THE RELATIONSHIP BETWEEN THE CRUSHING STRENGTH OF BRITTLE MATERIALS AND THE SIZE OF CUBICAL SPECIMENS TESTED

by

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LIST OF SYMBOLS USED IN THE THESIS

a	Cube edge dimension
b	Cube edge dimension
c	Half length of Griffith crack
d	Density
h	Height of specimen
m	Molecular spacing of material
n	Number of unit cubes
q	Number of basic elements
r	Number of basic elements
t	Time over which stress is applied
u	Mean strength
ω	Least lateral dimension
x	Random variable of strength
y	Number of unit cubes
E	Young's modulus of elasticity
M	Mass of specimen
P	Compressive stress
S	Tensile stress
W	Compressive load
α	Surface energy of specimen
γ	Size attribute; power of probability ratio
ε	Strain

φ	Random distribution function
η	Viscosity
ρ	Radius of curvature of crack
σ	Standard deviation
ξ	Retardation time
$\Phi(x)$	Density function

INTRODUCTION

The phenomenon of decreasing strength of materials with increase in size of the specimen tested has been of interest for the last sixty years. Most of the work which has been done in this field has been on coal, with a view to obtaining a method of correlating laboratory tests on small specimens with the supporting properties of mine pillars, or to examining the properties of coal with respect to mining the mineral, transportation, or preparation for selling it.

A consensus of the people who have worked with coals varying over a wide range of rank, from anthracite to semi-bituminous coal, and size, varying from specimens weighing 10^{-9} grams, to cubes measuring 64 inches on the side, confirm the existance of an inverse square root law relating the strength per unit area to the edge dimension, or related parameters. The application of this law to all coals has, however, been seriously contested by competent authorities.

Surprisingly, similar work on other geologic materials has been almost completely neglected. The work described in this thesis has been conducted upon cubes of shale, limestone and Plaster of Paris in order to examine the possibility of the existence of similar power laws relating strength to edge dimension for these materials. An attempt is made to review the various theories of failure which have been put forward in order to explain the phenomenon of decreasing strength with increasing size.

LITERATURE REVIEW

Although relatively little work has been done on the subject of variation of compressive strength with the size of sample tested, experiments, almost entirely on coal, have been carried out over the last sixty years.

The Scranton Engineers Club in July 1900 appointed a committee to make a general investigation of the compressive strength of anthracite (Griffith and Conner, 1912). Three sizes of specimen were used: one, two, and four inches high, all with a base two inches by two inches. The samples were prepared with their base parallel to the bedding planes of the seam. A total of 425 samples were tested at Cornell University, Lehigh University, and Pennsylvania State College, on behalf of the Committee.

From an inspection of the results of the tests, the committee reached the following conclusions: "In general, other things being equal, the crushing strength of mine pillars would vary inversely as the square root of the thickness of the bed."

A series of tests were conducted on anthracite specimens from Pennsylvania by Daniels and Moore (1907) at Lehigh University. The conclusions from these tests were: "The crushing strength per square inch of small cubes is greater than that for larger cubes".

Bunting (1907) concluded that "it is evident that coal prisms follow some law of strength relative to height and breadth", from a series of tests on anthracite specimens of different seams.

He also reports a series of tests by Professor Bauschinger on Swiss sandstone, for which Professor Johnson (1898) obtained the formula:

$$P = 5500 + 1565 \cdot \frac{b_1}{h}$$

where P is the strength in pounds per square inch.

b_1 is the least lateral dimension.

h is the height of the specimen in inches.

Professor Johnson also obtained the formula:

$$\frac{\text{Strength of prism}}{\text{Strength of cube}} = 0.778 + 0.222\,\frac{b_1}{h}$$

from an analysis of Professor Bauschinger's results on sandstone.

From his tests on 647 specimens of coal, Bunting proposed the formulae:

$$P = 1750 + 750\,\frac{b_1}{h}$$

and

$$\frac{\text{Strength of prism}}{\text{Strength of cube}} = 0.70 + 0.30\,\frac{b_1}{h}$$

since the curves of these formulae appeared to give the closest fit to his observed results.

In 1914 Rice and Smith of the United States Bureau of Mines carried out tests on 15 cubical specimens of coal ranging in size from $2\frac{1}{2}$ inches to 54 inches, obtained from the U. S. B. M. experimental mine at Bruceton, Pennsylvania. In testing the samples it was noted that cutting and transporting the samples, especially for the larger sizes weakened the structure.

Following these tests the Bureau of Mines conducted further tests (Greenwald, Howarth, Hartman, 1939) in situ, in the experimental mine, on samples of coal in the Pittsburgh seam. Seven small pillars were tested, with dimensions varying from about 30 inches to 70 inches. In subsequent tests five further pillars were tested.

From an analysis of the results of the Bureau of Mines proposed the following formula for the Pittsburgh coal:

$$P = 2800\, w^{\frac{1}{2}} \,.\, h^{-5/6}$$

where P is the crushing strength of the coal in pounds per square inch.

w is the width of the pillar in inches.

h is the height of the pillar in inches.

In 1937 Holland and Lawall reported tests on twenty-two coal beds in West Virginia. The compressive strengths and moduli of elasticity of the coals were determined. In summarizing the tests it was noted that "the unit compressive strength is less when determined on large cubes than when determined on small cubes".

In a paper published in 1954 Steart reports tests of small pillars of coal, nine inches square, and of different heights, from Durban Navigation Colliery, Natal, South Africa. The author explains the fracture of the specimens as that of a double wedging action, and, therefore, as the power of a wedge is proportional to its length when its width remains constant, it follows that the strength of pillars of coal with the same width is in inverse ratio to the height.

On the basis of the mechanical principle involved, as seen by Steart, and his observations from the laboratory tests and mine practice, the following rules relating size and strength of mine pillars were proposed:

1. The strengths per unit area of pillars of constant width vary as the square root of their heights.
2. The strengths per unit area of pillars of constant height vary as the square root of their widths.
3. The strengths per unit area of pillars of cube form vary in inverse ratio to the square root of their dimensions.

These rules are illustrated graphically in Figure 1, page 12.

Millard, Newman, and Phillips published a paper in 1955 reporting tests covering a wide range of rank in British coals. The tests were conducted on two ranges of size; lumps weighing 10^{-2} grams to 10^{3} grams, and 10^{-9} grams to 10^{-5} grams. Three different systems of loading were used; one platen remained a plane, while the other was either a plane, a rod or a sphere.

For three different coal types the law relating the crushing strength of the cubes to their weight was found to be respectively -0.52 ± 0.02, 0.51 ± 0.05, and -0.49 ± 0.04.

The authors explained the power law using Griffith's crack theory, which is reviewed in a later section of this thesis.

Gaddy (1956) published a paper in which he reported tests on cubes of coal from the Pittsburgh coal seam from the U.S. Bureau of Mines experimental mine at Bruceton, Pennsylvania, and also from the Clintwood, Pocahontas No.4,

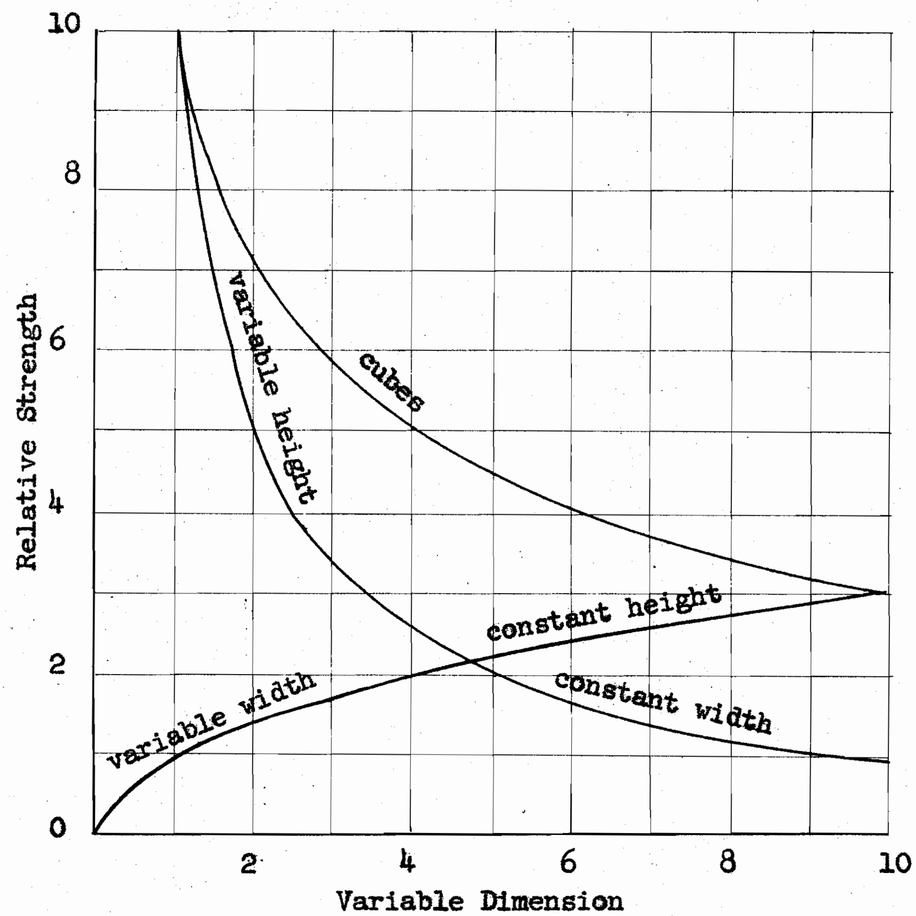

Figure 1. Chart showing Relationship of Strengths of Pillars to Their Heights and Widths in Varying Ratios, as Determined by Steart.

Harlan and Marker coal beds. The tests were conducted on cubes ranging in size from two inches to nine inches. The results of the tests on the Pittsburgh coal were correlated with the results mentioned above, obtained by the Bureau of Mines in earlier tests. The results were plotted in logarithmic form of strength per unit volume against volume of the specimen, and the power law obtained for the combined results was found to be -0.53, which was considered to be close, within the limits of experimental error, to -0.5, and also appeared to fit the results better than the power of -5/6 obtained by the Bureau of Mines, when plotted over a wider size range.

The results of the tests on the other coal seams gave powers respectively of -0.46, -0.42, -0.55 and -0.49, which were considered to be sufficiently close to one half to justify a universal inverse square root law for the types of coal tested.

In 1961, Evans, Pomeroy and Berenbaum, working at the Mining Research Establishment of the National Coal Board in Britain, published a paper on the compressive strength of coal.

Tests were conducted on specimens ranging in size from 1/8 inch to 2 inches, cut on a universal grinding machine. The coals were from two different seams with widely differeing characteristics, the Deep Duffryn coal and the Barnsley Hards. The results were plotted on a logarithmic scale; the mean strength against the average dimension of the side of the cube. The power laws obtained were respectively -0.32 ± 0.02, and -0.17 ± 0.02. These results contradict the one half power stated previously by several authorities. The authors explain the power law on the basis of a statistical theory, which is reviewed later.

EXPERIMENTAL PROCEDURE

Method

In all the work reported previously on the subject of reduction of strength with increase in size of test specimens, all except one series of experiments were concerned with coal. This study was, therefore, conducted in order to compare the behavior of limestone, shale, and Plaster of Paris with that of coal.

The limestone used was in the form of blocks quarried for building purposes. The shale, also in the form of large blocks, was roof rock obtained from above the Pocahontas No. 4 coal seam, from the U. S. Steel Company's coal mine at Gary, West Virginia. An analysis of the shale is given in Table I.

TABLE I

ANALYSIS OF POCAHONTAS NO. 4 ROOF SHALE

Constituent	Percentage
Sand (+1/16 mm)	40
Silt (1/16 to 1/256 mm)	47
Chlorite	2
Kaolinite (-1/256 mm)	2
Illite	9

As may be seen from the Table the shale has a high sand and silt content, with a low clay mineral and water content. It was thus little affected in the wet cutting process.

The Plaster of Paris was molded into large blocks in a wooden mold, approximately a one foot cube, though the blocks were cast only about five inches deep, due to the difficulties encountered in making a homogeneous mixture of large quantities of Plaster of Paris. The blocks were allowed to dry thoroughly at room temperature before cutting into smaller specimens for testing.

Since the materials under investigation were all obtained in large blocks without any particular correlation between the original positions in the geological column, although blocks were selected which resembled each other physically as closely as possible, the whole size range was cut from a single block. For the large shale specimens, however, this was not possible, since to cut four inches and five inches specimens requires very large blocks.

The specimens were cut on a Clipper wet type masonry saw, equipped with a 14 inch diamond impregnated blade. A cutting table capable of precise parallel movement was used, which enabled the materials to be cut in well shaped cubes.

The limestone and the plaster of Paris were relatively homogeneous and were easily cut into well shaped cubes. The shale, however, often had well marked fractures running through the blocks, and showed a tendency to split into concoidal fractures while being cut.

After cutting all the specimens were capped with Plaster of Paris on a glass sheet in order to present perfectly flat surfaces to the testing machine, thereby eliminating any large local stresses. After allowing a few days for the caps to dry out, the specimens were measured to the nearest 1/100 inch, across the middle of three faces mutually at right angles. The specimens were then crushed in a Tinius Olsen 400,000 pounds compression machine, the upper platen of which was mounted in a well lubricated bearing block, to ensure uniform application of the load.

The limestone specimens, in which signs of bedding were almost absent, were crushed in random directions with respect to the bedding. Bedding was totally absent in the Plaster of Paris. The shale had very marked bedding planes, and were therefore crushed in a direction perpendicular to the planes.

The load was applied to the specimens at a rate of approximately 700 pounds/square inch/second. This rate was judged by the operator. The justification for this approximation is that although the rate of loading has been shown (Hardy, 1959) to affect the elastic properties and the strength of materials, over the small range of loading rates within the limits of the judgment any variations in the final strength would be imperceptible.

Design of Experiments

In order to obtain consistent statistical information, the testing was designed according to the following theory proposed by Tucker (1945):

Let σ_1 be the standard deviation of compressive strengths of cubes of unit area, and let σ_b be the standard deviation of compressive strengths of cubes of b units in cross-section. Then

$$\sigma_b = \frac{\sigma}{\sqrt{b}} \tag{1}$$

Now, as $\sqrt{\text{area}}$ depends upon edge dimension, the standard deviation of compression strength of cubes is inversely proportional to the edge dimension. Thus, for tests upon cubes of different size, consider a universe of n cubes with mean strength μ. The standard deviation of cubes of unit size is $\frac{\sigma_1}{\sqrt{n}}$.

Then the standard deviation of cubes with edge dimensions of y units, for n cubes is $\frac{\sigma_1}{y\sqrt{n}}$.

An increase in precision is obtained for the same number of larger cubes, and in order to obtain the same accuracy it is only necessary to test $\frac{n}{y^2}$ cubes. Thus, for equal statistical information from each size of specimen tested, numbers are used in the ratio of the areas of the specimens.

In accordance with this theory a design was set up, shown in Table II, page 18.

In view, however, of the fact that the ranges of size were cut, as far as possible, from the same blocks, it was not possible to adhere to this design strictly. The general pattern was followed, and the results obtained from the tests appear to justify the pattern used.

TABLE II

NUMBERS OF CUBICAL SPECIMENS OF PARTICULAR SIZES TO BE TESTED, TO OBTAIN UNIFORM STATISTICAL INFORMATION

Edge Dimension of Cubes Inches	Number of Specimens
1	50
$1\frac{1}{2}$	23
2	13
$2\frac{1}{2}$	8
3	6
4	4
5	2

EXPERIMENTAL RESULTS

Limestone

A total of 68 specimens of limestone were crushed, ranging in size from one inch edge dimension to three inches. Initially, some half-inch specimens were also tested, but their small size and area in relation to the size and area of the spherically mounted upper platen of the testing machine caused the specimens to fail at stresses below the normal crushing strength. This was because the friction and inertia of the upper platen were too great to allow the platen to align itself exactly with the upper surface of the specimen, thus causing high local stresses along one edge of specimen. The upper size limit of three inches was the limit imposed by the testing machine, since its maximum capacity was 400,000 pounds.

The results of the tests of the limestone specimens, all cut from the same block, are shown in Table III below.

TABLE III

RESULTS OF CRUSHING TESTS IN CUBICAL SPECIMENS CUT FROM A SINGLE BLOCK OF LIMESTONE

Number of Specimens	Average Edge Dimension Inches	Average Crushing Strength lbs/in^2	Standard Deviation lbs/in^2
25	1.004	24,500	4241
27	1.517	24,400	3071
9	2.035	25,800	3063
6	2.510	24,100	3657
1	3.00	20,600	--

As may be seen from the Table, the variation in strength of specimens of any one size was quite considerable, giving rise to high standard deviations.

Evans, Pomeroy, and Berenbaum (1961) showed by a statistical analysis that the size attribute involved in the realtionship under examination is that of edge dimension for specimens of coal. In view of the experimental layout used for the experiments described in this thesis, however, good probability curves for the crushing strength of the larger sizes of the materials tested could not be obtained. As the distribution of results obtained were very similar to those obtained in testing coal, it was assumed that this same result applies for the materials tested, and the average edge dimensions of the cubes were used in analyzing the results of the tests.

The results are shown graphically on logarithmic paper in Figure 2, page 21. The slope of the line, obtained by linear regression, was found to be -0.11. However, statistical analysis shows that this slope is not significantly different from zero.

Shale

Three separate blocks of shale were tested on the Tinius Olsen testing machine. A total of 78 specimens ranging in size from one inch to three inches were cut from the first block, 83 specimens from one to three inches from the second block, and five specimens of four and five inches from the third block. The upper limit of five inches was again that imposed by the limitations of the available testing machine.

The results are shown in Tables IV, V, and VI, pages 22 and 23. The results from the separate tests were plotted on logarithmic graph paper, as shown in Figure 3, page 24. The slope of the lines from the different

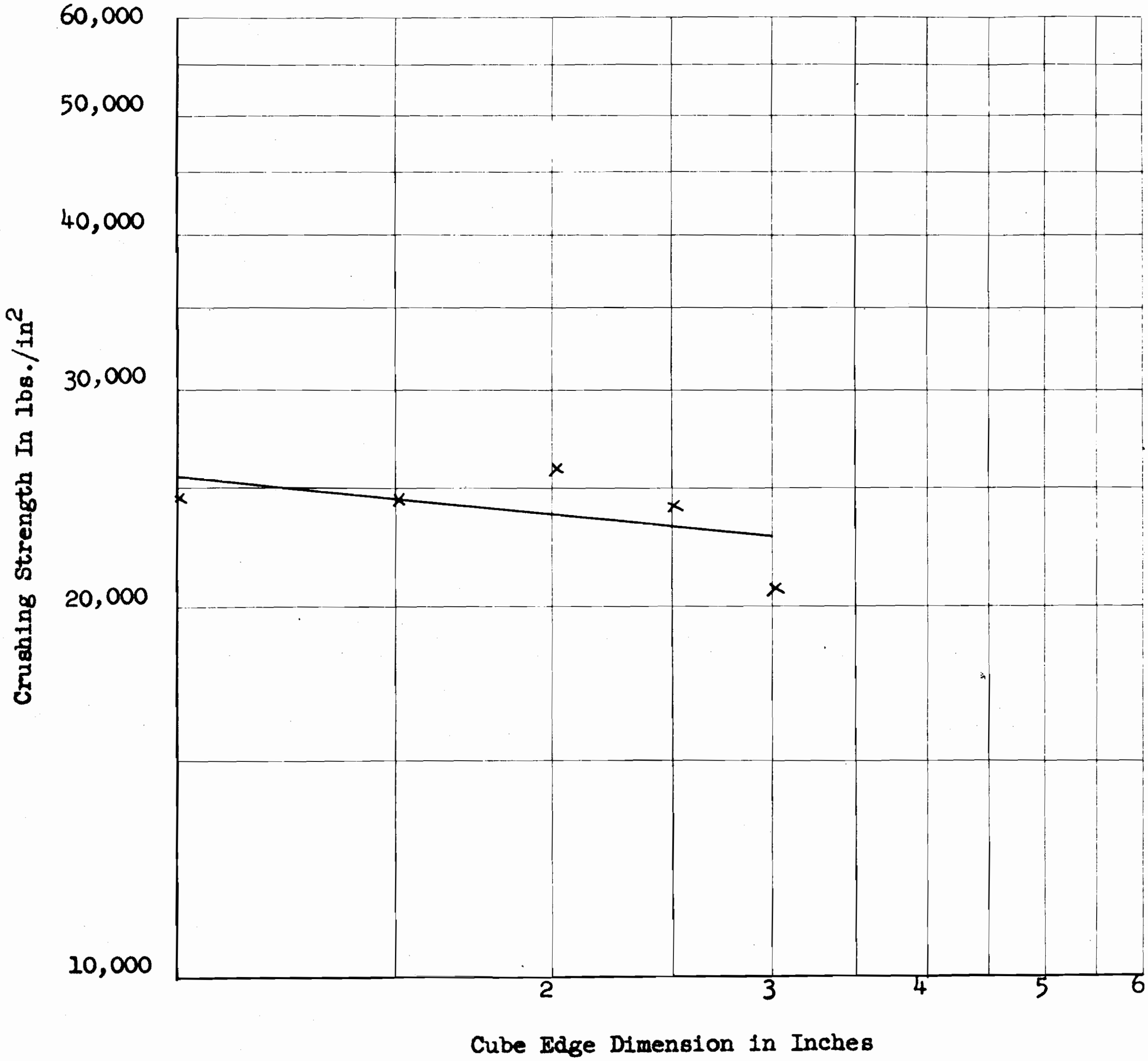

Figure 2. Graph Showing the Relationship Between the Crushing Strength of Limestone, and the Edge Dimension of Cubical Specimens Tested.

TABLE IV

RESULTS OF CRUSHING TESTS ON CUBICAL SPECIMENS CUT FROM THE FIRST BLOCK OF POCAHONTAS NO. 4 ROOF SHALE

Number of Specimens	Average Edge dimension Inches	Average Crushing Strength lbs/in^2	Standard Deviation lbs/in^2
36	1.020	16,100	2418
15	1.509	14,000	2192
17	2.007	13,000	2116
7	2.501	12,900	2750
3	2.998	14,800	1510

TABLE V

RESULTS OF CRUSHING TESTS ON CUBICAL SPECIMENS CUT FROM THE SECOND BLOCK OF POCAHONTAS NO. 4 ROOF SHALE

Number of Specimens	Average Edge Dimension Inches	Average Crushing Strength lbs/in^2	Standard Deviation lbs/in^2
45	1.014	14,400	1661
22	1.527	14,500	1885
9	2.053	13,800	1661
3	2.512	13,100	2794
4	3.030	10,600	924

TABLE VI

RESULTS OF CRUSHING TEST ON CUBICAL SPECIMENS CUT FROM THE THIRD BLOCK OF POCAHONTAS NO. 4 ROOF SHALE

Number of Specimens	Average Edge Dimension Inches	Average Crushing Strength lbs/in^2	Standard Deviation lbs/in^2
2	4.043	11,500	424
3	5.030	11,300	854

blocks are very close to each other, and therefore the data was combined, giving the values shown in Table VII, below.

TABLE VII

COMBINED RESULTS OF CRUSHING TESTS ON CUBICAL SPECIMENS OF POCAHONTAS NO. 4 ROOF SHALE

Number of Specimens	Average Edge Dimension Inches	Average Crushing Strength lbs/in^2	Standard Deviation lbs/in^2
81	1.017	15,200	2019
37	1.520	14,300	1985
26	2.023	13,300	1936
10	2.504	12,600	2603
7	3.016	12,400	1090
2	4.043	11,500	424
3	5.030	11,300	854

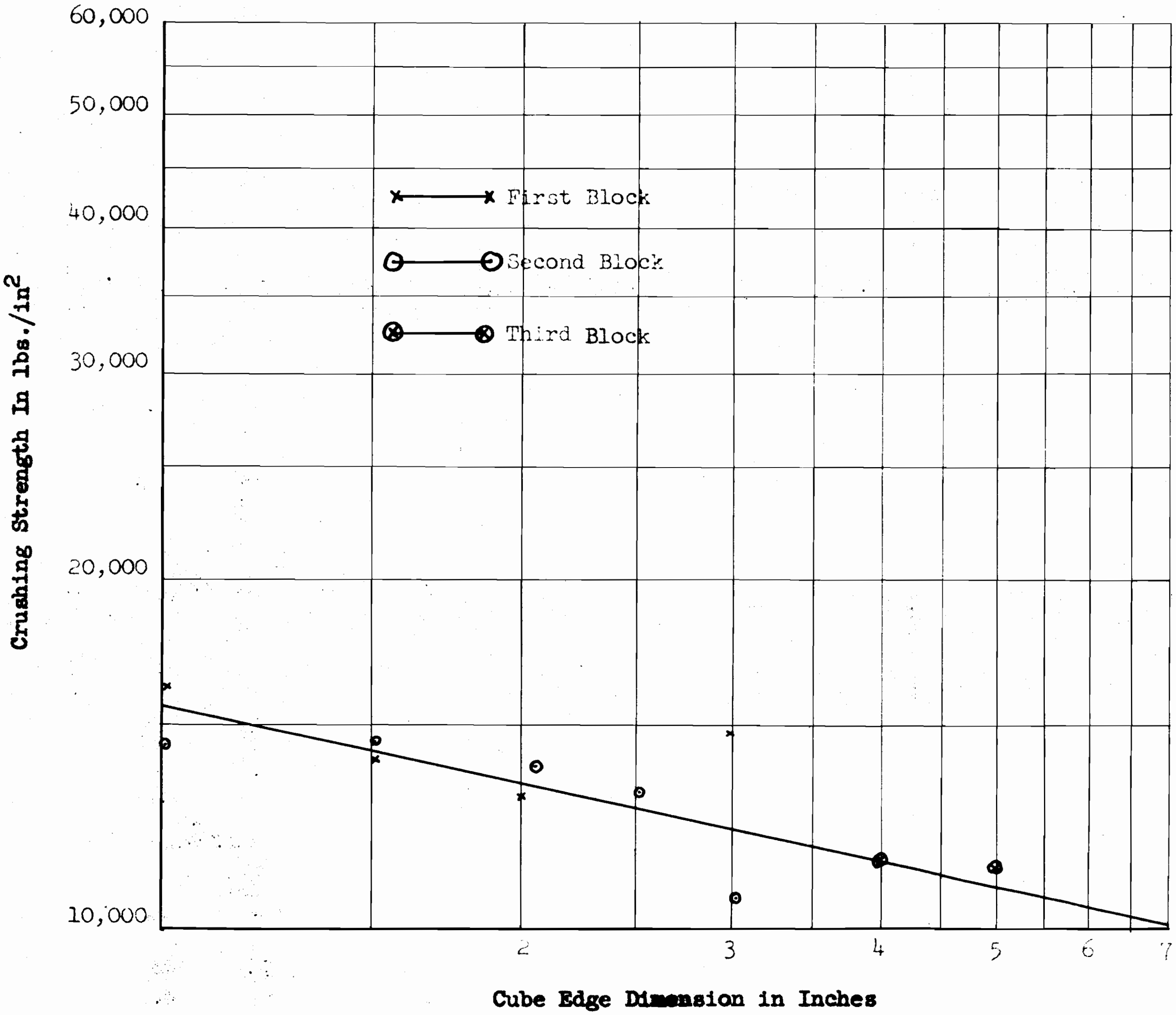

Figure 3. Graph Showing the Separate Relationships between Crushing Strength and Edge Dimension of Cubical Specimens of Three Blocks of Pocahontas No. 4 Roof Shale.

These results are plotted graphically in Figure 4, page 26. The slope of the line, again obtained by linear regression, is -0.20, which is significantly different from zero.

Plaster of Paris

Two separate blocks of Plaster of Paris were tested on a Baldwin 200,000 pounds testing machine. This machine was used due to its greater sensitivity at the much lower strengths of Plaster of Paris. The results of the first block, the specimens ranging in size from one inch to three inches, were so erratic that they were discarded. This was probably due to inadequate mixing, and inadequate drying period before testing. The second block was, therefore, given several weeks in which to dry out, after cutting the specimens. Seventy six specimens, ranging from one inch to three inches were tested. The upper limit was imposed by limited mixing facilities for the Plaster of Paris. To cast a five inch thick block it was found necessary to have very good mixing facilities, and to observe great care in eliminating lumps from the mixture.

The second block gave much more uniform results, which are shown in Table VIII, page 27.

The results are plotted on logarithmic paper in Figure 5, page 28. The slope of the line, obtained by linear regression is -0.10, but statistical analysis shows that this value is not significantly different from zero.

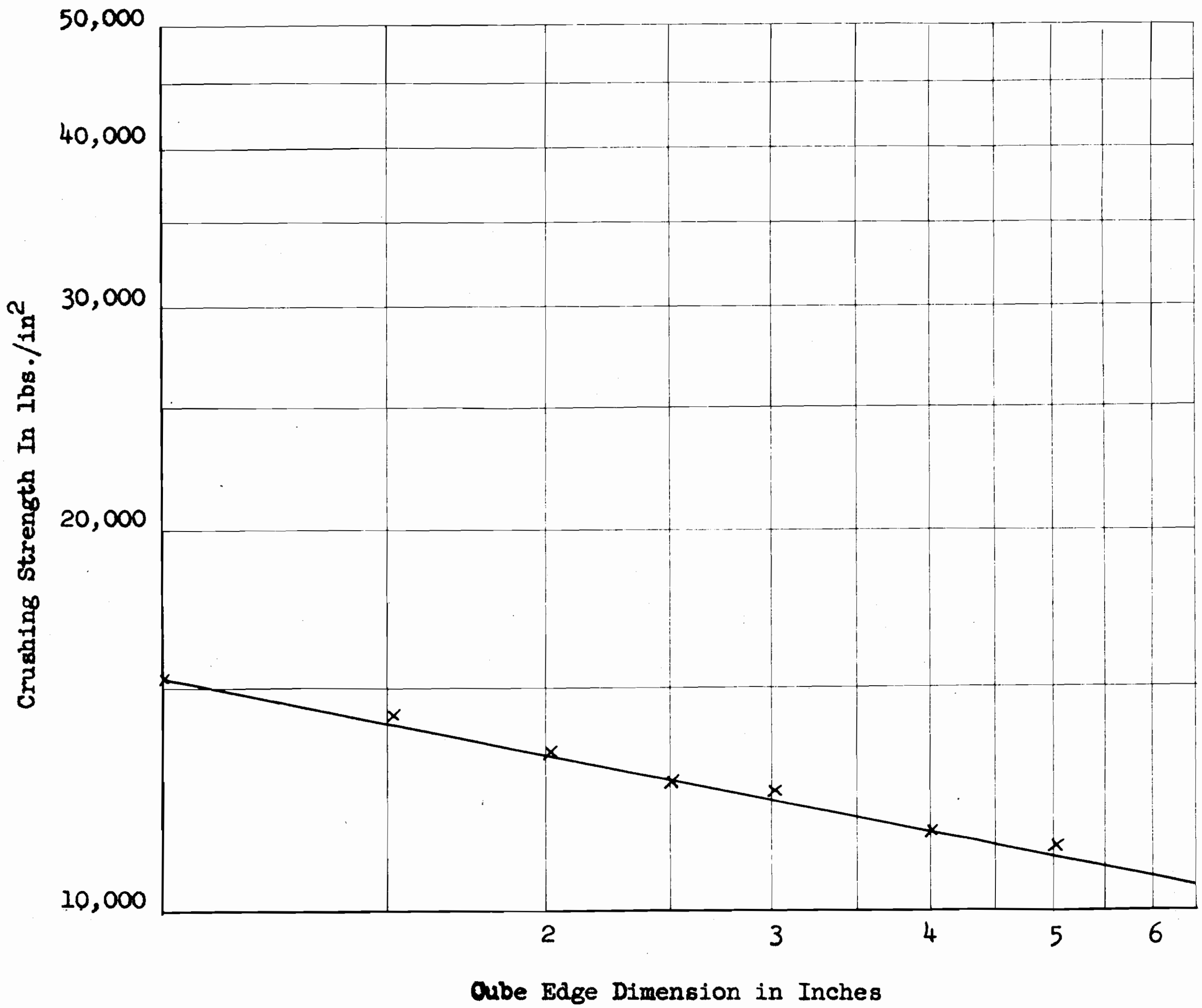

Figure 4. Graph Showing the Relationship between the Combined Results of Crushing Strength of Pocahontas No. 4 Roof Shale and Edge Dimension of Cubical Specimens Tested.

TABLE VIII

RESULTS OF CRUSHING TESTS ON CUBICAL

SPECIMENS OF PLASTER OF PARIS

Number of Specimens	Average Edge Dimension Inches	Average Crushing Strength lbs/in^2	Standard Deviation lbs/in^2
39	.993	1360	180
20	1.479	1460	154
10	1.983	1500	121
4	2.469	1520	77
3	2.991	1510	114

Analysis of Results

The Griffith crack theory of failure (Griffith, 1912), reviewed in the following section of this thesis, predicts that the value of the power law relating strength to edge dimensions of specimens tested will lie between zero and one half, the lower values for relatively unfissured materials, and the upper values for materials containing many random fissures.

The experimental work described above appears to support these predictions well. The limestone, and the Plaster of Paris, which have little or no natural cleating developed on a small scale, give values which are

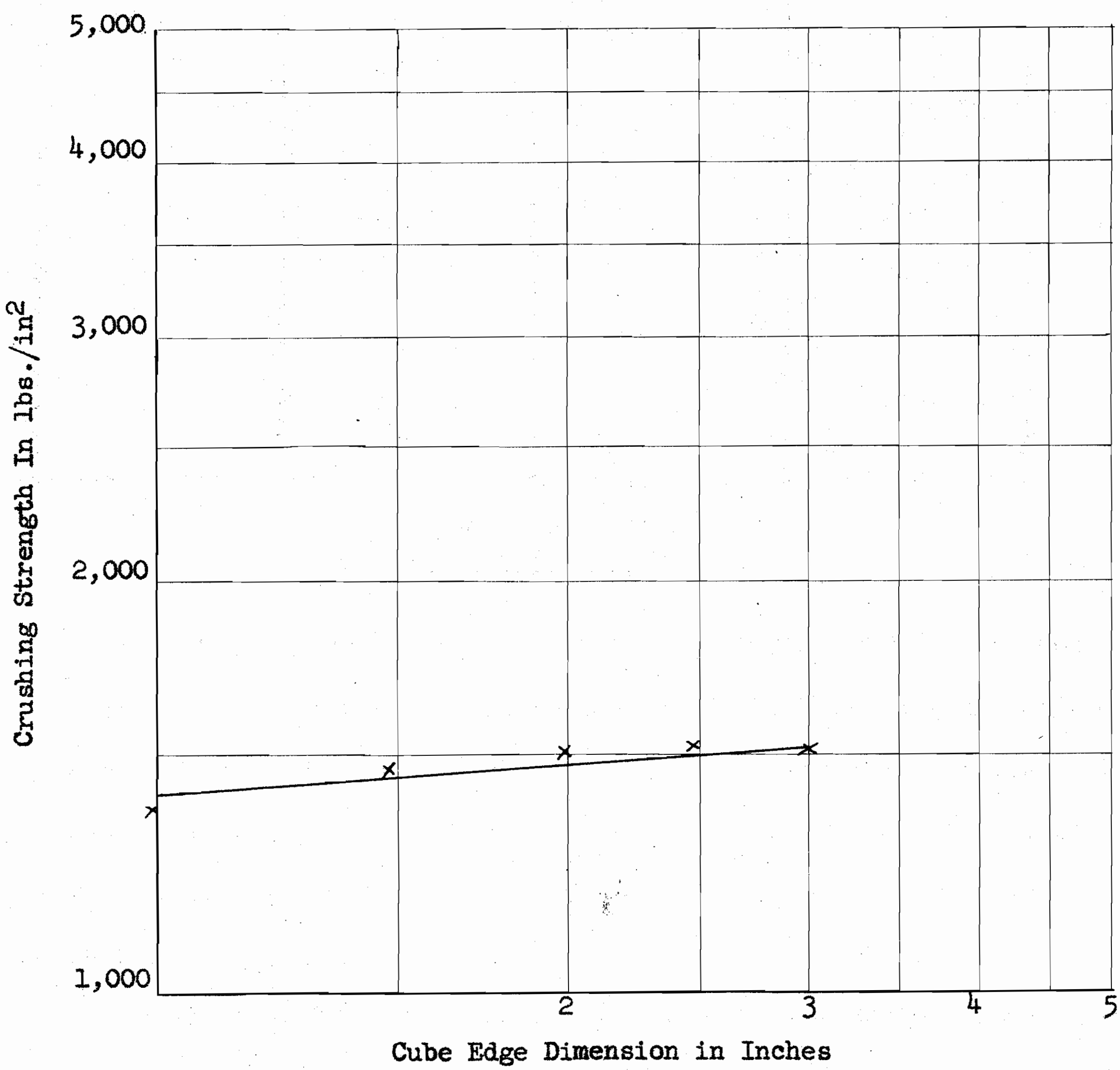

Figure 5. Graph Showing the Relationship between the Crushing Strength of Plaster of Paris and the Edge Dimension of Cubical Specimens Tested.

very close to zero. The shale, which usually has similar, though less well developed cleating to the coal seam over which it lies, gives a power of -0.20, higher than the limestone and shale, but lower than the values previously reported for coal, as would be expected from the theory.

In analyzing the results, however, two major points must be considered. Firstly, due to the small range of specimen sizes tested, limited by the available facilities, the accuracy of the power laws obtained is in some doubt. Ideally very much larger specimens should be tested, ranging up into tens and hundreds of inches on the side, but this would require exceedingly large testing machines. It is felt, however, that more useful results could be obtained if a machine capable of loads up to 4,000,000 pounds were available The second consideration is that of the relative size of the testing machine and the specimen. For the smaller sizes of specimen tested on both machines used the spherically mounted upper platen was considerably larger and heavier than the specimens, and both platens required a considerable force to move them in their mountings. This probably gave rise to high local stresses at the edges of the specimens, particularly in the case of the weaker Plaster of Paris, causing the smaller specimens to fail at lower values of stress than normally. This effect, in direct opposition to the reduction in strength effect in the larger sizes, probably masked the reduction in strength effect, and gave a lower magnitude for the power law than would otherwise have been obtained. Thus, in order to obtain true values of crushing strength for small specimens it is necessary to use small testing machines with small, well lubricated, sensitive spherically mounted upper platens.

An examination of the standard deviations obtained for each size of each material indicates the justification in using the lower number of specimens of the larger sizes, since the standard deviations are very close to each other for each material.

THEORETICAL EXPLANATION OF THE FAILURE OF ROCK

In several theoretical explanations of the behavior of coal and rock under stress it has been assumed that for all practical purposes rock is perfectly elastic, homogeneous, and isotropic. In actual fact, rock has none of these properties developed to a sufficient extent, as has been pointed out by several authorities (Corlett, Emery, 1959), to justify a theory based on these assumptions.

Any theory concerning rock behavior under stress must explain the following facts which have been observed by many people working with rock.

1. Rock is not usually perfectly elastic substance, but also exhibits a time dependent plastic flow. The rate of application of stress, therefore, affects the slope of the stress-strain relationship, and the final crushing strength. Due to the plastic effect, the stress-strain relationship is not a straight line.

2. Rock is not a homogeneous substance, but contains many flaws and irregularities, which may affect the strength considerably.
3. All geologic materials contain some form of regular jointing and bedding to a greater or smaller extent.
4. The crushing strength of rock varies over a wide range for the same sized specimens tested under the same conditions.
5. For coal and apparently for some rocks the crushing strength decreases with increase in the size of the specimen tested. This decrease often appears to follow an inverse power law.
6. The actual failure of the specimen takes place quite rapidly once a critical value of stress is reached.
7. An examination of the fragments of rock created by crushing shows that considerably energy is involved in the crushing process. A large quantity of fine dust is created in addition to quite a wide size distribution of fragments.
8. The larger fragments caused by crushing appear to be of two principal types. Wedge shaped fragments, with slickensided surfaces, and vertical fragments, the surfaces of which show no relative vertical movement, and have evidently broken in tension.

Phenomenological Explanation of Rock Behavior

A method of explaining rock behavior under stress, which appears to give a close representation of the observed phenomena is by creating an analogue, consisting of a combination of a number of simple physical models.

Where only slow variations of stress are involved the models consist of combinations of elastic and viscous elements. More rapid variations require an additional element, allowing for the inertia of the system. Time strain studies by Terry (1956) on coal, Ross (1958) on concrete, indicate that the time-strain behavior of these materials appears to follow the Burgers model. This model consists of the series combination of two basic units, the Maxwell unit, consisting of an elastic unit and a viscous element in series, and a Kelvin unit, and elastic and a viscous unit in parallel. This model is illustrated in Figure 6, page 33. The elastic unit consists of a spring, and the viscous element is a dashpot, with an equivalent viscosity of η .

For the Burgers model the strain at any time is given by the following equation:

$$\varepsilon = P\left[\frac{1}{Ea} + \frac{t}{\eta a} + \frac{1}{E}\left(1 - e^{-t/\xi}\right)\right] \qquad (2)$$

and

$$\xi = \frac{\eta}{E} \qquad (3)$$

where ε is the strain.

P is the applied stress.

t is the time after application of the stress.

ξ is the retardation time, necessary for the strain due to the third factor in equation (2) above, to go to within $\frac{1}{e}$ of its maximum strain.

E_a and E are the elastic elements.

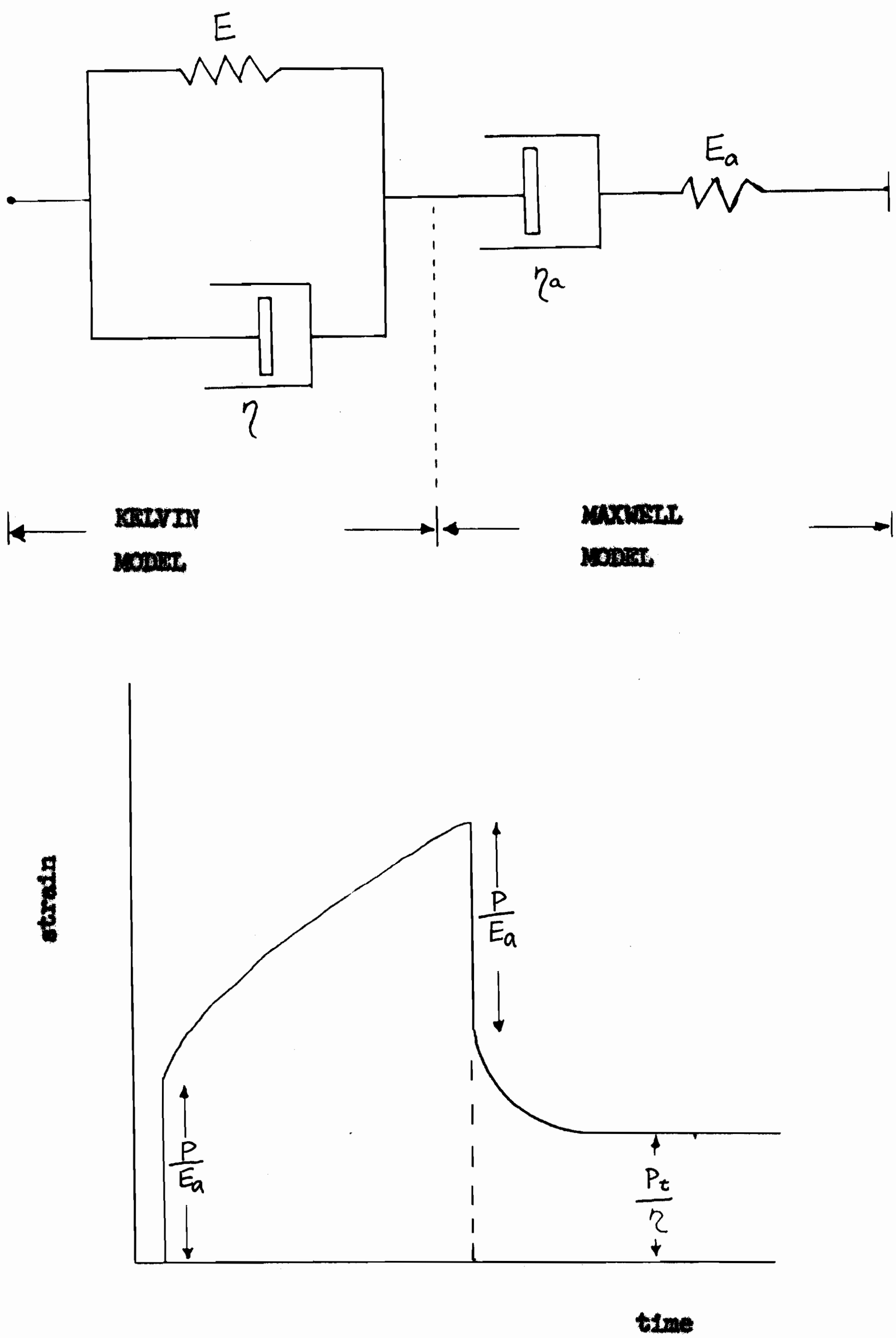

Figure 6. The Burgers Rheological Model

η_α and η are the viscous elements associated with the Maxwell and the Kelvin units respectively.

The viscoelastic constants, E_a, η_α , E, and η , are related to the properties of the material under study.

Tests by Hardy (1959) indicate that this model applies to several types of rock, in addition to the materials already mentioned.

The Application of Dimensional Analysis

From the work performed on rock, and from previous work on coal and on other materials, it appears that the factors most likely to affect the compressive strength of cubes of rock are:

The applied stress, P.
The edge dimension of the cube, a.
The rate of strain of the specimen, $\frac{de}{dt}$.
The surface energy contained in the specimen, α .
The elasticity of the material, E.
The solid viscosity of the material, η .

Applying the principles of dimensional analysis:

$$P = A\,(a)^b\,\left(\frac{de}{dt}\right)^c\,(\alpha)^d\,(E)^e\,(\eta)^f \tag{4}$$

Substituting the dimensions of the parameters:

$$\frac{M}{L\Theta} = A\,(L)^b\,\left(\frac{1}{\Theta}\right)^c\left(\frac{M}{\Theta^2}\right)^d\,\left(\frac{M}{L\Theta^2}\right)^e\,\left(\frac{M}{L\Theta}\right)^f \tag{5}$$

From the above equation:

$$1 = d - e - f \tag{6}$$

$$-1 = b - e - f \tag{7}$$

$$2 = c - 2d - 2e - f \tag{8}$$

Solving in terms of d and f:

$$e = 1 - d - f$$

$$b = -c$$

$$c = f$$

Therefore

$$P = E.\ A.\ \left(\frac{\alpha}{Ea}\right)^{d} \left(\frac{\eta}{E}\frac{de}{dt}\right)^{f} \tag{9}$$

Thus, 3 dimensionless groups are obtained. Now for specimens made from the same material the elasticity and the solid viscosity are the same, and if the specimens are tested on the same machine by the same operator the rates of strain will be approximately the same, and therefore, the dimensionless group, $\left(\frac{\eta}{E}\frac{de}{dt}\right)$ is a constant for specimens of the same material and is independent of the size of the specimen tested.

Therefore

$$P = K.\ E\ \left(\frac{\alpha}{E_a}\right)^{d} \tag{10}$$

If the result that the strength of specimens of rock depends upon the inverse square root of the depth of the specimen, obtained empirically by several independent people, is used, a value of $\frac{1}{2}$ for d is obtained; then:

$$P = K\ \left(\frac{E\alpha}{a}\right)^{\frac{1}{2}} \tag{11}$$

This value is the same as that obtained from Griffith's crack theory of failure, reviewed later.

Alternatively, the value which must be used for the power d in equation (10) is that obtained from the tests for the particular rock under consideration.

Griffith's Crack Theory of Failure

This theory was used by Griffith to explain the fact that the tensile strength of simple crystals when calculated by other theories gives a much higher value than that measured experimentally. He explained the phenomenon by the presence of large numbers of minute cracks in the material, and demonstrated their existence by experiments on glass.

The effect of the crack is to produce a very high concentration of stress at its edge. The solution for the stress produced obtained by C. E. Inglis was then used to obtain a mathematical analysis of the failure. The amount of the stress produced can be calculated from the result that the maximum tensile stress in a flat plate containing an elliptical hole of major axis 2c, and subjected to an average tensile stress, s, in a direction perpendicular to the major axis is given by:

$$T_o = 2s\left(\frac{c}{\rho}\right)^{\frac{1}{2}}$$

Where ρ is the radius of curvature at the ends of the major axis.

The maximum stress occurs at the ends of the major axis, and as $\rho \to 0$ i.e., the ellipse tends to a flat crack, the stress tends to infinity. For the type of crack under consideration, ρ may be estimated as the order of the intermolecular spacing, m.

The crack will spread if the stress given by the formula is equal to S_m, the maximum tensile stress in the material which can be sustained without cracking. To estimate this the process of cracking must be considered. This produces two new surfaces within the material whose distance apart is of the order of the intermolecular spacing, m, and which each possess surface energy x, per unit area which may be regarded as an intrinsic, measurable property of the material. This surface energy must be provided by the strain energy stored in the solid before cracking, and the quantity available will be of the order of that stored in the volume of the crack. This is $\frac{m, Sm}{2E}$ per unit area of the crack.

Equating the two energy expressions gives

$$Sm = 2 \left(\frac{E\alpha}{m}\right)^{\frac{1}{2}} \tag{12}$$

and putting $\rho = m$ and equating T_o and (12) gives:

$$To = \left(\frac{E\alpha}{c}\right)^{\frac{1}{2}} \tag{13}$$

where To is the tensile strength of the specimen.

Values of To calculated from this formula prove to be of the right order of magnitude.

In purely compressive tests between plane surfaces Griffith showed that the crushing strength should be eight times the plane tensile strength.

Phillips et al (1955) relate the above theory to the diminution of strength with increase in specimen size according to the following reason.

Assuming that c, the $\frac{1}{2}$ dimension of the largest cracks, is equal to 1/n times the specimen dimension, the following formula is obtained:

$$\log W = \log 8\left(\frac{nE\alpha}{d}\right)^{\frac{1}{2}} + \frac{1}{2}\log M \tag{14}$$

where W is the load at failure.

M is the mass of the specimen

d is the density of the material.

These quantities may all be measured except α and n.

Putting n equal to 1.5, implying that the cracks are nearly co-extensive with the lump. This is justifiable by the extensive cracking in all geologic materials.

Griffith's theory applies to a crack in an infinite body, and its application to a crack which is nearly coextensive with the body requires justification. The solution for a finite body is not available, but an indication of the error in truncating an infinte solution can be obtained; this may be done by considering what proportion of the extra load on the surrounding material, due to the presence of the crack is borne by the material in the immediate vicinity. In the interest of mathematical simplicity and since the estimate is in any case only approximate, the solution given by Sneddon (1951) for the two dimensional case is used.

From Sneddon's equations (102) and (104) the stresses at and normal to the plane of the crack ($x = 0$) are given by:

$$S_x = P_0 \left[\frac{y^2}{(y^2 - c^2)^{\frac{1}{2}}} - 1 \right] \qquad y > c \qquad (15)$$

Integrating over the plane of the crack in two stages

$$\frac{\int_a^{\alpha} S_x \cdot dy}{\int_c^{a} S_x \cdot dy} = \frac{a - (a^2 - c^2)^{\frac{1}{2}}}{c - \left[a - (a^2 - c^2)^{\frac{1}{2}} \right]} \qquad (16)$$

If the body is truncated at the planes $y = \pm a$, the load that is supported by the material that is left is increased in the ratio $1 + \beta$ over its former value, (the effect of the new boundary condition $S_y = 0$) at $y = \pm a$ is ignored). Thus, if $a/c = 1.5$ i.e., the crack is 2/3 the dimension of the specimen, the load on the uncracked portion is increased by only 60% compared with its share of the load in an infinite body. Phillips et al regard this as sufficient justification for applying Griffith's theory to an extensively cracked material.

Phillips and his co-workers explain their results by Griffith's crack theory. However, several responsible sources have obtained power laws for coal which are significantly different from one half, and the results of the present study are power laws for different materials which are also significantly different from one half.

In their analysis by Griffith's crack theory the authors make the assumption that the length of the cracks in the material (coal) under consideration bears a constant relationship, $2c = a/n$, to the dimension of the specimen, a. For extensively cracked materials such as soft coal this is probably the case. The blocks from which the specimens are cut will contain many cracks of all sizes. However, in cutting smaller specimens, those which have large cracks with the same or greater dimensions than the specimen dimension will break during the cutting process and be discarded immediately. Those specimens with cracks nearly co-extensive with the edge dimension will not break. Due to the random distribution of cracks in the block, in this way nearly all the specimens, regardless of size, will have cracks nearly co-extensive with the block. This argument is justified by the fact that many people who have worked with coal have reported the difficulty due to frequent breakage in cutting coal specimens. The one half power obtained by Phillips et al., Gaddy, and many others, thus justified by the Griffith crack theory, probably represents the highest power obtainable relating crushing strength to edge dimension, since it is not possible to obtain cracks more than co-extensive with the specimens tested.

At the other end of the scale, in solid materials such as glass, and possibly some hard rocks, it is possible that even though the cracks do exist, as shown by Griffith in glass, they are within a small range of size distribution, and far from co-extensive with the specimen. In this case the cracks would bear no relation at all to the size of the specimen (except possibly in very large specimens of hard rocks), and the Griffith crack theory would predict that the strength would be unaffected by the size of specimen since it is unrelated to crack size, i.e., that the power in the relationship would be zero. This would then represent the lower limit of the power. Thus, for all brittle materials the power would lie between zero and one half, the higher value for extensively cracked materials, and the lower value for materials which are more solid.

The results of this study on limestone, shale, and Plaster of Paris, and of the many previous studies on coal seem to be in close accord with this prediction. The limestone which was comparatively solid and uncleated gave a value of the power near to zero. The shale, which was more jointed gave a value of .20, and the Plaster of Paris which had no initial natural cracks also gave a value near to zero. In the work on coal, values ranging from .5 to .17 were obtained, the latter being for the Barnsley hards, which is very hard, grey coal, containing large quantities of durain, and which is relatively uncracked. The people who obtained the values of .5 usually reported the difficulty in cutting large specimens due to breakage of the coal.

Statistical Theory of Failure

This theory has been put forward by Evans and Berenbaum , and Pomeroy (1961) to explain their results of testing different types of coals. The power laws which they obtained were different for each type of coal and significantly different from one half. They rejected the Griffith crack theory put forward earlier by Phillips, Millard and Newman (1955), on the gronnds that their results did not agree with the half power law, and that the Griffith theory does not concern itself with the great variation of strength among cubes of the same size. They, therefore, put forward the weakest link theory to explain their results.

The theory is based onthe fact that the crushing strength of coal cubes of a particular size shows a wide variation in strength, and that the mean crushing strength decreases as the size of cube increases. This leads to a statistical analysis.

Consider a mechanical system of units joined end to end and let the probabilities of these units surviving the application of a particular stress by P_1, P_2, P_3, ..., P_n. Then if P is the probability of the system surviving the stress,

$$P = P_1 . P_2 . P_3 . P_4 . \dots\dots\dots . P_n \qquad (17)$$

If any probability is zero, then $P = 0$, and if $P_1 = P_2 = P_3 = P_4 = \dots = P_n$, then $P = P_o^n$, where P_o is the probability of a basic element surviving the stress.

Let P_a be the probability of cubes of side a surviving a particular stress. This cube is assumed to be comprised of r of the basic units. It is then desired to relate the probability of survival of the cube to another cube of side b, consisting of basic units. Then:

$$P_a = P_o^r \tag{18}$$

$$P_b = P_o^q \tag{19}$$

$$P_b = P_a^{q/r} \tag{20}$$

q/r if then related to the relative size of the cubes, say $(a/b)^{\gamma}$, where γ has a value near to 1, 2, or 3, according to whether the size attribute affecting the strength of the specimen is height, area, or volume.

Then $$P_b = p^{(b/a)^{\gamma}} \tag{21}$$

Therefore $$\gamma \log \frac{(b)}{(a)} = \log \frac{(\log P_a)}{(\log P_b)} \tag{22}$$

The general validity of this expression has been tested for specimens of coal. Histogram, as shown in Figure 7, page 43 were constructed. Probability curves are then constructed by summing the individual columns of the histogram. The curves are of the form of Figure 8, page 44. The extremities of the curves were neglected by the criterion of rejection. The values of P were then compared at particular stresses for which all the curves lie between 1 and .9. The value of P_a at a particular stress was taken to be the most accurate value, and values of P_b were read from the other curves. In this way corresponding values of $\frac{\log P_b}{\log P_a}$ and b/a were obtained, and when plotted on a log scale, linear regression gave a value of

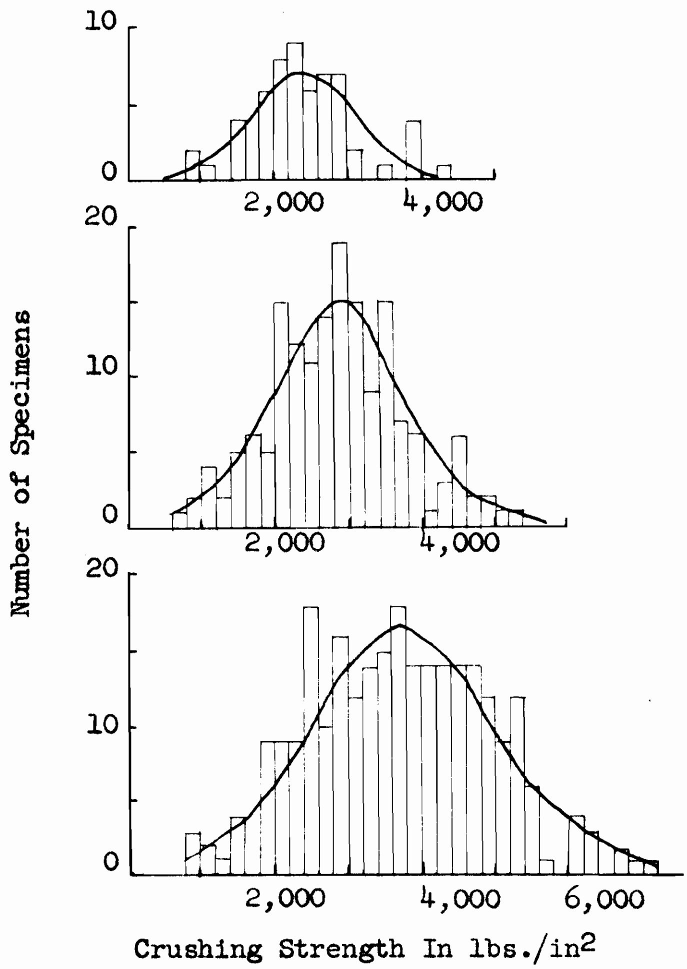

Figure 7. Histograms of Crushing Strength of Cubes of Deept Duffryn Coal, Determined by Evans, Pomeroy, and Berenbaum.

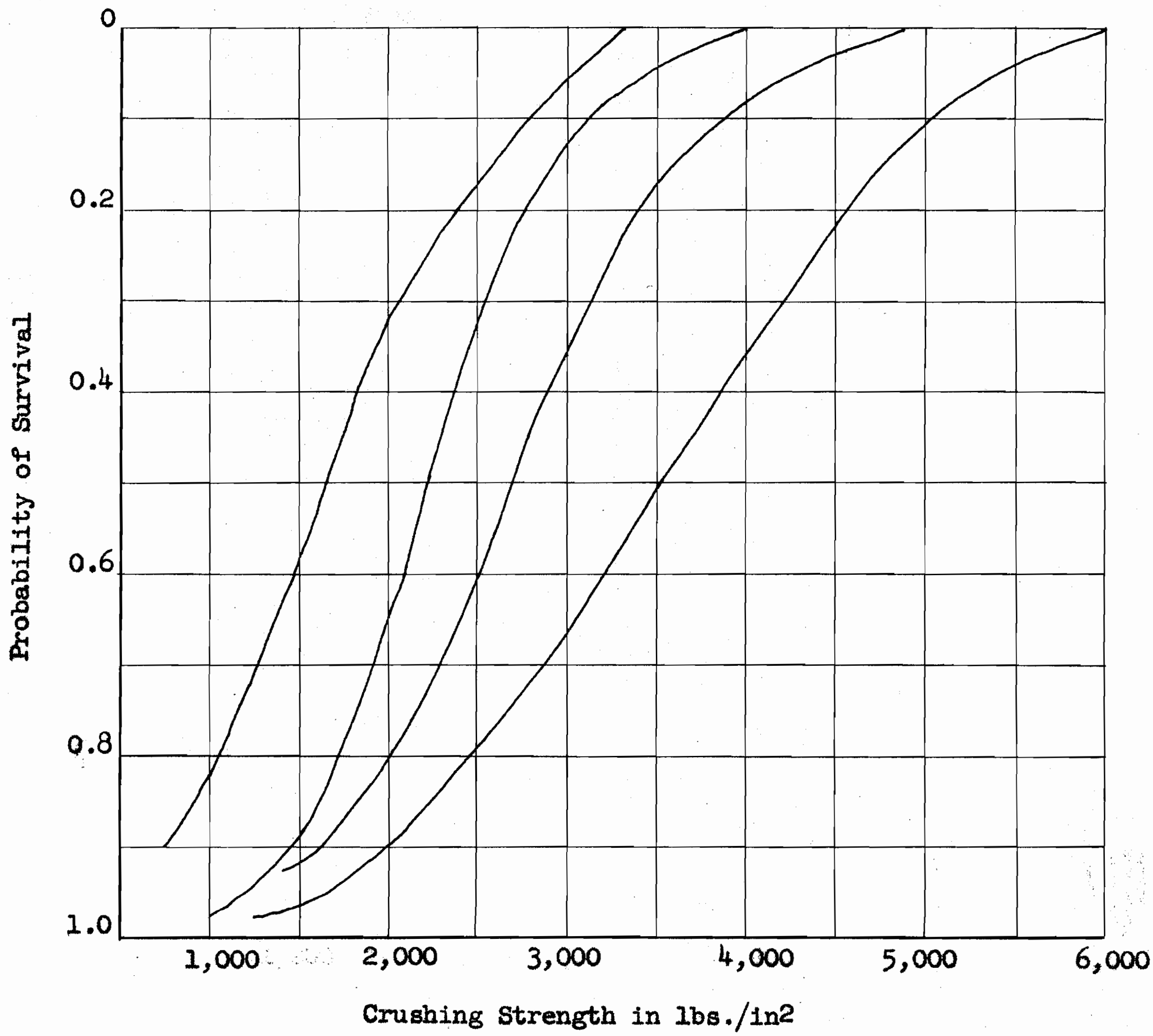

Figure 8. Probability-summation Curves for Crushing Strength of Deep Duffryn Coal, Determined by Evans, **Pomeroy, and Berenbaum.**

1, indicating that the size attribute is that of length of the side of the specimen.

Thus, if the cube consists of a basic element, then the probability of the cube surviving a particular stress is P_o^a .

Now consider P_o to be derived from a distribution function, $\phi(x)$ of the strength, x, of the elements, so that

$$P_o(x) = \int_x^\infty \phi(x)\,dx \qquad (23)$$

Let $\Phi(x)$ be the distribution function of the strength of the composite body, then:

$$\int_x^\infty \Phi(x)\,dx = \left[\int_x^\infty \phi(x)\,dx\right]^n \qquad (24)$$

Whence $\Phi(x)$ may be obtained by differentiating with respect to x:-

$$\Phi(x) = n\phi(x)\,P_o^{n-1}(x) \qquad (25)$$

The distribution curves or specimens of coal tested by Evans, Pomeroy and Berenbaum (1961) were approximately normal, and, therefore, Poisson's distribution may be used for an a sized cube:

$$\phi_a(x) = \frac{1}{\sqrt{2\pi\sigma}} \cdot \exp\left[\frac{-(x-u)^2}{2\sigma^2}\right] \qquad (26)$$

Where u is the mean strength, and σ is the standard deviation.

Equation (26) above may be conveniently written

$$\phi_a(x) = \frac{h}{\sqrt{\pi}} \cdot \exp(-h^2 y^2) \qquad (27)$$

Where $y = x - u$

and $h = 1/\sqrt{2\sigma}$

Equation (25) then becomes:

$$\Phi_b(x) = n \cdot \frac{h}{\sqrt{\pi}} \, exp(-h^2 y^2) \cdot \left[\frac{1 - erf(hy)}{2} \right]^{n-1} \tag{28}$$

Where erf(hy) is the error function, equal to:

$$\frac{2}{\sqrt{\pi}} \int_0^{hy} exp(-h^2 y^2) \cdot dhy$$

The model strength of b size cubes is obtained from:

$$\frac{d\,\Phi_b(x)}{dx} = 0 \tag{29}$$

Whence $$\frac{hy}{exp \cdot (-h^2 y^2)} \left[\frac{1 - erf(hy)}{2} \right] = -\frac{(n-1)}{2\sqrt{\pi}} \tag{30}$$

q The implication of this equation is that the modal strength decreases for multiple element specimens, and increase for sub-elements, i.e. fractional values of a. For small sub-elements, where the modal strength becomes large in relation to that of the element:

$$\left[\frac{1 - erf(hy)}{2} \right]_{y=\infty} = \frac{1}{\sqrt{\pi}} \int_{hy}^{\infty} exp(-h^2 y^2) \cdot dhy$$
$$= \frac{exp(-h^2 y^2)}{\sqrt{\pi}} \left[\frac{1}{2(hy)} - \frac{1}{4(hy^3)} + \cdots \right] \tag{31}$$

Substituting this in equation (30)

$$n = \frac{1}{2h^2 y^2} = \frac{\sigma^2}{(x-u)^2} \tag{32}$$

This approximation is allowable by the assumption already made that $x \gg u$, and it is likely that the assumption itself is valid for coal.

The sub-elements to which equation (31) refers may be thought of in the case of coal, as small cubical domains. If the length of the domain is a', and the length of the cube which shows a normal distribution is a, then $n = \frac{a'}{a}$, and so from equation (32)

$$x \propto (a')^{-\frac{1}{2}} \tag{33}$$

These domains are considered each to contain an innate weakness in the form of a crack. The physical size of the domain limits the crack it can contain and it is reasonable to assume that the modal length, c, of such cracks is directly related to a' i.e.

$$x \propto (c)^{-\frac{1}{2}} \tag{34}$$

This expression is identical to Griffith's equation.

In conclusion to their statistical theory of failure, Evans and Pomeroy make the following statements:

1. Given the strength distribution of any particular sized cube the distribution for any other size can be calculated from the equation $P_b = P_a{}^{b/a}$.

2. The relation between log modal stress and log side of cube which appears to be a straight line over the range studied, is probably a portion of a curve with a continuously changing slope. The straight line may be a tolerable approximation over a wide size range.

3. Griffith's theory, $x \propto a^{-\frac{1}{2}}$, is probably iniversally applicable only for small specimens. For larger specimens the crack length, 2c, must be used instead of the side of the cube. For large specimens the law would approximate to similar expressions in which the power has a value less than one half. Thus, the crushing load, W, for a cube of material that breaks by the propagation of a single Griffith crack would be expected to be related to size by an equation of the form:

$$W \propto a^{1.5}$$

and for an unflawed material:

$$W \propto a^2$$

For brittle materials the exponent of a would be between 1.5 and 2.

Through their statistical reasoning Evans and Pomeroy derive the same expression as postulated by Griffith, and as obtained by the dimensional analysis. They also reach the same conclusions as reasoned above directly from Griffith's theory, that the power relating crushing strength to edge dimension of cubes lies between zero and minus one half, a result which is firmly supported by the results of tests on coal, limestone and shale.

In their reasoning Evans and Pomeroy suggest that the power law for an "unflawed" material should be zero. Griffith, however, showed that even glass had minute flaws, sufficient to cause failure, and therefore it seems unlikely that an "unflawed" material exists at any rate in geologic materials. In this instance the reasoning above that the distribution of size of flaws determines the power law seems a more plausible explanation.

In experiments on glass Poncelet (1946) showed that during the stressing of specimens cracks were formed parallel to the direction of crushing even when the load was applied uniformly to a perfectly even surface. He also showed that it was impossible to eliminate the cracks. In work on concrete Jones (1958) showed that cracks are formed in concrete, parallel to the direction of loading, at loads considerably below the final crushing load.

Further, in comparing Griffith's theory of failure with other theories, such as Mohr's theory of failure, Griffith's theory predicts values of strength which are much nearer to the experimental values than any other theory (Jaeger, 1956, Clausing, 1959). It seems probable, therefore, that Griffith's theory gives a fairly close idea of the mechanism of failure. It also explains the decrease in strength with increase in size.

CONCLUSIONS

This study of the variation of compressive strength of limestone, shale, and Plaster of Paris indicates that the strength of cubes of shale decreases with increase in edge dimension, for specimens varying in size from one inch to five inches, following a power law of the form:

$$P = k.a^{-0.20}$$

where P is the compressive strength in pounds per square inch.

a is the edge dimension of the cube.

k is a constant

Over a size range from one inch to three inches the strength of limestone and Plaster of Paris appears to be little affected by the size of the cube tested, although it seems probable that over a larger size range, where the effect of the inertia and friction of the platen of the testing machine is less, some decrease in strength would occur.

The Griffith crack theory appears to give the best explanation of the mechanism of failure currently available, since it is substantiated by both quantitative and qualitative observations of the process of failure in brittle materials. An analysis of the Griffith theory indicates that the theory predicts that the power law relating the strength of specimens to their edge dimension should vary between one half and zero, the higher value for cracked materials, and the lower value for initially uncracked materials. This latter prediction is substantiated both by the results of these tests, and by the results of many previous studies of coal.

In order to substantiate the findings of this thesis it is necessary to test further specimens of these materials over a much larger size range, involving much larger testing machines than those used in this study.

The results of this study indicate that the upper value of the power law in the relationship between strength and size of brittle materials -0.5, and that value applies for cracked materials. Thus, if the value of -0.5 is used to design mine pillars the error involved will be on the side of safety.

SUMMARY

Cubes of coal have been tested in compression in the past, and it has been found that the following formula, relating the compressive strength to the size of the cube can be applied:

$$P = k.a^{-n}$$

Where P is the compressive strength in pounds per square inch.

a is the edge dimension of specimens tested.

n is a constant.

k is a constant.

The value of n has been found by a majority of people working on coal to be 0.5, however, lower values have also been found.

In this study limestone, shale and Plaster of Paris cubes, varying in size between one and three inches, and one and five inches in the case of the shale, were tested in compression. The results were converted to logarithmic form, and the value of n determined for each material. It was found for the limestone and the Plaster of Paris that the value of n was close to zero over the range of sizes tested, indicating that the strength is independent of the size of the specimen over the range one inch to three inch cubes. A value of 0.20 was found for the shale over the range one inch to five inches.

The Griffith crack theory of failure gives the following result:

$$P = k.c^{-0.5}$$

Where P is the compressive strength in pounds per square inch.

2c is the length of cracks in the material.

k is a constant.

Thus, depending upon the relationship between c and a, the Griffith theory predicts that the value of n should be 0.5 for cracked materials where the length of crack is directly proportional to the edge dimension of the specimen, and zero where the length of crack is independent of the edge dimension of the specimen, in relatively uncracked materials.

The Griffith theory is supported both by the results of compression tests, and by the results of the tests in this study and those previously conducted on coal.

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VITA

John Mills Noble was born on 14 July 1939, in Edinburgh, in Scotland. His parents subsequently moved to England, where he attended St. Hugh's School, Doncaster, and then the King's School, Pontefract, and Queen Elizabeth's Grammar School, Mansfield. In July, 1957, he passed the General Certificate of Education Advanced Level Examination in three subjects, and Scholarship level in two subjects, and was awarded a State Scholarship on the results of the examination. He obtained a place in the University of Sheffield in October, 1957, to study Mining Engineering. He graduated from the University of Sheffield in July, 1960 with a First Class Honours Degree in Mining Engineering.

He was awarded a Graduate Assistantship at Virginia Polytechnic Institute in September, 1960, to study for the Degree of Master of Science in Mining Engineering. On completion of this course he intends to return to England in September, 1961, to enter industry there.

He is a student member of the Institution of Mining Engineers, and a student member of the American Institute of Mining, Metallurgical and Petroleum Engineers.

ABSTRACT

Cubes of coal have been tested in compression in the past, and it has been found that the following formula, relating the compressive strength to the size of the cube can be applied:

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Where P is the compressive strength in pounds per square inch.

2c is the length of cracks in the material.

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