

A National Academy of Sciences Joint Panel on the San Fernando Earthquake [8] declared in 1971 that "present standard code requirements for earthquake design of highway bridges in high-risk areas are grossly inadequate and should be revised." Until recently, however, almost all considerations of earthquake forces on structures and relevant code provisions have been directed to the area of building construction rather than bridge construction [8].

The 1925 Santa Barbara earthquake caused several million dollars in damages and gave impetus to the inclusion of seismic design provisions in building codes. The Long Beach earthquake in 1933, with more than $50 million in damage, further demonstrated the need for consideration of earthquake forces. In 1937, the simple Newtonian concept that the lateral earthquake force on a structure is propotional to its mass was included in the Uniform Building Code; since then changes have continued in various building codes [8].

The first provisions in the United States for consideration of seismic loading in the design of highway bridges were included in the American Association of State Highway Officials (AASHO) 1958 Standard Specifications for Highway Bridges, and they remained unchanged for more than 15 years. In 1971, new seismic design criteria were being considered for adoption by the AASHO (now AASHTO) Bridge Committee when the San Fernando Earthquake occurred and demonstrated that the proposed provisions were inadequate.

The 12th edition (1977) of the AASHTO Standard Specification for Highway Bridges included a new approach for designing highway bridges to withstand earthquake forces. Article 1.2.20 of the specification requires that "in regions where earthquakes may be anticipated, structures shall be designed to resist earthquake motions by considering the relationship of the site to active faults, the seismic response of the soils at the site, and the dynamic response characteristics of the total structure." The Specifications call for seismic analysis by an equivalent static force method for simple structures and for a more rigorous response-spectra or transient analysis for more complex structures. These provisions are generally based on the 1973 Earthquake Design for Bridges in the State of California which was developed for conditions in that state, but were modified to permit their application to the other areas of the United States. California's design criteria evolved from technical information and code provisions that had been developed primarily for building design but represented the best guidance available for bridges at the time.

Recently, after about ten years of concentrated research effort (Section 2.2.3), two major achievements of immediate importance to engineers concerned with the protection of bridges against earthquake hazards were generated. Two reports by the Applied Technology Council (ATC) comprise the best guidance currently available on designing new bridges and retrofitting existing bridges. These two reports represent the first time that code provisions have been specifically developed for bridge design.

The first report, Seismic Design Guidelines for Highway Bridges [108], was unanimously adopted by the AASHTO Bridge Committee in 1982 and appeared as a Guide Specification for Seismic Design of Highway Bridges 1983 as a part of the 13th edition (1983) of the AASHTO Standard Specifications for Highway Bridges. These seismic guidelines contain design and construction requirements applicable to more than 85 percent of the highway bridges to be constructed in the United States. In most cases, all that is required is more attention to structural details that will considerably improve earthquake resistance without significantly adding to the complexity of design or the cost of construction.

The earthquake motions and forces specified in these guide specifications are based on an acceptably low probability of their being exceeded during the expected life of a bridge. Bridges and their components that are designed to resist these forces and are constructed using the design details contained in the guidelines may suffer damage, but the probability of collapse due to seismically induced ground shaking will be low. The development of the guidelines was carried out in light of the concepts upon which the United States Design Philosophy was based (see previous section).

The second report, Seismic Retrofitting Guidelines for Highway Bridges, contains the findings of the researches discussed in Section 2.2.3 and has helped to create an awareness of the relatively simple measures that can be taken to enhance the seismic resistance of existing bridges. It presents a systematic approach to identifying and correcting common seismic deficiencies in existing highway bridges.

2.3 SCOPE AND PURPOSE

The purpose of this study is to develop a computer code by which wall pier bridges can be analyzed for strong earthquake ground motions. In all the literature discussed in the previous section, little is mentioned concerning wall pier bridges except in the Seismic Design Guidelines for Highway Bridges, where it is suggested to design a wall pier as "a column in the weak direction of the pier", but gives no suggestion on how to design the wall pier in the strong direction. The reason behind this omission is not that wall pier bridges are unimportant or nonexistent, but because of all the analysis tools available, none contain plate elements which will model the wall piers.

Wall pier bridges (Figure 2.8) are frequently used in crossings of waterways and railroads. Thus they represent an important type of bridge which makes it more crucial to maintain their structural integrity after the occurrence of an earthquake. Also, wall pier bridges, due to their particular usage, mostly exist near the coastal regions of the United States (Figures 2.9 and 2.10) which are more susceptible to the destructive power of earthquakes. Something else that should be mentioned is that wall pier bridges cost more per mile of bridge than most other bridges currently being constructed in the United States.

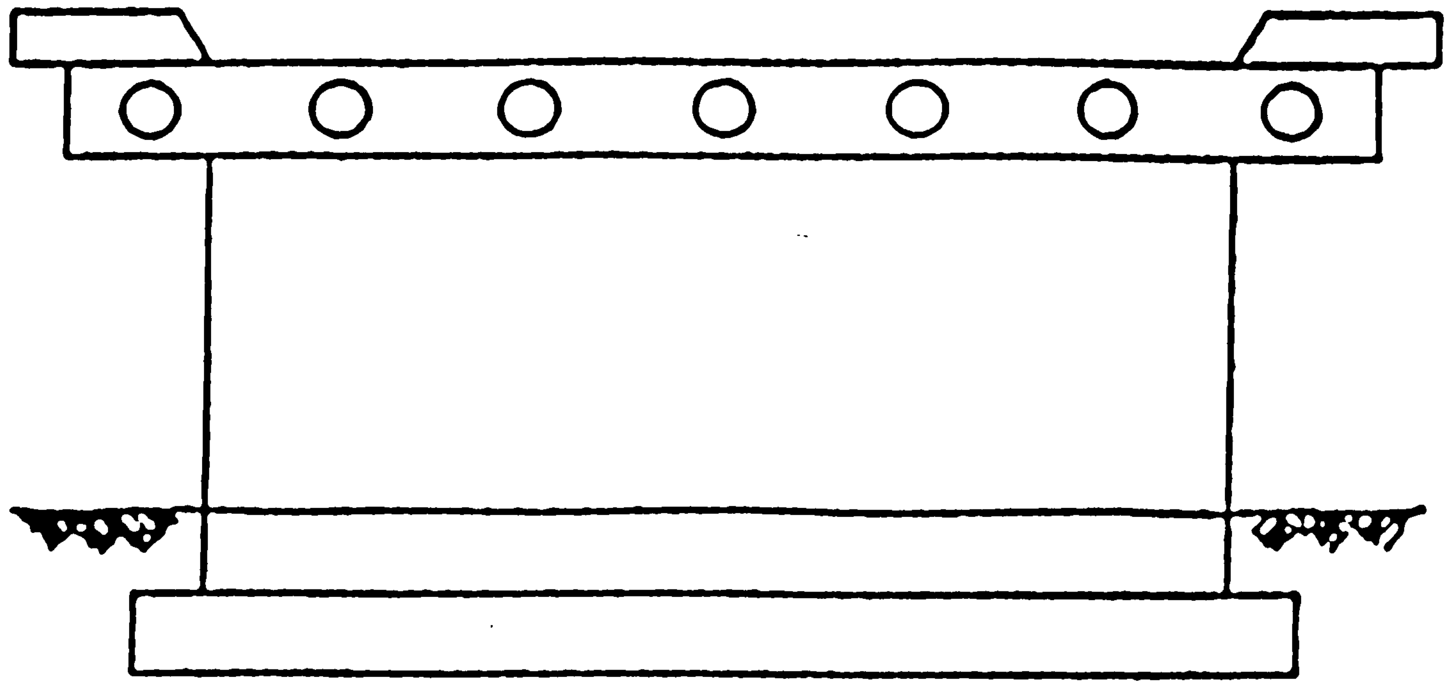


Figure 2.8: Typical Wall pier of a bridge.

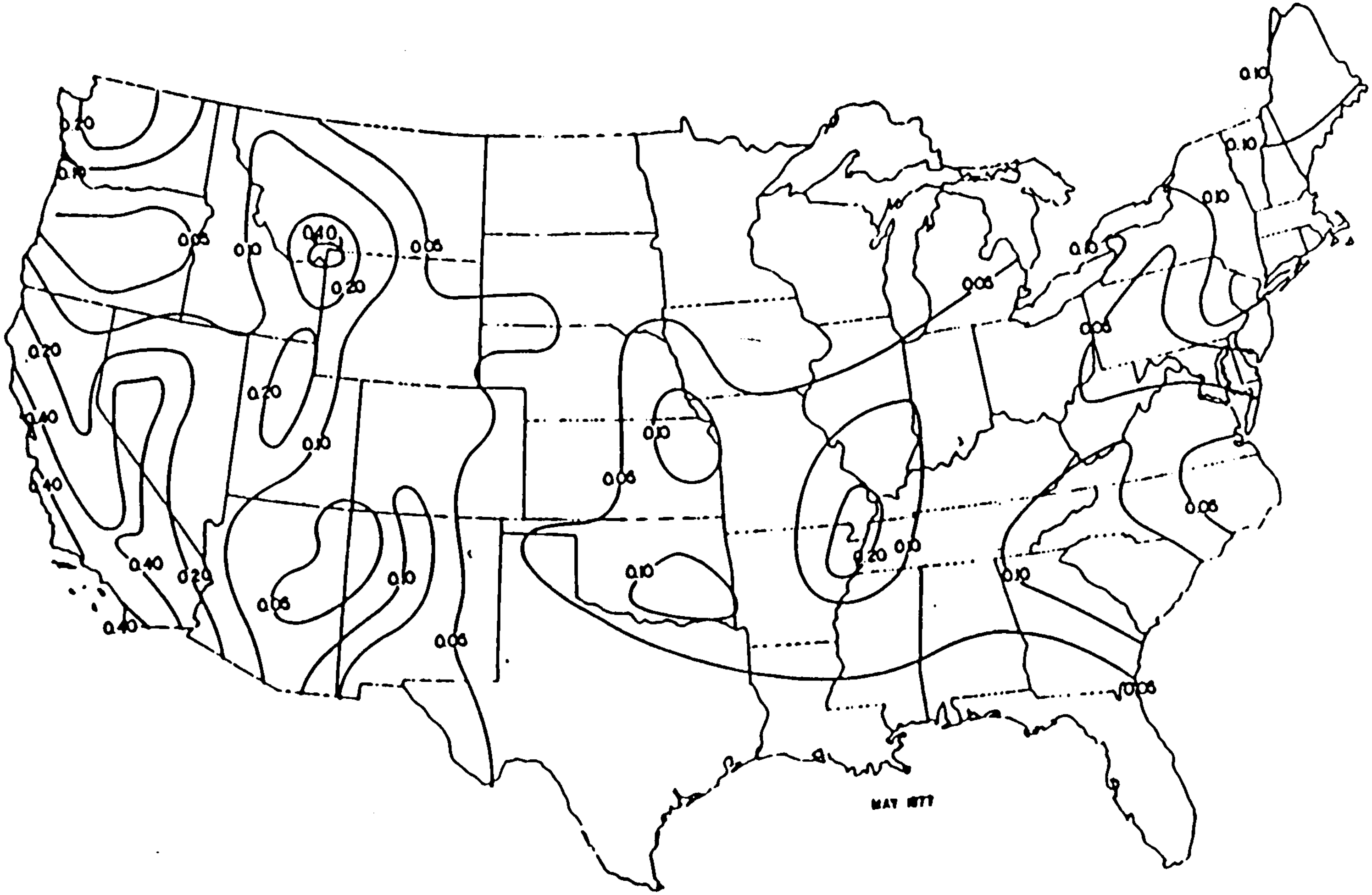


Figure 2.9: Acceleration Coefficent-Continental United States[108]

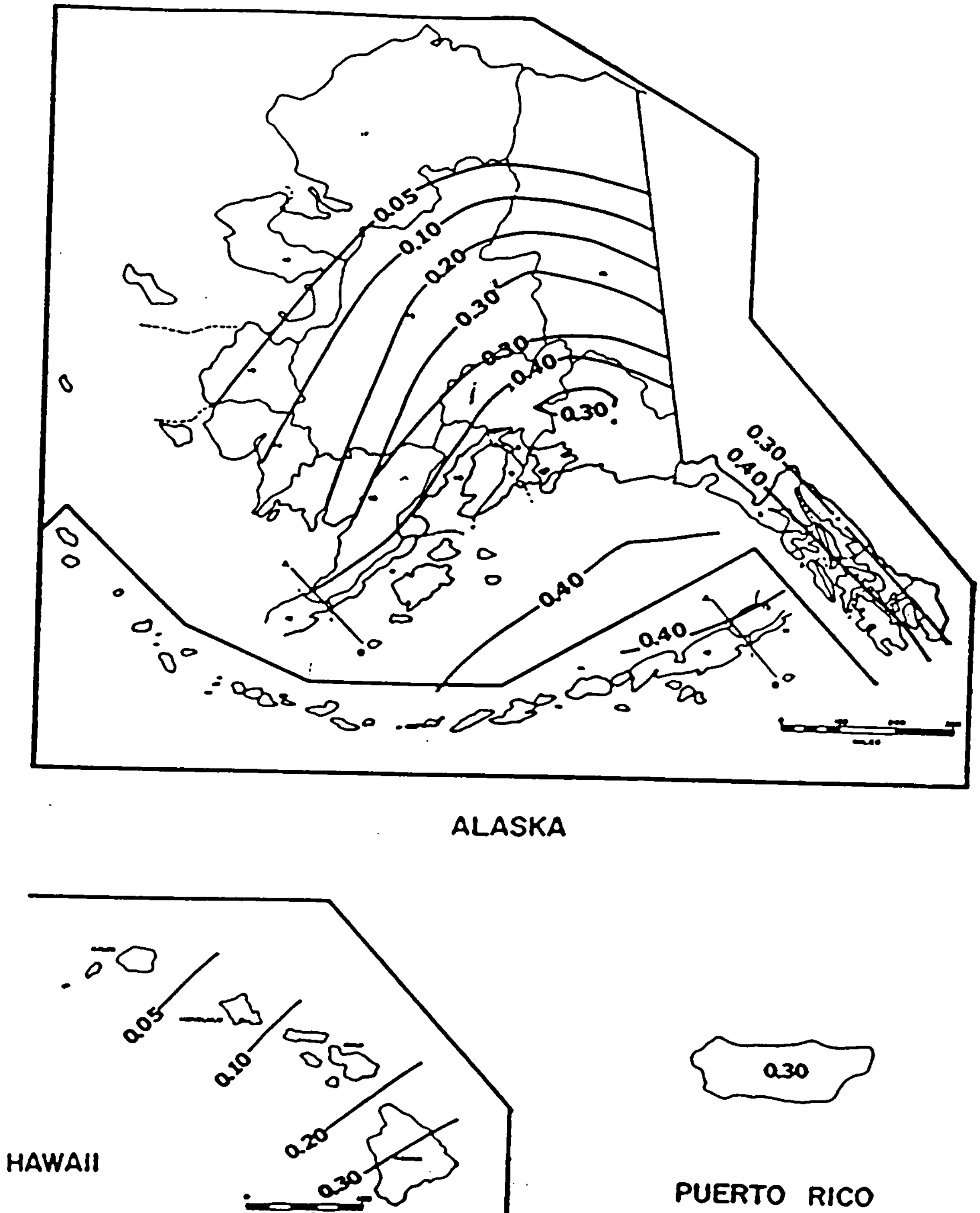


Figure 2.10: Acceleration coefficent-Alaska, Hawaii, and Puerto Rico[108]

The computer code developed in this study models a wall pier bridge using three types of elements:

1. A geometrically nonlinear plate element with shear deformation and transverse shear to represent the substructure wall pier.
2. A linear shear deformable beam element to represent the superstructure.
3. A transition element to connect between the plate and beam elements.

The computer code performs a transient analysis of the structure by reading time histories provided from digitized acceleration data [110]. From the output one can find:

1. The displacements and moments at the nodes of the plate elements.
2. The displacements and rotations at the nodes of the beam elements.

To keep the problem simple, it is reasonable to assume that the wall piers are fixed at the ground level, thus eliminating the need to use soil elements. Another assumption is to consider the whole structure to be subjected to the same ground motion intensity, which in reality might vary from one pier to another. This will not affect the results of this study considerably since the highest possible ground motion intensity will be used as the one affecting the whole structure.

Chapter III

FORMULATION OF THE PROBLEM

The objective of this chapter is to formulate the mathematical model of the wall pier bridge. The superstructure is represented by beam elements and the substructure is represented by plate elements. Transitional elements are used to connect the plate elements to the beam elements.

3.1 THE PLATE ELEMENT

3.1.1 General Considerations

The plate element that is selected to represent the wall pier of the bridge is formulated to account for the following characteristics:

1. Large deflection
2. Stretching of the plate
3. Bending of the plate
4. Transverse shear effect
5. Isotropic material

3.1.2 Review of Pertinent Literature

There are several papers (for example, Reissner and Stavsky [111], Dong and Taylor [129], and Bert and Mayberry [130]) in which classical Kirchhoff-Love kinematic assumptions are adopted and tranverse shear deformations are neglected. Refined plate theories which account for the transverse shear and normal stresses are available in the literature (see,for example, Reissner [112,113], Hencky [114], Mindlin [115], Kromm [116,117]). The Hencky-Mindlin theories are based on an assumed displacement field that accounts for linear or higher order variations of the displacements through the plate thickness. The Hencky-Mindlin theory itself accounts for linear variations of displacements through the thickness. In this theory the normals to the mid-plane before deformation remain straight but not necessarily normal to the mid-plane after deformation; consequently, a correction to transverse stiffnesses is required. The theory also does not satisfy the conditions of zero transverse shear stresses on the top and bottom faces of the plate. There are numerous studies involving the application of this theory to bending, vibration and transient analysis and some can be found in [119-120].

A higher-order theory that accounts for transverse shear deformation but also satisfies the zero conditions of transverse shear stresses on the top and bottom faces of the plate and does not require shear correction factors was suggested by Reddy [121]. The displacement field used by Reddy is similar to that of Levinson [122] which is limited to isotropic plates. Levison used the equilibrium equations of the classical plate theory which are inconsistent with the assumed displacement field. Reddy's modifications consist of a more systematic derivation of displacement field and associated, variationally consistent, equilibrium equations. The above theory has also appeared in Reissner [112,113,131], Hencky [114], Mindlin [115], Kromm [116,117], and Lo, Christensen and Wu [132].

The conventional variational formulation of the classical plate theory as well as higher-order theory involves higher order (i.e., second-order and above) derivatives of the transverse displacement. Therefore, in the finite-element modeling of such theories, one should impose the continuity of not only the transverse displacement but also its derivatives along the element boundary. In other words, a conforming plate bending element based on the displacement formulation of these theories requires continuity of transverse displacements and their derivatives across the inter-element boundaries. The construction of such an element is algebraically complicated, requiring, for example, a quintic polynomial with twenty-one degrees of freedom for a six-node triangular element. Computationally the element requires large storage and computer time.

To overcome the stringent continuity requirements placed by conventional variational formulations, several alternative formulations and associated elements have been developed (see [133] for a review). These include the hybrid finite elements, the mixed finite elements and the penalty (or Mindlin) elements. The hybrid elements are based on variational statements that use independent variation of displacements inside the domain of the element and tractions on the boundary of the element. The mixed elements use stationary variational principles, such as the Reissner variational principle or the Hu-Washizu variational principle [128], to construct independent variations of both displacements and bending moments in a plate. The Mindlin element is based on the conventional variational formulation of the first-order shear deformation theory of plates. Of course, the Mindlin element is relevant only in the case of the first-order shear deformation theory. The advantages of the mixed formulations over other formulations include: the relaxation of inter-element continuity requirements, accurate representation of stresses, reduced formulative effort and the ease with which the model can be applied to nonlinear and other complicated problems.

An independent approximation of displacements and bending moments (the so-called mixed formulations) in the classical theory of plates was first proposed independently by Herrmann [134] and Hellan [135]. Following Herrmann's work, a number of papers were published in the literature on mixed finite element models [136-140]. Reddy and his colleagues [141-143] used rectangular elements based on the Reissner type of variational principles to analyze bending, stability and vibration of linear, isotropic and orthotropic plates. In [143] a mixed plate element of the nonlinear bending and vibration of plates was presented. In all these works the transverse shear strains were not considered.

Reddy et al. [144] presented a mixed element for large deflection bending and free vibration of axisymmetric circular plates including shear deformation. Akay [145] presented results for large deflection dynamic analysis of plates using a four node mixed quadrilateral element. The analysis was based on dynamic von Karman plate equations [146], and includes transverse shear deformation. Pian et al. [147] presented a hybrid mixed finite element formulation using the Hellinger-Reissner variational principle for which the stress equilibrium conditions are not introduced initially but are brought in through additional internal displacement parameters. Noor and his colleagues [148-151] employed the mixed elements in the analysis of anisotropic plates. Apparently, these elements were based on the first-order shear deformation theory.

In the most recent published literature Soares and his colleagues [152-154] have developed a mixed finite element model for geometrically nonlinear transient analysis of plates subjected to arbitrary forces. This formulation was based on the Reissner functional presented by Noor [150] with the effect of transvere shear deformation, anisotropic material behavior and bending-extensional coupling included. In this study, the formulation presented by Soares [154] is the basis for the mixed finite element developed.

3.1.3 Mixed Formulation of the Plate Element

The Reissner functional for the plate [128,150,154-156] can be represented by:

$$\pi_R = V - U^C + \Omega + I_{S_\sigma} + I_{Su} \tag{3.1}$$

where

Π_R = Reissner functional for plate

V = Potential energy of the plate

U^c = Complementary energy

Ω = Work done by external forces; i.e., earthquake, static load,...etc.

I_{S_σ} = Work done by prescribed boundary stresses

I_{S_u} = Work done by prescribed boundary displacements

Referring to Figure 3.1 for the definition of the displacements, rotations, stress resultants, and moments, the terms on the right hand side of equation (3.1) can be written as:

$$V = \int_A \{ M_{xx} \frac{\partial\psi}{\partial x} - M_{yy} \frac{\partial\phi}{\partial y} + M_{xy}(\frac{\partial\psi}{\partial y} - \frac{\partial\phi}{\partial x}) + N_{xx}[\frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial w}{\partial x})^2]$$
$$+ N_{yy}[\frac{\partial v}{\partial y} + \frac{1}{2}(\frac{\partial w}{\partial y})^2] + N_{xy}[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + (\frac{\partial w}{\partial x})(\frac{\partial w}{\partial y})] \tag{3.2}$$
$$+ T_x(\frac{\partial w}{\partial x} + \psi) + T_y(\frac{\partial w}{\partial y} - \phi)\} dA$$

$$U^C = \frac{1}{2} \int_A \{ \frac{12}{h^3} (\frac{M_{xx}^2}{E} - \frac{2\nu}{E} M_{xx} M_{yy} + \frac{M_{yy}^2}{E} + \frac{M_{xy}^2}{G})$$
$$+ \frac{1}{h} (\frac{N_{xx}^2}{E} - \frac{2\nu}{E} N_{xx} N_{yy} + \frac{N_{yy}^2}{E} + \frac{N_{xy}^2}{G}) + \frac{6}{5hG}(T_x^2 + T_y^2)\} dA \tag{3.3}$$

$$\Omega = - \int_A (uf_x + vf_y + wf_z) dA \tag{3.4}$$

$$I_{S_\sigma} = - \int_{S_\sigma} \{ n_x(\bar{N}_{xx}u + \bar{N}_{xy}v + n_y(N_{xy}u + N_{yy}v) + n_x(\bar{M}_{xx}\psi - \bar{M}_{xy}\phi)$$
$$+ n_y(\bar{M}_{xy}\psi - \bar{M}_{yy}\phi) + w[n_x(\bar{T}_x + \bar{N}_{xx}\frac{\partial\bar{w}}{\partial x} + \bar{N}_{xy}\frac{\partial\bar{w}}{\partial y}) \tag{3.5}$$
$$+ n_y(\bar{T}_y + \bar{N}_{xy}\frac{\partial\bar{w}}{\partial x} + \bar{N}_{yy}\frac{\partial\bar{w}}{\partial y})]\} dS$$

$$I_{Su} = - \int_{S_u} \{ n_x[N_{xx}(u-\bar{u}) + N_{xy}(v-\bar{v})] + n_y[N_{xy}(u-\bar{u}) + N_{yy}(v-\bar{v})]$$
$$+ n_x[M_{xx}(\psi-\bar{\psi}) - M_{xy}(\phi-\bar{\phi})] + n_y[M_{xy}(\psi-\bar{\psi}) - M_{yy}(\phi-\bar{\phi})] \tag{3.6}$$
$$+ (w-\bar{w})[n_x(T_x + N_{xx}\frac{\partial w}{\partial x} + N_{xy}\frac{\partial w}{\partial y}) + n_y(T_y + N_{xy}\frac{\partial w}{\partial x} + N_{yy}\frac{\partial w}{\partial y})]\} dS$$

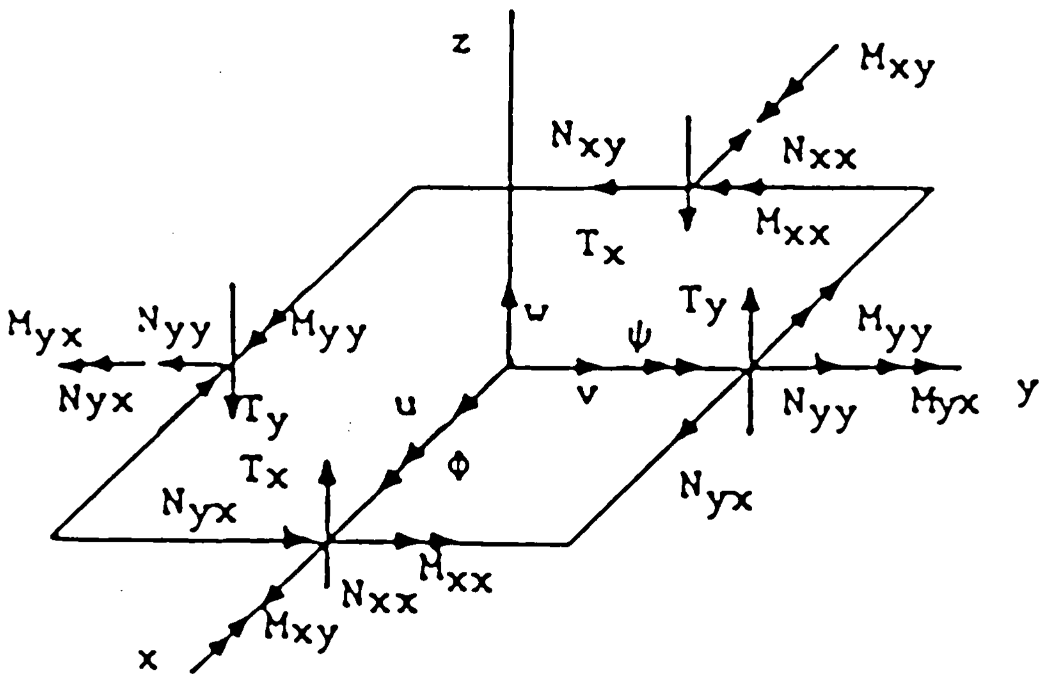


Figure 3.1: The Plate Positive Displacements, Rotations, Stress Resultant, and Moments.

where

u, v, w	are displacements in x, y, and z directions
ϕ, ψ	are rotations about x, y axis, respectively
M_{xx}, M_{yy}, M_{xy}	are moments
N_{xx}, N_{yy}, N_{xy}	are in-plane stress resultants
f_x, f_y, f_z	are applied pressures
T_x, T_y	are transverse shear stress resultants
n_x, n_y	are direction cosines of the outward normal to the plate boundary
E	is modulus of elasticity
ν	is Poisson's ratio
h	is thickness of the plate
G	is shear modulus
-	superscript indicating prescribed values at the plate boundary

Assume that

$$\{N\} = \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = h[C_{11}]^{-1}(\{\epsilon_{Lu}\}+\{\epsilon_N\}) \tag{3.7}$$

where

$$\{\epsilon_{Lu}\} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \tag{3.8}$$

$$\{\epsilon_N\} = \frac{1}{2}\begin{Bmatrix} (\frac{\partial w}{\partial x})^2 \\ (\frac{\partial w}{\partial y})^2 \\ 2(\frac{\partial w}{\partial x})(\frac{\partial w}{\partial x}) \end{Bmatrix} \tag{3.9}$$

$$[C_{11}] = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G} \end{bmatrix} \tag{3.10}$$

If the rotary inertia is neglected, the transverse shear stress resultants can be defined as follows:

$$\{T\} = \begin{Bmatrix} T_x \\ T_y \end{Bmatrix} = \begin{Bmatrix} \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} \\ \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} \end{Bmatrix} = [\Delta]\{M\} \tag{3.11}$$

where

$$[\Delta] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag{3.12}$$

and

$$\{M\} = \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} \tag{3.13}$$

Using matrix notation and Equations (3.7) and (3.11), the terms on the right hand side of Equation (3.1) are rewritten as follows:

$$V = \int_A \{M\}^T (\{\phi\} + [\Delta]^T(\{W\}+\{\psi\}))dA$$

$$+ \int_A h(\{\epsilon_{Lu}\}^T + \{\epsilon_N\}^T)[C_{11}]^{-1}(\{\epsilon_{Lu}\}+\{\epsilon_N\}) \tag{3.14}$$

$$U^C = \frac{1}{2}\int_A \{M\}^T(\frac{12}{h^3}[C_{11}] + \frac{6}{5h}[\Delta]^T[C_{22}][\Delta])\{M\}dA$$

$$+ \frac{1}{2}\int_A h(\{\epsilon_{Lu}\}^T+\{\epsilon N\}^T)[C_{11}]^{-1}(\{\epsilon_{Lu}\} + \{\epsilon_N\})dA \tag{3.15}$$

$$\Omega = -\int_A (\{U\}^T\{f\} + wf_z)dA \tag{3.16}$$

$$I_{S_\sigma} = -\int_{S_\sigma} \{n\}^T([\bar{N}]\{U\} + [\bar{M}]\{\psi\} + \{\bar{Z}\}w)dS \tag{3.17}$$

$$I_{S_u} = -\int_{S_u} \{n\}^T([\bar{N}](\{U\}-\{\bar{U}\}) + [\bar{M}](\{\psi\}-\{\bar{\psi}\}) + \{Z\}(w-\bar{w}))dS \tag{3.18}$$

where

$$\{\theta\} = \begin{Bmatrix} \frac{\partial\psi}{\partial x} \\ -\frac{\partial\phi}{\partial y} \\ \frac{\partial\psi}{\partial y} - \frac{\partial\phi}{\partial x} \end{Bmatrix} \tag{3.19}$$

$$\{W\} = \begin{Bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{Bmatrix} \tag{3.20}$$

$$\{\psi\} = \begin{Bmatrix} \psi \\ -\phi \end{Bmatrix} \tag{3.21}$$

$$[C_{22}] = \begin{bmatrix} \frac{1}{G} & 0 \\ 0 & \frac{1}{G} \end{bmatrix} \tag{3.22}$$

$$\{U\} = \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{3.23}$$

$$\{f\} = \begin{Bmatrix} f_x \\ f_y \end{Bmatrix} \tag{3.24}$$

$$\{n\} = \begin{Bmatrix} n_x \\ n_y \end{Bmatrix} \tag{3.25}$$

$$[N] = \begin{bmatrix} N_{xx} & N_{xy} \\ N_{xy} & N_{yy} \end{bmatrix} \tag{3.26}$$

$$[M] = \begin{bmatrix} M_{xx} & M_{xy} \\ M_{xy} & M_{xx} \end{bmatrix} \tag{3.27}$$

$$\{Z\} = \{T\} + [N]\{W\} \tag{3.28}$$

Integrating by parts the first term of Equation (3.1), the Reissner functional can be transformed to:

$$\pi_R = U_{Lu} + U_N + V_{wM} - U_M^C + \Omega + I_1 + I_2 + I_3 \tag{3.29}$$

where

$$U_{Lu} = \frac{1}{2} \int_A h\{\epsilon_{Lu}\}^T [C_{11}]^{-1} \{\epsilon_{Lu}\} dA \tag{3.30}$$

$$U_N = \int_A h\{\epsilon_N\}^T [C_{11}]^{-1} (\{\epsilon_{Lu}\} + \frac{1}{2}\{\epsilon_N\}) dA \tag{3.31}$$

$$V_{wM} = \int_A \{W\}^T \; [\Delta] \{M\} dA \tag{3.32}$$

$$U_M^C = \frac{1}{2} \int_A \{M\}^T \; (\frac{12}{h^3} \; [C_{11}] \; + \frac{6}{5h} \; [\Delta]^T [C_{22}] \, [\Delta]) \, \{M\} dA \tag{3.33}$$

$$I_1 = \int_{S_u} \{n\}^T [N] \, (\{U\} - \{\overline{U}\}) dS - \int_{S_u} \{n\}^T \{Z\} \, (w - \overline{w}) \, dS + \int_{S_M} \{n\}^T (\{M\} - \{\overline{M}\}) \; \{\psi\} dS \tag{3.34}$$

$$I_2 = - \int_{S_N} \{n\}^T [\overline{N}] \{U\} dS - \int_{S_Z} \{n\}^T \{\overline{Z}\} w dS \tag{3.35}$$

$$I_3 = \int_{S_\psi} \{n\}^T [M] \{\overline{\psi}\} dS \tag{3.36}$$

The integrals of Equation (3.34) vanish due to the prescribed boundary conditions:

$$\{U\} = \{\overline{U}\}; \quad w = \overline{w}; \quad [M] = [\overline{M}] \tag{3.37}$$

The kinetic energy of a plate considering transverse and in-plane inertia and neglecting rotary inertia is:

$$T = \frac{1}{2} \int_A \rho h \left((\frac{\partial u}{\partial t})^2 + (\frac{\partial v}{\partial t})^2 + (\frac{\partial w}{\partial t})^2 \right) dA \tag{3.38}$$

The plate is discretized in a manner such that the displacement and moment fields can be represented by:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [S]\ \{q\} \quad ; \quad \{M\} = [s]\ \{m\} \tag{3.39}$$

where {q} and {m} are the displacements and moments of the discretized model and [S] is the system shape function matrix. After substituting Equation (3.39) into Equations (3.29) and (3.38) and applying to them the Lagrange equations of motion [157,158]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\{\dot{q}\}}\right) - \frac{\partial L}{\partial\{q\}} = \frac{-\partial F}{\partial\{\dot{q}\}} \tag{3.40}$$

$$-\frac{\partial L}{\partial\{m\}} = 0 \tag{3.41}$$

in which

$$L = T - \Pi_R \tag{3.42}$$

and

$$F = 1/2\ \{\dot{q}\}^T\ [C]\ \{\dot{q}\} \tag{3.43}$$

where [C] is the damping matrix (Section 3.1.4), the following equations are obtained:

$$[M]\ \{\ddot{q}\} - \{Q\} + \{P\} = -\ [C]\ \{\dot{q}\} \tag{3.44}$$

$$-[G]\ \{m\} + [H]\ \{q\} + \{R\} = 0 \tag{3.45}$$

where [M] is the mass matrix, {Q} and {P} are the vectors of consistent applied forces and internal elastic forces, respectively. These matrices and vectors are given by:

$$[M] = \left[\frac{\partial^2 T}{\partial\{\dot{q}\}\partial\{\dot{q}\}}\right]; \quad \{Q\} = -\left\{\frac{\partial(\Omega+I_2)}{\partial\{q\}}\right\}$$

$$[G] = \left[\frac{\partial^2 U_M^C}{\partial\{m\}\partial\{m\}}\right]; \quad \{P\} = \left\{\frac{\partial(u_{Lu}+u_N+v_{wM})}{\partial\{q\}}\right\} \tag{3.46}$$

$$[H] = \left[\frac{\partial^2 V_{wM}}{\partial\{q\}\partial\{m\}}\right]; \quad \{R\} = -\left\{\frac{\partial I_3}{\partial\{m\}}\right\}$$

Equation (3.44) can be rewritten into a more familiar form:

$$[M]\,\{\ddot{q}\} + [C]\,\{\dot{q}\} + \{P\} = \{Q\} \tag{3.47}$$

which is the system dynamic equilibrium equation. The solution of this set of non-linear differential equations gives the nodal displacements. It should be noted that [M], [G] and [H] are constant matrices and only need to be calculated once. On the contrary, the vectors {P} and {Q} are functions of the nodal displacements and moments, and they need to be calculated for each deformed configuration of the plate.

From Equation (3.45) the nodal moments can be related to the nodal displacements by the following equation:

$$\{m\} = [G]^{-1}[H]\,\{q\} + [G]^{-1}\,\{R\} \tag{3.48}$$

But, for the most common boundary conditions the vector {R} is null, and Equation (3.48) becomes:

$$\{m\} = [G]^{-1}[H]\ \{q\} \tag{3.49}$$

3.1.4 The Plate Finite Element

The element used is a serendipity element with 8 nodes and 48 DOF (Figure 3.2). The degrees of freedoms per node are 3 displacements and 3 moments. The geometric, displacement and moment fields are represented by:

$$X = \sum_{i=1}^{8} S_i X_i; \quad Y = \sum_{i=1}^{8} S_i Y_i$$

$$U = \sum_{i=1}^{8} S_i U_i; \quad V = \sum_{i=1}^{8} S_i V_i; \quad W = \sum_{i=1}^{8} S_i w_i \tag{3.50}$$

$$M_{xx} = \sum_{i=1}^{8} S_i M_{xx_i}; \quad M_{yy} = \sum_{i=1}^{8} S_i M_{yy_i}; \quad M_{xy} = \sum_{i=1}^{8} S_i M_{xy_i}$$

where S_i are the element shape functions and are as follows:

$$S_1 = -\frac{1}{4}(1-\varepsilon)(1-\eta)(1+\varepsilon+\eta)$$

$$S_2 = \frac{1}{2}(1-\varepsilon^2)(1-\eta)$$

$$S_3 = \frac{1}{4}(1+\varepsilon)(1-\eta)(-1+\varepsilon-\eta)$$

$$S_4 = \frac{1}{2}(1+\varepsilon)(1-\eta^2)$$

$$S_5 = \frac{1}{4}(1+\varepsilon)(1+\eta)(-1+\varepsilon+\eta) \tag{3.51}$$

$$S_6 = \frac{1}{2} (1 - \varepsilon^2)(1 + \eta)$$

$$S_7 = \frac{1}{4} (1 - \varepsilon)(1 + \eta)(-1 - \varepsilon + \eta)$$

$$S_8 = \frac{1}{2} (1 - \varepsilon)(1 - \eta^2)$$

The matrices [G], [H], {Q}, {P}, and [M] in Equations (3.47) and (3.49) are integrated numerically using a 3*3 Gaussian mesh [160,161].

The mass matrix used is a diagonalized matrix since it is more efficient computationally to use lumped masses (Section 3.4). To get the lumped mass at each plate node, a technique developed by Rock and Hinton [159], which conserves the total mass of the element is used:

$$M_{ii} = \int_A \rho S_i S_i dA \int_A \rho dA / \int_A \rho \left(\sum_{j=1}^{8} S_j S_j \right) dA \qquad (3.52)$$

where ρ is the density of the material from which the plate is made. As for the damping matrix [C], it is represented using a popular scheme which combines a fraction α of the stiffness [K] and a fraction β of the the mass matrix [M] [160,161]. Thus

$$[C] = \alpha [K] + \beta [M] \qquad (3.53)$$

The damping matrix obtained is known as the Rayleigh or proportional damping matrix. For the plate element, the stiffness matrix [K] is a nonlinear stiffness matrix and is obtained in the following manner:

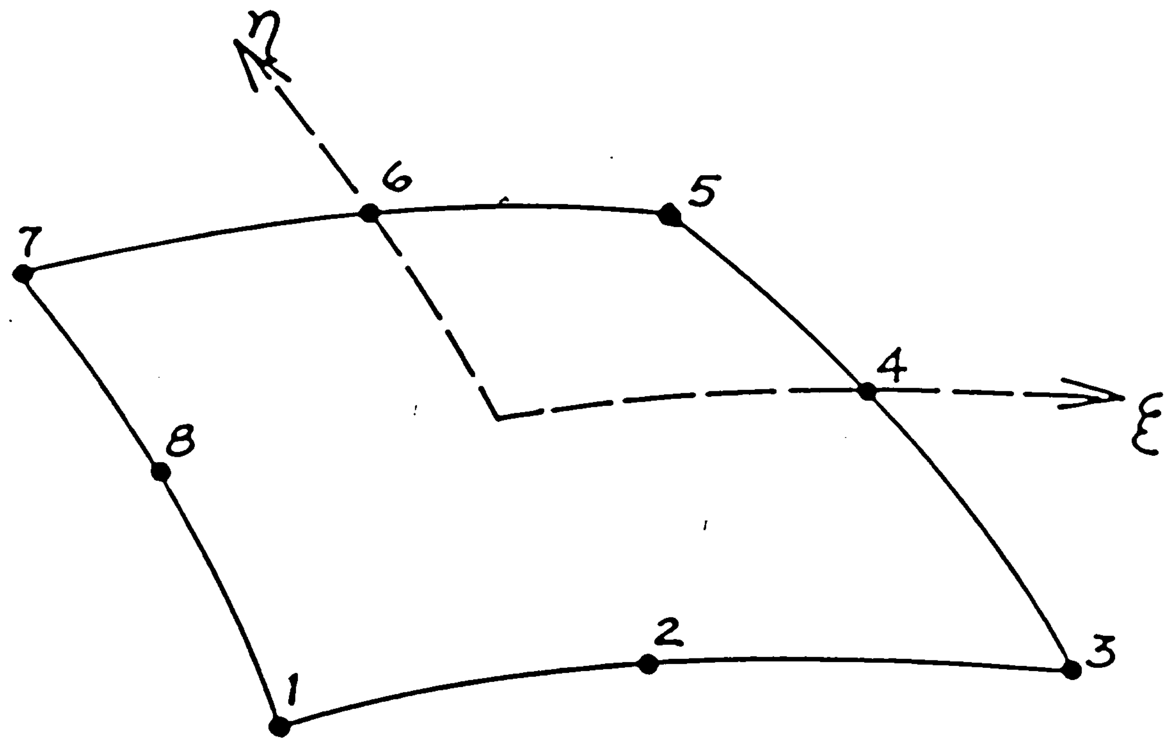


Figure 3.2: The Serendipity Plate Finite Element.

$$[K] = \partial\{P\} / \partial\{q\} \tag{3.54}$$

3.2 THE BEAM ELEMENT [162]

The beam element is assumed to be a straight bar of uniform cross section capable of resisting axial forces, bending moments about the two principal axes (X and Y axis) in the plane of its cross section, and twisting moments about its centroidal axis. At each node there are six DOF, i.e., three displacements and three rotations. The positive directions of these DOF are shown in Figure 3.3 . The effect of shear deformation is included in the formulation of the stiffness matrix which is presented in Figure 3.4 where

$$\Phi_x = \frac{12EI_x}{GA_{s_y} \ell^2} \tag{3.55}$$

and

$$\Phi_y = \frac{12EI_y}{GA_{s_x} \ell^2} \tag{3.56}$$

represent the shear deformation parameters in which A_s represents the beam cross sectional area effective in shear.

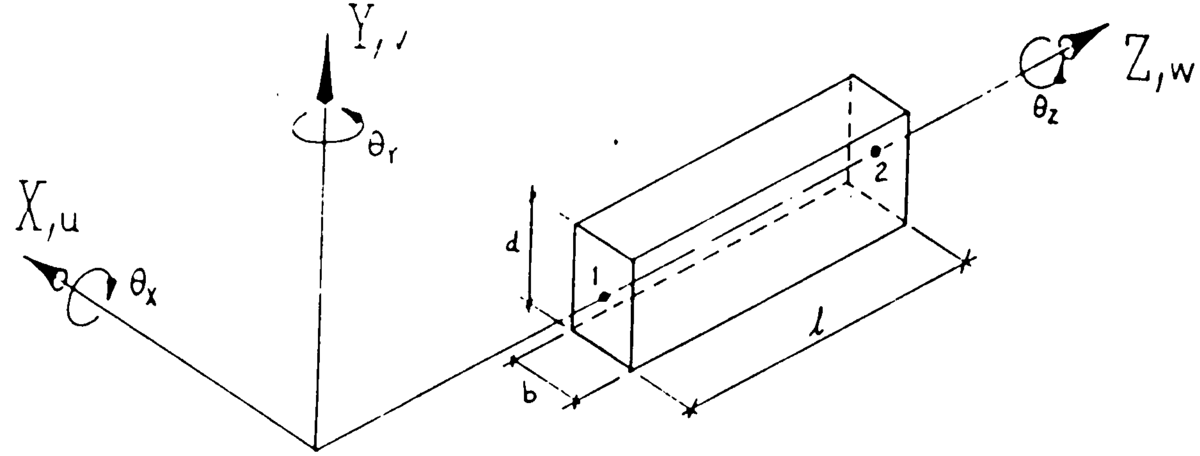


Figure 3.3: The Beam Element.

$$
[K_b] = \begin{bmatrix}
\frac{12EI_y}{\ell^3(1+\phi_x)} & 0 & 0 & 0 & \frac{6EI_y}{\ell^2(1+\phi_x)} & 0 & \frac{-12EI_y}{\ell^3(1+\phi_x)} & 0 & 0 & 0 & \frac{6EI_y}{\ell^2(1+\phi_x)} & 0 \\
 & \frac{12EI_x}{\ell^3(1+\phi_y)} & 0 & \frac{-6EI_x}{\ell^2(1+\phi_y)} & 0 & 0 & 0 & \frac{-12EI_x}{\ell^3(1+\phi_y)} & 0 & \frac{-6EI_x}{\ell^2(1+\phi_y)} & 0 & 0 \\
 & & \frac{AE}{\ell} & 0 & 0 & 0 & 0 & 0 & \frac{-AE}{\ell} & 0 & 0 & 0 \\
 & & & \frac{(4+\phi_y)EI_x}{\ell(1+\phi_y)} & 0 & 0 & 0 & \frac{6EI_x}{\ell^2(1+\phi_y)} & 0 & \frac{(2-\phi_y)EI_x}{\ell(1+\phi_y)} & 0 & 0 \\
 & & & & \frac{(4+\phi_x)EI_y}{\ell(1+\phi_x)} & 0 & \frac{-6EI_y}{\ell^2(1+\phi_x)} & 0 & 0 & 0 & \frac{(2-\phi_x)EI_y}{\ell(1+\phi_x)} & 0 \\
 & & & & & \frac{GJ}{\ell} & 0 & 0 & 0 & 0 & 0 & \frac{-GJ}{\ell} \\
 & & & & & & \frac{12EI_y}{\ell^3(1+\phi_x)} & 0 & 0 & 0 & \frac{-6EI_y}{\ell^2(1+\phi_x)} & 0 \\
 & & \text{SYMMETRIC.} & & & & & \frac{12EI_x}{\ell^3(1+\phi_y)} & 0 & \frac{6EI_x}{\ell^2(1+\phi_y)} & 0 & 0 \\
 & & & & & & & & \frac{AE}{\ell} & 0 & 0 & 0 \\
 & & & & & & & & & \frac{(4+\phi_y)EI_x}{\ell(1+\phi_y)} & 0 & 0 \\
 & & & & & & & & & & \frac{(4+\phi_x)EI_y}{\ell(1+\phi_x)} & 0 \\
 & & & & & & & & & & & \frac{GJ}{\ell}
\end{bmatrix}
$$

Figure 3.4: The Beam Stiffness Matrix[162].

In the case of a slender beam, both ϕ_x and ϕ_y can be taken as zero. This leads then to a force-displacement relationship in which the effects of shear deformations are neglected.

In the transient analysis, the mass matrix will be a lumped one and one in which, as in the case of the plate element, the rotary inertia is neglected. The total mass of the beam element (m) is divided equally between the two nodes. The matrix [M] represents the beam mass matrix:

$$[M_b] = \begin{bmatrix} [m_b] & 0 \\ 0 & [m_b] \end{bmatrix} \tag{3.57}$$

where

$$[m_b] = \begin{bmatrix} m/2 & & & & & \\ & m/2 & & & & \\ & & m/2 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \tag{3.58}$$

As for the damping matrix is derived using the popular scheme presented in Equation (3.53), except in this case all the entries in the beam damping matrix $[c_b]$ corresponding to rotations are replaced by zeroes.

3.3 THE TRANSITION ELEMENT

3.3.1 Purpose

The purpose of the transition element shown in Figure 3.5 is to join a beam element which represents the bridge superstructure to the upper nodes of the plate elements representing the solid wall pier. One of the problems faced is the fact that one beam node is connected to several plate nodes (at least three). Another problem is that the beam element has at each node three displacements and three rotations as DOF whereas at each plate node there are three displacements and three moments as DOF.

3.3.2 Formulation of the Transition Element

A suitable tranformation of the DOF is to assume rigid body motion and thus linearly interpolate the translational DOF and to consider the rotations of the beam node as being the same as the rotation of the upper edge of the wall pier [160]. As for the moments DOF, they are considered as loads applied on the beam node.

To illustrate the formulation of the transition element, consider a simple case in which a beam element is connected to one plate element as shown in Figure 3.6.
In this study, rotations are assumed to be less than 0.20 radians, thus the tangent of the angle is approximately

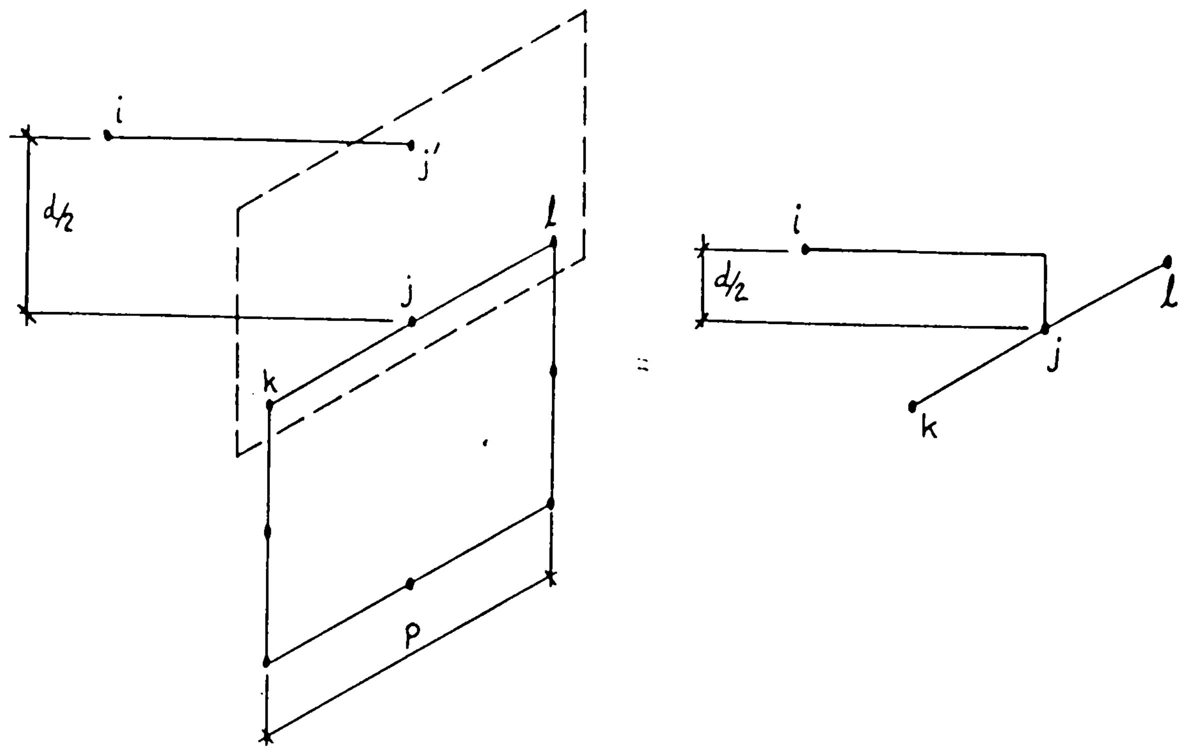


Figure 3.6: Example used to illustrate the formulation of the transition element.

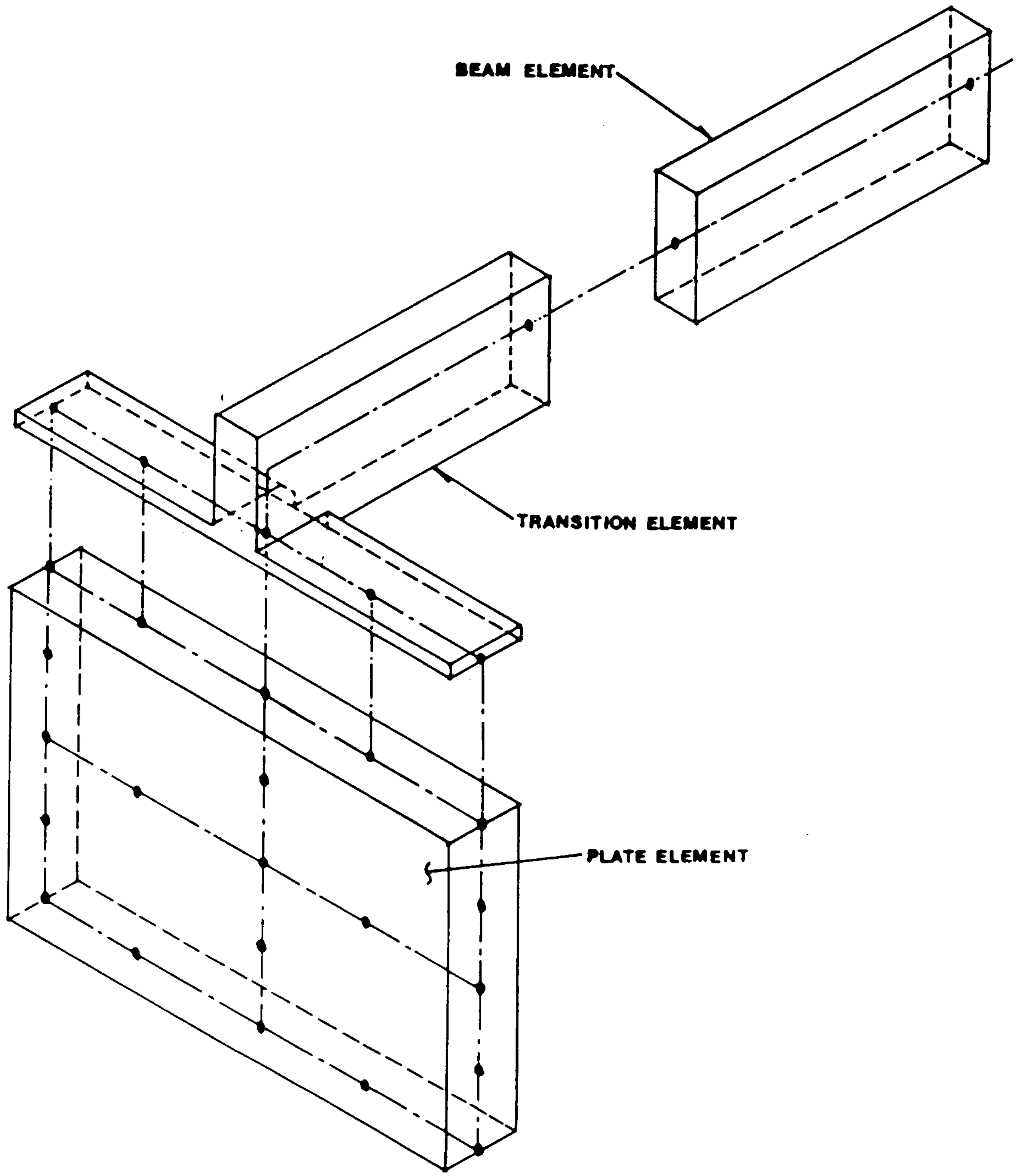


Figure 3.5: The Transition Element.

equal to the value of the angle in radians. Thus, the following relationship can be derived:

$$\{q_b\} = [T]\ \{q_p\} \tag{3.59}$$

where $\{q_b\}$ is the beam DOF , $\{q_p\}$ is the upper pier nodes DOF and [T] is the transformation matrix. If the beam element precedes the pier plate elements, then:

$$[T] = \begin{bmatrix} [I] & 0 \\ & \\ 0 & [\lambda] \end{bmatrix} \tag{3.60}$$

else:

$$[T] = \begin{bmatrix} [\lambda] & 0 \\ & \\ 0 & [I] \end{bmatrix} \tag{3.61}$$

where [I] is a 6-by-6 identity matrix and

$$[\lambda] = \begin{bmatrix} 1/2 & 0 & 0 & 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 1 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/p & 0 & 0 & 0 & 0 & 0 & -1/p \\ -1/d & -1/p & 0 & 0 & 0 & 0 & -1/d & 1/p & 0 \end{bmatrix} \tag{3.62}$$

Thus

$$
\begin{aligned}
u_j' &= \frac{1}{2} u_k + u_j + \frac{1}{2} u_\ell \\
v_j' &= \frac{1}{2} v_k + v_j + \frac{1}{2} v_\ell \\
w_j' &= \frac{1}{2} w_k + w_j + \frac{1}{2} w_\ell \\
\theta_y' &= (w_k - w_\ell)/P \\
\theta_z &= (v_\ell - v_k)/P - (u_j - u_i)/(d/2) \\
&= (v_\ell - v_k)/P - (u_k + u_\ell)/d
\end{aligned}
\tag{3.63}
$$

Based on the theory for transformation of axes [160], the following relation is deduced:

$$[K_T] = [T]^T \ [K_b] \ [T] \tag{3.64}$$

where $[K_T]$ is the transition element stiffness matrix, $[K_b]$ is the beam element stiffness matrix presented in Figure 3.4, and $[T]$ is the transformation matrix presented previously.

The mass matrix, $[M_T]$, is formulated in such a way that half the mass of the beam element is equally distributed among the plate nodes connected to the beam. In this case also, $[M_T]$ differs whether the beam element precedes or follows the pier. If the beam precedes the pier,

$$[M_T] = \begin{bmatrix} [m_b] & 0 \\ & \\ 0 & [m_p] \end{bmatrix} \tag{3.65}$$

If it follows the pier,

$$[M_T] = \begin{bmatrix} [m_p] & 0 \\ 0 & [m_b] \end{bmatrix} \tag{3.66}$$

where $[m_b]$ was defined previously and $[m_p]$ is a diagonal matrix in which each entry is equal to m/6 (in this case). As for the damping matrix the same approach used for the beam element is followed.

The above transition element was formulated based on one plate element per width of the bridge pier, and is easily generalized in the computer implementation. As for the moment DOF of the plate nodes, they are transformed into applied moments acting on the beam node i in the following manner:

$$\begin{Bmatrix} F_{x_i} \\ F_{y_i} \\ F_{z_i} \\ \\ M_{x_i} \\ M_{y_i} \\ M_{z_i} \end{Bmatrix} = \begin{bmatrix} [A] \\ \\ [B] \end{bmatrix} \begin{Bmatrix} m_{xx_k} \\ m_{yy_k} \\ m_{xy_k} \\ m_{xx_j} \\ m_{yy_j} \\ m_{xy_j} \\ m_{xx_\ell} \\ m_{yy_\ell} \\ m_{xy_\ell} \end{Bmatrix} \tag{3.67}$$

where [A] is a 3-by-9 zero matrix and

$$[B] = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ -1 & 0 & -1 & -1 & 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3.68)$$

which is based on Figure 3.7.

3.4 SOLUTION TECHNIQUE

The plate, beam and transition elements are combined together to model a wall pier bridge (see Chapter V). Thus the following global equation of motion is obtained:

$$[M]\ \{\ddot{q}\} + [C]\ \{\dot{q}\} + \{P\} = \{Q\} \qquad (3.69)$$

The matrix {P} for the beam and the transitional elements is obtained by the multiplication of their respective stiffness matrices by the displacement matrices.

3.4.1 Review of Solution Techniques [161-165]

The solution of transient dynamic problems of structural mechanics by the finite element method raises a certain number of difficulties. These are essentially related to the balance that has to be achieved between the accuracy of the results and the computational efficiency of the computer code governing the cost. When the problems are nonlinear, the question becomes even more crucial since it happens

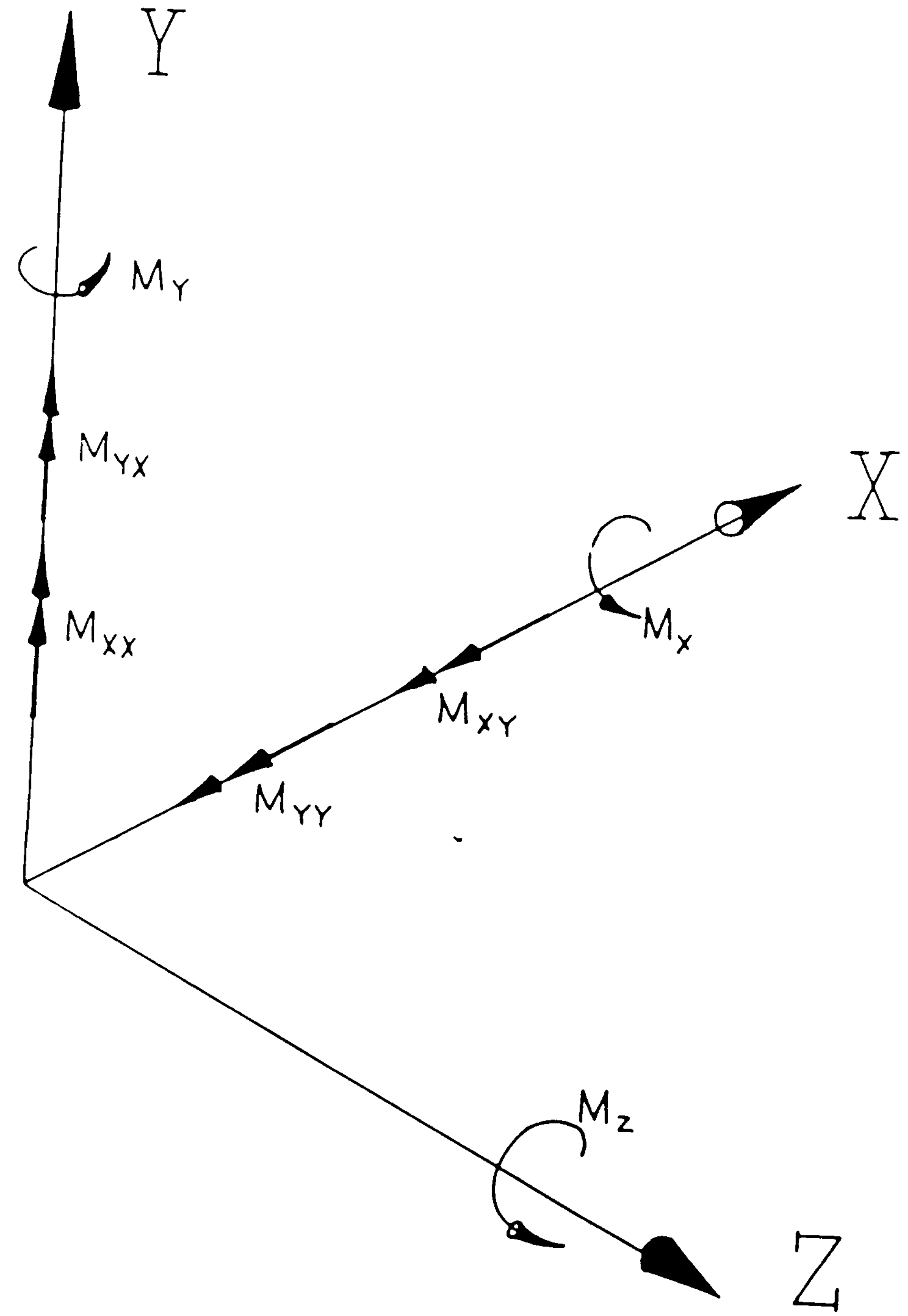


Figure 3.7: Figure showing the relationship between beam moments and plate moments.

frequently that the essential limitation is due to prohibitive computer times [165].

Usually the numerical solutions of the structural problems begin with a finite element discretization in space which leads to a set of ordinary differential equations in time. Two general classes of algorithms are used for their integration: explicit and implicit. Their relative advantages are well known [166,167]. Implicit algorithms tend to be numerically stable and thus allow for larger time steps. The amount of computation per step, however, is large and the organization of the program is more complex, especially if large structures are to be considered. Explicit algorithms lead to much simpler programs, especially if one accepts the approximation by a diagonal mass matrix, in which case the numerical operations per step do not involve the solution of a linear system of algebraic equations but only a linear combination of vectors. The numerical stability, however, requires significantly smaller steps. Except in special problems for which one method presents obvious and significant advantages, neither one of these algorithms is a priori superior to the other. The rational choice between them remains largely dependent upon the specific characteristics of the problem: its discretization, the loading history, the type of material behavior, the boundary conditions, etc.

In this study the model of the wall pier bridge involves a nonlinear earthquake analysis in which the digitized earthquake time history data is given at specific time increments. Thus, it is crucial to use an integration algorithm which is numerically stable and is not sensitive to the time step used. Consequently, the implicit Newmark method with modified Newton-Raphson type iteration at each time step is used to solve the equation of motion expressed by Equation (3.69).

3.4.2 The Newmark Method

From the Newmark relations:

$$\{\ddot{q}\}^{t+\Delta t} = \{t_d\} + \beta\Delta t^2\{\ddot{q}\}^{t+\Delta t}$$

$$\{\dot{q}\}^{t+\Delta t} = \{t_v\} + \gamma\Delta t\{\ddot{q}\}^{t+\Delta t} \tag{3.70}$$

we get at time $t+\Delta t$

$$\{\ddot{q}\}^{t+\Delta t} = \frac{1}{\beta\Delta t^2}\left[\{q\}^{t+\Delta t} - \{t_d\}\right]$$

$$\{\dot{q}\}^{t+\Delta t} = \{t_v\} + \frac{\gamma}{\beta\Delta t}\left[\{q\}^{t+\Delta t} - \{t_d\}\right] \tag{3.71}$$

where

$$\{t_d\} = \{q\}^t + \Delta t\{\dot{q}\}^t + (\tfrac{1}{2} - \beta)\Delta t^2\{\ddot{q}\}^t$$

$$\{t_v\} = \{q\}^t + (1-\gamma)\Delta t\{\ddot{q}\}^t \tag{3.72}$$

Substituting Equation (3.71) into the dynamic equilibrium Equation (3.69) gives

$$\frac{1}{\beta\Delta t^2}([M] + \gamma\Delta t[C])\{q\}^{t+\Delta t}$$
$$= \{Q\}^{t+\Delta t} - \{P\}^{t+\Delta t} + \frac{1}{\beta\Delta t}[M]\{t_d\} + [C]\left(\frac{\gamma}{\beta\Delta t}\{t_d\} - \{t_v\}\right) \tag{3.73}$$

which is a non-linear equation in displacements and thus is solved iteratively by a modified Newton-Raphson method.

Let $\{R\}_n^{t+\Delta t}$ be the residual force vector at the beginning of the n-th iteration of the time step t to t+Δt,

$$\{R\}_n^{t+\Delta t} = -\frac{1}{\beta\Delta t^2}([M] + \Delta t[C])\{q\}^{t+\Delta t} + \{Q\}^{t+\Delta t} - \{P\}_n^{t+\Delta t}$$
$$+ \frac{1}{\beta\Delta t}[M]\{t_d\} + [C]\left(\frac{1}{\beta\Delta t}\{t_d\} - \{t_v\}\right) \tag{3.74}$$

For each iteration it is necessary to solve

$$[K^*]_n^{t+\Delta t}\{\Delta q\}_n^{t+\Delta t} = \{R\}_n^{t+\Delta t} \tag{3.75}$$

where

$$[K^*]_n^{t+\Delta t} = [-\partial R_n^{t+\Delta t}/\partial\{q\}_n^{t+\Delta t}]$$
$$= \frac{1}{\beta\Delta t^2}([M] + \gamma\Delta t[C]) + [\partial\{P\}_n^{t+\Delta t}/\partial\{q\}_n^{t+\Delta t}] \tag{3.76}$$

Thus we can get $\{\Delta q\}_n^{t+\Delta t}$ at each time step. It follows that the accumulative displacements, velocities and accelerations are given by:

$$\{q\}_{n+1}^{t+\Delta t} = \{q\}_{n}^{t+\Delta t} + \{\Delta q\}_{n}^{t+\Delta t}$$

$$\{\dot{q}\}_{n+1}^{t+\Delta t} = \{\dot{q}\}_{n}^{t+\Delta t} + \frac{\gamma}{\beta \Delta t} \{\Delta q\}_{n}^{t+\Delta t} \tag{3.77}$$

$$\{\ddot{q}\}_{n}^{t+\Delta t} = \{\ddot{q}\}_{n}^{t+\Delta t} + \frac{1}{\beta \Delta t^{2}} \{\Delta q\}_{n}^{t+\Delta t}$$

For the first iteration of the time step t to $t+\Delta t$ the starting displacements are equal to the final displacements at time t,

$$\{q\}_{1}^{t+\Delta t} = \{q\}_{Final}^{t} \tag{3.78}$$

But in our case, the structure is suddenly exposed to the ground motion, causing the initial displacements, velocities and accelerations to be equal to zero. Also, in order to get a numerically stable solution the values of β=0.25 and γ= 0.5 are used [154,161] in the Newmark relations.

The fundamental steps of this solution technique at each time step are shown in Figure 3.8 and are as follows:

1. Compute of the applied forces {Q} at the end of time step t to $t+\Delta t$
2. Predict the displacements, velocities and accelerations at the end of the time step, using Equations (3.71) and (3.78).
3. Compute of the residual force, Equation (3.73).
4. If solution has converged (Sec. 3.4.2) update [K*] , Equation (3.76), and go to 1 for the next time step.

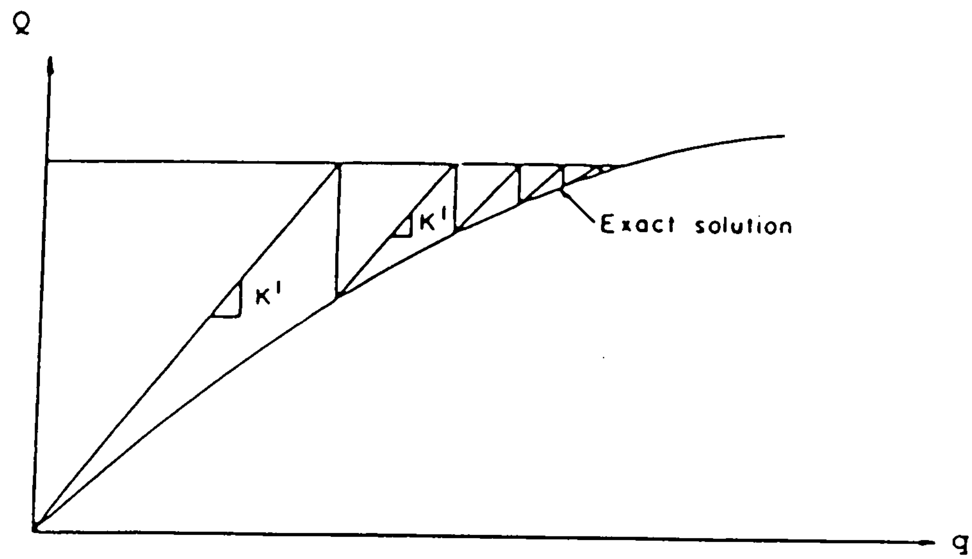


Figure 3.8: Figure showing the modified Newton-Raphson iteration process

5. Solve Equation (3.75) for $\{\Delta q\}_n^{t+\Delta t}$

6. Correct the displacements, velocities and accelerations at the end of the n-th iteration, using Equation (3.77).

7. Go to 3 for the next iteration.

3.4.3 Convergence Criteria

In this study, an implicit Newmark method with modified Newton-Raphson type iteration at each time step is used to solve the equation of motion in Section (3.4.1). Thus, for this solution technique to be effective, a realistic criterion should be used for the termination of the iteration. The incremental solution at the end of the iteration should be checked to see whether it has converged within preset tolerances. If the convergence tolerances are too loose, inaccurate results are obtained, and if the tolerances are too tight, much computational effort is spent to obtain needless accuracy.

Bathe discusses the whole subject of convergence criteria in great detail in References [161,169]. He mentions three different criteria and they are:

1. The displacement criterion.
2. The residual load criterion.
3. The energy criterion.

The second criterion contains some inconsistencies in units that can appear in the residual force vector, e.g., forces and moments in the beam element. The third contains some difficulties in that it involves a lot of computational work in this case, thus defeating its purpose. Therefore the first criterion represents a logical choice, and it is based on the Euclidean vector norm:

$$||\{\Delta q\}_n^{t+\Delta t}||_2 / ||\{q\}_n^{t+\Delta t}||_2 \leq \varepsilon_D \qquad (3.79)$$

where ϵ_D is the displacement convergence parameter. An appropriate value for ϵ_D is 0.001 which is used in the general nonlinear dynamic analysis computer program known as ADINA [170].

Chapter IV

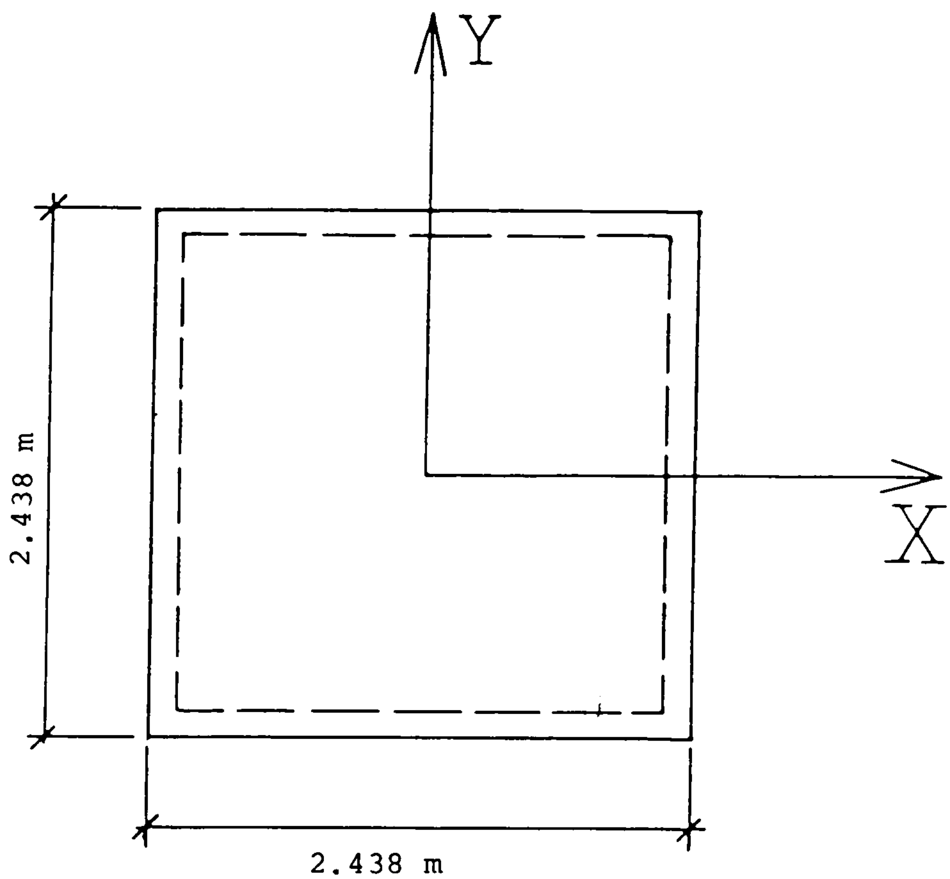
PROGRAM VERIFICATION

The formulation and solution technique presented in the previous chapter are implemented using programming concepts presented by Holzer [171] and Reddy [172]. To check the program and specifically the plate element, two plate example problems are analyzed.

4.1 EXAMPLE 1.

In this example, the simply supported square thin plate with unyielding supports, shown in Figure 4.1, is analyzed. The plate is subjected to a suddenly applied uniformly distributed force. The linear and nonlinear displacements and moments are calculated. Due to symmetry, only one quarter of the plate is analyzed, and it is represented by a 2*2 element mesh (Figure 4.2) with 44 moment DOF. In the linear and nonlinear analysis the number of displacement DOF are 12 and 34, respectively (Figure 4.2).

The central point displacements for the linear and nonlinear analysis are compared with the analytical solution [173] in Figures 4.3 and 4.4 respectively. The central point moments for the linear and nonlinear analysis are compared with the results presented by Akay [145] in Figures 4.5 and

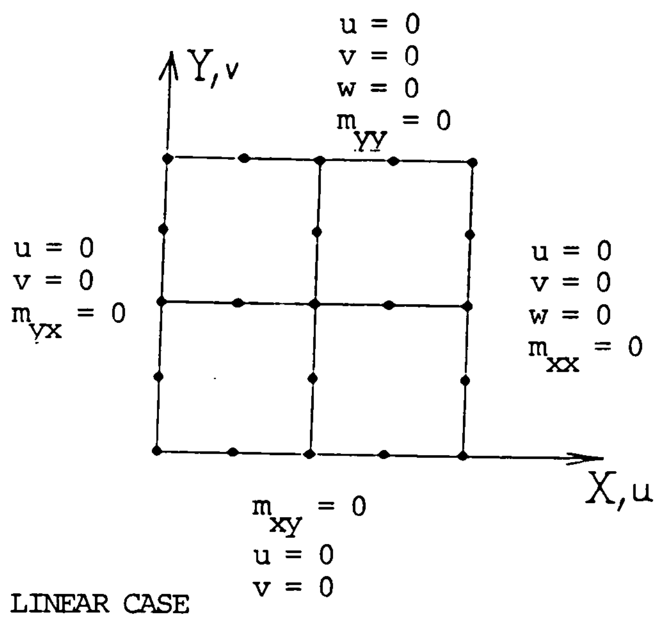


$h = 0.635$ cm $\qquad \rho = 2500 \ Kg/m^3$

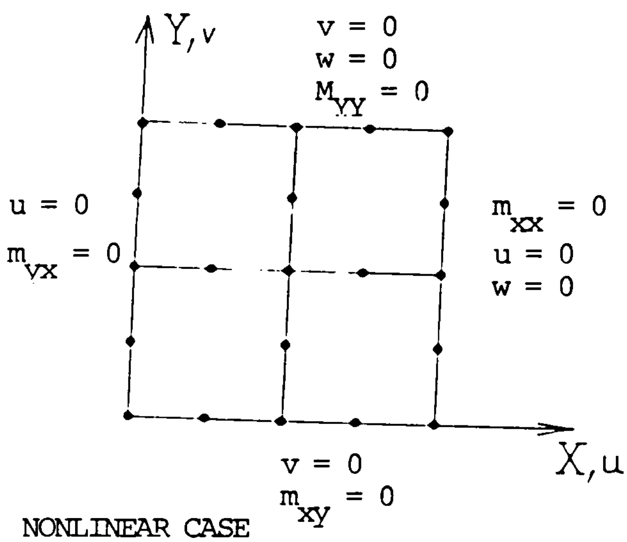
$\nu = 0.25 \qquad E = 6.9 \times 10^{11} \ N/m^2$

$f_z = 47.89 \ N/m^2 \qquad \Delta t = 0.005$ sec.

Figure 4.1: Example 1: Square Plate and Properties.



a) LINEAR CASE



b) NONLINEAR CASE

Figure 4.2: Example 1: Finite element Mesh and Boundary Conditions.

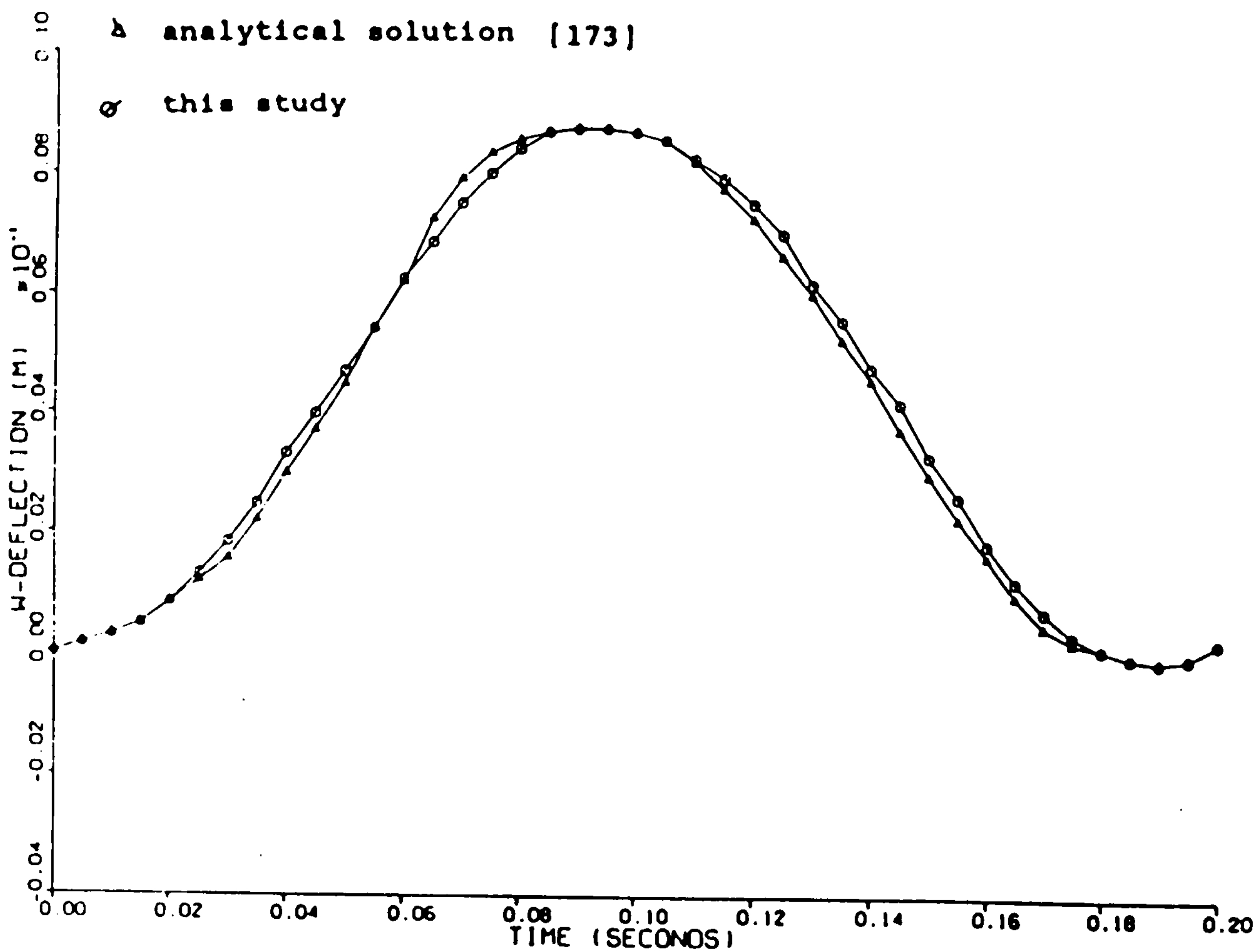


Figure 4.3: Example 1: Central Transverse Displacement in the Linear Analysis.

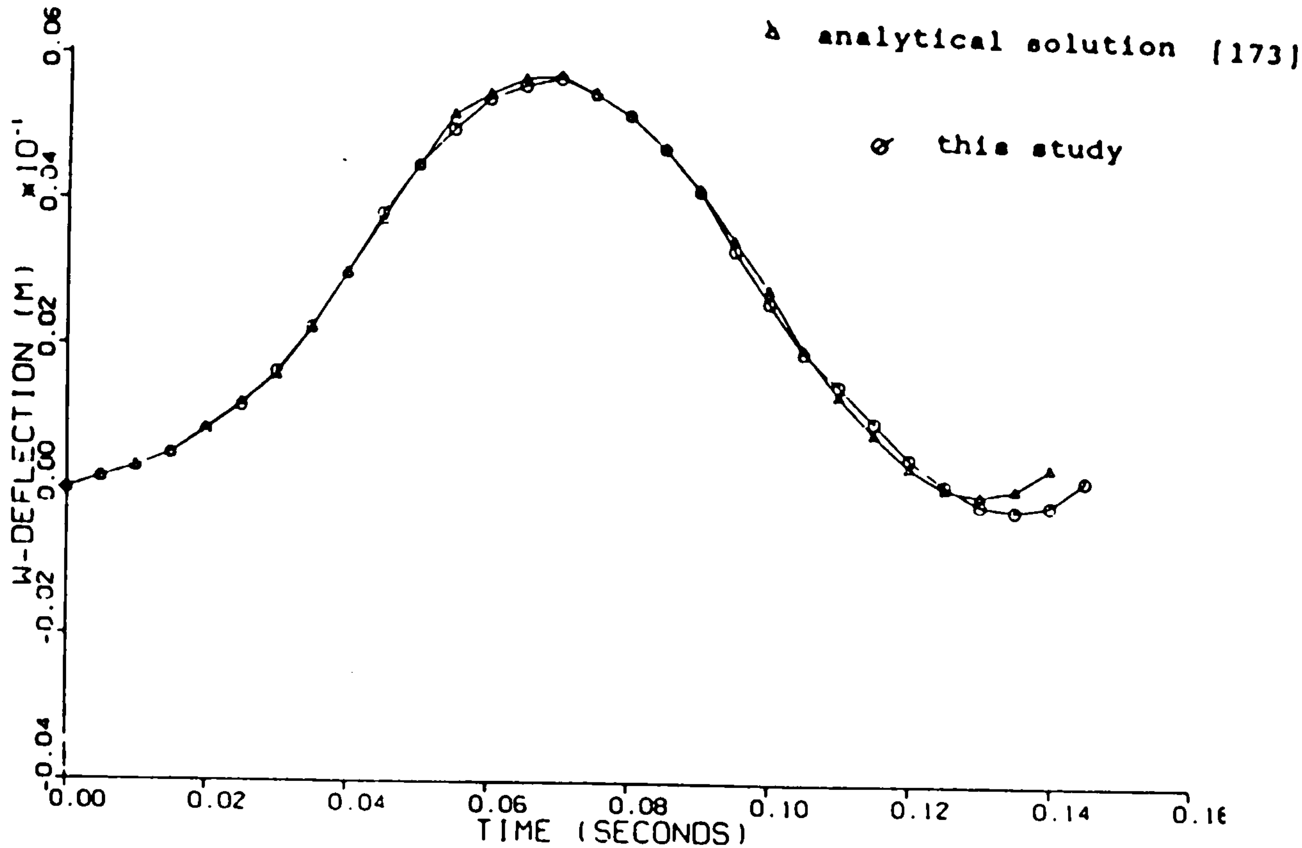


Figure 4.4: Example 1: Central Transverse Displacement in the Nonlinear Analysis.

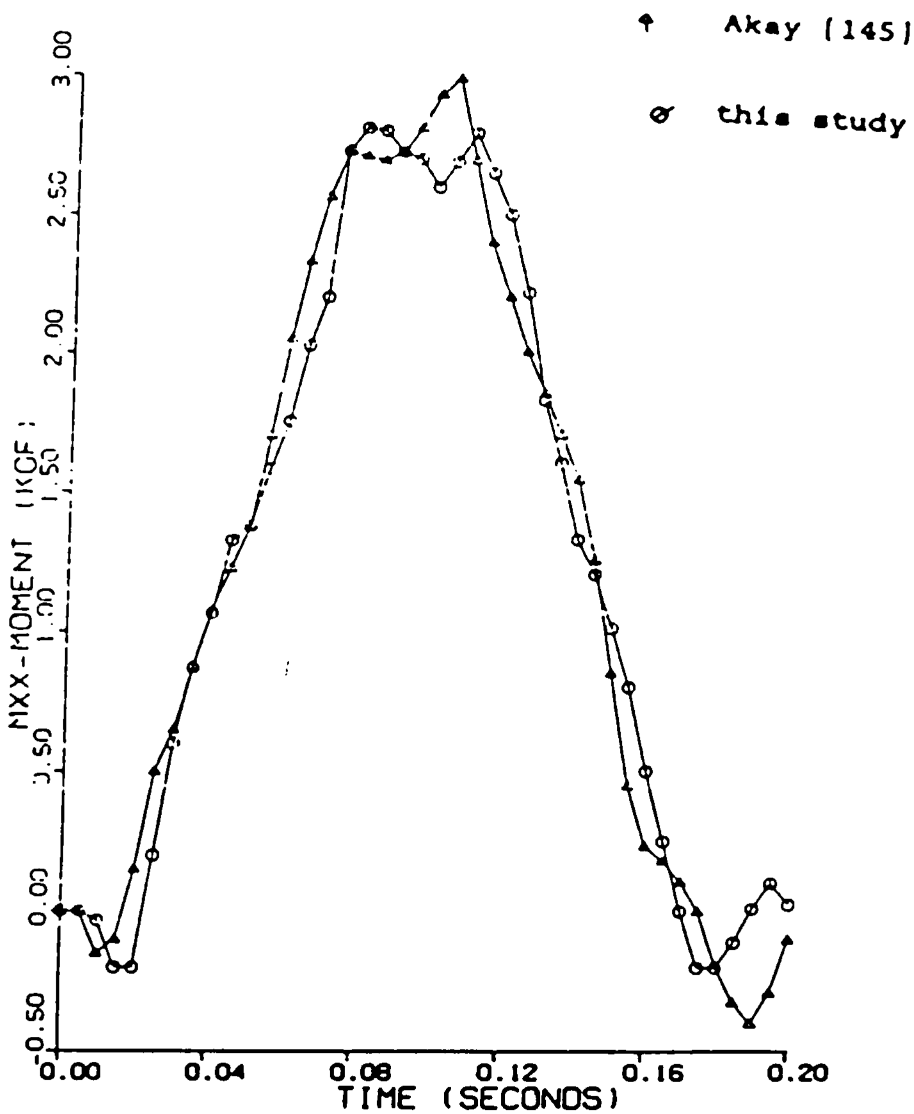


Figure 4.5: Example 1: Central Moment in the Linear Analysis.

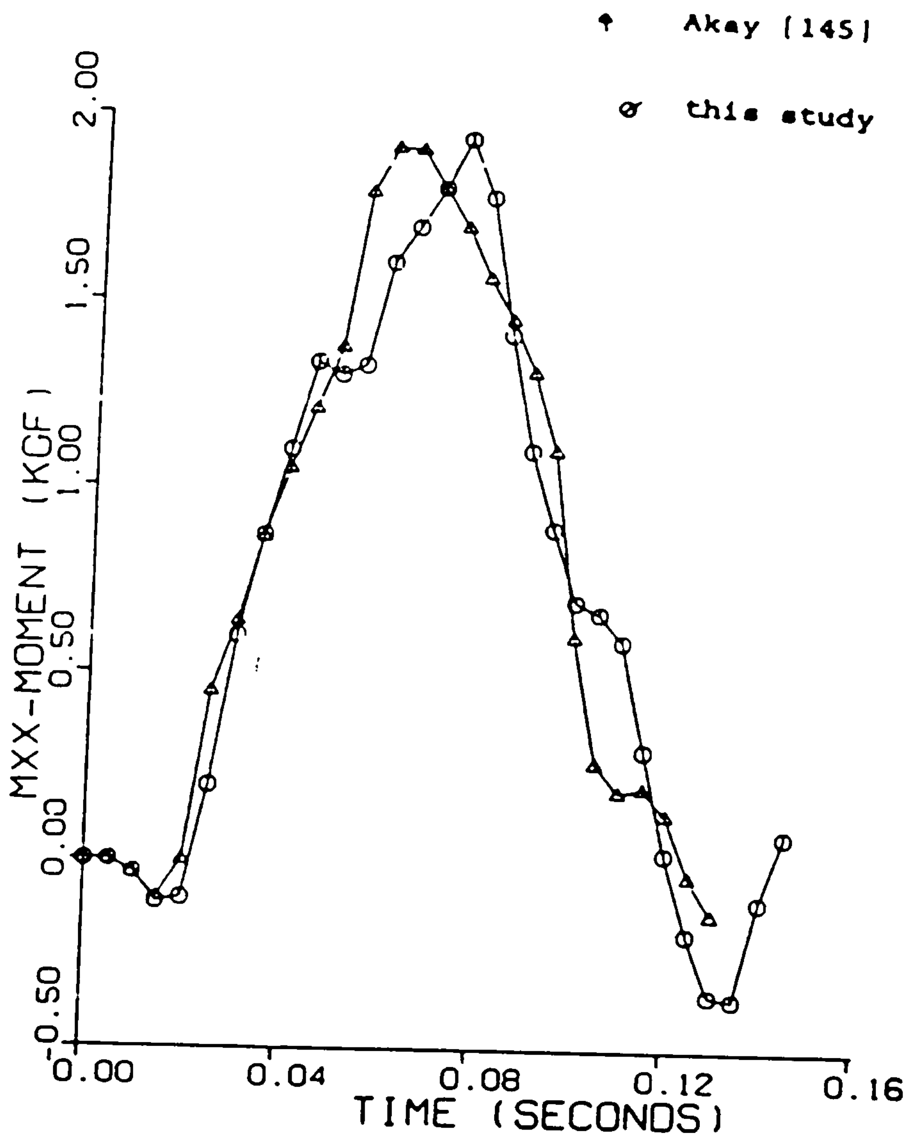


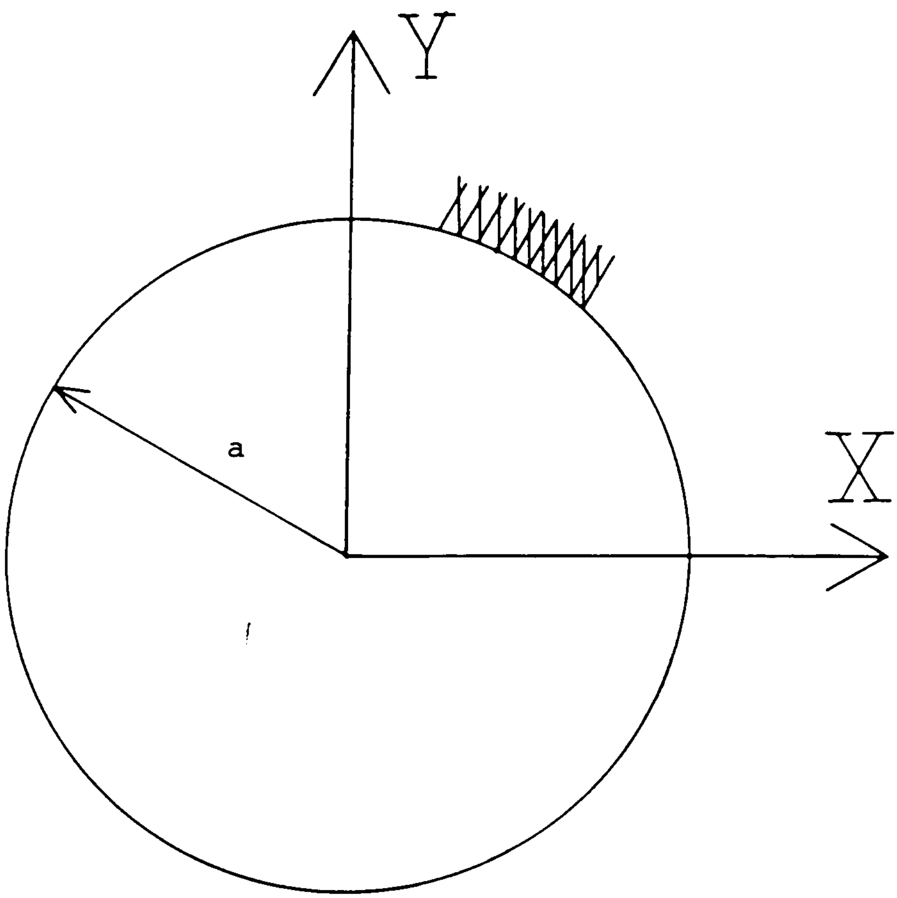
Figure 4.6: Example 1: Central Moment in the Nonlinear Analysis.

4.6, respectively. Thus, the results obtained using the mixed formulation compare favorably with the results of the mixed Galerkin displacement formulation used by Akay [145] where a 4*4 finite element mesh was used.

4.2 EXAMPLE 2.

In this example, the clamped circular thick plate, shown in Figure 4.7, is analyzed. The plate is subjected to a suddenly applied uniformly distributed force. The linear and nonlinear displacements and moments are calculated. Due to symmetry, only one quarter of the plate is analyzed, and it is represented by a five element mesh (Figure 4.8) with 59 moment DOF. In the linear and nonlinear analysis the number of displacement DOF are 19 and 45, respectively (Figure 4.8).

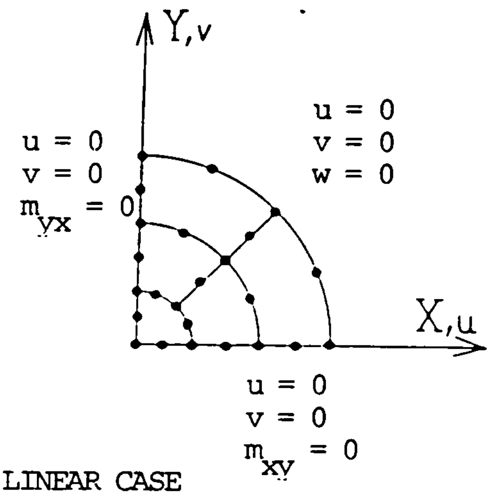
The central point displacements for the linear and nonlinear analysis are compared with the results by Pica and Hinton [174] in Figures 4.9 and 4.10, respectively. Also, the central point moments for the linear and nonlinear analysis are compared with the results presented by Pica and Hinton [174] in Figures 4.11 and 4.12, respectively. Thus, the results obtained using the mixed formulation compare favorably with the results of the displacement formulation used by Pica and Hinton [174] where the finite element mesh



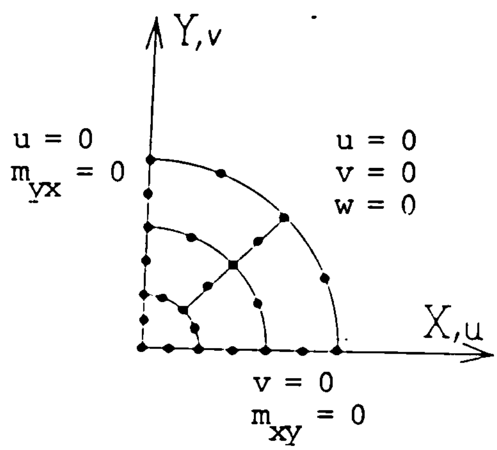
$a = 2.54$ m
$h = 0.508$ m
$\nu = 0.3$
$f_z = 6894$ N/m^2

$\rho = 1.08 \times 10^7$
$E = 70380$ Kgf/m^2
$\Delta t = 10$ seconds

Figure 4.7: Example 2: Circular Plate and Properties.



a) LINEAR CASE



b) NONLINEAR CASE

Figure 4.8: Example 2: Finite Element Mesh and Boundary Conditions.

had 60 and 96 displacement DOF in the linear and nonlinear cases, respectively.

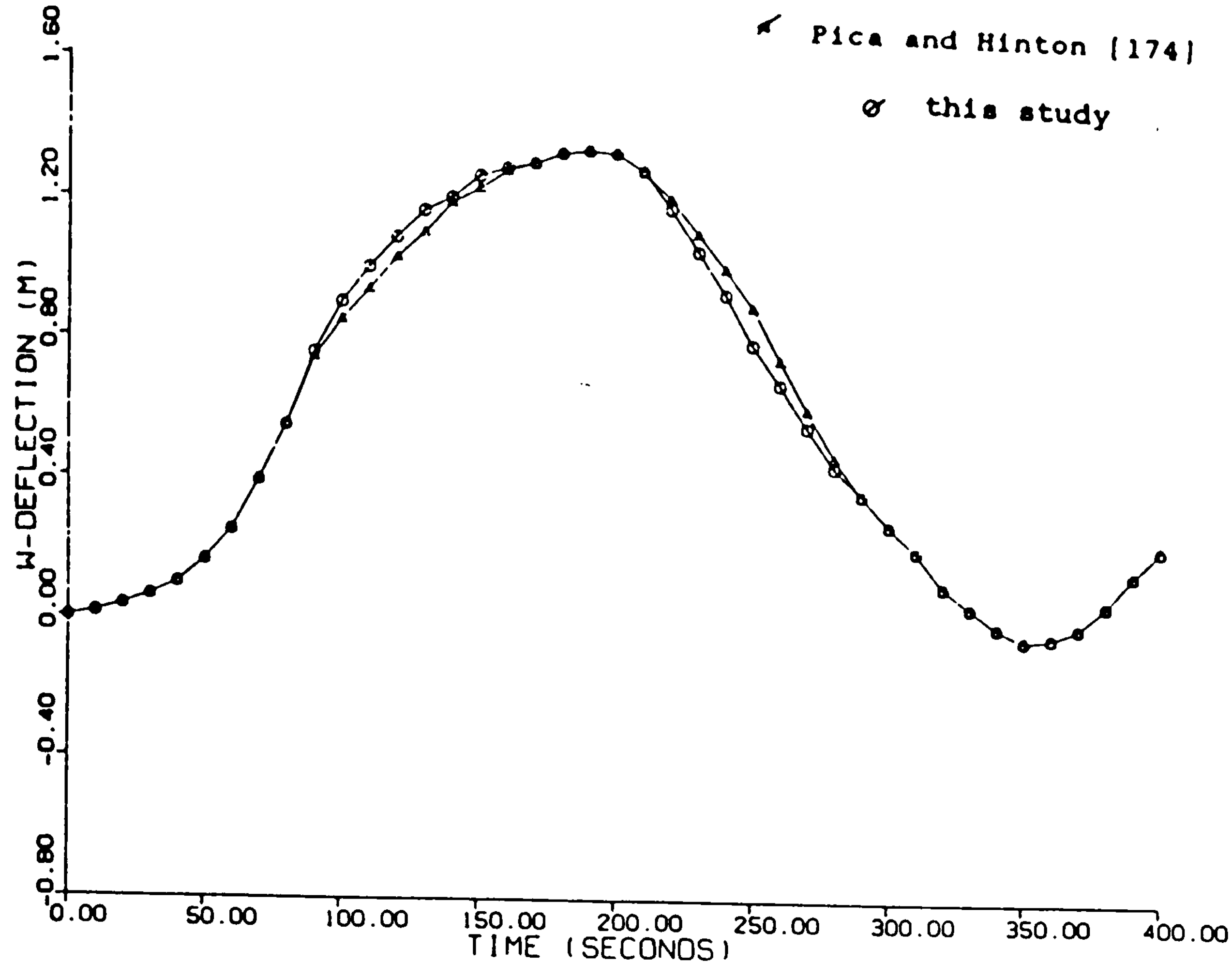


Figure 4.9: Example 2: Central Transverse Displacement in the Linear Analysis.

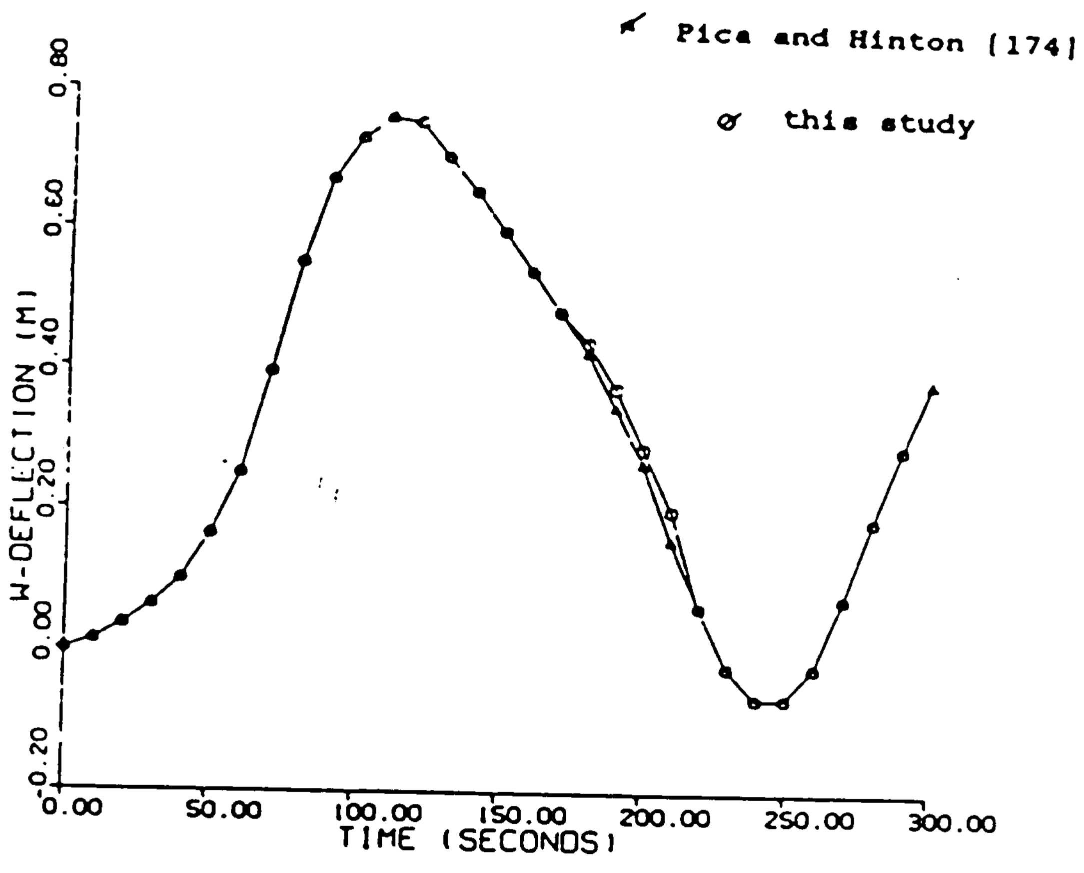


Figure 4.10: Example 2: Central Transverse Displacement in the Nonlinear Analysis.

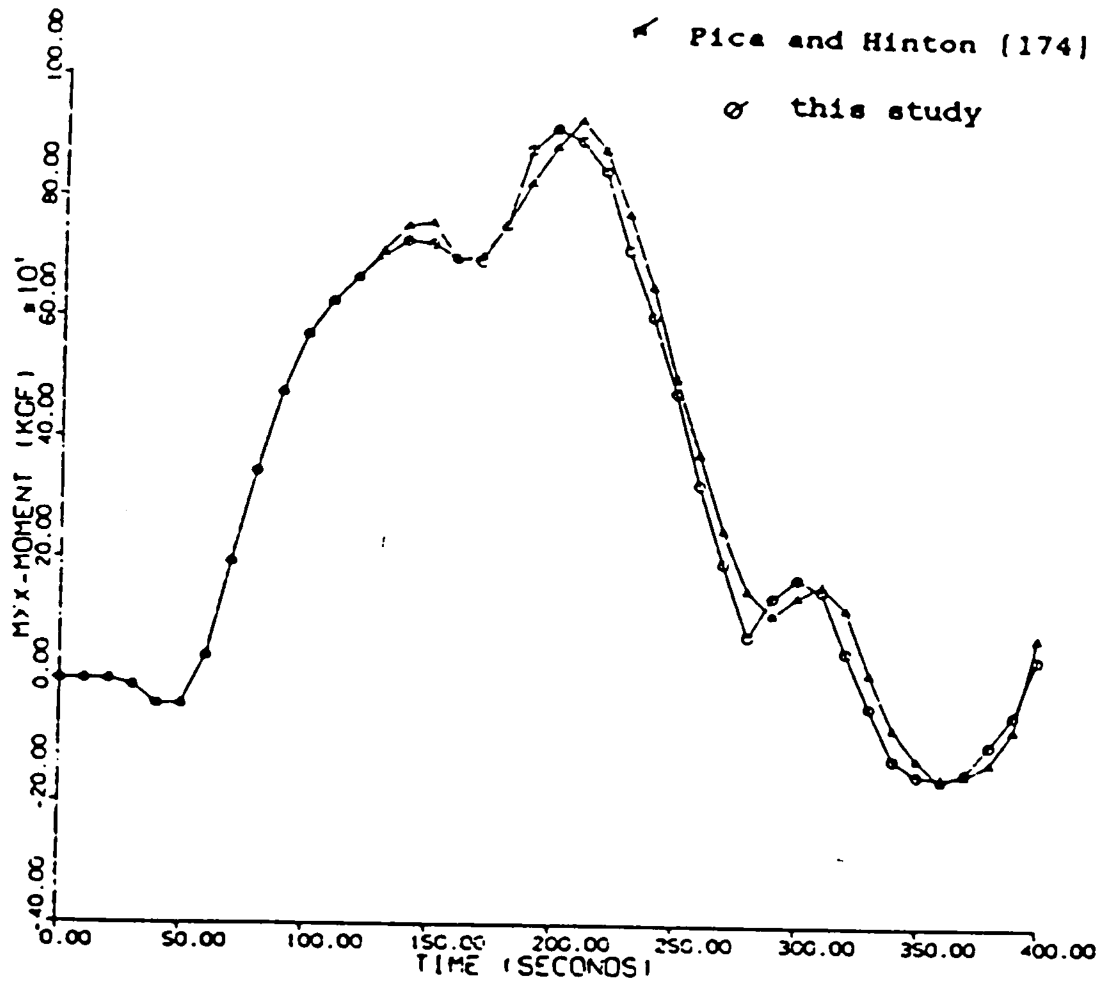


Figure 4.11: Example 2: Central Moment in the Linear Analysis.

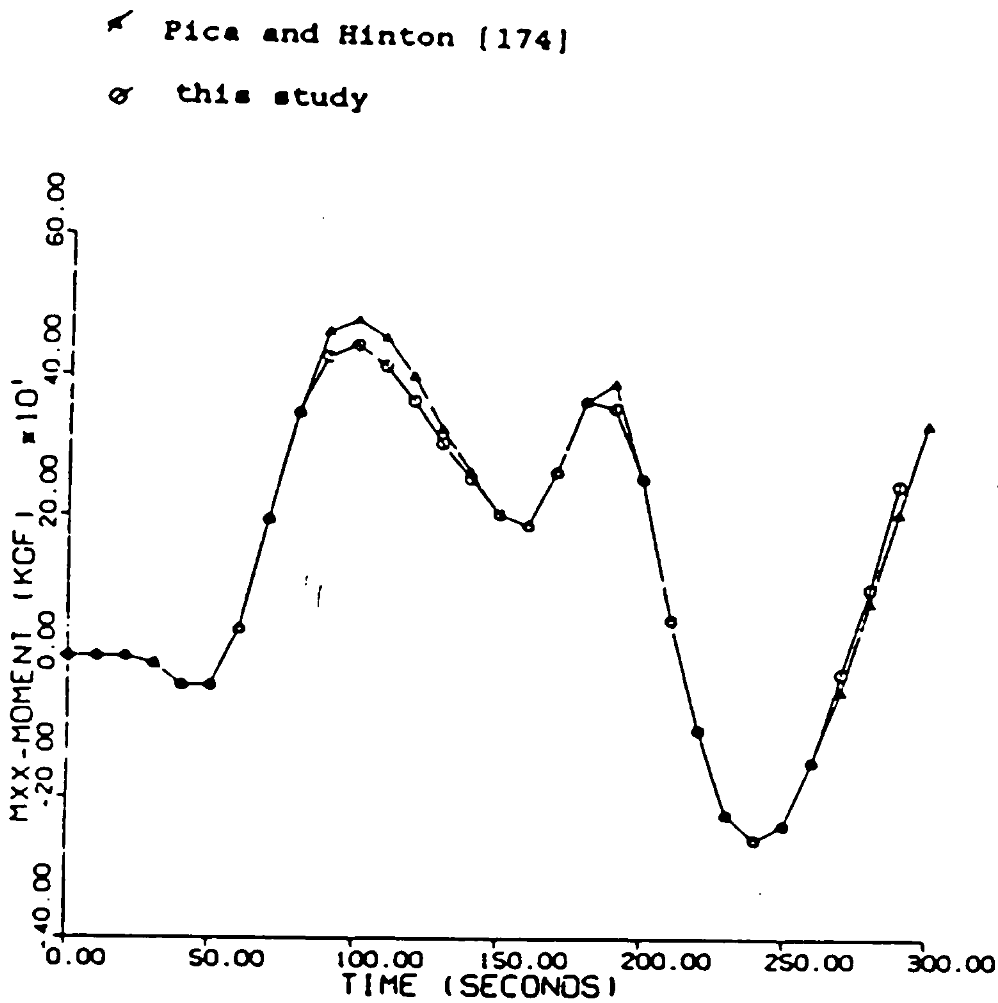


Figure 4.12: Example 2: Central Moment in the Nonlinear Analysis.

Chapter V

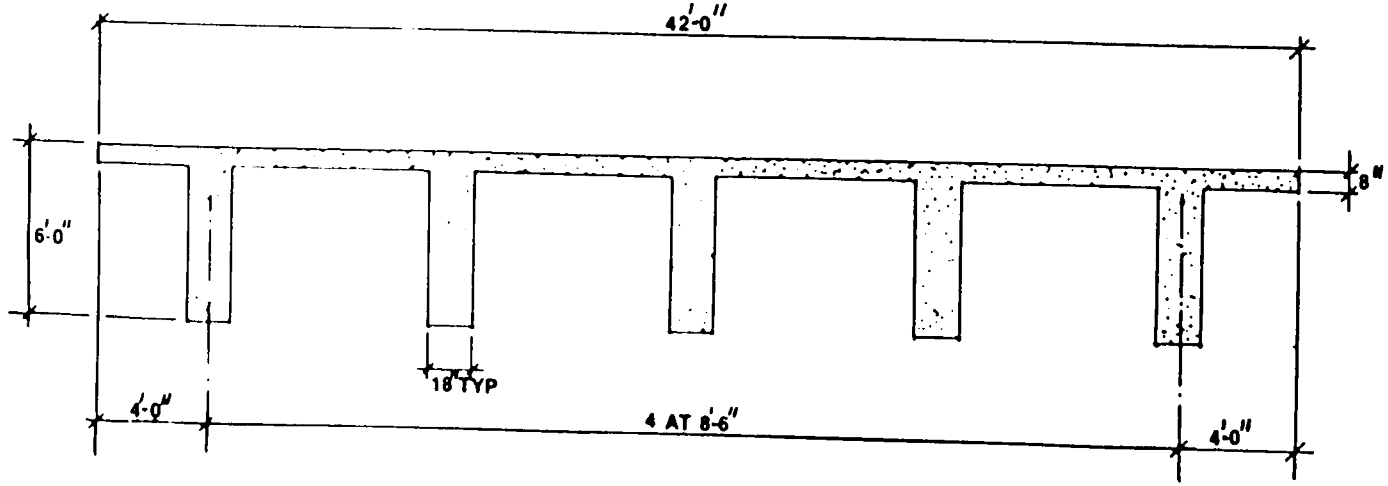
CASE STUDY

5.1 MODELING OF THE BRIDGE

The wall pier bridge shown in Figure 1.1 on page 5 has a total length of 174 ft consisting of two spans of 87 ft each. The cross-section of the superstructure and material properties are shown in Figure 5.1. The wall pier has a width of 43 ft and a height of 23 ft with a thickness of 20 inches. The density of concrete for the superstructure is considered to be 160 pounds per cubic foot to account for the weight of the bridge railings. The density of concrete for the substructure is the usual value of 150 pounds per cubic foot.

The wall pier bridge is modeled using the structural elements formulated in Chapter III. The model of this bridge, shown in Figure 5.2, is obtained based on the following concepts:

1. The wall pier is fixed at the ground level.
2. The mass is lumped at quarter points in the superstructure (Recommended by AASHTO [108]).
3. The supports at the abutments allow displacement only in the longitudinal direction and rotation around the X axis.



f'_c = 3250 PSI ν = 0.2 I_x = 244 ft^4

E_c = 432 x 10^6 lb/ft^2 A = 68 ft^2 I_y = 9903.5 ft^4

Figure 5.1: The Cross-Section of the Wall Pier Bridge.

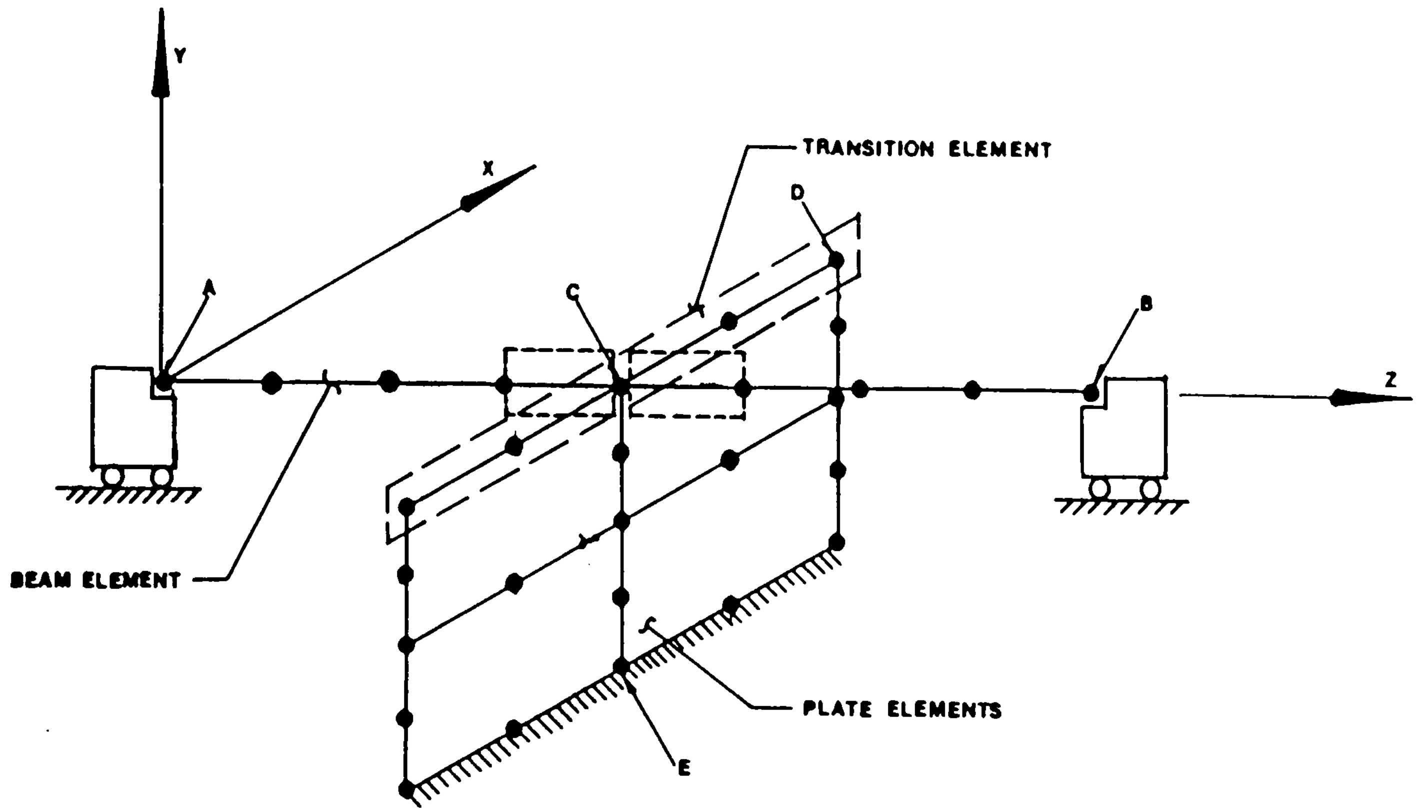


Figure 5.2: Model of the wall Bridge shown in Figure 1.1.

4. The superstructure is represented by beam elements, in which shear deformation is neglected.
5. The substructure is represented by nonlinear plate elements.
6. The bridge has an estimated period of 3.64 seconds and a damping ratio of 5 %, and the higher modes are assumed to be heavily damped. Thus, for construction of the damping matrix, $\alpha = 0.0289$ and $\beta = 0.0$.
7. Rigid support motion is assumed.

5.2 SEISMIC EXCITATION

The time history ground motion for the N-S component of the El Centro Earthquake that occurred on May 18, 1940 is applied to the bridge model in the longitudinal direction. The digitized data for this earthquake at time intervals of 0.02 seconds is obtained from Reference [110]. The time history ground motion is shown in Figure 5.3. The peak acceleration of 341.7 cm/sec/sec occurs at 2.12 seconds. Because of the costs involved in the nonlinear analysis and because the most critical ground motion occurs at the beginning of the earthquake, the nonlinear earthquake analysis of the bridge was executed for only the first 8.00 seconds.

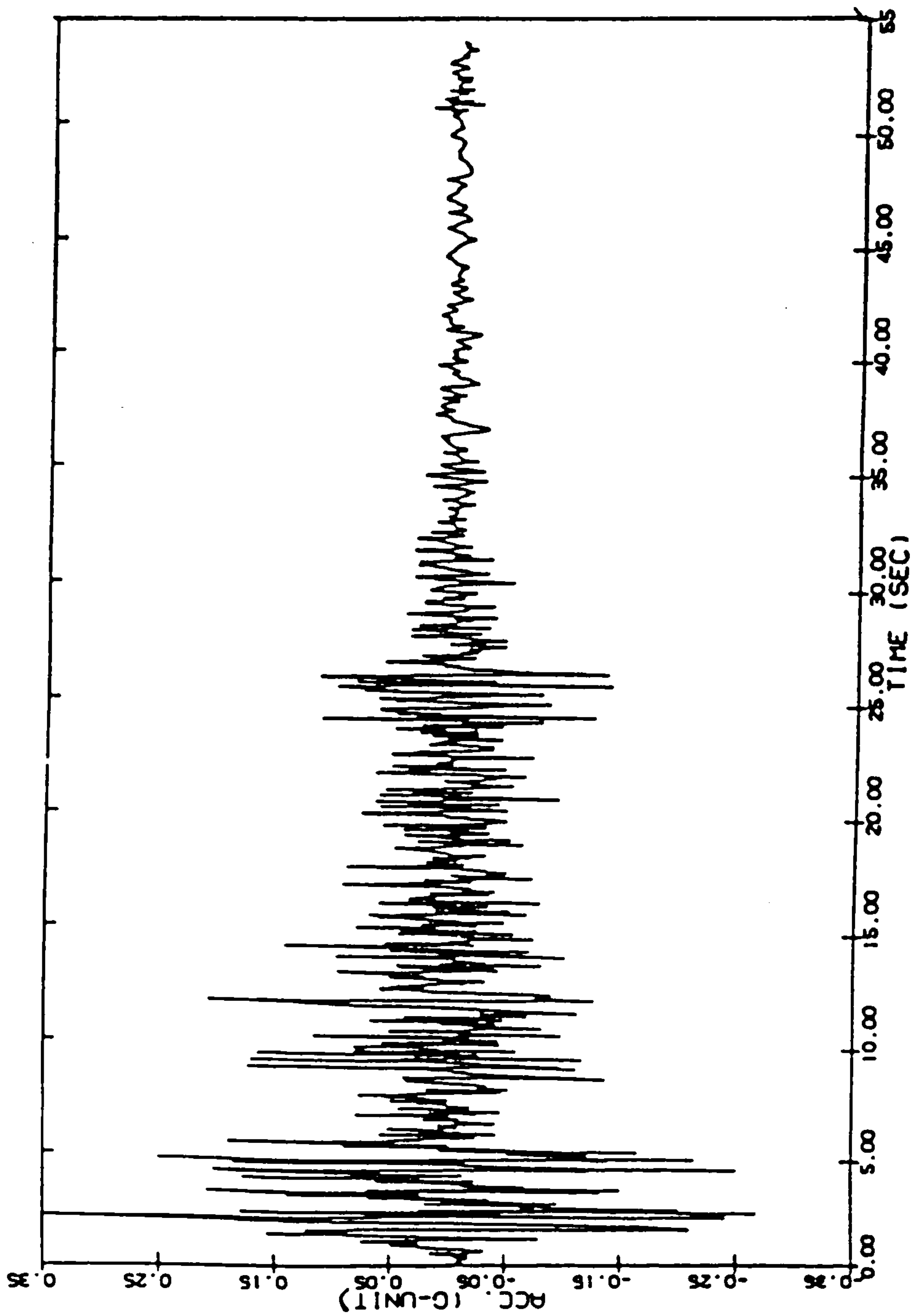


Figure 5.3: The Time History for N-S El Centro May 18, 1940.

5.3 RESULTS

The results for specific nodal points are presented in Figure 5.4 to Figure 5.7. In Figure 5.6 the w displacement for the upper middle node of the wall pier is compared to the edge w displacement.

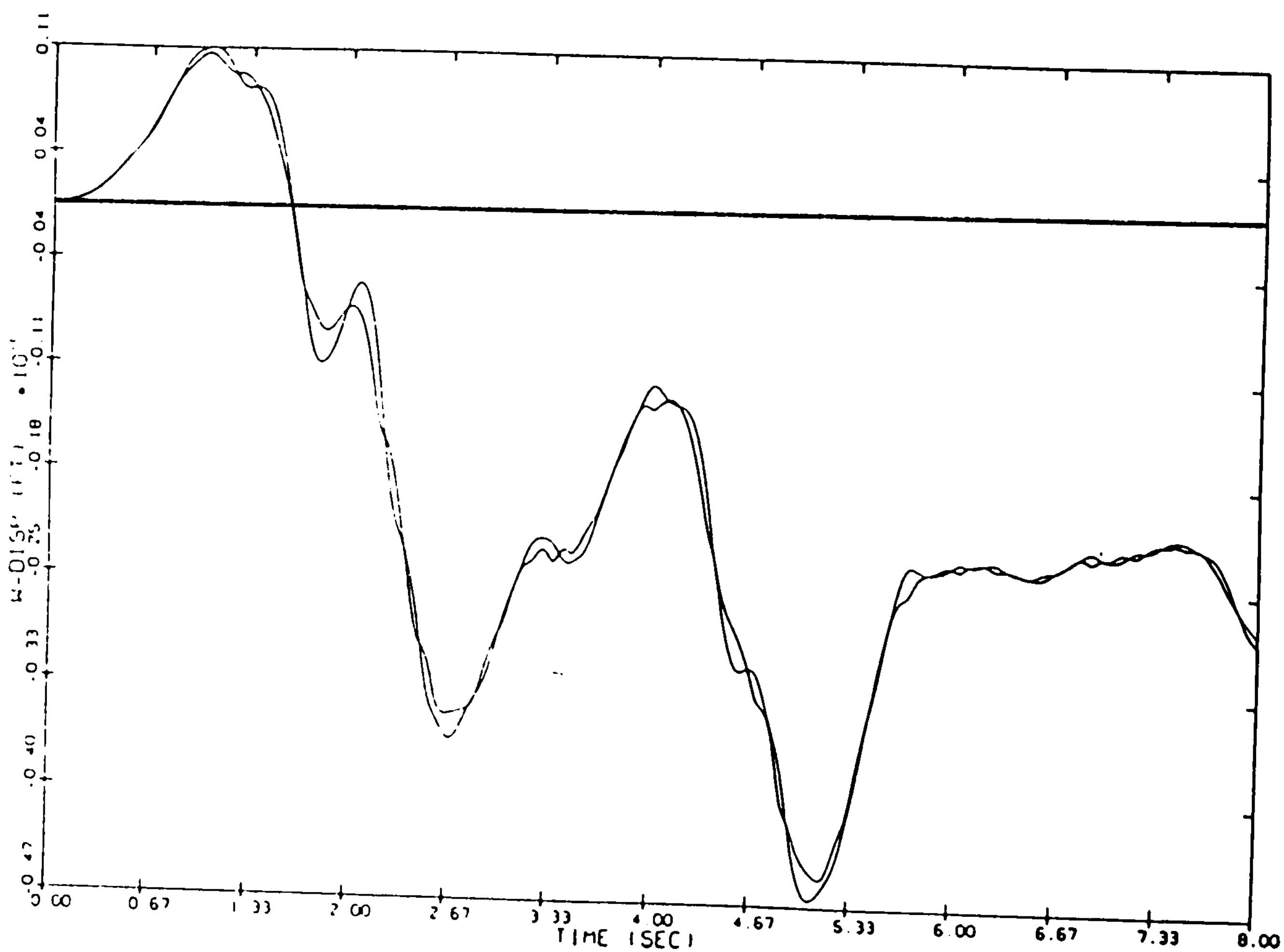


Figure 5.4: The Comparison of W-Displacement for Nodes A and B.

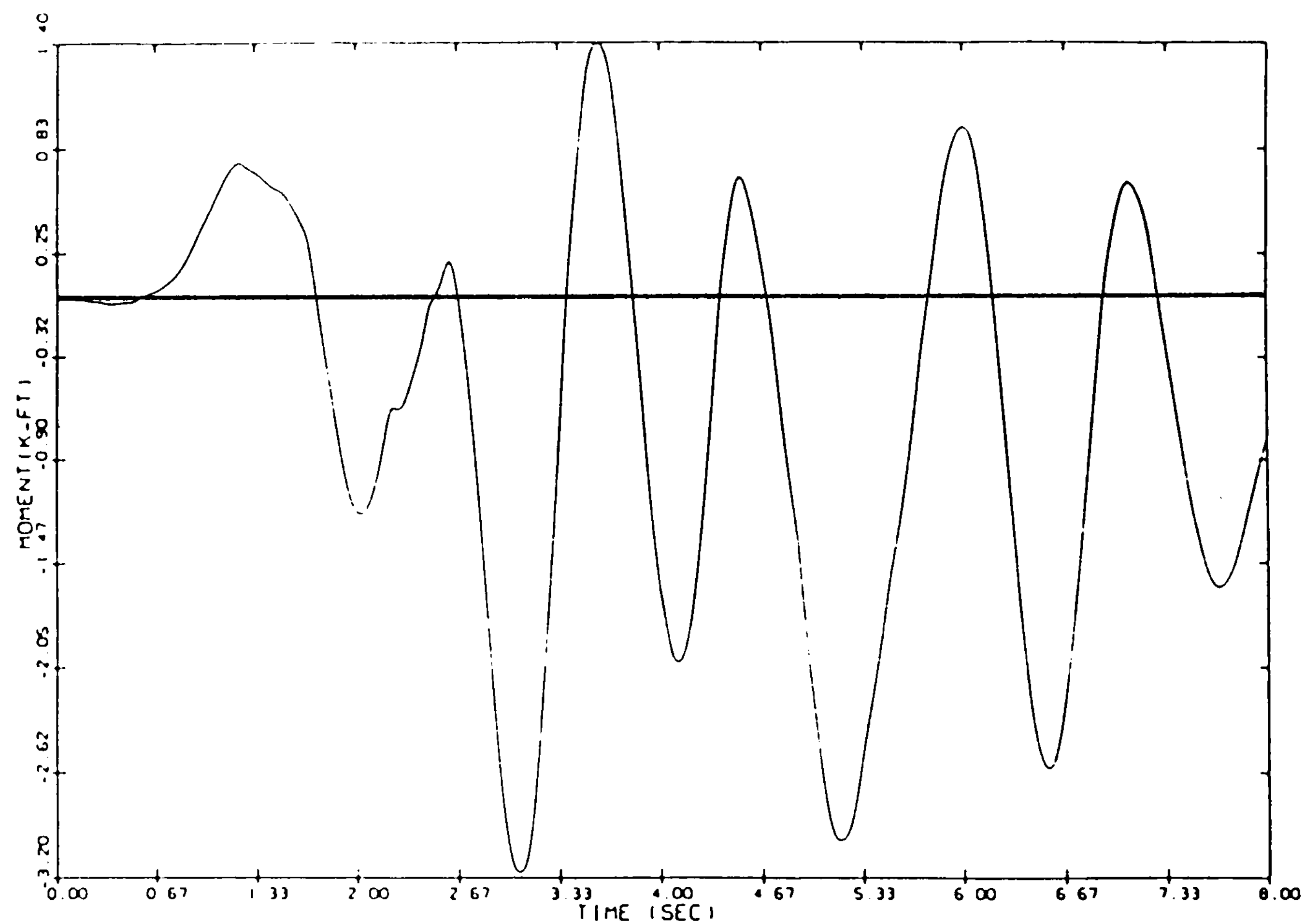


Figure 5.5: The Mxx-Moment at Node E at the Base of the Wall Pier.

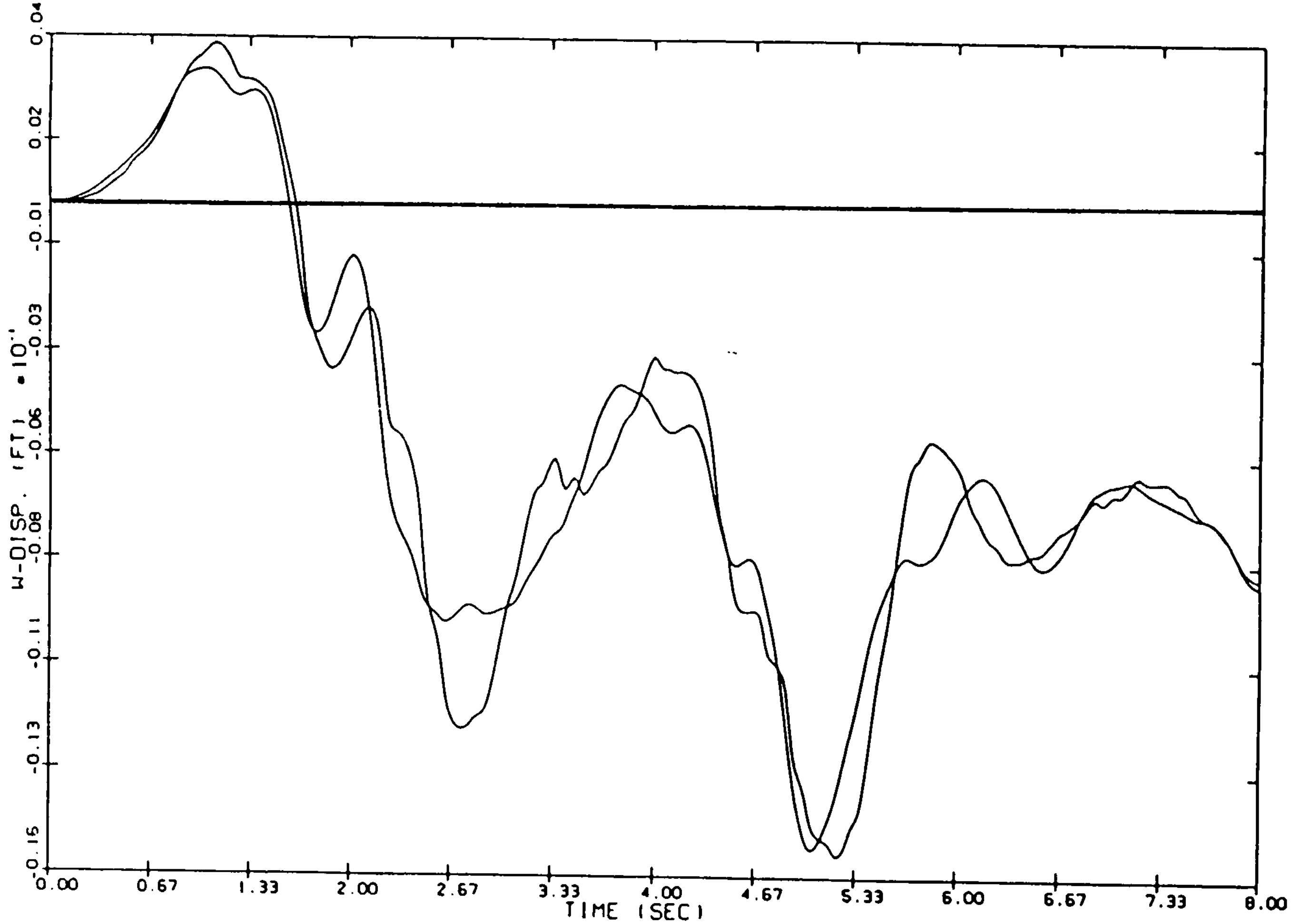


Figure 5.6: The Comparison of W-Displacement for Nodes C and D.

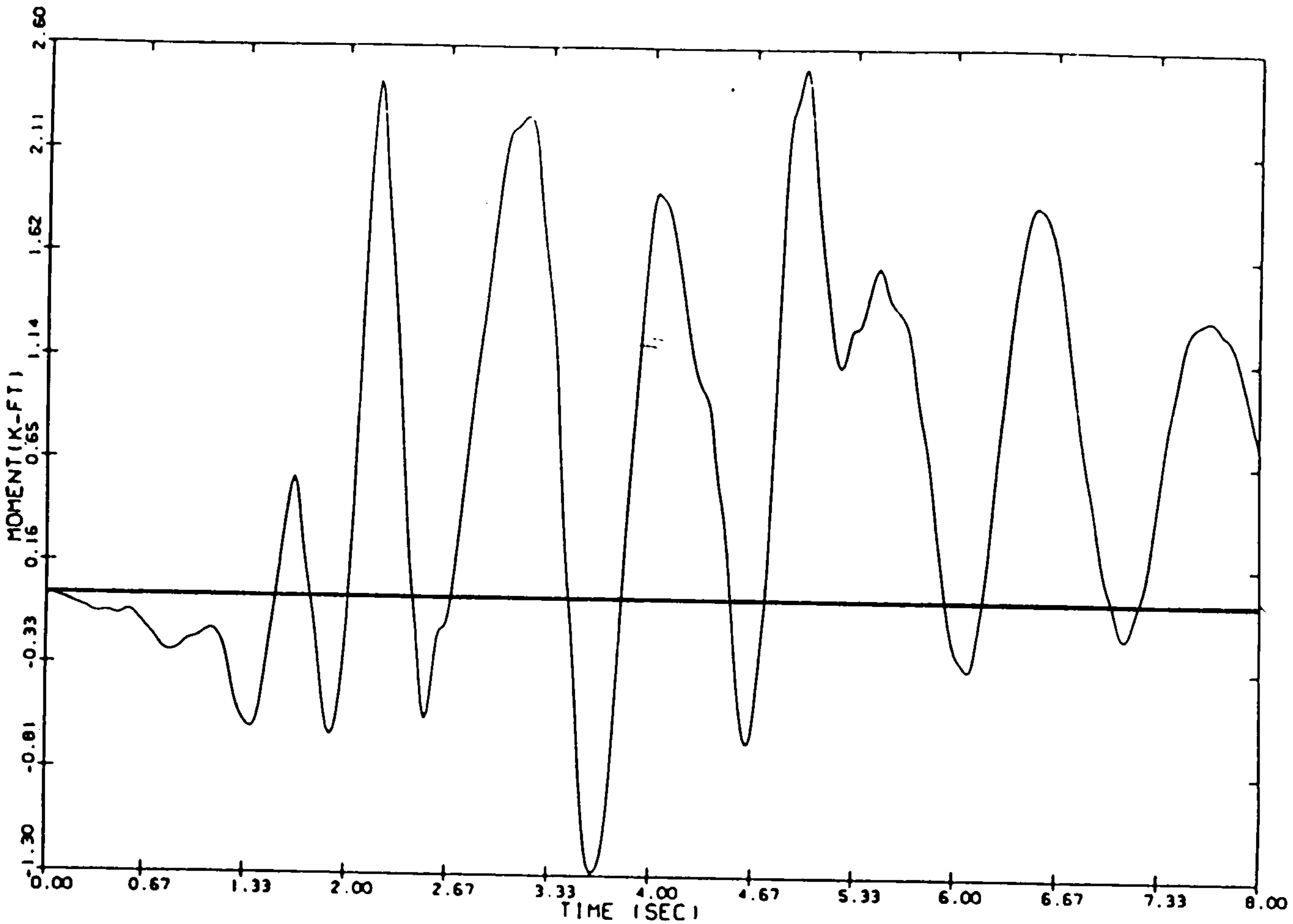


Figure 5.7: The Mxx-Moment for Node C.

Chapter VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

6.1.1 The Mixed Plate Element

The implementation of the mixed plate element has led to the following conclusions:

1. The mixed formulation developed is capable of predicting efficiently and accurately the linear and nonlinear transient displacements and moments of plate structures subjected to dynamic loadings.
2. The mixed formulation presented allows moment boundary conditions to be accurately satisfied.
3. The results obtained with the mixed formulation compare favorably with displacement finite element solutions, although mixed models with fewer displacement DOF were used.
4. The transient moments are calculated by a matrix transformation of the displacements, which is a more efficient and accurate computer operation than that required for displacement models.

6.1.2 The Bridge Mathematical Model

It has been shown from the results in CHAPTER V that the basic mathematical and analytical procedures presented in this study provide a rational and effective method for determining the dynamic response of monolithic concrete wall pier bridges under severe ground shaking conditions. In particular, the geometrically nonlinear mixed plate element has been shown to be effective in the modelling of the wall pier.

6.2 RECOMMENDATIONS

From the results in this study the following recommendations are made to improve the usefulness of the computer program developed and also to investigate in detail other aspects concerned with the nonlinear earthquake analysis of wall pier bridges:

1. Extend the program capability to include effects of material nonlinearity.
2. Implement an expansion joint element to model the response at the bridge abutments.
3. Implement nonlinear soil elements to model the soil structure interaction during an earthquake.
4. Extend the program to consider the effect of skewness between the deck and the wall pier.

5. Perform dynamic testing of bridge scale models to check the theoretical model.
6. Perform parametric studies on the effect of earthquakes on wall pier bridges by varying some of the wall pier bridge properties.

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