URANIUM DIOXIDE AND URANIA-THORIA FUEL CORES FOR THE UTR-10 REACTOR

by

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I. INTRODUCTION

The nuclear reactor at the Virginia Polytechnic Institute is a 10-kw Argonant-type thermal research reactor designated as the UTR-10. It consists of 2 fuel slabs imbedded in graphite which forms an internal and external reflector. The core is heterogeneous, consisting of UAl_4 fuel plates with aluminum cladding immersed in water, which acts as a moderator-coolant.

This thesis is a theoretical analysis of the critical parameters, transient response, and temperature characteristics of a proposed new fuel core for the UTR-10. The proposed core consists of a uniform lattice of slightly enriched UO_2 or urania-thoria fuel pins with aluminum cladding. The pins will be immersed in water, which will act both as moderator and coolant, as in the present core.

There are several advantages to be found in the selection of UO_2 or thoria-urania fuel pins. First, there is the comparative metallurgical simplicity and economy of processing and fabricating aluminum tubes loaded with oxide pellets. Another desirable feature of this type of core is its negative temperature coefficient of reactivity, which guarantees an inherently safe reactor. Finally, the time constants of an oxide-fueled core would make it adaptable to pulsed-neutron experiments.

In determining the critical masses of the proposed cores, the following procedure was employed: First, it was assumed that the leakage from the UO_2 or urania-thoria cores would be equal to the leakage from the present core. Then, for a critical fuel loading, the infinite multi-

plication constant would be the same for both cores. Values of k_∞ for various fuel enrichments were calculated and plotted as a function of moderator-to-fuel ratio. From these curves an optimum moderator-to-fuel ratio and fuel enrichment corresponding to the desired value of k_∞ were chosen. Then the critical masses of U-235 were easily calculated. The infinite multiplication constant for the present fuel loading was calculated from data obtained from ASAE - 19 (1).

In order to determine the inherent safety of any reactor system, it is necessary to examine the effects of temperature and reactivity transients on the multiplication of the core. To insure safe operation under all conditions, a thermal research reactor should exhibit a negative temperature coefficient of reactivity, and its response to a sudden insertion of reactivity should not be excessive. Consequently this thesis contains calculations of the theoretical temperature coefficient of reactivity for both UO_2 and urania-thoria fuel loadings. In addition, the period, maximum temperature, and maximum power have been calculated for a sudden insertion of 0.005 excess reactivity.

Another important consideration in reactor core design involves the generation and removal of heat in the fuel region. Since the thermal conductivity of both UO_2 and ThO_2 is low, fuel elements containing these materials would be subject to failure from thermal stresses if used in a reactor operating at high power. Therefore calculations have been made to ascertain that due to the small size of the fuel pins and the low power level of operation, no danger of failure due to thermal stresses will exist. Finally the approximate heating density in the core has been calculated.

In many of the methods and procedures used in reactor analysis it is necessary to make certain assumptions or approximations of varying degrees of refinement. Therefore it has not been possible to avoid making assumptions in the course of the calculations presented here. However, in each instance care has been taken to insure that only the most valid and justifiable approximations have been used.

II. REVIEW OF LITERATURE

The analysis of heterogeneous reactor systems consisting of a lattice of fuel elements interspaced in a moderating material forms an extensive and important part of reactor theory. A major portion of the analysis of heterogeneous configurations consists of determining the various factors which appear in the criticality relation as expressed by the fourfactor formula. Another important consideration in the design of any reactor core is the effect of temperature changes on the multiplication constant. In order to insure that the reactor system will be inherently safe, its response to reactivity transients and its heat transfer characteristics should be estimated as accurately as possible. In general, references (5), (7), (9), and (11) contain the basic concepts and techniques which are used in heterogeneous reactor analysis. In addition there are many other references which deal with theoretical approaches to many aspects of the problem. Much valuable information can be found in the proceedings of the United Nations International Conferences on the Peaceful Uses of Atomic Energy held in 1955 and 1958. Specific papers in this collection will be referred to when appropriate.

Nuclear cross sections used throughout this thesis were obtained from BNL 325 (1958). (13) Other nuclear parameters were obtained as far as possible from ANL-5800 (Reactor Physics Constants). (14) This was done in the hope that the use of a consistent set of constants would help to provide consistency in the results obtained.

The thermal utilization of the core was calculated from a relation derived from the application of integral transport theory to cylindrical geometry. This technique is discussed in reference (15).

Estimations of the fast fission factor ϵ were largely based on the work of Barringer (17). Klein, et. al (32) determined ϵ experimentally for slightly enriched UO_2 fuel pins and concluded that the fast fission factor is not a function of fuel pin diameter, fuel concentration, or fuel enrichment, but depends only on the water-to-metal ratio of the lattice. The fast fission factor in urania-thoria cores has been studied experimentally by Edlund (18), and by Ciuffa and Sani (19).

Theoretical values for the resonance escape probability of the fuel lattices were calculated from the conventional formulas, in which the resonance integrals were expressed by empirical relations (16). The subject of resonance capture in heterogeneous lattices has been extensively investigated, both theoretically and experimentally. CP-2062 (Resonance Escape Probability in Lattices) is an early theoretical treatment of the problem of determining the resonance escape probability (20). A large amount of more recent work on the subject is found in BNL-433 (Proceedings of the Brookhaven Conference on Resonance Absorption of Neutrons in Nuclear Reactors) (22).

The problems concerning a theoretical analysis of the effect of changes in temperature on the multiplication constant have been discussed by many authors. In addition to the material available in the aforementioned general references, valuable information may be found in the analyses of particular reactor configurations, such as the Brookhaven

reactor (24) and the UTR-10 with its present fuel loading (4). Two Russian papers (25), (26) consider factors which are affected by a temperature change. Empirical relation for expressing the effect of Doppler broadening on the resonance integral of UO_2 were obtained from Hellstrand, et al (27). A suggested set of values for the Doppler coefficient of the resonance integral of ThO_2 was obtained from (28).

The subject of reactor kinetics and temperature-reactivity power transients is discussed in most general references on reactor analysis. In particular, Walker (29) and Holmes and Meghreblian (10) contain thorough treatments of the problem of reactor kinetics. Murray (8) deals with the response of a reactor to a step increase in reactivity. Much experimental work on reactivity transients has been done on particular reactors and critical facilities.

Several references which discussed the phenomena of heat generation and removal in nuclear reactors were found; among these were (30) and (32). Physical properties of UO_2 and ThO_2 are discussed in (33). Thermal conductivities and temperature coefficients of linear expansion for reactor materials are listed in (34).

III. THEORY

I. Calculation of Critical Mass

The infinite multiplication constant, k_∞ for any reactor is defined as the ratio of the number of neutrons produced in any one generation to the number of neutrons produced in the preceding generation, neglecting leakage. The multiplication constant is often expressed by the formula:

$$k_\infty = \eta f P_{RE} \epsilon$$

where η = the number of primary neutrons produced per thermal neutron absorbed in the fissionable material

f = the fraction of thermal neutrons which are absorbed in the fissionable material

P_{RE} = the resonance escape probability, i.e., the fraction of neutrons that escape capture in the resonance energy region

ϵ = the ratio of the total number of fissions produced by neutrons at all energies to the number of fissions produced by thermal neutrons

The above relationship, often refereed to as the four-factor formula, applies to both homogeneous and heterogeneous reactor configurations. It is best suited to reactors with low fuel concentration, but can easily be adapted to highly-enriched systems.

In a thermal reactor, most of the fissions are caused by thermal neutrons. Neutrons which are produced are slowed down quickly, so that there is little chance of their being absorbed at high energies. For the UTR-10 the resonance escape probability and the fast fission factor

may be taken as unity, and the four-factor formula reduces to the expression

$$k_\infty = \eta f$$

this relationship may be expressed in the alternate form

$$k_\infty = \nu \bar{\Sigma}_f / \bar{\Sigma}_a$$

where ν = the number of fission neutrons produced per fission in the fuel

$\bar{\Sigma}_f$ = the average macroscopic fission cross-section for the fuel

$\bar{\Sigma}_a$ = the average macroscopic absorption cross-section for the reactor

In a finite reactor system, a certain fraction of the neutrons produced in each generation are lost through leakage out of the core. The effective multiplication constant k_{eff} for a finite reactor is the infinite-reactor multiplication constant k_∞ multiplied by a reduction factor which represents the non-leakage probability. Thus

$$k_{eff} = k_\infty \mathcal{L}$$

where $\mathcal{L}$ = non-leakage probability.

The leakage from a reactor depends upon the shape of the core and the presence or absence of reflecting materials around the core. The necessary condition that a reactor be critical is that $k_{eff} = 1$; in other words, for every neutron that is absorbed in the core, exactly one neutron is produced.

In calculating critical masses of reactor cores for the V. P. I. UTR-10 reactor, the following fundamental assumption has been made: The non-leakage probability for the proposed cores, consisting of a

lattice of UO_2 or urania-thoria fuel pins immersed in water, will be equal to the non-leakage probability for the present core, which consists of UAl_4 fuel plates immersed in water. This assumption appears reasonably valid, since no changes will be made to the reflector materials which surround the core, nor will the overall size or shape of the core be altered (17). If this assumption is valid, then with the condition that $k_{eff} = 1$ for criticality, it can easily be seen that the infinite multiplication constant of the new core will be equal to that of the present core, since

$$k_{eff} = k_{\infty} \mathcal{L}$$

where $\mathcal{L}$ = the probability of non-leakage.

A convenient criterion for selecting a heterogeneous configuration is the maximization of the multiplication constant k_{∞}. For any fuel enrichment, a plot of k_{∞} as a function of moderator-to-fuel ratio will yield a maximum k_{∞} for some particular value of the moderator-to-fuel ratio, i.e., since $k_{\infty} = \eta f P_{RE} \epsilon$ and η depends only on the fuel enrichment, k_{∞} varies as the product $f P_{RE} \epsilon$. The fast fission factor ϵ decreases slowly as the moderator-to-fuel ratio is increased, so that k_{∞} depends primarily on the product of the thermal utilization f and the resonance escape probability P_{RE}. As the moderator-to-fuel ratio decreases, i.e., the lattice becomes tighter, the neutrons have less chance to slow down in the moderator before being absorbed by a fuel element; consequently the resonance escape probability decreases. However, at the same time the thermal utilization is increasing, since the neutrons have less chance of being absorbed in the moderator. Where

the product of these two quantities is a maximum, the corresponding value of k_∞ will be a maximum.

To calculate the critical masses of slightly enriched UO_2 and urania-thoria lattices, the infinite multiplication constant for a critical fuel loading of UAl_4 plates was obtained from the UTR-10 (3). Values of k_∞ were calculated and plotted as a function of water-to-metal ratio, for both UO_2 and UO_2 - ThO_2 lattices. From the resulting curves, an optimum water-to-metal ratio could be determined. Curves were plotted for various fuel enrichments, and when the value of k_∞ at the optimum water-to-metal ratio reached the value of k_∞ for the present core, the total mass of U-235 in the core was calculated. The methods used in determining each of the factors in the calculations of k_∞, together with any assumptions made, are summarized below.

A. Thermal Utilization

For the calculation of thermal utilization, the unit cell model is employed. The lattice is divided into a number of identical unit cells consisting of a parallelepiped of square cross-section. The square cross-sections are then replaced by circular cross-sections of the same area. Each cell consists of a fuel rod surrounded by an annular region of moderator. The thermal utilization may be approximated by an expression derived from the application of integral transport theory to cylindrical geometry (15). The thermal utilization, f, is given by:

$$\frac{1}{f} - 1 = \frac{b^2 - c^2}{a^2} \frac{\Sigma_{MC}}{\Sigma_{UC}} \left[G + a \Sigma_{UC} \left(1 - \frac{a}{c} \right) \right] + K_M^2 b^2 C + \frac{b^2 - c^2}{c} \Sigma_{MC} \left(\frac{3}{2} \lambda - 1 \right)$$

where a = radius of fuel rod

b = radius of outer boundary of moderator cell

c = radius of outer boundary of cooling gap ($c = a$ in this lattice)

Σ_{MT} = total cross-section of moderator

Σ_{Mc} = capture cross-section of moderator

Σ_{Uc} = capture cross-section of fissionable material

Σ_{UT} = total cross-section of fissionable material

$$k_M^2 = 3 \Sigma_{MT} \Sigma_{Mc}$$

$$C = \frac{1}{2}\left[\frac{b^2/c^2}{(b^2/c^2)-1} \ln\left(\frac{b}{c}\right) - \frac{3}{4} + \frac{1}{4b^2/c^2}\right]$$

$G = \dfrac{\phi(a)}{\phi_U}$ where ϕ_U is the average flux in the uranium

where G is:

$$G = 1 + \frac{\Sigma_{Uc}}{\Sigma_{UT}} B$$

Values of B as a function of a Σ_{UT} are found in table 4 - 9 of (15). Values of λ as a function of $c\Sigma_{MT}$ are found in table 4 - 7 of (15).

The main problem in computing thermal utilization in heterogeneous reactors arises from the non-uniform spatial distribution of the thermal flux. The thermal flux is depressed in a heavily absorbing region. In

the above method of determining f, a zero source of thermal neutrons in the fuel rods is assumed.

B. Resonance Escape Probability

In a hetrogeneous reactor lattice the resonance escape probability P_{RE} is given by the conventional formula:

$$P_{RE} = \exp\left[-\frac{N_A V_A}{\bar{\xi}\,\Sigma_{sM} V_M}\,\frac{\bar{\phi}_A}{\bar{\phi}_M}\,(R.I.)_{eff}\right]$$

where V_A, V_M are the volume of absorber and of moderator, respectively, N_A is the number of absorber atoms per cubic centimeter, $\bar{\xi}$ is the average logarithmic energy decrement, and Σ_{sM} is the macroscopic scattering cross-section of the moderator. The ratio of the average flux in the moderator and absorber, $\bar{\phi}_M/\phi_A$ is called the resonance disadvantage factor. Several references advance the idea that the use of disadvantage factors in heterogeneous lattice calculations is untenable. The main argument is that the flux distribution in the moderator may generally be considered to be uniform and isotropic, while in the fuel the flux distribution is only important at low neutron energies where energy degradation becomes negligible. For low energy resonances in heavy absorbers, absorption predominates over scattering, and therefore the use of disadvantage factors is invalid (36), (37). In lattices composed of fuel pins of the size under consideration here, an analytical calculation of the resonance disadvantage factor is at best only an approximation, and could easily produce a larger error in the resonance

escape probability than if the factor were omitted. Therefore it was decided not to use the disadvantage factor in calculating values of P_{RE}.

The effective resonance integral, designated by $(R.I.)_{eff}$, is a complicated function of the geometry and composition of the heterogeneous system. It is usually given in a form that involves both the volume and the surface of the fuel lump (16). In these calculations the effective resonance integrals were expressed by empirical formulas of the form

$$(R.I.)_{eff} = A + B\sqrt{\gamma S/M}$$

where S/M is the surface-to-mass ratio of the fuel pin and

$$\gamma = \left\{ 1 + \frac{1}{4}\left(\frac{S}{V}\right)_A \frac{V_A}{\Sigma_s^M V_M} \right\}^{-1}$$

In the above expression:

$\left(\frac{S}{V}\right)_A$ = surface-to-volume ratio of absorbing material

V_A = volume fraction of absorbing material

V_M = volume fraction of moderator

Σ_s^M = moderator resonance scattering cross-section (average reciprocal mean free path in moderator)

C. The Fast Fission Factor ϵ

The fast fission factor ϵ is the most inaccurately determined quantity in the four-factor formula. The conventional method for determining ϵ in heterogeneous lattices involves the calculation of collision probabilities for neutrons created within a fuel element (6).

In water-moderated lattices, however, some neutrons which undergo collisions in the moderator may still be energetic enough to cause fast fissions in adjacent fuel elements. This interaction fast effect may be even larger than the ordinary fast effect if the fuel elements are small (12). Therefore the conventional approach is invalid for water-moderated lattices of small fuel elements.

It has been confirmed experimentally (38) that the fast fission factor in water-moderated lattices of slightly enriched UO_2 is not a function of rod size, fuel composition, or fuel enrichment, but only of the water-to-metal ratio of the lattice. Therefore it seems justifiable to use experimentally determined values for ϵ in lieu of a more accurate theoretical method of approach.

In slightly enriched $U^{235}O_2$-ThO_2 water-moderated lattices, the fast fission factor remains approximately constant in the interval 1.005 to 1.01 for a considerable range of water-to-metal ratios (17).

II. Calculation of Temperature Coefficient

The effect of temperature on reactivity is important from the point of view of reactor stability and the estimation of excess reactivity requirements. There are several reasons why a purely theoretical analysis of the effects of temperature on the multiplication constant of a reactor is difficult (26). First, there is insufficient data on the effect of energy on the cross-sections of fissionable materials. Second, one-velocity diffusion theory is inadequate for phenomena in which a change in the spectrum of the thermal neutrons plays an im-

portant part in the diffusion process. Furthermore, the overall temperature coefficient is the result of the combination of a number of effects operating in different directions. Inaccuracy in the evaluation of one of the components may result in a large degree of error in the resultant temperature coefficient.

There are several effects which contribute to the overall temperature coefficient of a reactor (39), (40). These include: (a) the Eta-Effect- caused by the change in the ratio of the effective cross-sections of U-235 to U-238 with neutron temperature, (b) the Doppler broadening of the U-238 or Th-232 resonances, (c) the change in thermal utilization arising from the flattening of the neutron flux distribution across a lattice cell as the neutron temperature rises, and (d) the thermal expansion of the reactor components.

In order to determine the effect of temperature on the infinite multiplication constant of the UO_2 and ThO_2 lattices under consideration, the various terms in the four-factor formula were calculated at the expected operating temperature of the reactor (100 degrees Fahrenheit) and the resulting k_∞ was compared with that calculated at room temperature (68 degrees Fahrenheit). The temperature coefficient $\frac{1}{k}\frac{dk}{dT}$ was then determined by assuming a uniform coefficient over the interval of 32 degrees Fahrenheit.

III. Transient Behavior

Once the temperature coefficient had been determined, it was possible to examine the transient behavior of the core. Specifically, the

response of the core to a sudden insertion of 0.5% excess reactivity was examined with respect to stable reactor period, maximum power level, and maximum temperature attained. In order to find the stable reactor period, it was necessary to solve the time-dependent thermal neutron diffusion equation, taking into account the contribution of delayed neutrons. By considering a single group of delayed neutrons with a decay constant λ equal to the properly weighted average for the 5 actual groups, a simplified expression for the stable reactor period was obtained. (See Appendix I). The solution is of the form $T = \frac{1}{\omega_0}$

where $\omega_0 \approx \frac{\lambda \rho}{\beta - \rho + l\lambda}$

In the above relation

T = stable reactor period

l = prompt neutron lifetime

β = fraction of neutrons which are delayed

ρ = excess reactivity which is inserted

λ = decay constant for delayed neutrons

When an amount of excess reactivity, δk_0, is added suddenly to a reactor, the power will rise until, at a temperature $-\frac{\delta k_0}{\alpha}$, where α is the negative temperature coefficient of reactivity, the multiplication constant again becomes equal to 1. At this time the power level ceases to rise but the temperature continues to increase, since by this time the power level is well above the cooling rate. As the temperature increases above $-\frac{\delta k_0}{\alpha}$, the multiplication constant decreases below 1 and the power level drops. When the power level reaches the cooling rate

it will continue to drop and the temperature will begin to decrease. Mathematical expressions for the maximum power level and maximum temperature of the core are derived in the Appendix. The final results are

$$P_{max} = P_o + \frac{C(\delta k_o)^2}{2\alpha \ell}$$

$$T_{max} = (\delta k_o) + \sqrt{(\delta k_o)^2 + \frac{2\alpha P_o \ell}{C}}$$

where P_o = power level before insertion of reactivity

C = total heat capacity of core

ℓ = average neutron lifetime

IV. Heat Generation and Removal

In a low-power research reactor the problems associated with cooling are usually not very significant. However, due to the low thermal conductivity of uranium dioxide and thorium dioxide, the possibility of the melting or deforming of a fuel element due to thermal stresses during an excursion should be considered. In addition, in order to predict the power response of a reactor upon the introduction of an amount of excess reactivity, the total heat capacity of the core must be known. Since the heat generated in the core is directly or indirectly obtained from the energy released in the nuclear fissions, the total heating density may be determined if the average flux in the core is known.

In order to determine the heat capacity C of the core, it was necessary to calculate the amount of fuel, moderator, and cladding in

the core region. Then, using the specific heats of the various constituents, the total heat capacity of the core in kilowatt-seconds per degree Centigrade was determined. It was assumed that heat transfer took place only through the aluminum cladding and the water, since during an excursion it would require a relatively long time for the graphite in the core region to heat up. The heat capacity of the core was used to calculate the maximum power and temperature as described earlier.

In order ot estimate the temperature gradient across the fuel elements, it was first necessary to determine the amount of heat generated per unit time per unit volume of fuel. This heating density, H_f, is a function of the local neutron flux density ϕ and the local fuel concentration, as shown in the following formula (31).

$$H_f = Q' \phi \Sigma_f$$

where Σ_f = macroscopic fission cross-section of fuel

Q' = the amount of energy released per fission

The average flux ϕ_{avg} may be expressed by the relation

$$\phi_{avg} = \frac{P_T F}{\Sigma_f V_f}$$

where P_T = power level of operation

F = no. of fissions per watt-second

V_f = volume of fuel

In the above relation the product $P_T F$ represents the fission rate.

If the above expression of ϕ_{avg} is substituted into the expression

for the heating density, the latter reduces to

$$H_f = Q' \left(\frac{P_T F}{V_f} \right)$$

which gives the average heating density of the core.

Having determined the average heating density in the fuel elements, it is now possible to find the temperature gradient across the pins. A complete analysis would include heat transfer through the fuel, the internal gap between the fuel and cladding, the cladding, the surface film on the cladding, and the bond between the cladding and the moderator. In the situation under consideration here, however, only the fuel and cladding are present. Temperature differences may be calculated from conventional heat transfer formulas, as shown below.

$$\text{Fuel: } \Delta T = \frac{H_f r_f^2}{4 k_f} \qquad \text{Clad: } \Delta T = \frac{H_f r_f^2 a_c}{2 r_c k_c}$$

where r_f, r_c = radius of fuel, mean radius of clad

k = thermal conductivity

a_c = thickness of clad

V. Epicadmium Fission in U-235

The possible influence of epicadmium fission in U-234 on the infinite multiplication constant of the water-moderated UO_2 and ThO_2 lattices under investigation should be considered.[1] In close-packed lattices made up of pin-type elements containing highly enriched uranium oxide mixed with thorium oxide, 10 to 25 % of the fissions are

[1] Epicadmium Fission refers to fission produced by neutrons having energies above the cadmium cut-off (o. 5 ev)

epicadmium (18). For a water-to-metal ratio much greater than one, however, the moderating effect of the water causes a severe reduction in the amount of U-235 resonance absorption, and consequently in the amount of epicadmium fission in U-235. Even for natural uranium, neutron absorptions in the U-235 resonances become important when the water-to-metal ratio of the lattice approaches one (41).

In any reactor in which non-thermal as well as thermal fission plays a substantial part, it is necessary to introduce an epithermal or fast multiplication constant, k_2. The infinite multiplication constant k_∞ is then the sum of the thermal and fast multiplication constants, i.e., $k_\infty = k_1 + k_2$ (18). The usual formulations of the critical equations in a reactor may be altered to include the contribution to the neutron economy of epithermal fissions in U-235. References (23) and (42), for example, derive critical equations in terms of several experimentally measured quantities. A measurement of the resonance integral for epicadmium neutrons in a graphite-uranium lattice has been made. The number obtained was 271 $\pm$ 25 barns (43).

In view of the fact that the optimum moderator-to-fuel ratio for the lattices under consideration in this report was about 2.5 to 1, and that the fuel enrichments were rather low, it was decided to neglect the effect of non-thermal fission in U-235 in calculating the multiplication constants. Interpretation of experimental data on slightly-enriched H_2O and D_2O lattices supports this decision (21), (44), (49).

IV. DATA AND RESULTS

A. Multiplication Constant and Critical Mass

The nuclear constants used in calculating the multiplication constant as given by the four-factor formula are shown in Table 1. These values were obtained from ANL-3800 (Reactor Physics Constants) with the exception of certain cross-sections, which were taken from BNL-325. All values are for thermal neutrons at 20°C unless otherwise stated.

Table 1

Fuel	
σ_f (U-235)	584 bn
σ_a (U-235)	689 bn
σ_a (U-238)	2.71 bn
ν (U-235)	2.46
Theoretical density of UO_2	10.96 g/cm^3
Theoretical density of ThO_2	10.03 g/cm^3
Moderator	
$\Sigma_s \xi$ (H_2O)*	1.353 cm^{-1}
Σ_{MT}	3.45 cm^{-1}
Σ_{Mc}	0.022 cm^{-1}

*Scattering cross-section averaged over slowing-down energy range.

Calculated values of the multiplication constant k_∞ for various fuel enrichments and moderator-to-fuel ratios are shown in Tables 2 and 3. The data in Table 2 is for uranium dioxide fuel rods, in which the

fuel has been compacted to within 92% theoretical density. Table 3 applies to urania-thoria fuel rods containing fuel compacted to within 95% theoretical density. The results are illustrated graphically in Figures 1 and 2.

Table 2

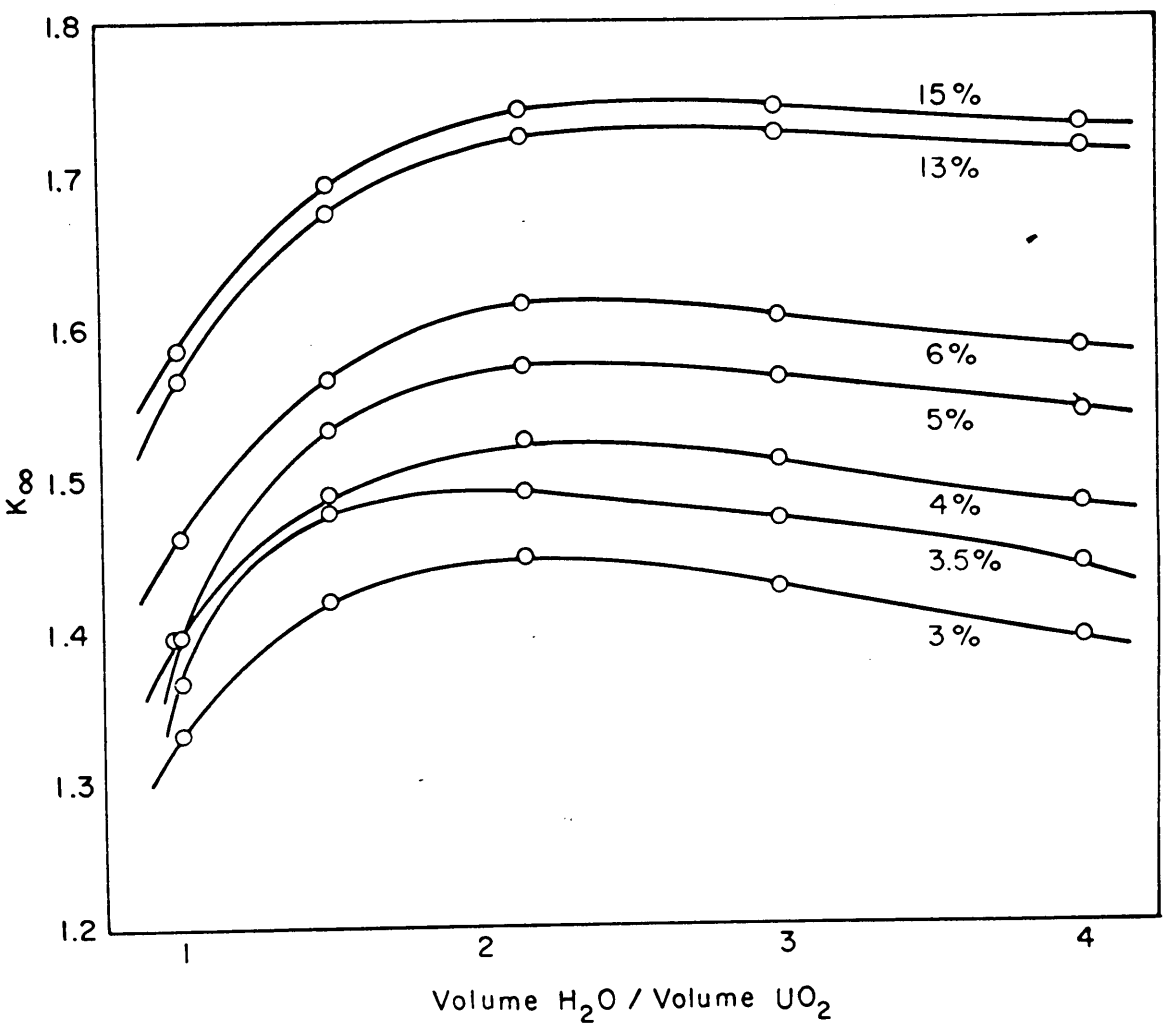
UO_2 Fuel Rods

Enrichment	Vol. H_2O/Vol. UO_2	η	f	P_{RE}	ϵ	k_∞
5%	1	1.940	0.951	0.720	1.05	1.394
5%	1.5	1.940	0.932	0.812	1.045	1.533
5%	2.14	1.940	0.907	0.860	1.04	1.574
5%	3	1.940	0.874	0.896	1.03	1.565
5%	4	1.940	0.838	0.920	1.03	1.540
6%	1	1.964	0.955	0.742	1.05	1.462
6%	1.5	1.964	0.938	0.814	1.045	1.566
6%	2.14	1.964	0.884	0.862	1.04	1.615
6%	3	1.964	0.884	0.897	1.03	1.605
6%	4	1.964	0.850	0.921	1.03	1.583
13%	1	2.032	0.969	0.759	1.05	1.567
13%	1.5	2.032	0.955	0.826	1.045	1.675
13%	2.14	2.032	0.937	0.871	1.04	1.726
13%	3	2.032	0.913	0.904	1.03	1.727
13%	4	2.032	0.884	0.926	1.03	1.714
15%	1	2.040	0.970	0.763	1.05	1.586
15%	1.5	2.040	0.957	0.830	1.045	1.692
15%	2.14	2.040	0.940	0.874	1.04	1.743
15%	3	2.040	0.916	0.906	1.03	1.744
15%	4	2.040	0.888	0.928	1.03	1.731
4%	1	1.905	0.944	0.737	1.05	1.392
4%	1.5	1.905	0.922	0.810	1.045	1.487
4%	2.14	1.905	0.895	0.859	1.04	1.524
4%	3	1.905	0.860	0.895	1.03	1.510
4%	4	1.905	0.820	0.919	1.03	1.480
3.5%	1	1.881	0.939	0.736	1.05	1.365
3.5%	1.5	1.881	0.916	0.809	1.045	1.478
3.5%	2.14	1.881	0.887	0.858	1.04	1.490
3.5%	3	1.881	0.850	0.894	1.03	1.471
3.5%	4	1.881	0.809	0.919	1.03	1.439
3%	1	1.850	0.933	0.735	1.05	1.332
3%	1.5	1.850	0.908	0.808	1.045	1.418
3%	2.14	1.850	0.877	0.858	1.04	1.447
3%	3	1.850	0.837	0.894	1.03	1.426
3%	4	1.850	0.794	0.918	1.03	1.389

Table 3

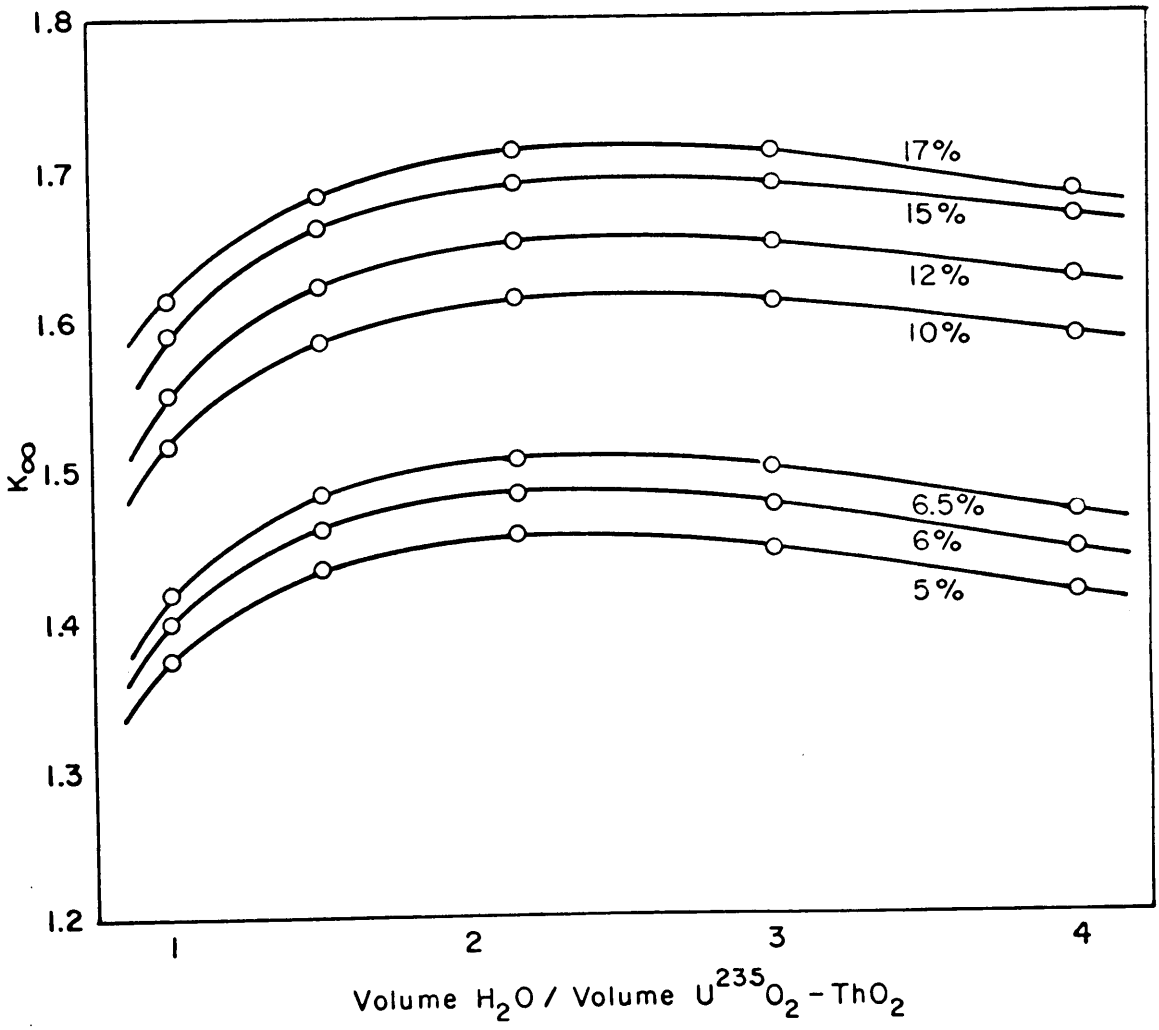
$U^{235}O_2$ - ThO_2 Fuel Rods

Enrichment	Vol. H_2O/Vol. Fuel	η	f	P_{FE}	k_∞
5.5%	1	1.754	0.955	0.812	1.374
5.5%	1.5	1.754	0.937	0.864	1.435
5.5%	2.14	1.754	0.914	0.899	1.457
5.5%	3	1.754	0.883	0.925	1.447
5.5%	4	1.754	0.848	0.942	1.416
6%	1	1.779	0.957	0.813	1.398
6%	1.5	1.779	0.940	0.865	1.461
6%	2.14	1.779	0.917	0.900	1.483
6%	3	1.779	0.887	0.925	1.475
6%	4	1.779	0.854	0.942	1.445
6.5%	1	1.801	0.958	0.814	1.418
6.5%	1.5	1.801	0.942	0.866	1.483
6.5%	2.14	1.801	0.920	0.900	1.507
6.5%	3	1.801	0.891	0.926	1.500
6.5%	4	1.801	0.858	0.943	1.470
10%	1	1.898	0.965	0.820	1.517
10%	1.5	1.898	0.951	0.870	1.586
10%	2.14	1.898	0.932	0.904	1.614
10%	3	1.898	0.906	0.928	1.612
10%	4	1.898	0.876	0.945	1.586
12%	1	1.930	0.968	0.823	1.552
12%	1.5	1.930	0.954	0.872	1.622
12%	2.14	1.930	0.936	0.906	1.652
12%	3	1.930	0.911	0.930	1.650
12%	4	1.930	0.882	0.946	1.626
15%	1	1.963	0.970	0.828	1.592
15%	1.5	1.963	0.957	0.876	1.662
15%	2.14	1.963	0.940	0.908	1.692
15%	3	1.963	0.916	0.932	1.691
15%	4	1.963	0.888	0.947	1.667
17%	1	1.979	0.972	0.832	1.616
17%	1.5	1.979	0.958	0.879	1.683
17%	2.14	1.979	0.941	0.910	1.713
17%	3	1.979	0.918	0.933	1.712
17%	4	1.979	0.888	0.948	1.683



U_2O Fuel Rods

K_∞ vs. Moderator-to-Fuel Ratio for Various Enrichments



$U^{235}O_2$-ThO Fuel Rods

K_∞ vs. Moderator-to-Fuel Ratio for Various Enrichments

In order to determine the critical mass of U-235 for the cores, it is first necessary to calculate the infinite multiplication constant k_∞ for the UTR-10 with its present loading of UAl_4 fuel plates. Because of the high enrichment of the fuel the resonance escape probability P_{RE} and the fast fission factor ϵ can be taken as unity. Then the four-factor formula reduces to

$$k = \eta f$$

which may also be written as

$$k = \nu \frac{\overline{\Sigma}_f}{\overline{\Sigma}_a}$$

The following values are given in Reference (3):

$\nu = 2.46$

$\overline{\Sigma}_f = 0.041$

$\overline{\Sigma}_a = 0.068$

Hence $k_\infty = 1.483$ for a loading of 90% enriched UAl_4 fuel plates.

The next step in finding the critical mass is to select a fuel enrichment and moderator-to-fuel ratio for the new core which gives a k_∞ of 1.483. From Figures 1 and 2 it can be seen that the maximum multiplication occurs at a moderator-to-fuel ratio of approximately 2.5 to 1. Also, a fuel consisting of 3.5% enriched UO_2 or a mixture of 6% $U^{235}O_2$ and 94% ThO_2 will produce approximately the desired k_∞.

From the total volume of the fuel boxes in the core and the optimum moderator-to-fuel ratio, the volume of fuel required is easily found to be 25,819 cubic centimeters. Then the critical masses may be found, using known densities and atomic weights. The final results are as follows:

Fuel	Critical Mass of U^{235}
3.5% enriched UO_2	8,031 grams
6% $U^{235}O_2$ and 94% ThO_2	13,045 grams

B. Temperature Coefficient of Reactivity

In calculating the temperature coefficient of reactivity, it was necessary to know the linear coefficient of thermal expansion for the materials in the core. The following values were obtained (33), (48).

Material	Temp. Coeff. of Linear Expansion/°C
UO_2	8.1×10^{-6}
UO_2-ThO_2	9×10^{-6}
Al	24×10^{-6}
H_2O	0.1023×10^{-3}

The effect of a change in temperature on the microscopic thermal neutron cross-sections had to be taken into account. The operating temperature of the reactor was considered to be 100 degrees Fahrenheit, identical to that of the UTR-10 with its present core. At this temperature the mean energy of thermal neutrons is 0.0268 ev. The desired neutron cross-sections for 0.0268 ev neutrons were obtained from BNL-325 (Neutron Cross Sections). Table 4 lists the values which were used in the calculation of the temperature coefficients.

Table 4

σ_f (U-235)	560 bn
σ_a (U-235)	660 bn
σ_a (U-238)	2.71 bn
ν (U-235)	2.46

The change in the resonance integral of uranium dioxide due to Doppler broadening may be estimated from an empirical relation which has been deduced from experimental data (27).

The following approximation holds:

$$R.I. = (R.I.)_o \left[1 + \beta\left(\sqrt{T} - \sqrt{T_o}\right)\right]$$

where $\beta = \left(0.58 + 0.5\,\frac{S}{M}\right)/0^{-2}$

Thus if the resonance integral $(R.I.)_o$ is known at a reference temperature T_o its value at any other temperature T may be computed as a function of T_o, T, and S/M, the surface-to-mass ratio of the fuel element at temperature T. It was found that the resonance integral for the UO_2 fuel pins under consideration increased by 0.082 barns when the temperature was increased from 20 degrees Centigrade (68 degrees Fahrenheit) to 37.78 degrees Centigrade (100 degrees Fahrenheit). The corresponding resonance excape probability increased by 0.0011.

The effect of a change in temperature on the quantities which comprise the four-factor formula may be seen in Table 5. The values listed are for 3.5% enriched UO_2 fuel rods with a moderator-to-fuel ratio of 2.14.

Table 5

Temperature, °C	η	f	$P_{R\epsilon}$	ϵ	k_∞
20	1.881	0.887	0.858	1.03	1.475
37.78	1.875	0.884	0.857	1.03	1.464

Using the values of k_∞ stated above, the temperature coefficient of reactivity $\frac{1}{k}\frac{dk}{dT}$ is found to be -4.35×10^{-4}/°C.

Table 6 contains corresponding data for 6% $U^{235}O_2$ - 96% ThO_2 fuel rods with a moderator-to-fuel ratio of 2.14.

Table 6

Temperature, °C	η	f	P_{RF}	ϵ	k_∞
20	1.779	0.917	0.900	1.01	1.483
37.78	1.770	0.915	0.897	1.01	1.468

From the above values of K_∞, the temperature coefficient of reactivity $\frac{1}{k}\frac{dk}{dT}$ is found to be -5.82×10^{-4}/°C.

C. Reactor Core Dimensions

The UTR-10 thermal research reactor consists of two fuel slabs imbedded in graphite which forms an internal and external reflector. The fuel slabs consist of boxes of 1/4 inch thick aluminum into which are placed assemblies of enriched UAl_4 fuel plates with aluminum cladding. The fuel slabs are reflected in the vertical direction on both top and bottom by a layer of water extending out of the fuel region of the tanks in which the water acts as a moderator-collant. The dimensions of the slabs are 5 27/32" x 19 15/16" x 58" high. The fuel region of the slabs, excluding the vertical layers of water reflector, is 26" high. Fuel plate dimensions are 26" x 3" x 0.080".

In the proposed core for the UTR-10 the UAl_4 fuel plates are replaced by aluminum fuel pins filled with enriched UO_2 or UO_2-ThO_2. The Babcock and Wilcox Company of Lynchburg, Virginia can supply fuel pins having an outer diameter of 0.436 inches and a thickness of 0.30 to 0.35 inches. These pins would be of the same height as the present

UAl_4 plates, and would be arranged in a square lattice in the fuel region of the core. It would be possible to remove single pins as well as pin clusters; consequently slight changes in the amount of fissionable material in the core could be made.

D. Response to a Step Insertion of Reactivity

1. Stable Reactor Period

The stable reactor period T following the sudden insertion of excess reactivity is given by the relation

$$T = \frac{\beta - \rho + \ell\lambda}{\lambda\rho}$$

in which

λ = the properly weighted average decay constant for the five actual groups of delayed neutrons

ρ = the amount of excess reactivity inserted

β = the fraction of neutrons which are delayed

ℓ = the average lifetime of prompt neutrons in the reactor

For U-235 $\lambda = 0.076$ and $\beta = 0.00755$. The prompt neutron lifetime was taken as 10^{-4} seconds, a typical value for uranium-water lattices. The excess reactivity ρ was taken as 0.5 , which is the maximum per-

mitted in the operating license of the V. P. I. UTR-10. The above constants resulted in a value for T of 6.7 seconds.

Both the UO_2 and the UO_2-ThO_2 fuel cores will have the same reactor period, since U-235 is the fissionable material in both cores, and the only change in any of the above values will be a slight difference in the prompt neutron lifetime according to which fuel loading is used.

2. Maximum temperature

The maximum temperature which the core will reach is given by the relation (See 7).

$$T_{max} = \frac{\delta k_0 + b}{|\alpha|}$$

where $$b = \sqrt{(\delta k_0)^2 + \frac{2|\alpha| P_0 \bar{l}}{C}}$$

In the above formulas,

δk_0 = the amount of excess reactivity introduced

α = the temperature coefficient of the reactor

P_0 = the power level before the insertion of the excess reactivity

$\bar{l}$ = average neutron lifetime in reactor, including delayed neutrons

C = the total heat capacity of the core

The amount of excess reactivity inserted is again taken as 0.5%. The initial operating power level P_0 is taken as 10 kilowatts, which is the maximum power at which the UTR-10 is designed to operate.

For U-235 the contribution of the delayed neutrons to the average neutron lifetime is approximately 0.0927 seconds. Therefore for the present system $\bar{\ell} = 10^{-4} + 0.0927$ or $\bar{\ell} = 0.0928$ seconds.

In calculating the total heat capacity of the core, it was assumed that during the excursion only the fuel elements and the surrounding water acted as a heat sink. Since the volumes, densities, and specific heats of each of these constituents was known, the total amount of heat absorbed in the core per degree rise in temperature could be determined. The quantities used in the calculations are found in Table 7.

Table 7

Material	Specific heat (at 20°C) cal/g °C	Density (at 20°C) g/cc	Heat capacity cal/°C
H_2O	0.99799	0.99823	185,072
Al	0.224	2.699	20,784
UO_2	0.056	10.955	14,572
UO_2-ThO_2	0.058	10.081	14,341

(Specific heats and densities were taken from References (31), (33), and (45).

For a reactor core made up of 3.5% enriched uranium dioxide fuel rods, the approximate total heat capacity was found to be 922.7 kilowatt-seconds per degree Centigrade. A core consisting of 6% $U^{235}O_2$ and 94% ThO_2 has a heat capacity of 921.7 kilowatt-seconds per degree Centigrade, approximately equal to that of the UO_2 core.

The temperature coefficient α has been estimated for the UO_2 fuel core as -4.35×10^{-4} per degree Centigrade, and for the UO_2-ThO_2 fuel core as -5.82×10^{-4} per degree Centigrade.

When the above values are substituted into the previously expressed relation for the maximum temperature of the reactor during the excursion, the following results are obtained:

Maximum temperature of UO_2 core = 174°Fahrenheit

Maximum temperature of UO_2-ThO_2 core = 163°Fahrenheit

3. Maximum power level

The maximum power level reached by the reactor during the excursion may be expressed in terms of the previously defined quantities as shown below

$$P_{max} = P_o + \frac{C(\delta k_o)^2}{2|\alpha| l}$$

Upon substitution of the appropriate quantities, the following results are obtained:

Maximum power level of UO_2 core = 286 kilowatts

Maximum power level of UO_2-ThO_2 core = 216 kilowatts

E. Heat Generation in Core

The average amount of heat $\bar{H}_f$ generated per unit time per unit volume of fuel was calculated from the relation (See page 21)

$$\bar{H}_f = Q' \frac{(P_t)(F)}{V_f}$$

in which the following values were used (31):

$Q' = 2.75 \times 10^{-14}$ Btu per fission

$P_t = 10{,}000$ watts

$F = 3.2 \times 10^{10}$ fissions/watt-second

$V_f = 25{,}819$ cubic centimeters

These values give an $\bar{H}_f$ of approximately 9.7 Btu per cubic foot per second.

At a power level of 286 kilowatts, which is the maximum power level reached following the step insertion of 0.5% excess reactivity (See page 21), the corresponding $\bar{H}_f$ is 276 Btu per cubic foot per second.

Once the average rate of heat generation in the fuel has been found, it is possible to estimate the temperature gradient across the fuel pins, according to the method discussed on page 21. The temperature difference between the center of the fuel pin and the outside surface of the aluminum cladding may be expressed as:

$$\Delta T = \frac{H_f r_f^2}{4k_f} + \frac{H_f r_f^2 a_c}{2r_c k_c}$$

where $H_f = 9.7$ Btu/ft^3 sec

$= 34{,}746$ Btu/ft^3 hr

$r_f = 0.0157$ foot

$k_f = 2$ Btu/hr ft °F (both UO_2 and ThO_2)

$a_c = 0.0025$ foot

$r_c = 0.017$ foot

$k_c = 121.68$ Btu/hr ft °F

Using the above values it is found that at a power level of 10 kilowatts the temperature gradient across the fuel pins is only about 1 degree Fahrenheit. When the power level is increased to 286 kilowatts, the corresponding temperature gradient is 31°F.

V. CONCLUSIONS

Calculations have shown that a fuel core for the V. P. I. UTR-10 reactor consisting of a lattice of enriched uranium dioxide or urania-thoria pins is feasible. The critical mass of such a core is several times larger than that of the present core, which consists of highly enriched UAl_4 fuel plates. This increase in critical mass is due to a reduction in neutron economy which occurs when the plates are replaced by pin-type elements. Fuel enrichments required to achieve a multiplication constant equal to that of the present core did not exceed 6% at an optimum water-to-metal ratio of 2.5.

The response of both the uranium dioxide and urania-thoria fuel loadings to a sudden insertion of excess reactivity indicated that both systems would be safe from damage caused by such an excursion. In addition, the theoretically calculated temperature coefficient of reactivity for both types of cores was negative, as required for inherently safe operation.

The temperature gradient across the fuel pins was estimated for both types of loadings, and the results that no danger of melting or failure due to thermal stresses would exist at any foreseeable power level of operation.

VI. SUMMARY

An analysis based on theoretical calculations has been carried out to determine the critical mass, transient behavior, and thermal characteristics of two proposed fuel cores for the V. P. I. UTR-10 Reactor. The cores would consist of lattices of aluminum rods containing enriched UO_2 or urania-thoria. It was found that for an optimum water-to-metal ratio of 2.5, the critical mass of U-235 for the uranium dioxide core was approximately 8,031 grams, while that for the urania-thoria core was approximately 13,045 grams.

The response of the proposed cores to a sudden insertion of excess reactivity was investigated. It was found that the stable reactor period, maximum core temperature, and maximum power level were all within reasonably safe limits. The temperature gradient across the fuel pins during the excursion was sufficiently small so as to eliminate the danger of melting within the fuel elements.

Theoretical calculations predict negative temperature coefficients of reactivity of approximately $5 \times 10^{-4}/°C$ for both the enriched UO_2 and urania-thoria cores. Therefore both systems would be inherently safe against excursions caused by increases in core temperature.

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X. APPENDIX I

The Stable Reactor Period T.

The thermal neutron diffusion equation in a non-equilibrium system can be written as

$$D\nabla^2\phi - \Sigma_a\phi + S = \frac{\partial n}{\partial t} = \frac{1}{v}\cdot\frac{\partial \phi}{\partial t} \qquad (1)$$

where v is the mean velocity of the thermal neutrons, so that $\phi = nv$ and S is the thermal neutron source term due to fissions. Since the number of thermal neutrons absorbed per cm^3 per second is $\Sigma_a\phi$ the over-all rate of production fission neutrons, including both prompt and delayed, is $\frac{k}{p}\Sigma_a\phi$ per cm^3 per second. Since β_i is the fraction of the total number of fission neutrons which are delayed neutrons belonging to the ith group, the rate of formation in the fission process of the nuclei which are the precursors of those neutrons is equal to $\beta_i\frac{k}{p}\Sigma_a\phi$ per cm^3 per second. If C_i is the concentration, in atomic nuclei per cm^3, of the precursors from which the ith group of delayed neutrons arise, then the normal rate of radio-active decay is $\lambda_i C_i$ nuclei per cm^3 per second, where λ_i sec^{-1} is the appropriate decay constant. The net rate of formation of delayed neutron precursors of the ith group is consequently given by

$$\frac{\partial C_i(r,t)}{\partial t} = -\lambda_i C_i(r,t) + \frac{k}{p}\beta_i\Sigma_a\phi(r,t) \qquad (2)$$

The source term of equation (1) must now be modified to allow for the delayed neutrons. The sum of the β_i values for the various

delayed groups is represented by β, and hence, $1 - \beta$ is the fraction of fission neutrons which are emitted promptly. The corresponding (prompt neutron) source term is then $(1 - \beta)k \Sigma_a \phi e^{-B^2\tau}$, which is the value given in (1) multiplied by $1 - \beta$.

The rate of formation of delayed neutrons in any group is equal to the rate of decay of the precursor, i.e., to $\lambda_i C_i$, and hence

total rate of formation of all the delayed neutrons = $\sum_{i=1}^{m} \lambda_i C_i$ per cm^3 per second, where the summation is taken for all the m groups of delayed neutrons. If this is multiplied by $e^{-B^2\tau}$, the non-leakage probability during the slowing down process, and also by the resonance escape probability p, the result, namely,

$$p e^{-B^2\tau} \sum_{i=1}^{m} \lambda_i C_i$$

is the source term due to delayed neutrons.

For the present case, therefore, the source term of (1) may be written as

$$S = (1-\beta) k \Sigma_a \phi e^{-B^2\tau} + p e^{-B^2\tau} \sum_{i=1}^{m} \lambda_i C_i$$

and consequently the appropriate thermal neutron diffusion equation is

$$D\nabla^2 \phi(r,t) - \Sigma_a \phi(r,t) + (1-\beta) k \Sigma_a \phi(r,t) e^{-B^2\tau} + p e^{-B^2\tau} \sum_{i=1}^{m} \lambda_i C_i(r,t) = \frac{1}{v} \frac{\partial \phi(r,t)}{\partial t}$$

Dividing through by Σ_a and substituting L^2 for D/Σ_a and l_0 for $1/\Sigma_a v$, as in (1), this becomes

$$L^2\nabla^2 \phi(r,t) + \left[(1-\beta) k e^{-B^2\tau} - 1\right] \phi(r,t) + \frac{p e^{-B^2\tau}}{\Sigma_a} \sum_{i=1}^{m} \lambda_i C_i(r,t) = l_0 \frac{\partial \phi(r,t)}{\partial t} \quad (3)$$

If it is postulated that the reactor has been operating in the steady state and that the effective multiplication factor undergoes a small step change at time t equal to zero, after which it remains constant, it can be shown that the space and time variables in (3) are separable. Suppose the reactor is near enough to critical so that the neutron flux can be calculated satisfactorily from the fundamental mode of the wave equation, and let

$$\phi(r,t) = \phi(r)\, T(t)$$

and $$C_i(r,t) = C_i(r)\, H_i(t)$$

Making these substitutions, (2) becomes

$$\frac{dH_i(t)}{dt} = -\lambda_i H_i(t) + \frac{k}{P}\beta_i \Sigma_a \frac{\phi_i(r)}{C_i(r)} T(t)$$

and consequently $C_i(\underline{r})/\phi(\underline{r})$ must be independent of $\underline{r}$.

With the same substitutions, for $\phi(\underline{r},t)$ and $C_i(\underline{r},t)$, equation (3) becomes

$$L^2 \frac{\nabla^2 \phi(r)}{\phi(r)} + \left[(1-\beta)k e^{-B^2\tau} - 1\right] + \frac{P e^{-B^2\tau}}{\Sigma_a} \sum_{i=1}^{m} \lambda_i \frac{C_i(r) H_i(t)}{\phi(r) T(t)} = \frac{l_o}{T(t)} \cdot \frac{dT(t)}{dt}$$

and, since $C_i(\underline{r})/\phi(\underline{r})$ is independent of $\underline{r}$, as shown above, it is seen that the variables have been separated. It will be noted that this separability is due to the fact that $C_i(\underline{r})$, the concentration of each delayed neutron precursor, is everywhere proportional to the flux, $\phi(r)$; this is, of course, to be expected on physical grounds.

Because the space and time variables can be separated, it is possible to write, as before, $\nabla^2 \phi(r) + B^2 \phi(r) = 0$

However, since the Laplacian operator has no effect on the time-variable function T(t), both sides of this equation may be multiplied by T(t)

to give the general wave equation

$$\nabla^2 \phi(r,t) + B^2 \phi(r,t) = 0 \tag{4}$$

where B^2 is the lowest eigenvalue resulting from the boundary condition that the thermal neutron flux shall be zero at the extrapolated boundary. Hence $\nabla^2 \phi(r,t)$ in (3) can be replaced by $-B^2 \phi(r,t)$.

The problem now is to solve the differential equations (2) and (3) taking into account (4). Since both of the former equations are linear and first order, and since the space and time variables are separable, solutions of the form

$$\phi(r,t) = \phi_0 e^{tw} \tag{5}$$

and

$$C_i(r,t) = C_{i0} e^{tw} \tag{6}$$

may be superimposed. It is then required to find the conditions on the parameters represented by w, having the dimensions of reciprocal time, which permit such solutions. It should be noted that ϕ_0 and C_{i0} are the values of the neutron flux and of the concentration of the precursor of the ith group of delayed neutrons, respectively, at time t = 0, at a point represented by the vector $\underline{r}$. The latter variable has been omitted from the symbols for simplicity of representation.

Upon substituting ϕ and C_i, as given by (5) and (6), respectively, into (2), the result is

$$C_{i0} w e^{tw} = -\lambda_i C_{i0} e^{tw} + \frac{k}{\ell} \beta_i \Sigma_a \phi_0 e^{tw}$$

and consequently

$$C_{io} = \frac{k \beta_i \Sigma_a \phi_o}{\rho (\omega + \lambda_i)} \tag{7}$$

Further, by introducing (4), (5), (6), and (7) into (3), the latter equation can be simplified to

$$-(L^2B^2+1) + (1-\beta) k e^{-B^2\tau} + k e^{-B^2\tau} \sum_{i=1}^{m} \frac{\lambda_i \beta_i}{\omega + \lambda_i} = l_o \omega$$

Dividing through by $1 + L^2B^2$, and recalling that $l_o/(1 + L^2B^2)$ is equal to l, the mean lifetime of the thermal neutrons in the finite reactor, it is seen that

$$(1-\beta) \frac{k e^{-B^2\tau}}{1+L^2B^2} - 1 + \frac{k e^{-B^2\tau}}{1+L^2B^2} \sum_{i=1}^{m} \frac{\lambda_i \beta_i}{\omega + \lambda_i} = l \omega$$

If the effective multiplication constant k_{eff} is defined by

$$k_{eff} = \frac{k e^{-B^2\tau}}{1+L^2B^2}$$

then the above equation becomes

$$(1-\beta) k_{eff} - 1 + k_{eff} \sum_{i=1}^{m} \frac{\lambda_i \beta_i}{\omega + \lambda_i} = l \omega \tag{8}$$

The total fraction β of delayed neutrons is equal to the sume of the individual β_i's by definition, and since $k_{eff} - 1$ is defined as the excess multiplication k_{ex}, it follows that (8) may be written as

$$k_{ex} + k_{eff} \sum_{i=1}^{m} \left(\frac{\lambda_i \beta_i}{\omega + \lambda_i} - \beta_i \right) = l \omega$$

or

$$k_{ex} = l \omega + k_{eff} \sum_{i=1}^{m} \frac{\omega \beta_i}{\omega + \lambda_i}$$

It is now convenient to introduce a quantity ρ, called the reactivity, and defined as

$$\rho \equiv \frac{k_{ex}}{k_{eff}} = \frac{k_{eff} - 1}{k_{eff}} \tag{10}$$

Hence, if (9) is divided through by k_{eff}, the result is

$$\rho = \frac{k_{ex}}{k_{eff}} = \frac{l\omega}{k_{eff}} + \sum_{i=1}^{m} \frac{\omega \beta_i}{\omega + \lambda_i} \tag{11}$$

From (10), $k_{eff} = 1/(1 - \rho)$ and, if this is introduced into (11), the latter becomes

$$\rho = \frac{l\omega}{l\omega + 1} + \frac{1}{l\omega + 1} \sum_{i=1}^{m} \frac{\omega \beta_i}{\omega + \lambda_i} \tag{12}$$

which is a convenient form for calculation purposes. Equation (12) is the characteristic equation which relates the parameters ω to the nuclear properties of the substances constituting the reactor. It is an algebraic equation of degree m + 1 in ω, and consequently there are, in general, m + 1 values of ω corresponding to each value of the reactivity ρ; of these m are negative and one is positive. Numerically each of the m negative values of ω is seen to be of the same order as one of the λ_i's i.e., the decay constants of the delayed neutron precursors. Hence, the neutron flux at a point $\underline{r}$, as a function of time, may be represented by a linear combination of m + 1 solutions of the form of (5); thus

$$\phi = A_0 e^{t\omega_0} + A_1 e^{t\omega_1} + \cdots + A_i e^{t\omega_i} + \cdots + A_m e^{t\omega_m} \tag{13}$$

where $\omega_0, \omega_1, \ldots \omega_i, \ldots \omega_m$ are the m + 1 roots of (12), ω_0 having a positive value and $\omega_1, \ldots \omega_m$ being approximately equal to $-\lambda_1, \ldots -\lambda_m$, re-

spectively. The A_0, A_1 , etc., are constants determined by the initial conditions in the given reactor.

Since all the terms beyond the first on the right-hand side of (13) have negative exponents, it is evident that as the time t following the sudden small change in reactivity increases, the contributions of these terms decrease rapidly to zero. These terms, therefore, make a transient contribution to the neutron flux, the values soon becoming negligible in comparison to the first term. Hence, after a short interval of time, which is of the order of something less than $\frac{1}{\lambda}$ where λ is the smallest decay constant, i.e., corresponding to the most-delayed neutrons, (13) reduces to the first term only, i.e.,

$$\phi = A_0 e^{t\omega_0}$$

In this case A_0 is evidently equal to ϕ_0 the flux at zero time, and so

$$\phi = \phi_0 e^{t\omega_0}$$

Further, since ω_0, as seen above, has the dimensions of a reciprocal time, it may be set equal to 1/T, so that

$$\phi = \phi_0 e^{t/T} \qquad (14)$$

It is evident, from the definition of the reactor period as the time required for the neutron flux to increase by a factor of e, that T is here the reactor period after the lapse of sufficient time to permit the contributions of the transient terms to damp out. Hence T, defined by

$$T = \frac{1}{\omega_0} \qquad (15)$$

is called the stable reactor period.

If all the groups of delayed neutrons are considered as a single group with a decay constant equal to the properly weighted average for the five actual groups, (11) becomes

$$\rho = \frac{l\omega}{k_{eff}} + \frac{\beta\omega}{\omega+\lambda} \qquad (16)$$

which is a quadratic equation in ω. It will be supposed that k_{ex} and ρ are small, so that k_{eff} is approximately equal to unity. Equation (16) may then be written as

$$\rho \approx l\omega + \frac{\beta\omega}{\omega+\lambda}$$

or

$$l\omega^2 + (\beta - \rho + l\lambda)\omega - \lambda\rho \approx 0$$

The solutions of this equation are

$$\omega = \frac{1}{2l}\left[-(\beta-\rho+l\lambda) \pm \sqrt{(\beta-\rho+l\lambda)^2 + 4l\lambda\rho}\right]$$

$$\omega = -\frac{(\beta-\rho+l\lambda)}{2l}\left[1 \pm \sqrt{1 + \frac{4l\lambda\rho}{(\beta-\rho+l\lambda)^2}}\right]$$

If $(\beta-\rho+l\lambda)^2 \gg |2l\lambda\rho|$, this becomes

$$\omega \approx -\frac{(\beta-\rho+l\lambda)}{2l}\left\{1 \pm \left[1 + \frac{2l\lambda\rho}{(\beta-\rho+l\lambda)^2}\right]\right\}$$

and the two solutions are

$$\omega_0 \approx \frac{\lambda\rho}{\beta-\rho+l\lambda} \qquad \text{and} \qquad \omega_1 \approx -\left(\frac{\beta-\rho+l\lambda}{l}\right)$$

The stable reactor period T can now be estimated, since by equation (15) $T = \frac{1}{\omega_0}$.

XI. APPENDIX II

Maximum Power Following a Step Insertion of Reactivity

Consider a reactor from which heat is removed at a constant rate and whose criticality factor changes linerly with the temperature of the reactor. Let P and T denote the average instantaneous power and temperature of the reactor, the temperature being measured from the steady-state temperature. Then the rate of change of the reactor's heat content Q is

$$\frac{dQ}{dt} = \frac{SdT}{dt} = P - P_o \tag{1}$$

where S is the thermal capacity of the system and P_o is the initial power level. The criticality factor k is

$$k = 1 + \delta k\,(t) + \alpha T \tag{2}$$

where α is the temperature coefficient of reactivity. The quantity $\delta k\,(t)$ in (2) represents the excess reactivity which is put into the reactor by extraneous means. For the reactor power, which is proportional to the neutron population,

$$\frac{dP}{dt} = \frac{\delta k + \alpha T}{\ell} P \tag{3}$$

ℓ being the generation time.

The pair of equations (1) and (3) are the simplest example of a kinetically non-linear reactor system. Whether the system is stable will depend on the value of $\delta k\,(t)$ and α. The changes resulting from a sudden insertion of an amount of excess reactivity k_o at time t = 0 are best analyzed by eliminating the time variable, that is, by dividing

(1) by (3).

$$\frac{dP}{dT} = \frac{(\delta k_o + \alpha T)\, P}{d\,(P - P_o)} \tag{4}$$

The integral of (4), which satisfies the initial condition P = P at T = 0, is

$$\frac{P - P_o}{P_o} - \ln \frac{P}{P_o} = \frac{\zeta}{\ell P_o} \left(T \delta k_o + \frac{\alpha}{2} T^2\right) \tag{5}$$

This equation gives the power as a function of the temperature.

After the insertion of the reactivity δk_o, the reactor power will increase to a maximum at $T = -\frac{\delta k_o}{\alpha}$; at this temperature the criticality factor again becomes 1. The corresponding values of the power level P are given by (5); they are the roots of

$$\frac{P - P_o}{P_o} - \ln \frac{P}{P_o} = -\frac{\zeta (\delta k_o)^2}{2\ell\, P_o \alpha} \tag{6}$$

If the maximum power is much larger than the steady power, the logarithmic term can be neglected, and one obtains

$$P_{max} = P_o + \frac{\zeta (\delta k)_o^2}{2\ell\, |\alpha|}$$

ABSTRACT

Theoretical calculations of critical mass, temperature coefficient, and transient behavior have been made for proposed uranium dioxide and urania-thoria fuel cores for the Virginia Polytechnic Institute UTR-10 Reactor. The fuel core consists of aluminum-clad pins immersed in a water lattice at a water-to-metal ratio of approximately 2.5 to 1. Criticality can be achieved with a loading of 8,031 grams of 3.5% enriched uranium dioxide of 92% theoretical density, or with 13,045 grams of a mixture of 6% $U^{235}O_2$ and 94% ThO_2. The temperature coefficients of both loadings are negative, indicating an inherently safe system. The response to a sudden insertion of 0.5% excess reactivity is examined with respect to stable reactor period, maximum power and maximum temperature rise. Thermal properties of the fuel cores are determined.