

ESSAYS ON INTELLECTUAL PROPERTY RIGHTS AND PRODUCT DIFFERENTIATION

by

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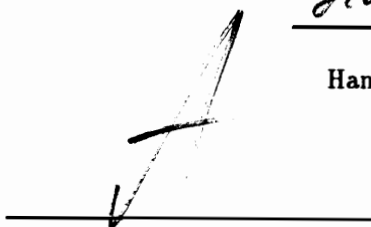
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(ABSTRACT)

This dissertation is a collection of essays on intellectual property rights and optimal product selection when innovation occurs sequentially. One of the highlights of this dissertation has been to show the possibility of full rent extraction by the patent holder when uncertainty in litigation is taken into consideration. The result of the theoretical model has practical policy implication regarding the design of an optimal patent system. The other highlight of this dissertation is to show the coexistence of maximal and minimal product differentiation in a sequentially growing market. This result sheds light on the simulation of a multi-dimensional product space.

Brief Summaries of Chapters:

Chapter 1 presents a survey of the historical, legal, and economic aspects of patents. The emphasis in this survey is to recognize the crucial elements in the current patent law practice and to initiate research projects thereof.

Chapter 2 considers a model of sequential innovation in which patent infringement occurs and the outcome of litigation is uncertain. By recognizing the “diminishing returns to litigation” exhibited in the winning probability distribution function for the plaintiff, it is shown that a basic researcher holding a patent is able to extract all the profit facilitated by the basic innovation. More intriguingly, under rather general circumstances, broader patent breadth may diminish the patent holder’s incentive to innovate.

Chapter 3 extends the previous model to include a rule on the reasonable royalty to determine the damage award. In addition to the full rent extraction results, the extended model further reveals that the second innovator has incentive to “invent around” with close imitation or “invent enough” with a much improved product. Comparative statics with

respect to parameters of litigation cost and granted patent breadth are performed. Among other things, it is demonstrated that an increase in patent breadth, and an increase of litigation costs may neutralize each other.

Chapter 4 analyzes a model of two-dimensional product differentiation in which sequential entry occurs and the potential entrant outperforms the incumbent in innovating a new dimension. For a three-stage entry-variety-price duopoly, a unique subgame-perfect equilibrium is obtained and fully characterized. Most importantly, the entrant will completely utilize its capacity to innovate and achieve the principle of maximum differentiation with respect to the innovated variety. However, it is shown that with a sequentially growing product space, firms will not choose extreme opposite positions in all dimensions in order to soften price competition; the principle of minimum differentiation persists with respect to the traditional variety.

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CHAPTER 0

SUCCINCT INTRODUCTION OF CHAPTERS

This dissertation contains four separate essays in the next four chapters. A common theme linking these chapters is the constant inquiry into the much neglected field of the law and economics of innovations. First of all, though with significant economic implication, patent law, the central legal mechanism on information transfer, is much too often a subject of discussion for the legal profession only. Chapter 1 through Chapter 3 constitute an attempt to bridge that gap between economic theorists and lawyers. Secondly, innovations generally take place in sequential stages and in multiple dimensions. These two features are worth exploring since a clear understanding on them will in turn furnishes important insights into the design of a more “economically sound” intellectual property rights system. While Chapters 2 and 3 emphasize the sequential nature of innovations, Chapter 4 specifically explores the innovative activities in a multi-dimensional setting. More precisely, the outline of the dissertation is as follows:

Chapter 1 presents a survey of the historical, legal, and economic aspects of patents. The emphasis in this survey is to recognize the crucial elements in the current patent law practice and to initiate research projects thereof.

Chapter 2 considers a model of sequential innovation in which patent infringement occurs and the outcome of litigation is uncertain. Between two strands of literature related to patent protection: the “fencepost” system literature and the “signpost” system literature, this model featuring vertical product differentiation is clearly more in line with the latter interpretation of patent protection. In particular, by recognizing the “diminishing returns to litigation”

exhibited in the winning probability distribution function for the plaintiff, it is shown that a basic researcher holding a patent is able to extract all the profit facilitated by the basic innovation. More intriguingly, under rather general circumstances, broader patent breadth may diminish the patent holder's incentive to innovate.

Chapter 3 extends the previous model in an unobtrusive, but important detail how profit is divided, if the patent holder prevails in court. A rule on the reasonable royalty is specified to determine the damage award. In addition to the full rent extraction results, the extended model further reveals that the second innovator has incentive to "invent around" with close imitation or "invent enough" with a much improved product. A more thorough investigation of efficacy of the patent system in terms of both effective patent protection and the average expected profits transferred is provided. Comparative statics with respect to parameters of litigation cost and granted patent breadth are performed. Among other things, it is demonstrated that an increase in patent breadth, and an increase of litigation costs may neutralize each other.

Chapter 4 analyzes a model of two-dimensional product differentiation in which sequential entry occurs and the potential entrant outperforms the incumbent in innovating a new product dimension. For a three-stage entry-variety-price duopoly, a unique subgame-perfect equilibrium is obtained and fully characterized. Most importantly, the entrant will completely utilize its capacity to innovate and achieve the principle of maximum differentiation with respect to the innovated variety. However, it is shown that with a sequentially growing product space, firms will not choose extreme opposite positions in all dimensions in order to soften price competition; the principle of minimum differentiation persists with respect to the traditional variety.

CHAPTER 1

DIMENSIONS OF INTELLECTUAL PROPERTY RIGHTS: AN INTRODUCTION TO PATENTS

1.1. What is A Patent ? A Survey

1.1.(i) *A brief historical overview*

The first recorded reference to the formation of intellectual property rights, especially patents, can be traced back to the fourth century B.C. In Aristotle's *Politics*, a proposal by one Hippodamus calls for a system of rewards to those who discover things useful to the state.

Opposing that view, however, Aristotle notes:

Concerning the matter of those who discover something advantageous for the city, to legislate that they receive some honor is not safe, though it sounds appealing; it would involve harassments and, it might well happen, change of regimes.

Although occurring in ancient time when technology was not advancing at the incredible speed that our modern civilization experiences, this historical debate symbolizes indeed the static vs. dynamic efficiency dichotomy surrounding patents. I shall defer a more thorough discussion of the law and economics of patents to the later subsections.

Open v. Exclusive

Patent originates from the term *letters patent* (a literal translation of the Latin *litterae patentes*), which means open letters. First of all, the “openness” encoded in patents does not

refer to the contemporary “disclosure” requirement but rather points to an open letter carrying an inside seal which conferred certain privileges, rights, ranks or titles. Thus the special rights can be “usable more than once” without breaking the inside seal of the sovereign grantor (David, 1993).

Invitation v. Invasion

A second intriguing historic fact is that patents started out as means to promote the introduction of foreign technologies (or rather, craftsmanship such as clock making and weaving) in fourteenth century England (Federico, 1929). Conversely, in modern times, the technologically lagging countries are not inclined to install strong patent systems at their own will unless threatened by technologically advanced countries.

Optimal v. Practical Patent

Moreover, it seems to be the case that many of the across-the-board features of the patent are more justifiable in its historical context than in the current market environment in which distribution of the appropriability is a crucial issue. For example, the duration of a patent in fourteenth century England was set for 14 years with possible 7-year extension while 7 years were the then conventional term of service of an apprentice. Thus the special privilege granted to the artisans would amount to at least two “generations”. The same reasoning seems hardly applicable to the patent length adopted in modern patent systems.¹ Similarly, the standards for patentability were not as stringent then as now most national patent systems demand. Not required to show prior art and originality, a person seeking patent protection needed only demonstrate the utility (usefulness) of the skills or products.

¹There are quite a few articles on optimal patent duration, for example, Nordhaus (1969), Gilbert and Shapiro (1990), Klemperer (1990) and Gallini (1992).

Strong v. Weak (or no) Patent Protection

The patent system has not been without oppositions ever since it took to the spotlight of economic policy. The first clear anti-patent swing came during the periods of 1860s and 1870s, when the free-trade movement was at its peak. The historical marks were made when the Netherlands completely abolished the patent system in 1869 and the installation of the Swiss patent was put on hold. The typical arguments against the patent at that time are not drastically different from what we have seen in recent history. It was always declared that a patent system is associated with various costs: the social deadweight loss caused by the monopoly power granted to a patent holder; the bureaucracy established along with the patent offices and all level of courts; deferral of socially desirable complementary invention and “wasteful” research in terms of “inventing around” in response to the patent system; concentration in some market fortified by a sequence of significant patents held by capable innovators. Concerned with these issues, the U.S. Supreme Court was once labeled as pro-antitrust but not pro-patent, notably during the period from 1930s through 1960s. Significant change in favor of patent seekers and patent holders came after the establishment of the Court of Appeals for the Federal Circuit.² From an international perspective, the World Trade Organization (WTO) has also elevated the discussion to a higher level in the Uruguay Round Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPs agreement). A uniform and effective (strong) patent protection is called for in the agreement (See Schott, 1994).

To conclude, although patents have been a legal institution for centuries, the exact and

²Some anecdotal evidence suggests that the CAFC is a good court for patentees. See, for example, Schmitt, *Business and the Law: Judicial Shift in Patent Cases*, N.Y. Times, Jan. 21, 1986, at D2 (citing Kodak-Polaroid dispute as the “most prominent example of an increasingly pro-patent sentiment in American courts” when the district judge refused to stay an injunction against Kodak while it appealed).

subtle features of the system are still being debated among philosophers, economists and lawyers. Moreover, historical studies reveal that the patent institutions are evolving over time to meet the varying needs of the national states and to fit the international trade environment. Thus fine tuning of the patent law through new legislation, court interpretation and establishment of international treaties appears to be crucial in achieving the goal of “improving welfare”. This leads us to the next section addressing the current legal practice of patent law.

1.1-(ii) *Legal aspects of patent*

To begin with, a subject matter sought to be patented needs to qualify the standards for patentability. As will be seen in the subsequent analysis, these requirements amount to a multi-dimensional test.³

Utility Requirement:

The patent code protects all inventions that are new and useful.⁴ The subtle meaning of “new invention” will be addressed in the discussion of novelty and nonobviousness requirements. Although a rare issue to be challenged, utility is not as a trivial a requirement as we tend to think. A closer look at the usefulness of an invention reveals a three-dimensional standard: to fulfill general utility, is it operable or capable of any use? to achieve

³To be specific, this discussion refers to American patent law. Administrative differences among countries exist, obviously, but the essence seems to be more similar than diversified. The lack of cross-section empirical work exemplifies this observation. For a detailed discussion on the intellectual property practice in UK and European Economic Community (EEC), see Cornish (1989) and van Dijk (1994).

⁴U.S.C. §101: Inventions patentable: Whoever invents or discovers any new and useful process, machine, manufacture, or composition of matter, or any new and useful improvement thereof, may obtain a patent therefor, subject to the conditions and requirements of the title.

specific utility, does it solve the problems that it claims? to qualify as a beneficial utility, does its intended purpose achieve minimal social good? Thus a pure speculative (or preemptive) invention that is of no general use may not be patentable under the general utility requirement. A typical example for the rejection of patent application based upon the specific utility requirement can be found in the case *Newman v. Quigg*, 877 F.2d 1575, 11 U.S.P.Q.2d (BNA) 1340 (Fed. Cir. 1989), in which a self-claimed “perpetual motion” energy-generating device did not pass the test performed by the National Bureau of Standards. In the phase of examining specific utility, new synthetic chemical compounds without particular uses often fail to obtain patents. Historically, inventions such as gambling “slot” machine and “Radar Signal Detector” have been rejected for patent protection according to the beneficial utility requirement.

Novelty Requirement:

This requirement deals with the “newness” of an invention. To be more specific, it contains exclusionary clauses for inventions that fall within the category of “prior art” by which known, used, abandoned or described (in printed publication) inventions are defined within a pre-specified period. Clearly, society wishes to reward only those technologies that are in fact new. In most cases the assessment of novelty seems to be straightforward. Nonetheless, this issue still requires further consideration when it comes to the inherent relationship among innovations. For example, an inventor may discover a new use for something already known. The “new use of old thing” patent application is subject to the test of doctrine of anticipation, i.e., whether an innovation would have been anticipated from the previously known technology which was properly disclosed. In some sense the novelty, nonobviousness and enablement requirements are interrelated since all refer to the concept of prior art, though from different perspectives as we shall see shortly.⁵

Nonobviousness Requirement:

This is what many patent lawyers consider the most important gatekeeper of patentability.⁶ Put simply, the nonobviousness test creates two scales: one is to establish the prior art as a whole while the other is to evaluate the invention from a fictional skilled artisan's perspective. The threshold of patentability implied by the nonobviousness requirement is that a trivial invention satisfying the utility and novelty requirements does not deserve a patent as a just reward.⁷ A cornerstone case for nonobviousness can be found in *Graham v. John Deere Co.* 383 U.S. 1, 148 U.S.P.Q. (BNA) 459 (1966), in which the court explicitly held the Graham patent invalid based on the two-faceted test. The invention's insignificant advance compared to prior art and its obviousness proved the patent to be invalid.⁸ It has also been argued that an implicit economic function of the nonobviousness test is to encourage the development, but not research of highly uncertain technology. When making reference to the uncertainties, nonobviousness requirement promotes an inventor's incentive to develop since the probability of being awarded a patent is positively correlated with the riskiness of the development process. However, the same logic can not be applied to the incentive to invent because research takes place before development and the success rates in research (experiments) are distinct from those in the later stage.⁹ There is also some indication that the courts use the cost of research as reference for the degree of nonobviousness of the innovation and vice versa.¹⁰

⁵For example, Green and Scotchmer (1990) construct a model where novelty and nonobviousness standards are of the same function.

⁶For instance, see Witherspoon, 1980.

⁷See U.S.C. § 103: A patent may not be obtained,... if the differences between the subject matter sought to be patented and the prior art are such that the subject matter as a whole would have been obvious at the time the invention was made...

⁸"Certainly a person having ordinary skill in the prior art,..., would immediately see that the thing to do was what Graham did."

⁹See Merges (1992): A note on the economic Function of § 103, pp. 411-21.

Disclosure and Enablement requirements:

In the late eighteenth century, the patent system evolved from a reward institution for introduction of finished (foreign) products to a information transfer mechanism for intellectual assets. The emphasis on dissemination of useful information stood out as the most important economic function of the patent system. Having passed the scrutiny of patentability, an inventor is asked to disclose the necessary information embodied in the patent to enable others skilled in the technical arts to perform.¹¹ An immediate concern that comes to mind is the possibility of patent value depleted by easy imitation through clear disclosure. Thus these requirements make the inventor's discovery public in exchange for a (monopoly) patent. They also delineate the boundaries of intellectual property rights. This brings us to the next category of patent law as stipulations.

Infringement and remedies:

After all, patent is the right to exclude.¹² In the course of a patent infringement litigation, three defenses could be taken by a alleged infringer: insisting that the claims in suit are not within the claims of the patent therefore not infringing; challenging the validity of the patent by reexamining the prior art; finding patent misuse by the patent holder. In some sense patent infringement is a matter of interpretation of the original claim. Noteworthy are the instruments frequently used by the court to discern the languages defining bounds of the intellectual property: doctrines of equivalents and reverse doctrine of equivalents. The former is used to stretch the patent protection beyond the scope of literal claim while the latter is

¹⁰See *Panduit Corp. v. Dennison Mfg.*, 774 F.2d 1082, 1089, 227 U.S.P.Q. 337 (Fed. Cir. 1985).

¹¹U.S.C. §112. Specification: The Specification shall... enable any person skilled in the art to which it pertains, or with which it is most nearly connected, to make and use the same, and shall set forth the best mode contemplated by the inventor...

¹²U.S.C. § 271: ... Whoever without authority makes, uses or sells any patented invention, during the term of the patent therefor, infringes the patent.

applied in the opposite direction to enforce patent protection only up to some proportion of the literal claim. It is clear that the patent system which has evolved into today's legal institution is a property right system. For example, the U.S. Patent Act [U.S.C. 261] states that "patents shall have the attributes of personal property." Parallel to the metes and bounds of real property are the literal claims described in patent specifications. To avoid confusion, note that the rule for damages for infringement on patents is a mixed version of property and liability rules. Temporary and permanent injunctions as well as damage awards equal to a reasonable royalty or lost profits are the regular means for remedies.

1.1.(iii) *Economic Issues*

Economists often depict the analysis of optimal patent as a tradeoff between static and dynamic efficiency. As Schumpeter observed:

"Any system that at every point of time fully utilizes possibilities to the best advantage may yet in the long run be inferior to a system that does so at no given point in time, because the latter's failure to do so may be a condition for the level of long run performance."

Criteria for static efficiency require widespread dissemination of intellectual assets at low marginal cost so that demand for valuable information can be met without the extra burden of monopoly pricing and profit.¹³ On the other hand, dynamic efficiency will not be attained if the patent institutions do not create sufficient incentive, i.e., rewards such as monopoly

¹³To draw an analogy between monopoly and patent privilege may be an oversimplified way of addressing the static efficiency problem. Alternative viewpoints have been proposed. See, for instance, Dam (1994) and Kobayashi (1986).

power, for the innovators to initiate R&D projects in the first place. Thus appropriability of information assets is associated with monopoly, and deadweight loss derived from monopolistic market is deemed as an acceptable second best solution (Dam, 1994, Nordhaus, 1969, Arrow, 1962).

Rent-seeking behavior through patent races is a topic which economists have extensively investigated. A patent race attempts to capture the competitive behavior between two (or more) firms pursuing the same grand prize, the patent. The common modeling approach has been a winner-takes-all R&D investment game. It is best described by Dasgupta (1986) in the following formulation:

N players ($N \geq 2$) bid for an indivisible object valued by each at $V (> 0)$. All bids are forfeited. The highest bidder wins the object. If there are $K (\leq N)$ highest bidders each of these (K) players wins the object, with probability $\frac{1}{K}$.

It turns out, throughout the patent race literature that the exact answers to the important policy questions such as the socially optimal number of firms involved in the innovative activity depend upon the specific environments that innovators are situated in.¹⁴ However, the patent race literature does not take into account the post patent competition, let alone the cumulative nature of innovation.

On the dynamic side, it has long been recognized that the engine behind economic growth is propelled by many different forces. Empirically, Solow (1957) observed that only 10 percent of per-capita growth (for the U.S. nonfarm sector over a 40 years span: 1909-1949) was associated with an increase in the ratio of capital to labor. This finding evokes economists' interest on the role of technological advancement in improving welfare. In recent endogenous

¹⁴The readers are referred to Reinganum (1989) for a detailed survey.

growth literature (Romer (1994), Grossman and Helpman (1991), Aghion and Howitt (1992), Stokey (1995)), the major forces behind growth have been identified as capital accumulation, skill acquisition and innovation. However, the role of intellectual property rights is not the central theme in this literature. Only very recently has there emerged a renewed interest in the interface of law and economics of patents. For instance, Hortsman, MacDonald and Slivinski (1985) look at patent as an information transfer mechanism while Scotchmer and Green (1990) specifically pin down the impacts on the pace of innovation from the stringency of novelty and disclosure standards. This new interest redirects many economic theorists' focus to the age-old question on how to design an optimal patent system. Thus there is discussion on the duration of monopoly power granted to the patentee, also known as patent length (Nordhaus, 1969). Furthermore, based upon an improved understanding of the procedure and legal practice of the patent law, significant attention has been paid to investigate the scope of the patent protection, often labeled as patent breadth (Gilbert and Shapiro, 1990; Klemperer, 1990; Waterson, 1990; Gallini, 1992).

To further the understanding of the incentive to innovate and to mitigate the tradeoff between static and dynamic efficiency, the problem of profit division between sequential innovators has also received a great deal of attention from economists recently. A few of the most interesting additions to this emerging literature have been provided by Green and Scotchmer (1995) and Chang (1995). Green and Scotchmer analyze the role of different legal mechanisms under which innovation takes place in two stages. They then address the issue of optimal patent breadth and duration in this framework. The authors argue that the potential patent holder may lack incentives to invest in the first place, because not all the social value facilitated by basic research can be transferred from the second generation products. Built upon this important seminal work, Chang (1995) further investigates the optimal patent

system where the allocation of profit is most efficient. Contrary to what Merges (1992) and Merges and Nelson (1990) believe to be the means to resolve a hold-up problem between blocking patents, Chang argues that the optimal patent policy would also extend broad protection to inventions with very little (stand-alone) value relative to the improvements that others may subsequently invent. The main result is that optimal patent breadth (patent protection) is not a monotonic function of the value of the invention. It is his conclusion that for the extreme cases in which the value of the second invention is tiny or quite large relative to the first invention the Patent Office and the court should grant the patent holder broad patent protection. When innovations are achieved sequentially, the (social) values of these innovations are correlated. Moreover, the stand-alone values of either the original or the improved technology cannot be used as the sole indicator for proper patent protection.

1.2 Reality Check

Several interesting aspects of the innovative activity related to patent will be surveyed in this section. A road map of the research projects completed in this dissertation is also sketched here.

1.2.(i) *Recognition of the Uncertainties in Defining Exact Patent Breadth, i.e., Implication of Imperfect Patent Protection.*

To start with, “patent law operates through legal doctrines, not through administrative means” (Dam, 1994). As stressed in the earlier discussion, these doctrines are clearly amenable to interpretation which in turn impairs uniformity and precision in patent disputes.

Some empirical estimates may confirm this observation. First, patent litigation is on the rise even after the installment of CAFC which was meant to reduce the number of litigation suits. During the 1980s, the number of patent infringement suits surged by 50%. Moreover, between 1978 and 1985 the winning percentage of the patent holders increased from 48% to 80% (Hylton 1993, Warshofsky. 1994).

1.2.(ii) *Inventing Around and Imitation*

Since most of the theoretical patent literature assumes perfect patent protection, one tends to conclude that monopoly power granted to the patent owners would be intact throughout their patents' duration. It is no mystery, however, to many legal practitioners that imitating or inventing around a patent impedes appropriability by the patent holder. Having fulfilled the disclosure and enablement requirements, a patent holder with (possibly) significant sunk cost is immediately confronted with the problem of knowledge spill-overs. Many market niches could be easily filled by swarming imitators who can either invent enough to escape the "web of infringement" with less time and investment or just daringly imitate the patent technology.¹⁵ Recent empirical studies also uphold this observation. For instance, Mansfield, Schwartz and Wagner (1981) concludes in a study of 48 new products that the ratios of imitation cost to innovation cost and imitation time to innovation time are about 65% and 70%, respectively. Furthermore, 60% of all patented and (commercially) successful innovations were imitated (from the original innovator's viewpoint) within 4 years after being disclosed. In another study by Levin, Klevorick, Nelson and Winter (1987), similar observations were made regarding the imitation phenomena.

¹⁵See, for instance, Scotchmer (1996).

1.2.(iii) *The Sequential Nature of Innovation*

Innovations are in nature sequential. One typical aspect of R&D is that a commercially profitable innovation results from basic research. Broadly speaking, basic research can be generated by individual researchers, independent research institutions such as universities, or industrial research laboratories. It has been noted by Jewkes et al. (1969) in their case study of seventy significant inventions that more than one half of them could be attributed to individual inventors who had no capacity in commercializing their achievements.

1.3 Research Topics

Several directions for research are suggested by the observations above.

1.3.(i) *Uncertainty in Litigation*

While contributing important insights into issues surrounding patents, most of the optimal patent literature has so far confined itself to the “fencepost” interpretation of the patent system. For example, Hortsmann et al (1985) study the propensity to patent for a successful innovator (the sole winner of a patent race) by specifying a “limited but exact coverage” patent system. Within a perfect fencepost system, as a consequence, important policy implications regarding the litigation process cannot be addressed. In reality, however, determination of the proper patent breadth is often a matter of interpretation by the Patent Office and the courts. In an attempt to capture this aspect of reality, Waterson (1990) looks at uncertainty in patent infringement litigation and employs the concept of “limited but

inexact patent coverage” in a horizontal product differentiation model. A model of vertical product differentiation with uncertain litigation outcome will be developed in this dissertation to investigate the differences between the fencepost and signpost interpretation of patent law. In a major contrast to most of the optimal patent literature, it is shown that full rent extraction is possible.

1.3.(ii) *Inventing Around and Inventing Enough*

Given a well-specified damage rule, the reasonable royalty, it is shown in this project that “effective” patent protection exists only in a certain interval within the granted scope of patent protection. Both low and high ends of an improvement would not induce actual patent litigation. This is so because given the combinations of {high winning probability for the patentee but small improvement by the infringing firm} or {large improvement but low winning probability}, the patentee’s expected gain from litigation is smaller than his litigation cost. Thus kind of “inventing around” and “inventing enough” behavior is observed in our model. Gallini (1992) also develops a model in which imitation is costly, but perfect substitute of the patented product. She shows that longer patent life induces more imitating rivals. Therefore optimal patent length should be sufficiently short to discourage imitation. In our model with focus on patent breadth, the firm has incentive to invent around and invent enough the patented technology because of lack of effective patent protection.

1.3.(iii) Scrutiny of Policy Instruments

The Patent Office and the courts have quite a few instruments at their disposal. Careful

examination of the effects when manipulating these instruments is an important task taken on in this project. Comparative statics will be extensively studied to understand the effectiveness of the patent protection and the impact upon the incentive to innovate. Let me emphasize that issues like interpretation of infringement, the use of doctrines of equivalents and reverse doctrine of equivalents, the novelty and disclosure requirements and anti-trust concerns related to collusive licensing agreements and research joint ventures (RJV) all play important roles in shaping the optimal patent system.

1.4 Directions for Future Research

Following is a tentative list of potential research directions unearthed during the studies pursued in this dissertation. Some of them are direct (or promising) extensions of the current modeling approach, while others add new elements required to improve further the realism of this line of research.

1.4.(i) *Optimal Complexity of the Patent System*

It has been proposed by Waterson (1990) that in industries where variety is highly valued copyright will be superior to patent. Klemperer (1990) also provides conditions under which either infinitely-live but narrowly focused or short-lived but broad patents would be socially efficient, respectively. But none of these articles evaluates the tradeoff between efficiency achieved by a more complex (and maybe differentiated) patent system and the costs associated with the functioning of the supporting rules. Theoretically, Kaplow (1995) has developed a model to analyze the tension between complexity of the law and the cost of

executing the legal rules.

1.4.(ii) *Strategic Role of the Patent Office*

La Manna (1992) has looked at the possible strategic roles played by the Patent Office. He shows that, depending on the time it takes for the Patent Office (a welfare-maximizer) to react, the optimal manipulations of patent life and minimum patentability standards would differ. More recently, concerned with the admissibility of some evidence during the settlement phase, Daughety and Reinganum (1995) investigate a two-receiver model in which the plaintiff is the sender and the defendant and the judge are the receivers. One of the major conclusions is that admissibility rules have distributional consequences: the plaintiff will prefer inadmissibility and the defendant will prefer admissibility. In the context of this dissertation, thorough comparative statics with respect to the instruments at the courts' disposal, e.g., the doctrine of equivalents and the reverse doctrine of equivalents, will also enhance further understanding on the possible roles of the patent system.

1.4.(iii) *Long Sequence of Innovations*

Recently there have been significant attempts to study a more general form of sequential innovation: a long sequence of cumulative improvements. O'Donoghue, Scotchmer, and Thisse (1995) are concerned with the tradeoff between granting leading and/or lagging patent breadth and granting finite or long patent life. O'Donoghue (1995) proposes alternatively structured patent protection so that firms are motivated to pursue "larger" innovations. The issue of uncertainty is not yet explored in this line of research.

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CHAPTER 2

THE DIVISION OF PROFIT IN SEQUENTIAL INNOVATION RECONSIDERED¹

I. Introduction

The design of an optimal patent system has re-emerged recently as the subject of economic inquiry. An intriguing fact is that the patent law does not and, perhaps, cannot circumscribe its objects, individual patents, in a precise and unquestionable way. For instance, the most important statutory criteria for patentability are “novelty” and “nonobviousness”. Novelty can be interpreted as the criterion to determine whether the new invention was not in “prior art”, i.e., whether the inventor has really invented something. The nonobviousness criterion excludes patentability of inventions for which it is “obvious” that they could be invented with sufficient effort, even though no one has bothered to do so so far. Infringement of patent can be generally defined as the nonsanctioned manufacture, use, making or sale of an invention for which a valid patent has been issued. Typically, infringement constitutes a situation where a new invention significantly overlaps with the patented technology. Significant overlap in turn is determined again in terms of novelty and nonobviousness — which are subject to qualification and interpretation. Therefore, legal determination of infringement can be a difficult task. It is not too far-fetched to imagine a complicated infringement case where the legal institutions are incapable of sound judgment.²

¹The research reported here is coauthored with Hans Haller. The authors thank Nancy Lutz and Frank Verboven for helpful comments.

²Dreyfuss (1989): “In general, the court (Court of Appeals for the Federal Circuit, the specialized court established in 1982 to focus on patent jurisdiction) has been successful with issues like obviousness... issues that arise mainly in enforcement proceedings have not been nearly as well explicated.... (the court) has yet to announce clear tests for many of the issues involved in the infringement question.” For instance, when asked to determine whether certain miniaturized calculators infringed Texas Instrument's pioneering calculator patent, the CAFC contradicted its statements by first recognizing significance of “pioneer status” of the patent but later rejecting the application of the doctrine of equivalents which favors the patentee in *Texas Instruments, Inc. v. United States International Trade Commission* (Federal Circuit, 1986). For a discussion of the case in detail, see also Merges and Nelson (1990).

One typical aspect of R&D is that a commercially profitable innovation results from basic research. Broadly speaking, basic research can be generated by individual researchers, independent research institutions such as universities, or industrial research laboratories. A rather unanticipated fact that has been noted by Jewkes et al. (1969) in their case study of seventy significant inventions is that more than one half of them could be attributed to individual inventors who had no capacity in commercializing their achievements. Thus the distribution of profit between basic researchers without directly marketable products and the vendors of marketable products derived from the basic technology should be of utmost importance and interest to economists. To illustrate, Robert W. Kearns, a former engineering professor who patented his intermittent wiper system in 1967, was awarded back royalties of \$10.2 million in a settlement with Ford Motor Company in 1990 and \$11.3 million by Federal Court in his patent infringing case against Chrysler Corporation in 1992. He has also sued the General Motors Corporation, the Toyota Motor Corporation, Fiat S.p.A and most large Japanese car manufacturers.³ These impressive episodes should not divert our attention from the fact that, as a rule, technological advancement nowadays demands more than just an ingeniously novel idea; that availability of sophisticated laboratory equipment and a substantial capital investment may also be essential. Nevertheless, companies and research institutions have realized by now that it can be profitable to sue for patent infringement for products they hold the patents to, but which they have never produced, never intended to produce, sometimes even considered non-producible or non-marketable. In a recent case involving commercial companies, Procter & Gamble sued Whitehall Laboratories and its parent, American Home Products Corporation over the cold remedy with ibuprofen although P.&G. has never had a product of this kind. Adopting similar strategies, Honeywell Inc. won a big patent case in 1992 against Minolta Camera Company of Japan over the auto-focus

³See *New York Times*, June 12, 1992.

camera lens, a technology Honeywell never itself developed commercially.⁴ Iowa State University, in yet another instance where independent research institutions try to claim the intellectual property right, was able to collect licensing fees of up to \$18 million from Sharp Corp. of Japan, NEC Corp., and Canon Inc. in 1992 on its 1973 patent covering an encoding process in the fax machine.⁵ Most research universities do have procedures and personnel to file patent applications and deal with licensing agreements as well as infringement suits.

Plausible theoretical models of cumulative innovation are still rare — despite the economic significance of this sort of innovation. A notable exception are Green and Scotchmer (1995) who address the issue of optimal patent breadth and duration, and the role of different legal mechanisms when innovation takes place in two stages. In their model, quality improvement is the only indicator for patent protection and infringement. Green and Scotchmer raise the concern that the potential patent holder may lack incentives to invest in the first place, because not all the social value facilitated by basic research can be transferred from the second generation products. In particular they point out the possibility that competition with derived products undermines the profitability of the initial product. Indeed, they show that, as a rule, not all the profit can be transferred to the first innovator. They suggest that under these circumstances there might be a rationale for longer lasting patents.

The present paper also deals with the division of labor and the division of profit in sequential innovation. Like in the model of Green and Scotchmer (1995), quality improvement is the only indicator for patent protection and infringement.⁶ Our paper focuses on the distribution of profit between basic researchers without directly marketable products and the vendors of marketable products derived from the basic technology. However, the most

⁴See Edmund L. Andrews, *New York Times*, Nov. 9, 1992 for these and other examples.

⁵See *Wall Street Journal*, Oct. 12, 1992.

distinctive and crucial features of our model are 1. asymmetric bargaining power and 2. uncertainty in litigation.

1. Asymmetric Bargaining Power

Green and Scotchmer (1995) assume that any agreement between the two parties means a fifty-fifty split of the available surplus among them. In contrast, we give virtually all bargaining power to the patent holder by allowing him to move first and make a take-it-or-leave-it licensing offer.

2. Uncertainty in Litigation

Moreover, we are concerned with the division of profit due to imperfect patent protection. By imperfect patent protection we mean here that the outcome of infringement litigation is uncertain. Indeed, both parties may agree privately whether or not an infringement occurs. Yet the court may come to a different conclusion. It is assumed that the court errs in favor of the subsequent innovator.

The two distinct modeling features work in opposite directions. Asymmetric bargaining power biases the outcome in favor of the initial patent holder. Uncertainty in litigation acts like a countervailing force that might prevent the patent holder from extracting all the surplus. Equally important is the quite realistic assumption that litigation is not free for either party. Litigation is not a credible threat available to the patent holder, if his own litigation costs exceed his expected profit transfer. This is the case, if the marketed product is not very

⁶In general, however, this need not be the case. Development can occur in one or more of the many dimensions of product characteristics. With heterogeneous consumers, it may be impossible to single out an unambiguous direction of quality improvement. For instance, motivated by Klemperer's (1990) product variety model, Lerner (1994) develops a proxy for patent scope based on the International Patent Classification scheme which is more in parallel to the concept of product differentiation.

profitable to start with. In our model this means that the quality of the marketed product is low. Another case where the expected profit transfer through litigation falls short of litigation costs arises when the patent holder's probability of winning is too small. In our model this occurs when the quality of the marketed product is high. *Ceteris paribus*, then, complete profit transfer can only be expected for intermediate size subsequent innovations. Despite the *prima facie* enormous bargaining advantage granted to him, the patent holder cannot always force a complete profit transfer.

Litigation can be used as a credible threat by the patent holder, if his own litigation costs do not exceed his expected transfer. In that case, there exists a subgame perfect equilibrium with complete profit transfer. In fact we can show that, *ceteris paribus*, the quality parameters of intermediate size innovations for which an equilibrium with complete profit transfer obtains, form an interval.

A major advantage of our modeling approach is that it allows for interesting comparative statics. We determine how the size or relative size of the interval where complete profit transfer obtains is affected by variations in litigation costs or patent breadth, respectively. First of all, we find that higher litigation costs for both parties may not unambiguously benefit the patent holder. Indeed, in a certain parameter range, higher litigation costs are detrimental to the patent holder. One salient feature (for economists) unearthed in the course of this investigation is the fact that the defendant's motion to postpone a patent infringement suit may have strategic reasons. The long lasting Intel-AMD suit over microprocessors and the recent Kodak-Sony patent dispute constitute typical examples.⁷

⁷For details, see "AMD wins 287 trial delay until 1994", *Electronic News*, Nov. 22, 1993 and "Sony filed a motion to postpone infringement suit", *Wall Street Journal*, March 23, 1994.

As for the optimal patent breadth, Green and Scotchmer present a special case in which unlimited patent protection may not be optimal when the uncertainty on the exact development is not resolved. Their key argument is based on two potential bargaining stages, *ex ante* and *ex post*. The parameters of development costs and realization of improvement are such that (1) the subsequent innovator would never enter the project without an *ex ante* agreement, if the patent breadth were infinite and an infringer had always to succumb to *ex post* licensing; (2) there is a finite patent breadth so that the subsequent innovator would break even when entering the project without an *ex ante* agreement. The second innovator's threat point in *ex ante* bargaining would be zero under (1) and (2). The first innovator's threat point, under further specific conditions, can be assumed to be zero under (1), but strictly positive under (2) where he could collect an *ex post* licensing fee with positive probability. Hence replacing infinite patent breadth by a particular finite patent breadth can benefit the patent holder in an *ex ante* agreement. In a quite different and conceivably more general context, we arrive at a further refutation of the argument that a broader patent unconditionally makes the patent holder better off. Unlike theirs, our conclusion is not derived from uncertainty in development, but from uncertainty in litigation.

To put our contribution into an even broader context, we distinguish between two strands of literature related to patent protection: the "fencepost" system literature and the "signpost" system literature. Adopting the fencepost interpretation of patent scope, Hortsmann et al. (1985) look at patents as information transfer mechanism and assume "limited but exact patent coverage". Within the second strand of literature, Waterson (1990) looks at uncertainty in patent infringement litigation from a different angle and develops a model where like in ours the concept of "limited but inexact patent coverage" is employed. Whereas we explicitly require concavity of the patent holder's winning probability, Waterson implicitly

imposes an equivalent property on the “court cost function” defining litigation costs and damage fees awarded to the patent holder. While Waterson is primarily concerned with the impact of patent protection on product variety — and the implied consumer welfare — in a horizontal product differentiation model, our emphasis lies on appropriability and incentives to innovate in a vertical product differentiation model with sequential innovation.

The paper is organized as follows. In Section 2, we specify a model to investigate the division of profit between an initial patent holder with no marketing power and a subsequent innovator of a derived product. For simplicity we assume that after the first innovation is made, the idea for each derivative improvement occurs to only one firm which is uniquely capable of developing it at a given cost. Section 3 is devoted to comparative statics. Several concluding remarks are made in Section 4. The more technical or elaborate proofs are collected in an Appendix.

II. The Model

There are one research institution and one firm. The research institution is called the patent holder (PH) hereafter. It has acquired a patent on its invention with quality x . We set $x = 0$ without loss of generality. The patent breadth granted is y^* with the definition that if the firm develops a product of $x+y$ with $y \in [0, y^*)$ then this product is deemed to infringe the patent x . Quality x is just a basic research outcome and has no market value *per se*. The firm is capable of developing a new product of quality y with $x \leq y \leq y^*$ so that it would surely infringe on the patent held by PH. The cost of developing quality y is c_y . Once developed, the new product can be produced at zero cost and has market value π_y .

The crucial elements of patent litigation can be described as follows: Each party incurs

the same litigation cost $L > 0$. There is an objective probability $f(y)$ of PH winning in litigation. The existence of such an $f(y)$ can be defended on the grounds that there is no perfect patent protection due to the nature of current patent law and the process of infringement litigation. While both parties may agree privately whether or not an infringement occurs, the court may come to a different conclusion. Occasionally, we treat y as variable and $f(y)$ as a decreasing function of $y \in [0, y^*]$ with $f(0) = 1$ and $f(y^*) = 0$. The further away from x a new invention is, the less likely is a verdict of infringement.

We model the strategic interaction as a strategic game between PH and the firm. The game lasts one period which is defined as the time interval beginning when PH makes the licensing offer and ending when the infringement issue is resolved. The two players take several steps during the period. There is no discounting within the period.

Both players enter the game with exogenously given and commonly known y . PH, as a first mover, makes a licensing agreement offer simply by specifying R with $R \in [0, \infty)$. We view R as a fixed-fee royalty: The number R represents the amount to be paid by the firm for the right to market its product. By offering $R=0$, PH tolerates the infringement without legal recourse. Facing the offer, the firm has three strategic alternatives: (i) quit the project; (ii) pay the royalty proposed by PH; (iii) challenge the patent infringement allegation. In the latter contingency, PH has to make one more move: take no action or litigate. In accordance with U.S. practice, we assume that even if it loses, the firm retains the profit from marketing this application while paying its litigation costs plus back royalties. Figure 1 summarizes the extensive form of the game, showing the order of decisions and the resulting (expected) payoffs.

[Figure 2.1 about here]

Set $M = \{\text{No-action, Litigation}\}$ and $N = \{\text{Take-it, Leave-it, Drop-out}\}$. Then the normal

form of the game has strategy spaces $S_{PH} = \mathbf{R} \times \mathbf{M}^{\mathbf{R}}$ for PH and $S_F = \mathbf{N}^{\mathbf{R}}$ for the firm. We consider strategy pairs that are Nash equilibria, i.e. each player chooses a strategy that maximizes its expected payoff given the other player's strategy. Moreover, we require subgame perfection: Equilibrium pairs of strategies induce equilibrium play in all subgames.

We distinguish four types of pure strategy equilibria. Which types occur, depends on the numerical specification of the model.

1. The Take-it equilibrium is characterized by an R_t with

$$\pi_y - c_y - R_t \geq 0,$$

$$\pi_y - c_y - R_t \geq \pi_y - c_y - f(y)R_t - L, \text{ and}$$

$$f(y)R_t - L \geq 0.$$

The firm responds with Take-it to this offer. Should the firm play Leave-it in response to this offer,

then PH would counter with Litigation.

2. The Leave-it equilibrium is characterized by an offer R_l with

$$\pi_y - c_y - f(y)R_l - L \geq 0,$$

$$\pi_y - c_y - f(y)R_l - L \geq \pi_y - c_y - R_l, \text{ and}$$

$$f(y)R_l - L \geq 0.$$

The firm responds with Leave-it to this offer and PH counters with Litigation.

3. The No-Action equilibrium is characterized by an offer R_n with

$$f(y)R_n - L \leq 0, \text{ and}$$

$$\pi_y - c_y \geq 0.$$

The firm responds with Leave-it to this offer and PH counters with No-action.

4. The Drop-out equilibrium is characterized by an offer R_d with

$$\pi_y - c_y - R_d \leq 0,$$

$$f(y)R_d - L \geq 0, \text{ and}$$

$$\pi_y - c_y - f(y)R_d - L \leq 0.$$

The firm responds with Drop-out to this offer. Should the firm respond with Leave-it to this offer,

then PH would counter with Litigation.

PH, as a leader in this game, has the sole interest in manipulating the offer R so as to collect the highest possible profit share from the firm. Therefore we will not pursue any further the No-Action and Drop-out equilibria where PH cannot generate any positive gain.⁸

We proceed with the following simplifying assumption:

(A1) $\pi_y = a \cdot y$, $c_y = c \cdot y$ where a and c are constants satisfying $a > c \geq 0$.

A constant marginal revenue occurs in a standard vertical (quality) differentiation problem. There consumers have utility functions of the form $U = \Theta y - P$ where Θ is a taste parameter and P is the price charged for the product of quality y . The distribution of tastes across consumers is given by the uniform distribution on the interval $[\theta', \theta]$ with $1 \geq \theta' \geq 0$ and $\theta = \theta' + 1$. Then, given y , the firm maximizes its gross profit by choosing the price level $P_y = \frac{y\theta}{2}$. The resulting gross profit is $\pi_y = \frac{y\theta^2}{4}$. Put $a = \theta^2/4$.

We first explore the possibility that PH can extract all the profit from the firm, i.e., where an offer $R_t = (a-c)y$ gets accepted in equilibrium. Necessary and sufficient conditions for such

⁸In a more complicated many-firm setting, however, these potential types of equilibria may significantly impact upon PH's decision-making.

an equilibrium outcome are described in Proposition 1. The intuition behind Proposition 1 can be directly developed by carefully observing the game tree. Notice that PH can never extract more than $(a - c)y$, since the firm is given the option to drop out. In addition to that, PH's ability to achieve full rent extraction relies crucially on (a) how large a licensing fee would make the firm indifferent between acceptance and litigation and (b) how credible is his threat to litigate in case the infringing firm were to turn down the offer. Conditions (a) and (b) derived from the proof of Proposition 1 precisely correspond to that reasoning.

Proposition 1. PH can extract all the profit it facilitates from the firm if and only if

$$(a) \quad y - yf(y) \leq \frac{L}{a - c} \text{ and}$$

$$(b) \quad yf(y) \geq \frac{L}{a - c}.$$

Proof: For a licensing agreement to prevail, i.e., a Take-it equilibrium to exist, the following conditions are necessary and sufficient:

$$\pi_y - c_y - R_t \geq 0,$$

$$\pi_y - c_y - R_t \geq \pi_y - c_y - f(y)R_t - L, \text{ and}$$

$$f(y)R_t - L \geq 0.$$

With the previous specification, they are equivalent to:

$$R_t \leq (a - c)y, \tag{1}$$

$$R_t \leq \frac{L}{1 - f(y)}, \text{ and} \tag{2}$$

$$R_t \geq \frac{L}{f(y)}. \tag{3}$$

Now assume (1) with equality. Then (2) and (3) are equivalent to

$$y[1 - f(y)] \leq \frac{L}{a - c} \text{ and} \tag{4}$$

$$yf(y) \geq \frac{L}{a - c}. \tag{5}$$

□□

Some simple comparative statics can help develop further intuition for this result. First of all, suppose a , c , y , and L are given such that $y \geq \frac{L}{a-c}$ and $f(y)$ is treated as a variable. Then (4) and (5) are simultaneously satisfied if and only if $f(y)$ is sufficiently large. This can be seen from the game tree in Figure 1. To achieve a Take-it equilibrium with $R=(a-c)y$, PH's threat of litigation in case the firm rejects the offer has to be credible, that is, $f(y) \cdot R - L \geq 0$. If $y \geq \frac{L}{a-c}$ or, equivalently, $(a-c)y \geq L$ and if $R=(a-c)y$, then the inequality $f(y)R - L \geq 0$ holds trivially for $f(y)=1$. By continuity, a high and only a high winning probability $f(y)$ helps PH achieve the licensing agreement where the firm obtains zero payoff. More specifically, we observe that $(a) \wedge (b)$ implies $f(y) \geq \frac{1}{2}$.

On the other hand, when a , c , y , and L are given with $y < \frac{L}{a-c}$, the situation changes drastically. Whereas (a) is always satisfied under this assumption, (b) breaks down for all $f(y) \in [0,1]$. Thus there does no longer exist the Take-it equilibrium PH is longing for. It is obvious from the game tree that PH is seriously concerned about the potential loss from litigation and therefore takes no action even if the firm dares to infringe.⁹ Under this particular specification, a smaller y does not benefit PH in achieving its goal of exploiting the firm — even with a high winning probability. To sum up, the preceding comments suggest there might exist lower and upper bounds for those y which permit the type of Take-it equilibrium in question.

To extend the analysis one step further and arrive at such bounds, we next consider a situation with exogenously given L , a , c , y^* , and a function $f: [0, y^*] \mapsto [0, 1]$ satisfying:

(A2) $f(0) = 1$, $f(y^*) = 0$, and f is twice differentiable with $f' < 0$, $f'' \leq 0$.

It is natural to assume f to be decreasing. It also appears plausible that PH's winning

⁹This could explain why some PHs never bother to file suit against the manufacturers of low-tech clones, targeting instead those subsequent prominent manufacturers whose products are sufficiently novel and making significant profit.

probability drops faster as y moves further away from $x=0$, or the more y approaches the delimiter of patent protection, y^* . From a different perspective, *ceteris paribus*, the firm is enjoying “increasing returns to litigation” as y varies.

Proposition 2. Suppose there exists a $y^* \in [0, y^*]$ with

$$(c) \quad y^* f(y^*) \geq \frac{L}{a-c}.$$

Then there exist y_l, y_r with the following properties:

- (i) $0 < y_l \leq y_r < y^*$;
- (ii) PH can extract all the profit it facilitates from the firm iff $y \in [y_l, y_r]$.

Having arrived at these conclusions, one still has to be very cautious in interpreting Propositions 1 and 2. First of all, the Take-it equilibrium described in the propositions is of a particular kind where an offer $R_t = (a - c)y$ gets accepted in equilibrium. When condition (c) holds as a strict inequality and y lies in the interval (y_l, y_r) , there is always sufficient slack for a Take-it equilibrium with $R_t < (a - c)y$.¹⁰ In other words, not all Take-it equilibria guarantee 100 per cent profit transfer. Secondly, one should wonder if only a Take-it equilibrium can guarantee complete profit transfer to PH. To clarify that matter, we present the following proposition.

Proposition 3. A Leave-it equilibrium always generates a profit less than $(a-c)y$ for PH.

¹⁰Namely, in the proof of Proposition 2, the bounds y_l and y_r are constructed such that $y \in (y_l, y_r)$ and $y f(y) > \frac{L}{a-c}$ imply $y f(y) > \frac{L}{a-c}$ or $(a - c)y > \frac{L}{f(y)}$ which in turn implies that $[\frac{L}{f(y)}, (a - c)y]$ is neither the empty set nor a singleton. Therefore there exists $R_t < (a - c)y$, say, $R_t = (a - c)y - \epsilon$ where ϵ is a small positive number, such that conditions (1), (2) and (3) are simultaneously satisfied.

Proof: A Leave-it equilibrium is subject to the following qualifications:

$$\pi_y - c_y - f(y)R_l - L \geq 0,$$

$$\pi_y - c_y - f(y)R_l - L \geq \pi_y - c_y - R_l, \text{ and}$$

$$f(y)R_l - L \geq 0.$$

With the previous specification, these are equivalent to:

$$(a - c)y - f(y)R_l - L \geq 0, \tag{6}$$

$$[1 - f(y)]R_l - L \geq 0, \text{ and} \tag{7}$$

$$f(y)R_l - L \geq 0. \tag{8}$$

The corresponding expected payoff for PH is at most $f(y)R_l - L$. From (6) we can infer the following inequality:

$$f(y)R_l - L < f(y)R_l + L \leq (a - c)y. \quad \square\square$$

By (A1) and Proposition 3, $(a - c)y$ is guaranteed to be greater than the payoffs that can be obtained via a Drop-out, a No-Action or a Leave-it equilibrium. Moreover, under (c) PH can obtain $(a - c)y$ as payoff in a Take-it equilibrium so that, indeed, $(a - c)y$ constitutes PH's best equilibrium payoff. These observations justifies our almost exclusive focus on the properties of the complete-profit-transfer Take-it equilibrium.

It is obvious that our conclusions about the distribution of profit rely crucially on condition (c). The mathematical interpretation of (c) is straightforward:

$$\text{Max}_{y \in [0, y^*]} yf(y) \geq \frac{L}{a - c}.$$

Note that the function $yf(y)$ is strictly concave and its maximizer is contained in the interval $[y_l, y_r]$. Hence this condition can be reformulated as:

$$\text{Max}_{y \in [y_l, y_r]} yf(y) \geq \frac{L}{a - c}.$$

Nonetheless, the economic meaning of (c) is more subtle than the mathematical

representation. Aiming at complete exploitation of the firm, PH has to balance two factors moving in opposite directions: what the firm is capable of, i.e., the magnitude of y , and how significant the chance is that he can win the infringement case, i.e., the range of $f(y)$. More specifically, while making a higher offer to cope with a higher y , PH strives to maintain a credible threat to avoid the opportunism of the firm as $f(y)$ declines. Under condition (c), PH masters this balancing act and extracts all profit.

So far we have shown in the previous propositions that PH is capable of achieving a Take-it equilibrium by offering a licensing agreement that transfers the entire profit $(a - c)y$ from the firm when condition (c) is satisfied. This outcome would provide PH the maximal incentive to invent under the specification of our model. Meanwhile, PH's payoff from this type of agreement is always greater than that from a Leave-it equilibrium — even if the latter is feasible.

III. Comparative Statics

In this section we focus on the comparative statics with respect to several key variables — within their most interesting range in our set-up. In the sequel we denote $\mathcal{L} \equiv y_r - y_l$, the length of the interval of y where a Take-it equilibrium with all the profit being transferred can be obtained. We first investigate how \mathcal{L} is responding to variations of the litigation cost L , the gross profit parameter a , and the product development cost parameter c . It suffices to see how \mathcal{L} depends on the compound parameter $k \equiv \frac{L}{a - c}$. For the ease of exposition, we denote \bar{y} as the median of the probability distribution function $f(y)$, i.e., $f(\bar{y}) = \frac{1}{2}$ and \hat{y} as the unique maximizer of the strictly concave function $yf(y)$, i.e., $f(\hat{y}) + \hat{y} \cdot f'(\hat{y}) = 0$.

Lemma 2. Suppose $\frac{\bar{y}}{2} \neq \hat{y}f(\hat{y})$, then $\frac{\partial \mathcal{L}}{\partial k} < 0$ for $k \in \left(\frac{\bar{y}}{2}, \hat{y}f(\hat{y})\right)$. Moreover, the corresponding intervals

$[y_l(k), y_r(k)]$ are strictly nested.

Intuitively, a higher litigation cost should have a stronger threatening effect on the firm who takes a chance when turning down the offer, since its net expected gain from litigating is eroded by the extra legal expenses. However, Lemma 2 says that even when the litigation cost is in the “favorable” range where a complete-profit-transfer via a Take-it equilibrium can be assured, higher litigation cost will damage the manipulative power of PH.¹¹ What we observe is that when k rises above $\frac{\bar{y}}{2}$ while staying below the bound $\hat{y}f(\hat{y})$, $yf(y)$ starts to effectively determine the boundaries of \mathcal{L} . Therefore the sensitivity of $\mathcal{L}(k)$ with respect to k depends only on the strict concavity of $yf(y)$. Recall that $g_2(y) \geq k$ is essentially the same as condition (3), $R \geq \frac{L}{f(y)}$. It is then obvious that a higher L makes it harder to satisfy (3). A similar interpretation can be applied to the parameters a and c .

Lemma 3. $\mathcal{L}(k)$ is strictly concave in $k \in [0, y^*]$ and there exists a unique $\tilde{k} \in (0, \frac{\bar{y}}{2})$ such that $\mathcal{L}(\tilde{k}) \geq \mathcal{L}(k)$ for all $k \in \mathbb{R}_+$.

Lemmata 2 and 3 convey a complete picture of the comparative statics with respect to k . The properties of $\mathcal{L}(k)$ follow directly from the concavity of $f(y)$, the probability of PH winning in litigation. In other words, the phenomenon observed here, that is, shrinkage of the interval of complete profit transfer with respect to certain k , can be attributed to a particular aspect of imperfect patent protection, increasing returns to litigation for the firm.

¹¹ Although not specifically modeled here, a longer litigation process can also be viewed as increasing litigation costs. Therefore a motion to postpone a patent infringement suit may have strategic reasons.

In a second type of comparative statics, we investigate how \mathcal{L} , the length of the interval where PH can extract all the surplus, is affected by a change of patent protection. Intuition may suggest that the best way to help PH transfer profit from the firm is to grant PH a broad patent protection. Intriguingly enough, this is a premature conclusion as the next proposition shows. One more simplifying assumption, (A3) is imposed to establish the result. Prior to that we have to extend the model appropriately by postulating that f takes the more general form $f(y; y^*)$, $0 \leq y \leq y^*$, where the patent breadth $y^* > 0$ is treated as variable in the sequel. The obvious notation $\mathcal{L}(k; y^*)$, $\tilde{k}(y^*)$, etc. will be used.

(A3) $f(y; y^*) = f(\frac{y}{y^*}; 1)$, i.e., $f(y; y^*)$ is homogeneous of degree 0.

(A3) stipulates that the winning probability for PH depends only on the ratio y/y^* , not on the absolute magnitude of y or y^* . An extremely high y^* might correspond to a very vague claim such as "All non-human transgenic mammals" or "All hand-use calculators."¹² The broader the patent protection, the easier is it for an allegedly infringing firm to challenge the patent claim. In a model with merely one-dimensional quality choice, imposing (A3) constitutes a simple attempt to capture that aspect of reality. (A3) has several immediate consequences:

Lemma 4. The functions $y_l(k; y^*)$, $y_r(k; y^*)$, and $\mathcal{L}(k; y^*)$ are homogeneous of degree 1 in $(k; y^*)$. The functions $\hat{y}(y^*)$ and $\bar{y}(y^*)$ are homogeneous of degree 1 in y^* .

Let us first state a result that conforms to intuition: As patent protection becomes

¹²See U.S. Patent No. 4,736,866, issued Apr. 12, 1988. This patent is granted to Doctors Philip Leder and Timothy Stewart of the Harvard Medical School for their successful work on transgenic mice. For the hand-held calculators case, see *Texas Instruments Inc. v. United States International Trade Commission* (Federal Circuit 1986.)

broadener, $\mathcal{L}(k; y^*)$ increases, i.e. the size of the interval where PH can extract all the surplus increases.

Proposition 4. The following three assertions hold:

- (I) $\frac{\partial}{\partial k} \mathcal{L}(k; y^*)$ is strictly increasing in $y^* > 0$ as long as $0 < k < \bar{y}(y^*)/2$.
- (II) $\mathcal{L}(k; y^*)$ is strictly increasing in $y^* > 0$ as long as $0 < k < \bar{y}(y^*)/2$.
- (III) $\tilde{k}(y^*)$ is strictly increasing in $y^* > 0$.

Let us now proceed to the promised, somewhat less intuitive result: As patent protection becomes broader, the relative size of the interval where PH can extract all the surplus may decrease.

Proposition 5. For any $0 < y^* < y^{**}$, there exists $\kappa(y^*, y^{**}) > 0$ such that

$$\frac{\mathcal{L}(k; y^{**})}{y^{**}} < \frac{\mathcal{L}(k; y^*)}{y^*} \quad \text{for all } 0 < k < \kappa(y^*, y^{**}).$$

Proposition 5 states that even though $\mathcal{L}(k, y^*)$ increases as patent protection becomes broader, \mathcal{L}/y^* , that is the relative size of the interval where PH can extract all the surplus, may be falling for certain k . The manipulative power of PH measured as the fraction of infringing y that provide maximal incentive to PH to innovate, can apparently diminish when the government institutes broader protection. Let us briefly explain how monotonicity, concavity and homogeneity of f can lead to such a conclusion. Homogeneity of degree zero of $f(y; y^*)$ yields homogeneity of degree one of $\mathcal{L}(k; y^*)$. The impact of higher y^* on the ratio \mathcal{L}/y^* is then immediate, since $\mathcal{L}(k; y^*)/y^*$ can be reduced to $\mathcal{L}(k/y^*; 1)$. Moreover, $f(y; y^*)$ strictly decreasing and strictly concave in y implies that $\mathcal{L}(k; y^*)$ is strictly increasing and

strictly concave in $k \leq \tilde{k}$. Thus for fixed k , the normalized $\mathcal{L}(\frac{k}{y^*}; 1)$ is greater than the normalized $\mathcal{L}(\frac{k}{y^{**}}; 1)$.

IV. Conclusions and Qualifications

In this article, we investigate the division of profit between a patent holder and a derived product producer in an environment with uncertainty about the outcome of infringement litigation. The two most distinctive features of our model, asymmetric bargaining power on the one hand and uncertainty in litigation on the other hand, work in opposite directions. Despite a *prima facie* bargaining advantage, the patent holder does not always achieve a complete profit transfer in equilibrium. Our analysis identifies the conditions on model parameters that permit a complete-profit-transfer equilibrium. Comparative statics with respect to several key parameters is performed.

On Asymmetric Bargaining Power. A situation where substantial bargaining power is enjoyed by the patent holder is worth exploring, since it reflects certain aspects of contemporary patent law practice. Historically, the situation has been somewhat different. The U.S. Supreme court was once labeled as antitrust and anti-patent, notably during the period from the 1930s through the 1960s. Significant change came after the establishment of the Court of Appeals for the Federal Circuit. The era of the CAFC has witnessed a dramatic increase in patent disputes. Some writers attribute this trend to the pro-patent attitude adopted by the court. More patents are upheld today than ever before. Some evidence suggests that, indeed, the CAFC is favorably inclined towards patentees. One example is presented by Schmitt (1986) who cited the Kodak-Polaroid dispute as the “most prominent example of an

increasingly pro-patent sentiment in American courts” when the district judge refused to stay an injunction against Kodak while it appealed. In terms of procedural and doctrinal changes, the new attitude manifests itself through the more frequent exercise of preliminary injunctions. This powerful instrument has the greatest impact when an alleged infringer uses the patented invention as an indispensable part in a larger production process. The latter was clearly the case when Gilbert Hyatt, a lone inventor won acceptance of a patent for the design of microchips which now are standard parts of most desktop computers. This lucrative opportunity attracted the attention of North American Philips Corporation who subsequently bought out the patent with the intention to collect licensing fees from major chip makers such as Intel, Texas Instruments, and Motorola. Another recent case pointed out by Merges (1995) is Ford Motor Company v. Lemelson in which the Lemelson bar coding patent was used to delay issuance of other patents of importance to the manufacturer. Since shutting down a manufacturer’s entire operation by means of an injunction is a credible threat nowadays, it appears not too unrealistic to assume that the patent holder makes the first move with a take-it-or-leave-it licensing offer. Though we should add that whereas the use of injunctions as a strategic tool constitutes an important fact deserving further analytic scrutiny, it is not among the strategic instruments available to the patent holder in our model.

On Imperfect Patent Protection. In the introduction, we have pointed out the important differences between the “signpost” and the “fencepost” interpretation of the patent system. In view of our theoretical analysis of a particular “signpost” system of patents, proposals to increase patent length in order to enhance profit transfer and, consequently, the incentives for basic research, do not sound too convincing. As for the optimal patent breadth, Green and Scotchmer (1995) present a special case in which unlimited patent protection may not be optimal when the uncertainty on the exact development y is not resolved. In a quite different

context, we arrive at another refutation of the argument that broader patent breadth unconditionally makes PH better off. However, unlike theirs our conclusion is not derived from uncertainty in development, but rests on a homogeneity assumption which renders the relative improvement measure y/y^* a main determinant of transferability of profit from the firm to the patent holder.

We believe that introducing uncertainty of the outcome of a patent infringement suit enriches and furthers the economic understanding of current patent systems. It opens a multifacet, widely unexplored research area of law and economics. Several elements might be added to our model: for instance, endogenous choice of y and y^* ; informational asymmetries discussed in the recent licensing literature (Gallini and Wright, 1990) and litigation literature (Bebchuk, 1984; Meurer, 1989; Reinganum and Wilde, 1986); different liability rules adopted by other countries; competition between PH and firm(s); etc.

A specific study along this line of research has been pursued by Aoki and Hu (1995). In their framework, a winner in a patent race is also a producer and is assumed to enjoy only imperfect patent protection, i.e., his winning probability in an infringement suit is $\theta < 1$. Sequential innovation is absent from their model. But there is a second firm that considers marketing the very same product. Again, the PH makes a take-it-or-leave-it licensing fee offer. If the second firm does not accept the offer, it can exit (drop out) or enter the market and infringe on the product. When entering the market without a licensing agreement, the second firm incurs an upfront "imitation cost". Upon infringement, PH may or may not take the entrant to court. If PH wins the litigation, the entrant is banned from the market. Aoki and Hu investigate under which parameter constellations licensing occurs and how the profit transferred to PH and overall welfare are affected by parameter changes. One of their main conclusions is that increasing both litigants' litigation costs favors licensing outcomes and, hence, leads to higher social welfare by avoiding the occurrence of patent monopoly, costly

imitation or costly litigation. We show in a different context that increasing litigation costs within a certain range will render licensing agreements with complete profit transfer less likely. This contrasting result is due to one of the most fundamental differences between the two papers as regards modelling of imperfect patent protection. In Aoki and Hu (1995), as a necessity, the winning probability θ for the patent holder cannot depend on the quality of a subsequent innovation. Since we aim at capturing some aspects of the sequential nature of innovation, we can treat the size of improvement y as an additional variable and make assumptions such as (A2) on how the winning probability $f(y)$ depends on y .

There is certainly an element of uncertainty that we intentionally ignore in the present formal analysis, concerning the allocation of litigation costs. As Dreyfuss (1989) points out, the Court of Appeals for the Federal Circuit — the specialized court established in 1982 to focus on patent jurisdiction — has failed to clarify the law on pecuniary damages.¹³ However, in our model we assume the smallest conceivable damages for PH: the licensing fee he is asking for. Given that PH can only do better under the prevalent practice, our qualitative results in favor of PH persist. Yet another qualification could be that outrageous licensing fee requests are corrected downward by the court or affect negatively f , the probability of winning litigation.

¹³The Patent Act permits the court to treble the damages, 35 U.S.C. § 284 (1982), and to award attorney's fees and cost in "exceptional cases" 35 U.S.C. § 285. These enhanced damages are typically awarded to penalize willful misbehavior. For example, Triarch Industries won treble damages against Trans Global Imports (*The Weekly Home Furnishings Newspaper*, Feb 21, 1994) and Exxon Corp. won award of \$18 million in attorney's fees in its infringement case against Lubrizol Corp (*Wall Street Journal*, Feb. 19, 1993.)

APPENDIX

Proof of Proposition 2: First, we perform comparative statics with respect to $y \in [0, y^*]$. For this purpose, we introduce the functions

$$g_1(y) = y[1 - f(y)]$$

and

$$g_2(y) = yf(y),$$

which appear in (4) and (5) and, obviously, play a critical role in our analysis. Notice that $g_1' = 1 - f - y \cdot f' > 0$ and $g_1'' = -2f' - y \cdot f'' > 0$ in the interval $(0, y^*)$. Hence g_1 is strictly increasing and strictly convex in y with $g_1(0) = 0$ and $g_1(y^*) = y^*$. Further notice that $g_2' = (y \cdot f)' = f + y \cdot f'$ and $g_2'' = (y \cdot f)'' = 2f' + y \cdot f'' < 0$. Thus g_2 is strictly concave in y with $g_2(0) = g_2(y^*) = 0$. Consequently, g_2 has a unique maximizer \hat{y} in $(0, y^*)$. This maximizer is given as the unique solution of the first order condition

$$g_2'(\hat{y}) = f(\hat{y}) + \hat{y} \cdot f'(\hat{y}) = 0.$$

Finally, let \bar{y} denote the ‘median’ of f , i.e. \bar{y} is implicitly given by the condition

$$f(\bar{y}) = \frac{1}{2}.$$

Claim: $\hat{y} \leq \bar{y}$ and $0 < y^*/2 < \bar{y} < y^*$.

To show this claim, recall that $g_2(y)$ is strictly concave in y with $g_2(0) = 0$ and $g_2(y^*) = 0$. By Takayama (1985) Theorem 1.C.3: f is concave on $(0, y^*)$ if and only if for any $x, y \in (0, y^*)$:

$$f'(y) \cdot (x - y) \geq f(x) - f(y).$$

Evaluate this inequality at $y = \bar{y}$ and let $x \rightarrow 0$. Then

$$-f'(\bar{y}) \cdot \bar{y} \geq 1 - \frac{1}{2} \text{ or } f'(\bar{y}) \cdot \bar{y} \leq -\frac{1}{2}.$$

Adding $f(\bar{y}) = \frac{1}{2}$ to the latter inequality yields

$$g_2'(\bar{y}) = \bar{y} \cdot f'(\bar{y}) + f(\bar{y}) \leq 0.$$

Strict concavity of g_2 and $g_2'(\hat{y}) = 0$ imply the assertion $\hat{y} \leq \bar{y}$. Further, (A2) has the immediate implication $0 < y^*/2 < \bar{y} < y^*$. This concludes the proof of the claim.

Next, recall that $g_1(y)$ is strictly increasing and strictly convex in y with $g_1(0) = 0$ and $g_1(y^*) = y^*$. Hypothesis (c) amounts to $g_2(\hat{y}) \geq \frac{L}{a-c}$.

Part (i):

$g_2(y)$ achieves its maximum at \hat{y} where $g_2'(\hat{y}) = f(\hat{y}) + \hat{y}f'(\hat{y}) = 0$. By the hypothesis, the continuity and other properties of g_2 , and the intermediate value theorem, there exist $z_l \in (0, \hat{y}]$ and $z_r \in [\hat{y}, y^*)$ such that $g_2(\hat{y}) \geq g_2(z_l) = g_2(z_r) = \frac{L}{a-c}$. If (c) holds with equality, then $z_l = z_r = \hat{y}$. If (c) holds with strict inequality, then $z_l < \hat{y} < z_r$.

Next note that (c) implies $y^* > g_2(\hat{y}) \geq \frac{L}{a-c} > 0$. Then, by the continuity and other properties of g_1 and the intermediate value theorem, there exists a unique $z \in (0, y^*)$ with $g_1(z) = \frac{L}{a-c}$. To compare the magnitudes of z_l and z , we consider two subcases.

Subcase (i-a): $\bar{y}/2 \geq \frac{L}{a-c}$. Now by definition, $f(\bar{y}) = \frac{1}{2}$ and therefore $\frac{L}{a-c} \leq \bar{y}/2 = g_1(\bar{y}) = g_2(\bar{y})$.

Therefore, $\bar{y} \in [z_l, z_r]$, by the strict concavity of g_2 . Also, $z \leq \bar{y}$, by the strict monotonicity of g_1 . Hence $z \leq z_r$. Moreover, $0 = g_1(0) = g_2(0)$, $\frac{1}{2} \cdot \bar{y} = g_1(\bar{y}) = g_2(\bar{y})$, strict convexity of g_1 and strict concavity of g_2 imply $g_1(y) < \frac{1}{2} \cdot y < g_2(y)$ for $y \in (0, \bar{y})$. If $\bar{y} = z_l$, then $z = \bar{y} = z_l$. If $\bar{y} > z_l$, then $g_1(z_l) < g_2(z_l) = \frac{L}{a-c}$. Thus $z > z_l$. In any case, therefore, $z \in [z_l, z_r]$. •••

Subcase (i-b): $\bar{y}/2 < \frac{L}{a-c}$. Then $\bar{y} \notin [z_l, z_r]$ and $z > \bar{y}$. By Lemma 1, $\bar{y} \geq \hat{y} \geq z_l$. Thus $z > z_l$. •••

Now set $y_l = z_l$ and $y_r = \min[z_r, z]$. Then (i) is satisfied.

Part (ii):

We commence with the sufficiency proof. When condition (c) holds and $y \in [y_l, y_r]$, then $y \in [z_l, z_r]$ and the strict concavity of g_2 implies (b) $yf(y) \geq \frac{L}{a-c}$. Further $y \in [y_l, y_r]$ implies $y \leq z$. Since $g_1(y)$ is an increasing function in $y \in [0, y^*]$, condition (a) $y[1 - f(y)] \leq \frac{L}{a-c}$ holds as well.

Now we turn to the necessity proof. (b) implies that $y \in [z_l, z_r]$. (a) implies that $y \leq z$. Together (a) and (b) imply $y \in [z_l, \min\{z_r, z\}] = [y_l, y_r]$. Note that we know from Proposition 1 that by offering $R_t = (a - c)y$, PH can extract all the profit it facilitates from the firm if and only if (a) and (b) both hold. We have shown that under the hypothesis (c),

the combination of (a) and (b) is equivalent to
 $y \in [y_l, y_r]$. This completes the proof. $\square\square$

Proof of Lemma 2: Suppose a, c, y , and $f(y)$ are given such that $k \in (\frac{\bar{y}}{2}, \hat{y}f(\hat{y}))$. This can be referred to subcase (i-b) in the proof of Proposition 2. Together with the supposition $\frac{\bar{y}}{2} \neq \hat{y}f(\hat{y})$ they imply that (c) holds as a strict inequality and thus $z_r > \hat{y} > z_l$. It is also known from (i-b) that $\bar{y} \notin [z_l, z_r]$ by the strict concavity of g_2 and $z > \bar{y}$ by the strict monotonicity of g_1 . By Lemma 1, $\bar{y} \geq \hat{y}$ and $\bar{y} \notin [z_l, z_r]$ imply $\bar{y} > z_r$. Thus $z > \bar{y} > z_r$. So $\mathcal{L} = y_r - y_l = \min[z_r, z] - z_l = z_r - z_l$. The strict concavity and the other properties of g_2 imply that for all k_1, k_2 such that $\frac{\bar{y}}{2} < k_1 < k_2 < \hat{y}f(\hat{y})$ the corresponding $z_r(k_1), z_l(k_1), z_r(k_2)$ and $z_l(k_2)$ have the following order:

$$z_l(k_1) < z_l(k_2) < \hat{y} < z_r(k_2) < z_r(k_1) \text{ or} \\
\mathcal{L}(k_1) = [z_r(k_1) - z_l(k_1)] > [z_r(k_2) - z_l(k_2)] = \mathcal{L}(k_2).$$

This implies the assertion. $\square\square$

We need a technical auxiliary result to proceed:

Lemma A. Suppose that $k = g(y)$ is strictly increasing, concave (convex) and twice continuously differentiable in the interval (a, b) and suppose that $g'(y) \neq 0$ for $y \in (a, b)$. Then $y = g^{-1}(k)$ exists and is monotone, convex (concave), and twice continuously differentiable with respect to k .

Proof: The existence, monotonicity, and twice continuous differentiability of g^{-1} are assured by the inverse function theorem; see Flett (1966; Th. 10.9.5). Moreover, we have

$$g^{-1'}(k) = \frac{1}{g'(g^{-1}(k))}.$$

Now, the only task left is to prove the concavity (convexity) conversion. Differentiation of and application of the chain rule to the foregoing formula for $g^{-1'}(k)$ yields

$$g^{-1'''}(k) = - \frac{g''(g^{-1}(k))}{[g'(g^{-1}(k))]^3},$$

which has sign opposite to that of $g''(g^{-1}(k))$. This implies convexity (concavity) of $g^{-1}(k)$.

$\square\square$

Proof of Lemma 3: We consider three cases where $\bar{k} = g_2(\hat{y}) = \frac{\bar{y}}{2}$.

Case(i): $k \in (\hat{y}f(\hat{y}), \infty)$. Then trivially $\mathcal{L}(k) = 0$, since condition (c) is violated, that is, there does not exist such interval $[y_l, y_r]$.

Case(ii): $k \in (\frac{\bar{y}}{2}, \hat{y}f(\hat{y}))$. Then, by Lemma 2, $\mathcal{L}(\bar{k}) \geq \mathcal{L}(k)$. (This is, however, a little more than what Lemma 2 states. When $k = \hat{y}f(\hat{y})$, $\mathcal{L}(k)$ is equal to zero since y_l and y_r coincide. So we include this boundary point in the statement.)

Case(iii): $k \in (0, \frac{\bar{y}}{2})$. Since both g_1 and g_2 are continuous, monotone, and twice differentiable, by the inverse function theorem, the following functions are well defined, unique, and twice differentiable:

$$h_1(k): [0, \frac{\bar{y}}{2}] \mapsto [0, \bar{y}] \text{ with } h_1(g_1(y)) = y \text{ for all } y \in [0, \bar{y}],$$

$$h_2(k): [0, \frac{\bar{y}}{2}] \mapsto [0, \hat{y}] \text{ with } h_2(g_2(y)) = y \text{ for all } y \in [0, \hat{y}].$$

Furthermore, by Lemma A, h_1 is monotone and strictly concave while h_2 is monotone and strictly convex. Therefore $\mathcal{L}(k) = h_1(k) - h_2(k)$ is strictly concave in k . Notice that h_1' is continuously decreasing from $h_1'(0) = \infty$ to $h_1'(\bar{k}) = \frac{1}{g_1'(\bar{y})}$ and h_2' is continuously increasing from $h_2'(0) = 1$ to $h_2'(\bar{k}) = \frac{1}{g_2'(\hat{y})}$. By Lemma 1 we already know $\bar{y} \geq \hat{y}$ which implies $g_2'(\bar{y}) = f(\bar{y}) + \bar{y}f'(\bar{y}) \leq 0$. Since $f(\bar{y}) = \frac{1}{2}$, $\bar{y}f'(\bar{y}) \leq -\frac{1}{2}$ or $-\bar{y}f'(\bar{y}) \geq \frac{1}{2}$. Then $g_1'(\bar{y}) = 1 - f(\bar{y}) - \bar{y}f'(\bar{y}) \geq 1 - \frac{1}{2} + \frac{1}{2} = 1$. Thus $\frac{1}{g_1'(\bar{y})} = h_1'(\bar{k}) \leq 1 < \frac{1}{g_2'(\hat{y})} = h_2'(\bar{k})$.

Set

$$H(k) = h_1'(k) - h_2'(k).$$

H is strictly decreasing and continuous with $H(0) > 0$ and $H(\bar{k}) < 0$. By the intermediate value theorem, there exists a unique $\tilde{k} \in (0, \frac{\bar{y}}{2})$ such that $H(\tilde{k}) = h_1'(\tilde{k}) - h_2'(\tilde{k}) = 0$, that is, $\mathcal{L}'(\tilde{k}) = 0$. By the strict concavity of $\mathcal{L}(k)$, such \tilde{k} will be the unique global maximizer in $k \in [0, \frac{\bar{y}}{2}]$.

Cases (i), (ii), and (iii) together imply $\mathcal{L}(\tilde{k}) \geq \mathcal{L}(k)$ for all $k \in \mathbf{R}_+$. This completes the proof. \square

Proof of Lemma 4: Consider $\lambda > 0$, $y^* > 0$ and $k \geq 0$. Then:

$$\eta \in [\lambda y_1(y^*; k), \lambda y_r(y^*; k)] \Leftrightarrow \eta = \lambda y \text{ and } y \in [y_1(y^*; k), y_r(y^*; k)]$$

$$\Leftrightarrow \eta = \lambda y \text{ and } y \cdot f(y; y^*) \geq k \text{ and } y \cdot (1 - f(y; y^*)) \leq k$$

$$\Leftrightarrow \eta = \lambda y \text{ and } y \cdot f(\lambda y; \lambda y^*) \geq k \text{ and } y \cdot (1 - f(\lambda y; \lambda y^*)) \leq k$$

$$\Leftrightarrow \eta = \lambda y \text{ and } \lambda y \cdot f(\lambda y; \lambda y^*) \geq \lambda k \text{ and } \lambda y \cdot (1 - f(\lambda y; \lambda y^*)) \leq \lambda k$$

$$\Leftrightarrow \eta \cdot f(\eta; \lambda y^*) \geq \lambda k \text{ and } \eta \cdot (1 - f(\eta; \lambda y^*)) \leq \lambda k \Leftrightarrow$$

$$\eta \in [y_1(\lambda y^*; \lambda k), y_r(\lambda y^*; \lambda k)].$$

This shows that in the relevant range, $y_1(y^*; k)$ and $y_r(y^*; k)$ and, consequently, $\mathcal{L}(y^*; k)$ are homogeneous of degree 1 in $(y^*; k)$. Moreover, $f(\lambda \bar{y}(y^*); \lambda y^*) = f(\bar{y}(y^*); y^*) = 1/2$ implies $\bar{y}(\lambda y^*) = \lambda \bar{y}(y^*)$.

Finally, (A3) implies $f(y; \lambda y^*) = f(y/\lambda; y^*)$ and, hence, $\frac{\partial}{\partial y} f(y; \lambda y^*) = \frac{1}{\lambda} \frac{\partial}{\partial y} f(y/\lambda; y^*)$. Therefore, $f(\hat{y}(y^*); y^*) + \hat{y}(y^*) \cdot \frac{\partial}{\partial y} f(\hat{y}(y^*); y^*) = 0$ if and only if $f(\lambda \hat{y}(y^*); \lambda y^*) + \lambda \hat{y}(y^*) \cdot \frac{\partial}{\partial y} f(\lambda \hat{y}(y^*); \lambda y^*) = 0$.

That means $\hat{y}(\lambda y^*) = \lambda \hat{y}(y^*)$. \square

Proof of Proposition 4: With (A3), $g_1(\lambda y; \lambda y^*) = \lambda y \cdot (1 - f(\lambda y; \lambda y^*)) = \lambda y(1 - f(y; y^*)) = \lambda g_1(y; y^*)$, i.e. g_1 is homogeneous of degree 1 in $(y; y^*)$. Similarly, g_2 is homogeneous of degree 1 in $(y; y^*)$. Furthermore h_1 , the inverse function of g_1 inherits the homogeneity of degree 1 in $(k; y^*)$, since $g_1(\lambda y; \lambda y^*) = \lambda g_1(y; y^*) = \lambda k$ implies $h_1(\lambda k; \lambda y^*) = \lambda y = \lambda h_1(k; y^*)$. Similarly, it can be demonstrated that h_2 , the inverse of g_2 , is homogeneous of degree 1 in $(k; y^*)$. Therefore, by Euler's theorem,

$$0 = \frac{\partial^2 h_1}{\partial k \partial y^*} \cdot y^* + \frac{\partial^2 h_1}{\partial k^2} \cdot k,$$

With the strict concavity of h_1 , we then have

$$\frac{\partial^2 h_1}{\partial k \partial y^*} = - \frac{\partial^2 h_1}{\partial k^2} \cdot \frac{k}{y^*} > 0. \quad (9)$$

Similarly, with the strict convexity of h_2 ,

$$\frac{\partial^2 h_2}{\partial k \partial y^*} = - \frac{\partial^2 h_2}{\partial k^2} \cdot \frac{k}{y^*} < 0. \quad (10)$$

Now $\mathcal{L}(k; y^*) = h_1(k; y^*) - h_2(k; y^*)$ with $\mathcal{L}(0; y^*) = 0$. Clearly, h_1 and h_2 are C^2 so that (9) and

(10) imply that $\frac{\partial^2 \mathcal{L}(k; y^*)}{\partial y^* \partial k} = \frac{\partial^2 \mathcal{L}(k; y^*)}{\partial k \partial y^*} > 0$; hence (I).

From $\mathcal{L}(0; y^*) \equiv 0$ follows $\frac{\partial}{\partial y^*} \mathcal{L}(0; y^*) \equiv 0$ which together with $\frac{\partial^2 \mathcal{L}(k; y^*)}{\partial k \partial y^*} > 0$ yields $\frac{\partial}{\partial y^*} \mathcal{L}(k; y^*) > 0$

for all $k > 0, y^* > 0$. Therefore (II).

Finally, $\frac{\partial}{\partial k} \mathcal{L}(\tilde{k}(y^*); y^*) = 0$ together with $\frac{\partial^2 \mathcal{L}(k; y^*)}{\partial y^* \partial k} > 0$ and strict concavity of \mathcal{L} in k implies

that $\tilde{k}(y^*) < \tilde{k}(y^{**})$ for $0 < y^* < y^{**}$, i.e. (III). $\square\square$

Proof of Proposition 5. We divide the proof into three parts:

(i) From Lemma 3 and its proof, we know that for any $y^* > 0$, there exists a unique $\tilde{k}(y^*) \in (0, \bar{y}(y^*)/2)$ such that $\mathcal{L}(n; y^*) < \mathcal{L}(m; y^*)$ for $0 \leq n < m \leq \tilde{k}(y^*)$.

(ii) For $0 < y^* < y^{**}$, set $\kappa(y^*, y^{**}) \equiv \tilde{k}(1) \cdot y^*$.

Then $0 < k < \kappa(y^*, y^{**})$ implies $0 < k/y^{**} < k/y^* < \tilde{k}(1)$. Hence by (i),

$$\mathcal{L}\left(\frac{k}{y^{**}}; 1\right) < \mathcal{L}\left(\frac{k}{y^*}; 1\right).$$

(iii) Let $0 < y^* < y^{**}$ and $0 < k < \kappa(y^*, y^{**})$.

Then by Lemma 4 and (ii),

$$\mathcal{L}(k; y^{**})/y^{**} = \mathcal{L}\left(\frac{k}{y^{**}}; 1\right) < \mathcal{L}\left(\frac{k}{y^*}; 1\right) = \mathcal{L}(k; y^*)/y^*. \quad \square\square$$

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CHAPTER 3

REASONABLE ROYALTY AND THE DIVISION OF PROFIT IN SEQUENTIAL INNOVATION¹

I. Introduction

The research documented in the current paper as well as work reported in Chou and Haller (1995) focuses on the distribution of profit between basic researchers without directly marketable products and the vendors of marketable products derived from the basic technology. However, the most distinctive and crucial features of our model are 1. uncertainty in litigation and 2. asymmetric bargaining power. The difference between Chou and Haller (1995) and the current paper lies in 3. the court decision on royalties and damages after a verdict of infringement.

The division of profit between sequential innovators has attracted recently a great deal of attention not only from economists but also from lawyers. Despite the fact that economists have been working on issues of intellectual property rights for a long time, the approach taken by the “patent race” literature so far cannot identify all the complex effects of patent protection when innovation is cumulative. The seminal contribution on the division of profit in sequential innovation has been made by Green and Scotchmer (1995). Green and Scotchmer analyze the role of different legal mechanisms under which innovation takes place in two stages. They then address the issue of optimal patent breadth and duration in this framework. The authors argue that the potential patent holder may lack incentives to invest in the first place, because not all the social value facilitated by basic research can be transferred from the second generation products.

¹The research reported here is coauthored with Hans Haller.

Green and Scotchmer (1995) — like all but one article of the patent literature — have confined themselves to the “fencepost” interpretation of the patent system. For example, Hortsmann et al (1985) study the propensity to patent for a successful innovator (the sole winner of a patent race) by specifying a “limited but exact coverage” patent system. Almost by definition, important policy implications regarding the litigation process cannot be addressed within a perfect fencepost system. In reality, however, patent breadth is *de facto* a matter of Patent Office and court interpretation. There are over 600 patent suits per year according to the American Intellectual Property Law Association (1986).

1.1. *Uncertainty in Litigation*

The “signpost” interpretation of the patent system takes into account the intriguing fact that patent protection is imperfect, that the patent system does not and, perhaps, cannot circumscribe its objects, individual patents, in a precise and unquestionable way. For instance, two of the most important statutory criteria for patentability are “novelty” and “nonobviousness”. Novelty can be interpreted as the criterion to determine whether the new invention was not in “prior art”, i.e., whether the inventor has really invented something. The nonobviousness criterion excludes patentability of inventions for which it is “obvious” that they could be invented with sufficient effort, even though no one has bothered to do so so far. Typically, infringement constitutes a situation where a new invention significantly overlaps with the patented technology. Significant overlap in turn is determined again in terms of novelty and nonobviousness — which are subject to qualification and interpretation. Therefore, legal determination of infringement can be a difficult task. It is not too far-fetched to imagine a complicated infringement case where the legal institutions are incapable of sound judgment. For instance, a patent is described by Kitti (1978) as a “lottery ticket”.

In a first attempt to capture this aspect of reality, Waterson (1990) has looked at uncertainty in patent infringement litigation and employed the concept of “limited but inexact patent coverage” in a horizontal product differentiation model. Pursuing this line of research one step further in a vertical product differentiation model, Chou and Haller (1995) find that when the inherent uncertainty of a patent infringement case is taken into consideration, the division of profit between sequential innovators will depend on the degree of the improvement made by the subsequent innovator. In particular, within a wide range of model parameters, full rent extraction by the initial patent holder is possible.

Uncertainty in litigation is a common feature of both our papers and the litigation literature. Also, the litigation process assumed here and in Chou and Haller (1995) is similar to the one most frequently used in the litigation literature. Among the precedents are Reinganum and Wilde (1986), Meurer (1989), and Aoki and Hu (1995). In their models, the plaintiff (the harmed party) makes a settlement offer to the defendant (the party who does harm). The defendant then responds by either accepting the offer (take-it) or refusing the settlement proposal (leave-it) after which a court action may be taken. The most striking modeling differences between Chou and Haller (1995) and the present paper on the one side and some of the litigation literature on the other side are twofold. First of all, the litigation literature focuses on asymmetric information between patentees and infringers about costs (harm, damages) or benefits (surplus, profits).² In the presence of asymmetric information, sequential equilibrium is the predominant solution concept. In contrast, our model assumes symmetric information. Then subgame perfect equilibrium is the appropriate solution concept. Secondly, the conventional litigation literature works with a given invention and a fixed probability that the patentee wins a patent infringement suit whereas in our approach,

²See, e.g., Bebchuck (1984), Reinganum and Wilde (1986), and Meurer (1989).

the degree of improvement upon a basic innovation determines the patentee's probability of winning infringement litigation.

1.2. Asymmetric Bargaining Power

Green and Scotchmer (1995) assume that any agreement between the two parties means a fifty-fifty split of the available surplus among them. In contrast, we give virtually all bargaining power to the patent holder by allowing him to move first and make a take-it-or-leave-it licensing offer.

The two distinctive features of our model, uncertainty in litigation and asymmetric bargaining power, work in opposite directions. Asymmetric bargaining power biases the outcome in favor of the initial patent holder. Uncertainty in litigation acts like a countervailing force that might prevent the patent holder from extracting all the surplus. Equally important is the quite realistic assumption that litigation is not free for either party.

Litigation is not a credible threat available to the patent holder, if his own litigation costs exceed his expected profit transfer. This is the case, if the marketed product is not very profitable to start with. In our model this means that the quality of the marketed product is low. Another case where the expected profit transfer through litigation falls short of litigation costs arises when the patent holder's probability of winning is too small. In our model this occurs when the quality of the marketed product is high. *Ceteris paribus*, then, complete profit transfer can only be expected for intermediate size subsequent innovations. Despite the *prima facie* enormous bargaining advantage granted to him, the patent holder cannot always force a complete profit transfer.

Litigation can be used as a credible threat by the patent holder, if his own litigation costs do not exceed his expected transfer. In that case, there exists a subgame perfect equilibrium with complete profit transfer. In fact we can show that, *ceteris paribus*, the quality parameters of intermediate size innovations for which an equilibrium with complete profit transfer obtains, form an interval.

1.3. Royalties and Damages

The model of the present paper differs from our previous one in an unobtrusive, but important detail: how profit is divided, if the patent holder prevails in court. In Chou and Haller (1995), after rendering the verdict of infringement, the court always grants the patent holder what she has demanded as a licensing fee at the very beginning of bargaining. By and large, the analysis of that model remains unaffected by the additional stipulation that the court imposed royalty must not exceed the total profit. As a matter of fact, prior to 1946 a patent holder was allowed to choose between the amount of damages she suffered and the amount of profit earned by the infringer. Since then, it is the courts rather than the plaintiffs who determine royalties and damages paid by infringers. One can argue, though, that in recent years the patent system has been leaning towards patent holders more heavily than ever [Dreyfuss (1989), Merges (1995)].

If determination of royalties and damages is left to the courts' discretion, then the issue of reasonable royalty arises and ought to be addressed. Since 1946, several doctrines of reasonable royalty have been applied. We furnish some details in Appendix A. In the current model, we postulate that after a verdict of infringement the court awards a royalty equal to the profit made by the infringer. Given the other specifications of our model, this assumption

proves consistent with the prevailing doctrines. See the discussion in Appendix A. One could be tempted to argue that if the court is capable of assessing the right amount of reasonable royalty, then the assumption of uncertain or unsound judgment associated with the legal system is unjustifiable. Notice, however, that both in our model and in practice the sequencing in infringement suits is such that damages are awarded after the finding of infringement. Therefore, we consider the two functions of the court, verifying the validity of a patent and ruling on infringement versus awarding the damages, as two distinct events.³ Whereas the model assumes uncertainty about who wins an infringement suit, there is no uncertainty about the amount of damages in case the court finds the defendant guilty of infringement: the court, according to the *ex post* factual findings, always grants the correct reasonable royalty.

I.4. *Summary of Results*

Three major results in Chou and Haller (1995) are still obtained under the new specification of reasonable royalty. That is, (1) a complete profit transfer equilibrium is attainable under a wide range of model parameters, (2) higher patent infringement litigation cost may dampen the patent holder's incentive to innovate, and (3) a broader patent breadth may not unconditionally improve the patent holder's incentive to innovate.

Unlike in Chou and Haller (1995) where technical intractability forced us to focus exclusively on the sub-interval where a complete-profit-transfer equilibrium exists, we are now able to partition the interval of proclaimed patent protection into three areas that can be characterized by three types of equilibria: complete-profit-transfer Take-it equilibrium,

³Typically, these functions are even exercised by two different courts, the CAFC and trial courts, respectively.

incomplete-profit-transfer Take-it equilibrium, and No-Action equilibrium, respectively. The strength (effectiveness) of the patent system can thus be accurately measured. The area where the No-action equilibria would emerge, provides the strongest incentive to the infringing firm to “trespass” the intellectual property of the patent holder. This area consists of an interval adjacent to the basic innovation and an interval adjacent to the upper end point of the interval of proclaimed patent protection. In other words, “inventing around” with close imitation or “inventing enough” with quite a novel (though still infringing) product may be observed in our model without the patent holder taking legal action.

In addition to the length of the complete-profit-transfer interval, the new framework allows to compute alternative measures of the efficacy of the patent system in transferring profits: the average expected payoff for the patent holder, both in absolute terms and relative to average expected profit.

The paper is organized as follows. In Section 2 we set forth the model to be used to investigate the profit division between sequential innovation. In Section 3 we present the comparative statics of raising and lowering infringement litigation costs. Moreover, we look at the instruments used by the courts to broaden (shorten) the patent breadth: doctrine of equivalents and reverse doctrine of equivalents. Some concluding remarks are offered in Section 4. Appendix A elaborates on doctrines of reasonable royalty. Appendix B contains more technical derivations of results.

II. The Model

We begin with a development of the basic model. There are one research institution and one firm. The research institution is called the patent holder (PH) hereafter. It has acquired a patent on its invention with quality x . We set $x = 0$ without loss of generality. The patent breadth granted is y^* with the definition that if the firm develops a product of $x+y$ with $y \in [0, y^*)$ then this product is deemed to infringe the patent x . Quality x is just a basic research outcome and has no immediate market value *per se*. The firm is capable of developing a new product of quality y with $x \leq y \leq y^*$ so that it would surely infringe on the patent held by PH. The cost of developing quality y is c_y . Once developed, the new product can be produced at zero cost and has market value π_y .

The crucial elements of patent litigation can be described as follows: Each party incurs the same litigation cost $L > 0$. There is an objective probability $f(y)$ of PH winning in litigation. The existence of such an $f(y)$ can be defended on the grounds that there is no perfect patent protection due to the nature of current patent law and the process of infringement litigation. While both parties may agree privately whether or not an infringement occurs, the court may come to a different conclusion. Occasionally, we treat y as variable and $f(y)$ as a decreasing function of $y \in [0, y^*]$ with $f(0) = 1$ and $f(y^*) = 0$. The further away from x a new invention is, the less likely is a verdict of infringement. If the court finds infringement in favor of the patent holder (the plaintiff), it assesses the extent of true damage (y), and orders compensation of a reasonable royalty, $R \geq 0$. According to the “analytical method” (see appendix A) and the model parameters, R would amount to the actual profit made by the infringer, that is, $R = \pi_y - c_y$.

We model the strategic interaction as a strategic game between PH and the firm. The

game lasts one period which is defined as the time interval beginning when PH makes the licensing offer and ending when the infringement and damage award issues are resolved. The two players take several steps during the period. There is no discounting within the period.

Both players enter the game with exogenously given and commonly known y . PH, as a first mover, makes a licensing agreement offer simply by specifying S with $S \in [0, \infty]$ although in equilibrium we would expect S to be bounded. We view S as a fixed-fee royalty: the number S represents the amount to be paid by the firm for the right to market its product. By offering $S=0$, PH tolerates the infringement without legal recourse. Facing the offer and knowing the amount of reasonable royalty, the firm has three strategic alternatives: (i) quit the project; (ii) pay the royalty proposed by PH; (iii) challenge the patent infringement allegation. In the latter contingency, PH then responds by making one of the following moves: take no action or litigate. In accordance with U.S. practice, we assume that even if it loses, the firm retains the profit from marketing this application while paying its litigation costs plus a reasonable royalty. Figure 1 summarizes the extensive form of the game, showing the order of decisions and the resulting (expected) payoffs.⁴

[Figure 3.1 about here]

Set $M=\{\text{No-action, Litigation}\}$ and $N=\{\text{Take-it, Leave-it, Drop-out}\}$. Then the normal form of the game has strategy spaces $S_{PH}=\mathbf{R} \times M^{\mathbf{R}}$ for PH and $S_F=N^{\mathbf{R}}$ for the firm. We consider strategy pairs that are Nash equilibria, i.e. each player chooses a strategy that maximizes its expected payoff given the other player's strategy. Moreover, we require subgame perfection: Equilibrium pairs of strategies induce equilibrium play in all subgames.

We distinguish three types of pure strategy equilibria. Which types occur, depends on

⁴The crucial difference to Chou and Haller (1995) lies in the award when PH wins litigation. Chou and Haller (1995) stipulate that PH receives the amount he asks for, i.e. S in our present notation. Notice that in equilibrium, PH never receives more than $\pi_y - c_y$, since the firm has the option to drop out.

the numerical specification of the model.

1. The Take-it equilibrium is characterized by S_t with

$$\pi_y - c_y - S_t \geq 0,$$

$$\pi_y - c_y - S_t \geq \pi_y - c_y - f(y)R - L, \text{ and}$$

$$f(y)R - L \geq 0.$$

The firm responds with Take-it to this offer. Should the firm play Leave-it in response to this offer,

then PH would counter with Litigation.

2. The Leave-it equilibrium is characterized by S_l with

$$\pi_y - c_y - f(y)R - L \geq 0,$$

$$\pi_y - c_y - f(y)R - L \geq \pi_y - c_y - S_l, \text{ and}$$

$$f(y)R - L \geq 0.$$

The firm responds with Leave-it to this offer and PH counters with Litigation.

3. The No-Action equilibrium is characterized by R with

$$f(y)R - L \leq 0, \text{ and}$$

$$\pi_y - c_y \geq 0.$$

The firm responds with Leave-it to all offers $S > 0$ and PH counters with No-action.

We proceed with the following simplifying assumption⁵:

(A1) $\pi_y = a \cdot y$, $c_y = c \cdot y$ where a and c are constants satisfying $a > c \geq 0$.

It is worth noting that since PH is solely interested in maximizing his (expected) payoff, i.e.

⁵Since PH's sole interest is in maximizing the profit which is transferred from the firm, he has no incentive to make an offer $S > \pi_y - c_y$. In other words, {Drop-out, Litigate} or {Drop-out, No-Action} can never be an equilibrium outcome since Drop-out is dominated by Take-it for the firm.

the profit transferred either via a licensing agreement in a Take-it equilibrium or via a reasonable royalty in a Leave-it equilibrium, we shall emphasize the particular type of equilibrium that yields the highest expected payoff for PH.

The interval $[0, y^*]$ can be partitioned into three areas each of which is characterized by a particular type of equilibrium. Necessary and sufficient conditions for these three types of equilibrium outcomes are described in Proposition 1. The intuition behind Proposition 1 can be developed by examining the game tree. Which type of equilibrium will prevail depends crucially on (a) how large a licensing fee would make the firm indifferent between acceptance and litigation and (b) how credible PH's threat is to litigate in case the infringing firm were to turn down the offer. Conditions (a)-(d) appearing in Proposition 1 directly follow from that reasoning.

Proposition 1. (i) A Complete-Profit-Transfer Take-it equilibrium will be attained, i.e., PH can extract all the profit it facilitates from the firm if and only if

$$(a) \quad y - yf(y) \leq \frac{L}{a-c} \text{ and}$$

$$(b) \quad yf(y) \geq \frac{L}{a-c}.$$

(ii) A No-Action equilibrium will be attained, i.e., the firm can retain all the profit if and only if

$$(c) \quad yf(y) \leq \frac{L}{a-c}$$

(iii) An Incomplete-Profit-Transfer Take-it equilibrium with $S = (a-c)yf(y) + L < \pi_y - c_y$ being accepted will be attained if and only if

$$(b) \quad yf(y) \geq \frac{L}{a-c} \text{ and}$$

$$(d) \quad y - yf(y) > \frac{L}{a-c}.$$

Some simple comparative statics can help develop intuition for the result in Proposition 1. First of all, suppose a , c , y , and L are given such that $y \geq \frac{L}{a-c}$ and $f(y)$ is treated as a variable. Then (a) and (b) are simultaneously satisfied if and only if $f(y)$ is sufficiently large. This can be seen from the game tree in Figure 1. To achieve a Take-it equilibrium with $R=(a-c)y$, PH's threat of litigation in case the firm rejects the offer has to be credible, that is, $f(y) \cdot R - L \geq 0$. If $y \geq \frac{L}{a-c}$ or, equivalently, $(a-c)y \geq L$ and if $R=(a-c)y$, then the inequality $f(y)R - L \geq 0$ holds trivially for $f(y)=1$. By continuity, a high and only a high winning probability $f(y)$ helps PH achieve the licensing agreement where the firm obtains zero payoff. Furthermore, if we examine the possibility of having a No-action equilibrium by using the same set of parameters, It is immediate that only small $f(y)$ can satisfy (d). Not surprisingly, it may as well be the case that there does not exist such $f(y)$ to sustain (b) and (d) simultaneously. One possible conjecture is that the existence of an incomplete-profit-transfer Take-it equilibrium depends upon the existence of appropriate 'intermediate' $f(y)$.

On the other hand, when a , c , y , and L are given with $y < \frac{L}{a-c}$, the situation changes drastically. Whereas (a) is always satisfied under this assumption, (b) breaks down for all $f(y) \in [0,1]$. Thus there does no longer exist the Take-it equilibrium PH is longing for. It is obvious from the game tree that PH is seriously concerned about the potential loss from litigation and therefore takes no action even if the firm dares to infringe. Under this particular specification, a smaller y does not benefit PH in achieving its goal of exploiting the firm — even with a high winning probability. Similarly, a Leave-it can never be obtained under this set of parameters. However, a No-Action equilibrium can be well expected since (d) holds trivially. To sum up, the preceding comments suggest there might exist specific bounds for those y which permit each type of equilibrium in question.

To extend the analysis one step further and arrive at such bounds, we next consider a

situation with exogenously given L , a , c , y^* , and a function $f: [0, y^*] \mapsto [0, 1]$ satisfying:

(A2) $f(0) = 1$, $f(y^*) = 0$, and f is twice differentiable with $f' < 0$, $f'' \leq 0$.

It is natural to assume f to be decreasing. It also appears plausible that PH's winning probability drops faster as y moves further away from $x=0$, or the more y approaches the delimiter of patent protection, y^* . From a different perspective, *ceteris paribus*, the firm is enjoying "increasing returns to litigation" as y varies. In reference to the "signpost" literature, Waterson (1990) looks at uncertainty in patent infringement litigation from a different angle and develops a horizontal product differentiation model. Whereas we explicitly require concavity of PH's winning probability, he implicitly imposes an equivalent property on the "court cost function" defining litigation costs and damage fees awarded to PH.

We first denote \bar{y} the 'median' of f , i.e. \bar{y} is implicitly given by the condition

$$f(\bar{y}) = \frac{1}{2}.$$

Proposition 2. Suppose

$$(M) \frac{\bar{y}}{2} \geq \frac{L}{a-c}.$$

Then there exist y_l , y_m , and y_r with the following properties:

- (i) $0 < y_l \leq y_m \leq y_r < y^*$;
- (ii) The best attainable equilibrium for the patent holder is a complete-profit-transfer Take-it equilibrium iff $y \in [y_l, y_m]$.
- (iii) A No-Action equilibrium is attainable iff $y \in [0, y_l] \cup [y_r, y^*]$.
- (iv) The best attainable equilibrium for the patent holder is an incomplete-profit-transfer Take-it equilibrium with $S_t = (a-c)yf(y) + L$ iff $y \in (y_m, y_r]$.

Corollary 1. (i) When $\frac{L}{a-c} \in (\hat{y}f(\hat{y}), \infty)$ there only exist No-Action equilibria for $y \in [0, y^*]$.
(ii) When $\frac{L}{a-c} \in (\frac{\bar{y}}{2}, \hat{y}f(\hat{y})]$ there exist both Take-it and No-Action equilibria for $y \in [0, y^*]$.

Note that even when litigation is not that costly relative to the profitability of the improved product, the “effective” patent protection exists only in the interval $[y_l, y_m]$. Both low and high ends of the improvement would not induce actual patent litigation (No-Action). This is so because given the combinations of {high $f(y)$ but low y } or {high y but low $f(y)$ }, PH’s expected gain from litigation is smaller than his litigation cost. In our model, as a consequence, the firm has incentives to invent around with close imitation or invent enough with a quite novel (though still infringing) product because of lack of effective patent protection. Empirically, as Mansfield et al (1981) point out, 60% of all patented and successful innovations were imitated within 4 years after introduction. Similar observations are also reported in Levin et al (1987).

It is obvious that our conclusions about the division of profit rely crucially on the relevant range that $\frac{L}{a-c}$ lies in. Under (M), with a relatively low $\frac{L}{a-c}$, three types of equilibria may emerge whereas a relatively high $\frac{L}{a-c}$ would only allow No-Action equilibria for all y in the proclaimed range of patent protection. Generally speaking, aiming at maximal exploitation of the firm, PH has to balance two factors moving in opposite directions: what the firm is capable of, i.e., the magnitude of y , and how significant the chance is that he can win the infringement case, i.e., the range of $f(y)$. More specifically, while making a higher offer to cope with a higher y , PH strives to maintain a credible threat to avoid the opportunism of the firm as $f(y)$ declines. Thus hypothesis (M) and conditions in Corollary 1 correspond to the extent by which PH can master this balancing act and extract profit from the firm.

III. Comparative Statics

III.1 Efficacy of the Patent System in Terms of Effective Patent Protection

In this sub-section we focus on the comparative statics with respect to several key variables — within their most interesting range in our set-up. In the sequel we denote $\mathcal{L} \equiv y_m - y_l$, the length of the interval of y where a complete-profit-transfer Take-it equilibrium can be obtained. \mathcal{L} may serve as a measure of the efficacy of the patent system. We first investigate how \mathcal{L} is responding to variations of the litigation cost L , the gross profit parameter a and the product development cost parameter c . It suffices to see how \mathcal{L} depends on the compound parameter $k \equiv \frac{L}{a-c}$.

Lemma 2. Suppose $\frac{\bar{y}}{2} \neq \hat{y}f(\hat{y})$, then $\frac{\partial \mathcal{L}}{\partial k} < 0$ for $k \in \left(\frac{\bar{y}}{2}, \hat{y}f(\hat{y})\right)$. Moreover, the corresponding intervals

$[y_m(k), y_l(k)]$ are strictly nested.

Intuitively, a higher litigation cost should have a stronger threatening effect on the firm who takes a chance when turning down the offer, since it will face a higher expected loss once it loses. However, Lemma 2 says that even when the litigation cost is in the “favorable” range where a complete-profit-transfer via a Take-it equilibrium can be assured, higher litigation cost will damage the manipulative power of PH. What we observe is that when k rises above $\frac{\bar{y}}{2}$ while staying below the bound $\hat{y}f(\hat{y})$, $g_2(y)$ starts to effectively determine the boundaries of \mathcal{L} . Therefore the sensitivity of $\mathcal{L}(k)$ with respect to k depends only on the strict concavity of $g_2(y)$. Recall that $g_2(y) \geq k$ is essentially the same as condition (3), $R \geq \frac{L}{f(y)}$. It is then obvious that a higher L makes it harder to satisfy (3). A similar interpretation can be applied to the parameters a and c .

Lemma 3. $\mathcal{L}(k)$ is strictly concave in k and there exists a unique $\tilde{k} \in (0, \frac{\bar{y}}{2})$ such that $\mathcal{L}(\tilde{k}) \geq \mathcal{L}(k)$ for all $k \in \mathbb{R}_+$.

In a second type of comparative statics, we investigate how \mathcal{L} , the length of the interval where PH can extract all the surplus, is affected by a change of patent protection. Intuition may suggest that the best way to help PH transfer profit from the firm is to grant PH a broad patent protection. Intriguingly enough, this is a premature conclusion as the next proposition shows. One more simplifying assumption, (A3) is imposed to establish the result. Prior to that we have to extend the model appropriately by postulating that f takes the more general form $f(y; y^*)$, $0 \leq y \leq y^*$, where the patent breadth $y^* > 0$ is treated as variable in the sequel. The obvious notation $\mathcal{L}(k; y^*)$, $\tilde{k}(y^*)$, etc. will be used.

(A3) $f(y; y^*) = f(\frac{y}{y^*}; 1)$, i.e., $f(y; y^*)$ is homogeneous of degree 0.

(A3) stipulates that the winning probability for PH depends only on the ratio y/y^* , not on the absolute magnitude of y or y^* . An extremely high y^* might correspond to a very vague claim such as “All non-human transgenic mammals” or “All hand-use calculators.” The broader the patent protection, the easier is it for an allegedly infringing firm to challenge the patent claim.

It has long been recognized that patent breadth is a viable instrument to influence inventors' incentives to innovate. Though not explicitly stated, theorists interested in the design of an optimal patent system seem to suggest to establish the optimal patent breadth through an adjustment in the written law, though not explicitly stated [Klemperer (1990), Gilbert and Shapiro (1990), Green and Scotchmer (1995)]. However, patent protection can

also be expanded (or conversely, contracted) overtime through legal doctrines without resorting to specific modification of the Patent Codes. In particular, the applications of the doctrine of equivalents and the reverse doctrine of equivalents constitute important instances of such flexibility. The doctrine of equivalents is “a creature of equity which expands upon this premise and allows a finding of infringement where the accused product does not literally infringe upon the claim, but is substantially equivalent to the entire claim” [Kayton (1985)]. Normally the doctrine of equivalents apply to the patent representing a “pioneer invention” - which the Supreme Court has defined as “a patent concerning a function never before performed, a wholly novel device, or one of such novelty and importance as to make a distinct step in the progress in the art,” [*Boyden Power-Brake Co. v. Westinghouse*, 170 U.S. 537, 569 (1898)]. As a symmetric counterpart to the doctrine of equivalents, the reverse doctrine of equivalents “is an equitable doctrine invoked in applying properly construed claims to an accused device. Just as the purpose of the ‘doctrine of equivalents’ is to prevent ‘pirating’ of the patentee’s invention, *Graver Tank*, 339 U.S. 605, 607, 608, (1950), so the purpose of the ‘reverse doctrine of equivalents’ is to prevent unwarranted extension of the claim beyond a fair scope of the patentee’s invention” [*Scripps Clinic & Research Fund. v. Genetech, Inc.*, 927 F.2d 1565, 18 U.S.P.Q.2d (BNA) 1001, 18 U.S.P.Q.2d (BNA) 1896 (Fed. Cir. 1991)].

Thus it is of economic significance to investigate how the specific features of imperfect patent protection, i.e. the signpost interpretation of the patent system, respond to variations in patent breadth. In the present paper, imposing (A3) is a simple attempt to capture the effect of broadening patent protection, be it through a change of the written law or the doctrine of equivalents, and the reverse doctrine of the equivalents. Notice that here we are not concerned with the controversy over the judicial standard for infringement analysis under the doctrine of

equivalents, e.g. tests like “element-by-element” and “invention as a whole” [See, for example, Lau (1989) and Merges (1992)]. Instead, we will look at the impact upon the PH’s incentive to innovate from adjusting the patent protection under the doctrine of equivalents – or otherwise.

Lemma 4. Suppose $k \in [0, \frac{\bar{y}}{2}]$, then the functions $y_l(k; y^*)$, $y_r(k; y^*)$, and $\mathcal{L}(k; y^*)$ are homogeneous of degree 1 in $(k; y^*)$. The functions $\hat{y}(y^*)$ and $\bar{y}(y^*)$ are homogeneous of degree 1 in y^* .

Let us first state a result that conforms to intuition: As patent protection becomes broader, $\mathcal{L}(k; y^*)$ increases, i.e. the size of the interval where PH can extract all the surplus increases.

Proposition 4. The following three assertions hold:

- (I) $\frac{\partial}{\partial k} \mathcal{L}(k; y^*)$ is strictly increasing in $y^* > 0$ as long as $0 < k < \bar{y}(y^*)/2$.
- (II) $\mathcal{L}(k; y^*)$ is strictly increasing in $y^* > 0$ as long as $0 < k < \bar{y}(y^*)/2$.
- (III) $\tilde{k}(y^*)$ is strictly increasing in $y^* > 0$.

Let us now proceed to the promised, somewhat less intuitive result: As patent protection becomes broader, the relative size of the interval where PH can extract all the surplus may decrease.

Proposition 5. For any $0 < y^* < y^{**}$, there exists $\kappa(y^*, y^{**}) > 0$ such that

$$\frac{\mathcal{L}(k; y^{**})}{y^{**}} < \frac{\mathcal{L}(k; y^*)}{y^*} \quad \text{for all } 0 < k < \kappa(y^*, y^{**}).$$

Proposition 5 states that even though $\mathcal{L}(k; y^*)$ increases as patent protection becomes broader, \mathcal{L}/y^* , that is the relative size of the interval where PH can extract all the surplus, may be falling for certain k . The manipulative power of PH measured as the fraction of infringing y that provide maximal incentive to PH to innovate, can apparently diminish when the court employs the doctrine of equivalents when the imperfect patent protection exhibits such feature as (A3).

III.2 Efficacy of the Patent System in Terms of the Average Expected Profits Transferred

In this sub-section we focus on the comparative statics with respect to k and y^* . In the sequel we denote

$$E\pi_p \equiv \frac{1}{y^*} \left\{ \int_{y_l}^{y_m} (a - c)y dy + \int_{y_m}^{y_r} [(a - c)yf(y) + L] dy \right\}.$$

$E\pi_p$ is then the expected payoff for PH given that y is uniformly distributed along the interval of proclaimed patent protection $[0, y^*]$. Recall that in Proposition 1 we have shown that the interval $[0, y^*]$ can be partitioned into three areas each of which is characterized by a particular type of equilibrium: the complete-profit-transfer Take-it equilibrium, the incomplete-profit-transfer Take-it equilibrium with $S_t = (a-c)yf(y)+L$ or the No-Action equilibrium. We first investigate how $E\pi_p$ is responding to variations of k (or equivalently, the litigation cost L).

Lemma 5. Suppose $\frac{\bar{y}}{2} \neq \hat{y}f(\hat{y})$, then $\frac{\partial E\pi_p}{\partial k} < 0$ for $k \in \left(\frac{\bar{y}}{2}, \hat{y}f(\hat{y})\right)$. Moreover, the corresponding intervals $[y_m(k), y_r(k)]$ are strictly nested.

Lemma 6. Suppose $\frac{\bar{y}}{2} = \hat{y}f(\hat{y})$, then there exists a unique $\hat{k} \in (\tilde{k}, \frac{\bar{y}}{2})$ such that $E\pi_p(k)$ is decreasing in $k \in [\hat{k}, \frac{\bar{y}}{2}]$.

Lemmata 5 and 6 are reminiscent of Lemmata 2 and 3. In particular, the properties of $L(k)$ and $E\pi_p$ follow directly from the concavity of $f(y)$, the probability of PH winning in litigation. In other words, the phenomenon observed here, that is, shrinkage of the interval of complete profit transfer and decrease in PH's average expected profit with respect to certain k , can be attributed to a particular aspect of imperfect patent protection, increasing returns to litigation for the firm.

Lemma 7. The efficiency ratio $\frac{E\pi_p(k;y^*)}{E\pi(k;y^*)}$ is homogeneous degree of zero in $(k; y^*)$.

Lemma 7, parallel to Proposition 5, reminds us of the potential drawback of an expanded patent protection: an increase in the relative profit transferred to PH as a result of an broadened patent breadth may be offset by the accompanying increase in litigation costs. Note that the application of the doctrine of equivalents may be associated with higher litigation cost in the following sense: (1) enforcement cost goes up as y^* is increased, and (2) the litigants have more burden of proof.

Some caution is warranted in interpreting Lemma 7. Namely, two alternative predictions can be made with respect to the absolute average expected profit transferred from the firm to PH when patent breadth is adjusted. First, observe that $E\pi(k;y^*)$, the average profit transfer to PH under perfect patent protection with y uniformly distributed over $[0, y^*]$, is just $(a-c)\frac{y^*}{2}$. Now suppose that under two patent protection regimes y^* and λy^* (w.l.o.g, $\lambda > 1$), the improvement parameter y are distributed with uniform density $\frac{1}{y^*}$ on $[0, y^*]$ and $\frac{1}{\lambda y^*}$ on $[0, \lambda y^*]$, respectively, and suppose that the efficiency ratio $\frac{E\pi_p(k;y^*)}{E\pi(k;y^*)}$ is equal to $\alpha < 1$. Then

Lemma 7 implies that

$$E\pi(\lambda k; \lambda y^*) = \alpha E\pi_p(\lambda k; \lambda y^*) = \alpha(a-c)\frac{\lambda y^*}{2} > \alpha(a-c)\frac{y^*}{2} = E\pi_p(k; y^*).$$

Alternatively, if the patent protection regime y^* can be viewed as a continuous contraction from the λy^* regime, then the improvement parameter y should be distributed with a uniform density $\frac{1}{\lambda y^*}$ on both $[0, y^*]$ and $[0, \lambda y^*]$. Then Lemma 7 implies

$$E\pi(\lambda k; \lambda y^*) = \alpha E\pi_p(\lambda k; \lambda y^*) = \alpha(a-c)\frac{\lambda y^*}{2} > \alpha(a-c)\frac{y^*}{2} \cdot \frac{y^*}{\lambda y^*} = E\pi_p(k; y^*).$$

While the former interpretation is better suited for the description of an adjustment in written law, the latter is more in line with the application of the reverse doctrine of equivalents. In either case, PH is enjoying an increase in his absolute average profit transferred from the infringing firm under a broader patent protection.

IV. Concluding Remarks

In this article, we investigate the division of profit between a patent holder and a derived product producer in an environment with uncertainty about the outcome of infringement but with certainty about the damage awards, a reasonable royalty granted to the prevailing plaintiff. Our analysis identifies the conditions on model parameters that permit a complete-profit-transfer Take-it equilibrium, an incomplete-profit-transfer Take-it equilibrium or a No-Action equilibrium. The interval of the proclaimed patent protection $[0, y^*]$ can be partitioned into three areas each of which is characterized by a particular type of equilibrium. Whereas the area in which complete-profit-transfer equilibria can be obtained gives a patent

holder maximal incentive to innovate, the area in which No-Action equilibria prevail encourage a subsequent innovator to imitate either with a close substitute or with a much advanced but still infringing product. Comparative statics with respect to important policy parameters such as litigation costs and patent breadth is performed.

In a broader context, while some litigation of the literature assumes litigation is always profitable for the plaintiff to rule out “nuisance” suits [Bebchuk (1984) and Reinganum and Wilde (1986)], the current model accommodates inaction as one of the strategies the patent holder may choose [Nalebuff (1987) and Meurer (1989)]. That is, a patent holder may not always be willing to use the court system. Consequently, the problem of making a credible threat has a significant effect on the equilibrium outcome. Recall that which type of equilibrium will prevail depends crucially on (a) how large a licensing fee would make the firm indifferent between acceptance and litigation and (b) how credible PH’s threat is to litigate in case the infringing firm were to turn down the offer. To maintain a credible litigation threat the patent holder must evaluate the potential gain and loss when the infringing firm refuses to settle. The issue of credibility may severely restrict the patent holder’s capacity of achieving the desired outcome – complete profit transfer. In addition, the particular phenomenon observed from the above comparative statics, that is, shrinkage of the interval of complete profit transfer and decrease in PH’s average expected profit with respect to certain k , is again a result of the dominance of effect (b) over effect (a).

Our analysis pays special attention to the effects of the variations in patent breadth, either by an adjustment in the written law or by application of the doctrine of the equivalents and the reverse doctrine of equivalents. It is clear that the current paper is not suggesting the “optimal” patent system. Rather it aims at assessing the efficacy of the current patent

system when uncertainty about the outcome of infringement litigation is taken into consideration. In particular, we are concerned with the incentive to innovate, not the harm (deadweight loss) caused by blocking patents as Merges and Nelson (1992) have emphasized. Therefore, it is not our intention to determine which y should be allowed to escape the “web of infringement”. [See, e.g., Scotchmer (1996).]

Appendix A

A Brief Review of the Doctrines on Reasonable Royalty

Prior to 1946, a successful patent claimant could choose between the amount of damages she suffered and the amount of profits earned by the infringer [See Chisum (1980)]. However, the high cost of determining an infringer’s profits eventually led Congress to drop infringer profits as an alternative measure of recovery [Act of August 1, 1946, ch. 726 s 1, 60 stat. 778 (current version at § 35 U.S.C. s 284 (1952))]. More sophisticated doctrines have been applied since. Despite the revision, an infringer’s profits continue to be crucial elements in computing the patent holder’s damages - either as a approximation for the patent holder’s “lost profits” or as a factor in determining a reasonable royalty. In general, a successful patent claimant is entitled to recover the profits she would have made but for the infringement; if lost profits cannot be proven, she is entitled to a reasonable royalty.

(A) Lost Profits:

(1) The Cornerstone case for lost profits: *Panduit Corp. v. Stahlin Bros. Fiber Works, Inc.* 575 F.2d 1152, 197 U.S.P.Q. (BNA) 726 (6th Cir.1978)

Four factors enumerated in Panduit have been used as the primary guidelines for determining whether a patent holder is entitled to recover lost profits.

(B) *Reasonable Royalty*:

(1) The Benchmark case: *Georgia-Pacific Corp v. United States Plywood Corp.* 318 F. Supp. 1116, 166 U.S.P.Q. (BNA) 235 (S.D.N.Y. 1970), modified, 446 F.2d 295, 170 U.S.P.Q. 369 (2d Cir.), cert. denied, 404 U.S. 870 (1971).

Georgia-Pacific has been relied upon heavily for its fifteen factors, among others, to be verified in determining a reasonable royalty.

(2) Variation of Georgia-Pacific: *Hanson v. Alpine Valley Ski Area, Inc.* 718 F.2d 1075, 219 U.S.P.Q. (BNA) 679 (Fed. Cir. 1983)

In *Hanson* the Federal Circuit has stated: 'The reasonable royalty may be based upon,...., a hypothetical royalty resulting from arm's length negotiation between a willing licensor and a willing licensee.'

Conceivably, problems with this hypothetical negotiation are manifold: First, the court is required to reconstruct a "fancy contract" based upon fantasy and flexibility [*Fromson v. Western Litho Plate & Supply Co.* 853 F.2d 1568, 1575-76, 7 U.S.P.Q.2d 1606 (Fed Cir. 1988)]. Secondly, it ignores the adverse impact upon converting property rule into liability rule at random wills [See Calabresi and Melamed (1972)]. That is, a flat reasonable royalty with no punitive effect would have reduced the incentive to innovate at the first place. Though an infringer may not be completely indifferent between an ex-ante licensing agreement and the ex-post damage award since there may be conceivable loss of goodwill and substantial sunk costs.

(3) What flows directly from the willing licensor/willing licensee model is the 'analytical method' (or accounting method). It computes a reasonable royalty based on the infringer's pre-infringement projection of profits. See, for instance, *TWM Mfg. Co. v. Dura Corp.*, 789 F.2d 895, 229 U.S.P.Q. (BNA) 525 (Fed. Cir. 1986).

In reference to the current modeling approach, the following features are worth further investigation.

To begin with, to be eligible for the lost profits compensation, the patent holder has to show she has capacity to produce the volume of potential sales lost due to infringement. However, cases have shown that no stringency has been maintained in this respect by the courts. For example, to prove its capacity to make the infringer's sales, the patent holder does not have to show that it had a plant ready and existing [See *Livesay Window Co. v. Livesay Industries, Inc.*, 251 F.2d 469, 473, 116 U.S.P.Q. 167, 171 (5th Cir. 1958); *W. L. Gore & Associated, Inc. v. Carlisle Corp.*, 198 U.S.P.Q. 353, 367 (D. Del. 1978).] The court even considered the possibility of subcontracting as the patent holder's potential capacity [see *Gyromat Corp. v. Champion Spark Plug Co.*, 735 F.2d 549, 554 222 U.S.P.Q. 4, 7 (Fed. Cir. 1984)]. As a result, the patent holder in our model, though without marketing power, would still be entitled to lost profits, which figure may be drawn according to the infringer's actual profits.

Secondly, despite the fact that the doctrines and their variations on reasonable royalty have been widely stated in patent damages cases, the analytical method seems to be the dominant guiding principle for computing reasonable royalty [See Conley (1987) citing that the courts giving only 'lip service to the willing licensor/willing licensee model')]. Furthermore, as a simple rule, the courts just subtract the infringer's usual profit from the profit earned by the

infringement, and award the entire difference to the patent holder. In a sense the analytical approach is a return to awarding to the patent holder the infringer's profits from the use of the invention.

Note that in the context of our model, the value of the second generation product is completely attributable to the basic patented technology. Thus the doctrines of 'entire market value' and 'apportionment' would yield the same compensation figure for the patent holder [See, e.g. *Westinghouse v. Wagner*, § 225 U.S. 604, 614 (1912)]. Specifically, in terms of the model parameters, the analytical approach amounts to a reasonable royalty $R = (a-c)y$ as we assume.

APPENDIX B

Proof of Proposition 1: (i) For a licensing agreement to prevail, i.e., a Take-it equilibrium to exist, the following conditions are necessary and sufficient:

$$\pi_y - c_y - S \geq 0,$$

$$\pi_y - c_y - S \geq \pi_y - c_y - f(y)R - L, \text{ and}$$

$$f(y)R - L \geq 0.$$

We first explore the possibility that PH can extract all the profit from the firm, i.e., where an offer $S = (a - c)y$ gets accepted in equilibrium. Thus the previous conditions are equivalent to:

$$S = (a - c)y, \quad (1)$$

$$R \leq \frac{L}{1 - f(y)} \quad (2)$$

$$R \geq \frac{L}{f(y)}. \quad (3)$$

(2) and (3) are then equivalent to

$$y[1 - f(y)] \leq \frac{L}{a - c} \quad \text{and} \quad (4)$$

$$yf(y) \geq \frac{L}{a - c}. \quad (5)$$

(ii) We proceed with the necessary and sufficient conditions for a No-Action equilibrium, i.e.,

$$\pi_y - c_y \geq 0, \quad \text{and}$$

$$f(y)R - L \leq 0.$$

With (A1) they are equivalent to:

$$yf(y) \leq \frac{L}{a - c}. \quad (6)$$

(iii) We commence with the necessity proof. Recall that for a licensing agreement to prevail, i.e., a Take-it equilibrium to exist, the following conditions must be satisfied:

$$\pi_y - c_y - S \geq 0,$$

$$\pi_y - c_y - S \geq \pi_y - c_y - f(y)R - L, \quad \text{and}$$

$$f(y)R - L \geq 0.$$

Note that in an incomplete-profit-transfer Take-it equilibrium the firm enjoys positive payoff after the transfer of profit through licensing agreement, i.e., $\pi_y - c_y - S > 0$. We then explore the possibility that PH can extract profit by offering $S_t = (a - c)yf(y) + L$ which gets accepted in equilibrium. With (A.1) and the assumption on reasonable royalty, the previous conditions can be rewritten as:

$$\pi_y - c_y - [(a - c)yf(y) + L] > 0,$$

$$0 \geq 0, \quad \text{and}$$

$$yf(y) \geq \frac{L}{a - c}.$$

These are just conditions (d) and (b)

$$y[1 - f(y)] > \frac{L}{a - c} \text{ and} \quad (d)$$

$$yf(y) \geq \frac{L}{a - c}. \quad (b)$$

Now we turn to the sufficiency proof. Suppose (b) and (d). Since only the Take-it or the Leave-it equilibrium has potential in generating payoffs for PH, we will focus on these two types of equilibria. We start with the possibility for PH to extract profit via a Take-it equilibrium.

Define

$$S_t \equiv (a - c)yf(y) + L$$

Thus we can have

$$\pi_y - c_y - S_t \geq \pi_y - c_y - [(a - c)yf(y) + L] \quad (7)$$

Also, by (d), we can infer

$$\pi_y - c_y > (a - c)yf(y) + L \quad (8)$$

or

$$\pi_y - c_y - [(a - c)yf(y) + L] > 0, \text{ that is}$$

$$\pi_y - c_y - S > 0. \quad (9)$$

Next note that By (A.1) and the assumption on the reasonable royalty, (b) can be rewritten as

$$f(y)R - L \geq 0. \quad (10)$$

(7), (9) and (10) establish that, indeed, S_t is a Take-it equilibrium offer. It is also obvious that such an licensing offer yields higher payoff for PH than any Leave-it equilibrium since

$$S_t = (a - c)yf(y) + L > (a - c)yf(y) - L.$$

We have shown that the combination of (b) and (d) is equivalent to the necessary and sufficient conditions for an Incomplete-Profit-Transfer equilibrium characterized by $S_t = (a - c)yf(y) + L$. Notice that strict inequality in (d) renders strict inequality in (8), that is incomplete-profit-transfer *strictu sensu*. This completes the proof.

□□

Proof of Proposition 2: First, we perform comparative statics with respect to $y \in [0, y^*]$. For this purpose, we introduce the functions

$$g_1(y) = y[1 - f(y)]$$

and

$$g_2(y) = yf(y),$$

which appear in (4) and (5) and, obviously, play a critical role in our analysis. Notice that $g_1' = 1 - f - y \cdot f' > 0$ and $g_1'' = -2f' - y \cdot f'' > 0$ in the interval $(0, y^*)$. Hence g_1 is strictly increasing and strictly convex in y with $g_1(0) = 0$ and $g_1(y^*) = y^*$. Further notice that $g_2' = (y \cdot f)' = f + y \cdot f'$ and $g_2'' = (y \cdot f)'' = 2f' + y \cdot f'' < 0$. Thus g_2 is strictly concave in y with $g_2(0) = g_2(y^*) = 0$. Consequently, g_2 has a unique maximizer \hat{y} in $(0, y^*)$. This maximizer is given as the unique solution of the first order condition

$$g_2'(\hat{y}) = f(\hat{y}) + \hat{y} \cdot f'(\hat{y}) = 0.$$

Claim: $\hat{y} \leq \bar{y}$ and $0 < y^*/2 < \bar{y} < y^*$.

To show this claim, recall that $g_2(y)$ is strictly concave in y with $g_2(0) = 0$ and $g_2(y^*) = 0$. By Takayama (1985) Theorem 1.C.3: f is concave on $(0, y^*)$ if and only if for any $x, y \in (0, y^*)$:

$$f'(y) \cdot (x - y) \geq f(x) - f(y).$$

Evaluate this inequality at $y = \bar{y}$ and let $x \rightarrow 0$. Then

$$-f'(\bar{y}) \cdot \bar{y} \geq 1 - \frac{1}{2} \text{ or } f'(\bar{y}) \cdot \bar{y} \leq -\frac{1}{2}.$$

Adding $f(\bar{y}) = \frac{1}{2}$ to the latter inequality yields

$$g_2'(\bar{y}) = \bar{y} \cdot f'(\bar{y}) + f(\bar{y}) \leq 0.$$

Strict concavity of g_2 and $g_2'(\hat{y}) = 0$ imply the assertion $\hat{y} \leq \bar{y}$. Further, (A2) has the immediate implication $0 < y^*/2 < \bar{y} < y^*$. This concludes the proof of the claim.

Next, recall that $g_1(y)$ is strictly increasing and strictly convex in y with $g_1(0) = 0$ and $g_1(y^*) = y^*$. Hypothesis (M) amounts to $g_1(\bar{y}) = g_2(\bar{y}) = \frac{\bar{y}}{2} \geq \frac{L}{a-c}$.

Part (i):

Recall that \bar{y} is defined as the intersection point of $g_1(y)$ and $g_2(y)$, i.e., $g_1(\bar{y}) = g_2(\bar{y}) = \frac{\bar{y}}{2}$.

By the hypothesis, the continuity and other properties of g_2 , and the intermediate value theorem, there exist $z_l \in (0, \bar{y}]$ and $z_r \in [\bar{y}, y^*)$ such that $g_2(\bar{y}) \geq g_2(z_l) = g_2(z_r) = \frac{L}{a-c}$.

Next note that (M) implies $y^* > g_2(\bar{y}) \geq g_2(\bar{y}) \geq \frac{L}{a-c} > 0$. Then, by the continuity and other properties of g_1 and the intermediate value theorem, there exists a unique $z \in (0, y^*)$ with $g_1(z) = \frac{L}{a-c}$. We then compare the magnitudes of z_l and z . Now recall that $f(\bar{y}) = \frac{1}{2}$ and therefore $\frac{L}{a-c} \leq \bar{y}/2 = g_1(\bar{y}) = g_2(\bar{y})$. Therefore, $\bar{y} \in [z_l, z_r]$, by the strict concavity of g_2 . Also, $z \leq \bar{y}$, by the strict monotonicity of g_1 . Hence $z \leq z_r$. Moreover, $0 = g_1(0) = g_2(0)$, $\frac{1}{2} \cdot \bar{y} = g_1(\bar{y}) = g_2(\bar{y})$, strict convexity of g_1 and strict concavity of g_2 imply $g_1(y) < \frac{1}{2} \cdot y < g_2(y)$ for $y \in (0, \bar{y})$. If $\bar{y} = z_l$, then $z = \bar{y} = z_l$. If $\bar{y} > z_l$, then $g_1(z_l) < g_2(z_l) = \frac{L}{a-c}$. Thus $z > z_l$. In any case, therefore, $z \in [z_l, z_r]$.

Now set $y_l = z_l$, $y_m = z$ and $y_r = z_r$. Then (i) is satisfied. • • •

Part (ii):

We commence with the sufficiency proof. When condition (M) holds and $y \in [y_l, y_m] = [z_l, z]$, then $y \in [z_l, z_r]$ and the strict concavity of g_2 implies (b) $yf(y) \geq \frac{L}{a-c}$. Further $y \in [y_l, y_m]$ implies $y \leq z$. Since $g_1(y)$ is an increasing function in $y \in [0, y^*]$, condition (a) $y[1 - f(y)] \leq \frac{L}{a-c}$ holds as well.

Now we turn to the necessity proof. (b) implies that $y \in [z_l, z_r]$. (a) implies that $y \leq z$. Together (a) and (b) imply $y \in [z_l, \min\{z_r, z\}] = [y_l, y_m]$. Note that we know from Proposition 1 that by offering $R_t = (a - c)y$, PH can extract all the profit it facilitates from the firm if and only if (a) and (b) both hold. We have shown that under the hypothesis (c), the combination of (a) and (b) is equivalent to

$y \in [y_l, y_m]$. • • •

Part (iii):

We commence with the sufficiency proof. When condition (M) holds and $y \in [0, y_l] \cup [y_r, y^*]$, then $y \in [0, y^*] \setminus (z_l, z_r)$ and the strict concavity of g_2 implies (c) $yf(y) \leq \frac{L}{a-c}$.

Now we turn to the necessity proof. By the strict concavity of g_2 , (c) implies that $y \in [0, y^*] \setminus (z_l, z_r)$ which in turn implies $y \in [0, y_l] \cup [y_r, y^*]$. Note that we know from Proposition 1 that a No-Action equilibrium is attainable if and only if (c) holds. We have shown that under the hypothesis (M), (c) is equivalent to $y \in [0, y_l] \cup [y_r, y^*]$. • • •

Part (iv):

We commence with the sufficiency proof. When condition (M) holds and $y \in (y_m, y_r]$, then $y \in (z, z_r]$ and the strict concavity of g_2 implies (b) $yf(y) \geq \frac{L}{a-c}$. Further $y \in (y_m, y_r]$ implies $y > z$. Since $g_1(y)$ is an strictly increasing function in $y \in [0, y^*]$, condition (d) $y[1 - f(y)] > \frac{L}{a-c}$ holds as well.

Now we turn to the necessity proof. (b) implies that $y \in [z_l, z_r]$. (d) implies that $y > z$. Together (b) and (d) imply $y \in (y_m, y_r]$. Note that we know from Proposition 1 that an incomplete-profit-transfer Take-it equilibrium is attainable if and only if (b) and (d) both hold. We have shown that under the hypothesis (M), the combination of (b) and (d) is equivalent to $y \in [y_m, y_r]$. This completes the proof. $\square\square$

Proof of Lemma 2: Suppose a, c, y , and $f(y)$ are given such that $k \in \left(\frac{\bar{y}}{2}, \hat{y}f(\hat{y})\right)$. Together with the supposition $\frac{\bar{y}}{2} \neq \hat{y}f(\hat{y})$ they imply that $g_2(\hat{y}) > k$ thus $z_r > \hat{y} > z_l$. It can also be inferred that $\bar{y} \notin [z_l, z_r]$ by the strict concavity of g_2 and $z > \bar{y}$ by the strict monotonicity of g_1 . By Lemma 1, $\bar{y} \geq \hat{y}$ and $\bar{y} \notin [z_l, z_r]$ imply $\bar{y} > z_r$. Thus $z > \bar{y} > z_r$. So $L = y_m - y_l = \min[z_r, z] - z_l = z_r - z_l$. The strict concavity and the other properties of g_2 imply that for all k_1, k_2 such that $\frac{\bar{y}}{2} < k_1 < k_2 < \hat{y}f(\hat{y})$ the corresponding $z_r(k_1), z_l(k_1), z_r(k_2)$ and $z_l(k_2)$ have the following order:

$$z_l(k_1) < z_l(k_2) < \hat{y} < z_r(k_2) < z_r(k_1) \text{ or}$$

$$L(k_1) = [z_r(k_1) - z_l(k_1)] > [z_r(k_2) - z_l(k_2)] = L(k_2).$$

This implies the assertion. $\square\square$

We need a technical auxiliary result to proceed:

Lemma A. Suppose that $k = g(y)$ is strictly increasing, concave (convex) and twice continuously differentiable in the interval (a, b) and suppose that $g'(y) \neq 0$ for $y \in (a, b)$. Then $y = g^{-1}(k)$ exists and is monotone, convex (concave), and twice continuously differentiable with respect to k .

Proof: The existence, monotonicity, and twice continuous differentiability of g^{-1} are assured

by the inverse function theorem; see Flett (1966; Th. 10.9.5). Moreover, we have

$$g^{-1'}(k) = \frac{1}{g'(g^{-1}(k))}.$$

Now, the only task left is to prove the concavity (convexity) conversion. Differentiation of and application of the chain rule to the foregoing formula for $g^{-1'}(k)$ yields

$$g^{-1''}(k) = -\frac{g''(g^{-1}(k))}{[g'(g^{-1}(k))]^3},$$

which has sign opposite to that of $g''(g^{-1}(k))$. This implies convexity (concavity) of $g^{-1}(k)$.

□□

Proof of Lemma 3: We consider three cases where $\bar{k} = g_2(\hat{y}) = \frac{\bar{y}}{2}$.

Case(i): $k \in (\hat{y}f(\hat{y}), \infty)$. Then trivially $\mathcal{L}(k) = 0$ by Corollary 1-(i), that is, there does not exist such interval $[y_l, y_m]$.

Case(ii): $k \in (\frac{\bar{y}}{2}, \hat{y}f(\hat{y})]$. Then, by Lemma 2, $\mathcal{L}(\bar{k}) \geq \mathcal{L}(k)$. (This is, however, a little more than what Lemma 2 states. When $k = \hat{y}f(\hat{y})$, $\mathcal{L}(k)$ is equal to zero since y_l and y_r coincide. So we include this boundary point in the statement.)

Case(iii): $k \in (0, \frac{\bar{y}}{2})$. Since both g_1 and g_2 are continuous, monotone, and twice differentiable, by the inverse function theorem, the following functions are well defined, unique, and twice differentiable:

$$h_1(k): [0, \frac{\bar{y}}{2}] \mapsto [0, \bar{y}] \text{ with } h_1(g_1(y)) = y \text{ for all } y \in [0, \bar{y}],$$

$$h_2(k): [0, \frac{\bar{y}}{2}] \mapsto [0, \hat{y}] \text{ with } h_2(g_2(y)) = y \text{ for all } y \in [0, \hat{y}].$$

Furthermore, by Lemma A, h_1 is monotone and strictly concave while h_2 is monotone and strictly convex. Therefore $\mathcal{L}(k) = h_1(k) - h_2(k)$ is strictly concave in k . Notice that h_1' is continuously decreasing from $h_1'(0) = \infty$ to $h_1'(\bar{k}) = \frac{1}{g_1'(\bar{y})}$ and h_2' is continuously increasing from $h_2'(0) = 1$ to $h_2'(\bar{k}) = \frac{1}{g_2'(\hat{y})}$. By Lemma 1 we already know $\bar{y} \geq \hat{y}$ which implies $g_2'(\bar{y}) = f(\bar{y}) + \bar{y}f'(\bar{y}) \leq 0$. Since $f(\bar{y}) = \frac{1}{2}$, $\bar{y}f'(\bar{y}) \leq -\frac{1}{2}$ or $-\bar{y}f'(\bar{y}) \geq \frac{1}{2}$. Then

$$g_1'(\bar{y}) = 1 - f(\bar{y}) - \bar{y}f'(\bar{y}) \geq 1 - \frac{1}{2} + \frac{1}{2} = 1. \text{ Thus } \frac{1}{g_1'(\bar{y})} = h_1'(\bar{k}) \leq 1 < \frac{1}{g_2'(\bar{y})} = h_2'(\bar{k}).$$

Set

$$H(k) = h_1'(k) - h_2'(k).$$

H is strictly decreasing and continuous with $H(0) > 0$ and $H(\bar{k}) < 0$. By the intermediate value theorem, there exists a unique $\tilde{k} \in (0, \frac{\bar{y}}{2})$ such that $H(\tilde{k}) = h_1'(\tilde{k}) - h_2'(\tilde{k}) = 0$, that is, $\mathcal{L}'(\tilde{k}) = 0$. By the strict concavity of $\mathcal{L}(k)$, such \tilde{k} will be the unique global maximizer in $k \in [0, \frac{\bar{y}}{2}]$.

Cases (i), (ii), and (iii) together imply $\mathcal{L}(\tilde{k}) \geq \mathcal{L}(k)$ for all $k \in \mathbb{R}_+$. This completes the proof. \square

Proof of Lemma 4: Consider $\lambda > 0$, $y^* > 0$ and $k \geq 0$. Then:

$$\eta \in [\lambda y_1(y^*; k), \lambda y_m(y^*; k)] \Leftrightarrow \eta = \lambda y \text{ and } y \in [y_1(y^*; k), y_m(y^*; k)]$$

$$\Leftrightarrow \eta = \lambda y \text{ and } y \cdot f(y; y^*) \geq k \text{ and } y \cdot (1 - f(y; y^*)) \leq k$$

$$\Leftrightarrow \eta = \lambda y \text{ and } y \cdot f(\lambda y; \lambda y^*) \geq k \text{ and } y \cdot (1 - f(\lambda y; \lambda y^*)) \leq k$$

$$\Leftrightarrow \eta = \lambda y \text{ and } \lambda y \cdot f(\lambda y; \lambda y^*) \geq \lambda k \text{ and } \lambda y \cdot (1 - f(\lambda y; \lambda y^*)) \leq \lambda k$$

$$\Leftrightarrow \eta \cdot f(\eta; \lambda y^*) \geq \lambda k \text{ and } \eta \cdot (1 - f(\eta; \lambda y^*)) \leq \lambda k \Leftrightarrow$$

$$\eta \in [y_1(\lambda y^*; \lambda k), y_m(\lambda y^*; \lambda k)].$$

This shows that in the relevant range, $y_1(y^*; k)$ and $y_m(y^*; k)$ and, consequently, $\mathcal{L}(y^*; k)$ are homogeneous of degree 1 in $(y^*; k)$. Moreover, $f(\lambda \bar{y}(y^*); \lambda y^*) = f(\bar{y}(y^*); y^*) = 1/2$ implies $\bar{y}(\lambda y^*) = \lambda \bar{y}(y^*)$.

Finally, (A3) implies $f(y; \lambda y^*) = f(y/\lambda; y^*)$ and, hence, $\frac{\partial}{\partial y} f(y; \lambda y^*) = \frac{1}{\lambda} \frac{\partial}{\partial y} f(y/\lambda; y^*)$. Therefore, $f(\hat{y}(y^*); y^*) + \hat{y}(y^*) \cdot \frac{\partial}{\partial y} f(\hat{y}(y^*); y^*) = 0$ if and only if $f(\lambda \hat{y}(y^*); \lambda y^*) + \lambda \hat{y}(y^*) \cdot \frac{\partial}{\partial y} f(\lambda \hat{y}(y^*); \lambda y^*) = 0$. That means $\hat{y}(\lambda y^*) = \lambda \hat{y}(y^*)$. \square

Proof of Proposition 4: With (A3), $g_1(\lambda y; \lambda y^*) = \lambda y \cdot (1 - f(\lambda y; \lambda y^*)) = \lambda y(1 - f(y; y^*)) = \lambda g_1(y; y^*)$, i.e. g_1 is homogeneous of degree 1 in $(y; y^*)$. Similarly, g_2 is homogeneous of degree 1 in $(y; y^*)$. Furthermore h_1 , the inverse function of g_1 inherits the homogeneity of degree 1 in $(k; y^*)$, since $g_1(\lambda y; \lambda y^*) = \lambda g_1(y; y^*) = \lambda k$ implies $h_1(\lambda k; \lambda y^*) = \lambda y = \lambda h_1(k; y^*)$. Similarly, it can be demonstrated that h_2 , the inverse of g_2 , is homogeneous of degree 1

in $(k; y^*)$. Therefore, by Euler's theorem,

$$0 = \frac{\partial^2 h_1}{\partial k \partial y^*} \cdot y^* + \frac{\partial^2 h_1}{\partial k^2} \cdot k,$$

With the strict concavity of h_1 , we then have

$$\frac{\partial^2 h_1}{\partial k \partial y^*} = - \frac{\partial^2 h_1}{\partial k^2} \cdot \frac{k}{y^*} > 0. \quad (9)$$

Similarly, with the strict convexity of h_2 ,

$$\frac{\partial^2 h_2}{\partial k \partial y^*} = - \frac{\partial^2 h_2}{\partial k^2} \cdot \frac{k}{y^*} < 0. \quad (10)$$

Now $\mathcal{L}(k; y^*) = h_1(k; y^*) - h_2(k; y^*)$ with $\mathcal{L}(0; y^*) = 0$. Clearly, h_1 and h_2 are C^2 so that (9) and

(10) imply that $\frac{\partial^2 \mathcal{L}(k; y^*)}{\partial y^* \partial k} = \frac{\partial^2 \mathcal{L}(k; y^*)}{\partial k \partial y^*} > 0$; hence (I).

From $\mathcal{L}(0; y^*) \equiv 0$ follows $\frac{\partial}{\partial y^*} \mathcal{L}(0; y^*) \equiv 0$ which together with $\frac{\partial^2 \mathcal{L}(k; y^*)}{\partial k \partial y^*} > 0$ yields

$$\frac{\partial}{\partial y^*} \mathcal{L}(k; y^*) > 0$$

for all $k > 0$, $y^* > 0$. Therefore (II).

Finally, $\frac{\partial}{\partial k} \mathcal{L}(\tilde{k}(y^*); y^*) = 0$ together with $\frac{\partial^2 \mathcal{L}(k; y^*)}{\partial y^* \partial k} > 0$ and strict concavity of \mathcal{L} in k implies

that $\tilde{k}(y^*) < \tilde{k}(y^{**})$ for $0 < y^* < y^{**}$, i.e. (III). \square

Proof of Proposition 5. We divide the proof into three parts:

(i) From Lemma 3 and its proof, we know that for any $y^* > 0$, there exists a unique $\tilde{k}(y^*) \in (0, \bar{y}(y^*)/2)$ such that $\mathcal{L}(n; y^*) < \mathcal{L}(m; y^*)$ for $0 \leq n < m \leq \tilde{k}(y^*)$.

(ii) For $0 < y^* < y^{**}$, set $\kappa(y^*, y^{**}) \equiv \tilde{k}(1) \cdot y^*$.

Then $0 < k < \kappa(y^*, y^{**})$ implies $0 < k/y^{**} < k/y^* < \tilde{k}(1)$. Hence by (i),

$$\mathcal{L}\left(\frac{k}{y^{**}}; 1\right) < \mathcal{L}\left(\frac{k}{y^*}; 1\right).$$

(iii) Let $0 < y^* < y^{**}$ and $0 < k < \kappa(y^*, y^{**})$.

Then by Lemma 4 and (ii),

$$\mathcal{L}(k; y^{**})/y^{**} = \mathcal{L}(\frac{k}{y^{**}}; 1) < \mathcal{L}(\frac{k}{y^*}; 1) = \mathcal{L}(k; y^*)/y^*. \quad \square\square$$

Proof of Lemma 5: By Corollary 1, when $k \in (\frac{\bar{y}}{2}, \hat{y}f(\hat{y})]$, only complete-profit-transfer Take-it equilibria yield positive payoffs for PH. Then

$$E\pi_p = \frac{1}{y^*} \int_{y_l}^{y_m} (a - c)y dy = \frac{1}{2y^*}(a - c)(y_m^2 - y_l^2)$$

$$\frac{\partial E\pi_p}{\partial k} = \frac{1}{y^*}(a - c)(y_m \frac{\partial y_m}{\partial k} - y_l \frac{\partial y_l}{\partial k})$$

$$= \frac{1}{y^*}(a - c)(\frac{y_m}{g_2'(y_m)} - \frac{y_l}{g_2'(y_l)}) < 0$$

given that $y_m \geq \hat{y} \geq \bar{y} \geq y_l$ and other properties of $g_2(y)$, i.e., $g_2'(y_m) < 0$ and $g_2'(y_l) > 0$.

□□

Proof of Lemma 6: When $k \in [0, \frac{\bar{y}}{2}]$, only complete-profit-transfer and incomplete-profit-transfer Take-it equilibria yield positive payoffs for PH. Thus

$$\begin{aligned} E\pi_p(k) &= \frac{1}{y^*} \left\{ \int_{y_l}^{y_m} (a - c)y dy + \int_{y_m}^{y_r} [(a - c)yf(y) + L] dy \right\} \\ &= \frac{1}{y^*} \left\{ \frac{1}{2}(a - c)(y_m^2 - y_l^2) + (a - c) \left[\int_{y_m}^{y_r} yf(y) dy + k(y_r - y_m) \right] \right\} \end{aligned}$$

It suffices to show $\frac{1}{2}(a - c)(y_m^2 - y_l^2)$ and $(a - c) \left[\int_{y_m}^{y_r} yf(y) dy + k(y_r - y_m) \right]$ are both decreasing in k to prove the claim.

Denote

$$\begin{aligned} D(k) &\equiv \frac{\partial(y_m^2 - y_l^2)}{\partial k} \\ &= 2(y_m \frac{\partial y_m}{\partial k} - y_l \frac{\partial y_l}{\partial k}) \\ &= 2(\frac{y_m}{g_1'(y_m)} - \frac{y_l}{g_2'(y_l)}) \end{aligned}$$

By the convexity of g_1 and concavity of g_2 we can infer $g_1''(y_m) > 0$, $g_2''(y_l) < 0$ and

$\frac{g'_1(y_m)}{[g'_1(y_m)]^3} - \frac{g'_2(y_l)}{[g'_2(y_l)]^3} < 0$. Thus it is clear that $D(k)$ is decreasing in $k \in [0, \frac{\bar{y}}{2}]$:

$$D'(k) = \frac{g'_1(y_m) - y_m g''_1(y_m)}{[g'_1(y_m)]^3} - \frac{g'_2(y_l) - y_l g''_2(y_l)}{[g'_2(y_l)]^3} < 0$$

Recall that $g'_1[y_m(\tilde{k})] = g'_2[y_l(\tilde{k})]$ at \tilde{k} , then

$$D(\tilde{k}) = \frac{2}{g'_1(y_m)}(y_m - y_l) > 0.$$

By lemma 3 we know $g'_1(\bar{y}) \geq 1$ and $g'_2(\bar{y}) = 0$ so that

$$D(\frac{\bar{y}}{2}) = 2\bar{y} \cdot \left(\frac{1}{g'_1(\bar{y})} - \frac{1}{g'_2(\bar{y})} \right) < 0$$

Thus by the intermediate value theorem, there exists a $\hat{k} \in (\tilde{k}, \frac{\bar{y}}{2})$ such that $D(\hat{k}) = 0$ and $D(k) \leq 0$ for $k \in [\hat{k}, \frac{\bar{y}}{2}]$. It is then immediate that $\frac{1}{2}(a - c)(y_m^2 - y_l^2)$ is decreasing in $[\hat{k}, \frac{\bar{y}}{2}]$ since its first order derivative is non-positive in the relevant range.

Now we verify that the second term, $(a - c)[\int_{y_m}^{y_r} yf(y)dy + k(y_r - y_m)]$ is decreasing in k . Observe that

$$\frac{\partial E\pi_p}{\partial k} = \frac{1}{y^*} \left\{ \frac{1}{2}(a-c)D(k) + (a-c) \left\{ \frac{g_2(y_r)}{g'_2(y_r)} - \frac{g_2(y_m)}{g'_1(y_m)} + k \left[\frac{1}{g'_2(y_r)} - \frac{1}{g'_1(y_m)} \right] \right\} \right\}$$

It is easy to verify that both $\frac{g_2(y_r)}{g'_2(y_r)} - \frac{g_2(y_m)}{g'_1(y_m)}$ and $[\frac{1}{g'_2(y_r)} - \frac{1}{g'_1(y_m)}]$ are both negative by the convexity

(concavity) of $g_1(g_2)$ for k in the relevant range. This, together with $\frac{1}{2}(a - c)(y_m^2 - y_l^2)$ decreasing, implies there exist $\hat{k} \in (0, \frac{\bar{y}}{2})$ such that $E\pi_p(k)$ is decreasing in $[\hat{k}, \frac{\bar{y}}{2}]$. $\square\square$

Proof of Lemma 7:

Denote

$$E(k; y^*) \equiv \frac{E\pi_p(k; y^*)}{E\pi(k; y^*)}, \text{ then}$$

$$\begin{aligned}
E(k; y^*) &= \frac{\frac{1}{y^*} \left\{ \int_{y_l}^{y_m} (a - c) y dy + \int_{y_m}^{y_r} [(a - c) y f(y; y^*) + L] dy \right\}}{\frac{1}{y^*} \int_0^{y^*} (a - c) y dy} \\
&= \frac{\frac{1}{2} (a - c) (y_m^2 - y_l^2) + (a - c) \left[\int_{y_m}^{y_r} y f(y; y^*) dy + k(y_r - y_m) \right]}{\frac{1}{2} (a - c) (y^*)^2} \\
&= \frac{(y_m^2 - y_l^2) + 2 \left[\int_{y_m}^{y_r} y f(y; y^*) dy + k(y_r - y_m) \right]}{(y^*)^2}
\end{aligned}$$

Let $\lambda > 0$. Then by the homogeneity of y_l , y_m , y_r and $f(y; y^*)$ from Lemma 4

$$\begin{aligned}
E(\lambda k; \lambda y^*) &= \frac{\lambda^2 (y_m^2 - y_l^2) + 2 \left[\int_{\lambda y_m}^{\lambda y_r} y f(y; \lambda y^*) dy + \lambda^2 k(y_r - y_m) \right]}{\lambda^2 (y^*)^2} \\
&= \frac{(y_m^2 - y_l^2) + 2 \left[\int_{y_m}^{y_r} y f(y; y^*) dy + k(y_r - y_m) \right]}{(y^*)^2} \\
&= E(k; y^*)
\end{aligned}$$

□□

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CHAPTER 4

THE INCENTIVE TO INNOVATE AND PRODUCT DIFFERENTIATION WITH SEQUENTIAL ENTRY IN A TWO-DIMENSIONAL SPACE

1. Introduction

The pioneering work of Hotelling (1929) has fostered the development of the theory of product differentiation. Although providing tremendous insights in economics and in political science, the original single-dimension model suffers from at least two major shortcomings which limit its potential applicability. First, attempts to incorporate prices into this basic model have encountered the difficulty of assuring the existence of an equilibrium in pure strategies. For instance, d'Aspremont et al (1979) demonstrate the possibility of the non-existence of pure strategy price equilibrium due to an undercutting strategy employed by duopolists when products offered are relatively similar. Special conditions, such as making the demand of each firm concave in its own price (Economides (1989), Gabszewicz and Thisse (1979), Lane (1980) and Shaked and Sutton (1982)) and seeking mixed-strategy equilibrium (Dasgupta and Maskin (1986) and Osborne and Pitchik (1987)), have been explored. Caplin and Nalebuff (1991) develop a set of very general conditions under which a price equilibrium does exist. Nevertheless, the problem of product equilibrium remains unsettled. Second, the existing results are not always applicable to multi-dimensional competition, which appears to be a better proxy for actual product differentiation. For example, wines vary in dryness, tartness, fruitiness, vintage, and alcoholic content. Cars vary in acceleration, roominess, fuel efficiency, quietness and in many other dimensions as well.

There is relatively little literature on multi-dimensional product differentiation probably because of its considerable complexity. From the perspective of multi-dimensional competition, Dixit and Stiglitz (1977) and Spence (1976) focus on answering Chamberlin's

question of whether the competitive market will provide the optimal amount of product variety. Economides (1986) studies a two-characteristic analogue of Hotelling's duopoly model and shows that symmetric locations yield price equilibrium even with linear utility functions. However, the complete symmetry assumption in this literature obviously does not allow us to address the issue of characteristics selection. Although proving the existence of a price equilibrium for any given location pair in an n -dimensional setting, Caplin and Nalebuff (1986) do not directly address the issue of uniqueness of product equilibrium. Instead, they look at the leader's optimal exclusionary or accommodating strategy in a sequential-entry duopoly with cost asymmetries. In their work, as in ours, Stackelberg leadership and rational expectations are assumed. The basic framework is similar to that of Neven and Thisse (1990) who consider price and product competition in the context of single variety and quality differentiation. Here we focus, instead, on a two-dimensional variety space with sequential entry in terms of the timing of innovation.

With an attempt to further capture some important aspects of the real competition, in the present paper we impose one more restriction on the action set of the incumbent's characteristics selection: the incumbent will not pursue a new dimension design even though she foresees its feasibility for the potential entrant. It is well recorded that a notable number of research-based new firms have been founded by frustrated researchers from industry giants such as IBM, Unisys, Hughes Aircraft, and Texas Instruments.¹ On the theoretical side of observation, it has been also noted by Scherer and Ross (1990), among others, that theory offers ambiguous predictions on the aspect of correlation between firm size and ability to innovate. Against the common belief that well-established firms have an edge in outperforming the entrants with relatively small size, there are factors like less bureaucracy

¹For example, see Pinchot (1986).

and higher propensity to take on risky projects (nothing to lose, much to gain; perhaps, less risk averse) that enable small firms to be successful. Therefore, our model appears to be appropriate for the early-stage, less concentrated industries such as computing equipment, control instruments, and semi-conductors (Acs and Audretsch (1987) and Dorfman (1987)) where an aggressive entrant innovates and thus enhances the possibility of increasing the dimensionality of product variety while the incumbent confines herself to the non-innovative variety space.

Another important issue that will be addressed in this paper is the endogenous determination of dimensionality induced by competition. There are a handful of examples that demonstrate the growth of dimensionality. For instance, 45 new features were added to VCRs by one manufacturer or another in a thirteen-year period from 1975 to 1987 (Swann (1990)). Swann does this in order to ease the computations necessary to simulate the outcome. These simulations shed light on how the dimensionality of a market expands given different levels of innovation costs. We reexamine the plausibility of his assumption about firms' choices of locations. In a sequentially growing product space, our two-dimensional model shows that firms will conform to the principle of minimal differentiation along the traditional dimension but at the same time pursue maximal differentiation along the newly invented dimension. In addition, the potential entrant's incentive to innovate can be addressed in terms of its product characteristics selection along the innovated dimension.

In order to understand the endogenous dimensionality decision and still make the analysis tractable, we employ a sequential-entry duopoly model in a two-dimensional variety space in which the price and product equilibria can be fully characterized. For simplicity, we interpret the traditional dimension as a condensed description of the previously developed (possibly

many) dimensions. Thus we may study the ensuing competition due to the introduction of an innovated new dimension.

This paper is organized as follows. In Section 2, a model is presented to investigate the three-stage product and price competition. In Section 3 we derive the demand functions. Section 4 and 5 deal with the characterization of price and product equilibrium respectively. Some concluding remarks are given in Section 6. The more technical or elaborate proofs are collected in an Appendix.

2. The Model

The demand and supply sides of the economy and the concept of equilibrium are formulated as a duopoly with feasible extension to a more general framework which may encompass higher dimensions.

Consumers

Products are characterized by a combination of two attributes, x and y in a unit square $[0,1] \times [0,1]$. This two-dimensional characteristic space not only represents the diversity of products but also distinguishes consumer preferences. A consumer is identified by a most preferred characteristic combination and by a quadratic transport cost function. Furthermore, a consumer of type (\bar{x}, \bar{y}) derives the following (indirect) utility from buying one unit of product (x, y) :

$$U(x, y; \bar{x}, \bar{y}) = R - (\bar{x} - x)^2 - (\bar{y} - y)^2 - P, \quad (1)$$

where P denotes the price of the product and R is the uniform reservation price across consumers. We assume that each consumer chooses only one unit of the products and R is large enough for all consumers to achieve positive utility (in equilibrium) with the purchase. Last, consumers as represented by the parameters (\bar{x}, \bar{y}) are assumed to be uniformly distributed over the unit square.²

Producers

On the supply side, we consider a duopoly. A three-stage game is modeled. In stage one, the incumbent firm **I** chooses its product $(z, 0)$ along the X axis only, with $z \in [0, 1]$. In stage two, based on the expected competition, the entrant **E** then responds with its product $(x, y) \in [0, 1] \times [0, 1]$. There is no cost of entry. Once chosen, product characteristics are fixed. After both product specifications are revealed, in stage three the prices are determined in a Bertrand-Nash equilibrium for the given products. Zero marginal production costs for both firms are assumed. Without loss of generality, we assume $z \leq x$. The case where $z \geq x$ can be dealt with in a symmetric way by reverting the X axis.

Equilibrium

For a given product selection pair $\{(z, 0), (x, y)\}$ we say that (p_i, p_e) is a *price equilibrium* if it is a Nash equilibrium of the price subgame. The quintuple $(z^*, x^*, y^*, p_i^*, p_e^*)$ is a *product-price equilibrium* if it is subgame perfect. In other words, we require subgame perfection: Equilibrium pairs of strategies induce equilibrium play in all subgames.

²The population is independent of the growth of dimensionality. If both firms compete in the one-dimensional variety line, the population can be viewed as the projection from the unit square onto the horizontal axis.

3. Demand Functions

The set of consumers who are indifferent between products $(z, 0)$ and (x, y) can be derived from (1). Given product characteristics $(z, 0)$ and (x, y) , the marginal consumers are represented by a function $\bar{y}(\bar{x})$:

$$\bar{y}(\bar{x}) = \frac{(p_e - p_i) + (x^2 - z^2) + y^2 - 2(x - z)\bar{x}}{2y} \quad (2)$$

Observing that $\bar{y}(\bar{x})$ is a linear and nonincreasing function of \bar{x} , we can partition the unit square into three subsets. Figure 1 illustrates a typical partition in which the upper right group prefers (x, y) to $(z, 0)$ and the lower left group prefers $(0, z)$ to (x, y) . With the supposition on unit demand for each consumer and uniform distribution of consumer preferences over the unit square, it is easily seen that the area of these partitions are equivalent to the respective aggregate market demands acquired by the firms, namely $D_i + D_e = 1$.

[Figure 4.1 about here]

Since the slope of $\bar{y}(\bar{x})$ will affect the aggregate demand function, we distinguish between two situations: *traditional dominance* and *innovational dominance*.³ The former refers to the case

$$\left| \frac{\partial \bar{y}}{\partial \bar{x}} \right| = \frac{(x - z)}{y} \geq 1 \quad (3)$$

³The case where $y=0$, i.e., the entrant elects to compete in X axis without innovation, will be regarded as a special case of traditional dominance. We can rule out the situation in which the exact same locations ($x=z$ and $y=0$) are chosen by both firms, since fierce Bertrand price competition would drive profit down to zero.

while the latter refers to the case

$$|\frac{\partial \bar{y}}{\partial \bar{x}}| = \frac{(x - z)}{y} \leq 1. \quad (4)$$

Condition (3) (respectively, (4)) implies that the (normalized) degree of differentiation with respect to the traditional variety, $(x - z)$, is greater (less) than that with respect to the innovated variety, y . In words, the term “dominance” indicates the entrant’s relative position as opposed to that chosen by the incumbent.

The next step is to derive the demand D_i (given p_e) based upon the prevailing types of dominance:

First, for both dominance types, we derive the “low” demand

$$D_i^{(l)} = \frac{[(p_e - p_i^{(1)}) + (x^2 - z^2) + y^2]^2}{8y(x - z)} \quad (5)$$

when

$$p_i^{(1)} \in [\max(\bar{p}_i^{(2)}, \tilde{p}_i^{(2)}), \bar{p}_i^{(1)}];^4$$

where

$$\bar{p}_i^{(1)} \equiv p_e + (x^2 - z^2) + y^2,$$

$$\bar{p}_i^{(2)} \equiv p_e + (x^2 - z^2) + y^2 - 2y \text{ and}$$

$$\tilde{p}_i^{(2)} \equiv p_e + (x^2 - z^2) + y^2 - 2(x - z).$$

Second, the functional form of the “medium” demand differs across two dominance types. If traditional dominance prevails then

$$\bar{D}_i^{(m)} = \frac{[(p_e - p_i^{(2)}) + (x^2 - z^2) + y^2 - y]}{2(x - z)} \quad (6)$$

⁴Alternatively, this condition can be formulated as: $p_i^{(1)} \in [\bar{p}_i^{(2)}, \bar{p}_i^{(1)}]$ when traditional dominance prevails and $p_i^{(1)} \in [\tilde{p}_i^{(2)}, \bar{p}_i^{(1)}]$ when innovational dominance prevails. In other words, the domain of (price) parameters for which the demand function is valid does depend on the dominance types.

when

$$p_i^{(2)} \in [\bar{p}_i^{(2)}, \bar{p}_i^{(3)}]$$

where

$$\bar{p}_i^{(3)} = \tilde{p}_i^{(2)} \equiv p_e + (x^2 - z^2) + y^2 - 2(x - z),$$

Similarly, when innovational dominance prevails we have

$$\tilde{D}_i^{(m)} = \frac{[(p_e - p_i^{(2)}) + (x^2 - z^2) + y^2 - (x - z)]}{2y} \quad (7)$$

when

$$p_i^{(2)} \in [\tilde{p}_i^{(3)}, \tilde{p}_i^{(2)}]$$

where

$$\tilde{p}_i^{(3)} = \bar{p}_i^{(2)} \equiv p_e + (x^2 - z^2) + y^2 - 2y;$$

Last, we derive the “high” demand

$$D_i^{(h)} = \frac{8y(x - z) - [(p_e - p_i^{(3)}) + (x^2 - z^2) + y^2 - 2(x - z) - 2y]^2}{8y(x - z)} \quad (8)$$

when

$$p_i^{(3)} \in [\bar{p}_i^{(4)}, \min(\bar{p}_i^{(3)}, \tilde{p}_i^{(3)})]$$

where

$$\bar{p}_i^{(4)} \equiv p_e + (x^2 - z^2) + y^2 - 2(x - z) - 2y.$$

the symbol l , m , and h stand for low, medium and high demand, respectively.⁵

It is easy to verify that at each kink ($\bar{p}_i^{(2)}$ and $\bar{p}_i^{(3)}$) demand is continuous. We show a

⁵Roughly speaking, a low demand $D_i^{(l)}$ corresponds to the situation where the incumbent's market share is less or equal to $\frac{1}{2}$. The high demand can be explained in a similar way.

typical graph of D_i in Figure 2. The convexity, linearity or concavity of three segments for D_i can be explained directly by the marginal increase (decrease) in demand induced by the changes in p_i . Since D_e is equal to $(1 - D_i)$ by assumptions, we observe that the shape of D_e will have corresponding concave, linear, and convex sections with respect to that of D_i . For each pair of the products, the profit function of firm k ($k = i, e$) is defined as $\pi_k(p_k, p_j) = p_k D_k(p_k, p_j)$ for $j \neq k$.

[Figure 4.2 about here]

4. Price equilibrium

We now turn to the characterization of the price equilibrium in the next section. The following Proposition, due to Caplin and Nalebuff (1991), provides sufficient conditions for existence and uniqueness of a price equilibrium in pure strategies within our framework.⁶

Proposition 1. If the transport cost is quadratic and the consumer distribution function is uniform, then for duopoly, in any number of dimensions, there exists a unique pure strategy equilibrium of the price subgame.

Next we characterize the equilibrium prices as a function of the given product characteristics. Since demand functions D_i and D_e depend on the types of dominance, we shall consider all *six* types of equilibrium ($D_i^{(l)} + D_e^{(h)}$, $D_i^{(m)} + D_e^{(m)}$, or $D_i^{(h)} + D_e^{(l)}$ combined with traditional or innovational dominance). Meanwhile, each price pair has to be within the relevant intervals as given in section 2 with respect to different market share decompositions. After checking these demand compatibility (boundary) conditions, we can rule out the cases where the incumbent takes over the majority of the total demand ($D_i^{(h)}$ vs. $D_e^{(l)}$) under either

⁶For existence, see Theorem 2 and for uniqueness, see Proposition 6, both in Caplin and Nalebuff (1991).

traditional or innovational dominance.⁷ Further notice that the demand functions are identical for $(D_i^{(l)}, D_e^{(h)})$ under either type of dominance so that it suffices to focus only on the remaining three types of equilibrium. We will then label these as Case (1), (2), and (3) respectively.

Case(1)

We start with $(\bar{D}_i^{(m)}, \bar{D}_e^{(m)})$, the linear segment, under traditional dominance. Solving the FOC for the relevant profit maximization yields a single solution given by

$$\bar{p}_i^* = \frac{[(x^2 - z^2) + 2(x - z) + y^2 - y]}{3}, \quad (9)$$

$$\bar{p}_e^* = \frac{[-(x^2 - z^2) + 4(x - z) - y^2 + y]}{3}, \quad (10)$$

when the following two boundary conditions are satisfied

$$(A) (x - z)[2 + (x + z)] \geq 4y - y^2,$$

$$(B) (x - z)[4 - (x + z)] \geq 2y + y^2.$$

Notice that only (A) is the binding constraint since (B) is always met as long as $x \leq 1$ when $x - z \leq y$ has to be satisfied. The profit functions derived from these equilibrium prices thus can be represented by the chosen product characteristics:

$$\pi_i(\bar{p}_i^*[(z, 0), (x, y)], \bar{p}_e^*[(z, 0), (x, y)]) = \frac{[(x^2 - z^2) + 2(x - z) + y^2 - y]^2}{18(x - z)}, \quad (11)$$

$$\pi_e(\bar{p}_i^*[(z, 0), (x, y)], \bar{p}_e^*[(z, 0), (x, y)]) = \frac{[-(x^2 - z^2) + 4(x - z) - y^2 + y]^2}{18(x - z)}. \quad (12)$$

⁷This result actually conforms with intuition. The incumbent is at a disadvantage in the sense of a limited choice set (X-axis only) with respect to the entrant's unconstrained options in the unit square. In fact, in order to achieve the high demand, the incumbent is so desperate to move closer to (or further away from) the entrant as to overturn the dominance type. See Appendix for details of the related computation.

Case (2)

Similarly, the equilibrium prices under innovational dominance for the linear segment are given by

$$\bar{p}_i^* = \frac{[(x^2 - z^2) - (x - z) + y^2 + 2y]}{3}, \quad (13)$$

$$\bar{p}_e^* = \frac{[-(x^2 - z^2) + (x - z) - y^2 + 4y]}{3}, \quad (14)$$

satisfying

$$(C) (x - z)[4 - (x + z)] \leq 2y + y^2,$$

$$(D) (x - z)[2 + (x + z)] \leq 4y - y^2.$$

Notice that now (C) becomes the binding constraint since (D) is again met as long as $x \leq 1$ when $x - z \geq y$ has to be satisfied. Consequently the profit function in terms of the product characteristics are

$$\pi_i(\bar{p}_i^*[(z, 0), (x, y)], \bar{p}_e^*[(z, 0), (x, y)]) = \frac{[(x^2 - z^2) - (x - z) + y^2 + 2y]^2}{18y}, \quad (15)$$

$$\pi_e(\bar{p}_i^*[(z, 0), (x, y)], \bar{p}_e^*[(z, 0), (x, y)]) = \frac{[-(x^2 - z^2) + (x - z) - y^2 + 4y]^2}{18y}. \quad (16)$$

Case (3)

We now turn to the third case where $(D_i^{(l)}, D_e^{(h)})$ is under either traditional or innovational dominance. Again, solving the FOC for the relevant profit maximization yields a single solution given by

$$\bar{p}_i = \frac{(x^2 - z^2) + y^2 + \left\{[(x^2 - z^2) + y^2]^2 + 32y(x - z)\right\}^{1/2}}{8}, \quad (17)$$

$$\bar{p}_e = \frac{-5[(x^2 - z^2) + y^2] + 3\left\{[(x^2 - z^2) + y^2]^2 + 32y(x - z)\right\}^{1/2}}{8}. \quad (18)$$

The corresponding boundary conditions are (D) when traditional dominance prevails and (B) when innovational dominance prevails. The profit functions are

$$\pi_i(\bar{p}_i[(z, 0), (x, y)], \bar{p}_e[(z, 0), (x, y)]) = \frac{[K_1 + K_2^{1/2}]^3}{1024y(x - z)}, \quad (19)$$

$$\pi_e(\bar{p}_i[(z, 0), (x, y)], \bar{p}_e[(z, 0), (x, y)]) = \frac{1}{8}[-5K_1 + 3K_2^{1/2}] \cdot \left\{1 - \frac{[K_1 + K_2^{1/2}]^2}{128y(x - z)}\right\} \quad (20)$$

$$\text{where } K_1 = (x^2 - z^2) + y^2,$$

$$K_2 = K_1^2 + 32y(x - z).$$

As Proposition 1 guarantees the existence and uniqueness of the equilibrium prices, we should point out that the above prices are indeed the equilibrium prices since the FOC's (the necessary conditions) yield single solution(s). Fig 3 illustrates the active boundary conditions inside the unit square for a given z .

[Figure 4.3 about here]

5. Product equilibrium

According to the order in which the game is played, the entrant enters the market and makes decision about her product characteristics in the second stage. Expecting that equilibrium prices will be chosen when proceeding into the third stage, e maximizes her profit by selecting the optimal product attributes (x^*, y^*) with a known z , that is,

$$\text{Max}_{(x, y) \in [0, 1]^2} \pi_e(x, y; z).$$

Case by case, we then derive the optimal product characteristics chosen by the entrant and arrive at our central result:

Proposition 2. When the entrant is about to make her decision on the product characteristics at the second stage of the game, she always finds $(\frac{1}{2}, 1)$ dominating any other location in the unit square characteristics space regardless what z has been chosen in the first stage.

Case (1)

(12) is the corresponding profit function under traditional dominance with linear demand segments for the entrant. The FOC's for maximization of (15) yield the following expression

$$\frac{\partial \pi_e}{\partial x} = \frac{(4x - 3x^2 - y + y^2 - 4z + 4xz - z^2)(4x - x^2 + y - y^2 - 4z + z^2)}{18(x - z)^2}, \quad (21)$$

$$\frac{\partial \pi_e}{\partial y} = \frac{(1 - 2y)(4x - x^2 + y - y^2 - 4z + z^2)}{9(x - z)}. \quad (22)$$

Observe that the component $(4x - x^2 + y - y^2 - 4z + z^2) = [(2 - z)^2 - (2 - x)^2] + [(\frac{1}{2})^2 - (y - \frac{1}{2})^2]$ is always positive when $z \leq x$ and $y \in [0, 1]$. Therefore the sign of (22) is determined by the expression $(1 - 2y)$. Meanwhile the other component $(4x - 3x^2 - y + y^2 - 4z + 4xz - z^2)$ in (21) is also positive since it can be rewritten as $[(x - z) - y] + y^2 + (x - z)(3 - 3x + z)$ with $x \geq z$ and $x \in [0, 1]$. These imply that $(1, \frac{1}{2})$ dominates the rest of the traditional dominance location when the demand is in linear segment for $z \in [0, \frac{1}{2}]$. However, when z exceeds $\frac{1}{2}$, as z moves along the X-axis, $(1, \frac{1}{2})$ no longer lies in the traditional dominance space. Since from (22) we know $\frac{\partial \pi_e}{\partial y} \geq 0$ for $y \leq \frac{1}{2}$, the FOC's yield the boundary solution $(1, 1 - z)$ which later on will be compared to other candidates for the optimal location.

Case (2)

Recall that (16) is the derived profit function under innovational dominance with linear demand segments for the entrant. The FOC's for maximization of (15) yield the following expressions

$$\frac{\partial \pi_e}{\partial x} = \frac{(1 - 2x)(x - x^2 + 4y - y^2 - z + z^2)}{9y}, \quad (23)$$

$$\frac{\partial \pi_e}{\partial y} = \frac{(-x + x^2 + 4y - 3y^2 + z - z^2)(x - x^2 + 4y - y^2 - z + z^2)}{18y^2}. \quad (24)$$

It is easy to verify that (24) is positive while the sign of (23) is the same as that of $(1 - 2x)$. Note that $(x - x^2 + 4y - y^2 - z + z^2) = [(x - z) + (y - y^2) + (3y - (x + z)(x - z))]$ $> (x - z)(3 - x + z) \geq 0$ since $z \leq x$, $y \geq x - z$ (innovational dominance), and $y \in [0, 1]$. A similar argument can be applied to the other component: $(-x + x^2 + 4y - 3y^2 + z - z^2) = [(x^2 - z^2) + 3(y - y^2) + y - (x - z)] > 0$. Again, one has to be cautious when z exceeds $\frac{1}{2}$. As explained earlier, we may assume without loss of generality that the entrant never locates to the left to the incumbent, i.e., $x \geq z$, then the best **e** can do with $z \geq \frac{1}{2}$ is to be located at $(z, 1)$. Notice that $\pi_e[(z, 1; z) = \frac{1}{2}$ for $z \in (\frac{1}{2}, 1)$. Both $(\frac{1}{2}, 1)$ and $(z, 1)$ will be taken into account when selecting the optimal location of **e**.

We will not directly pursue the FOC's for Case (3), the demand specification $(D_i^{(l)}, D_e^{(h)})$ under either dominance types because the solutions do not yield immediate intuition. Instead, we provide lemma 1 and lemma 2 to demonstrate the dominance of location $(\frac{1}{2}, 1)$ with respect to all the locations specified in Case (3).

Lemma 1. Given the transformation rule: $\hat{x} = y + z$ and $\hat{y} = x - z$, for every location (\hat{x}, \hat{y}) in *Case (3) under traditional dominance* we can symmetrically find a unique location (x, y) in *Case (3) under innovational dominance* such that the entrant makes greater or equal profit by choosing (x, y) over (\hat{x}, \hat{y}) .

By lemma 1, we can ignore the lower right-hand half of the area in *Case (3)* as the candidate for the consideration of a optimal location. The other important observation from the proof of lemma 1 is that if **e** were to choose (\hat{x}, \hat{y}) over (x, y) , both the price p_e and demand D_e would decrease so that her profit would diminish accordingly.

Lemma 2: The maximal profit the entrant can attain when she faces *medium demand under innovational dominance* is always greater than that in *Case (3) where she faces high demand*.

Lemma 2 excludes the possibility of finding a optimal characteristics combination in *Case (3)* since $(\frac{1}{2}, 1)$ or $(z, 1)$ can always make the entrant better off. Hence we are close to a complete proof of our main result, Proposition 2. The remainder of its proof is postponed to the Appendix.

One primary finding of Proposition 2 is that the optimal choice of attributes $(\frac{1}{2}, 1)$ for the entrant is independent of the attributes $(z, 0)$ chosen by the incumbent although **i**'s decision does affect **e**'s profit. More specifically, while utilizing the capacity to innovate up to a "safe" distance away from the incumbent so as to soften price competition, the entrant sticks to a central position which is favorable to the purchasing power of consumers. The entrant masters this balancing act by choosing $(\frac{1}{2}, 1)$. Another interesting feature of this type of competition is that **e** appears to be better off in an innovational dominance situation than in a traditional dominance circumstance for all possible market divisions (Lemma 1 and

Proposition 2). This is most easily seen when $\dot{\mathbf{i}}$ moves away from 0: Although enjoying the same payoff as at $(\frac{1}{2}, 1)$ under innovational dominance when $z=0$, the best variety combination $(1, \frac{1}{2})$ under traditional dominance induces fiercer price competition due to a larger shrinkage of the distance between her and the rival.⁸ In other words, we may interpret this phenomenon as an incentive to innovate in a weak sense.

Now we move one stage backwards to the characteristics decision made by the incumbent: fully anticipating the location chosen by the entrant and the last stage price competition, $\dot{\mathbf{i}}$ chooses her optimal location. Since \mathbf{e} 's second stage location decision is not affected by z , the optimization problem for $\dot{\mathbf{i}}$ is then

$$\text{Max}_{z \in [0,1]} \pi_i(\frac{1}{2}, 1; z).$$

We observe that $(\frac{1}{2}, 1)$ for \mathbf{e} always constitutes a $(\tilde{D}_i^{(m)}, \tilde{D}_e^{(m)})$ innovational dominance situation. So the only legitimate profit function for $\dot{\mathbf{i}}$ is (15). With $x=\frac{1}{2}$ and $y=1$, the straightforward optimizer is $z^*=\frac{1}{2}$.

To reiterate, we find a unique subgame perfect equilibrium in which the principle of minimal differentiation holds for the traditional variety spectrum (X) with $x^*=z^*=\frac{1}{2}$ while maximal differentiation prevails in the innovated variety dimension (Y) with $y^*=1$. Our intuition for this result is that two forces are at work here: On one hand being endowed with the capacity to differentiate products in an expanded dimension, the entrant fully exploits this advantage since the resulting much less fierce price competition promotes profitability.

⁸Suppose the incumbent moves δ away from 0, then the distance (or equivalently, the square of it) between $(\delta, 0)$ and $(1, \frac{1}{2})$ becomes smaller than that between $(\delta, 0)$ and $(\frac{1}{2}, 1)$ since $(\frac{1}{2})^2 + (1-\delta)^2 < 1^2 + (\frac{1}{2}-\delta)^2$.

On the other hand, both firms are inclined to be centrally located in the product space because they are approaching the mass of the consumers hence reducing the costs of delivering goods. Thus it appears to be a sensible result in this framework that the mixed version of the two opposite principles of product differentiation is unearthed. By the same token, it does not come as a surprise that the product selections with maximally differentiated firms on both dimensions are not an equilibrium although they yield the same payoffs as the unique equilibrium.⁹

6. Concluding Remarks

We have identified a unique price and product equilibrium in a two-dimensional product market where sequential expansion of dimensionality is endogenized. In contrast to the intuition predicting maximally differentiated duopoly in the standard one-dimensional model, our result indicates that both principles of maximum and minimum differentiation persist simultaneously in an industry where the potential entrant has a high tendency to take on the R&D project. Upon deriving the equilibrium, we also justify **e**'s incentive to innovate, that is, her willingness to be involved in an innovational dominance rather than a traditional dominance competition. In a similar context, we are able to overcome the multiplicity of product equilibria derived from a two-dimensional quality-and-variety model (for example, Neven and Thisse, 1990). Regarding the endogenous formation of dimensionality in a sequentially growing market, our results also demonstrate the need to search for a more adequate theoretic foundation for the location choices made by the firms, although simulations have yielded important insights (Swann, 1990).

⁹One might have also noticed that the same conclusion holds even if the game is modeled as a simultaneous move two-stage game due to the dominance of location ($x^* = \frac{1}{2}$, $y=1$) chosen by **e**.

Evidently, many other important elements could be incorporated into this simple model to resemble real world competition more closely. For instance, the uncertainty in successfully developing an innovation as well as firms' choices among different R&D technologies (involving different techniques and different degree of risk) have been the central issues in the patent race literature (Dasgupta and Stiglitz (1980), Harris and Vickers (1987), Reinganum (1983)). The entrant's decision on whether and how much to invest in R&D should be endogenized and be reflected by the (expected) payoffs. It then becomes natural to introduce informational asymmetries since firms do not always perfectly observe rivals' costs. The incumbent may signal its costs through locations in such an environment. Furthermore, there is uncertainty about the demand associated with a successful innovation. The resulting different reservation price schemes or the reformulation of population density (in terms of consumer types) reflect just a few of the many possible extensions of the basic model.

An important direction for future research will be to analyze whether these results will persist under less restrictive assumptions with respect to consumer utility function, namely, the quadratic transport cost function. For instance, Caplin and Nalebuff (1991) assert that a utility function which is linear in characteristics and exhibits ρ -concavity of the distribution of consumer taste parameters is sufficient for the existence of pure-strategy price equilibrium. However, it is unclear to what extent this result may be applied to obtain the product equilibrium in a two-stage setting. Meanwhile, the answer to the question how firms compete in a sequentially growing market with many variety and quality indices remains open since our result is basically confined to an abstract two-dimensional variety space.

APPENDIX

Derivation of the demand functions

Given p_e , the threshold prices $\bar{p}_i^{(1)}$ for D_i to be zero can be derived from the indifferent line which just passes through $(0, 0)$ in the unit square. From (2)

$$\bar{p}_i^{(1)} = p_e + (x^2 + y^2 - z^2). \quad (\text{A.1})$$

Similarly, $\bar{p}_i^{(2)}$ and $\tilde{p}_i^{(2)}$ are the prices for D_i to be just in the boundary of convex and linear regions, under traditional and innovational dominance respectively. Whereas $\bar{p}_i^{(2)}$ makes $\bar{y}(\bar{x})$ pass first through $(0, 1)$, $\tilde{p}_i^{(2)}$ makes $\bar{y}(\bar{x})$ pass first through $(1, 0)$. Therefore

$$\bar{p}_i^{(2)} = p_e + (x^2 + y^2 - z^2) - 2y, \quad (\text{A.2})$$

$$\tilde{p}_i^{(2)} = p_e + (x^2 + y^2 - z^2) - 2(x - z). \quad (\text{A.3})$$

In order to aggregate demand in the latter analysis, we denote \bar{x} as the intersection point of $\bar{y}(x)$ with the upper side of the unit square and \bar{x} with the lower side. When p_i is in the relevant range such that $D_i = D_i^{(1)}$, the indifferent line has the property of $\bar{x}=0$ and intercepts X-axis at

$$(\bar{x} = \frac{p_e - p_i + (x^2 + y^2 - z^2)}{2(x - z)}, y=0).$$

Correspondingly, $\bar{p}_i^{(3)}$ and $\tilde{p}_i^{(3)}$ are the prices for D_i to be in the boundary of linear and concave regions, under traditional and innovational dominance respectively. Again $\bar{p}_i^{(3)}$ passes through $(1, 0)$ and $\tilde{p}_i^{(3)}$ passes through $(0, 1)$. So we have

$$\bar{p}_i^{(3)} = \tilde{p}_i^{(2)} = p_e + (x^2 + y^2 - z^2) - 2(x - z), \quad (\text{A.4})$$

$$\tilde{p}_i^{(3)} = \bar{p}_i^{(2)} = p_e + (x^2 + y^2 - z^2) - 2y. \quad (\text{A.5})$$

When p_i is in the relevant range such that $D_i = \bar{D}_i^{(m)}$, the indifferent line intersects $Y=1$ at $(\bar{x} = \frac{p_e - p_i + (x^2 + y^2 - z^2) - 2y}{2(x - z)}, y=1)$ and X -axis at $(\bar{x} = \frac{p_e - p_i + (x^2 + y^2 - z^2)}{2(x - z)}, y=0)$.

Respectively,

for $D_i = \bar{D}_i^{(m)}$ we then have $\bar{x}=0$ and $\bar{x}=1$.

Last, $p_i^{(4)}$ for D_i to be equal to 1, under either dominance, can be derived as follow

$$p_i^{(4)} = p_e + (x^2 + y^2 - z^2) - 2y - 2(x - z). \quad (A.6)$$

\bar{x} for this price range is $\frac{p_e - p_i + (x^2 + y^2 - z^2) - 2y}{2(x - z)}$ whereas $\bar{x} = 1$.

With all the prices and \bar{x} specified above, i 's demand is just the aggregation of the area covered under (to the left of) the indifference curve inside the unit square, i.e., $\bar{x} + \int_{\bar{x}}^1 \bar{y}(x)dx$.

Thus we have

$$D_i^{(1)} = 0 + \int_0^{\bar{x}} \bar{y}(x)dx = \frac{[(p_e - p_i^{(1)}) + (x^2 - z^2) + y^2]^2}{8y(x - z)}, \quad (A.7)$$

$$\bar{D}_i^{(m)} = \bar{x} + \int_{\bar{x}}^1 \bar{y}(x)dx = \frac{[(p_e - p_i^{(2)}) + (x^2 - z^2) + y^2 - y]}{2(x - z)}, \quad (A.8)$$

$$\bar{D}_i^{(m)} = 0 + \int_0^1 \bar{y}(x)dx = \frac{[(p_e - p_i^{(2)}) + (x^2 - z^2) + y^2 - (x - z)]}{2y}, \quad (A.9)$$

$$D_i^{(h)} = \bar{x} + \int_{\bar{x}}^1 \bar{y}(x)dx = \frac{8y(x - z) - [(p_e - p_i^{(3)}) + (x^2 - z^2) + y^2 - 2(x - z) - 2y]^2}{8y(x - z)}. \quad (A.10)$$

Derivation of the boundary conditions

Recall (9) and (10) are the equilibrium prices for both demand to be in the linear segment, i.e., $(D_i^{(m)}, D_e^{(m)})$, under traditional dominance. To be compatible with these demand

specification, the equilibrium prices have to lie within the interval

$$\bar{p}_i^* \in [\bar{p}_i^{(3)}(\bar{p}_e^*), \bar{p}_i^{(2)}(\bar{p}_e^*)] \text{ and } \bar{p}_e^* \in [\bar{p}_e^{(3)}(\bar{p}_i^*), \bar{p}_e^{(2)}(\bar{p}_i^*)].$$

With (A.2), (A.4), we can show $\bar{p}_i^* \geq \bar{p}_i^{(3)}(\bar{p}_e^*)$ is satisfied if and only if

$$(A) (x - z)[2 + (x + z)] \geq 4y - y^2,$$

And $\bar{p}_i^* \leq \bar{p}_i^{(2)}(\bar{p}_e^*)$ is satisfied if and only if

$$(B) (x - z)[4 - (x + z)] \geq 2y + y^2.$$

Similar computation can show that $\bar{p}_e^* \in [\bar{p}_e^{(3)}(\bar{p}_i^*), \bar{p}_e^{(2)}(\bar{p}_i^*)]$ are satisfied if and only if both (A) and (B) hold. Therefore we conclude that (A) and (B) are the compatible (boundary) conditions for the demands to be in the linear segment under traditional dominance.

Once we repeat the process for the case of linear demand specification under innovational dominance, we find that $\tilde{p}_i^* \in [\tilde{p}_i^{(3)}(\tilde{p}_e^*), \tilde{p}_i^{(2)}(\tilde{p}_e^*)]$ and $\tilde{p}_e^* \in [\tilde{p}_e^{(3)}(\tilde{p}_i^*), \tilde{p}_e^{(2)}(\tilde{p}_i^*)]$ are satisfied if and only if

$$(C) (x - z)[4 - (x + z)] \leq 2y + y^2,$$

$$(D) (x - z)[2 + (x + z)] \leq 4y - y^2.$$

We then show the compatible condition for Case (3) where demands are specified as $(D_i^{(l)}, D_e^{(h)})$ under traditional dominance. It is easy to check that $\bar{p}_i \leq p_i^{(1)}(\bar{p}_e)$ and $\bar{p}_e \geq p_e^{(4)}(\bar{p}_i)$ are always satisfied and $\bar{p}_i \geq \bar{p}_i^{(2)}(\bar{p}_e)$ and $\bar{p}_e \leq \bar{p}_e^{(3)}(\bar{p}_i)$ both lead to condition (D). When innovational dominance prevails, it turns out that $\bar{p}_i \leq p_i^{(1)}(\bar{p}_e)$ and $\bar{p}_e \geq p_e^{(4)}(\bar{p}_i)$ are again satisfied and $\bar{p}_i \geq \bar{p}_i^{(2)}(\bar{p}_e)$ and $\bar{p}_e \leq \bar{p}_e^{(3)}(\bar{p}_i)$ lead to condition (B).

In a similar fashion, we claim that there is no location (characteristics) in the unit square compatible with demand specification $(D_i^{(h)}, D_e^{(l)})$ and the corresponding boundary conditions under either type of dominance. For instance, with $\bar{D}_i^{(h)}$ in mind, $\bar{p}_i \leq \bar{p}_i^{(3)}(\bar{p}_e)$ leads to (C)). Notice that (C) would never satisfy the traditional dominance constraint, $x - z \geq y$, unless x is located outside the unit square, i.e., $x \geq 1$. While in the $\tilde{D}_i^{(h)}$ case, $\bar{p}_i \leq \bar{p}_i^{(3)}(\bar{p}_e)$ leads to (A) which simply cannot satisfy the innovational dominance constraint, $x - z \leq y$. Thus we confirm our claim.

Proof for Lemma 1

Proof First we observe that (x, y) and (\hat{x}, \hat{y}) are symmetric to the line $y = x - z$ with given z in the unit square. Notice that when (C) and (D) hold as equalities, they are not only symmetric to the same line but also construct the bounds for $(D_i^{(l)}, D_e^{(h)})$. Therefore we have an one-to-one correspondence for (x, y) under innovational dominance and (\hat{x}, \hat{y}) under traditional dominance. Recall that the functional form of the profit function is not affected by the type of dominance so we can compare the following

(A.11)

$$\pi_e[(x, y); z] = \frac{1}{8}[-5K_1(x, y) + 3K_2(x, y)^{1/2}] \cdot \left\{ 1 - \frac{[K_1(x, y) + K_2(x, y)^{1/2}]^2}{128y(x - z)} \right\}$$

(A.12)

$$\pi_e[(\hat{x}, \hat{y}); z] = \frac{1}{8}[-5K_1(\hat{x}, \hat{y}) + 3K_2(\hat{x}, \hat{y})^{1/2}] \cdot \left\{ 1 - \frac{[K_1(\hat{x}, \hat{y}) + K_2(\hat{x}, \hat{y})^{1/2}]^2}{128\hat{y}(\hat{x} - \hat{z})} \right\}$$

We need to establish some preliminary results. Given the following conditions

$$x, y, z \in [0,1], \quad (\text{A.13})$$

$$x \geq z, \quad (\text{A.14})$$

$$y \geq (x - z), \text{ and} \quad (\text{A.15})$$

$$(x-z)(4 - x - z) \geq 2y + y^2 \quad (\text{A.16})$$

we can construct the implied inequalities:

$$x^2 + y^2 - z^2 \leq 2y, \text{ (from (A.15) and (A.16))} \quad (\text{A.17})$$

$$[(x^2 + y^2 - z^2)^2 + 32y(x-z)]^{1/2} \leq 6y, \text{ (from (A.15) and (A.17))} \quad (\text{A.18})$$

$$4y(x - z) \geq 2y^2 + y(x^2 + y^2 - z^2) \geq 2y^2 \text{ (from (A.15))} \quad (\text{A.19})$$

Note that both two terms in the profit function are always nonnegative. More precisely,

$$-5K_1 + 3K_2^{1/2} \geq 0$$

$$\Leftrightarrow 9 \times 32y(x - z) \geq 16(x^2 + y^2 - z^2)^2$$

This is implied by

$$32y(x - z) \geq 16y^2 \text{ and } (x^2 + y^2 - z^2) \leq 2y$$

which are directly deduced from (A.17) and (A.19). We thus have the desired nonnegativity of first term. As for the second term,

$$\left\{ 1 - \frac{[K_1 + K_2^{1/2}]^2}{128y(x - z)} \right\} \geq 0$$

$$\Leftrightarrow 128y(x - z) \geq (K_1 + K_2^{1/2})^2,$$

which can be shown to be the case by combining (A.17) and (A.18).

With substitution of variables, it is trivial that $y(x - z) = \hat{y}(\hat{x} - z)$. We then show with

$$y \geq (x - z)$$

$$\begin{aligned} K_1(\hat{x}, \hat{y}) &= \hat{x}^2 + \hat{y}^2 - z^2 \\ &= (\hat{x} + z)(\hat{x} - z) + \hat{y}^2 \\ &= (y + 2z)y + (x - z)^2 \\ &= x^2 + y^2 + z^2 + 2z(y - x) \\ &\geq x^2 + y^2 - z^2 = K_1(x, y). \end{aligned}$$

Similarly,

$$\begin{aligned} K_2(\hat{x}, \hat{y}) &= K_1(\hat{x}, \hat{y})^2 + 32\hat{y}(\hat{x} - z) \\ &= K_1(\hat{x}, \hat{y})^2 + 32y(x - z) \\ &\geq K_1(x, y)^2 + 32y(x - z) = K_2(x, y). \end{aligned}$$

It is now straightforward that the second term in the profit function

(A.20)

$$\left\{ 1 - \frac{[K_1(x, y) + K_2(x, y)^{1/2}]^2}{128y(x - z)} \right\} \geq \left\{ 1 - \frac{[K_1(\hat{x}, \hat{y}) + K_2(\hat{x}, \hat{y})^{1/2}]^2}{128\hat{y}(\hat{x} - \hat{z})} \right\}.$$

In the sequel we denote $k \equiv 2z^2 + 2z(y - x)$. Note that k is nonnegative in the specified region. We turn our attention to the first term in the profit function and claim the following inequality

$$(*) \quad [-5K_1(x, y) + 3K_2(x, y)^{1/2}] \geq [-5K_1(\hat{x}, \hat{y}) + 3K_2(\hat{x}, \hat{y})^{1/2}].$$

This holds if and only if

$$\left\{ 5k + 3[K_1(x, y)^2 + 32y(x - z)]^{1/2} \right\} \geq 3\{[K_1(x, y) + k]^2 + 32y(x - z)\}^{1/2}$$

$$\Leftrightarrow 144K_1(x, y)[k + K_1(x, y)] + 225 \times 32y(x - z) \geq 64k^2.$$

It is fairly easy to show that $k = 2z^2 + 2z(y - x) = 2z[y - (x - z)] \leq 2xy$. Furthermore, from the fact that (B) holds we can infer the following inequalities

$$\begin{aligned}
(B) \quad & (x - z)(4 - x - z) \geq y^2 + 2y \\
\Leftrightarrow & 4(x - z) \geq x^2 + y^2 - z^2 + 2y \\
\Leftrightarrow & 4y(x - z) \geq 2y^2 + yK_1(x, y) \\
\Rightarrow & 225 \times 32y(x - z) \geq 225 \times 16y^2 \geq 64 \times 4x^2y^2 \geq 64k^2.
\end{aligned}$$

This confirms claim (*).

With (A.20) and (*), we complete the proof. $\square\square$

Proof for Lemma 2

Proof First, we start with the case of $z=0$. In order to derive the optimal location within the Case (3) region in which (B) holds and $y \geq x$, we show that given (x, y) \mathbf{e} can make himself at least as well off by moving towards the line $y=x$, that is

$$\nabla(\pi_e[(x, y); z=0]) \cdot (1, -1) = \left(\frac{\partial \pi_e[(x, y); z=0]}{\partial x}, \frac{\partial \pi_e[(x, y); z=0]}{\partial y} \right) \cdot (1, -1) \geq 0$$

Simple manipulation shows that this is implied by $x \geq y^3$ and $x \geq y^5$ which are always satisfied when (B) holds. That is,

$$\begin{aligned}
\nabla \pi[(x, y); z] \cdot (1, -1) = & \frac{x-y}{256x^2y^2 \sqrt{(x^2+y^2)^2+32xy}} \times \{ [(x^8+16x^5y+6x^7y-1152x^2y^2+272x^4y^2 \\
& +4x^6y^2+32x^3y^3+18x^5y^3+272x^2y^4+6x^4y^4+16xy^5+18x^3y^5+4x^2y^6+6xy^7+y^8) + [(\sqrt{(x^2+y^2)^2+32xy}) \\
& \cdot (x^6+6x^5y-336x^2y^2+3x^4y^2+12x^3y^3+3x^2y^4+6xy^5+y^6)] \}.
\end{aligned}$$

Notice that with $y \geq x$ and $x \geq y^3$ and $x \geq y^5$ the coefficients of those two negative terms are large enough to offset the sum of the other terms so that we have nonnegative result. Thus the optimization problem can be reduced to

$$\text{Max}_{z \in [0,1]} \pi_e[(x, x; z=0)]$$

which yields the solution $x^* = \frac{\sqrt{-15+3\sqrt{33}}}{2}$. The corresponding profit is $\pi_e^* \simeq 0.544667 < \pi_e[(\frac{1}{2}, 1); z=0]$. We also observe that the difference between $\pi_e[(\frac{1}{2}, 1); z]$ and $\pi_e[(x, y); z]$ is increasing in z . So we conclude that $(\frac{1}{2}, 1; z)$ dominates $(\bar{x}, \bar{y}; z)$ for $z \in [0, \frac{1}{2}]$. Figure A.1 illustrates the increasing property of the maximum for the function $\pi(x, y, z) \equiv \pi_e[(\frac{1}{2}, 1); z] - \pi_e[(x, y); z]$.

[Figure A.1 about here]

Next, for large z ($z \geq \frac{1}{2}$) the comparison should be made between $\pi_e(z, 1; z)$ and $\pi_e(x, y; z)$. It is observed that for sufficiently large z the optimal solutions for $\text{Max } \pi_e$ are always corner solutions which can be included in Case (2). Figure A.2 depicts the contour graphs of $\pi_e(x, y; z)$ for different values of z . Therefore, $(z, 1; z)$ again dominates $(\bar{x}, \bar{y}; z)$.

[Figure A.2 about here]

Thus we complete the proof. □□

Proof for Proposition 2

Proof First we claim that the optimal location under innovational dominance $(\frac{1}{2}, 1; z)$ always achieves higher or equal profit than the optimal location under traditional dominance $(1, \frac{1}{2}; z)$ for \mathbf{e} in the interval of $z \in [0, \frac{1}{2}]$. That is, $\pi_e[(\frac{1}{2}, 1); z] \geq \pi_e[(1, \frac{1}{2}); z]$ when $z \in [0, \frac{1}{2}]$. Thus we have to show

$$\pi_e[(\frac{1}{2}, 1); z] = \frac{(z^2 - z + \frac{13}{4})^2}{18} \geq \pi_e[(1, \frac{1}{2}); z] = \frac{(z^2 - 4z + \frac{13}{4})^2}{18(1 - z)}$$

This inequality is implied by

$$z \in [0, \frac{1 - \sqrt{3} + \sqrt{6\sqrt{3} - 3}}{2}] \supseteq [0, \frac{1}{2}].$$

It is then straightforward to check that $\pi_e[(z, 1); z] = \frac{1}{2} \geq \pi_e[(1, 1 - z); z]$ when $z \in (\frac{1}{2}, 1)$.

Note that $\pi_e[(1, 1 - z); z] = \frac{1-z}{2}$ is decreasing in z for $z \in (\frac{1}{2}, 1]$. It can be easily shown that

$$\frac{1}{2} > \pi_e[(1, 1 - \frac{1}{2}); \frac{1}{2}] = \frac{1}{4}.$$

Note that in the case of $y=0$, namely, no innovation at all, the entrant can achieve at most $\pi_e[(x=1, y=0); z=0] = \frac{1}{2}$ which is always less or equal to the profit guaranteed to her by selecting

$(\frac{1}{2}, 1)$ or $(z, 1)$.

Together with lemma 2, we have shown that $(\frac{1}{2}, 1)$ and $(z, 1)$ weakly dominate all other characteristics combination in the unit square for $z \in [0, \frac{1}{2}]$ and $z \in (\frac{1}{2}, 1)$ respectively. Furthermore, without the artificial restriction of $x \geq z$, the entrant will always choose $(\frac{1}{2}, 1)$ over any other variety location for any given $z \in [0, 1)$. Thus we complete the proof. \square

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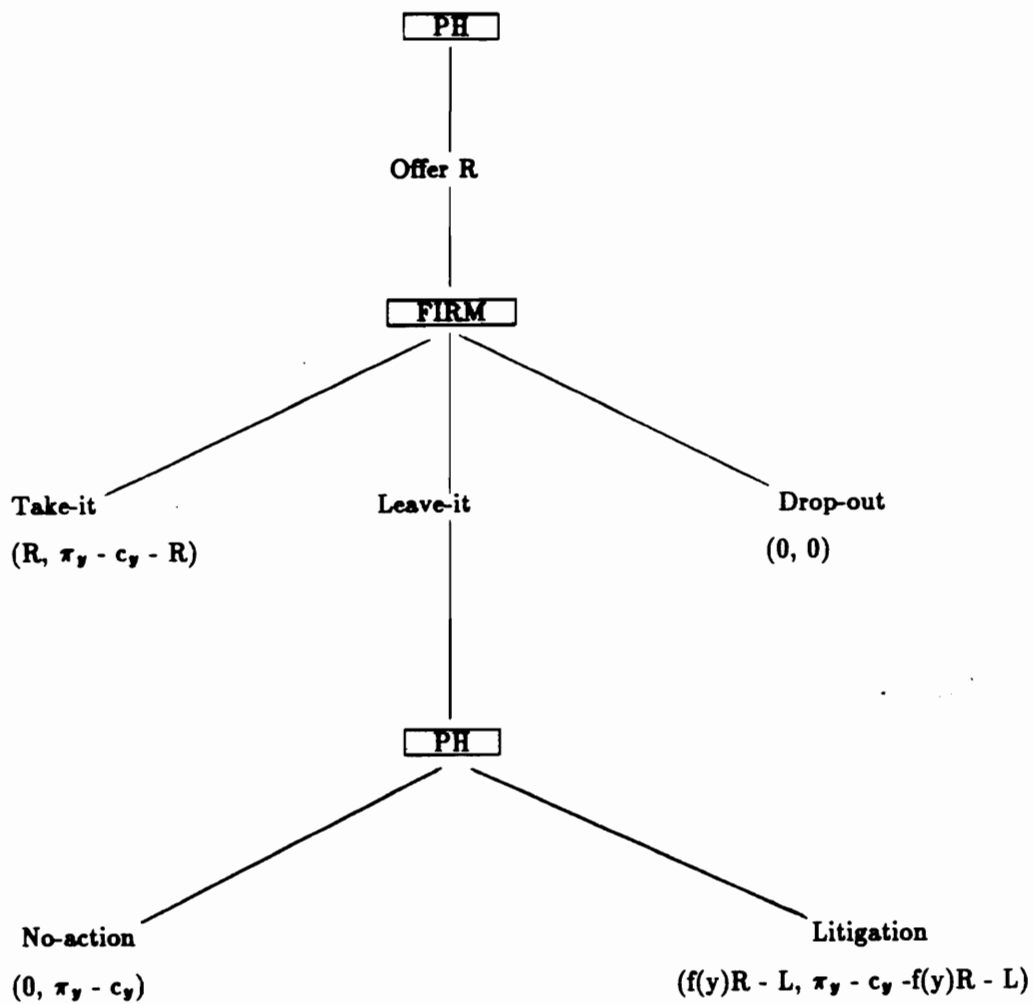


Figure 2.1: The Extensive Game

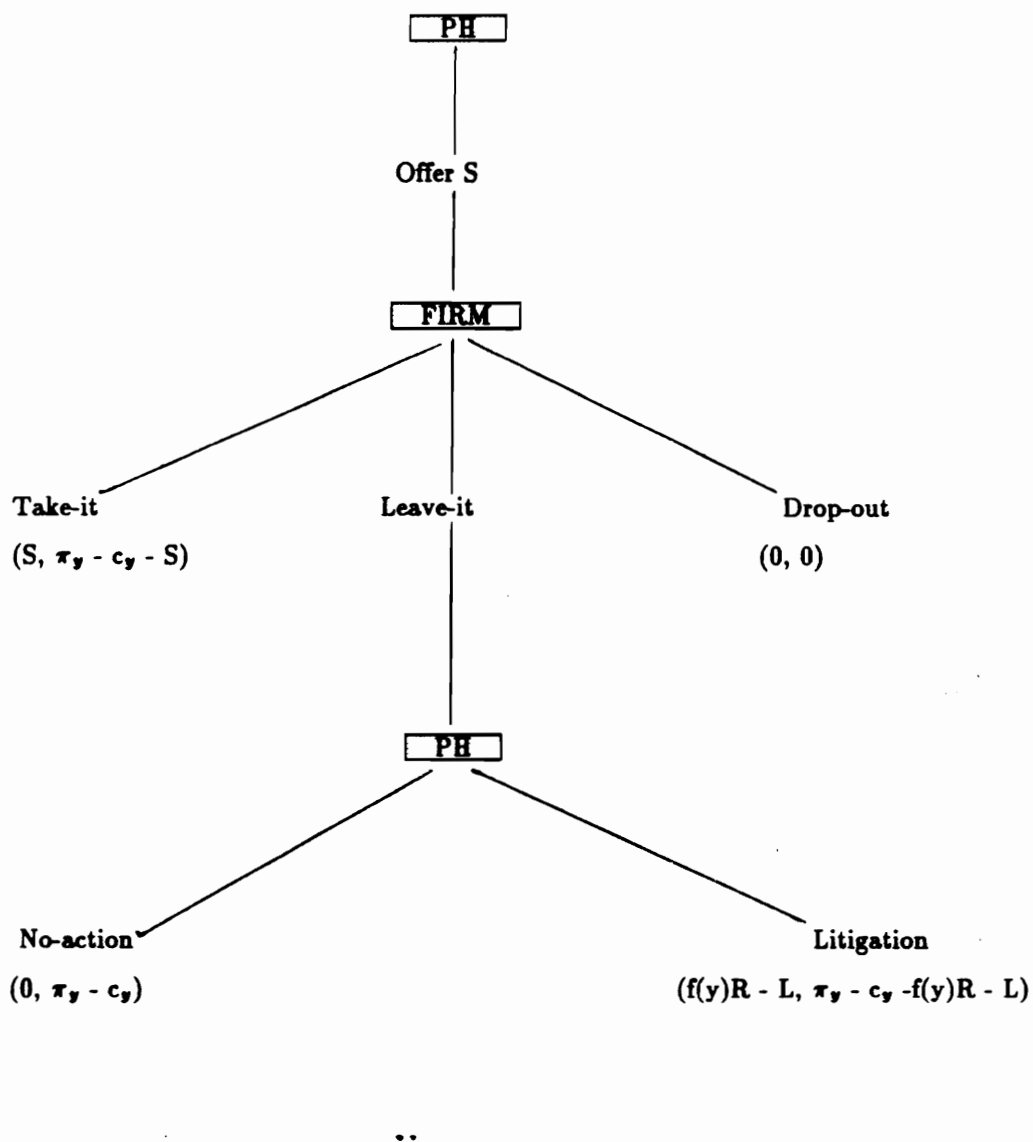


Figure 3.1: The Extensive Game

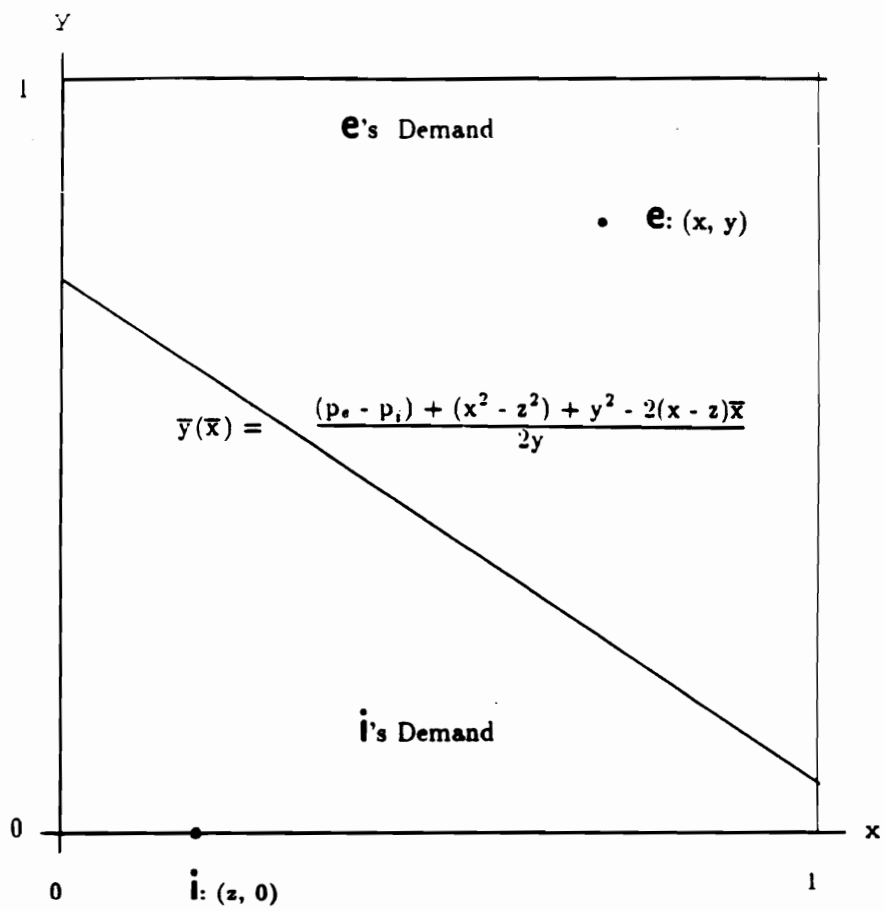


Figure 4.1: Demand for **i** and **e**

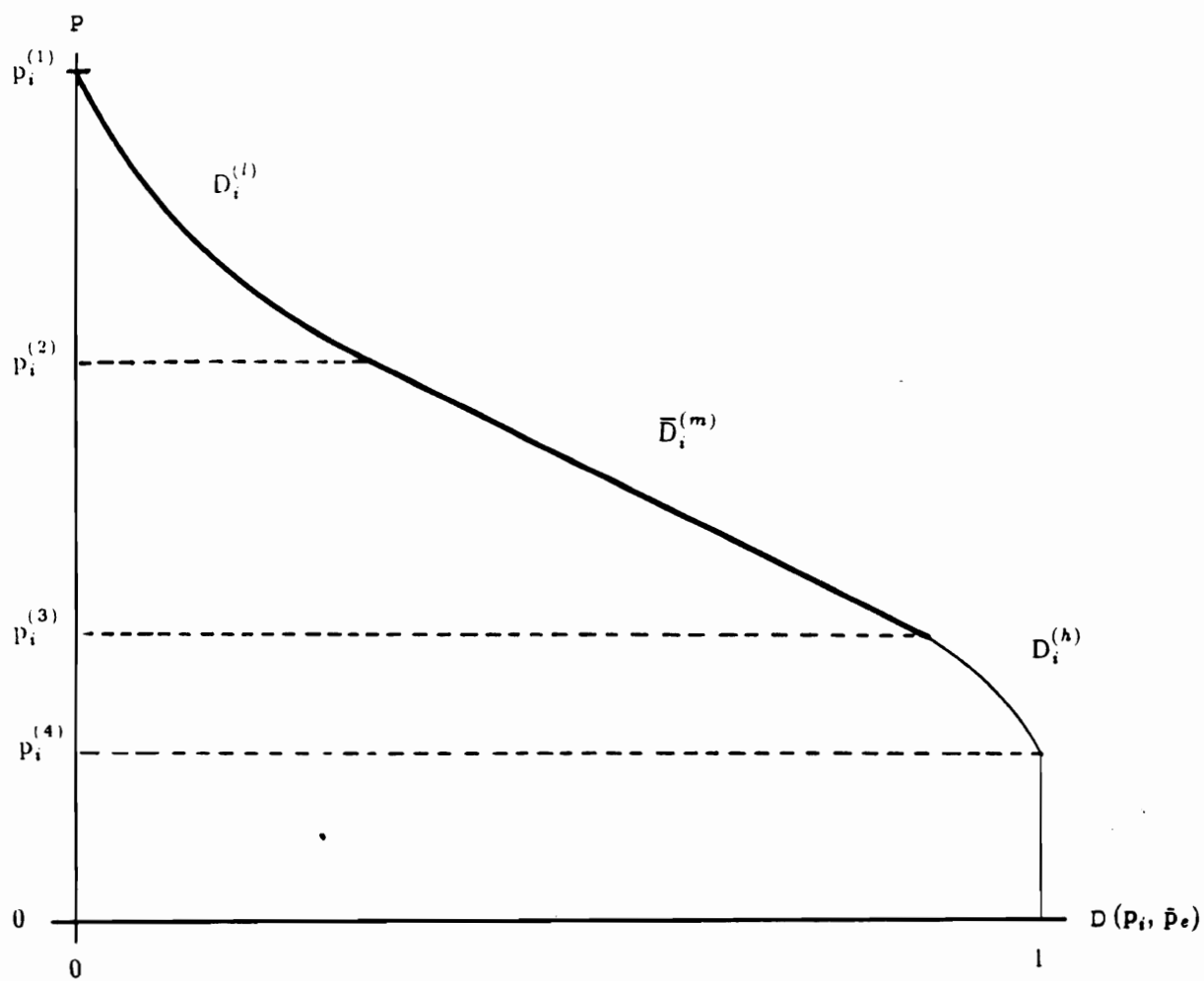


Figure 4.2: Demand functions D_i

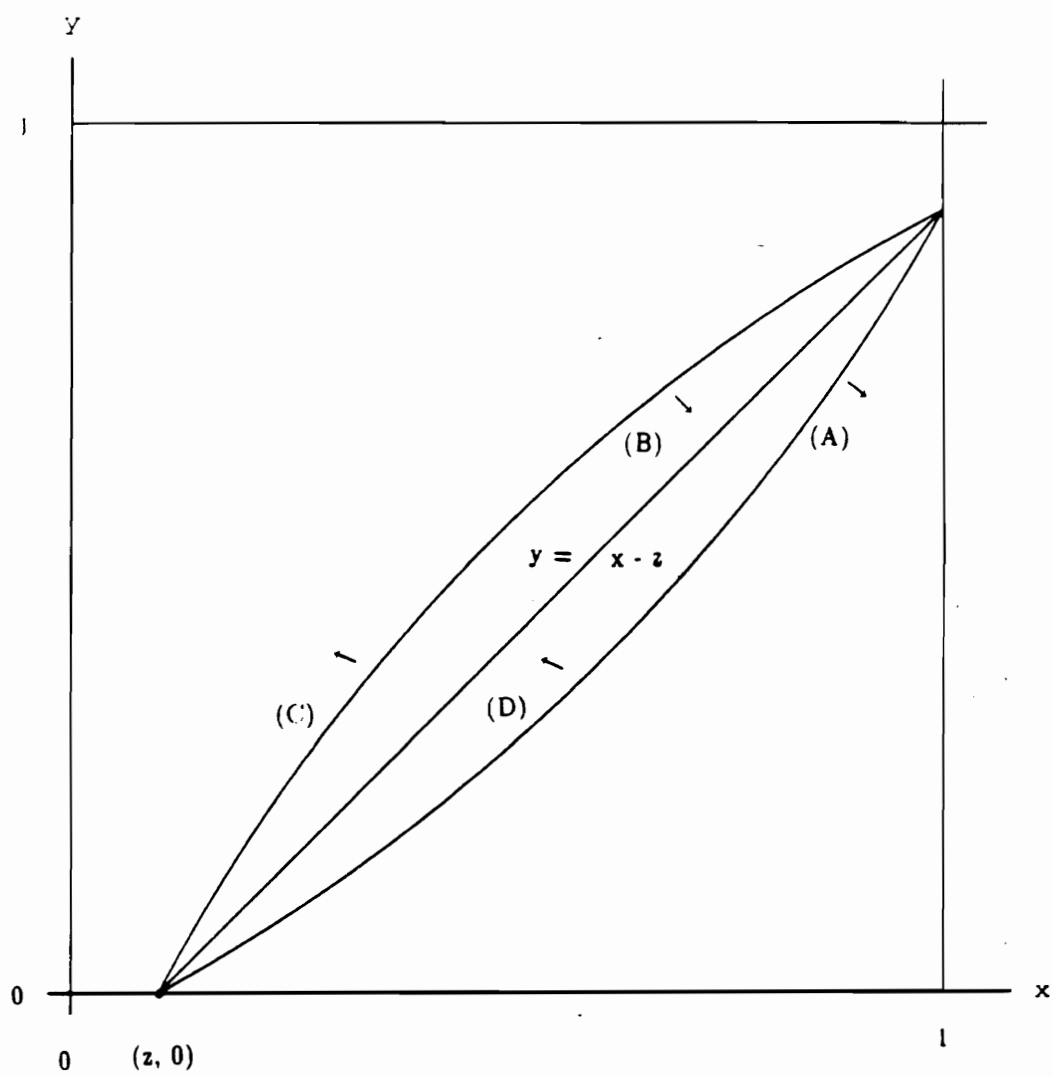
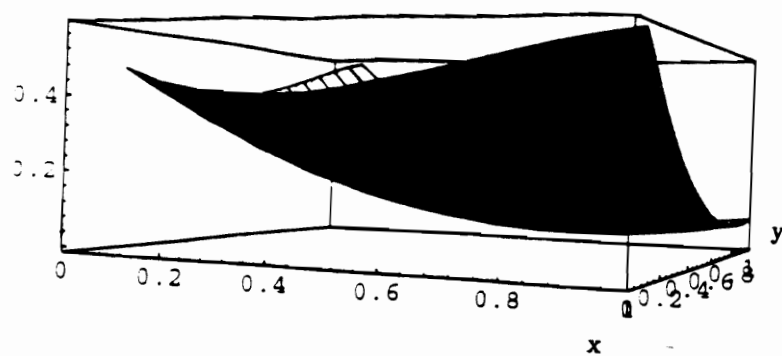
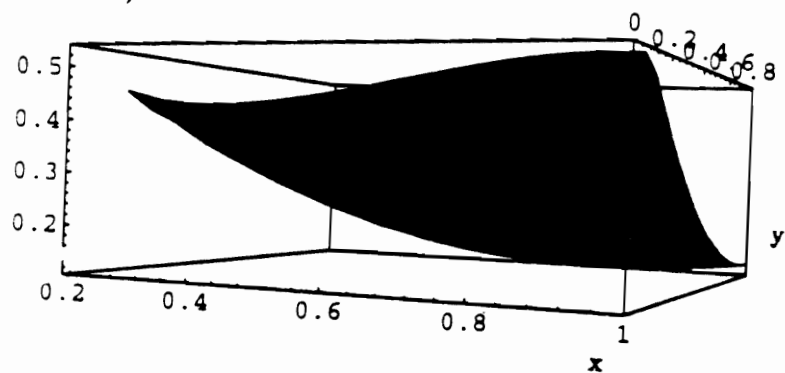


Figure 4.3: Boundary conditions for different dominance types

$$\pi(x,y; z=0)$$



$$\pi(x,y; z=.2)$$



$$\pi(x,y; z=.5)$$

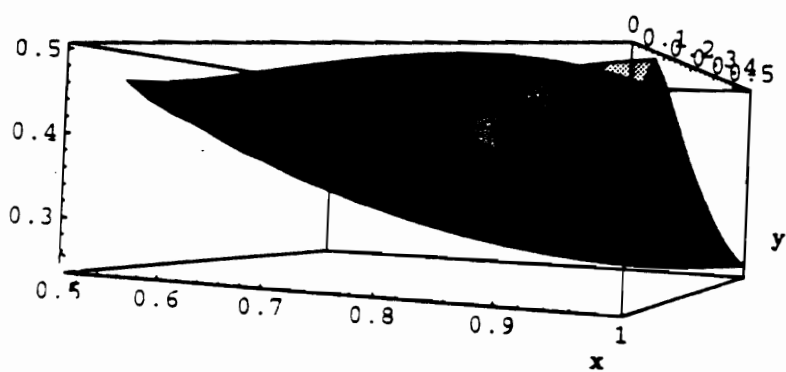


Figure A.1 $\pi(x,y; z)$ for selected z

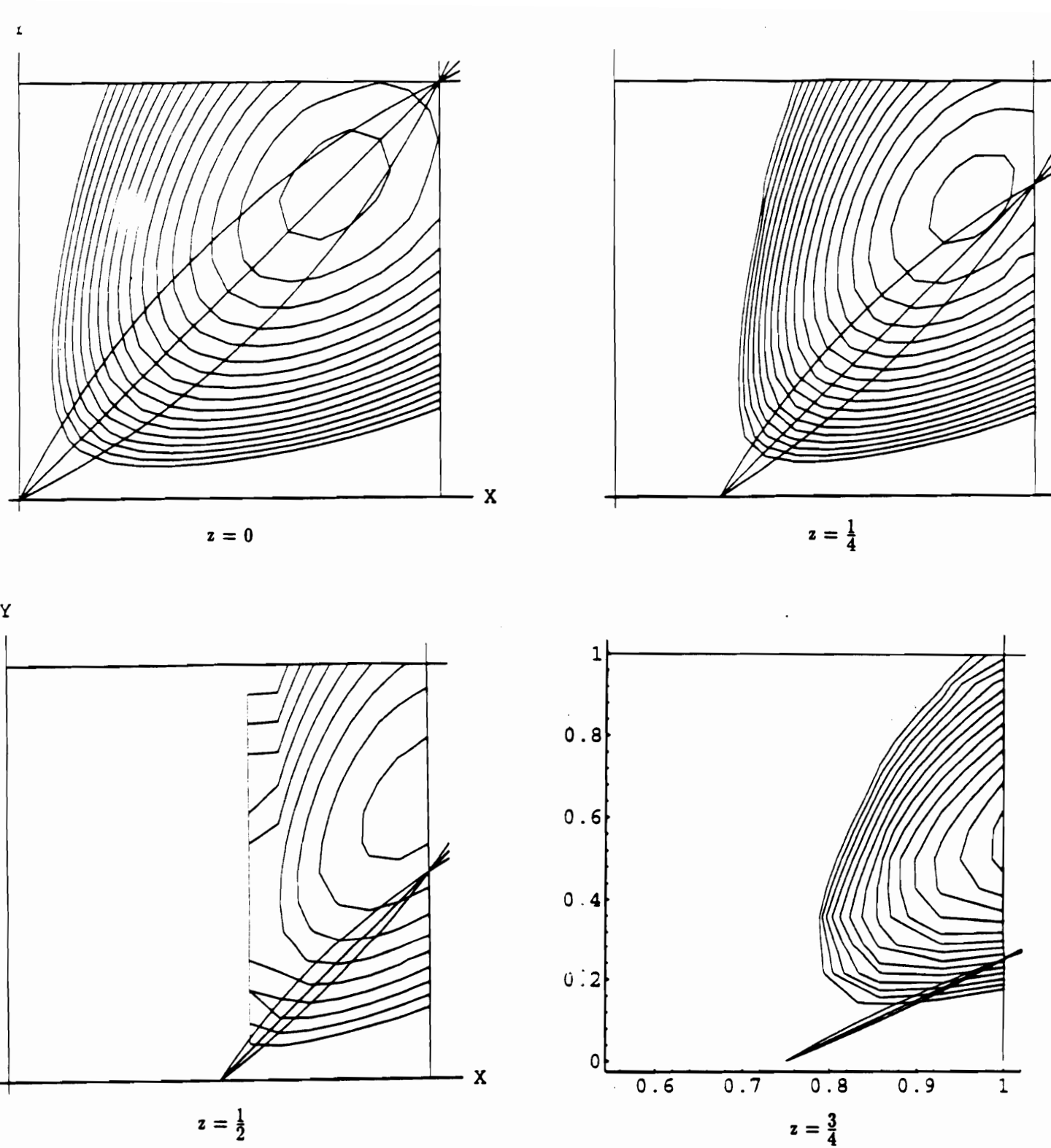
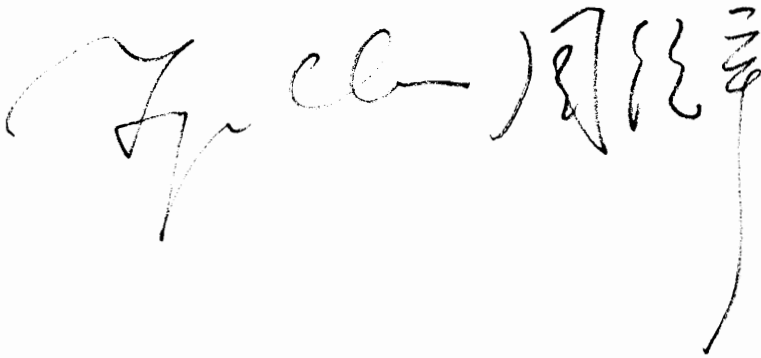


FIGURE A.2 Contour Graphs of $\phi_e(x, y; z)$ for selected z

VITA

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A handwritten signature in black ink, appearing to read 'Teyu Chou' in a stylized, cursive script. The signature is written on a white background.