THE EFFECT OF SPACE LAUNCH VEHICLE TRAJECTORY PARAMETERS ON PAYLOAD CAPABILITY AND THEIR RELATION TO INTERPLANETARY MISSION DESIGN

by

Julius Burt Lovell

Report submitted to the Graduate Faculty of the

Virginia Polytechnic Institute

in partial fulfillment of the requirements for the degree of

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a	Semi-major axis of heliocentric ellipse
C	Shortest distance from launch planet at launch to target planet at arrival
C_3	Twice the unit energy per unit mass - represents the vis-viva energy integral of orbit
e	Total energy of an orbit
F_r	Attracting force between two bodies
G	Universal gravity constant
h	Altitude (of body) above the planet surface
i	Orbit inclination
m_1	Mass of Earth
m_2	Mass of spacecraft
r	Radius vector from center of planet to center of spacecraft
R	Radius vector from center of Sun to spacecraft
R_1	Radius of Earth
R_L	Radius from Sun to launch planet at time of launch
R_P	Radius from Sun to target planet at time of arrival
S	Radial asymptote
T_F	Time of flight, days
V	Heliocentric velocity of spacecraft
V_e	Earth's orbital velocity about the sun, approximately 29.8 km/sec
V_{hL}	Hyperbolic excess velocity of spacecraft at launch
V_i	Inertial velocity
V_∞	Hyperbolic excess velocity of spacecraft at target planet

θ_L Right ascension of launch site

θ_S Right ascension of radial asymptote

μ Gravitational coefficient of Earth

ν_I True anomaly of injection

ν_S Angle from perigee of hyperbolic conic to the radial asymptote

Σ_L Launch azimuth

τ Angle from the departure asymptote to a radial line which passes through the perigee of the hyperbolic conic and the center of Earth

ϕ Central range angle

ϕ_L Launch site altitude

ϕ_S Declination of radial asymptote

SUMMARY

Three parameters must be controlled to assure that a near-Earth trajectory will coincide with a desired interplanetary trajectory. These parameters are C_3 (energy), ϕ_S (declination), θ_S (right ascension).

This paper shows that C_3 is an Earth-centered term proportional to the energy of a particle in orbit about the Earth but independent of the weight of the particle and the path taken to achieve the orbit. It then shows that there is a unique value of C_3 associated with each interplanetary trajectory. These characteristics of C_3 allow the presentation of vehicle performance in a parametric manner suitable for making vehicle comparisons and useful for preliminary planning of missions to any of the other planets in our solar system.

Declination, ϕ_S, is explained in terms of celestial equatorial coordinates and shown to be essentially equivalent to the latitude on Earth.

The right ascension is explained in terms of the celestial equatorial coordinate system and shown to be a measure of the Earth's orbital position.

Mission ground rules and vehicle and range constraints affect the performance capability for a specific mission. Some of these are the required declination range, the vehicle maximum parking orbit coast capability, the available azimuth range, duration of daily launch window and choice of launch site. These constraints are discussed briefly and an example given, for a 1973 mission to Mars which shows the effect on the launch period, of limiting parking orbit coast time and specifying a minimum daily launch period.

INTRODUCTION

Interplanetary mission studies require information interchanges between different areas such as mission analysis, launch vehicle, spacecraft and experiments.

This study attempts to define the space launch vehicle trajectory parameters and constraints which will affect the other design areas. The parameters are presented with figures and brief discussions which explain the parameters and their importance in interplanetary mission planning. It is assumed that the reader will be a technically educated person with an understanding of basic energy methods, but with no prior experience related to interplanetary trajectories or launch vehicle performance.

THE VIS-VIVA INTEGRAL

Launch vehicle capability is commonly presented parametrically as a curve of spacecraft weight versus C_3 where spacecraft or payload weight is defined as the total weight remaining after separation from the final stage of the launch vehicle. C_3 is an orbit energy parameter.

The parameter C_3 indicates the difficulty of achieving a particular interplanetary trajectory in the same manner that the number of levels in a building indicates the effort required to climb the stairs from bottom to top. The higher C_3 values are more difficult to attain and, therefore, for a given launch vehicle, result in a lower spacecraft weight capability.

C_3 was first recognized by Isaac Newton, in his consideration and expansion of Kepler's laws, as a vis-viva integral which relates the velocity of a body in an elliptical orbit about a larger body to the distance from the focus (center of the large body) of the ellipse. It can be developed by considering conservative forces acting on the body in orbit. This assumption is quite accurate for satellites in orbit about Earth with orbital altitudes somewhat greater than 180 km and for orbital coast periods of less than two orbits.

The orbital energy is determined as follows:

Potential energy is determined from Newton's law of Gravitation which states, "Any two particles attract each other with a force which acts along the line joining them and has a magnitude that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them." This is shown graphically in figure 1. For a spacecraft in orbit about Earth, this may be written as

$$-F_r = G \frac{m_1 m_2}{r^2}$$

The constant G is the universal gravitational constant and is required to make the terms equal.

m_1 represents the mass of the Earth

m_2 represents the mass of the spacecraft

r is the Earth radius plus spacecraft altitude. That is the distance from the center of mass of the Earth to the center of mass of the spacecraft.

$-F_r$ is the attracting force (negative sign signifies attraction)

The potential energy is equal to the attracting force multiplied by the distance through which it would act to achieve a zero potential. This may be written as $(-F_r)(r) = G \frac{m_1 m_2}{r}$

The kinetic energy is taken along the direction of travel and is

$$\frac{1}{2} m_2 V_i^2$$

V_i denotes inertial velocity or velocity relative to a nonrotating Earth.

The total orbital energy for the spacecraft is then

e = kinetic energy plus potential energy

or

$$e = \frac{1}{2} m_2 V_i^2 - G \frac{m_1 m_2}{r}$$

We now introduce the Earth's gravitational coefficient, μ, which equals the mass of the Earth multiplied by the universal gravitational constant

$$\mu = Gm_1$$

$$e = \frac{1}{2} m_2 V_1^{\ 2} - \frac{\mu m_2}{r}$$

We can operate on this further to get the desired form of C_3 which is twice the unit energy per unit spacecraft mass.

$$C_3 = 2 \frac{e}{m_2} = V_i^{\ 2} - \frac{2\mu}{r}$$

This is the classical definition for C_3 and gives C_3 in units of km^2/sec^2 when

V_i is expressed in km/sec

μ is expressed in km^3/sec^2

r is expressed in km

Note that it is possible to derive similar energy expressions for bodies in orbit about other celestial bodies by the proper definition of m_1 and m_2.

Examination of the equation for C_3 shows three possibilities:

1. C_3 will be negative. For this case the spacecraft is in an elliptic orbit. It cannot escape the Earth's gravity and must remain in orbit about the Earth or reenter the Earth's atmosphere. The special case of a circular Earth orbit exists when $C_3 = -\frac{\mu}{r}$.

2. C_3 will be zero. This is a borderline case between elliptic and hyperbolic orbits. It is called a parabolic orbit but has no significance for interplanetary missions.

3. C_3 will be positive. This is called a hyperbolic orbit because as r approaches infinity the potential function, $\frac{2\mu}{r}$, approaches zero. Thus for C_3 positive, C_3 represents the square of the magnitude of the spacecraft velocity (relative to Earth) which remains after the Earth's gravity ceases to affect the spacecraft.

$$C_{3(r \to \infty)} \Longrightarrow V_i^{\,2} + 0 \Longrightarrow V_{hL}^{\,2}$$

The velocity, relative to Earth, remaining upon escape from the Earth's gravity is called the hyperbolic excess velocity. It is designated by V_{hL} in this paper. Its magnitude is equal to the square root of C_3. Thus the velocity of the spacecraft in its orbit about the Sun (after Earth gravity effects are accounted for) is simply the vector sum of the hyperbolic excess velocity and the orbital velocity of the Earth.

This is a very important result because it establishes the hyperbolic excess velocity as one of three parameters necessary to patch the near-Earth trajectory into the heliocentric trajectory which considers only the Sun and the spacecraft as another two-body system.

RIGHT ASCENSION AND DECLINATION

If one were to attempt to hit a moving target with a baseball it would be necessary to control the direction of throw as well as the speed. A similar requirement exists for interplanetary travel where the speed is controlled by C_3 and the direction is established by two parameters called declination and right ascension of the radial asymptote.

This section attempts to explain what declination and right ascension mean in a physical sense.

Celestial Coordinate Systems

Astronomers have defined two celestial coordinate systems, both based on a celestial sphere, with the Earth as the center. An ecliptic coordinate system denoted by celestial latitude and celestial longitude is used for calculations when the Earth's rotation is not important. The equatorial coordinate system is used when the Earth's rotation must be considered. These two systems are depicted in figure 2 which has been taken from reference 4.

The Celestial Sphere.- The celestial sphere is defined in reference 4 and is essentially as follows:

> For the purpose of identifying directions in space, it is customary to image that all of the distant stars lie on a vast sphere, called the celestial sphere, which has its center at the Earth's center.

On the celestial sphere, two great circles known as the ecliptic and celestial equator are used as reference circles for the ecliptic and celestial coordinate systems, respectively. The ecliptic is formed by the intersection of the Earth's orbital plane with the celestial sphere. The celestial equator is formed by extending the plane of the Earth's equator until it intersects the celestial sphere.

The ecliptic and celestial equator intersect at two points called the equinoxes. The vernal equinox which is used as a reference point is that point of intersection at which the Sun, in its apparent motion along the ecliptic, crosses the celestial equator moving from south to north. This happens on about March 21 of each year.

In the <u>ecliptic coordinate system</u>, points on the celestial sphere are identified by <u>celestial latitude</u>, measured northward or southward from the ecliptic and by <u>celestial longitude</u>, measured eastward and westward along the ecliptic from the vernal equinox.

In the <u>equatorial coordinate system</u> a point on the celestial sphere is located by <u>declination</u> and by <u>right ascension</u>. Declination, measured in degrees northward or southward from the celestial equator, corresponds (except for minor effects of gravitational deviation and of the Earth's slightly non-spherical shape) to the parallel or latitude on the surface of the Earth along which the given celestial point passes directly overhead. Right ascension is measured eastward along the celestial equator from the vernal equinox. It is usually expressed in hours, minutes, and seconds from zero and 24 hours, where one hour equals 15 degrees of arc. Eastward in celestial coordinates is prograde to, or in the direction of, the Earth's orbit about the Sun; it is counterclockwise when viewing the equatorial plane from the north celestial pole.

Thus, right ascension designates a radial direction from the center of the Earth in the equatorial plane and declination defines an angle above or below the equatorial plane. A vector, which is directed radially from the Earth at a prescribed declination and has a projection onto the equatorial plane which is coincident with a prescribed right ascension, is uniquely oriented for a given day.

The spacecraft trajectory, from a few hours to a few days after injection, represents such a vector. Figures 3, 4, and 5 illustrate this condition.

Flight Path

Figure 3 shows the projection of the spacecraft flight path upon the surface of the Earth. In this figure the direction of vernal equinox is identified and the right ascension of the S vector, θ_S, is shown. The vector S is called the radial asymptote and is explained later.

The launch site is located by its right ascension, θ_L, declination (or latitude) ϕ_L, and the radius from the center of the Earth R_L.

The vehicle flight plane projection onto the surface of the Earth starts at the launch site along the direction of the launch azimuth, Σ_L. Azimuth is measured in degrees of arc with zero and 360 degrees towards the north, 90 degrees towards the east, and 180 degrees to the south. The inclination of the flight plane, i, is the angle between the plane of flight, projected to the equatorial plane and the equator. The angle through which the spacecraft would travel from launch until it appears to be going directly away from the Earth is called the central range angle, ϕ, and is measured in the plane of the trajectory.

The spacecraft appears to be going directly away from Earth as it approaches the limit of the Earth's sphere of influence. The Earth's sphere of influence may be considered a sphere whose radius is such that the Earth's gravity (determined by μ/r) is no greater than the gravity force of the Sun. This is at approximately 925,000 km.

The declination, ϕ_S, of the radial asymptote (also called the launch asymptote) is the angle measured between the radial asymptote, S, and the equatorial plane, positive towards north.

It can be considered to be the latitude of the ground trace of the spacecraft as the spacecraft escapes the Earth's sphere of gravity influence. Stated another way, if there were an observer at the center of the Earth looking at the spacecraft as it leaves the Earth's sphere of influence, the latitude at which his line of sight pierced the Earth's surface could be considered the declination of the departure asymptote. Actually this is not a correct representation, but it is sufficiently accurate for most mission planning.

In actuality the spacecraft will be traveling on a hyperbola and the asymptote of that hyperbolic trajectory will not pass through the center of the Earth.

The basic geometry for launch and ascent profile is shown in figure 4 which was taken from reference 3. This figure is drawn in the plane of the trajectory so it does not show the right ascension and declination of the radial asymptote. It does, however, show those events which occur within the plane of the trajectory.

The launch phase or boost phase starts at liftoff (at the launch site) and continues through the first powered flight phase, ϕ_1, the coast in parking orbit and the final stage burn, ϕ_2, terminating at injection into the interplanetary trajectory which will intercept the target planet on the desired arrival day.

After injection the spacecraft coasts in a hyperbolic orbit which continues to turn toward the Earth until it becomes parallel to the departure asymptote. The coast arc, ν_S, is measured from the perigee of the hyperbola to the radial asymptote. The radial asymptote is parallel to the departure asymptote and passes through the center of the Earth.

The point of injection is a function of the final stage flight direction and may vary from one vehicle to another. But, the departure hyperbola always has a theoretical point of closest approach to Earth called the perigee of the hyperbolic conic. So the coast arc, ν_S, is always defined from the perigee and not from the point of injection. Similarly, the point of injection is identified by the angle, ν_I, called the true anomaly of injection. The true anomaly is used to signify the position of a point on an orbit from the lowest point of the orbit; it is positive when taken in the direction in which the spacecraft is traveling.

Since the perigee and the departure hyperbola are fixed for a particular orbit, it is possible to define a line in the orbital plane which passes through the center of the Earth and the perigee of the orbit and intersects the departure asymptote. This line is called the line of apsides. For an elliptical Earth orbit, the line of apsides would pass through the point of closest approach (perigee) and the point of maximum altitude (apogee). For a hyperbolic orbit the angle between the line of apsides and the departure asymptote is designated by the symbol τ. It varies with the energy of the orbit, but is always greater than zero and less than 90 degrees.

There are an infinite number of departure trajectories for any specific declination of radial asymptote as is shown in figure 5. The asymptotic velocity and actual velocity relative to Earth are assumed identical in magnitude and direction for each of these trajectories. This assumption can be made with good accuracy because the radius of the Earth's sphere of influence, approximately 925,000 km, is very large when compared to the perigee radius (usually less than 200 km) and yet is quite small when compared to the Earth-Sun distance of approximately 150,000,000 km. The sphere of influence of any celestial body (other than the Sun) may be considered to be the radius at which the gravitational force of the body is just equal to the gravitational force of another body or bodies.

If the declination and right ascension of the asymptote of the spacecraft hyperbolic excess velocity and the day of the launch is known, then it is possible to determine with sufficient accuracy the initial conditions of the spacecraft orbit about the Sun.

The curve of payload versus C_3 must specify the applicable range of declinations because a particular launch vehicle and launch site will not allow all values of ϕ_S.

DETERMINATION OF LAUNCH OPPORTUNITY

It has been stated that C_3, ϕ_S, θ_S are the three parameters necessary to match the near-Earth trajectory to an interplanetary trajectory. The method for selecting a launch opportunity will now be described to illustrate the use of these parameters.

The complete trajectory problem will be broken down into three separate but dependent simplified parts as follows:

1. A near-Earth phase considering only the spacecraft and the Earth. This includes all of the powered launch phases and the early interplanetary coast phase.

2. A heliocentric (Sun-centered) phase which includes most of the trip time. An actual flight would require course correction maneuvers during this phase.

3. A near-target phase where the target planet and spacecraft are considered as a two-body system. This phase is handled in a manner similar to the near-Earth phase.

Heliocentric Trajectory Phase

Figure 4 shows the near-Earth phase and figure 6 shows the heliocentric or Sun-centered phase. If all planet orbits were in a single plane, then a minimum energy trajectory (one for which a given launch vehicle could deliver a maximum spacecraft weight to the transfer orbit) would be an ellipse about the Sun just large enough to include the Earth (at launch) at one end of the major axis and the target planet (at arrival) at the other end of the major axis. The spacecraft would travel 180 degrees around the Sun and there would be only one launch day and one arrival day corresponding to this minimum energy trajectory for each period of planet conjunction.

In reality, the other planet orbits are inclined slightly to the plane of the ecliptic and for practical reasons space missions require a launch period of several days. However, it is still desirable to launch near the minimum energy requirement to get the best use from the launch vehicle. To determine an acceptable period for mission consideration the heliocentric trajectory phase is examined parametrically about the period of planet conjunction to identify specific launch day-arrival day combinations for which the launch vehicle can deliver an acceptable spacecraft weight. These particular launch days selected for further mission studies are called the launch period or launch opportunity.

Use of Lambert's Theorem.- Once a launch day and trip time or arrival day has been selected Lambert's Theorem is solved to determine the trajectory which satisfies these conditions. Reference 2 was the source for much of the following: Lambert's Theorem states the transfer time between any two points on an elliptical orbit is a function of the sum of the distances of each point from the focus, distance between the points, and the semi-major axis of the ellipse. Functionally, this is represented as

$$T_F = T_F\ (R_L + R_P,\ C,\ a)$$

The Sun is located at one focus of the heliocentric orbit.

T_F = Trip time from launch planet to target planet, days

R_L = Radius from Sun to launch planet at time of launch

R_P = Radius from Sun to target planet at time of arrival

C = Distance from launch planet at time of launch to target planet at time of arrival (see figure 6)

For a specific launch day and arrival day, R_L and R_P are determined from the American Ephemeris Almanac, and C is determined by vector subtraction (see fig. 6, $C = R_L - R_P$). A digital computer is then used to solve for a , the semi-major axis of the heliocentric trajectory which intersects the center of the launch planet on the day of launch and the center of the target planet on the day of arrival.

These calculations are made in celestial equatorial coordinates. Velocities (magnitude and direction) are determined for the trajectory at launch and arrival times.

Near-Earth Phase

The heliocentric trajectory is matched to the planet centered trajectory as follows: The launch planets heliocentric velocity is subtracted vectorially from the heliocentric velocity of the transfer trajectory (see fig. 7) leaving V_{hL} the hyperbolic excess velocity of the spacecraft.

$(V_{hL})^2$ is equal to C_3 and this data can be plotted as C_3 contours for various launch day-arrival day combinations. These (C_3) energy contours are then examined and a launch period selected for further mission study. It is necessary to limit mission planning to near the minimum possible C_3 values to assure a reasonably efficient use of the launch vehicle (i.e., near maximum payload weight). But other mission objectives and requirements will tend to shift the launch day-arrival day choice away from the absolute minimum energy trajectory, so a total launch period of 60 to 120 days may be considered for early planning purposes.

Figure 8 is a C_3 contour plot for a 1973 mission to Mars, taken from reference 3. The launch period covered is from about June 13 to September 3, 1973, while arrival dates vary from November 1973 to May 1974. Declinations of the radial asymptotes are also shown. This particular planning chart is for type I transfer trajectories and shows that there are two arrival days for each launch day and each energy level.

A type I trajectory is a trajectory which travels less than 180 degrees around the Sun. Type II trajectories require the spacecraft to travel more than 180 degrees around the Sun.

The two different arrival dates for each launch day and C_3 level may be considered as different orientations of the trajectory ellipse. The heliocentric trajectory, except in the vicinity of the planets, is an elliptical trajectory; it contains the Sun at one focus and the position of the Earth at launch and the planet at arrival as points on the ellipse. For the shortest trip time, called class I trajectories, the trace of the ellipse would not cross the orbital trace of Mars before arrival. For the longest trip time, the ellipse must be reoriented such that the trajectory trace will cross the Mars orbital trace before the spacecraft intersects Mars; the spacecraft then intersects Mars on the second time its trajectory crosses the Mars orbit.

MISSION CONSTRAINTS

It has been shown so far that launch vehicle payload capability can be presented as a function of C_3 and that this manner of presentation is especially useful for interplanetary mission planning. The orientation of the hyperbolic excess velocity vector has been shown to be determined by right ascension and declination.

Since C_3 is independent of the ascent profile it can be used as a common parameter to compare launch vehicles. Each launch vehicle capability is determined for the optimum ascent trajectory and this capability is then plotted as spacecraft weight versus C_3.

There are, however, many constraints which will affect the launch vehicle capability for a particular mission. These include launch vehicle system limits, launch site and launch range constraints, and mission ground rules.

Some of these will be discussed briefly to identify their effect on launch vehicle capability. The Eastern Test Range (Cape Kennedy, Florida) will be assumed to be the launch site.

Range and Tracking Constraints.- The Eastern Test Range is responsible for the safety of property and personnel at the launch site and along the launch vehicle flight path. Launch azimuths between 45 degrees and 114 degrees east of north have been established for Eastern Test Range launches. In addition, both the spacecraft and the launch vehicle may have requirements for radar tracking and communications, such as receiving and sending flight data and guidance commands, during the ascent. These requirements, because of the location of tracking and data stations, may limit the azimuths available for a particular launch to approximately 65 degrees to 114 degrees east of north.

Mars Mission

Some aspects of a mission to Mars with a 1973 launch will be discussed to demonstrate the effects of various constraints.

Figure 8 shows C_3 requirements about the minimum energy point for type I trajectories and the declination associated with various arrival days. A maximum acceptable C_3 of 24 km^2/sec^2 will be assumed. For this case, the launch period appears to start on about August 25, with required declinations greater than +15 degrees and less than 55 degrees. The earliest arrival date is December 7, 1973. One other target related constraint to be considered is the spacecraft hyperbolic excess velocity at arrival at the target. This affects the speed of a flyby or impact mission and the orbit insertion velocity requirement for an orbital mission. The hyperbolic excess velocity at the target planet is designated as V_∞ to avoid confusion with the launch planet terms discussed previously. A line on figure 11 indicates a $V_\infty = 3.5$ km/sec. All points below and to the left of this line represent higher arrival velocities and are unacceptable.

The first possible launch day becomes June 27 to satisfy the target planet arrival velocity and the corresponding earliest arrival date is about January 15, 1974, instead of December 7, 1973.

Yaw Ascent.- Figure 3, which has been discussed previously, depicts the vehicle flight plane for a planar ascent. That is the trajectory from liftoff through injection. Another type of trajectory called a yaw or dogleg ascent is possible and is sometimes used to achieve a desired orbit inclination when range or other constraints would not allow a planar ascent. A yaw ascent occurs when the vehicle is launched along a preselected azimuth and then, sometime during the powered phase of flight, executes a left or right turn, called a yaw maneuver, to change the plane of the final trajectory. The yaw ascent is less efficient than an equivalent planar ascent, resulting in less spacecraft weight capability.

Planar Ascent.- A planar ascent to orbit will result in an orbital plane which contains the center of the Earth and the launch site latitude; during one complete orbit the spacecraft will have a ground trace which will include at least all latitudes from the launch site latitude to the corresponding latitude on the opposite side of the equator. Thus, if the capability existed to restart the launch vehicle after a coast in orbit, it would be possible to launch at any time of the day and select a parking orbit coast time such that a declination (ϕ_S) less than or equal to the magnitude of the launch site latitude could be achieved. However, if the required declination is greater than the launch site latitude, ϕ_L, then launch azimuths must be chosen to assure that the orbital plane will intersect the required declination or a yaw maneuver will be required during the ascent trajectory.

Launch Azimuths.- The choice of launch azimuth, Σ_L, affects spacecraft weight capability because the surface of the Earth is moving eastward, due to the Earth's rotation, at approximately 380 m/s at the Eastern Test Range. A due east launch (Σ_L = 90 degrees) takes full advantage of this rotational effect whereas launches at azimuths other than 90 degrees east of north will get correspondingly less assist.

Geometrical restrictions of the launch azimuth is discussed in reference 3 as follows:

"If the absolute value of the departure asymptotic declination is greater than the latitude of the launch site (28.3 degrees for the Eastern Test Range), then there is a band of azimuths symmetric about 90 degrees east of north which cannot be utilized for planar ascents. The limits of this band are determined by

$$\sin \Sigma_L = \frac{\cos \phi_S}{\cos \phi_L} \text{ "}$$

This effect is plotted on figure 9 which shows some geometrical relationships for the Eastern Test Range. Figure 9 is plotted for a hypothetical right ascension; however, due to the Earth's rotation, every right ascension is available for a launch at some time during each day. Launch is assumed to occur at 24 hours on the right-hand side of the figure with the vehicle flight path proceeding to the left along the launch azimuth. The time scale at the bottom of the figure represents 15 degrees of Earth rotation per hour; it could be considered degrees of longitude at the equator. Central range angles of 170 and 290 degrees have been plotted for later use. The central range angle trace is curved from vertical because of the inclination effect of orbits at different launch azimuths.

Declinations between plus and minus 50 degrees are available for the band of launch azimuths from 45 to 135 degrees east of north. The 50-degree declination limit to satisfy range safety constraints does not change the total launch opportunity assumed for a C_3 of 24 but does remove some arrival dates from our mission consideration.

Parking Orbit Coast.- Parking orbits are generally chosen to be circular at about 185 km altitude. The circular orbit velocity at 185 km is approximately four degrees of arc per minute. The circular orbit simplifies the design and control of the flight trajectory. The altitude represents a compromise, it must be of sufficient height to avoid excessive drag effects and allow adequate view periods for tracking, but it is desirable to keep the parking orbit as low as possible to maximize the launch vehicle's payload weight capability. Also the lower orbits are slightly faster in terms of coast arc per unit of time.

The maximum parking orbit coast time is determined by the capability of the spacecraft and launch vehicle systems. The current Centaur launch vehicle stage has a maximum parking orbit coast capability of about 30 minutes. This is equivalent to 120 degrees of orbital arc at a coast velocity of four degrees of arc per minute.

We will assume a Titan/Centaur launch vehicle with a maximum parking orbit coast capability of 120 degrees of arc. Figure 4 illustrated the various segments of the central range angle ϕ. For our launch vehicle choice the powered phases of flight, ϕ_1 and ϕ_2, are essentially constant at 18 and 25 degrees, respectively. We will define the true anomaly of injection to be a constant 11 degrees and, ν_S, the arc from the perigee of the hyperbolic conic to the radial asymptote will be approximately 136 degrees for a C_3 of 24 as shown in figure 10.

With these vehicle related assumptions the available central range angle values can be established as

$$\phi_{minimum} = \phi_1 + \phi_2 - \nu_I + \nu_S = 18 + 25 - 11 + 136 = 170°$$

$$\phi_{maximum} = \phi_{minimum} + \text{parking orbit coast capability}$$

$$= 170^\circ + 120^\circ = 290^\circ$$

Thus, for a Titan/Centaur launched from the Eastern Test Range we have available only declinations which can be achieved with a central range angle variation of 170 to 290 degrees. Figure 9 has been marked with these approximate range angle limits. Examination of figure 9 within the band of available central range angles shows that launch azimuths must be greater than 95 degrees to achieve the minimum required declination of +15 degrees while the maximum declination available to a planar ascent is about +30 degrees at a launch azimuth of 114 degrees east of north. To reach declinations greater than +30 degrees we may increase the parking orbit coast arc or utilize a yaw maneuver ascent to achieve equivalent azimuths greater than 114 degrees east of north. The true anomaly of injection may also be varied but the payload effect may be more severe than a yaw maneuver. (ν_I is assumed to be a constant 11 degrees for this example.).

Assume that we can utilize yaw maneuvers to an effective azimuth of 130 degrees east of north and that this allows a declination range of +15 degrees to +40 degrees. This change further restricts the available arrival days but still does not reduce the launch period. However, our launch azimuths must be greater than 95 degrees.

Daily Firing Window.- All of the preceding has ignored the amount of time available to accomplish a launch each day. It is possible to launch an interplanetary mission, with restraints on declination, right ascension and launch azimuth, only during a small part of the day. The time during which a launch is possible is called the daily launch window or daily firing window.

Reference 3 states:

"An adequate firing window should be provided during each launch date, because the launching of a vehicle at a precise instant in time is improbable due to the complexity of both the booster and the spacecraft.

"The available firing window for each launch day is a function of (1) launch site latitude, (2) launch azimuth interval, and (3) declination of the departure radial asymptote."

Figure 9 can be used to indicate the effect of a minimum daily window upon launch azimuth requirements and the available declination.

Assume a two-hour daily launch window, as a mission ground rule, and a required declination of +25 degrees. Referring to figure 9, it is seen that a central range angle of approximately 290 degrees (equivalent to a 30-minute parking orbit coast time) would limit the azimuth to 108 degrees minimum. The launch time for this case is 5.3 hours.

The time scale on figure 9 is for a hypothetical launch date and right ascension of the radial asymptote; however, the daily firing window for any interplanetary mission can be extracted as the change in time as follows: recognize that a chosen launch day-arrival day combination for the target planet specifies the trajectory in terms of C_3, declination and right ascension. The right ascension represents a radial direction in the celestial equatorial plane; but, the Earth is the center of the celestial coordinate systems. A given longitude will intersect every right ascension during the daily rotation of the Earth. We must select launch azimuths such that the launch trajectory intersects the required declination within the central range angle limits. The projection of the available central range angles onto the equatorial plane will sweep across the required right ascension as the Earth rotates. The daily launch window is that time during which the equatorial projection of an available range angle is aligned with the required right ascension. Thus, the daily launch window for any interplanetary mission is the projection of the available central range angles onto the equator, divided by the Earth's rotation rate; it can be extracted as the change in launch time on figure 9.

Having established the minimum azimuth to be 108°, we follow the 25° declination line in the direction of increasing azimuth and decreasing central range angle to a relative time of 7.3 hours. The corresponding azimuth is 123°.

Since range safety limits planar ascent trajectories to launch azimuths of about 114°, this example would require a yaw maneuver (dog leg) ascent trajectory.

It was previously shown that a declination of +30 degrees was available at an azimuth of 114 degrees with a 30-minute parking orbit coast capability when there were no requirements on daily window duration. Thus, for the example shown, the requirement for a two-hour daily window resulted in significant effects on available declination, useful azimuth range and ascent trajectory shaping.

Following the previous assumption that we could utilize a yaw maneuver ascent to an equivalent azimuth of 130 degrees, we can make an approximation of the maximum declination available with the two-hour daily window restraint. By following the 35 degree declination line from the limiting range angle (equivalent time of 5.6 hours) to the 130 degree azimuth line (equivalent time of 7.3 hours), we see that approximately 1.7 hours would be available as a daily window at a declination of 35 degrees. This is only a rough approximation of the daily window for the yaw ascent maneuver so assume that 35 degrees declination is available. This limits even further the choice of arrival days available for the early launch days.

The final launch period has been drawn on figure 11 to demonstrate how the constraints considered have reduced the available launch day-arrival day combinations.

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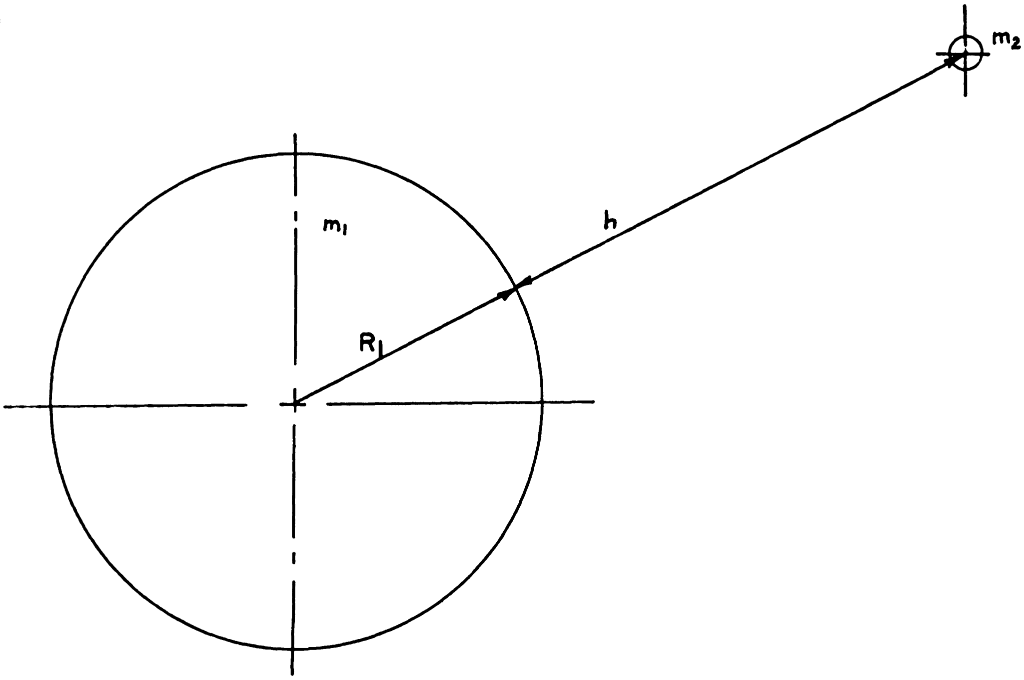
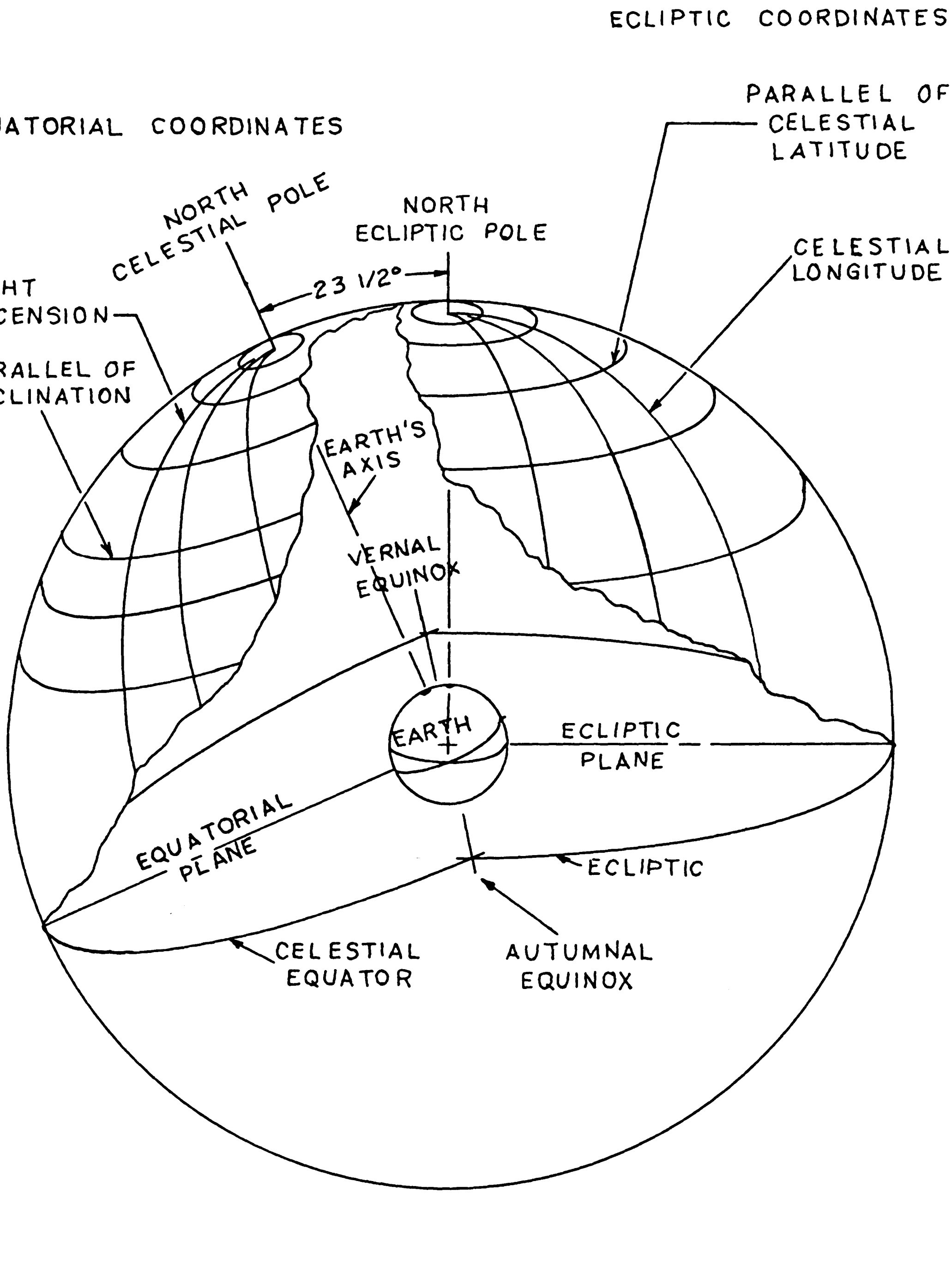
3. Kohlhase, Charles E.; and Bollman, Willard E.: Trajectory Selection Considerations for Voyager Missions to Mars During the 1971-1977 Time Period. Jet Propulsion Laboratory Engineering Planning Document No. 281, September 15, 1965.

4. Space Navigation Handbook. Bureau of Naval Personnel, NAVPERS 92988, 1962.



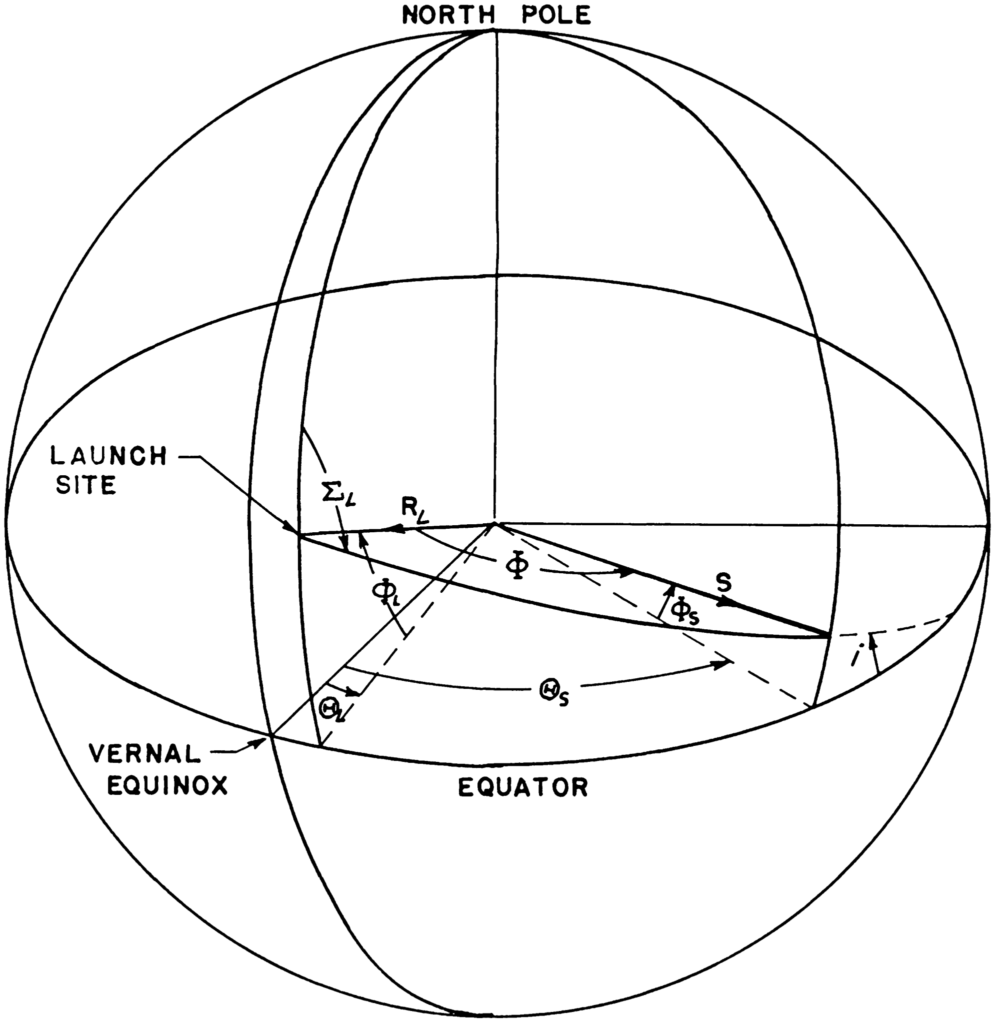
m_1 = MASS OF EARTH
m_2 = MASS OF SPACECRAFT
R_1 = EARTH'S RADIUS
h = SPACECRAFT ALTITUDE
r = RADIUS FROM CENTER OF EARTH TO CENTER OF SPACECRAFT
G = UNIVERSAL GRAVITY CONSTANT
μ = SYSTEM GRAVITATIONAL COEFFICIENT = $G(m_1 + m_2)$
IF $m_2 << m_1$. THEN $\mu \approx Gm_1$

FIGURE 1. A TWO BODY SYSTEM



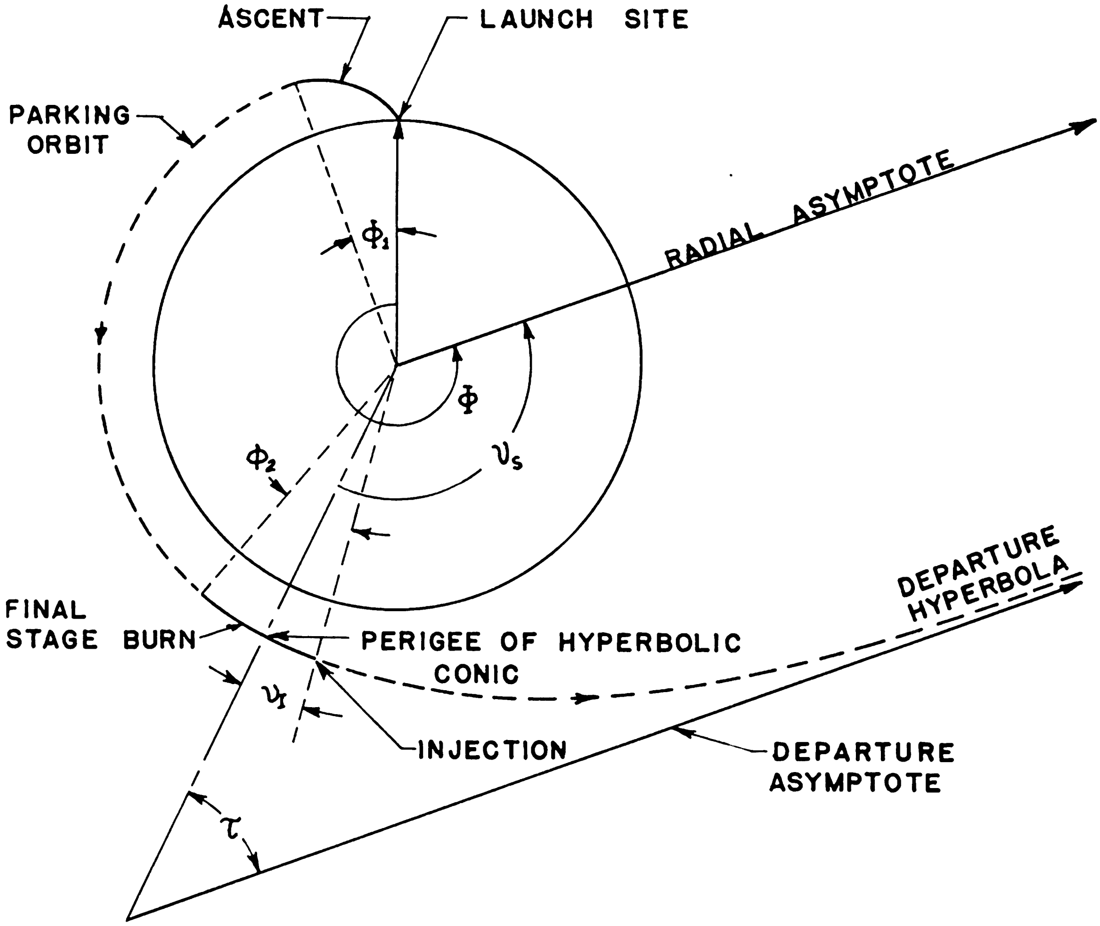
FROM REFERENCE 4

FIGURE 2. CELESTIAL SPHERE



Σ_L = LAUNCH AZIMUTH
Θ_L = RIGHT ASCENSION OF LAUNCH SITE
Φ_L = LATITUDE OF LAUNCH SITE
R_L = RADIUS TO LAUNCH SITE FROM EARTH'S CENTER
S = RADIAL ASYMPTOTE
Φ_S = DECLINATION OF RADIAL ASYMPTOTE
Θ_S = RIGHT ASCENSION OF RADIAL ASYMPTOTE
Φ = CENTRAL RANGE ANGLE
i = INCLINATION OF ORBIT

FIGURE 3 TRACE OF VEHICLE FLIGHT PLANE ON EARTH'S SURFACE



Φ_1 IS BURNING ARC OF LAUNCH VEHICLES INTO PARKING ORBIT

Φ_2 IS BURNING ARC OF FINAL STAGE

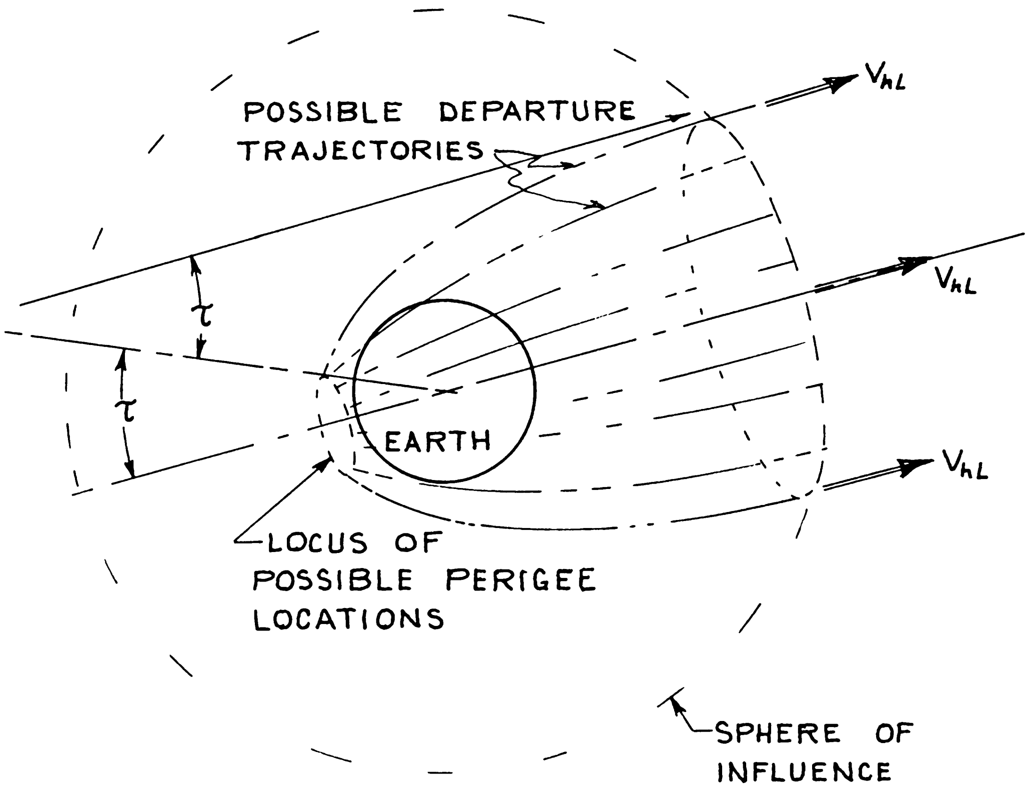
Φ IS CENTRAL ANGLE BETWEEN LAUNCH SITE AND RADIAL ASYMPTOTE

ν_s IS ANGLE BETWEEN PERIGEE AND RADIAL ASYMPTOTE

ν_I IS TRUE ANOMALY OF INJECTION

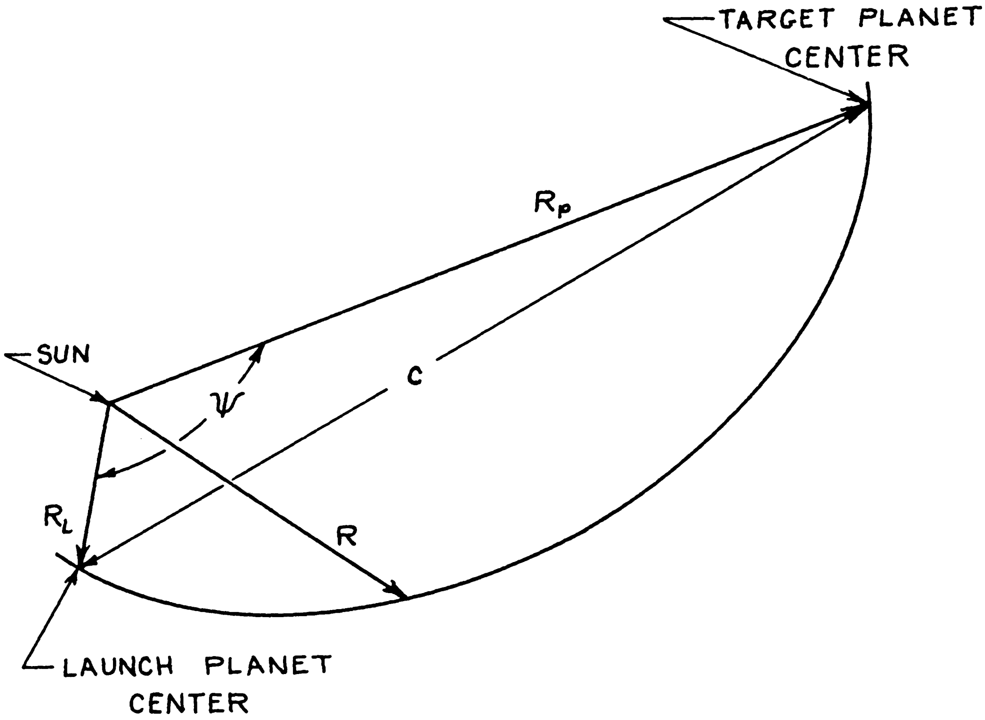
τ IS ANGLE BETWEEN DEPARTURE ASYMPTOTE AND LINE OF APSIDES

FIGURE 4 BASIC GEOMETRY FOR ASCENT PROFILE



V_{hL} IS HYPERBOLIC EXCESS VELOCITY OF SPACECRAFT RELATIVE TO EARTH

FIGURE 5. POSSIBLE DEPARTURE TRAJECTORIES



R_L IS HELIOCENTRIC RADIUS OF LAUNCH PLANET AT TIME OF LAUNCH

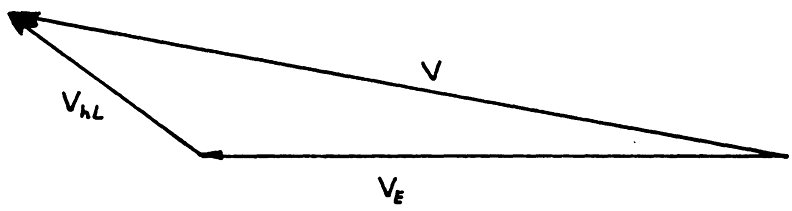
R_P IS HELIOCENTRIC RADIUS OF TARGET PLANET AT ARRIVAL

ψ IS HELIOCENTRIC CENTRAL ANGLE

R IS SPACECRAFT POSITION VECTOR

C IS DISTANCE BETWEEN LAUNCH PLANET AT LAUNCH AND TARGET PLANET AT ARRIVAL $\quad C = |\bar{R}_P - \bar{R}_L|$

FIGURE 6 IN-PLANE TRANSFER GEOMETRY



V IS HELIOCENTRIC VELOCITY OF SPACECRAFT AT LAUNCH

V_E IS EARTH'S ORBITAL VELOCITY ABOUT THE SUN APPROXIMATELY 29.8 KM/SEC.

V_{hL} IS SPACECRAFT HYPERBOLIC EXCESS VELOCITY
(IT IS THEORETICALLY POSSIBLE TO HAVE SPACECRAFT TRAJECTORIES TRAVEL ABOUT THE SUN IN THE DIRECTION OPPOSITE THE LAUNCH PLANETS ORBITAL ROTATION BUT THE V_{hL} REQUIRED FOR SUCH MISSIONS MAKES THEM IMPRACTICAL)

FIGURE 7. DETERMINATION OF HYPERBOLIC-EXCESS VELOCITY VECTOR

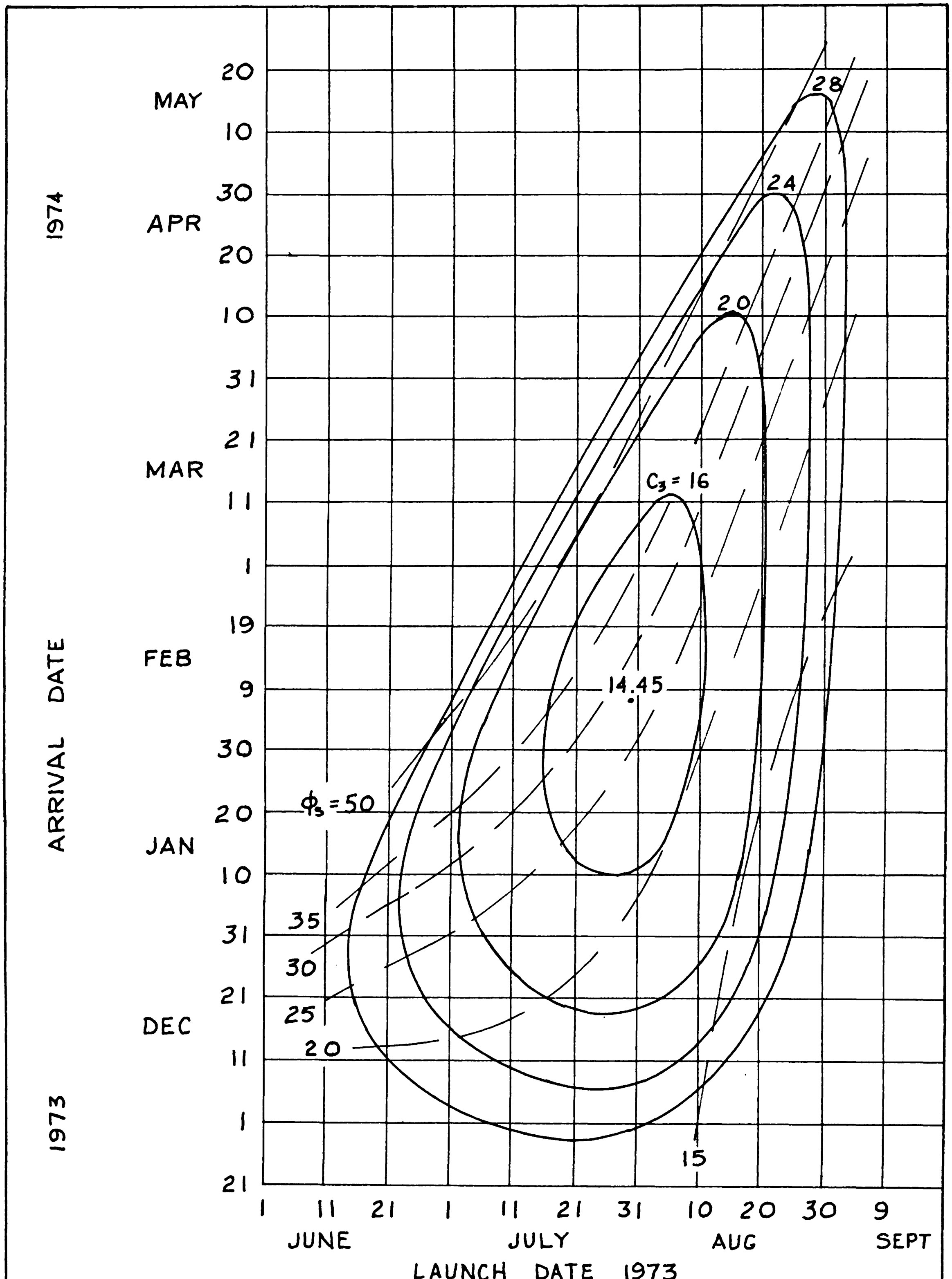


FIGURE 8 TRAJECTORY DESIGN, 1973 TYPE I

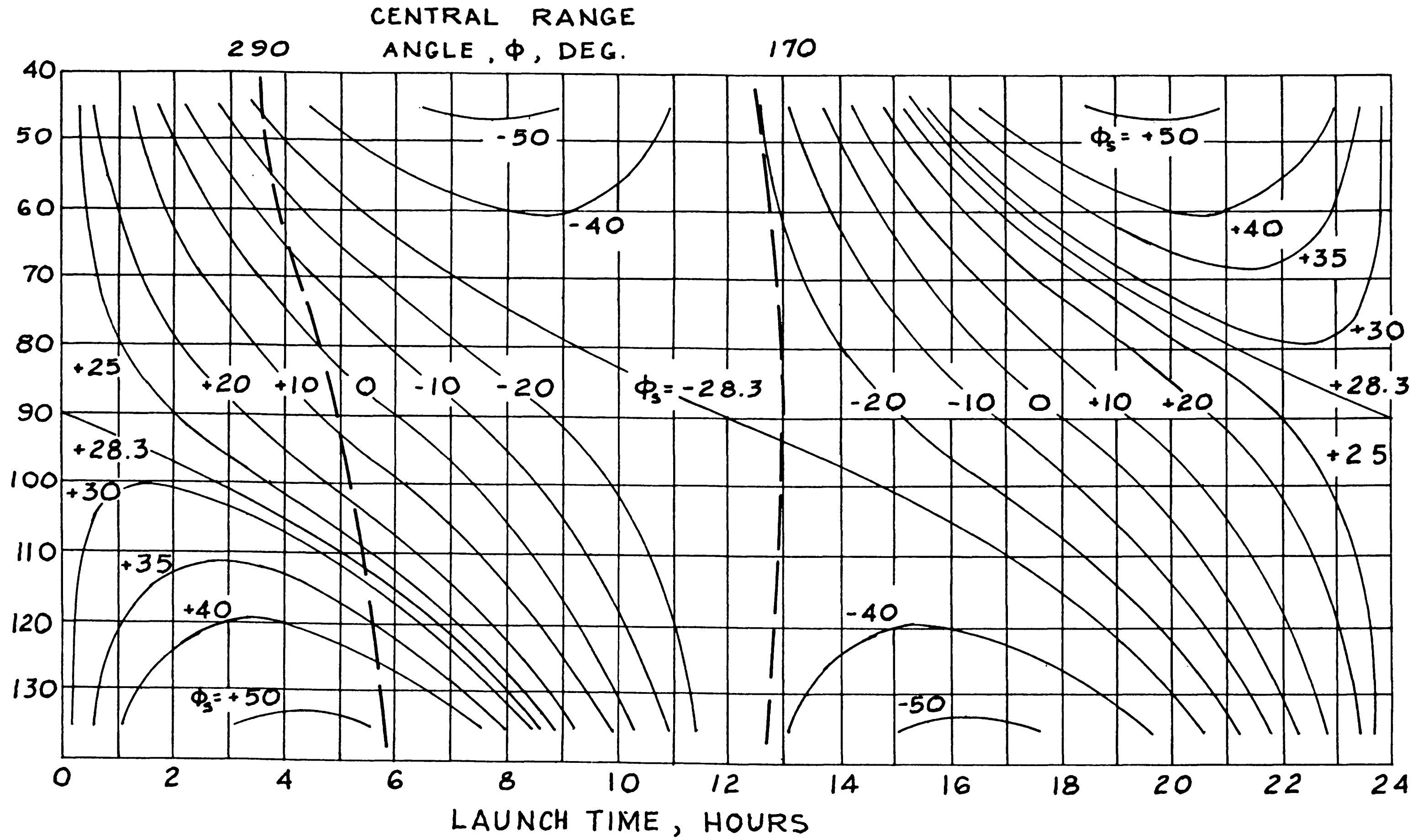


FIGURE 9 GEOMETRICAL RELATIONSHIPS
EASTERN TEST RANGE

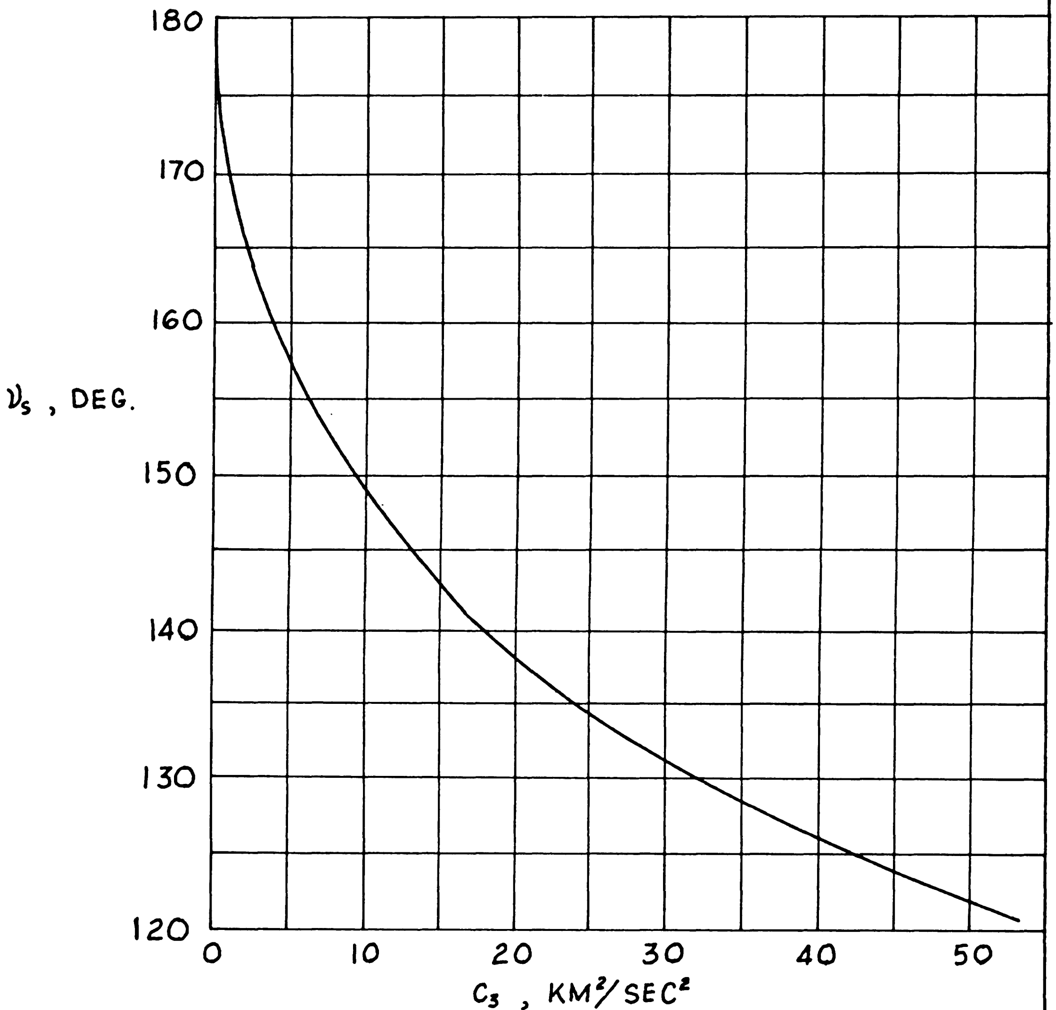


FIGURE 10. ANGLE FROM PERIGEE TO RADIAL ASYMPTOTE AS A FUNCTION OF C_3

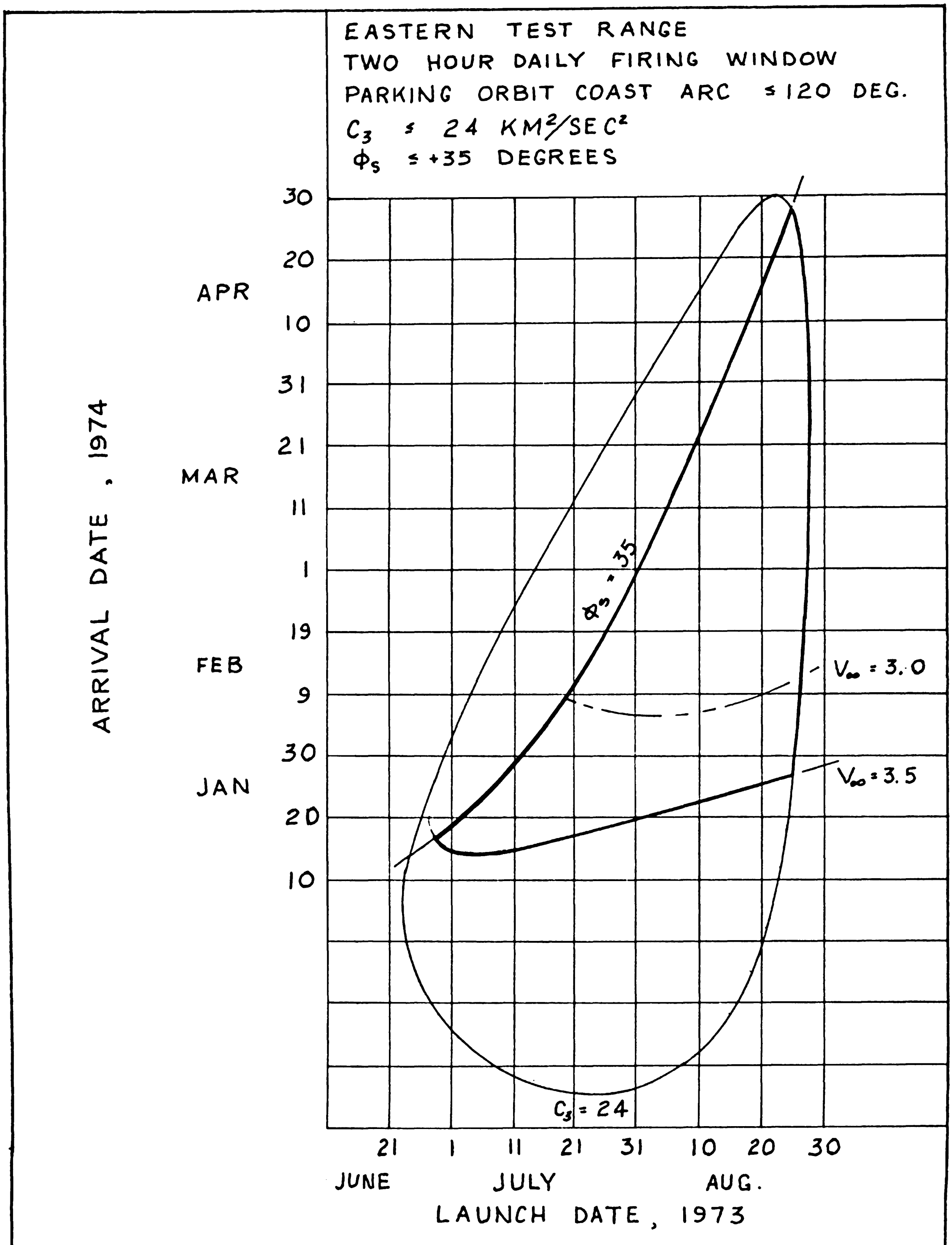


FIGURE II LAUNCH PERIOD, MARS 1973
TYPE I TRAJECTORIES