Experimental Identification of Nonlinear Systems

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(ABSTRACT)

A procedure is presented for using a primary resonance excitation in experimentally identifying the nonlinear parameters of a model approximating the response of a cantilevered beam by a single mode. The model accounts for cubic inertia and stiffness nonlinearities and quadratic damping. The method of multiple scales is used to determine the frequency-response function for the system. Experimental frequency- and amplitude-sweep data are compared with the prediction of the frequency-response function in a least-squares curve-fitting algorithm. The algorithm is improved by making use of experimentally known information about the location of the bifurcation points. The method is validated by using the parameters extracted to predict the force-response curves at other nearby frequencies.

We then compare this technique with two other techniques that have been presented in the literature. In addition to the amplitude- and frequency-sweep technique presented, we apply a second frequency-domain technique and a time-domain technique to the second mode of a cantilevered beam. We apply the restoring-force surface method assuming no a priori knowledge of the system and use the shape of the surface to guide us in assuming a form for the equation of motion. This equation is used in applying the frequency-domain techniques: a backbone curve-fitting technique based on the describing-function method and the amplitude- and frequency-sweep technique based on the method of multiple scales. We derive the equation of motion from a Lagrangian and discover that the form assumed based on the restoring-force surface is incorrect. All of the methods are reapplied with the new form for the equation of motion. Differences in the parameter estimates are discussed. We conclude by discussing the limitations encountered for each technique. These include the inability to separate the nonlinear curvature and inertia effects and problems in estimating the coefficients of small terms with the time-domain technique.

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To my Lord and Savior Jesus Christ

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Chapter 1

Introduction

1.1 Background and Motivation

Experimental identification is a necessary step toward improved modeling of nonlinear systems. It complements theoretical modeling by allowing the analyst to investigate several areas that cannot otherwise be explored. Experimental identification includes all efforts to move from observations and measurements of the responses of a system to a predictive model for the system. This may include qualitative observation to suggest a form for the model. It also includes estimating the parameters of an assumed form. Such an estimation can be done by comparing experimental data with theoretical predictions. Some of the areas that could not otherwise be explored include quantitative predictions of nonlinear vibration responses, the assessment of the effects of nonlinearities in the boundary conditions, and the validation of theoretical models.

Experimental identification enables quantitative modeling by providing a way to estimate parameters that cannot otherwise be accurately determined. Linear and nonlinear damping are examples of parameters that cannot be theoretically estimated. Tabaddor (1996) demonstrates that the effective cubic nonlinearity for a given mode of a beam will be the sum of the contributions of the beam and the boundary condition. So, even though the inertia and curvature effects can be estimated from the mode shapes, the effective cubic nonlinearity including the effects of the boundary conditions cannot be estimated theoretically. The quantitative effects of such parameters cannot be accounted for without experimental identification. In the same way, parameters that can be estimated theoretically can be compared with experimental values. This comparison allows theoretical estimation techniques to be validated.

Comparing theoretical and experimental estimates of parameters like the effective cubic nonlinearity allows the contribution of the boundary conditions to be assessed. Making use of the analysis of Tabaddor (1996), the contribution of the boundary conditions can be quantified by subtracting the theoretically known beam component from the experimentally determined effective cubic nonlinearity.

Once the influence of things like linear and nonlinear damping and boundary conditions has been quantified, quantitative modeling can be performed. This, in turn, makes validation of theoretical models viable. The validity of a model ultimately depends on its ability to accurately predict the response of a system. If a given theoretical model is able to accurately predict the quantitative response of a system, the soundness of assumptions like the system being weakly nonlinear and certain terms being considered "small" is proven.

Several experimental identification techniques for nonlinear systems have been presented in the literature. We developed a new identification technique that seemed to follow naturally from the multiple-scales analysis of the system. After proposing this technique, we began to search the literature for other techniques to compare it with. We were unable to find a technique in the literature that works completely for the cantilevered beam considered. While much has been done in the field of modal analysis, these techniques are almost exclusively suitable for linear system analysis. While time-domain techniques for linear systems can be fairly easily adapted, this is not the case for frequency-domain techniques.

The time-domain techniques that have been developed for nonlinear systems do not give satisfactory results for the beam considered. This results from the combination of two problems: scaling and differentiation. The scaling problems occur because the nonlinear terms in the equation of motion are considerably smaller than the other terms. Since all of these terms get added together, the noise in the larger terms may seriously obscure the nonlinear terms. The differentiation problem arises from the need to know all of the states of the beam. Using an accelerometer on this system would mean too much added mass. So, a strain gauge is used to sense the bending moment, which is linearly related to the strain. The bending moment is then converted to position and is differentiated to get the velocity and acceleration. Problems associated with differentiation lead to inaccuracies in the velocity and acceleration signals, which get compounded by the scaling issue.

The most promising frequency-domain technique in the literature is the backbone curve-fitting technique. However, this technique is unable to estimate the modal mass of the system.

The technique we present seems to avoid both of these problems. We compare the results from this technique with those from time-domain and backbone curve-fitting techniques. The technique presented uses the method of multiple scales to derive the frequency-response function for the system. The force- and frequency-response curves predicted by this frequency-response function are compared with experimental data so that a curve-fitting algorithm can be applied to estimate the parameters of the model.

By comparing the results of each of the identification techniques, we attempt to move toward determining the strengths and weaknesses of each technique and providing tools for determining which method to use on a given system.

1.2 Literature Review

1.2.1 Linear Analysis

The investigation of weakly nonlinear systems typically begins with analyzing the essentially linear behavior of the system for small response levels. This may be done both theoretically and experimentally. For continuous systems, this may lead to a closed-form solution or involve approximate methods, such as the assumed modes method. The foundations for such an analysis are discussed in Meirovitch (1986). It should be noted that the assumed modes method is also used in many nonlinear systems and that is the approach we follow.

The experimental side of such an investigation is modal analysis with "small" excitation levels. Many good references on the foundations of this developing field are available. We refer the reader to McConnell (1995). In attempting to perform such an analysis on nonlinear systems, the user needs to know how "small" the excitation needs to be. This necessitates being aware of some of the warning signs that nonlinearities are significantly affecting the results. Busby, Nopporn, and Singh (1986) demonstrate that the deterioration of coherence and the shifting of peaks in the frequency response (to the left for systems with softening-type springs and to the right for systems with hardening-type springs) with increasing forcing levels are signs of the onset of nonlinearity problems. Raman, Bajaj, and Davies (1996) discuss the response of nonlinear systems to a forcing similar to the burst chirp excitation used in modal testing. This nonstationary forcing leads to two deviations from the sine-dwell response. The jump phenomena will be delayed and the frequency-sweep plot will include amplitude oscillations after an amplitude jump. Understanding these effects may provide valuable insight into the response of a system at excitation levels where nonlinearities are significant.

1.2.2 Experimental Identification

The approaches to nonlinear identification presented in the literature can be separated into time-domain and frequency-domain techniques. Time-domain approaches include those based on using the restoring force and the states of a structure. Examples of this approach have been presented by Masri, Sassi, and Caughey (1982) and Crawley and Aubert (1986). Such approaches require the simultaneous measurement of the position, velocity, and acceleration. Crawley and Aubert present the results of a number of proof-of-concept experiments using separate sensors for the position, velocity, acceleration, and input force. Worden and Tomlinson (1991) apply this technique to several single-degree-of-freedom systems and demonstrate that this approach works well if the acceleration can be measured directly and then integrated to get the velocity and position. In attempting to identify an analog circuit representing a second-order single-degree-of-freedom system, they were unable to get satisfactory results by measuring the position and differentiating it. A similar time-domain method was applied to multiple-degree-of-freedom systems by Mohammad, Worden, and Tomlinson (1992) and Yasuda and Kamiya (1997). Yasuda and Kamiya present results based on measuring the position using a fiber optic sensor and differentiating it to get the velocity and acceleration.

These techniques are based on a least-squares curve fit for time-domain data, which is linearly related to the unknown coefficients. They have the advantages of requiring little a priori knowledge of the system and requiring less time and effort for data acquisition than the sine-dwell method used for the frequency-domain techniques. Potential drawbacks of these approaches include problems of differentiating noisy signals and being unable to accurately estimate the coefficients of terms which are small.

Frequency-domain techniques include approaches based on curve-fitting points along the backbone of the locus of frequency-response plots and methods based on balancing the Fourier coefficients in the equation of motion. A backbone curve-fitting technique based on the describing-function method is presented by Benhafsi, Penny, and Friswell (1995). Fahey and Nayfeh (1998) present a similar approach based on the method of multiple scales and apply it to a parametrically excited portal frame. Yasuda, Kamiya, and Komakine (1997) present a technique based on using the harmonic-balance method and applying a least-squares curve-fit to the Fourier coefficients of the sine and cosine terms for each harmonic in the equations of motion.

These frequency-domain techniques avoid the problems associated with differentiation and observation of small terms but may require considerably more theoretical effort.

1.2.3 Theoretical Modeling

The amplitude- and frequency-sweep identification technique we present requires more theoretical modeling than some of the other techniques in the literature. This is essentially done in two steps. The first is to come up with an equation of motion that captures the essential physics of the system. This involves including all terms suggested by known physical mechanisms. This may mean initially including some nonlinear terms that eventually prove to be negligible. For a continuous system like the beam we consider, we arrive at an equation of motion with the proper form for the cubic nonlinearities by starting with an appropriate Lagrangian and using the assumed modes method.

Once an appropriate equation of motion is found, the frequency-response function can be determined by applying perturbation techniques. Nayfeh (1973, 1981) describes several of these techniques. We apply the method of multiple scales and obtain force- and frequency-response curves typical of those described by Nayfeh and Mook (1979) for single-degree-of-freedom systems with softening-type springs.