The Edge Of Supersymmetry:
Stability Walls in Heterotic Theory

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Abstract

We explicitly describe, in the language of four-dimensional $\mathcal{N} = 1$ supersymmetric field theory, what happens when the moduli of a heterotic Calabi-Yau compactification change so as to make the internal non-Abelian gauge fields non-supersymmetric. At the edge of the region in Kähler moduli space where supersymmetry can be preserved, an additional anomalous $U(1)$ gauge symmetry appears in the four-dimensional theory. The D-term contribution to the scalar potential associated to this $U(1)$ attempts to force the system back into a supersymmetric configuration and provides a consistent low-energy description of gauge bundle stability.

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1 Introduction

Compactifications of the $E_8 \times E_8$ heterotic string [1, 2] and heterotic M-theory [3]–[6] on smooth, compact, three-dimensional manifolds have been studied for many years. The simplest way to preserve $\mathcal{N} = 1$ supersymmetry in the effective four-dimensional theory is to choose the compactification space to be a Calabi-Yau manifold, that is, to admit a metric with vanishing Ricci tensor. In this paper, we will adopt this approach. These compactifications necessarily involve the metric, $g_{a\bar{b}}$, where $a, \bar{b} = 1, 2, 3$ are the complex indices on the threefold. Additionally, one must specify the gauge fields A_a, with associated gauge group $G \subseteq E_8$, on the Calabi-Yau manifold. Whether or not these gauge fields preserve $\mathcal{N} = 1$ supersymmetry in the effective theory is determined by studying the variation of the higher-dimensional E_8 gauginos under supersymmetry transformations. The gaugino variations vanish, and, hence, $\mathcal{N} = 1$ supersymmetry is preserved, if and only if the gauge fields satisfy

$$F_{a\bar{b}}g^{a\bar{b}} = 0 \ , \ F_{ab} = F_{\bar{a}\bar{b}} = 0 \tag{1.1}$$

where F is the two-form field strength of the gauge field. These are known as the Hermitian Yang-Mills equations.

Specifying a Calabi-Yau threefold, X, requires the choice of its $h^{1,2}(X)$ complex structure moduli. These implicitly enter (1.1) by defining the holomorphic and anti-holomorphic coordinates. However, these moduli play no further role in this paper and we will henceforth ignore them. Crucially, we see that equations (1.1) depend explicitly on the metric of the Calabi-Yau manifold and, hence, on its $h^{1,1}(X)$ Kähler moduli. Indeed, whether these equations even have a solution, that is, whether it is possible for the gauge fields to preserve supersymmetry, is dependent on the values taken by the Kähler moduli. Generically, Kähler moduli space divides into regions where the gauge fields can preserve supersymmetry and regions where they cannot [7, 8, 9]. At each point in a supersymmetric region of Kähler moduli space the solution to Eqs. (1.1) depends on a number of arbitrary integration constants, the vector bundle moduli [10]–[13]. It is the combined Kähler/vector bundle moduli space in which we need to carry out our analysis.

For an Abelian internal gauge group, $G = U(1)$, it has been known for some time [16] that the supersymmetric part of the Kähler moduli space is a locus of co-dimension one; that is, effectively, one Kähler modulus is frozen if supersymmetry is to be preserved. In the four-dimensional $\mathcal{N} = 1$ effective theory, this is described by a Fayet-Iliopoulos (FI) D-term [14]–[18] associated with a $U(1)$ gauge symmetry which is anomalous in the Green-Schwarz sense. In this paper, we will be concerned with *non-Abelian internal gauge groups*, specifically $G = SU(n)$. In this case, the requirement that the internal gauge fields preserve supersymmetry does not fix any of the Kähler moduli. However, it is known from a mathematical analysis [7, 8] using methods of algebraic geometry, that the $h^{1,1}(X)$-dimensional Kähler moduli space generically decomposes into subspaces; some in which the non-Abelian internal gauge fields are supersymmetric, that is, where they satisfy Eq. (1.1), and others where they break supersymmetry. Traditionally in the literature, the associated four-dimensional effective theory is derived in the supersymmetric region of moduli space where Eq. (1.1) has a solution. In this paper, we will extend this analysis and describe, for the first time from a four-dimensional perspective, what happens as the moduli vary and the gauge fields start to break supersymmetry.

Of central importance is the following observation: as the fields are varied into the supersymmetry breaking region of moduli space, a new potential will appear for the scalar fields of the four-dimensional effective theory. As shown in Ref. [19], a straightforward dimensional reduction of the ten-dimensional

Yang-Mills term and the associated R^2 curvature term results in the following potential in the four-dimensional theory,[1]

$$V_{4d} = \frac{\alpha'}{4} \int_X \sqrt{-g} \left\{ \text{Tr}(F^{(1)}_{a\bar{b}} g^{a\bar{b}})^2 + \text{Tr}(F^{(2)}_{a\bar{b}} g^{a\bar{b}})^2 \right\}. \tag{1.2}$$

The notation here is standard [2] with the field strengths $F^{(1)}$ and $F^{(2)}$ being associated with the two E_8 factors in the gauge group and the integration taking over the Calabi-Yau manifold. For a supersymmetric field configuration satisfying Eq. (1.1), the terms in the integrand of Eq. (1.2) vanish. In this case, no potential is generated. However, if the Kähler moduli are varied so that Eq. (1.1) no longer has a solution, that is, so that our gauge fields are no longer supersymmetric, (1.2) no longer vanishes and we obtain a positive definite contribution to the potential energy as seen in four dimensions. We conclude that the region of moduli space in which the gauge connection satisfies (1.1) is *everywhere surrounded by a positive potential*. We refer to this as a *"wall of stability"*.

For the case of an Abelian internal gauge group, it can be explicitly seen that Eq. (1.2) leads to the FI D-term mentioned above. In the non-Abelian case, however, it might seem difficult to say more and present, for example, the potential as an explicit function of the four-dimensional moduli fields. After all, neither $F_{a\bar{b}}$ nor $g^{a\bar{b}}$ are known explicitly on a Calabi-Yau manifold. Nevertheless, an explicit form for this potential can indeed be derived. This is the main focus of this paper.

2 Bundle Supersymmetry in Kähler Moduli Space: An Example

It is clear from the preceding discussion that if an $\mathcal{N} = 1$ supersymmetric subregion of the Kähler moduli space exists, it is bounded by a positive definite potential wall, the boundary of Kähler moduli space or both. It is known from a mathematical analysis [7, 8] that "Kähler-cone" sub-structure exhibiting both boundaries can exist. We now discuss this in detail. For ease of exposition, we focus on a specific example in the present paper, leaving the general construction to Ref. [19]. The Calabi-Yau threefold, X, we consider is defined as the zero locus of a polynomial of degree $(2, 4)$ in the homogeneous coordinates of an ambient space $\mathbb{P}^1 \times \mathbb{P}^3$. A common notation [20] for this manifold is

$$X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^3 & 4 \end{array} \right]. \tag{2.1}$$

This Calabi-Yau threefold has $h^{1,1} = 2$ Kähler moduli and the Kähler form, J, can be written as

$$J = t^1 J_1 + t^2 J_2 , \tag{2.2}$$

where J_1 and J_2 are the Kähler forms of $\mathbb{P}^1$ and $\mathbb{P}^3$ respectively, and t^1, t^2 denote the Kähler moduli. The Kähler cone, that is, the set of allowed Kähler moduli for this Calabi-Yau manifold, is characterised by $t^1 > 0$ and $t^2 > 0$. The Kähler moduli pair up with two axions, χ^1, χ^2, into the complex fields

$$T^k = t^k + 2i\chi^k, \qquad k = 1, 2 . \tag{2.3}$$

These form the bosonic parts of four-dimensional $\mathcal{N} = 1$ chiral multiplets. The axions descend from the M-theory three-form or, in the weakly coupled heterotic string, from the NS two-form.

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[1]This formula assumes that the field strength F is a $(1, 1)$ form, even in the non-supersymmetric region where the first equation in Eq. (1.1) cannot be solved. In our context, the gauge fields are connections on holomorphic vector bundles for which this can always be arranged.

The gauge fields in this specific example are chosen as follows. First, note that, mathematically, gauge fields are defined to be connections on vector bundles which, in the present context, should be holomorphic. We would like to construct such a holomorphic vector bundle using line bundles on the Calabi-Yau manifold as building blocks. For the above Calabi-Yau manifold, line bundles are characterised by two integers, k, l, and are written as $\mathcal{O}_X(k,l)$. The first Chern class of these line bundles is given by $c_1(\mathcal{O}_X(k,l)) = kJ_1 + lJ_2$. The vector bundle we will consider is a monad bundle, a construction which has been used in the physics literature for some time [16], [21]–[26]. Such a bundle, V, is defined as the kernel of a map, f, between two sums of line bundles. The specific example we focus on in the present paper is [25, 26]

$$0 \to V \to \mathcal{O}_X(1,0) \oplus \mathcal{O}_X(1,-1) \oplus \mathcal{O}_X(0,1)^{\oplus 2} \xrightarrow{f} \mathcal{O}_X(2,1) \to 0 \ . \tag{2.4}$$

The bundle V has rank three and a vanishing first Chern class. Hence, it *generically* has a structure group $G = SU(3) \subset E_8$. What we would like to know is for which values of the Kähler moduli a gauge field connection on V exists satisfying the Hermitian Yang-Mills equations (1.1) and, hence, preserving $\mathcal{N} = 1$ supersymmetry? The general answer to this question was given by Donaldson, Uhlenbeck and Yau [27] who proved the following theorem: *For a fixed choice of Kähler moduli, there exists a solution of the Hermitian Yang-Mills equations if and only if the vector bundle is "slope-stable", that is, has no destabilizing sub-bundles.*[2] We postpone a detailed analysis of slope-stability to [19] and simply state the result for the above bundle V. The "maximally destabilizing" sub-bundle for the bundle V in (2.4) is a rank two bundle $\mathcal{F}$ with first Chern class

$$c_1(\mathcal{F}) = -J_1 + J_2 \ . \tag{2.5}$$

It is easy to see that the slope of this sub-bundle is given by

$$\mu(\mathcal{F}) = (4t^1 t^2 - (t^2)^2) \ . \tag{2.6}$$

This means that the slope is negative, and, hence, the bundle is supersymmetric, above the line $t^2 = 4t^1$, while supersymmetry is broken below this line. In Fig. 1, we have indicated these two regions in the Kähler cone of the Calabi-Yau manifold (2.1). Finally, note that the exact sequence in (2.4) actually defines a space of vector bundles, parametrized by a set of vector bundle moduli. In the supersymmetric region there are $h^1(X, V \otimes V^*) = 22$ such bundle moduli, which we denote by

$$\phi^\alpha, \qquad \alpha = 1, \ldots, 22 \ . \tag{2.7}$$

In all previous heterotic literature, the four-dimensional effective theory has been derived for Kähler moduli in the interior of the green (light shaded) region of Fig. 1, where the gauge fields preserve supersymmetry. In this paper, we will extend the four-dimensional description so that we can explicitly describe what happens as we approach and then cross over the $t^2 = 4t^1$ line, entering the red (shaded) region where (1.1) cannot be solved.

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[2]The slope of a sub-bundle $\mathcal{F}$ is defined as $\mu(\mathcal{F}) = \frac{1}{\text{rank}\mathcal{F}} \int_X c_1(\mathcal{F}) \wedge J \wedge J$, where J is the Kähler form. A bundle V is slope-stable if and only if $\mu(\mathcal{F}) < \mu(V)$ for all sub-bundles $\mathcal{F} \subset V$. We also note that the slope can be explicitly expressed in terms of the Kähler moduli t^i as $\mu(\mathcal{F}) = \frac{1}{\text{rank}\mathcal{F}} d_{ijk} c_1^i(\mathcal{F}) t^j t^k$, where $d_{ijk} = \int_X J_i \wedge J_j \wedge J_k$ are the triple intersection numbers of the Calabi-Yau manifold. For the Calabi-Yau manifold (2.1), the only non-vanishing intersection numbers are $d_{122} = 4$ and $d_{222} = 2$.
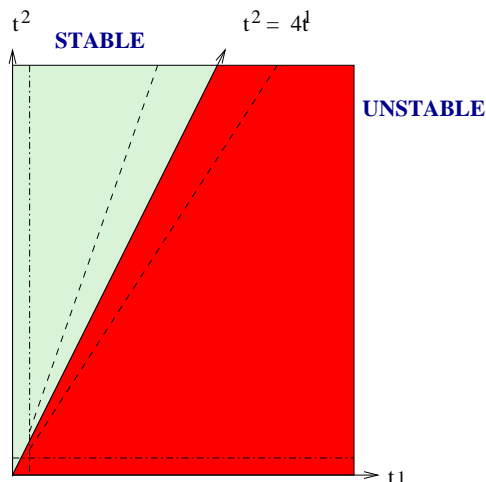
Figure 1: *The Kähler moduli space of the Calabi-Yau manifold* (2.1) *in terms of the moduli* $t^k = \mathrm{Re}(T^k)$. *The allowed set of Kähler moduli (the Kähler cone) is the positive quadrant. The supersymmetric region where* (1.1) *admits a solution is marked in green (light shading), whereas the non-supersymmetric region where it does not is marked in red (dark shaded). The boundary between them is the line* $t^2 = 4t^1$. *The dash-dotted lines parallel to the axes indicate where supergravity breaks down as the Kähler moduli become too small. The additional* $U(1)$ *vector and Higgs supermultiplets are light compared to the compactification scale between the two dashed lines.*

3   Four-Dimensional Effective Field Theory

The first observation we make in deriving the four-dimensional field theory is that, on the line between the supersymmetric and non-supersymmetric regions in Fig. 1, something special must happen to the gauge fields. Although this is a line in Kähler moduli space, if it is to support a solution to (1.1), we find that one is forced to a special locus in vector bundle moduli space. As shown in Ref. [19], the gauge fields must "split" into a direct sum; that is, whereas they were previously valued in the adjoint of $SU(3)$, they now must become valued in the adjoint representation of $S[U(2) \times U(1)]$ instead. This latter group is simply $U(2) \times U(1)$ where the determinant of the $U(2)$ matrix is constrained to be the inverse of the $U(1)$ phase. The gauge group of the four-dimensional theory is the commutant of the structure group in E_8. When the structure group is $SU(3)$, which it is in the supersymmetric part of the Kähler cone, this commutant is E_6. However, when $SU(3)$ changes to $S[U(2) \times U(1)]$ on the line between the supersymmetric and non-supersymmetric regions in Fig. 1, the low energy gauge group is enhanced to $E_6 \times U(1)$. In summary: *On the line between the supersymmetric and non-supersymmetric regions, an additional* $U(1)$ *appears in the four-dimensional gauge group.*[3] For the rest of this paper, we will work out the four-dimensional effective theory including this additional $U(1)$. We will show how this theory precisely reproduces the known physics in the supersymmetric region of Fig. 1. We will then use it to describe, for the first time, what happens in the non-supersymmetric region, as well as the smooth transition between these two regimes.

A prerequisite to writing down the four-dimensional theory is a knowledge of its spectrum. This can be calculated by computing the zero-modes of the Dirac operator twisted by the gauge fields. Equivalently, one can compute the cohomology of the various tensor products of the vector bundle V [28]–[31]. At

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[3]In general, on a boundary wall in the Kähler cone, an $SU(n)$ bundle can decompose into $S[U(n_1) \times U(n_2) \times \ldots]$ where $\sum_i n_i = n$. In any such case, the commutant symmetry of $S[U(n_1) \times U(n_2) \times \ldots]$ in E_8 will always be enhanced by at least one $U(1)$ symmetry. These, more general, cases are discussed in more detail in [19].

the stability wall, where the structure group of V becomes $S[U(2) \times U(1)]$, the results are presented in Table 1. A crucial observation is the following: *Only one sign of $U(1)$ charge appears in the low*

Fields	$E_6 \times U(1)$ charges	number of fields
ψ^β	1_0	7
B^I	$27_{-1/2}$	2
C^L	$1_{-3/2}$	16

Table 1: *The four-dimensional fields descending from higher-dimensional gauge fields. Shown are the vector bundle moduli, ψ^β, the E_6 **27**-matter fields, B^I, and the $U(1)$ charged, E_6 singlets, C^L. The $U(1)$ charge of each field is shown as a subscript.*

energy spectrum. This will be of central importance in what follows. Naively, both signs of $U(1)$ charge could have appeared. However, it turns out that, in examples of the kind we are discussing here, the relevant Dirac operators never have E_6 singlet zero modes which are positively charged under $U(1)$. This will be explicitly proven in Ref. [19]. The fields B^I are the usual matter fields, transforming in the **27** representation of E_6. There are no **$\overline{27}$** matter fields in the spectrum. Note that this model has only two generations. We have made no attempt to present a phenomenologically viable example here. Rather, we have chosen our model to be as simple as possible, while still illustrating the points we wish to make. More complicated, phenomenologically realistic theories simply mimic the structure we present here. The moduli ψ^β, $\beta = 1, \ldots 7$ of the $S[U(2) \times U(1)]$ bundle can be thought of as a subset of the $SU(3)$ bundle moduli described earlier in (2.7). Finally, we have the fields

$$C^L, \qquad L = 1, \ldots, 16. \tag{3.1}$$

These fields arise as a consequence of calculating the spectrum with the gauge fields valued in the adjoint of $S[U(2) \times U(1)]$ rather than $SU(3)$. Along with the Kähler moduli, they will play the key role in the rest of our discussion. Internal gauge bundles with a structure group that includes a $U(1)$ factor were constructed and analysed for the first time in Ref. [16]. It is also known for some time [17, 18] that such a $U(1)$ factor leads to an anomalous $U(1)$ gauge symmetry and an associated T-modulus dependent FI D-term in the four-dimensional $N = 1$ effective theory. Here, we will apply these result to our theory at the stability wall.

In addition to the fields of Table 1, there are also the usual heterotic moduli fields which include the complex structure and Kähler moduli already mentioned, as well as the dilaton, S, and possible M5 brane position moduli. How do these fields transform under the additional $U(1)$? The complex structure moduli are invariant. The imaginary parts of the Kähler moduli, dilaton and M5 brane position moduli, in contrast, all transform under $U(1)$. The $U(1)$ transformation of the Kähler moduli is of particular importance in our context. It follows directly from the heterotic Bianchi identity and is given by

$$\delta\chi^k = -\frac{3}{4} c_1^k(\mathcal{F}) \, \epsilon \,, \tag{3.2}$$

where the T-axions χ^k are defined in Eq. (2.3). Moreover,

$$c_1^k(\mathcal{F}) = (-1, 1) \tag{3.3}$$

are the components of the first Chern class (2.5) with respect to the basis $\{J_1, J_2\}$ of two-forms and ϵ is the transformation parameter. The result presented above is the lowest order contribution to the

transformation and will receive one-loop corrections. These one-loop terms do not affect the present discussion and will be neglected. However, their complete form is not without interest and will be discussed in Ref. [19].

It is known [16, 17] that low-energy $U(1)$ gauge symmetries in heterotic compactifications which arise from the presence of a $U(1)$ factor in the bundle structure group are generally anomalous in a Green-Schwarz sense. In such cases, the triangle anomaly is cancelled by the four-dimensional manifestation of the Green-Schwarz mechanism. More precisely, this cancellation involves a non-trivial transformation of the heterotic gauge kinetic function under the T-axion shifts (3.2) [4]. For our example, it is immediately obvious that the $U(1)$ symmetry must indeed be anomalous in this sense, since all of the $U(1)$ charges appearing in Table 1 are negative. As usual in the context of the Green-Schwarz mechanism, the $U(1)$ vector supermultiplet picks up a mass. From the point of view of the effective four-dimensional theory, this mass arises from a Higgs mechanism involving a linear combination of all non-trivially transforming fields. We only consider situations where the E_6 part of the gauge group is unbroken. Hence, $\langle B^I \rangle = 0$ and the B^I fields do not contribute to the $U(1)$ mass. However, the T and the C^L fields do contribute to the $U(1)$ mass, as we will later show.

We will now focus on the scalar potential of the four-dimensional theory. Since we have $\mathcal{N} = 1$ supersymmetry, this receives two types of contributions; those from F-terms and those from D-terms. It turns out that, in this simple example, only the D-term contributions are important. The fact that all of the fields C^L have the same charge means that they cannot appear in a perturbative superpotential in a gauge invariant manner. Since these are the fields which will be important here, we will not consider F-terms further. Using Table 1, Eq. (3.2) and the standard formulas of $\mathcal{N} = 1$ four-dimensional supergravity [33], one can write down the D-terms of our low energy theory. The E_6 D-terms are completely standard in form. Setting them to zero forces us to set $\langle B^I \rangle = 0$, consistent with the above assumption of an unbroken E_6 gauge group. We will thus discard these **27** family fields in our subsequent analysis. The $U(1)$ D-term is more interesting. We find, to quadratic order in the C^L fields, that

$$D^{U(1)} = f(t^i) + \frac{3}{2} G_{L\bar{M}} C^L \bar{C}^{\bar{M}} , \tag{3.4}$$

where the FI term $f(t^i)$ is given by

$$f(t^i) = \frac{3}{4} \frac{\mu(\mathcal{F})}{\mathcal{V}} . \tag{3.5}$$

Here, $G_{L\bar{M}}$ is the moduli space metric associated with the C^L fields and is generically a function of the t^k and ψ^β moduli. The only information we require about this quantity is that it is positive definite. $\mathcal{V}$ is the Calabi-Yau volume which, in general, is a cubic polynomial in the Kähler moduli t^k. For our specific example, it takes the form

$$\mathcal{V} = 2t^1(t^2)^2 + \frac{1}{3}(t^2)^3 . \tag{3.6}$$

Additionally, $\mu(\mathcal{F}) = 4t^1 t^2 - (t^2)^2$ is the slope of the destabilising sub-bundle $\mathcal{F}$, as in Eq. (2.6) . It is clear from the appearance of the slope in Eq. (3.5) that the FI term is *positive in the non-supersymmetric (red, dark shaded) region of Figure 1, negative in the supersymmetric (green, light shaded) region, and vanishes on the boundary line between these two.* The second term in (3.4) is the usual contribution to a $U(1)$ D-term from negatively charged fields and is positive semi-definite.

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[4]We emphasize that this is the case for a low-energy $U(1)$ gauge group that is also a factor in the vector bundle structure group [15, 16, 17]. For $U(1)$ symmetries for which this is not the case, the Green-Schwarz anomaly cancellation arises from the inhomogeneous shift of the dilatonic axion [14].
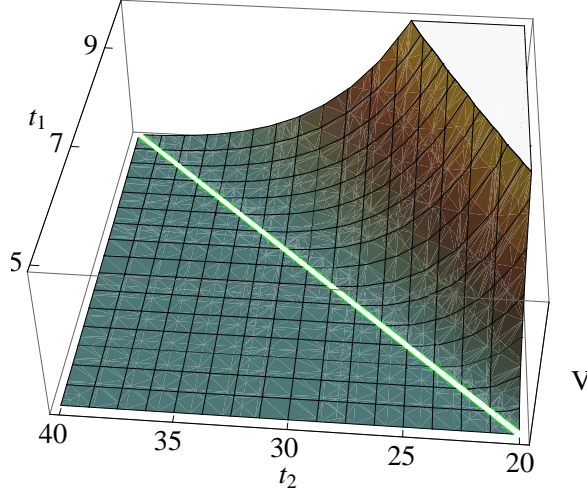
Figure 2: *The $U(1)$ D-term contribution to the scalar potential. The vertical axis is the potential V evaluated at the vacuum expectation values of the C^L fields, while the horizontal plane is the Kähler moduli space shown in Figure 1. The line $t^2 = 4t^1$, where the slope $\mu(\mathcal{F})$ vanishes, indicates the stability wall which separates the supersymmetric and non-supersymmetric regions. Note that for ease of viewing, the axes have been rotated relative to Figure 1.*

What happens to this D-term in the supposedly supersymmetric region of Kähler moduli space? *For any Kähler moduli in the slope stable region, the FI term is negative and so can be cancelled by choosing suitable vacuum expectation values (vevs) for the fields C^L. Therefore, in this region, one can have $D^{U(1)} = 0$. Hence, the vacuum energy vanishes and supersymmetry is indeed preserved.* Note that there are sixteen fields, C^L, but only one condition, $D^{U(1)} = 0$, needed to specify the vacuum. Thus there are fifteen flat directions, that is, fifteen massless fields, in the theory. *These fifteen massless fields plus the seven ψ^β fields in Table 1 form the complete set of twenty-two vector bundle moduli ϕ^α discussed in (2.7). The one remaining field, which is massive and, generically, a vacuum dependent linear combination of the T^k and C^L fields, plays the role of the $U(1)$ Higgs chiral superfield.*

Let us examine which field gets a mass in more detail. Expanding all fields around a vacuum, that is, $t^k = \langle t^k \rangle + \delta t^k$ and $C^L = \langle C^L \rangle + \delta C^L$, with the vevs chosen so that the D-term vanishes, we obtain the following expression to first order in the field fluctuations and leading order in $\langle C^L \rangle$;

$$D^{U(1)} = -\frac{3}{4}G_{jk}c_1^j(\mathcal{F})\delta t^k + \frac{3}{2}G_{L\bar{M}}\left(\langle C^L \rangle \delta \bar{C}^{\bar{M}} + \delta C^L \left\langle \bar{C}^{\bar{M}} \right\rangle\right) , \qquad (3.7)$$

where

$$G_{ij} = -\frac{\partial^2 \ln \mathcal{V}}{\partial t^i \partial t^j} \qquad (3.8)$$

is the Kähler moduli space metric, expressed in terms of the Calabi-Yau volume $\mathcal{V}$ as given in (3.6). Note that here, and henceforth, we will adopt the standard practice of denoting $\langle t^k \rangle$ simply as t^k. The D-term contribution to the potential is proportional to the square of this expression so that the linear combination (3.7) is, in fact, the Higgs field. Its bosonic superpartner, the corresponding linear combination of T-axions χ^i and phases of the fields C^L, is the Goldstone mode which is absorbed by the $U(1)$ vector field. The Higgs mass, m_H, which, from supersymmetry, must be equal to the $U(1)$ vector field mass $m_{U(1)}$ can be computed from Eq. (3.7) after canonically normalising the kinetic terms $\frac{1}{4}G_{ij}\partial \delta t^i \partial \delta t^j$ and $G_{L\bar{M}}\partial \delta C^L \partial \delta \bar{C}^{\bar{M}}$ of the fields involved. Neglecting higher-order terms in C and inverse powers of the

T-moduli one finds

$$m_H^2 = m_{U(1)}^2 = \frac{1}{s}\left(\frac{9}{16}c_1^i(\mathcal{F})c_1^j(\mathcal{F})G_{ij} + \frac{9}{4}G_{L\bar{M}}\langle C\rangle^L\langle\bar{C}\rangle^{\bar{M}}\right) , \tag{3.9}$$

where $s = \mathrm{Re}(S)$ is the real part of dilaton.

To make this more concrete, let us determine the mass and corresponding Higgs multiplet at a point on the stability wall in Kähler moduli space. At the stability wall, the slope of the destabilizing sub-sheaf, $\mathcal{F}$, and, hence, the Fayet-Iliopoulos term, vanish. This means that the C^L vevs also vanish in this region of the vacuum space. Using this fact in Eq. (3.7), we see that it is a particular combination of Kähler moduli, perpendicular to the stability wall in Kähler moduli space, which becomes massive on this locus. This can be explicitly verified for our example. In this case, the field fluctuation (3.7) of the D-term, evaluated at the stability wall by setting $\langle C^L\rangle = 0$ and $t^2 = 4t^1$, is

$$D^{U(1)} = \frac{9}{160}\frac{1}{(t^1)^2}(4\delta t^1 - \delta t^2) . \tag{3.10}$$

This shows that there is indeed one massive mode, given by the linear combination $4\delta t^1 - \delta t^2$, which represents the direction perpendicular to the stability wall, as expected. The corresponding linear combination of axions χ^i is the Goldstone mode. Evaluating Eq. (3.9) at the stability wall, one finds for the mass of the $U(1)$ vector field and the Higgs

$$m_H^2 = m_{U(1)}^2 = \frac{27}{128}\frac{1}{s(t^1)^2} . \tag{3.11}$$

As one moves away from this line into the supersymmetric region, the Goldstone boson becomes a linear combination of Kähler axions and C^L field phases. Further into the supersymmetric region, this becomes dominated by one of the C^L phases. *We recover, therefore, the usual spectrum of heterotic theory, with two massless T-moduli, the usual vector bundle moduli and no additional scalars. This is in agreement with the standard results for supersymmetric heterotic compactifications.*

We would now like to argue that near the stability wall it is consistent to keep both the $U(1)$ vector supermultiplet and the Higgs chiral superfield in the low-energy theory. For concreteness, we do this in the context of our specific example but the argument is of course general. Let us begin the analysis on the stability wall. The mass (3.11) should be compared to the squared mass of a typical gauge sector massive mode which is of the order $\mathcal{V}^{-1/3}/s$. Using Eqs. (3.6), (3.11) leads to

$$\frac{s\,m_{U(1)}^2}{\mathcal{V}^{-1/3}} \simeq \frac{1}{t^1} . \tag{3.12}$$

In the large radius limit, $t^1, t^2 \gg 1$, we have $m_{U(1)}^2 \ll \mathcal{V}^{-1/3}/s$ and the $U(1)$ mass is much smaller than typical heavy gauge sector masses on the stability line. Hence, in the regime where the supergravity approximation is valid it is always consistent to keep the $U(1)$ vector and Higgs supermultiplets in the low-energy theory. As a concrete example, for $t^1 \gtrsim 10^2$, the vector and Higgs supermultiplets masses are at least an order of magnitude below the mass scale of heavy gauge states. What happens as we move away from the stability line into the supersymmetric region of moduli space? In this case, the vevs of the C^L fields are no longer zero and the $U(1)$ mass, now given by expression (3.9), increases. Eventually it becomes of the order of the compactification scale and the vector and Higgs supermultiplets should no longer be kept in the low-energy theory. The region around the stability line for which $s\,m_{U(1)}^2/\mathcal{V}^{-1/3} < 10^{-1}$ is indicated in Fig. 1.

We can now go further and describe what happens in the non-supersymmetric region of Kähler moduli space. *In this region, the FI term given in (3.4) is positive. Since the second term in (3.4) is also positive,*

it is no longer possible to adjust the vevs of the C^L fields to cancel the FI term. Therefore, we find that $D^{U(1)} \neq 0$ at every point in this region of moduli space, reproducing the fact that supersymmetry is broken in the four-dimensional effective theory. The square of the D-term in the red (dark shaded) region of Fig. 1 gives rise to an everywhere positive four-dimensional potential. Note that this potential is minimized at each point in Kähler moduli space by setting $\langle C^L \rangle = 0$. The resulting potential for the Kähler moduli is plotted in Fig. 2. Close to the stability wall in Fig. 1, this potential is still relatively small and it makes sense to talk about a four-dimensional theory. However, *sufficiently far into the unstable region the energy density of the potential becomes comparable to the compactification scale and one would expect that no four-dimensional description exists.* Since, in the absence of other effects, there is no perturbative vacuum in the non-supersymmetric part of the Kähler cone, we will refrain from discussing masses in this region, except to note that the $U(1)$ vector supermultiplet continues to have the mass (3.9) induced by the Green-Schwarz mechanism.

To summarize: We have introduced a new D-term contribution to the potential of heterotic string and M-theory which is positive semi-definite and describes the supersymmetry properties of the internal non-Abelian gauge fields from the perspective of the four-dimensional effective theory. This D-term originates from an anomalous $U(1)$ gauge symmetry which arises in the low-energy theory. The corresponding vector supermultiplet has a mass induced by the Green-Schwarz mechanism. Associated with this enhanced $U(1)$ are charged light states in the spectrum. In the part of the Kähler moduli space where the internal vector bundle is supersymmetric, these states develop vacuum expectation values which cancel the Fayet-Iliopoulos term. In the part of Kähler moduli space where the bundle breaks supersymmetry, the Fayet-Iliopoulos term changes sign and can no longer be cancelled by vacuum expectation values of the charged states. Thus, the D-term vanishes in the region of Kähler moduli space where the internal gauge fields are supersymmetric, and is non-vanishing where the internal gauge fields break supersymmetry. For this mechanism to work, it is crucial that all of the charged states have the same sign of $U(1)$ charge. We have checked that this is indeed the case. Our picture provides, for the first time, a concrete four-dimensional description of supersymmetry breaking induced by non-Abelian heterotic gauge bundles.

The new potential we have described has many possible applications, from cosmology to moduli stabilization. One might imagine using non-perturbative effects to stabilise moduli a small way into the non-supersymmetric region, thus obtaining a naturally small scale of supersymmetry breaking. The global remnant of the $U(1)$ symmetry described here constrains the Lagrangian in the supersymmetric region. This may allow us to place restrictions on which vector bundles lead to realistic particle phenomenology. There are also more formal applications of our work. These concern, for example, linking what might be seen as different vector bundles in physical moduli space and proving bundle stability purely from four-dimensional field theoretical arguments. The authors hope to explore such topics in future publications.

Acknowledgments

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